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Study on the envelope process of the flexspline of harmonic reducer based on the angle of wave generator

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Abstract. Harmonic reducer is the core part of robot joint, its tooth shape design theory and meshing characteristics will directly affect the motion control accuracy and vibration characteristics of robot. This paper studies the meshing process of the circular spline and flexsplines of the harmonic reducer based on the double arc profile and the elliptical wave generator. The main innovation of this research is to take the input angle of the wave generator as an independent variable, which more clearly represents the enveloping process and meshing interval distribution of the tooth profile. The results show that the secondary meshing process of the double circular arc tooth profile exists on both sides of the long axis of the wave generator. But the second conjugate contact interval is small, so the optimization of tooth shape should focus on the first conjugate interval. This study provides a theoretical basis for the design of tooth profile parameters of harmonic reducer.

1. Introduction

Since the harmonic gear has the advantage of a high transmission accuracy and compact, it has been widely used in high-precision equipment. In recent years, more and more attention has been paid to circular arc tooth profile, especially double circular arc tooth profile. Compared with the involute profile, it has many advantages, such as more pairs of meshing teeth, larger meshing interval, more uniform stress distribution, etc., which improves the meshing performance of the harmonic reducer and makes the transmission more stable [1]. Xin has done a lot of research work in the tooth profile design and meshing theory of harmonic gear drive. Among them, innovative research results have been achieved in the design and device development of double arc tooth profile harmonic gear[2]. Dong studies on the movement of the tooth profile by the dynamic instantaneous center line of the flexible tooth relative to the rigid one and proposed the kinematic optimization model of backlash under different loads[3,4]. Wang found that the meshing interference can be effectively avoided by optimizing the radial deformation coefficient of the double circular arc tooth profile[5]. Chen et al. studied in detail the relationship between the design parameters of tooth profile and the meshing clearance and interference in the process of single tooth enveloping, and used the finite element simulation to verify[6]. Based on the image recognition algorithm and the laser ranging sensor, Ma et al. extracted the movement speed of the flexible tooth profile and the change rule of the neutral layer of the harmonic reducer under different wave generator speed conditions[7,8].

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2. Modeling of transmission mode with circular spline fixed and flexspline as output components

2.1. Deformation function of flexible bearing based on elliptical wave generator

The three core parts of harmonic reducer drive are circular spline, flexspline and wave generator. The wave generator rotates through the large deformation of the outer flexible bearing to drive the meshing transmission. The parameter equation of the wave generator can be expressed as

$$\begin{cases} x = a_T \sin(\varphi) \\ y = b_T \cos(\varphi) \end{cases} \tag{1}$$

The curve equation of the neutral layer after the flexible bearing is installed in the wave generator can be expressed as

$$\omega(\varphi) = \left|\vec{L}\right| - r_m \tag{2}$$

The research shows that it is feasible and convenient to use sine function to approximate the original function from the point of view of fitting results and error analysis. The fitting error is kept in the order of 0.001mm. So the radial deformation function can be expressed as

$$\omega = \omega_0 \cos(2\varphi) \tag{3}$$

Similarly , the tangent displacement and normal angle of the neutral layer curve can be expressed as

$$\begin{cases} v = -\int_0^{\varphi} \omega(\varphi) d\varphi = -\dfrac{\omega_0}{2}\sin(2\varphi) \\ u = \arctan(\dfrac{\rho'}{\rho}) \approx \dfrac{1}{r_m}\dfrac{d\omega}{d\varphi} = \dfrac{1}{r_m}(2\omega_0 \sin(2\varphi)) \end{cases} \tag{4}$$

2.2. Basic tooth shape of double circular arc of flexspline

The standard double arc tooth profile of the flexspline is composed of two arcs and a common tangent. The intersection of the center line of the flexible bearing and the center line of the tooth profile is the coordinate origin, and the center line of the tooth profile is the double arc tooth profile of the Y axis as shown in Figure 1. Considering that we study the process of the wave generator rotating anticlockwise, it is necessary to change the equation of the flexspline tooth profile into the left one so as to mesh with the right one of the circular spline.

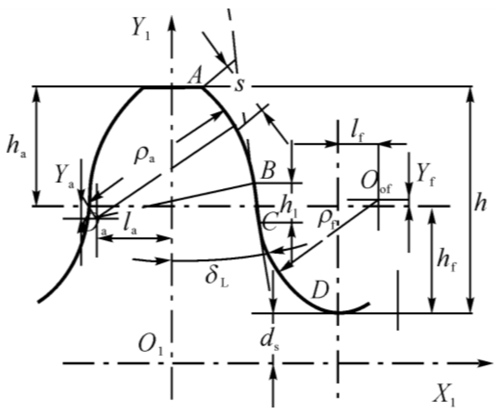

Figure 1. The parameters of double circular arc tooth profile.

The tooth profile equation of AB segment on the left side:

$$\begin{cases} x_{AB} = -(\rho_a \cos(\alpha_a - s/\rho_a) + x_{oa}) \\ y_{AB} = \rho_a \sin(\alpha_a - s/\rho_a) + y_{oa} \\ s \in (0, l_1), l_1 = \rho_a(\alpha_a - \delta_L) \end{cases} \tag{5}$$

The tooth profile equation of BC segment on the left side:

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$$\begin{cases} x_{BC} = -(\rho_a \cos\delta_L + x_{oa} + (s - l_1)\sin\delta_L) \\ y_{BC} = \rho_a \sin\delta_L + y_{oa} - (s - l_1)\cos\delta_L \\ s \in (l_1, l_2), l_2 = l_1 + h_l / \cos\delta_L \end{cases} \tag{6}$$

The tooth profile equation of CD segment on the left side:

$$\begin{cases} x_{CD} = -(x_{of} - \rho_f \cos(\delta_L + (s - l_2)/\rho_f)) \\ y_{CD} = y_{of} - \rho_f \sin(\delta_L + (s - l_2)/\rho_f) \\ s \in (l_2, l_3), l_3 = l_2 + \rho_f(\pi/2 - \arcsin((Y_f + h_f)/\rho_f) - \delta_L) \end{cases} \tag{7}$$

The meaning and calculation formula of tooth profile parameters can refer to reference [5]

2.3. Kinematic frame and tooth profile equation of circular spline

Suppose the tooth profile equation of the flexspline is:

$$\begin{cases} x_r = x(s) \\ y_r = y(s) \end{cases} \tag{8}$$

Transformation matrix from flexspline coordinate to circular spline coordinate is

$$M_{gr} = \begin{bmatrix} \cos(\beta) & \sin(\beta) & \rho\sin(\Delta\varphi) \\ -\sin(\beta) & \cos(\beta) & \rho\cos(\Delta\varphi) \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Enveloping tooth profile of flexspline:

$$\begin{cases} x_g = x_r \cos(\beta) + y_r \sin(\beta) + \rho\sin(\Delta\varphi) \\ y_g = -x_r \sin(\beta) + y_r \cos(\beta) + \rho\sin(\Delta\varphi) \end{cases} \tag{10}$$

According to envelope theory, the conjugate profile of a tooth of circular spline can be expressed as

$$\begin{cases} x_g = x_g(s, \varphi_H), y_g = y_g(s, \varphi_H) \\ \dfrac{\partial x_g}{\partial s} \cdot \dfrac{\partial y_g}{\partial \varphi_H} - \dfrac{\partial y_g}{\partial s} \cdot \dfrac{\partial x_g}{\partial \varphi_H} = 0 \end{cases} \tag{11}$$

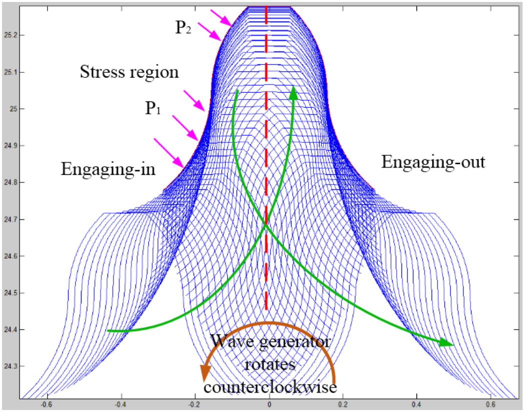
3. Analysis of meshing process based on the angle of wave generator

In order to better understand the engagement process, we will conduct research through detailed case analysis, and the design parameters of harmonic reducer are shown in Table 1.

Table 1. The design parameters of harmonic reducer

Parameters	Symbol	Value	Unit
Module	m	0.25	mm
Teeth number of the FS	z_1	200	--
Teeth number of the CS	z_2	202	--
Modification coefficient	w_0	1.12m	--
Radius of the FS neutral line	r_m	24.3	mm

The single tooth envelope process is shown in Figure 2 and Figure 3. When the wave generator rotates counterclockwise, the tooth profile of the flexspline rotates clockwise and then conjugate with the tooth top and tooth root arc of the circular spline. Considering that the direction of the external torque is counterclockwise, the left profile of the flexspline and the right profile of the circular spline contact in the whole enveloping process.



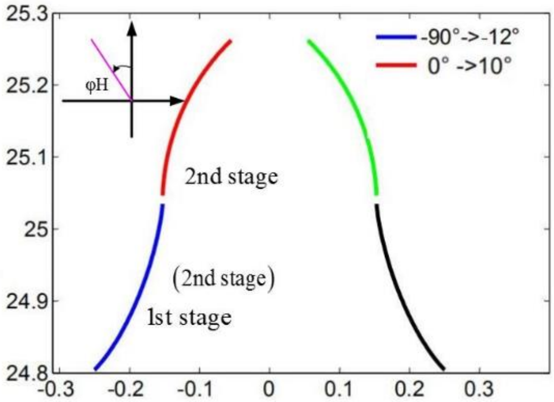


Figure 2. Enveloping process of a single tooth profile. **Figure 3.** Tooth profile of circular spline.

As shown in Figure 2, although the double arc tooth profile is composed of three curves, the conjugate contact between circular spline and flexspline only involves concave arc and convex arc. In the first stage of engagement, conjugate contact occurs in the convex tooth profile of flexspline and circular spline. At this time, the corresponding rotation angle interval of the wave generator is [-60°,-10°]. The angle of wave generator corresponding to the second stage of engagement is [0°,10°], the convex arc of the flexspline is conjugate with the concave arc of the circular spline, and the concave arc of the flexspline is conjugate with the convex arc of the circular spline in the meantime.

From the sign of the rotation angle of the wave generator, we can see that the first conjugate interval and the second conjugate interval are located on both sides of the long axis of the wave generator as shown in Figure 4. The secondary meshing process of the double circular arc tooth profile does exist, and exists on both sides of the long axis of the wave generator. But the second conjugate contact interval is small, so the optimization of tooth shape should focus on the first conjugate interval.

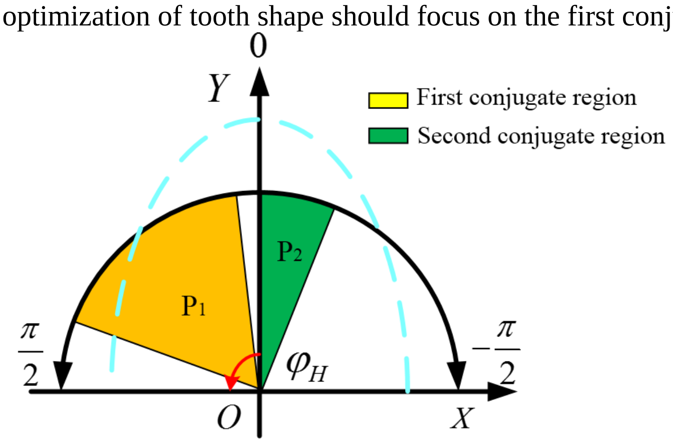


Figure 4. The contact area on both sides of the long axis of the wave generator.

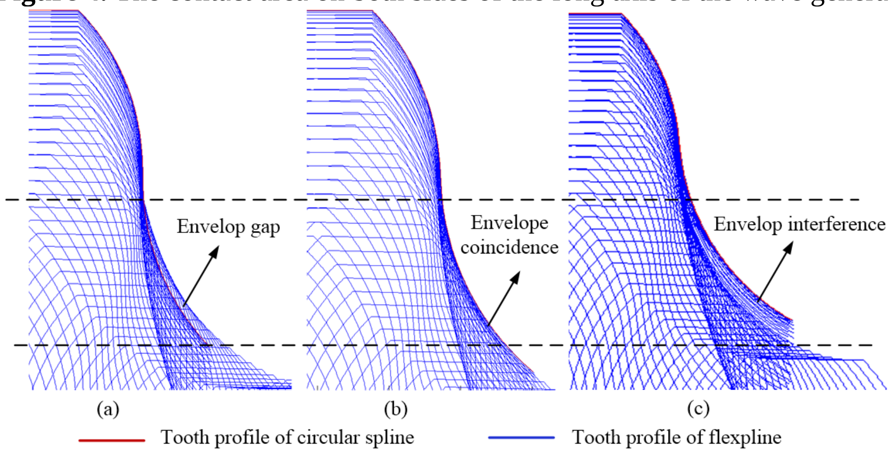


Figure 5. Possible forms of single tooth enveloping process.

Three different types of envelope results are described in Figure 5. Figure 5 (b) is the most appropriate envelope result, where the conjugate arc at the top of the flexspline tooth coincides with the conjugate arc at the root. If we can't realize the complete coincidence through optimization, then we must ensure that top circular envelope tooth profile of the flexspline is above the root circular envelope tooth profile as shown in Figure 5 (a), which is one of the most important conditions for proper transmission. When the enveloping results as shown in Figure 5(c) appear, it shows that the double arc tooth profile parameters of the flexspline need to be optimized to avoid enveloping interference.

4. Conclusion

In this paper, we establish the contact model of single tooth and multi tooth on the left side of the flexspline and the right side of the circular spline when the wave generator rotates anticlockwise. The meshing clearance, meshing interval and secondary meshing process have been studied. The conclusions obtained from the study are listed below:

(1) For the double arc tooth profile of flexspline, there are two conjugate intervals located on both sides of the long axis of the wave generator. One interval is called single-conjugate-contact region, which is 10 to 60 degrees ahead of the rotation angle of the wave generator. The other interval should be called single-conjugate-contact region, which is 0 to 10 degrees behand of the rotation angle of the wave generator.

(2) In the envelope process, if we can't realize the complete coincidence of the top circular envelope tooth profile and the root circular envelope tooth profile through optimization, then we must ensure that top circular envelope tooth profile of the flexspline is above the root circular envelope tooth profile, which is one of the most important conditions for proper transmission.

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