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17078

ECS 414

1. Consider G is a k-regular undirected network

$\Rightarrow$ The adjacency matrix A of G will contain k time '1' in each row

$\therefore$ By matrix multiplication $Ax = (k, k, \dots k)^T$

$\Rightarrow$ $Ax = kx$

$\Rightarrow$ k is eigen-value with $1 = (1, 1, 1 \dots 1)$ the eigenvector.

b) We know that eigen-vector are orthogonal for any other eigenvector $(a_1 a_2 \dots a_n)$

$(1, 1, 1 \dots 1) \cdot (a_1 a_2 \dots a_n) = 0$

$\Rightarrow$ $a_1 + a_2 + \dots + a_n = 0$

$\Rightarrow$ atleast one of the a_i must be negative

Now by Perron-Frobenius theorem k is the largest eigenvalue of adjacency matrix A.

C) Katz centrality of G is given by vector

$v = (I - \alpha A)^{-1} 1$

where $\alpha > 0$ $\alpha \neq 1/k$

This centrality give different value for different node in Regular network.

2. $$C_i = \frac{1}{l_i} = \frac{n}{\sum_j d_{ij}}$$

To prove : $$\frac{1}{C_1} + \frac{n_1}{n} = \frac{1}{C_2} = \frac{n_2}{n}$$

As per defination

$$C_1 = \frac{n}{\sum_j d_{1j}} \qquad C_2 = \frac{n}{\sum_j d_{2j}} \quad \text{—(A)}$$

$$C_1 = \frac{n}{\sum_j d_{1j}}$$

$$= \frac{n}{\sum_{j \in R_1}(d_{2j} - 1) + \sum_{j \in R_2}(d_{2j} + 1)}$$

$$= \frac{n}{\sum_{j \in R_1} d_{2j} - \sum_{j \in R_1} 1 + \sum_{j \in R_2}(d_{2j}) + \sum_{j \in R_2} 1}$$

$$= \frac{n}{n_2 - n_1 + (\sum_{j \in R_2} d_{2j} + \sum_{j \in R_2} d_{2j})}$$

$$= \frac{n}{n_2 - n_1 + \sum_j d_{2j}}$$

$$c_1 = \frac{n}{n_2 - n_1 + \sum_j d_{2j}}$$

$$\Rightarrow \quad \frac{n_2 - n_1 + \sum_j d_{2j}}{n} = \frac{1}{c_1}$$

$$\frac{n_2}{n} - \frac{n_1}{n} + \frac{\sum_j d_{2j}}{n} = \frac{1}{c_1}$$

from (A)

$$\frac{n_2}{n} - \frac{n_1}{n} + \frac{1}{c_2} = \frac{1}{c_1}$$

$$\boxed{\frac{1}{c_1} + \frac{n_1}{n} = \frac{1}{c_2} + \frac{n_2}{n}}$$

3.

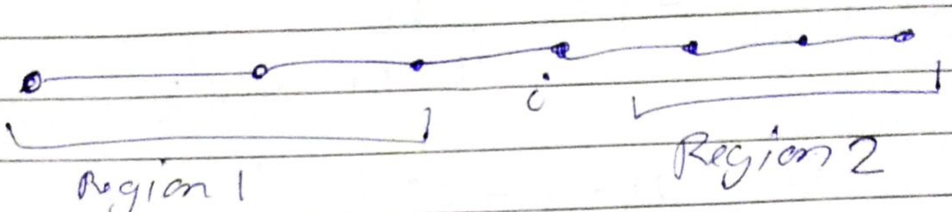
a) Betweeness is given by $= \sum_{s,t} \frac{n^i_{s,t}}{g_{s,t}}$

$$= \sum_{\substack{s \in \text{Region}_i \\ t \in \text{Region}_j \\ i \neq j}} 1$$

$$= \sum_{s,t} 1 - \sum_{\substack{s,t \\ \text{in same region}}} 1$$

$$= \cancel{n^2 - \sum_{m=1}^{k} n_m^2} = n^2 - \sum_{m=1}^{k} n_m^2$$

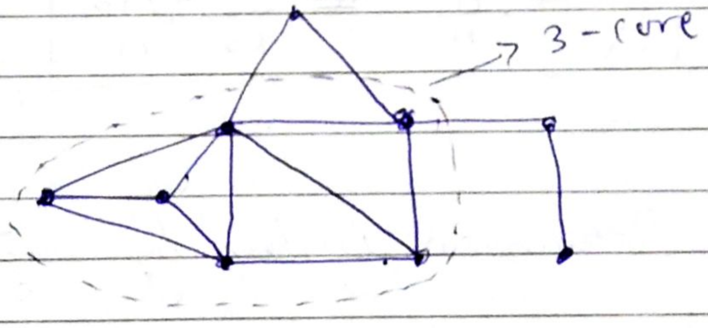
b)



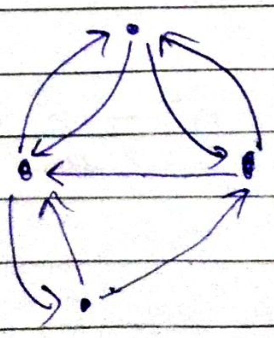
by using result from the first part

$$x_i = n^2 - [(i-1)^2 + (n-i)^2]$$
$$= n^2 - (i^2 + 1 - 2i + n^2 + i^2 - 2ni)$$
$$= 2in - 2i^2 + 2i - 1$$
$$= 2(in - i^2 + i) - 1$$
$$= 2i(n - i + 1) - 1$$

4. Find 3-core in followin network.

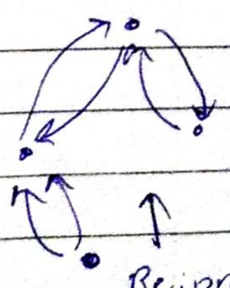


b) Reciprocity of a graph is defined if the vertex pair is reciprocal. A vertex pair is said to be reciprocal if there are edge in both direction between them.



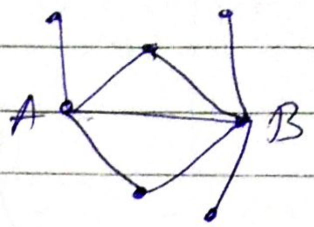
→ Total edge = 8

$$\text{Reciprocity} = \frac{6}{8} = \frac{3}{4}$$



Reciprocity of graph

c) Cosine:



A & B have 2-common neighbours

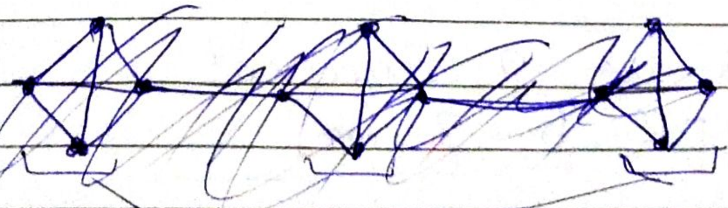
degree of A = 4
degree of B = 5

$$\sigma_{ij} = \frac{n_{ij}}{\sqrt{k_i}\sqrt{k_j}} = \frac{2}{\sqrt{4}\sqrt{5}}$$

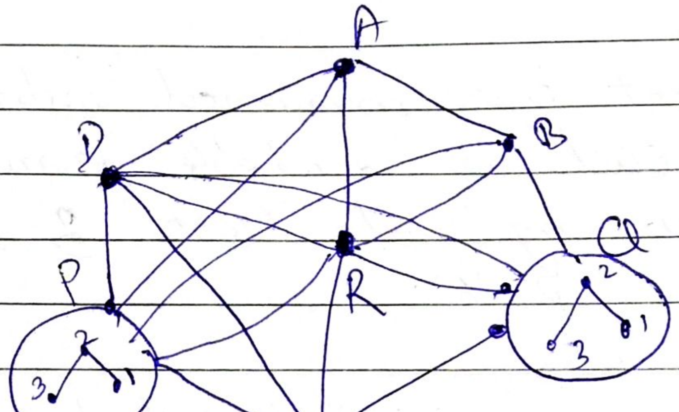
$$= \frac{1}{\sqrt{5}}$$

Q5. A 3-component is a maximal subset of vertices such that each is reachable from each of the other by at least 3-vertex independent path.

A 3-core is a maximal subset of vertices such that each is connected to atleast 3 other in the subset.



3 core



3-component 3 component

3 core

P & Q are 3 component
ABCDR is 3 core

Questions 6

Implementing the function without using built-in function

```
This is a GraphEdge class
self.node denotes node from where the node is generating
self.distance is the weight for the edge
```

In [2]:

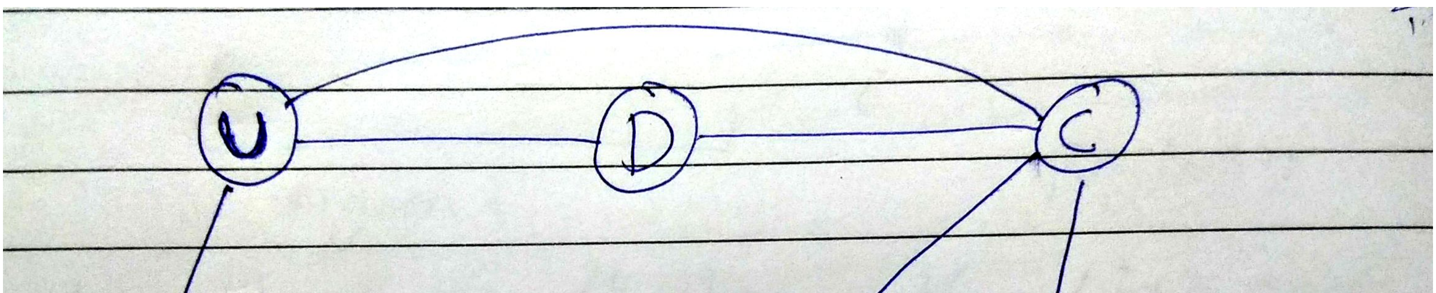
```
class GraphEdge(object):
    def __init__(self, node, distance):
        self.node = node
        self.distance = distance
```

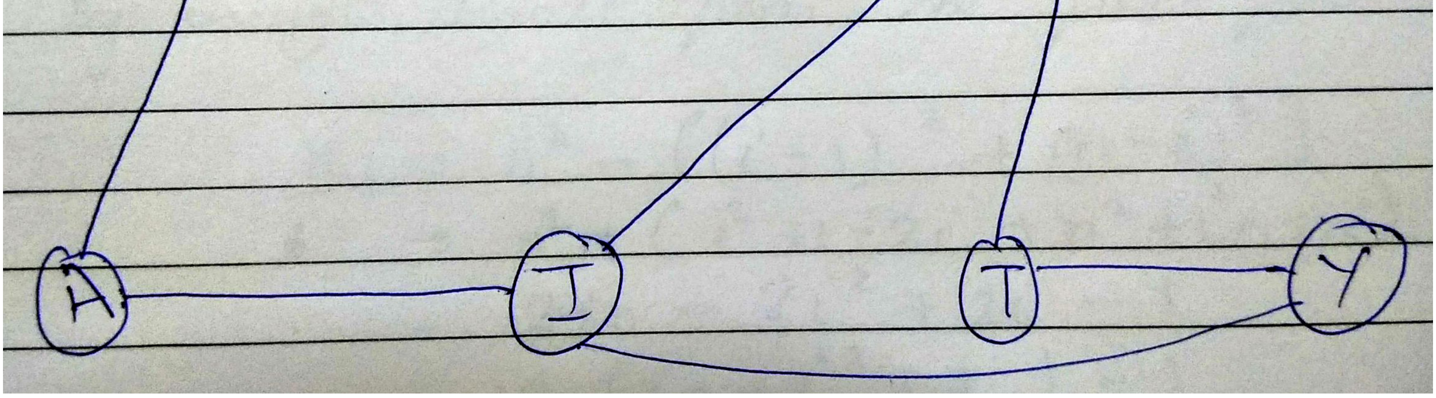
```
I have used math built-in fuction just to give 'infinity' weight to the edge
Here add_child is same as add node
Here remove_node is same as remove node
```

In [3]:

```
import math
class GraphNode(object):
    def __init__(self, val):
        self.value = val
        self.edges = []

    def add_child(self, node, distance):
        self.edges.append(GraphEdge(node, distance))

    def remove_child(self, del_node):
        if del_node in self.edges:
            self.edges.remove(del_node)

class Graph(object):
    def __init__(self, node_list):
        self.nodes = node_list

    def add_edge(self, node1, node2, distance):
        if node1 in self.nodes and node2 in self.nodes:
            node1.add_child(node2, distance)
            node2.add_child(node1, distance)

    def remove_edge(self, node1, node2):
        if node1 in self.nodes and node2 in self.nodes:
            node1.remove_child(node2)
            node2.remove_child(node1)
```

Graph used





Creating the graph as shown in the above image

In [4]:

```
node_u = GraphNode('U')
node_d = GraphNode('D')
node_a = GraphNode('A')
node_c = GraphNode('C')
node_i = GraphNode('I')
node_t = GraphNode('T')
node_y = GraphNode('Y')

graph = Graph([node_u, node_d, node_a, node_c, node_i, node_t, node_y])
graph.add_edge(node_u, node_a, 1)
graph.add_edge(node_u, node_c, 1)
graph.add_edge(node_u, node_d, 1)
graph.add_edge(node_d, node_u, 1)
graph.add_edge(node_d, node_c, 1)
graph.add_edge(node_a, node_u, 1)
graph.add_edge(node_a, node_i, 1)
graph.add_edge(node_c, node_d, 1)
graph.add_edge(node_c, node_u, 1)
graph.add_edge(node_c, node_i, 1)
graph.add_edge(node_c, node_t, 1)
graph.add_edge(node_i, node_a, 1)
graph.add_edge(node_i, node_c, 1)
graph.add_edge(node_i, node_y, 1)
graph.add_edge(node_t, node_c, 1)
graph.add_edge(node_t, node_y, 1)
graph.add_edge(node_y, node_i, 1)
graph.add_edge(node_y, node_t, 1)
```

Logic

I have used dijkstra alogrithm to find smallest distance between two node. Dijkstra uses edges weight to find the cheapest path. SO I have given weight of "1" to each edge. So weighted graph with all edgeweight of 1 is same as undirected unweighted graph

In [5]:

```
def dijkstra(start_node, end_node):
    distance_dict = {node: math.inf for node in graph.nodes}
    shortest_path_to_node = {}

    distance_dict[start_node] = 0
    while distance_dict:
        # Pop the shortest path
        current_node, node_distance = sorted(distance_dict.items(), key=lambda x: x[1])[
0]
        shortest_path_to_node[current_node] = distance_dict.pop(current_node)

        for edge in current_node.edges:
            if edge.node in distance_dict:
                new_node_distance = node_distance + edge.distance
                if distance_dict[edge.node] > new_node_distance:
                    distance_dict[edge.node] = new_node_distance
```

```
    return shortest_path_to_node[end_node]
```

In [6]:

```
def using_dijkstra():
    nodes = [node_a, node_c, node_d, node_i, node_t, node_u, node_y]
    closeness_centrality_dict = dict()
    for node1 in nodes:
        d=0
        for node2 in nodes:
            if node1 != node2:
                #print('Shortest Distance from {} to {} is {}'.format(node1.value, node2.
value, dijkstra(node1, node2)))
                d+=1/dijkstra(node1, node2)

        closeness_centrality_dict[node1.value] = (1/(len(nodes)-1))*d

        #print('d=',d)
    #print(closeness_centrality_dict)
    return closeness_centrality_dict
```

Creating the graph using built-in Library

In [7]:

```
import networkx as nx
g = nx.Graph()
g.add_edge('U','D')
g.add_edge('U','C')
g.add_edge('U','A')
g.add_edge('D','C')
g.add_edge('U','D')
g.add_edge('A','I')
g.add_edge('C','I')
g.add_edge('C','T')
g.add_edge('T','Y')
g.add_edge('I','Y')
```

This fuction return a dictionary of node with their closeness centrality

In [8]:

```
def using_built_in_library(graph):
    return nx.closeness_centrality(graph)
```

Sorting the node based on the centrality, if their is a tie then sorting those elements based on node name alphabatically

In [9]:

```
# array of sorted (node, centrality) which are created by using user defined function
#based on centrality
sorted_node_by_dij = sorted(using_dijkstra().items(), key=lambda x: (x[1],x[0]))
```

In [10]:

```
# array of sorted (node, centrality) which are created by using built-in function
#based on centrality
sorted_builtin_node = sorted(using_built_in_library(g).items(), key=lambda x: (x[1],x[0]
))
```

Final Testing

Checking if the list generate by user-defined function and built-in function are same or not

not

In [12]:

```
def print_node_with_closeness_centrality(node_list,node_dict):
    for node in node_list:
        print(f"Node {node} has closeness centrality {node_dict[node]}")


sorted_node_list_1 = [i[0] for i in sorted_node_by_dij]
sorted_node_list_2 = [i[0] for i in sorted_builtin_node]
if sorted_node_list_1==sorted_node_list_2:
    print('Test Pass ')
    print('')
    print('Closeness Centrality generated by user-defined Function')
    print('')
    print_node_with_closeness_centrality(sorted_node_list_1,dict(sorted_node_by_dij))
    print('----------------------------------------------------------------')
    print('')
    print('Closeness Centrality generated by built-in Function')
    print('')
    print_node_with_closeness_centrality(sorted_node_list_2,dict(sorted_builtin_node))

else:
    print('Test fail')
    print('')
    print('Closeness Centrality generated by user-defined Function')
    print('')
    print_node_with_closeness_centrality(sorted_node_list_1,dict(sorted_node_by_dij))
    print('----------------------------------------------------------------')
    print('')
    print('Closeness Centrality generated by built-in Function')
    print('')
    print_node_with_closeness_centrality(sorted_node_list_2,dict(sorted_builtin_node))
```

```
Test Pass

Closeness Centrality generated by user-defined Function

Node Y has closeness centrality 0.611111111111111
Node A has closeness centrality 0.6388888888888888
Node D has closeness centrality 0.6388888888888888
Node T has closeness centrality 0.6388888888888888
Node U has closeness centrality 0.7222222222222221
Node I has closeness centrality 0.75
Node C has closeness centrality 0.8333333333333333
----------------------------------------------------------------

Closeness Centrality generated by built-in Function

Node Y has closeness centrality 0.5
Node A has closeness centrality 0.5454545454545454
Node D has closeness centrality 0.5454545454545454
Node T has closeness centrality 0.5454545454545454
Node U has closeness centrality 0.6
Node I has closeness centrality 0.6666666666666666
Node C has closeness centrality 0.75
```

In [ ]:

In [ ]: