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17078

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Compulsary question

1. False

Q3. Average size of small component:

=> Size of small component to which node i belongs is simply sum of sizes of all the set of node reached by following on edge of node i plus 1 (node i itself.

-> If a node i has deg k and sizes of these set are $t_1 .. t_2 .. t_k$ then avg size of component to which node i belong is $1+k\langle t\rangle$

$$\langle n\rangle = 1 + \langle k\rangle_{small} * \langle t\rangle$$

$\langle k\rangle_{small}$ is avg deg of node in small component

$\langle t\rangle$ is avg size of the set of nodes reached by following on edge.

-> Prob that node belongs to small component given that it has deg k_i is u^k

$$P(\text{deg } k \mid \text{small component}) = \frac{P(\text{small component} \mid \text{deg } k) \times P(\text{deg } k)}{P(\text{small component})}$$

$P(\deg k) = p_k$

$\Rightarrow P(\text{small component}) = 1 - S = g_0(u)$

$P(\deg k \mid \text{small component}) = \dfrac{u^k \cdot p_k}{g_0(u)}$

$\langle k \rangle_{small}$ = Average of $P(\deg k \mid \text{small compo.})$

$$\langle k \rangle_{small} = \frac{1}{g_0(u)} \sum_{k=0}^{\infty} k p_k u^k$$

$$\boxed{\langle k \rangle_{small} = \frac{u g_0'(u)}{g_0(u)}} \quad - (1)$$

Avg. no. of neighbour reached along an edge

$$\langle t \rangle = 1 + \langle k \rangle_{neighbour} \langle t \rangle$$

$$\langle t \rangle = \frac{1}{1 - \langle k \rangle_{neighbour}}$$

$$\Rightarrow \langle k \rangle_{neighbour} = \frac{1}{u g_0'(u)} \sum_{k=0}^{\infty} k(k+1) p_{k+1} u^{k+1}$$

$$\Rightarrow \langle k \rangle_{neighbour} = \frac{1}{u g_0'(u)} \sum_{k=0}^{\infty} (k-1) k p_k u^k$$

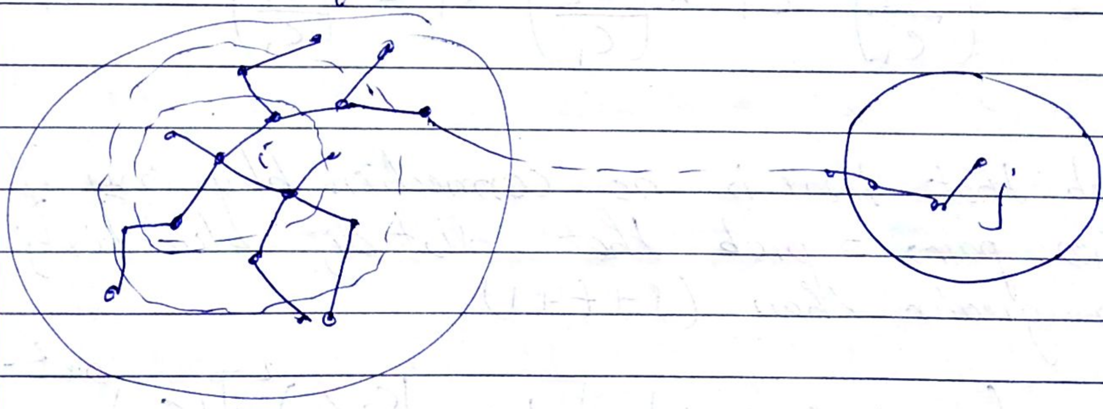
$$\langle k \rangle_{neighbour} = \frac{u g_0''(u)}{g_0'(u)}$$

$$\boxed{\langle t \rangle = \frac{g_0'(u)}{g_0(u) - u g_0''(u)}} \quad (2)$$

from (1) & (2).

$$\boxed{\langle s \rangle = 1 + \frac{u^2 g_0'(1)}{g_0(u)\left[1 - g_1'(u)\right]}}$$

Q4. <u>Diameter of Network.</u>



→ If there is a direct connection b/w surface of these 2 sets of nodes then shortest distance b/w i & j is no greater than s + t + 1

where s is a distance from node i
t is distance from ~~xx~~ node j

The ~~prob~~ of edge b/w individual pair of

node u, v on the two surface is
$$\frac{k_u k_v}{2m}$$

$\Rightarrow$ Avg. excess deg $= c_2/c_1$

Hence avg prob ~~between~~ of edge b/w any two pair of node on two surfaces is
$$\frac{\left(\frac{c_2}{c_1}\right)^2}{2m}$$

Avg total no. of pairs of nodes between ~~which such can~~ 2 surfaces are
$$\left(\frac{c_2}{c_1}\right)^{s-1} c_1 \times \left(\frac{c_2}{c_1}\right)^{t-1} c_1 = \left(\frac{c_2}{c_1}\right)^{s+t-2} c_1^2$$

Prob that there is no connection b/w any of these pairs = prob that dist d_{ij} b/w i & j is greater than $(s+t+1)$
$$p(d_{ij} > s+t+1) = \left[1 - \frac{(c_2/c_1)^2}{2m}\right]^{\left(\frac{c_2}{c_1}\right)^{s+t-2} c_1^2}$$

Taking log both side and noting that
$$\frac{\left(\frac{c_2}{c_1}\right)^2}{2m} \ll 1$$

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$$\ln\left(P(d_{ij} > s+t+1)\right) \simeq \left(\frac{c_2}{c_1}\right)^{s+t-2} c_1^2 \times (-1) \cdot \frac{\left(c_2/c_1\right)^2}{2m}$$

using $c_1 = \dfrac{2m}{n}$

$$P(d_{ij} > s+t+1) = e^{\left[-\frac{c_1}{n}\left(\frac{c_2}{c_1}\right)^{l-1}\right]}$$

$l = s+t+1$

$$\Rightarrow \boxed{l = A + \frac{\ln n}{\ln\left(\frac{c_2}{c_1}\right)}}$$

$l \Rightarrow$ diameter of config. model

$A \rightarrow$ constant.