

Compulsary question

1. False

Q3. Average size of small component:

⇒ Size of small component to which node i belongs is simply sum of sizes of all the set of node reached by following an edge of node i plus 1 (node i itself).

→ If a node i has deg k and sizes of these set are t_1, t_2, \dots, t_k then avg size of component to which node i belongs is $1 + k \langle t \rangle$

$$\langle n \rangle = 1 + \langle k \rangle_{\text{small}} * \langle t \rangle$$

$\langle k \rangle_{\text{small}}$ is avg deg of node in small component

$\langle t \rangle$ is avg size of the set of nodes reached by following an edge.

→ Prob that node belongs to small component given that it has deg k_i is u^{k_i}

$$P(\text{deg } k \mid \text{small component}) = \frac{P(\text{small component} \mid \text{deg } k) \times P(\text{deg } k)}{P(\text{small component})}$$

$$P(\text{small component})$$

$$P(\text{deg } k) = P_k$$

$$\Rightarrow P(\text{small component}) = 1 - S = g_0(u)$$

$$P(\text{deg } k \mid \text{small component}) = \frac{u^k \cdot p_k}{g_0(u)}$$

$$\langle k \rangle_{\text{small}} = \text{Average of } P(\text{deg } k \mid \text{small compo.})$$

$$\langle k \rangle_{\text{small}} = \frac{1}{g_0(u)} \sum_{k=0}^{\infty} k p_k u^k$$

$$\boxed{\langle k \rangle_{\text{small}} = \frac{u g_0'(u)}{g_0(u)}} \quad - (1)$$

Avg. no. of neighbours reached along an edge

$$\langle t \rangle = 1 + \langle k \rangle_{\text{neighbour}} \langle t \rangle$$

$$\langle t \rangle = \frac{1}{1 - \langle k \rangle_{\text{neighbour}}}$$

$$\Rightarrow \langle k \rangle_{\text{neighbour}} = \frac{1}{u g_0'(u)} \sum_{k=0}^{\infty} k(k+1) p_{k+1} u^{k+1}$$

$$\Rightarrow \langle k \rangle_{\text{neighbour}} = \frac{1}{u g_0'(u)} \sum_{k=0}^{\infty} (k-1) k p_k u^k$$

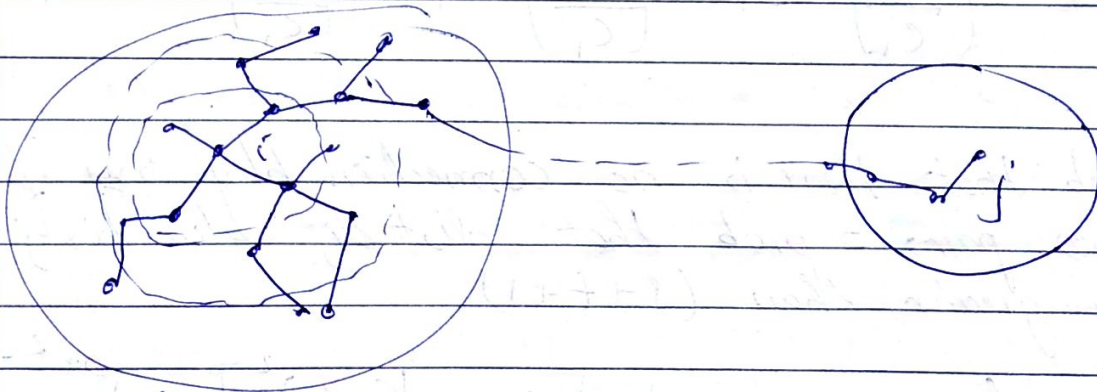
$$\langle k \rangle_{\text{neighbour}} = \frac{u g_0''(u)}{g_0'(u)}$$

$$\langle t \rangle = \frac{g_0'(u)}{g_0(u) - u g_0''(u)} \quad (2)$$

from ① & ②.

$$\langle S \rangle = 1 + \frac{u^2 g_0'(u)}{g_0(u) [1 - g_0'(u)]}$$

Q4. Diameter of network.



→ If there is a direct connection b/w surface of these 2 sets of nodes then shortest distance b/w i & j is no greater than $0 + t + 1$

where 0 is a distance from node i
 t is distance from node j

The probab of edge b/w individual pair of

node u, v on the two surface is
 $\frac{k_u k_v}{2m}$

$$\Rightarrow \text{Avg. inner deg} = c_2/c_1$$

Hence avg prob ~~between~~ of edge b/w any two pair of node on two surfaces is

$$\frac{\left(\frac{c_2}{c_1}\right)^2}{2m}$$

Avg total no. of pairs of nodes between which such an 2 surfaces are

$$\left(\frac{c_2}{c_1}\right)^{s-1} c_1 \times \left(\frac{c_2}{c_1}\right)^{t-1} c_1 = \left(\frac{c_2}{c_1}\right)^{s+t-2} c_1^2$$

Prob that there is no connection b/w any of these pairs = prob that $\text{dist } d_{ij}$ b/w i, j is greater than $(s+t+1)$

$$P(d_{ij} > s+t+1) = \left[1 - \frac{\left(\frac{c_2}{c_1}\right)^2}{2m} \right] \left(\frac{c_2}{c_1}\right)^{s+t-2} c_1^2$$

Taking log both side and noting that

$$\frac{\left(\frac{c_2}{c_1}\right)^2}{2m} \ll 1$$

$$\ln(P(\text{dij} > s+t+1)) \approx \left(\frac{c_2}{c_1}\right)^{s+t-2} \frac{c_1^2 \times (-1) \cdot \left(\frac{c_2}{c_1}\right)^2}{2m}$$

using $c_1 = \frac{2m}{n}$

$$P(\text{dij} > s+t+1) = e^{\left[-\frac{c_1}{n} \left(\frac{c_2}{c_1}\right)^{l-1} \right]}$$

$$l = s+t+1$$

$$\Rightarrow \boxed{l = A + \frac{\ln n}{\ln\left(\frac{c_2}{c_1}\right)}}$$

$l \Rightarrow$ diameter of
config. model

$A \rightarrow$ constant.

Q7

In [58]:

```
# importing libraries
import numpy as np
from math import sqrt
from matplotlib import pyplot as plt

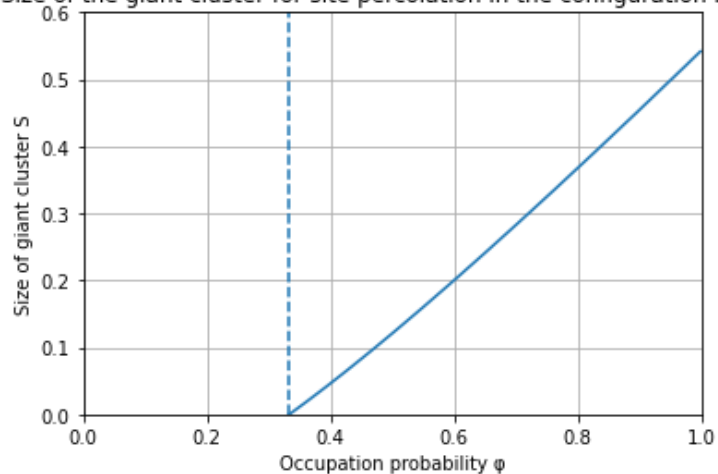
# Creating an array for  $\phi$  with [0.01, 0.02, 0.03 ..... 1.00]
phi = np.arange(0,1,0.001)

# Size of giant Cluster S
S = [(3/2)*x - sqrt((1/4)*(x**2) + x*((0.6**-1) - 1)) for x in phi]
```

In [60]:

```
plt.plot(phi,S)                                     # plotting phi vs. S
plt.xlim(0,1)                                       # Limiting yaxis from 0 to 1
plt.ylim(0,0.6)                                    # Limiting yaxis from 0 to 0.6
plt.axvline(1/3,ls='--')                           # plotting the verticle line
plt.ylabel("Size of giant cluster S")               # y axis label
plt.xlabel("Occupation probability  $\phi$ ")          # y axis label
plt.title("Size of the giant cluster for site percolation in the configuration model") # title
plt.grid()                                          # showing the grid
plt.show()                                         # plotting the plot.
```

Size of the giant cluster for site percolation in the configuration model



In []:

