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17078

ECS 414

1. Consider G is a k-regular undirected network

$\Rightarrow$ The adjacency matrix A of G will contain k time '1' in each row

$\therefore$ By matrix multiplication $Ax = (k, k, \dots k)^T$

$\Rightarrow Ax = kx$

$\Rightarrow k$ is eigen-value with $\mathbf{1} = (1, 1, 1 \dots 1)$ the eigenvector.

b) We know that eigen-vector are orthogonal for any other eigenvector $(a_1 a_2 \dots a_n)$

$$(1, 1, 1 \dots 1) \cdot (a_1 a_2 \dots a_n) = 0$$

$\Rightarrow a_1 + a_2 + \dots + a_n = 0$

$\Rightarrow$ atleast one of the a_i must be negative

Now by Perron-Frobenius theorem k is the largest eigenvalue of adjacency matrix A.

C) Katz centrality of G is given by vector

$$v = (I - \alpha A)^{-1} \mathbf{1}$$

where $\alpha > 0$ $\alpha \neq 1/k$

This centrality give different value for different node in Regular network.

2. $C_i = \frac{1}{l_i} = \frac{n}{\sum_j d_{ij}}$

To prove : $\frac{1}{C_1} + \frac{n_1}{n} = \frac{1}{C_2} = \frac{n_2}{n}$

As per defination

$$C_1 = \frac{n}{\sum_j d_{1j}} \qquad C_2 = \frac{n}{\sum_j d_{2j}} \quad - (A)$$

$$C_1 = \frac{n}{\sum_j d_{1j}}$$

$$= \frac{n}{\sum_{j \in R_1} (d_{2j} - 1) + \sum_{j \in R_2} (d_{2j} + 1)}$$

$$= \frac{n}{\sum_{j \in R_1} d_{2j} - \sum_{j \in R_1} 1 + \sum_{j \in R_2} (d_{2j}) + \sum_{j \in R_2} 1}$$

$$= \frac{n}{n_2 - n_1 + \left(\sum_{j \in R_2} d_{2j} + \sum_{j \in R_2} d_{2j}\right)}$$

$$= \frac{n}{n_2 - n_1 + \sum_j d_{2j}}$$

$$c_1 = \frac{n}{n_2 - n_1 + \sum_j d_{2j}}$$

$$\Rightarrow \quad \frac{n_2 - n_1 + \sum_j d_{2j}}{n} = \frac{1}{c_1}$$

$$\frac{n_2}{n} - \frac{n_1}{n} + \frac{\sum_j d_{2j}}{n} = \frac{1}{c_1}$$

from (A)

$$\frac{n_2}{n} - \frac{n_1}{n} + \frac{1}{c_2} = \frac{1}{c_1}$$

$$\boxed{\frac{1}{c_1} + \frac{n_1}{n} = \frac{1}{c_2} + \frac{n_2}{n}}$$

3.

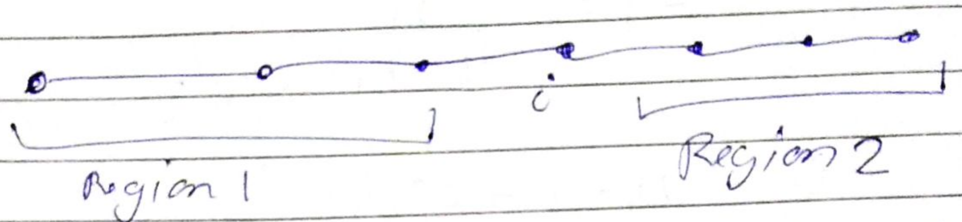
a) Betweeness is given by $= \sum_{s,t} \frac{n^i_{s,t}}{g_{s,t}}$

$$= \sum_{\substack{s \in \text{Region}_i \\ t \in \text{Region}_j \\ i \neq j}} 1$$

$$= \sum_{s,t} 1 - \sum_{\substack{s,t \\ \text{in same region}}} 1$$

$$= \cancel{n^2 - \sum_{m=1}^{k} n_m^2} = n^2 - \sum_{m=1}^{k} n_m^2$$

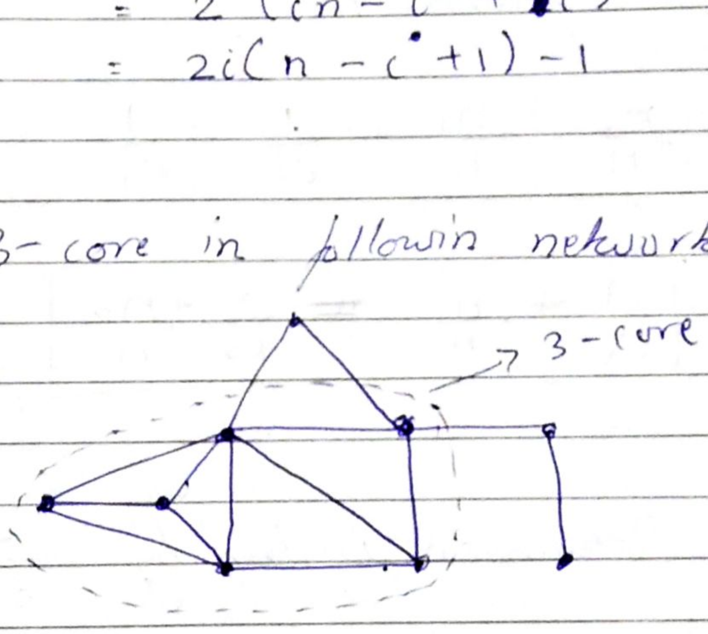
b)



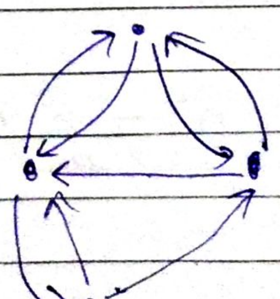
by using result from the first part

$$x_i = n^2 - [(i-1)^2 + (n-i)^2]$$
$$= n^2 - (i^2 + 1 - 2i + n^2 + i^2 - 2ni)$$
$$= 2in - 2i^2 + 2i - 1$$
$$= 2(in - i^2 + i) - 1$$
$$= 2i(n - i + 1) - 1$$

4. Find 3-core in followin network.

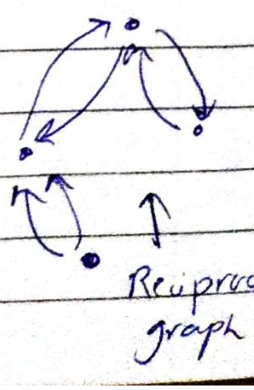


b) Reciprocity of a graph is defined if the vertex pair is reciprocal. A vertex pair is said to be reciprocal if there are edge in both direction between them.

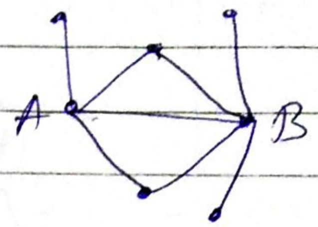


→ Total edge = 8

Reciprocity $= \frac{6}{8} = \frac{3}{4}$



c) Cosine:



A & B have 2-common neighbours

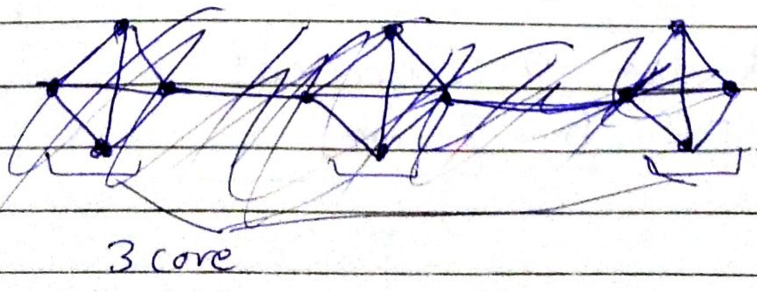
degree of A = 4
degree of B = 5

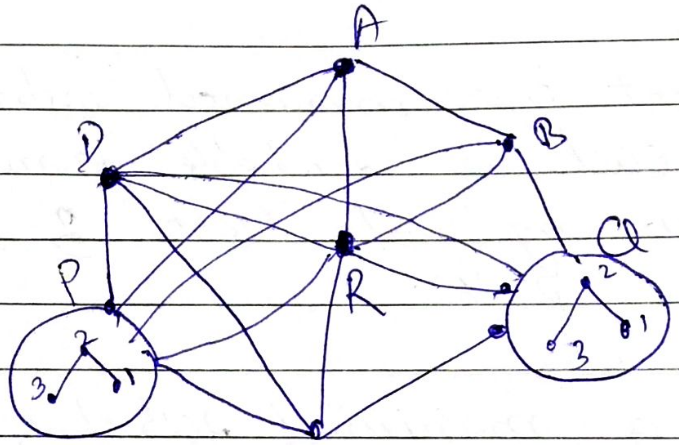
$$\sigma_{ij} = \frac{n_{ij}}{\sqrt{k_i}\sqrt{k_j}} = \frac{2}{\sqrt{4}\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}}$$

Q5. A 3- component is a maximal subset of vertices such that each is reachable from each of the other by at least 3- vertext independent path.

A 3-core is a maximal subset of vertices such that each is connected to atleast 3 other in the subset.





3-component C 3 component

3 core

P & Q are 3 component
ABCDR is 3 core