Pharmaceutical Multiphase Reactors
CHE.782

Design of Multiphase Flow Processes
669.266

General Background on Spatial Filtering Operations

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A part of this teaching material has been prepared for NanoSim (http://sintef.no/NanoSim/)

Statistical Analysis Techniques

Running statistics[1] allows to calculate variance and expected value of a collection of data with a significative memory saving.

The algorithm consists in updating the statistics every time a new sample is added.

Running average :

$$\langle x \rangle_{n+1} = \langle x \rangle_n + \frac{x_{n+1} - \langle x \rangle_n}{n}$$

Running variance :

$$S_{n+1} = S_n + \left(x_{n+1} - \langle x \rangle_n\right)\left(x_{n+1} - \langle x \rangle_{n+1}\right) \qquad {s_n}^2 = \frac{\left\langle \left(x - \langle x \rangle_n\right)^2 \right\rangle_n}{n\,(n-1)} = \frac{S_n}{n-1}$$

1) B. P. Welford (1962).["Note on a method for calculating corrected sums of squares and products"](). [Technometrics]() 4(3):419–420

Statistical Analysis Techniques

Referring to the phase *p,* the trace of the **two points velocity correlation** tensor is calculated as:

$$R(r\mathbf{e}_i,t) = \frac{1}{2}\frac{\left\langle \phi_p(\mathbf{x},t)\phi_p(\mathbf{x}+r\mathbf{e}_i,t)\mathbf{u}'_p(\mathbf{x},t)\cdot\mathbf{u}'_p(\mathbf{x}+r\mathbf{e}_i,t)\right\rangle}{\langle \phi_p(\mathbf{x},t)\phi_p(\mathbf{x}+r\mathbf{e}_i,t)\rangle}$$

ϕ_p : volume fraction of the phase *p*

$\mathbf{u}'_p$: fluctuating velocity of the phase *p*

It is used to quantify the correlation of velocity fluctuations at different locations.

Spatial Filtering Operations

The filtering process can be considered a subset of the general operation:

$$\overline{\psi}(\mathbf{x},t) = K * \psi = \int K(\mathbf{x}-\mathbf{z}, t-t')\psi(\mathbf{z},t')d\mathbf{z}dt'$$

Where $K(\mathbf{x}-\mathbf{z}, t-t')$ is the convoultion Kernel that defines the filtering methodology.

One of the most common Kernels is the top-Hat Kernel (box filter):

$$K(\mathbf{x}-\mathbf{z}, t-t') = \delta(t-t') \prod_{i=1}^{3} \frac{\mathcal{H}\left(\frac{\Delta_i}{2} - |x_i - z_i|\right)}{\Delta_i}$$

$\mathcal{H}$: Heaviside step function

Δ_i : spatial filter cut-off lenght on the i direction

Favre Average

Favre variables are mass-weighted variables. A Favre averaged variable is defined as:

$$\widetilde{\psi} = \frac{\overline{\rho\psi}}{\overline{\rho}}$$

In the case of an incompressible multiphase flow:

$$\widetilde{\psi}_p(\mathbf{x}, t) = \frac{\int K(\mathbf{x}-\mathbf{z}, t-t')\phi_p(\mathbf{z}, t')\psi_p(\mathbf{z}, t')d\mathbf{z}dt'}{\int K(\mathbf{x}-\mathbf{z}, t-t')\phi_p(\mathbf{z}, t')d\mathbf{z}dt'}$$

Where the subscript *p* indicates a phase variable and ϕ_p is the phase volume fraction.

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