

Pharmaceutical
Multiphase Reactors
CHE.782

Design of Multiphase
Flow Processes
669.266

General Background on Spatial Filtering Operations

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A part of this teaching material has been
prepared for NanoSim (<http://sintef.no/NanoSim/>)



NanoSim

NanoSim - A Multi-scale Simulation-Based Design Platform

Statistical Analysis Techniques

Running statistics¹ allows to calculate variance and expected value of a collection of data with a significative memory saving.

The algorithm consists in updating the statistics every time a new sample is added.

Running average :

$$\langle x \rangle_{n+1} = \langle x \rangle_n + \frac{x_{n+1} - \langle x \rangle_n}{n}$$

Running variance :

$$S_{n+1} = S_n + (x_{n+1} - \langle x \rangle_n) (x_{n+1} - \langle x \rangle_{n+1}) \quad s_n^2 = \frac{\langle (x - \langle x \rangle_n)^2 \rangle_n}{n(n-1)} = \frac{S_n}{n-1}$$

1) B. P. Welford (1962). "[Note on a method for calculating corrected sums of squares and products](#)". [Technometrics](#) 4(3):419–420

Statistical Analysis Techniques

Referring to the phase p , the trace of the **two points velocity correlation** tensor is calculated as:

$$R(r\mathbf{e}_i, t) = \frac{1}{2} \frac{\langle \phi_p(\mathbf{x}, t) \phi_p(\mathbf{x} + r\mathbf{e}_i, t) \mathbf{u}'_p(\mathbf{x}, t) \cdot \mathbf{u}'_p(\mathbf{x} + r\mathbf{e}_i, t) \rangle}{\langle \phi_p(\mathbf{x}, t) \phi_p(\mathbf{x} + r\mathbf{e}_i, t) \rangle}$$

ϕ_p : volume fraction of the phase p

\mathbf{u}'_p : fluctuating velocity of the phase p

It is used to quantify the correlation of velocity fluctuations at different locations.

Spatial Filtering Operations

The filtering process can be considered a subset of the general operation:

$$\bar{\psi}(\mathbf{x}, t) = K * \psi = \int K(\mathbf{x} - \mathbf{z}, t - t') \psi(\mathbf{z}, t') d\mathbf{z} dt'$$

Where $K(\mathbf{x} - \mathbf{z}, t - t')$ is the convolution Kernel that defines the filtering methodology.

One of the most common Kernels is the top-Hat Kernel (box filter):

$$K(\mathbf{x} - \mathbf{z}, t - t') = \delta(t - t') \prod_{i=1}^3 \frac{\mathcal{H}\left(\frac{\Delta_i}{2} - |x_i - z_i|\right)}{\Delta_i}$$

\mathcal{H} : Heaviside step function

Δ_i : spatial filter cut-off length on the i direction

Favre Average

Favre variables are mass-weighted variables. A Favre averaged variable is defined as:

$$\tilde{\psi} = \frac{\overline{\rho\psi}}{\bar{\rho}}$$

In the case of an incompressible multiphase flow:

$$\tilde{\psi}_p(\mathbf{x}, t) = \frac{\int K(\mathbf{x} - \mathbf{z}, t - t') \phi_p(\mathbf{z}, t') \psi_p(\mathbf{z}, t') d\mathbf{z} dt'}{\int K(\mathbf{x} - \mathbf{z}, t - t') \phi_p(\mathbf{z}, t') d\mathbf{z} dt'}$$

Where the subscript p indicates a phase variable and ϕ_p is the phase volume fraction.

Impressum & Disclaimer

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