QML Workshop

QML = Learning from data + Q.C. + classical data

$\mathcal{X} \xrightarrow{f(x)} \mathcal{Y}$ $\quad f(x) \approx \boxed{f_\theta(x)}$

Variational Circuits , Kernel methods

data x → Model(θ) Q.C.

expectation values

↓

E.V.S → Cost

update

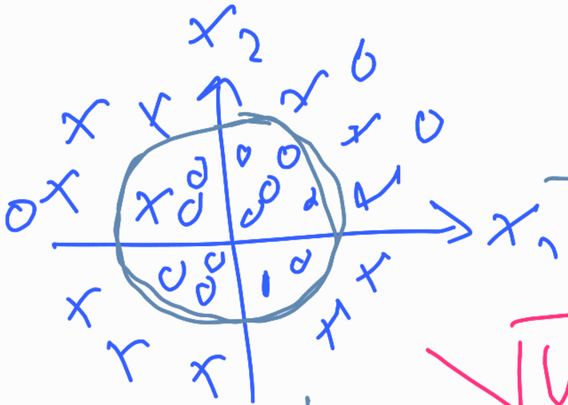
C.C.

Hybrid classical-Quantum

Kernel Methods: Q.K.

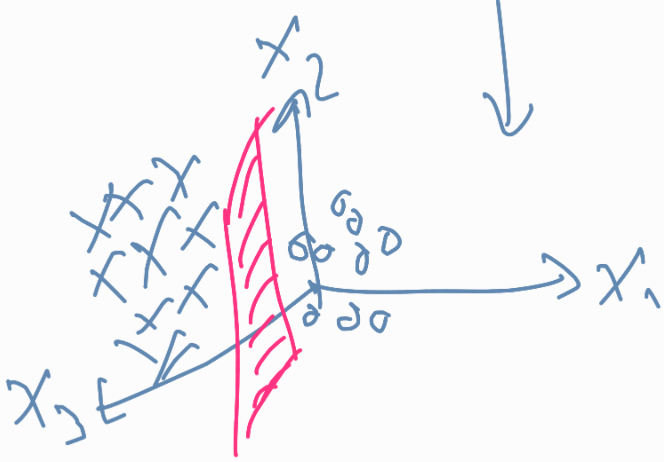
Suppose:

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$



map → $\begin{bmatrix} x_1 \\ x_2 \\ x_3 = x_1^2 + x_2^2 \end{bmatrix}$

$\boxed{U(x)}$



Today: → Kernels & QVC
→ Example: a Toy classification

2 → Data encoding / Feature maps

3 → VQC

4 → Quantum SVM

QML Model:
1. Deterministic
2. Probabalistic

Deterministic Models

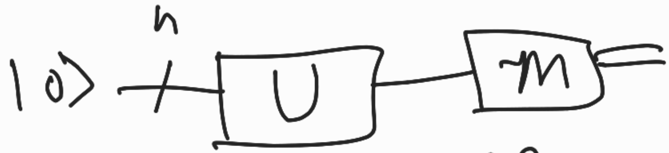
$\mathcal{X} \times \mathcal{Y}$

dom Range

$\mathcal{X} : (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_M) \xmapsto{f(x)} (y_1, y_2, \ldots, y_M)$

$$\boxed{f_\theta(x)}$$

$$|\psi(x, \theta)\rangle = U(\bar{x}, \bar{\theta})|0\rangle^{\otimes n}$$

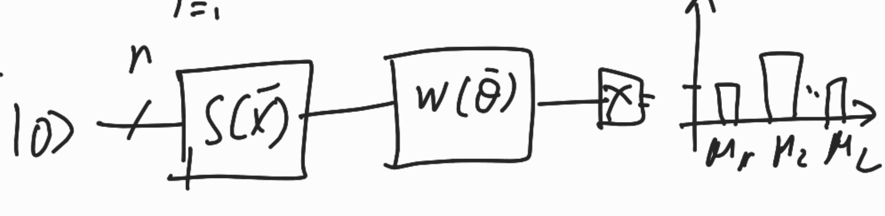


observable M

$$f_\theta(x) = \langle\psi(x,\theta)|M|\psi(x,\theta)\rangle$$

$$M = \sum_{i=1}^{L \le 2^n} \mu_i |\mu_i\rangle\langle\mu_i|$$

An example:



$$\boxed{M = \sum_i \mu_i f(\mu_i)}$$

$$\underline{C(x,\theta)} = F(f_\theta(x))$$

$$\boxed{f_{\tilde{\theta}}(x)}$$

$$\underline{U(x,\theta)} = \underbrace{W(\bar{\theta})}_{\text{ansatz}}\ \underbrace{S(\bar{x})}_{\text{feature map}}$$

Probabalistic Models:

$$M = \sum_i y_i |y_i\rangle\langle y_i|$$

$$p_\theta(y_i|\bar{x}) = |\langle y_i|\psi(\bar{x},\underline{\theta})\rangle|^2$$

$x_1 \to y_1$

$x_2 \to y_2$

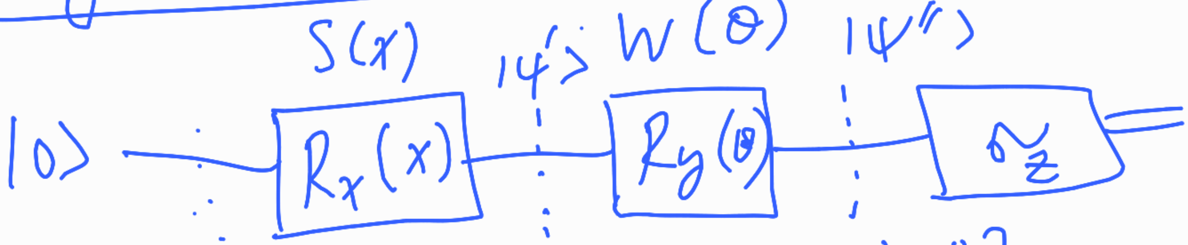
Supervised ML $\longrightarrow$ $(\bar{x}_1, \bar{x}_2, \dots \bar{x}_M)$

$$p_\theta(x_i) = |\langle x_i|\psi(\theta)\rangle|^2$$

Example: $\mathcal{X}: \{x_1, x_2, \cdots\cdots, x_m\}$

$Y: \{1, 1, \cdots, -1, -1, \cdots 1\} \in [-1, 1]$

Toy Variational Quantum Circuit (VQC)



$$M = \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$R_x(x) = \begin{bmatrix} \cos\frac{x}{2} & -i\sin\frac{x}{2} \\ -i\sin\frac{x}{2} & \cos\frac{x}{2} \end{bmatrix} \rightarrow e^{-i\frac{x}{2}\sigma_x}, \quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\psi'\rangle = \begin{bmatrix} \cos\frac{x}{2} \\ -i\sin\frac{x}{2} \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} = e^{-i\frac{\theta}{2}\sigma_y}$$

$$|\psi''\rangle = R_y|\psi'\rangle = \begin{bmatrix} \cos\frac{x}{2}\cos\frac{\theta}{2} + i\sin\frac{x}{2}\sin\frac{\theta}{2} \\ \cos\frac{x}{2}\sin\frac{\theta}{2} - i\sin\frac{x}{2}\cos\frac{\theta}{2} \end{bmatrix}$$

$$\boxed{|\psi(x,\theta)\rangle =}$$

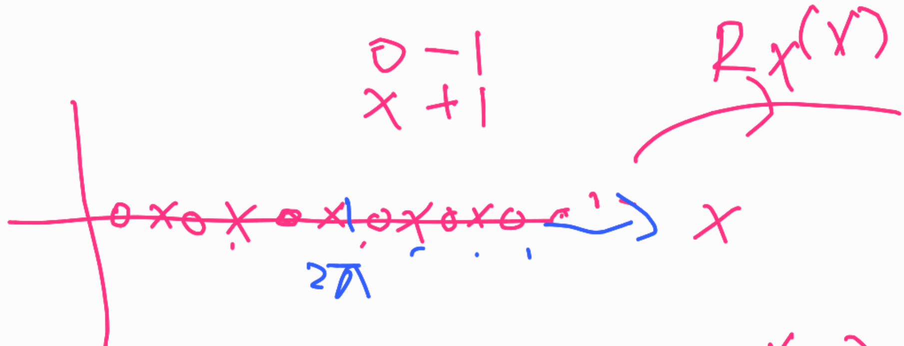
$$f_\theta(x) = \langle\psi|\sigma_z|\psi\rangle$$
$$= |\langle 0|\psi\rangle|^2 - |\langle 1|\psi\rangle|^2$$

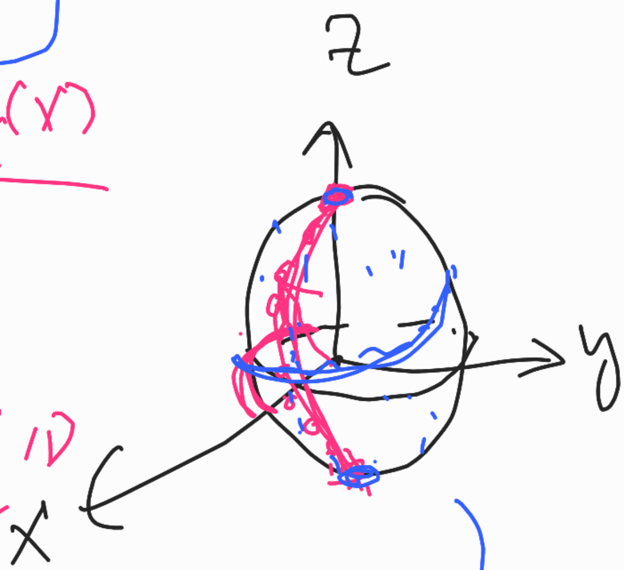
$$f_\theta(x) = \cos^2\frac{x}{2}\cos^2\frac{\theta}{2} + \sin^2\frac{x}{2}\sin^2\frac{\theta}{2}$$
$$- \left(\cos^2\frac{x}{2}\sin^2\frac{\theta}{2} + \sin^2\frac{x}{2}\cos^2\frac{\theta}{2}\right)$$

$$= \cos^2\frac{x}{2}\left(\cos\theta\right) - \sin^2\frac{x}{2}\left(\cos\theta\right)$$

$$\boxed{f_\theta(x) = \cos x \cos\theta}$$

o −1
x +1

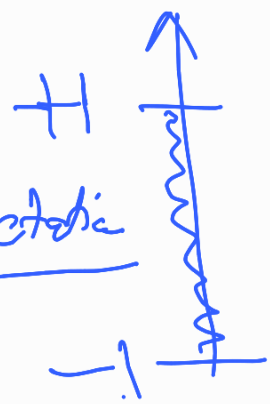




$$|\psi'\rangle = \cos\frac{x}{2}|0\rangle - i\sin\frac{x}{2}|1\rangle$$

$$= \cos\frac{x}{2}|0\rangle + e^{-i\frac{\pi}{2}}\sin\frac{x}{2}|1\rangle$$

$f_\theta = \underline{\cos x\cos\theta}$



Interpretation

f_θ

$f_\theta < 0 \longrightarrow y_p = -1$

$f_\theta > 0 \longrightarrow y_p = +1$

Lesson:

$\longrightarrow x \in (0, 2\pi)$

$\longrightarrow U(x, \theta)$

$\longrightarrow |\psi(x,\theta)\rangle$ should be in larger part of H

$$f_\theta(x) = \cos\theta \cos x$$

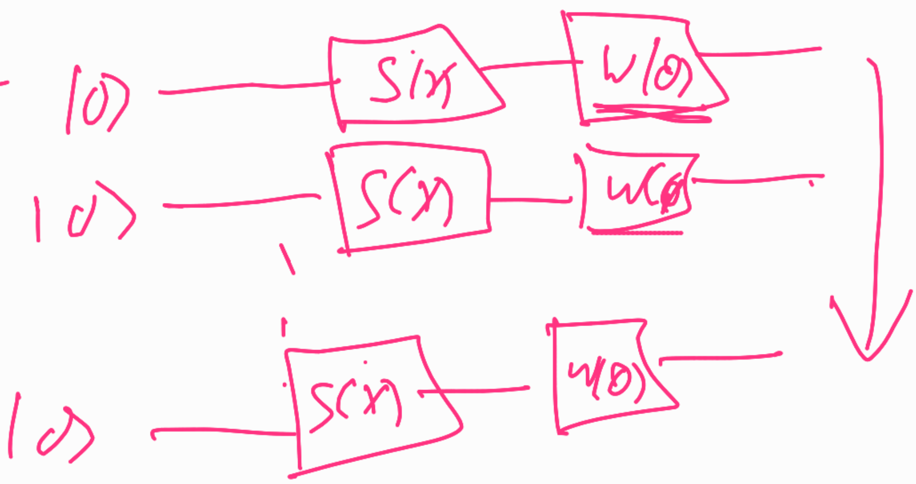
$$= \frac{1}{2} e^{-ix} \cos\theta + \frac{1}{2} e^{ix} \cos\theta$$

$$\boxed{f_\theta(x) = \sum_{n=-1}^{1} c_i e^{inx}}$$

$$f(x) = \sum_{n=-\infty}^{\infty} f_n e^{inx}$$

Repetition:

$$M = \sigma_z \otimes \sigma_z \otimes \dots \sigma_z$$



$$f_\theta(x) = (\cos\theta \cos x)^n$$

$$= \cos^n\theta \, \underbrace{\cos^n x}$$

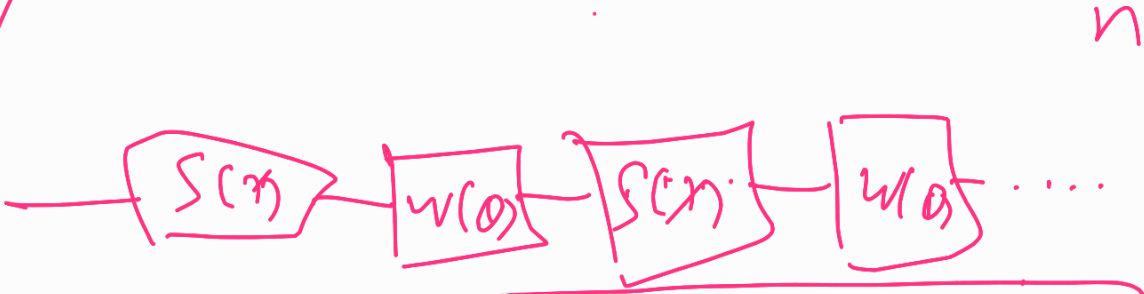
$$C_0 \cos nx + C_1 \cos(n-1)x + \cdots$$

$$\cos^n x = \left(\frac{e^{ix}+e^{-ix}}{2}\right)^n$$

$$= \frac{1}{2^n}\left\{ e^{inx},\ e^{i(n-1)x} \cdots e^{-i(n-1)x}\ e^{-inx} \cdots \right\}$$

$$\boxed{f_\theta(x) = \sum_{j=-n}^{n} c_j e^{ijx}}$$

$=$



$$\boxed{f_\theta(x) = \sum_{j=-n}^{n} c_j e^{ijx}}$$

expressivity of QML

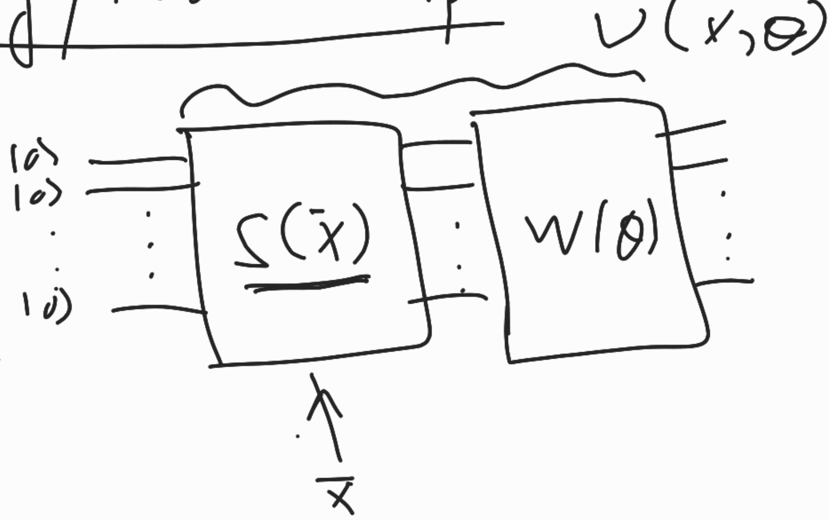
$$U \sim \underline{e^{-i\theta H}}$$

Data Encoding / Feature maps

$\underline{S(x) = \underline{U(\underline{x})}}$

$|\psi(x)\rangle = U(x)|\psi_0\rangle$

$|\psi_0\rangle = |0\rangle^{\otimes n}$



1. $|\psi(\bar{x})\rangle$ encoding directly or 2. Hamiltonian Simulation

$|\psi(x)\rangle = U(x)|\psi_0\rangle$: or $|\psi(x)\rangle = \underbrace{e^{-if(\bar{x})\hat{H}}}|\psi_0\rangle$

1. Basis Encoding:

$\bar{X} = \{\bar{x}^{(1)}, \bar{x}^{(2)}, \dots, \bar{x}^{(M)}\}$

$\bar{X}^{(j)}$

$x_k^{(j)}$

$\bar{x}^{(1)} = \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ \vdots \\ x_N^{(1)} \end{bmatrix}$

N - feature

M - examples

$\bar{x}^{(j)} = (x_1, x_2, \dots x_N)$

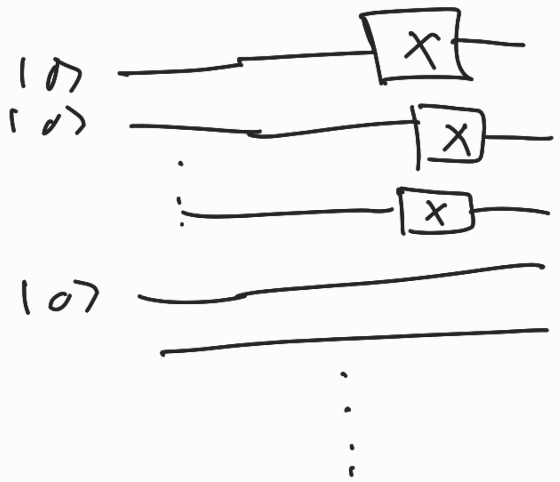
$x^{(j)} = (+3, -4, 7)$

$+3 \to 00011$, $-4 \to 10100$, $7 \to 00111$

$n = 4$

$\boxed{q = (n+1)N}$

$|\psi(x)\rangle = |x_1 x_2 x_3\rangle = \underbrace{|00011\,10100\,00111\rangle}$



1. Circuit depth =)
2. No need to normalize

2. Amplitude Encoding: $\bar{x} \in \mathbb{R}^N$

$$\bar{x}^{(j)} = [x_1, x_2, x_3, \cdots x_N]$$

$$\downarrow$$

$$\bar{a} = \frac{1}{\sqrt{x_1^2 + x_2^2 \cdots x_N^2}} [x_1, x_2, \cdots, x_N]$$

$$= [a_1, a_2, \cdots, a_N]$$

if $N \neq 2^n$

$$\bar{a} = [a_0, a_1, \cdots, a_{N-1}, 0, 0, \cdots 0]$$

$\tilde{N} = 2^n$

$$\boxed{n \text{ qubits: } = \log_2 \tilde{N} \leq \log_2 N}$$

$$|\psi(\bar{x})\rangle \approx \sum_{i=0}^{2^n} a_i |i\rangle$$

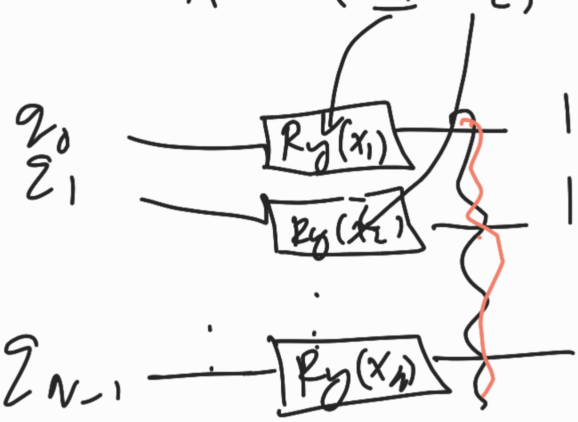
$$\boxed{|0\rangle^{\otimes n} \longrightarrow |\psi(x)\rangle}$$

3. Angle Encoding:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2} e^{i\phi}|1\rangle$$

For real amplitudes
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle = R_y(\theta)|0\rangle$$

$$X = (x_1, x_2, \dots x_N)$$



$$\left.\begin{aligned} |\psi(x_1)\rangle &= \cos\frac{x_1}{2}|0\rangle + \sin\frac{x_1}{2}|1\rangle \\ |\psi(x_2)\rangle &= \cos\frac{x_2}{2}|0\rangle + \sin\frac{x_2}{2}|1\rangle \\ &\vdots \end{aligned}\right\}$$

$$|\psi(\bar{X})\rangle = \bigotimes_i \underbrace{\left(\cos\frac{x_i}{2}|0\rangle + \sin\frac{x_i}{2}|1\rangle\right)}_{\boxed{n = N}}$$

Rescal $\bar{X} \longrightarrow \bar{X} \in [0, 2\pi]$

$\in [-\pi, \pi]$

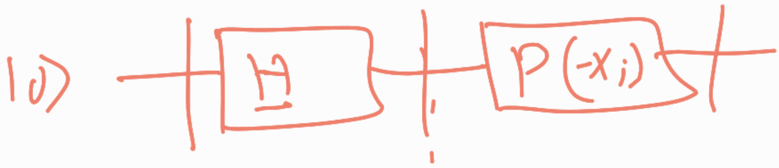
4. Phase Encoding / Z feature Map

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2} e^{i\phi}|1\rangle$$

$$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}} e^{i\phi}|1\rangle$$

$$\otimes_i |\psi(x_i)\rangle = \bigotimes_i \underbrace{\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}} e^{-ix_i}|1\rangle\right)} = |\psi(\bar{X})\rangle$$

$$|\psi(\bar{X})\rangle = \underline{|\psi(x_1)\rangle} \otimes \underline{|\psi(x_2)\rangle} \otimes \underline{\cdots}$$



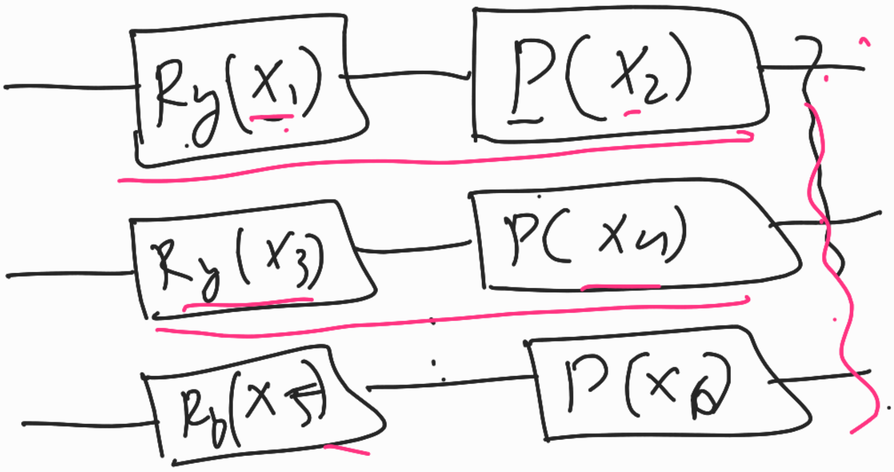
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$\frac{1}{\sqrt{2}}(|0\rangle + e^{-ix_i}|1\rangle)$

$\boxed{n = N}$

5. Dense Angle Encoding: $n = \frac{N}{2}$

$$|\psi(x_1, x_2)\rangle = \cos\frac{x_1}{2}|0\rangle + e^{ix_2}\sin\frac{x_1}{2}|1\rangle$$



Rescale $\vec{x} \in [0, 2\pi]$

6. Efficient SU2

$$|\psi\rangle_1 = (a' + ia'')|0\rangle + (b' + ib'')|1\rangle$$

$$\boxed{|a|^2 + |b|^2 = 1}$$

$$|\psi\rangle_2 = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$$

Hamiltonian Simulation View:

$$|\psi(x)\rangle = e^{-i f(x) \hat{H}} |\psi_0\rangle$$

$\boxed{t = f(x)}$

1. let $\boxed{\hat{H} = \sigma_z}$, $f(x) = \frac{x}{2}$

$$|\psi(x)\rangle = e^{-i \frac{x}{2} \sigma_z} |\psi_0\rangle$$

$$e^{-i \frac{x}{2} \sigma_z} = I - i \frac{x}{2} \sigma_z + \frac{1}{2}\left(-i \frac{x}{2} \sigma_z\right)^2 + \frac{1}{6}\left(-i \frac{x}{2} \sigma_z\right)^3 \cdots$$

$\boxed{\sigma_z^2 = I}$

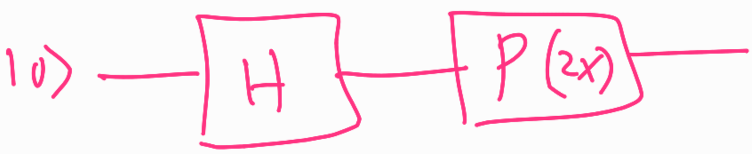
$\boxed{\sigma_z^3 = \sigma_z}$

$$= I \cos\frac{x}{2} - i \sigma_z \sin\frac{x}{2}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cos\frac{x}{2} - i \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \sin\frac{x}{2} = \begin{bmatrix} e^{-ix} & 0 \\ 0 & e^{ix} \end{bmatrix}$$

$$= R_z(x) = e^{-ix} \begin{bmatrix} 1 & 0 \\ 0 & e^{2ix} \end{bmatrix} = e^{-ix} P(2x)$$

$$|\psi_0\rangle = |+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i2x}|1\rangle\right)$$

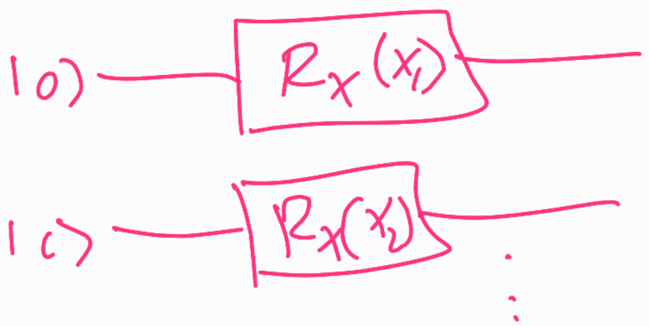


Phase encoding / Z FM

2. $f(x) = \frac{x}{2}$, $\hat{H} = \sigma_x$, $|\psi_0\rangle = |0\rangle$

$$|\psi(x)\rangle = e^{-i\frac{x}{2}\sigma_x}|0\rangle = R_x(x)|0\rangle$$

$$= \cos\frac{x}{2}|0\rangle - i\sin\frac{x}{2}|1\rangle$$



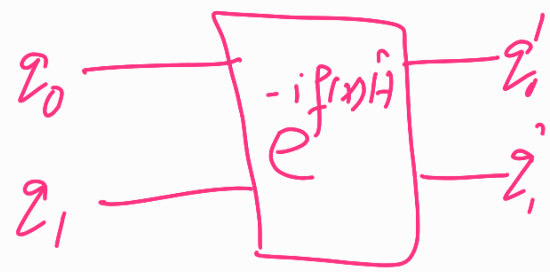
3. $|\psi(x)\rangle = e^{-i\frac{x}{2}\sigma_y}|0\rangle$

$$= R_y(x)|0\rangle$$

4. DAE $|\psi(x)\rangle = e^{-i\left[\frac{x_1}{2}\sigma_x + \frac{x_2}{2}\sigma_z\right]}|0\rangle$

5. Encoding data into correlations of qubits

$$H = \sigma_z \otimes \sigma_z$$



$$f(\bar{x}) = f(x_1, x_2)$$

$$e^{-if(x)\sigma_z\otimes\sigma_z} = I - if(x)\sigma_z\otimes\sigma_z + \frac{1}{2}(-if\sigma_z\sigma_z)^2 + \dots$$

$$e^{-if(x)\sigma_z\otimes\sigma_z} = I\cos[f(x)] - i\,\sigma_z\otimes\sigma_z\sin[f(x)]$$

$\sigma_z|0\rangle = |0\rangle$, $\sigma_z|1\rangle = -|1\rangle$

$q_0 q_1$	$q_0' q_1'$
$\vert 00\rangle$	$\vert 00\rangle e^{-if}$
$\vert 01\rangle$	$\vert 01\rangle e^{if}$
$\vert 10\rangle$	$\vert 10\rangle e^{if}$
$\vert 11\rangle$	$\vert 11\rangle e^{-if}$

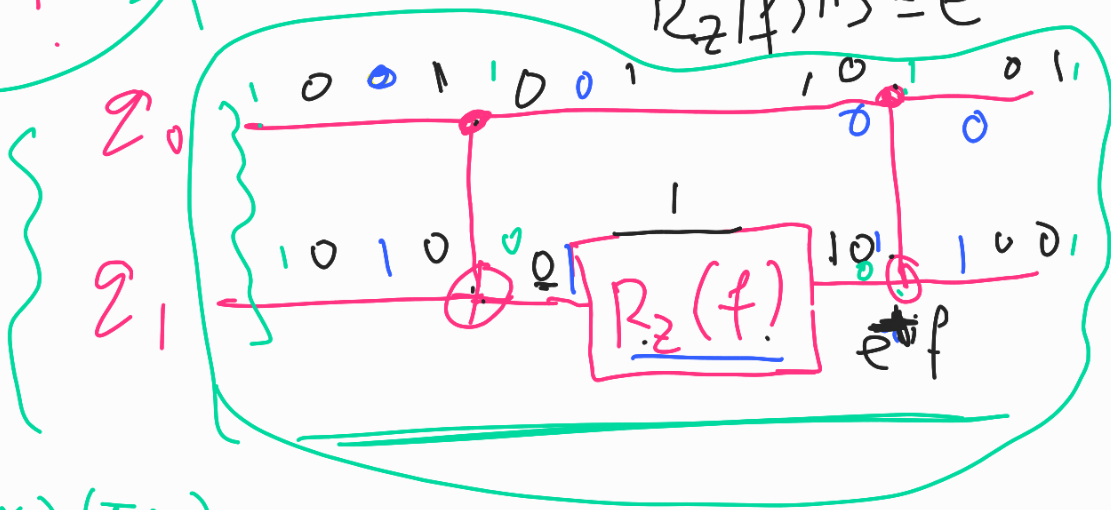
$$e^{-if\sigma_z\otimes\sigma_z}\,\vert 00\rangle$$

$$= \vert 00\rangle \cos f - i\,\vert 00\rangle \sin f$$

$$= \vert 00\rangle\, e^{+if}$$

$$R_z(f)\vert 0\rangle = e^{-if}$$

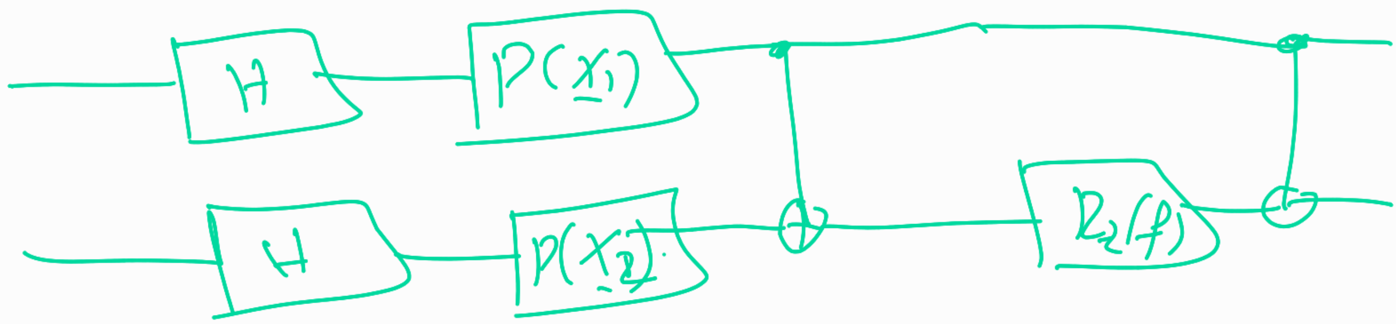
$$R_z(f)\vert 1\rangle = e^{if}$$



ZZ FM

$$f(\bar{x}) = 2(\pi - x_1)(\pi - x_2)$$

$$R_{ZZ}(x_1, x_2)$$

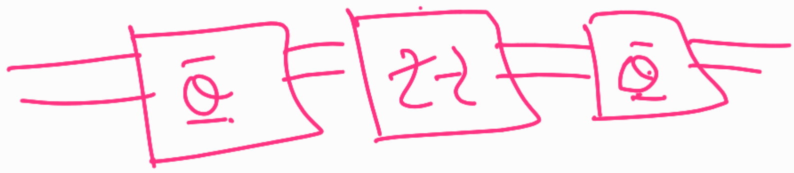


=

XY FM

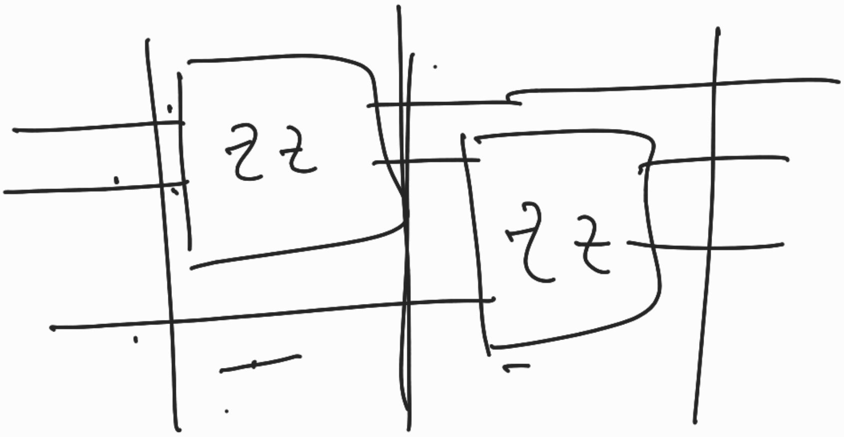
$$\hat{H} = \sigma_x \otimes \sigma_y$$

$$e^{-i f(x)\sigma_x\otimes\sigma_z} = H\otimes I\; e^{-i f(x)\sigma_z\otimes\sigma_z}\; H\otimes I$$



$$e^{H_1+H_2} \neq e^{H_1} e^{H_2}$$

$$\approx |e^{H_1}|e^{H_2}|$$



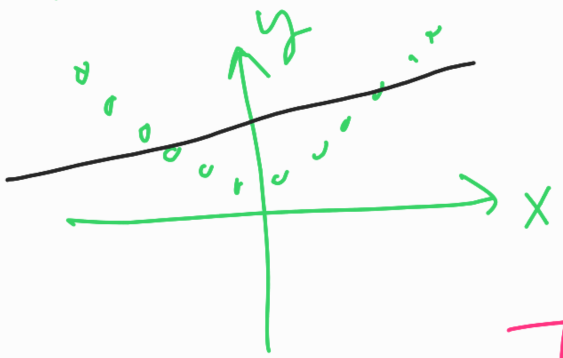
Quantum Variational Circuits

→ classification
→ Regression

→ Essentially Linear models higher dimensional feature space.

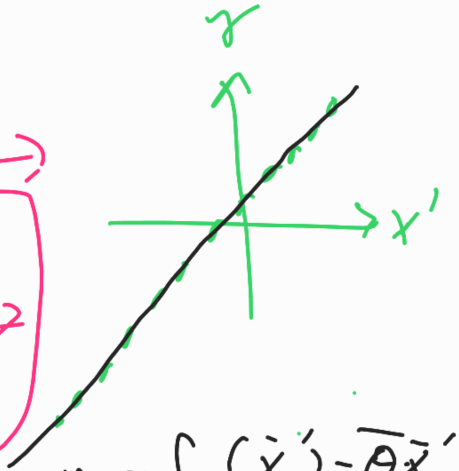
Classical Linear models:

$y = f_\theta(x) = \overline{\theta} \cdot \overline{X}$



$\bar{x}' = \phi(x) = x^2$

Feature map



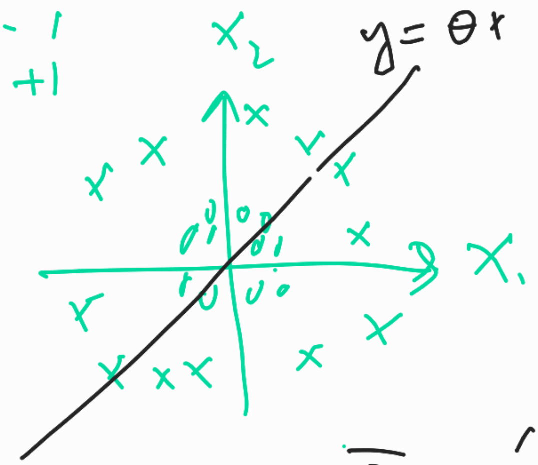
$y = f_{\bar{\theta}}(\bar{x}') = \bar{\theta}\bar{x}'$

$$\boxed{y = \bar{\theta} \cdot \phi(\bar{x})} = \sum_{i=1}^{N} \theta_i \phi_i(\bar{x}) = \sum_i \theta_i x_i'$$

$$= \theta_1 x_1' + \theta_2 x_2' + \theta_3 x_3' \dots$$

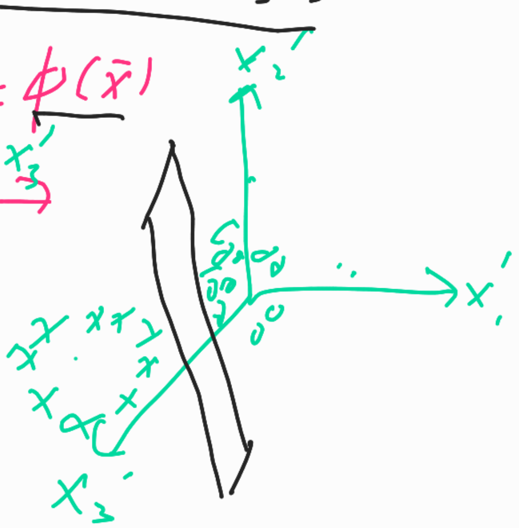
$x = -1$
$o = +1$

$y = \theta x$



$$x' = \begin{bmatrix} x_1 = x_1' \\ x_2 = x_2' \\ x_1^2 + x_2^2 \end{bmatrix} = \phi(\bar{x})$$

$= x_3'$



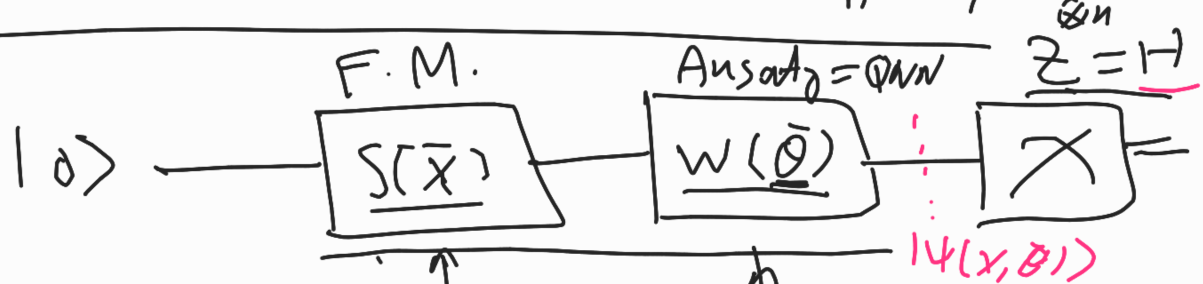
$$y = \bar{\theta} \cdot x' = \sum_i \theta_i \phi_i(x)$$

$$MSE = \frac{1}{2M} \sum_{i=1}^{M} (\hat{y}_i - y_i)$$

$\phi_{\bar{w}}(\bar{x})$

$$y = \sum f(\theta)\cdot\phi(x)$$
$$= \sum \theta_i\, \phi_i(x)$$

Variational Quantum Classifier/QNN



data = input paramete weight paramters.

$\longrightarrow$ $f = \langle H \rangle$ $\longrightarrow$ interpretation $f > 0,\ y = 1$
$f < 0,\ y = -1$

θ = Updated to minimize $C(x,\theta)$

Analysis:

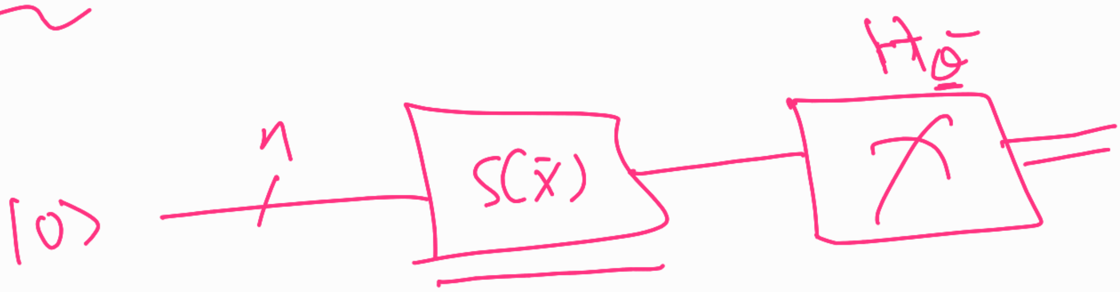
$$\hat{H} = Z^{\otimes n} = \sigma_z^{\otimes n}$$
$$= \sigma_z \otimes \sigma_z \otimes \sigma_z \otimes \cdots \otimes \sigma_z$$

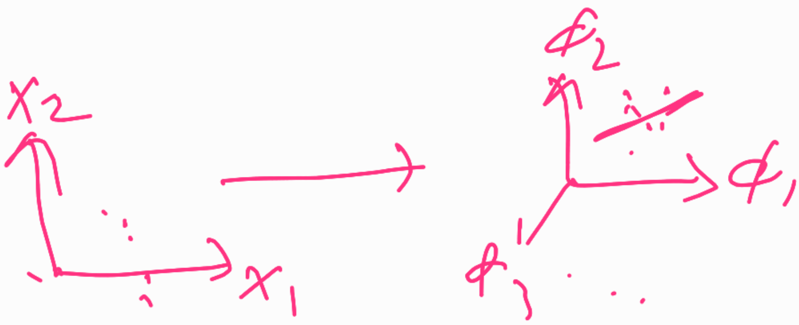
$$f_{\bar{\theta}}(\bar{x}) = \langle\psi|\hat{H}|\psi\rangle$$

$$f_{\bar{\theta}}(\bar{x}) = \langle 0| S^{\dagger}(\bar{x}) W^{\dagger}(\bar{\theta})\, Z^{\otimes n}\, \underbrace{W(\bar{\theta}) S(\bar{x})|0\rangle}_{|\psi(\bar{x},\bar{\theta})\rangle}$$

$$\underset{\sim}{H_{\bar\theta}} = \underline{W}^{\dagger}(\bar\theta)\, \underset{\sim}{Z}^{\otimes n}\, \underline{W}(\bar\theta)$$

$$\underset{\sim}{f_{\bar\theta}(\bar x)} = \langle \phi(\bar x) | H_{\bar\theta} | \phi(\bar x) \rangle$$





$$H_{\bar\theta} = \sum_{\alpha} \underline{c_\alpha(\bar\theta)}\, \underline{P_\alpha}$$

$$H_{\bar\theta}^{\otimes 3} = c_0 \underbrace{\sigma_z \sigma_z \sigma_z}_{P_1} + c_1 \underbrace{I}_{P_2} + c_2 \underbrace{I \otimes I \otimes Y \cdots}_{P_3}$$

$$f_{\bar\theta} = \langle \phi(\bar x) | H_{\bar\theta} | \phi(\bar x) \rangle$$

$$= \sum_{\alpha} c_\alpha \underset{\sim}{\langle \phi(x) | P_\alpha | \phi(x) \rangle}$$

$$\varphi_\alpha(\bar x) = \langle \phi(x) | P_\alpha | \phi(\bar x) \rangle$$

$$\boxed{f_{\bar{\theta}} = \sum_{\alpha} c_{\alpha}(\bar{\theta})\, \underline{\varphi_{\alpha}(\bar{x})}}$$

$$= \underline{c_1(\bar{\theta})}\, \underline{\varphi_1(x)} + \underline{c_2(\bar{\theta})}\, \underline{\varphi_2(\bar{x})} \cdots$$

$$= c_1(\bar{\theta})\, x_1' + c_2 x_2' + c_3 x_3' \cdots$$

$$\boxed{f_{\bar{\theta}} = \sum_i c_i x_i'}$$

Training of VQC:

$$f_{\bar{\theta}}(\bar{x}) = \langle 0 | U^{\dagger} M U | 0 \rangle$$

$$M = Z$$

$$U = W(\bar{\theta})\, S(\bar{x})$$

$$= \prod_{i=1} W_i(\bar{\theta}_i)\, S(\bar{x}_i)$$

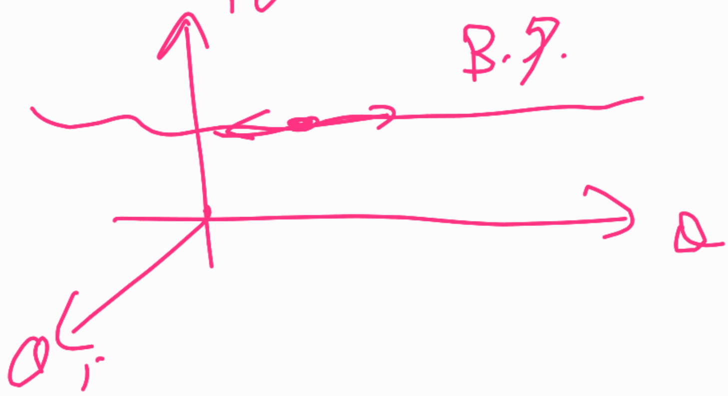
Barren plateaus:

$$\underline{\left\langle \frac{\partial f}{\partial \theta_i} \right\rangle \approx 0 \,, \quad \mathrm{Var}\left\{ \frac{\partial C}{\partial \theta_i} \right\}} \sim \frac{1}{2^n}$$

Shallow $\longrightarrow$ $\boxed{\text{depth} \le O(n^{1/2})}$

layers > n

$$\theta_i' = \theta_i - \alpha \frac{\partial f_\theta}{\partial \theta_i}$$



How to implement in Qiskit:

1. Dataset $\longrightarrow$ scale the data $\longrightarrow$ Tr. Set / Tst. Set
2. Choose E.M. to encode data X
3. Choose $W(\theta)$
4. Cost function $\longrightarrow$
5. Minimization Algorithm $\longrightarrow \theta^*$

$$\bar{X} = [-7, 2, 0.1, 10, 11] \xrightarrow{\text{rescaling}} [0-2\pi, 0-2\pi, \ldots]$$

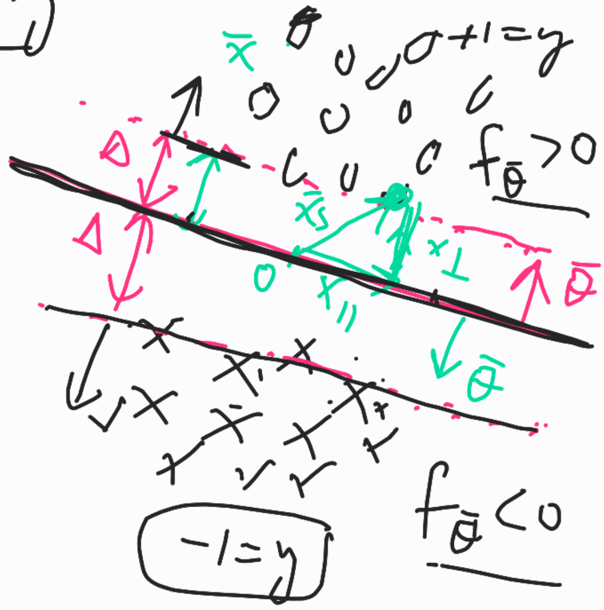
Quantum Support Vector Machines

Support Vector Machines' theory

$x = -1$
$o = +1$

$$f_{\bar\theta}(\bar x) = \bar\theta \cdot \bar x$$

s.t. maximum margin



$$\bar x_s = \bar x_{\parallel} + \bar x_{\perp}$$

$$\bar x_s = \bar x_{\parallel} + \Delta \frac{\bar\theta}{\theta}$$

$$\bar\theta = (\theta_1, \theta_2, \ldots, \theta_N)$$

$$\hat\theta = \frac{\bar\theta}{\theta}$$

$$f_{\bar\theta}(\bar x_s) = \bar\theta \cdot \bar x_s$$

$$= \bar\theta \cdot \bar x_{\parallel} + \Delta \frac{\bar\theta \cdot \bar\theta}{\theta}$$

$$f_{\bar\theta}(\bar x_s) = 0 + \Delta\theta \longrightarrow \boxed{\Delta = \frac{f_{\bar\theta}(\bar x_s)}{\theta}}$$

To maximize Δ, we can minimize θ, $\boxed{\frac{\theta}{\Delta} \longrightarrow \text{minimized}}$

Constraints

$$f_{\bar\theta}(\bar x_i) > \Delta, \quad y_i = 1$$

$$f_{\bar\theta}(\bar x_i) \le -\Delta, \quad y_i = -1$$

$$f(\theta) = \theta_1 x_1 + \theta_2 x_2 =$$

$$(\theta_1, \theta_2)$$

$$y_i f_{\bar\theta}(x_i) \geq \Delta$$

$$y_i f_{\bar\theta}(x_i) \geq \Delta$$

Constraint $\longrightarrow$ $\boxed{y_i f_\theta(x_i) - \Delta \geq 0}$

$$\boxed{C(\bar\theta) = \frac{1}{2}\left(\frac{\theta}{\Delta}\right)^2 - \sum_{i=1}^{M} \alpha_i \left[y_i f_{\bar\theta}(x_i) - \Delta\right]}$$

$$C(\bar\theta) = \frac{1}{2}\left(\frac{\theta}{\Delta}\right)^2 - \sum_i \alpha_i \Delta \left[y_i \frac{\bar\theta \cdot \bar x^{(i)}}{\Delta} - 1\right]$$

$\theta^2 = \theta_1^2 + \theta_2^2 + \dots + \theta_n^2$, ω $\frac{\theta}{\Delta} = \theta'$

$\alpha_i \Delta = \alpha_i'$

$$\boxed{C(\bar\theta') = \frac{\theta'^2}{2} - \sum_i \alpha_i' \left[y_i \underbrace{\bar\theta' \cdot x^{(i)}}_{\theta_1 x_1 + \theta_2 x_2 + \dots} - 1\right]}$$

Dual Form:

$$\frac{\partial C}{\partial \theta_j'} = 0 \quad : \quad \frac{\partial}{\partial \theta_j}(\theta_1^2 + \theta_2^2 \dots)$$

$$0 = \theta_j' - \sum_i \alpha_i' \left[y_i \quad x_j^{(i)}\right]$$

$$\theta_j' = \sum_i \alpha_i' \left[y_i \quad x_j^{(i)} \right]$$

$$\underline{\bar{\theta}'} = \sum_i \alpha_i' \left[y_i \quad \bar{x}^{(i)} \right]$$

$$C(\bar{\theta}, \bar{\alpha}) = \frac{1}{2} \bar{\theta}'^T \theta' - \sum_i \alpha_i \left[y_i \bar{\theta}^T \bar{x}^{(i)} - 1 \right]$$

$$C(\bar{\theta}, \bar{\alpha}) = \frac{1}{2} \sum_i \alpha_i' y_i \bar{x}^{iT} \cdot \sum_j \alpha_j' y_j \bar{x}^j$$

$$- \sum_i \alpha_i y_i \sum_j \alpha_j y_j \bar{x}^{jT} \cdot \bar{x}^i$$

$$+ \sum_i \alpha_i$$

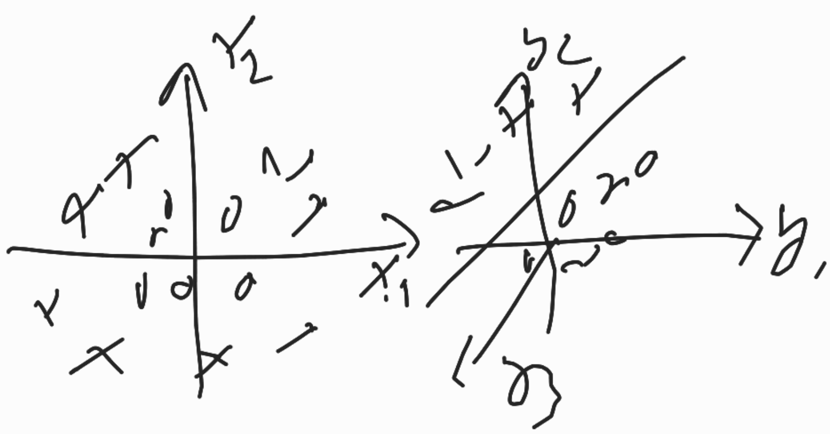
$$\boxed{C(\bar{\alpha}) = \sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i' \alpha_j' y_i y_j \bar{x}^{jT} \cdot \bar{x}^i}$$

$$\langle \bar{x}^j, \bar{x}^i \rangle$$

Kernel Methods

$$\bar{y} = f(\bar{x})$$

$$\underline{\bar{y}^{jT} \cdot y^i}$$



$$C(\alpha) = \sum \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i y_i y_j \, \bar{y}^{Tj}\bar{y}^i$$

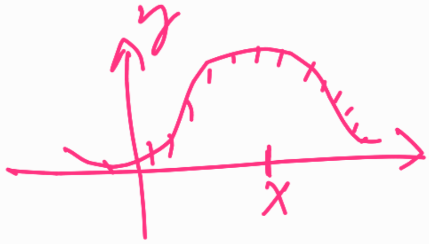
$$K(x^j, x^i) = \langle \bar{y}^j, \bar{y}^i \rangle$$

$$\bar{X}_{10} \longrightarrow \bar{y}_{20}, \quad \bar{y}_{100}$$

CML

Find $K(\bar{x}^j, \bar{x}^i)$ $\quad \boxed{y = f(\bar{x})}$ X

X



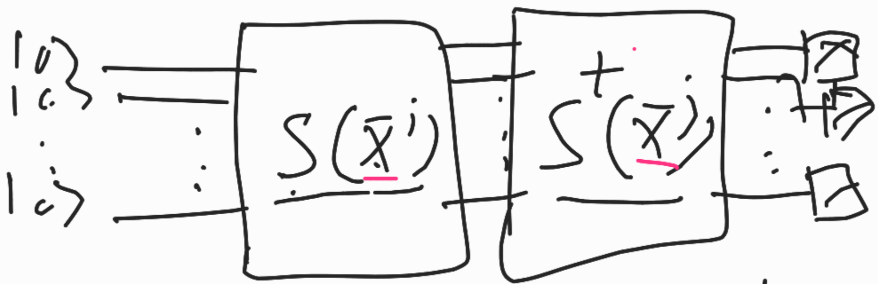
QML

$n = 10, \quad 2^{10}$

$$y = \phi(\bar{x})$$

$$|\phi(\bar{x})\rangle = S(\bar{x})|0\rangle$$

$$\langle \phi(x^j)|\phi(x^i)\rangle = \langle 0|S^\dagger(x^j)S(x^i)|0\rangle$$
$$= \langle 0|S^\dagger(\bar{x}^j)S(\bar{x}^i)|0\rangle$$



$$1 = \langle 0|S^\dagger(\bar{x}^j)|S(\bar{x}^i)|0\rangle \quad p(0) = \frac{N_0}{N} =$$

$$K = \begin{bmatrix} 1 & \cdots & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}_{2^m \times 2^m}$$

$\boxed{M = 2^n}$

To implement:

1. choose FM $|\phi(\bar{x})\rangle$
2. Compute $\underline{K(\bar{x}^i, \bar{x}^j)}$
3. Train $\underline{\text{SVM}}$

$\bar{x}^i = \bar{x}, \quad \bar{x}^j = \bar{y}$

A few examples

$x = x_{n-1} x_{n-2} \cdots x_1 x_0$

1. Basis encoding

$|\psi(x)\rangle = \underline{|x_{n-1} x_{n-2} \cdots x_1 x_0\rangle}$

$|\psi(y)\rangle = \underline{|y_{n-1} y_{n-2} \cdots y_1 y_0\rangle}$

$K(x,y) = \langle \psi(x) | \psi(y) \rangle = \underline{\langle x_{n-1} | y_{n-1} \rangle} \cdots \underline{\langle x_0 | y_0 \rangle}$

$$= \begin{cases} 0 & x \neq y \\ 1 & x = y \end{cases}$$

$$K(x,y) = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

2. Amplitude Encoding

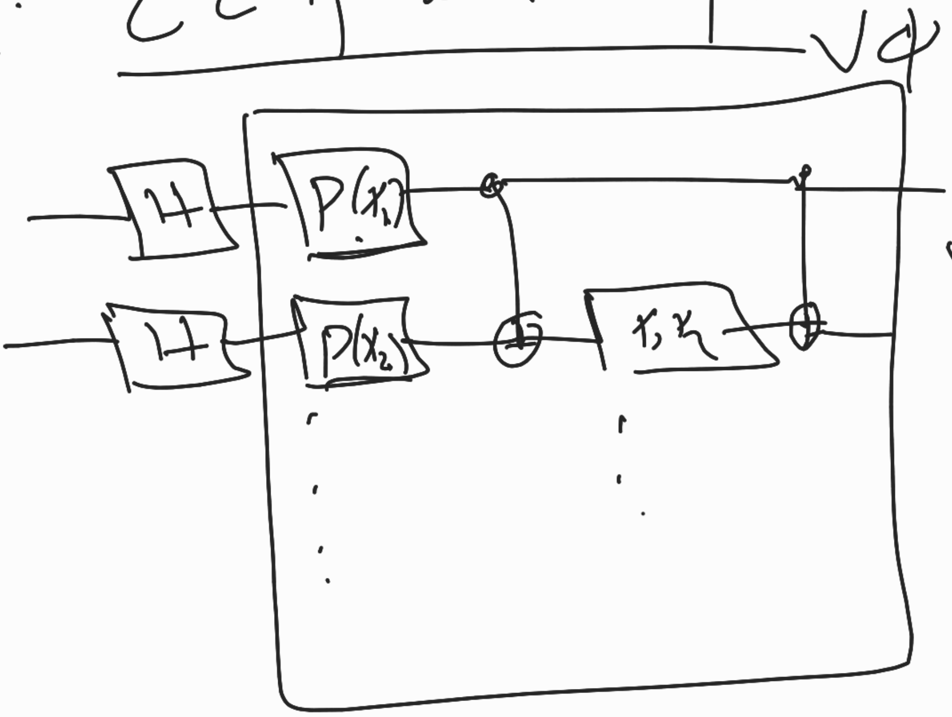
$$|\psi(\bar{x})\rangle = x_0|0\rangle + x_1|1\rangle + x_2|2\rangle \dots + x_n|n\rangle$$
$$|\psi(\bar{y})\rangle = y_0|0\rangle + y_1|1\rangle \dots + y_n|n\rangle$$

$$\langle\psi(x)|\psi(y)\rangle = x_0y_0 + x_1y_1 + x_2y_2 + \dots + x_ny_n$$

polynomial

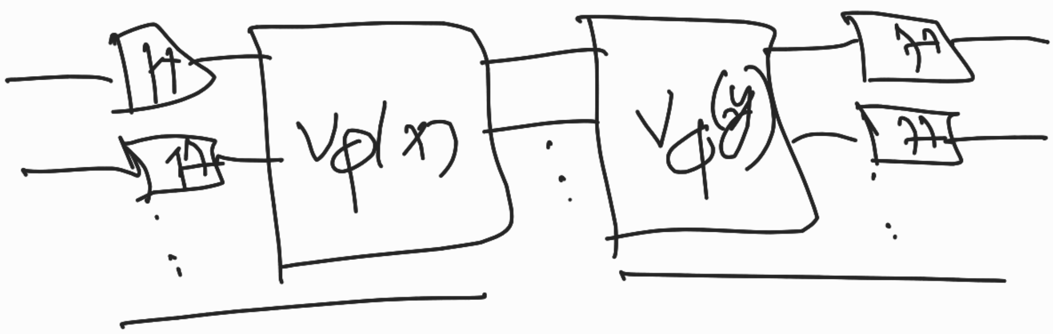
3. Product:

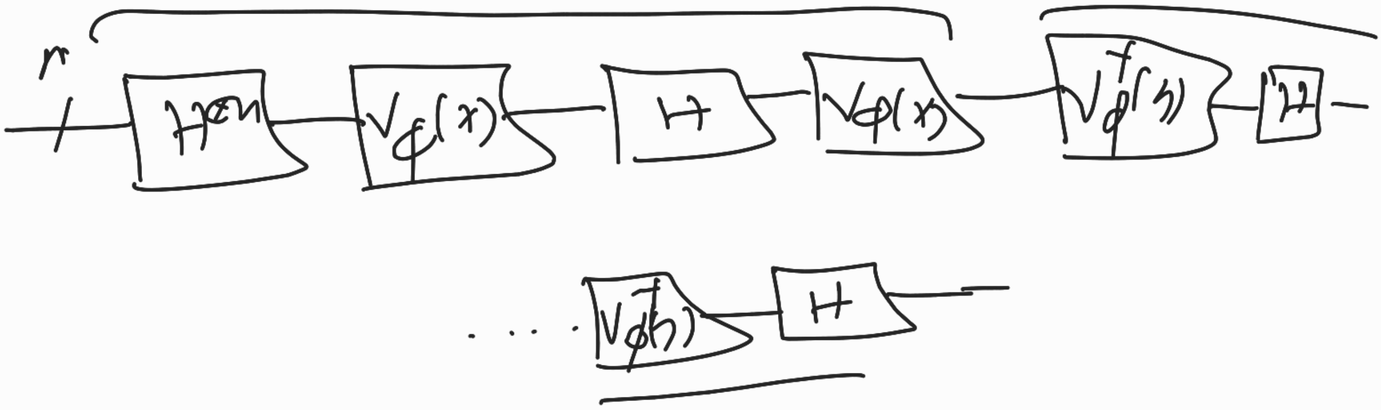
4. ZZ feature maps



$$V_\phi = e^{i\sum_i \phi_i(x) Z_i} \times e^{i\sum_{ij}\phi_{ij}(\bar{x}) Z_i Z_j}$$

$$(AB)^\dagger = B^\dagger A^\dagger$$





How to select a $|\phi(\bar{x})\rangle$

- start with a simple
- Add entangling layer
- repetition

$$|\phi(\bar{x}, \underline{\bar{\theta}})\rangle$$

Kernel Alignment