Sheet 2

1. Suppose that you want to measure a parameter ω_0. You choose a two-level system as your quantum probe. This parameter shows up in the Hamiltonian of your probe: $H = \frac{\hbar\omega_0}{2}\sigma_z$. You initialize the probe in the state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$. Here $\sigma_z\,|0\rangle = |0\rangle$ and $\sigma_z\,|1\rangle = -\,|1\rangle$.

   (a) What is the state of the probe at time t?

   (b) Suppose that at time t, I measure the observable $X = |0\rangle\langle 1| + |1\rangle\langle 0|$. What are the possible measurement results? What are the corresponding probabilities?

   (c) Given these probabilities, what is the Fisher information for the estimation of ω_0? Sketch this versus time.

   (d) Suppose now that the Hamiltonian is $H = \frac{\hbar\omega_0}{2}\cos(\omega t)$. This happens when we have to measure an oscillating field. What is the time at time t now?

   (e) We again perform a measurement of the observable X. What is the Fisher information now? Compare with the previous scenario and comment.