Sheet 3

1. Suppose that you want to measure a parameter ω_0. You choose a two-level system as your quantum probe. This parameter shows up in the Hamiltonian of your probe: $H = \frac{\hbar\omega_0}{2}\sigma_z$. You initialize the probe in the state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$. Here $\sigma_z\,|0\rangle = |0\rangle$ and $\sigma_z\,|1\rangle = -\,|1\rangle$. Now, however, after every time interval τ you apply a quantum NOT gate (or a X gate). What is the state just after applying N such gates (here N is even)? Explain.

2. Now consider the same situation as in the previous problem, except that we now have an oscillating field so that the Hamiltonian is now $H = \frac{\hbar\omega_0}{2}\cos(\omega t)\sigma_z$. You again apply the quantum NOT gates, but these are now applied whenever the field becomes zero. What is then the state at time $t = 2\pi N/\omega$? Here N is the number of gates applied.