

Graph-colorings: Sudoku game

To do this project I should learn some of the topics as,

1. What is Sudoku
2. What is Graph Coloring
3. Graph Coloring Algorithm
4. How Sudoku can be solved using Graph Coloring

1-What is Sudoku ?

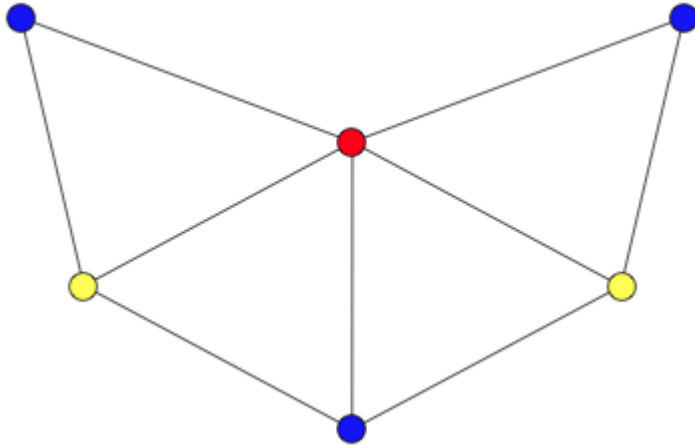
Sudoku is a single — player logic based puzzle. A Sudoku puzzle is a grid of 81 cells, which is divided into 9 rows, columns and regions(or blocks).

The goal is to place the numbers from 1- 9 into empty cells in such a way, that in every row, every column and every region (3 x 3 block) each number appears only once .

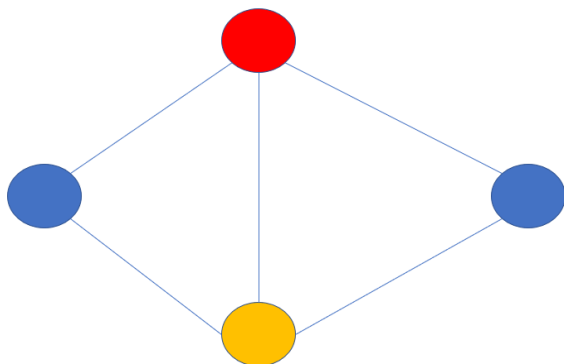
5	3			7					5	3	4	6	7	8	9	1	2
6			1	9	5				6	7	2	1	9	5	3	4	8
	9	8					6		1	9	8	3	4	2	5	6	7
8				6				3	8	5	9	7	6	1	4	2	3
4			8		3			1	4	2	6	8	5	3	7	9	1
7				2				6	7	1	3	9	2	4	8	5	6
	6					2	8		9	6	1	5	3	7	2	8	4
			4	1	9			5	2	8	7	4	1	9	6	3	5
				8			7	9	3	4	5	2	8	6	1	7	9

2-What is Graph Coloring ?

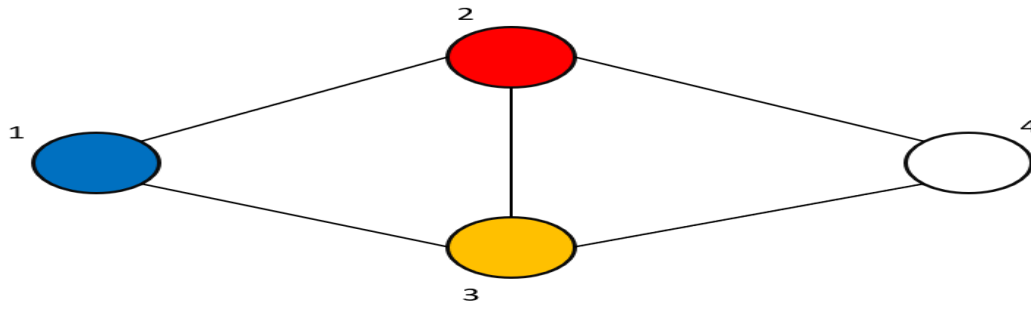
Graph coloring is an assignment of different colors (or labels) to the vertices of a graph, such that no 2 adjacent (connected) vertices have the same color



In **G- Graph Coloring Problem**, we have to find if a graph can be colored with a **minimum** of '**G**' colors. This '**G**' is also known as the **Chromatic Number** of a Graph, and is denoted as $\chi(G)$



for this graph, Chromatic Number, $G = 3 \{ \chi(3) \}$



3-Algorithm for Graph Coloring (m-Coloring Decision Problem)

The idea is to assign colors one by one to different vertices, starting from the vertex 0. Before assigning a color, check for safety by considering already assigned colors to the adjacent vertices i.e check if the adjacent vertices have the same color or not. If there is any color assignment that does not violate the conditions, mark the color assignment as part of the solution. If no assignment of color is possible then **backtrack** and return false.

Algorithm

1. Create a recursive function that takes current vertex index, number of vertices and output color array as arguments.
2. If the current vertex index is equal to number of vertices. Return True and print the color configuration in output array.
3. Assign color to a vertex (1 to m).
4. For every assigned color, check if the configuration is safe, (i.e. check if the adjacent vertices do not have the same color) recursively call the function with next index and number of vertices
5. If any recursive function returns true break the loop and return true.
6. If no recursive function returns true, then return false.

4-How Sudoku can be solved using Graph Coloring

Sudoku Graph is a graph with 81 vertices (or nodes) . Each cell in the Sudoku can be seen as a node of the graph. Each node (or cell) has an edge to every other node (cell) in its respective column, row, and 3 x 3 grid.

Sudoku can be viewed as Graph and thus can be solved by Graph Coloring with a Chromatic Number, $G = 9$.

