11.16.1.11

EE24BTECH11008 - Aslin Garvasis

Question:
Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Find the PMF of the random variable using the sum of three independent Bernoulli trials.
Take $P(N) = \frac{1}{2}$.

Solution:
Let X be a discrete random variable representing the number of non-defective bulbs from the three selected bulbs:

$$X = X_1 + X_2 + X_3 \tag{1}$$

where $X_1, X_2, X_3 \sim \text{Bernoulli}(p = 0.5)$.

$$X_i = \begin{cases} 1, & \text{Non-defective} \\ 0, & \text{Defective} \end{cases} \tag{2}$$

Thus, $X \sim \text{Binomial}(n = 3, p = 0.5)$.

Moment-Generating Function (MGF) Using the Z-Transform:

$$M_{X_i}(z) = (1 - p) + pz^{-1} \tag{3}$$

Since X_1, X_2, X_3 are independent,

$$M_X(z) = ((1 - p) + pz^{-1})^3 \tag{4}$$

$$= \sum_{n=0}^{3} \binom{3}{n} (1 - p)^{3-n} p^n z^{-n} \tag{5}$$

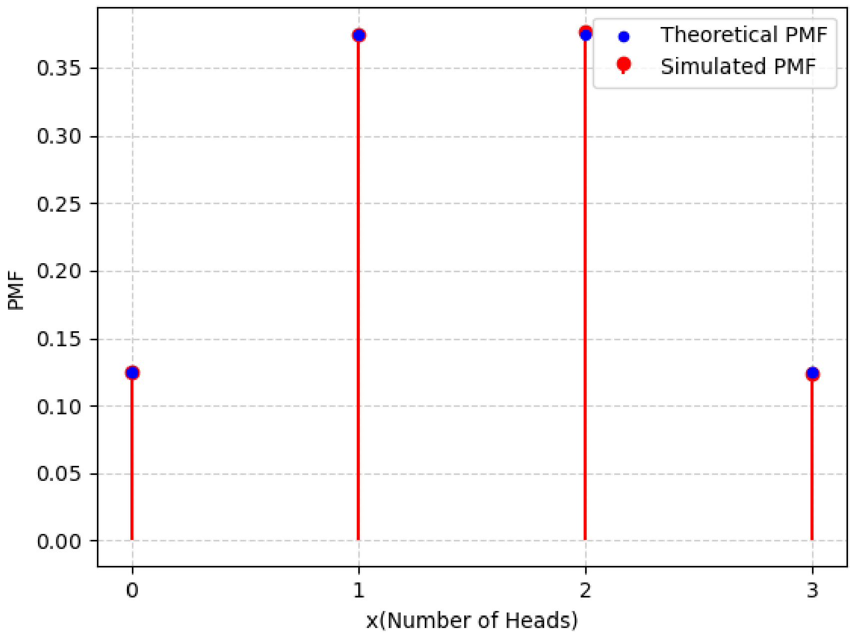
Substituting $p = \frac{1}{2}$:

$$p_X(n) = \frac{\binom{3}{n}}{8}, \quad n \in \{0, 1, 2, 3\} \tag{6}$$

Probability Mass Function (PMF):

$$p_X(n) = \begin{cases} \frac{1}{8}, & n = 0 \\ \frac{3}{8}, & n = 1 \\ \frac{3}{8}, & n = 2 \\ \frac{1}{8}, & n = 3 \end{cases} \tag{7}$$

The graph below compares the theoretically calculated and simulated PMF of the given random variable.



Theoretical PMF
Simulated PMF
PMF
x(Number of Heads)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0
1
2
3