2023-January Session-01-30-2023-shift-1

EE24BTECH11008-ASLIN GARVASIS

1) Let $\mathbf{A} = \begin{pmatrix} m & n \\ p & q \end{pmatrix}, d = |A| \neq 0 \, |A - D(AdjA)| = 0$.Then

   a) $(1+d)^2 = (m+q)^2$
   b) $1+d^2 = (m+q)^2$
   c) $(1+d)^2 = m^2+q^2$
   d) $1+d^2 = m^2+q^2$

2) The line l_1 passes through the point $(2,6,2)$ and is perpendicular to the plane $2x+y-2z=10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} + \frac{y+4}{-3} + \frac{z}{2}$ is :

   a) 7
   b) $\frac{19}{3}$
   c) $\frac{19}{2}$
   d) 9

3) If an unbiased die marked with $-2,-1,0,1,2,3$ on its faces, is through five times, then the probability that the product of the outcomes is positive is ...

   a) $\frac{881}{2592}$
   b) $\frac{521}{2592}$
   c) $\frac{440}{2592}$
   d) $\frac{27}{288}$

4) Let the system of linear equations

$$x+y+kz=2$$

$$2x+3y-z=1$$

$$3x+4y+2z=k$$

   have infinetely many solutions.Then the system

$$(k+1)x+(2k-1)y=7$$

$$(2k+1)x+(k+5)y=10 \text{ has :}$$

   a) infenitely many solutions
   b) unique solution satisfying $x-y=1$
   c) no solution
   d) unique solution satisfying $x+y=1$

5) If $\tan 15° + \frac{1}{\tan 75°} + \frac{1}{\tan 105°} + \tan 195° = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is :

   a) 4
   b) $4-2\sqrt{3}$
   c) 2
   d) $5-\frac{3}{2}\sqrt{3}$

6) Suppose $f : \mathbf{R} \to (0,\infty)$ be a differentiable function such that

$$5f(x+y) = f(x) \cdot f(y), \forall x,y \in \mathbf{R}$$

   If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to :

   a) 6875
   b) 6575
   c) 6825
   d) 6528

7) If $a_n = \frac{-2}{4n^2-16n+15}$, then $a_1 + a_2 + \cdots + a_25$ is equal to :

   a) $\frac{51}{144}$
   b) $\frac{49}{138}$
   c) $\frac{50}{141}$
   d) $\frac{52}{147}$

8) If the coefficient of x^{15} in the expansion of $\left(ax^2 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{\frac{1}{3}} - \frac{1}{bx^3}\right)$, where a and b are positive real numbers, then for each such ordered pair (a,b) :

   a) $a=b$
   b) $ab=1$
   c) $a=3b$
   d) $ab=3$

9) if $\overline{a}, \overline{b}, \overline{c}$ are three non-zero vectors and $\hat{n}$ is a vector perpendicular to $\overline{c}$ such that $\overline{a} = \alpha\overline{b} - \hat{n}, (\alpha \neq 0)$ and $\overline{b} \cdot \overline{c} = 12$, then $\left|\overline{c}X\left(\overline{a}X\overline{b}\right)\right|$ is equal to :

   a) 15

b) 9
c) 12
d) 6

10) The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to $x + 90y + 2 = 0$ is :

a) 2
b) 3
c) 4
d) 0

11) Let $y = x + 2, 4y = 3x + 6$ and $3y = 4x + 1$ be three tangent lines to the circle $(x-h)^2 + (y-k)^2 = r^2$. Then $h + k$ is equal to :

a) 5
b) $5\left(1+\sqrt{2}\right)$
c) 6
d) $5\sqrt{2}$

12) Let the solution curve $y = y(x)$ of the differential equation

$$\frac{dy}{dx} - \frac{3x^5 \tan^{-1} x^3}{(1+x^6)^{\frac{3}{2}}} y = 2x \cdot \exp \frac{x^3 - \tan^{-1} x^3}{\sqrt{(1+x^6)}}$$

pass through the origin. Then $y(1)$ is equal to :

a) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$
b) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$
c) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$
d) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$

13) Let a unit vector $\hat{\mathbf{OP}}$ make angle α, β, γ with the positive directions of the co-ordinate axes **OX,OY,OZ** respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$ $\hat{\mathbf{OP}}$ is perpendicular to the plane through points $(2,3,4)$ and $(1,5,7)$, then which of the following is true :

a) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$
b) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$
c) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$
d) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

14) If $[t]$ denotes the greatest integer $\leq t$, then the value of

$$\frac{3(e-1)^2}{e} \int_1^2 x^2 e^{[x]+[x^3]} dx \text{ is :}$$

a) $e^9 - e$
b) $e^8 - e$
c) $e^7 - e$
d) $e^8 - 1$

15) If $\mathbf{P}(h,k)$ be point on the parabola $x = 4y^2$, which is nearest to the point $\mathbf{Q}(0,33)$, then the distance of $\mathbf{P}$ from the directrix of the parabola $y^2 = 4(x+y)$ is equal to :

a) 2
b) 4
c) 8
d) 6