2021-March Session-03-16-2021-shift-1-16-30

EE24BTECH11008-ASLIN GARVASIS

16) Let $[x]$ denote greatest integer less than or equal to x. If for $n \in \mathbf{N}$, $\left(1-x+x^3\right)^n = \sum_{j=0}^{3n} a_j x^j$, then $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j} + 1$ is equal to :

    a) 2
    b) 2^{n-1}
    c) 1
    d) n

17) If $y = y(x)$ is the solution of the differential equation, $\frac{dy}{dx} + 2y\tan x = \sin x, y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function $y(x)$ over R is equal to :

    a) 8
    b) $\frac{1}{2}$
    c) $-\frac{15}{4}$
    d) $\frac{1}{8}$

18) The locus of the midpoints of the chord of the circle, $x^2+y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

    a) $\left(x^2+y^2\right)^2 - 16x^2 + 9y^2 = 0$
    b) $\left(x^2+y^2\right)^2 - 9x^2 + 144y^2 = 0$
    c) $\left(x^2+y^2\right)^2 - 9x^2 - 16y^2 = 0$
    d) $\left(x^2+y^2\right)^2 - 9x^2 + 16y^2 = 0$

19) The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to :

    a) 3
    b) 4
    c) 8
    d) 2

20) Let $\mathbf{S}_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$. Then $\lim_{k\to\infty} \mathbf{S}_k$ is equal to :

    a) $\tan^{-1}\left(\frac{3}{2}\right)$
    b) $\frac{\pi}{2}$
    c) $\cot^{-1}\left(\frac{3}{2}\right)$
    d) $\tan^{-1}(3)$

21) Consider an arithmetic series and a geometric series having four initial terms from the set $[11, 8, 21, 16, 26, 32, 4]$. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to . . .

22) Let $f : (0, 2) \to \mathbf{R}$ be defined as
$$f(x) = \log_2\left(1 + \tan\left(\frac{\pi x}{4}\right)\right).$$
Then, $\lim_{n\to\infty} \frac{2}{n}\left(f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \cdots + f(1)\right)$ is equal to . . .

23) Let $ABCD$ be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to . . .

24) If $\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$, then $a + b + c$ is equal to . . .

25) The total number of $3X3$ matrices A having entries from the set $(0, 1, 2, 3)$ such that the sum of all the diagonal entries of AA^T is 9, is equal to . . .

26) Let
$$\mathrm{P} = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix} \text{ and } \mathrm{A} = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$$
where $\omega = \frac{-1+i\sqrt{3}}{2}$, and $\mathbf{I}_3$ be the identity matrix of order 3. If the determinant of the matrix $\left(P^{-1}AP - \mathbf{I}_3\right)^2$ is $\alpha\omega^2$, then the value of α is equal to . . .

27) If the normal to the curve $y(x) = \int_0^x \left(2t^2 - 15t + 10t\right) dt$ at a point (a, b) is parallel to the line $x + 3y = -5, a > 1$, then the value of $|a + 6b|$ is equal to . . .

28) Let the curve $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} = 2(x + 1)$. If the numerical value of area bounded by the curve $y = y(x)$ and $x-$ axis is $\frac{4\sqrt{8}}{3}$, then the value of $y(1)$ is equal to . . .

29) Let $f : \mathbf{R} \to \mathbf{R}$ be a continous function such that $f(x) + f(x+1) = 2$, for all $x \in \mathbf{R}$. If $\mathbf{I}_1 = \int_0^x f(x)\, dx$ and $\mathbf{I}_2 = \int_{-1}^3 f(x)\, dx$, then the value of $\mathbf{I}_1 + 2\mathbf{I}_2$ is equal to . . .

30) Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2, \left|\frac{z+i}{z-3i}\right| = 1$ and $Re(w)$ has the minimum value. Then the minimum value of $n \in \mathbf{N}$ for which w^n is real, is equal to . . .