3.2.17

EE24BTECH11008 - Aslin Garvasis

Question:

Constuct a triangle whose sides are $3.6cm, 3.0cm$ and $4.8cm$. Bisect the smallest angle and measure each part.

Solution:

∵ The smallest angle is associated with the opposite smallest side.

Variable	value	Description
A	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	coordinates of first point
B	$\begin{pmatrix} 4.8 \\ 0 \end{pmatrix}$	coordinates of second point
C	$\begin{pmatrix} 2.81 \\ 2.24 \end{pmatrix}$	coordinates of third point
D	-	intersection of angle bisector of A in BC

TABLE 0: Input parameters

The angle bisector of a triangle of a triangle divides the opposite side into two parts proportional to the other two sides of the triangle.

$$\because \|AC\| = 3.6 \tag{1}$$

$$\because \|AB\| = 4.8 \tag{2}$$

$$\because \|BC\| = 3.0 \tag{3}$$

$$\cos \angle BAC = \left\| \frac{\mathbf{AC}^T \mathbf{AB}}{\|AC\| \, \|AB\|} \right\| = 0.78 \tag{4}$$

$$\angle BAC = 38.73° \tag{5}$$

$$\cos \angle ABC = \left\| \frac{\mathbf{BC}^T \mathbf{BA}}{\|BC\| \, \|BA\|} \right\| = 0.66 \tag{6}$$

$$\angle ABC = 48.70° \tag{7}$$

$$\cos \angle ACB = \left\| \frac{\mathbf{CA}^T \mathbf{CB}}{\|CA\| \, \|CB\|} \right\| = 0.05 \tag{8}$$

$$\angle ACB = 87.13° \tag{9}$$

$$\therefore \mathbf{D} = \frac{\|AC\| \cdot \mathbf{B} + \|AB\| \cdot \mathbf{C}}{\|AC\| + \|BC\|} \tag{10}$$

$$\implies \mathbf{D} = 3.6 \begin{pmatrix} 4.8 \\ 0 \end{pmatrix} + 4.8 \begin{pmatrix} 2.81 \\ 2.24 \end{pmatrix} = \begin{pmatrix} 3.66 \\ 1.28 \end{pmatrix} \tag{11}$$

$$\cos \angle BAD = \left\| \frac{\mathbf{AB}^T \mathbf{AD}}{\|AB\| \, \|AD\|} \right\| = 0.94 \tag{12}$$

$$\angle BAD = 19.28° \tag{13}$$

$$\implies \|BD\| = \frac{\|BC\| \, \|AB\|}{\|AB\| + \|AC\|} = 1.71 \tag{14}$$

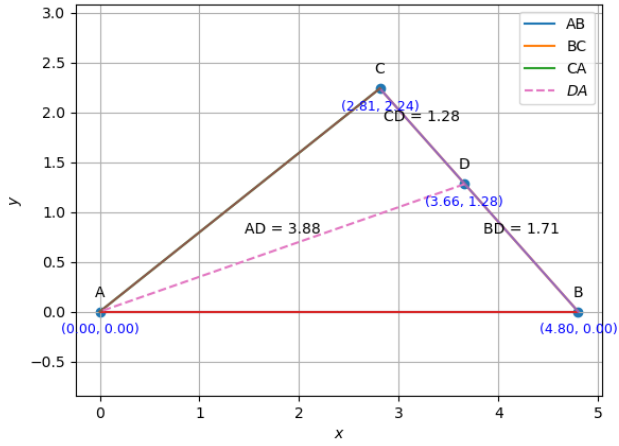$$\implies \|CD\| = \frac{\|BC\| \, \|AC\|}{\|AB\| + \|AC\|} = 1.28 \tag{15}$$

Fig. 0: Plot of points **A**, **B**, **C** and **D**