Chapter-16 Application of Derivatives

EE24BTECH11008-ASLIN GARVASIS

9) The slope of the tangent to a curve $y = f(x)$ at $[x, f(x)]$ is $2x+1$.If the curve passes through the point $(1, 2)$, then the area bounded by the curve, the x axis and the line $x = 1$ is (1995S)
   a) $\frac{5}{6}$
   b) $\frac{6}{5}$
   c) $\frac{1}{6}$
   d) 6
10) If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where$0 < x \le 1$, then in this interval (1997-2 Marks)
   a) both $f(x)$ and $g(x)$ are increasing functions
   b) both $f(x)$ and $g(x)$ are decreasing functions
   c) $f(x)$ is an increasing function
   d) $g(x)$ is an increasing function
11) The function $f(x) = \sin^4 x + \cos^4 x$ increases if (1999-2 Marks)
   a) $0 < x < \frac{\pi}{8}$
   b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$
   c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$
   d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$
12) Consider the following statements in S and R (2000S)

   S: Both $\sin x$ and $\cos x$ are decreasing functions in the interval $\left(\frac{\pi}{2}, \pi\right)$
   R: If a differentiable function decreases in an interval (a, b), then its derivative also decreases in (a, b)
   Which of the following is true ?
   a) Both S and R are wrong
   b) Both S and R are correct, but R is not the correct explanation of S
   c) S is correct and R is the correct explanation for S
   d) S is correct and R is wrong
13) Let $f(x) = \int e^x (x-1)(x-2)\,dx$ Then f decreases in the interval (2000S)
   a) $(-\infty, -2)$
   b) $(-2, -1)$
   c) $(1, 2)$
   d) $(2, +\infty)$
14) If the normal to the curve $y = f(x)$ at the point $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive $x-$ axis, then $f'(3) =$ (2000S)
   a) -1
   b) $\frac{-3}{4}$
   c) $\frac{4}{3}$
   d) 1
15)
$$Let f x = \begin{cases} |x| & \text{for } 0 < x \le 2, \\ 1 & \text{for } x = 0 \end{cases}$$
   Then at $x = 0, f$ has (2000S)
   a) a local maximum
   b) a local minimum
   c) no local maximum
   d) no extremum
16) For all $x \in (0, 1)$ (2000S)
   a) $e^x < 1 + x$
   b) $\log_e(1 + x) < x$
   c) $\sin x > x$
   d) $\log_e(x) > x$
17) If $f(x) = xe^{x(1-x)}$, then $f(x)$ is (2001S)
   a) increasing on $\left[\frac{-1}{2}, 1\right]$
   b) decreasing on R
   c) increasing on R
   d) decreasing on $\left[\frac{-1}{2}, 1\right]$
18) The tiangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point $(1, 1)$and the coordinate axes, lies in the first quadrant.If its area is 2, then the value of$b$ is (2001S)
   a) -1
   b) 3
   c) -3
   d) 1
19) Let $f(x) = \left(1 + b^2\right)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$.As b varies, the range of $m(b)$ is (2001S)
   a) $[0, 1]$
   b) $\left(0, \frac{1}{2}\right)$
   c) $\left[\frac{1}{2}, 1\right]$
   d) $(0, 1)$
20) The length of a longest interval in which the function $3\sin x - 4\sin^3 x$ is increasing, is (2002S)
   a) $\frac{\pi}{3}$
   b) $\frac{\pi}{2}$
   c) $\frac{3\pi}{2}$

d) π

21) The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is(are) (2002S)

a) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$
b) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$
c) $(0, 0)$
d) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

22) In $[0, 1]$ Lagranges Mean Value Theorem is NOT applicable to (2003S)

a)

$$f(x) = \begin{cases} \frac{1}{2} - x & x<\frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2 & x \geq \frac{1}{2} \end{cases}$$

b)

$$f(x) = \begin{cases} \frac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

c) $f(x) = x|x|$
d) $f(x) = |x|$

23) A tangent is drawn to ellipse $\frac{x^2}{27} + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ $\left(where\theta \in \left(0, \frac{\pi}{2}\right).\right)$ Then the value of θ such that sum of intercepts on axes made by this tangent is minimum, is (2003S)

a) $\frac{\pi}{3}$
b) $\frac{\pi}{6}$
c) $\frac{\pi}{8}$
d) $\frac{\pi}{4}$