Comput Optim Appl (2008) 39: 121–142
DOI 10.1007/s10589-007-9066-4

Bilevel optimization applied to strategic pricing in competitive electricity markets

M. Fampa · L.A. Barroso · D. Candal · L. Simonetti

Received: 23 November 2005 / Revised: 17 April 2006 / Published online: 19 September 2007

Abstract In this paper, we present a bilevel programming formulation for the problem of strategic bidding under uncertainty in a wholesale energy market (WEM), where the economic remuneration of each generator depends on the ability of its own management to submit price and quantity bids. The leader of the bilevel problem consists of one among a group of competing generators and the follower is the electric system operator. The capability of the agent represented by the leader to affect the market price is considered by the model. We propose two solution approaches for this non-convex problem. The first one is a heuristic procedure whose efficiency is confirmed through comparisons with the optimal solutions for some instances of the problem. These optimal solutions are obtained by the second approach proposed, which consists of a mixed integer reformulation of the bilevel model. The heuristic proposed is also compared to standard solvers for nonlinearly constrained optimization problems. The application of the procedures is illustrated in case studies with configurations derived from the Brazilian power system.

Keywords Electricity pool market · Strategic pricing · Bilevel programming · Mathematical program with equilibrium constraints

M. Fampa (✉)
Universidade Federal do Rio de Janeiro, Instituto de Matemática-DCC and COPPE, C.P. 68530, 21945-970, Rio de Janeiro, Brazil
e-mail: fampa@cos.ufrj.br

L.A. Barroso · D. Candal · L. Simonetti
Universidade Federal do Rio de Janeiro, COPPE, C.P. 68511, 21945-970, Rio de Janeiro, Brazil

L.A. Barroso
e-mail: barroso@cos.ufrj.br

D. Candal
e-mail: denise@cos.ufrj.br

L. Simonetti
e-mail: luidi@cos.ufrj.br

1 Introduction

The electricity supply industry throughout many countries has faced a significant restructuring process towards deregulation and competition. Even though the details of the restructuring process and regulatory framework may differ in each country [21], the overall organization in most cases follows the same principles: the generation of electricity becomes an unbundled (separated and deregulated) activity subject to liberalization in which the traditional centralized procedures—usually based on cost-minimization schemes—to both expansion and operation decisions have been replaced by market-oriented procedures.

One of the basic features of the deregulation process is the creation of a wholesale energy market (WEM), where all electric power purchase and sale transactions take place. On the WEM, all generators freely bid unit prices for their energy production (typically price-quantity bids on an hourly basis for the next day). Based on these bids, a system operator dispatches the generators by increasing price until demand is met. The dispatched generators are paid the unit price of the most expensive loaded generator, which corresponds to the system short-run marginal cost and is known as the system spot price. This corresponds to the well known uniform-price auction format, which is generally adopted in electricity markets and will be considered in this paper as well.

In the absence of non-convex cost functions, transmission losses and no unit commitment, after the dispatch, the market settlement is carried out: each generator receives from the WEM an hourly spot revenue that is the product of its energy production by the system spot price.

The existence of a bid-based dispatch/settlement has motivated several technical challenges that have been widely discussed and studied on the literature over the last years, see for example [10, 22, 23, 26].

In deregulated power markets the system dispatch and spot price depend on the price-quantity bids of each generating agent, i.e., these agents assume much more risk and become highly responsible for their own decisions: since their spot revenues depend on the result of the dispatch of their units and on the system spot price, the main question of a generation company is how to develop a bidding strategy that maximizes its profit. Conversely, regulators will be interested in analyzing such strategies in order to prevent market abuses.

This paper considers the problem of determining the profit-maximization bidding strategy for a generation company in a wholesale energy market. One of the difficulties of this problem is given by the fact that the spot price depends on the bids of all agents. Since this information is not available to any company at the time of its bid, the bidding strategy has to take into account uncertainties around these values.

The strategic bidding problem is a highly non-convex problem and due to its difficulty, there has been an intensive search for efficient algorithms to solve it. For example, Ramos et al. [31] presents a nonlinear formulation for the problem with complementarity conditions as constraints. Conejo and Priesto [10] and later, Conejo et al. [11] consider some heuristic procedures and Hobbs [18] uses linear complementarity models for the problem. Hobbs and Helman [19] present a complete overview of the application of complementarity-based models for power markets and Bushnell [9] presents an application of a complementarity model for hydrothermal systems, where hydro plants act as strategic players. More recently, the use of mixed integer linear programming (MILP) has been increasing: Torre [12] represented the non-convex residual demand function by a MILP model. Baíllo et al. [6] also represented the residual demand function by a MILP model but taking uncertainty into account, establishing scenarios of residual demand functions for the day-ahead market session.

In this paper we consider a static (single-stage) version of the strategic bidding problem with no transmission constraints, no unit commitment, linear cost functions and where competitors do not explicitly counteract to the price maker's bid (bids of the competitors are modelled based on scenarios). We propose a different approach to the problem. We formulate it as a special class of bilevel programs known as taxation problems [25].

Bilevel programming is appropriate for modelling optimization problems in which a subset of the decision variables is not controlled by the main optimizer, but by a second one, who optimizes his own objective function with respect to this subset of variables. The problem of determining the profit-maximizing bids of a generating agent in a wholesale energy market fits perfectly in this context, since the agent wants to maximize its profit while the system operator carries out a least-cost dispatch in order to minimize the overall dispatch cost. Bilevel programming has been used in the context of electric power markets to represent these contradictory interests. For example, Weber [35] and Hobbs [20] tackle the strategic bidding as a nested optimization problem, where the generator maximizes its individual welfare subject to a solution which maximizes the total social welfare based on all bids in the market. Both approaches consider the dynamic behavior of market players, which is modelled by iterative procedures that search for a Nash equilibria [27].

The differences between the bilevel programming formulation presented in this paper and the ones presented in [20, 35] are the representation of the bidding behavior (a scenario-based approach is used instead of a Nash equilibria), the fact that transmission network is not represented and the solution approaches.

We present two different solution approaches for the problem, the first one is based on heuristic techniques for the taxation problem presented in [8, 24]. The heuristic presented in this paper differs since in the strategic bidding problem all the dispatched competitors are paid the same unit price, which is the spot price. On the other hand, on the taxation problem, each competitor receives as a unit price for his services, the amount charged by himself. The good performance of the heuristic proposed is shown through the solution of test problems based on the Brazilian electricity system. The performance was greatly improved by using a specialized diversification strategy and a local search procedure.

On the second approach presented to the strategic bidding problem we consider it as a mixed integer program, using a binary decomposition of some variables. This approach leads to an exact solution and is used to validate the quality of the results presented by the heuristic for some instances of the problem.

This paper is organized as follows: Sect. 2 presents an overview of the strategic pricing in competitive electricity markets. In this section the mathematical formulation of the strategic bidding problem under uncertainty is presented and the Cournot and Bertrand schemes are introduced. Section 3 presents the primal-dual heuristic procedures and Sect. 4 introduces a MILP model for the problem. Section 5 presents the numerical results, in which the heuristic solutions are compared to the optimal solutions provided by the MILP model and are also compared to the solutions obtained by standard solvers for nonlinearly constrained optimization problems. Section 6 concludes.

2 Strategic pricing in electricity markets

In deregulated electricity markets, generators submit a set of hourly generation prices and available capacities for the following day. Based on these data and on an hourly load forecast, the system operator carries out the following economic dispatch at each time step:

$$
\begin{array}{llll}
 & & \text{dual variable} & \\
\min\limits_{g_j} & \sum\limits_{j\in J} \lambda_j g_j, & & \\
\text{s.t.} & \sum\limits_{j\in J} g_j = d, & \pi_d & \\
 & g_j \le \bar{g}_j, & \pi_{g_j}, & j \in J, \\
 & g_j \ge 0, & & j \in J,
\end{array}
\tag{1}
$$

where the input data d, λ_j and $\bar{g}_j$ represent, respectively, load (MWh), price bid ($\$/MWh$) and generation capacity bid (MWh) of generator j and the variables g_j represents the energy production of generator j (MWh). The optimal value of the dual variable π_d is considered as the system spot price and corresponds to the price paid by the system operator to each dispatched generator for each unit of energy production.

After the system dispatch is carried out, each plant receives a spot revenue which is given by the product of system spot price by its energy production. The profit R_j of each generator j, in each time step, corresponds to the difference between its spot revenue and its variable operating cost:

$$R_j = (\pi_d - c_j)g_j, \quad \text{for } j \in J,$$

where c_j represents its unit operating cost. Note that c_j differs from λ_j, its price bid.

The net profit R of a generation company E, which may be a utility or an independent power producer that owns several different generation units, is given by the sum of the profits from plants under its control:

$$R = \sum_{j\in E} R_j,$$

where E is also used to denote the set of indexes associated to the plants belonging to the company E ($E \subset J$).

Company E aims to solve the optimal bidding problem, which is the problem of determining a set of price bids $\lambda_E = \{\lambda_j : j \in E\}$ and quantity bids $\bar{g}_E = \{\bar{g}_j : j \in E\}$ that maximize its total net profit.

2.1 Bidding strategies under uncertainty

The complexity of the optimal bidding problem is greatly compounded by the fact that the calculation of π_d and g_j in the dispatch problem (1) depends on the knowledge of price vectors for all companies, as well as their generation availability and system load values. However, this information is not available to any single company at the time of its bid. Therefore, the bidding strategy has to take into account the uncertainty around these values.

One approach to model this simultaneous competition process is through a Nash equilibrium, where the competition process is simulated until a set of price/quantity equilibrium bids is obtained [7, 18–20, 35]. In this equilibrium, no agent can individually improve its net revenue by changing its bids, while the other bids remain the same. Another approach, which is adopted in this work, is to define a set of scenarios for the remaining agents' behavior and maximize the expected profit of the company over all scenarios. In this case, the bids from generators not belonging to company E and the load are considered uncertain, and represented by a set of scenarios indexed by s, which occur with exogenous probabilities $\{p_s : s = 1, \dots, S\}$. This approach was recently presented by Baíllo et al. [6], where it is assumed that, after the clearing of each market mechanism, information about the submitted aggregate offer and demand curves is made publicly available and agents can then build scenarios for its rivals bids.

Considering this approach based on scenarios, what the generator maximizes is not the deterministic value of its profit, but its expected value, $ER(\lambda_E, \overline{g}_E)$. If we let $\pi_d^s(\lambda_E, \bar{g}_E)$ and $g_j^s(\lambda_E, \bar{g}_E)$ represent, respectively, the system spot price and generation of plant j in scenario s for a given time step, the generator's problem become

$$\begin{aligned} \max_{\lambda_E, \bar{g}_E} \quad & ER(\lambda_E, \overline{g}_E) = \sum_{s \in S} p_s R^s(\lambda_E, \overline{g}_E), \\ \text{s.t.} \quad & \overline{g}_j \le g_j^*, \quad j \in E, \end{aligned} \tag{2}$$

where

$$R^s(\lambda_E, \overline{g}_E) = \sum_{j \in E} (\pi_d^s(\lambda_E, \bar{g}_E) - c_j) g_j^s(\lambda_E, \bar{g}_E) \tag{3}$$

is the profit of company E in scenario s and g_j^* is the maximum capacity of production of generator j.

Note that the values of the variables π_d^s and g_j^s result from the solution of the system dispatch (1) in each scenario, for the sets of price bids λ_E and quantity bids $\bar{g}_E$. Therefore, the problem of determining the optimal price-quantity bidding strategy for each plant $j \in E$ of a given company can be formulated as the following bilevel optimization problem:

$$
\begin{aligned}
\max_{\lambda_E,\bar{g}_E,g_j^s,\pi_d^s} \quad & \sum_{s\in S} p_s\, R^s(\lambda_E,\overline{g}_E), \\
\text{s.t.} \quad & \overline{g}_j \le g_j^*, \quad j\in E, \\
& (g_j^s,\pi_d^s)\in \arg\min_{g_j^s,\pi_d^s} \sum_{s\in S}\sum_{j\in E}\lambda_j g_j^s + \sum_{j\notin E}\lambda_j^{*s} g_j^s, \\
& \text{s.t.} \quad \sum_{j\in J} g_j^s = d^s, \quad s\in S, \\
& \qquad g_j^s \le \bar{g}_j, \quad j\in E,\ s\in S, \\
& \qquad g_j^s \le \bar{g}_j^{*s}, \quad j\notin E,\ s\in S, \\
& \qquad g_j^s \ge 0, \quad j\in J,\ s\in S.
\end{aligned}
\tag{4}
$$

The first level of problem (4) represents the interest of company E (maximize expected profit), while the second level represents the interest of the system operator (minimize operational cost). The company is classified as leader of the bilevel program, while the system operator is classified as follower.

2.2 Pure price and pure quantity bidding models

The well-known pure price and pure quantity bidding models are special cases of the joint price/quantity optimization framework presented previously. The former is a pure quantity bid model, that predefines price bids, usually as zero, and optimizes quantity bids, while the latter is a pure price bid model that predefines quantity bids, usually as the maximum capacity of generation, and optimizes price bids. Pure price and quantity bidding models are widely used in game theoretical models under the so-called Bertrand and Cournot models [15, 19, 20, 31]. Although we do not have a game-theory based approach in this work, we will also use the name Bertrand model to describe the pure price bidding model and the name Cournot model to reflect pure quantity bidding model.

In the following, we consider the Bertrand scheme for the optimal bidding problem. In this case, the quantity bid of each generator $j\in E$ is fixed as $\bar{g}_j^*$ and problem (4) can be rewritten as

$$
\begin{aligned}
\max_{\lambda_E,g_j^s,\pi_d^s} \quad & \sum_{s\in S} p_s \sum_{j\in E}(\pi_d^s - c_j) g_j^s, \\
\text{s.t.} \quad & (g_j^s,\pi_d^s)\in \arg\min_{g_j^s,\pi_d^s} \sum_{s\in S}\sum_{j\in E}\lambda_j g_j^s + \sum_{j\notin E}\lambda_j^{*s} g_j^s, \\
& \text{s.t.} \quad \sum_{j\in J} g_j^s = d^s, \quad s\in S, \\
& \qquad 0\le g_j^s \le \bar{g}_j^*, \quad j\in E,\ s\in S, \\
& \qquad 0\le g_j^s \le \bar{g}_j^{*s}, \quad j\notin E,\ s\in S.
\end{aligned}
\tag{5}
$$

In order to gain some insight into problem (5), we analyze the two-dimensional example presented in Fig. 1, where $E=\{1\}$. For simplicity we consider a

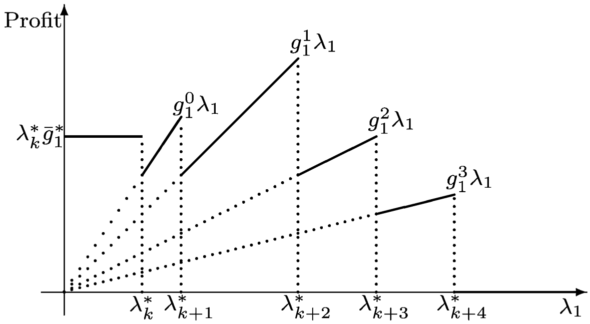

Fig. 1 Leader's profit

fixed scenario (its index is omited) and the operation cost c_1 equal to zero. Let $\{\lambda_2^* \le \lambda_3^* \le \cdots \le \lambda_{|J|}^*\}$ be the set of price bids of generators not belonging to company E.

At the optimal solution of the lower level problem, for a given price bid λ_1, generators are loaded by increasing price until demand is met.

Let k be the minimum index such that $\bar{g}_1^* + \sum_{j=2}^{k} \bar{g}_j^* > d$. For $\lambda_1 \le \lambda_k^*$, the generator of company E produces its maximum capacity at the optimal solution of the follower problem and the leader's objective is constant and equal to $\lambda_k^* \bar{g}_1^*$.

For $\lambda_l^* < \lambda_1 \le \lambda_{l+1}^*$, where $l \in [k, |J|-1]$, the production of company E at the optimal solution of the follower problem is given by $g_1^{k-l} \equiv \max\{0, d - \sum_{j=2}^{l} \bar{g}_j^*\}$ and the leader's objective is a linear function of λ_1 with slope g_1^{k-l}. Clearly $g_1^{k-l} \ge g_1^{k-l+1}$, for every l.

If λ_1 exceeds a sufficiently large value, $g_1 = 0$ at the optimal solution of the follower problem and the leader's profit is driven to zero.

We note that the leader's objective function is neither continuous nor concave. It is a piecewise linear function that presents local maxima at points where λ_1 assumes the value of the price bid of a generator not belonging to E.

Bilinear bilevel programs and its applications to the optimal setting of taxes on goods and services were considered in [8, 24, 25].

In this paper we consider algorithm approaches for the strategic bidding problem that exploit its structure and allow the development of very efficient extensions of heuristics proposed in [8, 24].

We assume that the system load can be met without the participation of the plants belonging to company E, in other words, the sum of the installed capacities of the remaining generators (excluding company E) is not smaller than the demand. This assumption is always valid since the energy deficit can be represented by a generator (that does not belong to any specific company) with "infinite" installed capacity and variable operating cost equal to the deficit cost. This assumption [24] constitutes a necessary and sufficient condition to assure that the bilevel problem has a bounded optimal solution. It also guarantees the equivalence of problem (5) and the following mathematical program with equilibrium constraints (MPEC), where the dependence on the dual variables are more explicitly shown. On the MPEC, the follower problem of (5) is replaced with its necessary and sufficient optimality conditions.

$$
\begin{aligned}
\max_{\lambda_E, g_j^s, \pi_d^s, \pi_{g_j}^s} \quad & \sum_{s \in S} p_s \sum_{j \in E} (\pi_d^s - c_j) g_j^s, \\
\text{s.t.} \quad & \sum_{j \in J} g_j^s = d^s, \quad s \in S, \\
& 0 \le g_j^s \le \bar{g}_j^*, \quad j \in E,\ s \in S, \\
& 0 \le g_j^s \le \bar{g}_j^{*s}, \quad j \notin E,\ s \in S, \\
& \pi_d^s + \pi_{g_j}^s - \lambda_j \le 0, \quad j \in E,\ s \in S, \\
& \pi_d^s + \pi_{g_j}^s \le \lambda_j^{*s}, \quad j \notin E,\ s \in S, \\
& \pi_{g_j}^s \le 0, \quad j \in J,\ s \in S, \\
& \sum_{s \in S} \left(\sum_{j \in E} \lambda_j g_j^s + \sum_{j \notin E} \lambda_j^{*s} g_j^s - d^s \pi_d^s - \sum_{j \in E} \bar{g}_j^* \pi_{g_j}^s - \sum_{j \notin E} \bar{g}_j^{*s} \pi_{g_j}^s \right) = 0.
\end{aligned} \tag{6}
$$

One approach to solve the strategic bidding problem is to directly apply bilinear programming algorithms ([1, 4, 33, 34]) to (6) or to its disjoint bilinear programming reformulation presented in the next section. Another approach is to use a mixed integer programming formulation for the problem, as considered in Sect. 4. However, due to the size of these problems in real applications, especially when a reasonable number of scenarios is considered, these approaches may not be suitable, as suggested in Sect. 5. This fact motivates the development of heuristic procedures, which is the subject of the next section.

3 Primal-dual heuristics

We introduce some heuristic procedures to solve the strategic bidding problem considering the Bertrand model. The procedures are extensions to problem (5) of the heuristics proposed in [8, 24] to the taxation problem. They differ from the ones presented in [8, 24], however, since in the strategic bidding problem, in each scenario, all the dispatched competitors are paid the same unit price.

We propose two procedures to improve the results obtained. The first one is a diversification procedure that considers different initial solutions for the problem and the second one is a local search that is implemented from the solution obtained from the heuristics.

The performance of the heuristics was greatly improved by the use of a very fast algorithm to solve the follower problem, which exploits its simple structure. The procedure is based on the decomposition of the problem into small subproblems, one for each scenario. At their optimal solutions the units are loaded by increasing price until demand is met.

Finally, we show that the convergence results for the heuristics presented for the taxation problem carry over to the strategic bidding problem.

Consider the following non-convex problem obtained when we penalize the nonlinear complementarity constraint of problem (6)

$$
\begin{aligned}
\max_{\lambda_E, g_j^s, \pi_d^s, \pi_{g_j}^s} \quad & \sum_{s \in S} p_s \sum_{j \in E} (\pi_d^s - c_j) g_j^s - \mu(\lambda_j g_j^s - \bar{g}_j^* \pi_{g_j}^s) \\
& \quad + \mu\left(\sum_{j \notin E} \lambda_j^{*s} g_j^s - \bar{g}_j^{*s} \pi_{g_j}^s - d^s \pi_d^s \right), \\
\text{s.t.} \quad & \sum_{j \in J} g_j^s = d^s, \quad s \in S, \\
& 0 \le g_j^s \le \bar{g}_j^*, \quad j \in E,\ s \in S, \\
& 0 \le g_j^s \le \bar{g}_j^{*s}, \quad j \notin E,\ s \in S, \\
& \pi_d^s + \pi_{g_j}^s - \lambda_j \le 0, \quad j \in E,\ s \in S, \\
& \pi_d^s + \pi_{g_j}^s \le \lambda_j^{*s}, \quad j \notin E,\ s \in S, \\
& \pi_{g_j}^s \le 0, \quad j \in J,\ s \in S,
\end{aligned}
\tag{7}
$$

where $\mu > 0$ is the penalty parameter.

In Theorem 1, we verify that the penalty scheme considered is an exact penalty scheme, i.e. if the penalty parameter is large enough in (7), then the complementarity constraint will be satisfied in its optimal solution. This result was proven by Anandalingam and White [2] for linear bilevel programs and by Labbé, Marcotte and Savard [24] for the model of taxation. We omit the proof of Theorem 1, since it is not difficult to extend the result to the strategic bidding in energy markets.

Theorem 1 *There is a penalty parameter $\bar{\mu} > 0$ such that problems* (5) *and* (7) *are equivalent for every $\mu > \bar{\mu}$.*

The heuristics schemes described in this paper present, iteratively, an approximate solution to the penalized problem (7). At each iteration, the penalty parameter is increased until the complementarity constraint is satisfied. In a given iteration and for a fixed value of μ, an approximate procedure is used to solve the nonlinear and non-convex problem (7). In this procedure, the variables are divided into two groups, one with the variables g_j^s and the other with the variables λ_E, π_d^s and $\pi_{g_j}^s$. The values of these groups of variables are then fixed alternately. Once fixed the values of one group, a linear programming problem is solved and the solution of this problem defines the values of the other group of variables. We note here that since we are considering the Bertrand model, the term $\bar{g}_j^* \pi_{g_j}^s$ on the objective function of (7) is a linear term. The choice of the linear program that is solved to obtain the values of the variables g_j^s defines two different heuristics. On the first one, the economic dispatch (1) is solved for each scenario and on the second one, the penalized problem (7) is solved. A pseudo-code for the first heuristic is presented in Fig. 2. In the sec-

procedure First_Heuristic ($\mu > 0$, $\tilde{g}_j^s, j \in J, s \in S$)
1 Read_Input();
2 **do**
3 Solve problem (7), considering $g_j^s = \tilde{g}_j^s$
and obtain a solution $\tilde{\lambda}_E$, $\tilde{\pi}_d^s$, $\tilde{\pi}_{g_j}^s, j \in J, s \in S$;
4 Solve problem (1) for each scenario $s \in S$, considering $\lambda_E = \tilde{\lambda}_E$
and obtain a solution $\tilde{g}_j^s, j \in J$;
5 Increase μ;
6 **until**
$\sum_{s\in S}\left(\sum_{j\in E}(\tilde{\lambda}_j \tilde{g}_j^s - \bar{g}_j^* \tilde{\pi}_{g_j}^s) + \sum_{j\notin E}(\lambda_j^{*s} \tilde{g}_j^s - \bar{g}_j^{*s} \tilde{\pi}_{g_j}^s) - d\tilde{\pi}_d^s\right) = 0$;
7 **return** $\tilde{\lambda}_j, j \in E$**;**
end First_Heuristic.

Fig. 2 Pseudo-code of the first heuristic

4 Solve problem (7), considering $\lambda_E = \tilde{\lambda}_E, \pi_d^s = \tilde{\pi}_d^s, \pi_{g_j}^s = \tilde{\pi}_{g_j}^s$
and obtain a solution $\tilde{g}_j^s, j \in J, s \in S$;

Fig. 3 Step 4 of the second heuristic

ond heuristic, the step 4 presented in Fig. 2 would be replaced with step 4 presented in Fig. 3.

In the heuristics presented, we first consider the equivalence between the bilevel problem (5) and the MPEC (6) and then approximate (6) by a sequence of penalized problems with increasing values for the penalty parameter. The convergence of the sequence produced by this procedure to the feasible set of the problem is guaranteed by Theorem 1. The idea of the heuristics is to start with the best solution for the leader of problem (5) and move from this solution to the feasible set of the problem, where the complementarity conditions of the follower problem are satisfied.

The heuristics consider the solution of each nonlinear penalized problem iteratively and approximately, through the solution of linear programs, so the non-convex optimization problem is replaced by a sequence of "easy" linear programs, where the primal variables g_j^s are separated from the dual variables π_d^s and $\pi_{g_j}^s$ and from the variables λ_j.

The procedures interact among the leader price bids λ_E and the energy production g_j^s, determined by the economic dispatch. They produce a sequence of basic feasible solutions to the follower problem that converge to a feasible solution of (5). However, the sequence may not converge to an optimal solution of the bilevel program. Since the problem is non-convex, the heuristics may converge to a local optimal solution, as illustrated by the example of Fig. 1. We note that since the leader's objective is a piecewise linear function of λ_E, all the stationary points of the function are either locally minimal or locally maximal. From step 3 on Fig. 2, the price bids obtained correspond to the maximum profit of company E for given energy productions for the generators. Therefore, the solution obtained by the heuristics is always a local maximum. In order to avoid the local optimum, two strategies are proposed next.

3.1 Diversification of the initial solution

We consider three different initial solutions for starting the heuristics. Based on the idea of initializing the procedures with the best solution for the leader, we first consider the solution corresponding to the generation of the maximum capacity of the plants belonging to company E. This solution is obtained by solving the economic dispatch problem (1) for each scenario, taking into account $\lambda_j = 0$ for $j \in E$.

The second initial solution corresponds to the case where no plant belonging to company E is dispatched. The idea is to start the heuristics from a completely different solution when compared to the first one, and therefore, enlarge the probability of converging to a new solution for the problem when applying the heuristics. This second solution is easily obtained when we solve the economic dispatch problem for each scenario $s \in S$, taking $\lambda_j > \max_{j \notin E}\{\lambda_j^*\}$, for all $j \in E$, which is equivalent to solve the linear programs

$$
\begin{aligned}
\min_{g_j} \quad & \sum_{j \notin E} \lambda_j^{*s} g_j^s, \\
\text{s.t.} \quad & \sum_{j \notin E} g_j^s = d^s, \\
& 0 \le g_j^s \le \bar{g}_j^*, \quad j \in E, \\
& 0 \le g_j^s \le \bar{g}_j^{*s}, \quad j \notin E,
\end{aligned}
$$

and establish $\tilde{g}_j^s = 0$, for $j \in E$. Since we assume that the remaining plants can always meet the demand, the problems above are never infeasible.

Finally, the third initial solution proposed is based on a polynomial algorithm to solve the deterministic problem, or the problem that considers only one scenario. In the following we prove that, in this case there is an optimal solution for the strategic bidding problem, where all plants belonging to company E bid the same price. It suffices to show the result for a company that owns two plants.

Theorem 2 *There exists an optimal solution to problem* (6), *with* $|E| = 2$ *and* $|S| = 1$, *such that the bid prices of both plants belonging to company E are equal.*

Proof Consider $E = \{1, 2\}$ and an optimal solution to (6) given by $\tilde{\pi}_d$, $\tilde{\pi}_{g_j}$, $\tilde{g}_j$, for $j \in J$ and $\tilde{\lambda}_j$, for $j \in E$.

Suppose that $\tilde{\lambda}_1 > \tilde{\lambda}_2$. In the following we show that in this case, it is possible to present another optimal solution to (6) such that $\lambda_1 = \lambda_2$.

We note that if $\tilde{\lambda}_1 > \tilde{\pi}_d$ then $\tilde{g}_1 = 0$. Therefore we consider, without loss of generality, that $\tilde{\lambda}_1 \le \tilde{\pi}_d$.

Let $\tilde{\tilde{\lambda}}_2 := \tilde{\lambda}_1$ and $\tilde{\tilde{\pi}}_{g_2} := \tilde{\pi}_{g_2} + \tilde{\lambda}_1 - \tilde{\lambda}_2$.

The solution obtained by replacing $\tilde{\lambda}_2$ with $\tilde{\tilde{\lambda}}_2$ and $\tilde{\pi}_{g_2}$ with $\tilde{\tilde{\pi}}_{g_2}$ at the given optimal solution to (6) is also a feasible solution to the problem, since

$$\tilde{\pi}_d + \tilde{\tilde{\pi}}_{g_2} - \tilde{\tilde{\lambda}}_2 = \tilde{\pi}_d + (\tilde{\pi}_{g_2} + \tilde{\lambda}_1 - \tilde{\lambda}_2) - \tilde{\lambda}_1 = \tilde{\pi}_d + \tilde{\pi}_{g_2} - \tilde{\lambda}_2 \leq 0, \tag{8}$$

and

$$\tilde{\tilde{\pi}}_{g_2} = \tilde{\lambda}_1 + \tilde{\pi}_{g_2} - \tilde{\lambda}_2 \leq \tilde{\pi}_d + \tilde{\pi}_{g_2} - \tilde{\lambda}_2 \leq 0. \tag{9}$$

Considering that $\tilde{\pi}_d \geq \tilde{\lambda}_1 > \tilde{\lambda}_2$, we have that $\tilde{g}_2 = \bar{g}_2^*$. Therefore

$$\tilde{\lambda}_2\tilde{g}_2 - \bar{g}_2^*\tilde{\pi}_{g_2} = \tilde{\lambda}_1\tilde{g}_2 - \bar{g}_2^*(\tilde{\pi}_{g_2} + \tilde{\lambda}_1 - \tilde{\lambda}_2) = \tilde{\tilde{\lambda}}_2\tilde{g}_2 - \bar{g}_2^*\tilde{\tilde{\pi}}_{g_2}.$$

From the last equality, it is straightforward to verify that the complementarity constraint in (6) also remains satisfied with the replacements on the values of the variables.

Finally, we note that the value of the objective function of (6) remains the same when we replace $\tilde{\lambda}_2$ and $\tilde{\pi}_{g_2}$. Therefore the solution obtained is also an optimal solution. □

Considering the result of Theorem 2, in the deterministic case, it is possible to model the problem considering just one plant belonging to the company, whose generation capacity is given by the sum of the capacities of all the plants that actually belong (we consider the same operation cost for all plants belonging to E).

It is not difficult to verify that there is an optimal solution to the strategic bidding problem, where all the plants belonging to E bid a price equal to the price-bid of one of the plants not belonging to E in some scenario, since otherwise the plants could always increase their bids without provoking any change on the generation of the plants of the system, given by the solution of the dispatch problem. On the other hand, the spot price of the system could only increase or remain the same with the bids increment, which means that the profit of company E could not get worse. This result leads us to a very efficient algorithm to the deterministic problem, where it is considered as a discrete optimization problem. It consists of solving the dispatch problem several times considering at each time, a different price-bid of one of the plants not belonging to E, as the bid of the company. The bid that brings the best profit for E among all of them is presented at the end.

The basic idea of this algorithm is used on the construction of the third initial solution. In this procedure, we assume that company E owns just one plant whose generation capacity is the sum of the capacities of all of its plants and with null operation cost. The procedure considers a fixed scenario and can be divided into two steps.

The first step aims to find the highest price bid λ_j that can be made by company E to let it produce all of its capacity. The production of the plants not belonging to E is given by the dispatch generation of the plants whose bid was not greater than λ_j. In this case, the profit of E is given by the product of λ_j and its total generation capacity.

In the second step, the price-bid of company E is increased and the corresponding profit is calculated for the cases where its bid is equal to each of the price-bids of the remaining agents. Therefore, the trade off between incrementing the spot price but reducing the energy production can be analyzed. At the end of this step, the best price is stored and the production of each generation unit can be easily obtained by solving the economic dispatch.

In the numerical results presented in this paper, we consider on the construction of this third initial solution, the scenario with highest probability of occurrence. Nevertheless, it is also possible to enlarge even more the diversification of the initial solution if we repeat the procedure several times, each of them randomly choosing a scenario among a list of candidates, that could be composed by all scenarios with probability of occurrence higher than a given parameter α. This idea of diversification is also used in meta-heuristics to avoid local optimums [13].

3.2 Local search

Once a solution to the strategic bidding problem is obtained by a heuristic, the next step is to carry out a local search starting from it.

Without lost of generality, let's consider that the indexes $j = 1, \dots, |E|$ are associated with the plants belonging to company E and satisfy the relation $\bar{g}_1^* \geq \bar{g}_2^* \geq \cdots \geq \bar{g}_{|E|}^*$.

The local search proposed consists of $|E|$ stages. For each stage, the economic dispatch (1) is solved for each scenario, n times, where n is equal to the number of different bids made by the agents not belonging to E in all scenarios considered.

During stage j, the price-bids of all the plants belonging to company E, except plant j, are kept fixed according to the best solution found until that step. The price-bid λ_j, of plant j, receives a different value at each time. This value is always equal to the price-bid of a plant not belonging to company E in some scenario.

At the end of the local search, the best price-bid obtained for each plant belonging to company E is stored. The production of each plant and the system spot prices for this set of bids can be easily found by solving (1) for each scenario.

Figure 4 presents the pseudo-code for the local search algorithm. Consider $\beta_1 < \beta_2 < \cdots < \beta_n$ as the different price-bids of the plants that do not belong to company E.

4 A mixed integer linear formulation

The idea of reformulating a bilevel program as a mixed integer program was first presented in [3], where the complementarity conditions for the follower problem are formulated with the introduction of binary variables, which are associated with the inequality constraints and determine whether they are active or not.

In this section an alternative MILP formulation will be presented and it will be used to obtain the global optimal solution of the strategic bidding problem. The main

procedure Local_Search($\tilde{\lambda}_E, \tilde{\pi}_d^s, \tilde{g}_j^s, j \in J, s \in S, \beta_k, k = 1, \dots, n$)
Read_Input();
$\lambda_E := \tilde{\lambda}_E$;
$z := \sum_{s \in S} p_s \sum_{j \in E} \tilde{\pi}_d^s \tilde{g}_j^s$;
do $i = 1, \dots, |E|$
  do $k = 1, \dots, n$
    $\lambda_i := \beta_k$;
    Solve problem (1) for each scenario $s \in S$
    and obtain the solution $g_j^s, \pi_d^s, j \in J$;
    if $z < \sum_{s \in S} p_s \sum_{j \in E} \pi_d^s g_j^s$ **then** update λ_i and z;
  end
end
return λ_E;
end Local_Search.

Fig. 4 Pseudo-code of the local search

idea of this formulation is to apply a binary decomposition scheme to the problem aiming at eliminating the nonlinearities. This idea was also considered in [8] for the freight tariff-setting problem. In [29] we also propose a binary decomposition scheme for this problem, but in that paper no heuristic was used to improve the lower bonds for the problem and the formulation proposed only considers a subset of all possible values for the price bids of the plants belonging to company E.

Our basic approach is to transform problem (6) into a mixed integer linear problem using a binary decomposition of the variables λ_j, $j \in E$.

From the complementarity conditions in (6), we have

$$\pi_d^s g_j^s = \lambda_j g_j^s - \pi_{g_j}^s \bar{g}_j^*,$$

for $j \in E$ and $s \in S$. Since we have assumed that the plants not belonging to E are capable of meeting the system load, we can consider the upper bound for λ_j, $j \in E$, given by the maximum price bid offered by all these plants on all possible scenarios, i.e.

$$\lambda_j \leq \bar{\lambda} := \max_{j \notin E, s \in S} \{\lambda_j^{*s}\}.$$

We can now substitute the variables λ_j, $j \in E$, by their binary decomposition $\lambda_j = \sum_{l=0}^{L} 2^l z_j^l$, where $z_j^l \in \{0, 1\}$ and $L := \lfloor \log_2(\bar{\lambda}) \rfloor$.

We define the variables $g_j^{ls} := g_j^s z_j^l$ and formulate (6) as the following mixed integer linear programming problem.

$$
\begin{aligned}
\max_{g_j^s,\pi_d^s,\pi_{g_j}^s,g_j^{ls},z_j^l} \quad & \sum_{s\in S} p_s \sum_{j\in E}\left(\sum_{l\in\bar{L}} 2^l g_j^{ls} - \pi_{g_j}^s \bar{g}_j^* - c_j g_j^s\right), \\
\text{s.t.} \quad & \sum_{j\in J} g_j^s = d^s, \quad s\in S, \\
& 0 \le g_j^s \le \bar{g}_j^*, \quad s\in S,\ j\in E, \\
& 0 \le g_j^s \le \bar{g}_j^{*s}, \quad s\in S,\ j\notin E, \\
& \pi_d^s + \pi_{g_j}^s - \sum_{l=0}^{L} 2^l z_j^l \le 0, \quad s\in S,\ j\in E,\ l\in\bar{L}, \\
& \pi_d^s + \pi_{g_j}^s \le \lambda_j^{*s}, \quad s\in S,\ j\notin E, \\
& \pi_{g_j}^s \le 0, \quad s\in S,\ j\in J, \\
& -\bar{g}_j^* z_j^l \le g_j^{ls} \le \bar{g}_j^* z_j^l, \quad s\in S,\ j\in E,\ l\in\bar{L}, \\
& -\bar{g}_j^*(1-z_j^l) \le g_j^{ls} - g_j^s \le \bar{g}_j^*(1-z_j^l), \quad s\in S,\ j\in E,\ l\in\bar{L}, \\
& \sum_{s\in S}\left(\sum_{j\in E}\left(\sum_{l\in\bar{L}} 2^l g_j^{ls} - \bar{g}_j^*\pi_{g_j}^s\right)\right. \\
& \qquad \left. + \sum_{j\notin E}(\lambda_j^{*s} g_j^s - \bar{g}_j^{*s}\pi_{g_j}^s) - d^s\pi_d^s\right) = 0.
\end{aligned}
\tag{10}
$$

It is important to highlight that even though this MILP approach can provide an optimal solution to the strategic bidding problem, it presents the drawback to deal with a large number of integer variables as the number of generators increase. This has motivated the development of alternatives solution approaches, such as the heuristics presented in this paper, which can also be used to generate bounds to be used in a branch-and-bound scheme.

5 Numerical results

The performance of the heuristics proposed for the strategic bidding pricing in competitive electricity markets is analyzed in this section for several configurations derived from the Brazilian Power System. The computational results of the heuristics are compared with the results of the MILP model presented, whenever it is possible. The heuristics are also compared to standard solvers for nonlinearly constrained optimization problems.

The Brazilian generation system has 138 plants with a total installed capacity of 82,6 GW. Hydro generation accounts for 90% of the installed capacity of the country, distributed in 110 hydro plants arranged in complex topologies over 12 main river basins. The generation technology for the remaining 28 plants includes nuclear power, natural gas, coal and diesel. Generation plants include a few very large plants, such as Itaipu (12,6 GW), several mid-sized plants (1–2 GW) and the remainder between 100 and 500 MW. The whole country is interconnected by over 70000 Km of high voltage transmission lines.

In order to evaluate the performance of the heuristics, we have considered several test-problems from the Brazilian System configuration. Each problem is composed by 138 plants ($|J| = 138$). The strategic bidder (to be optimized) controls a subset of these plants, whose size is denoted by $|E|$. Note that depending on the number and size of plants controlled by this company, its ability to be a strict price maker can be increased. We consider a set of bidding scenarios for the remaining $|J| - |E|$ plants with $|S|$ different scenarios. The time step is of one stage. No transmission network and unit commitment were considered.

When a hydro plant is considered, we have estimated its variable operating cost as its water-value, which is the opportunity cost of storing the energy and selling it in the future. The water values were taken from a stochastic hydrothermal scheduling model that is similar to the one used by the Brazilian System Operator [17, 28]. When a thermal plant is considered, we have used its operating cost.

The bidding scenarios for each test problem were generated by randomly sampling price bids for the remaining $|J| - |E|$ generators from an uniform probability distribution over the water value, in case of hydro plants, or variable operating cost, in case of thermal plants. The capacity of the generators is always equal to their installed capacity. No forced outage scenarios were considered.

For each test problem generated we have considered a Bertrand model, that is, only prices are decision variables for the strategic bidder (the quantity bids are the generators capacities). We applied the heuristics presented in Sect. 3 and the MILP model presented in Sect. 4. We also solved the instances using the standard solvers for nonlinearly constrained optimization problems, FILTER and SNOPT. The former was developed by Fletcher and Leyffer [14], and the latter, by Gill, Murray and Saunders [16]. Both solvers are available through NEOS server [32] at Argonne National Laboratory. When using the nonlinear solvers we considered the formulation for the problem given by the MPEC (6).

The solver Xpress (V.14) was used to find the optimal solution of the mixed integer model and of the linear programming problems considered in the heuristics. The computer used was a PC AMD XP 2.4 GHz with 1000 MB of RAM. Elapsed time of each algorithm and for each problem is presented next, as well as the performance of each solution approach.

In the following, columns 4 to 7 of Table 1 compares the heuristics solutions with the optimal solution obtained by the MILP model for the first eleven instances of the problem. For each of these tests, the value of the objective function given by the MILP model (z_I) and by the heuristics (z_H) are presented. These functions represent the expected profit of the bidding company when its price bids are given respectively by the solution of problems (10) and (7), with the proposed heuristics used for the latter. The elapsed CPU time is also provided for both procedures. The MILP model execution was aborted after 24 hours of CPU time without obtaining the optimal solution of instances P12 to P20. We note that to solve the instances by the MILP model, we have considered the lower bounds for the problem, given by the solution of the heuristics. The elapsed CPU time corresponding to the MILP model, does not

Table 1 (Integer/heuristic) bid-based solution × cost-based solution

P	$\|E\|$	$\|S\|$	Integer		Heuristic		Cost based		
			z_I	t	z_H	t	z_C	S_E	G_E
1	1	2	78200	0.8	78200	0.1	76245	3	3
2	1	2	110929	0.7	110929	0.2	110929	7	0
3	1	5	334109	5.7	334109	0.3	326670	19	2
4	2	2	45645	0.8	45645	0.1	45645	3	0
5	2	3	313535	21.9	313535	0.3	184230	10	70
6	2	5	213	0.2	213	0.8	213	0	0
7	2	100	411	51	411	19	411	0	0
8	5	2	202023	3.2	202023	0.2	200294	11	1
9	5	3	415896	4740	415896	0.6	369771	34	12
10	5	5	433582	6660	433582	0.6	417942	37	4
11	10	2	454181	27.4	454181	0.5	454181	29	0
12	5	10	–	–	393863	1.7	341214	38	15
13	5	100	–	–	3537	27	3471	1	2
14	5	250	–	–	5018	169	4888	3	3
15	10	5	–	–	405253	2.2	381577	42	6
16	10	10	–	–	411890	4.1	330484	51	25
17	10	15	–	–	298262	4	202459	50	47
18	10	20	–	–	372370	8.6	304804	50	22
19	15	10	–	–	291824	4.9	172685	50	69
20	15	15	–	–	336861	6.2	221031	63	52

consider the computation of these bounds. The use of the bounds provided by the heuristics reduced on an average of 7% the number of nodes on the branching trees and on 13% the CPU time needed to solve the instances. Furthermore, it allowed the solution of two more instances.

In column 6, z_H corresponds to the best solution found after the two heuristics presented in this paper were applied, each of them taking into account the three initial solution strategies and the implementation of the local search procedure. Therefore, in column 7, the CPU time contemplates all these procedures. The use of second heuristic with the first initial solution generated the best result for eleven instances. However, every combination of heuristics and initial solutions obtained the best solution for at least one instance, so we have decided to obtain the results applying all of them. The use of the local search improved considerable the results. The minor improvement was obtained when the second heuristic with the first initial solution was applied. In this case, we only generated the best solution for three more instances with the use of the local search. Nevertheless, the best solutions for these three instances were only obtained by this procedure, which generated the best solution for fourteen out of the twenty instances considered. We note that the elapsed CPU time in the local search procedure could be reduced if it was not repeated for the two heuristics in the cases where both found the same bids for the plants belonging to company E.

Columns 6 and 8 of the table compare the heuristics solution (z_H) with the profit of company E using cost-based bids (z_C), that is, bidding its own variable operating costs in case of thermal units or water values in case of hydro units.

The last two columns of Table 1 present in percentual values, the expected share of company E in the market (S_E) and the expected gain of company E when using the price bids given by the heuristic solution instead of cost-based bids (G_E).

The expected share of E is given by the ratio between the generation capacity of E and the expected generation capacity of the remaining plants,

$$S_E = \frac{\sum_{j \in E} \bar{g}_j^*}{\sum_{s \in S} p_s \sum_{j \notin E} \bar{g}_j^{*s}}.$$

The expected gain of E is given by the relative difference between the heuristic solution and the profit of company E when using cost-based bids,

$$G_E = \frac{z_H - z_C}{z_C}.$$

From the results presented in Table 1, we confirm the quality of the heuristics presented in this paper by noting that for all instances for which we could find the optimal solution for the strategic bidding problem through the MILP model (P1–P11), the heuristics also found the optimal solution, but in much smaller time.

Comparing the profits of company E given by the heuristics (z_H) with the ones obtained with cost-based bids (z_C), we can see that the heuristics propose bids for company E that lead to a higher profit in all instances, except in five of them (P2, P4, P6, P7, P11). We note, however, that for these five instances the solutions proposed by the heuristics are already the optimal solution, as shown by the MILP solution. Finally, we note that, specially for big instances of the problem, the opportunity of strategic bidding can indeed lead companies to much higher profits, as observed on the last five instances presented, where we have a medium increase of 43% on the profits. This observation is not difficult to understand, since in these problems, company E controls more and more generators on the market, which turns it more price maker and allows it to affect the market price through its bids more easily. Consequently, its profits increase.

In Table 2 we present the results obtained by FILTER and SNOPT. Columns 2 to 5 consider the solution obtained by the solvers when we do not use any initial solution as an input. Columns 6 to 9 present the results obtained when the three initial solutions presented in this paper are considered. In this case the value of the objective function provided by the solvers corresponds to the best solution found after they were applied taking into account each initial solution and the CPU time contemplates the three runs.

Comparing Tables 1 and 2 it is apparent that the heuristic procedure proposed for the strategic bidding problem is superior to the other solvers not only in solution terms but also in computation time terms. When we do not give an initial solution as an input, the solvers always obtain worse solutions than the heuristic procedure and when the initial solutions are given, the solvers obtain some improved results, but they are still worse than the ones obtained by the heuristics for most instances.

Table 2 Results obtained with FILTER and SNOPT

P	Filter[1]		Snopt[1]		Filter[2]		Snopt[2]	
	z_F	t	z_S	t	z_F	t	z_S	t
1	5465	0.3	64418	0.2	78200	0.6	78200	0.8
2	2405	0.3	74067	0.3	110929	0.6	110929	0.6
3	5450	1.5	74448	1.6	330163	0.6	334109	0.5
4	3802	0.3	28660	0.2	45645	0.4	45645	0.5
5	12778	0.6	226339	0.2	281603	1	226339	0.6
6	120	1.3	131	0.5	131	1.4	131	1.7
7	397	1128	152	151	397	1299	395	857
8	13394	0.3	46502	0.2	202023	0.4	202023	0.3
9	203950	0.7	54549	0.4	415535	0.9	415535	0.7
10	15191	1.7	60940	0.8	421304	2	430781	1
11	280185	0.3	27310	0.3	438581	0.5	438581	0.5
12	193770	7.8	105404	1	361877	9.1	361109	12.8
13	854	1124.1	2212	198	854	1200.8	907	986
14	–	–	–	–	–	–	–	–
15	243767	2	261571	1.3	365697	2.1	400703	4.8
16	312052	5.6	160612	3	388705	9	386084	9.6
17	223921	13.8	63906	70	265226	22.5	265513	110.5
18	316430	22	31223	10	351285	28.2	351285	30.7
19	231315	13.4	37495	25.3	279568	23.5	259334	18.2
20	208182	12.4	11545	5	272283	27.5	308691	47.1

Furthermore, when the heuristics do not obtain a better solution than the solvers (see instances 1–4 and 8), they obtain the same one, but with faster convergence. Both solvers could not obtain a solution for instance P14. Their execution were aborted due to lack of memory after more than one hour of CPU time. The results in Tables 1 and 2 are an evidence that nonlinear solvers may converge to suboptimal solutions of MPECs.

Finally, we have also attempted to obtain the global optimal solution of the instances 1–6, 8 and 9 considered in Table 1, using the algorithm described by Perron in [30], which is an improved version of the algorithm introduced in Audet et al. [5]. Perron ran his code on a Intel Xeon 3.06 GHz, 1 Mb cache memory, 2 GO RAM, considering the solution obtained by the heuristic as an initial solution. Instances 1–4 and 6 were solved after 0.31, 0.76, 0.23, 3.02 and 8.45 seconds of CPU time, respectively. The solutions obtained were, as expected, equal to the MILP solutions. The global optimal solution of the other instances considered, was not obtained after 200 000 seconds of CPU time. Once more these results show the importance of the development of heuristics with good performance to solve problems in real applications.

6 Conclusion

This paper presents a bilevel programming formulation to the problem of strategic bidding under uncertainty in electricity markets. A nongame approach is adopted and it is considered that the agent being optimized can obtain bidding scenarios (price-quantity) for its competitors. Although a price-bidding model is considered, the formulation can be easily applied to quantity-bidding models or to the joint price-quantity bidding model. We consider a simpler version of the strategic bidding problem: static (single-stage), with no transmission constraints, no unit commitment, only linear cost functions and where competitors do not counteract to the price maker's bid (bids of the competitors are modeled based on scenarios).

An heuristic approach to the non-convex and non-differentiable price-bidding model of the strategic bidding problem is proposed, which allows us to solve instances of significant size within reasonable computing times. The quality of the heuristic solution is confirmed by the comparison with the optimal solution for some instances of the problem, obtained by a mixed integer linear programming model and also by the comparison with the results obtained by the standard nonlinear solvers FILTER and SNOPT. An attempt to obtain the global optimal solution of the instances considered, using the algorithm described by Perron in [30], which is an improved version of the algorithm introduced in Audet et al. [5], has also been made.

The application of the proposed methodology is illustrated in case studies with configurations derived from the Brazilian power system. In all tests where we could obtain the optimal solution with the MILP model, we show that the heuristics also obtain the optimal solution. Furthermore, the heuristic results are superior to the ones obtained by the solvers both in solution terms and in computational time terms.

For all the instances considered, we show that the heuristics obtain price bids for the generators that lead them to a medium increment of 17% in their profits, when compared to the profits obtained with cost-based bids. In addition, the only instances where no increment was obtained, were the ones for which the cost-based bids already lead to the maximum profit.

We show that the performance of the heuristics proposed on the solution of the instances considered is superior to that of FILTER and SNOPT. Finally we report that the global optimal solution was only obtained for a few instances with the algorithm described in [30], confirming the importance of heuristic procedures to solve problems in real applications.

Acknowledgement We thank Mario V. Pereira of Power Systems Research for his helpful comments and for providing us with the data sets. We are also grateful to Sylvain Perron for running his code to solve our instances.

References

1. Alarie, S., Audet, C., Jaumard, B., Savard, G.: Concavity cuts for disjoint bilinear programming. Math. Program. **90**, 373–398 (2001)
2. Anandalingam, G., White, D.J.: A solution method for the linear Stackelberg problem using penalty functions. IEEE Trans. Autom. Control **35**, 1170–1173 (1990)

3. Audet, C., Hansen, P., Jaumard, B., Savard, G.: Links between linear bilevel and mixed 01 programming problems. J. Optim. Theory Appl. **93**(2), 273–300 (1997)
4. Audet, C., Hansen, P., Jaumard, B., Savard, G.: A symmetrical linear maxmin approach to disjoint bilinear programming. Math. Program. **85**, 573–592 (1999)
5. Audet, C., Hansen, P., Jaumard, B., Savard, G.: A branch and cut algorithm for nonconvex quadratically constrained quadratic programming. Math. Program. Ser. A **87**, 131–152 (2000)
6. Baíllo, A., Ventosa, M., Rivier, M., Ramos, A.: Optimal offering strategies for generation companies operating in electricity spot markets. IEEE Trans. Power Syst. **19**(2), 745–753 (2004)
7. Barroso, L.A., Fampa, M., Kelman, R., Pereira, M.V.F., Lino, P.: Market power issues in bid-based hydrothermal dispatch. Ann. Oper. Res. **117**, 247–270 (2002)
8. Brotcorne, L., Labbe, M., Marcotte, P., Savard, G.: A bilevel model and solution algorithm for a freight tariff-setting problem. Transp. Sci. **34**(3), 289–302 (2000)
9. Bushnell, J.: A mixed complementarity model of hydrothermal electricity competition in the western United States. Oper. Res. **51**(1), 80–93 (2003)
10. Conejo, A.J., Prieto, F.J.: Mathematical programming and electricity markets. Soc. Estad. Investig. Oper. TOP **9**(1), 1–53 (2001)
11. Conejo, A.J., Contreras, J., Arroyo, J.M., Torre, S.: Optimal response of an oligopolistic generating company to a competitive pool-based electric power market. IEEE Trans. Power Syst. **17**(2), 424–430 (2002)
12. de la Torre, S., Arroyo, J.M., Conejo, A.J., Contreras, J.: Price maker self-scheduling in a pool-based electricity market: a mixed-integer LP approach. IEEE Trans. Power Syst. **17**(4), 1037–1042 (2002)
13. Feo, A., Resende, M.G.C.: Greedy randomized adaptive search procedures. J. Glob. Optim. **6**, 109–133 (1995)
14. Fletcher, R., Leyffer, S.: Solving mathematical program with complementarity constraints as nonlinear programs. Optim. Methods Softw. **19**(1), 15–40 (2004)
15. Fudenberg, D., Tirole, J.: Game Theory, 5th printing. MIT Press, Cambridge (1996)
16. Gill, P.E., Murray, M., Saunders, M.A.: User's guide for SNOPT 5.3: a fortran package for large-scale nonlinear programming. Report NA 97-5, Department of Mathematics, University of California, San Diego (1997)
17. Granville, S., Barroso, L.A., Oliveira, G.C., Latorre, M.L., Campodónico, N., Thomé, L.M., Pereira, M.: Stochastic optimization of transmission constrained and large scale hydrothermal systems in a competitive framework. In: Proceedings of the IEEE General Meeting, Toronto (2003)
18. Hobbs, B.F.: Linear complementarity models of Nash-Cournot competition in bilateral and POOLCO power markets. IEEE Trans. Power Syst. **16**(2), 194–202 (2001)
19. Hobbs, B.F., Helman, U.: Complementarity-based equilibrium modeling for electric power markets. In: Bunn, D. (ed.) Modeling Prices in Competitive Electricity Markets, ch. 3. Wiley, New York (2004)
20. Hobbs, B.F., Metzler, C.B., Pang, J.: Strategy gaming analysis for electric power systems: an MPEC approach. IEEE Trans. Power Syst. **15**, 638–645 (2000)
21. Hunt, S.: Making Competition Work in Electricity. Wiley, New York (2003)
22. Kahn, E.: Regulation by simulation: the role of production cost models in electricity planning and pricing. Oper. Res. **43**(3), 388–398 (1995)
23. Kahn, E.: Numerical techniques for analyzing market power in electricity. Electr. J., 34–43 (1998)
24. Labbé, M., Marcotte, P., Savard, G.: A bilevel model of taxation and its application to optimal highway pricing. Manag. Sci. **44**, 1608–1622 (1998)
25. Labbé, M., Marcotte, P., Savard, G.: On a class of bilevel programs. In: di Pillo, Giannessi (eds.) Nonlinear Optimization and Related Topics, pp. 183–206. Kluwer Academic, Dordrecht (2000)
26. Lino, P., Barroso, L.A., Fampa, M., Pereira, M.V., Kelman, R.: Bid-based dispatch of hydrothermal systems in competitive markets. Ann. Oper. Res. **120**, 81–97 (2003)
27. Nash, J.F.: Equilibrium points in n-person games. In: Proceedings of NAS 36 (1950)
28. Pereira, M.V., Pinto, L.M.V.G.: Multi-stage stochastic optimization applied to energy planning. Math. Program. **52**, 359–375 (1991)
29. Pereira, M.V., Granville, S., Fampa, M., Dix, R., Barroso, L.A.: Strategic bidding under uncertainty: a binary expansion approach. IEEE Trans. Power Syst. **20**(1), 180–188 (2005)
30. Perron, S.: Applications jointes de l'optimisation combinatoire et globale, Ph.D. Thesis, École Polytechnique de Montréal (2004)
31. Ramos, A., Ventosa, M., Rivier, M.: Modeling competition in electric energy markets by equilibrium constraints. Util. Policy **7**(4), 233–242 (1999)
32. The NEOS guide. Available online at http://www-neos.mcs.anl.gov/

33. Thieu, T.V.: A note on the solution of bilinear problems by reduction to concave minimization. Math. Program. **41**, 249–260 (1988)
34. Visweswaran, V., Floudas, C.A.: New properties and computational improvement of the GOP algorithm for problems with quadratic objective function and constraints. J. Glob. Optim. **3**(4), 439–462 (1993)
35. Weber, J.D., Overbye, T.J.: An individual welfare maximization algorithm for electricity markets. IEEE Trans. Power Syst. **17**(3), 590–596 (2002)