A Shortest Path Algorithm Used to Solve Optimal Bids in an Electricity Spot Market

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Abstract

A good descriptive abstract functions to describe as accurately as possible the content and purpose of a research report. It doesnt provide results or conclusions of the research. It does incorporate key words found in the text and may include the purpose, methods, and scope of the research. Some people consider it an outline of the work, rather than a summary. Descriptive abstracts are usually very short100 words or less.

Introduction

A wholesale energy market is a market for electric power transactions. All the energy producers bid energy units depending on their production capacity. Then the system operator distributes hour shifts to some chosen generators(owned by the energy producers)by increasing value of their price until the demand is met. The really important point is that all generators unit price is that of the most expensive loaded generator. We will call this price, the spot price. Therefore as long as you are cheaper or equal to the final spot price you will be chosen by the operator. Companies try to predict the spot price to sell a maximum quantity of energy at the highest price under the future spot price so that they will be chosen over others. This is the result of the system operator that tries to minimise his expenses.

The problem is studied, among others, by Fampa et al.[Bilevel optimization applied to strategic pricing in competitive electricity markets]which presents two different approaches to solve the problem: The first one is a heuristic technique considering a generous dispatching. The second one is a mixed integer reformulation of the problem used to find the optimal solution and allows the validation of the heuristic approach.

The goal of this report is to find faster ways to solve it. By simplifying some of the constraints and changing the dispatching resolution a little bit, we can reduce the problem to a shortest path problem and apply faster algorithms.

The Optimal Bid/Bidding Problem

The problem studied is the following, we are a company that we will call the leader or L to separate our company from the others that wants to maximize its profit in a specific environment, a wholesale electric market like described in the introduction, where we compete against other companies. The main goal is to find what quantity bids we should make and at what prices to maximize our profit.

Mathematical presentation of the problem

Here is the data provided with the problem:

d	The total load needed
g_j	The energy production of generator j
J	Group of all generators
c_j	The operating cost of generator j

The variables used to model the problem are the following:

λ_j	The price bid of a generator j
$\overline{g}_j$	The generation capacity bid of generator j
π_d	The spot price
R_j	The profit of generator j
R	The net profit of a company E, group of generators $j \in E$

Companies related variables:

λ_E	set of price bids
$\overline{g}_E$	quantity bids

This problem can be formulated as the following linear program:

dual variable

$$min_{g_j} \sum_{j \in J} \lambda_j g_j \tag{1}$$

These are the quantity bids chosen by the system operator for each time step.

s.t.

$$\sum_{j \in J} g_j = d \tag{2}$$

The sum of all the quantity bids must fulfil the demand.

$$g_j \leq \overline{g}_j, j \in J \tag{3}$$

A quantity bid for a generator cannot exceed the generator's capacity.

$$g_j \geq 0, j \in J \tag{4}$$

The quantity bid for the generator j must be positive.

Let π_d denote the dual variable of constraint (2) and π_{g_j} that of constraint (3).
This model shows how the system operator wants to minimize his expenses. We can add another interesting value useful for future purposes, the profit for each generator j :

$R_j = (\pi_d - c_j)g_j, j \in J$

The profit for each generator is the amount of energy sold times the difference of the price it sold and its cost of production.
Then comes the objective function that a company wants to maximize and which represents its profit. First we have to introduce more variables to model the simultaneous competition between all the companies, the company L, uses a set of scenarios where competitors actions are represented. New variables are introduced for the bidding under uncertainty:

p_s	The probability of a scenario $s \in [1, ..., S]$
S	Set of scenarios
$ER(\lambda_E, \overline{g}_E)$	The expected profit
g_j^*	The maximum capacity of production of generator j
$\pi_d^s(\lambda_E, \overline{g}_E)$	Spot price for a specific scenario
$g_j^s(\lambda_E, \overline{g}_E)$	energy production of a generator j for a specific scenario

With the new variables the objective function for the generators becomes:

$$max_{\lambda_E, \overline{g}_E}\ ER(\lambda_E, \overline{g}_E) = \sum_{s \in S} p_s R^s(\lambda_E, \overline{g}_E)$$

where

$$R^s(\lambda_E, \overline{g}_E) = \sum_{j \in E} (\pi_d^s(\lambda_E, \overline{g}_E) - c_j) g_j^s(\lambda_E, \overline{g}_E)$$

is the profit of company E in scenario s and g_j^* is the maximum capacity of production of generator j.

s.t.

$\overline{g}_j \leq g_j^*, j \in E$

Fampa et al.[...] go on formulating the global problem with the objective function of the company L we represent and the system operator function. Afterwards they introduce two models based respectively on fixed prices and fixed quantities. Finally they present their different solutions to solve these models.

Presentation of the new formulation

$s \in S$	the set of all scenarios,
$j \in J$	the set of all generators we control,
$j \in J^c$	the set of all generators of the competitors,
$i \in I$	the set of all possible bid prices.

The model parameters:

p_s	the probability that the scenario s will be realised,
d_s	the total demand in the scenario s,
$\lambda_{s,j}^c$	the price bid by the competitors in the scenario s for their generator j,
$\bar{g}_{s,j}^c$	the quantity bid by the competitors in the scenario s for the generator j,
g_j^{c*}	the maximum quantity which can be produced by the competitors generator j,
g_j^*	the maximum quantity which can be produced by our generator j,
c_j	the operating cost to produce one unit of energy by our generator j.

The model variables:

R_s	our profit during scenario s,
R	our average profit for all scenarios,
π_s	the spot price during scenario s,
λ_j	the price bid by us for our generator j,
$\bar{g}_j$	the quantity bid by us for our generator j,
$g_{s,j}^c$	the actual quantity produced by the competitors generator j during scenario s,
$g_{s,j}$	the actual quantity produced by our generator j during scenario s.

The formulation is as follows:

$$\max R, \tag{5}$$

s.t.

$$R = \sum_{s \in S} p_s R_s \tag{6}$$

The total profit is equal to the sum of the scenario's profit associated with the probability.

$$R_s = \sum_{j \in J} (\pi_s - c_j)\, g_{s,j} \ \ \forall s \in S \tag{7}$$

The profit associated with a specific scenario is equal to the sum of our quantity bids sold for this scenario times the difference of the spot price for this scenario and the cost of production for the generator j.

$$\sum_{j \in J} g_{s,j} + \sum_{j \in J^C} g^c_{s,j} = d_s \ \ \forall s \in S \tag{8}$$

The sum of the quantity bids for our generators and those of our competitors must be equal to the demand.

$$g^c_{s,j} \geq 0 \ \ \forall s \in S, j \in J^c \tag{9}$$

$$g^c_{s,j} \leq \bar{g}^c_j \ \ \forall s \in S, j \in J^c \tag{10}$$

$$\bar{g}_{s,j} \geq 0 \ \ \forall s \in S, j \in J \tag{11}$$

$$\bar{g}_{s,j} \leq g^*_j \ \ \forall s \in S, j \in J \tag{12}$$

$$g_{s,j} \geq 0 \ \ \forall s \in S, j \in J \tag{13}$$

$$g_{s,j} \leq \bar{g}_j \ \ \forall s \in S, j \in J \tag{14}$$

The quantity bids must be positive and inferior or equal to the capacity of their associated generator j.

$$\lambda^c_{s,j} < \pi_s \to g^c_{s,j} = \bar{g}^c_j \quad \forall s \in S, j \in J^c \tag{15}$$

$$\lambda^c_{s,j} > \pi_s \to g^c_{s,j} = 0 \quad \forall s \in S, j \in J^c \tag{16}$$

$$\lambda_j < \pi_s \to g_{s,j} = \bar{g}_j \quad \forall s \in S, j \in J \tag{17}$$

$$\lambda_j > \pi_s \to g_{s,j} = 0 \quad \forall s \in S, j \in J \tag{18}$$

If a competitor's bid price is inferior to the spot price, then the actual quantity sold is the quantity bid made by the competitor for this generator. If the bid price is higher then the spot price, then no quantity is sold. Same applies for our bids.

$$\pi_s \in \left\{\lambda^c_{s,j} \forall j \in J^c\right\} \cup \{\lambda_j, \forall j \in J\} \quad \forall s \in S \tag{19}$$

The spot price for a specific scenario must be at one of the bid prices given by one of the companies for this scenario.

Before discussing the algorithm used to solve this problem, let us introduce two propositions to support the solution.
The first proposition tries to give more sight to where the optimal bid stands.

Proposition 1 - optimal bid price

The optimal value for each bid price λ_j is always equal to one of the possible competitors bid prices $\lambda^c_{s,j}$, assuming that ties are resolved in our advantage.[1] There exists an optimal bid price $\lambda_j \in \{\lambda^c_{s,j} : s \in S, j \in J^c\}$

[1] Mixed-Integer Programming Formulation, E. Marcott

Proof

First we consider that our bids are strictly between two other bids, there are no other bids in between:

$\exists s \in S : \lambda^c_{s,j1} < \lambda_j < \lambda^c_{s,j2}$

It is important to note that bid prices are in ascending order:
$\{\lambda_{s_1}, ..., \lambda^c_{s,j_1}, \lambda_j, \lambda^c_{s,j_2}, ..., \lambda_{s_n}\}$ where λ_{s_1} is the lowest bid price for a specific scenario s and λ_{s_n} is the highest bid price for the same scenario. Now lets prove that by increasing λ_j to $\lambda^c_{s,j2}$ the profit R would either increase or remain constant. We will prove this by analysing the three different possible cases. The reason there are only 3 cases which are $\lambda^c_{s,j2} \leq \pi_s$, $\lambda^c_{s,j1} \geq \pi_s$ and $\lambda_j = \pi_s$, is because the profit is mainly influenced by the spot price and for a given bid price the spot price can either be lower, higher or equal to the bid price.

- Case 1: $\lambda^c_{s,j2} \leq \pi_s$

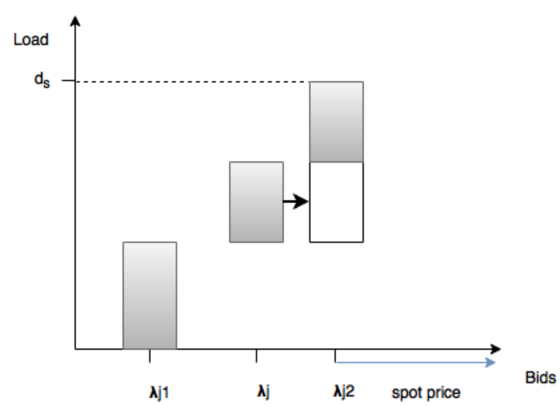

Figure 1: Spot price higher then $\lambda^c_{s,j2}$ bid. Ifλ_j becomes λ^c_{s,j_2} since the spot price is higher then the price offered by $\lambda_{s,j2}$, changing to $\lambda_{s,j2}$ will not change the profit R_s.

- Case 2: $\lambda^c_{s,j1} \geq \pi_s$

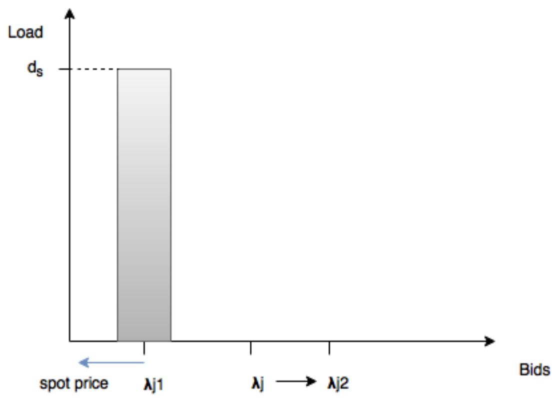

Figure 2: Spot price lower then $\lambda^c_{s,j1}$ bid. Since the spot price is lower than $\lambda_{s,j1}$ that means that the total load is achieved for the values under $\lambda_{s,j1}$, so changing $\lambda_{s,j1}$ to $\lambda_{s,j2}$ will have no effect on R_s.

- Case 3: $\lambda_j = \pi_s$

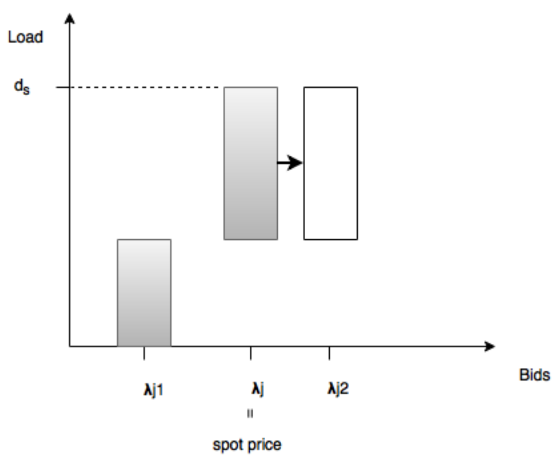

Figure 3: Spot price equals to λ_j bid. In this case, the production of λ_j is needed to fulfil the demand so if we increase it to $\lambda_{s,j2}$ the spot price will follow. Since the spot price follows our bid $\lambda_{s,j2}$ at least, that means that the profit R_s increases.

This proves that increasing the bid λ_j to $\lambda^c_{s,j2}$ results in a constant profit or an increasing one. Therefore it is always better to select bid prices amongst all possible competitors bid prices. Let us rewrite our problem with this new thought in mind. We introduce a competitor bid price λ^c_i from any scenario replacing s,j with i for an arbitrary scenario and generator. It is important to remember that those prices are sorted in ascending order:
$\lambda^c_1 < \lambda^c_2 < ... < \lambda^c_n$

This adds two new binary variables to the model:

$y_{s,i} = 1$ if the spot price in scenario s is set to λ^c_i
$x_{ij} = 1$ if our bid λ_j is equal to λ^c_i

This allows us to rewrite the model as follows:

$$\max R, \tag{20}$$

s.t.

$$R = \sum_{s\in S} p_s R_s \tag{21}$$

$$R_s = \sum_{j\in J}\left(\sum_{i\in I} y_{s,i}\lambda^c_i - c_j\right) g_{s,j} \quad \forall s \in S \tag{22}$$

The profit associated to a scenario equals the sum of the quantity bids multiplied by the difference of the bid prices chosen by the system operator and the production cost.

$$\bar{g}_j \geq 0 \quad \forall s \in S, j \in J \tag{23}$$

$$\bar{g}_j \leq g^*_j \quad \forall s \in S, j \in J \tag{24}$$

The bids must be positive and inferior to the capacity of the generator.

$$x_{ij} \in \{0,1\} \quad \forall i \in I, j \in J \tag{25}$$

$$\sum_{i\in I} x_{ij} = 1 \quad \forall j \in J \tag{26}$$

$$y_{s,i} \in \{0,1\} \quad \forall s \in S, i \in I \tag{27}$$

$$\sum_{i\in I} y_{s,i} = 1 \quad \forall s \in S \tag{28}$$

By definition those 2 variables are boolean. The sum for $y_{s,i}$ means that there can only be one spot price per scenario and it is set at a competitor's bid price. The sum for x_{ij} means that every bid λ_j that we do must be equal to one of the competitors bid prices.

$$g_{s,j} \geq 0 \quad \forall s \in S, j \in J \tag{29}$$

$$g_{s,j} \geq \bar{g}_j x_{ij} y_{s,i'} \quad \forall s \in S, j \in J \;\; i, i' \in I, i < i' \tag{30}$$

The quantity produced by a generator cannot be negative. If our bid price λ_j is equal to the bid price of a competitor's bid price which is inferior to the spot price then the actual production for our generator j for the scenario s is superior or equal to our quantity bid made for it.

$$g_{s,j} \leq \bar{g}_j \quad \forall s \in S, j \in J \tag{31}$$

$$g_{s,j} \leq \bar{g}_j \left(1 - x_{ij} y_{s,i'}\right) \quad \forall s \in S, j \in J \;\; i, i' \in I, i > i' \tag{32}$$

The quantity produced by our generator is inferior or equal to our quantity bid for that generator. If our bid price λ_j is equal to the bid price of a competitor's bid price which is superior to the spot price then the actual production for our generator j for the scenario s is zero.

$$\sum_{j \in J} g_{s,j} + \sum_{\substack{j \in J^c \\ \lambda^c_{s,j} < \lambda_i}} \bar{g}^c_{s,j} \leq d^s y_{s,i} + \left(\max_{s' \in S} d^{s'} \right) (1 - y_{s,i}) \tag{33}$$

$\forall s \in S, i \in I$

The sum of our quantities produced plus the sum of those from our competitors so that their price is inferior of this of λ_i is inferior or equal to the demand for the specific scenario if our price bid is equal to this of our competitor or to the maximum demand of another scenario.

$$\sum_{j \in J} g_{s,j} + \sum_{\substack{j \in J^c \\ \lambda^c_{s,j} \leq \lambda_i}} \bar{g}^c_{s,j} \geq d^s y_{s,i} \quad \forall s \in S, i \in I \tag{34}$$

The sum of our production plus the sum of quantities offered by the competitors so that their price is inferior or equal to λ_i is either greater then the demand for the scenario s if our bid price is equal to this of the competitor or is otherwise simply positive. We mentioned in our descriptions above that two of the constraints are not linear (30) and (32), luckily they involve binary variables and can be linearised:

$$g_{s,j} \geq \bar{g}_j - g^*_j (2 - x_{ij} - y_{s,i'}) \ \forall s \in S, j \in J, i, i' \in I, i < i' \tag{35}$$

$$g_{s,j} \leq g^*_j (2 - x_{ij} - y_{s,i'}) \ \forall s \in S, j \in J, i, i' \in I, i > i' \tag{36}$$

Since x_{ij} and $y_{s,i}$ are boolean variables we can verify their validity with the following tables. We compare the constraint (1)and its linearisation(2)
For the first linearisation:

(1) $g_{s,j} \geq \bar{g}_j \ x_{ij} \ y_{s,i'}$ (30)
(2) $g_{s,j} \geq \bar{g}_j - g^*_j (2 - x_{ij} - y_{s,i'})$ (35)

$y_{s,i'} \vert x_{ij}$	0	1
0	(1) $g_{s,j} \geq \bar{g}_j - 2g^*_j$ (2) $g_{s,j} \geq g_j - 2g^*_j$	(1) $g_{s,j} \geq 0$ (2) $g_{s,j} \geq \bar{g}_j - g^*_j$
1	(1) $g_{s,j} \geq 0$ (2)$g_{s,j} \geq \bar{g}_j - g^*_j$	(1)$g_{s,j} \geq \bar{g}_j$ (2)$g_{s,j} \geq \bar{g}_j$

For the second one:

(1) $g_{s,j} \leq \bar{g}_j \ (1 - x_{ij} y_{s,i'})$ (32)
(2) $g_{s,j} \leq g^*_j (2 - x_{ij} - y_{s,i'})$ (36)

$y_{s,i'} \vert x_{ij}$	0	1
0	(1) $g_{s,j} \leq \bar{g}_j$ (2) $g_{s,j} \leq 2g^*_j$	(1) $g_{s,j} \leq \bar{g}_j$ (2)$g_{s,j} \leq g^*_j$
1	(1) $g_{s,j} \leq \bar{g}_j$ (2)$g_{s,j} \leq g^*_j$	(1)$g_{s,j} \leq 0$ (2)$g_{s,j} \leq 0$

We notice that for each case in the table we have valid and similar inequalities.

We also have to replace another term in one of the constraints in the definition of R_s, $y_{s,i} g_{s,j}$ by $yg_{s,ij}$ which follows the following constraints:

$$yg_{s,ij} \geq 0 \tag{37}$$

The production must always be positive.

$$yg_{s,ij} \leq g_{s,j} \tag{38}$$

Since $y_{s,i}$ is boolean, after setting its minimum to 0 with the precedent constraint, its maximum is when $y_{s,i}$ is equal to 1, which is equal to the production of the generator.
QUESTION pourquoi ne pas écrire $0 < yg_{s,ij} \leq g_{s,j}$ au lieu d'écrire 2 contraintes ?

$$yg_{s,ij} \leq g^*_j y_{s,i} \tag{39}$$

The production of a generator must always be capped by its capacity.

$$yg_{s,ij} \geq g_{s,j} - g^*_j (1 - y_{s,i}) \tag{40}$$

If the spot price is set to λ^c_i then the production is equal to the quantity production decided. Otherwise our production must be inferior ro equal to the maximum capacity of the generator.
Je ne comprends pas tres bien l'interet de cette derniere contrainte vu qu'on dirait une combinaison des 3 autres.

A relaxation of the problem

Labbé etal[Mixed-Integer Programming Formulation] studied a new approach to the problem. Instead of considering that the generators can offer at most one amount at one price per generator, now the companies can offer bids with the capacities of all their generators combined.
Imagine 2 generators with the previous approach we could only do 2 bids, one per generator. With this approach we could do as many bids we wish without exceeding their capacity.
This already allows for improvement since we avoid the waste of generators not using their whole capacity.
Most importantly, they add another feature which is the stingy dispatching.
Generous dispatching used before meant that we would choose as spot price the first price corresponding to the quantity that would surpass the load needed. Now with this stingy dispatching we stop at the first price reaching the load needed.

This might not seem like a drastic change but this allows us to completely rewrite the problem. The new idea is that we can construct a solution satisfying M. Fampa's generous dispatching assumption by reducing infinitesimally our quantity corresponding to the spot prices:

The load dispatching

Generous solution

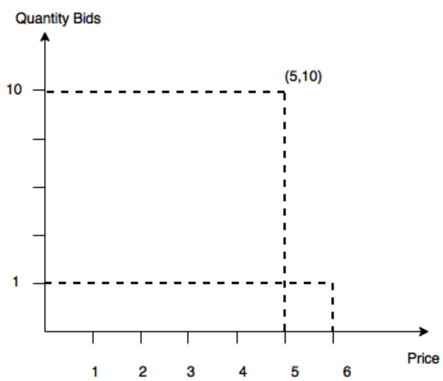

Figure 4: Load = 10, so the system controller will stop at the first price where the load is met. Here, one of the companies offers 10 quantities at the lowest price 5 and the other 1 quantity at 6. The first one has the lowest price and fulfils the needs of the operator therefore they sell their 10 quantities of energy at 5.

Stingy solution

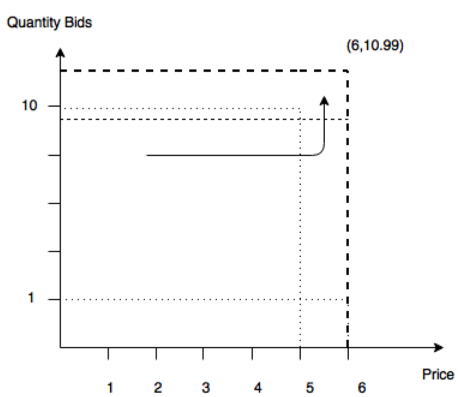

Figure 5: Same problem, but the company offers 9,99 instead of 10 therefore the spot price is fixed at the next price reaching the load which is 6, this way the company can sell the same amount of energy at a higher price.

The relaxation of the problem we consider assumes that for each generator can be any arbitrary number of bids. Bids are not related to specific generators any more. Each bid is now associated to a price λ_i. Therefore we need a new notations and variables:

g_i	The quantity bid on price λ_i
$G_i = \sum_{\substack{i' \in I \\ i' \leq i}} g_{i'}$	Cumulative bid quantities
$r_{s,i} = d_s - \sum_{\substack{j \in J^c \\ \lambda^c_{s,j} < \lambda_i}} \bar{g}^c_{s,j}$	The residual demands
$c^e_i = \sum_{\substack{j \in J \\ c_j < \lambda_i}} g^*_j$	The effective capacities

From now on, the G_i will be used instead of the g_i, with the following restrictions:

$G_{i'} \geq G_i$ if $i' > i$	Negative bid quantities are not allowed
$G_i \leq c^e_i$	Production at loss is not allowed

Proposition 2 - impact on profit

The impact on profit by modifying a consecutive list of cumulative bid quantities $\{G_i, ..., G_{i'}\}$ only depends on the modified quantities and their neighbouring quantities G_{i-1} and $G_{i'+1}$.

Proof

We are going to evaluate the effect on an arbitrary scenario of the modifications of those cumulative bid quantities. Let us see how the profit R_s reacts to those modifications. There are three possible cases:

- $G_{i-1} > r_{s,i}$:
  The demand is provided by the lower generators therefore the spot price π^s_d is lower than $\lambda_i \rightarrow R_s$ is constant.
- $G_{i'+1} \leq r_{s,i'+1}$:
  The last value of the packet of quantity bids is lower than the spot price $\pi^s_d \rightarrow \pi^s_d$ does not depend on them and the production neither because they are already all sold. R_s stays constant.
- $G_{i-1} \leq r_{s,i} \wedge G_{i'+1} > r_{s,i'+1}$:
  Since the demand is not satisfied under λ_i but is for λ'_i, the spot price is between those 2 values therefore a modification of those values does influence the profit.

We realize that the profit is a function of the modified cumulative bid quantities and their two neighbouring values. The function is the following:

$$R\left((\lambda_1, g_1), \ldots, (\lambda_n, g_n)\right) = R\left((\lambda_1, g_1)\right) + \sum_{k=2}^{n} \left[R\left((\lambda_{k-1}, G_{k-1}), (\lambda_k, g_k)\right) - R\left((\lambda_{k-1}, G_{k-1})\right)\right].$$

Proposition 3 - Thresholds

At least one of the set of optimal bids has all the G_i equal to one of the possible thresholds. The possible thresholds :

- The effective capacity c_i^e $\rightarrow$ we do not want to either be above or under it.
- The residual demand for the next higher bid price if lower than c_i^e.
- The bid quantity equal to following one is at threshold. $G_i = G_{i+1}$.

Proof

We can show that if we have a solution we can always improve it by moving one of the G_i as long as all the G_i are not at thresholds.
There are 2 cases to consider:

- $G_i > c_i^e$
  In this case we produce more than what effective, this means we can reduce G_i to c_i^e and increase our profit or leave it unchanged.
- $G_i < c_i^e$
  If G_i is not at any precedent threshold and $G_i < G_{i+1}$. we can always find i so that i' is not at threshold. In that case, the profit value obtained by increasing G_i to its next thresholds will depend on the spot price:
  - $\pi_s < \lambda_i$: All the demand is already covered, R_s stay unchanged after increasing G_i.
  - $\pi_s > \lambda_{i+1}$: We need at least to reach λ_{i+2} to reach the spot price. Since we cannot bid higher than G_{i+1} this means that increasing G_i will not change the value of the spot price. As a consequence R_s stays unchanged.
  - $\pi_s = \lambda_i$: The demand needs the production at λ_i. We face 2 cases:
    * $G_i < r_{s,i}$ $\rightarrow$ Increasing G_i increases our selling therefore increasing our profit.
    * $G_i \geq r_{s,i}$ $\rightarrow$ This means that we reached the maxima, implying that increasing will not increase our selling and our profit.

    In both cases the spot prices stays at λ_i.
  - $\pi_s = \lambda_{i+1}$: We visualize here a possible loss in profit. The spot price could decrease if the following happens, $G_i > r_{s,i}$. But $r_{s,i}$ is a threshold, and as a result G_i cannot be increased passed it. This means that the spot price remains constant and so increasing G_i does not impact the profit.

This proves that an optimal set of bids can be found where the G_i take specific values(thresholds). There are a finite number of thresholds and we can find the optimal solution in finite time.

Implementation of the shortest path

After what we learned from the third proposition, we are now looking to create the algorithm to solve the problem. The issue is that such algorithm takes an exponential time. Here is where the second proposition comes to play, "since the profit function R can be written as an incremental sum over individual bids, we can optimise each bid iteratively".
We define $R_i^{max}(G_i)$ $\rightarrow$ The maximum profit when bidding for a total quantity G_i at price up to λ_i.

$$R_i^{max}(G_i) = \max_{\substack{G_{i-1} \in \left\{c_{i-1}^e, G_i\right\} \cup \{r_{s,i} \forall s \in S\} \\ 0 \leq G_{i-1} \leq G_i}} R_{i-1}^{max}(G_{i-1}) + \left[R\left((\lambda_{i-1}, G_{i-1}), (\lambda_i, G_i - G_{i-1})\right) - R\left((\lambda_{i-1}, G_{i-1})\right)\right].$$

The optimal profit is obtained with $R_i^{max}(c_i^e)$ associated with the highest bid value of the problem.

"The single-scenario single-bid profit function $R_s((\lambda_i, G_i))$ has three regimes: none of the bid is sold, a partial amount of is sold at the value given, or all of it is sold at a potentially higher spot price."

$$R_s((\lambda_i, G_i)) = \begin{cases} 0 & \text{if } r_{s,i} < 0, \\ \lambda_i r_{s,i} - c(r_{s,i}) & \text{if } 0 \leq r_{s,i} < G_i, \\ G_i \left(\min_{i' \geq i} \left\{\lambda_{i'} : r_{s,i'+1} < G_i\right\}\right) & \\ -c(G_i) & \text{if } G_i \leq r_{s,i}. \end{cases}$$

Definition of the new variable and functions:

$c(G)$	The total of production for G units of energy.
$c(r_{s,i})$	The total cost of the residual demands.

The tactic used to speed up the calculation of our optimal profit is by pre-calculating these profit function($R_s((\lambda_i, G_i))$), afterwards we use those pre-calculations to calculate the profit increments.

$$R_s\left((\lambda_{i-1}, G_{i-1}), (\lambda_i, G_i - G_{i-1})\right) - R_s((\lambda_i, G_i)) = \begin{cases} 0 & \text{if } r_{s,i} < G_{i-1}, \\ R_s((\lambda_i, G_i)) - R_s((\lambda_{i-1}, G_{i-1})) & \text{if } G_{i-1} \leq r_{s,i}. \end{cases}$$

This is where we finally visualize the shortest path part of the algorithm. This last formula can be understood easily, there are two choices when adding an additional bid to a previous bid:

- Either there is no demand at the new bid price and the additional bid does not change the system ($r_{s,i} < G_{i-1}$).
- Or there is some demand to be fulfilled at the new bid price and the spot price will at least be at that value ($G_{i-1} \leq r_{s,i}$).

The algorithm