Ex # 14.1

(1) $f(x,y) = x^2 + xy^3$

(a) $f(0,0)$

$f(0,0) = (0)^2 + (0)(0)^3$

$= 0 + 0 = 0$

(b) $f(-1,1)$

$f(-1,1) = (-1)^2 + (-1)(1)^3$

$= 1 - 1 = 0$

(c) $f(2,3)$

$f(2,3) = (2)^2 + (2)(3)^3$

$= 4 + (2)(27)$

$= 4 + 54 = 58$

(d) $f(-3,-2)$

$f(-3,-2) = (-3)^2 + (-3)(-2)^3$

$= 9 + (-3)(-8)$

$= 9 + 24 = 33$

(2) $f(x,y) = \sin(xy)$

(a) $f(2, \frac{\pi}{6})$

$f(2, \frac{\pi}{6}) = \sin\left(\cancel{2} \times \frac{\pi}{\cancel{6}\,3}\right)$

$= \sin\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2}$

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(b) $f\left(-3, \frac{\pi}{12}\right)$

$f\left(-3, \frac{\pi}{12}\right) = \sin\left(-\cancel{3} \times \frac{\pi}{\cancel{12}_{4}}\right)$

$= -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$

(c) $f\left(\pi, \frac{1}{4}\right)$

$f\left(\pi, \frac{1}{4}\right) = \sin\left(\frac{\pi}{4}\right)$

$= \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

(d) $f\left(-\frac{\pi}{2}, -7\right)$

$f\left(-\frac{\pi}{2}, -7\right) = \sin\left(-\frac{\pi}{2} \times -7\right)$

$= \sin\left(\frac{7\pi}{2}\right)$

$= -1$

3) $f(x, y, z) = \frac{x - y}{y^2 + z^2}$

(a) $f(3, -1, 2) = \frac{3 - (-1)}{(-1)^2 + (2)^2}$

$= \frac{3 + 1}{1 + 4} = \frac{4}{5}$

$$\begin{array}{r|l} 2 & 4, 16 \\ \hline 2 & 2, 8 \\ \hline 2 & 1, 4 \\ \hline 2 & 1, 2 \\ \hline & 1, 1 \end{array}$$

(b) $f\left(1, \frac{1}{2}, -\frac{1}{4}\right)$

$f\left(1, \frac{1}{2}, -\frac{1}{4}\right) = \frac{1 - \frac{1}{2}}{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{4}\right)^2} = \frac{\frac{2-1}{2}}{\frac{1}{4} + \frac{1}{16}}$

$= \frac{\frac{1}{2}}{\frac{4+1}{16}} = \frac{1}{\cancel{2}} \times \frac{\cancel{16}^{8}}{5} = \frac{8}{5}$

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(c) $f(0, -\frac{1}{3}, 0) = \dfrac{0-(-\frac{1}{3})}{(-\frac{1}{3})^2+(0)^2}$

$= \dfrac{\frac{1}{3}}{\frac{1}{9}} = \frac{1}{\cancel{3}} \times \cancel{9}^{3} = 3$

(d) $f(2, 2, 100) = \dfrac{2-2}{(2)^2-(10)^2}$

$= \dfrac{0}{\cancel{804}} = 0$

(4) $f(x, y, z) = \sqrt{49 - x^2 - y^2 - z^2}$

(a) $f(0, 0, 0) = \sqrt{49-(0)^2-(0)^2-(0)^2}$

$= \sqrt{49-0-0-0} = \sqrt{49} = \cancel{4} \; 7$

(b) $f(2, -3, 6)$

$f(2, -3, 6) = \sqrt{49-(2)^2-(-3)^2-(6)^2}$

$\sqrt{49-4-9-36} = \sqrt{0} = 0$

(c) $f(0, -\frac{1}{3}, 0)$

$f(0, -\frac{1}{3}, 0) = \sqrt{49-(0)^2-(-\frac{1}{3})^2-(0)^2}$

$\sqrt{49-0-\frac{1}{9}-0} = \sqrt{\dfrac{441-1}{9}} = \sqrt{\dfrac{440}{9}} = \dfrac{2\sqrt{110}}{3}$

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(c) $f(-1,2,3)$

$$f(-1,2,3) = \sqrt{49-(-1)^2-(2)^2-(3)^2}$$

$$\sqrt{49-1-2-3} = \sqrt{43}$$

(d) $f\left(\frac{4}{\sqrt{2}}, \frac{5}{\sqrt{2}}, \frac{6}{\sqrt{2}}\right)$

$$f\left(\frac{4}{\sqrt{2}}, \frac{5}{\sqrt{2}}, \frac{6}{\sqrt{2}}\right) = \sqrt{49-\left(\frac{4}{\sqrt{2}}\right)^2-\left(\frac{5}{\sqrt{2}}\right)^2-\left(\frac{6}{\sqrt{2}}\right)^2}$$

$$\sqrt{49-\frac{16}{2}-\frac{25}{2}-\frac{36}{2}} = \sqrt{\frac{98-16-25-36}{2}}$$

$$\sqrt{\frac{21}{2}} = \frac{\sqrt{21}}{\sqrt{2}}$$

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Ex # 14.2

Q#1 $\lim_{(x,y)\to(0,0)} \frac{3x^2-y^2+5}{x^2+y^2+2}$

$= \frac{3(0)^2-(0)^2+5}{(0)^2+(0)^2+2}$

$= \frac{5}{2}$

Q#2 $\lim_{(x,y)\to(0,4)} \frac{x}{\sqrt{y}}$

$= \frac{0}{\sqrt{4}} = \frac{0}{2} = 0$

Q#3 $\lim_{(x,y)\to(3,4)} \sqrt{x^2+y^2-1}$

$= \sqrt{(3)^2+(4)^2-1} = \sqrt{9+16-1} = \sqrt{24} = 2\sqrt{6}$

Q#4 $\lim_{(x,y)\to(2,-3)} \left(\frac{1}{x}+\frac{1}{y}\right)^2$

$= \left(\frac{1}{2}+-\frac{1}{3}\right)^2 = \left(\frac{3-2}{6}\right)^2$

$= \left(\frac{1}{6}\right)^2 = \frac{1}{36}$

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Q#5 $\lim_{(x,y)\to(0,\frac{\pi}{4})} \sec x \tan y$

$\sec 0 \tan \frac{\pi}{4} = (1)(1) = 1$

Q#6 $\lim_{(x,y)\to(0,0)} \cos \frac{x^2+y^2}{x+y+1}$

$\cos \frac{(0)^2+(0)^2}{0+0+1} = \cos \frac{0}{0} = \cos 0 = 1$

Q#7 $\lim_{(x,y)\to(0,\ln 2)} e^{x-y}$

$= e^{0-\ln 2} = e^{-\ln 2} = \cancel{e}^{\cancel{\ln} 2^{-1}} = 2^{-1} = \frac{1}{2}$

Q#8 $\lim_{(x,y)\to(1,1)} \ln|1+x^2y^2|$

$= \ln|1+(1)^2(1)^2| = \ln|1+1| = \ln 2 = 0.693$

~~Q#9.~~

Q#9 $\lim_{(x,y)\to(0,0)} \frac{e^y \sin x}{x} = \frac{e^0 \sin 0}{0} = \frac{0}{0}$ → sandwich theorem

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$e^0 \cdot 1 = 1$

Q#10 $\lim_{(x,y)\to(1/27,\pi^3)} \cos \sqrt[3]{xy} = \cos \sqrt[3]{\left(\frac{1}{27}\right)(\pi)^3}$

$= \cos \frac{1}{3}\pi \quad \cos \frac{\pi}{3} = \frac{1}{2}$

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Q#11 $\lim_{(x,y)\to(1,\frac{\pi}{6})} \frac{x \sin y}{x^2+1}$

$= \frac{(1) \sin \pi/6}{(1)^2+1} = \frac{(1)(\frac{1}{2})}{1} = \frac{1}{2}$

Q#12 $\lim_{(x,y)\to(\frac{\pi}{2},0)} \frac{\cos y+1}{y-\sin x}$

$\frac{\cos 0+1}{0-\sin \frac{\pi}{2}} \quad \frac{1+1}{0-1} = \frac{2}{-1} = -2$

Q#13 $\lim_{(x,y)\to(1,1)} \frac{x^2-2xy+y^2}{x-y}$

$\frac{1+2(1)(1)+1}{1-1} = \frac{4}{0} = \infty$

$= \cancel{x^2-y} \quad \cancel{\frac{(x+y)^2}{x-y}}$

$= \frac{x^2-2xy+y^2}{x-y} = \frac{(x-y)^2}{(x-y)}$

$= \boxed{x-y}$

$\lim_{(x,y)\to(1,1)} x-y$

$= 1-1 = 0$

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Q#14 $\lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y} \to \frac{0}{0}$

$\frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}} = x+y$

$\lim_{(x,y)\to(1,1)} x+y = 1+1 = 2$

Q#15 $\lim_{(x,y)\to(1,1)} \frac{xy-y-2x+2}{x-1} \Rightarrow \frac{0}{0}$

$\frac{y(x-1)-2(x-1)}{(x-1)}$

$= \frac{(y-2)\cancel{(x-1)}}{\cancel{(x-1)}}$

$= y-2$

$\lim_{(x,y)\to(1,1)} y-2 = 1-2 = -1$

Q#16 $\lim_{(x,y)\to(2,-4)} \frac{y+4}{x^2y-xy+4x^2-4x} \Rightarrow \frac{0}{0}$

$= \frac{y+4}{xy(x-1)+4x(x-1)} = \frac{y+4}{(x-1)(xy+4x)}$

$= \frac{\cancel{y+4}}{(x-1)x\cancel{(y+4)}} = \frac{1}{x(x-1)}$

$\lim_{(x,y)\to(2,-4)} \frac{1}{x(x-1)} = \frac{1}{2(2-1)} = \frac{1}{2(1)} = \frac{1}{2}$

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Q#17 $\lim_{(x,y)\to(0,0)} \dfrac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

$\lim = \dfrac{(x-y)+2(\sqrt{x}-\sqrt{y})}{\sqrt{x}-\sqrt{y}}$

$= \dfrac{(\sqrt{x}-\sqrt{y})\ \dfrac{x-y}{\sqrt{x}-\sqrt{y}}+2}{\sqrt{x}-\sqrt{y}}$

$= \dfrac{x-y}{\sqrt{x}-\sqrt{y}}+2$

$= \dfrac{(x-y)^2}{x-y}+4$

$= x-y+4$

$\lim_{(x,y)\to(0,0)} x-y+4 = 0-0+4 = 4$

Q#17 solution#1

$= \dfrac{(x-y)+2(\sqrt{x}-\sqrt{y})}{\sqrt{x}-\sqrt{y}}$

$= \dfrac{x-y}{\sqrt{x}-\sqrt{y}} + \dfrac{2(\sqrt{x}-\sqrt{y})}{\sqrt{x}-\sqrt{y}}$

$= \dfrac{x-y}{\sqrt{x}-\sqrt{y}}+2$

$= \dfrac{(x-y)(\sqrt{x}+\sqrt{y})}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})}+2$

$\frac{0}{0}$

solution#2

$\dfrac{(x-y)+2(\cancel{\sqrt{x}-\sqrt{y}})}{\cancel{\sqrt{x}-\sqrt{y}}}$

$x-y+2$

$0-0+2$

$=2$

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$$\frac{\cancel{(x-y)}\,(\sqrt{x}+\sqrt{y})}{\cancel{(x-y)}}+2$$

$$\sqrt{x}+\sqrt{y}+2$$

$$\lim_{(x,y)\to(0,0)} \sqrt{x}+\sqrt{y}+2 = 0+0+2 = 2$$

Q#18 $$\lim_{(x,y)\to(2,2)} \frac{x+y-4}{\sqrt{x+y}-2} \Rightarrow \frac{0}{0}$$

$$\frac{(x+y-4)^2}{(\sqrt{x+y}-2)} = \frac{(\sqrt{x+y})^2-(\sqrt{4})^2}{(\sqrt{x+y})-(2)}$$

$$= \frac{(\sqrt{x+y}+\sqrt{4})(\sqrt{x+y}-\sqrt{4})}{\sqrt{x+y}-2}$$

$$= \frac{(\sqrt{x+y}+2)\cancel{(\sqrt{x+y}-2)}}{\cancel{(\sqrt{x+y}-2)}}$$

$$= \sqrt{x+y}+2$$

$$\lim_{(x,y)\to(2,2)} \sqrt{x+y}+2 = \sqrt{2+2}+2 = 2+2 = 4$$

Q#19 $$\lim_{(x,y)\to(2,0)} \frac{\sqrt{2x-y}-2}{2x-y-4} \Rightarrow \frac{0}{0}$$

$$\frac{\sqrt{2x-y}-2}{(\sqrt{2x-y})^2-(\sqrt{4})^2}$$

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$$\frac{\cancel{\sqrt{2x-y}-2}}{(\sqrt{2x-y}+2)\cancel{(\sqrt{2x-y}-2)}}$$

$$= \frac{1}{\sqrt{2x-y}+2}$$

$$\lim_{(x,y)\to(2,0)} \frac{1}{\sqrt{2x-y}+2} = \frac{1}{\sqrt{4-0}+2} = \frac{1}{2+2} = \frac{1}{4}$$

Q#20 $\lim_{(x,y)\to(4,3)} \dfrac{\sqrt{x}-\sqrt{y+1}}{x-y-1}$

$$\cancel{\frac{\sqrt{x}-\sqrt{y+1}}{(\sqrt{x})^2-(\sqrt{y-1})^2}} \quad \cancel{\frac{(x)^2-(\sqrt{y+1})^2}{(x)^2-(y-1)^2}}$$

$$\cancel{\frac{\sqrt{x}-\sqrt{y+1}}{\sqrt{x}-\sqrt{y-1}}}$$

$$\cancel{\frac{x-y+1}{x^2-}} \quad \frac{(\sqrt{x})^2-(\sqrt{y+1})^2}{(x-y)^2-(1)^2}$$

$$= \frac{\cancel{x-y-1}}{\cancel{(x-y-1)}(x-y+1)} = \frac{1}{x-y+1}$$

$$\cancel{\frac{(\sqrt{x})^2-(\sqrt{y+1})}{(\sqrt{x}-y)^2-(\sqrt{1})^2}}$$

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$$\frac{\cancel{\sqrt{x}-\sqrt{y+1}}}{(\sqrt{x}+\sqrt{y+1})\cancel{(\sqrt{x}-\sqrt{y+1})}}$$

$$= \frac{1}{\sqrt{x}+\sqrt{y+1}} = \lim_{(x,y)\to(4,3)} \frac{1}{\sqrt{x}+\sqrt{y+1}}$$

$$= \frac{1}{\sqrt{4}+\sqrt{3+1}} = \frac{1}{2.2} = \frac{1}{4}$$

Q#21 $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} \Rightarrow \frac{0}{0}$

Let $(x^2+y^2) = t$

$$\frac{\sin t}{t}$$ $\therefore$ sandwich Theorem $\frac{\sin x}{x} = 1$

$= 1$

$t = x^2 + y^2$

$$\lim_{(x,y)\to(0,0)} 1 = 1$$

Q#22 $\lim_{(x,y)\to(0,0)} \frac{1-\cos(xy)}{xy}$

Let $xy = \theta$

$$\frac{1-\cos\theta}{\theta}$$ $\therefore 1-\cos\theta = 2\sin^2\theta/2$

$$= \frac{2\sin^2\theta/2}{\theta}$$

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$$= \frac{\sin\frac{\theta}{2} \cdot \sin\frac{\theta}{2}}{\frac{\theta}{2}}$$

$$\lim_{\theta \to 0} \frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}} \cdot \sin\frac{\theta}{2}$$ ∴ sandwich Theorem

$$\cancel{\frac{\sin\theta}{\frac{\theta}{2}} \cdot \sin\theta}$$

$$\lim_{\theta \to 0} \frac{\sin\theta/2}{\theta/2} \cdot \lim_{\theta \to 0} \sin\frac{\theta}{2}$$

$$1 \cdot 0$$

$$= 0$$

now $\theta = xy$

$$\cancel{\frac{\sin xy/2}{xy/2}} \cdot \sin\frac{xy}{2}$$

$$= \lim_{(x,y) \to (0,0)}$$

Q# $\frac{1 - \cos t}{t}$ $\frac{(t)d(1-\cos t) - (1-\cos t)d(t)}{(t)^2}$

$$= \frac{(t)(\sin t) - 1 - \cos t}{t^2}$$

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Q#23 $\lim\limits_{(x,y)\to(1,-1)} \dfrac{x^3+y^3}{x+y}$

$$\frac{\cancel{(x+y)}(x^2-xy+y^2)}{\cancel{(x+y)}}$$

$$= \lim_{(x,y)\to(1,-1)} x^2-xy+y^2$$

$$= (1)^2-(1)(-1)+(-1)^2$$

$$= 1+1+1 = 3$$

Q#24 $\lim\limits_{(x,y)\to(2,2)} \dfrac{x-y}{x^4-y^4}$

$$\frac{x-y}{(x^2)^2-(y^2)^2} = \frac{x-y}{(x^2-y^2)(x^2+y^2)}$$

$$\frac{\cancel{x-y}}{\cancel{(x-y)}(x+y)(x^2+y^2)} = \frac{1}{(x+y)(x^2+y^2)}$$

$$\lim_{(x,y)\to(2,2)} \frac{1}{(x+y)(x^2+y^2)} = \frac{1}{(2+2)(4+4)}$$

$$= \frac{1}{(4)(8)} = \frac{1}{32}$$

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Q#25 $\lim_{P \to (1,3,4)} \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$

$= \frac{1}{1} + \frac{1}{3} + \frac{1}{4}$

$= \frac{12+4+3}{12} = \frac{19}{12}$

Q#26 $\lim_{P \to (1,-1,-1)} \frac{2xy + yz}{x^2 + z^2}$

$= \frac{2(1)(-1) + (-1)(-1)}{(1)^2 + (-1)^2}$

$= \frac{-2+1}{1+1} = \frac{-1}{2}$

Q#27 $\lim_{P \to (\pi,\pi,0)} \sin^2 x + \cos^2 y + \sec^2 z$

$\Rightarrow \sin^2\pi + \cos^2\pi + \sec^2\pi$

$= (0)^2 + (-1)^2 + (-1)^2$

$= 0 + 1 + 1$

$= 2$

Q#28 $\lim_{P \to (-1/4, \pi/2, 2)} \tan^{-1} xyz$

$\tan^{-1}\left[\left(-\frac{1}{4}\right)\left(\frac{\pi}{2}\right)(2)\right]$

$\tan^{-1}\left(-\frac{\pi}{4}\right) = -\tan^{-1}\frac{\pi}{4} = -1$

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Q#29 $\lim\limits_{P \to (\pi, 0, 3)} z e^{-2y} \cos 2x$

$= 3 e^{-2(0)} \cos 2(\pi)$

$= 3 e^{0} \cos 2\pi$

$= 3 \cdot 1 \cdot 1$

$= 3$

Q#30 $\lim\limits_{P \to (2, -3, 6)} \ln \sqrt{x^2 + y^2 + z^2}$

$= \ln \sqrt{(2)^2 + (-3)^2 + (6)^2}$

$= \ln \sqrt{4 + 9 + 36}$

$= \ln \sqrt{49}$

$= \ln 7$

$= 1.945$

Q#41 $f(x,y) = \dfrac{-x}{\sqrt{x^2+y^2}}$

$\lim\limits_{(x,y) \to (0,0)} \dfrac{-x}{\sqrt{x^2+y^2}}$

(i) $x = 0$

$\dfrac{0}{\sqrt{0+y^2}} = 0 \quad L_2 = 0$

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(ii) $y = 0$

$$\lim_{(x,y)\to(0,0)} \frac{-x}{\sqrt{x^2+y^2}}$$

$$\frac{-x}{\sqrt{x^2}} = \frac{-x}{x} = -1$$

$L_1 = -1$

$L_1 \neq L_2$ so, limit ~~doesn't~~ don't exist.

Q#42 $f(x,y) = \dfrac{x^4}{x^4+y^2}$

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}$$

(i) $y=0$

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+0}$$

$$= \frac{x^4}{x^4} = 1$$

$L_1 = 1$

(ii) $x=0$

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}$$

$$\frac{0}{0+y^2} = 0$$

$L_2 = 0$

$L_1 \neq L_2$

So, limit don't exist.

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Q#43 $f(x,y) = \dfrac{x^4 - y^2}{x^4 + y^2}$

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^2}{x^4 + y^2}$$

(i) $y = 0$

$$\frac{x^4 - 0}{x^4 + 0} = \frac{x^4}{x^4} = 1$$

$$\lim_{x\to 0} 1 = 1 \qquad L_1 = 1$$

(ii) $x = 0$

$$\frac{0 - y^2}{0 + y^2} = \lim_{y\to 0} \frac{-y^2}{y^2}$$

$$\cancel{\lim_{y\to 0} \frac{0}{0}} \quad \cancel{0 = L_2 = 0}$$

$$\lim_{y\to 0} \frac{-1}{1} = L_2 = -1$$

$L_1 \neq L_2$ so, limit doesn't exist.

Q#44 $f(x,y) = \dfrac{xy}{|xy|}$

(i) $\cancel{y=0}$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|}$$

(i) $y = 0$

$$\cancel{\lim_{x\to 0} \frac{x(0)}{|x(0)|}} = \frac{0}{0}$$

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(iii) $y = mx$

$$\lim_{(x,y)\to(0,0)} \frac{x \cdot mx}{|x \cdot mx|}$$

$$\Rightarrow \frac{mx^2}{|mx^2|}$$

$$\Rightarrow \frac{m\cancel{x^2}}{\cancel{x^2}|m|}$$

$$\lim_{(x,y)\to(0,0)} \frac{m}{|m|}$$

(iv) $y = mx^2$

$$\lim_{(x,y)\to(0,0)} \frac{x \cdot mx^2}{|x \cdot mx^2|}$$

$$\frac{mx^3}{|mx^3|}$$

$$\frac{mx^3}{x^3}$$

Q# 45 $g(x,y) = \dfrac{x-y}{x+y}$

$$\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y}$$

(i) $y = 0$

$$\lim_{x\to 0} \frac{x-0}{x+0}$$

$$\lim_{x\to 0} \frac{\cancel{x}\,1}{\cancel{x}} \Rightarrow \lim_{x\to 0} 1$$

$L_1 = 1$

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(ii) $x=0$

$$\lim_{y\to 0} \frac{0-y}{0+y}$$

$$\lim_{y\to 0} \frac{-\cancel{y}}{\cancel{y}} = \lim_{y\to 0} -1 = -1 \quad L_2=-1$$

$\Rightarrow L_1 \neq L_2$ so limit doesn't exist

Q#46 $g(x,y) = \dfrac{x^2-y}{x-y}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y}{x-y}$$

(i) $y=0$

$$\lim_{x\to 0} \frac{x^2-0}{x-0} = \lim_{x\to 0} \frac{x^{2-1}}{x} = \lim_{x\to 0} 1 = 1 \Rightarrow L_1=1$$

(ii) $x=0$

$$\lim_{y\to 0} \frac{0-y}{0-y} = \lim_{y\to 0} \frac{-\cancel{y}}{-\cancel{y}} = \lim_{y\to} 1 = 1$$

(iii) mx

$$\lim_{(x,y)\to(0,0)} \frac{x^2-mx}{x-mx} = \lim \frac{\cancel{x}(x-m)}{\cancel{x}(1-m)}$$

$$\lim_{(x,y)\to(0,0)} \frac{x-m}{1-m} = \frac{0-m}{1-m} = \frac{-m}{1-m}$$

$L_3 = \dfrac{-m}{1-m}$

$L_1 = L_2 \neq L_3$
so, Limit not exist

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Q#47 $h(x,y) = \dfrac{x^2+y}{y}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{y}$$

(i) $y = mx$

$$\cancel{\lim_{x\to 0} \frac{x^2+0}{0} =}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+mx}{mx} = \lim \frac{\cancel{x}(x+m)}{m\cancel{x}}$$

$$\lim_{x,y\to 0,0} \frac{x+m}{m} = \frac{0+m}{m} = \frac{\cancel{m}}{\cancel{m}} = 1$$

(ii) $y = mx^2$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+mx^2}{mx^2} = \lim_{(x,y)\to(0,0)} \frac{\cancel{x^2}(1+m)}{m\cancel{x^2}}$$

$$\lim_{(x,y)\to(0,0)} \frac{1+m}{m} = \frac{1+m}{m}$$

$L_3 \neq L_4$, so limit doesn't exist.

Q#48 $h(x,y) = \dfrac{x^2y}{x^4+y^2}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2}$$

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(i) $y=0$

$$\lim_{x\to 0} \frac{x^2(0)}{x^4+(0)^2} = \frac{0}{x^4} = 0$$

$L_1 = 0$

(ii) $x=0$

$$\lim_{y\to 0} \frac{(0)y}{(0)^4+y^2} = \frac{0}{y^2} = 0$$

$L_2 = 0$

(iii) $y = mx$

$$\lim_{(x,y)\to(0,0)} \frac{x^2(mx)}{x^4+mx} = \lim \frac{mx^3}{x^4+mx}$$

$$\lim_{(x,y)} \frac{mx^3}{x(x^3+m)} = \lim \frac{mx^2}{x^3+m}$$

$$\frac{m(0)}{0+m} = \frac{0}{m} = 0$$

$L_3 = 0$

(iv) $y = mx^2$

$$\lim_{(x,y)\to(0,0)} \frac{x^2(mx^2)}{x^4+mx^2} = \lim_{(x,y)\to(0,0)} \frac{mx^4}{x^4+mx^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{mx^4}{x^2(x^2+m)} = \lim_{(x,y)\to(0,0)} \frac{mx^2}{x^2+m}$$

$$= \frac{m(0)}{0+m} = \frac{0}{m} = 0 \quad L_4 = 0$$

$L_1 = L_2 = L_3 = L_4$, so, limit exists.

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Q#63 $f(x,y) = \dfrac{y^2}{x^2+y^2}$

$$\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$$

i) $y=0$

$$\lim_{x\to 0} \frac{0}{x^2+0} = \frac{0}{x^2} = 0 \Rightarrow L_1 = 0$$

ii) $x=0$

$$\lim_{y\to 0} \frac{y^2}{0+y^2} = \lim_{y\to 0} \frac{\cancel{y^2}}{\cancel{y^2}} = 1 \quad L_2 = 1$$

$L_1 \neq L_2$ so, limit not exist.

Q#64 $f(x,y) = \dfrac{2x}{x^2+x+y^2}$

$$\lim_{(x,y)\to(0,0)} \frac{2x}{x^2+x+y^2}$$

i) $y=0$

$$\lim_{x\to 0} \frac{2x}{x^2+x+0} = \lim_{x\to 0} \frac{2\cancel{x}}{\cancel{x}(x+1)} = \lim_{x\to 0} \frac{2}{x+1}$$

$$\frac{2}{1+1} = \frac{\cancel{2}}{\cancel{2}} = 1$$

$L_1 = 1$

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ii) $x = 0$

$$\lim_{y \to 0} \frac{2(0)}{0^2 + 0 + y} = \frac{0}{y} = 0$$

$L_2 = 0$

$L_1 \neq L_2$, so limit not exist

Q# 65 $f(x,y) = \tan^{-1}\left(\frac{|x| + |y|}{x^2 + y^2}\right)$

$$\lim_{(x,y) \to (0,0)} \tan^{-1}\left(\frac{|x| + |y|}{x^2 + y^2}\right)$$

i) $y = 0$

$$\lim_{x \to 0} \tan^{-1}\left(\frac{|x| + |0|}{x^2 + (0)^2}\right)$$

$$\lim_{x \to 0} \tan^{-1} \frac{|x|}{x^2}$$

ii) $y = mx$

$$\lim_{(x,y) \to (0,0)} \frac{|x| + |mx|}{x^2 + m^2x^2} = \lim \frac{x|1+m|}{x^2|1+m^2|}$$

$$\lim \frac{|1+m|}{x|1+m^2|} \Rightarrow \infty$$

undefined.

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Q#66 $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2+y^2}$$

(i) $y = 0$

$$\lim_{x\to 0} \frac{x^2 - 0^2}{x^2+0^2} = \frac{x^2}{x^2} = \lim_{x\to 0} 1 = 1 \Rightarrow L_1 = 1$$

(ii) $x = 0$

$$\lim_{y\to 0} \frac{0^2 - y^2}{0^2+y^2} = \lim_{y\to 0} \frac{-y^2}{y^2} = \lim_{y\to 0} -1 \Rightarrow L_2 = -1$$

$L_1 \neq L_2$, so limit not exist

Q#67 $f(x,y) = \ln\left(\dfrac{3x^2 - x^2y^2 + 3y^2}{x^2+y^2}\right)$

$$\lim_{(x,y)\to(0,0)} \ln\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2+y^2}\right)$$

(i) $y = 0$

$$\lim_{x\to 0} \ln\left(\frac{3x^2 - x^2(0)^2 + 3(0)^2}{x^2+(0)^2}\right)$$

$$\lim_{x\to 0} \ln\left(\frac{3x^2}{x^2}\right) \Rightarrow \ln 3$$

$L_1 = \ln 3$

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(ii) $x = 0$

$$\lim_{y \to 0} \ln\left(\frac{3(0)^2 - (0)^2 y^2 + 3y^2}{(0)^2 + y^2}\right)$$

$$\lim_{y \to 0} \ln\left(\frac{3\cancel{y^2}}{\cancel{y^2}}\right) = \ln 3$$

$$L_2 = \ln 3$$

(iii) $y = mx$

$$\lim_{(x,y)\to(0,0)} \ln\left(\frac{3x^2 - x^2(mx)^2 + 3(mx)^2}{x^2 + (mx)^2}\right)$$

$$= \ln\left(\frac{3x^2 - m^2x^4 + 3m^2x^2}{x^2 + m^2x^2}\right)$$

$$= \ln\left(\frac{\cancel{x^2}(3 - m^2x^2 + 3m^2)}{\cancel{x^2}(1+m^2)}\right)$$

$$= \ln\left(\frac{3 - m^2x^2 + 3m^2}{1+m^2}\right)$$

$$\ln\left(\frac{3 - 0 + 3m^2}{1+m^2}\right)$$

$$\ln\left(\frac{3\cancel{(1+m^2)}}{\cancel{1+m^2}}\right)$$

$$L_3 = \ln 3$$

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(iv) $y = mx^2$

$$\lim_{(x,y)\to(0,0)} \ln\left(\frac{3x^2 - x^2(mx^2)^2 + 3(mx^2)^2}{x^2+(mx^2)^2}\right)$$

$$= \ln\left(\frac{3x^2 - m^2x^6 + 3m^2x^4}{x^2+mx^4}\right)$$

$$= \ln\left(\frac{\cancel{x^2}(3 - m^2x^4 + 3m^2x^2)}{\cancel{x^2}(1+mx^2)}\right)$$

$$= \ln \frac{3 - m^2x^4 + 3m^2x^2}{1+mx^2}$$

$$\ln \frac{3-0+0}{1+0}$$

$L_4 = \ln 3$

$L_1 = L_2 = L_3 = L_4$, so limit exist.

Q# 68 $f(x,y) = \dfrac{3x^2y}{x^2+y^2}$

$$\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$$

i) $y = 0$

$$\lim_{x\to 0} \frac{3x^2(0)}{x^2+(0)} = \lim_{x\to 0} \frac{0}{x^2}$$

$L_1 = 0$

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(ii) $x = 0$

$$\lim_{y \to 0} \frac{3(0)^2 y}{(0)^2 + y^2} = \lim \frac{0}{y^2} = 0$$

$L_2 = 0$

(iii) $y = mx$

$$\lim_{(x,y) \to (0,0)} \frac{3x^2(mx)}{x^2 + (mx)^2}$$

$$= \frac{3mx^3}{x^2 + m^2x^2} = \frac{3mx^3}{x^2(1+m^2)}$$

$$= \frac{3mx}{1+m^2} = \frac{3m(0)}{1+m^2} = 0$$

$L_3 = 0$

(iv) $y = mx^2$

$$\lim_{(x,y) \to (0,0)} \frac{3x^2(mx^2)}{x^2 + (mx^2)^2}$$

$$= \frac{3mx^4}{x^2 + m^2x^4} = \frac{3mx^4}{x^2(1+m^2x^2)}$$

$$= \frac{3mx^2}{1+m^2x^2} = \frac{3m(0)}{1+m^2(0)} = 0$$

$L_4 = 0$

$L_1 = L_2 = L_3 = L_4$, so limit exists.

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Q#33 (a) $g(x,y) = \sin \frac{1}{xy}$

The given function is undefined at (0,0) or '$x=0$ and $y=0$' and defined for all other points of (x,y) and its limiting value exists. So, $f(x,y) = \sin \frac{1}{xy}$ is defined for all values of (x,y) except $x=0$ or $y=0$ / (0,0).

(b) $g(x,y) = \frac{x+y}{2+\cos x}$

The given function is continous for all real numbers so, $\frac{x+y}{2+\cos x}$ is defined for all real numbers and its limiting value exists.

so, $f(x,y) = \frac{x+y}{2+\cos x}$ is continous for all real numbers.

Q#34 (a) $g(x,y) = \frac{x^2+y^2}{x^2-3x+2}$ $(x-2)(x-1)=0$

The given function will be undefined at $x=2$ or $x=1$ and defined for all other points of (x,y) and its limiting value exists So, $f(x,y) = \frac{x^2+y^2}{x^2-3x+2}$ is continuous for all values of (x,y) except $x=2$ or $x=1$

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(b) $g(x,y) = \dfrac{1}{x^2 - y}$

The given function is undefined at $y = x^2$ and defined for all other points of (x,y) and its limiting value also exists, so, $g(x,y) = \dfrac{1}{x^2-y}$ is continous for all values of (x,y) except $y = x^2$.

Example #3

$$f(x,y) = \frac{2y}{y+\cos x}$$

y = constant

$$f_x = \frac{(y+\cos x)\frac{\partial}{\partial x}(2y) - 2y\frac{\partial}{\partial x}(y+\cos x)}{(y+\cos x)^2}$$

$$= \frac{(y+\cos x)(0) - 2y(0-\sin x)}{(y+\cos x)^2}$$

$$= \frac{2y\sin x}{(y+\cos x)^2}$$

$$f_y = \frac{(y+\cos x)\frac{\partial}{\partial y}(2y) - 2y\frac{\partial}{\partial y}(y+\cos x)}{(y+\cos x)^2}$$

$$= \frac{(y+\cos x)(2) - 2y(1+0)}{(y+\cos x)^2}$$

$$= \frac{2y + \cos x - \cancel{2y}}{(y+\cos x)^2} = \frac{\cos x}{(y+\cos x)^2}$$

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Ex # 14.3

Q#1 $f(x,y) = 2x^2 - 3y - 4$

(x const) $\frac{\partial f}{\partial y} = 2(0) - 3(1) - 0$

$= \cancel{-3y-4} \; -3y$

(y const) $\frac{\partial f}{\partial x} = 2(2x) - 0 - 0$

$= 4x \; \cancel{-4/4(x-1)}$

Q#2 $f(x,y) = x^2 - xy + y^2$

$\frac{\partial f}{\partial y} = 0 - 0(y) + 2y$

$= 2y$

$\frac{\partial f}{\partial x} = 2x - x(0) + 0$

$= 2x$

Q#1 $f(x,y) = 2x^2 - 3y - 4$

$\frac{\partial f}{\partial y} = 2(0) - 3(1) - 0$

$= 0 - 3 - 0 = -3$

$\frac{\partial f}{\partial x} = 2(2x) - 3(0) - 0$

$= 4x - 0 - 0 = 4x$

Q#2 $f(x,y) = x^2 - xy + y^2$

$\frac{\partial f}{\partial y} = 0 - (1)x + 2y$

$= 2y - x$

$\frac{\partial f}{\partial x} = 2x - y(1) + 0$

$= 2x - y$

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~~Q#3~~ $\cancel{f(x,y) = (x^2-1)(y+2)}$

$\cancel{\frac{\partial f}{\partial y}} = \cancel{x^2y + x^2 2 - y - 2}$

$\frac{\partial f}{\partial y} = \cancel{x^2(1) + x^2(1) - 1 - 0}$

$= x^2 + x^2 = 2x^2$

Q#3 $f(x,y) = (x^2-1)(y+2)$

$x^2y + 2x^2 - y - 2$

$\frac{\partial f}{\partial y} = x^2(1) + 0 - 1 - 0$

$= x^2 - 1$

$\frac{\partial f}{\partial x} = y(2x) + 2 \cdot 2x - 0 - 0$

$= 2xy + 4x \Rightarrow 2x(y+2)$

Q#4 $f(x,y) = 5xy - 7x^2 - y^2 + 3x - 6y + 2$

$\frac{\partial f}{\partial y} = 5x(1) - 0 - 2y + 0 - 6 + 0$

$= 5x - 2y - 6$

$\frac{\partial f}{\partial x} = 5y(1) - 7 \cdot 2x - 0 + 3 - 0 + 0$

$= 5y - 14x + 3$

Q#5 $f(x,y) = (xy-1)^2$

$\frac{\partial f}{\partial y} = 2(xy-1) \cdot \frac{\partial f}{\partial y}(xy-1)$

$= 2(xy-1)(x(1)-0)$

$= 2x(xy-1)$

$\frac{\partial f}{\partial x} = 2(xy-1) \frac{\partial f}{\partial x}(xy-1)$

$= 2(xy-1)(y)$

$= 2y(xy-1)$

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Q#6 $f(x,y) = (2x-3y)^3$

x-const $\frac{\partial f}{\partial y} = 3(2x-3y)^{3-1} \frac{\partial f}{\partial y}(2x-3y)$

$= 3(2x-3y)^2 \cdot (0-3(1))$

$= -9(2x-3y)^2$

y-const $\frac{\partial f}{\partial x} = 3(2x-3y)^{3-1} \frac{\partial f}{\partial x}(2x-3y)$

$= 3(2x-3y)^2 \cdot (2(1)-0)$

$= 6(2x-3y)^2$

Q#7 $f(x,y) = \sqrt{x^2+y^2}$

$\frac{\partial f}{\partial y} = \frac{1}{2} \times \frac{1}{\sqrt{x^2+y^2}} \frac{\partial f}{\partial y}(x^2+y^2)$ $\quad \therefore \frac{1}{2}(x^2+y^2)^{\frac{1}{2}-1}$

$\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}$

$= \frac{1}{\cancel{2}\sqrt{x^2+y^2}}(0+\cancel{2}y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

$\frac{\partial f}{\partial x} = \frac{1}{\cancel{2}\sqrt{x^2+y^2}} \cdot (\cancel{2}x+0)$

$= \frac{x}{\sqrt{x^2+y^2}}$

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Q#8 $f(x,y) = (x^3 + (y/2))^{2/3}$

$\frac{2}{3}-1$

$\frac{2-3}{3}$

$= -\frac{1}{3}$

$f_y = \frac{\partial f}{\partial y} = \frac{2}{3}(x^3 + (y/2))^{-1/3} \cdot \frac{\partial f}{\partial y}(x^3 + y/2)$

$$\frac{2}{3\sqrt[3]{x^3 + y/2}} \cdot 0 + \frac{1}{2}(1)$$

$$= \frac{1}{3\sqrt[3]{x^3 + y/2}}$$

$f_x = \frac{\partial f}{\partial x} = \frac{2}{3}(x^3 + (y/2))^{-1/3} \cdot \frac{\partial f}{\partial x}(x^3 + y/2)$

$$= \frac{2}{3\sqrt[3]{x^3 + y/2}} \cdot 3x^2 + 0$$

$$= \frac{2x^2}{\sqrt[3]{x^3 + y/2}}$$

Q#9 $f(x,y) = \frac{1}{x+y}$

$$\frac{\partial f}{\partial y} = \frac{(x+y)\frac{\partial f}{\partial y}(1) - \frac{\partial f}{\partial y}(x+y)(1)}{(x+y)^2}$$

$$= \frac{(x+y)(0) - 1}{(x+y)^2} = \frac{-1}{(x+y)^2} = \frac{-1}{(x+y)^2}$$

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$$\frac{\partial f}{\partial x} = \frac{(x+y)\frac{\partial f}{\partial x}(1) - (1)\frac{\partial f}{\partial x}(x+y)}{(x+y)^2}$$

$$= \frac{0-1}{(x+y)^2} = -\frac{1}{(x+y)^2}$$

Q#10 $f(x,y) = \dfrac{x}{x^2+y^2}$

$$\frac{\partial f}{\partial y} = \frac{(x^2+y^2)\frac{\partial f}{\partial y}(x) - x\frac{\partial f}{\partial y}(x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{(x^2+y^2)(0) - x(2y)}{(x^2+y^2)^2} = \frac{-2xy}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x} = \frac{(x^2+y^2)\frac{\partial f}{\partial x}(x) - x\frac{\partial f}{\partial x}(x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{(x^2+y^2)(1) - x(2x)}{(x^2+y^2)^2}$$

$$= \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

Q#11 $f(x,y) = (x+y)/(xy-1)$

$$\frac{\partial f}{\partial y} = \frac{(xy-1)\frac{\partial f}{\partial y}(x+y) - (x+y)\frac{\partial f}{\partial y}(xy-1)}{(xy-1)^2}$$

$$= \frac{(xy-1)(1) - (x+y)(x-0)}{(xy-1)^2}$$

$$= \frac{(\cancel{xy}-1) - (\cancel{x^2}}{(xy-1)^2} = \frac{1}{(xy-1)^2}$$

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$$\frac{(xy-1) - (x^2+2y)}{(xy-1)^2} = \frac{\cancel{xy}-1-x^2 \cancel{-xy}}{(xy-1)^2}$$

$$= \frac{-1-x^2}{(xy-1)^2} = \frac{-(x^2+1)}{(xy-1)^2}$$

$$\frac{\partial f}{\partial x} = \frac{(xy-1)\frac{\partial f}{\partial x}(x+y) - (x+y)\frac{\partial f}{\partial x}(xy-1)}{(xy-1)^2}$$

$$= \frac{(xy-1)(1) - (x+y)(y-0)}{(xy-1)^2}$$

$$= \frac{\cancel{xy}-1-\cancel{xy}-y^2}{(xy-1)^2}$$

$$= \frac{-1-y^2}{(xy-1)^2} = \frac{-(y^2+1)}{(xy-1)^2}$$

Q#12 $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$

$$\frac{\partial f}{\partial y} = \frac{1}{1+(y/x)^2} \cdot \frac{\partial f}{\partial y}\left(\frac{y}{x}\right) \qquad \therefore \frac{1}{1+x^2}$$

$$= \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x}(1)$$

$$= \frac{1}{x\left(1+\left(\frac{y}{x}\right)^2\right)} = \frac{1}{x+\frac{y^2}{x^2}\cdot x}$$

$$= \frac{1}{x+\frac{y^2}{x}} = \frac{1}{\frac{x^2+y^2}{x}} = \frac{x}{x^2+y^2}$$

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$$\frac{\partial f}{\partial x} = \frac{1}{1+(y/x)^2} \frac{\partial f}{\partial x} y/x$$

$$= \frac{1}{1+(y/x)^2} \cdot \frac{\partial f}{\partial x} y(x)^{-1}$$

$$= \frac{1}{1+(y/x)^2} \cdot y(-1x^{-2})$$

$$= \frac{-y}{x^2(1+(y/x)^2)}$$

$$= \frac{-y}{x^2 + \frac{y^2}{\cancel{x^2}} \cdot \cancel{x^2}}$$

$$\frac{\partial f}{\partial x} = -\frac{y}{x^2+y^2}$$

Q#13 $f(x,y) = e^{(x+y+1)}$

$$\frac{\partial f}{\partial y} = e^{(x+y+1)} \frac{\partial f}{\partial y}(x+y+1)$$

$$= e^{(x+y+1)} \cdot (1) = e^{(x+y+1)}$$

$$\frac{\partial f}{\partial x} = e^{(x+y+1)} \frac{\partial f}{\partial y}(x+y+1)$$

$$= e^{(x+y+1)} (1) = e^{(x+y+1)}$$

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Q#14 $f(x,y) = e^{-x}\sin(x+y)$

$$\frac{\partial f}{\partial y} = e^{-x}\frac{\partial f}{\partial y}\left(\sin(x+y)\right) + \sin(x+y)\frac{\partial f}{\partial y}(e^{-x})$$

$$= e^{-x}\left(\cos(x+y)\cdot(1)\right) + \sin(x+y)(e^{-x}\cdot 0)$$

$$= e^{-x}\cos(x+y)$$

$$\frac{\partial f}{\partial x} = e^{-x}\frac{\partial f}{\partial x}\left(\sin(x+y)\right) + \sin(x+y)\frac{\partial f}{\partial x}(e^{-x})$$

$$= e^{-x}\cos(x+y) + \sin(x+y)(-e^{-x})$$

$$= e^{x}\cos(x+y) - e^{-x}\sin(x+y)$$

Q#15 $f(x,y) = \ln(x+y)$

$$\frac{\partial f}{\partial y} = \frac{1}{x+y}\cdot(1)$$

$$= \frac{1}{x+y}$$

$$\frac{\partial f}{\partial x} = \frac{1}{x+y}(1)$$

$$= \frac{1}{x+y}$$

Q#16 $f(x,y) = e^{xy}\ln y$

$$\frac{\partial f}{\partial y} = e^{xy}\frac{\partial f}{\partial y}\ln y + \ln y\frac{\partial f}{\partial y}e^{xy}$$

$$= e^{xy}\left(\frac{1}{y}\right) + \ln y\left(e^{xy}\cdot x\right)$$

$$= \frac{e^{xy}}{y} + x e^{xy}\ln y$$

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$$\frac{\partial f}{\partial x} = e^{xy}\frac{\partial f}{\partial x}\ln y + \ln y\frac{\partial f}{\partial x}e^{xy}$$

$$= e^{xy}(0) + \ln y(e^{xy}.y)$$

$$= y e^{xy}\ln y$$

Q#17 $f(x,y) = \sin^2(x-3y)$

$$\frac{\partial f}{\partial y} = \sin(x-3y)^2$$

$$= 2\sin(x-3y)\frac{\partial f}{\partial y}\sin(x-3y)$$

$$= 2\sin(x-3y)\cos(x-3y)\frac{\partial f}{\partial y}x-3y$$

$$= 2\sin(x-3y)\cos(x-3y)(0-3)$$

$$= -6\sin(x-3y)\cos(x-3y)$$

$$\frac{\partial f}{\partial x} = 2\sin(x-3y)\frac{\partial f}{\partial x}\sin(x-3y)$$

$$= 2\sin(x-3y)\cos(x-3y)\frac{\partial f}{\partial x}(x-3y)$$

$$= 2\cancel{1}\sin(x-3y)\cos(x-3y)$$

Q#18 $f(x,y) = \cos^2(3x-y^2)$

$$\frac{\partial f}{\partial y} = 2\cos(3x-y^2)\frac{\partial f}{\partial y}\cos(3x-y^2)$$

$$= 2\cos(3x-y^2).-\sin(3x-y^2)\frac{\partial f}{\partial y}(3x-y^2)$$

$$= -2\cos(3x-y^2)\sin(3x-y^2)(0-2y)$$

$$= 4y\cos(3x-y^2)\sin(3x-y^2)$$

$$\frac{\partial f}{\partial x} = 2\cos(3x-y^2)\frac{\partial f}{\partial x}\cos(3x-y^2)$$

$$= 2\cos(3x-y^2).-\sin(3x-y^2)\frac{\partial f}{\partial x}(3x-y^2)$$

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$$= -2\cos(3x-y^2)\sin(3x-y^2)(3-0)$$

$$= -6\cos(3x-y^2)\sin(3x-y^2)$$

Q#22 $f(x,y) = \sum_{n=0}^{\infty} (xy)^n \quad (|xy| < 1)$

$(xy)^0 + (xy)^1 +$
$(xy)^2 + (xy)^3$
$1 + xy + x^2y^2 +$
$x^3y^3.$

$$f(x,y) = \frac{1}{1-xy}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial y}(1-xy)^{-1}$$

$$= \frac{-1}{(1-xy)^2} \cdot \frac{\partial f}{\partial y}(1-xy)$$

$$= \frac{x}{(1-xy)^2}$$

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$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x}(1-xy)^{-1}$$

$$= -1(1-xy)^{-2}\frac{\partial f}{\partial x}(1-xy)$$

$$= \frac{-1}{(1-xy)^2}(0-y(1))$$

$$= \frac{-1}{(1-xy)^2}(-y)$$

$$= \frac{y}{(1-xy)^2}$$

Q#22-34

Q#23 $f(x,y,z) = 1 + xy^2 - 2z^2$

$$f_x = \frac{\partial f}{\partial x} = 0 + y^2 - 0 = y^2$$

$$f_y = \frac{\partial f}{\partial y} = 0 + x(2y) - 0 = 2xy$$

$$f_z = \frac{\partial f}{\partial z} = 0 + 0 - 2(2z) = -4z$$

Q#24 $f(x,y,z) = xy + yz + xz$

$$f_x = \frac{\partial f}{\partial x} = y(1) + 0 + z(1) = y + z$$

$f_y = \frac{\partial f}{\partial y} = xy + yz + xz$
$= x(1) + z(1) + 0 = x + z$

$f_z = \frac{\partial f}{\partial z} = 0 + y(1) + x(1) = x + y$

Q#25 $f(x, y, z) = x - \sqrt{y^2 + z^2}$

$f_x = \frac{\partial f}{\partial x} = 1 - \frac{1}{2\sqrt{y^2+z^2}} \cdot \frac{\partial f}{\partial x}(y^2+z^2)$

$= 1 - \frac{1}{2\sqrt{y^2+z^2}}(0+0)$

$= 1$

$f_y = \frac{\partial f}{\partial y} = x - \sqrt{y^2+z^2}$

$= 0 - \frac{1}{2\sqrt{y^2+z^2}} \frac{\partial f}{\partial y}(y^2+z^2)$

$= -\frac{1}{\cancel{2}\sqrt{y^2+z^2}}(\cancel{2}y + 0)$

$= -\frac{y}{\sqrt{y^2+z^2}}$

$f_z = \frac{\partial f}{\partial z} = 0 - \frac{1}{2\sqrt{y^2+z^2}} \frac{\partial f}{\partial z}(y^2+z^2)$

$= -\frac{1}{\cancel{2}\sqrt{y^2+z^2}}(0+\cancel{2}z) = -\frac{z}{\sqrt{y^2+z^2}}$

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Q#26 $f(x,y,z) = (x^2+y^2+z^2)^{-1/2}$

$-\frac{1}{2}-1$
$\frac{-1-2}{2}$
$-\frac{3}{2}$

$f_x = \frac{\partial f}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} \frac{\partial f}{\partial x}(x^2+y^2+z^2)$

$= -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x+0+0)$

$= -x(x^2+y^2+z^2)^{-3/2}$

$f_y = \frac{\partial f}{\partial y} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} \frac{\partial f}{\partial y}(x^2+y^2+z^2)$

$= -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2} \cdot (0+\cancel{2}y+\cancel{0})$

$= -y(x^2+y^2+z^2)^{-3/2}$

$f_z = \frac{\partial f}{\partial z} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} \frac{\partial f}{\partial z}(x^2+y^2+z^2)$

$= -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(0+\cancel{0}+\cancel{2}z)$

$= -z(x^2+y^2+z^2)^{-3/2}$

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Q#27 $f(x,y,z) = \sin^{-1}(xyz)$ $\qquad \sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$$f_x = \frac{\partial f}{\partial x} = \frac{1}{\sqrt{1-(xyz)^2}} \frac{\partial f}{\partial x} xyz$$

$$= \frac{1}{\sqrt{1-x^2y^2z^2}} yz = \frac{yz}{\sqrt{1-x^2y^2z^2}}$$

$$f_y = \frac{\partial f}{\partial y} = \frac{1}{\sqrt{1-(xyz)^2}} \frac{\partial f}{\partial y}(xyz)$$

$$= \frac{1}{\sqrt{1-x^2y^2z^2}}(xz) = \frac{xz}{\sqrt{1-x^2y^2z^2}}$$

$$f_z = \frac{\partial f}{\partial z} = \frac{1}{\sqrt{1-(xyz)^2}} \frac{\partial f}{\partial z}(xyz)$$

$$= \frac{1}{\sqrt{1-x^2y^2z^2}} xy = \frac{xy}{\sqrt{1-x^2y^2z^2}}$$

Q#28 $f(x,y,z) = \sec^{-1}(x+yz)$

$$f_x = \frac{\partial f}{\partial x} = \frac{1}{x+yz\sqrt{(x+yz)^2-1}} \frac{\partial f}{\partial x}(x+yz) \qquad \sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}dx$$

$$= \frac{1}{x+yz\sqrt{(x+yz)^2-1}}(1) = \frac{1}{x+yz\sqrt{(x+yz)^2-1}}$$

$$f_y = \frac{\partial f}{\partial y} = \frac{1}{x+yz\sqrt{1+(x+yz)^2}} \frac{\partial f}{\partial y}(x+yz)$$

$$= \frac{1}{x+yz\sqrt{1+(x+yz)^2}} \cdot (0+z) = \frac{z}{x+yz\sqrt{1+(x+yz)^2}}$$

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$$f_z = \frac{1}{x+yz\sqrt{1+(x+yz)^2}} \frac{\partial f}{\partial z}(x+yz)$$

$$= \frac{1}{x+yz\sqrt{1+(x+yz)^2}}(0+y) = \frac{y}{(x+yz)\sqrt{1+(x+yz)^2}}$$

Q#29 $f(x,y,z) = \ln(x+2y+3z)$

$$f_x = \frac{\partial f}{\partial x} = \frac{1}{\ln(x+2y+3z)} \cdot \frac{\partial f}{\partial x}(x+2y+3z)$$

$$= \frac{1}{\ln(x+2y+3z)}(1+0+0) = \frac{1}{\ln(x+2y+3z)}$$

$$f_y = \frac{\partial f}{\partial y} = \frac{1}{\ln(x+2y+3z)} \frac{\partial f}{\partial y}(x+2y+3z)$$

$$= \frac{1}{\ln(x+2y+3z)}(0+2+0) = \frac{2}{\ln(x+2y+3z)}$$

$$f_z = \frac{\partial f}{\partial z} = \frac{1}{\ln(x+2y+3z)} \frac{\partial f}{\partial z}(x+2y+3z)$$

$$= \frac{1}{\ln(x+2y+3z)}(0+0+3)$$

$$= \frac{3}{\ln(x+2y+3z)}$$

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Q#30 $f(x,y,z) = yz \ln(xy)$

$f_x = \frac{\partial f}{\partial x} = yz \frac{\partial f}{\partial x}(\ln xy) + \ln(xy)\frac{\partial f}{\partial x}(yz)$

$= yz \frac{1}{xy} \frac{\partial f}{\partial x}(xy) + \ln(xy)(0)$

$= yz \frac{y}{xy} = \frac{yz}{x}$

$f_y = \frac{\partial f}{\partial y} = yz \frac{1}{xy} \frac{\partial f}{\partial y}(xy) + \ln(xy) \frac{\partial f}{\partial y}(yz)$

$= \frac{z}{x} \cdot x + \ln(xy)(z)$

$= z + z\ln(xy)$

$f_z = \frac{\partial f}{\partial z} = yz \frac{1}{xy} \frac{\partial f}{\partial z}(xy) + \ln(xy)\frac{\partial f}{\partial z}(yz)$

$= \frac{z}{x}(0) + \ln(xy)(y)$

$= y\ln(xy)$

Q#31 $f(x,y,z) = e^{-(x^2+y^2+z^2)}$

$f_x = \frac{\partial f}{\partial x} = e^{-(x^2+y^2+z^2)} \frac{\partial f}{\partial x}(-x^2-y^2-z^2)$

$= e^{-(x^2+y^2+z^2)}(-2x-0-0)$

$= -2x\, e^{-(x^2+y^2+z^2)}$

$f_y = \frac{\partial f}{\partial y} = e^{-(x^2+y^2+z^2)} \frac{\partial f}{\partial y}(-x^2-y^2-z^2)$

$= e^{-(x^2+y^2+z^2)}(-0-2y-0)$

$= -2y\, e^{-(x^2+y^2+z^2)}$

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$$f_z = e^{-(x^2+y^2+z^2)} \frac{\partial f}{\partial z}(-x^2-y^2-z^2)$$
$$= e^{-(x^2+y^2+z^2)}(-0-0-2z)$$
$$= -2z\, e^{-(x^2+y^2+z^2)}$$

Q#32 $f(x,y,z) = e^{-xyz}$

$$f_x = \frac{\partial f}{\partial x} = e^{-xyz} \frac{\partial f}{\partial x}(-xyz)$$
$$= e^{-xyz}(-yz) = -yz\, e^{-xyz}$$

$$f_y = \frac{\partial f}{\partial y} = e^{-xyz} \frac{\partial f}{\partial y}(-xyz)$$
$$= e^{-xyz}(-xz) = -xz\, e^{-xyz}$$

$$f_z = \frac{\partial f}{\partial z} = e^{-xyz} \frac{\partial f}{\partial z}(-xyz)$$
$$= e^{-xyz}(-xy) = -xy\, e^{-xyz}$$

Q#33 $f(x,y,z) = \tanh(x+2y+3z)$

$$f_x = \frac{\partial f}{\partial x} = \operatorname{sech}^2(x+2y+3z) \frac{\partial f}{\partial x}(x+2y+3z)$$ $\tanh x = \operatorname{sech}^2 x$
$$= \operatorname{sech}^2(x+2y+3z)$$

$$f_y = \frac{\partial f}{\partial y} = \operatorname{sech}^2(x+2y+3z) \frac{\partial f}{\partial y}(x+2y+3z)$$
$$= \operatorname{sech}^2(x+2y+3z)(2)$$
$$= 2\operatorname{sech}^2(x+2y+3z)$$

$$f_z = \frac{\partial f}{\partial z} = \operatorname{sech}^2(x+2y+3z) \frac{\partial f}{\partial z}(x+2y+3z)$$
$$= \operatorname{sech}^2(x+2y+3z)(3)$$
$$= 3\operatorname{sech}^2(x+2y+3z)$$

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Q#34 $\sinh(xy - z^2)$ $\qquad \sinh x = \cosh x$

$$f_x = \frac{\partial f}{\partial x} = \cosh(xy - z^2)\,\frac{\partial f}{\partial x}(xy - z^2)$$

$$= \cosh(xy - z^2).(y(1) - 0)$$

$$= y\cosh(xy - z^2)$$

$$f_y = \frac{\partial f}{\partial y} = \cosh(xy - z^2)\,\frac{\partial f}{\partial y}(xy - z^2)$$

$$= \cosh(xy - z^2).(x(1) - 0)$$

$$= x\cosh(xy - z^2)$$

$$f_z = \frac{\partial f}{\partial z} = \cosh(xy - z^2)\,\frac{\partial f}{\partial z}(xy - z^2)$$

$$= \cosh(xy - z^2).(0 - 2z)$$

$$= -2z\cosh(xy - z^2)$$

Ex # 14.4

Q#6 $w = z - \sin xy \qquad x = t,\ z = e^{t-1},\ t = 1,\ y = \ln t$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$\frac{dx}{dt} = \frac{dt}{dt} = 1$$

$$\frac{dy}{dt} = \frac{d}{dt}(\ln t) = \frac{1}{t}$$

$$\frac{dz}{dt} = \frac{d}{dt}(e^{t-1}) = e^{t-1}$$

$$\frac{\partial w}{\partial x} = z - \sin xy$$

$$= 0 - \cos xy\,\frac{\partial w}{\partial x}xy$$

$$= -y\cos xy$$

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$$\frac{\partial w}{\partial y} = z - \sin xy$$
$$= 0 - \cos xy\,(x)$$
$$= -x\cos xy$$

$$\frac{\partial w}{\partial z} = 1 - 0$$
$$= 1$$

sub in ⇒ $(-y\cos xy)(1) + (-x\cos xy)\left(\frac{1}{t}\right) + (1)(e^{t-1})$

$-\ln t\cos(t)(\ln t) + -\cancel{t}\cos(t)(\ln t)\frac{1}{\cancel{t}} + e^{t-1}$

$-\ln t\cos t\ln t - \cos t\ln t + e^{t-1}$

$-(\ln(1))\cos(1)(\ln 1) - \cos(1)(\ln 1) + e^{1-1}$

~~$-45\cos 45 - \cos 45 + e^{0}$~~

~~$-0 = 1+$~~

$= -(0)(\cos(1)(0)) - \cos(1)(0) + e^{0}$

$= 0 - 1 + 1$

$= 0$

Q#7 $z = 4e^{x}\ln y$, $x = \ln(u\cos v)$,
$y = u\sin v$, $(u, v) = (2, \pi/4)$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} \quad \text{—— (1)}$$

$$\frac{\partial z}{\partial x} = 4e^{x}\ln y$$
$$= 4\left[e^{x}\frac{\partial}{\partial x}\ln y + \ln y\frac{\partial}{\partial x}e^{x}\right]$$
$$= 4\left[e^{x}(0) + \ln y\, e^{x}\right]$$
$$= 4e^{x}\ln y$$

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$$\frac{\partial x}{\partial u} = \frac{\partial}{\partial u}\left(\ln(u\cos v)\right)$$

$$= \frac{1}{u\cos v}\frac{\partial}{\partial u} u\cos v$$

$$= \frac{1}{u\cos v}\cos v = \frac{1}{u} \quad \frac{1}{u\cos v}\cos v = \frac{1}{u}$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(4e^{x}\ln y)$$

$$= 4e^{x}\frac{1}{y} = \frac{4e^{x}}{y}$$

$$\frac{\partial y}{\partial u} = \frac{\partial}{\partial u}(u\sin v)$$

$$= \sin v$$

eq (i) $\Rightarrow \frac{\partial z}{\partial u} = (4e^{x}\ln y)\left(\frac{1}{\cos v}\right) + \left(\frac{4e^{x}}{y}\right)(\sin v)$

$$= (4e^{\ln u\cos v}\ln u\sin v)\left(\frac{1}{\cos v}\right) + \frac{4e^{\ln u\cos v}}{u\sin v}(\sin v)$$

$$= \frac{4u\cos v\ln u\sin v}{\cos v} + \frac{4u\cos v\sin v}{u\sin v}$$

$$= \boxed{4\cos v\ln u\sin v + 4\cos v} \text{ ——— (i)}$$

$$= 4\cos\frac{\pi}{4}\ln(2)\left(\frac{\pi}{4}\right)\sin\frac{\pi}{4} + 4\cos\frac{\pi}{4}$$

$$= 4\frac{\sqrt{2}}{2}\ln 2\frac{\sqrt{2}}{2} + 4\frac{\sqrt{2}}{2}$$

$$= 2\sqrt{2}\ln\sqrt{2} + 2\sqrt{2}$$

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$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} \quad \text{--- ii}$$

$$\frac{\partial x}{\partial v} = \frac{\partial}{\partial v}(\ln u\cos v)$$

$$= \frac{1}{u\cos v}\frac{\partial}{\partial v} u\cos v$$

$$= \frac{1}{\cancel{u}\cos v}\,\cancel{u} - \sin v$$

$$= -\frac{\sin v}{\cos v} = -\tan v$$

$$\frac{\partial y}{\partial v} = \frac{\partial}{\partial v}(u\sin v)$$

$$= u\cos v \,.$$

$$\Rightarrow \frac{\partial z}{\partial v} = (4e^{x}\ln y)(-\tan v) + \left(\frac{4e^{x}}{y}\right)(u\cos v)$$

$$\ln u\sin v$$

$$\frac{\partial z}{\partial v} = (4e^{\ln u\cos v})(-\tan v) + \frac{4e^{\ln u\cos v}}{u\sin v}(u\cos v)$$

$$\cancel{\ln u\sin v}$$

$$= (4u\cos v)(-\tan v) + 4\frac{\cancel{u}\cos v}{\cancel{u}\sin v}(u\cos v)$$

$$\cancel{\ln u\sin v}$$

$$= -4u\cos v\tan v + 4u\cos v\tan v$$

$$= -4u\cos v\tan v\ln u\sin v + 4u\cos v\tan v$$

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$$z = 4e^{x}\ln y$$

$$= 4e^{\ln(u\cos v)}\ln u\sin v$$

$$= 4u\cos v\ln u\sin v$$

$$\frac{\partial z}{\partial v} = 4\left[u\cos v\frac{\partial}{\partial v}\ln u\sin v + \ln u\sin v\frac{\partial}{\partial v}(u\cos v)\right]$$

$$= 4\left[u\cos v\frac{\sin v}{u\sin v} + \ln u\sin v\cdot\cos v\right]$$

$$= 4\cos v + 4\cos v\ln\sin v$$

$$\frac{\partial z}{\partial v} = 4u\cos v\ln u\sin v$$

$$= 4\left[u\cos v\frac{\partial}{\partial v}\ln u\sin v + \ln u\sin v\frac{\partial}{\partial v}u\cos v\right]$$

$$= 4\left[u\cos v\frac{1}{u\sin v}u\cos v + \ln u\sin v\, u\,\bar{}\sin v\right]$$

$$= 4u\tan v\cos v - 4u\sin v\ln u\sin v$$

$$\frac{\partial z}{\partial v} = 4\cos\frac{\pi}{4} + 4\cos\frac{\pi}{4}\ln(2)\sin\frac{\pi}{4}$$

$$\cancel{= 4\left(\frac{\sqrt{2}}{2}\right) + 4\left(\frac{\sqrt{2}}{2}\right)\ln\frac{\pi}{2}}$$

$$\cancel{= 2\sqrt{2} + 2\sqrt{2}\ln\sqrt{2}}$$

$$\cancel{= \sqrt{2}(2 + 2\ln 2}$$

$$= 4\frac{\sqrt{2}}{2} + 4\frac{\sqrt{2}}{2}\ln 2\frac{\sqrt{2}}{2}$$

$$= 2\sqrt{2} + 2\sqrt{2}\ln\sqrt{2}$$

$$= 2\sqrt{2} + 2\sqrt{2}\frac{1}{2}\ln 2$$

$$= 2\sqrt{2} + \sqrt{2}\ln 2$$

$$= \sqrt{2}(2 + \ln 2)$$

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$$\frac{\partial z}{\partial v} = 4u\tan v\cos v - 4u\sin v\,\cancel{\cos v}\ln u\sin v$$

$$= 4(2)\tan\frac{\pi}{4}\cos\frac{\pi}{4} - 4\cancel{0}(2)\sin\frac{\pi}{4}\ln 2\sin\frac{\pi}{4}$$

$$= 8(1)\left(\frac{\sqrt{2}}{2}\right) - 8\left(\frac{\sqrt{2}}{2}\right)\ln \cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}}$$

$$= 4\sqrt{2} - 4\sqrt{2}\ln\sqrt{2}$$

$$= 4\sqrt{2} - \cancel{4}^{2}\sqrt{2}\,\frac{1}{\cancel{2}}\ln 2$$

$$= 4\sqrt{2} - 2\sqrt{2}\ln 2$$

$$= \sqrt{2}\,(4 - 2\ln 2)$$

Q#8 $z = \tan^{-1}(x/y)$, $x = u\cos v$, $y = u\sin v$

$(u, v) = (1.3, \pi/6)$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} \quad \text{--- (i)}$$

$$\frac{\partial z}{\partial x} = \tan^{-1}(x/y)$$

$$= \frac{1}{1+(x/y)^2}\,\frac{\partial z}{\partial x}\,\frac{x}{y}$$

$$= \frac{1}{y\left(1+\frac{x^2}{y^2}\right)}$$

$$\frac{\partial z}{\partial y} = \tan^{-1}(x/y)$$

$$= \frac{1}{1+(x/y)^2}\,\frac{\partial z}{\partial y}\,\frac{x}{y}$$

$$\frac{\partial z}{\partial y}\,y^{-1}$$

$$-1\,y^{-2}$$

$$= \frac{1}{1+\left(\frac{x}{y}\right)^2}\,x\frac{\partial}{\partial y}(y)^{-1}\;-\frac{1}{y^2}$$

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$$= -\frac{x}{y^2\left(1+\frac{x^2}{y^2}\right)}$$

$$\frac{\partial x}{\partial u} = \frac{\partial}{\partial u} u\cos v$$

$$\frac{\partial x}{\partial u} = \cos v$$

$$\frac{\partial y}{\partial u} = \frac{\partial}{\partial u} u\sin v$$

$$= \sin v$$

eq (i) $\Rightarrow$ $$\frac{\partial z}{\partial u} = \left[\frac{y}{y^2\left(1+\frac{x^2}{y^2}\right)}\right]\left(\cos v\right) + \left(\frac{-x}{y^2\left(1+\frac{x^2}{y^2}\right)}\right)(\sin v)$$

$$= \frac{y\cos v}{y^2+x^2} - \frac{x\sin v}{y^2+x^2}$$

$$= \cancel{u\sin v\cos v}$$

$$= \frac{y\cos v - x\sin v}{y^2+x^2}$$

$$= \frac{\cancel{u\sin v\cos v} - \cancel{u\cos v\sin v}}{(u\sin v)^2+(u\cos v)^2} = \frac{0}{}$$

$$\boxed{\frac{\partial z}{\partial u} = 0}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} \quad \text{—— (ii)}$$

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$$\frac{\partial x}{\partial v} = u\cos v$$
$$= -u\sin v$$

$$\frac{\partial y}{\partial v} = u\sin v$$
$$= u\cos v$$

eq (i) $\Rightarrow$ $$\frac{\partial z}{\partial v} = \left(\frac{1}{y\left(1+\frac{x^2}{y^2}\right)}\right)(-u\sin v) + \left(\frac{-x}{y^2\left(1+\frac{x^2}{y^2}\right)}\right)(u\cos v)$$

$$= \frac{y}{y^2+\frac{x^2}{y^2}\times y^2}(-u\sin v) + -\frac{x}{y^2+x^2}(u\cos v)$$

$$= \frac{-y\,u\sin v}{y^2+x^2} - \frac{x\,u\cos v}{y^2+x^2}$$

$$= \frac{-y\,u\sin v - x\,u\cos v}{y^2+x^2}$$

$$= \frac{-(u\sin v)(u\sin v) - (u\cos v)(u\cos v)}{(u\sin v)^2+(u\cos v)^2}$$

$$= \frac{-\left[\cancel{(u\sin v)^2+(u\cos v)^2}\right]}{\cancel{(u\sin v)^2+(u\cos v)^2}}$$

$$\boxed{\frac{\partial z}{\partial v} = -1}$$

$$\cancel{\frac{\partial z}{\partial v} = \tan^{-1}\left(\frac{u\sin v}{u\cos v}\right)}$$

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$$\frac{\partial z}{\partial u} = \tan^{-1}\left(\frac{x}{y}\right) = \tan^{-1}\left(\frac{\cancel{u}\cos v}{\cancel{u}\sin v}\right)$$

$$\frac{\partial z}{\partial u} = \tan^{-1}(\cot v)$$

$$= \frac{1}{1+\cot v^2}\cancel{\tan^{-1}\cot v}\left(\frac{\partial z}{\partial u}\cot v\right)$$

$$\frac{1}{1+\cot^2 v}\cancel{\tan^{-1}(\cot v)}(0)$$

$$\boxed{\frac{\partial z}{\partial u} = 0}$$

$$\frac{\partial z}{\partial v} = \cancel{\tan^{-1}\left(\frac{\cos v}{\sin v}\right)}$$

$$= \cancel{\tan^{-1}(\cot v)\ \frac{\partial z}{\partial v}\cot v}$$

$$= \cancel{\tan^{-1}\cot v}$$

$$\frac{\partial z}{\partial v} = \tan^{-1}(\cot v)$$

$$= \frac{1}{1+\cot^2 v}\frac{\partial z}{\partial v}\cot v$$

$$= \frac{1}{1+\cot^2 v}(-\operatorname{cosec}^2 v)$$

$$= \frac{1}{1+\cot^2 v}\left(-\frac{1}{\sin^2 v}\right)$$

$$= -\frac{1}{1+\frac{\cos^2 v}{\sin^2 v}}\frac{1}{\sin^2 v}$$

$$= -\frac{1}{\sin^2 v+\cos^2 v} = \frac{-1}{1} = -1 \qquad \because \cos^2\theta+\sin^2\theta =$$

$$\boxed{\frac{\partial z}{\partial v} = -1}$$

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$$\frac{\partial z}{\partial u} = 0$$

$$\frac{\partial z}{\partial v} = -1$$

Q#9 $w = xy + yz + xz$, $x = u+v$, $y = u-v$, $z = uv$

$(u, v) = (\frac{1}{2}, 1)$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u} \quad \text{—(i)}$$

$$\frac{\partial w}{\partial x} = y + 0 + z = y + z$$

$$\frac{\partial w}{\partial y} = x + z + 0 = x + z$$

$$\frac{\partial w}{\partial z} = 0 + y + x = x + y$$

$$\frac{\partial x}{\partial u} = u + v = 1$$

$$\frac{\partial y}{\partial u} = u - v = 1.$$

$$\frac{\partial z}{\partial u} = v$$

Eq (i) $\Rightarrow \frac{\partial w}{\partial u} = (y+z)(1) + (x+z)(1) + (x+y)(v)$

$= y + z + x + z + vx + vy$

$= u - \cancel{v} + uv + u + \cancel{v} + uv + v(u+v) + v(u-v)$

$= 2u + 2uv + uv + v^2 + uv - v^2$

$$\boxed{\frac{\partial w}{\partial u} = 2u + 4uv}$$

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$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v} \quad \text{— (ii)}$$

$$\frac{\partial x}{\partial v} = u+v = 1$$

$$\frac{\partial y}{\partial v} = u-v = -1$$

$$\frac{\partial z}{\partial v} = u$$

Eq. (ii) $\Rightarrow$ $\frac{\partial w}{\partial v} = (y+z)(1) + (x+z)(-1) + (x+y)(u)$

$$= y + \cancel{z} - x - \cancel{z} + xu + yu$$

$$= y - x + xu + yu$$

$$= u - v - (u+v) + (u+v)u + (u-v)u$$

$$= \cancel{u} - v - \cancel{u} - v + u^2 + \cancel{uv} + u^2 - \cancel{uv}$$

$$\boxed{\frac{\partial w}{\partial v} = -2v + 2u^2}$$

$$\cancel{\frac{\partial w}{\partial v}} = xy + yz + xz$$

$$= (u+v)(u-v) + (u-v)(uv) + (u+v)uv$$

$$= u^2 - v^2 + u^2v - \cancel{v^2} + u^2v + \cancel{v^2}$$

$$= u^2 + 2u^2v - v^2$$

$$\frac{\partial w}{\partial u} = 2u + 2v(2u) - 0$$

$$\boxed{\frac{\partial w}{\partial u} = 2u + 4uv} \quad \text{— (I)}$$

$$\frac{\partial w}{\partial v} = 0 + 2u^2 - 2v$$

$$\boxed{\frac{\partial w}{\partial v} = -2v + 2u^2} \quad \text{— (II)}$$

eq (I) $\Rightarrow$ $\frac{\partial w}{\partial u} = 2\left(\frac{1}{2}\right) + 4\left(\frac{1}{2}\right)^2(1)$

$= 1 + 2$

$= 3$

$\frac{\partial w}{\partial v} = -2(1) + 2\left(\frac{1}{2}\right)^2$

$= -2 + 2\left(\frac{1}{4}\right)$

$= -2 + \frac{1}{2}$

$= \frac{-4+1}{2} = \frac{-3}{2}$

Q#10 $w = \ln(x^2+y^2+z^2)$, $x = ue^v \sin u$, $y = ue^v \cos u$

$z = ue^v$, $(u,v) = (-2,0)$

$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$ — (1)

$\frac{\partial w}{\partial x} = \ln(x^2+y^2+z^2)$

$= \frac{1}{x^2+y^2+z^2}\frac{\partial w}{\partial x}(x^2+y^2+z^2)$

$= \frac{2x}{x^2+y^2+z^2}$

$\frac{\partial w}{\partial y} = \frac{2y}{x^2+y^2+z^2}$

$\frac{\partial w}{\partial z} = \frac{2z}{x^2+y^2+z^2}$

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$$\cancel{\frac{\partial x}{\partial u} = ue^{v}\sin v = e^{v}\sin v}$$

$$\cancel{\frac{\partial y}{\partial u} = e^{v}\cos v\; ue^{v}\cos v = -e^{v}\sin u}$$

$$\cancel{\frac{\partial z}{\partial u} = e^{v}}$$

$$\frac{\partial x}{\partial u} = ue^{v}\sin u$$

$$= ue^{v}\frac{\partial x}{\partial u}\sin u + \sin u\frac{\partial x}{\partial u}ue^{v}$$

$$= ue^{v}\cos u + \sin u\, e^{v}$$

$$= ue^{v}\cos u + e^{v}\sin u$$

$$\frac{\partial y}{\partial u} = ue^{v}\cos u$$

$$= ue^{v}\frac{\partial y}{\partial u}\cos u + \cos u\frac{\partial y}{\partial u}ue^{v}$$

$$= -ue^{v}\sin u + e^{v}\cos u$$

$$e^{v}\cos u - ue^{v}\sin u$$

$$\frac{\partial z}{\partial u} = ue^{v} = e^{v}$$

eq(1) $\Rightarrow \left(\frac{2x}{x^2+y^2+z^2}\right)(e^{v}\sin u + ue^{v}\cos u) + \left(\frac{2y}{x^2+y^2+z^2}\right)(e^{v}\cos u - ue^{v}\sin u) + \left(\frac{2z}{x^2+y^2+z^2}\right)(e^{v})$

$$= \frac{\cancel{2(ue} \; 2ue^{v}\sin u}{(ue^{v}\sin u)^2 + (ue^{v}\cos u)^2 + (ue^{v})^2}(e^{v}\sin u + ue^{v}\cos u) + \frac{2(ue^{v}\cos u)}{(ue^{v}\sin u)^2 + (ue^{v}\cos u)^2 + (ue^{v})^2}(e^{v}\cos u - ue^{v}\sin u) +$$

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$$\frac{2ue^{v}}{(ue^{v}\sin u)^2+(ue^{v}\cos u)^2+(e^{v})^2} \quad (e^{v})$$

$$= \frac{2u(e^{v})^2(\sin u)^2 + 2u^2(e^{v})^2\sin u\cos u}{(ue^{v}\sin u)^2+(ue^{v}\cos u)^2+(e^{v})^2} +$$

$$\frac{2u(e^{v})^2(\cos u)^2 - 2u^2(e^{v})^2\sin u\cos u}{(ue^{v}\sin u)^2+(ue^{v}\cos u)^2+(e^{v})^2} +$$

$$= \frac{2u(e^{v})^2}{(ue^{v}\sin u)^2+(ue^{v}\cos u)^2+(e^{v})^2}$$

$$= \frac{2ue^{v^2}\sin^2 u + \cancel{2u^2e^{v^2}\sin u\cos u} + 2ue^{v^2}\cos^2 u + \cancel{2u^2e^{v^2}\sin u\cos u} + 2ue^{v^2}}{\cancel{ue^{v}\sin}\; u^2e^{v^2}\sin^2 u + u^2e^{v^2}\cos^2 u + u^2e^{v^2}}$$

$$= \frac{2ue^{v^2}\sin^2 u + 2ue^{v^2}\cos^2 u + 2ue^{v^2}}{e^{v^2}(u^2\sin^2 u + u^2\cos^2 u + u^2)}$$

$$= \frac{\cancel{e^{v^2}}(2u\sin^2 u + 2u\cos^2 u + 2u)}{\cancel{e^{v^2}}(u^2\sin^2 u + u^2\cos^2 u + u^2)}$$

$$= \frac{2u\,\cancel{(\sin^2 u + \cos^2 u + 1)}}{u^2\,\cancel{(\sin^2 u + \cos^2 u + 1)}}$$

$$\boxed{\frac{\partial w}{\partial u} = \frac{2}{u}}$$

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$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v} \quad \text{(ii)}$$

$$\frac{\partial x}{\partial v} = ue^v \sin u = u \sin u e^v$$

$$\frac{\partial y}{\partial v} = u \cos u e^v$$

$$\frac{\partial z}{\partial v} = ue^v$$

eq (ii) $\Rightarrow$

$$\frac{\partial w}{\partial v} = \frac{2x}{x^2+y^2+z^2}(u \sin u e^v) + \frac{2y}{x^2+y^2+z^2}(ue^v \cos u) + \frac{2z}{x^2+y^2+z^2}(ue^v)$$

$$= \frac{2(ue^v \sin u)(ue^v \sin u)}{(ue^v \sin u)^2+(ue^v \cos u)^2+(ue^v)^2} + \frac{2(ue^v \cos u)(ue^v \cos u)}{(ue^v \sin u)^2+(ue^v \cos v)^2+(ue^v)^2} + \frac{2(ue^v)(ue^v)}{(ue^v \sin u)^2+(ue^v \cos u)^2+(ue^v)^2}$$

$$= \frac{2u^2e^{v^2}\sin^2 u + 2u^2e^{v^2}\cos^2 u + 2u^2e^{v^2}}{u^2e^{v^2}\sin^2 u + u^2e^{v^2}\cos^2 u + u^2e^{v^2}}$$

$$= \frac{2\cancel{u^2e^{v^2}}(\cancel{\sin^2 u + \cos^2 u + 1})}{\cancel{u^2e^{v^2}}(\cancel{\sin^2 u + \cos^2 u + 1})}$$

$$\boxed{\frac{\partial w}{\partial v} = 2}$$

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$$z = \cancel{xy}\ln(x^2+y^2+z^2)$$

$$z = \ln(u^2e^{v^2}\sin^2 u + u^2e^{v^2}\cos^2 u + u^2e^{v^2})$$

$$\frac{\partial z}{\partial u} = \frac{1}{u^2e^{v^2}\sin^2 u + u^2e^{v^2}\cos^2 u + u^2e^{v^2}} \Big($$

$$(ue^v\sin u)^2 + (ue^v\cos u)^2 + (ue^v)^2$$

$$= \frac{(ue^v\sin u)}{2} \quad \frac{\partial z}{\partial u}(ue^v\sin u)$$

$$0e^v\sin u + ue^v\cos v$$

$$= 2\left(\frac{u^2e^{v^2}\sin^2 u + ue^v\cos u}{\cancel{2}}\right)$$

$$= \frac{ue^v\cos u}{2}(e^v\cos u - ue^v\sin u)$$

$$= 2\left(\frac{ue^{v^2}\cos^2 u - u^2e^{v^2}\cos u\sin u}{\cancel{4}}\right)$$

$$= 2\cancel{\phi}ue^{v^2}$$

$$\frac{u^2e^{v^2}\sin^2 u + u^2e^{v^2}\cancel{\cos u\sin u} + ue^{v^2}\cos^2 u - \cancel{u^2e^{v^2}\cos u\sin u} + ue^{v^2}}{(u^2e^{v^2}\sin^2 u + u^2e^{v^2}\cos^2 u + u^2e^{v^2})}$$

$$= \frac{2ue^{v^2}(\cancel{\sin^2 u + \cos^2 u + 1})}{u^2e^{v^2}(\cancel{\sin^2 u + \cos^2 u + 1})}$$

$$\boxed{\frac{\partial z}{\partial u} = \frac{2}{u}}$$

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$$\frac{\partial z}{\partial v} = \frac{1}{(ue^v \sin u)^2 + (ue^v \cos u)^2 + (ue^v)^2}$$

$$= \frac{ue^v \sin u + ue^v \cos v + ue^v}{u^2 e^{v^2} \sin^2 u + u^2 e^{v^2} \cos^2 u + u^2 e^{v^2}}$$

$$= ue^v(\sin u + \cos u + 1)$$

$$\frac{\partial z}{\partial v} = \frac{1}{(ue^v \sin u)^2 + (ue^v \cos u)^2 + (ue^v)^2}$$

$$\frac{\partial z}{\partial v} = \frac{2(ue^v \sin u)(ue^v \sin u) + 2(ue^v \cos u)(ue^v \cos u) + 2(ue^v)(ue^v)}{u^2 e^{v^2} \sin^2 u + u^2 e^{v^2} \cos^2 u + u^2 e^{v^2}}$$

$$= \frac{2u^2 e^{v^2} \sin^2 u + 2u^2 e^{v^2} \cos^2 u + 2u^2 e^{v^2}}{u^2 e^{v^2} \sin^2 u + u^2 e^{v^2} \cos^2 u + u^2 e^{v^2}}$$

$$\frac{\partial z}{\partial v} = 2$$

$$\frac{\partial z}{\partial u} = \frac{2}{u} = \frac{2}{-2} = -1$$

$$\frac{\partial z}{\partial v} = 2$$

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Ex # 14.4

Q#14

$$\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\,dx\,dy$$

$$= \int_0^1\int_0^1 \frac{1}{2}\, 2xy\,(x^2+y^2+1)^{-1/2}\,dx\,dy$$

$$= \int_0^1 \cancel{\frac{1}{2}}\, y\, \frac{(x^2+y^2+1)^{1/2}}{\cancel{1/2}}\Big|_0^1\,dy$$

$$= \int_0^1 y\sqrt{x^2+y^2+1}\Big|_0^1\,dy$$

$$= \int_0^1 y\sqrt{1+y^2+1} - y\sqrt{0+y^2+1}\,dy$$

$$= \int_0^1 y\sqrt{y^2+2} - y\sqrt{y^2+1}\,dy$$

$$= \frac{1}{2}\; 2y\sqrt{y^2+2} - 2y\sqrt{y^2+}\,dy$$

$$= \frac{1}{2}\;\frac{y^2+2}{3/\cancel{2}} - \frac{y^2+1}{3/\cancel{2}}\Big|_0^1 \cancel{dy}$$

$$= \frac{y^2+2}{3} - \frac{y^2+1}{3}\Big|_0^1$$

$$= \frac{y^2+2-y^2-1}{3}\Big|_0^1$$

$$= \frac{1}{3}\Big|^1 = \frac{1}{3}$$

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Ex # 14.1

Q#1 $\int_0^1\int_0^2 (x+3)\,dy\,dx$

$= \int_0^1 \left| xy + 3y \right|_0^2 dy\,dx$

$= \int_0^1 \left| x(2-0) + 3(2-0) \right| \cancel{dy}\,dx$

$= \int_0^1 \left| 2x + 6 \right| dx$

$= \left| 2\frac{x^2}{2} + 6x \right|_0^1$

$= \left| x^2 + 6x \right|_0^1$

$= (1-0)^2 + 6(1-0)$

$= 1 + 6$

$= 7$

Q#2 $\int_1^3\int_{-1}^1 (2x-4y)\,dy\,dx$

$\int_1^3 \left(2xy - 4\frac{y^2}{2} \right|_{-1}^1 dx$

$\int_1^3 \left| 2xy - 2y^2 \right|_{-1}^1 dx$

$\int_1^3 \left| 2x(1+1) - 2[1^2 - (-1)^2] \right| dx$

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$$\int_1^3 2x(2) - 2(1-1)\,dx$$

$$\int_1^3 4x\,dx$$

$$= \int \left| \frac{4^2 x^2}{2} \right|_1^3$$

$$= 2x^2 \Big|_1^3 = 2(9-1) = 16$$

4

$$\int_{-2}^{0} \int_{-1}^{2}$$

Q#3 $\int_2^4 \int_0^1 x^2 y\,dx\,dy$

$$\int_2^4 y \left| \frac{x^3}{3} \right|_0^1 dy = \int_2^4 \frac{y}{3} |1-0|\,dy$$

$$\int_2^4 \frac{y}{3}\,dy = \frac{1}{3} \cdot \left| \frac{y^2}{2} \right|_2^4 = \frac{1}{6}(16-4) = \frac{12}{6}$$

$$\frac{1}{6}(12) = 2$$

Q#4 $\int_{-2}^{0} \int_{-1}^{2} (x^2+y^2)\,dx\,dy$

$$\int_{-2}^{0} \left| \frac{x^3}{3} + xy^2 \right|_{-1}^{2} dy = \int_{-2}^{0} \left(\frac{(2)^3-(-1)^3}{3} + (2+1)y^2 \right) dy$$

$$\int_{-2}^{0} \left(\frac{8+1}{3} + 3y^2 \right) dy = \int_{-2}^{0} \left(\frac{9}{3} + 3y^2 \right) dy$$

$$\int_{-2}^{0} 3+3y^2\,dy = \cancel{\int_{-2}^{0} 3(1+y^2)} \left|3y + 3\frac{y^3}{3}\right|_{-2}^{0}$$

$$= \left|3y + y^3\right|_{-2}^{0} = 3(0+2) + (0+2)^3 = 6 + 8 = 14$$

Q#5 $\int_{0}^{\ln 3}\int_{0}^{\ln 2} e^{x+y}\,dy\,dx$

$$= \int_{0}^{\ln 3}\int_{0}^{\ln 2} e^{x}\,e^{y}\,dy\,dx = \int_{0}^{\ln 3} e^{x}\,e^{y}\Big|_{0}^{\ln 2} dx$$

$$\int_{0}^{\ln 3} e^{x}\left(e^{\ln 2} - e^{0}\right)dx = \cancel{\int_{0}^{\ln 3} 2e^{x}\,dx} \int_{0}^{\ln 3} e^{x}(2-1)\,dx$$

$$\int_{0}^{\ln 3} e^{x}\,dx = \left|e^{x}\right|_{0}^{\ln 3} = e^{\ln 3} - e^{0} = 3 - 1 = 2$$

Q#6 $\int_{0}^{2}\int_{0}^{1} y\sin x\,dy\,dx$

$$\int_{0}^{2}\left|\sin x\,\frac{y^2}{2}\right|_{0}^{1} dx = \frac{1}{2}\int_{0}^{2}\left|y^2 \sin x\right|_{0}^{1} dx$$

$$= \int_{0}^{2}\frac{1}{2}\left|1-0\right)^2 \sin x\,dx = \int_{0}^{2}\frac{1}{2}\sin x\,dx$$

$$= \frac{1}{2}\left|-\cos x\right|_{0}^{2} dx = \frac{1}{2}\; -\cos 2 + \cos 0$$

$$= \frac{1}{2}(-\cos 2 + 1) = \frac{1-\cos 2}{2}$$

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Q#7 $\int_{-1}^{0}\int_{2}^{5} dx\,dy$

$\int_{-1}^{0}\int_{0}^{0} x\Big|_{2}^{5} dy = \int_{-1}^{0} |5-2|\,dy = \int_{-1}^{0} 3\,dy$

$\int_{-1}^{0} |3y|_{-1}^{0} dy = 3(0+1) = 3$

Q#8 $\int_{4}^{6}\int_{-3}^{7} dy\,dx$

$\int_{4}^{6}\int_{-3}^{7} |y|_{-3}^{7} dx = \int_{4}^{6}(7+3)\,dx = \int_{4}^{16} 10\,dx$

$= 10|x|_{4}^{6} = 10|6-4| = 10(2) = 20$

Q#9 $\int_{0}^{1}\int_{0}^{1} \frac{x}{(xy+1)^2}\,dy\,dx$

$\int_{0}^{1}\int_{0}^{1} x(xy+1)^{-2}\,dy\,dx$

$\int_{0}^{1} \frac{(xy+1)^{-2+1}}{-2+1}\Big|_{0}^{1} dx = -\int_{0}^{1} (xy+1)^{-1}\Big|_{0}^{1} dx$

$= \int_{0}^{1} -\frac{1}{(xy+1)}\Big|_{0}^{1} dx = \int_{0}^{1}\left(\left(-\frac{1}{x(1)+1}\right) - \left(\frac{-1}{x(0)+1}\right)\right) dx$

$= \int_{0}^{1}\left(-\frac{1}{x+1} + 1\right) dx = \int_{0}^{1}\left(1 - \frac{1}{x+1}\right) dx$

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$\cancel{\left|x - \ln(x+1)\right|_0^1 = \left|(1-0) - \ln(1-0)\right| = 1 - \ln}$

$= \left|x - \ln(x+1)\right|_0^1 = (1-0) - \ln((1-0)+1) = 1 - \ln(1+1)$

$= 1 - \ln 2.$

Q#10 $\int_{\pi/2}^{\pi}\int_1^2 x\cos xy \, dy\, dx$

$\int_{\pi/2}^{\pi} \left| \sin xy \right|_1^2 dx$

$\cancel{\int_{\pi/2}^{\pi} \left| x \sin x(2-1) \right| dx = \int_{\pi/2}^{\pi} x \sin x \, dx}$

$= \int_{\pi/2}^{\pi} (\sin 2x - \sin x)\, dx$

$= \int_{\pi/2}^{\pi} \sin 2x \, dx - \int_{\pi/2}^{\pi} \sin x \, dx$

$= \left| -\frac{\cos 2x}{2} \right|_{\pi/2}^{\pi} - \left| \cos x \right|_{\pi/2}^{\pi} = -\cancel{\frac{\cos\pi}{2} + \frac{\cos\frac{\pi}{2}}{2} - \cos\pi - \cos\frac{\pi}{2}}$

$\cancel{= -\frac{1}{2} + \frac{0}{2} - 1 - 0 = -\frac{1}{2} - 1 = \frac{-1-2}{2} = -\frac{3}{2}}$

$= -\frac{\cos 2\pi}{2} + \frac{\cos 2\cdot\frac{\pi}{2}}{2} - \cos\pi + \cos\frac{\pi}{2}$

$= -\frac{1}{2} - \frac{1}{2} - 1 + 0 = \frac{-1-1-2}{2} = -\frac{4}{2}^{2} = -2$

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Q#11 $\int_0^{\ln 2}\int_0^1 xye^{y^2x}\,dy\,dx$

$= \int_0^{\ln 2}\frac{1}{2}\int_0^1 2xy\,e^{y^2x}\,dy\,dx$

$= \frac{1}{2}\int_0^{\ln 2}\left| e^{y^2x}\right|_0^1 dx = \frac{1}{2}\int_0^{\ln 2}\left(e^{(1)^2x} - e^{(0)^2x}\right)dx$

$= \frac{1}{2}\int_0^{\ln 2}(e^{x} - e^{0})\,dx = \frac{1}{2}\int_0^{\ln 2}(e^{x}-1)\,dx$

$= \frac{1}{2}\left|e^{x} - x\right|_0^{\ln 2} = \frac{1}{2}\left(e^{\ln 2} - e^{0} - \ln 2 + 0\right)$

$= \frac{1}{2}(2 - 1 - \ln 2) = \frac{1-\ln 2}{2}$

Q#12 $\int_3^4\int_1^2 \frac{1}{(x+y)^2}\,dy\,dx$

$\int_3^4\int_1^4 (x+y)^{-2}\,dy\,dx$

$\int_3^4 \left.\frac{(x+y)^{-2+1}}{-2+1}\right|_1^4 dx = \int_3^4 -\left.\frac{1}{x+y}\right|_1^2 dx$

$= \int_3^4\left(-\frac{1}{x+2} + \frac{1}{x+1}\right)dx$

$\left(-\ln(x+2) + \ln(x+1)\right)\Big|_3^4 = -\ln(4+2) + \ln(3+2) + \ln(4+1) - \ln(3+1)$

$= -\ln 6 + \ln 5 + \ln 5 - \ln 4 = 2\ln 5 - (\ln 4 + \ln 6)$

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$$\ln 5^{2} - \ln(4\times 6) = \ln 25 - \ln 24 = \ln\left(\frac{25}{24}\right)$$

Q#13 $\displaystyle\int_{-1}^{1}\int_{-2}^{2} 4xy^3\,dy\,dx$

$$\int_{-1}^{1}\left|\frac{\cancel{4}y^4}{\cancel{4}}(x)\right|_{-2}^{2}dx = \int_{-1}^{1}\left|xy^4\right|_{-2}^{2}dx$$

$$= \int_{-1}^{1} x(2)^4 - x(-2)^4\,dx = \int_{-1}^{1} 16-16 = \int_{-1}^{1} 0\,dx = 0$$

Q#14 $\displaystyle\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\,dy\,dx$

$$\int_0^1\int_0^1 xy\,(x^2+y^2+1)^{-1/2}\,dy\,dx$$

$$= \int_0^1\int_0^1 \frac{1}{2}\,x\,2y\,(x^2+y^2+1)^{-\frac{1}{2}}\,dy\,dx$$

$$= \int_0^1 \frac{1}{2}x\left.\frac{(x^2+y^2+1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right|_0^1 dx$$

$$= \int_0^1 \frac{1}{\cancel{2}}x\left.\frac{(x^2+y^2+1)^{\frac{1}{2}}}{\cancel{1/2}}\right|_0^1 dx = \int_0^1 x\left.(x^2+y^2+1)^{1/2}\right|_0^1 dx$$

$$= \cancel{x(x^2+(1)^2+1)^{1/2} - x(x^2+y^2+0}$$

$$\int_0^1\left(x(x^2+(1)^2+1)^{1/2} - x(x^2+(0)^2+1)^{1/2}\right)dx$$

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$$\int_0^1 \left(x(x^2+2)^{1/2} - x(x^2+1)^{1/2} \right) dx$$

$$\int_0^1 \frac{1}{2} \left(2x(x^2+2)^{1/2} - 2x(x^2+1)^{1/2} \right) dx$$

$$= \frac{1}{2} \left(\frac{(x^2+2)^{\frac{1}{2}+1}}{1/2+1} - \frac{(x^2+1)^{1/2+1}}{\frac{1}{2}+1} \right) \Big|_0^1$$

$$= \frac{1}{2} \cdot \frac{2}{3} \left((x^2+2)^{3/2} - (x^2+1)^{3/2} \right) \Big|_0^1$$

$$= \frac{1}{3} \left(((1)^2+2)^{3/2} - (0^2+2)^{3/2} - (1^2+1)^{3/2} + (0^2+1)^{3/2} \right)$$

$$= \frac{1}{3} \left((3)^{3/2} - (2)^{3/2} - (2)^{3/2} + (1)^{3/2} \right.$$

$$= \frac{1}{3} \left(3\sqrt{3} - 4\sqrt{2} + 1 \right)$$

Q#15 $\int_0^1 \int_2^3 \sqrt{1-x^2}\, dy\, dx$

$$= \int_0^1 \int_2^3 -\frac{1}{2}\, -2x \left(1-x^2 \right)^{1/2} dy\, dx$$

$$= \int_0^1 -\frac{1}{2} \frac{(1-x^2)^{\frac{1}{2}-1}}{\frac{1}{2}-1} \Big|_2^3 dx = \int_0^1 (1-x^2)^{-1/2}$$

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Q#15 $\int_0^1\int_2^3 x\sqrt{1-x^2}\, dy\, dx$

$= \int_0^1 x\sqrt{1-x^2}\; y\Big|_2^3 dx$

$= \int_0^1 x\sqrt{1-x^2}\,|3-2|\,dx = \int_0^1 x\sqrt{1-x^2}\,dx$

$= -\frac{1}{2}\int_0^1 -2x\,(1-x^2)^{1/2}\,dx =$

$= -\frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Bigg|_0^1 = -\frac{1}{2}\times\frac{2}{3}(1-x^2)^{3/2}\Big|_0^1$

$-\frac{1}{3}\left(\frac{1}{(1-1)^{3/2}} - \frac{1}{(1-0)^{3/2}}\right) \qquad -\frac{1}{\sqrt{1}} = 1$

$= -\frac{1}{3}\left(\frac{-1}{(1)^{3/2}}\right) = \frac{1}{3}.$

Q#16 $\int_0^{\pi/2}\int_0^{\pi/3}(x\sin y - y\sin x)\,dy\,dx$

$\int_0^{\frac{\pi}{2}} x(-\cos y)\Big|_0^{\pi/3} - \sin x\,\frac{y^2}{2}\Big|_0^{\pi/3} dx$

$\int_0^{\frac{\pi}{2}} x\left(-\cos\frac{\pi}{3} - \cancel{\cos 0}\right) - \sin x\left(\frac{(\pi/3)^2}{2} - \frac{0^2}{2}\right)dx$

$= \int_0^{\frac{\pi}{2}} x\left(\frac{1}{2}\right) + \frac{\pi^2}{18}\sin x = \int_0^{\frac{\pi}{2}} \frac{x}{2} + \frac{\pi^2}{18}\sin x$

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$$= \frac{1}{2}\frac{x^2}{2}\Big|_0^{\frac{\pi}{2}} - \frac{\pi^2}{18}\cos x\Big|_0^{\pi/2}$$

$$= \frac{1}{4}\left(\left(\frac{\pi}{2}\right)^2 - (0)^2\right) - \frac{\pi^2}{18}\cos\frac{\pi}{2} + \frac{\pi^2}{18}\cos 0$$

$$= \frac{1}{4}\left(\frac{\pi^2}{4}\right) - \frac{\pi^2}{18}$$

2	16, 18
2	8, 9
2	4, 9
2	2, 9
9	1, 9

$$= \frac{18\pi^2 - 16\pi^2}{288} = \frac{2\pi^2}{288} = \frac{\pi^2}{144}$$

Ex # 14.3

(1) $$\int_0^{\pi/2}\int_0^{\sin\theta} r\cos\theta\, dr\, d\theta$$

$$= \int_0^{\pi/2} \cos\theta \frac{r^2}{2}\Big|_0^{\sin\theta} d\theta$$

$$= \int_0^{\pi/2} \frac{\cos\theta}{2} (\sin\theta)^2 - (0)^2\, d\theta$$

$$= \int_0^{\pi/2} \frac{\cos\theta}{2} \sin^2\theta\, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2} \frac{(\sin\theta)^{2+1}}{2+1}\Big|_0^{\pi/2} d\theta$$

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$$= \frac{1}{2} \frac{\sin^3\theta}{3} \Big|_0^{\pi/2} d\theta$$

$$= \frac{1}{6} (\sin \frac{\pi}{2})^3 - (\sin 0)^3 = \frac{1}{6} ((1)^3 - (0)^3 = \frac{1}{6}$$

(2) $\int_0^{\pi} \int_0^{1+\cos\theta} r \, dr \, d\theta$.

$$\int_0^{\pi} \frac{r^2}{2} \Big|_0^{1+\cos\theta} d\theta = \int_0^{\pi} \frac{1}{2} ((1+\cos\theta)^2 - (0)^2 d\theta$$

$$= \int_0^{\pi} \frac{1}{2} \; 1 + \cos^2\theta + \cos\theta \, d\theta = \int_0^{\pi} \frac{1}{2} \, 1 + \frac{1+\cos\theta}{2} + \cos\theta \, d\theta$$

$$\int_0^{\pi} \frac{1}{2} + \frac{1+\cos 2\theta}{4} + \cos\theta \, d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{4}\left(\theta + \frac{\sin 2\theta}{2}\right) + \sin\theta \Big|_0^{\pi}$$

$$= \frac{1}{2}\theta + \frac{1}{8}\left(2\theta + \frac{\sin 2\theta}{2}\right) + \sin\theta \Big|_0^{\pi}$$

$$= \frac{1}{2}(\pi - 0) + \frac{1}{8}\left(2\pi - 0 + \sin 2\pi - \sin 0\right) + \sin \pi - \sin 0$$

$$= \frac{1}{2}\pi + \frac{1}{8}(2\pi - 0 + 0 - 0) + 0 + 0$$

$$= \frac{\pi}{2} + \frac{2\pi}{8} = \frac{4\pi + 2\pi}{8} = \frac{\cancel{6}^{3}\pi}{\cancel{8}_{4}} = \frac{3\pi}{4}$$

$$\begin{array}{r|l} 2 & 2,8 \\ 2 & 1,4 \\ & 1,2 \end{array}$$

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3 - $\int_0^{\pi/2}\int_0^{a\sin\theta} r^2\,dr\,d\theta$

$\int_0^{\pi/2} \frac{r^3}{3}\Big|_0^{a\sin\theta} d\theta = \int_0^{\pi/2} \frac{1}{3}(a\sin\theta)^3 - (0)^3\,d\theta$

$\frac{a^3}{3}\int_0^{\pi/2} \sin^3\theta\,d\theta = \frac{a^3}{3}\int_0^{\pi/2} \sin\theta\sin^2\theta\,d\theta$

$\frac{a^3}{3}\int_0^{\pi/2} \sin\theta\,\frac{1-\cos 2\theta}{2}\,d\theta$

$\frac{a^3}{3}\int_0^{\pi/2} \frac{\sin\theta - \sin\theta\cos 2\theta}{2}\,d\theta = \frac{a^3}{6}\int_0^{\pi/2} \sin\theta - \sin\theta\cos 2\theta\,d\theta$

$\frac{a^3}{6}\int_0^{\pi/2} \sin\theta - \sin\theta(2\cos^2\theta - 1)\,d\theta = \frac{a^3}{6}\int_0^{\pi/2} \sin\theta - 2\sin\theta\cos^2\theta + \sin\theta\,d\theta$

$\frac{a^3}{6}\int_0^{\pi/2} 2\sin\theta - 2\sin\theta\cos^3\theta\,d\theta = \frac{a^3 \times \cancel{2}}{\cancel{6}_3}\int_0^{\pi/2} \sin\theta - \sin\theta\cos^2\theta\,d\theta$

$\frac{a^3}{3}\int_0^{\pi/2} \sin\theta + (-\sin\theta)\cos^2\theta\,d\theta = \cancel{\frac{a^3}{3}\int_0^{\pi/2} \sin\theta}$

$\frac{a^3}{3}\Big| -\cos\theta + \frac{\cos^3\theta}{3}\Big|_0^{\pi/2} = \frac{a^3}{3}\left[-(\cos\tfrac{\pi}{2}) - (-\cos 0) + \left(\frac{(\cos\tfrac{\pi}{2})^3}{3} - \frac{(\cos 0)^3}{3}\right)\right]$

$\frac{a^3}{3}\left(-0+1+\frac{0}{3}-\frac{1}{3}\right) = \frac{a^3}{3}\left(\frac{3-1}{3}\right) = \frac{2a^3}{9}$

$a^3 - b^3 - 3ab(a-b)$
$a^3 - b^3 - 3a^2b + 3ab^2$

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(4) $\int_0^{\pi/6}\int_0^{\cos 3\theta} r\,dr\,d\theta$

$\int_0^{\pi/6} \frac{r^2}{2}\Big|_0^{\cos 3\theta} d\theta = \frac{1}{2}\int_0^{\pi/6} \cos^2 3\theta \, d\theta$

$\frac{1}{2}\int_0^{\pi/6} \frac{1+\cos 6\theta}{2} d\theta = \frac{1}{4}\int_0^{\pi/6} 1+\cos 6\theta \, d\theta$

$\frac{1}{4}\left(\theta + \frac{\sin 6\theta}{6}\right)\Big|_0^{\pi/6} = \frac{1}{4}\left(\frac{\pi}{6} - 0 + \frac{\sin 6(\frac{\pi}{6}) - \sin 6(0)}{6}\right)$

$= \frac{1}{4}\left(\frac{\pi}{6}\right) = \frac{\pi}{24}$

(5) $\int_0^{\pi}\int_0^{1-\sin\theta} r^2 \cos\theta \, dr\,d\theta$

$= \int_0^{\pi} \cos\theta \, \frac{r^3}{3}\Big|_0^{1-\sin\theta} d\theta = \frac{1}{3}\int_0^{\pi} \cos\theta \left((1-\sin\theta)^3 - (0)^3\right) d\theta$

$= \frac{1}{3}\int_0^{\pi} \cos\theta \, (1-\sin\theta)^3 d\theta$

$\frac{1}{3}\int_0^{\pi} \cos\theta\left(1 - \sin^3\theta - 3\sin\theta + 3\sin^2\theta\right) d\theta$

$\frac{1}{3}\int_0^{\pi} \left(\cos\theta - \cos\theta\sin^3\theta - 3\cos\theta\sin\theta + 3\cos\theta\sin^2\theta\right) d\theta$

$\frac{1}{3}\left| \sin\theta - \frac{\sin^4\theta}{4} - 3\frac{\sin^2\theta}{2} + 3\frac{\sin^3\theta}{3} \right|_0^{\pi}$

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$$\frac{1}{3}\left|\sin\theta - \frac{\sin^4\theta}{4} - \frac{3\sin^2\theta}{2} + \sin^3\theta\right|_0^{\pi}$$

$$\frac{1}{3}\left(\sin\pi - \sin 0 - \frac{(\sin\pi)^4 - \sin(0)}{4} - (\sin\pi)^3 + (\sin 0)^3\right)$$

$$\frac{1}{3}\left(0 - 0 - \frac{0 - 0}{4} - 0 + 0\right) = \frac{1}{3}(0) = 0$$

$= ①$

⑥ $$\int_0^{\pi/2}\int_0^{\cos\theta} r^3 \, dr\, d\theta$$

$$\int_0^{\pi/2} \frac{r^4}{4}\Big|_0^{\cos\theta} dr\, d\theta = \frac{1}{4}\int_0^{\pi/2} \cos^4\theta \, d\theta$$

$$= \frac{1}{4}\int_0^{\pi/2} \cos^2\theta \cdot \cos^2\theta \, d\theta = \frac{1}{4}\int_0^{\pi/2} \frac{1+\cos 2\theta}{2} \cdot \frac{1+\cos 2\theta}{2} \, d\theta$$

$$\frac{1}{16}\int_0^{\pi/2} (1+\cos 2\theta)^2 \, d\theta = \frac{1}{16}\int_0^{\pi/2} 1 + \cos^2 2\theta + \cos 2\theta \, d\theta$$

$$\frac{1}{16}\int_0^{\pi/2} 1 + \frac{1+\cos 4\theta}{2} + \cos 2\theta \, d\theta$$

$$\cancel{\frac{1}{16}\int_0^{\pi/2} 1 + \cos 2\theta + \cos 2\theta \cos 4\theta}$$

$$\frac{1}{16}\int_0^{\frac{\pi}{2}} \frac{2 + 1 + \cos 4\theta + 2\cos 2\theta}{2} \, d\theta$$

$$\frac{1}{32}\int_0^{\frac{\pi}{2}} 3 + \cos 4\theta + 2\cos 2\theta \, d\theta$$

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$$\frac{1}{32}\left(3\theta + \frac{\sin 4\theta}{4} \cancel{+2}\, \frac{\sin 2\theta}{2}\right|_0^{\pi/2}$$

$$= \frac{1}{32}\left(3(\pi/_2 - 0) + \frac{\sin 4(\pi/_2) - \sin(0)}{4} + \sin 2(\pi/_2) - \sin(0)\right)$$

$$= \frac{1}{32}\left(\frac{3\pi}{2} + 0 - 0 + 0 - 0\right) = \frac{3\pi}{64}$$

$r=0$ $y=\sqrt{1-x^2}$

$y^2 = 1-x^2$

(27) $\int_0^{\pi/2}\int_0^1 r^3\,dr\,d\theta$ $\quad \int_0^1\int_0^{\sqrt{1-x^2}} (x^2+y^2)\,dy\,dx$ $\quad \theta=0, \theta=\frac{\pi}{2}$ $\quad x^2+y^2=1$

$r=1$

$\int_0^{\pi/2} r^2\, r\,dr\,d\theta$

$\int_0^{\pi/2}\int_0^1 r^3\,dr\,d\theta$

$$= \int_0^{\pi/2} \frac{r^4}{4}\Big|_0^1 dr\,d\theta = \frac{1}{4}\int_0^{\pi/2} (1)^4 - (0)^4\,d\theta$$

$$\frac{1}{4}\int_0^{\pi/2} d\theta = \frac{1}{4}\,\theta\Big|_0^{\pi/2} = \frac{1}{4}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{8}$$

(28) $\int_0^{2\pi}\int_0^2 e^{-r^2}\, r\,dr\,d\theta$

$-r^2$

$-2r$

$\cancel{\int_0^{2\pi} \frac{e^{-r^2}}{-2r}\frac{r^2}{2}\Big|_0^2 d\theta = \int_0^{2\pi} \frac{r e^{-r^2}}{-4}\Big|_0^2 d\theta}$

$\cancel{\int_0^{2\pi} \frac{2e^4}{-4} = 2e}$

$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-(x^2+y^2)}\,dx\,dy$ $\quad y=\sqrt{4-y^2}$

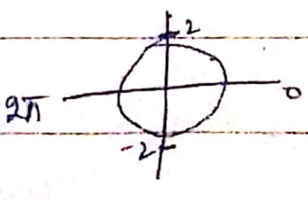
$x^2 = 4-y^2$

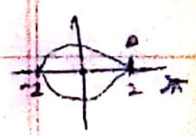
$\cancel{\int_0^{2\pi}\int_0^2 e^{-r^2} \cdot -2r}$

$x^2+y^2 = 2^2$

$\theta=0, \theta=2\pi$ $\quad \boxed{r=0 \quad r=2}$

$$\int_0^{2\pi}\int_0^{2} e^{-r^2}\, r\,dr\,d\theta$$





(28) $\int_0^{2\pi}\int_0^{2} e^{-r^2}\, r\, dr\, d\theta$

$\int_0^{2\pi}\int_0^{2} -\frac{1}{2} e^{-r^2} \cdot -2r \, dr\, d\theta = \int_0^{2\pi} -\frac{1}{2} e^{-r^2}\Big|_0^2 d\theta$

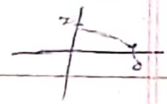
$-\frac{1}{2}\int_0^{2\pi}\left(e^{-2^2} - e^{0^2}\right) d\theta = -\frac{1}{2}\int^{2\pi}\left(e^{-4}-1\right) d\theta$

$= \frac{1}{2}\left(1-e^{-4}\right)\int_0^{2\pi} d\theta = \frac{1}{2}\left(1-e^{-4}\right)\theta\Big|_0^{2\pi}$

$= \frac{1}{2}\left(1-e^{-4}\right)2\pi = \pi\left(1-e^{-4}\right)$

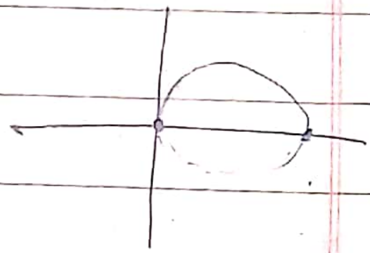
(29) $\int_0^{2}\int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$

$y = \sqrt{2x-x^2}$ $\qquad \theta = 0,\ \theta = \frac{\pi}{2}$



$y^2 = 2x - x^2$

$y^2 + x^2 = 2x$



$x^2 - 2x + y^2 = 0$

$x^2 - 2x + 1 - 1 + y^2 = 0$

$(x-1)^2 - 1 + y^2 = 0$

$(x-1)^2 + y^2 = 1$

$x = 0$

$x^2 + y^2 = r^2$

$r = 1$

$x - 1 = 1$

$x = 1 + 1$

$\boxed{x = 2}$

$\theta = 0,\ \theta = \frac{\pi}{2},\quad x = r\cos\theta$

$y^2 \pm 2(r\cos\theta) \mp x^2$

$x^2 + y^2 = 2r\cos\theta$ $\quad | \quad r = 2\cos\theta$

$r^2 = 2r\cos\theta$ $\quad | \quad r = 0$

$$\int_0^{\pi/2}\int_0^{2\cos\theta} \sqrt{r^2}\cdot r\,dr\,d\theta = \int_0^{\pi/2}\int_0^{2\cos\theta} r^2\,dr\,d\theta$$

$$= \int_0^{\pi/2} \frac{r^3}{3}\Big|_0^{2\cos\theta} d\theta = \frac{1}{3}\int_0^{\pi/2} 8\cos^3\theta\,d\theta$$

$$= \frac{1}{3}\int_0^{\pi/2} 8\cos^2\theta\cos\theta\,d\theta = \frac{8}{3}\int_0^{\pi/2}\cos\theta\cdot\cos^2\theta\,d\theta$$

$$= \frac{8}{3}\int_0^{\pi/2}\cos\theta(1-\sin^2\theta)\,d\theta = \frac{8}{3}\int_0^{\pi/2}\cos\theta - \cos\theta\sin^2\theta\,d\theta$$

$$= \frac{8}{3}\left|\sin\theta - \frac{\sin^3\theta}{3}\right|_0^{\pi/2} = \frac{8}{3}\left|\left(\sin\frac{\pi}{2} - \sin 0\right) - \frac{(\sin\frac{\pi}{2})^3}{3} - \right.$$

$$\frac{(\sin 0)^3}{3} = \frac{8}{3}\left(1 - 0 - \frac{1}{3} - 0\right)$$

$$= \frac{8}{3}\left(\frac{3-1}{3}\right) = \frac{16}{9}$$

30) $\int_0^1\int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\,dx\,dy$

$x = \sqrt{1-y^2}$

$x^2 = 1-y^2$

$x^2+y^2 = 1$

$r=1, \; r=0$

$\theta = 0, \; \theta = \frac{\pi}{2}$

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$$\int_0^{\pi/2}\int_0^1 r\cos r^2\,dr\,d\theta$$

$$\int_0^{\pi/2}\int_0^1 \frac{1}{2}\,2r\cos r^2\,dr\,d\theta = \frac{1}{2}\int_0^{\pi/2} \sin r^2\Big|_0^1 d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} [\sin 1^2 - \sin 0^2]\,d\theta = \frac{1}{2}\int_0^{\pi/2} \sin 1\,d\theta$$

$$= \frac{1}{2}\sin 1 \int_0^{\pi/2} 1\,d\theta = \frac{1}{2}\sin 1\;\theta\Big|_0^{\pi/2}$$

$$= \frac{1}{2}\sin 1\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{4}\sin 1$$

(31) $$\int_0^a\int_0^{\sqrt{a^2-x^2}} \frac{dy\,dx}{(1+x^2+y^2)^{3/2}} \quad (a>0)$$

$y = \sqrt{a^2-x^2}$

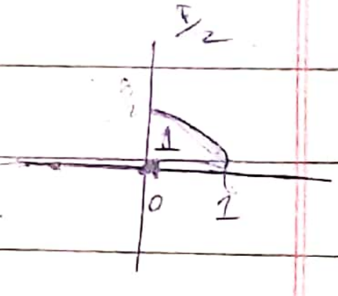
$y^2 = a^2 - x^2$

$x^2+y^2 = a^2$

$r = a$ $\qquad r = 0$

$\theta = 0$

$\theta = \pi/2$



$$\int_0^{\pi/2}\int_0^a \frac{r}{(1+r^2)^{3/2}}\,dr\,d\theta$$

$$\int_0^{\pi/2}\int_0^a r(1+r^2)^{3/2}\,dr\,\theta$$

$$= \int_0^{\pi/2}\int_0^a \frac{1}{2}\,2r\,(1+r^2)^{3/2}\,dr\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \frac{(1+r^2)^{-3/2+1}}{-\frac{3}{2}+1}\Bigg|_0^a d\theta$$

$$-\frac{3}{2}+1 \qquad \frac{-3+2}{2} = -\frac{1}{2}$$

$$= \frac{1}{\cancel{2}}\int_0^{\pi/2} \frac{-\cancel{2}}{1} (1+r^2)^{-1/2}\Big|_0^a d\theta$$

$$= \theta - \int_0^{\pi/2} (1+a^2)^{-1/2} - (1+0^2)^{-1/2} d\theta$$

$$= \theta - \int_0^{\pi/2} (1+a^2)^{-1/2} - 1 \, d\theta$$

$$= -(1+a^2)^{-1/2} - 1 \int_0^{\pi/2} d\theta$$

$$= 1 - (1+a^2)^{-1/2}\, \theta\Big|_0^{\pi/2} = 1 - \frac{1}{\sqrt{1+a^2}}\left(\frac{\pi}{2} - 0\right)$$

$$= \frac{\pi}{2}\left(1 - \frac{1}{\sqrt{1+a^2}}\right)$$