Bonus Project
Report
Effect of carrier frequency offset (CFO) on OFDM signal

Dr. Mohammed H. Karmoose
Communications & Electronics

Name: **Asmaa Gamal Abdel-Halem Mabrouk Nagy**
ID: 15010473
Section: 5
Name In Arabic: أسماء جمال عبد الحليم مبروك ناجي

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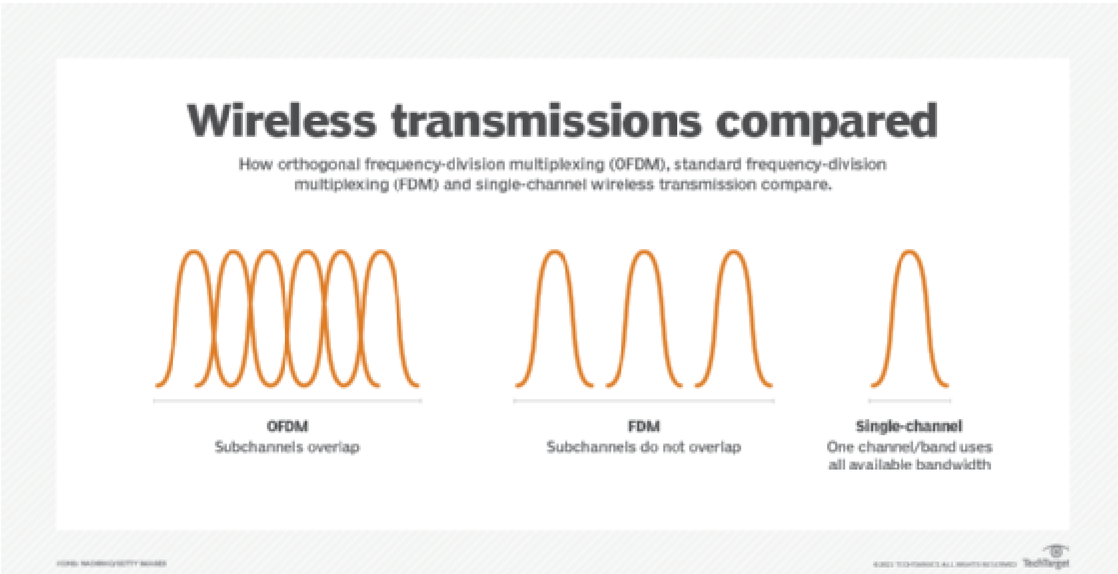
15010473

1. Brief Notes about OFDM:

What is OFDM ?

Orthogonal frequency-division multiplexing (OFDM) is a method of data transmission where a single information stream is split among several closely spaced narrowband subchannel frequencies instead of a single Wideband channel frequency. It is mostly used in wireless data transmission but may be employed in wired and fiber optic communication as well.

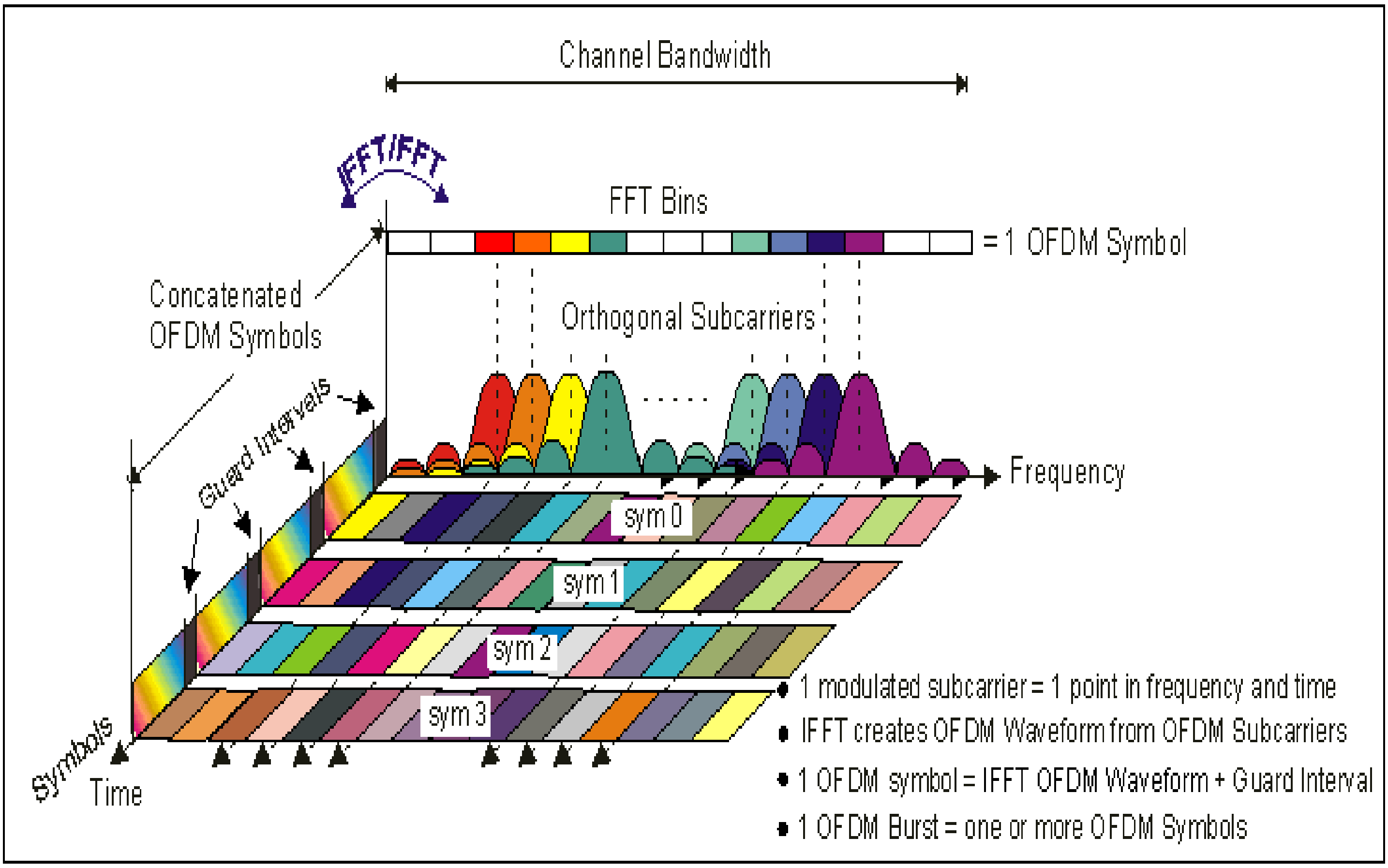
In a traditional single-channel modulation scheme, each data bit is sent serially or sequentially one after another. In OFDM, several bits can be sent in parallel, or at the same time, in separate sub-stream channels. This enables each sub-stream's data rate to be lower than would be required by a single stream of similar bandwidth. This makes the system less susceptible to interference and enables more efficient data bandwidth.



The differences among orthogonal frequency-division multiplexing, standard frequency-division multiplexing and a single wideband channel frequency wireless data transmission scheme.

How orthogonal frequency-division multiplexing works:

In the traditional stream, each bit might be represented by a 1 nanosecond segment of the signal, with 0.25 ns spacing between bits, for example. Using OFDM to split the signal across four component streams lets each bit be represented by 4 ns of the signal with 1 ns spacing between. The overall data rate is the same, 4 bits every 5 ns, but the signal integrity is higher.



Frequency-Time Representative of an OFDM signal

As an illustration, imagine you were sending a letter to your grandmother. You could write your letter on a single piece of paper and mail it to her in an envelope. This would be like using a single frequency (one piece of paper) to send your entire message. But, because your grandmother can't see well, you instead write the same message in larger letters (a slower data rate) on several pieces of paper (representing data streams on different channels) but put them all in the same envelope (using same overall frequency spectrum).

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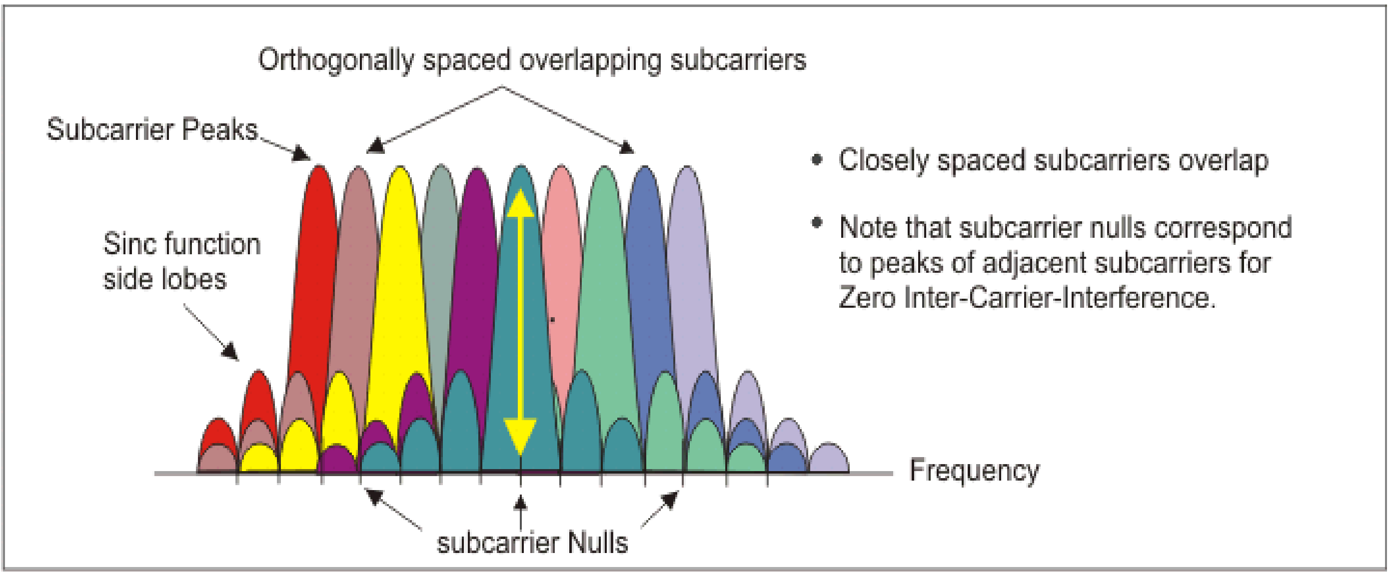
OFDM builds on simpler frequency-division multiplexing (FDM). In FDM, the total data stream is divided into several subchannels, but the frequencies of the subchannels are spaced farther apart so they do not overlap or interfere. With OFDM, the subchannel frequencies are close together and overlapping but are still orthogonal, or separate, in that they are carefully chosen and modulated so that the interference between the subchannels is canceled out.



OFDM Signal Frequency Spectra

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2. What is Carrier Frequency Offset (CFO) and How It Distorts the Rx Symbols:

In Physics, frequency in units of Hz is defined as the number of cycles per unit time. Angular frequency is the rate of change of phase of a sinusoidal waveform with units of radians/second.

$$2\pi f = \frac{\Delta\theta}{\Delta t}$$

where $\Delta\theta$ and Δt are the changes in phase and time, respectively.
A **Carrier Frequency Offset (CFO)** usually arises due to two reasons. The video below also explains this concept.

1. **[Frequency mismatch between the Tx and Rx oscillators]** No two devices are the same and there is always some difference between the manufacturer's nominal specification and the real frequency of that device. Moreover, this actual frequency keeps changing (slightly) with temperature, pressure, age, and some other factors.
2. **[Doppler effect]** A moving Tx, Rx or any kind of movement around the channel creates a Doppler shift that creates a carrier frequency offset as well. We will learn more about it during the discussion of a wireless channel in another post.

To see the effect of the carrier frequency offset $F\Delta$, consider again a received passband signal consisting of two PAM waveforms
in I and Q arms.

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$$
\begin{aligned}
r(t) &= v_I(t)\sqrt{2}\cos\left[2\pi(F_C+F_\Delta)t+\theta_\Delta\right] - v_Q(t)\sqrt{2}\sin\left[2\pi(F_C+F_\Delta)t+\theta_\Delta\right] \\
&= v_I(t)\sqrt{2}\cos\left[2\pi F_C t + 2\pi F_\Delta t+\theta_\Delta\right] - \\
&\qquad\qquad v_Q(t)\sqrt{2}\sin\left[2\pi F_C t + 2\pi F_\Delta t+\theta_\Delta\right] \qquad (1)
\end{aligned}
$$

Here, the impact of carrier offset can be seen as 2πFΔt which is changing the phase with time. Let us look into how to find FΔ,max, the maximum value CFO can take.

The accuracy of local oscillators in communication receivers is defined in terms of **ppm (parts per million)**. 1 ppm is just what it says: 1 out of 10^6 parts. To get a feel of how big or small this number is, 10^6 seconds translate into:

$$
\frac{10^6 \text{ seconds}}{24 \text{ hours/day} \times 3600 \text{ seconds/hour}} \approx 11.5 \text{ days}
$$

Hence, 11 ppm is equivalent to a deviation of 11 seconds every 11.5days. This might sound entirely harmless but for the purpose of typical wireless communication systems operating at several GHz of carrier frequency FC and several MHz of symbol rates RM, it is one of the major sources of signal distortion.
The ppm rating at the oscillator crystal indicates how much its frequency may deviate from the nominal value.

Consider an example of a wireless system operating at 2.4GHz and with ±20 ppm crystals, which is a more or less standard rating. The maximum deviation of the carrier frequency at the Tx or Rx can be:

$$\pm\frac{20}{10^6} \times 2.4 \times 10^9 = \pm 48\text{kHz}$$

However, in the worst-case scenario, the Tx can be 20 ppm above (or below) the nominal frequency, while the Rx can be 20 ppm below (or above) the nominal frequency, resulting in the overall difference of 40 ppm between the two. So, the worst-case CFO due to local oscillator mismatch in this example can be

$$\pm 2 \times 48 = \pm 96\text{kHz}$$

Keep in mind that this calculation is for basic precision and the actual frequency may vary depending on environmental factors, mainly the temperature and aging. Finally, a movement anywhere in the channel (whether by Tx, Rx or some other object within that environment) adds Doppler shift which can be up to several hundreds of Hz. Although this Doppler shift is much less than the oscillator generated mismatch, it severely distorts the Rx signal due to the changes it causes in the channel.

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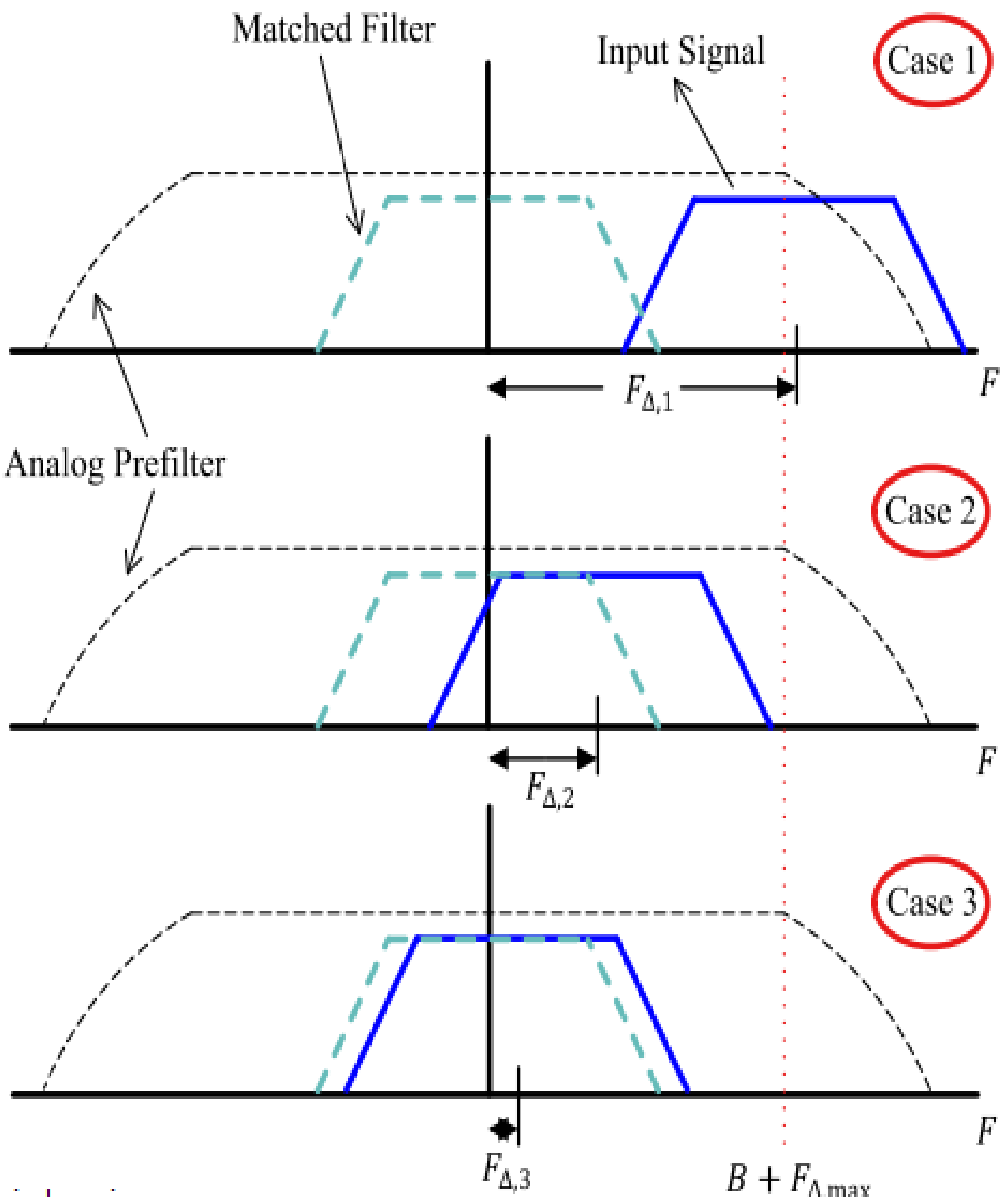
Now we turn our attention towards Figure below to differentiate between three possibilities in which FΔ can dictate the receiver design.



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Case 1: CFO > $F_{\Delta,max}$:

An input signal at the Rx is filtered by an analog prefilter to remove the out of band noise. Ideally, the frequency response of this prefilter G(F) should be flat within the frequency range

$$|F| \leq B + F_{\Delta,\max}$$

so that the incoming signal can pass through in an undistorted manner (we are assuming a flat wireless channel here as well. Actual wireless channels will be discussed later). The passband width of this prefilter is designed according to the maximum CFO FΔ,max expected at the Rx.
However, if the CFO is greater than FΔ,max then much of the actual intended signal will be filtered out by the analog prefilter and the Rx sampled signal will not even closely resemble the Tx signal as a linear function of Tx data. No amount of signal processing can then recover the signal. Since it is an outcome of poor system design, the only remedy is to redesign the system (particularly the analog frontend) with more accurate estimates and margins for CFO and other such random distortions.

Case 2: 15% of RM< CFO < $F_{\Delta,max}$:

Since CFO < FΔ,max, the Rx signal is within the passband of the analog prefilter G(F) and suffers no distortion. However, remember to maximize the SNR, the Rx signal must be passed through a matched filter. This is not possible in this case because much of the Rx signal bandwidth does not sufficiently overlap with the matched filter due to the CFO being greater than 15% of symbol rate RM. If applied, it would remove a significant portion of the incoming signal energy.
Since Rx signal cannot be matched filtered without distorting the signal, and the signal cannot be down sampled to 1 sample/symbol without

matched filtering, it is easy to deduce that more than 1 sample/symbol (say, L) is required to trace the frequency offset.

Case 3: CFO < 15% of RM:

When the signal is rotated by less than 15% RM, an offset cycle gets completed in less than 77 symbols, and hence **the effect of rotation on one symbol, although still significant, can now be tracked at symbol rate, or** 11 **sample/symbol**. In other words, matched filtering keeps most of the signal intact and as a result, symbol boundaries can be marked first (a problem known as symbol timing synchronization which we discuss in another article) before carrier frequency is estimated at the ISI-free symbol-spaced samples.

In the absence of noise, the sampled version of this mismatch in Eq(1) becomes $2\pi F\Delta\ nTS$. Now Eq (1) is very similar to phase rotation equation, and hence we can use directly use that result with proper substitution. The expression for the symbol-spaced samples in the presence of CFO $F\Delta$ can be obtained after replacing sample time index n with the

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symbol time index m.

$$z_I(mT_M) = a_I[m]\cos 2\pi F_\Delta mT_M - a_Q[m]\sin 2\pi F_\Delta mT_M$$
$$z_Q(mT_M) = a_I[m]\sin 2\pi F_\Delta mT_M + a_Q[m]\cos 2\pi F_\Delta mT_M$$

In the above equation,

$$2\pi F_\Delta mT_M = 2\pi\frac{F_\Delta}{R_M}m = 2\pi F_0 m$$

where the F_0 is defined as the **normalized Carrier Frequency Offset (nCFO)**: CFO normalized by the symbol rate.

$$F_0 = \frac{F_\Delta}{R_M}$$

This normalization is very important as we saw in the last subsection. The resulting expression takes the form

$$z_I(mT_M) = a_I[m]\cos 2\pi F_0 m - a_Q[m]\sin 2\pi F_0 m$$
$$z_Q(mT_M) = a_I[m]\sin 2\pi F_0 m + a_Q[m]\cos 2\pi F_0 m$$

In polar form, this expression can be written as

$$|z(mT_M)| = \sqrt{a_I^2[m] + a_Q^2[m]}$$
$$\measuredangle z(mT_M) = \measuredangle\Big(a_Q[m], a_I[m]\Big) + 2\pi F_0 m$$

Notice from the above equation that **a carrier frequency offset of** F∆ **keeps the magnitude unchanged but continually rotates the desired outputs** aI[m] **and** aQ[m] **on the constellation plane.** This is drawn for symbol-spaced samples in the scatter plot of Figures below for a 4-QAM and 16-QAM constellation.

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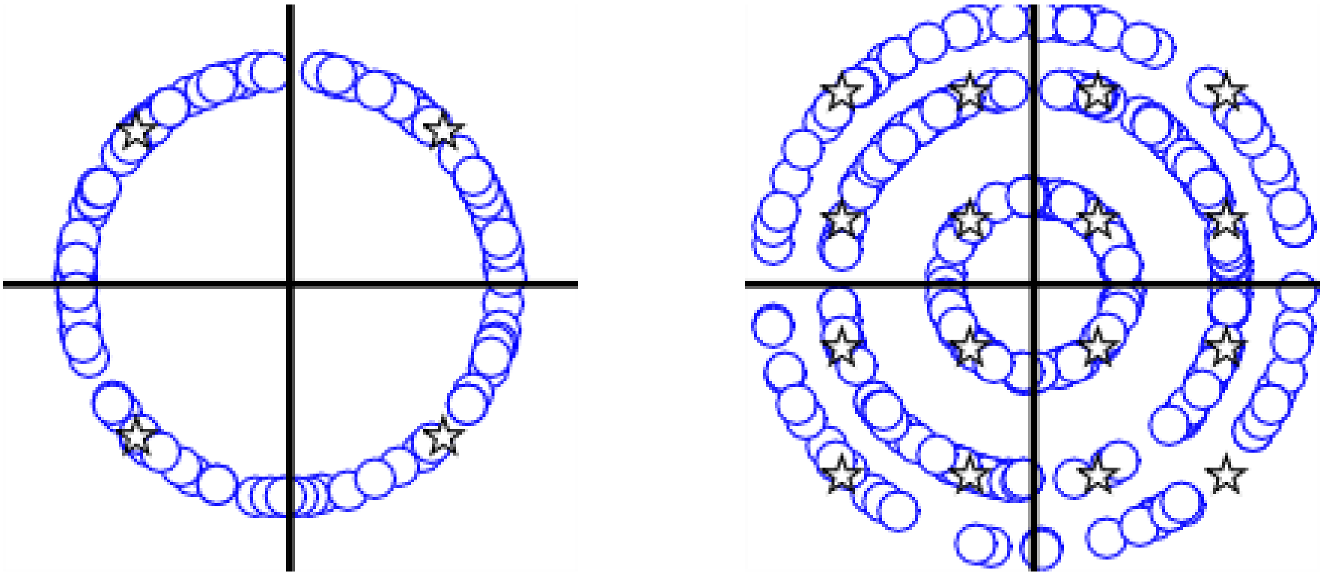

(a) 4-QAM constellation (b) 16-QAM constellation

Due to this reason, **a Carrier Frequency Offset (CFO), or** F∆**, in the received signal spins the constellation in a circle (or multiple circles for higher-order modulations)**. This is a natural outcome since the angular frequency is defined as the rate of change of phase.

The above results are summarized in Table below.

	Distortion	Matched Filter	Samples/Symbol
CFO $> F_{\Delta,\text{max}}$	Yes	No	None
$0.15 \cdot R_M <$ CFO $< F_{\Delta,\text{max}}$	No	No	L
CFO $< 0.15 \cdot R_M$	No	Yes	1

3. The Main Problem: The Effect Of the CFO Mismatch On The Decoding Of The OFDM Signal.

The carrier-frequency offset (CFO) refers to the mismatch between the frequency of the received signal and the frequency of the local oscillator at the receiver. Two factors contribute to the CFO: i) the frequency mismatch between the transmitter and the receiver oscillators; ii) the Doppler effect due to the relative mobility of the transmitter and the receiver. In practice, the oscillators at the transmitter and the receiver can never oscillate at identical frequencies; therefore, there always exists a CFO in the received baseband signal. Due to the CFO, the baseband signal is shifted in the frequency domain. As we go higher in carrier frequencies, the CFO is more pronounced due to both the oscillator frequency mismatch and the Doppler effect.

Multicarrier waveforms are more sensitive to the CFO than single-carrier waveform, since a subcarrier bandwidth is typically much smaller than the overall bandwidth in multicarrier waveforms. A small CFO can cause significant degradation in the symbol error rate performance. In an OFDM system, the CFO produces two effects: i) a common phase error (CPE), ii) an intercarrier interference (ICI). The CPE refers to a common phase rotation in all subcarriers and the ICI refers to an interference between the subcarriers due to the loss of subcarrier orthogonality. Let us define the CPE and the ICI, mathematically. Consider a received baseband signal subject to the CFO: $r[n] = x[n]e^{j2\pi\frac{\epsilon}{N}n}$, where ϵ is a normalized fractional CFO. The demodulation of the subcarrier l is then given by:

$$
\begin{aligned}
R_l &= \frac{1}{N}\sum_{n=0}^{N-1} r[n]e^{-j2\pi\frac{l}{N}n} \\
&\overset{(a)}{=} \frac{1}{N}\sum_{n=0}^{N-1}\sum_{i=0}^{N-1} X_i e^{j2\pi\frac{i}{N}n} e^{-j2\pi\frac{l}{N}n} e^{j2\pi\frac{\epsilon}{N}n} \\
&= X_l\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{\epsilon}{N}n} + \sum_{n=0}^{N-1}\sum_{i=0, i\neq l}^{N-1} X_i e^{j2\pi\frac{i-l+\epsilon}{N}n} \\
&\overset{(b)}{=} \alpha X_l \underbrace{e^{j\pi\epsilon\frac{N-1}{N}}}_{\text{CPE}} + \underbrace{\sum_{n=0}^{N-1}\sum_{i=0, i\neq l}^{N-1} X_i e^{j2\pi\frac{i-l+\epsilon}{N}n}}_{\text{ICI}_l},
\end{aligned}
\tag{6.10}
$$

where α is an attenuation factor common to all subcarriers, which is given by:

$$
\alpha = \frac{sinc(\epsilon)}{sinc(\epsilon / N)}. \tag{6.11}
$$

As an example, the two effects produced by CFO in an OFDM system (CPE and ICI) are illustrated in Fig. 6.23 for a 16-QAM constellation. CPE causes an identical phase rotation in all subcarriers within an OFDM symbol and ICI acts as an additive noise in the demodulated subcarrier. CPE can easily be compensated for as part of the channel equalization process.

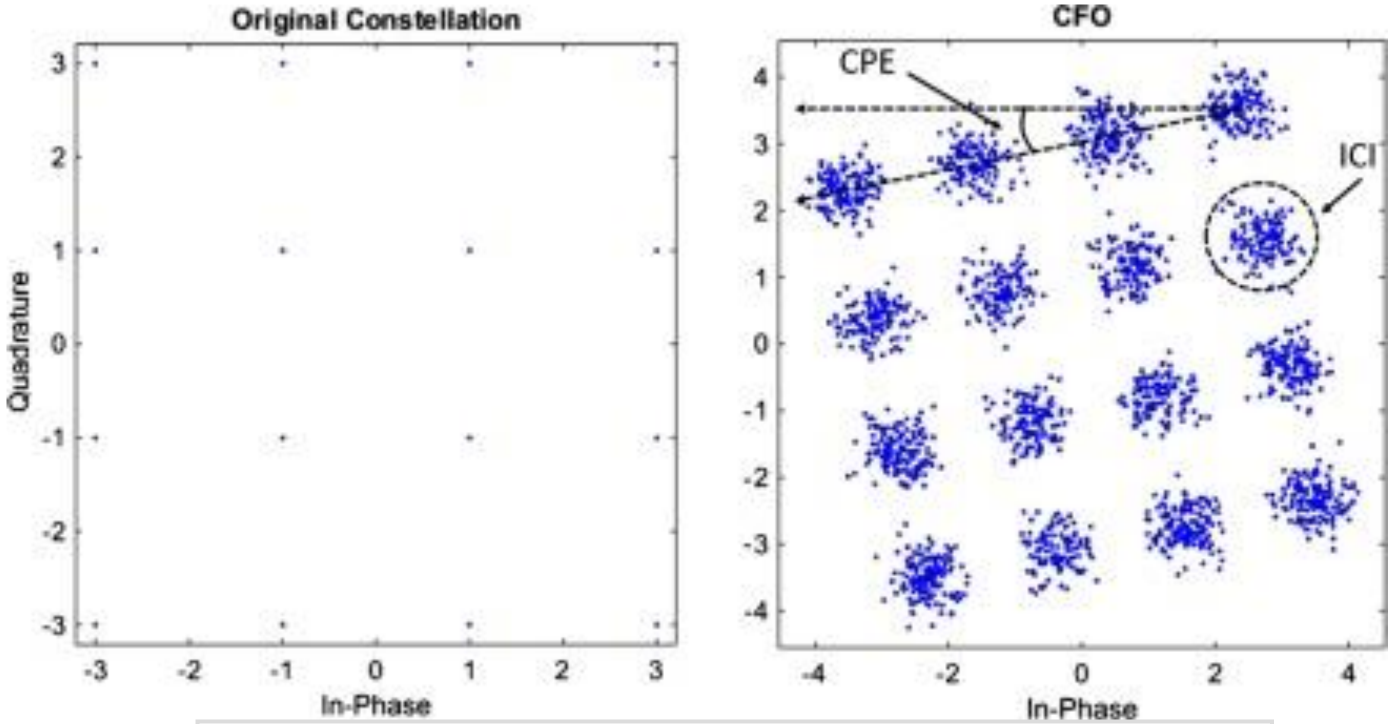

Figure 6.23. Effects of carrier-frequency offset in an OFDM system.

Assuming perfect CPE compensation, the achievable signal-to-interference ratio (SIR) due to ICI in the demodulated subcarrier *l* in the OFDM symbol subject to CFO is given by:

$$\mathrm{SIR}_l = \frac{\alpha^2 E\left[\,|X_l|^2\,\right]}{E\left[\,|\mathrm{ICI}_l|^2\,\right]}, \tag{6.12}$$

where $E[\cdot]$ is the expectation operator.

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4. <u>The Solution:</u> is by knowing how the receiver frequency shift Δf can be:

- **Estimated by the receiver.**
- **Compensated for.**
- **Practical Example: by using the Cyclic Prefix (CP) technique in the CFO Estimation of OFDM .**

CFO MODELING BASICS:

In the center of the OFDM spectrum the quantity of the Inter Carrier Interference (ICI), in comparison with their amount at the band edges, is almost double. This increase comes from the interfering of the subcarriers from both sides on the middle ones. However, the degradation of the SNR which is introduced by the frequency offset can be given by:

$$D_{freq} \cong \frac{10}{3 ln 10} (\pi \Delta f T)^2 \frac{E_s}{N_o}$$

Where Dfreq, T, Eb, No are in order; frequency offset, symbol duration, energy per bit (for OFDM signal) and one sided noise Power Spectrum Density (PSD).

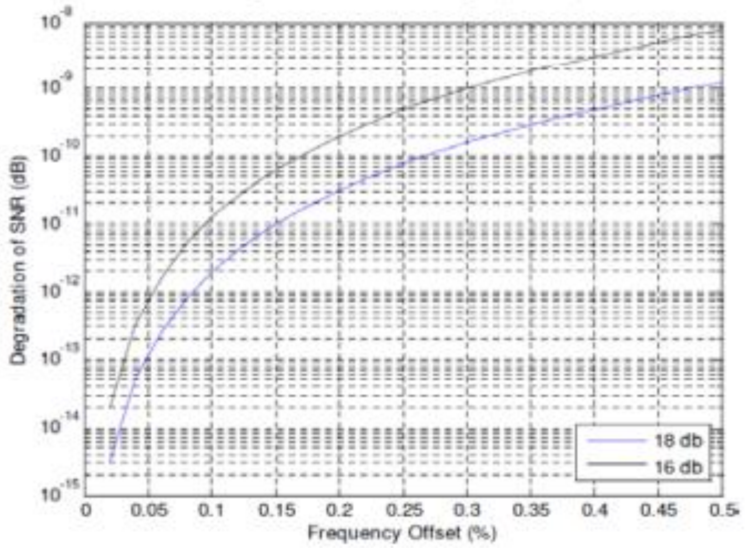

Fig. 7. SNR degradation as a function of the frequency offset

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If frequency offset is denoted as Δfc, the OFDM signal generated by the transmitter denoted as s(t) and y(t) is the signal received by the receiver, then:

$$s(t) = ej\omega tx(t)$$

$$y(t) = ej(\omega-\omega\hat{})t\ x(t)$$

$$\Delta\omega = \omega - \omega\hat{} = 2\pi\Delta fc$$

The frequency response of each sub-channel should be zero at all other sub-carrier frequencies, i.e., the sub-channels shouldn't interfere with each other. The effect of frequency offset is a translation of these frequency responses resulting in loss of orthogonality between the sub-carriers. CFO can produce Inter Carrier Interference (ICI) which can be much worse than the effect of noise on OFDM systems. That's why various CFO estimation and compensation algorithms have been proposed.

SOME "CFO" ESTIMATION & COMPANSATION ALGORITHMS:

For showing the importance of it, it is enough to mention that, by now the researchers have proposed numerous and various CFO estimation and compensation techniques and algorithms, which these methods can generally be categorized into two major branches:

A. Training based algorithm.

B. Blind algorithm and Semi-blind algorithm

A. Training based algorithm:

The training sequence can be designed the way that can limit the number of computations at the receiver side; therefore in general, these algorithms have a low computational complexity. On the other hand, the negative point of training based algorithm is the training sequences that must be transmitted from transmitter during its transmission. This can cause a reduction of the effectiveness of the data throughput. In this algorithm two repetitive OFDM symbols will be sent. This algorithm works on the base of knowing the start point of the OFDM symbol. This estimation for the small values of CFO is conditionally unbiased.

B. Blind and Semi-blind algorithms:

Another algorithm that has been used is called Blind CFO estimation algorithm. In these algorithms by using statistical properties of received signal, CFO will be estimated.

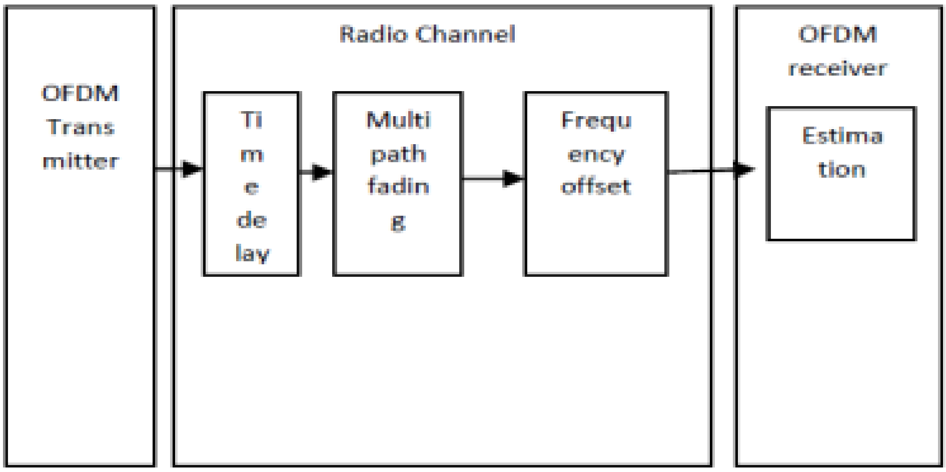

Fig. 8. Blind System Model

Since receiver doesn't have any knowledge of signal that transmitter has been sending, therefore, blind algorithms are considered to have a high computational complexity. Blind estimation is such technique which did not required any training symbol of pilot sub-carriers and performed well in frequency selective channels.

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This technique has low complexity due to the use of minimum number of operations of multiplication and division. The blind detection blind channel estimation based on the cyclic prefix is that this channel estimation concept is standard-compliant and can be applied to all commonly used OFDM systems that use a cyclic prefix. The blind detection without the necessity of pilot symbols for coherent detection is possible when joint equalization and detection is applied. This is possible by trellis decoding of differentially encoded PSK signals where the trellis decoding can efficiently be achieved by applying the Viterbi algorithm. The main design goal of a blind estimation is fast convergence to an operating point where the detection of information symbols is reliable as well as low computational complexity.

<u>Practical Example</u>

Using the Cyclic Prefix (CP) technique in the CFO Estimation of OFDM:

Cyclic prefix (CP) is a portion of an OFDM symbol used to absorb inter-symbol interference (ISI) caused by any transmission channel time dispersion and it can be used in CFO estimation. Figure (1) shows OFDM Symbol with CP. CP based estimation method exploits CP to estimate the CFO in time domain. Considering the channel effect is minimal and can be neglected, then, the lth OFDM symbol affected by CFO can be written as

$$y_l(n) = x_l(n)e^{\frac{j2\pi\varepsilon n}{N}} \tag{1}$$

Replacing n by $(n+N)$ in equation (1) the corresponding CP in the OFDM symbol can be written as

$$y_l(n+N) = x_l(n+N)e^{\frac{j2\pi\varepsilon\,(n+N)}{N}} \tag{2}$$

$$y_l(n+N) = x_l(n)e^{\left(\frac{j2\pi\varepsilon n}{N}+j2\pi\varepsilon\right)} \tag{3}$$

By comparing equations (1) and (3), we can find that the phase difference between CP and the OFDM symbol is 2πε. Therefore, the amount of CFO can be found from the argument of the multiplication of OFDM symbol by the conjugate of its CP:

$$\hat{\varepsilon} = \frac{1}{2\pi}\, arg\{y_l^*(n)y_l(n+N)\}, \mathrm{n} = -1, -2, \dots, -N_g \tag{4}$$

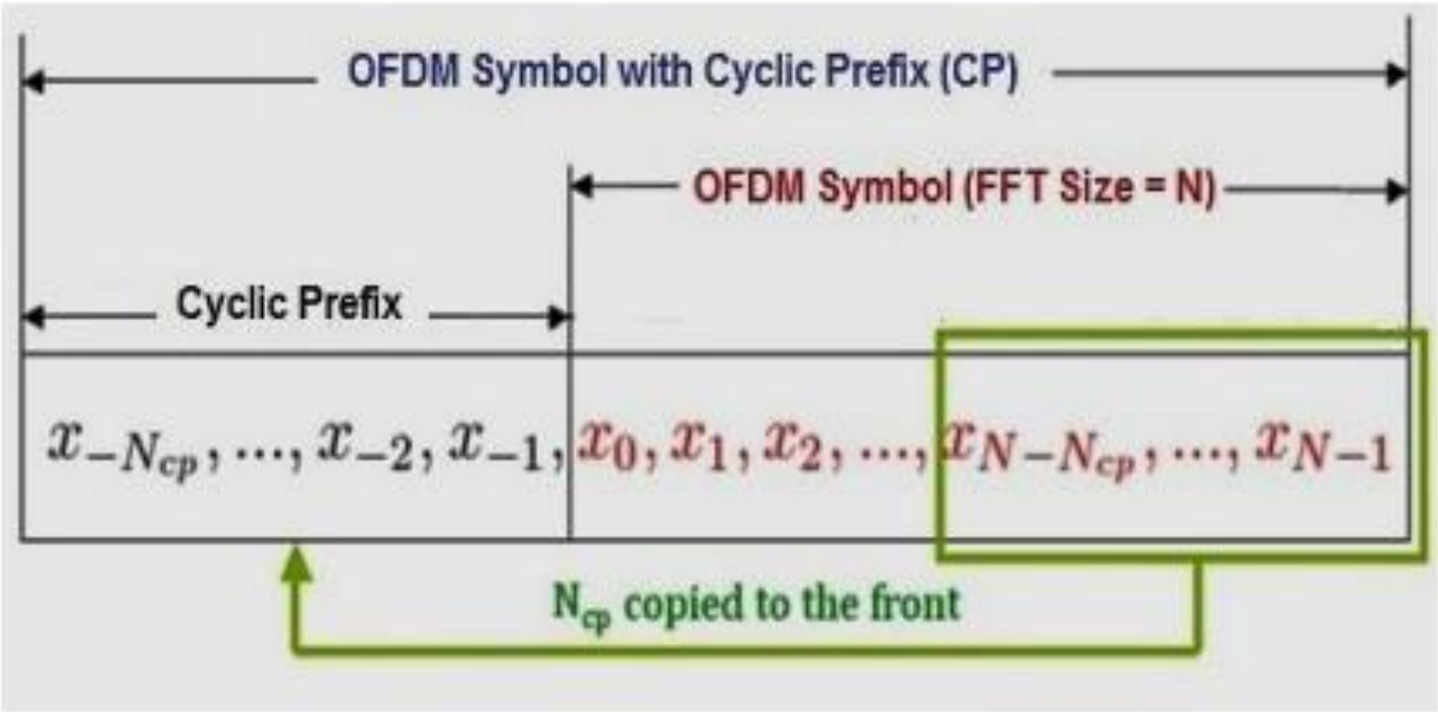

Fig. 1. OFDM Symbol with Cyclic Prefix

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In order to reduce the noise effect, its average can be taken over the samples in a CP interval as:

$$\hat{\varepsilon} = \frac{1}{2\pi} arg\left\{ \sum_{n=N_g}^{-1} y_l^*(n) y_l(n+N) \right\}, \mathrm{n} = -1, -2, \dots, -N_g \tag{5}$$

Since the argument operator arg( ) is performed by using $tan^{-1}()$, the range of CFO Estimation in equation (5) is $[-\pi,+\pi]/2\pi = [-0.5, +0.5]$ so that $|\hat{\varepsilon}| \leq 0.5$. Therefore, CP results CFO estimation in the range, $|\hat{\varepsilon}| \leq 0.5$. Hence, this technique is useful for the estimation of Fractional CFO (FFO). CFO estimation technique using CP does not estimate the integer offset [4]. To overcome this drawback, the training sequence technique is used to estimate CFO. This is helpful in increasing the range of the CFO estimation [16]-[22].

5. References:

1. Brief notes about OFDM (**click here**) :
    - What is OFDM?
    - How OFDM works.
2. Carrier frequency offset (CFO) in OFDM signal (**click here**):
    - What is Carrier Frequency Offset (CFO)?
    - How It Distorts the Rx Symbols.
3. The main problem (**click here**):

    the effect of the CFO mismatch on the decoding of the OFDM signal.
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