# REPORT -1

## WIRELESS COMMUNICATIONS


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**The Detailed Proof of The 2-Ray Ground Reflection Model**
**"Supplemented By: MATLAB Codes & graphs"**

# 1. The 2-Ray Ground Reflection Model

This model takes into account a line-of-sight and a ground reflection. It is a good approximation for propagation over a smooth well-reflecting terrain, as modeled by the plane-earth model, with:

$$P_R = \frac{\lambda^2}{(4\pi d)^2}\left[2\sin\frac{2\pi}{\lambda}\frac{h_T h_R}{d}\right]^2 G_T P_T G_R$$

# 2. The MATLAB Code & Graphs Representing The 2-Ray Ground Reflection Model

## The code:

- **Firstly: Pathloss Vs Distance:**

```
%--------------Assignment -1
%----------WIRELESS COMMUNICATIONS
%--------Prof. Dr. Said E. El-Khamy
%----Communication & Electronics department
%--Student Name: Asmaa Gamal Abdel-Halem Mabrouk Nagy %????? ???? ??? ?????? ????? ????
%--Student ID: 15010473
%--Title: The 2-Ray Ground Reflection Model MATLAB Codes & graphs

lambda = 0.3;
ht100=100;
ht30=30;
ht2=2;
hr=2;

axis=[];
p100=[];
p30=[];
p2=[];
pfsl=[];


for i=1000:5000
 d=10^(i/1000);
 axis =[axis d];
 fspower  = (lambda/(4*3.1415*d))^2 ;
 power100 = fspower * 4 *(sin(2*3.1415*hr*ht100/(lambda*d)))^2;
 power30  = fspower* 4 *(sin(2*3.1415*hr*ht30/(lambda*d)))^2;
 power2   = fspower * 4 *(sin(2*3.1415*hr*ht2/(lambda*d)))^2;

 p100 =[p100, 10*log10(power100)];
 p30 =[p30, 10*log10(power30)];
 p2 =[p2, 10*log10(power2)];
 pfsl=[pfsl, 10*log10(fspower)];
end

text('FontSize',18)
```

```
semilogx(axis,p100, 'g-',axis,p30, 'b-',axis,p2, 'r-',axis,pfsl,'y-')


xlabel('distance in m');
ylabel('pathloss');
text(1000,-66,'blue  : hr=30m');
text(1000,-74,'red   : hr=2m');
text(1000,-58,'red   : hr=100m');
text(1000,-50,'yellow: free space');

text(50,-180,'lambda = 0.30 m');
text(50,-190,'hr = 2 m');
```

- ## Secondly: Power vs Distance:

Here are the keys from the configuration relevant for positioning the hosts:

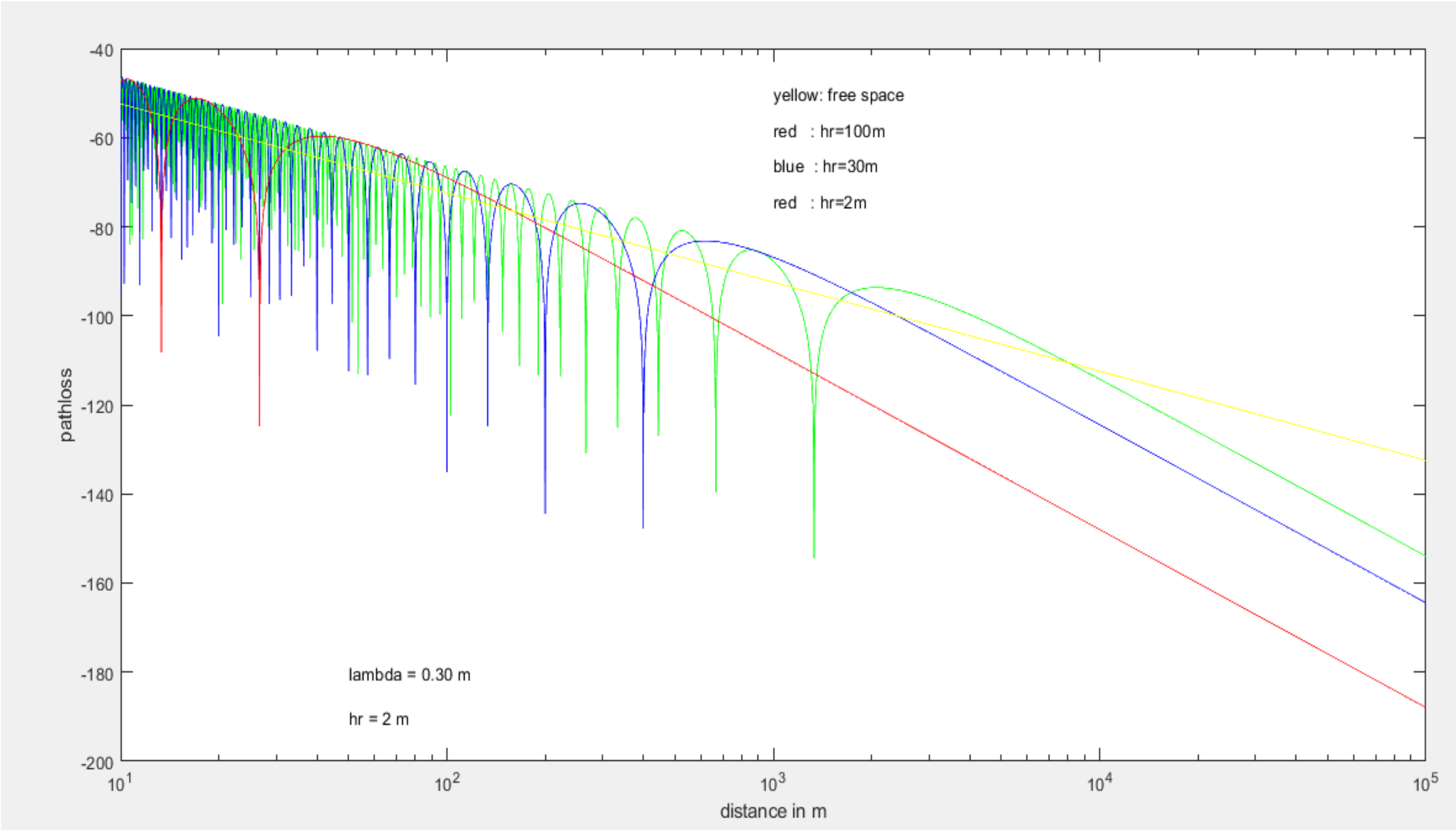
```
*.*.mobility.initFromDisplayString = false
*.*.mobility.typename = "StationaryMobility"
*.*.mobility.initialY = 200m
*.*.mobility.initialZ = 2m

*.source.mobility.initialX = 0m
*.destination.mobility.initialX = ${distance=0..50 step 0.25, 51..100 step 1, 105..200 step 5, 220..1000 step 20}m.
```

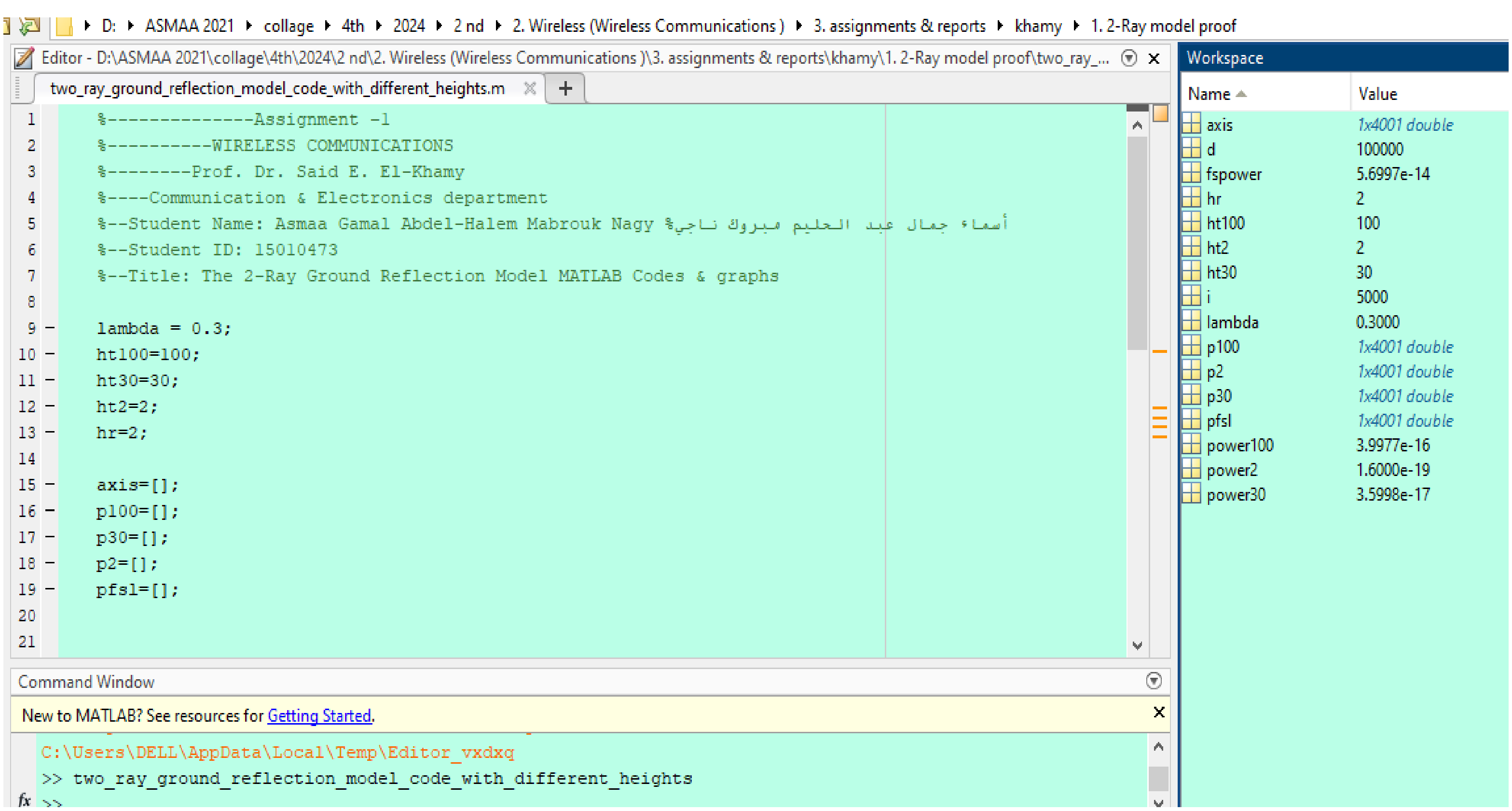
The other variable in the parameter study is the path loss type, which takes on the following values:
FreeSpacePathLoss, TwoRayGroundReflection, TwoRayInterference, RicianFading, LogNormalShadowing
The source host will transmit with the default power of 20mW. We will record the power of the received transmission, using the receptionPower statistic. The receptionPower statistic is declared in the NED file, and it uses the receptionMinSignalPower signal of the radio medium module as input:

```
@statistic[receptionPower](source="receptionMinSignalPower(radioMedium.signalArrivalStarted)"; record=last);
```

# The MATLAB Results Graphs:

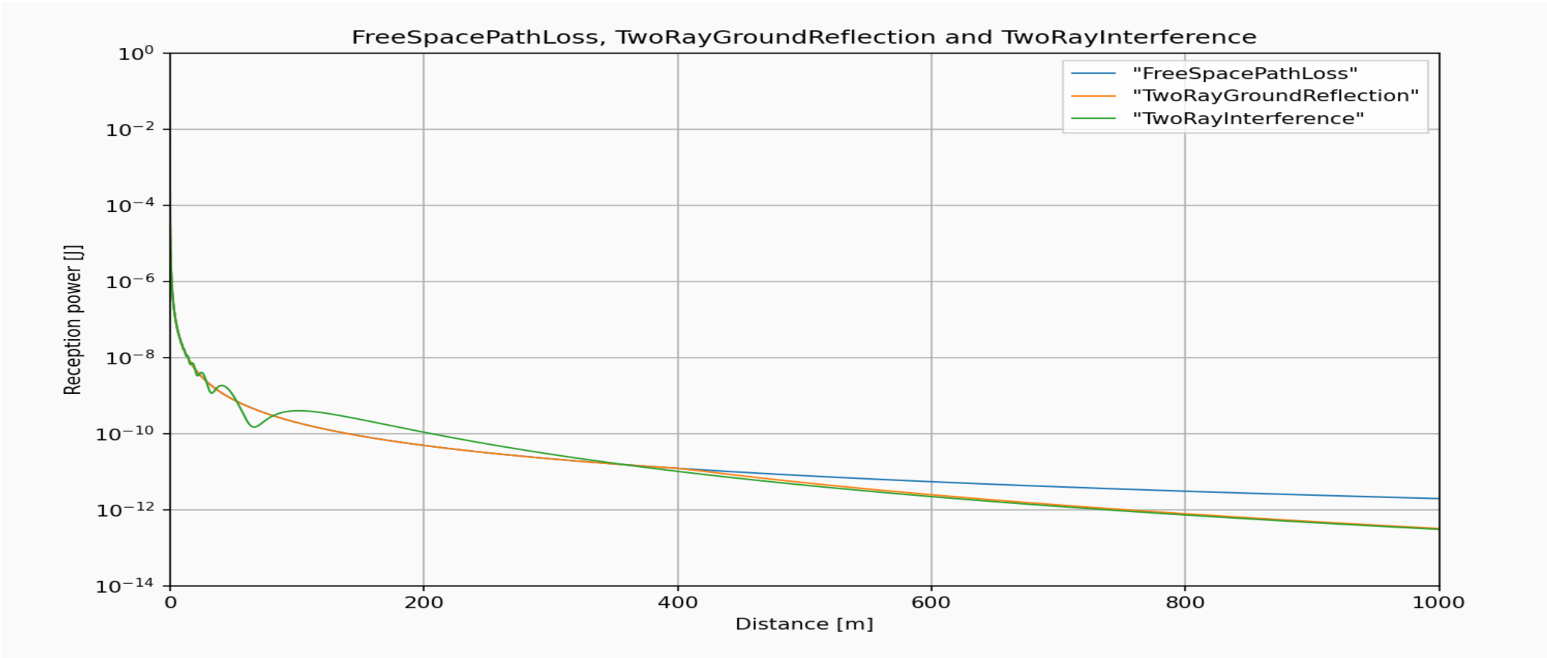
- ## Firstly: Pathloss Vs Distance:



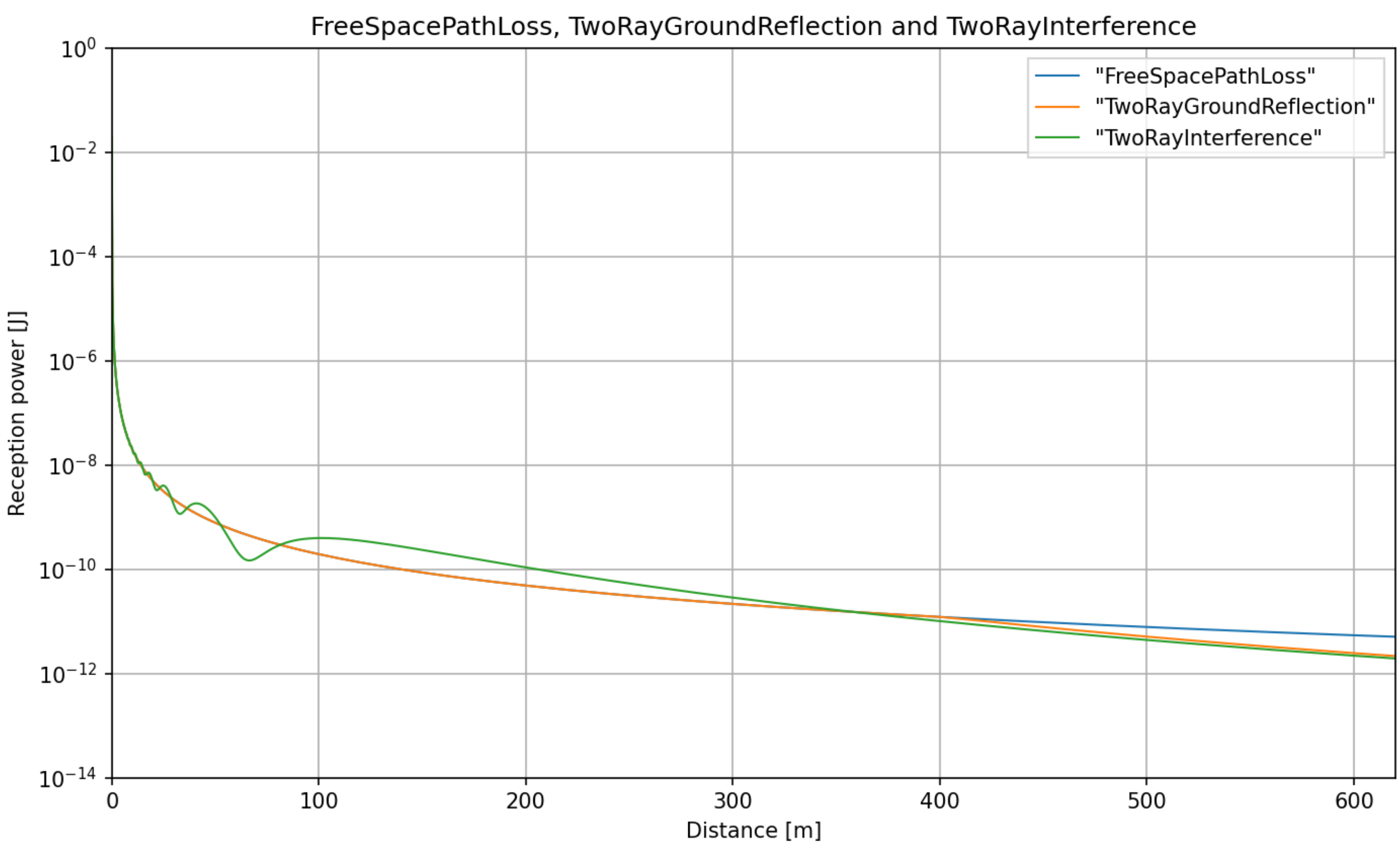



- ## **Secondly: Power vs Distance:**

The power of the received signal vs. distance, using FreeSpacePathLoss , TwoRayGroundReflection ,and TwoRayInterference path loss module types, is displayed on the following plot:

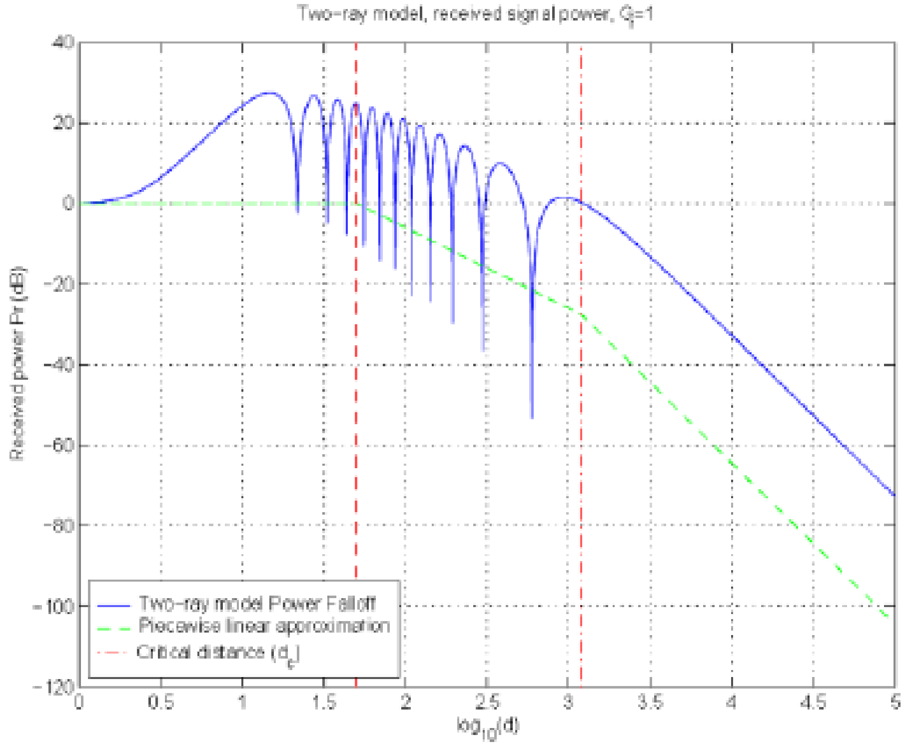



Here is the same plot zoomed in:



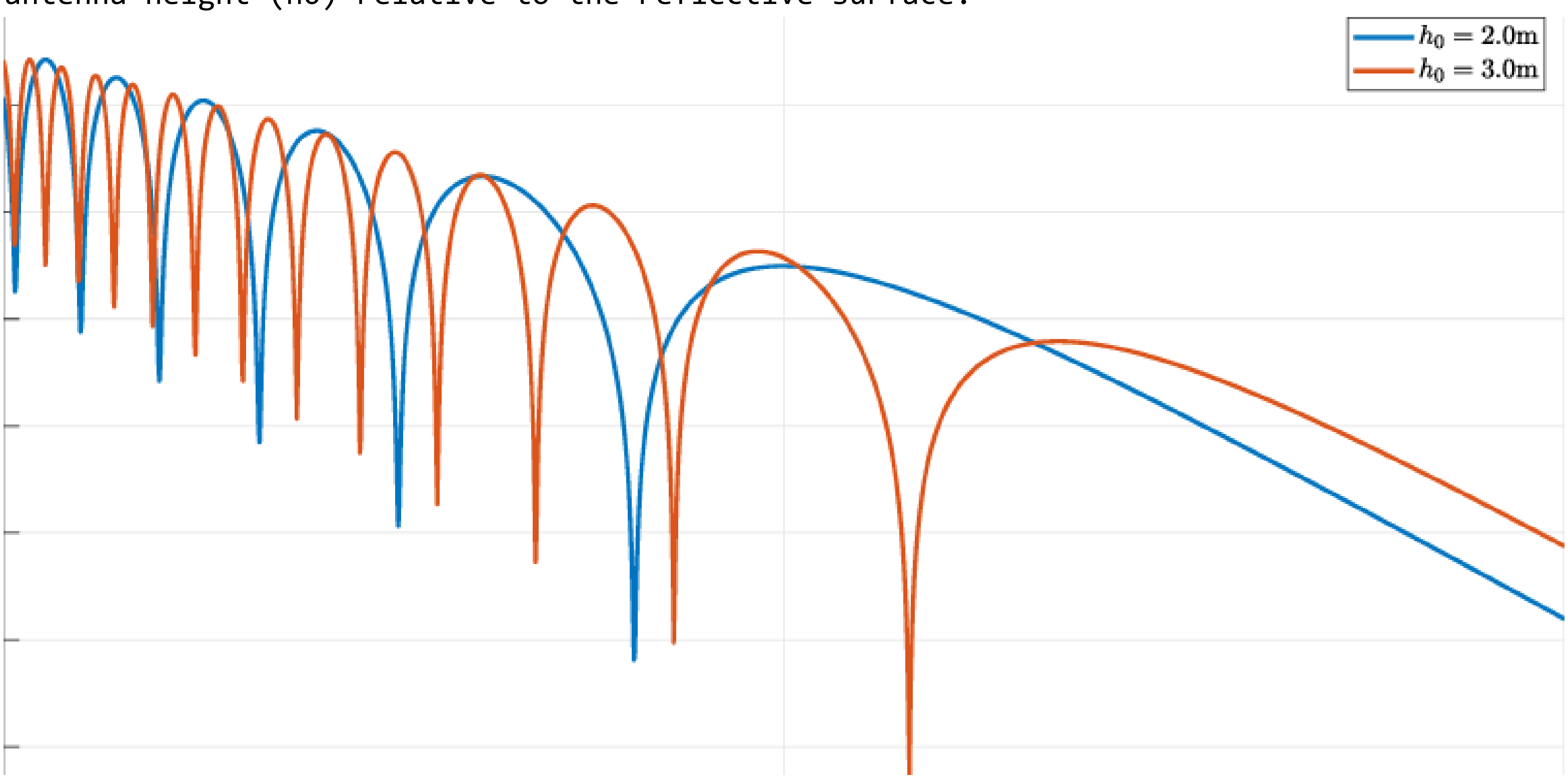

It is apparent that the two-ray ground reflection model yields the same values as the free space path loss model, up until the cross-over distance. After that point, the two curves diverge. The power of the two-ray interference model fluctuates in the near-field and converges to the two-ray ground reflection model in the far- field. Thus the two-ray interference model can be used for more realistic two-ray propagation simulations.

Then, the Received Power versus Distance for Two-Ray Model only can be shown as:




## The conclusion:

Two-ray model showing that received power is affected by both the link distance and the antenna height (h0) relative to the reflective surface:




Here in the above MATLAB sketch, We focus our attention on links of short-to-medium-range distances with antenna heights near-to-the-water-surface.

According to the below equation:

$$|E_{TOT}(d)| = 2\frac{E_0 d_0}{d}\sin\left(\frac{\theta_\Delta}{2}\right)$$

Where:

$$\frac{\theta_\Delta}{2} \approx \frac{2\pi h_t h_r}{\lambda d} < 0.3\ \mathbf{rad}$$

Which implies that:

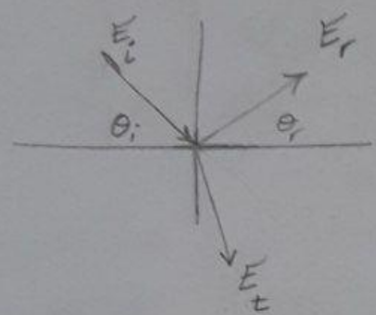
$$d > \frac{20\pi h_t h_r}{3\lambda} \approx \frac{20 h_t h_r}{\lambda}$$

## 3. The Detailed Proof of The 2-Ray Ground Reflection Model

③

# [2] Ground Reflection : (2 Ray Model)

* Reflection from (Perfect Conductor) :

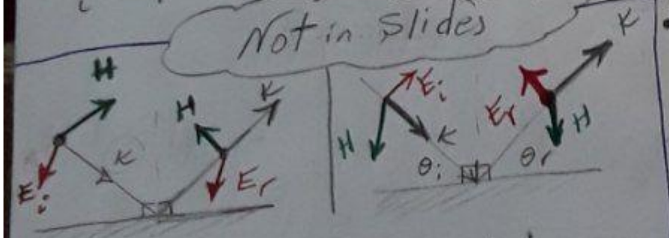



$E_i$ = incident Electric field .
$E_r$ = reflected " "
$E_t$ = transmitted " "

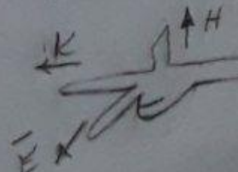
E

| ~~Parallel~~ - Vertical Polarization | ~~Perpendicular~~ horizontical Polarization |
|---|---|
| $\theta_i = \theta_r$ | $\theta_i = \theta_r$ |
| $E_i = E_r$ | $E_i = -E_r$ |

Not in Slides

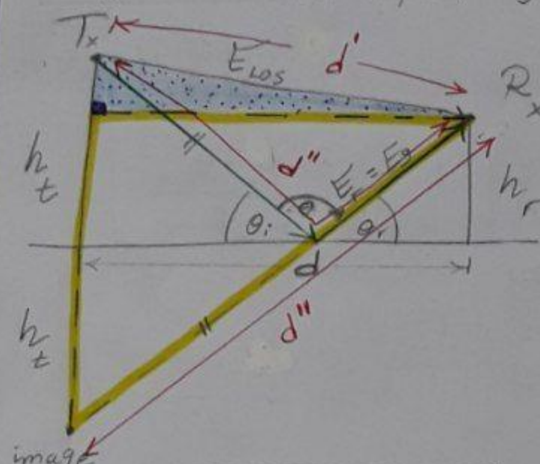



K → Propagation direction
E → Electric field
H → magnatic field ]⊕→ EMW




Right Hand Rule

* use it when: ① reflection from Ground only
① Los
→ $d >> h_t$ , $d >> h_r$
* 2 Ray Model is the 1st simple fading Model




* Given : Field equations

• d = several Kms < $d >> \frac{2 h_t h_r}{\lambda}$ , $d > 20 \frac{h_t h_r}{\lambda}$
• $d_o$ = ref distance @ $E = E_o$
• $h_t$ = 50 - 100 m
• $E_{tot} = E_{Los} + E_g$
• $|E|_{Los} = \frac{E_o d_o}{d}$ (inverse Proportional)
Free Space For Field
∴ $\boxed{P \propto \frac{1}{d^2}}$ [in Free space Los]

∴ $E_{tot}(d) = \frac{E_o d_o}{d}$ shortcut → * $\frac{d}{d}$ & Substitute: $d = 2\pi \frac{h_t h_r}{\lambda}$ numerator

* Required : Prove that in 2-Ray model Ground reflection :

① $E_{tot}(d) = \frac{2 E_o d_o (2\pi h_t h_r)}{\lambda d^2}$

② $\boxed{P_{r\,2\text{-}Ray} \propto \frac{1}{d^4}}$

③ $P_r = \frac{P_t G_t G_r h_t^2 h_r^2}{d^4}$

Reference: RaPPaPort Page. 97.

## Proof Solution

From the shape Geometry:

$$d' = \sqrt{d^2 + (h_t - h_r)^2} = d\sqrt{1 + \frac{(h_t - h_r)^2}{d^2}}$$ → from the blue dotted angle

$$\because \sqrt{1 + \text{small}} \simeq 1 + \frac{1}{2}\,\text{small}$$

$$\therefore d' = d\left[1 + \frac{1}{2}\frac{(h_t - h_r)^2}{d^2}\right]$$

Simillary

$$\therefore d'' = d\left[1 + \frac{1}{2}\frac{(h_t + h_r)^2}{d^2}\right]$$ → from the Yellow angle

$$\therefore \Delta d = d'' - d' = \cancel{d} + \frac{(h_t + h_r)^2}{2d} - \cancel{d} - \frac{(h_t - h_r)^2}{2d}$$

$$= \frac{\cancel{h_t^2} + \cancel{h_r^2} + 2h_t h_r - \cancel{h_t^2} - \cancel{h_r^2} + 2h_t h_r}{2d}$$

$$\therefore \boxed{\Delta d = \frac{2h_t h_r}{d}} = \text{Path difference}$$

$$\because \Delta\theta_{\text{2-Ray}} = \beta \cdot \Delta d = \frac{2\pi}{\lambda}\cdot \Delta d = \frac{4\pi h_t h_r}{\lambda d} = \text{Phase Shift} = \theta$$

$$\because E_{\text{Far Field}} = \frac{k}{r} e^{-j\beta r}, \quad E_{\text{reflect}} = \frac{k}{r}\rho e^{-j\beta d''}$$

Free space Los

$$\therefore E_{tot} = E_{Los} + E_g = \frac{k}{r} e^{-j\beta d'} + \frac{k}{r}\,\underset{\downarrow}{\rho}\, e^{-j\beta(d' + \Delta d)}$$

reflection Coef
For Perfect Conductor: $\rho = -1$

$$E_{tot} = \frac{k}{r} e^{-j\beta d'}\left[1 - e^{-j\overset{\Delta\theta}{\boxed{\beta\Delta d}}}\right]$$

$\boxed{r = d}$ $\neq d' \neq d''$

$\left[1 - e^{-j\beta\Delta d}\right] \to 2\sin\left(\frac{\theta}{2}\right)$

(later)

$\because d \gg \boxed{\frac{4h_t h_r}{\lambda}}$ $\therefore \sin\phi \simeq \phi$

$\because \phi \ll$

$$E_{tot} = |E_r| = \frac{2k}{d}\sin\left(\frac{2\pi h_t h_r}{\lambda d}\right)$$

$$\therefore \left|E\right|_{ToT} = \frac{K'}{d^2} \quad K' \to \text{const}$$

(Far field - 2 Ray Model - Ground Reflection)

$$E \propto \frac{1}{d} \xrightarrow[\text{for any: "Far field" except: < 2-Ray Model or higher Complexity}]{\therefore} P \propto \frac{1}{d^2}$$

$$\therefore E \propto \frac{1}{d^2} \xrightarrow[\substack{\cdot\ 2\text{Ray} \\ \cdot\ \text{Ground Reflect} \\ \cdot\ \text{Far field}}]{\therefore} P \propto \frac{1}{d^4}$$

$E_o d_o = K$ ← const related to $E_o$

$$\therefore \left|E_{tot}(d)\right| = \frac{2\pi 2 (K) h_t h_r}{\lambda d^2} = \left|E_r\right| = \frac{2 E_o d_o (2\pi h_t h_r)}{\lambda d^2}$$

$$\therefore \left|\frac{E_{tot}}{E_o d_o}\right| = \frac{4\pi h_t h_r}{\lambda d^2}$$

$$\because P_r \propto \left|E_{tot}\right|^2 \xrightarrow{\therefore} P_r \propto \left|\frac{E_{tot}}{E_o d_o}\right|^2 \xrightarrow{\therefore} P_r \propto \frac{1}{d^4}$$

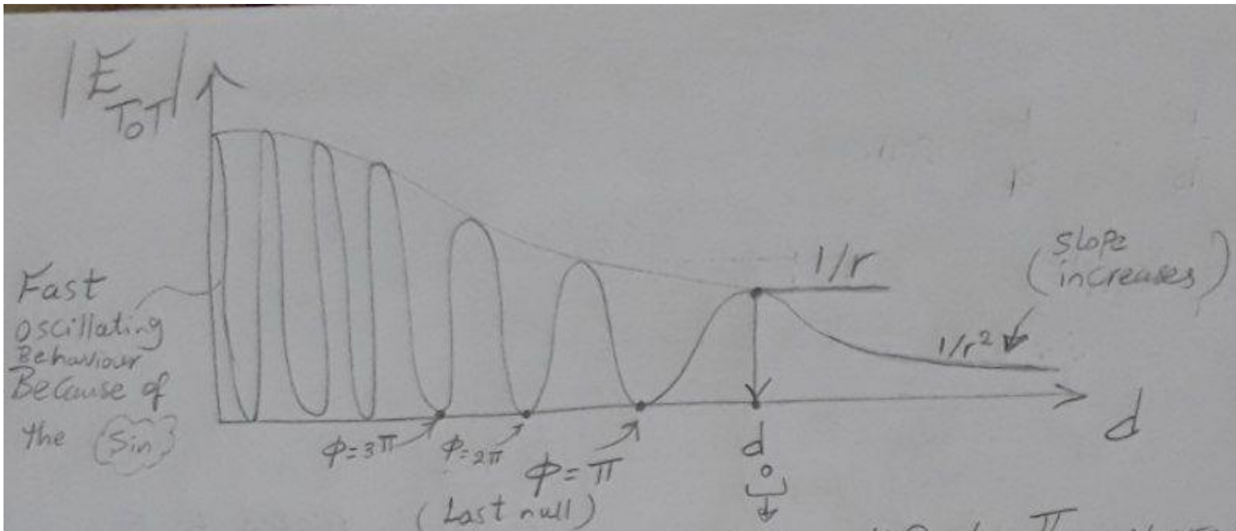
$$\because P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 \underbrace{d^2}_{\text{free space "LOS"}}}$$

← where $\boxed{\ell \geq 1}$ → assume $\ell = 1$

$$\boxed{\therefore P_{r\,(\text{2 Ray Ground Reflection})} = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2} \cdot \left|\frac{E_{tot}}{E_o d_o}\right|^2}$$

$$= \frac{P_t G_t G_r \cancel{\lambda^2}}{\cancel{(4\pi)^2}} \cdot \frac{\cancel{(4\pi)^2} h_t^2 h_r^2}{\cancel{\lambda^2} d^4}$$

$$\boxed{P_r = \frac{P_t G_t G_r h_t^2 h_r^2}{d^4}} \quad \#$$




$\uparrow \phi$ increases in the Left direction

Last peak @ $\phi = \frac{\pi}{2}$ where:

$$\boxed{\phi = \frac{2\pi h_t h_r}{\lambda d}}$$

$$\therefore \frac{\pi}{2} = \frac{2\pi h_t h_r}{\lambda d}$$

$$\therefore d = d_o = \frac{4 h_t h_r}{\lambda} = \text{reference distance @ } E = E_o$$

$\therefore$ for stable operation we choose $d > d_o$ to avoid instability in the Signal Level due to speedy Variation in the amplitude of $|E_{ToT}|$

$\therefore$ if $d \gg \frac{4 h_t h_r}{\lambda}$ $\quad \therefore \phi \ll$ $\quad \therefore \sin\phi = \phi$

$$\therefore |E_{tot}|_{2\text{-Ray}} = \frac{2 E_o d_o (2\pi h_t h_r)}{\lambda d^2}$$

$$\therefore P_{r\,2\text{-Ray}} = \frac{P_T G_T G_r h_t^2 h_r^2}{d^4}$$

(as Before)

\#

$$\therefore \boxed{P_{2\text{-Ray}} \propto \frac{1}{d^4}}$$

④ Prove That the 2-Ray Ground Reflection Power Decrease 40 dB Per decade [or] octave

Solution

$\because P \propto \frac{1}{d^4}$

$\therefore \frac{P}{P_0} = \frac{d_0^4}{d^4}$

$P_0$ reference

$\alpha = n$

$P_{dB} = P_0\Big|_{dB} - 10 \overset{(4)}{} \log_{10}\left(\frac{d}{d_0}\right) = P_0 - 10\alpha \log_{10}\frac{d}{d_0}$

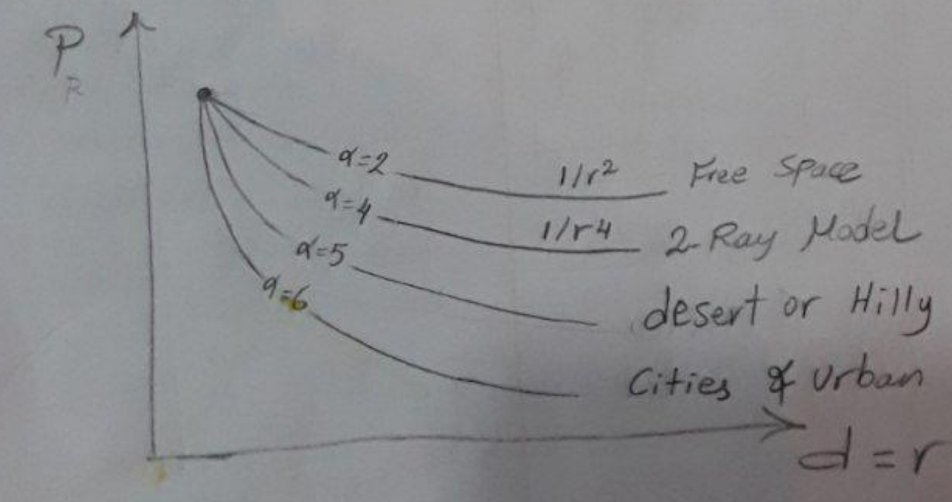
$P_{dB} = P_{0_{dB}} - 40 \log \frac{d}{d_0}$ $\qquad = P_0 \oplus 10\,\alpha \log\frac{d_0}{d}$

if: Power → ⊕
attenuation → ⊖

∴ In general:

$P \propto \frac{1}{d^{\boxed{n}}}$ → Parameter depends on the Environment

- أرض مدن مليئة بمباني وعوائق — $\alpha = 6$ destructive interference; (Some times) Constructive interference ← $\alpha = 3$
- أرض صحراوية — $\alpha = 5$
- 2-Ray — $\alpha = 4$: $\frac{E}{E_0} = \frac{d_0^2}{d^2}$, $\frac{P}{P_0} = \frac{d_0^4}{d^4}$
- Free space LOS — $n = \alpha = 2$: $\frac{E}{E_0} = \frac{d_0}{d}$, $\frac{P}{P_0} = \frac{d_0^2}{d^2}$




thank you


BYE