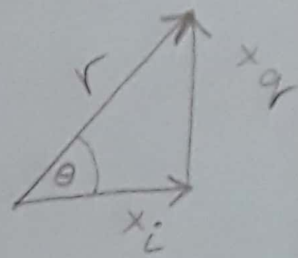
Proof ①: Rayleigh Fading Model of NLOS Multipaths Propagation

$\therefore x(t) = x_i + j x_q = r e^{j\theta}$



$\therefore r = \sqrt{x_i^2 + x_q^2}$, $\theta = \tan^{-1} \frac{x_q}{x_i}$

$\therefore x_i = r \cos\theta$, $x_q = r \sin\theta$

$\therefore$ to Change Variables in a Joint probability:

$P(r,\theta) = |J| \; P(x_i, x_q)$ where $|J| = \begin{vmatrix} \frac{\partial x_i}{\partial r} & \frac{\partial x_i}{\partial \theta} \\ \frac{\partial x_q}{\partial r} & \frac{\partial x_q}{\partial \theta} \end{vmatrix} = r$

$\therefore P(r,\theta) = r \; P(x_i, x_q)$

$\because x_i$ & x_q Gaussian (i.i.d) R.V. with Zero Mean

$\therefore P(x_i, x_q) = P(x_i)\, P(x_q) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x_i^2}{2\sigma^2}} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x_q^2}{2\sigma^2}} = \frac{1}{2\pi\sigma^2} e^{\frac{-r^2}{2\sigma^2}}$

$$\therefore \boxed{P(r,\theta) = \frac{r}{2\pi\sigma^2} \cdot e^{-r^2/2\sigma^3}}$$

$\therefore P(r)_{\text{Rayleigh}} = \int_0^{2\pi} P(r,\theta)\, d\theta =$

$$\boxed{PDF_{\text{Rayleigh}} = \frac{r}{\sigma^2} e^{\frac{-r^2}{2\sigma^2}}}$$

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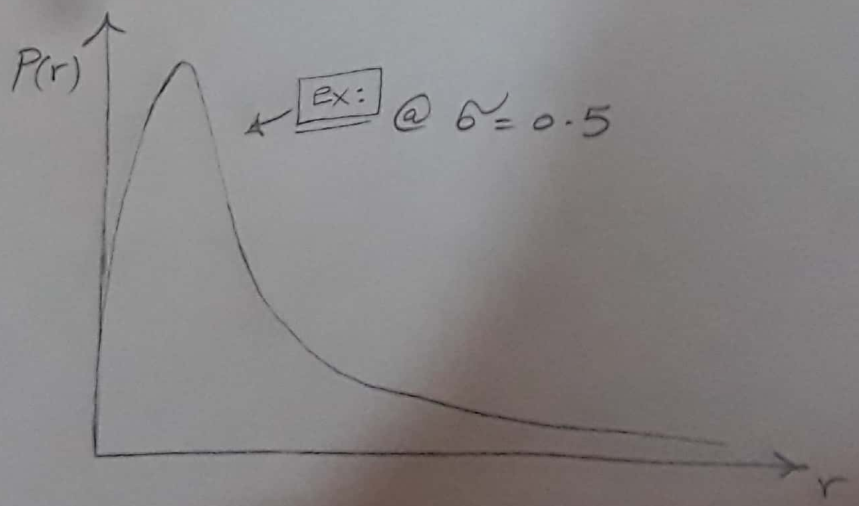
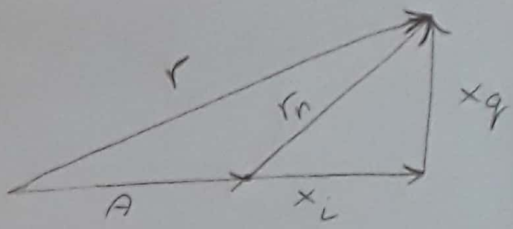


Proof ②:

Rician Fading Model of LOS & MultiPath Propagation

$\because X(t) = A + x_i + j\,x_q = r\,e^{j\theta}$



$\therefore r = \sqrt{(A+x_i)^2 + x_q^2}$

$= \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\dfrac{x_q}{x_i + A} = \tan^{-1}\dfrac{y}{x}$

$\therefore x = r\cos\theta \quad , \quad y = r\sin\theta$

$\because P(r,\theta) = |J|\,P(x,y) \xrightarrow{\text{where}} |J| = \begin{vmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial x}{\partial \theta} \\ \dfrac{\partial y}{\partial r} & \dfrac{\partial y}{\partial \theta} \end{vmatrix} = r$

$\because$ x is gaussian R.V. with mean A
y " " " " mean 0
and → x, y is indep

$\therefore P(x,y) = P(x)\,P(y) = \dfrac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-A)^2/2\sigma^2} \cdot \dfrac{1}{\sigma\sqrt{2\pi}} e^{-y^2/2\sigma^2}$

$= \dfrac{1}{2\pi\sigma^2} e^{-(r^2+A^2-2xA)/2\sigma^2}$

$$\boxed{\therefore P(r,\theta) = \frac{r}{2\pi\sigma^2}\, e^{-\frac{r^2+A^2}{2\sigma^2}} \cdot e^{\frac{rA\cos\theta}{\sigma^2}}}$$

$$\boxed{\therefore P(r)_{\text{Rician}} = \int_0^{2\pi} P(r,\theta)\,d\theta = \frac{r}{\sigma^2} \cdot e^{-\frac{(r^2+A^2)}{2\sigma^2}} \cdot I_0\left(\frac{rA}{\sigma^2}\right)}$$ #