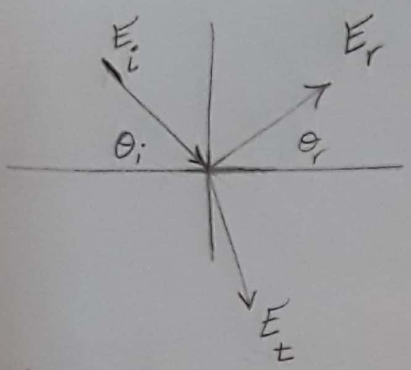
2 Ground Reflection (2 Ray Model)

* Reflection from Perfect Conductor:



E_i = incident Electric field.

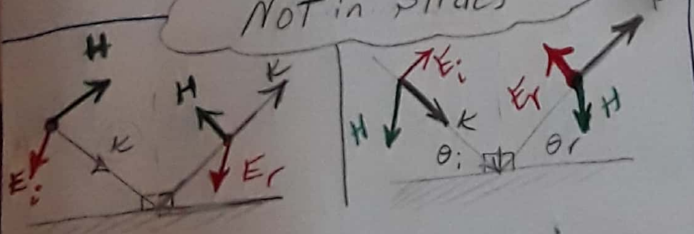
E_r = reflected " "

E_t = transmitted " "

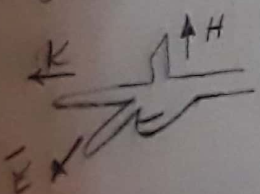
E

~~Parallel~~ - Vertical Polarization	~~Perpendicular~~ - horizontal Polarization
$\theta_i = \theta_r$	$\theta_i = \theta_r$
$E_i = E_r$	$E_i = -E_r$

Not in Slides



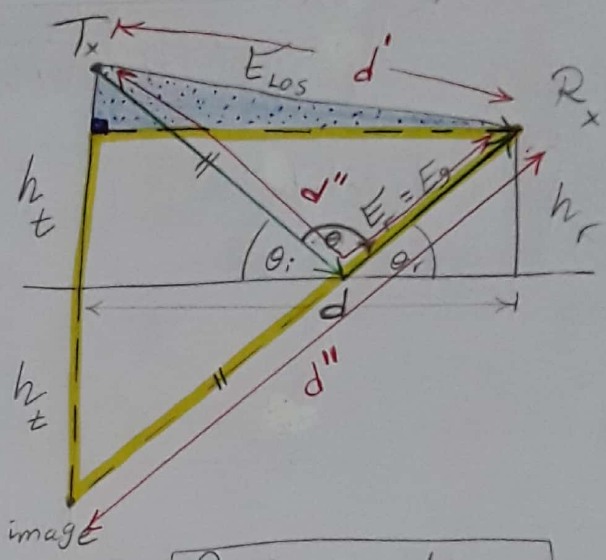
K → Propagation direction
E → Electric field
H → magnatic Field
] ⊕ → EMW



Right Hand Rule

* use it when: ① reflection from Ground only, ① LOS
 → $d \gg h_t$, $d \gg h_r$

* 2 Ray Model is the 1st simple fading Model



* Given: Field equations

- d = several Km_s < $d \gg \frac{2 h_t h_r}{\lambda}$, $d > 20 \frac{h_t h_r}{\lambda}$
- d_o = ref distance @ $E = E_o$
- h_t = 50 - 100 m
- $E_{tot} = E_{LOS} + E_g$
- $|E|_{LOS} = \frac{E_o d_o}{d}$ (inverse Proportional)
 Free space Far Field
 ⇒ $P \propto \frac{1}{d^2}$ [in Free space LOS]

∴ $E_{tot}(d) = \frac{E_o d_o}{d}$

shortcut → $* \frac{d}{d}$ & Substitute: $d = 2\pi \frac{h_t h_r}{\lambda}$ numerator

*Required: Prove that in 2-Ray model Ground reflection:

① $E_{tot}(d) = \frac{2 E_o d_o (2\pi h_t h_r)}{\lambda d^2}$

② $\boxed{P_{r\,\text{2-Ray}} \propto \frac{1}{d^4}}$

③ $P_r = \frac{P_t G_t G_r h_t^2 h_r^2}{d^4}$

Reference: RapPaPort Page. 97

Proof Solution

From the shape Geometry:

$$d' = \sqrt{d^2 + (h_t - h_r)^2} = d\sqrt{1 + \frac{(h_t-h_r)^2}{d^2}}$$ → From the blue dotted angle

$$\because \boxed{\sqrt{1 + \text{Small}} \simeq 1 + \frac{1}{2}\,\text{small}}$$

$$\therefore d' = d\left[1 + \frac{1}{2}\frac{(h_t-h_r)^2}{d^2}\right]$$

Similiary

$$\therefore d'' = d\left[1 + \frac{1}{2}\frac{(h_t+h_r)^2}{d^2}\right]$$ → from the Yellow angle

$$\therefore \Delta d = d'' - d' = d + \frac{(h_t+h_r)^2}{2d} - d - \frac{(h_t-h_r)^2}{2d}$$

$$= \frac{\cancel{h_t^2} + \cancel{h_r^2} + 2h_t h_r - \cancel{h_t^2} - \cancel{h_r^2} + 2h_t h_r}{2d}$$

$$\therefore \boxed{\Delta d = \frac{2h_t h_r}{d}} = \text{Path difference}$$

$$\because \underset{2\cdot\text{Ray}}{\Delta\theta} = \beta\cdot\Delta d = \frac{2\pi}{\lambda}\cdot\Delta d = \frac{4\pi h_t h_r}{\lambda d} = \text{Phase shift} = \theta$$

$$\because E_{\substack{\text{Far Field}\\\text{Free space}}} = \frac{k}{r}e^{-j\beta r}, \quad E_{\text{reflect}} = \frac{k}{r}\rho e^{-j\beta d''}$$

LOS

$$\therefore E_{tot} = E_{LOS} + E_g = \frac{k}{r}e^{-j\beta d'} + \frac{k}{r}\rho e^{-j\beta(d'+\Delta d)}$$

ρ: reflection Coef

For perfect Conductor: $\rho = -1$

$$E_{tot} = \frac{k}{r}e^{-j\beta d'}\underbrace{\boxed{\left[1 - e^{-j\overset{\Delta\theta}{\boxed{\beta\Delta d}}}\right]}}_{2\sin\left(\frac{\theta}{2}\right)}$$

$r = d$
$\neq d' \neq d''$

(Later)

$$\because d \gg \boxed{\frac{4h_t h_r}{\lambda}} \quad \therefore \sin\phi \simeq \phi$$

$$\therefore \phi \ll$$

$$E_r = |E_r| = \frac{2k}{d}\sin\left(\frac{2\pi h_t h_r}{\lambda d}\right)$$

$$\therefore |E|_{ToT} = \frac{k'}{d^2} \quad k' \rightarrow \text{Const}$$

(Far Field - 2 Ray Model - Ground Reflection)

$$E \propto \frac{1}{d} \xrightarrow{\therefore} P \propto \frac{1}{d^2}$$

for any: "far field" except: 2-Ray Model or higher Complexity

$$\therefore E \propto \frac{1}{d^2} \xrightarrow{\therefore} P \propto \frac{1}{d^4}$$

- 2Ray
- Ground Reflect
- Far field

$E_o d_o = K$ ← const related to E_o

$$\therefore |E_{tot}(d)| = \frac{2\pi 2 (K) h_t h_r}{\lambda d^2} = |E_r| = \frac{2 E_o d_o (2\pi h_t h_r)}{\lambda d^2}$$

$$\therefore \left| \frac{E_{tot}}{E_o d_o} \right| = \frac{4\pi h_t h_r}{\lambda d^2}$$

$$\because P_r \propto |E_{tot}|^2 \xrightarrow{\therefore} P_r \propto \left| \frac{E_{tot}}{E_o d_o} \right|^2 \xrightarrow{\therefore} P_r \propto \frac{1}{d^4}$$

$$\because P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$$

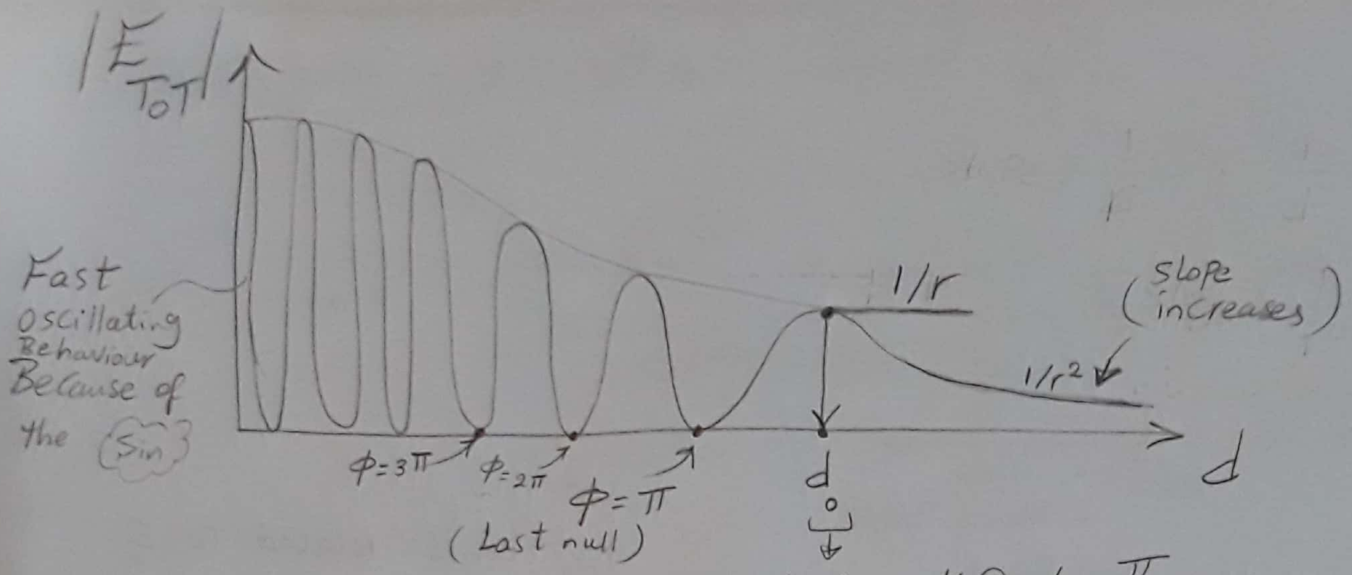
"free space LOS"

where: $L \geq 1$ — assume $L = 1$

$$\boxed{\therefore P_{r\,(2\text{ Ray Ground Reflection})} = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2} \cdot \left| \frac{E_{tot}}{E_o d_o} \right|^2}$$

$$= \frac{P_t G_t G_r \lambda^2}{(4\pi)^2} \cdot \frac{(4\pi)^2 h_t^2 h_r^2}{\lambda^2 d^4}$$

$$\boxed{P_r = \frac{P_t G_t G_r h_t^2 h_r^2}{d^4}} \quad \#$$



$\uparrow\phi$ increases in the Left direction

Last peak @ $\phi = \frac{\pi}{2}$ where:

$$\boxed{\phi = \frac{2\pi h_t h_r}{\lambda d}}$$

$$\therefore \frac{\pi}{2} = \frac{2\pi h_t h_r}{\lambda d}$$

$$\therefore d = d_0 = \frac{4 h_t h_r}{\lambda} = \text{reference distance @ } E = E_0$$

$\therefore$ for stable operation we choose $d > d_0$ to avoid instability in the Signal Level due to speedy Variation in the amplitude of $|E_{ToT}|$

$\therefore$ if $d >> \frac{4 h_t h_r}{\lambda}$ $\therefore \phi <<$ $\therefore \sin\phi = \phi$

$$\therefore |E_{tot}|_{2\text{-Ray}} = \frac{2 E_0 d_0 (2\pi h_t h_r)}{\lambda d^2}$$

$$\therefore P_{r\,2\text{-Ray}} = \frac{P_T G_T G_r h_t^2 h_r^2}{d^4}$$

(as Before)

#

$$\therefore \boxed{P_{2\text{-Ray}} \propto \frac{1}{d^4}}$$

④ Prove That the 2-Ray Ground Reflection Power Decrease 40 dB Per decade or octave

Solution

$$\because P \propto \frac{1}{d^4}$$

$$\therefore \frac{P}{P_o} = \frac{d_o^4}{d^4}$$

P_o reference

$$P_{dB} = P_o\Big|_{dB} - 10^{4} \log_{10}\left(\frac{d}{d_o}\right) = P_o - 10\,\alpha \log_{10}\frac{d}{d_o}$$

$\alpha = n$

$$= P_o + 10\,\alpha \log \frac{d_o}{d}$$

if: Power → ⊕
attenuation → ⊖

$$P_{dB} = P_{o\,dB} - 40 \log \frac{d}{d_o}$$

°° In General °°

$$P \propto \frac{1}{d^{\boxed{n}}}$$ → Parameter depends on the Environment

أرض مدن مليانة مباني وعوائق	أرض صحراوية	2-Ray	Free space LOS
destructive interference ← $\alpha = 6$ (Some times) Constructive interference ← $\alpha = 3$	$\alpha = 5$	$\alpha = 4$	$n = \alpha = 2$

$\alpha = 4$:

$$\frac{E}{E_o} = \frac{d_o^2}{d^2}$$

$$\frac{P}{P_o} = \frac{d_o^4}{d^4}$$

$n = \alpha = 2$:

$$\frac{E}{E_o} = \frac{d_o}{d}$$

$$\frac{P}{P_o} = \frac{d_o^2}{d^2}$$