TIME SERIES ANALYSIS & FORECASTING

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CLASSICAL DECOMPOSITION

THREE TYPES OF BASIC TIME SERIES DECOMPOSITION

1-The additive decomposition model:

$$X_t = T_t + S_t + C_t + I_t$$

X_t = Original data at time t

T_t = Trend value at time t

S_t = Seasonal fluctuation at time t

C_t = Cyclical fluctuation at time t

I_t = Irregular variation at time t.

2-The multiplicative decomposition model:

$$X_t = T_t \text{ x} S_t \text{x } C_t \text{x } I_t$$

3-The Mixed decomposition model:

For example, if T_t and C_t are correlated to each others, but they are independent from S_t and I I_t, the model will be

$$X_t = T_t \text{ x} S_t + C_t + I_t$$

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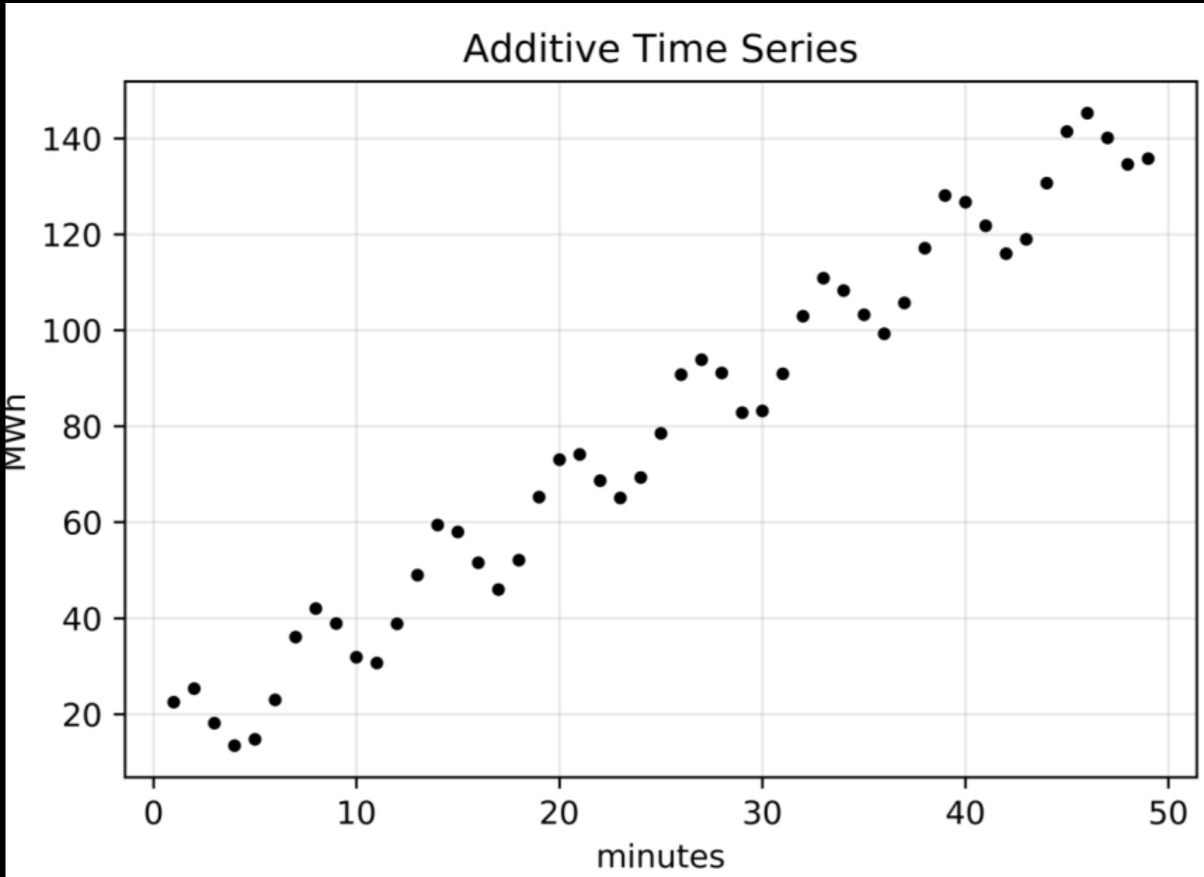
ADDITIVE MODEL

Additive models are used when the magnitudes of the seasonal and residual values are independent of trend.

$$X_t = T_t + S_t + C_t + I_t$$

[Intel]

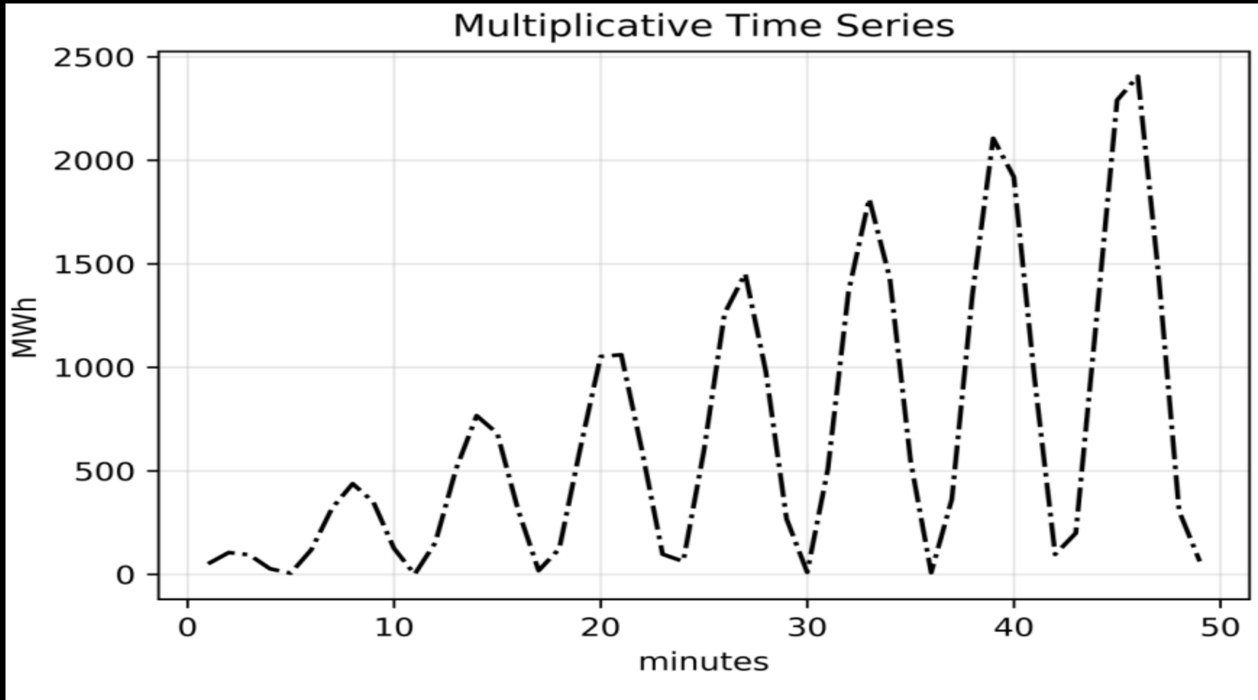


MULTIPLICATIVE MODEL

It is possible to transform a multiplicative model to an additive by applying a Log transformation.

- Multiplicative models are used when the magnitudes of the seasonal and residual values fluctuate with trend

$X_t = T_t \times S_t \times C_t \times I_t$



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TIME SERIES DECOMPOSITION IN PYTHON

In python , the function seasonal_decompose () estimates trend, seasonal, and irregular effects using the Moving Averages method (MA).

In this case, the series is decomposed into 3 components

trend-cycle component, seasonal component, and irregular component.

- **The additive decomposition model:**

$X_t = T_t + S_t + I_t$

- **The multiplicative decomposition model:**

$X_t = T_t \, x S_t x \, I_t$

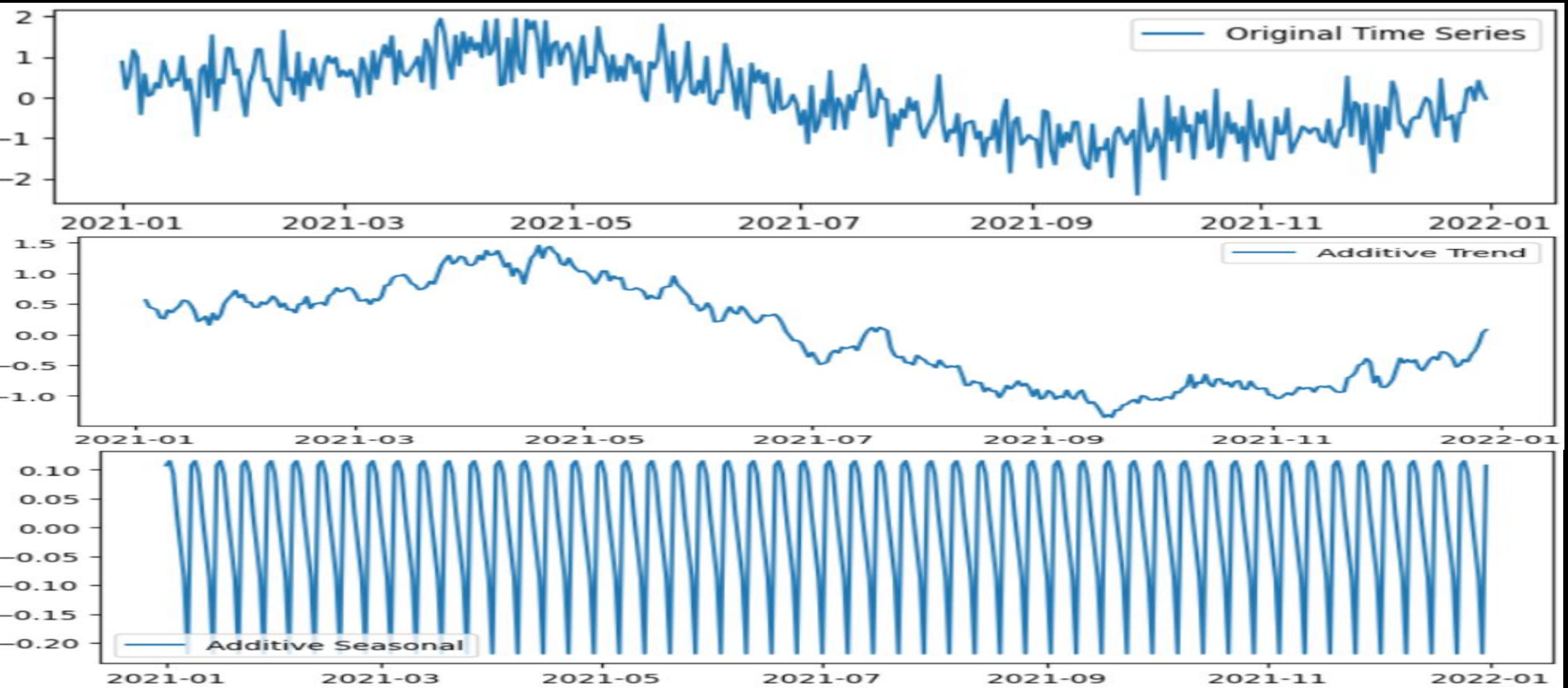
where T_t is the trend-cycle component (containing both trend and cycle components).

This is a naive decomposition. More sophisticated methods should be preferred. source

TIME SERIES DECOMPOSITION IN PYTHON

from statsmodels.tsa.seasonal import seasonal_decompose

result_add = seasonal_decompose(ts, model='additive')



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WHICH MODEL TO CHOOSE, ADDITIVE OR MULTIPLICATIVE?

- **The additive model is useful when the seasonal variation is relatively constant over time.**
- **The multiplicative model is useful when the seasonal variation increases over time.**
- ***We should use multiplicative models when the percentage change of our data is more important than the absolute value change (e.g., stocks).***

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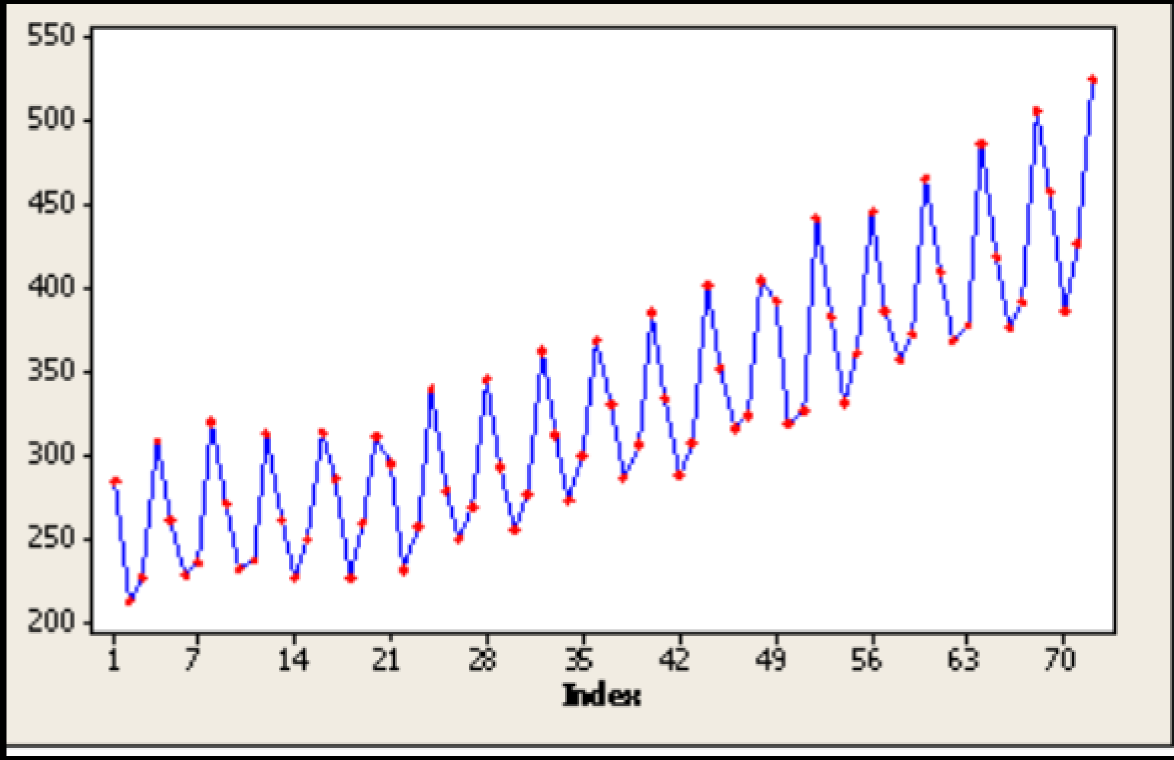
WHICH MODEL TO CHOOSE, ADDITIVE OR MULTIPLICATIVE?

- **Looked at quarterly product production in Australia. The seasonal variation looked to be about the same magnitude across time, so an additive decomposition might be good. Here's the time series plot:**



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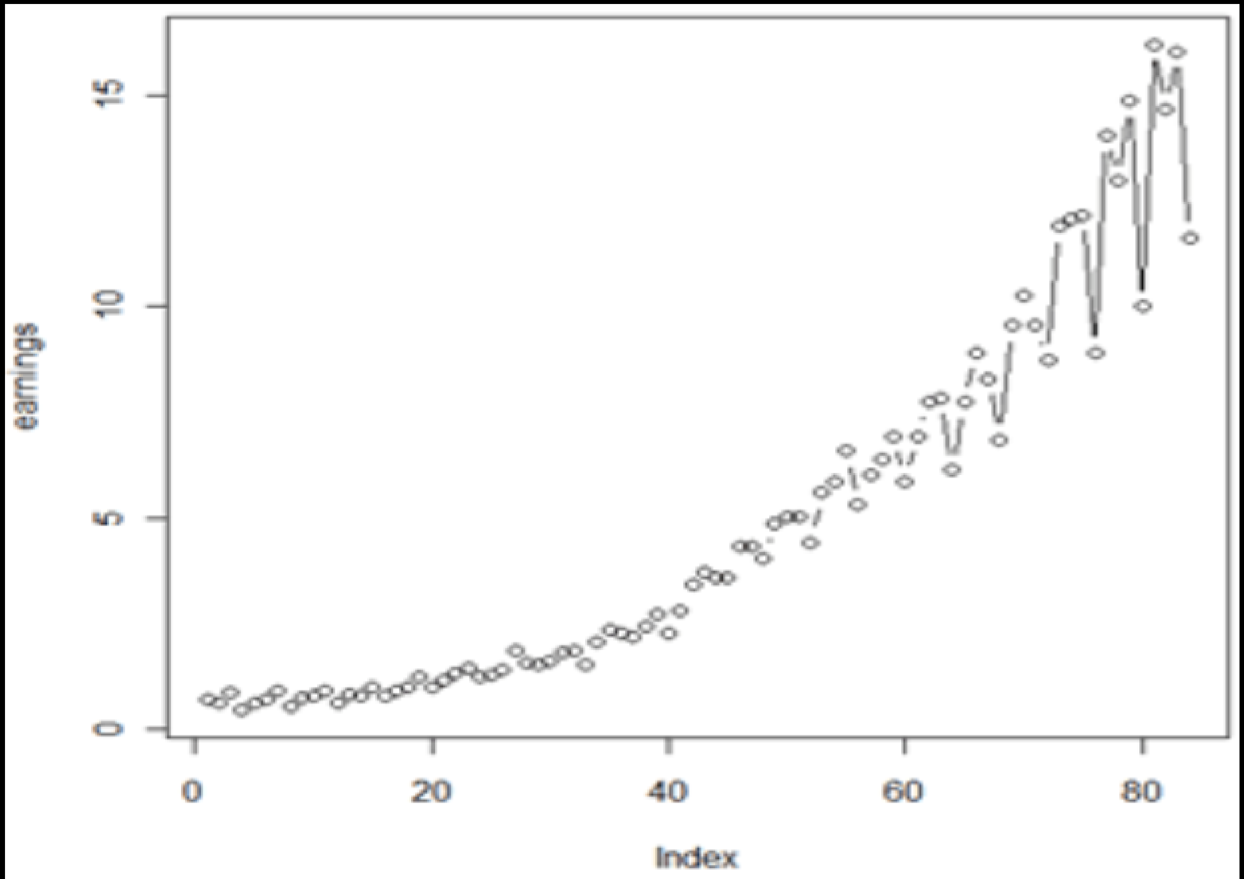
WHICH MODEL TO CHOOSE, ADDITIVE OR MULTIPLICATIVE?

The seasonal variation increases as we move across time. A multiplicative decomposition could be useful.



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HOW TO DECOMPOSE A TIME SERIES

Of the many ways to decompose a time series, these are the most common:

- **Single, double, or triple exponential smoothing**
- **Locally estimated scatterplot smoothing (LOWESS)**
- **Frequency-based methods common in signal processing**
- **More on these methods in future lessons!**

Intel

AUTOCOVARIANCE AND AUTOCORRELATION

BACKWARD (LAG) AND FORWARD (LEAD)

The lag, lead and difference notations are very convenient way to write linear time series models and to characterize their properties.

Definition :The lead (forward) operator denotes by F(.) on an element of a time series is used to shift the time index forward by one unit.

Definition: The lag (backshift) operator denotes by B(.) on an element of a time series is used to produce the previous element.

forward

T	Series	Lead 1
T1	5	10
T2	10	15
T3	15	20
T4	20	..
…	…	
		nan

backshift

T	series	Lag 1
T1	5	nan
T2	10	5
T3	15	10
T4	20	15
…	…	20

THE MEAN AND THE VARIANCE FUNCTIONS

Definition The **mean function** of a time series $\{X_t\}$ is denoted μ_t = E(X_t).

μ_t specifies the first order properties of the time series.

Definition The **variance function** of a time series $\{X_t\}$ is denoted

Var(X_t) =γx(0) = E[(X_t – μ_t)²].

- The sample mean is

$$\bar{x}=\frac{1}{n}\sum_{t=1}^{n} x_t$$

- The sample variance is

$$\bar{\gamma}x(0)=\frac{1}{n}\sum_{t=1}^{n}(x_t-\bar{x})^2$$

CORRELATION

What is correlation?

Correlation is a statistical measure that expresses the extent to which two variables are linearly related.

Correlation value:

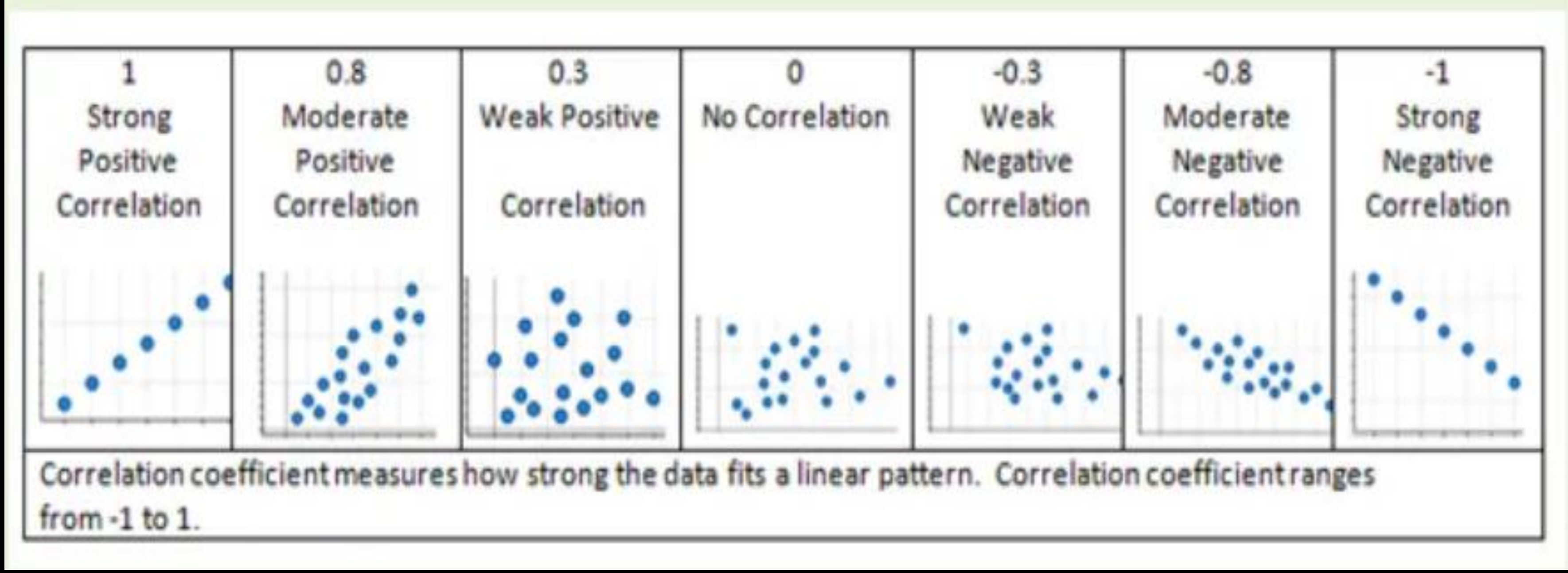
0 → no correlation

0 to 0.5 weak positive

0 to – 0.5 weak negative

1 t0 0.5 strong positive

-1 to -0.5 strong negative



AUTOCOVARIANCE AND THE AUTOCORRELATION FUNCTIONS

Definition The autocovariance function of a time series series $\{X_t\}$ is defined to be

$$\gamma x\ (s,t)=Cov(X_s\ ,X_t\ ) = E(X_s - \mu_s\ )(X_t - \mu_t\ )$$

for any time points s and t.

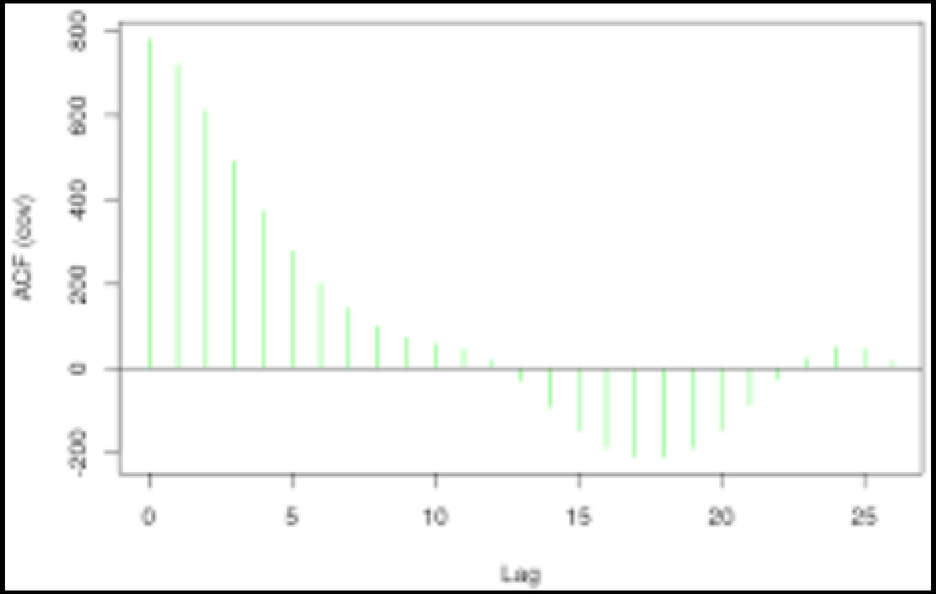
The function γ(s, t) specifies the second order properties of the time series.

Definition The autocorrelation function of a time series $\{X_t\}$ is defined as

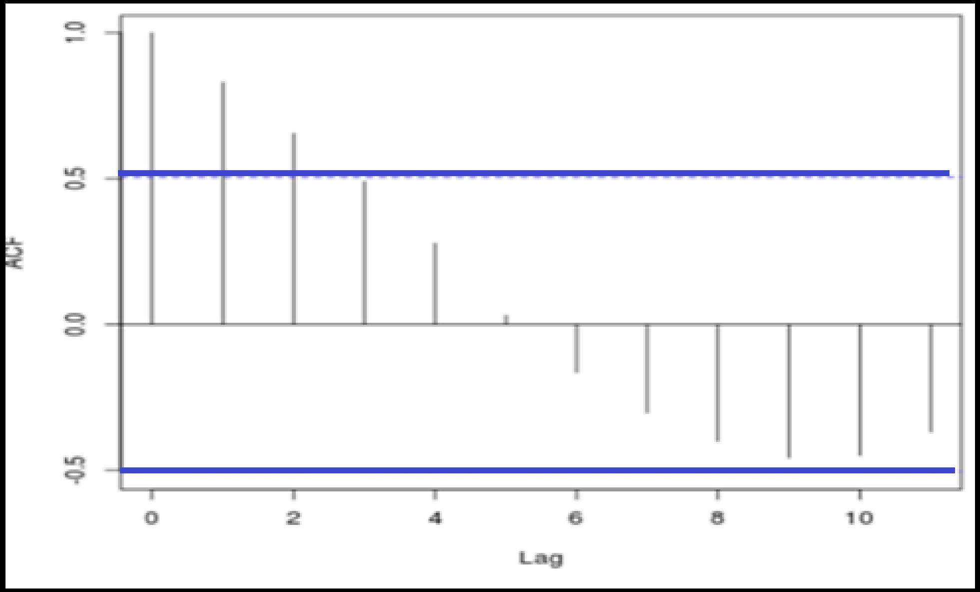
$$px=(s,t)=\frac{Cov(X_s\ ,X_t)}{\sqrt{Var(X_s)Var(X_t)}}$$

THE PLOT OF THE AUTOCOVARIANCE AND AUTOCORRELATION CORRELOGRAM) FUNCTIONS

- The plot of $\gamma(h)$ against the lag h values is called the **autocovariance function (ACVF).**
- The plot of ρ(h) against the lag h values is called the **autocorrelation function (ACF).**
- Note that ρx(0) = 1 and - 1 <= ρx(h) <= 1 for all h.



autocovariance function (ACVF).



autocorrelation function (ACF).

SAMPLE AUTOCOVARIANCE AND AUTOCORRELATION

- The **lag h sample autocovariance** is defined as

$$\hat{\gamma}x(h) = \frac{1}{n}\sum_{t=h+1}^{n}(X_t - \bar{x})(X_{t+h} - \bar{x})$$

- The **lag h sample autocorrelation** is defined as

$$-1 \leq \hat{p}x(h) = \frac{\hat{\gamma}x(h)}{\hat{\gamma}x(0)} = \frac{\sum_{t=1}^{n-h}(X_t - \bar{x})(X_{t+h} - \bar{x})}{\sum_{t=1}^{n}(X_t - \bar{x})^2} \leq 1$$

$\hat{\gamma}x(0)$ is autocovariance at lag h=0 it means variance

EXAMPLE - SAMPLE AUTOCORRELATION

- Find $\overline{p}$(h) where h = 1, 2 and 3

T	Z_t	Z_{t+1}	Z_{t+2}	Z_{t+3}	….	Z_{t-1}	Z_{t-2}
1	13	8	15	4		-	-
2	8	15	4	4		13	-
3	15	4	4	12		8	13
4	4	4	12	11		15	8
5	4	12	11	7		4	15
6	12	11	7	14		4	4
7	11	7	14	12		12	4
8	7	14	12	-		11	12
9	14	12	-	-		7	11
10	12	-	-	-		14	7

$$-1 \leq \hat{p}x(h) = \frac{\hat{\gamma}X(h)}{\hat{\gamma}X(0)} = \frac{\sum_{t=1}^{n-h}(X_t - \bar{x})(X_{t+h} - \bar{x})}{\sum_{t=1}^{n}(X_t - \bar{x})^2} \leq 1$$

The sample mean of these ten values is $\bar{X}$=10. Thus,

$$\rho x(1) = \frac{(13-10)(8-10)+(8-10)(15-10)+\dots+(7-10)\ (14-10)+(14-10)(12-10)}{(13-10)^2+(8-10)^2+\cdots+(14-10)^2+(12-10)^2} = \frac{-27}{144} =$$

- 0.188 Weak negative

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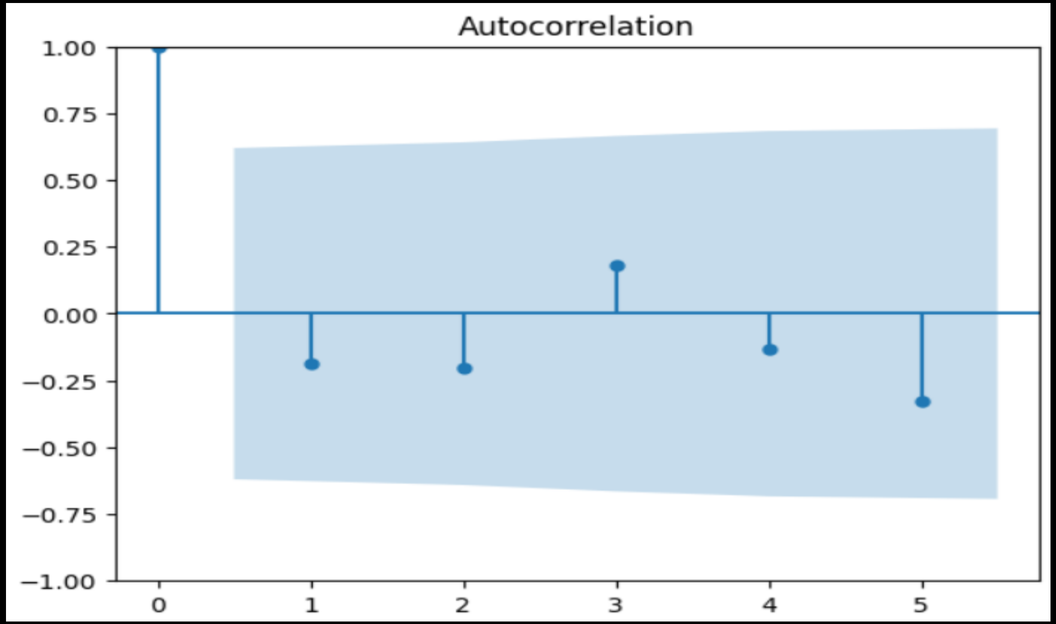
ACF PLOT

$\rho x(2) = \frac{(13-10)(15-10)+(8-10)(4-10)+\dots+(11-10)(14-10)+(7-10)(12-10)}{144} = \frac{-29}{144} =$ **- 0.201 weak negative**

$\rho x(3) = \frac{(13-10)(4-10)+(8-10)(4-10)+\dots+(12-10)(14-10)+(11-10)(12-10)}{144} = \frac{26}{144} =$ **0.181 weak positive**



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EXAMPLE – SAMPLE AUTOCOVARIANCE AND AUTOCORRELATION

T	X_t
1990	2
1991	1
1992	3
1993	5
1994	1
1995	6
1996	2
1997	6
1998	2
1999	4
2000	3
2001	7
2002	8
2003	1

Find $\hat{\gamma}x(h)$ and $\hat{p}$**x(h) where h=0,1 and 3**

$\bar{x}$**=3.642857**

$$\hat{\gamma}x(0) = \frac{1}{14}\sum_{t=1}^{14}(X_t - \bar{x})^2 = 5.23$$

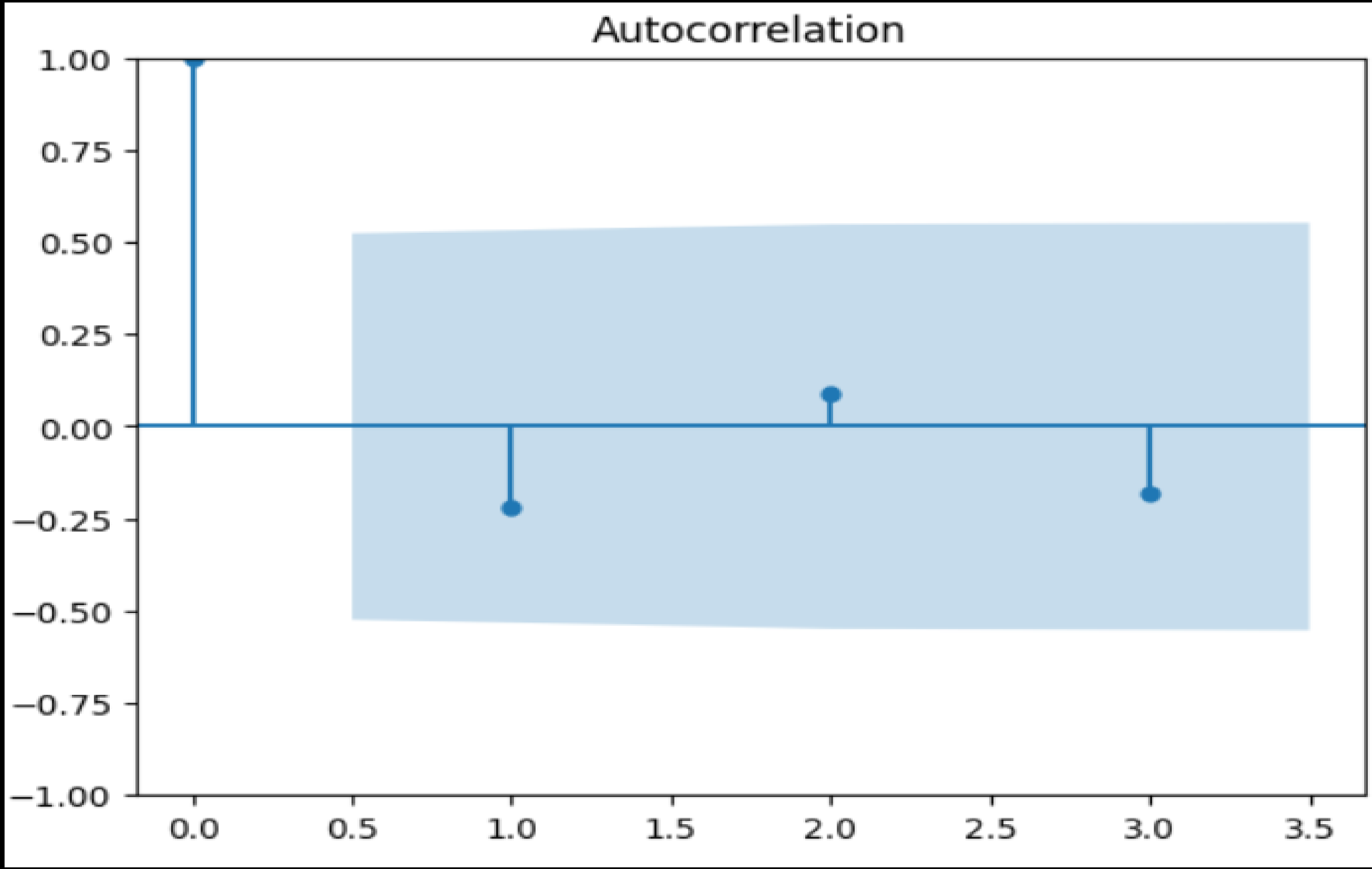
$$\hat{\gamma}x(1) = \frac{1}{14}\sum_{t=2}^{14}(X_t - \bar{x})(X_{t-1} - \bar{x}) = -1.152$$

$$\hat{\gamma}x(3) = \frac{1}{14}\sum_{t=4}^{14}(X_t - \bar{x})(X_{t-3} - \bar{x}) = -0.961$$

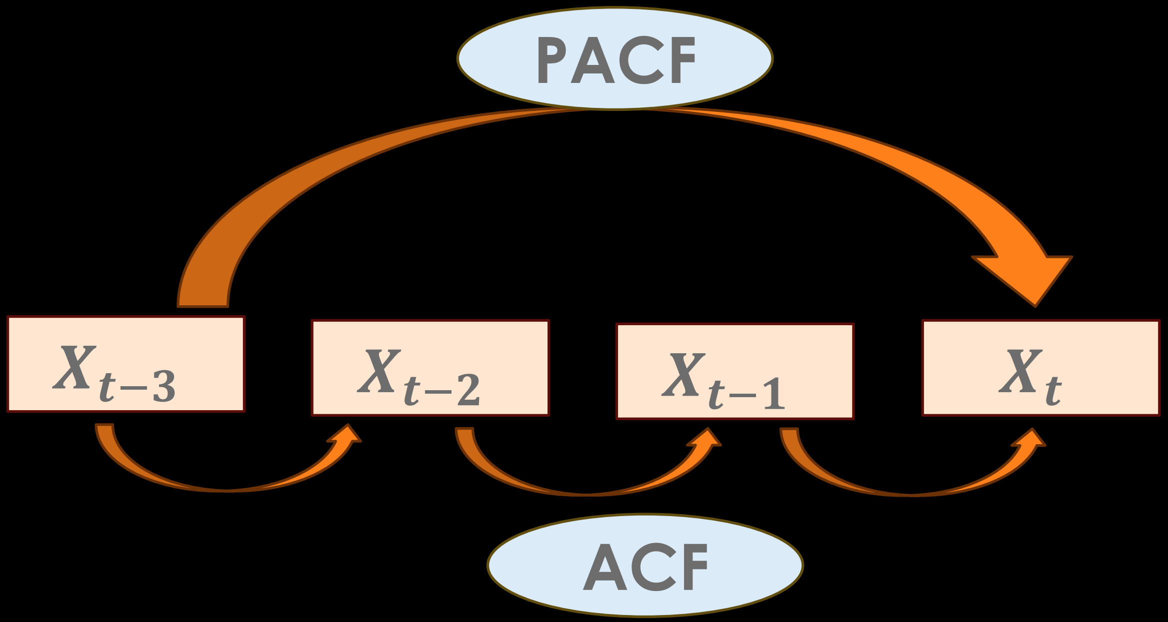
$$\hat{p}x(1) = \frac{\hat{\gamma}x(1)}{\hat{\gamma}x(0)} = -0.22$$

$$\hat{p}x(3) = \frac{\hat{\gamma}x(3)}{\hat{\gamma}x(0)} = -0.184$$

ACF PLOT


Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5

PACF



Partial autocorrelation removes the influence of intermediate lags, providing a clearer picture of the **direct relationship between a variable and its past values**. Unlike autocorrelation, partial autocorrelation focuses on the direct correlation at each lag.

SOURCE

PARTIAL AUTOCORRELATION PACF

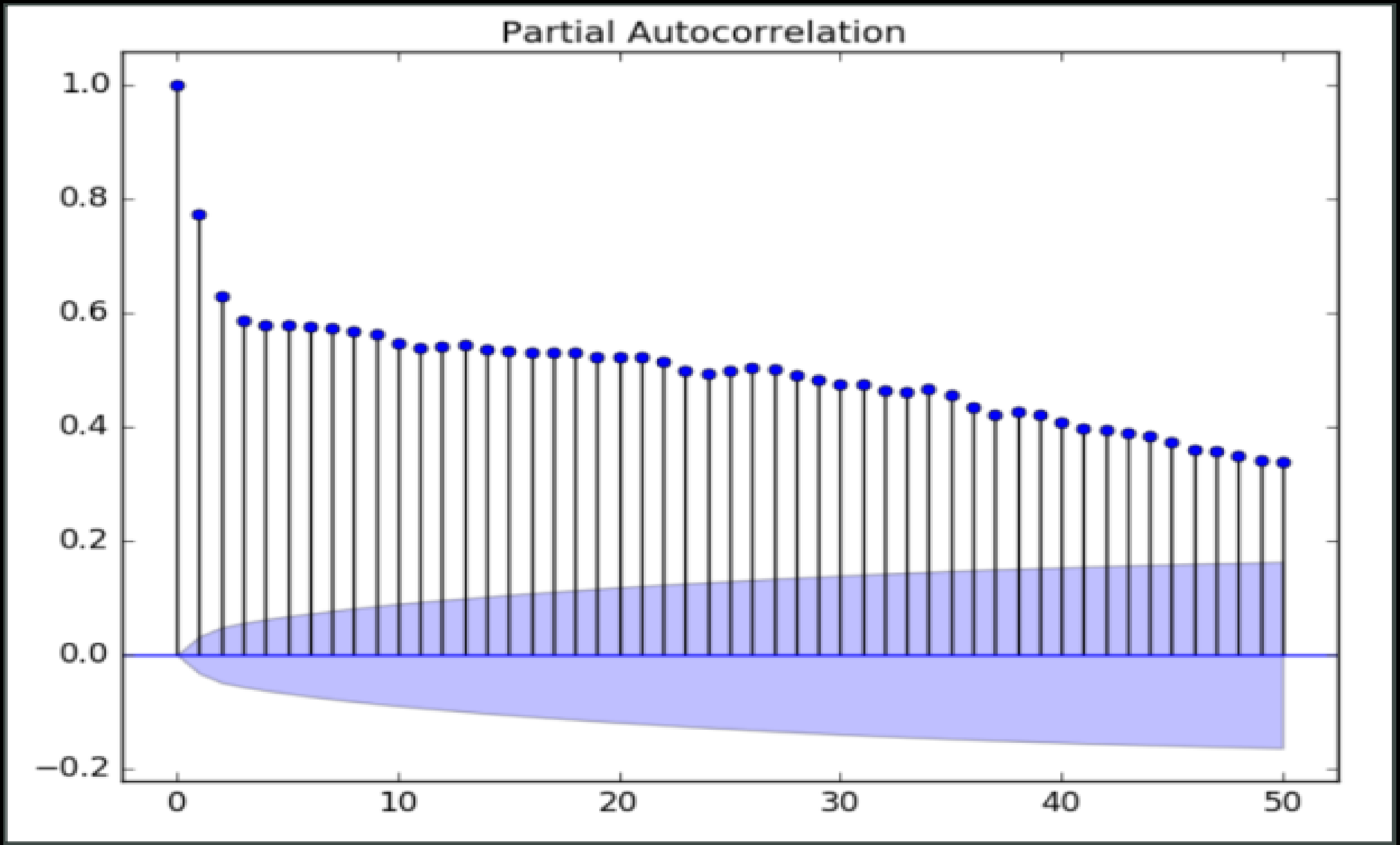
- The partial autocorrelation function (PACF) at lag k for a time series.
- $\phi k = \dfrac{\text{Cov}(X_t, X_{t-k} | X_{t-1}, X_{t-2}, \ldots, X_{t-k+1})}{\sqrt{Var(X_t | X_{t-1}, X_{t-2}, \ldots, X_{t-k+1}) Var(X_{t-k} | X_{t-1}, X_{t-2}, \ldots, X_{t-k+1})}}$

  X_t is the value of the time series at time t

  X_{t-k} is the value of the time series at time t-k

- $\text{Cov}(X_t, X_{t-k} | X_{t-1}, X_{t-2}, \ldots, X_{t-k+1})$ s the conditional covariance between X_t and X_{t-k} given the values of the intermediate lags
- $Var(X_t | X_{t-1}, X_{t-2}, \ldots, X_{t-k+1})$ is the conditional variance of X_t given the values of the intermediate lags.
- $Var(X_{t-k} | X_{t-1}, X_{t-2}, \ldots, X_{t-k+1})$ is the conditional variance of X_{t-k} given the values of the intermediate lags
- SOURCE

PACF



DIFFERENCE BETWEEN ACF AND PACF

Autocorrelation Function (ACF)	Partial Autocorrelation Function (PACF)
ACF measures the correlation between a data point and its lagged values, considering all intermediate lags. It gives a broad picture of how each observation is related to its past values.	PACF isolates the direct correlation between a data point and a specific lag, while controlling for the influence of other lags. It provides a more focused view of the relationship between a data point and its immediate past.
ACF does not isolate the direct correlation between a data point and a specific lag. Instead, it includes the cumulative effect of all intermediate lags.	PACF is particularly useful in determining the order of an autoregressive (AR) process in time series modeling. Significant peaks in PACF suggest the number of lag terms needed in an AR model.
ACF is helpful in identifying repeating patterns or seasonality in the data by examining the periodicity of significant peaks in the correlation values.	The point where PACF values drop to insignificance helps identify the cut-off lag, indicating the end of significant lags for an AR process.

SOURCE