__/__/____

3. Quick sort

Time complexiety

Average case :

For avg. case, consider the array gets divided into 2 parts of sizes K and (N-K)

$$\Rightarrow \quad T(N) = T(N-K) + T(K)$$

$$= \frac{1}{N}\left[\sum_{i=1}^{N-1} T(i) + \sum_{i=1}^{N-1} T(N-i)\right]$$

as both these ~~are~~ terms are equally likely functions,

$$T(N) = \frac{2}{N} \sum_{i=1}^{N-1} T(i)$$

$$N(T(N)) = 2 \sum_{i=1}^{N-1} T(i) \quad — 1$$

we can also write,

$$(N-1)(T(N-1)) = 2 \sum_{i=1}^{N-2} T(i) \quad — 2$$

subtracting 2 from 1,

$$N\,T(N) - (N-1)\,T(N-1) = 2 \sum_{i=1}^{N-1} T(i) - 2 \sum_{i=1}^{N-2} T(i)$$

$$= 2\,T(N-1) + N^2 c - (N-1)^2 c$$

where c is constant

$$\Rightarrow N\,T(N) = T(N-1)(2+N-1) + c + 2Nc - c$$

$$= (N+1)\,T(N-1) + 2Nc$$

__/__/____

divide both sides by N (N+1),

$$\frac{T(N)}{(N+1)} = \frac{T(N-1)}{N} + \frac{2Nc}{(N+1)} \quad \text{--- } (3)$$

if we put N = N-1,

$$\cancel{T(N)} \quad \frac{T(N-1)}{N} = \frac{T(N-2)}{(N-1)} + \frac{2c}{N}$$

⇒ (3) can be expressed as,

$$\frac{T(N)}{(N+1)} = \frac{T(N-2)}{(N-1)} + \frac{2c}{(N+1)} + \frac{2c}{N}$$

III^ly we can get the value of T(N-2) by replacing N by (N-2) in the eq^n (3).

$$\frac{T(N)}{(N+1)} = \frac{T(1)}{2} + 2c\left[1/2 + 1/3 + \cdots + 1/(N-1) + 1/N + 1/(N+1)\right]$$

$$\Rightarrow T(N) = 2c \log_2 N \, (N+1)$$

ignore the constant terms,

$$T(N) = \log_2 N * (N+1)$$

in asymptotic notation,

$$T(N) = N(\log_2 N)$$