42 YEARS OF EXCELLENCE

NARAYANA EDUCATIONAL INSTITUTIONS

NARAYANA LIVE CLASSES

Topic:Conic Section
Lecture No.:6

Date:12/01/2022

N LIVE IIT BATCH B1

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Various Forms of Equations of Tangents

VARIOUS FORMS OF TANGENTS

- Point Form
- Parametric Form
- Slope Form

Parabola	Parametric Co.od	Point Form(T=0)	Parametric Form	Slope Form (y = mx+c)
$y^2 = 4ax$	$(at^2, 2at)$	$yy_1 = 2a(x + x_1)$	$yt = x + at^2$	$y = mx + a/m$
$y^2 = -4ax$	$(-at^2, 2at)$	$yy_1 = -2a(x + x_1)$	$yt = -x + at^2$	$y = mx - a/m$
$x^2 = 4ay$	$(2at, at^2)$	$xx_1 = 2a(y + y_1)$	$xt = x + at^2$	$y = mx - am^2$
$x^2 = -4ay$	$(2at, -at^2)$	$xx_1 = -2a(y + y_1)$	$xt = -y + at^2$	$y = mx + am^2$

DOUBTS :

DPP#01 **PARABOLA**

Topics	**Equation of Parabola**

1.(E) The points on the parabola $y^2 = 36x$ whose ordinate is three times the abscissa are

(a)(0, 0), (4, 12) (b)(1, 3), (4, 12)

(c)(4, 12) (d)None of these

2.(E) The equation of the lines joining the vertex of the parabola $y^2 = 6x$ to the points on it whose abscissa is 24, is

(a) $y \pm 2x = 0$ (b) $2y \pm 3x = 0$

(c) $x \pm 2y = 0$ (d) $2x \pm y = 0$

3.(E) The points on the parabola $y^2 = 12x$ whose focal distance is 4, are

(a) $(2, \sqrt{3}), (2, -\sqrt{3})$ (b) $(1, 2\sqrt{3}), (1, -2\sqrt{3})$

(c)(1, 2) (d)None of these

4.(E) If the parabola $y^2 = 4ax$ passes through (–3, 2), then length of its latus rectum is

(a)2/3 (b)1/3

(c)4/3 (d)4

5.(E) A parabola has the origin as its focus and the line x =2 as the directrix. Then the vertex of the parabola is at **[AIEEE 2008]**

(a) (1, 0) (b) (0, 1)

(c) (2, 0) (d) (0, 2)

6.(E) $x - 2 = t^2, y = 2t$ are the parametric equations of the parabola

(a) $y^2 = 4x$ (b) $y^2 = -4x$

(c) $x^2 = -4y$ (d) $y^2 = 4(x-2)$

7.(E) The vertex, focus, directrix and length of the latus rectum of the parabola $y^2 - 4y - 2x - 8 = 0$ is

(a) V (6, 2), S (–11/2, 2) ,Eq. of directrix x = – 13/2, L.L.R. = 2

(b) V (– 6, 2), S (11/2, 2) ,Eq. of directrix x = – 13/2, L.L.R. = 3

(c)V (– 6, 2), S (–11/2, 2) ,Eq.of directrix x = – 13/2, L.L.R. = 2

(d) None of these

8.(E) The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is **[IIT- -2001]**

(a) x = – 1 (b) x = 1

(c) $x = -\frac{3}{2}$ (d) $x = \frac{3}{2}$

9.(E) The equation of the parabola whose axis is vertical and passes through the points (0, 0), (3, 0) and (–1, 4) is

(a) $x^2 - 3x - y = 0$ (b) $x^2 + 3x + y = 0$

(c) $x^2 - 4x + 2y = 0$ (d) $x^2 - 4x - 2y = 0$

10.(E) The equation of the parabola whose vertex is (–1, –2), axis is vertical and which passes through the point (3, 6), is

(a) $x^2 + 2x - 2y - 3 = 0$ (b) $2x^2 = 3y$

(c) $x^2 - 2x - y + 3 = 0$ (d)None of these

11.(M) The latus rectum of a parabola whose directrix is $x + y - 2 = 0$ and focus is (3, – 4), is

(a) $-3\sqrt{2}$ (b) $3\sqrt{2}$

(c) $-3/\sqrt{2}$ (d) $3/\sqrt{2}$

12.(E) The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum is

(a) 12 *sq. unit* (b)16 *sq. unit*

(c) 18 *sq. unit* (d)24 *sq. unit*

13.(E) The area of triangle formed inside the parabola $y^2 = 4x$ and whose ordinates of vertices are 1, 2 and 4 will be

(a) $\frac{7}{2}$ (b) $\frac{5}{2}$

(c) $\frac{3}{2}$ (d) $\frac{3}{4}$

14.(E) An equilateral triangle is inscribed in the parabola $y^2 = 4x$ one of whose vertex is at the vertex of the parabola, the length of each side of the triangle is

(a) $\sqrt{3}/2$ (b) $4\sqrt{3}/2$

(c) $8\sqrt{3}/2$ (d) $8\sqrt{3}$

15.(E) If a double ordinate of the parabola $y^2 = 4ax$ be of length 8a, then the angle between the lines joining the vertex of the parabola to the ends of this double ordinate is

(a) 30º (b) 60º

(c) 90º (d) 120º

16.(E) Let P be the point (1, 0) & Q be any point on $y^2 = 8x$ than locus of mid point of PQ is

(a) $x^2 + 4y + 2 = 0$ (b) $x^2 - 4y + 2 = 0$

(c) $y^2 - 4x + 2 = 0$ (d) $y^2 + 4x + 2 = 0$

17. PQ is a double ordinate of the parabola $y^2 = 4ax$. The locus of the points of trisection of PQ is

(a) $9y^2 = 4ax$ (b) $9x^2 = 4ay$

(c) $9y^2 + 4ax = 0$ (d) $9x^2 + 4ay = 0$

18.(E) The focal distance of a point on the parabola $y^2 = 16x$ whose ordinate is twice the abscissa, is

(a) 6 (b) 8

(c) 10 (d) 12

19.(M) Let A be the vertex and L the length of the latus rectum of the parabola $y^2 - 2y - 4x - 7 = 0$. The equation of the parabola with A as vertex, 2L the length of the latus rectum and the axis at right angles to that of the given curve is

(a) $x^2 + 4x + 8y - 4 = 0$ (b) $x^2 + 4x + 8y - 12 = 0$

(c) $x^2 + 4x - 8y + 12 = 0$ (d) $x^2 + 8x - 4y + 8 = 0$

20.(T) The circle $x^2 + y^2 + 2\lambda x = 0$, $\lambda \in R$ touches the parabola $y^2 = 4x$ externally, then

(a) $\lambda > 0$ (b) $\lambda < 0$

(c) $\lambda > 1$ (d) None of these

ANSWER KEY:

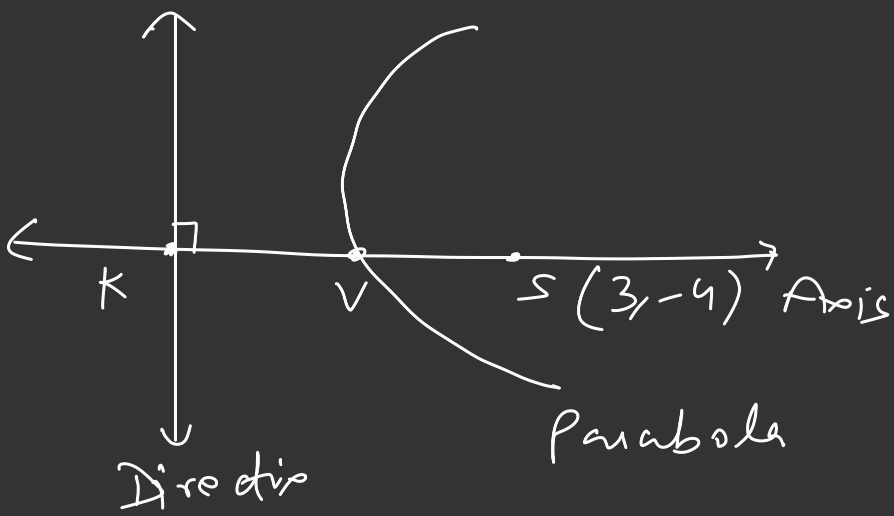
1	a	2	c	3	b	4	c	5	a
6	d	7	c	8	d	9	a	10	a
11	b	12	c	13	d	14	d	15	c
16	c	17	a	18	b	19	c	20	a

11.(M) The latus rectum of a parabola whose directrix is $x + y - 2 = 0$ and focus is (3, – 4), is

(a) $-3\sqrt{2}$ (b) $3\sqrt{2}$

(c) $-3/\sqrt{2}$ (d) $3/\sqrt{2}$

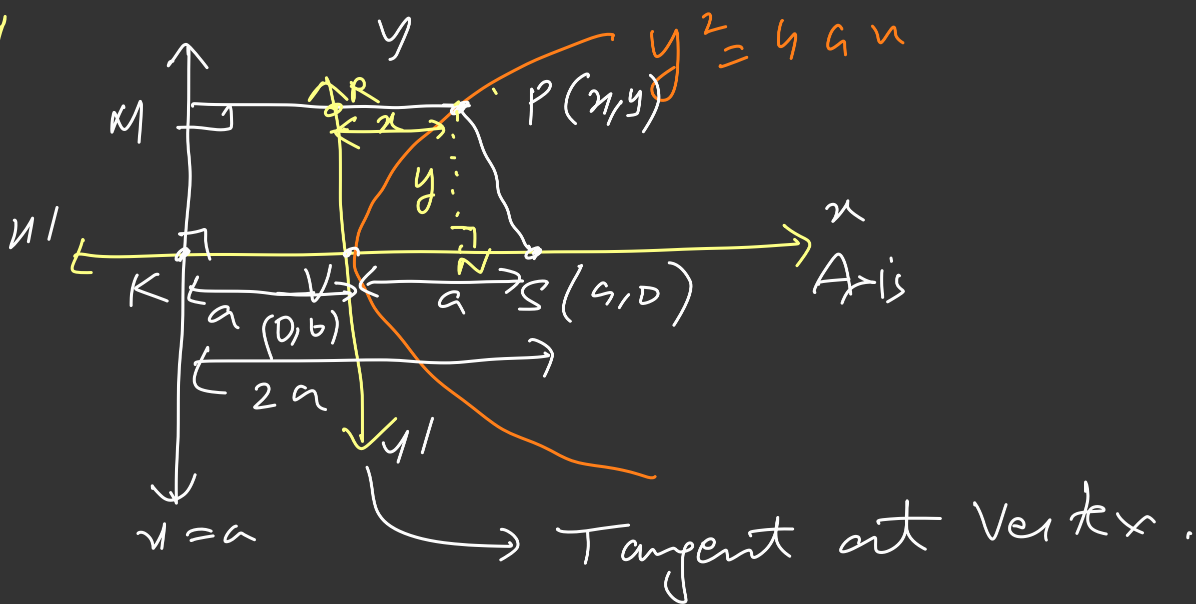
Sol:-



$y^2 = 4ax$

$L.R = 4a$

$= 4$



$L.R = 4a = 4(a)$

$$\boxed{L.R = 4\begin{pmatrix}\text{Distance of focus from} \\ \text{tangent at vertex}\end{pmatrix}}$$

$= 2(2a)$

Applicable to all parabolas

$$\boxed{L.R = 2(\text{Distance of Focus from Directrix})}$$

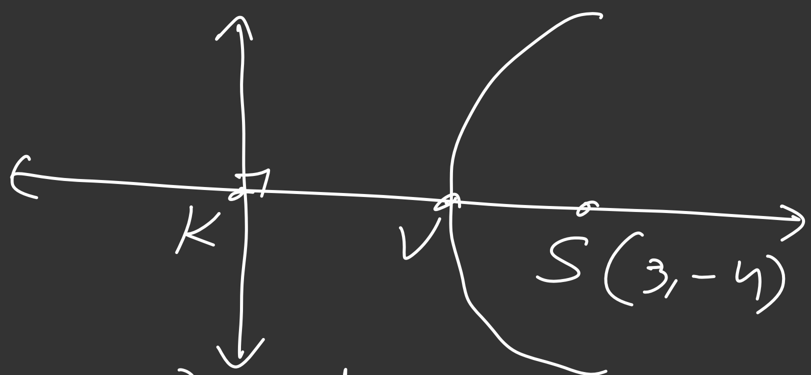
Now $y^2 = 4ax$

$\Rightarrow (PN)^2 = (4a)(PR)$

$$\Rightarrow \boxed{(\text{Distance of Point P on Parabola})^2 = (L\cdot R)\begin{pmatrix}\text{Distance of point P}\\ \text{from tangent at vertex}\end{pmatrix}}$$

Applicable to all parabolas having axis $\parallel$ to x axis.

$\therefore$ (II).



$L\cdot R = 4a = 2(2a)$

$$= 2\begin{pmatrix}\text{Distance of focus}\\ \text{from directix}\end{pmatrix}$$

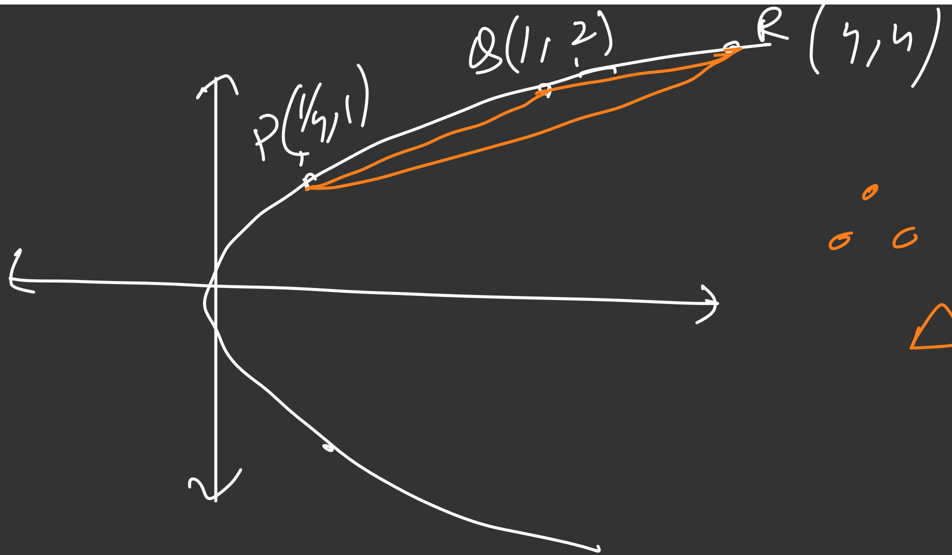
$\therefore$ Distance of $S(3,-4)$ from $x+y-2=0$

is : $\dfrac{|3+(-4)-2|}{\sqrt{2}} = \dfrac{3}{\sqrt{2}}$

$$\therefore L \cdot R = 2\left(\frac{3}{\sqrt{2}}\right) = \frac{6}{\sqrt{2}} = 3\sqrt{2} \text{ units}$$

13.(E) The area of triangle formed inside the parabola $y^2 = 4x$ and whose ordinates of vertices are 1, 2 and 4 will be

Sol:



$\therefore$ Area of $\triangle PQR$.

$y^2 = 4x$ As P, Q, R lies on this

$\therefore (1)^2 = 4x \Rightarrow x = 1/4$

$(2)^2 = 4x \Rightarrow x = 1$

$(4)^2 = 4x \Rightarrow x = 4$

$\therefore P\left(\frac{1}{4}, 1\right); Q(1, 2); R(4, 4)$

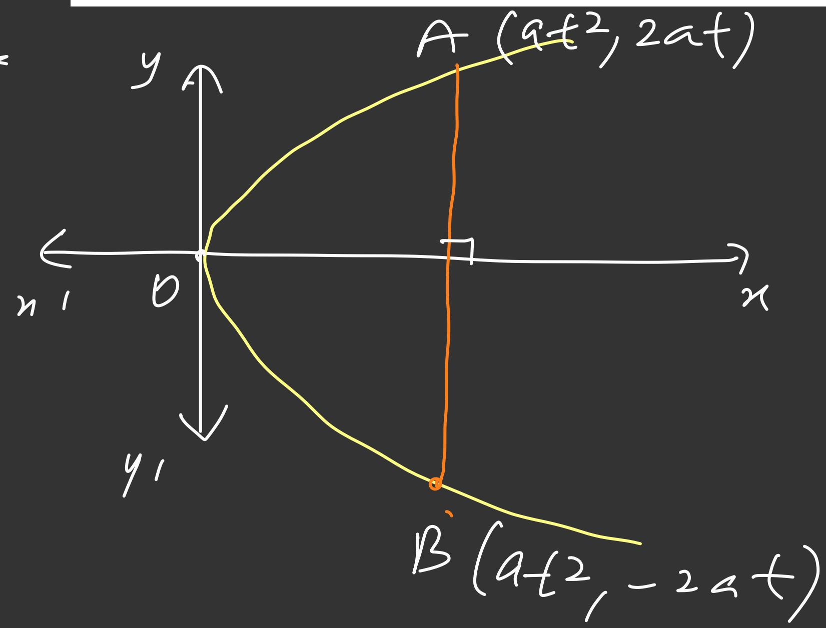
$\therefore$ Obtain Area of $\triangle PQR$

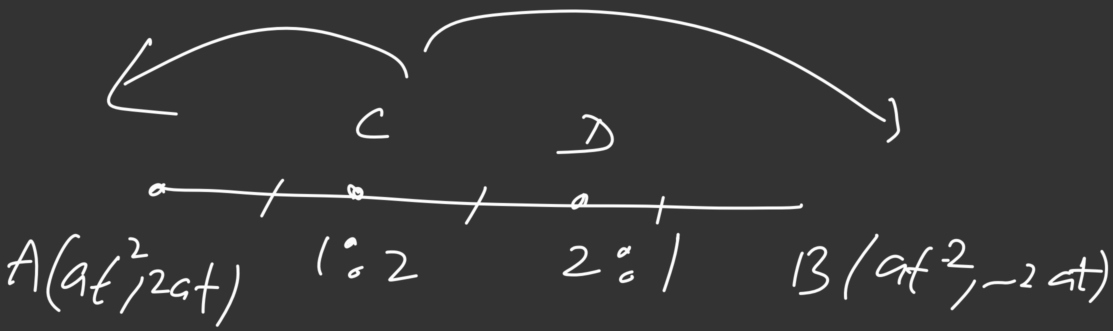
17. PQ is a double ordinate of the parabola $y^2 = 4ax$. The locus of the points of trisection of PQ is

(a) $9y^2 = 4ax$ (b) $9x^2 = 4ay$

(c) $9y^2 + 4ax = 0$ (d) $9x^2 + 4ay = 0$

Sol :-





C divides AB in 1 : 2

D divides AB in 2 : 1

$$\therefore C\left(\frac{3at^2}{3}, \frac{2at}{3}\right) \equiv C(h, k)$$

$$\therefore at^2 = h \; ; \; \frac{2at}{3} = k$$

$$t = 3k/2a$$

$$\therefore a\left(\frac{3k}{2a}\right)^2 = h$$

$$\Rightarrow a\left(\frac{9k^2}{4a^2}\right) = h$$

$$\Rightarrow 9k^2 = 4ah$$

$\therefore$ Locus is : $\boxed{9y^2 = 4ax}$

19.(M) Let A be the vertex and L the length of the latus rectum of the parabola $y^2 - 2y - 4x - 7 = 0$. The equation of the parabola with A as vertex, 2L the length of the latus rectum and the axis at right angles to that of the given curve is

(a) $x^2 + 4x + 8y - 4 = 0$ (b) $x^2 + 4x + 8y - 12 = 0$

(c) $x^2 + 4x - 8y + 12 = 0$ (d) $x^2 + 8x - 4y + 8 = 0$

Sol :-

$$y^2 - 2y - 4x - 7 = 0$$

$$\Rightarrow y^2 - 2y = 4x + 7$$

$$\Rightarrow y^2 - 2y + 1 = 4x + 8$$

$$\Rightarrow (y-1)^2 = 4(x+2)$$

$y - 1 = Y$;

$x + 2 = X$

$$\therefore Y^2 = 4X$$

$\boxed{a=1}$

$\therefore A(0,0)$: $X=0$; $Y=0$

$x=-2$; $y=1$

Axis: $Y=0$

$\boxed{y=1}$

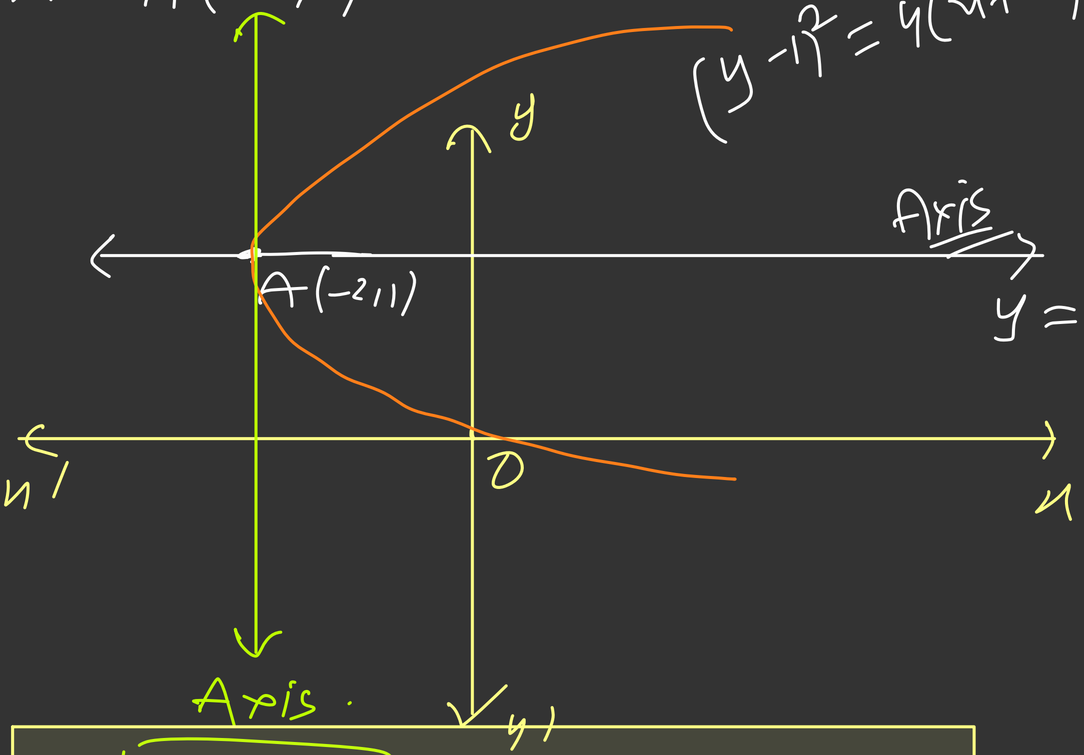
$\therefore A(-2,1)$

$L = L \cdot R = 4(1) = 4$ units.

Now New $L \cdot R = 2L = 2(4)$
$= 8$ units

Vertex is same ie $A(-2,1)$

$\therefore$ Axis lar to this and passing through $A(-2,1)$



$\boxed{x=-2}$ $L \cdot R = 8$ units

Vertex $(-2,1)$

Eq. of Upward parabola with vertex as $A(a,b)$ $L.R = L$ is

$(-2,1)$

$$(x-a)^2 = (L)(y-b)$$

$$\Rightarrow \boxed{(x+2)^2 = 8(y-1)}$$

is reqd. eq.

OR

$$(x-a)^2 = -L(y-b)$$

$$(x+2)^2 = -8(y-1)$$

... and a point on the parabola where the line segment makes an angle 'θ' to the x-axis is

1) $\frac{4\cos\theta}{\sin\theta}$ 2) $\frac{a\cos\theta}{\sin^2\theta}$

3) $\frac{4a\cos\theta}{\sin\theta}$ 4) $\frac{4a\cos\theta}{\sin^2\theta}$

8. If the join of ends of the latusrectum of $x^2 = 8y$ subtends an angle θ at the vertex of the parabola then $\cos\theta =$

1) $\frac{-4}{5}$ 2) $\frac{-2}{3}$ 3) $\frac{-3}{5}$ 4) $\frac{-1}{5}$

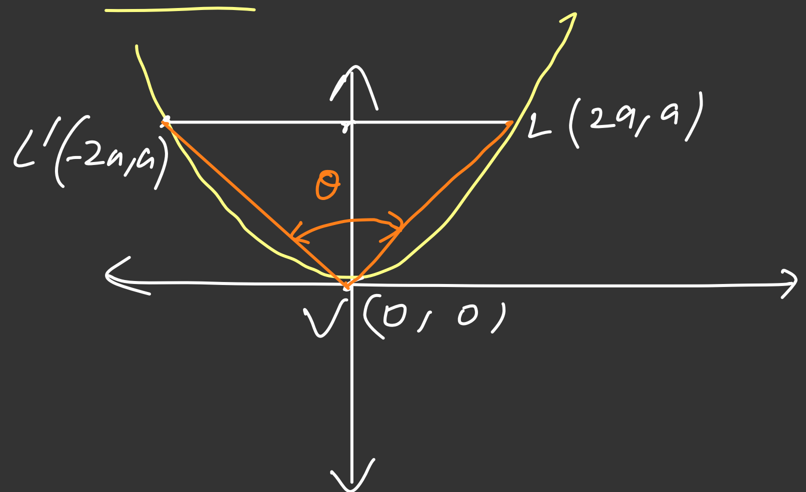
9. Area of the triangle formed by the vertex, focus and one end of latusrectum of the parabola $(x+2)^2 = -12(y-1)$ is

1) 18 2) 36 3) 12 4) 9

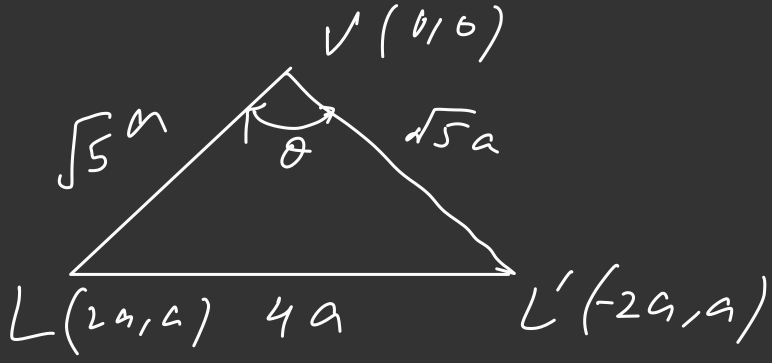
10. The abscissa of the orthocentre of the triangle formed by the lines $y = m_1x + \frac{a}{m_1}$,

Ex I:

8.



then $\cos\theta = ?$



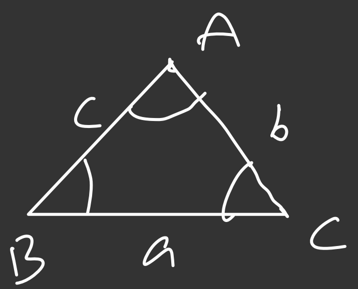
$LV = \sqrt{5}a$ $\qquad \cos\theta = ?$

$L'V = \sqrt{5}a$

$LL' = 4a$

$\therefore$ Apply cosine Law

$$\cos\theta = \frac{(LV)^2 + (L'V)^2 - (LL')^2}{2(LV)(L'V)}$$



$$= \frac{5a^2 + 5a^2 - 16a^2}{2(5)(a^2)} = \frac{-6}{10}$$

$$\boxed{\cos\theta = -3/5}$$

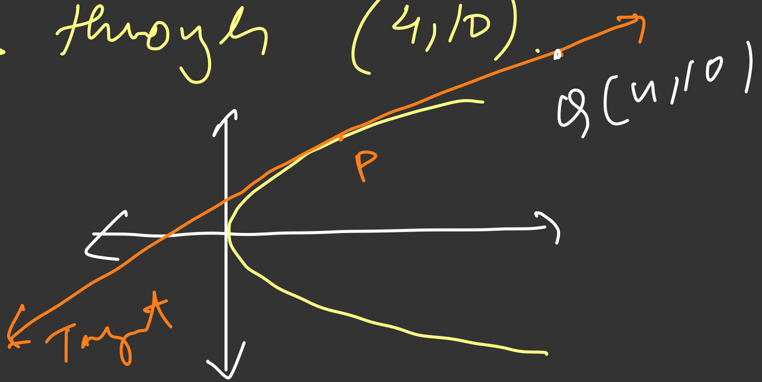
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$; $\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Examples:

1. Find eq. of tangent to $y^2 = 16x$ which passes through (4,10).

Sol:



$y = mx + \frac{a}{m}$ is eq. of tangent

$a = 4$

$\Rightarrow y = mx + \frac{4}{m}$ —i

This passes through $(4, 10)$

$\therefore 10 = m(4) + \frac{4}{m}$

$\Rightarrow 10m = 4m^2 + 4$

$\Rightarrow 5m = 2m^2 + 2$

$\Rightarrow 2m^2 - 5m + 2 = 0$

$$\boxed{m = \frac{1}{2}, 2}$$

$\therefore$ i) becomes: $y = \left(\frac{1}{2}\right)x + \frac{4}{\frac{1}{2}}$

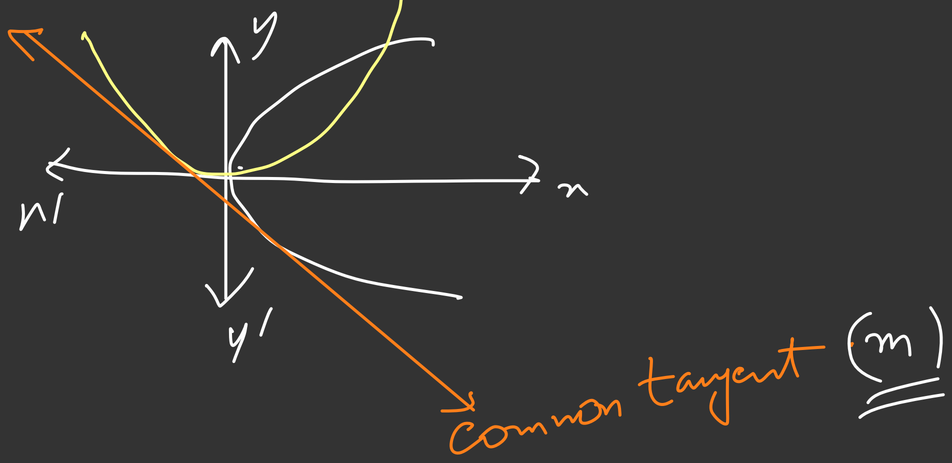
$$\boxed{y = \frac{x}{2} + 8}$$

and $y = 2x + \frac{4}{2}$

$$\boxed{y = 2x + 2}$$

2. Find eq. of common tangent to $y^2 = 32x$ and $x^2 = 108y$.

$a = 8$ $\qquad$ $a = 27$



Sol: Eq. of tangent to $y^2 = 4ax$ is

$$y = mx + a/m$$

$$\therefore \boxed{y = mx + 8/m} \quad (i)$$

Eq. of tangent to $x^2 = 4ay$ is

$$y = mx - am^2$$

$$\Rightarrow \boxed{y = mx - 27m^2} \quad (ii)$$

i) & ii) rep. same line

$$\therefore \frac{8}{m} = -27m^2$$

$$\Rightarrow \frac{-8}{27} = m^3$$

$$\Rightarrow \boxed{m = -2/3}$$

$\therefore$ i) becomes: $y = -\frac{2}{3}x + \frac{8}{(-2/3)}$

$$\boxed{y = -\frac{2}{3}x - 12}$$

is common tangent.

(3) Find Eq. of Common tangent to $y^2 = 4x$ and $x^2 = 32y$.

Sol:-

$$y = mx + \frac{1}{m} \quad \text{(i)}$$

$$y = mx - 8m^2 \quad \text{(ii)}$$

$$\therefore \frac{1}{m} = -8m^2 \Rightarrow m^3 = -1/8$$

$$\boxed{m = -1/2}$$

$$\therefore \text{i)} \Rightarrow y = \left(-\frac{1}{2}\right)x + \frac{1}{(-1/2)}$$

$$y = -\frac{x}{2} - 2$$

$$\Rightarrow 2y = -x - 4$$

$$\boxed{x + 2y + 4 = 0}$$

(4) Find eq. of common tangent to

$y^2 = 4x$ and $(x-3)^2 + y^2 = 9$.

Sol:-

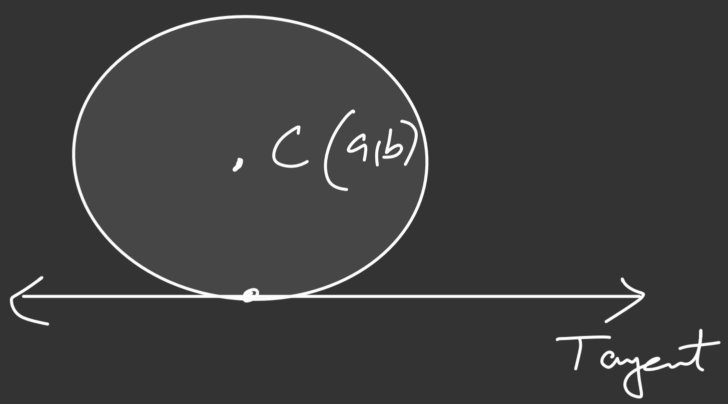
$C(3,0)$; $r = 3$

Eq. of tangent to : $y^2 = 4x$ is

$y = mx + \frac{1}{m}$

$\Rightarrow \boxed{m^2x - y + 1 = 0}$

Now this line is also tangent to circle : $C(3,0)$; $r = 3$



Various Forms of Equations of Tangents

1. Point of Intersection of Tangents:

Examples:

2. Normal to Parabola:

3. Slope Form: