Solving MaxCut with simulated quantum annealing - guide through the main theoretical concepts.

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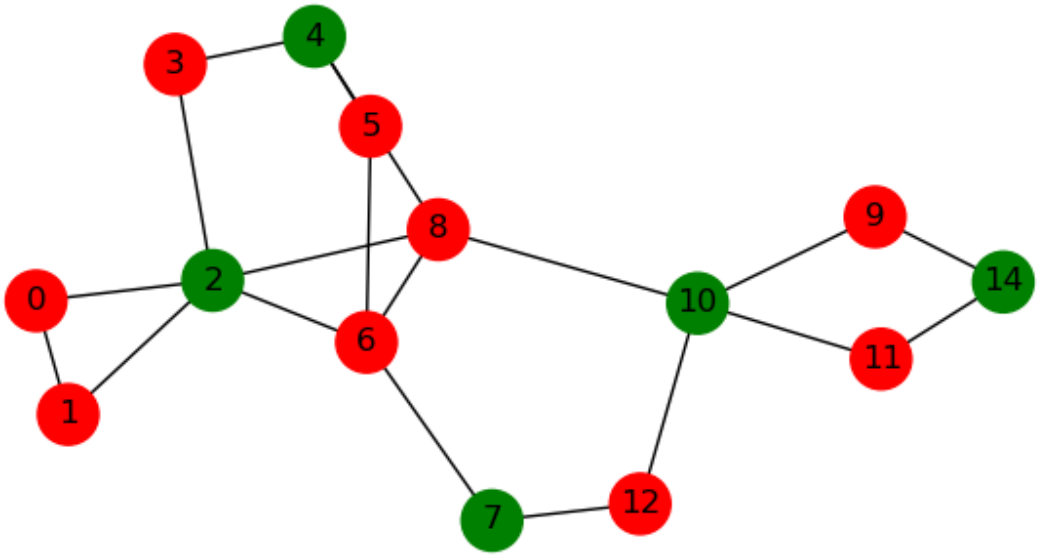
1 Ising mapping for the maximum cut problem.

The Maximum Cut (MaxCut) problem is a fundamental combinatorial optimization problem in graph theory. Given an undirected graph, the goal is to partition its vertices into two disjoint sets such that the number of edges connecting vertices from different sets is maximized. Formally, for a graph $G = (V, E)$, where V is the set of vertices and E is the set of edges, MaxCut seeks to find a partition $(S, \bar{S})$ of V that maximizes the number of edges with one endpoint in S and the other in $\bar{S}$. This problem has applications in various fields, including network designing efficient telecommunications networks, optimizing transportation routes, and maximizing the flow of goods or services in supply chain networks.



The MaxCut problem is NP-hard, meaning that no polynomial-time algorithm is currently known that can solve all instances of the problem. Therefore, finding efficient approximation algorithms for MaxCut is crucial for tackling a wide range of real-world optimization problems. The MaxCut problem can be formulated as a Quadratic Unconstrained Binary Optimization (QUBO) problem, where the goal is to minimize a quadratic function of binary variables subject to no constraints, a form particularly suitable for quantum annealing or other quantum optimization algorithms.

Indeed, the MaxCut problem can be mapped onto the search for the ground state of an Ising model. In this mapping, each vertex of the graph corresponds to a spin variable, and the interaction between spins is determined by the edges of the graph.

For further details and for an explicit construction of the Ising Hamiltonian associated to the MaxCut problem see [Max].

2 Quantum annealing.

Quantum annealing is a computational technique that exploits an adiabatic quantum dynamics to dynamically prepare the unknown ground state of a target Hamiltonian. At its core, quantum annealing relies on the adiabatic theorem, which ensures that if the evolution of a quantum system is slow enough compared to the energy gap between its ground state and the first excited state, the system remains in its instantaneous ground state throughout the evolution. In quantum annealing, this theorem is applied to the time-dependent Schrödinger equation, where the system's Hamiltonian evolves gradually from a simple, known initial Hamiltonian (H_0) to a problem-specific final Hamiltonian (H_f) over a certain time interval. This evolution is governed by an annealing schedule, typically a smooth function of time, which determines the rate at which the system transitions from H_0 to H_f. The simplest example in this direction is a linear interpolation:

$$H(t) = \frac{t-T}{T} H_0 + \frac{t}{T} H_f \ ,$$

where t runs from 0 to the annealing time T.

The energy spectrum of the system plays a crucial role in determining the annealing time T required to reach the ground state. A narrow energy gap between the ground state and the first excited state implies a slower annealing schedule to maintain adiabaticity, as the system needs more time to adjust to changes in the Hamiltonian without exciting to higher energy levels. Conversely, a wider energy gap allows for a faster annealing schedule, as the system can transition more rapidly while remaining in its ground state. Thus, the energy spectrum directly influences the efficiency of quantum annealing, with the annealing time balancing the need for adiabatic evolution and computational speed. By carefully designing the annealing schedule based on the system's energy spectrum, quantum annealing algorithms can effectively explore the solution space and find optimal solutions to complex optimization problems.

For further info see Ref. [ST06].

3 Simulated quantum annealing with Tensor network states.

The time evolution of Tensor network states can be simulated using a Time Dependent Variational Principle (TDVP) [HLO+16]. The main obstacle in this direction is the increasing correlation length that characterizes the evolution of a many-body system, and that is incompatible with any reasonable bond dimension for the Tensor network ansatz. However, simulating a quantum annealing process with a tensor network ansatz presents an intriguing perspective both in didactic contexts and for practical optimization applications [LTSC23]. This is because, in the adiabatic limit, the trajectory of states generated by the annealing dynamics forms a path of ground states of local Hamiltonians. When non-degenerate, these ground-states possess a finite correlation length [HK06], allowing for the utilization of tensor networks. Furthermore, the approximation introduced by the finite bond dimension can offer an advantage in searching for the ground state by suppressing non-adiabatic transitions to confine the dynamics to the space of states with a finite correlation length.

References

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