Probability

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Probability course contents

1 Permutation and Combination

- Basic counting principles
- Permutations
- Combinations
- The number of integer solutions of an equation

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Since 1995

3 Random Variables

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Poisson Approximation to Binomial Random variable

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For GATE $\{\underbrace{CE, EE}\}$

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Permutations and Combinations

Basic counting principles

Permutations

Combinations

The number of integer solutions of equations

2 pants   4 Shirts

$2 \times 4 = 8$

Basic Counting principles

Basic Counting principles | Product Rule

If one experiment can result in any of m possible outcomes and another experiment can result in any of n possible outcomes, then there are mn possible outcomes of the two experiments.

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A small community consists of 10 women, each of whom has 3 children. If one woman and one of her children are to be chosen as mother and child of the year, how many different choices are possible?

$10 \times 3 = 30$

2 pants 4 shirts 5 shoes

$2 \times 4 \times 5 = \underline{\underline{40}}$

The generalized basic principle of counting

If r experiments that are to be performed are such that the first experiment results in n_1 possible outcomes, second experiment results in n_2 possible outcomes, $\cdots$, r^{th} experiment in n_r possible outcomes then there is a total of $n_1 \times n_2 \times \cdots \times n_r$ possible outcomes of the r experiments.

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A college planning committee consists of 3 freshmen, 4 sophomores, 5 juniors, and 2 seniors. A subcommittee of 4, consisting of 1 person from each class, is to be chosen. How many different subcommittees are possible?

$$\underline{3} \times \underline{4} \times \underline{5} \times \underline{2} = \underline{\underline{120}}$$

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How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?

$\underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}$

26 letters

$0, 1, 2, \ldots 9 \Rightarrow 10$

$26^3 \times 10^4$

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How many license plates would be possible if repetition among letters or numbers were prohibited?

$\underline{26}$ $\underline{25}$ $\underline{24}$ $\underline{10}$ $\underline{9}$ $\underline{8}$ $\underline{7}$

$26 \times 25 \times 24 \times 10 \times 9 \times 8 \times 7$

Sum Rule

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Sum Rule

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If a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are n_1+n_2 ways to do the task.

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A student can choose a project from one of three lists. The three lists contain 23 Machine Learning, 15 IOT, and 19 Cyber Security possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

$$23 + 15 + 19 = \underline{\underline{57}}$$

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Suppose that either a member of the mathematics faculty or a student who is a mathematics major is chosen as a representative to a university committee. How many different choices are there for this representative if there are 17 members of the mathematics faculty and 43 mathematics majors and no one is both a faculty member and a student?

$$17 + 43 = \underline{\underline{60}}$$

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Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

At least → Complement Rule

Let P_6, P_7 and P_8 be the number of passwords of length 6, 7 and 8 resp where it contains at least one digit.

$36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 \cdot 36$

36^6

$26 \quad 26 \quad 26 \quad 26 \quad 26 \quad 26$ ⇒ passwords that contain only letters

26^6

26 letters → $26 \; 26 \; 26 \; 26 \; 26 \; 26$

0, 1, 2, .. 9 ⇒ 10 digits

Total characters = 36

$36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 \cdot 36$

$P_6 = 36^6 - 26^6$

$P_7 = 36^7 - 26^7$

$P_8 = 36^8 - 26^8$

Ans $P_6 + P_7 + P_8$

PERMUTATIONS

How many different ordered arrangements of the letters a, b, and c are possible?

a b c

a c b

b c a

b a c

c a b

c b a

6 arrangements are possible.

$\underline{3} \cdot \underline{2} \cdot \underline{1}$

$= 3 \times 2 \times 1$

$= 3!$

Each arrangement is called permutation

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Suppose now that we have n objects.
How many different ordered arrangements are possible?

$$\underline{n} \cdot \underline{n-1} \cdot \underline{n-2} \cdot \cdot \cdot \underline{1}$$

$$n(n-1)(n-2)\cdots 2\times 1 = n!$$

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How many different batting orders are possible for a baseball team consisting of 9 players?

Ans $\underline{\underline{9!}}$

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A class in probability theory consists of 6 men and 4 women. An examination is given, and the students are ranked according to their performance. Assume that no two students obtain the same score.

(a) How many different rankings are possible? $10!$

(b) If the men are ranked just among themselves and the women just among themselves, how many different rankings are possible?

Note: Consider the following order: men first and then next Women

men | women

$6!$ | $4!$

$\underline{\underline{6! \times 4!}}$

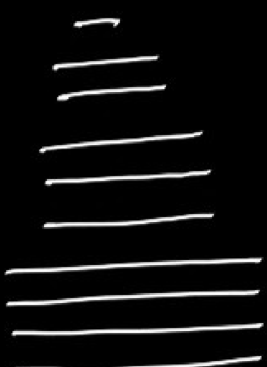
Ms. Jones has 10 books that she is going to put on her bookshelf. Of these, 4 are mathematics books, 3 are chemistry books, 2 are history books, and 1 is a language book. Ms. Jones wants to arrange her books so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

Books of same subject are different from each other.

$1 + 1 + 1 + 1 = 4$

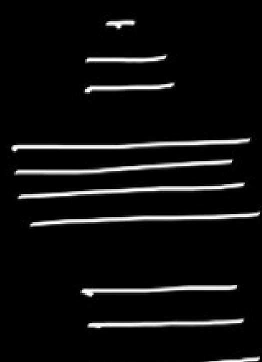
Ans $4! \times 4! \times 3! \times 2! \times 1!$

1



$4! \times 3! \times 2! \times 1!$

2



$3! \times 4! \times 2! \times 1!$

3

20

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How many different letter arrangements can be formed from the letters GATE?
(Only four letter words to be considered)

GATE

A, E, G, T $\Rightarrow$ 4 distinct objects

$4!$

How many different letter arrangements can be formed from the letters PEPPER?
(Only six letter words to be considered)

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A B C D E F ⑥

3 P's

2 E's

1 R

PPEPER

$P_1 P_2 E_1 P_3 E_2 R$

$P_2 P_1 E_2 P_3 E_1 R$

$P_3 P_1 E_2 P_2 E_1 R$

Arranging P's among themself and E's among themself will not result in new arrangement

EPRPPE

$E_1 P_1 R P_2 P_3 E_2$

$E_2 P_3 R P_1 P_2 E_1$

Ans $= \dfrac{6!}{3! \times 2!}$

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PPEPER

1

$P_1P_2E_1P_3E_2R$ $P_1P_2E_2P_3E_1R$

$P_1P_3E_1P_2E_2R$ $P_1P_3E_2P_2E_1R$

12 words

$P_2P_1E_1P_3E_2R$ $P_2P_1E_2P_3E_1R$

$P_2P_3E_1P_1E_2R$ $P_2P_3E_2P_1E_1R$

$P_3P_1E_1P_2E_2R$ $P_3P_1E_2P_2E_1R$

$P_3P_2E_1P_1E_2R$ $P_3P_2E_2P_1E_1R$

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In general, different permutations of n objects, of which n_1 are alike, n_2 are alike,$\cdots$, n_r are alike is given by

$$= \frac{n!}{n_1! \times n_2! \times \cdots \times n_r!}$$

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A chess tournament has 10 competitors, of which 4 are Russian, 3 are from the United States, 2 are from Great Britain, and 1 is from Brazil. If the tournament result lists just the nationalities of the players in the order in which they placed, how many outcomes are possible?

$$\frac{10!}{4! \times 3! \times 2!}$$

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How many different signals, each consisting of 9 flags hung in a line, can be made from a set of 4 white flags, 3 red flags, and 2 blue flags if all flags of the same color are identical?

$$\frac{9!}{4! \times 3! \times 2!}$$

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COMBINATIONS

COMBINATIONS

How many different groups of $\underline{\underline{r}}$ objects can be formed from a total of n objects? All objects are distinct.

Ans $^{n}C_{r}$

COMBINATIONS

Let us solve a simple question first.

How many different groups of three objects can be formed from a total of five objects (A,B,C,D,E)?

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ABC, ACB, BCA, BAC, CAB, CBA

BCD, BDC, CDB, CBD, DBC, DCB

CDE

$$\underline{5} \times \underline{4} \times \underline{3} = \underline{\underline{60}}$$ wrong answer

Every group of 3 objects has been counted 3! times.

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$$\frac{5 \times 4 \times 3}{3!} = \frac{5 \times 4 \times 3 \times 2!}{3! \times 2!} = \frac{5!}{3!(5-3)!} = {}^5C_3 = \underline{\underline{10}}$$

A committee of 3 is to be formed from a group of 20 people. How many different committees are possible?

Ans = $^{20}C_{3}$

From a group of 5 women and 7 men, how many different committees consisting of 2 women and 3 men can be formed? What if 2 of the men are feuding and refuse to serve on the committee together?

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① $^5C_2 \times {}^7C_3 = 10 \times 35 = 350$

m_1, m_2 → Feuding men

m_3, m_4, m_5, m_6, m_7

② The number of different groups of 3 men that are formed from 7 men is $^7C_3 = 35$

The number of groups of 3 men that are formed from 7 men such that feuding men are always together

m_1, m_2 $\underline{5 \text{ ways}} \Rightarrow 5$ groups

$m_1\ m_2\ m_3$ ✓

$m_1\ m_2\ m_4$ ✓

$m_1\ m_2\ m_5$ ✓

m_6 ✓

m_7 ✓

The number of groups of 3 men that are formed from 7 men such that feuding men are not always together $= 35 - 5 = \underline{\underline{30}}$

Ans $^5C_2 \times 30 = 10 \times 30 = \underline{\underline{300}}$

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$${}^5C_3 + {}^5C_2 \times 2 = \underline{\underline{30}}$$

$\left.\begin{array}{l} m_1 \\ m_2 \end{array}\right] \rightarrow$ Feuding men

m_3

m_4

m_5

m_6

m_7

Consider a set of n antennas of which m are defective and $n - m$ are functional and assume that all of the defectives and all of the functionals are considered distinguishable. How many linear orderings are there in which no two defectives are consecutive?

Consider a set of n antennas of which m are defective and $n - m$ are functional and assume that all of the defectives and all of the functionals are considered indistinguishable. How many linear orderings are there in which no two defectives are consecutive?

H/W

How many subsets are there of a set consisting of n elements?

NUMBER OF INTEGER SOLUTIONS OF EQUATIONS

Number of positive integer solutions to the equation

$$x_1 + x_2 + x_3 \cdots + x_r = n$$

NUMBER OF INTEGER SOLUTIONS OF EQUATIONS

Number of positive integer solutions to the equation

$x_1 + x_2 + x_3 \cdots + x_r = n$

Positive Integers $1, 2, 3, 4, \ldots$

Non-negative Integers $0, 1, 2, 3, \ldots$

$$\text{Ans} = {}^{n-1}C_{r-1}$$

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How many distinct positive integer-valued solutions of $x_1 + x_2 = 8$ are possible?

$1 + 7 = 8$ ✓

$2 + 6 = 8$ ✓

$3 + 5 = 8$ ✓

$4 + 4 = 8$ ✓

$5 + 3 = 8$ ✓

$6 + 2 = 8$ ✓

$7 + 1 = 8$ ✓

7 distinct positive integer solutions are possible.

How many distinct positive integer-valued solutions of $x_1 + x_2 + x_3 = 10$ are possible?

$2 + 3 + 5 = 10$

$1 + 8 + 1 = 10$

$${}^{9}C_{2} = {}^{10-1}C_{3-1} = 36$$

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Number of non-negative integer solutions to the equation $x_1 + x_2 + x_3 \cdots + x_r = n$ — 1

$$^{n+r-1}C_{r-1}$$

$x_1 = y_1 - 1$, $x_2 = y_2 - 1$, ... $x_r = y_r - 1$

when $y_1, y_2, \ldots y_r$ are positive integers then

$x_1, x_2, \ldots x_r$ are non-negative integers.

$$y_1 - 1 + y_2 - 1 + \cdots + y_r - 1 = n$$

$$y_1 + y_2 + \cdots + y_r = n + r \quad \text{— } 2$$

$$^{n+r-1}C_{r-1}$$

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How many distinct non-negative integer-valued solutions of $x_1 + x_2 = 3$ are possible?

$0 + 3 = 3$

$1 + 2 = 3$ Ans (4)

$2 + 1 = 3$

$3 + 0 = 3$

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How many distinct non-negative integer-valued solutions of $x_1 + x_2 + x_3 = 10$ are possible?

$n = 10$

$r = 3$

Ans $\;{}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

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An investor has 20 thousand dollars to invest among 4 possible companies. Each investment must be non-zero and be in units of a thousand dollars. If the total 20 thousand is to be invested, how many different investment strategies are possible?

Let x_1, x_2, x_3 and x_4 be the amounts invested in 4 companies in units of thousand dollars.

$$x_1 + x_2 + x_3 + x_4 = 20$$

$$^{20-1}C_{4-1} = {}^{19}C_3 = \underline{\underline{969}}$$

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How many distinct non-negative integer-valued solutions of $x_1 + x_2 = 3$ are possible?

$0 + 3 = 3$

$1 + 2 = 3$ Ans (4)

$2 + 1 = 3$

$3 + 0 = 3$

How many distinct non-negative integer-valued solutions of $x_1 + x_2 + x_3 = 10$ are possible?

$n = 10$

$r = 3$

Ans $\quad {}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

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Let x_1, x_2, x_3 and x_4 be the amounts invested in 4 companies in units of thousand dollars.

$$x_1 + x_2 + x_3 + x_4 = 20$$

$$^{20-1}C_{4-1} = {}^{19}C_3 = \underline{\underline{969}}$$

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How many subsets are there of a set consisting of n elements?

Sol Let us consider a set containing only two elements

$S = \{1, 2\}$

The possible subsets are

$\{\}$ $\{1\}$, $\{2\}$, $\{1, 2\}$

4 subsets are possible for a set 'S' which is containing 2 elements in it.

$\{\} \Rightarrow$ 0 elements $\{1\}, \{2\} \Rightarrow$ 1 element $\{1, 2\} \Rightarrow$ 2 elements

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In the same way if the set S contains 'n' elements then the total subsets are

$$\underbrace{{}^{n}C_{0}}_{\text{Null set}} + \underbrace{{}^{n}C_{1}}_{\text{subsets with exactly one element}} + \underbrace{{}^{n}C_{2}}_{\text{subsets with exactly two elements}} + \cdots + \underbrace{{}^{n}C_{n}}_{\text{Subset with n elements}} = 2^{n}$$

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How many different two digit numbers are formed by using the digits 2, 3 and 5.

2 3

2 5

3 2

3 5

5 2

5 3

Ans: 6

$$^{3}P_{2} = \frac{3!}{(3-2)!} = 6$$

How many different groups of two objects can be formed from a total of 3 objects A, B and C.

$\{A, B\}$ $\{A, C\}$ $\{B, C\}$

Ans $\underline{\underline{3}}$

$\underline{\underline{^{3}C_{2}}}$

Introduction to Probability

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Sample Space: The set of all possible outcomes of an experiment is known as the sample space of the experiment and is denoted by S.

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When a coin is tossed, the all possible outcomes are

$$S = \{H, T\}$$

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When a die is rolled, the all possible outcomes are

$$S = \{1, 2, 3, 4, 5, 6\}$$

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When a number is picked from first 200 natural numbers, all possible outcomes are

$$S = \{1, 2, 3, \ldots 200\}$$

Event: Any subset E of the sample space is known as an event.

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When a die is rolled, the even numbered outcomes possible are

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{2, 4, 6\} \subset S$$

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Consider the set of first 200 natural numbers, the numbers divisible by 6 are

$S = \{1, 2, 3, \ldots 200\}$

$E = \{6, 12, 18, \ldots 198\} \subset S$

$$\begin{array}{r|l} 6) & 200(33 \\ & \underline{18} \\ & \ \ 20 \\ & \ \ \underline{18} \\ & \ \ \ \ 2 \end{array}$$

$n(E) = 33$

If the experiment consists of flipping two coins, then the sample space consists of 4 points

$$S = \{HH, HT, TH, TT\}$$

If E is the event that a head appears on the first coin, then $E = \{HH, HT\}$

$$P(E) = \frac{2}{4}$$

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If the experiment consists of tossing two dice, then the sample space consists of 36 points

$$S = \begin{Bmatrix} (1,1) & (1,2) & \cdots & (1,6) \\ (2,1) & (2,2) & \cdots & (2,6) \\ \vdots & & & \\ (6,1) & (6,2) & \cdots & (6,6) \end{Bmatrix} \Rightarrow \text{Equally likely points.}$$

If E is the event that the sum of the dice equals 7, then $E = \{(1,6)\,(2,5)\,(3,4)\,(4,3)\,(5,2)\,(6,1)\}$

$(1,1)$ Sum = 2

$(6,6)$ Sum = 12

$$p(E) = \frac{6}{36}$$

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If the experiment consists of counting the number of calls received by a telephone station in a particular hour, then the sample space consists of non-negative integers.

$$S = \{x \mid x = 0, 1, 2, 3 \ldots\}$$

Countably infinite set

If E is the event that the number of calls received is more than three, then $E = \{x \mid x = 4, 5, 6, 7, \ldots\}$

$p(E) =$

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If the experiment consists of measuring (in hours) the lifetime of a Computer, then the sample space consists of non-negative real numbers

$$S = \{ x \mid x \geq 0 \} \Rightarrow$$ uncountably Infinite set

If E is the event that the Computer does not last longer than 5 hours, then $E = \{ x \mid 0 \leq x \leq 5 \}$

$$P(E) = \_\_\_\_$$

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Definition of Probability

If the sample space S of an experiment consists of finitely many outcomes (points) that are equally likely, then the probability $P(A)$ of an event A is given by

$$P(A) = \frac{\text{Number of points in } A}{\text{Number of points in } S}$$

Examples of equally likely events

Consider the experiment of tossing a coin. The possible outcomes are H,T.

$$p(H) = \frac{1}{2} \qquad p(T) = \frac{1}{2}$$

$$S = \{ \ldots \}$$

Consider the experiment of rolling a die. The possible outcomes are 1,2,3,4,5,6.

$$p(1) = p(2) = p(3) = p(4) = p(5) = p(6) = \frac{1}{6}$$

General Definition of Probability

Given a sample space S, with each event A of S (subset of S) there is associated a number $P(A)$ called the probability of A, such that the following axioms of probability are satisfied.

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Axioms of Probability

1. For every event A in S,

$$0 \le p(A) \le 1$$

$$A \subset S$$

$$n(A) \le n(S)$$

$$\frac{n(A)}{n(S)} \le 1$$

$$p(A) \le 1$$

$$p(A) \geq 0 \quad \text{non-negative}$$

$$p(A) = \frac{n(A)}{n(S)}$$

positive

$$\geq 0$$

$A = \{-1\}$

$n(A) = 1$

$A = \{0, 1\}$

$A = \{\ \}$

2. The entire sample space S has the probability equal to unity.

$$p(S) = \frac{n(S)}{n(S)} = 1$$

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3. For mutually exclusive events A and B $\{A \cap B = \phi\}$

$$p(A \cup B) = p(A) + p(B)$$

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Consider the experiment of rolling a die. Let E and O denote even and odd numbered outcomes of the sample space.

$p(E \cap O) = \frac{0}{6} =$

$S = \{1, 2, 3, 4, 5, 6\}$

$E = \{2, 4, 6\}$

Prime numbers $P = \{2, 3, 5\}$

$O = \{1, 3, 5\}$

$E \cap P \neq \phi$

$O \cap P \neq \phi$

$E \cap O = \phi$

E and O are mutually exclusive

If $A_1, A_2, \cdots, A_n$ are mutually exclusive events then

$$A_1 \cap A_2 = \phi \quad , \quad A_2 \cap A_3 = \phi$$

$$A_i \cap A_j = \phi$$

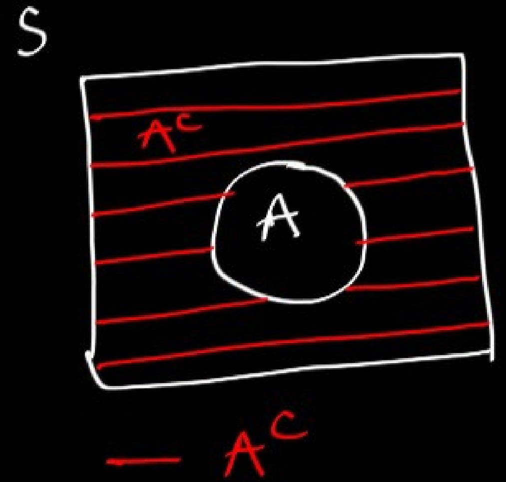
$$p(A_1 \cup A_2 \cup \cdots \cup A_n) = p(A_1) + p(A_2) + \cdots + p(A_n)$$

*** For mutually exclusive events, the probability of union is equal to sum of individual probabilities.

The probability of union of mutually exclusive events is equal to sum of individual probabilities.

Basic theorems

Complement Rule: For an event A and its complement A^c in a sample space S



$$A \cap A^c = \phi$$

$$A \cup A^c = S$$

$$p(A \cup A^c) = p(S)$$

$$p(A) + p(A^c) = 1$$

$$\boxed{p(A^c) = 1 - p(A)}$$ complement Rule

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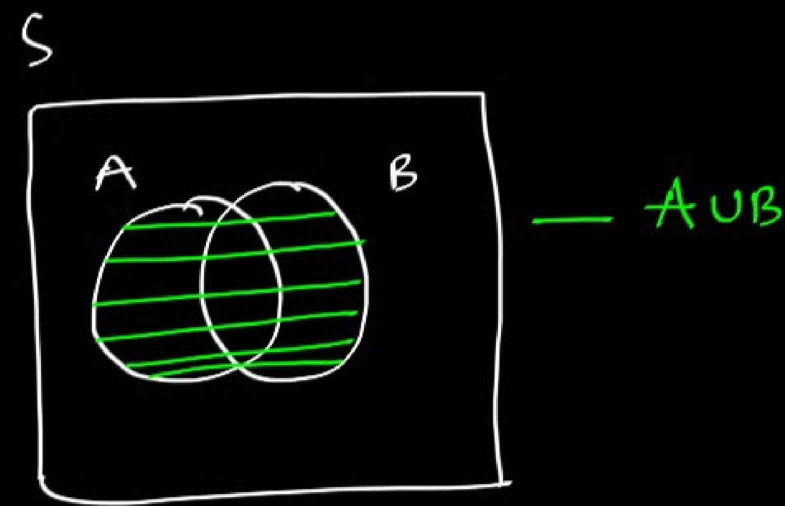
Addition Rule for Arbitrary Events: For events A and B in a sample space S

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

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For events A, B and C in a sample space S,

$$P(A \cup B \cup C) = p(A) + p(B) + p(C)$$
$$- p(A \cap B) - p(B \cap C) - p(C \cap A)$$
$$+ p(A \cap B \cap C)$$

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Similarly for events $A_1, A_2, \cdots, A_n$ in Sample space S

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum_{i=1}^{n} p(A_i) - \sum_{i_1 < i_2} p(A_{i_1} \cap A_{i_2}) + \sum_{i_1 < i_2 < i_3} p(A_{i_1} \cap A_{i_2} \cap A_{i_3}) + \cdots (-1)^{r+1} \sum_{i_1 < i_2 < \cdots i_r} p(A_{i_1} \cap A_{i_2} \cdots \cap A_{i_r}) + \cdots (-1)^{n+1} p(A_1 \cap A_2 \cap \cdots \cap A_n)$$

${}^nC_1 = n$

${}^nC_2$ terms

${}^nC_3$ terms

${}^nC_r$

Frequently used words in Probability course

At least: Greater than or equal to

$(\geq)$

What is the probability of having at least one error ?

$$p\left(\begin{array}{c}\text{At least}\\ \text{one error}\end{array}\right) = p(\text{error}=1) + p(\text{errors}=2) + p(\text{errors}=3) + \cdots$$

$$= 1 - p(\text{errors} < 1)$$

$$= 1 - p(\text{errors} = 0)$$

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At most: less than or equal to $(\leq)$

What is the probability of having at most two errors ?

$$p\left(\begin{matrix}\text{At most} \\ \text{two errors}\end{matrix}\right) = p(\text{errors} \leq 2)$$

$$= p(\text{errors} = 0) + p(\text{errors} = 1) + p(\text{errors} = 2)$$

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ACE

OR → Union → $\cup$

AND → Intersection → $\cap$

Break upto 4.20pm

ya Vangala

When you throw three fair dice, probability that the sum of the numbers on the top faces is 10 is

$n(S) = 6^3 = 216$

$S = \{(1,1,1)\ (1,1,2) \ldots\}$

Let x_1, x_2, x_3 be the outcomes on the three dices.

$x_1 + x_2 + x_3 = 10$ — (1)

N.o of positive integer solutions of eqn (1) $\quad {}^{10-1}C_{3-1} = {}^{9}C_{2} = 36$

Invalid solutions

$1+1+8$
$1+8+1$
$8+1+1$

$\frac{3!}{2!} = 3$

$1+2+7 \Rightarrow 3! = 6$

Valid solutions $= 36-(6+3)$
$= 27$

$P(\text{sum}=10) = \frac{27}{216}$

Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is $\frac{x}{1296}$. The value of x is

GATE-2014

A number is selected at random from first 200 natural numbers. Find the probability that the number is divisible by 6 or 8

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Sol

A = divisible by 6

B = divisible by 8

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$A = \{6, 12, 18, \ldots 198\}$

$6)\ 200\ (33$

18

20

18

2

$P(A) = \frac{33}{200}$

$B = \{8, 16, 24, \ldots 200\}$

$8)\ 200\ (25$

16

40

40

0

$P(B) = \frac{25}{200}$

$A \cap B = \{24, 48, \ldots 192\}$

$L.C.M(6, 8) = 24$

$24 \times 8 = 192$

$24)\ 200\ (8$

192

8

$P(A \cap B) = \frac{8}{200}$

$$P(A \cup B) = \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{50}{200} = \frac{1}{4}$$

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An integer is selected at random between 1 and 1000 (both inclusive). The probability that it is NOT divisible by 12 or 15 is

$$p\left(\begin{array}{c}\text{Not divisible}\\ \text{by 12 or 15}\end{array}\right) = 1 - p(12 \text{ or } 15)$$

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Suppose we want to pick two numbers from $\{1, 2, \cdots, 100\}$ randomly. The probability that sum of the two numbers is divisible by 5 is

$S = \{1, 2, 3, \ldots 100\}$

$S_0 = \{5, 10, 15, \ldots 100\}$

$n(S_0) = 20$

$S_1 = \{1, 6, 11, 16, \ldots\}$

$n(S_1) = 20$

$S_4 = \{4, 9, 14, 19, \ldots\}$

$n(S_4) = 20$

$S_2 = \{2, 7, 12, 17, \ldots\}$

$n(S_2) = 20$

$S_3 = \{3, 8, 13, 18, \ldots\}$

$n(S_3) = 20$

$$p(\text{Sum divisible by } 5) = \frac{{}^{20}C_2 + {}^{20}C_1 \cdot {}^{20}C_1 + {}^{20}C_1 \cdot {}^{20}C_1}{{}^{100}C_2}$$

$$= \frac{1}{5}$$

In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random has a sibling is ___ (GATE 2014)

Let the total number of families be N.

$$N \begin{cases} \frac{N}{2} \longrightarrow 1 \text{ child per family} \longrightarrow \frac{N}{2} \times 1 = \frac{N}{2} \\ \frac{N}{2} \longrightarrow 2 \text{ children per family} \longrightarrow \frac{N}{2} \times 2 = N \end{cases}$$

$$\text{Total} = \frac{N}{2} + N = \frac{3N}{2}$$

$$\text{Ans} = \frac{{}^{N}C_1}{{}^{3N/2}C_1} = \frac{N}{3N/2} = \frac{2}{3}$$

Consider the following speedy but inaccurate algorithm for guessing the median of a set S, where $n = |S|$ is odd and $n \geq 3$. Choose a three element subset R of S uniformly at random, then return the median of R. what is the probability that median of R is infact the median of S?

(a) $\frac{3(n+1)}{2n(n+2)}$

(b) $\frac{2(n-1)}{3n(n-2)}$

(c) $\frac{2(n+1)}{3n(n+2)}$

(d) $\frac{3(n-1)}{2n(n-2)}$

A die is loaded in such a way that the probability of the face with j dots turning up is proportional to j for $j = 1, 2, 3, 4, 5, 6$. What is the probability, in one roll of the die, that an odd number of dots will turn up?

(a) $\frac{1}{7}$

(b) $\frac{2}{7}$

(c) $\frac{3}{7}$

(d) $\frac{4}{7}$

A four digit number chosen at random. The probability that there are exactly two zeros in that number is

Six boys and six girls sit in a row. The probability that all the six girls sit together is

(a) $\frac{7! \times 6!}{12!}$

(b) $\frac{6! \times 6!}{12!}$

(c) $\frac{7!}{12!}$

(d) $\frac{7! \times 7!}{12!}$

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A card is selected at random from a pack of 52 cards. What is the probability that it is a

(1) Spade (or) Face card

(2) King (or) Red card

(3) King (or) Queen card

1) $P(S \cup F) = P(S) + P(F) - P(S \cap F)$

$$= \frac{^{13}C_1}{^{52}C_1} + \frac{^{12}C_1}{^{52}C_1} - \frac{^{3}C_1}{^{52}C_1}$$

$$= \frac{22}{52}$$

2) $P(K \cup R)$

$P(K) + P(R) - P(K \cap R)$

$$\frac{^{4}C_1}{^{52}C_1} + \frac{^{26}C_1}{^{52}C_1} - \frac{^{2}C_1}{^{52}C_1}$$

$$= \frac{28}{52} = 7/13$$

3) $P(K \cup Q)$

$P(K) + P(Q) - P(K \cap Q)$

$$\frac{^{4}C_1}{^{52}C_1} + \frac{^{4}C_1}{^{52}C_1} - 0 = \frac{8}{52}$$

Card Classification ♣♦♠♥

- 52 cards
  - 26 Red
    - 13 Diamounds A, 2, 3 .. 10, K, Q, J
    - 13 Hearts A, 2, 3 .. 10, K, Q, J
  - 26 Black
    - 13 Clubs A, 2, 3 .. 10, K, Q, J
    - 13 Spades A, 2, 3 .. 10, K, Q, J

Face cards $\Rightarrow$ 12

Two cards are drawn at random from a pack of 52 cards. What is the probability that

(1) both of them from same suit

(2) both of them from different suits

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$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

1) $$\frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2} = 4 \times \frac{^{13}C_2}{^{52}C_2} = \frac{4 \times 13 \times 12/2}{52 \times 51/2}$$

$$= \frac{4}{17}$$

2) $$p(\text{both from different suit}) = 1 - p(\text{both from same suit})$$

$$= 1 - \frac{4}{17} = \frac{13}{17} \; //$$

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Consider the following speedy but inaccurate algorithm for guessing the median of a set S, where $n = |S|$ is odd and $n \geq 3$. Choose a three element subset R of S uniformly at random, then return the median of R. what is the probability that median of R is infact the median of S?

(a) $\frac{3(n+1)}{2n(n+2)}$

(b) $\frac{2(n-1)}{3n(n-2)}$

(c) $\frac{2(n+1)}{3n(n+2)}$

(d) $\frac{3(n-1)}{2n(n-2)}$

$\{2, 3, 4\}$

median = 3

$\{2, 3, 4, 5\}$

median $= \frac{3+4}{2} = 3.5$

median of S

$S = \{ \underbrace{\_\ \_\ \_\ \_}_{\frac{n-1}{2}} \boxed{\_} \underbrace{\_\ \_\ \_\ \_}_{\frac{n-1}{2}} \}$

$R = \{ \frac{n-1}{2} \ \boxed{\_} \ \frac{n-1}{2} \}$

$$\text{Ans} = \frac{\left(\frac{n-1}{2}\right)(1)\left(\frac{n-1}{2}\right)}{{}^nC_3} = \frac{3(n-1)}{2n(n-2)}$$

11. An integer is selected at random between 1 & 1000 (both inclusive). The probability that it is NOT divisible by 12 or 15 is _______ [correct to 3 decimal places]

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Let A be the event that a number is divisible by 12 and B be the event that a number is divisible by 15.

$$12)\,1000\,(83$$
$$\frac{96}{40}$$
$$\frac{36}{(4)}$$

$\Rightarrow A = \{12, 24, \ldots 996\}$

$n(A) = 83$

$$15)\,1000\,(66$$
$$\frac{90}{100}$$
$$\frac{90}{10}$$

$\Rightarrow B = \{15, 30, \ldots 990\}$

$n(B) = 66$

L.C.M $(12, 15) = 60$

$\Rightarrow$ $$60)\,1000\,(16$$
$$\frac{60}{400}$$
$$\frac{360}{(40)}$$

$A \cap B = \{60, 120, 180, \ldots 960\}$

$\Rightarrow n(A \cap B) = 16$

$p(A \cup B) = p(A) + p(B) - p(A \cap B) = \frac{83}{1000} + \frac{66}{1000} - \frac{16}{1000} = 133/1000$

Required Ans $= 1 - \frac{133}{1000} = \frac{867}{1000}$

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02. A die is loaded in such a way that the probability of the face with j dots turning up is proportional to j for j = 1, 2, 3, 4, 5, 6. What is the probability, in one roll of the die, that an odd number of dots will turn up?

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

$p(X=j) \propto j$ ✓

$p(X=j) = kj$

$p(X=1) + p(X=2) + p(X=3) + \dots p(X=6) = 1$

$K + 2K + 3K + \dots 6K = 1$

$K(1+2+\dots 6) = 1$

$\frac{K(6)(6+1)}{2} = 1$

$K = \frac{1}{21}$

p(odd number of dots will turn up) $= p(X=1) + p(X=3) + p(X=5)$

$= \frac{1}{21} + 3\cdot\frac{1}{21} + 5\cdot\frac{1}{21} = \frac{9}{21} = \frac{3}{7}$

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04. A four digit number chosen at random. The probability that there are exactly two zeros in that number is _______ [correct to 3 decimal places]

__ __ __ __

Thousand's place can be filled in 9 ways as zero is not allowed.

In the remaining three places, we need two zeros which can be done in ${}^3C_2$ ways.

The remaining one place can be filled in 9 ways as we need exactly two zeros.

$$\text{Ans} = \frac{9 \times {}^3C_2 \times 9}{9 \times 10 \times 10 \times 10} = 0.027$$

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05. Six boys and six girls sit in a row. The probability that all the six girls sit together is

(a) $\frac{7!\times 6!}{12!}$ (b) $\frac{(6!)^2}{12!}$

(c) $\frac{7!}{12!}$ (d) $\frac{(7!)^2}{12!}$

Treat six girls as one unit

Total objects = six boys + 1 unit of girls

= 7 objects

$$\frac{7!\,6!}{12!}$$

(a)

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Conditional Probability

Conditional Probability

How do you read this $P(A|B)$?

Ans

Given that the outcomes are only from B what is the probability of A.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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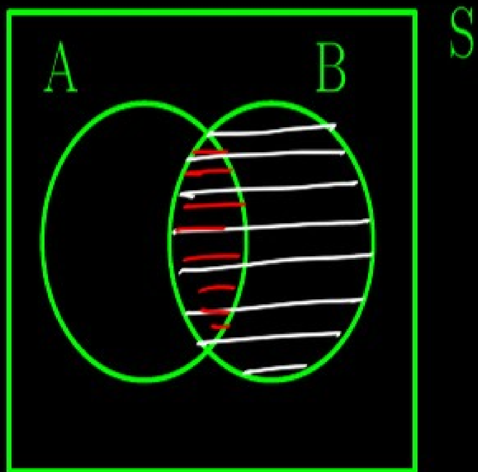
Conditional Probability



$$p(A|B) = \frac{n(A\cap B)}{n(B)}$$

$$= \frac{n(A\cap B)/n(S)}{n(B)/n(S)}$$

$$= \frac{p(A\cap B)}{p(B)}$$

B is called as Reduced Sample space.

Similarly the conditional probability $P(B|A)$ is given by

$$p(B|A) = \frac{p(A \cap B)}{p(A)}$$

A acts like Reduced sample space.

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A ticket is selected at random from 100 tickets numbered $\{00, 01, 02, \cdots, 99\}$. If X and Y denote the sum and product of the digits on the tickets respectively. The value of $P(X = 9 | Y = 0)$ is

(a) $\frac{1}{19}$

(b) $\frac{2}{19}$

(c) $\frac{3}{19}$

(d) $\frac{4}{19}$

$S = \{00, 01, \ldots 99\}$

$\underbrace{\{Y=0\} = \left\{\begin{matrix} 00, 01, \ldots 09 \\ 10, 20, \ldots 90 \end{matrix}\right\}}_{\text{Reduced sample space.}}$ # 19

$p(X=9 | Y=0) = \frac{2}{19}$

07

$X = 0 + 7$

$= 7$

$Y = 0(7)$

$= 0$

14

$X = 1+4 = 5$

$Y = 1 \times 4 = 4$

A die is thrown three times and sum of the numbers is found to be 16. The probability that 5 appears on the third throw is

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{2}{5}$

(d) $\frac{3}{10}$

$n(S) = 6^3 = 216$

$$\{\text{sum} = 16\} = \left\{\begin{array}{c} 4,6,6 \\ 6,4,6 \\ 6,6,4 \\ 5\ 5\ 6 \\ \boxed{\begin{array}{c} 5\ 6\ 5 \\ 6\ 5\ 5 \end{array}} \end{array}\right\} \Rightarrow \text{Reduced sample Space}$$

$\text{Ans} = \frac{2}{6}$

$= \frac{1}{3}$

$A \cap B = \phi$

If A and B are mutually exclusive then

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0}{P(A)} = 0$$

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Multiplication rule

$$p(A \cap B) = p(A|B)\,p(B)$$

$$= p(B|A)\,p(A)$$

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

$$p(B|A) = \frac{p(A \cap B)}{p(A)}$$

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Multiplication rule

$A \cap B = \phi$

If A and B are mutually exclusive then

$p(A \cap B) = 0 \quad , \quad p(A|B) = 0 \quad , \quad p(B|A) = 0$

$A \cap B = \phi \Rightarrow A$ and B are mutually exclusive.

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Since 1995

Multiplication rule

If A and B are independent then

$$p(A|B) = p(A)$$

$$p(B|A) = p(B)$$

$$p(A \cap B) = p(A|B)\,p(B) = p(A) \cdot p(B)$$

Independent events ex

The outcomes of the coin and outcomes of die are independent mutually exclusive events

$$A \cap B = \phi$$

$$p(A \cap B) = 0 \quad , \quad p(A|B) = 0 \quad p(B|A) = 0$$

Independent events A, B

$$p(A|B) = p(A) \quad , \quad p(B|A) = p(B)$$

$$p(A \cap B) = p(A) \cdot p(B)$$

For independent events, the probability of intersection is equal to product of individual probabilities.

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If $A_1, A_2, \cdots, A_n$ are independent then

$$p(A_1 \cap A_2 \cap A_3 \cdots \cap A_n) = p(A_1) \cdot p(A_2) \cdots p(A_n)$$

Independent events arise in the following cases:

- When a particular experiment is repeated multiple times
- When multiple experiments are taken together

$$p(H \cap 6) = p(H) \cdot p(6)$$

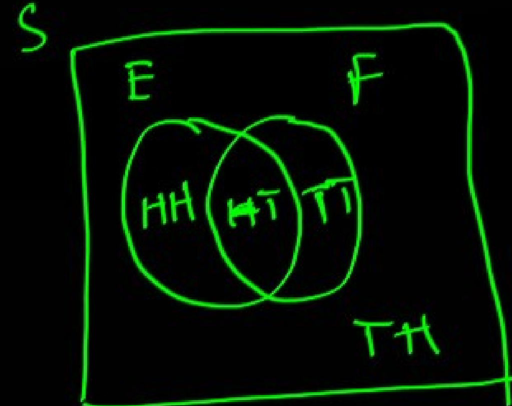
Two coins are flipped, and all 4 outcomes are assumed to be equally likely. Let E be the event that the first coin lands on heads and F the event that the second lands on tails. Are the events E and F independent?

$S = \{HH, HT, TH, TT\}$

$E = \{HH, HT\}$

$F = \{HT, TT\}$

$E \cap F = \{HT\}$



$P(E \cap F) = \frac{1}{4}$

$P(E) = \frac{2}{4}$

$P(F) = \frac{2}{4}$

$P(E \cap F) = P(E) \cdot P(F)$

$\frac{1}{4} = \frac{\cancel{2}}{\cancel{4}} \cdot \frac{2}{4}$

E and F are independent

A card is selected at random from an ordinary deck of 52 playing cards. Let E is the event that the selected card is an Ace and F be the event that it is a Spade. Are the events E and F independent?

$$p(E) = \frac{{}^{4}C_1}{{}^{52}C_1} = \frac{4}{52}$$

$$p(F) = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52}$$

$$p(E \cap F) = \frac{1}{52}$$

$$p(E \cap F) = p(E) \cdot p(F)$$

$$\frac{1}{52} = \frac{\cancel{4}}{\cancel{52}_{13}} \cdot \frac{\cancel{13}}{52}$$

E and F are $\underline{\text{independent}}$

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If A and B are two events such that the conditional probability $P(A|B) = 1$. Then $P(B^c|A^c)$ is

(a) $\frac{1}{4}$

(b) $\frac{1}{5}$

(c) 1

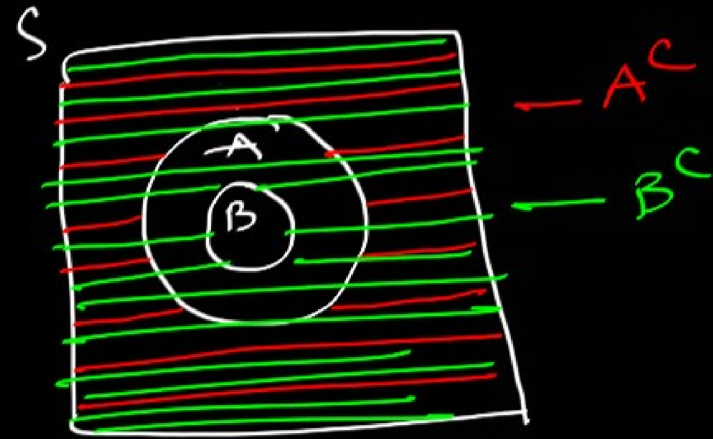
(d) $\frac{3}{4}$

$$p(A|B) = 1$$

$$\frac{p(A \cap B)}{p(B)} = 1$$

$$p(A \cap B) = p(B)$$

$$A \cap B = B$$



$$A^C \cap B^C = A^C$$

$$p(A^C \cap B^C) = p(A^C)$$

$$\frac{p(A^C \cap B^C)}{p(A^C)} = 1 \Rightarrow p(B^C/A^C) = 1$$

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Let E and F be any two events with $P(E) = 0.4$, $P(F) = 0.3$ and $P(F/E) = 3P(F/E^C)$. Then $P(E/F) =$

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A box contains the following three coins.

I. A fair coin with head on one face and tail on the other face

II. A coin with heads on both faces.

III. A coin with tails on both faces

A coin picked randomly from the box and tossed. Out of the two remaining coins in the box, one coin is then picked randomly and tossed. If the first toss results in a head, the probability of getting a head in the second toss is GATE-2021

A = Head in 1^{st} toss

B = Head in 2^{nd} toss

$$p(B|A) = \frac{p(A \cap B)}{p(A)}$$

$$p(A) = \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1$$

$$= \frac{1}{6} + \frac{1}{3}$$

$$= \frac{1}{2}$$

Break upto 4.20

$$p(A \cap B) = \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 1 + \frac{1}{3} \cdot 1 \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$$= 1/6$$

$$p(B|A) = \frac{1/6}{1/2} = \frac{1}{3}$$

H/w

A pair of dice is rolled. If the numbers appearing are different then the probability that sum is even is

(a) $\frac{1}{5}$

(b) $\frac{2}{5}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

H/w

Let A and B be independent events in Sample space S. It is known that $P(A \cap B) = 0.16$ and $P(A \cup B) = 0.64$ then $P(A)$ is

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H/W

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Player A plays two chess matches each with players B and C. A win, a draw and a loss are given 2, 1, 0 points respectively. The probability that A wins, draws or loses in any match is 0.6, 0.3 and 0.1 respectively. Outcomes of the matches are assumed to be independent. The probability that A gets at least 6 points is

H/w

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Let A, B and C be pairwise independent events such that $P(A) = P(B) = P(C) = \frac{1}{3}$ and $P(A \cap B \cap C) = \frac{1}{4}$. Then the probability that at least one of the events among A, B and C occurs is

(a) $\frac{11}{12}$

(b) $\frac{7}{12}$

(c) $\frac{5}{12}$

(d) $\frac{3}{4}$

H/W

Two biased coins C_1 and $C2$ have probabilities of getting heads $\frac{2}{3}$ and $\frac{3}{4}$ respectively. If both coins are tossed independently two times each, then the probability of getting exactly two heads out of these four tosses is

(a) $\frac{1}{4}$

(b) $\frac{37}{144}$

(c) $\frac{41}{144}$

(d) $\frac{49}{144}$

Bayes Rule

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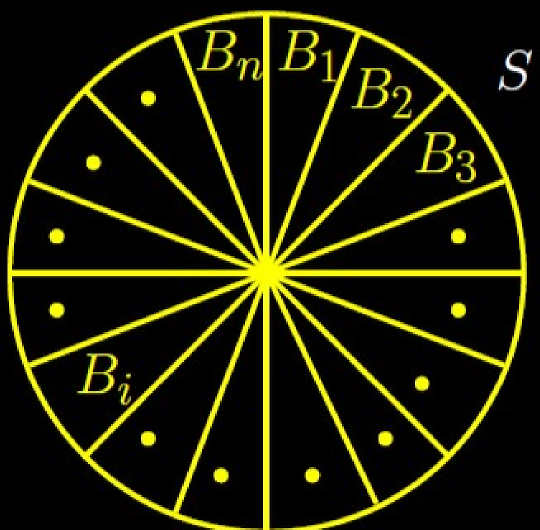
ACE

Bayes Rule

Consider the n mutually exclusive events $B_1, B_2, B_3, \cdots, B_n$ of Sample Space S.



$$B_i \cap B_j = \phi \quad \forall\ i \neq j$$

$$\underbrace{B_1 \cup B_2 \cup B_3 \cdots \cup B_n}_{\text{Exhaustive events.}} = S$$

Consider the experiment of tossing a fair dice. The possible outcomes are 1,2,3,4,5,6. Let E and O be the set of even and odd numbers.

$$E = \{2, 4, 6\}$$

$$O = \{1, 3, 5\}$$

$$E \cap O = \phi$$

$$E \cup O = S$$

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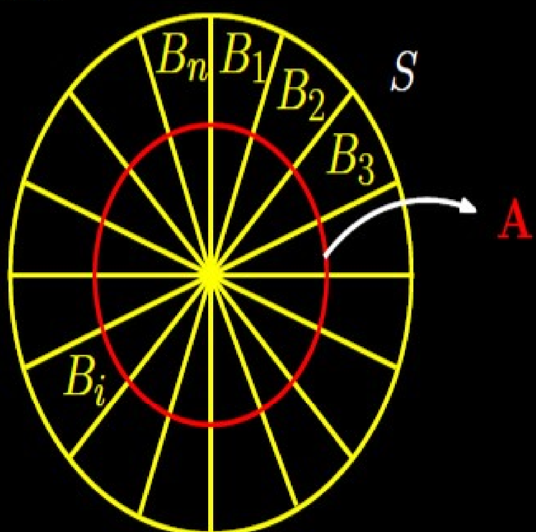
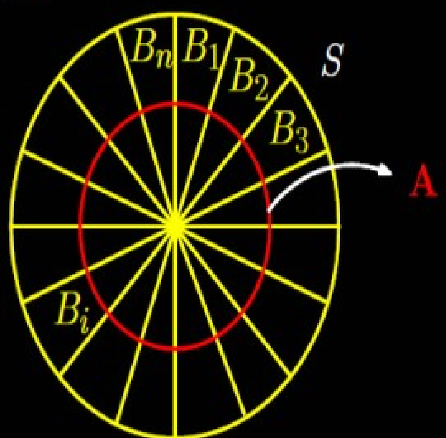
Bayes Rule

Consider the n mutually exclusive events $B_1, B_2, B_3, \cdots, B_n$ of Sample Space S. Let A be an event associated with the sample space S as shown below.



Bayes Rule

Consider the n mutually exclusive events $B_1, B_2, B_3, \cdots, B_n$ of Sample Space S. Let A be an event associated with the sample space S as shown below.



$$S = \left\{ \begin{matrix} (1,1) & (1,2) & \cdots & (1,6) \\ (2,1) & (2,2) & \cdots & (2,6) \\ & & \vdots & \\ (6,1) & (6,2) & \cdots & (6,6) \end{matrix} \right\}$$

A person throws two dice simultaneously. If the sum of the outcomes is 12, he offers lunch with probability $\frac{2}{3}$, if the sum is 7, he offers a lunch with probability $\frac{1}{2}$, in all others cases, he offers lunch with probability $\frac{1}{3}$. Identify the mutually exclusive events and the event A.

$$B_1 = \{\text{Sum} = 12\} = \{(6,6)\}$$

$$B_2 = \{\text{Sum} = 7\} = \left\{ \begin{matrix} (1,6) & (2,5) & (3,4) \\ (4,3) & (5,2) & (6,1) \end{matrix} \right\}$$

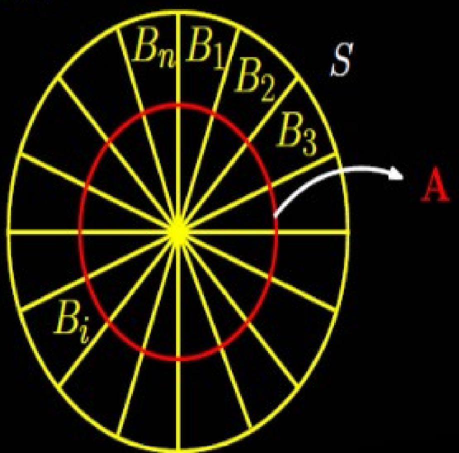
$$B_3 = \{\text{Sum} = \text{others}\} = \left\{ \begin{matrix} (1,1) & (1,2) & \cdots \\ & \vdots & \end{matrix} \right\}$$

29 points

A = offering lunch

Bayes Rule

Consider the n mutually exclusive events $B_1, B_2, B_3, \cdots, B_n$ of Sample Space S. Let A be an event associated with the sample space S as shown below.



The probability that a cell in a wireless system is over loaded is $\frac{1}{3}$. Given that it is overloaded, the probability of a blocked call is 0.3. Given that it is not overloaded the probability of a blocked call is 0.1. Identify the mutually exclusive events and the event A.

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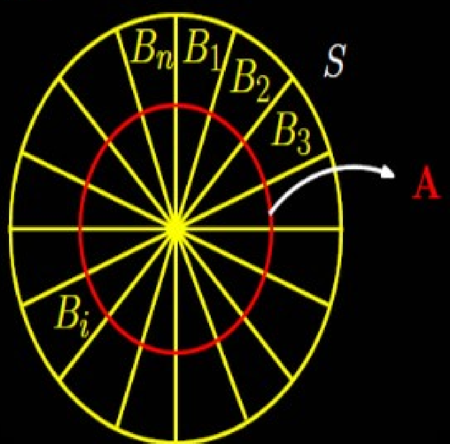
Bayes Rule

Consider the n mutually exclusive events $B_1, B_2, B_3, \cdots, B_n$ of Sample Space S. Let A be an event associated with the sample space S as shown below.



An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable. Identify the mutually exclusive events and the event A.

$B_1 = X$

$B_1 \cap B_2 = \phi$

$B_2 = Y$

A = Reliability

Bayes Rule

Consider the n mutually exclusive events $B_1, B_2, B_3, \cdots, B_n$ of Sample Space S. Let A be an event associated with the sample space S as shown below.

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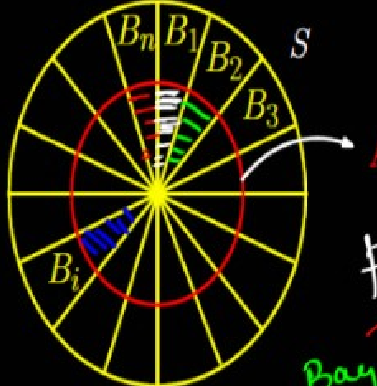

$$A = (A \cap B_1) \cup (A \cap B_2) \cup \cdots (A \cap B_i) \cdots \cup (A \cap B_n)$$

$$p(A) = p(A \cap B_1) + p(A \cap B_2) + \cdots p(A \cap B_i) + p(A \cap B_n)$$

$$p(A) = p(A|B_1)p(B_1) + \cdots + p(A|B_i)p(B_i) + \cdots p(A|B_n)p(B_n)$$

Total probability

Baye's Rule

$$\boxed{p(B_i|A) = \frac{p(A \cap B_i)}{p(A)} = \frac{p(A|B_i)\,p(B_i)}{p(A|B_1)p(B_1) + \cdots p(A|B_i)p(B_i) + \cdots p(A|B_n)p(B_n)}}$$

$$p(B_1|A) = \frac{p(A|B_1)\,p(B_1)}{p(A|B_1)p(B_1) + \cdots + p(A|B_i)p(B_i) + \cdots p(A|B_n)p(B_n)}$$

A person throws two dice simultaneously. If the sum of the outcomes is 12, he offers lunch with probability $\frac{2}{3}$, if the sum is 7, he offers lunch with probability $\frac{1}{2}$. In all others cases, he offers lunch with probability $\frac{1}{3}$

(1) Find the probability that lunch is offered

(2) If the lunch is offered then the probability that the sum of the outcomes is 12 i-

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$S = \begin{Bmatrix} (1,1)\,(1,2) \cdots (1,6) \\ (2,1)\,(2,2) \cdots (2,6) \\ \vdots \\ (6,1)\,(6,2) \cdots (6,6) \end{Bmatrix}$

$B_1 = \{Sum = 12\} = \{(6,6)\} \quad p(B_1) = 1/36$

$B_2 = \{Sum = 7\} = \begin{Bmatrix} (1,6)\,(2,5)\,(3,4) \\ (4,3)\,(5,2)\,(6,1) \end{Bmatrix} \quad p(B_2) = \frac{6}{36}$

$B_3 = \{Sum = others\} = \{(1,1)\,(1,2) \cdots\}$ 29 points $\quad p(B_3) = \frac{29}{36}$

A = offering lunch

$p(A|B_1) = \frac{2}{3} \quad p(A|B_2) = \frac{1}{2} \quad p(A|B_3) = 1/3$

1) $p(A) = p(A|B_1)p(B_1) + p(A|B_2)p(B_2) + p(A|B_3)p(B_3)$

$= \frac{2}{3} \cdot \frac{1}{36} + \frac{1}{2} \cdot \frac{6}{36} + \frac{1}{3} \cdot \frac{29}{36}$

$= 10/27$

(ii) $p(B_1|A) = \dfrac{\frac{2}{3} \cdot \frac{1}{36}}{10/27}$

$= \frac{1}{20}$

The probability that a cell in a wireless system is over loaded is $\frac{1}{3}$. Given that it is over loaded, the probability of a blocked call is 0.3. Given that it is not overloaded the probability of a blocked call is 0.1.

(i) find the probability that your call is blocked

(ii) find the conditional probability that the system is overloaded given that your call is blocked

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An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable. The probability that a randomly choosen shock absorber, which is found to reliable, is made by Y is

(GATE-12)

(a) 0.288

(b) 0.334

(c) 0.667

(d) 0.720

$B_1 = X \qquad B_2 = Y$

$P(B_1) = 0.6 \qquad P(B_2) = 0.4$

$P(A|B_1) = 0.96 \qquad P(A|B_2) = 0.72$

$$P(B_2|A) = \frac{(0.72)(0.4)}{(0.96)(0.6) + (0.72)(0.4)}$$

$$= 0.334$$

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A cab was involved in a hit and run accident at night. You are given the following data about the cabs in the city and the accident.

(i) 85% of cabs in the city are green and the remaining cabs are blue.

(ii) a witness identified the cab involved in the accident as blue.

(iii) it is known that a witness can correctly identify the cab colour only 80% of the time.

Which of the following options is closest to the probability that the accident was caused by a blue cab? (GATE 2018)

(a) 12% (c) 41% (b) 15% (d) 80%

$p(G) = 0.85$

$p(B) = 0.15$

$$= \frac{(0.15)(0.8)}{(0.15)(0.8) + (0.85)(0.2)}$$

$= 0.41$

Ans: 41%

$p\left(\begin{array}{c}\text{Accident} \\ \text{caused} \\ \text{by Blue}\end{array} \middle| \begin{array}{c}\text{witness identified} \\ \text{as Blue}\end{array}\right)$

Bag A contains 3 red and 7 white balls and Bag B contains 5 red and 4 white balls. One ball is drawn at random from the first bag and transferred to the second bag. Now, a ball is drawn from the second bag. Which of the following is/are CORRECT?

(a) The probability of drawn ball from the second bag is white is $\frac{47}{100}$

(b) The probability of drawn ball from the second bag is red is $\frac{53}{100}$

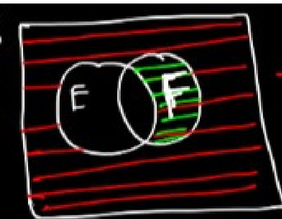
(c) If the drawn ball from the second bag is white then the probability that a red ball was transferred to bag B is $\frac{12}{47}$

(d) If the drawn ball from the second bag is red then the probability that a white ball was transferred to bag B is $\frac{35}{53}$

There are three urns labeled Urn 1, Urn 2 and Urn 3. Urn1 contains 2 white balls and 2 black balls, Urn 2 contains 1 white ball and three black balls and Urn 3 contains 3 white balls and 1 black ball. Consider two coins with probability of obtaining head in their single trials are 0.2 and 0.3. The two coins are tossed independently once, and an urn is selected according to the following scheme: Urn 1 is selected if 2 heads are obtained; Urn 3 is selected if 2 tails are obtained; otherwise Urn2 is selected. A ball is then drawn at random from the selected urn. Then P(Urn 1 is selected / the ball drawn is white) is equal to

Let E and F be any two events with $P(E) = 0.4$, $P(F) = 0.3$ and $P(F/E) = 3P(F/E^C)$. Then $P(E/F) =$

S



— E^C

Sol

$$p(E|F) = \frac{p(E \cap F)}{p(F)}$$

$$p(F|E) = 3\,p(F|E^c)$$

$$\frac{p(E \cap F)}{p(E)} = \frac{3\,p(F \cap E^c)}{p(E^c)}$$

$$p(F \cap E^c) = p(F) - p(E \cap F)$$

$$\frac{p(E \cap F)}{p(E)} = \frac{3\,(p(F) - p(E \cap F))}{1 - p(E)}$$

$$\frac{p(E \cap F)}{\cancel{0.4}\ 2} = \frac{\cancel{3}\,(0.3 - p(E \cap F))}{\cancel{0.6}\ \cancel{2}\ 1}$$

$$p(E \cap F) = 2(0.3) - 2\,p(E \cap F)$$

$$3\,p(E \cap F) = 2(0.3)$$

$$p(E \cap F) = 0.2$$

$$p(E|F) = \frac{0.2}{0.3}$$

$$= 2/3$$

A pair of dice is rolled. If the numbers appearing are different then the probability that sum is even is

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(a) $\frac{1}{5}$

(b) $\frac{2}{5}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

$$S = \left\{ \begin{matrix} (1,1), (1,2) & \cdots & (1,6) \\ (2,1)\ (2,2) & \cdots & (2,6) \\ \vdots & & \\ (6,1)\ (6,2) & \cdots & (6,6) \end{matrix} \right\}$$

$$\text{Reduced Sample Space} = \left\{ \begin{matrix} (1,2)\ (1,3) & \cdots & (1,6) \\ (2,1)\ (2,3) & \cdots & (2,6) \\ \vdots & & \\ (6,1)\ (6,2) & \cdots & (6,5) \end{matrix} \right\}$$

$$\{\text{Sum} = \text{even}\} = \left\{ \begin{matrix} (1,3) & (1,5) \\ (2,4) & (2,6) \\ (3,1) & (3,5) \\ (4,2) & (4,6) \\ (5,1) & (5,3) \\ (6,2) & (6,4) \end{matrix} \right\}$$

$$\text{Ans} = \frac{12}{30} = \underline{\underline{\frac{2}{5}}}$$

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16. Player A plays two chess matches each with players B and C. A win, a draw and a loss are given 2, 1, 0 points respectively. The probability that A wins, draws or loses in any match is 0.6, 0.3 and 0.1 respectively. Outcomes of the matches are assumed to be independent. The probability that A gets at least 6 points [Correct to 3 decimal places] is _______

$$p\left(\begin{array}{c}\text{A gets}\\ \text{atleast 6}\\ \text{points}\end{array}\right) = p\left(\begin{array}{c}\text{A gets}\\ \text{6 points}\end{array}\right) + p\left(\begin{array}{c}\text{A gets}\\ \text{7 points}\end{array}\right) + p\left(\begin{array}{c}\text{A gets}\\ \text{8 points}\end{array}\right)$$

6 points — 2 wins, 2 draws; 3 wins, 1 loss

7 points $\rightarrow$ 3 wins, 1 draw

8 points $\rightarrow$ 4 wins

$$= (0.6)^2(0.3)^2 + (0.6)^3(0.1)^1 + (0.6)^3(0.3) + (0.6)^4 = 0.2484$$

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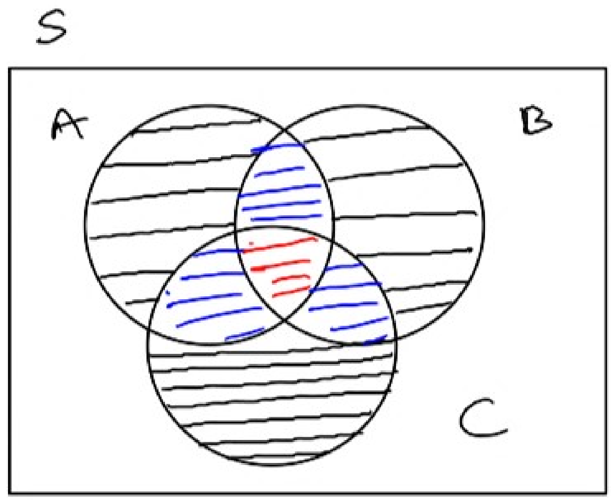
17. Let A, B and C be pairwise independent events such that P (A) = P (B) = P (C) = $\frac{1}{3}$ and $P(A \cap B \cap C) = \frac{1}{4}$. Then the probability that at least one of the events among A, B & C occurs is

(a) $\frac{11}{12}$ (b) $\frac{7}{12}$ (c) $\frac{5}{12}$ (d) $\frac{3}{4}$



$p\left(\begin{array}{l}\text{at least one of}\\ \text{the events}\end{array}\right) = p(A \cup B \cup C) = p(A) + p(B) + p(C) - p(A \cap B) - p(B \cap C) - p(C \cap A) + p(A \cap B \cap C)$

$= 3 \cdot \frac{1}{3} - 3 \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{4} = \frac{11}{12}$

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18. Two biased coins C_1 and C_2 have probabilities of getting heads $\frac{2}{3}$ and $\frac{3}{4}$, respectively, when tossed. If both coins are tossed independently two times each, then the probability of getting exactly two heads out of these four tosses is

(a) $\frac{1}{4}$ (b) $\frac{37}{144}$ (c) $\frac{41}{144}$ (d) $\frac{49}{144}$

When a coin is tossed two times we have 4 sample points

$\{HH, HT, TH, TT\}$

Exactly two heads

C_1 HH C_2 TT $\rightarrow \frac{2}{3}\times\frac{2}{3}\times\frac{1}{4}\times\frac{1}{4}$

C_1 HT C_2 — TH $\rightarrow \frac{2}{3}\times\frac{1}{3}\times\frac{1}{4}\times\frac{3}{4}$

— HT $\rightarrow \frac{2}{3}\times\frac{1}{3}\times\frac{3}{4}\times\frac{1}{4}$

C_1 TH $C_2 \rightarrow$ HT $\rightarrow \frac{1}{3}\times\frac{2}{3}\times\frac{3}{4}\times\frac{1}{4}$

$\rightarrow$ TH $\rightarrow \frac{1}{3}\times\frac{2}{3}\times\frac{1}{4}\times\frac{3}{4}$

C_1 TT $C_2 \rightarrow$ HH $\rightarrow \frac{1}{3}\times\frac{1}{3}\times\frac{3}{4}\times\frac{3}{4}$

$$\text{Ans} = \frac{4+6+6+6+6+9}{9\times16} = \frac{37}{144}$$

Let A and B be independent events in Sample space S. It is known that $P(A \cap B) = 0.16$ and $P(A \cup B) = 0.64$ then $P(A)$ is

$$p(A \cap B) = p(A) \cdot p(B)$$

$$0.16 = p(A) \cdot p(B)$$

$$\Rightarrow p(B) = \frac{0.16}{p(A)} \quad \text{— (1)}$$

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

$$0.64 = p(A) + \frac{0.16}{p(A)} - 0.16$$

$$0.64\,p(A) = (p(A))^2 + 0.16 - 0.16\,p(A)$$

$$(p(A))^2 - 0.8\,p(A) + 0.16 = 0$$

$$(p(A) - 0.4)^2 = 0$$

$$\Rightarrow p(A) = 0.4$$

The probability that a cell in a wireless system is over loaded is $\frac{1}{3}$. Given that it is over loaded, the probability of a blocked call is 0.3. Given that it is not overloaded the probability of a blocked call is 0.1.

(i) find the probability that your call is blocked

(ii) find the conditional probability that the system is overloaded given that your call is blocked

B_1 = overloaded $\quad p(B_1) = \frac{1}{3}$

B_2 = Not overloaded $\quad p(B_2) = \frac{2}{3}$

A = Call blocked

$$p(A|B_1) = 0.3$$

$$p(A|B_2) = 0.1$$

$$p(A) = p(A|B_1)p(B_1) + p(A|B_2)p(B_2)$$

$$= (0.3)\left(\frac{1}{3}\right) + (0.1)\left(\frac{2}{3}\right)$$

$$p(B_1|A) = \frac{(0.3)\left(\frac{1}{3}\right)}{(0.3)\left(\frac{1}{3}\right) + (0.1)\left(\frac{2}{3}\right)}$$

$$= \frac{3}{5}$$

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20. Bag A contains 3 red and 7 white balls and Bag B contains 5 red and 4 white balls one ball is drawn at random from the first bag and transferred to the second bag. Now, a ball is drawn from the second bag. Which of the following is/are CORRECT?

(a) The probability of drawn ball from the second bag is white is $\frac{47}{100}$

(b) The probability of drawn ball from the second bag is red is $\frac{53}{100}$

(c) If the drawn ball from the second bag is white then the probability that a red ball was transferred to bag B is $\frac{12}{47}$

(d) If the drawn ball from the second bag is red then the probability that a white ball was transferred to bag B is $\frac{35}{53}$

$$p\left(\begin{matrix}W\\ \text{from}\\ \text{Bag B}\end{matrix}\right) = p\left(\begin{matrix}W\\ \text{from}\\ \text{Bag B}\end{matrix} \middle| \begin{matrix}R\\ \text{from}\\ \text{Bag A}\end{matrix}\right) p\left(\begin{matrix}R\\ \text{from}\\ \text{Bag A}\end{matrix}\right) + p\left(\begin{matrix}W\\ \text{from}\\ \text{Bag B}\end{matrix} \middle| \begin{matrix}W\\ \text{from}\\ \text{Bag A}\end{matrix}\right) p\left(\begin{matrix}W\\ \text{from}\\ \text{Bag B}\end{matrix} \middle| \begin{matrix}W\\ \text{from}\\ \text{Bag A}\end{matrix}\right)$$

$$= \frac{4}{10}\cdot\frac{3}{10} + \frac{5}{10}\cdot\frac{7}{10} = \frac{47}{100}$$

a is true

$$p\left(\begin{matrix}R\\ \text{Trans}\\ \text{to}\\ \text{Bag B}\end{matrix} \middle| \begin{matrix}W\\ \text{from}\\ \text{Bag B}\end{matrix}\right) = \frac{\frac{12}{100}}{47/100} = \frac{12}{47}$$

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$$p\left(\begin{matrix}R\\ \text{from}\\ \text{Bag B}\end{matrix}\right) = p\left(\begin{matrix}R\\ \text{from}\\ \text{Bag B}\end{matrix}\middle|\begin{matrix}R\\ \text{from}\\ \text{Bag A}\end{matrix}\right) p\left(\begin{matrix}R\\ \text{from}\\ \text{Bag A}\end{matrix}\right) + p\left(\begin{matrix}R\\ \text{from}\\ \text{Bag B}\end{matrix}\middle|\begin{matrix}W\\ \text{from}\\ \text{Bag A}\end{matrix}\right) p\left(\begin{matrix}W\\ \text{from}\\ \text{Bag A}\end{matrix}\right)$$

$$= \frac{6}{10}\cdot\frac{3}{10} + \frac{5}{10}\cdot\frac{7}{10} = \frac{53}{100}$$ b is true

$$p\left(\begin{matrix}W\\ \text{Trans}\\ \text{to Bag B}\end{matrix}\middle|\begin{matrix}R\\ \text{from}\\ \text{Bag B}\end{matrix}\right) = \frac{\frac{35}{10\times10}}{\frac{53}{100}} = \frac{35}{53}$$

d is true

All are true

21. There are three urns labeled Urn 1, Urn 2 and Urn 3. Urn1 contains 2 white balls and 2 black balls, Urn 2 contains 1 white ball and three black balls and Urn 3 contains 3 white balls and 1 black ball. Consider two coins with probability of obtaining head in their single trials are 0.2 and 0.3. The two coins are tossed independently once, and an urn is selected according to the following scheme: Urn 1 is selected if 2 heads are obtained; Urn 3 is selected if 2 tails are obtained; otherwise Urn2 is selected. A ball is then drawn at random from the selected urn. Then P (Urn 1 is selected | the ball drawn is white) is equal to

(a) $\frac{6}{109}$ (b) $\frac{12}{109}$

(c) $\frac{1}{18}$ (d) $\frac{1}{9}$

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$$P\left(\begin{array}{l}\text{Ball drawn}\\ \text{is white}\end{array}\right) = 0.2 \times 0.3 \times \frac{2}{4} + 0.8 \times 0.7 \times \frac{3}{4} + (0.2 \times 0.7 + 0.8 \times 0.3)\frac{1}{4}$$

$$= 0.545$$

$$P\left(\begin{array}{l}\text{urn 1 is}\\ \text{selected}\end{array}\middle|\begin{array}{l}\text{Ball drawn}\\ \text{is white}\end{array}\right) = \frac{0.03}{0.545} = \frac{6}{109}$$

option (a)

Sampling

Sampling: Randomly drawing objects from a given set of objects.

Sampling with replacement:

The object that was drawn at random is placed back to the given set and the set is mixed thoroughly. Then we draw the next object at random.

Sampling without replacement:

The object that was drawn is put aside.

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Draw two objects one after the other with replacement.

Draw two objects one after the other without replacement.

Successively $\Rightarrow$ one after the other.

Draw two objects simultaneously with replacement

Draw two objects simultaneously without replacement

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A box contains 25 parts of which 10 are defective. Two parts are being drawn simultaneously in a random manner from the box. The probability of both the parts being good is GATE-14

$^{n}C_{2} = \frac{n(n-1)}{2}$

(a) $\frac{7}{20}$

(b) $\frac{42}{125}$

(c) $\frac{25}{29}$

(d) $\frac{5}{9}$

Total = 25

Defective = 10

Good = 25 - 10 = 15

$$\text{Ans} = \frac{^{15}C_2}{^{25}C_2} = \frac{\cancel{15}^{3} \times \cancel{14}^{7} / \cancel{2}}{\cancel{25}_{5} \times \cancel{24}_{8}4 / \cancel{2}} = \frac{7}{20}$$

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Four red balls, four green balls and four blue balls are put in a box. Three balls are pulled out of the box at random one after another without replacement. The probability that all the three balls are red is

GATE-18

(a) $\frac{1}{72}$

(b) $\frac{1}{55}$

(c) $\frac{1}{36}$

(d) $\frac{1}{27}$

$^{n}c_1 = n$

$4+4+4 = 12$

$\frac{4c_1}{12c_1} \cdot \frac{3c_1}{11c_1} \cdot \frac{2c_1}{10c_1} = \frac{\cancel{4}}{\cancel{12}} \times \frac{\cancel{3}}{11} \times \frac{\cancel{2}}{\cancel{10}\,5}$

$= \frac{1}{55}$

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A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be GATE-03

(a) 100%

(b) 50%

(c) 49%

(d) none

Total = 10

Defectives = 3

Good = 10 - 3 = 7

$$\frac{{}^{7}C_2}{{}^{10}C_2} \times 100 = \frac{7 \times 6}{10 \times 9} \times 100 = 46.66\%$$

(D)

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A box contains 10 screws, 3 of which are defective. Two screws are drawn at random one after the other with replacement. The probability that none of the two screws is defective will be

(a) 100%

(b) 50%

(c) 49%

(d) none

$$\frac{{}^{7}C_1}{{}^{10}C_1} \cdot \frac{{}^{7}C_1}{{}^{10}C_1} \times 100 = 49\%$$

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An urn contains 5 red and 7 green balls. A ball is drawn at random and its colour is noted. The ball is placed back into the urn along with another ball of the same colour. The probability of getting a red ball in the next draw is GATE-16

(a) $\frac{5}{12}$

(b) $\frac{5}{7}$

(c) $\frac{6}{12}$

(d) $\frac{8}{12}$

$$\text{Ans} = \frac{5}{12}$$

$$p\left(\substack{R \text{ in} \\ \text{next draw}}\right) = p\left(\substack{R \text{ in} \\ \text{next draw}} \middle| \substack{R \text{ in} \\ \text{first draw}}\right) p\left(\substack{R \text{ in} \\ \text{first draw}}\right)$$

$$+ \; p\left(\substack{R \text{ in} \\ \text{next draw}} \middle| \substack{G \text{ in} \\ \text{first draw}}\right) p\left(\substack{G \text{ in} \\ \text{first draw}}\right)$$

$$= \frac{6}{13} \cdot \frac{5}{12} + \frac{5}{13} \cdot \frac{7}{12} = \frac{5}{12}$$

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A bag has r red balls and b black balls. All balls are identical except for their colours. In a trial, a ball is randomly drawn from the bag, its colour is noted and the ball is placed back into the bag along with another ball of the same colour. Note that the number of balls in the bag will increase by one, after the trial. A sequence of four such trials is conducted. Which one of the following choices gives the probability of drawing a red ball in the fourth trial? GATE-2021

(a) $\left(\frac{r}{r+b}\right)\left(\frac{r+1}{r+b+1}\right)\left(\frac{r+2}{r+b+2}\right)\left(\frac{r+3}{r+b+3}\right)$

(b) $\frac{r}{r+b}$

(c) $\frac{r}{r+b+3}$

(d) $\frac{r+3}{r+b+3}$

Ans $= \frac{r}{r+b}$

Polya's Urn Model:

An urn contains w white and b blue chips. A chip is drawn at random and then is returned to the urn along with $c > 0$ chips of the same colour. We can prove that if $n = 2, 3, \cdots$ such experiments are made, then at each draw the probability of a white chip is still $\frac{w}{w+b}$ and the probability of a blue-chip is $\frac{b}{w+b}$.

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A bag contains 3 black and 2 red balls. Balls are drawn one after other three times without replacement. The probability that third draw is a red ball is

(a) $\frac{5}{8}$

(b) $\frac{5}{6}$

(c) $\frac{2}{5}$

(d) $\frac{3}{5}$

Problems related to Series

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Since 1995

Two players A and B alternately keep rolling a fair die. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is GATE-15

(a) $\frac{5}{11}$

(b) $\frac{1}{2}$

(c) $\frac{7}{13}$

(d) $\frac{6}{11}$

$p(6) = \frac{1}{6}$ $p(6^c) = \frac{5}{6}$

1 $\quad \frac{6}{A} \quad \frac{1}{6}$

3 $\quad \frac{6^c}{A} \; \frac{6^c}{B} \; \frac{6}{A} \quad \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}$

5 $\quad \frac{6^c}{A} \cdot \frac{6^c}{B} \cdot \frac{6^c}{A} \; \frac{6^c}{B} \; \frac{6}{A} \quad \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}$

$\vdots$

$$\frac{1}{6} + \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \cdots$$

Sum of infinite terms in G.P

$a = \frac{1}{6}$ $\quad r = (5/6)^2$

$$\frac{a}{1-r} = \frac{1/6}{1 - \frac{25}{36}} = \frac{6}{11}$$

A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is GATE-12

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(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

H ✓ $\frac{1}{2}$

T H ✗

T T H ✓ $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

T T T H ✗

T T T T H ✓ $\frac{1}{32}$

⋮

$$\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \frac{1}{2^7} + \cdots = \frac{1/2}{1 - 1/2^2}$$

$$= 2/3$$

Random Variables

Random variable is a function that maps the sample space to real numbers.

$$S = \{H, T\}$$

$$H \rightarrow X = 1 \qquad T \rightarrow X = -1$$

$$p(X=1) = \frac{1}{2} \qquad p(X=-1) = \frac{1}{2}$$

Let X represent the number of heads when a coin is tossed three times.

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$n(S) = 2^3 = 8$

TTT
TTH
THT
THH
HTT
HTH
HHT
HHH

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

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Let X represent the sum of outcomes when a die is rolled twice.

$n(S) = 6^2 = 36$

$$S = \left\{ \begin{matrix} (1,1) & (1,2) & \cdots & (1,6) \\ (2,1) & (2,2) & \cdots & (2,6) \\ & \vdots & & \\ (6,1) & (6,2) & \cdots & (6,6) \end{matrix} \right\}$$

$X = \text{Sum}$

$X = 2 \Rightarrow (1,1)$

$X = 12 \Rightarrow (6,6)$

X	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

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Discrete Random Variable (D.R.V)

A Random variable which takes finitely many values or Countably infinite values is called as Discrete R.V

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Let X represent the number of calls received by the telephone station in a particular hour. The probability distribution is given as follows

X	0	1	2	3	$\cdots$
$P(X=x)$	$e^{-\lambda}$	$\frac{e^{-\lambda}\lambda}{1!}$	$\frac{e^{-\lambda}\lambda^2}{2!}$	$\frac{e^{-\lambda}\lambda^3}{3!}$	$\cdots$

Countably infinite set

where $\lambda > 0$

(i) Comment on nature of the random variable $\Rightarrow$ D.R.V

$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots = e^x$$

(ii) Find the value of $\sum_{x=0}^{\infty} P(X=x)$

$$\sum_{x=0}^{\infty} p(X=x) = e^{-\lambda} + \frac{e^{-\lambda}\lambda}{1!} + \frac{e^{-\lambda}\lambda^2}{2!} + \cdots$$

$$= e^{-\lambda}\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right) = e^{-\lambda} \cdot e^{\lambda} = 1$$

Probability Mass Function

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Probability Mass Function

X	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$p(x=3) = \frac{1}{6}$$

$$f(x) = p(X=x) \Rightarrow PMF$$

$$p(x=7) = 0$$

$$p(x=1) = \frac{1}{6}$$

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Probability Mass Function

X	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$P(X=1) = 0$

$P(X=2) = \frac{1}{36}$

$P(X=12) = \frac{1}{36}$

$f(x) = P(X=x)$ pmf

$f(10) = P(X=10)$

$= \frac{3}{36}$

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Properties of Probability Mass Function

1. $f(x) = P(X = x)$

2. $f(x) \geq 0$

3. $\sum_{\forall x} f(x) = 1$

Cumulative Distribution Function

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Cumulative Distribution Function

X	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$P(X \leq \boxed{4}) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= \sum_{x=1}^{4} P(X=x)$$

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Cumulative Distribution Function

X	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$P(X \leq \boxed{10}) = \sum_{x=2}^{10} P(X = x)$$

$$F(x) = P(X \leq x) = \sum_{x_i=-\infty}^{x} P(X = x_i)$$

CDF

$$= \sum_{x_i=-\infty}^{x} f(x_i)$$

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Continuous Random Variable (C.R.V)

A random variable X and its distribution are of continuous type if its Cumulative distribution function $F(x)$ is given as

$$F(x) = p(X \le x) = \int_{-\infty}^{x} f(v)\,dv$$

where f is called Probability Density function (PDF)

$$F(x) = p(X \le x)$$

$$= \sum_{x_i=-\infty}^{x} f(x_i)$$

$f \Rightarrow$ PMF

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The relation between the CDF and PDF is given by

$$\boxed{\frac{d}{dx}F(x) = f(x)}$$

Properties of PDF and CDF

- Non-Negativity: $f(x) \geq 0$

- Normalization: Area under any probability density function is always $\longrightarrow$ unity

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

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- $P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$

- $P(X > x) = \int_{x}^{\infty} f(x)\,dx = 1 - p(x \leq x)$

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- $P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f(x)\,dx$

- $P(x_1 < X < x_2) = \int_{x_1}^{x_2} f(x)\,dx$

The above results are true only for C.R.V

Suppose that X is a continuous random variable whose probability density function is given by

$$f(x) = \begin{cases} C(4x - 2x^2) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the value of C?

(b) Find $P\{X > 1\}$.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^2 c(4x - 2x^2)\,dx = 1$$

$$c\left(\frac{4x^2}{2} - \frac{2x^3}{3}\right)_0^2 = 1$$

$$c\left(8 - \frac{16}{3}\right) = 1$$

$$c\left(\frac{8}{3}\right) = 1$$

$$\Rightarrow c = 3/8$$

$$P(X > 1) = \int_1^{\infty} f(x)\,dx$$

$$= \int_1^2 \frac{3}{8}(4x - 2x^2)\,dx + \int_2^{\infty} 0\,dx$$

$$= 0.5$$

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The amount of time in hours that a computer functions before breaking down is a continuous random variable with probability density function given by $f(x) = \begin{cases} \lambda e^{\frac{-x}{100}} & x \geq 0 \\ 0 & x < 0 \end{cases}$ What is the probability that

(a) a computer will function between 50 and 150 hours before breaking down?

(b) it will function for fewer than 100 hours?

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{\infty} \lambda e^{-\frac{x}{100}}\,dx = \left(\lambda \frac{e^{-x/100}}{-1/100}\right)_0^{\infty} = 1$$

$$\lambda \frac{e^{-\infty} - e^{-0}}{-1/100} = 1$$

$$\lambda \frac{(0-1)}{-1/100} = 1$$

$$\lambda = \frac{1}{100}$$

a) $\displaystyle\int_{50}^{150} \frac{1}{100} e^{-\frac{x}{100}}\,dx = \underline{0.383}$

b) $\displaystyle\int_{0}^{100} \frac{1}{100} e^{-\frac{x}{100}}\,dx = \underline{0.632}$

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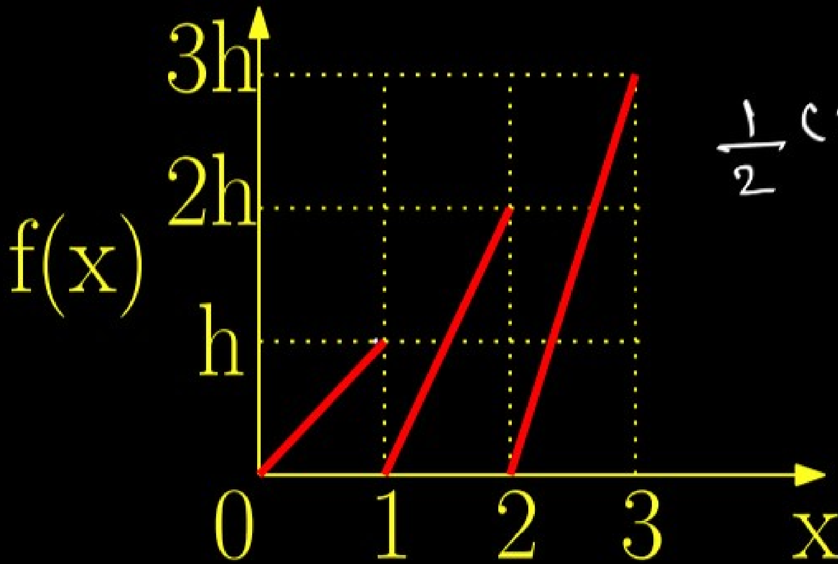
The graph of a function $f(x)$ is shown in figure . For $f(x)$ to be a valid probability density function, the value of h is (GATE 2018)



(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) 3

$$\frac{1}{2}(1)h + \frac{1}{2}(1)(2h) + \frac{1}{2}(1)(3h) = 1$$

$$\frac{h}{2}(1+2+3) = 1$$

$$\frac{6h}{2} = 1$$

$$\Rightarrow h = \frac{1}{3}$$

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Expectation of a Random variable

or

Mean of a Random variable

or

Average value of a Random variable

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If X is discrete random variable having a PMF $P(X = x)$, the expectation or the expected value of X, denoted by $E[X]$, is defined by

$$E[X] = \sum_{\forall x} x\, p(X=x)$$

$$= \sum_{\forall x} x\, f(x)$$

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If X is a continuous random variable having PDF $f(x)$ then it is easy to see that the analogous definition is to define the expected value of X by

$$E[X] = \int_{-\infty}^{\infty} x\, f(x)\, dx \qquad \text{First moment}$$

Expected value of Square of a Random variable

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx \quad \text{2nd moment}$$

Expected value of cube of a Random variable

$$E[X^3] = \int_{-\infty}^{\infty} x^3 f(x)\,dx \quad \text{3rd moment}$$

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Expected value of X^4

$$E[X^4] = \int_{-\infty}^{\infty} x^4 f(x)\,dx$$

Expected value of X^n

$$E[X^n] = \int_{-\infty}^{\infty} x^n f(x)\,dx \longrightarrow n^{th} \text{ moment}$$

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Expected value of a function of a Random Variable

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)\, f(x)\, dx$$

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Properties of Expectation

- $E[aX] = \int_{-\infty}^{\infty} ax\, f(x)\,dx = a \int_{-\infty}^{\infty} x\, f(x)\,dx = a\,E[x]$

where a is a constant.

$$E[ax+b] = a\,E[x] + b$$

- $E[b] = b$

where b is a constant.

If X and Y are two random variables then

$$E[X+Y] = E[X] + E[Y]$$

If a and b are two constants then

$$E[aX+bY] = a\,E[X] + b\,E[Y]$$

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Expectation is a linear operator

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$E[E[X]] = E[X]$

m	p	c
90	80	70

$$\text{Average mark} = \frac{90+80+70}{3} = 80$$

$$E[\text{Average mark}] = E[80] = 80$$

Consider a set of n antennas of which m are defective and $n - m$ are functional and assume that all of the defectives and all of the functionals are considered distinguishable. How many linear orderings are there in which no two defectives are consecutive?

Total antennas $= n$

defective $= m$

functioning $= n-m$

$| \; \underline{F_1} \; | \; \underline{F_2} \; | \; \underline{\quad} \; | \; . \; | \; . \; . \; | \; \underline{F_{n-m}} \; |$

$$\boxed{{}^{n-m+1}C_m \cdot (n-m)!\; m!}$$

Ex

$| \underline{\quad} | \underline{\quad} | \underline{\quad} |$

3 functioning

$\rightarrow$ $n-m$ functioning

06. A bag contains 3 black and 2 red balls. Balls are drawn one after other three times without replacement. The probability that third draw is a red ball is

(a) $\frac{5}{8}$ (b) $\frac{5}{6}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

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First draw	second draw	Third draw	
Black	Black	Red	$\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$
Black	Red	Red	$\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3}$
Red	Black	Red	$\frac{2}{5} \times \frac{3}{4} \times \frac{1}{3}$
Red	Red	Red not Possible	$\frac{2}{5} \times \frac{1}{4} \times \frac{0}{3}$

$$\frac{12 + 6 + 6 + 0}{60} = \frac{2}{5}$$

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The square root of the $Var(X)$ is called the standard deviation of X, and we denote this by σ_X.

$$\sigma_X = \sqrt{var(X)}$$

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$var(z) = E[z^2] - (E[z])^2$

Properties of Variance

- $Var[aX] =$ $E[(ax)^2] - (E[ax])^2$

where a is a constant. $= E[a^2x^2] - (aE[x])^2$

$= a^2(E[x^2] - (E[x])^2) = a^2 var(x)$

- $Var[b] =$ $E[b^2] - (E[b])^2 = b^2 - b^2 = 0$

where b is a constant.

$Var(ax+b) = a^2 var(x)$

- If a and b are two constants then

$Var[aX + b]$

$Var[aX - b]$ = $a^2 var(x)$

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$$var(z) = E[z^2] - (E[z])^2$$

If X and Y are two random variables then

$$Var[X \pm Y] = E[(x \pm y)^2] - (E[x \pm y])^2$$

$$= E[x^2] + E[y^2] \pm 2E[xy] - (E[x])^2 - (E[y])^2 \mp 2E[x]E[y]$$

$$= E[x^2] - (E(x))^2 + E(y^2) - (E(y))^2 \pm 2\big(\underbrace{E[xy] - E(x)E(y)}_{\text{covariance of } x, y}\big)$$

$$= var(x) + var(y) \pm 2\,cov(x, y)$$

$$var(x+y) = var(x) + var(y) + 2\,Cov(x, y)$$

$$Var(x-y) = var(x) + var(y) - 2\,Cov(x, y)$$

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Correlation Coefficient $r_{XY} = \dfrac{\text{Cov}(x, y)}{\sigma_x \, \sigma_y}$

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If X and Y are two Independent random variables then

$$E[XY] = E[X]\,E(Y)$$

$$Cov(X,Y) = E[XY) - E(X)E(Y)$$

$$= 0$$

$$var(X+Y) = var(X) + var(Y)$$

$$var(X-Y) = var(X) + var(Y)$$

A and B are independent

$$P(A \cap B) = P(A) \cdot P(B)$$

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If the difference between the expectation of the square of a random variable $E(X^2)$ and the square of the expectation of the random variable $[E(X)]^2$ is denoted by R, then (GATE-11)

(a) $R = 0$

(b) $R < 0$

(c) $R \geq 0$

(d) $R > 0$

$R = E[X^2] - (E(X))^2 \geq 0$

(c)

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Let X be a real valued random variable with $E[X]$ and $E[X^2]$ denoting the mean values of X and X^2, respectively. The relation which always holds true is

(GATE-14)

(a) $[E(X)]^2 > E(X^2)$

(b) $E(X^2) \geq [E(X)]^2$

(c) $[E(X)]^2 = E(X^2)$

(d) $E(X^2) > [E(X)]^2$

$$E[x^2] - (E[x])^2 \geq 0$$

$$\boxed{E[x^2] \geq (E[x])^2}$$

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A six-face fair dice is rolled a large number of times. The mean value of the outcomes is (GATE-17)

$$E[X] = \sum_{\forall x} x\, p(X=x)$$

Let X represent possible outcomes on the die

X	1	2	3	4	5	6
p(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E[X] = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + \cdots + 6\left(\frac{1}{6}\right)$$

$$= \frac{1}{6}(1+2+\cdots+6) = \frac{6(6+1)}{6(2)} = 3.5$$

$$1+2+\cdots n = \frac{n(n+1)}{2}$$

find the variance of X

$$\mathrm{var}(X) = E[X^2] - (E(X))^2$$

$$E[X^2] = 15.166 \qquad \mathrm{var}(X) = 2.91$$

$$E[X^2] = 1^2 \cdot \frac{1}{6} + 2^2 \cdot \frac{1}{6} + \cdots 6^2 \cdot \frac{1}{6}$$

$$= \frac{1}{6}(1^2 + 2^2 + \cdots + 6^2)$$

$$= \frac{1}{\cancel{6}} \cdot \frac{(\cancel{6})(6+1)(13)}{6}$$

$$= \frac{91}{6}$$

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A bag has
one counter marked as 1,
two counters marked as 4,
three counters marked as 9,
⋮
n counters marked as n^2.
If you draw one counter and are paid the amount shown on it (in Rs), then your expectation is

(a) n^2

(b) $\frac{n(n-1)}{2}$

(c) $\frac{n(n+1)}{2}$

(d) $\frac{(n-1)(n-2)}{2}$

Let the n.o of total counters be N

$$N = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

Amount	1	4	9	$\cdots$	n^2
p(Amount)	$\frac{1}{N}$	$\frac{2}{N}$	$\frac{3}{N}$		$\frac{n}{N}$

$$E[\text{Amount}] = 1\left(\frac{1}{N}\right) + 4\left(\frac{2}{N}\right) + 9\left(\frac{3}{N}\right) + \cdots + n^2\left(\frac{n}{N}\right)$$

$$= \frac{1}{N}\left(1^3 + 2^3 + 3^3 + \cdots + n^3\right)$$

Sum of cubes of 1st 'n' natural numbers

$$= \frac{1}{\frac{n(n+1)}{2}} \frac{(n(n+1))^2}{4} = \frac{n(n+1)}{2}$$

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Flip n independent fair coins, and let X be a random variable that counts how many coins come up heads. Let a be the constant. The expectation of a^X is

(a) $\left(\frac{a-1}{2}\right)^n$

(b) $\left(\frac{a+1}{3}\right)^n$

(c) $\left(\frac{a+1}{2}\right)^n$

(d) $\left(\frac{a-1}{3}\right)^n$

$$E[g(x)] = \sum_x g(x) f(x)$$

$n = 1$

$S = \{H, T\}$

	T	H
X	0	1
$p(x)$	$\frac{1}{2}$	$\frac{1}{2}$

$$E[a^X] = a^0 \cdot \frac{1}{2} + a^1 \cdot \frac{1}{2}$$

$$= \frac{a+1}{2} \quad \text{(c)}$$

$n = 2$

$S = \{HH, HT, TH, TT\}$

X	0	1	2
$p(x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

$$E[a^X] = a^0 \cdot \frac{1}{4} + a^1 \frac{2}{4} + a^2 \frac{1}{4}$$

$$= \frac{1}{4}(1 + 2a + a^2)$$

$$= \frac{(a+1)^2}{4}$$

The probability density function of a random variable X is $P_x(x) = e^{-x}$ for $x \geq 0$ and 0 otherwise. The expected value of the function $g_x(x) = e^{\frac{3x}{4}}$ is

GATE-15

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x) \, dx$$

$$= \int_{-\infty}^{0} e^{\frac{3x}{4}} \times 0 \, dx + \int_{0}^{\infty} e^{\frac{3x}{4}} \cdot e^{-x} \, dx$$

$$\frac{3}{4} - 1 = -\frac{1}{4}$$

$$= \int_{0}^{\infty} e^{-\frac{x}{4}} \, dx$$

$$= 4$$

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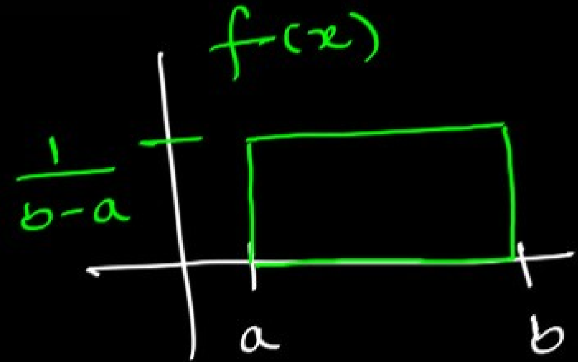
Uniform Distribution

The probability density function of a uniformly distributed random variable in the interval $[a, b]$ is given by

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$



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The value of $\int_a^b \frac{1}{b-a}\,dx =$ 1

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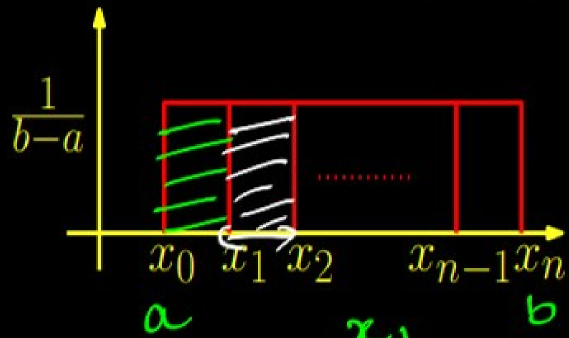
Let us divide the interval $[a, b]$ into n equal parts



Length of the interval = $b-a$

Length of each sub interval = $\frac{b-a}{n}$

$P(x_0 \leq X \leq x_1) = \int_{x_0}^{x_1} f(x)\,dx = \frac{b-a}{n} \cdot \frac{1}{b-a} = \frac{1}{n}$

$P(x_1 \leq X \leq x_2) = \frac{1}{n}$

$\vdots$

$P(x_{n-1} \leq X \leq x_n) = \frac{1}{n}$

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$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx = \int_a^b x \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}\left(\frac{x^2}{2}\right)_a^b$$

$$= \frac{1}{b-a}\,\frac{b^2-a^2}{2} = \frac{\cancel{(b-a)}(b+a)}{\cancel{(b-a)}(2)}$$

$$= \frac{b+a}{2}$$

$$E[x^2] = \int_a^b x^2 \cdot \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}\left(\frac{x^3}{3}\right)_a^b$$

$$= \frac{b^3-a^3}{3(b-a)} = \frac{\cancel{(b-a)}(b^2+ab+a^2)}{3\cancel{(b-a)}} = \frac{b^2+ab+a^2}{3}$$

$$\mathrm{Var}(x) = E(x^2) - [E(x)]^2$$

$$= \frac{b^2+ab+a^2}{3} - \left(\frac{b+a}{2}\right)^2$$

$$= \frac{4b^2+4ab+4a^2-3b^2-3a^2-6ab}{12}$$

$$= \frac{b^2-2ab+a^2}{12}$$

$$\mathrm{Var}(x) = \frac{(b-a)^2}{12}$$

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MSQ

A random variable X is uniformly distributed in the interval $[0, 1]$. Which of the following is/ are true?

(a) $E[X] = \frac{1}{2}$ T

(b) $E[X^2] = \frac{1}{3}$ T

(c) $E[X^3] = \frac{1}{4}$ T

(d) $Var(X) = \frac{1}{6}$ F

$$E[X] = \frac{b+a}{2} = \frac{1+0}{2} = \frac{1}{2}$$

$$E[X^2] = \frac{b^2+ab+a^2}{3} = \frac{1^2+0(1)+0^2}{3} = \frac{1}{3}$$

$$Var(X) = \frac{(b-a)^2}{12} = \frac{(1-0)^2}{12} = \frac{1}{12}$$

$$E[X^3] = \int_{-\infty}^{\infty} x^3 f(x)\,dx = \int_0^1 x^3 \cdot \frac{1}{1-0}\,dx = \frac{1}{4}$$

Ans a, b, C

Suppose Y is distributed uniformly in the open interval $(1, 6)$. The probability that the polynomial $3x^2 + 6xy + 3y + 6$ has only real roots is

GATE-19

Y is a U.R.V $(1,6)$

$$f(y) = \frac{1}{6-1} = \begin{cases} \frac{1}{5} & 1 < y < 6 \\ 0 & \text{elsewhere} \end{cases}$$

$$3x^2 + 6xy + 3y + 6 = 0$$

$$Ax^2 + Bx + C = 0$$

$A = 3,\ B = 6y\ \ C = 3y+6$

Condition for real roots

$$B^2 - 4AC \geq 0$$

$$(6y)^2 - 4(3)(3y+6) \geq 0$$

$$36y^2 - 36(y+2) \geq 0$$

$$y^2 - y - 2 \geq 0$$

$$y^2 - 2y + y - 2 \geq 0$$

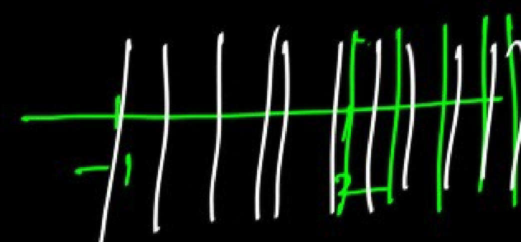
$$y(y-2) + 1(y-2) \geq 0$$

$$(y-2)(y+1) \geq 0$$

case 1

$y-2 \geq 0$ and $y+1 \geq 0$

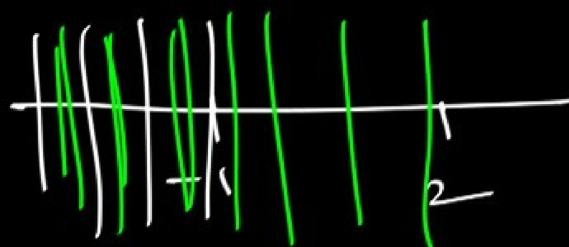
$y \geq 2$ and $y \geq -1$



$$y \geq 2$$

Case 2 $\quad y-2 \le 0$ and $y+1 \le 0$

$y \le 2$ and $y \le -1$



$y \le -1$

for real roots $\quad p(y \geq 2) + p(y \le -1)$

$$\int_2^6 \frac{1}{5}\,dx + 0 = \frac{4}{5} = \underline{\underline{0.8}}$$

Let X_1, X_2, and X_3 be independent and identically distributed random variables with the uniform distribution on $[0,1]$. The Probability $P(X_1$ is the largest) is $\frac{1}{3}$

i.i.d

GATE-14

2018 x_1, x_2, x_3 and x_4 i.i.d

$p(x_4 \text{ Smallest}) = \frac{1}{4}$

VK RS KL

$p(\text{VK Scoring more Runs}) = p(\text{RS Scoring more runs}) = p(\text{KL Scoring more runs}) = \frac{1}{3}$

Aditya Vangala

ACE

If X has uniform distribution in the interval from 0 to 10, then $P(X + \frac{10}{X} \geq 7) =$

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Let X_1,X_2,X_3 and X_4 be independent normal random variables with zero mean and unit variance. The probability that X_4 the smallest among the four is

GATE-18

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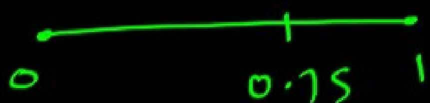
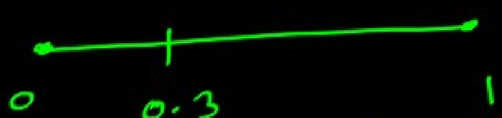
ACE

Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is

GATE-14



Length of shorter stick = 0.25



Length of shorter stick = 0.3

Length of the shorter stick is uniformly distributed in $(0, 0.5)$

$$E\left[\begin{matrix}\text{shorter}\\\text{stick}\end{matrix}\right] = \frac{0.5+0}{2}$$

$$= \underline{\underline{0.25}}$$

The variable X takes a value between 0 and 10 with uniform probability distribution. The variable Y takes a value between 0 and 20 with uniform probability distribution. The probability of the sum of variables $X + Y$ being greater than 20 is

(GATE 2019)

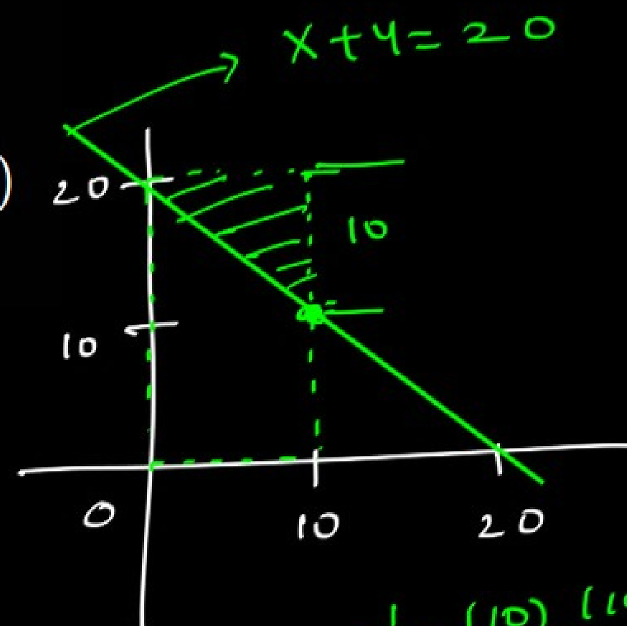
(a) 0.5 (b) 0 (c) 0.33 (d) 0.25

X is U.R.V in $[0, 10)$

Y is U.R.V in $[0, 20)$

$X + Y > 20$

$X + Y = 20 \Rightarrow \frac{X}{20} + \frac{Y}{20} = 1$



$$P(X+Y>20) = \frac{\frac{1}{2}(10)(10)}{10 \times 20}$$

$$= \frac{1}{4}$$

Aditya Vangala

Two independent random variables X and Y are uniformly distributed in the interval $[-1, 1]$. The probability that $max[X, Y]$ is less than $\frac{1}{2}$ is

(GATE-12)

Aditya Vangala

A point is randomly selected with uniform probability in the XY-plane with in the rectangle with corners at (0,0), (1,0), (1,2) and (0,2). If P is the length of the position vector of the point, the expected value of P^2 is

(a) $\frac{2}{3}$

(b) 1

(c) $\frac{4}{3}$

(d) $\frac{5}{3}$

A passenger arrives at a bus stop at 10 AM, knowing that bus will arrive at some time uniformly distributed between 10 AM and 10:30 AM

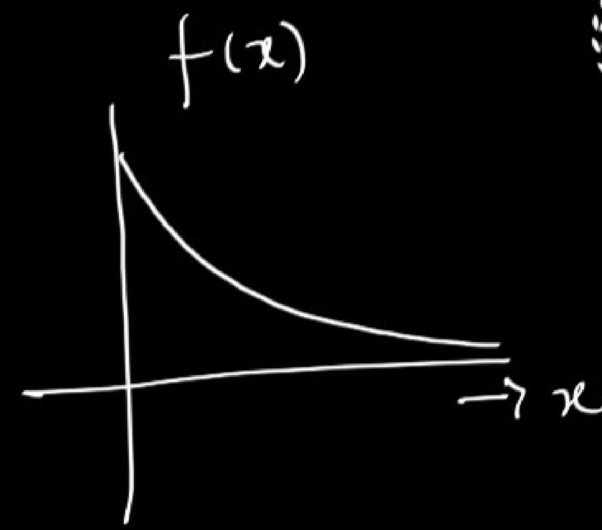
(i) What is the probability that he will have to wait longer than 10 minutes.

(ii) If at 10.15 AM the bus has not yet arrived, what is the probability that he will have to wait at least 10 additional minutes.

Exponential Distribution

The probability density function of an Exponential random variable is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$



where $\lambda > 0$ is a parameter of exponential distribution

$$\int_0^\infty \lambda e^{-\lambda x}\,dx = 1$$

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** $E[X] = \int_0^\infty x \lambda e^{-\lambda x} dx = \frac{1}{\lambda}$

$E[X^2] = \int_0^\infty x^2 \lambda e^{-\lambda x} dx = \frac{2}{\lambda^2}$

** $var[X] = E[X^2] - (E(X))^2$

$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$

$= \frac{1}{\lambda^2}$

For an Exponential random variable with parameter λ find $P(X > x)$, $x > 0$

$$p(X > x) = \int_x^\infty f(x)\,dx$$

$$= \int_x^\infty \lambda e^{-\lambda x}\,dx$$

$$= \left(\frac{\lambda e^{-\lambda x}}{-\lambda}\right)_x^\infty$$

$$\frac{e^{-\infty} - e^{-\lambda x}}{-1}$$

**

$$\boxed{p(X > x) = e^{-\lambda x}}$$

$$p(X > 2) = e^{-2\lambda}$$

$$p(X > 6) = e^{-6\lambda}$$

In practice, the exponential distribution often arises as the distribution of the amount of time until some specific event occurs. For instance, the amount of time (starting from now) until an earthquake occurs, or until a new war breaks out, or until a telephone call you receive turns out to be a wrong number are all random variables that tend in practice to have exponential distributions.

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A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The mean and variance of X are

(a) $\frac{1}{2}, \frac{1}{8}$

(b) $\frac{1}{2}, \frac{1}{4}$

(c) $1, \frac{1}{2}$

(d) $2, \frac{1}{2}$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\lambda = 2$$

$$E[X] = \frac{1}{\lambda} = \frac{1}{2}$$

$$\mathrm{Var}(X) = \frac{1}{\lambda^2} = \frac{1}{2^2} = \frac{1}{4}$$

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If X is exponentially distributed, the probability that X exceeds its expected value is

(GATE-21)

$$\boxed{p(X > x) = e^{-\lambda x}}$$

$$p(X > E[X])$$

$$= p\left(X > \frac{1}{\lambda}\right)$$

$$= e^{-\lambda \cdot \frac{1}{\lambda}} = e^{-1} = \underline{\underline{0.367}}$$

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ACE

The length of the shower on a tropical island during rainy season has an exponential distribution with parameter 2, time being measured in minutes. What is the probability that a shower will last more than 3 min?

$$p(X > x) = e^{-\lambda x}$$

$$\lambda = 2$$

$$p(X > 3) = e^{-3\lambda}$$

$$= e^{-3\times 2}$$

$$= e^{-6} = 0.0024$$

Let X_1 and X_2 be two independent exponentially distributed random variables with means 0.5 and 0.25, respectively. Then $Y = min(X_1, X_2)$ is

GATE-18

(a) exponentially distributed with mean $\frac{1}{6}$

(b) exponentially distributed with mean 2

(c) normally distributed with mean $\frac{3}{4}$

(d) normally distributed with mean $\frac{1}{6}$

Gaussian or Normal Random variable

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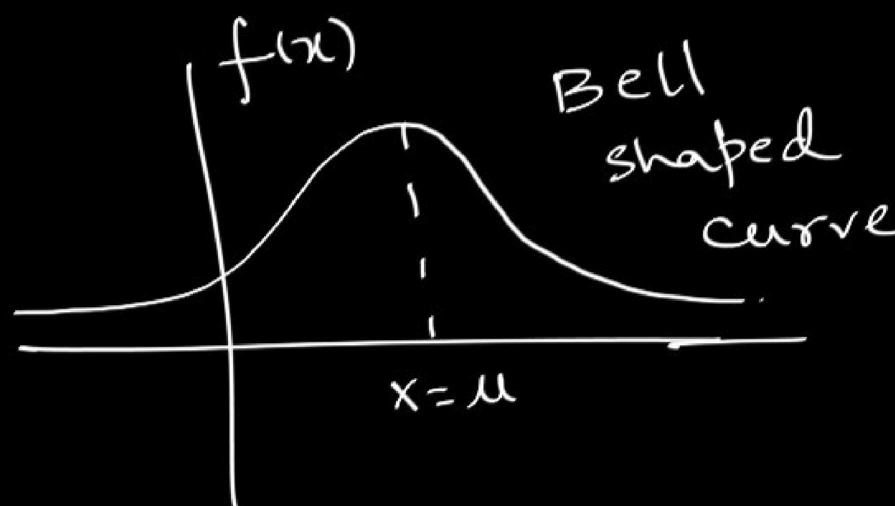
ACE

The probability density function of Gaussian or Normal Random variable is given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$$

$$E[X] = \mu$$

$$\text{var}(X) = \sigma^2$$



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$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx = 1$$

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Find the mean and variance of the following probability density function

1. $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-1)^2}{2}} \quad -\infty < x < \infty$

$\mu = 1, \ \sigma^2 = 1$

$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

2. $f(x) = \frac{1}{2\sqrt{2\pi}} e^{\frac{-x^2}{8}} \quad -\infty < x < \infty$

$\frac{1}{\sqrt{2\pi(4)}} e^{-\frac{(x-0)^2}{2(4)}}$

$\mu = 0 \qquad \sigma^2 = 4$

The shape of the cumulative distribution function of Gaussian distribution is

GATE-21

(A) Horizontal line

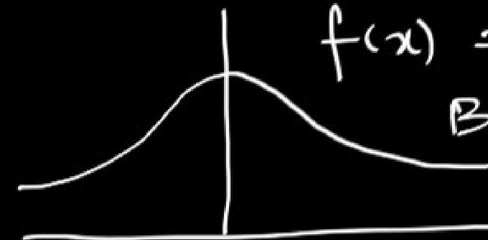
(B) Straight line at 45 degree angle

(C) Bell-shaped

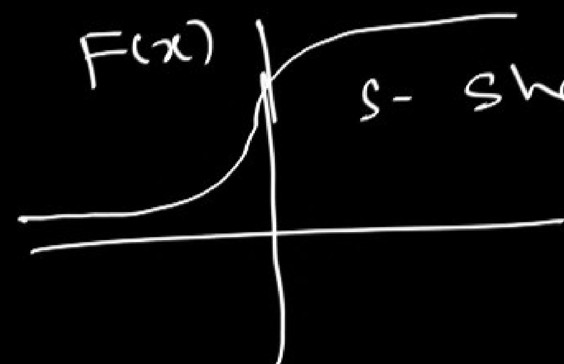
(D) S-shaped

$f(x) \Rightarrow$ PDF

Bell shaped



$$F(x) = p(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$

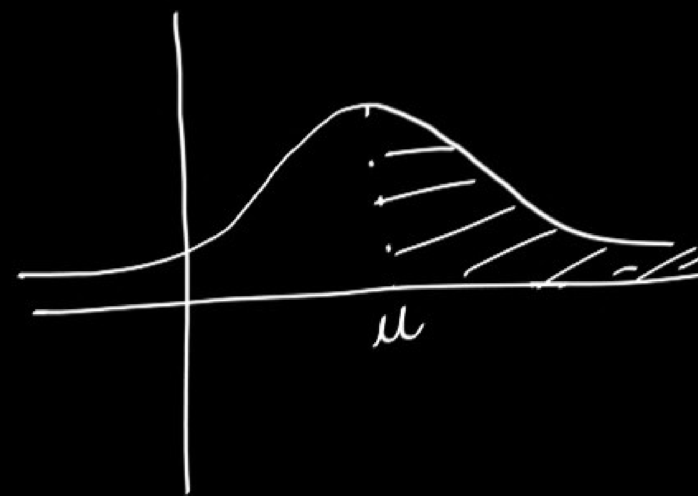


S- shaped

*

If X is Normal R.V with mean μ then

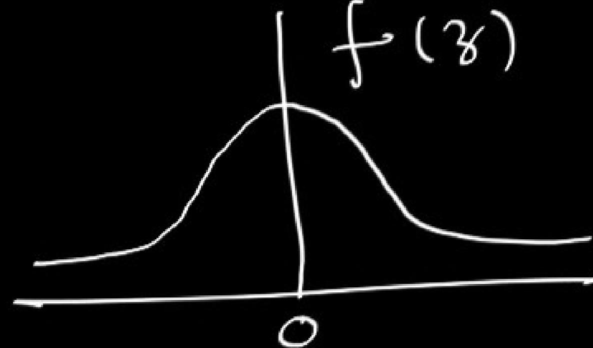
- $P(X > \mu) =$ 0.5
- $P(X \leq \mu) =$ 0.5



*

If Z is a normal R.V with mean=0 and variance=1 then Z is called as Standard Normal R.V.

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad -\infty < z < \infty$$



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If X is a normal R.V with mean$=\mu$ and variance$=\sigma^2$ then the nature of $Z = \frac{X-\mu}{\sigma}$ is standard Normal R.V

$E[Z] = 0$

$var(Z) = 1$

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ACE

Consider a standard normal r.v Z. Find the following values

$P(-1 < Z < 1) =$ $\int_{-1}^{1} f(z)\,dz = 0.6827$

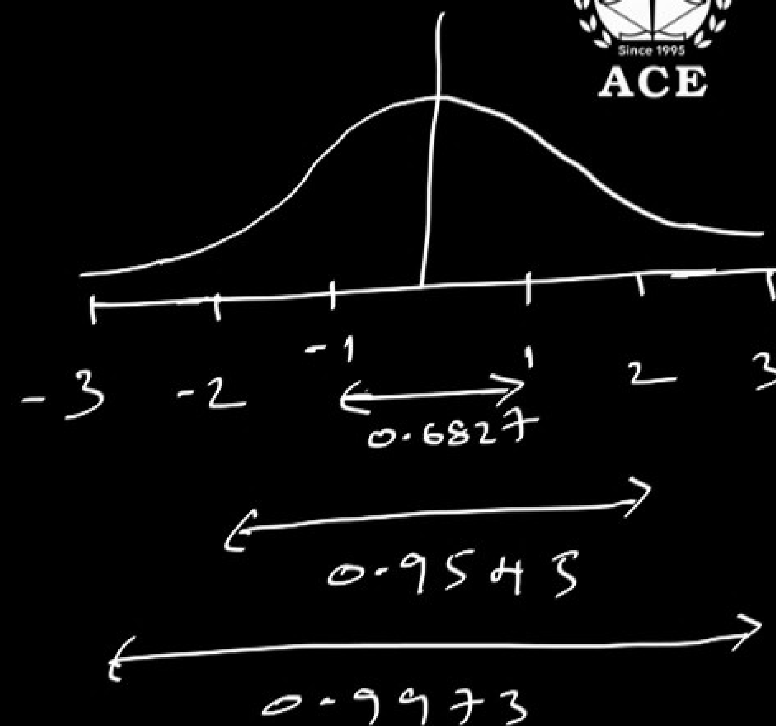
$P(0 < Z < 1) =$ $0.6827/2 = 0.3413$

$P(-2 < Z < 2) =$ 0.9545

$P(0 < Z < 2) =$ $0.9545/2$

$P(-3 < Z < 3) =$ 0.9973

$P(0 < Z < 3) =$ $0.9973/2$



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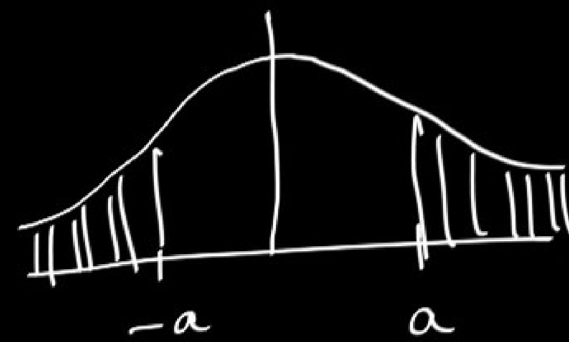
If $a > 0$ then the value of

$$P(Z > a) = \int_a^{\infty} f(z)\,dz$$

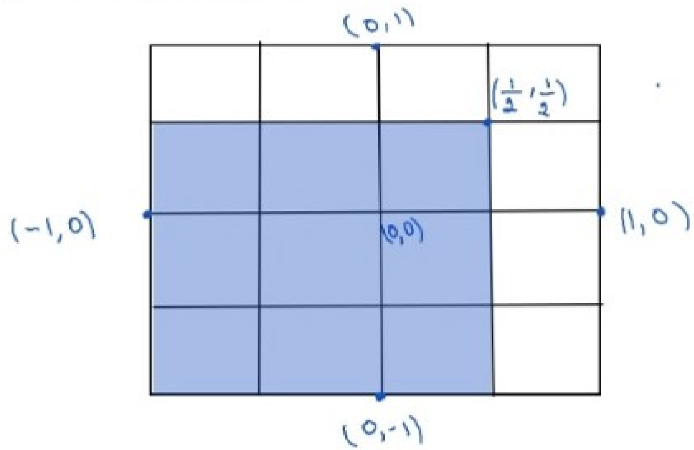
$$P(Z < -a) = p(z > a)$$



Two independent random variables X and Y are uniformly distributed in the interval $[-1, 1]$. The probability that $max[X, Y]$ is less than $\frac{1}{2}$ is

(GATE-12)

Given X and Y are uniform R.Vs in the interval $[-1, 1]$



$$P\left(\max(X,Y) < \tfrac{1}{2}\right) = \frac{\text{Area of shaded part}}{\text{Total Area}}$$

$$= \frac{(1.5)(1.5)}{2(2)}$$

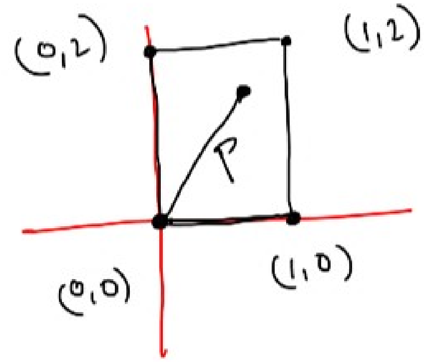
$$= \frac{9}{16}$$

Range : 0.55 to 0.57

42. A point is randomly selected with uniform probability in the XY-plane with in the rectangle with corners at (0,0), (1,0), (1,2) and (0,2). If P is the length of the position vector of the point, the expected value of P^2 is

(a) $\frac{2}{3}$ (b) 1

(c) $\frac{4}{3}$ (d) $\frac{5}{3}$



$0 < p < \sqrt{5}$

p is uniformly distributed in $(0, \sqrt{5})$

$$E[p^2] = \frac{b^2 + ab + a^2}{3} = \frac{(\sqrt{5})^2 + 0(\sqrt{5}) + 0^2}{3}$$

$$= 5/3$$

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43. A passenger arrives at a bus stop at 10 AM, knowing that bus will arrive at some time uniformly distributed between 10 AM and 10:30 AM

(i) What is the probability that he will have to wait longer than 10 minutes.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

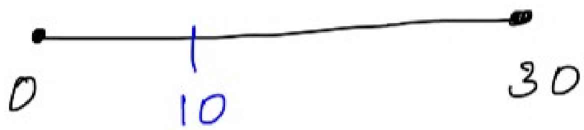
(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

(ii) If at 10.15 AM the bus has not yet arrived, what is the probability that he will have to wait atleast 10 additional minutes.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

Lets map 10am to 0 and 10.30 am to 30 minutes

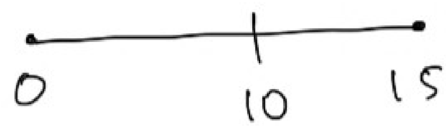


Passenger has to wait longer than 10 minutes implies that the bus has to arrive after 10.10 am

$$\int_{10}^{30} \frac{1}{30-0}\,dx = \frac{2}{3}$$

(ii) Given the bus has not yet arrived by 10.15 am.

we can work with reduced sample space.
map 10.15 am to 0 and 10.30 to 15



(ii) $$\int_{10}^{15} \frac{1}{15-0}\,dx = \frac{1}{3}$$

Let X_1 and X_2 be two independent exponentially distributed random variables with means 0.5 and 0.25, respectively. Then $Y = min(X_1, X_2)$ is

GATE-18

(a) exponentially distributed with mean $\frac{1}{6}$

(b) exponentially distributed with mean 2

(c) normally distributed with mean $\frac{3}{4}$

(d) normally distributed with mean $\frac{1}{6}$

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If x is an exponential R.v with parameter λ then

$$p(X > x) = e^{-\lambda x}$$

$X_1 \to$ expo $\frac{1}{\lambda_1} = 0.5 \Rightarrow \lambda_1 = 2$

$X_2 \to$ expo $\frac{1}{\lambda_2} = 0.25 \Rightarrow \lambda_2 = 4$

$Y = \min(X_1, X_2)$

$$p(Y > y) = p(\min(x_1, x_2) > y)$$

$$= p(x_1 > y \text{ and } x_2 > y)$$

$$= p(x_1 > y) \cdot p(x_2 > y)$$

$$= e^{-\lambda_1 y} \cdot e^{-\lambda_2 y}$$

$$= e^{-(\lambda_1 + \lambda_2) y}$$

$p(Y > y) = e^{-\lambda y}$, where $\lambda = \lambda_1 + \lambda_2$

$= 2 + 4$

$= 6$

Y is expo with mean $= \underline{\underline{1/6}}$

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ACE

The area (in percentage) under standard normal distribution curve of variable Z within limits from -3 to +3 is (GATE 2016)

$$P(-3 < Z < 3) = 0.9973$$

$$\text{Ans} = 99.73\%$$

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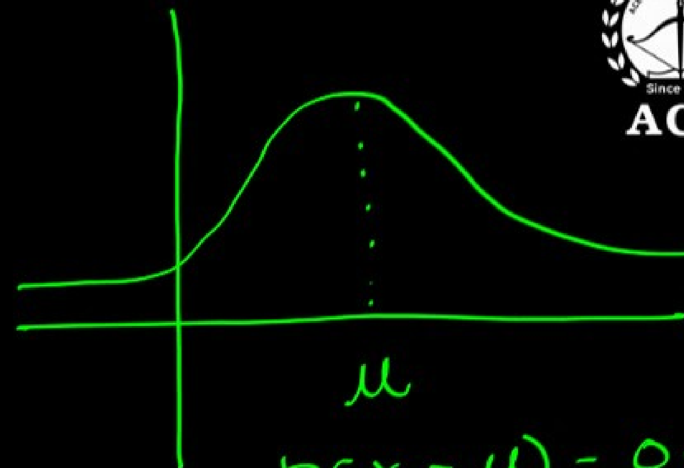
ACE

A nationalized bank has found that the daily balance available in its savings accounts follows a normal distribution with a mean of Rs. 500 and a standard deviation of Rs. 50. The percentage of savings account holders who maintain an average daily balance more than Rs. 500 is

GATE-14



$P(\underline{X} > \mu) = 0.5$

Balance available is Normal R.V

with mean = 500, $\sigma = 50$

$P(\text{Balance} > 500) = 0.5$

Ans: 0.5×100

$= 50\%$

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ACE

The lengths of a large stock of titanium rods follow a normal distribution with a mean of 440 mm and a standard deviation of 1mm. What is the percentage of rods whose lengths lie between 438 mm and 441 mm

GATE-19

(a) 81.85%
(b) 68.4%
(c) 99.75%
(d) 86.64%

$\frac{X-\mu}{\sigma} \Rightarrow$ standard normal R.V

Length is a Normal R.V

$\mu = 440mm \quad \sigma = 1mm$

$$p(438 < \text{Length} < 441)$$

$$= p\left(\frac{438-\mu}{\sigma} < \frac{\text{Length}-\mu}{\sigma} < \frac{441-\mu}{\sigma}\right)$$

$$= p(-2 < Z < 1)$$

$$= p(-2 < Z < 0) + p(0 < Z < 1)$$

$$= \frac{0.9545}{2} + \frac{0.6827}{2} = 0.8185$$

Ans 81.85%

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ACE

H/W

Consider two identically distributed zero mean random variables U and V. Let the cumulative distribution functions of U and $2V$ be $F(x)$ and $G(x)$ respectively. Then, for all values of x

GATE-13

(a) $F(x) - G(x) \leq 0$

(b) $F(x) - G(x) \geq 0$

(c) $(F(x) - G(x)).x \leq 0$

(d) $(F(x) - G(x)).x \geq 0$

For a random variable X $(-\infty < X < \infty)$ following normal distribution, the mean is $\mu = 100$. If the probability is $P = \alpha$ for $x \geq 110$. Then the probability of X lying between 90 and 110 i.e., $P(90 \leq X \leq 110)$ is equal to

GATE-08

(a) $1 - 2\alpha$

(b) $1 - \alpha$

(c) $1 - \frac{\alpha}{2}$

(d) 2α

Let X be a normal random variable with mean 1 and variance 4. The probability $P(X < 0)$ is

GATE-13

(a) 0.5

(b) greater than zero and less than 0.5

(c) greater than 0.5 and less than 1.0

(d) 1.0

Poisson Distribution

The Probability mass function of a Poisson random variable X is given by

$$p(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0, 1, 2, \ldots$$

where $\lambda > 0$ is the parameter of poisson r.v

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Find the value of the following $\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{x!} = 1$

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ACE

$$E[X] = \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^{x}}{x!} = \lambda$$

$$E[X^2] = \sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda}\lambda^{x}}{x!} = \lambda^2 + \lambda$$

$$var[X] = E[X^2] - (E[X])^2$$

$$= \lambda$$

mean and variance of poisson R.V are equal.

Some examples of random variables that generally obey the Poisson probability law as follows

- The number of misprints on a page of a book.
- The number of people in a community who survive to age 100.
- The number of wrong telephone numbers that are dialed in a day.
- The number of packages of dog biscuits sold in a particular store each day.
- The number of customers entering a post office on a given day.
- The number of vacancies occurring during a year in the federal judicial system.
- The number of α-particles discharged in a fixed period of time from some radioactive material.

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If calls arrive at a telephone exchange such that the time of arrival of any call is independent of the time of arrival of earlier or future calls, the probability distribution function of the total number of calls in a fixed time interval will be

GATE-14

(a) Poisson

(b) Gaussian

(c) Exponential

(d) Gamma

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Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by

GATE-14

(a) $\sqrt{\mu}$

(b) μ^2

(c) μ

(d) $\frac{1}{\mu}$

$$\text{mean} = \mu = \lambda$$

$$\text{variance} = \mu$$

$$\text{S.D} = \sqrt{\mu}$$

The second moment of a Poisson distributed random variables is 2. The mean of the random variable is

GATE-16

$$\lambda^2 + \lambda = 2$$

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda^2 + 2\lambda - \lambda - 2 = 0$$

$$\lambda(\lambda+2) - 1(\lambda+2) = 0$$

$$(\lambda+2)(\lambda-1) = 0$$

$$\lambda = -2, 1$$

Discard $\lambda = -2$

Ans $\lambda = 1$

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Since 1995

***

The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during randomly selected month is

GATE-14

$$p(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$\lambda = 5.2$

$$p(X<2) = p(X=0) + p(X=1)$$

$$= \frac{e^{-\lambda}\lambda^{0}}{0!} + \frac{e^{-\lambda}\lambda^{1}}{1!}$$

$$= e^{-\lambda}(1+\lambda) = e^{-5.2} \times 6.2 = \underline{\underline{0.034}}$$

An Observer counts 240 vehicles per hour at a specific highway location. Assume that the vehicle arrival at the location is Poisson distributed. The Probability of having one vehicle arriving over a 30 second time interval is

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GATE-14

240 vehicles — 1 hour

240 $\longrightarrow$ 60 × 60 sec

240 $\longrightarrow$ 60 × 2 × 30 sec

$\frac{240}{60 \times 2} = 2 \longrightarrow$ 30 sec

Average vehicles arrived over 30 sec = 2

$\lambda = 2$

$$p(x=1) = \frac{e^{-\lambda}\lambda^1}{1!}$$

$$= e^{-2} \times 2$$

$$= 0.2706$$

Bernoulli random variable

Consider an experiment which has only two possible outcomes namely success and failure.

$X = 0$ failure

$X = 1$ success

$P(X=0) = 1-p$

$P(X=1) = p$

Binomial random variable

Binomial random variable

Consider an experiment which has only two outcomes namely Success and Failure. The probability of success is p and probability of failure is $1 - p$. If such an experiment is performed n times and if X represent number of success then X is called Binomial R.V with parameters (n, p)

$$p(X=x) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots n$$

If a coin is tossed 3 times what is the probability of getting exactly 2 heads?

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TTT

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HHH

$$P(\text{exactly 2 heads}) = \frac{3}{8}$$

$n = 3$

$p = \frac{1}{2} \quad 1-p = \frac{1}{2}$

$$p(x=2) = {}^{3}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{1}$$

$$= (3)\frac{1}{8}$$

$$= \frac{3}{8}$$

If a coin is tossed 6 times what is the probability of getting exactly 4 heads?

$2^6 = 64$ outcomes

$n = 6 \quad p = \frac{1}{2} \quad 1-p = \frac{1}{2}$

$p(X=4) = {}^6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2$

$= \frac{15}{64}$

Find the value of the following $\sum_{x=0}^{n} {}^nC_x \, p^x (1-p)^{n-x} =$ 1

$$\sum_{x=0}^{n} p(X=x) = 1$$

$E[x] = np \Rightarrow$ mean

$Var(x) = np(1-p) =$ variance

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$E[X] =$

$E[X^2] =$

$var[X] =$

A fair (unbiased) coin is tossed 15 times. The probability of getting exactly 8 heads is

GATE-20

$n = 15 \quad p = \frac{1}{2} \quad 1-p = \frac{1}{2}$

$$p(x=8) = {}^{15}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^7$$

$$= {}^{15}C_8 \frac{1}{2^{15}}$$

$$= 0.1963$$

A fair coin is tossed 20 times. The probability that head will appear exactly 4 times in the first ten tosses, and tail will appear exactly 4 times in the next ten tosses is

GATE-20

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A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads ?

GATE-09

(a) $\left(\frac{1}{2}\right)^2$

(b) ${}^{10}C_2\left(\frac{1}{2}\right)^2$

(c) $\left(\frac{1}{2}\right)^{10}$

(d) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$

H H T T T T T T T T

$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdots\frac{1}{2}$

$= \frac{1}{2^{10}}$

A fair coin is tossed 10 times. what is the probability of getting exactly 2 heads?

$n = 10 \quad p = \frac{1}{2}$

${}^{10}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8$

$= {}^{10}C_2 \; 1/2^{10}$

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In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is

GATE-12

(a) $\frac{1}{32}$

(b) $\frac{2}{32}$

(c) $\frac{3}{32}$

(d) $\frac{6}{32}$

$p(+) = p(-) = \frac{1}{2}$

getting negative value is success

$p(x \leq 1) = p(x=0) + p(x=1)$

$= {}^5c_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 + {}^5c_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4$

$= \frac{6}{32}$

An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is

GATE-14

(a) 0.067

(b) 0.073

(c) 0.082

(d) 0.091

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Over a large set of inputs a program runs twice as often as it aborts. The probability that of the next 6 attempts, 4 or more will run is

$p(\text{runs}) = 2\,p(\text{Aborts})$

$p(\text{runs}) + p(\text{Aborts}) = 1$

$2\,p(\text{Aborts}) + p(\text{Aborts}) = 1$

$\Rightarrow p(\text{Aborts}) = \frac{1}{3}$

$p(\text{runs}) = \frac{2}{3}$

$p(X \geq 4) = p(X=4) + p(X=5) + p(X=6)$

$= {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + {}^6C_6 (2/3)^6 (1/3)^0$

$= 0.6803$

A certain type of missile hits its target with probability 0.3. The minimum number of missiles that should be fired so that probability of hitting the target at least once is greater than 75% is