1 Transient Heat Transfer

Problem 1.1 Two spheres are removed from a furnace and let to cool with air at 25°C under relatively low convection coefficient of 15 $\text{W.}\left(\text{m}^2.°\text{C}\right)^{-1}$. The spheres are made of copper, $\kappa_{\text{Cu}} = 401$ W. $(\text{m.}°\text{C})^{-1}$, and coal, $\kappa_{\text{Coal}} = 0.2$ W. $(\text{m.}°\text{C})^{-1}$. Can we apply the lumped capacitance method to both spheres?

Problem 1.2 A steel ball of 5 cm in diameter and at uniform temperature of 450°C is suddenly placed in a controlled environment where temperature is kept at 100°C. The prescribed convection heat transfer coefficient is 10 $\text{W.}\left(\text{m}^2.°\text{C}\right)^{-1}$. Calculate the time required for the ball to reach 150°C.
Given $C_{p,\text{steel}} = 0.46$ kJ.kg^{-1}, $\kappa_{\text{steel}} = 35$ W. $(\text{m.}°\text{C})^{-1}$ and $\rho_{\text{steel}} = 7.8 \times 10^3$kg.m^{-3}.

Problem 1.3 A long 20 cm diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 600°C. The shaft is then allowed to cool slowly in an environment chamber at 200°C with an average heat transfer coefficient of 80 $\text{W}/\left(\text{m}^2.°\text{C}\right)^{-1}$. Determine the temperature at the center of the shaft 45 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period. Given, for stainless steel 304 at room temperature:

κ	=	14.9	W.(m.°C)$^{-1}$	ρ	=	7900	kg.m^{-3}
C_p	=	477	J.(kg.°C)$^{-1}$	α	=	3.95×10^{-6}	m^2.s^{-1}

Problem 1.4 A new material is to be developed for bearing balls (spheres of 5 mm of radius) in a new rolling-element bearing. For annealing (heat treatment), each bearing ball is heated in a furnace until it reaches the thermal equilibrium at 400°C. Then, it is suddenly removed from the furnace and subjected to a two-step cooling process:

Stage 1: Cooling in an air flow of 20°C for a period of time t_{air} until the center temperature reaches 335°C. For this situation, the convective heat transfer coefficient of air is assumed constant and equal to 10 W.m^{-2}.K^{-1}. After the sphere has reached this specific temperature, the second step is initiated.

Stage 2: Cooling in a well-stirred water bath at 20°C, with convective heat transfer coefficient of water of 6000 $\text{W.}\left(\text{m}^2.\text{K}\right)^{-1}$.

The thermophysical properties of the material are $\rho = 3000$ kg.m^{-3}, $\kappa = 20$ W.(m.K)$^{-1}$ and $C_p = 1000$ J.(kg.K)$^{-1}$. Determine:

(a) The time t_{air} required for *Stage 1* of the annealing process to be completed;

(b) The time t_{water} required for *Stage 2* of the annealing process during which the center of the sphere cools from 335°C (the condition at the completion of *Stage 1*) to 50°C.

Problem 1.5 Carbon steel balls of 8 mm in diameter are thermally annealed. Firstly, by heating the balls to 900°C in a furnace and then allowing them to slowly cool to 100°C in ambient air at 35°C.

(a) Assuming that the average convective heat transfer coefficient is 75 W.m^{-2}.°C^{-1}, determine how long the annealing process will take;

(b) Also, if 2500 balls are to be annealed per hour, determine the total rate of heat transfer (in W) from the balls to the ambient air.

Given: $\rho = 7833$ kg.m^{-3}, $\kappa = 54$ W.(m.°C)$^{-1}$, $C_p = 0.465$ kJ.kg^{-1}.°C^{-1}, and $\alpha = 1.474\times10^{-6}$ m^2.s^{-1}.

Problem 1.6 A long 35 cm diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 400°C. The shaft is then allowed to cool slowly in a chamber at 150°C with an average convection heat transfer coefficient of 60 W.m^{-2}.°C^{-1}.

(a) Determine the temperature at the center of the shaft 20 min after the start of the cooling process.;

(b) Determine the heat transfer per unit-length of the shaft during this time period.

Given: $\kappa = 14.9$ W.m-1.°C^{-1}, $\rho = 7900$ kg.m^{-3}, $C_p = 477$ J.kg^{-1}.°C^{-1}, and $\alpha = 3.95\times10^{-6}$ m^2.s^{-1}.

Problem 1.7 Apples are left in the freezer at -15°C to cool from an initial uniform temperature of 20°C. The average convective heat transfer coefficient at the apples' surfaces is 8 W.m^{-2}.°C^{-1}. Treating the apples as 9 cm diameter sphere and taking their properties to be $\rho = 840$ kg.m^{-3}, $C_p = 3.81$ kJ.kg^{-1}.°C^{-1}, $\kappa = 0.418$ W.m^{-1}.°C^{-1}, and $\alpha = 1.3\times10^{-7}$ m^2.s^{-1}, determine the center and surface temperatures of thes apples after 1 h. Also, determine the amount of heat transfer from each apple.

Problem 1.8 Consider a large uranium plate of thickness $L = 4$ cm, $\kappa = 28$ W.m^{-1}.°C^{-1}, and $\alpha = 12.5\times10^{-6}$ m^2.s^{-1} that is initially at uniform temperature of 200°C. Heat is uniformlycgenerated in the plate at a constant rate of 5×10^6 W.m^{-3}. At time $t = 0$, one side of the plate is brought into contact with iced water and is maintained at 0°C at all times, while the other side is subjected to convection to an environment at $T_\infty = 30$°C with convective heat transfer coefficient of 45 W.m^{-2}.°C^{-1}. Considering a total of three equally spaced nodes in the medium, two at the boundaries and one at the middle, estimate the exposed surface temperature of the plate 2.5 min after the start of cooling using the finite difference method (FDM).

Problem 1.9 Consider three consecutive nodes $n-1$, n, $n+1$ in a plane wall. Using the finite difference form of the first derivative at the midpoints, show that the finite difference form of the second derivative can be expressed as

$$\frac{T_{n-1} - 2T_n + T_{n+1}}{\Delta x^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_N$$

Problem 1.10 Consider a large uranium plate of thickness 5 cm and thermal conductivity of 28 W.m^{-1}.°C^{-1} in which heat is generated uniformly at a constant rate of 6×10^5 W.m^{-3}. One side of the plate is insulated while the other side is subjected to convection to an environment at 30°C with average convective heat transfer coefficient of 60 W.m^{-2}.°C^{-1}. Considering six equally spaced nodes and steady conditions (*i.e.*, thermal equilibrium):

(a) obtain the finite difference formulation of this problem, and;

(b) determine the nodal temperatures.

Problem 1.11 Steel balls 12 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 400 K in an air environment for which $T_\infty = 325$ K and $h = 20$ W/(m^2.K).

(a) Estimate the time required for the cooling process.

(b) Now, assume that the air temperature increases linearly with time as

$$T_\infty = T_0 + \beta t \qquad [K],$$

where $T_0 = 325$ K, $\beta = 0.1875$ K/s and t is the time.

(i) Demonstrate that $T(t)$ for this new system configuration is given by the following differential equation:

$$\frac{d\theta}{dt} = -b\left(\theta - \beta t\right),$$

where $\theta = T_0 - T(t)$ and $b = \dfrac{hA_s}{\rho V C_p}$.

(ii) The solution for this differential equation is

$$T(t) = T_0 + \left(T_i - T_0 + \frac{\beta}{b}\right)\exp\left(-bt\right) + \beta\left(t - \frac{1}{b}\right),$$

where T_i is the inital temperature of the steel ball. Sketch the ball temperature versus time and air temperature versus time for $0 \leqslant t \leqslant 1$h.

Assume the following properties of the steel: $k = 40$ W/(m.K), $\rho = 7800$ kg/m^3, and $C_p =$ 600 J/(kg.K).

Problem 1.12 A long rod of 60 mm diameter and thermophysical properties $\rho = 8000$ kg/m^3, $C_p = 500$ J/(kg.K), and $k = 50$ W/(m.K) is initially at a uniform temperature and is heated in a forced convection furnace maintained at 750 K. The convection coefficient is estimated to be 1000 W/(m^2.K).

(a) What is the centerline temperature of the rod when the surface temperature is 550 K?

(b) In a heat-treating process, the centerline temperature of the rod must be increased from $T_i = 300$ K to $T = 500$ K. Compute and plot the centerline temperature histories for $h = 100$, 500, and 1000 W/(m^2.K). In each case the calculation may be terminated when $T = 500$ K.

Problem 1.13 A wall 0.12 m thick having a thermal diffusivity of 1.5$\times$10^{-6} m2/s is initially at a uniform temperature of 85°C. Suddenly one face is lowered to a temperature of 20°C, while the other face is perfectly insulated. Using the explicit finite difference method with space and time increments of 30 mm and 300 s, respectively, determine the temperature distribution at $t = 25$ min. The insulated (*i.e.,* adiabatic) face at node i can be treated as a symmetry plane, *i.e.,* $T_{i+1} = T_{i-1}$.

Problem 1.14 A granite sphere of 15 cm in diameter and at uniform temperature of 120°C is suddenly placed in a controlled environment where temperature is kept at 30°C. Average convective heat transfer coefficient is 350 W m$^{-2\circ}$C^{-1}. Calculate the temperature of the granite sphere at a radius of 4.5 cm after 21 minutes. Given properties of granite: κ = 3.2 W m$^{-1\circ}$C^{-1} and α = 13 $\times$ 10^{-7} m^{2}s^{-1}.

2 Initial Design of Heat Exchangers

Problem 2.1 Hot oil is to be cooled in a double-tube counter-flow heat exchanger. Copper inner tubes have diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg.s^{-1}, and the oil through the shell at a rate of 0.8 kg.s^{-1}. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this HE. Given,

(a) Water at 45°C: $\rho = 990$ kg.m^{-3}, $\kappa = 0.637$ W.(m.K)$^{-1}$, $Pr = 3.91$, $\nu = \mu/\rho = 0.602\times10^{-6}$ m^2.s^{-1};

(b) Oil at 80°C: $\rho = 852$ kg.m^{-3}, $\kappa = 0.138$ W.(m.K)$^{-1}$, $Pr = 490$, $\nu = 37.5 \times 10^{-6}$ m^2.s^{-1}.

The inner convective heat transfer coefficient, h_i, can be obtained from

$$Nu = \frac{h_i D_h}{\kappa} = \begin{cases} 4.36 & \text{(for laminar flows)}, \\ 0.023\text{Re}^{0.8}\text{Pr}^{0.4} & \text{(for turbulent flows)}, \end{cases}$$

and the outter convective heat transfer coefficient, h_0 is 75.2 W.$\left(\text{m}^2.\text{K}\right)^{-1}$.

Problem 2.2 A double-pipe (shell-and-tube) heat exchanger is constructed of stainless steel $\left(\kappa = 15.1\ \text{W.m}^{-1}.\text{K}^{-1}\right)$ inner tube of inner and outer diameters of 1.5 cm and 1.9 cm, respectively. The outer shell has inner diameter of 3.2 cm. The convective heat transfer coefficient is 800 W.m^{-2}.K^{-1} on the inner surface of the tube and 1200 W.m^{-2}.K^{-1} on the outer surface. For a fouling factor $\text{R}_{f,i} = 0.0004$ m^2.K.W^{-1} on the tube side and $\text{R}_{f,o} = 0.0001$ m^2.K.W^{-1} on the shell side, determine:

(a) The thermal resistance of the heat exchanger per unit length and;

(b) The overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.

Problem 2.3 Water at the rate of 68 kg.min^{-1} is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ.kg^{-1}.°C^{-1}. The fluids are used in a counterflow double-pipe HE, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W.m^{-2}.°C^{-1}. Given heat capacity of water (at constant pressure) of 4.18 kJ.kg^{-1}.°C^{-1},

(a) Calculate the HE area;

(b) Now assume that the HE is a shell-and-tube with water making one shell pass and the oil making two tube passes. Calculate the new HE. Assume that the overall heat-transfer coefficient remains the same.

Problem 2.4 For the HE of 3 with the same entering-fluid temperatures, calculate the exit water temperature when only 40 kg.min^{-1} of water is heated but the same quantity of oil is used. Also calculate the total heat transfer under such new conditions.

Problem 2.5 A finned-tube heat exchanger (Fig. 1) is used to heat 2.36 m^3.s^{-1} of air at 1 atm from 15.55 to 29.44°C. Hot water enters the tubes at 82.22°C, and the air flows across the tubes, producing an average overall heat-transfer coefficient of 227 W.m^{-2}.°C^{-1}. The total surface area of the exchanger 9.29 m^2. Calculate the exit water temperature and the heat-transfer rate. Assume air behaves as an ideal gas, *i.e.,*

$$\rho = \frac{P\overline{\text{MW}}}{RT},$$

with molar mass of $\overline{\text{MW}} = 28.97$ g.mol^{-1}, gas constant $R = 82.0573$ cm^3.atm.K^{-1}.mol^{-1} and heat capacity at constant pressure of 1005 J.kg^{-1}.K^{-1}.

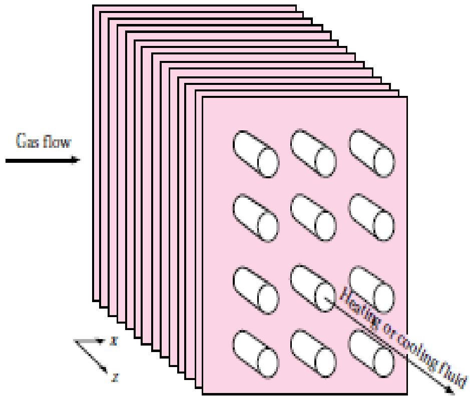

Figure 1: Cross-flow HE with unmixed fluids (Problem 2.5).

Problem 2.6 A double-pipe heat exchanger is constructed of copper, $\kappa = 380$ W.m^{-1}.°C^{-1}, with inner tube of internal diameter of 1.2 cm and external diameter of 1.6 cm. The outer tube has diameter of 3.0 cm. The convection heat transfer coefficient is 700 W.m^{-2}.°C^{-1} on the inner surface of the tube and 1400 W.m^{-2}.°C^{-1} on its outer surface. For a fouling factor $R_{f,i} = 0.0005$ m^2.°C.W^{-1} on the tube side and $R_{f,o} = 0.0002$ m^2.°C.W^{-1} on the shell side. Determine:

(a) Thermal resistance of the heat exchanger per unit-length and,

(b) Overall heat transfer coefficients (U_i and U_o) based on the inner and outer surface areas of the tube, respectively.

Problem 2.7 In a binary geothermal power plant, the working fluid isobutane is condensed by air in a condenser at 75°C $\left(h_{\text{fg}} = 255.7 \text{ kJ.kg}^{-1}\right)$ at a rate of 2.7 kg.s^{-1}. Air enters the condenser at 21°C and leaves at 28°C. The heat transfer surface area based on the isobutane side is 24 m^2. Determine the mass flow rate of air and the overall heat transfer coefficient. Given $C_{p,\text{air}} = 1005$ J.kg^{-1}.°C^{-1}.

Problem 2.8 A cross-flow air-to-water heat exchanger with effectiveness of 0.65 is used to heat water with hot air. Water enters the heat exchanger at 20°C at a rate of 4 kg.s^{-1}, while air enters at 100°C at a rate of 9 kg.s^{-1}. If the overall heat transfer coefficient based on the water side is 260 W.m^{-2}.°C^{-1}, determine the heat transfer surface area of the heat exchanger on the water side. Assume both fluids are unmixed. Given $C_{p,\text{water}} = 4180$ J.kg^{-1}.°C^{-1} and $C_{p,\text{air}} = 1010$ J.kg^{-1}.°C^{-1}.

Problem 2.9 A shell-and-tube process heater is to be selected to heat water from 20°C to 90°C by steam flowing on the shell side. The heat transfer load of the heater is 600 kW. If the inner diameter of the tubes is 1 cm and the velocity of water is not to exceed 3 m.s^{-1}, determine how many tubes need to be used in the heat exchanger. Given $C_{p,\text{water}} = 4180$ J.kg^{-1}.°C^{-1}.

Problem 2.10 A counterflow, concentric tube heat exchanger is designed to heat water from 20 to 80°C using hot oil, which is supplied to the annulus at 160°C and discharged at 140°C. The thin-walled inner tube has diameter of 20 mm, and the overall heat transfer coefficient is 500 W.m^{-2}.K^{-1}. The design condition calls for a total heat transfer rate of 3 kW.

(a) What is the length of the heat exchanger?

(b) After 3 years of operation, performance is degraded by fouling on the water side of the exchanger, and the water outlet temperature is just 65°C for the same fluid flow rates and inlet temperatures. What are the corresponding values of heat transfer rate, outlet temperature of the oil, overall heat transfer coefficient, and water-side fouling factor, R_f ?

Problem 2.11 Saturated steam at 1 atm and 100°C $\left(h_{fg} = 2257 \text{ kJ.kg}^{-1}\right)$ is condensed in a shell-and-tube heat exchanger (one shell, two tube passes). Cooling water enters the tubes at 15°C with an average velocity of 3.5 m.s^{-1}. The tubes are thin walled and made of copper with a diameter of 14 mm and length of 0.5 m. The convective heat transfer coefficient for condensation on the outer surface of the tubes is 21.8 kW.m^{-2}.K^{-1}. Determine:

(a) Number of tubes/pass required to condense 2.3 kg.s^{-1} of steam;

(b) Outlet water temperature;

(c) Maximum possible condensation rate that could be achieved with this heat exchanger using the same water flow rate and inlet temperature.

Given properties of:

- Saturated steam flow: T_{sat}=100°C and $h_{fg} = 2257$ kJ.kg^{-1};
- Cooling water: $\rho = 998$ kg.m^{-3}, $C_p = 4181$ J.kg^{-1}.K^{-1}, $\mu = 959 \times 10^{-6}$ N.s.m^{-2}, $\kappa = 0.606$ W.m^{-1}.K^{-1} and Pr $= 6.62$.

Also, cooling water convective heat transfer should be obtained from the Dittues-Boelter equation,

$$Nu = 0.023 \text{ Re}^{0.8}\text{Pr}^{1/3}$$