1 Transient Heat Transfer

Problem 1.1 Two spheres are removed from a furnace and let to cool with air at 25°C under relatively low convection coefficient of 15 $\text{W.}\left(\text{m}^2.^\circ\text{C}\right)^{-1}$. The spheres are made of copper, $\kappa_{\text{Cu}} = 401\ \text{W.}\left(\text{m.}^\circ\text{C}\right)^{-1}$, and coal, $\kappa_{\text{Coal}} = 0.2\ \text{W.}\left(\text{m.}^\circ\text{C}\right)^{-1}$. Can we apply the lumped capacitance method to both spheres?

Problem 1.2 A steel ball of 5 cm in diameter and at uniform temperature of 450°C is suddenly placed in a controlled environment where temperature is kept at 100°C. The prescribed convection heat transfer coefficient is 10 $\text{W.}\left(\text{m}^2.^\circ\text{C}\right)^{-1}$. Calculate the time required for the ball to reach 150°C.
Given $C_{p,\text{steel}} = 0.46\ \text{kJ.kg}^{-1}$, $\kappa_{\text{steel}} = 35\ \text{W.}\left(\text{m.}^\circ\text{C}\right)^{-1}$ and $\rho_{\text{steel}} = 7.8\times10^3\text{kg.m}^{-3}$.

Problem 1.3 A long 20 cm diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 600°C. The shaft is then allowed to cool slowly in an environment chamber at 200°C with an average heat transfer coefficient of 80 $\text{W}/\left(\text{m}^2.^\circ\text{C}\right)^{-1}$. Determine the temperature at the center of the shaft 45 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period. Given, for stainless steel 304 at room temperature:

κ	=	14.9	W.(m.°C)$^{-1}$	ρ	=	7900	kg.m^{-3}
C_p	=	477	J.(kg.°C)$^{-1}$	α	=	3.95$\times10^{-6}$	m^2.s^{-1}

Problem 1.4 A new material is to be developed for bearing balls (spheres of 5 mm of radius) in a new rolling-element bearing. For annealing (heat treatment), each bearing ball is heated in a furnace until it reaches the thermal equilibrium at 400°C. Then, it is suddenly removed from the furnace and subjected to a two-step cooling process:

Stage 1: Cooling in an air flow of 20°C for a period of time t_{air} until the center temperature reaches 335°C. For this situation, the convective heat transfer coefficient of air is assumed constant and equal to 10 W.m^{-2}.K^{-1}. After the sphere has reached this specific temperature, the second step is initiated.

Stage 2: Cooling in a well-stirred water bath at 20°C, with convective heat transfer coefficient of water of 6000 $\text{W.}\left(\text{m}^2.\text{K}\right)^{-1}$.

The thermophysical properties of the material are $\rho = 3000$ kg.m^{-3}, $\kappa = 20$ W.(m.K)$^{-1}$ and $C_p = 1000$ J.(kg.K)$^{-1}$. Determine:

(a) The time t_{air} required for *Stage 1* of the annealing process to be completed;

(b) The time t_{water} required for *Stage 2* of the annealing process during which the center of the sphere cools from 335°C (the condition at the completion of *Stage 1*) to 50°C.

Problem 1.5 Carbon steel balls of 8 mm in diameter are thermally annealed. Firstly, by heating the balls to 900°C in a furnace and then allowing them to slowly cool to 100°C in ambient air at 35°C.

(a) Assuming that the average convective heat transfer coefficient is 75 W.m^{-2}.°C^{-1}, determine how long the annealing process will take;

(b) Also, if 2500 balls are to be annealed per hour, determine the total rate of heat transfer (in W) from the balls to the ambient air.

Given: $\rho = 7833$ kg.m^{-3}, $\kappa = 54$ W.(m.°C)$^{-1}$, $C_p = 0.465$ kJ.kg^{-1}.°C^{-1}, and $\alpha = 1.474{\times}10^{-6}$ m^2.s^{-1}.

Problem 1.6 A long 35 cm diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 400°C. The shaft is then allowed to cool slowly in a chamber at 150°C with an average convection heat transfer coefficient of 60 W.m^{-2}.°C^{-1}.

(a) Determine the temperature at the center of the shaft 20 min after the start of the cooling process.;

(b) Determine the heat transfer per unit-length of the shaft during this time period.

Given: $\kappa = 14.9$ W.m-1.°C^{-1}, $\rho = 7900$ kg.m^{-3}, $C_p = 477$ J.kg^{-1}.°C^{-1}, and $\alpha = 3.95{\times}10^{-6}$ m^2.s^{-1}.

Problem 1.7 Apples are left in the freezer at -15°C to cool from an initial uniform temperature of 20°C. The average convective heat transfer coefficient at the apples' surfaces is 8 W.m^{-2}.°C^{-1}. Treating the apples as 9 cm diameter sphere and taking their properties to be $\rho = 840$ kg.m^{-3}, $C_p = 3.81$ kJ.kg^{-1}.°C^{-1}, $\kappa = 0.418$ W.m^{-1}.°C^{-1}, and $\alpha = 1.3{\times}10^{-7}$ m^2.s^{-1}, determine the center and surface temperatures of thes apples after 1 h. Also, determine the amount of heat transfer from each apple.

Problem 1.8 Consider a large uranium plate of thickness $L = 4$ cm, $\kappa = 28$ W.m^{-1}.°C^{-1}, and $\alpha = 12.5{\times}10^{-6}$ m^2.s^{-1} that is initially at uniform temperature of 200°C. Heat is uniformlycgenerated in the plate at a constant rate of $5{\times}10^6$ W.m^{-3}. At time $t = 0$, one side of the plate is brought into contact with iced water and is maintained at 0°C at all times, while the other side is subjected to convection to an environment at T$_\infty = 30$°C with convective heat transfer coefficient of 45 W.m^{-2}.°C^{-1}. Considering a total of three equally spaced nodes in the medium, two at the boundaries and one at the middle, estimate the exposed surface temperature of the plate 2.5 min after the start of cooling using the finite difference method (FDM).

Problem 1.9 Consider three consecutive nodes $n-1$, n, $n+1$ in a plane wall. Using the finite difference form of the first derivative at the midpoints, show that the finite difference form of the second derivative can be expressed as

$$\frac{T_{n-1} - 2T_n + T_{n+1}}{\Delta x^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_N$$

Problem 1.10 Consider a large uranium plate of thickness 5 cm and thermal conductivity of 28 W.m^{-1}.°C^{-1} in which heat is generated uniformly at a constant rate of 6×10^5 W.m^{-3}. One side of the plate is insulated while the other side is subjected to convection to an environment at 30°C with average convective heat transfer coefficient of 60 W.m^{-2}.°C^{-1}. Considering six equally spaced nodes and steady conditions (*i.e.*, thermal equilibrium):

(a) obtain the finite difference formulation of this problem, and;

(b) determine the nodal temperatures.

Problem 1.11 Steel balls 12 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 400 K in an air environment for which $T_\infty = 325$ K and $h = 20$ W/(m^2.K).

(a) Estimate the time required for the cooling process.

(b) Now, assume that the air temperature increases linearly with time as

$$T_\infty = T_0 + \beta t \qquad [K],$$

where $T_0 = 325$ K, $\beta = 0.1875$ K/s and t is the time.

(i) Demonstrate that $T(t)$ for this new system configuration is given by the following differential equation:

$$\frac{d\theta}{dt} = -b\left(\theta - \beta t\right),$$

where $\theta = T_0 - T(t)$ and $b = \dfrac{hA_s}{\rho V C_p}$.

(ii) The solution for this differential equation is

$$T(t) = T_0 + \left(T_i - T_0 + \frac{\beta}{b}\right)\exp\left(-bt\right) + \beta\left(t - \frac{1}{b}\right),$$

where T_i is the inital temperature of the steel ball. Sketch the ball temperature versus time and air temperature versus time for $0 \leqslant t \leqslant 1$h.

Assume the following properties of the steel: $k = 40$ W/(m.K), $\rho = 7800$ kg/m^3, and $C_p = 600$ J/(kg.K).

Problem 1.12 A long rod of 60 mm diameter and thermophysical properties $\rho = 8000$ kg/m^3, $C_p = 500$ J/(kg.K), and $k = 50$ W/(m.K) is initially at a uniform temperature and is heated in a forced convection furnace maintained at 750 K. The convection coefficient is estimated to be 1000 W/(m^2.K).

(a) What is the centerline temperature of the rod when the surface temperature is 550 K?

(b) In a heat-treating process, the centerline temperature of the rod must be increased from $T_i = 300$ K to $T = 500$ K. Compute and plot the centerline temperature histories for $h = 100$, 500, and 1000 W/(m^2.K). In each case the calculation may be terminated when $T = 500$ K.

Problem 1.13 A wall 0.12 m thick having a thermal diffusivity of 1.5$\times$10^{-6} m2/s is initially at a uniform temperature of 85°C. Suddenly one face is lowered to a temperature of 20°C, while the other face is perfectly insulated. Using the explicit finite difference method with space and time increments of 30 mm and 300 s, respectively, determine the temperature distribution at $t = 25$ min. The insulated (*i.e.,* adiabatic) face at node i can be treated as a symmetry plane, *i.e.,* $T_{i+1} = T_{i-1}$.

Problem 1.14 A granite sphere of 15 cm in diameter and at uniform temperature of 120°C is suddenly placed in a controlled environment where temperature is kept at 30°C. Average convective heat transfer coefficient is 350 W m$^{-2\circ}$C^{-1}. Calculate the temperature of the granite sphere at a radius of 4.5 cm after 21 minutes. Given properties of granite: κ = 3.2 W m$^{-1\circ}$C^{-1} and α = 13 $\times$ 10^{-7} m^{2}s^{-1}.

2 Initial Design of Heat Exchangers

Problem 2.1 Hot oil is to be cooled in a double-tube counter-flow heat exchanger. Copper inner tubes have diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg.s^{-1}, and the oil through the shell at a rate of 0.8 kg.s^{-1}. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this HE. Given,

(a) Water at 45°C: $\rho = 990$ kg.m^{-3}, $\kappa = 0.637$ W.(m.K)$^{-1}$, $Pr = 3.91$, $\nu = \mu/\rho = 0.602\times10^{-6}$ m^2.s^{-1};

(b) Oil at 80°C: $\rho = 852$ kg.m^{-3}, $\kappa = 0.138$ W.(m.K)$^{-1}$, $Pr = 490$, $\nu = 37.5 \times 10^{-6}$ m^2.s^{-1}.

The inner convective heat transfer coefficient, h_i, can be obtained from

$$Nu = \frac{h_i D_h}{\kappa} = \begin{cases} 4.36 & \text{(for laminar flows)}, \\ 0.023\text{Re}^{0.8}\text{Pr}^{0.4} & \text{(for turbulent flows)}, \end{cases}$$

and the outter convective heat transfer coefficient, h_0 is 75.2 W.$\left(\text{m}^2.\text{K}\right)^{-1}$.

Problem 2.2 A double-pipe (shell-and-tube) heat exchanger is constructed of stainless steel $\left(\kappa = 15.1\ \text{W.m}^{-1}.\text{K}^{-1}\right)$ inner tube of inner and outer diameters of 1.5 cm and 1.9 cm, respectively. The outer shell has inner diameter of 3.2 cm. The convective heat transfer coefficient is 800 W.m^{-2}.K^{-1} on the inner surface of the tube and 1200 W.m^{-2}.K^{-1} on the outer surface. For a fouling factor $\text{R}_{f,i} = 0.0004$ m^2.K.W^{-1} on the tube side and $\text{R}_{f,o} = 0.0001$ m^2.K.W^{-1} on the shell side, determine:

(a) The thermal resistance of the heat exchanger per unit length and;

(b) The overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.

Problem 2.3 Water at the rate of 68 kg.min^{-1} is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ.kg^{-1}.°C^{-1}. The fluids are used in a counterflow double-pipe HE, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W.m^{-2}.°C^{-1}. Given heat capacity of water (at constant pressure) of 4.18 kJ.kg^{-1}.°C^{-1},

(a) Calculate the HE area;

(b) Now assume that the HE is a shell-and-tube with water making one shell pass and the oil making two tube passes. Calculate the new HE. Assume that the overall heat-transfer coefficient remains the same.

Problem 2.4 For the HE of 3 with the same entering-fluid temperatures, calculate the exit water temperature when only 40 kg.min^{-1} of water is heated but the same quantity of oil is used. Also calculate the total heat transfer under such new conditions.

Problem 2.5 A finned-tube heat exchanger (Fig. 1) is used to heat 2.36 m^3.s^{-1} of air at 1 atm from 15.55 to 29.44°C. Hot water enters the tubes at 82.22°C, and the air flows across the tubes, producing an average overall heat-transfer coefficient of 227 W.m^{-2}.°C^{-1}. The total surface area of the exchanger 9.29 m^2. Calculate the exit water temperature and the heat-transfer rate. Assume air behaves as an ideal gas, *i.e.,*

$$\rho = \frac{P\overline{\text{MW}}}{RT},$$

with molar mass of $\overline{\text{MW}} = 28.97$ g.mol^{-1}, gas constant $R = 82.0573$ cm^3.atm.K^{-1}.mol^{-1} and heat capacity at constant pressure of 1005 J.kg^{-1}.K^{-1}.

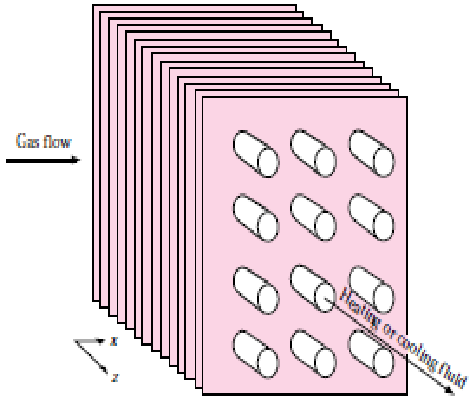


Figure 1: Cross-flow HE with unmixed fluids (Problem 2.5).

Problem 2.6 A double-pipe heat exchanger is constructed of copper, $\kappa = 380$ W.m^{-1}.°C^{-1}, with inner tube of internal diameter of 1.2 cm and external diameter of 1.6 cm. The outer tube has diameter of 3.0 cm. The convection heat transfer coefficient is 700 W.m^{-2}.°C^{-1} on the inner surface of the tube and 1400 W.m^{-2}.°C^{-1} on its outer surface. For a fouling factor $R_{f,i} = 0.0005$ m^2.°C.W^{-1} on the tube side and $R_{f,o} = 0.0002$ m^2.°C.W^{-1} on the shell side. Determine:

(a) Thermal resistance of the heat exchanger per unit-length and,

(b) Overall heat transfer coefficients (U_i and U_o) based on the inner and outer surface areas of the tube, respectively.

Problem 2.7 In a binary geothermal power plant, the working fluid isobutane is condensed by air in a condenser at 75°C $\left(h_{\text{fg}} = 255.7 \text{ kJ.kg}^{-1}\right)$ at a rate of 2.7 kg.s^{-1}. Air enters the condenser at 21°C and leaves at 28°C. The heat transfer surface area based on the isobutane side is 24 m^2. Determine the mass flow rate of air and the overall heat transfer coefficient. Given $C_{p,\text{air}} = 1005$ J.kg^{-1}.°C^{-1}.

Problem 2.8 A cross-flow air-to-water heat exchanger with effectiveness of 0.65 is used to heat water with hot air. Water enters the heat exchanger at 20°C at a rate of 4 kg.s^{-1}, while air enters at 100°C at a rate of 9 kg.s^{-1}. If the overall heat transfer coefficient based on the water side is 260 W.m^{-2}.°C^{-1}, determine the heat transfer surface area of the heat exchanger on the water side. Assume both fluids are unmixed. Given $C_{p,\text{water}} = 4180$ J.kg^{-1}.°C^{-1} and $C_{p,\text{air}} = 1010$ J.kg^{-1}.°C^{-1}.

Problem 2.9 A shell-and-tube process heater is to be selected to heat water from 20°C to 90°C by steam flowing on the shell side. The heat transfer load of the heater is 600 kW. If the inner diameter of the tubes is 1 cm and the velocity of water is not to exceed 3 m.s^{-1}, determine how many tubes need to be used in the heat exchanger. Given $C_{p,\text{water}} = 4180$ J.kg^{-1}.°C^{-1}.

Problem 2.10 A counterflow, concentric tube heat exchanger is designed to heat water from 20 to 80°C using hot oil, which is supplied to the annulus at 160°C and discharged at 140°C. The thin-walled inner tube has diameter of 20 mm, and the overall heat transfer coefficient is 500 W.m^{-2}.K^{-1}. The design condition calls for a total heat transfer rate of 3 kW.

(a) What is the length of the heat exchanger?

(b) After 3 years of operation, performance is degraded by fouling on the water side of the exchanger, and the water outlet temperature is just 65°C for the same fluid flow rates and inlet temperatures. What are the corresponding values of heat transfer rate, outlet temperature of the oil, overall heat transfer coefficient, and water-side fouling factor, R_f ?

Problem 2.11 Saturated steam at 1 atm and 100°C $\left(h_{fg} = 2257 \text{ kJ.kg}^{-1}\right)$ is condensed in a shell-and-tube heat exchanger (one shell, two tube passes). Cooling water enters the tubes at 15°C with an average velocity of 3.5 m.s^{-1}. The tubes are thin walled and made of copper with a diameter of 14 mm and length of 0.5 m. The convective heat transfer coefficient for condensation on the outer surface of the tubes is 21.8 kW.m^{-2}.K^{-1}. Determine:

(a) Number of tubes/pass required to condense 2.3 kg.s^{-1} of steam;

(b) Outlet water temperature;

(c) Maximum possible condensation rate that could be achieved with this heat exchanger using the same water flow rate and inlet temperature.

Given properties of:

- Saturated steam flow: T_{sat}=100°C and $h_{fg} = 2257$ kJ.kg^{-1};
- Cooling water: $\rho = 998$ kg.m^{-3}, $C_p = 4181$ J.kg^{-1}.K^{-1}, $\mu = 959 \times 10^{-6}$ N.s.m^{-2}, $\kappa = 0.606$ W.m^{-1}.K^{-1} and Pr $= 6.62$.

Also, cooling water convective heat transfer should be obtained from the Dittues-Boelter equation,

$$Nu = 0.023 \text{ Re}^{0.8}\text{Pr}^{1/3}$$

TRANSIENT HEAT TRANSFER

P1:

$$L_c = \frac{V}{A_s} = \frac{4/3\,\pi r^3}{4\pi r^2} = \frac{r}{3}$$

$$Bi_{cu} = \frac{h L_c}{K_{cu}} = \frac{15\, r_{cu}/3}{401}$$

$$Bi_{coal} = \frac{h L_c}{K_{coal}} = \frac{15\, r_{coal}/3}{0.2}$$

To use lumped method $Bi < 0.1$

$$Bi_{cu} = \frac{15\, r_{cu}/3}{401} \leq 0.1$$

$$\boxed{r_{cu} \leq 8.02\ m}$$

$$Bi_{coal} = \frac{15\, r_{coal}/3}{0.2} \leq 0.1$$

$$\boxed{r_{coal} \leq 4\times10^{-3}\, m}$$

P2: First, let's check if lumped-capacitance method can be used in this problem:

$$Bi = \frac{h L_c}{k} = \frac{h(\ell/3)}{k} = \frac{h(d/6)}{k}$$

$$Bi = \frac{10\,(5\times10^{-2}/6)}{35} = 2.38\times10^{-3} <<< 0.1$$

thus we can use lumped-~~capacity~~ capacitance method!

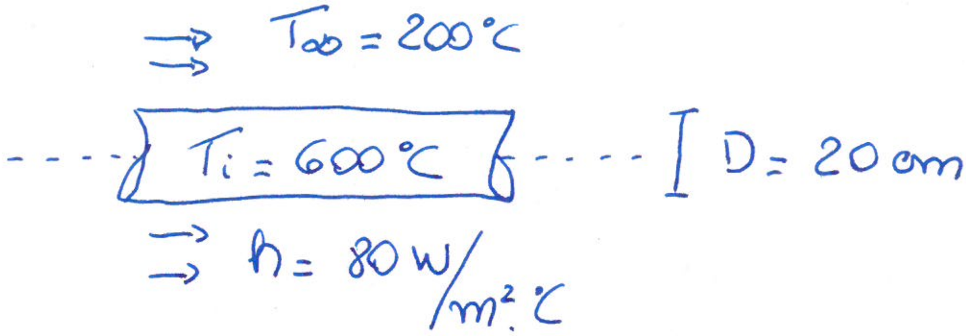
Therefore,

$$\frac{T(t)-T_\infty}{T_0-T_\infty} = e^{-bt} \quad \text{where } b = \frac{hA_s}{\rho V C_p} = \frac{h}{L_c \rho C_p}$$

$$\frac{150-100}{450-100} = \exp\left[\frac{-10t}{(5\times10^{-2}/6)\times 7.8\times10^{3}\times 0.46\times10^{3}}\right]$$

$$\boxed{t = 5818.27\ s \simeq 1.62\ h}$$

P3:



As the problem deals with a long geometry and the thermal symmetry is in the centerline, we can assume the problem as 1-D. The radius of the cylinder is 0.1 m, thus

$$Bi = \frac{h r_0}{k} = \frac{80 \times 0.1}{14.9} = 0.5369$$

$\hookrightarrow > 0.1 \therefore$ Lumped Method can not be used!

Thus using Heisler chart for cylindrical geometry with

$$\begin{cases} 1/Bi = 1.8625 \\ Fo = \tau = \dfrac{\alpha t}{r_0^2} = \dfrac{3.95 \times 10^{-6} \times (45 \times 60)}{0.1^2} = 1.0665 \end{cases}$$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = \boxed{0.40} = \frac{T_0 - 200}{600 - 200} \therefore \underline{T_0 = 360°C}$$

The actual heat transfer can be obtained from the Gröber chart with

$$\begin{cases} Bi^2\tau = 0.3074 \\ Bi = 0.5369 \end{cases} \Rightarrow \quad Q/Q_{MAX} = 0.62$$

Calculating Q_{MAX} for unit-length, i.e., $L = 1\,m$,

$$Q_{MAX} = m\,C_p\,(T_\infty - T_i)$$

with $m = \rho V = \rho(\pi r_0^2 L) = 248.1785\ Kg$

$$Q_{MAX} = 248.1785 \times 477 \times (600-200)$$

$$= 47352457.8000\ J = 47.35\ MJ$$

$$Q/Q_{MAX} = 0.62 \therefore \boxed{\begin{aligned} Q &= 29358523.8360\ J \\ Q &\cong 29.36\ MJ \end{aligned}}$$

Alternatively, we can solve this problem using the analytical method, with,

$Bi = 0.5369$ $\left\{ \begin{array}{l} \lambda_1 = 0.9694 \\ A_1 = 1.1218 \end{array} \right\}$ from "Coefficients & Bessel Functions" tables

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = 1.1218 \exp\left[-(0.9694)^2 \times 1.0665\right]$$
$$= 0.4118$$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = \frac{T_0 - 200}{600 - 200} = 0.4118 \therefore T_0 = 364.72°C$$

Now

$$\left(Q/Q_{MAX}\right)_{cyl} = 1 - 2\theta_0 \frac{J_1(\lambda_1)}{\lambda_1}$$

where $J_1(\lambda_1)$ is the 1st-order Bessel function in which λ_1 is the argument of the function, i.e., $J_1(\lambda_1) = J_1(0.9694) = 0.4296$

$$\left(Q/Q_{MAX}\right)_{cyl} = 1 - 2 \times 0.4118 \times \frac{0.4296}{0.9694} = 0.6350$$

$$Q = 30069418.3619 \text{ J} \cong 30.07 \text{ MJ}$$

where Q_{MAX} was calculated in page 4.

Note the difference (~ 2.42%) between the 2 methods!

P4: Stage 1: check if we can use the lumped-capacitance method:

$$Bi = \frac{h L_c}{k} = \frac{h\, r/3}{k} = \frac{10 \times 5\times10^{-3}/3}{20} = 8.3333\times10^{-4} < 0.1$$

Thus, lumped-capacitance method can be used for this stage, i.e., temperature changes uniformly throughout the sphere,

$$\frac{T(t)-T_\infty}{T_0 - T_\infty} = \exp\left[-\frac{h}{L_c \rho C_p} t\right]$$

$$\frac{335-20}{400-20} = \exp\left[-\frac{10t}{5\times10^{-3}/3 \times 3000 \times 1000}\right]$$

$$\boxed{t = 93.7993\ s} \quad (a)$$

Stage 2: checking Bi,

$$Bi = \frac{6000 \times 5\times10^{-3}/3}{20} = 0.5000 > 0.1$$

Thus, lumped-capacitance method can not be used!

Then, using the analytical method, we need to recalculate $\underline{Bi}$ number from Table (4.2). <u>Note</u> that, in order to use this Table, $\underline{Bi}$ for spheres is defined as (see top of the Table)

$$Bi = \frac{h r_0}{k} = \frac{6000 \times 5\times10^{-3}}{20} = 1.5000$$

$$\rightarrow \begin{cases} \lambda_1 = 1.7998 \\ A_1 = 1.3763 \end{cases}$$

$$\theta_{center} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 \exp\left(-\lambda_1^2 \tau\right)$$

where $\tau = \frac{\alpha t}{r_0^2}$ with $\alpha = \frac{k}{\rho C_p} = 6.6667\times10^{-6}$ m^2/s

$$\frac{50-20}{335-20} = 1.3763 \exp\left[-(1.7998)^2 \tau\right]$$

$$\tau = 0.8245 = \frac{\alpha t}{r_0^2} = \frac{6.6667\times10^{-6} \times t}{(5\times10^{-3})^2}$$

$$\boxed{t = 3.0919\ s}$$ (b)

P5

4-23 A number of carbon steel balls are to be annealed by heating them first and then allowing them to cool slowly in ambient air at a specified rate. The time of annealing and the total rate of heat transfer from the balls to the ambient air are to be determined.

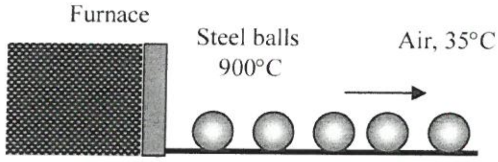
Assumptions **1** The balls are spherical in shape with a radius of $r_0 = 4$ mm. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the balls are given to be $k = 54$ W/m.°C, $\rho = 7833$ kg/m^3, and $C_p = 0.465$ kJ/kg.°C.

Analysis The characteristic length of the balls and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.008\text{ m}}{6} = 0.0013\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(75\text{ W/m}^2.°\text{C})(0.0013\text{ m})}{(54\text{ W/m.°C})} = 0.0018 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the time for the annealing process is determined to be

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{75\text{ W/m}^2.°\text{C}}{(7833\text{ kg/m}^3)(465\text{ J/kg.°C})(0.0013\text{ m})} = 0.01584\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{100-35}{900-35} = e^{-(0.01584\text{ s}^{-1})t} \longrightarrow t = \mathbf{163\text{ s} = 2.7\text{ min}}$$ (a)

The amount of heat transfer from a single ball is

$$m = \rho V = \rho\frac{\pi D^3}{6} = (7833\text{ kg/m}^3)\frac{\pi(0.008\text{ m})^3}{6} = 0.0021\text{ kg}$$

$$Q = mC_p[T_f - T_i] = (0.0021\text{ kg})(465\text{ J/kg.°C})(900-100)°\text{C} = 781\text{ J} = 0.781\text{ kJ (per ball)}$$

Then the total rate of heat transfer from the balls to the ambient air becomes

$$\dot{Q} = \dot{n}_{\text{ball}}Q = (2500\text{ balls/h}) \times (0.781\text{ kJ/ball}) = 1{,}953\text{ kJ/h} = \mathbf{543\text{ W}}$$ (b)

Y.A. Cengel, 'Heat Transfer: A Practical Approach', 2nd Edition

P6 Let's first calculate the Bi number for the long cylindrical shaft

$$Bi = \frac{h L_c}{k} = \frac{h r_o/2}{k}$$

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L} = r_o/2$$

$$= \frac{60 \times 0.1750/2}{14.9}$$

$$= 0.3523 > 0.1$$

Thus lumped-capacitance method can not be used.

Therefore, we need to rely on the analytical method with

$$Bi = \frac{h r_o}{k} = 0.7047$$

to be used in the Table 4.2 (see definition of Biot number to be used in this Table at the top caption!), leading to $\begin{cases} \lambda_1 = 1.0902 \\ A_1 = 1.1548 \end{cases}$

$$\theta_o = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 \exp[-\lambda_1^2 \tau]$$

where $\tau = \frac{\alpha t}{L_c^2} = \frac{3.95 \times 10^{-6} \times (20 \times 60)}{0.1750^2} = 0.1548$

$$\frac{T_o - 150}{400 - 150} = 1.1548 \exp[-(1.0902)^2 \times 0.1548]$$

$\boxed{T_0 = 390.1834\,^\circ C}$ (a)

The heat transfer per unit-length of the shaft can be calculated from Eqn. 10,

$$\left(\frac{Q}{Q_{MAX}}\right)_{cyl} = 1 - 2\,\theta_{0,cyl}\,\frac{J_1(\lambda_1)}{\lambda_1}$$

where the maximum heat (Q_{MAX}) that can be transferred from the cylindrical shaft per unit-length (i.e., per meter) is

$$Q_{MAX} = m\,C_p\,(T_\infty - T_i)$$

with

$$m = \rho V = \rho\,(\overset{3.1415}{\pi}\, r_0^2\, \overset{1\,m}{L}) = 760.0467\ Kg$$

$$Q_{MAX} = 90635568.9750\ J \cong 90.64\ MJ$$

Finally $J_1(\lambda_1)$ – 1st order Bessel function

$$J_1(\lambda_1) = J_1(\lambda_1 = 1.0902) = 0.4679$$

↳ Table 4.3

Thus,

$$\left(\frac{Q}{Q_{Max}}\right)_{cyl} = 1 - 2\,\theta_{0,cyl}\,\frac{J_1(\lambda_1)}{\lambda_1}$$

$$= 1 - 2\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1}$$

$$= 1 - 2\left(\frac{390 - 150}{400 - 150}\right)\frac{0.4679}{1.0902} = 0.1760$$

$$\boxed{Q = 15951860.1396\ J \cong 15.95\ MJ}$$ (b)

P7: Let's first check if the problem can be solved with the lumped-capacitance method, 11

$$Bi = \frac{h L_c}{k} = \frac{h (r_0/3)}{k} = 0.5742 > 0.1$$

↳ lumped-capacitance method can not be used!

Therefore, we need to use the analytical method to solve the problem. Using the definition of Biot number of Table 4.2 (caption at the top),

$$Bi = \frac{h r_0}{k} = 0.8612 \quad \begin{cases} \lambda_1 = 1.4763 \\ A_1 = 1.2390 \end{cases}$$

The temperature at the centre of the apples is

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 \exp\left[-\lambda_1^2 \tau\right]$$

where $\tau = \frac{\alpha t}{r_0^2} = \frac{1.3\times10^{-7} \times 3600}{0.045^2} = 0.2311$

$$\frac{T_0 - (-15)}{20 - (-15)} = \underbrace{\cancel{1.4763}^{\,1.2390} \exp\left[-(1.4763)^2 \times 0.2311\right]}_{0.7487}$$

$$\boxed{T_0 = 11.2058\,°C}$$

Now, ~~in~~ to calculate the temperature at the surface of the apples, we should use Eqn. 7

$$\theta_{sph} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 \exp[-\lambda_1^2 \tau] \frac{\sin(\lambda_1 r/r_0)}{\lambda_1 r/r_0}$$

~~at~~ ~~at~~ where $r = r_0$ (i.e., at the surface)

$$\frac{T(r=r_0,t) - T_\infty}{T_i - T_\infty} = A_1 \exp[-\lambda_1^2 \tau] \frac{\sin(\lambda_1 \cancel{r_0}/\cancel{r_0})}{\lambda_1 \cancel{r_0}/\cancel{r_0}}$$

$$\frac{T(r=r_0,t) - (-15)}{20 - (-15)} = 1.2390 \exp[-(1.4763)^2 \times 0.2311] \times \frac{\sin(1.4763 \text{ rad})}{1.4763}$$

$$= 0.5049$$

$$\boxed{T(r=r_0,t) = 2.6715\,°C}$$

to convert from rad to degrees, one should multiply rad by $180/\pi$!

The heat transferred from each apple to the environment (freezer) can be obtained from Eqn. 11:

$$(Q/Q_{MAX})_{sph} = 1 - 3\theta_{0,sph} \frac{\sin(\lambda_1) - \lambda_1 \cos(\lambda_1)}{\lambda_1^3}$$

$$\left(Q/Q_{max}\right)_{sph} = 1 - 3.(0.7487) \times \frac{\sin(1.4763\,rad) - 1.4763\cos(1.4763\,rad)}{1.4763^3}$$

$$\left(Q/Q_{max}\right)_{sph} = 0.4022$$

Q_{max} can be obtained from

$$Q_{max} = m\,C_p\,(T_i - T_\infty)$$

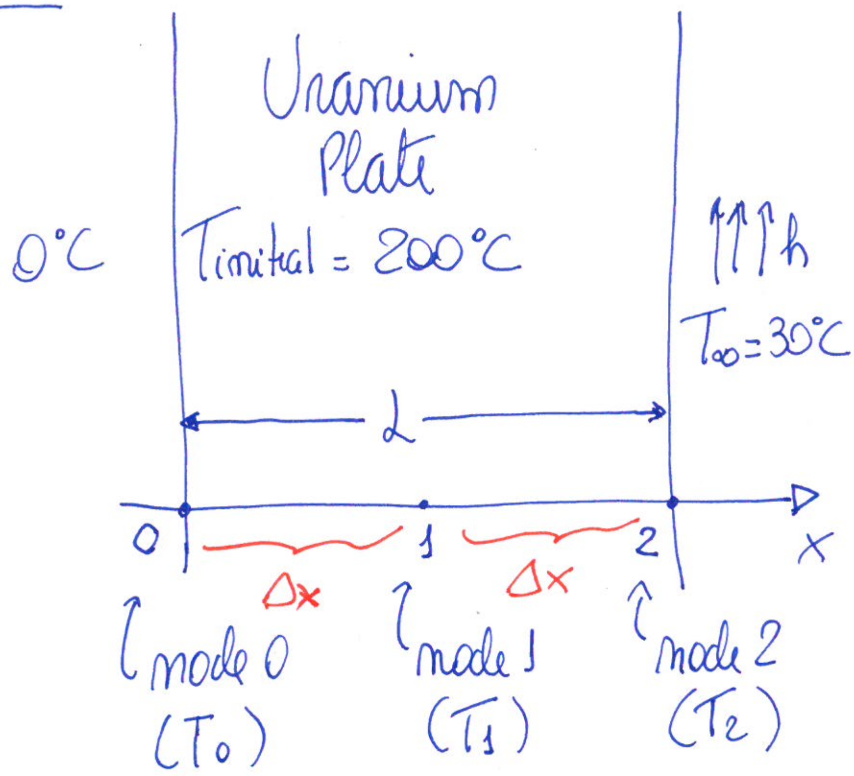
where $m = \rho V = \rho\left(\frac{4}{3}\pi r_0^3\right) = 0.3206\ Kg$

$$Q_{max} = 0.3206 \times 3.81 \times 10^3 \times [20-(-15)] = 42756.01\ J$$

$$\cong 42.76\ KJ$$

$$\boxed{Q = 17194.8584\ J \cong 17.19\ KJ}$$

P8 :



Dividing the domain in 3 nodes with spacing of

$$\Delta x = \frac{L}{M-1} = \frac{0.04}{3-1}$$

$$\Delta x = 0.02 \, m$$

With boundary conditions:

$$T(x=0,t) = T_0(t)$$

Our original thermal equation is

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2} + S$$

assuming K is constant

source/sink term

Discretising with FDM in space and time

$$\rho C_p \frac{T_i^{j+1} - T_i^j}{\Delta t} = K \frac{T_{i+1}^j - 2T_i^j + T_{i-1}^j}{(\Delta x)^2} + S_i^j$$

$\times \frac{1}{K}$

$$\frac{\rho C_p}{k}\,\frac{T_i^{j+1}-T_i^{j}}{\Delta t} = \frac{T_{i+1}^{j}-2T_i^{j}+T_{i-1}^{j}}{(\Delta x)^2} + \frac{S_i^{j}}{k}$$

$\to \alpha^{-1}$ (thermal diffusivity)

$$\underbrace{\frac{(\Delta x)^2}{\alpha\,\Delta t}}_{\tau^{-1}}\left(T_i^{j+1}-T_i^{j}\right) = \left[T_{i+1}^{j}-2T_i^{j}+T_{i-1}^{j}\right] + \frac{(\Delta x)^2}{k}S_i^{j}$$

$\tau^{-1} \to$ mesh Fourier number

$$T_i^{j+1} = T_i^{j} + \tau\left(T_{i+1}^{j}-2T_i^{j}+T_{i-1}^{j}\right) + \frac{\tau(\Delta x)^2}{k}S_i^{j}$$

$$\boxed{T_i^{j+1} = (1-2\tau)T_i^{j} + \tau\left(T_{i+1}^{j}+T_{i-1}^{j}\right) + \frac{\tau(\Delta x)^2}{k}S_i^{j} \qquad (1)}$$

with $\tau = \frac{\alpha\,\Delta t}{(\Delta x)^2} \leq 0.5$ (stability criterion)

$\Delta t \leq 16\,s$

$\to$ using $\Delta t = 15\,s$ in the above equation

$\downarrow$

$\tau = 0.4688$

~~For time t = 0s:~~ $\cancel{T_1^0 = 200°C = T_2^0}$

Equation 1 is designed for the central node(s),

1. For nodes in the borders of the domain:

(a) Imposed Dirichlet BC at $i=0$

$$T_0^0 = T_0^1 = T_0^2 = \dots = 0^\circ C$$

(b) Imposed ~~[illegible]~~ Neumann BC at $x = L$ (i.e., $i=2$)

$$-K \frac{\partial T}{\partial x} = h(T - T_\infty)$$

using a 2nd order accurate FD approximation at $x = L$:

$$-K \frac{T_{i+1}^j - T_{i-1}^j}{2\Delta x} = h(T_i^j - T_\infty)$$

$$\boxed{T_{i+1}^j = T_{i-1}^j - \frac{2\Delta x h}{K}(T_i^j - T_\infty)} \quad (2)$$

And initial conditions of:

$$T_1^0 = T_2^0 = \dots = 200^\circ C$$

For time $t=0s$ $(j=0)$:

- mode $i=1$:

$5\times10^{6}\ W/m^{3}$

$$T_1^1 = (1-2\tau)T_1^0 + \tau(T_2^0+T_0^0) + \frac{\tau(\Delta x)^2}{K} S_1^0$$

$$\boxed{T_1^1 = 139.73\,°C}$$

- mode $i=2$:

$$T_2^1 = (1-2\tau)T_2^0 + \tau(T_3^0+T_1^0) + \frac{\tau(\Delta x)^2}{K} S_2^0$$

T_3^0 is not within the mesh, but we can replace it from Eqn. (2):

$$T_3^0 = T_1^0 - 2\frac{h\Delta x}{K}(T_2^0 - T_\infty)$$

Leading to

$$T_2^1 = (1-2\tau)T_2^0 + \tau\left[T_1^0 - 2\frac{h\Delta x}{K}(T_2^0 - T_\infty) + T_1^0\right] + \frac{\tau(\Delta x)^2}{K} S_2^0$$

$$\boxed{T_2^1 = 228.51\,°C}$$

For time $t = 15s$ $(j = 1)$:

- mode $i = 1$:

$$T_1^2 = (1-2\tau)T_1^1 + \tau(T_2^1 + T_0^1) + \frac{\tau(\Delta x)^2}{k} S_1^1$$

$$\boxed{T_1^2 = 149.33°C}$$

- mode $i = 2$:

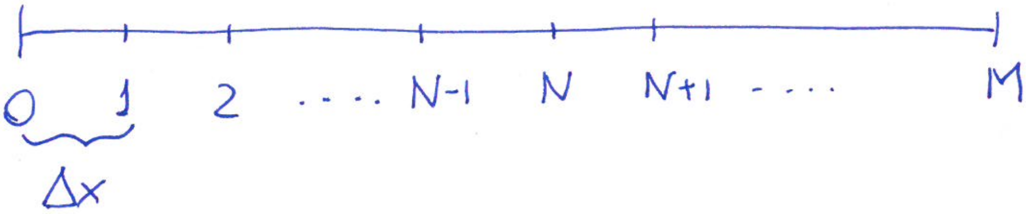
$$T_2^2 = (1-2\tau)T_2^1 + \tau\left[T_1^1 - 2h\Delta x/k\,(T_2^1 - T_\infty) + T_1^1\right] + \frac{\tau(\Delta x)^2}{k} S_2^1$$

$$\boxed{T_2^2 = 172.77°C}$$

Continuing with the same procedure until $t = 2.5\,min = 150\,s$ leads to

$$\boxed{T_2^{11} = 139°C}$$

P9: In a 1D plane wall divided in M nodes:



The expansion in Taylor series around node N leads to:

$$T_{N-1} = T_N - \Delta x \left(\frac{dT}{dx}\right)_N + \frac{(\Delta x)^2}{2!}\left(\frac{d^2T}{dx^2}\right)_N + O[(\Delta x)^3]$$

$$T_{N+1} = T_N + \Delta x \left(\frac{dT}{dx}\right)_N + \frac{(\Delta x)^2}{2!}\left(\frac{d^2T}{dx^2}\right)_N + O[(\Delta x)^3]$$

Summings these expansions and truncating in the second derivative:

$$\frac{T_{N+1} - 2T_N + T_{N-1}}{(\Delta x)^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_N$$

P10: Different from previous problems, in this case the system is assumed to have reached <u>thermal equilibrium</u>, i.e., <u>steady-state</u>. Thus, remembering the thermal energy conservation equation (Problem 8)

source /sink term

$$\rho C_p \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} + S$$

At <u>steady-state conditions</u>, this partial differential equation becomes

$$\kappa \frac{\partial^2 T}{\partial x^2} + S = 0$$

As the problem is <u>no longer time-dependent</u>, the j-index is dropped!

Discretising with FDM in space,

$$\kappa \frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2} + S_i = 0 \quad \times \frac{1}{\kappa}$$

$$\boxed{\frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2} + \frac{S_i}{\kappa} = 0 \quad \text{, for } i = 0, 1, 2, \ldots}$$

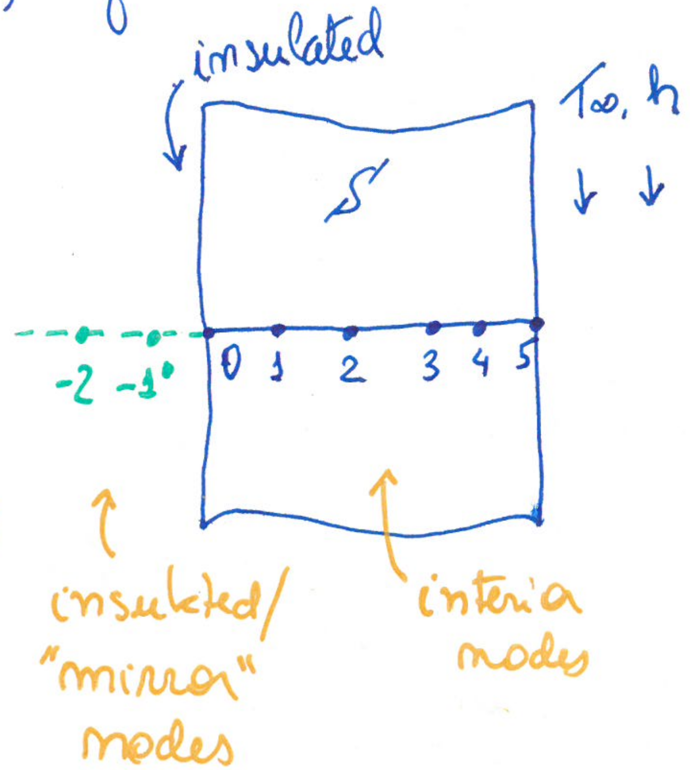
↳ discretised steady-state thermal energy equation.

The uranium plate is 5×10^{-2} m of thickness and should be discretised in 6 equally-spaced nodes, i.e.,

$$\begin{cases} M = 6 \\ L = 5\times10^{-2}\,\text{m} \end{cases} \Rightarrow \Delta x = \frac{L}{M-1} = 10^{-2}\,\text{m}$$

Thus,

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2} + \frac{\dot{S}_i}{k} = 0, \quad \text{for } i = 0, 1, 2, 3, 4, 5$$



$i = 0$ (left surface-insulated):

$$\frac{T_1 - 2T_0 + T_{-1}}{(\Delta x)^2} + \frac{\dot{S}_0}{k} = 0$$

We can treat insulated boundary nodes as interior nodes by assuming zero heat flux across the boundary. In such cases, the insulated domain is assumed to behave as a "mirror" image of the interior domain, thus

$$T_{-1} = T_1$$

and the discretised conservative equation for node $i=0$ is

$$\frac{T_1 - 2T_0 + T_1}{(\Delta x)^2} + \frac{\dot{S}_0}{k} = 0 \qquad (i)$$

i=1 (interior node): $$\frac{T_2 - 2T_1 + T_0}{(\Delta x)^2} + \frac{S_1}{k} = 0 \qquad \text{(ii)}$$

i=2 (interior node): $$\frac{T_3 - 2T_2 + T_1}{(\Delta x)^2} + \frac{S_2}{k} = 0 \qquad \text{(iii)}$$

i=3 (interior node): $$\frac{T_4 - 2T_3 + T_2}{(\Delta x)^2} + \frac{S_3}{k} = 0 \qquad \text{(iv)}$$

i=4 (interior node): $$\frac{T_5 - 2T_4 + T_3}{(\Delta x)^2} + \frac{S_4}{k} = 0 \qquad \text{(v)}$$

i=5 (right surface - convection): Neumann boundary condition is imposed to the rhs surface node:

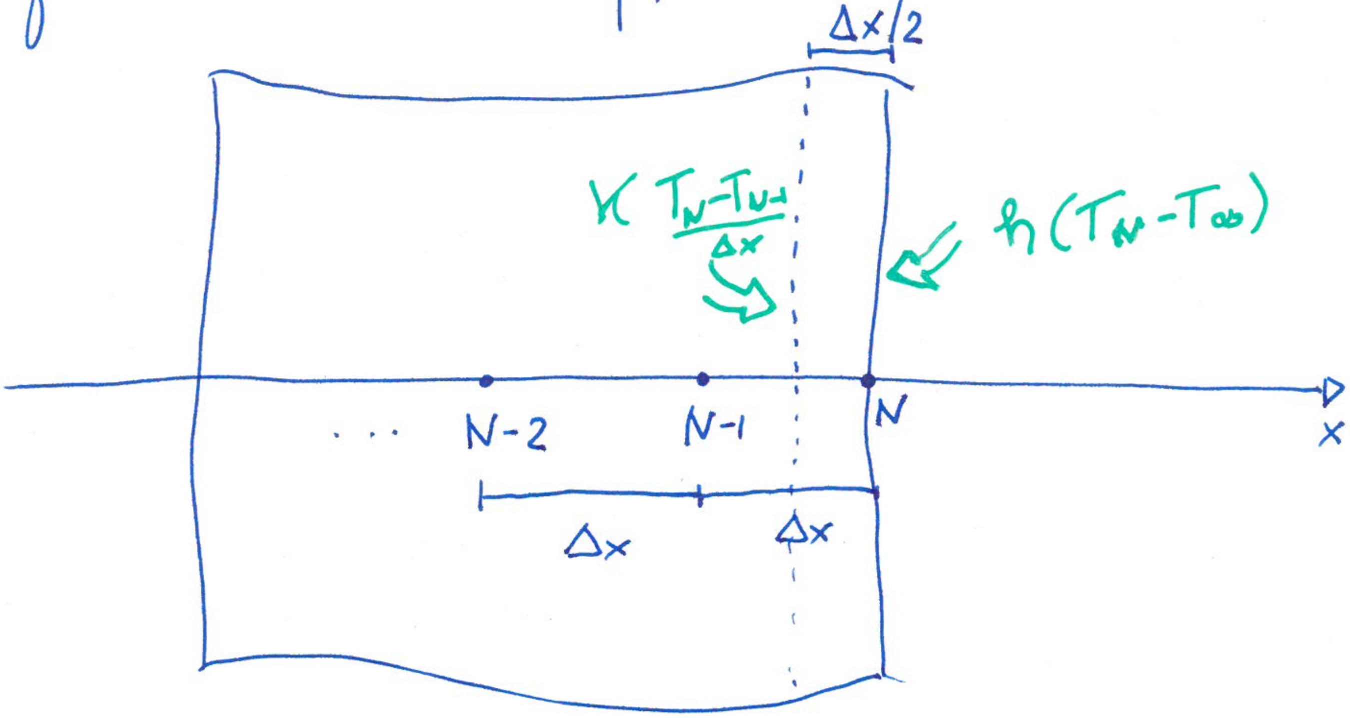
$$-k \frac{\partial T}{\partial x} = h(T - T_\infty) + S,$$

using 1st-order accurate FD approximation at $i=5$:

$$-k \frac{T_i - T_{i-1}}{\Delta x} = h(T_i - T_\infty) + S_i$$

$$h(T_5 - T_\infty) + k \frac{T_5 - T_4}{\Delta x} + S_5 = 0$$

Heat (S) is uniformly distributed across the domain, however the rhs surface ($i=5$) is exposed to convective heat flux. Schematically,



Therefore, source terms for the domain are

$$S_0 = S_1 = S_2 = S_3 = S_4 = \overset{S_5 =}{6} \times 10^5 \ \mathrm{W/m^3} = S$$

however the source term for $\underline{i=5}$ corresponds to just half of the cell, thus it does need to be ~~scaled~~ scaled as

$$S_5 \left(\frac{\Delta x}{2} \right)$$

The same would be applied to S_0, however as the left-hand side border is an insulated node, the "mirror image" scheme takes this (scaling) into account. Finally, for $\underline{i=5}$:

$$h(T_5 - T_\infty) + \kappa \frac{T_5 - T_4}{\Delta x} + S\left(\frac{\Delta x}{2}\right) = 0 \qquad (vi)$$

Summarising $\underbrace{\text{Eqns. (i)-(vi)}}_{(a)}$, after some rearranging

$$\left\{\begin{array}{lr}
-2T_0 + 2T_1 & = -S\Delta x^2/\kappa \\
T_0 - 2T_1 + T_2 & = -S\Delta x^2/\kappa \\
T_1 - 2T_2 + T_3 & \\
T_2 - 2T_3 + T_4 & = -S\Delta x^2/\kappa \\
T_3 - 2T_4 + T_5 & = -S\Delta x^2/\kappa \\
-T_4/\Delta x + \left(\frac{1}{\Delta x} + \frac{h}{\kappa}\right)T_5 & = \frac{hT_\infty}{\kappa} - \frac{S}{\kappa}\left(\frac{\Delta x}{2}\right)
\end{array}\right.$$

Here, there are 6 algebraic equations with $\underline{6}$ unknowns (T_0, T_1, T_2, T_3, T_4 & T_5). Thus, the system of equations is determined. Equations above were arranged in this format to ease matricial representation, i.e.,

$$\underline{\underline{A}}\ \underline{x} = \underline{b}$$

$$\begin{pmatrix} -2 & 2 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -1/\Delta x & \left(1/\Delta x + \frac{h}{k}\right) \end{pmatrix} \begin{pmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{pmatrix} = \begin{pmatrix} G \\ G \\ G \\ G \\ G \\ \frac{hT_\infty}{k} - \frac{S}{k}\left(\frac{\Delta x}{2}\right) \end{pmatrix}$$

where $G = -S(\Delta x)^2/k$

Also, replacing some terms in the matrices:

$-1/\Delta x = -100 \text{ m}^{-1}$

$1/\Delta x + h/k = 102.1429 \text{ m}^{-1}$

$G = -2.1429 \text{ °C}$

$\frac{hT_\infty}{k} - \frac{S}{k}\left(\frac{\Delta x}{2}\right) = -42.8571 \text{ °C/m}$

leading to:

$$\boxed{\begin{array}{ll} T_0 = 456.7867 \text{ °C} & T_2 = 452.5009 \text{ °C} \\ T_1 = 455.7152 \text{ °C} & T_3 = 447.1436 \text{ °C} \end{array}}$$

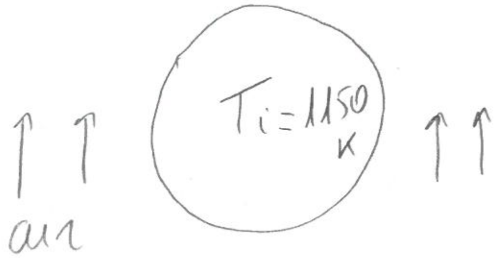
(b)

$$\boxed{\begin{aligned} T_4 &= 439.6435\,°C \\ T_5 &= 430.0004\,°C \end{aligned}}$$ (b)

~~Example 01~~ : P11

$T_\infty = 325K$

$h = 20 W/m^2K$



$D = 12\times10^{-3} m$

$T_i = 1150 K$

$K = 40 W/m.K$

$\rho = 7800 Kg/m^3$

$C_p = 600 J/Kg.K$

$T(t) = 400 K$

$\boxed{t = ?}$

(a) Calculating Bi :

$$Bi = \frac{h L_c}{K} = \frac{h(r/3)}{K} = \frac{20\times(12\times10^{-3}/6)}{40}$$

$Bi = 0.001 < 0.1$

→ We <u>can</u> use lumped method

$$\frac{T(t) - T_\infty}{T_0 - T_\infty} = e^{-bt}$$

where $b = \frac{hA_s}{\rho V C_p} = \frac{h}{L_c \rho C_p}$

$b = 2.1368\times10^{-3}\ s^{-1}$

$$\frac{400-325}{1150-325} = e^{-2.1368\times10^{-3}t}$$

$\boxed{t = 1122.19 s}$ (a)

(b) From the Bi number we know that the temperature remains <u>uniform</u> during the cooling. From the energy balance (Eq. 1 of Lecture Notes)

$$h A_s (T_\infty - T) dt = m C_p dT \qquad (1)$$

replacing $m = \rho V$:

$$h A_s (T_\infty - T) dt = \rho V C_p dT \qquad (2)$$

Now air temperature increases linearly with time:

$$T_\infty = T_0 + \beta t \qquad (3)$$

Replacing (3) in (2)

$$h A_s (T_0 + \beta t - T) dt = \rho V C_p dT \qquad (4)$$

Defining $\theta = T_0 - T$ and $\frac{d\theta}{dT} = -1$ and replacing in (4)

$$h A_s (\theta + \beta t) dt = -\rho V C_p d\theta$$

$$\boxed{\frac{d\theta}{dt} = -\frac{hA_s(\theta+\beta t)}{\rho V C_p} = -b(\theta+\beta t)} \quad (5)$$

(b.i)

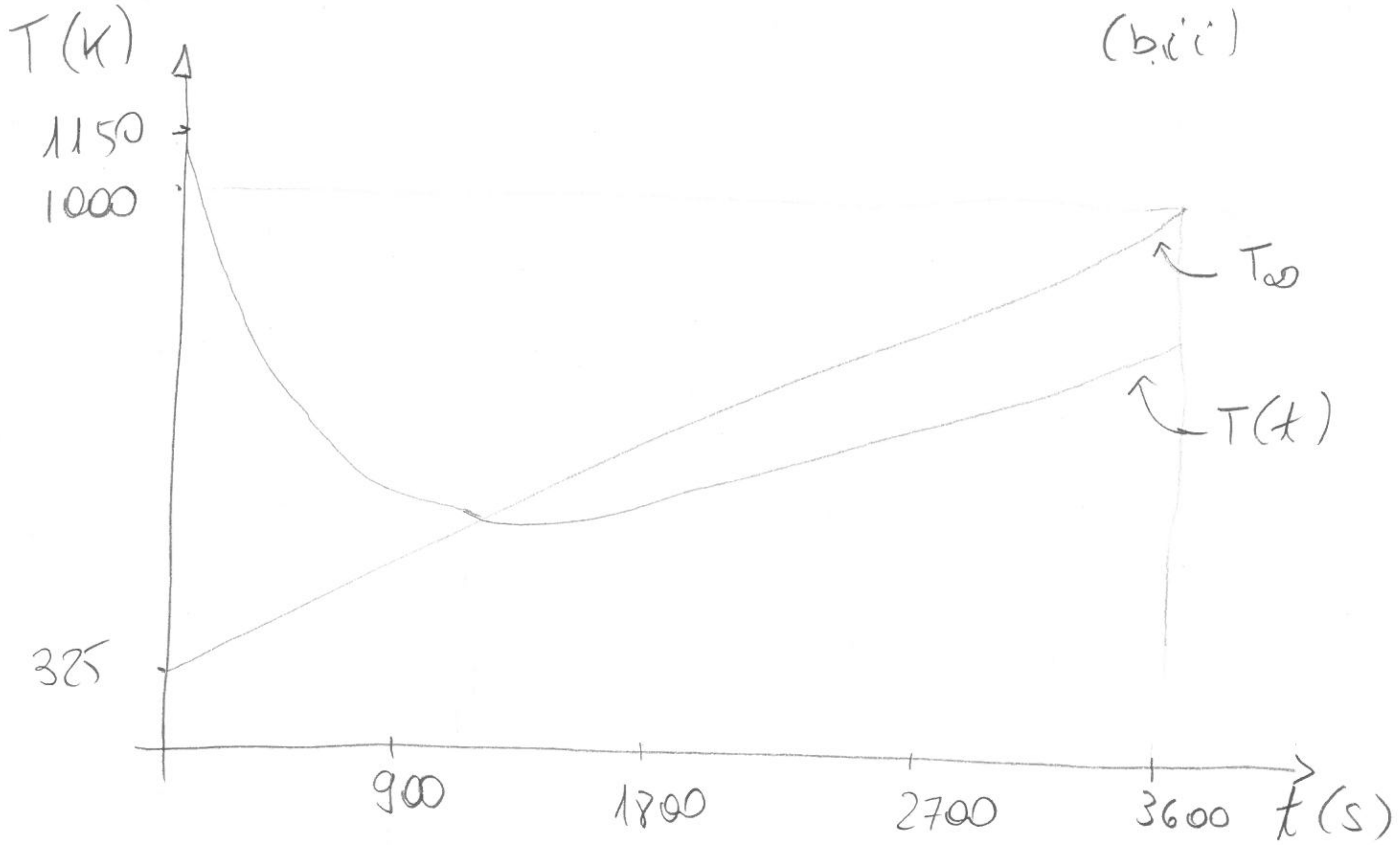
(c) The (analytical) solution for Eqn. 5 is

$$T(t) = T_0 + \left(T_i - T_0 + \frac{\beta}{b}\right)\exp(-bt) + \beta\left(t - 1/b\right) \quad (6)$$

Thus for $t = [0, 3600s]$:

t (s)	$T_\infty(t)$ (K)	$T(t)$ (K)
0	325.00	1150
10	326.88	1132.58
50	334.38	1066.89
100	343.75	993.14
500	418.75	644.58
1000	512.50	532.49
2000	700.00	624.96
3600	1000.00	912.66

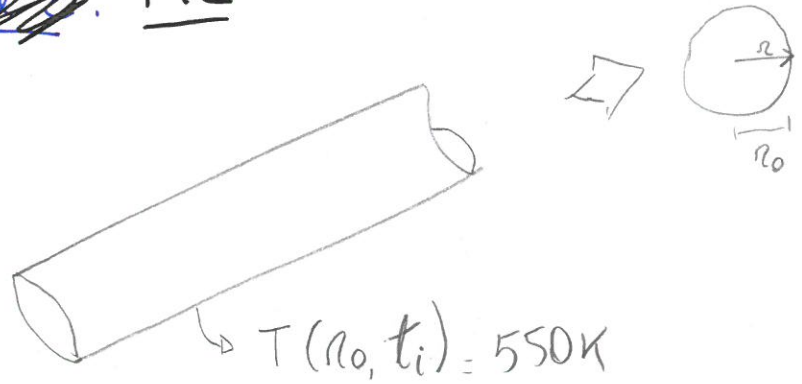
(b.ii)



T_∞: linear function

$T(t)$: non-linear function

~~Example 2~~: P12



$T_\infty = 750\,K$

$D = 60\times10^{-3}\,m$

$\rho = 8000\,Kg/m^3$

$C_p = 500\,J/Kg\,K$

$K = 50\,W/mK$

$T(r_0, t_i) = 550\,K$

$100 \leq h \leq 1000\,W/m^2K$

$T(0, t_i) = ?$

$0 \leq r \leq r_0$

- Assuming Fourier number larger than 0.2;
- 1-D problem & constant properties;

(a)

From Eqn. 10 of the Lecture notes:

$$\theta_{cyl} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0\left(\frac{\lambda_1 r}{r_0}\right) \qquad (10)$$

and at centre of the geometry

$$\theta_{cyl} = \frac{T(0,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \qquad (12)$$

We want to determine $T(0,t)$ at time $t = t_i$ such that $T(r_0, t_i) = 550\,K$. Replacing Eqn. 12 in 10,

$$\frac{T(r_0,t_i)-T_\infty}{T_i-T_\infty} = \frac{T(0,t_i)-T_\infty}{T_i-T_\infty} J_0\left(\frac{\lambda_1 r_0}{r_0}\right) \qquad (*)$$

In order to solve ~~this~~ expression, we need to determine λ_1 from Table 4.2 (Appendix 1). For this table, Bi number is defined as

$$Bi = \frac{h r_0}{K} = \frac{1000 \times (60 \times 10^{-3})/2}{50} = 0.60$$

$$\lambda_1 = 1.0184$$

From Table 4-3, we can obtain $J_0(\lambda_1 r_0/r_0)$:

$$\frac{\lambda_1 r_0}{r_0} = \lambda_1 \therefore J_0(\lambda_1) = 0.7571$$

Replacing in (*):

$$(550-750) = (T(0,t_i)-750) \times 0.7571$$

$$\boxed{T(0,t_i) = 485.83\ K} \quad (a)$$

(b)

h (W/m²K)	Bi	λ_1	A_1
100	0.06	0.3438	1.0148
500	0.30	0.7465	1.0712
1000	0.60	1.0184	1.1345

At the center of the cylinder

$$\theta_{cyl} = \frac{T(0,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$T(0,t) = T_\infty + (T_i - T_\infty) A_1 e^{-\lambda_1^2 \tau}$$

where $\tau = \frac{\alpha t}{r_0^2}$ with $\alpha = \frac{k}{\rho C_p} = 1.25 \times 10^{-5}\ m^2/s$

$\tau = 1.39 \times 10^{-2}\, t$

$$T(0,t) = 750 + (300 - 750) \times A_1 e^{-\lambda_1^2 \times (1.39 \times 10^{-2}) t}$$

$$T(0,t) = 750 - 450 A_1 \exp(-1.39 \times 10^{-2} \lambda_1^2 t)$$

Using λ_1 & A_1 from the table above:

$h_1 = 100\ W/m^2K$

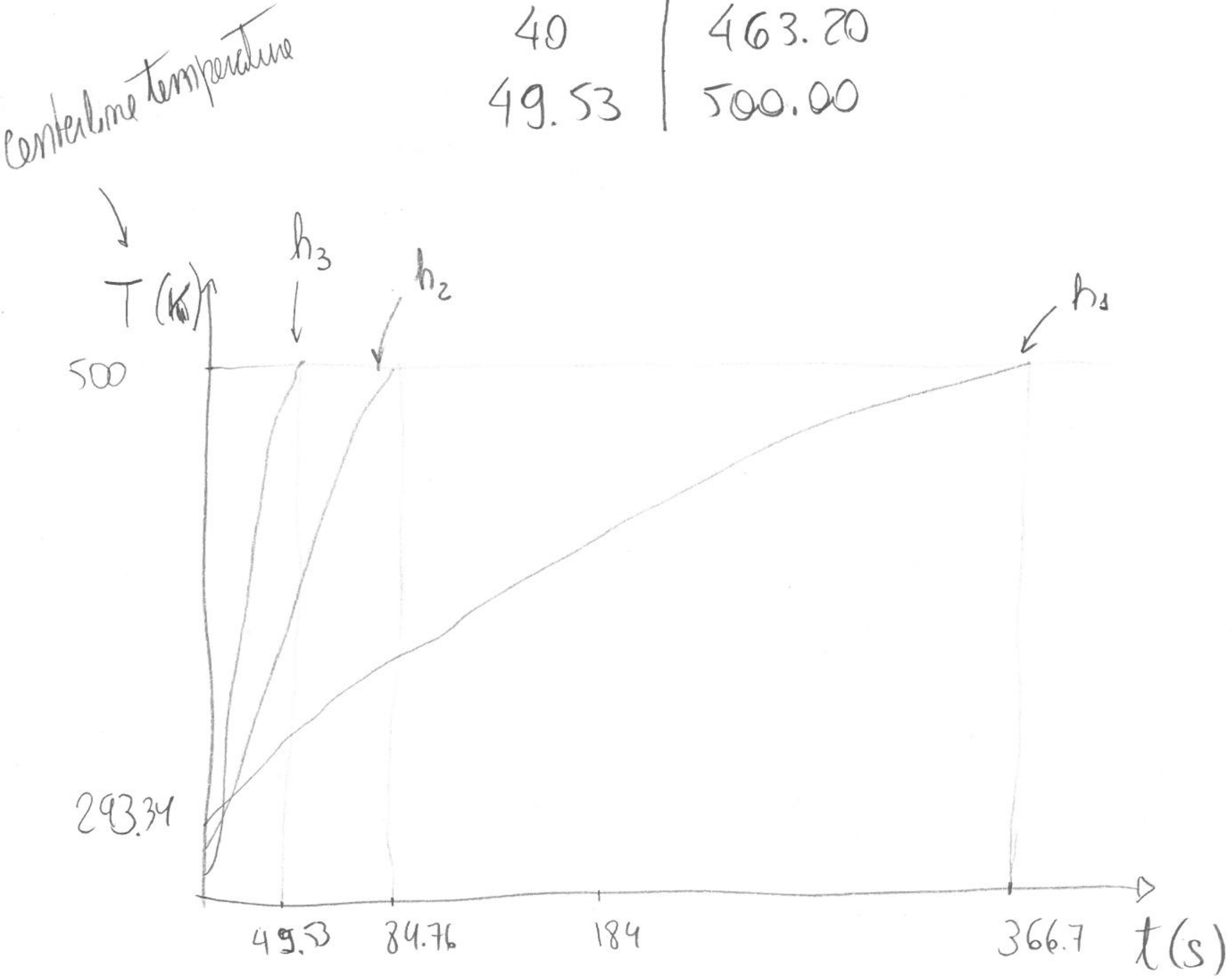
t (s)	T (K)	t (s)	T (K)
0	293.34	300	471.04
50	329.35	360	497.23
100	362.53	366.70	500.00
200	421.23		

$h_2 = 500\ W/m^2K$

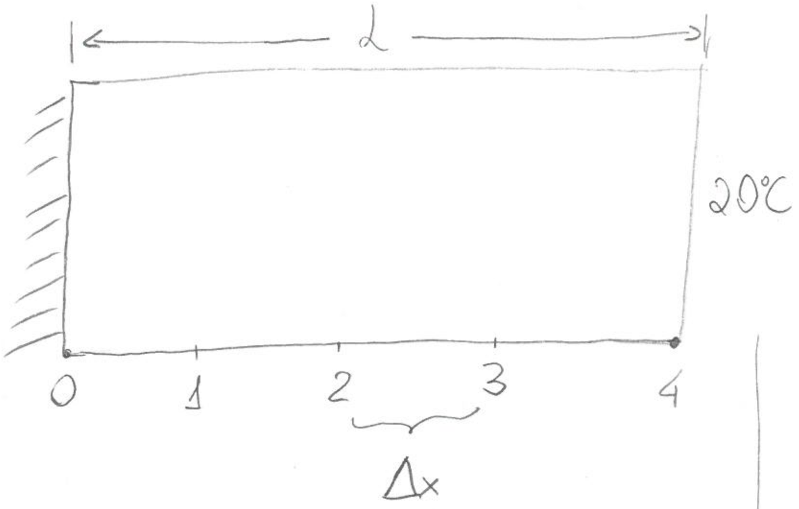
$t(s)$	$T(K)$
0	267.96
20	337.14
40	396.39
50	422.74
84.76	500.00

$h_3 = 1000\ W/m^2K$

$t(s)$	$T(K)$
0	239.48
20	367.35
35	441.76
40	463.20
49.53	500.00



~~Example 03~~ P13



$L = 0.12\,m$

$T(x, t=0) = 85°C$

$\alpha = 1.5 \times 10^{-6}\, m^2/s$

$T(L, t) = 20°C$ (BC)

$\Delta x = 30 \times 10^{-3}\, m$

$\Delta t = 300\, s$

$T(x, t = 1500s) : ?$

$\Delta x = \dfrac{L}{M-1}$

$30 \times 10^{-3} = \dfrac{0.12}{M-1}$

$\underline{M = 5 \text{ nodes}}$

• Our original thermal equation is

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2} + S$$

Discretising with FDM in space and time

$$\rho C_p \frac{T_i^{j+1} - T_i^j}{\Delta t} = K \frac{T_{i+1}^j - 2T_i^j + T_{i-1}^j}{(\Delta x)^2} + S_i^j \qquad \times \frac{(\Delta x)^2}{K}$$

$$\underbrace{\frac{\rho C_p}{K}}_{\alpha^{-1}} \frac{(\Delta x)^2}{\Delta t} \left(T_i^{j+1} - T_i^j\right) = \left[T_{i+1}^j - 2T_i^j + T_{i-1}^j\right] + \frac{(\Delta x)^2}{K} S_i^j$$

Defining $\tau = \dfrac{\alpha \Delta t}{(\Delta x)^2}$ (mesh Fourier number)

$$\boxed{T_i^{j+1} = T_i^j + \tau\left(T_{i+1}^j - 2T_i^j + T_{i-1}^j\right) + \frac{\tau (\Delta x)^2}{K} S_i^j} \qquad (1)$$

Checking the stability criteria ($\tau \leq 1/2$)

$$\tau = \frac{\alpha \Delta t}{(\Delta x)^2} = 0.5$$ (stability criteria is satisfied with $\Delta t = 300\,s$)

- Boundary conditions:

(a) Dirichlet BC at $x = L$:

$T(x=L, t) = 85°C \therefore T_4^j = 85°C \quad (j = 0, 1, \dots)$

(b) Adiabatic plane at $x=0$ can be treated as a symmetry plane, i.e., $T_{i+1}^j = T_{i-1}^j$ for $\underline{i=0}$ $(T_1^j = T_{-1}^j)$

- The problem can be defined as

$$T_i^{j+1} = T_i^j + \tau\left(T_{i+1}^j - 2T_i^j + T_{i-1}^j\right) + \frac{\tau(\Delta x)^2}{K}\cancel{S_i^j} \quad (1)$$

(no source) $\rightarrow 0$

$$\begin{cases} i = 0, 1, 2, 3 \\ j = 0, 1, \dots \end{cases}$$

BC1: $T_4^j = 20°C \quad (j = 0, 1, \dots)$

BC2: $T_1^j = T_{-1}^j \quad (j = 0, 1, \dots)$

Initial Conditions: $T_i^0 = 85°C \quad (i = 0, 1, 2, 3)$

(a) $j=0$:

(a.1) node $i=0$:

T_1^0 (BC2)

$$T_0^1 = T_0^0 + \tau\left(T_1^0 - 2T_0^0 + \cancel{T_{-1}^0}\right)$$

$$T_0^1 = 85 + 0.5(85 - 2\times 85 + 85) = 85\,^\circ C$$

(a.2) node $i=1$:

$$T_1^1 = T_1^0 + \tau\left(T_2^0 - 2T_1^0 + T_0^0\right)$$

$$T_1^1 = 85\,^\circ C$$

(a.3) node $i=2$:

$$T_2^1 = T_2^0 + \tau\left(T_3^0 - 2T_2^0 + T_1^0\right)$$

$$T_2^1 = 85\,^\circ C$$

(a.4) node $i=3$:

$$T_3^1 = T_3^0 + \tau\left(T_4^0 - 2T_3^0 + T_2^0\right)$$

$$T_3^1 = 52.50\,^\circ C$$

(a.5) node $i=4$: (BC 1)

$$T_4^1 = 20\,^\circ C$$

(b) $j = 1$:

(b.1) mode $i = 0$:

T_1^1 (BC2)

$$T_0^2 = T_0^1 + \tau(T_1^1 - 2T_0^1 + \cancel{T_{-1}^1})$$

$$T_0^2 = 85\,°C$$

(b.2) mode $i = 1$:

$$T_1^2 = T_1^1 + \tau(T_2^1 - 2T_1^1 + T_0^1)$$

$$T_1^2 = 85\,°C$$

(b.3) mode $i = 2$:

$$T_2^2 = T_2^1 + \tau(T_3^1 - 2T_2^1 + T_1^1)$$

$$T_2^2 = 68.75\,°C$$

(b.4) mode $i = 3$:

$$T_3^2 = T_3^1 + \tau(T_4^1 - 2T_3^1 + T_2^1)$$

$$T_3^2 = 52.50\,°C$$

(b.5) mode $i = 4$: (BC1)

$$T_4^2 = 20\,°C$$

(c) $j = 2$:

(c.1) mode $i = 0$:

T_1^2 (BC2)

$$T_0^3 = T_0^2 + \tau(T_1^2 - 2T_0^2 + \cancel{T_{-1}^2}) = 85\,°C$$

(c.2) mode $i=1$:

$$T_1^3 = T_1^2 + \tau(T_2^2 - 2T_1^2 + T_0^2) = 76.88°C$$

(c.3) mode $i=2$:

$$T_2^3 = T_2^2 + \tau(T_3^2 - 2T_2^2 + T_1^2) = 68.75°C$$

(c.4) mode $i=3$:

$$T_3^3 = T_3^2 + \tau(T_4^2 - 2T_3^2 + T_2^2) = 44.38°C$$

(c.5) mode $i=4$: (BC1)

$$T_4^3 = 20°C$$

(d) $j=3$:

T_1^3 (BC2)

(d.1) mode $i=0$:

$$T_0^4 = T_0^3 + \tau(T_1^3 - 2T_0^3 + \cancel{T_{-1}^3}) = 76.88°C$$

(d.2) mode $i=1$:

$$T_1^4 = T_1^3 + \tau(T_2^3 - 2T_1^3 + T_0^3) = 76.88°C$$

(d.3) mode $i=2$:

$$T_2^4 = T_2^3 + \tau(T_3^3 - 2T_2^3 + T_1^3) = 60.63°C$$

(d.4) mode $i=3$:

$$T_3^4 = T_3^3 + \tau(T_4^3 - 2T_3^3 + T_2^3) = 44.38°C$$

(d.5) mode $i=4$: (BC1)

$$T_4^4 = 20°C$$

(e) $j = 4$:

(e.1) node $i = 0$:

T_1^4 (BC2)

$$T_0^5 = T_0^4 + \tau(T_1^4 - 2T_0^4 + \cancel{T_{-1}^4}) = 76.88\,°C$$

(e.2) node $i = 1$:

$$T_1^5 = T_1^4 + \tau(T_2^4 - 2T_1^4 + T_0^4) = 68.76\,°C$$

(e.3) node $i = 2$:

$$T_2^5 = T_2^4 + \tau(T_3^4 - 2T_2^4 + T_1^4) = 60.63\,°C$$

(e.4) node $i = 3$:

$$T_3^5 = T_3^4 + \tau(T_4^4 - 2T_3^4 + T_2^4) = 40.32\,°C$$

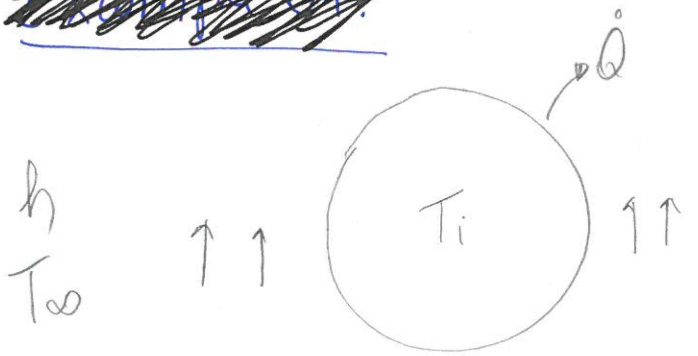
(e.5) node $i = 4$: (BC1)

$$T_4^5 = 20\,°C$$

j	$t(s)$	T_0^j	T_1^j	T_2^j	T_3^j	T_4^j
0	0	85	85	85	85	20
1	300	85	85	85	52.50	20
2	600	85	85	68.75	52.50	20
3	900	85	76.88	68.75	44.38	20
4	1200	76.88	76.88	60.63	44.38	20
5	1500	76.88	68.76	60.63	40.32	20

[redacted]

P14



$D = 15\times10^{-2}\,m$

$T_i = 120\,°C$

$T_\infty = 30°C$; $K = 3.2\,W/m.°C$

$h = 350\,W/m^2.°C$

$\alpha = 13\times10^{-7}\,m^2/s$

$\left\{ \begin{array}{l} T(r = 0.045\,m,\ t = 21\,min): ? \\ Q(t = 21\,min): ? \end{array} \right.$

(a) Graphical Method: For the Heisler charts (Fig. 4.17) of the Lecture Notes:

$$\tau = \frac{\alpha t}{r_0^2} = \frac{13\times10^{-7}\times1260}{(15\times10^{-2}/2)^2} = 0.2921$$

$$Bi^{-1} = \frac{K}{h r_0} = 0.1219 \qquad \frac{r}{r_0} = \frac{0.045}{0.075} = 0.6$$

From Fig 4.17 (b): $\theta = \dfrac{T(r,t) - T_\infty}{T_0 - T_\infty} = 0.59$

From Fig 4.17 (a): $\theta_0 = \dfrac{T_0 - T_\infty}{T_i - T_\infty} = 025$

$$\frac{T(r,t) - T_\infty}{025(T_i - T_\infty)} = 0.59 \quad \therefore \quad \boxed{T(r,t) = 43.28°C}$$

(b) Using the analytical solution

$$\Theta_{sph} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_0)}{\lambda_1 r/r_0}$$

For Bi = 8.2031 and using Table 4.2 (Lecture Notes):

$$\begin{cases} \lambda_1 = 2.7733 \\ A_1 = 1.8958 \end{cases} \qquad \frac{\lambda_1 r}{r_0} = 1.6640$$

$$\frac{T(r,t) - 30}{120 - 30} = 1.8958 \exp\left[-2.7733^2 \times 0.2921\right] \frac{\sin(1.6640\,\text{rad})}{1.6640}$$

$$\boxed{T(r,t) = 40.80\,°C}$$

$\pi = 3.1415926$

$\pi\,\text{rad} = 180°$

$1.6640 - Y$

$Y = 180 \times \frac{1.664}{\pi}$ rad

Now, in order to calculate the heat, we can use the Gröber chart (Fig 4.17 c),

$$Bi^2\tau = 19.66 \Rightarrow Q/Q_{MAX} \cong 0.92$$

$$Q = 0.92\,Q_{MAX} = 0.92\left[\rho V C_p (T_\infty - T_i)\right] \qquad (\alpha = K/\rho C_p)$$

$$= 0.92\left[K/\alpha \left(\tfrac{4}{3}\pi r_0^3\right)(T_\infty - T_i)\right]$$

$\boxed{Q = -359.99\ KJ}$ (negative: heat leaving the system)

Alternatively we can use Eqn. 15 (Lecture Notes)

$$Q/Q_{MAX} = 1 - 3\theta_0 \frac{\sin\lambda_1 - \lambda_1 \cos\lambda_1}{\lambda_1^3}$$

Where $\theta_0 = A_1 e^{-\lambda_1^2 \tau} = 0.2005$

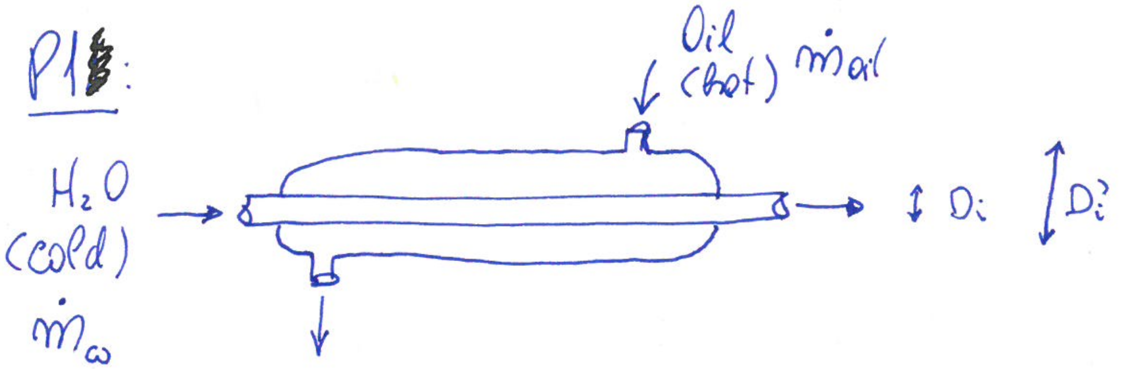
$$Q/Q_{MAX} = 0.9169$$

$$Q = 0.9169 \left[\rho V C_p (T_\infty - T_i)\right]$$

$$\boxed{Q = -358.78 \text{ kJ}}$$

HEAT EXCHANGERS

P1:



$D_i = 2\times10^{-2}$ m

$D_i' = 3\times10^{-2}$ m

$\dot{m}_w = 0.5$ Kg/s

$\dot{m}_{oil} = 0.8$ Kg/s

$T_w^{aver} = 45°C$

$T_{oil}^{aver} = 80°C$

$h_o = 75.2\ W/m^2K$

$\boxed{U = ?}$

The overall HT coefficient is expressed as:

$$U^{-1} = h_i^{-1} + h_o^{-1}$$

h_o: convective HT coef. outside the tube

h_i: convective HT coef. inside the tube

We need to calculate $\underline{h_i}$ from the Nusselt number definition:

$$Nu = \frac{h_i D_h}{K}$$

D_h: hydraulic diameter $= D_i = 2\times10^{-2}$ m

In order to calculate $\underline{Nu}$, we first need to know the ~~flow~~ water flow regime in the inner tube through the Reynolds number:

$$Re_D = \frac{\rho v D}{\mu} = \frac{v D}{\nu}$$

ν: kinematic viscosity

The flow velocity can be obtained from

the mass flow rate, diameter of the tube and density of the fluid:

$$\dot{m}_\omega = v \rho A$$

$$v = \frac{\dot{m}_\omega}{\rho A} = \frac{\dot{m}_\omega}{\rho \, \pi D^2/4}$$

$$v = \frac{0.5 \text{ Kg/s}}{990 \text{ Kg/m}^3 \times \pi \times (2\times10^{-2} \text{ m})^2/4} = 1.6077 \text{ m/s}$$

with Re_D:

$$Re_D = \frac{1.6077 \times 2\times10^{-2}}{0.602\times10^{-6}} = 53411.9601$$

As $Re_D >>> 4000$, the water flow is <u>turbulent</u> and we can use any expression ~~for~~ ~~the~~ to determine the <u>Nu</u> number, e.g., Dittus-Boelter:

$$Nu = 0.023 \, Re_D^{0.8} \, Pr^{0.4}$$

$$Nu = 0.023 \, (53411.9601)^{0.8} \, (3.91)^{0.4} = 240.2754$$

Therefore

$$Nu = \frac{h_i D_h}{K} \therefore h_i = \frac{K\,Nu}{D_h} = \frac{0.637 \times 240.2754}{2 \times 10^{-2}}$$

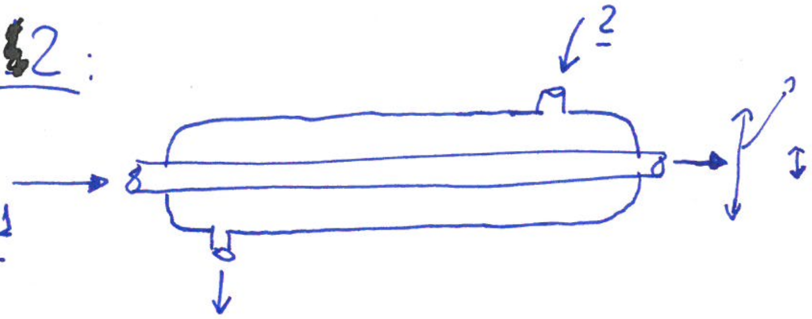
$$h_i = 7652.7715\, W/m^2 \cdot K$$

Now, calculating $\underline{U}$:

$$U^{-1} = h_i^{-1} + h_o^{-1} = (7652.7715)^{-1} + (75.2)^{-1}$$

$$\boxed{U = 74.4682\, W/m^2 K}$$

P2:



$D_o' = 3.2\times10^{-2}\,m$

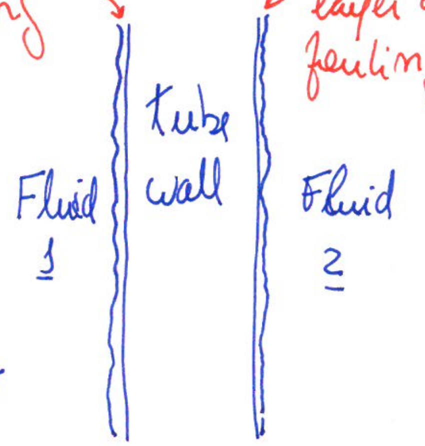
$D_i = 1.5\times10^{-2}\,m$
$D_o = 1.9\times10^{-2}\,m$

$K = 15.1\ W/m.K$
$h_i = 800\ W/m^2.K$
$h_o = 1200\ W/m^2.K$
$R_{f,i} = 4\times10^{-4}\ m^2.K/W$
$R_{f,o} = 10^{-4}\ m^2.K/W$

(a) R : ?
(b) U_i & U_o : ?

The thermal resistance for a shell-and-tube HE with fouling on both HT surfaces is expressed as:

$$R = \frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o}.$$



$$R = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi KL} + \frac{R_{f,o}}{A_o} + 1/h_oA_o$$

Assuming the tube has a length of 1m,

$$A_i = \pi D_i L = 0.04712\ m^2$$
$$A_o = \pi D_o L = 0.05969\ m^2$$

$$R = \frac{1}{800 \times 0.04712} + \frac{4\times10^{-4}}{0.04712} + \frac{\ln\left(1.9\times10^{-2}/1.5\times10^{-2}\right)}{2\pi \times 15.1 \times 1} +$$

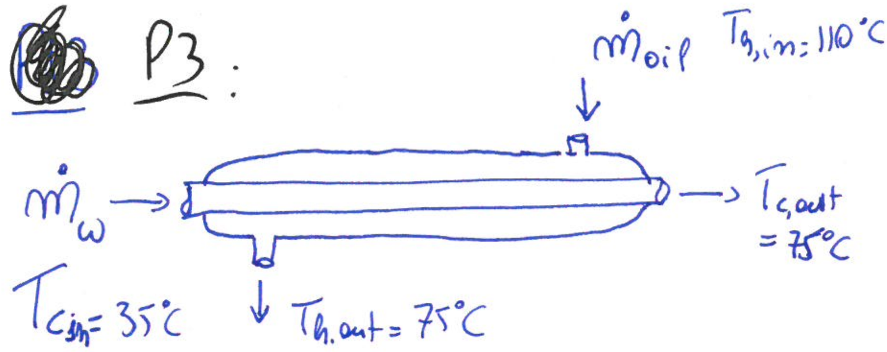
(a)

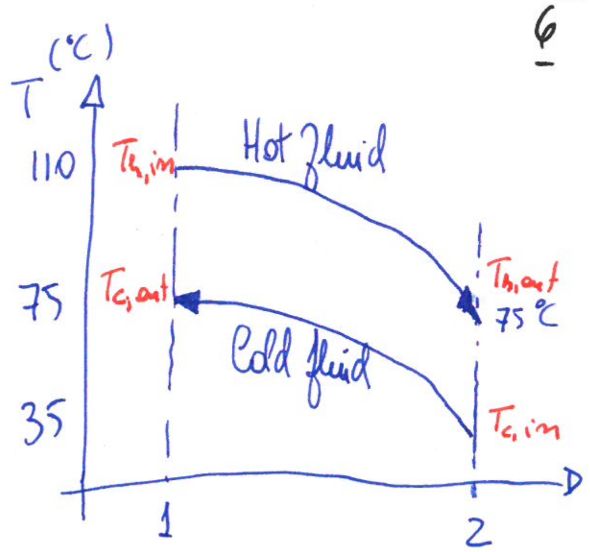
$$+ \frac{10^{-4}}{0.05969} + \frac{1}{1200 \times 0.05969} = \underline{5.3145\times10^{-2}\ K/W}$$

(b) $$U_i = \frac{1}{RA_i} = 399.33\ W/m^2 \cdot K$$

$$U_o = \frac{1}{RA_o} = 315.24\ W/m^2 \cdot K$$

P3:





The total heat transfer can be obtained from:

$$Q = \dot{m}_w C_{p_w} \Delta T_w$$

$$Q = 68 \frac{Kg}{min} \times 4.18 \times 10^3 \frac{J}{Kg.°C} \times (75-35)°C = 189.49 \, KJ/s$$

And the HE surface area is

$$Q = U A \Delta T_{lm} \quad \text{where } U = 320 \, W/m^2 °C, \text{ and}$$

$$\Delta T_{lm} = \frac{(T_{h,out} - T_{c,out}) - (T_{h,in} - T_{c,in})}{\ln \left(\frac{T_{h,out} - T_{c,out}}{T_{h,in} - T_{c,in}} \right)} \quad \Big\} \text{ parallel-flow}$$

$$\Delta T_{lm} = \frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln \left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}} \right)} \quad \Big\} \text{ Counter-flow}$$

For counter-flow HE:

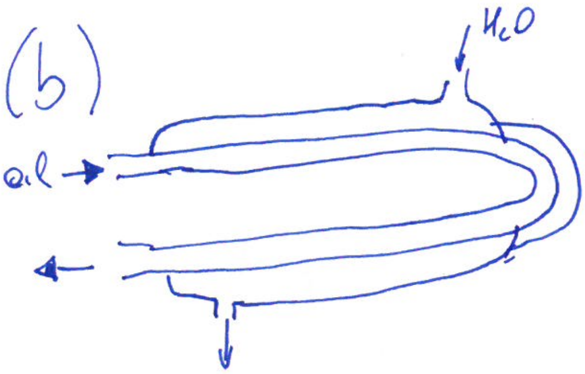
$$\Delta T_{lm} = \frac{(110-75)-(75-35)}{\ln\left(\frac{110-75}{75-35}\right)} = 37.44°C$$

Thus,

$$Q = UA\Delta T_{lm} = 189.49$$

$$A = \frac{189.49 \times 10^3 \, J/s}{320 \, W/m^2 \cdot °C \times 37.44°C} = 15.82 \, m^2$$

(b)



For cross-flow & multipass shell-and-tube HE:

$$Q = UA_s F \Delta T_{lm}$$

From Fig 10.8 (Appendix of Lecture Notes), with

$$R = \frac{T_{c,in} - T_{c,out}}{T_{h,out} - T_{h,in}} = \frac{35-75}{75-110} = 1.1429$$

$$P = \frac{T_{h,out} - T_{h,in}}{T_{c,in} - T_{h,in}} = \frac{75-110}{35-110} = 0.4667$$

From the plot : $F \cong 0.8$

$$Q = U A F \Delta T_{lm}$$

$$A = 19.77 m^2$$

3

P14: From P13 with the original water flow rate of 68 Kg/min,

$$Q = 189.49 \frac{kJ}{s} = -\dot{m}_{oil} C_{P_{oil}} \Delta T_{oil}$$

$$\dot{m}_{oil} = -\frac{189.49 \times 10^3 J/s}{1.9 \times 10^3 J/_{kg\,°C} \times (75-110)°C} = 2.8495 \; Kg/s$$

For this problem: $T_{h,in} = 110°C$

$T_{c,in} = 35°C$

$\dot{m}_w = 40 \; Kg/min$

The energy balance for both fluid streams:

$$Q_w + Q_{oil} = 0$$

$T_{c,out}$ & $T_{h,out}$ are unknown!

$$\dot{m}_w C_{p_w} (T_{c,out} - T_{c,in}) = -\dot{m}_{oil} C_{P_{oil}} (T_{h,out} - T_{h,in})$$

$$\frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{h,out}} = \overbrace{\frac{\dot{m}_{oil} C_{p_{oil}}}{\dot{m}_w C_{p_w}}}^{\phi}$$

$$\boxed{T_{c,out} = T_{c,in} + \phi (T_{h,in} - T_{h,out})}$$

↳ we have $T_{c,out}$ as a function of $T_{h,out}$!

With $\dot{Q} = UA\Delta T_{lm}$

counter-flow

$$\dot{m}_w C_{p_w}(T_{c,out} - T_{c,in}) = UA\,\frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln\left[\dfrac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right]}$$

Substituting $T_{c,out}$ in this expression by

$$T_{c,out} = T_{c,in} + \phi\,(T_{h,in} - T_{h,out})$$

and solving for $T_{h,out}$

$$\hookrightarrow \boxed{T_{h,out} = 79.70\,°C}$$

Now

$$\frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{h,out}} = \frac{m_{oil} C_{p_{oil}}}{m_w C_{p_w}} = 1.9427$$

$$\boxed{T_{c,out} = 93.86\,°C}$$

And the HT:

$$\dot{Q} = \dot{m}_w C_{p_w}(T_{c,out} - T_{c,in})$$

$$\dot{Q} = 164023.2\ J/s = 164.02\ KJ/s = 164.02\ KW$$

Alternative method using NTU method:

Calculating the capacity rates, C_{min}/C_{max}

$$\phi = \frac{\dot{m}_{oil} C_{p\,oil}}{\dot{m}_{w} C_{pw}} = 1.9427$$

$$\frac{C_{min}}{C_{max}} = \frac{1}{\phi} = 0.5147$$

$$NTU_{max} = \frac{UA}{C_{MIN}} = \frac{320 \times 15.82}{0.6667 \times 4.18 \times 10^3} = 1.8167$$

From Fig. 10-13 (Appendix of Lecture Notes):

$\hookrightarrow \varepsilon \cong 0.75$

Or from Table 10.3 (also in Appendix of Lecture Notes):

$\hookrightarrow \varepsilon = 0.7446$

Because the cold fluid is the minimum,

$$\varepsilon = \frac{\Delta T_{cold}}{\Delta T_{max}} = \frac{\Delta T_{cold}}{110 - 35} = 0.7446$$

$\Delta T_{cold} = 55.8450 \therefore T_{c,out} = 90.8450°C$

And the heat transfer is

$$\dot{Q} = \dot{m}_w C_{p_w} \Delta T_{COLD} = 155.62 \text{ KW}$$

P~~4~~5: The cold fluid is air at 1atm with temperature ranging from 15.55°C to 29.44°C. The heat transfer rate is,

$$\dot{Q}_{air} = \dot{m}_{air} C_{p\,air} \Delta T_{air} = \dot{m}_c C_{p_c} \Delta T_c$$

We know the volumetric flow rate of air - 2.36 m³/s, but we need to obtain the mass flow rate, $\dot{m}_{air}$,

volumetric flow rate

$$\dot{m}_{air} = \rho_{air} \dot{v}_{air}$$

Assuming ideal gas behaviour

$$\rho_{air} = \frac{P\overline{MW}}{RT} = 4.2212 \times 10^{-5}\ mol/cm^3 \times \overline{MW}$$

molar mass

$$\rho_{air} = 4.2212 \times 10^{-5} \frac{mol}{cm^3} * 28.97 \frac{g}{mol}$$

$$\rho_{air} = 1.223\ Kg/m^3$$

Thus

$$\dot{m}_{air} = 2.8863 \text{ Kg/s}$$

The heat transfer rate is:

$$\dot{Q}_{air} = 2.8863 \times 1005 \times (29.44 - 15.55)$$

$$\dot{Q}_{air} = 40291.16 \text{ J/s} = 40.29 \text{ KW}$$

The problem does not state which fluid (air or water) can be considered <u>minimum</u>. If air is the minimum fluid, we can calculate NTU and with the help of Fig. 10-15 (Appendix of the Lecture Notes) calculate $\dot{m}_w$ and the exit water temperature. However if the water is the minimum fluid, we may need to use an iterative method.

<u>Assumption 1:</u> Air is the minimum fluid:

$$C_{MIN} = \dot{m}_c C_{p_c} = 2.8863 \times 1005 = 2900.73 \text{ W/°C}$$

and

$$NTU_{MAX} = \frac{UA}{C_{MIN}} = \frac{227 \times 9.29}{2900.73} = 0.7270$$

And the effectiveness is

$$\mathcal{E} = \frac{\Delta T \text{ (MINIMUM FLUID)}}{\text{Max Temp Diff in HE}}$$

$$\mathcal{E} = \frac{\Delta T_{air}}{\Delta T_{MAX}} = \frac{29.44 - 15.55}{82.22 - 15.55} = 0.2083$$

Using $\mathcal{E}$ & NTU_{MAX} in Fig. 10-15, we can not find a match with the existing curves, therefore water is the minimum fluid.

Assumption 2: Water is the minimum fluid.

Therefore we need to estimate C_{MIN}; calculate $NTU_{MAX} \Rightarrow \Delta T_h$

Thus:

$$C_{MAX} = \dot{m}_{air} c_{P_{air}} = 2900.73 \text{ W/°C}$$

$$NTU_{MAX} = UA / C_{MIN}$$

$$\mathcal{E} = \frac{\Delta T_\omega}{\Delta T_{MAX}} = \frac{\left| T_\omega^{out} - T_\omega^{in} \right|}{82.22 - 15.55} \qquad (1)$$

With

$$\dot{Q}_{air} = -\dot{Q}_w = \underbrace{\dot{m}_w C_{pw}}_{C_{MIN}} \Delta T_w \therefore \Delta T_w = -\frac{40291.16}{C_{MIN}} \quad (2)$$

C_{MIN}/C_{MAX}	C_{MIN}	NTU_{MAX}	$\lvert\Delta T_w\rvert$ (Eqn. 2)	ε_{TABLE} (from Fig 10-15 a Table 10-3)	ε_c (from Eqn. 1)	Error
0.5	1450.37	1.4540	27.78	0.65	0.4167	35.89
0.25	725.18	2.9080	55.56	0.89	0.8334	6.36
0.22	638.16	3.3045	63.14	0.92	0.9471	2.95

CONVERGED!

$$\text{Error } (\%): \frac{\lvert \varepsilon_{TABLE} - \varepsilon_c \rvert}{\varepsilon_{TABLE}} \times 100$$

Thus for $C_{MIN} = \dot{m}_w C_{pw} = 638.16$ (C_{pw}: 4180 J/kg K)

$$\dot{m}_w = 0.1527 \text{ kg/s}$$

$$\Delta T_w = T_w^{OUT} - T_w^{IN} = -\frac{40291.16}{638.16} \therefore T_w^{OUT} = 19.08\,°C$$

P86: Thermal resistance of the HE per <u>unit-length</u> is given by Eqn. 6 (Lecture Notes)

$$R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi K L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

where
$$\begin{cases} A_i = \pi D_i L = 0.03770 \text{ m}^2 \\ A_o = \pi D_o L = 0.05026 \text{ m}^2 \end{cases}$$

$$R = \frac{1}{700 \times 0.03770} + \frac{5\times10^{-4}}{3.77\times10^{-2}} + \frac{\ln(1.6/1.2)}{2\pi \times 380 \times 1} + \frac{2\times10^{-4}}{5.026\times10^{-2}} + \frac{1}{700 \times 5.026\times10^{-2}}$$

$$\boxed{R = 8.3679\times10^{-2} \ °C/W}$$ (a)

The overall heat transfer coefficients based on inner and outer areas of the tube per <u>unit-length</u> are

$$R = \frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$, thus

$$U_i = 1/RA_i = \frac{1}{R \times \pi D_i L} = 316.9875 \ W/m^2 \cdot °C$$ (b)

$$U_o = 1/RA_o = \frac{1}{R \times \pi D_o L} = 237.7722 \ W/m^2 \cdot °C$$

P 87

13

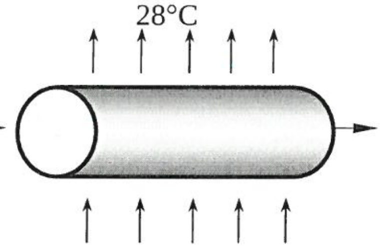
13-64 Isobutane is condensed by cooling air in the condenser of a power plant. The mass flow rate of air and the overall heat transfer coefficient are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The heat of vaporization of isobutane at 75°C is given to be h_{fg} = 255.7 kJ/kg and specific heat of air is given to be C_p = 1005 J/kg.°C.

Analysis First, the rate of heat transfer is determined from

$$\dot{Q} = (\dot{m}h_{fg})_{\text{isobutane}} = (2.7\text{ kg/s})(255.7\text{ kJ/kg}) = 690.39\text{ kW}$$



The mass flow rate of air is determined from

$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{air}}$$

$$\dot{m}_{\text{air}} = \frac{\dot{Q}}{C_p(T_{\text{out}} - T_{\text{in}})} = \frac{690.39\text{ kJ/s}}{(1.005\text{ kJ/kg.°C})(28°\text{C} - 21°\text{C})} = \mathbf{98.14\ kg/s}$$

The temperature differences between the isobutane and the air at the two ends of the condenser are

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}} = 75°\text{C} - 21°\text{C} = 54°\text{C}$$
$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}} = 75°\text{C} - 28°\text{C} = 47°\text{C}$$

and

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{54 - 47}{\ln(54/47)} = 50.4°\text{C}$$

Then the overall heat transfer coefficient is determined from

$$\dot{Q} = UA_s\Delta T_{\text{lm}} \longrightarrow 690{,}390\text{ W} = U(24\text{ m}^2)(50.4°\text{C}) \longrightarrow U = \mathbf{571\ W/m^2.°C}$$

Y.A. Cengel, 'Heat Transfer: A Practical Approach', 2nd Edition

P88

19

13-89 Water is heated by hot air in a heat exchanger. The mass flow rates and the inlet temperatures are given. The heat transfer surface area of the heat exchanger on the water side is to be determined.

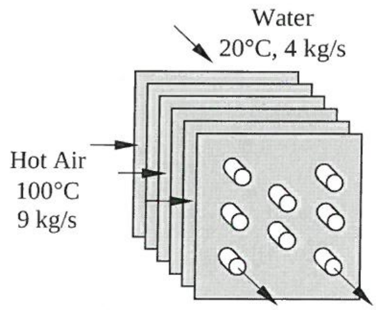
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and air are given to be 4.18 and 1.01kJ/kg.°C, respectively.

Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h C_{ph} = (4\text{ kg/s})(4.18\text{ kJ/kg.°C}) = 16.72\text{ kW/°C}$$

$$C_c = \dot{m}_c C_{pc} = (9\text{ kg/s})(1.01\text{ kJ/kg.°C}) = 9.09\text{ kW/°C}$$



Therefore, $C_{\min} = C_c = 9.09\text{ kW/°C}$

and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{9.09}{16.72} = 0.544$

Then the NTU of this heat exchanger corresponding to $C = 0.544$ and $\varepsilon = 0.65$ is determined from Fig. ~~13-26~~ 10.15 to be

$$\text{NTU} = 1.5$$

Then the surface area of this heat exchanger becomes

$$\text{NTU} = \frac{UA_s}{C_{\min}} \longrightarrow A_s = \frac{\text{NTU}\,C_{\min}}{U} = \frac{(1.5)(9.09\text{ kW/°C})}{0.260\text{ kW/m}^2\text{.°C}} = \mathbf{52.4\ m^2}$$

P89

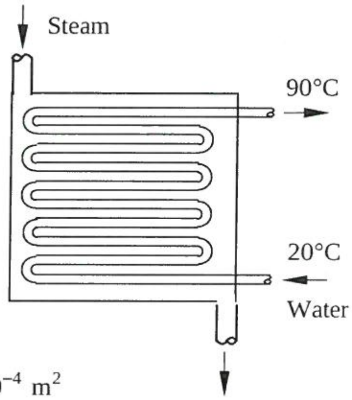
20

3-107 Water is to be heated by steam in a shell-and-tube process heater. The number of tube passes need to be used is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of the water is given to be 4.19 kJ/kg.°C.

Analysis The mass flow rate of the water is



$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in})$$

$$\dot{m} = \frac{\dot{Q}}{C_{pc}(T_{c,out} - T_{c,in})} = \frac{600 \text{ kW}}{(4.19 \text{ kJ/kg.°C})(90°\text{C} - 20°\text{C})} = 2.046 \text{ kg/s}$$

The total cross-section area of the tubes corresponding to this mass flow rate is

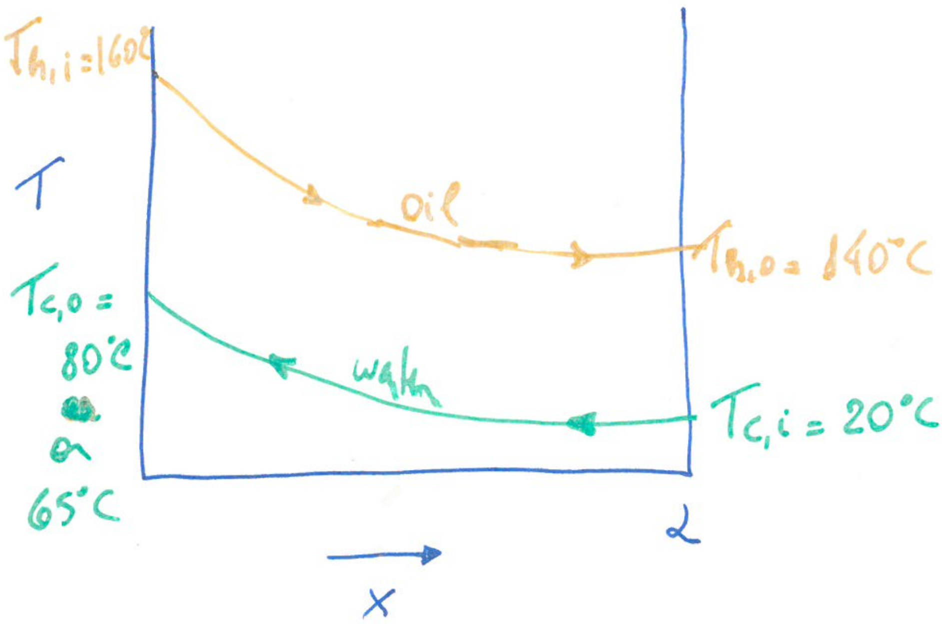
$$\dot{m} = \rho V A_c \rightarrow A_c = \frac{\dot{m}}{\rho V} = \frac{2.046 \text{ kg/s}}{(1000 \text{ kg/m}^3)(3 \text{ m/s})} = 6.82 \times 10^{-4} \text{ m}^2$$

Then the number of tubes that need to be used becomes

$$A_s = n\frac{\pi D^2}{4} \longrightarrow n = \frac{4A_s}{\pi D^2} = \frac{4(6.82 \times 10^{-4} \text{ m}^2)}{\pi (0.01 \text{ m})^2} = 8.68 \cong \mathbf{9}$$

Therefore, we need to use at least 9 tubes entering the heat exchanger.

P10:



Main assumptions:

- no heat loss to the surroundings;
- negligible tube wall conduction resistance

(a) Tube length: L?

Tube length can be obtained Eqn. 8 (Lecture Notes),

$$\dot{Q} = U A_s \Delta T_{lm} = U(\pi D L)\Delta T_{lm}$$

where $\Delta T_{lm} = \dfrac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln \dfrac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}}$

counter-flows

$$\Delta T_{lm} = 98.6521\,°C$$

$$\boxed{L = \frac{\dot{Q}}{U \pi D \Delta T_{lm}} = 0.9680\ \text{m}}$$

(b) 3 yrs of operation with fouling led to

$T_{c,o} = 65\,°C \Rightarrow \left\{ \begin{array}{l} \dot{Q}' \\ T'_{h,o} \\ U' \\ R_f \end{array} \right\} ?$

With $\dot{Q} = C_c\,(T_{c,o} - T_{c,i})$, the following ratio may be formed in terms of the design and 3 years of conditions

$$\frac{\dot{Q}}{\dot{Q}'} = \frac{C_c\,(T_{c,o} - T_{c,i})}{C_c\,(T'_{c,o} - T_{c,i})} = \frac{60}{45} = 1.3333$$

Therefore

$$\boxed{\dot{Q}' = \dot{Q} / 1.3333 = 2250.0563\ \mathrm{W}}$$

and with the ratio of heat rates

$$\frac{\dot{Q}}{\dot{Q}'} = \frac{C_h\,(T_{h,i} - T_{h,o})}{C_h\,(T_{h,i} - T'_{h,o})} = 1.3333$$

$$\boxed{T'_{h,o} = 144.9996\,°C}$$

And U'

$$U' = \frac{\dot{Q}'}{A_s \Delta T'_{lm}}$$

with $\Delta T'_{lm} = 109.3146\,°C$

$$\boxed{U' = 338.4331\ W/m^2 \cdot °C}$$

At the original conditions:

$$U^{-1} = [h_i^{-1} + h_o^{-1}],$$

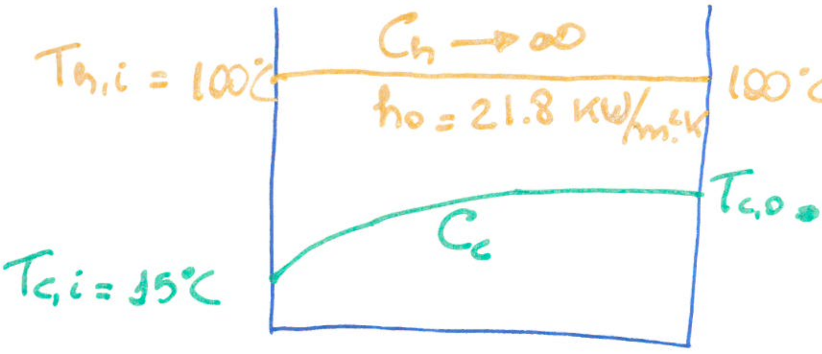
and after 3yrs of operations

$$(U') = [h_i^{-1} + h_o^{-1} + R_f]^{-1}$$

$$R_f = (U')^{-1} - \underbrace{(h_i^{-1} + h_o^{-1})}_{U^{-1}}$$

$$\boxed{R_f = 9.5479 \times 10^{-4}\ \frac{m^2 \cdot K}{W}}$$

P11:



saturated steam
$P = 1 atm$; $\dot{m}_h = 2.3\ kg/s$

$\hookrightarrow$ water, $u = 3.5\ m/s$

Tubes: $\begin{cases} D = 14 \times 10^{-3} m \\ L = 0.5 m \end{cases}$

Assumptions:

- no heat loss to the surroundings;
- negligible thermal resistance due to the tube walls.

(a) number of tubes/pass to condense 2.3 kg/s of steam: ?

Effectiveness of the thermal system

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{MAX}} \therefore \dot{Q} = \varepsilon\, C_{MIN} (T_{h,i} - T_{c,i})$$

where $C_{MIN} = C_c = \dot{m}_c c_{p_c}$,

with

$$\dot{m}_c = \rho \pi \frac{D^2}{4} u \underbrace{N}_{\text{number of tubes}}$$

Thus

$$C_{MIN} = \rho \pi \frac{D^2}{4} u N c_p = 2248.0807 N$$

And the heat transfer rate for the HE is

$$\dot{Q} = \dot{m}_h h_{fg} = 2.3 \times 2257 \times 10^3 = 5191100 \text{ W}$$

Thus,

$$\dot{Q} = \varepsilon C_{MIN} (T_{h,i} - T_{c,i})$$

$$5191100 = \varepsilon \times (2248.0807 N)(100 - 15)$$

$$\boxed{\varepsilon N = 27.1662} \quad (*)$$

Now, we need to calculate the effectiveness, ε.

As $C_r = \frac{C_c}{\underset{\hookrightarrow \infty}{C_h}} \rightarrow 0$, from Table 10.3 (Lecture

Notes):

$$\varepsilon = 1 - \exp(-NTU)$$

where

$$NTU = \frac{U A_s}{C_{MIN}}$$

Overall HT coefficient, U, is calculated from

$$U = (1/h_i + \underset{\uparrow\, 21.8 \times 10^{+3}}{1/h_o})^{-1}$$

h_i can be obtained from Colburn equation,

$$Nu = \frac{hD}{K} = 0.023\, Re^{0.8}\, \underbrace{Pr^{1/3}}_{\to\, 6.62}$$

$$\hookrightarrow Re = \frac{\rho u D}{\mu} = 50992.7007$$

$$\frac{h_i \cdot 14\times10^{-3}}{0.606} = 251.9728$$

$$h_i = 10906.8226 \; W/m^2K$$

$$\boxed{U = 7269.6983 \; W/m^2K}$$

Finally, surface heated area is

$$A_s = \pi D L N P$$

number of passes per tube
$\hookrightarrow 2$

$$A_s = 0.04398\, N$$

Thus,

$$\boxed{NTU = \frac{UA_s}{C_{MIN}} = 0.1422}$$

We can finally obtain ε

$$\varepsilon = 1 - \exp(-NTU) = 0.1326$$

Now, using Eqn (*), we can finally obtain the required number of tubes

$$\mathcal{E}N = 27.1662$$

$$\boxed{N = 204.8733 \approx 205 \text{ tubes}}$$

with total surface area of

$$\boxed{A_s = \pi D N L P = 9.0161 \text{ m}^2}$$

(b) Outlet water temperature ($T_{c,out}$) with

$$C_{MIN} = 2248.0807N = 460856.5435 \text{ W/K}$$

with

$$\dot{Q} = C_{MIN}(T_{c,out} - T_{c,in})$$

$$\boxed{T_{c,out} = 26.2640\,°C}$$

(c) Maximum condensation rate occurs when

$$\dot{Q} = \dot{Q}_{MAX}$$

$$\dot{m}_{R,max} = \dot{Q}_{MAX} / h_{fg} = \frac{C_{MIN}(T_{R,in} - T_{c,in})}{h_{fg}} = 17.3561 \text{ kg/s}$$