Hello World!

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1 HDTN:Introduction

This is a LaTeX document backed with Haskell, displaying a brute force method of computing the divisors of highly divisible triangle numbers. Two methods will be used, initally brute force search, and second a mathematical distillation of the sequence to optimize the required computations towards a minimal point.

2 Brute Force Search

The following section describes the process of searching for divisors for HDTNs with a brute force approach. Through constructive mathematics a small set of functions can be denoted for finding HDTN, and the structure should be similar to the unoptimized prime finder as essentially it is a prime check for non prime numbers. But the key difference is calculating the divisors is essential as the divisors are being seached for, not for numbers that lack divisors.

2.1 BFS: General Intuitions

1+2 = 3
1+2+3= 6
1+2+3+4 = 10
1+2+3+4+5 = 15
1+2+3+4+5+6 = 21
1+2+3+4+5+6+7 = 28

A HDTN is the sum of consecutive natural numbers, not all natural numbers in the summing sequence is a divisor of the HDTN. The minimum multiple is two as a double of x. Therefore all possible divisors lie withing y/2. Looking at the above sequence, the difference between the last number in the summing sequence and the sum increases by an exponential degree. For now a bound y/2 is sufficient, but an optimized search would vary the bound dependant on y, possible allowing it to be y/4 and even lower, but implimenting such a bound would require some precision as if the variable bound is incorrect then divisors will be missed during the search.
1) HDTN lie in either the odd or even domain. Encapsulating both sides. Due to symetry being present in both odd and even domains.
2) If a HDTN is odd it cannot have even divisors.
3) However a even HDTN can have odd divisors.
4) Hence even HDTN have more divisors as they lie in both the odd and even domain.
5) The set of possible divisors is from 2 to B where B is the bound.
6) If the HDTN is odd then the set of possible divisors only contains odd numbers.
7) If the HDTN is even then the set of possible divisors contain all natural numbers from 2 to B.

2.2 BFS: Distillation of Intuitions

First a classification function will be definied.

$$\sigma^A(x) = y \begin{cases} 1 \mid (x \mod 2 = 0) \\ 0 \mid (x \mod 2 \neq 0) \end{cases} \tag{1}$$

Hence a HDTN can be classified as odd or even. Based on this classification two different search methods are used. If odd, only odd factors are generated from 3 to the factor where y/3 = b. If even, all factors are checked. From two, to the factor where y*2 = b.
The bounds can denoted as such;

$$B^O \wedge B^E \tag{2a}$$

$$B^O(y) = y/3 \tag{2b}$$

$$B^E(y) = y/2 \tag{2c}$$

$$[F_{B^O}] = [F \in [3...\frac{y}{3}] \mid F] \tag{3a}$$

$$[F_{B^E}] = [F \in [3...\frac{y}{2}] \mid F] \tag{3b}$$

$$[F_{B^O}]_{|F_{B^O}|} * 3 = y \tag{3c}$$

$$[F_{B^E}]_{|F_{B^E}|} * 2 = y \tag{3d}$$

hence the maximal factor is computed, it is then infered that the respective set infers all divisors. constructing a set of ones equal to the size of the respective set yields a state that when summed is equal to the amount of divisors respective of the HDTN. Simple, ez.
Written in full form it follows as such;

3 BFS: Formalization

$$\mathbb{B}(x) = y \begin{cases} 1 \mid x \mod 2 = 0 \\ 0 \mid x \mid 2 \neq 0 \end{cases} \tag{4a}$$

$$[F(\mathbb{B}(x))] = [a...b] \begin{cases} a = 2, b = \frac{x}{2} \mid \mathbb{B}(x) = 0 \\ \\ a = 3, b = \frac{x}{3} \mid \mathbb{B}(x) \neq 0 \end{cases} \tag{4b}$$

$$\sum_{i=0}^{|[F(\mathbb{B}(x))]|} = [F(\mathbb{B}(x))]_i \tag{4c}$$

The above has been written into a Haskell program to show proof of concept and function.