Answers

The questions and example answers that appear in this resource were written by the author. In examination, the way marks would be awarded to answers like these may be different.

Chapter 1

Getting started

1 **a** Student answers will vary based on what they already know and feel confident doing.

b Some students will select the things they are less confident in, but other may select things they enjoy doing or are good at. Encourage them to say why they have made each selection.

2 **a** There are many possible answers for each value. For example, (a) could be 9^2 or $9(2 + 7)$ or $8 \times 10 + 1$. Let students use calculators to check that each other's clues work.

b Twenty-one thousand, eight hundred and thirty-seven

3 **a** 9^3

b 12^2

c 7^5

d $\left(\frac{1}{3}\right)^2$

e $\left(\frac{4}{3}\right)^0$

f $9^{\frac{1}{2}}$

g 14 000 010 019

4 **a** Any real-world measurement problems involve a level of approximation, as do problems where you have to work out if you have enough money, or have catered enough food, estimated times of arrivals, estimates for building materials and costs of doing different jobs.

b Encourage students to share ideas and discuss their own methods of deciding.

c Answers will vary, but could include that estimating allows you find errors and judge the size an answer should be, avoid mistakes due to button push or place value errors.

Exercise 1.1

1 **a** $\{3, 4, 6, 11, 16, 19, 25\}$

b $\{4, 6, 16\}$

c $\{3, 11, 19, 25\}$

d $\{-4, -1, 0, 3, 4, 6, 11, 16, 19, 25\}$

e $\{-4, -1\}$

f $\left\{\frac{1}{2}\right\}$

g $\{4, 16, 25\}$

h $\{3, 11, 19\}$

i $\{-4, -1, 0, \frac{1}{2}, 0.75, 6\}$

2 **a** $\{109, 111, 113, 115\}$

b Various, e.g. $\{2010, 2012, 2014, 2016\}$ or $\{2020, 2022, 2024, 2026\}$ etc.

c $\{995, 997, 999, 1001, 1003, 1005\}$

d $\{1, 4, 9, 16, 25\}$

e Various, e.g. $\{0.49, 048, 0.47, 0.46, 0.45\}$ or $\{0.4, 0.3, 0.2, 0.1\}$

f Various, e.g. $\frac{1}{3}, \frac{3}{5}, \frac{7}{12}, \frac{2}{3}, \frac{11}{20}, \frac{13}{20}, \frac{7}{10}$

3 **a** Even

b Even

c Odd

d Odd

e Even

f Even

4 **a** A perfect number is one where the sum of its factors, including 1, but excluding the number itself, is that number. 6 is perfect number because $1 + 2 + 3 = 6$.

b A palindromic number is a 'symmetrical' number like 16461 that remains the same when its digits are reversed.

c A narcissistic number is one that is the sum of its own digits each raised to the power of the number of digits, e.g. $371 = 3^3 + 7^3 + 1^3$.

Exercise 1.2

1 a $19 < 45$
b $12 + 18 = 30$
c $0.5 = \frac{1}{2}$
d $0.8 \neq 8.0$
e $-34 < 2 \times -16$
f $\therefore x = \sqrt{72}$
g $x \leqslant -45$
h π is approximately equal to 3.14
i $5.1 > 5.01$
j $3 + 4 \neq 3 \times 4$
k $12 - (-12) > 12$
l $(-12) + (-24) < 0$
m $12x$ is approximately equal to -40

2 a False
b True
c True
d True
e True
f True
g False
h True
i True
j True
k False
l False
m True
n False

3 Students' own discussions.

Exercise 1.3

1 a 2, 4, 6, 8, 10
b 3, 6, 9, 12, 15
c 5, 10, 15, 20, 25
d 8, 16, 24, 32, 40
e 9, 18, 27, 36, 45
f 10, 20, 30, 40, 50
g 12, 24, 36, 48, 60
h 100, 200, 300, 400, 500

2 a 29, 58, 87, 116, 145, 174, 203, 232, 261, 290
b 44, 88, 132, 176, 220, 264, 308, 352, 396, 440
c 75, 150, 225, 300, 375, 450, 525, 600, 675, 750
d 114, 228, 342, 456, 570, 684, 798, 912, 1026, 1140
e 299, 598, 897, 1196, 1495, 1794, 2093, 2392, 2691, 2990
f 350, 700, 1050, 1400, 1750, 2100, 2450, 2800, 3150, 3500
g 1012, 2024, 3036, 4048, 5060, 6072, 7084, 8096, 9108, 10 120
h 9123, 18 246, 27 369, 36 492, 45 615, 54 738, 63 861, 72 984, 82 107, 91 230

3 a 32, 36, 40, 44, 48, 52
b 50, 100, 150, 200, 250, 300, 350
c 4100, 4200, 4300, 4400, 4500, 4600, 4700, 4800, 4900

4 576, 396, 792, 1164

5 816 and 1116

Exercise 1.4

1 a 10
b 40
c 12
d 9
e 385
f 66

2 No – the common multiples are infinite.

Exercise 1.5

1 a $F_4 = 1, 2, 4$
b $F_5 = 1, 5$
c $F_8 = 1, 2, 4, 8$
d $F_{11} = 1, 11$
e $F_{18} = 1, 2, 3, 6, 9, 18$
f $F_{12} = 1, 2, 3, 4, 6, 12$
g $F_{35} = 1, 5, 7, 35$
h $F_{40} = 1, 2, 4, 5, 8, 10, 20, 40$
i $F_{57} = 1, 3, 19, 57$
j $F_{90} = 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90$
k $F_{100} = 1, 2, 4, 5, 10, 20, 25, 50, 100$
l $F_{132} = 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132$
m $F_{160} = 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160$
n $F_{153} = 1, 3, 9, 17, 51, 153$
o $F_{360} = 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360$

2 a 4
  b 45
  c 14
  d 22
  e 8

3 a false
  b true
  c true
  d true
  e true
  f true
  g true
  h false

4 The smallest factor is 1 and the largest factor is the number itself.

Exercise 1.6

1 a 3
  b 8
  c 5
  d 14
  e 4
  f 2
  g 22
  h 6

2 a Any two from: 4, 6, 10, 14
  b 12 and 18 are the only possible two, less than 20

3 1 because each prime number has only 1 and itself as factors.

4 18 m

5 20 students

6 150 bracelets

Why do mathematicians find prime numbers exciting?

1 a Every even integer greater than 2 can be written as the sum of two prime numbers.
  b The weak conjecture is that every odd integer greater than 5 can be written as the sum of three odd prime numbers. Harald Helfgott's proof uses complicated mathematics to prove that this is correct. His proof is largely accepted by the mathematics community but they also acknowledge (as does he) that the strong conjecture is much more difficult to prove and that the method used to prove the weak conjecture won't work for the strong one.

2 a The prime number theorem shows that prime numbers become less common as they get bigger using the rate at which prime numbers occur.
  b Yes. Euclid (325–265BCE) proved there are infinitely many prime numbers. This proof is known as Euclid's theorem.

3 If you write prime backwards you get emirp. An emirp is a prime number that when you write it backwards gives you a different prime. For example, 17 and 71. The first few emirps are: 13, 17, 31, 37, 71, 73, 79, 97, 107, 113, 149, 157.

Exercise 1.7

1 2

2 14

3 a 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28
  b $6 = 3 + 3$, $8 = 3 + 5$,
  $9 = 2 + 7$, $10 = 5 + 5$,
  $12 = 5 + 7$, $14 = 3 + 11$,
  $15 = 2 + 13$, $16 = 5 + 11$,
  $18 = 5 + 13$, $20 = 3 + 17$,
  $21 = 2 + 19$, $22 = 5 + 17$,
  $24 = 5 + 19$ or $17 + 7$, $25 = 2 + 23$,
  $26 = 3 + 23$ or $13 + 13$, $27 =$ not possible,
  $28 = 5 + 23$

4 3 and 5, 5 and 7, 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73

5 149 is prime. Determined by trial division by all integers from 2 to $\sqrt{149}$

Exercise 1.8

1 a $30 = 2 \times 3 \times 5$
  b $24 = 2 \times 2 \times 2 \times 3$
  c $100 = 2 \times 2 \times 5 \times 5$
  d $225 = 3 \times 3 \times 5 \times 5$
  e $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
  f $504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7$
  g $650 = 2 \times 5 \times 5 \times 13$
  h $1125 = 3 \times 3 \times 5 \times 5 \times 5$
  i $756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7$
  j $9240 = 2 \times 2 \times 2 \times 3 \times 5 \times 7 \times 11$

Exercise 1.9

1 a 12
b 24
c 18
d 26
e 25
f 22
g 78
h 5

2 a 540
b 216
c 360
d 240
e 360
f 2850
g 270
h 360

3 a HCF = 36
LCM = 216
b HCF = 25
LCM = 200
c HCF = 5
LCM = 2280
d HCF = 12
LCM = 420

4 120 listeners

5 36 minutes

6 a 8
b 16
c 2^n

Exercise 1.10

1 a +\$100
b −25 km
c −10 marks
d +2 kg
e −1.5 kg
f 8000 m
g −10 °C
h −24 m
i −\$2000
j +\$250
k −2 h
l +400 m

Exercise 1.11

1 a $2 < 8$
b $4 < 9$
c $12 > 3$
d $6 > -4$
e $-7 < 4$
f $-2 < 4$
g $-2 > -11$
h $-12 > -20$
i $-8 < 0$
j $-2 < 2$
k $-12 < -4$
l $-32 < -3$
m $0 > -3$
n $-3 < 11$
o $12 > -89$
p $-3 < 0$

2 a −12, −8, −1, 7, 10
b −10, −8, −4, −3, 4, 9
c −12, −11, −7, −5, 0, 7
d −94, −90, −83, −50, 0

3 a −4
b 10
c −14
d −3
e −2.7
f 5
g −6
h −6
i −27
j −4
k −4
l −5

4 a 1°C
b 1°C
c −3 °C
d 12 °C
e −3 °C

5 \$28.50

6 a −\$420
b \$920
c −\$220

7 −11 m

8 −8°C

9
- **a** 8 p.m.
- **b** 12 p.m.
- **c** 10 p.m.
- **d** 1 a.m.

10
- **a** 17.1 litres per day
- **b** 578 litres

Exercise 1.12

1
- **a** 9
- **b** 49
- **c** 121
- **d** 144
- **e** 10 000
- **f** 196
- **g** 1
- **h** 27
- **i** 64
- **j** 1000

2
- **a** 441
- **b** 361
- **c** 1024
- **d** 4624
- **e** 216
- **f** 729
- **g** 1 000 000
- **h** 5832
- **i** 27 000
- **j** 8 000 000

3
- **a** $x = 5$
- **b** $x = 2$
- **c** $x = 11$
- **d** $x = 9$
- **e** $x = 18$
- **f** $x = 20$
- **g** $x = 20$
- **h** $x = 15$
- **i** $x = 1$
- **j** $x = 81$
- **k** $x = 1$
- **l** $x = 6561$
- **m** $x = 8$
- **n** $x = 1$
- **o** $x = 4$

4
- **a** 3
- **b** 8
- **c** 1
- **d** 2
- **e** 10
- **f** 0
- **g** 9
- **h** 20
- **i** 36
- **j** 42
- **k** 2
- **l** 1
- **m** −3
- **n** 4
- **o** 10
- **p** −6
- **q** 8
- **r** 9
- **s** −12
- **t** 18

5 **a**
$$\begin{aligned} 324 &= \underbrace{2\times 2}\times\underbrace{3\times 3}\times\underbrace{3\times 3}\\ \sqrt{324} &= 2\times 3\times 3\\ \sqrt{324} &= 18 \end{aligned}$$

b
$$\begin{aligned} 225 &= 3\times 3\times 5\times 5\\ \sqrt{225} &= 3\times 5\\ \sqrt{225} &= 15 \end{aligned}$$

c
$$\begin{aligned} 784 &= \underbrace{2\times 2}\times\underbrace{2\times 2}\times\underbrace{7\times 7}\\ \sqrt{784} &= 2\times 2\times 7\\ \sqrt{784} &= 28 \end{aligned}$$

d
$$\begin{aligned} 2025 &= \underbrace{3\times 3}\times\underbrace{3\times 3}\times\underbrace{5\times 5}\\ \sqrt{2025} &= 3\times 3\times 5\\ \sqrt{2025} &= 45 \end{aligned}$$

e
$$\begin{aligned} 19\,600 &= \underbrace{2\times 2}\times\underbrace{2\times 2}\times\underbrace{5\times 5}\times 7\times 7\\ \sqrt{19\,600} &= 2\times 2\times 5\times 7\\ \sqrt{19\,600} &= 140 \end{aligned}$$

f
$$\begin{aligned} 250\,000 &= \underbrace{2\times 2}\times\underbrace{2\times 2}\times\underbrace{5\times 5}\times\underbrace{5\times 5}\times\underbrace{5\times 5}\\ \sqrt{250\,000} &= 2\times 2\times 5\times 5\times 5\\ \sqrt{250\,000} &= 500 \end{aligned}$$

6 **a**
$$\begin{aligned} 27 &= \underbrace{3\times 3\times 3}\\ \sqrt[3]{27} &= 3 \end{aligned}$$

b
$$\begin{aligned} 729 &= \underbrace{3\times 3\times 3}\times\underbrace{3\times 3\times 3}\\ \sqrt[3]{729} &= 3\times 3\\ \sqrt[3]{729} &= 9 \end{aligned}$$

c
$$\begin{aligned} 2197 &= \underbrace{13\times 13\times 13}\\ \sqrt[3]{2179} &= 13 \end{aligned}$$

d
$$\begin{aligned} 1000 &= \underbrace{2\times 2\times 2}\times\underbrace{5\times 5\times 5}\\ \sqrt[3]{1000} &= 2\times 5\\ \sqrt[3]{1000} &= 10 \end{aligned}$$

e
$$\begin{aligned} 15\,625 &= \underbrace{5\times 5\times 5}\times\underbrace{5\times 5\times 5}\\ \sqrt[3]{15\,625} &= 5\times 5\\ \sqrt[3]{15\,625} &= 25 \end{aligned}$$

f
$$\begin{aligned} 32\,768 &= \underbrace{2\times 2\times 2}\times\underbrace{2\times 2\times 2}\times\underbrace{2\times 2\times 2}\times\underbrace{2\times 2\times 2}\times\underbrace{2\times 2\times 2}\\ \sqrt[3]{32\,768} &= 2\times 2\times 2\times 2\times 2\\ \sqrt[3]{32\,768} &= 32 \end{aligned}$$

7 **a** 25
b 49
c 64
d 32
e 7
f 5
g 14
h 10
i 8
j 4
k 10
l 10
m 6
n 6
o 3
p $\frac{3}{2}$

8 **a** 10 cm
b 27 cm
c 41 mm
d 40 cm

9 **a** 31
b 17
c 65
d 17
e 68
f 24
g 730
h 82
i 33
j 129

Exercise 1.13

1 **a** 128
b 486
c 85
d 96
e 320
f 512

2 **a** $2^4 \times 3^4$ is greater by 1040
b $\sqrt[4]{625} \times 3^6$ is greater by 2877

3 **a** 2^6
b 3^5
c $2^4 \times 5^2$
d $2^6 \times 5^2$
e 2^{14}
f $2^8 \times 3^4$
g 3^{10}
h 5^8

4 $25 = 5^2$
$36 = 2^2 \times 3^2$
$64 = 2^6$
The index is always even.

Exercise 1.14

1 **a** True
b False: 3^6
c True
d False: 8^6
e True
f True
g True
h False: 10^5
i False: 5^{-8}
j False: -2^8
k True
l False: -1

2 **a** 10^7
b 3^5
c 2^5
d 10^{-3}
e 10
f 12^0
g 3^{-7}
h 4^{-7}
i 3^{12}
j 5^{-4}
k 4^{-6}
l 4^0

3 **a** $\frac{5}{6}$ or 0.833
b $\frac{1}{36}$ or 0.0278
c $\frac{1}{2}$ or 0.5
d $\frac{1}{12}$ or 0.0833

4 **a** $\frac{1}{3}$
b $\frac{1}{4}$
c $\frac{1}{2}$
d $\frac{1}{16}$
e $\frac{1}{16}$

5 **a** 4^{-1}
b 5^{-1}
c 7^{-1}
d 9^{-1}
e $10\,000^{-1}$
f 256^{-1}
g 49^{-1}
h 18^{-1}

6 **a** 5.0625
b 1000
c 2.25
d 0.015 625
e 36
f 8
g 13
h 17

7 **a** 3^1
b 3^2
c 3^6
d 3^{-3}
e 3^{-1}
f 3^0
g 3^{-5}
h $-(3^2)$

Exercise 1.15

1 **a** $\sqrt{25}$
b $\sqrt[3]{3}$
c $\sqrt{40}$
d $\sqrt{6}$
e $\sqrt[8]{3}$
f $\left(\sqrt[4]{2}\right)^3$
g $\left(\sqrt[3]{12}\right)^2$
h $\left(\sqrt[9]{5}\right)^2$

2 **a** $5^{\frac{1}{2}}$
b $8^{\frac{1}{3}}$
c $13^{\frac{1}{3}}$
d $11^{\frac{1}{4}}$
e $9^{\frac{2}{3}}$
f $6^{\frac{4}{3}}$
g $32^{\frac{3}{4}}$
h $2\left(12^{\frac{7}{5}}\right)$

3 **a** 5
b 3
c 4
d 8
e 36
f 0.5
g 6.78
h 0.0016
i 0.5
j 16
k 36
l 64

4 **a** $c = 70 \times \left(\sqrt[4]{m}\right)^3$
b 251.40 calories
c 41 622.25 calories

Exercise 1.16

1 **a** $(4 + 7) \times 3$
$= 11 \times 3$
$= 33$
b $(20 - 4) \div 4$
$= 16 \div 4$
$= 4$
c $50 \div (20 + 5)$
$= 50 \div 25$
$= 2$
d $6 \times (2 + 9)$
$= 6 \times 11$
$= 66$
e $(4 + 7) \times 4$
$= 11 \times 4$
$= 44$
f $(100 - 40) \times 3$
$= 60 \times 3$
$= 180$

g $16 + (25 \div 5)$
$= 16 + 5$
$= 21$

h $19 - (12 + 2)$
$= 19 - 14$
$= 5$

i $40 \div (12 - 4)$
$= 40 \div 8$
$= 5$

j $100 \div (4 + 16)$
$= 100 \div 20$
$= 5$

k $121 \div (33 \div 3)$
$= 121 \div 11$
$= 11$

l $15 \times (15 - 15)$
$= 15 \times 0$
$= 0$

2 **a** 108
b 72
c 3
d 10
e 32
f 9
g 5
h 1
i 140

3 **a** $5 \times 10 + 3$
$= 50 + 3$
$= 53$

b $5 \times (10 + 3)$
$= 5 \times 13$
$= 65$

c $2 + 10 \times 3$
$= 2 + 30$
$= 32$

d $(2 + 10) \times 3$
$= 12 \times 3$
$= 36$

e $23 + 7 \times 2$
$= 23 + 14$
$= 37$

f $6 \times 2 \div (3 + 3)$
$= 12 \div 6$
$= 2$

g $\frac{15 - 5}{2 \times 5}$
$= \frac{10}{10}$
$= 1$

h $(17 + 1) \div 9 + 2$
$= 18 \div 9 + 2$
$= 2 + 2$
$= 4$

i $\frac{16 - 4}{4 - 1}$
$= \frac{12}{3}$
$= 4$

j $17 + 3 \times 21$
$= 63 + 17$
$= 80$

k $48 - (2 + 3) \times 2$
$= 48 - 5 \times 2$
$= 48 - 10$
$= 38$

l $12 \times 4 - 4 \times 8$
$= 48 - 32$
$= 16$

m $15 + 30 \div 3 + 6$
$= 15 + 10 + 6$
$= 31$

n $20 - 6 \div 3 + 3$
$= 20 - 2 + 3$
$= 21$

o $10 - 4 \times 2 \div 2$
$= 10 - 4 \div 1$
$= 10 - 4$
$= 6$

4 **a** 7
b 7
c 3
d 0
e 3
f 10

5
- **a** 13
- **b** 8
- **c** 58
- **d** 192
- **e** 12 000
- **f** 1660
- **g** 260
- **h** 868

6
- **a** 18
- **b** 3
- **c** 3
- **d** 8
- **e** 4
- **f** 9

7
- **a** False
- **b** True
- **c** False
- **d** True

8
- **a** $3 \times (4 + 6) = 30$
- **b** $(25 - 15) \times 9 = 90$
- **c** $(40 - 10) \times 3 = 90$
- **d** $(14 - 9) \times 2 = 10$
- **e** $(12 + 3) \div 5 = 3$
- **f** $(19 - 9) \times 15 = 150$
- **g** $(10 + 10) \div (6 - 2) = 5$
- **h** $(3 + 8) \times (15 - 9) = 66$
- **i** $(9 - 4) \times (7 + 2) = 45$
- **j** $(10 - 4) \times 5 = 30$
- **k** $6 \div (3 + 3) \times 5 = 5$
- **l** BODMAS means that brackets are not needed.
- **m** $(1 + 4) \times (20 \div 5) = 20$
- **n** $(8 + 5 - 3) \times 2 = 20$
- **o** $36 \div (3 \times 3 - 3) = 6$
- **p** $3 \times (4 - 2) \div 6 = 1$
- **q** $(40 \div 4) + 1 = 11$
- **r** BODMAS means that brackets are not needed.

9
- **a** $2 - 10 \div 5 = 0$
- **b** $13 - 18 \div 9 = 11$
- **c** $8 \div (16 - 14) - 3 = 1$
- **d** $(9 + 5) - (6 - 4) = 12$
  or $(9 + 5) - (12 - 4) = 6$

Exercise 1.17

1
- **a** -10
- **b** 8.86
- **c** 13
- **d** 29
- **e** -22
- **f** 8.75
- **g** 20
- **h** 0
- **i** 4
- **j** 70
- **k** 12
- **l** 20
- **m** 8
- **n** 15
- **o** 20

2
- **a** Correct
- **b** Incorrect = 608
- **c** Correct
- **d** Correct
- **e** Incorrect = 368
- **f** Incorrect = 10

3
- **a** $12 \div (28 - 24) = 3$
- **b** $84 - 10 \times 8 = 4$
- **c** $3 + 7(0.7 + 1.3) = 17$
- **d** $23 \times 11 - 22 \times 11 = 11$
- **e** $40 \div 5 \div (7 - 5) = 4$
- **f** $9 + 15 \div (3 + 2) = 12$

4
- **a** 0.5
- **b** 2
- **c** 0.183
- **d** 0.5
- **e** $\frac{1}{3}$ is approximately equal to 0.333 (3 s.f.)
- **f** 1
- **g** 2
- **h** $\frac{2}{3}$ is approximately equal to 0.667 (3 s.f.)

5 Correct to 3 significant figures
- **a** 0.0112
- **b** 0.0950
- **c** -0.317

6 Correct to 3 significant figures
- a 89.4
- b 20.8
- c 7.52
- d 19.6
- e 2.94
- f 1.45
- g 0.25
- h 2.16

7
- a 1
- b 0.5
- c −26.94
- d 0.28
- e 14.5
- f 6.54
- g 1728.69
- h −1999
- i 0.339

You may find that your calculator gives an exact answer rather than a decimal. This may include a root or a fraction. Check your calculator manual to find out how to change this to a decimal.

Exercise 1.18

1
- a 3.19
- b 0.06
- c 38.35
- d 2.15
- e 1.00

2
- a 500
- b 53 400
- c 3000
- d 0
- e 10 100

3
- a 630 000
- b 100 000
- c 10 000
- d 10 000
- e 160 000

4
- a i 4512
  - ii 4510
  - iii 5000
- b i 12 310
  - ii 12 300
  - iii 10 000
- c i 65 240
  - ii 65 200
  - iii 70 000
- d i 320.6
  - ii 321
  - iii 300
- e i 25.72
  - ii 25.7
  - iii 30
- f i 0.0007650
  - ii 0.000765
  - iii 0.0008
- g i 1.009
  - ii 1.01
  - iii 1
- h i 7.349
  - ii 7.35
  - iii 7
- i i 0.009980
  - ii 0.00998
  - iii 0.01
- j i 0.02814
  - ii 0.0281
  - iii 0.03
- k i 31.01
  - ii 31.0
  - iii 30
- l i 0.006474
  - ii 0.00647
  - iii 0.006

5
- a 2.556
- b 2.56
- c 2.6
- d 2.56
- e 2.6
- f 3

Exercise 1.19

1
- a $\frac{49}{10} = 4.9$, which is close to 5, so not sensible
- b $4 \times 3 \times 9 = 108$, so not sensible
- c $5 \times 8 = 40$, so not sensible
- d $50 \times 8 = 400$, so sensible
- e $3 \times 300 = 900$, so not sensible

f $6 \times \sqrt{20} = 6 \times 4.5$ (approximate root between $\sqrt{16}$ and $\sqrt{25}$) $= 27$, so sensible

2 a $\frac{23.6}{6.3}$ is approximately equal to $\frac{24}{6} = 4$

b $\frac{4}{0.09 \times 4}$ is approximately equal to $\frac{4}{0.36}$ is approximately equal to 11

c $\frac{7 \times 0.5}{9}$ is approximately equal to $\frac{3.5}{9}$ is approximately equal to 0.39

d $\frac{5 \times 6}{2.5 + 1}$ is approximately equal to $\frac{30}{3.5}$ is approximately equal to 8.6

e $\frac{\sqrt{49}}{2.5 + 4}$ is approximately equal to $\frac{7}{6.5}$ is approximately equal to 1

f $(0.5 + 2)(6.5 - 2)$ is approximately equal to $(2.5)(4.5)$ is approximately equal to 11.3

g $\frac{24 + 20}{5 + 6}$ is approximately equal to $\frac{44}{11} = 4$

h $\frac{110 - 45}{19 - 14}$ is approximately equal to $\frac{65}{5} = 13$

i $3^2 \times \sqrt{49}$ is approximately equal to $9 \times 7 = 63$

j $\sqrt{224 \times 45}$ is approximately equal to $\sqrt{10\,080}$ is approximately equal to 100

k $\sqrt{9} \times \sqrt{100}$ is approximately equal to $3 \times 10 = 30$

l $4^3 \times 2^4$ is approximately equal to $64 \times 16 = 1024$

3 Answers given to 1 d.p.

a 3.7
b 12.7
c 0.4
d 8.0
e 1.0
f 10.8
g 4.2
h 11.7
i 44.4
j 100.5
k 30.4
l 898.2

Making decisions about accuracy

1 a Whole numbers
b 2 d.p.
c Millions
d 4 d.p.
e 3 s.f.

2 a Zaf changed decimals to fractions to easily divide by 2. Marwan cancelled before rounding to have fewer numbers to deal with.

b Once you have rounded, you are calculating exact values, so even if 2 and 3 are rounded values, $2 + 3$ is equal to 5, not approximately equal to 5.

3 Possible examples:

a Overestimate the cost of buying several items to make sure you definitely have enough money

b Underestimate the size of a doorway to make sure you have enough room to move furniture though it.

Practice questions

1 $49 - 30 = 19$

2 9 and −4 or −9 and 4

3 15

4 216 216

5 735
736
737
738
739
741
742
743
744

6 $1080 = 2^3 \times 3^3 \times 5$

1080 is not a cube number. Not all the factors are powers with indices that are multiples of 3.

7 a 33 and 61
b 26 and 45

8 a 32
b 340
c 25

d 33

e 9

f −48

9 **a** $5 \times 7 - 3 \times 8 = 11$

b $(5 - 3^2) \times 6 + 8 \div (-2) = -28$

10 $(7 + 14) \div (4 - 1) \times 2 = 14$

11 1.16

12 **a** −4

b 0.276 to 3 s.f.

13 D, C, B, A

14 **a** $\sqrt{338}$

b $\frac{17}{2}$

c 5

d $\frac{216}{125}$

e $\frac{3}{2}$

15 **a** $60 = 2^2 \times 3 \times 5$

$36 = 2^2 \times 3^2$

b $\text{LCM} = 2^2 \times 3^2 \times 5$

$= 180$

c 28 August 2023

16 BAD

Practice questions worked solutions

1 The prime numbers smaller than 20 are: 2, 3, 5, 7, 11, 13, 17, 19

Sum of the three largest prime numbers smaller than 20
$= 13 + 17 + 19$
$= 49$

Product of the three smallest prime numbers
$= 2 \times 3 \times 5$
$= 30$

Difference $= 49 - 30$
$= 19$

2 Product = −36, which is negative ⇒ one number is positive and the other is negative.

Factor pairs of 36: 1×36
2×18
3×12
4×9
6×6

You can make a difference of 13 with either 9 and −4 or −9 and 4.

3 The number if one fifteenth of its own square.

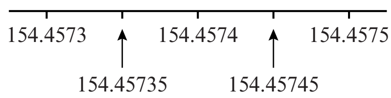
⇒ The number must be multiplied by 15 to square it.

⇒ The number is 15.

4 $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 11 \times 13$

$= 216\,216$

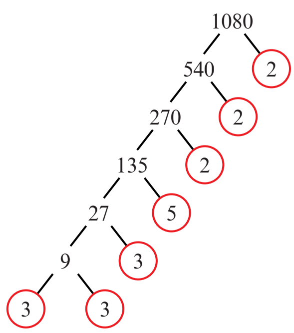
5 Look at 154.4574 on a number line and you will see that the number 154.45ABC must lie between 154.45735 and 154.45744. (154.45745 rounds up to 154.45735.)



So, A must be 7. The possibilities are:

A	B	C
7	3	5
7	3	6
7	3	7
7	3	8
7	3	9
7	4	1
7	4	2
7	4	3
7	4	4

6



$1080 = 2^3 \times 3^3 \times 5$

The power of 5 is not a multiple of 3 so 1080 is **not** a cube number.

7 **a** $2013 = 3 \times 11 \times 61$
$= 33 \times 61$

$33 + 61 = 94$ so the numbers are 33 and 61.

b $1170 = 2 \times 3 \times 3 \times 5 \times 13$
$= (3 \times 3 \times 5) \times (2 \times 13)$
$= 45 \times 26$

$45 - 26 = 19$ so the numbers are 45 and 26.

8 **a** $12 + 20 = 32$

b $4 \times 85 = 340$

c $11 \times 2 + (15 - 6) - 6$
$= 22 + 9 - 6$
$= 25$

d $-15 - (-48)$
$= -15 + 48$
$= 33$

e $-3 \times (-11) + (-24)$
$= 33 - 24$
$= 9$

f $(-4)^3 + 16$
$= -64 + 16$
$= -48$

9 **a** $5 + 7 - 3 - 8 = 1$

b $(5 - 3^2) \times 6 + 8 \div (-2)$
$= -4 \times 6 + (-4)$
$= -24 - 4$
$= -28$

10 $(7 + 14) \div (4 - 1) \times 2 = 14$

11 1.16 (to 3 s.f.)

12 **a** is approximately equal to

$$\frac{5 - 5^2}{\sqrt{25}} = \frac{5 - 25}{5} = -\frac{20}{5} = -4$$

b Calculator answer $= -4.276\,348\,739\ldots$

Difference $= 0.276\,348\,739\ldots$
$= 0.276$ (to 3 s.f.)

13 $A = 4 \times (4 + 16) = 4 \times 20 = 80$

$B = \frac{64}{16} + 4 = 4 + 4 = 8$

$C = \frac{16 - 4}{4} = 3$

$D = 16 - 16 \times 4 + 1$
$= 16 - 64 + 1$
$= -47$

The order is D, C, B, A.

14 **a** $\sqrt{98} + \sqrt{72} = \sqrt{49 \times 2} + \sqrt{36 \times 2}$
$= 7\sqrt{2} + 6\sqrt{2}$
$= 13\sqrt{2}$

b $(3^{-2} + 2^{-3}) \times 216^{\frac{2}{3}}$

$= \left(\frac{1}{3^2} + \frac{1}{2^3}\right) \times \left(\sqrt[3]{216}\right)^2$

$= \left(\frac{1}{9} + \frac{1}{8}\right) \times 6^2$

$= \frac{17}{72} \times 36 = \frac{17}{2}$

c $\left(\left(\sqrt{2}\right)^2 + 23\right)^{\frac{1}{2}}$

$= \sqrt{(2 + 23)}$

$= \sqrt{25}$

$= 5$

d $\left(\frac{36}{25}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{36}{25}}\right)^3$

$= \left(\frac{6}{5}\right)^3$

$= \frac{216}{125}$

e $\left(\frac{16}{81}\right)^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{\frac{16}{81}}}$

$= \frac{1}{\left(\frac{2}{3}\right)}$

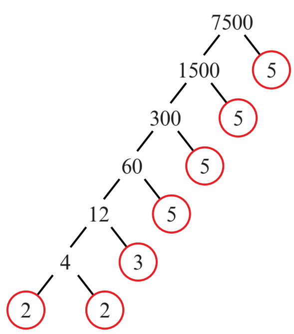
$= \frac{3}{2}$

15 **a** $60 = 2 \times 2 \times 3 \times 5$
$36 = 2 \times 2 \times 3 \times 3$

b $2 \times 2 \times 3 \times 3 \times 5 = 180$

c 180 days after 1 March 2023 is 28 August 2023.

16



$7500 = 2^2 \times 3^1 \times 5^4$
$= \text{B A D}$

Chapter 2

Getting started

1 a C
  b A
  c A

2 Possible answers are:
  a a^0
  b $\frac{a^m}{a^n}$
  c $a^m \times a^n$
  d $(a^m)^n$

3 Example 1:
  a Sign error
  b $3x - x + 2$

  Example 2:
  a Multiplied both numerator and denominator by 3 instead of just the numerator. $\left(3 = \frac{3}{1} \text{ and not } \frac{3}{3}\right)$
  b $\frac{3x + 12}{5}$

  Example 3:
  a Cancelled part of a term, but both x and 2 need to be divided by 2.
  b $\frac{x + 2}{2}$ cannot be simplified further, but can be written as $\frac{x}{2} + 1$.

4 a There are different options, But in general, if you let one number be x, the consecutive number is $x + 1$. The sum of the numbers is $x + x + 1 = 2x + 1$. Any multiple of 2 is even, so if you add 1, it will be odd.
  b Using the same argument $x + x + 1 + x + 2 = 3x + 3$. $3x$ can be odd or even (depending on the value of x) so the answer can be either odd or even.

Exercise 2.1

1 a $6xy$
  b $7ab$
  c xyz
  d $2y^2$
  e $4ab$
  f $12xy$
  g $5ab$
  h yz^2
  i $\frac{6}{x}$
  j $\frac{4x}{2y} = \frac{2x}{y}$
  k $\frac{x + 3}{4}$
  l $\frac{m^3}{m^2} = m$
  m $4x + 5y$
  n $7a - 2b$
  o $2x(x - 4)$
  p $\frac{3(x + 1)}{2x}$
  q $\frac{2(x + 4)}{3}$
  r $\frac{4x}{6x} = \frac{2}{3}$

2 a $m + 13$
  b $m + 5$
  c $25 - m$
  d m^3
  e $\frac{m}{3} + 3$
  f $4m - 2m = 2m$

3 a $x + 3$
  b $x - 6$
  c $10x$
  d $-8 + x$
  e $x + x^2$
  f $x + 2x = 3x$
  g $\frac{2x}{x + 4}$

4 a $\$(x - 10)$
  b $\$\frac{x}{4}$
  c \$15

5 a $m + 10$ years
  b $m - 10$ years
  c $\frac{m}{2}$ years

6 **a** $\$\frac{p}{3}$

b $\$\frac{p}{5}$, $\$\frac{p}{5}$ and $\$\frac{3p}{5}$

Exercise 2.2

1 **a** 9
b 30
c 10
d 27
e 18
f 7
g 16
h 36
i 4
j 6
k 6
l 30
m 5
n 2
o $\frac{1}{2}$

2 **a** 30
b 45
c 16
d 5
e 13
f 16
g 31
h 450
i 24
j 8
k 24
l 5
m $\frac{26}{3}$
n 10
o 4
p 3
q 6
r 225
s 12
t −10
u 129 600

3 **a** **i** $y = 0$
ii $y = 12$
iii $y = 16$
iv $y = 40$
v $y = 200$

b **i** $y = 1$
ii $y = 10$
iii $y = 13$
iv $y = 31$
v $y = 151$

c **i** $y = 100$
ii $y = 97$
iii $y = 96$
iv $y = 90$
v $y = 50$

d **i** $y = 0$
ii $y = 1.5$
iii $y = 2$
iv $y = 5$
v $y = 25$

e **i** $y = 0$
ii $y = 9$
iii $y = 16$
iv $y = 100$
v $y = 2500$

f **i** undefined
ii $y = 33.3$ (3 s.f.)
iii $y = 25$
iv $y = 10$
v $y = 2$

g **i** $y = 4$
ii $y = 10$
iii $y = 12$
iv $y = 24$
v $y = 104$

h **i** $y = -6$
ii $y = 0$
iii $y = 2$
iv $y = 14$
v $y = 94$

i **i** $y = 0$
ii $y = 81$
iii $y = 192$
iv $y = 3000$
v $y = 375\,000$

4 a $\$(3x + 2y)$
b i $18
ii $100
iii $350

5 a $P = 42$ cm
b $P = 8$ m
c $P = 60$ cm
d $P = 20$ cm

6 a i 43
ii 53
iii 71
iv 151
b They're all prime numbers.
c When $n = p$, $n^2 + n + p$ becomes $n(n + 2)$; in other words, it has factors n and $n + 2$, so is not prime.

What is the point of algebra?

Students' own work. There are many accessible examples that students could use, for example, conversion formulae, using algebra to find break even points (finance and economics), calculating doses of medicine based on person's mass or other factors, working out trajectories in sports (such as basketball or snooker), using BMI and other factors to work out health and fitness and so on.

Exercise 2.3

1 a $6x, 4x, x$
b $-3y, \frac{3}{4}y, -5y$
c $ab, -4ba$
d $-2x, 3x$
e $5a, 6a$ and $5ab, ab$
f $-1xy, -yx$

2 a $8y$
b $7x$
c $13x$
d $22x$
e $5x$
f 0
g $-x$
h $-3y$
i $4x$
j $7xy$
k $4pq$
l $13xyz$
m $2x^2$
n $5y^2$
o $-y^2$
p $12ab^2$
q $5x^2y$
r $2xy^2$

3 a $5x + y$
b $4x + 2y$
c $7x$
d $4 + 4x$
e $6xy - 2y$
f $-x^2 + 2x$
g $-x + 4y$
h $3x + 3y$
i $8x + 6y$
j $8x - 2y$
k $14x^2 - 4x$
l $10x^2$
m $12xy - 2x$
n $8xy - 2xz$
o $-x^2 - 2y^2$
p $8x^2 y - 2xy$
q $6xy - x$
r $6xy - 2$

4 a $2y - 8$
b $4x^2 - 5x$
c $7x + 4y$
d $y^2 + 5y - 7$
e $x^2 - 5x + 3$
f $x^2 + 5x - 7$
g $3xyz - 3xy + 2xz$
h $8xy - 10$
i $-3x^2 + 6x - 4$

5 a $P = 8x$
b $P = 4x + 14$
c $P = 6x + 3$
d $P = 5x + 4$
e $P = 12y - 6$
f $P = 8y^2 + 2y + 14$
g $P = 12y - 4$
h $P = 18x - 1$

Magic squares

1 No, the sum of numbers in a 3 × 3 magic square is always 3 times the centre number. In this case, that would be $12a + 15y$ for each row, column and diagonal and they do not all sum to that.

2 For example:

$a - c$	$a + b + c$	$a - b$
$a - b + c$	a	$a + b - c$
$a + b$	$a - b - c$	$a + c$

3 a

$x - 1$	$x + 1$	x
$x + 1$	x	$x - 1$
x	$x - 1$	$x + 1$

x	$x + 1$	$x - 1$
$x - 1$	x	$x + 1$
$x + 1$	$x - 1$	x

x	$x - 1$	$x + 1$
$x + 1$	x	$x - 1$
$x - 1$	$x + 1$	x

$x + 1$	$x - 1$	x
$x - 1$	x	$x + 1$
x	$x + 1$	$x - 1$

b Yes. Student's own magic squares or explanation.

Exercise 2.4

1 a $12x$
b $8y$
c $12m$
d $6xy$
e $8xy$
f $27xy$
g $24yz$
h $12xy$
i $8x^2y^2$
j $8x^2y$
k $27xy^2$
l $24xy^2$
m $8a^2b$
n $12ab^2c$
o $12a^2bc$
p $16a^2b^2c$
q $24abc$
r $72x^2y^2$

2 **a** $24x$
b $30x^2y$
c $12x^2y^2$
d x^3yz
e $48x$
f $24x^3y$
g $4x^2y^2$
h $12a^2bc$
i $60xy$
j $8xy$
k $9x^3y$
l $8x^3y^3$
m $42x^2y^2z^2$
n $56x^3y^2$
o $36x^2y^2z$
p $18x^4y^4$
q $54x^4y$
r $6x^3y^3$

3 **a** $5r$
b $4r$
c $3r$
d $6s$
e $7r$
f $2s$
g $\frac{s}{4}$
h $\frac{1}{4s}$
i $\frac{t}{2}$
j $6s$
k $\frac{1}{4}$
l $\frac{1}{9}$

4 **a** $4x$
b $6y$
c $\frac{4x}{y}$
d 8
e $\frac{7x^2}{y^2}$
f $3x$
g $\frac{x}{3}$
h $\frac{1}{4y}$
i $7y$
j $\frac{9y}{4}$
k $4xy$
l $\frac{4y}{x}$

5 **a** $\frac{ab}{6}$
b $\frac{a^2}{12}$
c $\frac{5a^2b}{6}$
d $\frac{10a}{3b}$
e $\frac{3ab}{8}$
f $\frac{25a^2}{4}$
g 2
h $\frac{a^2}{3}$
i $2ab$
j $\frac{8a}{3}$
k $\frac{1}{4}$
l a^2

Exercise 2.5

1 **a** $2x + 12$
b $3x + 6$
c $8x + 12$
d $10x - 60$
e $4x - 8$
f $6x - 9$
g $5a + 20$
h $24 + 6a$
i $9a + 18$
j $14c - 14d$
k $6c - 4d$
l $4c + 16d$
m $10x - 10y$
n $18x - 12y$
o $12y - 6x$
p $4s - 16t^2$
q $9t^2 - 9s$
r $28t + 7t^2$

2 **a** $2x^2 + 2xy$
b $3xy - 3y^2$
c $2x^2 + 4xy$
d $12x^2 - 8xy$
e $x^2y - xy^2$
f $12xy + 6y$
g $18ab - 8ab^2$
h $6a^2 - 4a^2b$
i $12a^2 - 12a^3$
j $36a - 8ab$
k $10b - 5ab$
l $12a - 3ab$
m $2x^2y^2 - 4x^3y$
n $12xy^2 - 8x^2y^2$
o $3x^2y^2 + 3xy^3$
p $2x^3y + x^2y^2$
q $81x^2 - 18x^3$
r $12xy^2 - 4x^2y^2$

3 **a** $A = x^2 + 7x$
b $A = 2x^3 - 2x$
c $A = 4x^2 - 4x$

Maths jokes

Student's own discussions, but could include puns, play on words, misinterpretation of concepts.

You could extend this by asking students to develop their own funny maths memes to share with the class.

Exercise 2.6

1 **a** $10 + 5x$
b $7y - 6$
c $4x - 8$
d $6x - 6$
e $2t^2 + 8t - 5$
f $4x + 1$
g $3x$
h $8x + 6$
i $6x + 9$
j $3h + 2$
k $8d + 6$
l $3y + xy - 4$
m $2x^2 + 8x - 4$
n $-4y^2 + 4xy + 8y$
o $10s - 12s^2$
p $6x^2 + 12x - 9$
q $-y^2 + 6y$
r $6x - 6$

2 **a** $6x + 154$
b $4x + 2$
c $7x + 26$
d 92
e $2x^2 + 16$
f $6p^2 + 10px$
g $24pq + 4p$
h $2xy + 4x$
i $-3x - 18xy$
j $21x - 12y - 2xy$
k $22x^2 - 7x^3$
l $x^2 - xy + 6x - 3y$
m $16s - 3st - 8$
n $2x^2$
o $4x^2 + 8xy$
p $2x^2 - 3x + 15$
q $9k - 17$
r $7xy + 9x$

Exercise 2.7

1 **a** $-30p - 60$
b $-15x - 21$
c $-20y - 1$
d $-3q + 36$
e $-24t + 84$
f $-12z + 6$
g $-6x - 15y$
h $-24p - 30q$
i $-27h + 54k$
j $-10h - 10k + 16j$
k $-8a + 12b + 24c - 16d$
l $-6x^2 - 36y^2 + 12y^3$

2 **a** $-5x - 8$
b $-5x + 12$
c $10x - 38$
d $-13f$
e $-36g + 37$
f $12y - 20$
g $-26x^2 - 76x$
h $-x^2 + 77x$
i $-9x^2 + 30x$

j $24q$
k $-42pq + 84p$
l $-48m + 48n$

3 **a** $12x - 6$
b $13x - 6$
c $-2x + 17$
d $x + 13$
e $23 - 7x$
f $10x - 8$
g $7x - 5$
h $x^2 - 5x + 8$
i $3x^2 - 7x + 2$
j $2x^2 + 3x + 6$
k $2x - 18$
l $6x^2 + 6x - 6$

Exercise 2.8

1 **a** x^8
b a^{10}
c y^2
d x^{13}
e y^9
f y^7
g y^6
h t^5
i $6x^7$
j $9y^6$
k $2m^4$
l $6s^7$
m $15x^3$
n $8x^7$
o $8z^7$
p $4x^7$

2 **a** x^2
b g^9
c y
d k^2
e s^4
f x^2
g $3x^2$
h $3p^3$
i $4y$
j $\frac{x}{2}$
k 3
l $3b$
m $\frac{1}{3x}$
n $4ab$
o 1

3 **a** a^4
b y^6
c f^{12}
d y^6
e $32x^{10}$
f $9c^4d^4$
g 1
h $125x^6$
i a^6b^6
j $x^{10}y^{20}$
k x^3y^{12}
l $16g^2h^4$
m $81x^8$
n x^4y^{24}
o 1

4 **a** $12x^6$
b $24x^3y$
c $4k^4$
d $\frac{x^2}{4}$
e $44x^3a^4b^2$
f $4x^3 + 28x$
g $4x^3 - x^5$
h x^2
i $\frac{7}{x^4}$
j $2x^2$
k $\frac{a^{12}}{b^6}$
l $\frac{x^4y^8}{16}$
m 1
n $8x^5$
o $2xy^3$

Exercise 2.9

1 a true
  b false
  c false
  d false

2 a $\frac{1}{x^2}$
  b $\frac{1}{y^3}$
  c $\frac{1}{x^2y^2}$
  d $\frac{2}{x^2}$
  e $\frac{12}{x^3}$
  f $\frac{7}{y^3}$
  g $\frac{8x}{y^3}$
  h $\frac{12}{x^3y^4}$

3 a b
  b $\frac{6}{x^6}$
  c $\frac{1}{3s^4}$
  d $\frac{1}{h^{11}}$
  e $\frac{1}{8x^6}$
  f $\frac{1}{c^6}$
  g x
  h $\frac{1}{x^5}$
  i $\frac{a^7}{b^5}$
  j $\frac{x^{10}}{y^{12}}$
  k $\frac{2y^{11}}{x^4}$
  l 1
  m $\frac{12n^{15}}{m^3}$

4 a $x = 4$
  b $x = 5$
  c $x = 2$
  d $x = -3$
  e $x = 3$
  f $x = 3$
  g $x = 2$
  h $x = 4$

Exercise 2.10

1 a $x^{\frac{2}{3}}$
  b $x^{\frac{7}{6}}$
  c $\frac{1}{y^3}$
  d $\frac{x^3}{y}$
  e $a^{\frac{4}{7}}$
  f $\frac{7b^2}{4}$
  g $\frac{2}{x^2}$
  h $\frac{3}{4k}$
  i $3x^{\frac{7}{4}}$
  j $\frac{1}{4x^{\frac{3}{2}}}$
  k $\frac{s}{4}$
  l $\frac{3x^{\frac{3}{4}}}{2}$
  m $\frac{x}{8}$
  n $\frac{1}{4x^{\frac{3}{2}}}$
  o $x^{\frac{13}{12}}$
  p $\frac{x^{\frac{1}{6}}}{y^{\frac{7}{6}}}$

2 a $x = 6$
  b $x = \frac{1}{2}$
  c $x = 16\,807$
  d $x = 257$
  e $x = 4$
  f $x = 4$
  g $x = 6$
  h $x = 5$
  i $x = 2$
  j $x = -4$
  k $x = \frac{1}{6}$
  l $x = \frac{3}{4}$

Practice questions

1 **a** $n + 12$
b $2n - 4$
c $(nx)^2$
d $(n^2)^3$ or $(n^3)^2$ or n^6

2 **a** Any of $2n$, $4n$, $6n$,…
b $2n$ is always even because it is a multiple of 2. Every even number is 'next to' an odd number and $2n + 1$ is 'next to' $2n$.
c $p = 2n + 3$
d $2n + 1 + 2n + 3 = 4n + 4$ = even + even = even

3 **a** $p + 2q + \mathrm{r}$
b $8q$, $3r + 4q$
c Top brick = $2h + 2j + 2k$ = even + even + even = even

4 **a** $15xy + x$
b $5xy + 3y$

5 **a** a^2b
b $2x^6$
c $6x^4y^2$
d 1
e $4x^5y^3$
f $15x^2$
g x^3
h $16x^{-10}$ or $\dfrac{16}{x^{10}}$
i $\dfrac{27x^3}{64y^3}$
j $\dfrac{x^{19}}{3y^{12}}$
k $\dfrac{7p^6}{6q^{20}}$

6 0

7 **a** 10
b 10
c 10

8 7.35

9 **a** $8x - 4$
b $x^2 + 37xy$

10 **a** $m^2 - n^2$
b 0

11 **a** x^3
b $\dfrac{4}{x^2}$
c $\dfrac{1}{(2x-2)^3} = \dfrac{1}{8x^3 - 24x^2 + 24x - 8}$

12 **a** 3
b 3
c 3

13 18

14 **a** $15x$
b $9y^3$
c $4x$

15 **a** -3
b -3
c $\dfrac{1}{3}$
d $-\dfrac{1}{3}$

16 **a** pq
b $\dfrac{pq}{4}$
c p^3

17 **a** 4
b $\dfrac{15}{8}$

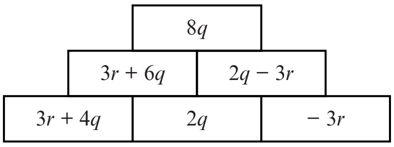
Practice questions worked solutions

1 **a** $n + 12$
b $2n - 4$
c $(nx)^3$
d $(n^2)^3$

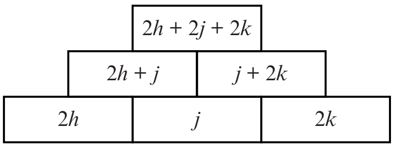
2 **a** $2n$ (or $2n$ + any even number)
b $2n$ is always even and $2n + 1$ is one more than an even number. Whole numbers alternate
odd even odd even …
so $2n + 1$ must be odd.
c $p + 2 = 2n + 1 + 2$
$= 2n + 3$
d $(2n + 1) + (2n + 3)$
$= 4n + 4$
$= 2(2n + 2)$, which is a multiple of 2 and hence even.

3 a $p + q + q + r$
$= p + 2q + r$

b



c



Top brick $= 2h + 2j + 2k$
$=$ three even numbers added together
$=$ even number

4 a $9xy + 3x + 6xy - 2x$
$= 3x - 2x + 9xy + 6xy$
$= x + 15xy$

b $6xy - xy + 3y$
$= 5xy + 3y$

5 a $\dfrac{a^{\cancel{3}2}\,b^{\cancel{4}1}}{{}^{1}\cancel{a^{1}}\cancel{b^{3}}} = a^2b$

b $2(x^3)^2 = 2x^6$

c $3x \times 2x^3y^2$
$= (3 \times 2) \times x \times x^3 \times y^2$
$= 6x^4y^2$

d $(4ax^2)^0 = 1$

e $4x^2y \times x^3y^2$
$= 4x^2 \times x^3 \times y \times y^2$
$= 4x^5y^3$

f $3x^{-4} \times 5x^6$
$= 3 \times 5 \times x^{-4} \times x^6$
$= 15 \times x^{-4+6}$
$= 15x^2$

g $\dfrac{3x^5}{7x^4} \div \dfrac{6x^{-6}}{14x^{-4}}$

$= \dfrac{{}^{1}\cancel{3}x^5}{{}_{1}\cancel{7}x^4} \times \dfrac{{}^{2}\cancel{14}x^{-4}}{{}_{2}\cancel{6}x^{-6}}$

$= x^{5-4-4-(-6)}$

$= x^3$

h $(4x^{-5})^2 = 4^2(x^{-5})^2$
$= 16x^{-10}$

i $\left(\dfrac{3x}{4y}\right)^3 = \dfrac{(3x)^3}{(4y)^3}$

$= \dfrac{27x^3}{64y^3}$

j $\dfrac{4x^{12}y^{-3}}{12x^{-7}y^9} = \dfrac{1}{3}x^{12--(7)}y^{-3-9}$

$= \dfrac{1}{3}x^{19}y^{-12}$

$= \dfrac{x^{19}}{3y^{12}}$

k $\dfrac{{}^{7}\cancel{14}p^5q^{-4}}{{}_{6}\cancel{30}p^4q^4} \times \dfrac{{}^{1}\cancel{5}pq^{-7}}{{}_{1}\cancel{2}p^{-4}q^5}$

$= \dfrac{7}{6}p^{5+1-4-(-4)}q^{-4-4-7-5}$

$= \dfrac{7}{6}p^{10}q^{-20}$

$= \dfrac{7p^{10}}{6q^{20}}$

6 $7x^3y^2 \times (2x)^3 - (4x^3y)^2 - 4xy^2 \times 10x^5$
$= 7x^3y^2 \times 8x^3 - 16x^6y^2 - 40x^6y^2$
$= 56x^6y^2 - 16x^6y^2 - 40x^6y^2$
$= 0$

7 $x + 5 - (x - 5)$
$= x + 5 - x + 5$
$= 10$ for *all* values of x.
So, **a** 10 **b** 10 **c** 10

8 $s = \dfrac{1}{2}(u + v)t$

$= \dfrac{1}{2}\left(\dfrac{2}{5} + \dfrac{9}{2}\right) \times 3$

$= \dfrac{3}{2}\left(\dfrac{4 + 45}{10}\right) \times 3$

$= \dfrac{3 \times 49}{20}$

$= \dfrac{147}{20}$

9 a $5(x - 2) + 3(x + 2)$
$= 5x - 10 + 3x + 6$
$= 8x - 4$

b $5x(x + 7y) - 2x(2x - y)$
$= 5x^2 + 35xy - 4x^2 + 2xy$
$= x^2 + 37xy$

10 a $m(m - n) - n(n - m)$
$= m^2 - mn - n^2 + mn$
$= m^2 - n^2$

b $x(y - z) + y(z - x) + z(x - y)$
$= \cancel{xy} - \cancel{xz} + \cancel{yz} - \cancel{xy} + \cancel{xz} - \cancel{yz}$
$= 0$

11 **a** $x^5 \times x^{-2} = x^{5-2} = x^3$

b $\frac{{}^{4}\cancel{8}x^{\cancel{2}1}}{{}_{1}\cancel{2}x^{\cancel{4}2}} = \frac{4}{x^2}$

c $(2x-2)^{-3} = \frac{1}{(2x-2)^3}$

$= \frac{1}{2^3(x-1)}$

$= \frac{1}{8(x-1)}$

12 **a** $4^x = 43 \Rightarrow x = 3$

b $3^x - 5 = 22$

$3^x = 27$

$= 3^3$

$x = 3$

c $4 \times 6^p = 864$

$6^p = 216$

$= 6^3$

$p = 3$

13 $a^b - c^a + b^a$

$= 3^2 - (-1)^3 + 2^3$

$= 9 - (-1) + 8$

$= 17 + 1$

$= 18$

14 **a** $3x^{\frac{1}{2}} \times 5x^{\frac{1}{2}}$

$= 15x^{\frac{1}{2}+\frac{1}{2}}$

$= 15x$

b $(81y^6)^{\frac{1}{2}} = 81^{\frac{1}{2}}(y^6)^{\frac{1}{2}}$

$= \sqrt{81}y^3$

$= 9y^3$

c $(64x^3)^{\frac{1}{3}} = 64^{\frac{1}{3}}(x^3)^{\frac{1}{3}}$

$= \sqrt[3]{64}x^1$

$= 4x$

15 **a** $\left(\frac{1}{2}\right)^x = 8$

$(2^{-1})^x = 2^3$

$2^{-x} = 2^3$

$-x = 3$

So, $x = -3$

b $3^x = \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$

$x = -3$

c $125^x = 5$

$(5^3)^x = 5$

$5^{3x} = 5^1$

$3x = 1$

So, $x = \frac{1}{3}$

d $125^x = \frac{1}{5}$

$(5^3)^x = 5^{-1}$

$5^{3x} = 5^{-1}$

$3x = -1$

$x = -\frac{1}{3}$

16 **a** $2^{x+y} = 2^x2^y$

$= pq$

b $2^{x+y-2} = \frac{2^x2^y}{2^2} = \frac{pq}{4}$

c $2^{3x} = (2^x)^3 = p^3$

17 **a** $n^{-1} = 2^{-2}$

$n^{-1} = (2^2)^{-1}$

$n = 2^2 = 4$

b $4^n = \left(\sqrt[4]{32}\right)^3$

$(2^2)^n = \left[(2^5)^{\frac{1}{4}}\right]^3$

$2^{2n} = 2^{\frac{5}{4}\times 3}$

$2n = \frac{15}{4}$

So $n = \frac{15}{8}$

Chapter 3

Diagrams provided as answers are NOT TO SCALE and are to demonstrate construction lines or principles **only**.

Getting started

Each student will produce a different spider diagram depending on their prior knowledge. The information shown might include facts about angles in triangles and quadrilaterals, names and properties of special triangles and quadrilaterals, how to measure lines and angles, and so on.

You can ask students to complete their own and then to compare with others and add any points they have missed out (but know). You could also use the student contributions to develop a class spider diagram, which will give you some idea of what students already know so that you can focus the work on the new concepts in this chapter.

Shapes and solids

Answer suggestions:

1 Students can show each other where they have found the different elements. It is possible to find them all.

2 Answers will vary, but students should be able to find rectangles, trapezia and general quadrilaterals, pentagons, hexagons and some triangles inside the building, formed by the structures.

3 a (Hexagonal) prism

b Yes, if the base is a hexagon, the prism has six rectangular faces.

4 a It speeds up the process and allows different members of the team to put pieces together on one model to check for overlaps/errors and gives a view of the finished process. Building information modelling (BIM) allows them to strip the model down to beams and walls so that they can decide where to install or place infrastructural elements.

b Computer models can be moved, changed and rescaled as needed digitally. It is easy to share and collaborate ideas and different members of the team can work on the design at the same time.

Exercise 3.1

1

	i	ii	iii
a	acute	Answers will vary	40°
b	acute		70°
c	obtuse		130°
d	acute		30°
e	obtuse		170°
f	right		90°
g	acute		70°
h	acute		60°
i	obtuse		140°

2 290°

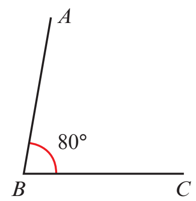
3 a This protractor is able to measure angles from 0° to 360°.

b Student's own answer. Something like: ensure that the 0°/360° marking of the protractor is aligned with one of the arms of the angle you are measuring, and the vertex of the angle is aligned with the centre of the protractor. Whether you use the inner or outer scale will be determined by which arm you aligned with 0. Use the scale that gives an angle $<180°$.

c You would use the scale that gives you an angle $>180°$.

Exercise 3.2

1 a



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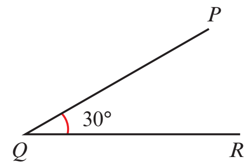

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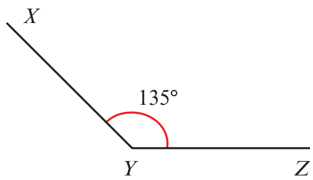
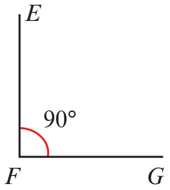
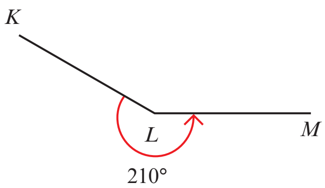
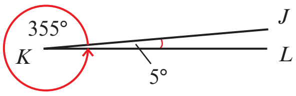


d 

e 

f 

Exercise 3.3

1 **a** EBF and FBC; or ABD and DBE

b ABE and EBC; or DBA and CBG; or DBC and ABG

c ABD and DBC; or ABE and EBC;
or ABF and FBC;
or ABG and CBG;
or DBE and EBG;
or DBF and FBG;
or DBC and CBG;
or DBA and ABG;
or ABG and GBC

d DBE, EBF, FBC and CBG
or DBA and ABG
or DBF, FBC and CBG
or DBF and FBG
or DBC and CBG
(and combinations of these)

e FBC

f EBA

2 **a** $x = 68°$

b $x = 40°$

c $x = 65°$; $y = 115°$

d $x = 59°$; $y = 57°$

e $x = 16°$; $y = 82°$; $z = 16°$

f $x = 47°$; $y = 43°$; $z = 133°$

g $x = 57°$

h $x = 71°$

i $x = 38°$

3 **a** 30°

b 15°

c 30°

4 60° and 120°

5 53°, 127° and 53°.

Exercise 3.4

1 **a** $a = 112°$ (alternate angles equal)
$b = 112°$ (vertically opposite angles equal)

b $x = 105°$ (alternate angles equal)
$y = 30°$ (sum of triangle)
$z = 45°$ (alternate angles equal)

c $c = 40°$ (vertically opposite angles equal)
$b = 72°$ (corresponding angles equal)
$a = 68°$ (angles on a line)
$d = 68°$ (vertically opposite angles equal)
$e = 40°$ (alternate angles equal)

d $a = 39°$ (corresponding angles equal)
$b = 102°$ (angle sum of triangle)

e $x = 70°$ (angle on a line)
$y = 70°$ (corresponding angles equal)
$z = 85°$ (corresponding angles equal (180 − 95 = 85°, angles on a line, z is corresponding angle equal to 85°)

f $x = 45°$ (alternate angles equal)
$y = 60°$ (alternate angles equal)

g $x = 82°$ (co-interior angles supplementary)
$y = 60°$ (corresponding angles equal)
$z = 82°$ (angles on a line)

h $x = 42$ (alternate angles equal)
$y = 138°$ (angles on a line)
$z = 65°$ (alternate angles equal)

i $a = 40°$ (alternate angles equal)
$b = 140°$ (angles on a line)
$d = 75°$ (angles on a line)
$c = 75°$ (corresponding angles equal)
$e = 105°$ (corresponding angles equal)

2 **a** AB $\parallel$ DC (alternate angles equal)

b AB $\nparallel$ DC (co-interior angles not supplementary)

c AB $\parallel$ DC (co-interior angles supplementary)

General results

1 a y
b $180 - x$
c y is equal to z.
d x and z are equal.

2 a Angle ABF is equal to angle CDF (corresponding angles since lines AB and CE are parallel). Angle CDF is equal to angle CEG (corresponding angles since lines BF and EG are parallel). So angle ABC = angle CEG, so $x = y$.
b Angle AFE is equal to angle HFB (vertically opposite angles). Angle BFH is equal to angle DGH (corresponding angles since lines AB and CD are parallel). So angle AFE = angle DGH, so $x = y$.

Exercise 3.5

1 a $x = 54°$ (angle sum of triangle)
b $x = 66°$ (base angle isosceles Δ)
c $x = 115°$ (angle sum of triangle)
$y = 65°$ (exterior angle of triangle equal to sum of the opposite interior angles OR angles on a line)
$z = 25°$ (angle sum of triangle)

2 a $x = 60°$ (exterior angle of Δ equal to sum of opposite interior angles), so $x + x = 120°$, $x = 60°$
b $4x = 86° + (180° - 2x)$
(exterior angle equals sum of opposite interior angles, and angle on straight line)
$6x = 266$
$x = 44.3°$

3 a angle $BAC = 180° - 95°$
(angles on a straight line) = 85°
angle $ACB = 180° - 105°$
(angles on a straight line) = 75°
$180° = x + 75° + 85°$
(angle sum of triangle)
$x = 180° - 160°$
$x = 20°$
b angle $CAB = 56°$
(vertically opposite angles equal)
$180° = 56° + 68° + x$
(angle sum of triangle)
$x = 180° - 124°$
$x = 56°$
c angle $ACE = 53°$ (angles on straight line)
$x = 53°$ (alt angles equal)
OR
angle $CDE = 59°$ (alt angles equal)
$180° = 68° + 59° + x$ (angle sum of Δ)
$x = 180° - 127°$
$x = 53°$
d $180° = 58°$ + angle ACB + angle CBA (angle sum of triangle)
angle ACB = angle CBA (isosceles Δ)
$\Rightarrow 180° = 58° + 2y$
$2y = 122°$
$y = 61°$
$x = 180° - 61°$
(exterior angles of a triangle equal to sum of opposite interior angles)
$x = 119°$
e angle $AMN = 180° - (35° + 60°)$
(angle sum of Δ)
angle $AMN = 85°$
$x = 85°$
(corresponding angles equal)
f angle $ACB = 360° - 295°$
(angles around a point)
angle $ACB = 65°$
angle $ABC = 65°$ (isosceles Δ)
$x = 180° - (2 \times 65°)$ (angle sum of Δ)
$x = 50°$

Exercise 3.6

1 a Rhombus, kite or square
b Square

2 a angle $QRS = 112°$ (vertically opposite angles equal)
$x = 112°$ (opposite angles in parallelogram)
b $x = 62°$ (isosceles Δ)
c $360° = 110° + 110° + 2x$
(angle sum of quadrilateral)
$140° = 2x$
$x = 70°$

d angle $MLQ = 180° - 110°$
(angles on a straight line)
angle $LMN = 180° - 98°$
(angles on a straight line)
$360° = 70° + 82° + 92° + x$
(angle sum of quadrilateral)
$x = 116°$

e $360° = 3x + 4x + 2x + x$
(angle sum of quadrilateral)
$360° = 10x$
$x = 36°$

f $360° = (180° - x) + 50° + 110° + 90°$
(angles on a straight line, and angle sum of quadrilateral)
$360° = (180° - x) + 250°$
$110° = 180° - x$
$x = 70°$

3 a $180° = 70° + 2y$ (angle sum of a Δ, isosceles Δ to give $2y$)
$110° = 2y$
$y = 55°$
$\therefore$ angle $PRQ = 55°$
angle $MRS = 180° - (55° + 55°)$
(angles on a straight line, and isosceles triangle)
$x = 70°$

b angle $MNP = 98°$
(opposite angles in parallelogram)
angle $RNM = 180° - 98°$
(angles on a straight line)
$= 82°$
$180° = 2x + 82°$ (angle sum of a triangle, and isosceles triangle)
$2x = 98°$
$x = 49°$

c angle $QRP = 55°$
(angle sum of a triangle, and isosceles triangle)
angle $QRP = x$
(alternate angles equal)
$x = 55°$

Exercise 3.7

1

Number of sides	5	6	7	8
Angle sum	540°	720°	900°	1080°

Number of sides	9	10	12	20
Angle sum	1260°	1440°	1800°	3240°

2 **a** 108°
b 120°
c 135°
d 144°
e 150°
f 165.6°

3 **a** 2340°
b 360°
c 156°
d 24°

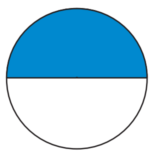
4 24 sides

5 **a** $x = 135°$
b $x = 110°$
c $x = 72°$

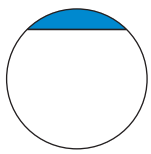
Exercise 3.8

1 **a** Diameter
b Major arc
c Radius
d Minor sector
e Chord
f Major segment

2 **a**



b



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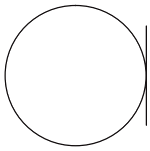

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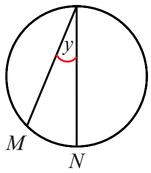


3 **a** Radius

b Diameter

c Minor arc

d DO, FO or EO

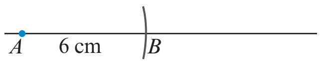
e Major arc

f Sector

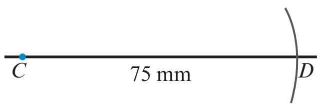
Exercise 3.9

NOT TO SCALE

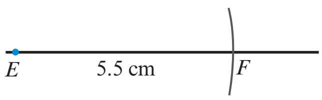
1 **a**



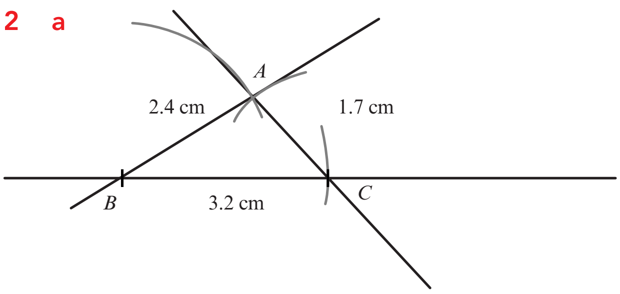
b



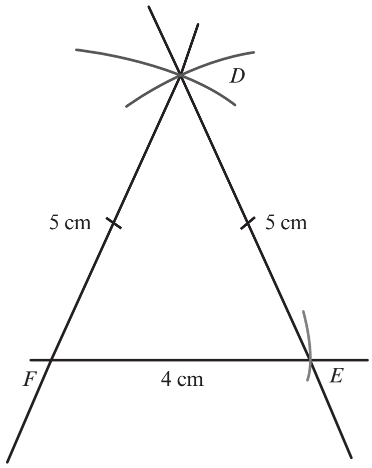
c



2 **a**



b



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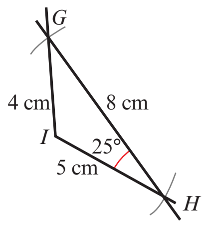
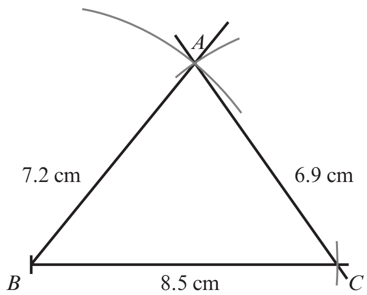
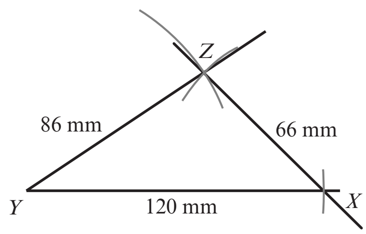


3 **a**



b



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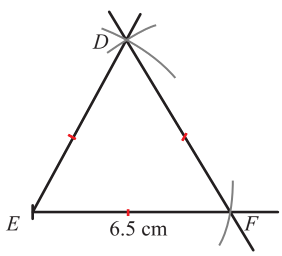

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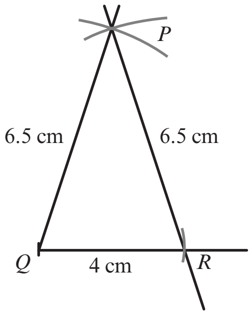


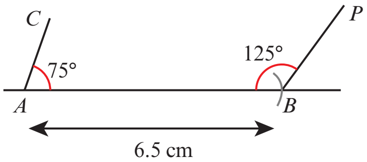
4 **a** Accurate drawing

b **i** 5.5 cm

ii 4.2 cm

Practice questions

1 NOT TO SCALE



2 **a** $x = 99°$ (co-interior angles supplementary)

b $x = 65°$ (corresponding angles equal)

c $x = 75°$ (angle sum of isosceles Δ)

d $x = 112°$ (opposite angles of $\|$ gram)

e $x = 110°$

If y = angle AEC

$\Rightarrow 360° = 90° + 110° + 90° + y$

$y = 70°$

$\therefore$ angle $AEC = 70°$

angle $ADE = 70°$ (isosceles triangle)

$x = 180° - 70°$ (angles on a line)

$x = 110°$

f $x = 72.5°$

Let y stand for base angles of isosceles Δ.

$2y + 35° = 180°$ (base angles isosceles Δ and angle sum of Δ)

$y = 72.5°$

$\Rightarrow$ angle $QRP = 72.5°$

angle $NRQ = 35°$ (alternate angles equal)

$180° = x + 72.5° + 35°$

$x = 72.5°$

3 **a** Angles in a triangle add up to 180°. $x + y + 90° = 180°$, so $x + y = 90°$

b $y = 53°$

4 $a = 70°$, $b = 110°$, $c = 100°$

5 720°

6 Let angle $MQN = x$

Then angle $PMQ = x$ (isosceles triangle)

So angle $MPQ = 180° - 2x$ (angles in a triangle add up to 180 degrees)

Therefore angle $MPN = 180° - (180° - 2x) = 2x$ (angles on a straight line)

So angle $PMN = 2x$ (isosceles triangle)

and angle $NMQ = x + 2x = 3x$

7 **a** 360°

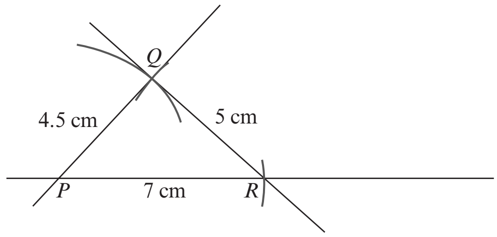
b 24° if a regular polygon

c 156°

8 **a** Exterior angle of triangle is equal to the sum of two opposite interior angles $\frac{x}{2} + \frac{x}{2} = x$.

b Opposite angles of parallelogram equal, and vertically opposite angles equal.

9 NOT TO SCALE



10 **b** They all intersect at the same point.

d The circle will always pass through all three vertices if drawn correctly.

11 32

12 **a** 108°

b 36°

13 18

14 **a** Angle $UVP = x$ (alternate angles are equal)

Angle $WVQ = c$ (alternate angles are equal)

b Angle UVP + angle PVQ + angle WVQ = 180° (angles on a straight line sum to 180°)

Therefore $a + b + c = 180°$, so the interior angles of a triangle sum to 180°

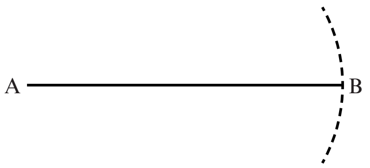
c Angle $RQV = 180° - c$

d $a + b = 180° - c$ = angle RQV

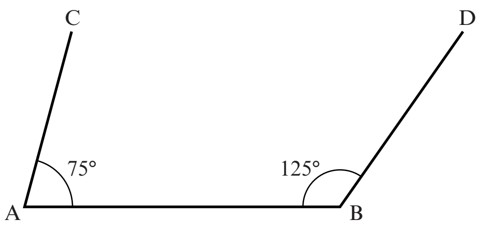
Therefore the exterior angle of a triangle is equal to the sum of the two interior opposite angles.

Practice questions worked solutions

1 a



b



2 a Angle CFG corresponds to angle AEF so angle $CFG = 81°$

x + angle $CFG = 180°$ because the angles on a straight line add up to 180°.

$x + 81° = 180° \Rightarrow x = 99°$

b $x + 65°$ because angle NSR corresponds to angle QTS.

c Angle $ABC = x$ because base angles of an isosceles triangle are equal

$x + x + 30° = 180°$ because the sum of the interior angles of a triangle is 180°

$2x = 150°$

$x = 75°$

d Angle $PNM = 180° - 112°$
$= 78°$ (supplementary angles)

Angle $PNM + x = 180°$ (supplementary angles)

$x = 180° - 78°$
$= 112°$

e Angle $AEC + 90° + 90° + 110° = 360°$ (angles sum in a quadrilateral = 360°)

Angle $AEC = 360° - 290°$
$= 70°$

Angle $ADE = 70°$ (base angles of an isosceles triangle are equal)

$x = 180° - 70° = 110°$ (angles on a straight line add to 180°)

f Angle PRQ = angle RPQ (base angles of an isosceles triangle are equal)

$2 \times$ angle $RPQ + 35° = 180°$ (angles in a triangle add up to 180°)

Angle $RPQ = \dfrac{180° - 35°}{2}$

$= \dfrac{145°}{2} = 72.5°$

x = angle $RPQ = 72.5°$ (alternate angles are equal)

3 a $x + y + 90° = 180°$ (angles in a triangle add up to 180°)

So $x + y = 90°$

b $37 + y = 90°$

$y = 90° - 37°$
$= 53°$

4 $a + 110° = 180°$

$a = 70°$

$a + b = 180°$

$b = 180° - 70°$
$= 110°$

$c + 100° + 100° + 60° = 360°$

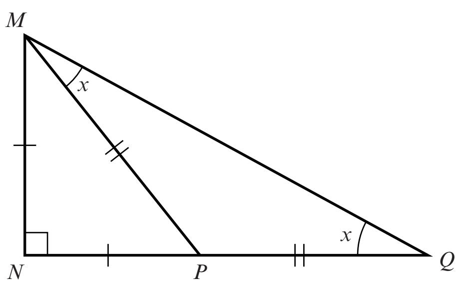
$c = 100°$

5 Exterior angle $= \dfrac{360°}{6} = 60°$

Interior angle $= 180° - 60° = 120°$

Sum of interior angles $= 120° \times 6 = 720°$

6



Let angle $MQN = x$

Triangle MPQ is isosceles $\Rightarrow$ angle $PMQ = x$ (base angles are equal)

Angle $MPQ = 180° - 2x$ (angles in a triangle add up to 180°)

Therefore, angle $MPN = 180° - (180° - 2x) = 2x$ (angles on a straight line add up to 180°)

Angle $PMN = 2x$ (base angles of an isosceles triangle are equal)

So, angle $NMQ = 2x + x = 3x = 3 \times$ angle MQN

7 **a** 360° (true for all polygons)

b $\frac{360°}{15} = 24°$

c $180° - 24° = 156°$

8 **a** Angles in a triangle add to 180°

Angle $BAC = 180° - \left(\frac{x}{2} + \frac{x}{2}\right)$

$= 180° - x$

Angles on a straight line add up to 180°

so, angle $CAD + (180° - x) = 180°$

Therefore, angle $CAD = y = x$

b Angle $PSN = x$ (opposite angles in a parallelogram are equal)

y is vertically opposite angle PSN so $y = x$

9 Check by measurement.

10 **a** Check by measurement.

b They all intersect at the same point.

c Check by drawing.

d The point of intersection is always the centre of the circle passing through all three points.

11 If n = number of sides, then the total interior angle is $180°(n - 2) = 5400$

$n - 2 = 30$

$n = 32$

So, the number of sides is 32

12 **a** Exterior angle $\frac{360°}{5} = 72°$

x = interior angle $= 180° - 72° = 108°$

b Triangle ABC is isosceles

$\Rightarrow x + 2y = 180°$

$108° + 2y = 180°$

$2y = 72°$

$y = 36°$

13 $3x = \frac{360°}{10} = 36°$

$x = 12°$

Therefore, exterior angle of $B = \frac{5}{3} \times 12° = 20°$

$\frac{360°}{20°} = 18$ sides

14 **a** Angle UVP is alternate with angle $VPQ = a$

so, angle $UVP = a$

Similarly, $WVQ = c$

b Angles on a straight line add up to 180°

so, angle UVP + angle PVQ + angle WVQ = 180°

$\Rightarrow a + b + c = 180°$

c angle $RQV = 180° - c$

d $a + b + c = 180°$

so, $180° - c = a + b$

Therefore, exterior angle RQV = sum of two opposite interior angles

Chapter 4

Getting started

1 Answers could include: observations, experiments and measurements, research using secondary sources, questionnaires and sampling.

2 Some possible answers are:

	Pictogram	Bar chart	Pie chart	Line graph
a	Number of medals that each country won	Total medal count for some countries	Proportion of total medals awarded to each continent	Number of medals each country won over a period of time
b	Use the key and number of circles to find number of medals	Read frequency from vertical axis and country name from horizontal axis	Use size of sector to compare countries	Read frequency from vertical axis and use slope of line to see if the number increases or decreases for different times (given on horizontal axis)
c	Mostly for effect/ decorative Used for discrete data in categories	To compare data sets Used for discrete data in categories	To compare data sets Used for discrete data in categories	To show trends or patterns Data that changes over time; best for continuous data (although this data is discrete)
d	Use of different symbols or different sized symbols can make data difficult to compare/interpret (size and surface area of symbols can be used to mislead, especially with circular symbols	Scale and/or axis manipulation can be misleading, for example, not starting at 0, reducing or increasing intervals Using 3D bars can also make the categories look different	% might not sum to 100, circles with different sizes can give different impressions, total numbers may not be given	Vertical scale can be manipulated, may not start at 0 and can jump in different sized steps, labelling might not be clear

Exercise 4.1

1 a, b Students' answers will vary, below are possible answers.

Categorical data	Numerical data
Hair colour	Number of people in household
Eye colour	Hours spent doing homework
Favourite subject	Hours spent watching TV
Mode of transport to school	Number of books read in a month
Brand of toothpaste used	Shoe size
Make of cell phone	Test scores

2 a Continuous
b Discrete
c Continuous
d Continuous
e Discrete
f Continuous
g Continuous
h Discrete
i Continuous
j Discrete
k Discrete
l Discrete

3 a i Experiment
ii Primary
iii Numerical
iv Discrete

b i Survey
ii Primary
iii Categorical
iv Discrete

c i Use existing data
ii Secondary
iii Numerical
iv Continuous

d i Survey
ii Primary
iii Categorical
iv Discrete

e i Use existing data
ii Secondary
iii Numerical
iv Discrete

f i Experiment
ii Primary
iii Numerical
iv Discrete

g i Survey
ii Primary
iii Numerical
iv Continuous

h i Use existing data
ii Secondary
iii Categorical
iv Discrete

i i Use existing data
ii Secondary
iii Numerical
iv Discrete

j i Survey
ii Primary
iii Numerical
iv Discrete

Exercise 4.2

1

Score	Tally	Total
1	~~||||~~ |||	8
2	~~||||~~ ~~||||~~ ||	12
3	~~||||~~ ||	7
4	~~||||~~ |||	8
5	~~||||~~ |||	8
6	~~||||~~ ||	7
		50

2 Students' own answers.

3 a 7

b 2 and 12

c It is impossible to score 1 with two dice.

d There are three ways of getting each of these scores.

Exercise 4.3

1 a

Number of coins	0	1	2	3	4	5	6	7	8
Frequency	6	2	6	4	4	2	4	1	1

b 8

c 2

d None or two coins

e 30: add column and total the frequencies.

2 a

Amount ($)	0–9.99	10–19.99	20–29.99	30–39.99	40–49.99	50–59.99
Frequency	7	9	5	2	1	1

b 16

c 1

d $10–$19.99

3

Call length	Frequency
0–59 s	0
1 min–1 min 59 s	4
2 min–2 min 59 s	3
3 min–3 min 59 s	6
4 min–4 min 59 s	4
5 min–5 min 59 s	3

Exercise 4.4

1

4	5 8 9 9
5	3 3 4 5 5 5 6 6 6 8 9
6	0 0 3 7 8

Key
Key 4|5 represents 45 kg

2 a

Branch A		Branch B
5	11	
	12	
4 2	13	
9 9 0	14	2
5 2	15	9
9 8 6 4	16	0 5 9
9 9 5 2	17	7
9 8 8 6 0 0	18	0 5 6 7 7 8 8 8 8
9 8 0	19	0 0 1 1 3 6 8
1 0 0	20	0 0 0 1 4 5

Key
Branch A 5 | 11 represents 115 pairs
Branch B 14 | 2 represents 142 pairs

b Branch B: 205 pairs

c Branch B, as the data are clustered round the bottom of the diagram where the higher values are located.

3 a 26

b 12 cm

c 57 cm

d 6

e i More data clustered round top of diagram; possibly need to add 0 as a stem.

ii Data clustered round bottom of the diagram, possibly need to add more stems (i.e. higher than 5).

4 a 7

b 101 beats per minute

c 142 beats per minute

d Exercise raised the heart rate of everyone in the group. Data moved down the stems after exercise, indicating higher values for all people.

Exercise 4.5

1 a 15

b 33

c Mostly right-handed

d 90

2 Student's own answers.

3 a

	Checks on phone	Does not check on phone
Checks on computer	5	3
Does not check on computer	3	1

b Most people surveyed check their email on their phone (8 out of 12), of these five of them check both. Only three people don't check email on their phones and only one person does not check email on a phone or a computer.

Exercise 4.6

1 a i 3695 miles

ii 8252 miles

iii 4586 miles

b Istanbul to Montreal

c 21 128 miles

d 4 hours

e Blanks match a city to itself so there is no flight distance.

Exercise 4.7

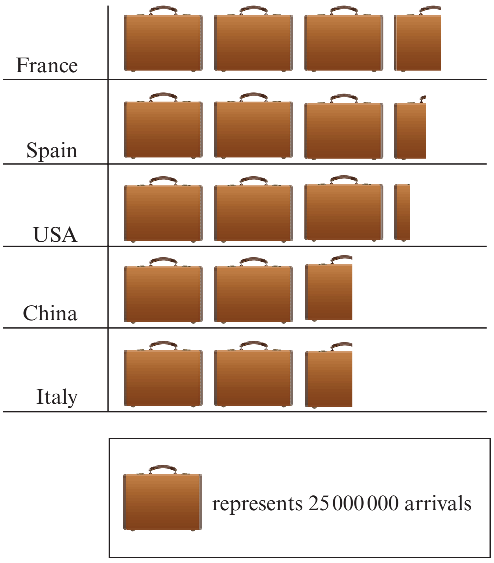
1 a 250 000

b 500 000

c 125 000

d 375 000

2 Answers may vary depending on the scale students choose. For example:



3 a Reel deal

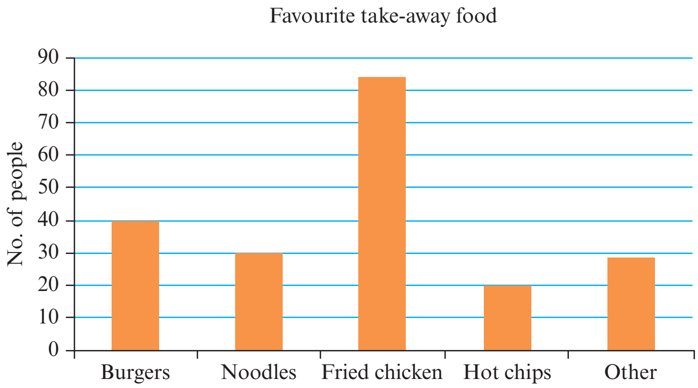
b Fish tales

c Golden rod – 210 fish;
Shark bait – 420 fish;
Fish tales – 140 fish;
Reel deal – 490 fish;
Bite-me – 175 fish

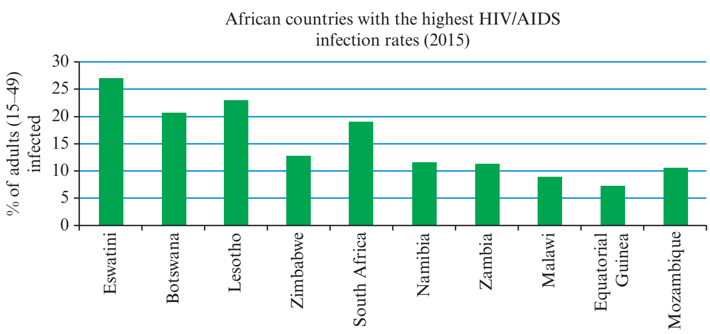
d 1435 fish

Exercise 4.8

1 a



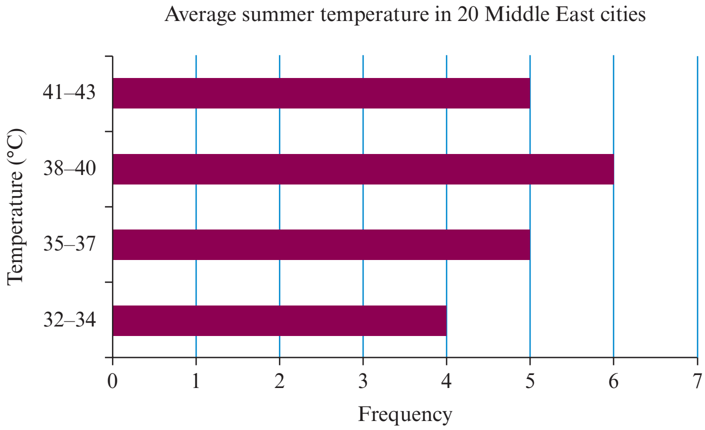
b



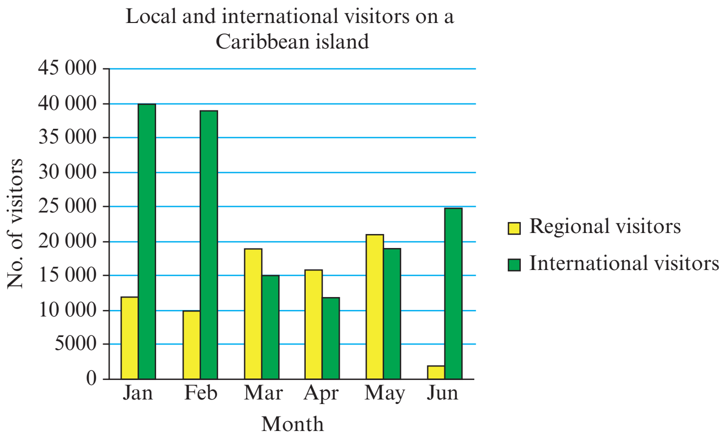
2 **a**

Temperature (°C)	32–34	35–37	38–40	41–43
Frequency	4	5	6	5

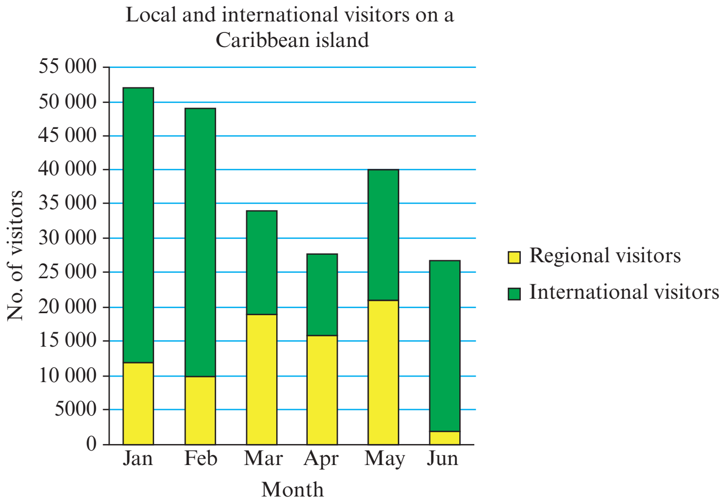
b



3 **a**

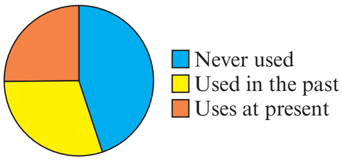


b

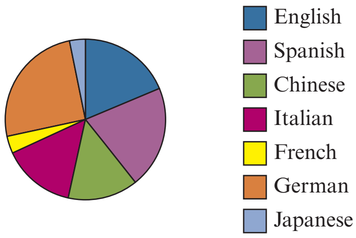


Exercise 4.9

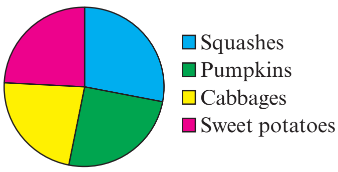
1 Students' use of online support



2 Home language of people passing through an international airport



3 Land used on a farm to grow vegetables



4 **a** $\frac{1}{4}$

b $\approx 11\%$

c 0.25

d **i** 225

ii 100

iii 200

iv 150

Graphs can tell a story

1 Students' own discussions. Should indicate that change in the graph is in response to events and that lines sloping up indicate noise levels increasing, while lines sloping down indicate noise levels dropping.

2 Students' own discussions. For example, the spectators get happier as the game starts, then their team scores a goal and the happiness level goes up quickly and stays high, then the other team scores a goal and the level drops quickly, staying low. The other team scores a second goal and the happiness level drops really low and stays low until their team scores a goal, then it increases to about the same level as at the start of the match. It stays at that level until their team scores a third goal and then it goes up quickly. At the end of the match they are even happier because their team won; after that, the level of happiness drops slowly, but remains high.

3 Students draw their own graph to show their happiness levels over a school day. If they seem willing, they can share these with their groups, but do not force them to do this.

Exercise 4.10

1 **a** **i** approximately 33 °C

ii approximately 65 °C

b It heats about 14 degrees in 30 seconds, so assuming a 1 degree heating in 2 seconds, it will reach 100 degrees in approximately 188 seconds.

2 **a** 2 °C

b Between 07:00 and 09:00

3 **a** 62.5 bpm

b At 6.30 a.m., as her pulse rate started steadily increasing after a short rest.

c 150 bpm at 6.50 a.m.

d It dropped fairly steadily, returning to the starting rate after about 10 minutes.

Practice questions

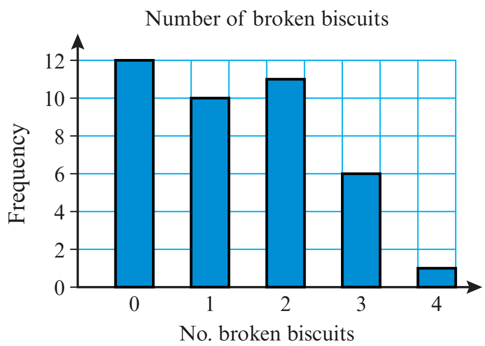
1 **a** Primary data – it is data collected she collected herself by counting.

b Discrete data – the data can only take certain values.

c

No. of broken biscuits	Tally	Frequency
0	卌 卌 ||	12
1	卌 卌	10
2	卌 卌 |	11
3	卌 |	6
4	|	1
		40

d



2 **a** Heathrow

b 15 397

c

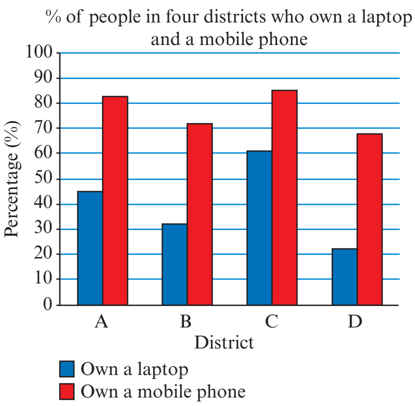
Gatwick	24 000
Heathrow	40 000
London City	6 000
Luton	11 000
Stansted	15 000

d

3 **a** 4980

b District C – it has the highest percentage of laptops.

c



4 **a** Sport played by students.

b Five

c Baseball

d $\frac{1}{4}$

e 28 (to nearest whole number)

f 83 (to the nearest whole number)

5 **a** Pictogram

b Each stick person represents 1 billion people.

c $\frac{1}{2}$ billion = 500 million

d 200 years

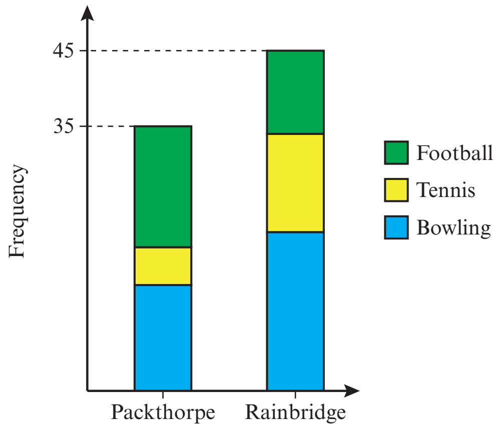
e 2012

f 9 full stick people and $\frac{1}{5}$ of a stick person.

6 **a**

	Football	Tennis	Bowling	Total
Packthorpe	14	5	16	35
Rainbridge	21	13	11	45
Total	35	18	27	80

b



Practice questions worked solutions

1 **a** The data is collected directly through an experiment, so it is PRIMARY data.

b The data can only be numbers of biscuits

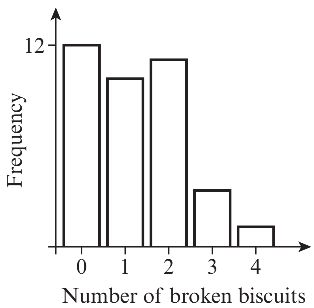
$\Rightarrow$ it can only take whole number values

$\Rightarrow$ it is discrete data.

c

No. of broken biscuits	Tally	Frequency
0	𝍸 𝍸 ||	12
1	𝍸 𝍸	10
2	𝍸 𝍸 |	11
3	𝍸 |	6
4	|	1

d

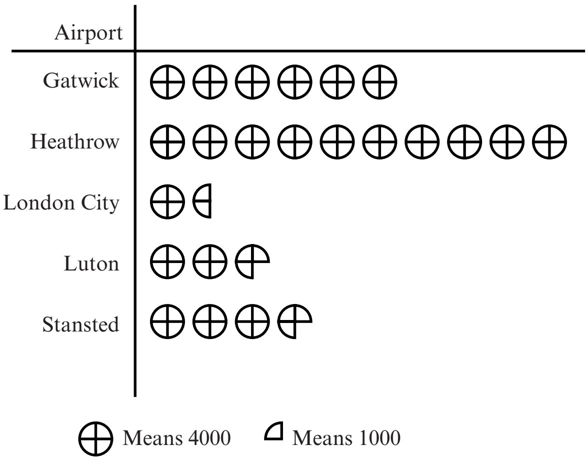


2 **a** Heathrow

b 15 397

c 24 000, 40 000, 6000, 11 000, 15 000

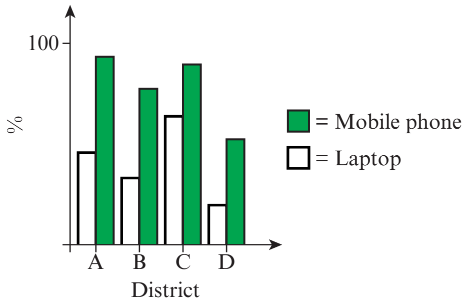
d



3 **a** $0.83 \times 6000 = 4980$ people

b District C because a large proportion of people own laptops.

c



4 **a** Discrete

b 5

c Baseball

d $\frac{1}{4} \times 200 = 50$

e 28

f 83

5 **a** Pictogram

b 1 full symbol represents 1 billion people

c 1 billion

d $1930 - 1650 = 280$ years

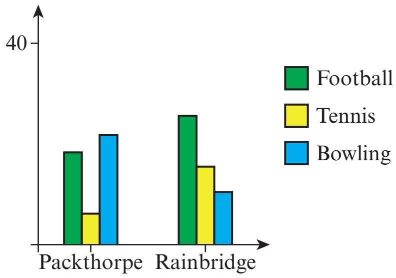
e In 2012

f 9 whole symbols and $\frac{1}{5}$ of another

6 **a**

	Football	Tennis	Bowling	Total
Packthorpe	14	5	16	35
Rainbridge	21	13	11	45
Total	35	18	27	80

b



Past paper questions

1 $60 = 2 \times 2 \times 3 \times 5$

$90 = 2 \times 3 \times 3 \times 5$

So $3 \times 5 = 15$

2 $\simeq \dfrac{40 \times 3}{20 - 4^2} = \dfrac{40 \times 3}{4} = 30$

3 **a** Pentagon

b exterior angle $= \dfrac{360°}{18} = 20°$

$\therefore$ interior angle $= 180° - 20° = 160°$

4 **a** $\frac{1}{2^5} = 2^{-5}$

b **i** $3^{18-t} = 3^6$

$18 - t = 6$

$t = 12$

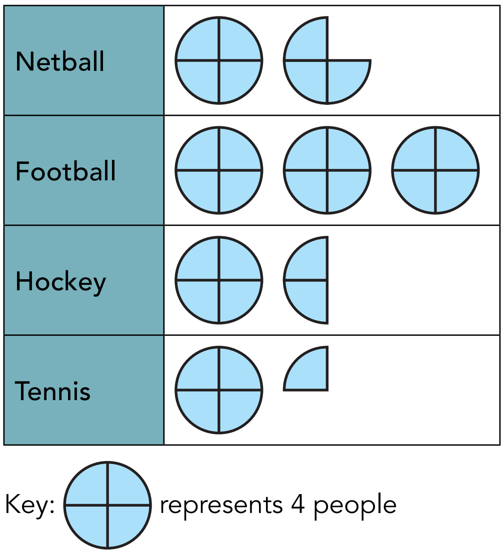
ii $8 \times 6 \times w^{10} \times w^5 = 48w^{15}$

5 $a = 59^{\circ}$ (vertically opposite)

$b = 37^{\circ}$ (corresponding)

$c = 180^{\circ} - 59^{\circ} - 37^{\circ} = 84^{\circ}$ (angle sum in a triangle)

6



7 Angle $ACB = \frac{180^{\circ} - 38^{\circ}}{2} = 71^{\circ}$

Angle $ACD = 180^{\circ} - 71^{\circ} = 109^{\circ}$

8 $4p^7q^{-1}$

9 **a** **i** $17 \times 4 = 68$

ii 61, 67, 71, 73

b **i** 67

ii 64

iii 65

iv 72

c $\frac{1}{7}$

d 3.722

e $8 = 2 \times 2 \times 2$ $\quad 14 = 2 \times 7$

LCM $= 2 \times 2 \times 2 \times 7 = 56$

f **i** 12 °C

ii 17 °C

10 **a** $(\sqrt[4]{81})^3 y^{16 \times \frac{3}{4}} = 3^3 y^{12} = 27y^{12}$

b $2^3 = (2^2)^p$

$\Rightarrow 2^3 = 2^{2p}$

$2p = 3$

$p = \frac{3}{2}$

Chapter 5

Getting started

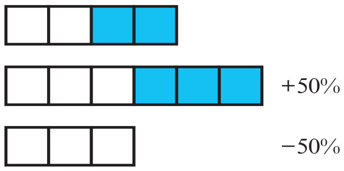
1

A = F	B = Q
C = H	D = I
E = U	G = O
J = V	K = L
M = P	N = Y
R = T	W = X

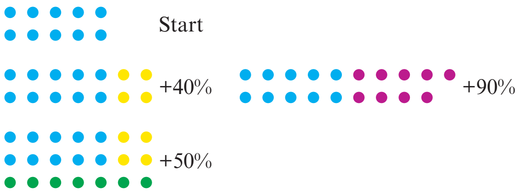
S has no matching value.

2 Students' own drawings. For example:

a



b



Exercise 5.1

1

a $\frac{5}{9}=\frac{10}{18}=\frac{15}{27}=\frac{20}{36}$

b $\frac{3}{7}=\frac{6}{14}=\frac{9}{21}=\frac{12}{28}$

c $\frac{12}{18}=\frac{6}{9}=\frac{2}{3}=\frac{8}{12}$

d $\frac{18}{36}=\frac{1}{2}=\frac{2}{4}=\frac{3}{6}$

e $\frac{110}{128}=\frac{55}{64}=\frac{165}{192}=\frac{220}{256}$

2

a $\frac{1}{3}$

b $\frac{1}{3}$

c $\frac{3}{4}$

d $\frac{3}{5}$

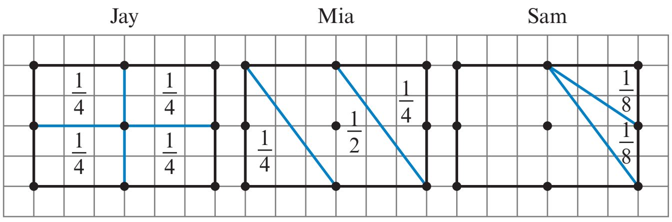
e $\frac{1}{5}$

f $\frac{2}{3}$

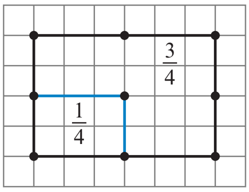
g $\frac{3}{10}$

Fraction diagrams

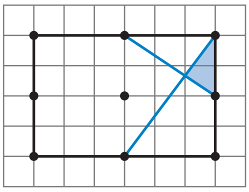
1 a



b It is possible. For example:

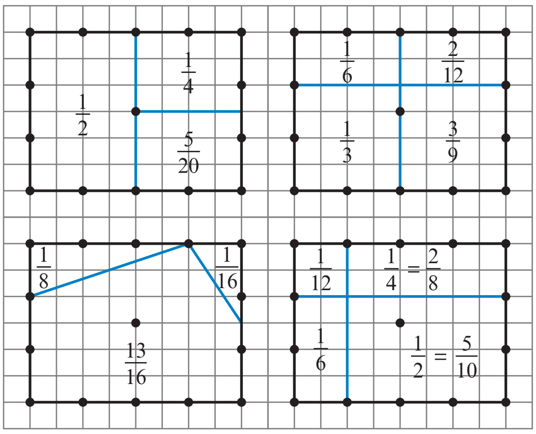


c



$\frac{1}{24}$ (area of the shaded triangle is $\frac{1}{2} \times 2 \times 1 = 1$, area of entire rectangle is $4 \times 6 = 24$, so shaded triangle is $\frac{1}{24}$ of the rectangle)

2 Students' own work. There are many possible solutions and using equivalent fractions will produce more options. For example:



Exercise 5.2

1 a $\frac{10}{27}$

b $\frac{3}{14}$

c $\frac{2}{9}$

d $\frac{1}{4}$

2 a $\frac{1}{5}$

b $\frac{8}{21}$

c 2

d $\frac{92}{35} = 2\frac{22}{35}$

e 32

f $\frac{319}{8} = 39\frac{7}{8}$

g 180

h $80\frac{1}{2}$ or $\frac{161}{2}$

Exercise 5.3

1 a $\frac{2}{3}$

b $\frac{5}{7}$

c $\frac{1}{4}$

d $\frac{13}{9} = 1\frac{4}{9}$

e $\frac{11}{30}$

f $\frac{1}{24}$

g $\frac{7}{8}$

h $2\frac{1}{16}$

2 a $3\frac{1}{3}$

b $6\frac{5}{11}$

c $18\frac{1}{4}$

d $3\frac{3}{4}$

e $-\frac{5}{6}$

f $12\frac{11}{16}$

g $6\frac{13}{16}$

h $2\frac{29}{60}$

i $1\frac{25}{42}$

j $\frac{1}{2}$

k $9\frac{5}{12}$

l $3\frac{7}{60}$

Fraction patterns

1 0.4, 0.333333…, 0.625, 0.428…, 0.444…, 0.1875

2 Terminating: $\frac{2}{5}$ $\frac{5}{8}$ $\frac{3}{16}$

Non-terminating: $\frac{1}{3}$ $\frac{3}{7}$ $\frac{4}{9}$

3 Terminating: the denominators are all products of powers of 2 and 5 only.

4 They all have the same numbers in the same order, but starting in different places.

Exercise 5.4

1 $\frac{3}{7}$

2 $\frac{14}{15}$

3 $\frac{4}{63}$

4 $\frac{2}{11}$

5 $\frac{147}{5} = 29\frac{2}{5}$

6 $\frac{48}{85}$

7 $\frac{189}{122} = 1\frac{67}{122}$

8 $\frac{13}{14}$

9 a $\frac{7}{10}$

b $\frac{77}{60}$

Exercise 5.5

1 $\frac{1}{40}$

2 $\frac{4}{5}$

3 $\frac{60}{7} = 8\frac{4}{7}$

4 5

5 24

6 $\frac{1}{8}$

7 $\frac{3}{8}$

8 $9\frac{3}{5}$

Exercise 5.6

1 90 people

2 $\frac{4}{21}$

3 98

4 $\frac{3}{7}$

5 $\frac{1}{4}$

6 3 cups and $3\frac{3}{4}$ cups of water

Exercise 5.7

1 a $\frac{7}{10}$

b $\frac{3}{4}$

c $\frac{1}{5}$

d $\frac{9}{25}$

e $\frac{3}{20}$

f $\frac{1}{40}$

g $\frac{43}{20}$

h $\frac{33}{25}$

i $\frac{47}{40}$

j $\frac{271}{250}$

k $\frac{1}{400}$

l $\frac{1}{50\,000}$

2 a 60%

b 28%

c 85%

d 30%

e 4%

f $416.67\% = 416\frac{2}{3}\%$

Thinking about percentages

1 a Method D.

b There are different reasons why the others won't work, mostly because they all use the incorrect method. Individual reasons might include: A – no % included in the calculation, B – multiplying by the answer cannot give the answer, C − 0.013 is not 13% and E – 13 is given as fraction of 500 rather than 100.

2 a Most will show 0.25 because 25% is equivalent to 0.25.

b 12% × 650 or 650 × 12% (the other methods involve converting the percentage and don't use the percentage function, for example, 0.12 × 650, 12 ÷ 100 × 650, 650 ÷ 100 × 12).

c You are dealing with percentages here, so you include the sign.

d Students will compare calculators and probably discover that they don't all work in exactly the same way.

3 Students' explanation of different methods for calculating 15% of 500.

Exercise 5.8

1 a 15

b 12

c 135

d 360

e 75

f 45

g 0.078 m

h 0.275 L

2 a 40%

b 25%

c 27.0% (3 s.f.)

3 16 397 batteries (16396.8)

4 77.8 (3 s.f.)%

5 79.2 (3 s.f.)%

6 25%

7 0.025%

8 177.33%

Exercise 5.9

1 4%

2 21%

3 7%

4 19%

5 25%

6 44%

Exercise 5.10

1 a 44

b 46

c 50

d 42

e 41.6

2 a 79.5

b 97.52

c 60.208

d 112.36

e 53.265

3 a 111.6

b 105.4

c 86.8

d 119.04

e 115.32

4 a 3.62

b 23.3852

c 36.0914

d 0

e 36.019

5 33 h

6 \$13.44

7 26 199

8 126 990

9 10 h 34 min

10 174.90%

11 No. 100% − 12% − 12% = 74.44% which is a 22.56% decrease

12 14.9

13 Students' reasoning may vary.

- a No, on day 8 the points will have doubled but on day 7 they will have halved.
- b Increasing by 40% and then 50% as the increase is cumulative, so you are actually increasing 140% by 50%.
- c It is cheaper to take the 50% discount. For example, if 400 MB of data cost \$100, then you would pay \$50 for 400 MB, if you take the extra 50% free, then you get 600 MB for \$100 and this means you pay \$50 for only 300 MB.

Exercise 5.11

1 175

2 362.857

3 1960

4

Sale price (\$)	% reduction	Original price (\$)
52.00	10	57.78
185.00	10	205.56
4700.00	5	4 947.37
2.90	5	3.05
24.50	12	27.84
10.00	8	10.87
12.50	7	13.44
9.75	15	11.47
199.50	20	249.38
99.00	25	132.00

5
- a \$20.49
- b \$163.93
- c \$11.89
- d \$19.66
- e \$12.95
- f \$37.54
- g \$24.39
- h \$105.90
- i \$0.81
- j \$0.66

6
- a 40 students
- b 33 students

7 \$20

8 80 kg

9 210 litres (3 s.f.)

Exercise 5.12

1
- a 3.8×10^2
- b 4.2×10^6
- c 4.56×10^{10}
- d 6.54×10^{13}
- e 2×10^1
- f 1×10^1
- g 1.03×10^1
- h 5×10^0
- i 4×10^{-3}
- j 5×10^{-5}
- k 3.2×10^{-5}
- l 5.64×10^{-8}

2
- a 2 400 000
- b 310 000 000
- c 10 500 000
- d 9 900
- e 71
- f 0.000 36
- g 0.000 000 016
- h 0.000 000 203
- i 0.0088

3
- a 0.000 025 kg
- b 2.5×10^{-5} kg

4 4.0208×10^{13} km

5 8.4×10^{-2} mm

Investigation: Standard form on a calculator

1 Learners' own answers.

2 a i 1.09×10^5
ii 2.876×10^{-6}
iii 4.012×10^9
iv 1.89×10^7
v 3.123×10^{13}
vi 2.876×10^{-4}
vii 9.02×10^{15}
viii 8.076×10^{-12}
ix 8.124×10^{-11}
x 5.0234×10^{19}

b 8.076×10^{-12}
8.124×10^{-11}
2.876×10^{-6}
2.876×10^{-4}
1.09×10^5
1.89×10^7
4.012×10^9
3.123×10^{13}
9.02×10^{15}
5.0234×10^{19}

Exercise 5.13

1 Display will vary according to the calculator used.

a 4.2×10^{12}
b 0.000 018
c 2 700 000
d 0.0134
e 0.000 000 001
f 42 300 000
g 0.000 310 2
h 3 098 000 000
i 2.076×10^{-23}

2 a 1.3607×10^{18}
b 1.0274×10^{-15}
c 1.0458×10^0
d 1.6184×10^{11}
e 5.2132×10^{19}
f 3.0224×10^{-16}
g 2.3141×10^{12}
h 1.5606×10^{17}

3 a 2.596×10^6
b 7.569×10^{-5}
c 4.444×10^{-3}
d 1.024×10^{-7}
e 3.465×10^{-4}
f 2.343×10^7
g 5.692×10^3
h 3.476×10^{-3}
i 1.040×10^{-3}

4 Volume of space is $(3.27 \times 10^{-7}) \times (2 \times 10^{-7}) \times (1.16 \times 10^{-4}) = 7.5864 \times 10^{-17}$.
$7.927 \times 10^{-17} > 7.5864 \times 10^{-17}$, so it will not fit.

5 a 1.07×10^9
b 1×10^{12}

Exercise 5.14

1 a 8×10^{30}
b 4.2×10^{12}
c 2.25×10^{26}
d 1.32×10^9
e 1.4×10^{32}
f 3×10^1
g 2×10^1
h 3×10^3
i 3×10^{42}
j 1.2×10^3
k 5×10^2
l 1.764×10^{15}

2 a 3.4×10^4
b 3.7×10^6
c 5.627×10^5
d 7.057×10^9
e 5.7999973×10^9

3 a 8×10^{-10}
b 6.4×10^{-12}
c 3.15×10^{-9}
d 3.3×10^{-2}
e 2×10^{33}
f 7×10^{-37}
g 5×10^{12}
h 1.65×10^1

4 **a** 2.731×10^{-2}
b 2.88×10^{-1}
c 7.01056×10^{3}
d 1.207×10^{-5}

5 8.64×10^{4} seconds

6 **a** 3×10^{9} metres
b 6×10^{9} metres
c 3.06×10^{10} metres

Practice questions

1 $\frac{5}{16}$

2 **a** 5%
b $\frac{6}{25}$
c 17 822

3 29 975

4 7.5%

5 The 20% increase is not an increase on the original salary, it is an increase on the larger salary at the end of year 1. Timur's salary will be multiplied by 1.1 × 1.2 = 1.32. Timur's salary is increased by 32% overall.

6 $n = 1.88 \times 10^{7}$

7 **a** 9.46×10^{12}
b 0.423
c 1.88×10^{5}

8 3 299 000

9 $\frac{ab}{100} \times 10^{m+n+2}$

10 **a** $4400 (2 s.f.)
b $5000 (2 s.f.)
c $V\left(1 - \frac{x}{100}\right)^{n}$

11 On 1 January 2038 the painting is due to be worth $2520. So the value of the painting will first be worth more than $2500 during 2037.

Practice questions worked solutions

1 $\frac{5}{6}\left(\frac{1}{4} + \frac{1}{8}\right) = \frac{5}{6}\left(\frac{2}{8} + \frac{1}{8}\right)$

$= \frac{5}{{}_2\cancel{6}} \times \frac{\cancel{3}^1}{8}$

$= \frac{5}{16}$

2 **a** 19 + 24 + 31 + 10 + 11 = 95
100% − 95% = 5% gained a grade U

b $\frac{24}{100} = \frac{6}{25}$

c 19% of 93 800 = 0.19 × 93 800
= 17 822

3 27 500 × 1.09 = 29 975 cases

4 $\frac{172 - 160}{160} \times 100\% = 7.5\%$

5 The second increase is 20% on an already increased salary.
1.1 × 1.2 = 1.32 ⇒ 32% increase

6 $n = \frac{3 \times 10^{8} \times 2 \times 10^{7}}{3 \times 10^{8} + 2 \times 10^{7}}$

$= \frac{6 \times 10^{15}}{300\,000\,000 + 20\,000\,000}$

$= \frac{6 \times 10^{15}}{320\,000\,000}$

$= \frac{6 \times 10^{15}}{3.2 \times 10^{8}}$

$= 1.875 \times 10^{7}$

$= 1.88 \times 10^{7}$ (to 3 s.f.)

7 **a** $3.0 \times 10^{5} \times 60 \times 60 \times 24 \times 365$
$= 9.46 \times 10^{12}$ km

b $\frac{4.0 \times 10^{13}}{9.46 \times 10^{12}} = 4.23$ light years

c $3.0 \times 10^{5} \times 0.625 = 1.88 \times 10^{5}$ miles per second

8 $\frac{3.352 \times 10^{6}}{1.016} = 3\,299\,000$

9 $xy = ab \times 10^{m+n}$

$= \frac{ab}{100} \times 10^{m+n+2}$

10 **a** $\frac{3875}{0.88} = \$4403$

b Divide by 0.88 again = $5004

c $V\left(\frac{100 - x}{100}\right)^{n}$

11 $\$1800 \times 1.02 = \$1836$

$\times 1.02 = \$1873$

$\times 1.02 = \$1910$

$\times 1.02 = \$1948$

$\times 1.02 = \$1987$

$\times 1.02 = \$2027$

$\times 1.02 = \$2068$

$\times 1.02 = \$2109$

$\times 1.02 = \$2151$

$\times 1.02 = \$2194$

$\times 1.02 = \$2238$

$\times 1.02 = \$2283$

$\times 1.02 = \$2328$

$\times 1.02 = \$2375$

$\times 1.02 = \$2423$

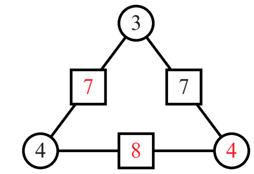
$\times 1.02 = \$2471$

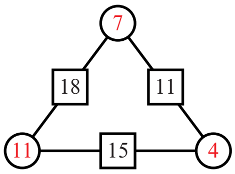
$\times 1.02 = \$2520$

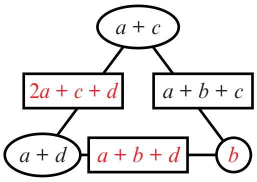
Therefore, the amount in 2038 is \$2520.

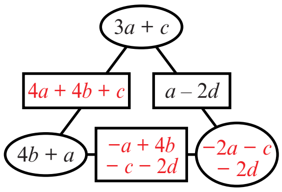
Chapter 6

Getting started









Exercise 6.1

1 **a** $x = 7$
b $x = -5$
c $x = 9$
d $x = -\frac{62}{7}$
e $x = 5$
f $n = 11$
g $q = 1.75$
h $t = 0.5$

2 **a** $x = 2$
b $x = -10$
c $y = -3$
d $x = \frac{11}{15}$
e $p = 1$
f $x = 60$

3 **a** $x = 2$
b $p = 3$
c $t = 1$
d $m = 5$
e $n = 10$
f $x = -\frac{13}{6}$

4 **a** $x = 2$
b $x = 2$
c $x = 12$
d $x = 1$
e $x = \frac{15}{4}$
f $p = -\frac{5}{2}$
g $p = \frac{20}{13}$
h $x = -1$

5 **a** $x = 11.5$
b $x = 10.5$
c $x = 16.7$
d $x = 3$
e $x = -\frac{1}{7}$
f $x = 10$

6 **a** $x = 3$
b $x = \frac{1}{2}$
c $x = \frac{5}{2}$
d $x = 0$, so there is no solution
e $x = 1$
f $x = -13$

7 **a** $x = 1$
b $x = \frac{1}{3}$
c $x = -\frac{3}{4}$
d $x = \frac{1}{3}$
e $x = \frac{1}{5}$
f $x = -\frac{1}{6}$

Exercise 6.2

1 **a** $3(x + 2)$
b $3(5y - 4)$
c $8(1 - 2z)$
d $5(7 + 5t)$
e $2(x - 2)$
f $3x + 7$
g $2(9k - 32)$
h $11(3p + 2)$
i $2(x + 2y)$
j $3(p - 5q)$
k $13(r - 2s)$
l $2(p + 2q + 3r)$

2 a $7(3u - 7v + 5w)$
b $3x(y + 1)$
c $3x(x + 1)$
d $3p(5q + 7)$
e $3m(3m - 11)$
f $10m^2(9m - 8)$
g $12x^3(3 + 2x^2)$
h $4pq(8p - q)$

3 a $2m^2n^2(7 + 2mn)$
b $abc(17 + 30b)$
c $m^2n^2(49m + 6n)$
d $\frac{1}{2}(a + 3b)$
e $\frac{1}{8}x(6x^3 + 7)$
f $8(x - 4)$
g $(x + 1)^2 (1 - 4x)$
h $2x^3(3 + x + 2x^2)$
i $7xy(x^2 - 2xy + 3y)$
j $(y + 3)(x + 2)$

4 a $4(x + 3y)$
b $15(3x + y)$
c $a^2b(3a - 4b)$
d $3(17z^3 + 7x^2)$
e $4x^3y^4(3 - 5x^2y)$
f Fully factorised

5 a $3(3x + 4y)$
b $3(3x - 4y)$
c $10a(x + y)$
d $(2x^3 - 3)(5x^2 - 2)$
e $(x + y)(x - y - 2)$
f $(x - 2m + 1)(x + 2m - 1)$
g $(4x + 5y)(2a + 3b)$
h $(abx + bcy)(1 + c)$
i $(ax - by)(1 + k)$

6 a $4(6x + 35)$
b $240(2x + 5)$

7 $3(60 - x) - 4y$

8 a 9003×10^m
b This is not in standard form because 9003 is greater than 10.
c In standard form: $9.003 \times 10^{m+3}$.

Sums of consecutive numbers

1 Students' choice of numbers.

2 They are all divisible by 3.

3 a $n + 1$ and $n + 2$.
b $n + n + 1 + n + 2 = 3n + 3 = 3(n + 1)$. This is divisible by 3. This confirms the answer to question 2.

4 $n + n + 1 + n + 2 + n + 3 + n + 4 = 5n + 10 = 5(n + 2)$. Divisible by 5.
Seven numbers $7n + 21 = 7(n + 3)$. Divisible by 7.

5 Sum of four numbers is divisible by 2, not 4. Sum of six numbers is divisible by 3 not 6.

6 When the number of integers is p, where p is prime, the answer is always divisible by p. This doesn't work for non-primes.

Exercise 6.3

1 a $a = c - b$
b $r = p - q$
c $h = \frac{g}{f}$
d $b = \frac{d - c}{a}$
e $a = bc$
f $n = \frac{t + m}{a}$

2 a $m = an - t$
b $a = \frac{t}{n - m}$
c $x = \frac{tz}{y}$
d $x = bc + a$
e $y = c - \frac{d}{x}$
f $b = a - c$

3 a $r = q(p - t)$
b $b = \frac{x - a}{c}$
c $m = n - \frac{t}{a}$
d $a = \frac{bc}{d}$
e $a = x - bc$
f $z = \frac{xy}{t}$

4 a $b = c^2$
b $b = \frac{c^2}{a}$
c $b = \left(\frac{c}{a}\right)^2$

d $b = c^2 - c$

e $b = x - c^2$

f $y = \left(\frac{x}{c}\right)^2$

5 a $w = \frac{P}{2} - l$

b $w = 35.5$ cm

6 a $a = \frac{(v-u)}{t}$

b 10.72 m/s

7 $l = g\left(\frac{T^2}{4\pi^2}\right)$

Practice questions

1 9

2 a Temperature will be 19 °C.

b You will need to climb to 1500 m.

c 52.9

3 $y = \frac{qx}{p}$

4 $x^2y(x^2y^2 + 7 - 3xy^2)$

5 $-\frac{5}{2}$

6 $2n + 1 + 2n + 3 + 2n + 5 = 69$

$6n + 9 = 69$

$n = 10$

Smallest number is 21.

7 a $14 - n$

b $3n - (14 - n) = 4n - 14 = 22$

$n = 9$

8 a $F = \frac{9T}{5} + 32$

b 17.6 °F

c −40 degrees

9 a $3x + 42$ cm

b $x + 3$ cm

c $x + 7$ cm

d $8x + 30 = 2(4x + 15)$ cm

e $8x + 30 = 3x + 42$

$x = 2.4$ cm

f Perimeter of each shape is 49.2 cm.

Area is 80.28 cm².

10 a $12x + 13$

b 0.5

c 19 cm

d 4.27 cm

11 $b = \frac{1}{(c-a)^2}$

12 $2(2p^2 + a)(pq^3 + 2)$

13 $\frac{5}{2}$

Practice questions worked solutions

1 $T = \frac{3p-5}{2}$

$12 = \frac{3p-5}{2}$

$3p - 5 = 24$

$3p - 5 = 29$

$p = \frac{29}{3}$

2 a Increase in height $= 1300 - 500$
$= 800$ m

$\frac{800}{200} = 4$ °C

So, the new temperature = 23 °C − 4° °C
= 19 °C

b $\frac{\text{increase in height}}{200} = 5$

Therefore, increase in height $= 5 \times 200$
$= 1000$ m

c $13 = \frac{700 - 12}{q}$

$13 = \frac{688}{q}$

$13q = 688$

$q = \frac{688}{13} = 52.9$

3 $\frac{x}{y} = \frac{p}{q}$

$py = qx$

$y = \frac{qx}{p}$

4 $x^4y^3 + 7x^2y - 3x^3y^3$

$= x^2y(x^2y^3 + 7 - 3xy^2)$

5 $0.8x + 3 = 2(0.6x + 2)$

$0.8x + 3 = 1.2x + 4$

$0.4x = -1$

$x = -\frac{1}{0.4} = -2.5$

6 $2n + 1 + 2n + 3 + 2n + 5 = 69$

$6n + 9 = 609$

$6n = 60$

$n = 10$

The smallest number is $2n + 1 = 21$

7 **a** $14 - n$

b $3n + (14 - n)(-1)$

$= 3n - 14 + n$

$= 4n - 14$

$4n - 14 = 22$

$4n = 36$

$n = 9$

8 **a** $T = \frac{5}{9}(F - 32)$

$\frac{9T}{F} = F - 32$

Therefore, $F = \frac{9T}{5} + 32$

b $F = \frac{9}{5}(-8) + 32$

$= -\frac{72}{5} + \frac{160}{5}$

$= \frac{88}{5}$

$= 17.6\,°\text{C}$

c $x = \frac{5}{9}(x - 32)$

$9x = 5(x - 32)$

$9x = 5x - 160$

$x = -40$

So, $-40\,°\text{F} = -40\,°\text{C}$

9 **a** $x + 14 + x + 14 + x + 14$

$= 3x + 42$

b $EF = 2x + 3 - x$

$= x + 3$

c $FG = 12 + 2x - (x + 5)$

$= 12 + 2x - x - 5$

$= x + 7$

d $x + 12 + 2x + 2x + 3 + x + 5 + x + 3 + x + 7$

$= 8x + 30$

e $3x + 42 = 8x + 30$

$5x = 12$

$x = \frac{12}{5} = 2.4$

f Perimeter of $ABC = 3x + 42$

$= 7.2 + 42$

$= 49.2\,\text{cm}$

Perimeter of $DEFGHI = 8x + 30$

$= 19.2 + 30$

$= 49.2\,\text{cm}$

Area $DEFGHI = (2x + 3)(12 + 2x)$

$- (x + 3)(x + 7)$

$= 7.8 \times 16.8 - 5.4 \times 9.4$

$= 80.28\,\text{cm}^2$

10 **a** $3 + 2(x + 1) + 2(3x + 2) + 4(x + 1)$

$= 3 + 2x + 2 + 6x + 4 + 4x + 4$

$= 12x + 13$

b $4(x + 1) = 3 + 2(x + 1)$

$4x + 4 = 3 + 2x + 2$

$2x = 1$

$x = \frac{1}{2}$

c $3x + 2 = \frac{3}{2} + 2 = \frac{7}{2}$

$4(x + 1) = 4 \times \frac{3}{2} = 6$

Perimeter $= 2\left(\frac{7}{2} + 6\right)$

$= 7 + 12$

$= 19\,\text{cm}$

d Area of rectangle $= \frac{7}{2} \times 6 = 21\,\text{cm}^2$

Area of square $\times 1.15 = 21$

Therefore, area of square $= \frac{21}{1.15}$

Side of square $= \sqrt{\frac{21}{1.15}} = 4.27\,\text{cm}$

11 $a + \frac{1}{\sqrt{b}} = c$

$\frac{1}{\sqrt{b}} = c - a$

$\frac{\sqrt{b}}{1} = \frac{1}{c - a}$

$b = \left(\frac{1}{c - a}\right)^2$

12 $4p^3q^4 + 8p^2q + 2apq^3 + 4a$

$= 2(2p^3q^4 + 4p^2q + apq^3 + 2a)$

13 $4^{2x-3} = 2^{5+2(x-3)}$

$(2^2)^{2x-3} = 2^{5+2x-6}$

$(2^2)^{2x-3} = 2^{2x-1}$

$2(2x - 3) = 2x - 1$

$4x - 6 = 2x - 1$

$2x = 5$

$x = \frac{5}{2}$

Chapter 7

Getting started

a $\frac{1}{24}$

b $\frac{1}{4}$

c $\frac{1}{10}$

d $\frac{3}{8}$

e $\frac{1}{2}$

f $\frac{1}{2}$

Exercise 7.1

1 **a** 12.5 cm
b 9 cm

2 **a** 25 cm
b 35 m
c 23 km

3 **a** 16 m^2
b 42 m^2
c 8 cm^2
d 54 cm^2

4 **a** **i** 5850 cm^2
ii 0.585 m^2
b 360 cm

5 **a** 50 m^2
b 52.29 m^2
c 33.1 cm^2 (3 s.f.)
d 37.8 cm^2
e 36 cm^2
f 145.16 cm^2
g 55.7 cm^2 (3 s.f.)

6 **a** $h = 6$ cm
b $b = 17$ cm
c $a = 2.86$ cm (3 s.f.)
d $b = 5$ cm
e $h = 10.2$ cm (3 s.f.)

7 183 tiles

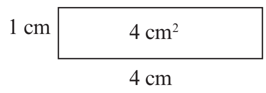
8 **a** 74.8 cm^2
b $\frac{133}{162}xy$

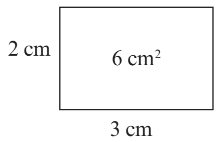
9 14.14 cm × 14.14 cm

10 **a** 0.3458 m^2
b $\frac{1}{2}$
c Consider this as a red/black triangle on top of a yellow/white triangle. The base of each triangle is the same but the height of the white/yellow triangle is twice the height of the red/black triangle, so its area will be twice as big. Therefore the area of the red/black triangle is half of the area of the yellow/white triangle which means that the area of the yellow and white region of the flag is the same as the area of the red and black region.

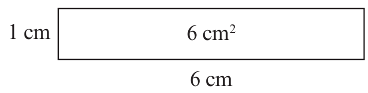
11 Students' answers will vary; the following are just examples.

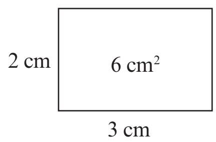
a



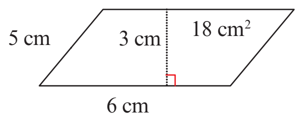


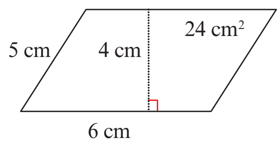
b



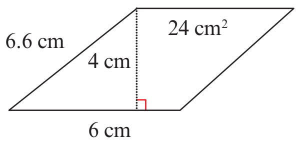


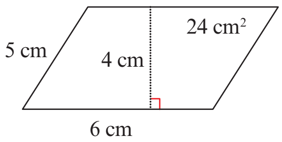
c





d





12 Area = 440 square units and perimeter = 102 units

13 32 cm^2

14 72 cm

Increasing areas

1 4 cm^2

2 Area of 6 cm square is 36 cm^2. This is nine times larger than the 2 cm square.

3 Area of 10 cm square is 100 cm^2. This is 25 times larger than the 2 cm square.

4 When we multiply the side length by k the area is multiplied by k^2.

5 14 cm^2

6 The same result applies: multiplying the side length by k means the area is multiplied by k^2.

Exercise 7.2

1 Answers are correct to 3 s.f.

a $A = 50.3$ m^2 $C = 25.1$ m
b $A = 7.55$ mm^2 $C = 9.74$ mm
c $A = 0.503$ m^2 $C = 2.51$ m
d $A = 0.785$ cm^2 $C = 3.14$ cm
e $A = 1.57$ km^2 $C = 4.44$ km
f $A = 1.27$ m^2 $C = 4$ m (exact)

2 Answers correct 3 s.f.

a $A = 250$ cm^2
b $A = 13.7$ cm^2
c $A = 68.3$ m^2
d $A = 55.4$ cm^2
e $A = 154$ m^2
f $A = 149$ cm^2

3 23 bags

4 White = 0.1 m^2 Red = 1.0 m^2

5 0.03 m^2

6 2 × 12 cm pizza ≈ 226.2 cm^2 and 24 cm pizza ≈ 452.4 cm^2, so two small pizzas is not the same amount of pizza as one large pizza.

Exercise 7.3

1 a $C = 9\pi$ cm $A = 20.25\pi$ cm^2
b $C = 74\pi$ cm $A = 1369\pi$ cm^2
c $C = 120\pi$ mm $A = 3600\pi$ mm^2
d $C = \frac{14\pi}{2} + 14$ cm $A = \frac{49\pi}{2}$ cm^2
e $C = \frac{12\pi}{2} + 12$ cm $A = \frac{36\pi}{2}$ cm^2
f $C = \frac{18.4\pi}{2} + 18.4$ cm $A = \frac{84.64\pi}{2}$ cm^2

2 a $C = 10\pi$ cm
b $C = 14\pi$ cm
c $A = 0.9025\pi$ cm^2
d $A = \frac{9\pi}{2}$ cm^2

3 a 12 cm
b $A = 144 - 36\pi$ cm^2

4 $A = 32\pi$ mm^2

Exercise 7.4

1 a $A = 12.6$ cm^2 $P = 16.2$ cm
b $A = 25.1$ cm^2 $P = 22.3$ cm
c $A = 1.34$ cm^2 $P = 7.24$ cm
d $A = 116$ cm^2 $P = 44.2$ cm
e $A = 186$ m^2 $P = 55.0$ m
f $A = 0.185$ cm^2 $P = 1.88$ cm
g $A = 36.3$ cm^2 $P = 24.6$ cm
h $A = 98.1$ m^2 $P = 43.4$ m

2 a $A = 198$ m^2 $l = 22.0$ m
b $A = 70.4$ cm^2 $l = 17.2$ cm
c $A = 94.7$ cm^2 $l = 29.6$ cm
d $A = 14.5$ m^2 $l = 9.69$ m

3 a $A = 16.4$ m^2 $P = 6.54$ m
b $A = 243$ cm^2 $P = 62.5$ cm

4 1856 m

5 70.0 m

6 14 cm

7 9 cm

8 **a** $A = 40 - \frac{25}{8}\pi\,\text{cm}^2$ $P = 21 + \frac{5}{2}\pi\,\text{cm}$

b $A = 90 - 4\pi\,\text{cm}^2$ $P = 30 + 2\pi\,\text{cm}$

c $A = 60 - \frac{25}{6}\pi\,\text{m}^2$ $P = 34 + \frac{5}{3}\pi\,\text{m}$

d $A = 70.56 - 17.64\pi\,\text{cm}^2$ $P = 16.8 + 8.4\pi\,\text{cm}$

e $A = 324 - 81\pi\,\text{m}^2$ $P = 18\pi\,\text{m}$

9 **a** $P = 144\,\text{cm}$ $A = 1400\,\text{cm}^2$

b $P = 7.07\,\text{cm}$ $A = 3.63\,\text{cm}^2$

c $P = 15.6\,\text{cm}$ $A = 17.0\,\text{cm}^2$

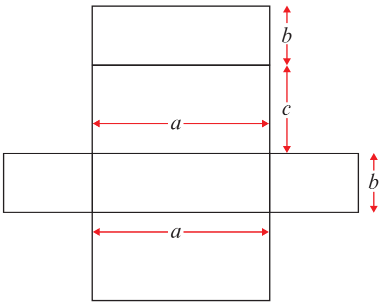
d $P = 26.6\,\text{cm}$ $A = 32.6\,\text{cm}^2$

e $P = 61.1\,\text{cm}$ $A = 181\,\text{cm}^2$

10 6.77 cm

Exercise 7.5

1



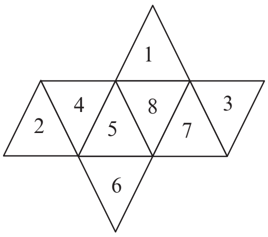
2 **a** Trapezium-based prism

b O and S

c $PQ = RQ = UV = VW$

3 **a** is correct

4



5 **a** $344\,\text{cm}^2$

b $0.000\,24\,\text{m}^3$

Exercise 7.6

1 Volume = $66\,\text{cm}^3$ Surface area = $144\,\text{cm}^2$

2 **a** **i** $720\,\text{cm}^3$

ii $548\,\text{cm}^2$

b **i** $13.8\,\text{mm}^3$ (3 s.f.)

ii $40.3\,\text{mm}^2$ (3 s.f.)

3 $432\,000\,\text{cm}^3$

4 **a** $768\,\text{cm}^3$

b $816\,\text{cm}^2$

5 $3.39\,\text{m}^3$ (3 s.f.)

6 **a** $200\pi\,\text{cm}^3$

b $542\,\text{cm}^2$ (3 s.f.)

7 $241\,\text{cm}^3$ (3 s.f.)

8 **a** 320 boxes

b $8.5\,\text{m}^2$

c $48\,\text{m}^3$

9 Volume = $264\,\text{cm}^3$

Surface area = $306\,\text{cm}^2$

Exercise 7.7.

1 **a** $1600\pi\,\text{cm}^2$ (3 s.f.)

b $\frac{32\,000}{3}\pi\,\text{cm}^3$ (3 s.f.)

2 $5300\,\text{cm}^3$ (3 s.f.)

3 $549\,000\,000\,000\,\text{km}^3$ (3 s.f.)

4 $2\,600\,000\,\text{m}^3$ (3 s.f.)

5 **a** $340\pi\,\text{m}^2$ (3 s.f.)

b $725\pi\,\text{m}^3$ (3 s.f.)

6 $1110\,\text{cm}^3$ (3 s.f.)

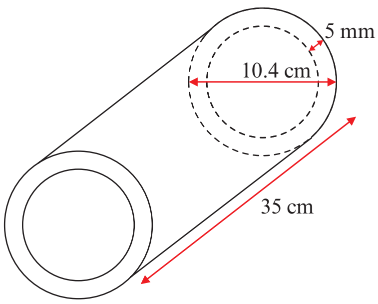
7 **a** $754\,\text{cm}^3$ (3 s.f.)

b $415\,\text{cm}^2$ (3 s.f.)

8 2.29 cm (3 s.f.)

9 $\frac{R}{r} = \sqrt[3]{2}$

10 **a**



b Volume of metal in the tube

$$= \left(\pi \times \left(\frac{10.4}{2}\right)^2 \times 35\right) - \left(\pi \times \left(\frac{10.4 - 1}{2}\right)^2 \times 35\right)\,\text{cm}^3$$

c $544\,\text{cm}^3$ (3 s.f.)

d Total surface area of tube = 2 × area of ring + area of outer tube + area of inner tube
(Note 'ring' is the 5 mm thick end of the cylinder.)

$$2 \times \left[\pi \times \left(\frac{10.4}{2}\right)^2 - \pi\left(\frac{10.4-1}{2}\right)^2\right] + (\pi \times 10.4 \times 35) + [\pi \times (10.4-1) \times 35]\ \text{cm}^2$$

11 260 cm^3

12 a i 205 cm^2

ii 197.47 cm^2

b i 225 cm^3

ii 254.47 cm^2

Practice questions

1 33 900 mm (3 s.f.)

2 P = 32.3 cm (3 s.f.) Area = 47.7 cm^2

3 2.31 m^3 (3 s.f.)

4 $\frac{550}{3}\pi$ cm

5 4.02 m^2

6 a 62.8 cm

b 125.7 cm^2

c 37.5%

7 a 168 cm^3

b 0.000 168 m^3

8 Volume = 336 cm^3

Surface area = 360 cm^2

Practice questions worked solutions

1 $600 \times \pi \times 18 = 33\,929$ mm

$= 33.9$ m

2 Perimeter $= 9 + 6 + 9 + 1 + 1 + \frac{1}{2}(\pi + 4)$

$= 32.3$ cm

Area $= 9 \times 6 - \frac{1}{2}\pi \times 4^2$

$= 28.9$ cm^2

3 Volume $= \pi \times \left(\frac{1.4}{2}\right)^2 \times 1.5 = 2.31$ m^3

4 $\frac{1}{3} \times \pi \times 5^2 \times 12 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$

$= 100\pi + \frac{250}{3}\pi$

$= \frac{550\pi}{3}$ cm^3

5 $8 \times \pi \times 1.2^2 \times \frac{40}{360}$

$= 4.02$ m^2

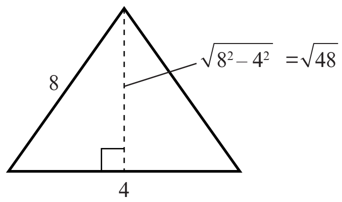
6 a The triangle removes $\frac{60°}{360°} = \frac{1}{6}$ of each circle

Therefore, perimeter $= 3 \times \frac{5}{6} \times \pi \times 8$

$= 20\pi$ cm

b Area $= 3 \times \frac{5}{6} \times \pi \times 4^2$ + area of triangle

$= 40\pi$ + area of triangle



Area of triangle $= \frac{1}{2} \times 8 \times \sqrt{48}$

Therefore, total area $= 40\pi + 2\sqrt{48}$

$= 40\pi + 16\sqrt{3}$ cm^2

c Scale factor of lengths $= \frac{11}{8} = 1.375$

Therefore, increase in perimeter = 37.5%.

7 a Volume $= 6 \times 5 \times 7 - \frac{1}{2} \times 3 \times 4 \times 7$

$= 210 - 42$

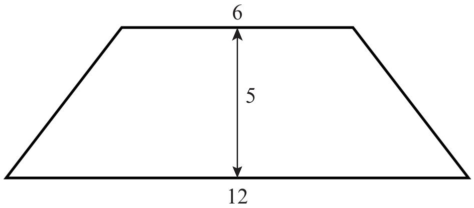
$= 168$ cm^3

b 1 m^3 = 100 × 100 × 100 cm^3

= 1 000 000 cm^3

Volume in m^3 $= \frac{168}{1\,000\,000} = 0.000\,168$ m^3

8 Each sloping face is a trapezium.



Therefore, surface area

$=12^2 + 6^2 + 4 \times \left(\frac{12+6}{2}\right) \times 5$

$= 360\,\text{cm}^2$

Volume $= \frac{1}{3} \times 12^2 \times 8 - \frac{1}{3} \times 6^2 \times 4$

$= 336\,\text{cm}^2$

Chapter 8

Getting started

1, 2 Students' individual work.

3 It is sensible to choose 'higher' for any number where more than half the numbers are higher (so any decimal between 0 and 0.5). It is sensible to choose 'lower' for any number where more than half the numbers are lower (so any decimal between 0.5 and 1).

Talking about probability

1 Probability only tells us the expected, or 'on average' proportion of 'true' answers. It does not tell us exact proportions.

2 The three possible outcomes are not necessarily equally likely.

3 Statistical experiments always give a range of outcomes. Different outcomes are possible, so they also have a non-zero probability.

4 Any random experiment like this can produce unexpected outcomes, because each of the combinations of red and blue socks is possible.

5 We should only give probabilities for outcomes of experiments that have not already been completed. It makes more sense to talk about the probability that it rains *tomorrow*.

Exercise 8.1

1 **a** $\frac{7}{50}$

b 0.14

2 **a** $\frac{1}{10}$

b $\frac{3}{20}$

c $\frac{131}{260}$

d $\frac{141}{260}$

3 **a** $\frac{235}{300} = 0.783$

b 233

4 **a** 80

b $\frac{7}{20}$

c 30

5 5 750

6 **a** $\frac{1}{77}$

b $\frac{76}{77}$

7 **a** $\frac{4}{9}$

b $\frac{5}{9}$

c $\frac{0}{9}$

d 1

8 9 blue balls

9 **a** $\frac{1}{13}$

b $\frac{1}{4}$

c $\frac{1}{2}$

d $\frac{3}{13}$

Exercise 8.2

1 **a**

		First throw	
		H	T
Second throw	H	HH	TH
	T	HT	TT

b **i** $\frac{1}{2}$

ii $\frac{1}{4}$

iii $\frac{3}{4}$

iv $\frac{1}{4}$

2 **a**

		First dice					
	×	1	2	3	4	5	6
Second dice	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

b **i** $\frac{1}{36}$

ii $\frac{0}{36}$

iii $\frac{2}{9}$

iv $\frac{7}{9}$

v $\frac{1}{6}$

vi $\frac{2}{9}$

3 **a**

		Spinner				
		1	2	4	6	8
Tetrahedral dice	3	3	3	4	6	8
	5	5	5	5	6	8
	7	7	7	7	7	8
	9	9	9	9	9	9

b **i** $\frac{6}{20} = \frac{3}{10}$

ii $\frac{14}{20} = \frac{7}{10}$

iii $\frac{9}{20}$

iv $\frac{9}{20}$

v $\frac{8}{20} = \frac{2}{5}$

4 **a**

		First throw					
		4	6	10	12	15	24
Second throw	4	4	2	2	4	1	4
	6	2	6	2	6	3	6
	10	2	2	10	2	5	2
	12	4	6	2	12	3	12
	15	1	3	5	3	15	3
	24	4	6	2	12	3	24

b **i** $\frac{5}{18}$

ii $\frac{2}{3}$

iii 1

iv $\frac{17}{18}$

v $\frac{2}{9}$

vi $\frac{8}{18}$

5 **a** Set A

+	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12

Set B

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b Set B, but set A is not far away from being sensible

Exercise 8.3

1 **a** $\frac{1}{36}$

b $\frac{1}{4}$

c $\frac{1}{6}$

d $\frac{5}{6}$

2 **a** Red, red; red, blue; blue, red; blue, blue

b **i** 58.3%

ii 41.7%

iii 24.3%

iv 51.4%

v 48.6%

vi 34.0%

vii 66.0%

3 **a** $\frac{1}{169}$

b $\frac{1}{2704}$

c $\frac{1}{52}$

d $\frac{3}{8}$

e $\frac{1}{2}$

f $\frac{1}{2}$

4 a 0.24
b 0.24
c 0.36
d 0.76
e 0.52

Practice questions

1 a $\frac{2}{5}$
b $\frac{3}{5}$
c 0

2 a $\frac{1}{3}$
b 0
c $\frac{5}{6}$
d $\frac{1}{3}$

3 a

Face	1	2	3	4
Probability	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{5}{18}$	$\frac{1}{6}$
	$\frac{4}{18}$	$\frac{6}{18}$	$\frac{5}{18}$	$\frac{3}{18}$

b 2
c $\frac{13}{18}$

4 a

		Josh						
	+	$5	$1	$1	50c	20c	20c	20c
Soumik	$5	$10	$6	$6	$5.50	$5.20	$5.20	$5.20
	$5	$10	$6	$6	$5.50	$5.20	$5.20	$5.20
	$5	$10	$6	$6	$5.50	$5.20	$5.20	$5.20
	$2	$7	$3	$3	$2.50	$2.20	$2.20	$2.20
	50c	$5.50	$1.50	$1.50	$1	70c	70c	70c
	50c	$5.50	$1.50	$1.50	$1	70c	70c	70c
	50c	$5.50	$1.50	$1.50	$1	70c	70c	70c

b $\frac{6}{49}$
c $\frac{18}{49}$
d $\frac{25}{49}$

5 **a** $\frac{1}{8}$

b $\frac{1}{9}$

c $\frac{11}{96}$

6 **a**

		Square spinner			
		1	2	3	4
Pentagon spinner	1	1, 1	2, 1	3, 1	4, 1
	2	1, 2	2, 2	3, 2	4, 2
	3	1, 3	2, 3	3, 3	4, 3
	4	1, 4	2, 4	3, 4	4, 4
	5	1, 5	2, 5	3, 5	4, 5

b **i** $\frac{4}{20} = \frac{1}{5}$

ii $\frac{7}{20}$

iii $\frac{2}{20} = \frac{1}{10}$

iv $\frac{9}{20}$

c **i** $P(A \text{ or } B) = \frac{7}{20}$

but $P(A) + P(B) = \frac{1}{5} + \frac{7}{20} = \frac{11}{20}$

ii Events A and B are not mutually exclusive.

Practice questions worked solutions

1 **a** $\text{P(banana)} = \frac{4}{10} = \frac{2}{5}$

b $\frac{2+4}{10} = \frac{6}{10} = \frac{3}{5}$

c 0

2 **a** $\frac{2}{6} = \frac{1}{3}$

b $\frac{0}{6} = 0$

c $\frac{5}{6}$

d $\frac{2}{6} = \frac{1}{3}$

3 **a** $\frac{4}{18}, \frac{4}{16}, \frac{5}{18}, \frac{3}{18}$

b Face 2

c $\frac{4+6+3}{18} = \frac{13}{18}$ or $1 - \frac{5}{18} = \frac{13}{18}$

4 Josh: \$1, 50c, \$5, 20c, 20c, 20c

Soumik: \$5, \$5, \$5, \$2, 50c, 50c, 50c

a

		Josh					
	+	\$5	\$1	50c	20c	20c	20c
Soumik	\$5	\$10	\$6	\$5.50	\$5.20	\$5.20	\$5.20
	\$5	\$10	\$6	\$5.50	\$5.20	\$5.20	\$5.20
	\$5	\$10	\$6	\$5.50	\$5.20	\$5.20	\$5.20
	\$2	\$7	\$3	\$2.50	\$2.20	\$2.20	\$2.20
	50c	\$5.50	\$1.50	\$1	70c	70c	70c
	50c	\$5.50	\$1.50	\$1	70c	70c	70c
	50c	\$5.50	\$1.50	\$1	70c	70c	70c

b $\frac{3}{42} = \frac{1}{14}$

c $\frac{15}{42} = \frac{5}{14}$

d $\frac{22}{42} = \frac{11}{21}$

5

	1	2	3	4	5	6
Probability	x	x	x	x	$\frac{1}{6}$	$\frac{1}{3}$

a $4x + \frac{1}{6} + \frac{1}{3} = 1$

$4x = \frac{6}{6} - \frac{1}{6} - \frac{2}{6} = \frac{1}{2}$

$x = \frac{1}{8}$

$\text{P(rolling 3)} = \frac{1}{8}$

b $\text{P(6 and 6)} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

c $\text{P}(2, 6) + \text{P}(6, 2) + \text{P}(3, 4) + \text{P}(4, 3)$

$= \frac{1}{8} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{8} + \frac{1}{8} \times \frac{1}{8} + \frac{1}{8} \times \frac{1}{8}$

$= \frac{1}{24} + \frac{1}{24} + \frac{1}{64} + \frac{1}{64}$

$= \frac{1}{12} + \frac{1}{32} = \frac{11}{96}$

6 a

		Square			
		1	2	3	4
Pentagon	1	1, 1	2, 1	3, 1	4, 1
	2	1, 2	2, 2	3, 2	4, 2
	3	1, 3	2, 3	3, 3	4, 3
	4	1, 4	2, 4	3, 4	4, 4
	5	1, 5	2, 5	3, 5	4, 5

b **i** P(total of the scores is 5) $= \frac{4}{20} = \frac{1}{5}$

ii P(the scores have a difference of 1)

$= \frac{7}{20}$

iii P(total of scores is 5 and scores have a difference of 1)

= P(2, 3) + P(3, 2)

$= \frac{2}{20} = \frac{1}{10}$

iv P(total of scores is 5 or scores have a difference of 1 or both)

P(1, 4) + P(4, 1) + P(2, 3) + P(3, 2) + P(4, 3) + (3, 4) + P(4, 5) + P(5, 4) + P(1, 2) + P(2, 1)

$= \frac{10}{20} = \frac{1}{2}$

c **i** $P(A \text{ or } B) = \frac{1}{2}$

$P(A) = \frac{4}{20}$

$P(B) = \frac{7}{20}$

$P(A) + P(B) = \frac{11}{20} \neq \frac{1}{2}$

ii A and B overlap so $P(A) + P(B)$ counts some possibilities twice.

Past paper questions

1 $\frac{13}{201} = 0.06467...$

$5.6\% = 0.056$

$0.065 = 0.065$

$\frac{5}{89} = 0.056179$

So $5.6\%, \frac{5}{89}, \frac{13}{201}, 0.065$

2 $1 - 0.15 = 0.85$

3 $\frac{1}{3}\pi \times 4.5^2 \times 10.4 = \frac{351}{5}\pi$

4 $7a(3a + 4b)$

5 **a** $1 - 0.38 = 0.62$

b 0

6 **a** 6.4×10^5

b 6×10^{-4}

7 **a**

Colour	Blue	Red	Yellow	Green
Probability	0.15	0.2	0.22	0.43

b $200 \times 0.2 = 40$ times

8 $\frac{1.2}{1.6} \times 100 = 7.5\%$

9 **a** 6.05×10^{-2}

b 4.0261×10^{11}

10 **a** isosceles

b 4.4 cm

c $\frac{1}{2} \times 5 \times 4.4 = 11\,\text{cm}^2$

d $\frac{1}{2} \times 5 \times 4.4 \times 6 = 66\,\text{cm}^3$

11 **a** $6a + 4b$

b $4 \times 9 + (3 \times -2) = 36 - 6 = 30$

c **i** $x = 80$

ii $3x = 21$

$x = 7$

iii $10x + 5 = 27$

$x = 22$

$x = 2.2$

d $3r = p + 5$

$r = \frac{p + 5}{3}$

12 **a** $3 \times 1.2 + \pi \times \left(\frac{1.2}{2}\right)^2 = 4.73\,\text{m}^2$

b $4.73097 \ldots \times 0.2 = 0.946\,\text{m}^3$

$0.946 \times 1000 = 946$ litres

c increase = 60.805 … litres

$= 0.0608 \ldots \text{m}^3$

$\frac{0.0608}{4.73} = 0.0128\,\text{m} = 1.29\,\text{cm}$

13 **a** $\pi \times 2.4 \times 6.3 + \pi \times 2.4^2 = \frac{4\pi R^2}{2} + \pi R^2$

$3\pi R^2 = \frac{522\pi}{25}$

$R^2 = \frac{522}{25 \times 3}$

$R = \sqrt{\frac{522}{75}} = 2.64\,\text{cm}$

b $\frac{1}{3} \times \pi \times 7.6^2 \times 16 - \frac{1}{3} \times \pi \times \left(\frac{7.6}{4}\right)^2 \times 4$

$= 953\,\text{cm}^3$

Chapter 9

Getting started

1 **a** Students' own description of a number sequence.

b The term in position n, in other words, the term in any position.

c Next term is 11 because you subtract 4 each time.

d Yes, once you know the rule for a sequence, you can use it to work out the value of the term in any position. In this sequence: $T_n = 27 - 4(n - 1)$, so $T_{20} = 27 - 4(20 - 1) = -49$.

2 **a** Rational number can be written as fractions in the form $\frac{a}{b}$ or the decimal equivalent. Decimals must terminate or be recurring. Irrational numbers cannot be written as fractions and they result in non-terminating, non-recurring decimals.

b π is irrational because it cannot be expressed as a fraction.

c $0.6666\ldots = \frac{2}{3}$, so it is rational.

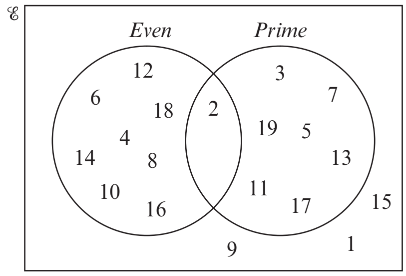
3 **a** 5

b 7

c 2

d 1

4



Exercise 9.1

1 **a** 5, 7, 9, 11, 13, **15**, **17**, **19**, ...
+2 +2 +2 +2 +2 +2 +2 +2

b 3, 8, 13, 18, 23, **28**, **33**, **38**, ...
+5 +5 +5 +5 +5 +5 +5 +5

c 3, 9, 27, 81, 243, **729**, **2187**, **6561**, ...
×3 ×3 ×3 ×3 ×3 ×3 ×3 ×3

d 0.5, 2, 3.5, 5, 6.5, **8**, **9.5**, **11**, ...
+1.5 +1.5 +1.5 +1.5 +1.5 +1.5 +1.5 +1.5

e 8, 5, 2, –1, –4, **–7**, **–10**, **–13**, ...
–3 –3 –3 –3 –3 –3 –3 –3

f 13, 11, 9, 7, 5, **3**, **1**, **–1**, ...
–2 –2 –2 –2 –2 –2 –2 –2

g 6, 4.8, 3.6, 2.4, 1.2, **0**, **–1.2**, **–2.4**
–1.2 –1.2 –1.2 –1.2 –1.2 –1.2 –1.2 –1.2

h 2.3, 1.1, –0.1, –1.3, **–2.5**, **–3.7**, **–4.9**, ...
–1.2 –1.2 –1.2 –1.2 –1.2 –1.2 –1.2

2 **a** 81, −243, 729; rule = multiply previous term by −3.

b Fr, Sa, Su; rule = days of the week.

c u, b, j; rule = skip 1 extra letter of the alphabet each time.

3 **a** $x = \frac{1}{2}$ or any combination of an integer and a half.

b $x = 5$ or any number with 5 in the units place.

c x = any number <0.

Exercise 9.2

1 **a** 23, 27, 31
b 49, 64, 81
c −17, −31, −47

2 **a** 1, −2, −5, …; −56
b 1, 0, −1, …; −18
c $\frac{1}{2}$, 2, 4.5, …; 200
d 0, 6, 24, …; 7980
e $\frac{3}{2}$, 1, $\frac{3}{4}$ …; $\frac{1}{7}$
f 2, 16, 54, …; 16 000

3 **a** **i** 33
ii $2n + 3$
b **i** 73
ii $5n - 2$
c **i** 14 348 907
ii 3^n
d **i** 21.5
ii $1.5n - 1$
e **i** −34
ii $-3n + 11$
f **i** −15
ii $-2n + 15$
g **i** −10.8
ii $-1.2n + 7.2$
h **i** 450
ii $2n^2$

4 **a** $4(2n - 1)$
b 3996
c 30
d Rule is $8n - 4$, so $8n - 4 = 154$ should give integer value of n if 154 is a term:

$8n - 4 = 158$
$8n = 158$
$n = 19.75$

OR

19th term = 148 and 20th term = 156, therefore 154 is not a term.

5 3, 6
6, 9

6 **a** 1st difference: 5, 7, 9, 11
2nd difference: 2, 2, 2; so, sequence is quadratic.
b $T_6 = 48$
c $n^2 + 2n$
d $T_{20} = 440$

7 $x = -2$

8 $n = 5$

9 **a** 2, 8, 18, 32, 50
b **i** $2n^2 + 1$
ii $4n^2$
iii $6n^2 + 1$
c

n	1	2	3
Sequence	5	6	11
$2n^2$	2	8	18
Sequence − $2n^2$	3	−2	−7

n	4	5
Sequence	20	33
$2n^2$	32	50
Sequence − $2n^2$	−12	−17

d $-5n + 8$
e $2n^2 - 5n + 8$

10 **a** $2n^2 + n + 2$
b $n^2 + n + 11$

11 **a** It has a constant 2nd difference of 4, so sequence is quadratic.
b 68
c $T_n = 2n^2 - 4n - 2$
d 4798

12 **a** $T_5 = 30$, $T_6 = 42$
b $T_n = n^2 + n$
c 240
d $n = 10$

13 **a** 2^{n-1}
b 2^{63}
c 3^{n-1}

Fibonacci patterns

1 a The number of clockwise and the number of anticlockwise spirals will often be consecutive terms of a Fibonacci sequence.

b Students will need to physically count and keep track of the sections they have counted to find the pattern.

2 There are many examples including seeds on a sunflower, sections on the skin of a pineapple, the arrangement of leaves on the stems of plants. An online search will give a selection of answers.

3 a An investigation will show that many artists, including Salvador Dali and Leonardo da Vinci, often produced work using this ratio. It can also been seen in the relationships between height and width in buildings, for example in the Acropolis and the Great Pyramid at Giza. The golden ratio is also used to space out facial features. Advertising logos and visually appealing layouts often reflect the golden ratio.

b In simple terms, the golden ratio can be worked out using the dimensions of a rectangle where the ratio $a : a + b$ is equivalent to $b : a$. There are many examples of diagrams that show this.

Exercise 9.3

a

Pattern number n	1	2	3	4	5	6	n	300
Number of matches m	4	7	10	13	16	19	$m = 3n + 1$	901

b

Pattern number p	1	2	3	4	5	6	p	300
Number of circles c	1	3	5	7	9	11	$c = 2p - 1$	599

c

Pattern number p	1	2	3	4	5	6	p	300
Number of triangles t	5	8	11	14	17	20	$t = 3p + 2$	902

d

Pattern number p	1	2	3	4	5	6	p	300
Number of squares s	5	10	15	20	25	30	$s = 5p$	1500

Exercise 9.4

1 a $5, 9, 13, \ldots 101$

b $-2, 1, 4, \ldots 70$

c $4\frac{1}{2}, 9\frac{1}{2}, 14\frac{1}{2}, \ldots 124\frac{1}{2}$

d $-1, -3, -5, \ldots -49$

e $1\frac{1}{2}, 2, 2\frac{1}{2}, \ldots 13\frac{1}{2}$

f $1, 7, 17, \ldots 1\,249$

g $1, 4, 9, \ldots 625$

h $6, 7, 9, \ldots 16\,777\,221$

2 30 is T_6 and 110 is T_{11}.

3 T_9

4 a 153

b $n = 6$

5 a The subscript $n + 1$ means the term after u_n, so this rule means that to find the term in a sequence, you have to add 2, to the current term (u_n). So, if the term is 7, then $u_n + 1$ is $7 + 2 = 9$

b $-8, -6, -4, -2, 0$

Exercise 9.5

1 a Rational

b Rational

c Rational

d Rational

e Irrational

f Irrational

g Rational

h Rational

i Rational

j Rational

k Rational

l Rational

m Rational

n Irrational

o Irrational

p Irrational

2 a $\frac{6}{1}$

b $\frac{19}{8}$

c $\frac{427}{1000}$

d $\frac{8}{9}$

e $\frac{427}{10\,003}$

f $\frac{311}{99}$

3 Possible answers include:

a 2

b $\sqrt{5}$

c 1

d 2

4 The set of rational numbers and the set of irrational numbers are both infinite sets. But the set of rational numbers is 'countable' whereas the set of irrational numbers is 'uncountable'. This might suggest that there are more irrational numbers than rational numbers.

The term 'countable' does not mean finite.

In this context we mean that, if you tried to pair up every rational number with exactly one irrational number, you would have a lot of irrational numbers left over that you couldn't pair up but no rational numbers would be upaired.

5 Students' own answers. Example: An 'imaginary number' is a quantity of the form ix, where x is a real number and i is the positive square root of -1, e.g. $\sqrt{-3} = i\sqrt{3}$.

Exercise 9.6

1 a Let $x = 0.\dot{6}$

Then $10x = 6.\dot{6}$

Subtracting:

$$10x = 6.\dot{6}$$
$$-x = 0.\dot{6}$$
$$9x = 6$$
$$\text{So } x = \frac{6}{9}$$
$$\text{Simplify } x = \frac{2}{3}$$

b Let $x = 0.\dot{1}\dot{7}$

Then $100x = 17.\dot{1}\dot{7}$

Subtracting:

$$100x = 17.\dot{7}$$
$$-x = 0.1\dot{7}$$
$$\text{So } 99x = 17$$
$$x = \frac{17}{99}$$

2 **a** $\frac{5}{9}$

b $\frac{1}{9}$

c $\frac{8}{9}$

d $\frac{8}{33}$

e $\frac{61}{99}$

f $\frac{32}{99}$

g $\frac{206}{333}$

h $\frac{233}{999}$

i $\frac{208}{999}$

j $\frac{1}{45}$

k $\frac{17}{90}$

l $\frac{31}{990}$

m $\frac{27}{11}$

n $\frac{1034}{333}$

o $\frac{248}{99}$

p $\frac{9990}{999} = 10$

q $\frac{5994}{999} = 6$

r $\frac{8}{9}$

s $\frac{999}{999} = 1$

t $\frac{900}{9} = 100$

Recurring decimals

1 **a** **i** 0.1

ii 0.01

iii 0.001

iv 0.000 000 001

b As the number subtracted tends to 1, the answer tends to 0. Yes it will reach 0.

c $\frac{2}{3}, \frac{2}{9}$

d $0.\dot{8}$

e $\frac{8}{9}$

f $\frac{4}{9}, \frac{5}{9}, 0.\dot{9}, 1$

g As the fractions represent infinite 9 there is no 1 at the end of the infinite 0 and so 0.999… = 1

2 **a** 4.41 > 4.1 but 4.1 < 4.5

b Another 9 could be added to the end of 4.49999.

c Yes. $X = 4.4\dot{9}$

$10x = 44.\dot{9}$

$9x = 40.5$

$x = \frac{40.5}{9} = \frac{9}{2} = 4.5$

No.

Exercise 9.7

1 **a** $2\sqrt{7}$

b $\sqrt{7}$

c $10\sqrt{3}$

d $3\sqrt{11}$

e $2\sqrt{6}$

f $5\sqrt{10}$

g $15\sqrt{2}$

h $24\sqrt{2}$

i $-15\sqrt{7}$

j $-12\sqrt{6}$

k $14\sqrt{2}$

l $-10\sqrt{15}$

2 **a** $\sqrt{54}$

b $\sqrt{40}$

c $\sqrt{98}$

d $-\sqrt{24}$

e $\sqrt{108}$

f $-\sqrt{72}$

3 **a** Student discussion; could include changing them all to the form $\sqrt{n}$.

A $3\sqrt{5},\ 3\sqrt{3},\ 2\sqrt{5}$

B $4\sqrt{6},\ 3\sqrt{6},\ \sqrt{24}$

C $2\sqrt{15},\ 3\sqrt{6},\ 4\sqrt{3}$

b Student discussion and ideas. Comparing like surds first can mean you don't need to convert all of them.

4 **a** Possible errors are:

1: Added coefficients correctly but also added roots.

2: Subtracted and added unlike surds.

b 1: $11\sqrt{3} + 3\sqrt{2}$

2: $4\sqrt{5} + 3\sqrt{2} - 2\sqrt{3}$

5 Any numerical examples to show the expression are unequal. For example:

a $\sqrt{4} + \sqrt{9} = 2 + 3 = 5$ and $\sqrt{4+9} = \sqrt{13} = 3.605...$

b $\sqrt{9} - \sqrt{4} = 3 - 2 = 1$ and $\sqrt{9-4} = \sqrt{5} = 2.236...$

6 **a** $5\sqrt{3} + 3\sqrt{7}$

b $2\sqrt{11} + 3\sqrt{5}$

c $-2\sqrt{2} - 3\sqrt{5}$

d $6\sqrt{2} - 2\sqrt{7}$

e $8\sqrt{5} - 4\sqrt{2}$

f $8\sqrt{3} + 8\sqrt{2}$

7 **a** $2\sqrt{5} + \sqrt{5} = 3\sqrt{5}$

b $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

c $4\sqrt{3} - 6\sqrt{3} = -2\sqrt{3}$

d $6\sqrt{2} + 6\sqrt{2} = 12\sqrt{2}$

e $5\sqrt{3} - 8\sqrt{3} = -3\sqrt{3}$

f $9\sqrt{3} + 4\sqrt{3} = 13\sqrt{3}$

8 **a** $1 + 5\sqrt{3}$

b $3\sqrt{3} - 1$

c 1

d $\sqrt{3} + 5$

e 8

f $3\sqrt{7} - 3\sqrt{2}$

9 $4 + 2\sqrt{10} + 2\sqrt{5}$

10 $x^2 = (2\sqrt{35})^2 - (4\sqrt{7})^2$

$= 4(35) - 16(7)$

$= 140 - 112$

$= 28$

So, $x = \sqrt{28}$

$= \sqrt{4} \times \sqrt{7}$

$= 2\sqrt{7}$

Checking for errors

a 2D and 4C are incorrect.

2A and 4D are correct.

b Possible mistakes are:

In 2D the student has added the numbers instead of multiplying.

In 4C the student has only multiplied the numerator by $\sqrt{2}$.

Exercise 9.8

1 **a** $\sqrt{35}$

b $\sqrt{33}$

c $24\sqrt{6}$

d $9\sqrt{30}$

e $-4\sqrt{30}$

f 54

g $6\sqrt{6}$

h $6\sqrt{2}$

i $4\sqrt{10}$

2 **a** $\sqrt{2}$

b $\sqrt{6}$

c $-\sqrt{10}$

d $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

e $\sqrt{15}$

f $\frac{1}{3}$

g $6\sqrt{3}$

h $-2\sqrt{3}$

i $4\sqrt{3}$

j $\frac{\sqrt{5}}{3}$

k $3\sqrt{10}$

l $\frac{9}{2}$

3 **a** $2\sqrt{5} + 8$

b $3\sqrt{2} + 3$

c $-2\sqrt{3} - 8$

d $-4\sqrt{5} - 24$

e $6\sqrt{11} - 4$

f $4 - 8\sqrt{3}$

g $2\sqrt{2} + \sqrt{10}$

h $2\sqrt{3} + 3\sqrt{6}$

i $2\sqrt{5} - 8$

j $4\sqrt{5} + 8\sqrt{15}$

k $2\sqrt{6} - 4\sqrt{21}$

l $5\sqrt{10} - 20$

4 **a** $3\sqrt{2}$

b $8\sqrt{3}$

c $6\sqrt{2}$

d 1

e $\frac{5\sqrt{3}}{2}$

f 3

5 **a** $\frac{5\sqrt{3}}{3}$

b $-\frac{2\sqrt{11}}{11}$

c $\frac{\sqrt{15}}{5}$

d $\frac{2-\sqrt{2}}{2}$

e $-\frac{3\sqrt{5}}{10}$

f $\frac{4+\sqrt{6}}{2}$

g $\frac{2-\sqrt{2}}{10}$

h $2-\sqrt{5}$

i $\frac{12-3\sqrt{3}}{13}$

j $\frac{2\sqrt{3}+1}{11}$

k $8+3\sqrt{7}$

l $\frac{2\sqrt{2}-4\sqrt{6}}{-11}$ or $\frac{4\sqrt{6}-2\sqrt{2}}{11}$

6 $\frac{10\sqrt{7}+2\sqrt{12}-2\sqrt{21}+14}{3}$

7 $\frac{21-\sqrt{2}}{439}$

Exercise 9.9

1 $20-4\sqrt{7}$ cm

2 **a** 4 cm

b 4 cm, 3 cm and 2.65 cm (students to explain that they have rounded the decimal in 2.65)

3 **a** $4\sqrt{59}$

b 944 cm^2

4 **a** $h^2 = \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2$

$= \frac{36}{2} + \frac{36}{2}$

$= \frac{72}{2}$

$= 36$

So, $h = \sqrt{36} = 6$ cm

b $P = 6\sqrt{2} + 6$ cm

c $\frac{1}{2} \times \frac{6}{\sqrt{2}} \times \frac{6}{\sqrt{2}} = 9$ cm^2

5 **a** 7π mm^2

b $L = 50 + 2\sqrt{7}$ mm and $W = 30 + 2\sqrt{7}$ mm

c $1528 + 160\sqrt{7}$

6 **a** Length $= 297\sqrt{2}$

$d = \sqrt{(297\sqrt{2})^2 + 297^2}$

$= \sqrt{297^2 \times 2 + 297^2}$

$= \sqrt{297^2 \times 3}$

$= 297\sqrt{3}$

Alternatively,

$d = \sqrt{(297\sqrt{2})^2 + 297^2}$

$= \sqrt{264\,627}$

$= \sqrt{9 \times 9 \times 9 \times 11 \times 11 \times 3}$

$= (3 \times 9 \times 11)\sqrt{3}$

$= 297\sqrt{3}$

Exercise 9.10

1 **a** {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

b {Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec}

c {1, 2, 3, 4, 6, 9, 12, 18, 36}

d {red, orange, yellow, green, blue, indigo, violet}

e {7, 14, 21, 28, 35, 42, 49}

f {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

g {TOY, OYT, YTO, YOT, OTY, TYO}

2 Various answers are possible. Examples include:

a hamster, mouse

b peas, beans

c Dublin, Amsterdam

d Rhine, Yangtze

e redwood, palm

f soccer, rugby

g Italy, Spain

h Aconcagua, Kilimanjaro

i Bach, Puccini

j lily, orchid

- **k** 12, 15
- **l** flatback, Olive Ridley
- **m** Uranus, Neptune
- **n** surprised, mad
- **o** African, American
- **p** pentagon, quadrilateral

3 **a** square numbers
- **b** continents of the world
- **c** even numbers less than 10
- **d** multiples of 2
- **e** factors of 12

4 **a** false
- **b** true
- **c** true
- **d** false
- **e** true

Exercise 9.11

1 **a** **i** $A \cap B = \{6, 8, 10\}$
- **ii** $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$
- **b** **i** 3
- **ii** 8

2 **a** **i** $C \cap D = \{a, g, u, w, z\}$
- **ii** $C \cup D = \{a, b, g, h, u, w, x, y, z\}$
- **b** Yes, u is an element of C and D.
- **c** No, g is an element of both sets and will be an element of the union of the sets.

3 **a** Equilateral triangles have two sides equal.
- **b** All equilateral triangles are isosceles, so F is entirely contained within G. The intersection is simply F.

4 **a** **i** $T \cup W = \{1, 2, 3, 6, 7, 9, 10\}$
- **ii** $T \cap W = \{1, 3\}$
- **b** Yes; 5 is not listed in T.

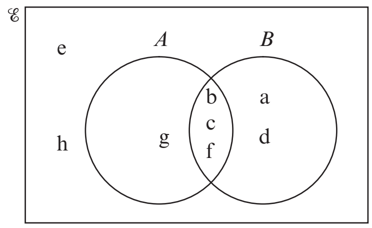
5 **a** {cat, bird, turtle, aardvark}
- **b** {rabbit, emu, turtle, mouse, aardvark}
- **c** {rabbit, cat, bird, emu, turtle, mouse, aardvark}
- **d** { } or ∅
- **e** {rabbit, emu, mouse }
- **f** {rabbit, cat, bird, emu, turtle, mouse, aardvark}

Exercise 9.12

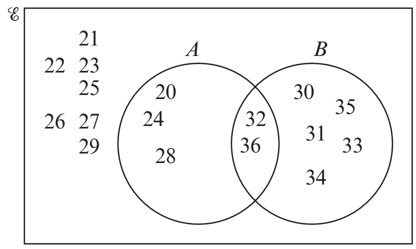
1 **a** $A = \{6, 12, 18, 24\}$ and $B = \{4, 8, 12, 16, 20, 24\}$
- **b** $A \cap B = \{12, 24\}$
- **c** $A \cup B = \{4, 6, 8, 12, 16, 18, 20, 24\}$

2 **a** **i** $P = \{a, b, c, d, e, f\}$
- **ii** $Q = \{e, f, g, h\}$
- **b** $P \cap Q = \{e, f\}$
- **c** **i** $(P \cup Q)' = \{i, j\}$
- **ii** $P \cap Q' = \{a, b, c, d\}$

3 **a**

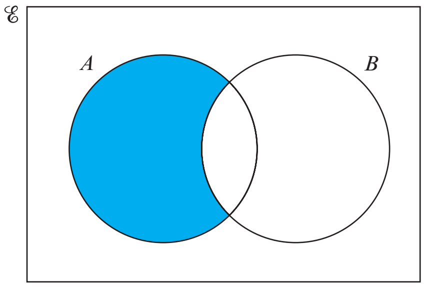


b

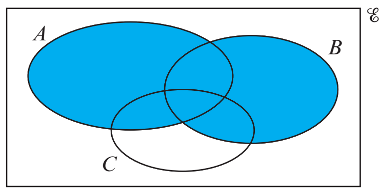


4 **a** $x = 6$
- **b** $n(V) = 16$
- **c** $n(S)' = 16$

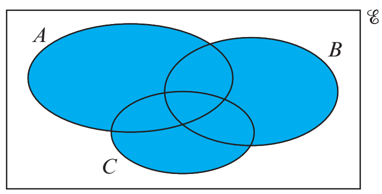
5



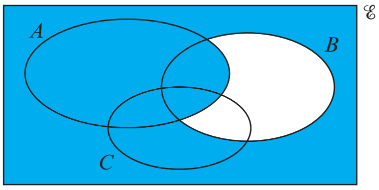
6 **a**



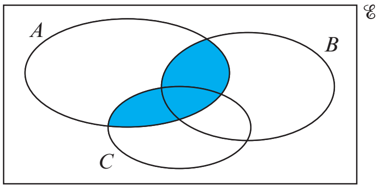
b



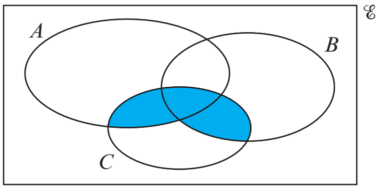
c



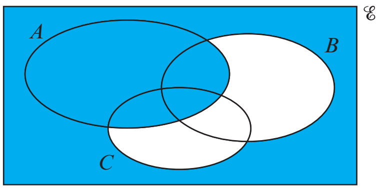
d



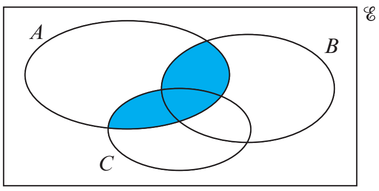
e



f



g



7 9

8 6

Exercise 9.13

1 **a** $\{x : x \text{ is a square number less than } 101\}$
b $\{x : x \text{ is a day of the week}\}$
c $\{x : x \text{ is an integer}, x < 0\}$
d $\{x : 2 < x < 10\}$
e $\{x : x \text{ is a month of the year}, x \text{ has } 30 \text{ days}\}$

2 **a** $\{x : x \text{ is an integer}, 1 < x < 9\}$
b $\{x : x \text{ is a letter of the alphabet}, x \text{ is a vowel}\}$
c $\{x : x \text{ is a letter of the alphabet}, x \text{ is a letter in the name Nicholas}\}$
d $\{x : x \text{ is an even number}, 1 < x < 21\}$
e $\{x : x \text{ is a factor of } 36\}$

3 **a** {41, 42, 43, 44, 45, 46, 47, 48, 49}
b {equilateral triangle, square, regular pentagon, regular hexagon}
c {18, 21, 24, 27, 30}

4 $\{x : x \text{ is a multiple of 3 and 5}\}$

5 **a** **i** {5}
ii {1, 2, 3, 4, 5}
iii {1, 2, 3, 4, 5}
iv {6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17}
v {1, 2, 3, 4, 5}
b ℰ
c {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17}

6 **a** $A = \{x, y : y = 2x + 4\}$ is the set of ordered pairs on a straight line. The set is infinite, so you cannot list all the points on the line.
b $B = \{x : x^3 \text{ is negative}\}$ is the set of negative cubes; any negative number cubed will result in a negative cubed number, so the set is infinite.

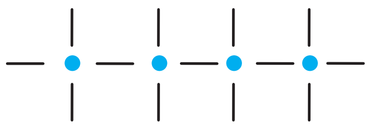
Practice questions

1 **a**

Pattern number (n)	1	2	3	4
Number of dots (d)	5	8	11	14

b $d = 3n + 2$
c 182
d 29

2 **a**



b

Dots	1	2	3	4	5	6
Lines	4	7	10	13	16	19

c 298
d $3n + 1$
e 28

3
$$\begin{aligned} 5\sqrt{2} - 2\sqrt{8} &= 5\sqrt{2} - 2\sqrt{4 \times 2} \\ &= 5\sqrt{2} - 2\sqrt{4} \times \sqrt{2} \\ &= 5\sqrt{2} - 2 \times 2 \times \sqrt{2} \\ &= 5\sqrt{2} - 4\sqrt{2} \\ &= \sqrt{2} \end{aligned}$$

4 45

5 $\dfrac{17\sqrt{3}}{6}$

6 $5\sqrt{6}$ cm

7 **a** $a = 29, b = 12$

b $p = 29, q = 180$

8 $\dfrac{40 + 4\sqrt{7}}{9}$

9 **a** 1

b 3

c {a, b, c, e, f, g}

d {e, g}

e {a, b, c, d, e, g}

f {a, b, c, d, f}

g {a, b, c, d, e, g}

10 4

11 $\dfrac{5029}{1665}$

12 $x = 0.999\,999$

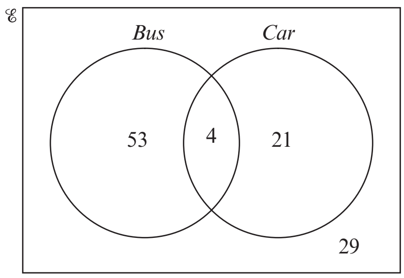
$10x = 9.999\,99$

$9x = 9$

$x = 1$

13 $\dfrac{1}{3}$

14



a 82

b 21

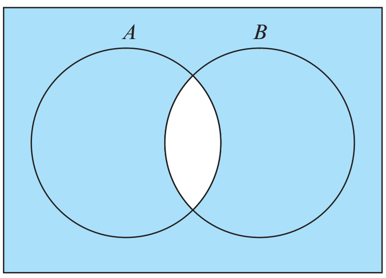
c 29

15 **a** $A = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

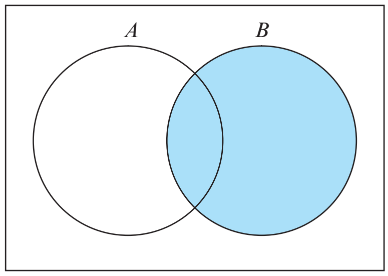
b 2

c $(A \cap B)' = \{$integers NOT including 3 or 6$\}$

16 **a**



b



17 **a** 1, 1, 2, 3, 5, 8, 13, 21

b Fibonacci numbers

c u_{13}

18 **a** 1, 3, 6, 10, 15

b Triangular numbers

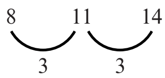
c $n = 20$

d $u_{n-1} = \frac{1}{2}(n - 1)n$

Practice questions worked solutions

1 **a** 8, 11, 14

b



$d = 3n + 5$

c $d(60) = 3 \times 60 + 5$

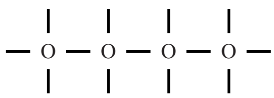
$= 185$

d $89 = 3n + 5$

$3n = 84$

$n = 28$

2 **a**



b 13, 16, 19

c If d = number of dots and L = number of lines.

$L = 3d + 1$ $\qquad L(99) = 3 \times 99 + 1$

$= 298$

d $3n + 1$

e $85 = 3n + 1$

$3n = 84$

$n = 28$

There are 85 lines in the pattern with 28 dots.

3 $5\sqrt{2} - 2\sqrt{8} = 5\sqrt{2} - 2\sqrt{4}\sqrt{2}$
$= 5\sqrt{2} - 4\sqrt{2}$
$= \sqrt{2}$

4 $\dfrac{3\sqrt{3} \times 5\sqrt{45}}{\sqrt{15}} = \dfrac{3\cancel{\sqrt{3}} \times 5\sqrt{9}\cancel{\sqrt{5}}}{\cancel{\sqrt{3}}\cancel{\sqrt{5}}}$
$= 3 \times 5 \times 5 \times 3$
$= 225$

5 $\dfrac{\sqrt{3}}{2} + \dfrac{7}{\sqrt{3}} = \dfrac{\sqrt{3}}{2} + \dfrac{7\sqrt{3}}{(\sqrt{3})^2}$
$= \dfrac{\sqrt{3}}{2} + \dfrac{7\sqrt{3}}{3}$
$= \dfrac{3\sqrt{3} + 14\sqrt{3}}{6}$
$= \dfrac{17\sqrt{3}}{6}$

6 Area $= \dfrac{1}{2} \times 3\sqrt{2} \times \dfrac{10}{\sqrt{3}}$
$= \dfrac{1}{2} \times 3\sqrt{2} \times \dfrac{10\sqrt{3}}{(\sqrt{3})^2}$
$= 5\sqrt{2}\sqrt{3}$
$= 5\sqrt{6}$

7 **a** $(3 + 2\sqrt{5})^2 = 9 + 12\sqrt{5} + 4 \times 5$
$= 29 + 12\sqrt{5}$
Therefore, $a = 29$ and $b = 12$

b $(3 + 2\sqrt{5})^2 = 29 + 12\sqrt{5}$
$= 29 + \dfrac{12\sqrt{5}\sqrt{45}}{\sqrt{45}}$
$= 29 + \dfrac{12\sqrt{225}}{\sqrt{45}}$
$= 29 + \dfrac{12 \times 15}{\sqrt{45}}$
$= 29 + \dfrac{180}{\sqrt{45}}$
Therefore, $p = 29$ and $q = 180$.

8 $\dfrac{3}{4 + \sqrt{7}} + \dfrac{7}{4 - \sqrt{7}} = \dfrac{3(4 - \sqrt{7})}{(4 + \sqrt{7})(4 - \sqrt{7})} + \dfrac{7(4 + \sqrt{7})}{(4 + \sqrt{7})(4 - \sqrt{7})}$
$= \dfrac{12 - 3\sqrt{7} + 28 + 7\sqrt{7}}{4^2 - (\sqrt{7})^2}$
$= \dfrac{40 + 4\sqrt{7}}{16 - 7}$
$= \dfrac{40 + 4\sqrt{7}}{9}$

9 **a** $M \cap N = \{f\} \Rightarrow n(M \cap N) = 1$

b 3

c 6

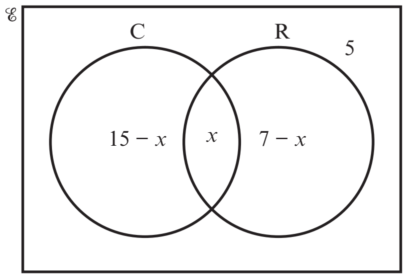
d 2

e 6

f 5

g 6

10



$15 - x + x + 7 - x + 5 = 24$

$27 - x = 24$

$x = 3$

$\text{n}(R \cap C') = 7 - x = 7 - 3 = 4$

11 $3.0\dot{2}0\dot{4} = x$

$10x = 30.\dot{2}0\dot{4}$

$10\,000x = 30\,204.\dot{2}0\dot{4}$

$9990x = 30\,174$

$x = \frac{30\,174}{9990} = \frac{5029}{1665}$

12 $x = 0.\dot{9}$

$10 = 9.\dot{9}$

$9x = 9$

$x = 1$

13 $0.\dot{3}00\dot{3} + 0.\dot{0}33\dot{0}$

Let $x = 0.\dot{3}00\dot{3}$

$10\,000x = 3003.\dot{3}00\dot{3}$

$9999x = 3003 \Rightarrow x = \frac{3003}{9999}$

Similarly, $0.\dot{0}33\dot{0} = y$

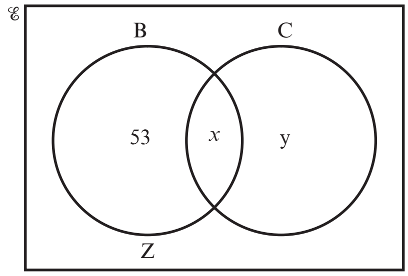
$10\,000y = 330.\dot{0}33\dot{0}$

Therefore, $9999y = 330 \Rightarrow y = \frac{330}{9999}$

So, $0.\dot{3}00\dot{3} + 0.\dot{0}33\dot{0} = \frac{3003}{9999} + \frac{330}{9999}$

$= \frac{3333}{9999} = \frac{1}{3}$

14



$x + y + 53 = 78$ (78 travelled by car or bus or both)

$53 + x + y + z = 107$ (There are 107 in total)

so, $z = 107 - 53 - 25$ (Combining both pieces of information)

$= 29$

$y + z = 50$ (50 did not travel by bus)

$y = 21$

$x = 78 - 53 - 21$

$= 4$

a $53 + x + z = 53 + 4 + 29$

$= 86$

b $y = 21$

c $z = 29$

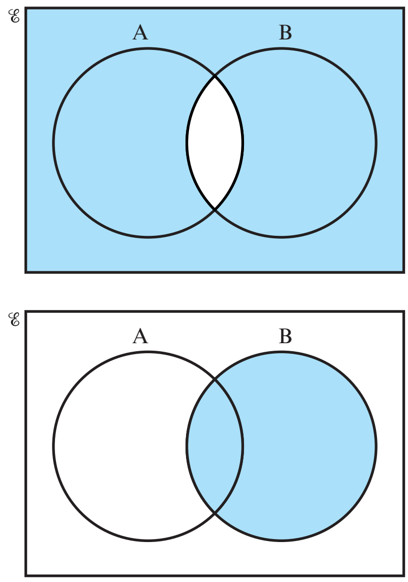
15 **a** $A = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

b $A \cap B = \{3, 6\}$

$\text{n}(A \cap B) = 2$

c The elements not in both A and B = the integers that are not positive multiples of 3 between −4 and 7.

16



17 $u_{n+2} = u_{n+1} + u_n$

a $u_1 = u_2 = 1$

1, 1, 2, 3, 5, 8, 13, 21

b Th e Fibonacci sequence.

c

34	55	89	144	233
9th	10th	11th	12th	13th

It is in the 13th position.

18 $u_n = \frac{1}{2}n(n + 1)$

a 1, 3, 6, 10, 15

b Triangular numbers

c
$$210 = \frac{1}{2}n(n + 1)$$
$$n(n + 1) = 420$$
$$n^2 + n - 420 = 0$$
$$(n + 21)(n - 20) = 0$$
$$\Rightarrow n = 20 \text{ or } n = -21$$

But $n > 0$ so $n = 20$

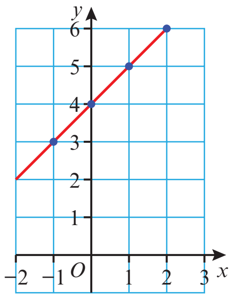
d $u_{n-1} = \frac{1}{2}(n - 1)(n - 1 + 1)$

$= \frac{1}{2}n(n - 1)$

Chapter 10

Getting started

1



2 **a** y-value is 4 more than the x-value.

b $y = x + 4$

3 Gradient = 3

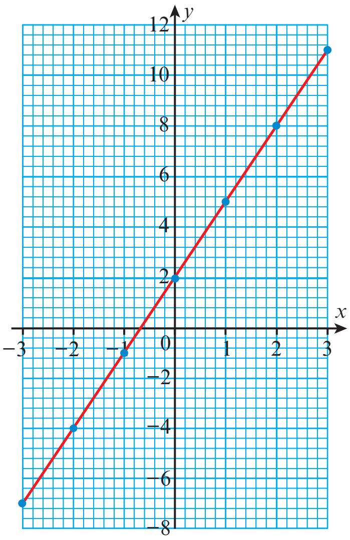
y-intercept = −2

Equation of the line is $y = 3x - 2$

Exercise 10.1

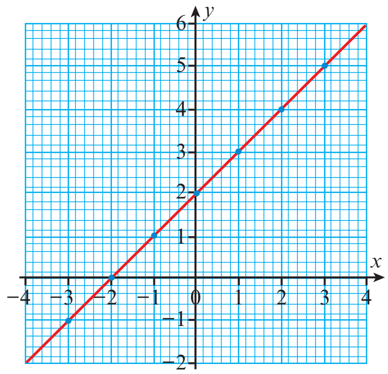
1 **a**

x	−3	−2	−1	0	1	2	3
y	−7	−4	−1	2	5	8	11



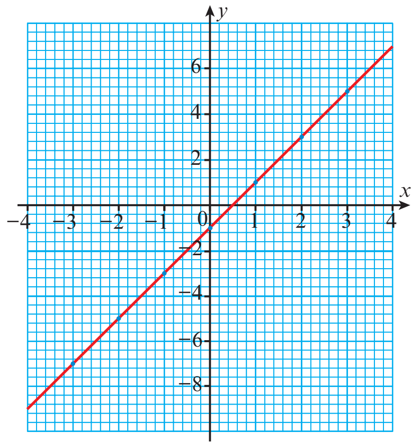
b

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5



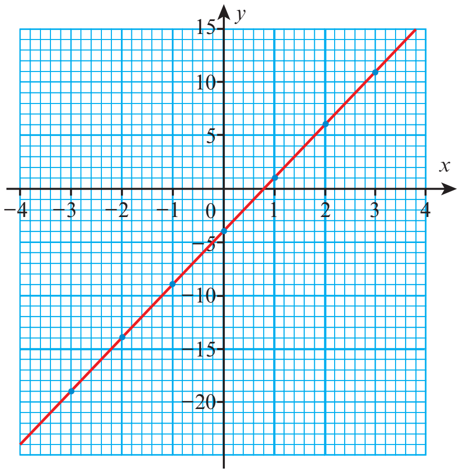
c

x	−3	−2	−1	0	1	2	3
y	−7	−5	−3	−1	1	3	5



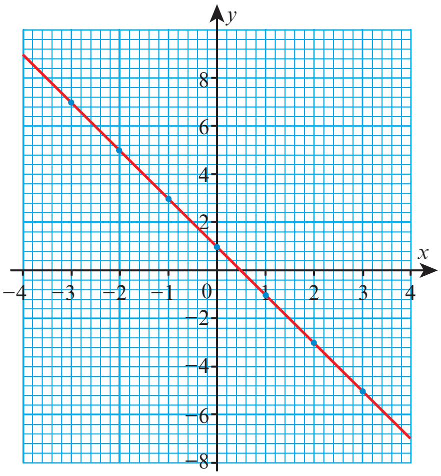
d

x	−3	−2	−1	0	1	2	3
y	−19	−14	−9	−4	1	6	11



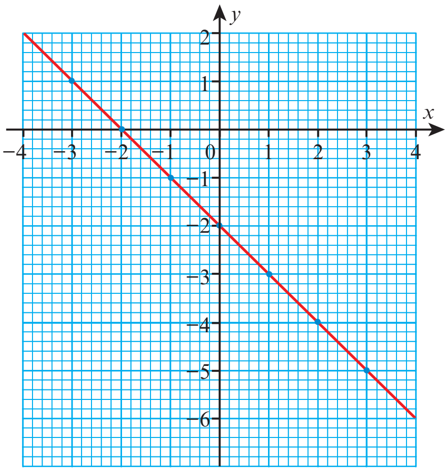
e

x	−3	−2	−1	0	1	2	3
y	7	5	3	1	−1	−3	−5



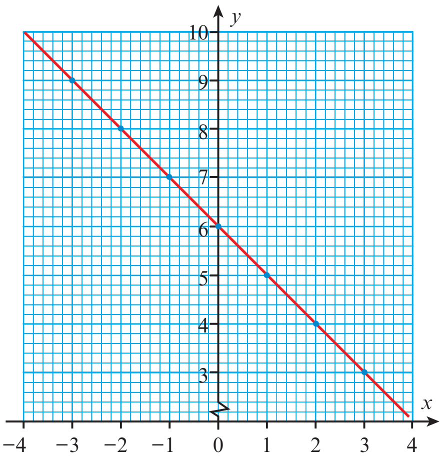
f

x	−3	−2	−1	0	1	2	3
y	1	0	−1	−2	−3	−4	−5



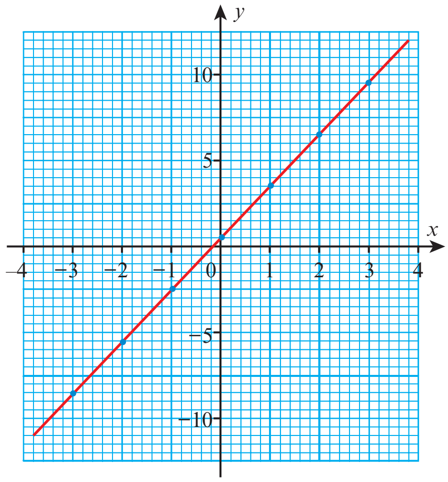
g

x	−3	−2	−1	0	1	2	3
y	9	8	7	6	5	4	3



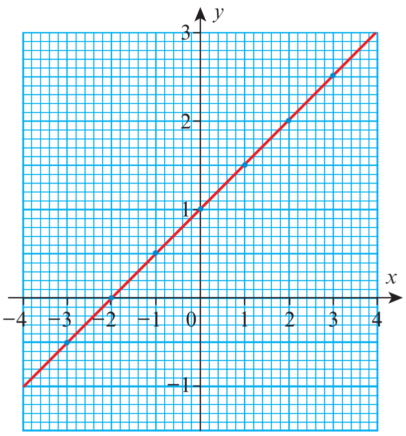
h

x	−3	−2	−1	0	1	2	3
y	−8.5	−5.5	−2.5	0.5	3.5	6.5	9.5



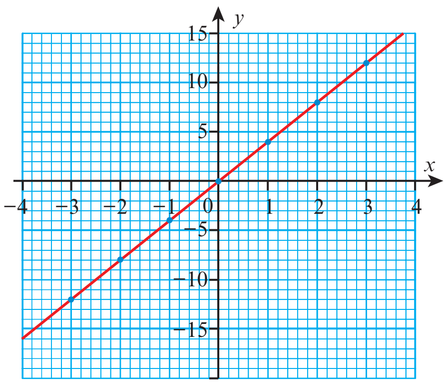
i

x	−3	−2	−1	0	1	2	3
y	−0.5	0	0.5	1	1.5	2	2.5



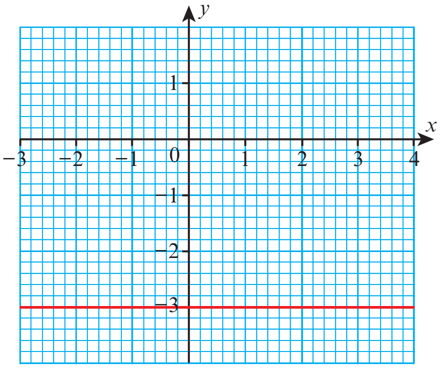
j

x	−3	−2	−1	0	1	2	3
y	−12	−8	−4	0	4	8	12



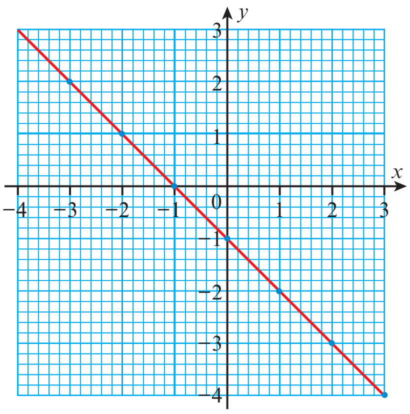
k

x	−3	−2	−1	0	1	2	3
y	−3	−3	−3	−3	−3	−3	−3



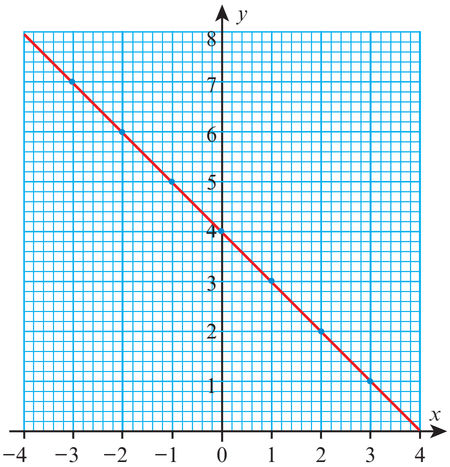
l

x	−3	−2	−1	0	1	2	3
y	2	1	0	−1	−2	−3	−4



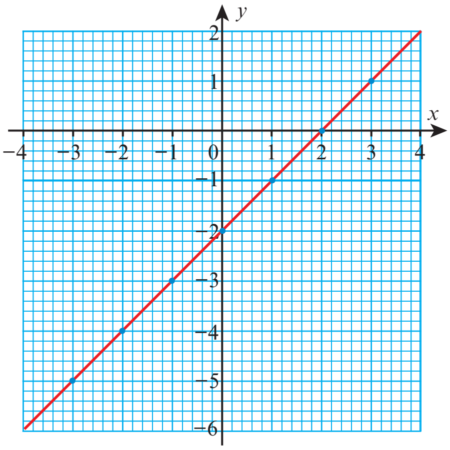
m

x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1



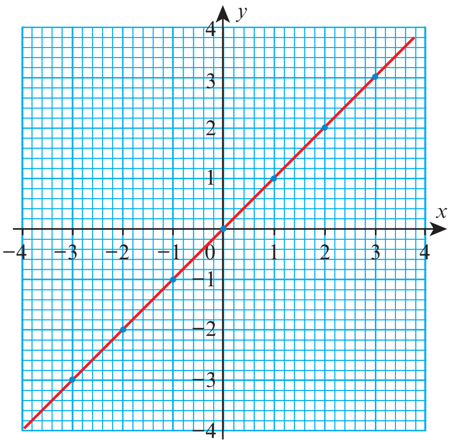
n

x	−3	−2	−1	0	1	2	3
y	−5	−4	−3	−2	−1	0	1



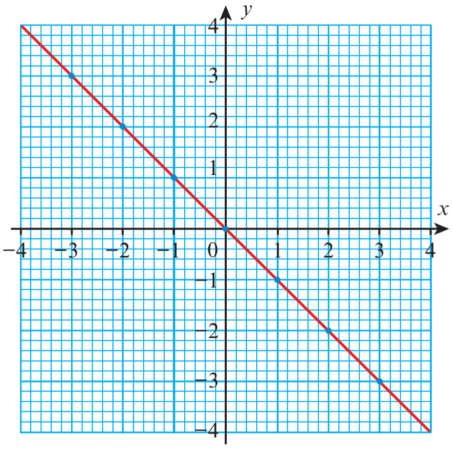
o

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

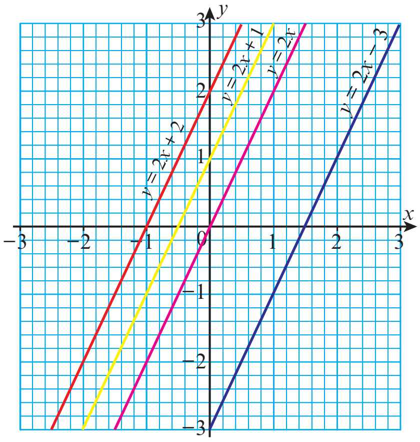


p

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3



2



The lines are parallel.

3 **a**

x	−3	0	3
$y = x + 2$	−1	2	5

b

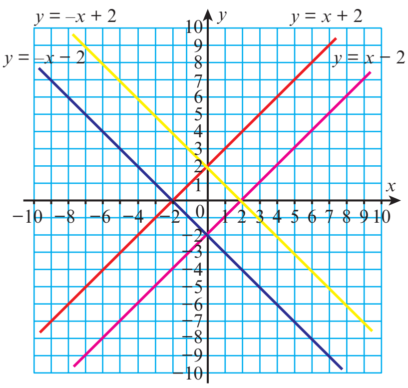
x	−3	0	3
$y = -x + 2$	5	2	−1

c

x	−3	0	3
$y = x - 2$	−5	−2	1

d

x	−3	0	3
$y = -x - 2$	1	−2	−5



4 **a** $y = x + 2$ cuts the x-axis at $x = -2$
$y = -x + 2$ cuts the x-axis at $x = 2$
$y = x - 2$ cuts the x-axis at $x = 2$
$y = -x - 2$ cuts the x-axis at $x = -2$

b $y = x + 2$ and $y = x - 2$

c $-x + 2$ and $-x - 2$

d $y = x + 2$ and $y = -x + 2$

e $y = x - 2$ and $y = -x - 2$

f None of the graphs

g $y = x + 2$ is parallel to $y = x - 2$
$y = -x + 2$ is parallel to $y = -x - 2$

h Same coefficients of x but different constant values.

Exercise 10.2

1 **a** $x = -4$

b $x = 2$

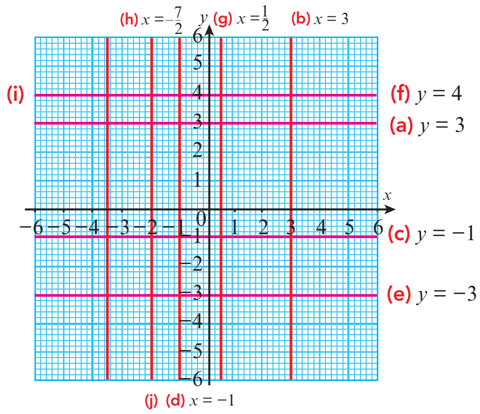
c $x = 7$

d $y = 7$

e $y = 3$

f $y = -6$

2



Exercise 10.3

1 **a** 3

b 2

c 1

d −3

e −5

f −1

g $\frac{1}{3}$

h $\frac{2}{3}$

i $-\frac{1}{4}$

2 **a** 3

b 1

c 2

d −3

e −3

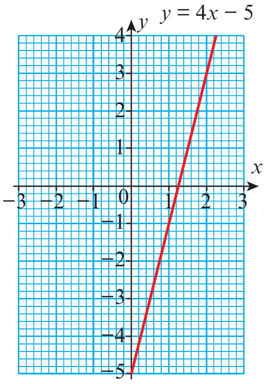
f $\frac{17}{4}$

3 450 m

The gradient is equal to the coefficient of x in the equation of the line.

Exercise 10.4

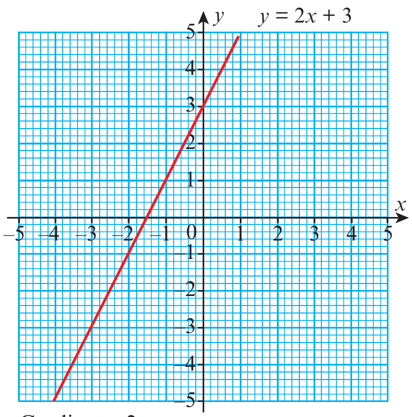
1 a



Gradient = 4

y-intercept = −5

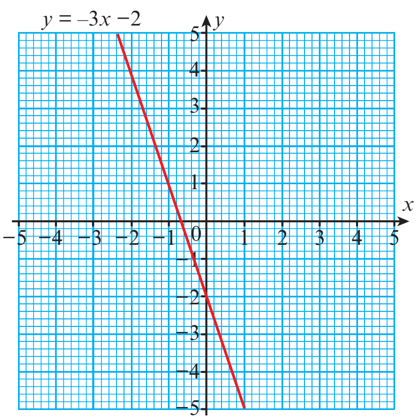
b



Gradient = 2

y-intercept = 3

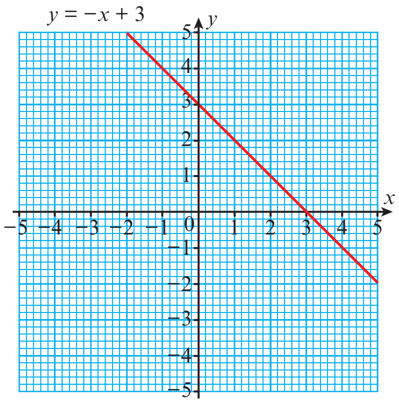
c



Gradient = −3

y-intercept = −2

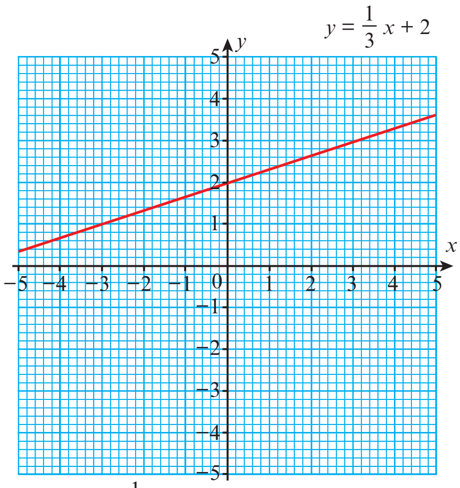
d



Gradient = −1

y-intercept = 3

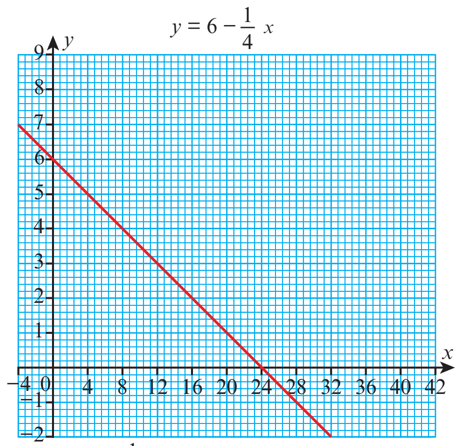
e



Gradient = $\frac{1}{3}$

y-intercept = 2

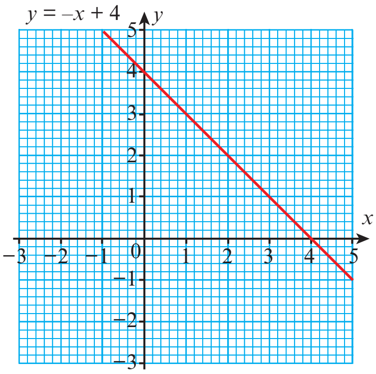
f



Gradient = $\frac{1}{4}$

y-intercept = 6

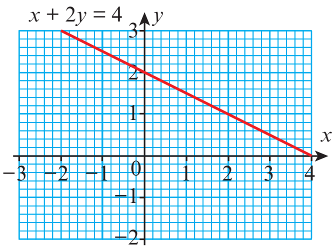
2 **a**



x-intercept $= -1$

y-intercept $= 4$

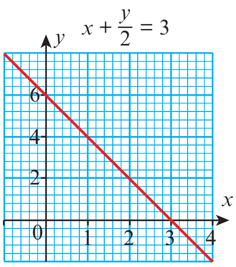
b



x-intercept $= -\frac{1}{2}$

y-intercept $= 2$

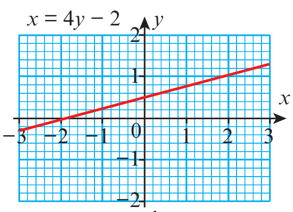
c



x-intercept $= -2$

y-intercept $= 6$

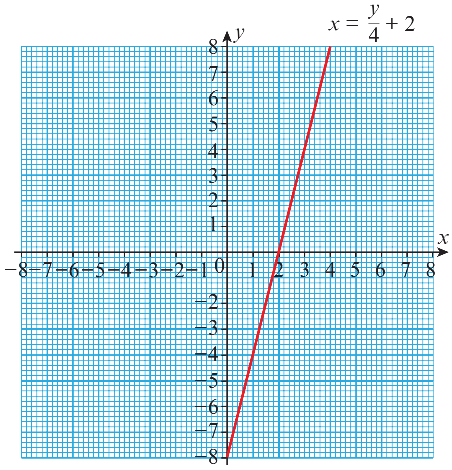
d



x-intercept $= \frac{1}{4}$

y-intercept $= \frac{1}{2}$

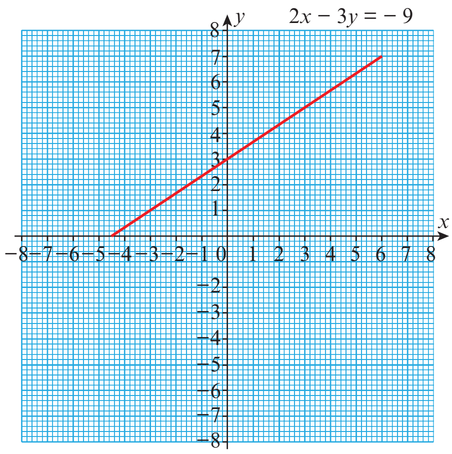
e



x-intercept $= 4$

y-intercept $= -8$

f



x-intercept $= -4.5$

y-intercept $= 3$

3

	$y = mx + c$	Gradient	y-intercept
a	$y = \frac{1}{2}x - 2$	$\frac{1}{2}$	-2
b	$y = -2x + 1$	-2	1
c	$y = 2x + 4$	2	4
d	$y = 2x - 5$	2	-5
e	$y = 2x + 5$	2	5
f	$y = -\frac{1}{3}x + 2$	$-\frac{1}{3}$	2
g	$y = 3x - 2$	3	-2
h	$y = -4x + 2$	-4	2
i	$y = 2x + 4$	2	4
j	$y = 6x - 12$	6	-12
k	$y = \frac{1}{8}x - 3$	$\frac{1}{8}$	-3
l	$y = -12x + 6$	-12	6

4 **a** $y = 2x + 3$
b $y = -3x - 2$
c $y = 3x - 1$
d $y = 0.75x - 0.75$
e $y = -2$
f $y = 4$

5 **a** $y = -\frac{3}{2}x - 0.5$
b $y = -\frac{3}{4}x + 2$
c $y = \frac{1}{2}x - 3$

6 **a** $y = -4x - 1$
b $y = \frac{1}{3}x + 1$
c $y = -3x + 2$
d $y = 5x + 2$
e $y = 3x + 1$
f $y = -x + 2$
g $y = 2x - 3$
h $y = \frac{2}{3}x - 1$
i $y = \frac{1}{4}x - 2$

7 **a** $y = 4x - 5$
b $y = -3x + 17$
c $y = \frac{9}{5}x - \frac{6}{5}$
d $y = \frac{17}{4}x - \frac{71}{4}$

8 Any line with the same gradient, e.g.
a $y = -3x - 5$
b $y = 2x + 13$
c $y = \frac{x}{2} - 3$
d $y = -x - 4$
e $x = -8$
f $y = 6$

9 **a, c**

10 **a** $y = 2x - 2$
b $y = 2x$
c $y = 2x - 4$
d $y = 2x + \frac{1}{2}$

11 **a** Any line with gradient $\frac{2}{3}$, e.g. $y = \frac{2}{3}x - 5$
b Any line with same y-intercept, e.g. $y = 2x + 3$
c $y = 3$

Investigation

No. Imagine a straight road with a straight footbridge crossing above it. Both the road and the bridge follow a straight line path, but they do not cross because there is vertical distance between them.

Exercise 10.5

1 $y = -5x + 8$

2 **a** Gradient $AB = -2$;
Gradient $PQ = \frac{1}{2}$; $-2 \times \frac{1}{2} = -1$,
so AB is perpendicular to PQ.
b Gradient $MN = \frac{1}{2}$; $\frac{1}{2} \times -2 = -1$,
so MN is perpendicular to AB.

3 $y = -\frac{1}{3}x + 5$

4 **a** $y = -\frac{1}{2}x + \frac{1}{2}$ or $x + 2y - 1 = 0$
b $y = x - 3$

5 Gradient $A = -2$, gradient $B = \frac{1}{2}$:
$-2 \times \frac{1}{2} = -1$, so A is perpendicular to B.

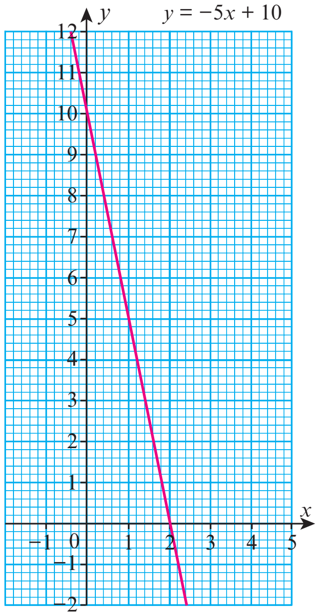
6 Gradient $AB = \frac{10}{9}$; gradient $AC = -1$, so AB is not perpendicular to AC and figure cannot be a rectangle.

7 **a** $y = 5x - 18$
b $y = x - 4$
c $y = 1$

8 $2x - y + 6 = 0$

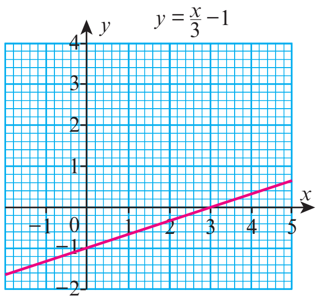
Exercise 10.6

1 a



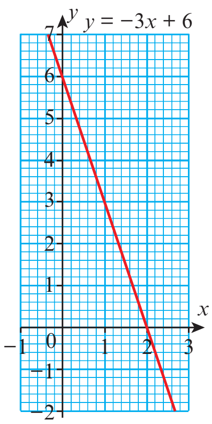
x-intercept = 2
y-intercept = 10

b



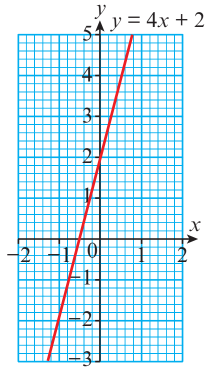
x-intercept = 3
y-intercept = −1

c



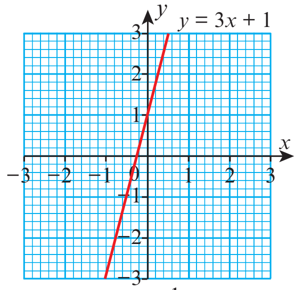
x-intercept = 2
y-intercept = 6

d



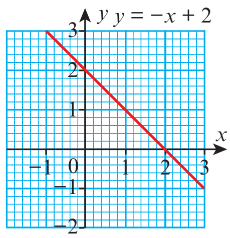
x-intercept = –0.5
y-intercept = 2

e



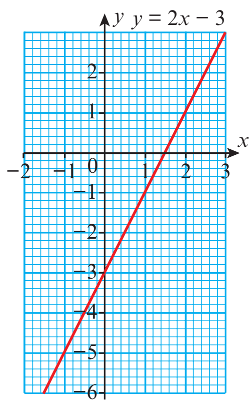
x-intercept = $-\frac{1}{3}$
y-intercept = 1

f



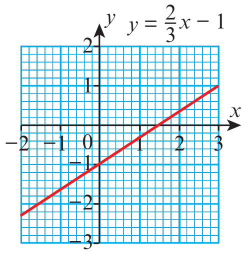
x-intercept = 2
y-intercept = 2

g



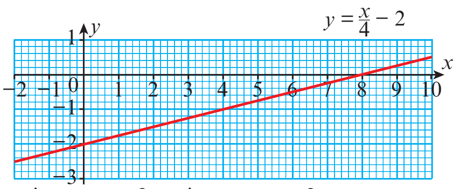
x-intercept = 1.5
y-intercept = –3

h



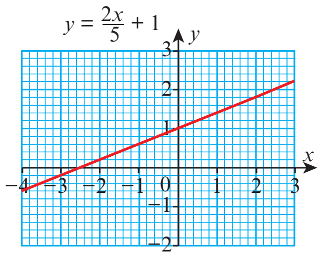
x-intercept = 1.5
y-intercept = −1

i



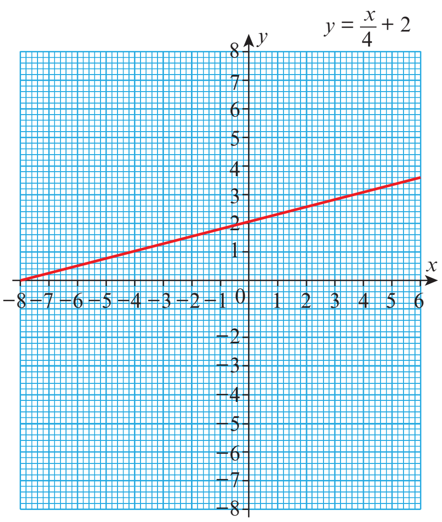
x-intercept = 8 y-intercept = −2

j



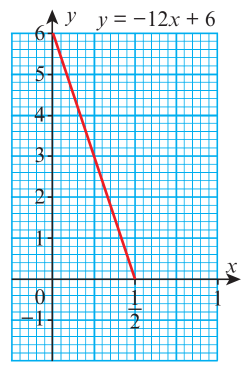
x-intercept = −2.5
y-intercept = 1

k



x-intercept = −8 y-intercept = 2

l



x-intercept = 0.5
y-intercept = 6

2 **a** $c = 2$
b $c = -4$
c $c = -9$
d $c = -8$
e $c = 4$
f $c = 3$
g $c = -2$
h $c = 2$

Exercise 10.7

1 **a** Length = 8.49 midpoint = (6, 9)
b Length = 4.47 midpoint = (3, 8)
c Length = 5.66 midpoint = (6, 5)
d Length = 3.16 midpoint = (4.5, 9.5)
e Length = 5 midpoint = (2.5, 5)
f Length = 1.41 midpoint = (11.5, 3.5)
g Length = 5 midpoint = (1, 3.5)
h Length = 6.08 midpoint = (4.5, 2)
i Length = 11.05 midpoint = (−2.5, 1.5)

2 AB = 5.39 midpoint = (3, 4.5)
CD = 4.47 midpoint = (−4, 6)
EF = 8.60 midpoint = (−2.5, 2.5)
GH = 7.07 midpoint = (3.5, 0.5)
IJ = 5.10 midpoint = (2.5, −3.5)
KL = 12.6 midpoint = (1, −3)
MN = 5.39 midpoint = (−3.5, −2)
OP = 7.81 midpoint = (−4.5, −4)

3 5.83

4 B

5 B

6 $AB = 6.40$
$AC = 4.24$
$BC = 7.28$

7 $a = 7$

8 $E = (-6, -2)$

Exercise 10.8

1 **a** $x^2 + 4x + 3$
b $x^2 + 10x + 24$
c $x^2 + 19x + 90$
d $x^2 + 15x + 36$
e $x^2 + 2x + 1$
f $x^2 + 9x + 20$
g $x^2 - 3x - 28$
h $x^2 + 5x - 24$
i $x^2 - 1$
j $x^2 - x - 72$
k $x^2 - 13x + 42$
l $x^2 - 9x - 52$
m $y^2 - 11y - 42$
n $z^2 - 64$
o $t^2 + 13t - 68$
p $h^2 - 6h + 9$
q $g^2 + 3\frac{1}{2}g - 2$
r $d^2 - \frac{9}{16}$

2 **a** $12 - 7x + x^2$
b $3 + 7x - 6x^2$
c $6m^2 - 17m + 7$
d $-8x^2 + 2x + 3$
e $8a^2 - 2b^2$
f $-8m^2 - 2mn + 3n^2$
g $x^2 + \frac{3}{4}x + \frac{1}{8}$
h $2x^2 - \frac{2}{3}x - \frac{1}{6}$
i $-36b^2 - 26b + 42$
j $6x^2 + 9x - 15$
k $6x^3 + 9x^2 + 2x + 3$
l $15x^4 - 18x^2 + 3$

3 **a** $2x^2 + 9x + 9$
b $3y^2 + 10y + 7$
c $7z^2 + 15z + 2$
d $4t^2 + 17t - 15$
e $2w^2 - 23w + 56$
f $16g^2 - 1$
g $72x^2 + 23x - 4$
h $360c^2 - 134c + 12$
i $-2m^2 + 10m - 12$

4 **a** $-2x^4 + 6x^2y - 4y^2$
b $-4x^4 + 2xy^2 - 4x^3y + 2y^3$
c $6x^3 + 9x^2y - 2xy - 3y^2$

5 **a** $15x^3 + 21x^2 - 24x - 12$
b $x^3 - 5x^2 - 25x + 125$
c $12x^3 + x^2 - 9x + 2$
d $4x^3 + 32x^2 + 80x + 64$
e $12x^3 - 32x^2 + 25x - 6$
f $18x^3 - 33x^2 + 20x - 4$
g $x^3 + 6x^2 + 12x + 8$
h $8x^3 - 24x^2 + 24x - 8$
i $x^4y^4 - x^4$
j $\frac{1}{81} - \frac{x^2}{18} + \frac{x^4}{16}$

6 **a** $V = \left(2x + \frac{1}{2}\right)(x - 2)^2\,\text{cm}^3$
b $2x^3 - 7.5x^2 + 6x + 2$
c $0.196\,\text{cm}^3$

Exercise 10.9

1 **a** $x^2 - 2xy + y^2$
b $a^2 + 2ab + b^2$
c $4x^2 + 12xy + 9y^2$
d $9x^2 - 12xy + 4y^2$
e $x^2 + 4xy + 4y^2$
f $y^2 - 8x^2y + 16x^4$
g $x^4 - 2x^2y^2 + y^4$
h $4 + 4y^3 + y^6$
i $4x^2 + 16xy^2 + 16y^4$
j $\frac{1}{4x^2} - \frac{1}{4xy} + \frac{1}{16y^2}$
k $\frac{9x^2}{16} - \frac{3xy}{4} + \frac{y^2}{4}$
l $a^2 + ab + \frac{b^4}{4}$
m $a^2b^2 + 2abc^4 + c^8$
n $9x^4y^2 - 6x^2y + 1$

o $\frac{4x^2}{9} + \frac{16xy}{3} + 16y^2$

p $x^2 - 6x + 9$

2 a $4x - 12$

b $2x^2 + 2x - 19$

c $2y^2 + 8x^2$

d $\frac{x^2}{2} + \frac{8x}{3} - 2$

e $6x^2 + 13.8x + 3.6$

f $-16x^2 + 8xy + 2x - 2y^2$

g $-x^2 + 3x - 22$

h $4x^2 - 12xy - 19y^2$

i $-2x^3 - x^2 - 17x$

j $4x^2 - 13x - 1$

3 a -49

b 9

c 66

d 36

e 0

f 321

Discussion

The two numbers still multiply to give the constant term, but no longer add to give the coefficient of x because they will be multiplied by the coefficients of x in the brackets.

For example, $(2x + 1)(3x + 2) = 6x^2 + 4x + 3x + 2 = 6x^2 + 7x + 2$. The 1 and 2 multiply to give the constant term of 2, but they do not add to the give the coefficient of x (which is 7). This is because the 1 and 2 are multiplied by $2x$ and $3x$ respectively.

Exercise 10.10

1 a $(x + 12)(x + 2)$

b $(x + 2)(x + 1)$

c $(x + 4)(x + 3)$

d $(x + 7)(x + 5)$

e $(x + 9)(x + 3)$

f $(x + 6)(x + 1)$

g $(x + 6)(x + 5)$

h $(x + 8)(x + 2)$

i $(x + 10)(x + 1)$

j $(x + 7)(x + 1)$

k $(x + 20)(x + 4)$

l $(x + 7)(x + 6)$

2 a $(x - 6)(x - 2)$

b $(x - 4)(x - 5)$

c $(x - 4)(x - 3)$

d $(x - 4)(x - 2)$

e $(x - 8)(x - 4)$

f $(x - 7)(x - 7)$

g $(x - 10)(x + 2)$

h $(x - 9)(x + 2)$

i $(x - 8)(x + 4)$

j $(x + 3)(x - 2)$

k $(x + 11)(x - 3)$

l $(x + 12)(x - 2)$

3 a $(y + 17)(y - 10)$

b $(p - 6)(p + 14)$

c $(x - 12)(x - 12)$

d $(t + 18)(t - 2)$

e $(v + 15)(v + 5)$

f $3(x + 4)(x + 3)$

g $5(x + 1)(x - 2)$

h $3(x - 5)(x + 2)$

i $3(x - 1)^2$

j $2(x - 9)(x + 2)$

k $-2(x + 3)(x + 4)$

l $(x - 10)(x + 10)$

Exercise 10.11

1 a $(x + 6)(x - 6)$

b $(p + 9)(p - 9)$

c $(w + 4)(w - 4)$

d $(q + 3)(q - 3)$

e $(k + 20)(k - 20)$

f $(t + 11)(t - 11)$

g $(x + y)(x - y)$

h $(9h + 4g)(9h - 4g)$

i $4(2p + 3q)(2p - 3q)$

j $(12s + c)(12s - c)$

k $(8h + 7g)(8h - 7g)$

l $3(3x + 4y)(3x - 4y)$

m $2(10q + 7p)(10q - 7p)$

n $5(2d + 5e)(2d - 5e)$

o $(x^2 + y^2)(x^2 - y^2)$

p $x(y - x)(y + x)$

2 a $(x + 3)^2$

b $(x + 2)^2$

c $(x - 7)^2$

d $(x - 9)^2$

e $(6 + x)^2$
f $(7 + x)^2$
g $(2 - x)^2$
h $(5 - x)^2$
i $(2x + 5)^2$

3 71

4 6

Exercise 10.12

1 a $x = 0$ or $x = 9$
b $x = 0$ or $x = -7$
c $x = 0$ or $x = 21$
d $x = 4$ or $x = 5$
e $x = -7$ or $x = -1$
f $x = -3$ or $x = 2$
g $x = -2$ or $x = -1$
h $x = -10$ or $x = -1$
i $x = 3$ or $x = 4$
j $x = 6$ or $x = 2$
k $x = 10$ or $x = -10$
l $t = -18$ or $t = 2$
m $y = -17$ or $y = 10$
n $p = -14$ or $p = 6$
o $w = 12$

2 a $x = -5$ or $x = 2$
b $x = -2$ or $x = 1$
c $x = 1$ or $x = -10$
d $x = 4$ or $x = -4$
e $x = -9$ or $x = 4$
f $x = -4$ or $x = 4$
g $x = 3$ or $x = 1$
h $x = 12$ or $x = 2$
i $x = 2$ or $x = 1$

Practice questions

1 a $15x^2 + 2x - 8$
b $x^2 + 20x + 36$
c $4x^2 - 9$
d $12y^4 - 5y^2 - 3$

2 $x^2 - \frac{1}{x^2}$

3 a $4x^2 - 2x + 25$
b $2x^2 + 8$

4 a $7 + 4\sqrt{3}$
b $4 + 2\sqrt{3}$
c $\frac{2 + \sqrt{3}}{2}$

5 a $y = 1.5x + 3$
b $y = 3$
c $y = -4x - 4$
d $y = -\frac{x}{10} - 3\frac{1}{2}$
e $x = 3$
f $y = \frac{2x}{3} - 3$
g $y = -x$

6 a $y = 4x + 4$
b $y = -3x + 13$
c $y = 0.5x + 0.9$

7 a $(m - n)(m + n)$
b $10\,000 - 9$
c $(100 - 3)(100 + 3) = 97 \times 103$

8 a $(x - 17)(x + 2)$
b $(4x - 7y)(4x + 7y)$

9 $x = 2$ or $x = -14$

10 $27x^3 + 54x^2 + 36x + 8$

11 a i $6x(2x - 1)$
ii $(y - 6)(y - 7)$
iii $(d + 14)(d - 14)$
b i $x = 0$ or $x = \frac{1}{2}$
ii $y = 6$ or $y = 7$
iii $d = 14$ or $d = -14$

12 20 cm or 380 cm

13 2 or 4

14 $x = 7$ or $x = -2$

15 **a** $(x - 21)(x - 29)$

b $x = 21$ or $x = 29$

c $AD = 100 - 2x$ m

d Area $= 100x - 2x^2$

e $100x - 2x^2 = 1218$

$x = 21$ or $x = 29$

If $x = 21$, width $= 21$ m and length $= 58$ m

If $x = 29$, width $= 29$ m and length $= 42$ m

16 **a** $b = 6$

b $(7, 4)$

c $y = -2x + 8$

d 5 units^2

Practice questions worked solutions

1 **a** $(3x - 2)(5x + 4)$
$= 15x^2 - 10x + 12x - 8$
$= 15x^2 + 2x - 8$

b $(x + 2)(x + 18)$
$= x^2 + 2x + 18x + 36$
$= x^2 + 20x + 36$

c $(2x + 3)(2x - 3)$
$= 4x^2 + 6x - 6x - 9$
$= 4x^2 - 9$

d $(4y^2 - 3)(3y^2 + 1)$
$= 12y^4 - 9y^2 + 4y^2 - 3$
$= 12y^4 - 5y^2 - 3$

2 $\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right) = x^2 - 1 + 1 - \frac{1}{x^2}$
$= x^2 - \frac{1}{x^2}$

3 **a** $(2x - 5)^2 = (2x - 5)(2x - 5)$
$= 4x^2 - 10x - 10x + 25$
$= 4x^2 - 20x + 25$

b $(x + 2)^2 + (x - 2)^2$
$= x^2 + 4x + 4 + x^2 - 4x + 4$
$= 2x^2 + 8$

4 **a** $(2 + \sqrt{3})(2 + \sqrt{3}) = 4 + 2\sqrt{3} + 2\sqrt{3} + 3$
$= 7 + 4\sqrt{3}$

b $(1 + \sqrt{3})(1 + \sqrt{3}) = 1 + \sqrt{3} + \sqrt{3} + 3$
$= 4 + 2\sqrt{3}$

c $\frac{(2 + \sqrt{3})^2}{(1 + \sqrt{3})^2} = \frac{7 + 4\sqrt{3}}{4 + 2\sqrt{3}}$
$= \frac{(7 + 4\sqrt{3})(4 - 2\sqrt{3})}{(4 + 2\sqrt{3})(4 - 2\sqrt{3})}$
$= \frac{28 - 14\sqrt{3} + 16\sqrt{3} - 8 \times 3}{16 - 4 \times 3}$
$= \frac{4 + 2\sqrt{3}}{4}$
$= \frac{2 + \sqrt{3}}{2}$

5 **a** $y = \frac{3}{2}x + 3$

b $y = 3$

c $y = -4x - 4$

d $y = -\frac{1}{10}x - \frac{7}{2}$

e $x = 3$

f $y = \frac{2}{3}x - 3$

g $y = -x$

6 **a** $y = 4x + c$
At $x = 3$, $y = 16$,
$16 = 12 + c$
$c = 4$
So, $y = 4x + 4$

b $y = -3x + c$
At $x = 7$, $y = -8$,
$-8 = -21 + c$
$c = 13$
So, $y = -3x + 13$

c $y = 0.5x + c$
At $x = 3$, $y = 2.4$,
$2.4 = 1.5 + c$
$c = 0.9$
So, $y = 0.5x + 0.9$

7 $AB^2 = (9 - 2)^2 + (6 - 10)^2$
$= 49 + 16$
$= 65$

$AC^2 = (-6 - 2)^2 + (-4 - 10)^2$
$= 64 + 196$
$= 260$

$BC^2 = (-6 - 9)^2 + (-4 - 6)^2$
$= 225 + 100$
$= 325$

$AB^2 + AC^2 = 65 + 260$
$= 325$
$= BC^2$

Therefore the lengths satisfy Pythagoras' theorem

$\Rightarrow$ the triangle is right angled.

8 **a** $m^2 - n^2 = (m + n)(m - n)$

b $9991 = 10\,000 - 9$
$= 100^2 - 3^2$

c $9991 = 100^2 - 3^2$
$= (100 + 3)(100 - 3)$
$= 103 \times 97$

9 **a** $(x - 17)(x + 2)$

b $(4x)^2 - (7y)^2 = (4x + 7y)(4x - 7y)$

10 $x^2 + 12x = 28$

$x^2 + 12x - 28 = 0$

$(x + 14)(x - 2) = 0$

$x = -14$ or $x = 2$

11 $(3x + 2)^3 = (3x + 2)(3x + 2)(3x + 2)$
$= (9x^2 + 6x + 6x + 4)(3x + 2)$
$= (9x^2 + 12x + 4)(3x + 2)$
$= 27x^3 + 54x^2 + 36x + 8$

12 **a** **i** $12x^2 - 6x = 6x(2x - 1)$

ii $y^2 - 13y + 42 = (y - 7)(y - 6)$

iii $d^2 - 196 = d^2 - 14^2$
$= (d + 14)(d - 14)$

b Using previous answers:

i $6x(2x - 1) = 0$
$x = 0$ or $2x - 1 = 0$
$x = 0$ or $x = \frac{1}{2}$

ii $(y - 7)(y - 6) = 0$
$y = 6$ or $y = 7$

iii $(d + 14)(d - 14) = 0$
$d = -14$ or $d = 14$

13 Square $\Rightarrow$ all sides are equal in length

So, $2x^2 + 3x - 9 = x^2 - 3x + 7$

$x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

$x = -8$ or $x = 2$

If $x = -8$, $x^2 - 3x + 7 = 64 + 24 + 7 = 95 \Rightarrow$ square perimeter $= 4 \times 95$

$= 380$ cm

If $x = 2$, $x^2 - 3x + 7 = 4 - 6 + 7 = 5 \Rightarrow$ square perimeter $= 4 \times 5$

$= 20$ cm

14 $AB^2 = (a - 3)^2 + (4 - 3)^2$
$= a^2 - 6a + 9 + 1$
$= a^2 - 6a + 10$

But $AB = \sqrt{2} \Rightarrow a^2 - 6a + 10 = (\sqrt{2})^2$
$= 2$

Therefore, $a^2 - 6a + 8 = 0$

$(a - 4)(a - 2) = 0$

$a = 4$ or $a = 2$

15 $5^{x(x+3)} = (5^2)^{x^2 - x - 7}$

Therefore, $x(x + 3) = 2(x^2 - x - 7)$

$x^2 + 3x = 2x^2 - 2x - 14$

So, $x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$

$x = 7$ or $x = -2$

16 **a** $x^2 - 50x + 609 = (x - 29)(x - 21)$

b $2x^2 - 100x + 1218 = 0$

$x^2 - 50x + 609 = 0$

$(x - 29)(x - 21) = 0$

$x = 29$ or $x = 21$

c $2AD = 100 - 2x$

so $AD = 50 - x$

d Area $= x(50 - x)$

e $x(50 - x) = 609$

$50x - 50x^2 = 609$

$x^2 - 50x - 609 = 0$

Therefore, $x = 29$ or $x = 21$

So, the rectangle is $21 \times (50 - 21) = 21 \times 29$

(or $29 \times (50 - 21) = 29 \times 21$)

17 **a** $\frac{b - 2}{11 - 3} = \frac{1}{2}$

$2b - 4 = 8$

$2b = 12$

$b = 6$

b $\left(\frac{3 + 11}{2}, \frac{6 + 2}{2}\right)$, i.e. (7, 4)

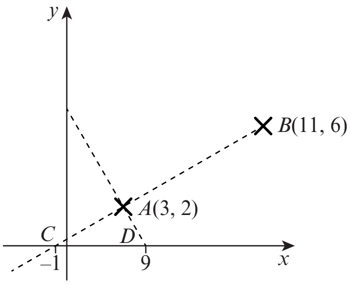
c Gradient of perpendicular line

$= -\dfrac{1}{\left(\frac{1}{2}\right)} = -2$

$y = -2x + c$ passes through (7, 4)

so, $4 = -14 + c \Rightarrow c = 18$

Therefore, $y = -2x + 18$



Line through A and B has equation

$y = \frac{1}{2}x + c$

and passes through (3, z),

so $2 = \frac{3}{2} + c \Rightarrow c = \frac{1}{2}$

Therefore, $y = \frac{1}{2}x + \frac{1}{2}$

This passes through x-axis. When $y = 0$,

$\frac{1}{2}x + \frac{1}{2} = 0 \Rightarrow x = -1$

Line through A perpendicular to AB passes through the x-axis when

$-2x + 18 = 0$

Therefore, $x = 9$.

Area $= \frac{1}{2}AD \times AC$

$= \frac{1}{2}\sqrt{(9-3)^2 + (2-0)^2}\sqrt{(3--1)^2 + (2-0)^2}$

$= \frac{1}{2}\sqrt{40}\sqrt{20}$

$= \frac{1}{2}\sqrt{8}\sqrt{5}\sqrt{4}\sqrt{5} = 10\sqrt{2}$

Chapter 11

Getting started

1 Student activity

2 a 11
 b 2.5
 c 2.38
 d 7
 e 26
 f 27.78

3 Student investigation into Pythagorean triples.

4 a

	a	$\frac{a^2-1}{2}$	$\frac{a^2+1}{2}$
i	3	4	5
ii	5	12	13
iii	7	24	25
iv	9	40	41

 b All the sets of numbers are Pythagorean triples. They all satisfy the relationship $a^2 + b^2 = c^2$.
 c Other odd values of a also generate Pythagorean triples.
 When $a = 1$ the other values are 0 and 1. These values satisfy the relationship $a^2 + b^2 = c^2$, but they are not a Pythagorean triple because 0 is not a positive integer.

Exercise 11.1

1 a $x = 10$ cm
 b $y = 13.4$ cm
 c $h = 2.59$ cm
 d $p = 1.62$ cm
 e $t = 7.21$ m

2 a $x = 7.42$ m
 b $y = 3.63$ cm
 c $t = 8.66$ cm
 d $p = 12$ m
 e $a = 6$ cm

3 a $x = 2.80$ cm
 b $y = 4.47$ cm
 c $h = 4.28$ cm
 d $p = 8.54$ km
 e $k = 10.4$ cm
 f $h = 8.06$ cm
 g $d = 6.08$ m
 h $f = 13$ m

4 a Right-angled
 b Not right-angled
 c Not right-angled
 d Right-angled
 e Right-angled

Exercise 11.2

1 53.2 inches

2 3.03 m

3 277 m

4 3.6 m

5 0.841 m

6 a 5.39
 b 3.16
 c 9.90
 d 10.30

7 $P = 42.4$ cm

8 6.02 cm

9 Height = 13.9 cm and area = 111 cm^2

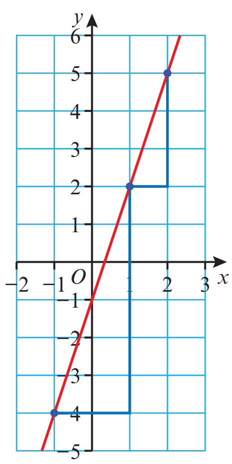
10 23.4 m

11 Area is 45.0 cm^2, so there is enough paint.

12 4.24 cm

Gradients and triangles

1–3



Gradient of $AB = \frac{6}{2} = 3$

Gradient of $BC = \frac{3}{1} = 3$

4 The vertical length is always 3 × the horizontal length. The gradient of the line is 3. They are the same.

5 It doesn't matter where you draw the triangles, the gradient of the line is the same all the way along.

6 The length of the hypotenuse divided by the shortest side is always equal to 3.16 (to 2 d.p.).

7 The ratios of corresponding sides are the same for all the triangles and the internal ratios of sides are the same for all the triangles.

Exercise 11.3

1 **a** Similar; all angles equal
b Similar; sides in proportion
c Not similar; angles not equal
d Not similar; sides not in proportion
e Similar; angles equal
f Similar; sides in proportion
g Not similar; sides not in proportion
h Similar; sides in proportion
i Similar; angles equal
j Similar; all angles equal

2 **a** $x = 12$
b $y = 5$
c $p = 12$
d $a = 12$
e $b = 5.25$
f $c = 5.14$

3 $AC = 8.75$ cm

4 Angle BAC = Angle EDC (alternate angles)
Angle ABC = Angle DEC (alternate angles)
Angle ACB = Angle DCE (vertically opposite angles)
All three angles are equal so the triangles are similar.
$CE = 4.51$ cm

5 $BC = 2.97$ m

6 Lighthouse = 192 m

7 $r = 8$

8 $x = 60$

Exercise 11.4

1 **a** $\frac{4}{2} = 2 \quad \frac{6}{5} = 1.2$
The ratio of corresponding sides are not the same so the shapes are not similar.
b All sides of shape 1 have length x and all sides of shape 2 have length y so the ratio of corresponding sides will be equal and the shapes are similar.
c $\frac{5}{4} = 1.25 \quad \frac{4}{3} = 1.\dot{3}$
Ratios not equal, so not similar.
d $\frac{80}{60} = 1.\dot{3} \quad \frac{60}{45} = 1.\dot{3}$
Ratios of corresponding sides equal, therefore they are similar.
e $\frac{12}{8} = 1.5 \quad \frac{9}{6} = 1.5$
Ratios of corresponding sides equal, therefore they are similar.
f They are not similar because not all corresponding angles are equal.

2 **a** $x = 9$
b $y = 14$
c $p = 3.30$
d $y = 7.46$
e $x = 50$, $y = 16$
f $x = 22.4$, $y = 16.8$
g $x = 7.5$, $y = 12.5$
h $x = 178$

3 $x = 10$

4 **a** All the angles on any square are 90°, so all corresponding angles are equal.
All squares have four equal sides, so the ratio of corresponding sides will always be equal.
b The ratio of corresponding sides of different rectangles will not always be equal.
c Circles may be different sizes, but they are all identical in shape, so are therefore similar to each other.
d All regular shapes are similar. For example, all regular pentagons are similar to each other. Irregular shapes do not behave in the same way.

Exercise 11.5

1 a 421.88 cm^2
  b 78.1 m^2
  c 1562.5 m^2
  d 375 cm^2

2 a $x = 24\text{ cm}$
  b $x = 30\text{ m}$
  c $x = 2.5\text{ cm}$
  d $x = 15\text{ cm}$

3 a Area will be 4 times larger.
  b Area will be 9 times larger.
  c Area will be smaller by a factor of 4.

4 8 : 3

Exercise 11.6

1 k^2; k^3

2 a 4
  b 16 : 1
  c 64 : 1

3 216 cm^2

4 172 cm^2

5 a 16 mm
  b 157.9 cm^2
  c 83.2 cm^3

6 a $20.8\dot{3}\text{ cm}^3$
  b $21.\dot{3}\text{ mm}^3$
  c 0.75 m^3
  d 56.64 m^3

7 a 525 cm^2
  b 6860 cm^3
  c 36 cm
  d 14.15 cm

8

Height	13 cm	11 cm	9 cm
Surface area	$x\text{ cm}^2$	$\frac{121x}{169}\text{ cm}^2$	$\frac{81x}{169}\text{ cm}^2$
Volume	$y\text{ cm}^3$	$\frac{1331y}{2197}\text{ cm}^3$	$\frac{729y}{2197}\text{ cm}^3$

9 $x = 3.72$

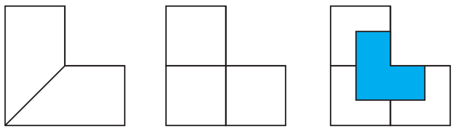
Exercise 11.7

1 a i SM
    ii PQ
    iii BC
  b i MSR
    ii EFG
    iii OPQ
  c $ABCDEFG$ is congruent to $SMNOPQR$

2 a A, C
  b D, F
  c B, G
  d E, H, L

3 a DEF similar GHI
  b $ABCD$ similar $EFGH$
  c $MNOP$ congruent $STQR$
  d $ABCDEFGH$ congruent $PIJKLMNO$ and both similar to $WXQRSTUV$
  e ABC similar MON

4



a b c

5 Since triangle FAB and FED are congruent:

Angle FAB = angle FED and that makes triangle CAE a right angled isosceles triangle.

It follows that $AC - BC = EC - DC$, so $BC = CD$.

$BF = DF$ (corr sides of congruent triangles)

Therefore $BFCD$ is a kite (two pairs of adjacent equal sides).

6 The two turns are the same so all the corresponding angles in the triangles must be the same. Since they both walk 1000 metres before turning then all three corresponding side lengths must be the same and so the triangles are congruent.

Practice questions

1 215 m further

2 4.21 m

3 a 35 cm
  b 37 cm

4 **a** $a^2 + b^2 = c^2$

$(7x)^2 + (24x)^2 = 150^2$

$49x^2 + 576x^2 = 22\,500$

$625x^2 = 22\,500$

$x^2 = 36$

b 336 cm

5 3.7 km

6 4.5 cm

7 **a** Let angle $CAB = x$

Then angle $ABC = 90° - x$ (angles in triangle sum to 180)

Angle CAB = angle DCB = angle $DAC = x$

Angle BCA = angle BDC = angle CDA = 90°

Angle ABC = angle CBD = angle ACD = $90° - x$

All corresponding angles are equal, so triangles ABC, CBD and ACD are similar.

b By comparing ratios of corresponding sides in triangles ABC and CBD:

$\frac{AB}{CB} = \frac{BC}{BD}$

$\frac{c}{a} = \frac{a}{e}$

$ce = a^2$

By comparing ratios of corresponding sides in triangles ABC and ACD:

$\frac{AB}{AC} = \frac{AC}{AD}$

$\frac{c}{b} = \frac{b}{d}$

$cd = b^2$

c $a^2 + b^2 = ce + cd$

$= c(d + e)$

$= c^2$

8 **a** 16.8 cm

b 103 cm²

9 **a** $a^2 + b^2 = c$

$(uv)^2 + \left(\frac{u^2 - v^2}{2}\right)^2 = \left(\frac{u^2 + v^2}{2}\right)^2$

$\Rightarrow \quad u^2v^2 + \frac{u^4 - 2u^2v^2 + v^4}{4} = \frac{u^2 + 2u^2v^2 + v^2}{4}$

$\Rightarrow \quad 4u^2v^2 + u^4 - 2u^2v^2 + v^4 = u^4 + 2u^2v^2 + v^4$

$\Rightarrow \quad u^4 + 2u^2v^2 + v^4 = u^4 + 2u^2v^2 + v^4$

b 17, 144, 145

(If $a = 17$ then $uv = 1 \times 17$ because 17 is a prime number and its only factors are 1 and itself. If you substitute the values of 1 and 17 into the formulae for b and c then you get $b = 144$ and $c = 145$.)

c Let a = prime (p), then $a = p \times 1$ (so $u = p$, $v = 1$)

If b and c differ by 1 then:

$\Rightarrow b + 1 = c$

$b = \frac{p^2 - 1^2}{2}$

$c = \frac{p^2 + 1^2}{2}$

The difference between b and c is:

$c - b$

$= \frac{p^2 + 1^2}{2} - \frac{p^2 - 1^2}{2}$

$= \frac{p^2}{2} + \frac{1}{2} - \left(\frac{p^2}{2} - \frac{1}{2}\right)$

$= 1$

Practice questions worked solutions

1 $HC = \sqrt{521^2 - 320^2} = \sqrt{169\,041}$

Difference $= 350 + \sqrt{169\,041} - 521$

$= 240.1$ m

2 $\sqrt{4.5^2 - 1.6^2} = 4.21$ m

3 **a** $\sqrt{21^2 + 28^2} = 35$ cm $= AC$

b $AD^2 + AC^2 = 12^2 + 35^2$

$AD = \sqrt{1369} = 37$ cm

4 **a** $(7x)^2 + (24x)^2 = 150^2$

$49x^2 + 576x^2 = 22\,500$

$625x^2 = 22\,500$

$x = \frac{22\,500}{625} = 36$

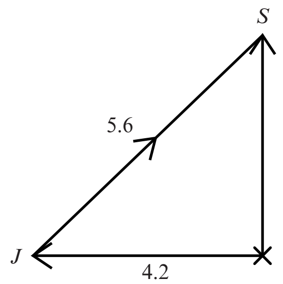
b $x = \sqrt{36} = 6$

So, perimeter $= 7 \times 6 + 24 \times 6 + 150$

$= 42 + 144 + 150$

$= 336$ cm

5



$\sqrt{5.6^2 - 4.2^2} = 3.70$ km

6 $\frac{3}{8} = \frac{r}{12}$

So $r = \frac{3 \times 12}{8} = \frac{9}{2}$ cm

7 **a** All three have the same angles.

b $\frac{a}{e} = \frac{c}{a} \Rightarrow a^2 = ce$

$\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = cd$

c $a^2 + b^2 = ce + cd$

$= c(e + d)$

$= c^2$

8 **a** $\sqrt[3]{\frac{343}{125}} \times 12 = 16.8$ cm

b $\left(\sqrt[3]{\frac{343}{125}}\right)^2 \times A = 201.6$

$A = 103$ cm^2

9 **a** $a^2 + b^2 = (uv)^2 + \left(\frac{u^2 - v^2}{2}\right)^2$

$= \frac{4(uv)^2 + (u^2 - v^2)^2}{4}$

$= \frac{(u^2)^2 + 2(uv)^2 + (v^2)^2}{4}$

$= \left(\frac{u^2 + v^2}{2}\right)^2 = c^2$

So, $c^2 = a^2 + b^2$

b $uv = 17$ so $u = 17$ and $v = 1$

$a = 17$

$b = \frac{17^2 - 1^2}{2} = 144$

$c = \frac{17^2 + 1^2}{2} = 145$

$145^2 = 21\,025 = 144^2 + 17^2$

so (17, 144, 145) is a Pythagorean triple.

c If a is prime, then $a = uv$ and u and v are 1 and a or a and 1

To make b positive, $u > 1$, so $u = a$ and $v = 1$.

$b = \frac{a^2 - 1}{2}$ and $c = \frac{a^2 + 1}{2}$

$c - b = \frac{a^2 + 1}{2} - \frac{a^2 - 1}{2}$

$= \frac{a^2 + 1 - a^2 + 1}{2}$

$= \frac{2}{2}$

$= 1$

Chapter 12

Getting started

1 He has used the 'middle' value of the ordered list. This is called the *median*.

2 Most of the numbers are fairly close to the value given by Rohan, so it is reasonable.

3 The actual values for each of the 7 weeks are lost. Instead, they are represented by a single value, which cannot give the full picture.

4 Jess has not arranged the times in order but has picked the middle value in the list. She has considered the position only and not how the value compares with the other values in the list.

5 Write the list in order and then take the middle value:

1 2.5 3 5.5 7 7.5 9.5

The middle value is 5.5 hours.

6 It would not change, because the middle number has not changed.

7 You can add the numbers up and divide by 7, for example. This is called the *mean*. This can be useful if you want the average to change when *any* of the values change.

Exercise 12.1

1 a i Mode = 12
ii Median = 9
iii Mean = 8
b i Mode = 8
ii Median = 6
iii Mean = 5.7
c i Mode = 2.1 and 8.2
ii Median = 4.15
iii Mean = 4.79
d i Mode = 12
ii Median = 9
iii Mean = 11.7

2 Mean increased from 8 to 11.7 because of the extreme value of 43 in(d). No change to mode or median.

3 a i 54
ii 48.5
b i 84.25
ii 98.875

4 For example, 1, 2, 3, 4, 15

5 Mode = none; mean = 96.4; median = 103

He will choose the median because it's the highest.

6 4451.6 cm

7 2.38 kg

8 $91.2\dot{6}$ °C

9 For example, 3, 4, 4, 6, 8

10 For example, 2, 3, 4, 7, 9

11 $\dfrac{mX + nY}{m + n}$

Units for averages

1 Mean = 3.22 metres

Range = 2.8 metres

2 320 cm 280 cm 410 cm
160 cm 440 cm

3 Mean = 322 cm

Range = 280 cm

They have been multiplied by 100.

The mean and range are both multiplied by the same number as the original data.

4 Changing the units changes the numerical value of both the mean and range. This means that you can make any mean or range bigger or smaller than any other just by changing the units of either value. If you fix the units for both sets of data, then you are comparing 'like-for-like'.

5 If you add m to each value, you also add m to the mean BUT the range remains unchanged. Because the values are all m units larger, then the overall mean will be the same m units larger. But the range measures the *difference* between the largest and smallest values, both of which have increased by the same value and so are the same distance apart.

Exercise 12.2

1 **a** Ru

  i Mean = 0.152

  ii Range = 0.089

 Oli

  i Mean = 0.139

  ii Range = 0.059

 b Ru

 c Oli

2 **a** Archimedes median = 13

 Bernoulli median = 15

 b Archimedes range = 16

 Bernoulli range = 17

 c Bernoulli

 d Archimedes

3 Backlights. Footlights has the best mean but the range is large, whereas Backlights and Brightlights have the same range but Backlights has a higher mean.

Exercise 12.3

1 **a** Mean = 4.5

 b Median = 4

 c Mode = 4 and 5

 d Range = 8

2 **a**

Price	Frequency	Total
$6.50	180	$1170
$8	215	$1720
$10	124	$1240
		$4130

 b $7.96

3 **a** Mode = no letters

 b Median = 1 letter

 c Mean = 0.85 letters

 d Range = 5

4 **a** Mode = 1 child

 b Median = 2 children

 c Mean = 2.12 children

5 **a** **i** Mode = 8 marks

  ii Median = 6.5 marks

  iii Mean = $6.0\dot{3}$ marks

 b If she wants to suggest that the class is doing better than it really is, she would use the mode and say something like: most students got 8 of 10.

6 **a**

Stem	Leaf
4	6
5	0 0 4 5 7 8 9
6	0 1 1 2 3 3 6 6 8 9
7	0 4

Key
4 | 6 represents 46 kilograms

 b 12

 c The data has many modes.

 d 74 − 46 = 28

 e 61 kg

7 **a**

Stem	Leaf
12	1 5 6 6 8 8 8 9 9
13	0 1 2 3 3 4 6 8
14	0 0 2 2 3 6
15	0

Key
12 | 1 represents 121 components per hour

 b 29

 c 132.5

Exercise 12.4

1 **a** 141.7 cm

 b $140 < h \leqslant 145$

2 **a** 5.28 min

 b 5 min 17 s

 c $2 < t \leqslant 4$

3 57.36 °C

4 **a** Hawks: 76.7 kg

 Eagles: 78.4 kg

 b $75 \leqslant M < 85$ for both

5 39.2 cm

6 42.23 years

Exercise 12.5

1 a Median = 6, $Q_1 = 4$, $Q_3 = 9$, IQR = 5

b Median = 17, $Q_1 = 12$, $Q_3 = 21$, IQR = 9

c Median = 14, $Q_1 = 5$, $Q_3 = 18$, IQR = 13

d Median = 3.4, $Q_1 = 2.45$, $Q_3 = 4.95$, IQR = 2.5

e Median = 15.65, $Q_1 = 13.9$, $Q_3 = 18.42$, IQR = 4.53

2 a $40.25 - 2.35 = 37.9$ hectares

b Q1 = 3.55, Q2 = 7.2 and Q3 = 23.83

c In this case, the high range shows that the data is very spread out but this is skewed by one high value, so the IQR is more representative of the spread of sizes.

3 Median = 6, $Q_1 = 4$, $Q_3 = 8$, IQR = 4

4 a Summer: median = 18.5, $Q_1 = 15.5$, $Q_3 = 23.5$

Winter: median = 11.5, $Q_1 = 9.25$, $Q_3 = 12.75$

b Summer: IQR = 8 Winter: IQR = 3.5

c The lower IQR in winter shows that car numbers are more consistent. In poor weather people either use their own transport or take transport more consistently.

5 a Julia: median = 23, $Q_1 = 13$, $Q_3 = 24$

Aneesh: median = 18, $Q_1 = 14$, $Q_3 = 20$

b Julia: IQR = 11 Aneesh: IQR = 6

c The IQR for the *Algebraist* is more consistent than that for the *Statistician* and is therefore more likely to have a particular audience while the variation is greater for the *Statistician* and therefore could appeal to a varying audience.

6 a i 6.5

ii 5.9

b i 10.85

ii 14.05

c i 3.275

ii 3.65

d At first glance it seems like country driving gets much better fuel consumption as it appears that the data is distributed more towards the higher end of the stems. However, the smaller interval and the decimal nature of the data mean that when you look at IQR, there is not such a massive difference in consumption given that the difference between the two IQRs is only 0.375.

7 a Test 1: range = 55; IQR = 82 − 39.5 = 42.5; median = 60.5

Test 2: range = 69; IQR = 81.5 − 45 = 36.5; median = 61.5

Test 3: range = 44; IQR = 71.5 − 45 = 26.5; median = 62

b Interpretations will vary, but generally the students performed worst on Test 3.

Practice questions

1 a 294 g

b 15.2 g

c 5

2 5

3 a 38

b

Sunshine		Shade
4	0	2 3 6
8 3 2 1	1	0 2 2 2 4 9 9
8 3 1	2	0 5 8 9
7 6 4 2 1 0	3	4 9
9 5 4 3	4	5 5 9
1	5	

Key
Sunshine 9|4 represents 4.9g
Shade 3|4 represents 3.4g

c 3.1 g

d 1.9 g

e $5.1 - 0.4 = 4.7$ g

f $4.9 - 0.2 = 4.7$ g

g Those collected in sunshine were, on average, larger, but both were equally spread out.

h $4.3 - 1.8 = 2.5$ g

i $3.4 - 1.2 = 2.2$ g

j There are no extreme values, which means that the range gives a sensible measure of spread as well as the interquartile range.

4 a 1 and 21

b $\frac{135 + 5n}{42 + n}$

c 8

d 2

5 a 24

b $25 < T \leqslant 30$

c 30

Practice questions worked solutions

1 **a** $14.7 \times 20 = 294$ g

b $\text{Total} = 294 + 30 \times 15.6$
$= 762$

$\text{Mean} = \frac{762}{50} = 15.24$ g

c $\frac{762 + 184}{50 + n} = 17.2$

so $50 + n = \frac{762 + 184}{17.2}$

$n = 55 - 50 = 5$ sweets

2 Total tremors $= 18 \times 4.5 = 81$

Total in first 10 years $= 10 \times 4.1 = 41$

In the last 8 years, mean $= \frac{81 - 41}{8}$

$= 5$ per year

3 **a** $19 + 19 = 38$ insects

b

Sunshine		Shade
4	0	2 3 6
8 3 2 1	1	0 2 2 2 4 9 9
8 3 1	2	0 5 8 9
7 6 4 2 1 0	3	4 9
9 5 4 3	4	5 5 9
1	5	

Key
Sunshine 9|4 represents 4.9g
Shade 3|4 represents 3.4g

c $\frac{19 + 1}{2} = 10\text{th is } 31$ g

d 19 g

e $51 - 4 = 47$ g

f $49 - 2 = 47$ g

g On average, those collected in the sunshine are heavier.

h Need the $\frac{19 + 1}{4} = 5\text{th}$ and

$3 \times \frac{19 + 1}{4} = 15\text{th}$

$43 - 18 = 25$ g

i $34 - 12 = 22$ g

j No real outliers and data is fairly symmetrical.

Both IQRs are similar *and* both ranges are similar.

4 **a** Total frequency $= 42 + n$

$2 + n + 5 + 2 + 1 < 32$

i.e. $n < 22$

and $11 + 2 + n + 5 + 2 + 1 > 21$

$n > 0$

So $1 \leqslant n \leqslant 21$

b

$$\frac{2 \times 21 + 3 \times 11 + 4 \times 2 + 5n + 6 \times 5 + 7 \times 2 + 8 \times 1}{42 + n}$$

$$= \frac{135 + 5n}{42 + n}$$

c $\frac{135 + 5n}{42 + n} = 3.5$

$135 + 5n = 147 + 3.5n$

$1.5n = 12$

$n = 8$

d Size 2

5 **a** $16 + 5 + 2 + 1 = 24$

b $25 < T \leqslant 30$

c $\frac{16 \times 27.5 + 5 \times 32.5 + 2 \times 37.5 + 1 \times 42.5}{24}$

$= 30\,°\text{C}$

Past paper questions

1 **a** 2, 8, 14

b $6k - 4 = 422$

$6k = 426$

$k = 71$

2 $x^2 - 7x - 5x + 35 = x^2 - 12x + 35$

3 $\frac{x}{12} = \sin 35°$

$x = 12 \sin 35°$

$= 6.88$ cm

4 **a** $(-3, -1)$

b $\frac{3}{2}$

c $y = \frac{3}{2}x - 1$ or $y = 1.5x - 1$

5 **a** All three angles are the same.

b $\frac{AC}{27} = \frac{6}{18}$

$AC = 27 \times \frac{1}{3} = 9$ cm

6 **a** 17

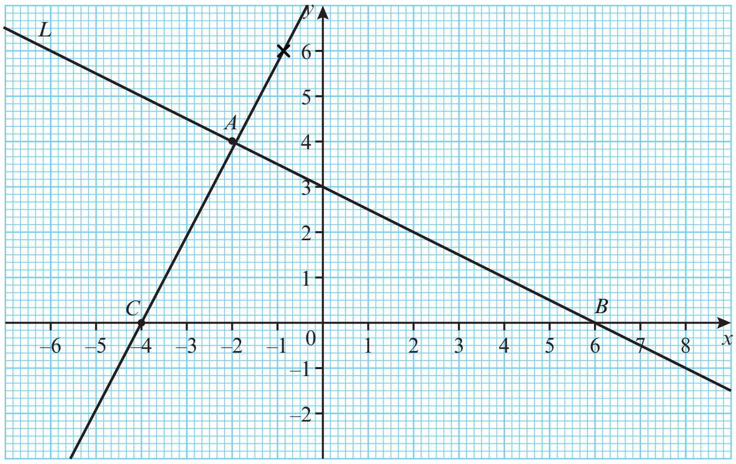
b $3n + 2$

7 **a** $(-2, 4)$

b **i** $-\frac{4}{8} = -\frac{1}{2}$

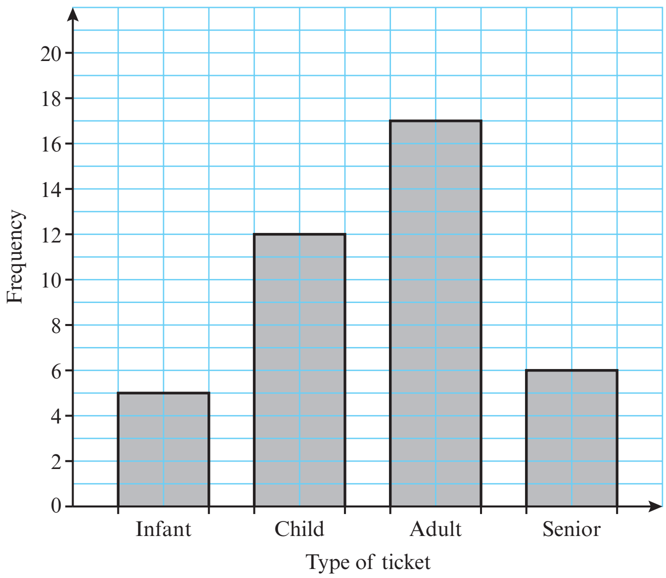
ii $y = -\frac{1}{2}x + 3$

c **i** and **ii**



iii 9.25 units

8 **a** **i**



ii $17 - 12 = 5$

iii adult

iv $\frac{12}{40} = \frac{3}{10} = 0.3$

b **i** $104 - 18 = 86$

ii 18 27 31 45 60 72 104

median is 45

iii $\frac{\sum x}{7} = 51$

9 **a** 4, 10, 18, 28

b $-7n + 32$

10

$$x = 0.4\dot{7}$$
$$10x = 4.\dot{7}$$
$$100x = 47.\dot{7}$$
$$90x = 43$$
$$x = \frac{43}{90}$$

11 **a** $\left(\frac{5+9}{2}, \frac{-5+3}{2}\right) = (7, -1)$

b $\sqrt{4^2 + 8^2} = \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5} = 8.94$

12 **a** $7a(3a + 4b)$

b $5(4x^2 - 9y^2) = 5(2x - 3y)(2x + 3y)$

13 **a** 20°

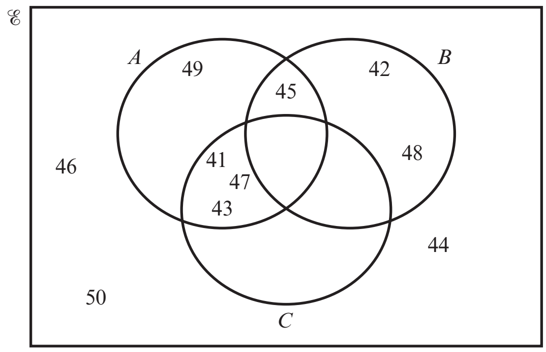
b $\frac{BE}{5.2} = \frac{6.75}{7.8} \Rightarrow BE = 4.5\,\text{cm}$

c volume multiplier $= \frac{780}{32}$

height multiplier $= \sqrt[3]{\frac{780}{32}}$

height $= 2 \times \sqrt[3]{\frac{780}{32}} = 5.80\,\text{cm}$

14 **a**



b **i** {41, 43, 47}

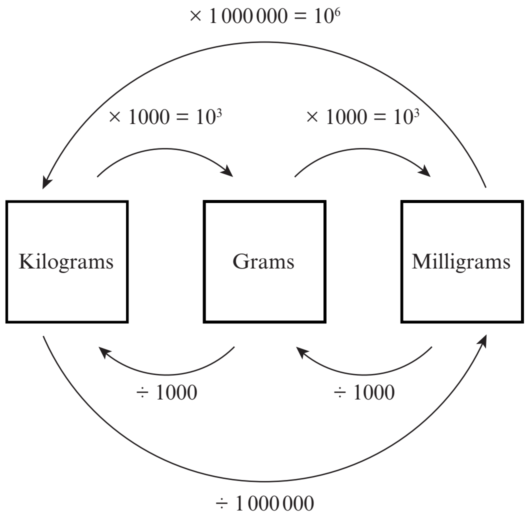
ii {44, 46, 49, 50}

iii ∅

Chapter 13

Getting started

1



2 Kilo = thousands
Milli = thousandths
Centi = hundredths

3 Students' conversion diagrams (examples could include: tonnes–kilograms–grams, or amps–milliamps)

Exercise 13.1

1 **a** 4000 g
b 5000 m
c 3.5 cm
d 8.1 cm
e 7300 mg
f 5.760 t
g 210 cm
h 2000 kg
i 1.40 m
j 2.024 kg
k 0.121 g
l 23 000 mm
m 35 mm
n 8036 m
o 9.077 g

2 $3.22\text{ m} < 3\frac{2}{9}\text{ m} < 324\text{ cm}$

3 $125\text{ ml} < \frac{1}{2}\text{ litre} < 0.65\text{ litres} < 780\text{ ml}$

4 60

5 **a** 14 230 mm, 0.014 23 km
b 19 060 mg, 0.000 019 06 t
c 2750 ml, 275 cl
d 4 000 000 mm^2, 0.0004 ha
e 1300 mm^2, 0.000 000 13 ha
f 10 000 mm^3, 0.000 01 m^3

6 **a** 27 m^3
b 27 000 000 cm^3
c $2.7 \times 10^{10}\text{ mm}^3$

7 **a** $1.09 \times 10^{12}\text{ km}^3$
b $1.09 \times 10^{21}\text{ m}^3$
c $1.09 \times 10^{30}\text{ mm}^3$

8 **a** $1.13 \times 10^{2}\text{ cm}^3$
b $1.13 \times 10^{5}\text{ cm}^3$
c $1.13 \times 10^{-13}\text{ km}^3$

9 **a** 6
b 20 g

10 **a** No
b No
c Yes

11 **a** Computers use a binary number system and 'mega' stands for 2^{20}. This is equal to 1 048 576.
b Examples include:
Pico – a millionth of a millionth ($\div 10^{12}$)
Nano – a thousandth of a millionth ($\div 10^{9}$)
Deca – 10 times
Hecto – 100 times
Giga – a thousand million times ($\times 10^{9}$)

Babylonian mathematics

1 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

2 Answers will depend on students' research. Possible reasons are that 60 is a convenient number because it has a lot of factors. It is possible to count to 12 using the bones in the fingers of one hand. So a base 60 system might have come about by multiplying the 12 on one hand by the 5 fingers on the other. Another possibility is that the number 60 comes from the length of a growing season for certain crops. So, a year would be approximately 6 × 60 days.

3 History appears to suggest that there are 360 degrees in a circle because that is approximately the number of days in a full year. In reality, 360 is a very convenient number because it has so many whole number factors. This means a circle can be divided into smaller parts without having to use fractions in any way.

Exercise 13.2

1 a i 22 30 to 23 30

ii 09 15 to 10 45

iii 19 45 to 21 10

b 09 30

2 3 h 39 min

3 9 min 47 s

4 Monday 10 February 02 30

5 a

Day	Mon	Tues	Wed
Total time worked	7 h 55 min	7 h 55 min	7 h 25 min

Day	Thurs	Fri
Total time worked	7 h 53 min	8 h 24 min

b 39 h 32 min

c $223.36

d i yes

ii He entered 5 [DMS] instead of 17 [DMS] and then subtracted 12°45′ and got a negative time for the afternoon.

Exercise 13.3

1 a 20 02

b 45 min

c 23 min

2 a 1 h 7 min

b

Aville	11 10
Beeston	11 45
Crossway	11 59
Darby	12 17

c 14 25

3 a 00 17

b 12 h 40 min

c 5 h 46 min

d i 01 29 or 13 34

ii unlikely to be 01 29 because it is in the middle of the night – in the dark.

e i 1–6 February (Wed–Mon)

ii 1–4 February (Wed–Sat)

Exercise 13.4

1 a $11.5 \leqslant 12 < 12.5$

b $7.5 \leqslant 8 < 8.5$

c $99.5 \leqslant 100 < 100.5$

d $8.5 \leqslant 9 < 9.5$

e $71.5 \leqslant 72 < 72.5$

f $126.5 \leqslant 127 < 127.5$

2 a $2.65 \leqslant 2.7 < 2.75$

b $34.35 \leqslant 34.4 < 34.45$

c $4.95 \leqslant 5.0 < 5.05$

d $1.05 \leqslant 1.1 < 1.15$

e $-2.35 \leqslant -2.3 < -2.25$

f $-7.25 \leqslant -7.2 < -7.15$

3 a $131.5 \leqslant 132 < 132.5$

b $250 \leqslant 300 < 350$

c $402.5 \leqslant 405 < 407.5$

d 14.5 million $\leqslant$ 15 million $<$ 15.5 million

e $32.25 \leqslant 32.3 < 32.35$

f $26.65 \leqslant 26.7 < 26.75$

g $0.45 \leqslant 0.5 < 0.55$

h $12.335 \leqslant 12.34 < 12.345$

i $131.5 \leqslant 132 < 132.5$

j $0.1335 \leqslant 0.134 < 0.1345$

4 250 kg $\leqslant$ 300 kg < 350 kg

5 **a** 99.5 m $\leqslant$ 100 m < 100.5 m

b 15.25 seconds $\leqslant$ 15.3 seconds < 15.35 seconds

6 4.45 m $\leqslant L <$ 4.55 m

Exercise 13.5

1 **a** $30.8 \leqslant a^2 < 31.9$

b $13\,900 \leqslant b^3 < 14\,100$

c $5.43 \leqslant cd^3 < 5.97$

d $609 \leqslant (a^2 + b^2) < 615$

e $0.248 < \frac{c}{b^2} < 0.252$

f $2.66 < \frac{ab}{cd} < 2.82$

g $-43.5 < \frac{c}{a} - \frac{b}{d} < -46.5$

h $2.66 < \left(\frac{a}{d} \div \frac{c}{b}\right) < 2.82$

i $48.9 < \left(dc + \sqrt{\frac{a}{b}}\right) < 50.7$

j $47.9 < \left(dc - \sqrt{\frac{a}{b}}\right) < 49.7$

2 37 kg $\leqslant$ mass left < 39 kg

3 **a** 3.605 cm $\leqslant$ length < 3.615 cm; 2.565 cm $\leqslant$ width < 2.575 cm

b 9.246825 cm^2 $\leqslant$ area < 9.308 625 cm^2

c 9.25 cm^2 $\leqslant$ area < 9.31 cm^2

4 **a** 511 105 787 km^2 $\leqslant$ surface area 511 266 084 km^2

b $1.086\,525\,72 \times 10^{12}$ km^3 $\leqslant$ volume of Earth < $1.087\,036\,906 \times 10^{12}$ km^3

5 The smallest number of cupfuls is 426.4, and the largest is 433.6.

6 Maximum gradient = 0.0739

Minimum gradient = 0.06

7 **a** 8.1 cm^2 $\leqslant$ area of Δ < 8.5 cm^2

b 5.76 cm $\leqslant$ hypotenuse < 5.90 cm

8 $63.4° \leqslant x° < 63.6°$

9 $45.2\% \leqslant \left(\frac{45}{98} \times 100\right) < 46.7\%$

10 332 kg $\leqslant$ mean mass < 335 kg (3 s.f.)

11 117.36 $\leqslant$ number of 5s < 117.84

12 **a** Max = 232.875; min = 128.625

b **i** Max 5.32 and min 4.86

ii Only 1 can be used. The value of a is 5 to 1 s.f. If we find the maximum and minimum values to 2 s.f. we get 5.3 and 4.9. This doesn't tell us any more than the answer is 5 to 1 s.f.

13 Upper bound is 0.910 m/s

Lower bound is 0.769 m/s

Exercise 13.6

1 **a** 140 °F

b 60 °F

c −16 °C

d 38 °C

2 **a** 4 lb

b 4 kg

c 36 kg

d 132 lbs

e **i** Correct

ii 18 lb = 8 kg

iii 60 lb = 27 kg

iv Correct

3 **a** \$16

b \$64

c £36

d £24

4 **a** 165 min

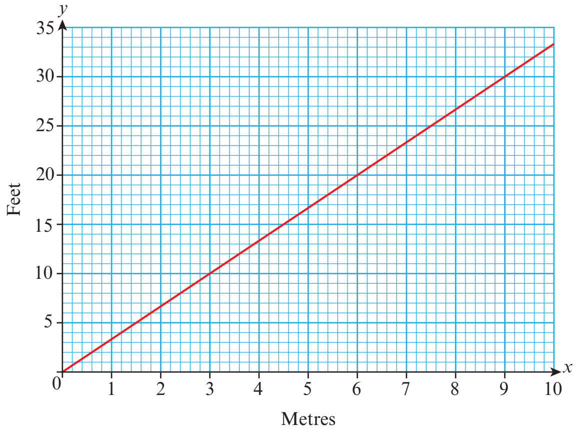
b 4.8 kg

c $(40m) + 30 = 25$

$\Rightarrow m = -0.125$ kg

You cannot have a negative mass of meat. As the graph assumes it will always take at least 30 minutes to cook any piece of meat, you cannot use this graph for meat with a very small mass that will take less than 30 minutes to cook.

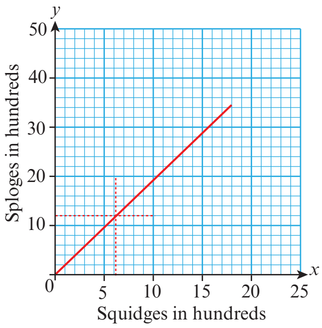
5 a



b 3600 ft (answer may vary +/− 100 foot)

c 1050 m (answer may vary slightly if answer to (b) varies from that shown)

6 a



b 625 *squidges* (answer may vary)

c 224 000 *ploggs* (answer may vary: 220 000 − 228 000)

Exercise 13.7

1 \$18.50

2 \$4163.00

3 £7960

4 \$384.52

5 \$2589.20

6 \$113.77

Practice questions

1 a 104 km/h

b 69 mph

2 a Monday: 8 hours 25 minutes
Tuesday: 8 hours 43 minutes
Wednesday: 8 hours 42 minutes
Thursday: 8 hours 38 minutes
Friday: 10 hours 17 minutes

b 44 hours 45 mins

c \$400.51

d \$352.45

3 a 33.5, 34.5

b 550, 650

c 12.685, 12.695

d 665, 675

4 12.25 kg, 12.75 kg

5 a i 7540

ii 49 692

iii 9238.50

iv 25 426.50

b 1232.61

6 a 3.78×10^{11}

b $378\,500 \leqslant$ area of Japan $< 377\,500$

c 335 people per square kilometre

7 a Approximately 18 litres

b Approximately 6 gallons

c $L = 4.5G$. The formula is not exact because the values read from the graph are approximate.

8 a −1.55

b 1.53

c 0.62

9 a $4.116 \times 10^3\,\text{cm}^3 \leqslant$ volume of cube $< 4.038 \times 10^3\,\text{cm}^3$

b $4.116 \times 10^6\,\text{mm}^3 \leqslant$ volume of cube $< 4.038 \times 10^6\,\text{mm}^3$

10 a $11.94\,\text{cm}^2$

b 7.09 cm

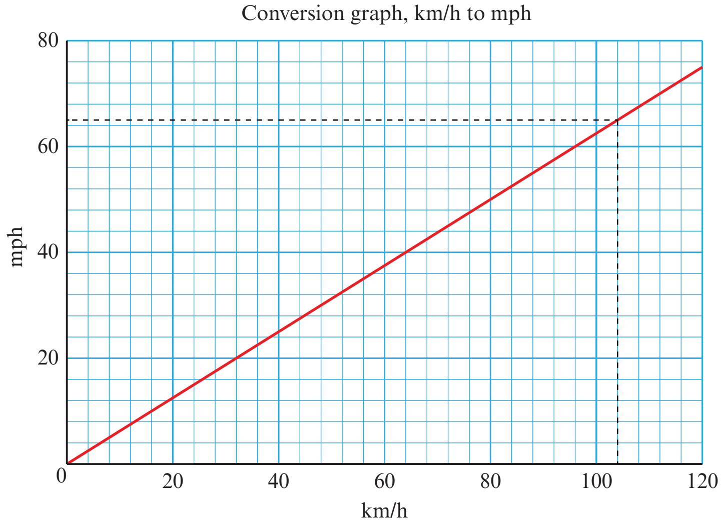
c 0.89

11 a $759\,\text{cm}^3$

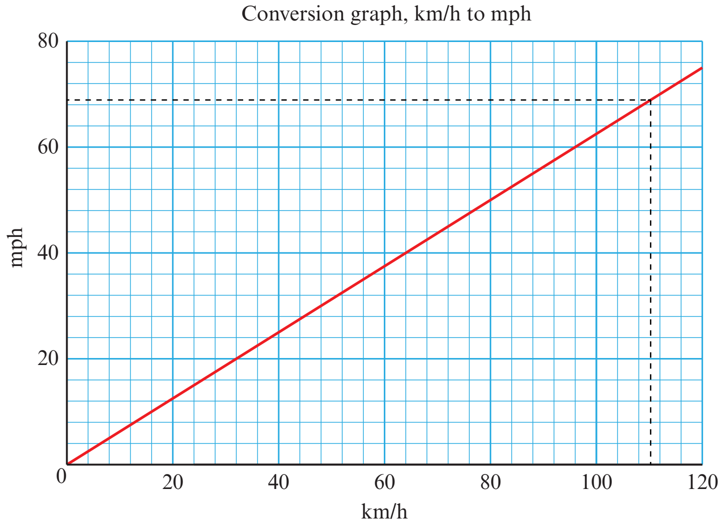
b $4.47\,\text{cm} \leqslant a < 4.95\,\text{cm}$

Practice questions worked solutions

1 **a**



b



2 **a** 8.25
8.43
8.42
8.38
10.17

b 42 + 2 + 45 minutes = 44 hours and 45 minutes

c $44.75 \times \$8.95 = \$400.51$

d $0.88 \times \$400.51 = \$352.45$

3

	upper bound	lower bound
a	34.5	33.5
b	650	550
c	12.695	12.685
d	675	665

4 Greatest = 12.75 kg
Least = 12.25 kg

5 **a** **i** 1 US \$ = 75.40 Indian Rupees
100 US \$ = 100 × 75.40 Indian Rupees
= 7540 Indian Rupees

ii €1 = 82.82 Indian Rupees
€600 = 600 × 82.82 Indian Rupees
= 49 692 Indian Rupees

iii 1 Dhs = 20.53 Indian Rupees
450 Dhs = 450 × 20.53 Indian Rupees
= 9238.5 Indian Rupees

iv 1 SR = 20.10 Indian Rupees
1265 SR = 1265 × 20.10 Indian Rupees
= 25 426.50 Indian Rupees

b 1 Australian dollar = 56.79 Indian Rupees

1 Indian Rupee = $\frac{1}{56.79}$ Australian dollar

14 000 Indian Rupees = $\frac{14\,000}{56.79}$ Australian dollars

= 246.52 Australian dollars

6 **a** $378\,000 \times 1000 \times 1000 = 3.78 \times 10^{11}\ \text{m}^2$

b Upper bound = 378 500
Lower bound = 377 500

c 126 500

$\frac{126\,500\,000}{377\,500} = 335$

7 **a** 18.5 litres

b 5.95 gallons

c $L = kG$

$18.5 = k \times 4$

$k = \frac{18.5}{4} = 4.625$

so, L is approximately equal to $4.625G$

The value of k was calculated using estimates from the graph.

8 **a** $2.55 + 4.35 - 8.45 = -1.55$

b $\frac{2.55^2}{4.25} = 1.53$

c $\frac{2.55}{8.55 - 4.25} = 0.59$

9 **a** Upper bound = 14.55 × 13.25 × 21.25
= 4116 cm^3

Lower bound = 14.45 × 13.15 × 21.25
= 4038 cm^3

b Upper bound = 145.5 × 132.5 × 213.5
= 4 116 013 mm^3

Lower bound = 144.5 × 131.5 × 212.5
= 4 037 872 mm^3

10 **a** $\frac{1}{2} \times 5.25 \times 4.55 = 11.94\ \text{cm}^2$

b $\sqrt{5.35^2 + 4.65^2} = 7.09\ \text{cm}$

c $\frac{4.65}{5.25} = 0.89$

11 **a** $7.5^2 \times 13.5 = 759.375\ \text{cm}^3$

b $A = \sqrt{\frac{\text{volume}}{h}}$

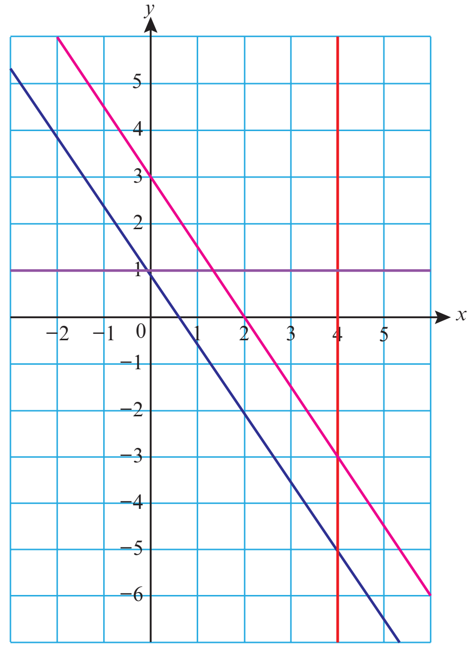
Upper bound = $\sqrt{\frac{325}{13.25}} = 4.95\ \text{cm}$

Lower bound = $\sqrt{\frac{275}{13.75}} = 4.47\ \text{cm}$

Chapter 14

Getting started

1, 2, 4



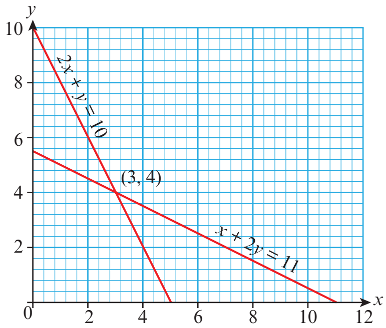
3 3 square units

5 12 square units

6 $3 \leqslant A < 12$

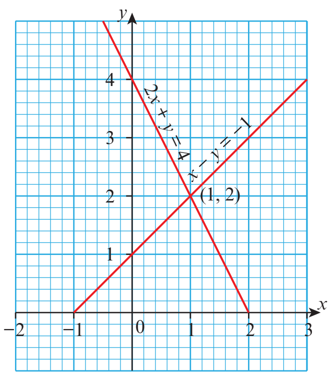
Exercise 14.1

1 a



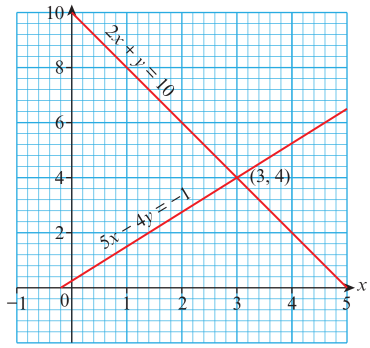
Solution is $x = 3$ and $y = 4$

b



Solution is $x = 1$ and $y = 2$

c



Solution is $x = 3$ and $y = 4$

2 a $x = -2,\ y = -2$

b $x = 3,\ y = 3$

c $x = 3,\ y = -2$

d $x = -1,\ y = 6$

e $x = \frac{1}{7},\ y = -2$

f $x = \frac{4}{3},\ y = \frac{4}{3}$

3 a $x = \frac{9}{11},\ y = -\frac{1}{11}$

b $x = \frac{5}{4},\ y = -\frac{3}{4}$

c $x = \frac{7}{4},\ y = 1$

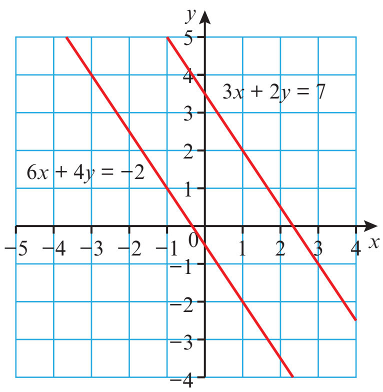
d $x = \frac{25}{17},\ y = \frac{22}{17}$

4 a The scale can sometimes make it difficult to read off certain values, such as fractions, accurately.

b The equations must be solved algebraically.

When is there a solution?

1



2 The lines are parallel and never meet, so there will be no solutions that work in both equations at the same time.

3 The coefficients of x and y in the first equation have been multiplied by the same number to get the second equation, BUT this is not true for the right-hand sides.

4 If $\frac{c}{a}$ is not the same as $\frac{d}{b}$ there will be solutions. If they ARE the same, then $\frac{c}{a}$ and $\frac{d}{b}$ must both be equal to $\frac{4}{3}$.

Exercise 14.2

1 a $x = 2, y = 5$
b $x = 3, y = -2$
c $x = -10, y = 6$
d $x = \frac{4}{3}, y = -\frac{10}{3}$
e $x = -2, y = 4$
f $x = -\frac{11}{3}, y = 17$
g $x = \frac{1}{2}, y = \frac{1}{2}$
h $x = \frac{19}{17}, y = \frac{10}{17}$

2 a $x = 4, y = 4$
b $x = 2, y = 6$
c $x = 1, y = 2$
d $x = 5, y = -1$
e $x = 3, y = 4$
f $x = 1, y = 3$
g $x = 6, y = 3$
h $x = 5, y = 4$
i $x = 4, y = 3$
j $x = 4, y = 6$
k $x = 6, y = 6$
l $x = 4, y = 2$

3 a $x = 2, y = 4$
b $x = 4, y = 3$
c $x = -5, y = -10$
d $x = 5, y = 5$
e $x = \frac{7}{4}, y = \frac{9}{4}$
f $x = 5, y = 3$
g $x = \frac{6}{5}, y = \frac{9}{10}$
h $x = \frac{7}{3}, y = -\frac{6}{13}$
i $x = -\frac{118}{55}, y = -\frac{5}{11}$
j $x = \frac{29}{4}, y = \frac{35}{12}$
k $x = 1, y = -4$
l $x = -1, y = -4$
m $x = 5, y = -7$
n $x = -\frac{7}{3}, y = \frac{3}{2}$
o $x = \frac{3}{5}, y = \frac{29}{5}$

4 a $x = 3, y = 4$
b $x = 2, y = 4$
c $x = -3, y = 5$
d $x = 6, y = 3$
e $x = 3, y = 5$
f $x = 3, y = -4$
g $x = 5, y = 3$
h $x = 2, y = 4$
i $x = 2, y = 3$
j $x = -2, y = 1$
k $x = -3, y = -2$
l $x = \frac{1}{2}, y = 2$
m $x = -\frac{1}{2}, y = 3$
n $x = -3, y = 4$
o $x = 5, y = 8$

5 a $x = \frac{112}{25}, y = -\frac{504}{25}$
b $x = 3, y = -2$
c $x = -8, y = -2$
d $x = 6, y = -18$
e $x = -0.739, y = -8.217$
f $x = 5.928, y = -15.985$ (3 d.p.)

6 a 90 and 30
b −14.5 and −19.5
c 31.5 and 20.5
d 14 and 20

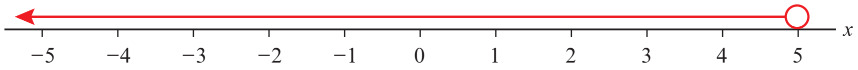
7 Pen drive $10 and hard drive $25

8 48 blocks (36 of 450 seats and 12 of 400 seats)

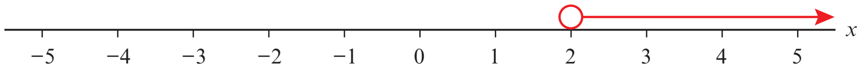
9 Students will create their own problem for each other.

Exercise 14.3

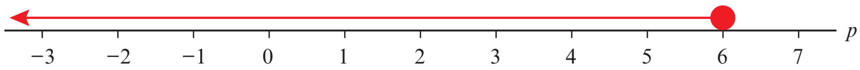
1 **a**



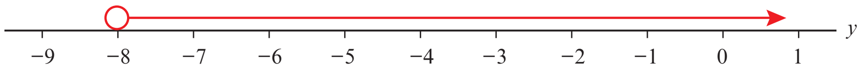
b



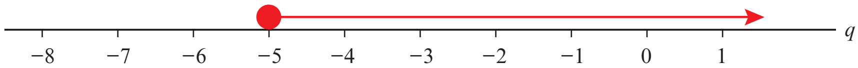
c



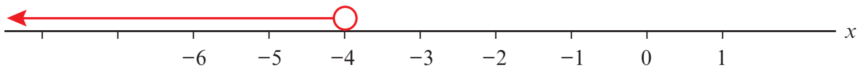
d



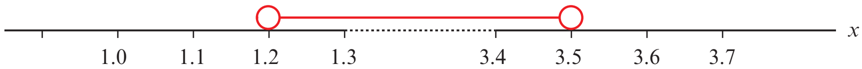
e



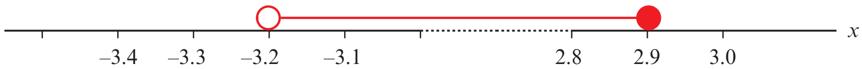
f



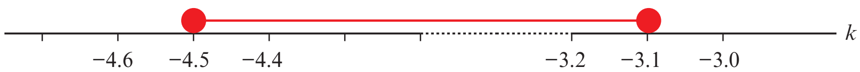
g



h



i



2 **a** $\{4, 5, \ldots, 31, 32\}$

b $\{8, 9, \ldots, 18, 19\}$

c $\{18, 19, \ldots, 26, 27\}$

d $\{-3, -2, -1\}$

e $\{-3, -2, -1, 0\}$

f $\{3, 4, \ldots, 10, 11\}$

g $\{-6, -5, -4\}$

h $\{4, 5, 6\}$

i $\{3, 4\}$

Exercise 14.4

1 **a** $x < 2$

b $x > 3$

c $y \leqslant \frac{14}{15}$

d $y > -2$

e $c \geqslant 2$

f $x \leqslant -1$

g $x < 6$

h $p > 3$

i $x > -15$

j $g \geqslant 4$

k $w < 8$

l $k < \frac{7}{10}$

2 **a** $y > 30$

b $y \leqslant 30$

c $z > 62$

d $k > 33$

e $r < \frac{10}{3}$

3 **a** $q < 12$

b $g \leqslant \frac{11}{2}$

c $h \geqslant -\frac{3}{2}$

d $h < 19$

e $y \geqslant -\frac{44}{3}$

f $n < 48$

g $v \leqslant -\frac{13}{6}$

h $e > \frac{31}{28}$

i $t > 9\frac{1}{4}$

j $t > \frac{109}{4}$

k $t > \frac{763}{4}$

4 **a** $-\frac{11}{2} < x < -2$

b $-3 < x \leqslant 9$

c $6 < x \leqslant 13$

d $\frac{16}{3} < x \leqslant 7$

e $-\frac{16}{5} < x \leqslant -\frac{4}{17}$

5 **a**

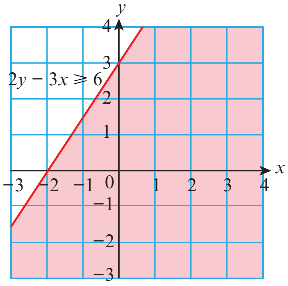
b $p \geqslant 1\,700\,000$

6 $78 + 28b \leqslant 630$

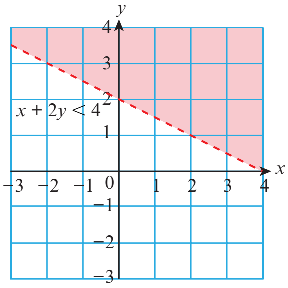
19 boxes (cannot load a fraction of a box)

Exercise 14.5

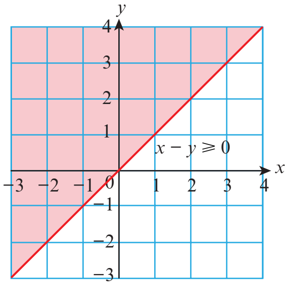
1



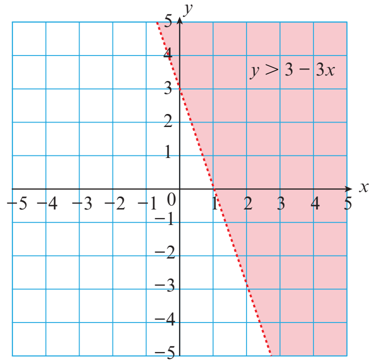
2



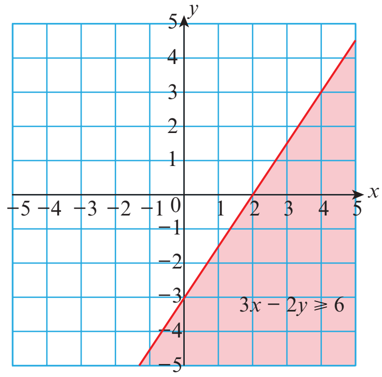
3



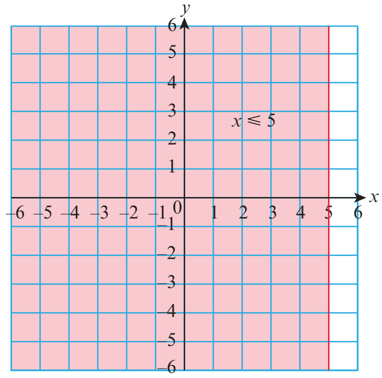
4 **a**



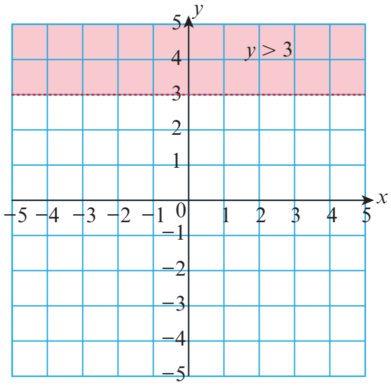
b



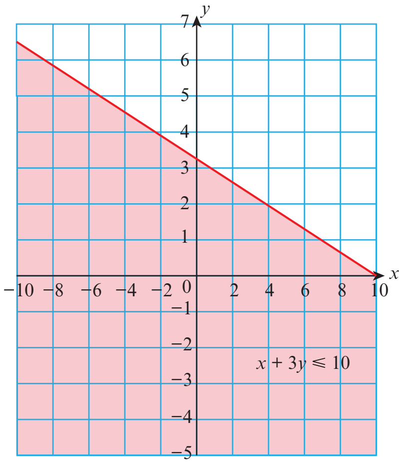
c



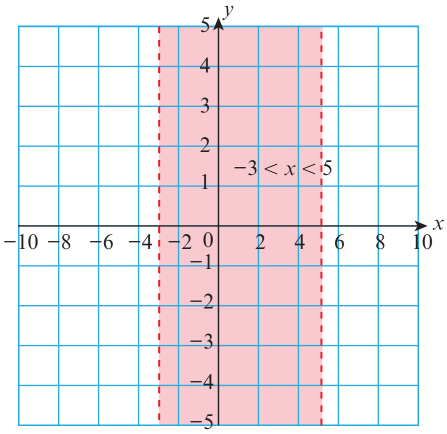
d



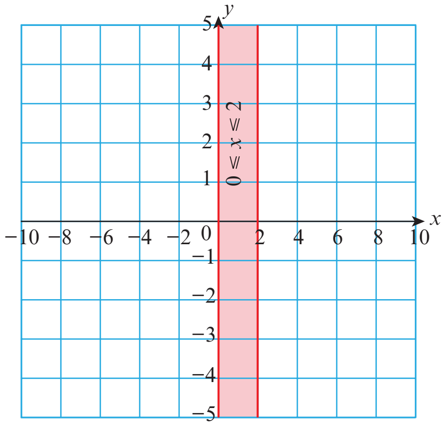
e



f



g



5 **a** Above

b Below

c Above and below

6 **a** $y \leqslant 4x + 5$

b $x + y < 3$

c $y \geqslant \frac{1}{3}x + 1$

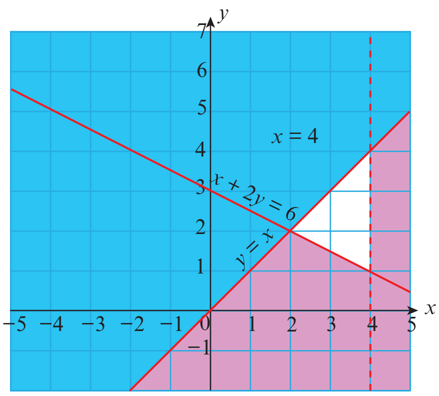
d $y \leqslant -\frac{3}{2}x$

Shading the wanted region

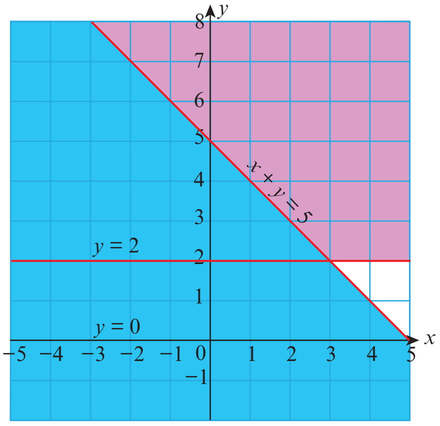
Shading the wanted region for a single inequality works well, but when there is more than one inequality then it is more difficult because it is hard to see the region that satisfies all the inequalities since shaded regions overlap. Only the region that is shaded for all the inequalities will count as the wanted region.

Exercise 14.6

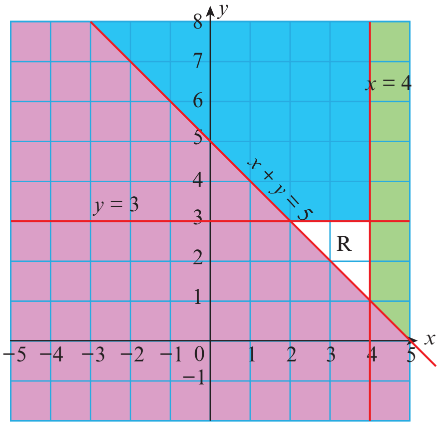
1



2



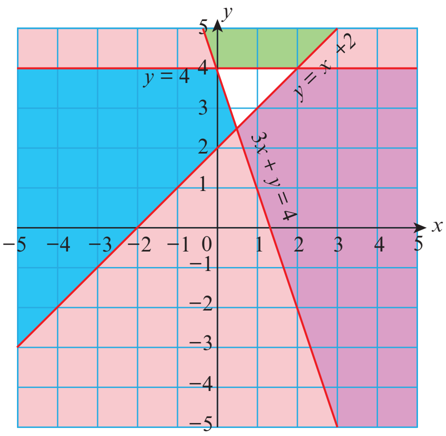
3



4 $y \leqslant -x + 4$, $y > 2x + 1$, $x \leqslant 2$

5 (3, 0), (2, 0), (2, 1), (1, 1), (1, 2), (1, 0), (0, 3), (0, 2), (−1, 4)

6



(0, 4) (1, 4) (2, 4) (1, 3)

Exercise 14.7

1 a $(x + 3)^2 + 5$
  b $(x + 4)^2 - 15$
  c $(x + 6)^2 - 16$
  d $(x + 3)^2 - 4$
  e $(x - 2)^2 + 8$
  f $(x - 1)^2 + 16$
  g $\left(x + \frac{5}{2}\right)^2 - \frac{21}{4}$
  h $\left(x + \frac{7^2}{2}\right) - \frac{57}{4}$
  i $\left(x - \frac{3}{2}\right)^2 - \frac{21}{4}$
  j $\left(x + \frac{7^2}{2}\right) - \frac{81}{4}$
  k $\left(x - \frac{13}{2}\right)^2 - \frac{165}{4}$
  l $(x - 10)^2 + 300$

2 a $x = 0.74$ or -6.74
  b $x = -0.54$ or -7.46
  c $x = 3.41$ or 0.59
  d $x = 1.14$ or -6.14
  e $x = 2$ or 1
  f $x = 11.92$ or 0.08

3 a $x = 3.70$ or -2.70
  b $x = 1.37$ or -4.37
  c $x = 0.16$ or -6.16
  d $x = 2$ or 4
  e $x = 1.89$ or 0.11
  f $x = 5.37$ or -0.37
  g $x = 1.30$ or -2.30
  h $x = 3$ or -1
  i $x = 1.62$ or -0.62

4 Students' own choice of question.

Exercise 14.8

1 a $3(x + 1)^2 + 11$
  b $2(x + 2)^2 - 7$
  c $6(x + 1)^2 + 14$
  d $2\left(x + \frac{3}{2}\right)^2 + \frac{1}{2}$
  e $4(x - 1)^2 + 7$
  f $2(x - 1)^2 + 15$
  g $5(x + 1)^2 - 4$
  h $3\left(x + \frac{7}{6}\right)^2 - \frac{73}{12}$
  i $2\left(x - \frac{3}{4}\right)^2 - \frac{33}{8}$

j $5\left(x+\frac{7}{10}\right)^2-\frac{209}{20}$

k $2\left(x-\frac{13}{4}\right)^2-\frac{161}{8}$

l $3\left(x-\frac{10}{3}\right)^2+\frac{1100}{3}$

2 a 108
b 120
c 135
d 144
e 150
f 165.6

3 a 2340
b 360
c 156
d 24

Exercise 14.9

1 a $x = -3$ or -4
b $x = -6$ or -2
c $x = -7$ or -4
d $x = -5$ or 1
e $x = -8$ or 2
f $x = 8$ or -20
g $x = 4$ or 2
h $x = 7$ or -4
i $x = 8$ or -3
j $x = 8$ or 4
k $x = 11$ or -9
l $x = 12$ or -3
m $x = 6$ or 4
n $x = 5$ or 7
o $x = -3$ or 12

2 a $x = 0.162$ or -6.16
b $x = -1.38$ or -3.62
c $x = -2.38$ or -4.62
d $x = -0.586$ or -3.41
e $x = 3.30$ or -0.303
f $x = 3.41$ or 0.586
g $x = 7.16$ or 0.838
h $x = 2.73$ or -0.732
i $x = 6.61$ or -0.606
j $x = 8.24$ or -0.243
k $x = 8.14$ or -0.860
l $x = -0.678$ or -10.3

3 a $x = 3.73$ or 0.268
b $x = 3.30$ or -0.303
c $x = 0.896$ or -1.40
d $x = -0.851$ or 2.35
e $x = -1.37$ or 0.366
f $x = 0.681$ or -0.881

4 a $x = 4.79$ or 0.209
b $x = 0.631$ or 0.227
c $x = 0.879$ or -0.379
d $x = 1.35$ or -2.95
e $x = -2.84$ or -9.16
f $x = 6.85$ or 0.146

5 a $x=\frac{5+\sqrt{21}}{2}$ or $x=\frac{5-\sqrt{21}}{2}$
b $x=\frac{-3+\sqrt{65}}{4}$ or $x=\frac{-3-\sqrt{65}}{4}$
c $x=\frac{5+\sqrt{37}}{6}$ or $x=\frac{5-\sqrt{37}}{6}$
d $x=-2+\sqrt{3}$ or $x=-2-\sqrt{3}$
e $x=-4+\sqrt{7}$ or $x=-4-\sqrt{7}$
f $x=\frac{5+\sqrt{15}}{2}$ or $x=\frac{5-\sqrt{15}}{2}$

6 $x = 1.61$ cm (-5.61 is not a solution because length cannot be negative)

7 a 4.53 metres
b 248 months

Exercise 14.10

1 a $(3x+2)(x+4)$
b $(2x+3)(x-1)$
c $(3x+2)(2x-1)$
d $(3x+8)(x+2)$
e $(2x-5)(x+2)$
f $(4x-1)(4x+9)$
g $(3x+1)(x+5)$
h $(4x-1)(2x+1)$
i $(2x+3)(x-2)$
j $(2x+3)(x+3)$
k $(3x+8)(x-2)$
l $(5x-3)(2x+1)$
m $(5x+1)(x+1)$
n $(2x-1)(x-9)$
o $(6x-5)(2x+3)$

Exercise 14.11

1 As Exercise 14.10

2 a $(3x-7)(2x+3)$
b $-(2x+3)(x+5)$
c $(2x+3y)(2x+3y)$
d $(3x+y)(2x-7y)$
e $x(x-9)(x-4)$
f $2(3x-4y)(x-5y)$
g $(3x+2)(2x+1)$
h $(3x-4)(x-3)$
i $3(x-5)(x-8)$
j $p(x+3)(x+4)$
k $x(5x-6)(x-2)$
l $3x(4-x)^2$
m $(x-1)(x-2)$
n $4(x-2)(x-1)$
o $(2x)(6x+13)$

Exercise 14.12

1 a $\frac{x}{2}$
b $\frac{y}{4}$
c 5
d 10
e $\frac{t}{6}$
f $\frac{u}{3}$
g $\frac{t}{10}$
h $\frac{y}{2}$
i $\frac{3z}{4}$
j $\frac{4t}{3}$

2 a $\frac{xy}{3}$
b $\frac{x}{4y}$
c $\frac{1}{2}$
d $\frac{y}{2}$
e $5x$
f $3b$
g $\frac{2x}{3y}$
h $3b$
i $\frac{2}{3de}$
j $\frac{1}{4b^2}$

3 a $\frac{a}{5b}$
b ab
c $\frac{b}{2}$
d $\frac{ac}{4}$
e $\frac{abc}{2}$
f $\frac{9b}{4c}$
g $(abc)^2$
h $\frac{3y}{4x}$
i $\frac{4x^2z}{3y}$
j 9

4 a $\frac{18}{17z^3}$
b $\frac{x^3z}{2y}$
c $\frac{3v}{7u^2w^4}$
d $\frac{x+3}{x+4}$
e $\frac{x}{x+4}$
f $\frac{y^3}{y+1}$
g $\frac{x-6}{x-4}$
h $\frac{x+5}{x-3}$
i 8
j $\frac{3x+2}{3x-2}$
k $\frac{x+3}{x+8}$
l $\frac{2x-3}{x+1}$
m $\frac{7x-1}{x-4}$
n $\frac{5y-4}{y-7}$
o $\frac{3x-7}{5x-4}$

5 **a** $\frac{3x-4}{7x+1}$

b x^2+y^2

c $\frac{1}{x}$

d x^2+1

e 1

f $\sqrt{x^3+y^3}$

Exercise 14.13

1 **a** $\frac{x^2}{4}$

b $\frac{3y^2}{14}$

c $\frac{3z^2}{14}$

d $\frac{t^2}{3}$

e 1

f $\frac{1}{6}$

g $\frac{3f}{2e}$

h $\frac{gh^2}{32}$

i 2

j $\frac{1}{2y^2}$

k $\frac{2d}{7c}$

l $\frac{r}{2pq}$

2 **a** $\frac{3z^2t^2}{x^3}$

b $\frac{2xt}{3}$

c $\frac{3}{4xy}$

d $\frac{64t^4y^4}{27}$

e $\frac{3}{4(x+y)^5(x-y)}$

f $\frac{1}{4(a-b)}$

g $\frac{\left(\sqrt{z^2+t^2}\right)^3}{432(x^2+y^2)}$

h $\frac{z-t}{z-y}$

Exercise 14.14

1 **a** $\frac{3y}{4}$

b $\frac{8t}{15}$

c $\frac{12u}{35}$

d $\frac{z}{14}$

e $\frac{5(x+y)}{12}$

f $\frac{3x}{2}$

g $\frac{11y}{8}$

h $\frac{a}{40}$

i $\frac{a}{2}$

j $\frac{7x+18y}{63}$

2 **a** $\frac{19(x+1)^2}{56}$

b $\frac{29pqr}{136}$

c $\frac{93p}{70}$

d $\frac{71x}{84}$

e $\frac{62x^2}{63}$

f $\frac{33-5x}{18}$

3 **a** $\frac{x+3}{a}$

b $\frac{23}{12a}$

c $\frac{19x}{6y}$

d $\frac{3a+2}{a^2}$

e $\frac{17}{6x}$

f $\frac{7}{5e}$

4 **a** $\frac{2x+5}{(x+1)(x+4)}$

b $\frac{5x-7}{(x-1)(x-2)}$

c $\dfrac{7x + 39}{(x + 2)(x + 7)}$

d $\dfrac{5}{2x}$

e $\dfrac{7}{6xy}$

f $\dfrac{2 + x^2}{x}$

g $\dfrac{x^2 + 2x + 5}{2(x + 1)}$

h $\dfrac{(x^2 - 1)(27y - 14)}{63y^2}$

i $\dfrac{2y - x^3}{2x^2y}$

j $\dfrac{4x^2y + 4xy - yz^2 - z^3}{12xyz^2}$

k $\dfrac{1}{x + 3}$

l $\dfrac{2}{x + 2}$

Practice questions

1 $x = 2, y = 3$

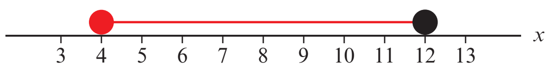
2 **a** $18x - 4 < 88$

$18x < 92$

$x < 5.11$

b 432 cm^2

3



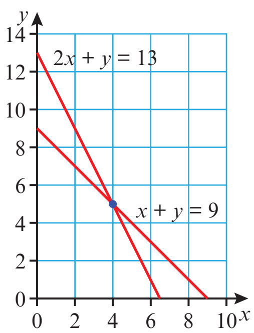
4 **a**

x	0	3	9
y	9	6	0

b

x	0	1	5
y	13	11	3

c



d $x = 4, y = 5$

5 14

6 **a** $50 - x$

b $x(50 - x) = 50x - x^2$

c $50x - x^2$

$= -(x^2 - 50x)$

$= -\{(x - 25)^2 - 625\}$

$= 625 - (x - 25)^2$

$x = 25$

d 625 m^2

7 $(x + 4)^2 - 33$

$x = -4 \pm \sqrt{33}$

8 $x = 1.41$ or $x = -6.41$

9 4, 5, 6, 7, 8, 9

10 $\dfrac{6y^3}{x^2}$

11 **a** $100 - x$

b $\left(\dfrac{x}{4}\right)^2 = \dfrac{x^2}{16}$ and $\left(\dfrac{100 - x}{4}\right)^2 = \dfrac{(100 - x)^2}{16}$

c $\dfrac{x^2}{16} + \dfrac{(100 - x)^2}{16} = 325$

$x = 40$ or $x = 60$

d 40 cm

12 **a** $(3x - 1)(x - 2)$

b $(x - y)(x + y - 2z)$

13 $\dfrac{5 + x}{(x + 2)(x - 2)}$

14 $5(x - 3)^2 + 6$

$5(x - 3)^2 \geqslant 0$

$5(x - 3)^2 + 6 \geqslant 6 > 0$

15 $\dfrac{x - 7}{x + 3}$

16 **a** $1 - p$

b $p - p^2$

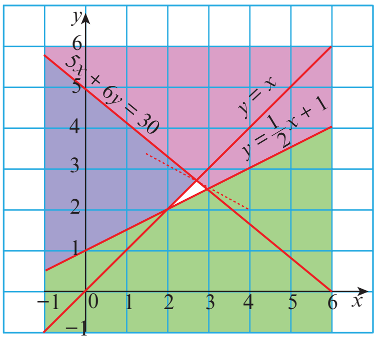
c $p = \dfrac{1}{4}$ or $p = \dfrac{3}{4}$

d $\dfrac{15}{16}$

17 **i** $\sqrt{37}$

ii 5

18 a



b Greatest value for $x + 2y = 8\frac{2}{11}$

(occurs at intersection of $x = y$ and $5x + 6y = 30$)

Practice questions worked solutions

1 $6x - 5y = -3$ ①

$5x + 4y = 22$ ②

$4 \times ① \rightarrow 24x - 20y = -12$ ③

$5 \times ② \rightarrow 25x + 20y = 110$ ④

$③ + ④ \rightarrow 49x = 98$

$x = \frac{98}{49} = 2$

$① \rightarrow 6(2) - 5y = -3$

$12 + 3 = 5y$

$5y = 15$

$y = 3$

Check in ②: $5(2) + 4(3) = 10 + 12 = 22$ ✓

So, $x = 2$ and $y = 3$.

2 a $2(3x + 1 + 6x - 3) < 88$

$2(9x - 2) < 88$

$9x - 2 < 44$

$9x < 46$

$x < \frac{46}{9}$

b $\frac{46}{9} = 5\frac{1}{9}$ and $x < 5\frac{1}{9}$

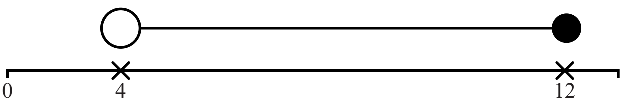
Largest possible $x = 5$ if x is an integer.

Largest area $= (3(5) + 1)(6(5) - 3)$

$= 16 \times 27$

$= 432\,\text{cm}^2$

3



4 a

x	0	3	9
y	9	6	0

b

x	0	1	5
y	13	11	3

c graph showing $x + y = 9$ and $2x + y = 13$ for $0 \leqslant x \leqslant 10$, $0 \leqslant y \leqslant 15$

d $x = 4$, $y = 5$

5 $\frac{1}{2}n(n + 1) = 105$

$n(n + 1) = 210$

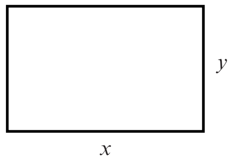
$n^2 + n - 210 = 0$

$(n + 15)(n - 14) = 0$

$\Rightarrow n = 14$ or -15

So, $n = 14$ because $n > 0$.

6 a



$2x + 2y = 100$

$2y = 100 - 2x$

$y = 50 - x$

Width $= 50 - x$

b Area $= xy = x(50 - x) = 50x - x^2$

c $50x - x^2 = -(x^2 - 50x)$

$= -\{(x - 25)^2 - 625\}$

$= 625 - (x - 25)^2$

Maximum when $x = 25\,\text{m}$

d Area $= 25 \times 25 = 625\,\text{m}^2$

7 $x^2 + 8x - 17$

$= (x + 4)^2 - 16 - 17$

$= (x + 4)^2 - 33$

$x^2 + 8x - 17 = 0$

$(x + 4)^2 - 33 = 0$

$(x + 4)^2 = 33$

$x + 4 = \pm\sqrt{33}$

$x = -4 \pm \sqrt{33}$

8 $x^2 + 5x - 9 = 0$

$ax^2 + bx + c = 0$

$a = 1$, $b = 5$, $c = -9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25^2 - 4 \times 1 \times -9}}{2}$$

$$= \frac{-5 \pm \sqrt{61}}{2}$$

So, $x = -\frac{5}{2} \pm \frac{\sqrt{61}}{2}$

9 $\frac{1}{4}(3x - 2) > 2$

$3x - 2 > 8$

$3x > 10$

$x > \frac{10}{3}$

$3x - 12 < 17$

$3x < 29$

$x < \frac{29}{3}$

So, $\frac{10}{3} < x < \frac{29}{3}$

10 $\frac{4x}{3y} \div \frac{3x^2}{9y^4}$

$= \frac{4\cancel{x}^1}{{}^1\cancel{3}\cancel{y}^1} \times \frac{{}^1\cancel{9}\,y^{\cancel{4}3}}{{}^1\cancel{3}\,x^{\cancel{2}2}}$

$= \frac{4y^3}{x^2}$

11 a $100 - x$

b Squares have side $\frac{x}{4}$ and $\frac{100 - x}{4}$

so areas are $\left(\frac{x}{4}\right)^2$ and $\left(\frac{100 - x}{4}\right)^2$

c $\frac{x^2}{16} + \frac{(100 - x)^2}{16} = 325$

$x^2 + x^2 - 200x + 10\,000 = 5200$

$2x^2 - 200x + 4800 = 0$

$x^2 - 100x + 2400 = 0$

$(x - 60)(x - 40) = 0$

$x = 60$ or $x = 40$

d Smaller square: $x = 40$ cm
= total wire used
= perimeter

12 a $3x^2 - 7x + 2$
$= (3x - 1)(x - 2)$

b $x^2 - y^2 = (x - y)(x + y)$
$2yz - 2xz = 2z(y - x)$
So, $x^2 - y^2 - 2xz + 2yz$
$= (x - y)(x + y) + 2z(y - x)$
$= (x - y)(x + y) - 2z(x - y)$
$= (x - y)(x + y - 2z)$

13 $\frac{3}{(x - 2)(x + 2)} + \frac{1}{x - 2}$

$= \frac{3}{(x - 2)(x + 2)} + \frac{x + 2}{(x - 2)(x + 2)}$

$= \frac{x + 5}{(x - 2)(x + 2)}$

14 $5x^2 - 30x + 51$
$= 5(x^2 - 6x) + 51$
$= 5\{(x - 3)^2 - 9\} + 51$
$= 5(x - 3)^2 - 45 + 51$
$= 5(x - 3)^2 + 6$
So, $5(x - 3)^2 \geqslant 0$ for all x
Therefore, $5(x - 3)^2 + 6 \geqslant 6 > 0$ so it is always positive.

15 $\frac{\cancel{(x - 4)}\cancel{(x - 3)}}{\cancel{(x + 7)}\cancel{(x - 4)}} \times \frac{\cancel{(x - 7)}(x + 7)}{\cancel{(x - 3)}(x + 3)}$

$= \frac{(x + 7)}{(x + 3)}$

16 a P(tails) = P(not heads)
= 1 − P(heads)
= $1 - p$

b $p(1 - p)$

c $p(1 - p) = \frac{3}{16}$

$p - p^2 = \frac{3}{16}$

$16p - 16p^2 = 3$

$16p^2 - 16p + 3 = 0$

$(4p - 1)(4p - 3) = 0$

$p = \frac{1}{4}$ or $p = \frac{3}{4}$

d $p = \frac{3}{4}$

P(at least one head) = 1 − P(no heads)
= 1 − P(tails) × P(tails)
$= 1 - \frac{1}{4} \times \frac{1}{4}$
$= 1 - \frac{1}{16}$
$= \frac{15}{16}$

17 $x^2 - 5x - 3 = 0$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times (-3)}}{2}$

$= \frac{5 \pm \sqrt{25 + 12}}{2}$

$= \frac{5 \pm \sqrt{37}}{2}$

So, $a = \frac{5 + \sqrt{37}}{2}$ and $b = \frac{5 - \sqrt{37}}{2}$

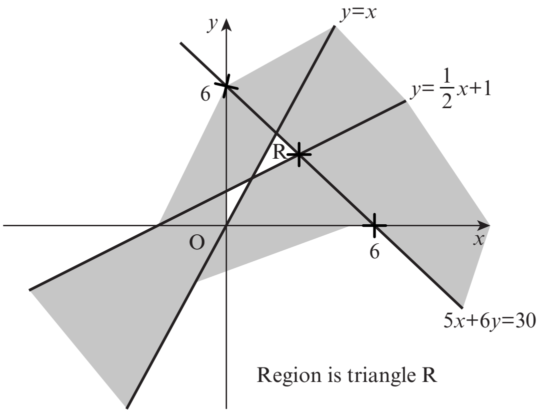
i $a - b = \frac{5}{2} + \frac{\sqrt{37}}{2} - \left(\frac{5}{2} - \frac{\sqrt{37}}{2}\right)$

$= \sqrt{37}$

ii $a + b = \frac{5}{2} + \frac{\sqrt{37}}{2} + \frac{5}{2} - \frac{\sqrt{37}}{2}$

$= 5$

18 **a**



b 6

Chapter 15

Getting started

1 a Student C is correct.

b Student A's solution suggests that 1 cm on the map is $\frac{1}{50\,000}$ cm in the real world. This would mean the real world is smaller than the map. Instead, student A needed to use the fact that 1 cm on the map is 50 000 cm in the real world. Student A has also converted cm to km incorrectly.

Student B found the distance 1 950 000 mm correctly, but has converted to km incorrectly. There are 1000 m in each km and 1000 mm in each m. This means there are 1 000 000 mm in 1 km, so the answer is 1.95 km.

2 It would also have 3000 seats, they would just be smaller! It's important to understand that a scale model has exactly the same features in the same numbers. They are just a different size.

3 $0.32 \times \frac{14.4}{0.08} = 57.6$ cm

Exercise 15.1

1 6.8 m × 5.2 m

2 a 3 cm

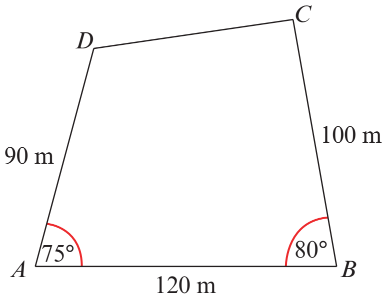
b 2.4 cm

3 a 5.6 cm

b 15°

Exercise 15.2

1 a



b $BCD = 92°$; $ADC = 113°$

c 80 m

2 a 20°

b 3.4 m

3 a 20 m

b 34.8 m

Exercise 15.3

1 a 270°

b 135°

c 045°

2 a 082°

b 315°

3 a 110°

b 050°

c 230°

d 025°

e 280°

4 a 108°

b 288°

c 147 km

5 a 9.6 km

b 090°

6 a 121° to 123°

b 471.7 m

Exercise 15.4

1

	Hypotenuse	Opposite *A*	Adjacent *A*
a	*c*	*a*	*b*
b	*y*	*z*	*x*
c	*p*	*q*	*r*
d	*l*	*n*	*m*
e	*c*	*d*	*e*
f	*e*	*f*	*g*

2 a opp(30°) = 5.7 cm

b opp(40°) = x cm

adj(50°) = x cm

c opp(65°) = q m or adj(25°)

opp(25°) = p m or adj(65°)

hypotenuse = r m

Lengths and angles in triangles

1–3 Students' own drawings and measurements.

4–5 The answers are all the same (0.577).

6 For angles of 30° and 60°, the value of $\frac{\text{opp}(30°)}{\text{adj}(30°)}$ is 0.577 for all triangles.

7 For right-angled triangles with angles A and B, the value of $\frac{\text{opp}(A)}{\text{adj}(A)}$ will be the same for any angle A whatever the side lengths of the triangle.

8 If you divide a different pair of sides the value will be different, but for a given angle and a given pair of sides the value will be the same whatever the side lengths of the triangle.

Exercise 15.5

1 a 0.700
b 1.04
c 0.325
d 1
e 0.279
f 0.323
g 0.00873
h 0

2 a $\tan A = \frac{1}{2}$
b $\tan A = \frac{3}{2}$
c $\tan A = \frac{1}{4}$
$\tan B = 4$
d $\tan x = \frac{3}{2}$
e $\tan z = \frac{n}{m}$
$\tan y = \frac{m}{n}$
f $\tan C = a$
g $\tan D = p^2$

3 a 5.20 cm
b 4.62 m
c 35.7 m
d 3.54 km
e 18 cm
f 10.3 cm

4 a 20.8 cm
b 16.1 cm
c 9.17 cm
d 7.85 cm
e 40.6 cm
f 115 m
g 2.61 m
h 95.8 km
i 39.8 m

5 a 1.0724
b 32.2 m

6 32.3 m

7 a 1.73
b 2

8 0.45 m

9 36.18 m

Exercise 15.6

1 a 40.4°
b 60.0°
c 74.3°
d 84.3°

2 a 22°
b 38°
c 38°
d 70°

3 a $a = 35.0°$
b $b = 77.5°$
c $c = 38.7°$
$d = 51.3°$
d $e = 18.4°$
e $f = 30°$

4 71.8° (1 d.p.)

5 21.2° (1 d.p.)

6 a 13.3 (3 s.f.)
b 26.7 (3 s.f.)

7 $AB = 6.32$ (3 s.f.)
$ACB = 64.6°$ (1 d.p.)

Exercise 15.7

1 a i $\frac{4}{5}$ ii $\frac{3}{5}$ iii $\frac{4}{3}$
b i $\frac{7}{25}$ ii $\frac{24}{25}$ iii $\frac{7}{24}$
c i $\frac{12}{13}$ ii $\frac{5}{13}$ iii $\frac{12}{5}$

d **i** $\frac{20}{29}$ **ii** $\frac{21}{29}$ **iii** $\frac{20}{21}$

e **i** $\frac{8}{17}$ **ii** $\frac{15}{17}$ **iii** $\frac{8}{15}$

f **i** $\frac{4}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{4}{3}$

g **i** $\frac{13}{85}$ **ii** $\frac{84}{85}$ **iii** $\frac{13}{84}$

2 **a** 0.0872
b 0.9962
c 0.5000
d 0.8660
e 0.8660
f 0.5000
g 0.9962
h 0.0872

3 **a** $\cos 42° = \frac{g}{e}$
b $\sin 60° = \frac{c}{a}$
c $\cos 25° = \frac{RQ}{RP}$
d $\sin \theta = \frac{y}{r}$
e $\cos 48 = \frac{q}{r}$
f $\sin 30° \frac{e}{f}$
g $\cos 35° = \frac{HI}{JI}$
h $\cos \theta = \frac{x}{r}$

4 **a** 0.845 m
b 4.5 m
c 10.6 km
d 4.54 cm
e 10.6 cm
f 9.57 cm
g 14.1 cm
h 106 cm
i 4.98 cm
j 42.9 m
k 2.75 m
l 137 m

5 **a** 81.9°
b 57.1°
c 22.0°
d 30°

6 **a** 25.9°
b 44.9°
c 69.5°
d 79.6°
e 26.9°
f 11.5°

7 1.93 m (2 d.p.)

8 **a** 10.1 km (3 s.f.)
b 14.9 km (3 s.f.)

9 **a** 2.11 km
b 5.87 km
c 054°
d 7.98 km

10 **a** 473 m
b 1608 m

11 **a** 14.1 m (3 s.f.)
b 5.13 m (3 s.f.)

12 552 m (3 s.f.)

13 **a** $x = 14.82$ cm
b $y = 10.09$ cm
c $z = 44.99$ m
d $a = 29.52$ cm
$b = 52.80$ cm

14 **a** **i** 0.577
ii 0.577
b **i** 1.11
ii 1.11
c **i** −1.73
ii −1.73
d **i** 0.249
ii 0.249
$\therefore \tan x = \frac{\sin x}{\cos x}$

15 **a** **i** 1
ii 1
b 1
c $\sin^2 x + \cos^2 x = 1$

16 Students' posters

Exercise 15.8

1 **a** 2 cm
b 9 cm
c 8 cm

2 **a** 16 cm

b $\frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ cm

c $\frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$ cm

3 103.68 cm^2

4 $24\sqrt{3}$ m

Exercise 15.9

1 **a** $ABC = 16.2°$

b $BC = 17.9$ m

2 $AB = 13.856$ cm (3 d.p.)

3 **a** $ABC = 59.0°$

b $AB = 1.749$ (3 d.p.)

c Capacity = 4.05 m^3

4 $ABC = ACB = 38.9°$ and $BAC = 102.1°$

5 **a** 020°

b 281.9 m

c 98 668 m^2

6 **a** 3.5 m (1 d.p.)

b $DE = 6.1$ m (1 d.p.)

7 $QT = 16$ cm

8 **a** $AOE = 72°$

b $AOM = 36°$

c $OM = 1.376$ cm (3 d.p.)

d 0.688 cm^2

e 6.882 cm^2 (3 d.p.)

9 77.255 cm^2

10 $6.882a^2$ cm^2

11 $\dfrac{na^2}{\tan\left(\dfrac{360°}{2n}\right)}$

Investigation

Sin 30 and sin 150 = $\frac{1}{2}$
Sin 10 and sin 170 = 0.174
Sin 60 and sin 120 = 0.866
Sin 5 and sin 175 = 0.0872

The two angles add up to 180 degrees. Sin x and sin $(180 - x)$ are the same each time.
Cos 30 and cos 330 = 0.866

Cos 60 and cos 300 = $\frac{1}{2}$
Cos 50 and cos 310 = 0.643
Cos 15 and cos 345 = 0.0644

The two angles add up to 360 degrees. Cos x and cos $(360 - x)$ are the same each time.
Tan 30 and tan 210 = 0.577
Tan 60 and tan 240 = 1.73
Tan 15 and tan 195 = 0.268
Tan 100 and tan 280 = −5.67

The two angles *differ* by 180 degrees. Tan x and tan $(x + 180)$ are the same each time.

Exercise 15.10

1 **a** −cos 60°

b sin 145°

c −cos 44°

d sin 10°

e −cos 92°

f cos 40°

g sin 59°

h sin 81°

i cos 135°

j cos 30°

2 **a** 30, 150

b 90

c 45, 315

d 78.7, 258.7

e 150, 210

f 191.5, 348,5

g 109.5, 250.5

h 60, 240

i 104, 284

3 **a** 45

b 120

c 55

d 45

e 270

f 120

g 270

h 90

i 696, 384

4 30, 150, 210, 330

5 a 30°, 150°
b 120°, 240°
c 18.4°, 198.4°
d No solutions
e 60°, 300°
f 30°, 150°, 330°

6 41.4, 60, 300, 318.6

Exercise 15.11

1 a 6.96
b 8.58
c 25.3
d 38.8°

2 a 10.6 cm
b 5.73 cm
c 4.42 cm
d 5.32 cm
e 6.46 cm
f 155 mm

3 a 54.7°
b 66.8° or 113.2°
c 69.8° or 110.2°
d 25.3° or 154.7°
e 52.7° or 127.3°
f 50.5°

4 $C = 63°$
$AC = 15.9$ cm
$CB = 21.3$ cm

5 $F = 25°$
$DE = 9.80$
$EF = 14.9$ cm

6 $R = 32.2°$
$P = 27.8°$
$QR = 7.0$ cm

7 a Y is opposite a side shorter than X, so $Y < X$ and therefore $<40°$.
b $Y = 30.9°$ and $Z = 109.1°$
c $XY = 22.1$ cm

8 a $ACB = 51°$
b $ABC = 52°$
c $AC = 32.25$ mm

Exercise 15.12

1 $AC = 8.62$ cm

2 $DE = 22.3$ cm

3 $P = 53.8°$

4 a $TU = 18.7$ m
b $U = 32.1°$
c $T = 52.9°$

5 a $X = 60°$
b $Y = 32.2°$
c $Z = 87.8°$

6 a Return = 14.4 km
b 296°

7 51.6 m on a bearing of 274°

Exercise 15.13

1 a 10.0 cm^2
b 15.0 cm^2
c 52.0 cm^2
d 17.2 cm^2
e 22.7 cm^2
f 24.2 cm^2

2 108 cm^2

3 0.69 m^2

4 42.1 cm^2

5 a 30.6 cm^2
b 325.9 cm^2
c 1.74 m^2

6 a 174 cm^2
b 8.7 cm and 21.5 cm

7 a $Q = 22.6°$
b $P = 53.1°$

Exercise 15.14

1 a $AC = 25$ cm
b $EC = 13.0$ cm
c 27.5°

2 a $EG = \sqrt{50}$ m
b $AG = \sqrt{75}$ m
c $AGE = 35.3°$

3 **a** $ACB = 53.1°$
b $BC = 5$ m
c $CD = 4.2$ m
d $BM = 4.5$ m
e $BCD = 65°$

4 **a** 14.9 cm
b 15.2 cm
c $\theta = 11.4°$

5 **a** $\sqrt{x^2 + y^2}$
b 90°
c $\sqrt{x^2 + z^2}$
d $\sqrt{x^2 + y^2 + z^2}$

Practice questions

1 2.9 cm

2 $AC = 9.8$ m, $BC = 6.9$ m

3 $DAB = 47.9°$

4 9.9 m

5 **a** $X = 10.1$ m (to 3 s.f.)
b $y = 20.6°$

6 **a** **i** QX = 60 tan 4° = 50.3 m
ii 78.3 m
b **i** 250.3 m
ii 257.4 m
iii 077°

7 **a** 5.16 m
b 3.11 m²

8 **a** 7 cm
b 51.1°

9 **a** (90°, 1)
b −1
c 2 solutions

10 30, 150

11 **a** **i** $AB = 107.3$ km
ii $PAB = 66.6$
iii 143.4°
b **i** 5 h
ii 12 km/h

12 $\frac{1}{4\sqrt{3}}$ or $\frac{\sqrt{3}}{12}$

13 **a** 30°, 330°
b 88.4°, 268.4°
c 153.4°, 333.4°
d 180°
e 30°, 210°, 150°, 330°
f 30°, 41.8°, 138.2°, 150°

Practice questions worked solutions

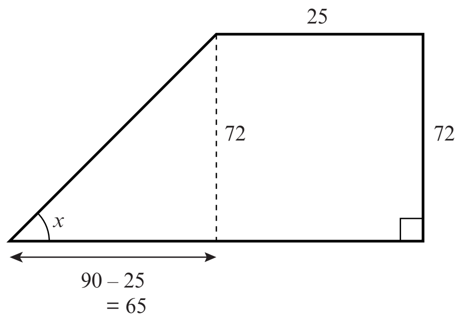
1 Window measures 250 cm by 350 cm

Drawing measures $\frac{250}{150}$ cm by $\frac{350}{150}$ cm

$$\text{Diagonal length} = \sqrt{\left(\frac{250}{150}\right)^2 + \left(\frac{350}{150}\right)^2}$$
$$= 2.87\text{ cm}$$

2 12 cos 35° = 9.83 m
12 sin 35° = 6.88 cm

3



$$\tan x = \frac{72}{65} \Rightarrow x = \tan^{-1}\left(\frac{72}{65}\right) = 47.9°$$

4 12 tan 35° + 1.5 = 9.90 m

5 **a** $\frac{12}{x} = \tan 50°$
$$x = \frac{12}{\tan 50°} = 10.07\text{ m}$$

b Horizontal distance $RS = 72 - 30 - 10.07$
$= 31.93$ m

$$\tan y = \frac{12}{31.93}$$
$$\Rightarrow y = \tan^{-1}\left(\frac{12}{31.93}\right) = 20.60°$$

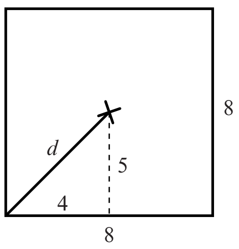
6 **a** **i** 60 × tan 40° = 50.3 m
ii $\frac{60}{\cos 40°} = 78.3$ m
b **i** 250.3 m
ii $\sqrt{60^2 + 250.3^2} = 257.4$ m
iii $\tan^{-1}\left(\frac{250.3}{60}\right) = 77°$ to nearest degree

7 **a** $AB^2 = 3^2 + 8^2 - 2 \times 3 \times 8 \times \cos 15°$

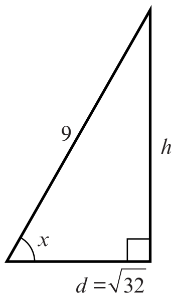
$AB = 5.16\,\text{m}$

b $\frac{1}{2} \times 3 \times 8 \times \sin 15° = 3.11\,\text{m}^2$

8 **a**



$d^2 = \sqrt{4^2 + 4^2} = \sqrt{32}$



$h^2 = 9^2 - (\sqrt{32})^2$

$= 81 - 32$

$= 49$

$h = \sqrt{49} = 7\,\text{cm}$

b $\tan^{-1}\left(\frac{7}{9}\right) = 37.9°$

9 **a** $(90°, 1)$

b -1

c There are two, at $x = 210°$ and $330°$.

10 $\cos 300° = \frac{1}{2}$

$\Rightarrow x = 30°$ or $180° - 30°$

$= 30°$ or $150°$

11 **a** **i**

$AB = \sqrt{100^2 + 60^2 - 2 \times 100 \times 60 \times \cos 80°}$

$= 107.3\,\text{km}$

ii $\frac{\sin PAB}{100} = \frac{\sin 80°}{107.3}$

$PAB = \sin^{-1}\left(\frac{100 \sin 80°}{107.3}\right)$

$= 66.6°$

iii $360° - 150° - 66.6° = 143.4°$

b **i** $\frac{100}{20} = 5$ hours

ii $\frac{60}{5} = 12\,\text{km/h}$

12 $\frac{\frac{1}{2} \times \left(\frac{\sqrt{2}}{2}\right)^2}{\sqrt{3} \times 1} = \frac{\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{4}}{\sqrt{3}}$

$= \frac{1}{4\sqrt{3}}$

13 **a** $\cos x = \frac{\sqrt{3}}{2} \Rightarrow x = 30°$

b $2 \tan x = 72$

$\tan x = 36$

$x = 88.4°$

c $2(\tan x + 2) = 3$

$\tan x + 2 = \frac{3}{2}$

$\tan x = -\frac{1}{2}$

$x = 153.4°$

d $\cos x = -1$

$x = 180°$

e $\tan x = \pm\frac{1}{\sqrt{3}}$

$x = -30° + 180°$ or $30°$

$= 30°$ or $150°$

f $(3 \sin x - 2)(2 \sin x - 1) = 0$

$\sin x = \frac{2}{3}$ or $\sin x = \frac{1}{2}$

$x = 41.8°$. $180° - 41.8°$ or $x = 30°$, $180° - 30°$

so, $x = 41.8°, 138.2°, 30°, 150°$

Chapter 16

Getting started

1 It is likely to show positive correlation, because temperatures are lower at higher altitudes, so there would be greater snowfall near the top of the mountain. There are lots of other factors, such as the direction of the prevailing wind, the gradient of each slope and the overall climate of the region. The correlation may be very weak; the points could be scattered much more.

2 Students will come up with their own ideas, but possible examples include:

The heights and masses of 100 people. This is likely to be a positive correlation.

Temperature and sales of rain jackets. This is likely to be a negative correlation, because higher temperatures usually go with drier weather.

Taking all of the schools in Japan, you could plot the number of classrooms against the number of miles from Mount Fuji. It is very unlikely that these two values would show any correlation.

3 Just because there is a correlation between two variables, it does not mean that one of them 'causes' the other to change. In this situation it is likely that higher temperatures will mean that ice creams sell better, but also that more people are on the beaches, leading to shark attacks.

Exercise 16.1

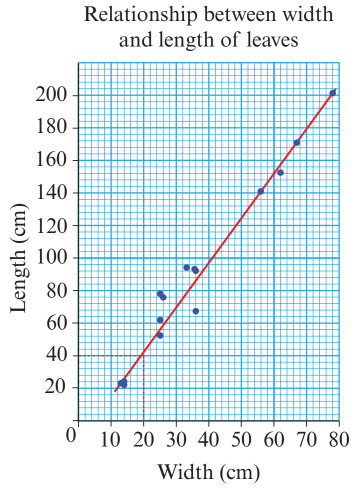
1 a Positive; weak

b Zero correlation

c Negative; weak

d Negative; strong

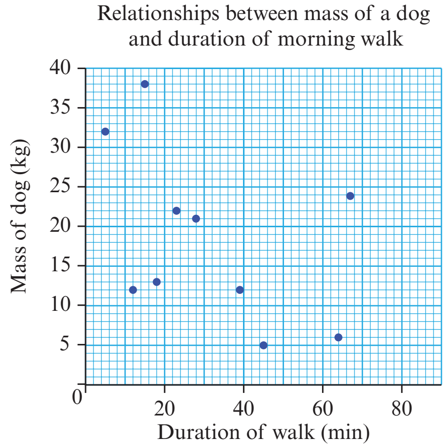
2 a, c



b Strong positive correlation.

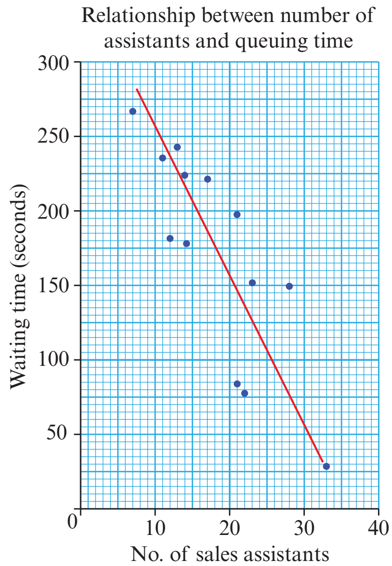
d 40 cm

3 a



b Zero correlation

4 a, c



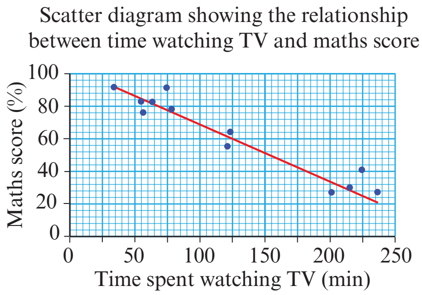
b Strong negative correlation.

d Value is outside the range of the collected data and waiting time will be negative time!

5 **a** $A = 122$, $B = 92$, $C = 56$, $D = 28$, $E = 200$

b Strong negative correlation.

c

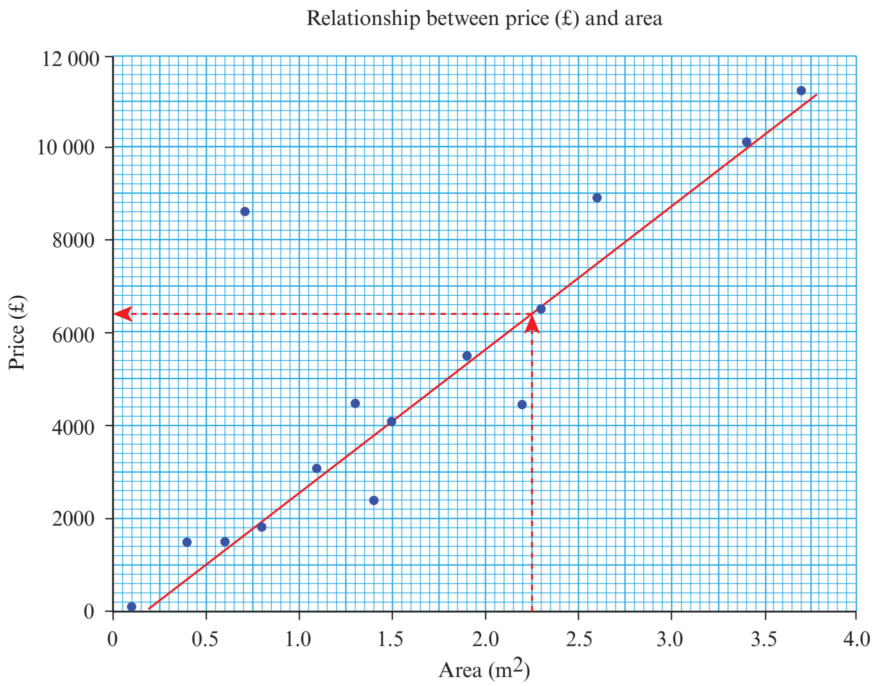


d 105 mins

e The correlation is strong and Aneesh's score is within the range of the collected data. This means the estimate is likely to be reliable. It can never be exact, but it is expected to be close to the actual value.

Practice questions

1 **a, c**

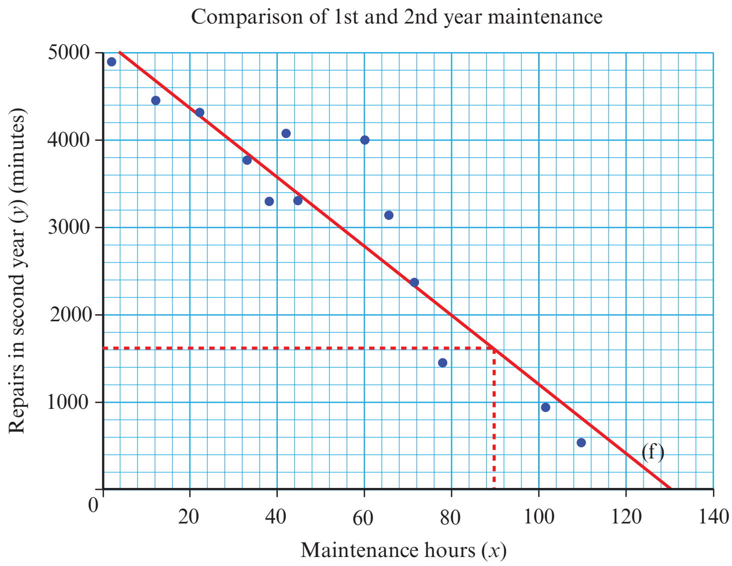


b Painting E because other paintings of a similar size are much cheaper.

d $6400

e Value is outside the range of the collected data.

2 a, c



b Strong negative correlation.

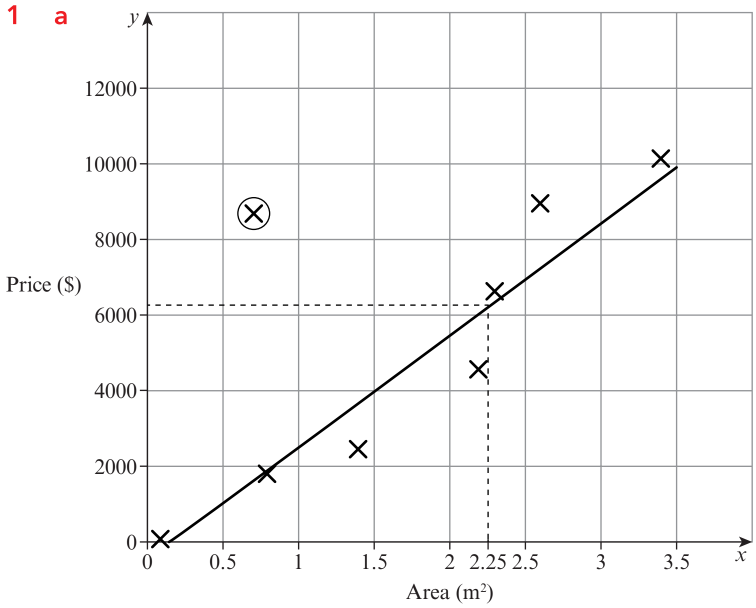
d 1600 minutes

e Repair time is a negative number – value is outside the range of the collected data.

f Approximately 130 hours – this is an extrapolated value so might not be accurate.

Practice questions worked solutions

1 a



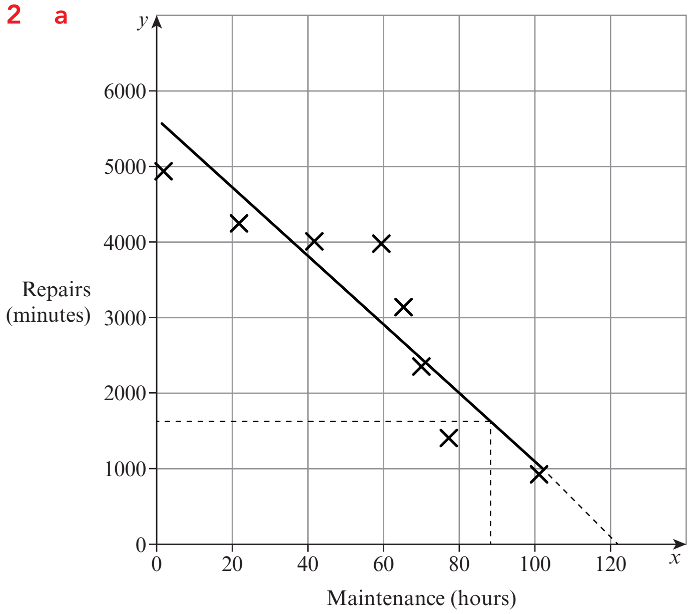
b E (circled above)

c See graph.

d $1.5 \times 1.5 = 2.25\,\text{m}^2$ is approximately equal to $6100

e $2.1 \times 2.1 > 4$, which is outside the range of the data, where the pattern may change.

2 **a**



b Strong, negative, linear correlation

c See graph

d approximately 1750

e You would need to extend the line, but it would then predict a negative repair time.

f approximately 12.2 hours

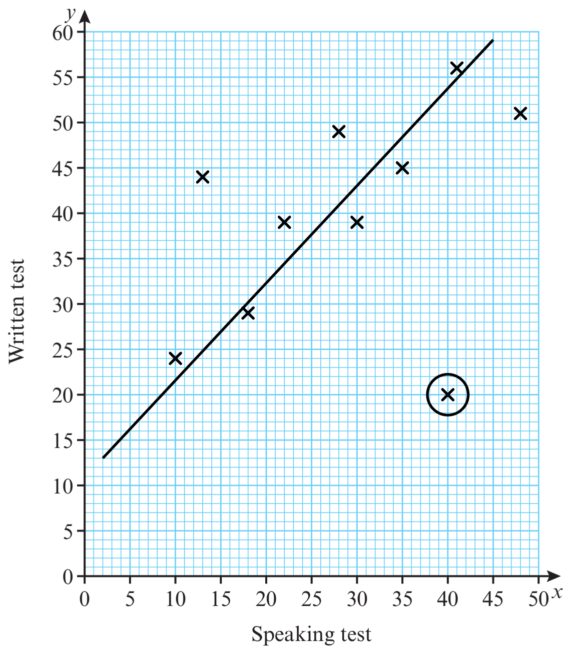
Past paper questions

1 $5.665 \leqslant l < 5.675$

2
$$\begin{aligned} 10x + 5y &= 15 \\ x - 5y &= 40 \\ 11x &= 55 \\ x &= 5 \\ 10 + y &= 3 \\ y &= -7 \end{aligned}$$

$x = 5, y = -7$

3 **a, c, d**



b strong positive linear correlation

e 38

4 **a** correct position of town B 9 cm from A at angle of 140° from A

b **i** triangle constructed using points A and B from part **a** and C 7 cm from A and 5 cm from B, bearing of C from A less than 140°

ii between 38° and 42°

5 **a** $2c - 3d$

b $12x = 30$

$x = \frac{5}{2}$

c $27 - 9x$

d $\frac{2A}{a+b} = \frac{2 \times 38.64}{5.5 + 3.7} = \frac{77.28}{9.2} = 8.4$

e **i** $3x = 5y$
$2y = x + 4$

ii $3x = 5y$

$3x = 6y - 12$

$6y - 12 = 5y$

$y = 12,\ x = 20$

6 $P_{upper} = 2(12.5 + 4.5) = 2 \times 17 = 34$

7 $(x + 2)^2 - 4 - 9$

$a = 2$ and $b = -13$

8 area factor $= 24\,000^2$

Area in cm $= 32 \times 24\,000^2$

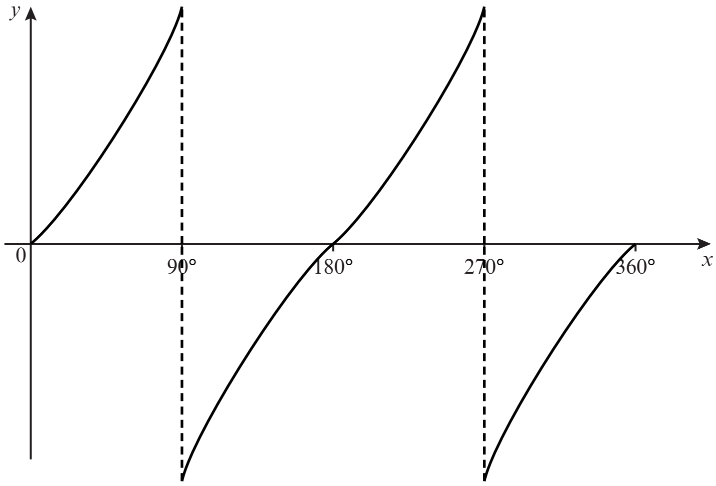
$= 1.8432 \times 10^{10}\ \text{cm}^2$

$1\ \text{km}^2 = (1000 \times 100) \times (1000 \times 100)\ \text{cm}^2 = 1 \times 10^{10}\ \text{cm}^2$

Area $= 1.8432\ \text{km}^2$

9 $\frac{3x - 1 - 2x - 4}{(x + 2)(3x - 1)} = \frac{x - 5}{(x + 2)(3x - 1)}$

10 **a**



b $\tan x = \frac{1}{3}$

$x = 11.3°, 11.3° + 180°$

$= 11.3°$ or $191.3°$

11 **a** $AC^2 = 6.4^2 + 10.6^2 - 2 \times 6.4 \times 10.6 \times \cos 102°$

$AC = 13.5$ cm

b $\frac{BX}{\sin 44°} = \frac{10.6}{\sin 58°} \Rightarrow BX = 8.6$ cm

c $\frac{1}{2} \times (8.68 + 6.4) \times 10.6 \sin 78° = 78.2\text{ cm}^2$

12 **a** $13 \times 24 + 20 \times 24 + \sqrt{231} \times 24 + 2 \times \frac{1}{2} \times \sqrt{231} \times 13 = 1354\text{ cm}^2$

b $\phi = \cos^{-1}\left(\frac{150}{170}\right) = 28.1° \frac{1}{2} \times \sqrt{231} \times 13 \times 24 = 2371\text{ cm}^3$

c 24.6°

13 **a** 75.6

b **i** $2a - 3b$

ii $\frac{3}{4}$

c **i** $x = -5$

ii $20 - 12x = 23$

$12x = -3$

$x = -\frac{1}{4}$

d $3^2x^6 = 9x^6$

e $6x^2 + 3xy - 10xy - 5y^2 = 6x^2 - 7xy - 5y^2$

14 **a** $130 + 80 + 170 + 50 = 530$ m

b $\cos\theta = \frac{100^2 + 150^2 - 120^2}{2 \times 100 \times 150} \Rightarrow \theta = 52.9°$

c **i** $\phi = \cos^{-1}\left(\frac{150}{170}\right) = 28.1°$

ii $360 - 28.1 = 332°$

d $14\,982\text{ m}^2 = 1.498$ hectares

Chapter 17

Getting started

1 Student answers will depend on how much they already know. They can use the key words in this chapter, the glossary at the end of the book, a dictionary or online sources (such as Investopedia) to find the meanings of words they don't know.

2 a $40 + 5\% = 42$ or $1.05 \times 40 = 42$

b $40 - 10\% = 36$ or $0.9 \times 40 = 36$

3 a $P = \frac{100I}{RT}$

b $R = \frac{100I}{PT}$

c $T = \frac{100I}{PR}$

Exercise 17.1

1 $49.50

2 $428.75

3 a $13.50

b $6.45

c 9.35

d $12.15

e $13.68

4 $2085.75

5 $474.30

6 $8250

7

Annie	$319.20
Bonnie	$315.00
Connie	$300.30
Donny	$403.20
Elizabeth	$248.85

Exercise 17.2

1

Employee	a Net income ($)	b % $\left(\frac{\text{net}}{\text{gross}}\right)$
B Willis	317.00	47
M Freeman	158.89	35
J Malkovich	557.20	43
H Mirren	383.13	42
M Parker	363.64	43

2 a Mean weekly earnings: $836.63

b Median weekly earnings: $853.30

c Range of earnings: $832.50

3 a Difference between gross and net income:

M Badru: 3954.52

B Singh: 724.79

b Percentage of gross income that each takes home as net pay:

M Badru: 69.3%

B Singh: 57%

Exercise 17.3

1

	Taxable income	Annual tax	Monthly tax
a	$98 000.00	$21 149.25	$1762.44
b	$120 000.00	$27 309.25	$2275.77
c	$129 000.00	$29 829.25	$2485.77
d	$135 000.00	$31 509.25	$2625.77
e	$178 000.00	$43 856.75	$3654.73

2 a i Yes

ii No – he pays $6181.25

iii $\$6181.25 = \$4681.25 + (40\,000 - 34\,000) \times 0.25$

b $67 616.75

c i She owes additional tax.

ii $238.25

Types of tax

Investigation

Answers will vary from country to country, but the basic definitions of terms are:

a Value added tax – tax added at each stage of a product's lifespan as value is added, in other words a consumption tax levied on a product repeatedly at every point of sale.

b General sales tax – tax levied on goods when they are sold at retail outlets, calculated as a percentage of the value of the goods.

c Customs and excise duties – customs duties are also called import taxes, they are levied on good imported to a country to try and protect local industry that makes the same items; excise duties are indirect taxes on the sale of locally made goods or services, such as alcohol, tobacco and energy.

d Capital gains tax – a tax on profit from the sale of assets.

e Estate duties – a tax on wealth or assets that are inherited.

f Property taxes – rates and other taxes levied on real estate.

g Air passenger tax – a charge levied on passengers (often older than 16) who fly out of different airports, varies from place.

h Corporate tax – direct tax on the income or capital of some businesses.

Exercise 17.4

1 a $15.00
  b $12.19
  c $62.50
  d $312.00
  e $144.38
  f $108.00
  g $190.04
  h $72.00
  i $21 375.00

2 a 545.00
  b 715.00
  c $1120.00
  d $1416.00
  e $1071.88
  f $1305.00
  g $803.85
  h $1370.00
  i $190 500.00

3 4 years

4 7% p.a.

5 33 years 4 months

6 a $32
  b $96
  c i $40.80
    ii $136.80

7 a $11 700
  b £3700
  c 15.4% (1 d.p.)

Exercise 17.5

1 a $100
  b $60
  c $460

2 $2850

3 a $141.83
  b $2072

4 a £301
  b 33.5% (1 d.p.)

5 a $3657.80
  b 13.09% (2 d.p.)

Exercise 17.6

1 a $10 035.20
  b $9920.00

2 $88 814.66

3 $380 059.62 (2 d.p.)

4 a $4998.09
  b $5077.92

Personal finances

Students' own discussion.

Exercise 17.7

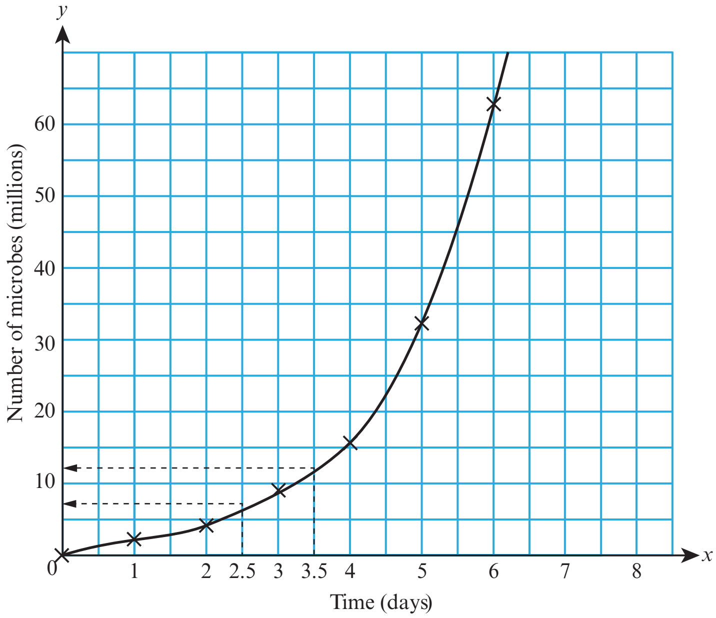
1 a 8.207 billion
  b 8.642 billion
  c 10.629 billion

2 a 1882
  b 1721

3 **a**

Time (days)	0	1	2	3	4	5	6	7	8
Total number of microbes (millions)	1	2	4	8	16	32	64	128	256

b



c **i** Approximately 6 million

ii Approximately 12 million

d Just over 4 days

4 **a** 6.5 minutes

b 12 grams

5 $27 085.85

6 **a** $10 120

b $8565.57

c $5645.41

d $\$11\,000(0.92)^n$

7 $2903.70

8 **a** 7 137 564

b 10 years

9 15 hours

Is it worth it?

a Around 20%

b Students' discussion, could include model and make of car, colour, mileage done, whether it is maintained and serviced or not, whether or not it is in accident.

c $12 600

d Students' own answers. Good advice is to buy a low mileage one year old car. This has already depreciated by about 20%, so you get good value for your money.

Exercise 17.8

1 **a** **i** $5.00

ii 25.00%

b **i** $50.00

ii 10.00%

c **i** $0.30

ii 20.00%

d **i** $0.05

ii 16.67%

2 **a** **i** $100.00

ii 25.00%

b **i** $0.10

ii 13.33%

c **i** $0.25

ii 5.00%

d **i** $0.65

ii 10.00%

3 Percentage profit = 66.67%

Exercise 17.9

1 a $156.00
  b $400.00
  c $399.15
  d $500

2 $840

3 $3225

4 $360

5 $220.80

6 $433.55 for 10 and $43.36 each

7 28%

8 a $41.32
  b 37%

Exercise 17.10

1

Original price ($)	% discount	Savings ($)	Sale price ($)
89.99	5	4.50	85.49
125.99	10	12.60	113.39
599.00	12	71.88	527.12
22.50	7.5	1.69	20.81
65.80	2.5	1.65	64.16
10 000.00	23	2300.00	7700.00

2

Original price ($)	Sale price ($)	% discount
89.99	79.99	11
125.99	120.00	5
599.00	450.00	25
22.50	18.50	18
65.80	58.99	10
10000.00	9500.00	5

Practice questions

1 a $451.95
  b 2.75 hours

2 a $12
  b $14.40

3 8.5%

4 a $18.20
  b 71.2%

5 $33.60

6 $647.51

7 a $30 000.00
  b $2 977.53
  c $2 307.59

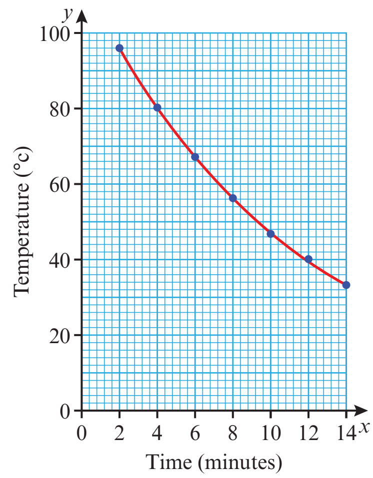
8 28.07%

9 11%

10 a $24 300
   b 25.9%

11 a 619 173.64
   b 13 years

12 a Decreases by 16% every 2 minutes, so exponential decay.
   b



   c 7.25 minutes

Practice questions worked solutions

1 a $36 \times 10.48 + 1.5 \times 10.48 \times 4.75 = \$451.95$
  b $\dfrac{420.75 - 36 \times 10.48}{1.5 \times 10.48} \approx 2\dfrac{3}{4}$ hours

2 a $\$15 \times 0.8 = \$12$
  b $15 \times 0.8 \times 1.2 = \$14.4$

3 $\dfrac{102}{94} = 1.085 \Rightarrow 8.5\%$ increase

4 a $35 \times 1.25 - 25.55 = \$18.20$
  b $\dfrac{18.20}{25.55} = 0.712\ldots$ so 71% profit

5 $(160 \times 0.07 \times 3) = \$33.60$

6 $500 \times 1.09^3 = \$647.51$

7 **a** $\frac{35\,730.48}{1.063^3} = \$30\,000$

b $\frac{35\,730.48}{12} = \$2977.54$

c $\frac{35\,730.48}{12} \times 0.775 = \$2307.59$

8 $\frac{14\,875 - 10\,700}{14\,875} \times 100 = 28.1\%$

9 $\frac{2200 - 1950}{2200} \times 100 = 11.4\%$

10 **a** $18\,000 \times 1.35 = 24\,300$

b $\frac{14\,300 - 18\,000}{24\,300} \times 100 = 25.9\%$

11 **a** $100\,000 \times 1.2^{10} = \$619\,173.64$

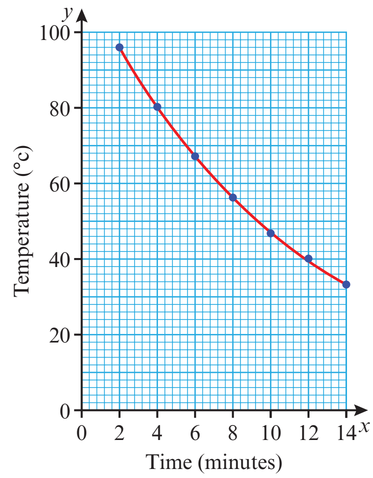
b $100\,000 \times 1.2^{13} = \$1\,069\,932$ after 13 years

12 **a** $\frac{96 - 80.6}{96} \times 100 = 16.04\%$

$\frac{80.6 - 67.7}{80.6} \times 100 = 16.00\%$

Same decay (16%) each time.

b



c 7.8, 7.9 minutes

Chapter 18

Getting started

1 a AB: Penguin moving downwards through air towards surface of the water; BC: Enters water and continues downwards; CD: At C, penguin turns and starts to swim back upwards towards the surface.

b 6 m

c 1 second

d 3.5 metres below the surface

e 5.5 seconds

f Students' answers will vary. However, it is likely to curve up above the x-axis again as the bird surfaces and then curve back down as it dives under the surface again.

2 a Set A: all symmetrical about y, all pass through origin (0, 0) and all ∪-shaped.

Set B: all symmetrical about y, all pass through origin (0, 0) and all ∩-shaped.

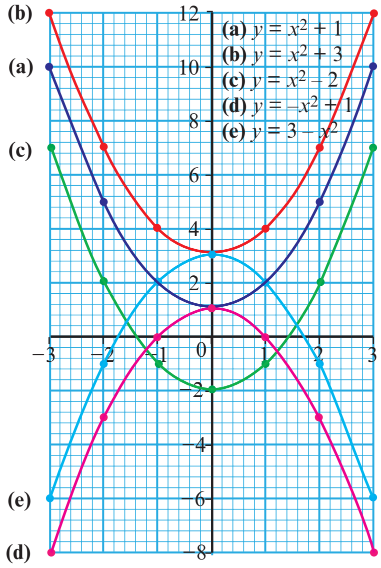
Set C: all symmetrical about y and all ∪-shaped, all have a y-intercept.

b Set A and B: main difference is the width of the graph, Set C, main difference is the y-intercept and that affects whether or not graph has x-intercepts.

c If the coefficient of x is > 0 (positive) the graph is ∪-shaped, if the coefficient is < 0 (negative) the graph is ∩ shaped. The value of the coefficient also affects the width of the graph. As the coefficient increases, the graph becomes narrower.

d i a is the y-intercept.

ii For $a > 0$, the graph is shifted vertically upwards by a units and the turning point is above the x-axis. If $a < 0$, the graph is moved a units down and the turning point is below the x-axis.



f When the value of the constant term changes the graph moves up or down the y-axis.

2 a C

b B

c A

d D

e E

Exercise 18.1

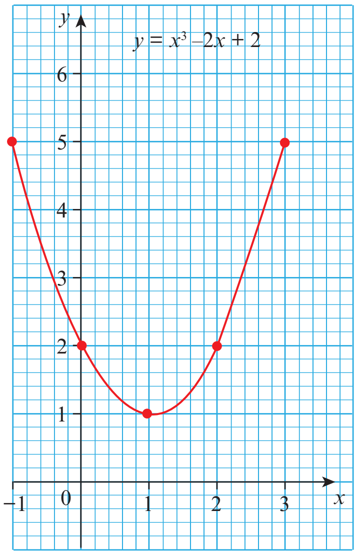
1

	x	−3	−2	−1	0	1	2	3
a	$y = x^2 + 1$	10	5	2	1	2	5	10
b	$y = x^2 + 3$	12	7	4	3	4	7	12
c	$y = x^2 - 2$	7	2	−1	−2	−1	2	7
d	$y = -x^2 + 1$	−8	−3	0	1	0	−3	−8
e	$y = 3 - x^2$	−6	−1	2	3	2	−1	−6

Exercise 18.2

1

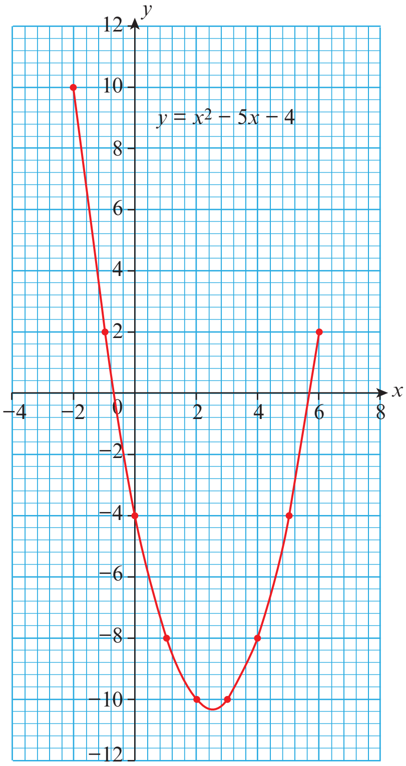
x	−1	0	1	2	3
$y = x^2 - 2x + 2$	5	2	1	2	5



2

x	−2	−1	0	1	2
x^2	4	1	0	1	4
$-5x$	10	5	0	−5	−10
-4	−4	−4	−4	−4	−4
$y = x^2 - 5x - 4$	10	2	−4	−8	−10

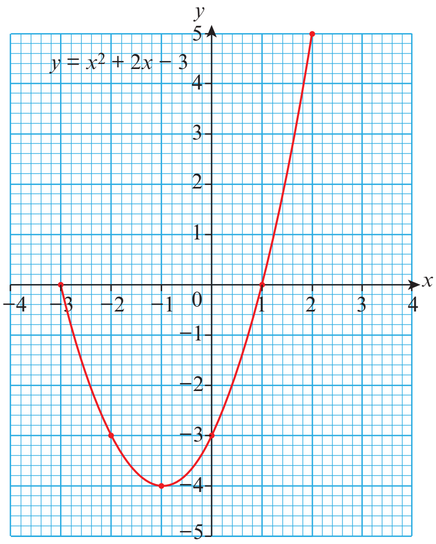
x	3	4	5	6
x^2	9	16	25	36
$-5x$	−15	−20	−25	−30
-4	−4	−4	−4	−4
$y = x^2 - 5x - 4$	−10	−8	−4	2



3

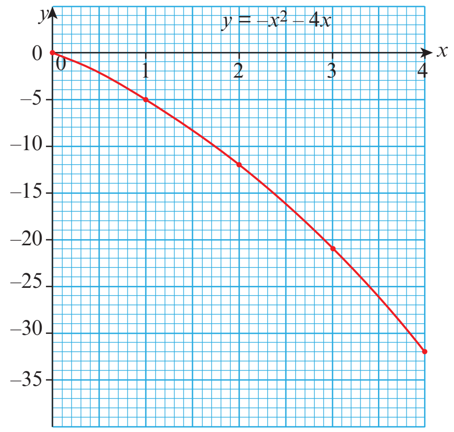
x	−3	−2	−1
$y = x^2 + 2x - 3$	0	−3	−4

x	0	1	2
$y = x^2 + 2x - 3$	−3	0	5



4

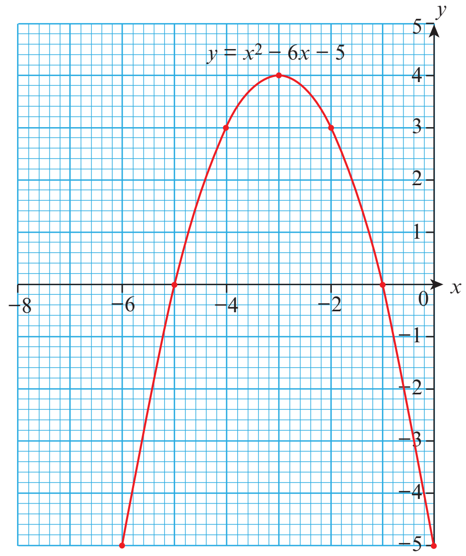
x	0	1	2	3	4
$y = -x^2 - 4x$	0	−5	−12	−21	−32



5

x	−6	−5	−4	−3
$y = -x^2 - 6x - 5$	−5	0	3	4

x	−2	−1	0
$y = -x^2 - 6x - 5$	3	0	−5



6 **a** 6 m

b 2 seconds

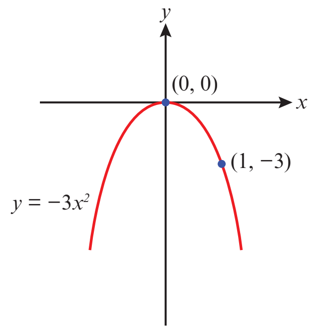
c 3 seconds

d 4.5 m

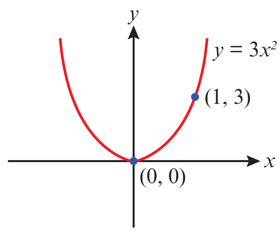
e The water surface is at $h = 0$.

Exercise 18.3

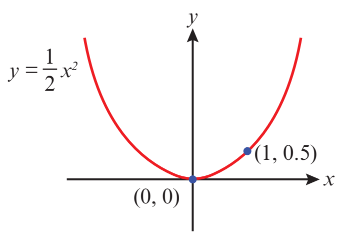
1 **a**



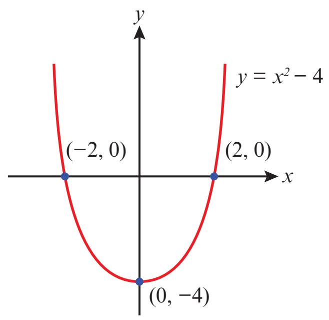
b



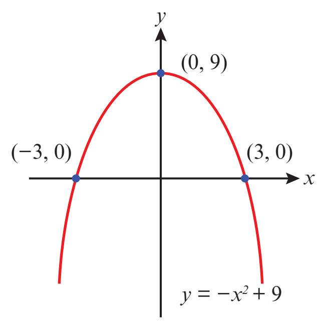
c



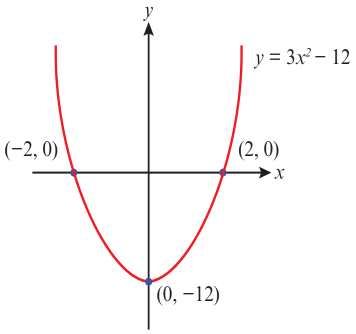
d



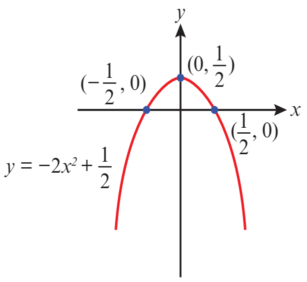
e



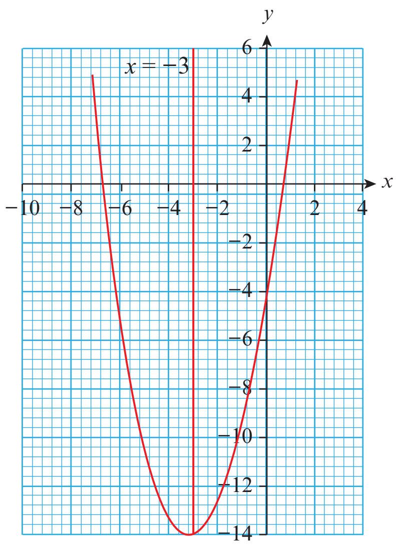
f



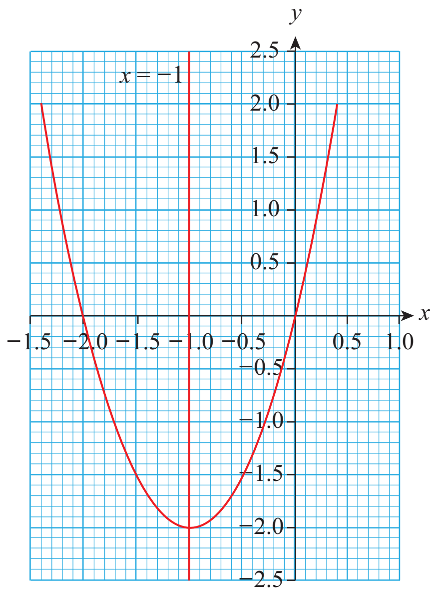
g



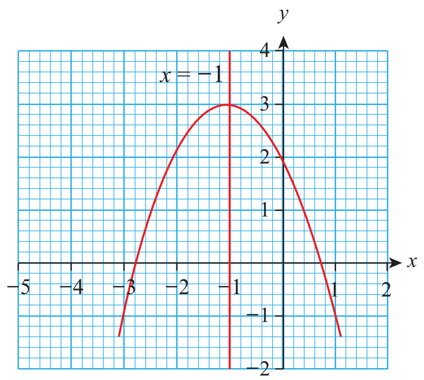
2 **a**



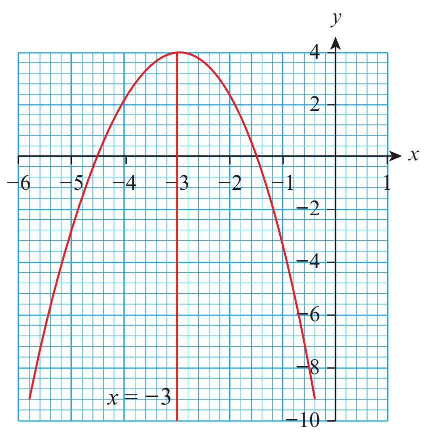
b



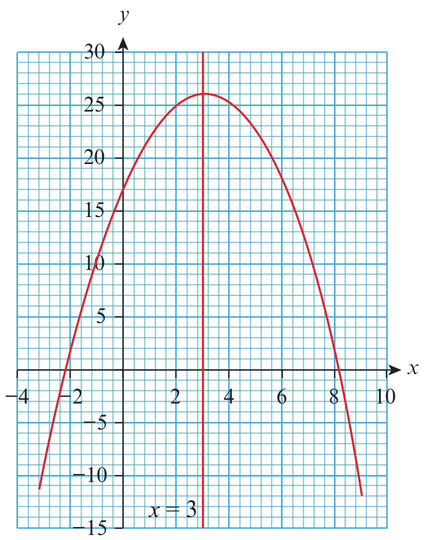
c



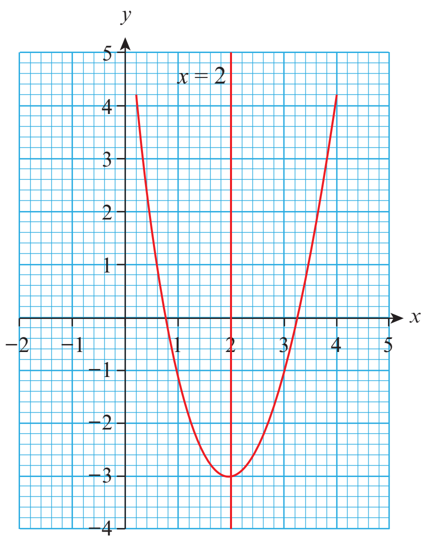
d



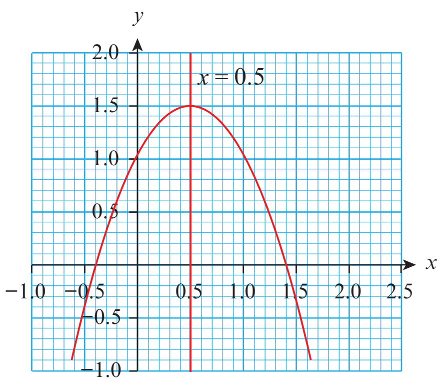
e



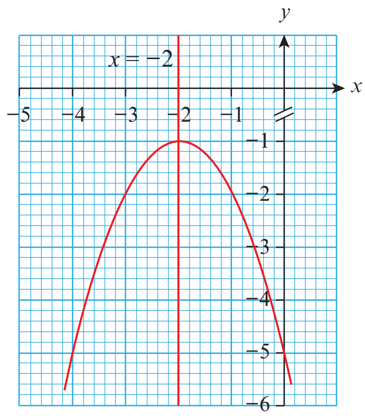
f



g



h



3 **a** $y = -x^2 - 4x + 5$

b $y = 4 - x^2$

c $y = x^2 - 3x - 4$

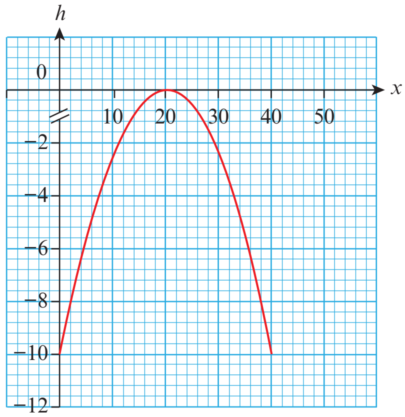
d $y = x^2 - 2x - 3$

4 **a** (20, 0)

b $0 \leqslant x \leqslant 20$

c $-10 \leqslant h \leqslant 0$

d



e Width = 40 m

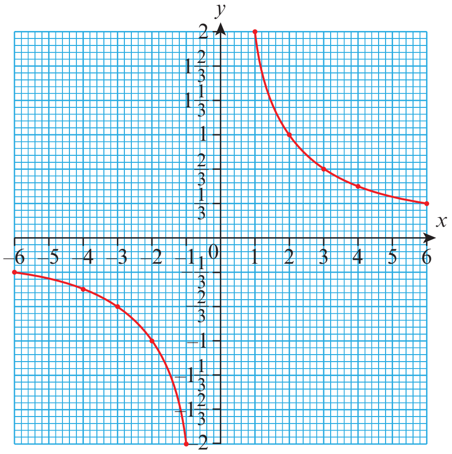
f Max height = 10 m

Exercise 18.4

1 **a**

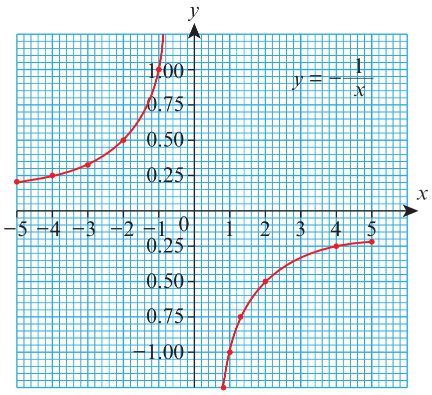
x	−6	−4	−3	−2	−1
$y = \frac{2}{x}$	$-\frac{1}{3}$	−0.5	$-\frac{2}{3}$	−1	−2

x	1	2	3	4	6
$y = \frac{2}{x}$	2	1	$\frac{2}{3}$	0.5	$\frac{1}{3}$



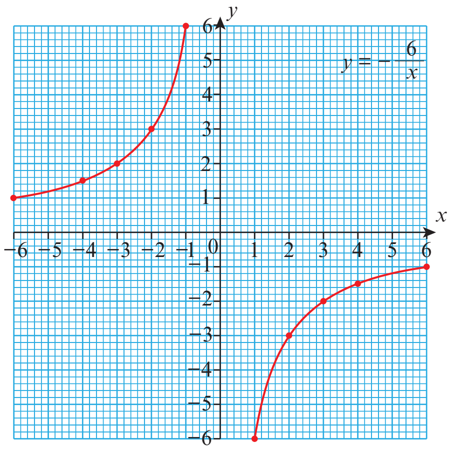
b

x	−5	−4	−3	−2	−1	1	2	3	4	5
$y = -\frac{1}{x}$	0.2	0.25	$\frac{1}{3}$	0.5	1	−1	−0.5	$-\frac{1}{3}$	−0.25	−0.2



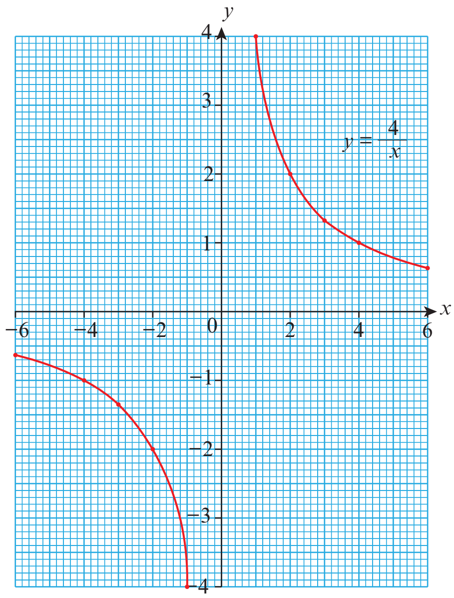
c

x	−6	−4	−3	−2	−1	1	2	3	4	6
$y = -\frac{6}{x}$	1	1.5	2	3	6	−6	−3	−2	−1.5	−1

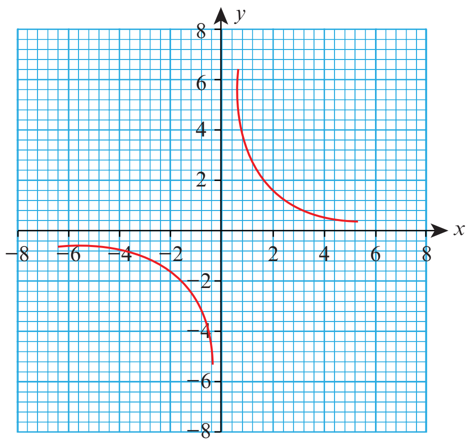


d

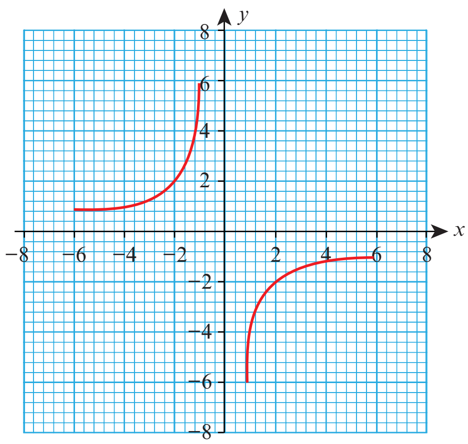
x	-6	-4	-3	-2	-1	1	2	3	4	6
$y = \frac{4}{x}$	$-\frac{2}{3}$	-1	$-1\frac{1}{3}$	-2	-4	4	2	$1\frac{1}{3}$	1	$\frac{2}{3}$



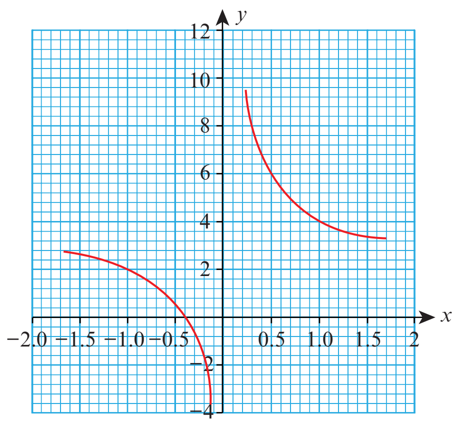
2 a



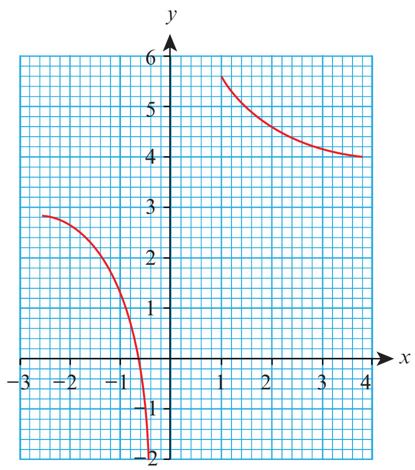
b



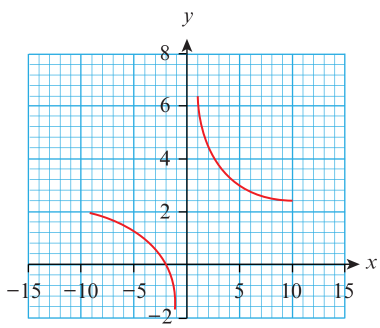
c



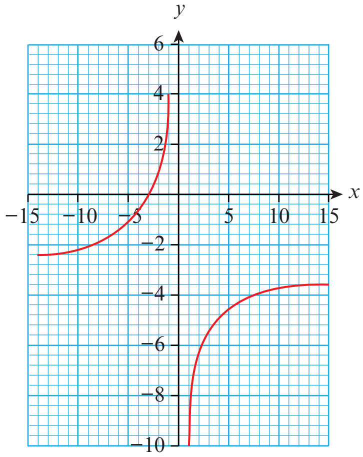
d



e



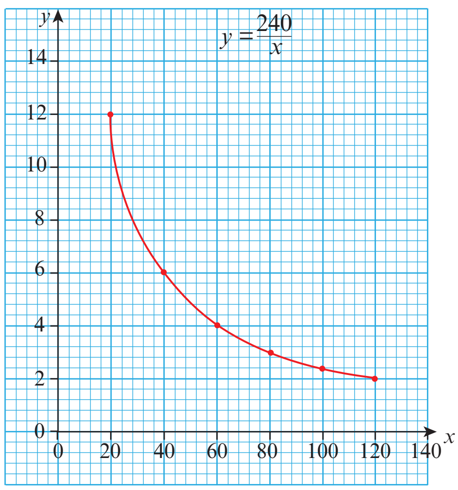
f



3 a

x	20	40	60	80	100	120
y	12	6	4	3	2.4	2

b



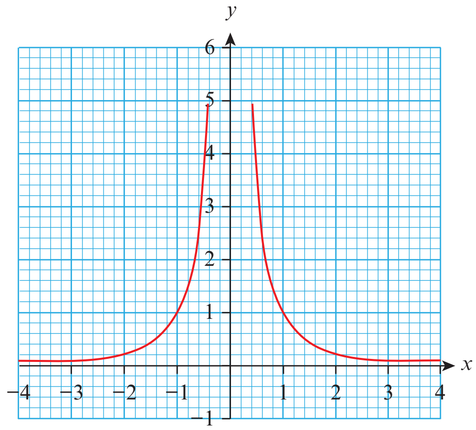
c $y = \frac{240}{x}$

4 a

x	−4	−3	−2	−1	$-\frac{1}{2}$
y	$\frac{1}{16}$	$\frac{1}{9}$	$\frac{1}{4}$	1	4

x	$\frac{1}{2}$	1	2	3	4
y	4	1	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{16}$

b



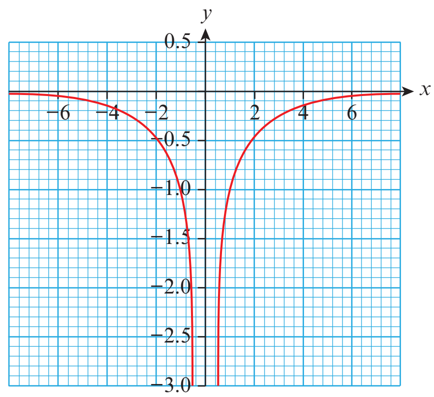
c Graph is still disjoint but both curves are above the x-axis on opposite sides of the y-axis.

d Division by 0 is undefined.

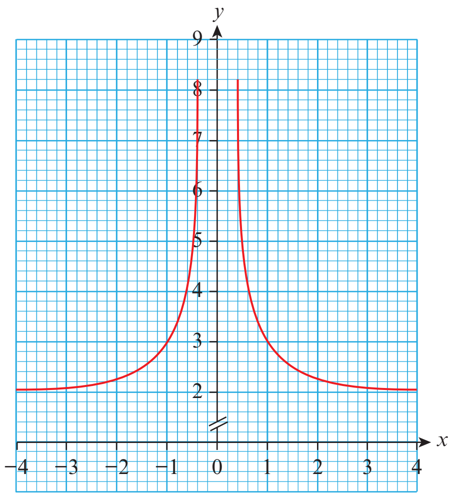
e $y = 0$ (the x-axis) and $x = 0$ (the y-axis)

f $x = 0$ and $y = 3$

g i



ii



h Suggested answer: The equation $y = ax^{-1}$ can be written with positive indices as $y = \frac{a}{x}$. This is the standard form of a reciprocal graph and it will give a hyperbola with two curves in opposite quadrants. The equation $y = ax^{-2}$ can be written with positive indices as $y = \frac{a}{x^2}$. The range for this function is all positive numbers, so the two curves of the graph will be above the x-axis. If $y = -ax^{-2}$, the equation becomes $y = -\frac{a}{x^2}$ and the range will be negative numbers, meaning that the two curves will be below the x-axis.

Exercise 18.5

1 a $x = -1$ and $x = 2$

b $x = -2.4$ and $x = 3.4$

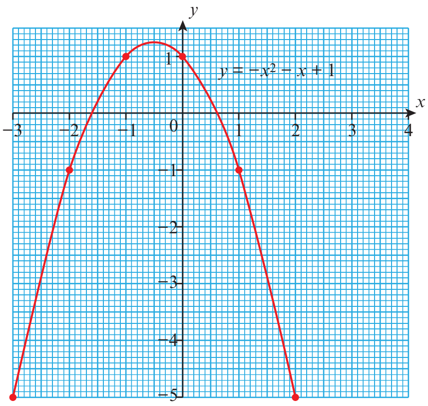
c $x = -2$ and $x = 3$

2 a

x	-3	-2	-1
$y = -x^2 - x + 1$	-5	-1	1

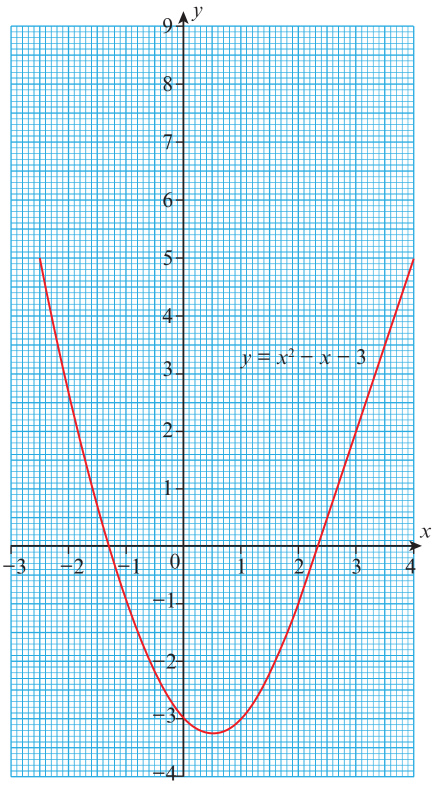
x	0	1	2
$y = -x^2 - x + 1$	1	-1	-5

b

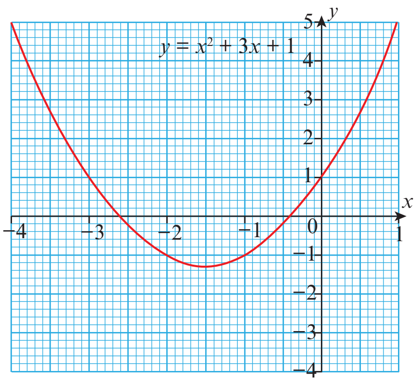


c $x = -1.6$ and $x = 0.6$

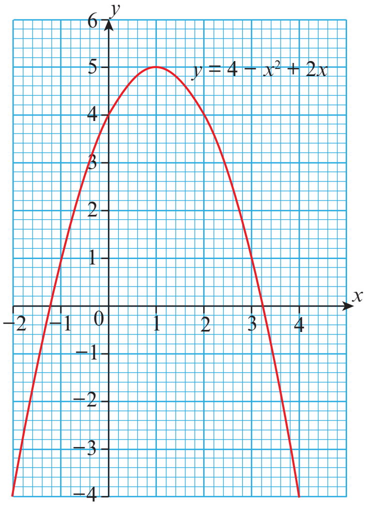
3 a $x = -1.3$ and $x = 2.3$



b $x = -2.6$ and $x = -0.4$



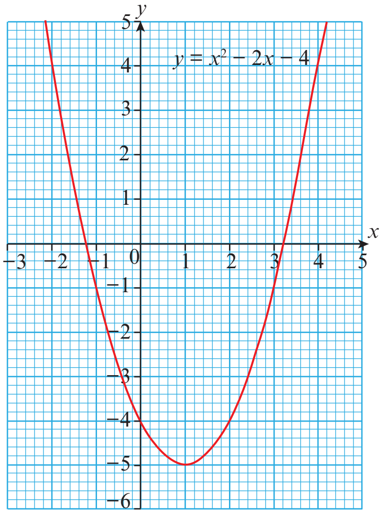
4 **a**



b **i** $x = -1.2$ and $x = 3.2$

ii $x = 0$ or $x = 2$

5 **a**



b **i** $x = -1.2$ and $x = 3.2$

ii $x = -1.8$ and $x = 3.8$

iii $x = -1$ and $x = 3$

Exercise 18.6

1 **a** $x = 2$ and $x = -1$

b $x = 2$ and $x = -2$

c $x = -2$ and $x = 1$

d $x = 1.2$ and $x = -0.4$

2 Students' own graphs.

a $(0, 0)$ and $(3, 9)$

b $(-1.4, -1.4)$ and $(1.4, 1.4)$

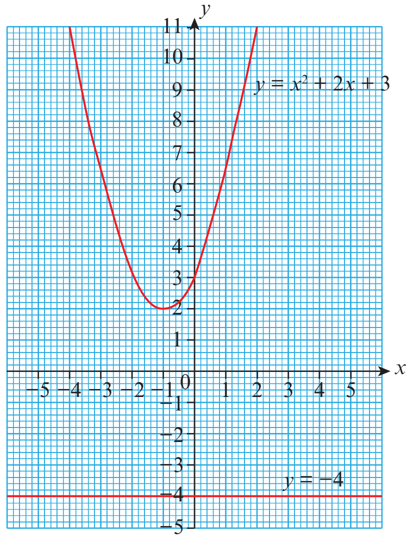
c $(2, 0)$

3 **a** $x = 9.1$ and $x = 0.9$

b $x = -2$ and $x = 4$

c $x = 3.8$ and $x = -1.8$

4



There are no points of intersection.

5 **a** $x = -1$, $y = 9$ and $x = 7$, $y = 17$

b $x = -1.64$, $y = -0.27$ and $x = 2.14$, $y = 7.27$

c $x = 2$, $y = 5$

6 **a** $(1, 6)$ and $(5, 14)$

b $\left(\frac{\sqrt{14}}{3}, \frac{8 + 5\sqrt{14}}{3}\right)$ and $\left(\frac{-\sqrt{14}}{3}, \frac{8 - 5\sqrt{14}}{3}\right)$

c $\left(\frac{1 + 3\sqrt{5}}{2}, 5 + 3\sqrt{5}\right)$ and

$\left(\frac{1 - 3\sqrt{5}}{2}, 5 - 3\sqrt{5}\right)$

Plotting simple cubic graphs

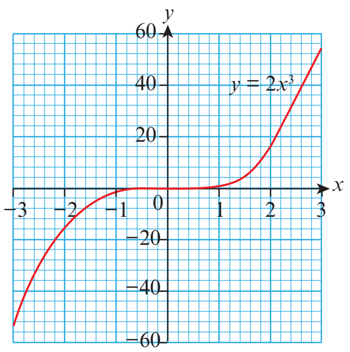
Students should find that increasing the value of a makes the graph narrower and the value of d is the y-intercept. There is only ever one x-intercept and this is at the point $\sqrt[3]{-\frac{d}{a}}$.

Exercise 18.7

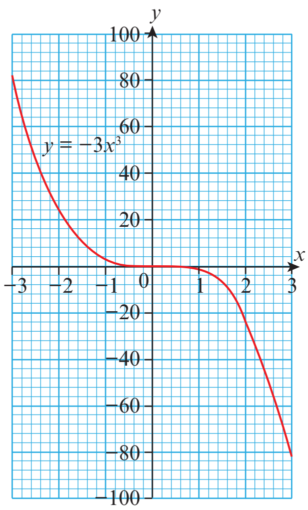
1

	x	−3	−2	−1	0	1	2	3
a	$y = 2x^3$	−54	−16	−2	0	2	16	54
b	$y = -3x^3$	81	24	3	0	−3	−24	−81
c	$y = x^3 - 2$	−29	−10	−3	−2	−1	6	25
d	$y = 3 + 2x^3$	−51	−13	1	3	5	19	57
e	$y = x^3 - 2x^2$	−45	−16	−3	0	−1	0	9
f	$y = 2x^3 - 4x + 1$	−41	−7	3	1	−1	9	43
g	$y = -x^3 + x^2 - 9$	27	3	−7	−9	−9	−13	−27
h	$y = x^3 - 2x^2 + 1$	−44	−15	−2	1	0	1	10

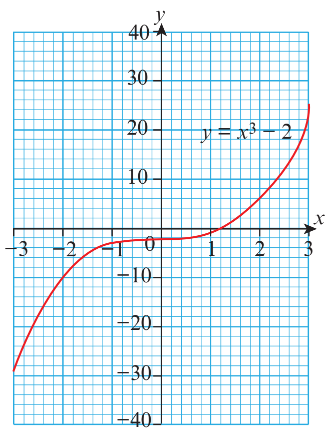
a



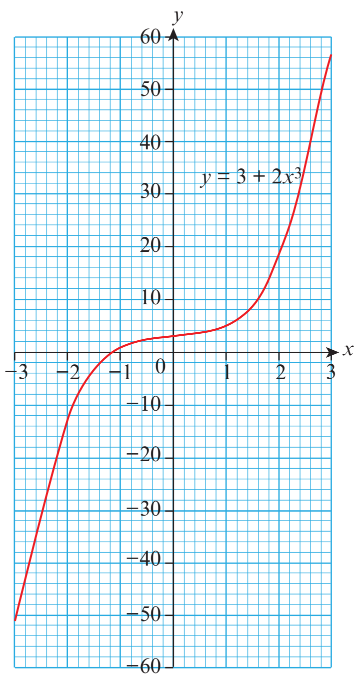
b



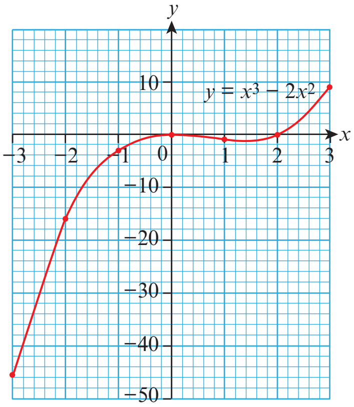
c



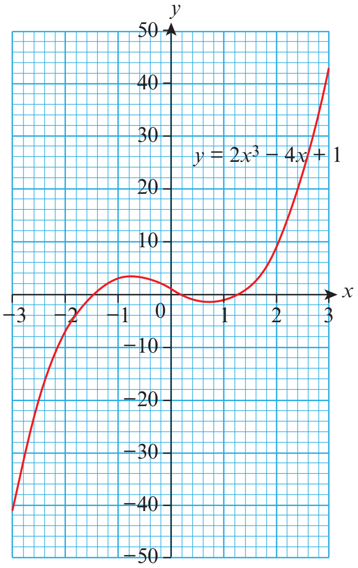
d



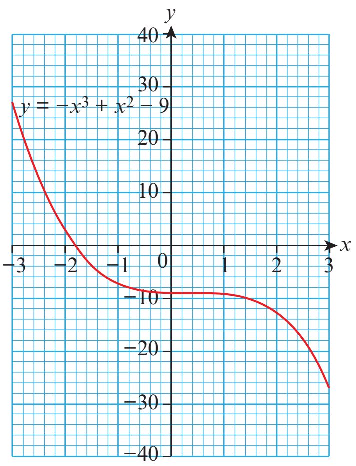
e



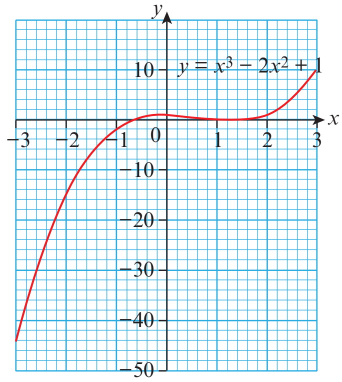
f



g



h



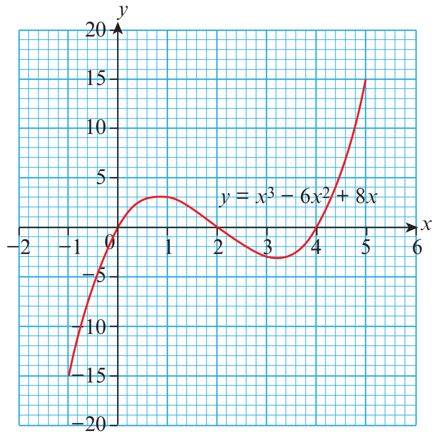
2 **a**

x	−1	−0.5	0	0.5	1
$y = x^3 - 6x^2 + 8x$	−15	−5.6	0	2.6	3

x	1.5	2	2.5	3	3.5
$y = x^3 - 6x^2 + 8x$	1.9	0	−1.9	−3	−2.6

x	4	4.5	5
$y = x^3 - 6x^2 + 8x$	0	5.6	15

b



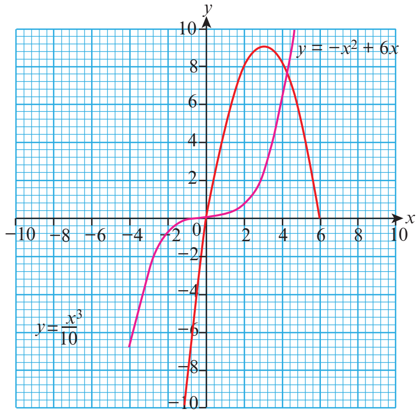
c **i** $x = 0$, $x = 2$ and $x = 4$

ii $x = 0.7$, 1, and $x = 4.3$

3 **a**

x	−4	−3	−2	−1	0	1
$y = \frac{x^3}{10}$	−6.4	−2.7	−0.8	−0.1	0	0.1
$y = 6x - x^2$	−40	−27	−16	−7	0	5

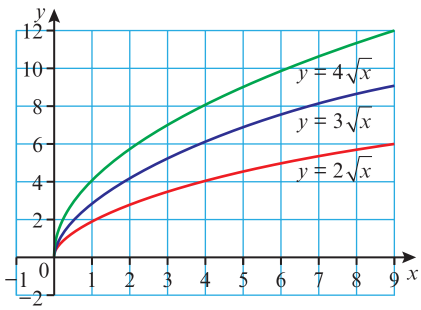
x	2	3	4	5	6
$y = \frac{x^3}{10}$	0.8	2.7	6.4	12.5	21.6
$y = 6x - x^2$	8.1	9.1	8.1	5.1	0



b $x = 0$ and $x = 4.2$

4 **a** The square root of a negative number is undefined, so x cannot be negative and the domain is $x \geqslant 0$.

b **i**



ii As a increases, the curve moves further from the x-axis.

iii Negative values of a mean that the curve is reflected in the x-axis.

c Students' sketches and notes will vary, but the graph will be a reciprocal graph in the first quadrant. Larger values of a move the curve further from the origin and negative values of a produce a reflection in the x-axis.

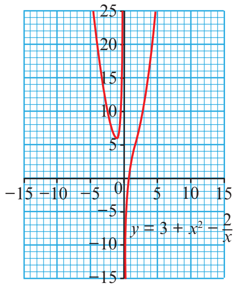
Exercise 18.8

1

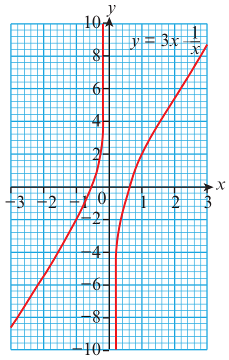
	x	−3	−2	−1	−0.5	−0.2	0	0.2	0.5	1	2	3
a	$y = 3 + x^2 - \frac{2}{x}$	12.7	8	6	7.3	13.0	N/A	−7.0	−0.8	2	6.5	11.3
b	$y = 3x - \frac{1}{x}$	−8.7	−5.5	−2	0.5	4.4	N/A	−4.4	−0.5	2	5.5	8.7
c	$y = -x + x^2 + \frac{2}{x}$	11.3	5	0	−3.3	−9.8	N/A	9.8	3.8	2	3.5	6.7
d	$y = -x^3 - 2x + 1$	34	13	4	2.1	1.4	1	0.6	−0.1	−2	−11.5	−32

Note: The y-values are rounded to 1 decimal place.

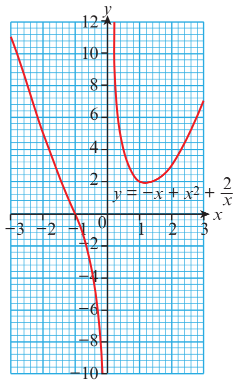
a $y = 3 + x^2 - \dfrac{2}{x}$



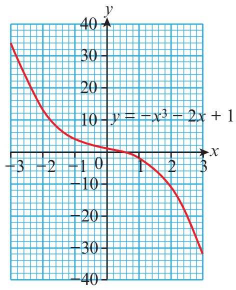
b $y = 3x - \dfrac{1}{x}$



c $y = -x + x^2 + \dfrac{2}{x}$

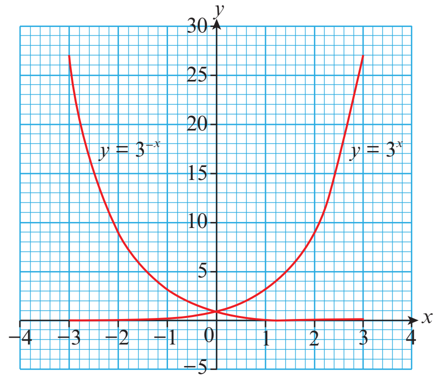


d $y = -x^3 - 2x + 1$



Exercise 18.9

1 **a, b**

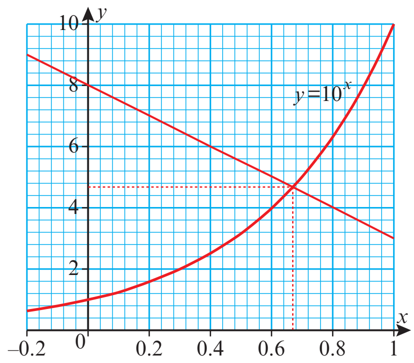


c The two graphs are symmetrical about the y-axis.

2 **a** **i** 2

ii 0.8

b



$x = 0.67$ (but allow $0.66 - 0.68$)

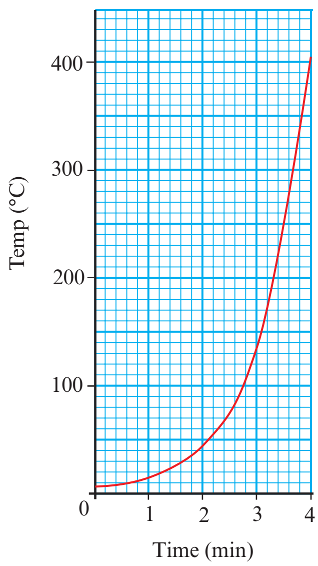
3 **a** 2

b 5.3 hours

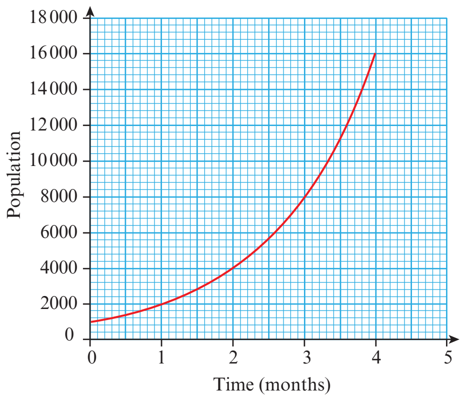
c 64

d 20 hours

4



5 **a**

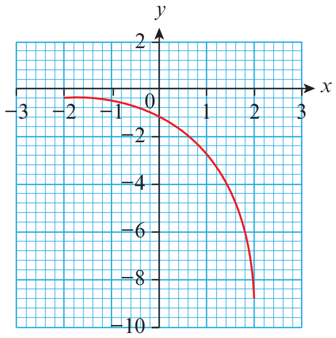


b 3 months

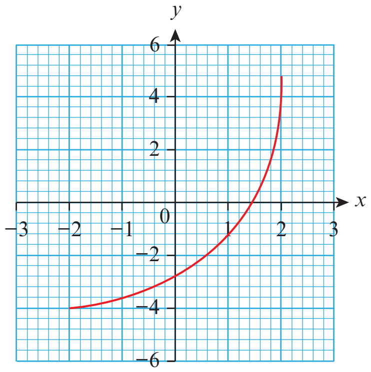
c 64 000

6 **a** Instructions will vary, but should include determining whether the graph is increasing or decreasing using the value of a. If a is positive, the graph is decreasing; if a is negative, the graph is increasing.
Use $a + q$ to determine the y-intercept.
Work out the asymptote by finding the line $y = q$.
If $a < 0$, the graph is below the asymptote and if $a > 0$, the graph is above the asymptote.

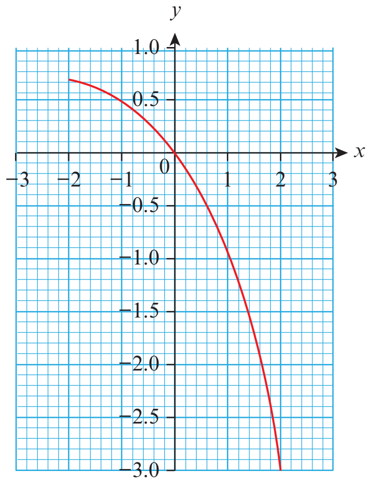
b **i**



ii



iii

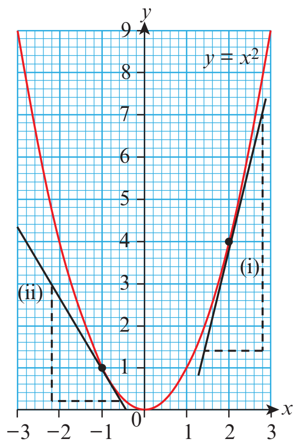


c **i** Many possible answers. For example, $y = 4 \times 2^{(x+0)} + (-6)$, with y-intercept at -2.

ii Greatest possible intercept for these values is 9. Many possible equations, including $y = y = 4 \times 2^{(x+0)} + 5$

Exercise 18.10

1 **a**

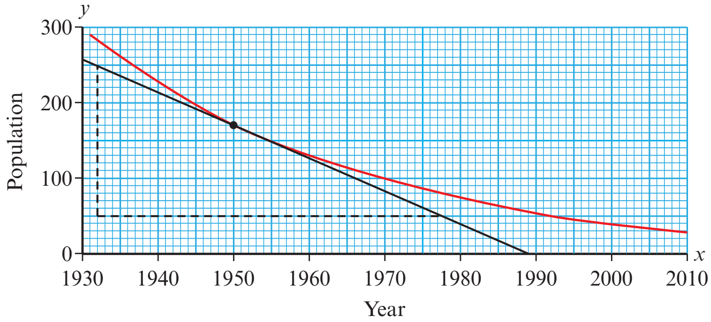


i 4

ii −1.75

b (−1.5, 2.25)

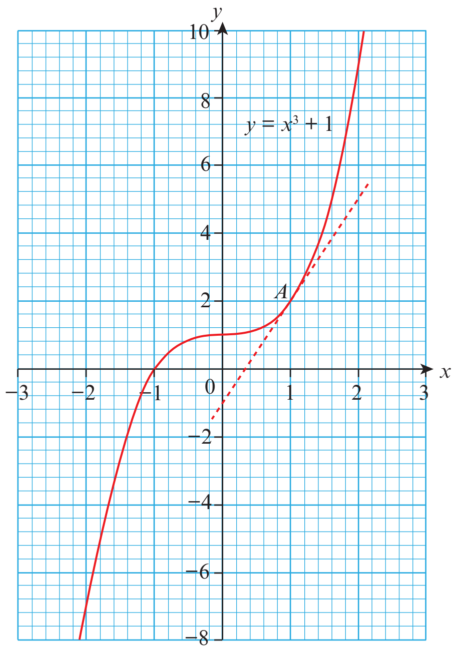
2 **a**



The gradient at point (1950, 170) is −4.4 people per year.

b Rate of change of population in the village in 1950.

3 **a**



b 3

Gradients of tangents

1 For the curve x^2 the gradient of the tangent at any point is twice the value of x at that point.

2 For the curve x^3 the gradient of the tangent at any point is three times the value of x^2 at that point.

Exercise 18.11

1 **a** $4x^3$

b $6x^5$

c $9x^8$

d $12x^2$

e $24x$

f $49x^6$

g $-16x^3$

h $84x^{11}$

i $-80x^4$

2 **a** 6

b 3

c 32

d −8

e −36

f 960

3 (3, 27)

Exercise 18.12

1 **a** $4x^3 + 5x^4$

b $9x^2 - 20x^3$

c $42x^5 + 18x$

d $x^2 - 28x^6$

e $30x^4 - \frac{32}{11}x^3$

f $-14x + 18x^5$

g $36x^2 + \frac{16}{3}x^7$

h $-120x^{11} - 80x^9$

i $8x - 36x^2 + 20x^3$

j $-\frac{32}{11}x^3 + \frac{6}{7}x^2 - \frac{3}{2}x$

2 **a** 93

b 52

c 12

3 (1, 5) and (−2, −4)

4 (0, 0) or $\left(\sqrt{3}, -\frac{9}{4}\right)$

Exercise 18.13

1 a 5
b -4
c 0
d 7
e -3
f $8x - 4$
g $21x^2 + 2$
h $x^2 + x$
i m

2 a $2x + 2$
b $5x^4 + 8x^3$
c $2x - 1$
d $2x - 9$
e $16x^3 + 24x^2$
f $-10x + 20$
g $4x + 5$
h $6x - 7$
i $24x + 23$
j $12x - 13$
k $42x - 44$
l $2x + 6$
m $8x + 4$
n $18x - 12$
o $\frac{3}{5}x^2 + \frac{6}{5}x$
p $\frac{14}{3}x^6 + x^5$
q $10x - 20$
r $2x$

3 67

4 $\left(\frac{2}{3}, \frac{1}{3}\right)$

5 $\left(\frac{1}{3}, -3\right)$

6 $(1, 5)$ and $(2, -4)$

7 $(2, 11)$ and $(-2, -5)$

8 a $a = 2$, gradient at $x = 4$ is 92
b $\frac{dy}{dx}$ at $x = -3$ is 50

Exercise 18.14

1 a $y = 6x - 9$
b $y = -4x - 4$
c $y = 56x - 144$
d $y = 18.25x - 19.25$
e $y = \frac{9}{20}x - \frac{1}{16}$

2 $\left(\frac{34}{19}, 0\right)$

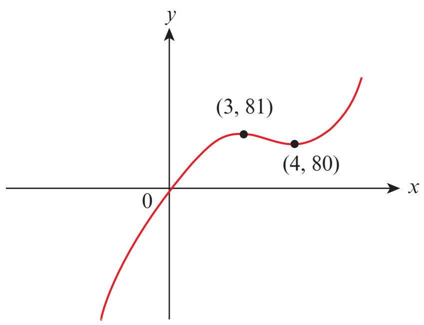
3 0.25

4 $\left(\frac{26}{9}, 2\right)$

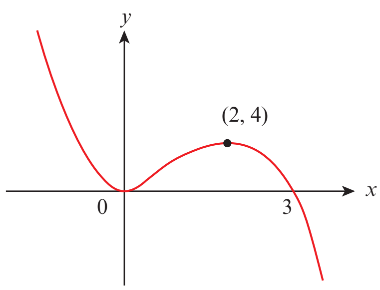
Exercise 18.15

1 a $(2, -3)$ min
b $(-3, -13)$ min
c $(4, 14)$ max
d $(2, -8)$ min
e $(1, -1)$ max
f $\left(-\frac{3}{2}, -\frac{13}{4}\right)$ min
g $\left(\frac{3}{10}, \frac{89}{20}\right)$ max
h $(-2, 15)$ max and $(2, -17)$ min
i $(0, 3)$ min and $(4, 35)$ max
j $(2, -4)$ min
k $(0, -25)$ min
l $\left(\frac{3}{4}, -\frac{9}{8}\right)$ min
m $(3, 81)$ max and $(4, 80)$ min
n $(0, 0)$ min and $(2, 4)$ max

2 m



n



3 **a** $\frac{dh}{dt} = 7 - 10t$

b 2.45 m

4 **a** 54 thousand

5 **a** Length = $2 - 2x$ and
width = $1 - 2x$
V = length x width x
depth = $x(2 - 2x)(1 - 2x)$

b The width is only 1 m and we are subtracting two lots of x from this length. So we can only subtract something less than 0.5.

c $x = 0.211,\ V = 0.192$

Practice questions

1 **a** A: $x = -2$
B: $y = -x$
C: $y = x^2 - 2$
D: $y = 2x + 1$

b **i** $(-2, 2)$

ii $(3, 7)$ and $(-1, -1)$

c $\left(-\frac{1}{3}, \frac{1}{3}\right)$

d D

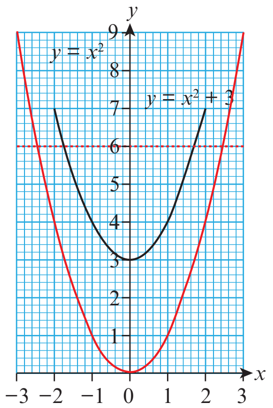
e C

2 **a**

x	−2	−1.5	−1	−0.5	0
y	7	5.25	4	3.25	3

x	0.5	1	1.5	2
y	3.25	4	5.25	7

b



c No, x^2 will never equal $x^2 + 3$

d **i** $x = +2.4$ or -2.4

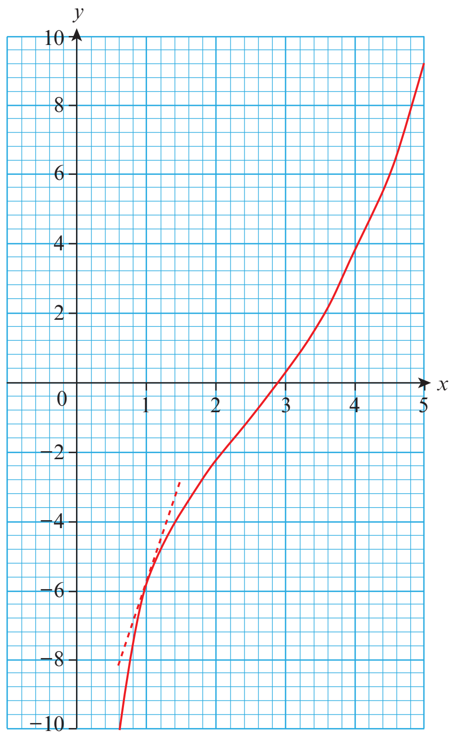
ii $x = +1.7$ or -1.7

3 **a** $p = -10,\ q = 6.3,\ r = 9.2$

b

x	0.6	1	1.5	2	2.5
y	−10	−5.9	−3.7	−2.3	−1.1

x	3	3.5	4	4.5	5
y	0.3	1.9	3.8	6.3	9.2



c $x = 2.9$

d Gradient = 6

4 **a** vi

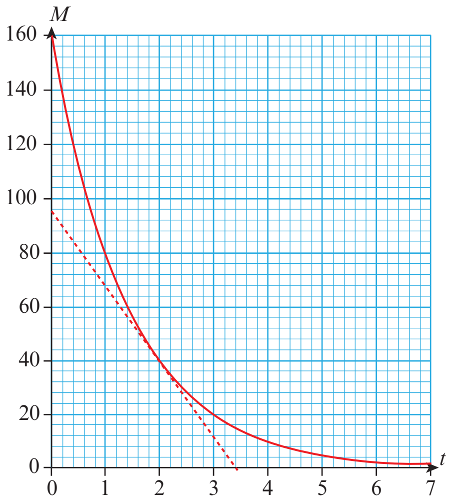
b ii

c i

d iv

5 **a** **i** $p = 160, q = 10, r = 2.5$

ii



iii Rate of change = 28.2

b $t = 1$

6 $x = 2.31$ and $y = 2.77$

$x = -0.65$ and $y = 1.78$

7 **a** $y = 3(x + 1)^2 - 4$

b $x = -1$

c $(-1, -4)$

8 **a** 17

b $6x + 2$

c 14

d $y = 14x - 11$

e $\left(\frac{11}{14}, 0\right)$

9 $y = \dfrac{-x - 21}{4}$

10 **a** $y = 2a - x$

b Area $= x(2a - x)$

c $A = x(2a - x) = 2ax - x^2$

$\dfrac{dA}{dx} = 2a - 2x$

$2a - 2x = 0 \Rightarrow x = a$

$y = 2a - x = a$

So all sides have length a and the rectangle is a square.

Practice questions worked solutions

1 **a** A: $x = -2$

B: $y = -x$

C: $x^2 - 2$

D: $y = 2x + 1$

b **i** $(-2, 2)$

ii $(-1, -1)$ and $(3, 7)$

c $2x + 1 = -x$

$3x = -1$

$x = \frac{1}{3}$

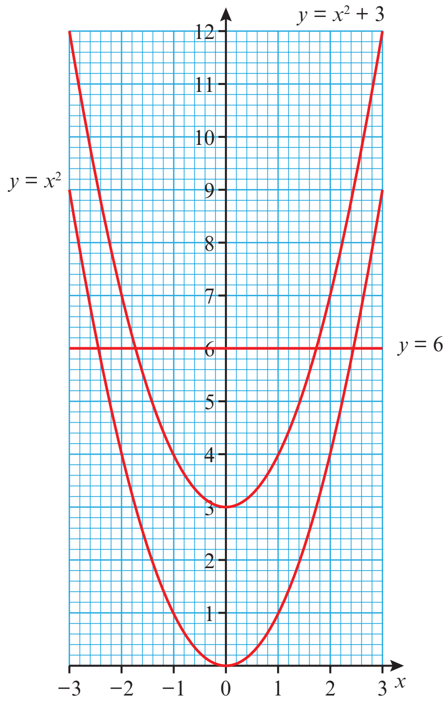
$\left(-\frac{1}{3}, \frac{1}{3}\right)$

d Graph D

e C is symmetrical about the y-axis

2 **a** Missing values are 7, 3.25

b, d $y = x^2$



c Meet when $x^2 + 3 = x^2$

$3 = 0$ which never occurs

d Solutions are:

i 2.4

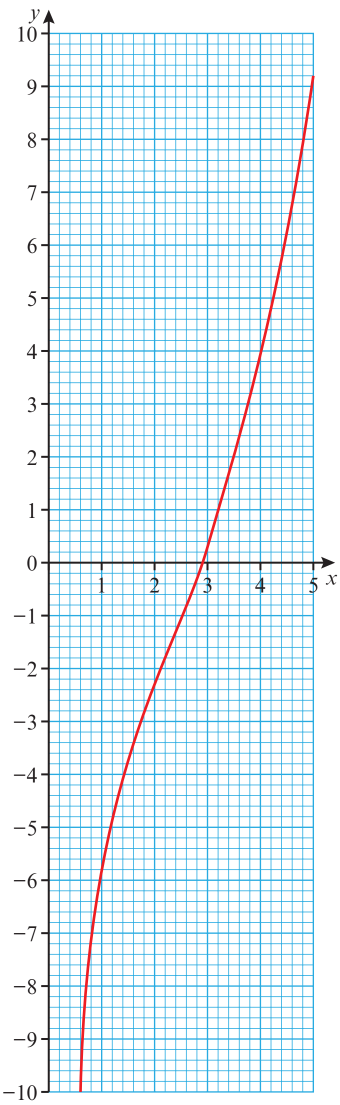
ii 1.7

3 **a** $p = \frac{0.3^3}{12} - \frac{6}{0.6} = -9.982 = -10.00$ to 1 d.p.

b $q = \frac{4.5^3}{12} - \frac{6}{4.5} = 6.3$

c $r = \frac{5^3}{12} - \frac{6}{5} = 9.2$

b



c Point of intersection with x-axis is when $x = 2.9$

d Gradient is approximately equal to 6.3

4 **a** (vi) Negative quadratic curve

b (ii) Exponential growth curve

c (i) Cubic graph with positive y-intercept

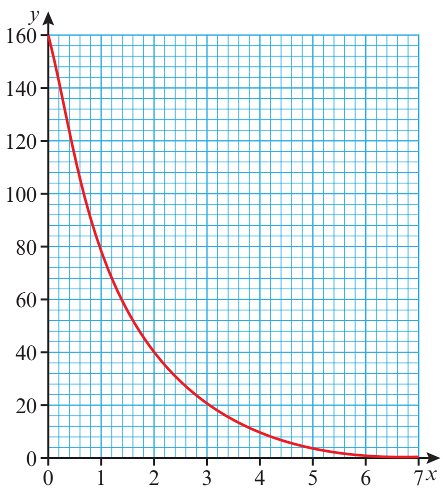
d y-values are always negative

5 **a** **i** $p = \frac{160}{2^0} = 160$

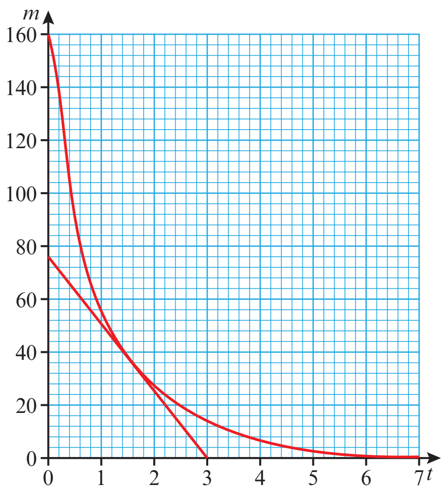
$q = \frac{160}{2^4} = \frac{160}{16} = 10$

$r = \frac{160}{2^6} = \frac{160}{64} = \frac{5}{2}$

ii



iii



Estimated gradient = −26

b $m = 160 - M$

$\Rightarrow 160 - M = M$

$2M = 160$

$M = 80$

So, $80 = \frac{160}{2^t} \Rightarrow 2^t = 2$

$\Rightarrow t = 1$

6 $y = 2x^2 - 3x - 1$ ①

$3y - x = 6$ ②

Substituting ① in ②

$3(2x^2 - 3x - 1) - x = 6$

$6x^2 - 9x - 3 - x = 6$

$6x^2 - 10x - 9 = 0$

$$x = \frac{10 \pm \sqrt{100 - 4 \times 6 \times -9}}{12}$$

$$= \frac{5}{6} \pm \frac{\sqrt{79}}{6}$$

$$y = \frac{41}{18} \pm \frac{\sqrt{79}}{18}$$

7 **a** $y = 3x^2 + 6x - 1$

$= 3(x^2 + 2x) - 1$

$= 3[(x + 1)^2 - 1] - 1$

$= 3(x + 1)^2 - 4$

b $x = -1$

c $(-1, -4)$

8 **a** $x = 2 \Rightarrow y = 3 \times 4 + 2 \times 2 + 1$

$= 17$

b $\frac{dy}{dx} = 6x + 2$

c $\frac{dy}{dx}$ at $x = 2 = 6 \times 2 + 2 = 14$

d Gradient 14 at (2, 17)

$y = mx + c$

$y = 14x + c$

$17 = 28 + c$

$c = -11$

e $y = 0 \Rightarrow 14x = 11$

$x = \frac{11}{14}$

so, $\left(\frac{11}{14}, 0\right)$

9 $x = 5 \Rightarrow y = (5 - 3)^2 = 4$

Now $y = (x - 3)^2 = x^2 - 6x + 9$

$\frac{dy}{dx} = 2x - 6$

At $x = 5$, $\frac{dy}{dx} = 10 - 6 = 4$

Gradient of tangent = 4

Gradient of perpendicular normal = $-\frac{1}{4}$

because $4 \times -\frac{1}{4} = 1$

$y = -\frac{1}{4}x + c$

$4 = -\frac{5}{4} + c \Rightarrow c = \frac{16}{4} + \frac{5}{4} = \frac{21}{4}$

So, $y = -\frac{1}{4}x + \frac{21}{4}$

10 **a** Perimeter $= 2x + 2y = 4a$

So, $2y = 4a - 2x$

$y = 2a - x$

b Area $= xy = 2(2a - x) = A$

$A = 2ax - x^2$

c $\frac{dA}{dx} = 2a - 2x = 0$ when A is maximum

So, $x = a$ when A is maximum

$x = a$ and $y = 2a - x$

$= 2a - a$

$= a$

$x = y = a$ and the rectangle is a square.

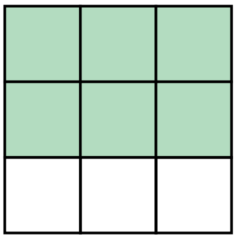
Chapter 19

Getting started

1 It fits onto itself as it turns round a point, so it has rotational symmetry.

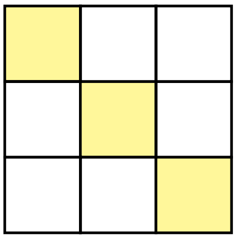
3 a 2

b Possible solution:



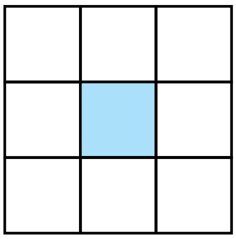
c 1

d Possible solution:



e Answers will vary; check each other's answers.

f Many options, but simplest solution is:



Exercise 19.1

1 a None

b *CD*, *HG*

c *CD*, *HG*

d *AB*

e *AB*, *EF*

f *AB*, *CD*

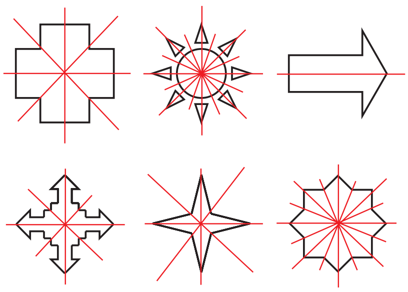
g *CD*

h *AB*, *CD*, *GH*

2

Shape	Number of lines of symmetry
Square	4
Rectangle	2
Equilateral triangle	3
Isosceles triangle	1
Scalene triangle	0
Kite	1
Parallelogram	0
Rhombus	2
Regular pentagon	5
Regular hexagon	6
Regular octagon	8

3



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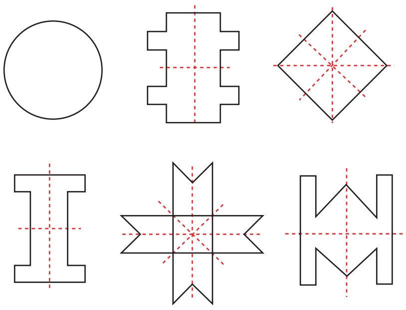


5 Students' own answers.

Exercise 19.2

1 a 2

b 5

c 2

d 6

e 2

f 1

g 1

h 1

2 **b**

Regular polygon	Lines of symmetry	Order of rotational symmetry
Triangle	3	3
Quadrilateral	4	4
Pentagon	5	5
Hexagon	6	6
Octagon	8	8
Decagon	10	10

c Lines of symmetry = order of rotational symmetry in regular polygons

d Number of sides = lines of symmetry = order of rotational symmetry in regular polygons

3 Students' own answers.

4 There may be some variation, depending on the font chosen, but the most likely answers are:

a ABCDEMUVWY

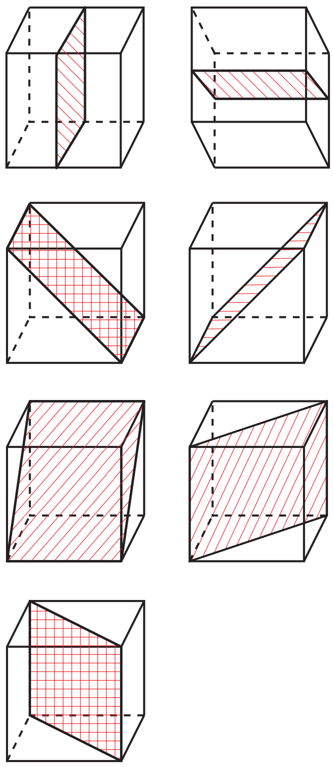
b HIOX

c HINOSX

5 Students' own answers.

Exercise 19.3

1



2 **a** 4

b Infinite

c Infinite

d 2 if base is a right-angled, isosceles triangle

e 2

f 2

g Infinite

h 7

i 2

Exercise 19.4

1 Each has a rotational symmetry of 2

2 **a** Infinite

b 1

c 2

d 8

e Infinite

f 1

Exercise 19.5

1 **a** $AB = 5\text{ cm}$

b $AB = 30\text{ cm}$

c $AB = 2.4\text{ m}$

2 Join OP and construct a line at right angles to OP that will be the chord.

3 $x = 43°$

4 13.5 cm

5 $AO = 9\text{ cm}$

Area $AOCB = 108\text{ cm}^2$

6 O is the centre of both concentric circles.

Construct OX perpendicular to AD.

$\therefore X$ is the mid-point of AD and BC

$\therefore BX = XC$ and $AX = XD$

$AB = AX - BX = XD - XC = CD$

7 **a** 17.3 cm

b 4.25 m

c 31.1 mm

8 $10\sqrt{3} \approx 17.3\text{ cm}$

Exercise 19.6

1 a $x = 43°$, $y = 43°$, $z = 94°$
  b $x = 124°$, $y = 34°$
  c $x = 35°$
  d $x = 48°$

2 a $x = 41.5$
  b $x = 38°$

3 a Tangents subtended from the same point are equal in length.
  b i $CAB = 70°$
    ii $DAC = 20°$
    iii $ADC = 70°$

Exercise 19.7

1 a $p = 50°$, $q = 65°$, $r = 65°$
  b $b = 80°$
  c $c = 30°$, $d = 55°$, $e = 45°$, $f = 45°$
  d $p = 85°$, $q = 105°$
  e $b = 60°$
  f $x = 94°$, $y = 62°$, $z = 24°$
  g $p = 85°$, $q = 65°$

2 a $AOB = 2x$
  b $OAB = 90° - x$
  c $BAT = x$

3 a $a = 70°$
  b $b = 125°$
  c $c = 60°$, $d = 60°$, $e = 80°$, $f = 40°$

4 a $90° - x$
  b $180° - 2x$
  c $2x - 90°$

5 a Draw the chords AD and BC. ADX and BCX are angles in the same segment, so they are equal. Similarly, angle DAX is the same as angle CBX. AXD and BXC are vertically opposite angles, so they are the same, too. This means that both triangles contain the same three angle and so they are similar.
  b Using similarity, $\frac{DX}{CX} = \frac{AX}{BX}$. You can then multiply through by CX and BX.

Understanding the alternate segment theorem

1 Students should notice that the angle between the chord and the angle in the alternate segment is equal. They can compare their diagrams to the one given to check they used the correct angles.

2 a Any two angles drawn in the same segment are equal, this means that you can draw another angle in the alternate segment, using the diameter as one of the lines forming the angle and know that it is still equal to y.
  b Triangle PAB is right angled as it is the angle in a semicircle.
  c Angle APB must be $90 - y$.
  d The angle between the diameter and a tangent is 90, so $90 - y + x = 90$, so $x = y$.

Exercise 19.8

1 a 120°
  b 85°
  c 80°
  d 120°
  e 90°
  f 90°

2 Angle $BTC = 180° - 30° - (180° - 60°) = 30°$ because angles in a triangle add up to 180°.
  So angle $TDC = 30°$ by the alternate segment theorem.
  $CTD = 180° - 60° - 30° = 90°$ (angle sum in a triangle)
  So CD is diameter because the angle in the segment is 90°.

3 $CTD = 90°$
  So $TDC = 180° - 90° - x = 90° - x$
  So by the alternate segment theorem $CTB = 90° - x$
  But $BCT = 180° - x$
  So $y + 180° - x + 90° - x = 180°$
  So $2x - y = 90°$

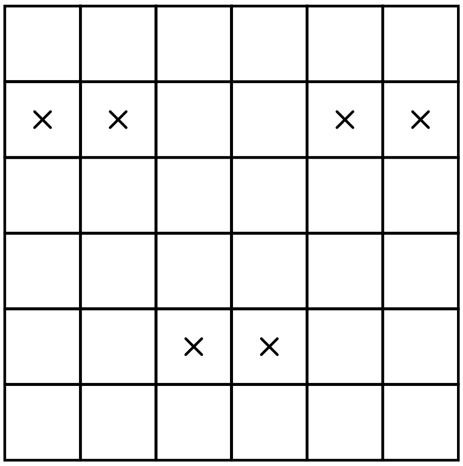
4 103°

Practice questions

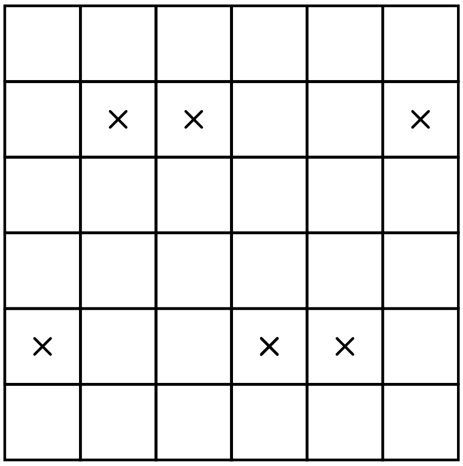
1 **a** and **e**

2 Order 3

3 **a** For example:



b For example:



4 $a = 90°$, $b = 53°$, $c = 90°$, $d = 53°$

5 **a** 6

b 7

c 7

6 $OPX = OQX = 90°$ (tangent perpendicular to radius)

$POQ = 150°$

So, $PXQ = 360° - 90° - 90° - 150°$ (angle sum of quad)

Angle $PXQ = 30°$

7 $x = 26$

Angle at centre = 2 × angle at circumference

$360 - 4x = 2(3x + 50)$

$360 - 4x = 6x + 100$

$10x = 260$

$x = 26$

8 **a** angle $QSP = 60°$ (alternate segment theorem)

b angle $SQP = 60°$ (angle sum of triangle)

c angle $PBQ = 60°$ (angle sum of triangle)

d angle $QRS = 140°$ ($PQRS$ is cyclic quadrilateral)

9 **a** They are vertically opposite angles.

b Angles in the same segment

c From parts (a) and (b), angle BXA = angle DXC and angle XAB = angle XDC. This means that angle ABX = angle DCX. So all three angles are the same and the triangles are similar.

d Triangles ABX and DCX are similar so the ratio of sides AX and BX is equal to the ratio of sides DX and CX.

$$\frac{AX}{BX} = \frac{DX}{CX}$$

so $(AX)(CX) = (BX)(DX)$

10 **a** **i** Angle $BAD = \frac{1}{2} \times$ angle $= BOD = \frac{1}{2} \times 86° = 43°$

ii Angle at centre = 2 × angle at circumference.

b **i** Angle BCD = $180° - 43° = 137°$

ii Opposite angles of a cyclic quadrilateral add up to 180°.

Practice questions worked solutions

1 **a** Yes. The dotted lines are lines of symmetry. There is rotational symmetry of order 2 about the intersection of the lines of symmetry.

b Neither

c Rotational symmetry only

d Reflection symmetry only

e Yes. Exactly the same as a. but with rotational symmetry of order 4.

2 3

3 **a**

×	×			×	×
		×	×		

b

	×	×			×
×			×	×	

4 **a** 90°, because a diameter and tangent always meet at a right angle.

b $90° - 37° = 53°$

c 90°, because the angle in a semicircle is always 90°.

d $180° - 37° - 90° = 53°$

5 **a** 6

b One through end faces.
One through centre of each rectangular face.
$6 + 1 = 7$

c 7

6 $XPQ = XQP = 90°$ because a radius meets a tangent at 90°.

Total angles in $OXPQ = 360°$

So $PXQ = 360° - 90° - 90° - 150°$
$= 30°$

7 Reflex angle $AOC = 360° - 4x°$ because angles at a point add up to 360°.

Angle at centre = 2 × angle at circumference

$$360° - 4x = 2(4x - 50°)$$
$$360° - 4x = 8x + 100°$$
$$12x = 260°$$
$$x = \frac{260°}{12} = 21.7°$$

8 **a** 60° by the alternate segment theorem.

b $180° - 40° - 60° = 80°$
(Angles in a triangle add up to 180°.)

c $QPB = 60°$ (Base angles of an isosceles triangle are equal.)
$PBQ = 180° - 60° - 60° = 60°$ (Angles in a triangle add up to 180°.)

d $180° - 40° = 140°$ (Opposite angles in a cyclic quadrilateral add up to 180°.)

9 **a** Vertically opposite angles

b Vertically opposite angles

c $ABX = DCX$ (Angles in the same segment are equal.)
and
$BAX = CDX$
So triangles ABX and DCX have the same angles.

d $\frac{AX}{DX} = \frac{BX}{CX}$ because ratios of corresponding sides are equal.
So $(AX)(CX) = (BX)(DX)$

10 **a** **i** $BAD = \frac{1}{2} \times 86° = 43°$

ii Angle at centre = 2 × angle at circumference

b **i** $BCD + 43° = 180°$
$BCD = 137°$

ii Opposite angles in a cyclic quadrilateral adds up to 180°.

Chapter 20

Getting started

1 a fx means number of passengers (x) multiplied by the frequency (f).

b Find the product of x and f for each row.

c 32

d Sum of fx

e 1.625

f 1

g 2

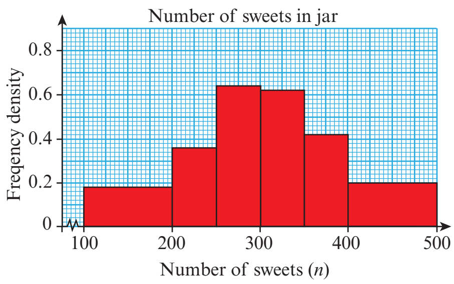
2 Answers will vary but should include that the diagram shows a frequency diagram (histogram) and a frequency polygon. The histogram shows non-overlapping class intervals on the horizontal axis and the frequency on the vertical axis. The frequency polygon is plotted at the class midpoints and shows the shape of the distribution.

Exercise 20.1

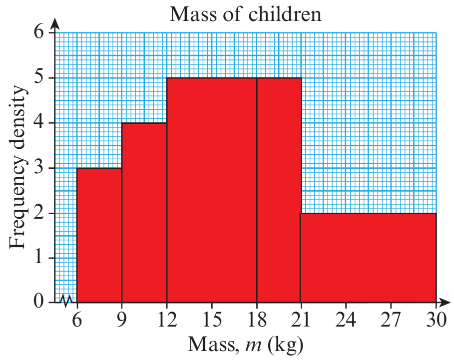
1 a

No. of sweets (n)	Frequency (f)	Class width	Frequency density
$100 \leqslant n < 200$	18	100	0.18
$200 \leqslant n < 250$	18	50	0.36
$250 \leqslant n < 300$	32	50	0.64
$300 \leqslant n < 350$	31	50	0.62
$350 \leqslant n < 400$	21	50	0.42
$400 \leqslant n < 500$	20	100	0.2

b



2



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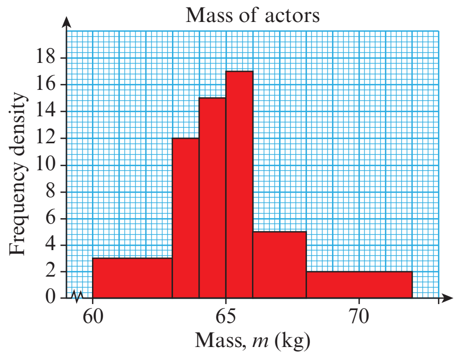


4 a Students' ideas.

b They both show the shape of the distribution. In the histogram the larger the area the greater the frequency for that class interval, and in the stem-and-leaf diagram the longer the leaves the greater the frequency for that stem value.

c When you have too many individual items of data to list separately on a stem-and-leaf diagram.

5 a 80

b 73

c 7

d Body fat is too low for intense physical activity.

e No – the expectation is that soldiers are physically active and therefore keep their body fat at a satisfactory level.

6 **a**

Age (a) in years	Frequency
$0 < a \leqslant 15$	12
$15 < a \leqslant 25$	66
$25 < a \leqslant 35$	90
$35 < a \leqslant 45$	90
$45 < a \leqslant 70$	50

b 156

7 **a** No – frequency density and not frequency given.

b Yes – most of the bars are with the boundaries of the speed limits.

c **i**

Speed (km/h)	Frequency	Class width	Frequency density
$0 \leqslant s < 50$	240	50	4.8
$50 \leqslant s < 65$	320	15	21.3
$65 \leqslant s < 80$	500	15	33.3
$80 \leqslant s < 95$	780	15	52
$95 \leqslant s < 110$	960	15	64
$110 \leqslant s < 125$	819	15	54.6
$125 \leqslant s < 180$	638	55	11.6

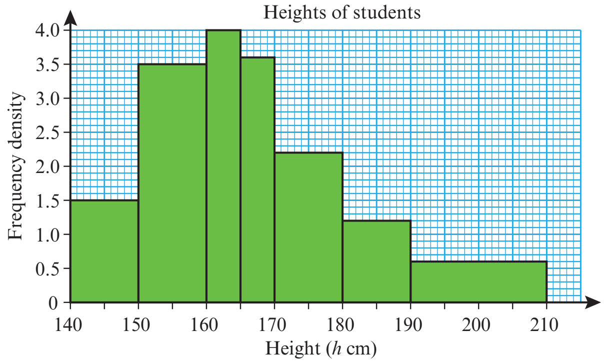
ii 240 below the minimum speed limit

d 15%

8 **a**

Height (h cm)	Frequency
$140 \leqslant h < 150$	15
$150 \leqslant h < 160$	35
$160 \leqslant h < 165$	20
$165 \leqslant h < 170$	18
$170 \leqslant h < 180$	22
$180 \leqslant h < 190$	12
$190 \leqslant h < 210$	12

b



c 150–160

d 75.7

9 Answers will vary depending on the data that students collect.

Check that students measure time in seconds and collect the raw data by experiment before they organise it into a frequency distribution. A suitable scale might have a wider class at the start (0–15 seconds) and end (>50 seconds) with varied intervals between those values.

Histograms could be drawn on graph paper to make it easier to work with frequency density.

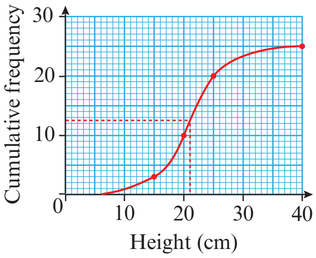
Exercise 20.2

1 **a**

Height in cm	$5 \leqslant h < 15$	$15 \leqslant h < 20$	$20 \leqslant h < 25$	$25 \leqslant h < 40$
Number of plants	3	7	10	5
Cumulative frequency	3	10	20	25

b 21–25 cm

c

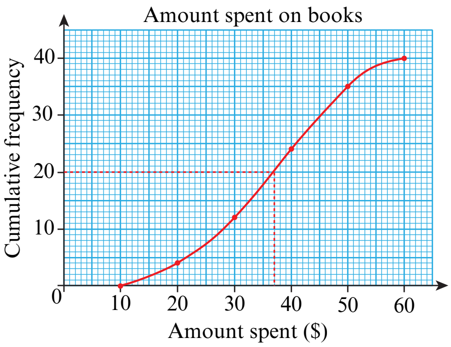


Median = 21 cm

2 **a** \$36.25

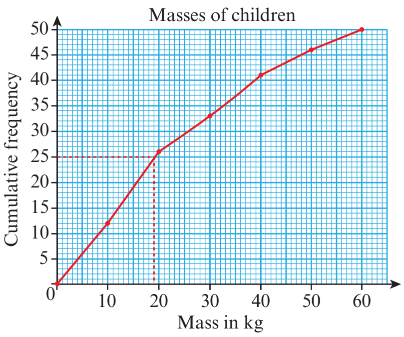
b $p = 12$, $q = 24$, $r = 35$

c



d Median amount spent \$37

3 **a**



b 19 kg

c 7

Exercise 20.3

1 **a** 30.0 cm

b 27.5 cm

c 33.5 cm

d 6 cm

e 29.5 cm

2 **a** **i** Paper 1: 48% Paper 2: 60%

ii Paper 1: 28% Paper 2: 28%

iii Paper 1: 52% Paper 2: 65%

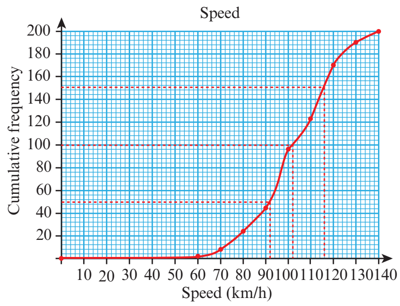
b Paper 1: >66% Paper 2: >79%

3 **a** **i** 45 kg

ii 330 girls

b 10%

4 **a**



b Median = 102 km/h

$Q_1 = 92$ km/h

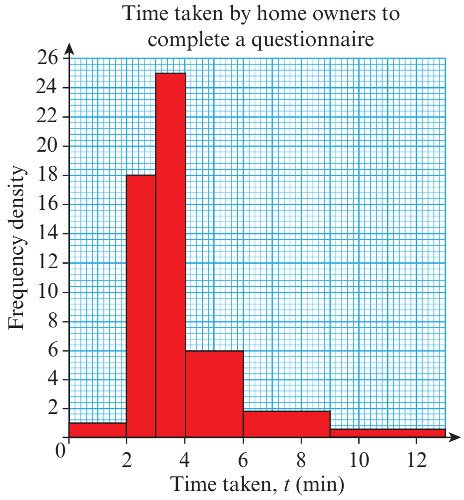
$Q_3 = 116$

c IQR = 24 km/h

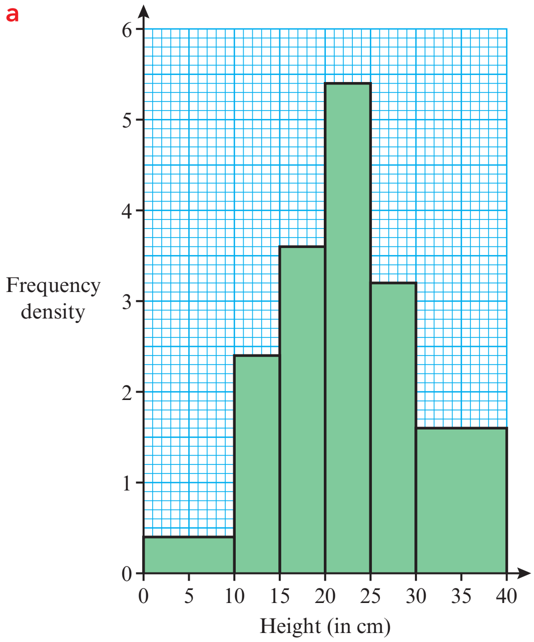
d 14.5%

Practice questions

1



2 **a**



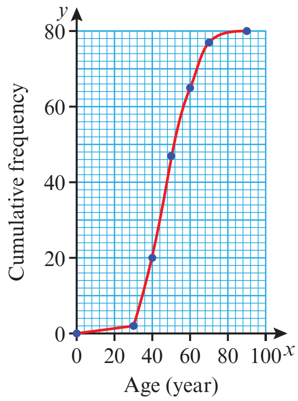
b

Height (h cm)	Frequency
$0 \leqslant h < 10$	4
$10 \leqslant h < 15$	12
$15 \leqslant h < 20$	18
$20 \leqslant h < 25$	27
$25 \leqslant h < 30$	16
$30 \leqslant h < 40$	16

c 22.5 cm

3 **a** We say we are 29 right up until the day we become 30.

b



c Approximately 43

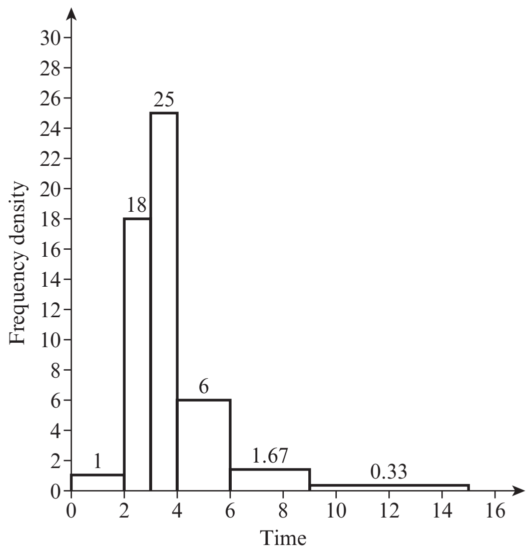
d Approximately $56 - 40 = 18$

e Approximately 12.5%

Practice questions worked solutions

1

Time	Number	Frequency density
$0 \leqslant t < 2$	2	$\frac{2}{2} = 1$
$2 \leqslant t < 3$	18	$\frac{18}{1} = 18$
$3 \leqslant t < 4$	25	$\frac{25}{1} = 25$
$4 \leqslant t < 6$	12	$\frac{12}{2} = 6$
$6 \leqslant t < 9$	5	$\frac{5}{3} = 1.67$
$9 \leqslant t < 15$	2	$\frac{2}{6} = 0.33$



2 a Additional bars have heights 0.4, 2.4, 3.6

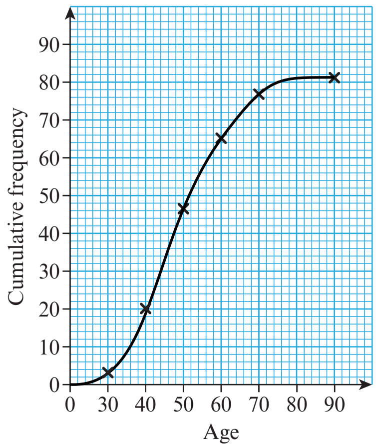
b $5.4 \times 5 = 22$
$3.2 \times 5 = 16$
$1.6 \times 10 = 16$

c $\dfrac{4 \times 5 + 12 \times 12.5 + 18 \times 17.5 + 22 \times 22.5 + 16 \times 27.5 + 16 \times 35}{88} = 22.5$

3 a We know there are no further ages until past 30.

b

Age	Cumulative frequency
$0 \leqslant t < 30$	2
$0 \leqslant t < 40$	20
$0 \leqslant t < 50$	47
$0 \leqslant t < 60$	65
$0 \leqslant t < 70$	77
$0 \leqslant t < 90$	80



c approximately 52

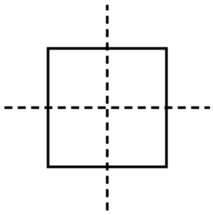
d approximately 25

e approximately 28%

Past paper questions

1 a 4

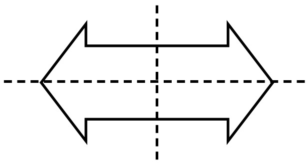
b



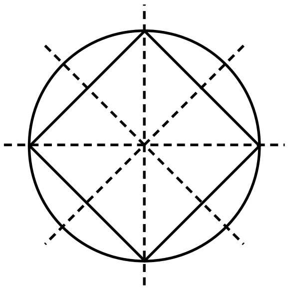
2 0.892€ to the dollar

3 $8300 \times 1.056^6 = \$11\,509.64$

4 a i



ii

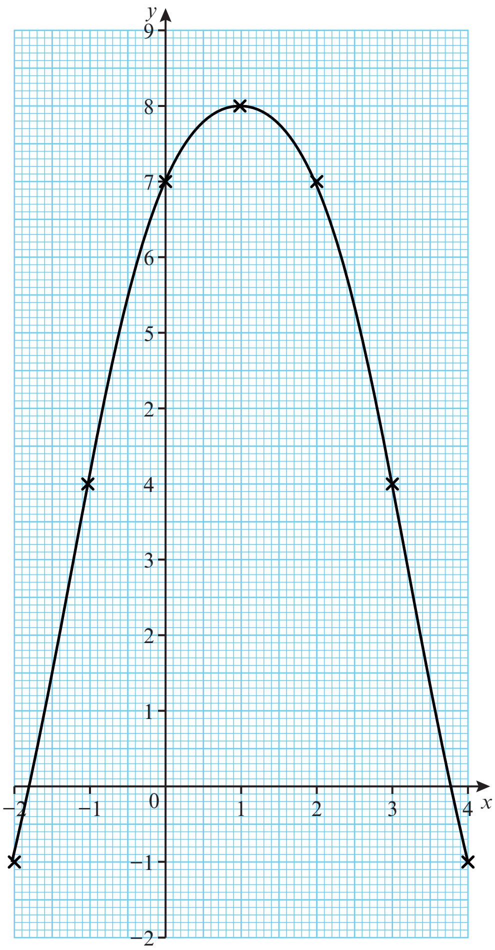


b rhombus

5 a

x	−2	−1	0	1	2	3	4
y	−1	4	7	8	7	4	−1

b



c $x = 1$

d $x = -1.8, 3.8$

6 **a** $2.35 + 4.45 = 6.80$

b $10 - 3.4 - 2 \times 0.85 = 4.90$

c **i** $34 \times \$8.25 = \$280.50$

ii $\$280.25 + 1.5 \times \$8.25 \times 8 = \$379.50$

d $3.5 + 7 + 8 + 10.5 = 33$ hours

e \$85.20

f \$13 891.50

7 **a** $x = 2$

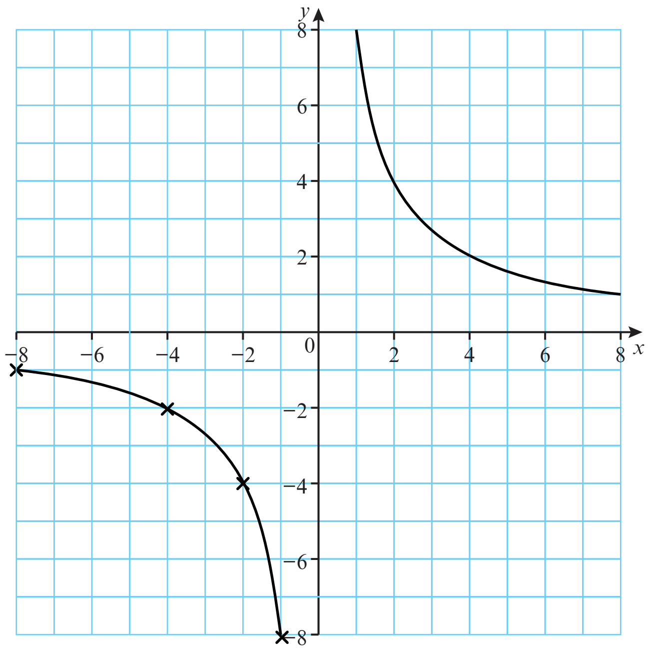
b **i** $4 = \frac{k}{2} \Rightarrow k = 8$

ii $y = \frac{8}{x} = \frac{8}{250} = 0.032$

c **i**

x	−8	−4	−2	−1
y	−1	−2	−4	−8

ii



d $y = x$

8 **a** $180 - 2 \times 62 = 180 - 124 = 56°$

b $180 - \frac{360}{10} = 144°$

c $x = 90° - 58° = 32°$

$y = 90° - x = 58°$

d 28° CED is alternate with the given 28° angle

e $\sqrt{21^2 + 28^2} = 35$ cm

9 **a** **i** $14a + 4a = 18a$

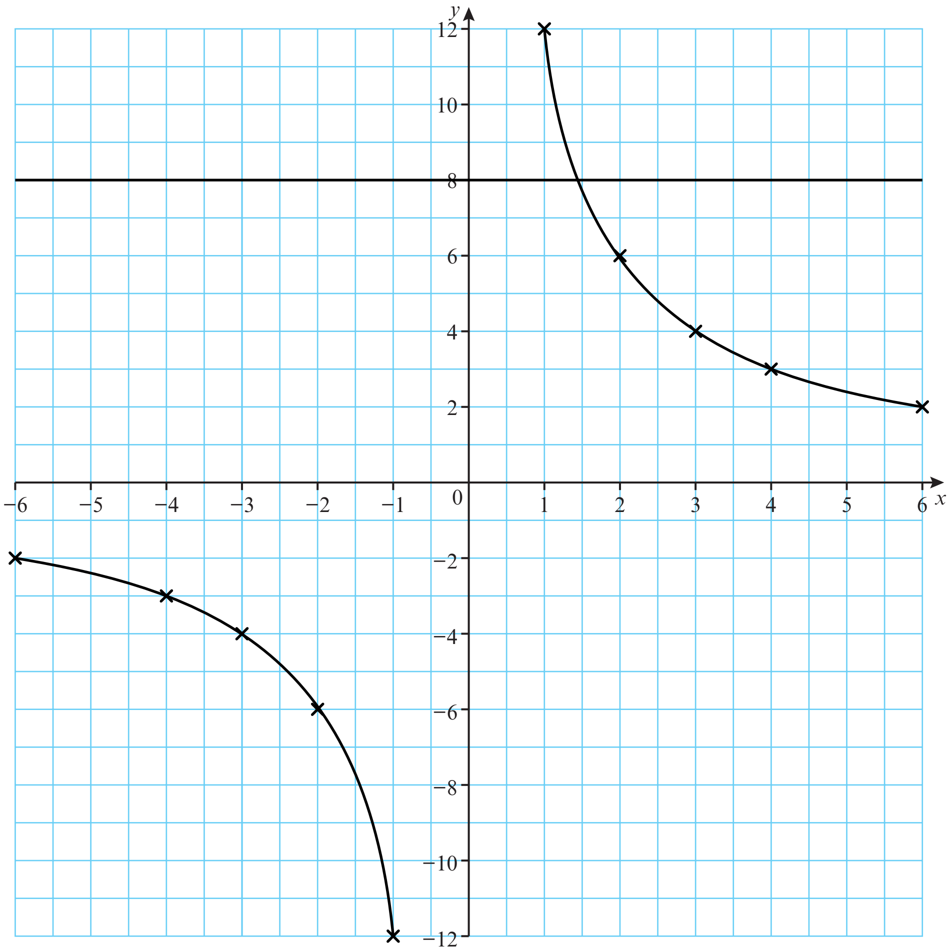
ii $14a^2$

b 6, 9, 14

c **i**

x	−6	−4	−3	−2	−1	1	2	3	4	6
y	−2	−3	−4	−6	−12	12	6	4	3	2

ii and **iii**



iv $x = 1.5$

10 $x = \frac{110°}{2} = 55° y = 24°$

11 $x^2 - 7x + 5 = 0$

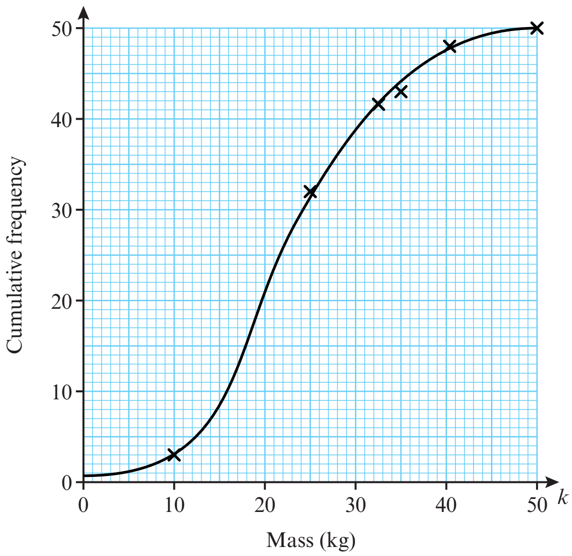
$x^2 - 2x + 1 = 5x - 4$

$y = 5x - 4$

12 **a** **i**

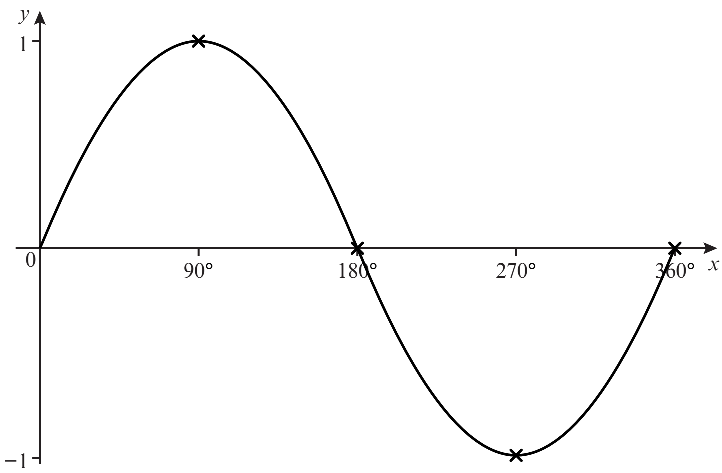
Mass (k kg)	$k \leqslant 10$	$k \leqslant 25$	$k \leqslant 35$	$k \leqslant 40$	$k \leqslant 50$
Cumulative frequency	3	22	43	48	50

ii



iii ≈42

13 a i

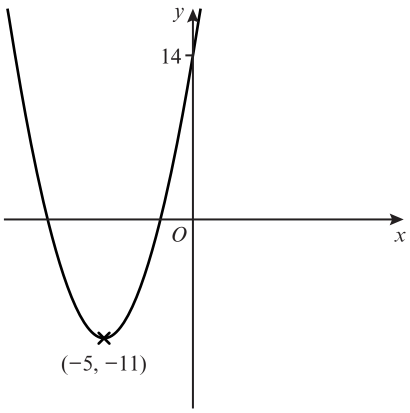


ii rotational symmetry order 2 about (180, 0)

b $\sin x = \frac{3}{4} \Rightarrow x = 48.6°, 180 - 48.6° = 131.4°$

c i $(x + 5)^2 - 25 + 14 = (x + 5)^2 - 11$

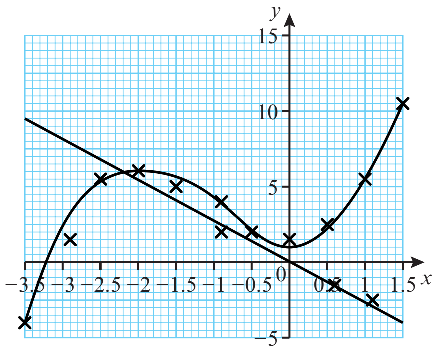
ii



14 a

x	−3.5	−3	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5
y	−4.1	2	5.1	6	5.4	4	2.6	2	2.9	6	12.1

b



c $x = -3.3$

d $x^3 + 3x^2 + 2 = -2x$

$x = 2.2$

e $2 < k < 6$

15 a $\frac{2.65}{2.5} \times 100 - 100 = 6\%$

b $500 \times 1.015^7 = \$554.92$

c $1.106^{20} = 1.3736\ldots$ so 37.4 % increase

d $r = \sqrt[22]{\frac{2607}{6400}} \times 100 = 96\%$

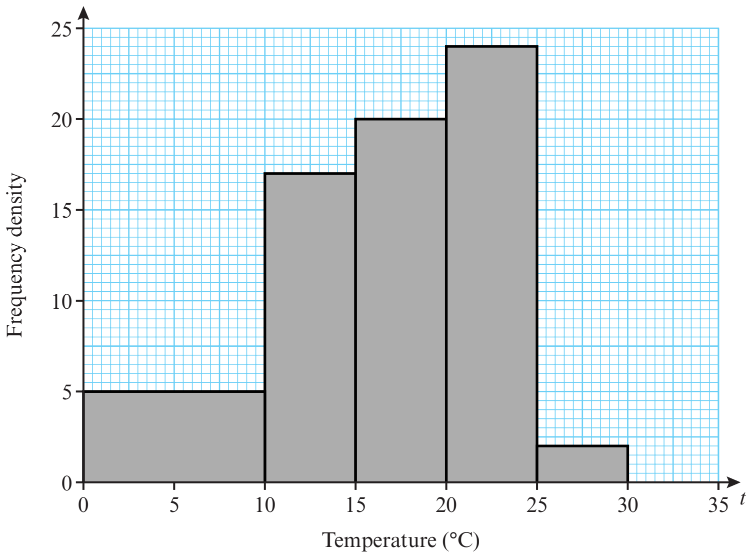
16 a i range = 27 − 20 = 7, mode = 21, mean = 22.7

ii $\frac{3}{14} = 0.2143$

b $nx - (n - 1)(x + 1) = nx - (nx + n - x - 1) = -n + x + 1$

c i 16.54

ii



17 a $\dfrac{(x-5)(x+5)}{(x-5)(x+4)} = \dfrac{x+5}{x+4}$

b $\dfrac{(x+5)(x-1)+x(x+8)}{x(x+1)} = \dfrac{2x^2+12x-5}{x(x+1)}$

c **i** $\dfrac{dy}{dx} = 6x^2 - 8x$

ii $6(4^2) - 8(4) = 6 \times 16 - 32 = 64$

iii $6x^2 - 8x = 0$

$2x(3x-4) = 0$

$x = 0$ or $x = \dfrac{4}{3}$

$y = 6$ or $y = \dfrac{98}{27}$

Chapter 21

Getting started

1 a $3:2$
  b $4:9$
  c $3:10$

2 a 36°, 72°, 108° and 144°
  b $\frac{108}{360} = 30\%$

3 a Answers will vary but could include: cost of flooring or building; water flow/usages; salary or wages earned; cost of hiring; speed of wind or athlete; cricket scoring; heart rate for fitness; pressure exerted; exchange rates.
  b Examples could include: litres/km; pressure per square inch (psi); words per minute; gigabytes per second and so on.

4 a $a = 1$, $b = 20$, $c = 12$, $d = 6$ and $e = 3.5$
  b $y = 4x$

5 a $1:10$
  b $1:100$
  c $100:1$
  d $1:1\,000$
  e $1\,000:1$
  f $1:60$

6 a $1:2$
  b $1:8$
  c $3:8$
  d $3:25$
  e $3:200$
  f $1:20$
  g $8:5$
  h $2:15$

7 a Length : width of a screen
  b $19.5:9 = 39:18$; $16:10 = 8:5$; $21:9 = 7:3$
  c 24 cm
  d Students' own investigation and measurements.

Exercise 21.1

1 a $1:1$
  b $1:5$
  c $25:3$
  d $3:10$
  e $3:20$
  f $1:5$
  g $10:4:8 = 5:2:4$

2 a $12:5$
  b $5:12$

3 a $2:3$
  b $3:4$
  c $11:16$
  d $1:2$

4 a $1:12$
  b $1:2$
  c $1:8$
  d $7:6$
  e $10:3$
  f $5:12$

Exercise 21.2

1 a $x = 9$
  b $y = 24$
  c $y = 2$
  d $x = 6$
  e $x = 176$
  f $y = 65$
  g $x = 35$
  h $y = 180$
  i $y = 1\,400$
  j $x = 105$
  k $x = 1.25$
  l $y = 4$

2 a $x = 15$
  b $x = 8$
  c $y = 20$
  d $x = 2.4$
  e $x = 0.6$
  f $y = 3.25$
  g $x = 5.6$
  h $y = 7.2$

3 **a** False: The order matters, so you cannot just reverse the ratios.

b True

c False: $\frac{20}{15} = \frac{4}{3}$ and not $\frac{3}{4}$

d False: 48 is 6 times 8, so the daughter's age must be 6.

e True

4 **a** 1 g

b 1.33 g

c 7 : 5

d 3 : 5

5 **a** 18 : 25 : 5

b 1.67 g

c 4.17 g

6 **a** 20 ml

b 2.5 ml

7 15 750 kg

Exercise 21.3

1 **a** 40 : 160

b 1 200 : 300

c 15 : 35

d 12 : 48

e 150 : 450

f 22 : 16

g 220 : 80

h 230 : 460 : 1 610

2 0.31 = 300 ml

3 Josh gets 27, Ahmed gets 18

4 Annie gets $50, Andrew gets $66.67 and Amina gets $83.33

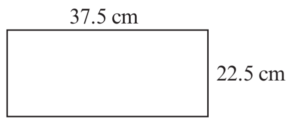
5 Students should draw a 16 cm line with 6 cm marked and 10 cm marked.

6

	N (kg)	P (kg)	K (kg)
a	0.25	0.375	0.375
b	1.25	1.875	1.875
c	5	7.5	7.5
d	6.25	9.375	9.375

7 1.8 m : 2.25 m : 1.35 m

8



9 1200 people

10 **a** $\pi r^2 : 2\pi r$

$= \pi r \times r : \pi r \times 2$

$= r : 2$

b $\frac{4}{3}\pi r^3 : 4\pi r^2$

$= 4\pi r^3 : 12\pi r^2$

$= 4\pi r^2 \times r : 4\pi r^2 \times 3$

$= r : 3$

Exercise 21.4

1

	(i)	(ii)
a	1 : 200	0.005 : 1
b	1 : 250	0.004 : 1
c	1 : 25 000	0.00 004 : 1
d	1 : 200 000	0.000 005 : 1
e	1 : 28.6	0.035 : 1
f	1 : 16 700 000	0.000 000 06 : 1

2 **a** 4 m

b 6 m

c 14 m

d 48 m

3 **a** 0.0012 m = 0.12 cm = 1.2 mm

b 0.0003 km = 300 mm

c 0.0024 km = 2400 mm

d 0.00151 km = 1510 mm

4 **a** Rectangle with dimensions: 100 mm × 250 mm

b Rectangle with dimensions: 80 mm × 200 mm

5 5.5 mm

6 12 : 1

7 0.9625 mm

8 **a** 1740 km

b 1640 km

c 1520 km

9 **a** 0.9 m
b **i** 5.65 m
ii 4.05 m
iii 1.6 m
iv 1.98 m
c **i** 11.34 m^2
ii 9.3312 m^2
iii 8.019 m^2
d 1.44 m^2
e $59.39

Population density

Students may find information online, in social studies textbooks or in an atlas.

1 **a** Greenland is the least densely populated area on Earth with 0.0259 people/km^2, the Sahel area in Africa has a population density of 2 people/km^2. The most densely populated places on Earth are usually small islands. Macau, for example, has 19 737 people/km^2 and Singapore has 8000 people/km^2.

b Small areas with lots of people will automatically have a high population density. Generally, though, high population densities are linked to 'pull factors' that attract people to live in a particular area.

c Population density is a simple relative measure that doesn't tell you how spread out people are in the area mentioned, so social scientists consider distribution patterns as well. For example, Nepal has a population density of 203 people per km^2, but Nepal is in the Himalaya, so people are not evenly spread throughout the country. In fact, the capital Kathmandu has a population density of 19 726 people per km^2.

2 **a** Examples will vary. A healthy coral reef would be densely populated with marine animals, but if the coral bleached and died or there was a tsunami, the animals that lived on the reef would move away.

b Not generally in the modern world as people tend to be limited in their choices of moving or not. But worldwide, there is a trend towards urbanisation as people move to cities because they think the conditions there will be good for them. Similarly, there are examples where conditions become terrible, such as in the event of a natural disaster or war when people will move away and population density may be reduced.

3 Answers will vary.

a Ecologists study how increasing or decreasing population density impacts biodiversity and use of resources.

b Epidemiologists may study how quickly infectious diseases spread in areas of high population density.

c Planners may use population density to make decisions about energy use and supply, fibre and data connections needed and transport links.

Exercise 21.5

1 **a** 2.4 kg/$
b 0.12 l/km
c $105/night
d 0.25 km/min
e 27 students/teacher
f 3 hours/hole dug

2 **a** 9600 t
b 48 000 t

3 **a** 120 l
b 840 l

4 7.4 minutes

5 12.75 km

6 **a** 805 km
b 76.67 km

7 **a** 3000 km
b 312.5 km

8 **a** 2 hours 40 minutes
b 2 hours 30 minutes
c 4 hours 26 minutes 40 seconds
d 1 hour 40 minutes
e 34 days 17 hours 20 minutes

9 110 km/h

10 18.7 km/h

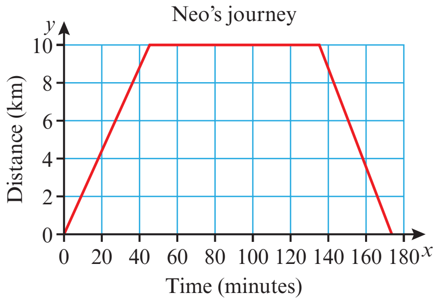
11 **a** 37.578 km/h
b 40.236 s

Exercise 21.6

1 a 700 m
  b 7 min
  c 09 07 and 09 21
  d Going to the library

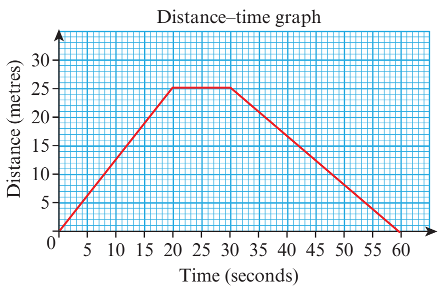
2 a 45 min
  b 17 54
  c 17 15

3 a



  b 18 minutes
  c 16 minutes

4 a



  b 15 m
  c 5 m

Exercise 21.7

1 a and b

Answers will vary, examples:

i The object is moving in the direction of y at a constant speed. Example: a helium-filled children's balloon released in a large hall (with no breeze).

ii The object is stationary. Example: a parked car.

iii The object is moving in the direction of y at a constant speed, then suddenly changes direction, moving at a much faster speed. Example: a squash ball travelling towards the court wall, hitting it then bouncing back.

iv The object is moving very quickly in the direction of y at a constant speed, then stops and is stationary for a while, then continues in the same direction at the same speed as before, then stops and is stationary again. Example: a train travelling from Valladolid to Madrid, stopping at Segovia on the way.

v The object travels slowly at first, then very quickly, then slowly again in the direction of y. Example: an Olympic runner doing interval training.

vi The object is moving at a constant speed in the opposite direction to y, then it suddenly changes direction and travels at a slightly faster speed in the direction of y.

2 a 6 min
  b 10 km/h
  c 3 min
  d 3.33 m/s

3 a For the first 50 minutes the taxi travelled a distance of 10 km at 12 km/h, then was stationary for 50 minutes, then took 20 minutes to return to starting point at 30 km/h. The taxi was then stationary for 40 minutes, then travelled 5 km in 40 minutes at a speed of 7.5 km/h and was then stationary for 40 minutes.
  b 130 minutes – the graph is horizontal.
  c 25 km
  d i 12 km/h
    ii 10 km/h
    iii 6 km/h
    iv 6.25 km/h

4 a 1500 m
  b 2 m/s
  c He was stationary.
  d 0.5 m/s

5 a Other questions are possible, these are just examples:

What is the total time taken to attain a height of 16 m?

When was the helicopter descending?

When was the helicopter ascending?

During what time period was the vertical speed the greatest?

At what speed was the helicopter travelling between 2 and 4 seconds?

Exercise 21.8

1 a i 0 km/h
ii 3 km/h
iii 8 km/h
iv 2 km/h
b From 10 00 until 10 20 the speed remains constant at 8 km/h.
From 10 20 to 10 30 the speed drops uniformly to 2 km/h.
c i 0.375 km
ii 0.833 km
iii 7.58 km
d 3.64 km/h

2 a 2 m/s^2
b 35 m
c 3.5 m/s

3 a 1 m/s^2
b 100 m
c 15 m/s

Exercise 21.9

1 c, d, e, f, i

2 a $a = 6, b = 15$
b $y = \frac{6}{5}x$

Exercise 21.10

1 $6.75

2 60 min

3 70 s

4 172.5 kg

5 10.5 km

6 a 320 g flour, 64 g sultanas, 80 g margarine, 99 ml milk, 32 g sugar, 16 g salt
b 4 : 1

7 250 g

8 a 550 km
b 27 litres

9 a 13 ft
b i 4 m
ii 6.5 m
c i 30 ft
ii 6.59 m
d 6.49 m

Exercise 21.11

1 a i 100
ii 25
iii 8
b 250 cm

2

Number of people	120	150	200	300	400
Days the water will last	40	32	24	16	12

3 a 8 days
b 2 days

4 722.86 km/h

5 3 h 36 min

Exercise 21.12

1 a 1.5
b 15
c 8

2 p and q are not inversely proportional because $p \times q$ is not constant.

3 a $y = \frac{4.5}{x}$
b $y = \frac{62.5}{x}$
c $y = \frac{2}{x}$
d $y = \frac{0.28}{x}$
e $y = \frac{4.8}{x}$

4 a $k = 5120$
b $y = 10$
c $y = 23.70$
d $x = 5.98$

5

x	0.1	0.25	0.5	0.0625
y	25	4	1	64

6

x	25	100	3.70	1
y	10	5	26	50

7 **a** 2.5
b 1000
c 0.125

8 **a** $F = \frac{864}{p^3}$
b $p = 4$
c $F = 32$

9 400

10 6.4

11 60

12 **a** False
b False
c True

13 5 h

14 16 666. 7N (16.7 kN to 3 s.f.)

15 **a** 2 °C
b As temperature varies inversely it will never reach −1 °C.

16 **a** 40
b 6

17 25%

Practice questions

1 Raja receives $40

2 300 cm = 3 m

3 **a** 1.6 kg raisins
b 1.2 kg dates

4 9 cups

5 **a** Bahram. He has $\frac{6}{15} = \frac{2}{5}$ of the set at the start and $\frac{10}{25} = \frac{2}{5}$ of the set at the end.
b 8% increase
c The number of cards must be divisible by both 4 + 6 + 5 = 15 and 6 + 10 + 9 = 25. The LCM of 15 and 25 is 3 × 5 × 5 = 75.

6 **a** 1.31×10^{-6} metres.
b 1.31 micrometres.

7 **a** 90 km/h
b 1080 km/h^2
c 15 km
d $2\frac{1}{2}$ min
e 18 km/h
f 17.5 km

8 **a** $d = \frac{20}{9}t^2$
b 35.6 m
c 1.25 s

9 **a** F, because it is a straight line through the origin.
b B, because the y-coordinate of every point on the line is the same.
c E.
d A. y decreases as x increases and it is a curve. The answer is not C because this graph shows a straight line.
e Also A. y decreases as x increases and it is a curve. The answer is not C because this graph shows a straight line.
f C could be e.g. $y = 4 - 3x$. It will be an equation in the form $y = mx + c$, where c is positive and m is negative.
D could be $y = x^2 + 3$ or $y = x^n + c$ where c is positive and $m > 1$.

10 **a** $x = 0.06$ m
b $x = 0.72$ m
c $m = 9$ kg
d $E = \frac{Px^2}{h^2}$
e $m = 116.7h$ kg

11 **a** $R = \frac{1.536}{d^2}$
b $R = 0.0423$
c $d = \sqrt{\frac{1.536}{R}}$
d 226 mm
e 1.15 ohms

Practice questions worked solutions

1 7 + 5 = 12
$\frac{\$96}{12} = \$8$
Raja receives 5 × 8 = $40

2 25 × 12 = 300 cm
= 3 m

3 $4 + 5 + 3 = 12$

$\frac{4.8}{12} = 0.4$

a $4 \times 0.4 = 1.6\,\text{kg}$

b $3 \times 0.4 = 1.2\,\text{kg}$

4 $\times 3 \left(\begin{matrix} 3 : 4 \\ 9 : 12 \end{matrix} \right) \times 3$

9 cups wholemeal

5 **a** Initially distribution in fractions:

$\frac{4}{15} : \frac{6}{15} : \frac{5}{15}$ i.e. $\frac{4}{15} : \frac{2}{5} : \frac{1}{3}$

Final distribution in fractions:

$\frac{6}{25} : \frac{10}{25} : \frac{9}{25}$ i.e. $\frac{6}{25} : \frac{2}{5} : \frac{9}{25}$

Bahram has the same fraction in both.

b $\dfrac{\frac{9}{25}}{\frac{1}{3}} = 1.08$, i.e. 8% increase

c Must be divisible by 15 and 25.

LCM of 15 and 25 = 75 cards.

6 **a** $\frac{118}{9 \times 10^4} = 0.00131\,\text{mm}$

$= 0.000\,001\,31\,\text{m}$

$= 1.31 \times 10^{-6}\,\text{m}$

d $1.31 \times 10^{-6} \times 10^{6} = 1.31$ micrometres

7 **a** $1.5\,\text{km/minute} = 1.5 \times 60$

$= 90\,\text{km/h}$

b $\frac{1.5}{5} = 0.3\,\text{km/minute/minute}$

c $\left(\frac{15 + 5}{2}\right) \times 1.5 = 15\,\text{km}$

d $2\frac{1}{2}$ minutes

e $\frac{4}{5} \times 0.5 = 0.4\,\text{km/minute}$

f $\left(\frac{7.5 + 2.5}{2}\right) \times 0.5 + 15 = 2.5\,\text{km} + 15\,\text{km}$

$= 17.5\,\text{km}$

8 **a** $d \propto t^2$

$d = kt^2$

$20 = k \times 9$

$k = \frac{20}{9}$

$\Rightarrow d = \frac{20}{9}t^2$

b $d = \frac{20}{9} \times 4^2 = \frac{20 \times 16}{9} = \frac{320}{9}\,\text{m}$

c $\frac{20}{9}t^2 = 3.5 \Rightarrow t = \sqrt{\frac{9 \times 3.5}{20}} = 1.25$ seconds

9 **a** F Straight line through (0, 0).

b B y-coordinate the same for every point.

c E $y \propto x^2 \Rightarrow y = kx^2$

$x = 0 \Rightarrow y = 0$, so E

d $y \propto \frac{1}{x}$ or $y = \frac{k}{x}$ is a curve.

As x increases, y decreases, so A.

e $y = \frac{k}{x^3}$ Also A.

f C $y = ax + b$, with $a > 0$ and $b < 0$.

D $y = bx^2 + c$, with $b > 0$ and $c > 0$.

10 $x = km$

$30 = k \times 5 \Rightarrow k = 6$

a $x = 6m$

b $x = 12 \times 6 = 72\,\text{cm}$

c $0.54 = 6m$

$\Rightarrow m = \frac{0.54}{6} = 0.09\,\text{kg}$

d $E = kx^2$

$P = kh^2 \Rightarrow k = \frac{P}{h^2}$

So $\Rightarrow E = \frac{Px^2}{h^2}$

e $49P = \frac{Px^2}{h^2} \Rightarrow x^2 = 49h^2$

so $x = 7h$ metres

$6m = 7h$ so $m = \frac{7h}{6}\,\text{kg}$

11 $R = \frac{k}{d^2}$

a $0.096 = \frac{k}{16}$

$k = 16 \times 0.096$

$= 1.536$

so $R = \frac{1.536}{d^2}$

b $R = \frac{1.536}{36} = 0.0427$ ohms

c $d^2 = \frac{1.536}{R} \Rightarrow d = \sqrt{\frac{1.536}{R}}$

d $d = \sqrt{\frac{1.536}{3 \times 10^{-5}}} = 226\,\text{mm}$

e $R = \frac{1.536}{R^2} \Rightarrow R^3 = 1.536$

$R = \sqrt[3]{1.536} = 1.15$ ohms

Chapter 22

Getting started

1 a It means you should keep $\frac{1}{10}$ of your speed + 2 vehicle lengths between you and the vehicle in front.
  b 11 truck lengths
  c 10 km/hr
  d There is a partial variation; the faster you go, the more distance you need between you and the vehicle in front.

2 a Cost of the USB port
  b 1.8 is the cost of 1 metre of cable
  c 12.25 m

3 a $h = 3a - 12$
  b $a = \frac{12 + h}{3}$
  c $h = 10.5$ m
  d 3 m

Exercise 22.1

1 a $4x = 32$
  $x = 8$
  b $12x = 96$
  $x = 8$
  c $x + 12 = 55$
  $x = 43$
  d $x + 13 = 25$
  $x = 12$
  e $x - 6 = 14$
  $x = 20$
  f $9 - x = -5$
  $x = 14$
  g $\frac{x}{7} = 2.5$
  $x = 17.5$
  h $\frac{28}{x} = 4$
  $x = 7$

2 a $y = 3$
  b $y = 12$
  c $y = 46$
  d $y = 70$

3 a $x = 13$
  b $x = 9$
  c $x = 2$
  d $x = 11$

4 a i $(s + 2q)$ cm
   ii $(4p + 3r)$ cm
  b i $3rs$ cm^2
   ii $(4ps + 8pq + 6rq)$ cm^2

5 a $t = \frac{x}{320} + \frac{x}{240} = \frac{7x}{960}$
  b 52.5 minutes

Heart rates

1 Each value compares two different quantities: ml/min, ml/beat, bpm.

2 Students' own heart rates.

3 a Answers will vary. But for a HR of 70, $C = 70 \times 80 = 5600$ ml; The norm for C is between 4 and 7 litres, so check that students get an answer in that range.
  b Students' own answers
  c 85.7 beats per minute

4 As R increases, C increases; as R decreases C decreases.

5 a Inversely proportional; as one value increases the other decreases.
  b Artery: 0.0016
  Venae cavae: 0.000 316
  Arteriole: 1
  c As the diameter decreases the resistance increases. The power of four means that if the diameter is halved the resistance will becomes 16 times as great.
  d When the arteries become narrower the resistance increases, which means that blood flow is reduced and the risk of heart-related health problems increases.

Exercise 22.2

1 Child = 15.5 years and parent = 46.5 years

2 Silvia has 70 marbles; Jess has 350 marbles.

3 Kofi has \$51.25 and Soumik has \$46.25

4 \$250 and \$500

5 9 years

6 Width = 15 cm and length = 22 cm

7 48 km

8 Pam = 12 years and Amira = 24 years

9 6.30 p.m.

10 50 km

Exercise 22.3

1 −8 and −5 or 5 and 8

2 $t = 2$ seconds

3 12

4 4 and 7

5 6 cm

6 8 cm

7 a 12 sides

b n not an integer when the equation is solved

8 a Width of smaller rectangle = $(x - 1)$ cm

Since two rectangles are similar:

$1 : x = (x - 1) : 1$

$x(x - 1) = 1$

$x^2 - x - 1 = 0$

b $x = 1.62$ or $x = -0.62$

c Negative solution can't work as a length must be positive

d Perimeter = 5.24 cm

9 0.836 seconds

10 (−7, −6, −5), (4, 5, 6)

11 7 or −2

12 3 cm by 8 cm

13 1.96 seconds

14 6 or −4

15 2.75 cm

16 7 and 8

Exercise 22.4

1 a $x = m - bp$

b $x = pr - n$

c $x = \frac{m}{4}$

d $x = \sqrt{\frac{c+b}{a}}$

e $x = \frac{d - 2b - c}{m}$

f $x = 3by$

g $x = \frac{p}{m}$

h $x = \frac{np}{m}$

i $x = \frac{mk}{2}$

j $x = \frac{20}{p}$

2 a $x = \frac{m - 3y}{3}$

b $x = \frac{4t - c}{4}$

c $x = \frac{y + 15}{3}$

d $x = \frac{5}{2}$

e $x = \frac{m}{4c} + y$

f $x = 2r - \frac{a}{\pi r}$

3 a $m = \frac{E}{c^2}$

b $R = \frac{100I}{PT}$

c $m = \frac{2k}{v^2}$

d $b = \frac{2A}{h} - a$

e $h = \frac{3V}{A}$

f $h = \frac{3V}{\pi r^2}$

4 a $x = \sqrt{\frac{m}{a}}$

b $x = \sqrt{m + y}$

c $x = \sqrt{n - m}$

d $x = \sqrt{ay}$

e $x = \sqrt{\frac{ac}{b}}$

f $x = \sqrt{a + b^2}$

g $x = \sqrt{\frac{n}{m}}$

h $x = \frac{m^2}{y}$

i $x = \frac{a^2}{5}$

j $x = y^2 + z$

k $x = (y + z)^2$

l $x = \left(\frac{c}{a-b}\right)^2$

m $x = \left(\frac{a-m}{b}\right)^2$

n $x = \frac{y^2+1}{3}$

o $x = \frac{y-a^2}{2}$

p $x = \frac{a^2+by^2}{4y^2}$

5 **a** 38 °C

b 100 °C

c 0 °C

6 **a** 2.11

b 6.18

c 0.40

Exercise 22.5

1 **a** $a = \frac{b-x}{1-x}$

b $a = \frac{L}{B+1+C}$

c $a = \frac{5b}{b-1}$

d $a = \frac{x(y+1)}{y-1}$

e $a = \frac{3-y}{y-1}$

f $a = \sqrt{\frac{2}{m-n}}$

2 $c = \sqrt{\frac{E}{m}}$

3 $a = \sqrt{c^2 - b^2}$

4 **a** $y = \frac{2x}{3} + 2$

b $y = 3x - c$

c $\frac{4x+z}{3}$

d $y = \frac{2(b-a)}{3}$

5 $a = \frac{2y}{1-y}$

6 $r = \sqrt{\frac{2A}{\theta}}$

7 $x = \sqrt{\frac{yz^2}{k}}$

8 **a** $E = 49$

b $v = \sqrt{\frac{2E}{m}}$

9 **a** $V = 2\,010\,619\text{ cm}^3$

b $r = \sqrt{\frac{V}{\pi h}}$

10 **a** $A = 1.13\text{ m}^2$

b $A = 1.13\text{ m}^2$

c $d = \sqrt{\frac{4A}{\pi}}$

11 **a** $\text{SA} = \pi r^2 + 2\pi rh$

b $\text{Sealant} = \frac{\text{SA}}{6} = \frac{1}{6}(\pi r^2 + 2\pi rh)$

c 5.608 litres

Shadow maths

1 It means that as the time of day changes, the length of the shadow (L) changes.

The function for this is $L = \frac{H}{\tan a}$, where a is the angle of the light (usually the Sun)

2 Students may remember how to do this from primary science lessons, but they can find out how to use a shadow stick (basically a vertical ruler) to develop a ratio that they can apply to other objects. They may also discover online calculators that use coordinates, time and shadow length to determine heights.

Exercise 22.6

1

	i f(2) =	ii f(−2) =	iii f(0.5) =	iv f(0) =
a	8	−4	3.5	2
b	8	−12	0.5	−2
c	3	−5	0	−1
d	11	11	3.5	3
e	0	8	−0.75	0
f	6	−10	−1.875	−2

2 **a** −5

b −1

c 5

d −17

3 **a** 0

b −4

c 5

d −3.9375

4 **a** 0
b −9
c −2
d 5

5 **a** 16
b 16
c 1

6 $x = \frac{4}{3}$

7 $x = \frac{1}{3}$

8 **a** $x = 6$
b Domain: $x \geqslant -\frac{1}{4}$
Range: $y \geqslant 0$

9 **a** $x = -2$ or 3
b $x = -6$

10 **a** $2a$
b $2a + 4$
c $8a$
d $8a$

11 **a** 9
b $x = 2$

12 **a** 15
b 3
c 1

Exercise 22.7

1 **a** $fg(x) = x + 3$; $gf(x) = x + 3$
b $fg(x) = 50x^2 - 15x + 1$;
$gf(x) = 10x^2 - 15x + 5$
c $fg(x) = 27x^2 - 48x + 22$;
$gf(x) = 9x^2 - 12x + 4$
d $fg(x) = \frac{4x^2 - 36}{3}$;
$gf(x) = \frac{16x^2}{9} - 9$

2 **a** $-2x$
b −4
c 16
d −2

3 **a** $9x + 4$
b $18x^2 + 1$
c 3456
d 150
e $\frac{726}{25}$

4 **a** 26
b 7
c 26
d 29

5 $gh(4) = 5$, $hg(4) = \frac{4}{5}$

6 **a** $-56 + 16x^2 - x^4$
b $56 - 16x^2 + x^4$
c $-56 + 16x^2 - x^4$
d $56 - 16x^2 + x^4$

7 **a** −25
b $\frac{3}{2}$
c $-\frac{7}{34}$
d $\frac{1}{3}$
e −15

8 **a** $(x^2 + 36)^2$
b $\sqrt{x^8 + 36}$
c 0
d $\sqrt{76}$

9 $hgf(1) = \frac{1}{0}$, which is undefined.

Exercise 22.8

1 **a** $\frac{x}{7}$
b $\sqrt[3]{\frac{1}{7x}}$
c $\sqrt[3]{x}$
d $\frac{x - 3}{4}$
e $2(x - 5)$
f $2x - 2$
g $\frac{x}{3} + 2$
h $\frac{2x - 9}{2}$
i $\frac{4x - 2}{2 + x}$
j $\sqrt[3]{x - 5}$

k $\frac{x^2-8}{3}$

l $\frac{x+1}{x-1}$

2 $g^{-1}(x) = 3(x+44)$

3 **a** **i** $f^{-1}(x) = \frac{x}{5}$

ii $ff^{-1}(x) = x$

iii $f^{-1}f(x) = x$

b **i** $f^{-1}(x) = x - 4$

ii $ff^{-1}(x) = x$

iii $f^{-1}f(x) = x$

c **i** $f^{-1}(x) = \frac{x+7}{2}$

ii $ff^{-1}(x) = x$

iii $f^{-1}f(x) = x$

d **i** $f^{-1}(x) = \sqrt[3]{x-2}$

ii $ff^{-1}(x) = x$

iii $f^{-1}f(x) = x$

e **i** $f^{-1}(x) = \frac{x^2+1}{2}$

ii $ff^{-1}(x) = x$

iii $f^{-1}f(x) = x$

f **i** $f^{-1}(x) = \frac{9}{x}$

ii $ff^{-1}(x) = x$

iii $f^{-1}f(x) = x$

g **i** $f^{-1}(x) = \sqrt[3]{x+1}$

ii $ff^{-1}(x) = x$

iii $f^{-1}f(x) = x$

4 **a** $f^{-1}(x) = g(x)$

b $f^{-1}(x) = g(x)$

c $f^{-1}(x) \neq g(x)$

d $f^{-1}(x) = g(x)$

5 **a** 8

b 20

c 11

6 **a** −10

b $\frac{5x+2}{20}$

c $x = 1.54$

d **i** $-56\frac{2}{5}$

ii 3

iii $-7\frac{4}{5}$

Practice questions

1 \$2

2 16 5c coins and 34 10c coins

3 **a** $12T$

b $\frac{11}{6}$ hours

c $\text{Time} = \frac{11}{6} - T$

$\text{Distance} = 48\left(\frac{11}{6} - T\right)$

d $12T + 48\left(\frac{11}{6} - T\right) = 64$

$T = \frac{2}{3} = 40$ minutes

Lana cycled 8 km

4 9 cm × 13 cm

5 16

6 **a** $\frac{30}{x} - 2 = \frac{30}{x+4}$

$30(x+4) - 2x(x+4) = 30x$

$30x + 120 - 2x^2 - 8x = 30x$

$2x^2 + 8x - 120 = 0$

$x^2 + 4x - 60 = 0$

b $2x + 4 = 16$

7 84 cm

8 $x = \frac{1}{\sqrt{y}} - 1$

9 $x = \frac{z^2(y+1)^2}{(y-1)^2}$

10 **a** False

b True

c True

d False

11 **a** 14

b $x = 1.26$ or -0.26

c $x = 1.76$ or -0.76

d $x = 1$

e $\frac{4-x}{3}$

f $63x^2 - 99x - 32$

g $x = \frac{25}{9}$

h $18x^4 - 36x^3 - 39x^2 + 57x + 40$

12 a $$\mathrm{ff}(x) = \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = \frac{\frac{1+x-1+x}{1+x}}{\frac{1+x+1-x}{1+x}} = \frac{2x}{2} = x$$

b $\mathrm{f}^{-1}(x) = \dfrac{1-x}{1+x}$

c $\mathrm{ffff}(x) = \dfrac{1-x}{1+x}$

13 a 7

b $\dfrac{3-x}{4}$

c 4

14 $-\dfrac{3}{4}$

Practice questions worked solutions

1 $6x + 3(x + 2) = 24$

$6x + 3x + 6 = 24$

$9x = 18$

$x = 2$

White paint costs \$2 per litre.

2 Let x = number of 5c coins.

Let y = number of 10c coins.

$x + y = 50 \quad \Rightarrow \quad 5x + 5y = 250 \quad ①$

$5x + 10y = 420 \quad \Rightarrow \quad \underline{5x + 10y = 420} \quad ②$

② − ① $\quad 5y = 170$

$y = 34$

$x = 16$

3 a Distance = speed × time

$= 12T$

b $1\frac{5}{6}$ hours $= \frac{11}{6}$ hours

c Remaining distance $= 64 - 12T$

Time taken $= \frac{11}{6} - T$

d $48 = \dfrac{64 - 12T}{\frac{11}{6} - T}$

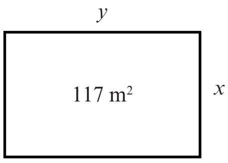
so $88 - 48T = 64 - 12T$

$36T = 24$

$T = \frac{24}{36} = \frac{2}{3} = 40$ minutes

Distance to station $= 12 \times \frac{2}{3} = 8$ km

4



$xy = 13 \times 9$

$2(x + y) = 44 \Rightarrow x + y = 22$

$x = 13$ and $y = 9$

Rectangle is 13 m by 9 m

5 $n^2 + (n + 1)^2 = 545$

$n^2 + n^2 + 2n + 1 = 545$

$2n^2 + 2n - 544 = 0$

$n^2 + n - 272 = 0$

$(n + 17)(n - 16) = 0 \Rightarrow n = 16$

6 $\dfrac{30}{x+4} = \dfrac{30}{x} - 2$

a $\Rightarrow 30x = 30(x + 4) - 2x(x + 4)$

$\Rightarrow 30x = 30x + 120 - 2x^2 - 8x$

$2x^2 + 8x - 120 = 0$

$x^2 + 4x - 60 = 0$

b $(x + 10)(x - 6) = 0 \Rightarrow x = 6$ or $x = -10$

so, $x = 6$ because $x > 0$

Therefore, total $= 6 + 6 + 4$

$= 16$

7 $(2x - 1)(x - 1) = \frac{1}{2}(x + 1)(3x + 2)$

$2x^2 - 3x + 1 = \frac{1}{2}(3x^2 + 5x + 2)$

$4x^2 - 6x + 1 = 3x^2 + 5x + 2$

$x^2 - 11x - 1 = 0$

$$x = \frac{11 \pm \sqrt{121 - 4 \times 1 \times -1}}{2} = \frac{11 \pm \sqrt{125}}{2}$$

But $\dfrac{11 - \sqrt{125}}{2} < 0 \Rightarrow x - 1$ would be negative

So, $x = \dfrac{11 + \sqrt{125}}{2}$

Perimeter $= x + 1 + 3x + 2 + \sqrt{(x + 1)^2 + (3x + 2)^2}$

$= 84.6$ cm

8 $\sqrt{x} + \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}\sqrt{y}}$

$(\sqrt{x})^2 + 1 = \frac{1}{\sqrt{y}}$

$x + 1 = \frac{1}{\sqrt{y}}$

$x = \frac{1}{\sqrt{y}} - 1$

9 $y = \frac{\sqrt{x} + z}{\sqrt{x} - z}$

$(\sqrt{x} - z)y = \sqrt{x} + z$

$y\sqrt{x} - yz = \sqrt{x} + z$

$y\sqrt{x} - \sqrt{x} = z + yz$

$\sqrt{x}(y - 1) = z(1 + y)$

$\sqrt{x} = \frac{z(1 + y)}{y - 1}$

$x = \frac{z^2(1 + y)^2}{(y - 1)^2}$

10 a $fg(x) = (5 - x) - 5$
$= -x \neq x$

So, $f^{-1}(x) \neq g(x)$

b $g(5 - x) = 5 - (5 - x)$
$= 5 - 5 + x$
$= x$

$g^{-1}(x) \to 5 - x$

c $fg(x) = -x$

d $gf(x) = 5 - (x - 5)$
$= 10 - x \neq fg(x)$

11 a $f(-2) = 3(-2)^2 - 3(-2) - 4$
$= 12 + 6 - 4$
$= 14$

b $3x^2 - 3x - 4 = -3$

$3x^2 - 3x - 3 = 0$

$x = \frac{3 \pm \sqrt{9 - 4 \times 3 \times -1}}{6}$

$= \frac{3 \pm \sqrt{21}}{6}$

$x = -0.264$ or 1.264

c $3x^2 - 3x - 4 = 0$

$x = \frac{3 \pm \sqrt{9 - 4 \times 3 \times -4}}{6}$

$= \frac{3 \pm \sqrt{57}}{6}$

$x = -\,0.758$ or 1.758

d $g(x) = 2g(x) - 1$

$g(x) = 1$

$4 - 3x = 1$

$3x = 3$

$x = 1$

e $y = 4 - 3x$

$3x = 4 - y$

$x = \frac{4 - y}{3}$

$x \leftrightarrow y$

So $f^{-1}(x) = \frac{4 - x}{3}$

f $3(4 - 3x)^2 - 3(4 - 3x) - 4 - 4\{4 - 3(3x^2 - 3x - 4)\}$

$= 3(16 - 24x + 9x^2) - 12 + 9x - 4 - 16 + 36x^2 - 36x - 48$

$= 63x^2 - 99x - 32$

g $4 - 3(4 - 3x) = 17$

$4 - 12 + 9x = 17$

$9x = 25$

$x = \frac{25}{9}$

h $3(3x^2 - 3x - 4)^2 - 3(3x^2 - 3x - 4) - 4 - (3x^2 - 3x - 4)^2$

$= 2(3x^2 - 3x - 4)^2 - 9x^2 + 9x + 12 - 4$

$= 2(9x^4 + 9x^2 + 16 - 18x^3 - 24x^2 + 24x) - 9x^2 + 9x + 12 - 4$

$= 18x^4 - 36x^3 - 39x^2 + 57x + 40$

12 a $ff(x) = \frac{1 - \frac{1 - x}{1 + x}}{1 + \frac{1 - x}{1 + x}} = \frac{1 + x - 1 + x}{1 + x + 1 - x} = \frac{2x}{2} = x$

b $f^{-1}(x) = \frac{1 - x}{1 + x}$

c $fffff(x) = f(x) = \frac{1 - x}{1 + x}$ because $ff(x) = x$

13 $f(x) = 3 - 4x$

a $f(-1) = 3 - 4(-1) = 7$

b $y = 3 - 4x$

$4x = 3 - y$

$x = \frac{3 - y}{4}$

$x \leftrightarrow y$

So, $f^{-1}(x) = \frac{3 - y}{4}$

c $ff^{-1}(4) = 4$

14 $\dfrac{5}{2x-1} = -2$

$5 = -4x + 2$

$4x = -3$

$x = -\dfrac{3}{4}$

Chapter 23

Getting started

1 **a** A – flipped (reflected) across the y-axis
B – rotated 90° clockwise about the origin
C – moved right and down; D – enlarged

b A – reflection
B – rotation
C – translation
D – enlargement

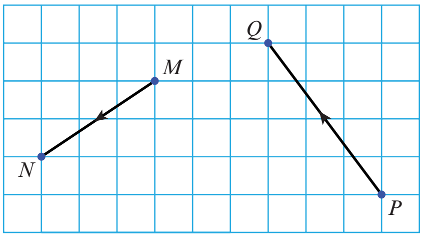
c **i** A, B and C
ii D

2 **a**

	Direction of movement	
	x	y
If number is positive	right	up
If number is negative	left	down

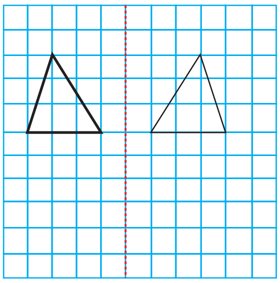
b $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

c

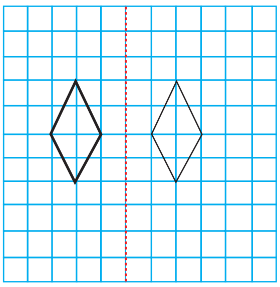


Exercise 23.1

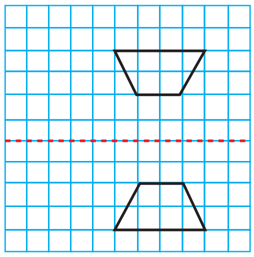
1 **a**



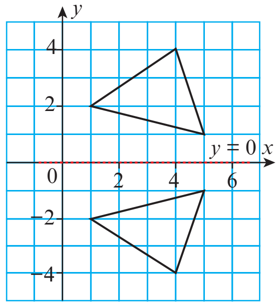
b



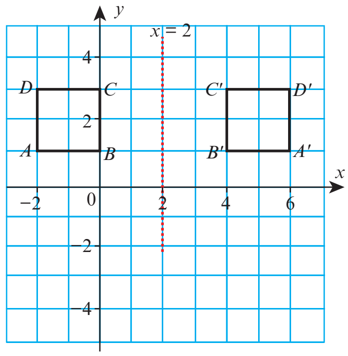
c



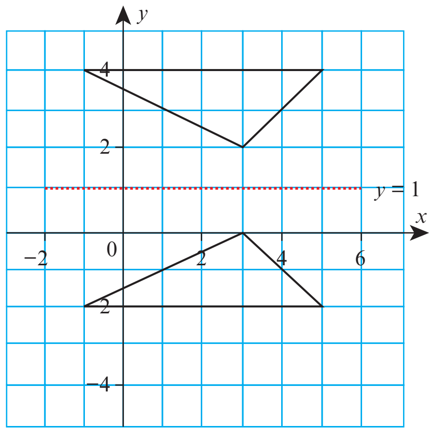
2 **a**



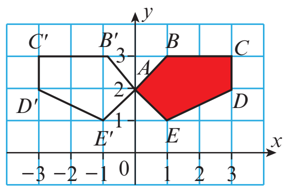
b



c



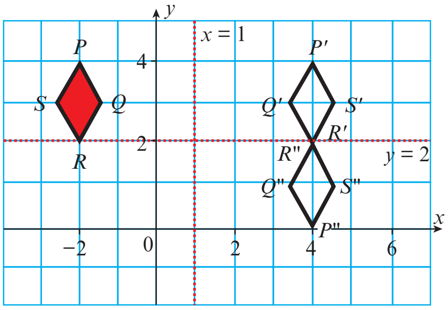
3 **a**



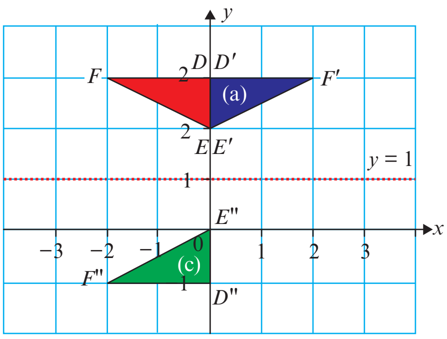
b $B' = (-1, 3)$

c A is invariant – A and A' are the same point.

4 **a** and **b**



5 **a** and **c**

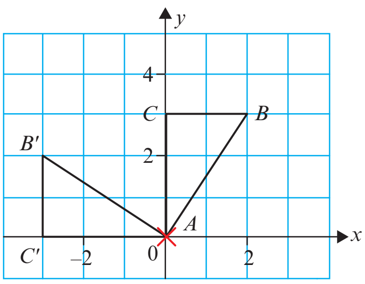


b F is at $(-2, 3)$

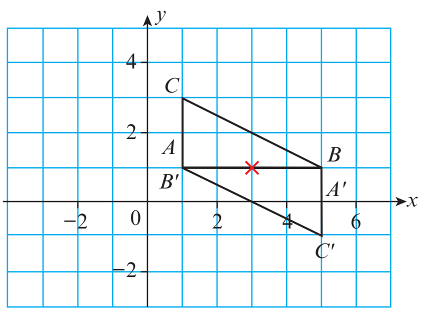
F′ is at $(2, 3)$

Exercise 23.2

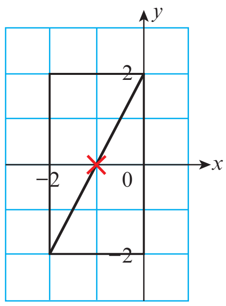
1 **a**



b



c



2 **a** 90° clockwise about $(-6, 2.5)$

b 180° about $(3.5, 2)$

c 90° clockwise about $(4, 0)$

d 180° about $(0, 0)$

e 90° clockwise about $(-4, -1)$

3 **a** Centre of rotation A; angle of rotation 90° clockwise

b Centre of rotation point on line AC; angle of rotation 180°

c Centre of rotation point on line AC; angle of rotation 90° clockwise

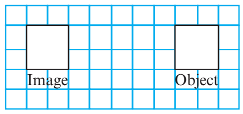
4 **a** No

b No

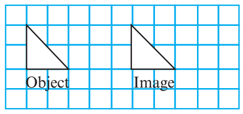
c Yes

Exercise 23.3

1 **a**



b

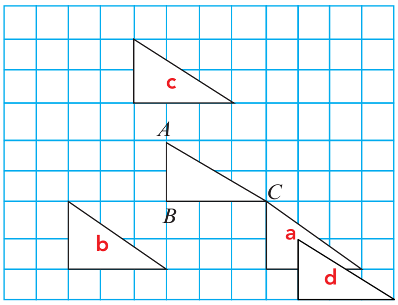


2 **a** $A \rightarrow B\begin{pmatrix}-6\\0\end{pmatrix}$ $\quad A \rightarrow C\begin{pmatrix}3\\6\end{pmatrix}$

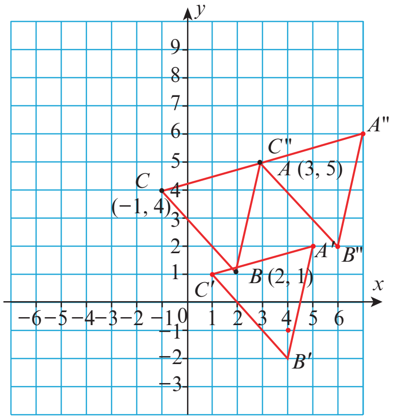
b $A \rightarrow B\begin{pmatrix}0\\-7\end{pmatrix}$ $\quad A \rightarrow C\begin{pmatrix}-6\\1\end{pmatrix}$

c $A \rightarrow B\begin{pmatrix}0\\5\end{pmatrix}$ $\quad A \rightarrow C\begin{pmatrix}6\\-3\end{pmatrix}$

3

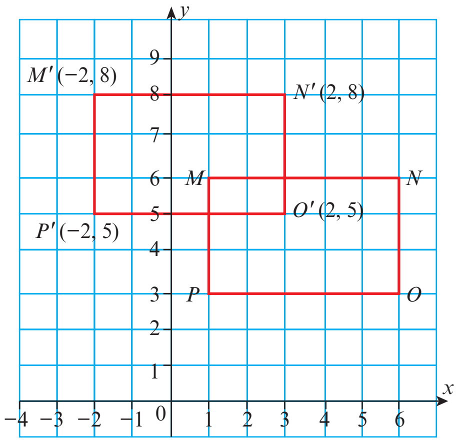


4



5 X' (7, −1)
Y' (6, 4)
Z' (3, −7)

6 **a** and **b**



Exercise 23.4

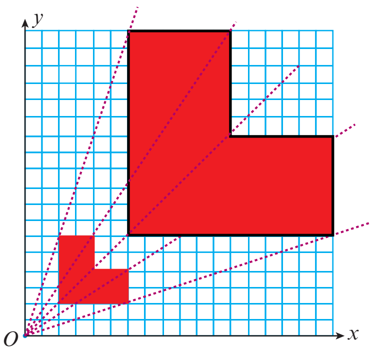
1 **a** Scale factor 2; centre of enlargement = (8, 0)

b Scale factor 2; centre of enlargement = (3, −2)

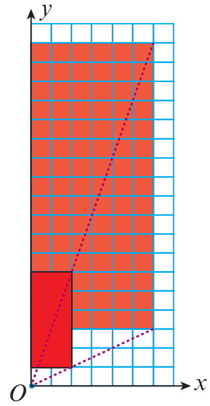
c Scale factor 2; centre of enlargement (−3, 4)

d Scale factor $\frac{1}{2}$; centre of enlargement (0, 0)

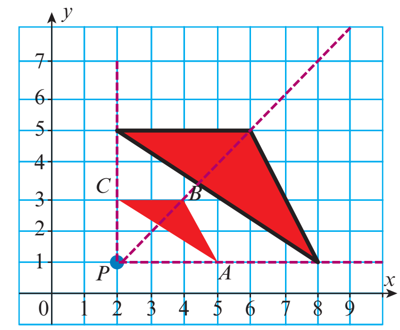
2 **a**



b



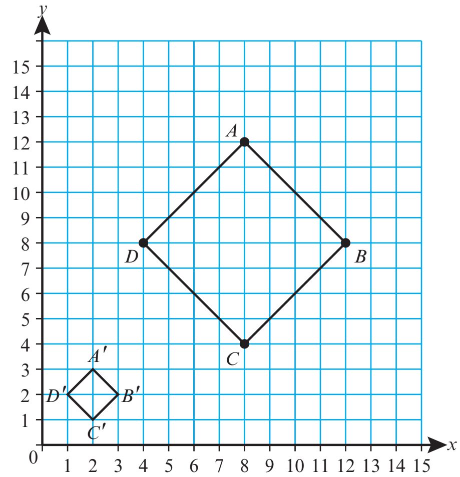
3



4 Scale factor $\frac{1}{2}$; centre of enlargement (0, −1)

5 Scale factor 1.5; centre of enlargement (4, 2)

6



7 **a** 9.6 cm wide

b Length will be tripled.

c No; the image will not be in proportion.

d 2.5 cm long and 1.5 cm wide

8 **a** Scale factor is 0.75

b 1.78 times smaller

9 **a** Answers will vary depending on how students view the shapes. There are translations – the large coloured stars are moved to the right (or left) along a line. There are enlargements of both the star shapes and the frames. There are reflections – for example, the large coloured stars could be reflections of each other. The smaller shapes containing the blue, orange, green and turquoise stars bottom centre of the design could also be rotations around the centre of the space in the middle.

b Answers will vary, but could include translating the larger shapes with the coloured stars down to fit the spaces in the bottom row, or reflecting that entire row so it fits into the open space.

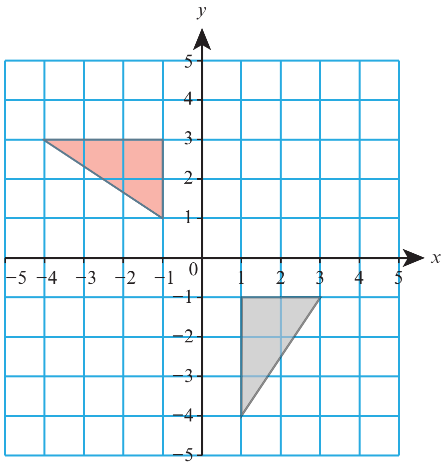
Exercise 23.5

1 **a** $y = -x$

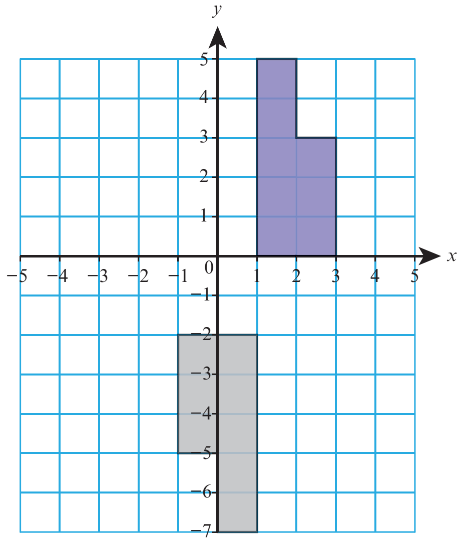
b $y = x - 1$

c $y = 2 - x$

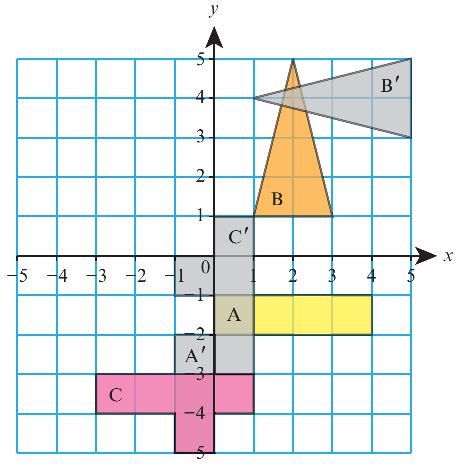
2 **a**



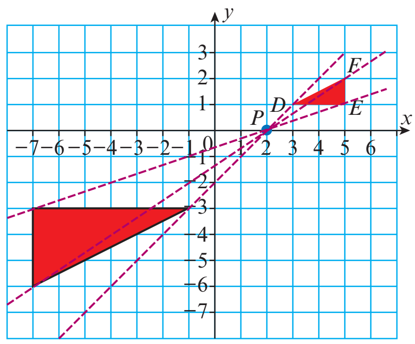
b



3 **a–c**



4



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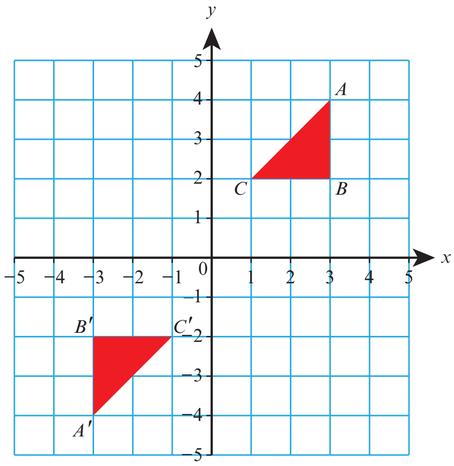

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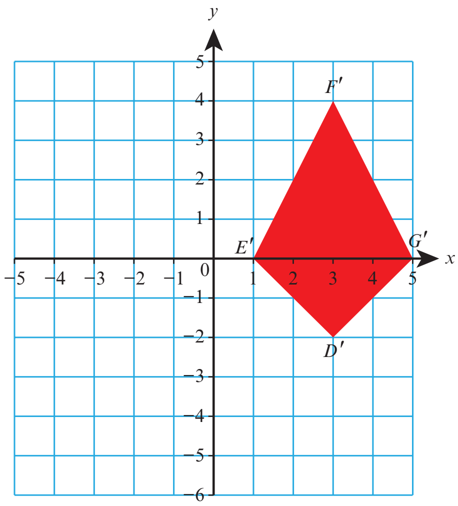

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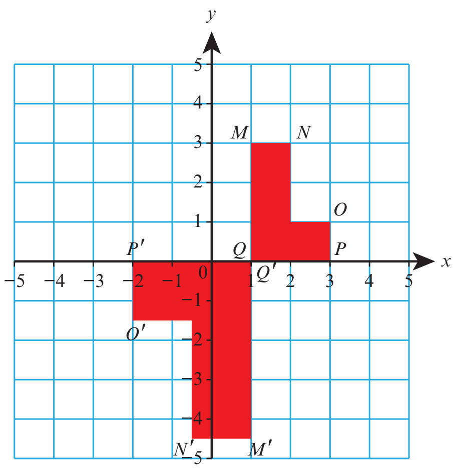

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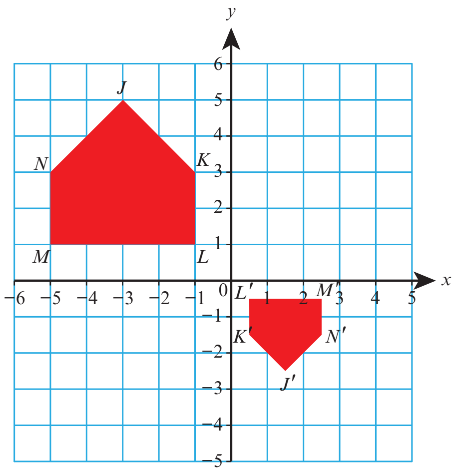
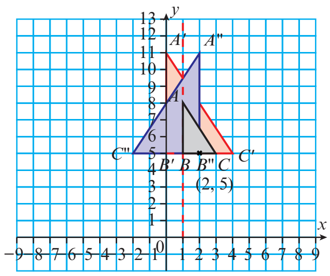


9



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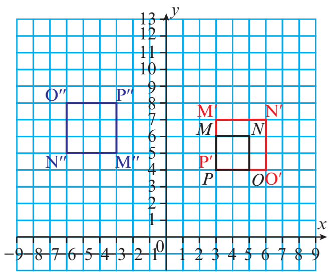


Exercise 23.6

1 **a** $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

b $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

c $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

d $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

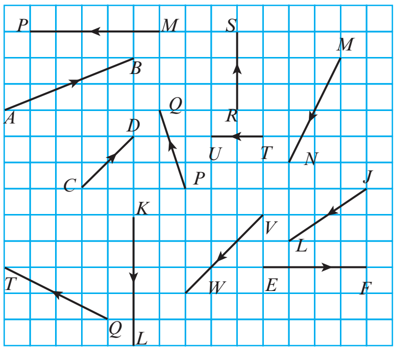
e $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$

f $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

g $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$

h $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

2



3 **a** **i** $\overrightarrow{AB} = \begin{pmatrix}4\\0\end{pmatrix}$ $\overrightarrow{DC} = \begin{pmatrix}4\\0\end{pmatrix}$

ii $\overrightarrow{BC} = \begin{pmatrix}1\\3\end{pmatrix}$ $\overrightarrow{AD} = \begin{pmatrix}1\\3\end{pmatrix}$

b They are equal.

4 **a** $\begin{pmatrix}4\\2\end{pmatrix}$

b $\begin{pmatrix}5\\-1\end{pmatrix}$

c $\begin{pmatrix}6\\-1\end{pmatrix}$

d $\begin{pmatrix}0\\-3\end{pmatrix}$

e $\begin{pmatrix}-4\\3\end{pmatrix}$

f $\begin{pmatrix}5\\2\end{pmatrix}$

Exercise 23.7

1 **a** $\begin{pmatrix}9\\-21\end{pmatrix}$

b $\begin{pmatrix}\frac{3}{2}\\-\frac{7}{2}\end{pmatrix}$

c $\begin{pmatrix}-6\\14\end{pmatrix}$

d $\begin{pmatrix}-3\\7\end{pmatrix}$

e $\begin{pmatrix}-\frac{9}{4}\\\frac{21}{4}\end{pmatrix}$

f $\begin{pmatrix}4.5\\-10.5\end{pmatrix}$

2 **a** $\overrightarrow{DF} = 2\overrightarrow{JK}$

b $\overrightarrow{JQ} = \frac{1}{4}\overrightarrow{JF}$

c $\overrightarrow{HP} = \frac{1}{2}\overrightarrow{HF}$

d $2\overrightarrow{GO} = \frac{1}{2}\overrightarrow{GC}$

e $3\overrightarrow{DG} = 1\overrightarrow{CL}$

f $6\overrightarrow{BE} = 2\overrightarrow{CL}$

3 **a** $\begin{pmatrix}2\\8\end{pmatrix}$

b $\begin{pmatrix}9\\21\end{pmatrix}$

c $\begin{pmatrix}4.5\\10.5\end{pmatrix}$

d $\begin{pmatrix}0.75\\3\end{pmatrix}$

e $\begin{pmatrix}1.5\\6\end{pmatrix}$

f $\begin{pmatrix}-36\\-84\end{pmatrix}$

g $\begin{pmatrix}1.5\\6\end{pmatrix}$

h $\begin{pmatrix}-\frac{5}{3}\\-\frac{35}{9}\end{pmatrix}$

Exercise 23.8

1 **a** $\begin{pmatrix}12\\-6\end{pmatrix}$

b $\begin{pmatrix}3\\-5\end{pmatrix}$

2 $\begin{pmatrix}12\\-7\end{pmatrix}$

3 **a** $\begin{pmatrix}12\\8\end{pmatrix}$

b $\begin{pmatrix}8\\24\end{pmatrix}$

c $\begin{pmatrix}-4\\-12\end{pmatrix}$

d $\begin{pmatrix}2\\0\end{pmatrix}$

e $\begin{pmatrix}0\\12\end{pmatrix}$

f $\begin{pmatrix}16\\21\end{pmatrix}$

g $\begin{pmatrix}10\\9\end{pmatrix}$

h $\begin{pmatrix}-2\\-7\end{pmatrix}$

4 **a** $2\mathbf{q} - 2\mathbf{p}$

b $2\mathbf{p} + \mathbf{q}$

c $\mathbf{p} - \mathbf{q}$

5 a $\mathbf{x} + \mathbf{y}$

b $\frac{3}{4}(\mathbf{x} + \mathbf{y})$

c $-\frac{1}{4}\mathbf{x} + \frac{3}{4}\mathbf{y}$

6 a $2\mathbf{a} + 3\mathbf{b}$

b $\mathbf{a} + \frac{3\mathbf{b}}{2}$

c $\mathbf{b}$

d $\mathbf{a} + \frac{\mathbf{b}}{2}$

Exercise 23.9

1 a 4.12

b 3.61

c 4.24

d 5

e 4.47

f 5

g 5.83

2 a 10.30

b 13.04

c 5

d 10

3 a 5

b 13

c 17

4 a $A(4, 2)$, $B(-1, 3)$, $C(6,-2)$

b $\overrightarrow{AB} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$, $\overrightarrow{CB} = \begin{pmatrix} -7 \\ 5 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

5 a 10

b 8.60

6 100 km/h

7 6.71 km/h (3 s.f.)

8 a $\mathbf{b} - \mathbf{a}$

b 3

c $\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD}$

So $CD = -2\mathbf{a} + 3\mathbf{b} - \mathbf{a} = 3\mathbf{b} - 3\mathbf{a} = 3\overrightarrow{AB}$

So CD is parallel to AB, so the triangles are similar.

9 a $-\mathbf{p} + \mathbf{q}$

b $\frac{2}{3}(-\mathbf{p} + \mathbf{q})$

c $\frac{2}{3} + \frac{1}{3}$

d $\mathbf{q} + \frac{1}{2}\mathbf{p}$

10 a $\frac{\mathbf{a}}{2}$

b $-\frac{\mathbf{b}}{2}$

c $\frac{\mathbf{a} - \mathbf{b}}{2}$

d $\frac{3\mathbf{a} + 3\mathbf{b}}{4}$

11 (4, 5)

12 a $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

b $\overrightarrow{AM} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$

c $\overrightarrow{ON} = \frac{1}{3}\mathbf{a}$

13 a i $\mathbf{b} - \mathbf{a}$

ii $\frac{1}{3}(\mathbf{b} - \mathbf{a})$

iii $\frac{1}{6}\mathbf{b} - \frac{2}{3}\mathbf{a}$

iv $\frac{1}{6}\mathbf{b} - \frac{2}{3}\mathbf{a}$

b $\overrightarrow{MN} = \overrightarrow{NC}$ (they are both $\frac{1}{6}\mathbf{b} - \frac{2}{3}\mathbf{a}$) and they share a common point N, so the points M, N and C are collinear.

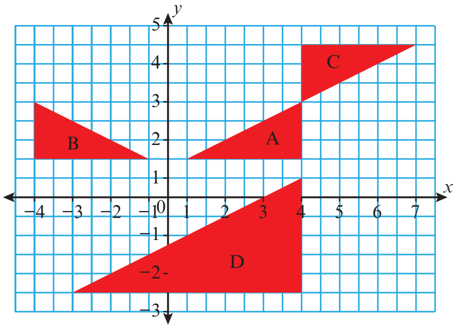
Perpendicular vectors

1 Students can draw any perpendicular vectors to use.

2 If your vectors are $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$ work out ac and bd. You will notice that for all perpendicular pairs $ac = -bd$. You can also write this as $ac + bd = 0$. The quantity $ac + bd$ is known as the scalar product of the two vectors.

Practice questions

1 a



2 A: reflection about $y = 0$ (x-axis)

B: translation $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

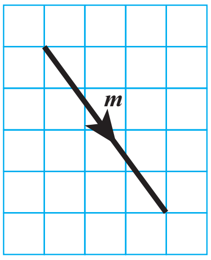
C: enlargement scale factor 2, centre origin

D: rotation 90° anticlockwise about the origin

3 a i $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

ii $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$

b



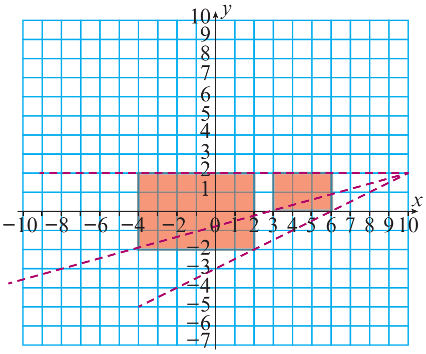
4 a $(-1, 2)$

b Scale factor -2

5 a $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

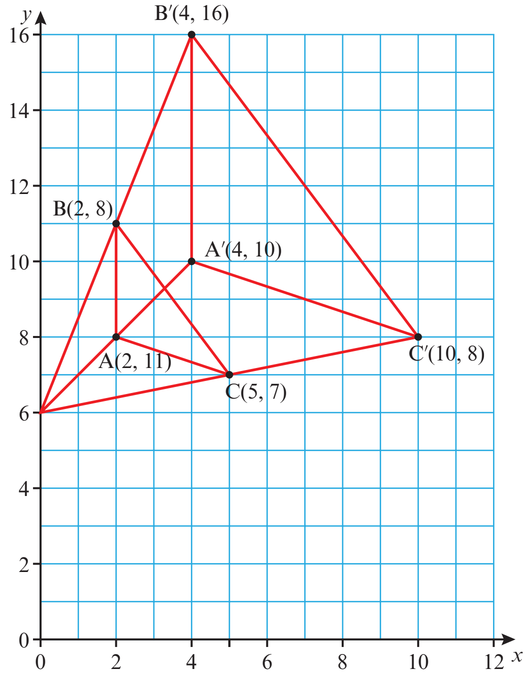
b Rotation 180° about centre (6, 0)

c i



ii 4 : 1

6 a

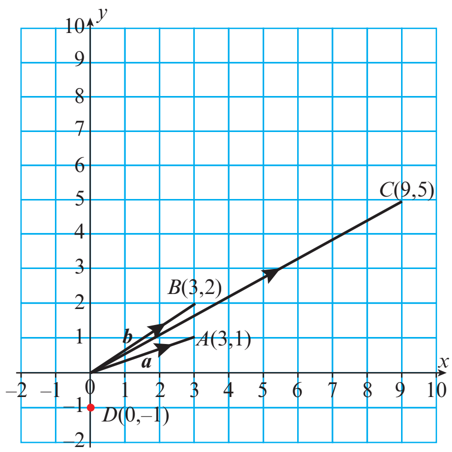


b (0, 6)

c 2

d 4

7 a



b $\mathbf{a} - \mathbf{b}$

c $|\mathbf{a}| = 3.16$

8 **a** **i** Translation $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

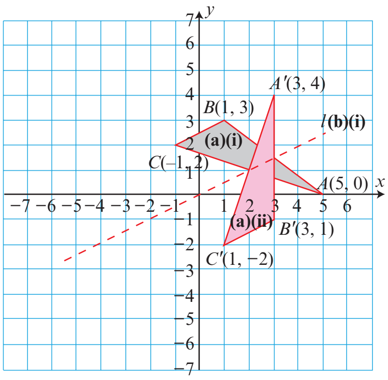
ii Enlargement scale factor 3, centre origin

iii Rotation 90°, centre (2, 1) and translation $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

iv Enlargement scale factor −2, centre (0, 4)

b Shapes B, D

9 **a**



b **ii** $y = \frac{x}{2}$

10 **a** $\mathbf{b} + \mathbf{c}$

b $\mathbf{b} + \mathbf{c} + \mathbf{d}$

c $\overrightarrow{DE} = 2\mathbf{b}$ and $\overrightarrow{EC} = -(\mathbf{b} + \mathbf{c})$

So, following the path from B to A to D to E to C,

$\mathbf{c} = -\mathbf{b} + (\mathbf{b} + \mathbf{c} + \mathbf{d}) + 2\mathbf{b} - (\mathbf{b} + \mathbf{c})$

$\mathbf{c} = \mathbf{d} + \mathbf{b}$

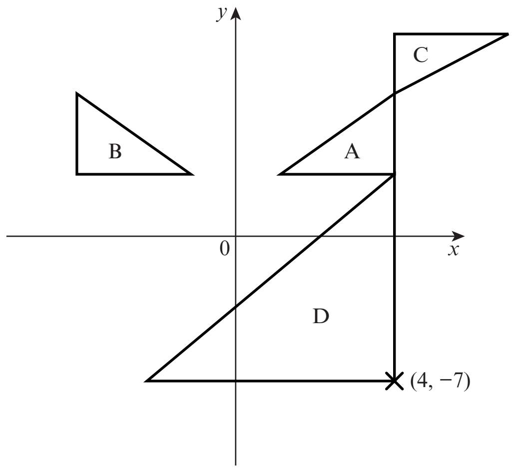
d $\overrightarrow{AD} = \mathbf{b} + \mathbf{c} + \mathbf{d}$

$= \mathbf{c} + \mathbf{c} = 2\mathbf{c}$

So AD is parallel to BC and the quadrilateral is a trapezium.

Practice questions worked solutions

1



2 A: Reflection in x-axis

B: Translation with column vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

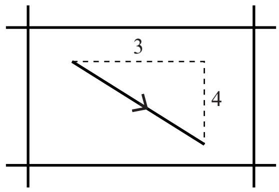
C: Enlargement, scale factor 2, centre (0, 0)

D: Rotation 90° anticlockwise about (0, 0)

3 **a** **i** $\mathbf{m} + \mathbf{n} = \begin{pmatrix} 3 - 2 \\ -4 + 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

ii $3\mathbf{n} = \begin{pmatrix} 3 \times -2 \\ 3 \times 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$

b



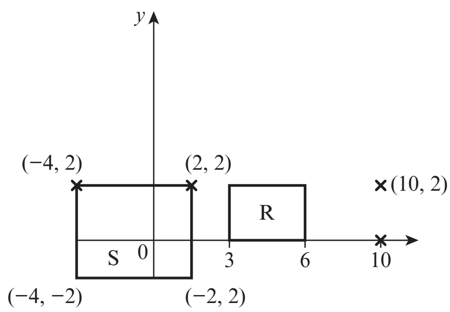
4 **a** Centre is (−1, 2).

b Scale factor is −2.

5 **a** $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

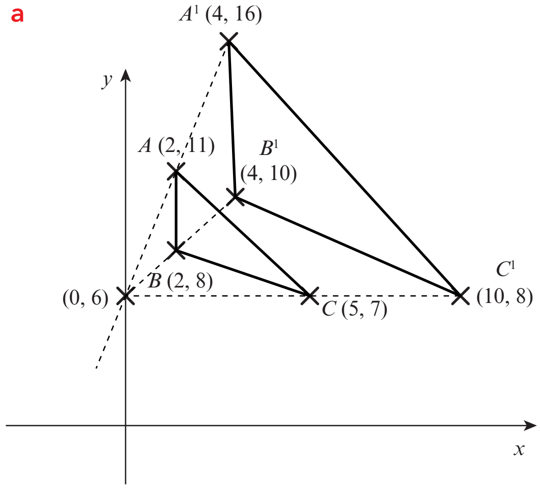
b Rotation, centre (6, 0), 180°

c **i**



ii $2^2 = 4$

6 **a**

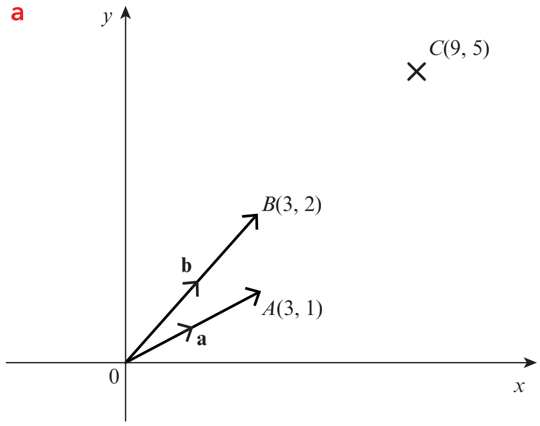


b Centre is (0, 6), where the ray lines meet.

c 2

d $2^2 = 4$

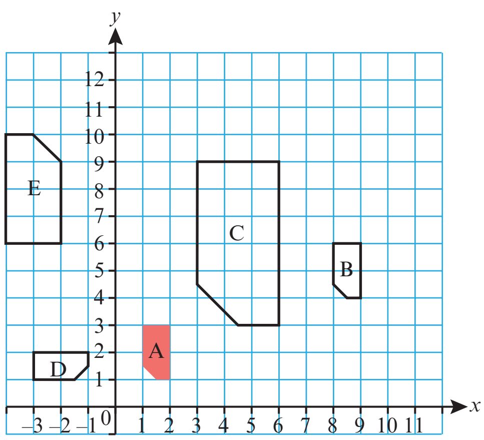
7 **a**



b $\mathbf{a} - \mathbf{b} = \begin{pmatrix}3\\1\end{pmatrix} - \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$

c $|\mathbf{a}| = \sqrt{3^2 + 1^1} = \sqrt{10} = 3.16$

8 **a**



i Translation $\begin{pmatrix}7\\3\end{pmatrix}$

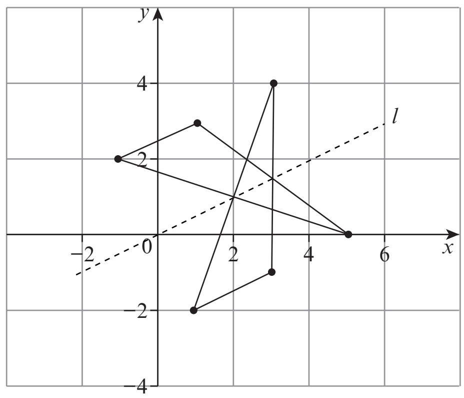
ii Enlargement, scale factor 3, centre (0, 0).

iii Rotation 90° anticlockwise about (0, 0).

iv Enlargement, scale factor $-\frac{2}{3}$, centre (0, 7).

b B, D

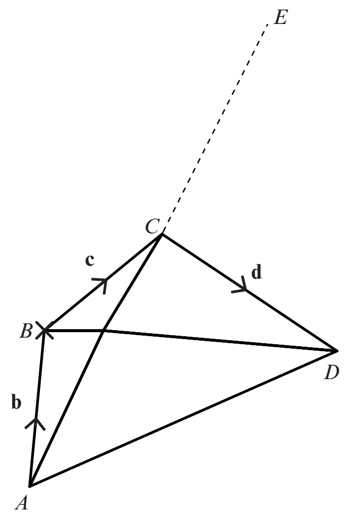
9 **a** i and ii.



Equation of l is $y = \frac{1}{2}x$

10 **a** $\mathbf{b} + \mathbf{c}$

b $\mathbf{b} + \mathbf{c} + \mathbf{d}$



$\overrightarrow{DE} = -\mathbf{d} + \mathbf{b} + \mathbf{c} = 2\overrightarrow{AB} = 2\mathbf{b}$

$2\mathbf{b} = -\mathbf{d} + \mathbf{b} + \mathbf{c}$

So, $\mathbf{a} + \mathbf{b} = \mathbf{c}$

d $\overrightarrow{AD} = \mathbf{b} + \mathbf{c} + \mathbf{d}$

$= \mathbf{c} + (\mathbf{b} + \mathbf{d})$

$= \mathbf{c} + \mathbf{c}$

$= 2\mathbf{c} = 2\overrightarrow{BC}$

So, BC is parallel to AD.

$\Rightarrow ABCD$ is a parallelogram.

Chapter 24

Getting started

1 **Statement A:** Not correct. You cannot assume that both outcomes are equally likely. Think about what factors affect the weather.

Statement B: Not correct. It is a misconception that a probability gives the proportion of outcomes that will actually happen. You can experiment to check this statement, but you can also think about a simpler example, the probability of heads is 0.5, but this does not mean you will get 10 heads if you toss a coin 20 times.

Statement C: Not correct. The probability of each number on a dice is $\frac{1}{6}$. It is no harder to roll a 6 than any other number. You may never have rolled four 6s in a row personally, but you cannot base probability on personal experience.

Statement D: Not correct. Probability is not based on patterns of recent events.

2 a $\mathscr{E}$ symbol for universal set (sample space)

b Elements only in set A

c Elements shared by set A and set B

d Elements not in A or B but contained in the universal set

e Elements only in set B

3 a $n(A)$: number of elements in set A

$A \cup B$: union of set A and B

$A \cap B$: intersection of set A and B

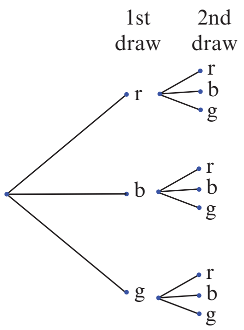
b $n(A)$: number of elements in region A and any intersections

$A \cup B$: combine elements of set A and set B with none repeated

$A \cap B$: elements in the overlapping region

Exercise 24.1

1 a



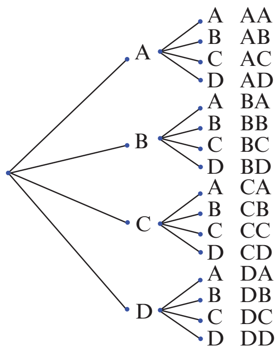
b 9 possible outcomes

c 3

d 5

e 4

2 a

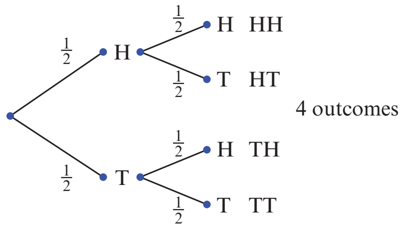


b 16

c $\frac{1}{16}$

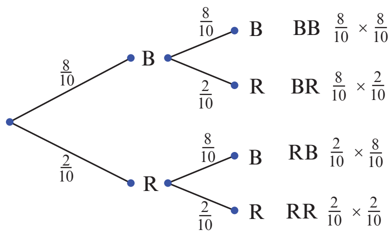
Exercise 24.2

1



$P(TT \text{ or } HH) = \frac{2}{4} = \frac{1}{2}$

2 a

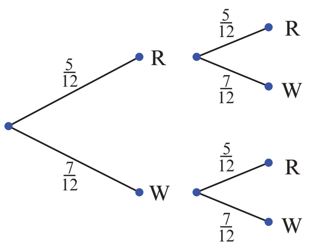


b i $P(RR) = \frac{1}{25}$

ii $P(RB) + P(BR) = \frac{8}{25}$

iii $P(BB) = \frac{16}{25}$

3 **a**



b **i** $P(RR) = \frac{25}{144}$

ii $P(WW) = \frac{49}{144}$

4 **a** 0.49

b 0.09

c 0.21

d 0.42

e 0.51

5 **a** 4

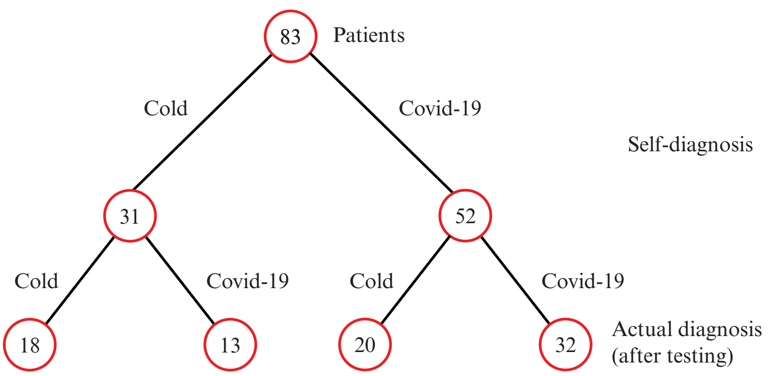
b $\frac{4}{9}$

c $\frac{1}{9}$

d He is equally likely either to buy two birds, or to buy one of each.

Frequency trees

1 **a**



b It clearly shows the outcomes.

c A frequency tree shows the actual data values and a probability tree shows the probabilities.

2 **a**

Self \ Actual	Cold	COVID-19
Cold	18	13
COVID-19	20	32

b Students' own opinion with justification.

3 **a** 0.62

b 41.94%

Exercise 24.3

1 **a** $\frac{1}{2}$

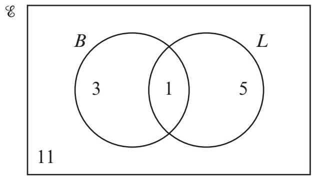
b $\frac{2}{3}$

c $\frac{1}{6}$

d $\frac{1}{3}$

e 1

2

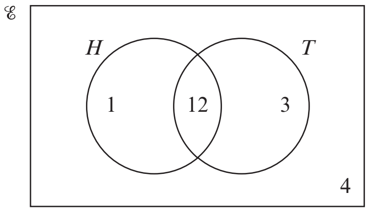


a $\frac{4}{5}$

b $\frac{1}{4}$

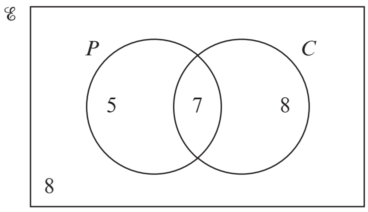
c $\frac{11}{20}$

3 **a**



b $\frac{3}{5}$

4 **a**

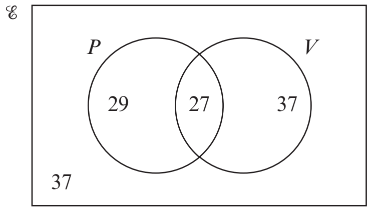


b **i** $\frac{5}{28}$

ii $\frac{5}{7}$

iii $\frac{1}{4}$

5 **a**



b **i** $\frac{32}{65}$

ii $\frac{93}{130}$

iii $\frac{27}{130}$

iv $\frac{37}{130}$

6 **a** 12

b 3

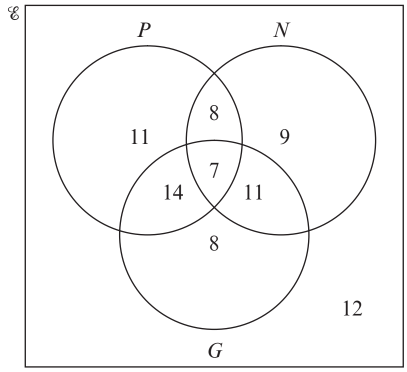
c 21

d 12

e $\frac{7}{12}$

f $\frac{12}{19}$

7 **a**



b 7

c 14

d $\frac{12}{80} = \frac{3}{20}$

8 **a** $x = 9$

b **i** 102

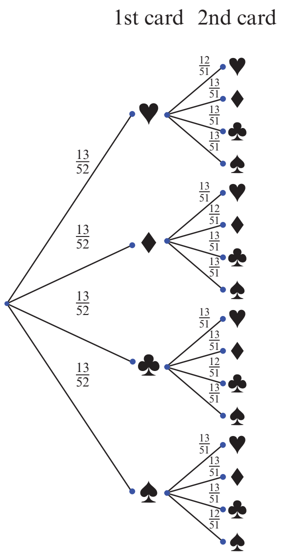
ii 17

ii 23

c $\frac{17}{130}$

Exercise 24.4

1 **a**



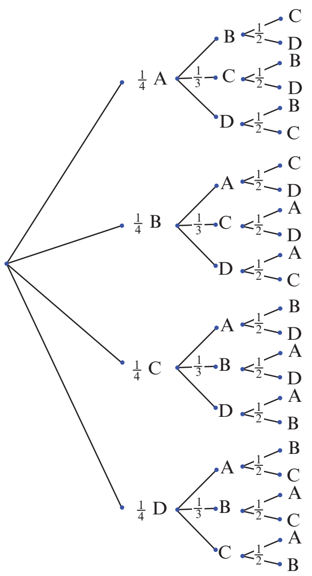
b **i** $P(\heartsuit\heartsuit) = \frac{13}{52} \times \frac{12}{51} = \frac{3}{51}$

ii $P(\clubsuit\clubsuit) = \frac{13}{52} \times \frac{12}{51} = \frac{3}{51}$

iii $P(\text{red, black}) = \frac{26}{52} \times \frac{26}{51} = \frac{13}{51}$

iv $P(\heartsuit\heartsuit \text{ given first card is } \heartsuit) = \frac{12}{51} = \frac{4}{17}$

2 **a**



b **i** $\frac{1}{24}$

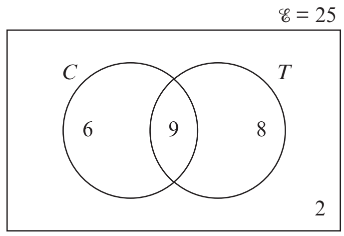
ii $\frac{1}{24}$

iii 0

c $\frac{1}{4}$

d $\frac{1}{24}$

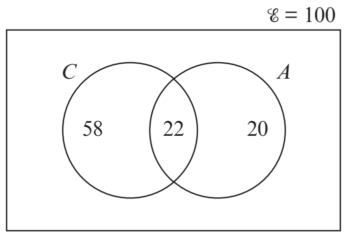
3



a $\frac{3}{5}$

b $\frac{9}{17}$

4 **a**



b **i** 0.58

ii $\frac{11}{40}$ or 0.275

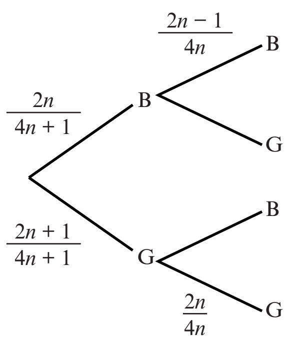
5 **a** $\frac{60}{243} = 0.247$

b $\frac{37}{60} = 0.617$

c $\frac{48}{137} = 0.350$

d $\frac{71}{160} = 0.444$

6



P(2 counters same colour)

$= P(BB \text{ or } GG)$

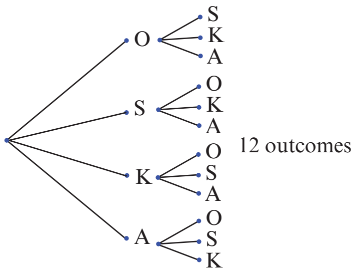
$= \dfrac{2n}{4n+1} \times \dfrac{2n-1}{4n} + \dfrac{2n+1}{4n+1} \times \dfrac{2n}{4n}$

$= \dfrac{4n^2 - 2n}{4n(n+1)} + \dfrac{4n^2 + 2n}{4n(n+1)}$

$= \dfrac{8n^2}{4n(n+1)}$

$= \dfrac{2n}{n+1}$

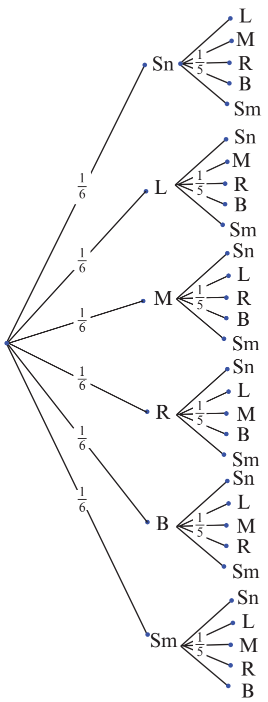
7 **a**



b $\dfrac{1}{12}$

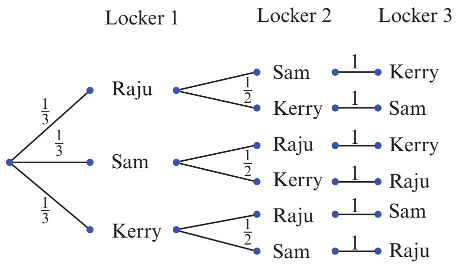
c $\dfrac{1}{12}$

8 **a**



b $\dfrac{1}{30}$

9 **a**



b Conditional – once the first name is chosen it cannot be chosen again, so the second choice depends on the first, and so on.

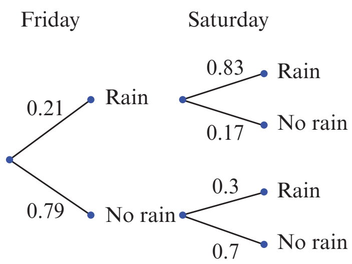
c 1 way

d 6 ways

e $\dfrac{1}{6}$

10 $\dfrac{4}{15}$

11 **a**



b **i** 0.1743

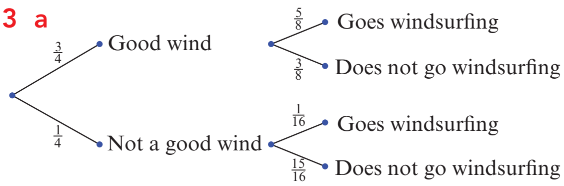
ii 0.4113

12 $P(\text{rain both days}) = \dfrac{1}{50}$

$P(\text{sun both days}) = \dfrac{96}{125}$

$P(\text{1 day sun and 1 day rain}) = \dfrac{53}{250}$

13 **a**



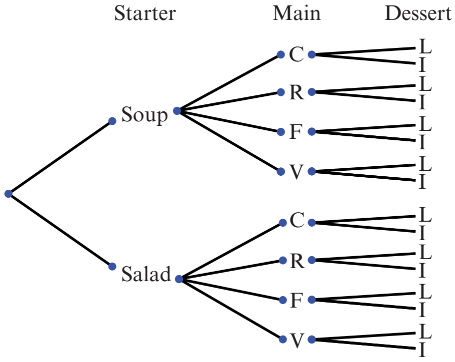
b $\dfrac{15}{32}$

c $\dfrac{33}{64}$

d $\dfrac{31}{128}$

Practice questions

1 **a**



b $\frac{1}{16}$

2 **a** 10

b $P(\text{maths}) = 0.84$

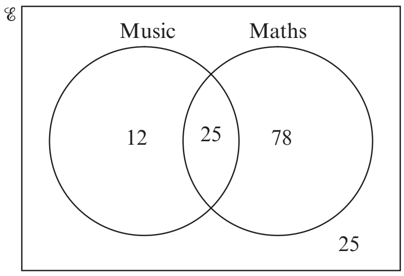
c $P(\text{maths or physics}) = 0.96$

3 **a** **i** $\frac{84}{210} = \frac{2}{5}$

ii $\frac{148}{210} = \frac{74}{105}$

b $\frac{1320}{2730} = \frac{44}{91}$

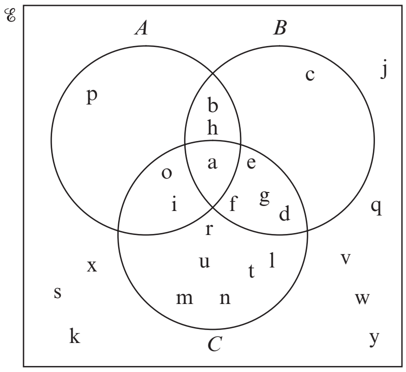
4 **a**



b P(music) = 0.264

c P(music given maths) = 0.243

5 **a**

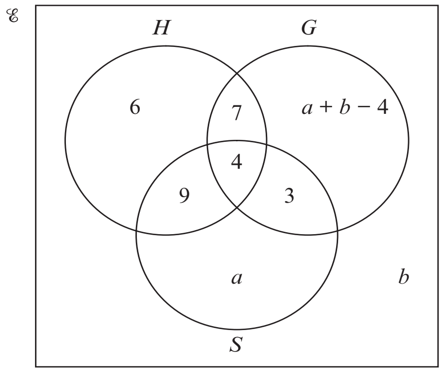


b **i** {a}

ii {b, c, p}

iii {p}

6



19

7 **a** **i** $\frac{n}{n+5}$

ii $\frac{n^2 - n}{n^2 + 9n + 20}$ or equivalent

b $\frac{n^2 - n}{n^2 + 9n + 20} = \frac{7}{22}$

$\Rightarrow 22(n^2 - n) = 7(n^2 + 9n + 20)$

$\Rightarrow 22n^2 - 22n = 7n^2 + 63n + 140$

$\Rightarrow 22n^2 - 22n - 7n^2 - 63n - 140 = 0$

$\Rightarrow 15n^2 - 85n - 140 = 0$

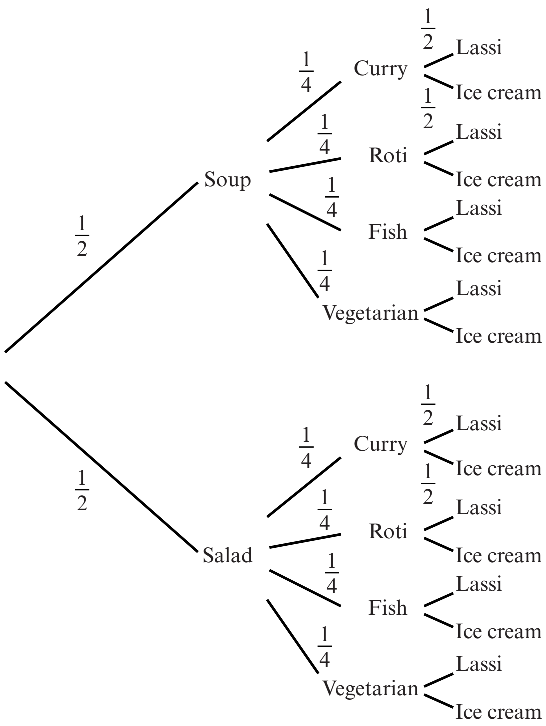
$\Rightarrow 3n^2 - 17n - 28 = 0$

c $n = 7$

$P(\text{exactly one black and one white}) = \frac{35}{66}$

Practice questions worked solutions

1 **a**



b P(soup, curry, ice cream) $= \frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{16}$

2 **a** 10

b $\frac{130 + 80}{250} = \frac{210}{250} = \frac{21}{25}$

c $\frac{240}{250} = \frac{24}{25}$

3 **a** **i** P(at least one green) = 1 − P(no green)

$$= 1 - \frac{9}{15} \times \frac{8}{14}$$

$$= 1 - \frac{72}{210}$$

$$= \frac{148}{210} = \frac{74}{105}$$

ii $1 - \text{P(same colour)} = 1 - \left(\frac{5}{15} \times \frac{4}{14} + \frac{4}{15} \times \frac{3}{14} + \frac{6}{15} \times \frac{5}{14}\right)$

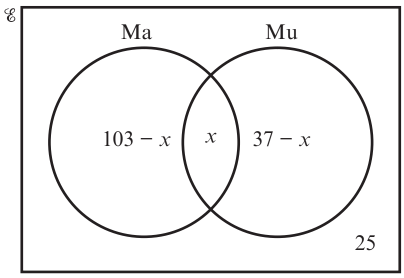
$$= 1 - \frac{20 + 12 + 30}{210}$$

$$= \frac{148}{210} = \frac{74}{105}$$

b P(brown, not brown, not brown) × 3 $= 3 \times \frac{4}{15} \times \frac{11}{14} \times \frac{10}{13}$

$$= \frac{44}{91}$$

4 **a**



$25 + 103 - x + x + 37 - x = 140$

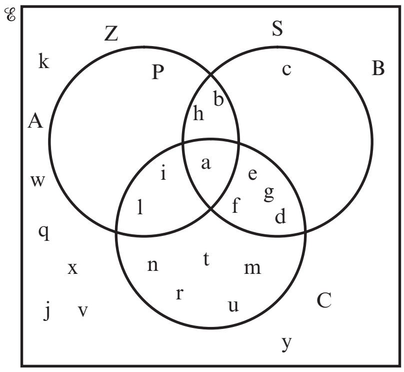
$154 - x = 140$

$x = 25$

b $25 + 12 = 37$

c $\frac{25}{103}$

5 **i**

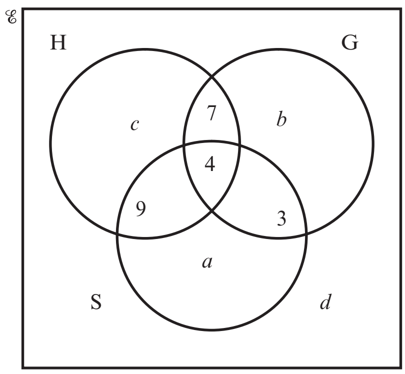


ii **a** {a}

b {b, c, h, p}

c {p}

6



$a + d = b + c - 2$

$a + b + c + d + 29 = 59$ (Total number)

$a + b + c + d = 30$

We need $b + c = a + d$

$= 32 - (b + 2)$

$b + c = 16$

7 **a** **i** $\frac{n}{n+5}$

ii $\frac{n}{n+5} \times \frac{n-1}{n+4} = \frac{n(n-1)}{(n+5)(n+4)}$

b $\frac{n(n-1)}{(n+5)(n+4)} = \frac{7}{22}$

$22n(n-1) = 7(n+4)(n+5)$

$22n^2 - 22n = 7n^2 + 63n + 140$

$15n^2 - 85n - 140 = 0$

$3n^2 - 17n - 28 = 0$

c $(3n + 4)(n - 7) = 0$

$n = -\frac{4}{3}$ or 7 but $n > 0$

so, $n = 7$

Therefore, P(exactly one of each)

= P(W, B) + P(B, W)

$= \frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11}$

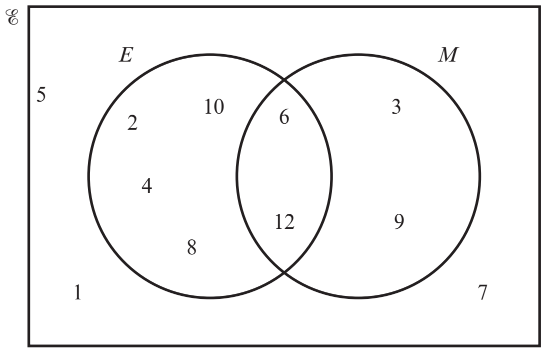
$= \frac{70}{121}$

Past paper questions

1 **a** stationary

b The student is travelling fastest between the time 1300 and 1320 because the graph is steepest then.

2 **a** **i**



ii 2

iii $\frac{2}{12} = \frac{1}{6}$

b No. 2 is prime *and* even.

3 Maxi travels 20 km before Pippa starts.

Now 110 km apart.

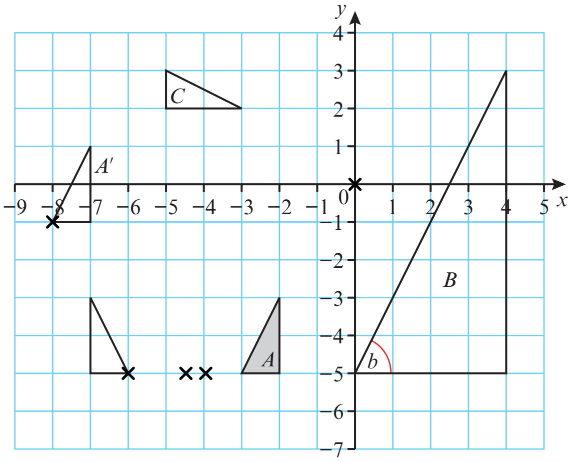
Both travel $\frac{110}{2} = 55$ km further.

$20 + 55 = 75$ km

4 **a** **i** enlargement, scale factor 4, centre of enlargement $(-4, -5)$

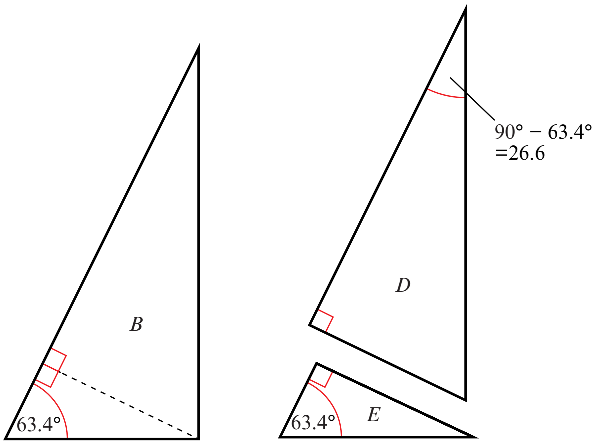
ii rotation through 90° clockwise about origin

b **i** and **ii**



c $\tan b = \frac{8}{4} = 2 \Rightarrow b = \tan^{-1}(2) = 63.4°$

d Yes, they have the same three angles.



5 **a** **i** $1.125 \times 152 = 171$

ii $152 + 171 + \frac{3}{8} \times 152 = 380$

b 152 : 171 : 57

c 8 rows

d **i** \$6 is 4 parts so 1 part is \$1.50.
$7 \times 1.5 = \$10.50$

ii $9 \times 1.5 = \$13.50$

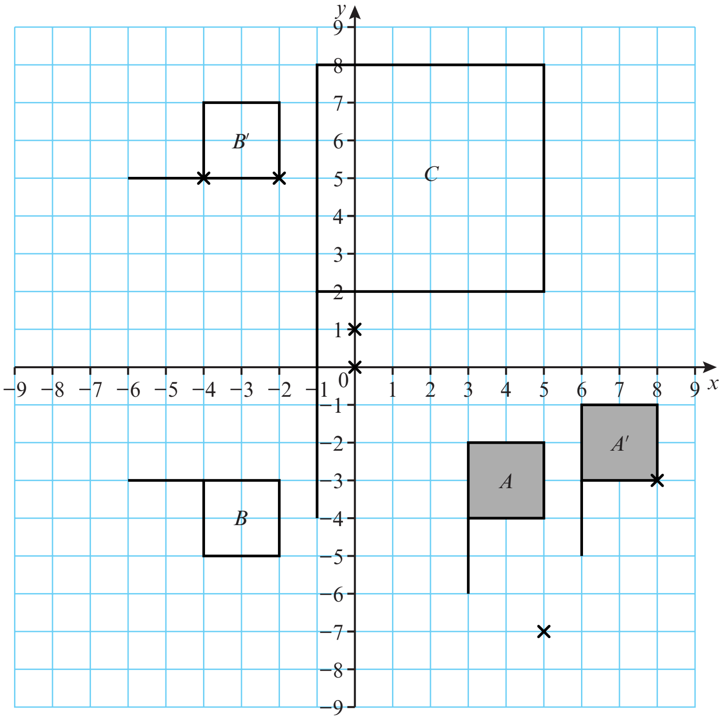
iii $120 \times \$13.50 + 136 \times \$10.50 + 30 \times 6 = \$3228$

iv $\frac{3228}{4500} = 71.7\%$

6 **a** rotation 90º anticlockwise about (0, 0)

b enlargement, scale factor 3, centre (5, −7)

c and **d**



7 **a** $\begin{pmatrix} 14 \\ -6 \end{pmatrix}$

b $\begin{pmatrix} -12 \\ 21 \end{pmatrix}$

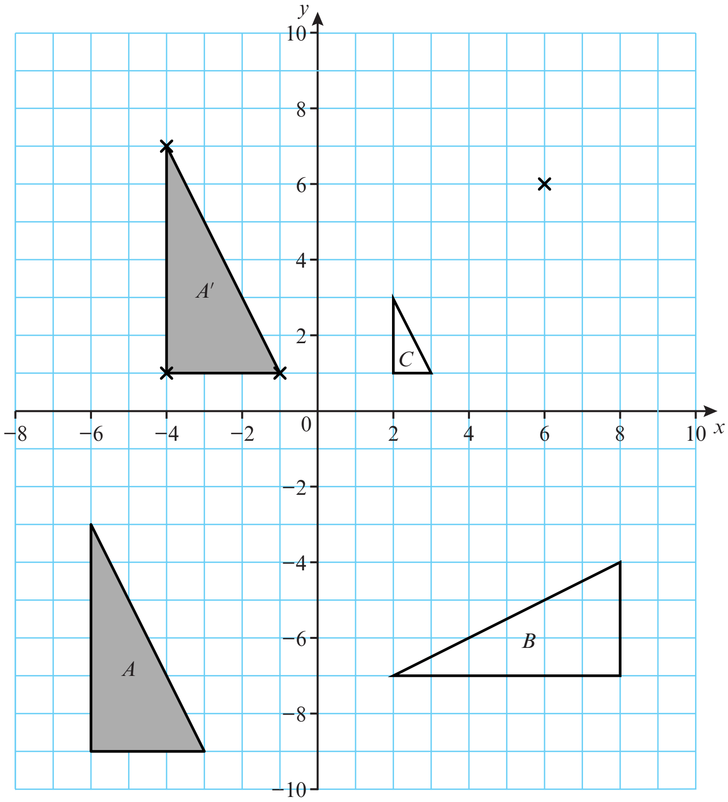
8 $20 = k\sqrt{25} = 5k \Rightarrow k = 4$

$y = 4\sqrt{36} = 24$

9 **a** **i** rotation 90º anticlockwise about (0, −1)

ii enlargement, scale factor $\frac{1}{3}$ centre (6, 6)

b



10 $\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS}$

$$= \mathbf{a} + \frac{4}{9}(\overrightarrow{PQ})$$

$$= \mathbf{a} + \frac{4}{9}(-\mathbf{a} + \mathbf{b})$$

$$= \frac{5}{9}\mathbf{a} + \frac{4}{9}\mathbf{b}$$

11 $2 \times \frac{1}{5} \times \frac{1}{4} = \frac{1}{20} \times 2 = \frac{1}{10}$

12 a $\mathrm{fg}(x) = 4(5x - 4) + 3 = 20x - 13$

$p = -13$

b $y = \frac{5x - 1}{3}$

$3y = 5x - 1$

$x = \frac{3y + 1}{5}$

$\mathbf{h}^{-1}(x) = \frac{3x + 1}{5}$

13 $m = \dfrac{k}{(p-1)^2}$ $\quad 5 = \dfrac{k}{9} \therefore k = 45$

$m = \dfrac{45}{(p-1)^2}$

$m = \dfrac{45}{25}$

$= \dfrac{9}{5}$

14 a i $3\mathbf{m} = \begin{pmatrix} 15 \\ 21 \end{pmatrix}$

ii $\sqrt{10^2 + 24^2} = \sqrt{676} = 26$

b $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE} = \mathbf{p} + \dfrac{3}{4}\mathbf{q}$

15 a i \$7680 **ii** \$34 240

b \$5306.04

c 26.7%

d $\dfrac{36\,515}{1.09} = \$33\,500$

e $\dfrac{9}{22}(x + 290) = \dfrac{5}{12}x$

$x = 15\,660$

$\therefore$ Arun paid $\dfrac{5}{12} \times 15\,660 = \$6525$

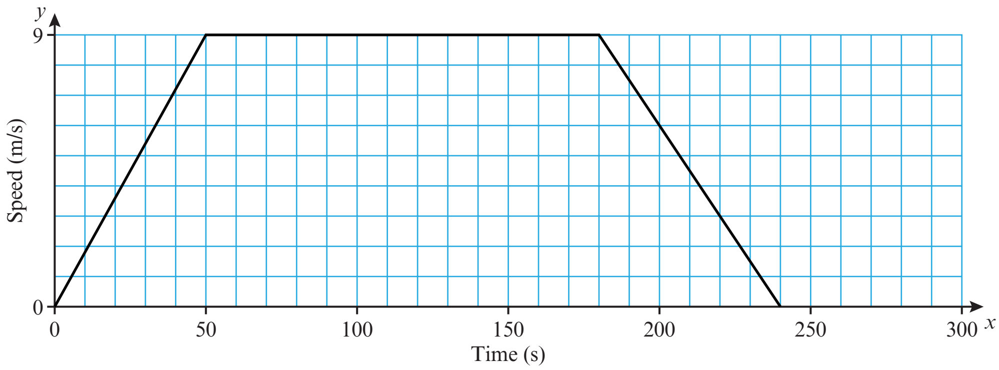
16 a $9\,\text{m s}^{-1}$ in $50\,\text{s}$ $\dfrac{9}{50} = 0.18\,\text{m/s}^2$

b deceleration $= 1944 \times \dfrac{1000}{3600 \times 3600}$

$= 0.15\,\text{m/s}^2$

time $= \dfrac{9}{0.15} = 60\,\text{s}$

c

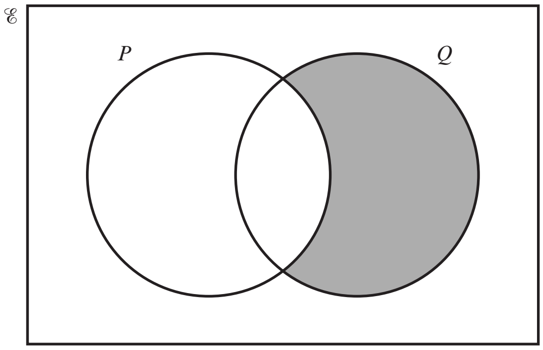


d distance $= \dfrac{1}{2} \times 50 \times 9 + (130 \times 9) + \dfrac{1}{2} \times 60 \times 9 = 225 + 1395 + 270 = 1890\,\text{m}$

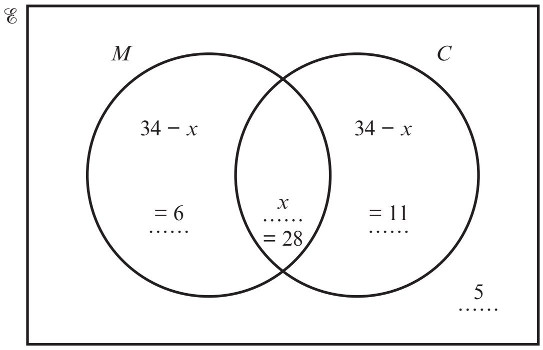
time $= 240\,\text{s}$

average speed $= \dfrac{1890}{240} = 7.88\,\text{m/s}$

17 **a**



b



$73 - x + 5 = 50$

$x = 28$

c **i** 8

ii 11

iii 29

iv $(C \cap S) \cap B'$

v $\frac{19}{30}$

vi $\frac{4}{19} \times \frac{3}{18} = \frac{12}{342} = \frac{2}{57}$

vii $p(A) = \frac{15}{30} = \frac{1}{2}$ $\quad p(B) = \frac{8+7}{30} = \frac{1}{2}$ Same probability means they are equally likely.

18 **a** **i** 1.991×103

ii $\frac{1}{2}at^2 = s - ut \Rightarrow a = \frac{2(s-ut)}{t^2}$

b **i** $(2x + 3)(x - 1) - (x + 1)(x - 2) = 62$

$2x^2 + x - 3 - (x^2 - x - 2) = 62$

$x^2 + 2x - 1 = 62$

$x^2 + 2x - 63 = 0$

ii $(x + 9)(x - 7)$

iii $x = -9$ or $x = 7$

19 **a** $BC = \sqrt{80^2 + 115^2 - 2 \times 80 \times 115 \times \cos 72°} = 118.1$ m

b $\frac{\sin ABC}{115} = \frac{\sin 72}{118.1}$

$ABC = 67.8°$

c **i** 255°

ii 7.2°

d 11.8 km/h

e $\frac{\text{distance}}{80} = \sin 72$

distance = 80 sin 72 = 76.1

76.1 m

20 **a** $(3^2)^2 + 1 = 82$

b $\frac{x+2}{7}$

c $gg(x) = (x^2 + 1)^2 + 1 = x^4 + 2x^2 + 2$

$a = 1, b = 2, c = 2$

d $3^{7x-2} = 81$

$3^{7x-2} = 3^4$

$7x - 2 = 4$

$7x = 8$

$x = \frac{8}{7}$

21 **a** **i** $1.5 < h \leqslant 1.6$

ii 1.62 m

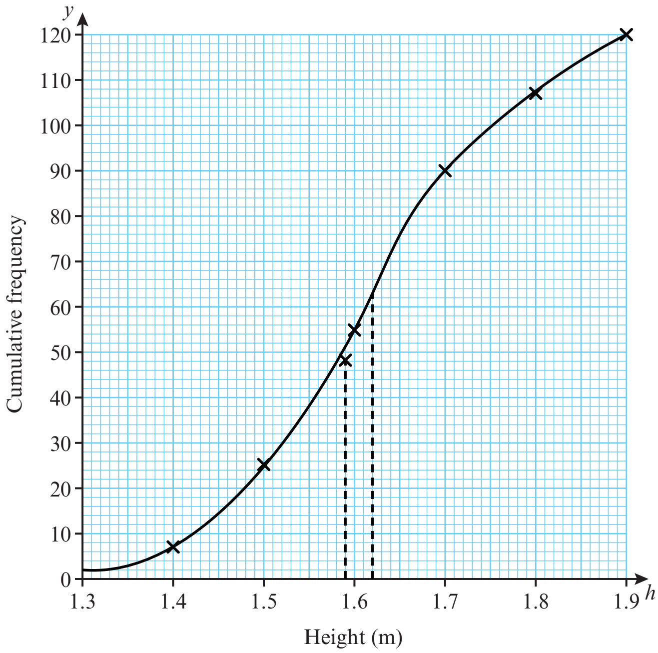
b **i** $\frac{14}{120} = \frac{7}{60}$

ii $3\left(\frac{7}{60} \times \frac{7}{119} \times \frac{6}{118}\right) = \frac{21}{20\,060}$

c **i**

Height (h metres)	$h \leqslant 1.4$	$h \leqslant 1.5$	$h \leqslant 1.6$	$h \leqslant 1.7$	$h \leqslant 1.8$	$h \leqslant 1.9$
Cumulative frequency	7	25	55	79	106	120

ii



d i approximately equal to 1.63 m

ii 1.59 m