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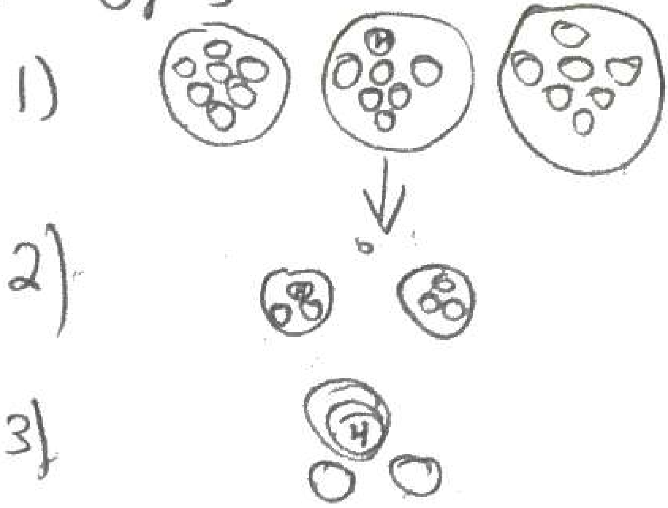
* do we know the weight of a coin?
No but can logically reason it quickly.

18.1.3] there are 21 coins, one is heavier. how many weightings are required to find the odd coin?

1) 3 groups of 7, select heaviest.
2) 2 groups of 3, one odd. If two groups balance, the heaviest is the odd coin. otherwise pick heaviest set of 3...
3) select heaviest from group of 3

$\boxed{\text{3 moves are required.}}$



~~18.1.4)~~ given a 7oz & 3oz Bucket, Make 5oz.

did not actually work backwards... maybe not optimal?

\# oz

7B	3B	steps
5	0	9
5	3	8
7	1	7
0	1	6
1	0	5
1	3	4
4	0	3
4	3	2
7	0	1

1) fill 7 to 7
2) pour 3 into 3, 4 remains in 7
3) pour out 3
4) pour 3 into 3 from 7, 1 remains
5) pour out 3
6) pour 1 from 7 into 3
7) fill 7,
8) pour 7 into 3
9) 7 has 5, 3 has 3.

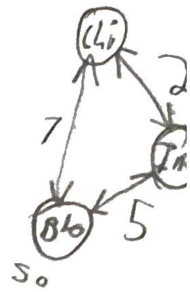
18.2.27] HW4 2

Home: Bloomington (monday) destination: Indianapolis (thursday)

profit	Ind	Blo	chi
	12	16	17

f_{day}(from loc)

cost	Ind	Blo	Chi
Ind	-	5	2
Blo	5	-	7
Chi	2	7	-



Stages: Monday (1), Tuesday (2), Wednesday (3)

States: Ind, Blo, chi

$f_3(chi) = 17 - 2 = 15^*$, $f_3(Blo) = 16 - 5 = 11$, $f_3(Ind) = 12$

$$f_2(chi) = \max \begin{cases} 17 + f_3(chi) = 32^* - 0 = 32^* \\ 17 + f_3(Blo) = 17 - 7 + 16 = 28 \\ 17 + f_3(Ind) = 17 - 2 + 12 = 29 \end{cases}$$

$$f_2(Blo) = \max \begin{cases} 16 + f_3(chi) = 16 + 15 - 7 = 24 \\ 16 + f_3(Blo) = 16 + 11 - 0 = 27^* \\ 16 + f_3(Ind) = 16 + 12 - 5 = 23 \end{cases}$$

$$f_2(Ind) = \max \begin{cases} 12 + f_3(chi) = 12 + 15 - 2 = 25^* \\ 12 + f_3(Blo) = 12 + 11 - 5 = 18 \\ 12 + f_3(Ind) = 12 + 12 - 0 = 24 \end{cases}$$

$$f_1(chi) = \max \begin{cases} 17 + f_2(chi) = 17 + 32 - 0 = 49^* \\ 17 + f_2(Blo) = 17 + 27 - 7 = 37 \\ 17 + f_2(Ind) = 17 + 25 - 2 = 40 \end{cases}$$

$$f_1(Blo) = \max \begin{cases} 16 + f_2(chi) = 16 + 32 - 7 = 41 \\ 16 + f_2(Blo) = 16 + 27 - 0 = 43^* \\ 16 + f_2(Ind) = 16 + 25 - 5 = 36 \end{cases}$$

$$f_1(Ind) = \max \begin{cases} 12 + f_2(chi) = 12 + 32 - 2 = 42^* \\ 12 + f_2(Blo) = 12 + 27 - 5 = 34 \\ 12 + f_2(Ind) = 12 + 25 - 0 = 37 \end{cases}$$

$$f_0(Blo) = \max \begin{cases} f_1(chi) - 7 = 42 \\ f_1(Blo) - 0 = 43^* \\ f_1(Ind) - 5 = 37 \end{cases}$$

Stay in Bloomington, travel to Indianapolis on wednesday.

43 units Blo → Blo → Blo → Ind

18.2.3 Blo → Cle

Find shortest path

Working backwards:

$f(\text{Tol}) = 3$ $\quad f(\text{Gary}) = \overset{3+3}{6}$

$f(\text{Col}) = 3$

$f(\text{Day}) = \min \begin{cases} 1 + f(\text{Tol}) = 4^* \\ 2 + f(\text{Col}) = 5 \end{cases}$

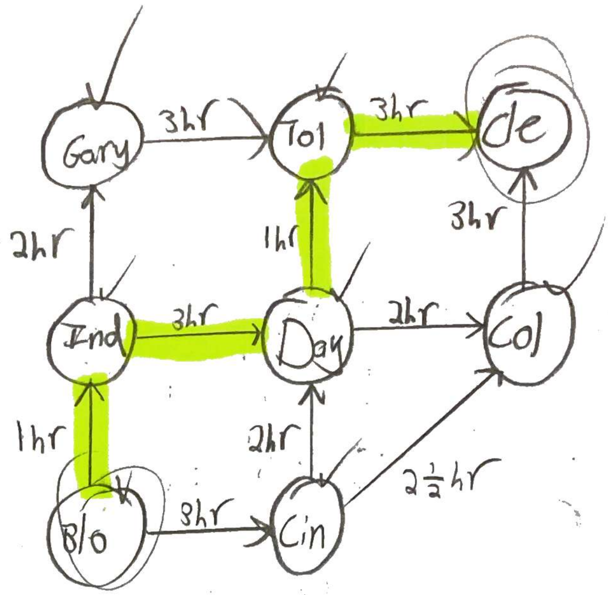
$f(\text{Cin}) = \min \begin{cases} f(\text{Day}) + 2 = 6 \\ f(\text{Col}) + 2.5 = 5.5^* \end{cases}$

$f(\text{Ind}) = \min \begin{cases} f(\text{Gary}) + 2 = 8 \\ f(\text{Day}) + 3 = 7^* \end{cases}$

$f(\text{Blo}) = \min \begin{cases} f(\text{Ind}) + 1 = 8^* \\ f(\text{Cin}) + 3 = 8.5 \end{cases}$

8 hours

Blo → Ind → Day → Tol



18.3.1]

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18.3.1) given Ex 4, find optimal Production Schedule 3/21/23
if $I_0 = 3$.

month:	1	2	3	4
demand:	1	3	2	4
real :	0	3	0	4

15

Setup cost $A = 3$
Production cost $p = 1$
holding cost $h = 0.50$
Max Prod/mo $U = 5$
Max monthly close Inventory $I_{max} = 4$

Meet demand & minimize costs:
Solution:
Stage: month 1:4 (t)
State: beginning inventory given stage (i)
$c(x)$ is cost of producing x units during a period
$f_t(i)$ is min cost to meet demand in t with i units on hand.

Stage 4
$f_4(4) = c(0) = 0 + .5 \cdot 4$ $\quad x_4(4) = 0$
$f_4(3) = c(1) = 3+1=4$ $\quad x_4(3) = 1$
$f_4(2) = c(2) = 3+2=5$ $\quad x_4(2) = 2$
$f_4(1) = c(3) = 3+3=6$ $\quad x_4(1) = 3$
$f_4(0) = c(4) = 3+4=7$ $\quad x_4(0) = 4$

⋮

looking at ex 4 table 5:
$f_1(3) = 15$ $\quad x_1(3) = 0$
month 0: produce 0
month 1: produce 0
month 2: $f_2(2) = 14$, $x_2(2) = 3$
month 3: $f_3(2) = 12$ $\quad x_3(2) = 0$
month 4: $f_4(0) = 7$ $\quad x_4(0) = 4$

$\boxed{0-3-0-4}$ 15

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18.3.3 Using figure 5 and tables in Ex 4 determine production level & production cost for

a) $(2,3) \rightarrow (3,1)$

month:	1	2	3	4
demand:	1	3	2	4
on hand:	0	0	0	0
Prod:	1			

@ month 2, 3 on hand, demand is 3.
Need to produce 1 at a cost of
$3 + 1 \cdot 1 = \$4$

$\boxed{\begin{array}{l} \text{Units: } 1 \\ \text{Cost: } \$4 \end{array}}$

b) $(4,2) - (5,0)$

@ month 4, 2 on hand, demand is 4.
Need to produce 2 units. cost is $\$3 + 2(1) = \$5$

$\boxed{\begin{array}{l} \text{Units} = 2 \\ \text{Cost} = \$5 \end{array}}$

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18.4.2] max $z = 5x_1 + 4x_2 + 2x_3$
st $4x_1 + 3x_2 + 2x_3 \le 8$

x_1, x_2, x_3 nonnegative Int

$f_3(8) = 8^*$, $x_3(8) = 4$; $f_3(4) = 4$, $x_3(4) = 2$; $f_3(0) = 0$, $x_3 = 0$
$f_3(7) = 6$, $x_3(7) = 3$; $f_3(3) = 2$, $x_3(3) = 1$; Benefit = 8
$f_3(6) = 6$, $x_3(6) = 3$; $f_3(2) = 2$, $x_3(2) = 1$
$f_3(5) = 4$, $x_3(5) = 2$; $f_3(1) = 0$, $x_3(1) = 0$

$f_2(8) = 2\cdot 3 + f_3(2) = 6 + 2 = 8^*$ $x_2(8) = 2$ Benefit = 10.
$f_2(7) = 2\cdot 3 + f_3(1) = 6 + 0 = 6$ $x_2(7) = 2$
$f_2(6) = 2\cdot 3 + f_2(0) = 6 + 0 = 6$ $x_2(6) = 2$
$f_2(5) = 1\cdot 3 + f_2(2) = 3 + 2 = 5$ $x_2(5) = 1$
$f_2(4) = 1\cdot 3 + f_2(1) = 3 + 0 = 3$ $x_2(4) = 1$
$f_2(3) = 1\cdot 3 + f_2(0) = 3 + 0 = 3$ $x_2(3) = 1$
$f_2(2) = 0\cdot 3 + f_2(2) = 0 + 2 = 2$ $x_2(2) = 0$
$f_2(1) = 0\cdot 3 + f_2(1) = 0 + 0 = 0$ $x_2(1) = 0$
$f_2(0) = 0\cdot 3 + f_2(0) = 0 + 0 = 0$ $x_2(0) = 0$

$f_1(8) = 2\cdot 4 + f_2(0) = 8 + 0 = 8^*$ $x_3(8) = 2$ Benefit = 10
$f_1(7) = 1\cdot 4 + f_2(3) = 4 + 3 = 7$ $x_3(7) = 1$
$f_1(6) = 1\cdot 4 + f_2(2) = 4 + 2 = 6$ $x_3(6) = 1$
$f_1(5) = 1\cdot 4 + f_2(1) = 4 + 0 = 4$ $x_3(5) = 1$
$f_1(4) = 1\cdot 4 + f_2(0) = 4 + 0 = 4$ $x_3(4) = 1$
$f_1(3) = 0\cdot 4 + f_2(3) = 0 + 3 = 3$ $x_3(3) = 0$
$f_1(2) = 0\cdot 4 + f_2(2) = 0 + 2 = 2$ $x_3(2) = 0$
$f_1(1) = 0\cdot 4 + f_2(1) = 0 + 0 = 0$ $x_3(1) = 0$
$f_1(0) = 0\cdot 4 + f_2(0) = 0 + 0 = 0$ $x_3(0) = 0$

$$\boxed{\begin{array}{l}\text{Max } Z = 10 \\ \text{by either } x_2 = 2,\ x_3 = 1 \\ \text{or } x_1 = 2\end{array}}$$

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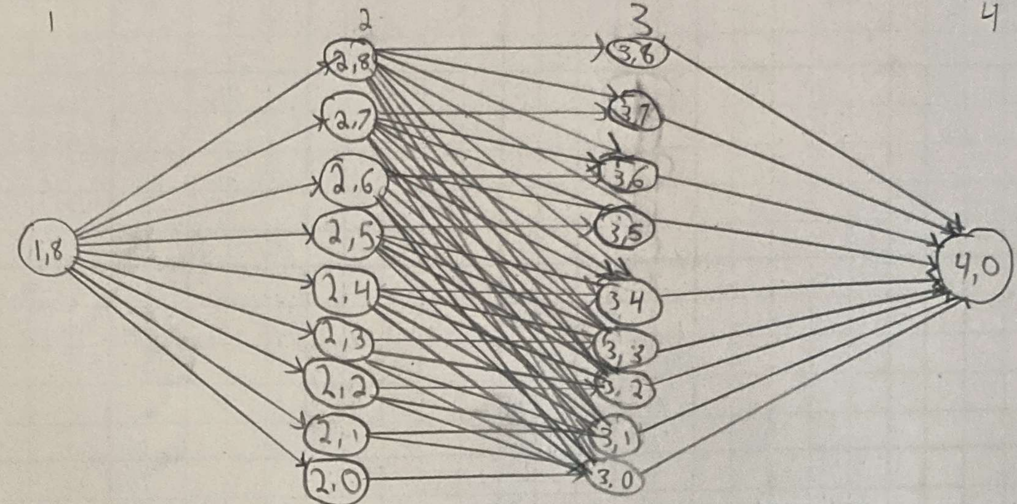
18.4.3] Using problem 18.4.2

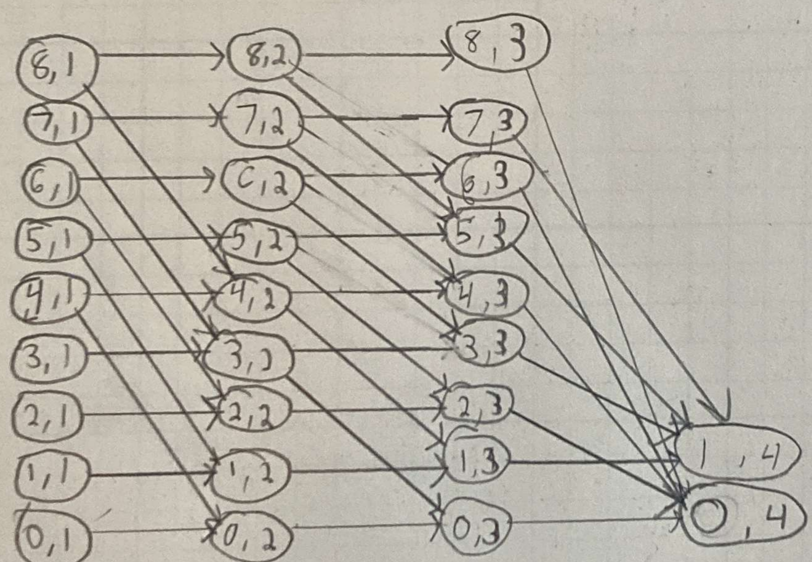
a) draw Network diagram corresponding to

$F_{T+1}(d) = 0$

$F_t(d) = \max_{x_t} \{ r_t(x_t) + f_{t+1}[d - g_t(x_t)] \}$



b) $g(w) = \max_j \{ b_j + g(w - w_j) \}$



3/9/23

5) Use DP to solve a Knapsack problem in which the Knapsack can hold 13 lb

x_i : integer

Solution: 3 items. let x_i indicate Quantity of item i to bring, g_i be the weight of item i, and r_i be the benefit of item i.

item	Weight (lb)	Benefit
1	3	12
2	5	25
3	7	50

Stage 3 Calculations

$f_3(13) = \max\{50x_3\}$ $\quad 7x_3 \le 13$

$= 50$

$f_3(7) = f_3(8) = f_3(9) = f_3(10) = f_3(11) = f_3(12) = f_3(13) = 50$

$f_3(0) = f_3(1) = f_3(2) = f_3(3) = f_3(4) = f_3(5) = f_3(6) = 0$

$x_3(p) = 1 \quad 7 \le p \le 13$

$x_3(p) = 0 \quad 0 \le p < 7$

Stage 2 Calculations

$f_2(13) = \max\{25x_2 + f_3(d - 5x_2)\}$

$f_2(13) = M\begin{cases} 25(0) + f_3(13) = 50 & x_2 = 0 \\ 25(1) + f_3(8) = 75^* & x_2 = 1 \\ 25(2) + f_3(3) = 50 \end{cases}$

$f_2(13) = 75, \ x_2 = 1$

$f_2(12) = \text{Max}\begin{cases} 25(0) + f_3(12) = 50 & x_2 = 0 \\ 25(1) + f_3(7) = 75^* & x_2 = 1 \\ 25(2) + f_3(2) = 50 & x_2 = 2 \end{cases}$

$f_2(12) = 75, \ x_2 = 1$

$f_2(11) = \text{Max}\begin{cases} 25(0) + f_3(11) = 50^* & x_2 = 0 \\ 25(1) + f_3(6) = 25 & x_2 = 1 \\ 25(2) + f_3(1) = 50^* & x_2 = 2 \end{cases}$

$f_2(11) = 50, \ x_2 = 0 \mid x_2 = 2$

$f_2(10) = \text{Max}\begin{cases} 25(0) + f_3(10) = 50^* & x_2 = 0 \\ 25(1) + f_3(5) = 25 & x_2 = 1 \\ 25(2) + f_3(0) = 50^* & x_2 = 2 \end{cases}$

$f_2(10) = 50 \quad x_2 = 0 \mid x_2 = 2$

$f_2(9) = \max\begin{cases} 25(0) + f_3(9) = 50^* & x_2 = 0 \\ 25(1) + f_3(4) = 25 & x_2 = 1 \end{cases}$

$f_2(9) = 50 \quad x_2 = 0$

$f_2(8) = \max\begin{cases} 25(0) + f_3(8) = 50^* & x_2 = 0 \\ 25(1) + f_3(3) = 25 & x_2 = 1 \end{cases}$

$f_2(8) = 50 \quad x_2 = 0$

Stage 2 Calculations cont...

$f_2(7) = \max \begin{cases} 25(0) + f_3(7) = 50^* & x_2 = 0 \\ 25(1) \end{cases}$ $f_2(7) = 50$ $x_2 = 0$

$f_2(6) = \max \begin{cases} 25(0) + f_3(6) = 0 & x_2 = 0 \\ 25(1) + f_3(1) = 25^* & x_2 = 1 \end{cases}$ $f_2(6) = 25$ $x_2 = 1$

$f_2(5) = \max \begin{cases} 25(0) + f_3(5) = 0 & x_2 = 0 \\ 25(1) + f_3(0) = 25^* & x_2 = 1 \end{cases}$ $f_2(5) = 25$ $x_2 = 1$

$f_2(0) = f_2(1) = f_2(2) = f_2(3) = f_2(4) = 0$

Stage 3 Calculations:

$$f_1(13) = \max \begin{cases} 12(0) + f_2(13) = 75^* & x_1 = 0 \\ 12(1) + f_2(10) = 62 & x_1 = 1 \\ 12(2) + f_2(7) = 74 & x_2 = 2 \\ 12(3) + f_2(4) = 36 & x_2 = 3 \\ 12(4) + f_2(1) = 48 & x_2 = 4 \end{cases}$$

Bring one item 2 and one item 3 for a Benefit of 75.

) Consider a Knapsack Problem for which $\frac{c_1}{w_1} > \frac{c_2}{w_2}$. Show that if the Knapsack can hold d pounds, and $d \geq d^*$ where $d^* = \frac{c_1 w_1}{c_1 - w_1\left(\frac{c_2}{w_2}\right)}$, Then The Optimal solution must use atleast 1 item 1.

What? Solving for Problem 5: $\frac{c_3}{w_3} \geq \frac{c_2}{w_2} \geq \frac{c_1}{w_1}$

$$d^* = \frac{c_3 w_3}{c_3 - w_3\left(\frac{c_2}{w_2}\right)} = \frac{7 \cdot 50}{80 - 7 \cdot \frac{25}{5}} = 23.333$$

when $d = 24$,

$x_1 = 1$ $x_2 = 0$ $x_3 = 3$

$\text{Max} = 162$