EIND 464: Homework 3

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Section 9.2

2. Cleaning pollutants from a river.... want to minimize cost while still meeting pollution removal requirements. no pollutants can be removed from a site that hasn't been built. 80,000 p_1 and 50,000 p_2 units need to be removed from the Momiss River. Let x_{ij} indicate tons of water treated by site i for pollutant j, and y_i indicate the construction of a treatment station. M must be greater than the maximum feasible tonnage of water pumped, which is 400,000 for p_1 and 250,000 for p_2, which is 650,000 tons total.

$$\begin{aligned}\text{Min z } &= 100,000y_1 + 60,000y_2 + 40,000y_3 + 20x_{11} + 20x_{12} + 30x_{21} + 30x_{22} + 40x_{31} + 40x_{32} \\ \text{s.t.} \quad & .4x_{11} + .25x_{21} + .2x_{31} \geq 80,000 \\ & .3x_{12} + .2x_{22} + .25x_{32} \geq 50,000 \\ & x_{11} + x_{12} \leq My_1 \\ & x_{21} + x_{22} \leq My_2 \\ & x_{31} + x_{32} \leq My_3 \\ & \forall\{x_{ij}|y_i\} \geq 0 \text{ , } y_i \text{ Bin}\end{aligned}$$

12. Warehouse problem... Shipping x_{ij} units from four cities (i = 1, 2, 3, 4) to three regions (j = 1, 2, 3). Need to build a warehouse (y_i) before it can be used, If city 1 (NY) then City 4 (Atl), Only 2 warehouses total, and either city 4 (Atl) or city 2 (LA) (Denoted with variable B). Minimize weekly cost as an IP.

$$\begin{aligned}\text{Min z } &= 20x_{11} + 40x_{12} + 50x_{13} + 48x_{21} + 15x_{22} + 26x_{23} + 26x_{31} + 35x_{32} + 18x_{33} + 24x_{41} + 50x_{42} + 35x_{43} \\ & +400y_1 + 500y_2 + 300y_3 + 150y_4 \\ \\ \text{s.t.} \quad & x_{11} + x_{21} + x_{31} + x_{41} \geq 80 \\ & x_{12} + x_{22} + x_{32} + x_{42} \geq 70 \\ & x_{13} + x_{23} + x_{33} + x_{43} \geq 40 \\ \\ & x_{11} + x_{12} + x_{13} \leq 100 \\ & x_{21} + x_{22} + x_{23} \leq 100 \\ & x_{31} + x_{32} + x_{33} \leq 100 \\ & x_{41} + x_{42} + x_{43} \leq 100 \\ \\ & y_4 \geq y_1 \\ & y_1 + y_2 + y_3 + y_4 \leq 2 \\ & y_4 \leq MB \\ & y_2 \leq M(1 - B) \\ & y_i\{i = 1, 2, 3, 4\} \text{ Bin} \\ & B \text{ Bin}\end{aligned}$$

20. “WSP Publishing” selling textbooks.... assignment of two reps in 7 states. Each X_{ij} represents a feasible pair of states to visit. the coefficient is how many students are in those two states. The first batch of constraints sets the maximum representatives per state to 1 (it is always “better” to not visit one state twice). The last objective allows only two assignments total.

$$\text{Max z } = 72X_{AB} + 85X_{AC} + 71X_{BC} + 50X_{BD} + 63X_{CD} + 77X_{DE} + 39X_{DF} + 92X_{DG} + 74X_{EF} + 89X_{FG}$$

$$\text{s.t.} \quad X_{AB} + X_{AC} <= 1 \quad \textit{city A}$$

$$X_{AB} + X_{BC} + X_{BD} <= 1 \quad \textit{city B}$$

$$X_{AC} + X_{BC} <= 1 \quad \textit{city C}$$

$$X_{BD} + X_{CD} + X_{DE} + X_{DF} + X_{DG} <= 1 \quad \textit{city D}$$

$$X_{DE} + X_{EF} <= 1 \quad \textit{city E}$$

$$X_{DF} + X_{EF} + X_{FG} <= 1 \quad \textit{city F}$$

$$X_{DG} + X_{FG} <= 1 \quad \textit{city G}$$

$$X_{AB} + X_{AC} + X_{BC} + X_{BD} + X_{CD} + X_{DE} + X_{DF} + X_{DG} + X_{EF} + X_{FG} <= 2 \quad \textit{two workers}$$

Solution: reach 177 students by visiting $X_{AC} = 85$ and $X_{DG} = 92$.

29. You have been assigned to arrange the songs on the cassette version of Madonna’s latest album. A cassette tape has two sides (1 and 2). The songs on each side of the cassette must total between 14 and 16 minutes in length. The length and type of each song are given in Table 28. The assignment of songs to the tape must satisfy the following conditions:

(a) Each side must have exactly two ballads.
(b) Side 1 must have at least three hit songs.
(c) Either song 5 or song 6 must be on side 1.
(d) If songs 2 and 4 are on side 1, then song 5 must be on side 2.

Explain how you could use an integer programming formulation to determine whether there is an arrangement of songs satisfying these restrictions. **Solution:** The decision variables could be X_{ijk} where X is a binary representation of song i, which is of type j, existing on side k of the cassette. It is not clear what optimization is supposed to occur, but a feasible solution can be found regardless.

$$\text{Max Z} = \sum_i \sum_j \sum_k X_{ijk}$$

Now with ijk, constraints can be set so that each of the conditions is satisfied.

$$s.t. \quad X_{\bullet,Ballad,1} = 2$$

$$X_{\bullet,Ballad,2} = 2$$

(This sets both sides of the cassette to have two ballads)

An “if-then” constraint is needed, which might look like

$$X_{2,\bullet,1} \cdot X_{4,\bullet,1} \geq X_{5,\bullet,2}$$

as well as a pair of “either-or” constraints:

$$X_{5,\bullet,1} \geq My$$

$$X_{6,\bullet,1} \geq M(1-y)$$

38. A company sells seven types of boxes, ranging in volume from 17 to 33 cubic feet. The demand and size of each box are given in Table 39. The variable cost (in dollars) of producing each box is equal to the box’s volume. A fixed cost of $1,000 is incurred to produce any of a particular box. If the company desires, demand for a box may be satisfied by a box of larger size. Formulate and solve (with LINDO, LINGO, or Excel Solver) an IP whose solution will minimize the cost of meeting the demand for boxes.

This problem doesn’t make sense to me... We were given the same problem in Assignment 2 and there is nothing to optimize (and there is no posted solution). Make the smallest box that meets demand. Don’t mess with using bigger boxes for smaller boxes, it’s more expensive and gives no benefit.

Solution:

let x_i denote the quantity of box i to deliver.

$$\text{Min Z } = 1033x_1 + 1030x_2 + 1026x_3 + 1024x_4 + 1019x_5 + 1018x_6 + 1017x_7$$

$$s.t. \quad x_1 >= 400$$

$$x_1 + x_2 >= 700$$

$$x_1 + x_2 + x_3 >= 1200$$

$$x_1 + x_2 + x_3 + x_4 >= 1900$$

$$x_1 + x_2 + x_3 + x_4 + x_5 >= 2100$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 >= 2500$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 >= 2700$$

$$\forall x_i >= 0$$

Z = \$2,766,400 $x_1 = 400$, $x_2 = 300$, $x_3 = 500$, $x_4 = 700$, $x_5 = 200$, $x_6 = 400$, $x_7 = 200$,

Section 9.3

3. Solve Dorian with B&B

6. solve with B&B:

9. During the next five periods, the demands in Table 58 must be met on time. At the beginning of period 1, the inventory level is 0. Each period that production occurs a setup cost of \$250 and a per-unit production cost of \$2 are incurred. At the end of each period a per-unit holding cost of \$1 is incurred.

   (a) Solve for the cost-minimizing production schedule using the following decision variables: x_t = units produced during month t and $y_t = 1$ if any units are produced during period t, $y_t = 0$ otherwise.
   Solution: *Not sure how to incorporate holding costs with just these two DV's.* Adding an Inventory variable I makes sense and allows me to keep track of extra inventory. but that isn't the question.

$$\text{Min Z } = 2\left(x_1 + x_2 + x_3 + x_4 + x_5\right) + 250\left(y_1 + y_2 + y_3 + y_4 + y_5\right)$$

$$s.t. \quad x_1 >= 220$$

$$x_1 + x_2 >= 500$$

$$x_1 + x_2 + x_3 >= 860$$

$$x_1 + x_2 + x_3 + x_4 >= 1000$$

$$x_1 + x_2 + x_3 + x_4 + x_5 >= 1270$$

$$x_1 <= My_1$$

$$x_2 <= My_2$$

$$x_3 <= My_3$$

$$x_4 <= My_4$$

$$x_5 <= My_5$$

   $x_1 = 1270$, z = 2790

   (b) Solve for the cost-minimizing production schedule using the following variables: y_t's defined in part (a) and x_{it} = number of units produced during period i to satisfy period t demand.
   Solution:

   (c) Which formulation took LINDO or LINGO less time to solve?
   Solution: The x_{it} formulation is faster.

   (d) Give an intuitive explanation of why the part (b) formulation is solved faster than the part (a) formulation.
   Solution: While both formulations model the same problem, the 2nd method passes more valuable information to the algorithm and doesn't need to iterate over its old answers as much.

Section 9.4

2.

$$
\begin{aligned}
\text{Min z } &= 3x_1 + x_2 \\
\text{s.t.} \quad & x_1 + 5x_2 \geq 8 \\
& x_1 + 2x_2 \geq 4 \\
\forall \; & x_i >= 0
\end{aligned}
$$

Because the relaxed IP has an integer optimal solution, that is also the IP optimum solution. $Z = 2$, $x_2 = 2$

Section 9.5

2. **Solution:** Bring the Dining room set & the Sofa. Modeling this problem as a knapsack problem assumes that each item has a solid volume and cannot be placed inside of another item. We also assume that if the total volume of items is less than the capacity of the truck, the items will fit. There is no spacial awareness of how volume is distributed.

```julia
using JuMP
using HiGHS
using DataFrames

m = Model(HiGHS.Optimizer);
set_silent(m)

MaxVol= 1100
@variable(m, x[1:5], Bin)
df = DataFrame( "Item" => ["Bedroom set", "Dining room set", "Stereo", "Sofa", "TV set"],
                "Value" => [60, 48, 14, 31, 10],
                "Volume" => [800, 600, 300, 400, 200])


@constraint(m, sum(df.Volume[i] * x[i] for i in 1:5) <= MaxVol)
@objective(m, Max, sum(df.Value[i] * x[i] for i in 1:5))

println(m);
optimize!(m)
# println(solution_summary(m))
items_chosen = [df.Item[i] for i in 1:5 if value(x[i]) > 0]
println("Items to bring: ")
for i in 1: length(items_chosen)
    println(items_chosen[i])
end

```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL Julia REPL (v1.8.5)

```
Max 60 x[1] + 48 x[2] + 14 x[3] + 31 x[4] + 10 x[5]
Subject to
 800 x[1] + 600 x[2] + 300 x[3] + 400 x[4] + 200 x[5] <= 1100.0
 x[1] binary
 x[2] binary
 x[3] binary
 x[4] binary
 x[5] binary

Items to bring:
Dining room set
Sofa
```