Homework 3 Solution

Monday, April 19, 2021 11:29 PM

1. (a) Define $\divideontimes$ as multiplication of entries of two matrices in corresponding position.

$$g(1,1) = \begin{bmatrix} 12 & 10 & 8 \\ 10 & 8 & 10 \\ 8 & 10 & 4 \end{bmatrix} \divideontimes \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 8$$

$$g(1,2) = \begin{bmatrix} 10 & 8 & 6 \\ 8 & 10 & 4 \\ 10 & 4 & 2 \end{bmatrix} \divideontimes \cdots = -28$$

$$g(2,1) = \begin{bmatrix} 10 & 8 & 10 \\ 8 & 10 & 4 \\ 6 & 4 & 2 \end{bmatrix} \divideontimes \cdots = -28$$

$$g(2,2) = \begin{bmatrix} 8 & 10 & 4 \\ 10 & 4 & 2 \\ 4 & 2 & 0 \end{bmatrix} \divideontimes \cdots = 8$$

b)

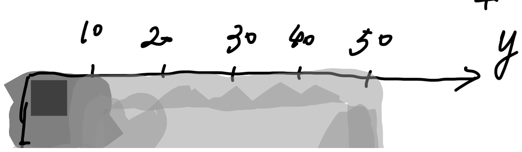
$$h(1,1) = \text{median}\left(\begin{bmatrix} 12 & 10 & 8 \\ 10 & 8 & 10 \\ 8 & 10 & 4 \end{bmatrix} \right) = 10$$

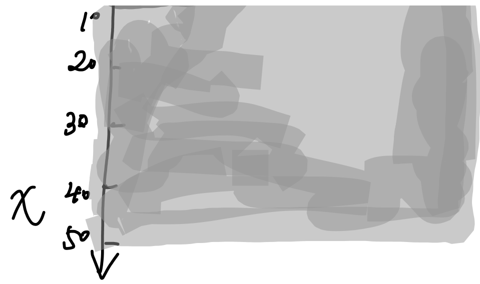
$$h(1,2) = \text{median}\left(\begin{bmatrix} 10 & 8 & 6 \\ 8 & 10 & 4 \\ 10 & 4 & 2 \end{bmatrix} \right) = 8$$

$$h(2,1) = \text{median}\left(\begin{bmatrix} 10 & 8 & 10 \\ 8 & 10 & 4 \\ 6 & 4 & 2 \end{bmatrix} \right) = 8$$

$$h(2,2) = \text{median}\left(\begin{bmatrix} 8 & 10 & 4 \\ 10 & 4 & 2 \\ 4 & 2 & 0 \end{bmatrix} \right) = 4$$

2. a)





b) Around $x=10, y=10$:

	9	10	11
9	0	0	100
10	0	0	100
11	100	100	100

$f(x,y)$

After applying median filter:

	9	10	11
9	0	0	100
10	0	100	100
11	100	100	100

$g(x,y)$

$f(x,y)$ and $g(x,y)$ are different at $x=10$, $y=10$

3. (a) For image 1: at $y=255$, the pixel should be:

y: 254	255	256	257
0	0	200	200

$\Downarrow$ apply the averaging filter

y: 254	255	256	257
0	67	133	200

For image2, at corners the image is:

0	0	200	200
0	0	200	200
200	200	0	0
200	200	0	0

$0 \rightarrow 89$

Image has a value 89 which image1 doesn't.
So the histogram is different.

(b) Matlab code.

4. a) Yes.

First, convolution is linear.

Second, the linear transformation of a linear transformation is linear.

b).

0,0 1	0,1 2	0,2 3	0,3 2	0,4 1
1,0 2	1,1 4	1,2 6	1,3 4	1,4
2,0 3	2,1 6	2,2 9	2,3	2,4
3,0 2	3,1	3,2	3,3	3,4
4,0 1	4,1	4,2	4,3	4,4

Apply once:

$$f'(2,2) = \frac{1}{9}\big(f(1,1) + f(1,2) + f(1,3) + f(2,1) + f(2,2) + f(2,3) + f(3,1) + f(3,2) + f(3,3)\big)$$

Apply twice.

$$f''(2,2) = \frac{1}{9}\left(f'(1,1) + f'(1,2) + f'(1,3) + f'(2,1) + f'(2,2) + f'(2,3) + f'(3,1) + f'(3,2) + f'(3,3)\right)$$

Substitute f' into the equation of f'',

we get

$$\frac{1}{81}\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 4 & 6 & 4 & 2 \\ 3 & 6 & 9 & 6 & 3 \\ 2 & 4 & 6 & 4 & 2 \\ 1 & 2 & 3 & 2 & 1 \end{bmatrix}$$

(c) Using the double filtering,

the image would be more blurry

applying $f(x,y)$ once.