Homework 2 Solution

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1. (a)

$$T(r_k) = \sum_{i=0}^{r_k} h(i) = \sum_{i=0}^{r_k} (10+i)$$

$$= 10(r_k+1) + \frac{r_k(r_k+1)}{2}.$$

$$= \frac{r_k^2}{2} + \frac{21}{2} r_k + 10$$

(b) When $r_k = 49$

$$T(49) = 1725$$

The desired histogram:

$$h_d(r_k') = \frac{1725}{49+1} = 34.5$$

The desired cumulative distribution:

$$T_d(r_k') = \sum_{i=0}^{r_k'} h_d(r_k) = \sum_{i=0}^{r_k'} 34.5 = 34.5(r_k'+1)$$

So:

$$T(r_k) = \frac{r_k^2}{2} + \frac{21}{2} r_k + 10 = T_d(r_k') = 34.5(r_k'+1)$$

$$r_k' = \frac{2}{69}\left(\frac{r_k^2}{2} + \frac{21}{2} r_k + 10\right) - 1$$

$$= \frac{1}{69}(r_k^2 + 21 r_k + 20) - 1$$

$$= \frac{1}{69} r_k^2 + \frac{\cancel{2}7}{23} r_k \cancel{\phi} - \frac{49}{69}$$

$$M(r_k) = r_k' = \frac{1}{69} r_k^2 + \frac{7}{23} r_k - \frac{49}{69} \quad \text{(round)}$$

(c) Although the desired histogram is:

$$h_d(M(r_k)) = 34.5$$

The transformed image cannot exactly

follow the equation.

To find the new histogram. map each r_k in the original image to the closest r_k' in the new image according to:

$M(r_k) = \frac{1}{69} r_k^2 + \frac{7}{23} r_k - \frac{48}{69}$

and get $h(M(r_k))$

2. (a) $E[g'(x,y)] = E[0.5 g_1(x,y) + 0.25 g_2(x,y)]$

$= 0.5 E[g_1(x,y)] + 0.25 E[g_2(x,y)]$

$= 0.5 E[f(x,y) + n_1(x,y)]$

$+ 0.25 E[2f(x,y) + n_2(x,y)]$

$= 0.5 f(x,y) + 0.5 E[n_1(x,y)]$

$+ 0.5 f(x,y) + 0.25 E[n_2(x,y)]$

$= f(x,y)$

(b) $Var[g'(x,y)] = Var[0.5 g_1(x,y) + 0.25 g_2(x,y)]$

$= Var[0.5 f(x,y) + 0.5 n_1(x,y) + 0.25 \times 2 f(x,y) + 0.25 n_2(x,y)]$

Because of the independence assumption

$= 0.5^2 Var[f(x,y)] + 0.5^2 Var[n_1(x,y)]$

$+ 0.5^2 Var[f(x,y)] + 0.25^2 Var[n_2(x,y)]$

$= 0 + \frac{1}{4}\sigma^2 + 0 + \frac{1}{16}\sigma^2$

$= \frac{5}{16}\sigma^2$