EECS 203A Exam #1 April 30, 2020

Name:

I.D.:

This is an Open Book and Open Notes exam. Calculators are allowed. Collaboration with other people is not allowed. GOOD LUCK!

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

Question 10:

Question 11:

Question 12:

TOTAL:

Question 1 (12 points) Let H be a spatial domain operator that maps an input image $I(x, y)$ to an output image $O(x, y)$ where

$$O(x, y) = \sum_{k=0}^{y} I(x-1, k)$$

For all parts of this problem, you may ignore boundary effects.

a) Is H a linear operator?

b) Prove your answer to part a).

c) Can H be represented by a single filter mask?

d) Prove your answer to part c).

Question 2 (12 points) Consider the 4×4 image

2	5	8	11
4	8	12	16
6	11	16	21
8	14	20	26

We would like to shrink the image to 3×3 using bilinear interpolation. Use the method described in class to determine the resulting 3×3 image.

Question 3 (6 points) Let $*$ denote the convolution of an image $f(x, y)$ with a filter $h(x, y)$. Let

$$g(x, y) = h(x, y) * f(x, y)$$

where $h(x, y)$ is known to be

$$h(x, y) = Ae^{-2\pi^2(x^2+y^2)}$$

a) Is it possible to recover $f(x, y)$ from $g(x, y)$?

b) If you answered yes, explain how. If you answer no, explain why not.

Question 4 (6 points) Suppose that the DFT of the 64×64 image $h(x, y)$ is given by

$$H(u, v) = 2\delta(u, v) + 0.5\delta(u, v - 8) + 0.5\delta(u, v - 56) \quad u = 0, 1, 2, \ldots, 63 \quad v = 0, 1, 2, \ldots, 63$$

Find $h(x, y)$.

Question 5 (8 points) Consider the averaging operator T that generates the value at each pixel (x, y) of the output image by averaging the nine pixels centered at (x, y) in the input image. Let Process 1 be the application of a 3×3 median filter to the input image followed by applying the T operator to the image that results from the median filtering. Let Process 2 be the application of the operator T to the input image followed by applying the 3×3 median filter to the output image generated by T.

a) In general, will Process 1 and Process 2 give the same output image for a given input image? Ignore boundary effects.

b) If you answered Yes to part a, prove your answer. If you answered No to part a, provide a counterexample.

Question 6 (14 points) Consider a digital image f with 25 gray levels from 0 to 24. Suppose that the gray level histogram for f is given by

$$h(r_k) = 100 - r_k \qquad\qquad r_k = 0, 1, 2, \ldots, 24$$

a) Find the cumulative distribution $T(r_k)$.

b) Use the method described in class to determine the gray level transformation $M(r_k)$ for $r_k = 0, 1, 2, \ldots, 24$ that corresponds to histogram equalization. You may leave your answer in the form of an equation that can be solved to obtain $M(r_k)$ for each r_k.

c) Will the transformation $M(r_k)$ darken or brighten the image? Explain.

Question 7 (10 points) Consider the filter

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

where $D(u,v) = \sqrt{u^2 + v^2}$

a) Is this filter isotropic? Explain.

b) In words, what will be the effect of applying this filter to an image?

c) Plot $H(u,0)$ as a function of u.

d) Plot $h(x,0)$ as a function of x.

e) In words, what will be the effect of increasing D_0 for this filter?

Question 8 (6 points) Explain why we might use a Butterworth lowpass filter instead of an ideal lowpass filter.

Question 9 (6 points) Consider the gray level transformation

$$s = c\ \ln(1 + r)$$

for mapping an 8-bit image to another 8-bit image.
a) What value should we assign to c so that the dynamic range of r maps to the full dynamic range of s?

b) In words, explain how this transformation will change the appearance of an input image.

Question 10 (8 points) Consider an image $g(x, y)$ defined by

$$g(x, y) = f + n(x, y)$$

where f is a constant and $n(x, y)$ is a zero-mean noise source with variance σ^2 that is independent from pixel to pixel. Suppose that we filter $g(x, y)$ with a 5×5 averaging filter where each of the 25 mask coefficients is $\frac{1}{25}$ to obtain the output image $o(x, y)$. Ignore boundary effects.

a) What is the expected value of $o(x, y)$ at pixel (x, y)?

b) What is the variance of $o(x, y)$ at pixel (x, y)?

Question 11 (6 points) Assume the textbook image coordinate system (x increases going down, y increases going to the right).

```
   ------> y
   |
   |
x  v
```

Consider a spatial domain operator that generates a $M \times N$ output image $g(x, y)$ by shifting each vertical column in the $M \times N$ input image $f(x, y)$ one column to the left. We represent the operator by

$$g(x,y) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s,t) f(x+s, y+t)$$

for $x = 0, 1, 2, \ldots, M-1$ and $y = 0, 1, 2, \ldots, N-1$.

Find $w(s,t)$. Ignore boundary effects.

Question 12 (6 points) Consider a spatial domain operator that generates the value at each pixel (x, y) of a $M \times N$ output image $g(x, y)$ by averaging the 4 (N_4) neighbors of pixel (x, y) in the $M \times N$ input image $f(x, y)$. We represent the operator by

$$g(x, y) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s, t) f(x + s, y + t)$$

for $x = 0, 1, 2, \ldots, M - 1$ and $y = 0, 1, 2, \ldots, N - 1$.

Find $w(s, t)$. Ignore boundary effects.