EECS 203A Exam #1 April 30, 2020

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This is an Open Book and Open Notes exam. Calculators are allowed. Collaboration with other people is not allowed. You have until 12:50 pm California time to submit your exam electronically. Show all of your work. I am available at ghealey@uci.edu if you have questions. GOOD LUCK!

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

Question 10:

Question 11:

Question 12:

TOTAL:

Question 1 (12 points) Let H be a spatial domain operator that maps an input image $I(x, y)$ to an output image $O(x, y)$ where

$$O(x, y) = \sum_{k=0}^{y} I(x-1, k)$$

For all parts of this problem, you may ignore boundary effects.

2 a) Is H a linear operator? Yes

4 b) Prove your answer to part a).

$$H[af(x,y) + bg(x,y)] = \sum_{K=0}^{Y} [af(x-1,K) + bg(x-1,K)]$$

$$= \left(\sum_{K=0}^{Y} af(x-1,K)\right) + \left(\sum_{K=0}^{Y} bg(x-1,K)\right)$$

$$= aH[f(x,y)] + bH[g(x,y)]$$ Linear

2 c) Can H be represented by a single filter mask? No

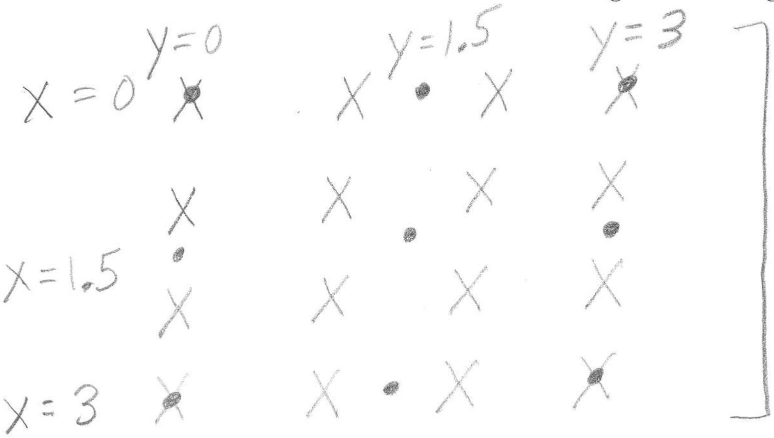
4 d) Prove your answer to part c).

For any value of y, H can be represented by a mask with $y+1$ elements. Since the size of the mask changes with y, H can not be represented by a single filter mask.

Question 2 (12 points) Consider the 4×4 image

$$\begin{array}{c|cccc} x=0 & 2 & 5 & 8 & 11 \\ 1 & 4 & 8 & 12 & 16 \\ 2 & 6 & 11 & 16 & 21 \\ 3 & 8 & 14 & 20 & 26 \\ \hline & 0 & 1 & 2 & 3 \; y \end{array}$$

We would like to shrink the image to 3×3 using bilinear interpolation. Use the method described in class to determine the resulting 3×3 image.



4

X = original samples
• = new samples

Original image is $f(x,y) = 2x + 3y + xy + 2$

Bilinear interpolation is

$$\begin{array}{ccc} f(0,0) & f(0,1.5) & f(0,3) \\ f(1.5,0) & f(1.5,1.5) & f(1.5,3) \\ f(3,0) & f(3,1.5) & f(3,3) \end{array}$$

Resulting image is

$$\begin{array}{ccc} 2 & 6.5 & 11 \\ 5 & 11.75 & 18.5 \\ 8 & 17 & 26 \end{array}$$

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Question 3 (6 points) Let $*$ denote the convolution of an image $f(x,y)$ with a filter $h(x,y)$. Let

$$g(x,y) = h(x,y) * f(x,y)$$

$G(u,v) = H(u,v)\,F(u,v)$

where $h(x,y)$ is known to be

$$h(x,y) = Ae^{-2\pi^2(x^2+y^2)}$$

2 a) Is it possible to recover $f(x,y)$ from $g(x,y)$? Yes.

4 b) If you answered yes, explain how. If you answer no, explain why not.

$F(u,v) = \dfrac{G(u,v)}{H(u,v)}$ where $H(u,v) = \mathcal{F}[h(x,y)]$

$= \text{Gaussian}$

$f(x,y) = \mathcal{F}^{-1}[F(u,v)]$

6 **Question 4 (6 points)** Suppose that the DFT of the 64×64 image $h(x,y)$ is given by

$$H(u,v) = 2\delta(u,v) + 0.5\delta(u,v-8) + 0.5\delta(u,v-56) \quad u = 0,1,2,\ldots,63 \quad v = 0,1,2,\ldots,63$$

Find $h(x,y)$.

$H(u,v) = 2\delta(u,v) + 0.5[\delta(u,v-8) + \delta(u,v+8)]$

$h(x,y) = 2 + \cos\left(\dfrac{2\pi y}{8}\right) \quad x = 0,1,\ldots,63 \quad y = 0,1,\ldots,63$

This uses

$$\cos(2\pi u_0 x + 2\pi v_0 y) \xleftrightarrow{\mathcal{F}} \frac{[\delta(u+Mu_0, v+Nv_0) + \delta(u-Mu_0, v-Nv_0)]}{2}$$

Question 5 (8 points) Consider the averaging operator T that generates the value at each pixel (x, y) of the output image by averaging the nine pixels centered at (x, y) in the input image. Let Process 1 be the application of a 3×3 median filter to the input image followed by applying the T operator to the image that results from the median filtering. Let Process 2 be the application of the operator T to the input image followed by applying the 3×3 median filter to the output image generated by T.

a) In general, will Process 1 and Process 2 give the same output image for a given input image? Ignore boundary effects.

No.

b) If you answered Yes to part a, prove your answer. If you answered No to part a, provide a counterexample.

Suppose the input image is

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Process 1 gives an image of all zeros because the median filter removes the 9.

Applying T to the input image gives

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Process 2 gives

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Process 1 $\neq$ Process 2

Question 6 (14 points) Consider a digital image f with 25 gray levels from 0 to 24. Suppose that the gray level histogram for f is given by

$$h(r_k) = 100 - r_k \qquad r_k = 0, 1, 2, \ldots, 24$$

3 a) Find the cumulative distribution $T(r_k)$.

$$T(r_K) = \sum_{i=0}^{r_K} (100 - i) = 100(r_K + 1) - \frac{r_K(r_K+1)}{2}$$

$$= -\frac{r_K^2}{2} + 99.5 r_K + 100$$

$T(24) = 2{,}200$

$d(r_K) = \frac{2{,}200}{25} = 88$

8 b) Use the method described in class to determine the gray level transformation $M(r_k)$ for $r_k = 0, 1, 2, \ldots, 24$ that corresponds to histogram equalization. You may leave your answer in the form of an equation that can be solved to obtain $M(r_k)$ for each r_k.

Input

r_K	$h(r_K)$	$T(r_K)$
0	100	100
1	99	199
2	98	297
⋮	⋮	⋮
r_K	$100 - r_K$	$-r_K^2/2 + 99.5 r_K + 100$
⋮	⋮	

Desired

r_K	$d(r_K)$	$G(r_K)$
0	88	88
1	88	176
2	88	264
⋮	⋮	⋮
r_K	88	$88(r_K + 1)$
⋮	⋮	⋮

For any r_K, $M(r_K)$ is the value r_K' for which $88(r_K' + 1)$ is closest to $-\frac{r_K^2}{2} + 99.5 r_K + 100$

c) Will the transformation $M(r_k)$ darken or brighten the image? Explain.

1 Brighten.

2 The input image has more pixels at darker gray levels than a uniform distribution.

Question 7 (10 points) Consider the filter

$$H(u, v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

where $D(u, v) = \sqrt{u^2 + v^2}$

a) Is this filter isotropic? Explain.

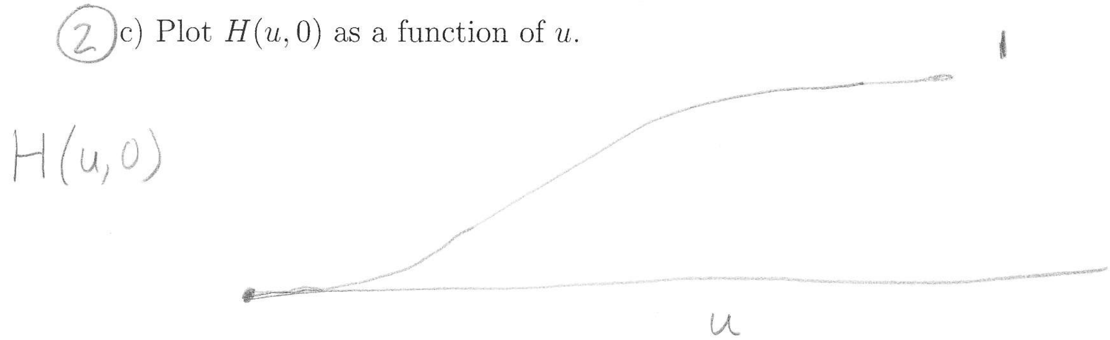
1 Yes.

1 The only dependence on u or v is in $D(u,v)$ which depends only on distance from $(u,v) = (0,0)$ and not on direction.

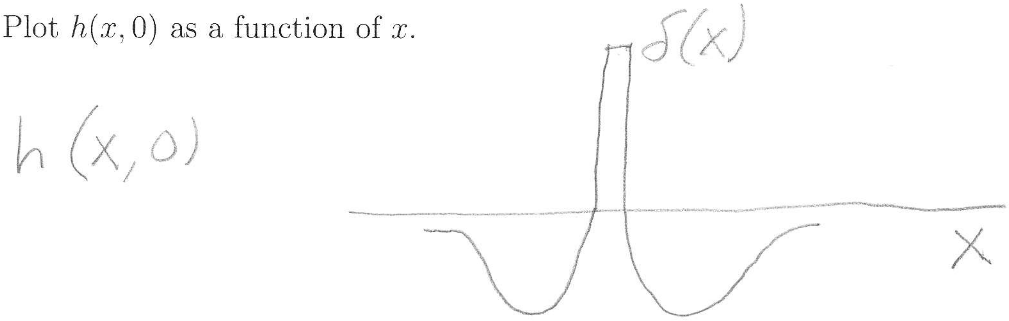
b) In words, what will be the effect of applying this filter to an image?

2 This is a one minus a Gaussian highpass filter.

2 c) Plot $H(u, 0)$ as a function of u.



2 d) Plot $h(x, 0)$ as a function of x.



2 e) In words, what will be the effect of increasing D_0 for this filter?

As D_0 increases, the transition of the highpass filter moves to a higher frequency.

6 **Question 8 (6 points)** Explain why we might use a Butterworth lowpass filter instead of an ideal lowpass filter.

An ideal lowpass filter can cause ringing and negative values in the filtered image.

A Butterworth lowpass filter of low order can avoid ringing and negative values in the filtered image.

Question 9 (6 points) Consider the gray level transformation

$$s = c \ln(1 + r)$$

for mapping an 8-bit image to another 8-bit image.

3 a) What value should we assign to c so that the dynamic range of r maps to the full dynamic range of s?

$r = 0 \rightarrow s = 0$

$r = 255 \rightarrow s = c \ln(256)$

$$c = \frac{255}{\ln(256)}$$

3 b) In words, explain how this transformation will change the appearance of an input image.

The log GLT provides more dynamic range for dark gray levels and less dynamic range for bright gray levels. The log GLT will make the image brighter.

Question 10 (8 points) Consider an image $g(x, y)$ defined by

$$g(x, y) = f + n(x, y)$$

where f is a constant and $n(x, y)$ is a zero-mean noise source with variance σ^2 that is independent from pixel to pixel. Suppose that we filter $g(x, y)$ with a 5×5 averaging filter where each of the 25 mask coefficients is $\frac{1}{25}$ to obtain the output image $o(x, y)$. Ignore boundary effects.

a) What is the expected value of $o(x, y)$? (at pixel (x,y))

$$o(x,y) = \frac{1}{25}\left[\sum_{s=-2}^{2}\sum_{t=-2}^{2}\left(f + n(x+s,y+t)\right)\right]$$

$$= \frac{1}{25}\left[25f + \sum_{s=-2}^{2}\sum_{t=-2}^{2} n(x+s,y+t)\right]$$

Since $n(x,y)$ is zero-mean, $E[o(x,y)] = f$.

b) What is the variance of $o(x, y)$? (at pixel (x,y))

$$VAR[o(x,y)] = VAR\left[\frac{1}{25}\left[\sum_{s=-2}^{2}\sum_{t=-2}^{2}\left(f+n(x+s,y+t)\right)\right]\right]$$

$$= \frac{1}{25^2} VAR\left[\sum_{s=-2}^{2}\sum_{t=-2}^{2} n(x+s,y+t)\right]$$

$$= \frac{1}{25^2}(25\sigma^2) = \frac{\sigma^2}{25}$$ Since noise independent

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Question 11 (6 points) Assume the textbook image coordinate system (x increases going down, y increases going to the right).

```
   ------> y
   |
   |
x  v
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Consider a spatial domain operator that generates a $M \times N$ output image $g(x, y)$ by shifting each vertical column in the $M \times N$ input image $f(x, y)$ one column to the left. We represent the operator by

$$g(x,y) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s,t) f(x+s, y+t)$$

for $x = 0, 1, 2, \ldots, M-1$ and $y = 0, 1, 2, \ldots, N-1$.

Find $w(s, t)$. Ignore boundary effects.

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) \\ f(1,0) & f(1,1) & f(1,2) \\ f(2,0) & f(2,1) & f(2,2) \end{bmatrix}$$

$$g(x,y) = \begin{bmatrix} g(0,0) & g(0,1) & g(0,2) \\ g(1,0) & g(1,1) & g(1,2) \\ g(2,0) & g(2,1) & g(2,2) \end{bmatrix} = \begin{bmatrix} f(0,1) & f(0,2) & * \\ f(1,1) & f(1,2) & * \\ f(2,1) & f(2,2) & * \end{bmatrix}$$

$$g(x,y) = f(x, y+1)$$

$w(0,1) = 1$ all other $w(s,t) = 0$

6 **Question 12 (6 points)** Consider a spatial domain operator that generates the value at each pixel (x, y) of a $M \times N$ output image $g(x, y)$ by averaging the 4 (N_4) neighbors of pixel (x, y) in the $M \times N$ input image $f(x, y)$. We represent the operator by

$$g(x, y) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s, t) f(x + s, y + t)$$

for $x = 0, 1, 2, \ldots, M - 1$ and $y = 0, 1, 2, \ldots, N - 1$.

Find $w(s, t)$. Ignore boundary effects.

$g(x,y) = \frac{1}{4}\left[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)\right]$

$w(1,0) = w(-1,0) = w(0,1) = w(0,-1) = \frac{1}{4}$

all other $w(s,t) = 0$