

EECS 203A: HOMEWORK #3

Due: April 22, 2021

1. Let $f(x, y)$ be the 4×4 digital image

$$\begin{array}{cccc} f(0,0) & f(0,1) & f(0,2) & f(0,3) \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) \end{array} = \begin{array}{cccc} 12 & 10 & 8 & 6 \\ 10 & 8 & 10 & 4 \\ 8 & 10 & 4 & 2 \\ 6 & 4 & 2 & 0 \end{array}$$

a) Let $g(x, y)$ be the digital image that results after filtering $f(x, y)$ with the Laplacian mask with -8 in the center. Find $g(1, 1)$, $g(1, 2)$, $g(2, 1)$, $g(2, 2)$.

b) Let $h(x, y)$ be the digital image that results after filtering $f(x, y)$ with a 3×3 median filter. Find $h(1, 1)$, $h(1, 2)$, $h(2, 1)$, $h(2, 2)$.

2. Consider the 50×50 digital image $f(x, y)$ defined by

$$f(x, y) = \begin{cases} 0 & \text{if } 0 \leq x \leq 10 \text{ and } 0 \leq y \leq 10 \\ 100 & \text{otherwise} \end{cases}$$

a) Draw the image.

b) Let $g(x, y)$ be the output image that results after processing $f(x, y)$ with a 3×3 median filter? Ignore boundary cases. Is $g(x, y)$ the same as $f(x, y)$? Explain.

3. Consider two images. Image1 is 512×512 pixels where the first 256 columns have brightness 0 and the last 256 columns have brightness 200. Image2 is 512×512 pixels with the pattern of a chess board with an 8×8 pattern of 64×64 pixel squares that are alternatively brightness 0 and brightness 200. The histograms of Image1 and Image2 are the same. Suppose that each image is filtered by a 3×3 averaging mask where each weight in the mask is $1/9$. Use pixel replication for the boundaries.

a) Is the histogram of filtered Image1 the same as the histogram of filtered Image2? Explain.

b) If your answer is no, submit a plot of the two histograms.

4. Let $f(x, y)$ be the 3×3 smoothing filter with nine elements each having a value of $1/9$.

a) Suppose that we filter an input image using $f(x, y)$ and then filter the result with $f(x, y)$ again. Is this double filtering process a linear operation on the input image? Explain.

b) If you answered yes to part a, derive the filter mask that corresponds to the double filtering process. If you answered no to part a, explain why not.

c) What will be the difference in the appearance of the filtered image when using $f(x, y)$ versus using the double filtering process?

Computer Problems:

a) Apply an 11×11 linear averaging filter to the triangle image and the cat image. For this filter, you can let all 121 coefficients be 1 and scale the result by $1/121$. Use pixel replication for the boundaries so that the input and output images are the same size. Submit your code and the two displayable filtered images. Also submit a plot of the gray level histogram for the triangle and the cat image and the gray level histogram for the two filtered images.

b) Repeat part a) for an 11×11 median filter. Thus, for parts a) and b), you should generate a total of four displayable images and a total of six distinct plots.