EECS 203A Exam #2 June 9, 2020

Name:

I.D.:

This is an Open Book and Open Notes exam. Calculators are allowed. Collaboration with other people is not allowed. Show all of your work. GOOD LUCK!

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

Question 10:

Question 11:

Question 12:

Question 13:

Question 14:

TOTAL:

Question 1 (6 points) Let $f(x, y)$ be the 4×4 digital image

```
f(0,0) f(0,1) f(0,2) f(0,3)       6   4   2   4
f(1,0) f(1,1) f(1,2) f(1,3)   =   6   4   2   4
f(2,0) f(2,1) f(2,2) f(2,3)       6   4   2   4
f(3,0) f(3,1) f(3,2) f(3,3)       6   4   2   4
```

Find the DFT $F(u, v)$ of $f(x, y)$ for $u = 0, 1, 2, 3$ and $v = 0, 1, 2, 3$. Simplify your answer.

Question 2 (5 points) Consider the three filters arithmetic mean filter (AMF), geometric mean filter (GMF), and the contraharmonic mean filter with Q=1 (CMF1).

a) Which of these filters is linear?

b) Which of these filters is highpass?

c) Which of these filters is the best for salt noise reduction?

d) Which of these filters is the best for pepper noise reduction?

e) Which of these filters is the worst for pepper noise reduction?

Question 3 (4 points) Consider a digital color image $C(x, y)$ represented in terms of its RGB component images $R(x, y), G(x, y), B(x, y)$. We perform histogram equalization on each of the three component images separately to obtain a new color image $C'(x, y)$ with the equalized bands. Can the hue at a pixel (x, y) change when we transform from $C(x, y)$ to $C'(x, y)$? Explain.

Question 4 (6 points) Let $h(x, y)$ be the 64×64 filter defined by

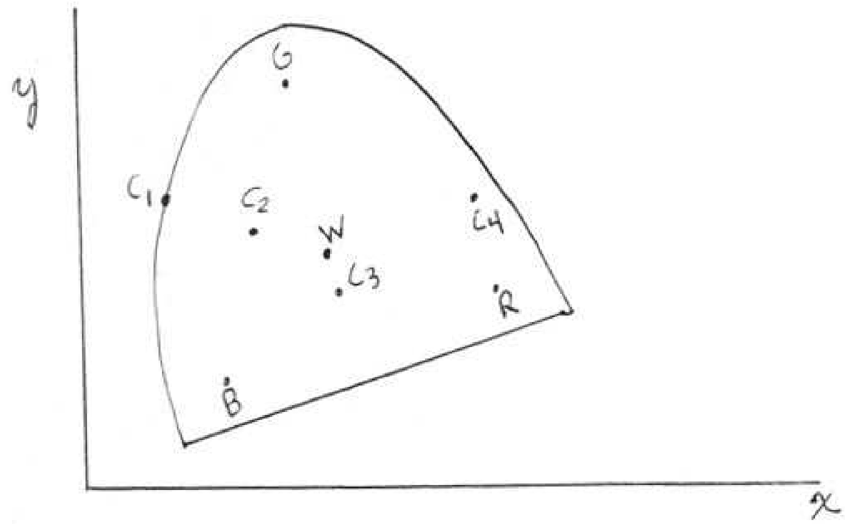
$$h(x, y) = 1 + 2\cos(0.125\pi x) \qquad x = 0, 1, 2, \ldots, 63 \quad y = 0, 1, 2, \ldots, 63$$

a) Compute the DFT $H(u, v)$ for $u = 0, 1, 2, \ldots, 63 \quad v = 0, 1, 2, \ldots, 63$.

b) Suppose that $H(u, v)$ from part a is used to filter a 64×64 input image using multiplication in the frequency domain. Describe the set of all input images for which applying the filter $H(u, v)$ will not change the input image.

Question 5 (8 points) Suppose that a rectangular area in image $f(x, y)$ with vertices $(x, y) = \{(6, 2), (6, 10), (11, 10), (11, 2)\}$ appears distorted in image $f'(x', y')$ with corresponding vertices $(x', y') = \{(5, 4), (5, 12), (11, 10), (11, 2)\}$ Determine the functions $x'(x, y)$ and $y'(x, y)$ using a bilinear model for the distortion. You can specify equations that determine the parameters of the model. You do not need to solve the equations.

Question 6 (5 points) Consider a chromaticity diagram labeled with the chromaticities R,G,B of three monitor primaries and the white point W. Consider the 4 chromaticities c_1, c_2, c_3, c_4. Assume that points that appear to lie on a straight line actually lie on the line.



a) Which chromaticity corresponds to monochromatic light?

b) Which 2 chromaticities have the same hue?

c) Which chromaticity has the highest saturation?

d) Which chromaticities can be matched using combinations of the the 3 primaries?

e) Which chromaticity can be matched using only 2 primaries?

Question 7 (8 points) Consider the 2×2 image defined by

```
f(0,0) f(0,1)        5  11
               =
f(1,0) f(1,1)        2  17
```

We would like to zoom this image to 4×4 using the sampling strategy discussed in class with bilinear interpolation. Find the resulting 4×4 image.

Question 8 (10 points) Let $h1$ and $h2$ be linear spatial filters defined by the masks

```
        1  2  1                      1  2  1
h1 =    2  4  2              h2 =    1  2  1
        1  2  1                      1  2  1
```

a) Suppose that filtering an input image with $h1$ and then filtering the result with $h2$ is equivalent to filtering the input image with $h3$. Find the mask for $h3$.

b) Does $h3$ represent an isotropic filter? Explain.

c) Is $h3$ best described as a smoothing filter or a sharpening filter? Explain.

Question 9 (12 points) Assume the textbook image coordinate system

```
------> y
|
|

x
```

Consider a spatial domain filter that transforms an input image $f(x, y)$ with M rows and N columns into an output image $g(x, y)$ according to

$$g(x, y) = \frac{1}{3}\left[f(x+1, y+1) + f(x, y+1) + f(x-1, y+1)\right]$$

a) Explain in words how the filter transforms the input image.

b) Based on the spatial form of this filter, is it a lowpass or highpass filter? Explain your answer.

c) Given the input image

$$f(x, y) = 50(1 - (-1)^x)$$

What is the output image $g(x, y)$? Ignore boundary effects.

d) Given the input image

$$f(x, y) = 50(1 - (-1)^y)$$

What is the output image $g(x, y)$? Ignore boundary effects.

Question 10 (6 points) Suppose that the pixels $z = (z_r\ z_g\ z_b)^T$ in a color image have a 3×1 mean vector μ and a 3×3 covariance matrix Σ given by $\Sigma = E[(z-\mu)(z-\mu)^T]$

where $\Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

a) Find an orthogonal 3×3 matrix A that implements the transform

$$w = (w_1\ w_2\ w_3)^T = Az$$

so that w_1, w_2, and w_3 are uncorrelated with each other and V_1 and V_2 are both maximized where $V_1 = \text{variance}[w_1]$ and $V_2 = \text{variance}[w_1] + \text{variance}[w_2]$.

b) Find V_1 and V_2 for the matrix A that solves part a.

Question 11 (6 points) Suppose that a color image is defined by

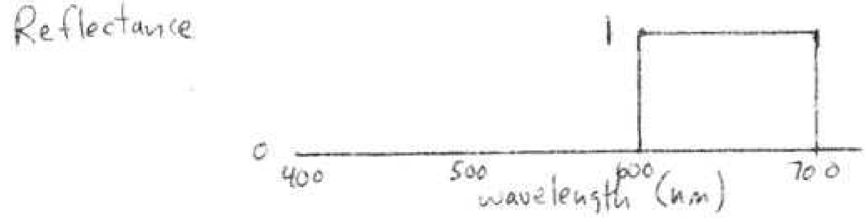
$$\begin{aligned} R(x,y) &= 100 + n_r(x,y) \\ G(x,y) &= 100 + n_g(x,y) \\ B(x,y) &= 100 + n_b(x,y) \end{aligned}$$

where $(n_r\ n_g\ n_b)^T$ is a Gaussian noise vector at each pixel and each of the three components of the noise vector are uncorrelated with each other. Let $E[(n_r\ n_g\ n_b)^T] = (0\ 0\ 0)^T$ and let the variances of the three noise components be $\sigma_r^2 = 1, \sigma_g^2 = 2$, and $\sigma_b^2 = 4$. Let $p(r,g,b)$ be the pdf for the pixels in the color image.

a) Find $p(r,g,b)$.

b) Find the possible values of g and b for which $p(98, 98, 96) = p(100, g, b)$.

Question 12 (6 points) Consider a small area A on a white sheet of paper. Suppose that red ink has the reflectance as a function of wavelength given by



a) Plot the average reflectance as a function of wavelength for the area A if half of the area of A is covered with red ink.

b) Plot the average reflectance as a function of wavelength for the area A if a fraction f of the area of A is covered with red ink.

c) What color will the area A appear if A is completely covered with red ink?

Question 13 (12 points) Consider a spatial domain filter that transforms an input image $f(x, y)$ with M rows and N columns (M and N are even integers) into an output image $g(x, y)$ according to

$$g(x, y) = \frac{1}{3}\left[f(x+1, y+1) + f(x, y+1) + f(x-1, y+1)\right]$$

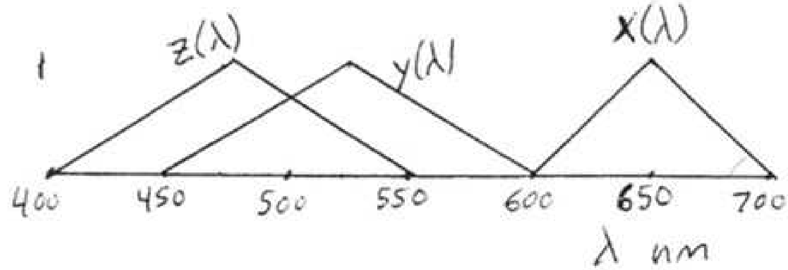
a) Let $G(u, v)$ be the DFT of $g(x, y)$ and let $F(u, v)$ be the DFT of $f(x, y)$. For the filter defined above, find $H(u, v)$ so that $G(u, v) = H(u, v)F(u, v)$. Simplify your answer.

b) Find $|H(u, v)|$

c) Plot $|H(u, v)|$ as a function of u for $v = N/2$.

d) Is this filter both linear and shift invariant? Explain your answer.

Question 14 (6 points) Consider a creature with a trichromatic color vision system having the color matching functions $x(\lambda), y(\lambda), z(\lambda)$ defined by



where $x(\lambda)$ corresponds to primary $R(\lambda)$, $y(\lambda)$ corresponds to primary $G(\lambda)$, and $z(\lambda)$ corresponds to primary $B(\lambda)$.

a) Determine a linear combination of $R(\lambda), G(\lambda), B(\lambda)$ that will match a unit energy monochromatic stimulus at 650nm.

b) Determine a linear combination of $R(\lambda), G(\lambda), B(\lambda)$ that will match a unit energy monochromatic stimulus at 500nm.

c) Is it possible that a monochromatic light at 620nm will look the same as a monochromatic light at 650nm to this creature? Explain your answer.