Trinomial Tree Model for Option Pricing

The trinomial tree model is an upgraded version of the binomial tree model used for option pricing. Simply put, the binomial tree model allows the stock price to either rise or fall at each step, whereas the trinomial tree model introduces a third possibility: the price remains unchanged. This is like predicting future stock movements with not just two extreme outcomes—"up" or "down"—but also considering the possibility that the price stays the same. This improvement makes the calculations more accurate and better reflects real market conditions.

This essay will use an AAPL option to demonstrate the trinomial tree models, implementing them for both European and American styles (with and without dividends). Our model assumes a current stock price (S_0) of $214.10, a strike price (K) of $215.00, a risk-free interest rate (r) of 4.5%, a volatility (σ) of 24.46%, and a time to maturity (T) of 3 months.

To analyze the pricing behavior of AAPL call options under different market assumptions, I applied a trinomial tree model to compare four scenarios: European vs. American options, with and without a 2.0% dividend yield. In the no-dividend case, both European and American options have the same price ($10.750952), indicating no advantage to early exercise—consistent with the theory that American call options on non-dividend-paying stocks should not be exercised early. When a 2.0% dividend yield is introduced, the prices of both types of options drop to $10.175183, a decrease of approximately 5.4%. This reflects the negative impact of dividends on call option values by reducing the expected future stock price. Even with dividends, the American option does not exceed the European in value, suggesting that early exercise remains suboptimal under these parameters. Figures 1–4 show the trinomial trees, displaying stock prices (S) and option values (O) at each node. In the dividend-paying models, the stock price paths shift downward, illustrating the effect of expected dividends. Overall, early exercise does not provide added value for the American option under the given settings.

Following the previous analysis of option pricing under four different scenarios, I further examine the speed of convergence and computational complexity of the trinomial tree model to assess its practicality and efficiency. As shown in picture 5, all models exhibit rapid convergence in the initial steps, with option prices stabilizing after around 10 steps. Models without dividends converge to higher values (around $10.75), while dividend-paying models settle at lower values (around $10.50), reflecting the dampening effect of dividends on option value. The convergence paths of American and European options are nearly identical within each category, indicating that early exercise has little impact on convergence under the current parameters. In terms of computational complexity, execution time increases notably with the number of steps, especially beyond 10 steps. American options consistently require slightly more computation time, with the American model with dividends showing the steepest rise—highlighting its greater computational burden. pictures 6 and 7 provide a closer look at the convergence behavior for the non-dividend and dividend-paying models, respectively. Both show fast early convergence, while execution time fluctuates more at higher step counts, possibly due to limitations in algorithm performance. Picture 8 illustrates how increasing dividend yields gradually reduce option prices. The early exercise premium becomes noticeable only at higher dividend yields (4% to 5%). At a 5% yield, the American call option shows a modest premium of about $0.03 over the European call, indicating that the early exercise feature starts to add tangible value.

After evaluating the performance of the trinomial model itself, we extend the analysis by introducing the binomial tree model to provide a comprehensive comparison of convergence speed and computational efficiency between the two lattice approaches. As shown in Picture 10, European and American options priced under the binomial model are consistently higher than their trinomial counterparts, especially at lower step counts, where the binomial model exhibits a more pronounced tendency to overestimate. A structural comparison between Picture 11 and 1–4 reveals that binomial trees have only two branches per node, resulting in fewer nodes overall. While this reduces computational burden, it also introduces greater pricing deviations. Picture 12 presents a direct comparison of convergence behavior and computational complexity. At the same number of steps, trinomial models demonstrate more stable and smoother convergence, whereas binomial models show a characteristic zigzag pattern in error reduction. Although both models reach similar error levels by around 30 steps, the trinomial model converges in a more consistent and predictable manner. Theoretical complexity analysis supports this observation: at n = 20, a binomial tree has approximately 231 nodes, while a trinomial tree has 441 nodes—nearly double the computational load. Picture 13 further illustrates this trend through a relative error analysis, confirming the trinomial model's superior accuracy and smoother convergence path. This makes the trinomial approach particularly suitable for pricing tasks where stability and precision are critical.

To sum up, In the comparative study of pricing AAPL options, the trinomial tree model demonstrated a more stable convergence, typically stabilizing after 10-15 steps, and it can capture more nuanced market states, including dividend payments. Particularly, when the dividend yield is relatively high (around 4-5%), American call options may gain additional value from early exercise, whereas in low or no dividend scenarios, there is almost no difference between European and American pricing. In contrast, although the binomial tree model requires less computational effort, it often produces higher option valuations, indicating that the model's structure has a substantial impact on the final price. It is also noteworthy that the trinomial tree model requires roughly twice as many nodes as the binomial tree, leading to higher computational costs, but its error convergence is smoother, thereby offering more consistent pricing results to some extent.