

# The Welfare Economics of Default Options in 401(k) Plans

## Appendix

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## 1 Proofs

### Proof of Theorem 1

Let  $m$  denote a monetary transfer, and let  $X(m)$  and  $f$  denote the individual's opportunity set and decision frame, respectively. For any alternative bundle  $x$ ,<sup>1</sup>

$$EV_A(x) = \inf \{m \mid yP^*x \text{ for all } m' \geq m \text{ and } y \in C(X(m'), f)\}$$

and

$$EV_B(x) = \sup \{m \mid xP^*y \text{ for all } m' \leq m \text{ and } y \in C(X(m'), f)\}$$

First we show that if  $P_i^*$  is transitive, then  $zP_i^*x$  implies  $EV_{Ai}(z) \geq EV_{Ai}(x)$  and  $EV_{Bi}(z) \geq EV_{Bi}(x)$ . Choose any  $\varepsilon > 0$ . By definition,  $yP_i^*z$  for all  $m' \geq EV_{Ai}(z) + \varepsilon$  and  $y \in C(X(m'), f)$ . Thus, by transitivity,  $yP_i^*x$  for all  $m' \geq EV_{Ai}(z) + \varepsilon$  and  $y \in C(X(m'), f)$ , which implies  $EV_{Ai}(x) \leq EV_{Ai}(z)$ . Similarly, by definition,  $xP_i^*y$  for all  $m' \leq EV_{Ai}(x) - \varepsilon$  and  $y \in C(X(m'), f)$ . Thus, by transitivity,  $zP_i^*y$  for all  $m' \leq EV_{Ai}(x) - \varepsilon$  and  $y \in C(X(m'), f)$ , which implies  $EV_{Bi}(z) \geq EV_{Bi}(x)$ .

Next choose any  $x' \in X_M$ . If  $x'$  is a weak generalized Pareto optimum we are done, so suppose it is not. Consider the (necessarily) non-empty set  $U = \{y \in X \mid yP_i^*x' \text{ for all } i\}$ .

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<sup>1</sup>The definitions given here are special cases of the definitions in Bernheim and Rangel (2009), in that here the alternative to the status quo is a specific bundle  $x$ , rather than an alternative opportunity set.

Choose any individual  $j$  and consider some  $z'$  and  $f$  such that  $(U, f) \in \mathcal{G}$  and  $z' \in C_j(U, f)$ .<sup>2</sup> We claim that  $z'$  is a weak generalized Pareto optimum in  $X$ . If it were not, then there would be some  $w$  such that  $wP_i^*z'$  for all  $i$ . By the transitivity of  $P_i^*$ , we would then have  $w \in U$ , which contradicts  $z' \in C_j(U, f)$  (because in particular  $wP_j^*z'$ ). From our first step, we then have  $EV_{Ai}(z') \geq EV_{Ai}(x')$  and  $EV_{Bi}(z') \geq EV_{Bi}(x')$  for all  $i$ , from which it follows that

$$\sum_i (\lambda_{Ai}EV_{Ai}(z') + \lambda_{Bi}EV_{Bi}(z')) \geq \sum_i (\lambda_{Ai}EV_{Ai}(x') + \lambda_{Bi}EV_{Bi}(x'))$$

Consequently,  $z' \in X_M$ .  $\square$

### Proof of Theorem 2

Let  $m^0(d, \theta)$  be the equivalent variation associated with choosing the default; i.e., the solution to:

$$V(0, 1 + m^0(d, \theta), \theta) = V(d, 1 - \tau(d), \theta). \quad (1)$$

Also let  $m^1(\theta, \gamma)$  be the equivalent variation associated with opting out; i.e., the solution to:

$$V(0, 1 + m^1(\theta, \gamma), \theta) = V(x^*(\theta), 1 - \tau(x^*(\theta)), \theta) - \gamma. \quad (2)$$

Our assumptions on  $V$  guarantee existence and uniqueness of the solutions, as well as continuity of the resulting functions.

Given the compactness of  $[0, \bar{\gamma}] \times \Theta$ , there exists  $(\gamma', \theta')$  that minimizes  $m^1(\theta, \gamma)$  on that domain; moreover, because  $V(0, 0, \theta') = -\infty$  while  $V(x^*(\theta'), 1 - \tau(x^*(\theta')), \theta') - \gamma'$  is finite, we know that  $m_L \equiv m^1(\gamma', \theta') > -1$ . Likewise, given the compactness of  $[0, \bar{x}] \times \Theta$ ,  $m^0(d, \theta)$  achieves a maximum,  $m_H$ , on its domain. Trivially,  $m_L < m_H$ . Because  $V$  is continuously differentiable and  $[m_L, m_H] \times \Theta$  is compact,  $V_z(0, 1 + m, \theta)$  has a minimum,  $v_L > 0$  (recall that  $V$  is strictly increasing in  $z$ ) and a maximum,  $v_H$ , on that domain.

Define  $Q(d)$  as the set of values of  $(\theta, \gamma)$  for which the worker elects the default; i.e.,  $(\theta, \gamma)$  such that

$$V(d, 1 - \tau(d), \theta) \geq V(x^*(\theta), 1 - \tau(x^*(\theta)), \theta) - \gamma,$$

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<sup>2</sup>Here we are employing the assumptions, stated in BR, that (i)  $C(G)$  is non-empty for all  $G \in \mathcal{G}^*$ , and (ii) for every set  $Z$  there exists a frame  $f$  such that  $(Z, f) \in \mathcal{G}$ .

or equivalently

$$m^0(d, \theta) \geq m^1(\theta, \gamma).$$

Aggregate worker surplus is given by:

$$\int_{\Omega} m^1(\theta, \gamma) dH(\xi) + \int_{Q(d)} [m^0(d, \theta) - m^1(\theta, \gamma)] dH(\xi).$$

Only the second term, which measures the incremental benefit received by workers who elect the default, varies with  $d$ . Thus the worker-surplus maximization problem is:

$$\max_d \int_{Q(d)} [m^0(d, \theta) - m^1(\theta, \gamma)] dH(\xi) \quad (3)$$

Let  $\phi(x)$  denote the fraction of individuals for whom  $x^*(\theta) = x$ . Note that  $\phi(x)$  is strictly positive for  $x \in \mathcal{A}$  and zero otherwise. Let  $\phi^* \equiv \max_{d \in \mathcal{A}} \phi(d)$ .

Consider any  $d \in \mathcal{A}$ . For any individual with  $x^*(\theta) = d$ , we have

$$V(0, 1 + m^0(d, \theta), \theta) - V(0, 1 + m^1(\theta, \gamma), \theta) = \gamma.$$

It follows that

$$[m^0(d, \theta) - m^1(\theta, \gamma)] v_H \geq \gamma.$$

Consequently, we have

$$\int_{Q(d)} [m^0(d, \theta) - m^1(\theta, \gamma)] dH^\theta(\theta) dH_k^\gamma(\gamma) \geq \frac{\phi(d)\gamma_k}{v_H}. \quad (4)$$

Now consider any  $d \notin \mathcal{A}$ . From equations (1) and (2), we see that, for all  $(\gamma, \theta) \in Q(d)$ ,

$$V(0, 1 + m^0(d, \theta), \theta) - V(0, 1 + m^1(\theta, \gamma), \theta) \leq \gamma$$

(where we have used the fact that  $V(x^*(\theta), 1 - \tau(x^*(\theta)), \theta) \geq V(d, 1 - \tau(d), \theta)$ ). It follows that

$$[m^0(d, \theta) - m^1(\theta, \gamma)] v_L \leq \gamma.$$

Consequently,

$$\int_{Q(d)} [m^0(d, \theta) - m^1(\theta, \gamma)] dH^\theta(\theta) dH_k^\gamma(\gamma) \leq \frac{\bar{\gamma}}{v_L} \int_{Q(d, \bar{\gamma})} dH^\theta(\theta). \quad (5)$$

where  $Q(d, \gamma) \subset \Theta$  denotes the opt-in set for a fixed value of  $\gamma$ , and where we have used the fact that an increase in  $\gamma$  expands the set  $Q(d, \gamma)$ .

Now suppose the theorem is false. Then there is some sequence  $H_k^\gamma$  with  $\bar{\gamma}_k \rightarrow 0$  and  $\gamma_k/\bar{\gamma}_k > e^* > 0$ , and an associated sequence of optimal defaults  $d_k \notin \mathcal{A}$  with  $d_k \rightarrow d^* \notin \mathcal{A}$ . Plainly, from (4) and (5), we must have, for all  $k$ ,

$$\int_{Q(d_k, \bar{\gamma}_k)} dH^\theta(\theta) \geq \frac{v_L}{v_H} \phi^* e^* > 0.$$

Accordingly, we will introduce a contradiction by demonstrating that  $\int_{Q(d_k, \bar{\gamma}_k)} dH^\theta(\theta) \rightarrow 0$ .

We claim that, if  $d_k \rightarrow d^* \notin \mathcal{A}$ , then for all  $\varepsilon > 0$  there exists  $K^\varepsilon$  such that for  $k > K^\varepsilon$  all those with ideal points outside  $(d^* - \varepsilon, d^* + \varepsilon)$  opt out. We prove this claim in four steps.

Step 1: With a default of  $d^* - \frac{\varepsilon}{2}$ , there exists  $K_L^\varepsilon$  such that for  $k > K_L^\varepsilon$ , all workers for whom  $x^*(\theta) \leq d^* - \varepsilon$  opt out.

Because  $x^*(\theta)$  is continuous and  $\Theta$  compact, we know that  $\{\theta \mid x^*(\theta) \leq d^* - \varepsilon\}$  is compact. Thus, we can define

$$\vartheta_L = \max_{\theta \in \{\theta' \mid x^*(\theta') \leq d^* - \varepsilon\}} \left[ V(x^*(\theta), 1 - \tau(x^*(\theta)), \theta) - V(d^* - \frac{\varepsilon}{2}, 1 - \tau(d^* - \frac{\varepsilon}{2}), \theta) \right].$$

Furthermore, because  $x^*(\theta)$  is unique, we necessarily have  $\vartheta_L > 0$  (otherwise we would have  $x^*(\theta) = d^* - \frac{\varepsilon}{2}$  for some  $\theta \in \{\theta' \mid x^*(\theta') \leq d^* - \varepsilon\}$ ). Step 1 then follows from the fact that there exists  $K_L^\varepsilon$  such that  $\bar{\gamma}_k < \vartheta_L$  for all  $k > K_L^\varepsilon$ .

Step 2: With a default of  $d^* + \frac{\varepsilon}{2}$ , there exists  $K_H^\varepsilon$  such that for  $k > K_H^\varepsilon$ , all workers for whom  $x^*(\theta) \geq d^* + \varepsilon$  opt out.

The proof mirrors that of Step 1. The set  $\{\theta \mid x^*(\theta) \geq d^* + \varepsilon\}$  is also compact, so we define

$$\vartheta_H = \max_{\theta \in \{\theta' \mid x^*(\theta') \geq d^* + \varepsilon\}} \left[ V(x^*(\theta), 1 - \tau(x^*(\theta)), \theta) - V(d^* + \frac{\varepsilon}{2}, 1 - \tau(d^* + \frac{\varepsilon}{2}), \theta) \right],$$

and observe that  $\vartheta_H > 0$ . Step 2 then follows from the fact that there exists  $K_H^\varepsilon$  such that  $\bar{\gamma}_k < \vartheta_H$  for all  $k > K_H^\varepsilon$ .

Step 3: With any default  $d \in [d^* - \frac{\varepsilon}{2}, d^* + \frac{\varepsilon}{2}]$  and  $k > \max\{K_L^\varepsilon, K_H^\varepsilon\}$ , all workers for whom  $x^*(\theta) \notin (d^* - \varepsilon, d^* + \varepsilon)$  opt out.

Consider a worker for whom  $x^*(\theta) \leq d^* - \varepsilon$ . By Step 1, for  $k > K_L^\varepsilon$  we know that

$$V(x^*(\theta), 1 - \tau(x^*(\theta)), \theta) - \bar{\gamma}_k > V(d^* - \frac{\varepsilon}{2}, 1 - \tau(d^* - \frac{\varepsilon}{2}), \theta) \quad (6)$$

With  $d \in [d^* - \frac{\varepsilon}{2}, d^* + \frac{\varepsilon}{2}]$ , we also have

$$V(d^* - \frac{\varepsilon}{2}, 1 - \tau(d^* - \frac{\varepsilon}{2}), \theta) \geq V(d, 1 - \tau(d), \theta) \quad (7)$$

To see why, let  $q \in (0, 1)$  satisfy  $qx^*(\theta) + (1 - q)d = d^* - \frac{\varepsilon}{2}$ , and define  $\tilde{z} = 1 - q\tau(x^*(\theta)) - (1 - q)\tau(d)$ . Because  $V$  is quasiconcave,

$$V(d^* - \frac{\varepsilon}{2}, \tilde{z}, \theta, 0) \geq \min\{V(x^*(\theta), 1 - \tau(x^*(\theta)), \theta), V(d, 1 - \tau(d), \theta)\} = V(d, 1 - \tau(d), \theta)$$

Because  $\tau$  is convex,  $V(d^* - \frac{\varepsilon}{2}, 1 - \tau(d^* - \frac{\varepsilon}{2}), \theta) \geq V(d^* - \frac{\varepsilon}{2}, \tilde{z}, \theta, 0)$ . Combining these inequalities yields (7). Combining (6) and (7), we obtain

$$V(x^*(\theta), 1 - \tau(x^*(\theta)), \theta) - \bar{\gamma}_k > V(d, 1 - \tau(d), \theta),$$

which implies that the worker opts out of  $d$ , as desired.

The case of any worker for whom  $x^*(\theta) \geq d^* + \varepsilon$  is completely analogous, but employs Step 2 instead of Step 1.

Step 4: Now we prove the claim. Because  $d_k \rightarrow d^*$ , there exists  $K_I^\varepsilon$  such that, for  $k > K_I^\varepsilon$ , we have  $d_k \in [d^* - \frac{\varepsilon}{2}, d^* + \frac{\varepsilon}{2}]$ . Defining  $K^\varepsilon = \max\{K_L^\varepsilon, K_H^\varepsilon, K_I^\varepsilon\}$ , we see that for  $k > K^\varepsilon$  and with a default rate of  $d_k$ , all workers for whom  $x^*(\theta) \notin (d^* - \varepsilon, d^* + \varepsilon)$  opt out.

Having established the claim, we now complete the proof of the theorem. If  $d^* \notin \mathcal{A}$ , then the measure of workers with ideal points in  $(d^* - \varepsilon, d^* + \varepsilon)$ , call it  $y(\varepsilon)$ , converges to zero along with  $\varepsilon$ . But plainly  $y(\varepsilon) \geq \int_{Q(d_k, \bar{\gamma}_k)} dH^\theta(\theta)$  for  $k > K^\varepsilon$ . Consequently, we have  $\int_{D(d_k, \bar{\gamma}_k)} dH^\theta(\theta) \rightarrow 0$ , and thus the desired contradiction.  $\square$

### Proof of Theorem 3

With zero opt-out costs,  $EV$  evaluated in frame  $f$  is given by the value of  $m_A^1$  satisfying

$$V(0, 1 + m_A^1, \theta, f) = V(x^*(\theta, d), 1 - \tau(x^*(\theta, d)), \theta, f)$$

Because  $V$  is strictly increasing in  $z$ , the value of  $d$  that maximizes the RHS also maximizes  $EV$  evaluated in frame  $f$ . By definition, the solution to  $\max_{x \in X} V(x, 1 - \tau(x), \theta, f)$  is  $x = x^*(\theta, f)$ . It follows immediately that the solution to  $\max_{d \in X} V(x^*(\theta, d), 1 - \tau(x^*(\theta, d)), \theta, f)$  is  $d = f$ . Thus, because  $EV_A$  is evaluated from the perspective of frame  $f = \bar{x}$ , it is maximized by setting  $d = \bar{x}$ , and because  $EV_B$  is evaluated from the perspective of frame  $f = 0$ , it is maximized by setting  $d = 0$ .

We complete the proof by showing that  $EV_A$  and  $EV_B$  are respectively non-decreasing and non-increasing in  $d$  on  $[0, \bar{x}]$ . First observe that, as a consequence of our monotonicity assumption,  $x^*(\theta, d)$  is non-decreasing in  $d$ . Second, note that  $V(x, 1 - \tau(x), \theta, 0)$  is non-increasing and  $V(x, 1 - \tau(x), \theta, \bar{x})$  non-decreasing in  $x$  on  $[x^*(\theta, 0), x^*(\theta, \bar{x})]$ . To see why, consider any  $x', x''$  with  $x^*(\theta, \bar{x}) \geq x'' > x' \geq x^*(\theta, 0)$ . Let  $z' = 1 - \tau(x')$ ,  $z'' = 1 - \tau(x'')$ , and

$$\tilde{z} = (1 - \tau(x'')) \frac{x' - x^*(\theta, 0)}{x'' - x^*(\theta, 0)} + (1 - \tau(x^*(\theta, 0))) \frac{x'' - x'}{x'' - x^*(\theta, 0)}.$$

Because  $V$  is quasiconcave,

$$V(x', \tilde{z}, \theta, 0) \geq \min \{V(x^*(\theta, 0), 1 - \tau(x^*(\theta, 0)), \theta, 0), V(x'', 1 - \tau(x''), \theta, 0)\} = V(x'', 1 - \tau(x''), \theta, 0).$$

Because  $\tau$  is convex,

$$V(x', 1 - \tau(x'), \theta, 0) \geq V(x', \tilde{z}, \theta, 0).$$

Combining these inequalities, we have

$$V(x', 1 - \tau(x'), \theta, 0) \geq V(x'', 1 - \tau(x''), \theta, 0),$$

as desired. An analogous argument establishes  $V(x', 1 - \tau(x'), \theta, \bar{x}) \leq V(x'', 1 - \tau(x''), \theta, \bar{x})$ .

Third, it follows as a consequence of the first two steps that  $V(x^*(\theta, d), 1 - \tau(x^*(\theta, d)), \theta, 0)$

is non-increasing and  $V(x^*(\theta, d), 1 - \tau(x^*(\theta, d)), \theta, \bar{x})$  non-decreasing in  $d$  on  $[0, \bar{x}]$ . The desired properties then follow from the fact that  $V(0, 1 + m_A^1, \theta, f)$  is non-decreasing in  $m_A^1$ .

□

### Proof of Theorem 4

Throughout this proof, we use  $i$  to denote a particular individual. BR define the relation  $R_i^*$  as follows:  $xR_i^*y$  iff  $y \in C_i(X, f)$  implies  $x \in C_i(X, f)$  for all  $(X, f) \in \mathcal{G}$ . Also,  $x$  is a weak generalized Pareto improvement over  $y$  iff  $xR_i^*y$  for every individual and  $xP_i^*y$  for some individual.

Part 1: Regardless of whether the welfare-relevant domain is restricted or unrestricted, offering a plan with the  $d = 0$ , where choices are made in frame  $f_D \geq f_M$  for cases of frame-dependent weighting, yields a weak generalized Pareto improvement over no plan.

Partition the set of employees into two groups, those who opt out and those who do not (both of which have positive measure under our assumptions). Those who do not opt out receive the bundle  $(e, x, z) = (0, 0, 1)$  both with and without the plan. By definition,  $(0, 0, 1)R_i^*(0, 0, 1)$ . A worker who opts out chooses some bundle  $(e', x', z')$ , where  $x' > 0$  and  $z' < 1$ , over the bundle  $(0, 0, 1)$ . With anchoring, the choice is made in frame  $f = d = 0$ , and our monotonicity assumption implies that the same worker would choose  $(e', x', z')$  over  $(0, 0, 1)$  in any frame  $f > 0$ . With frame-dependent weighting, the choice is made in some  $f_D \geq f_M$ , and  $\delta^i(f') \leq \delta^i(f_M) \leq \delta^i(f_D)$  for any welfare-relevant frame  $f'$  implies that the same worker would choose  $(e', x', z')$  over  $(0, 0, 1)$  in  $f'$ . Thus, we have  $(e', x', z')P_i^*(0, 0, 1)$  for those who opt out.<sup>3</sup> The desired conclusion follows directly.

Part 2: Regardless of whether the welfare-relevant domain is restricted or unrestricted, and regardless of the prevailing choice frame for the cases of frame-dependent weighting, offering a plan with  $d > 0$  does not yield a weak generalized Pareto improvement over no plan.

Consider the set of workers for whom  $x^*(\theta^i, \bar{x}) = 0$  in the case of anchoring, and  $x^*(\theta^i) = 0$

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<sup>3</sup>The same reasoning implies that, for those who are willing to either opt out or choose the default, we have  $(e', x', z')R_i^*(0, 0, 1)$ .

in the case frame-dependent weighting (both of which have positive measure under our assumptions). In the prevailing choice frame, call it  $f'$ , such workers either opt out to  $x = 0$  and receive the bundle  $(e', 0, 1)$  (in the case of anchoring, any workers opting out would choose  $x = 0$  because, by our monotonicity requirement,  $x^*(\theta^i, \bar{x}) = 0$  implies  $x^*(\theta^i, f) = 0$  for all  $f$ , including  $f'$ ), or fail to opt out and receive the bundle  $(0, d, 1 - \tau(d))$ . In the first case  $(0, 0, 1)P_i^*(e', 0, 1)$ , and in the second  $(0, 0, 1)P_i^*(0, d, 1 - \tau(d))$  (in the cases of frame-dependent weighting because  $x^*(\theta^i) = 0$ , and in the case of anchoring because  $x^*(\theta^i, \bar{x}) = 0$  implies  $x^*(\theta^i, f) = 0$  for all  $f$ ). The desired conclusion follows directly.

Part 3: For models with frame-dependent weighting, a plan with  $d = 0$  does not achieve a weak generalized Pareto improvement over no plan if choices are made in some frame  $f' < f_M$ .

Suppose  $d = 0$  and that choices are made in some frame  $f' < f_M$ . Consider the set of workers for whom  $\gamma^i \in \left(\frac{1}{\delta^i(f_M)}\Delta(\theta^i, d, \pi), \frac{1}{\delta^i(f')} \Delta(\theta^i, d, \pi)\right)$ , which has positive measure (because the interval is open for all  $\delta^i$  and  $\theta^i$ ). Because the choice frame is  $f'$ , any such worker opts out and receives the bundle  $(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i)))$ . But the same worker would choose the bundle  $(0, 0, 1)$  over  $(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i)))$  in frame  $f_M$ . Thus, we do *not* have  $(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i))) R_i^*(0, 0, 1)$ .

Part 4: For models with frame-dependent weighting, fixing  $d = 0$ , a plan with choices made in frame  $f_M$  achieves a weak generalized Pareto improvement over any plan with choice made in frame  $f' > f_M$ .

Suppose  $d = 0$  and consider the choice frames  $f'$  and  $f_M$  with  $f' > f_M$ . We partition the set of workers as follows: for group  $L$ ,  $\gamma^i < \frac{1}{\delta^i(f')} \Delta(\theta^i, d, \pi)$ ; for group  $I$ ,  $\gamma^i \in \left(\frac{1}{\delta^i(f')} \Delta(\theta^i, d, \pi), \frac{1}{\delta^i(f_M)} \Delta(\theta^i, d, \pi)\right)$ ; and for group  $H$ ,  $\gamma^i > \frac{1}{\delta^i(f_M)} \Delta(\theta^i, d, \pi)$ . (We will consider workers at the boundaries between these groups separately below.) For the same reasons as in Part 3, each of these groups has positive measure. Those in group  $L$  opt out and receive the bundle  $(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i)))$  in both frames, and by definition  $(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i))) R_i^*(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i)))$ . Those in group  $H$  end up with



$(0, d, 1 - \tau(d))$  in both frames because they do not opt out, and by definition  $(0, d, 1 - \tau(d)) R_i^*(0, d, 1 - \tau(d))$ . Those in group  $I$  opt out in frame  $f_M$ , receiving bundle  $(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i)))$ , and do not opt out in frame  $f'$ , receiving bundle  $(0, d, 1 - \tau(d))$ . Moreover, all such workers would choose  $(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i)))$  over  $(0, d, 1 - \tau(d))$  in all frames  $f < f_M$ . Thus,  $(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i))) P_i^*(0, d, 1 - \tau(d))$ . We treat workers at the boundary between groups  $L$  and  $I$  the same as members of group  $I$  if they opt out in frame  $f'$ , and the same as members of group  $L$  if they do not opt out in frame  $f'$ . We treat workers at the boundary between groups  $I$  and  $H$  the same as members of group  $H$  if they do not opt out in frame  $f_M$ ; if they do opt out in frame  $f_M$ , we still have  $(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i))) R_i^*(0, d, 1 - \tau(d))$  because they choose  $(e', x^*(\theta^i), 1 - \tau(x^*(\theta^i)))$  over  $(0, d, 1 - \tau(d))$  strictly in all frames  $f < f_M$ , and weakly in frame  $f_M$ . The desired conclusion follows directly.  $\square$

## 2 Additional simulation results

In this section we provide the following supplementary figures, all of which pertain to models of frame-dependent weighting. Figures A.1 through A.4 show  $EV_A$  and  $EV_B$  as functions of the default rate for, respectively, decisions made in the naturally occurring frame with an employee match, decisions made in the naturally occurring frame without an employee match, decisions made in the alternative frame (99% reduction in as-if opt-out costs) with an employee match, and decisions made in the alternative frame without an employee match. Figure A.5 shows the overall opt-out frequencies as functions of the default rate for decisions made in the naturally occurring and alternative frames with an employee match; Figure A.6 shows the same opt-out frequencies without an employee match. Figures 2, 3, 6, and 7 in the text contain the same information as the figures below, except that here we have extended the range of the default rates to 90%.

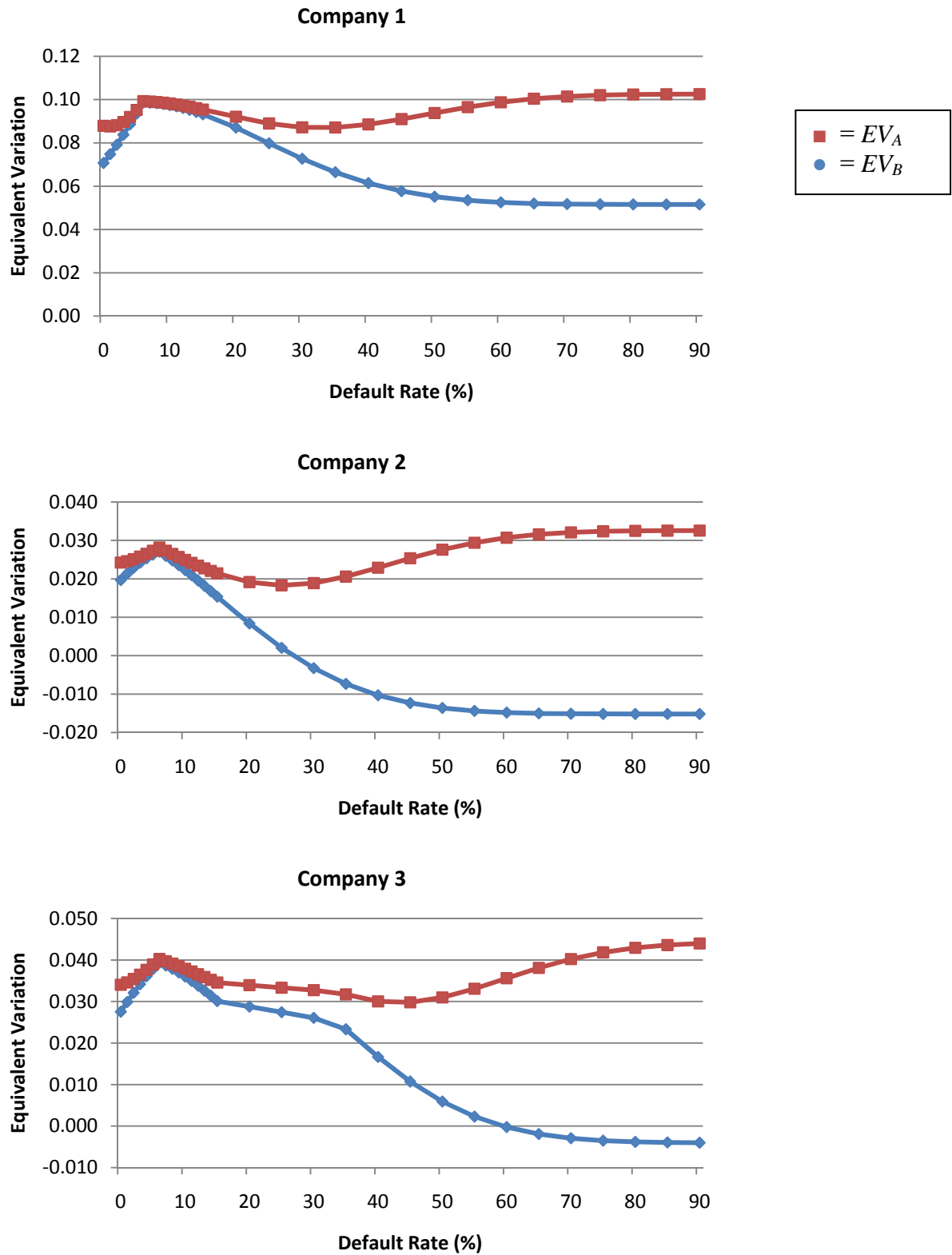


Figure A.1: Average  $EV_A$  and  $EV_B$  for choices made in naturally occurring frame, with an employer match

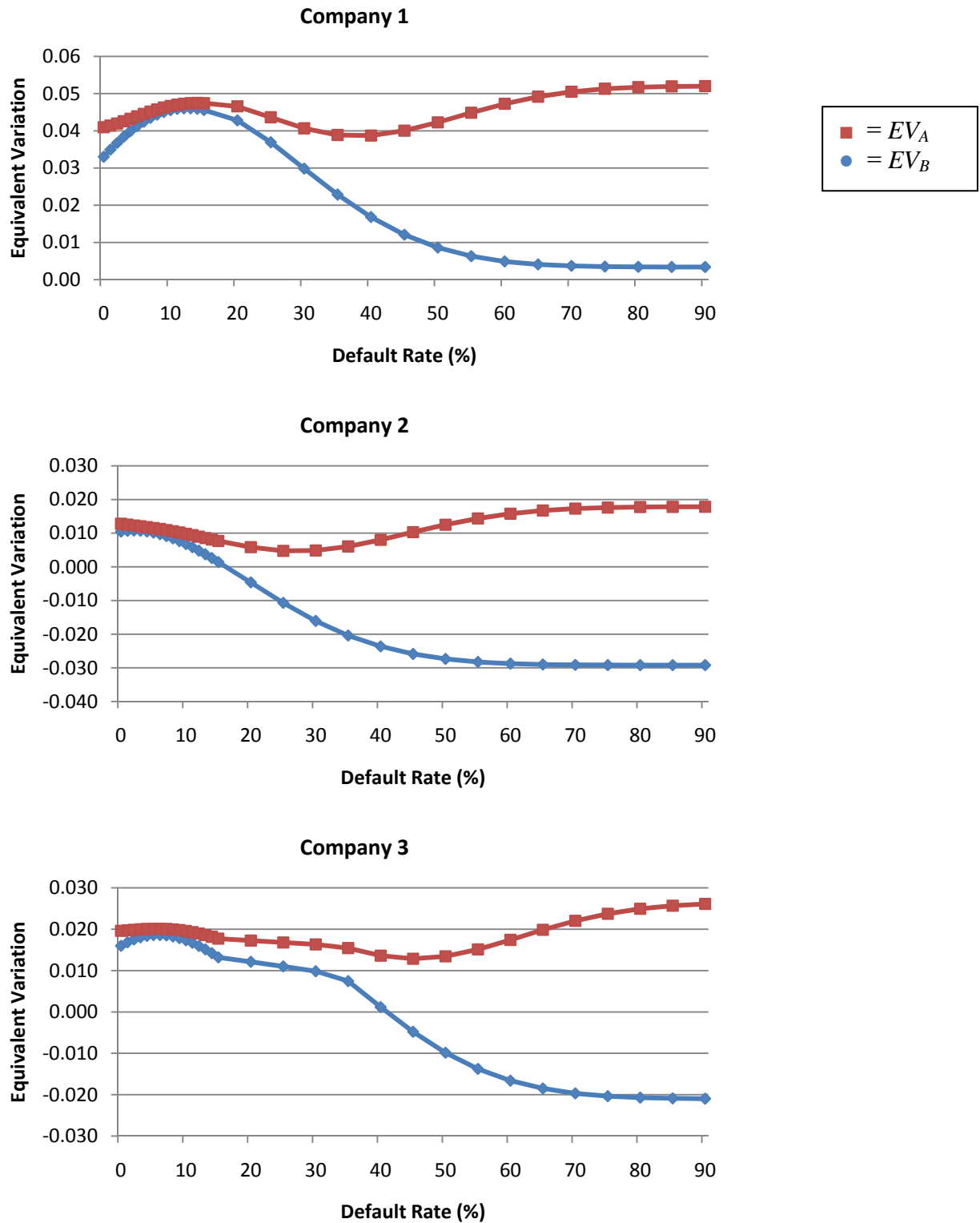


Figure A.2: Average  $EV_A$  and  $EV_B$  for choices made in naturally occurring frame, without an employer match.

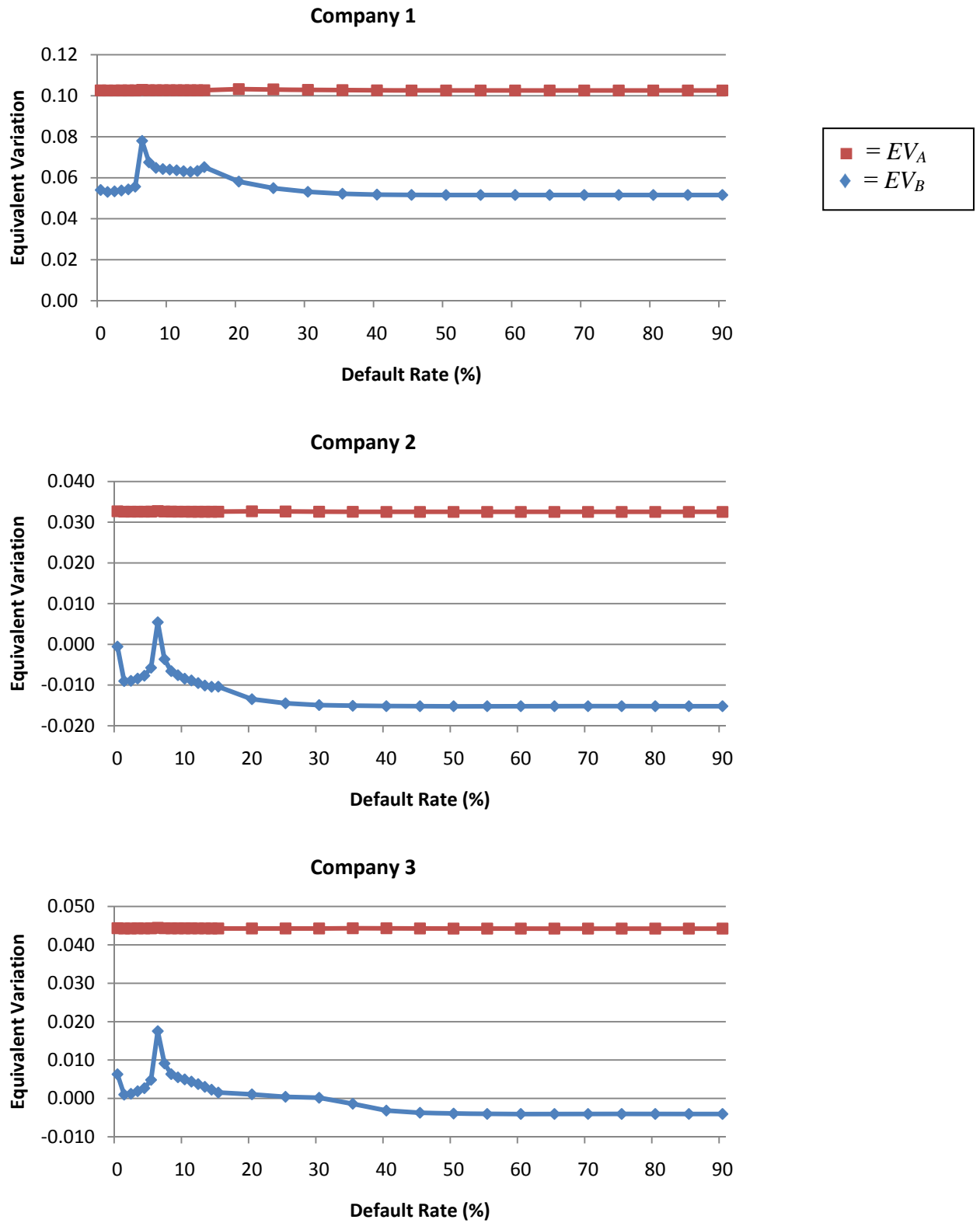


Figure A.3: Average  $EV_A$  and  $EV_B$  for choices made in the alternative frame (99% reduction in opt-out costs), with an employer match

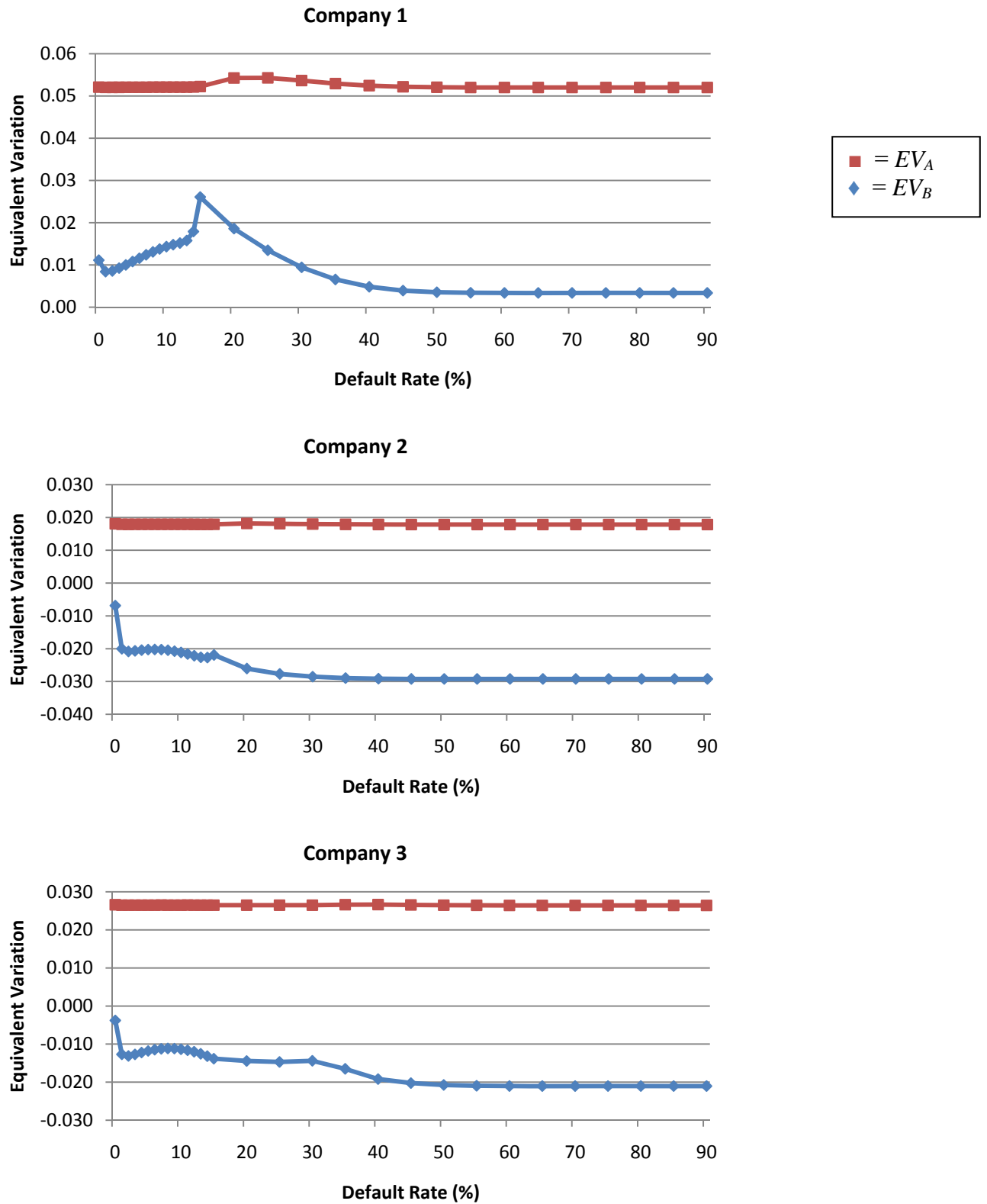


Figure A.4: Average  $EV_A$  and  $EV_B$  for choices made in the alternative decision frame (99% reduction in opt-out costs), without an employer match

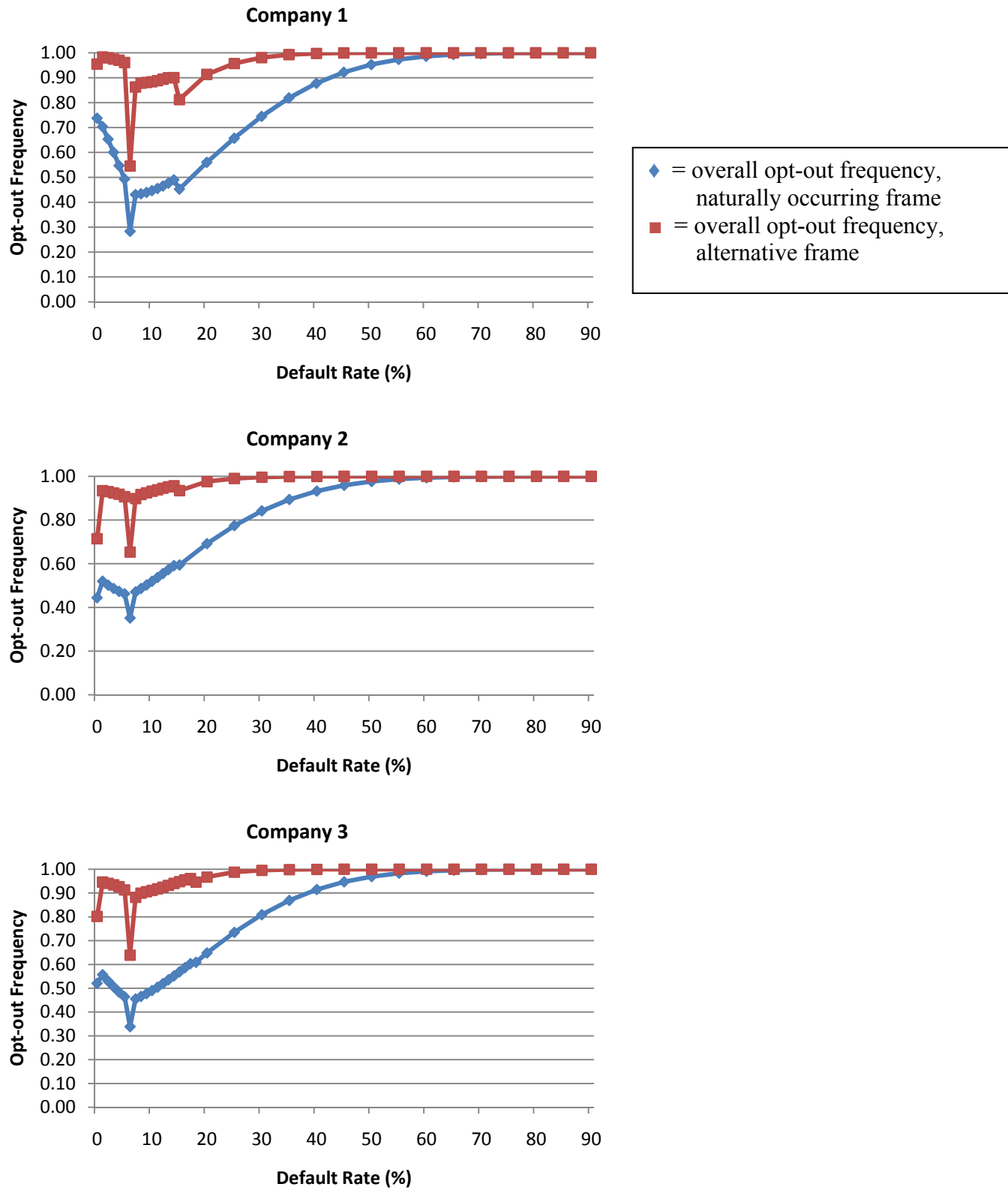


Figure A.5: Opt-out frequencies for various decision frames, with an employer match

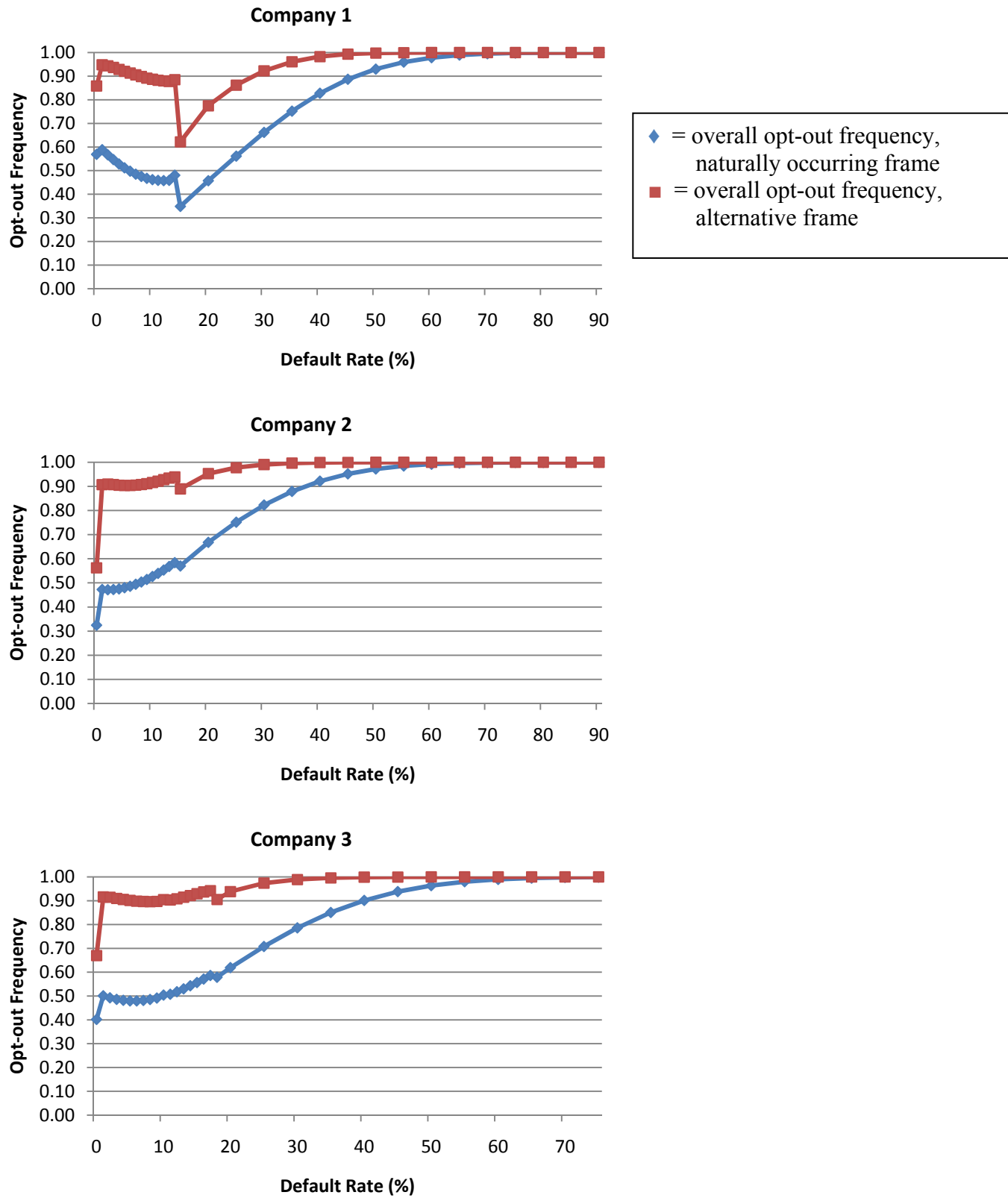


Figure A.6: Opt-out frequencies for various decision frames, without an employer match