- **Concepts and Calculations**

parametric relations for θ_1 and θ_3 as a function of θ_2

- **Geometry of the problem**

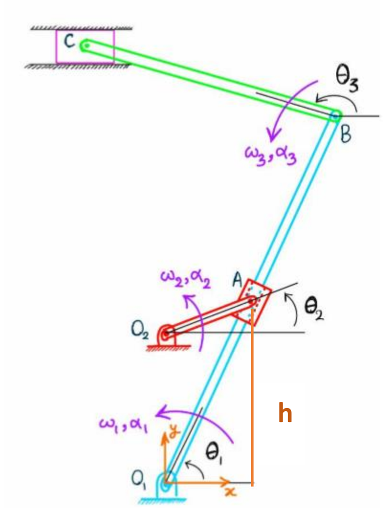

Figure 1: Main mechanism

a) θ_1 with respect to θ_2

To determine h, first we need to find the vertical coefficient of O_2A and then add O_1O_2:

$O_1O_2 + O_2A.\sin(\theta_2) = h$ (1)

Due to figure 1, below is the relation between θ_1 and θ_2:

$\text{Sin}(\theta_1) = \frac{h}{AO1}$ (2)

By applying the Cosine law in the O_1O_2A triangle we can obtain AO_1:

$$AO1 = \sqrt{\left(O1O2^2 + O2A^2 + 2.O1O2.O2A.\cos\left(\theta 2 + \frac{\pi}{2}\right)\right)} \qquad (3)$$

Since ω_2 = 1 rad/s and it's constant:

$\theta_2 = \omega_2 . t + \theta_0$, notice how theta is assumed to be zero initially

$\theta_2 = t$ (rad) (4)

There are two approaches, one is that we could substitute the value in (4) in equation (3), then substitute the result in (2) for AO_1. This way we could simply plot **θ_1 with respect to time**.

The other approach is **to differentiate both sides of the equation (2) with respect to time** and plot the result. The same approach could be used to determine α_3.

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b) θ_3 with respect to θ_2 :

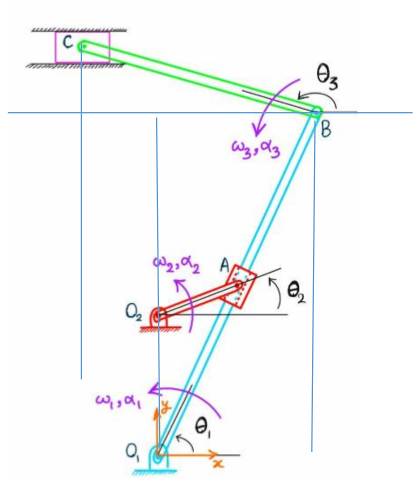

Figure 2: Motion mechanism. θ_3 with respect to θ_1

Due to the geometry provided in figure 2, we can derive the below relation :

$\mathrm{Sin}(\pi - \theta_3) = (CO_1)_y - BO_1 . \mathrm{Sin}(\theta_1) / BC$

By differentiating both sides of the above equation we can simply derive ω_3 with respect to θ_1 and ω_1. Notice how we have derived the relation between θ_1 with respect to θ_2 in section a.

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c) Deriving V_c

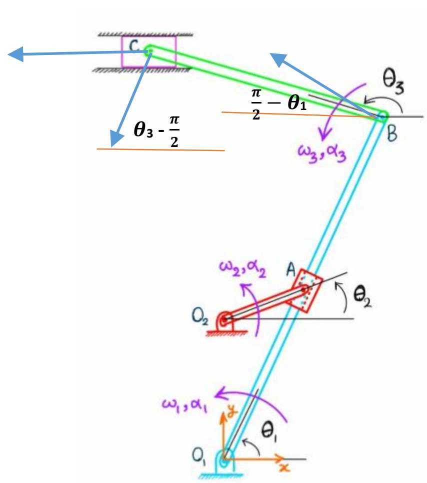


Figure 3: Kinematics-Slider C

In this section, the kinematics of the problem is studied for the slider C.

$V_c = V_B + V_{C/B}$

$\mathbf{V_{C/B} = BCw_3}$

$V_B = V_{O1} + V_{B/O1}$, $V_{O1} = 0$ (m/s)

$\mathbf{V_B = BO_1.\ \omega_1}$

And thus simply by noticing figure 3 we can conclude that:

$\mathbf{V_C = V_{B.}\cos(\frac{\pi}{2} - \theta_1) + V_{C/B}.\cos(\theta_3 - \frac{\pi}{2})}$

Part a)

1. plotting θ_1 and θ_3 with respect to time

Part a) The below relations are derived by studing the geometry of the problem which is discussed in the report.

```
%constants, units are in meters
AO2 = 5
O1O2 = 12
BO1 = 32
BC = 16
CO1_Y = 32
w2 = 1 %The angular velocity of theta 2, unit: rad/s


syms t1(t2) t3(t2) t2(t) AO1(t)
%theta 2, the angular velocity is given to be 1 rad/s
t2(t) = w2 * t
AO1(t) = sqrt(AO2.^2 + O1O2.^2 - 2*AO2*O1O2*cos(t2+pi/2))
t1(t) = asin((O1O2 + AO2*sin(t2))/AO1)
t3(t) = simplify(asin((32 -BO1*t1)/BC))

end
```

Output:

```
AO2 = 5
O1O2 = 12
BO1 = 32
BC = 16
CO1_Y = 32
w2 = 1
```

t2(t) = t

AO1(t) =

$$\sqrt{169 - 120\cos\left(t + \frac{\pi}{2}\right)}$$

t1(t) =

$$\operatorname{asin}\left(\frac{5\sin(t) + 12}{\sqrt{169 - 120\cos\left(t + \frac{\pi}{2}\right)}}\right)$$

t3(t) =

$$-\operatorname{asin}\left(2\operatorname{asin}\left(\frac{5\sin(t) + 12}{\sqrt{120\sin(t) + 169}}\right) - 2\right)$$

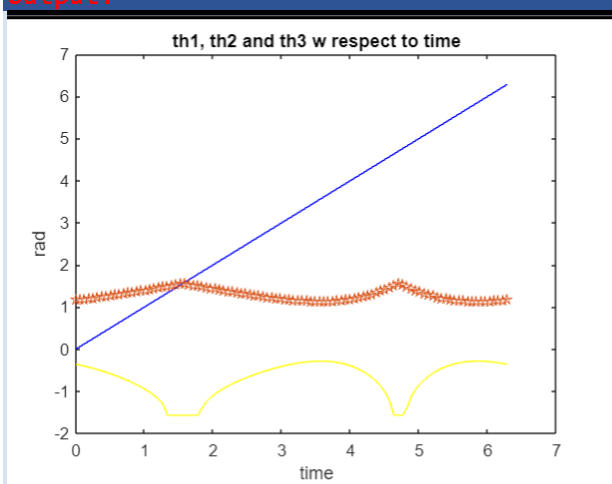
Generating points and plotting them:

Plotting the t1, t2 and t3 with respect to time

```
%Since w2 is 1 rad/s, one revolution takes 2*pi seconds
time = linspace(0, 2*pi, 100);


th1 = t1(time);
th2 = t2(time);
th3 = t3(time);
%plotting the values
%the blue one is theta2, pink is theta 1 and yellow is theta 3
plot(time, th2, 'b', time, th1, 'P', time, th3, 'y')
title ("th1, th2 and th3 w respect to time");
xlabel("time");
ylabel("rad");
```

Output:



Blue : theta2

Pink : th1

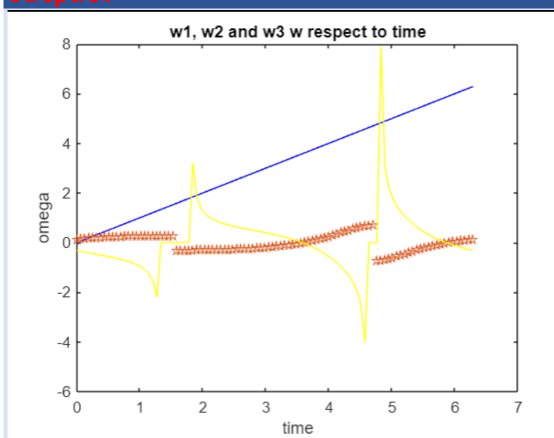
Yellow: theta3

Part B)

Plotting w1, w2 and w3 with respect to time

```
syms w1(t) w2(t) w3(t)
w2(t) = t
w1(t) = diff(t1)
w3(t) = diff(t3)
omega2 = w2(time);
omega1 = w1(time);
omega3 = w3(time);
%plotting the values
%the blue one is w2, pink is w1 and yellow is w3
 plot(time, omega2, 'b', time, omega1, 'P', time, omega3, 'y')
 title ("w1, w2 and w3 w respect to time");
 xlabel("time");
 ylabel("omega");
```

Output:



The more points generated, the more precise the graph

The discontinuities are related to the asine

Function, however the more points generated, the

The more precise the result

w1(t) =

$$\frac{\frac{5\cos(t)}{\sqrt{\sigma_1}} - \frac{60\sin\left(t+\frac{\pi}{2}\right)(5\sin(t)+12)}{\sigma_1^{3/2}}}{\sqrt{\frac{(5\sin(t)+12)^2}{120\cos\left(t+\frac{\pi}{2}\right)-169}+1}}$$

where

$$\sigma_1 = 169 - 120\cos\left(t+\frac{\pi}{2}\right)$$

w3(t) =

$$-\frac{2\left(\frac{5\cos(t)}{\sqrt{\sigma_1}} - \frac{60\cos(t)\,\sigma_2}{\sigma_1^{3/2}}\right)}{\sqrt{1-\frac{\sigma_2^2}{\sigma_1}}\,\sqrt{1-\left(2\,\mathrm{asin}\left(\frac{\sigma_2}{\sqrt{\sigma_1}}\right)-2\right)^2}}$$

where

$$\sigma_1 = 120\sin(t) + 169$$

$$\sigma_2 = 5\sin(t) + 12$$

Part B) velocity if the slider

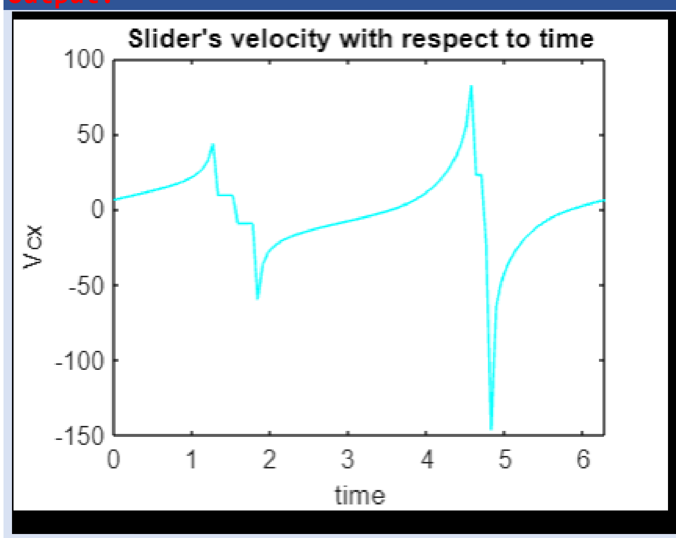
V_C

Ploting the velocity of the slider with respect to time

```
syms Vb(t) Vcb(t) Vc(t)
Vb(t) = BO1.*w1;
Vcb(t) = BC.*w3;
Vc(t) = Vb.*cos(pi/2 - t1) + Vcb.*cos(t3 - pi/2)


vcx = Vc(time)
%ploting the values
plot(time, Vc, 'c')
title ("Slider's velocity with respect to time");
xlabel("time");
ylabel("Vcx");
```

Output:




```
Vc(t) =
```

$$\frac{32\cos\left(\frac{\pi}{2}-\operatorname{asin}\left(\frac{\sigma_3}{\sqrt{169-\sigma_1}}\right)\right)\left(\frac{5\cos(t)}{\sqrt{169-\sigma_1}}-\frac{60\,\mathrm{si}}{(1\ldots}\right.}{\sqrt{\frac{\sigma_3^2}{\sigma_1-169}+1}}$$

where

$$\sigma_1 = 120\cos\left(t+\frac{\pi}{2}\right)$$

$$\sigma_2 = 2\operatorname{asin}\left(\frac{\sigma_3}{\sqrt{\sigma_4}}\right)-2$$

$$\sigma_3 = 5\sin(t)+12$$

$$\sigma_4 = 120\sin(t)+169$$

<u>The more points generated, the more precise the graph</u>

The discontinuities are related to the nonlinearity of the previous equations, however the more points generated, the more precise the results.

Part C)

Part c) Determinig alpha 1, alpha2 and alpha2

clearly they can be achieved by differentiating the angular velocity with respect to time

```
syms a1(t) a2(t) a3(t)
a2(t) = diff(w2)
a1(t) = diff(w1)
a3(t) = diff(w3)


alpha2 = a2(time);
alpha1 = a1(time);
alpha3 = a3(time);


%plotting the values
%the blue one is a2, pink is a1 and yellow is a3
 plot(time, alpha2, 'b', time, alpha1, 'r', time, alpha3, 'y')
 title ("a1, a2 and a3 w respect to time");
 xlabel("time");
 ylabel("alpha");
```

Output:

a2(t) = 1

a1(t) =

$$-\frac{\frac{5\sin(t)}{\sigma_4}+\frac{600\cos(t)\,\sigma_1}{\sigma_3}-\frac{10800\,\sigma_1{}^2\,\sigma_5}{(169-120\,\sigma_6)^{5/2}}+\frac{60\,\sigma_6\,\sigma_5}{\sigma_3}}{\sqrt{\sigma_2}}-\frac{\left(\frac{5\cos(t)}{\sigma_4}-\frac{60\,\sigma_1\,\sigma_5}{\sigma_3}\right)\left(\frac{120\,\sigma_1\,\sigma_5{}^2}{(120\,\sigma_6-169)^2}+\frac{10\cos(t)\,\sigma_5}{120\,\sigma_6-169}\right)}{2\,\sigma_2{}^{3/2}}$$

where

$$\sigma_1 = \sin\left(t + \frac{\pi}{2}\right)$$

$$\sigma_2 = \frac{\sigma_5^2}{120\,\sigma_6 - 169} + 1$$

$$\sigma_3 = (169 - 120\,\sigma_6)^{3/2}$$

$$\sigma_4 = \sqrt{169 - 120\,\sigma_6}$$

$$\sigma_5 = 5\sin(t) + 12$$

$$\sigma_6 = \cos\left(t + \frac{\pi}{2}\right)$$

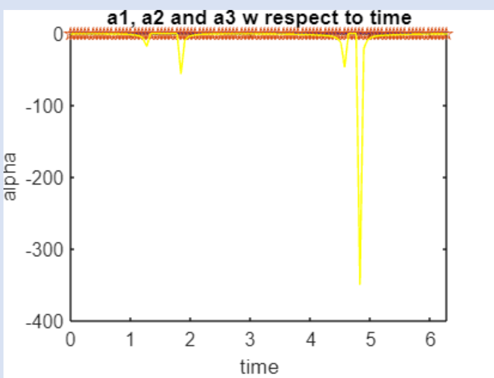
And alpha 3 is:

a3(t) =

$$\frac{2\left(\frac{5\sin(t)}{\sqrt{\sigma_5}} + \frac{600\cos(t)^2}{\sigma_5^{3/2}} - \frac{10800\cos(t)^2\,\sigma_6}{\sigma_5^{5/2}} - \frac{60\sin(t)\,\sigma_6}{\sigma_5^{3/2}}\right)}{\sqrt{\sigma_2}\,\sigma_3} + \frac{\sigma_1\left(\frac{120\cos(t)\,\sigma_6^2}{\sigma_5^2} - \frac{10\cos(t)\,\sigma_6}{\sigma_5}\right)}{\sigma_2^{3/2}\,\sigma_3} + \frac{4\,\sigma_4\,\sigma_1^2}{\left(\frac{\sigma_6^2}{\sigma_5} - 1\right)\left(1 - \sigma_4^2\right)^{3/2}}$$

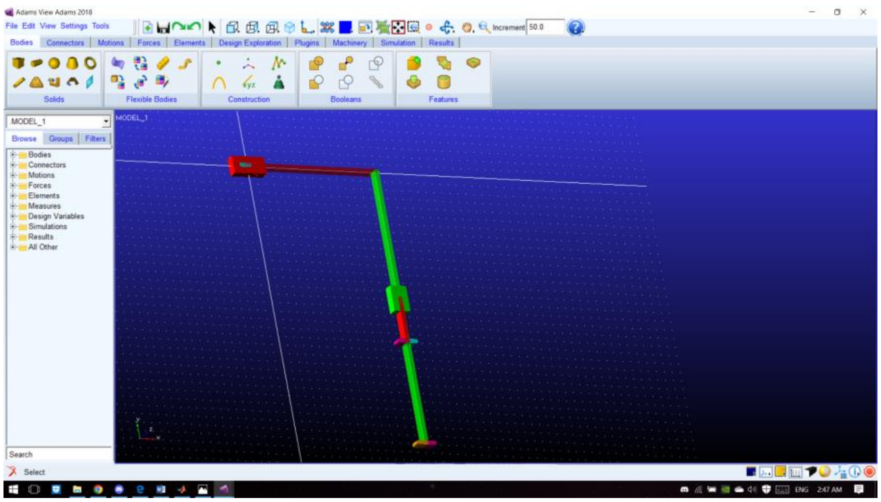
where

$$\sigma_1 = \frac{5\cos(t)}{\sqrt{\sigma_5}} - \frac{60\cos(t)\,\sigma_6}{\sigma_5^{3/2}}$$

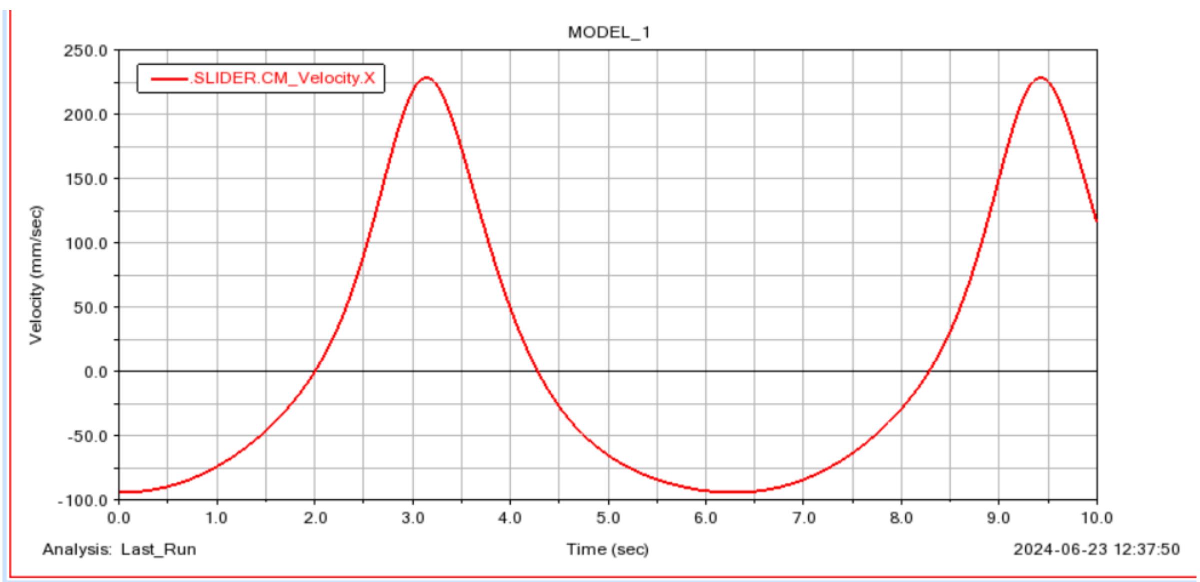


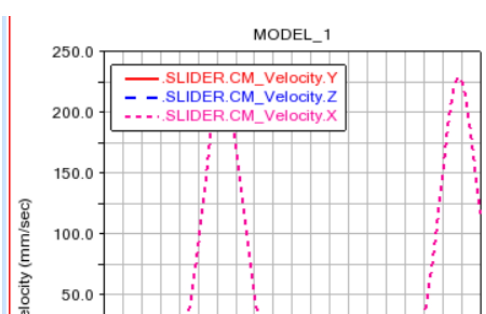
Took hours, still so much noise and not at all accurate

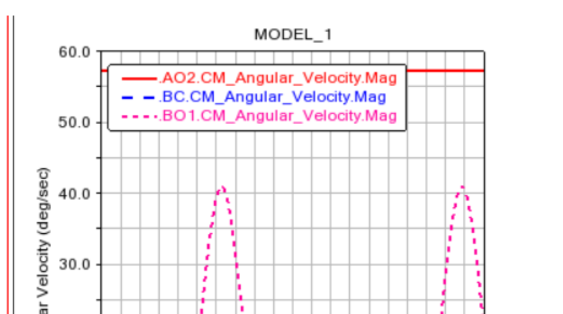
Part D) MCS ADAMS : Model



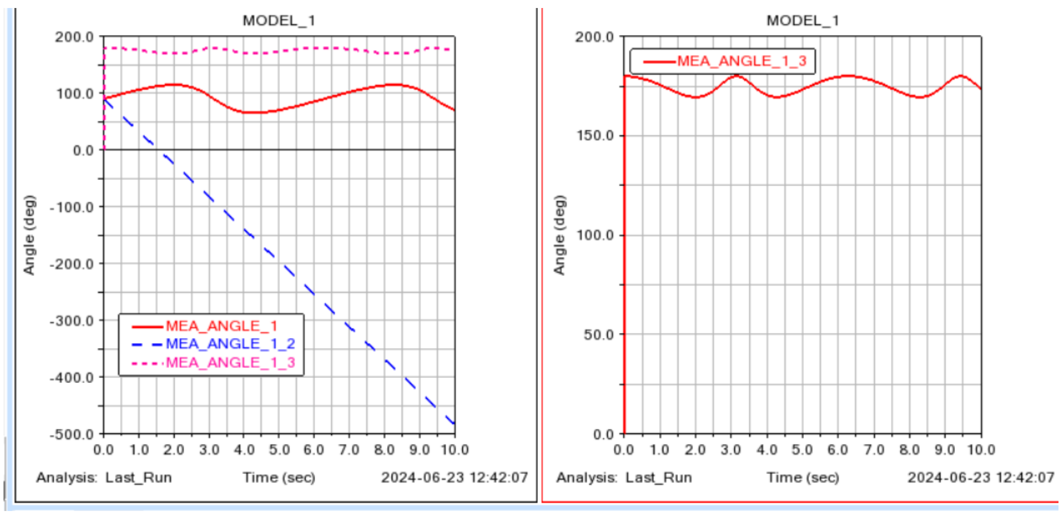
Simulation:





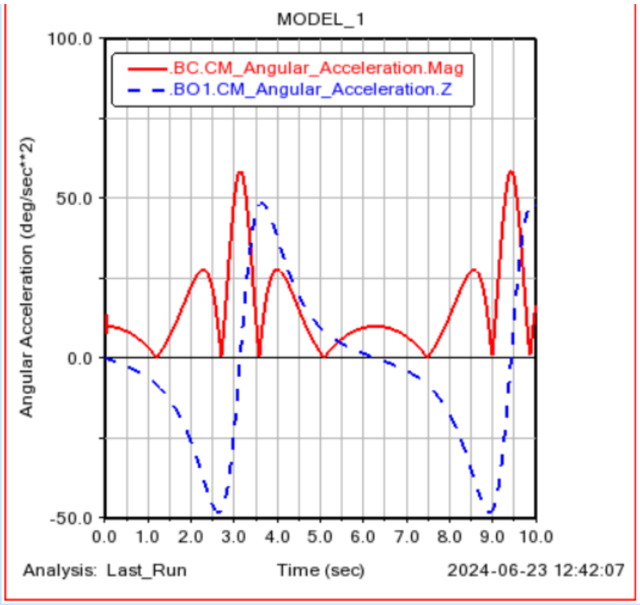


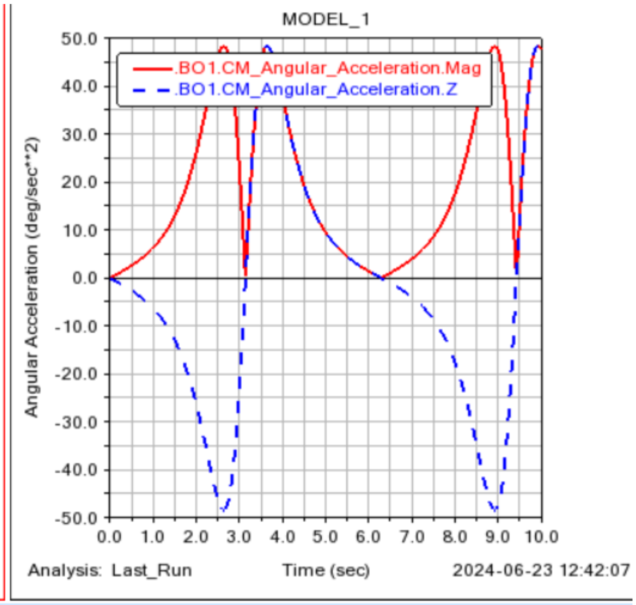
The above graphs obtained by ADAMS are quite different from the ones obtained by matlab, first because the matlab ones, starts when theta2 is zero, but in ADAMS the initial value for theta 2 is 90 degrees. The other reason is that in matlab only 100 points are generated and the results have a number of noticeable discontinuities which can be decreased by increasing the number of generated dots. (The program runtime for 100 points is somewhat 2 hours!)



The above figure represents the w1, w2 and w3 and by comparing these graphs with the ones obtained in part a, it can be perceived that their somewhat similar(ignoring the discontinuities in matlab results).

It is worth noting that the values obtained in ADAMS are reported in degrees while in matlab the values are provided in radians.


MODEL_1
.BC.CM_Angular_Acceleration.Mag
.BO1.CM_Angular_Acceleration.Z
Angular Acceleration (deg/sec**2)
Analysis: Last_Run
Time (sec)
2024-06-23 12:42:07


MODEL_1
.BO1.CM_Angular_Acceleration.Mag
.BO1.CM_Angular_Acceleration.Z
Angular Acceleration (deg/sec**2)
Analysis: Last_Run
Time (sec)
2024-06-23 12:42:07