Instrumentation*

*Note: Instrumentation — Homework I

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***Abstract*—The current document contains the solutions to the first assignment prepared for the class Instrumentation, 8106126-01 . Knowledge of Instrumentation is required for deep understanding of the materials presented in this report.**

***Index Terms*—Instrumentation,LS, RLS**

I. Introduction

A. List of Problems

1) Least Squares Algorithm
2) Sensor Design and Selection
3) Sensor Analysis
4) Insulin Injection

II. Problem 1— Least Squares Algorithm

A. Part A

LS (Least Square) Algorithm:

$$\phi = \begin{bmatrix} x_0^2 & x_0 & 1 \\ x_1^2 & x_1 & 1 \\ & \dots & \\ x_n^2 & x_n & 1 \end{bmatrix}, \theta = \begin{bmatrix} \alpha_2 \\ \alpha_1 \\ \alpha_0 \end{bmatrix} \tag{1}$$

Notice how by the above definition, n+1 samples are available.

$$Y = \phi \; \theta$$

(2)

Since ϕ is not necessarily a square matrix, an inverse doesn't necessarily exist and thus both sides are multiplied by ϕ^T since $\phi^T \phi$ is a square matrix and therefore invertible.

$$\phi^T Y = \phi^T \phi \; \theta, \quad (\phi^T \phi)^{-1} \phi^T Y = \theta \tag{3}$$

For this problem due to the provided data the matrices are defined as below:

$$\phi = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}, Y = \begin{bmatrix} -2 \\ 2 \\ 4 \\ 4 \\ 2 \end{bmatrix} \tag{4}$$

Substituting the above matrices in the equation we get:

$$\theta = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} \tag{5}$$

Substituting θ back into $Y = \phi \; \theta$, we can see the derived θ matrix is correct.

B. Part B

RLS (Recursive Least Square) Algorithm given that:

$$P(0) = 10^{\alpha} I_{n \times n} \qquad a_n(0) = 0 \tag{6}$$

RLS Algorithm:

step 1. $P_{t+1} = P_t - \frac{P_t \phi_{t+1} \phi_{t+1}^T P_t}{1 + \phi_{t+1}^T P_t \phi_{t+1}}$ step 2. $K_{t+1} = P_{t+1} \phi_{t+1}$ step 3. $e_{t+1} = y_{t+1} - \phi_{t+1} \theta_t$ step 4. $\theta_{t+1} = \theta_t + K_{t+1} e_{t+1}$

- Results for $\alpha = 100$:

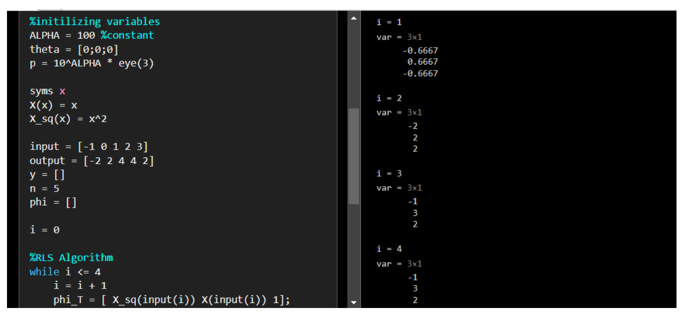

Fig. 1. RLS algorithm with α = 100

- Steps to converge: 2

C. Part C

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1) $\alpha = 1$: : Results:
Steps to converge: ¡5
- $\alpha = 2$: Results:

Steps to converge: 5
- $\alpha = 5$: Results: Steps to converge: 2

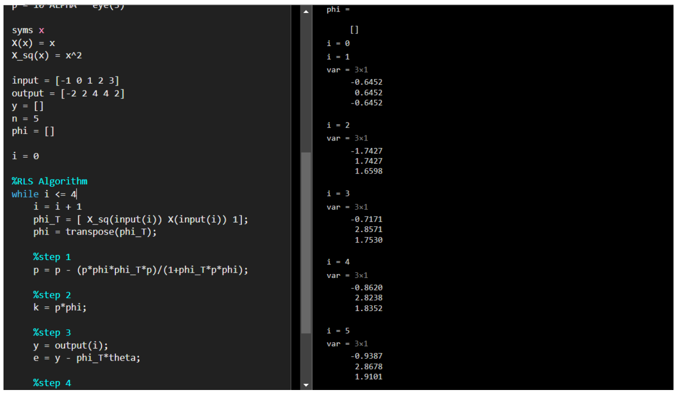

Fig. 2. RLS algorithm with α = 1

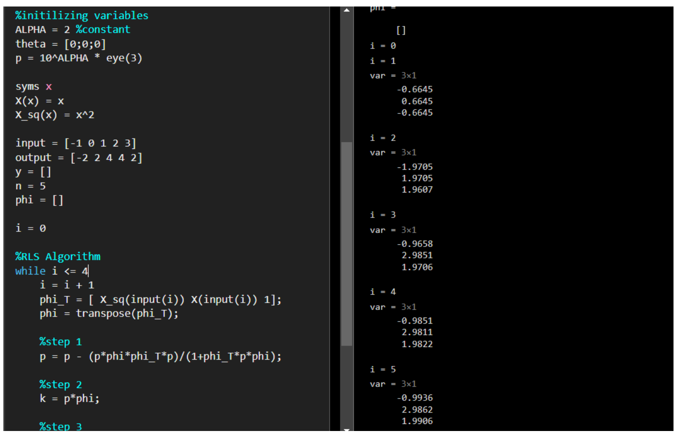

Fig. 3. RLS algorithm with α = 2

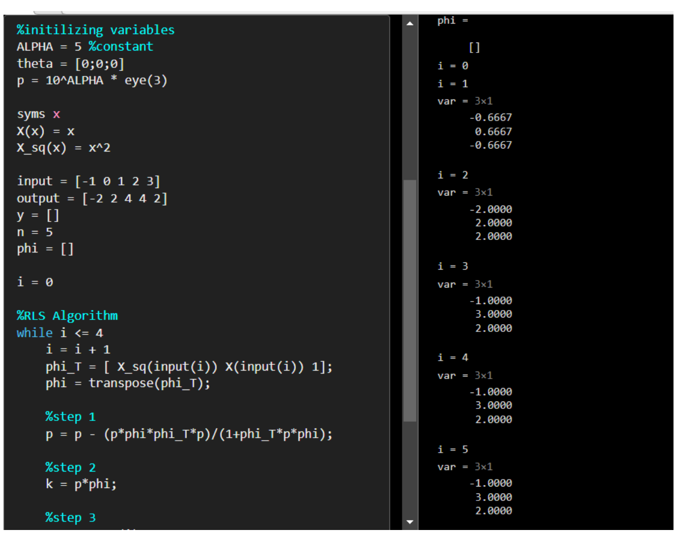

Fig. 4. RLS algorithm with α = 5

D. Part D

1) 1. Is RLS algorithm stable for all values of α: ?

- If α is too large, P(0) becomes too large, which can lead to numerical instability during updates and may adversely affect the convergence of the algorithm.
- If α is too small, the initial estimates of the parameters may yield poor performance initially, slowing convergence but generally maintaining stability.

while different values of α can affect convergence speed and performance, the RLS algorithm remains stable. There's no inherent maximum value for α that causes instability.

2) 2. Is the number of steps required for convergence the same for all values of α?: No, the smaller the α, the slower the convergence speed.

3) 3. Under the assumption that noise is added to the output of the model, which model's performance is more stable against data noise?: Notice how the the first step of the algorithm is equivalent to:

$$P_{t+1} = (P_t^{-1} + \phi_{t+1}\phi_{t+1}^T)^{-1} \tag{7}$$

For the model to ignore the changes caused by noise, P_t must be more dominant than $\phi_{t+1}\phi_{t+1}^T$ in determining P_{t+1} and thus P_t^{-1} must be large, consequently P_t must be small. Therefore, smaller values of α is better for achieving stability against noise. *(And yes I just wrote that myself, and no, I didn't use ChatGPT the great:))*

III. Problem 2— Sensor Design and Selection

A. Part A

The sensor is not quite accurate in terms of accuracy since it is off by 7 mmHg and mistakenly labels the patience blood pressure as elevated leading to further misdiagnosis.

Since human blood pressure sensors are often designed to measure up to 258 mmHg, accuracy for such a sensor can be reported as below:

$$accuracy = \frac{127 - 120}{258 - 0} = 2.71 \quad \%FS \tag{8}$$

B. Part B

Precision of the sensor is quite better in the first experiment.

$$precision_{exp1} = \frac{122 - 118}{258 - 0} = 1.55 \quad \%FS \tag{9}$$

$$precision_{exp2} = \frac{127 - 119}{258 - 0} = 3.10 \quad \%FS \tag{10}$$

C. Part C

- Repeatability: This reflects the sensor's ability to produce consistent results when measuring the same condition multiple times
- Reproducibility: This indicates the sensor's performance under different conditions or setups (i.e., the second test with a different calibration device).

Repeatability is generally more critical for ensuring stable performance in a hospital environment. This consistency is crucial for monitoring patients where accurate readings are expected across multiple measurements during the same scenario. Strong repeatability **ensures reliability** when clinicians interpret vital signs.

D. Part D

Accuracy: The variability observed in accuracy and broader measurement range in the second test would impact clinical decisions and must be fixed. How? Frequent calibration against standardized reference pressure values. minimizing human error and establishing a feedback loop for continuous adjustment based on observed discrepancies in accuracy during routine clinical use.

IV. Problem 3— Sensor Analysis

A. Part A

A large hysteresis error may indicate significant nonlinearity or lag in the system, which is crucial in many applications. For example, in thermocouples and temperature sensors, understanding the hysteresis behavior is essential for accurate measurements.

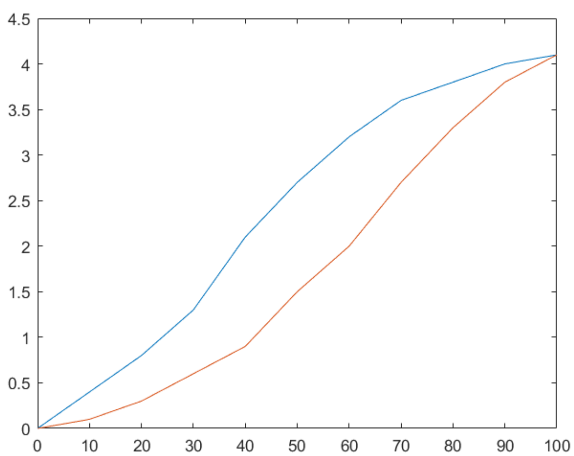

Fig. 5. hysteresis phenomenon

$$hysteresis\_error = output\_increment - output\_decrement \tag{11}$$

$$HE_{FS} = \frac{hysteresis\_error}{max\_output} \tag{12}$$

Results: maximum HE = 29.27 %FS

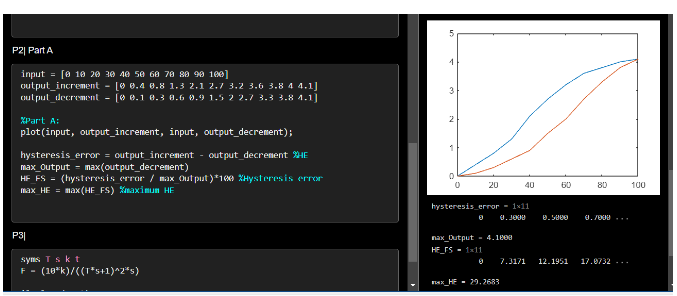

Fig. 6. Results in Matlab

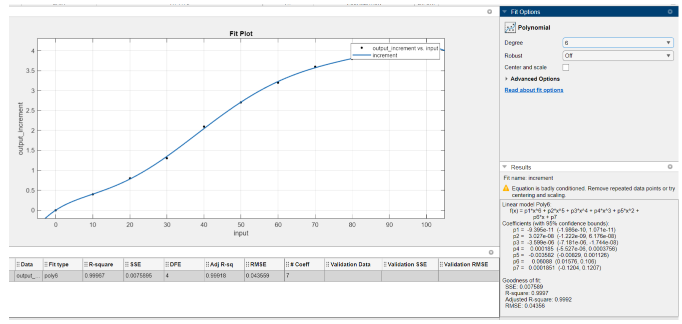

Fig. 7. Increment curve, since SSE and RMSE are minimized, this is the best curve. (degree: 6)

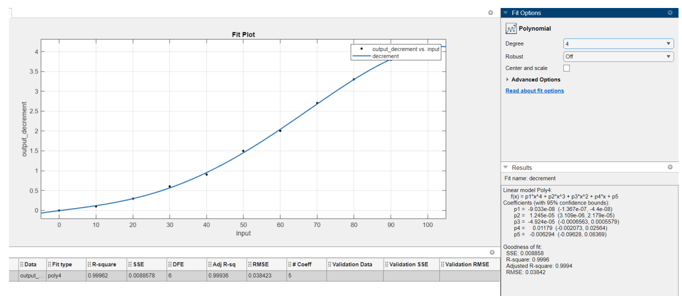

Fig. 8. decrement curve, since SSE and RMSE are minimized, this is the best curve. (degree: 4)

B. Part B

C. Part C

Zero drift:

$$ZeroDrift = MeasuredOutputat0CExpectedOutputat0C \tag{13}$$

$$Zero\ \ Drift\ \ (Increasing) = 0.00.0 = 0.0\ V \tag{14}$$

$$Zero\ \ Drift(Decreasing) = 0.00.0 = 0.0\ V \tag{15}$$

Sensitivity drift:

$$S_{expected} = \frac{4.1 - 0}{100 - 0} = 0.041\ v/c \tag{16}$$

$$S_{observed(increasing)} = S_{observed(decreasing)} = \frac{4.1 - 0}{100 - 0} = 0.041\ v/c \tag{17}$$

$$Sensitivity\ \ Drift = S_{expected} - S_{observed} = 0 \tag{18}$$

but do notice if we judge by the data, $S_{observed(increasing)}$ is 0.1/10 (least sensitive measurement) and $S_{observed(decreasing)}$ is 0.1/10, the other approach is to measure this for each data sample and take average. **Under the above assumptions, the sensitivity drift is 0.041 - 0.01 = 0.031**.

V. Problem 4— Insulin Injection

1) a) Analytical Solution for Plasma Insulin Concentration:

$$y(s) = g_p(s).u(s) \tag{19}$$

notice since P = 10U:

$$y(s) = \frac{k_p \ .10}{(\tau s + 1)^2} \tag{20}$$

to obtain y(t), inverse laplace transform is required.

$$y(t) = \frac{10 k_p t}{\tau^2} e^{-t/\tau} \tag{21}$$

2) b) Calculate the Time for Maximum Concentration: To find the time at which the plasma insulin concentration reaches its maximum, differentiate (y(t)) with respect to time and set the derivative to zero.

$$\frac{dy}{dt} = \frac{10 k_p}{\tau^2} e^{-t/\tau} - \frac{10 k_p t^2}{\tau^3} e^{-t/\tau} \tag{22}$$

$$\frac{dy}{dt} = \frac{10 k_p}{\tau^2} e^{-t/\tau} (1 - \frac{t}{\tau}) = 0 \rightarrow t = \tau \tag{23}$$

3) c) Maximum Plasma Insulin Concentration:

$$y_{max}(t) = \frac{10 k_p \sqrt{\tau}}{\tau^2} e^{-\sqrt{\tau}/\tau} \tag{24}$$

4) d) Numerical Calculation and Plotting: Calculate the Time for Maximum Concentration = 30 min Maximum Plasma Insulin Concentration = 6.13 mU / L

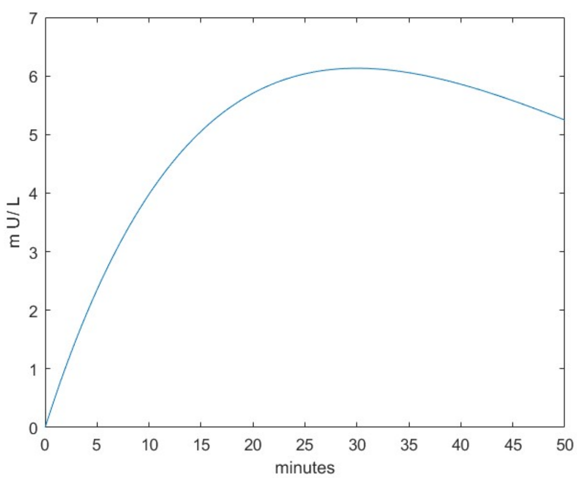

Fig. 9. As expected the maximum occurs after 30min

A. e) Simulation in Matlab

Fig. 10. simulation for when u = 1U

As it is shown in the figure, Y is increasing by time but the speed seems to be slowing down as time passes.