



University of Dhaka

Department of Applied Mathematics

First Year B.S. (Honors), Academic Session: 2024-2025

Course Title: Math Lab I (Mathematica), Course Code: AMTH 150

Assignment No.: 3A (Coordinate & Vector Geometry with its real-life applications)

Name:

Roll No.:

Group:

Instruction: Write an appropriate programming code using **MATHEMATICA** software to get the output of each problem and hence visualize them properly.

1. (a) Find the rectangular coordinates of the points with the following polar coordinates:

(i). $\left(3, \frac{5\pi}{3}\right)$; (ii). $\left(3, -\frac{3\pi}{4}\right)$; (iii). $\left(-2, \frac{7\pi}{6}\right)$; (iv). $\left(-2, -\frac{\pi}{6}\right)$.

- (b) Find the polar coordinates of the points with the following rectangular coordinates:

(i). $(-2, -2\sqrt{3})$; (ii). $(\sqrt{3}, 1)$; (iii). $(2, -2)$; (iv). $(0, 3)$.

- (c) Plot the point P with polar coordinates $\left(4, \frac{3\pi}{4}\right)$, and find other polar coordinates

(r, θ) of this same point for which:

(i). $r > 0, -2\pi \leq \theta < 0$; (ii). $r < 0, 0 \leq \theta < 2\pi$; (iii). $r > 0, 2\pi \leq \theta < 4\pi$.

2. (a) Transform the following equations from polar coordinates to rectangular coordinates. Hence, sketch and identify each of the graphs.

(i). $r = \frac{10}{5 + 4\cos\theta}$; (ii). $r = \frac{9}{3 - 6\cos\theta}$; (iii). $r = \frac{8}{2 - \sin\theta}$; (iv). $r = \frac{6}{8 + 2\sin\theta}$.

- (b) Find the distance between the pair of points $(2, \pi)$ and $\left(3, -\frac{3\pi}{4}\right)$.

3. (a) Analyze the following equations and sketch them showing the relevant information.
- (i). $4x^2 + 3y^2 + 8x - 6y - 5 = 0$; (ii). $x^2 + 6x - 4y + 1 = 0$; (iii). $2y^2 - x^2 + 2x + 8y + 3 = 0$.
- (b) Find an equation for the ellipse with foci at $(1,2)$ and $(-3,2)$, and one vertex at $(-4,2)$. Graph the equation.
4. (a) A reflecting telescope contains a mirror shaped like a paraboloid of revolution. If the mirror is 4 inches across at its opening and is 3 inches deep, where will the collected light be concentrated?
- (b) An arch for a bridge over a highway is in the form of half an ellipse. The top of the arch is 20 feet above the ground level. The highway has four lanes, each 12 feet wide; a center safety strip 8 feet wide; and two side strips, each 4 feet wide. What should the span of the bridge be if the height 28 feet from the center is to be 13 feet?
- (c) Two recording devices are set 2400 feet apart, with the device at point A to the west of the device at point B. At a point between the devices 300 feet from point B, a small amount of explosive is detonated. The recording devices record the time until the sound reaches each. How far directly north of point should a second explosion be done so that the measured time difference recorded by the devices is the same as that for the first detonation?
5. (a) Remove the first-degree terms from each of the following equations:
- (i). $3x^2 + 4y^2 - 12x + 4y + 13 = 0$;
- (ii). $x^2 + 3y^2 + 2x + 12y + 1 = 0$;
- (iii). $3x^2 - 4y^2 + 6x + 24y - 135 = 0$.
- (b) For each of the following equations, determine the appropriate rotation formula so that once the rotation of axes is made the equation contains no xy -term. Also, sketch and analyze the new equations.
- (i). $10x^2 + 12xy + 4y^2 - x - y + 10 = 0$;

(ii). $16x^2 + 24xy + 9y^2 - 60x + 80y = 0;$

(iii). $x^2 - 6xy - 5y^2 + 4x - 22 = 0.$

6. (a) Sketch the graphs of the parabola $y^2 + 12y + x - 1 = 0$ and show the focus and directrix. Also, find the length of the latus rectum and the equation of the tangent at the vertex.
- (b) Find the value of constant λ for which the ellipse $9x^2 + \lambda y^2 = 9\lambda$ passes through the point $(6, 2)$. Hence,
- (i). Sketch the ellipse and locate the foci and vertices in the same figure with blue heavy dots. Also, find its eccentricity.
- (ii). Draw the integer points inside the ellipse with red heavy dots. How many integer points are there inside the ellipse?
7. (a) Determine whether the second-degree equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of straight line or not. If so, then
- (i). Separate the lines, and find their point of intersection and angle between them.
- (ii). Find the equation of the bisectors of the angles between the lines.
- (iii). Plot the lines in the same set of axes using different styles.
- (iv). Find the area of the triangle formed by the lines and x -axis.
- (v). Mark the vertices of the triangle with heavy orange dots and fill the area with blue color.
- (b) Find the value of λ for which the general equation of second degree $12x^2 + \lambda xy + 2y^2 + 11x - 5y + 2 = 0$ represents pair of straight lines. Also, find the point of intersection and the angle between them.
- (c) Find the value of k , if the lines joining the origin to the points of intersections of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and $x + 2y = k$ are mutually perpendicular. Verify this conjecture graphically.