Problem Set #5

Econ 103

Part I – Problems from the Textbook

Chapter 4: 1, 3, 5, 7, 9, 11, 13, 15, 25, 27, 29

Part II – Additional Problems

1. Suppose X is a random variable with support $\{-1, 0, 1\}$ where $p(-1) = q$ and $p(1) = p$.
   (a) What is $p(0)$?
   (b) Calculate the CDF, $F(x_0)$, of X.
   (c) Calculate $\mathbb{E}[X]$.
   (d) What relationship must hold between p and q to ensure $\mathbb{E}[X] = 0$?
2. Fill in the missing details from class to calculate the variance of a Bernoulli Random Variable *directly*, that is *without* using the shortcut formula.
3. Prove that the Bernoulli Random Variable is a special case of the Binomial Random variable for which $n = 1$. (Hint: compare pmfs.)
4. Suppose that X is a random variable with support $\{1, 2\}$ and Y is a random variable with support $\{0, 1\}$ where X and Y have the following joint distribution:

$$p_{XY}(1,0) = 0.20, \qquad p_{XY}(1,1) = 0.30$$
$$p_{XY}(2,0) = 0.25, \qquad p_{XY}(2,1) = 0.25$$

   (a) Express the joint distribution in a 2×2 table.
   (b) Using the table, calculate the marginal probability distributions of X and Y.
   (c) Calculate the conditional probability distribution of $Y|X = 1$ and $Y|X = 2$.
   (d) Calculate $\mathbb{E}[Y|X]$.
   (e) What is $\mathbb{E}[\mathbb{E}[Y|X]]$?
   (f) Calculate the covariance between X and Y using the shortcut formula.