Midterm Examination #2

Econ 103, Statistics for Economists

November 3rd, 2014

You will have 70 minutes to complete this exam. Graphing calculators, notes, and textbooks are not permitted.

I pledge that, in taking and preparing for this exam, I have abided by the University of Pennsylvania's Code of Academic Integrity. I am aware that any violations of the code will result in a failing grade for this course.

Name: ______________________________

Student ID #: ______________________________

Signature: ______________________________

Question:	1	2	3	4	5	6	7	Total
Points:	24	16	20	20	20	20	20	140
Score:								

Instructions: Answer all questions in the space provided, continuing on the back of the page if you run out of space. Show your work for full credit but be aware that writing down irrelevant information will not gain you points. Be sure to sign the academic integrity statement above and to write your name and student ID number on *each page* in the space provided. Make sure that you have all pages of the exam before starting.

Warning: If you continue writing after we call time, even if this is only to fill in your name, twenty-five points will be deducted from your final score. In addition, two points will be deducted for each page on which you do not write your name and student ID.

1. Mark each statement as True or False. If you mark a statement as False, provide a brief explanation. If you mark a statement as True, no explanation is needed.

   (a) (4 points) If $Cov(X,Y) = 0$ then $E[XY] = E[X]E[Y]$

   (b) (4 points) Unlike a probability mass function, a probability density function *can* take on values greater than one.

   (c) (4 points) If X and Y are uncorrelated then $Var(X - Y) = Var(X) - Var(Y)$

   (d) (4 points) The pdf $f(x)$ of a continuous random variable X gives $P(X = x)$.

   (e) (4 points) For any random variable X and any function g, $E[g(X)] = g(E[X])$.

   (f) (4 points) Even if $\widehat{\theta}$ is a biased estimator of θ_0, it can *still* be consistent for θ_0.

2. Unless otherwise specified, answer each part with a *single line* of R code.
   (a) (4 points) Calculate the median of an F random variable with numerator degrees of freedom 3 and denominator degrees of freedom 8.
   (b) (4 points) Calculate the probability that a Binomial $n = 20$, $p = 2/3$ random variable takes on a value strictly greater than 15.
   (c) (4 points) Make 100 iid draws from a t distribution with 5 degrees of freedom.
   (d) (4 points) Write code to plot the pdf of a standard normal random variable between -3 and 3. You may use more than one line of code in your answer to this part.

3. (20 points) Write an R function called `CI.normal.mean` that constructs a 90% confidence interval for the mean of a normal population with *known* population variance. Your function should take two arguments: `x` is a vector of sample data, assumed to be a sequence of iid draws from a normal population, and `s` is the population standard deviation, assumed known. Your function should return a vector with two elements: the first is the lower confidence limit and the second is the upper confidence limit.

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4. Rodrigo has a bowl containing 10 balls: five of them are red and the rest are blue. Rossa wants to know the fraction of red balls in the bowl so he draws four balls at random with replacement from the bowl.

   (a) (5 points) Let X be the number of red balls that Rossa draws. What kind of random variable is X? Write down its support set, its pmf, and the values of its parameters.

   (b) (5 points) To estimate the fraction of red balls in the bowl, Rossa divides X, from the preceding part, by four. Is this estimator unbiased? What is its variance?

   (c) (5 points) What is the probability that Rossa's estimator from the preceding part will *exactly equal* the true fraction of red balls in the bowl?

   (d) (5 points) Now suppose that Rossa decides to make his random draws *without* replacement. Will your answer to the preceding part change? If so, how?

5. Let X and Y be independent, normally distributed random variables representing the returns of two stocks such that $\mu_x = \mu_y = 1$ and $\sigma_x^2 = \sigma_y^2 = 2$. A portfolio $\Pi(\omega)$ is a linear combination of the form $\omega X + (1 - \omega)Y$ where $\omega \in [0, 1]$ is the fraction of your total funds that are invested in X.

   (a) (10 points) If you wish to construct the portfolio with the lowest possible variance, what value of ω should you choose? Prove your answer.

   (b) (5 points) Calculate the expected value and the variance of the portfolio from the preceding part.

   (c) (5 points) Approximately what is the probability that the minimum variance portfolio will make a *negative* return?

6. Let X be a continuous RV with CDF $F(x_0) = (x_0^3 + 1)/2$ and support set $[-1, 1]$.
   (a) (5 points) Calculate the probability that X takes on a value *outside* $(-0.5, 0.5)$.

   (b) (5 points) Calculate the pdf of X.

   (c) (5 points) Calculate the expected value of X.

   (d) (5 points) Calculate the quantile function of X.

7. Let $Y_1, \ldots, Y_9 \sim$ iid $\mathrm{N}(\mu = 1, \sigma^2 = 4)$, $X = \frac{1}{9}\sum_{i=1}^{9} Y_i$ and $Z = \frac{1}{8}\sum_{i=1}^{9}(Y_i - X)^2$. Specify the *precise* distribution of each of the random variables listed below, including the values of any and all relevant parameters.

   (a) (5 points) X

   (b) (5 points) $\dfrac{3}{2}(X-1)$

   (c) (5 points) $2Z$

   (d) (5 points) $\dfrac{3(X-1)}{\sqrt{Z}}$

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