

FIRST MIDTERM EXAMINATION
ECON 103, STATISTICS FOR ECONOMISTS
FEBRUARY 16TH, 2016

You will have 70 minutes to complete this exam. Graphing calculators, notes, and textbooks are not permitted.

I pledge that, in taking and preparing for this exam, I have abided by the University of Pennsylvania's Code of Academic Integrity. I am aware that any violations of the code will result in a failing grade for this course.

Name: _____

Signature: _____

Student ID #: _____ Recitation #: _____

| | | | | | | |
|-----------|----|----|----|----|----|-------|
| Question: | 1 | 2 | 3 | 4 | 5 | Total |
| Points: | 35 | 15 | 25 | 30 | 35 | 140 |
| Score: | | | | | | |

Instructions: Answer all questions in the space provided, continuing on the back of the page if you run out of space. Show your work for full credit but be aware that writing down irrelevant information will not gain you points. Be sure to sign the academic integrity statement above and to write your name and student ID number on *each page* in the space provided. Make sure that you have all pages of the exam before starting.

Warning: If you continue writing after we call time, even if this is only to fill in your name, twenty-five points will be deducted from your final score. In addition, two points will be deducted for each page on which you do not write your name and student ID.

1. The dataframe `Austin` describes 4499 students who entered the UT Austin in 2000:

| | SATv | SATq | School | GPA |
|---|------|------|-----------------|--------|
| 1 | 690 | 580 | BUSINESS | 3.8235 |
| 2 | 530 | 710 | NATURAL SCIENCE | 3.5338 |
| 3 | 610 | 700 | NATURAL SCIENCE | 3.3679 |
| 4 | 730 | 700 | ENGINEERING | 3.3437 |
| 5 | 700 | 710 | NATURAL SCIENCE | 3.7205 |
| 6 | 540 | 690 | LIBERAL ARTS | 2.6851 |

Columns 1–2 are SAT *verbal* and *quantitative* scores (out of 800), `School` indicates if a student is in the school of BUSINESS, ENGINEERING, LIBERAL ARTS or NATURAL SCIENCE, and `GPA` gives GPA at graduation (out of 4.0).

- 2 (a) The first thing I did was compute some basic descriptive statistics:

| | SATv | SATq | School | GPA |
|----------|--------|---------------|----------------------|---------------|
| Min. | :270.0 | Min. :320.0 | BUSINESS : 832 | Min. :1.837 |
| 1st Qu.: | 540.0 | 1st Qu.:570.0 | ENGINEERING : 695 | 1st Qu.:2.846 |
| Median : | 590.0 | Median :630.0 | LIBERAL ARTS :1847 | Median :3.238 |
| Mean : | 595.6 | Mean :624.9 | NATURAL SCIENCE:1125 | Mean :3.195 |
| 3rd Qu.: | 650.0 | 3rd Qu.:680.0 | | 3rd Qu.:3.582 |
| Max. : | 800.0 | Max. :800.0 | | Max. :4.000 |

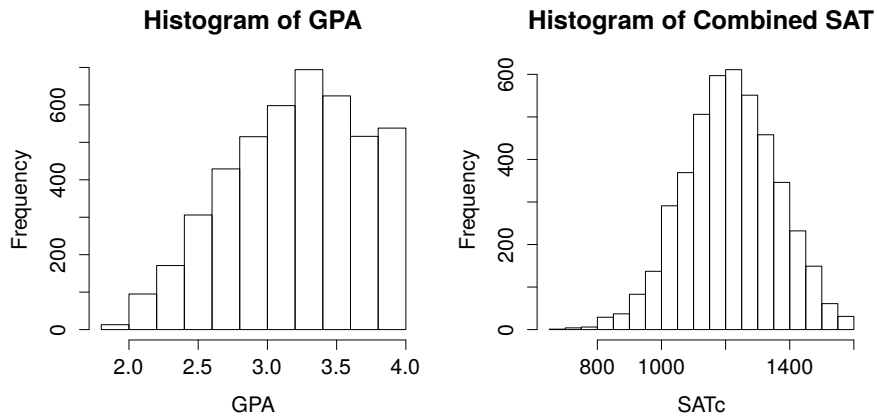
I created these results with a single line of R code. What was it?

- 3 (b) I decided to calculate each student's *combined* SAT score and add it as an additional column in `Austin`. After doing this, the first few rows of `Austin` were:

| | SATv | SATq | School | GPA | SATc |
|---|------|------|-----------------|--------|------|
| 1 | 690 | 580 | BUSINESS | 3.8235 | 1270 |
| 2 | 530 | 710 | NATURAL SCIENCE | 3.5338 | 1240 |
| 3 | 610 | 700 | NATURAL SCIENCE | 3.3679 | 1310 |
| 4 | 730 | 700 | ENGINEERING | 3.3437 | 1430 |
| 5 | 700 | 710 | NATURAL SCIENCE | 3.7205 | 1410 |
| 6 | 540 | 690 | LIBERAL ARTS | 2.6851 | 1230 |

Write a single line of R code to compute `SATc` and add it to `Austin`.

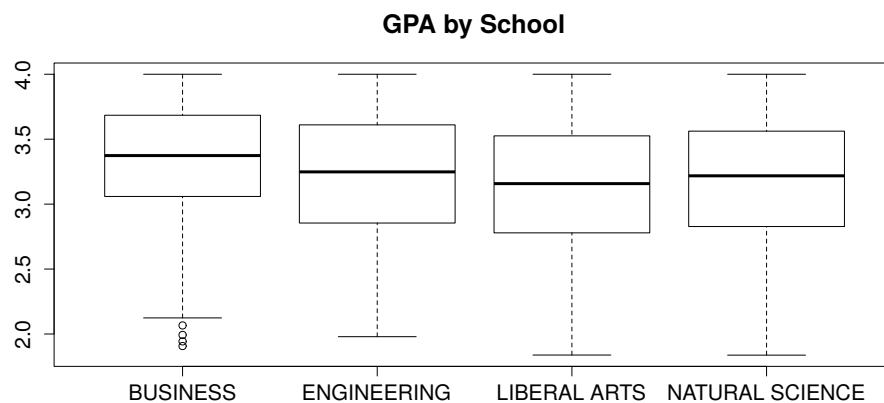
- 3 (c) The next thing I did was to create histograms of GPA and combined SAT scores:



Give the full R command I used to make the histogram for *GPA*. Don't forget to include the title and label for the *x*-axis.

- 3 (d) Is there any evidence of skewness in GPA or SATc? Explain briefly.

- 3 (e) I was interested to see whether GPA varies by school so I produced this plot:



Write out the full R command that I used. Don't forget the plot title.

Name: _____

Student ID #: _____

3 (f) What are the “little circles” that appear in the preceding plot? Explain briefly.

3 (g) Next I calculate the average *combined* SAT score for students of each school:

```
Austin$School: BUSINESS
```

```
[1] 1230.288
```

```
-----  
Austin$School: ENGINEERING
```

```
[1] 1281.554
```

```
-----  
Austin$School: LIBERAL ARTS
```

```
[1] 1187.163
```

```
-----  
Austin$School: NATURAL SCIENCE
```

```
[1] 1230.338
```

What command did I use?

Use these statistics to answer the remaining parts:

| | GPA | SATc |
|------|-----|------|
| Mean | 3.2 | 1220 |
| S.D. | 0.5 | 145 |
| Cov. | 29 | |

3 (h) Based on the sample z-scores, which is more “extreme” in this dataset: a GPA of 3.7 or a combined SAT score of 1400? Explain briefly.

3 (i) Calculate the sample correlation between GPA and SATc.

Name: _____

Student ID #: _____

- 3 (j) Calculate the slope of the regression line when SATc is used to predict GPA
- 3 (k) Continuing from the previous part, calculate the *intercept* of the regression line.
- 3 (l) Hazel's combined SAT score was 1450. Based on the regression from the preceding two parts, what GPA would we predict for her?
- 15 2. Write an R function called `skew` that calculates the skewness of a sample of observations. Your function should take a single argument, the vector of data `x`, and return the skewness of this vector. Your function should account for the possibility that `x` may have missing values and drop them before carrying out the calculation.

3. This problem asks you to construct arbitrage strategies. While the example from class involved buying only, in general an arbitrage strategy can involve buying *and or* selling. This question concerns the ongoing election to become the presidential candidate for the Democratic Party (who will ‘win the nomination’). To decide who wins the nomination, there are a series of votes in each state. Parts (a) and (b) concern who wins the overall nomination, and parts (c) and (d) concern who will win the next two state votes, in Nevada and South Carolina.

- 5 (a) Rodrigo wonders which of the two remaining Democratic candidates, Sanders or Clinton, will win the nomination so he checks his two prediction markets. On one he finds contract C trading at \$0.63 that pays \$1 if Clinton wins; on the other he finds contract S trading at \$0.48 that pays \$1 if Sanders wins. Explain the probability rule these prices violate.
- 5 (b) Continuing from the previous part, construct an arbitrage strategy that Rodrigo could use to exploit the market mis-pricing. Explain exactly what he should buy and/or sell and how much he will earn for each possible outcome.
- 5 (c) Later Rodrigo again checks both prediction markets. On one he finds contract B trading at \$0.62 that pays \$1 if Clinton wins *both* the Nevada and South Carolina primaries. On the other he finds contract NV trading at \$0.55 that pays \$1 if Clinton wins the *Nevada* primary. Explain the probability rule these prices violate.

| |
|----|
| 10 |
|----|

- (d) Continuing from the previous part, construct an arbitrage strategy that Rodrigo could use to exploit the market mis-pricing of contracts B and NV. Explain exactly what he should buy and/or sell and how much he will earn for each possible outcome. Write no more than 5 bullet points.

Name: _____

Student ID #: _____

This was an assigned homework problem from the text. I have re-phrased it slightly to make it easier to understand but the substance and solution are unchanged.

4. A plane has crashed in one of three possible locations: the mountains (M), the desert (D), or the sea (S). Based on its flight path, experts have calculated the following prior probabilities that the plane is in each location: $P(M) = 0.5$, $P(D) = 0.3$ and $P(S) = 0.2$. If we search the mountains then, given that the plane is actually there, we have a 30% chance of *failing* to find it. If we search the desert then, given that the plane is actually there, we have a 20% chance of *failing* to find it. Finally, if we search the sea then, given that the plane is actually there, we have a 90% chance of *failing* to find it. Naturally if the plane is *not* in a particular location but we search for it there, we will not find it. You may assume that searches in each location are independent. Let F_M be the event that we *fail* to find the plane in the mountains. Define F_D and F_S analogously.

10

- (a) We started by searching the mountains. We did not find the plane. What is the conditional probability that the plane is nevertheless in the mountains? Explain.

20

- (b) After failing to find the plane in the mountains, we searched the desert, and the sea. We did not find the plane in either location. After this more exhaustive search what is the conditional probability that the plane is in the mountains? Explain.

5. Let X be a random variable that equals the total number of heads that you get over four tosses of a fair coin and define $Y = 10X + 25$.

5 (a) Write out the support set and probability mass function of X .

5 (b) Write out the cumulative distribution function of X .

5 (c) Calculate $E[X]$.

5 (d) Calculate $E[X^2]$.

5 (e) Calculate $Var(X)$.

5 (f) Calculate $E[Y]$.

5 (g) Calculate $Var(Y)$.

Name: _____

Student ID #: _____