

Problem Set #5

Econ 103

Part I – Problems from the Textbook

Chapter 4: 1, 3, 5, 7, 9, 11, 13, 15, 25, 27, 29

Part II – Additional Problems

1. Suppose X is a random variable with support $\{-1, 0, 1\}$ where $p(-1) = q$ and $p(1) = p$.
 - (a) What is $p(0)$?
 - (b) Calculate the CDF, $F(x_0)$, of X .
 - (c) Calculate $E[X]$.
 - (d) What relationship must hold between p and q to ensure $E[X] = 0$?
2. Fill in the missing details from class to calculate the variance of a Bernoulli Random Variable *directly*, that is *without* using the shortcut formula.
3. Prove that the Bernoulli Random Variable is a special case of the Binomial Random variable for which $n = 1$. (Hint: compare pmfs.)
4. Suppose that X is a random variable with support $\{1, 2\}$ and Y is a random variable with support $\{0, 1\}$ where X and Y have the following joint distribution:

$$\begin{aligned}p_{XY}(1, 0) &= 0.20, & p_{XY}(1, 1) &= 0.30 \\p_{XY}(2, 0) &= 0.25, & p_{XY}(2, 1) &= 0.25\end{aligned}$$

- (a) Express the joint distribution in a 2×2 table.
- (b) Using the table, calculate the marginal probability distributions of X and Y .
- (c) Calculate the conditional probability distribution of $Y|X = 1$ and $Y|X = 2$.
- (d) Calculate $E[Y|X]$.
- (e) What is $E[E[Y|X]]$?
- (f) Calculate the covariance between X and Y using the shortcut formula.