

# Problem Set #5

Econ 103

## Part I – Problems from the Textbook

Chapter 4: 1, 3, 5, 7, 9, 11, 13, 15, 25, 27, 29

## Part II – Additional Problems

1. Suppose  $X$  is a random variable with support  $\{-1, 0, 1\}$  where  $p(-1) = q$  and  $p(1) = p$ .
  - (a) What is  $p(0)$ ?
  - (b) Calculate the CDF,  $F(x_0)$ , of  $X$ .
  - (c) Calculate  $\mathbb{E}[X]$ .
  - (d) What relationship must hold between  $p$  and  $q$  to ensure  $\mathbb{E}[X] = 0$ ?
2. Fill in the missing details from class to calculate the variance of a Bernoulli Random Variable *directly*, that is *without* using the shortcut formula.
3. Prove that the Bernoulli Random Variable is a special case of the Binomial Random variable for which  $n = 1$ . (Hint: compare pmfs.)
4. Suppose that  $X$  is a random variable with support  $\{1, 2\}$  and  $Y$  is a random variable with support  $\{0, 1\}$  where  $X$  and  $Y$  have the following joint distribution:

$$\begin{aligned} p_{XY}(1, 0) &= 0.20, & p_{XY}(1, 1) &= 0.30 \\ p_{XY}(2, 0) &= 0.25, & p_{XY}(2, 1) &= 0.25 \end{aligned}$$

- (a) Express the joint distribution in a  $2 \times 2$  table.
- (b) Using the table, calculate the marginal probability distributions of  $X$  and  $Y$ .
- (c) Calculate the conditional probability distribution of  $Y|X = 1$  and  $Y|X = 2$ .
- (d) Calculate  $\mathbb{E}[Y|X]$ .
- (e) What is  $\mathbb{E}[\mathbb{E}[Y|X]]$ ?
- (f) Calculate the covariance between  $X$  and  $Y$  using the shortcut formula.