Problem Set #11

Econ 103

Part I – Problems from the Textbook

Chapter 9: 23 (use the "textbook" procedure), 27, 29

Part II – Additional Problems

1. This question revisits the data from this semester's anchoring experiment. In a previous assignment you used these data to construct confidence intervals. In this question you will carry out hypothesis tests. You may assume throughout that the sample size is large enough to use the Central Limit Theorem. Details of how to load the data from my website appear in Lecture 19. Be sure to properly account for missing values.

   (a) Suppose we want to test the null hypothesis of equality of population means across the two groups. What is the value of our test statistic?

   (b) Suppose we want to test the equality of population means against the one-sided alternative that the the "Hi" group has a higher mean at the 10% level. What is our critical value, and what is our decision rule? Do we reject the null hypothesis?

   (c) Calculate the p-value for a test of the equality of population means against the one-sided alternative that the "Hi" group has a higher mean.

   (d) Suppose we wanted to test the equality of population means against the two-sided alternative at the 10% level. What is our critical value, and what is our decision rule? Do we reject the null hypothesis?

   (e) Calculate the p-value for a test of the equality of population means against the two-sided alternative.

2. This problem uses a dataset that investigates the relationship between schizophrenia and the volume (in cm^3) of a particular region of the brain (the left hippocampus) measured using an MRI machine. In a past assignment you used this dataset to construct confidence intervals. In this question you will carry out hypothesis tests

   The dataset contains 15 sets of monozygotic (i.e. identical) twins, one of whom has schizophrenia ("Affected") and the other who does not ("Unaffected"). The idea of

using identical twins is to hold constant unobserved genetic and socioeconomic confounding variables that might influence whether someone develops schizophrenia. You can download the data from my website as follows:

```
data.url <- "http://www.ditraglia.com/econ103/case0202.csv"
twins <- read.csv(data.url)
head(twins)

##   Unaffected Affected
## 1       1.94     1.27
## 2       1.44     1.63
## 3       1.56     1.47
## 4       1.58     1.39
## 5       2.06     1.93
## 6       1.66     1.26
```

For this question you may assume that the sample differences between the left hippocampus volume of the "Affected" and "Unaffected" twins are drawn from a normal population with unknown variance.

(a) Carry out a one-sided test at the 5% level of the null hypothesis of no difference against the alternative that the affected twin has a larger left hippocampus, on average. What is your test statistic? What is your critical value? What is your decision rule? What is your decision?

(b) Repeat part (a) for a test against the *opposite* one-sided alternative.

(c) Repeat part (a) but test against the *two-sided* alternative.

(d) Explain the differences between your results in parts (a), (b), and (c).

(e) Calculate the p-values corresponding to parts (b) and (c).

3. This problem concerns a dataset comparing the scores of men and women on the Armed Forces Qualifying Test (AFQT). In an earlier assignment you used this dataset to construct confidence intervals. In this question you will carry out hypothesis tests instead. Throughout you may assume that the sample size is large enough for the CLT to apply. As before, the data are available from my website:

```
data.url <- "http://www.ditraglia.com/econ103/ex0222.csv"
test.scores <- read.csv(data.url)
head(test.scores)

##   Gender Arith Word Parag Math AFQT
```

```
## 1   male    19   27    14   14 70.3
## 2 female    23   34    11   20 60.4
## 3   male    30   35    14   25 98.3
## 4 female    30   35    13   21 84.7
## 5 female    13   30    11   12 44.5
## 6 female     8   15     6    4  4.0
```

Each row is an individual who took the test. The first column gives that individual's sex, while the second through fifth columns give the individual's score on four parts of the test. The final column is an overall percentile score for the test.

(a) For each section of the exam, as well as for overall percentile scores, test the null hypothesis that the population mean scores are equal for men and women at the 5% level. In which cases do you reject, and in which cases do you fail to reject?

(b) How do your results from part (a) relate to the CIs you constructed using this dataset in an earlier assignment?

(c) Calculate the two-sided p-values for each test from part (a).

4. In April of 2013, Public Policy Polling carried out a survey of 1247 registered voters to determine whether Republicans and Democrats differ in their beliefs about various conspiracy theories. To answer this question, you'll need to download the full results of their survey which I've posted on my website for convenience:
`http://www.ditraglia.com/econ103/conspiracy.pdf`
In an earlier assignment you used these data to construct confidence intervals. In this question you'll use them to carry out hypothesis tests. Throughout you may assume that the sample size is large enough for the approximate based on the central limit theorem to be valid.

(a) Suppose we wanted to test the null hypothesis that 20% of registered voters believe that a UFO crashed at Roswell, New Mexico in 1947 and the US Government covered it up. There are two possible test statistics we could use. Calculate them both and explain the difference. Which is preferable?

(b) Suppose that we wanted to test the null hypothesis from the preceding part against the one-sided alternative that more than 20% of registered voters believe in the UFO conspiracy. Calculate the p-value for this test.

(c) Repeat the preceding part for the *two-sided* alternative.

(d) Calculate the p-value for a test of the null hypothesis that equal proportions of Romney and Obama voters believe in the UFO conspiracy against the two-sided alternative. There are two test statistics you could use. Calculate the p-value using each and explain the difference. Which should we prefer?

5. Suppose $X_1, \ldots, X_n \sim$ iid $N(\mu_x, \sigma_X^2)$ independently of $Y_1, \ldots, Y_m \sim$ iid $N(\mu_Y, \sigma_Y^2)$.

   (a) Suppose $n = m = 10$ and you know that $\sigma_X^2 = \sigma_Y^2 = 10$. Express the power of a two-sided test of $H_0\colon \mu_X = \mu_Y$ at the 5% level in terms of the true, unknown difference of population means $\Delta = \mu_X - \mu_Y$.

   (b) Evaluate the power formula you derived in part (a) using R by setting

```
delta <- seq(from = -10, to = 10, by = 0.01)
```

   Plot your results. Approximately how large would the true difference of population means have to be for you to have at least a 50% chance of rejecting the null at the 5% level? What is the power when $\Delta = 0$? Why?

   (c) Repeat parts (a) and (b) with $n = m = 100$. How do your results change? Explain.

6. Professor Neil is interested in determining whether viewing different colors affects subjects' mental states in a way that alters their athletic ability. As a part of her research she carries out the following experiment. Each subject is randomly assigned to wait in one of two rooms: a room in which all of the walls have been painted pink or another in which all of the walls have been painted red. After waiting for five minutes, each subject is taken to a track and asked to run a 5K as fast as possible. Using the data from this experiment, Professor Neil carries out a statistical test of the null hypothesis that the population mean 5K time is equal across groups (those who waited in the pink room versus those who waited in the red room). Testing at the 1% level, she finds a statistically significant difference. For each of the following, answer True or False. If false, explain.

   (a) The p-value for the null hypothesis that population means are equal across groups is greater than 0.01.

   (b) Professor Neil would also have found a statistically significant difference had she carried out her test at the 5% level.

   (c) If there were really no difference in population means across the two groups, the chance of observing a test statistic at least as extreme as that observed by Professor Neil would be 0.01 or less.

   (d) Professor Neil's findings have important practical implications for sports regulatory organizations such as the International Olympic Committee: all locker rooms should be painted exactly the same color to keep from throwing off the outcomes of sporting events.