

# Problem Set #3

Econ 103

## Part I – Problems from the Textbook

Chapter 11: 1, 3

Chapter 15: 1(a), 5(a)

## Part II – Additional Problems

1. What value of  $a$  minimizes  $\sum_{i=1}^n (y_i - a)^2$ ? Prove your answer.

2. Let

$$z_{x_i} = \frac{x_i - \bar{x}}{s_x}, \text{ and } z_{y_i} = \frac{y_i - \bar{y}}{s_y}.$$

Show that if we carry out a regression with  $z_{y_i}$  in place of  $y$  and  $z_{x_i}$  in place of  $x$ , the intercept  $a$  will equal zero while the slope  $b$  will equal  $r$ , the sample correlation.

3. Let  $\hat{y}$  denote our prediction of  $y$  from a linear regression model:  $\hat{y} = a + bx$  and let  $r$  be the correlation coefficient between  $x$  and  $y$ .

(a) Express  $b$  in terms of  $s_{xy}$  and  $s_x$ .

(b) Express  $a$  in terms of  $b$  and the sample means of  $x$  and  $y$ .

(c) Express  $r$  in terms of the  $s_{xy}$ ,  $s_x$  and  $s_y$ .

(d) Show that

$$\frac{\hat{y} - \bar{y}}{s_y} = r \left( \frac{x - \bar{x}}{s_x} \right)$$

(e) (3 points) Using the equation derived in (d), briefly explain “regression to the mean.”

4. Lothario, an unscrupulous economics major, runs the following scam. After the first midterm of Econ 103 he seeks out the students who did extremely poorly and offers to sell them “statistics pills.” He promises that if they take the pills before the second midterm, their scores will improve. The pills are, in fact, M&Ms and don’t actually improve one’s performance on statistics exams. The overwhelming majority of Lothario’s former customers, however, swear that the pills really work: their scores improved on the second midterm. What’s your explanation?