Problem Set #4

Econ 103

Part I – Problems from the Textbook

Chapter 3: 1, 3, 5, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29

Part II – Additional Problems

1. Suppose you flip a fair coin twice.
   (a) List all the basic outcomes in the sample space.
   (b) Let A be the event that you get at least one head. List all the basic outcomes in A.
   (c) What is the probability of A?
   (d) List all the basic outcomes in A^c.
   (e) What is the probability of A^c?

2. Suppose I deal two cards at random from a well-shuffled deck of 52 playing cards. What is the probability that I get a pair of aces?

3. Suppose everyone in a class of one hundred students flips a fair coin five times.
   (a) What is the probability that John Smith, a particular student in the class, gets five heads in a row?
   (b) What is the probability that at least one person gets five heads in a row?

4. (Adapted from Mosteller, 1965) A jury has three members: the first flips a coin for each decision, and each of the remaining two independently has probability p of reaching the correct decision. Call these two the "serious" jurors and the other the "flippant" juror (pun intended).
   (a) What is the probability that the serious jurors both reach the same decision?
   (b) What is the probability that the serious jurors each reach different decisions?
   (c) What is the probability that the jury reaches the correct decision? Majority rules.

5. This question refers to the prediction market example from lecture. Imagine it is October 2012. Let O be a contract paying \$10 if Obama wins the election, zero otherwise, and R be a contract paying \$10 if Romney wins the election, zero otherwise. Let Price(O) and Price(R) be the respective prices of these contracts.

   (a) Suppose you *buy* one of each contract. What is your profit?

   (b) Suppose you *sell* one of each contract. What is your profit?

   (c) What must be true about Price(O) and Price(R), to prevent an opportunity for statistical arbitrage?

   (d) How is your answer to part (c) related to the Complement Rule?

   (e) What is the implicit assumption needed for your answers to parts (a)–(c) to be correct? How would your answers change if we were to relax this assumption?

6. "Odd Question" # 6, from Hacking (2001):

   > You are a physician. You think it is quite likely that one of your patients has strep throat, but you aren't sure. You take some swabs from the throat and send them to a lab for testing. The test is (like nearly all lab tests) not perfect. If the patient has strep throat, then 70% if the time the lab says yes. But 30% of the time it says NO. If the patient does not have strep throat, then 90% of the time the lab says NO. But 10% of the time it says YES. You send five successive swabs to the lab, from the same patient. and get back these results in order: YES, NO, YES, NO, YES.

   Let S be the event that the patient has strep throat, and S^c be the even that she does not. Let Y be the event that a given test says YES and $N = Y^c$ be the event that a given test says NO. You may assume that the tests are independent.

   (a) Calculate the probability that your patient has strep throat. (Hint, there is a missing piece of information and you should express your answer *in terms of it.*)

   (b) Based on your answer to part (b) do you think the patient has strep throat? Explain.

Part III – "Challenge" Problems

These are optional. If you're up for a challenge you'll learn quite a bit from these.

7. (Adapted from Mosteller, 1965) "What is the least number of persons required if the probability exceeds 1/2 that two or more of then have the same birthday? (Year of birth need not match)." [Hint: Use the Complement Rule.]

8. Formally prove the Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ using the basic notions of set theory we learned in class and the axioms of probability. [Hint: Try to translate the intution from the Venn diagram into an equation.]

9. Weren't stumped by the Monte Hall problem? Try this example from Mosteller (1965):

   > Three prisoners, A, B, and C, with apparently equally good records have applied for parole. The parole board has decided to release two of the three, and the prisoners know this but not which two. A warder friend of prisoner A knows who are to be released. Prisoner A realizes that it would be unethical to ask the warder if he, A, is to be released, but thinks of asking for the name of *one* prisoner *other than himself* who is to be released. He thinks that before he asks, his chances of release are 2/3. He thinks that if the warder says "B will be released," his own chances have now gone down to 1/2, because either A and B or B and C are to be released. And so A decides not to reduce his chances by asking. However, A is mistaken in his calculations. Explain.