# Problem Set #6

## Econ 103

## Part I – Problems from the Textbook

Chapter 4: 19, 21, 23 (*When necessary, use R rather than the Normal tables in the front of the textbook.*)

## Part II – Additional Problems

1. Suppose that $X$ is a random variable with the following PDF

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Graph the PDF of $X$.

   (b) Show that $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

   (c) What is $P(0.5 < X < 1.5)$?

2. A random variable is said to follow a Uniform$(a, b)$ distribution if it is equally likely to take on any value in the range $[a, b]$ and never takes a value outside this range. Suppose that $X$ is such a random variable, i.e. $X \sim$ Uniform$(a, b)$.

   (a) What is the support of $X$?

   (b) Explain why the PDF of $X$ is $f(x) = 1/(b - a)$ for $a \le x \le b$, zero elsewhere.

   (c) Using the PDF from part (b), calculate the CDF of $X$.

   (d) Verify that $f(x) = F'(x)$ for the present example.

   (e) Calculate $E[X]$.

   (f) Calculate $E[X^2]$. *Hint:* recall that $b^3 - a^3$ can be factorized as $(b - a)(b^2 + a^2 + ab)$.

   (g) Using the shortcut formula and parts (e) and (f), show that $Var(X) = (b - a)^2/12$.

3. Suppose that $X \sim N(0, 16)$ independent of $Y \sim N(2, 4)$. Recall that our convention is to express the normal distribution in terms of its mean and variance, i.e. $N(\mu, \sigma^2)$. Hence, $X$ has a mean of zero and variance of 16, while $Y$ has a mean of 2 and a variance of 4. In completing some parts of this question you will need to use the R function `pnorm` described in class. In this case, please write down the command you used as well as the numeric result.

   (a) Calculate $P(-8 \leq X \leq 8)$.

   (b) Calculate $P(0 \leq Y \leq 4)$.

   (c) Calculate $P(-1 \leq Y \leq 6)$.

   (d) Calculate $P(X \geq 10)$.

   **Note:** *In the following five questions* $X_1, X_2 \sim$ *iid* $N(\mu, \sigma^2)$, $Y = (X_1 - \mu)/\sigma$, $Z = (X_2 - \mu)/\sigma$.

4. (a) What is the distribution of $X_1 + X_2$?

   (b) Use R to calculate $P(X_1 + X_2 > 5)$ if $\mu = 5$ and $\sigma^2 = 50$.

   (c) Use R to calculate the 10th percentile of the distribution of $X_1 + X_2$.

5. (a) What is the distribution of $Y^2$?

   (b) Use R to calculate $P(Y^2 \geq 1)$.

6. (a) What is the distribution of $Y^2 + Z^2$?

   (b) Use R to calculate the 95th percentile of the distribution of $Y^2 + Z^2$.

7. (a) What is the distribution of $Z/\sqrt{Y^2}$?

   (b) What value of $c$ satisfies $P(-c \leq Z/\sqrt{Y^2} \leq c) = 0.95$?

   (c) How does the interval in part (b) compare to the corresponding interval for $Z$?

8. (a) What is the distribution of $Y^2/Z^2$?

   (b) Use R to calculate the 95th percentile of the distribution of $Y^2/Z^2$.