1 Algebraic form and conjugation of a complex number

Exercise 1.1

Simplify:

1. $(-2+3i)+(7-8i)$
2. $(4i-3)-(1+10i)$
3. $(\sqrt{2}+i)\cdot(3-\sqrt{3}i)$
4. $\frac{2-3i}{5+4i}$

Exercise 1.2

Find the real numbers x, y satisfying the given equations:

1. $x(2+3i)+y(4-5i)=6-2i$
2. $(x-i)\cdot(2-yi)=11-23i$
3. $\frac{x}{2-3i}+\frac{y}{3+2i}=1$

Exercise 1.3

Solve the given equations in the set of complex numbers:

1. $z^2+3\bar{z}=0$
2. $2z+(1+i)\bar{z}=1-3i$
3. $z^2-z+1=0$
4. $\frac{z+1}{\bar{z}-1}=-1$
5. $(z+\bar{z})+i(z-\bar{z})=2i-6$
6. $(i-3)z=5+i-z$
7. $\frac{1-3i}{3z+2i}=\frac{2i-3}{5-2iz}$

Exercise 1.4

For which values of the real parameters a, b the equation $3\bar{z}-2z=a+bi$ has a solution?

Exercise 1.5

On the complex plane, draw sets of numbers z that satisfy the given conditions:

1. $\operatorname{Im}[(1+2i)z-3i]<0$
2. $\operatorname{Re}(z-i)^2\geq 0$
3. $z^2=2\operatorname{Re}(iz)$
4. $\operatorname{Re}\left(z^3\right)\geq\operatorname{Im}\left(z^3\right)$

Exercise 1.6

Sketch the set of all complex numbers z for which the number $\omega=\frac{z}{z+i}$ is

1. real
2. purely imaginary

Exercise 1.7

Points $z_1=-1+2i$, $z_2=i$, and $z_4=2+4i$ are the vertices of the parallelogram. Find the position of vertex z_3 of this parallelogram.

2 Modulus and argument of a complex number. The trigonometric form of a complex number.

Exercise 2.1

Calculate the modules of the given complex numbers:

1. $4i$
2. $12i-5$
3. $\sqrt{7}+\sqrt{29}i$
4. $(\sqrt{5}-\sqrt{3})+(\sqrt{5}+\sqrt{3})i$
5. $\sin\alpha+i\cos\alpha$, and $\alpha\in\mathbb{R}$

Exercise 2.2

Give a geometric interpretation of the modulus of difference of complex numbers. Using this interpretation, draw sets of complex numbers a satisfying the given conditions:

1. $|z+1-2i|=3$
2. $2\leq|z+i|<4$
3. $|(1+i)z-2|\geq 4$
4. $\left|\frac{z+3}{z-2i}\right|\geq 1$
5. $\operatorname{Re}(z+1)<0$ and $|i-z|\leq 3$
6. $\left|z^2+4\right|\leq|z-2i|$