



Learning Objectives

This review session has one objective, and that is to *Work in groups to identify course objectives that you are not comfortable with.*

- The following sentences have errors or are missing something. Correct them or add the missing details.

(a) **(Subspaces & Bases)**

A basis for the subspace V is a set S such that $\dim S = \dim V$ and $\text{span}(S) = V$.

(b) **(Vectors)**

\vec{w} is a convex combination of u and v if:

$$\vec{w} = \{ \vec{w} : \vec{w} = a\vec{u} + b\vec{v} \text{ for all } a, b \in [0, 1] \}$$

(c) **(Determinants, Dot products)**

If \vec{a} , \vec{b} , and \vec{c} are pairwise orthogonal vectors in \mathbb{R}^3 , then $\det[\vec{a}|\vec{b}|\vec{c}] = 1$.

- (Linear Transformations)**

A transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is called *almost linear* if for all $x, y \in \mathbb{R}^n$, we have $\|T(x+y)\| \leq \|T(x) + T(y)\|$, and for all $\alpha \geq 0$ and $x \in \mathbb{R}^n$, we have $\|T(\alpha x)\| \leq \alpha \|T(x)\|$.

- Write down a clear and concise explanation of what one must do in order to show that a transformation is almost linear.
- Show that the transformation $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ is not almost linear.
- Show that any linear transformation is almost linear.

- (Inverses)**

Mohammed has partially row-reduced the matrix A using the following steps. Use his work to write A as a product of elementary matrices. Use this to compute $\det A$.

$$A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_4 \rightarrow r_4 - r_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_4 \leftrightarrow r_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

- (Determinants)**

For which values of a is the matrix A invertible? What is the rank of A when it is not invertible?

$$A = \begin{bmatrix} 0 & 3 & 2 \\ a & 0 & 0 \\ 8 & 2 & 0 \end{bmatrix}$$

- (Eigenvalues & Diagonalization, Similar Matrices)**

Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the transformation which projects vectors onto the subspace $V = \text{span}\{\vec{v}\}$, where $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

- (a) Find an appropriate basis to express T as a matrix.
- (b) What are the eigenvectors of T ? Is T diagonalizable?

6. **(Computational Objectives, Representations of Lines)**

Let $\vec{a} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Define:

$$V = \{\vec{x} \in \mathbb{R}^2 \mid \vec{x} \perp \vec{a} \text{ and } \vec{x} \perp \vec{b}\}$$

- Express V as the solution to an augmented matrix
- Express V as a linear system
- Express V in parametric form
- Express V as the span of vectors
- Sketch V .