



Learning Objectives

This review session has one objective, and that is to *work in groups to identify learning objectives that you are not comfortable with*.

- The following sentences have errors or are missing something. Correct them or add the missing details.

(a) **(Subspaces & Bases; Span; Linear Independence)**

A basis for the subspace V is a set S such that $\dim S = \dim V$ and $\text{span } S = V$.

(b) **(Vectors)**

Vector \vec{w} is a convex combination of \vec{u} and \vec{v} if:

$$\vec{w} = \{ \vec{w} : \vec{w} = a\vec{u} + b\vec{v} \text{ for all } a, b \in [0, 1] \}$$

(c) **(Determinants; Dot products)**

If \vec{a} , \vec{b} , and \vec{c} are pairwise orthogonal vectors in \mathbb{R}^3 , then $\det[\vec{a} \mid \vec{b} \mid \vec{c}] = 1$.

(d) **(Eigenvectors & Diagonalization)**

Every matrix A has a basis of eigenvalues.

- (Linear Transformations)**

A transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is called *almost linear* if for all $x, y \in \mathbb{R}^n$, and $\alpha \geq 0$, we have

$$\begin{aligned} \|T(x + y)\| &\leq \|T(x) + T(y)\| \\ \|T(\alpha x)\| &\leq \alpha \|T(x)\| \end{aligned}$$

- Write down a clear and concise explanation of what one must do in order to show that a transformation is almost linear.
- Show that the transformation $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ is not almost linear.
- Show that any linear transformation is almost linear.

- (Inverses)**

Mohammed has partially row-reduced the matrix A using the following steps. Use his work to write A as a product of elementary matrices. Use this to compute $\det A$.

$$A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_4 \rightarrow r_4 - r_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_4 \leftrightarrow r_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

4. (Determinants and Inverses; Subspaces)

We define:

$$A = \begin{bmatrix} 0 & 3 & 2 \\ a & 0 & 0 \\ 8 & 2 & 0 \end{bmatrix}$$

- (a) For which values of a is the matrix A invertible?
- (b) What is the rank of A when it is not invertible?
- (c) When A is not invertible, find a basis for the range of the transformation $T_A(x) = Ax$.

5. (Eigenvalues & Diagonalization; Similar Matrices; Projections)

Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the transformation which projects vectors onto the subspace $V = \text{span}\{\vec{v}\}$, where $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

- (a) Write T in the standard basis, and in one other basis. Which one do you prefer?
- (b) Is T one-to-one? Is T onto?
- (c) What are the eigenvectors of T ? Is T diagonalizable?
- (d) Is T invertible?

6. (Computational Objectives; Representations of Lines; SLE)

Let $\vec{a} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Define the set V as

$$V = \left\{ \vec{x} \in \mathbb{R}^3 : (\vec{x} - \vec{e}_3) \perp \vec{a} \text{ and } (\vec{x} - \vec{e}_3) \perp \vec{b} \right\}$$

- (a) Write down a system of linear equations where V is the complete solution set.
- (b) Write down the corresponding augmented matrix.
- (c) Express V in vector form.
- (d) Express V as the span or translated span of vectors.
- (e) Sketch V .