Learning Objectives

> This review session has one objective, and that is to *work in groups to identify learning objectives that you are not comfortable with*.

1. The following sentences have errors or are missing something. Correct them or add the missing details.

   (a) **(Subspaces & Bases; Span; Linear Independence)**
   A basis for the subspace V is a set S such that $\dim S = \dim V$ and $\operatorname{span} S = V$.

   (b) **(Vectors)**
   The vector $\vec{w}$ is a convex combination of $\vec{u}$ and $\vec{v}$ if:

   $$\vec{w} = \left\{ \vec{w} : \vec{w} = a\vec{u} + b\vec{v} \text{ for all } a, b \in [0,1] \right\}$$

   (c) **(Determinants; Dot products)**
   If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are pairwise orthogonal vectors in $\mathbb{R}^3$, then $\det\left[\vec{a} \mid \vec{b} \mid \vec{c}\right] = 1$.

   (d) **(Eigenvectors & Diagonalization)**
   Every matrix A has a basis of eigenvalues.

2. **(Working with New Definitions)**
   A transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is called *almost linear* if for all $\vec{x}$, $\vec{y} \in \mathbb{R}^n$, and $\alpha \geq 0$, we have

   $$\|T(\vec{x} + \vec{y})\| \leq \|T(\vec{x}) + T(\vec{y})\|$$
   $$\|T(\alpha\vec{x})\| \leq \alpha\|T(\vec{x})\|$$

   (a) Write down a numbered list of what one must do in order to show that a transformation is almost linear.

   (b) Show that the transformation $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is not almost linear.

   (c) Show that any linear transformation is almost linear.

3. **(Inverses & Elementary Matrices)**
   Mohammed has partially row-reduced the matrix A using the following steps. Use his work to write A as a product of elementary matrices. Use this to compute $\det A$.

$$A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 2 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_4 \to r_4 - 2r_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_4 \leftrightarrow r_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

4. **(Determinants and Inverses; Subspaces)**
We define:

$$A = \begin{bmatrix} 0 & 3 & 2 \\ x & 0 & 0 \\ 8 & 2 & 0 \end{bmatrix}$$

(a) For which values of x is the matrix A invertible?

(b) What is the rank of A when it is not invertible?

(c) When A is not invertible, find a basis for the range of the transformation $T_A(\vec{v}) = A\vec{v}$.

5. **(Eigenvalues & Diagonalization; Similar Matrices; Projections)**
Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which projects vectors onto the subspace $X = \operatorname{span}\{\vec{v}\}$, where $\vec{v} = \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right]$.

(a) Write T in the standard basis, and in one other basis. Which one do you prefer?

(b) Is T one-to-one? Is T onto?

(c) What are the eigenvectors of T? Is T diagonalizable?

(d) Is T invertible?

6. **(Computational Objectives; Representations of Lines; SLE)**
Let $\vec{a} = \left[\begin{smallmatrix} -1 \\ 0 \\ 1 \end{smallmatrix}\right]$ and $\vec{b} = \left[\begin{smallmatrix} 1 \\ 0 \\ 0 \end{smallmatrix}\right]$. Define the set V as

$$V = \left\{ \vec{x} \in \mathbb{R}^3 : (\vec{x} - \vec{e}_3) \cdot \vec{a} = 0 = (\vec{x} - \vec{e}_3) \cdot \vec{b} \right\}$$

(a) Write down a system of linear equations where V is the complete solution set to the system.

(b) Write down the corresponding augmented matrix to the system.

(c) Express V in vector form.

(d) Express V as the span or translated span of vectors.

(e) Sketch V.