Learning Objectives

Is writing all the learning objectives necessary? In this session you will be working with linear algebra. Recall the course learning objectives:

- *Work independently to understand concepts and procedures that have not been previously explained to you.*
- *Clearly and correctly express the mathematical ideas of linear algebra to others, and understand and apply logical arguments and definitions that have been written by others.*
- *Translate between algebraic and geometric viewpoints to solve problems.*
- *Use matrices, matrix arithmetic, matrix inverses, systems of linear equations, row reduction, determinants, and eigenvalues and eigenvectors to solve problems. As well, write vectors in different bases and pick an appropriate basis when working on problems.*

1. The following sentences have errors or are missing something. Correct them or add the missing details.

   (a) A transformation $f:\mathbb{R}^n \to \mathbb{R}^m$ is linear if it sends sums to sums, $f(\vec{0}) = \vec{0}$, and $f(\alpha\vec{x}) = \alpha f(\vec{x})$.

   (b) An $n \times m$ matrix represents a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.

   (c) A basis for the subspace V is a set S such that $\dim S = \dim V$ and $\operatorname{span}(S) = V$.

   (d) $\vec{w}$ is a convex combination of u and v if:

   $$\vec{w} = \left\{ \vec{w} : \vec{w} = a\vec{u} + b\vec{v} \text{ for all } a, b \in [0,1] \right\}$$

   (e) If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are pairwise orthogonal vectors in $\mathbb{R}^3$, then $\det\left[\vec{a}|\vec{b}|\vec{c}\right] = 1$.

2. Sam has a function $T:\mathbb{R}^n \to \mathbb{R}^m$, which has the following property:

   For any subspace $V \subset \mathbb{R}^n$, $T(V)$ is a subspace of $\mathbb{R}^m$.

   (a) What are the possible values of $T(\vec{0})$?

   (b) Sam says that their function is not linear. Can this be true?

3. Orthogonality.

4. Mohammed has partially row-reduced the matrix A using the following steps. Use his work to write A as a product of elementary matrices. Use this to compute $\det A$.

$$A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_4 \to r_4 - r_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_4 \leftrightarrow r_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

5. For which values of a is the matrix A invertible? Compute the inverse of A when it exists (your answer may depend on a).

$$A = \begin{bmatrix} 0 & 3 & 2 \\ a & 0 & 0 \\ 8 & 2 & 0 \end{bmatrix}$$

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation which projects vectors onto the subspace $V = \text{span}\{\vec{v}\}$, where $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

   (a) Show that T is linear.

   (b) What are the eigenvectors of T? Is T diagonalizable?

   (c) How do the eigenvectors of T relate to V and $V^{\perp}$?