



## Learning Objectives

This review session has one objective, and that is to *Work in groups to identify course objectives that you are not comfortable with.*

- The following sentences have errors or are missing something. Correct them or add the missing details.

(a) **(Subspaces & Bases)**

A basis for the subspace  $V$  is a set  $S$  such that  $\dim S = \dim V$  and  $\text{span}(S) = V$ .

(b) **(Vectors)**

$\vec{w}$  is a convex combination of  $u$  and  $v$  if:

$$\vec{w} = \{ \vec{w} : \vec{w} = a\vec{u} + b\vec{v} \text{ for all } a, b \in [0, 1] \}$$

(c) **(Determinants; Dot products)**

If  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$  are pairwise orthogonal vectors in  $\mathbb{R}^3$ , then  $\det[\vec{a} \mid \vec{b} \mid \vec{c}] = 1$ .

- (Linear Transformations)**

A transformation  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is called *almost linear* if for all  $x, y \in \mathbb{R}^n$ , we have  $\|T(x+y)\| \leq \|T(x) + T(y)\|$ , and for all  $\alpha \geq 0$  and  $x \in \mathbb{R}^n$ , we have  $\|T(\alpha x)\| \leq \alpha \|T(x)\|$ .

- Write down a clear and concise explanation of what one must do in order to show that a transformation is almost linear.
- Show that the transformation  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^2$  is not almost linear.
- Show that any linear transformation is almost linear.

- (Inverses)**

Mohammed has partially row-reduced the matrix  $A$  using the following steps. Use his work to write  $A$  as a product of elementary matrices. Use this to compute  $\det A$ .

$$A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_4 \rightarrow r_4 - r_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{r_4 \leftrightarrow r_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

- (Determinants)**

For which values of  $a$  is the matrix  $A$  invertible? What is the rank of  $A$  when it is not invertible?

$$A = \begin{bmatrix} 0 & 3 & 2 \\ a & 0 & 0 \\ 8 & 2 & 0 \end{bmatrix}$$

- (Eigenvalues & Diagonalization; Similar Matrices)**

Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the transformation which projects vectors onto the subspace  $V = \text{span}\{\vec{v}\}$ , where  $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ .

- (a) Find an appropriate basis to express  $T$  as a matrix.
- (b) What are the eigenvectors of  $T$ ? Is  $T$  diagonalizable?

6. **(Computational Objectives, Representations of Lines)**

Let  $\vec{a} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$  and  $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ . Define:

$$V = \{\vec{x} \in \mathbb{R}^2 \mid \vec{x} \perp \vec{a} \text{ and } \vec{x} \perp \vec{b}\}$$

- Express  $V$  as the solution to an augmented matrix
- Express  $V$  as a linear system
- Express  $V$  in parametric form
- Express  $V$  as the span of vectors
- Sketch  $V$ .