Below is a list of content-level learning outcomes and anti-outcomes. That is, below is a list of what you should know, and occasionally what you don't need to know. Please use this list as a study guide.

Objectives that are included on Midterm 1 are in Fuchsia.
Objectives that are included on Midterm 2 are in Burnt Orange.

Math Notation

- Define *subset*, *set equality*
- Sets, set builder notation, unions, intersections, special sets $\mathbb{R}$, $\mathbb{Z}$, $\mathbb{Q}$
- Find the sum of sets (e.g., $A + B$ where A and B are sets). (*Not in the textbook*)
- Given two sets, determine if one is a subset of the other.
- Draw subsets of $\mathbb{R}$ or $\mathbb{R}^2$.

Vectors

- Define *linear combination*, *norm*, and *unit vector*.
- Define *non-negative linear combination*, *convex linear combination* (*Not in the textbook*)
- Explain the connection between vectors and points.
- Draw vectors as arrows or points, choosing the appropriate representation for the situation.
- Convert between vectors and points.
- Write vectors in column form.
- Compute linear combinations of vectors.
- Write a vector as a linear combination of others.
- Compute the norm of a vector.
- Compute the distance between two vectors.
- Compute or draw the image of a set of vectors under a transformation.

Representations of Lines and Planes

- Describe a line or plane as (i) a set, (ii) vector form, and (iii) a coordinate equation (e.g., $y = mx + b$).
- Convert between representations of lines and planes.
- Determine if two lines/planes are identical.
- Determine if two lines/planes are intersect.
- Express simple geometric figures (e.g., squares and triangles) as sets with set builder notation.

Spans

- Define *span*
- Express translated spans in set notation. (*Not in the textbook*)
- Describe the link between vector form of lines/planes and translated spans. (*Not in the textbook*)
- Determine if a vector is in a translated span.

SLE's (Systems of Linear Equations; Mainly covered in online homework)

- Define *consistent/inconsistent system*.

- Use row reduction to solve a system.
- Convert between a system and its representation as an augmented matrix.
- Describe solutions as intersecting hyperplanes
- Find particular & general solutions to SLE's in vector form or as translated spans.
- Produce systems with 0,1, or infinitely many solutions.

Linear Independence/Dependence

- Define *linear independence/linear dependence* of vectors both algebraically and geometrically.
- Explain in plain words what linear independence/dependence means.
- Determine if vectors are linearly independent/dependent.
- Given a set of vectors, find multiple linearly independent spanning sets (if possible).

Dot Products

- Define *dot product* geometrically and algebraically, *orthogonal*
- Compute dot products of vectors described graphically or numerically.
- Determine the sign of a dot product from a picture.
- Determine whether one vector points in the direction of another vector and to what degree.

Projections

- Define *projection* and *component in the direction of* (*Also called 'projection' in the textbook*)
- Compute projections onto points, lines, and simple sets.
- State the relationship between projection and orthogonality.

Subspace & Bases

- Define *subspace*, *basis*, *dimension*
- Determine if a set is a subspace or a translated subspace
- Find bases for subspaces & their dimension
- Find the dimension of translated subspaces
- create examples and non-examples of subspaces

Matrices

- Define matrix-vector multiplication as a linear combination of columns and dot product with rows, the *identity matrix*, the *zero matrix*.
- Compute matrix-vector products
- Compute matrix-matrix products
- Express SLE's as matrix equations
- Express linear combination problems as matrix equations

Change of Basis & Coordinates

- Define what it means to *represent a vector in terms of a basis*.

- Use notation $[\vec{v}]_{\mathcal{B}}$ and $\begin{bmatrix}1\\2\end{bmatrix}_{\mathcal{B}}$ where $\mathcal{B}$ is a basis.
- Write a vector in another basis.
- Find a basis so a vector has a particular representation.
- Distinguish between left-handed and right-handed bases in $\mathbb{R}^1$, $\mathbb{R}^2$, and $\mathbb{R}^3$.
- Find a matrix for a change of basis.

Linear Transformations

- Define *linear transformation*, *one-to-one*, *onto*, *invertible*, *range*, *null space*, *column space*, *row space*, *image* of a set, *rank*.
- Determine if a transformation is linear.
- Produce examples and non-examples of linear transformations.
- Find a matrix representation of a linear transformation in a basis.
- Describe connection between matrix multiplication and composition of functions.
- Explain why matrix products aren't commutative.
- Determine if a transformation is one-to-one/onto/invertible
- Determine the rank of a linear transformation
- Fundamental Subspaces
  - Prove the fundamental subspaces are subspaces.
  - Find bases for the fundamental subspaces (by row reduction)
  - Identify which subspaces change during row reduction

(Theme from here on out is $T : \mathbb{R}^n \to \mathbb{R}^n$)

Inverses

- Define *inverse matrix*, *elementary matrix*.
- Explain in words that invertible matrices correspond to reversible linear transformations
- Given a description of a linear transformation, determine if its corresponding matrix is invertible.
- Determine if a matrix is invertible
- Find the inverse of a matrix
- Decompose a matrix into a product of elementary matrices
- Use matrix inverses to solve SLE's.
- Know and apply the formula for $(AB)^{-1}$.

Similar Matrices

- Define *similar matrices*
- Write a linear transformation in different bases
- Find bases so that linear transformations (e.g., projections) have nice representations.

Determinants

- Define *determinant* as an oriented volume.
- Compute determinants of a linear transformation geometrically (without a matrix).
- Compute determinants of 2×2, 3×3, diagonal, and triangular matrices quickly with a formula.
- Compute determinants of $n \times n$ using row reduction.
- Compute determinants of compositions via multiplication.
- Compute volumes of images.
- State relationship between determinants and invertibility.

Eigenvectors & Diagonalization

- Define *eigenvector*, *eigenvalue*, *eigen space*, *characteristic polynomial*, *algebraic multiplicity*, *geometric multiplicity*
- Find eigenvalues and eigenvectors geometrically
- Find eigenvalues and eigenvectors algebraically
- Compute characteristic polynomials
- Determine if a transformation has a basis of eigenvectors
- Diagonalize a matrix
- Use diagonalization to compute large powers of a matrix
- Given eigenvectors and eigenvalues, produce a transformation with those eigenvectors and eigenvalues

Computational Objectives

- Find the norm of a vector
- Find the distance between two vectors
- Compute a given linear combination of vectors
- Find the angle between two vectors
- Determine if two lines intersect
- Convert between a SLE and its augmented matrix
- Row reduce a matrix
- Write the complete solution to SLEs
- Classify sets of explicit vectors (i.e., column vectors with numbers as entries) as linearly independent/dependent
- Compute a matrix-vector product
- Compute a matrix-matrix product
- Find a basis for the row space/column space/null space of a matrix
- Find the inverse of a matrix
- Find the rank of a matrix

- Find the determinant of a 2×2 or 3×3 matrix
- Compute eigen values/eigen vectors of explicit matrices