Logic Practical Week 13

Question 3:

Show that the following propositional formulae are tautologies without using truth tables:

1. $(\neg q \wedge (p \Rightarrow q)) \Rightarrow \neg q$
2. $((p \vee q) \wedge \neg p) \Rightarrow q$

1. (¬q ∧ (p ⇒ q)) ⇒ ¬q

$$\begin{array}{rl}
(\neg q \wedge (p \Rightarrow q)) \Rightarrow \neg q & \text{(Implication Law)} \\
(\neg q \wedge (\neg p \vee q)) & \text{(Distribution Law)} \\
(\neg q \wedge \neg p) \vee (\neg q \wedge q) & \text{(De Morgan's)} \\
\neg(q \vee p) \Rightarrow \neg q & \text{(Implication Law)} \\
(q \vee p) \vee \neg q & \\
1 \ \checkmark &
\end{array}$$

$$2.((p \vee q) \wedge \neg p) \Rightarrow q$$

$$\begin{array}{r}
((p \vee q) \wedge \neg p) \Rightarrow q \\
(\neg p \wedge p) \vee (\neg p \wedge q) \\
(\neg p \wedge q) \Rightarrow q \\
\neg(\neg p \wedge q) \vee q \\
(p \vee \neg q) \vee q \\
1 \ \checkmark
\end{array}$$

Question 4:

Show that $(a \vee c) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a)$ and $(b \Rightarrow c) \wedge a$ are logically equivalent without using truth tables.

Equation 1:

$$\begin{array}{rl}
(a \vee c) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) & \text{(Implication Law)} \\
(a \vee c) \wedge (\neg b \vee c) \wedge (\neg c \vee a) & \text{(Commutativity)} \\
(a \vee c) \wedge (a \vee \neg c) \wedge (\neg b \vee c) & \text{(Distribution Law)} \\
(a \vee (c \wedge \neg c)) \wedge (\neg b \vee c) & \\
a \wedge (\neg b \vee c) &
\end{array}$$

Equation 2:

$$\begin{array}{rl}
(b \Rightarrow c) \wedge a & \text{(Implication Law)} \\
(\neg b \vee c) \wedge a & \text{(Commutative Law)} \\
a \wedge (\neg b \vee c) &
\end{array}$$

$\therefore$ **Equation 1** $\equiv$ **Equation 2**

Question 9: Conjunctive Normal Form

Question 9: Conjunctive normal form

Reduce the following formulae φ to conjunctive normal form without using truth tables.

1. $((p \wedge \neg q) \vee r) \Rightarrow (\neg p \wedge \neg r)$
2. $(p \wedge (q \Rightarrow r)) \Rightarrow s$
3. $(p_1 \wedge p_2) \vee (p_3 \wedge (p_4 \vee p_5))$

1. ((p ∧ ¬q) ∨ r) ⇒ (¬p ∧ ¬r)

$$
\begin{aligned}
((p \land \neg q) \lor r) \Rightarrow (\neg p \land \neg r) &\qquad \text{(Implication Law)}\\
\neg((p \land \neg q) \lor r) \lor (\neg p \land \neg r) &\qquad \text{(De Morgan's)}\\
(\neg(p \land \neg q) \land \neg r) \lor (\neg p \land \neg r) &\qquad \text{(Distribution Law)}\\
\neg r \land (\neg p \lor (\neg(p \land \neg q))) &\qquad \text{(De Morgan's)}\\
\neg r \land (\neg p \lor (\neg p \lor q)) &\\
\neg r \land (\neg p \lor q) \checkmark &
\end{aligned}
$$

2. (p ∧ (q ⇒ r)) ⇒ s

$$
\begin{aligned}
(p \land (q \Rightarrow r)) \Rightarrow s &\qquad \text{(Implication Law)}\\
\neg(p \land (q \Rightarrow r)) \lor s &\qquad \text{(Implication Law)}\\
\neg(p \land (\neg q \lor r)) \lor s &\qquad \text{(De Morgan's)}\\
(\neg p \lor \neg(\neg q \lor r)) \lor s &\qquad \text{(De Morgan's)}\\
(\neg p \lor (q \land \neg r)) \lor s &\qquad \text{(Distribution Law)}\\
((\neg p \lor q) \land (\neg p \lor \neg r)) \lor s &
\end{aligned}
$$

General Formula: (Distribution Law)

$$(A \land B) \lor C) \equiv (C \lor A) \land (C \lor B)$$

Where $A = (\neg p \lor q), B = (\neg p \lor \neg r), C = s$

Applying the General Formula:

$$
\begin{aligned}
((\neg p \lor q) \land (\neg p \lor \neg r)) \lor s\\
(s \lor (\neg p \lor q)) \land (s \lor (\neg p \lor \neg r))\\
(s \lor \neg p \lor q) \land (s \lor \neg p \lor \neg r) \checkmark
\end{aligned}
$$