

# Resolution

## Resolution

### Resolution Rule:

$$\frac{p \vee q \quad \neg p \vee r}{q \vee r}$$

### Concept:

- If you have a long clause containing  $x$ , and another long clause containing  $\neg x$ , you can remove the  $x$  and  $\neg x$ , and join all the remaining literals with ORs ( $\vee$ )

### Clauses:

- A clause ( $C_i$ ) is a disjunction of literals (a sequence of literals joined by ORs,  $\vee$ ) of literals  
For conjunctive normal form ( $C_1 \wedge C_2 \wedge \dots \wedge C_m$ ) each  $C$  is a collection of literals joined with ORs  $\vee$ .
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## Soundness & Completeness

Resolution is both sound and complete

- **Soundness:** If Resolution announces that  $\phi$  is a theorem, then  $\phi$  is a tautology
- **Completeness:** If  $\phi$  is a tautology, then Resolution announces that  $\phi$  is a theorem

**Note:** In the worst case, Resolution involves an exponential number of applications

## Satisfiability & Tautologies

- **Sat-Solvers:** Checks if a given formula of propositional logic is satisfiable
  - **Proof Systems:** Systems like Resolution aim to prove theorems
  - **Link:** A formula  $\phi$  is satisfiable if, and only if, its negation  $\neg\phi$  is not a tautology
    - If  $\phi$  is satisfiable, there exists a truth assignment making  $\phi$  true
    - This means there exists a truth assignment making  $\neg\phi$  false, meaning that  $\neg\phi$  is not a tautology
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## The Resolution Algorithm

- Natural Deduction tries to prove theorems from scratch, but Resolution takes a different approach by taking a given formula and working with it to decide if it is a theorem

### **Method:**

1. Start with a given formula  $\phi$
2. Negate it (because Resolution works by showing the negation is unsatisfiable)
3. Convert the negated formula to C.N.F ( $C_1 \wedge C_2 \wedge \dots \wedge C_m$ )
4. Extract the individual Clauses  $C_1, C_2, \dots, C_m$
5. Repeatedly resolve each clause

### **Halting Conditions:**

- **Proving a Theorem:** If we ever infer the empty clause  $\emptyset$ , then we halt and output that  $\phi$  is a theorem
- **Disproving a Theorem:** If we get to the point where we have not inferred the empty clause and we cannot infer any new clauses, then we halt and output that  $\phi$  is not a theorem

### **Extra Rule:**

- **Deleting Literals:** When resolving, we are also allowed to delete repeated literals in any clause
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### **Example:**

- **Question:** Let  $\phi$  be the formula  $\neg((p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q))$ . Is  $\phi$  a theorem?

Negate the Formula:  $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$

- The Formula is already in Conjunctive Normal Form

List the Clauses:

Clause 1:  $p \vee q$

Clause 2:  $\neg p \vee q$

Clause 3:  $p \vee \neg q$

Clause 4:  $\neg p \vee \neg q$

Resolution Rule: Combine the clauses with clashing literals

Clause 1 and 2: (The  $p$  and  $\neg p$  clash)

$$(p \vee q) \quad (\neg p \vee q) \longrightarrow q \vee q \longrightarrow q$$

Clause 3 and 4: (The  $p$  and  $\neg p$  clash)

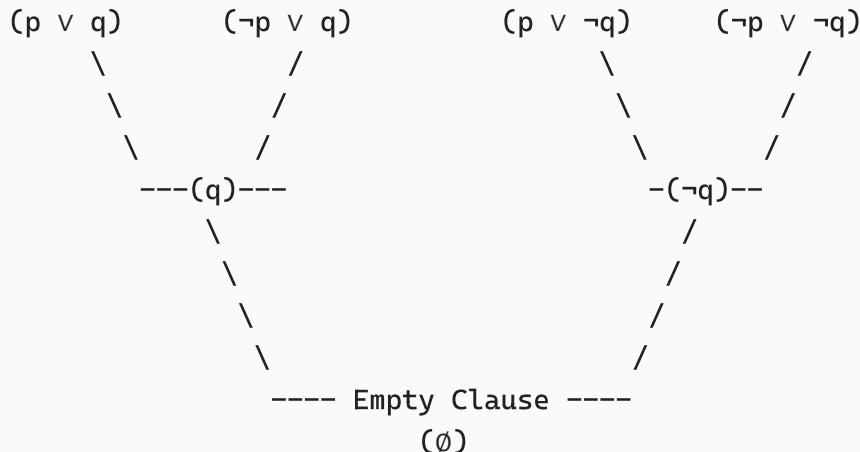
$$(p \vee \neg q) \quad (\neg p \vee \neg q) \longrightarrow \neg q \vee \neg q \longrightarrow \neg q$$

Now the Clauses  $p$  and  $\neg p$  clash, leaving nothing left ( $\emptyset$ )

Since we have inferred the Empty Clause, it means  $\neg\phi$  is a contradiction

$\therefore \phi$  is a Theorem

**Diagram:**



## Shortcut when converting to c.n.f. $(\neg(A \Rightarrow B)) \equiv A \wedge \neg B$

- Used when Implications are involved ( $\Rightarrow$ )

The negation of  $A \Rightarrow B$  is equivalent to  $A \wedge \neg B$

**Proof:**

Start with the Negation:

$$\neg(A \Rightarrow B)$$

Implication Law

$$\neg(\neg A \vee B)$$

De Morgan's

$$(A \wedge \neg B)$$

Truth Table:

A		B		$A \Rightarrow B$		$\neg(A \Rightarrow B)$		$\neg B$		$A \wedge \neg B$
T		T		T		F		F		F
T		F		F		T		T		T
F		T		T		F		F		F
F		F		T		F		T		F

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- [Logic Practical Week 15](#)