

**Master Networks-IoT.  
Operations Research.**

**Quiz  
November 2024**

In the following, we consider that the number of edges (resp. arcs) in an undirected (resp. directed) graph is  $m$ , and the number of vertices is  $n$ .

1. Let  $T(n) = 8n^4 + 3n^2$ . Which of the following statements are true?  
(There might be more than one correct answer.)
  - A)  $T(n) = O(n)$
  - B)  $T(n) = \Omega(n^2)$
  - C)  $T(n) = \Theta(n)$
  - D)  $T(n) = O(n^4)$
2. What is the maximum number of edges does a path of a connected acyclic graph with  $n$  vertices contain?
  - A)  $n$
  - B)  $n - 1$
  - C) It depends on the paths
  - D)  $\frac{n(n-1)}{2}$
3. Consider an undirected graph with  $n$  vertices. Assume that the graph is connected. What are the minimum and maximum numbers of edges, respectively, that the graph could have?
  - A)  $n - 1$  and  $\frac{n(n-1)}{2}$
  - B)  $n - 1$  and  $n^2$
  - C)  $n$  and  $2^n$
  - D)  $n$  and  $n^n$
4. We consider two algorithms A and B solving the same problem on a graph  $G$ . A is in  $O(m)$  and B is in  $O(n \log n)$ . Which algorithm to use if  $G$  is a dense graph?
  - A) Algorithm A
  - B) Algorithm B

5. We consider two algorithms A and B solving the same problem on a graph  $G$ . A is in  $O(m)$  and B is in  $O(n \log n)$ . Which algorithm to use if  $G$  is a tree?
- Algorithm A
  - Algorithm B
6. How much space does the adjacency list representation of a graph require?
- $O(n)$
  - $O(m)$
  - $O(n + m)$
  - $O(n^2)$
7. How much space does the adjacency matrix of a graph require?
- $O(n)$
  - $O(n^2)$
  - $O(n + m)$
  - $O(m^2)$
8. What is the time complexity of a graph search algorithm (choose the most accurate possible answer)?
- $O(n^2)$
  - $O(m)$
  - $O(nm)$
  - $O(n + m)$
9. In the graph  $G_1$ , what is  $\{A, B, E\}$ ?

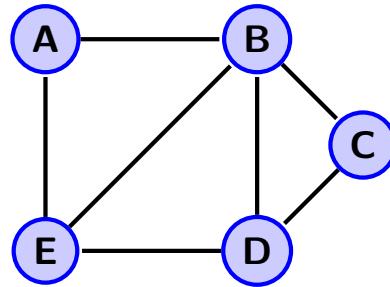
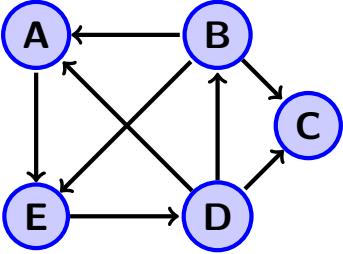


Figure 1: Graph  $G_1$ .

- A) an arc  
 B) an edge  
 C) a path  
 D) a cycle
10. In the graph  $G_1$ , propose a list of explored nodes using the generic graph search algorithm from  $E$ . Represent the selected edges in the graph to show the exploration.
11. In the graph  $G_2$ , propose a list of explored nodes in BFS order starting from  $D$ . Represent the selected arcs in the graph to show the BFS exploration.
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graph TD
    A((A)) --> B((B))
    A((A)) --> E((E))
    B((B)) --> C((C))
    B((B)) --> D((D))
    C((C)) --> D((D))
    D((D)) --> E((E))
    E((E)) --> D((D))
    E((E)) --> B((B))
  
```
- Figure 2: Graph  $G_2$ .
12. In the graph  $G_2$ , propose a list of explored nodes in DFS order starting from  $D$ . Represent the selected arcs in the graph to show the DFS exploration.
13. In the graph  $G_2$ , the number of strongly connected components is:

- A) 1
- B) 2
- C) 3
- D) 5

14. In the graph  $G_2$ , the number of connected components is:

- A) 1
- B) 2
- C) 3
- D) 5

15. How many connected components does a tree with  $n$  vertices ( $n \geq 4$ ) contain after deleting 2 edges?

- A)  $n$
- B) 3
- C) It depends on the trees
- D)  $n - 1$

16. Consider an undirected graph  $G$  represented by an adjacency matrix. Given a vertex  $v$ , how many operations are required to compute the degree of  $v$ ?

- A)  $O(n)$
- B)  $O(\min(n, m))$
- C)  $O(m)$
- D)  $O(n + m)$

17. Consider a directed graph  $G$  represented by adjacency lists (each storing the outgoing arcs of a vertex). Given a vertex  $v$ , how many operations are required to identify the indegree of  $v$ ?

- A)  $O(n)$
- B)  $O(\min(n, m))$
- C)  $O(m)$
- D)  $O(n + m)$

## Answers

1. B,D.
  2. B.
  3. A.
  4. B.
  5. A.
  6. C.
  7. B.
  8. C.
  9. C.
10.  $(E, A, B, C, D)$ . Arcs of the tree (root note  $E$ ):  $(E, A)$ ,  $(E, B)$ ,  $(B, C)$ ,  $(C, D)$ .  
Other answers are possible.
11.  $(D, A, B, C, E)$ . Arcs of the tree (root note  $D$ ):  $(D, A)$ ,  $(D, B)$ ,  $(D, C)$ ,  $(A, E)$ .  
Other answers are possible.
12.  $(D, B, A, E, C)$ . Arcs of the tree (root note  $D$ ):  $(D, B)$ ,  $(B, A)$ ,  $(A, E)$ ,  $(B, C)$ .  
Other answers are possible.
13. B.
  14. A.
  15. B.
  16. A.
  17. D.