Comprehensive Quiz: Operations Research

Network Algorithms and Graph Theory

Course: USEEN3 - Operations Research
Master: Networks-IoT
Academic Year: 2025-2026
Total Questions: 64

Instructions

In the following quiz, we consider that:

- The number of edges (resp. arcs) in an undirected (resp. directed) graph is **m**
- The number of vertices is **n**
- For multiple choice questions, choose the best answer unless stated otherwise
- Some questions may have multiple correct answers

Section 1: Algorithm Analysis & Complexity

1. Let $T(n) = 5n^3 + 2n^2 + 7n$. Which of the following statements are TRUE? *(Multiple answers possible)*

A) $T(n) = O(n^2)$
B) $T(n) = \Omega(n^2)$
C) $T(n) = \Theta(n^3)$
D) $T(n) = O(n^3)$

Answer: B, C, D

2. What is the running time of BFS (Breadth-First Search) algorithm?

A) $O(n^2)$
B) $O(m)$
C) $O(n + m)$
D) $O(n \log n)$

Answer: C

3. What is the running time of DFS (Depth-First Search) algorithm?

A) $O(n^2)$
B) $O(m)$
C) $O(n + m)$
D) $O(n \log n)$

Answer: C

4. Which notation describes an upper bound on the running time?

A) O notation
B) Ω notation
C) Θ notation
D) All of the above

Answer: A

5. Which notation describes both upper and lower bounds?

A) O notation
B) Ω notation
C) Θ notation
D) None of the above

Answer: C

Section 2: Graph Theory Basics

6. Consider a connected undirected graph with n vertices. What are the minimum and maximum numbers of edges, respectively?

A) n-1 and n(n-1)/2
B) n and n^2
C) n-1 and n^2
D) n and 2n

Answer: A

7. How many edges does a tree with n vertices contain?

A) n
B) n-1
C) It depends on the tree
D) n(n-1)/2

Answer: B

8. What is the maximum number of edges in a simple undirected graph with n vertices?

A) n
B) n-1
C) n^2
D) n(n-1)/2

Answer: D

9. In a directed graph with n vertices and m arcs, what is the sum of all indegrees?

A) n
B) m
C) 2m
D) n^2

Answer: B

10. In an undirected graph with n vertices and m edges, what is the sum of all degrees?

A) n
B) m
C) 2m
D) n+m

Answer: C

11. Which of the following statements is TRUE for a tree with n vertices?

A) It has n-1 edges and is connected
B) It has n edges and contains no cycles
C) It has n-1 edges and may contain cycles
D) It has n edges and is disconnected

Answer: A

12. A path is called a cycle if:

A) All vertices are distinct
B) The first and last vertices are the same and k > 2
C) It contains exactly n edges
D) It visits all vertices

Answer: B

Section 3: Graph Representations

13. How much space does the adjacency list representation of a graph require?

A) O(n)
B) O(m)
C) O(n+m)
D) $O(n^2)$

Answer: C

14. How much space does the adjacency matrix representation of a graph require?

A) O(n)
B) O(m)
C) O(n+m)
D) $O(n^2)$

Answer: D

15. Consider an undirected graph G represented by an adjacency matrix. Given a vertex v, how many operations are required to compute the degree of v?

A) O(1)
B) O(k) where k is the degree of v
C) O(n)
D) O(m)

Answer: C

16. Consider a directed graph G represented by adjacency lists (storing outgoing arcs). Given a vertex v, how many operations are required to identify the indegree of v?

A) O(1)
B) O(k) where k is the degree of v
C) O(m)
D) O(n+m)

Answer: D

17. Which representation is more space-efficient for sparse graphs?

A) Adjacency matrix
B) Adjacency list
C) Both are equally efficient
D) Neither is efficient

Answer: B

18. Which representation allows checking if two vertices are adjacent in O(1) time?

A) Adjacency matrix
B) Adjacency list
C) Both
D) Neither

Answer: A

Section 4: Algorithm Complexity Analysis

19. We consider two algorithms A and B solving the same problem. A is in O(n+m) and B is in O(n log n). Which algorithm to use if G is a tree?

A) Algorithm A
B) Algorithm B
C) Both are equal
D) Cannot determine

Answer: A

20. We consider two algorithms A and B solving the same problem. A is in O(m) and B is in O(n log n). Which algorithm to use if G is a dense graph?

A) Algorithm A
B) Algorithm B
C) Both are equal
D) Cannot determine

Answer: B

21. We consider two algorithms A and B solving the same problem. A is in O(n+m) and B is in O(n log n). Which algorithm to use if G is a complete graph?

A) Algorithm A
B) Algorithm B
C) Both are equal
D) Cannot determine

Answer: B

22. For a tree with n vertices, what is the value of m (number of edges)?

A) n
B) n-1
C) n log n
D) n^2

Answer: B

23. For a complete graph with n vertices, what is the value of m (number of edges)?

A) n
B) n-1
C) n(n-1)/2
D) n^2

Answer: C

Section 5: Graph Search Algorithms

24. Which data structure is used in BFS (Breadth-First Search)?

A) Stack
B) Queue
C) Priority Queue
D) Heap

Answer: B

25. Which data structure is used in DFS (Depth-First Search)?

A) Stack
B) Queue
C) Priority Queue
D) Heap

Answer: A

26. BFS explores vertices in which order?

A) Random order
B) Layer by layer from the source
C) Deepest vertices first
D) By vertex index

Answer: B

27. DFS explores vertices by:

A) Layer by layer
B) Going as deep as possible before backtracking
C) Random selection
D) Shortest path first

Answer: B

28. Which algorithm naturally computes shortest paths (in terms of number of edges)?

A) BFS
B) DFS
C) Both
D) Neither

Answer: A

29. BFS can be used to:

A) Compute shortest paths in unweighted graphs
B) Check graph connectivity

C) Find connected components
D) All of the above

Answer: D

30. DFS can be used to:

A) Compute topological ordering
B) Detect cycles
C) Find connected components
D) All of the above

Answer: D

Section 6: Graph Connectivity

31. A connected component in an undirected graph is:

A) A maximal subset where every pair has a path
B) A minimal subset with all vertices
C) A subset with maximum edges
D) Any subset of vertices

Answer: A

32. A strongly connected component in a directed graph means:

A) Every vertex can reach every other vertex in the component
B) There exists at least one path between some vertices
C) All vertices have the same degree
D) The component has no cycles

Answer: A

33. How many connected components does a tree with n vertices ($n \geq 4$) contain after deleting 2 edges?

A) 1
B) 2
C) 3
D) n

Answer: C

34. A graph with n vertices and n-1 edges is:

A) Always a tree
B) A tree if it is connected
C) Always disconnected
D) Always contains a cycle

Answer: B

Section 7: Topological Ordering

35. A topological ordering is defined for:

A) Any graph
B) Directed acyclic graphs (DAG)
C) Undirected graphs only
D) Graphs with cycles

Answer: B

36. In a topological ordering of a DAG, for every arc (v,w):

A) f(v) < f(w)
B) f(v) > f(w)
C) f(v) = f(w)
D) No relation

Answer: A

37. Every DAG has:

A) No source vertex
B) At least one source vertex
C) Exactly one source vertex
D) No topological ordering

Answer: B

38. Which algorithm can compute a topological ordering?

A) BFS
B) DFS
C) Dijkstra
D) Bellman-Ford

Answer: B

39. A source vertex in a directed graph is a vertex with:

A) Indegree = 0
B) Outdegree = 0
C) Degree = 0
D) Maximum degree

Answer: A

Section 8: Shortest Path Algorithms

40. Dijkstra's algorithm is used for:

A) Graphs with negative edge weights
B) Graphs with non-negative edge weights
C) Unweighted graphs only
D) All types of graphs

Answer: B

41. What data structure is essential for efficient implementation of Dijkstra's algorithm?

A) Stack
B) Queue
C) Priority Queue
D) Array

Answer: C

42. Bellman-Ford algorithm can handle:

A) Only positive weights
B) Negative weights but no negative cycles
C) Only unweighted graphs
D) Cannot detect negative cycles

Answer: B

43. In Bellman-Ford algorithm, SP(v,i) represents:

A) Shortest path to v using exactly i edges
B) Shortest path to v using at most i edges
C) Shortest path from v using i edges
D) None of the above

Answer: B

44. The maximum number of iterations in Bellman-Ford for a graph with n vertices is:

A) n
B) n-1
C) n+1
D) 2n

Answer: B

45. RIP (Routing Information Protocol) uses which algorithm?

A) Dijkstra
B) Bellman-Ford

C) BFS
D) DFS

Answer: B

46. OSPF (Open Shortest Path First) uses which algorithm?

A) Dijkstra
B) Bellman-Ford
C) BFS
D) Floyd-Warshall

Answer: A

47. What is the maximum hop count in RIP?

A) 10
B) 15
C) 20
D) 255

Answer: B

48. Floyd-Warshall algorithm computes:

A) Single-source shortest paths
B) All-pairs shortest paths
C) Minimum spanning tree
D) Maximum flow

Answer: B

Section 9: Routing Protocols

49. OSPF is classified as a:

A) Distance-vector protocol
B) Link-state protocol
C) Path-vector protocol
D) Hybrid protocol

Answer: B

50. RIP is classified as a:

A) Distance-vector protocol
B) Link-state protocol
C) Path-vector protocol
D) Hybrid protocol

Answer: A

51. In OSPF, each router broadcasts:

A) Its routing table
B) Information about its local neighbors
C) Distance vectors
D) All network topology

Answer: B

52. In RIP, each router broadcasts:

A) Link state advertisements
B) Full routing tables
C) Local neighbor information
D) Network topology

Answer: B

53. An Autonomous System (AS) is:

A) A single router
B) An IP network typically owned by an ISP
C) A collection of end hosts
D) A physical network cable

Answer: B

54. BGP (Border Gateway Protocol) is used for:

A) Routing within an AS
B) Routing between AS
C) Local network routing
D) Device-to-device communication

Answer: B

55. OSPF Areas are used to:

A) Increase routing table size
B) Reduce computational complexity
C) Support more routers
D) Enable multicasting

Answer: B

56. In OSPF, Area 0 is called:

A) Border area
B) Backbone area

C) Stub area
D) Transit area

Answer: B

Section 10: Minimum Spanning Trees

57. A spanning tree of a graph G must:

A) Include all vertices of G
B) Include all edges of G
C) Have maximum total weight
D) Contain cycles

Answer: A

58. Prim's algorithm is a:

A) Divide-and-conquer algorithm
B) Greedy algorithm
C) Dynamic programming algorithm
D) Backtracking algorithm

Answer: B

59. Kruskal's algorithm is a:

A) Divide-and-conquer algorithm
B) Greedy algorithm
C) Dynamic programming algorithm
D) Backtracking algorithm

Answer: B

60. In Prim's algorithm, we start with:

A) All vertices
B) One arbitrary vertex
C) The heaviest edge
D) Multiple disconnected components

Answer: B

61. In Kruskal's algorithm, edges are considered in order of:

A) Arbitrary order
B) Increasing weight
C) Decreasing weight
D) Vertex index

Answer: B

62. The running time of Prim's algorithm (heap-based) is:

A) $O(n^2)$
B) O(m log m)
C) O((n+m) log n)
D) O(n log n)

Answer: C

63. Kruskal's algorithm can grow:

A) Only one tree at a time
B) Multiple trees in parallel
C) No trees
D) Trees from leaves to root

Answer: B

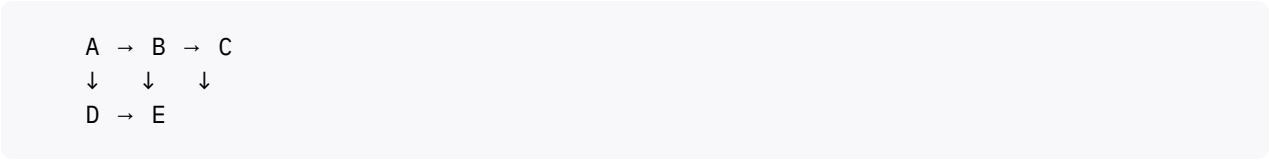
64. For a graph with n vertices, a minimum spanning tree has how many edges?

A) n
B) n-1
C) n+1
D) n^2

Answer: B

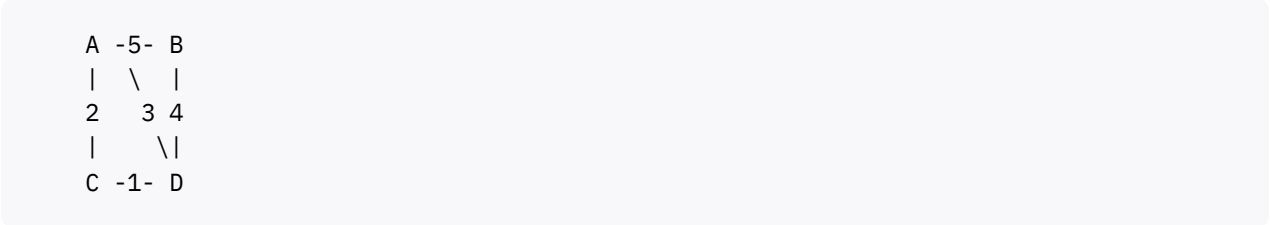
Additional Open-Ended Questions

65. Given the following directed graph G:



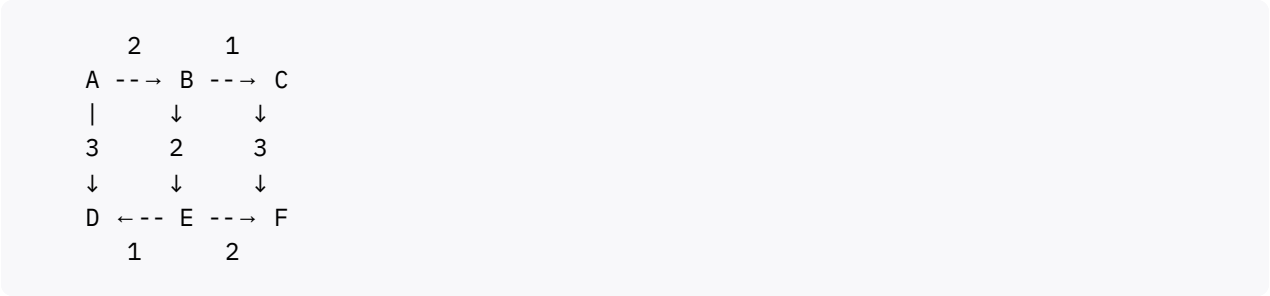
a) Propose a list of explored nodes in BFS order starting from A.
b) Propose a list of explored nodes in DFS order starting from A.
c) Determine the number of strongly connected components.
d) Determine the number of connected components (treating it as undirected).

66. Given the graph below with weighted edges:



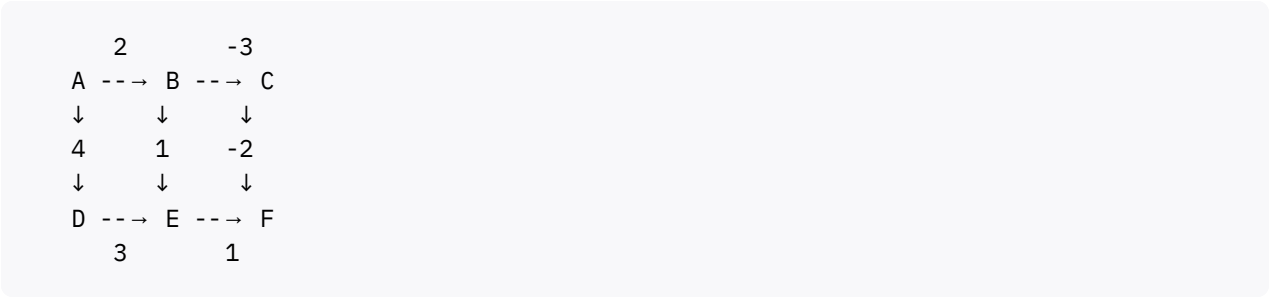
a) Compute the minimum spanning tree using Prim's algorithm starting from A.
b) Compute the minimum spanning tree using Kruskal's algorithm.
c) What is the total weight of the MST?

67. Consider the following weighted directed graph:



a) Compute the shortest path from A to all other vertices using Dijkstra's algorithm.
b) Show the evolution of distances at each iteration.

68. Given a graph with negative edge weights:



a) Can Dijkstra's algorithm be used? Why or why not?
b) Apply Bellman-Ford algorithm to find shortest paths from A.
c) Show SP(v,i) for i=1,2,3.

69. Design questions:

a) Explain the difference between link-state and distance-vector routing protocols.
b) Why does OSPF use Dijkstra while RIP uses Bellman-Ford?
c) What are the advantages and disadvantages of each approach?

70. Algorithmic comparison:

Complete the following table comparing different algorithms:

Algorithm	Graph Type	Time Complexity	Space Complexity	Use Case
BFS				
DFS				
Dijkstra				
Bellman-Ford				
Floyd-Warshall				

Algorithm	Graph Type	Time Complexity	Space Complexity	Use Case
Prim				
Kruskal				

End of Quiz

Note: This comprehensive quiz covers all major topics from the Operations Research course including:

- Algorithm complexity analysis
- Graph theory fundamentals
- Graph representations
- Graph search algorithms (BFS, DFS)
- Connectivity and components
- Topological ordering
- Shortest path algorithms (Dijkstra, Bellman-Ford, Floyd-Warshall)
- Routing protocols (OSPF, RIP)
- Minimum spanning trees (Prim, Kruskal)

Good luck with your preparation!