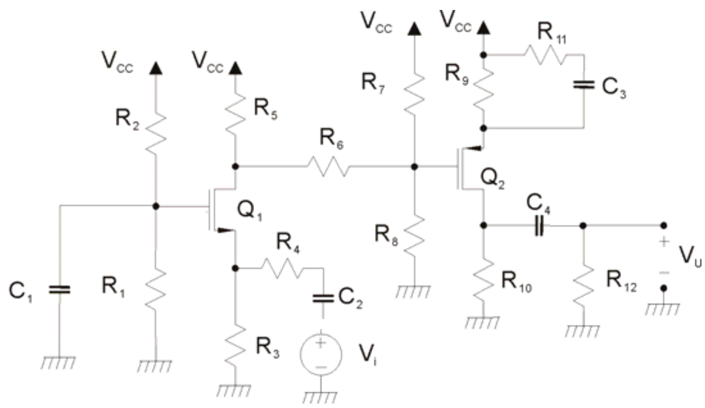
ELETTRONICA DIGITALE
Corso di Laurea in Ingegneria Informatica

Prova scritta 09 giugno 2014

Esercizio A

R_1 =10 kΩ	R_{10} = 5 kΩ
R_2 = 10 kΩ	R_{11} = 100 Ω
R_4 = 100 Ω	R_{12} = 20 kΩ
R_5 = 3 kΩ	C_1= 1 μF
R_6 = 1 kΩ	C_2= 68 nF
R_7 = 10 kΩ	C_3 = 33 nF
R_8 = 40 kΩ	C_4 = 10 nF
R_9 = 1.5 kΩ	V_{CC} = 18 V



Q_1 è un transistore MOS a canale n resistivo con V_{T1} = 1 V; Q_2 è un transistore MOS a canale p resistivo con V_{T2} = - 1 V. Per entrambi la corrente di drain in saturazione è data da $I_D=k(V_{GS}-V_T)^2$ con k = 0.5mA/V^2.
Con riferimento al circuito in figura:

1) Calcolare il valore della resistenza R_3 in modo che, in condizioni di riposo, la tensione del source di Q_2 sia 15 V. Determinare, inoltre, il punto di riposo dei due transistori e verificarne la saturazione. (R: R_3 = 2420.42 Ω)
2) Determinare V_U/Vi alle frequenze per le quali C_1, C_2, C_3 e C_4 possono essere considerati dei corto circuiti. (R: V_U/Vi = -23.42)
3) (**Solo per 12 CFU**) Determinare la funzione di trasferimento V_U/V_i e tracciarne il diagramma di Bode quotato asintotico del modulo. (R: f_{z1} = f_{p1}; f_{z2} = 0 Hz; f_{p2} = 4835.75 Hz; f_{z3} = 3014.3 Hz; f_{p3} = 10153.4 Hz; f_{z4} = 0 Hz; f_{p4} = 636.6 Hz)

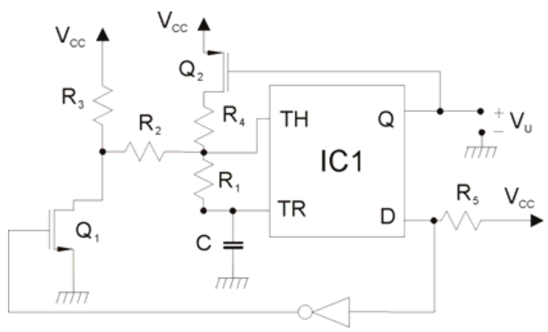
Esercizio B
Progettare una porta logica in tecnologia CMOS, utilizzando la tecnica della pull-up network e della pull-down network, che implementi la funzione logica:

$$Y = \overline{\overline{A}B}\left(\overline{C}+\overline{D}\right)+\overline{DE}\left(B+A\overline{C}\right)$$

Determinare il numero dei transistori necessari e disegnarne lo schema completo. Dimensionare inoltre il rapporto (W/L) di tutti i transistori, assumendo, per l'inverter di base, W/L pari a 2 per il MOS a canale *n* e pari a 5 per quello a canale *p*. Si specifichino i dettagli della procedura di dimensionamento dei transistori.

Esercizio C

R_1 =500 Ω	R_4 = 900 Ω
R_2 = 100 Ω	R_5 = 1 k Ω
R_3 = 1 kΩ	C = 100 nF
V_{CC} = 5 V	

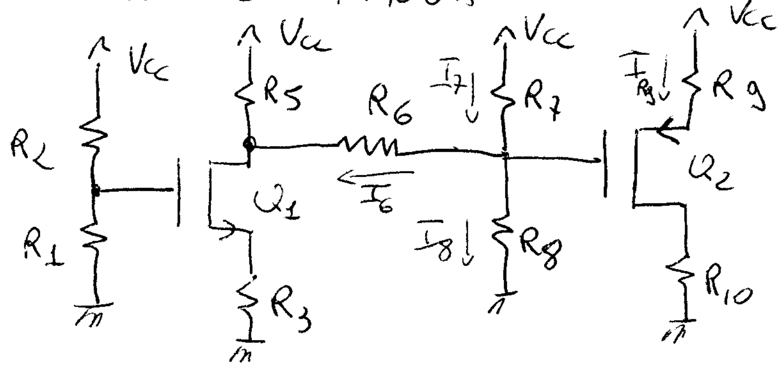


Il circuito IC_1 è un NE555 alimentato a V_{CC} = 5V, Q_1 ha una R_{on} = 0 e V_T = 1 V, Q_2 ha una R_{on} = 0 e V_T = -1 V. Determinare la frequenza del segnale di uscita del multivibratore. (R: f = 11772 Hz).

...OVA SCRITTA 09/06/2014

ES. A

CIRCUITO IN CONTINUA



$I_D = K(V_{GS} - V_{T1})^2$

$K = 0.5 \times 10^{-3} \frac{A}{V^2}$

$V_{T1} = 1V$

$V_{T2} = -1V$

1) CALCOLARE R_3 per $V_{S2} = 15V$

$$I_{R9} = \frac{V_{cc} - V_{S2}}{R_9} = \frac{18 - 15}{1500} = 2\,mA$$

$$I_{R10} = I_{R9} = I_{D2} = 2\,mA$$

$$(V_{GS2} - V_{T2}) = -\sqrt{\frac{I_{D2}}{K}} = -2V$$

$$V_{GS2} = -2 + V_{T2} = -3V$$

$$V_{D2} = R_{10} I_{D2} = 10\,V$$

$$V_{DS2} = V_{D2} - V_{S2} = 10 - 15 = -5V$$

$$V_{DS2} = -5 \leq -2V = (V_{GS2} - V_T) \Rightarrow Q_2 \text{ è SATURO}$$

$$Q_2: \begin{cases} I_{D2} = 2\,mA \\ V_{DS2} = -5\,V \\ V_{GS2} = -3\,V \\ g_{m2} = 2K|V_{GS2} - V_T| = \\ = 2 \times 10^{-3} \frac{A}{V} \end{cases}$$

$$V_{G2} = V_{GS2} + V_{S2} = -3 + 15 = +12\,V$$

$$I_7 = \frac{V_{cc} - V_{G2}}{R_7} = \frac{18 - 12}{10^4} = 0.6\,mA$$

$$I_8 = \frac{V_{G2}}{R_8} = \frac{12}{40 \times 10^3} = 0.3\,mA$$

$$I_6 = I_7 - I_8 = 0.6 \times 10^{-3} - 0.3 \times 10^{-3} = 0.3\,mA$$

$$V_{D1} = V_{G2} - R_6 I_6 = 11.7\,V$$

$$I_5 = \frac{V_{cc} - V_{D1}}{R_5} = \frac{18 - 11.7}{3000} = 2.1\,mA$$

$$I_{D1} = I_5 + I_6 = 2.4\,mA$$

$$(V_{GS1} - V_{T1}) = +\sqrt{\frac{I_{D1}}{K}} = 2.131\ V$$

$$V_{GS1} = 3.131\ V$$

$$V_{G1} = \frac{V_{CC}\ R_1}{R_1 + R_2} = \frac{18 \cdot 10^4}{20 \cdot 10^3} = 9\ V$$

$$V_{S1} = V_{G1} - V_{GS1} = 9 - 3.191 = 5.809\ V$$

$$\underline{\underline{R_3}} = \frac{V_{S1}}{I_{D1}} = \frac{5.809}{2.4 \times 10^{-3}} = \underline{\underline{2420.42\ \Omega}}$$

$$V_{DS1} = 11.7 - 5.809 = 5.891\ V$$

$$V_{DS1} = 5.891 > 2.191 = (V_{GS1} - V_T) \Rightarrow Q_1 \text{ è SATURO}$$

$$Q_1 : \begin{cases} I_{D1} = 2.4\ mA \\ V_{DS1} = 5.891\ V \\ V_{GS1} = 3.191\ V \\ g_{m1} = 2K(V_{GS1} - V_{T1}) = \\ \quad = 2.191 \times 10^{-3}\ \frac{A}{V} \end{cases}$$

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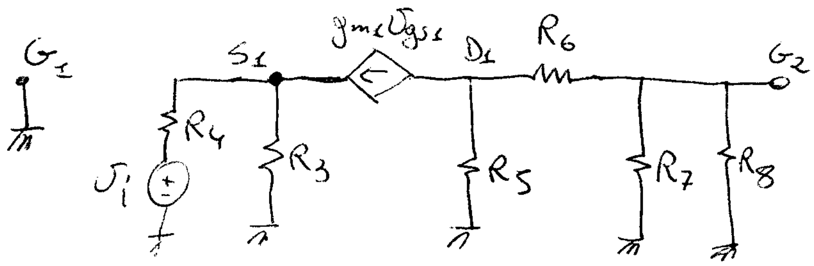
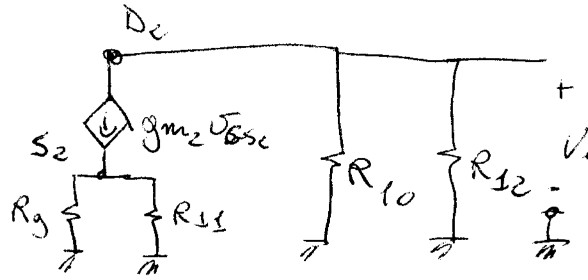
2) DET. V_u/V_i con C_i CORTOCIRCUITATI





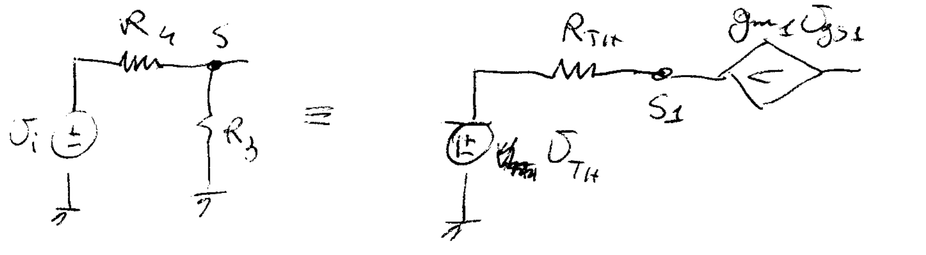
$$V_u = -(g_{m2} v_{gs2})\ R_{10} \| R_{12}$$

$$V_{S2} = (g_{m2} v_{gs2})\ R_9 \| R_{11}$$

$$V_{GS2} = V_{G2} - V_{S2} = V_{G2} - (g_{m2} V_{gs2})(R_9 \| R_{11}) \Rightarrow v_{GS2} = \frac{v_{g2}}{1 + g_{m2}(R_9 \| R_{11})}$$

$$v_{g2} = -(g_{m1} v_{gs1}) \cdot \frac{R_5}{R_5 + R_6 + (R_7 \| R_8)} \cdot (R_7 \| R_8)$$

Per il calcolo di V_{S1} possiamo fare così:



$$v_{TH} = v_i \frac{R_3}{R_3 + R_4}$$

$$R_{TH} = R_3 \| R_4$$

$V_{s1} = V_{TH} + R_{TH}\, g_{m1} V_{gs1}$

$V_{g1} = 0$

$$\Rightarrow V_{s1} = V_{TH} - R_{TH}\, g_{m1} V_{s1}$$

$$\Rightarrow V_{s1} = \frac{V_{TH}}{1+g_m R_{TH}} = \frac{V_i\, R_3}{R_3+R_4} \cdot \frac{1}{1+g_{m1}(R_3 \| R_4)}$$

In alternativa considerando che la resistenza vista dal source è pari a $\frac{1}{g_{m1}}$ si ha

$$\cancel{V_{s1} = \frac{V_i}{R_4+(R_3 \| \frac{1}{g_{m1}})} \cdot \left(R_3 \| \frac{1}{g_{m1}}\right) = \frac{V_i}{R_4+\frac{R_3}{1+R_3 g_{m1}}} \cdot \frac{R_3}{1+R_3 g_{m1}}} =$$

$$= \cancel{\frac{V_i\, R_3}{R_4+R_3+R_4 R_3 g_{m1}} = \frac{V_i\, R_3}{R_3+R_4} \cdot \frac{1}{1+g_{m1}(R_3 \| R_4)}}$$

$$V_{s1} = \frac{V_i}{R_4+\left(R_3 \| \frac{1}{g_{m1}}\right)} \cdot \left(R_3 \| \frac{1}{g_{m1}}\right) = V_i \frac{\frac{R_3}{1+g_m R_3}}{R_4+\frac{R_3}{1+g_m R_3}} = V_i \frac{R_3}{R_3+R_4+g_{m1} R_3 R_4}$$

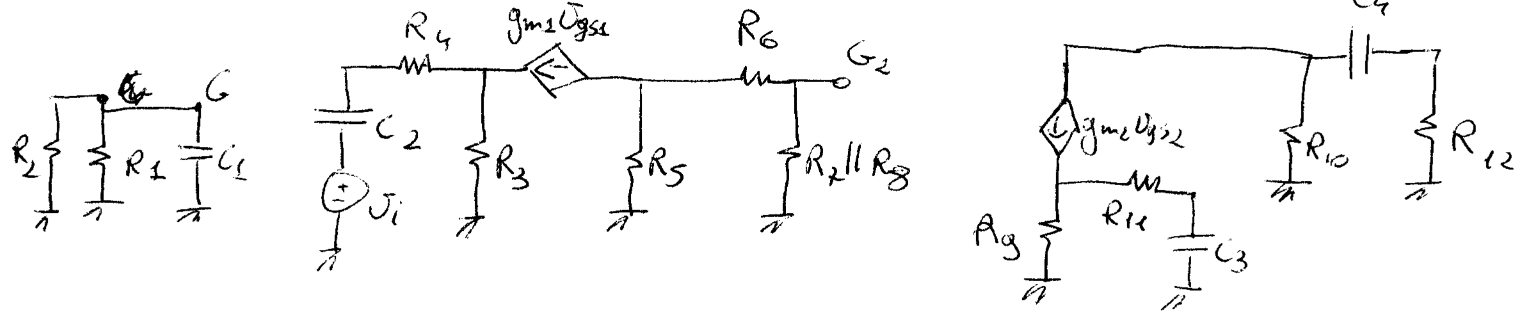
Raccogliendo tutti i termini si ottiene:

$$\frac{V_u}{V_i} = \underbrace{-(R_{10} \| R_{12})\, g_{m2} \frac{1}{1+g_{m2}(R_9 \| R_{11})}}_{6.73684} \underbrace{(-g_{m1}) \frac{R_5}{R_5+R_6+(R_7 \| R_8)} (R_7 \| R_8)}_{4.382} \underbrace{\left[-\frac{R_3}{R_3+R_4+g_{m1} R_3 R_4}\right]}_{0.793389}$$

$A_{CB} \cong -23.4215$

$$\left|\frac{V_u}{V_i}\right|_{dB} = 27.392 \text{ dB}$$

3) $\cancel{V_u(s)}$ $\frac{V_u(s)}{V_i(s)}$



…: polo e zero coincidenti

C_2: $f_{z_2} = \phi$

$$f_{p_2} = \frac{1}{2\pi C_2 R_{VC2}} = 4835.75\ Hz$$

$$R_{VC2} = R_4 + \left(R_3 \parallel \frac{1}{g_{m1}}\right) = 484\ \Omega$$

C_3: $$f_{z_3} = \frac{1}{2\pi C_3 (R_3 + R_{11})} = 3014.298\ Hz$$

(frequenza per la quale l'impedenza di $R_4 \parallel \left(R_{11} + \frac{1}{C_3 s}\right)$ diventa infinita)

$$f_{p_3} = \frac{1}{2\pi C_3 \underbrace{\left[R_{11} + R_3 \parallel \frac{1}{g_{m2}}\right]}_{475\ \Omega}} = 10153.425\ Hz$$

C_4: $f_{z_4} = \phi$

$$f_{p_4} = \frac{1}{2\pi C_4 (R_{10} + R_{12})} = 636.62\ Hz$$

Quindi si ha:

$$\frac{V_u(s)}{V_i(s)} = A_{CB} \frac{s^2 (s + \omega_{z3})}{(s + \omega_{p2})(s + \omega_{p3})(s + \omega_{p4})}$$

$\omega_{z3} = 18939.39$ rad/sec ($f_{z3} = 3014.298$ Hz)

$\omega_{p2} = 30383.31$ rad/sec ($f_{p2} = 4835.75$ Hz)

$\omega_{p3} = 63795.85$ rad/sec ($f_{p3} = 10153.425$ Hz)

$\omega_{p4} = 4000.00$ rad/sec ($f_{p4} = 636.62$ Hz)

$|A_{CB}|_{dB} = 23.392\ dB$

ERCIZIO B

$$Y = \overline{\bar{A}B}\,(\bar{C}+\bar{D}) + \overline{DE}\,(B + A\bar{C}) =$$

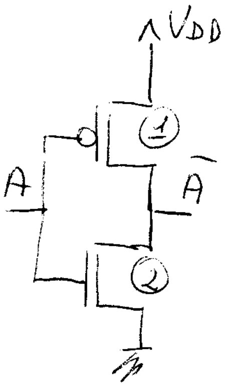
$$= (A+\bar{B})(\bar{C}+\bar{D}) + (\bar{D}+\bar{E})(B + A\bar{C}) =$$

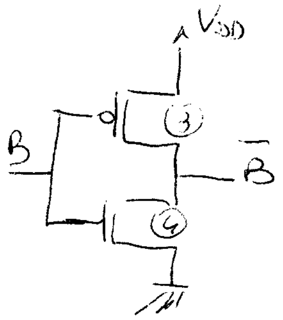
$$= A\bar{C} + A\bar{D} + \bar{B}\bar{C} + \bar{B}\bar{D} + B\bar{D} + A\bar{C}\bar{D} + B\bar{E} + A\bar{C}\bar{E} =$$

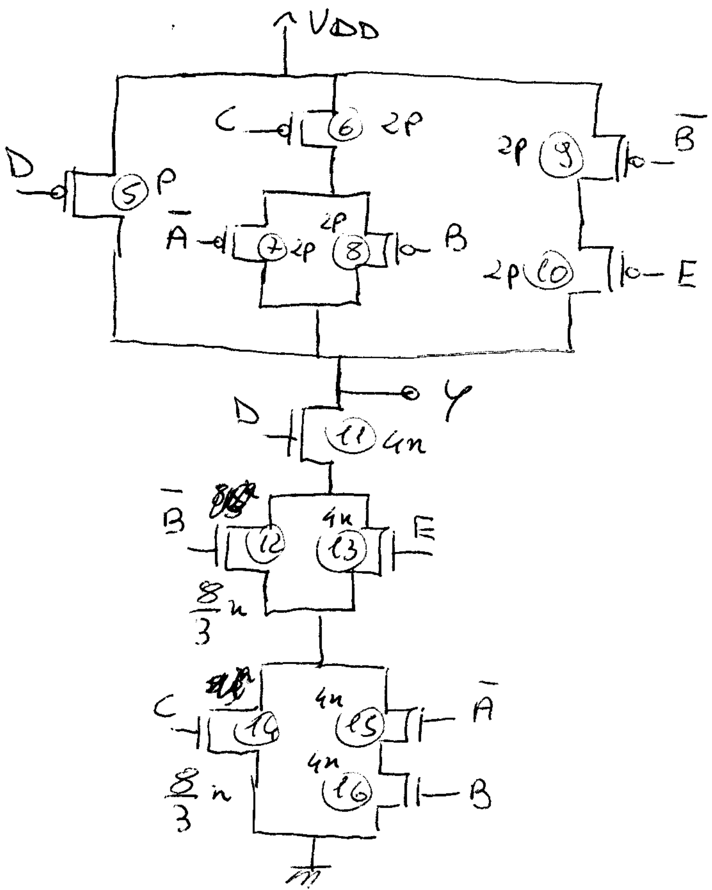
$$= A\bar{C} + A\bar{D} + \bar{B}\bar{C} + \bar{D} + B\bar{E} =$$

$$= \bar{D} + \bar{C}(A+\bar{B}) + B\bar{E}$$

Occorrono 16 MOSFET (12 + 4 per gli inverter)







$Q_1 = Q_3 = p = 5$

$Q_2 = Q_4 = n = 2$

) PUN: $Q_6 - Q_7$ oppure $Q_6 - Q_8$ oppure $Q_9 - Q_{10}$ $\Rightarrow$ $Q_6, Q_7, Q_8, Q_9, Q_{10} = 2p = 10$

$Q_5 = p = 5$

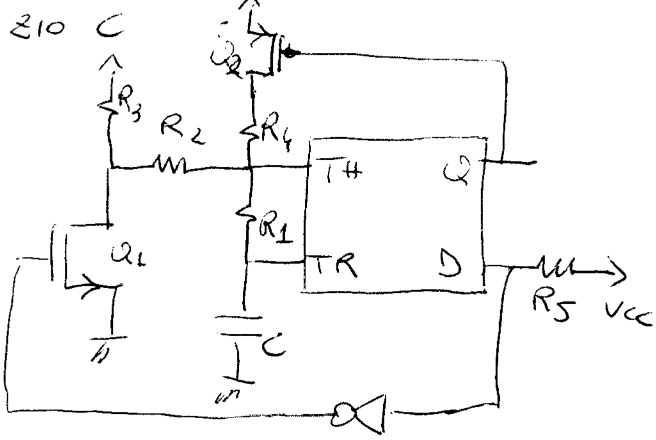
) PDN: $Q_{11} - Q_{13} - Q_{15} - Q_{16}$ $\Rightarrow$ $Q_{11}, Q_{13}, Q_{15}, Q_{16} = 4n = 8$

$Q_{11} - Q_{12} - Q_{14} \Rightarrow \frac{1}{4n} + \frac{2}{x} = \frac{1}{n} \Rightarrow \frac{2}{x} = \frac{3}{4n} \Rightarrow x = \frac{8}{3}n = \frac{16}{3}$

$Q_{12}, Q_{14} = \frac{8}{3}n = \frac{16}{3}$

L'altra soluzione (Q_{11}, Q_{13}, Q_{14}) non è ad area minima $\left(\begin{matrix} Q_{14} = 2n \\ Q_{12} = 4n \end{matrix} \Rightarrow 6n = 12 > \frac{32}{3}\right)$

ERCIZIO C



$$v_C = v_f + (v_i - v_f) e^{-\frac{t}{\tau}}$$

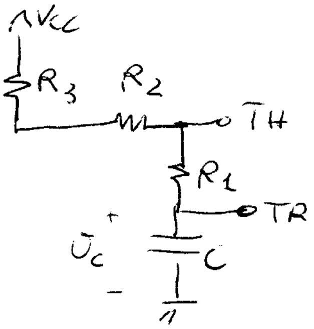
$$e^{-\frac{T}{\tau}} = \frac{v_{con} - v_f}{v_i - v_f} \qquad e^{\frac{T}{\tau}} = \frac{v_i - v_f}{v_{con} - v_f}$$

$$T = \tau \ln\left(\frac{v_i - v_f}{v_{con} - v_f}\right)$$

1° CASO)

$Q = 1 \Rightarrow v_{G2} = V_{cc} \quad v_{S2} = V_{cc} \Rightarrow v_{GS2} = \phi V > -1 = V_{T2} \Rightarrow Q_2$ OFF

$D = HI \Rightarrow v_{G1} = \phi V \quad v_{S1} = \phi V \Rightarrow v_{GS1} = \phi V < 1V = V_{T1} \Rightarrow Q_1$ OFF



$$v_i = \frac{1}{3} V_{cc} = 1.\bar{6}\ V$$

$$v_f = V_{cc} = 5V$$

$$v_{TH} = \frac{2}{3} V_{cc}$$

$$I_{R2} = I_{R1} = \frac{V_{cc} - 2/3 V_{cc}}{R_2 + R_3} = 1.\bar{51}\ mA$$

$$v_{CORR} = \frac{2}{3} V_{cc} - R_1 I_{R1} = 2.\bar{57}\ V$$

$v_i < v_{con} < v_f \Rightarrow$ AVVIENE LA COMMUTAZIONE

$1.6V < 2.\bar{57}V < 5V$ ok

$\tau_1 = C \cdot R_{VC1}$

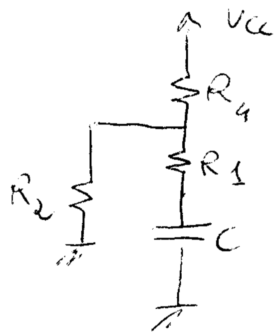
$R_{VC1} = R_1 + R_2 + R_3 = 1600\ \Omega$ $\Big] \Rightarrow \tau_1 = 160\ \mu s$

$$\underline{\underline{T_1}} = \tau_1 \ln\left(\frac{v_i - v_f}{v_{con} - v_f}\right) = 50.95\ \mu s$$

2° CASO

$Q = 0 \Rightarrow v_{G2} = \phi V \quad v_{S2} = V_{cc} \Rightarrow v_{GS2} = -V_{cc} < -1V = V_{T2} \Rightarrow Q_2$ ON

$D = 0 \Rightarrow v_{G2} = 1 V_{cc} \quad v_{S2} = \phi V \Rightarrow v_{GS1} = V_{cc} > 1V = V_{T1} = Q_1$ ON



$$v_i = 2.\bar{57}\ V$$

$$v_f = V_{cc} \frac{R_2}{R_2 + R_4} = 0.5V$$

$$v_{con} = \frac{1}{3} V_{cc} = 1.\bar{6}\ V$$

$v_i > v_{con} > v_f \Rightarrow$ AVVIENE LA COMMUTAZIONE

$2.\bar{57}V > 1.\bar{6}V > 0.5V$ OK

$\tau_2 = C R_{vc2}$

$R_{vc2} = R_1 + (R_2 \| R_4) = 590\,\Omega$ $\Bigg] \Rightarrow \tau_2 = 59\,\mu s$

$\underline{T_2} = \tau_2 \ln\left(\dfrac{v_i - v_f}{v_{con} - v_f}\right) = \underline{33.99\,\mu s}$

$T = T_1 + T_2 = 84.94695 \times 10^{-6}\, s$

$f = 11772.05\ Hz$