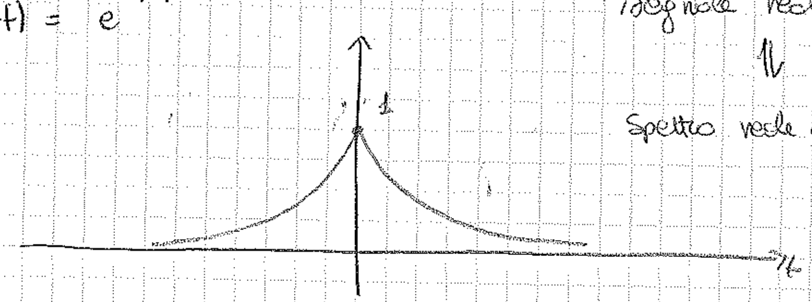
TCF - Esercizio 1

$x(t) = e^{-|t|}$

segnale reale e pari

$\Updownarrow$

Spettro reale e pari



$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt =$$

$$= \int_{-\infty}^{0} e^{t}\, e^{-j2\pi f t}\, dt + \int_{0}^{+\infty} e^{-t}\, e^{-j2\pi f t}\, dt =$$

$$= \int_{-\infty}^{0} e^{t(1-j2\pi f)}\, dt + \int_{0}^{+\infty} e^{-t(1+j2\pi f)}\, dt =$$

$$= \left. \frac{e^{t(1-j2\pi f)}}{1-j2\pi f} \right|_{-\infty}^{0} - \left. \frac{e^{-t(1+j2\pi f)}}{1+j2\pi f} \right|_{0}^{+\infty} =$$

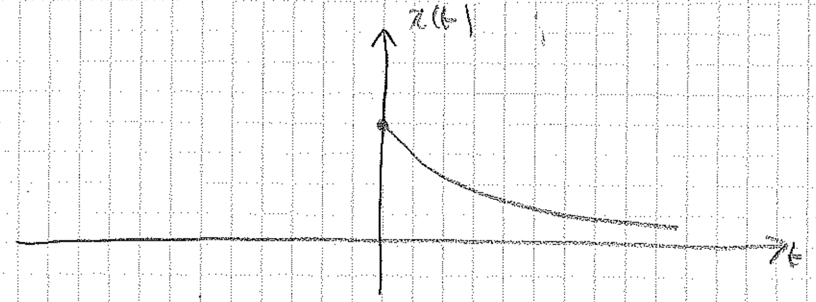
$$= \frac{1}{1-j2\pi f} + \frac{1}{1+j2\pi f} = \frac{1+\cancel{j2\pi f} + 1 - \cancel{j2\pi f}}{1+4\pi^2 f^2} = \frac{2}{1+4\pi^2 f^2}$$

TCF - Esercizio 2

$x(t) = e^{-t} u(t)$

segnale reale $\rightleftarrows$ spettro a simmetria hermitiana

$X(-f) = X^*(f)$



$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt = \int_{0}^{+\infty} e^{-t} e^{-j2\pi f t}\, dt =$$

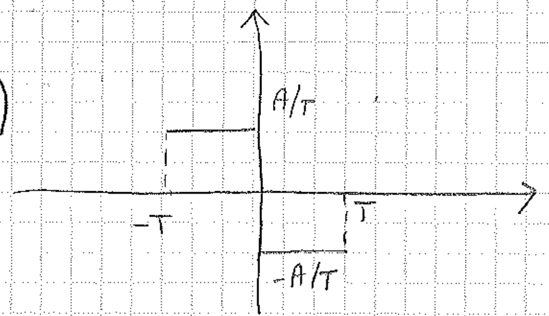
$$= \int_{0}^{+\infty} e^{-t(1+j2\pi f)}\, dt = - \left. \frac{e^{-t(1+j2\pi f)}}{1+j2\pi f} \right|_{0}^{+\infty} = \frac{1}{1+j2\pi f}$$

TCF - Esercizio 3 - triangolo con teorema

$$x(t) = A\left(1 - \frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right)$$ segnale reale e pari

•) teorema di derivazione

$$y(t) = \frac{d}{dt}x(t) = \frac{A}{T}\mathrm{rect}\left(\frac{t+T/2}{T}\right) - \frac{A}{T}\mathrm{rect}\left(\frac{t-T/2}{T}\right)$$



⚠ $$\int_{-\infty}^{+\infty} y(t)\,dt = 0 \rightleftarrows Y(f)\Big|_{f=0} = 0$$

$$Y(f) = j2\pi f\, X(f) \Rightarrow X(f) = \frac{Y(f)}{j2\pi f} \quad \text{se } Y(0) = 0\,!$$

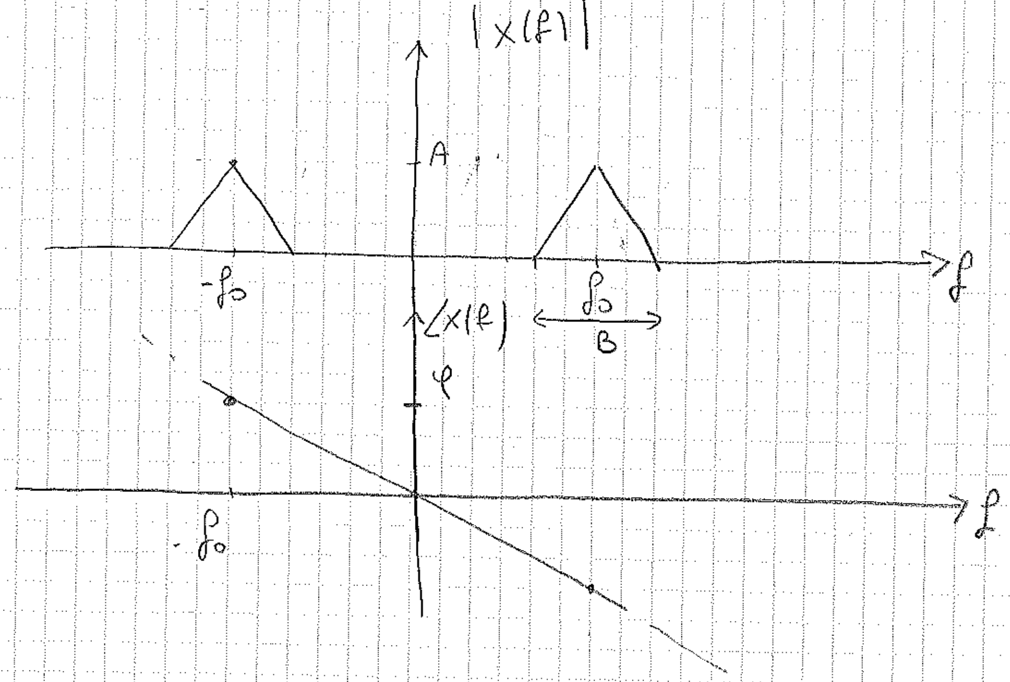
$$Y(f) = \frac{A}{T}T\,\mathrm{sinc}(fT)\,e^{j2\pi f\frac{T}{2}} - \frac{A}{T}T\,\mathrm{sinc}(fT)\,e^{-j2\pi fT/2} =$$

$$= A\,\mathrm{sinc}(fT)\left[e^{j2\pi fT/2} - e^{-j2\pi fT/2}\right] = 2j\,A\,\mathrm{sinc}(fT)\sin(2\pi fT/2)$$

$$X(f) = \frac{\cancel{2}\cancel{j}\,AT\,\mathrm{sinc}(fT)\sin(\pi fT)}{\cancel{j}\cancel{2}\pi fT} = AT\,\mathrm{sinc}^2(fT)$$

Si può anche fare con il teorema di convoluzione sapendo che il triangolo è il risultato della convoluzione tra due rect.

ITCF - Esercizio 4 - Modulazione



$$X(f) = |X(f)|\, e^{-j\varphi(f)}$$

$$|X(f)| = A\left(1 - \frac{|f+f_0|}{B/2}\right) \mathrm{rect}\left(\frac{f+f_0}{B}\right) + A\left(1 - \frac{|f-f_0|}{B/2}\right) \mathrm{rect}\left(\frac{f-f_0}{B}\right)$$

$$\varphi(f) = -a \cdot f \qquad a = \frac{\varphi}{f_0}$$

$$X(f) = A(f) \cdot e^{+j\frac{\varphi}{f_0} \cdot f} = A(f) \cdot e^{j 2\pi \frac{\varphi}{f_0} f \cdot \frac{1}{2\pi}} = A(f)\, e^{j 2\pi f t_0}$$

$$t_0 = \frac{\varphi}{2\pi f_0}$$

•) teorema del ritardo

$$x(t) = a(t-t_0)$$

•) teorema della modulazione per ottenere $a(t)$

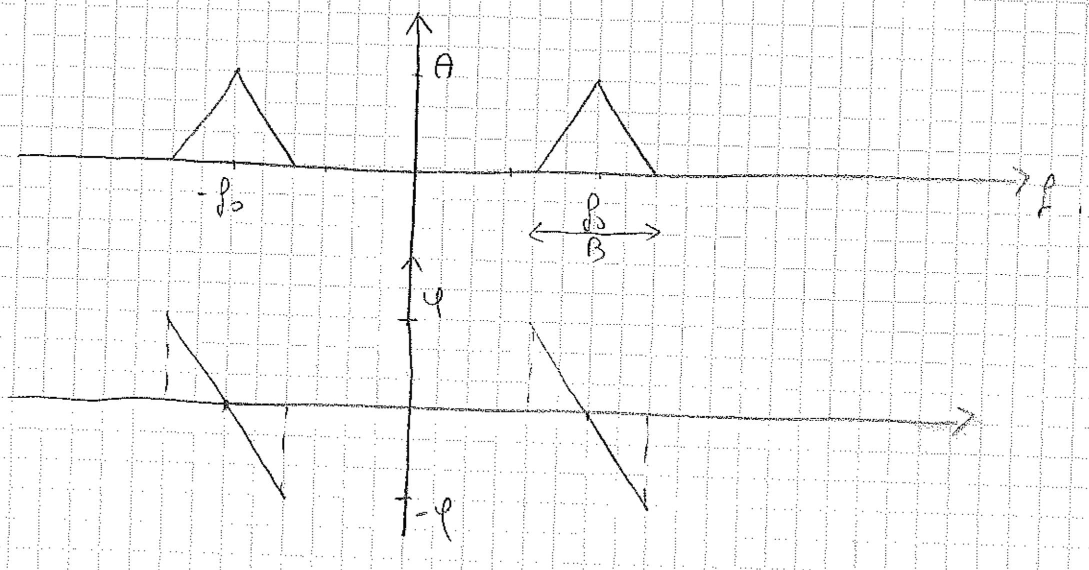
$$\boxed{y(t) = x(t)\cdot\cos(2\pi f_0 t) \rightleftarrows Y(f) = \frac{X(f-f_0)+X(f+f_0)}{2}}$$

$$a(t) = 2\,x_0(t)\cos(2\pi f_0 t)$$

$$x_0(t) = \frac{AB}{2}\,\mathrm{sinc}^2\left(t\frac{B}{2}\right)$$

$$x(t) = AB\,\mathrm{sinc}^2\left(\frac{B}{2}(t-t_0)\right)\cdot\cos\left(2\pi f_0(t-t_0)\right) \qquad t_0 = \frac{\varphi}{2\pi f_0}$$

ITCF - Esercizio 5 - Modulazione



$$X_0(f) = A\left(1-\frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right)e^{j\frac{2\varphi}{B}f}$$

$$X(f) = X_0(f - f_0)\, e^{j\varphi_0(f-f_0)} + X_0(f+f_0)\, e^{j\varphi_0(f+f_0)}$$

$$X_0(f) = A\left(1 - \frac{|f|}{B/2}\right) \mathrm{rect}\left(\frac{f}{B}\right)$$

$$\varphi_0(f) = \frac{2\varphi}{B}$$

$$Y_0(f) = X_0(f) \cdot e^{j\varphi_0(f)}$$

$$X(f) = Y_0(f + f_0) + Y_0(f - f_0)$$

·) teorema della modulazione

$$x(t) = 2\, y_0(t) \cos(2\pi f t)$$

$$Y_0(f) = X_0(f)\, e^{-j\frac{2\varphi}{B} f} = X_0(f)\, e^{-j 2\frac{\pi\varphi}{\pi B} f} = X_0(f)\, e^{-j 2\pi f t_0}$$

$$t_0 = \frac{\varphi}{\pi B}$$

·) teorema del ritardo

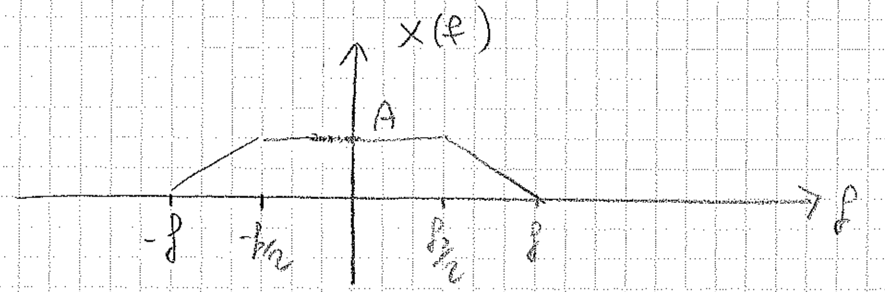
$$y_0(t) = x_0(t - t_0) \qquad x_0(t) = \frac{AB}{2} \mathrm{sinc}^2\left(t \frac{B}{2}\right)$$

$$y_0(t) = \frac{AB}{2} \mathrm{sinc}^2\left((t - t_0)\frac{B}{2}\right)$$

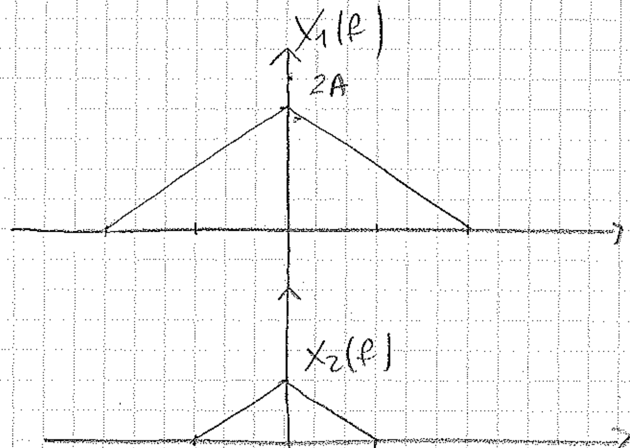
$$x(t) = AB\, \mathrm{sinc}^2\left((t - t_0)\frac{B}{2}\right) \cos(2\pi f_0 t)$$

nel 1° caso il segnale modulato è traslato di t_0, nel secondo caso invece il segnale modulante è traslato di t_0.

ITCF - Esercitazione 6 - trapezio



I° soluzione



$$X(f) = X_1(f) - X_2(f)$$

$$X_1(f) = 2A\left(1 - \frac{|f|}{f_0}\right) \mathrm{rect}\left(\frac{f}{2f_0}\right)$$

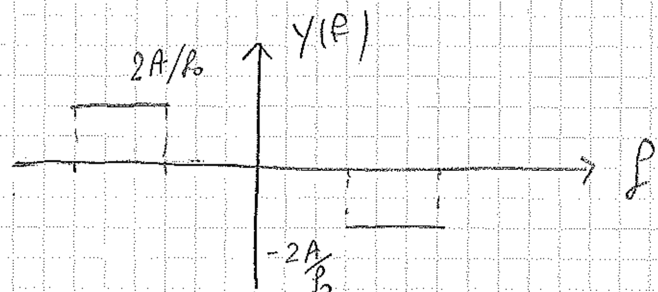
$$X_2(f) = A\left(1 - \frac{|f|}{f_0/2}\right) \mathrm{rect}\left(\frac{f}{f_0}\right)$$

$$x(t) = x_1(t) - x_2(t) = 2Af_0 \,\mathrm{sinc}^2(t f_0) - \frac{f_0 A}{2} \mathrm{sinc}^2\left(t \frac{f_0}{2}\right)$$

II° soluzione

teorema di integrazione e dualità.

$$X(f) = \int_{-\infty}^{f} Y(f)\, df$$



$$\int_{-\infty}^{+\infty} Y(f)\, df = 0$$

$$x(t) = \int_{-\infty}^{t} y(t)\,dt \rightleftharpoons X(f) = \frac{Y(f)}{j2\pi f} \quad \text{se } Y(0) = 0$$

per il teorema di dualità:

$$X(f) = \int_{-\infty}^{f} Y(f)\,df \rightleftharpoons x(t) = -\frac{y(t)}{j2\pi t} \quad \text{se } y(t)\Big|_{t=0} = 0$$

$$\int Y(f)\,df = 0$$

$$Y(f) = \frac{2A}{f_0}\,\text{rect}\left(\frac{f + 3f_0/2}{f_0/2}\right) - \frac{2A}{f_0}\,\text{rect}\left(\frac{f - 3f_0/2}{f_0/2}\right)$$

$$y(t) = A\,\text{sinc}\left(t\frac{f_0}{2}\right)e^{-j2\pi t\frac{3f}{2}} - A\,\text{sinc}\left(t\frac{f_0}{2}\right)e^{j2\pi t\frac{3f}{2}} =$$

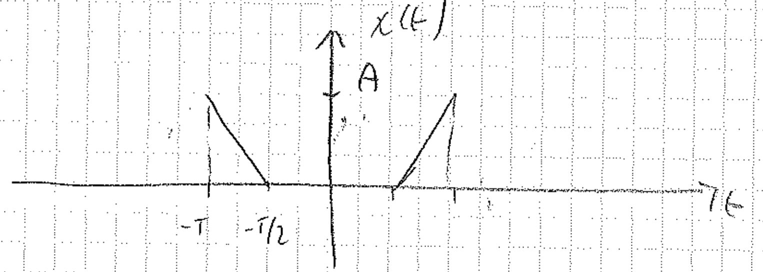
~~sinc~~

$$= -2jA\,\text{sinc}\left(t\frac{f}{2}\right)\sin\left(2\pi t\frac{3f}{2}\right)$$ immaginario puro e dispari.

$$x(t) = A\,\text{sinc}\left(t\frac{f_0}{2}\right)\frac{\sin(\pi t 3f)}{t\,3\pi f_0}\cdot 3f_0\pi =$$

$$= A\,3f_0\pi\,\text{sinc}\left(t\frac{f_0}{2}\right)\text{sinc}(t\,3f_0)$$

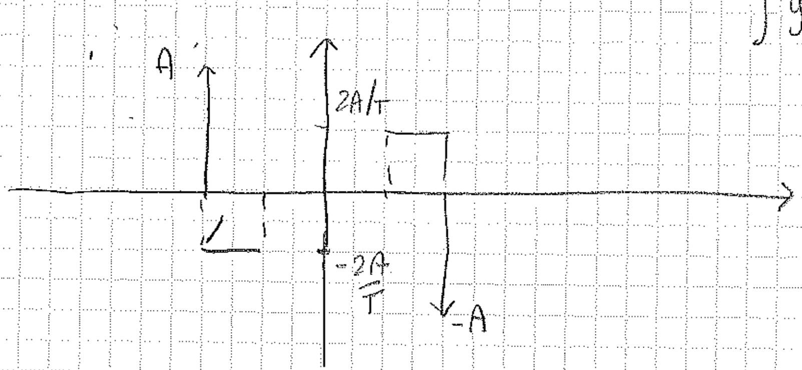
TCFG - 1 - funzione lineare a tratti



•) teorema di derivazione ⚠ $\delta(t) = \frac{\partial}{\partial t} u(t)$

$y(t) = \frac{\partial}{\partial t} x(t)$

$\int y(t)\, dt = 0 \Rightarrow Y(f)\Big|_{f=0} = 0$



$Y(f) = j2\pi f\, X(f)$ e $X(f) = \frac{Y(f)}{j2\pi f}$

$y(t) = A\,\delta(t+T) - A\,\delta(t-T) - \frac{2A}{T} \mathrm{rect}\left(\frac{t+3T/2}{T/2}\right) + \frac{2A}{T} \mathrm{rect}\left(\frac{t-3T/2}{T/2}\right)$

$Y(f) = A e^{j2\pi f T} - A e^{-j2\pi f T} - \frac{2A}{T}\frac{T}{2} \mathrm{sinc}\left(f\frac{T}{2}\right) e^{j2\pi f \frac{3T}{2}} + \frac{2A}{T}\frac{T}{2} \mathrm{sinc}\left(f\frac{T}{2}\right) e^{-j2\pi f 3T/2} =$

$= 2jA \sin(2\pi f T) - 4jA\,\mathrm{sinc}\left(f\frac{T}{2}\right) \sin(2\pi f\, 3T/2)$

$X(f) = \frac{A}{\pi f} \sin(2\pi f T) - \frac{A}{\pi f} \mathrm{sinc}\left(f\frac{T}{2}\right) \sin(2\pi f\, 3T/2)$

TCFG - Esercizio 2 - Modulazione di ampiezza

$$y(t) = x(t) \cos(2\pi f_0 t)$$

$$x(t) = AB \operatorname{sinc}^2(Bt)$$

Si risolve utilizzando la trasformata del coseno

$$Y(f) = X(f) \otimes TF\{\cos(2\pi f_0 t)\} =$$

$$= X(f) \otimes \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right] =$$

$$= \frac{1}{2} X(f-f_0) + \frac{1}{2} X(f+f_0)$$

la $\delta(t)$ è l'elemento neutro rispetto alla convoluzione.

TCFG - 4 - AM con fase

$$y(t) = x(t) \cos(2\pi f_0 t + \varphi)$$

$$x(t) = AB \operatorname{sinc}^2(Bt)$$

$$Y(f) = X(f) \otimes TF\{\cos(2\pi f_0 t + \varphi)\} =$$

$$= X(f) \otimes \left[\frac{\delta(f-f_0)e^{j\varphi} + \delta(f+f_0)e^{-j\varphi}}{2}\right] =$$

$$= \frac{1}{2} X(f-f_0)e^{j\varphi} + \frac{1}{2} X(f+f_0)e^{-j\varphi}$$

TCFG-4- Sinusoidi multicomponenti

$$x(t) = A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t) \qquad \frac{f_1}{f_2} \in \mathbb{R}$$

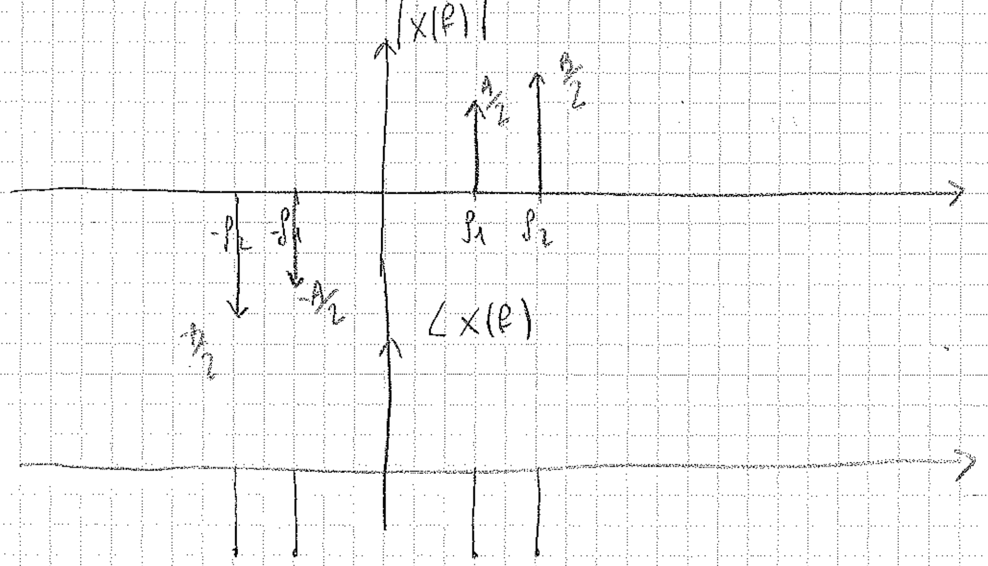
$$X(f) = \frac{A}{2}\left[\delta(f-f_1) + \delta(f+f_1)\right] + \frac{B}{2}\left[\delta(f-f_2) + \delta(f+f_2)\right]$$

Disegno.

Caso con ~~seni~~ funzioni seno.

$$x(t) = A\sin(2\pi f_1 t) + B\sin(2\pi f_2 t)$$

$$X(f) = \frac{A}{2j}\left[\delta(f-f_1) - \delta(f+f_1)\right] + \frac{B}{2j}\left[\delta(f-f_2) - \delta(f+f_2)\right)$$



$$j = e^{j\pi/2} = \cos(\pi/2) + j\sin(\pi/2)$$

$$-j = e^{-j\pi/2} = e^{-j\pi/2}$$

TCFG - 5 -

$$x(t) = \begin{cases} \cos^2(\pi t / T) & |t| < T/2 \\ 0 & \text{altrove} \end{cases}$$

$$x(t) = \cos^2\left(2\pi t \frac{1}{2T}\right) \text{rect}\left(\frac{t}{T}\right)$$

$$\cos(2\alpha) = 1 - 2\cos^2(\alpha)$$

$$\cos^2(\alpha) = \frac{1}{2} - \frac{1}{2}\cos(2\alpha)$$

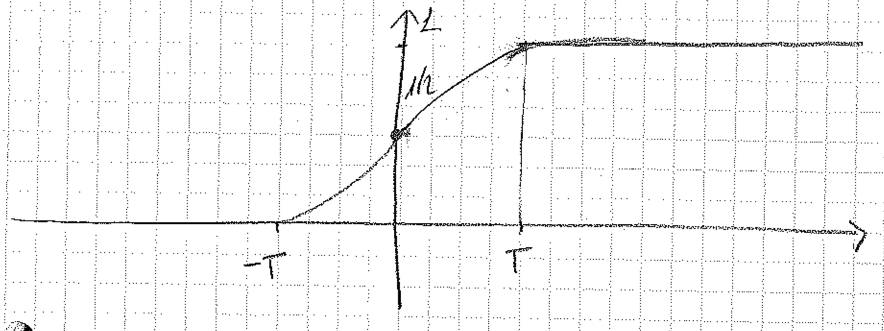
$$x(t) = \left(\frac{1}{2} - \frac{1}{2}\cos\left(2\pi t \frac{1}{T}\right)\right) \text{rect}\left(\frac{t}{T}\right)$$

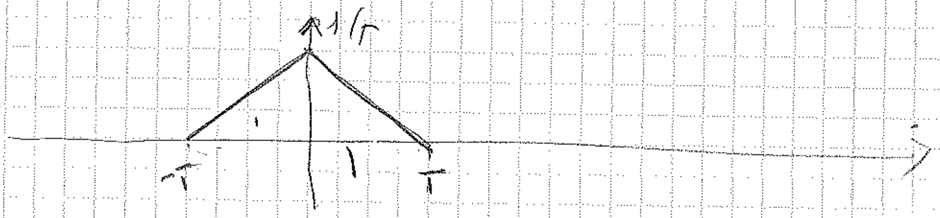
$$X(f) = \left[\frac{1}{2}\delta(f) - \frac{1}{4}\left(\delta\left(f - \frac{1}{T}\right) + \delta\left(f + \frac{1}{T}\right)\right)\right] \otimes T\,\text{sinc}(fT) =$$

$$= \frac{T}{2}\,\text{sinc}(fT) - \frac{T}{4}\,\text{sinc}\left(T\left(f - \frac{1}{T}\right)\right) + \frac{T}{4}\,\text{sinc}\left(T\left(f + \frac{1}{T}\right)\right)$$

TCFG-6

$$x(t) = \begin{cases} 0 & t < -T \\ \frac{1}{2}\left(\frac{t}{T}+1\right)^2 & -T \le t < 0 \\ 1-\frac{1}{2}\left(\frac{t}{T}-1\right)^2 & 0 \le t < T \\ 1 & t > T \end{cases}$$





$$y(t) = \frac{1}{T}\left(1-\frac{|t|}{T}\right) \text{rect}\left(\frac{t}{2T}\right) \qquad \int_{-\infty}^{+\infty} y(t)\,dt \neq 0$$

É necessario usare il Teorema di integrazione generalizzato.

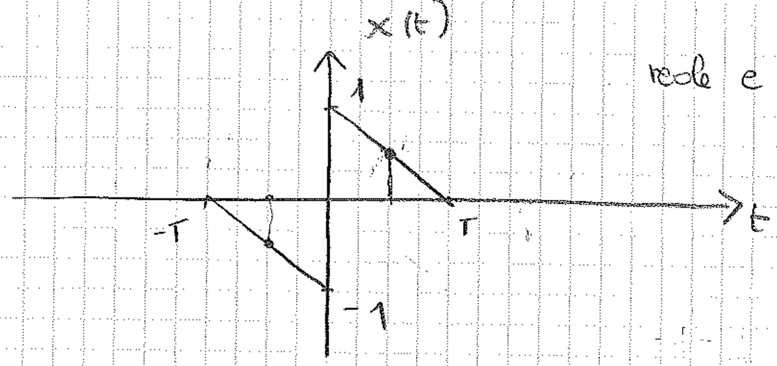
$$x(t) = \int_{-\infty}^{t} y(\alpha)\,d\alpha = y(t) \otimes u(t)$$

$$X(f) = Y(f) \cdot U(f) \qquad U(f) = \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$$

$$X(f) = \frac{1}{2}Y(0) + \frac{Y(f)}{j2\pi f}$$

$$Y(f) = \text{sinc}^2(fT) \qquad Y(0) = 1$$

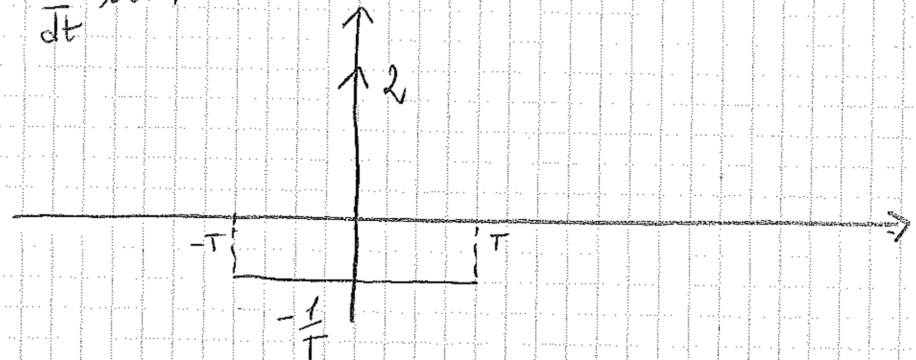
TCFG - 7 -



reale e dispari

.) teorema di derivazione

$$y(t) = \frac{d}{dt} x(t)$$



$$\int_{-\infty}^{+\infty} y(t)\, dt = 2 + \left(-\frac{1}{\cancel{T}} \cdot 2\cancel{T}\right) = 0$$

$$Y(f) = j2\pi f \, X(f)$$

$$X(f) = \frac{Y(f)}{j2\pi f}$$

$$y(t) = -\frac{1}{T} \operatorname{rect}\left(\frac{t}{2T}\right) + 2\delta(t)$$

$$Y(f) = -2 \operatorname{sinc}(f 2T) + 2$$

$$X(f) = \frac{1}{j2\pi f}\left[-2\,\text{sinc}(f2T) + 2\right] =$$

$$= -\frac{\text{sinc}(f2T)}{j\pi f} + \frac{2\,1}{j2\pi f}$$

TCFG-8

$$x(t) = B\,\text{sinc}^2(Bt) \cdot \cos(2\pi f_0 t + \varphi_1)$$

$$y(t) = B\,\text{sinc}(Bt) \cdot \cos(2\pi f_0 t - \varphi_2)$$

$z(t) = x(t) \otimes y(t)$ trovare la trasformata di $z(t)$

·) teorema dello convoluzione

$$Z(f) = X(f) \cdot Y(f)$$

$$X(f) = \left(1 - \frac{|f|}{B}\right) \text{rect}\left(\frac{f}{2B}\right) \otimes \left[\frac{1}{2}\delta(f-f_0)e^{j\varphi_1} + \frac{1}{2}\delta(f+f_0)e^{-j\varphi_1}\right]$$

$$Y(f) = \text{rect}\left(\frac{f}{B}\right) \otimes \left[\frac{1}{2}\delta(f-f_0)e^{-j\varphi_2} + \frac{1}{2}\delta(f+f_0)e^{j\varphi_2}\right]$$

$$X(f) = \frac{1}{2}X_0(f-f_0)e^{j\varphi_1} + \frac{1}{2}X_0(f+f_0)e^{-j\varphi_1}$$

$$Y(f) = \frac{1}{2}Y_0(f-f_0)e^{-j\varphi_2} + \frac{1}{2}Y_0(f+f_0)e^{j\varphi_2}$$

$$Z(f) = \frac{1}{4}X_0(f-f_0)Y_0(f-f_0)e^{j(\varphi_1-\varphi_2)} + \frac{1}{4}X_0(f+f_0)Y_0(f+f_0)e^{-j(\varphi_1-\varphi_2)} =$$

$$= \frac{1}{4}Z_0(f-f_0)e^{j\varphi} + \frac{1}{4}Z_0(f+f_0)e^{-j\varphi}$$

$$z(t) = \frac{1}{2} z_0(t) \cdot \cos\left(2\pi f_0 t + \varphi\right)$$

$$Z_0(f) = \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right) \quad \mathrm{rect}\left(\frac{f}{B}\right)$$

$$Z_0(f) = \frac{1}{2} \mathrm{rect}\left(\frac{f}{B}\right) + \frac{1}{2}\left(1 - \frac{|f|}{B/2}\right) \mathrm{rect}\left(\frac{f}{B}\right)$$

$$z_0(t) = \frac{B}{2} \mathrm{sinc}\left(tB\right) + \frac{B}{4} \mathrm{sinc}^2\left(t\frac{B}{2}\right)$$