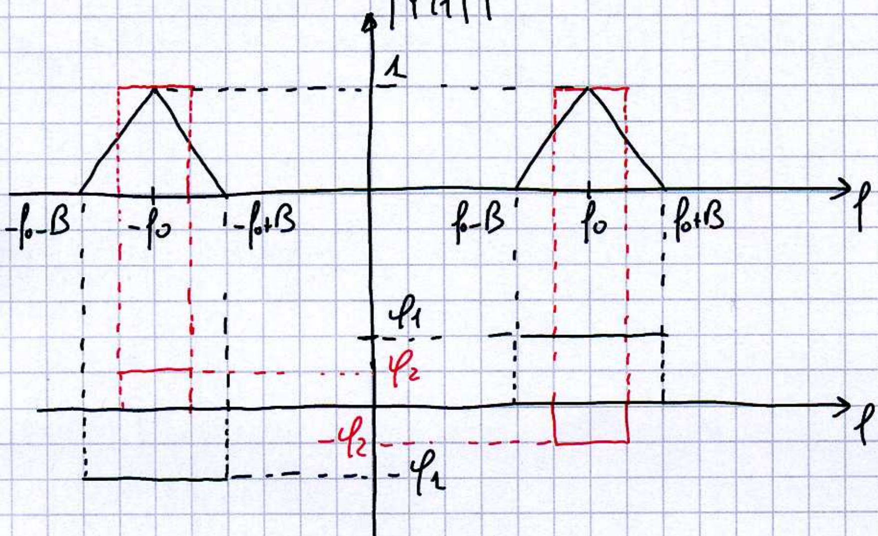
SOLUZIONE PROVA IN ITINERE COM. NUM 2012 A/B

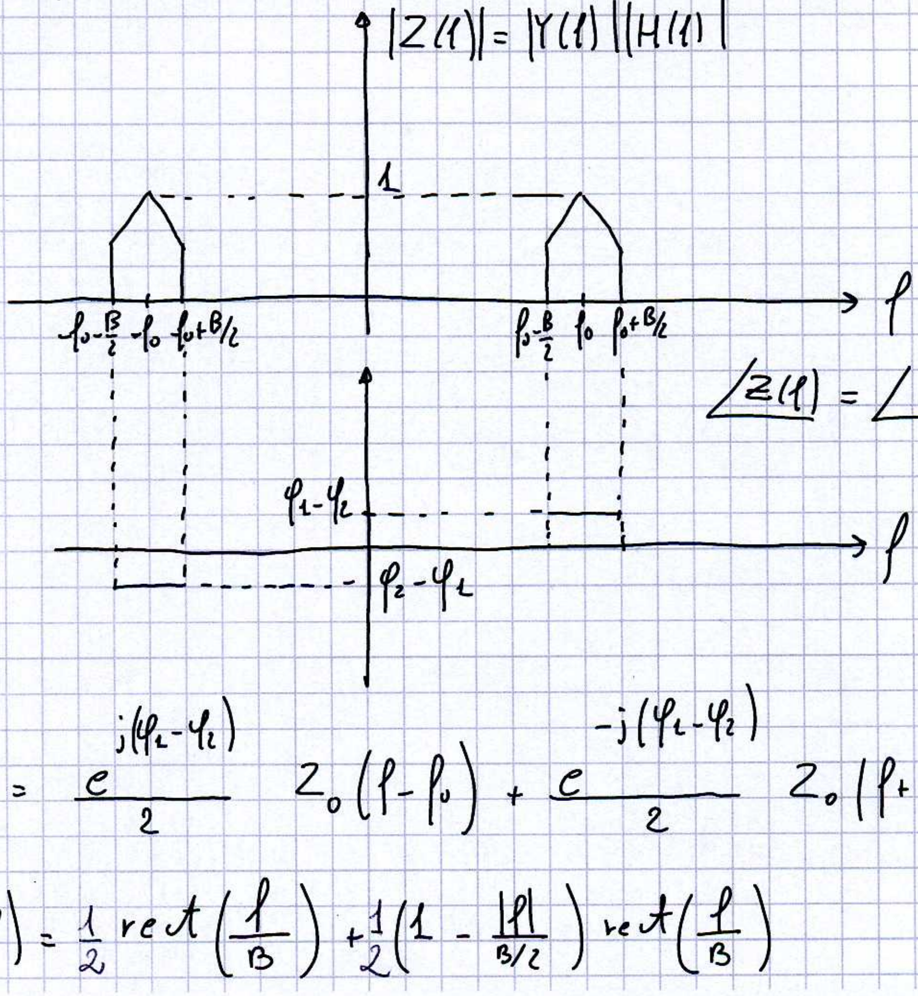
Es ①

$$X(f) = \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$Y(f) = \frac{e^{j\varphi_1}}{2} X(f - f_0) + \frac{e^{-j\varphi_1}}{2} X(f + f_0)$$



Per via grafica si ottiene la $Z(f)$



$$\angle Z(f) = \angle Y(f) + \angle H(f)$$

$$Z(f) = \frac{e^{j(\varphi_1-\varphi_2)}}{2} Z_0(f - f_0) + \frac{e^{-j(\varphi_1-\varphi_2)}}{2} Z_0(f + f_0)$$

$$Z_0(f) = \frac{1}{2} \mathrm{rect}\left(\frac{f}{B}\right) + \frac{1}{2}\left(1 - \frac{|f|}{B/2}\right) \mathrm{rect}\left(\frac{f}{B}\right)$$

$$z(t) = z_0(t) \cos(2\pi f_0 t + \varphi_1 - \varphi_2)$$

$$z_0(t) = \frac{1}{2} B \,\mathrm{sinc}(Bt) + \frac{B}{4} \mathrm{sinc}^2\left(\frac{B}{2} t\right)$$

$$E_z = \int_{-\infty}^{+\infty} z^2(t)\, dt = \int_{-\infty}^{+\infty} |Z(f)|^2 df$$

$$= \frac{2}{4} \int_{f_0-\frac{B}{2}}^{f_0+\frac{B}{2}} \left[\frac{1}{2} \mathrm{rect}\left(\frac{f}{B}\right) + \frac{1}{2}\left(1 - \frac{|f|}{B/2}\right) \mathrm{rect}\left(\frac{f}{B}\right)\right]^2 df$$

$$= \frac{1}{2}\left[\frac{1}{4} B + \frac{1}{4} \cdot \frac{2}{3} \frac{B}{2} + 2 \cdot \frac{1}{4} \cdot \frac{B}{2}\right] = \frac{1}{8}\left[B + \frac{B}{3} + B\right] = \frac{7}{24} B$$

$P_z = 0$

La condizione per cui $z(t)$ sia reale e pari è che

$$\varphi_1 - \varphi_2 = K\pi \Rightarrow \varphi_2 = \varphi_1 - K\pi \quad , \quad K = 0, \pm 1, \pm 2, \dots$$