Prova scritta di Elettrotecnica

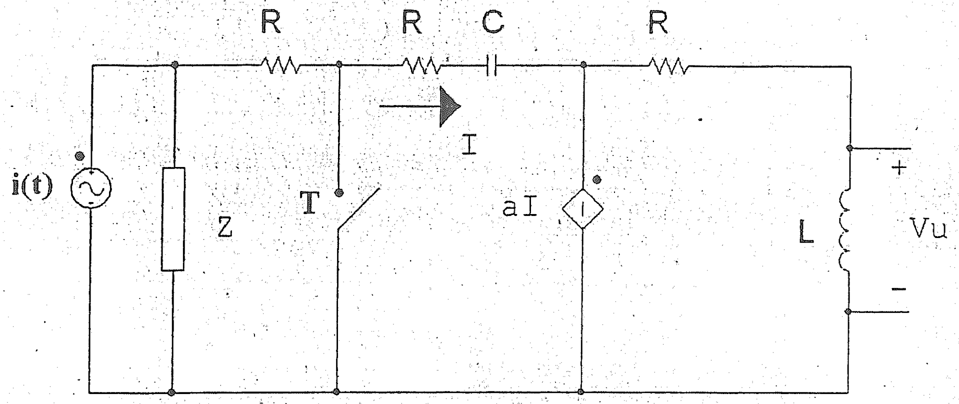
(A)

Corso di Laurea in Ingegneria Informatica

(<u>12 cr.</u>: 1, 3, 4,5; <u>9 cr.</u>: 1, 2 o 5, 3, 6; <u>6 cr.</u>: 2, 5, 6)

Pisa 18/07/03 Allievo: ....

1) Supponendo il circuito di figura in condizioni stazionarie per t<0, determinare l'andamento temporale della tensione V_u per t>0 quando il tasto T si chiude.



$\overline{Z} = 5 + j10\ \Omega;$

$R = 10\ \Omega;$

$L = 10 mH;$

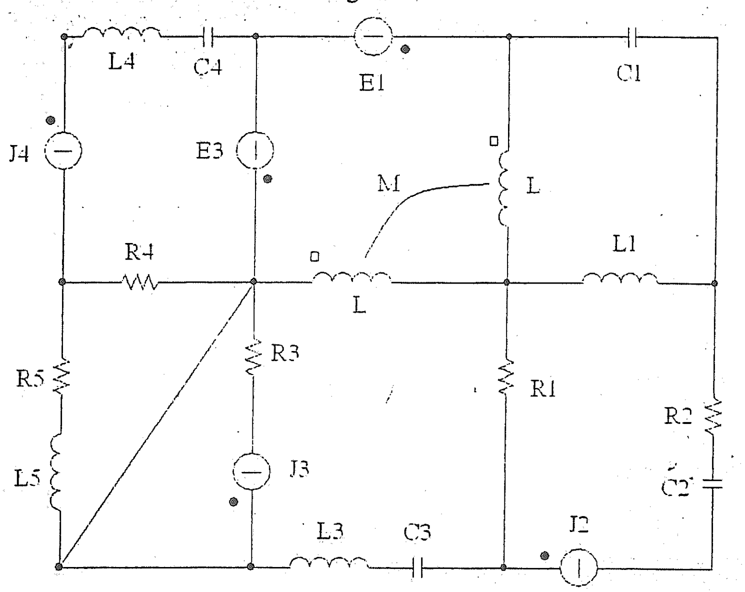
$C = 200 \mu F;$

$a = 0.5\ V/A;$

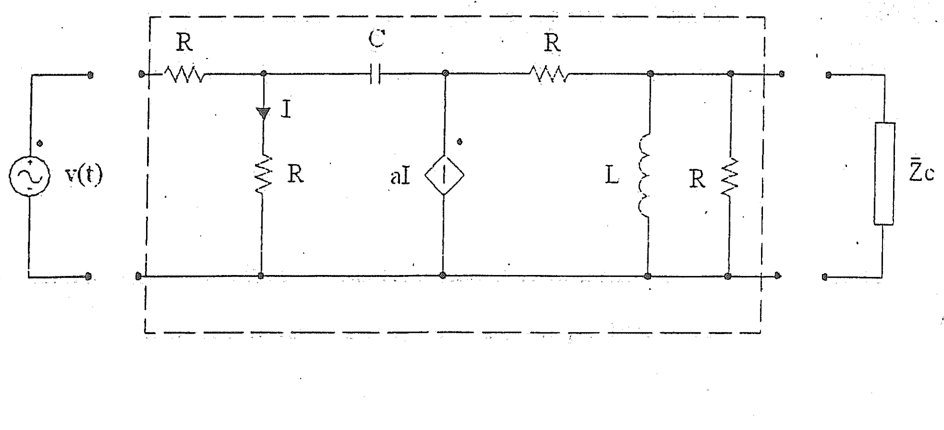
$f = 50\ Hz;$

$i(t) = 10\sqrt{2}\sin(\omega t + \pi/4)\ A$

2) Per il circuito in figura scrivere un sistema di equazioni di equilibrio con il metodo delle tensioni nodali, supponendo il circuito stesso in condizioni di regime sinusoidale.



3) Determinare i parametri Z del doppio bipolo di figura e calcolare la potenza attiva e reattiva erogata dal generatore di tensione quando a valle del doppio bipolo è collegato il carico Z_c.



$\overline{Z}_c = 20 + j5\ \Omega;$

$R = 2\ \Omega;$

$L = 10 mH;$

$C = 200 \mu F;$

$a = 0.5\ V/A;$

$v(t) = 220\sqrt{2}\sin(314t)\ V$

Prova scritta di Elettrotecnica

versione provvisoria

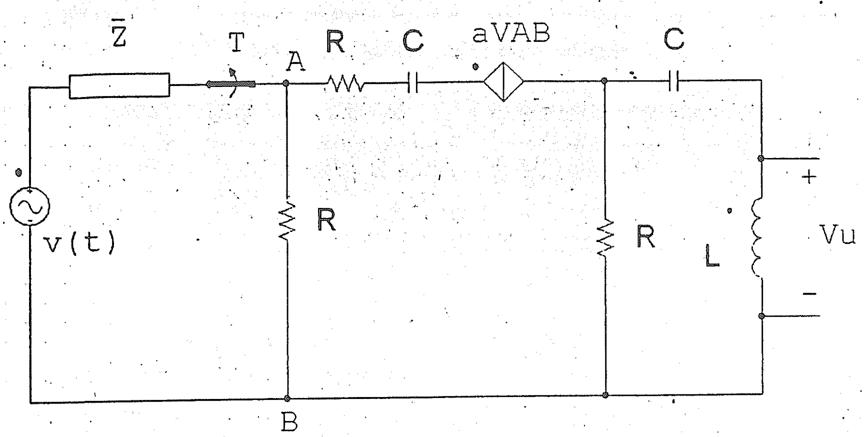
(B)

Corso di Laurea in Ingegneria Informatica

(12 cr.: 1, 3, 4,5; 9 cr.: 1, 2 o 5, 3, 6; 6 cr.: 2, 5, 6)

Pisa 18/07/03 Allievo: ....

1) Supponendo il circuito di figura in condizioni stazionarie per t<0, determinare l'andamento temporale della tensione V_u per t>0 quando il tasto T si apre.



$\overline{Z} = 5 + j10\ \Omega;$

$R = 10\ \Omega;$

$L = 10\,mH;$

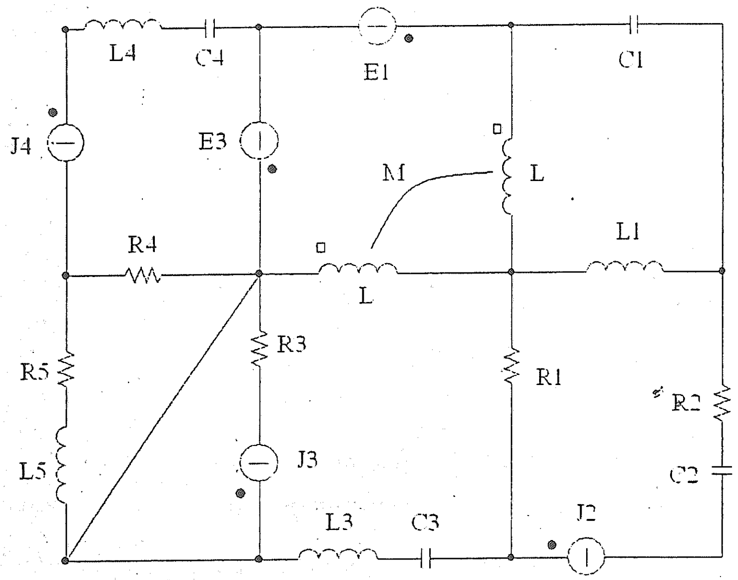
$C = 100\,\mu F;$

$a = 0.5\ A/V;$

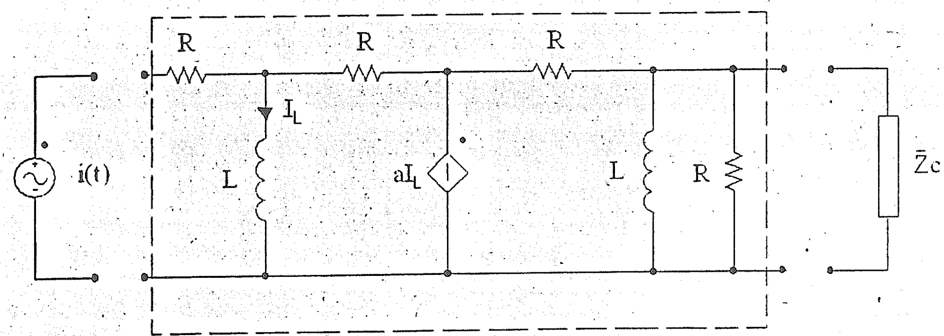
$f = 50\ Hz;$

$v(t) = 100\sqrt{2}\sin(\omega t + \pi/6)\ V;$

2) Per il circuito in figura scrivere un sistema di equazioni di equilibrio con il metodo delle correnti di maglia, supponendo il circuito stesso in condizioni di regime sinusoidale.



3) Determinare i parametri Z del doppio bipolo di figura e calcolare la potenza attiva e reattiva erogata dal generatore di corrente quando a valle del doppio bipolo è collegato il carico Z_c.



$\overline{Z}_c = 10 + j15\ \Omega;$

$R = 1\ \Omega;$

$L = 100\,mH;$

$C = 100\,\mu F;$

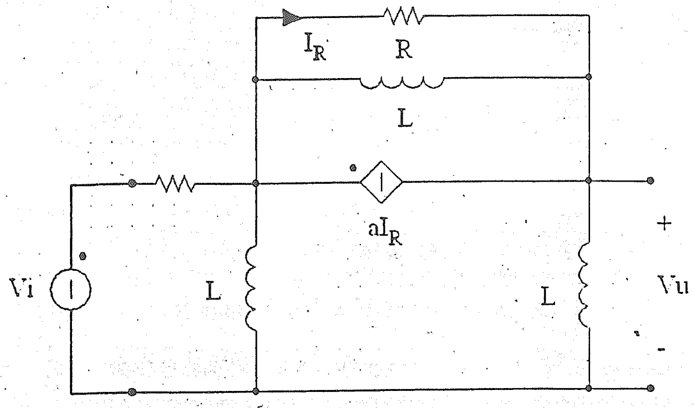
$a = 0.5\ V/A;$

$i(t) = 2\sqrt{2}\sin(1000t)\ A$

TIPOLOGIA A e B (comune)

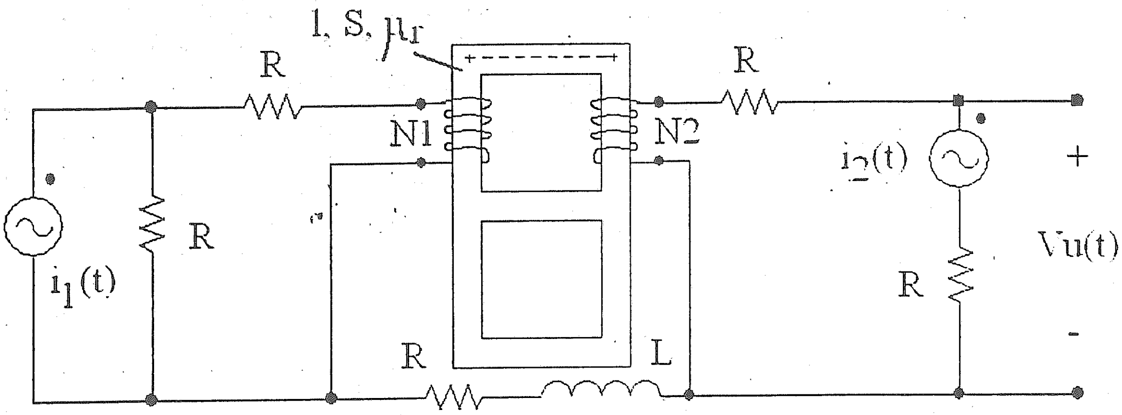
4) Determinare la funzione di trasferimento V_u/V_i per il seguente circuito e tracciare i diagrammi di Bode per l'ampiezza e la fase della relativa risposta in frequenza.

18/7/03



$R = 20\ \Omega$;
$L = 15\,mH$;
$a = 0.5$;

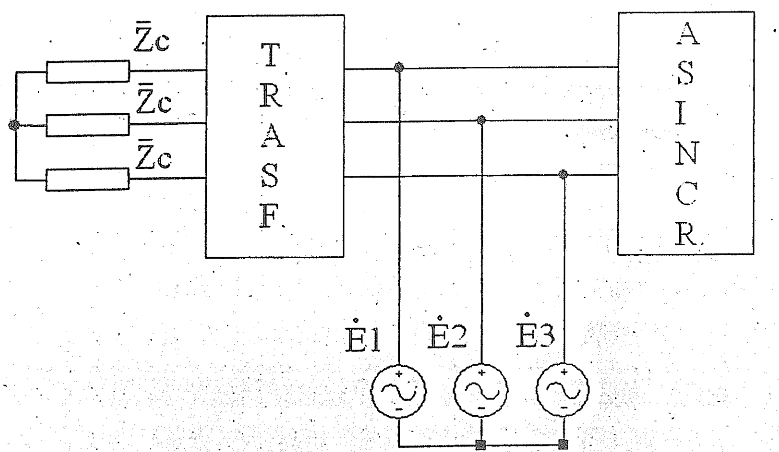
5) Il circuito in figura è da considerarsi in condizioni di regime per effetto dei generatori inseriti. Determinare l'andamento temporale della tensione V_u e l'energia elettromagnetica media immagazzinata nei due induttori mutuamente accoppiati.



$R = 10\ \Omega$;
$L = 10\,mH$;
$N_1 = 400$;
$N_2 = 300$;
$l = 10\,cm$;
$S = 3\,cm^2$;
$\mu_r = 1000$;

$i_1(t) = 5 + 10\sqrt{2}\sin(500t)\quad A$; $\qquad i_2(t) = 5\sqrt{2}\cos(1000t + \pi/3)\ \ A$;

6) Nel sistema trifase di figura, determinare la potenza attiva e reattiva erogata dal generatore di tensione trifase. Si determinino inoltre le perdite nel ferro del motore e del trasformatore.

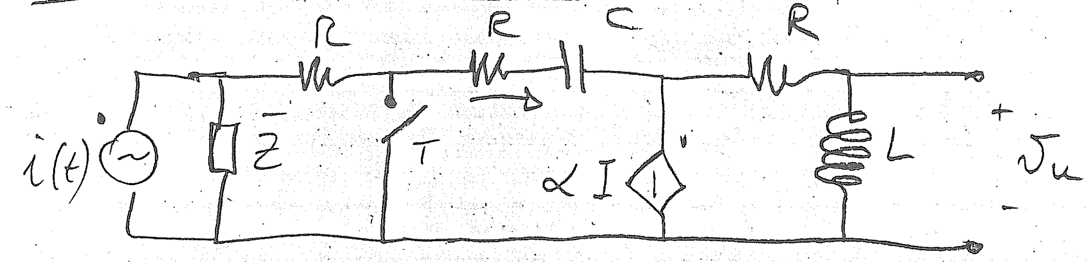


Motore asincrono	Trasformatore
Prova a vuoto	Prova a vuoto
$V_{10} = 400\ V$;	$V_{10} = 380\ V$;
$I_{10} = 15\ A$;	$I_{10} = 12\ A$;
$P_{10} = 1250\ W$;	$P_{10} = 1500\ W$;
Prova in cc	Prova in cc
$V_{1cc} = 40\ V$;	$V_{1cc} = 30\ V$;
$I_{1cc} = 7\ A$;	$I_{1cc} = 5\ A$;
$P_{1cc} = 150\ W$;	$P_{1cc} = 170\ W$;
$k = 0.5$; $(E_1^A = kE_2^A)$;	
$s = 0.75$;	
$R_{1s} = 0.5\ \Omega$;	
$X_{1s} = 1.25\ \Omega$;	

$\bar{Z}_c = 25 + j10\ \Omega$; $\quad \dot{E}_1 = 240\ V$; $\quad f = 50\ Hz$;

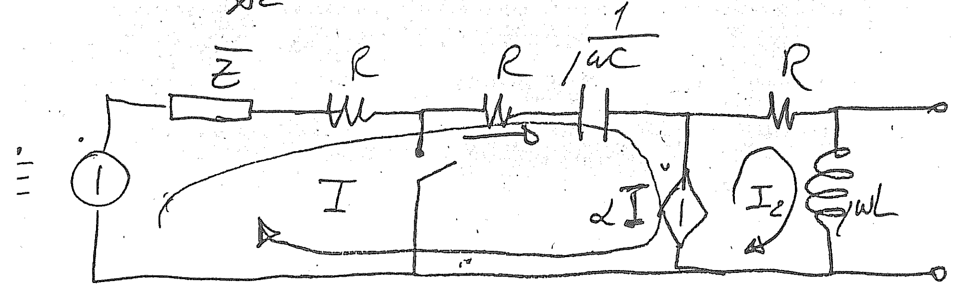
(DEL 18-07-03)

1A del 18/7/03 versione provvisoria



Per $t<0$ studio c. i. (tensione sul condensatore, corrente sull'induttore)

$$\dot I_g = \frac{10\cdot\sqrt2}{\sqrt2}\cdot e^{j\frac{\pi}{4}} = 7.07 + j\,7.07 \text{ A}$$



$$\dot E = \bar Z \cdot \dot I_g = -35.35 + j\,106 \text{ V}$$

$$\begin{cases} \dot E - \alpha \dot I = \left[\bar Z + 2R + \frac{1}{j\omega C}\right]\dot I \\ \alpha \dot I = (R + j\omega L)\,\dot I_2 \end{cases}$$

$$\dot I = \frac{\dot E}{\bar Z + 2R + \frac{1}{j\omega C} + \alpha} = -4.27 + j\,5.21 \text{ A}$$

$$\dot I_2 = \frac{\alpha \dot I}{R + j\omega L} \cong -0.12 + j\,0.3 \text{ A}$$

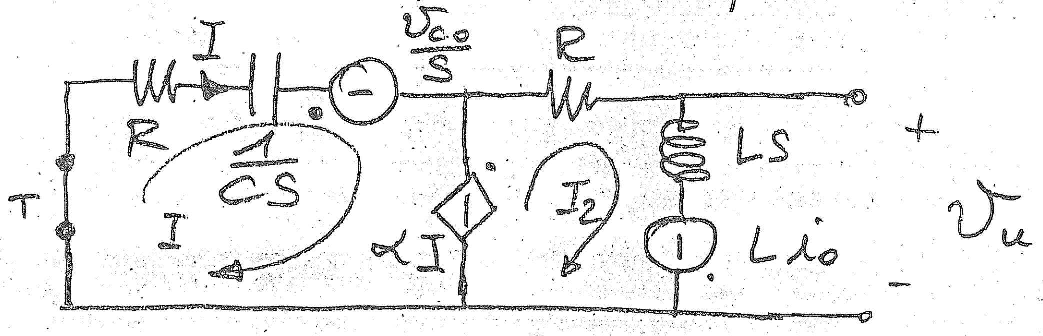
$$\dot V_c = \frac{1}{j\omega C}\dot I \cong 83 + j\,68 \text{ V}$$

$$v_c(t) = 107.25\cdot\sqrt2\,\sin(\omega t + 0.68)\text{ V} \Rightarrow \boxed{v_c(0) = v_c(t)\big|_{t=0} \cong 96\text{ V}}$$

$$i_2(t) = 0.32\cdot\sqrt2\,\sin(\omega t + 1.95)\text{ A}$$

$$i_L(t) = i_2(t) \Rightarrow \boxed{i_{L0} = i_2(0) = 0.42 \text{ A} = i_0}$$

studiamo il circuito L-trasformato per $t>0$ quando si chiude il tasto T



$$\begin{cases} -\dfrac{v_{c0}}{s} - \alpha I = \left(R + \dfrac{1}{Cs}\right) I \\ \alpha I + L i_0 = (R+Ls)\, I_2 \end{cases}$$

dalla prima:

$$I = \frac{-\dfrac{v_{c0}}{s}}{R + \dfrac{1}{Cs} + \alpha} = -\frac{v_{c0}\cdot C}{(\alpha+R)Cs+1}$$

sostituendo nella seconda equ.

$$-\frac{\alpha v_{c0} C}{(\alpha+R)Cs+1} + L i_0 = (R+Ls)\, I_2$$

$$I_2 = \frac{1}{R+Ls}\cdot\left[L i_0 - \frac{\alpha v_{c0} C}{(\alpha+R)Cs+1}\right]$$

$$v_u(s) = Ls\, I_2 - L i_0$$

$$= \frac{Ls}{R+Ls}\cdot\left[L i_0 - \frac{\alpha v_{c0} C}{(\alpha+R)Cs+1}\right] - L i_0$$

$$= \frac{Ls\cdot L i_0\left[(\alpha+R)Cs+1\right] - \alpha v_{c0} C L s - L i_0 (R+Ls)\left[(\alpha+R)Cs+1\right]}{(R+Ls)\left[(\alpha+R)Cs+1\right]}$$

6s 1A 18/7/03 (3)

$$v_u(s) = \frac{L i_o \left[(\alpha+R)Cs+1\right]\left(\cancel{Ls} - R \cancel{+Ls}\right) - \alpha v_{co} L C s}{(R+Ls)\left[(\alpha+R)Cs+1\right]}$$

$$v_u(s) = -\frac{\left[R i_o(\alpha+R) + \alpha v_{co}\right] LCs + RL i_o}{(R+Ls)\left[(\alpha+R)Cs+1\right]}$$

$$v_u(s) = -\frac{1.85 \cdot 10^{-4} s + 0.0422}{(10+10^{-2}s)(0.0021s+1)}$$

$$v_u(s) = -\frac{1.85 \cdot 10^{-4}}{10^{-2} \cdot 0.0021} \cdot \frac{(s+228.38)}{(s+1000)(s+476.2)}$$

$= -8.8$

$$v_u(s) = -8.8 \cdot \left[\frac{A}{s+1000} + \frac{B}{s+476.2}\right]$$

$$A = v_u(s) \cdot (s+1000)\Big|_{s=-1000} = 1.4731$$

$$B = v_u(s) \cdot (s+476.2)\Big|_{s=-476.2} = -0.4731$$

$$v_u(t) = -8.8 \cdot \left[1.4731 \cdot e^{-1000t} - 0.4731\, e^{-476.2t}\right] u(t)$$

Es 2A

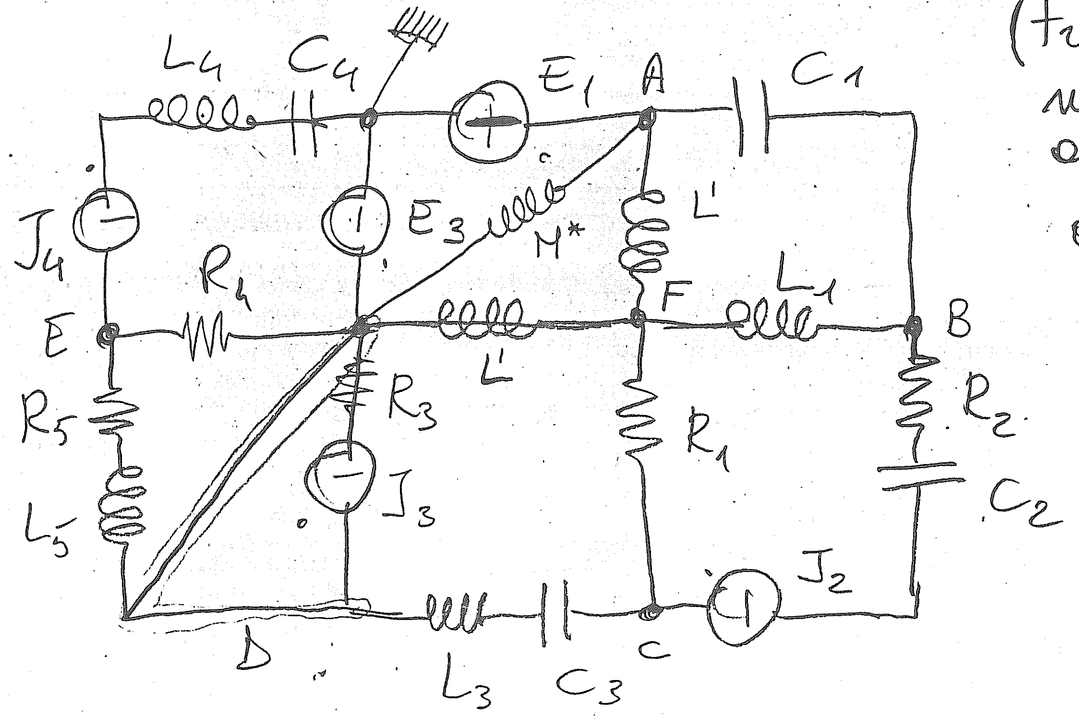
18/7/03

versione provvisoria

(4)

Metodo tensioni nodali

(trasformo mutuo accopp. a π)



$$L' = \frac{\Delta}{L-M} \qquad M^* = \frac{\Delta}{M} \qquad \text{con } \Delta = L^2 - M^2$$

$n°$ equ. $= n - 1 - n_{gE} = 7 - 1 - 2 = 4$ eq

A) $V_A = \dot{E}_1$

D) $V_D = \dot{E}_3$

B) $-\dot{J}_2 = \left(j\omega C_1 + \frac{1}{j\omega L_1}\right)\dot{V}_B - \dot{V}_A j\omega C_1 - \frac{\dot{V}_F}{j\omega L_1}$

C) $\dot{J}_2 = \left(\frac{1}{R_1} + \frac{1}{j\omega L_3 + \frac{1}{j\omega C_3}}\right)V_C - \frac{V_D}{j\omega L_3 + \frac{1}{j\omega C_3}} - \frac{V_F}{R_1}$

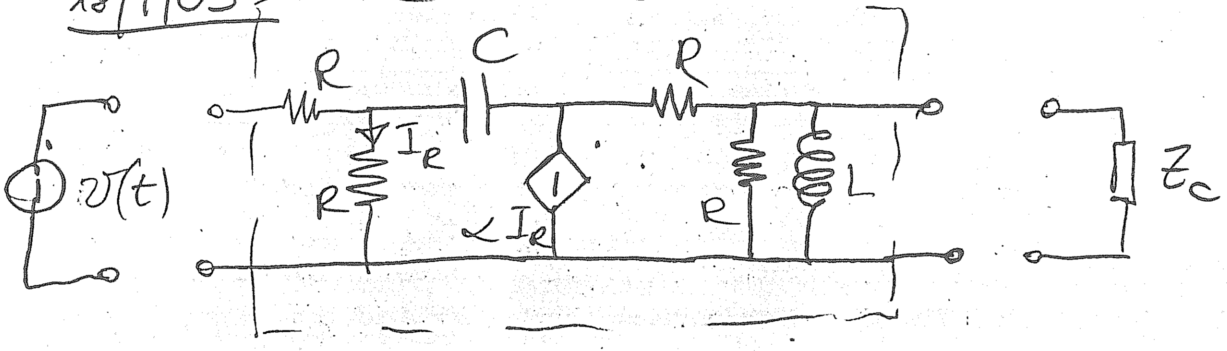
E) $-J_4 = \left(\frac{1}{R_4} + \frac{1}{R_5 + j\omega L_5}\right)V_E - \frac{V_D}{R_5 + j\omega L_5} - \frac{V_D}{R_4}$

F) $0 = \left(\frac{1}{j\omega L_1} + \frac{1}{j\omega L'} + \frac{1}{j\omega L'} + \frac{1}{R_1}\right)V_F - \frac{V_C}{R_1} - \frac{V_A}{j\omega L'} - \frac{V_D}{j\omega L'} - \frac{V_B}{j\omega L_1}$

iS 3A
18/7/03

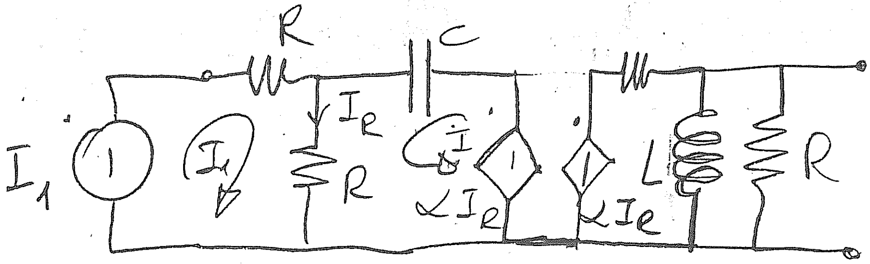
Parametri $\bar{Z}$
versione provvisoria

(5) ~~(1)~~



$$\boxed{\omega = 314 \ \frac{rad}{s}}$$

$$\begin{cases} \dot{V}_1 = \bar{Z}_{11}\dot{I}_1 + \bar{Z}_{12}\dot{I}_2 \\ \dot{V}_2 = \bar{Z}_{21}\dot{I}_1 + \bar{Z}_{22}\dot{I}_2 \end{cases} ; \quad \bar{Z}_{11} = \frac{\dot{V}_1}{\dot{I}_1}\Bigg|_{I_2=0} ; \quad \bar{Z}_{21} = \frac{\dot{V}_2}{\dot{I}_1}\Bigg|_{I_2=0}$$



Sdoppio generatore di tensione:

$$\begin{cases} \alpha I_R = \left(R + \frac{1}{j\omega C}\right)\dot{I} + R\dot{I}_1 \\ I_R = I_1 + I \end{cases} \Rightarrow (\alpha - R) I_1 = \left(R + \frac{1}{j\omega C} - \alpha\right)\dot{I}$$

$$\Downarrow$$

$$\dot{I} = \frac{(\alpha - R)}{R + \frac{1}{j\omega C} - \alpha}\dot{I}_1$$

$$\dot{V}_1 = R\dot{I}_1 + R\dot{I}_1 + \frac{R(\alpha - R)}{(R-\alpha) + \frac{1}{j\omega C}}\dot{I}_1 \Rightarrow \bar{Z}_{11} = \frac{\dot{V}_1}{\dot{I}_1} = 2R + \frac{R(\alpha - R)}{(R-\alpha) + \frac{1}{j\omega C}}$$

$$\boxed{\bar{Z}_{11} = 3.98 - j0.18\ \Omega}$$

$$\dot{V}_2 = \bar{Z}_p \cdot \dot{I}_2 ; \text{ con } \bar{Z}_p = \frac{Rj\omega L}{R + j\omega L} \text{ e } \dot{I}_2 = \frac{\alpha I_R}{R + \bar{Z}_p}$$

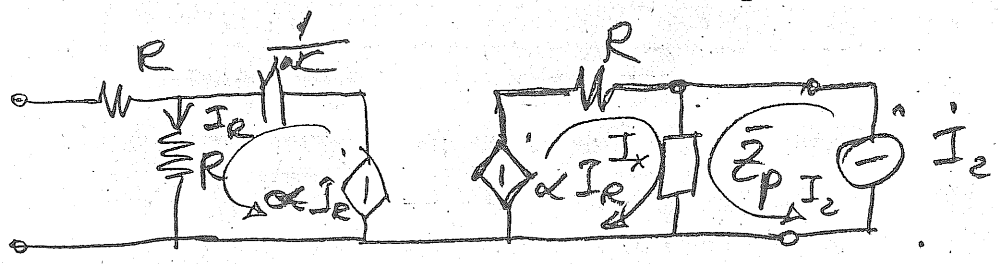
$$I_R = I_1 + I = I_1\left(1 + \frac{(\alpha - R)}{R - \alpha + \frac{1}{j\omega C}}\right)$$

$$\dot{V}_2 = \bar{Z}_p \cdot \frac{\alpha}{R + \bar{Z}_p} \cdot \left(1 + \frac{(\alpha - R)}{R - \alpha + \frac{1}{j\omega C}}\right)\dot{I}_1 \Rightarrow \boxed{\bar{Z}_{21}} = \frac{\dot{V}_2}{\dot{I}_1} = \frac{\alpha \bar{Z}_p}{R + \bar{Z}_p} \cdot \left(1 + \frac{\alpha - R}{R - \alpha + \frac{1}{j\omega C}}\right) = \boxed{0.23 + j0.05}$$

; 3A
8/7/03

6 9

$$\bar{Z}_{12} = \frac{\dot{V}_1}{\dot{I}_2}\bigg|_{I_1=0} \quad ; \quad \bar{Z}_{22} = \frac{\dot{V}_2}{\dot{I}_2}\bigg|_{I_1=0}$$



$$\alpha I_R = \left(R + \frac{1}{j\omega C}\right) I_R \Rightarrow I_R = 0$$

$$\dot{V}_2 = \bar{Z}_p(\dot{I}_2 + \dot{I}_x) \quad \boxed{\bar{Z}_{22}} = \frac{\dot{V}_2}{\dot{I}_2} = \frac{\bar{Z}_p \cdot R}{R + \bar{Z}_p} = \boxed{0.9 + j0.28\ \Omega}$$

$$\dot{V}_1 = 0 \Rightarrow \boxed{\bar{Z}_{12} = \frac{\dot{V}_1}{\dot{I}_2} = 0}$$

$$\bar{Z} = \begin{bmatrix} 3.98 + j0.18 & 0 \\ 0.23 + j0.05 & 0.9 + j0.28 \end{bmatrix}$$

$$\dot{V} = 220\ V$$

$$\bar{S} = \dot{V}\dot{I}_1^*$$

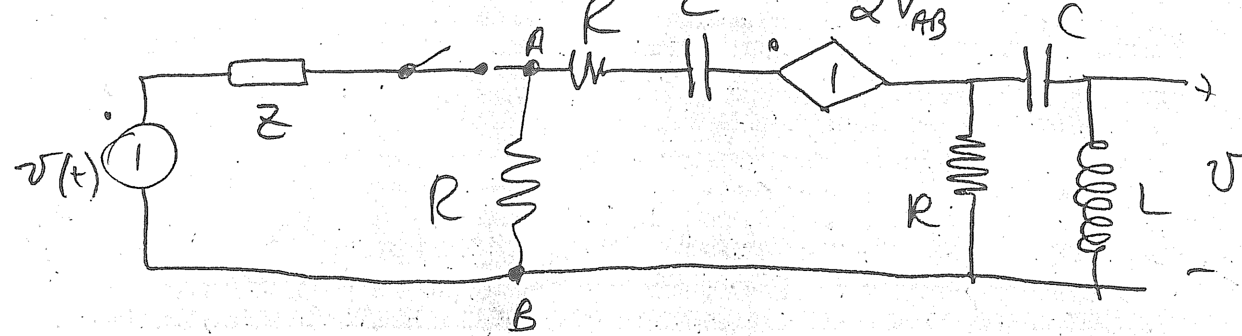
$$\begin{cases} \dot{V}_1 = \bar{Z}_{11}\dot{I}_1 + 0 \cdot \dot{I}_2 \Rightarrow \dot{I}_1 = \dfrac{\dot{V}}{\bar{Z}_{11}} = 55.12 + j2.6\ A \\ \dot{V}_2 = \bar{Z}_{21}\dot{I}_1 + \bar{Z}_{22}\dot{I}_2 \\ \dot{V}_1 = \dot{V} \end{cases}$$

$$\Downarrow$$

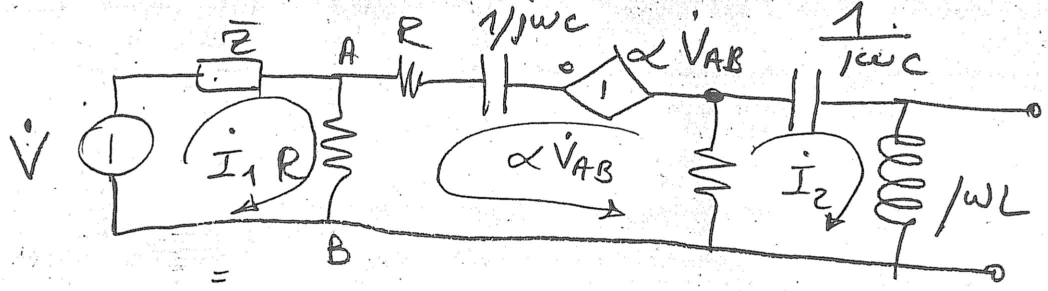
$$\bar{S} = \dot{V} \cdot \dot{I}_1^* = 12.1 - j0.57\ kVA$$

$$\boxed{P = 12.1\ KW \ ; \ Q = -0.57\ KVAR}$$

s 1B del 18/7/03 versione provvisoria



Per $t<0$ studiamo c. i.



$$\begin{cases} \dot{V} = (\bar{Z} + R)\dot{I}_1 + R\alpha \dot{V}_{AB} \\ \dot{V}_{AB} = R\dot{I}_1 + \alpha R \dot{V}_{AB} \Rightarrow \dot{V}_{AB} = \dfrac{R}{1-\alpha R}\dot{I}_1 \end{cases}$$

$$\dot{V} = \left[(\bar{Z}+R) + \frac{\alpha R^2}{1-\alpha R}\right]\dot{I}_1$$

$$\dot{I}_1 = \frac{\dot{V}(1-\alpha R)}{(\bar{Z}+R)(1-\alpha R)+\alpha R^2} \cong 6.75 - j7 \text{ A}$$

con $\dot{V} = \frac{100\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} \cdot e^{j\frac{\pi}{6}} = 86.6 + j50$ V

$$\dot{V}_{AB} = \frac{R}{1-\alpha R} \cdot \dot{I}_1 = -16.86 + j17.43 \text{ V}$$

Per trovare la corrente $\dot{I}_2$ applico partitore di corrente:

$$\dot{I}_2 = -\frac{\alpha \dot{V}_{AB} \cdot R}{R + j\omega L + \frac{1}{j\omega C}} = 3.62 + j1.67 \text{ A}$$

Poichè uno dei 2 condensatori è in serie ad un generatore di corrente per quel condensatore

$$\dot V_{c2} = \frac{\dot I_2}{j\omega C} = 53.3 - j\,115.3 \ \text{V}$$

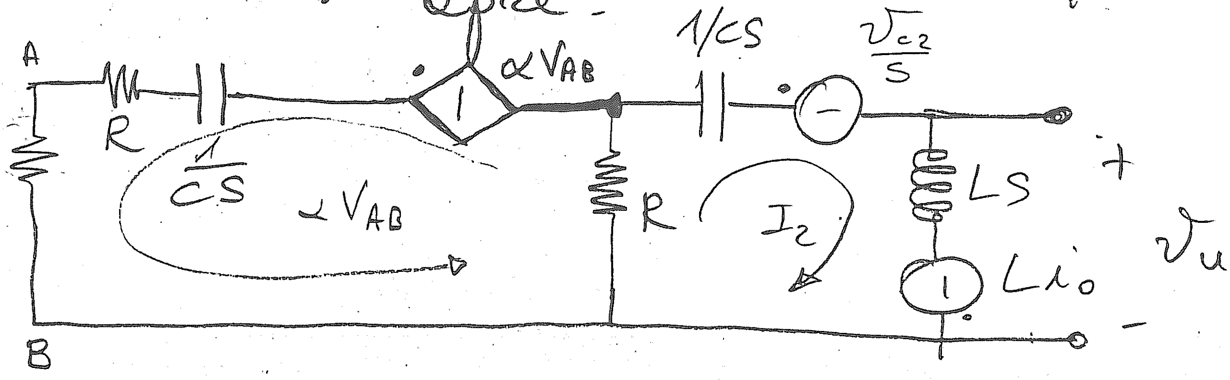
$$v_{c2}(t) \cong 127 \cdot \sqrt{2} \cdot \sin(\omega t - 1.13) \ \text{V}$$

$$\boxed{v_{c20} \cong -163 \ \text{V}}$$

$$i_L(t) = i_2(t) \cong 4\sqrt{2}\,\sin(\omega t + 0.43) \ \text{A}$$

$$\boxed{i_0 = i_2(0) = 2.37 \ \text{A}}$$

Studiamo il circuito per $t > 0$ quando il tasto T si apre.



$$V_{AB} = \alpha V_{AB} \cdot R \Rightarrow V_{AB} = 0$$

$$L i_0 - \frac{v_{c2}}{s} = \left(R + LS + \frac{1}{CS}\right) I_2$$

$$I_2 = \frac{L i_0 - \frac{v_{c2}}{s}}{R + LS + \frac{1}{CS}} = \frac{LCS i_0 - v_c C}{LCS^2 + RCS + 1}$$

$$v_u(s) = LS\, I_2 - L i_0$$

$$= LS \cdot \frac{LCS i_0 - v_c C}{LCS^2 + RCS + 1} - L i_0 \Rightarrow$$

$\not{8}$S 1B
18/7/03

9

$$v_u(s) = \frac{L^2 \cancel{CS^2} i_0 - v_c LCS - L^2 \cancel{CS^2} i_0 - LRC i_0 s - L i_0}{LCS^2 + RCS + 1}$$

$$v_u(s) = -\frac{(v_c LC + LRC i_0) s + L i_0}{LCS^2 + RCS + 1} = -\frac{-1.4 \cdot 10^{-4} s + 0.0237}{10^{-6} s^2 + 0.001 s + 1}$$

$$v_u(s) = \frac{1.4 \cdot 10^{-4}}{10^{-6}} \cdot \frac{s - 170}{(s + 500 - j866)(s + 500 + j866)}$$

$\simeq 140$

$$v_u(s) = 140 \left[\frac{A}{s + 500 - j866} + \frac{A^*}{s + 500 - j866} \right]$$

$$A = v_u(s) \cdot (s + 500 - j866) \Big|_{s = -500 + j866} = 0.5 + j\,0.3868$$

$A = M + jN \quad ; \quad A^* = M - jN$

$$v_u(t) = \left[2 \cdot e^{-500t} (M \cos 866t + N \sin 866t) \right] u(t)$$

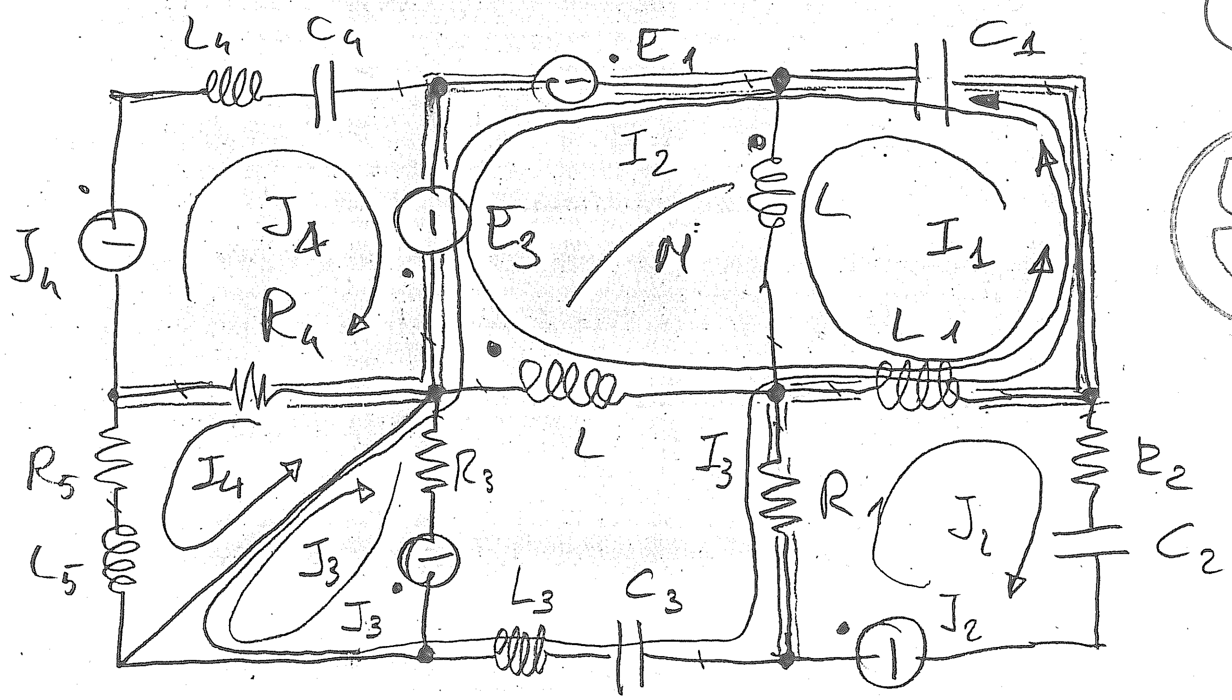
$$\boxed{v_u(t) = 2 e^{-500t} \cdot \left[0.5 \cos(866t) + 0.3868 \sin(866t) \right] u(t)}$$

Es 2B

18/7/03

Metodo correnti di maglia

10



N° eq: $r - n + 1 - n_{g\,corr} = 13 - 7 + 1 - 3 = 4$ eq

Scelto l'albero di figura, con le correnti indicate:

1) $0 = \left(j\omega L_1 + j\omega L + \frac{1}{j\omega C_1}\right)\dot{I}_1 + \left(j\omega L_1 + \frac{1}{j\omega C_1}\right)\left(\dot{I}_2 + \dot{I}_3\right) + j\omega L_1 \dot{J}_2 + j\omega M \dot{I}_2$

) $\dot{E}_3 - \dot{E}_1 = \left(j\omega L_1 + j\omega L + \frac{1}{j\omega C_1}\right)\dot{I}_2 + \left(j\omega L_1 + \frac{1}{j\omega C_1}\right)\left(\dot{I}_1 + \dot{I}_3\right) + j\omega L_1 \dot{J}_2 + j\omega M \dot{I}_2$

) $\dot{E}_3 - \dot{E}_1 = \left(j\omega L_1 + j\omega L_3 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_3} + R\right)\dot{I}_3 + \left(j\omega L_1 + \frac{1}{j\omega C_1}\right)\left(\dot{I}_1 + \dot{I}_2\right) + \left(R_1 + j\omega L_1\right)\dot{J}_2$

) $0 = \left(R_4 + R_5 + j\omega L_5\right)\dot{I}_4 - R_4 \dot{J}_4$

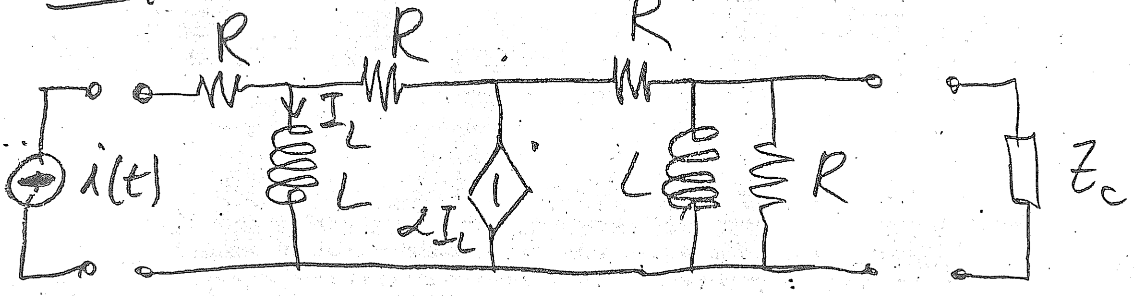
OS 3.B

18/7/03

Parametri $\bar{Z}$

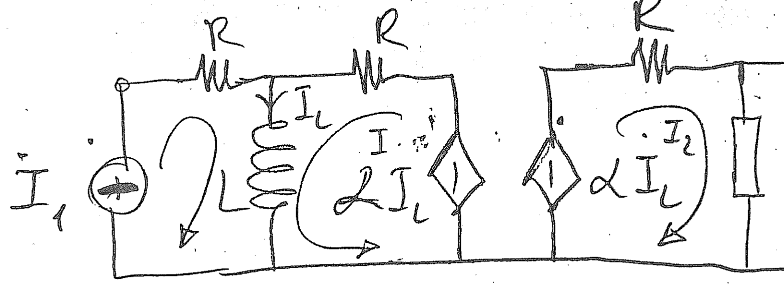
11

~~1~~



$\omega = 1000 \frac{rad}{s}$

$$\begin{cases} \dot{V}_1 = \bar{Z}_{11}\dot{I}_1 + \bar{Z}_{12}\dot{I}_2 \\ \dot{V}_2 = \bar{Z}_{21}\dot{I}_1 + \bar{Z}_{22}\dot{I}_2 \end{cases} \Rightarrow \bar{Z}_{11} = \frac{\dot{V}_1}{\dot{I}_1}\bigg|_{\dot{I}_2=0} ; \quad \bar{Z}_{21} = \frac{\dot{V}_2}{\dot{I}_1}\bigg|_{\dot{I}_2=0}$$



$$\bar{Z}_p = \frac{R \cdot j\omega L}{R + j\omega L} \cong 1 + j\,0.010\ \Omega$$

Sdoppio generatore di tensione:

$$\begin{cases} \alpha \dot{I}_L = (R + j\omega L)\dot{I} + j\omega L \dot{I}_1 \\ \dot{I}_L = \dot{I} + \dot{I}_1 \end{cases} \Rightarrow \left| \begin{array}{c} (R + j\omega L - \alpha)\dot{I} = (\alpha - j\omega L)\dot{I}_1 \\ \Downarrow \\ \dot{I} = \dfrac{\alpha - j\omega L}{R + j\omega L - \alpha}\dot{I}_1 \end{array} \right.$$

$$\dot{V}_1 = R\dot{I}_1 + j\omega L \dot{I}_1 + \frac{j\omega L \cdot (\alpha - j\omega L)}{R + j\omega L - \alpha}\dot{I}_1$$

$$\boxed{\bar{Z}_{11}} = \frac{\dot{V}_1}{\dot{I}_1} = R + j\omega L + \frac{j\omega L(\alpha - j\omega L)}{R - \alpha + j\omega L} = \boxed{2 + j\,0.005\ \Omega}$$

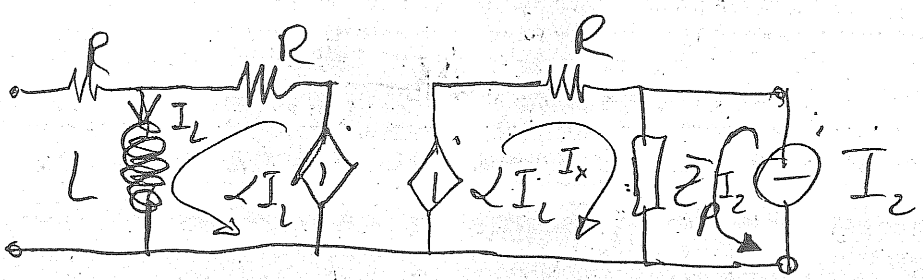
$$\dot{V}_2 = \bar{Z}_p \cdot \dot{I}_2 \quad \text{con} \quad \dot{I}_2 = \frac{\alpha \dot{I}_L}{R + \bar{Z}_p} = \frac{\alpha}{R + \bar{Z}_p} \cdot \left[\frac{\alpha - j\omega L}{R + j\omega L - \alpha} + 1\right]\dot{I}_1$$

$$\boxed{\bar{Z}_{21}} = \frac{\dot{V}_2}{\dot{I}_1} = \frac{\alpha \bar{Z}_p}{R + \bar{Z}_p} \cdot \left[\frac{\alpha - j\omega L}{R + j\omega L - \alpha} + 1\right] = \boxed{2.5 \cdot 10^{-5} + j\,0.0025}$$

Es. 3B
18/7/03

(2) 12

$$\bar{Z}_{12} = \frac{\dot V_1}{\dot I_2}\Big|_{\dot I_1=0} \; ; \; \bar{Z}_{22} = \frac{\dot V_2}{\dot I_2}\Big|_{\dot I_1=0}$$



$$\alpha \dot I_L = (R + j\omega L)\,\dot I_L \Rightarrow \dot I_L = 0$$

$$\alpha I = 0 = (R + \bar{Z}_p)\,\dot I_x + \bar{Z}_p \dot I_2$$

$$I_x = -\frac{\bar{Z}_p}{R+\bar{Z}_p}\,\dot I_2$$

$$\dot V_2 = \bar{Z}_p \cdot (I_x + I_2) = \bar{Z}_p\left[-\frac{\bar{Z}_p}{R+\bar{Z}_p} + 1\right]\dot I_2 = \bar{Z}_p\left(\frac{\cancel{-\bar{Z}_p}+\cancel{\bar{Z}_p}+R}{R+\bar{Z}_p}\right) I_2$$

$$\boxed{\bar{Z}_{22}} = \frac{\dot V_2}{\dot I_2} = \left(\frac{\bar{Z}_p \cdot R}{R+\bar{Z}_p}\right) = \boxed{0.5 + j\,0.0025\ \Omega}$$

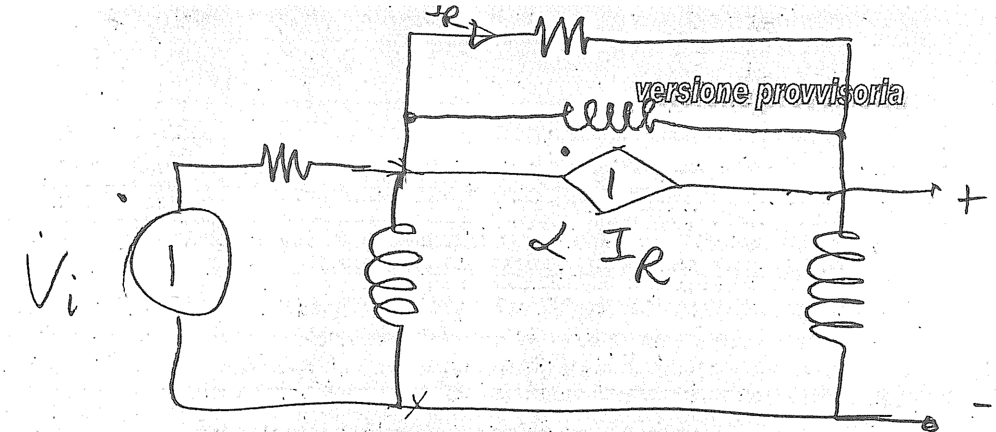
$$\boxed{\bar{Z}_{12}} = \frac{\dot V_1}{\dot I_2} = \boxed{0} \qquad (\dot V_1 = j\omega L \cdot \dot I_L = 0)$$

$$\bar{Z} = \begin{bmatrix} 2 + j0.005 & 0 \\ 2.5\cdot 10^{-5} - j0.0025 & 0.5 + j0.0025 \end{bmatrix}$$

$$\bar{S} = \dot V_1 \cdot \dot I^* \; ; \; \dot I = 2\ A$$

$$\begin{cases} \dot V_1 = \bar{Z}_{11}\dot I_1 + 0\,\dot I_2 \\ \dot V_2 = \bar{Z}_{21}\dot I_1 + \bar{Z}_{22}\dot I_2 \\ \dot I_1 = \dot I \end{cases} \Rightarrow \quad \begin{aligned} &\dot V_1 = \bar{Z}_{11}\dot I = 4 + j0.01\ V \\ &\bar{S} = \dot V_1 \cdot \dot I^* = 8 + j\,0.02\ VA = P + jQ \end{aligned}$$

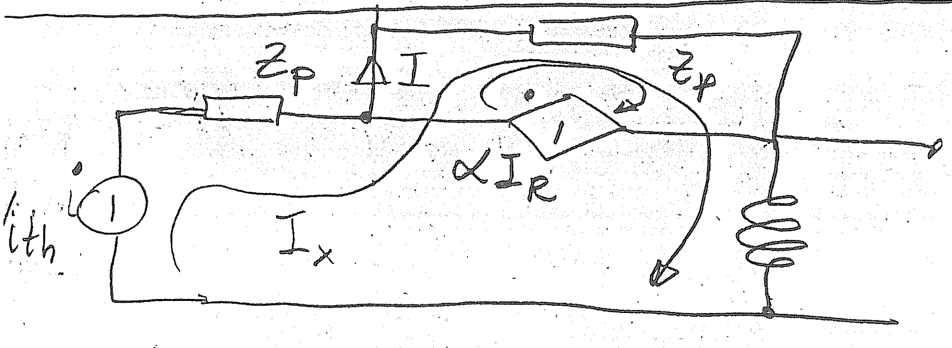
$\boxed{P = 8\ W \quad Q = 0.02\ VAR}$



ds 4
18/7/03

13

$$Z_p = \frac{RLS}{R+LS}$$



$$\dot V_{ith} = \frac{\dot V_i LS}{R+LS} = V_i \frac{Z_p}{R}$$

$$I_R = \frac{I \cdot LS}{R+LS}$$

$$\Downarrow$$

$$I = \frac{R+LS}{LS} I_R = \left(\frac{R}{LS}+1\right) I_R$$

$$\begin{cases} \dot V_{th} = (2Z_p + LS)\, I_x + \alpha I_R \cdot Z_p \\ \dot I = \dot I_x + \alpha \dot I_R \Rightarrow \left(\frac{R}{LS} + 1 - \alpha\right) I_R = I_x \end{cases}$$

$$\dot I_R = \frac{LS\, I_x}{R+(1-\alpha)LS}$$

$$V_{th} = \left[2 Z_p + Ls + \alpha Z_p \frac{LS}{R+(1-\alpha)LS}\right] I_x$$

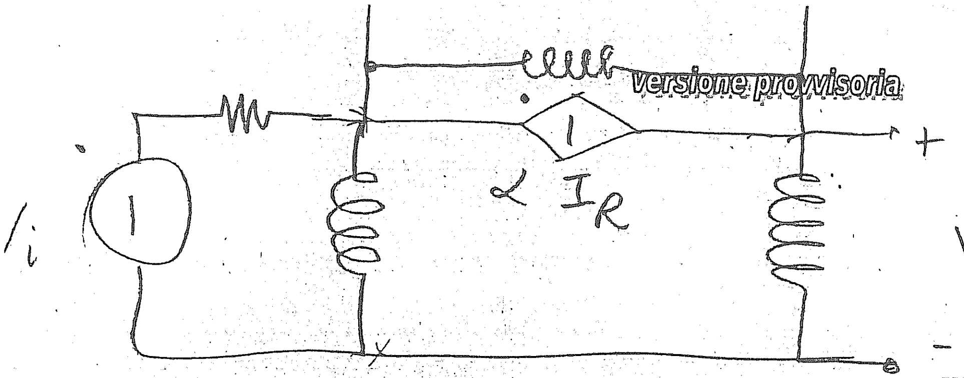
$$\dot I_x = V_{th} \cdot \frac{R+(1-\alpha)LS}{(2Z_p+LS)(R+(1-\alpha)LS)+\alpha Z_p LS} =$$

$$= \frac{V_i \cancel{Z_p}}{R} \cdot \frac{R+(1-\alpha)LS}{2 \cancel{Z_p}(R+(1-\alpha)LS) + \frac{RLS+(1-\alpha)L^2S^2}{Z_p} + \alpha \cancel{Z_p} LS} = \quad 3;$$

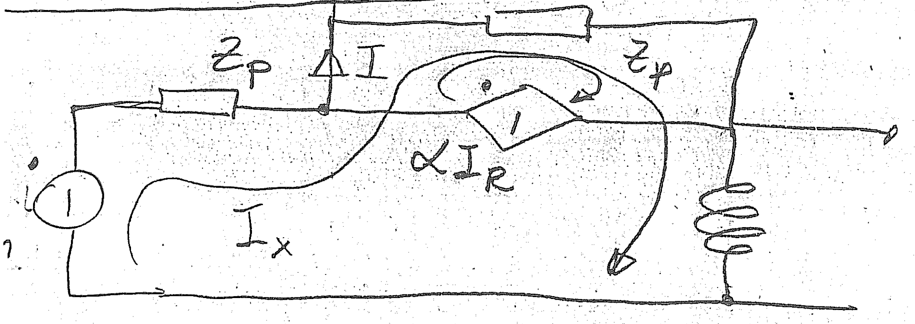
$$= \frac{V_i}{\ldots} \cdot \frac{R+(1-\alpha)LS}{\ldots} \qquad \boxed{V_u = LS\, I_x}$$

OS 4 18/7/03

14



$$Z_p = \frac{RLS}{R+LS}$$



$$\dot V_{th} = \frac{\dot V_i\, LS}{R+LS} = V_i \frac{Z_p}{R}$$

$$I_R = \frac{I \cdot LS}{R+LS}$$

$$\Downarrow$$

$$I = \frac{R+LS}{LS} I_R = \left(\frac{R}{LS}+1\right) I_R$$

$$\dot V_{th} = \left(2Z_p + LS\right) I_x + \alpha I_R \cdot Z_p$$

$$\dot I = \dot I_x + \alpha \dot I_R \Rightarrow \left(\frac{R}{LS} + 1 - \alpha\right) I_R = I_x$$

$$\dot I_R = \frac{LS\, I_x}{R+(1-\alpha)LS}$$

$$V_{th} = \left[2 Z_p + LS + \alpha Z_p \frac{LS}{R+(1-\alpha)LS}\right] I_x$$

$$\dot I_x = V_{th} \cdot \frac{R+(1-\alpha)LS}{(2Z_p+LS)(R+(1-\alpha)LS) + \alpha Z_p LS} =$$

$$= \frac{V_i \cancel{Z_p}}{R} \cdot \frac{R+(1-\alpha)LS}{2\cancel{Z_p}(R+(1-\alpha)LS) + \frac{RLS+(1-\alpha)L^2S^2}{Z_p} + \alpha \cancel{Z_p} LS} =$$

$$= \frac{V_i}{R} \cdot \frac{R+(1-\alpha)LS}{2R(R+(1-\alpha)LS) + (R+(1-\alpha)LS)(R+LS) + \alpha LS}$$

$$\boxed{V_u = LS\, I_x}$$

$$= \frac{V_i\,LS}{R+LS} \cdot \frac{R+(1-\alpha)LS}{\left(2\dfrac{RLS}{R+LS}+LS\right)\left(R+(1-\alpha)LS\right)+\alpha\dfrac{RLS}{R+LS}\cdot LS}$$

$$= V_i\,\frac{\cancel{LS}}{\cancel{R+LS}}\;\frac{R+(1-\alpha)LS}{\cancel{LS}\left[\dfrac{2R+R+LS}{R+\cancel{LS}}\right]\left(R+(1-\alpha)LS\right)+\dfrac{\alpha RLS}{R+\cancel{LS}}}$$

$$I_x = V_i\,\frac{R+(1-\alpha)LS}{(3R+LS)\left(R+(1-\alpha)LS\right)+\alpha RLS}$$

$$I_x = V_i\,\frac{R+(1-\alpha)LS}{3R^2+3R(1-\alpha)LS+RLS+(1-\alpha)L^2S^2+\alpha RLS}$$

$$I_x = V_i\,\frac{R+(1-\alpha)LS}{(1-\alpha)L^2S^2+\left[3R-3\alpha R+R+\alpha R\right]LS+3R^2}$$

$$I_x = V_i\,\frac{R+(1-\alpha)LS}{(1-\alpha)L^2S^2+2R(2-\alpha)LS+3R^2}$$

$$V_u = LS\,I_x = V_i\,\frac{LS\left[R+(1-\alpha)LS\right]}{(1-\alpha)L^2S^2+2R(2-\alpha)LS+3R^2}$$

$$= \frac{V_i LS}{R+LS} \cdot \frac{R+(1-\alpha)LS}{\left(2\frac{RLS}{R+LS}+LS\right)\left(R+(1-\alpha)LS\right)+\alpha\frac{RLS}{R+LS}\cdot LS}$$

$$= V_i \frac{\cancel{LS}}{\cancel{R+LS}} \frac{R+(1-\alpha)LS}{\cancel{LS}\left[\frac{2R+R+LS}{\cancel{R+LS}}\right]\left(R+(1-\alpha)LS\right)+\frac{\alpha RLS}{\cancel{R+LS}}}$$

$$= V_i \frac{R+(1-\alpha)LS}{(3R+LS)\left(R+(1-\alpha)LS\right)+\alpha RLS}$$

$$= V_i \frac{R+(1-\alpha)LS}{3R^2+3R(1-\alpha)LS+RLS+(1-\alpha)L^2S^2+\alpha RLS}$$

$$= V_i \frac{R+(1-\alpha)LS}{(1-\alpha)L^2S^2+\left[3R-3\alpha R+R+\alpha R\right]LS+3R^2}$$

$$= V_i \frac{R+(1-\alpha)LS}{(1-\alpha)L^2S^2+2R(2-\alpha)LS+3R^2}$$

$$V_u = LS\, I_x = V_i \frac{LS\left[R+(1-\alpha)LS\right]}{(1-\alpha)L^2S^2+2R(2-\alpha)LS+3R^2}$$

18/7/03 17 05 $\cancel{4}$ 3

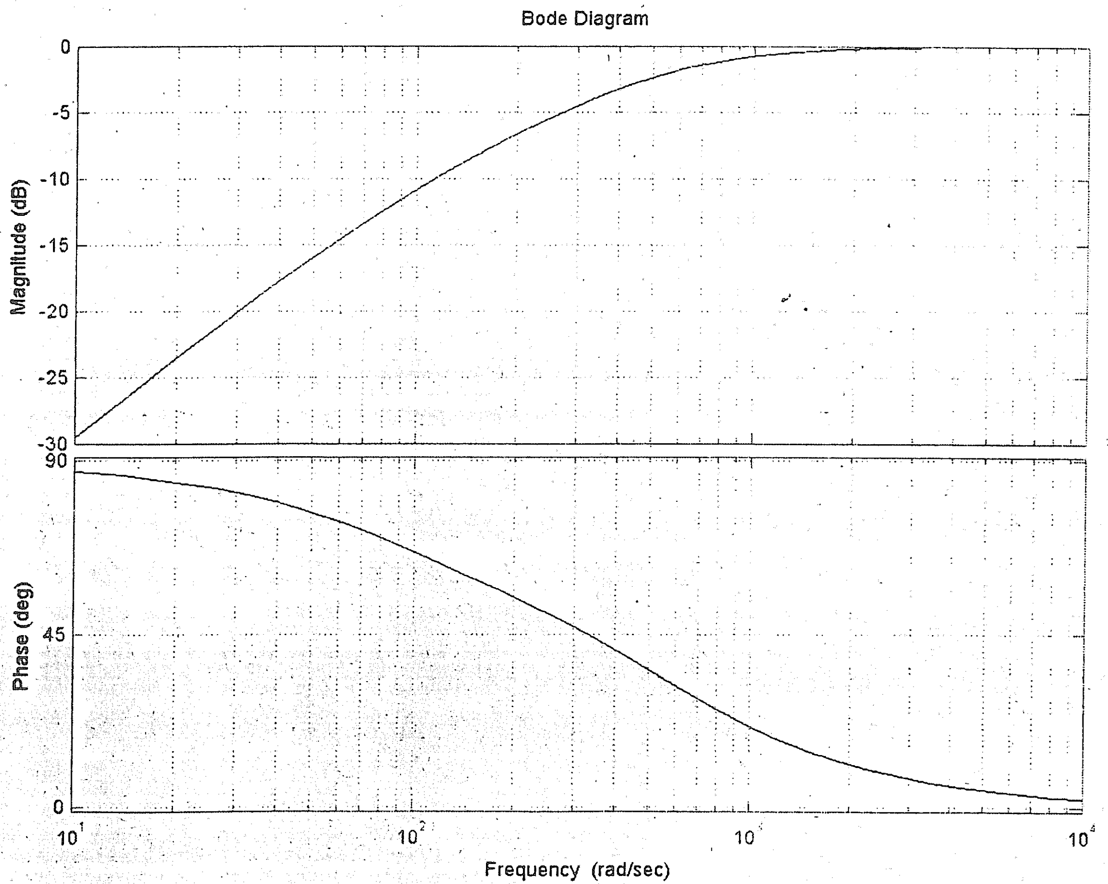
$$W = \frac{LR}{3R^2} \cdot \frac{j\omega \left[\frac{[illegible]}{\;\cdots R\;}\right]}{-\left[\frac{(1-\alpha)}{3R^2}\right]\omega^2 L^2 + \frac{2R(2-\alpha)}{3R^2}\, j\omega L + 1}$$

zeri : $j\omega_{z1} = 0 \;;\; j\omega_{z2} = -200$

poli : $j\omega_{p1} = -473.2 \;;\; j\omega_{p2} = -126.8$

$K = 0.0033$

$$W = 0.0033 \, \frac{j\omega \cdot \left[\frac{j\omega}{200} + 1\right]}{\left[\frac{j\omega}{473.2} + 1\right]\left[\frac{j\omega}{126.8} + 1\right]}$$



$$W = \frac{LR}{3R^2} \cdot \frac{j\omega \left[\frac{j\omega L(1-\alpha)}{\ldots} + 1 \right]}{-\left[\frac{(1-\alpha)}{3R^2}\right]\omega^2 L^2 + \frac{2R(2-\alpha)}{3R^2} j\omega L + 1}$$

versione provvisoria

18/7/03

8s C 3

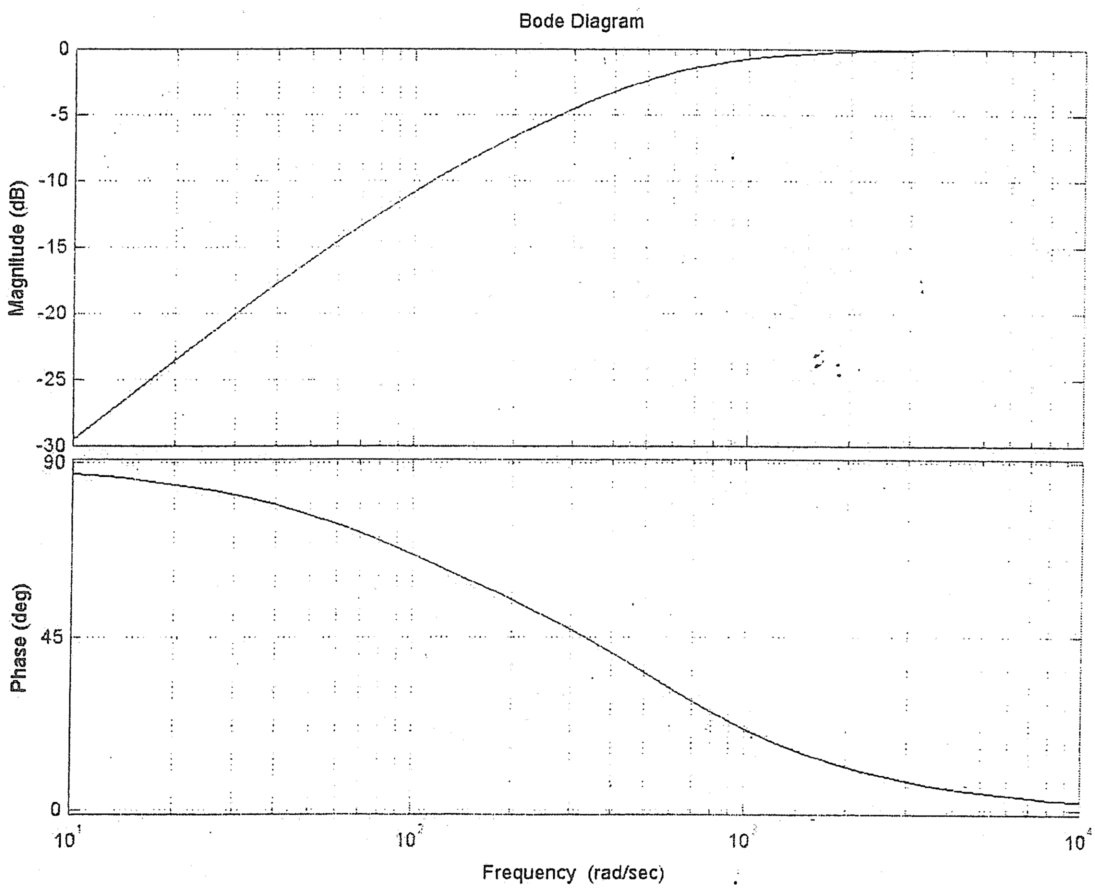
18

zeri : $j\omega_{z1} = 0$; $j\omega_{z2} = -200$

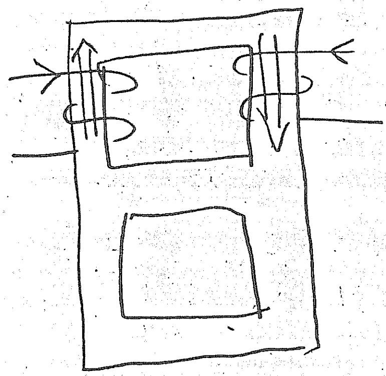
poli : $j\omega_{p1} = -473.2$; $j\omega_{p2} = -126.8$

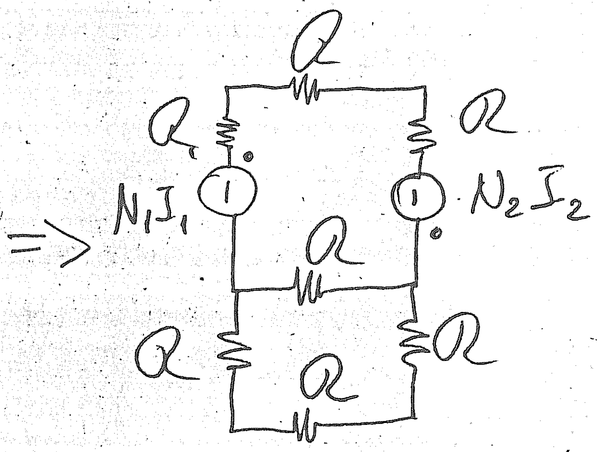
$K = 0.0033$

$$W = 0.0033 \, \frac{j\omega \cdot \left[\frac{j\omega}{200} + 1\right]}{\left[\frac{j\omega}{473.2} + 1\right]\left[\frac{j\omega}{126.8} + 1\right]}$$



ES 5
18/7/03





$$\mathcal{R} = \frac{l}{\mu_0 \mu_R \cdot S} = 2.65 \cdot 10^5 \ H^{-1}$$

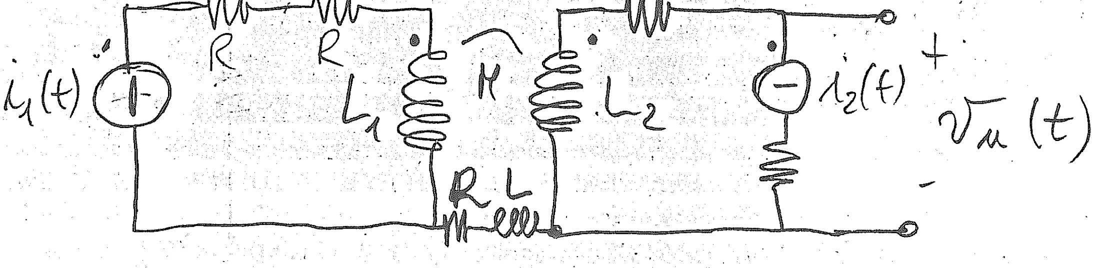
$$L_1 = \left.\frac{\Phi_{1c}}{I_1}\right|_{I_2=0} = \frac{N_1^2}{\mathcal{R}_V} = 0.161 \ H \quad \text{con } \mathcal{R}_V = 3\mathcal{R} + 3\mathcal{R} // \mathcal{R} =$$

$$= 3\mathcal{R} + \frac{3\mathcal{R}\cdot\mathcal{R}}{4\mathcal{R}} = \frac{15}{4}\mathcal{R}$$

$$L_2 = \left.\frac{\Phi_{2c}}{I_2}\right|_{I_1=0} = \frac{N_2^2}{\mathcal{R}_V} = 0.0905 \ H$$

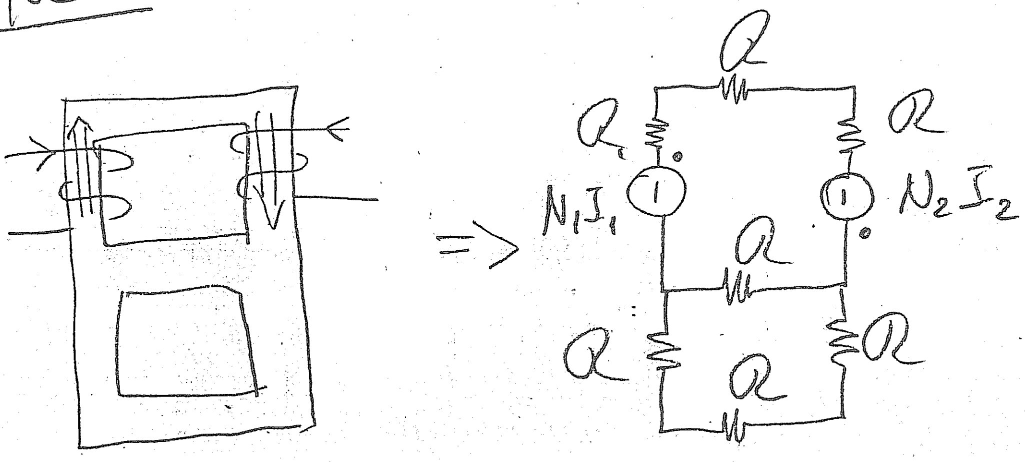
$$M = \left.\frac{\Phi_{1-2c}}{I_1}\right|_{I_2=0} = \frac{N_1 \cdot N_2}{\mathcal{R}_V} = 0.1206 \ H \quad (+)$$

Disegniamo il circuito elettrico:
(sostituiamo il gen. di corrente $i_1(t)$ con il generatore di tensione equivalente)



ES 5
18/7/03

2 1



$$\mathcal{R} = \frac{l}{\mu_0 \mu_r \cdot S} = 2.65 \cdot 10^5 \ H^{-1}$$

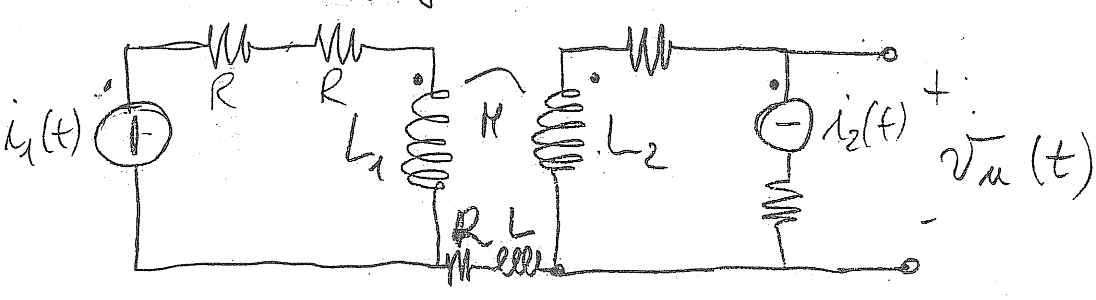
$$L_1 = \left.\frac{\Phi_{1c}}{I_1}\right|_{I_2=0} = \frac{N_1^2}{\mathcal{R}_V} = 0.161 \ H$$

con $\mathcal{R}_V = 3\mathcal{R} + 3\mathcal{R} // \mathcal{R} =$
$= 3\mathcal{R} + \frac{3\mathcal{R}\cdot\mathcal{R}}{4\mathcal{R}} = \frac{15}{4}\mathcal{R}$

$$L_2 = \left.\frac{\Phi_{2c}}{I_2}\right|_{I_1=0} = \frac{N_2^2}{\mathcal{R}_V} = 0.0905 \ H$$

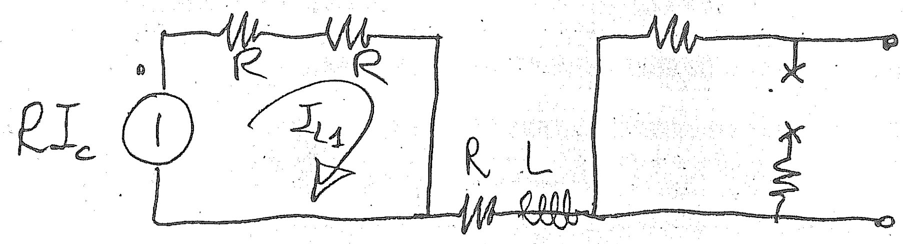
$$M = \left.\frac{\Phi_{1-2c}}{I_1}\right|_{I_2=0} = \frac{N_1 \cdot N_2}{\mathcal{R}_V} = 0.1206 \ H \quad (+)$$

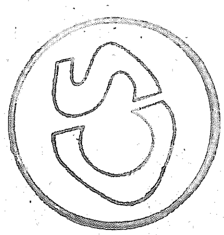
Disegniamo il circuito elettrico:
(sostituiamo il gen. di corrente $i_1(t)$ con il generatore di tensione equivalente)



Applichiamo sovrapp. effetti: ES 5 18/7/03 ~~12~~ 21

Agisce la componente in continua di $i_1(t)$:

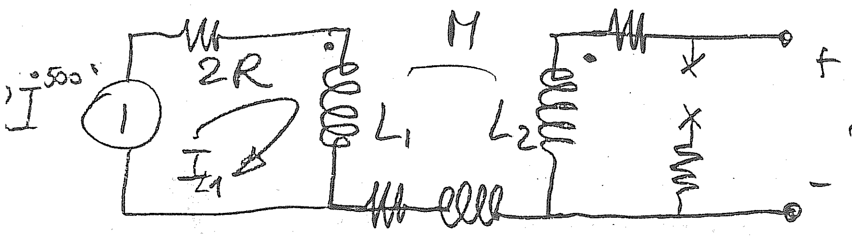




$I_{L1}^c = \frac{RI_c}{2R} = \frac{5}{2} = 2.5\ A$; $\underline{\underline{v_u^c = 0}}$; $I_{L2}^c = 0$

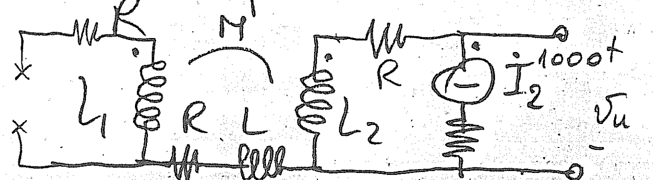
Agisce comp. sinusoidale a pulsazione $500\ \frac{rad}{s}$

$\dot{I}^{500} = 10\ A$



$\dot{I}_{L1}^{500} = \frac{R\dot{I}^{500}}{2R+j\omega L_1} = 0.3 - j1.17\ A$; $I_{L2}^{500} = 0$; $\dot{v}_u^{500} = j\omega M \dot{I}_{L1}^{500} = 70.6 + j17.5\ V$

Agisce comp. sinusoidale a pulsazione $1000\ \frac{rad}{s}$



$\dot{I}_2^{1000} = 5 \cdot e^{j\left(\frac{\pi}{3}+\frac{\pi}{2}\right)} = -4 + j2.5\ A$

$\dot{I}_{L1}^{1000} = 0$; $\dot{I}_{L2}^{1000} = \dot{I}_2^{1000} = -4 + j2.5\ A$

$\dot{v}_u^{1000} = (R + j\omega L_2)\dot{I}_2^{1000} \cong -270 - j366.8\ V$

$v_u(t) = |\dot{v}_u^{500}| \cdot \sqrt{2} \cdot \sin(500t + \varphi_{500}) + |\dot{v}_u^{1000}| \sqrt{2} \sin(1000t + \varphi_{1000}) \Rightarrow$

$$\boxed{v_u(t) = 72.8 \cdot \sqrt{2} \sin(500t + 0.24) + 455.2 \cdot \sqrt{2} \sin(1000t - 2.2)\ V}$$

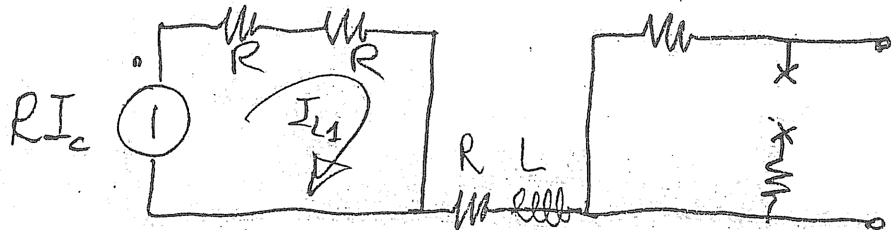
$$\boxed{\bar{W} = \frac{1}{2} L_1 \cdot I^{c^2} + \frac{1}{2} L_1 (I_{L1e}^{500})^2 + \frac{1}{2} L_2 (I_{L2e}^{1000})^2 = 1.75\ \text{Joule}}$$

Applichiamo sovrapp. effetti :

Es 5
18/7/03

~~2~~ 22

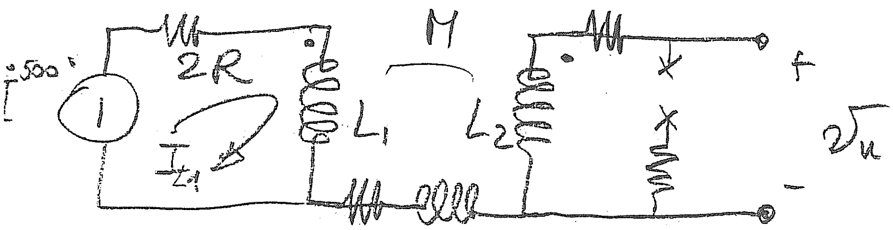
Agisce la componente in continua di $i_1(t)$:



$$I_{L1}^{c} = \frac{RI_c}{2R} = \frac{5}{2} = 2.5\ A; \quad \underline{\underline{v_u^{c} = 0}}; \quad I_{L_2}^{c} = 0$$

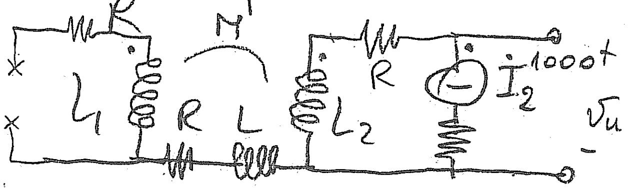
Agisce comp. sinusoidale a pulsazione 500 $\frac{rad}{s}$

$\dot{I}^{500} = 10\ A$



$$\dot{I}_{L_1}^{500} = \frac{R\dot{I}^{500}}{2R+j\omega L_1} = 0.3 - j1.17\ A; \quad \dot{I}_{L_2}^{500} = 0; \quad \dot{V}_u^{500} = j\omega M \dot{I}_{L_1}^{500} = 70.6 + j17.5\ V$$

Agisce comp. sinusoidale a pulsazione 1000 $\frac{rad}{s}$



$$\dot{I}_2^{1000} = 5 \cdot e^{j\left(\frac{\pi}{3}+\frac{\pi}{2}\right)} = -4 + j2.5\ A$$

$$\dot{I}_{L_1}^{1000} = 0; \quad \dot{I}_{L_2}^{1000} = \dot{I}_2^{1000} = -4 + j2.5\ A$$

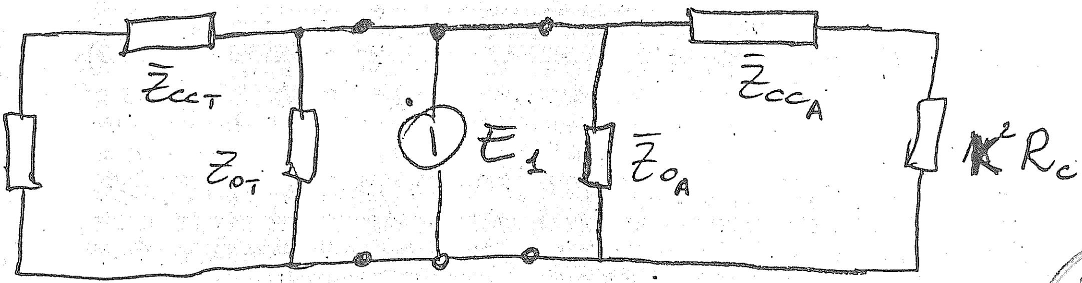
$$\dot{V}_u^{1000} = (R + j\omega L_2)\,\dot{I}_2^{1000} \cong -270 - j366.8\ V$$

$$v_u(t) = |\dot{V}_u^{500}| \cdot \sqrt{2} \cdot \sin(500t + \varphi_{500}) + |\dot{V}_u^{1000}| \sqrt{2} \sin(1000t + \varphi_{1000}) \Rightarrow$$

$$\boxed{v_u(t) = 72.8 \cdot \sqrt{2} \sin(500t + 0.24) + 455.2 \cdot \sqrt{2} \sin(1000t - 2.2)\ V}$$

$$\boxed{W = \frac{1}{2} L_1 \cdot I^{c^2} + \frac{1}{2} L_1 \left(I_{L_1}^{500}\right)^2 + \frac{1}{2} L_2 \left(I_{L_2}^{1000}\right)^2 = 1.75\ \text{Joule}}$$

Monofase equivalente:



Parametri Trasformatore

Dalla prova a vuoto:

$R_0 = \dfrac{V_{10}^2}{P_{10}} = 96.26\ \Omega$; $G_0 = \dfrac{1}{R_0} = 0.0104\ \Omega$;

$Y_0 = \dfrac{\sqrt{3}\, I_{10}}{V_{10}} = 0.0547\ \Omega$; $X_0 = \dfrac{1}{\sqrt{Y_0^2 - G_0^2}} = 18.62\ \Omega$;

$$\boxed{Z_0 = \frac{R_0 \cdot jX_0}{R_0 + jX_0} = 3.47 + j\,17.95\ \Omega}$$

Dalla prova in c.c.

$|Z_{cc}| = \dfrac{V_{1cc}}{\sqrt{3}\, I_{1cc}} = 3.46\ \Omega$; $\cos\varphi_{cc} = \dfrac{P_{1cc}}{\sqrt{3}\, V_{1cc}\, I_{1cc}} = 0.6543$;

$$\bar{Z}_{cc} = |Z_{cc}| \cdot [\cos\varphi_{cc} + j\sin\varphi_{cc}] = \boxed{2.26 + j\,2.62\ \Omega}$$

Parametri macchina asincrona

(stesse formule di prima): $R_0 = 128\ \Omega$

$\bar{Z}_0 = 1.85 + j\,15.28\ \Omega$

$\bar{Z}_{cc} = 1.02 + j\,3.13\ \Omega$

$\bar{Z}_{cc} = R_{cc} + jX_{cc}$; $R_{cc} = R_{1s} + K^2 R_{2r} \Rightarrow R_{2r} = \dfrac{R_{cc} - R_{1s}}{K^2}$

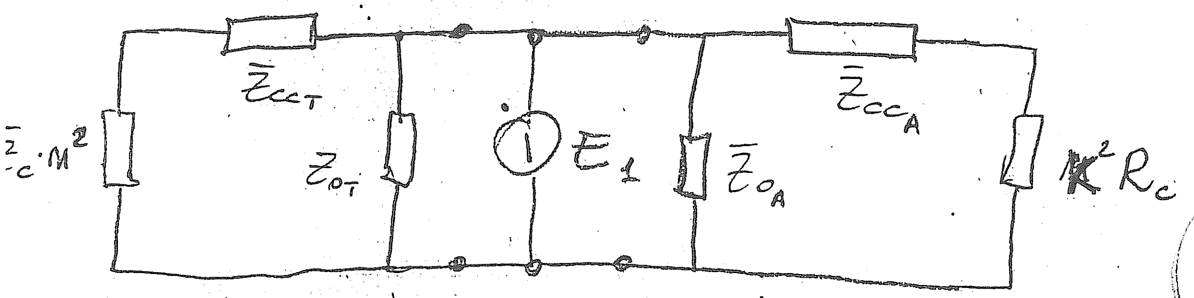
$$\boxed{R_c = R_{2r} \cdot \frac{1-s}{s} \cong 0.7\ \Omega}$$

Es. 6 del 18/7/03

versione provvisoria

(1) (24)

, mofase equivalente:



rametri Trasformatore

lla prova a vuoto:

$_0 = \frac{V_{10}^2}{P_{10}} = 96.26\ \Omega$; $G_0 = \frac{1}{R_0} = 0.0104\ \Omega$

$= \frac{\sqrt{3}\, I_{10}}{V_{10}} = 0.0547\ \Omega$; $X_0 = \frac{1}{\sqrt{Y_0^2 - G_0^2}} = 18.62\ \Omega$;

$_0 = \frac{R_0 \cdot jX_0}{R_0 + jX_0} = 3.47 + j\,17.95\ \Omega$

lla prova in c.c.

$| = \frac{V_{1cc}}{\sqrt{3}\, I_{1cc}} = 3.46\ \Omega$; $\cos\varphi_{cc} = \frac{P_{1cc}}{\sqrt{3}\, V_{1cc}\, I_{1cc}} = 0.6543$;

$= |Z_{cc}| \cdot [\cos\varphi_{cc} + j\sin\varphi_{cc}] = \boxed{2.26 + j\,2.62\ \Omega}$

metri macchina asincrona

se formule di prima): $R_0 = 128\ \Omega$

$= 1.85 + j\,15.28\ \Omega$

$= 1.02 + j\,3.13\ \Omega$

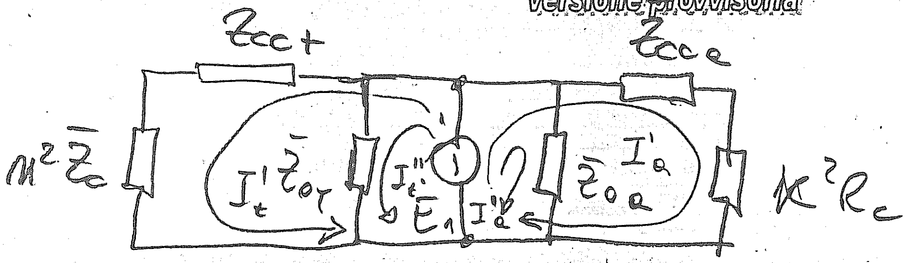
$= R_{cc} + jX_{cc}$; $R_{cc} = R_{1S} + K^2 R_{2R} \Rightarrow R_{2R} = \frac{R_{cc} - R_{1S}}{K^2}$

$= R_{2R} \cdot \frac{1-s}{s} \simeq 0.1\ \Omega$

Es 6 ~~2~~

18/7/03

25



$$\dot E_1 = 240\ V$$

$$I'_t = \frac{\dot E_1}{\bar Z_{cct} + m^2 \bar Z_c} = 20.7 - j\,12.45\ A$$

$$I''_t = \frac{\dot E_1}{\bar Z_{0T}} = 2.5 - j\,12.8\ A$$

3

$$I'_a = \frac{\dot E_1}{\bar Z_{cca} + k^2 R_c} = 25.42 - j\,66.82\ A$$

$$I''_a = \frac{\dot E_1}{\bar Z_{0a}} = 1.87 - j\,15.47\ A$$

$$\dot I_g = I'_t + I''_t + I'_a + I''_a = 50.5 - j\,107.6\ A$$

$$\bar S_g = 3 \cdot \dot E_1 \cdot \dot I_g^* = 36.36 + j\,77.5\ KVA$$

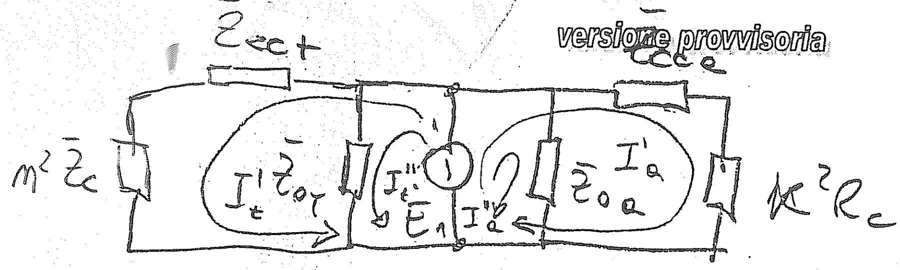
$$\boxed{P = 36.36\ KW;\quad Q = 77.5\ KVAR}$$

$$P_{fe_t} = 3 \cdot \frac{E_1^2}{R_{0t}} \cong 1800\ W$$

$$P_{fe_a} = 3 \cdot \frac{E_1^2}{R_{0a}} = 1350\ W$$

6s 6 ~~2~~ 26

18/7/03



$$\dot{E}_1 = 240 \text{ V}$$

$$I'_t = \frac{\dot{E}_1}{\bar{Z}_{cct} + n^2 \bar{Z}_c} = 20.7 - j\,12.45 \text{ A}$$

$$I''_t = \frac{\dot{E}_1}{\bar{Z}_{0T}} = 2.5 - j\,12.8 \text{ A}$$

$$I'_a = \frac{\dot{E}_1}{\bar{Z}_{cca} + K^2 R_c} = 25.42 - j\,66.82 \text{ A}$$

$$I''_a = \frac{\dot{E}_1}{\bar{Z}_{0a}} = 1.87 - j\,15.47 \text{ A}$$

$$\dot{I}_g = I'_t + I''_t + I'_a + I''_a = 50.5 - j\,107.6 \text{ A}$$

$$\bar{S}_g = 3 \cdot \dot{E}_1 \cdot \dot{I}_g^* = 36.36 + j\,77.5 \text{ KVA}$$

$$\boxed{P = 3\cancel{6}.36 \text{ KW}; \quad Q = 77.5 \text{ KVAR}}$$

$$P_{fe_t} = 3 \cdot \frac{E_1^2}{R_{0t}} \cong 1800 \text{ W}$$

$$P_{fe_a} = 3 \cdot \frac{E_1^2}{R_{0a}} = 1350 \text{ W}$$