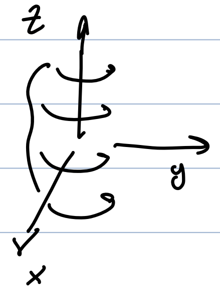
SUPERFICIE DI ROTAZIONE $\gamma = \begin{pmatrix} \gamma_1(t) \\ 0 \\ \gamma_3(t) \end{pmatrix}$ $\gamma_1 > 0$ $t \in [0,b]$



$$\phi(u,v) = \begin{cases} x = \gamma_1(u)\cos(v) \\ y = \gamma_1(u)\sin(v) \\ z = \gamma_3(u) \end{cases} \qquad u \in [0,b] \quad v \in [0,2\pi]$$

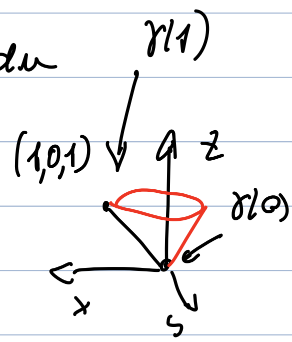
$$\left\| \frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v} \right\| = \gamma_1(u)\sqrt{[\gamma_1'(u)]^2 + [\gamma_3'(u)]^2}$$

$$A(\Sigma) = \int_0^{2\pi}\int_0^b du\, dv\; \gamma_1(u)\sqrt{[\gamma_1'(u)]^2 + [\gamma_3'(u)]^2}$$

$$A(\Sigma) = 2\pi\int_0^b \gamma_1(u)\sqrt{[\gamma_1'(u)]^2 + [\gamma_3'(u)]^2}\, du$$

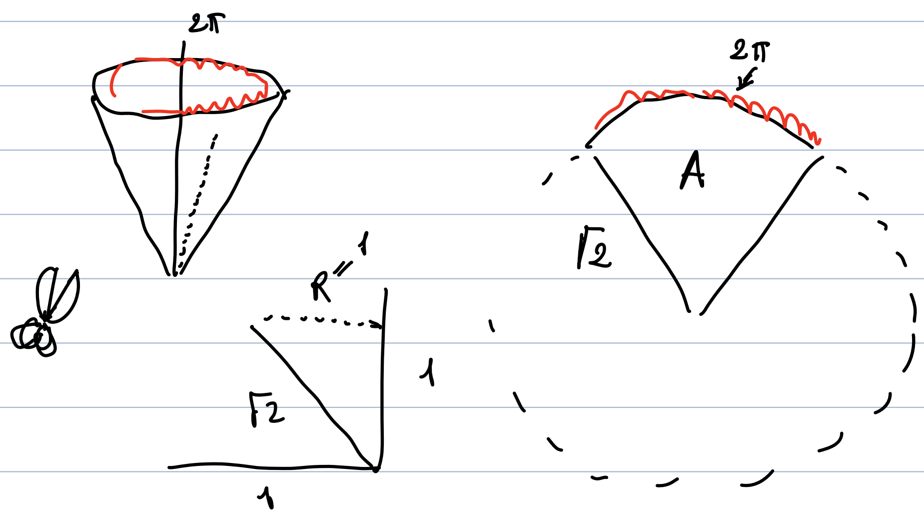
ES $\gamma_1(t) = t$ $t \in [0,1]$

$\gamma_3(t) = t$



$$A(\Sigma) = 2\pi\int_0^1 u\sqrt{1^2 + 1^2}\, du$$

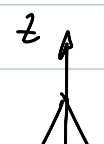
$$= 2\sqrt{2}\pi\int_0^1 u\, du = 2\sqrt{2}\pi \left.\frac{u^2}{2}\right|_0^1 = \sqrt{2}\pi$$



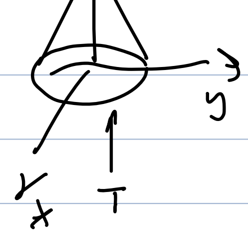
$2R\pi = 2\pi$
lunghezza della circonferenza di base del cono

$$\frac{A}{(\sqrt{2})^2\pi} = \frac{A}{A_{TOT}} = \frac{2\pi}{\ell(circ)} = \frac{\cancel{2\pi}}{2\sqrt{2}\cancel{\pi}}$$

$$\frac{A}{2\pi} = \frac{1}{\sqrt{2}} \qquad A = \sqrt{2}\pi$$



$f = 1 - \sqrt{x^2+y^2}$



$\sigma = \begin{pmatrix} x \\ y \\ f(x,y) \end{pmatrix}$

$\sigma(u,v) = u\,i + v\,j + f(u,v)\,k \qquad (u,v) \in T$

$A = \iint_T \sqrt{1+(f_u)^2+(f_v)^2}\, du\, dv \qquad \frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v} = -f_u i - f_v j + k$

$A = \iint_{B(0,1)} \sqrt{1 + \left(-\frac{u}{\sqrt{u^2+v^2}}\right)^2 + \left(\frac{-v}{\sqrt{u^2+v^2}}\right)^2}\, du\, dv$

$A = \iint_{B(0,1)} \sqrt{1 + \frac{u^2}{u^2+v^2} + \frac{v^2}{u^2+v^2}} = \iint_{B(0,1)} \sqrt{\frac{u^2+v^2+u^2+v^2}{u^2+v^2}} =$

$\iint_{B(0,1)} \sqrt{2}\, du\, dv = \pi \cdot 1^2 \cdot \sqrt{2}$

$V = \iint_{B(0,1)} \int_0^{1-\sqrt{x^2+y^2}} 1\, dz \qquad \text{Cono} = \left\{ (x,y,z) : \begin{array}{l} (x,y) \in B(0,1) \\ 0 \le z \le 1-\sqrt{x^2+y^2} \end{array} \right\}$

$= \iint_{B(0,1)} 1 - \sqrt{x^2+y^2}\, dx\, dy = \int_0^{2\pi} d\theta \int_0^1 \rho\, d\rho\, (1-\rho)$

$= 2\pi \int_0^1 \rho - \rho^2\, d\rho = 2\pi \left(\frac{\rho^2}{2} - \frac{\rho^3}{3} \Big|_0^1 \right)$

$= 2\pi \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{2\pi}{6} = \frac{\pi}{3}$

---

(1) $\sigma(u,v) = \sigma_1(u,v)\, i + \sigma_2(u,v)\, j + \sigma_3(u,v)\, k$

(2) $\phi(x,y) = x\, i + y\, j + f(x,y)\, k$ CARTESIANA

(3) $F = 0 \qquad x^2+y^2+z^2-1=0$ IMPLICITA

$\frac{\partial F}{\partial z} \neq 0 \qquad \exists\, f \quad : \quad z = f(x,y)$

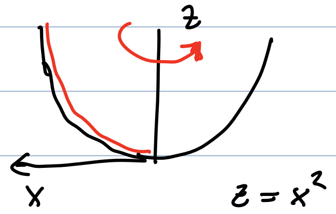
$$F(x,y,z)=0 \quad \Rightarrow \quad F(x,y,f(x,y))=0$$

$$\frac{\partial f}{\partial x} = -\frac{F_x}{F_z} \qquad \frac{\partial f}{\partial y} = -\frac{F_y}{F_z}$$

$$\sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2} = \sqrt{1+\frac{(F_x)^2}{(F_z)^2}+\frac{(F_y)^2}{(F_z)^2}} =$$

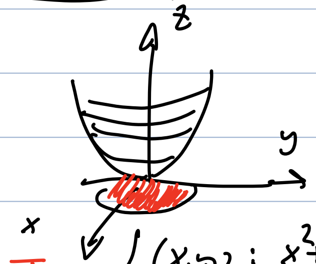
$$= \sqrt{\frac{(F_z)^2+(F_x)^2+(F_y)^2}{(F_z)^2}} = \frac{|\nabla F|}{|F_z|}$$

$z = x^2+y^2$



$\{y=0\}$

$z=x^2$



$T = \{(x,y) : x^2+y^2 \le 1\}$

$t\in[0,1]$ $\quad x=\sqrt{z}$

$$\gamma(t) = \begin{pmatrix} \sqrt{t} \\ 0 \\ t \end{pmatrix}$$

$$\sigma(u,v) = \sqrt{u}\cos(v)\, i + \sqrt{u}\sin(v)\, j + u\, k$$

$$A = 2\pi \int_0^1 \sqrt{u}\sqrt{1^2+\left(\frac{1}{2\sqrt{u}}\right)^2}\, du$$

$$= 2\pi \int_0^1 \frac{\sqrt{4u}}{\sqrt{4}}\sqrt{1+\frac{1}{4u}}\, du = \frac{\cancel{2}\pi}{\cancel{2}}\int_0^1 \sqrt{4u+\frac{\cancel{4u}}{\cancel{4u}}}\, du$$

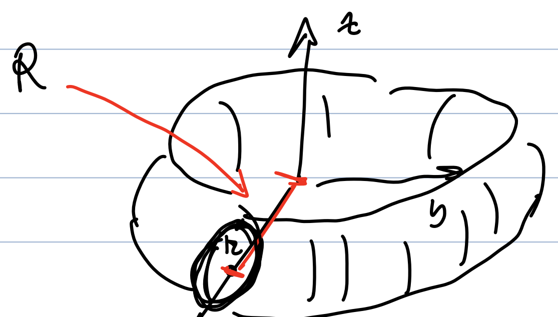
$$= \pi \int_0^1 \sqrt{1+4u}\, du = \frac{\pi}{4}\,\frac{1}{1+\frac{1}{2}}\,(1+4u)^{1+\frac{1}{2}}\Big|_0^1$$

$$= \frac{\pi}{2\cancel{4}}\,\frac{\cancel{2}}{3}\,(1+4u)^{3/2}\Big|_0^1$$

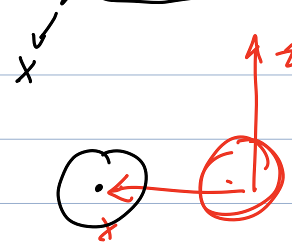
$$= \frac{\pi}{6}\left(5^{3/2}-1\right)$$

$t\in[0,2\pi]=[a,b]$

SUPERFICIE TORO



$$\begin{cases} \gamma_1(t) = r\cos(t) + R \\ \gamma_2(t) = 0 \\ \gamma_3(t) = r\sin(t) \end{cases}$$



$$\sigma(u,v) = (r\cos(u)+R)\cos(v)\, i + (r\cos(u)+R)\sin(v)\, j + r\sin(u)\, k$$

$$A = 2\pi\int_0^{2\pi} \gamma_1(u)\sqrt{[\gamma_1'(u)]^2 + [\gamma_2'(u)]^2}\, du$$

$$= 2\pi\int_0^{2\pi} (r\cos(u)+R)\sqrt{(-r\sin(u))^2 + (r\cos(u))^2}\, du$$

$$= 2\pi\int_0^{2\pi} (r\cos(u)+R)\sqrt{r^2(\cos^2(u)+\sin^2(u))}\, du$$

$$= 2\pi\int_0^{2\pi} (r\cos(u)+R)\, r\, du$$

$$= 2\pi\Big(\underbrace{\int_0^{2\pi} r^2\cos(u)\, du}_{=0} + \int_0^{2\pi} rR\, du\Big)$$

$$= 2\pi\, 2\pi r R = (2\pi r)(2\pi R)$$

$$\mathrm{Vol}(\text{Toro}) = \pi r^2\, 2\pi R = 2\pi^2 r^2 R$$

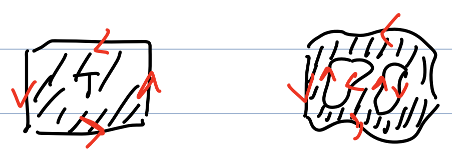
BORDO DI UNA SUPERFICIE

Σ PORZIONE DI SUPERFICIE REGOLARE

$\phi : T \to \mathbb{R}^3$, $\phi \in C^1$, ϕ iniettiva $\mathring{T}$ rank $J\phi = 2$

T LIMITATO, CHIUSO $T = \overline{\mathring{T}}$, MISURABILE

∂T È UNIONE DI UN NUMERO FINITO DI CURVE REGOLARI



$\phi(\partial T) \subseteq \Sigma$ BORDO DI Σ

∂T È ORIENTATO IN MODO CHE T STIA ALLA NOSTRA

SINISTRA ($\partial^+ T$)

BORDO È ORIENTATO POSITIVAMENTE $\quad \phi(\partial^+ T) = \partial^+ \Sigma$

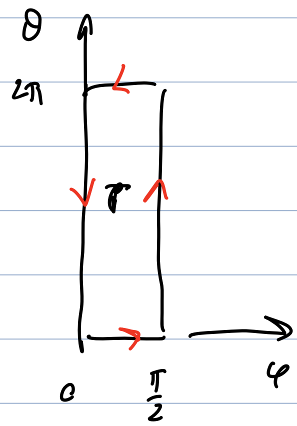
$\partial T \qquad \partial \Sigma$

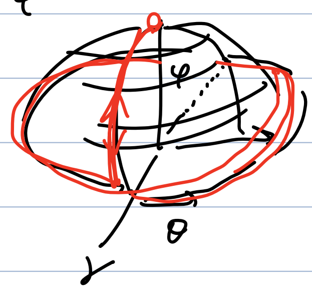
ϕ È BIUNIVOCA $\Rightarrow$ $\partial \Sigma = \phi(\partial T)$

ϕ NON È BIUNIVOCA $\Rightarrow$ $\partial \Sigma \subseteq \phi(\partial T)$

BORDO DELLA SEMISFERA È LA CIRCONF. EQUATORIALE

$$\begin{cases} x = R \sin\varphi \sin\theta \\ y = R \sin\varphi \cos\theta \\ z = R \cos\varphi \end{cases} \qquad \begin{matrix} \varphi \in [0, \pi/2) \\ \theta \in [0, 2\pi] \end{matrix}$$





$\downarrow$

$\varphi \in [0, \pi]$

BORDO DELLA SFERA È VUOTO

CURVE		SUPERFICI
$\gamma : [a,b] \to \mathbb{R}^3$, $[a,b] \subseteq \mathbb{R}$		$\sigma : T \to \mathbb{R}^3$, $T \subseteq \mathbb{R}^2$
$f : \gamma([a,b]) \to \mathbb{R}$		$f : \sigma(T) \to \mathbb{R}$
$\int_a^b f(\gamma(t)) \lVert \gamma'(t) \rVert \, dt$	I SPECIE	$\int_\Sigma f \, dS = \int_T f(\phi(u,v)) \left\lVert \frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v} \right\rVert du\, dv$
$f = 1 \quad \int_a^b \lVert \gamma'(t) \rVert = \ell(\gamma)$		$f = 1 \quad \int_\Sigma 1 \, dS = A(\Sigma)$
$\vec{f} : \gamma[a,b] \to \mathbb{R}^3$	II SPECIE	$\vec{f} : \Sigma \to \mathbb{R}^3$
$\int_\gamma \vec{f} \cdot dx = \sum \int_a^b f_i(\gamma(t)) \gamma_i'(t) \, dt$		$\int_\Sigma \vec{f} \cdot \hat{n} \, dS = \int_T \vec{f}(\phi(u,v)) \cdot \left(\frac{\partial \phi}{\partial u} \wedge \frac{\partial \sigma}{\partial v} \right) dS$
		($\hat{n}$ versore normale

LAVORO DI UNA FORZA

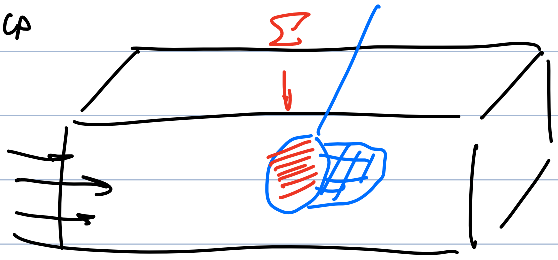
$$\hat{n} = \frac{\frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v}}{\left\| \frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v} \right\|}$$

$$\int_T \vec{f}(\phi(u,v)) \cdot \underbrace{\left(\frac{\frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v}}{\left\| \frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v} \right\|} \right)}_{\hat{n}} \cdot \underbrace{\left\| \frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v} \right\| du\, dv}_{= dS}$$

$$\int_T \vec{f} \cdot \hat{n} \, dS$$

FLUSSO DI UNA QUANTITÀ

MECCANICA



$\vec{v}$

$$\phi = \int_\Sigma \vec{v} \, \hat{n} \, dS$$

QUANTITÀ DI ACQUA CHE PASSA DA Σ IN UNA UNITÀ DI TEMPO

ELETTRODINAMICA $\vec{j}$ VETTORE DENSITÀ DI CORRENTE

FLUSSO DI $\vec{j}$ SU Σ È LA CORRENTE CHE PASSA PER UNITÀ DI TEMPO

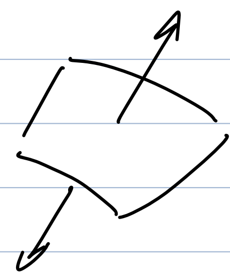
TERMODINAMICA $\vec{J}$ VETTORE FLUSSO DI CALORE

$J = k \nabla T$

FLUSSO DI J È LA QUANTITÀ CALORE CHE PASSA SULLA SUPERFICIE IN UNA UNITÀ DI TEMPO

$$\phi(u,v) = \phi_1(u,v)\, i + \phi_2(u,v)\, j + \phi_3(u,v)\, k$$

$$+ \frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v} \qquad - \frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v}$$

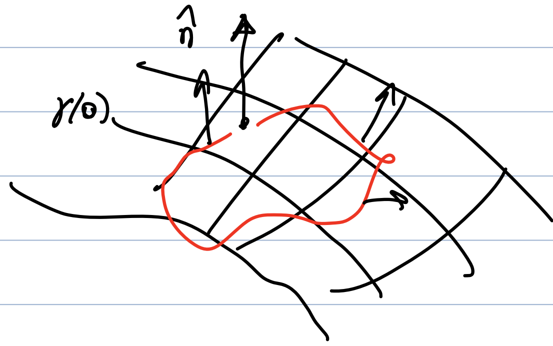


$$\hat{n} = \frac{\frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v}}{\left\| \frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v} \right\|}$$ normale esterna

$$\hat{n} = - \frac{\frac{\partial \phi}{\partial u} \wedge \frac{\partial \phi}{\partial v}}{\| \quad \|}$$ normale interna

Σ ORIENTABILE $\quad \forall\, \gamma : [0,b] \to \Sigma \quad \gamma(0) = \gamma(b)$

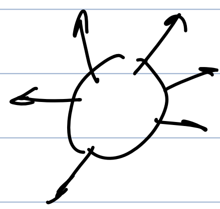
$$\lim_{t \to b} \hat{n}(\gamma(t)) = \hat{n}(\gamma(0))$$



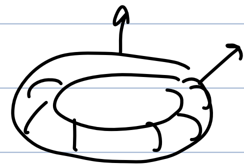
CARTESIANA

$$\phi(u,v) = u\, i + v\, j + f(u,v)\, k$$

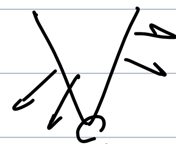
$$\hat{n} = \frac{-f_u\, i - f_v\, j + k}{\sqrt{1 + (f_u)^2 + (f_v)^2}}$$



SFERA



TORO



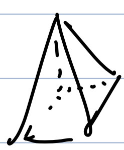
CONO

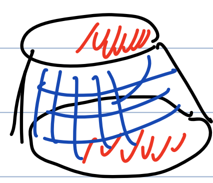
rank $J\phi \neq 2$

DEF. Σ REGOLARE A TRATTI $\quad \exists\, \sigma_1, \dots, \sigma_n$ curve regolari $\sigma_i \subseteq \Sigma$ (SPIGOLI)

CHE DELIMITANO DELLE SUPERFICI Σ_j REGOLARI (FACCE)

$$\Sigma \subseteq \cup\, \Sigma_j$$





$$\int_{\Sigma} \vec{f} \cdot \hat{n}\, dS = \sum_{j=1}^{M} \int_{\Sigma_j} \vec{f} \cdot \hat{n}\, dS$$