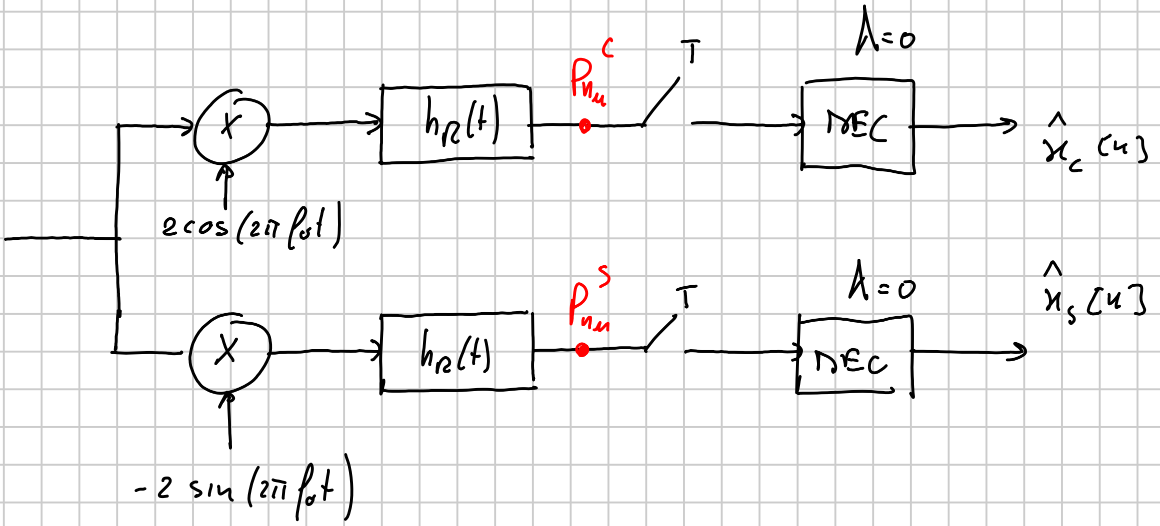
28/05/2014

Es #2 del 3/7/2012



$$s(t) = \sum_k x_c[n]\, p(t-kT)\cos(2\pi f_0 t) - \sum_k x_s[n]\, p(t-kT)\sin(2\pi f_0 t)$$

$x_c[n] \in A_s^c = \{-1, 2\}$ ind. ed equipr.

$x_s[n] \in A_s^s = \{-1, 1\}$

$$p(t) = 2B\,\mathrm{sinc}(2Bt) + B\,\mathrm{sinc}\left[2B\left(t-\frac{1}{2B}\right)\right] + B\,\mathrm{sinc}\left[2B\left(t+\frac{1}{2B}\right)\right]$$

$f_0 \gg B$

$B = \frac{1}{T}$

$c(t) = \delta(t)$, DSP del rumore

$$S_n(f) = \frac{N_0}{2}\left[\mathrm{rect}\left(\frac{f-f_0}{B}\right) + \mathrm{rect}\left(\frac{f+f_0}{B}\right)\right]$$

$h_R(t)$ è un filtro passa-basso ideale di banda B

$\lambda = 0$ sia sul ramo in fase che quadratura

1) Energia media per simbolo trasmesso
2) $P_{n_u}^c$, $P_{n_u}^s$
3) $P_E(b)$

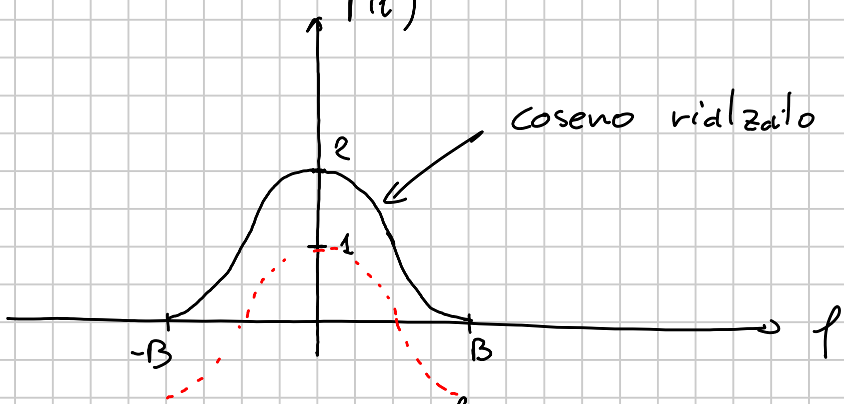
Soluzione

1) $E_s = E\left[\int_{-\infty}^{+\infty} s_n^2(t)\,dt\right] = E\left[\int_{-\infty}^{+\infty}\left[x_c[n]\,p(t-nT)\cos(2\pi f_0 t) + - x_s[n]\,p(t-nT)\sin(2\pi f_0 t)\right]^2 dt\right.$

I) $E\left[\int_{-\infty}^{+\infty} x_c^2[n]\,\underbrace{p^2(t-nT)\cos^2(2\pi f_0 t)}\,dt\right] =$

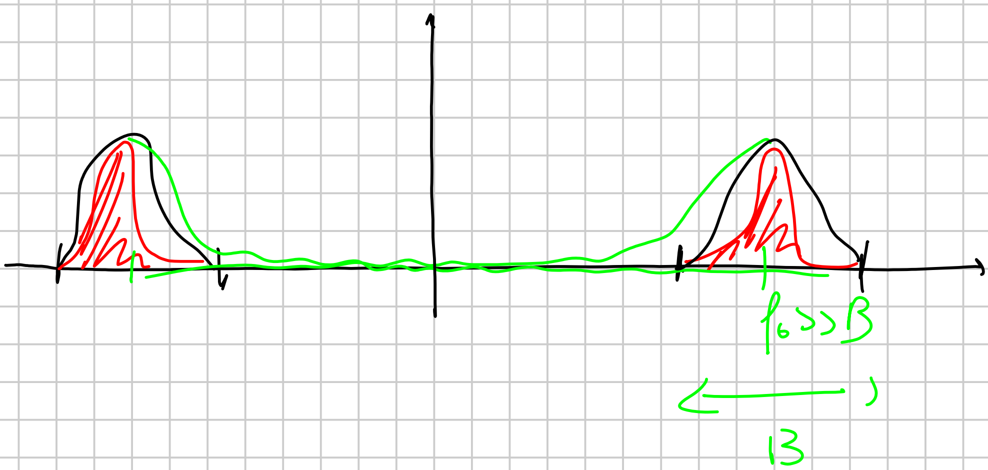
$= E\left[x_c^2[n]\right]\underbrace{\int_{-\infty}^{+\infty} p^2(t-nT)\cos^2(2\pi f_0 t)\,dt}_{\frac{E_p}{2}}$ $\qquad B_p << f_0$

$p(t) \overset{TCF}{\Longleftrightarrow} P(f)$



$P(f) = \text{rect}\left(\frac{f}{2B}\right) + \frac{1}{2}\text{rect}\left(\frac{f}{2B}\right)e^{-j\frac{\pi f}{B}} + \frac{1}{2}\text{rect}\left(\frac{f}{2B}\right)e^{j\frac{\pi f}{B}}$

$= \text{rect}\left(\frac{f}{2B}\right)\left[1 + \cos\left(\frac{\pi f}{B}\right)\right]$



$$\mathrm{I})\quad E\left[x_c^2[k]\right]\int_{-\infty}^{+\infty} p^2(t-kT)\cos^2(2\pi f_0 t)\,dt = \frac{5}{2}\cdot\frac{3B}{2} = \frac{15}{4}B$$

$$E_p = \int_{-B}^{B}\left(1+\cos\left(\frac{\pi f}{B}\right)\right)^2 df = \int_{-B}^{B} 1 + \cos^2\left(\frac{\pi f}{B}\right) + 2\cos\left(\frac{\pi f}{B}\right) df$$

$$= 2B + B = 3B$$

$$E\left[x_c^2[k]\right] = \frac{1}{2}(-1)^2 + \frac{1}{2}\cdot(2)^2 = \frac{1}{2}+\frac{4}{2} = \frac{5}{2}$$

$$\mathrm{II})\quad E\left[x_s^2[k]\right]\underbrace{\int_{-\infty}^{+\infty} p^2(t-kT)\sin^2(2\pi f_0 t)\,dt}_{E_p/2} = \frac{3}{2}B$$

$$E\left[x_s^2[k]\right] = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = 1$$

$$\mathrm{III})\quad 2\underbrace{E\left[x_c[k]\,x_s[k]\right]}_{=0}\underbrace{\int_{-\infty}^{+\infty} p^2(t-kT)\cos(2\pi f_0 t)\sin(2\pi f_0 t)\,dt}_{=0}$$

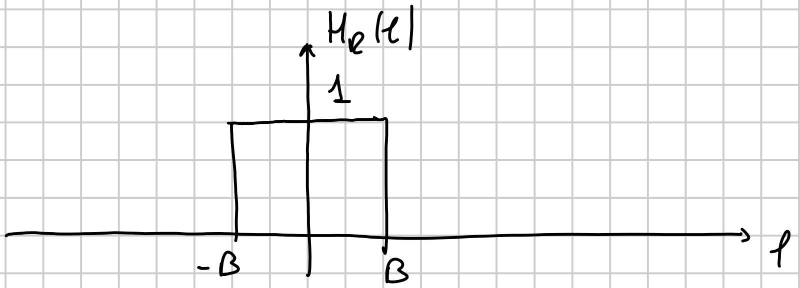
$$E\left[x_c[k]\,x_s[k]\right] = \underbrace{C_{x_c x_s}}_{=0} + \overset{0}{\eta_{x_c}}\,\overset{0}{\eta_{x_s}} = 0$$

sim) ind. $\Rightarrow$ incorrelati $\Rightarrow$ $C_{x_c x_s} = 0$

$$E_s = \frac{15}{4}B + \frac{3}{2}B = \frac{21}{4}B$$

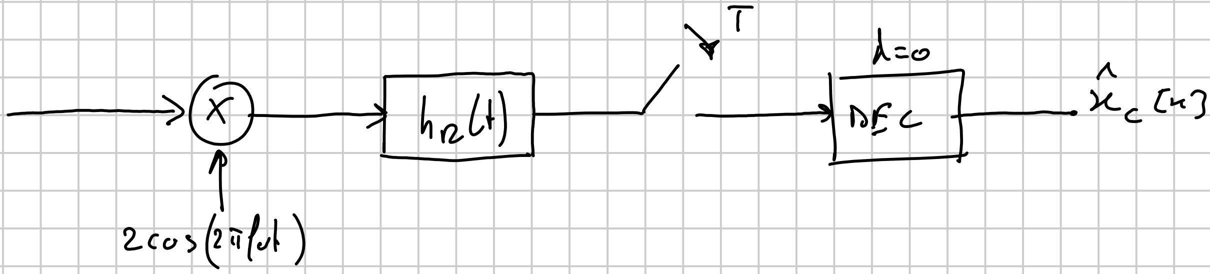
$$2)\quad P_{n_u}^c = \int_{-\infty}^{+\infty} S_{n_u}^c(f)\,df = \int_{-\infty}^{+\infty} S_{n_i}^c(f)\,|H_R(f)|^2\,df = 2BN_0 = P_{n_u}^s$$

$$S_{n_i}^c = N_0\,\mathrm{rect}\left(\frac{f}{2B}\right)$$



3) $P_E(b) = ?$

$P_E^c(b) = ?$



$$h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, H_R(f) = P(f) \quad \text{è un coseno rialzato}$$

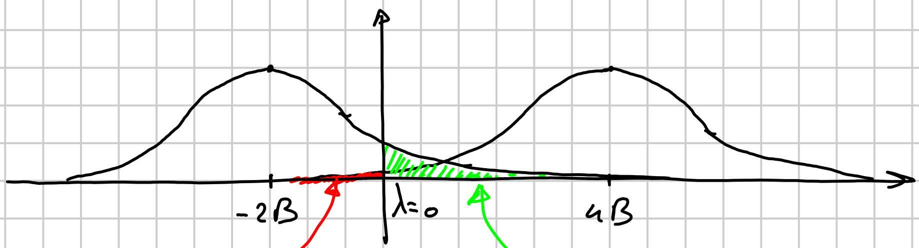
$\Downarrow$

ASSENZA DI ISI

$$h(0) = p(0) = \int_{-\infty}^{+\infty} P(f)\, df = 2B$$

$$y_c[n] = h(0)\, x_c[n] + n_c[n]$$

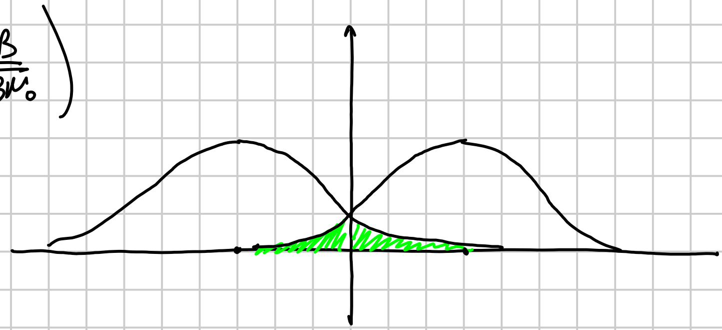
$$\sigma_{n_c}^2 = P_{n_u}^c$$



$$P_E^c(b) = P\{\hat{x} = -1 \mid x = +2\}\, P\{x = +2\} + P\{\hat{x} = +2 \mid x = -1\}\, P\{x = -1\}$$

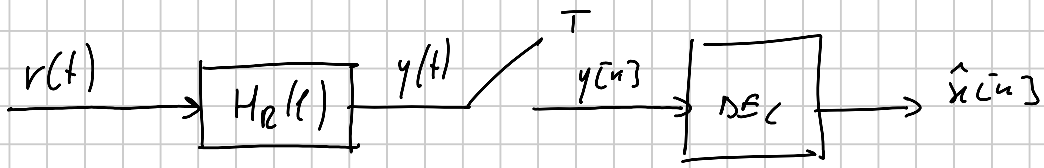
$$Q\left(\frac{4B}{\sqrt{2BN_0}}\right) \qquad P\{x=+2\} = 1/2 \qquad Q\left(\frac{2B}{\sqrt{2BN_0}}\right) \qquad P\{x=-1\} = 1/2$$

$$P_E^S(b) = Q\left(\frac{2B}{\sqrt{2BN_0}}\right)$$



$$P_E(b) = \frac{1}{2} P_E^S(b) + \frac{1}{2} P_E^C(b)$$

ES. # 2 del 5/12/2013



$$s(t) = \sum_n x[n]\, p(t - nT)$$

$x[n] \in A_s = \{-1, +3\}$, $P\{x = -1\} = \frac{1}{3}$, $P\{x = +3\} = \frac{2}{3}$

$$P(f) = \sqrt{1 - |fT|}\; \mathrm{rect}\left(\frac{fT}{2}\right)$$

$c(t) = \delta(t)$ $\qquad\qquad$ $\lambda = \frac{1}{T}$

$S_n(f) = \frac{N_0}{2}$

$H_R(f) = P(f)$

1) E_s
2) Cond. Nyquist
3) P_{n_u}
4) $P_E(b)$

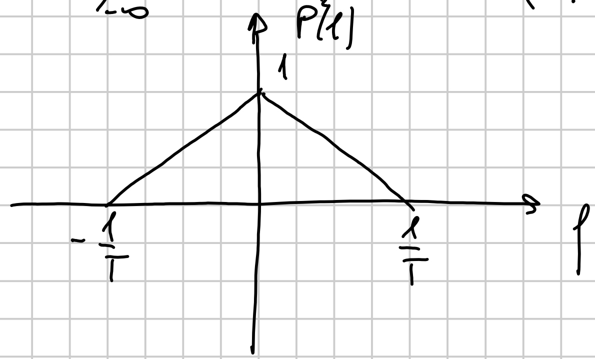
SOLUZIONE

1) $E_s = E\left[\int_{-\infty}^{+\infty}\left[x[n]\, p(t-nT)\right]^2 dt\right] =$

$= E\left[x^2[n]\right] E_p$

$\frac{1}{3}\cdot(-1)^2 + \frac{2}{3}(+3)^2 = \frac{1}{3} + \frac{18}{3} = \frac{19}{3}$

$E_p = \int_{-\infty}^{+\infty} P^2(f)\, df = \int_{-\infty}^{+\infty}\left(1 - \frac{|f|}{1/T}\right) \mathrm{rect}\left(\frac{f}{2/T}\right) df = \frac{1}{T}$

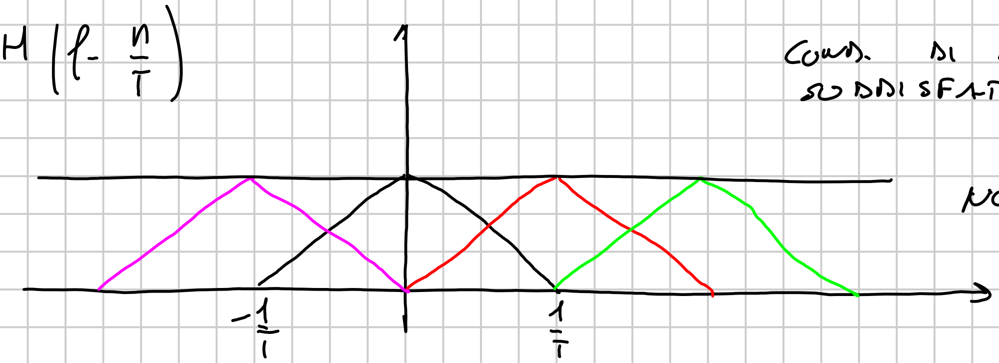


$E_s = \frac{19}{3}\cdot\frac{1}{T} = \frac{19}{3T}$

2) $h(t) = p(t) \otimes c(t) \otimes h_R(t)$

$H(f) = P(f)\, C(f)\, H_R(f) = P^2(f)$

$\bar{H}(f) = \frac{1}{T}\sum_n H\left(f - \frac{n}{T}\right)$



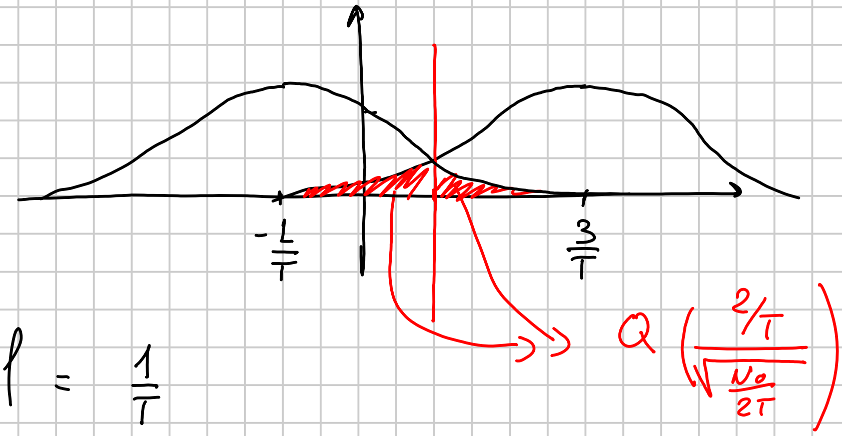
COND. DI NYQUIST SODDISFATTA

⇓

NO ISI

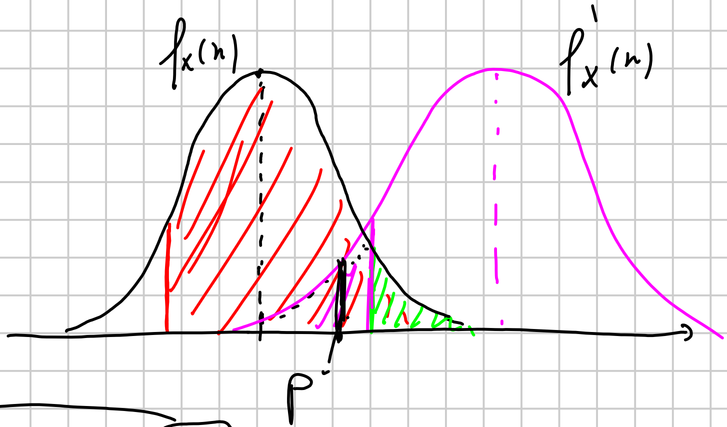
3) $P_{n_u} = \int_{-\infty}^{+\infty} S_{n_u}(f)\, df = \frac{N_0}{2}\int_{-\infty}^{+\infty} |H_R(f)|^2\, df = \frac{N_0}{2} E_p = \frac{N_0}{2T}$

1) $P_E(b)$



$$h(0) = \int_{-\infty}^{+\infty} H(f)\, df = \frac{1}{T}$$

$$P_E(b) = \frac{1}{3} Q\left(\frac{2/T}{\sqrt{\frac{N_0}{2T}}}\right) + \frac{2}{3} Q\left(\frac{2/T}{\sqrt{\frac{N_0}{2T}}}\right) = Q\left(\frac{2/T}{\sqrt{\frac{N_0}{2T}}}\right)$$

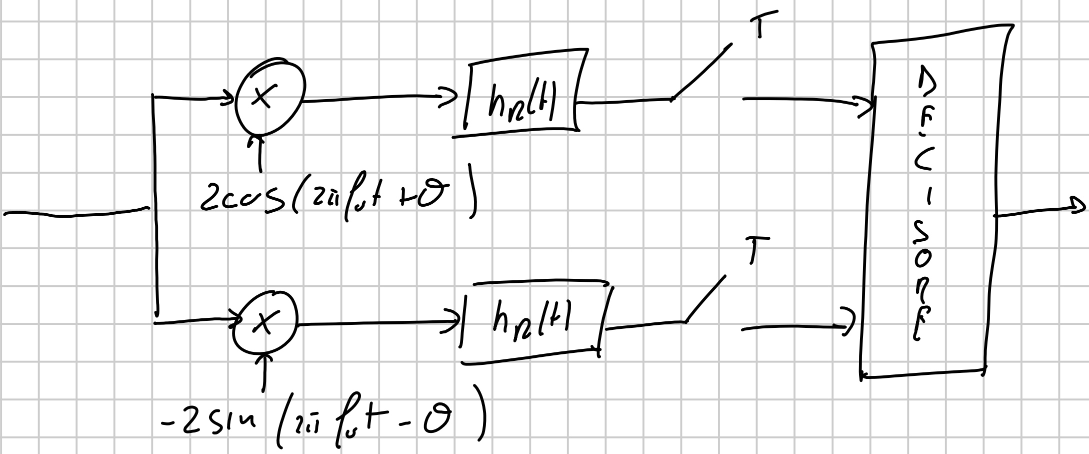


$$\boxed{\begin{array}{c} Q(\lambda) = \int_{\lambda}^{+\infty} f_x(x)\, dx \\ X \in \mathcal{N}(0,1) \end{array}}$$

$$\frac{\lambda - \mu}{\sigma}$$

$$\int_{-\infty}^{\lambda} f'_x(x)\, dx = Q\left(\frac{-\lambda + \mu}{\sigma}\right)$$

Es #3 COMPITINO PARTE II 2013



$$s(t) = \sum_n x_c[n]\, p(t-nT)\cos(2\pi f_0 t) - \sum_n x_s[n]\, p(t-nT)\sin(2\pi f_0 t)$$

$$x_c[n] \in A_s^c = \{-1, +2\}$$
$$x_s[n] \in A_s^s = \{-1, +1\}$$

$n(t)$ è un processo Gauss Bianco DP: $\frac{N_0}{2}$

$$P(f) = \mathrm{rect}\left(\frac{f}{2B}\right)\sqrt{1-\frac{|f|}{B}}$$

$f_0 \gg B$

$$c(t) = \delta(t-t_0)$$

$B = \frac{1}{T}$

$$h_R(t) = p(t+t_0)$$

$\lambda = 0$

1) E_s
2) $P_{n_u}^{c,s}$
3) Determinare σ tale che sia assente cross-talk
4) P_E sul simbolo QAM

SOLUZIONE

$$E_s = E[x_c^2[n]]\frac{E_p}{2} + E[x_s^2[n]]\frac{E_p}{2}$$

$f_0 \gg B$

$$\frac{1}{2}(-1)^2 + \frac{1}{2}(+2)^2 = \frac{5}{2}$$

$$1$$

$$E_p = \int_{-\infty}^{+\infty} P^2(f)\, df = \int_{-B}^{B} 1 - \frac{|f|}{B}\, df = B$$

$$E_s = \left(\frac{5}{2}+1\right)\frac{B}{2} = \frac{7}{4}B$$

2) $$P_{n_u}^{c,s} = \int_{-\infty}^{+\infty} S_{n_i}^{c,s}(f)\,|H_R(f)|^2\, df = N_0\int_{-\infty}^{+\infty}|P(f)|^2\, df = N_0 B$$

$$p(t+t_0) \overset{TCF}{\Longleftrightarrow} \left|P(f)\, e^{j2\pi f t_0}\right| = P(f)$$

3) $s_n(t) = x_c[n]\, p(t-nT) \cos(2\pi f_0 t) - x_s[n]\, p(t-nT) \sin(2\pi f_0 t)$

$$s_n'(t) = s_n(t-t_0) =$$

$$= x_c[n]\, p(t-nT-t_0) \cos[2\pi f_0 (t-t_0)] - x_s[n]\, p(t-nT-t_0) \sin[2\pi f_0 (t-t_0)] =$$

$$= x_c[n]\, p(t-nT-t_0) \cos[2\pi f_0 t - \varphi_0] - x_s[n]\, p(t-nT-t_0) \sin(2\pi f_0 t - \varphi_0)$$

$$\varphi_0 = 2\pi f_0 t_0$$

$$s_n^c(t) = s_n'(t) \cdot 2\cos(2\pi f_0 t - \vartheta) =$$

$$= 2 x_c[n]\, p(\cdots) \cos(2\pi f_0 t - \varphi_0) \cos(2\pi f_0 t - \vartheta) +$$

$$- 2 x_s[n]\, p(\cdot\cdot) \sin(2\pi f_0 t - \varphi_0) \cos(2\pi f_0 t - \vartheta)$$

$$\frac{1}{2} \cos(4\pi f_0 t - \varphi_0 - \vartheta) + \frac{1}{2} \cos(\varphi_0 - \vartheta) \qquad (\cos \cdot \cos)$$

$$\frac{1}{2} \sin(4\pi f_0 t - \varphi_0 - \vartheta) + \frac{1}{2} \sin(-\varphi_0 + \vartheta) \qquad (\cos \cdot \sin)$$

$\Downarrow$

VA ANNULLATO $\Rightarrow$ $\varphi_0 = \vartheta$

$$\boxed{\vartheta = 2\pi f_0 t_0}$$

4) •) ISI

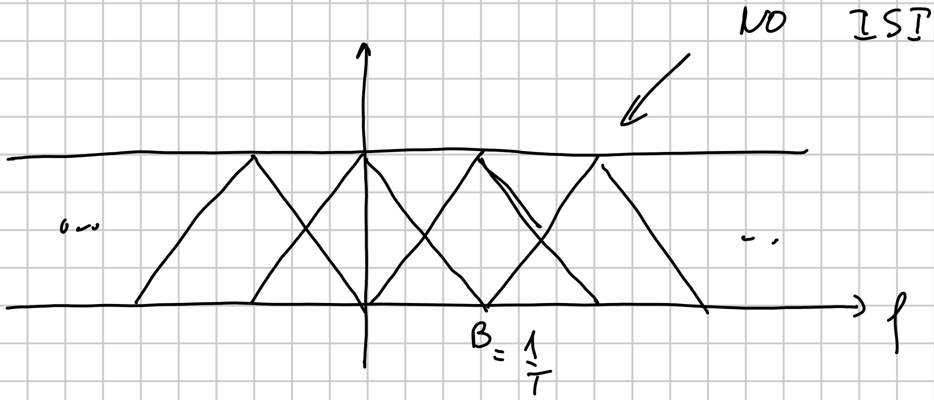
$$h(t) = p(t) \otimes c(t) \otimes h_R(t)$$

$$H(f) = P(f)\, C(f)\, H_R(f)$$

$$= P(f)\, \cancel{e^{-j2\pi f t_0}}\, P(f)\, \cancel{e^{j2\pi f t_0}} = P^2(f)$$

$$P^2(f) = \left(1 - \frac{|f|}{B}\right) \text{rect}\left(\frac{f}{2B}\right)$$

$$B = \frac{1}{T}$$



$$\cdot)\ h(0) = \int_{-\infty}^{+\infty} H(f)\, df = \int_{-\infty}^{+\infty} P^2(f)\, df = B$$

$$P_E^c(b) = \frac{1}{2} Q\left(\frac{B}{\sqrt{N_0 B}}\right) + \frac{1}{2} Q\left(\frac{2B}{\sqrt{N_0 B}}\right)$$

$$P_E^s(b) = Q\left(\frac{B}{\sqrt{N_0 B}}\right)$$

$$P_E^{QAM} = P\{\hat{x}_c = -1 \mid x_c = +2\}\, P\{x_c = +2\} + \ldots$$

$$\boxed{P_E^{QAM} = P_E^c(b)\left(1 - P_E^s(b)\right) + P_E^s(b)\left(1 - P_E^c(b)\right) + P_E^c(b)\, P_E^s(b)}$$