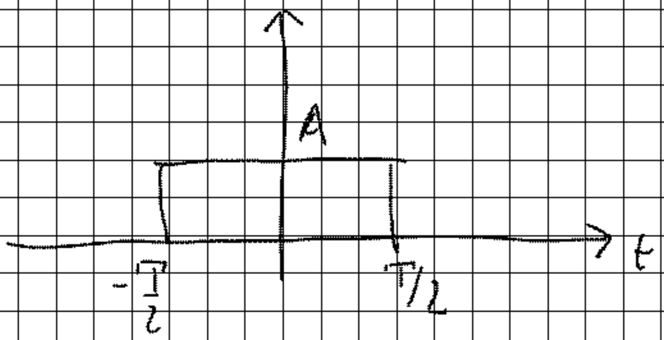
07 / 03 / 2012

Esercizio #1



$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt$$

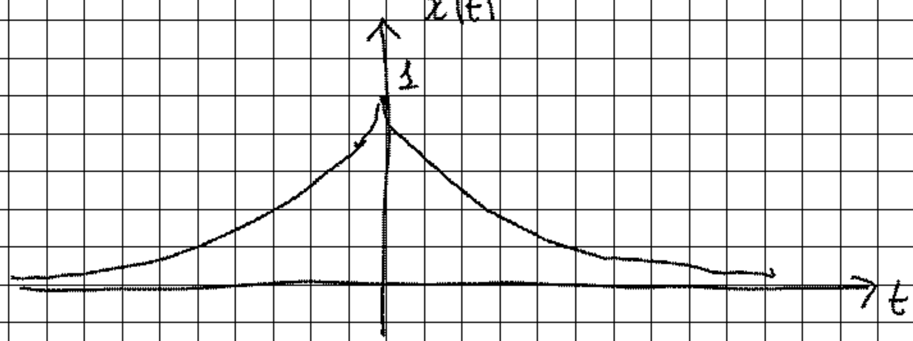
$$X(f) = \int_{-\frac{T}{2}}^{T/2} A\, e^{-j2\pi f t}\, dt = A \left. \frac{e^{-j2\pi f t}}{(-j2\pi f)} \right|_{-\frac{T}{2}}^{T/2}$$

$$= \frac{A}{(-j2\pi f)} \left(e^{-j\pi f T} - e^{+j\pi f T} \right) = \frac{AT}{\pi f T} \sin(\pi f T) =$$

$$= AT \operatorname{sinc}(fT)$$

$$x(t) = \operatorname{rect}\left(\frac{t}{T}\right) \rightleftharpoons X(f) = T \operatorname{sinc}(fT)$$

Esercizio #2



$$x(t) = e^{-|t|}$$

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt =$$

$$= \int_{-\infty}^{0} e^{t}\, e^{-j2\pi f t}\, dt + \int_{0}^{+\infty} e^{-t}\, e^{-j2\pi f t}\, dt =$$

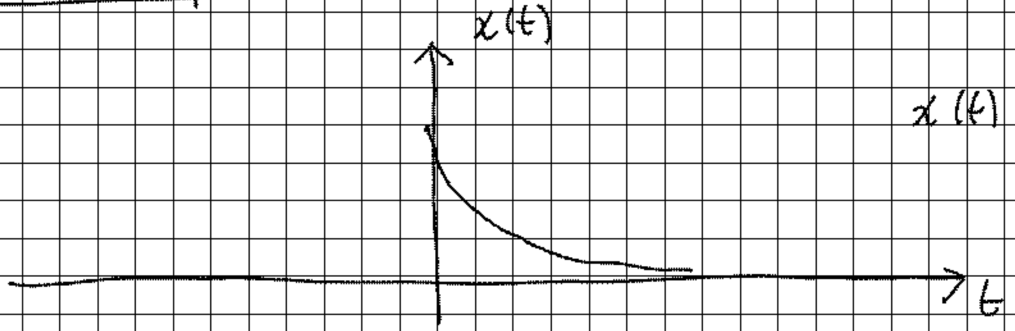
$$= \int_{-\infty}^{0} e^{t(1-j2\pi f)}\, dt + \int_{0}^{+\infty} e^{-t(1+j2\pi f)}\, dt =$$

$$= \left.\frac{e^{t(1-j2\pi f)}}{(1-j2\pi f)}\right|_{-\infty}^{0} + \left.\frac{e^{-t(1+j2\pi f)}}{-(1+j2\pi f)}\right|_{0}^{+\infty}$$

$$= \left(\frac{1}{1-j2\pi f} - 0\right) + \left(0 + \frac{1}{1+j2\pi f}\right) =$$

$$X(f) = \frac{1}{1-j2\pi f} + \frac{1}{1+j2\pi f} = \frac{1+\cancel{j2\pi f}+1-\cancel{j2\pi f}}{1+4\pi^2 f^2} = \frac{2}{1+4\pi^2 f^2}$$

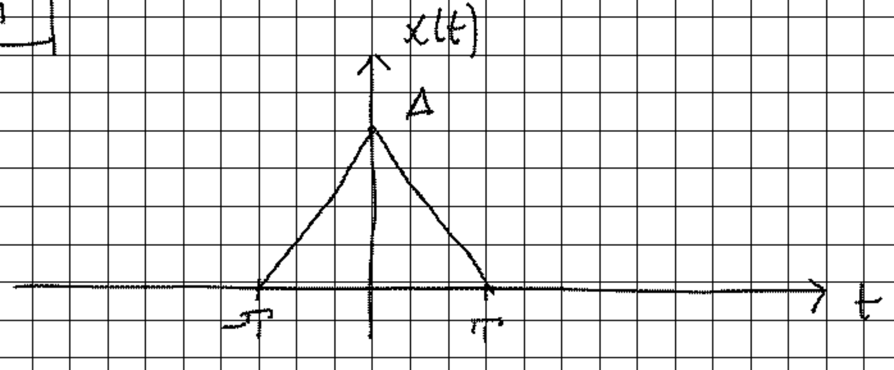
Esercizio #3



$$x(t) = e^{-t}\, u(t)$$

$$X(f) = \int_{0}^{+\infty} e^{-t}\, e^{-j2\pi f t}\, dt = \frac{1}{1+j2\pi f}$$

Esercizio #4



$$x(t) = A\left(1-\frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right)$$

I) $$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt =$$

$$= \int_{-T}^{0} A\left(1+\frac{t}{T}\right) e^{-j2\pi f t}\, dt + \int_{0}^{T} A\left(1-\frac{t}{T}\right) e^{-j2\pi f t}\, dt =$$

$$\int_{-T}^{0} A\left(1+\frac{t}{T}\right) e^{-j2\pi f t}\, dt = A\left(1+\frac{t}{T}\right)\frac{e^{-j2\pi f t}}{(-j2\pi f)}\Bigg|_{-T}^{0} - \int_{-T}^{0} \frac{A}{T}\frac{e^{-j2\pi f t}}{(-j2\pi f)}\, dt =$$

$$= \frac{A}{(-j2\pi f)} - 0 - \frac{A}{T}\frac{e^{-j2\pi f t}}{(-j2\pi f)^2}\Bigg|_{-T}^{0} =$$

$$= -\frac{A}{j2\pi f} + \frac{A\left(1-e^{2j\pi f T}\right)}{T\,4\pi^2 f^2}$$

$$\int_{0}^{T} A\left(1-\frac{t}{T}\right) e^{-j2\pi f t}\, dt = A\left(1-\frac{t}{T}\right)\frac{e^{-j2\pi f t}}{(-j2\pi f)}\Bigg|_{0}^{T} + \frac{A}{T}\int_{0}^{T} \frac{e^{-j2\pi f t}}{(-j2\pi f)}\, dt =$$

$$= 0 + \frac{A}{j2\pi f} + \frac{A}{T}\frac{e^{-j2\pi f t}}{(-j2\pi f)^2}\Bigg|_{0}^{T} =$$

$$= \frac{A}{j2\pi f} - \frac{A}{T\,4\pi^2 f^2}\left(e^{-j2\pi f T} - 1\right)$$

$$X(f) = -\cancel{\frac{A}{i2\pi f}} + \frac{A}{T4\pi^2 f^2}\left(1-e^{i2\pi fT}\right) + \cancel{\frac{A}{i2\pi f}} - \frac{A}{T4\pi^2 f^2}\left(e^{-i2\pi fT}-1\right) =$$

$$= \frac{A}{4\pi^2 f^2 T}\left[1-e^{i2\pi fT} + 1 - e^{-i2\pi fT}\right] =$$

$$= \frac{A}{4\pi^2 f^2 T}\left(1-e^{i2\pi fT}\right)\left(1-e^{-i2\pi fT}\right) =$$

$$= \frac{A}{4\pi^2 f^2 T}\cancel{e^{i\pi fT}}\left(e^{-i\pi fT}-e^{+i\pi fT}\right)\cancel{e^{-i\pi fT}}\left(e^{+i\pi fT}-e^{-i\pi fT}\right) =$$

$$= \frac{A}{\pi^2 f^2 T\,(2i)(-2i)}\left(e^{-i\pi fT}-e^{+i\pi fT}\right)\left(e^{+i\pi fT}-e^{-i\pi fT}\right) =$$

$$= \frac{A}{\pi^2 f^2 T}\left(\frac{e^{-i\pi fT}-e^{+i\pi fT}}{-2i}\right)\left(\frac{e^{+i\pi fT}-e^{-i\pi fT}}{2i}\right) =$$

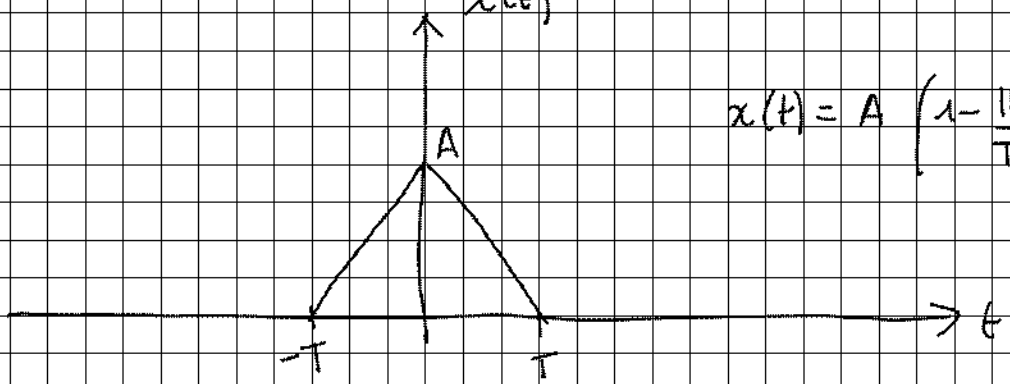
$$= \frac{A}{\pi^2 f^2 T}\sin(\pi fT)\cdot\sin(\pi fT)$$

$$= AT \frac{\sin(\pi f T)}{\pi f T} \cdot \frac{\sin(\pi f T)}{\pi f T} = AT\,\mathrm{sinc}^2(fT)$$

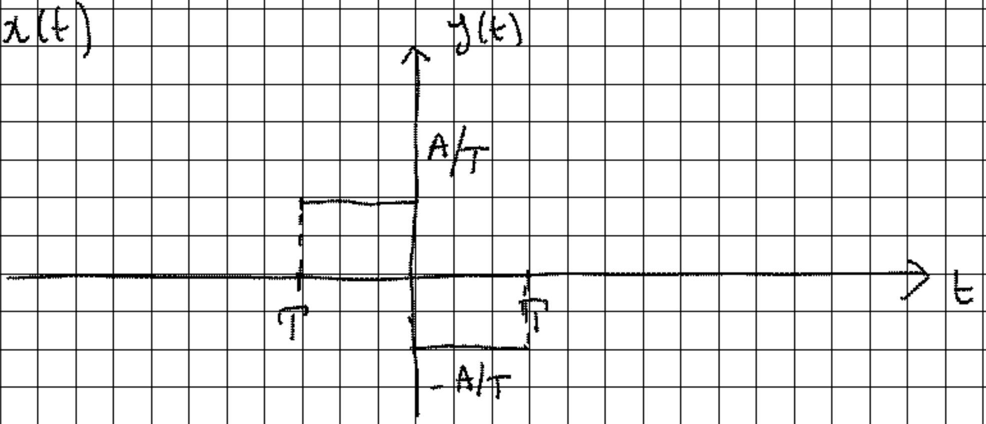
$$x(t) = \left(1-\frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right) \rightleftharpoons X(f) = T\,\mathrm{sinc}^2(fT)$$

II)

$$x(t) = A\left(1-\frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right)$$



$$y(t) = \frac{\partial}{\partial t} x(t)$$



·) Teorema di derivazione

$$y(t) = \frac{\partial}{\partial t} x(t) \rightleftharpoons Y(f) = j2\pi f\, X(f)$$

$$X(f) = \frac{Y(f)}{j2\pi f}$$

$$y(t) = \frac{A}{T} \operatorname{rect}\left(\frac{t+T/2}{T}\right) - \frac{A}{T} \operatorname{rect}\left(\frac{t-T/2}{T}\right)$$

$$y_0(t) = \operatorname{rect}\left(\frac{t}{T}\right) \rightleftharpoons Y_0(f) = T \operatorname{sinc}(fT)$$

$$y(t) = \frac{A}{T}\left[y_0(t+T/2) - y_0(t-T/2) \right]$$

) Teorema del ritardo

$$Y(f) = \frac{A}{T} Y_0(f)\, e^{+j2\pi f\frac{T}{2}} - \frac{A}{T} Y_0(f)\, e^{-j2\pi f\frac{T}{2}}$$

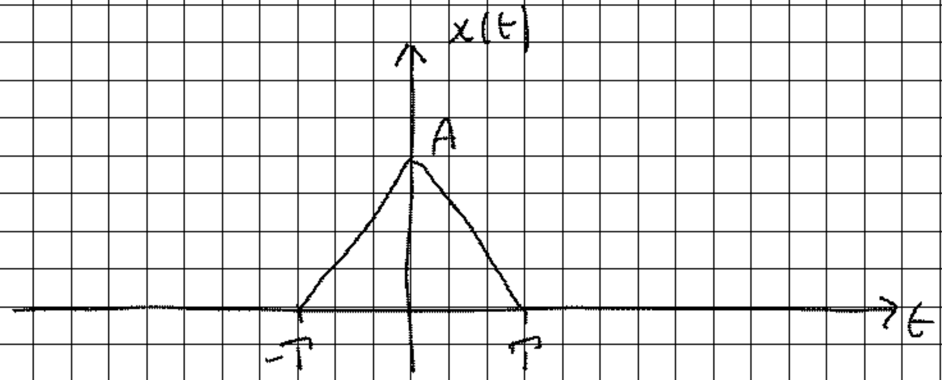
$$Y(f) = \frac{A}{T} T \operatorname{sinc}(fT) \left(e^{+j\pi f T} - e^{-j\pi f T} \right) \cdot \frac{2j}{2j} =$$

$$= A\, 2j \operatorname{sinc}(fT) \cdot \sin(\pi f T)$$

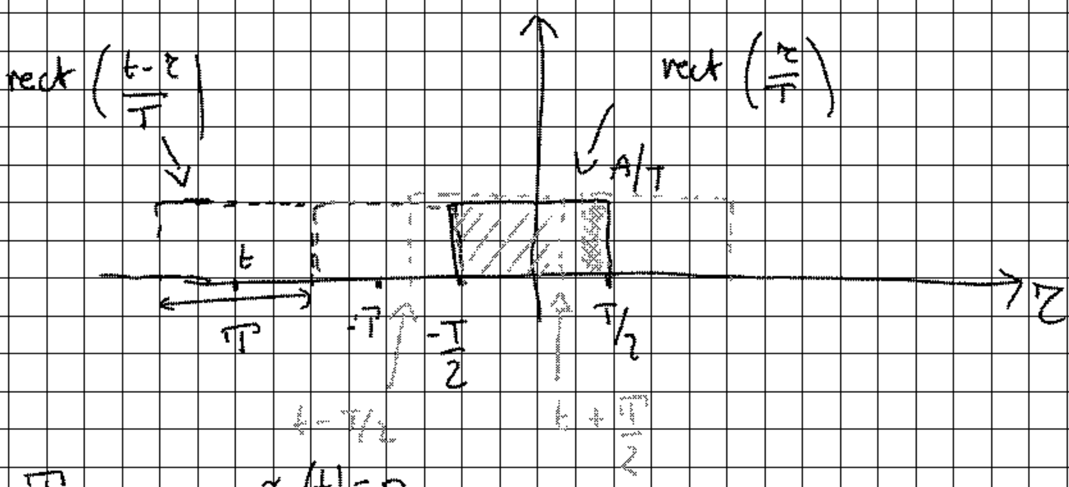
$$X(f) = \frac{Y(f)}{j2\pi f} = \frac{A}{\pi f} \operatorname{sinc}(fT) \sin(\pi f T) =$$

$$= AT \operatorname{sinc}^2(fT)$$

III)



$$x(t) = \frac{A}{T}\left(\mathrm{rect}\left(\frac{t}{T}\right) \otimes \mathrm{rect}\left(\frac{t}{T}\right)\right)$$



$t < -T \qquad x(t) = 0$

$-T \leq t < 0 \qquad \left(t + \frac{T}{2} - \left(-\frac{T}{2}\right)\right) \cdot \frac{A}{T} = (t+T)\frac{A}{T} = A\left(1 + \frac{t}{T}\right)$

$0 \leq t < T \qquad \left(\frac{T}{2} - \left(t - \frac{T}{2}\right)\right)\frac{A}{T} = (T - t)\frac{A}{T} = A\left(1 - \frac{t}{T}\right)$

$t \geq T \qquad x(t) = 0$

$$x(t) = A\left(1 - \frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right)$$

$$x(t) = \frac{A}{T}\,\mathrm{rect}\left(\frac{t}{T}\right) \otimes \mathrm{rect}\left(\frac{t}{T}\right)$$

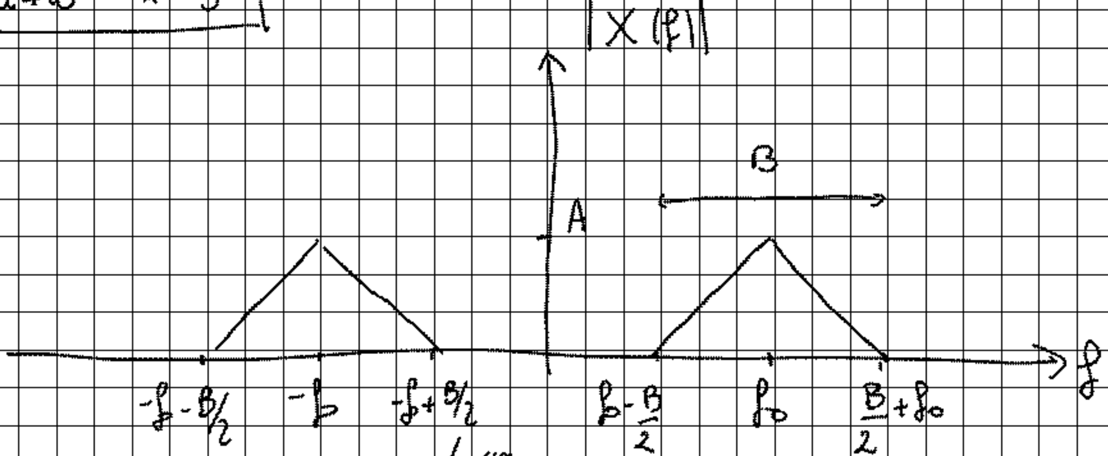
) teorema della convoluzione

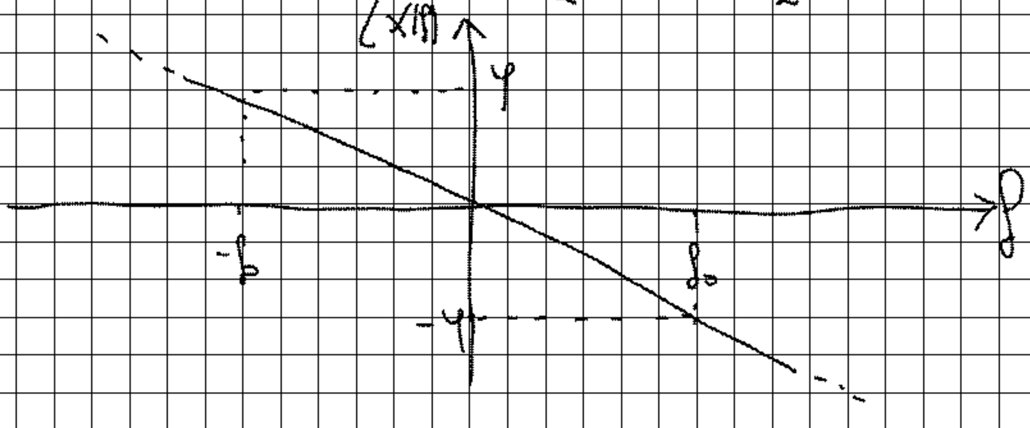
$$z(t) = x(t) \otimes y(t) \rightleftharpoons Z(f) = X(f) \cdot Y(f)$$

$$X(f) = \frac{A}{T} T \,\mathrm{sinc}(fT) \cdot T \,\mathrm{sinc}(fT) =$$

$$= AT \,\mathrm{sinc}^2(fT)$$

Esercizio #5





$$X(f) = |X(f)| \, e^{j\angle X(f)}$$

$$|X(f)| = X_1(f-f_0) + X_1(f+f_0)$$

$$X_1 = A\left(1 - \frac{|f|}{B/2}\right) \mathrm{rect}\left(\frac{f}{B}\right)$$

$\angle X(f) = a f + b$

$b = 0$

$a = -\frac{\varphi}{f_0}$

$\angle X(f) = -\frac{\varphi}{f_0} f$

$$X(f) = \underbrace{\left[X_1(f - f_0) + X_1(f + f_0)\right]}_{A(f)} e^{-j\frac{\varphi}{f_0} f}$$

$$X(f) = A(f)\, e^{-j 2\pi f t_0}$$

$t_0 = \frac{\varphi}{2 f_0 \pi}$

·) teorema del ritardo

$x(t) = a(t - t_0)$

·) $A(f) = X_1(f - f_0) + X_1(f + f_0)$

$X_1(f) = A\left(1 - \frac{|f|}{B/2}\right) \text{rect}\left(\frac{f}{B}\right)$

·) teorema della modulazione

$a(t) = 2\cos(2\pi f_0 t)\, x_1(t)$

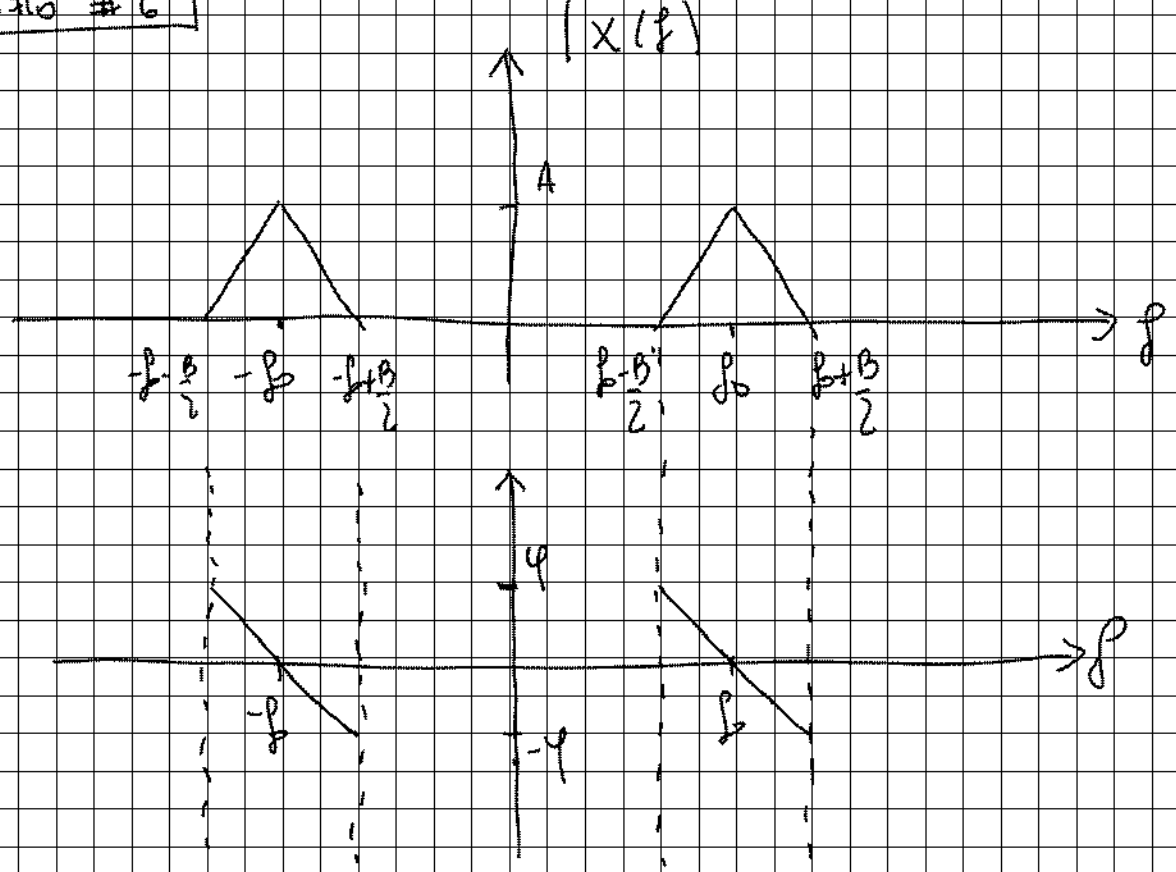
$y(t) = x(t) \cdot \cos(2\pi f_0 t)$

$Y(f) = \frac{X(f - f_0) + X(f + f_0)}{2}$

$x_1(t) = A \cdot \frac{B}{2} \text{sinc}^2\left(t \frac{B}{2}\right)$

$$x(t) = AB\cos\left(2\pi f_0(t-t_0)\right)\,\mathrm{sinc}^2\left(\frac{B}{2}(t-t_0)\right)$$

Esercizio # 6



$$X(f) = X_0(f-f_0)\,e^{j\varphi_1(f)} + X_0(f+f_0)\,e^{j\varphi_2(f)}$$

$$X_0(f) = A\left(1-\frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right)$$

$$\varphi_1(f) = \varphi_0(f-f_0) = -\frac{2\varphi}{B}(f-f_0)$$

$$\varphi_2(f) = \varphi_0(f+f_0) = -\frac{2\varphi}{B}(f+f_0)$$

$$X(f) = X_0(f-f_0)\,e^{j\varphi_0(f-f_0)} + X_0(f+f_0)\,e^{j\varphi_0(f+f_0)}$$

$$Y(f) = X_0(f) e^{i\varphi_0(f)}$$

$$X(f) = Y(f - f_0) + Y(f + f_0)$$

·) teorema della modulazione

$$\boxed{x(t) = 2\cos(2\pi f_0 t)\, y(t)}$$

$$Y(f) = X_0(f) \cdot e^{-j\frac{2\varphi}{B}f} = X_0(f)\, \boxed{e^{-j2\pi f t_0}} \qquad t_0 = \frac{\varphi}{B\pi}$$

·) teorema del ritardo

$$z(t - t_0) \rightleftarrows Z(f)\, e^{-j2\pi f t_0}$$

$$y(t) = x_0(t - t_0)$$

$$x_0(t) = \frac{AB}{2} \operatorname{sinc}^2\left(t\frac{B}{2}\right)$$

$$x(t) = AB\cos(2\pi f_0 t) \operatorname{sinc}^2\left((t - t_0)\frac{B}{2}\right)$$

elisa.giusti@iet.unipi.it

050 - 22 17 673