SOLUZIONE COMPITINO 18/04/2009

Es. 1

$$y(t) = y_2(t) - y_1(t)$$

$$y_2(t) = K_2\left[K_1 x(t) + x(t-T)\right]$$

$$y_1(t) = K_1 x(t-T)$$

$$y(t) = K_1 K_2 x(t) + (K_2 - K_1)\, x(t-T)$$

$$h(t) = T\left[\delta(t)\right]$$

$$\boxed{\begin{aligned} h(t) &= K_1 K_2 \delta(t) + (K_2 - K_1)\,\delta(t-T) \\ H(f) &= TCF\left[h(t)\right] = K_1 K_2 + (K_2 - K_1)\, e^{-j2\pi f T} \end{aligned}}$$

j) STAZIONARIETÀ

$$y'(t) = T\left[x(t-t_0)\right] = K_1 K_2\, x(t-t_0) + (K_2 - K_1)\, x(t - t_0 - T) =$$
$$= y(t-t_0)$$

<u>È STAZIONARIO</u>

jj) LINEARITÀ

$$x(t) = a\, x_1(t) + b\, x_2(t)$$

$$y(t) = K_1 K_2\left[a\, x_1(t) + b\, x_2(t)\right] + (K_2 - K_1)\left[a\, x_1(t-T) + b\, x_2(t-T)\right] =$$

$$= a\left[K_1 K_2\, x_1(t) + (K_2 - K_1)\, x_1(t-T)\right] + b\left[K_1 K_2\, x_2(t) + (K_2 - K_1)\, x_2(t-T)\right]$$

<u>È LINEARE</u>

jjj) CAUSALITÀ

L'uscita all'istante t non dipende da valori dell'ingresso successivi all'istante t $\Rightarrow$ <u>È CAUSALE</u>

iv) MEMORIA

L'uscita all'istante t dipende anche da valori dell'ingresso precedenti a t $\Rightarrow$ HA MEMORIA

v) STABILITÀ BIBO

$$|y(t)| \leq N \quad \text{se} \quad |x(t)| \leq M \quad \forall t$$

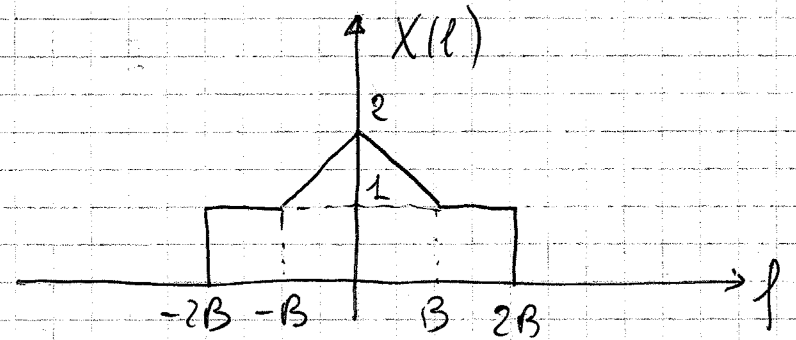
$$|y(t)| = |K_1 K_2\, x(t) + (K_2 - K_1)\, x(t-T)| \leq$$

$$\leq |K_1 K_2|\,|x(t)| + |K_2 - K_1|\,|x(t-T)| \leq$$

$$\leq K' M + K'' M = K''' M = N$$

ES. 2

$$X(f) = \mathrm{rect}\left(\frac{f}{4B}\right) + \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$



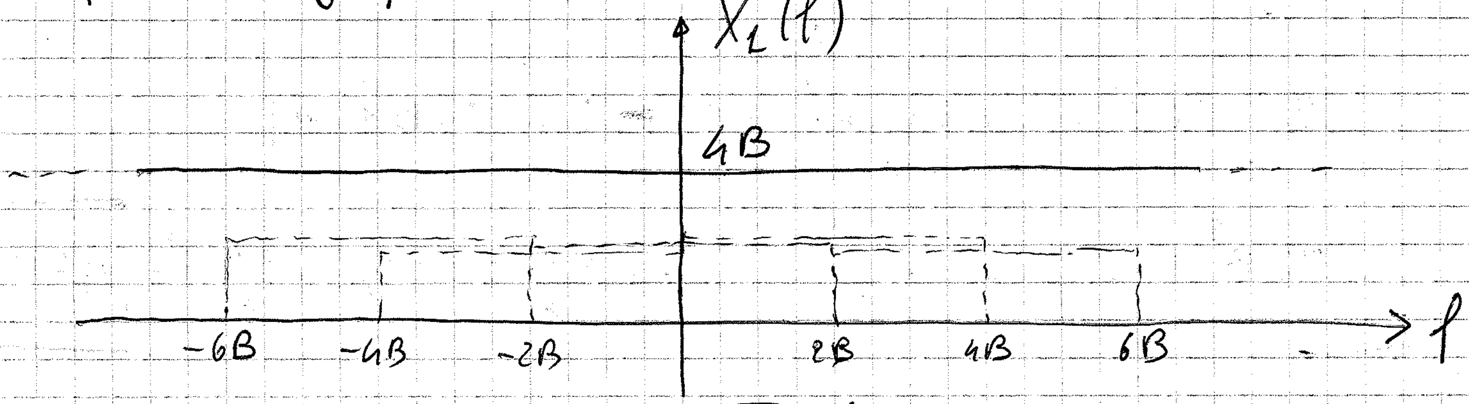
$$\bar{X}(f) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X\left(f - \frac{n}{T}\right) = 2B \sum_{n=-\infty}^{+\infty} X(f - n2B) =$$

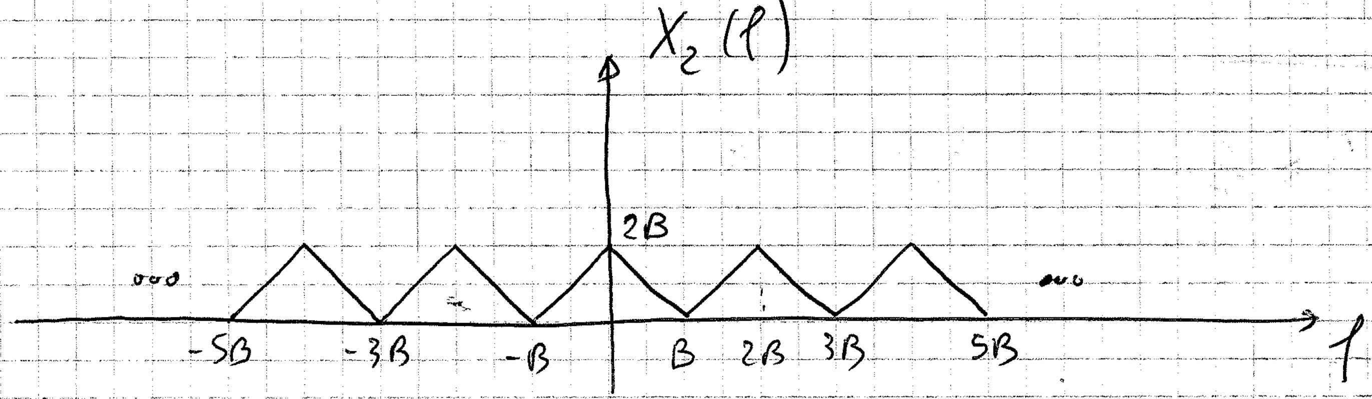
$$= 2B \sum_{n=-\infty}^{+\infty} X_1(f - n2B) + 2B \sum_{n=-\infty}^{+\infty} X_2(f - n2B)$$

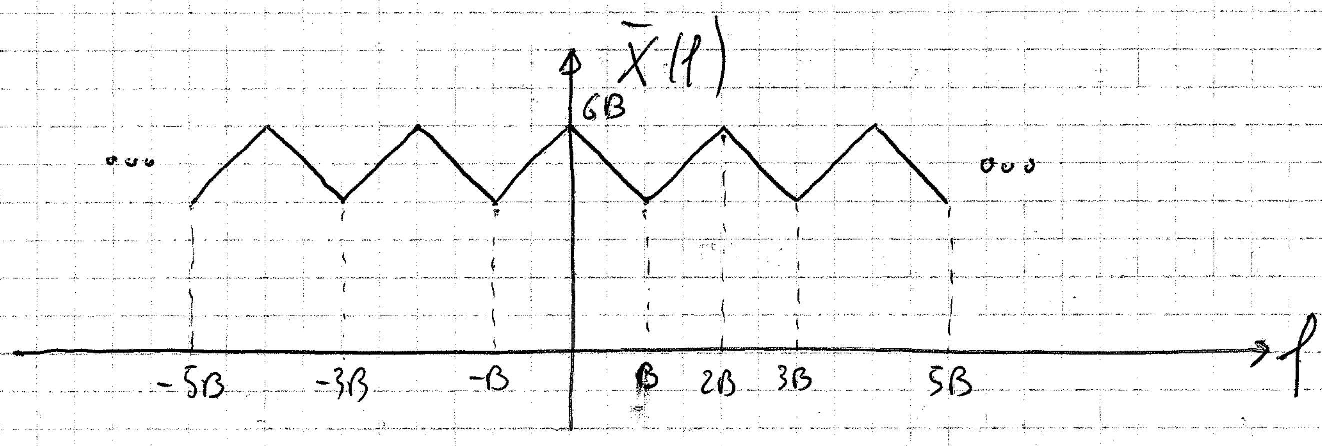
$$X_1(f) = \mathrm{rect}\left(\frac{f}{4B}\right), \quad X_2(f) = \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$\bar{X}(f) = \bar{X}_1(f) + \bar{X}_2(f)$$

·) per via grafica







$$\boxed{\begin{aligned} \bar{X}(f) &= \sum_{n=-\infty}^{+\infty} X_0(f - n2B) \\ X_0(f) &= 4B\,\mathrm{rect}\left(\frac{f}{2B}\right) + 2B\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) \end{aligned}}$$ 1)

$$Y(f) = \bar{X}(f)\,H_1(f) = X_0(f) = 4B\,\mathrm{rect}\left(\frac{f}{2B}\right) + 2B\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$\boxed{y(t) = 8B^2\,\mathrm{sinc}(2Bt) + 2B^2\,\mathrm{sinc}^2(Bt)}$$

$y(t)$ non è una replica fedele di $x(t)$ 2)

$$E_y = \int_{-\infty}^{+\infty} |Y(f)|^2 \, df = 2\int_0^B (6B - 2f)^2 \, df =$$

$$= 2\int_0^B 36B^2 \, df + 2\int_0^B 4f^2 \, df - 2\int_0^B 24Bf \, df =$$

$$= 72B^3 + 8\frac{B^3}{3} - 48\frac{B^3}{2} = \frac{432 + 16 - 144}{6} B^3 = \frac{304}{6} B^3 = \boxed{\frac{152}{3} B^3}$$

$$\boxed{P_y = 0} \quad 3)$$

$$\boxed{T_2 \leq \frac{1}{2B} \quad , \quad h_2(t) = 2B \, \text{sinc}(2Bt)} \quad 4)$$

ES 3

$$x_0(t) = A \cos\left(2\pi \frac{t}{4T}\right) \text{rect}\left(\frac{t}{2T}\right)$$

$$x(t) = \sum_{n=-\infty}^{+\infty} x_0(t - n4T)$$

$$X_n = \frac{1}{T_0} X_0\left(\frac{n}{T_0}\right) \quad , \quad T_0 = 4T \quad , \quad \boxed{X(f) = \sum_{n=-\infty}^{+\infty} X_n \, \delta\left(f - \frac{n}{T_0}\right)}$$

$$X_0(f) = \text{TCF}[x_0(t)] = \frac{A}{2}\left[\delta\left(f - \frac{1}{T_0}\right) + \delta\left(f + \frac{1}{T_0}\right)\right] \otimes$$

$$\otimes \, 2T \, \text{sinc}(2Tf) = AT\left\{\text{sinc}\left[2T\left(f - \frac{1}{T_0}\right)\right] + \text{sinc}\left[2T\left(f + \frac{1}{T_0}\right)\right]\right.$$

$$X_n = \frac{1}{4T} AT\left\{\text{sinc}\left[2T\left(\frac{n}{4T} - \frac{1}{4T}\right)\right] + \text{sinc}\left[2T\left(\frac{n}{4T} + \frac{1}{4T}\right)\right]\right\} =$$

$$\boxed{= \frac{A}{4}\left[\text{sinc}\left(\frac{n-1}{2}\right) + \text{sinc}\left(\frac{n+1}{2}\right)\right]}$$