$$f(x) = \overbrace{2yz}^{f_1}\, i + \overbrace{2z(x+3y)}^{f_2}\, j + \overbrace{[y(2x+3y) + 2z]}^{f_3}\, k$$

$$\operatorname{rot} f = 0 \qquad (x,y,z) \in \mathbb{R}^3$$

$$f = \nabla U$$

$$f_1 = \frac{\partial U}{\partial x} = 2yz$$

φ funzione arbitraria C^1 dipendente solo da y e z

$$U(x,y,z) = 2yzx + \varphi(y,z)$$

$$f_2 = \frac{\partial U}{\partial y} = 2z(x+3y)$$

$$\frac{\partial U}{\partial y} = \overbrace{\cancel{2zx} + \frac{\partial \varphi}{\partial y}}^{\partial_y U} = \overbrace{2z(x+3y)}^{f_2} = \cancel{2zx} + 2\cdot z\cdot 3y$$

$$\frac{\partial \varphi(y,z)}{\partial y} = 6\cdot z\cdot y \implies \varphi(y,z) = 3zy^2 + \psi(z)$$

$$\psi \in C^1$$

$$U(x,y,z) = 2yzx + 3zy^2 + \psi(z)$$

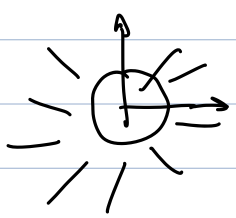
$$\frac{\partial U}{\partial z} = \overbrace{2yx + 3y^2 + \psi'(z)}^{\partial_z U} = \overbrace{y(2x+3y) + 2z}^{f_3}$$

$$\cancel{2yx} + \cancel{3y^2} + \psi'(z) = \cancel{2xy} + \cancel{3y^2} + 2z \qquad \psi(z) = z^2 + c$$

$$\boxed{U(x,y,z) = 2yxz + 3zy^2 + z^2 + c}$$

$$f(x) = g(\|x\|)\, x \qquad x \in \mathbb{R}^n$$

CAMPO RADIALE $\quad f \parallel x \qquad g: \mathbb{R}^+ \to \mathbb{R} \quad g \in C^1$



$$f = \nabla U$$

$$U(x) = G(\|x\|)$$

$$\frac{\partial U}{\partial x_i} = f_i(x) = g(\|x\|)\, x_i \qquad i = 1, \dots, n$$

$$\frac{\partial U}{\partial x_i} = \frac{\partial}{\partial x_i} G(\|x\|) = G'(\|x\|) \cdot \frac{\partial}{\partial x_i} \|x\| = G'(\|x\|) \frac{\partial}{\partial x_i} \left(\sum_{k=1}^{n} x_k^2\right)^{1/2}$$

$$= G'(\|x\|) \cdot \frac{1}{\cancel{2}} \left(\sum x_k^2\right)^{-1/2} \cdot \cancel{2} x_k \underbrace{\frac{\partial}{\partial x_i} x_k}_{= \delta_{ik}}$$

$$= G'(\|x\|) \cdot \frac{x_i}{\|x\|}$$

$$\frac{\partial U}{\partial x_i} = G'(\|x\|) \frac{x_i}{\|x\|} = g(\|x\|)\, x_i \qquad i = 1, \dots n$$

$$G'(\|x\|) = \|x\|\, g(\|x\|)$$

$$G'(s) = s\, g(s) \quad \Rightarrow \quad G(s) = \int_0^s t\, g(t)\, dt + c$$

$$\boxed{f(x) = \frac{1}{\sqrt{1 + \|x\|^2}}\, x}$$

$$g(\|x\|) = \frac{1}{\sqrt{1 + \|x\|^2}}$$

$$g(s) = \frac{1}{\sqrt{1 + s^2}}$$

$$G'(s) = \frac{s}{\sqrt{1+s^2}} = s\, g(s)$$

$$G(s) = \sqrt{1+s^2}$$

$$\boxed{U(x) = \sqrt{1 + \|x\|^2} + c}$$

$$U(x) = \frac{k}{\|x\|} \qquad x \in \mathbb{R}^3 \setminus \{0\}$$

$$F_i = \frac{\partial}{\partial x_i} U = \frac{\partial}{\partial x_i} k \|x\|^{-1} = k \frac{\partial}{\partial x_i} \left(\sum_{k=1}^{3} x_k^2\right)^{-1/2} =$$

$$= k \cdot -\frac{1}{2} \left(\sum x_k^2\right)^{-3/2} \cdot 2 x_k \delta_{ik} = -k \frac{x_i}{\|x\|^3}$$

$$\operatorname{div} v = \sum_{i=1}^{3} \frac{\partial}{\partial x_i} v_i$$

$$\operatorname{div} F = -k \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \frac{x_i}{\|x\|^3} = -k \sum_{i=1}^{3} \frac{1\, \|x\|^3 - x_i \frac{\partial}{\partial x_i} \left(\sum x_k^2\right)^{\frac{1}{2} \cdot 3}}{\|x\|^6}$$

$$= -k \sum_{i=1}^{3} \frac{\|x\|^3 - x_i \, \frac{3}{\cancel{2}} \left(\sum_{k=1}^{3} x_k^2\right)^{1/2} \cdot \cancel{2} x_k \delta_{ik}}{\|x\|^6}$$

$$= -k \sum_{i=1}^{3} \frac{\|x\|^3 - 3 x_i \, x_i \, \|x\|}{\|x\|^6}$$

$$= -k \sum \frac{\|x\|^3 - 3 x_i^2 \|x\|}{\|x\|^6} =$$

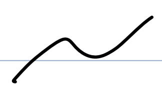
$$= \frac{-k}{\|x\|^6} \left(3 \|x\|^3 - 3 \|x\| \sum x_i^2 \right)$$

$$= -\frac{k}{\|x\|^6} \left(3\|x\|^3 - 3\|x\|^3 \right) = 0$$

$F : \mathbb{R}^3 \to \mathbb{R}^3 \qquad F \in C^2(\mathbb{R}^3)$

$\mathrm{div}(\mathrm{rot}(F)) = 0$ ES. PER CASA

$\int_\gamma F \cdot dx \qquad \gamma$ curva $\qquad \gamma : [a,b] \to \mathbb{R}^3$



Σ superficie $\qquad \phi : T \to \mathbb{R}^3$, $T \subset \mathbb{R}^2$ $\qquad$ (1) $\phi \in C^1$

$T = \overline{\mathring{T}}$ $\qquad T$ limitato

$(\phi_1(u,v), \phi_2(u,v), \phi_3(u,v))$ $\qquad$ (T non è "strano")

$(u_0,v_0) \in T \longmapsto (\phi_1(u_0,v_0), \phi_2(u_0,v_0), \phi_3(u_0,v_0))$

(2) ϕ È INIETTIVA SU $\mathring{T}$

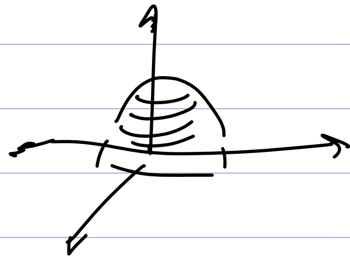
(3) $\operatorname{rank}\left(\frac{\partial \phi}{\partial u}, \frac{\partial \phi}{\partial v}\right) = 2$ in $\mathring{T}$

SUPERFICIE CARTESIANA

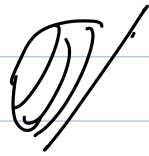
$(\phi_1, \phi_2, \phi_3) = (u, v, f(u,v))$

$\phi = (x, y, \sqrt{1-x^2-y^2})$

$f(x,y) = \sqrt{1-x^2-y^2}$



$(u, v, \sqrt{1-u^2-v^2})$



γ curva