$d(\vec{x},0) = \|\vec{x}\|$ $\qquad G = \{ x^4+y^4+z^4 = 1 \} \subseteq \mathbb{R}^3$

$\vec{x} = (x,y,z)$

$d^2(\vec{x},0) = x^2+y^2+z^2$ $\qquad f(x,y,z) = x^2+y^2+z^2$

$$\mathcal{L}(x,y,z,\lambda) = x^2+y^2+z^2 - \lambda\left(x^4+y^4+z^4-1\right)$$

$$\nabla\mathcal{L} = \left(\frac{\partial}{\partial x}\mathcal{L}, \frac{\partial}{\partial y}\mathcal{L}, \frac{\partial}{\partial z}\mathcal{L}, \frac{\partial}{\partial \lambda}\mathcal{L}\right)$$

$$\begin{cases} \frac{\partial\mathcal{L}}{\partial x} = 2x - 4\lambda x^3 = 2x(1-2\lambda x^2) \\ \frac{\partial\mathcal{L}}{\partial y} = 2y - 4\lambda y^3 = 2y(1-2\lambda y^2) \\ \frac{\partial\mathcal{L}}{\partial z} = 2z - 4\lambda z^3 = 2z(1-2\lambda z^2) \end{cases}$$

$$\frac{\partial\mathcal{L}}{\partial \lambda} = x^4+y^4+z^4-1$$

$\vec{x}_0 = (x,y,z) \in G$

$\vec{x}_1 = (-x,y,z) \in G$

$\|\vec{x}_0\| = \|\vec{x}_1\|$

POSSIAMO RESTRINGERE

$x \geq 0,\ y \geq 0,\ z \geq 0,\ \lambda \geq 0$

$\nabla\mathcal{L} = 0$

1) $x = 0$ oppure $x^2 = \frac{1}{2\lambda}$ $\qquad$ 1) $x = 0$ oppure $x = \frac{1}{\sqrt{2\lambda}}$

2) $y = 0$ oppure $y^2 = \frac{1}{2\lambda}$ $\qquad$ 2) $y = 0$ " $y = \frac{1}{\sqrt{2\lambda}}$

3) $z = 0$ oppure $z^2 = \frac{1}{2\lambda}$ $\qquad$ 3) $z = 0$ " $z = \frac{1}{\sqrt{2\lambda}}$

$P_1 = (x,y,z) = (0,0,0) \notin G$ $\qquad 0^4+0^4+0^4 = 0 \neq 1$

$\underbrace{\left(\frac{1}{\sqrt{2\lambda}},0,0\right)}_{P_2}$ $\qquad \underbrace{\left(0,\frac{1}{\sqrt{2\lambda}},0\right)}_{P_3}$ $\qquad \underbrace{\left(0,0,\frac{1}{\sqrt{2\lambda}}\right)}_{P_4}$

$\frac{1}{(\sqrt{2\lambda})^4} + 0^4 + 0^4 = 1$ $\qquad \frac{\partial\mathcal{L}}{\partial\lambda} = 0$ $\qquad P_2 \in G$

$\frac{1}{4\lambda^2} = 1$ $\qquad \lambda^2 = \frac{1}{4}$ $\qquad \lambda = \frac{1}{2}$

$\left(\frac{1}{\sqrt{\ }}, 0, 0, \frac{1}{2}\right)$ $\qquad \left(0, \frac{1}{\ }, 0, \frac{1}{\ }\right)$ $\qquad \left(0, 0, \frac{1}{\ }, \frac{1}{\ }\right)$

$\left(\frac{1}{\sqrt{2\lambda}}, \ldots, \frac{1}{2}\right) \quad \left(\ldots, \frac{1}{\sqrt{2\lambda}}, \ldots, \frac{1}{2}\right) \quad \left(\ldots, \frac{1}{\sqrt{2\lambda}}, \frac{1}{2}\right)$

$\frac{1}{\sqrt{2\lambda}} = 1$

$(1, 0, 0, 1/2) \qquad (0, 1, 0, 1/2) \qquad (0, 0, 1, 1/2)$

$f(P_2) = f(P_3) = f(P_4) = 1$ Minimo assoluto della distanza al quadrato

$P_5 = \left(\frac{1}{\sqrt{2\lambda}}, \frac{1}{\sqrt{2\lambda}}, 0\right) \qquad P_6 = \left(\frac{1}{\sqrt{2\lambda}}, 0, \frac{1}{\sqrt{2\lambda}}\right) \qquad P_7 = \left(0, \frac{1}{\sqrt{2\lambda}}, \frac{1}{\sqrt{2\lambda}}\right)$

$P_5 \in G \qquad \frac{1}{(\sqrt{2\lambda})^4} + \frac{1}{(\sqrt{2\lambda})^4} + 0 = 1 \iff \frac{2}{(\sqrt{2\lambda})^4} = \frac{2}{4\lambda^2} = \frac{1}{2\lambda^2} = 1$

$\lambda^2 = \frac{1}{2} \Rightarrow \lambda = \frac{1}{\sqrt{2}}$

né max né min

$f(P_5) = f(P_6) = f(P_7) = 2\left(\frac{1}{\sqrt{2\lambda}}\right)^2 = \frac{2}{2\lambda} = \frac{1}{\lambda} = \sqrt{2}$ $\quad P_5, P_6, P_7$ STAZIONARI

$P_8 = \left(\frac{1}{\sqrt{2\lambda}}, \frac{1}{\sqrt{2\lambda}}, \frac{1}{\sqrt{2\lambda}}\right) \qquad \frac{3}{(\sqrt{2\lambda})^4} = \frac{3}{4\lambda^2} = 1 \qquad \lambda^2 = \frac{3}{4}$

$\lambda = \frac{\sqrt{3}}{2}$

$f(P_8) = \frac{3}{2\lambda} = \frac{3}{2\frac{\sqrt{3}}{2}} = \sqrt{3}$ massimo assoluto della distanza al quadrato

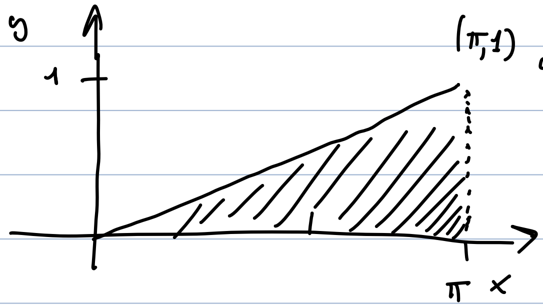
(2) $\displaystyle\int_0^1 \left(\int_{\pi y}^{\pi} \frac{\sin(x)}{x}\,dx\right) dy \qquad f(x,y) = \begin{cases} \frac{\sin(x)}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

$\displaystyle\int_0^1 (\sin I(\pi) - \sin I(\pi y))\,dy \qquad \iint_D f(x,y)\,dx\,dy \quad f$ continua

NORMALE RISPETTO A ASSE Y

$D = \{ (x,y) \in \mathbb{R}^2 \quad 0 \le y \le 1, \quad \pi y \le x \le \pi \}$

$\pi y = x$



$y = \frac{x}{\pi}$

$G(x) = \displaystyle\int_0^x \frac{\sin(t)}{t}\,dt$

$\displaystyle\int^{\pi} \left(\int^{x/\pi} \sin(x)\,dy \right) dx$

$D = \{ (x,y) \in \mathbb{R}^2 : \quad 0 \le x \le \pi \quad 0 \le y \le \frac{x}{\pi} \}$

NORMALE RISPETTO A X

$$\int_0^{\pi}\left(\frac{\sin(x)}{x}\int_0^{x/\pi} 1\,dy\right)dx = \int_0^{\pi}\frac{\sin(x)}{x}\left(y\Big|_0^{x/\pi}\right)dx$$

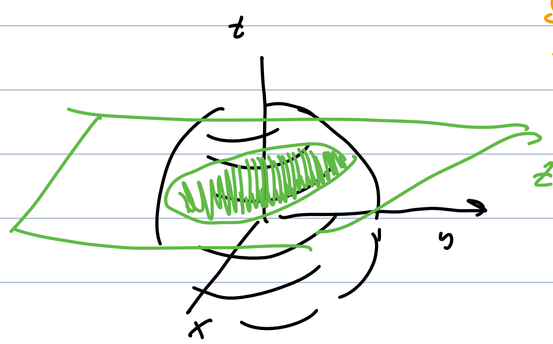
$$= \int_0^{\pi}\frac{\sin(x)}{x}\left(\frac{x}{\pi}-0\right)dx = \int_0^{\pi}\frac{\sin(x)}{\pi}\,dx = \frac{1}{\pi}\left(-\cos(x)\Big|_0^{\pi}\right)$$

$$= \frac{1}{\pi}\left(-\cos(\pi)+\cos(0)\right) = \frac{2}{\pi}$$

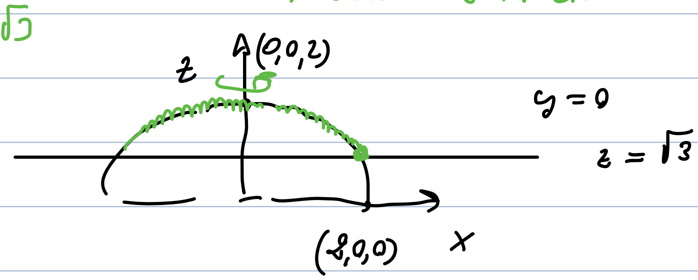
(3)

$$\Sigma = \{(x,y,z)\in\mathbb{R}^3 \quad x^2+y^2+z^2=4 \quad : \quad \sqrt{3}\le z\le 2\}$$

SFERA DI CENTRO $(0,0,0)$
E RAGGIO $R=2$



CALOTTA SFERICA



$$\Sigma = \{(x,y,z)\in\mathbb{R}^3 \quad (x,y)\in T \quad xi+yj+f(x,y)k$$

$\{x^2+y^2\le 1\}$ $\quad xi+yj+\sqrt{4-x^2-y^2}\,k$

$\underbrace{\sqrt{4-x^2-y^2}}_{f(x,y)}$

PUNTI DI INTERSEZIONE FRA SFERA E PIANO $z=\sqrt{3}$

$$x^2+y^2 = 4-z^2 = 4-(\sqrt{3})^2 = 1$$

$$A(\Sigma) = \iint_T 1\cdot\sqrt{1+f_x^2+f_y^2}\,dx\,dy \qquad \sqrt{1+|\nabla f|^2}\,dx\,dy = dS$$

$$f_x = \frac{1}{2}\frac{-2x}{\sqrt{4-x^2-y^2}} = -\frac{x}{\sqrt{4-x^2-y^2}}$$

$$= \iint_T\sqrt{1+\frac{x^2}{4-x^2-y^2}+\frac{y^2}{4-x^2-y^2}}$$

$$= \iint_T\sqrt{\frac{4-\cancel{x^2}-\cancel{y^2}+\cancel{x^2}+\cancel{y^2}}{4-x^2-y^2}}\,dx\,dy = \iint_T\frac{2}{\sqrt{4-(x^2+y^2)}}\,dx\,dy$$

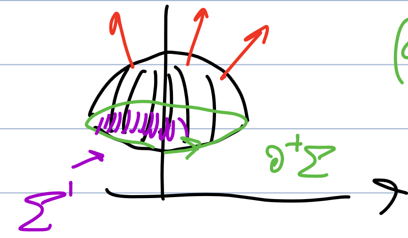
$$= \int_0^{2\pi}d\theta\int_0^1 d\rho\,\rho\frac{2}{\sqrt{4-\rho^2}} = -2\pi\int_0^1\frac{-2\rho}{\sqrt{4-\rho^2}}\,d\rho = -4\pi\sqrt{4-\rho^2}\Big|_0^1$$

$$\frac{d}{d\rho}\sqrt{4-\rho^2} = \frac{d}{d\rho}(4-\rho^2)^{1/2} = \frac{1}{2}(4-\rho^2)^{-1/2}\cdot(-2\rho)$$

$$= -4\pi\left(\sqrt{4-3}-\sqrt{4}\right) = -4\pi\left(\sqrt{3}-2\right) = 4\pi\left(2-\sqrt{3}\right)$$

$(\cos(t), \sin(t))$

$\vec{F} = (0,0,1)$ FLUSSO



$$\iint_{\Sigma} \vec{F}\cdot\hat{n}\, dS \overset{\text{"}}{=} \iint_{\Sigma'} \vec{F}\cdot\hat{n}\, d\vec{S}$$

$$T \Downarrow \quad (x,y) \longrightarrow x i + y j + f(x,y) k$$

$$\det\begin{vmatrix} i & j & k \\ 1 & 0 & f_x \\ 0 & 1 & f_y \end{vmatrix} = -f_x i - f_y j + k$$

$$\hat{n} = \frac{-f_x i - f_y j + k}{\sqrt{1+f_x^2+f_y^2}}$$

$$\iint_T (0i+0j+1k)\cdot\frac{(-f_x i - f_y j + k)}{\cancel{\sqrt{1+|\nabla f|^2}}}\cancel{\sqrt{1+|\nabla f|^2}}\, dx\,dy$$

$$\iint_T 1\, dx\,dy = \pi$$

$\vec{F} = (0,0,1)$ $\qquad F = \text{rot}\, G = (G_1, G_2, 0)$

$$\text{rot}\, G = \det\begin{pmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ G_1 & G_2 & 0 \end{pmatrix} = (-\partial_z G_2, \partial_z G_1, \partial_x G_2 - \partial_y G_1)$$

$$\phantom{\text{rot}\, G = } \quad (0 \;,\; 0 \;,\; 1)$$

$$\boxed{\begin{matrix} G_1 = G_1(x,y) \\ G_2 = G_2(x,y) \end{matrix}}$$

$G_2 = x \qquad G_1 = 0$

$$\vec{F} = \text{rot}\,(0,x,0)$$

$$\iint \vec{F}\cdot\hat{n}\, dS = \iint \text{rot}\, G\cdot\hat{n}\, dS = \int G\cdot\hat{\tau}\, ds$$

Σ Σ $\partial^+\Sigma$

$$= \int_0^{2\pi} \left(0 + \cos(t)\cos(t) + 0\right) dt = \int_0^{2\pi} \cos^2(t)\, dt = \pi$$