Soluzione Esercizio 2

1)

$$E_s = \frac{1}{2} E_{s1} + \frac{1}{2} E_{s2} = \frac{1}{2} E_p + \frac{9}{2} E_p$$

$$E_p = \int_{-\frac{1}{T}}^{+1/T} (1 - |fT|)\, df = \frac{1}{T}$$

$$E_s = \frac{9}{2T}$$

2) $G(f) = P(f) \cdot C(f)\, G_R(f) = (1 - |fT|)(1 + j2\pi fT) \quad |fT| \leq 1$

La condizione di Nyquist

$$\sum_k G(f - k/T) = T$$

basta verificarlo in un periodo $[0; 1/T]$

~~Nel~~ Nell'intervallo $[0 - 1/T]$

$$\sum_k G(f - k/T) = G(f) + G(f - 1/T) =$$

$$= (1 - fT)(1 + j2\pi fT) + (1 + (f - \tfrac{1}{T})T)(1 + j2\pi (f - \tfrac{1}{T})T) =$$

$$= 1 + j2\pi fT - fT(1 + j2\pi fT) + (1 + fT - 1)(1 + j2\pi fT - j2\pi)$$

$$= 1 + \cancel{j2\pi fT} - \cancel{fT(1 + j2\pi fT)} + \cancel{fT(1 + j2\pi fT)} - \cancel{j2\pi fT} = 1$$

$$\sum_k G(f - k/T) = 1 \quad \Rightarrow \quad g(0) = \frac{1}{T}$$

3) $$P_{nu} = \frac{N_0}{2} \int_{-\frac{1}{T}}^{1/T} \left(1 - |fT|\right) df = \frac{N_0}{2T}$$

4) $$P_e(b) = \frac{1}{2} Q\left(\frac{\lambda - 1}{\sigma_n}\right) + \frac{1}{2} Q\left(\frac{3 - \lambda}{\sigma_n}\right) =$$

$$\cancel{= \frac{1}{2} Q\left(\lambda - 1 \sqrt{2T}\right)}$$

$$= \frac{1}{2} Q\left((\lambda - 1)\sqrt{\frac{2T}{N_0}}\right) + \frac{1}{2} Q\left(3 - \lambda \sqrt{\frac{2\pi}{N_0}}\right)$$

5) $$\frac{\partial P_e(b)}{\partial \lambda} = 0$$

$$\frac{1}{2} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(\lambda - 1)^2}{2\sigma_n^2}} \cdot 1 + \frac{1}{2} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(3 - \lambda)^2}{2\sigma_n^2}} \cdot (-1) = 0$$

$$e^{-(\lambda - 1)^2 / 2\sigma_n^2} = e^{-(3 - \lambda)^2 / 2\sigma_n^2}$$

$$(\lambda-1)^2 = (3-\lambda)^2$$

$$\lambda^2+1-2\lambda = 9+\lambda^2-6\lambda$$

$$4\lambda = 8$$

$$\lambda = 2$$