Es. 1

15-02-2011

$z(t) = z_1(t) + z_2(t)$

$z_1(t) = \mathrm{rect}\left(\frac{t - T/2}{T}\right) e^{-\frac{t}{\tau}}$

$z_2(t) = \mathrm{sinc}\left(\frac{t}{T}\right) \cos\left(2\pi f_0 t + \theta\right)$

$T \gg 1/f_0 \quad , \quad \tau \gg T$

Calcolo dello spettro

$Z(f) = Z_1(f) + Z_2(f)$

$Z_1(f) \overset{TCF}{\Longleftrightarrow} z_1(t) \quad , \quad Z_2(f) \overset{TCF}{\Longleftrightarrow} z_2(t)$

$$e^{-\frac{t}{\tau}} u(t) \overset{TCF}{\Longleftrightarrow} \int_0^{\infty} e^{-\frac{t}{\tau}} e^{-j2\pi f t} \, dt$$

$$e^{-\frac{t}{\tau}} \mathrm{rect}\left(\frac{t - T/2}{T}\right) \overset{TCF}{\Longleftrightarrow} \int_0^{T} e^{-\frac{t}{\tau}} e^{-j2\pi f t} \, dt =$$

$$= \int_0^T e^{-\left(\frac{1}{\tau} + j2\pi f\right)t} \, dt = \int_0^T e^{-\alpha t} \, dt =$$

$\alpha \triangleq \frac{1}{\tau} + j2\pi f$

$$= -\frac{1}{\alpha} e^{-\alpha t} \Big|_0^T = -\frac{1}{\alpha} e^{-\alpha T} + \frac{1}{\alpha} =$$

$$= \frac{1}{\alpha}\left(1 - e^{-\alpha T}\right) = \frac{1}{\frac{1}{T}+j2\pi f}\left[1 - e^{-\left(\frac{1}{T}+j2\pi f\right)T}\right]$$

$$Z_2(f) = T\,\mathrm{rect}\left(\frac{f}{\frac{1}{T}}\right) \otimes \left[\frac{e^{j\vartheta}}{2}\,\delta(f-f_0) + \frac{e^{-j\vartheta}}{2}\,\delta(f+f_0)\right] =$$

$$= \frac{T}{2} e^{j\vartheta}\,\mathrm{rect}\left(\frac{f-f_0}{\frac{1}{T}}\right) + \frac{T}{2} e^{-j\vartheta}\,\mathrm{rect}\left(\frac{f+f_0}{\frac{1}{T}}\right)$$

Calcolo dell'energia

$$E_z = \int_{-\infty}^{+\infty} |z(t)|^2\, dt = \int_{-\infty}^{+\infty} |Z(f)|^2\, df$$

$z(t)$ è reale

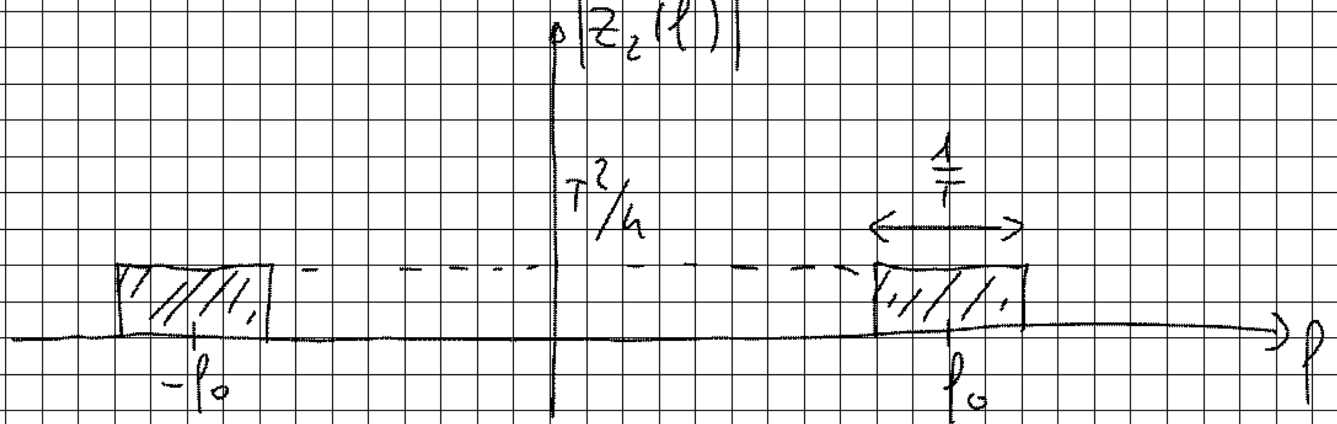
$$E_z = \int_{-\infty}^{+\infty} \left[z_1(t) + z_2(t)\right]^2 dt =$$

$$= \underbrace{\int_{-\infty}^{+\infty} z_1^2(t)\, dt}_{E_{z_1}} + \underbrace{\int_{-\infty}^{+\infty} z_2^2(t)\, dt}_{E_{z_2}} + \underbrace{2\int_{-\infty}^{+\infty} z_1(t)\, z_2(t)\, dt}_{E_{z_{1,2}}}$$

$$E_{z_1} = \int_{-\infty}^{+\infty} \left[\mathrm{rect}\left(\frac{t-T/2}{T}\right) e^{-\frac{t}{T}}\right]^2 dt = \int_{-\infty}^{+\infty} \mathrm{rect}\left(\frac{t-T/2}{T}\right) e^{-\frac{2t}{T}}\, dt$$

$$= \int_0^T e^{-\frac{2t}{T}}\, dt = -\frac{T}{2} e^{-\frac{2t}{T}}\Big|_0^T = -\frac{T}{2} e^{-\frac{2T}{T}} + \frac{T}{2}$$

$$E_{z_2} = \int_{-\infty}^{+\infty} z_2^2(t)\, dt = \int_{-\infty}^{+\infty} |Z_2(f)|^2\, df = 2\,\frac{1}{T}\cdot\frac{T^2}{4} = \frac{T}{2}$$



$$E_{z_{12}} = 2\int_{-\infty}^{+\infty} z_1(t)\, z_2(t)\, dt = 2\int_{-\infty}^{+\infty} z_1(t)\, z_2^*(t)\, dt$$

$$= 2\int_{-\infty}^{+\infty} Z_1(f)\, Z_2^*(f)\, df$$

$$Z_1(f) = \frac{\tau}{1+j2\pi f\tau}\left[1 - e^{-\left(\frac{T}{\tau}+j2\pi fT\right)}\right] \overset{\tau \gg t}{\simeq}$$

$$\simeq \frac{\tau}{1+j2\pi f\tau}\left(1 - e^{-j2\pi fT}\right)$$

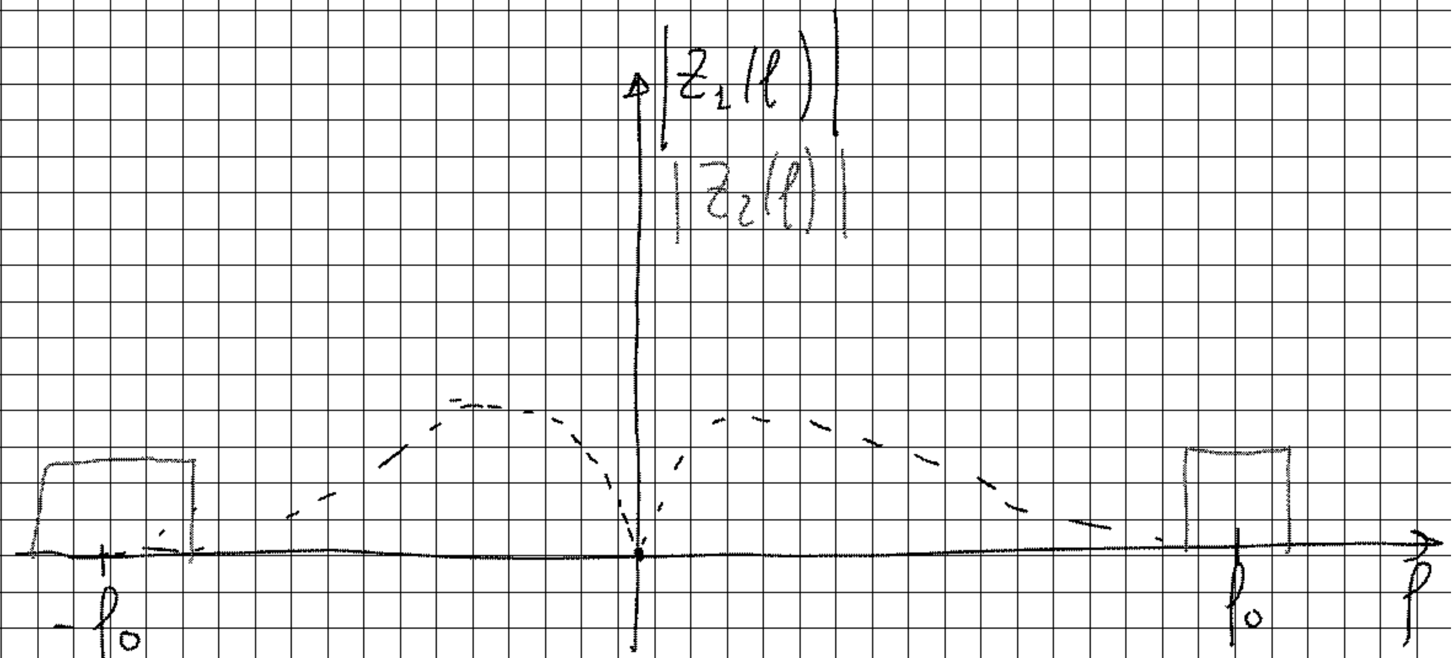
$$|Z_1(f)| = \sqrt{Z_1(f)\, Z_1^*(f)} =$$

$$\left[\frac{\tau}{1+j2\pi f\tau}\cdot\frac{\tau}{1-j2\pi f\tau}\left(1-e^{-j2\pi fT}\right)\left(1-e^{j2\pi fT}\right)\right]^{\frac{1}{2}}$$

$$\left[\frac{\tau^2}{1+4\pi^2 f^2\tau^2}\left(1-e^{j2\pi fT}-e^{-j2\pi fT}+1\right)\right]^{\frac{1}{2}} =$$

$$\left[\frac{T^2}{1+4\pi^2 f^2 T^2}\left[2-2\left(\frac{e^{j2\pi fT}+e^{-j2\pi fT}}{2}\right)\right]\right]^{1/2} =$$

$$=\left[\frac{T^2}{1+4\pi^2 f^2 T^2}\left(2-2\cos 2\pi fT\right)\right]^{\frac{1}{2}}$$



$$|Z_1(f)|\,|Z_2(f)| \simeq 0$$

$$Z_1(f)\,Z_2^*(f) = |Z_1(f)|e^{j\varphi_1(f)}\cdot|Z_2(f)|e^{-j\varphi_2(f)} =$$

$$= \underbrace{|Z_1(f)|\,|Z_2(f)|}_{\simeq 0} e^{-j(\varphi_1(f)-\varphi_2(f))} \simeq 0$$

$$\Rightarrow E_{z_{12}} \simeq 0$$