Soluzione Compitino - Fila C

$\boxed{\text{Es. 3}}$

$$x(t) = x_1(t) + x_2(t)$$

$$T_0 = 3T$$

$$x_1(t) = 1$$

$$x_2(t) = \sum_{k=-\infty}^{+\infty} \cos\left[\frac{2\pi(t-kT_0)}{4T}\right] \mathrm{rect}\left(\frac{t-kT_0}{2T}\right)$$

$$X_n = X_{1n} + X_{2n}$$

$$X_{1n} = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$$

$$X_{2n} = \frac{1}{T_0}\int_{-T}^{T} \cos\left(\frac{2\pi t}{4T}\right) e^{-j2\pi n f_0 t}\, dt$$

$$= \frac{1}{2T_0}\int_{-T}^{T} e^{j\frac{2\pi t}{4T}} e^{-j2\pi n f_0 t}\, dt \; +$$

$$+ \frac{1}{2T_0}\int_{-T}^{T} e^{-j\frac{2\pi t}{4T}} e^{-j2\pi n f_0 t}\, dt =$$

$$X_{2n} = \frac{1}{2T_0}\int_{-T}^{T} e^{-j2\pi\left(nf_0 - \frac{1}{4T}\right)t}\,dt \; +$$

$$+ \frac{1}{2T_0}\int_{-T}^{T} e^{-j2\pi\left(nf_0 + \frac{1}{4T}\right)t}\,dt$$

$$= \frac{1}{2T_0}\int_{-T}^{T} e^{-j2\pi\left(\frac{n}{3T} - \frac{1}{4T}\right)t}\,dt + \frac{1}{2T_0}\int_{-T}^{T} e^{-j2\pi\left(\frac{n}{3T} + \frac{1}{4T}\right)t}\,dt =$$

$$= \frac{1}{2T_0}\,\frac{1}{-j2\pi\left(\frac{4n-3}{12T}\right)}\, e^{-j2\pi\left(\frac{4n-3}{12T}\right)}\Bigg|_{-T}^{T} \; +$$

$$+ \frac{1}{2T_0}\,\frac{1}{-j2\pi\,\frac{4n+3}{12T}}\, e^{-j2\pi\,\frac{4n+3}{12T}}\Bigg|_{-T}^{T} =$$

$$= 2\,\frac{j}{2\pi(4n-3)}\left[e^{-j\frac{\pi}{6}(4n-3)} - e^{j\frac{\pi}{6}(4n-3)}\right]$$

$$+ 2\,\frac{j}{2\pi(4n+3)}\left[e^{-j\frac{\pi}{6}(4n+3)} - e^{j\frac{\pi}{6}(4n+3)}\right]$$

$$X_{2n} = \frac{2}{\pi(4n-3)} \left[\frac{e^{j\frac{\pi}{6}(4n-3)} - e^{-j\frac{\pi}{6}(4n-3)}}{2j} \right] +$$

$$+ \frac{2}{\pi(4n+3)} \left[\frac{e^{j\frac{\pi}{6}(4n+3)} - e^{j\frac{\pi}{6}(4n+3)}}{2j} \right]$$

$$= \frac{\sin\left[\frac{\pi}{6}(4n-3)\right]}{\frac{\pi}{2}(4n-3)} + \frac{\sin\left[\frac{\pi}{6}(4n+3)\right]}{\frac{\pi}{2}(4n+3)} =$$

$$= \frac{1}{3}\operatorname{sinc}\left(\frac{4n-3}{6}\right) + \frac{1}{3}\operatorname{sinc}\left(\frac{4n+3}{6}\right)$$

$$\boxed{X_n = \delta[n] + \frac{1}{3}\left[\operatorname{sinc}\left(\frac{4n-3}{6}\right) + \operatorname{sinc}\left(\frac{4n+3}{6}\right)\right]}$$

$\boxed{E_x = \infty}$ segnale periodico

$$P_x = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |x(t)|^2 dt = \frac{1}{3T}\int_{-T}^{T}\left[1+\cos\left(\frac{2\pi t}{4T}\right)\right]^2 dt +$$

$$+\frac{1}{3T}\int_{-\frac{3}{2}T}^{-T} 1\, dt + \frac{1}{3T}\int_{+T}^{+\frac{3}{2}T} 1\, dt =$$

$$= \frac{1}{3T}\int_{-T}^{T} \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi t}{2T}\right) + 1 + 2\cos\left(\frac{2\pi t}{4T}\right) dt + \frac{1}{3}$$

$$= \frac{1}{3T}\left[T + 0 + 2T\right] + \frac{1}{3T}\,\frac{4T}{2\pi}\,\sin\frac{2\pi t}{4T}\Big|_{-T}^{T} + \frac{1}{3} = 1 + \frac{2}{3\pi}(2) + \frac{1}{3} = \boxed{\frac{4}{3} + \frac{4}{3\pi} = P_x}$$

$$\boxed{x_{eff} = \sqrt{P_x} = \sqrt{\frac{4}{3} + \frac{4}{3\pi}}}$$

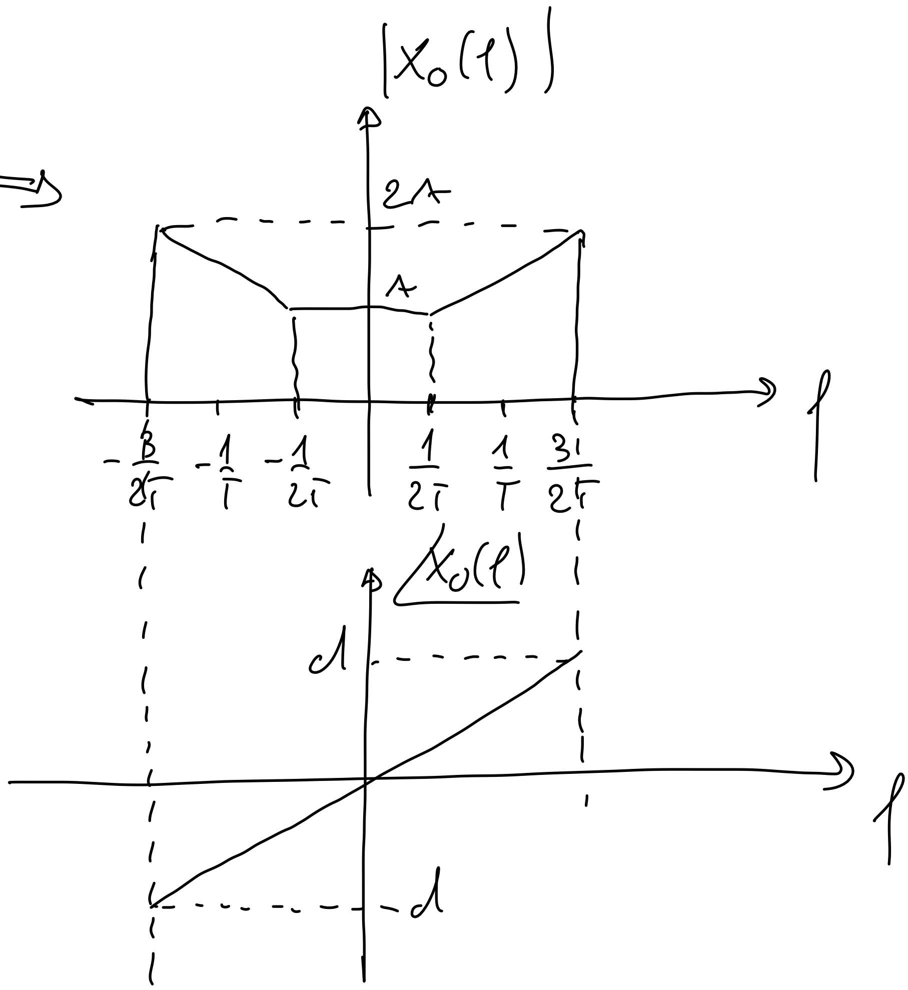
$$X_m = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\, dt = \frac{1}{3T}\int_{-T}^{T}\left[1+\cos\left(\frac{2\pi t}{4T}\right)\right] dt + 2\frac{1}{3T}\int_{-\frac{3}{2}T}^{T} 1\, dt$$

$$= \frac{1}{3T}\left[2T + \frac{4T}{2\pi}\left|\sin\left(\frac{2\pi t}{4T}\right)\right|_{-T}^{T}\right] + \frac{1}{3} = 1 + \frac{2}{3\pi}\cdot 2 =$$

$$= \boxed{1 + \frac{4}{3\pi} = x_m}$$

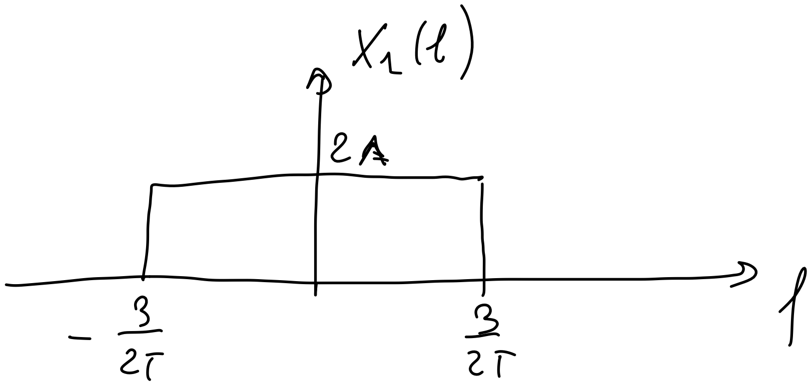
Es. 4

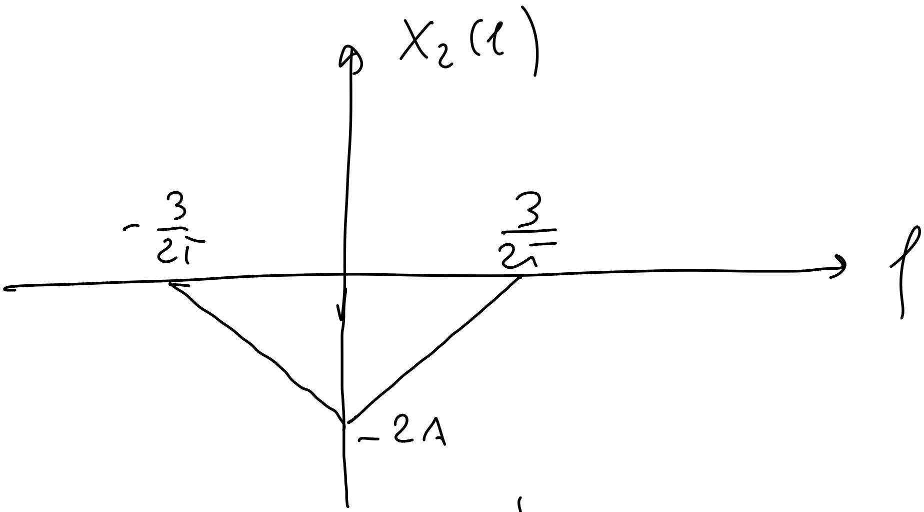
Si definisce $X_0(f) \Rightarrow$

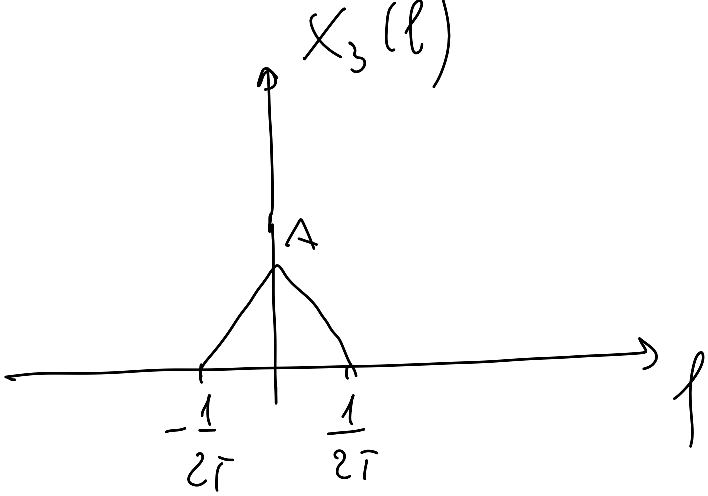


$$X_0(f) = |X_0(f)|\, e^{j \angle X_0(f)}$$

$$|X_0(f)| = X_1(f) + X_2(f) + X_3(f)$$







$$x_1(t) = \text{ATCF}\left[X_1(f)\right]$$

$$x_2(t) = \text{ATCF}\left[X_2(f)\right]$$

$$x_3(t) = \text{ATCF}\left[X_3(f)\right]$$

$$X_0(f) = \left[X_1(f) + X_2(f) + X_3(f) \right] e^{j \angle X_0(f)}$$

$$\angle X_0(f) = 2\pi f t_0$$ bisogna determinare t_0

$$\left. \angle X_0(f) \right|_{f = \frac{3}{2T}} = d = 2\pi \frac{3}{2T} t_0$$

$$\Rightarrow t_0 = \frac{Td}{3\pi}$$

$$\angle X_0(f) = 2\pi f \frac{Td}{3\pi}$$

$$x_0(t) = x_1(t+t_0) + x_2(t+t_0) + x_3(t+t_0)$$

$$X_1(f) = 2A \, \mathrm{rect}\left(\frac{f}{3/T}\right) \Leftrightarrow x_1(t) = \frac{6A}{T} \mathrm{sinc}\left(\frac{3t}{T}\right)$$

$$X_2(f) = -2A \left(1 - \frac{|f|}{3/2T}\right) \mathrm{rect}\left(\frac{f}{3/T}\right) \Leftrightarrow x_2(t) = -\frac{3A}{T} \mathrm{sinc}^2\left(\frac{3t}{2T}\right)$$

$$X_3(f) = A \left(1 - \frac{|f|}{1/T}\right) \mathrm{rect}\left(\frac{f}{2/T}\right) \Leftrightarrow x_3(t) = \frac{A}{T} \mathrm{sinc}^2\left(\frac{t}{T}\right)$$

$$X(f) = X_0\left(f - \frac{9}{2T}\right) + X_0\left(f + \frac{9}{2T}\right)$$

$$x(t) = x_0(t)\, e^{j2\pi \frac{9}{2T} t} + x_0(t)\, e^{-j2\pi \frac{9}{2T} t}$$

$$= 2x_0(t) \cos\left(9\pi \frac{t}{T}\right)$$

$$\boxed{\begin{aligned} x(t) &= 2\left[x_1(t+t_0) + x_2(t+t_0) + x_3(t+t_0)\right]\cos\left(9\pi\frac{t}{T}\right) \\ t_0 &= \frac{Td}{3\pi} \\ x_1(t) &= \frac{6A}{T}\,\mathrm{sinc}\left(\frac{3t}{T}\right) \\ x_2(t) &= -\frac{3A}{T}\,\mathrm{sinc}^2\left(\frac{3t}{2T}\right) \\ x_3(t) &= \frac{A}{T}\,\mathrm{sinc}^2\left(\frac{t}{T}\right) \end{aligned}}$$