1) Calcolare la DSP di $X(t) \Rightarrow S_X(f)$
e la sua potenza P_X

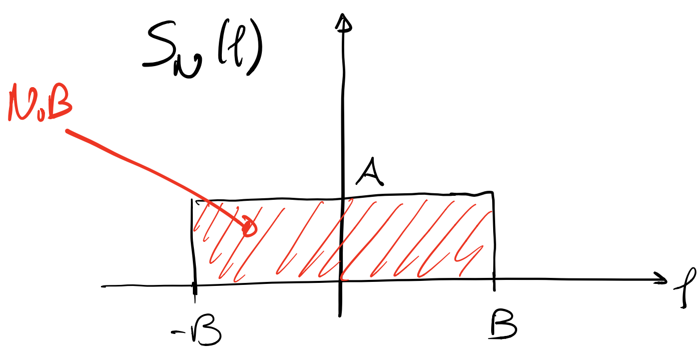
2) Si campioni a $t=0$ e si scriva la ddp di X

3) $$X \longrightarrow \boxed{(\cdot)^2} \longrightarrow Y = X^2$$

Si calcoli la ddp di Y

Soluzione

1) $S_X(f) = S_N(f)\,|H(f)|^2$



$R_N(\tau) = 2AB\,\text{sinc}(2B\tau)$

$R_X(\tau) = R_N(\tau) \otimes h(\tau) \otimes h(-\tau)$

$$A = ? \quad \Rightarrow \quad P_N = N_0 B = \int_{-\infty}^{+\infty} S_N(f)\,df = 2A\cancel{B} = N_0 \cancel{B}$$

$$\Downarrow$$

$$A = \frac{N_0}{2}$$

$$S_N(f) = \frac{N_0}{2}\,\text{rect}\left(\frac{f}{2B}\right)$$

$$H(f) = \int_{-\infty}^{+\infty} e^{-2t} u(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_0^{+\infty} e^{-(2+j2\pi f)t}\, dt$$

$$= \frac{1}{2+j2\pi f}$$

$$|H(f)|^2 = H(f)\,H^*(f) = \frac{1}{2+j2\pi f}\cdot\frac{1}{2-j2\pi f} = \frac{1}{4+4\pi^2 f^2}$$

$$\boxed{S_x(f) = \frac{N_0}{2}\,\frac{1}{4+4\pi^2 f^2}\,\mathrm{rect}\left(\frac{f}{2B}\right)}$$

$$P_x = \int_{-\infty}^{+\infty} S_x(f)\,dx = \frac{N_0}{2}\int_{-B}^{B}\frac{1}{4+4\pi^2 f^2}\,df$$

$$\int_{-\infty}^{x}\frac{1}{1+z^2}\,dz = \mathrm{arctg}\,x$$

$$\Downarrow$$

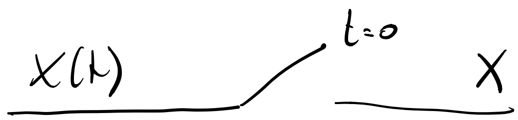
$$\int_{-\infty}^{x}\frac{1}{1+(\alpha z)^2}\,dz = \frac{1}{\alpha}\,\mathrm{arctg}\,\frac{x}{\alpha}$$

$$P_x = \frac{N_0}{8}\int_{-B}^{B}\frac{1}{1+\pi^2 f^2}\,df = \frac{N_0}{8}\,\frac{1}{\pi}\,\mathrm{arctg}\,\frac{f}{\pi}\Bigg|_{-B}^{B}$$

$$= \frac{N_0}{8\pi}\left[\mathrm{arctg}\left(\frac{B}{\pi}\right) - \mathrm{arctg}\left(-\frac{B}{\pi}\right)\right] = \frac{N_0}{4\pi}\,\mathrm{arctg}\left(\frac{B}{\pi}\right)$$

$$\boxed{P_X = \frac{N_0}{4\pi} \, \text{arctg}\left(\frac{B}{\pi}\right)}$$

2)



$X(t)$ è proc. Gaussiano

X è una V.A. Gaussiana

$$X \in \mathcal{N}(\eta_X, \sigma_X^2)$$

$$\Downarrow$$

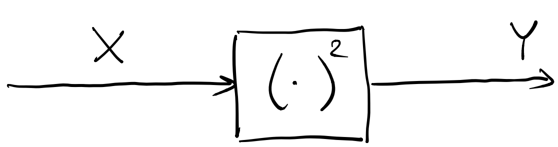
$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\frac{(x-\eta_X)^2}{2\sigma_X^2}}$$

$\eta_X(t) = 0 \Rightarrow \eta_X = 0$ proc è bianco

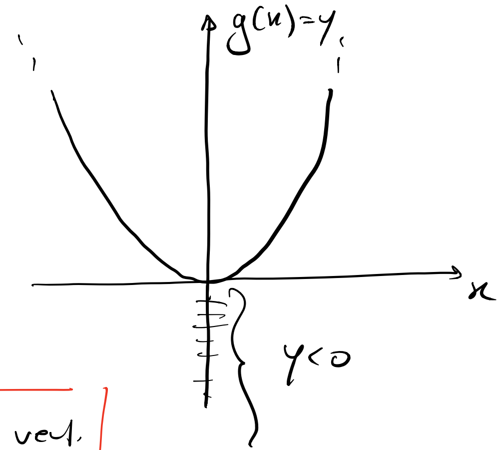
$$\sigma_X^2 = P_X - \eta_X^2 = P_X$$

$$\boxed{f_X(x) = \frac{1}{\sqrt{2\pi P_X}} e^{-\frac{x^2}{2P_X}}}$$

3)



$g(x) = x^2$



$f_Y(y) = ?$

$\boxed{y<0 \Rightarrow f_Y(y) = 0}$

$\boxed{y=0 \Rightarrow g'(x) = 0 \Rightarrow \text{asintoto vert.}}$

$y>0 \Rightarrow \text{due sol} \Rightarrow \pm\sqrt{y}$

$$f_Y(y) = \frac{f_X(\sqrt{y})}{|g'(\sqrt{y})|} + \frac{f_X(-\sqrt{y})}{|g'(-\sqrt{y})|} = \frac{\frac{1}{\sqrt{2\pi P_x}} e^{-\frac{y}{N_0}}}{2\sqrt{y}} + \frac{\frac{1}{\sqrt{2\pi P_x}} e^{-\frac{y}{N_0}}}{|2(-\sqrt{y})|}$$

$g'(x) = 2x$

$$= \boxed{\frac{1}{\sqrt{2\pi P_x\, y}} e^{-\frac{y}{N_0}} \qquad y>0}$$

$$\lim_{y \to 0^+} f_Y(y) = +\infty$$