Antitrasf Z:

$$Y(s) = \frac{K_1}{s+P_1} + \frac{K_2}{s+P_2} + \dots \quad \text{nel T.C.}$$

$$\text{Se} \quad y(z) = \frac{2z^2+2z}{z^2+2z-3} \Rightarrow \frac{Y(z)}{z} = \frac{2z+2}{z^2+2z-3} = \frac{A}{z+3} + \frac{B}{z-1}$$

Radici (-3,1)

• preparare la $y(z)$ dividendo tutto per z ed antitrasf $\frac{Y(z)}{z}$

$$\Rightarrow A = (z+3)\frac{y(z)}{z}\Big|_{z=-3} = \cancel{(z+3)}\frac{(2z+2)}{\cancel{(z+3)}(z-1)} = \frac{-4}{-4} = 1$$

$$\Rightarrow B = (z-1)\frac{y(z)}{z}\Big|_{z=1} = \cancel{(z-1)}\frac{(2z+2)}{(z+3)\cancel{(z-1)}} = 1$$

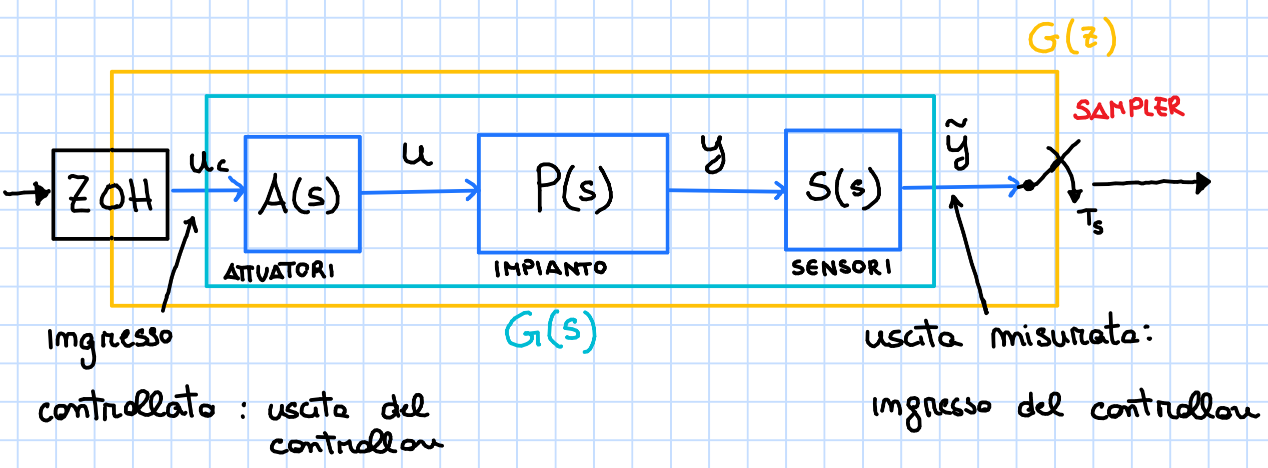
$$\frac{y(z)}{z} = \frac{1}{z+3} + \frac{1}{z-1} = y(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

$$\alpha^k \xrightarrow{z} \frac{z}{z-\alpha}$$

$$Y_k = Z^{-1}\{Y(z)\} = -3^k H(k) + 1 \cdot H(k)$$

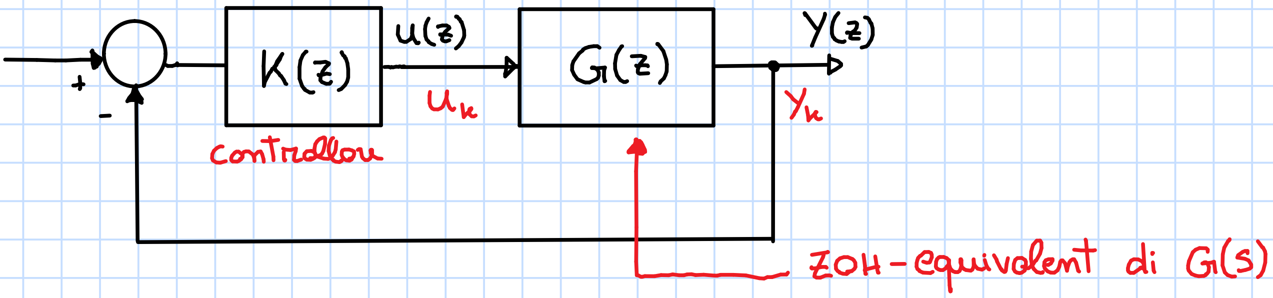
PROGETTAZIONE DI SISTEMI DI CONTROLLO

IMPORTANTE !



$G(s) \rightarrow G(z)$

$A, B, C, D \rightarrow F, G, C, D$ con $F = e^{AT_s}$ $\quad G = \int_0^{T_s} e^{A\tau} B \, d\tau$



Ex: PROGETTARE CONTROLLORE PROPORZ $K(z) = K$ PER L'IMPIANTO

$$G(z) = \frac{1}{z(z-1)} \qquad \text{F.d.t.} \rightarrow \frac{G(k)}{1+G(k)}$$

Calcolo la f.d.t. a ciclo chiuso

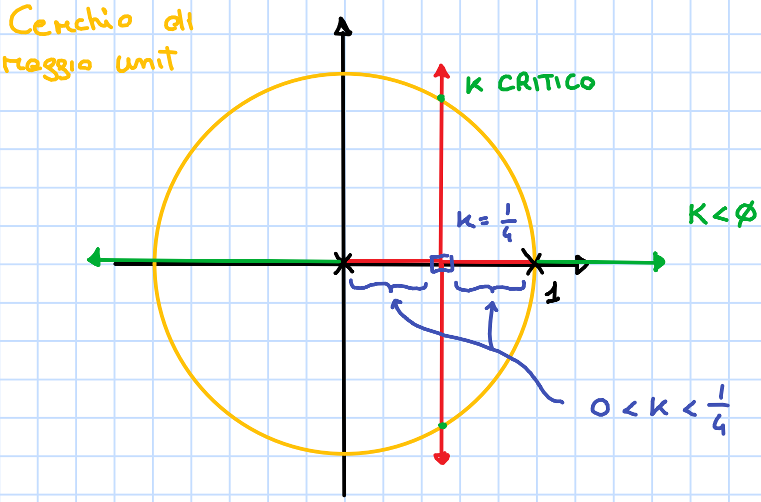
$$G_{cc}(z) = \frac{G(z)K(z)}{1+G(z)K(z)} = \frac{K/z(z-1)}{1+K/z(z-1)} = \frac{K}{z(z-1)+K} = \frac{K}{z^2-z+K}$$

al variare di K il polo del sist a ciclo chiuso si spostano

$$z_{1,2} = \frac{1 \pm \sqrt{1-4K}}{2} \qquad K=0 \Rightarrow \begin{cases} z=0 \\ z=1 \end{cases}$$

gli stessi del ciclo aperto

Disegno il luogo delle radici (al variare di un param, come cambiano)



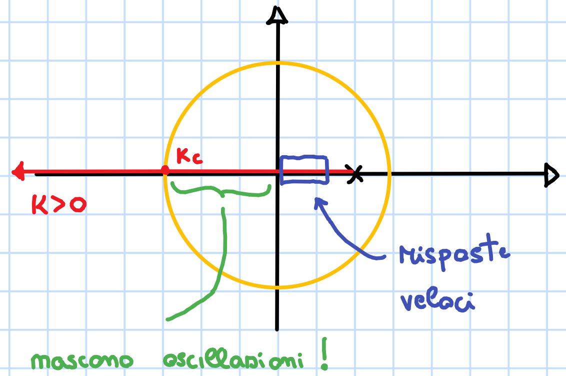
$$z_{1,2} = \frac{1 \pm \sqrt{1-4K}}{2}$$

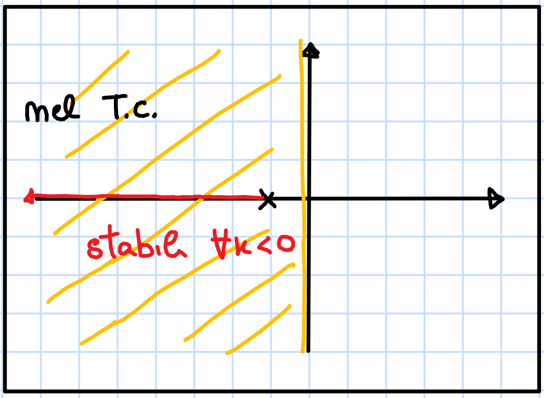
finchè $k \le \frac{1}{4}$ restano reali poi vanno nella parte complessa!

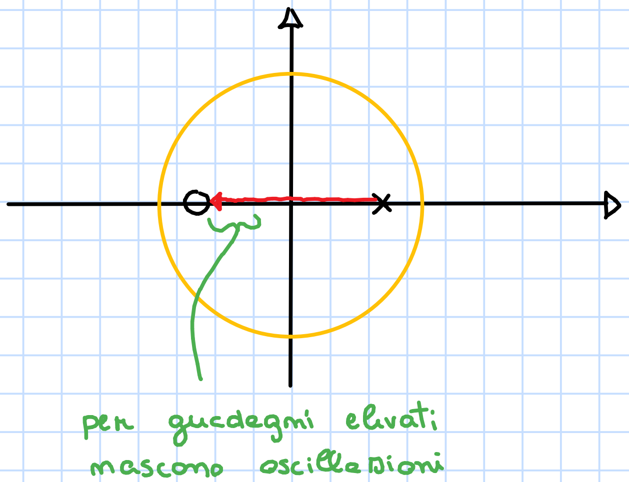
SISTEMA ASINTOT STABILE A CICLO CHIUSO

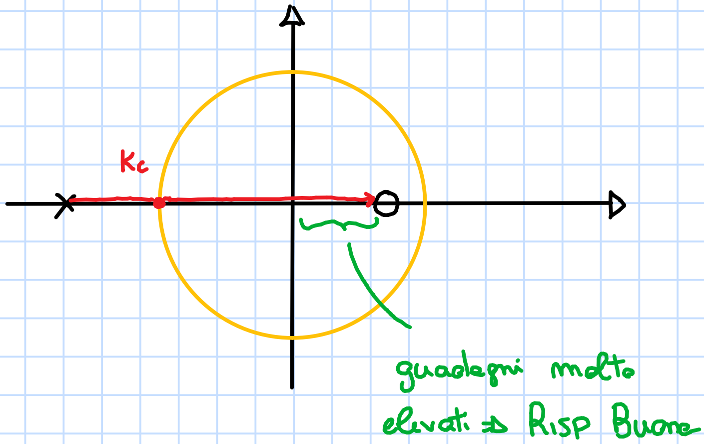
$$\begin{cases} K = 1/4 & \text{2 radici Reali coincidenti} \\ 0 < K < \frac{1}{4} & \text{2 radici Reali differenti} \\ 1/4 < K < K_{critico} & \text{2 radici complesse e } |z| < 1 \end{cases}$$

Esempi di luoghi

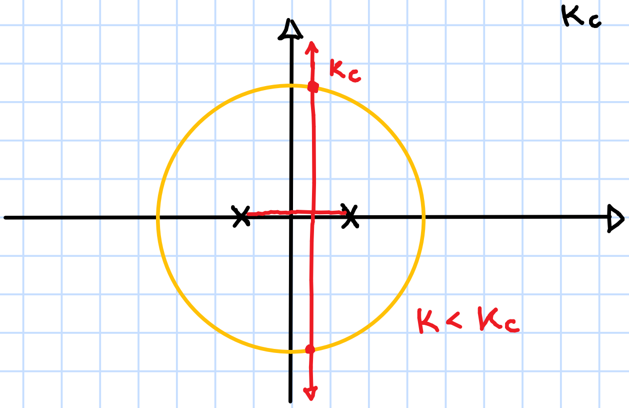


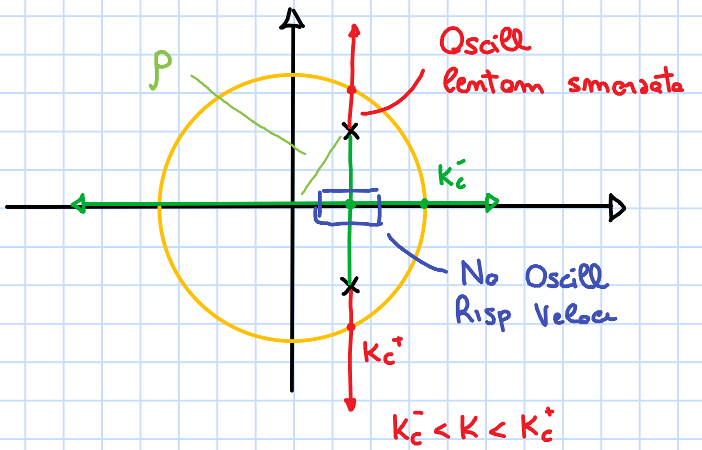


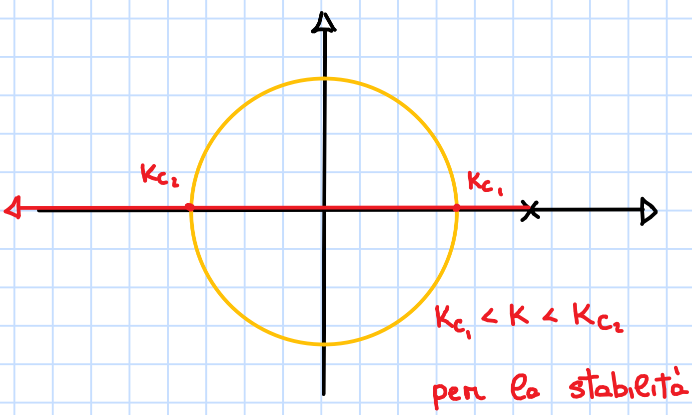


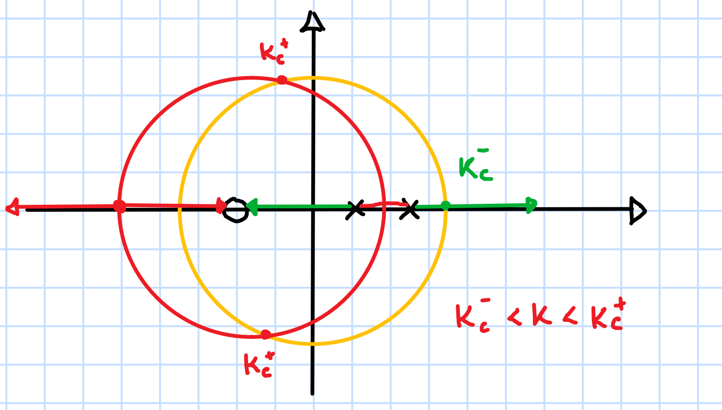


SE CI SONO PIÙ POLI
CHE ZERI C'È SEMPRE
K_c

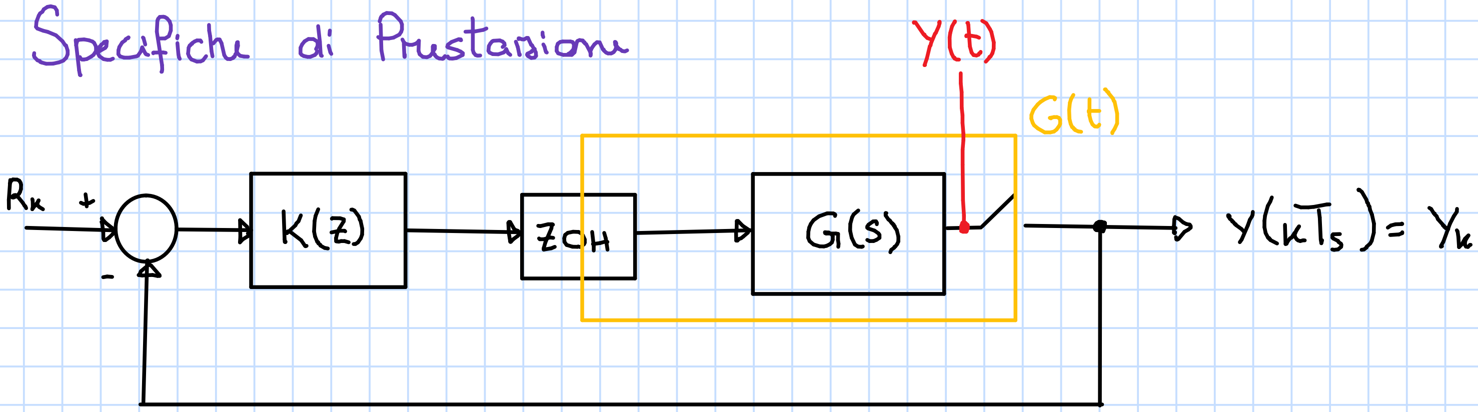






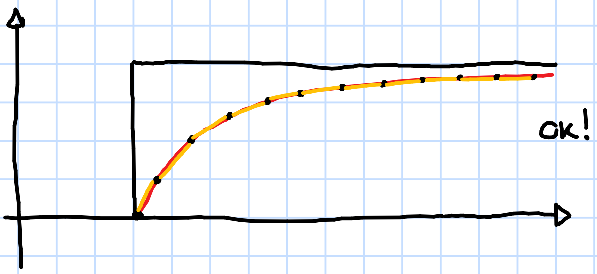


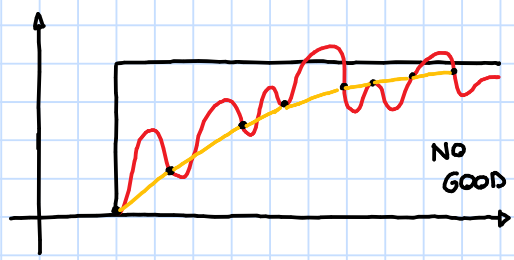
Specifiche di Prestazioni



L'obiettivo è regolare l'uscita $Y(t)$ a R, questo è ottenuto regolando Y_k e R.

Se T_s è adeguato allora "interpolando Y_k ottengo una buona approssimazione di $Y(t)$





Quindi le specifiche di prestazioni son definite per $Y(t)$ e non per Y_k.

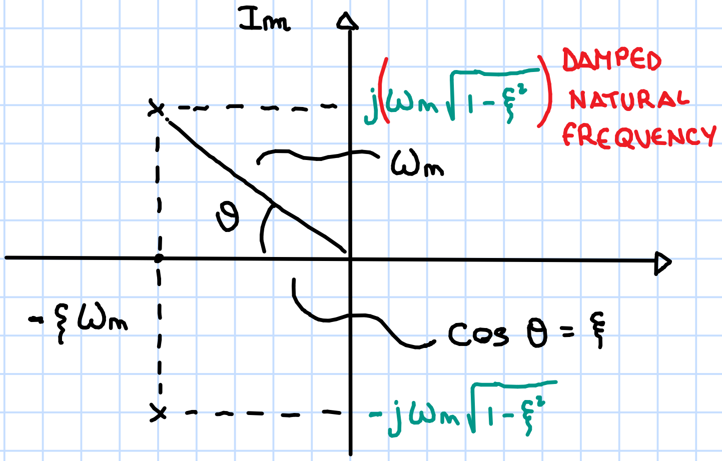
Modello di Riferimento (Sist II ordine) a ciclo chiuso

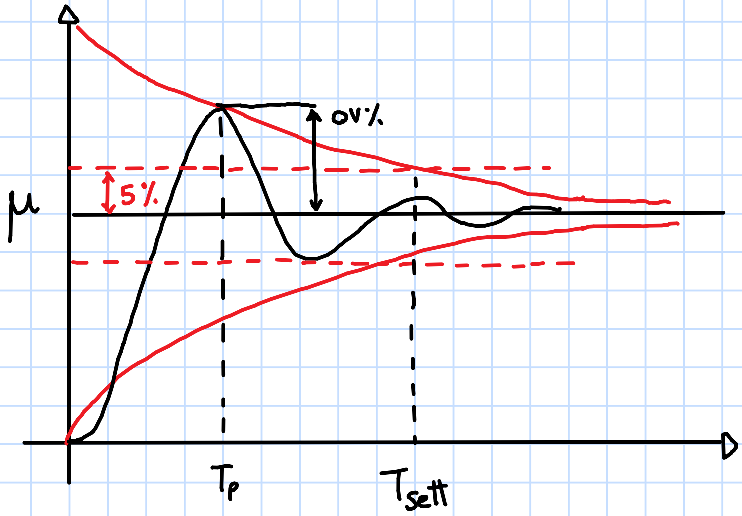
Complesso!

$$G(s) = \mu \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\downarrow$$

$$Y(t) = \mu \left[1 - \frac{1}{\sqrt{1-\xi^2}} \cdot e^{-\xi\omega_n t} \cdot \sin\left(\omega_n \sqrt{1-\xi^2}\, t + \arccos \xi \right) \right]$$





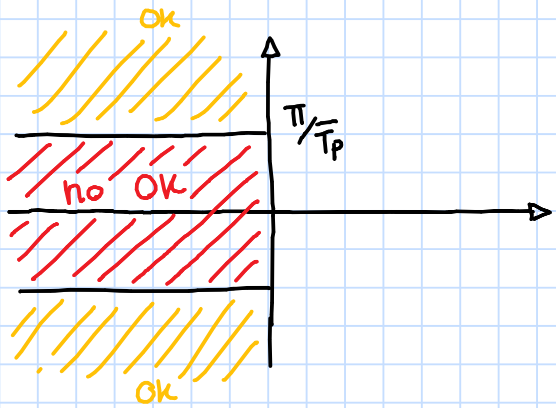
$T_M = T_p \cong \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$ $\quad T_{sett.\,5\%} \cong \dfrac{3}{\xi\omega_n}$ $\quad T_{sett\,2\%} \cong \dfrac{4}{\xi\omega_n}$ $\Rightarrow \begin{cases} T_{assest} \\ T_{a2} \\ T_{a5} \end{cases}$

Tempo MAX sovraelong

$OV\% = e^{-\xi\omega_n \cdot T_p} \cdot 100 = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \cdot 100$ + M_p = sovraelong MAX

- **Peak Time** $< \overline{T}_p$

$\dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} < \overline{T}_p \Rightarrow \boxed{\omega_n\sqrt{1-\xi^2} > \dfrac{\pi}{\overline{T}_p}}$

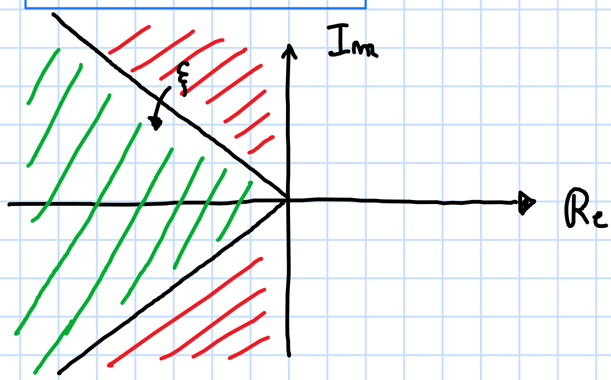


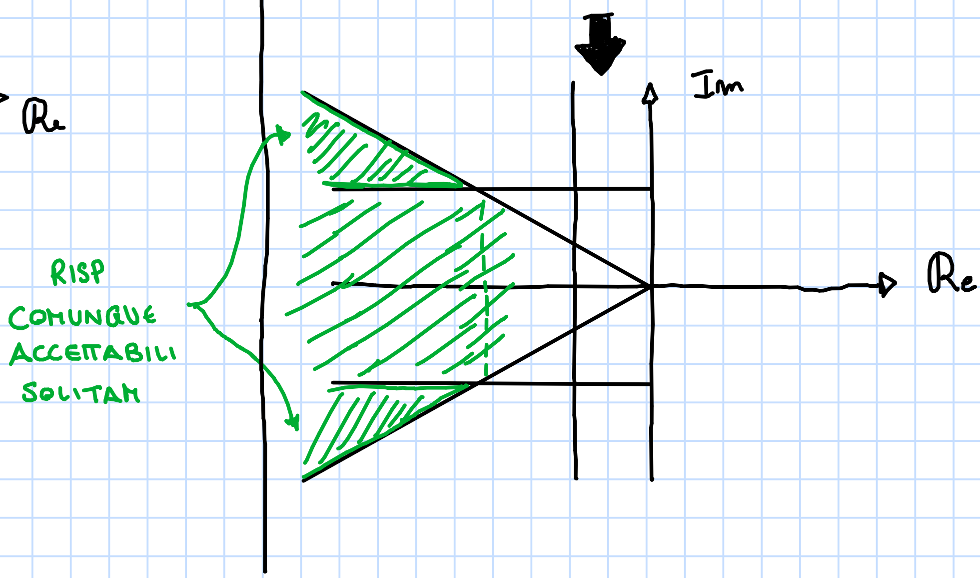
- **Settling Time** $< \overline{T}_s$

$\boxed{\xi\omega_n > \dfrac{3}{\overline{T}_s}}$

- **Overshoot** $e^{-\frac{3\pi}{\sqrt{1-\xi^2}}} < \overline{OS}$ ....

$\Rightarrow \boxed{\xi > \dfrac{-\ln\left(\frac{\overline{OS}}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\overline{OS}}{100}\right)}}}$





Come passo trasform specifiche per il tempo continuo in specifiche per il tempo discreto?

IDEA: se $\dot{x} = Ax$ ho come ZOH equiv $x_{k+1} = e^{AT_s} x_k$

allora se λ è autovalore di A

e $e^{\lambda T_s}$ è autovalore di e^{AT_s}

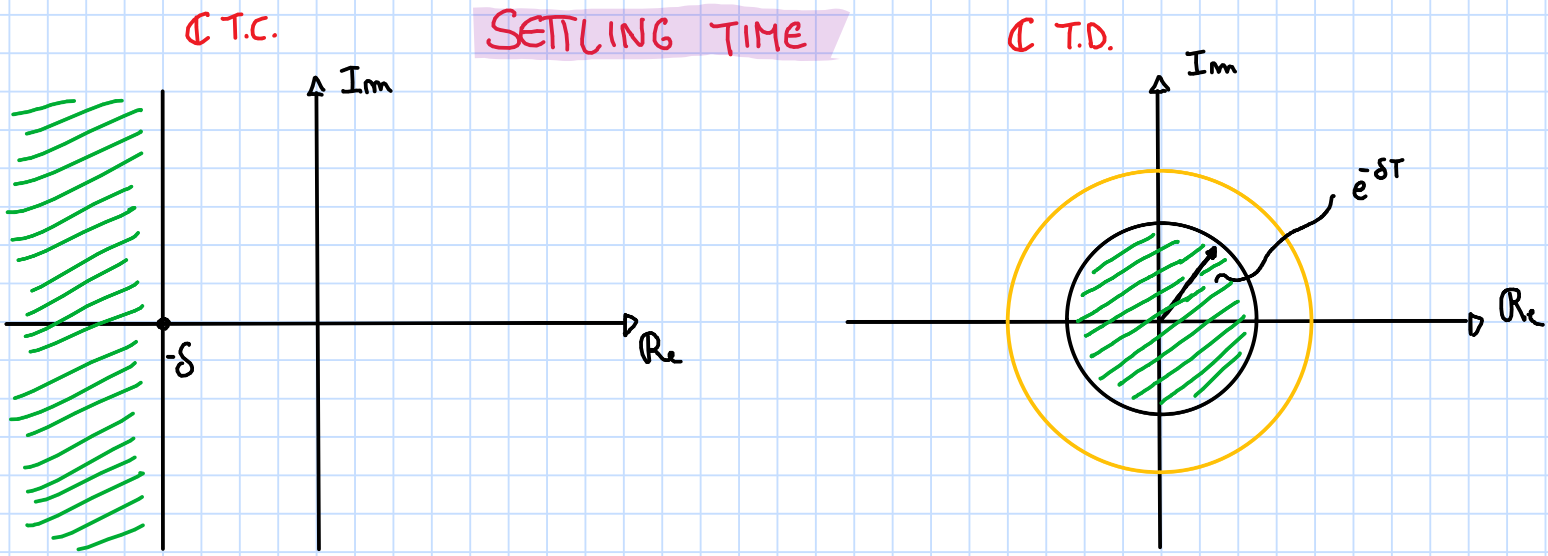
T_s SAMPLING TIME

$\Downarrow$

Possiamo mappare aree del piano complesso del TC in aree del piano complesso TD con la formula:

$$\Rightarrow \quad z = e^{sT_s}$$ SAMPLING TRANFORMATION

SETTLING TIME



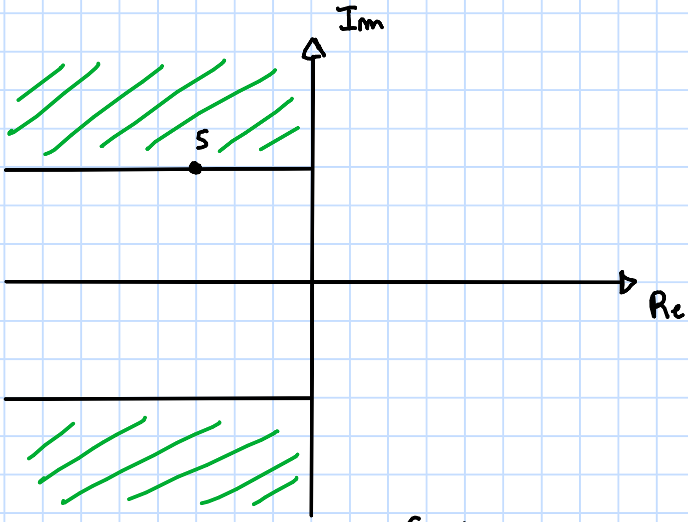
retta:

$s = \delta \pm j\omega \xrightarrow{z = e^{sT_s}} z = e^{(-\delta \pm j\omega) T_s} = \underbrace{e^{-\delta T_s}}_{\text{DIST DAL CENTRO COSTANTE}} \cdot e^{+j\omega T_s}$

$\omega \in [+\infty, -\infty]$

PEAK TIME

$\mathbb{C}$ T.C.

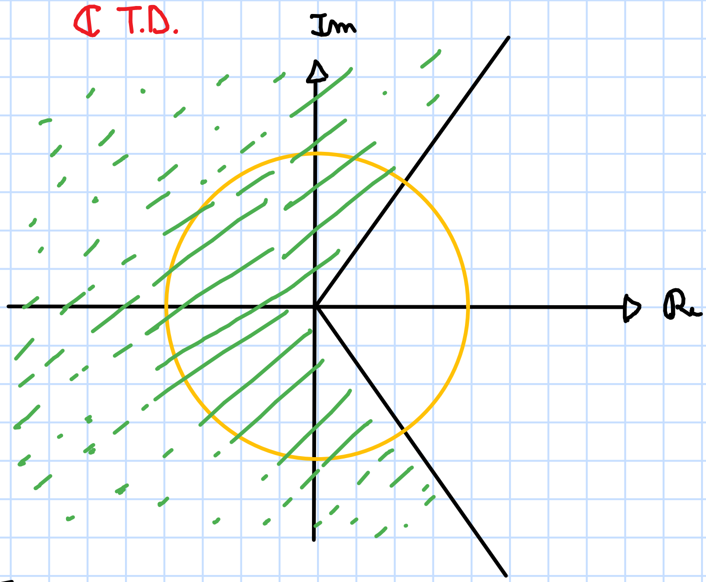


$s = \delta + j\omega$

$-\infty < \delta < 0$

$s = \delta - j\omega$

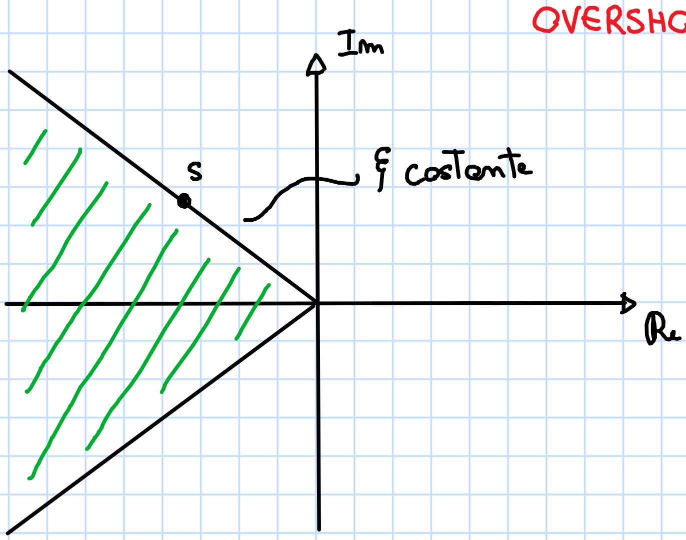
$\mathbb{C}$ T.D.

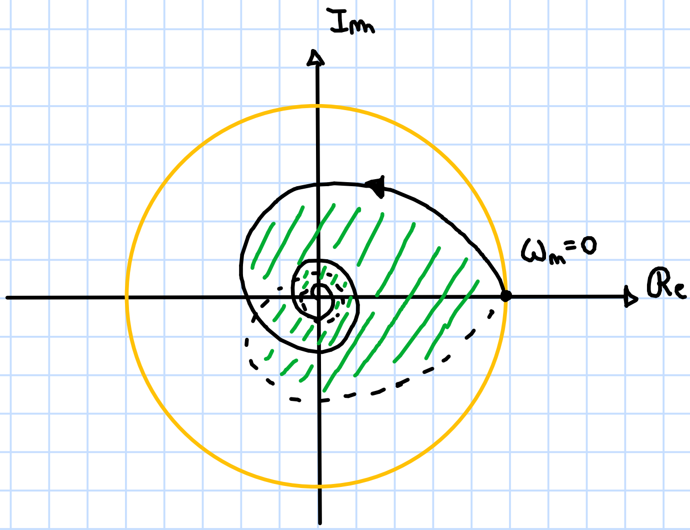


$$z = e^{\delta T_s} \cdot \underbrace{e^{j\omega T_s}}_{\text{fase costante}}$$

OVVIAMENTE DEVE STARE IL POLO DENTRO IL CERCHIO UNITARIO PER LA STABILITÀ

OVERSHOOT





$$s = -\underbrace{\xi}_{\text{COSTANTE}}\omega_n \pm j\omega_n\underbrace{\sqrt{1-\xi^2}}_{\text{COSTANTE}}$$

$\omega_n \in [0, +\infty]$

$$z = \underbrace{e^{-\xi\omega_n T_s}}\cdot e^{\pm j\omega_n\sqrt{1-\xi^2}\,T_s}$$

$(\omega_n = 0) \Rightarrow 1$

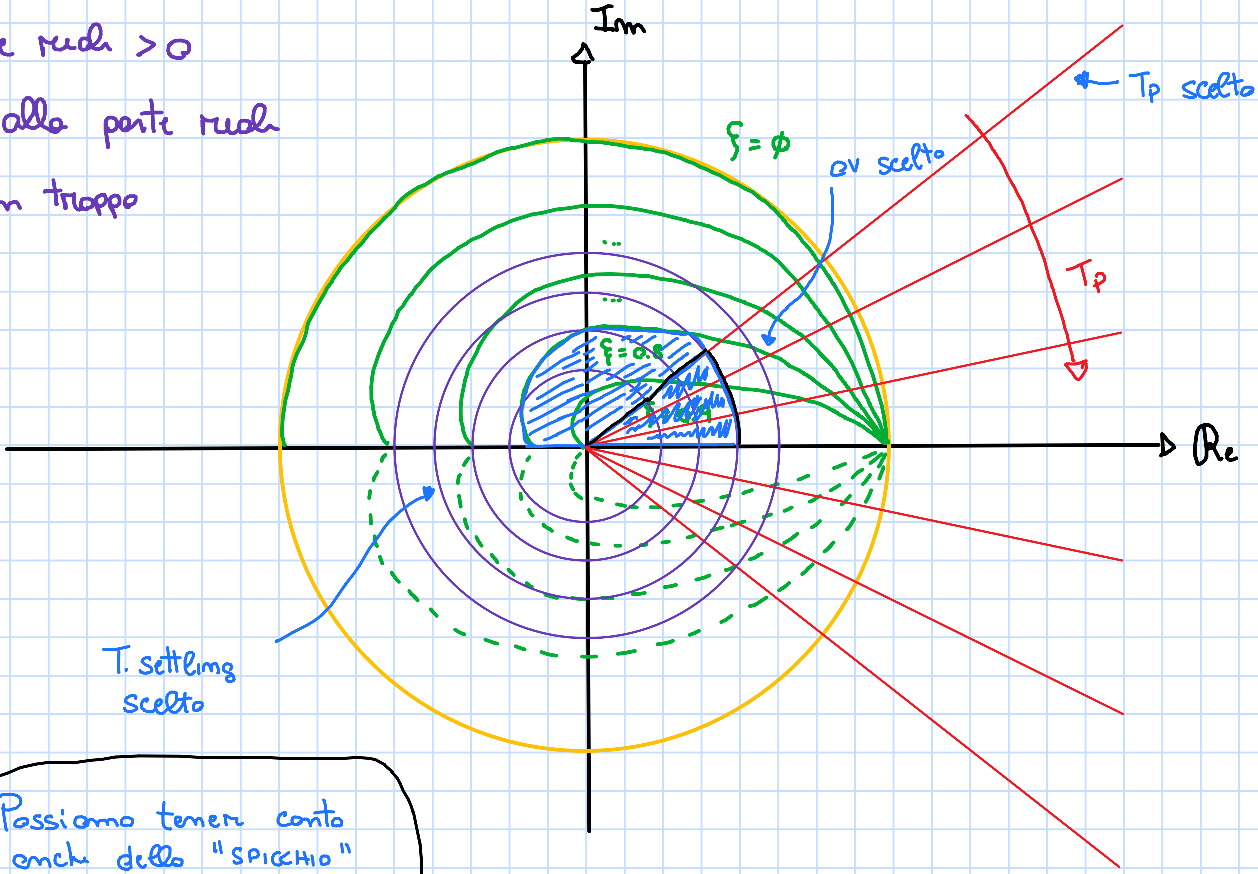
$(\omega_n \text{ cresce}) \Rightarrow < 1$

$(\omega_n \to +\infty) \Rightarrow \emptyset$

Ideale:

- Vicino all'origine
- a parte reale > 0
- vicino alla parte reale

ma non troppo



Possiamo tener conto anche dello "SPICCHIO" restante del T settling!

Disegno importante per capire le varie caratteristiche!