Soluzione Compitino - Fila B

Es 3

$$x(t) = x_1(t) + x_2(t)$$

$$T_0 = 3\bar{T}$$

$$x_1(t) = \sum_n \text{rect}\left(\frac{t - nT_0}{2\bar{T}}\right)$$

$$x_2(t) = \sum_n \cos\left[\frac{2\pi(t - nT_0)}{4\bar{T}}\right] \text{rect}\left(\frac{t - nT_0}{2\bar{T}}\right)$$

$$X_n = X_{1n} + X_{2n}$$

$$X_{1n} = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x_1(t)\, e^{-j2\pi n f_0 t}\, dt$$

$$= \frac{1}{3\bar{T}}\int_{-\bar{T}}^{\bar{T}} e^{-j2\pi n f_0 t}\, dt =$$

$$= \frac{1}{3\bar{T}}\,\frac{1}{-j2\pi n\frac{1}{3\bar{T}}}\, e^{-j2\pi n f_0 t}\Big|_{-\bar{T}}^{\bar{T}}$$

$$= \frac{1}{-j2\pi n} \left[e^{-j2\pi n \frac{1}{3\tau}\tau} - e^{+j2\pi n \frac{1}{3\tau}\tau} \right]$$

$$= \frac{1}{\pi n} \sin\left(\frac{2}{3}n\pi\right) = \frac{2}{3} \operatorname{sinc}\left(\frac{2}{3}n\right)$$

vedi sol fila C

$$X_{2n} = \frac{1}{3} \operatorname{sinc}\left(\frac{4n-3}{6}\right) + \frac{1}{3} \operatorname{sinc}\left(\frac{4n+3}{6}\right)$$

$$X_n = \frac{2}{3} \operatorname{sinc}\left(\frac{2}{3}n\right) + \frac{1}{3}\left[\operatorname{sinc}\left(\frac{4n-3}{6}\right) + \operatorname{sinc}\left(\frac{4n+3}{6}\right)\right]$$

$E_x = \infty$ segnale periodico

$$P_x = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |x(t)|^2 dt = \frac{1}{3\tau} \int_{-\tau}^{T} \left(1 + \cos\frac{2\pi t}{4\tau}\right)^2 dt$$

$$= \frac{1}{3\tau} \int_{-\tau}^{T} 1 + \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi t}{2\tau}\right) + 2\cos\left(\frac{2\pi t}{4\tau}\right) dt$$

$$= \frac{1}{3\tau}\left(2\tau + T\right) + \frac{2}{3\tau} \frac{4\tau}{2\pi} \sin\left(\frac{2\pi t}{4\tau}\right)\Bigg|_{-\tau}^{T} =$$

$$= 1 + \frac{4}{3}\frac{2}{\pi} = \boxed{1 + \frac{8}{3\pi} = P_x}$$

$$x_M = \sqrt{1 + \frac{8}{3\pi}}$$

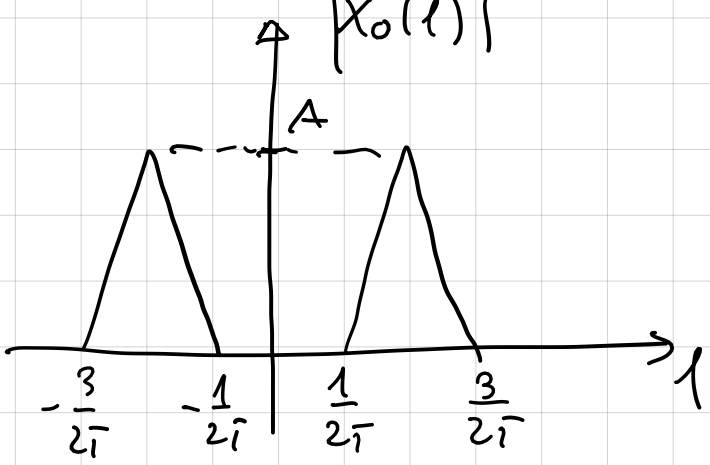
$$x_m = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\,dt = \frac{1}{3T}\int_{-T}^{T}\left(1 + \cos\frac{2\pi t}{4T}\right)dt$$

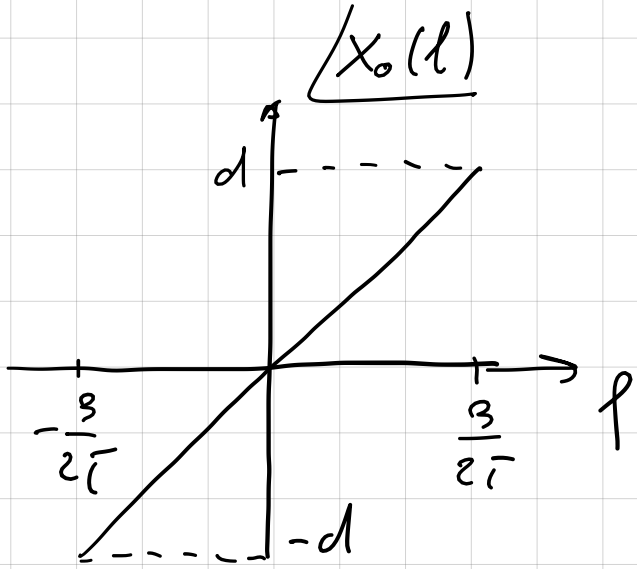
$$= \frac{1}{3T}\cdot 2T + \frac{1}{3T}\cdot\frac{4T}{2\pi}\sin\frac{2\pi t}{4T}\Big|_{-T}^{T}$$

$$= \boxed{\frac{2}{3} + \frac{4}{3\pi} = x_m}$$

Es 4

Definisco $X_0(f)$





$$X(f) = X_0\left(f - \frac{9}{2\tau}\right) + X_0\left(f + \frac{9}{2\tau}\right)$$

$$x(t) = x_0(t)\, e^{j 2\pi \frac{9}{2\tau} t} + x_0(t)\, e^{-j 2\pi \frac{9}{2\tau} t}$$

$$= 2 x_0(t) \cos\left(9\pi \frac{t}{\tau}\right)$$

$$X_0(f) = |X_0(f)|\, e^{j \angle X_0(f)}$$

$$|X_0(f)| = X_1\left(f - \frac{1}{T}\right) + X_1\left(f + \frac{1}{T}\right)$$

dove $X_1(f) = A\left(1 - \frac{|f|}{1/2\tau}\right) \mathrm{rect}\left(\frac{f}{1/\tau}\right)$

$$\angle X_0(f) = 2\pi f t_0 \quad , \quad t_0 = \frac{T d}{3\pi} \quad \left(\begin{array}{c}\text{vedi}\\ \text{sol. filo C}\end{array}\right)$$

$$X_0(f) = \left[X_1\left(f - \frac{1}{T}\right) + X_1\left(f + \frac{1}{T}\right)\right] e^{j 2\pi f t_0}$$

$$x_0'(t) = \mathrm{ATCF}\left[X_1\left(f - \frac{1}{T}\right) + X_1\left(f + \frac{1}{T}\right)\right]$$

$$x_0(t) = x_0'(t + t_0)$$

$$x_0'(f) = x_1(t)\, e^{j 2\pi \frac{t}{T}} + x_1(t)\, e^{-j 2\pi \frac{t}{T}}$$

$$= 2\, x_1(t) \cos\left(2\pi \frac{t}{T}\right)$$

$$x_1(t) = ATCF\left[X_1(1)\right] = \frac{A}{2T} \operatorname{sinc}^2\left(\frac{t}{2T}\right)$$

$$x_0'(f) = \frac{A}{T} \operatorname{sinc}^2\left(\frac{t}{2T}\right) \cos\left(2\pi \frac{t}{T}\right)$$

$$x_0(f) = \frac{A}{T} \operatorname{sinc}^2\left(\frac{t+t_0}{2T}\right) \cos\left[2\pi \frac{(t+t_0)}{T}\right]$$

$$x(f) = \frac{2A}{T} \operatorname{sinc}^2\left(\frac{t+t_0}{2T}\right) \cos\left[2\pi \frac{(t+t_0)}{T}\right] \cdot$$

$$\cdot \cos\left(3\pi \frac{t}{T}\right)$$