ESERCIZIO 1 | A

$$x(t) = \sum_n \mathrm{rect}\left(\frac{t - \frac{2}{B}n}{\frac{1}{2B}}\right) = \sum_n x_0(t - nT_0)$$

$$h(t) = B\,\mathrm{sinc}^2(Bt) \Rightarrow H(f) = \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

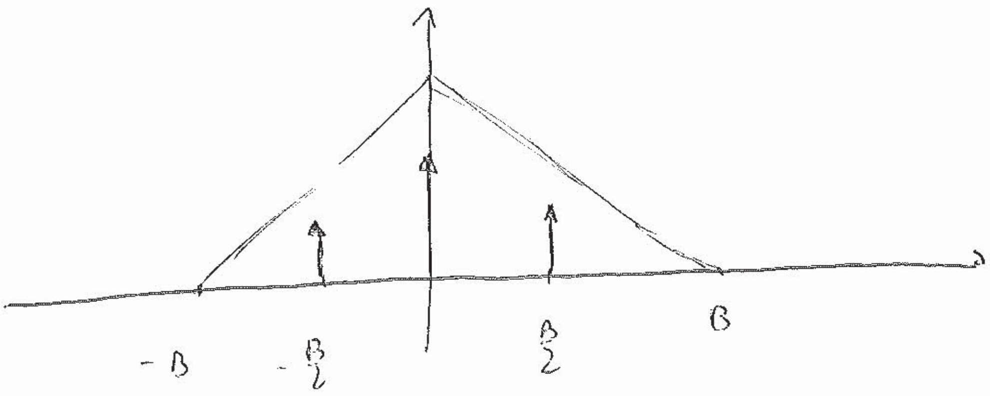
$$X_0(f) = \frac{1}{2B}\,\mathrm{sinc}\left(\frac{f}{2B}\right)$$

$$\boxed{T_0 = \frac{2}{B}}$$

$$X(f) = \frac{1}{T_0}\sum_k X_0\left(\frac{k}{T_0}\right)\delta\left(f - \frac{k}{T_0}\right) =$$

$$= \frac{B}{2}\sum_k \frac{1}{2B}\,\mathrm{sinc}\left(k\frac{B}{2}\,\frac{1}{2B}\right)\delta\left(f - k\frac{B}{2}\right)$$

Dal filtro passano solo le componenti per $k = 0, \pm 1$



$$X_0 = \frac{1}{4} \qquad X_1 = X_{-1} = \frac{1}{4}\,\mathrm{sinc}\left(\frac{1}{4}\right) = \frac{1}{4}\,\frac{4}{\pi}\sin\left(\frac{\pi}{4}\right) = \frac{1}{\pi}\,\frac{\sqrt{2}}{2}$$

$$Y(f) = \frac{1}{4}\delta(f) + \underbrace{\frac{1}{2}}_{H(\frac{B}{2})}\cdot\frac{\sqrt{2}}{2\pi}\,\delta\left(f - \frac{B}{2}\right) + \underbrace{\frac{1}{2}}_{H(-\frac{B}{2})}\cdot\frac{\sqrt{2}}{2\pi}\,\delta\left(f + \frac{B}{2}\right)$$

$$y(t) = \frac{1}{4} + \frac{\sqrt{2}}{2\pi}\cos\left(2\pi\frac{B}{2}t\right)$$

$$P_y = \frac{1}{16} + \frac{2}{16\pi^2} + \frac{2}{16\pi^2} = \frac{\pi^2 + 4}{16\pi^2} = \frac{4 + \pi^2}{16\pi^2} \qquad E_y = \infty$$

FILA B

$$x_0(t) = \left(1 - \frac{|t|}{1/2B}\right) \mathrm{rect}\left(\frac{t}{1/B}\right)$$

$$X_0(f) = \frac{1}{2B} \mathrm{sinc}^2\left(\frac{f}{2B}\right)$$

$$X(f) = \frac{B}{2} \sum_k X_0\left(\frac{k}{T_0}\right) \delta\left(f - \frac{k}{T_0}\right) =$$

$$= \frac{B}{2} \sum_k \frac{1}{2B} \mathrm{sinc}^2\left(k \frac{B}{2} \frac{1}{2B}\right)$$

$$H(f) = \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$X_0 = \frac{1}{4} \qquad X_1 = \frac{1}{4} \mathrm{sinc}^2\left(\frac{1}{4}\right) = \frac{1}{4} \frac{\sin^2\left(\frac{\pi}{4}\right)}{\left(\frac{\pi}{4}\right)^2} =$$

$$= \frac{1}{4} \frac{16}{\pi^2} \frac{1}{2} = \frac{2}{\pi^2}$$

$$X_{-1} = \frac{2}{\pi^2}$$

$$Y(f) = \frac{1}{4} \delta(f) + \frac{1}{2} \frac{2}{\pi^2} \delta\left(f - \frac{B}{2}\right) + \frac{1}{2} \frac{2}{\pi^2} \delta\left(f + \frac{B}{2}\right)$$

$$y(t) = \frac{1}{4} + \frac{2}{\pi^2} \cos\left(2\pi \frac{B}{2} t\right)$$

$$P_y = \frac{1}{16} + \frac{1}{\pi^4} + \frac{1}{\pi^4} \qquad E_y = \infty$$

FILA C

$$x_0(t) = rect\left(\frac{t}{1/2B}\right) \cdot \left(1 - \frac{|t|}{1/2B}\right) rect\left(\frac{t}{1/B}\right)$$

$$X_0(f) = \frac{1}{2B} sinc\left(\frac{f}{2B}\right) - \frac{1}{2B} sinc^2\left(\frac{f}{2B}\right)$$

$$X(f) = \frac{B}{2} \sum_k \left[\frac{1}{2B} sinc\left(k \frac{B}{2} \frac{1}{2B}\right) - \frac{1}{2B} sinc^2\left(k \frac{B}{2} \frac{1}{2B}\right)\right] \delta\left(f - k\frac{B}{2}\right)$$

$$H(f) = \left(1 - \frac{|f|}{B}\right) rect\left(\frac{f}{2B}\right)$$

$$X_0 = \frac{B}{2}\left[\frac{1}{2B} - \frac{1}{2B}\right] = 0$$

$$X_1 = \frac{B}{2}\left(\frac{1}{2B} \frac{\sin(\pi/4)}{\frac{\pi}{4}} - \frac{1}{2B} \frac{\sin^2(\pi/4)}{(\pi/4)^2}\right) =$$

$$= \frac{1}{4}\left[\frac{4}{\pi}\frac{\sqrt{2}}{2} - \frac{16}{\pi^2}\frac{1}{2}\right] = \frac{1}{4}\left[\frac{2\sqrt{2}}{\pi} - \frac{8}{\pi^2}\right] = X_{-1}$$

$$Y(f) = \frac{1}{8}\left[\frac{2\sqrt{2}}{\pi} - \frac{8}{\pi^2}\right] \delta\left(f - \frac{B}{2}\right) + \frac{1}{8}\left[\frac{2\sqrt{2}}{\pi} - \frac{8}{\pi^2}\right] \delta\left(f + \frac{B}{2}\right)$$

$$y(t) = \frac{1}{4}\left(\frac{2\sqrt{2}}{\pi} - \frac{8}{\pi^2}\right) \cos\left(2\pi \frac{B}{2} t\right)$$

$$P_y = 2 \cdot \frac{1}{64}\left(\frac{2\sqrt{2}}{\pi} - \frac{8}{\pi^2}\right)^2$$

$$E_y = \infty$$

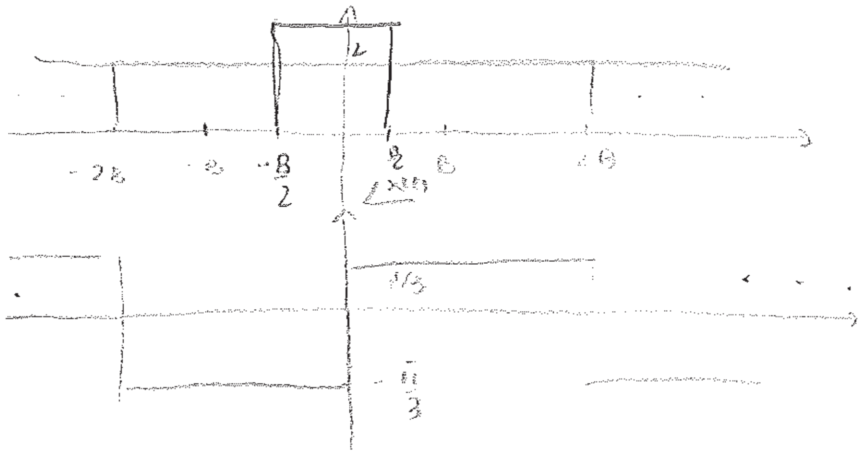
ESERCIZIO 2 | FILA A

$$x(t) = 2\,\mathrm{sinc}(2Bt)\cos\left(2\pi B t + \frac{\pi}{3}\right)$$

$$X(f) = \frac{1}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right) \otimes \left[\delta(f-B)e^{j\frac{\pi}{3}} + \delta(f+B)e^{-j\frac{\pi}{3}}\right] =$$

$$= \frac{1}{2B}\,\mathrm{rect}\left(\frac{f-B}{2B}\right)e^{j\frac{\pi}{3}} + \frac{1}{2B}\,\mathrm{rect}\left(\frac{f+B}{2B}\right)e^{-j\frac{\pi}{3}}$$

$$p(t) = B\,\mathrm{sinc}(Bt) \rightleftharpoons P(f) = \mathrm{rect}\left(\frac{f}{B}\right)$$



$$\frac{1}{2B} \quad 4B$$

$$\bar{X}(f) = \frac{1}{T_c}\sum_n X\left(f - \frac{n}{T}\right)$$

$$Y(f) = 2\,\mathrm{rect}\left(\frac{f - B/4}{B/2}\right)e^{j\frac{\pi}{3}} + 2\,\mathrm{rect}\left(\frac{f + B/4}{B/2}\right)e^{-j\frac{\pi}{3}}$$

$$y(t) = 2B\,\mathrm{sinc}\left(\frac{B}{2}t\right)\cos\left(2\pi\frac{B}{4}t + \frac{\pi}{3}\right)$$

$$E_y = 8B$$

$$P_y = 0$$

ESERCIZIO 2 FILA B

$$x(t) = 2\,\mathrm{sinc}(2Bt)\,\sin\left(2\pi Bt + \pi/3\right)$$

$$X(f) = \frac{1}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right) \otimes \left(\frac{1}{2j}\delta(f-B)\,e^{j\frac{\pi}{3}} - \frac{1}{2j}\delta(f+B)\,e^{-j\frac{\pi}{3}}\right)$$

$$= \frac{1}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right) \otimes \left(\delta(f-B)\,e^{j\frac{\pi}{3}}e^{-j\frac{\pi}{2}} - \delta(f+B)\,e^{-j\frac{\pi}{3}}e^{-j\frac{\pi}{2}}\right)$$

$$= \frac{1}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right) \otimes \left(\delta(f-B)\,e^{-j\frac{\pi}{6}} + \delta(f+B)\,e^{+j\frac{\pi}{6}}\right)$$

$$= \frac{1}{2B}\,\mathrm{rect}\left(\frac{f-B}{2B}\right) e^{-j\frac{\pi}{6}} + \frac{1}{2B}\,\mathrm{rect}\left(\frac{f+B}{2B}\right) e^{+j\frac{\pi}{6}}$$

$$Y(f) = 2\,\mathrm{rect}\left(\frac{f-B/4}{B/2}\right) e^{-j\frac{\pi}{6}} + 2\,\mathrm{rect}\left(\frac{f+B/4}{B/2}\right) e^{+j\frac{\pi}{6}}$$

$$y(t) = 2B\,\mathrm{sinc}\left(\frac{B}{2}t\right)\cos\left(2\pi\frac{B}{4}t - \frac{\pi}{6}\right)$$

ESERCIZIO 2) FILA C

$$x(t) = 2\,\text{sinc}(2Bt)\cos\left(2\pi Bt + \frac{2}{3}\pi\right)$$

$$X(f) = \frac{1}{2B}\,\text{rect}\left(\frac{f}{2B}\right) \otimes \left[\delta(f-B)\,e^{j\frac{2}{3}\pi} + \delta(f+B)\,e^{-j\frac{2}{3}\pi}\right] =$$

$$= \frac{1}{2B}\,\text{rect}\left(\frac{f-B}{2B}\right)e^{j\frac{2}{3}\pi} + \frac{1}{2B}\,\text{rect}\left(\frac{f+B}{2B}\right)e^{-j\frac{2}{3}\pi}$$

$$H(f) = \cancel{4B}\;\text{rect}\left(\frac{f}{B}\right)$$

$$Y(f) = 2\,\text{rect}\left(\frac{f-B/4}{B/2}\right)e^{j\frac{2}{3}\pi} + 2\,\text{rect}\left(\frac{f+B/4}{B/2}\right)e^{-j\frac{2}{3}\pi}$$

$$y(t) = 2B\,\text{sinc}\left(\frac{B}{2}t\right)\cos\left(2\pi\frac{B}{4}t + \frac{2}{3}\pi\right)$$

ESERCIZIO 3 | FILA A ~~B~~ C

$$T[\cdot] = \int_a^t x(\alpha - t_1)\, d\alpha$$

. lineare

$$c_1 x_1(t) + c_2 x_2(t) \Longrightarrow c_1 y_1(t) + c_2 y_2(t)$$

$$\int_a^t [c_1 x_1(\alpha - t_1) + c_2 x_2(\alpha - t_2)]\, d\alpha =$$

$$= c_1 \int_a^t x_1(\alpha - t_1)\, d\alpha + c_2 \int_a^t x_2(\alpha - t_1)\, d\alpha$$ OK

. causale

l'uscita dipende solo da istanti passati OK

. stazionario

$$x(t - t_0) \Rightarrow y(t - t_0)$$

$$\alpha - t_0 = \beta \Rightarrow \alpha = \beta - t_0$$

$$\int_a^t x(\alpha - t_1 - t_0)\, d\alpha = \int_{a - t_0}^{t - t_0} x(\beta - t_1)\, d\beta \neq$$

$$E\, y(t - t_0) = \int_a^{t - t_0} x(\alpha)\, d\alpha \Rightarrow$$ NO STAZIONARIO

. è con memoria: l'uscita dipende da istanti ~~pre~~ precedenti a t

ESERCIZIO 3 | B

$$T[\cdot] = \int_a^{t-t_1} x(\alpha)\, d\alpha = y(t)$$

lineare **SÌ**

$c_1 x_1(t) + c_2 x_2(t) \Rightarrow c_1 y_1(t) + c_2 y_2(t)$

Causale sì

con memoria sì

Stazionario NO

$x(t - t_0)$

$\alpha - t_0 = \beta$

$$\int_a^{t-t_1} x(\alpha - t_0)\, d\alpha = \int_{a-t_0}^{t-t_1-t_0} x(\beta)\, d\beta = y(t-t_0) + \int_{a-t_0}^{a} x(\beta) d\beta$$