4 MAGGIO 2023

LINEAM $\quad 0<p\leq n$

$p<n$

$$y^{(n)}(t) = \sum_{i=0}^{n-1} -\alpha_i \, y^{(i)}(t) + \sum_{j=0}^{p} \beta_j \, u^{(j)}(t)$$

$$z^{(n)}(t) = \sum_{i=0}^{n-1} -\alpha_i \, z^{(i)}(t) + u(t)$$

$$y(t) = \sum_{j=0}^{p} \beta_j \, z^{(j)}(t)$$

$$x = \begin{bmatrix} z \\ \dot{z} \\ \vdots \\ z^{(n-1)} \end{bmatrix}$$

$$\underbrace{\dot{x} = Ax + Bu}_{}$$

$$y(t) = \sum_{i=1}^{p+1} \beta_{i-1} x_i$$

$$y = Cx + Du$$

$$\left[C = \underbrace{\begin{bmatrix} \beta_0 & \beta_1 & \cdots & \beta_p & 0 & \cdots & 0 \end{bmatrix}}_{n} \qquad D = 0 \right]$$

$$p < n$$

P = n SISTEMA PROPRIO (NON STRETTAMENTE)

$$y(t) = \sum_{i=1}^{n} \beta_{i-1} x_i + \beta_n \overset{n}{z} =$$

$$\overset{n}{z}(t) = \sum_{i=0}^{n-1} -\alpha_i \underbrace{\overset{i}{z}(t)}_{x_{i+1}} + u(t)$$

$$= \sum_{i=1}^{n} \beta_{i-1} x_i - \beta_n \sum_{i=1}^{n} \alpha_{i-1} x_i + \beta_n u =$$

$$= \sum_{i=1}^{n} \left(\beta_{i-1} - \beta_n \alpha_{i-1} \right) x_i + \beta_n u = g(x,u,t)$$

$$y = Cx + Du$$

$$C = [\beta_0 - \beta_n \alpha_0 \quad \beta_1 - \beta_n \alpha_1 \quad \dots \quad \beta_{n-1} - \beta_n \alpha_{n-1}]$$

$$D = \beta_n$$

$p = n$

ESEMPI

$$\ddot{y} + y = 2u + \dot{u} \Rightarrow \ddot{y} = -y + 2u + \dot{u}$$

$$\dot{x} = \begin{array}{cc} \left[\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}\right] \\ \begin{array}{cc} -\alpha_0 & -\alpha_1 \end{array} \end{array} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{array}{c} \begin{bmatrix} 2 & 1 \end{bmatrix} \\ \begin{array}{cc} \beta_0 & \beta_1 \end{array} \end{array} x + \begin{bmatrix} 0 \end{bmatrix} u$$

$$\dddot{y} + y = 2u + \dot{u} \qquad \dddot{y} = -y + 2u + \dot{u}$$

$$\dot{x} = \left[\begin{array}{c|cc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline -1 & 0 & 0 \end{array}\right] x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} x + 0 \, u$$

$$\dddot{y} + 2\ddot{y} + 3\dot{y} + y = 2u + \dot{u} + 3\dddot{u}$$

$$\dddot{y} = -y - 3\dot{y} - 2\ddot{y} + 2u + \dot{u} + 3\dddot{u}$$

$$\dot{x} = \left[\begin{array}{c|cc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline -1 & -3 & -2 \end{array}\right] x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & -8 & -6 \end{bmatrix} x + 3u$$

$$\beta_0 - \beta_n\alpha_0 \qquad \beta_1 - \beta_n\alpha_1 \qquad \beta_2 - \beta_n\alpha_2 \qquad \beta_n$$

$$2 - 3 \cdot 1 = -1 \qquad 1 - 3 \cdot 3 \qquad 0 - 3 \cdot 2$$

SCELTA DELLO STATO NON È UNICA

↳ CAMBIAMENTO COORDINATE DI STATO

X DESCRIVE LO STATO

$\hat{x} = \phi(x)$ È UNA RAPPRESENTAZ. EQUIVALENTE

$\Leftrightarrow$

$\phi(\cdot)$ BIIETTIVA (ESISTE LA FUNZ. INVERSA $\phi^{-1}(\cdot)$)

CAMBIAMENTI DI COORDINATE LINEARI

$\phi(\cdot)$ LINEARE

→ ESPRIMIBILE IN FORMA MATRICIALE

(RAPPRESENTATA DA MATRICE T)

$$\hat{x} = \phi(x) = Tx \qquad T_{n \times n}$$

$$\det(T) \neq 0$$

$\Rightarrow$ ESISTE T^{-1} : $\underbrace{x = T^{-1} \hat{x}}$

x:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$\longrightarrow$ $\hat{x}$

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$$
$$y = \hat{C}\hat{x} + \hat{D}u$$

$$\dot{\hat{x}} = T\dot{x} = TAx + TBu$$

$$= \underbrace{TAT^{-1}}_{\hat{A}}\hat{x} + \underbrace{TB}_{\hat{B}}u$$

$$y = Cx + Du =$$

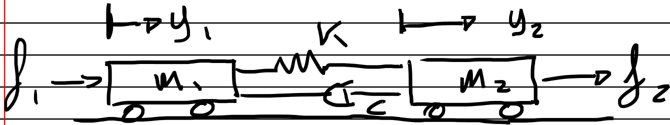
$$= \underbrace{CT^{-1}}_{\hat{C}}\hat{x} + \underbrace{D}_{\hat{D}}u$$

$$\hat{A} = TAT^{-1}$$

$$\hat{B} = TB$$

$$\hat{C} = CT^{-1}$$

$$\hat{D} = D$$



$$X = \begin{bmatrix} y_1 \\ y_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} \qquad U = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K}{m_1} & \frac{K}{m_1} & -\frac{C}{m_1} & \frac{C}{m_1} \\ \frac{K}{m_2} & -\frac{K}{m_2} & \frac{C}{m_2} & -\frac{C}{m_2} \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} y_1 \\ y_2 - y_1 \\ \dot{y}_1 \\ \dot{y}_2 - \dot{y}_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 - x_1 \\ x_3 \\ x_4 - x_3 \end{bmatrix} \qquad x = \begin{bmatrix} y_1 \\ y_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{bmatrix}$$

$$\hat{x} = T x = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}}_{T} x$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{c}{m_1} & \frac{c}{m_1} \\ \frac{k}{m_2} & -\frac{k}{m_2} & \frac{c}{m_2} & -\frac{c}{m_2} \end{bmatrix}$$

$$\hat{A} = TAT^{-1}$$

$$\hat{B} = TB$$

$$\hat{C} = CT^{-1}$$

$$\hat{D} = D$$

$$AT^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & \frac{k}{m_1} & 0 & \frac{c}{m_1} \\ 0 & -\frac{k}{m_2} & 0 & -\frac{c}{m_2} \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & k/m_1 & 0 & c/m_1 \\ 0 & -k/m_1 - k/m_2 & 0 & -c/m_1 - \frac{c}{m_2} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\hat{B} = T B = \; = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & 0 \\ -\frac{1}{m_1} & \frac{1}{m_2} \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix}$$

$$\hat{C} = C T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \qquad \hat{D} = D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$y = \hat{C}\,\hat{x} + \cancel{\hat{D}}\,u$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\hat{C}\,\hat{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 - y_1 \\ \dot{y}_1 \\ \dot{y}_2 - \dot{y}_1 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\longleftrightarrow$$

SOLUZIONI DELLE EQUAZIONI IN FORMA DI STATO

$$\dot{x} = f(x, u, t) \qquad x(0) = x_0$$

$\tilde{x}(t)$ SOLUZIONE NON ESISTE SEMPRE E

SE ESISTE NON È DETTO CHE SIA UNICA

$$\dot{\tilde{x}}(t) = f(\tilde{x}, u, t)$$

$$\tilde{x}(0) = x_0$$

$\tilde{x}(t)$ È SOLUZIONE

$\Downarrow$

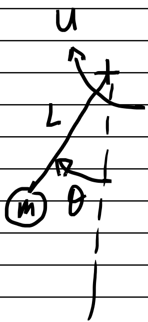
MOVIMENTO DELLO STATO

$$y = \underline{g}(\tilde{x}, u, t)$$

TRA TUTTI I MOVIMENTI DELLO STATO CE NE SONO ALCUNI PARTICOLARI?

$$\dot{\tilde{x}}_E = 0 = f(\tilde{x}_E, u_E, t) \qquad x(0) = x_{E0}$$

$$y_E = g(\tilde{x}_E, u_E, t)$$



$$\dot{x} = \underbrace{\begin{bmatrix} x_2 \\ \frac{1}{mL^2}\left(u - c x_2 - mgL \operatorname{sen} x_1\right) \end{bmatrix}}_{f(x,u,t)} \qquad x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

$$y = \theta$$

$$g(x,u,t)$$

$\dot{x} = 0$

$x_2 = 0$

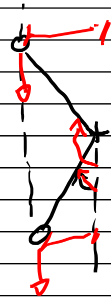
$f(x, u, t) = 0 \rightarrow \cancel{\frac{1}{mL^2}} \left(u - \cancel{c x_2}^{\,\rightarrow 0} - mgL \sin x_1 \right) = 0$

$\sin x_1 = \frac{u}{mgL} \rightarrow x_1 = \arcsin\left(\frac{u}{mgL}\right)$

$x_2 = 0$

$u = 0$

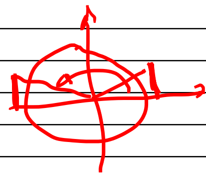
$x_1 = 0 \quad \vee \quad x_1 = \pi$



$\frac{u}{mgL} < 1$

$x_2 = 0$

2 CONF. DI EQUILIBRIO



$u = \frac{1}{2} mgL$

$\frac{u}{mgL} = \frac{1}{2}$

$\frac{u}{mgL} > 1$ NON ESISTONO SOLUZIONI DI EQUILIBRIO

PERTURBAZIONE DEL MOVIMENTO

$$x(t) = \tilde{x}(t) + \delta x(t)$$

MOVIMENTO PERTURBATO

PERTURBAZIONE

$$\dot{x}(t) = \dot{\tilde{x}}(t) + \dot{\delta x}(t) = f(x, u, t)$$

CONSIDERANDO $\tilde{x}$ MOVIMENTO DI EQUILIBRIO $(\tilde{x}_E)$

$$\dot{x}(t) = \underbrace{\cancel{\dot{\tilde{x}}_E(t)}}_{0} + \dot{\delta x}(t) = f(x, u, t)$$

$$\dot{x}(t) = \dot{\delta x}(t) = f(x, u, t)$$

PERTURBAZ DI MOVIMENTO DI EQUILIBRIO

DINAMICA DEL MOVIMENTO PERTURBATO COINCIDE CON LA DINAMICA DELLA PERTURBAZIONE