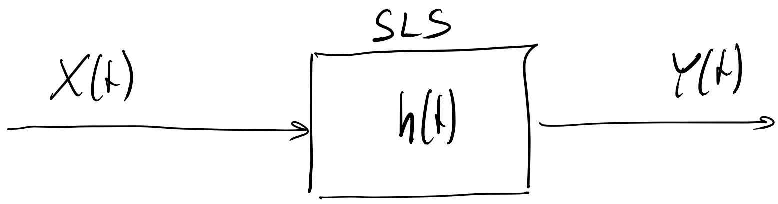
ESERCIZIO - 18/07/2017

$X(t)$

$$S_X(f) = \frac{\sigma_X^2}{2B} \operatorname{rect}\left(\frac{f}{2B}\right)$$



$$h(t) = \frac{1}{2}\delta(t) + \delta(t-T) + \delta(t-2T)$$

1) Calcolare $S_Y(f)$
2) Disegnare il grafico di $S_Y(f)$ per $B = \frac{1}{2T}$
3) Calcolare $R_Y(\tau)$
4) Disegnare il grafico di $R_Y(\tau)$ per $B = \frac{1}{2T}$

Soluzione

1) Due strade

$$S_X(f) \Rightarrow R_X(\tau) \Rightarrow R_Y(\tau) = R_X(\tau) \otimes h(\tau) \otimes h(-\tau)$$

$$\Downarrow$$

$$S_Y(f) = TCF\left[R_Y(\tau)\right]$$

$$h(t) \Rightarrow H(f) \Rightarrow S_Y(f) = S_X(f)\,|H(f)|^2$$

• I strada

$$R_X(\tau) = ATCF\left[S_X(f)\right] = ATCF\left[\frac{\sigma_X^2}{2B} \operatorname{rect}\left(\frac{f}{2B}\right)\right]$$

$$= \sigma_X^2 \operatorname{sinc}(2B\tau)$$

$$R_Y(\tau) = R_X(\tau) \otimes \underbrace{h(\tau) \otimes h(-\tau)}$$

$$C_h(\tau) = \int_{-\infty}^{+\infty} h(t)\, h(t-\tau)\, dt$$

$$h(t) = \frac{1}{2}\delta(t) + \delta(t-T) + \delta(t-2T)$$

$$h(-t) = \frac{1}{2}\delta(t) + \delta(t+T) + \delta(t+2T)$$

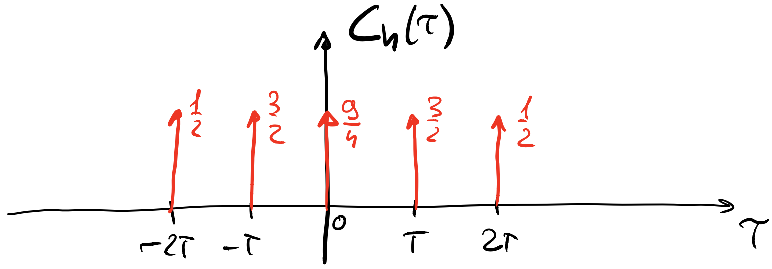
$$h(t) \otimes h(-t) = \frac{1}{4}\delta(t) + \frac{1}{2}\delta(t+T) + \frac{1}{2}\delta(t+2T) +$$

$$+ \frac{1}{2}\delta(t-T) + \delta(t) + \delta(t+T) +$$

$$+ \frac{1}{2}\delta(t-2T) + \delta(t-T) + \delta(t)$$

$$= \frac{9}{4}\delta(t) + \frac{3}{2}\delta(t+T) + \frac{1}{2}\delta(t+2T) +$$

$$+ \frac{3}{2}\delta(t-T) + \frac{1}{2}\delta(t-2T)$$



$$R_Y(\tau) = R_X(\tau) \otimes C_h(\tau)$$

$$= \frac{1}{2}R_X(\tau-2T) + \frac{3}{2}R_X(\tau-T) + \frac{9}{4}R_X(\tau) + \frac{3}{2}R_X(\tau+T) + \frac{1}{2}R_X(\tau+2T)$$

$$R_Y(\tau) = \boxed{\sigma_X^2 \left[\frac{1}{2}\,\mathrm{sinc}\left[2B(\tau+2T)\right] + \frac{3}{2}\,\mathrm{sinc}\left[2B(\tau+T)\right] + \frac{9}{4}\,\mathrm{sinc}\left[2B\tau\right] + \frac{3}{2}\,\mathrm{sinc}\left[2B(\tau-T)\right] + \frac{1}{2}\,\mathrm{sinc}\left[2B(\tau-2T)\right] \right]}$$

$$S_Y(f) = \frac{\sigma_X^2}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right)\left[\underbrace{\frac{1}{2} e^{j2\pi f 2T}} + \frac{3}{2} e^{j2\pi f T} + \frac{9}{4} + \frac{3}{2} e^{-j2\pi f T} + \underbrace{\frac{1}{2} e^{-j2\pi f 2T}} \right]$$

riconosciamo il coseno!

$$= \boxed{\frac{\sigma_X^2}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right)\left[\frac{9}{4} + \cos(4\pi f T) + 3\cos(2\pi f T) \right]}$$

- II^a strada

$$h(t) = \frac{1}{2}\delta(t) + \delta(t-T) + \delta(t-2T)$$

$$H(f) = \frac{1}{2} + e^{-j2\pi f T} + e^{-j4\pi f T}$$

$$|H(f)|^2 = H(f)\,H^*(f) =$$

$$= \left(\frac{1}{2} + e^{-j2\pi f T} + e^{-j4\pi f T}\right)\left(\frac{1}{2} + e^{j2\pi f T} + e^{j4\pi f T}\right)$$

$$= \frac{1}{4} + \frac{1}{2} e^{j2\pi f T} + \frac{1}{2} e^{j4\pi f T} + \frac{1}{2} e^{-j2\pi f T} + 1 + e^{j2\pi f T} + \frac{1}{2} e^{-j4\pi f T} + e^{-j2\pi f T} + 1$$

$$= \frac{9}{4} + \frac{3}{2} e^{j2\pi f T} + \frac{1}{2} e^{j4\pi f T} + \frac{3}{2} e^{-j2\pi f T} + \frac{1}{2} e^{-j4\pi f T}$$

$$= \frac{9}{4} + 3\cos(2\pi f T) + \cos(4\pi f T)$$

$$S_Y(f) = S_X(f)\,|H(f)|^2$$

$$= \boxed{\frac{\sigma_X^2}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right)\left[\frac{9}{4} + 3\cos(2\pi f T) + \cos(4\pi f T)\right]}$$

$$R_Y(\tau) = \mathrm{ATCF}\left[S_Y(f)\right]$$

$$= \frac{9}{4}\sigma_X^2\,\mathrm{sinc}(2B\tau) + \frac{3}{2}\Big[\sigma_X^2\,\mathrm{sinc}\left[2B(\tau-T)\right] +$$

$$+ \sigma_X^2\,\mathrm{sinc}\left[2B(\tau+T)\right]\Big] + \frac{1}{2}\Big[\sigma_X^2\,\mathrm{sinc}\left[2B(\tau-2T)\right] +$$

$$+ \sigma_X^2\,\mathrm{sinc}\left[2B(\tau+2T)\right]\Big]$$

$\Rightarrow$ Si ottengono gli stessi risultati

$B = \frac{1}{2T}$