CALCOLO DELLE PRESTAZIONI NEL CASO DI MODULAZIONE PSK

$$x[k] \equiv e^{j\vartheta_m} \qquad \vartheta_m = \frac{2\pi}{M}(m-1) \qquad (PSK)$$

$$x[k] = \underbrace{\cos(\vartheta_m)}_{x_c[k]} + j\underbrace{\sin(\vartheta_m)}_{x_s[k]}$$

Poiché il simbolo $x[k]$ si può scrivere nello stesso modo di un simbolo QAM, il calcolo delle prestazioni segue quello fatto per la QAM.

Ricaviamo adesso la densità spettrale di Potenza delle componenti in fase e quadratura del rumore in banda passante.

Supponiamo di avere una modulazione QAM.

Si è visto che il segnale all'uscita del ricevitore in assenza di cross-talk, è:

$$y_c(t) = \sum_{k=-\infty}^{+\infty} x_c[k]\, h(t-kT_s) + n_c(t) \otimes h_n(t) \qquad \text{(RAMO IN FASE)}$$

$$y_s(t) = \sum_{k=-\infty}^{+\infty} x_s[k]\, h(t-kT_s) + n_s(t) \otimes h_n(t) \qquad \text{(RAMO IN QUADRATURA)}$$

$$h(t) = p(t) \otimes c(t) \otimes h_n(t) = p(t) \otimes h_n(t)$$

Per calcolare le prestazioni del sistema di comunicazione è necessario conoscere la DSP dei processi di rumore $n_c(t)$ e $n_s(t)$.

<u>Per processi stazionari</u>

$$\begin{cases} R_{n_c n_c}(t) = R_{n_s n_s}(t) \\ R_{n_s n_c}(t) = - R_{n_s n_c}(-t) = - R_{n_c n_s}(t) \end{cases}$$

$$\Downarrow \; \text{TF}$$

$$\begin{cases} S_{n_c n_c}(f) = S_{n_s n_s}(f) & \text{reale e pari} \\ S_{n_s n_c}(f) = -j\, S_{n_s n_c}(-f) & \text{immaginaria pura e dispari} \end{cases}$$

N.B. $\quad R_{n_s n_c}(t) = -R_{n_s n_c}(-t)$

per $t=0 \Rightarrow R_{n_s n_c}(0) = 0 \Rightarrow$

$$R_{n_s n_c}(0) = E\{ n_s(t_1) \cdot n_c(t_1) \} = 0$$

$n_s(t_1)$ e $n_c(t_1)$ sono V.A. incorrelate e se congiuntamente Gaussiane anche indipendenti

Affinché le rumore in fase e quadratura siano indipendenti, non ci deve essere cross-talk e le V.A. di rumore devono essere incorrelate.

$$R_n(t) = R_{n_c n_c}(t)\cos(2\pi f_0 t) - R_{n_s n_c}(t)\sin(2\pi f_0 t) =$$

$$= \mathrm{Re}\{ R_{\tilde{n}}(t)\, e^{j 2\pi f_0 t} \}$$

$$R_{\tilde{n}}(t) = R_{n_c n_c}(t) + j\, R_{n_s n_c}(t)$$

TRASFORMATA DI FOURIER

$$S_{\tilde{n}}(f) = S_{n_c n_c}(f) + j\, S_{n_s n_c}(f) \Rightarrow$$

$$\Downarrow$$

$$\begin{cases} S_{n_c n_c}(f) = \dfrac{S_{\tilde{n}}(f) + S_{\tilde{n}}(-f)}{2} \\[2ex] j\, S_{n_s n_c}(f) = \dfrac{S_{\tilde{n}}(f) - S_{\tilde{n}}(-f)}{2} \end{cases}$$

$$S_n(f) = \frac{1}{2} S_{\tilde{n}}(f-f_0) + \frac{1}{2} S_{\tilde{n}}(f+f_0)$$

Osservazioni

(1) per un processo passa-banda

$$S_n(f) = 0 \qquad |f| \geq 2f_0$$

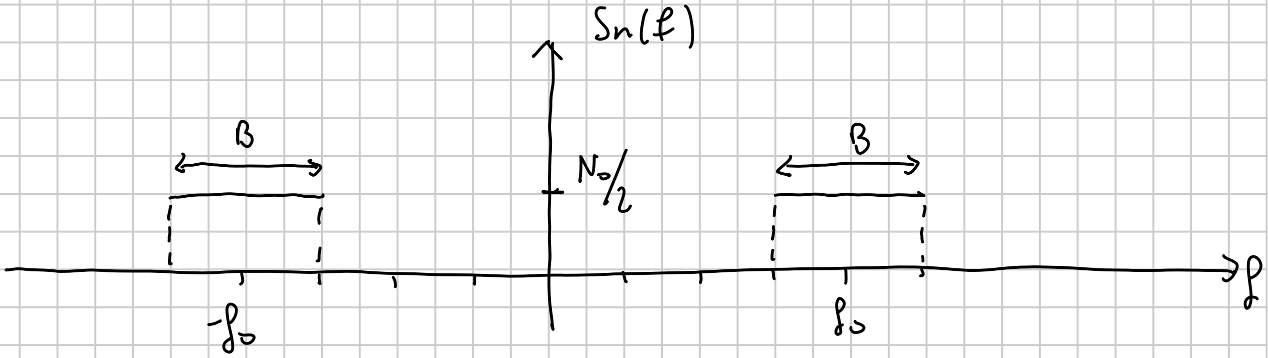
(2) Per definizione le DSP dell'inviluppo complesso $\tilde{n}(t)$

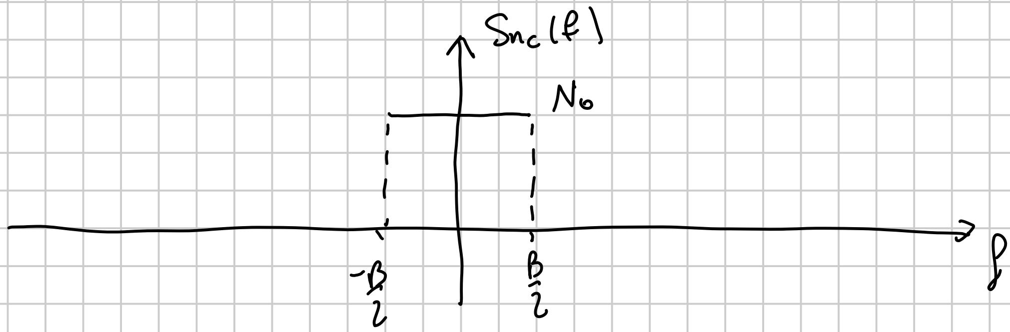
$$S_{\tilde{n}}(f) = 0 \qquad \text{per } f < -f_0$$

$\Downarrow$ $S_n(f)$ è reale e pari.

$$S_{n_c}(f) = S_{n_s}(f) = \begin{cases} S_n(f+f_0) + S_n(f-f_0) & |f| \leq f_0 \\ 0 & \text{altrove} \end{cases}$$

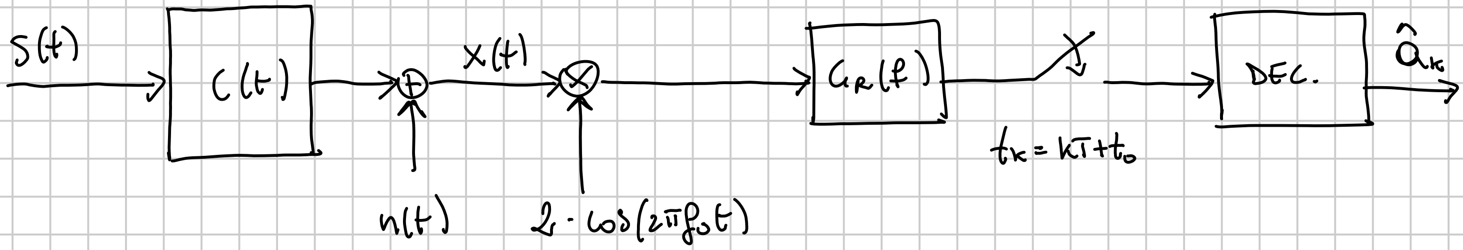
$$j S_{n_s n_c}(f) = -j S_{n_c n_s}(f) = \begin{cases} S_n(f+f_0) - S_n(f-f_0) & |f| \leq f_0 \\ 0 & \text{altrove} \end{cases}$$



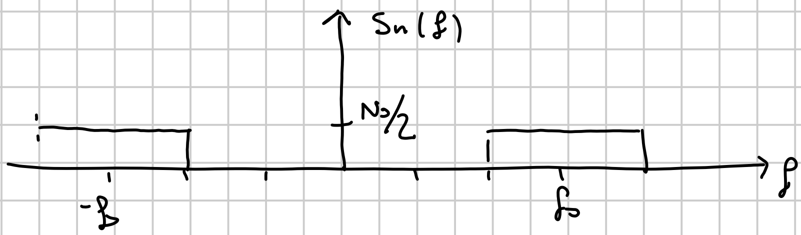


$$S_{n_s n_c}(f) = - S_{n_c n_s}(f) = 0$$

Esercizio – PAM banda passante



n(t)



$$n(t) = \sum_{k=-\infty}^{+\infty} a_k \, p(t - kT) \cos(2\pi f_0 t)$$

$a_i \in A_s = \{\pm 1\}$ indip. ed equiprobabili

$$P(f) = \begin{cases} \sqrt{T} \cos(\pi f T/2) & |fT| \leq 1 \\ 0 & \text{altrove} \end{cases}$$

$$G_R(f) = P(f)$$

$$C(f) = \begin{cases} e^{-j[2\pi(f-f_0)\tau + \pi/6]} & |f-f_0| \leq \frac{1}{T} \\ e^{-j[2\pi(f+f_0)\tau - \pi/6]} & |f+f_0| \leq \frac{1}{T} \\ 0 & \text{altrove} \end{cases}$$

Calcoliamo:

1) $P_E(M) = P_E(b)$

2) E_s energia media per simbolo trasmesso

<u>Soluzione</u>:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k \underbrace{\left[p(t-kT)\cos(2\pi f_0 t)\right] \otimes c(t)}_{g(t)} + n(t)$$

$$g(t) = \left[p(t-kT)\cos(2\pi f_0 t)\right] \otimes c(t)$$

$$G(f) = \left[P(f)\, e^{-j2\pi kTf} \otimes \left(\frac{\delta(f-f_0)+\delta(f+f_0)}{2}\right)\right] \cdot C(f)$$

$$G(f) = \left[\frac{1}{2} P(f-f_0)\, e^{-j2\pi kT(f-f_0)} + \frac{1}{2} P(f+f_0)\, e^{-j2\pi kT(f+f_0)}\right] \cdot C(f)$$

$$= \frac{1}{2} P(f-f_0)\, e^{-j2\pi kT(f-f_0)}\, e^{-j2\pi(f-f_0)\tau}\, e^{-j\pi/6} +$$

$$+ \frac{1}{2} P(f+f_0)\, e^{-j2\pi kT(f+f_0)}\, e^{-j2\pi(f+f_0)\tau}\, e^{+j\pi/6}$$

$$G(f) = \frac{1}{2} P(f-f_0)\, e^{-j2\pi(f-f_0)(kT+\tau)}\, e^{-j\pi/6} +$$

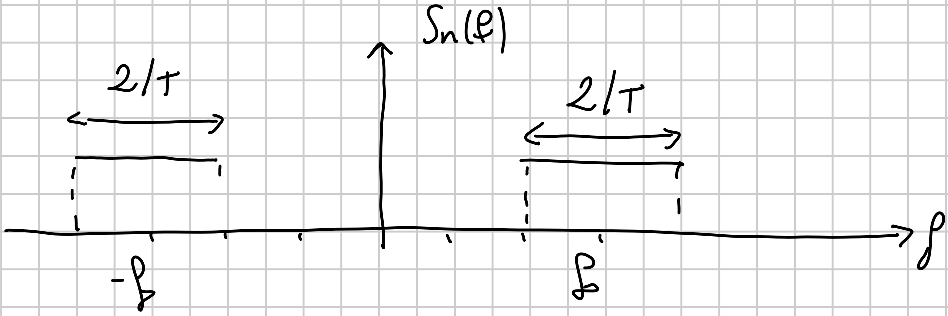
$$\frac{1}{2} P(f+f_0)\, e^{-j2\pi(f+f_0)(kT+\tau)}\, e^{+j\pi/6}$$

$$g(t) = p(t-kT-\tau)\cos(2\pi f_0 t + \pi/6)$$

$$g(t) = p(t-kT-\tau)\left[\cos(2\pi f_0 t)\cos(\pi/6) - \sin(2\pi f_0 t)\sin(\pi/6)\right]$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k\, p(t-kT-\tau)\left[\cos(2\pi f_0 t)\cos(\pi/6) - \sin(2\pi f_0 t)\sin(\pi/6)\right] + n(t)$$

$n(t)$ processo Gaussiano stazionario additivo passa-banda



$$n(t) = n_c(t)\cos(2\pi f_0 t) - n_s(t)\sin(2\pi f_0 t)$$

Il segnale demodulato $z(t)$ sarà:

$$z(t) = x(t)\cdot 2\cos(2\pi f_0 t) = z_s(t) + z_n(t)$$

$$z_s(t) = 2\sum_{k=-\infty}^{+\infty} a_k\, p(t-kT-\tau)\left[\cos^2(2\pi f_0 t)\cos(\pi/6) - \sin(2\pi f_0 t)\cdot\cos(2\pi f_0 t)\sin(\pi/6)\right]$$

$$2\cos^2(\alpha) = 1 + \cos(2\alpha)$$

$$2\sin(\alpha)\cos(\alpha) = \sin(2\alpha)$$

$$\boxed{z_s(t) = \sum_k a_k\, p(t-kT-\tau)\left[\left(1+\cos(4\pi f_0 t)\right)\cos(\pi/6) - \sin(4\pi f_0 t)\sin(\pi/6)\right]}$$

$$z_n(t) = n(t)\cdot 2\cos(2\pi f_0 t) \qquad n(t) = n_c(t)\cos(2\pi f_0 t) - n_s(t)\sin(2\pi f_0 t)$$

$$\boxed{z_n(t) = n_c(t)\left(1+\cos(4\pi f_0 t)\right) - n_s(t)\sin(4\pi f_0 t)}$$

$G_R(f)$ è un filtro adattato e passa-basso

$$y(t) = z(t)\otimes g_R(t) = z_s(t)\otimes g_R(t) + z_n(t)\otimes g_R(t)$$

$$\boxed{y_s(t) = z_s(t)\otimes g_R(t) = \sum_{k=-\infty}^{+\infty} a_k \cos(\pi/6)\, p(t-kT-\tau)\otimes g_R(t)}$$

$$\boxed{y_n(t) = z_n(t)\otimes g_R(t) = n_c(t)\otimes g_R(t)}$$

$$q(t) = p(t) \otimes g_R(t) = p(t) \otimes p(t)$$

$$Q(f) = P^2(f) = \begin{cases} T \cos^2\left(\pi f \, T/2\right) & |fT| \leq 1 \\ 0 & \text{altrove} \end{cases}$$

$$= \begin{cases} \frac{T}{2}\left(1 + \cos\left(\pi f T\right)\right) & |fT| \leq 1 \\ 0 & \text{altrove} \end{cases}$$

FUNZIONE COSENO RIALZATO CON $\alpha = 1$

$\Downarrow$

Soddisfa il I° criterio di Nyquist

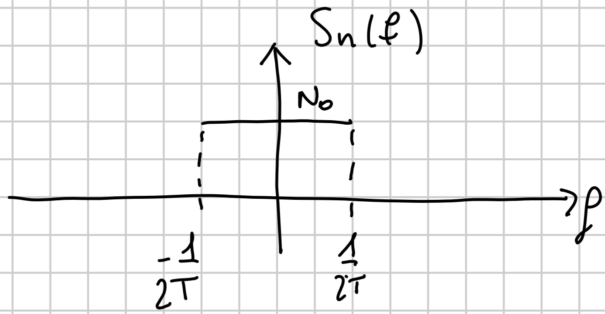
$$\sum_k Q\left(f - \frac{k}{T}\right) = T \Rightarrow q(0) = 1$$

$$y_s(t) = \sum_k a_k \cos(\pi/6) \, q(t - kT - \tau)$$

$$t_i = iT + t_0 = iT + \tau \Rightarrow t_0 = \tau$$

$$\boxed{y_s[n] = a_k \, q(0) \cos(\pi/6) = a_k \cos(\pi/6)}$$

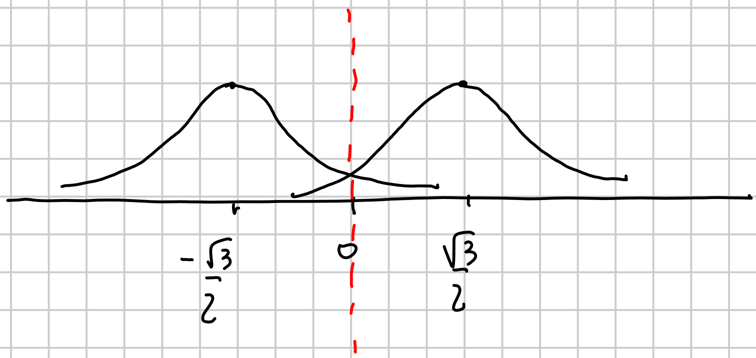
$$g_n(t) = n_c(t) \otimes g_R(t)$$



$$\sigma_{cp}^2 = N_0 \int g_R(t)^2\, dt = N_0 \int G_R^2(f)\, df = N_0 \int P^2(f)\, df = N_0$$

$$y[k] = a_k \cos(\pi/6) + n_{cp}[k] \qquad a_k \in \{\pm 1\}$$

$$\lambda = 0$$



$$P_E(M) = P_E(b) = \frac{1}{2} \Pr\{\hat{a}_k = 1 \mid a_k = -1\} + \frac{1}{2} \Pr\{\hat{a}_k = 1 \mid a_k = 1\}$$

$$\Pr\{\hat{a}_k = 1 \mid a_k = -1\} = \Pr\{\hat{a}_k = -1 \mid a_k = 1\}$$

$$P_E(M) = \Pr\{\hat{a}_k = 1 \mid a_k = -1\} = Q\left(\frac{\sqrt{3}/2}{\sqrt{N_0}}\right) = Q\left(\sqrt{\frac{3}{4N_0}}\right)$$

E_s per simbolo Tx

$$s(t) = \sum_i a_i \, p(t - iT) \cos(2\pi f_0 t)$$

$E_{s1} \Rightarrow \quad s_1(t) = 1 \cdot p(t) \cos(2\pi f_0 t) \qquad (a_k = 1)$

$$E_{s1} = \int_{-\infty}^{+\infty} s_1^2(t) \, dt = \int_{-\infty}^{+\infty} p^2(t) \cos^2(2\pi f_0 t) \, dt \qquad (a_k = -1)$$

$$= \int_{-\infty}^{+\infty} p^2(t) \left(\frac{1}{2} + \frac{1}{2} \cos(4\pi f_0 t) \right) dt \cong \frac{1}{2} \int_{-\infty}^{+\infty} p^2(t) \, dt = \frac{1}{2} E_p$$

$E_{s2} \Rightarrow \quad s_2(t) = -1 \cdot p(t) \cos(2\pi f_0 t)$

$$E_{s2} = \int_{-\infty}^{+\infty} p^2(t) \cos^2(2\pi f_0 t) \, dt = E_{s1}$$

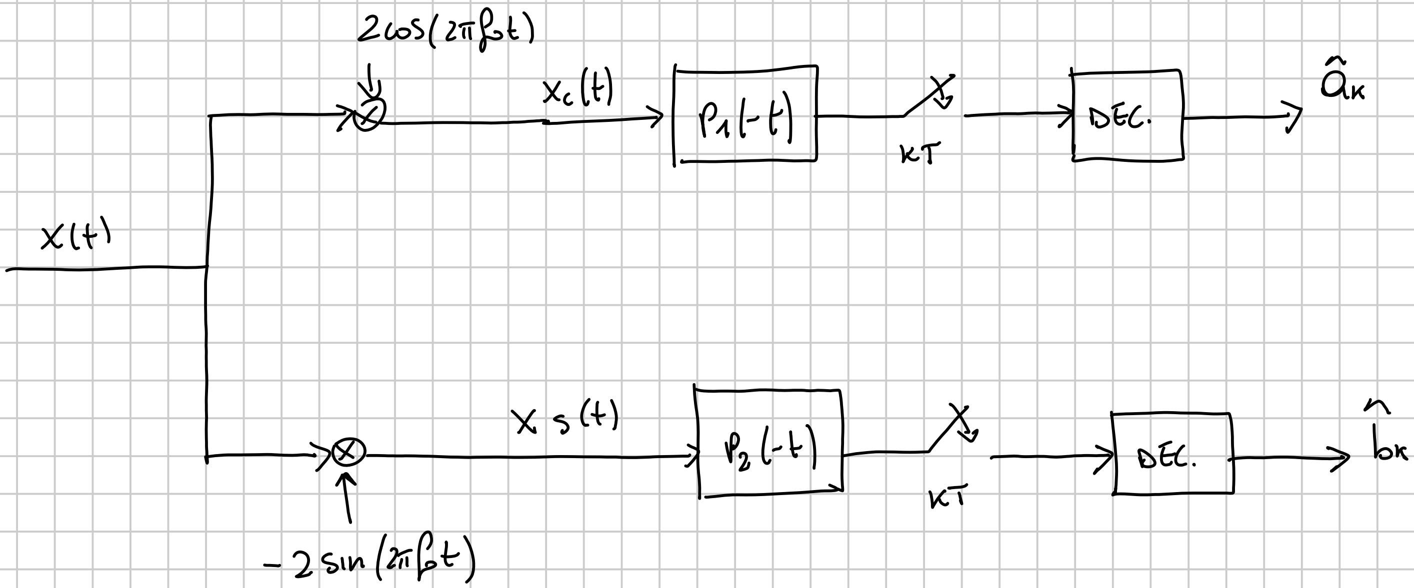
$$E_s = \frac{1}{2} E_{s1} + \frac{1}{2} E_{s2} = E_{s1} = \frac{1}{2} E_p = \frac{1}{2}$$

$E_s \Rightarrow$ energia media per intervallo di segnalazione del segnale Tx

1) $E_s = P_s T_s \qquad P_s = \int_{-\infty}^{+\infty} S_s(f) \, df$

2) $E_s = \int_0^T s^2(t) \, dt$

QAM in banda passante

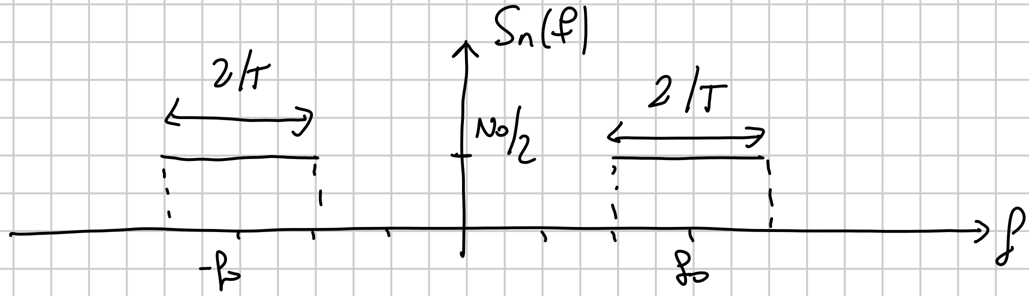


$$x(t) = \sum_{i=-\infty}^{+\infty} a_i\, p_1(t-iT)\cos(2\pi f_0 t) - \sum_{i=-\infty}^{+\infty} b_i\, p_2(t-iT)\sin(2\pi f_0 t) + n(t)$$

$a_i \in \{\pm 2\}$ $\qquad$ $b_i \in \{\pm 1\}$ $\qquad$ indip. ed equiprobabili

$$P_1(f) = \begin{cases} T\sqrt{|f|} & |fT| \le 1 \\ 0 & \text{altrove} \end{cases}$$

$$P_2(f) = \begin{cases} \sqrt{T}\,\sin(\pi |f| T/2) & |fT| \le 1 \\ 0 & \text{altrove} \end{cases}$$



RAMO IN FASE

$$x_c(t) = x(t) \cdot 2\cos(2\pi f_0 t) =$$

$$= \sum_i a_i\, p_1(t-iT)\, 2\cos^2(2\pi f_0 t) - \sum_i b_i\, p_2(t-iT)\sin(2\pi f_0 t)\, 2\cos(2\pi f_0 t) +$$

$$n(t)\cdot 2\cos(2\pi f_0 t)$$

$n(t) = n_c(t)\cos(2\pi f_0 t) - n_s(t)\sin(2\pi f_0 t)$

$$= \sum_i a_i\, p_1(t-iT)\left(1+\cos(4\pi f_0 t)\right) - \sum_i b_i\, p_2(t-iT)\sin(4\pi f_0 t) +$$

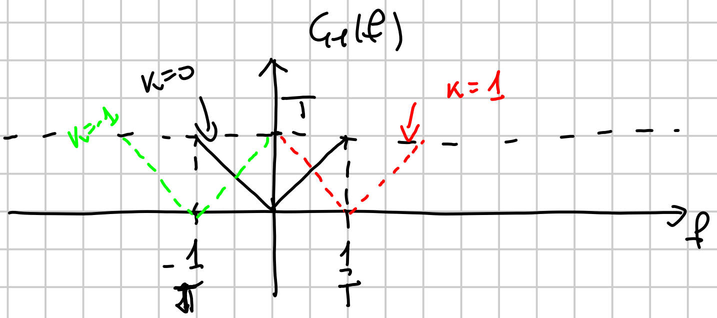
$$n_c(t)\left(1+\cos(4\pi f_0 t)\right) - n_s(t)\sin(4\pi f_0 t)$$

Il filtro in ricezione $p_1(-t)$ è F.A. e passa-basso

$$y_c(t) = \sum_i a_i\, p_1(t-iT) \otimes p_1(-t) + n_c(t) \otimes p_1(-t)$$

$$g_1(t) = p_1(t) \otimes p_1(-t)$$

$$G_1(f) = P_1(f)\cdot P_1^*(f) = |P_1(f)|^2 = P_1^2(f) = \begin{cases} T^2\,|f| & |fT| \le 1 \\ 0 & \text{altrove} \end{cases}$$

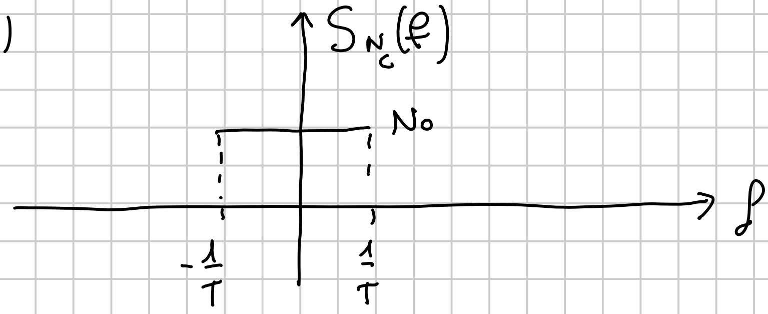


$$\sum_k G_1(f - k/T) = T \Rightarrow \begin{array}{ll} g_1(0) = 1 & \\ g_1(k) = 0 & k \neq 0 \end{array}$$

All'uscita del campionatore

$$y_c[k] = a_k + n_{c_{p_1}}[k]$$

$$n_{c_{p_1}}(t) = n_c(t) \otimes p_1(-t)$$



RAMO IN QUADRATURA

$$x_s(t) = x(t) \cdot \left(-2 \sin(2\pi f_0 t)\right) =$$

$$= \sum_i a_i \, p_1(t - iT) \left(-2 \sin(2\pi f_0 t) \cdot \cos(2\pi f_0 t)\right) + \sum_i b_i \, p_2(t - iT) \, 2 \sin^2(2\pi f_0 t)$$

$$+ n(t) \left(-2 \sin(2\pi f_0 t)\right) \qquad n(t) = n_c(t) \cos(2\pi f_0 t) - n_s(t) \sin(2\pi f_0 t)$$

$$= -\sum_i a_i \, p_1(t - iT) \sin(4\pi f_0 t) + \sum_i b_i \, p_2(t - iT) \left(1 - \cos(4\pi f_0 t)\right)$$

$$- n_c(t)\sin(4\pi f t) + n_s(t)\left(1-\cos(4\pi f t)\right)$$

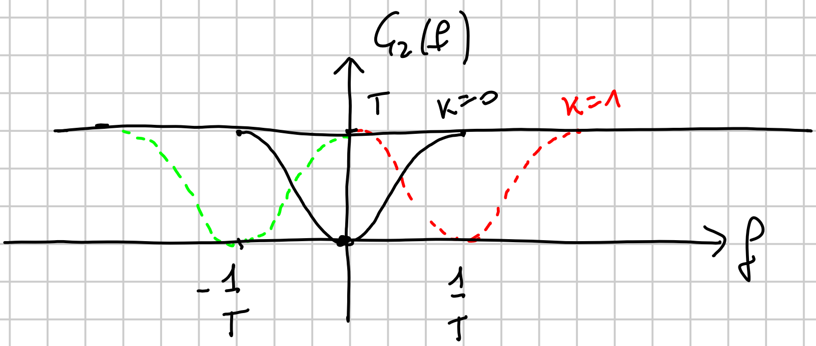
Il filtro in ricezione è $p_2(-t) \Rightarrow$ F.A. e passa basso

$$y_s(t) = \sum_i b_i\, p_2(t-iT) \otimes p_2(-t) + n_s(t) \otimes p_2(-t)$$

$$g_2(t) = p_2(t) \otimes p_2(-t)$$

$$G_2(f) = P_2(f)\cdot P_2^*(f) = |P_2(f)|^2 = P_2^2(f) = \begin{cases} T\sin^2\left(\pi |f| T/2\right) & |fT| \le 1 \\ 0 & \text{altrove} \end{cases}$$

$$G_2(f) = \begin{cases} \frac{T}{2}\left(1-\cos(\pi f T)\right) & |fT| \le 1 \\ 0 & \text{altrove} \end{cases}$$



$$\sum G_2\left(f-k/T\right) = T \quad \Rightarrow \quad g_2(0) = 1$$

$$\frac{T}{2}\left(1-\cos(\pi f T)\right) + \frac{T}{2}\left(1-\cos\left(\pi\left(f-\frac{1}{T}\right)T\right)\right) =$$

$$= \frac{T}{2} - \frac{T}{2}\cancel{\cos(\pi f T)} + \frac{T}{2} - \frac{T}{2}\underbrace{\cancel{\cos(\pi f T - \pi)}}_{= -\cos(\pi f T)} = T$$

All'uscita del campionatore abbiamo:

$$y_s[k] = b_k + n_{sp}[k]$$

$$\begin{cases} y_c[k] = a_k + n_{cp}[k] = a_k + \nu_c \\ y_s[k] = b_k + n_{sp}[k] = b_k + \nu_s \end{cases}$$

$$S_{\nu_c}(f) = S_{n_c}(f)\,|P_1(f)|^2 \qquad\qquad S_{n_c}(f) = S_{n_s}(f)$$

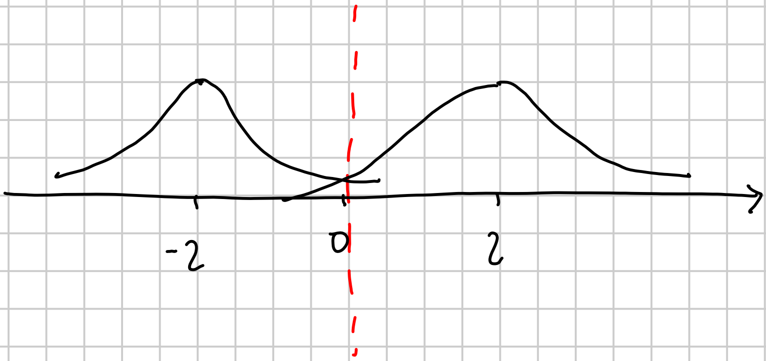
$$S_{\nu_s}(f) = S_{n_s}(f)\,|P_2(f)|^2$$

$$\sigma^2_{\nu_c} = \int_{-\infty}^{+\infty} S_{n_c}(f)\,|P_1(f)|^2\,df = N_0\int_{-\infty}^{+\infty} P_1^2(f)\,df = N_0\int_{-\infty}^{+\infty} G_1(f)\,df = N_0\,g_1(0) = N_0$$

$$\sigma^2_{\nu_s} = \int_{-\infty}^{+\infty} S_{n_s}(f)\,|P_2(f)|^2\,df = N_0\int_{-\infty}^{+\infty} P_2^2(f)\,df = N_0\int_{-\infty}^{+\infty} G_2(f)\,df = N_0\,g_2(0) = N_0$$

RAMO IN FASE

$$y_c[k] = a_k + \nu_c$$

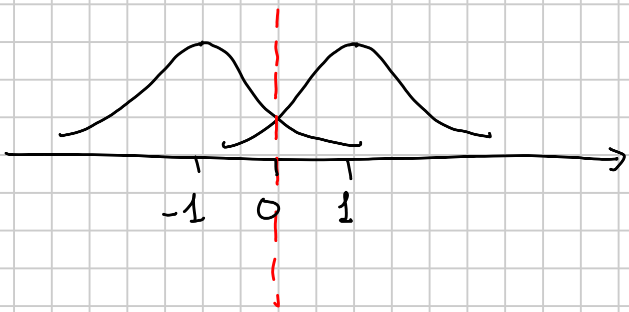


$$P_c = \frac{1}{2} \Pr\{\hat{a}_k = -2 \mid a_k = 2\} + \frac{1}{2} \Pr\{\hat{a}_k = 2 \mid \hat{a}_k = -2\} = \Pr\{\hat{a}_k = -2 \mid a_k = 2\}$$

$$P_c = Q\left(\frac{2}{\sqrt{N_0}}\right)$$

RAMO IN QUADRATURA

$$y_s[k] = b_k + \nu_s$$



$$P_s = \frac{1}{2} \Pr\{\hat{b}_k = -1 \mid b_k = 1\} + \frac{1}{2} \Pr\{\hat{b}_k = 1 \mid b_k = -1\} = \Pr\{\hat{b}_k = -1 \mid b_k = 1\}$$

$$P_s = Q\left(\frac{1}{\sqrt{N_0}}\right)$$

$$P_E^{QAM}(4) = P_c(1-P_s) + P_s(1-P_c) + P_s P_c$$

Calcoliamo adesso l'energia media per simbolo trasmesso.

$$s_1(t) = 2p_1(t)\cos(2\pi f_0 t) - p_2(t)\sin(2\pi f_0 t) \qquad (2,1)$$

$$s_2(t) = 2p_1(t)\cos(2\pi f_0 t) + p_2(t)\sin(2\pi f_0 t) \qquad (2,-1)$$

$$s_3(t) = -2p_1(t)\cos(2\pi f_0 t) - p_2(t)\sin(2\pi f_0 t) \qquad (-2,1)$$

$$s_4(t) = -2p_1(t)\cos(2\pi f_0 t) + p_2(t)\sin(2\pi f_0 t) \qquad (-2,-1)$$

$$E_{s1} = \int_{-\infty}^{+\infty} s_1^2(t)\,dt = \int_{-\infty}^{+\infty} 4p_1^2(t)\cos^2(2\pi f_0 t)\,dt + \int_{-\infty}^{+\infty} p_2^2(t)\sin^2(2\pi f_0 t)\,dt =$$

$$= 4\int_{-\infty}^{+\infty} p_1^2(t)\left(\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0 t)\right)dt + \int_{-\infty}^{+\infty} p_2^2(t)\left(\frac{1}{2} - \frac{1}{2}\cos(4\pi f_0 t)\right)dt =$$

$$= 4\cdot\frac{1}{2}\int_{-\infty}^{+\infty} p_1^2(t)\,dt + \frac{1}{2}\int_{-\infty}^{+\infty} p_2^2(t)\,dt = 2E_{p1} + \frac{1}{2}E_{p2}$$

$$E_{s2} = E_{s3} = E_{s4} = E_{s1} = 2E_{p1} + \frac{1}{2}E_{p2}$$

$$E_s = \frac{1}{4}E_{s1} + \frac{1}{4}E_{s2} + \frac{1}{4}E_{s3} + \frac{1}{4}E_{s4} = 2E_{p1} + \frac{1}{2}E_{p2}$$

E_s energia media per intervallo di segnalazione del segnale Trasmess.

$$E_s = E\left\{\int_0^T s^2(t)\, dt\right\}$$

$$E\{a_i \cdot a_{i+k}\} = \begin{cases} E\{a_i^2\} & k=0 \\ (E\{a_i\})^2 & k\neq 0 \end{cases} = \begin{cases} 4 & k=0 \\ 0 & k\neq 0 \end{cases}$$

$$E\{a_i^2\} = \frac{1}{2}(4) + \frac{1}{2}(4) = 4$$

$$R_a[k] = 4\,\delta[k]$$

$$E\{b_i \cdot b_{i+k}\} = \begin{cases} E\{b_i^2\} & k=0 \\ (E\{b_i\})^2 & k\neq 0 \end{cases} = \begin{cases} 1 & k=0 \\ 0 & k\neq 0 \end{cases}$$

$$R_b[k] = \delta[k]$$

$$E\{a_i\, b_{i+k}\} = E\{a_i\} \cdot E\{b_{i+k}\} = 0 \qquad \forall k$$

$$E_s = E\left\{ \int_0^T s^2(t)\, dt \right\} =$$

$$= E\left\{ \int_0^T \left(\sum_i a_i\, p_1(t-iT)\cos(2\pi f_0 t) - \sum_i b_i\, p_2(t-iT)\sin(2\pi f_0 t) \right) \cdot \right.$$

$$\left. \left(\sum_k a_k\, p_1(t-kT)\cos(2\pi f_0 t) - \sum_k b_k\, p_2(t-kT)\sin(2\pi f_0 t) \right) dt \right\}$$

$$= \int_0^T \sum_i \sum_k E\{a_i a_k\}\, p_1(t-iT)\, p_1(t-kT)\cos^2(2\pi f_0 t)\, dt \;+$$

$$- \int_0^T \sum_i \sum_k E\{a_i b_k\}\, p_1(t-iT)\, p_2(t-kT)\cos(2\pi f_0 t)\sin(2\pi f_0 t)\, dt \;+$$

$$- \int_0^T \sum_i \sum_k E\{a_k b_i\}\, p_1(t-kT)\, p_2(t-iT)\cos(2\pi f_0 t)\sin(2\pi f_0 t)\, dt$$

$$+ \int_0^T \sum_i \sum_k E\{b_i b_k\}\, p_2(t-iT)\, p_2(t-kT)\sin^2(2\pi f_0 t)\, dt$$

$$E_S = \int_0^T \sum_i 4\, p_1^2(t-iT) \cos^2(2\pi f t)\, dt + \int_0^T \sum_i p_2^2(t-iT) \sin^2(2\pi f t)\, dt =$$

$$= \frac{1}{2} \int_0^T \sum_i 4\, p_1^2(t-iT)\, dt + \frac{1}{2} \int_0^T \sum_i p_2^2(t-iT)\, dt$$

$$\int_0^T \sum_i p(t-iT)\, dt = \sum_i \int_0^T p(t-iT)\, dt = \qquad \alpha = t - iT$$

$$= \sum_i \int_{iT}^{iT+T} p(\alpha)\, d\alpha = \int_{-\infty}^{+\infty} p(\alpha)\, d\alpha$$

$$E_S = 4 \frac{1}{2} \int_{-\infty}^{+\infty} p_1^2(t)\, dt + \frac{1}{2} \int_{-\infty}^{+\infty} p_2^2(t)\, dt = 2\, E_{p_1} + \frac{1}{2} E_{p_2}$$