# Soluzione Compitino del 31/05/2019 - Fila B

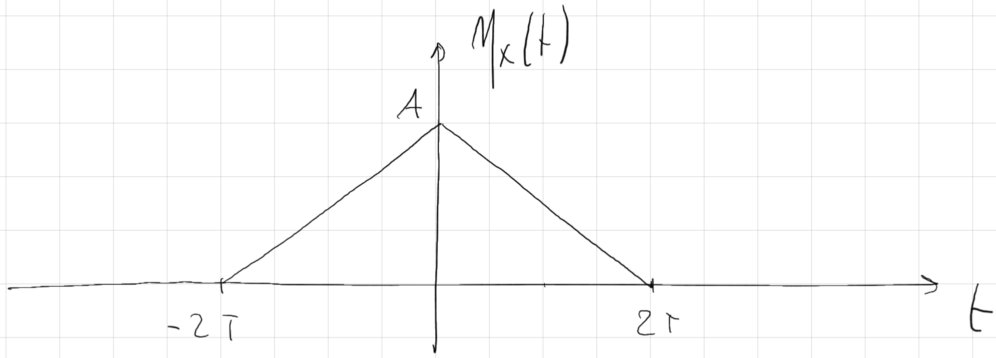
## Es #1

a) $$\eta_X(t) = E\left[X(t)\right] = \int_{-\infty}^{+\infty} A \,\mathrm{rect}\left(\frac{t+t_0}{2T}\right) \frac{1}{2T}\,\mathrm{rect}\left(\frac{t_0}{2T}\right) dt_0$$

$$= \frac{A}{2T}\int_{-\infty}^{+\infty} \mathrm{rect}\left(\frac{t_0}{2T}\right) \mathrm{rect}\left(\frac{t_0+t}{2T}\right) dt_0 =$$

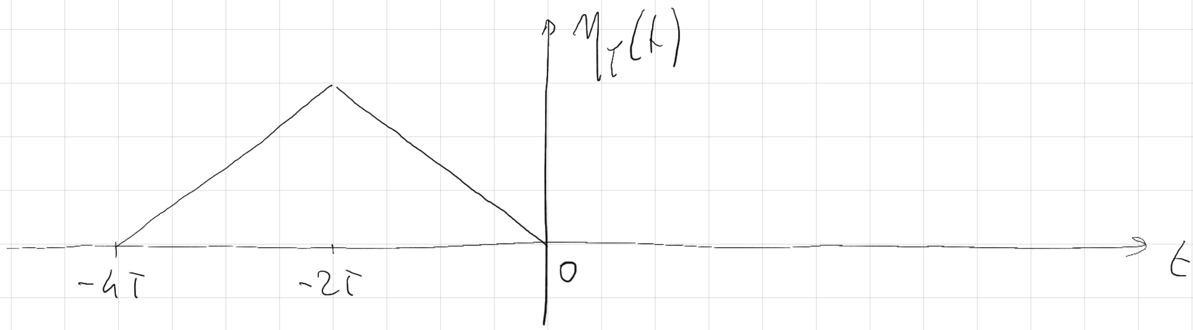
$$= \frac{A}{2T}\,\mathrm{rect}\left(\frac{t}{2T}\right) \otimes \mathrm{rect}\left(\frac{-t}{2T}\right)$$

$$= \frac{A}{2T}\,\mathrm{rect}\left(\frac{t}{2T}\right) \otimes \mathrm{rect}\left(\frac{t}{2T}\right) = \frac{A}{\cancel{2T}}\,\cancel{2T}\left(1-\frac{|t|}{2T}\right)\mathrm{rect}\left(\frac{t}{4T}\right)$$




b) $$\eta_Y(t) = E\left[Y(t)\right] = \eta_X(t) \otimes h(t) = \frac{1}{A}\eta_X(t+2T)$$

$$= \left(1 - \frac{|t+2T|}{2T}\right)\mathrm{rect}\left(\frac{t+2T}{4T}\right)$$



# Es #2

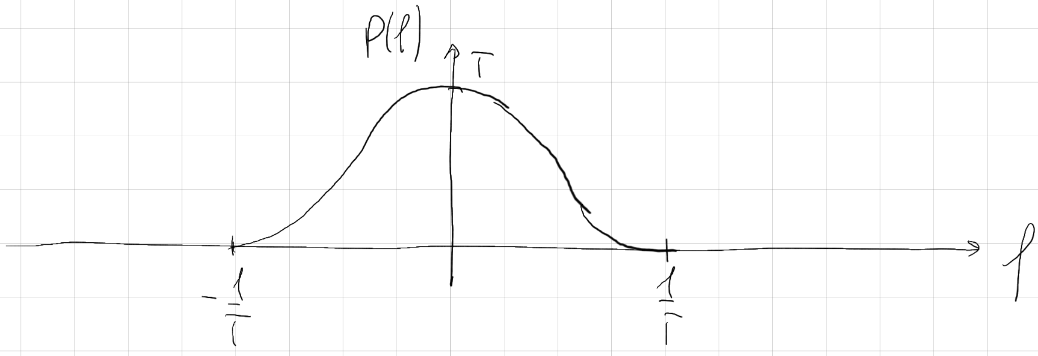
a) $E_s = E[x^2]\, E_p = \frac{31}{4} \cdot \frac{3}{2} T = \boxed{\frac{93}{8} T}$

$E[x^2] = \frac{1}{4}(-2)^2 + \frac{3}{4}(3)^2 = 1 + \frac{27}{4} = \frac{31}{4}$

$p(t) = 2\,\mathrm{sinc}\left(\frac{2t}{T}\right) + \mathrm{sinc}\left(\frac{2}{T}\left(t - \frac{T}{2}\right)\right) + \mathrm{sinc}\left(\frac{2}{T}\left(t + \frac{T}{2}\right)\right)$

$P(f) = 2 \cdot \frac{T}{2}\,\mathrm{rect}\left(\frac{fT}{2}\right) + \frac{T}{2}\,\mathrm{rect}\left(\frac{fT}{2}\right)\left[e^{-j\pi f T} + e^{j\pi f T}\right]$

$= T\,\mathrm{rect}\left(\frac{fT}{2}\right)\left[1 + \cos(\pi f T)\right]$




$E_p = \int_{-\infty}^{+\infty} |P(f)|^2\, df = T^2 \int_{-\frac{1}{T}}^{\frac{1}{T}} \left[1 + \cos^2(\pi f T) + 2\cos(\pi f T)\right] df$

$= T^2 \cdot \frac{1}{T} + T^2 \cdot \frac{1}{2} \cdot \frac{1}{T} = T + \frac{T}{2} = \frac{3}{2} T$

b) $S_s(f) = \frac{1}{T} S_x(f) \, |P(f)|^2$

$S_x(f) = TFS\left[R_x[m]\right]$

$R_x[m] = C_x[m] + \eta_x^2$

$$\eta_x = E[x] = \frac{1}{4}(-2) + \frac{3}{4}(3) = -\frac{2}{4} + \frac{9}{4} = \frac{7}{4}$$

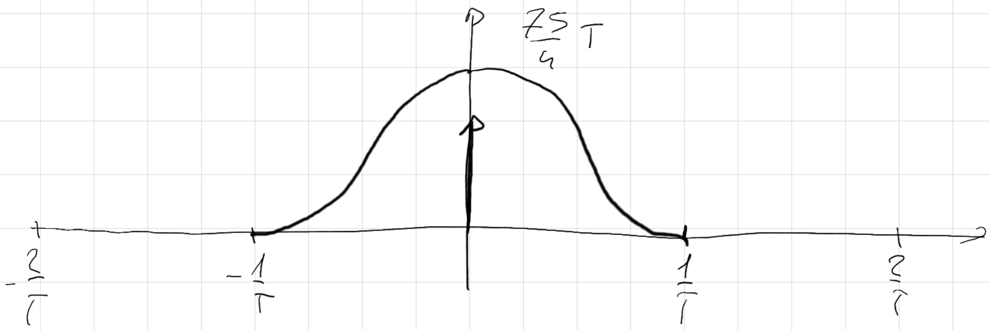
$$C_x[m] = \sigma_x^2 \, \delta[m]$$

$$\sigma_x^2 = E\left[(x-\eta_x)^2\right] = \frac{1}{4}\left(-2-\frac{7}{4}\right)^2 + \frac{3}{4}\left(3-\frac{7}{4}\right)^2$$

$$= \frac{225}{16} + \frac{75}{16} = \frac{300}{16} = \frac{75}{4}$$

$$R_x[m] = \frac{75}{4}\delta[m] + \frac{7}{4}$$

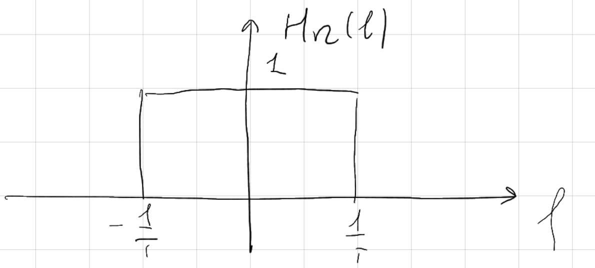
$$S_x(f) = \frac{75}{4} + \frac{7}{4T}\sum_n \delta\left(f - \frac{n}{T}\right)$$




$$\boxed{S_s(f) = \frac{75}{4}T \, \mathrm{rect}\left(\frac{fT}{2}\right)\left[1+\cos(\pi f T)\right]^2 + \frac{7}{4}\delta(f)}$$

Sopravvive solo una $\delta(\cdot)$

c) $$P_{n_u} = \int_{-\infty}^{+\infty} S_N(f)\,|H_R(f)|^2\,df = \frac{N_0}{2}\frac{2}{T} = \frac{N_0}{T}$$



d) $H(f) = P(f)\, H_R(f) = P(f)$

$h(t) = p(t)$

$$h(nT) = 2\,\mathrm{sinc}(2n) + \mathrm{sinc}(2n-1) + \mathrm{sinc}(2n+1)$$

$= 2\,\delta[n]$ ASSENZA DI ISI

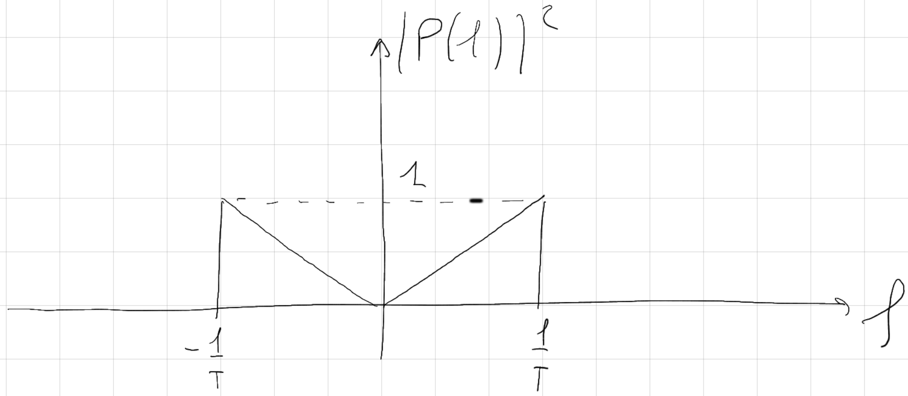
e) $P_E(b) = \frac{1}{4}\, Q\left(\frac{4}{\sqrt{N_0/T}}\right) + \frac{3}{4}\, Q\left(\frac{6}{\sqrt{N_0/T}}\right)$

Es #3

a) $E_s = \frac{1}{2}\left[E[x_c^2] + E[x_s^2]\right] E_p = \frac{1}{2}\left(2 + \frac{7}{4}\right)\frac{1}{T} = \boxed{\frac{15}{8T}}$

$E[x_c^2] = \frac{1}{3}(-2)^2 + \frac{2}{3}(1)^2 = \frac{4}{3} + \frac{2}{3} = 2$

$E[x_s^2] = \frac{3}{4}(1)^2 + \frac{1}{4}(2)^2 = \frac{3}{4} + \frac{4}{4} = \frac{7}{4}$




$E_p = \frac{1}{T}$

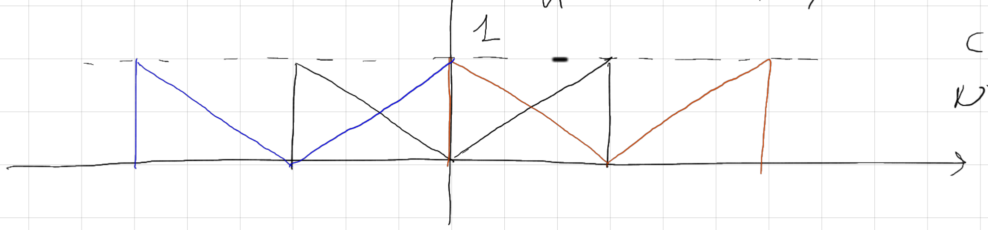
b) $P_{n_{u_c}} = P_{n_{u_s}} = P_{n_u} = N_0 \, E_{H_R} = \frac{N_0}{T}$

$E_{H_R} = E_p = \frac{1}{T}$

c) verifico la cond. di Nyquist

$H(f) = P(f) \, H_R(f) = |fT| \, \text{rect}\left(\frac{fT}{2}\right)$

$\sum_n H\left(f - \frac{n}{T}\right) = \text{cost}$




COND. DI NYQUIST VERIF.

$H(f) = \text{rect}\left(\frac{fT}{2}\right) - \left(1 - |f|T\right) \text{rect}\left(\frac{fT}{2}\right)$

$h(t) = \frac{2}{T} \text{sinc}\left(\frac{2t}{T}\right) - \frac{1}{T} \text{sinc}^2\left(\frac{t}{T}\right)$

$h(0) = \frac{1}{T}$

$$P_E(a) = P_E^c(b)\left(1 - P_E^s(b)\right) + P_E^s(b)\left(1 - P_E^c(b)\right) + P_E^c(b) \, P_E^s(b)$$

$$P_E^c(b) = \frac{1}{3} Q\left(\frac{\frac{2}{T}}{\sqrt{\frac{N_0}{T}}}\right) + \frac{2}{3} Q\left(\frac{\frac{1}{T}}{\sqrt{\frac{N_0}{T}}}\right)$$

$$P_E^s(b) = \frac{3}{4} Q\left(\frac{\frac{1}{T}}{\sqrt{\frac{N_0}{T}}}\right) + \frac{1}{4} Q\left(\frac{\frac{2}{T}}{\sqrt{\frac{N_0}{T}}}\right)$$