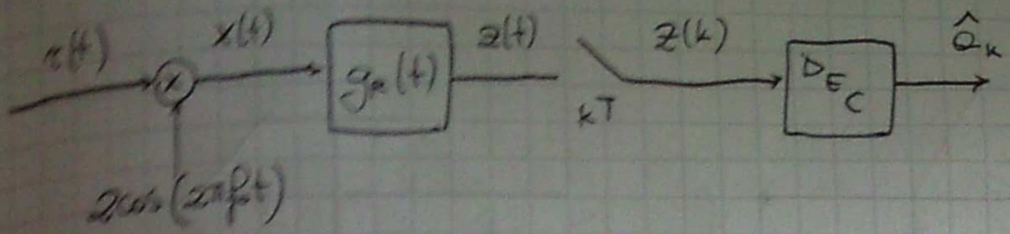
2° COMPITINO 2010

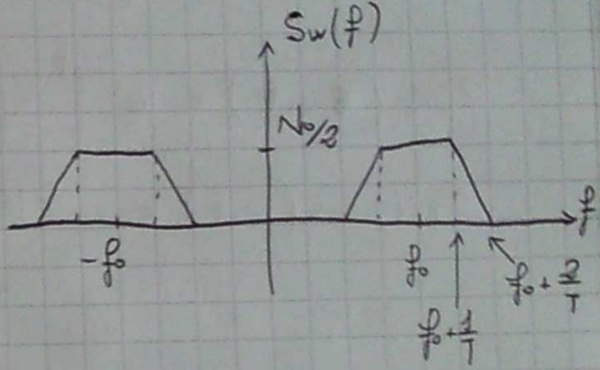


$r(t) = \sum_i a_i\, g_T(t - iT) \cos(2\pi f_0 t) + w(t)$

$a_i \in A \equiv [-1, 1]$
indip. ed equiprob.

$G_T(f) = \mathrm{rect}\left(\frac{f}{3/T}\right)$

$G_R(f) = T \cdot \left(1 - \frac{|f|}{1/T}\right) \cdot \mathrm{rect}\left(\frac{f}{2/T}\right)$



1) $\overline{E}_R$ 2) Eq. b.b. rx 3) $S_{n_c}(f)$ e P_{n_c}
4) $g(t)$ 5) $P(e)$ con $\lambda = 1/4$

① $\overline{E}_R = \overline{E}_T = \sum_k E_k P_k$

$$E_k = \int_{-\infty}^{+\infty} m_k^2 \cdot g_T^2(t) \cdot \cos^2(2\pi f_0 t)\, dt$$

$$= m_k^2 \int_{-\infty}^{+\infty} \frac{1}{2} g_T^2(t)\, dt + \overbrace{m_k^2 \int_{-\infty}^{+\infty} \frac{1}{2} g_T^2(t) \cos(4\pi f_0 t)\, dt}^{=0 \text{ perché } g_T^2(t) \text{ e } \cos() \text{ disgiunti in banda}} =$$

$$= \frac{1}{2} m_k^2 \cdot E_{g_T}$$

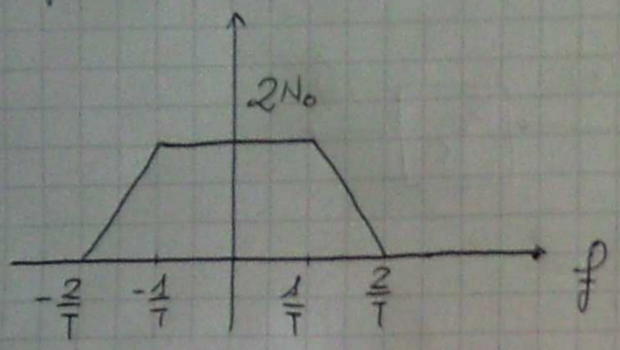
$$E_{-1} = E_1 = \frac{1}{2} \cdot \frac{3}{T} = \frac{1}{T} \overset{\text{equip.}}{=} \overline{E}_R$$

② $\tilde{r}(t) = s(t) + \tilde{w}(t)$

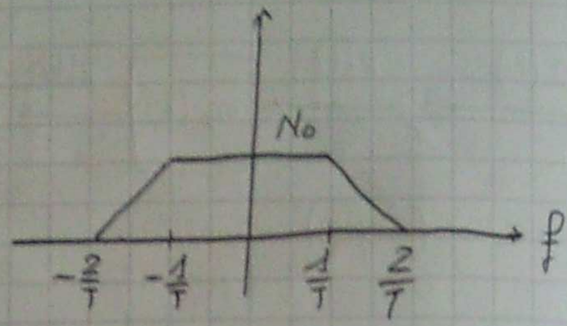
con $s(t) = \sum_i a_i\, g_T(t - iT)$

$\tilde{w}(t) = w_c(t) + j\, w_s(t)$

$S_{\tilde{w}}(f) \rightarrow$



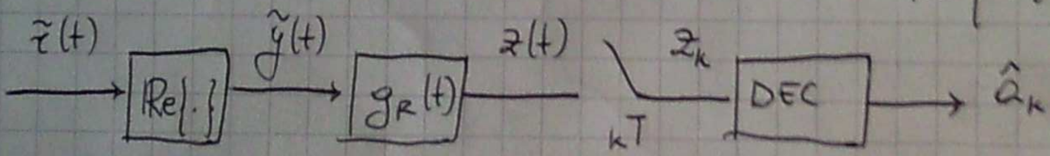
$S_{w_c} = S_{w_s}$



$$r(t) = \mathrm{Re}\left\{\tilde{r}(t)\cdot e^{j\omega_0 t}\right\} = \frac{\tilde{r}(t)\cdot e^{j\omega_0 t} + \tilde{r}^*(t)\cdot e^{-j\omega_0 t}}{2}$$

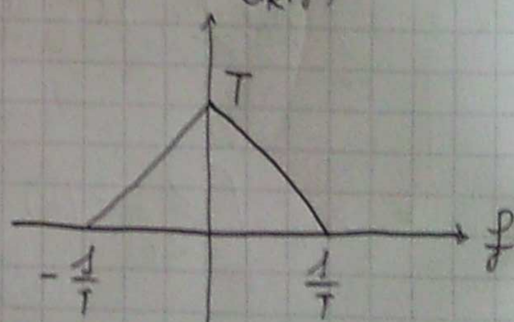
$$2\cos(\omega_0 t) = e^{j\omega_0 t} + e^{-j\omega_0 t}$$

$$2r(t)\cos(\omega_0 t) = \tilde{r}(t)\cdot\frac{e^{j2\omega_0 t}}{2} + \underbrace{\frac{\tilde{r}(t) + \tilde{r}^*(t)}{2}}_{\mathrm{Re}\{\tilde{r}(t)\}} + \frac{e^{-j2\omega_0 t}}{2}$$



③ $S_{n_c}(f) = S_{w_c}(f)\cdot|G_R(f)|^2$

$= N_0\cdot|G_R(f)|^2$



valore medio nullo

$$P_{n_c} = \sigma_{n_c} = \int_{-\infty}^{+\infty} S_{n_c}(f)\cdot df$$

$$= N_0\int_{-\frac{1}{T}}^{\frac{1}{T}} |G_R(f)|^2\, df = N_0\cdot\frac{2}{3}\cdot T^{\cancel{2}}\cdot\frac{1}{\cancel{T}} = \frac{2N_0T}{3}$$

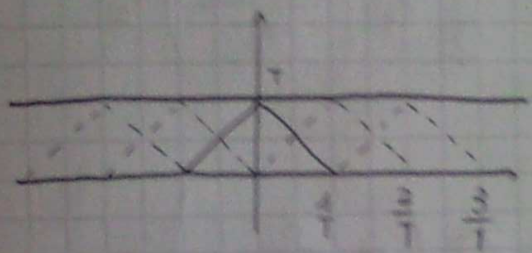
④ $g(t) = g_R(t) \otimes g_T(t) = \int_{-\infty}^{+\infty} g_T(\tau) \cdot g_R(t-\tau)\, d\tau$

$G(f) = G_T(f) \cdot G_R(f) = G_R(f)$

$g(t) = \text{sinc}^2\left(t/T\right)$

$g_o(0) = \int_{-\infty}^{+\infty} G(f)\, df = \frac{2}{T} \cdot T \cdot \frac{1}{2} = 1$

$\sum G\left(f - \frac{m}{T}\right)$ deve essere uguale a T



Ok, è un impulso di Nyquist

⑤

$$y(t) = \underbrace{r(t) \cdot 2\cos(2\pi f_0 t)}_{y'(t)} + w(t)$$

$$y'(t) = \sum_i a_i\, g_T(t - iT) \cos(2\pi f_0 t) \cdot 2\cos(2\pi f_0 t)$$

$$= \sum_i a_i\, g_T(t - iT) \cdot \left[2\cos(\overbrace{2\pi f_0 t}^{\alpha}) \cdot \cos(\overbrace{2\pi f_0 t}^{\beta})\right]$$

Semplifico usando la formula $\left[2\cos(\alpha) \cdot \cos(\beta) = \cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$

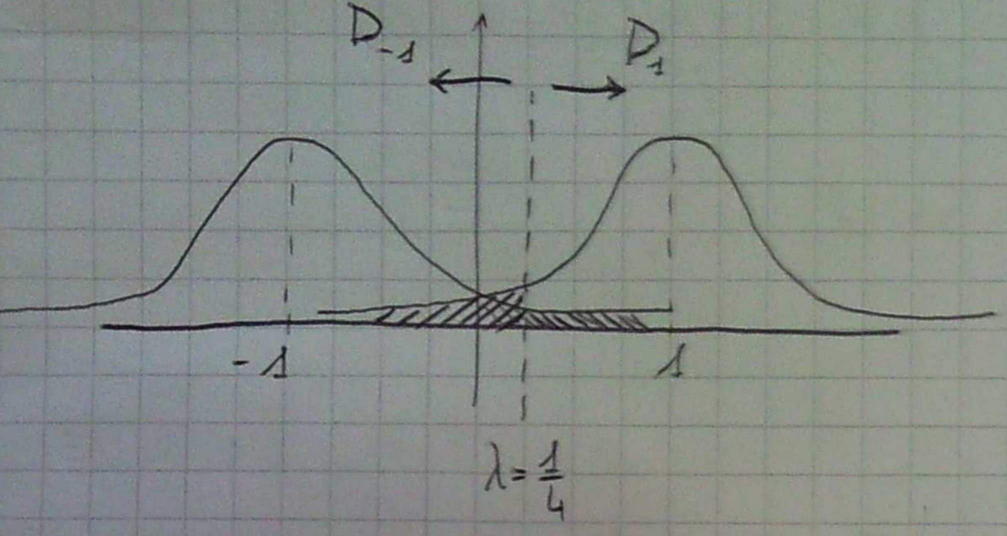
$$= \sum_i a_i\, g_T(t - iT) \left[\cos(4\pi f_0 t) + \underbrace{\cos(0)}_{=1}\right]$$

La componente utile in banda base che passerà dal filtro $g_R(t)$ è:

$$z'(t) = y'(t) \otimes g_R(t) = \sum_i a_i\, g(t - iT).$$

$$z_k = a_k \cdot \underbrace{g(0)}_{=1} + n_k$$

⟶



$$P(e) = P(e|1)\cdot P(1) + P(e|-1)\cdot P(-1)$$

$$= \frac{1}{2} Q\left(\frac{1-1/4}{\sigma_{n_c}}\right) + \frac{1}{2} Q\left(\frac{\frac{1}{4}-(-1)}{\sigma_{n_c}}\right) =$$

$$= \frac{1}{2} Q\left(\frac{3}{4\sigma_{n_c}}\right) + \frac{1}{2} Q\left(+\frac{5}{4\sigma_{n_c}}\right) =$$

$$= \frac{1}{2} Q\left(\frac{3}{4\sqrt{\frac{2N_0T}{3}}}\right) + \frac{1}{2} Q\left(+\frac{5}{4\sqrt{\frac{2N_0T}{3}}}\right)$$