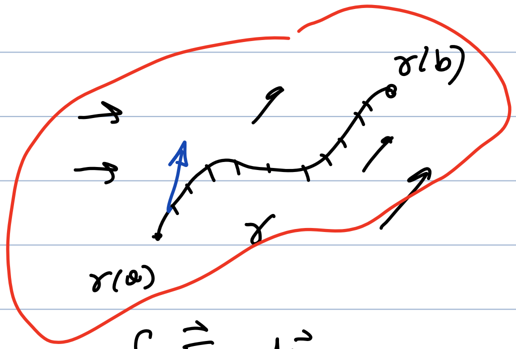
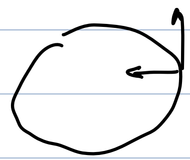
$F = (F_1, F_2, F_3) \qquad F: \mathbb{R}^3 \to \mathbb{R}^3$

$dL = F \cdot dz$



$F \perp dz$

$dL = 0$



$F \in C(\Omega) \qquad \gamma(t) \subset \Omega \qquad \forall t \in [a,b]$

$\gamma \in C^1([a,b])$ semplice

$\gamma(t) = (\gamma_1(t), \gamma_2(t), \gamma_3(t)) \qquad \gamma(t) \neq \gamma(s) \quad t,s \in]a,b[$

$\gamma(a) = \gamma(b)$ chiusa

$$\int_\gamma \vec{F} \cdot d\vec{x} \overset{\text{def}}{=} \int_a^b F(\gamma(t)) \cdot \dot{\gamma}(t)\, dt = \int_a^b F(\gamma(t)) \cdot \gamma'(t)\, dt$$

$$\int_a^b F \cdot T$$

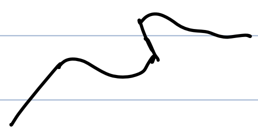
$$\int_a^b F_1(\gamma(t))\gamma_1'(t) + F_2(\gamma(t))\gamma_2'(t) + F_3(\gamma(t))\gamma'(t)\, dt$$

$$= \int_\gamma F_1 dx + F_2 dy + F_3 dz$$

LAVORO PER SPOSTARE PUNTO DI APPLICAZIONE DI F DA $\gamma(a)$ A $\gamma(b)$

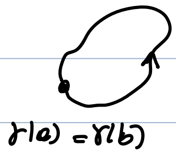
γ è C^1 A TRATTI $\quad \gamma \in C([a,b]) \quad a = t_0 < t_1 \dots < t_n = b$

$$\gamma|_{t_{k-1}, t_k} \in C^1([t_{k-1}, t_k])$$



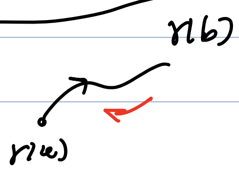
$$\int_\gamma F \cdot dx = \sum_{k=1}^{n} \int_{t_{k-1}}^{t_k} F(\gamma(t)) \cdot \gamma'(t)\, dt$$

$\oint_\gamma F \cdot dx$ CIRCUITAZIONE $\quad \gamma(a) = \gamma(b)$



CURVA CHIUSA SEMPLICE

$\tilde{\gamma}$ È OPPOSTA A γ



$$\int_{\tilde{\gamma}} F \cdot dx = -\int_{\gamma} F \cdot dx$$

DEF. F CAMPO CONSERVATIVO SE ESISTE U $\quad F = \nabla U \qquad U \in C^1(\Omega)$

$F = (F_1, F_2, F_3) \qquad F_i = \partial_i U \qquad F \in C^0(\Omega)$

- **LEMMA** $\quad \int_{\gamma} F \cdot dx = U(\gamma(b)) - U(\gamma(a)) \quad$ SE $\quad F = \nabla U$

- COROLLARIO $\quad \oint_{\gamma} F \cdot dx = 0 \quad$ SE $\quad F = \nabla U$

DIM (LEMMA)

$$\int_{\gamma} F \cdot dx = \int_a^b F(\gamma(t)) \cdot \gamma'(t)\, dt = \int_a^b \nabla U(\gamma(t)) \cdot \gamma'(t)\, dt$$

$$= \int_a^b \frac{d}{dt} U(\gamma(t))\, dt = U(\gamma(b)) - U(\gamma(a))$$

$= \langle \nabla U(\gamma(t)), \gamma'(t) \rangle$

$$\frac{d}{dt} U(\gamma(t)) = \nabla U(\gamma(t)) \cdot \gamma'(t) = \partial_1 U(\gamma(t))\gamma_1'(t) + \partial_2 U(\gamma(t))\gamma_2'(t) + \partial_3 U(\gamma(t))\gamma_3'(t)$$

FORMULA DERIV FUNZ COMPOSTA

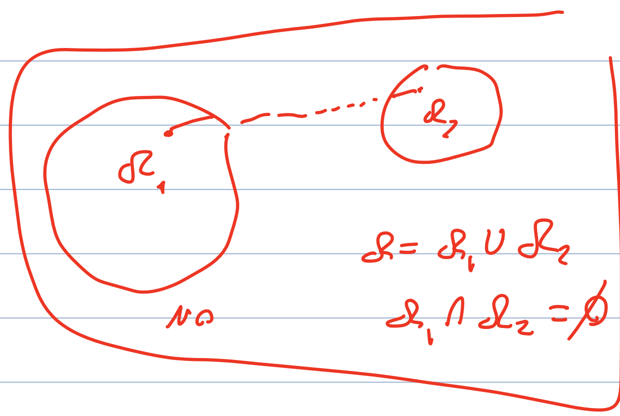
$$\int_a^b f'(t)\,dt = f(b) - f(a)$$

TEOREMA $\quad F \in C^1(\Omega) \quad \Omega$ aperto connesso per archi

SONO EQUIVALENTI

a) γ_1, γ_2 regolari a tratti $\quad \gamma_1(a) = \gamma_2(a) \quad \gamma_1(b) = \gamma_2(b)$

$\int_{\gamma_1} F \cdot dx = \int_{\gamma_2} F \cdot dx \qquad \gamma_i(t) \subset \Omega \quad \gamma_1(t) \neq \gamma_2(t) \quad t \in]a,b[$

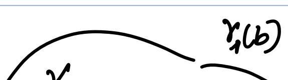


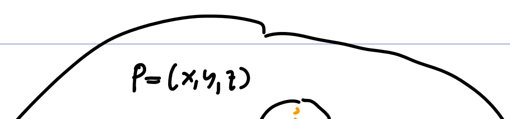
b) γ_1 chiusa $\quad \oint_{\gamma_1} F \cdot dx = 0$

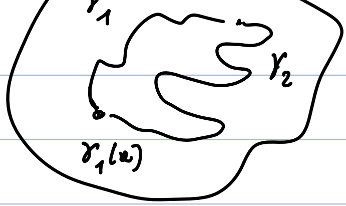
c) F CONSERVATIVO

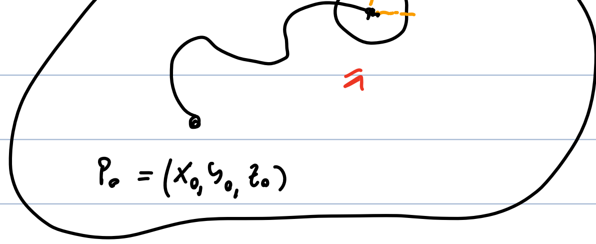
DIM $\quad$ c) $\Rightarrow$ b) $\quad$ APPENA VISTO

a) $\Rightarrow$ c)









$$\boxed{\int_\gamma F \cdot dx = U(x,y,z)} \qquad (x,y,z) \longrightarrow U(x,y,z) = \int_\gamma F \cdot dx$$

$$\boxed{F = \nabla U} \qquad F(x,y,z) = \nabla U(x,y,z)$$

$$F_1(x,y,z) = \partial_1 U(x,y,z) \qquad (x,y,z) \in \Omega \Rightarrow \exists\, h > 0 : B((x,y,z),h) \subset \Omega$$

$$\boxed{\Gamma \quad x(t), y(t), z(t) = (x + th,\ y,\ z)} \qquad t \in [0,1] \qquad (x'(t), y'(t), z'(t)) = (h, 0, 0)$$

$$U(x+th,y,z) = U(x,y,z) + \int_\Gamma F \cdot dx = U(x,y,z) + \int_0^1 F_1(x+th,y,z)\,h\,dt + \cancel{\int_0^1 F_2(\ )\cdot 0\,dt + \int_0^1 F_3(\ )\cdot 0\,dt}$$

$$\frac{U(x+th,y,z) - U(x,y,z)}{h} = \frac{1}{1}\int_0^1 F_1(x+th,y,z)\,dt = F_1(x+\tau h, y, z)$$

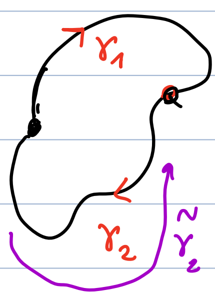
$$0 < |\tau| < t$$

$$h \to 0 \quad \downarrow \qquad\qquad \downarrow$$

$$\partial_1 U(x,y,z) \quad = \quad F_1(x,y,z)$$

b) $\Rightarrow$ a)

γ



$$\oint_\gamma F \cdot dx = 0 = \int_{\gamma_1} F \cdot dx + \int_{\gamma_2} F \cdot dx$$

$$\int_{\gamma_2} F \cdot dx = -\int_{\gamma_1} F \cdot dx$$

$$\int_{\tilde{\gamma}_2} F \cdot dx = -\int_{\gamma_2} F \cdot dx$$

$$\int_{\gamma_1} F \cdot dx = \int_{\tilde{\gamma}_2} F \cdot dx \qquad \gamma_1(a) = \tilde{\gamma}_2(a) \qquad \gamma_1(b) = \tilde{\gamma}_2(b)$$

def

$\gamma(a) = P_0$

$\gamma(0) = P_0$

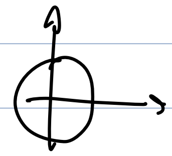
FISSO P_0 $\quad U(x,y,z) = \int_\gamma F \cdot dx \quad \gamma(1) = P = (x,y,z)$

$F = \nabla U \qquad F = \nabla(U + c) \qquad c$ costante

$\Omega = \mathbb{R}^2 \setminus \{0,0\} \qquad F(x,y) = -\frac{y}{x^2+y^2}\, i + \frac{x}{x^2+y^2}\, j$

NON CONSERVATIVO

$$\oint_\gamma F \cdot dx \neq 0$$



$$\oint_\gamma F \cdot dx = 2\pi \qquad \gamma(t) = (\cos(t), \sin(t))$$