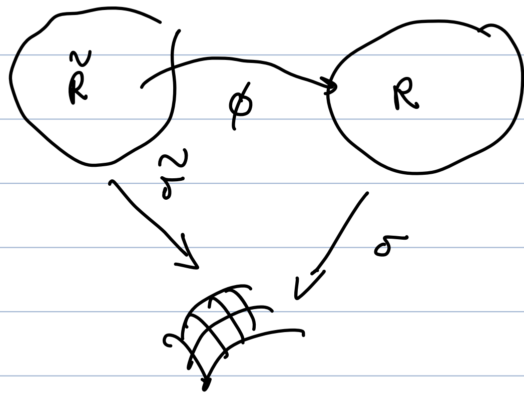
PARAMETRIZZAZIONI

$\sigma \in C^1$ $\quad \tilde{R} \subset \mathbb{R}^2$

$\sigma : \underset{(u,v)}{R} \to \Sigma \subseteq \mathbb{R}^3$ $\qquad \tilde{\sigma} : \underset{(\tilde{u},\tilde{v})}{\tilde{R}} \to \Sigma$

σ È CONGRUENTE A $\tilde{\sigma}$ $\qquad \exists\, \phi : \tilde{R} \to R$ cambio di variabile invertibile $\phi \in C^1$

$\tilde{\sigma}(\tilde{u},\tilde{v}) = \sigma(\phi(\tilde{u},\tilde{v}))$



$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \phi$

$\det(J\phi) \neq 0$

a) $\det J\phi(\tilde{u},\tilde{v}) > 0 \quad \forall\, \tilde{u},\tilde{v} \in \overset{\circ}{\tilde{R}}$

b) $\det J\phi(\tilde{u},\tilde{v}) < 0$

a) EQUIVALENTI

b) ANTI-EQUIVALENTI

$\frac{\partial \sigma}{\partial u} \wedge \frac{\partial \sigma}{\partial v} \neq 0$ $\qquad \frac{\partial \sigma}{\partial u}$ è tg. a Σ, $\quad \frac{\partial \sigma}{\partial v}$ è tg a Σ

PIANO TG. IN $(u_0, v_0, \sigma(u_0,v_0))$

$$\Pi(u,v) = \sigma(u_0,v_0) + \frac{\partial \sigma}{\partial u}(u_0,v_0)(u-u_0) + \frac{\partial \sigma}{\partial v}(u_0,v_0)(v-v_0)$$

$$\left(\frac{\partial \sigma}{\partial u} \,\middle|\, \frac{\partial \sigma}{\partial v} \right) = \begin{pmatrix} \frac{\partial \sigma_1}{\partial u} & \frac{\partial \sigma_1}{\partial v} \\ \frac{\partial \sigma_2}{\partial u} & \frac{\partial \sigma_2}{\partial v} \\ \frac{\partial \sigma_3}{\partial u} & \frac{\partial \sigma_3}{\partial v} \end{pmatrix}$$

$$\sqrt{\Pi_1^2 + \Pi_2^2 + \Pi_3^2} = \left\| \frac{\partial \sigma}{\partial u} \wedge \frac{\partial \sigma}{\partial v} \right\| = \sqrt{\det \left(\frac{\partial \sigma}{\partial u}\ \frac{\partial \sigma}{\partial v} \right)^T \left(\frac{\partial \sigma}{\partial u}\ \frac{\partial \sigma}{\partial v} \right)}$$

LIN. INDIPENDENZA

$$\alpha \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \beta \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \iff \alpha = \beta = 0$$

$$\begin{cases} \boxed{\begin{matrix} \alpha v_1 + \beta w_1 = 0 \\ \alpha v_2 + \beta w_2 = 0 \end{matrix}} \\ \alpha v_3 + \beta w_3 = 0 \end{cases}$$

$\alpha v_1 + \beta w_1 = 0$
$\alpha v_2 + \beta w_2 = 0$

$\det \begin{vmatrix} v_1 & w_1 \\ v_2 & w_2 \end{vmatrix} \neq 0$

$$\begin{cases} \alpha v_1 + \beta w_1 = 0 \\ \alpha v_3 + \beta w_3 = 0 \end{cases} \quad \det \begin{pmatrix} v_1 & w_1 \\ v_3 & w_3 \end{pmatrix} \neq 0$$

$\alpha v_2 + \beta w_2 = 0 \qquad \det \begin{vmatrix} v_2 & w_2 \\ \dots \end{vmatrix} \neq 0$

$\alpha v_3 + \beta w_3 = 0$

$$\begin{array}{c} j \rightarrow \\ \\ k \rightarrow \end{array} \begin{pmatrix} v_1 & w_1 \\ \vdots & \vdots \\ v_n & w_n \end{pmatrix} \qquad \begin{cases} \alpha v_j + \beta w_j = 0 \\ \alpha v_k + \beta w_k = 0 \end{cases} \Rightarrow \alpha = \beta = 0$$

$n \geq 3$

$$\begin{pmatrix} v_1 & w_1 & z_1 \\ \vdots & \vdots & \vdots \\ v_n & w_n & z_n \end{pmatrix}$$

$$\alpha v + \beta w + \gamma z = 0 \iff \alpha = \beta = \gamma = 0$$

$\tilde{\sigma}(\tilde{u}, \tilde{v}) = \sigma(\phi(\tilde{u}, \tilde{v}))$

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{u}} \wedge \frac{\partial \tilde{\sigma}}{\partial \tilde{v}} \neq 0$$

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{u}} = \frac{\partial}{\partial \tilde{u}} \sigma(\phi(\tilde{u}, \tilde{v}))$$

$$= \frac{\partial \sigma}{\partial u} \frac{\partial \phi_1}{\partial \tilde{u}} + \frac{\partial \sigma}{\partial v} \frac{\partial \phi_2}{\partial \tilde{u}}$$

$$= m_{11} \frac{\partial \sigma}{\partial u} + m_{12} \frac{\partial \sigma}{\partial v}$$

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = J\phi$$

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{v}} = \frac{\partial}{\partial \tilde{v}} \sigma(\phi(\tilde{u}, \tilde{v}))$$

$$= \frac{\partial \sigma}{\partial u} \frac{\partial \phi_1}{\partial \tilde{v}} + \frac{\partial \sigma}{\partial v} \frac{\partial \phi_2}{\partial \tilde{v}}$$

$$= m_{21} \frac{\partial \sigma}{\partial u} + m_{22} \frac{\partial \sigma}{\partial v}$$

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{v}} \in \operatorname{Span}\left\{ \frac{\partial \sigma}{\partial u}, \frac{\partial \sigma}{\partial v} \right\}$$

$$\tilde{\sigma} \approx \tilde{\sigma}(\tilde{u}_0, \tilde{v}_0) + \frac{\partial \tilde{\sigma}}{\partial \tilde{u}}(\tilde{u}_0, \tilde{v}_0)(\tilde{u} - \tilde{u}_0) + \frac{\partial \tilde{\sigma}}{\partial \tilde{v}}(\tilde{u}_0, \tilde{v}_0)(\tilde{v} - \tilde{v}_0)$$

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{u}} \in \operatorname{Span}\left\{ \frac{\partial \sigma}{\partial u}, \frac{\partial \sigma}{\partial v} \right\}$$

$$\frac{\partial \tilde{\sigma}}{\partial \tilde{u}} \wedge \frac{\partial \tilde{\sigma}}{\partial \tilde{v}} = \left(m_{11} \frac{\partial \sigma}{\partial u} + m_{12} \frac{\partial \sigma}{\partial v} \right) \wedge \left(m_{21} \frac{\partial \sigma}{\partial u} + m_{22} \frac{\partial \sigma}{\partial v} \right)$$

$$(a + b) \wedge c = a \wedge c + b \wedge c$$

$a\wedge b = -\,b\wedge a \qquad a\wedge a = 0$

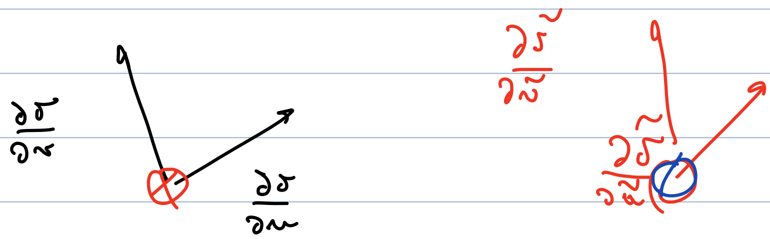
$$\cancel{m_{11}\frac{\partial\sigma}{\partial u}\wedge m_{21}\frac{\partial\sigma}{\partial u}} + m_{11}\frac{\partial\sigma}{\partial u}\wedge m_{22}\frac{\partial\sigma}{\partial v} + m_{12}\frac{\partial\sigma}{\partial v}\wedge m_{21}\frac{\partial\sigma}{\partial u} + \cancel{m_{12}\frac{\partial\sigma}{\partial v}\wedge m_{22}\frac{\partial\sigma}{\partial v}}$$

$$= m_{11}\,m_{22}\,\frac{\partial\sigma}{\partial u}\wedge\frac{\partial\sigma}{\partial v} + m_{12}\,m_{21}\,\frac{\partial\sigma}{\partial v}\wedge\frac{\partial\sigma}{\partial u}$$

$$= \left(m_{11}\,m_{22} - m_{12}\,m_{21}\right)\frac{\partial\sigma}{\partial u}\wedge\frac{\partial\sigma}{\partial v}$$

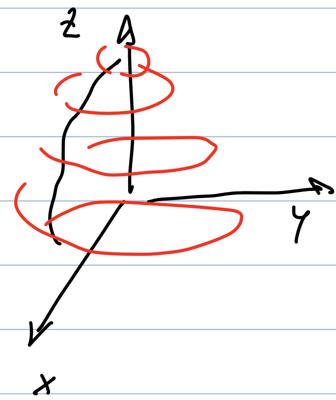
$$\frac{\partial\tilde\sigma}{\partial\tilde u}\wedge\frac{\partial\tilde\sigma}{\partial\tilde v} = (\det J\phi)\,\frac{\partial\sigma}{\partial u}\wedge\frac{\partial\sigma}{\partial v}$$

$$A(\Sigma) = \int_R \left\|\frac{\partial\sigma}{\partial u}\wedge\frac{\partial\sigma}{\partial v}\right\| du\,dv = \int_{\tilde R}\left\|\frac{\partial\tilde\sigma}{\partial\tilde u}\wedge\frac{\partial\tilde\sigma}{\partial\tilde v}\right\| d\tilde u\,d\tilde v$$



vettore normale

SUPERFICIE DI ROTAZIONE



$\gamma(t) = (\gamma_1(t), 0, \gamma_3(t)) \subseteq$ PIANO-XZ

$t\in[a,b]$

$\gamma_1 \geq 0$

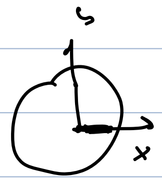
$\sigma(u,v) = \gamma_1(u)\cos(v)\,i + \gamma_1(u)\sin(v)\,j + \gamma_3(u)\,k$

$u \in [a,b]$ $\quad v \in [0,2\pi]$

$xi + yj + f(x,y)k$

$z = x^2+y^2 \qquad \sigma(x,y) = xi + yj + (x^2+y^2)k$

$R = \{(x,y) : x^2+y^2 \le 1\}$



$$A(\Sigma) = \int_R \sqrt{1+f_x^2+f_y^2}\,dxdy$$

$$= \int_{x^2+y^2\le 1} \sqrt{1+4x^2+4y^2}\,dxdy$$

$$= \int_0^{2\pi} d\theta \int_0^1 \rho\, d\rho \sqrt{1+4\rho^2}$$

$$= \frac{2\pi}{8}\int_0^1 8\rho\,(1+4\rho^2)^{1/2}\,d\rho = \frac{\pi}{4}\,\frac{(1+4\rho^2)^{3/2}}{\frac{3}{2}}\Big|_0^1$$

$\gamma_1(t) = t$

$\gamma_2(t) = t$

$\gamma(t) = (t,0,t)$

$t \in [0,1]$