$$s(t) = \sum_n x_c[n]\, p(t-nT)\cos(2\pi f_0 t)$$
$$\quad - \sum_n x_s[n]\, p(t-nT)\sin(2\pi f_0 t)$$

$x_c[n] \in A_s^c = \{-1, 2\}$ ind. ed equiprob.

$x_s[n] \in A_s^s = \{-2, 1\}$

$p(t) = \mathrm{sinc}\left[B\left(t - \frac{1}{2B}\right)\right] + \mathrm{sinc}\left[B\left(t + \frac{1}{2B}\right)\right]$

$B = \frac{2}{T}$

$c(t) = \delta(t)$

$n(t)$ bianco in banda $\Rightarrow \frac{N_0}{2} = S_n(f)$

$h_R(t) = p(t)$

$\lambda = 0$

1) E_s

2) $P_{n_{u_c}}$ e $P_{n_{u_s}}$

3) $P_E(M)$

Soluzione

1) $E_s = \frac{1}{2}\left[E[x_c^2] + E[x_s^2]\right] E_p$

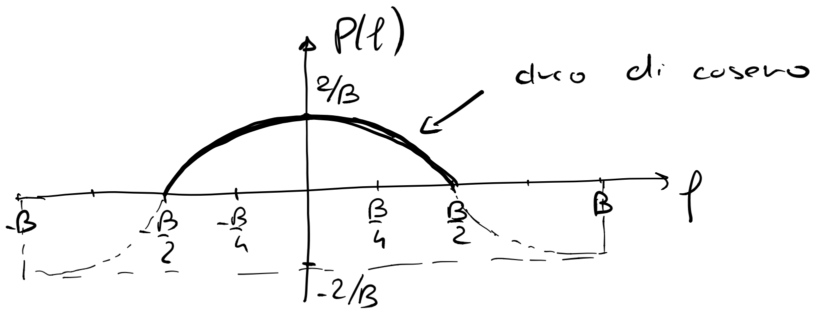
$\Rightarrow E[x_c^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(+2)^2 = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$

$$\Rightarrow E[x_s^2] = \frac{1}{2}(-2)^2 + \frac{1}{2}(+1)^2 = \frac{5}{2}$$

$E_p = ?$

$$P(f) = \frac{1}{B}\,\mathrm{rect}\left(\frac{f}{B}\right)\left[e^{-j2\pi f\frac{1}{2B}} + e^{j2\pi f\frac{1}{2B}}\right]$$

$$= \frac{2}{B}\,\mathrm{rect}\left(\frac{f}{B}\right)\cos\left(\frac{\pi f}{B}\right)$$



$$E_p = \frac{4}{B^2}\int_{-\frac{B}{2}}^{\frac{B}{2}} \cos^2\left(\frac{\pi f}{B}\right) df = \frac{4}{B^2}\int_{-\frac{B}{2}}^{\frac{B}{2}}\left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi f}{B}\right)\right]df$$

$$= \frac{4}{B^2}\cdot B + \underbrace{\frac{2}{B^2}\int_{-\frac{B}{2}}^{\frac{B}{2}} \cos\left(\frac{2\pi f}{B}\right) df}_{=0} = \frac{4}{B}$$

in realtà è un errore di calcolo e viene $\frac{2}{B}$

$$E_s = \frac{1}{2}\left(\frac{5}{2} + \frac{5}{2}\right)\frac{4}{B} = \boxed{\frac{10}{B}}$$

2) $P_{n_{u_c}} = P_{n_{u_s}} = N_0 E_{H_R} = N_0 E_p = \frac{4N_0}{B}$

3) Assenza di ISI

$$h(t) = p(t) \otimes \tilde{c}(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$

$$H(f) = P(f)\, H_R(f) = P^2(f)$$

$$P^2(f) = \frac{4}{B^2}\,\mathrm{rect}\left(\frac{f}{B}\right)\cos^2\left(\frac{\pi f}{B}\right) = H(f) = \frac{2}{B^2}\,\mathrm{rect}\left(\frac{f}{B}\right)\cdot\left[1+\cos\left(\frac{2\pi f}{B}\right)\right]$$

$$h(t) = \frac{2}{B^2}\, B\,\mathrm{sinc}(Bt) \otimes \left[\delta(t) + \frac{1}{2}\delta\left(t-\frac{1}{B}\right) + \frac{1}{2}\left(t+\frac{1}{B}\right)\right]$$

$$= \frac{2}{B}\,\mathrm{sinc}(Bt) + \frac{1}{B}\,\mathrm{sinc}\left[B\left(t-\frac{1}{B}\right)\right] + \frac{1}{B}\,\mathrm{sinc}\left[B\left(t+\frac{1}{B}\right)\right]$$

$$\Rightarrow h(nT) = h\left(n\frac{2}{B}\right) = \frac{2}{B}\,\mathrm{sinc}\left(B\, n\frac{2}{B}\right) + \frac{1}{B}\,\mathrm{sinc}\left[B\left(n\frac{2}{B}-\frac{1}{B}\right)\right] + \frac{1}{B}\,\mathrm{sinc}\left[B\left(n\frac{2}{B}+\frac{1}{B}\right)\right]$$

$$= \frac{2}{B}\,\mathrm{sinc}(2n) + \frac{1}{B}\,\mathrm{sinc}(2n-1) + \frac{1}{B}\,\mathrm{sinc}(2n+1)$$

$$= \frac{2}{B}\,\delta[n]$$ cond. di Nyquist soddisfatta

$$h(0) = \frac{2}{B}$$

$$P_E(r) = P_E^I(b)\left(1 - P_E^q(b)\right) + P_E^q(b)\left(1 - P_E^I(b)\right) + P_E^I(b)\,P_E^q(b)$$

$$P_E^I(b) = \underbrace{P(\hat{\alpha}_1 | \alpha_2)} P(\alpha_2) + \underbrace{P(\hat{\alpha}_2 | \alpha_1)} P(\alpha_1)$$

$$y_c[n] = h(0)\, x_c[n] + n_{u_c}[n]$$

$$= \frac{1}{2} Q\left(\frac{h(0)\cdot 2}{\sqrt{P_{n_{u_c}}}}\right) + \frac{1}{2} Q\left(\frac{h(0)\cdot 1}{\sqrt{P_{n_{u_c}}}}\right)$$

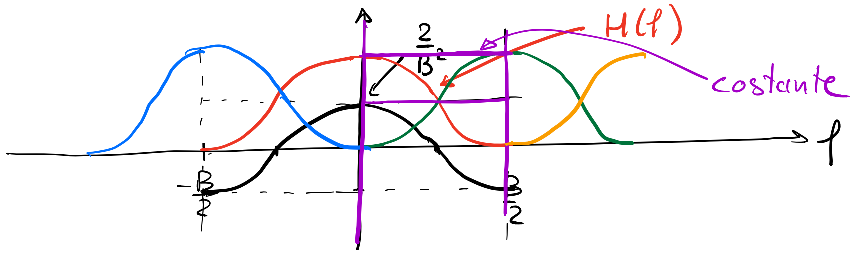
$$= \frac{1}{2} Q\left(\frac{\frac{4}{B}}{\sqrt{\frac{4N_0}{B}}}\right) + \frac{1}{2} Q\left(\frac{\frac{2}{B}}{\sqrt{\frac{4N_0}{B}}}\right)$$

$$\boxed{\begin{aligned} P_E^Q(b) &= \frac{1}{2} Q\left(\frac{\frac{4}{B}}{\sqrt{\frac{4N_0}{B}}}\right) + \frac{1}{2} Q\left(\frac{\frac{2}{B}}{\sqrt{\frac{4N_0}{B}}}\right) \\ P_E^Q(b) &= P_E^I(b) = P_E(b) \end{aligned}}$$

$$\boxed{P_E(M) = 2P_E(b)\left(1 - P_E(b)\right) + P_E^2(b)}$$

-) Verifica dell'assenza di ISI in frequenza

$$H(f) = \frac{2}{B^2} \text{rect}\left(\frac{f}{B}\right) \cdot \left[1 + \underbrace{\cos\left(\frac{2\pi f}{B}\right)}\right] \qquad \frac{1}{T} = \frac{B}{2}$$



$0 \leq f \leq \frac{B}{2} \Rightarrow$ 2 componenti non nulle
componente rossa + verde

$$\frac{2}{B^2}\left[1+\cos\left(\frac{2\pi f}{B}\right)\right]+\frac{2}{B^2}\left[1+\cos\left(\frac{2\pi\left(f-\frac{B}{2}\right)}{B}\right)\right]$$

$$= \frac{2}{B^2}+\frac{2}{B^2}\cos\left(\frac{2\pi f}{B}\right)+\frac{2}{B^2}+\frac{2}{B^2}\cos\left(\frac{2\pi f}{B}-\frac{2\pi \cancel{B}/2}{\cancel{B}}\right)$$

$$\cos\left(\frac{2\pi f}{B}-\frac{2\pi B/2}{B}\right) = \cos\left(\frac{2\pi f}{B}-\pi\right) = -\cos\left(\frac{2\pi f}{B}\right)$$

$$= \frac{2}{B^2}+\cancel{\frac{2}{B^2}\cos\left(\frac{2\pi f}{B}\right)}+\frac{2}{B^2}-\cancel{\frac{2}{B^2}\cos\left(\frac{2\pi f}{B}\right)} = \frac{4}{B^2} = \text{cost.}$$

$\Downarrow$

ASSENZA DI ISI