# Raccolta domande orali Analisi I

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$$\int_{-1}^{2} \frac{x}{|x|}\,dx \tag{1}$$

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^3}(x-1)^n \tag{2}$$

$$A = \left\{x \in \mathbb{R} : x^3 - x^2 < 0\right\} \quad \inf, \min, \sup, \max \ ? \tag{3}$$

$$\int_{-1}^{1} x^3 \cdot e^{-x^4}\,dx \tag{4}$$

$$f(x) = x^{\ln x} \quad f'(x) = ? \tag{5}$$

$$\sqrt[4]{e^{i\frac{\pi}{2}}} \tag{6}$$

$$\sum_{n=1}^{\infty} (-1)^n \cdot \cos\left(\frac{1}{n}\right) \tag{7}$$

$$\lim_{x\to+\infty} \sqrt{x} \cdot \sin\left(\frac{1}{x}\right) \tag{8}$$

$$\lim_{x\to 0} \frac{e^x - e^{-2x}}{x} \tag{9}$$

$$y''(x) + y(x) = 1 \tag{10}$$

$$\left\{f(z) = \frac{(1+z)^2}{1+z^2}, z \geq 0\right\} \tag{11}$$

$$\sqrt{x+y} \leq \sqrt{x} + \sqrt{y} \quad x, y \geq 0 \tag{12}$$

$$z = \frac{1}{2} + \frac{i}{2} \quad z^n = ? \ \ n \in \mathbb{N} \tag{13}$$

$$\lim_{n\to+\infty} z^n \tag{14}$$

$$f(x) = e^{|x|^3} \quad f'(1) = ? \tag{15}$$

$$\int_{0}^{+\infty} \frac{x}{1+x^2}\,dx \tag{16}$$

$$\lim_{x\to 0^+} \frac{\sin x}{x + \sqrt{x}} \tag{17}$$

$$\sum_{n=0}^{+\infty} \frac{x^n}{(n!)^n} \tag{18}$$

$$\lim_{x\to+\infty} \frac{\sin x}{x + \sqrt{x}} \tag{19}$$

$$y'(x) + y(x) = e^x \tag{20}$$

$$\left(\frac{1+i}{1-i}\right)^4 \frac{i^4}{1+2i} \tag{21}$$

$$f(x) = e^{-x^3} \quad \text{è convessa?} \tag{22}$$

$$\int_{a}^{1} \ln x\,dx \tag{23}$$

$$y'(x) + y(x) = x \tag{24}$$

$$\lim_{x\to+\infty} \frac{\{x\}}{x} \tag{25}$$

$$\left\{n \in \mathbb{N} : \sin\left(n \cdot \frac{\pi}{2}\right) > 0\right\} \quad \inf, \min, \sup, \max \ ? \tag{26}$$

$$\int_{-1}^{1} \arctan x\,dx \tag{27}$$

$$e^z \ \text{ con } z = 1 + i \quad \mathrm{Re}(e^z) = ? \tag{28}$$

$$f(x) = x \ \ln x \ \text{ è convessa?} \tag{29}$$

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{(\sqrt{n})^n} \quad \text{insieme di convergenza?} \tag{30}$$

$$\sum_{n=1}^{\infty} e^{d\,h} \quad d = ? \tag{31}$$

$$y'(x) = \tan(y(x)) \tag{32}$$

$$A = \left\{x \in \mathbb{R} : y = e^{-x^2}(x^2 - x) > 0\right\} \tag{33}$$

$$\sum_{k=1}^{\infty} \frac{1}{\int_0^k x\,dx} \quad \text{converge?} \tag{34}$$

$$\int_{-1}^{1} [x]\,dx \tag{35}$$

$$f(x) = |x|^{\cos x} \quad f'_{\pm}(0) =? \tag{36}$$

$$y'(x) + y(x) = \ln x\, e^{-x} \tag{37}$$

$$\int e^x \cos x\,dx \tag{38}$$

$$f(x) = e^{-\frac{1}{x^2}} \quad \lim_{x\to 0^+} f(x) =? \tag{39}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + n + \ln n} \quad \text{converge?} \tag{40}$$

$$z \cdot z - z \cdot \overline{z} = 0 \tag{41}$$

$$\sum_{n=1}^{\infty} (2n)!\, x^n \quad \text{converge?} \tag{42}$$

$$\left\{x^4 - 4x^3 + 6x^2 - 4x + 1 > 0\right\} \tag{43}$$

$$y''(x) - 5y'(x) + 6y(x) = x \tag{44}$$

$$\sum_{n=1}^{+\infty} \frac{\ln n}{n^2 \ln(n^2)} \tag{45}$$

$$\text{punto di flesso di } e^{-x^2} \tag{46}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \quad \text{è crescente?} \tag{47}$$

$$\begin{cases} y'(x) = y^3(x) \\ y(0) = 1 \end{cases} \tag{48}$$

$$\sum_{n=1}^{\infty} \left(e^{\frac{1}{n}} - e^{-\frac{1}{n}}\right) \tag{49}$$

$$\int_0^1 \ln(x^2)\,dx \tag{50}$$

$$\int_0^{\frac{\pi}{4}} \tan x\,dx \tag{51}$$

$$z = 1 + i \qquad z^{\frac{3}{2}} =? \tag{52}$$

$$\begin{cases} y'(x) = 1 + y^2(x) \\ y(0) = 1 \end{cases} \tag{53}$$

$$\int_2^{+\infty} \frac{1}{x\,\ln^2 x}\,dx \tag{54}$$

$$\frac{d}{dx}\left(x^{\ln x}\right) \tag{55}$$

$$\int x\,\cos(2x)\,dx \tag{56}$$

$$\sum_{n=1}^{\infty} (2^n + 3^n)(x-3)^n \tag{57}$$

$$\left\{z \in \mathbb{C} : ||z||^2 = 1\right\} \cap \left\{\text{Re } z = \frac{1}{2}\right\} \tag{58}$$

$$A = \left\{x \in \mathbb{R} : \sqrt{x^2 - x^3} > 0\right\} \tag{59}$$

$$\int_a^{+\infty} \frac{1}{x^4}\,dx \tag{60}$$

$$\left\{x \in \mathbb{R} : \sin^2 x - 2\sin x + 1 > 0\right\} \tag{61}$$

$$\lim_{x\to 0} \frac{2\sin x - \sin(2x)}{x^3} \tag{62}$$

$$\text{Re}[(1+i)^4] \tag{63}$$

$$f(x) = e^{|x|} \quad f'(1) =? \tag{64}$$

$$\frac{z\,\overline{z} + 1}{z^2} = 1 \tag{65}$$

$$\int_0^{+\infty} e^{ax}\,dx \quad \text{per quali } a \text{ converge?} \tag{66}$$

$$\int_0^{\frac{\pi}{4}} \sin^2 x\,dx \tag{67}$$

$$\left\{z \in \mathbb{C} : (z^2 - 1)(||z||^2 - 1) = 0\right\} \tag{68}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\left(\frac{\ln n}{100}\right)}} \tag{69}$$

$$\lim_{x\to 0} \frac{1}{x} \int_0^x \sin(\cos(t^2))\,dt \tag{70}$$

$$\int_{-1}^{1} \{x\}\, dx \tag{71}$$

$$f(x) = e^{(e^x)} \quad f''(0) =? \tag{72}$$

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{1+n+n^2} \tag{73}$$

$$\frac{d}{dx}(f \circ f)(x) = \frac{d}{dx} f(f(x)) \tag{74}$$

Quando $f(f(x))$ è convessa? (75)

$$\lim_{x \to +\infty} \frac{\cos(\ln(x)) - 1}{\ln(\ln(\ln(x)))} \tag{76}$$

$$\int_0^{\pi} \cos^2(x)\, dx \tag{77}$$

$$\frac{d}{dx} \sqrt{x}^{\sqrt{x}} \tag{78}$$

$$\sum_{n=1}^{\infty} \frac{x^n}{\int_n^{n+1} e^t\, dt} \tag{79}$$

$$\left\{ z \in \mathbb{C} : (z^2 - i)(\overline{z}^2 - 1) = 0 \right\} \tag{80}$$

$$\int \sin^3(x)\, dx \tag{81}$$

$$z^2 + 6z - 3 < 0 \quad z \in \mathbb{C} \tag{82}$$

$$\begin{cases} y'(x) = |x| \\ y(0) = 0 \end{cases} \tag{83}$$

$$\frac{d}{dx}\left( \sqrt[n]{1+x^2} \right) \tag{84}$$

$$\sum_{n=0}^{\infty} (1 + x + x^2)^n \tag{85}$$

$$\int_a^1 \frac{1}{x \ln x}\, dx \tag{86}$$

$$z + \overline{z}^2 = z^2 - \overline{z} \tag{87}$$

$$y'' + y' + y = 1 \tag{88}$$

$$f(x) = \begin{cases} \frac{\pi}{3} & x < 0 \\ \cos x & x \geq 0 \end{cases} \quad f'(0) =? \tag{89}$$

$$\lim_{x \to 0} \frac{\sqrt[5]{1+x} - \sqrt[5]{1-x}}{x} \tag{90}$$

$$y'(x) = \frac{1 - y^2(x)}{y(x)}\, x^2 \tag{91}$$

$$f(x) = |x+1| \quad \text{convessa?} \tag{92}$$

$$\sum_{n=1}^{+\infty} \ln \left| \ln \left( \frac{1}{n^a} \right) \right| \tag{93}$$

$$i^{2021} =? \tag{94}$$

$$\int \cos 2x\, \cos 3x\, dx \tag{95}$$

$$\{x : \cos(|x|) < 0\} \quad \inf =? \tag{96}$$

$$\lim_{x \to +\infty} \frac{\ln(x) - \ln(x^2)}{\ln(x^3)} \tag{97}$$

$$A = \left\{ z \in \mathbb{C} : (z^2 - i) = 0 \right\} \cup \{ \ln(||z+1||) = 0 \} \tag{98}$$

$$\sum_{n=0}^{+\infty} x^{n+1} \tag{99}$$

$$\lim_{x \to 0} \frac{\sin(x) - \sin(-x)}{x} \tag{100}$$

$$\int_{-2}^{1} |x+1|\, dx \tag{101}$$

$$\sqrt[3]{i} \tag{102}$$

$$\left\{ x : \cos^2(x) + \sin(x) > 1 \right\} \tag{103}$$

$$\lim_{x \to +\infty} \frac{e^{-x} \ln x + x}{x - e^{-x}} \tag{104}$$

$$y'(x) + y(x) = e^{2x} \tag{105}$$

$\tan x$ dove è convessa? (106)

$$\arg \sqrt{\frac{1}{i}} \tag{107}$$

$$\max_A x - x^2 \quad A = \{x \in [0, 2\pi] : \cos x < 0\} \tag{108}$$

$$\sum_{n=M}^{+\infty} n^M x^n \quad M \in \mathbb{N} \tag{109}$$

$$\lim_{x \to 0^-} e^{\frac{|\sin x|}{x^2}} \tag{110}$$

$$f(x) = |x|^{20} \tag{111}$$

$$z^3 = \frac{2}{i} \tag{112}$$

$$\int_0^1 x \ln x \, dx \tag{113}$$

$$\sum_{n=0}^{\infty} \frac{(2x)^n}{n!} \quad x \in \mathbb{R} \tag{114}$$

$$A = \{z \in \mathbb{C} : \mathrm{Re}(z) < 0\} \cap \left\{z^2 + 1 = -3\right\} \tag{115}$$

$$\lim_{x \to 0^-} \frac{\sqrt[7]{1+x} - \sqrt[7]{1-x}}{e^x - 1} \tag{116}$$

$$\sum_{n=e^4} \frac{n \ln n}{(1+n^2)e^n} \tag{117}$$

$$\begin{cases} y'(x) = \sqrt{y(x)} \\ y(0) = 1 \end{cases} \tag{118}$$

$$f(x) = x^2 - x^3 \quad \max_A f \quad A = \left\{x : x^2 < 4\right\} \tag{119}$$

$$f(x) = x^2(2 + \sin x) \quad \text{studiare la convessità} \tag{120}$$

$$\begin{cases} y'(x) = \sqrt[3]{y(x)} \\ y(0) = 1 \end{cases} \tag{121}$$

- $\lim_{x\to+\infty} \frac{\int_0^x e^{-t^2}dt}{\ln x}$

- $\lim_{x\to+\infty} \frac{\int_0^x e^{-2t}dt}{\ln x}$

- $\sum_{n=0}^{+\infty}(1+x+x^2)^n$ per quali $x$ converge?

  $|1+x+x^2|<1$

- $\int_0^2 \frac{x-1}{|x-1|}dx$

  $\begin{cases} x-1>0 & x>0 \\ -x-1>0 & x<0 \end{cases}$

- $f(x)=\begin{cases} x+1 & x\geq 0 \\ x & x<0 \end{cases}$ è derivabile in 0?

- $A=\{z\in\mathbb{C} : (z^2-1)(\|z\|^2-1)(\|\bar{z}\|^2-1)=0\}$

- $\int_a^{+\infty} \frac{1}{x^3}dx$ per quali $a$ converge?

- $\sum_{n=1}^{\infty} ne^{\alpha n}$ $\alpha\in\mathbb{R}$ per quali $\alpha$ la serie converge?

- $\int_0^1 \ln x\,dx$

- $f(x)=\cos(|x|)$ $f'(0)$ esiste?

- $\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0}$ relativo alla funzione $f(x)=\begin{cases} x & x<0 \\ x+1 & x\geq 0 \end{cases}$

- $\sum_{n=1}^{\infty} n^2 e^{\alpha n}$ $\alpha\in\mathbb{R}$

- $\mathrm{Re}[(1+i)^4]$

- Studiare la derivabilità in 0 di $f(x) = \begin{cases} x \ln|x| & x \neq 0 \\ 0 & x = 0 \end{cases}$

- $\int_0^{\pi} e^x \sin(2x)\, dx$

- $\sum_{n=0}^{+\infty} \frac{1}{2^n + n} (x-2)^n$

- Taylor $\sin(4x)$ $\quad x = \frac{\pi}{2}$

- max $f'$ $\qquad f(x) = \sin(x^2)$

- $y^{(n)}(x) = 1 \qquad n \in \mathbb{N}$

- $\lim_{x \to +\infty} \frac{\cos(\ln(x)) - 1}{\ln(\ln(x^2))}$

- $\{ x \in \mathbb{R} : \sqrt{x^2 - x^3} > 0 \}$

- $\lim_{x \to +\infty} \frac{\int_0^x e^{-t^2} dt}{x}$

- $\frac{y'}{3} + x^2 y = e^{-x^3}$

- $\sum \log_2 \left| \log_2 \left( \frac{1}{n^\alpha} \right) \right| \qquad \alpha \in \mathbb{R} \setminus \{0\}$

- $z + \bar{z}^2 = z^2 - 1$

- $\int_0^1 \frac{x^2}{1 + x^6} dx$

- calcolare $\sqrt[3]{\frac{1}{2+2i}}$

- se si prende $n \in \mathbb{N}$, $n$ dispari cosa posso dire di $n^2+n$

- $f(x) = \ln x - e^x$ , $\mathcal{D}: x>0$ la funzione ha un max?

- $\int_0^{+\infty} \frac{1}{x^x-1}\,dx$

- $\begin{cases} y'' - 5y' + 4y = 0 \\ y(1) = 0 \\ y'(1) = 0 \end{cases}$

- $\{ z \in \mathbb{C} : |z-i| = 1 \} \cap \{ z \in \mathbb{C} : \|z\| < 4 \}$

- $f(x) = x \ln^2(|x|)$

  $f'(0)$

- $f(x) = \begin{cases} x & x \le 0 \\ x+1 & x > 0 \end{cases}$ è derivabile in 0?

- $\int_0^{+\infty} \frac{x}{1+x^4}$

- $\sum (-1)^{n+1} \frac{n \sin(n)}{n^4+1}$

- $\int_{-2}^{2} \frac{x-1}{|x-1|}\,dx$

- $\frac{d}{dx}\, x^{(\ln x)}$

- $\begin{cases} y'(x) = 1 - y^2(x) \\ y(0) = 0 \end{cases}$

- $\sum_{n=0}^{+\infty} x^{n+2}$
- convessità $x^2 \ln(x^3)$
- $\lim_{x \to 0^+} \sin\left(\frac{1}{x}\right) \cdot x^{4/9}$
- max e min $\{x \in \mathbb{R} : x^4 - 2x^2 + 1 < 4\}$
- $f(x) = \sqrt{9 - x^2}$ convessità
- $\lim_{x \to +\infty} \frac{e^x + e^{-\frac{x}{2}}}{x}$
- $\operatorname{Arg}\left(\sqrt{\frac{1}{i}}\right)$
- $\lim_{x \to 0^+} \frac{e^x - e^{-\frac{x}{2}}}{x}$ $\left(= \frac{3}{2}\right)$
- $\lim_{x \to 0^+} \frac{e^x + e^{-\frac{x}{2}}}{x}$ $(= +\infty)$
- $\sum (-1)^n \frac{\arctan(n^2+1)}{n^2+1}$
- $y'(x) + y(x) = e^{2x}$
- $\int_1^5 \frac{dt}{t^2+t}$
- $\lim_{x \to 0^+} \frac{\sin(x)}{x + \sqrt[3]{x}}$
- punti di flesso $f(x) = x^2 \ln x$
- $\sqrt[5]{e^{\frac{i\pi}{4}}}$
- $y''(x) + y'(x) = 1$
- $\left\{x \in [0, 2\pi] : \frac{1}{\sin^2(x) + \sin(x)} < 1\right\}$

- $\lim_{x \to +\infty} \frac{\ln(e^x + x)}{x^2 + x + 1}$

- $\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$ , così è $\sum_{k=0}^{\infty} \frac{(-x)^k}{k!} = e^{-x}$ ?

  quanto fa $e^x \cdot e^{-x}$ ?

  verificare che $\left(\sum_{0}^{\infty} \frac{x^k}{k!}\right)\left(\sum_{0}^{\infty} \frac{(-x)^k}{k!}\right)$ sia uguale ad 1

- $\sinh(x)$ verificare se è strettamente crescente