Esame 09/06/2008

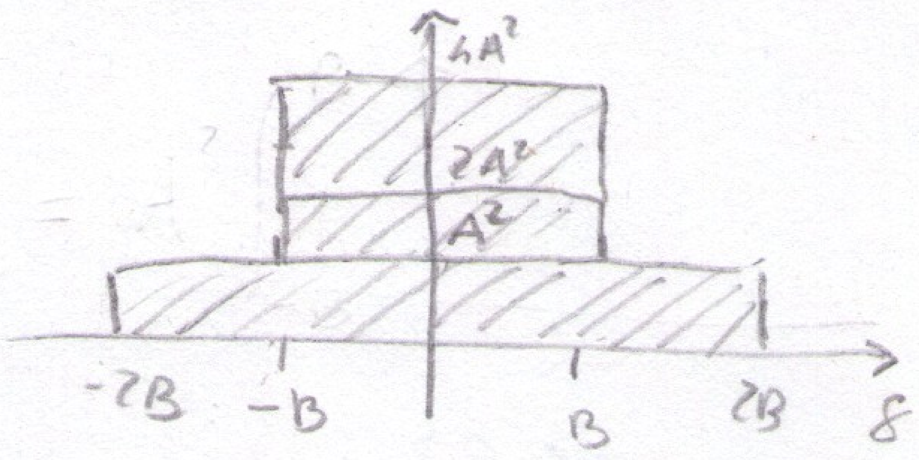
Esercizio 1

$x(t) = x_1(t) + x_2(t) = 2AB\,\text{sinc}(2Bt) + 4AB\,\text{sinc}(4Bt)$

$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt \overset{\text{Parseval}}{=} \int_{-\infty}^{\infty} |X(f)|^2\,df$

$X(f) = A\,\text{rect}\left(\frac{f}{2B}\right) + A\,\text{rect}\left(\frac{f}{4B}\right)$ ②

$E_x = \int_{-\infty}^{\infty} \left|A\,\text{rect}\left(\frac{f}{2B}\right) + A\,\text{rect}\left(\frac{f}{4B}\right)\right|^2 df = \int_{-\infty}^{\infty}\left[A^2\,\text{rect}\left(\frac{f}{2B}\right) + A^2\,\text{rect}\left(\frac{f}{4B}\right) + 2A^2\,\text{rect}\left(\frac{f}{2B}\right)\text{rect}\left(\frac{f}{4B}\right)\right] df$

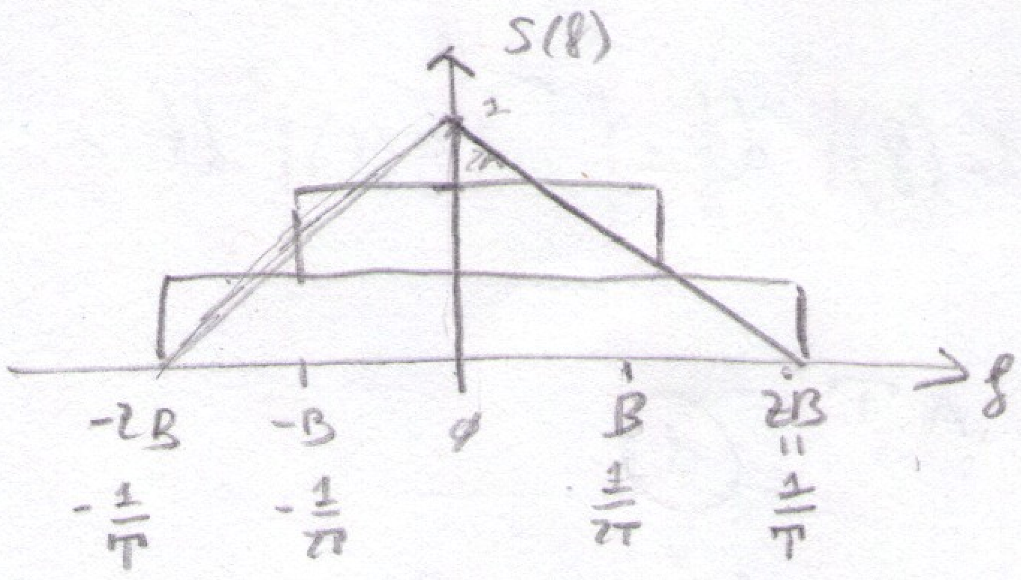


$= 2A^2 \cdot B + 4A^2 \cdot 2B = 2A^2(B + 4B) = 10A^2B$

$P_x = 0$ perchè $E_x = K < +\infty$ ①

$y[n] = x_\alpha[n] + x_\beta[n]$; $x_\alpha[n] = \sum_n [x(nT) \otimes h(nT)]$; $x_\beta[n] = \sum_n x(nT)$

$s(t) = x(t) \otimes h(t) \rightleftharpoons S(f) = X(f)H(f) = \left[A\,\text{rect}\left(\frac{f}{2B}\right) + A\,\text{rect}\left(\frac{f}{4B}\right)\right]\cdot\left(1 - \frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right) =$



$X_\alpha(f) = \frac{1}{T}\sum_n S\left(f - \frac{n}{T}\right) = 2A\delta(f)$

$X_\beta(f) = \frac{1}{T}\sum_n X\left(f - \frac{n}{T}\right) = A\delta\left(f + \frac{1}{T}\right) + 2A\,\delta(f) + A\delta\left(f - \frac{1}{T}\right)$

$Y(f) = A\delta\left(f - \frac{n}{T}\right) + 4A\,\delta(f) + A\delta\left(f - \frac{1}{T}\right)$; $P(f) = \text{rect}\left(\frac{f}{2B}\right)$; $Y(f) = 4A\delta(f) \rightleftharpoons y(t) = 4A$ ③

$E_y = \int_{-\infty}^{\infty} |y(t)|^2\,dt = +\infty$; $P_y = \lim_{T\to\infty} \frac{1}{T} E_{X_T} = \lim_{T\to\infty} \frac{E_{X_T}}{T} = 0$

Esercizio 2

$s_T(t) = \sum_i a_i\, g_T(t - iT)\cos^2(2\pi f_0 t + \theta) = \sum_i \frac{a_i}{2} g_T(t - iT) + \sum_i \frac{a_i}{2} g_T(t - iT)\cos(4\pi f_0 t + 2\theta) =$

$= \sum_i \frac{a_i}{2} g_T(t - iT) + \sum_i \frac{a_i}{2} g_T(t - iT)\cos(4\pi f_0 t)$

$E_T = \sum_{K=0}^{1} E_K P_K = E_0 P_0 + E_1 P_1$

$E_0 = \int_{-\infty}^{\infty} |S_0(t)|^2\,dt = \int_{-\infty}^{\infty}\left|\frac{1}{2}g_T(t) + \frac{1}{2}\cos(4\pi f_0 t)g(t)\right|^2 dt$

$= \int_{-\infty}^{\infty}\left[\frac{1}{4}g_T^2(t) + \frac{1}{4}\cos^2(4\pi f_0 t)\,g_T^2(t) + \frac{2}{4}\cos(4\pi f_0 t)\,g_T^2(t)\right] dt =$

$= \frac{1}{4}\int_{-\infty}^{\infty} g_T^2(t)\,dt + \frac{1}{8}\int_{-\infty}^{\infty} g_T^2(t)\,dt + \underbrace{\frac{1}{8}\int_{-\infty}^{\infty}\cos(8\pi f_0 t)\,g_T^2(t)}_{\simeq 0} + \underbrace{\frac{2}{4}\int_{-\infty}^{\infty}\cos(4\pi f_0 t)\,g_T^2(t)\,dt}_{\simeq 0}$

$= \frac{1}{4}\cdot\frac{T}{2} + \frac{1}{8}\cdot\frac{T}{2} = \frac{T}{8} + \frac{T}{16} = \frac{3}{16}T = E_1$