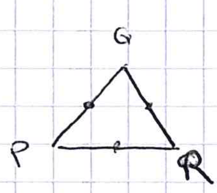
HW #5

Problema 1

$$L(1/2) = R + \tfrac{1}{2}(Q-R) = \frac{Q+R}{2}$$

Formula del punto medio



$\frac{1}{2}(P+Q+R-Z)$

Formula del punto medio per il lato opposto Z

$$L_i(t) = y_i + t\left(\tfrac{1}{2}(y_1+y_2+y_3-y_i) - y_i\right)$$
$$= \frac{t}{2}(y_1+y_2+y_3) + y_i\left(1-\tfrac{3}{2}t\right)$$
$$\therefore \quad t = \tfrac{2}{3} \Rightarrow L_i(t) = \tfrac{1}{3}(y_1+y_2+y_3) = \beta.$$

Problema 2:

$$\beta(T-\{y_k\}) = \frac{1}{N}(y_1+\cdots+y_{N+1} - y_k)$$ ← omettere y_k

$$L_k(t) = y_k + t\left(\beta(T-\{y_k\}) - y_k\right)$$
$$= \frac{t}{N}(y_1+\cdots+y_{N+1}) + y_k\left(1 - \frac{N+1}{N}t\right)$$
$$\Rightarrow L_k\left(\frac{N}{N+1}\right) = \frac{1}{N+1}(y_1+\cdots+y_n) + y_k\underbrace{(1-1)}_{0} = \beta(T).$$
$$\Rightarrow \beta(T) \in L_k, \quad k=1,\dots,N+1.$$
$$\Rightarrow \beta(T) \in L_1 \cap L_2 \cap \cdots \cap L_{N+1}$$

Problema 3

Sia $H = \{(x_1,\dots,x_N) \in \mathbb{R}^N \mid a_1x_1 + a_2x_2 + \cdots + a_Nx_N = d\}$

$p = (p_1,\dots,p_N), \quad q = (q_1,\dots,q_N)$

$\therefore \quad p \in H \Rightarrow a_1p_1 + \cdots + a_Np_N = d$

$\quad\;\; q \in H \Rightarrow a_1q_1 + \cdots + a_Nq_N = d$

$$L(t) = p + t(q-p) = (x_1(t),\dots,x_n(t))$$
$$a_1x_1(t) + a_2x_2(t) + \cdots + a_Nx_N(t)$$
$$= a_1(p_1 + t(q_1-p_1)) + \cdots + a_n(p_n + t(q_n-p_n))$$
$$= \underbrace{(a_1p_1 + \cdots + a_Np_N)}_{d} + t\left(\underbrace{(a_1q_1+\cdots+a_Nq_N)}_{d} + \underbrace{(a_1p_1+\cdots+a_np_n)}_{d}\right)$$
$$= d + t(d-d) = d \Rightarrow L \subset H$$

Problem 3

Usando le formule a pagina 2 della lezione 5

$L(t) = p + t v \quad , \quad v = q - p$

$A = (a_1, \dots, a_n) \quad , \quad H = \{ A_1 x_1 + \dots + A_N x_N = d \}$

$(A, v) = (A, q-p) = (A, q) - (A, p) = d - d = 0$

$(A, p) = d$

$\Longrightarrow L \subseteq H.$

Problema 4:

$H = \{ (x_1, x_2, x_3, x_4) \mid A_1 x_1 + A_2 x_2 + A_3 x_3 + A_4 x_4 = D \}$

(a) Sostituisci i punti dati nell'equazione per H per ottenere un sistema lineare di equazioni per i coefficienti.

$(0,0,0,0) \in H \Longrightarrow D = 0$ (sempre $0 \in H \Longrightarrow D = 0$)

$A_1 - A_2 = 0 \quad , \quad A_1 - A_3 = 0 \quad , \quad 2A_1 - A_2 - A_3 - A_4 = 0$

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 2 & -1 & -1 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & -1 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} +1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$\therefore A_4 = 0, \quad A_2 - A_3 = 0 \Longrightarrow A_2 = A_3 \quad , \quad A_1 - A_2 = 0$

$\Longrightarrow A_1 = A_2 = A_3$

$\therefore (A_1, A_2, A_3, A_4) = c(1, 1, 1, 0) \qquad \underline{c = 1}$

$\boxed{x_1 + x_2 + x_3 = 0}$

(b) Lo stesso modo $x_1 + x_2 + x_4 = 0$

(c) Lo stesso modo $x_1 + x_3 + x_4 = 0$

(d) I punti $(0,0,0,0)$ e $(2,1,1,1)$ appartengono a ciascun iperpiano, quindi ogni iperpiano contiene la linea che passa per $(0,0,0,0)$ e $(2,1,1,1)$

Problema 5

ⓐ $p(1) = \sum_{k=1}^{1} 2k+1 = 3$

$p(2) = \sum_{k=1}^{2} 2k+1 = 3+5 = 8$

$p(3) = \sum_{k=1}^{3} 2k+1 = 3+5+7 = 15$

Interpolazione di Lagrange

$$p_1(x) = \frac{(x-2)(x-3)}{(1-2)(1-3)} = \frac{1}{2}(x-2)(x-3)$$

$$p_2(x) = \frac{(x-1)(x-3)}{(2-1)(2-3)} = -(x-1)(x-3)$$

$$p_3(x) = \frac{(x-1)(x-2)}{(3-1)(3-2)} = \frac{1}{2}(x-1)(x-2)$$

$$\Rightarrow p(x) = 3\,p_1(x) + 8\,p_2(x) + 15\,p_3(x)$$

$$= x^2 + 2x$$

Verificare : $p(x) = x^2 + 2x \Rightarrow p(1) = 3$

$p(2) = 8$

$p(3) = 15$

ⓑ $S(1) = p(1)$

$$S(N) = P(N) \Longrightarrow \sum_{k=1}^{N} 2k+1 = N^2 + 2N.$$

$$S(N+1) = \sum_{k=1}^{N+1} 2k+1 = \{2(N+1)+1\} + \sum_{k=1}^{N} 2k+1$$

$$= 2(N+1)+1 + \underbrace{N^2+2N}_{S(N)=P(N)}$$

$$= N^2 + 2N + 1 + 2(N+1)$$

$$= (N+1)^2 + 2(N+1) = P(N+1)$$

∴ per il principio di induzione $S(N) = P(N)$

$N = 1, 2, \ldots$

Problema 6 $\quad V = \mathbb{R}^S = \{f : S \to \mathbb{R}\}$

① $f, g \in V \Rightarrow (f+g)(s) = f(s)+g(s) = g(s)+f(s)$
$= (g+f)(s)$

$\Rightarrow f+g = g+f$

② $f, g, h \in V \Rightarrow ((f+g)+h)(s) = f(s)+g(s)+h(s)$
$= f(s) + (g+h)(s)$

$\Rightarrow (f+g)+h = f+(g+h)$

③ $z : S \to \mathbb{R} \quad z(s) = 0$ per ogni $s \in S$
$f : S \to \mathbb{R} \Rightarrow (f+z)(s) = f(s) + z(s) = f(s)$
$\Rightarrow f+z = f$
$(z+f)(s) = z(s) + f(s) = f(s)$
$\Rightarrow z+f = f. \qquad \boxed{z = 0}$

④ $f : S \to \mathbb{R}$, $(-f) : S \to \mathbb{R}$, $(-f)(s) = -f(s)$
$\Rightarrow (f + (-f))(s) = f(s) - f(s) = 0$
$\Rightarrow f + (-f) = 0$ (oppure z da ③)

$r, s \in \mathbb{R}$, $x \in S$

⑤ $((rs)f)(x) = rsf(x) = r(s f(x)) \Rightarrow (rs)f = r(sf)$

⑥ $((r+s)f)(x) = (r+s) f(x)$
$= r f(x) + s f(x)$

$\Rightarrow (r+s)f = rf + sf.$

⑦ $(r(f+g))(x) = r(f+g)(x) = rf(x) + rg(x)$
$\Rightarrow r(f+g) = rf + rg$

⑧ $(1f)(x) = f(x) \Rightarrow 1f = f$

Problema 7

$$M_{n \times m} \longleftrightarrow \mathbb{R}^{n \times m}$$

$$A = (a_{ij}) \longleftrightarrow v = v_e$$

$$v_{n(i-1)+j} = a_{ij}$$

$$n = 3, \quad m = 2$$

$$\begin{array}{ccc} v_1, & v_2, & v_3 \\ \parallel & \parallel & \parallel \\ a_{11} & a_{12} & a_1 \end{array}$$

Con questa osservazione, la ~~ver~~ verifica diventa uguale alla verifica che $\mathbb{R}^k$ è uno spazio vettoriale