Esercizio 3 FILA A

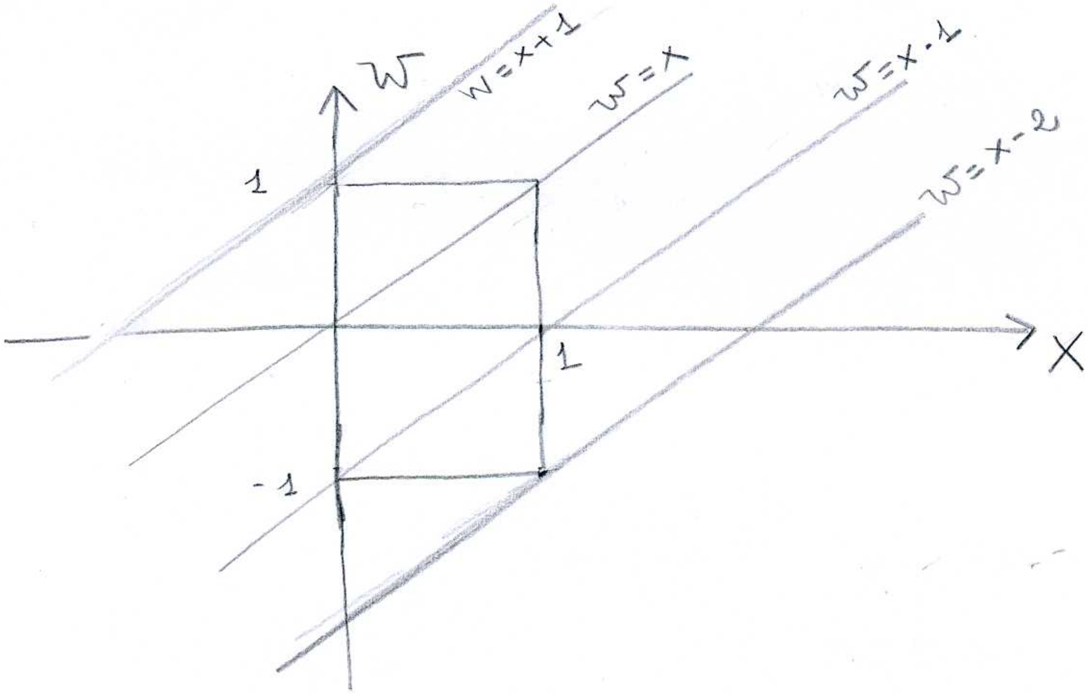
$X \in \mathcal{U}(0,1)$ $\quad f_X(x) = \text{rect}\left(\frac{x-1/2}{1}\right)$

$Y \in \mathcal{U}(0,1)$ $\quad f_Y(y) = \text{rect}\left(\frac{y-1/2}{1}\right)$

$Z = X - 2Y + 1$

$W = 2Y - 1 \in \mathcal{U}(-1,1)$ $\quad f_W(w) = \frac{1}{2}\text{rect}\left(\frac{w}{2}\right)$

$Z = X - W$ e X e W sono ancora indipendenti

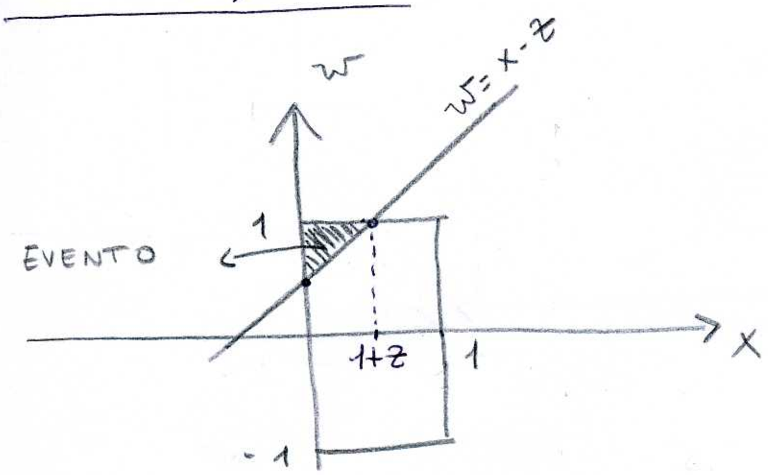


$$F_Z(z) = \Pr\{Z \le z\} = \Pr\{X - W \le z\} =$$
$$= \Pr\{W \geq X - z\}$$

Se $z \leq -1$ $\quad F_Z(z) = 0$

Se $z \geq 2$ $\quad F_Z(z) = 1$

Se $0 \geq z \geq -1$



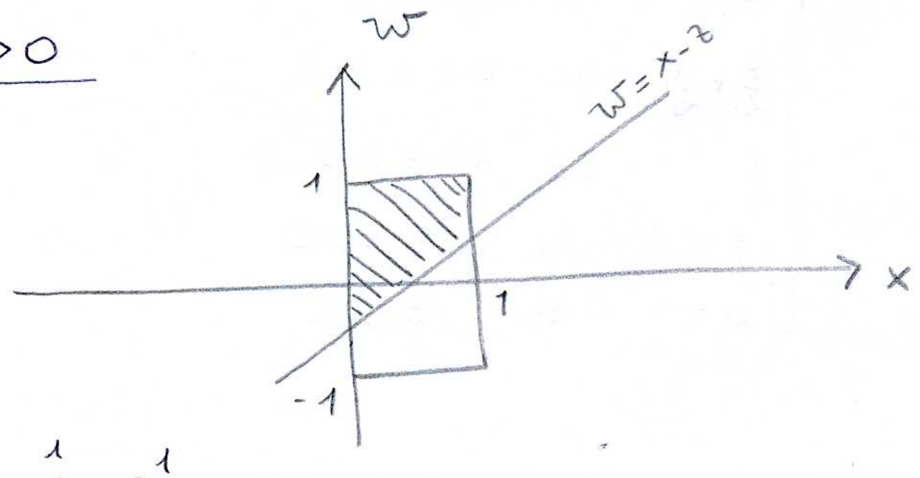
$y = x - z$

$$Pr\{Z \leq z\} = \int_{x=0}^{1+z} \int_{w=x-z}^{1} f_{XW}(x,w)\,dx\,dw =$$

$$= \int_0^{1+z} dx \int_{x-z}^{1} \frac{1}{2}\,dy = \frac{1}{2} \int_0^{1+z} (1 - x + z)\,dx = \frac{1}{2} \left[(1+z)x - \frac{x^2}{2} \right]_0^{1+z} =$$

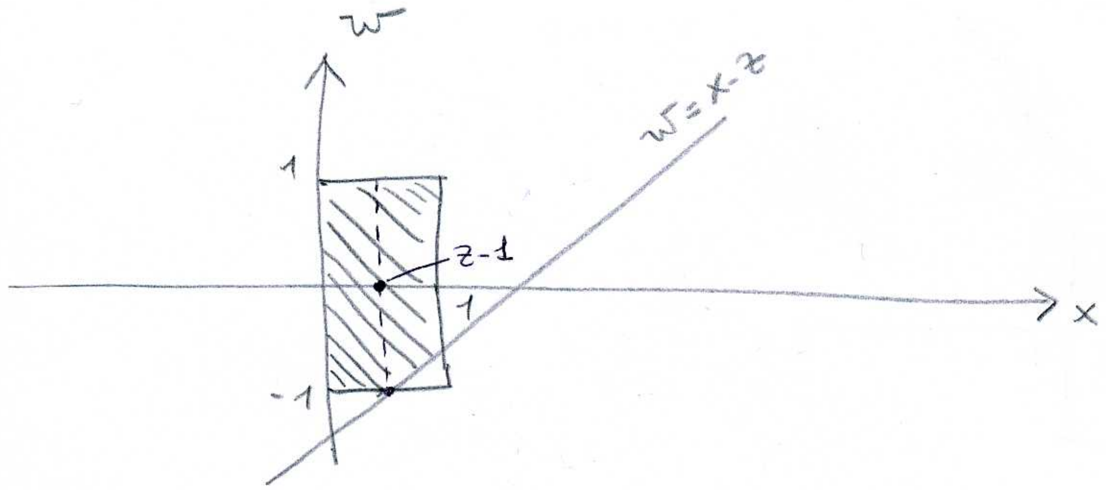
$$= \frac{z^2}{4} + \frac{z}{2} + \frac{1}{4}$$

Se $1 \geq z > 0$



$$Pr\{Z \leq z\} = \int_{x=0}^{1} \int_{w=x-z}^{1} \frac{1}{2}\,dw\,dx = \frac{z}{2} + \frac{1}{4}$$

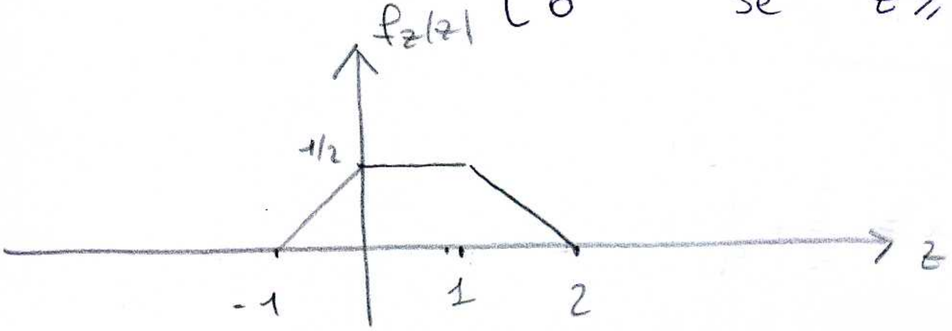
Se $\quad 2 \geq z > 1$



$$Pr\{Z \leq z\} = \frac{(z-1)^2}{2} + \int_{x=z-1}^{1} \int_{w-z}^{1} \frac{1}{2}\, dx\, dw = -\frac{z^2}{4} + z$$

$$F_Z(z) = \begin{cases} 0 & \text{se } z \leq -1 \\ \frac{z^2}{4} + \frac{z}{2} + \frac{1}{4} & \text{se } 0 \geq z > -1 \\ \frac{z}{2} + \frac{1}{4} & \text{se } 1 \geq z > 0 \\ 1 & \text{se } z \geq 2 \end{cases}$$

$$f_Z(z) = \frac{\partial}{\partial z} F_Z(z) = \begin{cases} 0 & \text{se } z \leq -1 \\ \frac{z}{2} + \frac{1}{2} & \text{se } 0 \geq z > -1 \\ \frac{1}{2} & \text{se } 1 \geq z > 0 \\ -\frac{z}{2} + 1 & \text{se } 2 \geq z > 1 \\ 0 & \text{se } z \geq 2 \end{cases}$$



Si poteva anche svolgere senza calcolarsi la $F_Z(z)$.

$$Z = X - 2Y + 1$$

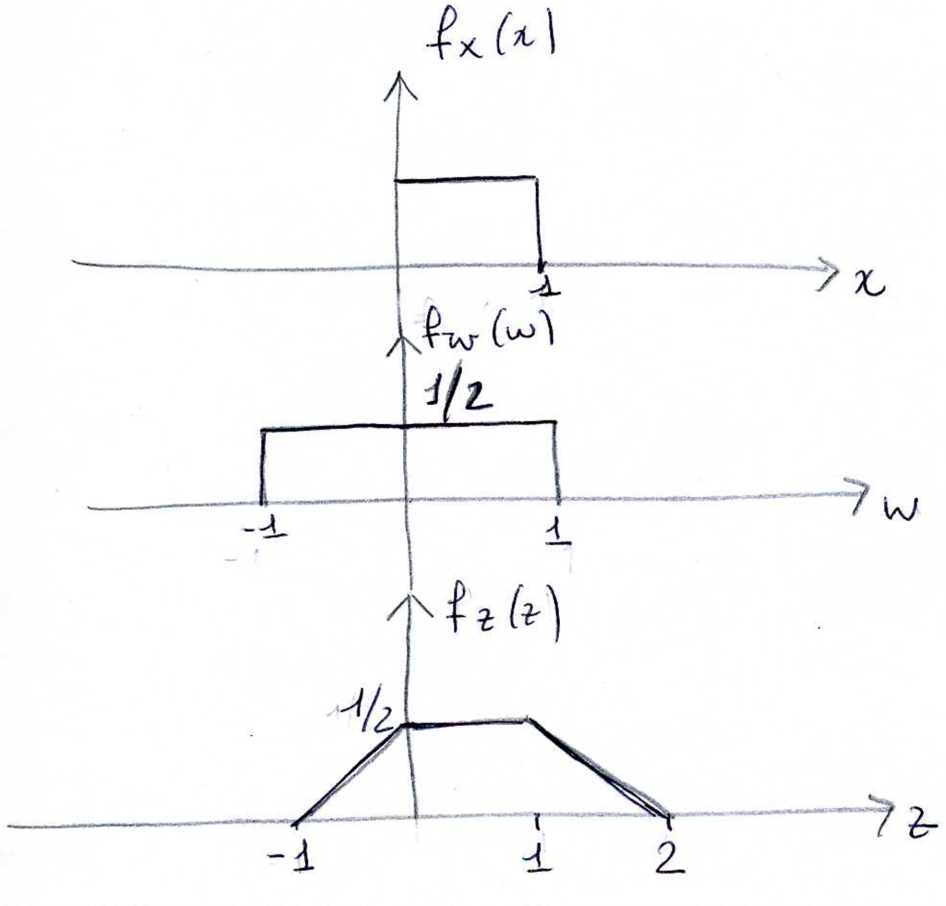
$W = -2Y + 1 \quad \in \mathcal{U}(-1,1) \qquad f_W(w) = \frac{1}{2}\,\text{rect}\left(\frac{w}{2}\right)$

poiché X e Y sono indipendenti lo sono anche X e W.

Poiché Z si può riscrivere come

$$Z = X + W$$

$$f_Z(z) = f_X(x) \otimes f_W(w)$$



Esercizio 3 FILA B

$X \in \mathcal{U}(0,1)$ $\quad f_X(x) = \text{rect}\left(\frac{x-1/2}{1}\right)$

$Y \in \mathcal{U}(0,1)$ $\quad f_Y(y) = \text{rect}\left(\frac{y-1/2}{1}\right)$

$Z = X + 2Y - 1$

$W = -2Y + 1 \in \mathcal{U}(-1,1)$ $\quad f_W(w) = \frac{1}{2}\,\text{rect}\left(\frac{w}{2}\right)$

$Z = X - W$

Vedi soluzione per la FILA A