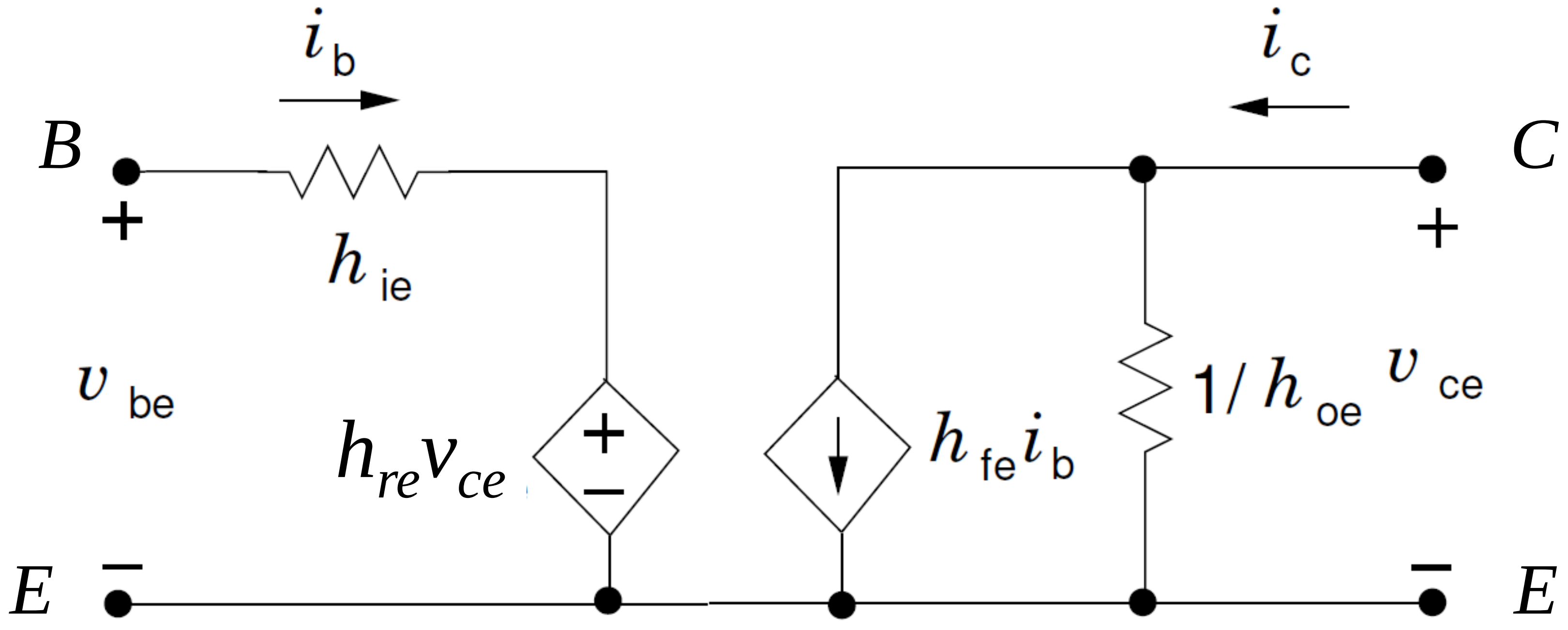
Elettronica Digitale
A.A. 2020-2021

Lezione 08/04/2021

Transistore BJT– Modello linearizzato per piccoli segnali

$$\begin{cases} v_{be} = h_{ie} i_b + h_{re} v_{ce} \\ i_c = h_{fe} i_b + h_{oe} v_{ce} \end{cases}$$

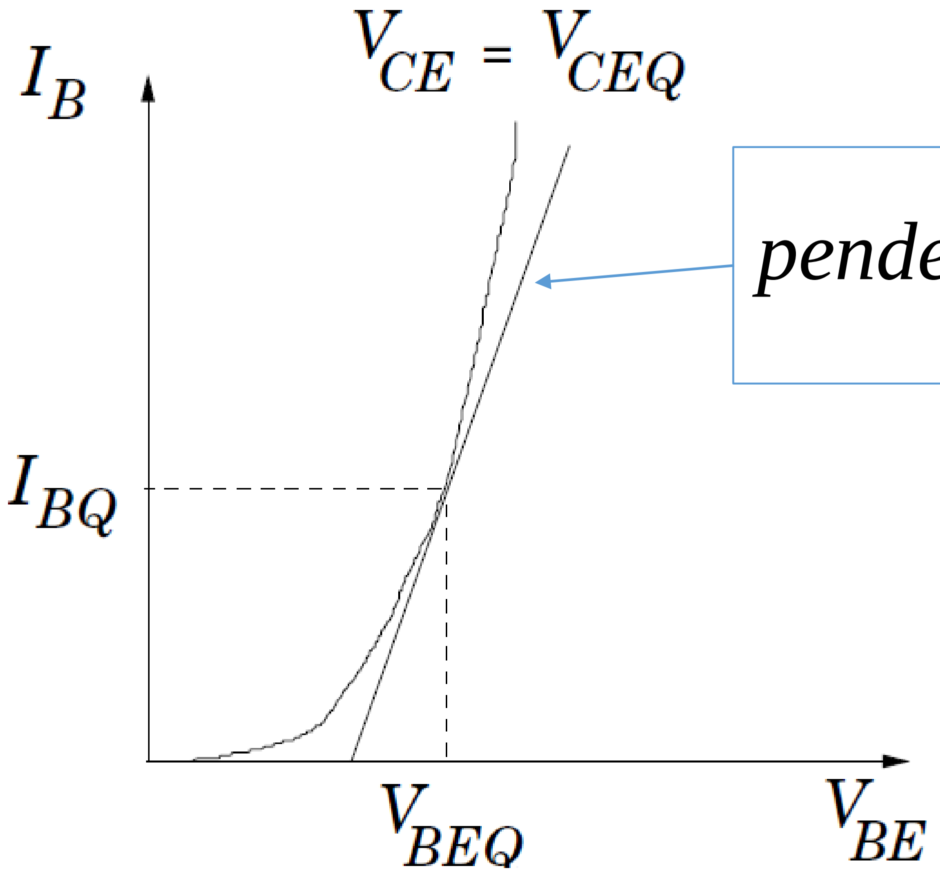
$$\boxed{h_{ie} = \left.\frac{\partial v_{BE}}{\partial i_B}\right|_Q} \quad \boxed{h_{re} = \left.\frac{\partial v_{BE}}{\partial v_{CE}}\right|_Q} \quad \boxed{h_{fe} = \left.\frac{\partial i_C}{\partial i_B}\right|_Q} \quad \boxed{h_{oe} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_Q}$$



Transistore BJT– Modello linearizzato per piccoli segnali

$$\begin{cases} v_{be} = h_{ie} i_b + h_{re} v_{ce} \\ i_c = h_{fe} i_b + h_{oe} v_{ce} \end{cases}$$

$$\boxed{h_{ie} = \left. \frac{\partial v_{BE}}{\partial i_B} \right|_Q}$$

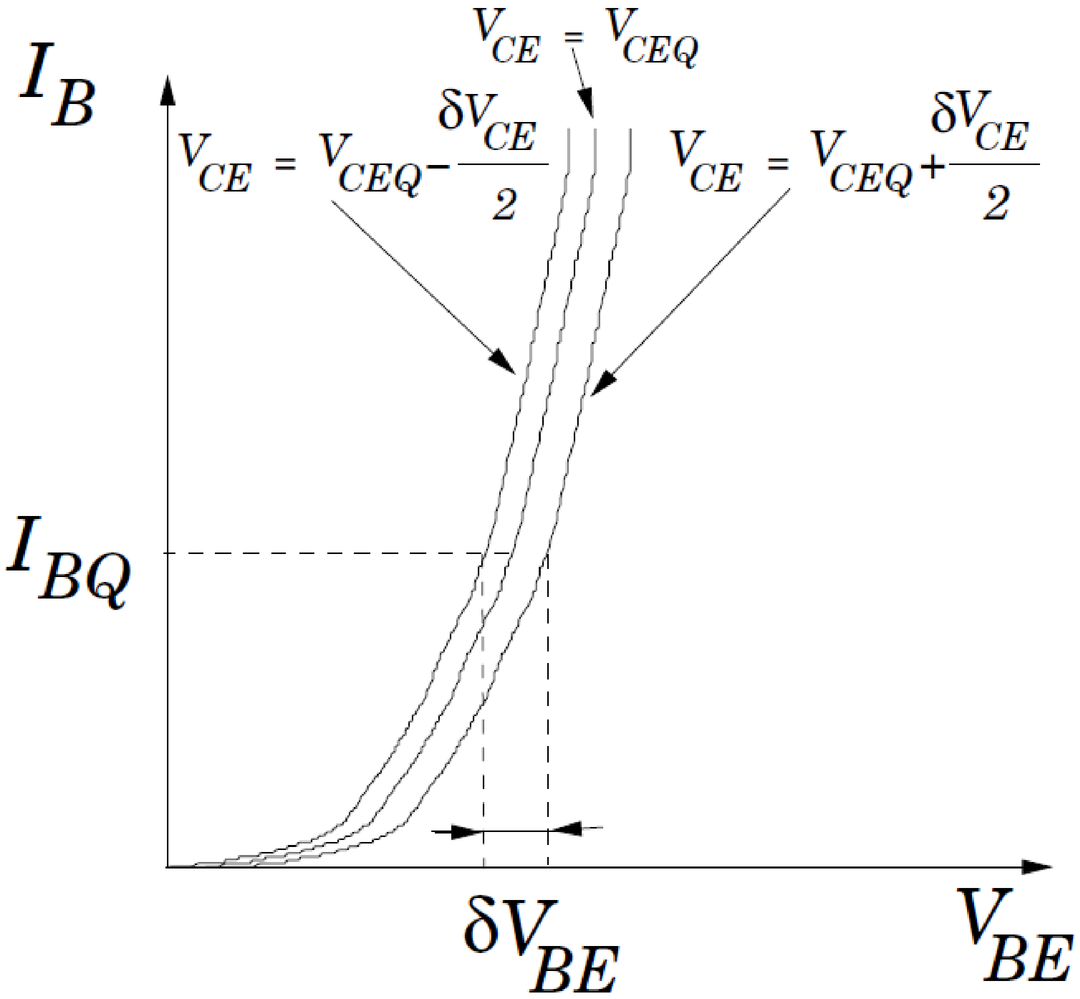


$$\boxed{pendenza = \frac{1}{h_{ie}}}$$

Transistore BJT– Modello linearizzato per piccoli segnali

$$\begin{cases} v_{be} = h_{ie} i_b + h_{re} v_{ce} \\ i_c = h_{fe} i_b + h_{oe} v_{ce} \end{cases}$$

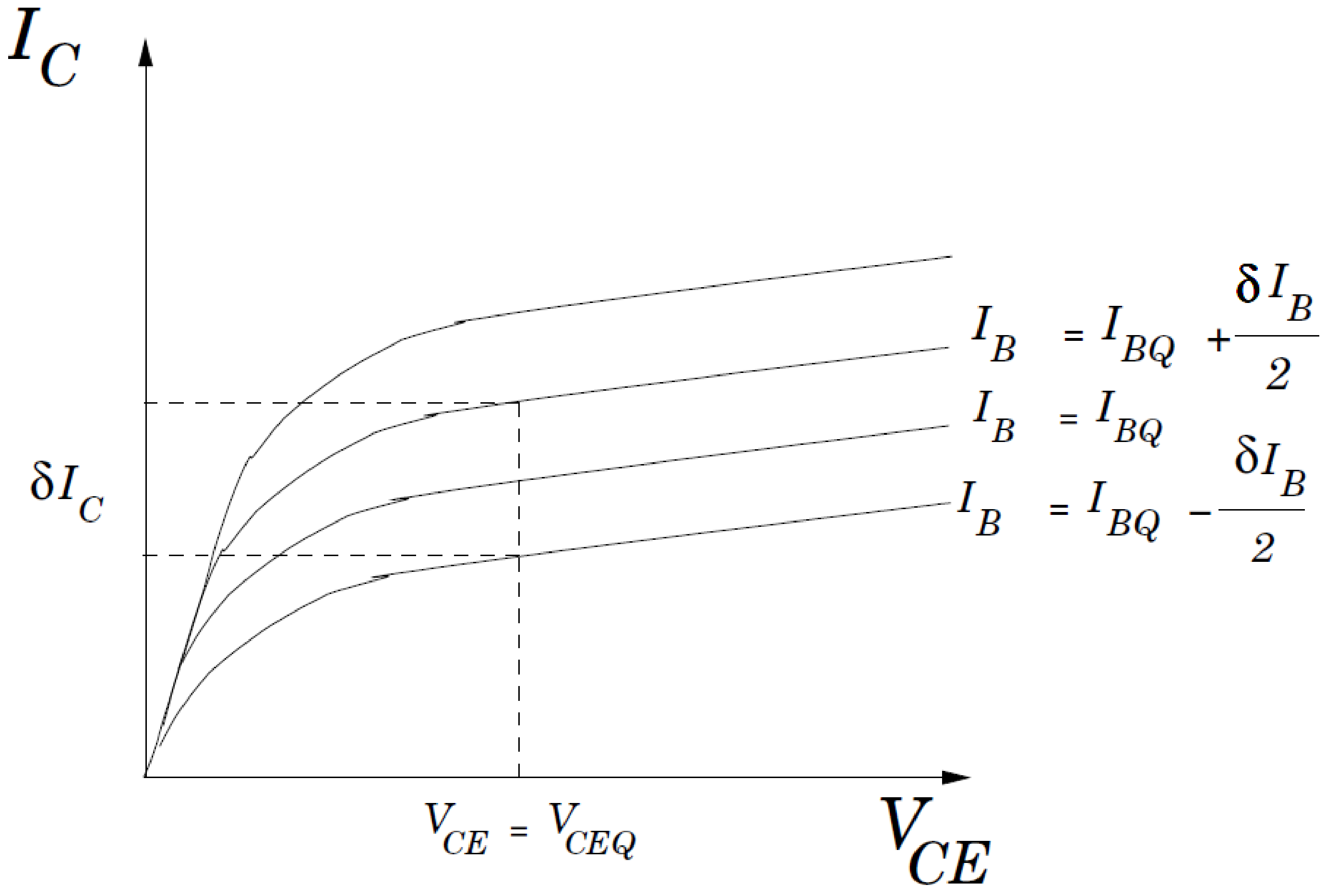
$$\boxed{h_{re} = \left.\frac{\partial v_{BE}}{\partial v_{CE}}\right|_Q = \lim_{\delta v_{CE} \to 0} \left.\frac{\delta v_{BE}}{\delta v_{CE}}\right|_{i_B = I_{BQ}}}$$



Transistore BJT– Modello linearizzato per piccoli segnali

$$\begin{cases} v_{be} = h_{ie} i_b + h_{re} v_{ce} \\ i_c = h_{fe} i_b + h_{oe} v_{ce} \end{cases}$$

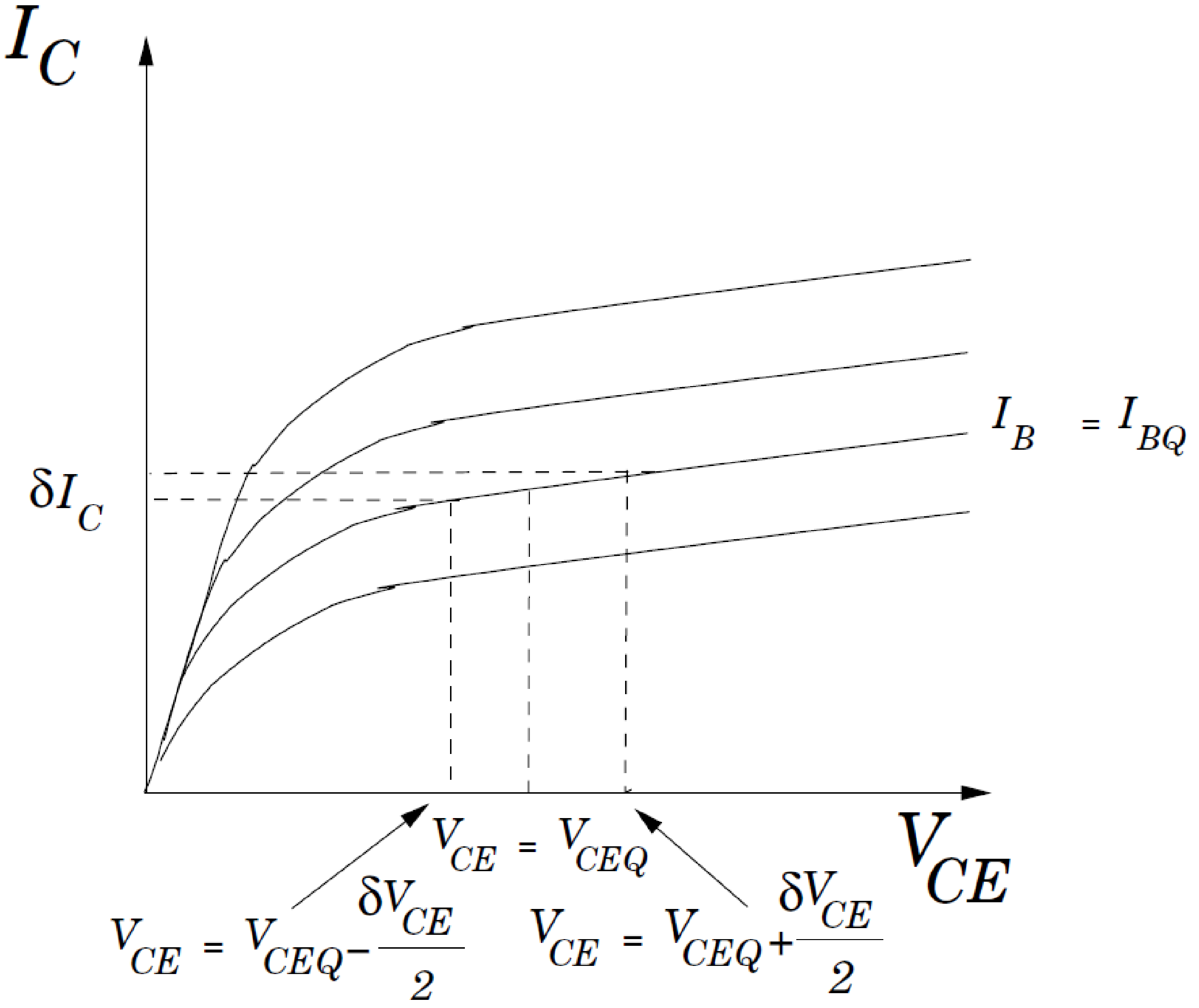
$$\boxed{h_{fe} = \left.\frac{\partial i_C}{\partial i_B}\right|_Q = \lim_{\delta i_B \to 0} \left.\frac{\delta i_C}{\delta i_B}\right|_{v_{CE} = V_{CEQ}}}$$



Transistore BJT– Modello linearizzato per piccoli segnali

$$\begin{cases} v_{be} = h_{ie} i_b + h_{re} v_{ce} \\ i_c = h_{fe} i_b + h_{oe} v_{ce} \end{cases}$$

$$\boxed{h_{oe} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_Q = \lim_{\delta v_{CE} \to 0} \left.\frac{\delta i_C}{\delta v_{CE}}\right|_{i_B = I_{BQ}}}$$



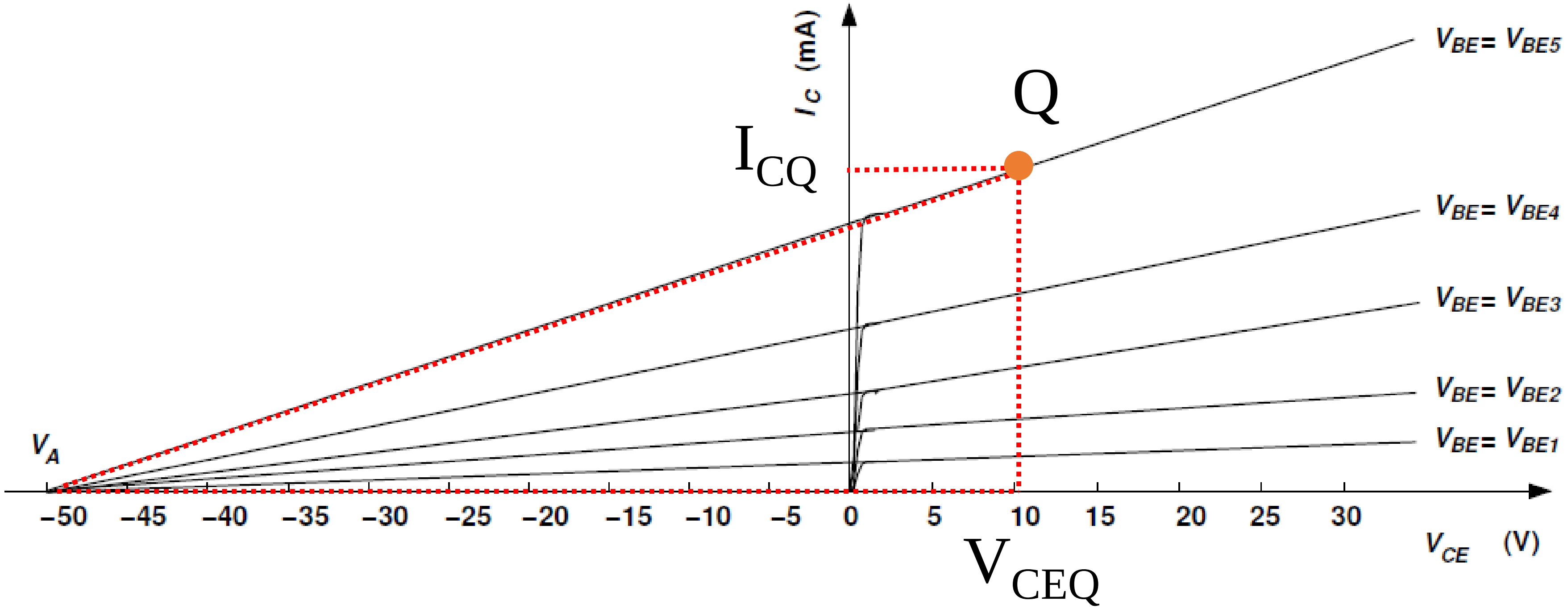
Transistore BJT– Modello linearizzato per piccoli segnali

Determinazione dei valori dei parametri

$$h_{re} = \left.\frac{\partial v_{BE}}{\partial v_{CE}}\right|_Q \approx 0$$

$$h_{oe} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_Q$$

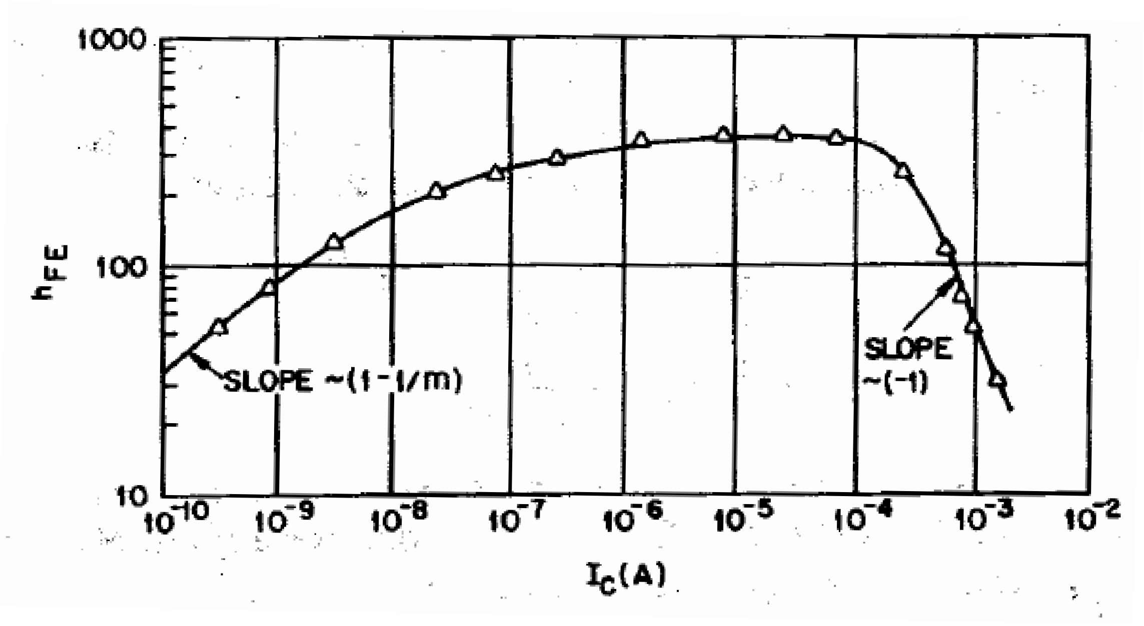
$$h_{oe} \approx \frac{I_{CQ}}{V_A + V_{CEQ}}$$



Transistore BJT– Modello linearizzato per piccoli segnali

Determinazione dei valori dei parametri

$$h_{fe} = \left.\frac{\partial i_C}{\partial i_B}\right|_Q \qquad i_C = \beta_F i_B \qquad \beta_F = f(i_C)$$



$$\frac{\partial i_C}{\partial i_B} = \beta_F + i_B \frac{\partial \beta_F}{\partial i_B} = \beta_F + i_B \frac{\partial \beta_F}{\partial i_C}\frac{\partial i_C}{\partial i_B}$$

$$\frac{\partial i_C}{\partial i_B} = \frac{\beta_F}{1 - i_B \dfrac{\partial \beta_F}{\partial i_C}} \qquad h_{fe} = \left.\frac{\partial i_C}{\partial i_B}\right|_Q = \frac{\left.\beta_F\right|_Q}{1 - i_B \left.\dfrac{\partial \beta_F}{\partial i_C}\right|_Q} = \frac{\left.\beta_F\right|_Q}{1 - \dfrac{i_C}{\beta_F}\left.\dfrac{\partial \beta_F}{\partial i_C}\right|_Q}$$

$$\Rightarrow \boxed{h_{fe} \approx \beta_F}$$

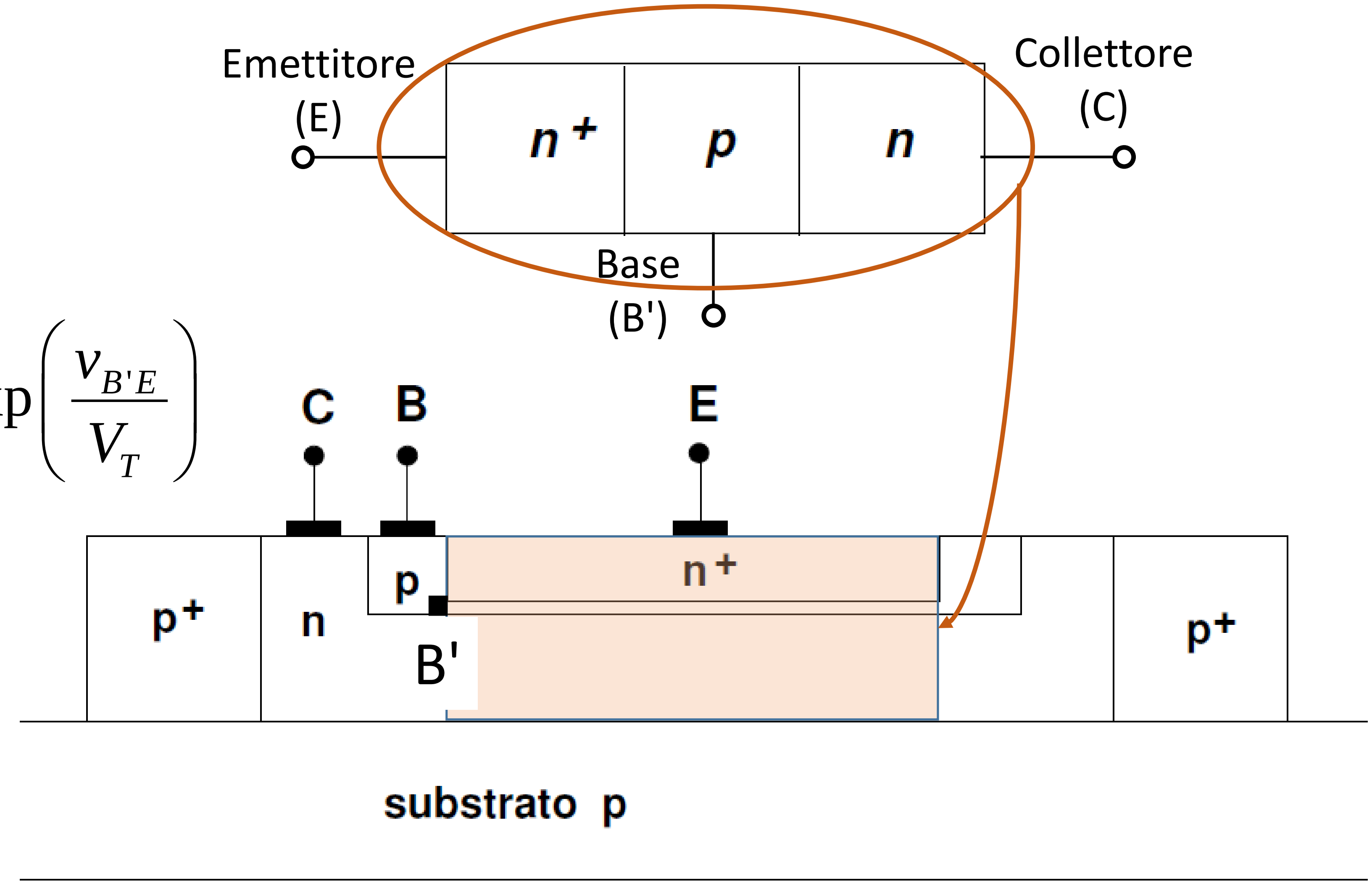
Transistore BJT– Modello linearizzato per piccoli segnali

Determinazione dei valori dei parametri

$$h_{ie} = \left.\frac{\partial v_{BE}}{\partial i_B}\right|_Q = r_{bb'} + r_{b'e}$$

$$i_C \approx \alpha_F I_{ES} \exp\left(\frac{v_{B'E}}{V_T}\right) \approx I_{ES} \exp\left(\frac{v_{B'E}}{V_T}\right)$$

$$i_B \approx \frac{i_C}{\beta_F} \approx \frac{I_{ES} \exp\left(\frac{v_{B'E}}{V_T}\right)}{\beta_F}$$



Transistore BJT– Modello linearizzato per piccoli segnali

Determinazione dei valori dei parametri

$$h_{ie} = \left.\frac{\partial v_{BE}}{\partial i_B}\right|_Q = r_{bb'} + r_{b'e}$$

$$i_B \approx \frac{i_C}{\beta_F} \approx \frac{I_{ES} \exp\left(\frac{v_{B'E}}{V_T}\right)}{\beta_F}$$

$$\frac{1}{r_{b'e}} = \left.\frac{\partial i_B}{\partial v_{B'E}}\right|_Q = \frac{1}{V_T}\frac{I_{ES} \exp\left(\frac{v_{B'E}}{V_T}\right)}{\beta_F} - \frac{1}{\beta_F^2}\frac{\partial \beta_F}{\partial v_{B'E}} I_{ES} \exp\left(\frac{v_{B'E}}{V_T}\right)$$

$$\frac{1}{r_{b'e}} = \frac{1}{V_T}\frac{i_C}{\beta_F} - \frac{1}{\beta_F^2}\frac{\partial \beta_F}{\partial i_C}\frac{\partial i_C}{\partial v_{B'E}} i_C = \frac{1}{V_T}\frac{i_C}{\beta_F} - \frac{i_C}{\beta_F^2}\frac{i_C}{V_T}\frac{\partial \beta_F}{\partial i_C}$$

Transistore BJT– Modello linearizzato per piccoli segnali

Determinazione dei valori dei parametri

$$h_{ie} = \left.\frac{\partial v_{BE}}{\partial i_B}\right|_Q = r_{bb'} + r_{b'e}$$

$$i_B \approx \frac{i_C}{\beta_F} \approx \frac{I_{ES} \exp\left(\frac{v_{B'E}}{V_T}\right)}{\beta_F}$$

$$\frac{1}{r_{b'e}} = \frac{1}{V_T}\frac{i_C}{\beta_F} - \frac{i_C}{\beta_F^2}\frac{i_C}{V_T}\frac{\partial \beta_F}{\partial i_C} = \frac{1}{V_T}\frac{i_C}{\beta_F}\left(1 - \frac{i_C}{\beta_F}\frac{\partial \beta_F}{\partial i_C}\right)$$

$$h_{fe} = \frac{\left.\beta_F\right|_Q}{1 - \left.\frac{i_C}{\beta_F}\frac{\partial \beta_F}{\partial i_C}\right|_Q}$$

$$\frac{1}{r_{b'e}} = \left.\frac{1}{V_T}\frac{i_C}{\beta_F}\frac{\beta_F}{h_{fe}}\right|_Q = \left.\frac{1}{V_T}\frac{i_C}{h_{fe}}\right|_Q \Rightarrow \boxed{r_{b'e} = \frac{V_T}{I_{CQ}} h_{fe}}$$

Transistore BJT– Modello linearizzato per piccoli segnali

Determinazione dei valori dei parametri

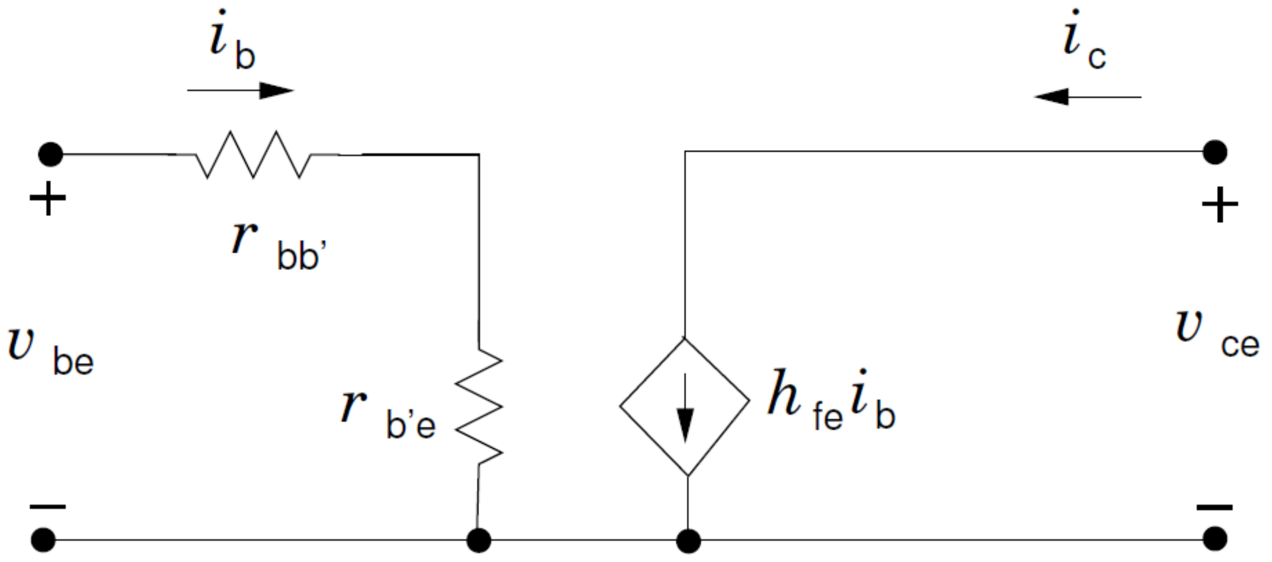
$$h_{ie} = \left.\frac{\partial v_{BE}}{\partial i_B}\right|_Q = r_{bb'} + r_{b'e} \qquad\qquad r_{b'e} = \frac{V_T}{I_{CQ}} h_{fe}$$

Se il costruttore fornisce i parametri per un particolare punto di riposo si può determinare il valore di $r_{bb'}$ considerando che tale resistenza è indipendente dal punto di riposo

$$h^*_{ie} = r^*_{bb'} + r^*_{b'e} \qquad\qquad r^*_{bb'} = h^*_{ie} - r^*_{b'e} = r_{bb'}$$

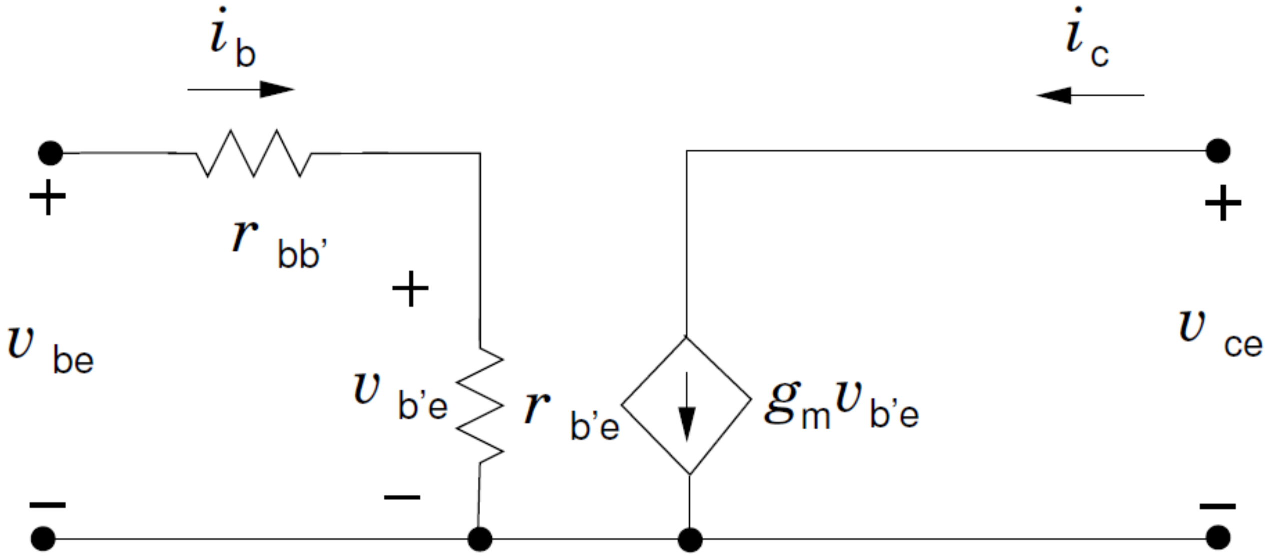
$$h_{ie} = r_{bb'} + r_{b'e} = \left(h^*_{ie} - r^*_{b'e}\right) + \frac{V_T}{I_{CQ}} h_{fe}$$

Transistore BJT– Modello linearizzato per piccoli segnali



$$h_{ie} = r_{bb'} + r_{b'e} \qquad r_{b'e} = \frac{V_T}{I_{CQ}} h_{fe}$$

$$h_{fe} \approx \beta_F$$

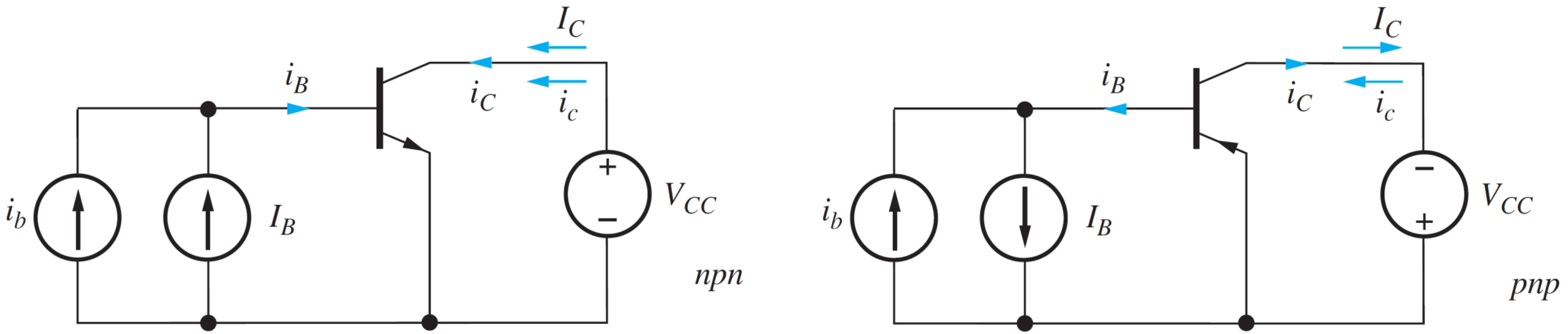


$$g_m v_{b'e} = g_m r_{b'e} i_b = h_{fe} i_b$$

$$g_m = \frac{h_{fe}}{r_{b'e}} = \frac{I_{CQ}}{V_T h_{fe}} h_{fe}$$

$$\boxed{g_m = \frac{I_{CQ}}{V_T}}$$

Transistore BJT– Equivalenza modello linearizzato per piccoli segnali NPN e PNP



$$i_B = I_B + i_b$$

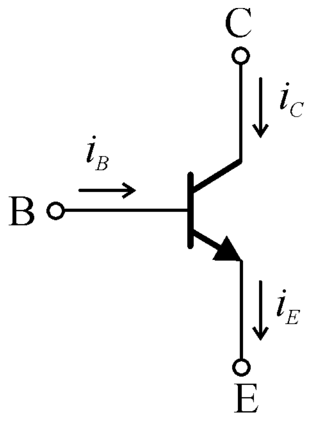
$$i_B = I_B - i_b$$

$$i_C = I_C + i_c = \beta_F I_B + h_{fe} i_b$$

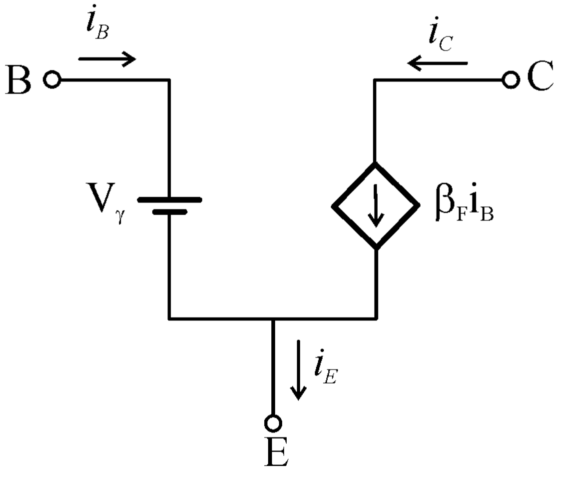
$$i_C = I_C - i_c = \beta_F I_B - h_{fe} i_b$$

Transistore BJT– Modelli NPN e PNP in zona attiva diretta

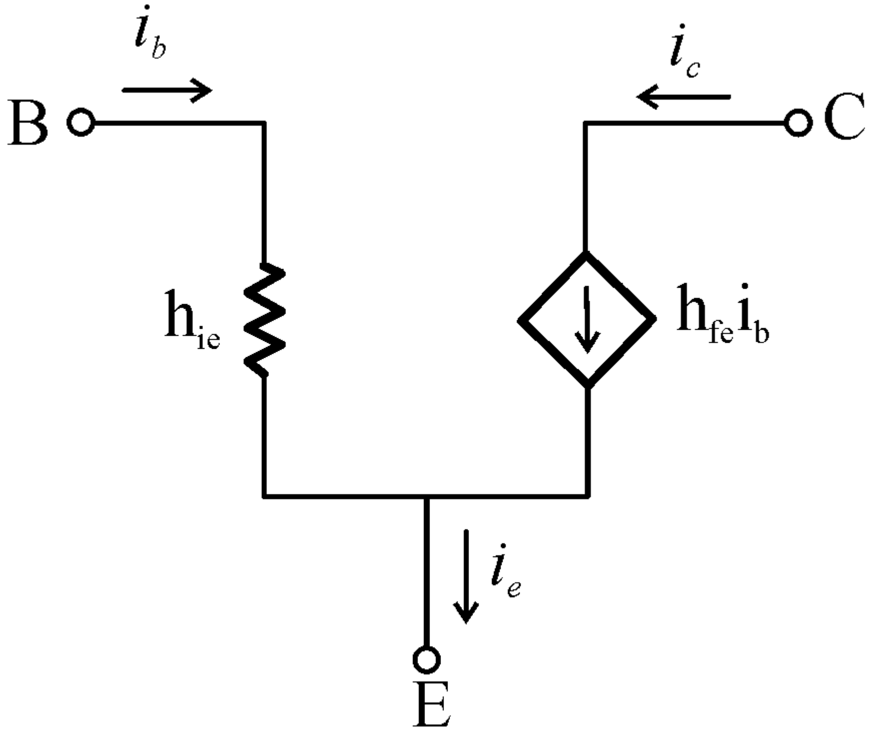
NPN



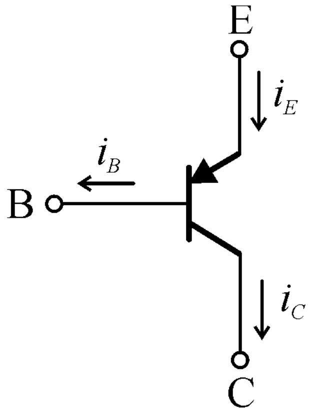
Ampi segnali

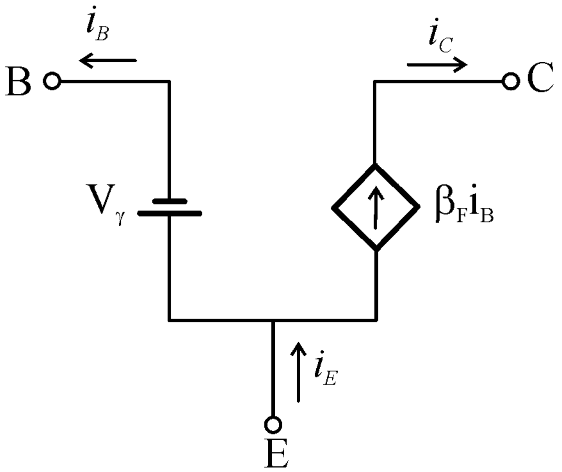


Piccoli segnali semplificato



PNP





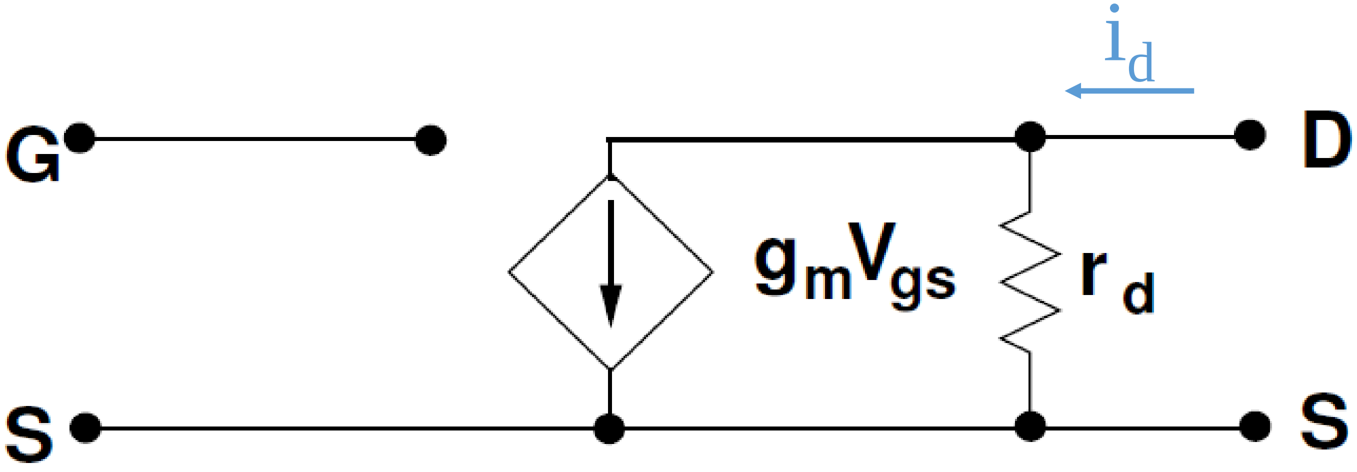
Transistore MOSFET– Modelli linearizzato per piccoli segnali in saturazione

$$i_D = k\frac{W}{L}\left(v_{GS} - V_T\right)^2\left(1+\lambda v_{DS}\right) = f\left(v_{GS}, v_{DS}\right)$$

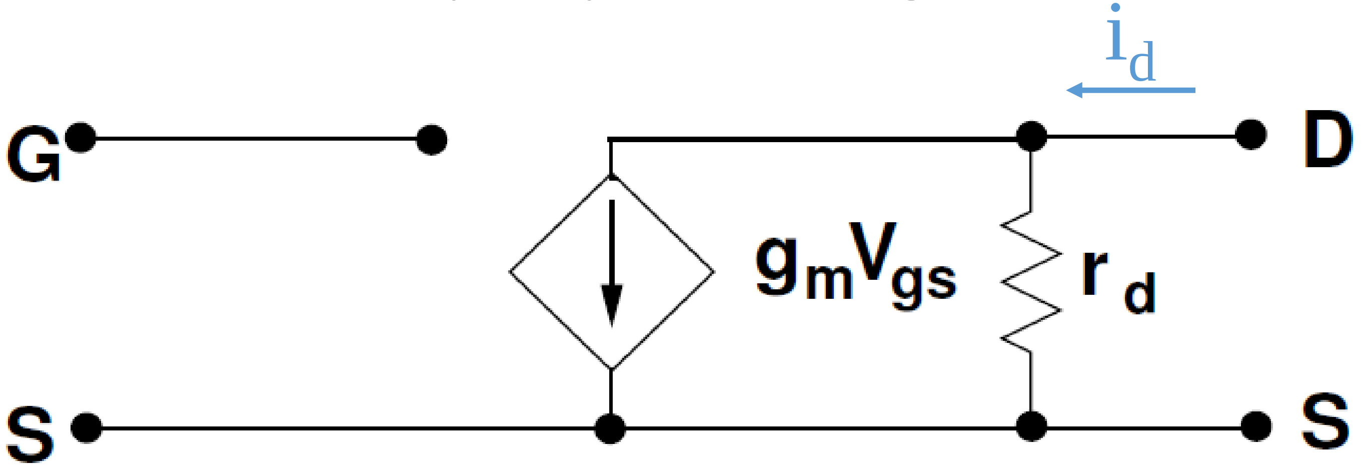
$$i_D = I_{DQ} + i_d(t) = f\left(V_{GSQ} + v_{gs}, V_{DSQ} + v_{ds}\right) = f\left(V_{GSQ}, V_{DSQ}\right) + \left.\frac{\partial f}{\partial v_{GS}}\right|_Q v_{gs} + \left.\frac{\partial f}{\partial v_{DS}}\right|_Q v_{ds} + \dots$$

$$i_d = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q v_{gs} + \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q v_{ds}$$

$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_d}$$



Transistore MOSFET– Modelli linearizzato per piccoli segnali in saturazione



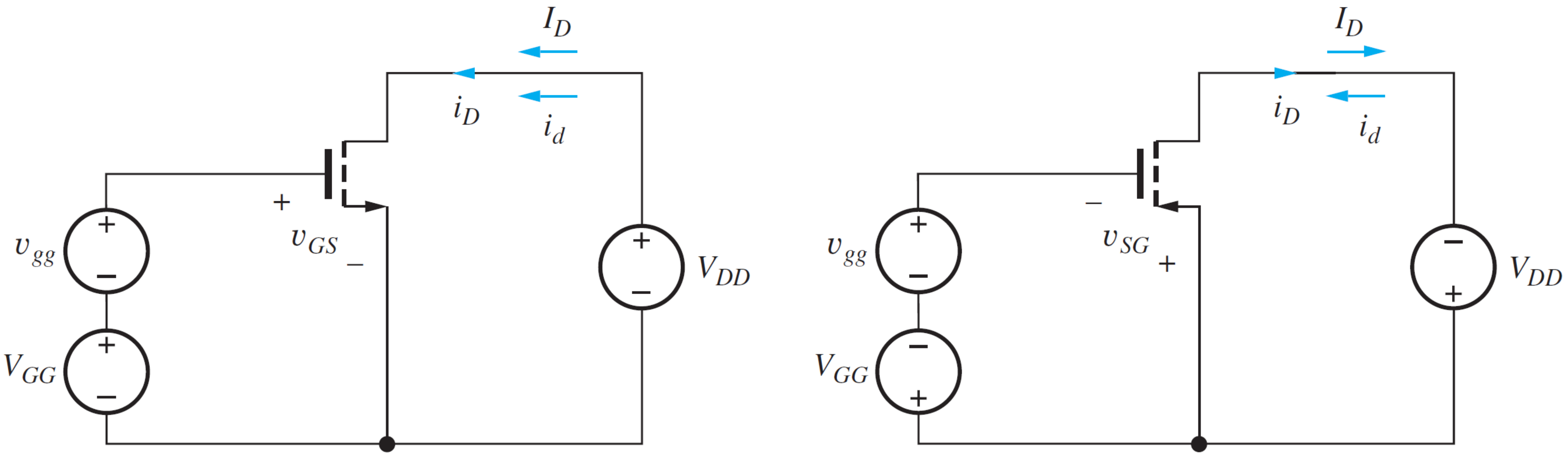
$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_d}$$

$$i_D = k\frac{W}{L}\left(v_{GS} - V_T\right)^2\left(1 + \lambda v_{DS}\right)$$

$$\boxed{g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q = 2k\frac{W}{L}\left(v_{GS} - V_T\right)\left(1 + \lambda v_{DS}\right)\Big|_Q = \frac{2I_{DQ}}{\left(V_{GSQ} - V_T\right)}}$$

$$\frac{1}{r_d} = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q = k\frac{W}{L}\left(v_{GS} - V_T\right)^2 \lambda\Big|_Q = \frac{I_{DQ}\lambda}{\left(1 + \lambda V_{DSQ}\right)} = \frac{I_{DQ}}{\left(\frac{1}{\lambda} + V_{DSQ}\right)} \approx I_{DQ}\lambda \quad \Rightarrow \quad \boxed{r_d \approx \frac{1}{\lambda I_{DQ}}}$$

Transistore MOSFET– Equivalenza modello linearizzato per piccoli segnali NMOS e PMOS



$$v_{GS} = V_{GG} + v_{gg}$$

$$i_D = I_{DQ} + i_d = k\frac{W}{L}(V_{GG} - V_T)^2 + g_m v_{gg}$$

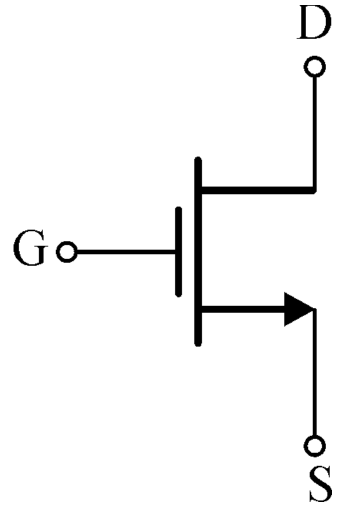
$$v_{SG} = V_{GG} - v_{gg}$$

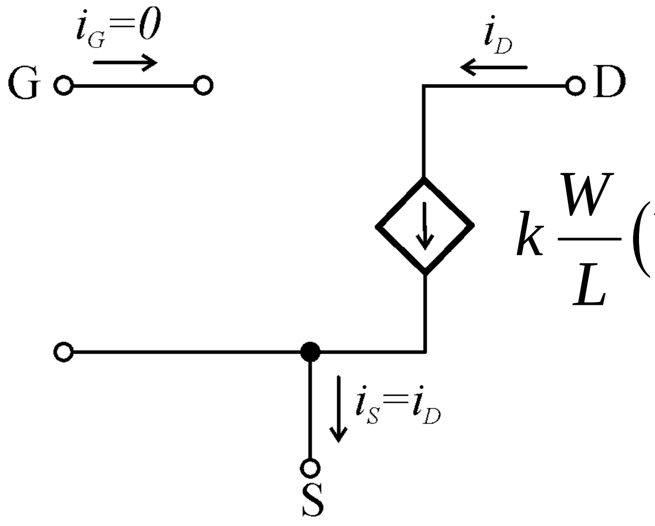
$$i_D = I_{DQ} - i_d = k\frac{W}{L}(-V_{GG} - V_T)^2 - g_m v_{gg}$$

Transistore MOSFET– Modelli NMOS e PMOS in zona di saturazione

Canale N

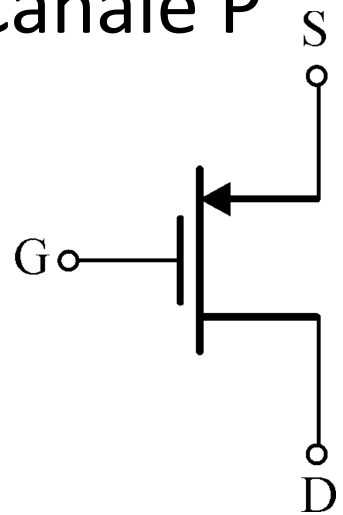
Ampi segnali

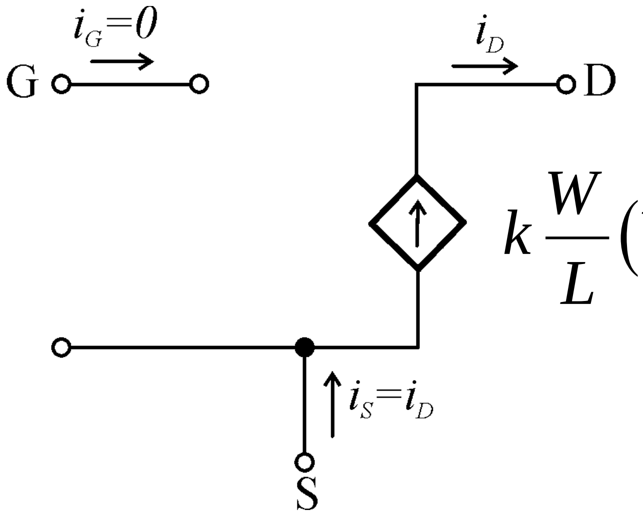




$$k\frac{W}{L}\left(V_{GS}-V_T\right)^2$$

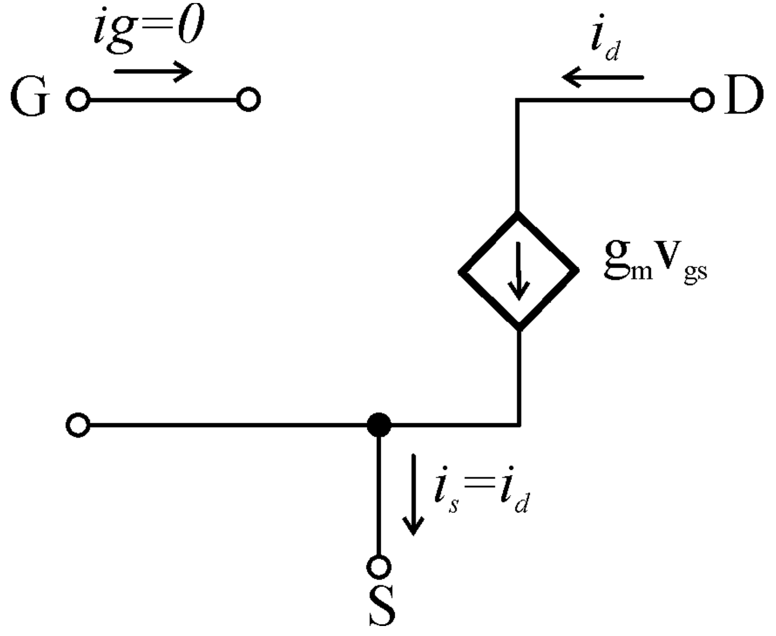
Canale P





$$k\frac{W}{L}\left(V_{GS}-V_T\right)^2$$

Piccoli segnali semplificato



$$g_m = 2k\frac{W}{L}\left|V_{GS}-V_T\right|$$