FLUSSO $\vec{F}$ Σ superficie regolare, orientata

$$\phi(\vec{F},\Sigma) = \iint_{\Sigma} \vec{F}\cdot\hat{n}\, dS \qquad \|\hat{n}\| = 1$$

$$T \ni (u,v) \longrightarrow \phi(u,v) \in \mathbb{R}^3$$

$$\phi(\vec{F},\Sigma) = \iint_{T} \vec{F}(\phi(u,v)) \circ \left(\frac{\partial\phi}{\partial u} \wedge \frac{\partial\phi}{\partial v}\right) du\,dv$$

$$\boxed{\vec{F} = x\,i + 0\,j + y\,k} \qquad \Sigma = \{ x^2+y^2+z^2 = R^2 \}$$

$$\phi(\vec{F},\Sigma) = \iint_{S^+} \vec{F}\cdot\hat{n}\, dS + \iint_{S^-} \vec{F}\cdot\hat{n}\, dS$$

$$S^+ = \Sigma \cap \{z \geq 0\} \qquad S^- = \Sigma \cap \{z \leq 0\}$$

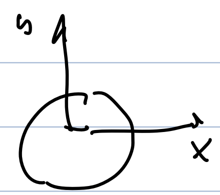
$$S^- = \{(x,y) : x^2+y^2 \leq R^2 \quad z = -\sqrt{R^2-x^2-y^2}\} \qquad S^+ = \{(x,y) : x^2+y^2 \leq R^2 \quad z = \underbrace{+\sqrt{R^2-x^2-y^2}}_{= f(x,y)}\}$$

$$n = -f_x\, i - f_y\, j + k$$

$$\hat{n} = \frac{-f_x\, i - f_y\, j + k}{\sqrt{1+|\nabla f|^2}} \qquad f_x = \frac{1}{\cancel{2}\sqrt{R^2-x^2-y^2}}\,(-\cancel{2}x) = \frac{-x}{\sqrt{R^2-x^2-y^2}}$$

$$\phi(\vec{F},S^+) = \iint_{x^2+y^2 \leq R^2} \left[\frac{x\;x}{\sqrt{R^2-x^2-y^2}} + 0\,(\ ) + y \right] dx\,dy$$

$$= \iint_{x^2+y^2 \leq R^2} \frac{x^2}{\sqrt{R^2-x^2-y^2}}\, dx\,dy + \underbrace{\iint_{x^2+y^2 \leq R^2} y\, dx\,dy}_{=0}$$



$$= \int_0^{2\pi} d\theta \int_0^R \rho\, d\rho\, \frac{\rho^2 \cos^2(\theta)}{\sqrt{R^2-\rho^2}}$$

$$= \int_0^{2\pi} \cos^2(\theta)\, d\theta \int_0^R \rho^2\, \frac{\rho}{\quad}\, d\rho$$

$\pi \qquad \sqrt{R^2-\rho^2}$

FORMULA EULERO

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad \cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$

$$\int_0^{2\pi} \cos^2\theta = \int_0^{2\pi} \frac{e^{i2\theta} + 2 + e^{-2i\theta}}{4}\, d\theta \qquad \int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$= \frac{1}{4}\cancel{\frac{e^{i2\theta}}{2i}} + \frac{1\theta}{2} + \frac{1}{4}\frac{1}{\cancel{(-2i)}}\cancel{e^{2i\theta}}\Big|_0^{2\pi} = \pi \qquad \int e^{i2\theta}\,d\theta = \frac{1}{2i}e^{2i\theta}$$

$$= \pi\int_0^R \rho^2 \frac{\rho}{\sqrt{R^2-\rho^2}}\,d\rho = -\pi\, \overset{f}{\rho^2}\, \overset{g}{\cancel{\sqrt{R^2-\rho^2}}}\Big|_0^R$$

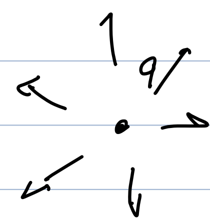
$$\quad f \qquad g' \qquad + \pi\int_0^R \underset{f'}{2\rho}\, \underset{g}{\sqrt{R^2-\rho^2}}\,d\rho$$

$$\frac{d}{d\rho}(R^2-\rho^2)^{1/2} = \frac{1}{\cancel{2}}(R^2-\rho^2)^{-1/2}\cdot -\cancel{2}\rho$$

$$= \pi\int_0^R 2\rho\sqrt{R^2-\rho^2}\,d\rho = -\frac{2}{3}\pi(R^2-\rho^2)^{3/2}\Big|_0^R = \frac{2}{8}\pi R^3$$

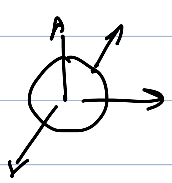
$$\frac{d}{d\rho}(R^2-\rho^2)^{3/2} = \frac{3}{2}(R^2-\rho^2)^{1/2}(-2\rho)$$

$$\boxed{\phi(\vec F, \Sigma) = \frac{4}{3}\pi R^3}$$



$$\vec E = k q \frac{\vec x}{\|\vec x\|^3}$$

$$\phi(\vec E, \Sigma) \qquad \Sigma = \{x^2+y^2+z^2 = R^2\}$$



$$n(x) = x \qquad \hat n(x) = \frac{\vec x}{\|x\|}$$

$$\phi(\vec{E},\Sigma) = \int_{\|\vec{x}\|=R} kq \frac{\vec{x}}{\|\vec{x}\|^3}\cdot\frac{\vec{x}}{\|\vec{x}\|}\, dS = \int_{\|\vec{x}\|=R} kq \frac{\|x\|^2}{\|x\|^4}$$

$$\int_{\|x\|=R} kq \frac{1}{\underset{R^2}{\underset{\|}{\|x\|^2}}} = \frac{kq}{R^2}\int_{\|x\|=R} dS = \frac{kq}{R^2}\, 4\pi R^2 = 4\pi kq$$

TEO (GAUSS) $\rho(x,y,z)$ DISTRIBUZIONE DI CARICA

FLUSSO USCENTE DA SUPERFICIE CHIUSA Σ È
$4k\pi\, q_{TOT}$ $\quad q_{TOT}$ È LA CARICA CONTENUTA IN Σ

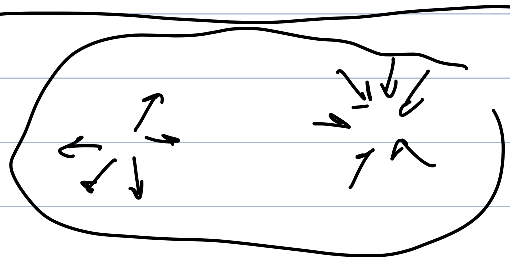
TEOREMA (DIVERGENZA) $D \subseteq \mathbb{R}^3$ limitato e normale rispetto a tutti e tre gli assi

$\vec{F} = F_1 i + F_2 j + F_3 k$ $\quad \partial D$ SUPERFICIE REGOLARE (A TRATTI)

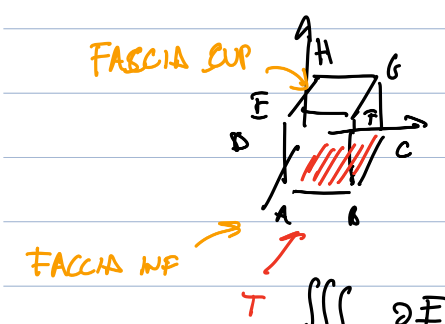
$\vec{F} \in C^1(D)$ $\quad$ ORIENTATA CON NORMALE $\hat{n}$

$\operatorname{div}\vec{F} = \frac{\partial}{\partial x_1}F_1 + \frac{\partial}{\partial y}F_2 + \frac{\partial}{\partial z}F_3$

$$\iiint_D \operatorname{div}\vec{F}\, dx\,dy\,dz = \iint_{\partial D} \vec{F}\cdot\hat{n}\, dS$$



$Q \{D\}$ $\quad [0,1]^3 = [0,1]^2 \times [0,1] = T \times [0,1]$



$$\vec{F} = 0i + 0j + Ek$$

$$\operatorname{div}\vec{F} = \frac{\partial}{\partial x}0 + \frac{\partial}{\partial y}0 + \frac{\partial E}{\partial z}$$

$$\iiint_Q \frac{\partial E}{\partial z}\, dx\,dy\,dz = \iint_T dx\,dy \int_0^1 \frac{\partial E}{\partial z}\, dz$$

$$= \iint_T \left(E(x,y,1) - E(x,y,0)\right) dx\,dy$$

$$\vec{F}\cdot\vec{n}\big|_{SUP} = E$$

$$= \iint_T E(x,y,1)\,dx\,dy - \iint_T E(x,y,0)\,dx\,dy$$

$$\vec{F}\cdot\vec{n}\big|_{INF} = -E$$

$$= \iint_T E(x,y,1)\cdot 1\,dx\,dy + \iint_T E(x,y,0)(-1)\,dx\,dy$$

FACCIA SUPERIORE $\{(x,y)\in T \,,\, z=1\}$ $\hat{n} = (0,0,1)$

FACCIA INFERIORE $\{(x,y)\in T : z=0\}$ $\hat{n} = (0,0,-1)$

$$\iint_{\partial Q} \vec{F}\cdot\hat{n}\,dS$$

$$B(0,R) = \{x^2+y^2+z^2 \le R^2\} \qquad \partial B = \Sigma$$

$$\vec{F} = x i + 0 j + y k \qquad \Sigma = \{x^2+y^2+z^2 = R^2\}$$

$$\underset{\underset{\partial B(0,R)}{\parallel}}{\iint_{\Sigma}} \vec{F}\cdot\hat{n}\,dS = \iiint_{B(0,R)} \operatorname{div} F\,dx\,dy\,dz = \iiint_{B(0,R)} 1\,dx\,dy\,dz = \frac{4\pi R^3}{3}$$

$$\operatorname{div} F = \frac{\partial}{\partial x}x + \frac{\partial}{\partial y}0 + \frac{\partial}{\partial z}y = 1$$

$$D = B(P_0, R)$$

$$\frac{1}{m(B(P_0,R))}\iiint_{B(P_0,R)} \operatorname{div}\vec{F}\,dx = \iint_{\partial B(P_0,R)} \vec{F}\cdot\hat{n}\,dS\,\frac{1}{m(B(P_0,R))}$$

$$\downarrow R\to 0$$

$$\operatorname{div} F(P_0) = \lim_{R\to 0}\frac{1}{m(B(P_0,R))}\iint_{\partial B} \vec{F}\cdot\hat{n}\,dS$$

densità di flusso del campo uscente per unità di volume

$$\iint_{\partial D} \vec{E}\cdot\hat{n}\,dS = 4\pi k\, q_{TOT}$$

q_{TOT} è la carica totale dentro a D

$$\parallel$$

$$\iiint \operatorname{div}\vec{E} = 4\pi k \iiint \rho(x,y,z)\,dx\,dy\,dz$$

$$\iiint_D \left(\operatorname{div} \vec{E} - 4k\pi \rho\right) dx\,dy\,dz = 0 \quad \Rightarrow \quad \operatorname{div} \vec{E} = 4\pi k \rho$$

$$\vec{E} = \nabla U \qquad U \text{ potenziale}$$

$$\Delta U(x) = \operatorname{div}(\nabla U)(x) = 4\pi k \rho(x) \qquad \text{EQ. POISSON}$$

$$\Delta U(x) = \sum_{i=1}^{3} \frac{\partial^2 U}{\partial x_i^2}$$

$$\vec{F} = \nabla \times V = \operatorname{rot} V \quad \Rightarrow \quad \operatorname{div} \vec{F} = 0$$

$$\nabla \times V = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ V_1 & V_2 & V_3 \end{vmatrix} = (\partial_y V_3 - \partial_z V_2)\, i - (\partial_x V_3 - \partial_z V_1)\, j + (\partial_x V_2 - \partial_y V_1)\, k$$

$$\operatorname{div}(\nabla \times V) = \partial_x (\partial_y V_3 - \partial_z V_2) - \partial_y (\partial_x V_3 - \partial_z V_1) + \partial_z (\partial_x V_2 - \partial_y V_1) = 0$$

$$V \in C^2(\Omega) \qquad \operatorname{div}(\nabla \times V) = 0$$

$\operatorname{div} \vec{F} = 0$ ALLORA ESISTE $\vec{V}$ T.C. $\vec{F} = \nabla \times \vec{V}$

$\vec{V}$ SI CHIAMA POTENZIALE VETTORE

TEO $\vec{F} \in C^1(\Omega)$ Ω CONVESSO $\operatorname{div} \vec{F} = 0$

$\exists\, \vec{V} : \vec{F} = \nabla \times \vec{V}$

DIM $\vec{V} = 0\, i + V_2\, j + V_3\, k \qquad \nabla \times \vec{V} = \vec{F}$

$$\nabla \times \vec{V} = (\partial_y V_3 - \partial_z V_2)\, i - \partial_x V_3\, j + \partial_x V_2\, k \overset{?}{=} F_1\, i + F_2\, j + F_3\, k$$

$$\partial_x V_2 = F_3 \qquad V(x,y,z) = \int^{x} F_3(s,y,z)\, ds \;\; + \varphi(y,z)$$

FISSO UGUALE A ZERO

$$\partial_x V_3 = -F_2 \qquad V_3(x,y,z) = -\int_{x_0}^{x} F_2(t,y,z)\,dt + \psi(y,z)$$

$$\partial_y V_3 - \partial_z V_2 = F_1$$

$$\frac{\partial}{\partial y}\left(-\int_{x_0}^{x} F_2(t,y,z)\,dt + \psi(y,z)\right) - \frac{\partial}{\partial z}\int_{x_0}^{x} F_3(s,y,z)\,ds = F_1(x,y,z)$$

$$-\int_{x_0}^{x} \frac{\partial}{\partial y} F_2(t,y,z)\,dt + \frac{\partial}{\partial y}\psi - \int_{x_0}^{x} \frac{\partial}{\partial z} F_3(s,y,z)\,ds = F_1(x,y,z)$$

$$\int_{x_0}^{x} -\frac{\partial}{\partial y} F_2(s,y,z) - \frac{\partial}{\partial z} F_3(s,y,z)\,ds + \partial_y \psi = F_1(x,y,z)$$

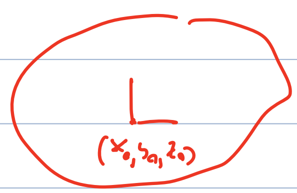
$$\partial_x F_1 + \partial_y F_2 + \partial_z F_3 = 0 \qquad \partial_x F_1 = -\partial_y F_2 - \partial_z F_3$$

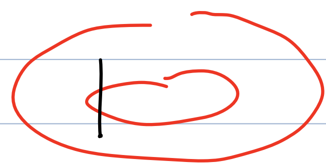
$$\int_{x_0}^{x} \frac{\partial}{\partial x} F_1(s,y,z) + \frac{\partial}{\partial y}\psi = F_1(x,y,z)$$

$$\cancel{F_1(x,y,z)} - F_1(x_0,y,z) + \frac{\partial}{\partial y}\psi = \cancel{F_1(x,y,z)}$$

$$\frac{\partial}{\partial y}\psi(y,z) = F_1(x_0,y,z) \qquad \boxed{\psi(y,z) = \int_{y_0}^{y} F_1(x_0,t,z)\,dt}$$

- $V_1 = 0 \qquad V_2 = \int_{x_0}^{x} F_3(s,y,z)\,ds \qquad V_3 = -\int_{x_0}^{x} F_2(s,y,z)\,ds + \int_{y_0}^{y} F_1(x_0,t,z)\,dt$





BEN DEFINITI SE
Ω È CONVESSO

$$\vec{F} = \left(x + y^2,\ y - z,\ -2z + \sin(y)\right) \qquad x,y,z \in \mathbb{R}^3$$

$$\operatorname{div} F = \frac{\partial}{\partial x}(x+y^2) + \frac{\partial}{\partial y}(y - z) + \frac{\partial}{\partial z}(-2z + \sin(y))$$

$$= 1 + 1 + (-2) = 0$$

$$V_2 = \int_{x_0}^{x} \left(-2z + \sin(y)\right) ds = (x - x_0)\left(-2z + \sin(y)\right)$$

$$V_3 = -\int_{x_0}^{x} \underbrace{(y - z)}_{F_2(s,y,z)} ds + \int_{y_0}^{y} \underbrace{\left(x_0 + t^2\right)}_{F_3(x_0,t,z)} dt = -(x-x_0)(y-z) + x_0 t + \frac{t^3}{3}\Big|_{y_0}^{y}$$

$$= -(x-x_0)(y-z) + x_0(y-y_0) + \frac{y^3}{3} - \frac{y_0^3}{3}$$

$J(x)$ dist. courb.

$$\vec{B}(x) = \frac{\mu_0}{4\pi} \iiint_{\mathbb{R}^3} \vec{J}(y) \wedge \frac{\vec{x}-\vec{y}}{|x-y|^3} dy$$

$$\operatorname{div}\left(\frac{x}{\|x\|^3}\right) = 0 \qquad x \neq 0 \qquad \operatorname{div}_x\left(\frac{x-y}{\|x-y\|^3}\right) = 0 \qquad x \neq y$$

$$\operatorname{div}_x \vec{B}(x) = \operatorname{div}_x \frac{\mu_0}{4\pi} \iiint_{\mathbb{R}^3} \vec{J}(y) \wedge \frac{\vec{x}-\vec{y}}{\|x-y\|^3} dy$$

$$= \frac{\mu_0}{4\pi} \iiint_{\mathbb{R}^3} \underbrace{\operatorname{div}_x \left(\vec{J}(y) \wedge \frac{\vec{x}-\vec{y}}{\|x-y\|^3}\right)}_{= 0} dy = 0$$

$y \neq x$

$$\operatorname{div} \vec{B} = 0 \qquad \exists \vec{A} : \quad \vec{B} = \nabla \times \vec{A}$$

$$\boxed{\vec{A}(x) = \frac{\mu_0}{4\pi} \iiint_{\mathbb{R}^3} \frac{\vec{J}(y)}{|x-y|} dy}$$