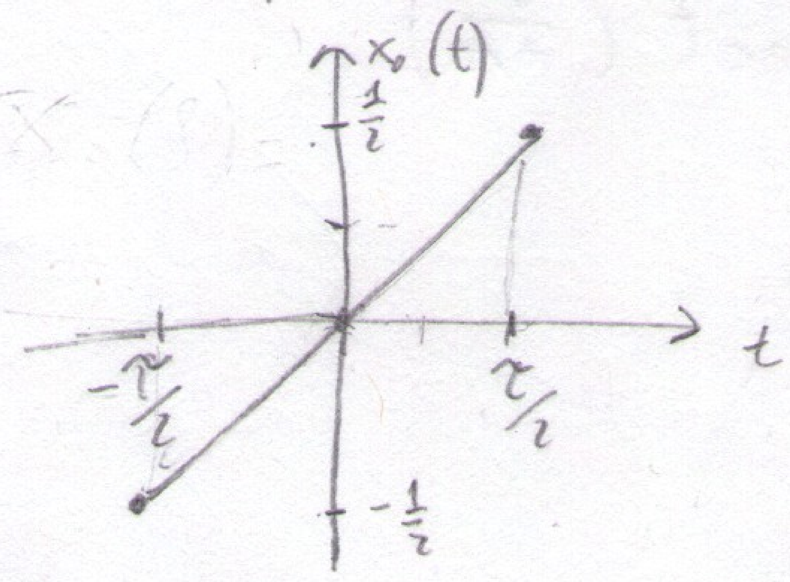
Esame 18/09/08 ⑨

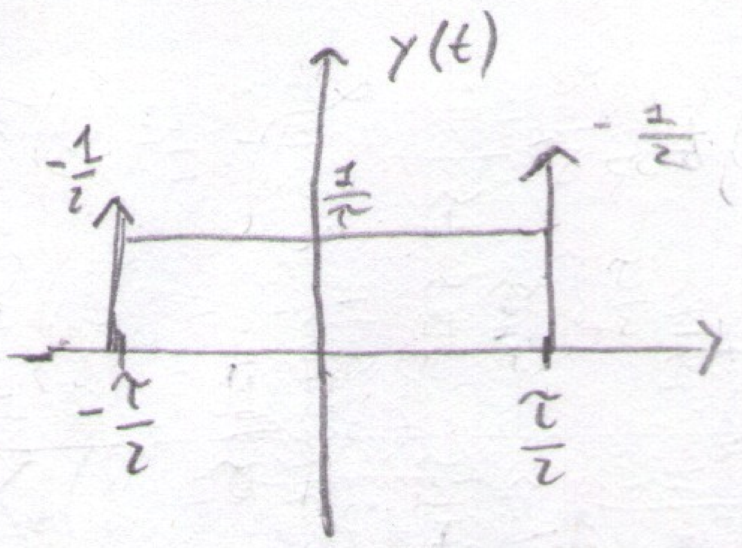
Esercizio 1

$x(t)=\sum_n x_0(t-nT)$; $x_0(t)=\frac{t}{\tau}\,\mathrm{rect}\left(\frac{t}{\tau}\right)$; $\tau << T$



$$X(f)=\frac{1}{T}\sum_n X_0\left(\frac{n}{T}\right)\delta\left(f-\frac{n}{T}\right)$$

$$X_0(f)\rightleftharpoons x_0(t)$$



$$y(t)=\frac{d}{dt}x_0(t)$$

$$y(t)=-\frac{1}{2}\delta\left(t+\frac{\tau}{2}\right)+\frac{1}{\tau}\,\mathrm{rect}\left(\frac{t}{\tau}\right)-\frac{1}{2}\delta\left(t-\frac{\tau}{2}\right)$$

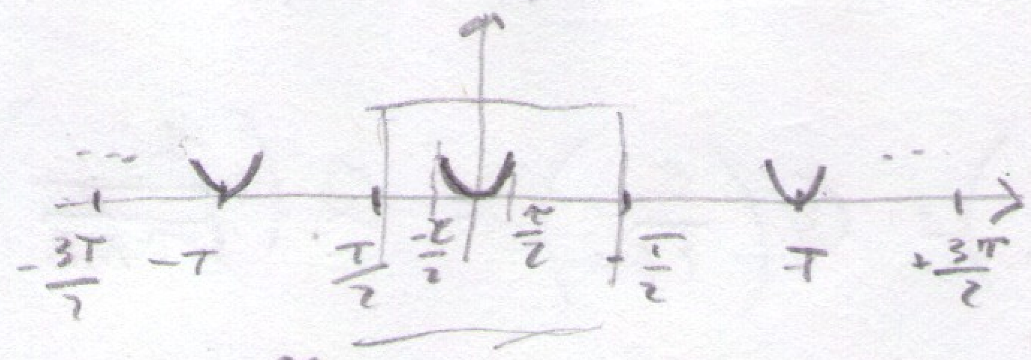
$$Y(f)=-\frac{1}{2}e^{j2\pi f\frac{\tau}{2}}+\mathrm{sinc}(f\tau)-\frac{1}{2}e^{-j2\pi f\frac{\tau}{2}}$$

$$Y(f)=\mathrm{sinc}(f\tau)-\cos\left(2\pi f\frac{\tau}{2}\right)$$

$$X_0(f)=\frac{Y(f)}{j2\pi f}+\frac{Y(0)}{2}\delta(f)=\frac{\mathrm{sinc}(f\tau)-\cos\left(2\pi f\frac{\tau}{2}\right)}{j2\pi f}$$

$$X(f)=\frac{1}{T}\sum_n\frac{\mathrm{sinc}\left(\frac{n}{T}\tau\right)-\cos\left(2\pi\frac{n}{T}\frac{\tau}{2}\right)}{j2\pi\frac{n}{T}}\delta\left(f-\frac{n}{T}\right)=\sum_n\frac{\mathrm{sinc}\left(\frac{n}{T}\tau\right)-\cos\left(\pi\tau\frac{n}{T}\right)}{j2\pi n}\delta\left(f-\frac{n}{T}\right)$$ ①

$$P_x=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|x(t)|^2\,dt$$



$$=\frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}|x_0(t)|^2\,dt=$$

$$=\frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\left|\frac{t}{\tau}\,\mathrm{rect}\left(\frac{t}{\tau}\right)\right|^2dt=\frac{\tau^2}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}t^2\,dt=\frac{\tau^2}{T}\left|\frac{t^3}{3}\right|_{-\frac{\tau}{2}}^{\frac{\tau}{2}}=\frac{\tau^2}{T}\left(\frac{\frac{\tau^3}{8}}{3}+\frac{\frac{\tau^3}{8}}{3}\right)=$$

$$=\frac{\tau^2}{T}\,2\cdot\frac{\tau^3}{8_4}\cdot\frac{1}{3}=\frac{\tau^5}{T\,12}$$

Esercizio 2

$$E_T=P_{S_T}T=\int_{-\infty}^{\infty}S_{ST}(f)\,df\cdot T\ ;\ S_{ST}=\frac{E\{a_i^2\}}{T}|G_T(f)|^2$$

$$E\{a_i^2\}=\sum_{k=\emptyset}^{1}m_k^2p_k=(-1)^2\cdot\frac{1}{2}+1^2\frac{1}{2}=1\quad\Rightarrow S_{ST}=\frac{|G_T(f)|^2}{T}=\frac{\sqrt{T}^2\,\mathrm{rect}(fT)}{T}=\mathrm{rect}\left(\frac{f}{1/T}\right)$$

$$E_T=\int_{-\infty}^{\infty}\mathrm{rect}\left(\frac{f}{1/T}\right)df\cdot T=T\cdot\frac{1}{T}=1$$ ①

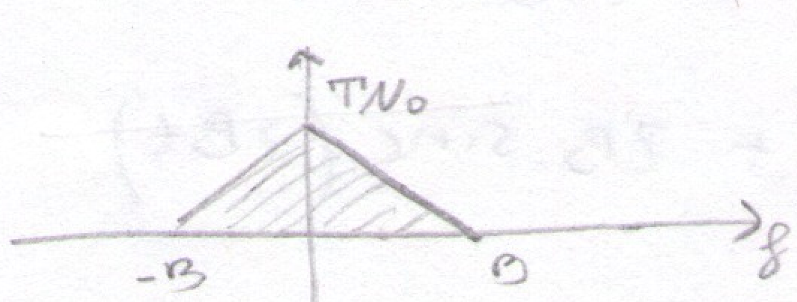
$n(t)\triangleq\{w(t)\cdot 2\cos(2\pi f_0t)\}\otimes g_R(t)$, $\tilde{n}_c(t)\triangleq\tilde{w}_c(t)\otimes g_R(t)$

$$P_{\tilde{n}_c}=\int_{-\infty}^{\infty}S_{\tilde{n}_c}(f)\,df\ ;\ S_{\tilde{n}_c}(f)=S_{\tilde{w}_c}(f)|G_R(f)|^2$$

$$S_{\tilde{w}}(f)=\begin{cases}4S_w(f+f_0) & f\geq -f_0\\ \emptyset & \text{altrove}\end{cases}=2N_0\,\mathrm{tri}\left(\frac{f}{2B}\right)\ ;\ S_{\tilde{w}_c}=\frac{S_{\tilde{w}}(f)+S_{\tilde{w}}(-f)}{4}=N_0\,\mathrm{tri}\left(\frac{f}{2B}\right)$$

$$S_{\tilde{n}_c}(f)=N_0\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)\cdot T\,\mathrm{rect}\left(\frac{f}{1/T}\right)=TN_0\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$P_{\tilde{n}_c}=\int_{-\infty}^{\infty}TN_0\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)df=$$



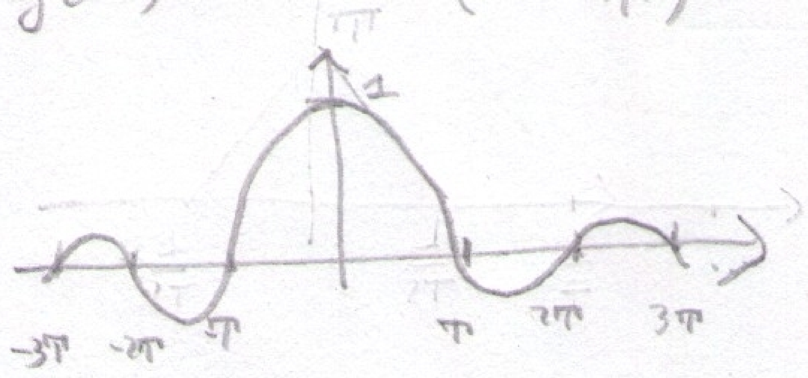
$$=TN_0B$$

$$BER = \frac{P(e)}{1} = P(e)$$

$$z(t) = \tilde{y}(t) \otimes g_R(t) = \Re\{\tilde{r}(t)\} \otimes g_R(t)$$

$$g(t) \triangleq g_{Tc}(t) \otimes g_R(t) \Rightarrow G(f) = G_{Tc}(f)\, G_R(f) = G_T^2(f) \Big|= T\, \mathrm{rect}\left(\frac{f}{1/T}\right)$$

$$g(t) = \mathrm{sinc}\left(t\, \frac{1}{T}\right)$$

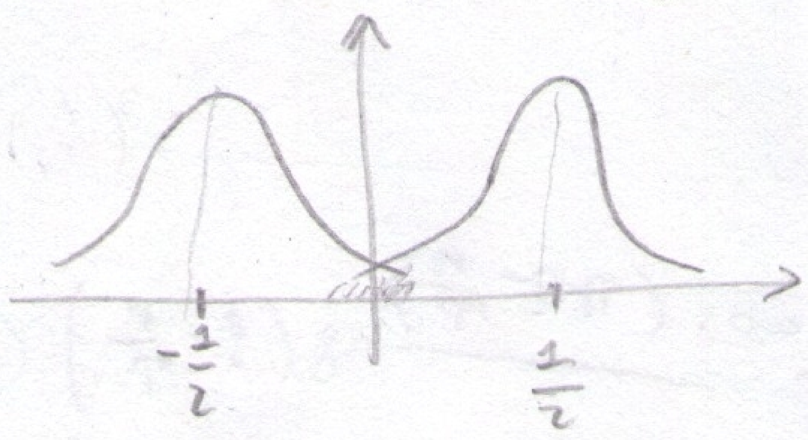


$$g(mT) = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases} \quad \text{Nyquist!!!}$$

$$r(t) \triangleq \Re\{\tilde{r}(t)\, e^{j2\pi f_0 t}\} = \Re\{e^{j\theta}\, a_i\, e^{j2\pi f_0 t}\} \Rightarrow \tilde{r}(t) = a_i e^{j\theta}$$

$$y(t) = a_i \cos(\theta) \; ; \quad z(t) = \sum_i a_i \cos(\theta)\, g(t - iT) + n(t)$$

$$z_k = a_k \cos(\theta)\, g(0) + n_k = a_k \cos(\theta) + n_k = a_k \cos\left(\frac{\pi}{3}\right) + n_k = \frac{1}{2} a_k + n_k$$



$$z_k\Big|_{k=0} = -\frac{1}{2} + n_k \quad ; \quad z_k\Big|_{k=1} = \frac{1}{2} + n_k \qquad \in \mathcal{N}\left(\mp\frac{1}{2}, \sigma_n\right)$$

$$P_r[\hat{1} \mid -1] = Q\left(\frac{0 + \frac{1}{2}}{\sigma_n}\right) = Q\left(\frac{1}{2\sigma_n}\right)$$

$$P_r[-\hat{1} \mid 1] = 1 - Q\left(-\frac{1}{2\sigma_n}\right) = Q\left(\frac{1}{2\sigma_n}\right)$$

$$P_r(e) = \frac{1}{2} Q\left(\frac{1}{2\sigma_n}\right) + \frac{1}{2} Q\left(\frac{1}{2\sigma_n}\right) = Q\left(\frac{1}{2\sigma_n}\right) = Q\left(\frac{1}{\sqrt{T N_0 B}}\right) = Q\left(\sqrt{\frac{1}{T N_0 B}}\right)$$

$$BER = Q\left(\sqrt{\frac{1}{T N_0 B}}\right)$$