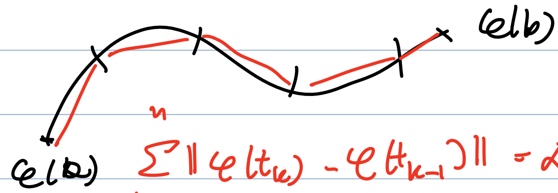
$\varphi : [a,b] \to \mathbb{R}^n$ $\quad \varphi \in C^1$ $\quad$ iniettiva $\quad [a,b[$

$$\mathcal{L}(\varphi) \stackrel{def}{=} \int_a^b \|\varphi'(t)\|\, dt \qquad \varphi \in C^1[a,b]$$

$$\mathcal{L}(\varphi) = \sup_{\mathcal{P}} \mathcal{L}(\mathcal{P})$$



$$\sum_{k=1}^{n} \|\varphi(t_k) - \varphi(t_{k-1})\| = \mathcal{L}(\mathcal{P})$$

$\vec{f} : [a,b] \to \mathbb{R}^n$ $\qquad f : [a,b] \to \mathbb{R}$

$$\left\|\int_a^b \vec{f}\, dt\right\| \le \int_a^b \|\vec{f}\|\, dt \qquad \left|\int_a^b f(t)\,dt\right| \le \int_a^b |f(t)|\,dt$$

$$\vec{f}(t) = \begin{pmatrix} f_1(t) \\ \vdots \\ f_n(t) \end{pmatrix} \qquad \int_a^b \vec{f} = \begin{matrix} \int_a^b f_1 \\ \vdots \\ \int_a^b f_n \end{matrix}$$

$u \in \mathbb{R}^n$

$$\left\langle \int_a^b \vec{f}, \vec{u} \right\rangle = \sum_{i=1}^{n} \left(\int_a^b f_i(t)\,dt\right) u_i = \sum_{i=1}^{n} \int_a^b f_i(t)\, u_i \, dt = \int_a^b \langle \vec{f}(t), u \rangle$$

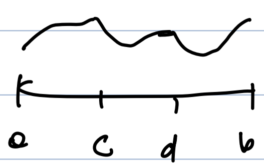
$$\to \left|\left\langle \int_a^b \vec{f}, \vec{u} \right\rangle\right| = \left|\int_a^b \langle \vec{f}, \vec{u} \rangle\right| \le \int_a^b |\langle \vec{f}, \vec{u}\rangle| \le \int_a^b \|\vec{f}\|\,\|\vec{u}\|\, dt$$

$$= \|\vec{u}\| \int_a^b \|\vec{f}\|\, dt$$

$$\vec{u} = \int_a^b \vec{f}\, dt$$

$$\left|\left\langle \int_a^b \vec{f}, \int_a^b \vec{f} \right\rangle\right| \le \left\|\int_a^b \vec{f}\right\| \int_a^b \|\vec{f}\|$$

$$\left\|\int_a^b \vec{f}\right\|^{\cancel{2}} \le \cancel{\left\|\int_a^b \vec{f}\right\|} \int_a^b \|\vec{f}\|$$



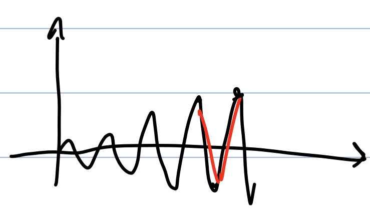
$\varphi_{|[a,c]} \in C^1 \qquad \varphi \in C[a,b]$

$\varphi_{|[c,d]} \in C^1$

$\varphi_{|[d,b]} \in C^1$

$$\int_a^c \|\varphi'\| + \int_c^d \|\varphi'\| + \int_d^b \|\varphi'\|$$

$$f(t) = \begin{cases} t \sin \frac{1}{t} & t \ne 0 \\ 0 & t = 0 \end{cases}$$

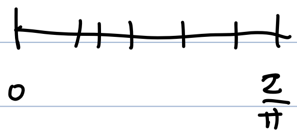


$$\varphi(t) = (t, f(t)) \qquad t \in \left[0, \frac{2}{\pi}\right]$$

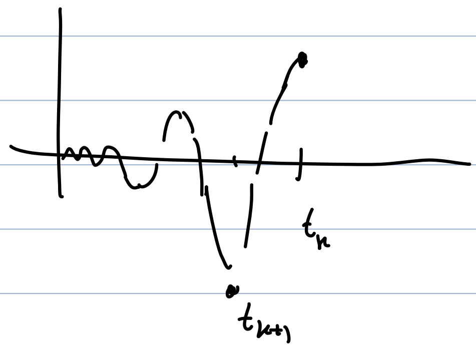
$\varphi \in C^1 \left]0, \frac{2}{\pi}\right]$ $\qquad \varphi'(0)$ N.E. $\qquad \mathcal{L}(\varphi) = \int_0^{2/\pi} \|\varphi'(t)\|$

$\sup_{P} \mathcal{L}(P) = +\infty$

$\sin\left(\frac{1}{t}\right) = \pm 1$ $\quad \frac{1}{t} = \frac{\pi}{2} + k\pi$ $\quad t_k = \frac{1}{\frac{\pi}{2}+k\pi}$ $\quad k=0,\dots,n$



$\sum_{k=1}^{n} \|\varphi(t_k)-\varphi(t_{k-1})\|$



$\|\varphi(t_k)-\varphi(t_{k-1})\|^2 = (t_k-t_{k-1})^2 + |f(t_k)-f(t_{k-1})|^2$

$\geq |f(t_k)-f(t_{k-1})|^2$

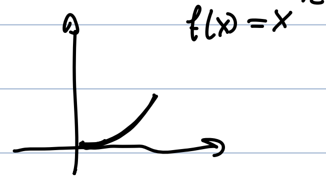
$\|\varphi(t_k)-\varphi(t_{k-1})\| \geq |f(t_k)-f(t_{k-1})|$

$\geq \left|t_k \sin\frac{1}{t_k} - t_{k-1}\sin\left(\frac{1}{t_{k-1}}\right)\right|$

$t_k + t_{k-1} \geq t_k$

$\sum_1^n \|\varphi(t_k)-\varphi(t_{k-1})\| \geq \sum_{k=1}^n \frac{1}{\frac{\pi}{2}+k\pi} \sim \sum_1^n \frac{1}{k} = \ln(n) \to +\infty$

$\varphi(t) = (t, t^{3/2})$ $\quad t\in[0,1]$

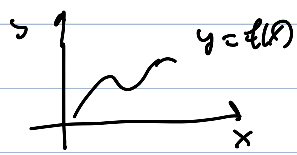


$\varphi'(t) = 1, \frac{3}{2} t^{1/2}$

$\int_0^1 \sqrt{1^2 + \left(\frac{3}{2}\right)^2 t^{1/2 \cdot 2}}\,dt$

$\int_0^1 \sqrt{1+\frac{9}{4} t}\,dt = \int_0^1 \left(1+\frac{9}{4} t\right)^{1/2} dt = \frac{1}{1+\frac{1}{2}}\left(1+\frac{9}{4} t\right)^{1+\frac{1}{2}}\frac{4}{9} \Big|_0^1 = \frac{1}{\frac{3}{2}}\left(1+\frac{9}{4}\right)\cdot\frac{4}{9}$

$\varphi(t) = (t, f(t))$

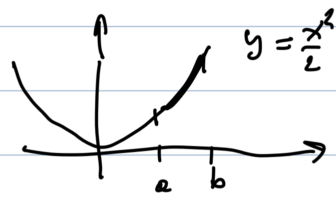


$\varphi(t) = (\cos(t), \sin(t))$ $\quad \|\varphi'(t)\| = 1$

$\varphi(t) = (a\cos(t), b\sin(t))$ $\quad \varphi'(t) = (-a\sin(t), b\cos(t))$

$\|\varphi'(t)\| = \sqrt{a^2\sin^2(t)+b^2\cos^2(t)}$

$\int_0^{2\pi}\sqrt{a^2\sin^2(t)+b^2\cos^2(t)}\,dt$



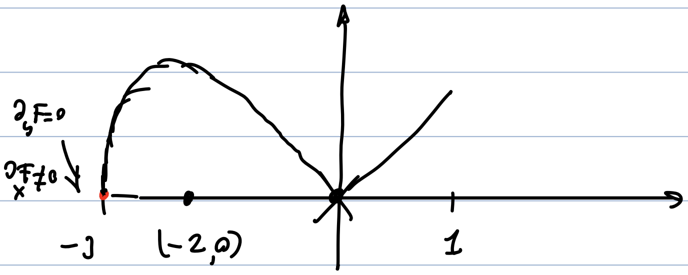
$\varphi(t) = \left(t, \frac{t^2}{2}\right)$

$$\int_a^b \sqrt{1+t^2}\,dt \qquad t = \sinh x$$

$$\int_{\text{settsinh}(a)}^{\text{settsinh}(b)} \cosh^2 t\,dt$$

$$\text{settsinh}(x) = \ln\left(x + \sqrt{1+x^2}\right)$$

$$0 = F(x,y) = \underbrace{(y^2 - 3x^2 - x^3)}_{f}\underbrace{((x+2)^2 + y^2)}_{g} \qquad -3 \le x \le 1$$



$f = 0$

$y^2 = 3x^2 + x^3$

$y = \pm\sqrt{3x^2+x^3} = \pm|x|\sqrt{x+3}$

$\nabla f = (-6x - 3x^2, 2y) = (0,0)$ PT CRITICI

$(3x(-2x-1), 2y)$

$\nabla f(0,0) = (0,0)$

$$Hf = \begin{pmatrix} -6-6x & 0 \\ 0 & 2 \end{pmatrix}$$

$$Hf(0,0) = \begin{pmatrix} -6 & 0 \\ 0 & 2 \end{pmatrix}$$

$-3 \le x \le 0$ $\quad g(x) = |x|\sqrt{x+3} = -x\sqrt{x+3}$

$$g'(x) = -\left[\sqrt{x+3} + \frac{x}{2\sqrt{x+3}}\right] = -\frac{3(x+2)}{2\sqrt{x+3}}$$

$$g'' = -\frac{3}{4}\frac{4+x}{\sqrt{(x+3)^2}} < 0$$

$0 < x < 1$

$g(x) = |x|\sqrt{x+3} = x\sqrt{x+3}$ $\qquad g' > 0 \quad x > 0$

$$g'' = \frac{3}{4}\frac{4+x}{\sqrt{(x+3)^2}} > 0$$