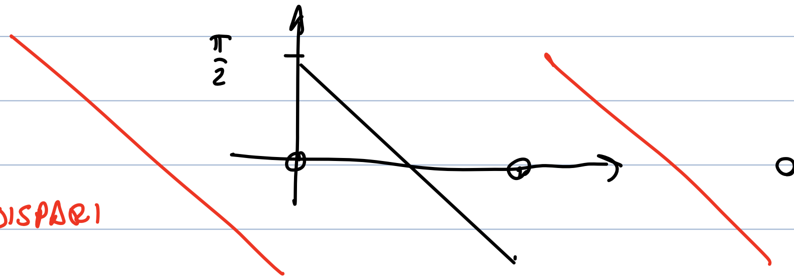
$f(x) = \frac{\pi - x}{2}$ $\quad [0, 2\pi[$



f PROLUNGATA 2π - PERIODICA DISPARI

C^1 A TRATTI

$f(x) = \sum_{k=1}^{\infty} b_k \sin(kx) \qquad x \neq 2k\pi \qquad k \in \mathbb{Z}$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin(kx)\, dx = \frac{2}{\pi}\int_0^{\pi} f(x) \sin(kx)\, dx$$

DISPARI DISPARI

PARI

$$\frac{2}{\pi}\int_0^{\pi} \frac{\pi - x}{2} \sin(kx)\,dx = \frac{2}{\pi}\left[-\frac{\pi - x}{2}\,\frac{\cos(kx)}{k}\Big|_0^{\pi} - \int_0^{\pi} +\frac{1}{2}\left(+\frac{\cos(kx)}{k}\right)dx\right]$$

$f \quad g' \qquad\qquad f \quad g$

$$\frac{2}{\pi}\left[-\frac{\pi-\pi}{2}\cancel{\cos(k\pi)} + \frac{\pi - 0}{2}\,\frac{\cos(0)}{k} - \frac{1}{2k}\int_0^{\pi}\cancel{\cos(kx)\,dx}\right]$$

$$-\frac{1}{2k^2}\sin(kx)\Big|_0^{\pi}$$

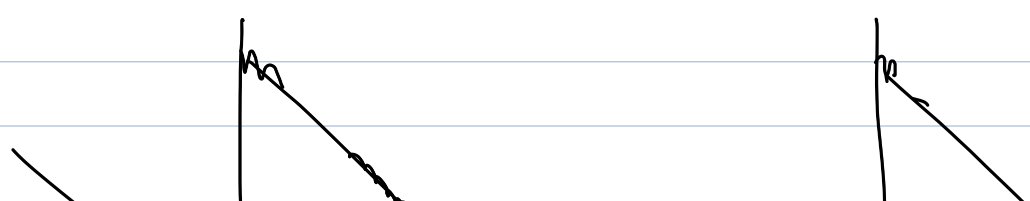
$$= \frac{\cancel{2}}{\cancel{\pi}}\,\frac{\cancel{\pi}}{\cancel{2}}\,\frac{1}{k}$$

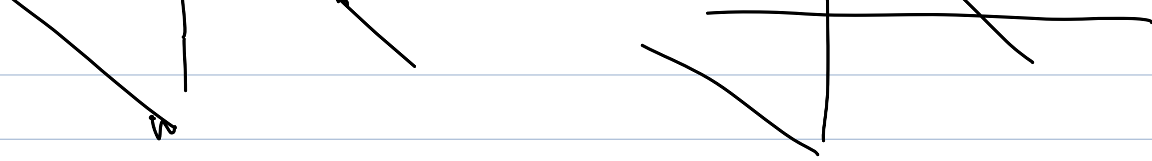
$$f(x) = \underline{f(x)} = \sum_{k=1}^{\infty} \frac{1}{k}\sin(kx) = \qquad x \neq 2k\pi \qquad k \in \mathbb{Z}$$

$$\left|\sum_{k=1}^{\infty} \frac{1}{k}\sin(kx)\right| \le \sum_{1}^{\infty} \frac{1}{k}|\sin(kx)| \le \sum_{1}^{\infty} \frac{1}{k} = +\infty$$

$\sin(kx)$ non ha segno costante

$\sum_{1}^{\infty} \frac{1}{k}\sin(kx)$ FENOMENO DI GIBBS

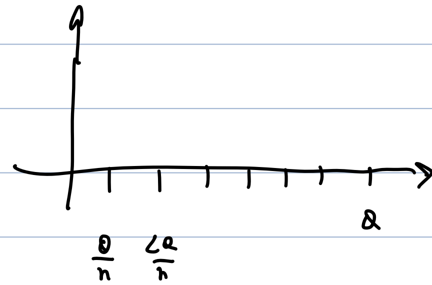




$$g(x) = \begin{cases} \dfrac{\sin(x)}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$\int_0^{+\infty} \frac{\sin(x)}{x} = \frac{\pi}{2}$$

$$\int_0^{a} \frac{\sin(x)}{x}\,dx \approx \sum_{k=1}^{n} \frac{\sin\left(\frac{ka}{n}\right)}{\frac{k\cancel{a}}{\cancel{n}}} \cancel{\frac{a}{n}}$$

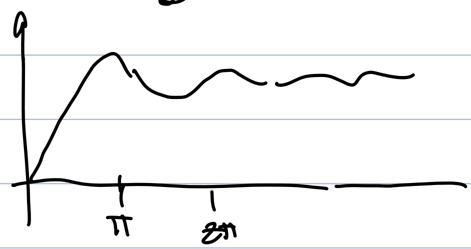


$$\sum_{k=1}^{n} \frac{\sin \frac{ka}{n}}{k}$$

$$\phi(a) = \int_0^{a} \frac{\sin(x)}{x}\,dx \qquad \phi(0) = 0 \qquad \phi(\infty) = \frac{\pi}{2}$$

$$\phi'(a) = \frac{\sin(a)}{a} \qquad \phi'(a) = 0 \qquad a = k\pi \qquad k \geq 1 \qquad k \in \mathbb{N}$$

$\phi(a)$



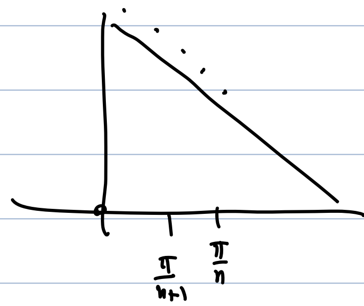
$$\int_0^{\pi} \frac{\sin(x)}{x}\,dx \geq \int_0^{a} \frac{\sin(x)}{x}\,dx$$

$$\approx 1{,}85194$$

$$\frac{\pi}{2} \approx 1{,}570$$

$$S_n(f)(x) = \sum_{k=1}^{n} \frac{\sin(kx)}{k} \approx \int_0^{x} \frac{\sin(t)}{t}\,dt$$

$$S_n f\left(\frac{\pi}{n}\right) = \sum \frac{\sin(k\pi/n)}{k} \sim 1{,}85$$

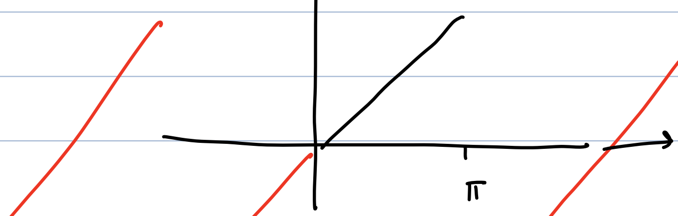


$$S_n f(0) = \frac{\pi}{2} = 1{,}57$$

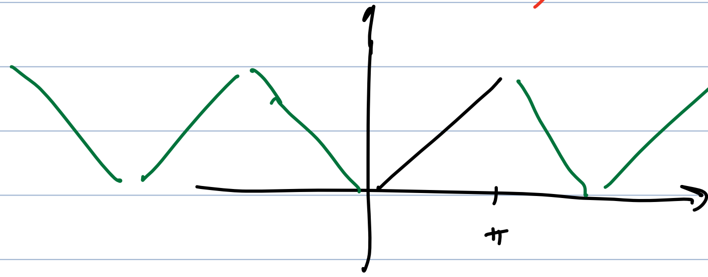
$$\lim_{x \to 0^+} \frac{S_n f(x)}{S_n(f(0))} \approx \frac{1{,}87}{1{,}57} \qquad 17\ \%$$

$$f(x) = x \qquad x \in [0, \pi]$$

$$\bar{f}(x) = x \qquad x \in]-\pi, \pi]$$



$\hat{f}(x) = |x|$ $\quad x \in]-\pi, \pi]$



$f(x) = x = \sum_{k=1}^{\infty} b_k \sin(kx)$

f È DISPARI SI SVILUPPA SOLO CON $\sin(kx)$ PERCHÉ $\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx = 0$

Disp Pari

f È PARI " " " " $\cos(kx)$ $\quad \int_{-\pi}^{\pi} f(x)\sin(kx)\,dx = 0$

Pari Disp

$\bar{f} = \sum_{k=1}^{\infty} b_k \sin(kx)$ $\quad b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} \bar{f}(x)\sin(kx)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin(kx)\,dx$

Disp Disp

$$= \frac{2}{\pi}\int_0^{\pi} x \sin(kx)\,dx$$

f g'

$$\boxed{\int_{-a}^{a} \varphi(x)\,dx = 2\int_0^{a} \varphi(x) \quad \text{SE } \varphi \text{ È PARI}}$$

$$= \frac{2}{\pi}\left[-x \frac{\cos(kx)}{k}\Big|_0^{\pi} + \int_0^{\pi} 1 \frac{\cos(kx)}{k} \right]$$

$$= \frac{2}{\pi}\left[-\pi \frac{\cos(k\pi)}{k} + 0 + \cancel{\frac{\sin(kx)}{k^2}}\Big|_0^{\pi} \right]$$

$$= -\frac{2}{k}\cos(k\pi) = -\frac{2}{k}(-1)^{k} = \frac{2}{k}(-1)^{k+1}$$

$$x = \sum_{k=1}^{\infty} \frac{2}{k}(-1)^{k+1}\sin(kx) \qquad x \neq k\pi \qquad k \neq 0$$

$$\hat{f}(x) = |x| = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(kx)$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} |x| \cos(kx)\,dx = \frac{2}{\pi}\int_0^{\pi} x \cos(kx)\,dx$$

P P

$k=0$ $\quad \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\frac{x^2}{2}\Big|_0^{\pi} = \frac{2}{\pi}\frac{\pi^2}{2} = \pi = a_0$

$k \neq 0 \quad \frac{2}{\pi} \int_0^\pi \underset{f}{x} \underset{g'}{\cos(kx)}\,dx = \frac{2}{\pi}\left[x \cancel{\frac{\sin(kx)}{k}}\Big|_0^\pi - \int_0^\pi 1 \, \frac{\sin(kx)}{k}\,dx \right]$

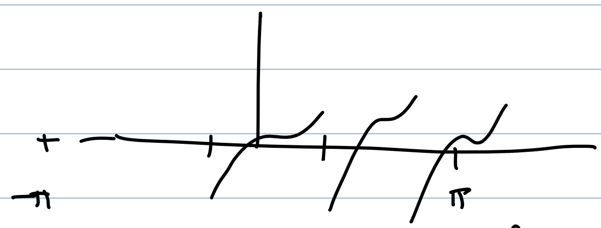
$= -\frac{2}{\pi}\,\frac{1}{k}\left(-\frac{\cos(kx)}{k}\right)\Big|_0^\pi = \frac{2}{\pi k^2}\left(\cos(k\pi) - \cos(0)\right)$

$= \frac{2}{\pi k^2}\left((-1)^k - 1\right) = \begin{cases} 0 & k \text{ PARI} \\ \frac{2(-2)}{\pi k^2} & k \text{ DISPARI} \end{cases}$

$$\boxed{|x| = \frac{\pi}{2} + \sum_{k=1}^{\infty} \frac{2}{\pi k^2}\left((-1)^k - 1\right)\cos(kx) \qquad x \in \mathbb{R}}$$

$|b_k| \leq \frac{C}{k^2}$ $\qquad \left| \sum \frac{2}{\pi k^2}\left((-1)^k - 1\right)\cos(kx) \right| \leq \sum \frac{4}{\pi k^2} \cdot 1$

$f :]-\ell, \ell] \to \mathbb{R} \qquad \ell > 0 \qquad$ qualsiasi e prolungata 2ℓ - PERIODICA



$$f(x) = \frac{1}{2\ell}\int_{-\ell}^{\ell} f(t)\,dt + \sum_{k=1}^{\infty} \frac{1}{\ell}\int_{-\ell}^{\ell} f(t)\cos\left(\frac{k\pi}{\ell}t\right)\cos\left(\frac{k\pi}{\ell}x\right) +$$

$$+ \sum \frac{1}{\ell}\int_{-\ell}^{\ell} f(t)\sin\left(\frac{k\pi}{\ell}t\right)\sin\left(\frac{k\pi}{\ell}x\right)$$

2π-PER $\cos(kx) \longmapsto \cos\left(\frac{k\pi}{\ell}x\right) \qquad 2\ell$ - PERIODICO

$\sin(kx) \longmapsto \sin\left(\frac{k\pi}{\ell}x\right)$

$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(kt)\,dt \longrightarrow a_k = \frac{1}{\ell}\int_{-\ell}^{\ell} f(t)\cos\left(\frac{k\pi}{\ell}t\right)$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(kt)\,dt \longrightarrow b_k = \frac{1}{\ell}\int_{-\ell}^{\ell} f(t)\sin\left(\frac{k\pi}{\ell}t\right)$$

$\ell \to +\infty$ LIMITE FORMALE

A ESSERE PRECISI

SUPPONIAMO $\int_{\mathbb{R}} f(t)\,dt = \int_{-\infty}^{+\infty} f(t)\,dt$ ESISTE $\int_{\mathbb{R}} |f(t)|\,dt < +\infty$

FINITO

$$\left|\frac{1}{\ell}\int_{-\ell}^{\ell} f(t)\,dt\right| \leq \frac{1}{\ell}\int_{-\ell}^{\ell} |f(t)|\,dt = 0$$

$+\infty$

$$\frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\pi}{\ell}\int_{-\ell}^{\ell} f(t)\cos\left(\frac{k\pi}{\ell}(t-x)\right)dt$$

$$F(\xi) = \frac{1}{\pi}\int_{-\ell}^{\ell} f(t)\cos[\xi(t-x)]\,dt \qquad \xi \in \mathbb{R}$$

$$\xi_k = \frac{k\pi}{\ell} \qquad \Delta\xi = \frac{\pi}{\ell}$$

$$\sum F\left(\frac{k\pi}{\ell}\right)\frac{\pi}{\ell} \rightsquigarrow \int_0^{\infty} F(\xi)\,d\xi \qquad \ell \to +\infty$$

$$f(x) = \frac{1}{\pi}\int_0^{+\infty} d\xi \int_{-\infty}^{+\infty} f(t)\cos(\xi(t-x))\,dt$$

$$+\frac{1}{\pi}\int_0^{+\infty} d\xi\int_{-\infty}^{+\infty} f(t)\sin(\xi(t-x))\,dt$$

$$a_\xi = \frac{1}{\pi}\int_{\mathbb{R}} f(t)\cos(\xi t)\,dt$$

$$f(x) = \int_0^{+\infty} a_\xi\cos(\xi x) + b_\xi\sin(\xi x)\,d\xi \qquad b_\xi = \frac{1}{\pi}\int_{\mathbb{R}} f(t)\sin(\xi t)\,dt$$

$$\frac{a_0}{2} + \sum_{1}^{\infty} a_k\cos(kx) + b_k\sin(kx)$$

FORMULA DI INVERSIONE DI FOURIER

$f(x) : \mathbb{R} \to \mathbb{R}$ $\qquad \hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-ix\xi}\,dx$

TRASFORMATA DI FOURIER

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\xi) e^{ix\xi} d\xi$$

$$\frac{d}{d\xi} \hat{f}(\xi) = \frac{d}{d\xi} \int_{-\infty}^{\infty} f(x) e^{-ix\xi} dx = \int_{-\infty}^{+\infty} \frac{d}{d\xi}\left(f(x) e^{-ix\xi}\right) dx$$

$$= \int_{-\infty}^{+\infty} f(x) \quad e^{-ix\xi}(-ix)\, dx = -i \int_{-\infty}^{+\infty} x\, f(x)\, e^{-ix\xi} dx$$

$$\widehat{f'(x)} = \int_{-\infty}^{+\infty} f'(x) e^{-ix\xi} dx = \cancel{f(x) e^{-ix\xi} \Big|_{-\infty}^{+\infty}} - \int_{-\infty}^{+\infty} f(x) \; e^{-ix\xi} (-i\xi)$$

$f' \quad g \qquad\qquad f \quad g'$

$$= i\xi \int_{-\infty}^{+\infty} f(x) e^{-ix\xi} = i\xi \hat{f}(\xi)$$

$$\widehat{f''(x)} = -|\xi|^2 \hat{f}(\xi)$$

Neumann $\quad \vec{B}$ campo magnetico variabile nel tempo

induce $\vec{E}$

ϕ_B FLUSSO USCENTE DA

Σ SUPERFICIE ORIENTATA

$C = \partial^+ \Sigma$

$$-\frac{d}{dt} \phi_B = \oint_C \vec{E} \cdot \tau\, ds$$

$$\oint_C \vec{E} \cdot \tau\, ds = \iint_\Sigma (\nabla \times \vec{E}) \cdot \hat{n}\, dS$$

$$-\frac{d}{dt} \iint_\Sigma \vec{B} \cdot \hat{n}\, dS = -\iint_\Sigma \frac{\partial \vec{B}}{\partial t} \cdot \hat{n}\, dS = \iint_\Sigma (\nabla \times \vec{E}) \cdot \hat{n}\, dS$$

$$\frac{\partial \vec{B}}{\partial t} = -\operatorname{rot} \vec{E}$$

$P_0 \in \mathbb{R}^3$ $D(P_0, r)$ DISCO 2-DIMENSIONALE

$$\frac{1}{\pi r^2} \iint_{D(P_0,r)} (\nabla \times \vec{F}) \hat{n} \, dS = \frac{1}{\pi r^2} \oint_{\partial^+ D} \vec{F} \cdot \hat{\tau} \, ds$$

$r \to 0^+$

$$\sigma = x_i i + y_j j + 1k \quad \leftarrow f$$

$$\sqrt{1 + f_x^2 + f_y^2} = 1$$

$$\frac{1}{\pi r^2} \iint_D (\nabla \times \vec{F})_3 \, dx dy$$

$$\downarrow$$

$$(\nabla \times \vec{F}(P_0))_3 = \lim_{r \to 0^+} \frac{1}{\pi r^2} \oint_{\partial^+ D} \vec{F} \cdot \hat{\tau} \, ds$$

COMPONENTE DI rot F RISPETTO A $\hat{n}$ NEL PUNTO P_0 È UGUALE ALLA DENSITÀ DI CIRCUITAZIONE ATTORNO A P_0 NEL PIANO PERPENDICOLARE A $\hat{n}$