Part 1 Recap

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```
#| default_exp part_1_recap
```

Please fill up the questionnaire at: https://esami.unipi.it

Jupyter and Python

Installing Anaconda and dependencies to run the notebooks

- Install Anaconda: Anaconda Download Page
- Refer to the Anaconda documentation for more information here
- Open a terminal window:
  - Windows: From the Start menu, search for and open "Anaconda Prompt"
  - MacOS/Linux: Click the terminal icon.
- Once in a terminal,
  - create an environment: `conda create --name env-automatic-control python=3.9`
  - activate it: `conda activate env-automatic-control`
- Install everything that is needed
  - Jupyter notebook:

    `conda install -c conda-forge notebook`
  - Standard engineering libraries:

    `python -m pip install matplotlib numpy pandas`

    `conda install numpy scipy matplotlib` # if not yet installed

    `conda install -c conda-forge control`

    `conda install -c anaconda sympy`

You are set up!

Going back to work (opening your Jupyter notebooks)

- Open a terminal window:
    - Windows: From the Start menu, search for and open “Anaconda Prompt”
    - MacOS/Linux: Click the terminal icon.
- Move to yur project folder (e.g. `cd /Users/<USERNAME>/MyProject/`)
  (if you need to find out how to find your folder path in Windows read below: "Note: To find out where is your folder in Windows")
- Activate your conda environment: `conda activate env-automatic-control`
- Run: `jupyter notebook`

A new tab in your browser should open up automatically showing the jupyter home folder (your current folder). If it does not open, open your favourite brower and type: `localhost:8888` in the browser address bar. Sometime jupyter requires a token - this is automatically created when jupyter is started. Look back to your terminal window right after the command `jupyter notebook` there should be a line similar to:

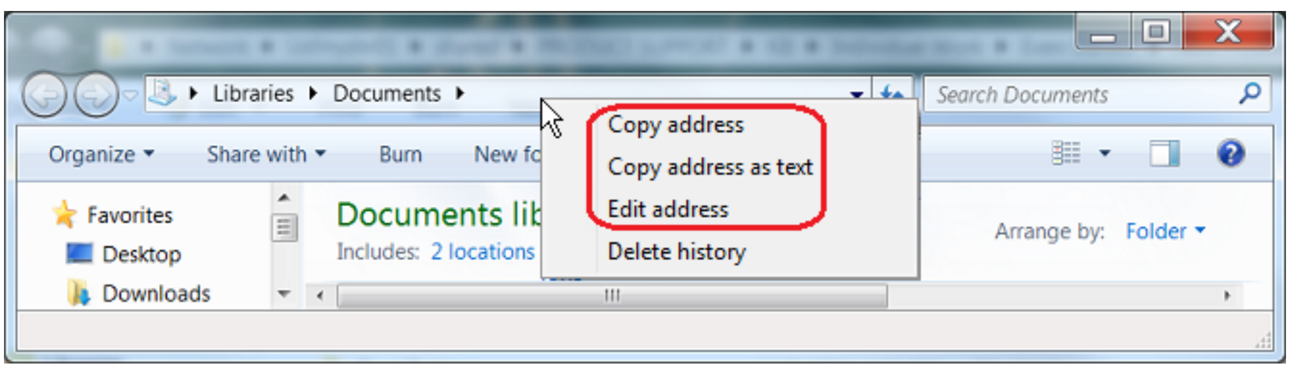
```
To access the notebook, open this file in a browser:
    file:///Users/<YOUR_USERNAME>/Library/Jupyter/runtime/nbserver-73848-
open.html
Or copy and paste one of these URLs:
    http://localhost:8888/?
token=66781622f3ed474ccf1f0f61e5708b035acfcf4e41cc7c8a
 or http://127.0.0.1:8888/?
token=66781622f3ed474ccf1f0f61e5708b035acfcf4e41cc7c8a
```

Copy and paste one of the last two addresses in your browser to start your jupyter session.

You are ready to start coding!

Note: To find out where is your folder in Windows:

- Open Windows Explorer and go inside the folder you need
- Right click on the address path and right-click
- Select "copy address as text" (see picture below)
- Go back to your Anaconda prompt and paste it (remember to type `cd` first!)



References

- Installing Jupyter Software
- Python Control Library

The Control Problem

- Systems and control systems
- Three problems:
    - system identification problem
    - simulation problem
    - control Problem
- Why do we need feedback control

LTI systems

- Homogeneity
- Superposition (Additivity)
- Time Invariance

- Impulse Response

    > LTI systems can be characterised by their response to an impulse function (the output of the system when presented with a brief input signal)

Transfer Functions

- Laplace domain representation of a system

> Definition: Transfer function is the Laplace transform of the impulse response of a linear, time-invariant system with a single input and single output when you set the initial conditions to zero.

- State space vs Transfer functions
- Laplace Transform

The Laplace transform is important because it is a tool for solving differential equations: *it transforms linear differential equations into algebraic equations, and convolution into multiplication!*

- Derivatives and integrals become algebric operations

From a system perspective, the Transfer function expresses the relation between the Laplace Transform of the input and that of the output:

$$Y(s) = \mathbf{c}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}U(s)$$

- Properties of the Laplace Transform (relationships between time and frequency)
  - $\frac{df}{dt}(t) \to sF(s)$
  - $af(t) + bg(t) \to aF(s) + bG(s)$
- Laplace transforms of known functions:
  - $\delta(t) \to 1$
  - $1(t) \to \frac{1}{s}$
  - $e^{-at} \to \frac{1}{s+a}$
  - ...

Stability and other properties

- System $G(s)$ is asymptotically stable if all its poles have $Re < 0$
- System $G(s)$ is stable if all its poles have G(s) $Re \leq 0$ and if those with $Re = 0$ have multiplicity one.
- System $G(s)$ is unstable if it has at least one pole with $Re > 0$ or with $Re = 0$ and multiplicity ≥ 2.

- BIBO stability:
  - A system is BIBO stable (Bounded Input, Bounded Output) if for each limited input, there is a limited output
  - For linear systems: BIBO stability if and only if poles of the transfer functions have $Re < 0$

- Stability of a system in a state space representation
- Controllability (Reachability)
- Observability

Block diagrams

- Standard representation of interconnected systems and subsystems using Transfer Functions
- Block diagram algebra (serie, parallel, feedback) and equivalent block diagrams
- Zero-poles cancellations
- Impact on stability and other properties

System Response

- Dominant pole approximation
- First-order and second-order systems
    - First-order systems: step response and performance
    - time constant (τ) - Characterises completely the response of a first order system
    - settling time: $t_s = -\tau ln(0.05)$ (time to get to 95% of steady state)

    - Second order systems: step response and performance

    - More parameters to define performance: max overshoot, time of max overshoot, settling time, oscillation period

Frequency response

- Assuming that the system is asymptotically stable,
- If we input a sinusoid $Asin(\omega t)$
- The output of the system converges $y(t) = A|G(j\omega)|sin(\omega t + \angle G(j\omega))$
- $G(j\omega)$ is the frequency response
- If we know magnitude and phase of $G(j\omega)$ (for varying ω) then we know how the system behaves for all sinusoidal inputs with different driving frequencies.

Bode plots

- Frequency Response and Bode Plots
  - analysing the gain and the phase shift across the full frequency spectrum
  - gain diagram and phase diagram
- Leverages the properties of transfer functions and of logarithms:
  - The transfer function of linear systems are polynomial fractions in s
  - It is always possible to factor polynomials in terms of their roots (zeros and poles)
  - Each polynomial is the product of first-order or second-order (potentially with multiplicity $>$ 1)
  - To build the frequency response of the system is useful to have simple rules to represent each term
  - The logarithm of a product is equal to a sum of logarithms
    - remember to convert gain to dB: $|G(j\omega)|_{dB} = 20log_{10}|G(j\omega)|$
- Sketching Bode Plots
  - Given a transfer function $G(s)$ we can obtain the (steady state) frequency response setting $s = j\omega \rightarrow G(j\omega)$
  - $G(j\omega) = \text{real} + \text{imag}j$
  - Gain: $\sqrt{\text{real}^2 + \text{img}^2} = |G(j\omega)|$
  - Phase: $atan2(\text{img}, \text{real}) = arg(G(j\omega)) = \angle G(j\omega)$
  - Our building blocks:
    - constant gain
    - integrator
    - simple pole
    - complex poles
    - specular behaviour for the zeros (properties of the logarithms)
    - Transform the trasnfer function to the Bode form before drawing the Bode plots!

Final value theorem and steady state error

- Meaning of final value
- Theorem: for an asymptotically stable LTI system: $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$
- System type: number of poles at the origin
- Analysis of the final value for different system types and different inputs
- For a feedback system: the final value of the error is a much better indicator of the performance of our controller

Diving deeper into the feedback loop

- Included load disturbances and measurement noise in the feedback loop
- Sensitivity function (feedback and disturbances / low frequency specifications)
- Complementary sensitivity function (feedback and measurements / high frequency specifications)
- Design requirements on the loop gain function $|R(s)G(s)|$

Routh-Hurwitz Criterion

- Necessary and sufficient condition to analyse the stability of the systems without solving for the roots explicitely
- Usage can be extended to beyond stability
  - Gain margins
  - Deal with requirement on the speed of the response (time to half)

Coming up next

- Polar plots and the Nyquist plot
- Stability margins
- Root locus
- PID controllers
- Lead-lag/lad-lead compensators

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