ESERCIZIO 1

$$x[n] = a^{|n|} \qquad 0 < a < 1$$

$$x[n] = \underbrace{a^{n} u[n]}_{y[n]} + a^{-n} u[-n] - x[0]\,\delta[n] =$$

$$= y[n] + y[-n] - x[0]\,\delta[n]$$

$$Y(f) = \sum_{n=0}^{\infty} a^{n} e^{-j2\pi f n T} = \sum_{n=0}^{+\infty} \left(a e^{-j2\pi f T}\right)^{n} =$$

$$= \frac{1}{1 - a e^{-j2\pi f T}}$$

$$\boxed{\sum_{n=0}^{\infty} q^{n} = \frac{1}{1-q} \qquad |q| < 1}$$

$$Y(-f) = Y^{*}(f)$$

$$X(f) = Y(f) + Y^{*}(f) - x[0] =$$

$$= 2\,\mathrm{Re}\{Y(f)\} - x[0] =$$

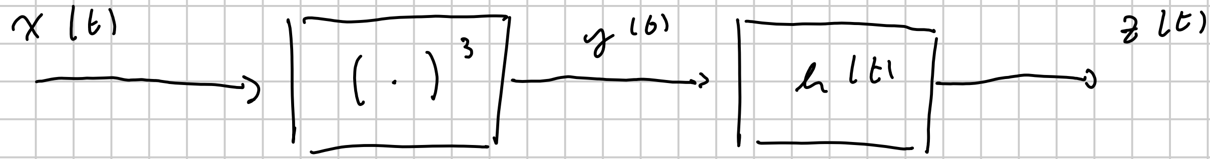
$$= \frac{1}{1 - a e^{-j2\pi f T}} + \frac{1}{1 - a e^{+j2\pi f T}} - 1 =$$

$$= \frac{1 - a e^{+j2\pi f T} + 1 - a e^{-j2\pi f T} - (1 - a e^{-j2\pi f T})(1 - a e^{+j2\pi f T})}{(1 - a e^{-j2\pi f T})(1 - a e^{+j2\pi f T})}$$

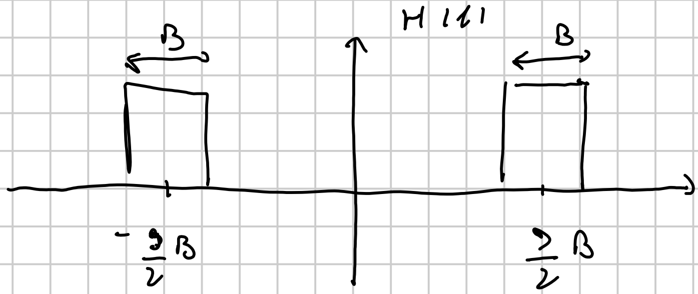
$$= \frac{1 - \cancel{a e^{+j2\pi f T}} + 1 - \cancel{a e^{-j2\pi f T}} - 1 - a^2 + \cancel{2a \cos 2\pi f T}}{1 + a^2 - 2a\cos(2\pi f T)} =$$

$$= \frac{1 - a^2}{1 - a^2 - 2a\cos(2\pi f T)}$$

# ESERCIZIO 2




$$x(t) = A\cos(3\pi B t + \theta)$$




$$y(t) = (x(t))^3 = A^3 \cos^3(3\pi B t + \theta) =$$

$$= A^3 \cos(3\pi B t + \theta)\cos^2(3\pi B t + \theta) =$$

$$= A^3 \cos(3\pi B t + \theta)\left(\frac{1}{2} + \frac{1}{2}\cos(6\pi B t + 2\theta)\right) =$$

$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$= \frac{A^3}{2}\cos(3\pi B t + \theta) + \frac{A^3}{2}\cos(3\pi B t + \theta)\cos(6\pi B t + 2\theta) =$$

$$= \frac{A^3}{2}\cos(3\pi B t + \theta) + \frac{A^3}{2}\left(\frac{1}{2}\cos(9\pi B t + 3\theta) + \frac{1}{2}\cos(3\pi B t + \theta)\right) =$$

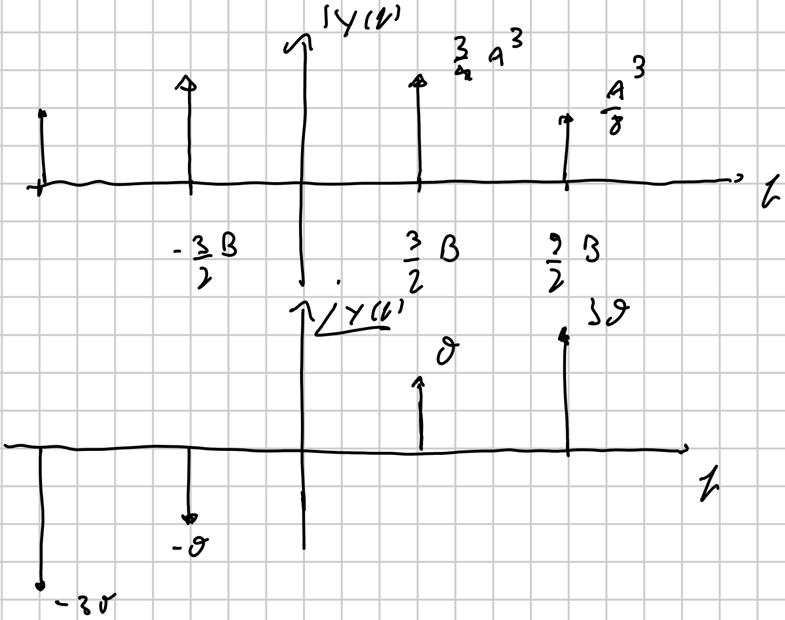
$$= \underbrace{\left(\frac{A^3}{2} + \frac{A^3}{4}\right)}_{\frac{3}{4}A^3}\cos(\underbrace{3\pi B t + \theta}_{2\pi\frac{3}{2}Bt+\theta}) + \frac{A^3}{4}\cos(\underbrace{9\pi B t + 3\theta}_{2\pi\frac{9}{2}Bt+3\theta})$$

$$Y(f) = \frac{3}{4}A^3\left(\frac{\delta(f - f_{01})e^{j\theta} + \delta(f + f_{01})e^{-j\theta}}{2}\right) +$$

$$\frac{A^3}{4}\left(\frac{\delta(f - f_{02})e^{j3\theta} + \delta(f + f_{02})e^{-j3\theta}}{2}\right)$$

$f_{01} = \frac{3}{2} B$

$f_{02} = \frac{9}{2} B$




$$Z(f) = \frac{A^3}{8}\left[\delta\left(f - \frac{9}{2}B\right)e^{+j3\theta} + \delta\left(f + \frac{9}{2}B\right)e^{-j3\theta}\right]$$

$$z(t) = \frac{A^3}{4}\cos\left(2\pi \frac{9}{2}B t + 3\theta\right)$$

## ESERCIZIO 3

$$y(t) = |x(t)| + \int_a^t x(\alpha)\,d\alpha$$

1) LINEARE?

2) STAZIONARIO?

3) CON MEMORIA?

4) STABILE BIBO?

5) CAUSALE?

1) $x(t) = a\,x_1(t) + b\,x_2(t)$

$$T[x(t)] = |a\,x_1(t) + b\,x_2(t)| + \int_a^t (a\,x_1(\alpha) + b\,x_2(\alpha))\,d\alpha \neq$$

$$a|x_1(t)| + b|x_2(t)| + a\int_a^t x_1(\alpha)\,d\alpha + b\int_a^t x_2(\alpha)\,d\alpha$$

<u>NO LINEARE</u>

2) $x(t-t_0) \Rightarrow y(t-t_0)$

$$T\left[x(t-t_0)\right] = |x(t-t_0)| + \int_a^t x(\alpha - t_0)\, d\alpha =$$

$$= |x(t-t_0)| + \int_{a-t_0}^{t-t_0} x(\alpha')\, d\alpha'$$

$$\neq$$

$$y(t-t_0) = |x(t-t_0)| + \int_a^{t-t_0} x(\alpha)\, d\alpha$$

NO STAZIONARIO

3) CON MEMORIA

4) CAUSALE

5) $|x(t)| < M \quad \forall t$

$$y(t) = |x(t)| + \int_a^t x(\alpha)\, d\alpha$$

$$|y(t)| = \left|\, |x(t)| + \int_a^t x(\alpha)\, d\alpha \,\right| \leq$$

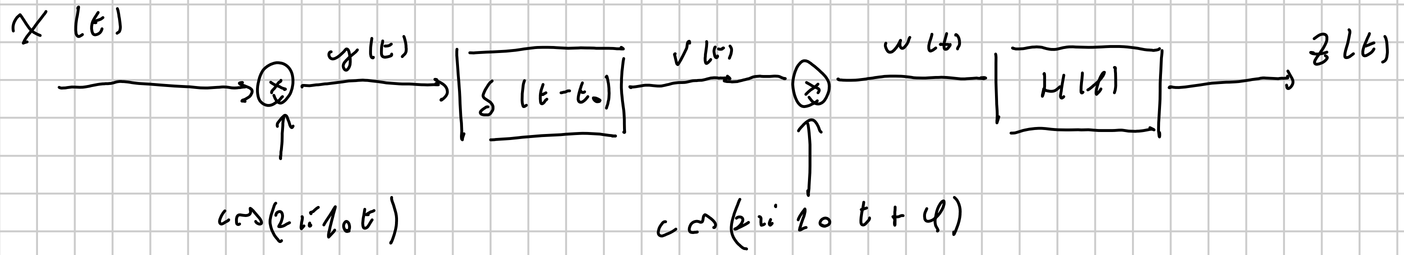
$$\leq |x(t)| + \left| \int_a^t x(\alpha)\, d\alpha \right| \leq$$

$$\leq M + \int_a^t |x(\alpha)|\, d\alpha = M + M(t-a) \leq H$$

$t \leq \infty$

NON È STABILE

ESERCIZIO 4 SISTEMA WIRELESS




$$x(f) = \left[1 + \cos\left(\pi \frac{f}{B}\right)\right] \mathrm{rect}\left(\frac{f}{2B}\right)$$

$H(f)$ filtro LP di banda $B$

1) $Y(f)$
2) $W(f)$
3) $Z(f)$
4) $\varphi$ : $z(t)$ $E_z$ max

$$y(t) = x(t)\cos(2\pi f_0 t)$$

$$\Updownarrow$$

$$Y(f) = \frac{X(f-f_0) + X(f+f_0)}{2} =$$

$$= \frac{1}{2}\left(1 + \cos\left(\pi \frac{f-f_0}{B}\right)\right)\mathrm{rect}\left(\frac{f-f_0}{2B}\right) + \frac{1}{2}\left(1 + \cos\left(\pi \frac{f+f_0}{B}\right)\right)\mathrm{rect}\left(\frac{f+f_0}{2B}\right)$$

$$v(t) = y(t) \otimes \delta(t-t_0)$$

$$\Updownarrow$$

$$V(f) = Y(f)\, e^{-j2\pi f t_0}$$

$$w(t) = v(t)\cos(2\pi f_0 t + \varphi)$$

$$W(f) = V(f) \otimes \left( \frac{1}{2}\delta(f-f_0)e^{j\varphi} + \frac{1}{2}\delta(f+f_0)e^{-j\varphi} \right) =$$

$$= \frac{1}{2}V(f-f_0)e^{j\varphi} + \frac{1}{2}V(f+f_0)e^{-j\varphi}$$

$$W(f) = \frac{1}{2}Y(f-f_0)e^{-j2\pi(f-f_0)t_0}e^{j\varphi} +$$

$$+ \frac{1}{2}Y(f+f_0)e^{-j2\pi(f+f_0)t_0}e^{-j\varphi} =$$

$$= \frac{1}{2}e^{-j2\pi f t_0}\left\{ \left[ \frac{1}{2} + \frac{1}{2}\cos\left(\pi\frac{(f-2f_0)}{B}\right) \mathrm{rect}\left(f - \frac{2f_0}{B}\right) + \right.\right.$$

$$\left. + \left(\frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi f}{B}\right)\right)\mathrm{rect}\left(\frac{f}{2B}\right)\right] e^{j2\pi f_0 t_0 + j\varphi} +$$

$$+ \left[\left(\frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi f}{B}\right)\right)\mathrm{rect}\left(\frac{f}{2B}\right) + \left(\frac{1}{2} + \frac{1}{2}\cos\left(\frac{f+2f_0}{2B}\right)\right)\mathrm{Rect}\left(\frac{f+2f_0}{2B}\right)\right] \cdot$$

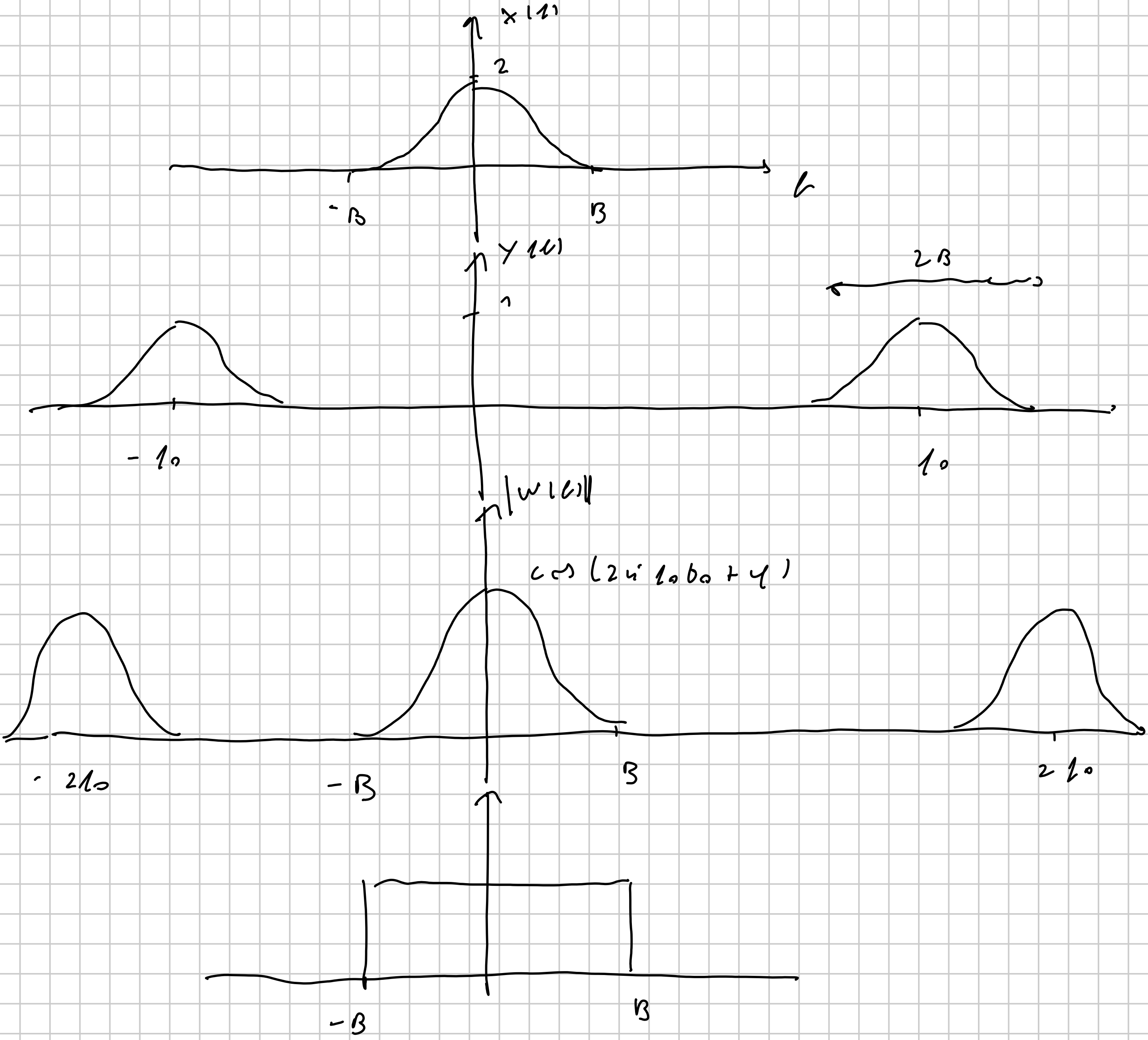
$$\left. \cdot\, e^{-j2\pi f_0 t_0 - j\varphi} \right\}$$

DEFINISCO

$$W_{2f_0} = \frac{1}{4}e^{-j2\pi f t_0}e^{j2\pi f_0 t_0 + j\varphi}\left(1 + \cos\left(\pi\frac{(f-2f_0)}{B}\right)\right)\mathrm{rect}\left(\frac{f-2f_0}{2B}\right)$$

$$W_{-2f_0} = \frac{1}{4}e^{-j2\pi f t_0}e^{-j2\pi f_0 t_0 - j\varphi}\left(1 + \cos\left(\pi\frac{(f+2f_0)}{B}\right)\right)\mathrm{rect}\left(\frac{f+2f_0}{2B}\right)$$

$$W(f) = W_{2f_0} + W_{-2f_0} + \frac{1}{2}e^{-j2\pi f t_0}\left(1 + \cos\left(\frac{\pi f}{B}\right)\right)\cos(2\pi f_0 t_0 + \varphi)\,\mathrm{rect}\left(\frac{f}{2B}\right)$$




$$z(f) = \frac{1}{2} e^{-j 2\pi f t_0} \cos(2\pi f_0 t_0 + \varphi) \underbrace{\left(1 + \cos\left(\frac{\pi f}{B}\right)\right) \mathrm{rect}\left(\frac{f}{2B}\right)}_{X(f)}$$

$$z(t) = \frac{1}{2} \cos(2\pi f_0 t + \varphi)\, x(t - t_0)$$

$$X(f) = \mathrm{rect}\left(\frac{f}{2B}\right) + \mathrm{rect}\left(\frac{f}{2B}\right) \cos\left(\frac{\pi f}{B}\right)$$

$$\Updownarrow$$

$$x(t) = 2B\, \mathrm{sinc}(t\, 2B) + B\, \mathrm{sinc}(t\, 2B) \otimes \left[ \delta\left(t - \frac{1}{2B}\right) + \delta\left(t + \frac{1}{2B}\right) \right]$$

$$E_z = \int_{-\infty}^{+\infty} |z(t)|^2 \, dt = \int_{-\infty}^{+\infty} |Z(f)|^2 \, df$$

$$z(t) = \frac{1}{2} \cos(2\pi f_0 t_0 + \varphi) \, x(t - t_0)$$

$$E_z = \frac{1}{4} \cos^2(2\pi f_0 t_0 + \varphi) \, E_x$$

$$\cos^2(2\pi f_0 t_0 + \varphi) = 1 \implies 2\pi f_0 t_0 + \varphi = k\pi$$

$$\boxed{\varphi = -2\pi f_0 t_0 + k\pi}$$

$$E_{z\,max} = \frac{1}{4} E_x$$

ESERCIZIO 5

$$x(t) \xrightarrow{nT} x[n] \longrightarrow \boxed{p(t)} \xrightarrow{z(t)} \boxed{h(t)} \longrightarrow y(t) \; ?$$

$$x(t) = \text{sinc}\left(\frac{t}{T}\right)$$

$$p(t) = \text{rect}\left(\frac{t - T/2}{T}\right)$$

$$H(f) = \frac{\pi f T}{\sin(\pi f T)} \, \text{rect}(fT)$$

$$x[n] = \text{sinc}\left(n\frac{T}{T}\right) = \text{sinc}(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$z(t) = \sum_n x[n] \, p(t - nT) = p(t)$$

$$y(t) = z(t) \otimes h(t)$$

$$\Updownarrow$$

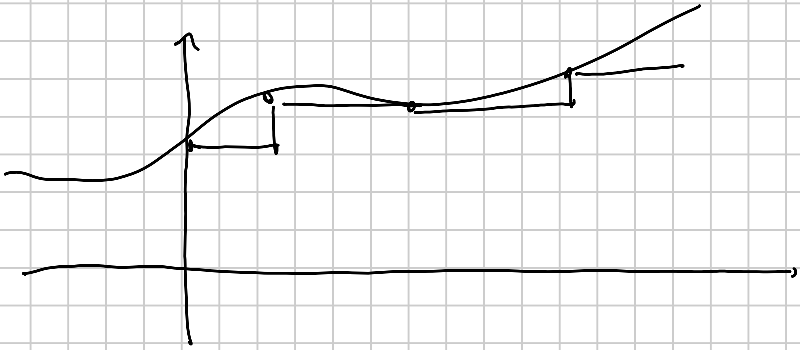
$$Y(f) = Z(f)\, H(f)$$

$$P(f) = T \,\mathrm{sinc}(fT)\, e^{-j2\pi f \frac{T}{2}} = T\,\mathrm{sinc}(fT)\, e^{-j\pi f T}$$

$$Y(f) = T\,\mathrm{sinc}(fT)\, e^{-j\pi f T} \underbrace{\frac{\pi f T}{\sin(\pi f T)}} \mathrm{rect}(fT) =$$

$$= T \frac{\sin(\cancel{\pi f T})}{\cancel{\pi f T}} e^{-j\pi f T} \frac{\cancel{\pi f T}}{\cancel{\sin}(\pi f T)} \mathrm{rect}(fT)$$

$$y(t) = \mathrm{sinc}\left(\frac{t - T/2}{T}\right)$$



<u>ESERCIZIO 1</u> 15/02/10

$$z(t) = \underbrace{\mathrm{rect}\left(\frac{t - T/2}{T}\right) e^{-t/\tau}}_{z_1(t)} + \overbrace{\mathrm{sinc}\left(\frac{t}{T}\right) \cos(2\pi f_0 t + \theta)}^{z_2(t)}$$

$$\boxed{\begin{array}{l} T \gg \dfrac{1}{f_0} \\ \tau \gg T \end{array}}$$

$$z(f) = z_1(f) + z_2(f)$$

$$z_1(f) = \int_0^T e^{-\frac{t}{2}} e^{-j2\pi f t} \, dt = \left. \frac{e^{-\frac{t}{2} - j2\pi f t}}{-\frac{1}{2} - j2\pi f} \right|_0^T =$$

$$= \frac{e^{-\frac{T}{2} - j2\pi f T} - 1}{-\frac{1}{2} - j2\pi f}$$

$$z_2(f) = \frac{1}{T} \operatorname{rect}\left(\frac{f}{1/T}\right) \otimes \left[\delta(f - f_0)\frac{e^{j\theta}}{2} + \delta(f + f_0)\frac{e^{-j\theta}}{2}\right] =$$

$$= \frac{1}{2T} \operatorname{rect}\left(\frac{f - f_0}{1/T}\right) e^{j\theta} + \frac{1}{2T} \operatorname{rect}\left(\frac{f + f_0}{1/T}\right) e^{-j\theta}$$

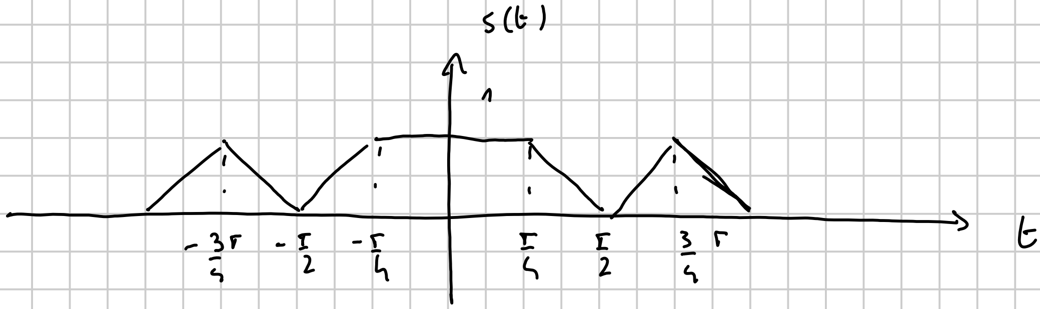
$$E_z = E_{z_1} + E_{z_2}$$

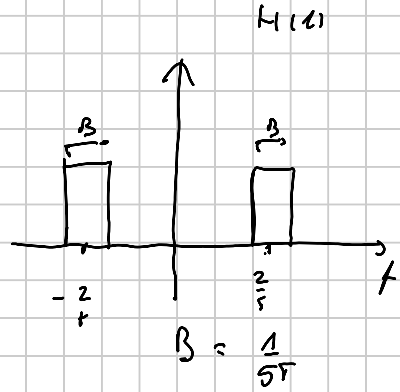
$$E_{z_2} = 2 \frac{1}{4T^2 T} = \frac{1}{2T^3}$$

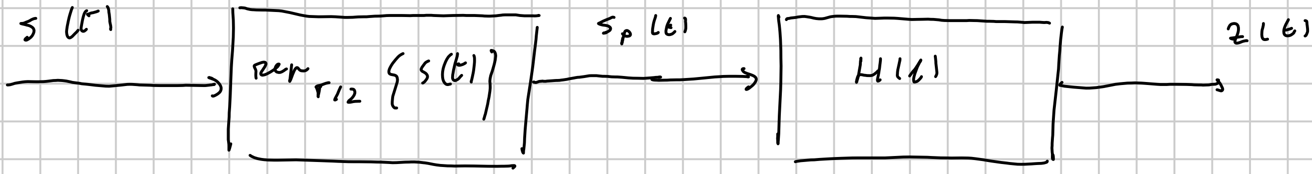
$$E_{z_1} = \int_0^T e^{-2t/2} \, dt = -\frac{2}{2} e^{-2t/2} \Big|_0^T =$$

$$= -\frac{2}{2}\left(e^{-2T/2} - 1\right)$$

ESERCIZIO 20/04/2004




$$s_p(t) = \sum_n s\left(t - n\frac{T}{2}\right)$$

$$s_1(t) = 2\left(1 - \frac{|t|}{T/2}\right) rect\left(\frac{t}{T}\right) \rightleftharpoons S_1(f) = \cancel{2}\frac{T}{\cancel{2}}\, sinc^2\left(f\frac{T}{2}\right)$$

$$s_2(t) = \left(1 - \frac{|t|}{T/4}\right) rect\left(\frac{t}{T/2}\right) \rightleftharpoons S_2(f) = \frac{T}{4}\, sinc^2\left(f\frac{T}{4}\right)$$

$$s(t) = s_1(t) - s_2(t) + s_2\left(t - \frac{3}{4}T\right) + s_2\left(t + \frac{3}{4}T\right)$$

$$S(f) = S_1(f) - S_2(f)\left[1 - e^{-j2\pi f\frac{3}{4}T} - e^{+j2\pi f\frac{3}{4}T}\right] =$$

$$= S_1(f) - S_2(f)\left[1 - 2\cos 2\pi f\frac{3}{4}T\right]$$

$$T_0 = \frac{T}{2}$$

$$S_p(f) = \frac{1}{T_0}\sum_k S\left(\frac{k}{T_0}\right)\delta\left(f - \frac{k}{T_0}\right) = \frac{2}{T}\sum_k S\left(\frac{2k}{T}\right)\delta\left(f - \frac{2k}{T}\right)$$

$$k = \pm 1$$

$$S\left(\frac{2}{T}\right) = S\left(-\frac{2}{T}\right) = T\,\cancel{sinc^2\left(\frac{2}{T}\frac{T}{2}\right)} - \frac{T}{4}\, sinc^2\left(\frac{2}{T}\frac{T}{4}\right)\cdot$$

$$\cdot\left(1 - 2\cos 3\pi\right) =$$

$= -\frac{T}{4} \cdot \frac{4}{\pi^2} (1+2) = -3\frac{T}{\pi^2}$

$\frac{2}{T} S\left(\frac{2}{T}\right) = \frac{2}{T} S\left(-\frac{2}{T}\right) = -\frac{6}{\pi^2}$

$Z(f) = -\frac{6}{\pi^2}\left[\delta\left(f-\frac{2}{T}\right) + \delta\left(f+\frac{2}{T}\right)\right]$

$\Updownarrow$

$z(t) = -\frac{12}{\pi^2}\cos\left(2\pi \frac{2}{T} t\right)$

## ESERCIZIO 2

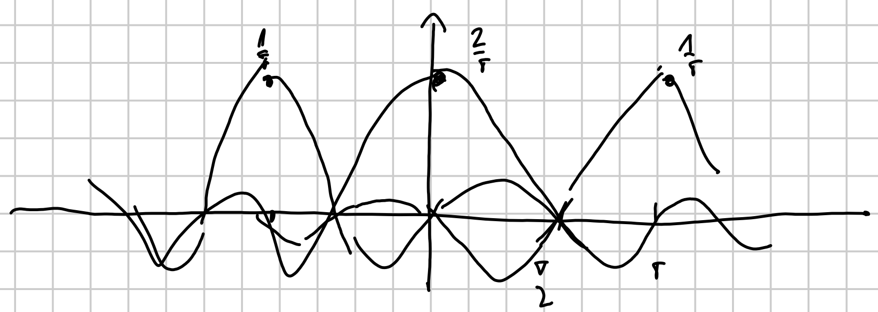
$X(f) = \left[1 + \cos(2\pi f T)\right] \mathrm{rect}\left(f\frac{T}{2}\right)$

$F_c = \frac{2}{T}$

$x(t) = \left\{\delta(t) + \frac{1}{2}\left[\delta(t-T) + \delta(t+T)\right]\right\} \otimes \frac{2}{T}\mathrm{sinc}\left(\frac{2t}{T}\right)$

$= \frac{2}{T}\mathrm{sinc}\left(\frac{2t}{T}\right) + \frac{1}{T}\left(\mathrm{sinc}\left(\frac{2(t-T)}{T}\right) + \mathrm{sinc}\left(\frac{2(t+T)}{T}\right)\right)$

$x[n] = x(nT_c) = x\left(n\frac{T}{2}\right) = \begin{cases} \frac{2}{T} & n=0 \\ \frac{1}{T} & n=\pm 2 \end{cases}$



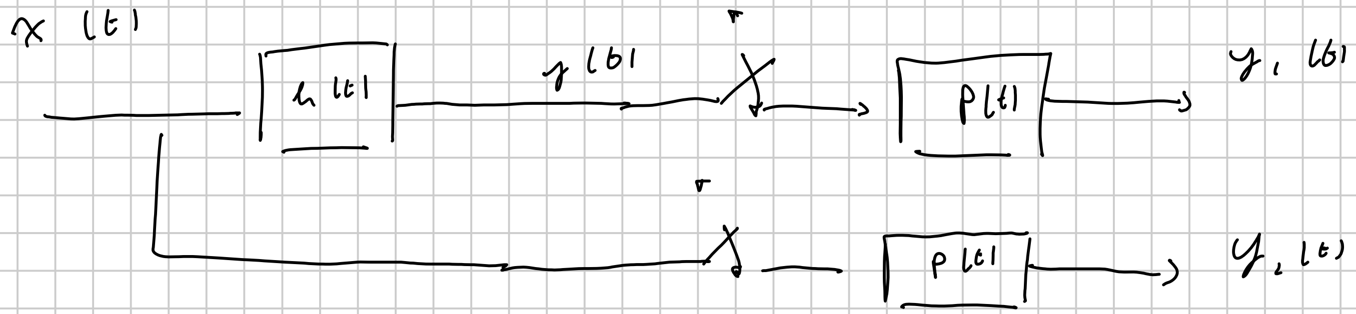
$$\bar{X}(f) = \frac{T}{2} \sum_n x\left(n\frac{T}{2}\right) e^{-j2\pi n f \frac{T}{2}} =$$

$$= \frac{T}{2}\left\{\frac{2}{T} + \frac{1}{T}\left[e^{-j2\pi f 2\frac{T}{2}} + e^{+j2\pi f 2\frac{T}{2}}\right]\right\} =$$

$$= 1 + \cos(2\pi f T)$$

$$X(f) = \bar{X}(f)\,\mathrm{rect}\left(f\frac{T}{2}\right)$$

ESERCIZIO 2 del 19/04/10




$x(t) = 2AB\,\mathrm{sinc}^2(2Bt)$

$h(t) = 2B\,\mathrm{sinc}(2Bt)$

$p(t)$ INTERP. CARD. DI BANDA B

$T = \frac{1}{2B}$

$y_1(t)$

$y_2(t)$

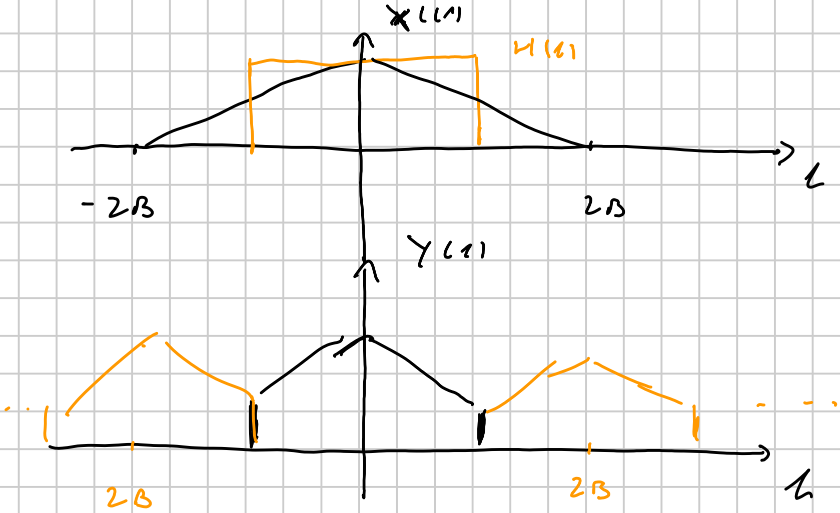
$E_{y_1}$, $P_{y_1}$

$E_{y_2}$, $P_{y_2}$

$$X(f) = A\left(1 - \frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{4B}\right)$$

$$H(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$Y(f) = X(f)\,H(f)$$




$$\bar{Y}(f) = \frac{1}{T}\sum_n Y\left(f - \frac{n}{T}\right)$$

$$= 2B \sum_n Y(f - n\,2B)$$

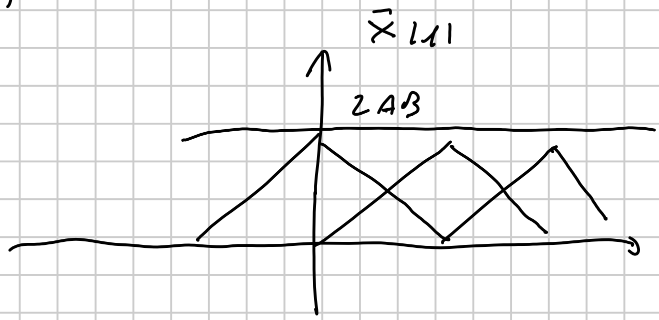
$$Y_1(f) = 2AB\left(1 - \frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{4B}\right)\underbrace{\mathrm{rect}\left(\frac{f}{2B}\right)}_{p(f)} =$$

$$= 2AB\left(1 - \frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) =$$

$$= AB\,\mathrm{rect}\left(\frac{f}{2B}\right) + AB\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$y_1(t) = 2AB^2\,\mathrm{sinc}(t\,2B) + AB^2\,\mathrm{sinc}^2(t\,B)$$

$$\bar{X}(f) = 2B\sum_n X(f - 2nB)$$




$$Y_2(f) = \bar{X}(f)\,p(f) = 2AB\,\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$y_2(t) = 4AB^2\,\mathrm{sinc}(2Bt)$$

$$E_{y_1} = \int_{-\infty}^{+\infty} |Y_1(\ell)|^2 \, d\ell = \int_{-B}^{B} (AB)^2 \, d\ell + 2\int_{0}^{B} (AB^2)\left(1-\frac{\ell}{B}\right)^2 d\ell +$$

$$+ \int_{-B}^{B} 2(AB^2)\left(1-\frac{|\ell|}{B}\right) d\ell =$$

$$\int_{-B}^{B} (AB)^2 \, d\ell = 2B\,A^2B^2$$

$$2\int_{0}^{B} A^2B^2 \left(1-\frac{\ell}{B}\right)^2 d\ell = 2A^2B^2 \int_{0}^{B} \left(1+\frac{\ell^2}{B^2}-\frac{2\ell}{B}\right) d\ell =$$

$$= 2A^2B^2 \left(\ell + \frac{\ell^3}{3B^2} - \frac{\ell^2}{B}\right)\Bigg|_{0}^{B} = 2A^2B^2\left[\cancel{B} + \frac{B}{3} - \cancel{B}\right] =$$

$$= \frac{2}{3} A^2 B^3$$

$$4\int_{0}^{B} A^2B^2 \left(1-\frac{\ell}{B}\right) d\ell = 4A^2B^2\left[\ell - \frac{\ell^2}{2B}\right]\Bigg|_{0}^{B} = 4A^2B^2\left(B-\frac{B}{2}\right) =$$

$$= 2A^2B^3$$

$$E_{y_1} = \frac{14}{3} A^2 B^3$$

$$E_{y_2} = \int_{-\infty}^{+\infty} |Y_2(\ell)|^2 = \int_{-B}^{B} 4A^2B^2 \, d\ell = 4A^2B^2\,2B = 8A^2B^3$$

$$P_{y_1} = P_{y_2} = 0$$