I Compitino Com. Num. 07/04/2018

Sol. es. 4 - Fila A

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$

$$x_1(t) = 2\left(1 - \frac{|t|}{2T}\right) \mathrm{rect}\left(\frac{t}{4T}\right)$$

$$x_2(t) = \mathrm{rect}\left(\frac{t}{2T}\right)$$

$$x_3(t) = \left(1 - \frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right)$$

$$X(f) = X_1(f) + X_2(f) + X_3(f)$$ per la linearità della TCF

$$X_1(f) = \mathrm{TCF}\left[x_1(t)\right] = 4T \,\mathrm{sinc}^2(2Tf)$$

$$X_2(f) = \mathrm{TCF}\left[x_2(t)\right] = 2T \,\mathrm{sinc}(2Tf)$$

$$X_3(f) = \mathrm{TCF}\left[x_3(t)\right] = T \,\mathrm{sinc}^2(Tf)$$

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt = \int_{-\infty}^{+\infty} x^2(t) \, dt = \int_{-\infty}^{+\infty} \left(x_1(t) + x_2(t) + x_3(t)\right)^2 dt$$

$$= \int_{-\infty}^{+\infty} x_1^2(t) \, dt + \int_{-\infty}^{+\infty} x_2^2(t) \, dt + \int_{-\infty}^{+\infty} x_3^2(t) \, dt + 2\int_{-\infty}^{+\infty} x_1(t)\, x_2(t) \, dt$$

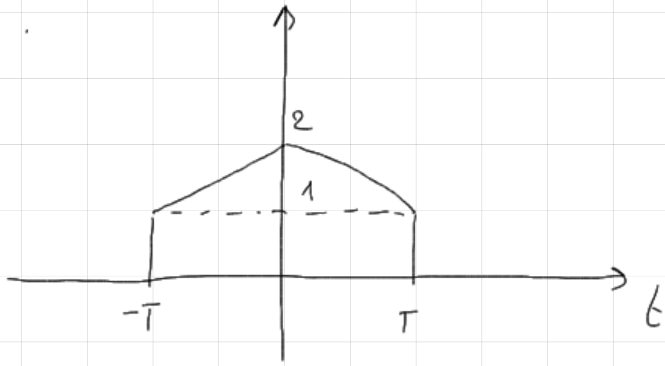
$$+ \int_{-\infty}^{+\infty} x_1(t)\, x_3(t) \, dt + \int_{-\infty}^{+\infty} x_2(t)\, x_3(t) \, dt$$

$$\int_{-\infty}^{+\infty} x_1^2(t)\,dt = \frac{2}{3}4 \cdot 2T = \frac{16}{3}T$$

$$\int_{-\infty}^{+\infty} x_2^2(t)\,dt = 2T$$

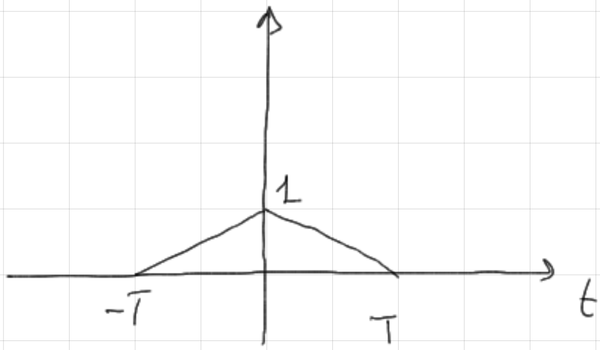
$$\int_{-\infty}^{+\infty} x_3^2(t)\,dt = \frac{2}{3}T$$

$x_1(t)\,x_2(t) \Rightarrow$



$$2\int_{-\infty}^{+\infty} x_1(t)\,x_2(t)\,dt = 2(2T + T) = 6T$$

$x_2(t)\,x_3(t) \Rightarrow$



$$2\int_{-\infty}^{+\infty} x_2(t)\,x_3(t)\,dt = 2T$$

$$x_1(t)\,x_3(t) = \left(1 - \frac{|t|}{2T}\right)\left(1 - \frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right)$$

$$2\int_{-\infty}^{+\infty} x_1(t)\,x_3(t)\,dt = 4\int_0^T 2\left(1 - \frac{t}{2T}\right)\left(1 - \frac{t}{T}\right) dt$$

$$= 8\int_0^T 1 - \frac{t}{T} - \frac{t}{2T} + \frac{t^2}{2T^2}\,dt =$$

$$= 8T - 8\int_0^T \frac{3t}{2T}\,dt + 8\int_0^T \frac{t^2}{2T^2}\,dt$$

$$= 8T - \frac{12}{T}\frac{t^2}{2}\Big|_0^T + \frac{4}{T^2}\frac{t^3}{3}\Big|_0^T$$

$$= 8T - \frac{6}{T}T^2 + \frac{4}{3T^2}T^3 = 8T - 6T + \frac{4}{3}T =$$

$$= 2T + \frac{4}{3}T = \frac{10}{3}T$$

$$E_x = \left(\frac{16}{3} + 2 + \frac{2}{3} + 6 + 2 + \frac{10}{3}\right)T = \frac{58}{3}T$$

$$E_x < \infty \Rightarrow P_x = 0 \Rightarrow x_{eff} = 0\ ,\ x_m = 0$$

Sol. Es. 4 - Fila B

$$x(t) = x_1(t) + x_2(t) - x_3(t)$$

$$x_1(t) = 2\left(1 - \frac{|t|}{2T}\right) \mathrm{rect}\left(\frac{t}{4T}\right)$$

$$x_2(t) = \mathrm{rect}\left(\frac{t}{2T}\right)$$

$$x_3(t) = \left(1 - \frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right)$$

$$X_1(f) = TCF\left[x_1(t)\right] = 4T \,\mathrm{sinc}^2(2Tf)$$

$$X_2(f) = TCF\left[x_2(t)\right] = 2T \,\mathrm{sinc}(2Tf)$$

$$X_3(f) = TCF\left[x_3(t)\right] = T \,\mathrm{sinc}^2(Tf)$$

$$X(f) = X_1(f) + X_2(f) - X_3(f)$$

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt = \int_{-\infty}^{+\infty} \left[x_1(t) + x_2(t) - x_3(t)\right]^2 dt$$

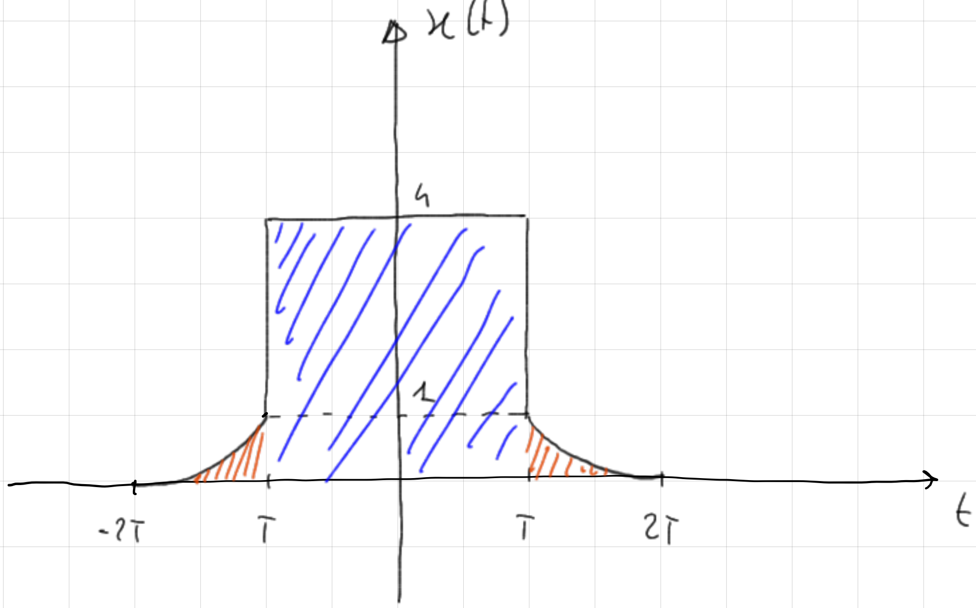
$$= \int_{-\infty}^{+\infty} x_1^2(t)\, dt + \int_{-\infty}^{+\infty} x_2^2(t)\, dt + \int_{-\infty}^{+\infty} x_3^2(t)\, dt +$$

$$+ \int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt - \int_{-\infty}^{+\infty} x_1(t)\, x_3(t)\, dt - \int_{-\infty}^{+\infty} x_2(t)\, x_3(t)\, dt$$

I calcoli dei vari integrali sono gli stessi fatti per la fila A

$$E_x = \left(\frac{16}{3} + 2 + \frac{2}{3} + 6 - 2 - \frac{10}{3}\right)T = \frac{26}{3}T$$

Si poteva anche calcolare direttamente osservando che:



$$E_x = 2\frac{T}{3} + 4 \cdot 2T = \left(\frac{2}{3} + 8\right)T = \frac{26}{3}T$$

$$E_x < \infty \Rightarrow P_x = 0 \Rightarrow x_{eff} = x_m = 0$$

Es. 5 fila A

$$x(t) = A \cos\alpha \sin\beta$$

$$\alpha = 6\pi f_0 t \quad , \quad \beta = 4\pi f_0 t$$

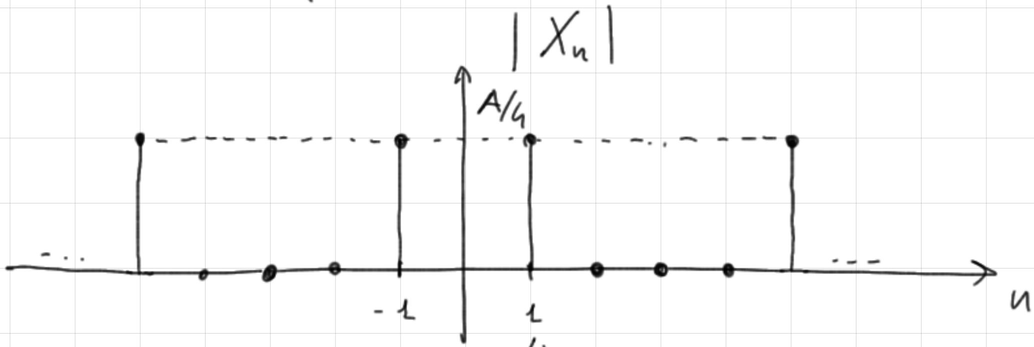
$$x(t) = \frac{A}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right] = \frac{A}{2}\left[\sin(10\pi f_0 t) - \sin(2\pi f_0 t)\right]$$

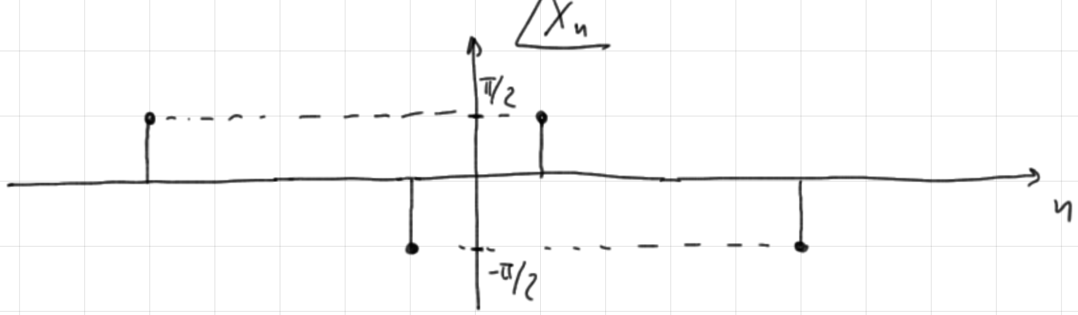
$$x(t) = \frac{A}{2}\sin(10\pi f_0 t) - \frac{A}{2}\sin(2\pi f_0 t)$$

$$X_n = \begin{cases} \frac{A}{4j} \quad (n=5) \quad , -\frac{A}{4j} \quad (n=-5) \\ 0 \quad (n \neq \pm 1, \pm 5) \\ -\frac{A}{4j} \quad (n=1) \quad , \frac{A}{4j} \quad (n=-1) \end{cases}$$

$$\frac{A}{4j} = -j\frac{A}{4} = \frac{A}{4}e^{-j\frac{\pi}{2}}$$

$$-\frac{A}{4j} = j\frac{A}{4} = \frac{A}{4}e^{j\frac{\pi}{2}}$$





Segnale periodico $\Rightarrow$ $E_x = \infty$

$$P_x = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x^2(t)\, dt \qquad T_0 = \frac{1}{f_0}$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \left[\frac{A}{2}\sin(10\pi f_0 t)\right]^2 dt + \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \left[\frac{A}{2}\sin(2\pi f_0 t)\right]^2 dt$$

$$- \underbrace{\frac{2}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \frac{A^2}{4}\sin(10\pi f_0 t)\sin(2\pi f_0 t)\, dt}_{= 0 \; *}$$

$$= \frac{1}{T_0}\frac{A^2}{4}T_0 \cdot \frac{1}{2} + \frac{1}{T_0}\frac{A^2}{4}T_0 \cdot \frac{1}{2} = \frac{A^2}{4}$$

$$* \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \sin(10\pi f_0 t)\sin(2\pi f_0 t)\, dt =$$

$$= \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \frac{1}{2}\left[\cos(8\pi f_0 t) - \cos(12\pi f_0 t)\right] dt$$

$$= \underbrace{\frac{1}{2}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos(8\pi f_0 t)\, dt}_{= 0} - \underbrace{\frac{1}{2}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos(12\pi f_0 t)\, dt}_{= 0}$$

sono oscillazioni complete in T_0

$P_x = \frac{A^2}{4} \Rightarrow x_{eff} = \frac{|A|}{2}$

$x_m = 0$ poiché è un valor medio di sinusoidi

Es 5 fila B

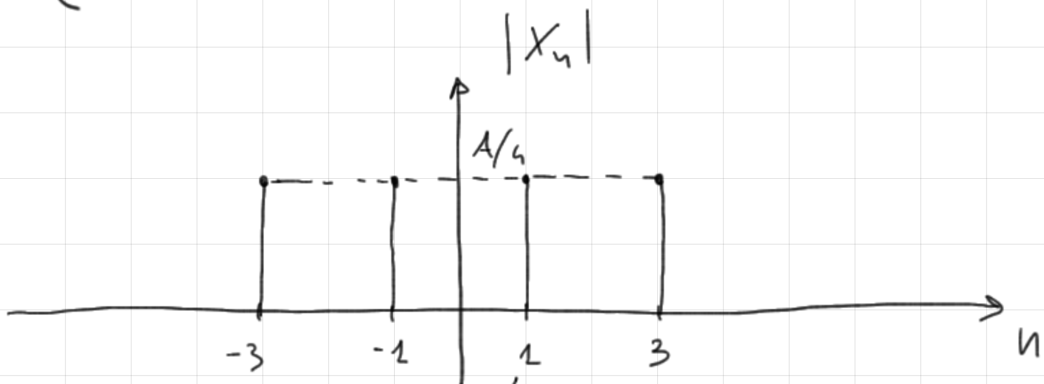
$x(t) = A \sin\alpha \sin\beta$

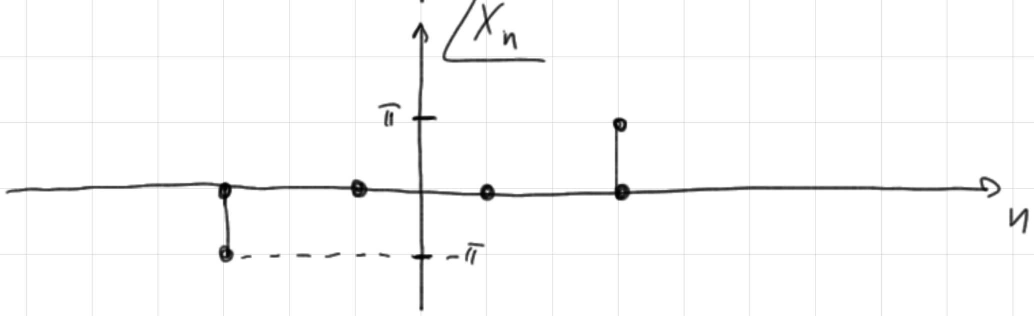
$\alpha = 4\pi f_0 t \quad , \quad \beta = 2\pi f_0 t$

$$x(t) = \frac{A}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$= \frac{A}{2}\left[\cos(2\pi f_0 t) - \cos(6\pi f_0 t)\right]$$

$$X_n = \begin{cases} \frac{A}{4} & n = \pm 1 \quad , \quad -\frac{A}{4} \quad n = \pm 3 \\ 0 & \text{altrove} \end{cases}$$





Segnale periodico $\Rightarrow$ $E_x = \infty$

$$P_x = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x^2(t)\, dt \qquad f_0 = \frac{1}{T_0}$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \left[\frac{A}{2}\cos(2\pi f_0 t)\right]^2 dt + \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \left[\frac{A}{2}\cos(6\pi f_0 t)\right]^2 dt$$

$$\underbrace{- \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \frac{A^2}{4}\cos(2\pi f_0 t)\cos(6\pi f_0 t)\, dt}_{=0 \quad *}$$

$$= \frac{1}{T_0}\frac{A^2}{4}\cdot\frac{1}{2}T_0 + \frac{1}{T_0}\frac{A^2}{4}\cdot\frac{1}{2}T_0 = \frac{A^2}{4}$$

$$* \quad \cos(2\pi f_0 t)\cos(6\pi f_0 t) = \frac{1}{2}\left[\cos(8\pi f_0 t) - \cos(4\pi f_0 t)\right]$$

$$\Rightarrow \underbrace{\frac{1}{2}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos(8\pi f_0 t)\, dt}_{=0} - \underbrace{\frac{1}{2}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos(4\pi f_0 t)\, dt}_{=0}$$

$$P_x = \frac{A^2}{4} \Rightarrow x_{eff} = \frac{|A|}{2}$$

$x_m = 0$ poiché il v.m. di cosinusoidi è zero

Es. 6 - Fila A

TSF:
$$X_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\, e^{-j 2\pi n f_0 t}\, dt \qquad f_0 = \frac{1}{T_0}$$

ATSF:
$$x(t) = \sum_{n=-\infty}^{+\infty} X_n e^{j 2\pi n f_0 t}$$

Dimostrazione della biunivocità

$$X_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\, e^{-j 2\pi n f_0 t}\, dt =$$

$$= \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \sum_{m=-\infty}^{+\infty} X_m e^{j 2\pi m f_0 t}\, e^{-j 2\pi n f_0 t}\, dt$$

$$= \sum_{m=-\infty}^{+\infty} X_m \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} e^{j 2\pi (m-n) f_0 t}\, dt$$

$$\Rightarrow \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} e^{j 2\pi (m-n) f_0 t}\, dt = \begin{cases} T_0 & m = n \\ 0 & m \neq n \end{cases} = T_0\, \delta[m-n]$$

$$X_n = \frac{1}{T_0} \sum_{m=-\infty}^{+\infty} X_m T_0 \delta[m-n] = X_n \qquad \text{c.v.d.}$$

Es. 6 - Fila B

$$x(t) \overset{TCF}{\Longleftrightarrow} X(f)$$

$$y(t) = \frac{d}{dt} x(t) \quad \overset{TCF}{\Longleftrightarrow} \quad j2\pi f X(f)$$

Dim.

$$y(t) = \frac{d}{dt} x(t) = \frac{d}{dt} \int_{-\infty}^{+\infty} X(f) e^{j2\pi f t} df =$$

$$= \int_{-\infty}^{+\infty} X(f) \frac{d}{dt}\left[e^{j2\pi f t}\right] df = \int_{-\infty}^{+\infty} \underbrace{j2\pi f X(f)}_{Y(f)} e^{j2\pi f t} df$$

$$y(t) = ATCF\left[Y(f)\right]$$

$$\Downarrow$$

$$TCF\left[y(t)\right] = Y(f) = j2\pi f X(f)$$

c.v.d.