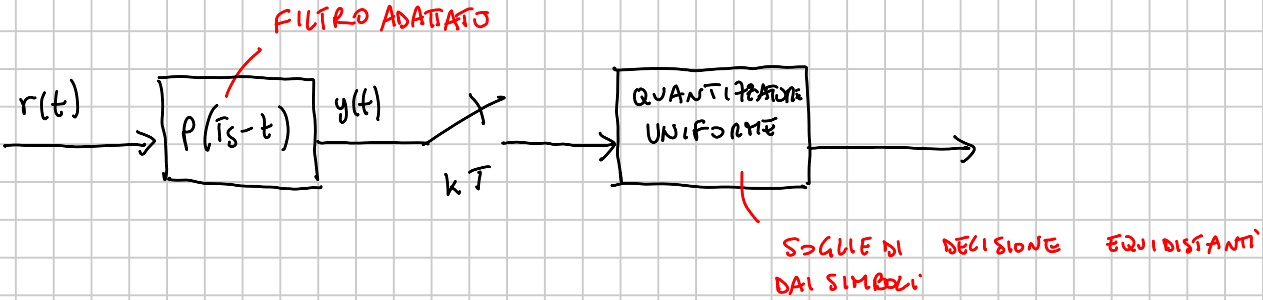
RICEVITORE OTTIMO PER UN SISTEMA DI COMUNICAZIONE PAM

Per un sistema di comunicazione PAM con simboli equiprobabili e nel caso di rumore AWGN, il ricevitore ottimo è

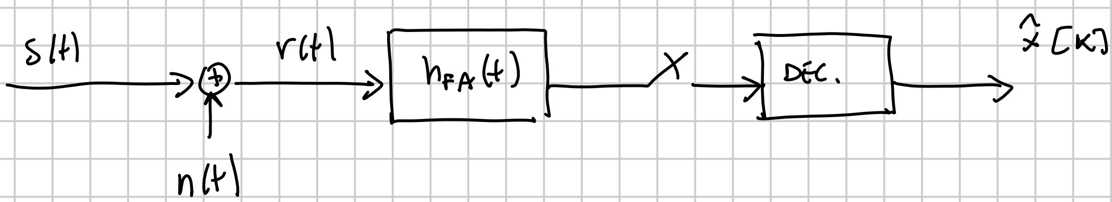


$$P_E(b) \triangleq \Pr\{\hat{b}[k] \neq b[k]\}$$

$$P_E(M) \triangleq \Pr\{\hat{x}[k] \neq x[k]\}$$

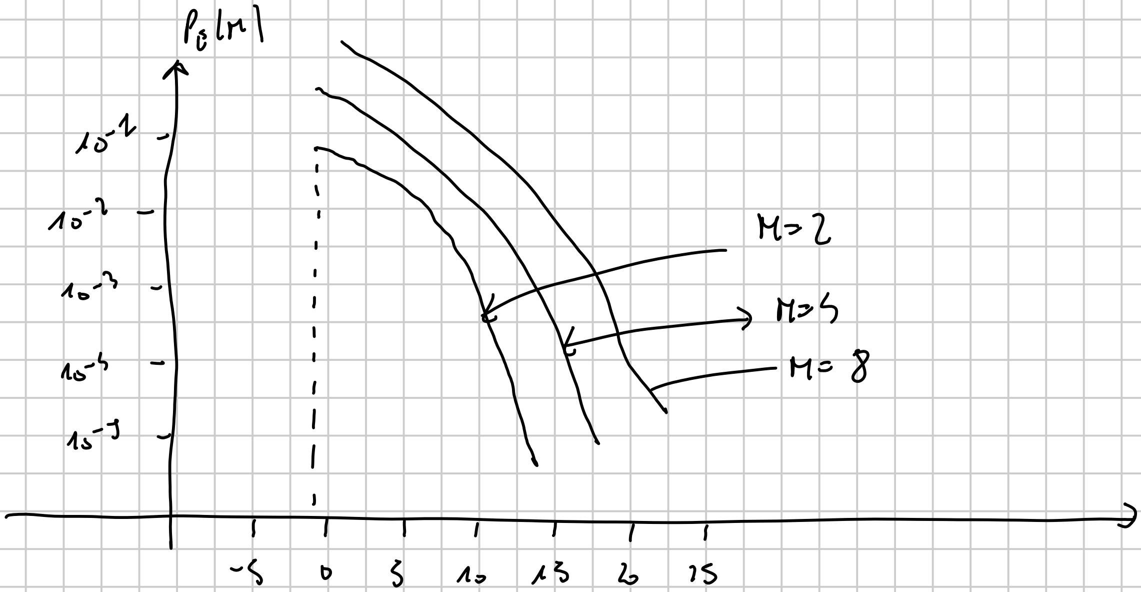
$$\boxed{\frac{P_E(M)}{\log_2 M} \leq P_E(b) \leq \frac{M/2}{M-1} P_E(M)}$$

PRESTAZIONI DI UNA M-PAM IDEALE, IN PRESENZA DI SOLO RUMORE



$$s(t) = \sum_{k=-\infty}^{+\infty} x[k]\, p(t-kT_S)$$

$$P_E(M) = \left(\frac{M-1}{M}\right) \operatorname{erfc}\left(\sqrt{\frac{3 \log_2 M \ \mathrm{SNR}}{(M^2-1)}}\right)$$



Per SNR > 10 dB e Gray

$$P_E(b) \simeq \frac{P_E(M)}{\log_2 M}$$

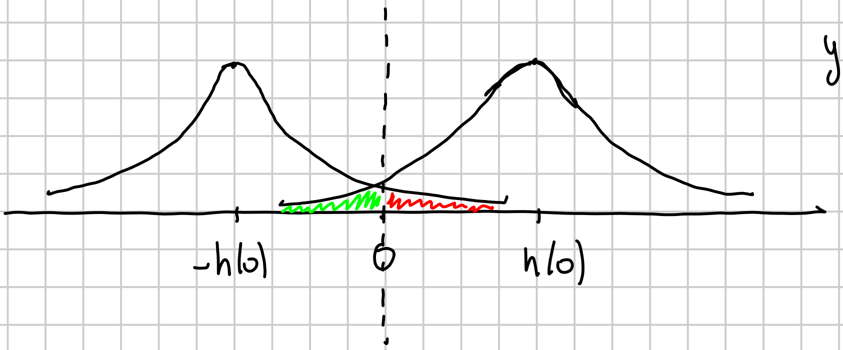
2-PAM

$$P_E(M) = P_E(b) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{SNR}\right)$$

$$y[k] = x[k]\,h(0) + n[k]$$

$$y[k] = x[k] + n[k]$$

$$y[k] \in \mathcal{N}\left(x[k], \sigma_n^2\right)$$



$$P_E(b) = \underbrace{Pr\{x_k = +1\}\,Pr\{\hat{x}_k = -1 \mid x_k = 1\}}_{Pr\{\hat{x} = -1,\ x = 1\}} + \underbrace{Pr\{x_k = -1\}\cdot Pr\{\hat{x}_k = 1 \mid x_k = -1\}}_{Pr\{\hat{x} = 1,\ x = -1\}}$$

$$\Pr\{\hat{x}_k = -1 \mid x_k = 1\} = \int_{-\infty}^{0} f_y(y \mid x_k = 1)\, dy$$

$h(0) = 1$

$$f_y(y \mid x_k = 1) = \frac{1}{\sqrt{2\pi\, \sigma_n^2}}\, e^{-\frac{(y - h(0)x_k)^2}{2\sigma^2_n}}$$

$$y[k] = x[k]\, h(0) + n_u[k]$$

$$\sigma^2_{nu} = \frac{N_0}{2} E_{hr}$$

$$S_w(f) = S_N(f) \cdot |H_R(f)|^2$$

$$\sigma^2_{nu} = \int S_w(f)\, df$$

$$S_N(f) = \frac{N_0}{2}$$

$$H_R(f) = H_{FA}(f)$$

$$E_{hR} = h(0)$$

$$H_{FA}(f) = P(f)$$

$$\int H^2_{FA}(f)\, df = \int P^2(f)\, df$$

NTC con $C(f) = 1 \Rightarrow H(f) = P(f) \cdot H_{FA}(f)$

$$\int P^2(f)\, df = \int H_{FA}^2(f)\, df = \int H(f)\, df = h(0)$$

$\sigma_{nu}^2 = \frac{N_0}{2} h(0)$

$E_s = h^2(0)$

$$\Rightarrow SNR = \frac{h^2(0)}{\frac{N_0}{2} h(0)} = \frac{2\, h(0)}{N_0} = \frac{2E_s}{N_0}$$

$$Pr\{\hat{x} = -1 \mid x = 1\} = \int_{-\infty}^{0} f_y(y \mid x = 1)\, dy =$$

$$= \Phi\left(\frac{h(0) - 0}{\sigma_{nu}}\right) = 1 - Q\left(\frac{0 - h(0)}{\sigma_{nu}}\right) =$$

$$= Q\left(\frac{h(0)}{\sigma_{nu}}\right) = Q\left(\sqrt{\frac{2\, h(0)}{N_0}}\right) \triangleq \frac{1}{2}\, \mathrm{erfc}\left(\sqrt{SNR}\right)$$

$$SNR = \frac{2E_s}{N_0} = \frac{2\, h(0)}{N_0}$$

$$Pr\{\hat{x}_k = 1 \mid x_k = -1\} = Pr\{\hat{x}_k = -1 \mid x_k = 1\}$$

$$P_E(b) = \frac{1}{2} \cdot Q\left(\sqrt{SNR}\right) + \frac{1}{2}\, Q\left(\sqrt{SNR}\right) = Q\left(\sqrt{SNR}\right)$$

PRESENZA DI ISI E DI RUMORE (Canale $c(f)$ non ideale)

$$y(t) = \sum_{k=-\infty}^{+\infty} x[k]\, h(t - kT_s) + n_u(t)$$

$$h(t) = p(t) \otimes c(t) \otimes h_R(t)$$

$$n_u(t) = n(t) \otimes h_R(t)$$

$$y[k] = x[k]\, h(0) + I[k] + n_u[k]$$

$$I[k] = \sum_{\substack{n=-\infty \\ n \neq k}}^{+\infty} x[n]\, h((k-n)T_s)$$

Il filtro $P(f)$ e $h_R(t)$ devono essere tali da eliminare ISI e massimizzare SNR

Massimizzo SNR con il vincolo $I(k) = 0 \quad \forall k$

RADICE DI COSENO RIALZATO

$$\boxed{\cdot)\ I[k] = 0 \quad \text{quando} \quad h(t) = h_{RC}(t)}$$

$$P(f)\, C(f)\, H_R(f) = H_{RC}(f)\, e^{-j 2\pi f T_s}$$

$$\boxed{\begin{aligned} |P(f)| &= |H_R(f)| = \sqrt{\frac{H_{RC}(f)}{|C(f)|}} \\ \angle P(f) &= \angle H_R(f) = -\pi f T_s - \frac{\angle C(f)}{2} \end{aligned}}$$

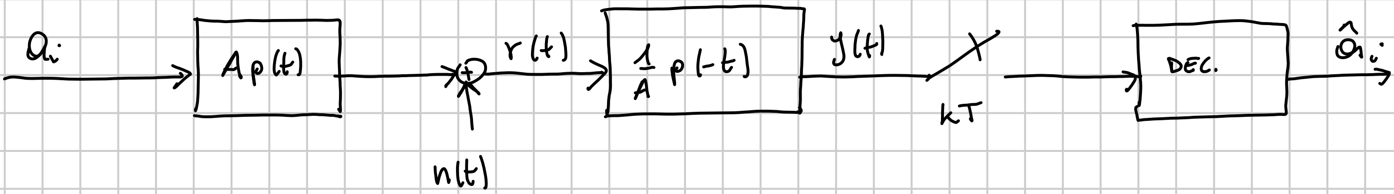
Se $C(f) = 1 \Rightarrow P(f) = H_R(f) = \sqrt{H_{RC}(f)}\, e^{-j 2\pi f T_s}$

$$SNR = \frac{E\left[x^2[k]\right] \cdot h^2(0)}{\frac{N_0}{2} \int_{-B_c}^{B_c} \frac{H_{RC}(f)}{|C(f)|}\, df}$$

Se $C(f) = 1 \Rightarrow SNR = \frac{2E_s}{N_0}$

Esercizio #1

4-PAM

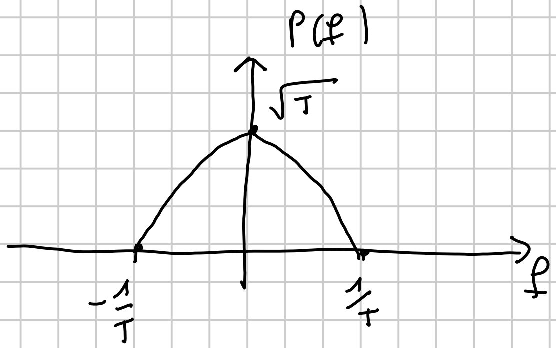


$$r(t) = \sum_{i=-\infty}^{+\infty} a_i\, A\, p(t - iT) + n(t)$$

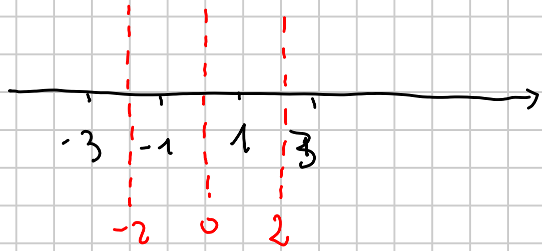
$a_i \in A_s \{\pm 3, \pm 1\}$ indipendenti ed equiprobabili

$n(t)$ è AWGN $\quad S_N(f) = \frac{N_0}{2}$

$$P(f) = \begin{cases} \sqrt{T}\,\sqrt{1-|fT|} & |fT| \le 1 \\ 0 & \text{altrove} \end{cases}$$



$$\hat{a}_k = \begin{cases} 3 & \text{se} \quad y[k] \ge 2 \\ 1 & \text{se} \quad 0 \le y[k] < 2 \\ -1 & \text{se} \quad -2 \le y[k] < 0 \\ -3 & \text{se} \quad y[k] < -2 \end{cases}$$



1) E_s energia media per simbolo Tx

2) $P_E(M)$ in funzione di $\frac{E_s}{N_0}$

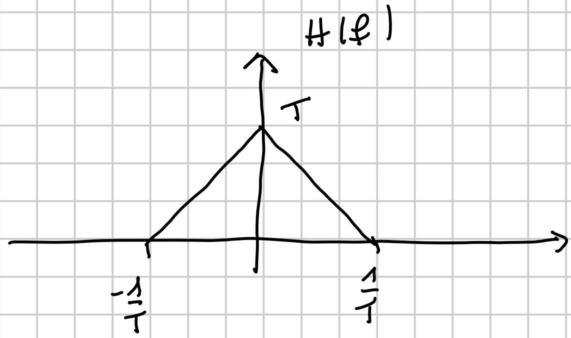
Segnale all'uscita del filtro adattato

$$y(t) = \sum_i a_i\, h(t - iT) + w(t)$$

$$h(t) = A\,p(t) \otimes h_{FA}(t) = A\,p(t) \otimes \frac{1}{A}\,p(-t) = p(t) \otimes p(-t)$$

$$H(f) = P(f)\,P^*(f) = |P(f)|^2$$

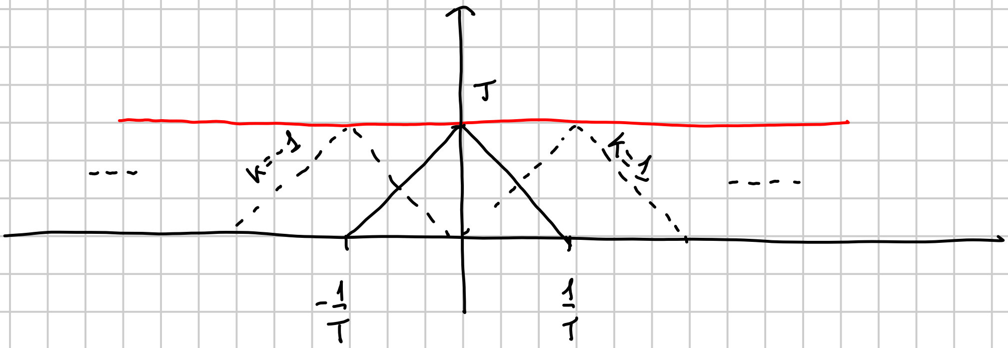
$$H(f) = \begin{cases} T\,(1-|fT|) & |fT| \le 1 \\ 0 & \text{altrove} \end{cases}$$



$$h(0) = \int_{-\infty}^{+\infty} H(f)\,df = \frac{2}{T}\cdot T\cdot\frac{1}{2} = 1$$

Dobbiamo controllare & si verifica ISI

Dopo il campionamento $\Rightarrow \underbrace{\sum_{K} H\left(f-\frac{K}{T}\right)} = T \Rightarrow h(0) = 1$



Calcolare la potenza di rumore in uscita al filtro in ricezione

$$S_W(f) = S_N(f)\,|H_{FA}(f)|^2 \qquad h_{FA}(t) = \frac{1}{A}\,p(-t)$$

$$\sigma_W^2(f) = \int_{-\infty}^{+\infty} S_W(f)\,df = \frac{N_0}{2}\frac{1}{A^2}\int_{-\infty}^{+\infty} |H_{FA}(f)|^2\,df =$$

$$= \frac{N_0}{2}\frac{1}{A^2}\int_{-\infty}^{+\infty} |P(f)|^2\,df = \frac{N_0}{2}\frac{1}{A^2}\int_{-\infty}^{+\infty} H(f)\,df = \frac{N_0}{2A^2}h(0) = \frac{N_0}{2A^2}h(0) = \frac{N_0}{2A^2}$$

1) Energia media per simbolo Tx

2) Energia media per intervallo di segnalazione del segnale Tx

(1) $E_s = \sum_k pr\{\alpha_k\} \cdot E_{pk} = \sum_k pr\{\alpha_k\} \, \alpha_k^2 \cdot E_p$

(2) $E_s = E\left\{ \int_0^T s^2(t) \, dt \right\} = P_s \cdot T$

$$P_s = \int_{-\infty}^{+\infty} S_s(f) \, df$$

Se $E\{\alpha_i\} = 0$ e/o vale l'ortogonalità

$$\int_{-\infty}^{+\infty} s_i(t) \, s_j(t) \, dt = 0 \qquad i \neq j$$

$$E_s = E\left[\int_0^T s^2(t) \, dt \right] = E\left[\int_0^T \sum_k a_k \, p(t-kT) \cdot \sum_n a_n \, p(t-nT) \right] dt =$$

$$= \int_0^T \sum_k \sum_n E\{a_k \cdot a_n\} \cdot p(t-kT) \, p(t-nT) \, dt$$

Se $E\{a_n\} = 0$

$$E_s = \int_0^T \sum_k E\{a_k^2\} \cdot p^2(t-kT) = \sum_k E\{a_k^2\} \cdot \int_0^T p^2(t-kT) \, dt$$

$$E_S = \frac{1}{4} \cdot E_{n\,-3} + \frac{1}{4} \cdot E_{n\,-1} + \frac{1}{4} \cdot E_{n\,1} + \frac{1}{4} \cdot E_{n\,3}$$

$$E_{n\,-3} = \int_{-\infty}^{+\infty} (-3)^2 \; p^2(t)\,dt = 9 \cdot E_p = E_{n\,3}$$

$$E_{n\,-1} = \int_{-\infty}^{+\infty} (-1)^2 \; p^2(t)\,dt = 1 \cdot E_p = E_{n\,1}$$

$$E_n = \left(\frac{1}{4} (-3)^2 + \frac{1}{4} (-1)^2 + \frac{1}{4} (1)^2 + \frac{1}{4} (3)^2 \right) E_p =$$

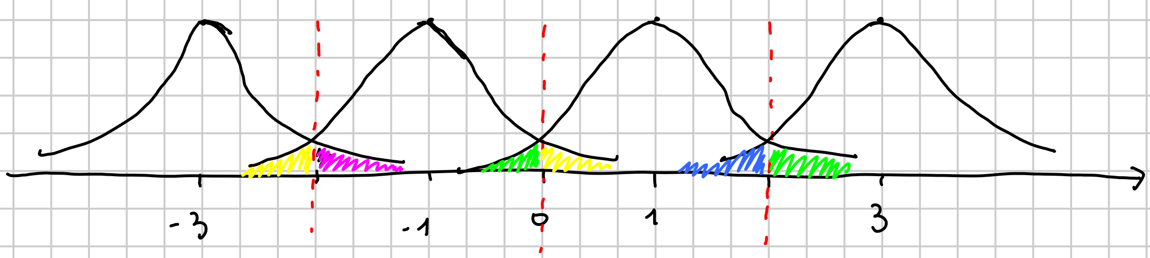
$$= \frac{1}{4} \left(9 + 1 + 1 + 9 \right) E_p = 5\, E_p$$

$$E_p = \int_{-\infty}^{+\infty} p^2(t)\,dt = A^2 \cdot \frac{2}{T} \cdot T \cdot \frac{1}{2} = A^2$$

$$E_S = 5A^2$$

Calcoliamo la probabilità di errore

$$y[k] = a_k\, h(0) + w_k = a_k + w_k \in \mathcal{N}\left(a_k, \sigma_w^2\right)$$



$$P_E(M) = \frac{1}{4} Pr\left\{ \hat{a}_k \neq -3 \mid a_k = -3 \right\} + \frac{1}{4} Pr\left\{ \hat{a}_k \neq -1 \mid a_k = -1 \right\} + \frac{1}{4} Pr\left\{ \hat{a}_k = 1 \mid a_k = 1 \right\}$$

$$+ \frac{1}{4} Pr\left\{ \hat{a}_k \neq 3 \mid a_k = 3 \right\}$$

$$Pr\left\{ \hat{a}_k \neq 3 \mid a_k = 3 \right\} = \int_{-\infty}^{2} f_Y(y \mid a_k = 3)\, dy = \int_{-\infty}^{2} \frac{1}{\sqrt{2\pi\sigma_w^2}} e^{-\frac{(y-3)^2}{2\sigma_w^2}} dy =$$

$$= Q\left(\frac{3-2}{\sigma_w}\right) = Q\left(\frac{1}{\sigma_w}\right)$$

$$Pr\left\{ \hat{a}_k \neq -3 \mid a_k = -3 \right\} = Q\left(\frac{1}{\sigma_w}\right)$$

$$Pr\left\{ \hat{a}_k \neq -1 \mid a_k = -1 \right\} = Pr\left\{ y[k] < -2 \mid a_k = -1 \right\} + Pr\left\{ y[k] > 0 \mid a_k = 1 \right\} =$$

$$= Q\left(\frac{-1+2}{\sigma_w}\right) + Q\left(\frac{0+1}{\sigma_w}\right) = 2Q\left(\frac{1}{\sigma_w}\right)$$

$$Pr\left\{ \hat{a}_k \neq 1 \mid a_k = 1 \right\} = 2Q\left(\frac{1}{\sigma_w}\right)$$

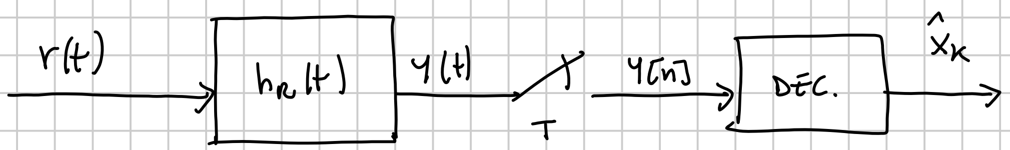
$$P_E(M) = \frac{1}{4} Q\left(\frac{1}{\sigma_w}\right) + \frac{1}{4} \cdot 2Q\left(\frac{1}{\sigma_w}\right) + \frac{1}{4} \cdot 2Q\left(\frac{1}{\sigma_w}\right) + \frac{1}{4} Q\left(\frac{1}{\sigma_w}\right)$$

$$P_E(M) = \frac{3}{2} Q\left(\frac{1}{\sigma_w}\right)$$

$$\sigma_N^2 = \frac{N_0}{2A^2} = \frac{5}{5}\,\frac{N_0}{2A^2} = \frac{5}{2}\,\frac{N_0}{E_S}$$

$$P_E(M) = \frac{3}{2}\,Q\left(\sqrt{\frac{2}{5}\,\frac{E_S}{N_0}}\right)$$

Esercizio #2

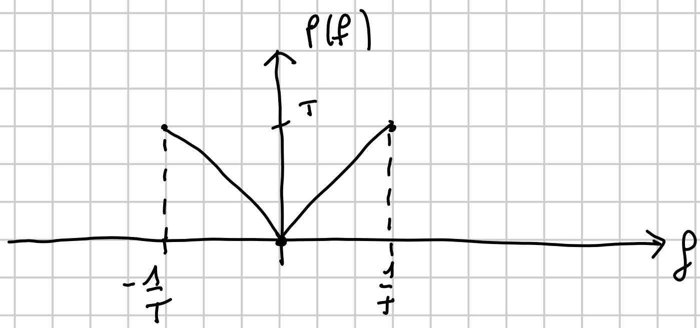


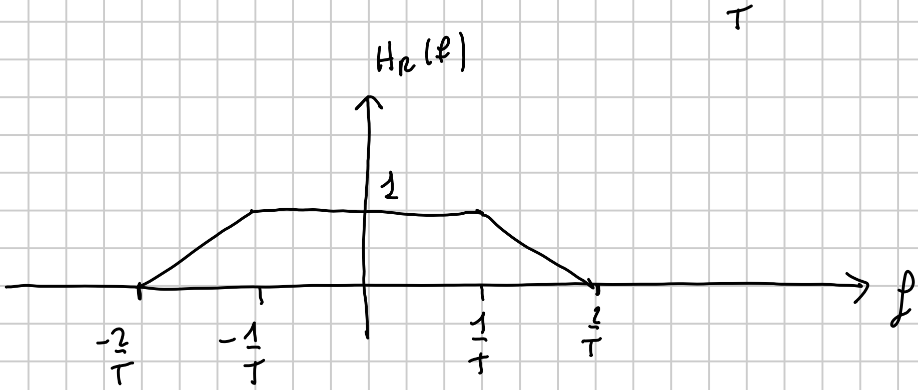
$$r(t) = \sum_{k=-\infty}^{+\infty} x_k\, p(t-kT) + n(t)$$

$x_k \in A_S(0,4)$

$n(t)$ è AWGN $\qquad S_N(f) = \frac{N_0}{2}$

$$P(f) = T\,|fT|\,\mathrm{rect}\left(\frac{fT}{2}\right)$$





$$\hat{x}_k = \begin{cases} 0 & \text{se } \ y[k] \leq \lambda \\ 4 & \text{se } \ y[k] > \lambda \end{cases} \qquad \lambda = 3/2$$

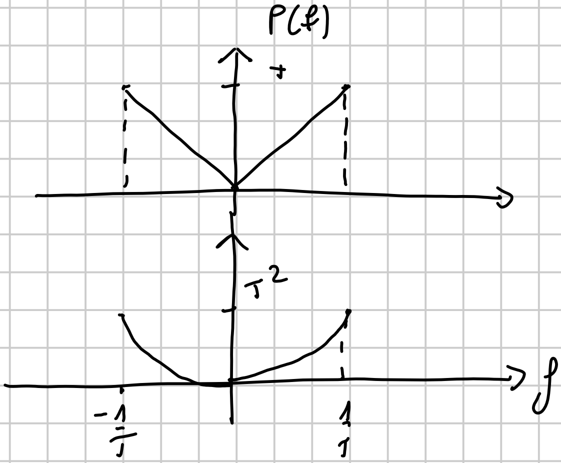
1) E_s energia media per simbolo trasmesso

2) DSP di $s(t)$

3) P_{na}

4) $P_E(b)$

(1) $E_s = \frac{1}{2} \cdot E_{s0} + \frac{1}{2} E_{s4}$



$E_{s0} = 0$

$$E_{s4} = (4)^2 \int_{-\infty}^{+\infty} p^2(t)\,dt = 16 \cdot \frac{2}{3} T^2 \frac{1}{T} = \frac{32}{3} T$$

$$E_s = \frac{1}{2} E_{s4} = \frac{16}{3} T$$

$$s(t) = \sum_k x_k \, p(t - kT)$$

$$S_s(f) = \frac{1}{T_s} S_x(f) \, |P(f)|^2$$

$S_x(f)$ è la TF della funzione di autocorrelazione dei simboli

$$S_x(f) = \sum_{m=-\infty}^{+\infty} R_x[m]\, e^{-j2\pi f m T}$$

$$R_x[m] = E\{x_i \cdot x_{i+m}\} = \begin{cases} E\{x_i^2\} & m=0 \\ E\{x_i \cdot x_{i+m}\} & m\neq 0 \end{cases}$$

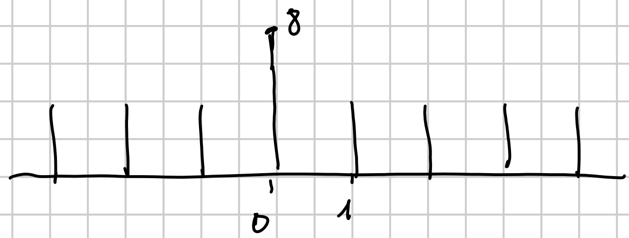
Se i simboli sono indipendenti

$$R_x[m] = \begin{cases} E\{x_i^2\} & m=0 \\ E^2\{x_i\} & m\neq 0 \end{cases}$$

$$E\{x_i\} = \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 4 = 2$$

$$E\{x_i^2\} = \frac{1}{2}\cdot (0)^2 + \frac{1}{2}\cdot (4)^2 = 8$$

$$R_x[m] = \begin{cases} 8 & m=0 \\ 4 & m\neq 0 \end{cases}$$

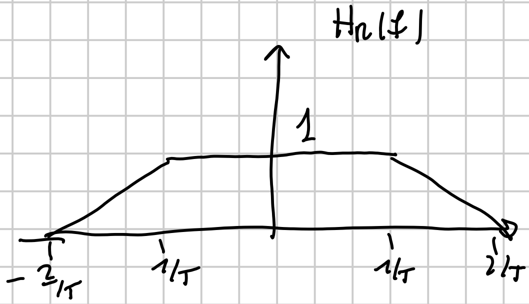


$$R_x[m] = 4\,\delta[m] + 4$$

$$S_x(f) = 4 + \frac{4}{T} \sum_k \delta\left(f - \frac{k}{T}\right)$$

$$S_s(f) = \frac{1}{T} |P(f)|^2 \left[4 + \frac{4}{T} \sum_k \delta\left(f - \frac{k}{T}\right) \right] =$$

$$= \frac{4}{T} |P(f)|^2 + \frac{4}{T^2} \sum_k \left|P\left(\frac{k}{T}\right)\right|^2 \delta\left(f - \frac{k}{T}\right)$$

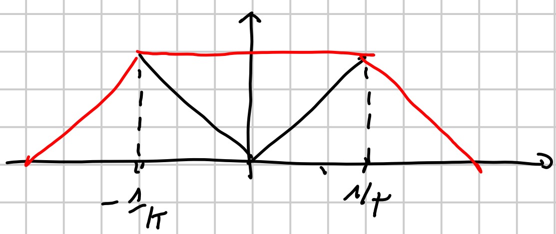


$P_{nu} = ?$

$$S_{nu}(f) = S_n(f) \, |H_R(f)|^2 = \frac{N_0}{2} |H_R(f)|^2$$

$$P_{nu} = \int S_{nu}(f)\, df = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H_R(f)|^2 \, df = \frac{N_0}{2} \left[\frac{2}{T} + \frac{2}{3} \frac{1}{T} \cdot 1 \right] = \frac{4}{3} \frac{N_0}{T}$$

$$y(t) = \sum_k x_k \, h(t - kT) + n_u(t)$$

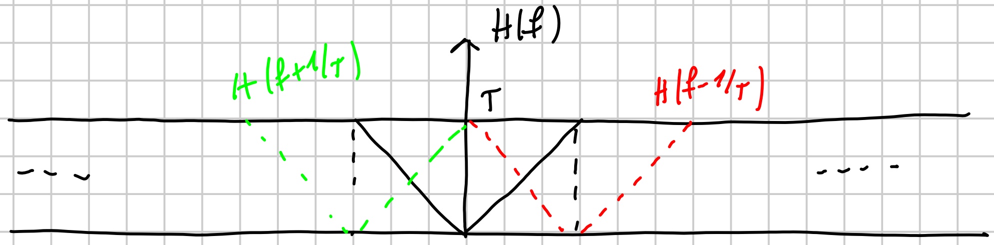


$n_u(t) = n(t) \otimes h_R(t)$

$h(t) = p(t) \otimes h_R(t)$

$H(f) = P(f) \cdot H_R(f) = P(f)$

Dobbiamo verificar se $\sum_n H(f - n/T) =$ costante



$$\sum_n H(f - n/T) = T \quad \Rightarrow \quad h(0) = 1$$

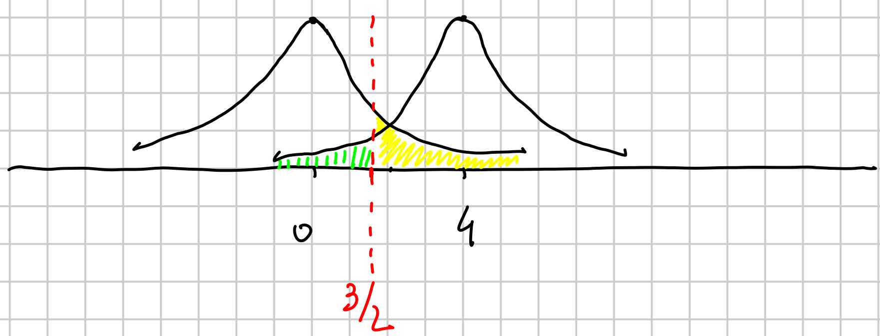
$$h(0) = \int H(f)\, df = \frac{2}{T} \cdot T \cdot \frac{1}{2} = 1$$

$$y_k = x_k \cdot h(0) + n_{uk} = x_k + n_{uk} \quad \in \mathcal{N}(x_k, \sigma^2_{nu})$$

$$\sigma^2_{nu} = P_{nu}$$

$$P_r\{x_k = 0\} = P_r\{x_k = 4\} = \frac{1}{2}$$

$$P_E(b) = P_E(M) = \underbrace{P_r\{x_k = 0\}}_{=1/2} \cdot P_r\{\hat{x}_k = 4 \mid x_k = 0\} + \underbrace{P_r\{x_k = 4\}}_{=1/2} \cdot P_r\{\hat{x}_k = 0 \mid x_k = 4\}$$

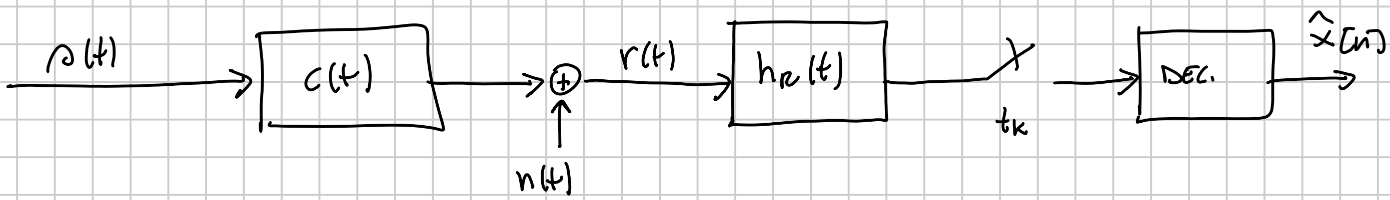


$$Pr\left\{\hat{x}_k = 4 \mid x_k = 0\right\} = Q\left(\frac{3/2 - 0}{\sigma_{nu}}\right) = Q\left(\frac{3}{2\sigma_{nu}}\right)$$

$$Pr\left\{\hat{x}_k = 0 \mid x_k = 4\right\} = Q\left(\frac{4 - 3/2}{\sigma_{nu}}\right) = Q\left(\frac{5}{2\sigma_{nu}}\right)$$

$$P_E(M) = P_E(b) = \frac{1}{2} Q\left(\frac{3}{2\sigma_{nu}}\right) + \frac{1}{2} Q\left(\frac{5}{2\sigma_{nu}}\right)$$

Esercizio #3



$$c(t) = \delta(t)$$

$$s(t) = \sum_k x_k \, p(t - kT) \qquad x_k \in A_s = \{0, 1\}$$

$$p(t) = \begin{cases} \sqrt{1/T} & \frac{T}{8} \le t \le \frac{T}{8} + T \\ 0 & \text{altrove} \end{cases}$$

$$S_N(f) = \frac{N_0}{2} \qquad \lambda = \frac{1}{4}$$

1) E_s energia media per simbolo trasmesso

2) Sia $h_R(t) = A\,p(t_0 - t)$ determinare t_0 in modo che $h_R(t)$ sia causale

3) Calcola P_{nu}

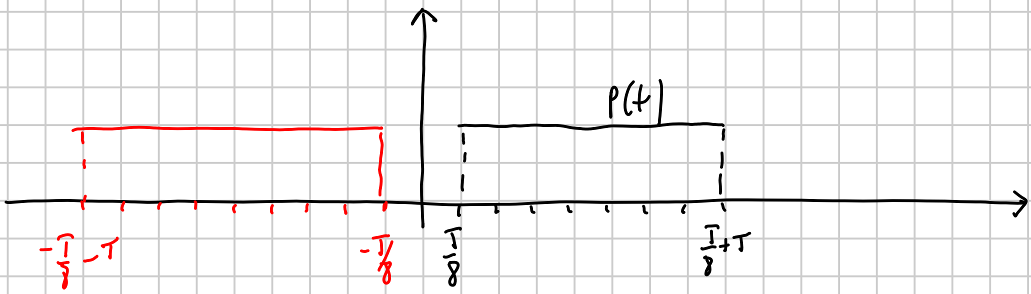
4) Determinare l'istante di campionamento ottimo

5) Determinare A in modo che $P_E(b)$ sia minima

(1) $E_s = \frac{1}{2} E_{s0} + \frac{1}{2} E_{s1} = \frac{1}{2} E_{s1} = \frac{1}{2}$

$$E_{s1} = (1)^2 \int_{-\infty}^{+\infty} p^2(t)\,dt = \frac{1}{T} \cdot T = 1$$

(2) $h_R(t) = A\,p(t_0 - t)$



Affinché sia causale $t_0 = \frac{9T}{8}$

$$h_R(t) = A\,p\left(\frac{9T}{8} - t\right)$$

(3) $S_{n_u}(f) = S_n(f)\,|H_R(f)|^2$

$$P_{n_u} = \sigma^2_{n_u} = \frac{N_0}{2}\int |H_R(f)|^2\, df = \frac{N_0}{2} A^2 \cancel{T}\,\frac{1}{\cancel{T}} = \frac{N_0}{2} A^2$$

$$h_R(t) = A\, p(t_0 - t) \qquad p(t) = \sqrt{\frac{1}{T}}\ \mathrm{rect}\left(\frac{t - 5T/8}{T}\right)$$

(4) $C(f) = 1$

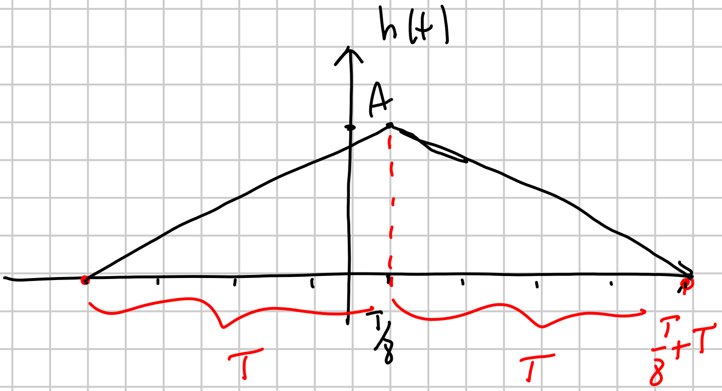
$$H(f) = P(f)\, H_R(f)$$

$$P(f) = \frac{1}{\sqrt{T}}\, T\ \mathrm{sinc}(fT)\, e^{-j2\pi f\, 5T/8}$$

$$H_R(f) = A\, P(f)\, e^{j2\pi f\, 9T/8} = A\sqrt{T}\ \mathrm{sinc}(fT)\, e^{j2\pi f T/2}$$

$$H(f) = AT\ \mathrm{sinc}^2(fT)\, e^{-j2\pi f T/8}$$

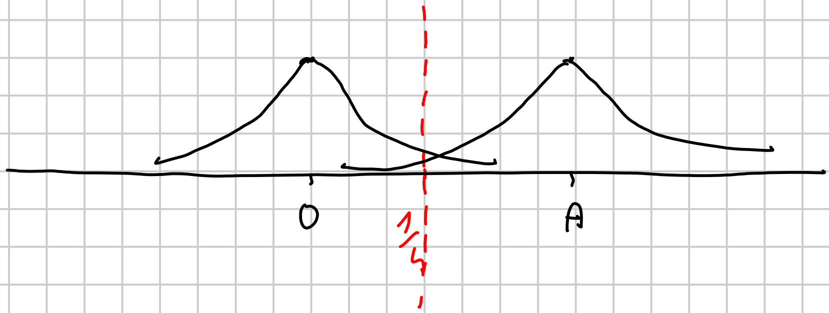
$$h(t) = A\left(1 - \frac{|t - T/8|}{T}\right)\ \mathrm{rect}\left(\frac{t - T/8}{2T}\right)$$



$$y(t) = \sum_k x_k\, h(t - kT) + n_u(t)$$

$$h(t)\Big|_{t=t_k} = \begin{cases} K = A & k=0 \\ 0 & \text{altrove} \end{cases} \quad \text{se} \quad \boxed{t_k = \frac{T}{8} + kT}$$

$$y_k = A x_k + n_{uk} \in \mathcal{N}(A x_k, \sigma_{nu}^2) \qquad h(0) = A$$



$\lambda = 1/4$

Si suppone $A > 1/4$

$$\Pr\{\hat{x}_k = 0 \mid x_k = 1\} = Q\left(\frac{A-\lambda}{\sigma_{nu}}\right) = Q\left(\frac{A-1/4}{\sigma_{nu}}\right)$$

$$\Pr\{\hat{x}_k = 1 \mid x_k = 0\} = Q\left(\frac{1/4}{\sigma_{nu}}\right)$$

$$P_E(b) = \frac{1}{2} Q\left(\frac{1/4}{\sigma_{nu}}\right) + \frac{1}{2} Q\left(\frac{A-1/4}{\sigma_{nu}}\right) \qquad \sigma_{nu} = \sqrt{\frac{2A^2}{N_0}}$$

$$P_E(b) = \frac{1}{2} Q\left(\sqrt{\frac{1}{8A^2 N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{2(A-1/4)^2}{N_0 A^2}}\right)$$

Sapendo che il rumore è AWGN, canale è ideale e ricevitore ottimo il decisore ottimo è quello a minima distanza euclidea e quindi la soglia deve essere equidistante dai simboli

Poiché $\lambda = \frac{1}{4} \Rightarrow A = \frac{1}{2}$

$P_E(A)$

$$\frac{\partial P_E(b)}{\partial A} = 0$$

$$\frac{\partial Q}{\partial A}\left(\sqrt{\frac{1}{8A^2N_0}}\right) + \frac{\partial}{\partial A} Q\left(\sqrt{\frac{2(A-1/4)^2}{N_0A^2}}\right) = 0$$

$$\frac{1}{2} e^{-\frac{1}{4\sigma_n(A)}} \cdot \frac{\partial}{\partial A}\left(\frac{1}{4\sigma_n(A)}\right) + \frac{1}{2} e^{-\frac{(A-1/4)}{\sigma_n(A)}} \cdot \frac{\partial}{\partial A}\left(\frac{A-1/4}{\sigma_n(A)}\right) = 0$$

$$\frac{1}{4\sigma_n(A)} = \frac{1}{4A}\sqrt{\frac{2}{N_0}} \Rightarrow \frac{\partial}{\partial A}\left(\frac{1}{4\sigma_n(A)}\right) = -\frac{1}{A^2}\frac{1}{4}\sqrt{\frac{2}{N_0}}$$

$$\frac{A-1/4}{\sigma_n(A)} = \frac{A-1/4}{A\sqrt{\frac{N_0}{2}}} \Rightarrow \frac{\partial}{\partial A}\left(\frac{A-1/4}{A\sqrt{\frac{N_0}{2}}}\right) = \frac{1}{4A^2}\sqrt{\frac{2}{N_0}}$$

$$e^{-\frac{1}{4\sigma_n(A)}}\left(-\frac{1}{4A^2}\sqrt{\frac{2}{N_0}}\right) + e^{-\frac{(A-1/4)}{4\sigma_n(A)}}\left(\frac{1}{4A^2}\sqrt{\frac{2}{N_0}}\right) = 0$$

$$e^{-\frac{1}{4\sigma_n(A)}} = e^{-\frac{(A-1/4)}{4\sigma_n(A)}}$$

$$+\frac{1}{4\sigma_n(A)} = +\frac{(A-1/4)}{4\sigma_n(A)}$$

$$\frac{1}{8A^2 \cancel{N_0}} = \frac{2(A-1/4)^2}{\cancel{N_0 A^2}} \Rightarrow A = \frac{1}{2}$$

$$\cancel{\frac{1}{8}} = 2A^2 + \cancel{\frac{1}{8}} - A \Rightarrow A(2A-1) = 0$$

$$A = 1/2$$