Part 2 Recap

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Nyquist Stability Criterion

- **Goal**
  - When we want to study the stability of the closed loop system:
    - We want to find the roots of $1 + G(s)H(s) = 0$,
    - this corresponds to take the open loop transfer function $G(s)H(s)$, add 1 and find its zeros.
- **Why is this difficult**
  - The system can be high order (e.g. order 50),
  - Finding zeros would only give us stability information,
  - Other information could be useful (e.g. stability margins).

Cauchy's argument principle

> - We can tell the relative difference between the number of poles and zeros inside of a contour by counting how many time the plot circles the origin and in which direction

- Apply Caushy's argument principle to know if there are zeros of $1 + GH$ in the right half plane (in which case the system is unstable)

- Plot GH and shift the origin to the left by 1: we can look at how many circling of the point $-1 + 0j$ the plot of GH does.

The Nyquist Plot

Steps:

- Take the open loop transfer function GH
- Plot the Nyquist plot of GH
  - 1. Set $s = j\omega$ in the transfer function
  - 1. Sweep $\omega \in [0, \infty]$ and plot the resulting complex numbers
  - 1. Draw the reflection about the real axis to account for negative ω
  - Note that the Nyquist contour is traced in the **clock-wise** direction (by convention)

Key is step #2:

- For simple transfer functions there are only four points that we need to solve for
  - 1. $|G|$ and $\angle G$ at $\omega = 0$ (start of the plot)
  - 1. $|G|$ and $\angle G$ at $\omega = \infty$ (mid point of the plot)
  - 1. Intersections with the imaginary axis
  - 1. Intersections with the real axis

- Count the number of times the point -1 is encircled and in which direction
- Determine the relative number of poles and zeros inside the nyquist contour.

Therefore:

$$Z = N + P$$

where

- Z is the number of zeros in the right half plane (or poles in closed loop)
- N is the number of clockwise encirclements of -1
- P is the number of open loop right half plane poles

Stability margins

- Gain and Phase margin: extra gain or phase that we have available before the system starts to oscillates or become unstable

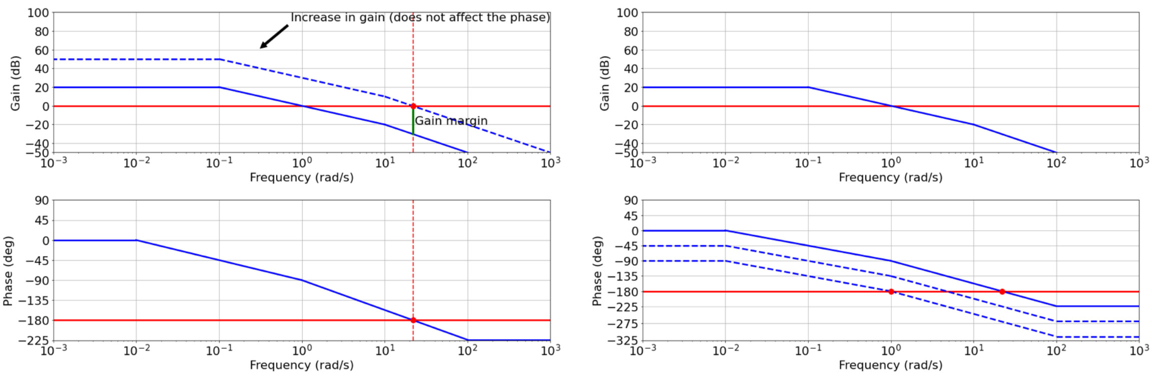
- Robustness of the system

- Makes it possible to quantify how stable a system is

    - Systems (or designs) that have less margin could be considered as being "less" stable: smaller variations in the system could lead to instability
- We want our controller to be robust to these because we cannot know our system perfectly and hence we need margins.

- The more uncertainty we have the more margin we should design in.

- **Gain margin**: The gain required to cross the 0dB line at the frequency where phase is -180^o
- **Phase margin**: how much phase delay takes to make -180^o phase at the 0 dB gain frequency.



- Uncertainty in one specific parameters can affect you more than you think.

Loop analysis and loop shaping

Feedback Goals

- Stability of the closed loop system
- Performance while tracking inputs
  - tracking error, typically specified in terms of steady state error to specific inputs:
    - e.g., error to step inputs less than 5%
  - behaviour of the transient, typically specified in terms of bandwidth, rise time, settling time, damping ratio, overshoot (these are requirements for the transfer function between Y and Y_{ref}).
- Robustness to noise measurement and disturbances

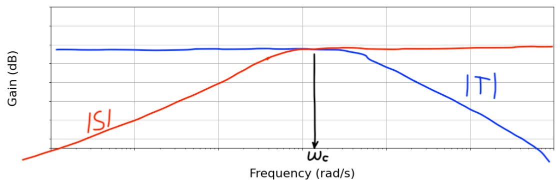
- One advantage of the Nyquist stability theorem is that it is based on the loop transfer function $L = GR$,
- Easy to see how the controller influences the loop transfer function.
- **Loop shaping**: Choose a compensator that gives a loop transfer function with a desired shape.

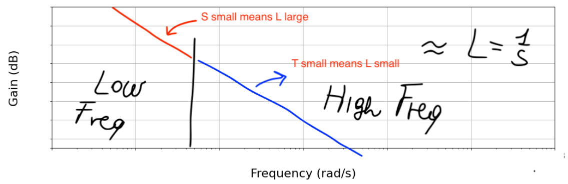
Sensitivity and Complementary Sensitivity Functions

We want to design R so that all transfer functions have good properties

- tracking (at freq. where this is important)
- disturbance rejection (at freq. where this is important)
- noise attenuation (at freq. where this is important)

- $L(s) = G(s)R(s)$
- $S(s) = \frac{1}{1+L(s)}$
- $T(s) = \frac{L}{1+L(s)}$





- Is a trial-and-error procedure
- We typically start with a Bode plot of the process transfer function G
- Choosing the gain crossover frequency ω_{gc} is a major design decision: a compromise between attenuation of load disturbances and injection of measurement noise
- Finally shape the loop transfer function by changing the controller gain and adding poles and zeros to the controller transfer function
- The controller gain at low frequencies can be increased by so-called *lag compensation*, and the behavior around the crossover frequency can be changed by so-called *lead compensation*.

Nominal sensitivity peak

$$M_s = \max_{0 \le \omega \le \infty} |S(j\omega)| = \max_{0 \le \omega \le \infty} \left| \frac{1}{1 + G(j\omega)R(j\omega)} \right|$$

- M_s gives us an indication of how far L is from -1

Relationship between loop function harmonic response and closed loop

- Relationship between the harmonic response of the loop transfer function $L(j\omega)$ and the closed loop $H(j\omega)$

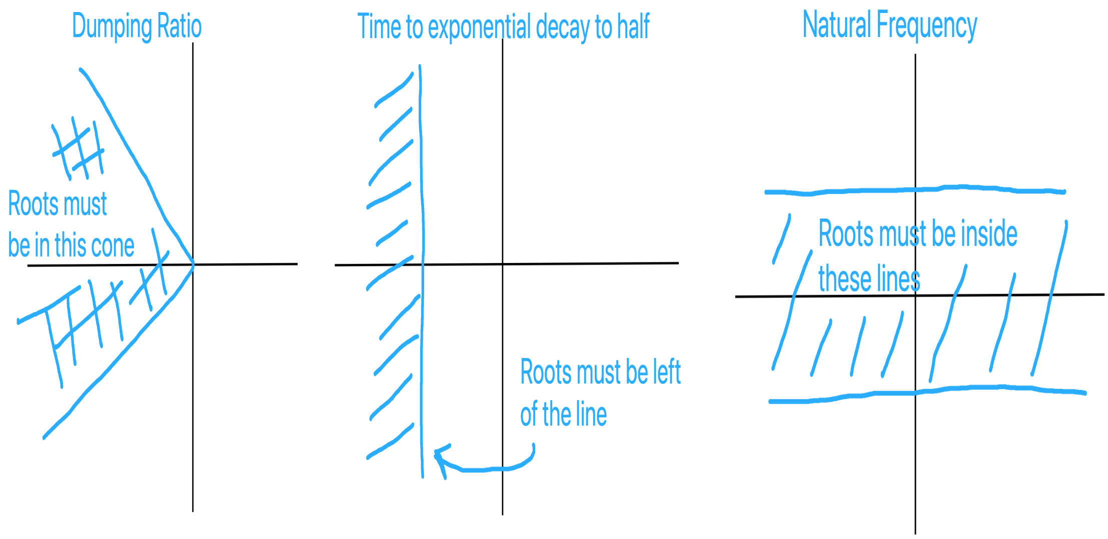
$$|H(j\omega)| = \frac{|L(j\omega)|}{|1 + L(j\omega)|} = \frac{|L(j\omega)|}{\sqrt{1 + |L(j\omega)|^2}}$$

- Translation of design requirements into requirements on the Bode plot of the loop funtion (barriers)
- The "barriers" on the Bode plots are there to help us shape the desired harmonic response of the loop function
- The controller is designed so that the loop function always stays within the admissible regions

Root Locus

- System with multiple known parameters, and one unknown or varying parameter (K)
- Changing K changes the locations of the poles
- How to design a system that meets the requirements:
  - What value of K should I choose to meet the requirements (i.e., that places the poles at the correct location in the s plane)
- What is the effect of variations on a control system that has been already designed:
  - How sensitive is the system to a value of K that is slightly different than what we have estimated (or predicted).
- Rules to sketch the root locus

- Translate requirements into s-plane requirements:



- **The angle condition:** $\angle(G(s)) = (2n+1)\pi$
- **The magnitude condition:** $|KG(s)| = 1$

Phase Lead/Phase Lag Compensators

- A lead compensator can increase the stability or speed of reponse of a system;
- A lag compensator can reduce (but not eliminate) the steady-state error.

$$R(s) = \frac{\frac{s}{w_z} + 1}{\frac{s}{w_p} + 1} = \frac{w_p}{w_z}\frac{s + w_z}{s + w_p}$$

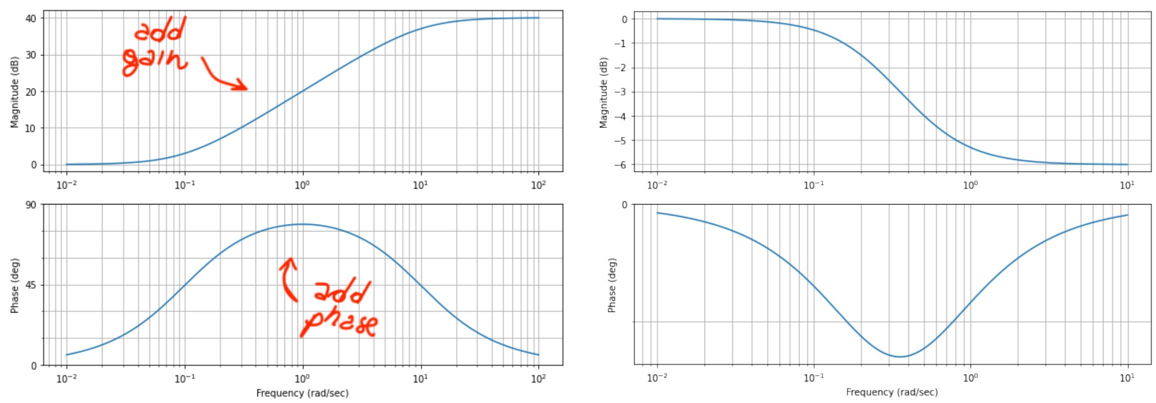
Lead compensator:

- $w_z < w_p$
- $K = \frac{w_p}{w_z}$ (gain)

Lag compensator

- $w_z > w_p$
- $K = \frac{w_p}{w_z}$ (gain)

- Multiplying the two T.F. together means adding everything together on the Bode plot
- Lead/Lag compensator:
  - Behaves like a real zero early on, at low frequency
  - Until the real pole pulls it back at high frequency
  - See blue line for its approximate representation

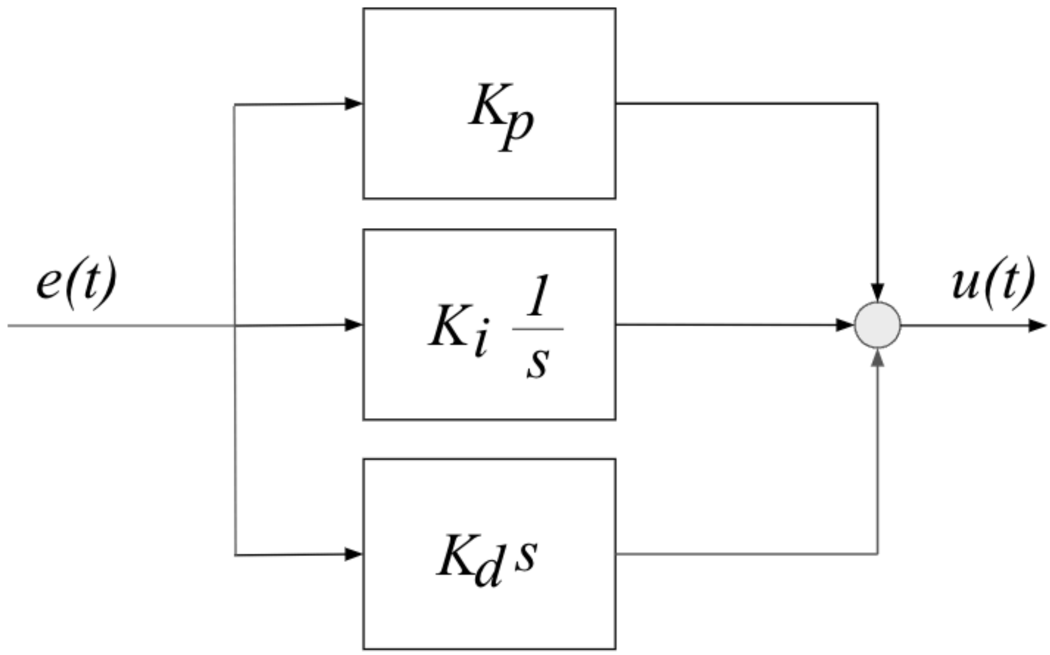


- Using the root locus and Bode plots to place the zero and pole

PID Controllers

- PID = Proportional-Integral-Derivative
    - Describes how the error term is treated before being sum and sent into the plant
- It is a simple and effective controller in a wide range of applications
- Majority of controllers in industrial applications are PIDs

The general structure of a PID controller is:



- The three gains K_p, K_i, K_d are adjustable and can be tuned to the specific application
- Varying K_p, K_i, K_d means adjusting how sensitive the system is across the three paths

- Ziegler Nichols tuning rules
- Practical problems:
    - Derivative is sensitive to noise
        - Filtering and Set Point Weighting
    - If the integral of the error grows too much, the control output might hit actuation limits
        - Integral windup: Initializing the controller integral to a desired value or zeroing the integral value every time the error is equal to, or crosses zero reduces the problem.

- Solving the Cruise Control Problem deploying a PID controller on a car

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