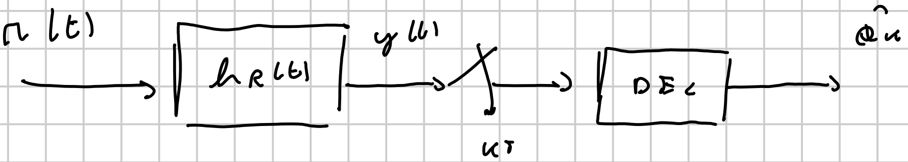
ESERCIZIO 1

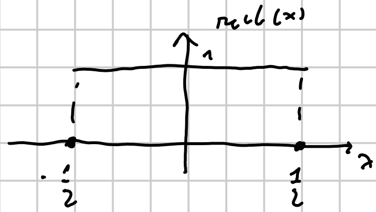


$a_i \in \{-e \,;\, 2e\}$ indipendenti ed equiprobabili

$r(t) = \sum_i a_i \, p(t - iT) + n(t) = s(t) + n(t)$

$S_n(f) = \frac{n_0}{2} \quad \forall f$

$P(f) = T \, |f| \, e^{-|f|} \, rect\left(f \frac{T}{2}\right)$ $\qquad rect(x) = 0$ se $x = \pm \frac{1}{2}$



$H_R(f) = e^{|f|} \, rect\left(f \frac{T}{2}\right)$

$$\hat{a}_k = \begin{cases} -e & \text{se } x_k \le \lambda \\ 2e & \text{se } x_k > \lambda \end{cases}$$

$\lambda = \frac{3}{2} e$

1) E_s ?

2) $P_E(\lambda)$?

3) λ_{ott} ?

1) $E_s = P_r \, T$

$$P_r = \int_{-\infty}^{+\infty} S_s(f) \, df$$

$$S_s(f) = \frac{1}{T} S_a(f) \, |P(f)|^2$$

$$S_a(f) \triangleq \sum_m R_a[m]\, e^{-j2\pi f m T}$$

$$R_a[m] = E\{a_i\, a_{i+m}\} = \begin{cases} E\{a_i^2\} & m = 0 \\ E\{a_{i+m}\}\, E\{a_i\} & m \neq 0 \end{cases}$$

$$E\{a_i\} = \frac{1}{2}(-e) + \frac{1}{2}(2e) = \frac{e}{2}$$

$$E\{a_i^2\} = \frac{1}{2}(-e)^2 + \frac{1}{2}(2e)^2 = \frac{5}{2}e^2$$

$$R_a[m] = \begin{cases} \frac{5}{2}e^2 & m = 0 \\ \frac{e^2}{4} & m \neq 0 \end{cases} = \frac{5}{2}e^2\,\delta[m] + \frac{1}{4}e^2$$

$$S_a(f) = \frac{5}{2}e^2 + \frac{e^2}{4T}\sum_k \delta\left(f - \frac{k}{T}\right)$$

$$S_s(f) = \frac{1}{T}|P(f)|^2\left[\frac{5}{2}e^2 + \frac{e^2}{4T}\sum_k \delta\left(f - \frac{k}{T}\right)\right]$$

$$E_s = \int_{-\infty}^{+\infty} |P(f)|^2\left[\frac{5}{2}e^2 + \frac{e^2}{4T}\sum_k \delta\left(f - \frac{k}{T}\right)\right]df =$$

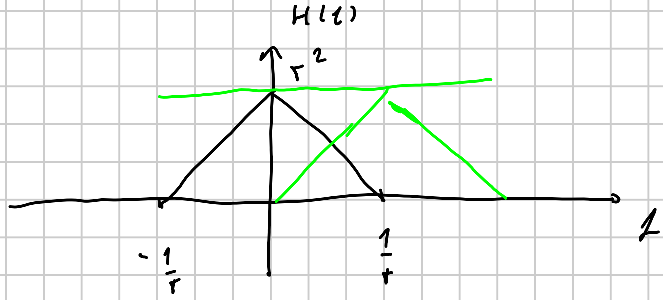
$$= \frac{5}{2}e^2 \underbrace{\int_{-\infty}^{+\infty}|P(f)|^2\,df}_{1} + \frac{e^2}{4T}\sum_k \underbrace{\int_{-\infty}^{+\infty}|P(f)|^2\,\delta\left(f - \frac{k}{T}\right)df}_{2} =$$

1 $$\int_{-\infty}^{+\infty}|P(f)|^2\,df = \frac{1}{4}\left[1 - e^{-2/T}\left(1 + \frac{2}{T} + \frac{2}{T^2}\right)\right] = E_p$$

2 $$\int_{-\infty}^{+\infty}|P(f)|^2\,\delta\left(f - \frac{k}{T}\right)df = |P(0)|^2 = 0$$

$$E_s = \frac{9}{5} c^2 E_p$$

$$H(f) = P(f)\, H_R(f) = T^2 |f| \, rect\left(f \frac{T}{2}\right)$$



$$\sum_k H\left(f - \frac{k}{T}\right) = T^2$$

$$\Downarrow$$

$$h(0) = T$$

NO ISI

$$y_k = h(0)\, a_k + n_k = T a_k + n_k$$

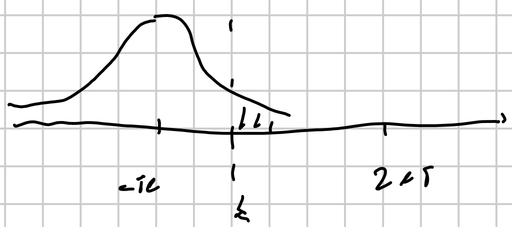
$$n_n(t) = n(t) \otimes h_R(t)$$

$$S_{nn}(f) = \frac{N_0}{2} |H_R(f)|^2$$

$$\sigma^2_{nn} = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H_R(f)|^2 \, df = \frac{N_0}{2} \, 2 \int_0^{+\infty} e^{2f} \, df = \frac{N_0}{2} \left(e^{2/T} - 1 \right)$$

$$P_E(b) = \frac{1}{2} Pr\{e \mid a_k = -c\} + \frac{1}{2} Pr\{e \mid a_k = 2c\}$$

$$Pr\{e \mid a_k = -c\} = Q\left(\frac{\lambda + cT}{\sigma_{nn}} \right)$$



$$P_r\left\{ e \mid a_k = 2e \right\} = Q\left(\frac{h + 2e\sigma}{\sigma_{nn}} \right)$$

$$P_E(h) = \frac{1}{2} Q\left(\frac{-h + 2e\sigma}{\sigma_{nn}} \right) + \frac{1}{2} Q\left(\frac{h + e\sigma}{\sigma_{nn}} \right)$$

$$h = \frac{3}{2} e$$

3) h att ?

$$\frac{d P_E}{d h} = 0$$

$$Q'(x) \doteq \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$\frac{d P_E(h)}{d h} = -\frac{1}{2} \cancel{\left(+ \frac{1}{\sqrt{2\pi}} \right)} e^{-\frac{(-h + 2e\sigma)^2}{2\sigma^2_{nn}}} + \frac{1}{2} \cancel{\left(\frac{1}{\sqrt{2\pi}} \right)} e^{-\frac{(h + e\sigma)^2}{2\sigma^2_{nn}}} = 0$$

$$e^{-\frac{(-h + 2e\sigma)^2}{2\sigma^2_{nn}}} = e^{-\frac{(h + e\sigma)^2}{2\sigma^2_{nn}}}$$

$$(-h + 2e\sigma)^2 = (h + e\sigma)^2$$

$$\boxed{h = \frac{e}{2}\sigma}$$

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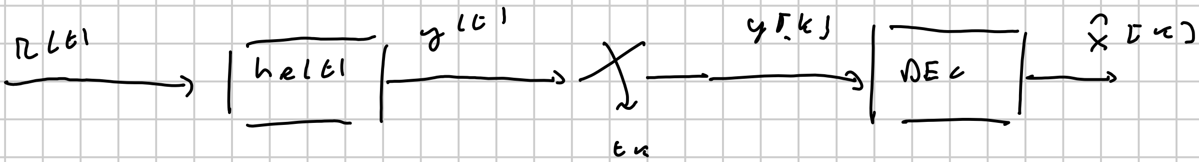


$x[i] \in \{0, 2\}$

$w(t)$ AWGN $\qquad S_w(f) = \frac{N_0}{2}$

$p(t) = 2B \,\text{sinc}(2Bt) + B \,\text{sinc}(Bt)$

$H_R(f) = \text{rect}\left(\frac{f}{2B}\right)$

$$\hat{x}_k = \begin{cases} 0 & \text{se } y[k] \leq \lambda \\ 2 & \text{se } y[k] > \lambda \end{cases} \qquad \lambda = 1$$

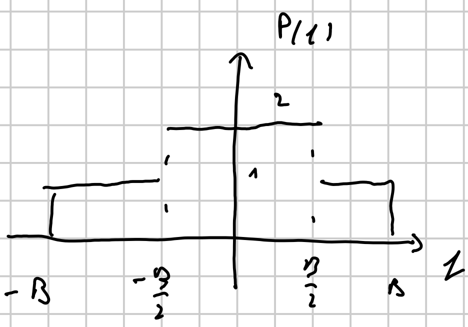
1) E_s

2) ISTANTE DI CAMP. OTTIMO (NO ISI)

3) $P_E(b)$

$p(t) = 2B \,\text{sinc}(2Bt) + B \,\text{sinc}(Bt)$

$P(f) = \text{rect}\left(\frac{f}{2B}\right) + \text{rect}\left(\frac{f}{B}\right)$

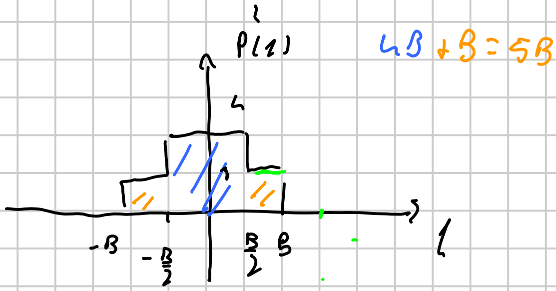


$H_R(f) = \text{rect}\left(\frac{f}{2B}\right)$

1)

$$E_s = \left[\frac{1}{2}(2)^2 + \frac{1}{2}(0)^2\right] E_p = 2E_p$$

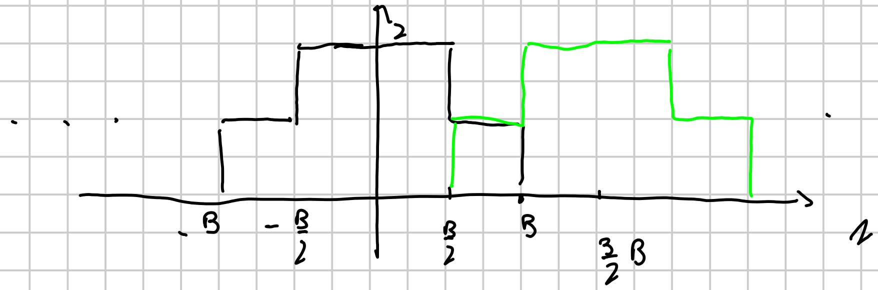
$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2 \, df = 5B$$



$$E_s = 10 \, B$$

2) NO ISI $\Leftarrow$ COND. Nyq.

$$\sum_m H\left(f - \frac{m}{T_c}\right) = T_c \, h(0)$$



$$\boxed{T_c = \frac{2}{3B}} \Rightarrow t_k = k \frac{2}{3B}$$

VERIFICHIAMO NYQ NEL TEMPO

$$h(t) = p(t) = 2B \, \text{sinc}(2Bt) + B \, \text{sinc}(Bt)$$

$$h[m] = 2B \, \text{sinc}\left(2B \frac{2}{3B} m\right) + B \, \text{sinc}\left(B \frac{2}{3B} m\right) =$$

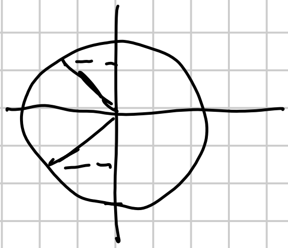
$$= \begin{cases} h[0] & m = 0 \\ 0 & m \neq 0 \end{cases}$$

$$h[m] = 2B \frac{\sin\left(\frac{4}{3} m \pi\right)}{\frac{4}{3} m \pi} + B \frac{\sin\left(\frac{2}{3} m \pi\right)}{\frac{2}{3} m \pi} =$$

$$= \frac{3}{2} \frac{B}{\pi i} \left[\sin\left(\frac{4}{3} \pi i\right) + \sin\left(\frac{2}{3} \pi i\right) \right]$$

$$h(0) = 3B$$

$$h[m]\Big|_{m \neq 0} = 0 \qquad \sin\left(\frac{4}{3} \pi i\right) = - \sin\left(\frac{2}{3} \pi i\right)$$



3)

$$S_{nn}(f) = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H_R(f)|^2 \, df = N_0 B$$

$$h(0) = 3B$$

$$y[n] = 3B \, x[n] + n[n]$$

$$P_b(e) = \underbrace{\frac{1}{2} Q\left(\frac{1}{\sqrt{N_0 B}} \right)}_{P(e \mid a_k = 0)} + \underbrace{\frac{1}{2} Q\left(\frac{6B - 1}{\sqrt{N_0 B}} \right)}_{P(e \mid a_k = 2)}$$

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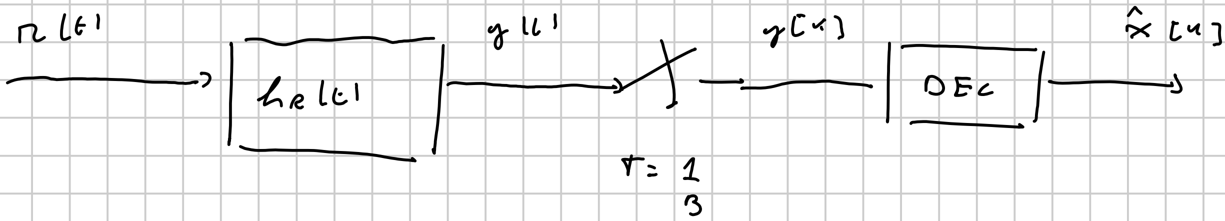


$$s(t) = \sum_k x[k]\, p(t - kT)$$

$x[k] \in \{-2 \,;\, +3\}$ und. equip.

$p(t) = 2B \operatorname{sinc}(2Bt)$

$\boxed{c(t) = \operatorname{sinc}^2(Bt)}$ $\quad C(f) \neq 1$

$S_n(f) = \frac{N_0}{2}$ AWGN

$h_R(t) = 2B \operatorname{sinc}(2Bt)$

1) E_s

2) P_{nn} (Potenza media di rumore dopo $h_R(t)$)

3) $P_E(b)$

1) $E_s = \frac{1}{2} E_{s_1} + \frac{1}{2} E_{s_2}$

$$E_{s_1} = \int_{-\infty}^{+\infty} s_1^2(t)\, dt = \int_{-\infty}^{+\infty} (-2\, p(t))^2$$

$$E_{s_2} = \int_{-\infty}^{+\infty} (+3\, p(t))^2\, dt$$

$$E_p = \int_{-\infty}^{+\infty} p^2(t)\, dt = \int_{-\infty}^{+\infty} P^2(f)\, df = 2B$$

$$E_s = \frac{1}{2} 4 E_p + \frac{1}{2} 9 E_p = 13 B$$

2) $y(t) = r(t) \otimes h_R(t) =$

$$= \sum_k x[k]\, h(t - kT) + w(t)$$

$$h(t) = p(t) \otimes c(t) \otimes h_R(t)$$

$$P_{nn} = \sigma^2_{nn} = \int_{-\infty}^{+\infty} S_w(f)\, df = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H_R(f)|^2\, df = \frac{N_0}{2} 2B = N_0 B$$

3) $P_E(b)$

Verifica Nyq.

$$h(t) = p(t) \otimes c(t) \otimes h_R(t)$$

$$\Updownarrow$$

$$H(f) = P(f)\, C(f)\, H_R(f) = \underbrace{\mathrm{rect}\left(\frac{f}{2B}\right)}_{P(f)} \underbrace{\frac{1}{B}\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)}_{C(f)} \cdot$$

$$\cdot \underbrace{\mathrm{rect}\left(\frac{f}{2B}\right)}_{H_R(f)} =$$

$$= \frac{1}{B}\left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

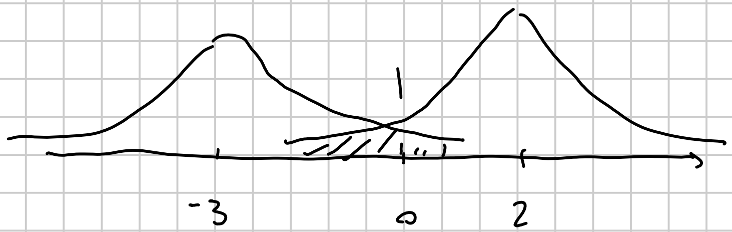
$$\sum_k H(f - kB) = \frac{1}{B} = T \Rightarrow h(0) = 1$$

$$y_k = h(0)\, x[k] + w[k] = x[k] + w[k]$$

$$h = 0$$

$$P_E(n) = \frac{1}{2} P_r\{\hat{x}[k] = -2 \mid x[k] = 3\} + \frac{1}{2} P_r\{\hat{x}[k] = 3 \mid x[k] = 2\}$$

$$= \frac{1}{2} Q\left(\frac{3}{\sqrt{N_0 T}}\right) + \frac{1}{2} Q\left(\frac{2}{\sqrt{N_0 T}}\right)$$



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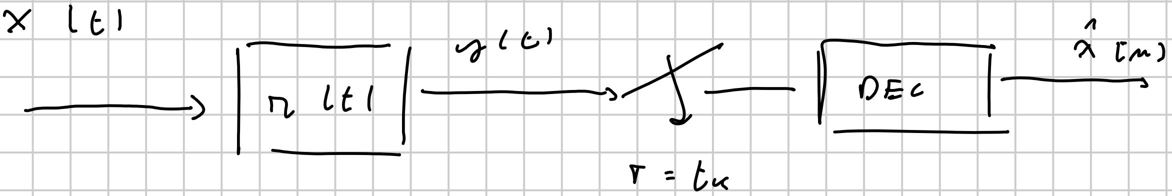
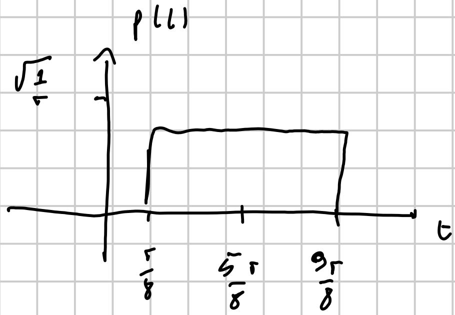


$$x[k] \in \{0, +1\}$$

$$p(t) = \begin{cases} \sqrt{\frac{1}{T}} & \frac{T}{8} < t < \frac{T}{8} + T \\ 0 & \text{altrove} \end{cases}$$



$S_n(f) = \frac{N_0}{2}$ AWGN $\qquad h = \frac{1}{4}$

1) E_s

2) $r(t) = p(t_0 - t)$ determinare t_0 $r(t)$ causale

3) P_{nn} in uscita da $r(t)$

4) $t_{k\,ott}$

1)

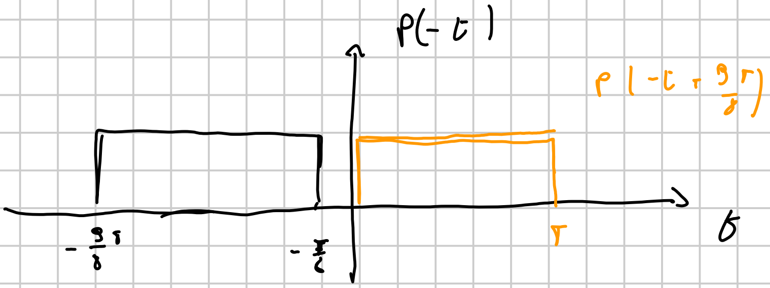
$$E_s = \frac{1}{2} E_{s_0} + \frac{1}{2} E_{s_1} = 0 + \frac{1}{2} \cdot (1) = \frac{1}{2}$$

2)

$$r(t) = A\, p(t_0 - t)$$

$$p(t) = \sqrt{\frac{1}{T}}\ \mathrm{rect}\left(\frac{t - 5T/8}{T}\right)$$

$p(-t)$



$$r(t) = p\left(-t + \frac{9T}{8}\right)$$

$$\sigma^2_{nn} = \frac{N_0}{2}\int_{-\infty}^{+\infty} |R(f)|^2\, df = \frac{N_0}{2}\int_{-\infty}^{+\infty} r^2(t) = A^2 \frac{N_0}{2}$$

4)

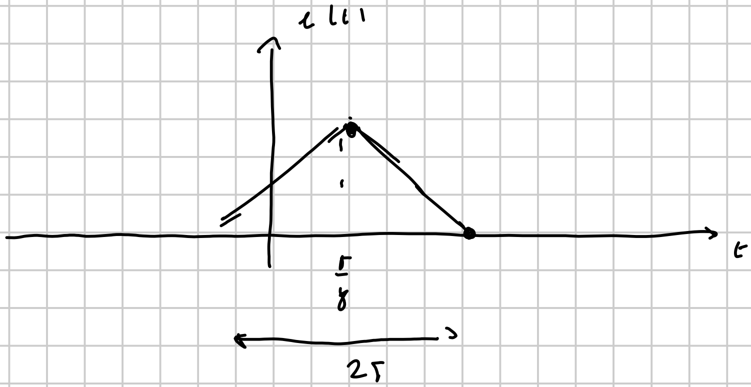
$$H(f) = P(f)\, R(f)$$

$$P(f) = \sqrt{\frac{1}{T}}\ T\ \mathrm{sinc}(fT)\, e^{-j 2\pi f \frac{5T}{8}}$$

$$R(f) = A\, P(f)\, e^{j2\pi f 3T/8} = A\sqrt{T}\,\mathrm{sinc}(fT)\, e^{j2\pi f \frac{T}{2}}$$

$$H(f) = A\,T\,\mathrm{sinc}^2(fT)\, e^{-j2\pi f \frac{T}{8}}$$

$$g(t) = A\left(1 - \frac{|t - T/8|}{T}\right)\mathrm{rect}\left(\frac{t - T/8}{2T}\right)$$



$$t_k = \frac{T}{8} + kT$$

$P_E(b)$

$$y[k] = A\,x[k] + n_k$$

$$P(e \mid x[k]=0) = Q\left(\frac{1/4}{\sqrt{A^2 \frac{N_0}{2}}}\right) = Q\left(\sqrt{\frac{1}{8A^2 N_0}}\right)$$

$$P(e \mid x[k]=1) = Q\left(\sqrt{\frac{(A - 1/4)^2}{A^2 N_0/2}}\right)$$

A t.c. $P(e)$ minima

$$P(e \mid x[k]=0) = P(e \mid x[k]=1)$$

$A : \frac{1}{2}$