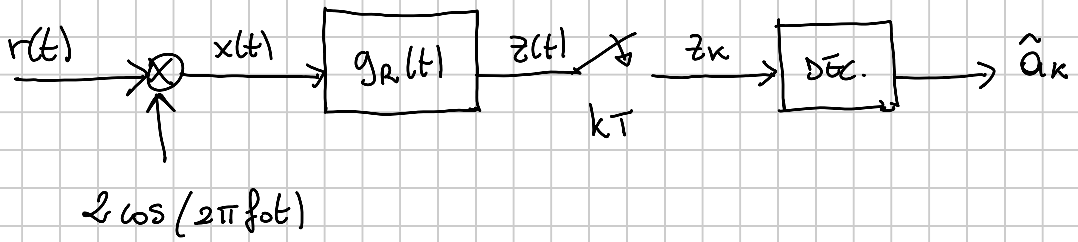
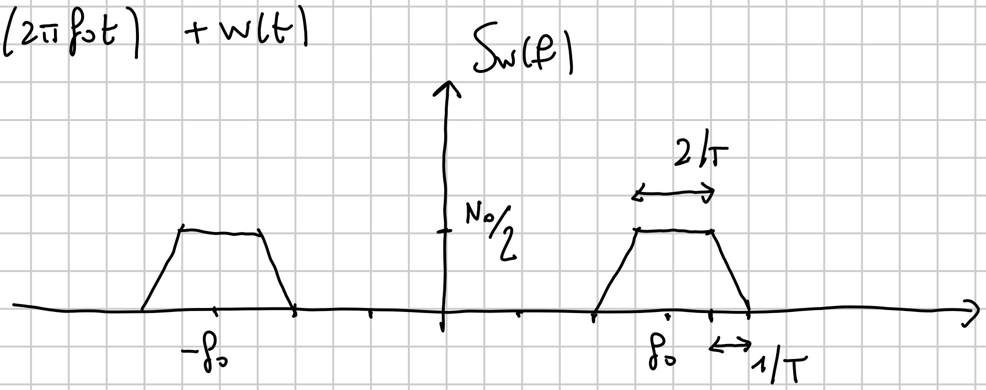
Compito 04/06/2010 PAM in P.B.



$$r(t) = \sum_i a_i \, g_T(t-iT) \cos(2\pi f_0 t) + w(t)$$

w(t) processo Gaussiano



$$G_T(f) = \text{rect}\left(f\frac{T}{2}\right)$$

$$a_i \in A = [\pm 1]$$

$$G_R(f) = T\left(1 - |f|T\right) \text{rect}\left(f\frac{T}{2}\right)$$

1. E_s per intervallo di segnalazione del segnale trasmesso
2. DSP del rumore in uscita al filtro in ricezione
3. Risposta impulsiva del sistema di comunicazione
4. $P_E(b)$ se $\lambda = 1/4$

Soluzione

(1) $$E_s = E\left\{ \int_0^T s^2(t)\, dt \right\} \qquad s(t) = \sum_i a_i \, g_T(t-iT)$$

$$E\{a_i \cdot a_{i+m}\} = \begin{cases} E\{a_i^2\} & m=0 \\ E^2\{a_i\} & m\neq 0 \end{cases} = \begin{cases} 1 & m=0 \\ 0 & m\neq 0 \end{cases}$$

$$R_a[m] = \delta[m]$$

$$E_s = E\left\{ \int_0^T \sum_i a_i\, g_T(t-iT) \cdot \sum_k a_k\, g_T(t-kT)\, dt \right\} =$$

$$= \int_0^T \sum_i \sum_k E\{a_i a_k\}\, g_T(t-iT)\, g_T(t-kT)\, dt =$$

$$= \int_0^T \sum_i g_T^2(t-iT)\, dt = \int_{-\infty}^{+\infty} g_T^2(\alpha)\, d\alpha = E_{g_T} = \frac{2}{T}$$

$$G_T(f) = \text{rect}\left(\frac{fT}{2}\right)$$

2) $$x(t) = r(t) \cdot 2\cos(2\pi f_0 t) =$$

$$= \sum_i a_i\, g_T(t-iT) \cos(2\pi f_0 t) \cdot 2\cos(2\pi f_0 t) + w(t) \cdot 2\cos(2\pi f_0 t) =$$

$$= \sum_i a_i\, g_T(t-iT)\left(1 + \cos(4\pi f_0 t)\right) + w_c(t)\, 2\cos^2(2\pi f_0 t) - w_s(t)\, 2\cos(2\pi f_0 t)\sin(2\pi f_0 t)$$

$$w(t) = w_c(t)\cos(2\pi f_0 t) - w_s(t)\sin(2\pi f_0 t)$$

$$= \sum_i a_i\, g_T(t-iT)\left(1 + \cos(4\pi f_0 t)\right) + w_c(t)\left(1 + \cos(4\pi f_0 t)\right) - w_s(t)\sin(4\pi f_0 t)$$

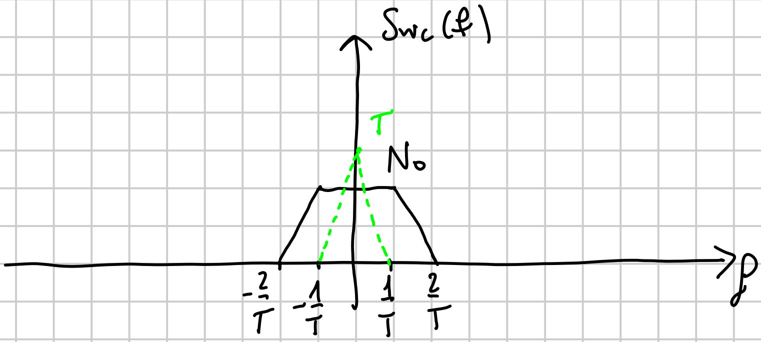
Il filtro in ricezione e' passa basso e quindi:

$$z(t) = \sum_i a_i \, g(t-iT) + w_c(t) \otimes g_R(t)$$

$$g(t) = g_T(t) \otimes g_R(t)$$

$$n_c(t) = w_c(t) \otimes g_R(t)$$

$$S_{nc}(f) = S_{wc}(f) \, |G_R(f)|^2$$



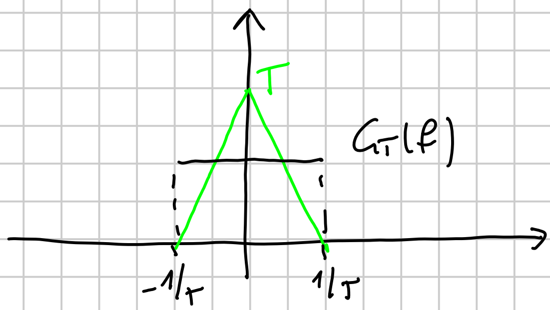
$$S_{wc}(f) = \begin{cases} S_w(f+f_0) + S_w(f-f_0) & |f| < f_0 \\ 0 & \text{altrove} \end{cases}$$

$$S_{nc}(f) = N_0 \, |G_R(f)|^2 \qquad G_R(f) = T\left(1-|f|T\right) \operatorname{rect}\left(\frac{fT}{2}\right)$$

$$\sigma_n^2 = N_0 \, \frac{2}{3} \cdot T^{\cancel{2}} \cdot \frac{1}{\cancel{T}} = \frac{2}{3} N_0 T$$

(3) $g(t) = g_T(t) \otimes g_R(t)$

$$G(f) = G_T(f) \cdot G_R(f) = G_R(f)$$

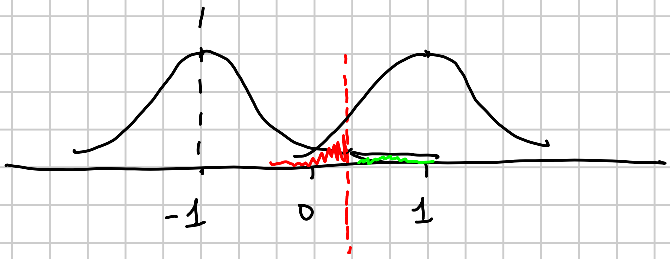


$$\sum_k G\left(f - k/T\right) = T$$

si dimostra per via grafica che e' soddisfatta la condizione di Nyquist.

$g(0) = 1$

5) $z_k = a_k\, g(0) + n_{ck} = a_k + n_{ck} \quad \in \mathcal{N}(a_k, \sigma_n^2)$



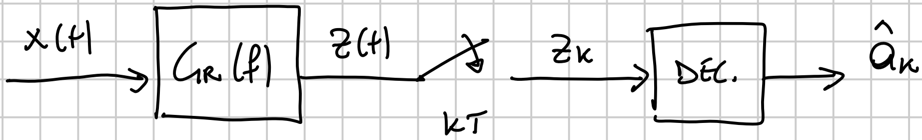
$\lambda = 1/4$

$$P_E(b) = \frac{1}{2} \cdot \Pr\{\hat{a}_k = -1 \mid a_k = 1\} + \frac{1}{2} \Pr\{\hat{a}_k = 1 \mid a_k = -1\}$$

$$\Pr\{\hat{a}_k = -1 \mid a_k = 1\} = Q\left(\frac{1 - 1/4}{\sigma_n}\right)$$

$$\Pr\{\hat{a}_k = 1 \mid a_k = -1\} = Q\left(\frac{1 + 1/4}{\sigma_n}\right)$$

Compitino 24/05/2005 PAM B.B



$$x(t) = \sum_i a_i\, g_T(t - iT) + w(t)$$

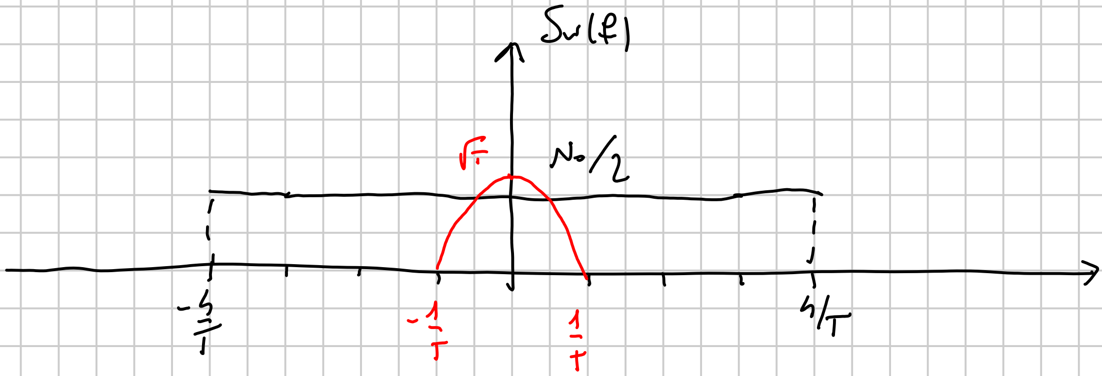
$$S_w(f) = \frac{N_0}{2} \operatorname{rect}\left(\frac{fT}{8}\right) \qquad a_i \in A = \{\pm 1\}$$

$$G_T(f) = \sqrt{T}\cos\left(\frac{\pi f T}{2}\right)\text{rect}\left(\frac{fT}{2}\right)$$

1) Determinare l'espressione del filtro in ricezione $G_R(f)$ in modo che

·) ISI è nullo ·) massimo SNR

2) Probabilità di errore se $\lambda = -1$



Il rumore è bianco nella banda del segnale

$$g_R(t) = g_T(T-t) \quad \text{Filtro adattato}$$

$$G_R(f) = G_T^*(f)\, e^{i 2\pi f T} = G_T(f)\, e^{i 2\pi f T}$$

$$z(t) = \sum_i a_i\, g(t - iT) + n(t)$$

$$g(t) = g_T(t) \otimes g_R(t)$$

$$n(t) = w(t) \otimes g_R(t)$$

$$G(f) = G_T(f) G_T(f) e^{i 2\pi f T} = G_T^2(f)\, e^{i 2\pi f T}$$

$$G(f) + G(f - 1/T) = \sum_k G(f - k/T) = K$$

$$= \frac{T}{2}\left(1 + \cos(\pi f T)\right) e^{i 2\pi f T} + \frac{T}{2}\left(1 + \cos\left(\pi \left(f - \frac{1}{T}\right) T\right)\right) e^{i 2\pi \left(f - \frac{1}{T}\right) T} =$$

$$= \left[\frac{T}{2} + \frac{T}{2}\cos(\pi f T)\right] e^{i 2\pi f T} + \left[\frac{T}{2} + \frac{T}{2}\underbrace{\cos(\pi f T - \pi)}_{= -\cos(\pi f T)}\right] e^{i 2\pi f T} \underbrace{e^{-i 2\pi}}_{= 1} =$$

$$= \frac{T}{2} e^{i 2\pi f T} + \frac{T}{2} e^{i 2\pi f T} = T e^{i 2\pi f T}$$

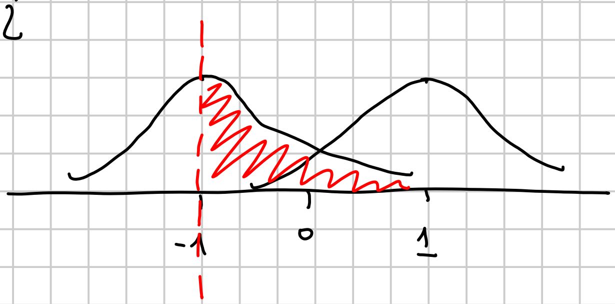
$$g(t)\Big|_{t = kT} = \delta(t - T)$$

$$z(t) = \sum_i a_i \, \delta(t - T - iT) + n(t)$$

$k = i + 1$ $\quad z_k = a_k + n_k$

(3) $$S_n(f) = S_w(f) \, |G_T(f)|^2 = \frac{N_0}{2} |G_T(f)|^2$$

$$\sigma_n^2 = \frac{N_0}{2} g(T) = \frac{N_0}{2}$$



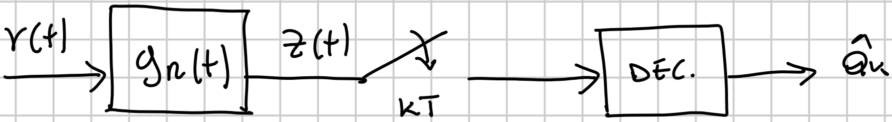
$\lambda = -1$

$Pr\{\hat{a}_k = 1 \,|\, a_k = -1\} = 1/2$

$Pr\{\hat{a}_k = -1 \,|\, a_k = 1\} = Q\left(\frac{2}{\sigma_n}\right)$

$$P_E(b) = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2} Q\left(\frac{2}{\sigma_n}\right) = \frac{1}{4} + \frac{1}{2} Q\left(\frac{2}{\sigma_n}\right)$$

Compito del 25/06/2008 PAM B.B



$$r(t) = \sum_i a_i \, g_T(t - iT) + w(t) \qquad S_w(f) = \frac{N_0}{2}$$

$a_i \in \{\pm 1\}$

$$g_T(t) = e^{-|t|/T} \, \mathrm{rect}\left(\frac{t}{T}\right)$$

$$g_R(t) = g_T(t)$$

1. Energia media per intervallo di segnalazione del segnale trasmesso
2. Verificare le condizione di Nyquist
3. Potenza medio di rumore
4. $P_E(b)$ $\quad \lambda = 0$

(1) $E_s = E\left\{ \int_0^T s^2(t)\, dt \right\} =$

$$= E\left\{ \int_0^T \sum_i a_i\, g_T(t-iT) \sum_k a_k\, g_T(t-kT)\, dt \right\}$$

$$E\{a_i\, a_{i+m}\} = \begin{cases} E\{a_i^2\} = 1 & m=0 \\ E\{a_i\}\cdot E\{a_{i+m}\} = 0 & m \neq 0 \end{cases}$$

$R_a[m] = \delta[m]$

$$E_s = \int_0^T \sum_i g_T^2(t-iT)\, dt = \sum_i \int_0^T g_T^2(t-iT)\, dt = \int_{-\infty}^{+\infty} g_T^2(\alpha)\, d\alpha =$$

$$= \int_{-\infty}^{+\infty} e^{-2|t|/T}\, \mathrm{rect}\left(\frac{t}{T}\right) dt = 2\int_0^{T/2} e^{-2t/T}\, dt =$$

$$= 2\left[\frac{e^{-2t/T}}{(-2/T)}\right]_0^{T/2} = \cancel{2}\left[-\frac{T}{\cancel{2}} e^{-1} + \frac{T}{\cancel{2}}\right] = T\left(1 - e^{-1}\right) > 0$$

(2) $z(t) = \sum_i a_i\, g(t-iT) + n(t)$

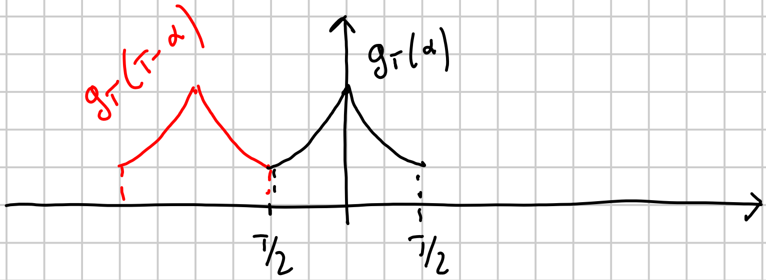
$g(t) = g_T(t) \otimes g_R(t)$

$n(t) = w(t) \otimes g_R(t)$

$$g(t) = \int_{-\infty}^{+\infty} g_T(\alpha)\, g_T(t-\alpha)\, d\alpha$$

$$g(0) = \int_{-\infty}^{+\infty} g_T(\alpha)\, g_T(-\alpha)\, d\alpha = \int_{-\infty}^{+\infty} g_T^2(\alpha)\, d\alpha = T\left(1-e^{-1}\right)$$

$$g(kT) = \int_{-\infty}^{+\infty} g_T(\alpha)\, g_T(kT-\alpha)\, d\alpha = 0 \qquad \forall\ k \neq 0$$



Soddisfa la condizione di Nyquist

(3) $S_n(f) = S_w(f)\, |G_R(f)|^2$

$$\sigma_n^2 = \frac{N_0}{2} \int_{-\infty}^{+\infty} |G_R(f)|^2\, df = \frac{N_0}{2} \int_{-\infty}^{+\infty} g_T^2(t)\, dt = \frac{N_0}{2} \underbrace{T\left(1-e^{-1}\right)}_{g(0)}$$

(4) $z_k = a_k\, g(0) + n_k \qquad \lambda = 0$

$$P_E(b) = \frac{1}{2} \Pr\{\hat{a}_k = 1 \mid a_k = -1\} + \frac{1}{2} \Pr\{\hat{a}_k = -1 \mid a_k = 1\}$$

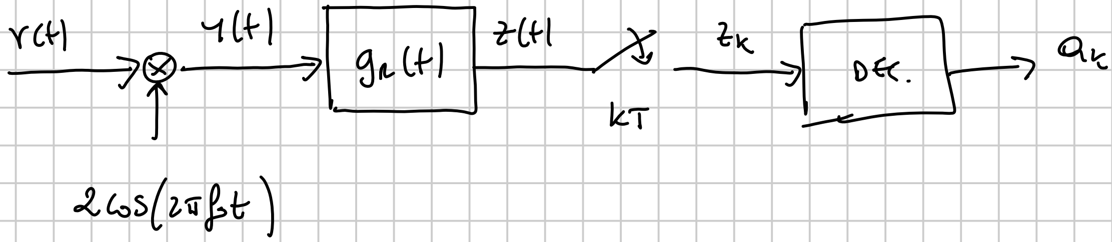
Poiché i simboli sono equidistanti dalla soglia

$$Pr\{\hat{a}_k = 1 \mid a_k = -1\} = Pr\{\hat{a}_k = -1 \mid a_k = 1\}$$

$$P_E(b) = Pr\{\hat{a}_k = 1 \mid \hat{a}_k = -1\} = Q\left(\frac{g(0)}{\sigma_n}\right) = Q\left(\sqrt{\frac{2g(0)}{N_0}}\right) =$$

$$= Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

Esercizio su PSK P.B.



$$r(t) = \sum_n g_T(t - nT)\cos(2\pi f_0 t + \theta_n) + w(t)$$

$$\theta_n \in \left\{\frac{\pi}{4}, -\frac{3\pi}{4}\right\}$$

$$g_T(t) = \left(1 - \frac{2|t|}{T}\right) \mathrm{rect}\left(\frac{t}{T}\right)$$

$g_R(t) = A \,\mathrm{rect}\left(\frac{t}{T}\right)$ B coincide con la banda di $g_R(t)$

$w(t)$ $R_w(\tau) = 2N_0 B \,\mathrm{sinc}(2\tau B)\cos(2\pi f_0 \tau)$ $f_0 \gg 1/T$

B coincide con la banda del filtro in ricezione.

(1) E_s medio per simbolo trasmesso

(2) DSP del rumore in uscita al filtro in ricezione

(3) Verificare la condizione di Nyquist

(4) $P_E(b)$ se $\lambda = 0$

(1) $E_s = \frac{1}{2} E_{s_1} + \frac{1}{2} E_{s_2}$

$$s_1(t) = g_T(t) \cos(2\pi f_0 t + \pi/4)$$

$$s_2(t) = g_T(t) \cos(2\pi f_0 t - 3\pi/4)$$

$$s_1(t) = g_T(t) \left[\cos(2\pi f_0 t) \cos(\pi/4) - \sin(2\pi f_0 t) \sin(\pi/4) \right]$$

$$E_{s_1} = \int_{-\infty}^{+\infty} s_1^2(t)\, dt = \int_{-\infty}^{+\infty} g_T^2(t) \left[\cos^2(\pi/4) \cdot \frac{1}{2} + \sin^2(\pi/4) \cdot \frac{1}{2} \right] dt =$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} g_T^2(t)\, dt = \frac{1}{\cancel{2}} \frac{\cancel{2}}{3} \frac{T}{2} = \frac{T}{6}$$

$E_{s_2} = E_{s_1}$ MODULAZIONE EQUI ENERGIA

$$E_s = \frac{1}{2} E_{s_1} + \frac{1}{2} E_{s_2} = \frac{T}{6}$$

$$y(t) = r(t) \cdot 2\cos(2\pi f_0 t) =$$

$$= 2\sum_n g_T(t-nT)\cos(2\pi f_0 t + \vartheta_n)\cos(2\pi f_0 t) + w(t)\cdot 2\cos(2\pi f_0 t)$$

$$2\cos\alpha \cdot \cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$w(t) = w_c(t)\cos(2\pi f_0 t) - w_s(t)\sin(2\pi f_0 t)$$

$$y(t) = \sum_n g_T(t-nT)\left[\cos(\vartheta_n) + \cos(4\pi f_0 t + \vartheta_n)\right] +$$

$$w_c(t)\left(1 + \cos(4\pi f_0 t)\right) - w_s(t)\sin(4\pi f_0 t)$$

All'uscita del filtro in ricezione che e' passa-basso

$$z(t) = \sum_n g_T(t-iT) \otimes g_R(t)\cos(\vartheta_n) + \underbrace{w_c(t) \otimes g_R(t)}_{n_c(t)}$$

$$S_{w_c}(f) = N_0 \operatorname{rect}\left(\frac{f}{2B}\right)$$

$$S_{n_c}(f) = N_0 \operatorname{rect}\left(\frac{f}{2B}\right) \cdot |G_R(f)|^2$$

$$g_R(t) = A\operatorname{rect}\left(\frac{t}{T}\right)$$

$$G_R(f) = AT\operatorname{sinc}(fT)$$

$$S_{n_c}(f) = N_0 A^2 T^2 \operatorname{sinc}^2(fT)\operatorname{rect}\left(\frac{f}{2B}\right)$$

$$\sigma_n^2 = \int_{-\infty}^{+\infty} S_{n_c}(f)\, df = N_0 \int_{-\infty}^{+\infty} |G_R(f)|^2 df = N_0 \int_{-\infty}^{+\infty} g_R^2(t)\, dt$$

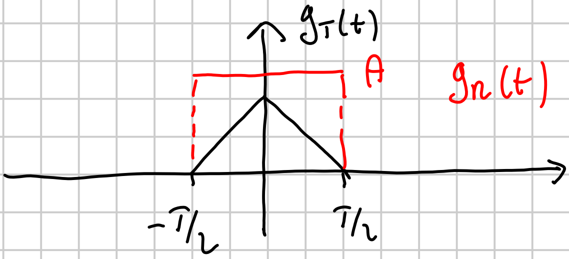
$$g_R(t) = A \,\mathrm{rect}\left(\frac{t}{T}\right)$$

$$\sigma_n^2 = N_0 A^2 T$$

(3) $$z(t) = \sum_n g_T(t-nT) \otimes g_R(t) \cos(\theta_n) + n_c(t)$$

$$g(t) = g_T(t) \otimes g_R(t)$$

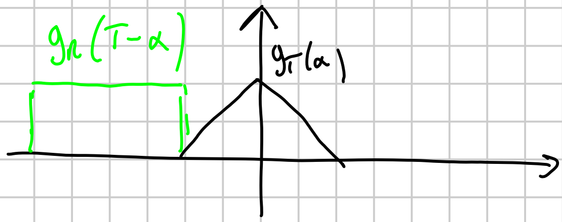
$$g(t) = \int_{-\infty}^{+\infty} g_T(\alpha)\, g_R(t-\alpha)\, d\alpha$$



$$g(0) = \int_{-\infty}^{+\infty} g_T(\alpha)\, g_R(-\alpha)\, d\alpha$$

$$= A\frac{T}{2}$$

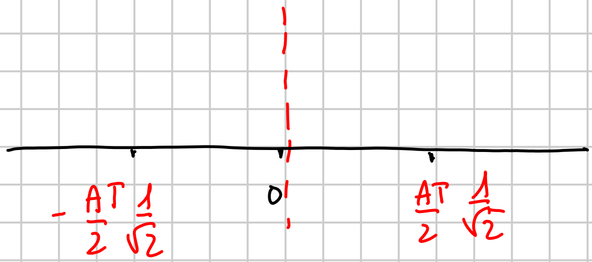
$$g(kT) = \int_{-\infty}^{+\infty} g_T(\alpha)\, g_R(kT-\alpha) = 0 \qquad k \neq 0$$



(4) $$z_k = g(0)\cdot \cos\theta_k + n_{ck} = \frac{AT}{2}\cos\theta_k + n_{ck}$$

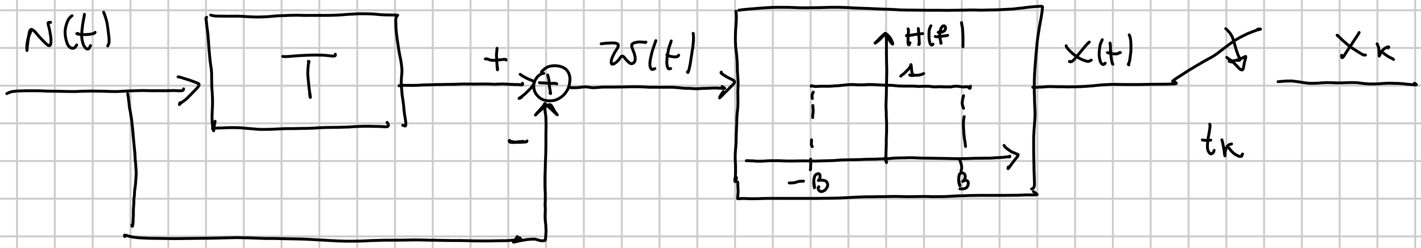
$$\cos\left(\pi/4\right) = \frac{1}{\sqrt{2}} \qquad \cos\left(-\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$z_k\Big|_{\vartheta_1} = \frac{AT}{2}\cdot\frac{1}{\sqrt{2}} + n_{ck} \qquad z_k\Big|_{\vartheta_2} = -\frac{AT}{2}\frac{1}{\sqrt{2}} + n_{ck}$$



$$P_E(b) = P_r\left\{\hat{\vartheta}_k = \vartheta_1 \mid \vartheta_k = \vartheta_2\right\} = Q\left(\frac{AT}{2\sqrt{2}}\frac{1}{\sigma_n}\right)$$

FILTRAGGIO PROCESSI



N(t) è Gaussiano con $S_n(f) = \frac{N_0}{2}$

$X_k = x(2kT) \qquad t_k = 2kT$

Determinare le DDP congiunte delle variabili aleatorie x_k

$w(t) = N(t-T) - N(t)$ è Gaussiano

$X(t)$ è Gaussiano

$E\{w(t)\} = 0$

$E\{w(t+\tau)\, w(t)\} = E\Big\{ \big(N(t+\tau-T) - N(t+\tau)\big) \cdot \big(N(t-T) - N(t)\big) \Big\} =$

$= E\{ N(t+\tau-T)\, N(t-T) \} - E\{ N(t+\tau-T)\, N(t) \} - E\{ N(t+\tau)\, N(t-T) \}$
$+ E\{ N(t+\tau)\, N(t) \} =$

$= R_N(\tau) - R_N(\tau-T) - R_N(\tau+T) + R_N(\tau) = 2R_N(\tau) - R_N(\tau-T) -$
$- R_N(\tau+T)$

$R_N(\tau) = \frac{N_0}{2}\, \delta(\tau)$

$R_w(\tau) = N_0\, \delta(\tau) - \frac{N_0}{2}\, \delta(\tau-T) - \frac{N_0}{2}\, \delta(\tau+T)$

$w(t)$ è SSL

$X(t) = w(t) \otimes h(t)$

$$E\{x(t)\} = \eta_w(t) \otimes h(t) = 0 = \eta_w(t)\, H(0)$$

$$E\{x(t)\, x(t+\tau)\} = R_w(\tau) \otimes \underbrace{h(\tau) \otimes h(-\tau)}_{f(\tau)}$$

$$f(\tau) = h(\tau) \otimes h(-\tau) = h(\tau) \otimes h(\tau)$$

$$F(f) = |H(f)|^2 = \text{rect}\left(\frac{f}{2B}\right)$$

$$f(\tau) = 2B \,\text{sinc}\,(2B\tau) = \qquad \boxed{B = \frac{1}{T}}$$

$$= \frac{2}{T}\,\text{sinc}\left(\frac{2}{T}\tau\right)$$

$$R_x(\tau) = \left[N_0\,\delta(\tau) - \frac{N_0}{2}\,\delta(\tau - T) - \frac{N_0}{2}\,\delta(\tau+T)\right] \otimes \frac{2}{T}\,\text{sinc}\left(\frac{2}{T}\tau\right) =$$

$$= \frac{2N_0}{T}\,\text{sinc}\left(\frac{2}{T}\tau\right) - \frac{N_0}{T}\,\text{sinc}\left(\frac{2}{T}(\tau - T)\right) - \frac{N_0}{T}\,\text{sinc}\left(\frac{2}{T}(\tau+T)\right)$$

$x(t)$ è SSL e Gaussiano

X_k estratte dal processo agli istanti $2kT$ sono

V.A. congiuntamente Gaussiane

$$R_x(k2T) = ? \qquad \boxed{k = 1, 2, .., N.}$$

$$R_x(k2T) = \frac{N_0 2}{T} \operatorname{sinc}(4k) - \frac{N_0}{T} \operatorname{sinc}\left(\frac{2}{T}(2kT - T)\right) -$$

$$- \frac{N_0}{2} \operatorname{sinc}\left(\frac{2}{T}(2kT + T)\right) =$$

$$= \frac{2N_0}{T} \operatorname{sinc}(4k) - \frac{N_0}{T} \operatorname{sinc}(4k - 2) - \frac{N_0}{T} \operatorname{sinc}(4k + 2) = \emptyset$$

$k = 1, 2, \dots, N$

le variabili X_k sono incorrelate e quindi indipendenti.

$$\rho_{xy} = \frac{C_{xy}}{\sigma_x \sigma_y} = \frac{R_{xy} - \eta_x \eta_y}{\sigma_x \sigma_y}$$

$$f_{\underline{x}}(x_1, x_1, \dots, x_n) = \frac{1}{\sqrt{(2\pi)^N \sigma_x^{2N}}} e^{-\frac{\sum_i x_i^2}{2\sigma_x^2}}$$

$$= \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi \sigma_x^2}} e^{-\frac{x_i^2}{2\sigma_x^2}}$$

FILTRAGGIO PROCESSI #2

$$X(t) \longrightarrow \boxed{H(f)} \longrightarrow Y(f)$$

$$H(f) = 2\,\text{rect}\left(\frac{f}{4}\right)$$

$$X(t) = S + w(t)$$

$w(t)$ è Gaussiano con $S_w(f) = \frac{N_0}{2}$

S è V.A. discreta $\quad f_S(s) = \frac{1}{2}\delta(s) + \frac{1}{2}\delta(s-1)$

$w(t)$ e S sono indipendenti

(1) la ddp di 1° ordine del process in uscita $Y(t)$

(2) $E\{Y(t)\}$

(3) funzione di autocorrelazione

(1) $\quad Y(t) = X(t) \otimes h(t) = A + D(t)$

$D(t)$ è Gaussiano

$$E\{D(t)\} = E\{w(t)\} \otimes h(t) = 0$$

$$S_D(f) = S_W(f)\,|H(f)|^2 = 4\,\frac{N_0}{2}\,\mathrm{rect}\left(\frac{f}{4}\right)$$

$$P_D = \sigma_D^2 = \int_{-\infty}^{+\infty} S_D(f)\,df = 4\,\frac{N_0}{2}\cdot 4 = 8N_0$$

perché

valor medio nullo

$$H(f) = 2\,\mathrm{rect}\left(\frac{f}{4}\right)$$

$$A = S \otimes h(t)$$

$$a = s \otimes h(t) = s \cdot H(0) = 2s$$

$$A = 2S \qquad f_a(a) = \frac{1}{2}\delta(a) + \frac{1}{2}\delta(a-2)$$

$$y(t) = A + D(t)$$ campiono il processo $t = t_0$

$$Y = A + D$$ devo trovare la ddp di Y

$$f_Y(y) = f_A(a) \otimes f_D(d)$$

$$f_Y(y|a=0) = D \in \mathcal{N}(0, \sigma_D^2) \qquad \sigma_D^2 = 8N_0$$

$$f_Y(y|a=2) = 2+D \in \mathcal{N}(2, \sigma_D^2)$$

$$f_Y(y) = \Pr\{a=0\}\cdot f_Y(y|a=0) + \Pr\{a=2\}\cdot f_Y(y|a=2)$$ TEO. PROB. TOTALE

$$f_Y(y) = \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma_D^2}} e^{-\frac{y^2}{2\sigma_D^2}} + \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma_D^2}} e^{-\frac{(y-2)^2}{2\sigma_D^2}}$$

(2) $$E\{Y(t)\} = E\{A + D(t)\} = E\{A\} + \underbrace{E\{D(t)\}}_{=0} = E\{A\} = 1$$

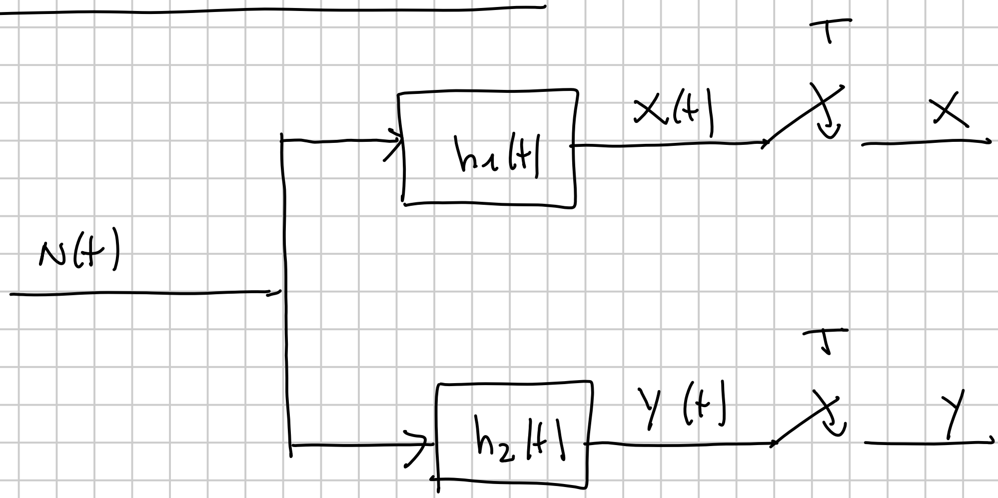
(3) $$E\{Y(t)\,Y(t+\tau)\} = E\{(A+D(t))\cdot(A+D(t+\tau))\} =$$

$$= E\{A^2\} + E\{A\,\cancel{D(t+\tau)}\} + E\{\cancel{D(t)}\,A\} + E\{D(t)\,D(t+\tau)\} =$$

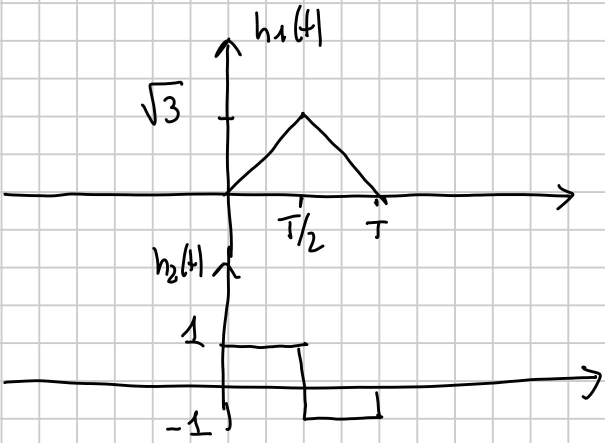
$$= \left(0\cdot\frac{1}{2} + \frac{1}{2}(2)^2\right) + R_D(\tau) = 2 + R_D(\tau) =$$

$$= 2 + 8N_0 \,\mathrm{sinc}(4\tau)$$

FILTRAGGIO PROCESSI #3



N(t) è Gaussiano $\quad S_N(f) = \frac{N_0}{2}$



① $\Pr\{x > y\}$

② $E\{xy\}$

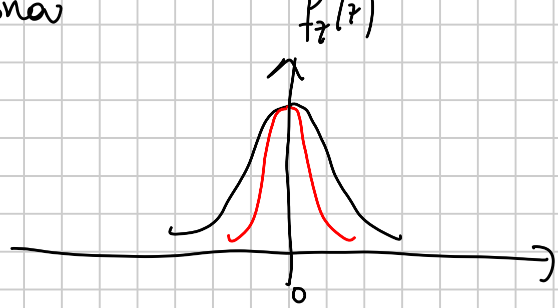
① $$X = x(T) = \left[N(t) \otimes h_1(t)\right]_{t=T} = \int_{-\infty}^{+\infty} N(\tau)\, h_1(T-\tau)\, d\tau$$

$$Y = y(T) = \left[N(t) \otimes h_2(t)\right]_{t=T} = \int_{-\infty}^{+\infty} N(\tau)\, h_2(T-\tau)\, d\tau$$

$$X \in \mathcal{N}(\eta_x, \sigma_x^2) \qquad Y \in \mathcal{N}(\eta_y, \sigma_y^2)$$

$$\Pr\{X > Y\} = \Pr\{\underbrace{X - Y}_{z} > 0\} = \Pr\{z > 0\}$$

$z = X - Y$ é Gaussiana



$\eta_z = E\{z\} = \emptyset$

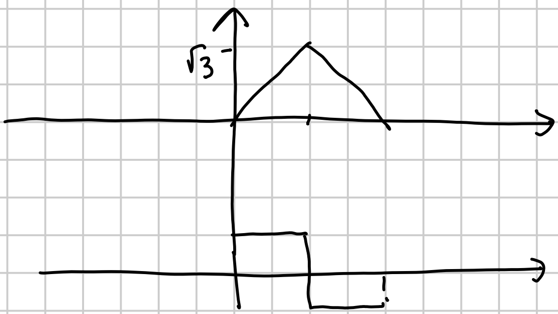
$\Pr\{z > 0\} = 1/2$

② $E\{xy\} = E\left\{ \int_{-\infty}^{+\infty} N(\tau)\, h_1(T-\tau)\, d\tau \int_{-\infty}^{+\infty} N(\alpha)\, h_2(t-\alpha)\, d\alpha \right\}$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \underbrace{E\{N(\tau)\,N(\alpha)\}}_{R_N(\tau-\alpha)} \cdot h_1(t-\tau)\, h_2(t-\alpha)\, d\tau\, d\alpha$$

$$R_N(\tau-\alpha) = \frac{N_0}{2}\,\delta(\tau-\alpha)$$

$$E\{xy\} = \int_{-\infty}^{+\infty} \frac{N_0}{2}\, h_1(t-\tau)\, h_2(t-\tau)\, d\tau = \emptyset \qquad \alpha = \tau$$



V.A. sono incorrelate e indipendenti perché Gaussiane.