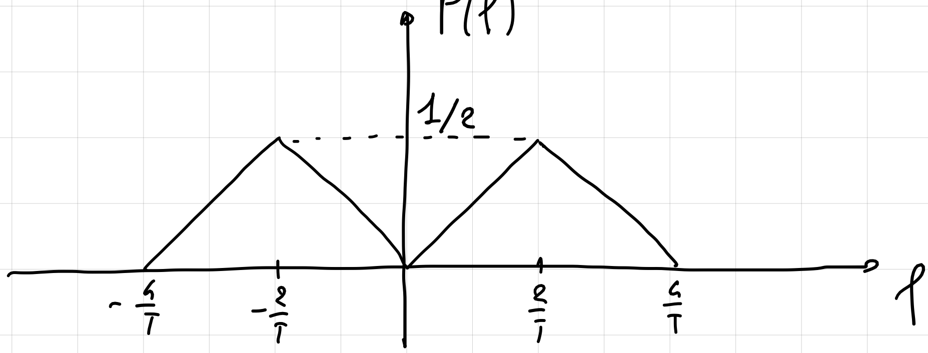
Soluzione testo esame del 06/02/2017 – Es. 2

1) $E_s = E[x_n^2]\, E_p$

$$E[x_n]^2 = \frac{1}{2}\cdot(-1)^2 + \frac{1}{2}(2)^2 = \frac{1}{2} + 2 = \frac{5}{2}$$

$$P(f) = \left(1 - \frac{|f|}{2/T}\right)\mathrm{rect}\left(\frac{f}{4/T}\right) \otimes \frac{1}{2}\left[\delta\left(f - \frac{2}{T}\right) + \delta\left(f + \frac{2}{T}\right)\right]$$

$$= \frac{1}{2}\left(1 - \frac{|f - 2/T|}{2/T}\right)\mathrm{rect}\left(\frac{f - 2/T}{4/T}\right) + \frac{1}{2}\left(1 - \frac{|f + 2/T|}{2/T}\right)\mathrm{rect}\left(\frac{f + 2/T}{4/T}\right)$$



$$E_p = \frac{1}{4}\cdot\frac{2}{T}\cdot\frac{1}{3}\cdot 4 = \frac{2}{3T}$$

$$E_s = \frac{5}{2}\cdot\frac{2}{3T} = \frac{5}{3T}$$

2) $$P_{n_u} = \frac{N_0}{2}\int_{-\infty}^{+\infty} |H_R(f)|^2\, df = \frac{N_0}{2}\cdot\frac{4}{T} = \frac{2N_0}{T}$$

$$H_R(f) = \mathrm{rect}\left(\frac{1}{4/T}\right)$$

3) $$S_s(f) = \frac{1}{T}\,\bar{S}_x(f)\,|P(f)|^2$$

$$\bar{S}_x(f) = TFS\left[R_x[m] \right]$$

$$R_x[m] = C_x[m] + E^2[x[m]]$$

$$E[x[m]] = \frac{1}{2}$$

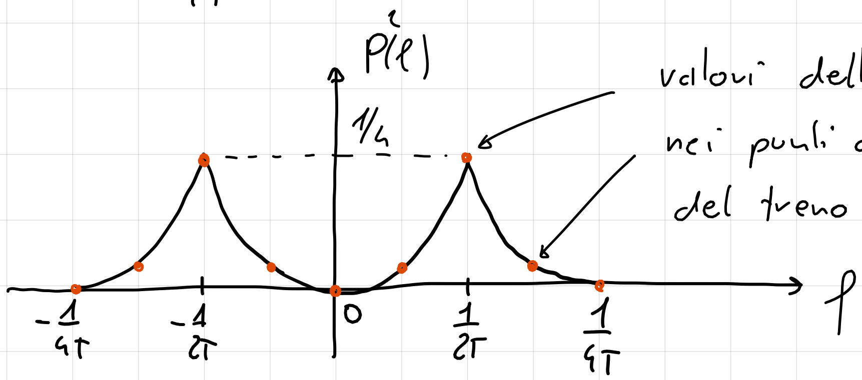
$$C_x[m] = \begin{cases} \frac{1}{2}\left(-\frac{1}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{4} & m=0 \\ 0 & m \neq 0 \end{cases}$$

simboli indipendenti $\Rightarrow$ incorrelati

$$R_x[m] = \frac{1}{4}\delta[m] + \frac{1}{4}$$

$$\bar{S}_x(f) = \frac{1}{4} + \frac{1}{4T}\sum_n \delta\left(f - \frac{n}{T}\right)$$

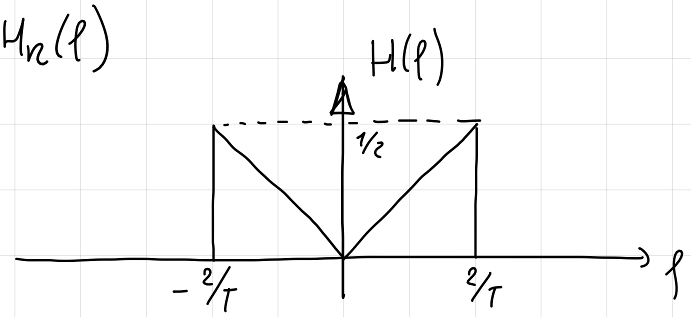
$$S_s(f) = \frac{1}{4}|P(f)|^2 + \frac{1}{4T}\sum_n \delta\left(f - \frac{n}{T}\right)|P(f)|^2$$



valori della $|P(f)|^2$ nei punti di applicazione del treno di $\delta(f)$

4) $h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$

$$H(f) = P(f)\, H_R(f)$$



$$H(f) = \frac{1}{2}\left[\mathrm{rect}\left(\frac{f}{4/T}\right) - \left(1 - \frac{|f|}{2/T}\right)\mathrm{rect}\left(\frac{f}{4/T}\right)\right]$$

$$h(t) = \frac{1}{2}\left[\frac{4}{T}\,\mathrm{sinc}\left(\frac{4t}{T}\right) - \frac{2}{T}\,\mathrm{sinc}^2\left(\frac{2t}{T}\right)\right]$$

$$h[n] = \frac{2}{T}\,\mathrm{sinc}(4n) - \frac{1}{T}\,\mathrm{sinc}^2(2n) = \begin{cases} \frac{1}{T} & n = 0 \\ 0 & n \neq 0 \end{cases}$$

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5) $$P_E(b) = \frac{1}{2}\,Q\left(\frac{1/T}{\sqrt{\frac{2N_0}{T}}}\right) + \frac{1}{2}\left(\frac{2/T}{\sqrt{\frac{2N_0}{T}}}\right)$$