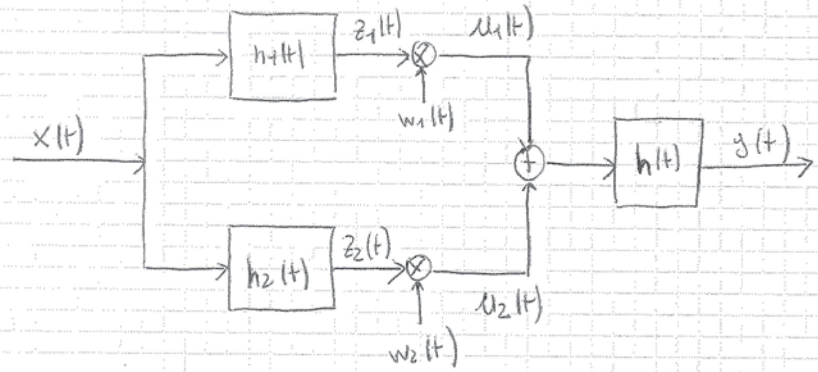
Esercizio 1



$$x(t) = B \operatorname{sinc}(Bt)\, e^{j\pi Bt} + \frac{B}{2} \operatorname{sinc}^2\left(\frac{B}{2} t\right) e^{-j\pi Bt}$$

$$h_1(t) = B \operatorname{sinc}(Bt)\, e^{j\pi Bt}$$

$$h_2(t) = B \operatorname{sinc}(Bt)\, e^{-j\pi Bt}$$

$$w_1(t) = 2 \cos(\pi Bt)$$

$$w_2(t) = -2 \sin(\pi Bt)$$

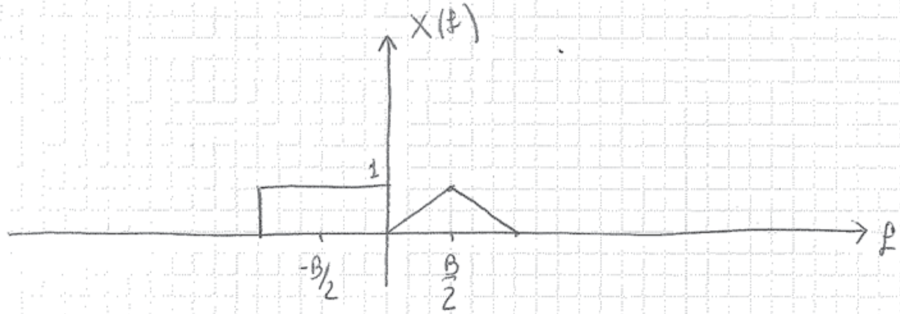
$$h(t) = B \operatorname{sinc}(Bt)$$

teorema del ritardo e teorema di dualità:

$$x(t) = x_1(t) + x_2(t)$$

$$x_1(t) = B\,\text{sinc}(Bt)\, e^{j\pi B t} \rightleftharpoons X_1(f) = \text{rect}\left(\frac{f + B/2}{B}\right)$$

$$x_2(t) = \frac{B}{2}\,\text{sinc}^2\left(\frac{B}{2}t\right) e^{-j\pi B t} \rightleftharpoons X_2(f) = \left(1 - \frac{|f - B/2|}{B/2}\right)\text{rect}\left(\frac{f - B/2}{B}\right)$$



$$Z_1(f) = X(f) \cdot H_1(f)$$

$$H_1(f) = X_1(f)$$

quindi:

$$Z_1(f) = X_1(f)$$

$$Z_2(f) = X(f) \cdot H_2(f)$$

$$H_2(f) = \mathrm{rect}\left(\frac{f - B/2}{B}\right)$$

quindi:

$$Z_2(f) = X_2(f)$$

$$u_1(t) = z_1(t) \cdot w_1(t) = x_1(t) \cdot w_1(t) =$$

$$= B\,\mathrm{sinc}(Bt)\, e^{j\pi Bt} \cdot \left(e^{j\pi Bt} + e^{-j\pi Bt} \right) =$$

$$= B\,\mathrm{sinc}(Bt)\, e^{j2\pi Bt} + B\,\mathrm{sinc}(Bt)$$

$$u_2(t) = z_2(t) \cdot w_2(t) = x_2(t) \cdot w_2(t) =$$

$$= \frac{B}{2}\,\mathrm{sinc}^2\left(\frac{B}{2}t\right) e^{-j\pi Bt} \left(j\, e^{j\pi Bt} - j\, e^{-j\pi Bt} \right) =$$

$$= j\frac{B}{2}\,\mathrm{sinc}^2\left(\frac{B}{2}t\right) - j\frac{B}{2}\,\mathrm{sinc}^2\left(\frac{B}{2}t\right) e^{-j2\pi Bt}$$

$$H(f) = \text{rect}\left(\frac{f}{B}\right)$$

$$y(t) = \left(u_1(t) + u_2(t)\right) \otimes h(t) =$$

$$= B\,\text{sinc}\left(Bt\right) + j\,\frac{B}{2}\,\text{sinc}^2\left(\frac{B}{2}t\right) = y_1(t) + j\,y_2(t)$$

$$E_y = \int_{-\infty}^{+\infty} |y(t)|^2\,dt = \int_{-\infty}^{+\infty} y(t)\cdot y^*(t)\,dt =$$

$$= \int_{-\infty}^{+\infty} \left(y_1^2(t) + y_2^2(t)\right)dt = E_{y1} + E_{y2}$$

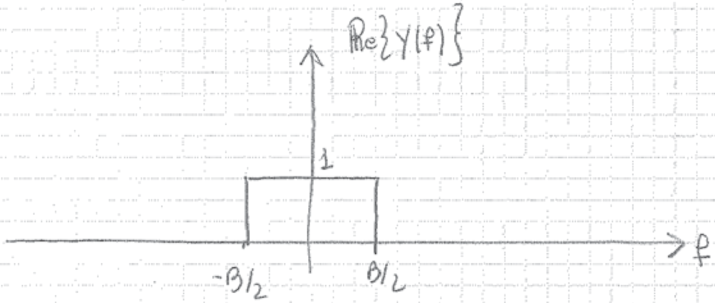
$$E_{y1} = \int_{-\infty}^{+\infty} y_1^2(t)\,dt = \int_{-\infty}^{+\infty} Y_1^2(f)\,df = B$$

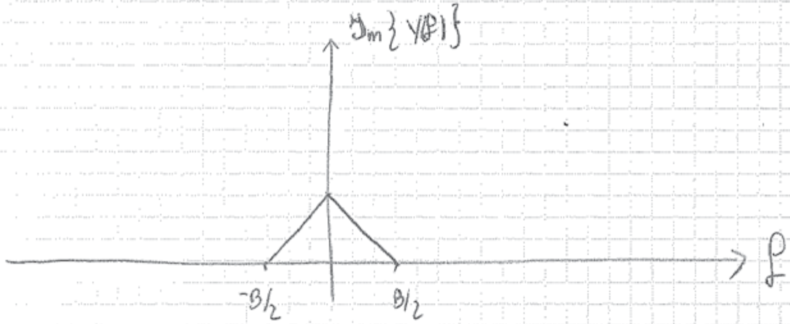
$$E_{y2} = \int_{-\infty}^{+\infty} y_2^2(t)\,dt = \int_{-\infty}^{+\infty} Y_2^2(f)\,df = 2\cdot\int_0^{B/2} \left(1 - \frac{f}{B/2}\right)^2 df = \frac{B}{3} =$$

$$E_y = B + \frac{B}{3} = \frac{4B}{3}$$

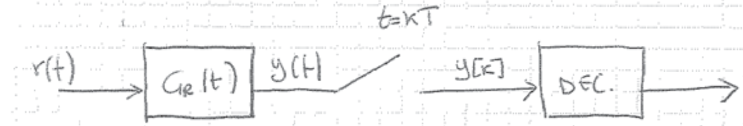
=

$$Y(f) = \mathrm{rect}\left(\frac{f}{B}\right) + j\left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right)$$





Esercizio 2



$$r(t) = s(t) + n(t)$$

$$s(t) = \sum_{k} x[k]\, p(t - kT)$$

$$x[k] \in \{-1, 3\}$$

$$P\{x=-1\} = \frac{1}{3}$$

$$P\{x=3\} = \frac{2}{3}$$

$$P(f) = \sqrt{1-|fT|}\ \text{rect}\left(\frac{fT}{2}\right)$$

$$S_n = \frac{N_0}{2}$$

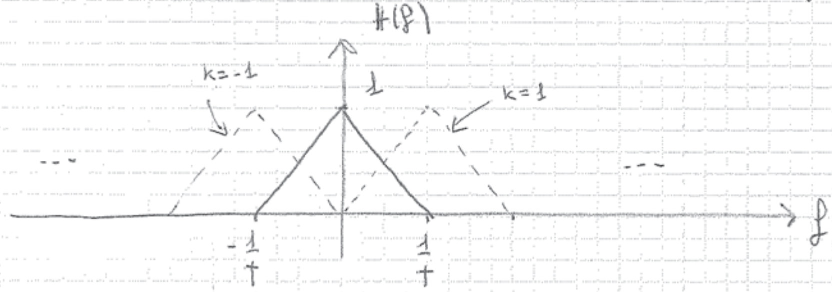
$$G_R(f) = P(f)$$

1) $$E_s = \frac{1}{3}(1)^2 \cdot E_p + \frac{2}{3}(3)^2 \cdot E_p = \left(\frac{1}{3} + 6\right) E_p = \frac{19}{3} E_p$$

$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2\, df = \frac{2}{T} \cdot 1 \cdot \frac{1}{2} = \frac{1}{T}$$

$$E_s = \frac{19}{3T}$$

2) $H(f) = P(f) \cdot G_R(f) = (1-|fT|)\ \text{rect}\left(\frac{fT}{2}\right)$



$$\sum_{k=-\infty}^{+\infty} H(f - k/T) = 1$$

la condizione di Nyquist è verificata.

$$h(0) = \int_{-\infty}^{+\infty} H(f) = \frac{1}{T}$$

3) $S_w(f) = S_n(f) \cdot |G_R(f)|^2$

$$P_w = \int_{-\infty}^{+\infty} S_w(f)\,df = \frac{N_0}{2}\int_{-\infty}^{+\infty} G_R^2(f)\,df = \frac{N_0}{2T}$$

4) $y[k] = h(0)\,x[k] + w_k$ $\qquad w_k \in \mathcal{N}\left(0, \frac{N_0}{2T}\right)$

$$P_E(b) = \frac{1}{3} Q\left(\frac{2/T}{\sqrt{N_0/2T}}\right) + \frac{2}{3} Q\left(\frac{2/T}{\sqrt{N_0/2T}}\right) = Q\left(\frac{2}{T}\sqrt{\frac{2T}{N_0}}\right) =$$

$$= Q\left(\sqrt{\frac{8}{N_0 T}}\right)$$