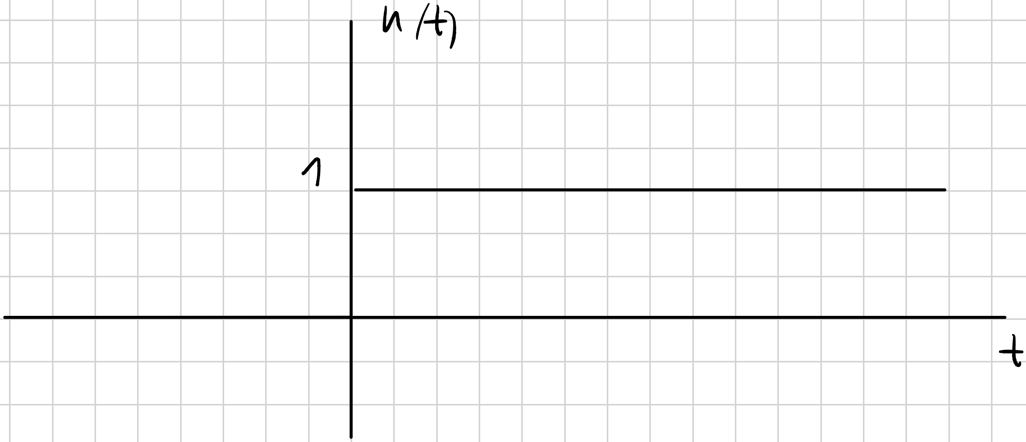
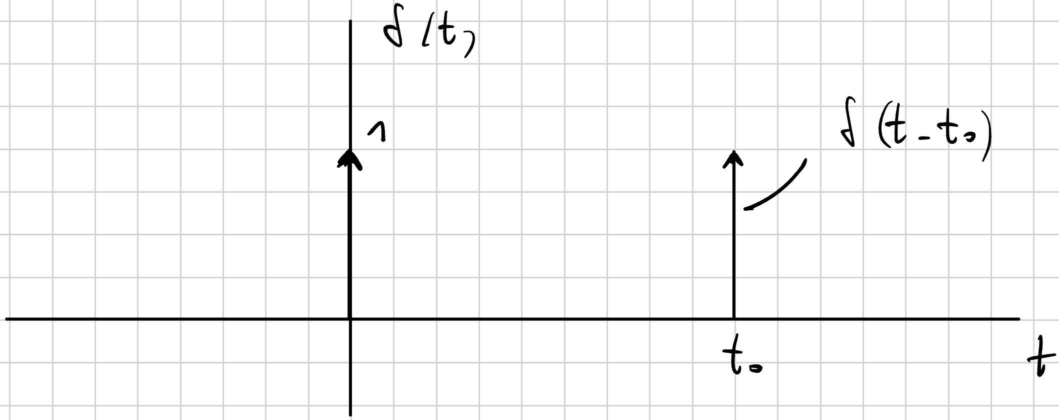
FUNZIONE DELTA DI DIRAC

$$\delta(t) \triangleq \frac{d\,u(t)}{dt}$$



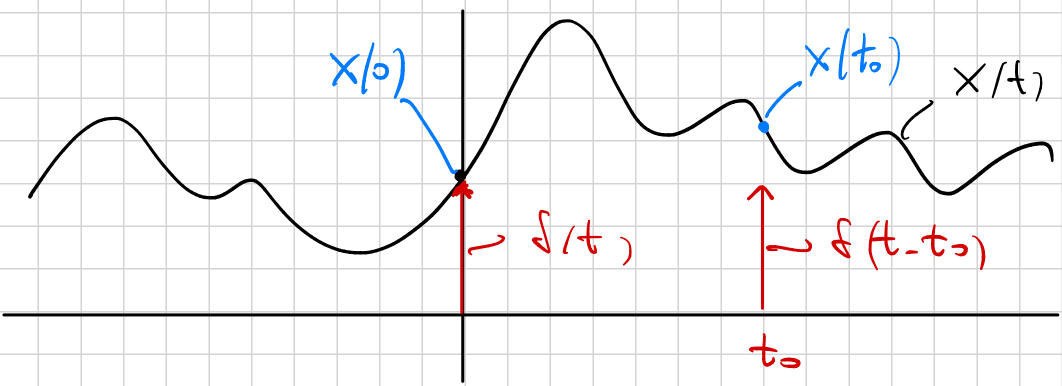


$$u(t) = \int_{-\infty}^{t} \delta(\alpha)\, d\alpha$$

PROPRIETA'

- CAMPIONATRICE

$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$$



$$\underline{\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)}$$

- PARI $\quad \underline{\delta(t) = \delta(-t)}$

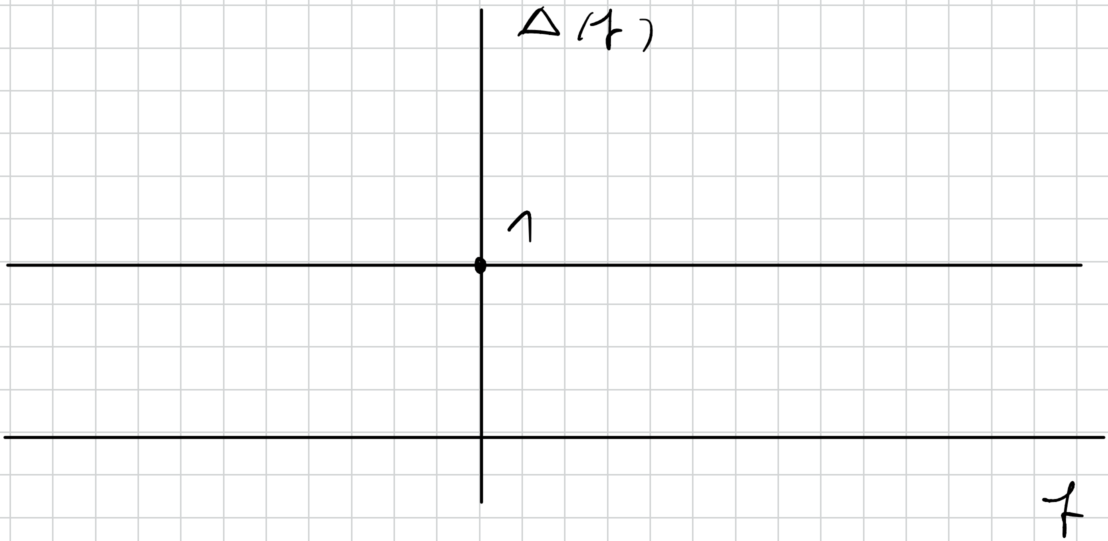
- INVARIANZA AL PRODOTTO CONVOLUZIONE

$$\underline{x(t) \otimes \delta(t) = x(t)}$$

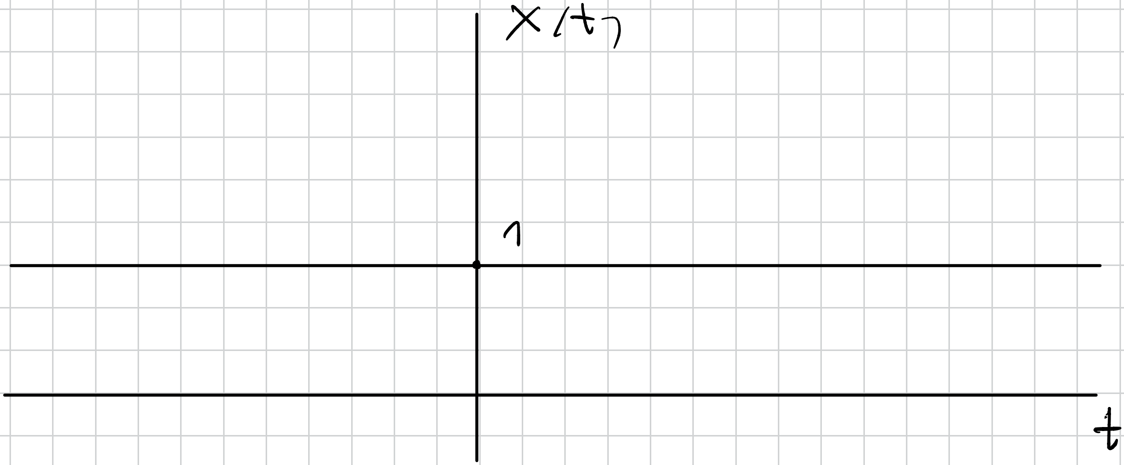
TRASFORMATA DI FOURIER

$$\delta(t) \rightleftarrows \Delta(f) = 1$$

$$\Delta(f) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j 2\pi f t}\, dt = 1$$

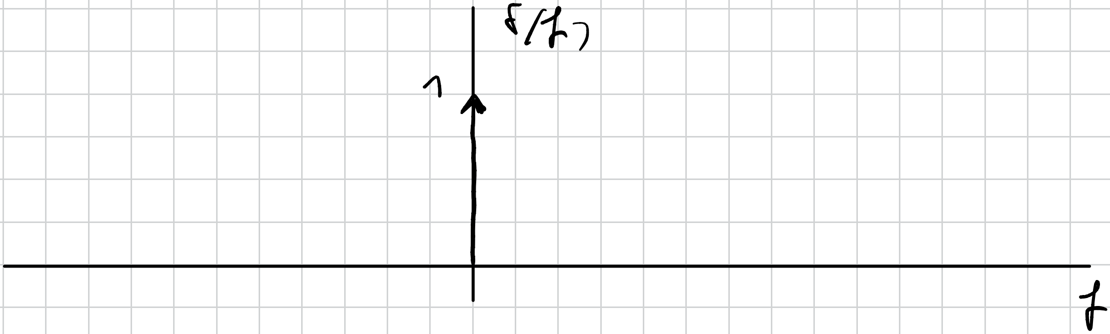


$$\delta(t-t_0) \rightleftarrows 1 \cdot e^{-j 2\pi f t_0}$$



$$\delta(t) \rightleftharpoons 1$$

$$1 \rightleftharpoons \delta(f)$$



DUALITA'

$$\boxed{\begin{array}{c} x(t) \rightleftharpoons X(f) \\ X(t) \rightleftharpoons x(-f) \end{array}}$$

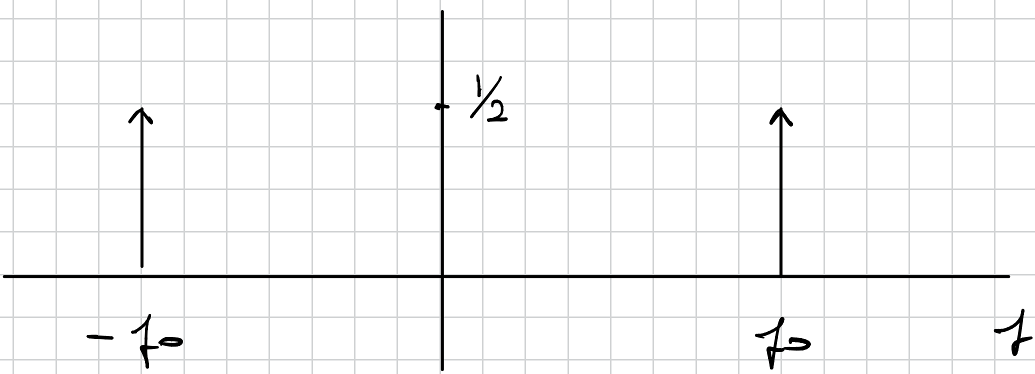
TRASFORMATA COSENO

$$\cos(2\pi f_0 t) \rightleftharpoons \frac{\delta(f-f_0)+\delta(f+f_0)}{2}$$

dimo

$$\cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}$$

$$\boxed{\begin{aligned} \delta(t-t_0) &\rightleftharpoons e^{-j2\pi f t_0} \\ e^{-j2\pi f_0 t} &\rightleftharpoons \delta(f+f_0) \end{aligned}}$$



TRASFORMATA SENO

$$\sin(2\pi f_0 t) \rightleftarrows \frac{\delta(f-f_0) - \delta(f+f_0)}{2j}$$

dimo

$$\sin(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j}$$

APPLICANDO LA DUALITÀ DELLA DELTA DIRAC.

TEOREMA MODULAZIONE (RIVISTO)

$$x(t)\cos(2\pi f_0 t) \rightleftharpoons \frac{X(f-f_0)+X(f+f_0)}{2}$$

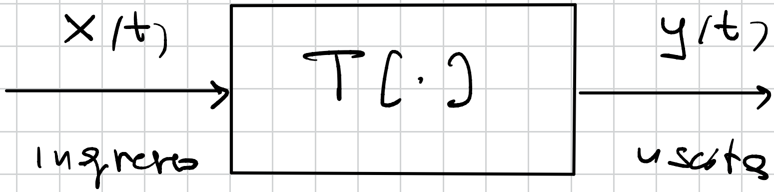
<u>dimo</u>

APPLICANDO IL TEOREMA DEL PRODOTTO

$$X(f) \otimes \frac{\delta(f-f_0)+\delta(f+f_0)}{2}$$

$$\stackrel{!}{=} \frac{X(f-f_0)+X(f+f_0)}{2}$$

SISTEMI



$$y(t) = T[\,x(\alpha)\,;\,t\,]$$

- **STAZIONARIETÀ**

$$y(t) = T[\,x(t)\,]$$

$$T[\,x(t-t_0)\,] = y(t-t_0)$$

- **CAUSALITÀ**

$$y(t) = T[\,x(\alpha),\ \alpha \le t\ ;\ t\,]$$

LINEARITA'

$$y_1(t) = T[x_1(t)]$$

$$y_2(t) = T[x_2(t)]$$

$$T[x_1(t) + x_2(t)] = T[x_1(t)] + T[x_2(t)]$$

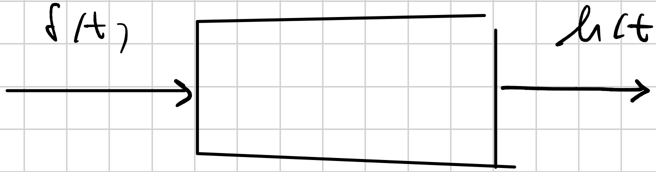
$$= y_1(t) + y_2(t)$$

STABILITA' (BIBO)

$$|x(t)| \leq M \rightarrow |y(t)| \leq K$$

SISTEMI LTI

$$h(t) \triangleq T[\delta(t)]$$ RISPOSTA IMPULSIVA



$$y(t) = T[x(t)]$$

$$= T[x(t) \otimes \delta(t)]$$

$$= T\left[\int_{-\infty}^{\infty} x(\alpha)\,\delta(t-\alpha)\,d\alpha\right]$$

linearità

$$= \int_{-\infty}^{\infty} T[x(\alpha)\,\delta(t-\alpha)]\,d\alpha$$

$$= \int_{-\infty}^{\infty} x(\alpha)\,T[\delta(t-\alpha)]\,d\alpha$$

stazionarità
$$= \int_{-\infty}^{\infty} x(\alpha)\, h(t-\alpha)\, d\alpha$$

$$= x(t) \otimes h(t)$$

$$\boxed{\begin{aligned} y(t) &= x(t) \otimes h(t) \\ Y(f) &= X(f)\, H(f) \end{aligned}}$$

$H(f)$ RISPOSTA IN FREQUENZA DEL SISTEMA