# Soluzione compitino - fila A

## $\boxed{\text{Es. } 3}$

$$x(t) = x_1(t) + x_2(t)$$

$$x_1(t) = \sum_n \mathrm{rect}\left(\frac{t - nT_0}{4T}\right) \qquad T_0 = 5T$$

$$x_2(t) = \sum_n \cos\left(\frac{2\pi\,(t-nT_0)}{4T}\right) \mathrm{rect}\left(\frac{t-nT_0}{2T}\right)$$

$$X_n = X_{1n} + X_{2n}$$

$$X_{1n} = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \mathrm{rect}\left(\frac{t}{4T}\right) e^{-j2\pi n f_0 t}\,dt$$

$$= \frac{1}{5T}\int_{-2T}^{2T} e^{-j2\pi n f_0 t}\,dt = \frac{1}{\cancel{5T}}\,\frac{1}{-j2\pi \frac{n}{\cancel{5T}}}\, e^{-j2\pi \frac{n}{5T} t}\Bigg|_{-2T}^{2T}$$

$$= \frac{1}{\pi n}\,\sin\left(\frac{4}{5}\pi n\right) = \frac{4}{5}\,\mathrm{sinc}\left(\frac{4}{5} n\right)$$

$$X_{2n} = \frac{1}{5T}\int_{-T}^{T} \cos\left(\frac{2\pi t}{4T}\right) e^{-j2\pi n f_0 t}\, dt$$

$$= \frac{1}{10T}\int_{-T}^{T} e^{-j2\pi\left(\frac{n}{5T}-\frac{1}{4T}\right)t}\, dt \; +$$

$$+ \frac{1}{10T}\int_{-T}^{T} e^{-j2\pi\left(\frac{n}{5T}+\frac{1}{4T}\right)t}\, dt$$

$$= \frac{1}{10T}\,\frac{1}{-j2\pi\left(\frac{4n-5}{20T}\right)}\, e^{-j2\pi\left(\frac{4n-5}{20T}\right)t}\Bigg|_{-T}^{T} \; +$$

$$+ \frac{1}{10T}\,\frac{1}{-j2\pi\left(\frac{4n+5}{20T}\right)}\, e^{-j2\pi\left(\frac{4n+5}{20T}\right)t}\Bigg|_{-T}^{T}$$

$$= \frac{1}{10\cancel{T}}\,\frac{1}{\pi\left(\frac{4n-5}{20\cancel{T}}\right)} \cdot \sin\left[2\pi\left(\frac{4n-5}{20}\right)\right] \; +$$

$$+ \frac{1}{10\cancel{T}}\,\frac{1}{\pi\left(\frac{4n+5}{20\cancel{T}}\right)} \sin\left[2\pi\left(\frac{4n+5}{20}\right)\right]$$

$$= \frac{2}{5}\,\mathrm{sinc}\left(\frac{4n-5}{10}\right) + \frac{2}{5}\,\mathrm{sinc}\left(\frac{4n+5}{10}\right)$$

$$\boxed{X_n = \frac{4}{5}\,\mathrm{sinc}\left(\frac{4}{5}n\right) + \frac{2}{5}\left[\mathrm{sinc}\left(\frac{4n-5}{10}\right) + \mathrm{sinc}\left(\frac{4n+5}{10}\right)\right]}$$

$$\boxed{E_x = \infty}$$

$$P_x = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \left[x_1(t) + x_2(t)\right]^2 dt =$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x_1^2(t) + x_2^2(t) + 2x_1(t)\,x_2(t)\,dt$$

$$= \frac{1}{T_0}\int_{-2T}^{2T} 1\,dt + \frac{1}{T_0}\int_{-T}^{T}\left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi t}{2T}\right)\right]dt +$$

$$+ \frac{2}{T_0}\int_{-T}^{T}\cos\left(\frac{2\pi t}{4T}\right)dt$$

$$= \frac{4\cancel{T}}{5\cancel{T}} + \frac{\cancel{T}}{5\cancel{T}} + \frac{2}{5\cancel{T}}\,\frac{4\cancel{T}}{\cancel{2}\pi}\cdot\cancel{2} = \boxed{1 + \frac{8}{5\pi} = P_x}$$

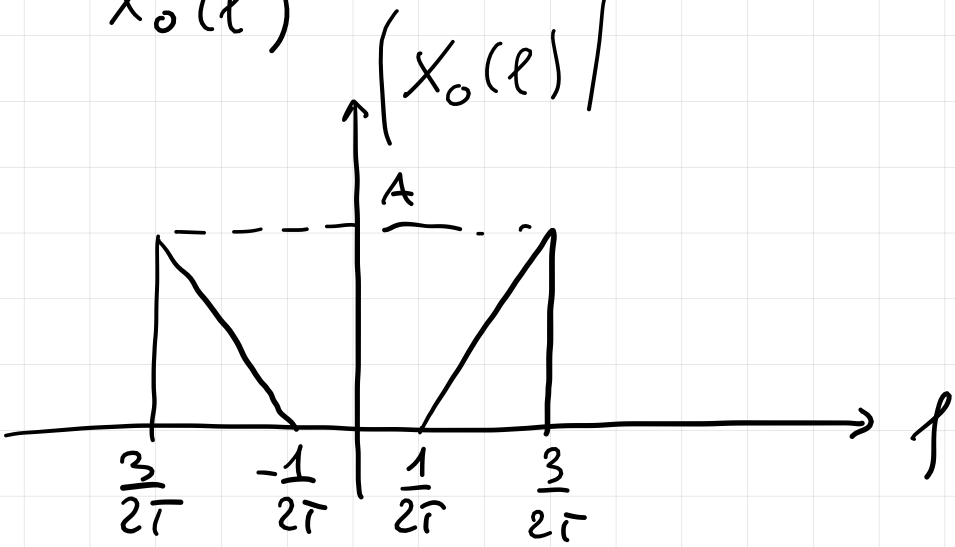
$$x_{eff} = \sqrt{1 + \frac{8}{5\pi}}$$

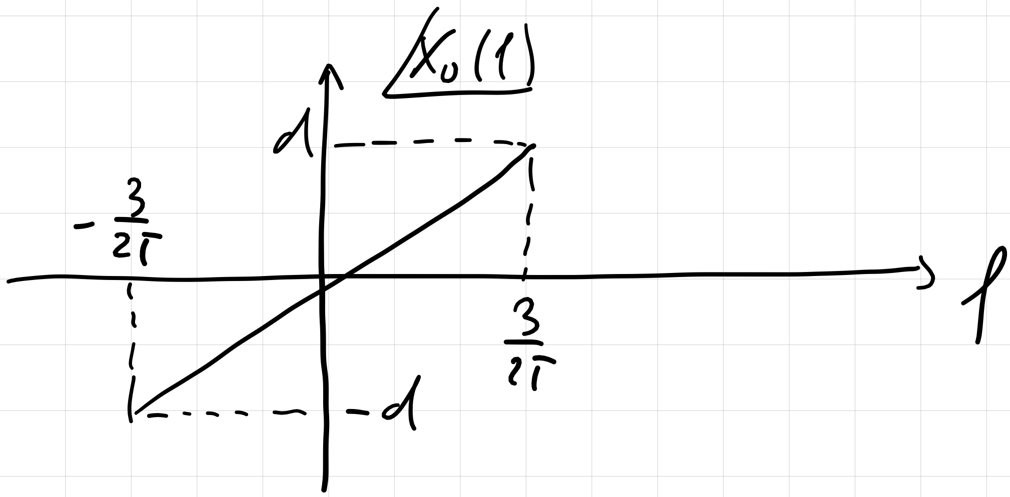
$$x_m = \frac{1}{5T}\int_{-2T}^{2T} 1 \cdot dt + \frac{1}{5T}\int_{-T}^{T} \cos\frac{2\pi t}{4T}\, dt$$

$$= \frac{4}{5} + \frac{1}{5\cancel{T}}\,\frac{4\cancel{T}}{\cancel{2}\pi}\,\cancel{2} = \boxed{\frac{4}{5} + \frac{4}{5\pi} = x_m}$$

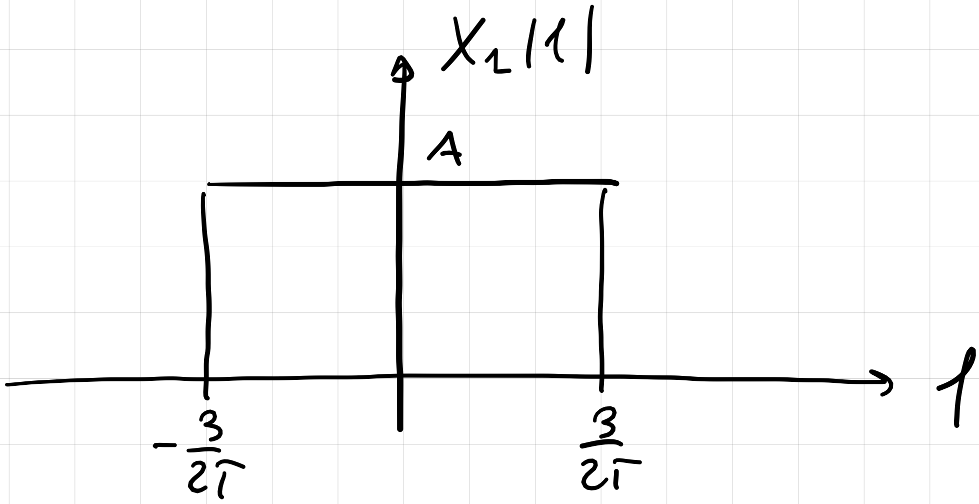
## $\boxed{Es\ 4}$

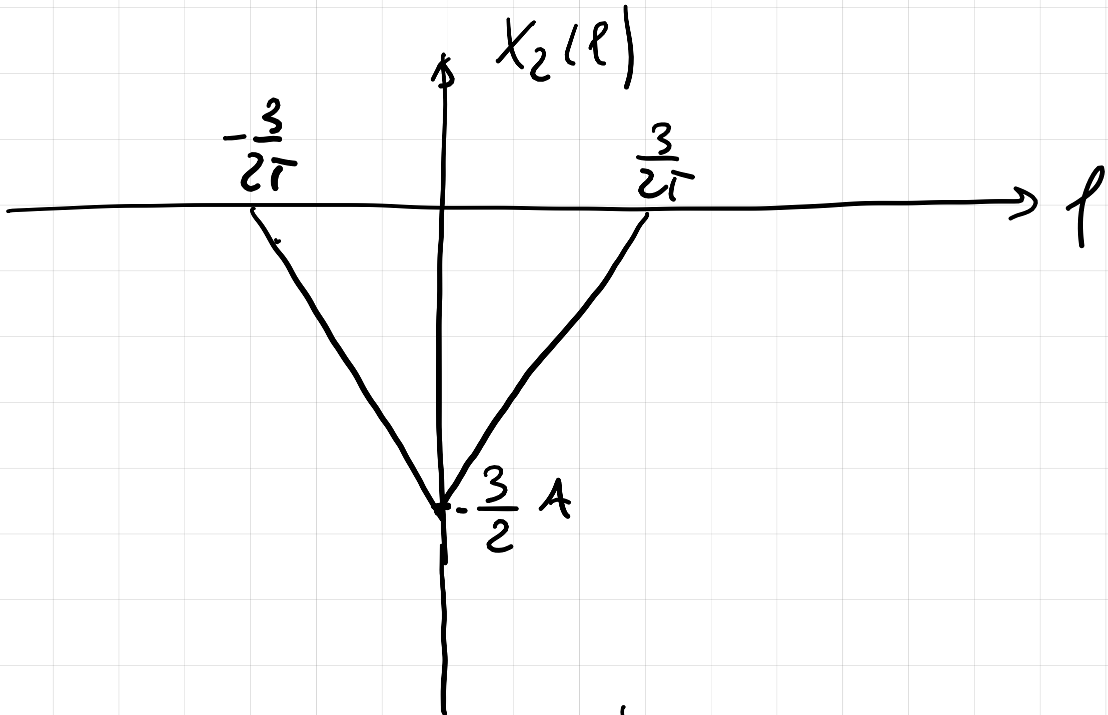
Definisco $X_0(f)$

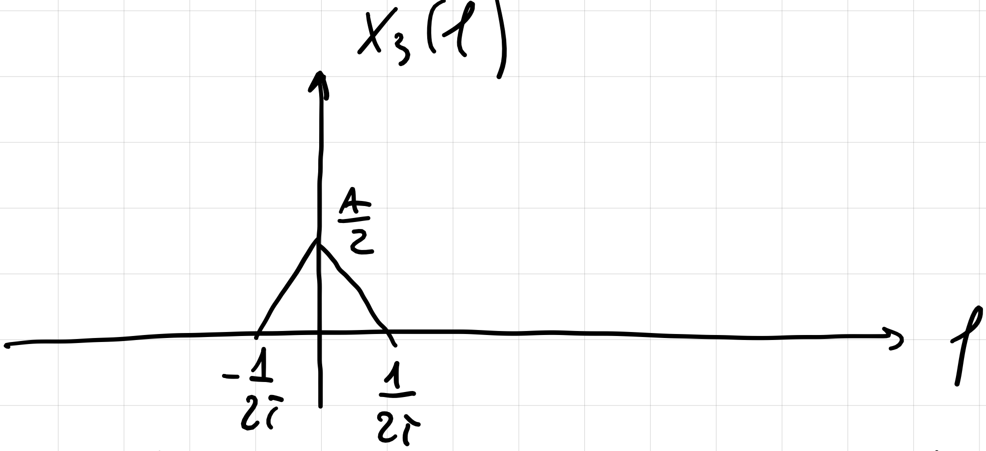



$$|X_0(f)| = X_1(f) + X_2(f) + X_3(f)$$




$$X_1(f) = A\,\mathrm{rect}\left(\frac{t}{3/T}\right) \Leftrightarrow x_1(t) = \frac{3A}{T}\,\mathrm{sinc}\left(\frac{3t}{T}\right)$$

$$X_2(f) = -\frac{3}{2}A\left(1 - \frac{|f|}{3/2T}\right)\mathrm{rect}\left(\frac{t}{3/T}\right) \Leftrightarrow x_2(t) = -\frac{9A}{4T}\,\mathrm{sinc}^2\left(\frac{3t}{2T}\right)$$

$$X_3(f) = \frac{A}{2}\left(1 - \frac{|f|}{1/2T}\right)\mathrm{rect}\left(\frac{t}{T}\right) \Leftrightarrow x_3(t) = \frac{A}{4T}\,\mathrm{sinc}^2\left(\frac{t}{2T}\right)$$

$$\angle X_0(f) = 2\pi f t_0 \quad , \quad t_0 = \frac{Td}{3\pi} \quad \left(\text{vedi sol fila C}\right)$$

$$X_0(f) = \left[X_1(f) + X_2(f) + X_3(f)\right] e^{j2\pi f t_0}$$

$$X(f) = X_0\left(f - \frac{9}{2T}\right) + X_0\left(f + \frac{9}{2T}\right)$$

$$x_0(t) = \text{ATCF}\left[X_0(f)\right]$$

$$= x_1(t+t_0) + x_2(t+t_0) + x_3(t+t_0)$$

$$x(t) = x_0(t)\, e^{j2\pi \frac{9}{2T} t} + x_0(t)\, e^{-j2\pi \frac{9}{2T} t}$$

$$= 2\, x_0(t) \cos\left(9\pi \frac{t}{T}\right)$$

$$\boxed{\begin{aligned} x(t) &= 2\left[x_1(t+t_0) + x_2(t+t_0) + x_3(t+t_0)\right] \cdot \\ &\quad \cdot \cos\left(9\pi \frac{t}{T}\right) \\ x_1(t) &= \frac{3A}{T} \operatorname{sinc}\left(\frac{3t}{T}\right) \\ x_2(t) &= \frac{9A}{4T} \operatorname{sinc}^2\left(\frac{3t}{2T}\right) \\ x_3(t) &= \frac{A}{4T} \operatorname{sinc}^2\left(\frac{t}{2T}\right) \end{aligned}}$$