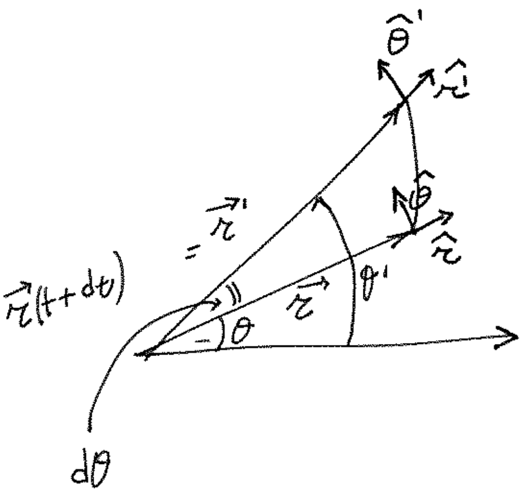
# Accelerazione in coord. POLARI

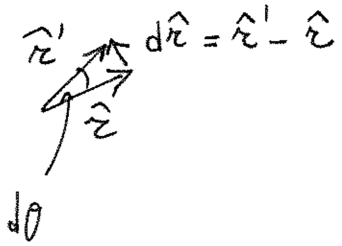



$\left(\hat{\theta}\right.$ orientato nel verso di $\theta$ positivi, $\left.\hat{\theta}\cdot\hat{r}=0\right)$

$$\vec{r} = r\,\hat{r}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\,\hat{r} + r\,\frac{d\hat{r}}{dt}$$

$$\hat{r}^2 = 1 \quad \Rightarrow \quad \cancel{2}\,\hat{r}\cdot d\hat{r} = 0 \qquad d\hat{r} \perp \hat{r}$$




$$d\hat{r} = |\hat{r}|\cdot d\theta\;\hat{\theta} = 1\cdot d\theta\;\hat{\theta} = d\theta\;\hat{\theta}$$

$$\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\,\hat{\theta} = \omega\,\hat{\theta} \qquad \omega = \frac{d\theta}{dt}$$

$$\vec{v} = \frac{dr}{dt}\,\hat{r} + r\omega\,\hat{\theta}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2r}{dt^2}\,\hat{r} + \frac{dr}{dt}\,\frac{d\hat{r}}{dt} + \frac{dr}{dt}\,\omega\,\hat{\theta} + r\,\frac{d\omega}{dt}\,\hat{\theta} + r\omega\,\frac{d\hat{\theta}}{dt}$$


$$d\hat{\theta} = |\hat{\theta}|\cdot d\theta\,(-\hat{r})$$

$$\frac{d\hat{\theta}}{dt} = -\,\omega\,\hat{r}$$

$$\vec{a} = \left[\frac{d^2r}{dt^2} + r\omega(-\omega)\right]\hat{r} + \left[\frac{dr}{dt}\,\omega + \frac{dr}{dt}\,\omega + r\,\frac{d\omega}{dt}\right]\hat{\theta} =$$

$$= \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(2\omega\,\frac{dr}{dt} + r\,\frac{d\omega}{dt}\right)\hat{\theta} = \vec{a}_r + \vec{a}_\theta =$$

$$= a_r\,\hat{r} + a_\theta\,\hat{\theta}$$

Dove $\quad a_r = \dfrac{d^2r}{dt^2} - \omega^2 r \qquad\qquad |\vec{a}| = \sqrt{a_r^2 + a_\theta^2}$

$$a_\theta = r\,\frac{d\omega}{dt} + 2\omega\,\frac{dr}{dt}$$

## Casi particolari

1. Moto circolare : $r$ = costante $\left(\frac{dr}{dt} = 0\right)$

$$\vec{v} = r\omega\,\hat{\theta}$$

$$\vec{a} = \underbrace{-r\omega^2\,\hat{r}}_{\vec{a}_{centripeta}} + \underbrace{r\frac{d\omega}{dt}\,\hat{\theta}}_{\vec{a}_{tang.}}$$

1.a moto circolare uniforme $\omega$ = costante $\left(\frac{d\omega}{dt} = 0\right)$

$$\vec{a} = -r\omega^2\hat{r}$$

1. Sia $\vec{a}_{\hat{\theta}} = 0$ ovvero $\vec{a} = a_r\,\hat{r}$

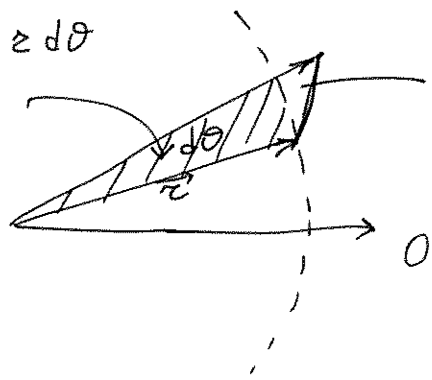
Allora :

$a_\theta = 0 \qquad r\frac{d\omega}{dt} + 2\omega\frac{dr}{dt} = 0$ molt. per $r$

$$r^2\frac{d\omega}{dt} + 2\omega r\frac{dr}{dt} = 0$$

$$\frac{d}{dt}\left(r^2\omega\right) = 0$$

$dS = \frac{1}{2}r\,r\,d\theta$




Area infinitesima al II ordine in $d\theta$

$$0 = \frac{d}{dt}\left(\frac{1}{2}r^2\omega\right) = \frac{d}{dt}\left(\frac{1}{2}r^2\frac{d\theta}{dt}\right) =$$

$$= \frac{d}{dt}\left(\frac{dS}{dt}\right)$$

ovvero $\frac{dS}{dt} = \frac{1}{2}r^2\omega$ è costante .

$\uparrow$

Velocità AREOLARE