(1.a)

$$f(x,y) = \int_x^y \cos(t^2)\,dt \qquad (x,y) \in \mathbb{R}^2$$

$$\boxed{\frac{d}{dx}\int_0^x f(t)\,dt = f(x) \quad f \text{ continue}}$$

f_{xxy} $\quad f_{yy}$

$$f_x = \frac{\partial f}{\partial x} = -\cos(x^2) \qquad f_{xxy} = f_{xyx}$$

$$f_{xy} = \frac{\partial^2 f}{\partial y \partial x} = 0 \qquad \bullet\, f_{xyx} = 0$$

$$f_y = \cos(y^2)$$

$$\bullet\, f_{yy} = -\sin(y^2)\cdot 2y$$

$$F(x,y) = \int_{\varphi(x,y)}^{\psi(x,y)} f(t,x,y)\,dt$$

$$\frac{\partial F}{\partial x} = \int_{\varphi(x,y)}^{\psi(x,y)} \frac{\partial f}{\partial x}(t,x,y)\,dt + f(\psi(x,y),x,y)\frac{\partial \psi}{\partial x} - f(\varphi(x,y),x,y)\frac{\partial \varphi}{\partial x}$$

(1.b) $\quad f(x,y) = 8x^2 + \int_1^y e^{-t^2}\,dt \qquad$ Piano TG. $\quad P=(3,1)$

$$\varphi(x,y) = \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0) + f(x_0,y_0) = f(x_0,y_0) + \left(\nabla f(x_0,y_0), \begin{pmatrix} x-x_0 \\ y-y_0 \end{pmatrix}\right)$$

$$\varphi(x,y) - f(x,y) = o\left(\sqrt{(x-x_0)^2+(y-y_0)^2}\right) \quad \Leftarrow f \in C^1$$

$$f(3,1) = 8\cdot 3^2 + \int_1^1 e^{-t^2}\,dt = 8\cdot 9 = 72$$

$$\frac{\partial f}{\partial x}(3,1) = 16x\Big|_{x=3} = 48 \qquad \frac{\partial f}{\partial y}(x,y)\Big|_{y=1} = e^{-y^2}\Big|_{y=1} = \frac{1}{e}$$

$$\varphi(x,y) = 72 + 48(x-3) + \frac{1}{e}(y-1)$$

2.)

$$f(x,y,z) = xyz + \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \qquad x,y,z \neq 0$$

$$f \text{ è } C^\infty(\mathbb{R}^3 \setminus \{x,y,z=0\})$$

$$\inf_D f = -\infty \qquad \sup_D f = +\infty \qquad\qquad D'$$

$$\nabla f = 0$$

$$\begin{cases} \dfrac{\partial f}{\partial x} = yz - \dfrac{1}{x^2} \\ \dfrac{\partial f}{\partial y} = xz - \dfrac{1}{y^2} \\ \dfrac{\partial f}{\partial z} = xy - \dfrac{1}{z^2} \end{cases} \qquad \begin{cases} \dfrac{\partial f}{\partial x} = 0 \\ \dfrac{\partial f}{\partial y} = 0 \\ \dfrac{\partial f}{\partial z} = 0 \end{cases} \qquad \begin{matrix} yz = \dfrac{1}{x^2} \\ xz = \dfrac{1}{y^2} \\ xy = \dfrac{1}{z^2} \end{matrix} \qquad \boxed{\begin{matrix} x^2yz = 1 \\ xy^2z = 1 \\ xyz^2 = 1 \end{matrix}}$$

$$\begin{cases} x\,(xyz) = 1 \\ y\,(xyz) = 1 \\ z\,(xyz) = 1 \end{cases} \qquad xyz = \frac{1}{x} \qquad xyz = \frac{1}{y} \qquad xyz = \frac{1}{z}$$

$$\frac{1}{x} = \frac{1}{y} \qquad \frac{1}{x} = \frac{1}{z}$$

$$\boxed{x = y = z} \qquad x^2 \cdot x \cdot x = 1 \qquad x^4 = 1 \qquad x = \pm 1$$

$$x^4 - 1 = 0$$

$$(x^2-1)(x^2+1) = 0$$

$P = (1,1,1)$ $\qquad Q = (-1,-1,-1)$ $\qquad$ UNICI PT. STAZIONARI

$$f_{xx} = \frac{2}{x^3} \qquad f_{xy} = z \qquad f_{xz} = y$$

$$f_{yx} = z \qquad f_{yy} = \frac{2}{y^3} \qquad f_{yz} = x$$

$$f_{zx} = y \qquad f_{zy} = x \qquad f_{zz} = \frac{2}{z^3}$$

$$Hf(x,y,z) = \begin{pmatrix} \frac{2}{x^3} & z & y \\ z & \frac{2}{y^3} & x \\ y & x & \frac{2}{z^3} \end{pmatrix}$$

$$Hf(1,1,1) = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \qquad Hf(-1,-1,-1) = \begin{pmatrix} -2 & -1 & -1 \\ -1 & -2 & -1 \\ -1 & -1 & -2 \end{pmatrix}$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 2-\lambda \end{vmatrix} + \begin{vmatrix} 1 & 2-\lambda \\ 1 & 1 \end{vmatrix}$$

$$= (2-\lambda)\left[(2-\lambda)^2-1\right] - \left[2-\lambda-1\right] + \left[1-(2-\lambda)\right]$$

$$= (2-\lambda)^3 - (2-\lambda) - 1 + \lambda - 1 + \lambda$$

$$= (2-\lambda)^3 - 2 + \lambda - 1 - 1 + 2\lambda$$

$$= (2-\lambda)^3 + 3\lambda - 4 = 0$$

$$-\lambda^3 + 3\cdot 2\lambda^2 - 12\lambda + 8 - 4 + 3\lambda = 0$$

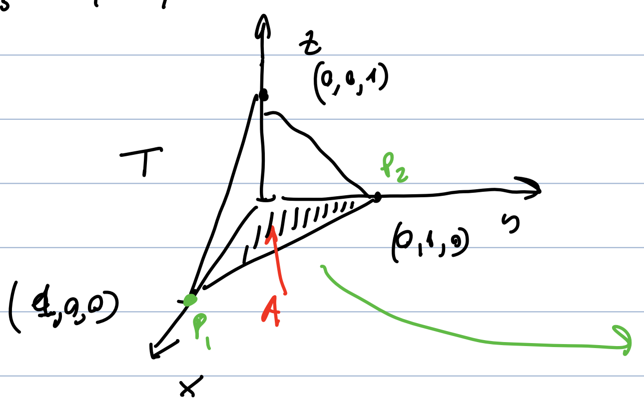
$$-\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$$

$$-\lambda^3 + 6\lambda^2 - 9\lambda + 4 \;\Big|\; \frac{\lambda - 4}{-\lambda^2}$$

$$P(\lambda) = (\lambda^2 - 2\lambda + 1)(4-\lambda) = (\lambda-1)^2(4-\lambda)$$

$\lambda_{1,2,3} = 1, 1, 4$ $\quad P_1$ min relativo

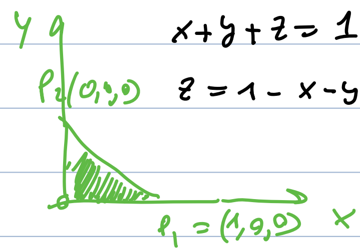
(3) $\quad T$



$x = 0 \qquad y = 0 \qquad z = 0$

$x + y + z = 1$

$z = 1 - x - y$



$$\bar{x} = \frac{\int_T x\,dx\,dy\,dz}{\int_T dx\,dy\,dz} = \frac{\frac{1}{24}}{\frac{1}{6}} = \frac{6}{24} = \frac{1}{4}$$

$m(A) = \frac{1}{2}$

$V(T) = \frac{1}{2}\cdot 1 \cdot \frac{1}{3} = \frac{1}{6}$

$$T = \{ (x,y) \in D \qquad 0 \le z \le 1 - x - y \}$$

$$D = \{ (x,y) \in \mathbb{R}^2 : \quad 0 \le x \le 1 \quad 0 \le y \le 1 - x \}$$

$$\int_T x\,dx\,dy\,dz = \int_D dx\,dy \int_0^{1-x-y} x\,dz$$

$$= \int_D x\,dx\,dy \int_0^{1-x-y} 1\,dz = \int_D x\,dx\,dy \; z\Big|_0$$

$$\frac{\partial}{\partial y} -\frac{(1-x-y)^3}{2} = 1-x-y$$

$$= \int_D x\,dx\,dy\,(1-x-y)$$

$$= \int_0^1 dx\, x \int_0^{1-x} dy\,(1-x-y) = -\int_0^1 x\,dx \;\frac{(1-x-y)^2}{2}\Big|_0^{1-x}$$

$$= -\frac{1}{2}\int_0^1 x \left\{ \left[1-x\cancel{-(1-x)}\right]^2 - (1-x-0)^2 \right\}$$

$$= +\frac{1}{2}\int_0^1 x\,(1-x)^2\,dx$$

$$= \frac{1}{2}\int_0^1 x(1-2x+x^2)\,dx = \frac{1}{2}\int_0^1 x-2x^2+x^3\,dx$$

$$= \frac{1}{2}\left(\frac{x^2}{2} - 2\frac{x^3}{3} + \frac{x^4}{4}\right)\Big|_0^1 = \frac{1}{2}\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right) = \frac{1}{2}\,\frac{6-8+3}{12} = \frac{1}{24}$$