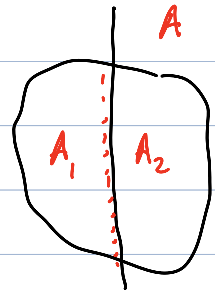
$$\int_a^b f'(x)\,dx = f(b) - f(a) \qquad f \in C^1$$

$v = \nabla f$ POTENZIALE



$$\int_A f = \int_{A_1} f + \int_{A_2} f$$

$$\int_A f'(x) \qquad \int_a^b \int_{\varphi(x)}^{\psi(x)} \frac{\partial f}{\partial x_j} = \int_A \frac{\partial f}{\partial x_i}$$

$$= \int_a^b f(x,\psi(x)) - f(x,\varphi(x))\,dx$$

$A_1, A_2 \subseteq \mathbb{R}^n$ misurabili $\iff m(\partial A_1) = 0 \quad m(\partial A_2) = 0$

$\Rightarrow \quad \mathring{A} \subseteq A \subseteq \bar{A}$

$m(\mathring{A}) = m(A) = m(\bar{A})$

$A_1 \cap A_2 \qquad A_1 \cup A_2$ misurabili

$\Rightarrow \quad \int_A f = \int_{\mathring{A}} f = \int_{\bar{A}} f$

$\bigcap_{k=1}^{N} A_k \qquad \bigcup_{k=1}^{N} A_k$ misurabili

$q_i \in [0,1] \qquad q_i \in \mathbb{Q} \qquad \overline{\bigcup \{q_i\}} = [0,1]$

non misurabile

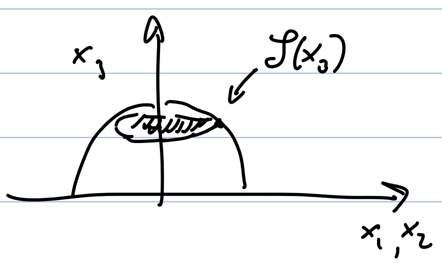
$$\lim_{n\to+\infty} \int_A f_n(x)\,dx \overset{?}{=} \int_A \lim_{n\to+\infty} f_n(x)\,dx$$

$$S^2 = \{x \in \mathbb{R}^3 \quad \|x\| = 1\} \qquad B = \{x \in \mathbb{R}^3 \mid \|x\| \leq 1\}$$

$$C = B \cap \{x_3 > 0\} = \{x \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1 \quad x_3 > 0\}$$

$$x_3 = \sqrt{1 - x_1^2 - x_2^2}$$

$$\int_C dx_1\,dx_2\,dx_3 = \int_0^1 dx_3 \int_{S(x_3)} 1\,dx_1\,dx_2 =$$



$$x_1^2 + x_2^2 = 1 - x_3^2$$

$$= \int_0^1 dx_3\; m(S(x_3)) = \int_0^1 dx_3\; \pi\,(1 - x_3^2)$$

$$= \pi \left. x_3 - \frac{x_3^3}{3} \right|_0^1 = \pi\left(1 - \frac{1}{3}\right) = \frac{2}{3}\pi$$

$m(B) = \frac{4\pi}{3}$ $\qquad$ $m(B(0,R)) = \frac{4}{3}\pi R^3$

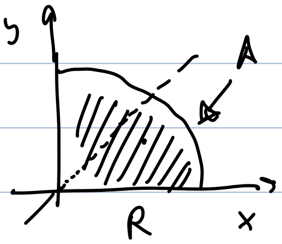
CENTROIDE $\qquad \rho = 1$

$A \subset \mathbb{R}^2 \qquad \bar{x} = \frac{1}{m(A)} \int_A x \, dx\, dy \qquad \bar{y} = \frac{1}{m(A)} \int_A y \, dx\, dy$

$\rho(x,y) \qquad M = \int_A \rho(x,y)\, dx\, dy$ massa totale

$\bar{x} = \frac{1}{M} \int_A x\, \rho(x,y)\, dx\, dy \qquad \bar{y} = \frac{1}{M} \int y\, \rho(x,y)\, dx\, dy$

BARICENTRO



$\rho = 1 \qquad \bar{x} = \bar{y}$

$m(A) = \frac{\pi R^2}{4} \qquad A = \{ x \in [0,R] \quad 0 \le y \le \sqrt{R^2 - x^2} \}$

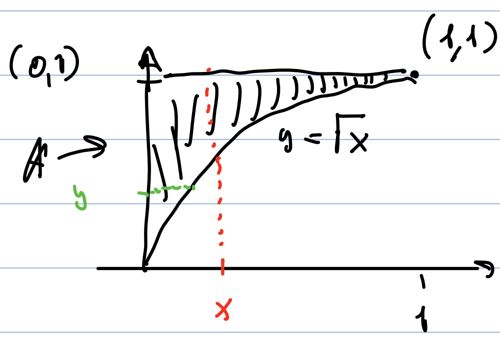
$m(A) \cdot \bar{x} = \int_A x\, dx\, dy = \int_0^R dx \int_0^{\sqrt{R^2-x^2}} \cancel{x}\, dy = \int_0^R dx\, x \int_0^{\sqrt{R^2-x^2}} 1\, dy$

$= \int_0^R dx\, x \left(y \Big|_0^{\sqrt{R^2-x^2}} \right) = \int_0^R dx\, x\sqrt{R^2-x^2} = \int_0^R dx \frac{2x}{2} (R^2-x^2)^{1/2}$

$= -\frac{1}{2} \int_0^R -2x\, (R^2-x^2)^{1/2}\, dx = -\frac{1}{2} \frac{(R^2-x^2)^{3/2}}{\frac{3}{2}} \Big|_0^R$

$= -\frac{1}{3} (R^2-x^2)^{3/2} \Big|_0^R = \frac{R^3}{3}$

$\bar{x} = \frac{R^3}{3\pi R^2} 4 = \frac{4}{3\pi} R$



$\int_A \sin(y^3)\, dx\, dy$ $\qquad$ ESISTE f continua, A misurabile

$\frac{d}{dy}\left(-\cos(y^3)\right) = \sin(y^3) \cdot 3y^2$

$\int^1 dx \int^1 dy\, \sin(y^3) = \int \sin(y^3)\, dx\, dy$

$$\int_0^1 dy \int_0^{y^2} \sin(y^3)\,dx = \int_0^1 dy\, \sin(y^3) \int_0^{y^2} dx = \int_0^1 dy\, \sin(y^3)\left(x\Big|_0^{y^2}\right)$$

$$= \frac{1}{3}\int_0^1 dy\, \sin(y^3)\, 3y^2 = -\frac{1}{3}\left(\cos(y^3)\Big|_0^1\right) = -\frac{1}{3}\cos(1) + \frac{1}{3}$$

LEMMA $\int_R f(x,y)\,dx\,dy$ $\qquad R = [a,b]\times[c,d]$

$f(x,y) = F(x)\,G(y)$

■ $$\int_R F(x)\,G(y)\,dx\,dy = \int_a^b dx \int_c^d dy\; F(x)\,G(y)$$

$$= \left(\int_a^b F(x)\,dx\right)\left(\int_c^d G(y)\,dy\right)$$

$$\int_R f(x_1, x_2, \dots x_n)\,dx_1 \dots dx_n$$

$[a,b]\times[c,d]\times[e,f]\dots$

$f(x_1 \dots x_n) = F(x_1)\,G(x_2)\,H(x_3)\dots$

$$\int_a^b F(x_1)\,dx_1 \int_c^d G(x_2)\,dx_2 \dots$$

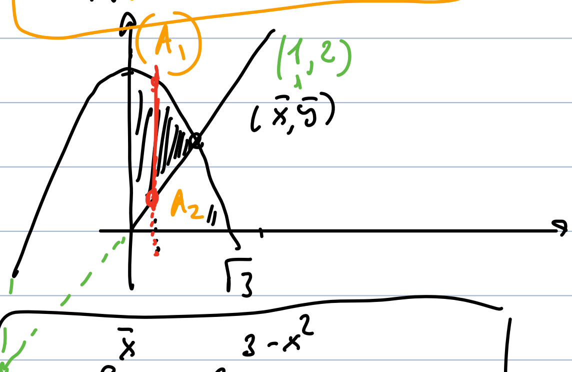
$$\int_a^b f = \int_a^c f + \int_c^b f$$

$$\boxed{\int_{A_1} (x+2y)\,dx\,dy}$$

A REGIONE DI PIANO DELIMITATA

$y = 2x$

$y = 3 - x^2$ NEL I QUADRANTE



$2\bar x = 3 - \bar x^2$ $\qquad \bar x^2 + 2\bar x - 3 = 0$

$\bar y = 2\bar x$ $\qquad \bar x = -1 \pm \sqrt{1+3} = -1 \pm 2 \begin{cases} 1 \\ -3 \end{cases}$

$$\rightarrow \boxed{\int_0^{\bar x} dx \int_{2x}^{3-x^2} (x+2y)\,dy}$$

$A_1 = \{ x \in [0, \bar x] \quad 2x < y < 3 - x^2 \}$

INSIEME NORMALE RISPETTO A Y

$\varphi : [a,b] \to [c,d]$

$t = \varphi(x) \qquad dt = \varphi'(x)\,dx$

$\phi : \Omega' \longrightarrow \Omega$

$\Omega' \subseteq \mathbb{R}^2_{(u,v)} \qquad \Omega \subseteq \mathbb{R}^2_{(x,y)}$

$\begin{cases} x = \varphi(u,v) \\ y = \psi(u,v) \end{cases}$

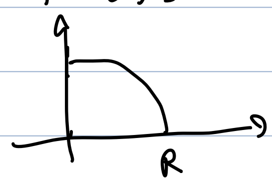
- $\displaystyle\int_{\Omega} f(x,y)\,dx\,dy = \int_{\Omega'} f(\varphi(u,v), \psi(u,v))\,|\det J\phi|\,du\,dv$

$J\phi = \begin{pmatrix} \frac{\partial \varphi}{\partial u} & \frac{\partial \varphi}{\partial v} \\ \frac{\partial \psi}{\partial u} & \frac{\partial \psi}{\partial v} \end{pmatrix}$

$\det J\phi \neq 0 \Rightarrow \phi$ é localmente invertibile

$A = \{ (x,y) \in \mathbb{R}^2 , \ 0 \leq x \leq R^2 \ 0 \leq y \leq \sqrt{R^2 - x^2} \}$

$\displaystyle\int_A x\,dx\,dy$



$\begin{cases} x = \rho \cos\theta \\ y = \rho \sin\theta \end{cases}$

$A = \{ (\rho, \theta) \in \mathbb{R}^2 : \ 0 \leq \rho \leq R \quad \theta \in [0, \frac{\pi}{2}] \}$

$\det J = \rho \geq 0$

$\displaystyle\int_0^{\pi/2} d\theta \int_0^R d\rho \ \rho\cos\theta\ \rho = \int_0^{\pi/2} d\theta \int_0^R \rho^2 \cos\theta\, d\rho$

$\displaystyle = \left(\int_0^{\pi/2} \cos\theta\, d\theta\right)\left(\int_0^R \rho^2 d\rho\right)$

$\displaystyle = \left(+ \sin\theta \Big|_0^{\pi/2} \right) \left(\frac{\rho^3}{3} \Big|_0^R \right)$

$e^{-\|(x,y)\|^2}$

$\displaystyle\int_{B(0,R)} e^{-x^2-y^2}\,dx\,dy = \int_{B(0,R)} e^{-(x^2+y^2)}\,dx\,dy$

$\displaystyle = \iint e^{-x^2} e^{-y^2}\,dx\,dy$

$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$

$\displaystyle\int_{-R}^{R} dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} e^{-x^2} e^{-y^2}\,dy$

$2\pi \quad R \quad 2 \quad R \quad 2 \quad R \quad 2$

$$\int_0 d\theta \int_0 d\rho \; e^{-\rho^2}\rho = \frac{2\pi}{-2}\int_0 -2\rho e^{-\rho^2} d\rho = -\pi \int_0 \frac{d}{d\rho} e^{-\rho^2} d\rho$$

detJ

$$= -\pi\left(e^{-R^2} - e^{-0}\right) = \pi\left(1-e^{-R^2}\right)$$

$$\int_{\mathbb{R}^2} e^{-(x^2+y^2)} dx\,dy \overset{def}{=} \lim_{R\to+\infty}\left(\int_{B(0,R)} e^{-(x^2+y^2)} dx\,dy\right) = \lim_{R\to+\infty}\left[\pi\left(1-e^{-R^2}\right)\right]$$

$$= \pi$$

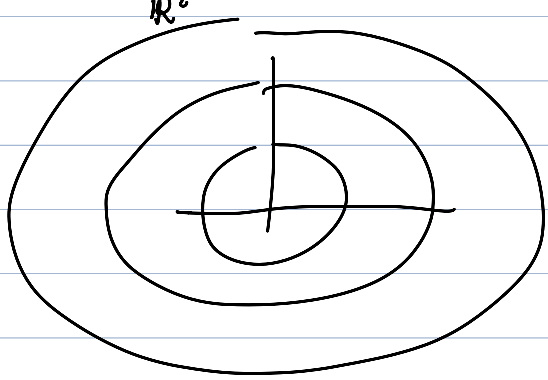
$$I^2 = \Big(\underbrace{\int_{-\infty}^{+\infty} e^{-x^2}dx}_{I}\Big)\Big(\underbrace{\int_{-\infty}^{+\infty} e^{-y^2}dy}_{I}\Big) = \int_{\mathbb{R}^2} e^{-x^2}e^{-y^2}dx\,dy = \pi$$

$$\int_{-\infty}^{+\infty} e^{-x^2}dx = \sqrt{\pi} \qquad\qquad \int_0^{+\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$$

$$\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \qquad\qquad \mathrm{Erf}(x) \underset{x\to+\infty}{\longrightarrow} 1$$

DEf $\quad f \geq 0 \quad f:\mathbb{R}^2\to\mathbb{R} \quad f$ continua

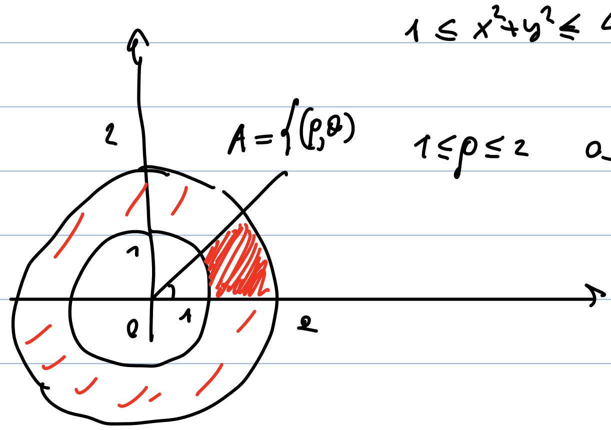
$$\int_{\mathbb{R}^2} f(x,y)\,dx\,dy = \lim_{R\to+\infty}\left(\int_{B(0,R)} f(x,y)\,dx\,dy\right)$$



$$A = \left\{ (x,y)\in\mathbb{R}^2 : 0\leq y\leq \sqrt{3}x,\ 1\leq x^2+y^2\leq 4 \right\}$$

$$\int_A \frac{1}{1+x^2+y^2}dx\,dy$$

$A = \{(\rho,\theta) \quad 1\leq\rho\leq 2 \quad 0\leq\theta\leq\frac{\pi}{3}\}$



$$\int_0^{\pi/3} d\theta \int_1^2 \frac{1}{1+\rho^2}\rho\, d\rho$$

$$\parallel$$

$$\pi \int^{2} 2\rho \; d\rho$$

$2 \cdot 3 \int_1 \quad 1+\rho^2$

$$\frac{\pi}{6}\int_1^2 \frac{d}{d\rho}\ln(1+\rho^2)\,d\rho = \frac{\pi}{6}\left[\ln(1+2^2)-\ln(1+1^2)\right]$$

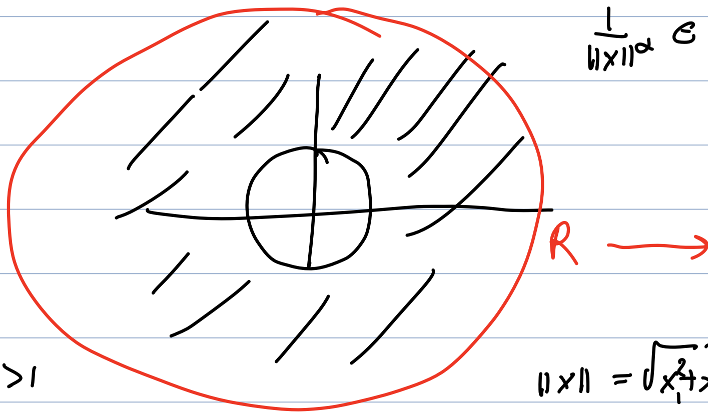
$$= \frac{\pi}{6}(\ln 5 - \ln 2) = \frac{\pi}{6}\ln\left(\frac{5}{2}\right)$$

$$\int_{\mathbb{R}^2\setminus B(0,1)} \frac{1}{\|x\|^\alpha}\,dx \qquad \|x\| = \sqrt{x_1^2+x_2^2}$$

$\frac{1}{\|x\|^\alpha} \geq 0$

$\frac{1}{\|x\|^\alpha} \in C(\mathbb{R}^2\setminus\{0\})$

$$\lim_{R\to+\infty}\int_{B(0,R)\cap\{\mathbb{R}^2\setminus B(0,1)\}} \frac{1}{\|x\|^\alpha}\,dx$$



$B(0,R)\cap\{\mathbb{R}^2\setminus B(0,1)\}$ $\qquad R>1$

$\|x\| = \sqrt{x_1^2+x_2^2} = \rho$

$\|x\|^\alpha = \rho^\alpha$

$\shortparallel$

$\{\theta\in[0,2\pi] \quad 1\leq\rho\leq R\}$

$$\int_0^{2\pi} d\theta \int_1^R d\rho\,\rho\,\frac{1}{\rho^\alpha} = 2\pi\int_1^R \rho^{1-\alpha}\,d\rho \xrightarrow{R\to+\infty} 2\pi\int_1^\infty \rho^{1-\alpha}\,d\rho$$

$$2\pi\int_1^\infty \frac{1}{\rho^{\boxed{\alpha-1}}}\,d\rho$$

$\alpha-1>1$

$\boxed{\alpha>2}$

$$\int_1^{+\infty}\frac{1}{x^\alpha}\,dx<+\infty \qquad \alpha>1 \qquad \mathbb{R}^1$$

$$\int_{\mathbb{R}^2\setminus B(0,1)}\frac{1}{\|x\|^\alpha}\,dx<+\infty \qquad \alpha>2 \qquad \mathbb{R}^2$$

(1)

(2)

$a,b,c>0$

$$A = \left\{\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\leq 1\right\} =$$

$$\mathrm{Vol}(A) = \frac{4\pi}{3}abc \qquad A\subseteq\mathbb{R}^3$$