SOLUZIONE ES. 2

1) $$E_s = E\left[\int_{-\infty}^{+\infty} s_n^2(t)\, dt\right] = E\left[\int_{-\infty}^{+\infty} x_c^2[n]\, p^2(t) \cos^2(2\pi f_0 t)\, dt\right.$$

$$\left. + \int_{-\infty}^{+\infty} x_s^2[k]\, p^2(t) \sin^2(2\pi f_0 t)\, dt - \int_{-\infty}^{+\infty} x_s[n]\, x_c[n]\, p^2(t) \cos(2\pi f_0 t) \cdot \sin(2\pi f_0)\, dt \right]$$

$$= \int_{-\infty}^{+\infty} E[x_c^2]\, p^2(t) \cos^2(2\pi f_0 t)\, dt +$$

$$+ \int_{-\infty}^{+\infty} E[x_s^2]\, p^2(t) \sin^2(2\pi f_0 t)\, dt +$$

$$- \int_{-\infty}^{+\infty} E[x_c]\, \underbrace{E[x_s]}_{=0}\, p^2(t) \cos(2\pi f_0 t) \sin(2\pi f_0 t)\, dt$$

$$= \frac{1}{2} E[x_c^2] \int_{-\infty}^{+\infty} \overbrace{\cos(4\pi f_0 t)\, p^2(t)}^{\simeq 0}\, dt + \frac{1}{2} E[x_c^2] \int_{-\infty}^{+\infty} p^2(t)\, dt +$$

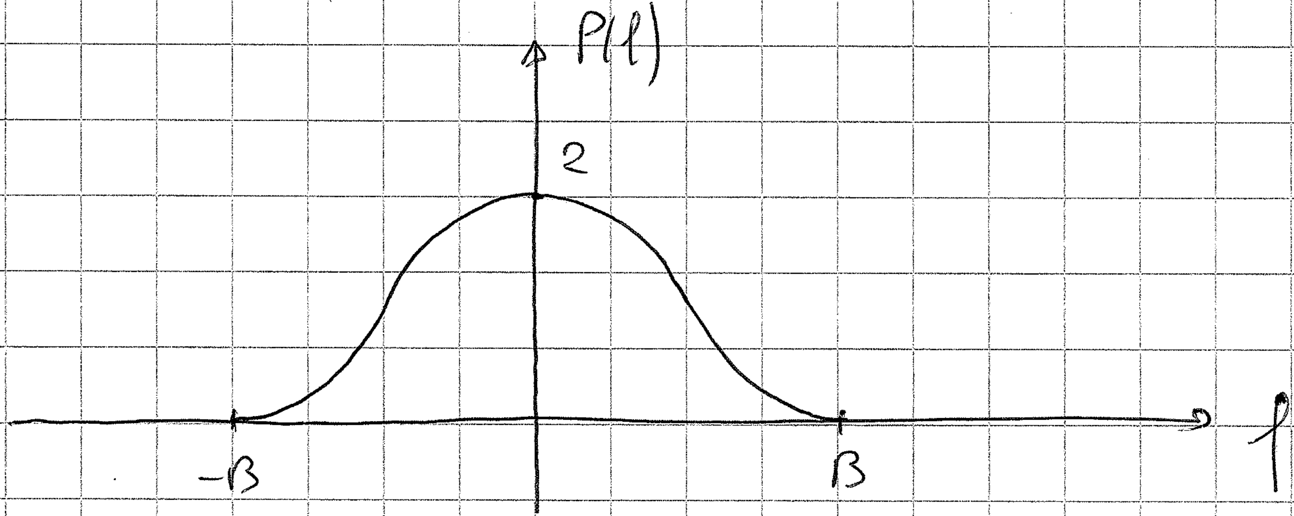
$$+ \frac{1}{2} E[x_s^2] \int_{-\infty}^{+\infty} \underbrace{\cos(4\pi f_0 t)\, p^2(t)}_{\simeq 0}\, dt + \frac{1}{2} E[x_s^2] \int_{-\infty}^{+\infty} p^2(t)\, dt =$$

$$= \frac{1}{2} E[x_c^2]\, E_p + \frac{1}{2} E[x_s^2]\, E_p = \frac{1}{2} \cdot \frac{5}{2} \cdot E_p + \frac{1}{2} \cdot 1 \cdot E_p$$

$$= \frac{7}{4} E_p$$

$$E_p = \int_{-\infty}^{+\infty} p^2(t)\,dt = \int_{-\infty}^{+\infty} P^2(f)\,df$$

$$P(f) = \mathrm{rect}\left(\frac{f}{2B}\right) + \mathrm{rect}\left(\frac{f}{2B}\right)\cos(\pi f/B)$$



$$E_p = \int_{-B}^{B} \left[1 + \cos(\pi f/B)\right]^2 df =$$

$$= 2B + \underbrace{\int_{-B}^{B} 2\cos(\pi f/B)\,df}_{=0} + \frac{1}{2}\cdot 2B + \underbrace{\frac{1}{2}\int_{-B}^{B} \cos(2\pi f/B)\,df}_{=0}$$

$$= 3B$$

$$\boxed{E_s = \frac{7}{4}\cdot 3B = \frac{21}{4}B}$$

2) $S_{n_c}(f) = N_0 \, \mathrm{rect}\left(\frac{f}{B}\right)$

$S_{n_s}(f) = N_0 \, \mathrm{rect}\left(\frac{f}{B}\right)$

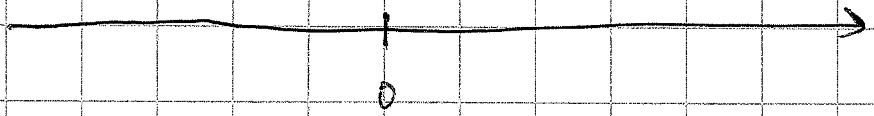
$$S_{n_{uc}}(f) = S_{n_c}(f)\left|H_R(f)\right|^2 = S_{n_c}(f)$$

$$S_{n_{us}}(f) = S_{n_s}(f)\left|H_R(f)\right|^2 = S_{n_s}(f)$$

$$P_{n_{uc}} = \int_{-\infty}^{+\infty} S_{n_c}(f)\,df = \int_{-\infty}^{+\infty} S_{n_s}(f)\,df = P_{n_{us}} = N_0 B$$

3)

3) $P_E(b)$ sul ramo in fase =



in assenza di ISI

$$y_c[n] = x_c[n]\, h(0) + n_{u_c}[n]$$

$$h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$

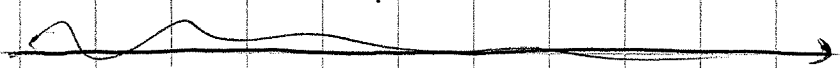
$$H(f) = P(f)\, H_R(f) = P(f) \quad \text{è di Nyquist} \Rightarrow \text{no ISI}$$

$$h(t) = p(t) \Rightarrow h(0) = p(0) = \int_{-\infty}^{+\infty} P(f)\, df =$$

$$= \int_{-B}^{B} \left[1 + \cos\left(\pi f B\right)\right] df = 2B = h(0)$$

$$y_c[n] = 2B\, x_c[n] + n_{u_c}[n]$$

$$P_E^{(c)}(b) = P\left\{\hat{x}_c = -2 \,\middle|\, x_c = 1\right\} P\left\{x_c = 1\right\} + P\left\{\hat{x}_c = 1 \,\middle|\, x_c = -2\right\} P\left\{x_c = -2\right\}$$



3) $P_E(b)$ sul ramo in fase $\left(P_E^c(b)\right)$

$$h(t) = p(t) \otimes c(t) \otimes h_R(t) = p(t) \otimes h_R(t)$$

$H(f) = P(f)\, H_R(f) = P(f)$ è di Nyquist $\Rightarrow$ NO ISI

$$h(t) = p(t) \Rightarrow h(0) = p(0) = \int_{-\infty}^{+\infty} P(f)\, df$$

$$y_c[n] = x_c[n]\, h(0) + n_{x_c}[n]$$

$$h(0) = \int_{-B}^{B} \left[1 + \cos\left(\pi f / B\right)\right] df = 2B$$

$$P_E^c(b) = P\{\hat{x}_c = -2 \mid x_c = 1\}\, P\{x_c = 1\} +$$

$$P\{\hat{x}_c = 1 \mid x_c = -2\}\, P\{x_c = -2\}$$

$$P\{x_c = 1\} = P\{x_c = -2\} = \frac{1}{2}$$

$$P\{\hat{x}_c = -2 \mid x_c = 1\} = Q\left(\frac{2B}{\sqrt{BN_0}}\right) = Q\left(\sqrt{\frac{4B}{N_0}}\right)$$

$$P\{\hat{x}_c = 1 \mid x_c = -2\} = Q\left(\frac{4B}{\sqrt{BN_0}}\right) = Q\left(\sqrt{\frac{16B}{N_0}}\right)$$

$$P_E^s(b) = P\{\hat{x}_s = -1 \mid x_s = 1\}\, P\{x_s = 1\} +$$

$$P\{\hat{x}_s = 1 \mid x_s = -1\}\, P\{x_s = -1\}$$

$$= P\{\hat{x}_s = 1 \mid x_s = -1\} = Q\left(\sqrt{\frac{4B}{N_0}}\right)$$