591AA 21/22 – ELENCO DEI PROBLEMI 1

Problema 1. Calcolare

(a) $(1, 2, -3) + (2, 0, 1)$;
(b) $(2, -1, -1, 3) - (-1, 0, 0, 2)$.

$\mathbb{R}^n$ vs $\mathbb{R}$

$(1,2,-3)+(2,0,1) = (3,2,-2)$

$\mathbb{R}^1$ = spazio vettoriale $= \{(x) \mid x \in \mathbb{R}\}$

$\mathbb{R}^1$

double a[1],
double x;

$(2,-1,-1,3) - (-1,0,0,2) = (3,-1,-1,1)$

$\mathbb{R}$

(a[0], a[1])

double a[2]

$\mathbb{R}^2$

Problema 2. Trova a, b tali che

$\frac{1}{2}R_1 = a + b/2 = 2$

$$a(2, 1, 1) + b(1, 1, 2) = (4, 3, 5)$$

$$\begin{array}{ll} R_1 & 2a + b = 4 \\ R_2 & a + b = 3 \\ R_3 & a + 2b = 5 \end{array}$$

$\longrightarrow$ $R_2 = R_2 - R_1/2$, $R_3 = R_3 - R_1/2$

$2a + b = 4$
$a + b/2 = 1$
$a + 3b/2 = 3$

double a[3]

$\mathbb{R}^3$

$\Longrightarrow b = 2$, $2a + b = 4 \Rightarrow a = 1$

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Problema 3. Trova a, b, c tale che:

$$a(1,1,1)+b(0,1,1)+c(0,0,1)=(1,2,3)$$

$= (a, a+b, a+b+c) = (1,2,3)$

$\Longrightarrow \begin{cases} a=1 \\ a+b=2 \\ a+b+c=3 \end{cases} \Longrightarrow b=1 \Longrightarrow c=1$

$(a,b,c)=(1,1,1)$

Problema 4. Risolvere

$$x+y+z=1$$
$$x+2y+2z=1$$
$$x+2y+3z=1$$

$R_2 = R_2 - R_1$

$R_3 = R_3 - R_1$

$$\begin{array}{l} R_1 \quad x+y+z=1 \\ R_2 \quad x+2y+2z=1 \\ R_3 \quad x+2y+3z=1 \end{array} \xrightarrow{\substack{R_2=R_2-R_1 \\ R_3=R_3-R_1}} \begin{cases} x+y+z=1 \\ y+z=0 \\ y+2z=0 \end{cases}$$

$$\xrightarrow{R_3=R_3-R_2} \begin{cases} x+y+z=1 \\ y+z=0 \\ z=0 \end{cases} \Longrightarrow y=0 \Longrightarrow x=1$$

$(x,y,z)=(1,0,0)$

Problema 5. Mostrare che il seguente sistema di equazioni lineari non ha soluzioni:

$$-x + 3y - 2z = 1$$
$$-x + 4y - 3z = 0$$
$$-x + 5y - 4z = 0$$

$$\cancel{\begin{aligned} x + y &= 1 \\ y &= 1 \\ y &= 0 \end{aligned}}$$

$$\begin{aligned} R_1 \quad -x + 3y - 2z &= 1 \\ R_2 \quad -x + 4y - 3z &= 0 \\ R_3 \quad -x + 5y - 4z &= 0 \end{aligned} \xrightarrow[R_3 = R_3 - R_1]{R_2 = R_2 - R_1} \begin{aligned} -x + 3y - 2z &= 1 \\ 2\,(\,y - z &= -1 \\ 2y - 2z &= -1 \end{aligned}$$

$$\xrightarrow{R_3 = R_3 - 2R_2} \begin{aligned} -x + 3y - 2z &= 1 \\ y - z &= -1 \\ 0 &= 1 \end{aligned}$$

Problema 6. Mostrare che il seguente sistema di equazioni lineari ha un numero infinito di soluzioni

$$-x + 3y - 2z = 4$$
$$-x + 4y - 3x = 5 \qquad z$$
$$-x + 5y - 4x = 6$$

Esempio

$$\begin{aligned} x + y + z &= 1 \\ y + z &= 0 \end{aligned}$$

$$\Rightarrow y = -z$$

$$x = 1 - y - z = 1 - (-z) - z = 1$$

$$x = 1,\ y = -z,$$

$$(1, -z, z)$$

$$\begin{aligned} R_1 \quad -x + 3y - 2z &= 4 \\ R_2 \quad -x + 4y - 3z &= 5 \\ R_3 \quad -x + 5y - 4z &= 6 \end{aligned} \xrightarrow[R_3 = R_3 - R_1]{R_2 = R_2 - R_1} \begin{aligned} -x + 3y - 2z &= 4 \\ y - z &= 1 \\ 2y - 2z &= 2 \end{aligned}$$

$$\xrightarrow{R_3 = R_3 - 2R_2} \begin{aligned} -x + 3y - 2z &= 4 \\ y - z &= 1 \\ 0 &= 0 \end{aligned}$$

$$y = 1 + z$$

$$-x = 4 - 3y + 2z$$

$$x = -4 + 3(1 + z) + 2z = \cdots$$

Problema 7. Calcolare

(a)

$$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$$

In generale, se A e B sono matrici 2x2, allora AB non è uguale a BA.

$$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$AB \neq BA$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 2 & -2 \end{pmatrix}$$

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$\det\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = (1)(4) - (2)(3) = -2$$

Problema 8.

(a) Calcolare

$$\det\begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}$$

(b) Trova t tali che

$$\det\begin{pmatrix} 1 & 2 \\ 3 & t \end{pmatrix} = 0$$

(c) Trova t tali che

$$\det\begin{pmatrix} 1-t & 2 \\ 4 & 8-t \end{pmatrix} = 0$$

(a) $\det\begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix} = (2)(8) - (3)(5) = 1$

(b) $\det\begin{pmatrix} 1 & 2 \\ 3 & t \end{pmatrix} \overset{?}{=} 0 \quad \det = (1)(t) - (2)(3) = 0$

$\Rightarrow t - 6 = 0 \Rightarrow t = 6$

(c) $\det\begin{pmatrix} 1-t & 2 \\ 4 & 8-t \end{pmatrix} = (1-t)(8-t) - (2)(4) = 0$

$\Rightarrow 8 - 9t + t^2 - 8 = t^2 - 9t = 0$

$\Rightarrow t(t-9) = 0 \Rightarrow t = 0 \text{ o } t = 9$