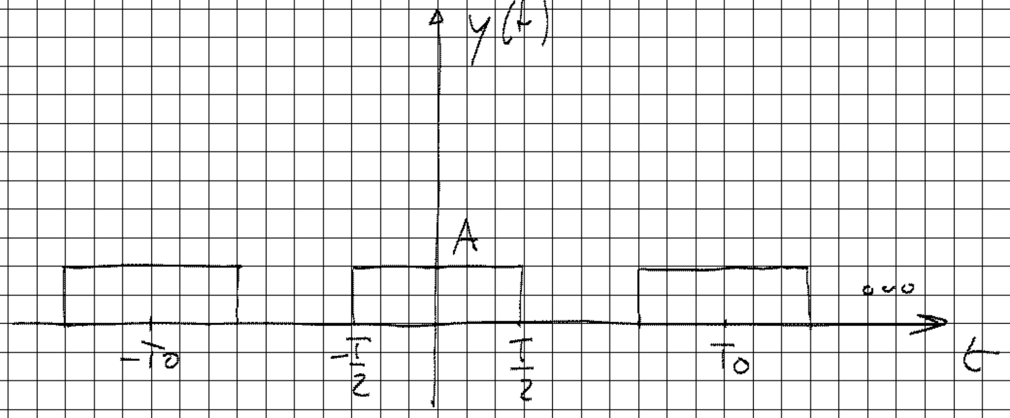
ESERCIZIO – TRENO DI RETTANGOLI



Calcolare Y_n e $Y(f)$

Soluzione

$$y(t) = \sum_{k=-\infty}^{+\infty} x(t - kT_0)$$

$$Y(f) = \frac{1}{T_0} \sum_{k=-\infty}^{+\infty} X\left(\frac{k}{T_0}\right) \delta\left(f - \frac{k}{T_0}\right)$$

$$Y_n = \frac{1}{T_0} X\left(\frac{k}{T_0}\right)$$

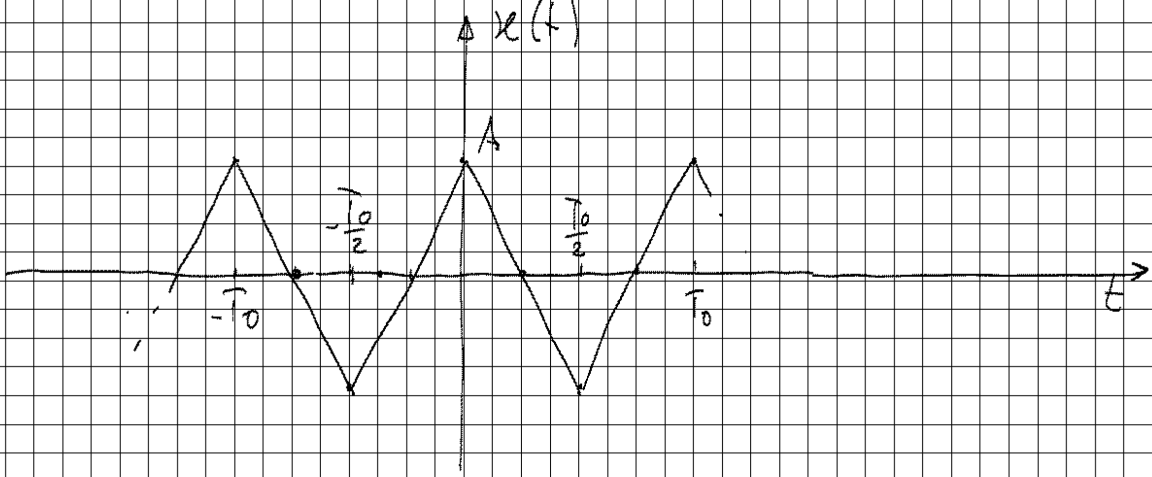
$$X(f) = AT\,\text{sinc}(Tf)$$

$$Y(f) = \frac{A}{T_0} \sum_{k=-\infty}^{+\infty} T\,\text{sinc}\left(T\frac{k}{T_0}\right) \delta\left(f - \frac{k}{T_0}\right) =$$

$$= \frac{AT}{T_0} \sum_{k=-\infty}^{+\infty} \text{sinc}\left(\frac{T}{T_0}k\right) \delta\left(f - \frac{k}{T_0}\right)$$

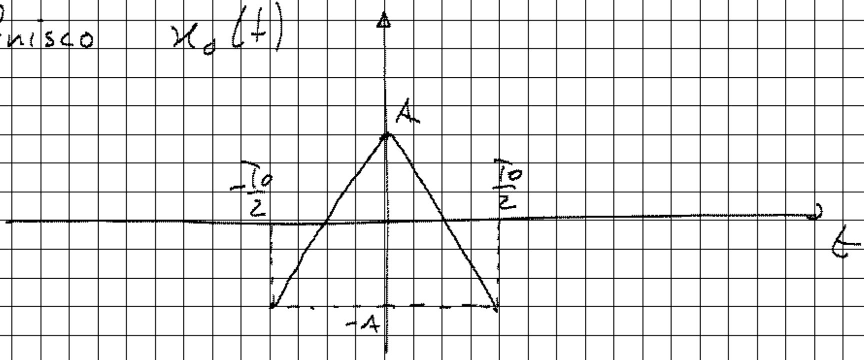
$$Y_n = \frac{AT}{T_0}\,\text{sinc}\left(\frac{T}{T_0}k\right)$$

ESERCIZIO - ONDA TRIANGOLARE



Calcolare $X(f)$ e X_n

Definisco $x_0(t)$



$$x_0(t) = \left[2A\left(1 - \frac{|t|}{T_0/2}\right) - A\right] \mathrm{rect}\left(\frac{t}{T_0}\right)$$

$$X(f) = \frac{1}{T_0} \sum_{k=-\infty}^{+\infty} X_0\left(\frac{k}{T_0}\right) \delta\left(f - \frac{k}{T_0}\right)$$

$$X_n = \frac{1}{T} X_0\left(\frac{k}{T_0}\right)$$

$$X_0(f) = 2A\frac{T_0}{2}\,\text{sinc}^2\left(\frac{T_0}{2}f\right) - AT_0\,\text{sinc}(T_0 f) =$$

$$= AT_0\left[\text{sinc}^2\left(\frac{T_0}{2}f\right) - \text{sinc}(T_0 f)\right]$$

$$X(f) = \frac{1}{T_0}\sum_{k=-\infty}^{+\infty} AT_0\left[\text{sinc}^2\left(\frac{T_0}{2}\frac{k}{T_0}\right) - \text{sinc}\left(T_0\frac{k}{T_0}\right)\right]\delta\left(f-\frac{k}{T_0}\right)$$

$$= A\sum_{k=-\infty}^{+\infty}\left[\text{sinc}^2\left(\frac{k}{2}\right) - \text{sinc}(k)\right]\delta\left(f-\frac{k}{T_0}\right)$$

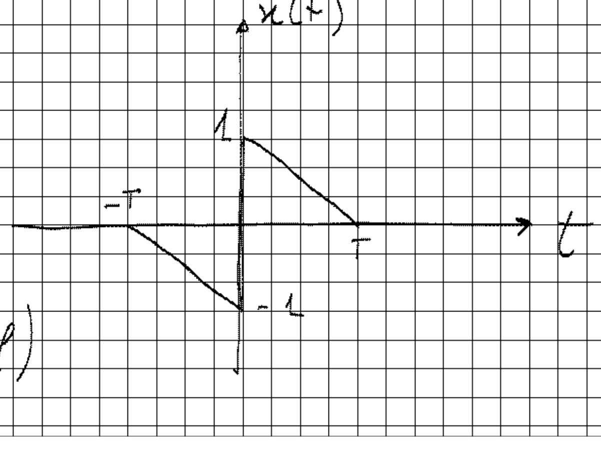
$$X_k = A\left[\text{sinc}^2\left(\frac{k}{2}\right) - \text{sinc}(k)\right] = \begin{cases} 0 & k \text{ pari} \\ \dfrac{4A}{k^2\pi^2} & k \text{ dispari} \end{cases}$$

$$\text{sinc}^2\left(\frac{k}{2}\right) = \frac{\sin^2\left(k\frac{\pi}{2}\right)}{\left(k\frac{\pi}{2}\right)^2} = \begin{cases} 0 & k \text{ pari} \\ \dfrac{4}{k^2\pi^2} & k \text{ dispari} \end{cases}$$

$$X(f) = \sum_{k=-\infty}^{+\infty} X_k\,\delta\left(f-\frac{k}{T_0}\right)$$

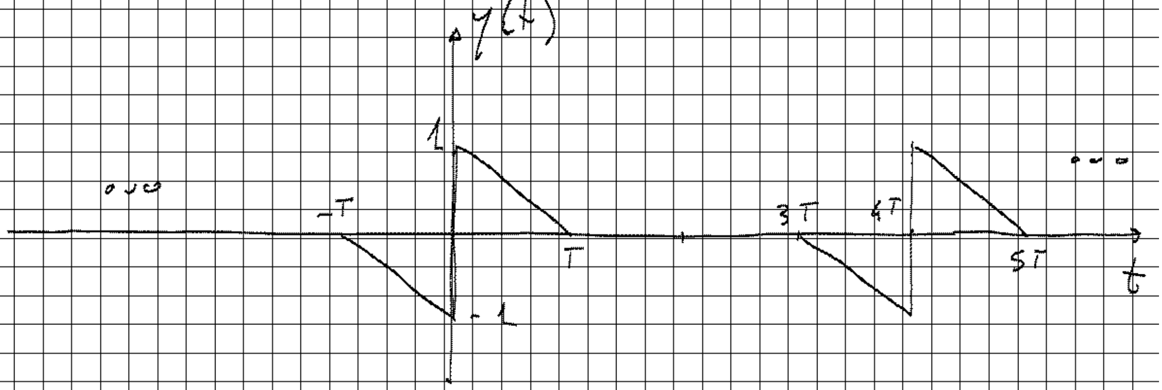
ESERCIZIO

$$y(t) = \sum_{n=-\infty}^{+\infty} x(t-4nT)$$



Calcolare $Y(f)$

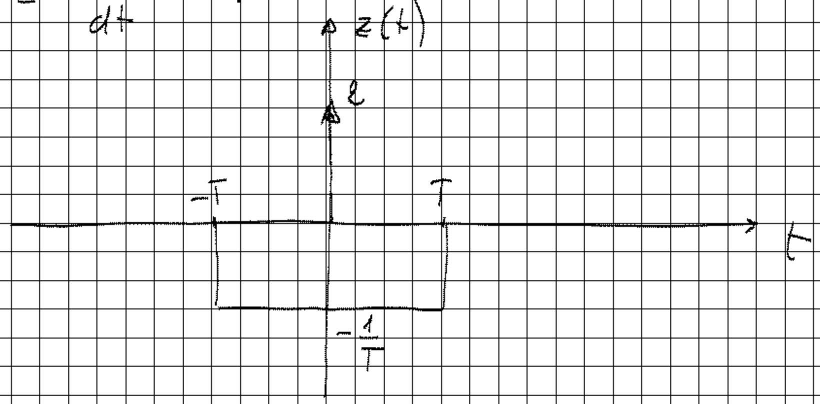
Solu**z**ione



$$Y(f) = \frac{1}{T_0} \sum_{k=-\infty}^{+\infty} X\left(\frac{k}{T_0}\right) \delta\left(f - \frac{k}{T_0}\right) \quad , \quad T_0 = 4T$$

$$Y(f) = \frac{1}{4T} \sum_{k=-\infty}^{+\infty} X\left(\frac{k}{4T}\right) \delta\left(f - \frac{k}{4T}\right)$$

$$z(t) = \frac{d}{dt} x(t)$$



$$z(t) = -\frac{1}{T} \operatorname{rect}\left(\frac{t}{2T}\right) + 2\delta(t)$$

$$Z(f) = -\frac{1}{T} 2T \operatorname{sinc}(2Tf) + 2 = 2\left[1 - \operatorname{sinc}(2Tf)\right]$$

$$Z(0) = 0$$

$$X(f) = \frac{Z(f)}{j2\pi f} = \frac{2\left[1 - \text{sinc}(2Tf)\right]}{j2\pi f} =$$

$$= \frac{1 - \text{sinc}(2Tf)}{j\pi f}$$

$$Y(f) = \frac{1}{4T} \sum_{k=-\infty}^{+\infty} \frac{1 - \text{sinc}\left(2T\frac{k}{4T}\right)}{\frac{j\pi k}{4T}} \, \delta\left(f - \frac{k}{4T}\right) =$$

$$= \sum_{k=-\infty}^{+\infty} \frac{1 - \text{sinc}(k/2)}{j\pi k} \, \delta\left(f - \frac{k}{4T}\right)$$

$$Y_k = \frac{1 - \text{sinc}(k/2)}{j\pi k}$$

ESERCIZIO

Dato $s(t) = \text{sinc}(2Bt - 1/2) + \text{sinc}(2Bt + 1/2)$

Si calcolino Energia e Potenza dei segnali

$$x(t) = s(t)\sin(2\pi Bt)$$

$$y(t) = \sum_{k=-\infty}^{+\infty} s\left(t - \frac{2k}{B}\right)$$

Soluzione

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt = \int_{-\infty}^{+\infty} |X(f)|^2 \, df$$

$$X(f) = \frac{1}{2j}\left[S(f - f_0) - S(f + f_0)\right] =$$

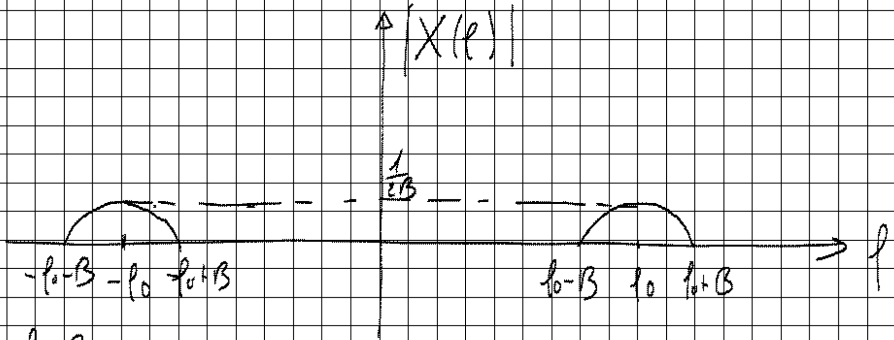
$$s(t) = \text{sinc}\left[2B\left(t - \frac{1}{4B}\right)\right] + \text{sinc}\left[2B\left(t + \frac{1}{4B}\right)\right]$$

$$S(f) = \frac{1}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right) e^{-j\frac{2\pi f}{4B}} + \frac{1}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right) e^{j\frac{2\pi f}{4B}} =$$

$$= \frac{1}{\cancel{2}B}\,\mathrm{rect}\left(\frac{f}{2B}\right)\cancel{2}\cos\left(\frac{\pi f}{2B}\right) = \frac{1}{B}\,\mathrm{rect}\left(\frac{f}{2B}\right)\cos\left(\frac{\pi f}{2B}\right)$$

$$X(f) = \frac{1}{2j}\left[\frac{1}{B}\,\mathrm{rect}\left(\frac{f-f_0}{2B}\right)\cos\left[\frac{\pi(f-f_0)}{2B}\right] - \frac{1}{B}\,\mathrm{rect}\left(\frac{f+f_0}{2B}\right)\cos\left[\frac{\pi(f+f_0)}{2B}\right]\right]$$

$$|X(f)|^2 = \frac{1}{4B^2}\left[\cos^2\left[\frac{\pi(f-f_0)}{2B}\right]\mathrm{rect}\left(\frac{f-f_0}{2B}\right) + \cos^2\left[\frac{\pi(f+f_0)}{2B}\right]\mathrm{rect}\left(\frac{f+f_0}{2B}\right)\right]$$



$$E_x = \frac{2}{4B^2}\int_{f_0-B}^{f_0+B}\cos^2\left[\frac{\pi(f-f_0)}{2B}\right]df = \quad (f - f_0 = f')$$

$$= \frac{1}{2B^2}\int_{-B}^{B}\cos^2\left(\frac{\pi f'}{2B}\right)df = \frac{1}{2B^2}\int_{-B}^{B}\left[\frac{1}{2}+\frac{1}{2}\cos\left(\frac{2\pi f'}{2B}\right)\right]df'$$

$$= \frac{1}{2B^{\cancel{2}}}\cdot\frac{1}{\cancel{2}}\cdot\cancel{2B} = \frac{1}{2B}$$

$$P_x = 0$$

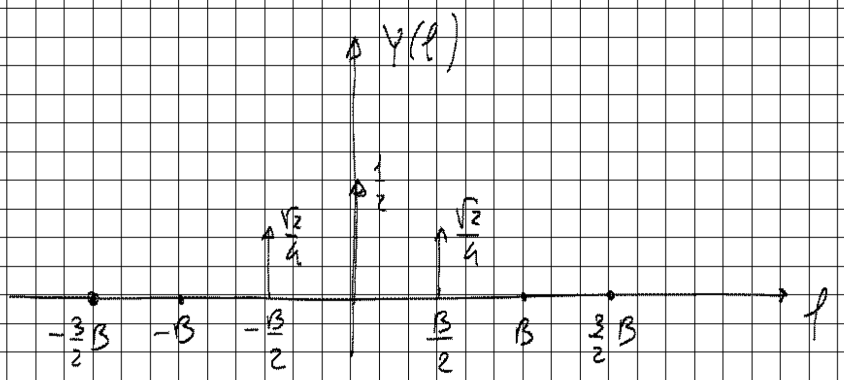
$E_y = \infty$, $y(t)$ è un segnale periodico

$$P_y = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |y(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |Y_k|^2$$

$$Y_k = \frac{1}{T_0} S\left(\frac{k}{T_0}\right) = \frac{B}{2} S\left(\frac{B}{2}k\right) =$$

$$= \frac{\cancel{B}}{2}\,\frac{1}{\cancel{B}}\,\mathrm{rect}\left(\frac{\frac{Bk}{2}}{2B}\right)\cos\left(\frac{\frac{\pi B k}{2}}{2B}\right) = \frac{1}{2}\,\mathrm{rect}\left(\frac{k}{4}\right)\cos\left(\frac{\pi}{4}k\right)$$

$$Y_k = \begin{cases} \frac{1}{2} & k = 0 \\ \frac{\sqrt{2}}{4} & k = \pm 1 \\ 0 & k = \pm 2, \pm 3, \ldots \end{cases}$$



$$P_y = \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \frac{1}{2}$$