$$\omega''(\rho) + \frac{(n-1)\,\omega'(\rho)}{\rho} = 0 \qquad \omega(\|x\|) = \omega(\rho) \qquad \rho = \|x\|$$

$$\rho > 0 \qquad \Delta\omega = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}\omega = 0$$

$$\omega'(\rho) = f(\rho)$$

$$f'(\rho) = -(n-1)\frac{f(\rho)}{\rho} \qquad n = 2 \qquad \mathbb{R}^2$$

EQ. VARIABILI SEPARABILI

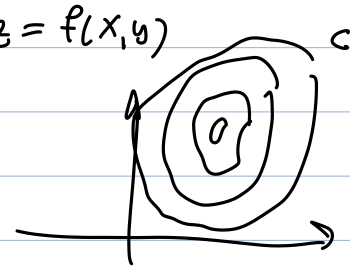
$$\frac{df}{d\rho} = f'(\rho) = -\frac{f(\rho)}{\rho} \Rightarrow \frac{df}{f} = -\frac{d\rho}{\rho} \qquad \ln f = -\ln\rho + c$$

$$\ln f = \ln\frac{1}{\rho} + c$$

$$\omega(\rho) = c_1 \ln\rho + c_2 \Leftarrow \omega'(\rho) = f = \frac{c}{\rho}$$

$$\boxed{n = 3 \qquad \omega(\rho) = \frac{c_1}{\rho} + c_2}$$

CURVE DI LIVELLO $\qquad f : A \to \mathbb{R}$, $A \subseteq \mathbb{R}^2$ $\qquad z = f(x,y)$

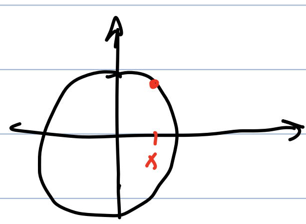


$$\mathcal{C}_c = \{ (x,y) \in A : f(x,y) = c \}$$

CURVA (CONTINUA) $\qquad \gamma : [0,1] \to \mathbb{R}^n \qquad \gamma$ continua $\qquad \gamma(t) = \begin{pmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \vdots \\ \gamma_n(t) \end{pmatrix}$

$$f(x,y) = x^2 + y^2$$

$$\mathcal{C}_1 = \{ x^2 + y^2 = 1 \}$$



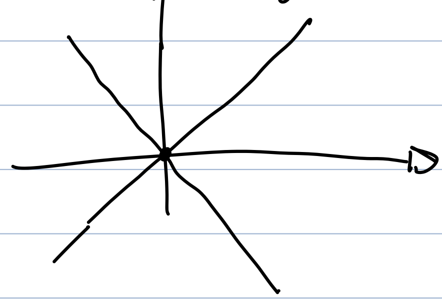
$$x^2 + (g(x))^2 = 1$$
$$g^2(x) = 1 - x^2$$
$$g(x) = \pm\sqrt{1-x^2}$$

$$\mathcal{C}_0 = \{ (x,y) : x^2 + y^2 = 0 \} = \{(0,0)\}$$

$$\mathcal{C}_{-1} = \{ (x,y) : x^2 + y^2 = -1 \} = \emptyset$$

$$f_1(x,y) = x^2 - y^2$$

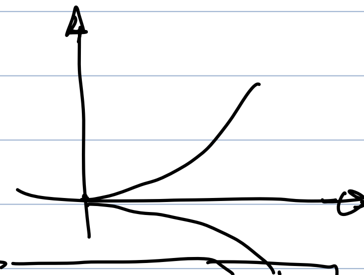
$\mathcal{C}_0 = \{ (x,y) : \; x^2 - y^2 = 0 \}$ $\quad x^2 = y^2 \quad y = \pm x$



$f_2(x,y) = x^3 - y^2 \in C^\infty$

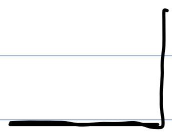
$x^3 = y^2 \qquad x \geq 0$

$\mathcal{C}_0 = \{ (x,y) : \; x^3 - y^2 = 0 \}$ $\qquad y = \pm\sqrt{x^3}$



CURVA REGOLARE DI CLASSE C^1

$\gamma : [0,1] \to \mathbb{R}^n \qquad \gamma_i \in C^1[0,1] \quad 0 \neq |\gamma'(t)| = \sqrt{(\gamma_1'(t))^2 + (\gamma_2'(t))^2 + \dots + (\gamma_n'(t))^2} \neq 0$



$f \in C^1(A) \qquad A \subseteq \mathbb{R}^2 \qquad f(x,y) = 0$ RISOLVERE

$\exists\, g : I \to \mathbb{R} \qquad y = g(x) \qquad f(x, g(x)) = 0$

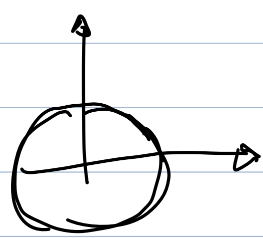
$\min\limits_{\overline{\Omega}} f \qquad \Omega$ aperto limitato $\qquad \min\limits_{x^2+y^2 \leq 1} x^2 + \sin(x) y$

$\nabla f(\bar{x}) = 0 \qquad \bar{x} \in \Omega \qquad Hf(\bar{x})$ $\qquad \overline{B} = \{ x^2 + y^2 \leq 1 \}$

$\partial \overline{\Omega} = \{ g(x,y) = 0 \}$

$\partial B = \{ x^2 + y^2 = 1 \} = \{ (x,y) : \; g(x,y) = 0 \}$

$g(x,y) = x^2 + y^2 - 1$



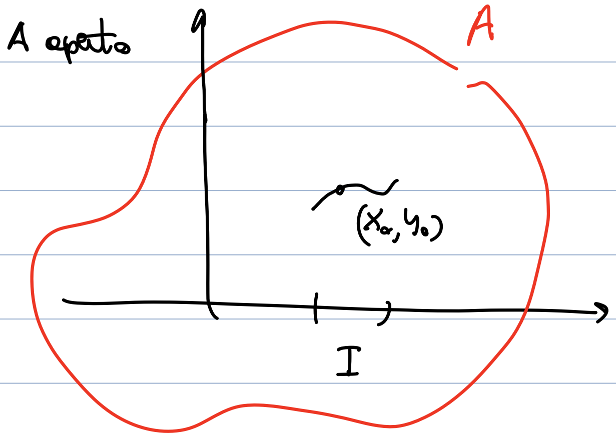
$|(\cos(t), \sin(t))| \qquad t \in [0, 2\pi]$

$f|_{\partial B} = \cos^2(t) + \sin(\cos(t)) \sin(t) \qquad t \in [0, 2\pi]$

TEOREMA DELLA FUNZIONE IMPLICITA (DINI)

$A \subseteq \mathbb{R}^2$ $\quad f \in C^1(A) \quad (x_0, y_0) \in A \quad$ t.c. $\quad f(x_0,y_0) = 0$

A aperto $\qquad \dfrac{\partial f}{\partial y}(x_0,y_0) \neq 0$



$\Rightarrow \exists\, I \ni x_0 \qquad g : I \to \mathbb{R}$

$g \in C^1(I) \qquad g(x_0) = y_0$

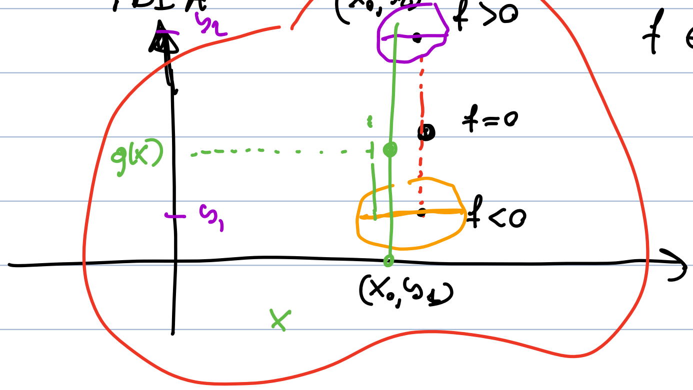
t.c. $\quad f(x, g(x)) = 0 \quad \forall x \in I$

- $g'(x) = - \dfrac{\frac{\partial f}{\partial x}(x, g(x))}{\frac{\partial f}{\partial y}(x, g(x))}$

$g : I \to \mathbb{R}$ supponiamo che sia derivabile

$\forall x \in I \quad f(x, g(x)) \equiv 0 \quad \Rightarrow \quad \dfrac{d}{dx} f(x,g(x)) = 0 = \dfrac{\partial f}{\partial x}(x,g(x)) + \dfrac{\partial f}{\partial y}(x,g(x))\, g'(x)$

IDEA



$f \in C^0(A) \qquad f(x_0,y_0) = 0$

SUPPONIAMO CHE $f(x_0, y)$ sia monotona (crescente in senso stretto)

f monotona in senso stretto

$y \to f(x,y)$

PERM. SEGNO | VICINO a (x_0, y_1) $\quad f < 0$

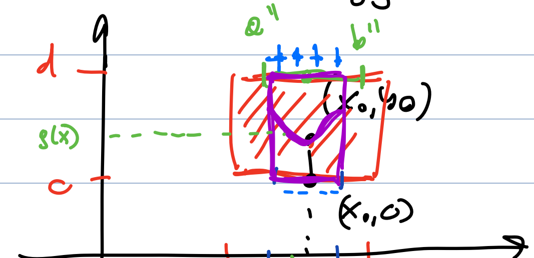
VICINO (x_0, y_2) $\quad f > 0$

$x \to g(x) \in C^0 \quad : \quad f(x, g(x)) = 0$

$f \in C^1(A) \qquad f(x_0,y_0) = 0 \qquad \dfrac{\partial f}{\partial y}(x_0,y_0) > 0$

PERM. SEGNO

$f \in C^1(A) \Rightarrow \dfrac{\partial f}{\partial y} \in C^0$ e se $\dfrac{\partial f}{\partial y}(x_0,y_0) > 0 \Rightarrow \exists\, Q \ni (x_0,y_0)$



t.c. $\quad \dfrac{\partial f}{\partial y}(x,y) > 0 \quad \forall (x,y) \in Q$

$Q = [a,b] \times [c,d]$

$a\ a'\ x\ b'\ b$

$f(x_0,c)<0 \qquad f(x,c)<0 \qquad x\in[a',b']$

$f(x_0,d)>0 \qquad f(x,d)>0 \qquad x\in[a'',b'']$

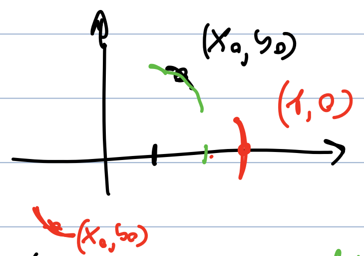
$I=[a,b]\cap[a',b']\cap[c',d']$

$x\mapsto g(x)$

$g(x)$ È UNICO VALORE TRA c E d DOVE $f(x,t)$ SI ANNULLA

$t\in[c,d]$

OSSERVAZIONI



$f(x,y)=x^2+y^2-1$

$f(x_0,y_0)=0$

$\frac{\partial f}{\partial y}(x_0,y_0)\neq 0 \qquad \frac{\partial f}{\partial y}(x,y)=2y\neq 0 \qquad y\neq 0 \qquad f(x,g(x))=0$

$\frac{\partial f}{\partial y}(1,0)=0 \qquad \frac{\partial f}{\partial x}=2x$

$f(x_0,y_0)=0$

$\frac{\partial f}{\partial y}(x_0,y_0)\neq 0 \qquad x\in I$

$I\ni x_0$

$\exists g:\ f(x,g(x))=0$

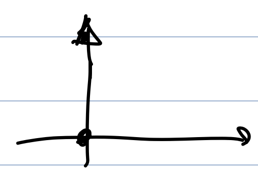
$g(x)$

$f(x_0,y_0)=0$

$\frac{\partial f}{\partial x}(x_0,y_0)\neq 0 \qquad h(y)=x$

$f(h(y),y)=0$

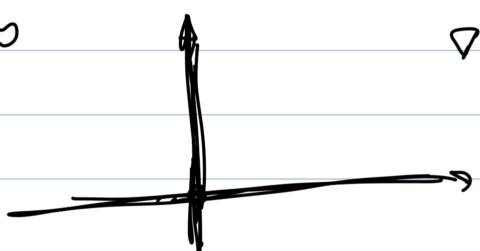
$f\in C^1 \qquad f(x_0,y_0)=0 \qquad \nabla f(x_0,y_0)\neq 0$



$f_1=x^2+y^2 \qquad (x_0,y_0)=(0,0) \qquad f_1(x_0,y_0)=0$

$\nabla f_1=(2x,2y) \qquad \nabla f_1(x_0,y_0)=(0,0)$

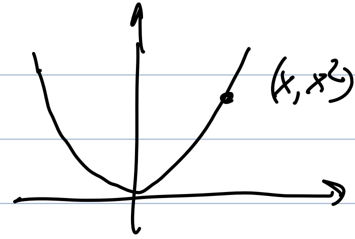
$f_2(x,y)=xy \qquad \nabla f_2(x,y)=(y,x) \qquad (x_0,y_0)=(0,0)$



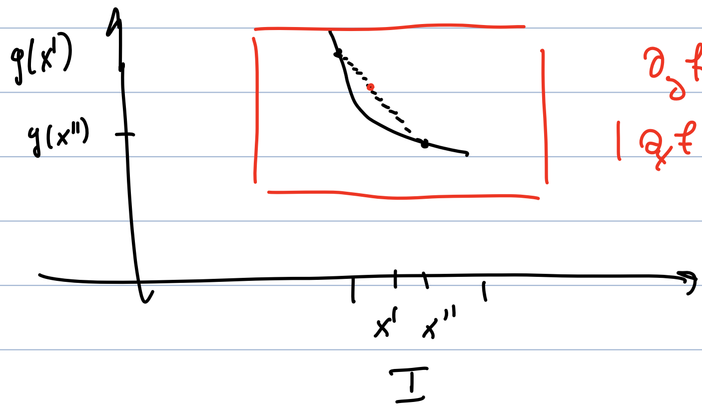
$f_2(0,0)=0$

$\nabla f_2(0,0)=0$

$f_g(x,y) = (y-x^2)^2$ $\qquad f_g = 0 \iff y - x^2 = 0$



$\nabla f = \left(2(y-x^2)(-2x),\ 2(y-x^2)\,1\right)$



$\partial_y f > 0$

$|\partial_x f| \leq \max|\partial_x f|$

$f(x, g(x)) = 0 \qquad f(x', g(x')) = 0 = f(x'', g(x''))$

$|g(x') - g(x'')| < \varepsilon$ se $|x' - x''| < \delta$ continuità della g

$$0 = f(x', \cancel{g(x')}) = f(x'', \cancel{g(x'')}) + \frac{\partial f}{\partial x}(\xi,\eta)(x''-x') + \frac{\partial f}{\partial y}(\xi,\eta)\left(g(x') - g(x'')\right)$$

$$f(x,y) = f(x_0,y_0) + \frac{\partial f}{\partial x}(\xi,\eta)(x-x_0) + \frac{\partial f}{\partial y}(\xi,\eta)(y-y_0)$$

$$|g(x') - g(x'')| = + \left| \frac{\frac{\partial f}{\partial x}(\xi,\eta)(x''-x')}{\underset{\neq 0}{\frac{\partial f}{\partial y}(\xi,\eta)}} \right|$$

$\frac{\partial f}{\partial y}(x_0,y_0) \neq 0$

$$|g(x') - y''(x)| \leq \frac{\max\left|\frac{\partial f}{\partial x}\right|}{\min\left|\frac{\partial f}{\partial y}\right|} |x'' - x'| \leq c\,|x'' - x'|$$

$f(x) \qquad f'(x_0) \neq 0 \qquad f$ derivabile $\Rightarrow f$ monotona

$f \in C^1$

$f'(x_0) \neq 0 \Rightarrow f'(x_0) > 0 \rightarrow$ PERM. SEGNO $f'(x) > 0$ VICINO A x_0

$f'(x_0) < 0 \rightarrow f'(x) < 0$ " A x_0

$f \in C^1$ $\quad f'(x_0) \neq 0$ $\quad$ f è invertibile vicino a x_0

$F(x,y) = y - f(x)$ $\qquad F(x_0,y_0) = y_0 - f(x_0) = 0 \qquad y_0 = f(x_0)$

$\frac{\partial F}{\partial y}(x,y) = 1 \neq 0$

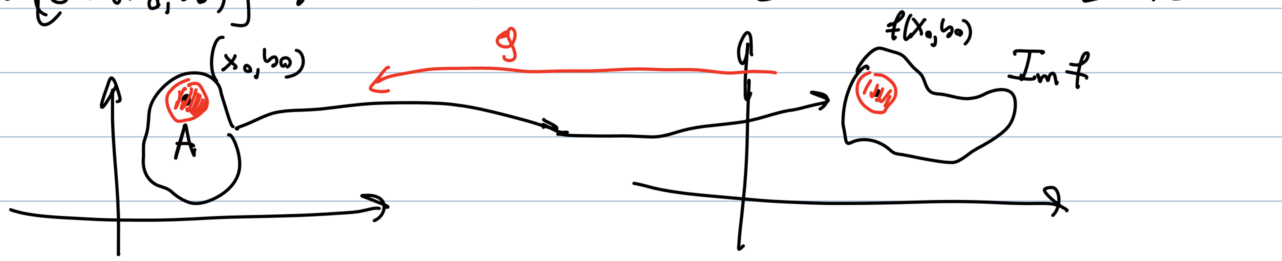
$\frac{\partial F}{\partial x} = -f'(x) \qquad \frac{\partial F}{\partial x}(x_0,y_0) = -f'(x_0) \neq 0$

TEOREMA INVERSIONE LOCALE

$f: A \to \mathbb{R}^2$, $A \subset \mathbb{R}^2$ $\qquad f(x,y) = \begin{pmatrix} f_1(x,y) \\ f_2(x,y) \end{pmatrix}$ $\qquad f \in C^1(A)$

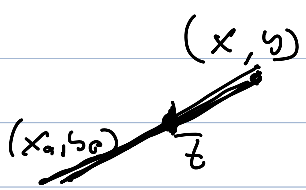
$Jf(x,y) = \begin{pmatrix} \partial_x f_1 & \partial_y f_1 \\ \partial_x f_2 & \partial_y f_2 \end{pmatrix}$ matrice Jacobiana

$\det[Jf(x_0,y_0)] \neq 0$ ALLORA f È INVERTIBILE LOCALMENTE



$g(f(x,y)) = (x,y) \qquad \forall (x,y) \in B(x_0,y_0,R)$

$(\rho,\theta) \xrightarrow{f} \begin{pmatrix} \rho\cos\theta \\ \rho\sin\theta \end{pmatrix}$ $\qquad \det Jf = \rho \neq 0 \qquad (x,y) \neq (0,0)$



$f(x,y) = f(x_0,y_0) + \partial_x f(\xi,\eta)(x-x_0)$
$+ \partial_y f(\quad)(y-y_0)$

$t(x_0,y_0) + (1-t)(x,y)$