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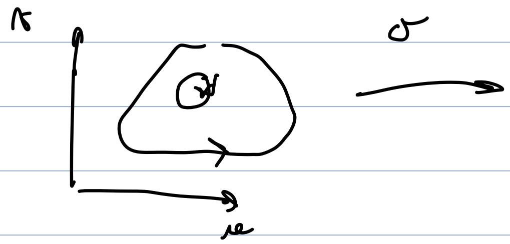
Σ (porzione) di superficie regolare orientata, bordo $\partial^+\Sigma$ è una curva (unione) regolare

T limitato, connesso $T = \overset{\circ}{\bar{T}}$, misurabile ∂T È PERCORSA "LASCIANDO" T A SX CON CURVA REGOLARE



$\sigma = \sigma_1(u,v)\,i + \sigma_2(u,v)\,j + \sigma_3(u,v)\,k$

$\partial\Sigma \subseteq \sigma(\partial T)$

$\vec{F} \in C^1(\Omega) \qquad \Omega \supset \Sigma$

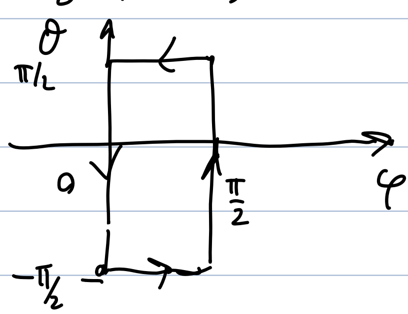
$$\iint_{\Sigma} (\nabla \times \vec{F}) \cdot \hat{n}\, dS = \oint_{\partial^+\Sigma} \vec{F} \cdot \vec{\tau}\, ds = \int_{\gamma} \vec{F} \cdot dx = \int_{\gamma} \omega$$

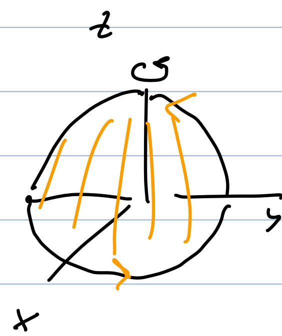
$\partial^+\Sigma$ è descritta $\gamma : I \to \mathbb{R}^3 \qquad \vec{\tau} = \dfrac{\gamma_1'(t)\,i + \gamma_2'(t)\,j + \gamma_3'(t)\,k}{\|\gamma'(t)\|}$

$\omega = F_1\,dx + F_2\,dy + F_3\,dz$

$\vec{F} = x\,i + y\,k \qquad \Sigma$ quarto di sfera di Raggio R

$$\sigma \begin{cases} x = R\sin(\varphi)\cos(\theta) \\ y = R\sin(\varphi)\sin(\theta) \\ z = R\cos(\varphi) \end{cases} \qquad \begin{array}{l} \varphi \in [0, \pi/2] \\ \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \end{array}$$





$\nabla \times \vec{F} = (\partial_y F_3 - \partial_z F_2)\,i - (\partial_x F_3 - \partial_z F_1)\,j + (\partial_x F_2 - \partial_y F_1)\,k$

$= (\partial_y y - 0)\,i - (0 - 0)\,j + (0 - 0)\,k = 1i$

$\nabla \times \vec{F} = (1, 0, 0)$

$$\underbrace{\frac{\partial\sigma}{\partial\varphi}}_{\|}\wedge\underbrace{\frac{\partial\sigma}{\partial\theta}}_{\|} = \sin(\varphi)\cos(\theta)\, i + \sin\varphi\sin\theta\, j + \cos\varphi\, k$$

$$\Phi(\nabla\times\vec{F},\Sigma) = \int_{-\pi/2}^{\pi/2} d\theta \int_0^{\pi/2} d\varphi \; \sin(\varphi)\cos(\theta)\; R^2\sin\varphi$$

$$= R^2\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta \int_0^{\pi/2}\sin^2(\varphi)\,d\varphi$$

$$= R\;2\int_0^{\pi/2}\sin^2\varphi\, d\varphi$$

$$= R^2\;2\;\frac{\pi}{4} = R^2\frac{\pi}{2}$$

$$(\sin\varphi)^2 = \left(\frac{e^{i\varphi}-e^{-i\varphi}}{2i}\right)^2 =$$

$$\int_0^{\pi/2} \frac{e^{2i\varphi}-2+e^{-2i\varphi}}{-4}$$

γ_1

$$x = R\cos(t) \qquad y = R\sin(t) \qquad z = 0 \qquad t\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \qquad F = x\,i + 0 + y\,k$$

$$\int_{-\pi/2}^{\pi/2} \underbrace{R\cos(t)}_{x}\,\underbrace{(-R\sin(t))}_{x'} + 0 + R^2\sin(t)\cos(t) = 0$$

γ_2

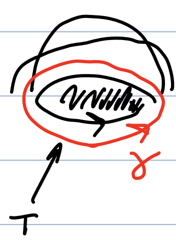
$$x = 0 \qquad y = R\cos(t) \qquad z = R\sin(t) \qquad t\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

$$\int_{-\frac{\pi}{2}}^{\pi/2} R\cos(t)\,R\cos(t) = R^2\int_{-\frac{\pi}{2}}^{\pi/2}\cos^2(t)\,dt = \frac{\pi R^2}{2}$$

$$\Sigma = \{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+2z^4 = 4 \quad z\geq 0\}$$

$$z = \sqrt[4]{\frac{4-(x^2+y^2)}{2}}$$

$$\vec{E} = (x,\, z-x,\, y^2-z).$$



$x^2+y^2=4$

$$T\ni(x,y)\to\left(x,y,\sqrt[4]{\frac{4-(x^2+y^2)}{2}}\right) \qquad \iint_\Sigma \vec{E}\cdot\hat{n}\,dS$$

$$\vec{E} = \mathrm{rot}\,\vec{F}$$

$$\iint_{\Sigma} \vec{E}\cdot\hat{n}\,dS = \iint_{\Sigma} (\nabla\times\vec{F})\cdot\hat{n}\,dS = \int_{\partial^+\Sigma} \vec{F}\,\vec{\tau}\,ds$$

$$\mathrm{div}\,E = \frac{\partial}{\partial x}E_1 + \frac{\partial}{\partial y}E_2 + \frac{\partial}{\partial z}E_3 = 1 + 0 - 1 = 0$$

$$F = (F_1, F_2, 0) \qquad \nabla\times F = \underbrace{-\partial_z F_2}_{E_1} i + \underbrace{\partial_z F_1}_{E_2} j + \underbrace{(\partial_x F_2 - \partial_y F_1)}_{E_3} k$$

$$-\partial_z F_2 = x \qquad F_2 = -xz + \varphi(x,y)$$

$$\partial_z F_1 = z - x \qquad F_1 = \frac{z^2}{2} - xz + \cancel{\psi}(x,y)$$

$$\boxed{\mathrm{rot}\,(\nabla U) = 0}$$

$$\partial_x F_2 - \partial_y F_1 = \boxed{y^2 - z}$$

$$\partial_x\left(-xz + \varphi(x,y)\right) - \partial_y\left(\frac{z^2}{2} - xz\right) = \overbrace{-z + \partial_x\varphi}^{E_3} \qquad \varphi_x = xy^2$$

$$= -z + y^2$$

$$F = \left(\frac{z^2}{2} - xz\right) i + \left(xy^2 - xz\right) j + 0k$$

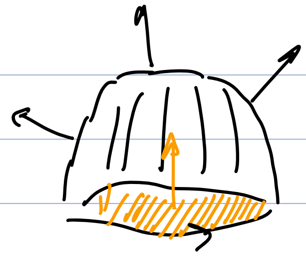
γ $\begin{cases} x = 2\cos(t) \\ y = 2\sin(t) \end{cases}$ $t \in [0, 2\pi)$

$$\boxed{\int_0^{2\pi} \underbrace{2\cos(t)}_{x(t)}\,\underbrace{(2\sin(t))^2}_{y^2(t)}\,\underbrace{2\cos(t)}_{y'(t)}\,dt} = 8\int_0^{2\pi} (\sin(t)\cos(t))^2\,dt$$

$$= \frac{8}{4}\int_0^{2\pi} \sin^2(2t)\,dt = 4\pi$$

$$\Sigma : T \to \mathbb{R}^3 \qquad \Sigma_1 : T \to \mathbb{R}^3$$

$$\int_{\Sigma} \mathrm{rot}\,\vec{F}\cdot\hat{n}\,dS = \oint_{\partial^+\Sigma} \vec{F}\cdot\vec{\tau}\,ds = \oint_{\partial^+\Sigma_1} \vec{F}\cdot\vec{\tau}\,ds = \iint_{\Sigma_1} \mathrm{rot}\,\vec{F}\cdot\hat{n}\,dS$$



$$\int_{B(0,2)} (y^2 - \underset{0}{\underset{\parallel}{z}}) \, dx\,dy = \int_0^{2\pi}\int_0^{2} d\theta \; \rho \, d\rho \; \rho^2 \sin^2\theta$$

$$= \int_0^{2\pi} \sin^2\theta) \int_0^{2} \rho^3 d\rho \; = 4\pi$$