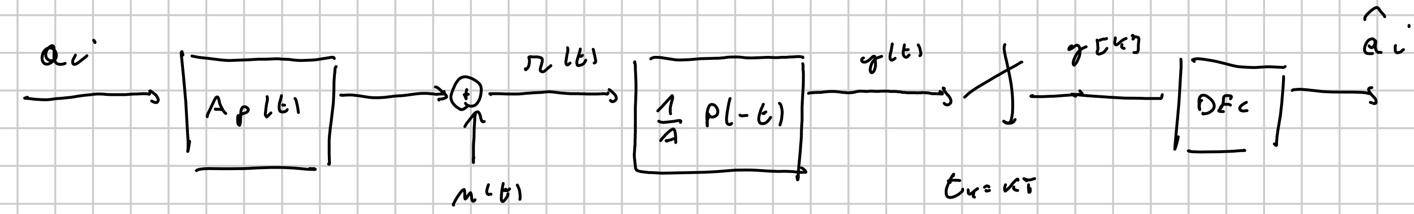
ESERCIZIO PAM 4 LIVELLI



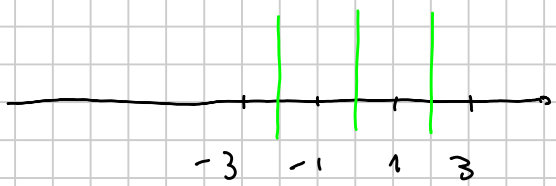
$$r(t) = \sum_i a_i \, A \, p(t - iT) + n(t)$$

$$a_i \in \{ \pm 3 \; ; \; \pm 1 \}$$

$n(t)$ AWGN $\qquad S_n(f) = \frac{N_0}{2} \quad \forall f$

$$P(f) = \begin{cases} \sqrt{T}\,\sqrt{1 - |fT|} & |fT| \le 1 \\ 0 & \text{altrimenti} \end{cases}$$

$$\hat{a}_k = \begin{cases} +3 & \text{se} \quad y[k] \ge 2 \\ 1 & \text{se} \quad 0 \le y[k] < 2 \\ -1 & \text{se} \quad -2 \le y[k] < 0 \\ -3 & \text{se} \quad y[k] < -2 \end{cases}$$



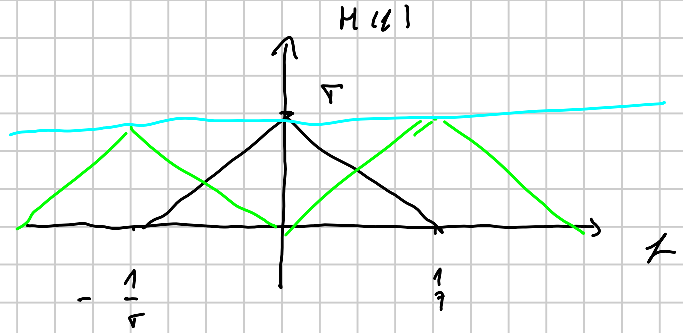
$SNR = \frac{E_s}{N_0}$?

$P_E(b)$?

$$y(t) = \sum_i a_i \, h(t - iT) + n_u(t)$$

$$h(t) = p(t) \otimes p(-t)$$

$$H(f) = P(f) \cdot P^*(f) = |P(f)|^2 = \begin{cases} T\,(1 - |fT|) & |fT| \le 1 \\ 0 & \text{altrove} \end{cases}$$



$$\sum_n H\left(f - \frac{n}{T}\right) = T \quad \Rightarrow \quad h(n) = 1$$

$$n_n(t) = n(t) \otimes h_R(t) = \frac{1}{A}\, n(t) \otimes p(-t)$$

$$\sigma^2_{nn} = \frac{N_0}{2}\int_{-\infty}^{+\infty} |H_R(f)|^2\, df = \frac{N_0}{2A^2}\int_{-\infty}^{+\infty} \underbrace{|P(f)|^2}_{H(f)}\, df =$$

$$= \frac{N_0}{2A^2}\, h(0) = \frac{N_0}{2A^2}$$

$$E_s = E\left\{\int_0^T s^2(t)\, dt\right\} = A^2 E\left\{\int_0^T \sum_i \sum_m a_i a_m\, p(t-iT)\, p(t-mT)\, dt\right\} =$$

$$= A^2 \sum_i \sum_m E\{a_i a_m\} \int_0^T p(t-iT)\, p(t-mT)\, dt =$$

$$E\{a_i a_m\} = \begin{cases} E\{a_i^2\} & \text{se } i = m \\ E\{a_i\}\, E\{a_m\} & \text{se } i \neq m \end{cases} =$$

$$E\{a_i\} = \frac{1}{4}(-3) + \frac{1}{4}(-1) + \frac{1}{4}(1) + \frac{1}{4}(3) = 0$$

$$E\{a_i^2\} = \frac{1}{4}(-3)^2 + \frac{1}{4}(-1)^2 + \frac{1}{4}(1)^2 + \frac{1}{4}(3)^2 = 5$$

$$E\{a_i a_m\} = \begin{cases} \sigma & i = m \\ 0 & \text{altrimenti} \end{cases}$$

$$E_s = \sigma A^2 \sum_i \int_0^T p^2(t - iT) = \sigma A^2 \int_{-\infty}^{+\infty} p^2(t)\, dt =$$

$$= \sigma A^2 \int_{-\infty}^{+\infty} p^2(t)\, dt = \sigma A^2$$

$$P_e(h) = \frac{1}{4} P_r\{e \mid a_k = -3\} + \frac{1}{4} P_r\{e \mid a_k = -1\} +$$

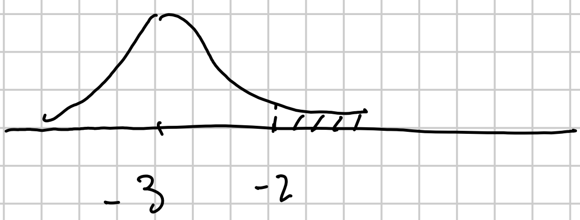
$$+ \frac{1}{4} P_r\{e \mid a_k = 1\} + \frac{1}{4} P_r\{e \mid a_k = 3\}$$

$$P_r\{e \mid a_k = -3\}$$

$$y_k \mid a_k = -3 \in \mathcal{N}(-3, \sigma^2_{nn})$$

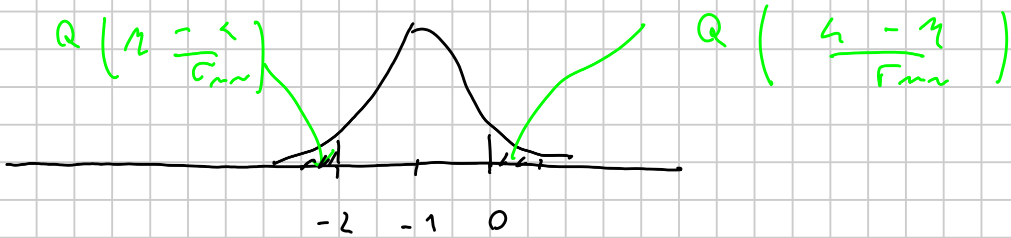
$$P_r\{-3 + n_k > 2\} = Q\left(\frac{h - \eta}{\sigma_{nn}}\right) = Q\left(\frac{-2 + 3}{\sigma_{nn}}\right) =$$

$$= Q\left(\frac{1}{\sigma_{nn}}\right)$$



$$P_r\{e \mid a_k = 1\} = P_r\{e \mid a_k = -1\} =$$

$$= P_r\{-1 + n_k > 0 \,,\; -1 + n_k \leq -2\} =$$



η : v.m

λ : soglia

$$= P_r\{-1 + n_k > 0\} + P_r\{-1 + n_k \leq -2\} =$$

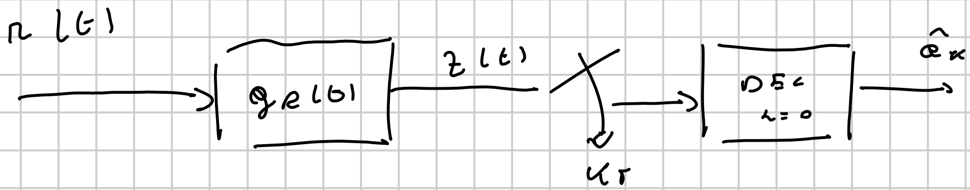
$$= Q\left(\frac{0+1}{\sigma_{nn}}\right) + Q\left(\frac{2-1}{\sigma_{nn}}\right) = 2\,Q\left(\frac{1}{\sigma_{nn}}\right)$$

$$P_e = \frac{1}{4}\, 6\, Q\left(\frac{1}{\sigma_{nn}}\right) = \frac{3}{2}\, Q\left(\frac{1}{\sigma_{nn}}\right)$$

$$\sigma^2_{nn} = \frac{N_0}{2A^2} = \frac{5}{5}\,\frac{N_0}{2A^2} = \frac{5}{2}\,\frac{N_0}{E_s}$$

$$P_e(b) = \frac{3}{2}\, Q\left(\sqrt{\frac{2}{5}\,\frac{E_s}{N_0}}\right)$$

ESERCIZIO 1



$$r(t) = \sum_i a_i \, g_T(t - iT) + w(t)$$

$w(t)$ AWGN $\qquad S_w(f) = \frac{N_0}{2}$

$a_i \in \{-1, 1\}$ ind. equiprobabili

$$g_T(t) = e^{-|t|/T} \, \text{rect}\left(\frac{t}{T}\right)$$

$$g_R(t) = g_T(t)$$

1) Energia media di $r(t)$

2) Verificare assenza ISI

3) Potenza media di rumore dopo $g_R(t)$

4) BER ($P_E(b)$)

$$E_s = E\left\{ \int_0^T r^2(t)\, dt \right\} = E\left\{ \int_0^T \sum_i a_i \, g_T(t - iT) \sum_n a_n \, g_T(t - nT)\, dt \right\} =$$

$$= \int_0^T \sum_i \sum_n E\{a_i a_n\} \, g_T(t - iT) \, g_T(t - nT) \, dt$$

$$E\{a_i a_n\} = \begin{cases} E\{a_i^2\} = 1 & i = n \\ E^2\{a_i\} = 0 & i \neq n \end{cases}$$

$$E\{ a_i^2 \} = \frac{1}{2} (1)^2 + \frac{1}{2} (-1)^2 = 1$$

$$E\{ a_i \} = \frac{1}{2} 1 + \frac{1}{2} (-1) = 0$$

$$= \sum_i \int_0^T g_T^2 (t - iT) \, dt = \int_{-\infty}^{+\infty} g_T^2 (t) \, dt =$$

$$= \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{2|t|/T} = 2 \int_0^{\frac{T}{2}} e^{-2t/T} \, dt =$$

$$= 2 \left(-\frac{2}{T} \right) e^{-2t/T} \Big|_0^{\frac{T}{2}} = \frac{2}{T} \left(1 - \frac{1}{e} \right) = E_s$$

2)

$$z(t) = r(t) \otimes g_R(t) = \sum_i a_i \, g(t - iT) + n(t)$$

$$g(t) = g_T(t) \otimes g_R(t)$$

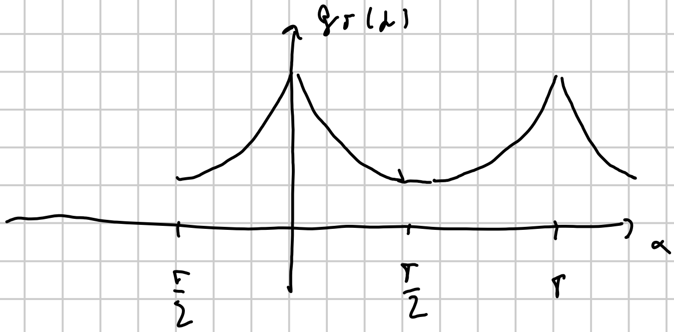
$$n(t) = w(t) \otimes g_R(t)$$

NO ISI SE $g(kT) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$

$$g(t) = \int_{-\infty}^{+\infty} g_T(\alpha) \, g_T(t - \alpha) \, d\alpha$$

$$g(0) = \int_{-\infty}^{+\infty} g(\alpha) \, g(-\alpha) \, d\alpha = \int_{-\infty}^{+\infty} g_T^2(\alpha) \, d\alpha$$

$$g(kT) = \int_{-\infty}^{+\infty} g_T(\alpha) \, g_T(kT - \alpha) \, d\alpha = 0$$



MO ISI

3) σ^2_{nn} ?

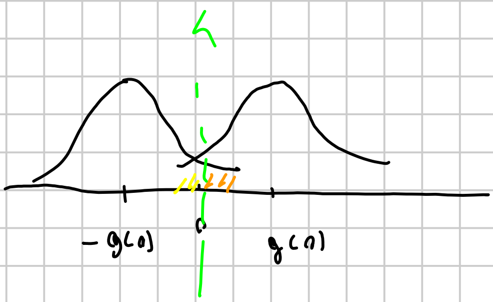
$$\sigma^2_{nn} = \frac{N_0}{2} \int_{-\infty}^{+\infty} |G_R(f)|^2 \, df = \frac{N_0}{2} \, g(0)$$

4)

$$z_k = a_k \, g(0) + n_k$$

$$P(e) = \frac{1}{2} \Pr\{e \mid a_k = -1\} + \frac{1}{2} \Pr\{e \mid a_k = 1\} =$$

$$= \Pr\{g(0) + n_k < 0 \mid a_k = 1\} = Q\left(\frac{g(0)}{\sqrt{\frac{N_0}{2} g(0)}}\right) =$$



$$= Q\left(\sqrt{\frac{2E_s}{N_0}}\right) = Q\left(\sqrt{SNR}\right)$$

ESERCIZIO 1

$$x(t) \longrightarrow \boxed{G_R(f)} \xrightarrow{z(t)} \underset{kT}{\times} \xrightarrow{z_k} \boxed{DEC} \longrightarrow \hat{a}_k$$

$$x(t) = \sum_i a_i \, g_T(t - iT) + w(t)$$

$$G_T(f) = \sqrt{T} \cos\left(\pi f \frac{T}{2}\right) \mathrm{rect}\left(f \frac{T}{2}\right)$$

$a_i \in \{-1, 1\}$ ind. eq.

$w(t)$ GAUSSIANO con $S_w(f) = \frac{N_0}{2} \mathrm{rect}\left(f \frac{T}{8}\right)$

⚠ NO AWGN

1) $G_R(f)$ OTTIMO?

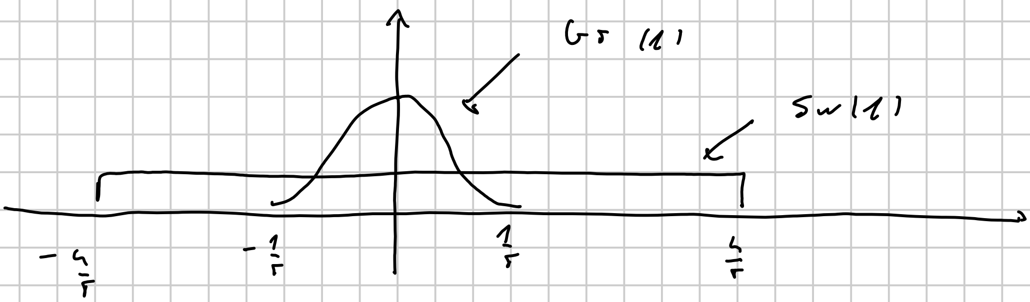
↳ NO ISI

MAX SNR

2) $P(e)$ $a = -1$

$$G_T(f) = \sqrt{T} \cos\left(\pi f \frac{T}{2}\right) \mathrm{rect}\left(\frac{f}{2/T}\right)$$

$$S_w(f) = \frac{N_0}{2} \mathrm{rect}\left(\frac{f}{8/T}\right)$$



IL RUMORE È BIANCO NELLA BANDA DEL SISTEMA

IL FILTRO OTTIMO È IL FILTRO ADATTATO

$$g_R(t) = g_T(-t) = g_T(t)$$

$\downarrow$

$G_T(f)$ REALE E PARI

$$G_R(f) = G_T^*(f) = G_T(f)$$

$$z(t) = \sum_i a_i \, g(t - iT) + n(t)$$

$$g(t) = g_T(t) \otimes g_R(t) = g_T(t) \otimes g_T(t)$$

$$n(t) = w(t) \otimes g_R(t)$$

$$G(f) = T \cos^2\left(\pi f \frac{T}{2}\right) \operatorname{rect}\left(f \frac{T}{2}\right) =$$

$$= \frac{T}{2}\left(1 + \cos(\pi f T)\right) \operatorname{rect}\left(f \frac{T}{2}\right)$$

$$g(kT) = \begin{cases} g(0) = 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

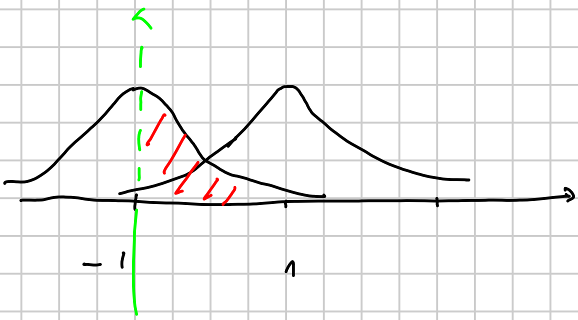
$$S_n(f) = S_w(f) \, |G_T(f)|^2 = \frac{N_0}{2} |G_T(f)|^2$$

$$\sigma^2_{nn} = \frac{N_0}{2} \int_{-\infty}^{+\infty} |G_T(f)|^2 \, df = \frac{N_0}{2} g(0) = \frac{N_0}{2}$$

$$z_k = a_k + n_k$$

$$z_k \mid a_k = 1 \in \mathcal{N}(1, \sigma_n^2)$$

$$z_k \mid a_k = -1 \in \mathcal{N}(-1, \sigma_n^2)$$



$$Pr\{e \mid a_k = 1\} = Pr\{z_k < \lambda \mid a_k = 1\} =$$

$$= Pr\{1 + n_k < \lambda \mid a_k = 1\} = Q\left(\frac{1-\lambda}{\sigma_n}\right)$$

$$Pr\{e \mid a_k = -1\} = \frac{1}{2} = Q(0)$$

ESERCIZIO X CASA

$$r(t) \longrightarrow \boxed{h_R(t)} \xrightarrow{y(t)} \underset{kT}{\diagup} \longrightarrow \boxed{DEC} \longrightarrow \hat{a}_k$$

$a_i \in \{-e ; 2e\}$ indipendenti equiprobabili

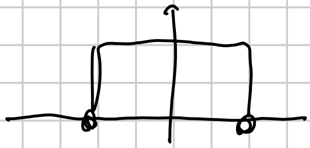
$$r(t) = \sum_i a_i \, p(t - iT) + n(t)$$

$n(t)$ AWGN $\qquad S_n(f) = \frac{N_0}{2}$

$p(t) = T\,|t|\,e^{-|t|}\ \mathrm{rect}\left(t\,\frac{T}{2}\right)$

$\mathrm{rect}(x) = 0$

$x = \pm\frac{1}{2}$



$H_R(f) = e^{|t|}\ \mathrm{rect}\left(t\,\frac{T}{2}\right)$

$$\hat{Q}_n = \begin{cases} -e & x_n < \lambda \\ 2e & x_n > \lambda \end{cases}$$

$\lambda = \frac{3}{2}\,e$

E_s ?

$P_r\{e\}$

λ ott. (SOGLIA CHE MINIMIZZA LA $P_r\{e\}$)