$f \in C^1(A)$ $\quad A \overset{\cup}{\ } $ aperto $\quad f(x) = f(x^0) + (\nabla f(x^0), x - x^0) + \omega(x)$

$x^0 \in \mathring{A}$ $\quad \lim_{x \to x^0} \frac{\omega(x)}{\|x - x^0\|} = 0$

$f : A \to \mathbb{R}$

$f \in C^2(A)$ $\quad \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} f(x^0) = \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_i} f(x^0)$

$Hf(x^0) = \frac{\partial^2 f(x^0)}{\partial x_i \partial x_j}$ $\quad i,j = 1 \dots n$ $\quad H$ simmetrica

$f : A \to \mathbb{R}^m$, $A \subseteq \mathbb{R}^n$ $\quad f(x) = \begin{pmatrix} f_1(x) \\ \vdots \\ f_m(x) \end{pmatrix}$ $\quad Jf(x^0) = \begin{pmatrix} \partial_1 f_1 & \dots & \partial_n f_1 \\ \vdots & & \\ \partial_1 f_m & \dots & \partial_n f_m \end{pmatrix}$

$A \xrightarrow{f} B \xrightarrow{g} \mathbb{R}^k$, $A \subseteq \mathbb{R}^n$, $B \subseteq \mathbb{R}^m$ $\quad h(x) = g(f(x))$

$h : \mathbb{R}^n \to \mathbb{R}^k$

$\mathbb{R} \ni x_0$ $\quad y_0 = f(x_0)$ $\quad h'(x_0) = g'(f(x_0))\, f'(x_0)$

$Jf \in \mathcal{M}(m \times n)$ $\quad$ m righe, n colonne $\quad JG \in \mathcal{M}(k \times m)$ $\quad$ k - righe, m - colonne

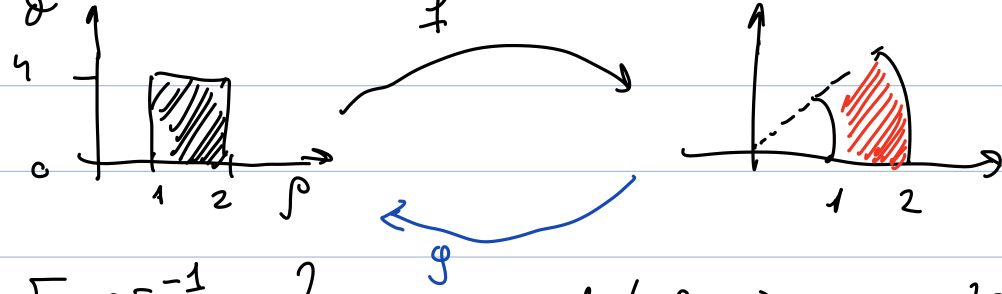
$Jh(x^0) = Jg(y^0)\, Jf(x^0) = Jg(f(x^0))\, Jf(x^0)$

$$\frac{\partial h_i}{\partial x_j}(x^0) = \sum_{s=1}^{m} \frac{\partial}{\partial y_s} g_i(y^0) \frac{\partial f_s}{\partial x_j}(x^0)$$

$$f : \underset{\rho}{[1,2]} \times \underset{\vartheta}{[0, \pi/4]} \to \begin{pmatrix} \rho \cos\vartheta \\ \rho \sin\vartheta \end{pmatrix} = \begin{pmatrix} f_1(\rho,\vartheta) \\ f_2(\rho,\vartheta) \end{pmatrix} \begin{matrix} x \\ y \end{matrix} \in \mathbb{R}^2$$

$[1,2] \times [0,\pi/4] \subseteq \mathbb{R}^2$

$$Jf(\rho,\vartheta) = \begin{pmatrix} \frac{\partial}{\partial \rho} f_1 & \frac{\partial f_1}{\partial \vartheta} \\ \frac{\partial}{\partial \rho} f_2 & \frac{\partial f_2}{\partial \vartheta} \end{pmatrix} = \begin{pmatrix} \cos\vartheta & -\rho \sin\vartheta \\ \sin\vartheta & \rho\cos\vartheta \end{pmatrix}$$



$[Jf]^{-1}$?

$$\det(Jf(\rho,\theta)) = \rho\cos^2\theta + \rho\sin^2\theta = \rho \geq 0$$

$$g(f(\rho,\theta)) = \begin{pmatrix}\rho\\ \theta\end{pmatrix}$$

$$\begin{matrix} x \xrightarrow{\varphi} x \\ y \to y \end{matrix} \qquad J\varphi = \begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$Jg \cdot Jf = I$$

$\longrightarrow$ $Jg = [Jf]^{-1}$

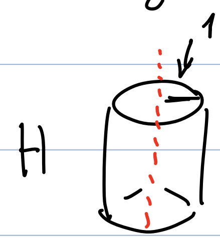
$g(f(x)) = x$ $\qquad$ f, g sono inverse una dell'altra

$g'(f(x))\, f'(x) = 1$

$$g'(f(x_0)) = \frac{1}{f'(x_0)}$$

• $f: \underset{\rho}{\underset{\Downarrow}{[0,1]}} \times \underset{\theta}{\underset{\Downarrow}{[0,2\pi]}} \times \underset{z}{\underset{\Downarrow}{[0,H]}} \longrightarrow \begin{pmatrix}\rho\cos\theta\\ \rho\sin\theta\\ z\end{pmatrix} = \begin{pmatrix} f_1(\rho,\theta,z)\\ f_2(\rho,\theta,z)\\ f_3(\rho,\theta,z)\end{pmatrix}$

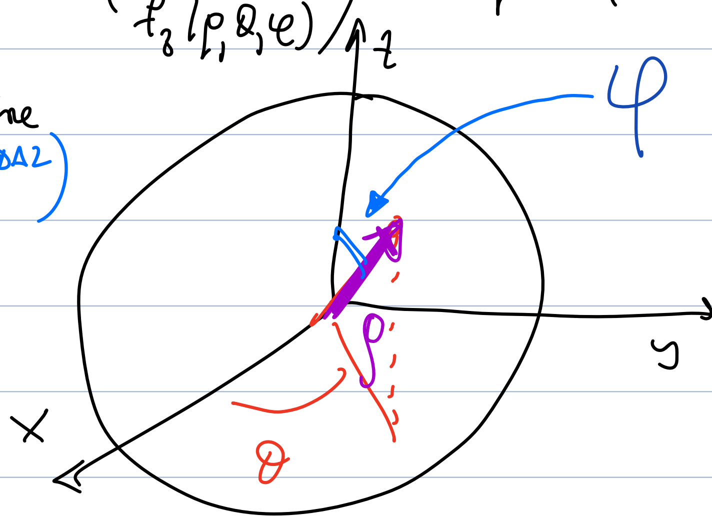
$\det J = \rho$



• $f: \underset{\rho}{\underset{\Downarrow}{[0,1]}} \times \underset{\theta}{\underset{\Downarrow}{[0,2\pi]}} \times \underset{\varphi}{\underset{\Downarrow}{[0,\pi]}} \longrightarrow \begin{pmatrix} f_1(\rho,\theta,\varphi)\\ f_2(\rho,\theta,\varphi)\\ f_3(\rho,\theta,\varphi)\end{pmatrix} = \begin{pmatrix}\rho\sin\varphi\cos\theta\\ \rho\sin\varphi\sin\theta\\ \rho\cos\varphi\end{pmatrix}$

longitudine $\quad$ latitudine (MISURATA DAL POLO NORD)

$\det Jf = \rho^2 \sin\varphi$



$$f(x,y) = x^y \qquad \{(x,y) \in \mathbb{R}^2 \quad x > 0\}$$
$$= e^{y \ln x}$$

$$\partial_x f(x,y) = e^{y\ln x} \cdot \frac{y}{x} = \frac{x^y y}{x} = x^{y-1} \cdot y$$

$$\nabla f(x,y) = (x^{y-1} y, x^y \ln x)$$

$$\partial_y f(x,y) = e^{y \ln x} \ln x = x^y \ln x$$

f è differenziabile in $\{(x,y) \quad x>0\}$

$$\partial_x f, \partial_y f \in C^0$$

$$Hf = \begin{pmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{pmatrix} = \begin{pmatrix} e^{y\ln x}\left(\frac{y}{x}\right)^2 + e^{y \ln x} y\left(-\frac{1}{x^2}\right) & e^{y\ln x} \frac{y}{x} \ln x + \frac{e^{y \ln x}}{x} \\ & e^{y\ln x} \ln^2 x \end{pmatrix}$$

$$y \in \mathbb{R}^n \qquad x \in \mathbb{R}^n \qquad x^0 \neq 0$$

$$\frac{\partial}{\partial y} \|x\| \Big|_{x=x^0} = \lim_{t\to 0} \frac{\|x^0+ty\| - \|x^0\|}{t} = \lim_{t\to 0} \frac{(\|x^0+ty\| - \|x^0\|)(\|x^0+ty\| + \|x^0\|)}{t\,(\|x^0+ty\| + \|x^0\|)}$$

$$= \lim_{t\to 0} \frac{\|x^0+ty\|^2 - \|x^0\|^2}{t\,(\|x^0+ty\| + \|x^0\|)} = \lim_{t\to 0} \frac{\cancel{\|x^0\|^2} + t^{\cancel{2}}\|y\|^2 + 2(x^0,y) - \cancel{\|x^0\|^2}}{\cancel{t}\,(\|x^0+ty\| + \|x^0\|)}$$

$$= \lim_{t\to 0} \frac{2(x^0,y) + t\|y\|^2}{\|x^0+ty\| + \|x^0\|} = \frac{2(x^0,y)}{2\|x^0\|} = \frac{(x^0,y)}{\|x^0\|}$$

$t\|y\|^2 \to 0$, $\quad x^0+ty \to x^0$

$$x^0 \neq 0$$

$$f(x,y) = (\operatorname{arctg} y)^x \qquad \text{differenziabile in } (1,1)$$
$$= e^{x \ln(\operatorname{arctg}(y))}$$

$$\partial_x f(x,y) = (\operatorname{arctg}(y))^x \cdot \ln(\operatorname{arctg}(y))$$

$$\partial_y f(\;) = (\operatorname{arctg}(y))^x \cdot \frac{x}{\operatorname{arctg}(y)} \frac{1}{1+y^2} = (\operatorname{arctg} y)^{x-1} \cdot \frac{x}{1+y^2}$$

$\partial_x f(1,1)$

$\partial_y f(1,1)$

$$df(1,1)\begin{pmatrix} x \\ y \end{pmatrix} = \partial_x f(1,1)\, x + \partial_y f(1,1)\, y$$

$$= \frac{\pi}{4} \ln \frac{\pi}{4}\, x + \frac{1}{1+1}\, y$$

$$= \frac{\pi}{4} \ln \frac{\pi}{4}\, x + \frac{1}{2}\, y$$

$f(x) = \sqrt{1-\|x\|^2}$ $\quad x \in \mathbb{R}^3 \quad \|x\| \leq 1$

scrivere piano tg. in x^0, $\|x^0\| < 1$