SOLUZIONE PROVA DI COM. NUM.

DEL 07/02/2019

Es (1)

$$x_1(t) = x(t)\cos(2\pi f_0 t + \varphi)$$

$$x_2(t) = x(t)\sin(2\pi f_0 t + \varphi)$$

$$X_1(f) = B\,\text{sinc}(Bt)\cos(2\pi f_0 t)\cos(2\pi f_0 t + \varphi) =$$

$$= \frac{B}{2}\,\text{sinc}(Bt)\left[\cos(4\pi f_0 t + \varphi) + \cos\varphi\right]$$

$$X_2(f) = B\,\text{sinc}(Bt)\cos(2\pi f_0 t)\sin(2\pi f_0 t + \varphi) =$$

$$= \frac{B}{2}\,\text{sinc}(Bt)\left[\sin(4\pi f_0 t + \varphi) + \sin\varphi\right]$$

$$y_1(t) = x_1(t) \otimes h(t)$$

$$y_2(t) = x_2(t) \otimes h(t)$$

$$Y_1(f) = X_1(f)\,H(f)$$

$$Y_2(f) = X_2(f)\,H(f), \qquad H(f) = \left(1 - \frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right)$$

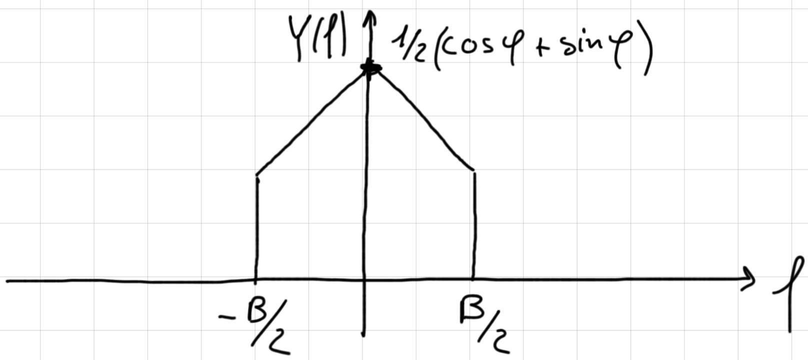
Le componenti di $X_1(f)$ e $X_2(f)$ ad alta frequenza ($2f_0$) vengono cancellate, rimangono pertanto solo le comp in banda base

$$Y_1(f) = \frac{1}{2}\,\mathrm{rect}\left(\frac{f}{B}\right)\cos\varphi \cdot \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$Y_2(f) = \frac{1}{2}\,\mathrm{rect}\left(\frac{f}{B}\right)\sin\varphi \cdot \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$y(t) = y_1(t) + y_2(t)$$

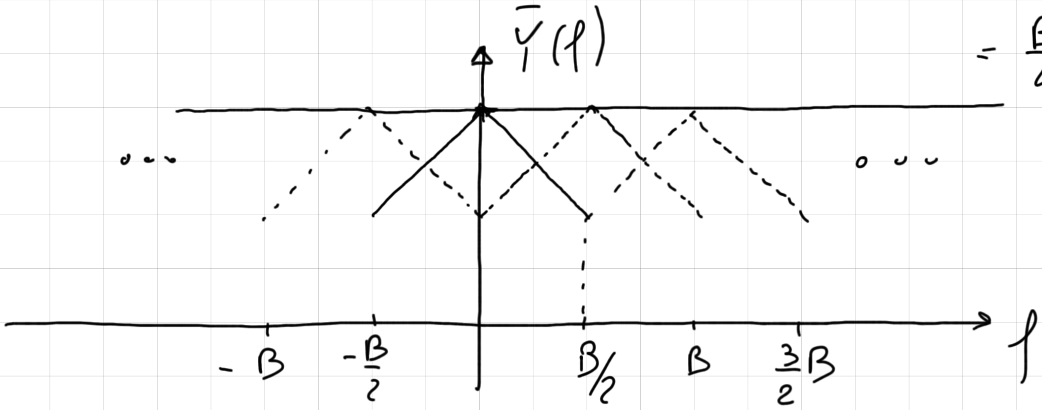
$$Y(f) = Y_1(f) + Y_2(f) = \frac{1}{2}(\cos\varphi + \sin\varphi)\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{B}\right)$$



$$y[n] = y(nT) = y\left(\frac{2n}{B}\right)$$

$$\bar{Y}(f) = \frac{B}{2}\sum_{n=-\infty}^{+\infty} Y\left(f - n\frac{B}{2}\right) = \frac{B}{2}\cdot\frac{1}{2}(\cos\varphi + \sin\varphi) =$$

$$= \frac{B}{4}(\cos\varphi + \sin\varphi)$$



$$Z(f) = \bar{Y}(f)P(f) = \frac{B}{4}(\cos\varphi + \sin\varphi)\,\mathrm{rect}\left(\frac{f}{B}\right)$$

$$\boxed{z(t) = \frac{B^2}{4}(\cos\varphi + \sin\varphi)\,\mathrm{sinc}(Bt)}$$

$$\boxed{E_2 = \int_{-\infty}^{+\infty} |Z(f)|^2 \, df = \frac{B^5}{16} (\cos\varphi + \sin\varphi)^2}$$

$$\max_{\varphi} E_2 \Rightarrow (\cos\varphi + \sin\varphi)^2 \text{ deve essere max}$$

$$K(\varphi) = (\cos\varphi + \sin\varphi)^2$$

$$\frac{d}{d\varphi} K(\varphi) = 0 \Rightarrow 2(\cos\varphi + \sin\varphi)(-\sin\varphi + \cos\varphi) = 0$$

$$\cos^2\varphi - \sin^2\varphi = 0 \Rightarrow \cos^2\varphi = \sin^2\varphi$$

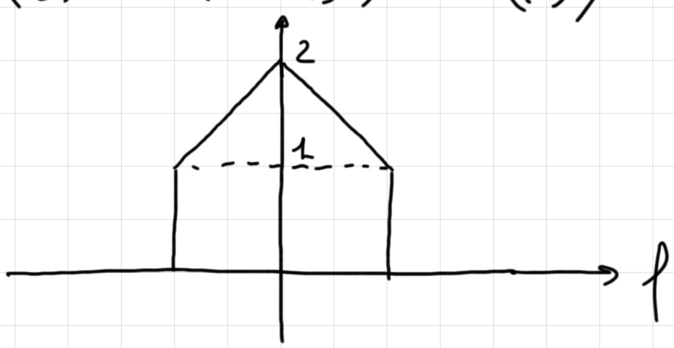
$$\Rightarrow |\cos\varphi| = |\sin\varphi| \Rightarrow \boxed{\varphi = \frac{\pi}{4} + k\pi}$$

Es. 2

$$E_s = \frac{1}{2} E[x^2] E_p$$

$$E[x^2] = \frac{1}{2}(-1)^2 + \frac{1}{2}(2)^2 = \frac{5}{2}$$

$$P(f) = \mathrm{rect}\left(\frac{f}{2B}\right) + \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$



$$P^2(f) = \text{rect}\left(\frac{f}{2B}\right) + \left(1 - \frac{|f|}{B}\right)^2 \text{rect}\left(\frac{f}{2B}\right) + 2\left(1 - \frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right)$$

$$E_p = 2B + \frac{2}{3}B + 2B = \frac{14}{3}B$$

$$\boxed{E_s = \frac{1}{2} \cdot \frac{5}{2} \cdot \frac{14B}{3} = \frac{35}{6}B}$$

$$\boxed{P_{n_u} = N_0 \int_{-\infty}^{+\infty} |H_R(f)|^2 \, df = 2N_0B}$$

Calcolo della $h(t)$

$\Rightarrow$ attenzione all'errore di sincronizzazione di fase $\varphi = \frac{\pi}{3}$

$\Rightarrow$ dopo la demodulazione rimane un $\cos\left(\frac{\pi}{3}\right)$

$$h(t) = p(t) \otimes c(t) \otimes h_R(t) \cdot \cos\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) p(t)$$

$$h(nT) = h\left(\frac{n}{B}\right) = \left[2B \,\text{sinc}(2n) + B\,\text{sinc}^2(n)\right] \cos\left(\frac{\pi}{3}\right)$$

$$= \begin{cases} 3B\cos\left(\frac{\pi}{3}\right) = \frac{3}{2}B & n = 0 \\ 0 & n \neq 0 \end{cases}$$

- COND. DI NYQUIST VERIFICATA
- $h(0) = \frac{3}{2}B$

$$P_E(b) = \frac{1}{2} Q\left(\frac{\frac{3}{2}B}{\sqrt{2N_0B}}\right) + \frac{1}{2}\left(\frac{3B}{\sqrt{2N_0B}}\right)$$