ESERCITAZIONE

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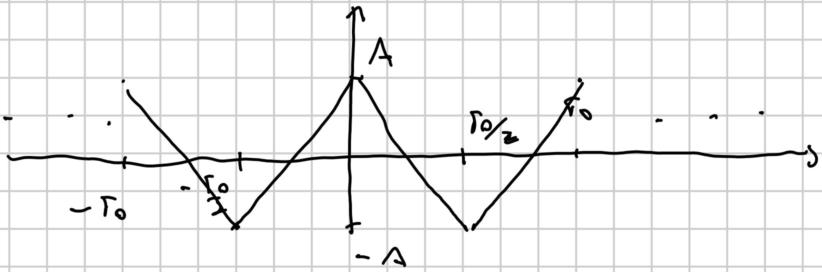


$$x(t) = \sum_k x_0(t - kT_0)$$

$$x(t) = -x\left(t - \frac{T_0}{2}\right)$$

$$x_0(t) = \left(A - \frac{4A|t|}{T_0}\right) \mathrm{rect}\left(\frac{t}{T_0}\right)$$

$x(t)$ PERIODICO, REALE, PARI, ALTERNATIVO

$$X_n = \begin{cases} 0 & n \text{ PARI} \\ \frac{2}{T_0} \int_0^{\frac{T_0}{2}} x(t)\, e^{-j2\pi n f_0 t}\, dt & \text{per } n \text{ DISP.} \end{cases}$$

$$X_n = \frac{2}{T_0} \int_0^{T_0/2} \underbrace{\left(A - \frac{4At}{T_0}\right)}_{g} \underbrace{e^{-j2\pi n f_0 t}}_{h'} dt =$$

$$= \frac{2}{T_0} \left[\underbrace{\left(A - \frac{4At}{T_0}\right)}_{g} \underbrace{\frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0}}_{h} \right]_0^{T_0/2} + \int_0^{T_0/2} \underbrace{\frac{4A}{T_0}}_{g'} \underbrace{\frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0}}_{h} dt$$

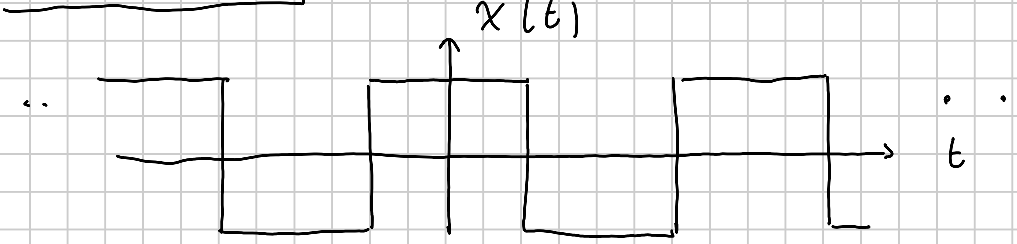
$$= \frac{2}{\cancel{T_0}} \left[(A - 2A) \frac{e^{-j\pi n}}{-j\cancel{2}\pi n \cancel{f_0}} - A + \frac{4A}{T_0} \frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0} \Bigg|_0^{T_0/2} \right.$$

$$= -\frac{A}{j\pi n} \left[e^{-j\pi n} - 1 - \frac{2}{j\pi n} \left(e^{-j\pi n} - 1 \right) \right] =$$

$$= \frac{A}{j\pi n} \left(\underbrace{e^{-j\pi n}}_{-1} + 1 \right) - \frac{2A}{\pi n^2} \left(\underbrace{e^{-j\pi n}}_{-1} - 1 \right) \Rightarrow$$

$$X_n = \frac{4A}{\pi n^2}$$

ESERCIZIO



$x(t)$ PARI, REALE, ALTERNATIVO $\Rightarrow$ $X_n = X_{-n}$

$\mathrm{Im}\{X_n\} = 0$

$X_n = 0$ n pari

$$X_n = \begin{cases} 0 & n \text{ PARI} \\ \frac{2}{T_0} \int_0^{\frac{T_0}{2}} x(t)\, e^{-j2\pi n f_0 t}\, dt & n \text{ DISP} \end{cases}$$

$$= \frac{2}{T_0} \left[\int_0^{T_0/4} A\, e^{-j2\pi n f_0 t} + \int_{\frac{T_0}{4}}^{\frac{T_0}{2}} (-A)\, e^{-j2\pi n f_0 t}\, dt \right.$$

$$= \frac{2A}{T_0} \left[\frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0} \Bigg|_0^{\frac{T_0}{4}} - \frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0} \Bigg|_{\frac{T_0}{4}}^{\frac{T_0}{2}} \right] =$$

$$= \frac{2A}{-j\cancel{T_0}\, 2\pi n \cancel{f_0}} \left[e^{-j2\pi n f_0 \frac{T_0}{4}} - 1 - \left[e^{-j2\pi n f_0 \frac{T_0}{2}} - e^{-j2\pi n f_0 \frac{T_0}{4}} \right] \right] =$$

$$= j \frac{A}{\pi n} \left[e^{-j\frac{\pi n}{2}} - 1 - e^{-j\pi n} + e^{-j\frac{\pi n}{2}} \right] =$$

$$= -j \frac{A}{\pi n} \left[-2\underbrace{e^{-j\frac{\pi n}{2}}} + \left(\cancel{e^{-j\pi n}}_{-1} + 1 \right) \right]$$

$$e^{-j\pi\frac{n}{2}} = -j$$

$$X_n = \frac{2A}{\pi n}\, \underbrace{j\,(-j)^n}_{e^{j\frac{\pi}{2}}\left(e^{-j\frac{\pi}{2}}\right)^n = e^{j\frac{\pi}{2}}\, e^{-jn\frac{\pi}{2}} = e^{-j(n-1)\frac{\pi}{2}} = \left(e^{-j\pi}\right)^{\frac{n-1}{2}} = (-1)^{\frac{n-1}{2}}} \qquad n \text{ DISP.}$$

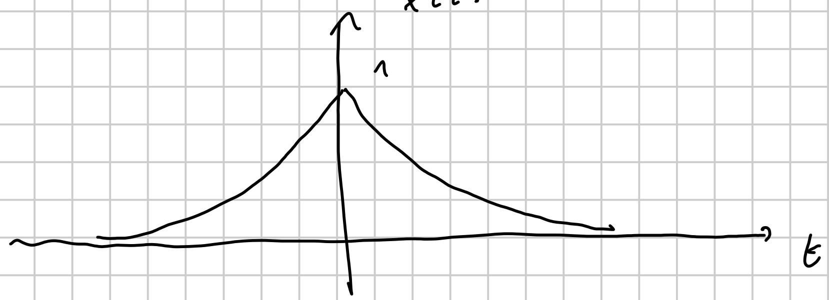
$$X_1 = \frac{2A}{\pi}(-1)^0 = \frac{2A}{\pi}$$

$$X_3 = \frac{2A}{3\pi}(-1)^1 = -\frac{2A}{3\pi}\ ; \quad X_5 = \frac{2A}{5\pi}(-1)^2 = \frac{2A}{5\pi}$$

$$\boxed{\text{TCF}}$$

ESERCIZIO ESPONENZIALE BILATERO

$$x(t) = e^{-|t|}$$



$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt =$$

$$= \int_{-\infty}^{0} e^{t}\, e^{-j2\pi f t}\, dt + \int_{0}^{+\infty} e^{-t}\, e^{-j2\pi f t}\, dt =$$

$$= \left.\frac{e^{t(1-j2\pi f)}}{1-j2\pi f}\right|_{-\infty}^{0} + \left.\frac{e^{-t(1+j2\pi f)}}{-(1+j2\pi f)}\right|_{0}^{+\infty}$$

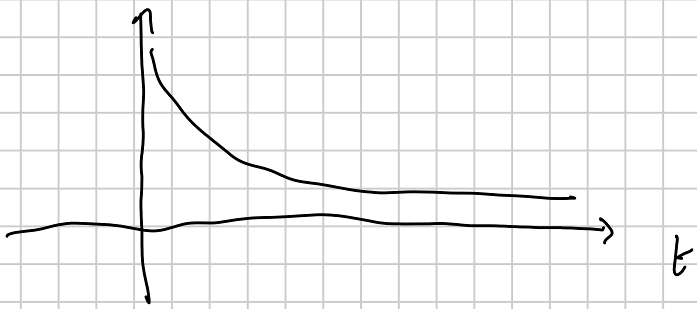
$$= \left(\frac{1}{1-j2\pi f} - 0\right) + \left(0 + \frac{1}{1+j2\pi f}\right) =$$

$$X(f) = \frac{1}{1-j2\pi f} + \frac{1}{1+j2\pi f} = \frac{1+\cancel{j2\pi f}+1-\cancel{j2\pi f}}{1+4\pi^2 f^2} =$$

$$= \frac{2}{1+4\pi^2 f^2}$$

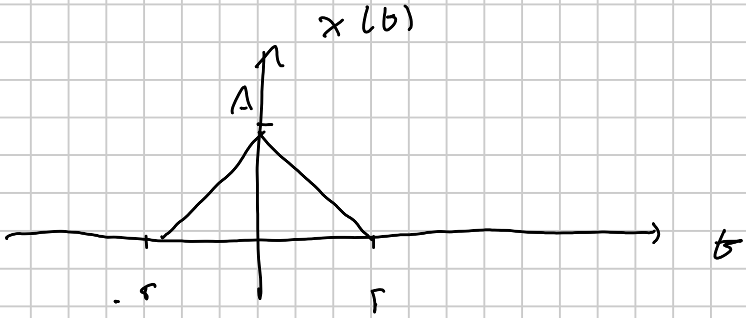
<u>ESERCIZIO 2</u> ESPONENZIALE MONOLATERO

$$x(t) = e^{-t}\, u(t)$$



$$X(f) = \int_0^{+\infty} e^{-t}\, e^{-j2\pi f t}\, dt = \frac{1}{1+j2\pi f}$$

ESERCIZIO IMPULSO TRIANGOLARE



$$x(t) = A\left(1 - \frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{T}\right)$$

I) $$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt =$$

$$= \int_{-T}^{0} A\left(1 + \frac{t}{T}\right) e^{-j2\pi f t}\, dt + \int_{0}^{T} A\left(1 - \frac{t}{T}\right) e^{-j2\pi f t}\, dt =$$

$$\int_{-T}^{0} \underbrace{A\left(1 + \frac{t}{T}\right)}_{g} \underbrace{e^{-j2\pi f t}}_{f'}\, dt = \underbrace{A\left(1 + \frac{t}{T}\right)}_{g} \underbrace{\frac{e^{-j2\pi f t}}{(-j2\pi f)}}_{f}\Bigg|_{-T}^{0} +$$

$$- \int_{-T}^{0} \underbrace{\frac{A}{T}}_{g'} \underbrace{\frac{e^{-j2\pi f t}}{(-j2\pi f)}}_{f}\, dt =$$

$$= \frac{A}{-j2\pi f} - 0 - \frac{A}{T} \frac{e^{-j2\pi f t}}{(-j2\pi f)^2}\Bigg|_{-T}^{0} =$$

$$= -\frac{A}{j2\pi f} + \frac{A}{T}\left(\frac{1 - e^{j2\pi f T}}{4\pi^2 f^2}\right)$$

$$\int_{0}^{T} A\left(1 - \frac{t}{T}\right) e^{-j2\pi f t}\, dt = \frac{A}{j2\pi f} - \frac{A}{T4\pi^2 f^2}\left(e^{-j2\pi f T} - 1\right)$$

$$X(f) = -\cancel{\frac{A}{j2\pi f}} + \frac{A}{\tau 4\pi^2 f^2}\left(1 - e^{j2\pi f\tau}\right) + \cancel{\frac{A}{j2\pi f}} - \frac{A}{\tau 4\pi^2 f^2}\left(e^{-j2\pi f\tau} - 1\right) =$$

$$= \frac{A}{4\pi^2 f^2 \tau}\left[1 - e^{j2\pi f\tau} + 1 - e^{-j2\pi f\tau}\right] =$$

$$= \frac{A}{4\pi^2 f^2 \tau}\left(1 - e^{j2\pi f\tau}\right)\left(1 - e^{-j2\pi f\tau}\right) =$$

$$= \frac{A}{4\pi^2 f^2 \tau}\, \cancel{e^{+j\pi f\tau}}\left(e^{-j\pi f\tau} - e^{+j\pi f\tau}\right)\cancel{e^{-j\pi f\tau}} \cdot \left(e^{+j\pi f\tau} - e^{-j\pi f\tau}\right) =$$

$$4 = -2j \cdot 2j$$

$$= \frac{A}{\pi^2 f^2 \tau}\left(\frac{e^{-j\pi f\tau} - e^{+j\pi f\tau}}{-2j}\right)\left(\frac{e^{+j\pi f\tau} - e^{-j\pi f\tau}}{-2j}\right) =$$

$$= \frac{A}{\pi^2 f^2 \tau} \sin(\pi f\tau) \sin(\pi f\tau) =$$

$$= A\tau \frac{\sin(\pi f\tau)}{\pi f\tau} \frac{\sin(\pi f\tau)}{\pi f\tau} = A\tau\, \mathrm{sinc}^2(f\tau)$$

II)

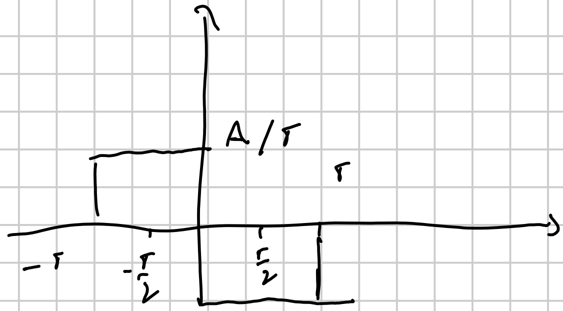
$$x(t) = \frac{A}{\tau}\left(\mathrm{rect}\left(\frac{t}{\tau}\right) \otimes \mathrm{rect}\left(\frac{t}{\tau}\right)\right)$$

$$z(t) = x(t) \otimes y(t) \rightleftharpoons Z(f) = X(f)\, Y(f)$$

$$X(f) = \frac{A}{\tau}\, \tau\, \mathrm{sinc}(f\tau)\, \tau\, \mathrm{sinc}(f\tau) = A\tau\, \mathrm{sinc}(f\tau)$$

III)

$$y(t) = \frac{\partial}{\partial t} x(t)$$



$$y(t) = \frac{d}{dt} x(t) \rightleftarrows Y(f) = j 2\pi f X(f)$$

$$X(f) = \frac{Y(f)}{j 2\pi f}$$

$$y(t) = \frac{A}{T} \operatorname{rect}\left(\frac{t + T/2}{T} \right) - \frac{A}{T} \operatorname{rect}\left(\frac{t - T/2}{T} \right)$$

$$y_0(t) = \operatorname{rect}\left(\frac{t}{T} \right) \qquad y(t) = \frac{A}{T}\left[y_0(t + T/2) - y_0\left(t - \frac{T}{2}\right)\right]$$

$$Y(f) = \frac{A}{T} Y_0(f) e^{+j 2\pi f \frac{T}{2}} - \frac{A}{T} Y_0(f) e^{-j 2\pi f \frac{T}{2}} =$$

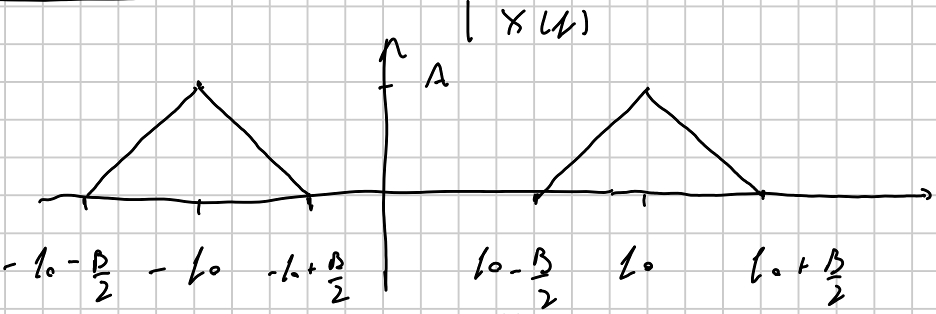
$$= \frac{A}{T} T \operatorname{sinc}(fT) \left[e^{+j \cancel{2}\pi f \frac{T}{\cancel{2}}} - e^{-j\pi f T} \right] \frac{2j}{2j}$$

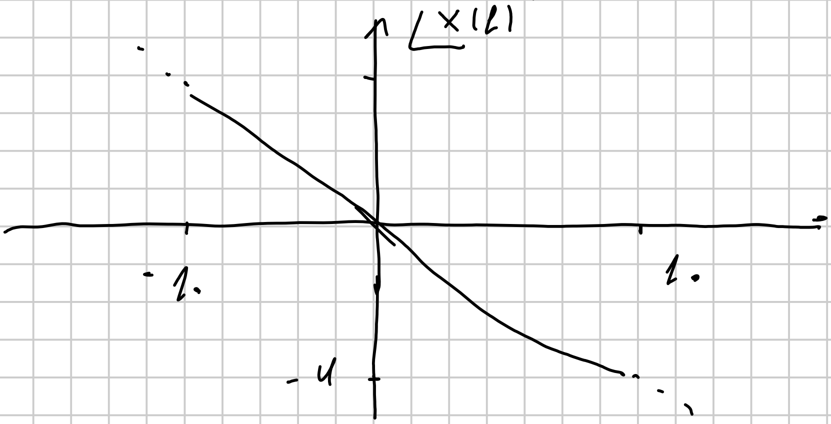
$$= A\, 2j \operatorname{sinc}(fT) \sin(\pi f T) = Y(f)$$

$$X(f) = \frac{Y(f)}{j 2\pi f} = \frac{AT}{\pi f T} \operatorname{sinc}(fT) \sin(\pi f T) =$$

$$= AT \operatorname{sinc}^2(fT)$$

ESERCIZIO





$$X(f) = |X(f)| e^{j\angle X(f)}$$

$$|X(f)| = x_1(f-f_0) + x_1(f+f_0)$$

$$x_1(f) = A\left(1 - \frac{|f|}{B/2}\right) \operatorname{rect}\left(\frac{f}{B}\right)$$

$$\angle X(f) = af + b$$

$$b = 0$$

$$a = -\frac{\varphi}{f_0}$$

$$X(f) = \underbrace{\left[x_1(f-f_0) + x_1(f+f_0)\right]}_{A(f)} e^{-j\frac{\varphi}{f_0}f}$$

$$X(f) = A(f)\, e^{-j2\pi f t_0} \qquad t_0 = \frac{\varphi}{2 f_0 \pi}$$

.) teorema del ritardo

$$x(t) = a(t - t_0)$$

.) $A(f) = x_1(f-f_0) + x_1(f+f_0)$

$$a(t) = x_1(t)\; 2\cos(2\pi f_0 t)$$

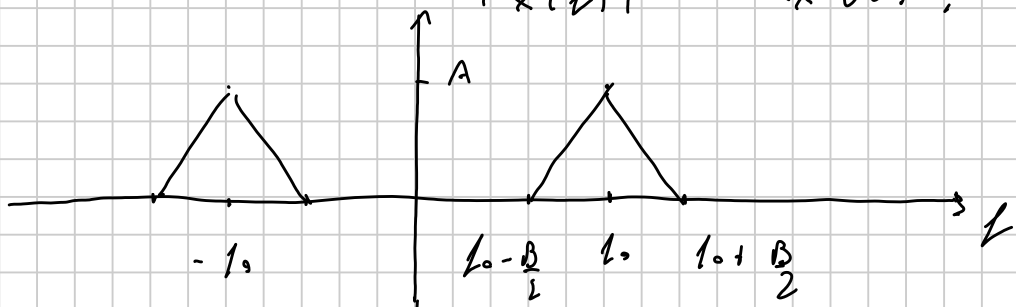
$$\boxed{\begin{aligned} y(t) &= x(t)\cos 2\pi f_0 t \\ Y(f) &= \frac{X(f-f_0) + X(f+f_0)}{2} \end{aligned}}$$

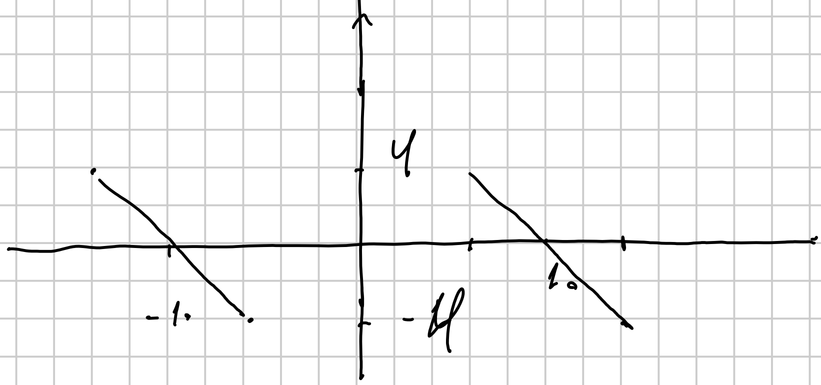
$$x_1(t) = A\,\frac{B}{2}\,\mathrm{sinc}^2\left(t\,\frac{B}{2}\right)$$

$$x(t) = AB\cos\big(2\pi f_0 (t-t_0)\big)\,\mathrm{sinc}^2\left(\frac{B}{2}(t-t_0)\right)$$

ESERCIZIO X CASA

$|X(f)|$ $\qquad$ $x(t)$?





$$X(f) = X_0(f-f_0)\,e^{j\varphi_1(f)} + X_0(f+f_0)\,e^{j\varphi_2(f)}$$

$$X_0(f) = A\left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right)$$

$$\varphi_1(f) = \varphi_0(f-f_0) = -\frac{2\varphi}{B}(f-f_0)$$

$$\varphi_2(f) = \varphi_0(f+f_0) = -\frac{2\varphi}{B}(f+f_0)$$

$$X(f) = X_0(f - f_0)\, e^{j\varphi_0(f-f_0)} + X_0(f+f_0)\, e^{j\varphi_0(f+f_0)}$$

$$Y(f) = X_0(f)\, e^{j\varphi_0(f)}$$

TEOREMA DELLA MODULAZIONE

$$x(t) = 2\cos(2\pi f_0 t)\, y(t)$$

TEOREMA DEL RITARDO

$$y(t) = x_0(t - t_0) \qquad \text{con} \qquad t_0 = \frac{\varphi}{B\pi}$$

$$x_0(t) = \frac{AB}{2}\, \mathrm{sinc}^2\left(\frac{tB}{2}\right)$$

QUINDI

$$x(t) = AB \cos(2\pi f_0 t)\, \mathrm{sinc}^2\left((t - t_0)\frac{B}{2}\right)$$