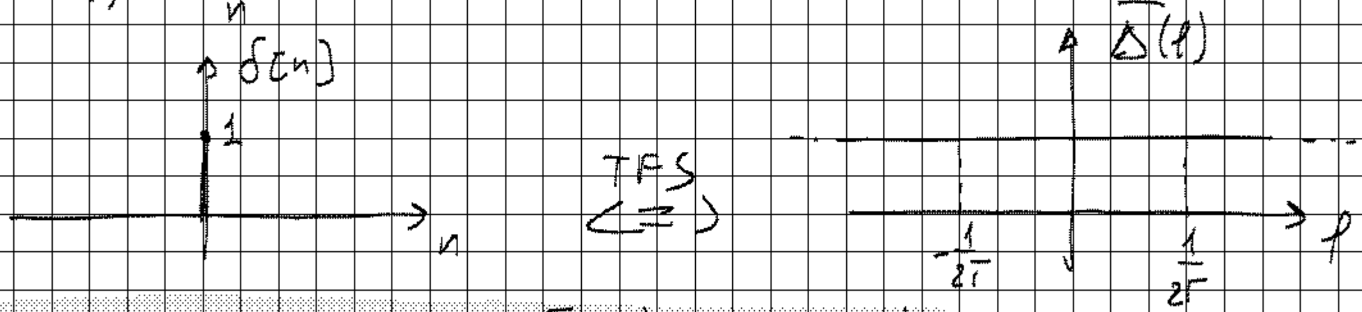
ESEMPIO – TFS DI UNA $\delta[n]$

$$\overline{\Delta}(f) = \sum_n \delta[n]\, e^{-j2\pi n f T} = 1$$



RELAZIONE TRA $\overline{X}(f)$ e $X(f)$

$x[n] = x(nt)$ ottenuta per campionamento di $x(t)$ con intervallo di campionamento T

$$x(t) \overset{TCF}{\Longleftrightarrow} X(f) \qquad , \qquad x[n] \overset{TFS}{\Longleftrightarrow} \overline{X}(f)$$

$$\overline{X}(f) = \frac{1}{T} \sum_n X\left(f - \frac{n}{T}\right)$$

Dimostrazione:

$$\overline{X}(f) = \sum_n x[n]\, e^{-j2\pi n f T} = \sum_n x(nT)\, e^{-j2\pi n f T} =$$

$$= \sum_n \int_{-\infty}^{+\infty} X(\alpha)\, e^{j2\pi \alpha n T}\, d\alpha \; e^{-j2\pi n f T} =$$

$$= \int_{-\infty}^{+\infty} X(\alpha) \sum_n e^{-j2\pi (f-\alpha) nT}\, d\alpha =$$

$$= \int_{-\infty}^{+\infty} X(\alpha)\, \frac{1}{T} \sum_n \delta\left(f - \alpha - \frac{n}{T}\right) d\alpha =$$

$$= \frac{1}{T} \sum_n \int_{-\infty}^{+\infty} X(\alpha)\, \delta\left[\left(f - \frac{n}{T}\right) - \alpha\right] d\alpha =$$

$$= \frac{1}{T} \sum_n X\left(f - \frac{n}{T}\right)$$