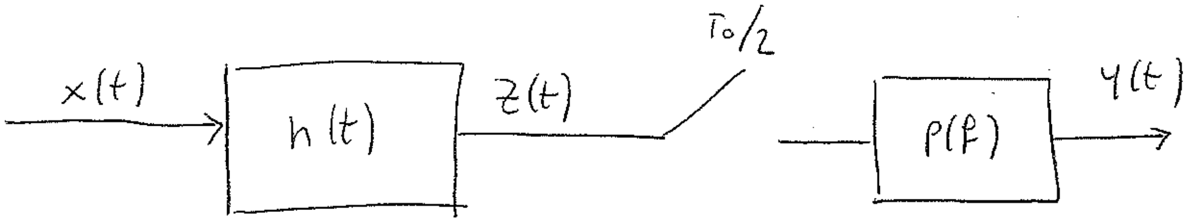
Esercizio 1

$$x(t) = \sum_{n=-\infty}^{+\infty} x_0(t - nT_0)$$

$$x_0(t) = \left(1 - \frac{8|t|}{T_0}\right) \mathrm{rect}\left(\frac{4t}{T_0}\right)$$

$$X_0(f) = \frac{T_0}{8} \,\mathrm{sinc}^2\left(f \frac{T_0}{8}\right)$$

$$X(f) = \frac{1}{8} \sum_k \mathrm{sinc}^2\left(\frac{k}{8}\right) \delta\left(f - \frac{k}{T_0}\right)$$



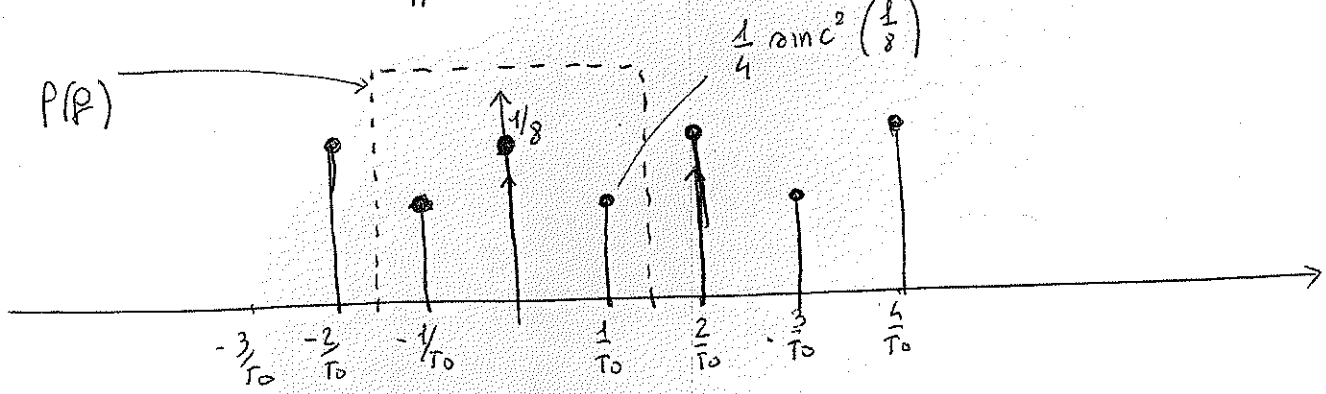
$$H(f) = \mathrm{rect}\left(\frac{f\, T_0}{3}\right)$$

$$Z(f) = H(f) \cdot X(f)$$

$$Z(f) = \frac{1}{8}\delta(f) + \frac{1}{8}\mathrm{sinc}^2\left(\frac{1}{8}\right)\delta\left(f - \frac{1}{T_0}\right) + \frac{1}{8}\mathrm{sinc}^2\left(\frac{1}{8}\right)\delta\left(f + \frac{1}{T_0}\right)$$

Il segnale $z(t)$ e' poi campionato con passo di campionamento $\frac{T_0}{2}$ quindi lo spettro del segnale campionato sarà

$$\bar{Z}(f) = \frac{2}{T_0} \sum_{n=-\infty}^{+\infty} Z\left(f - \frac{2n}{T_0}\right)$$



$$Y(f) = \bar{Z}(f) \cdot P(f) =$$

$$= \frac{1}{8}\,\delta(f) + \frac{1}{4}\,\mathrm{sinc}^2\left(\frac{1}{8}\right)\left[\delta\left(f - \frac{1}{T_0}\right) + \delta\left(f + \frac{1}{T_0}\right)\right]$$

$$y(t) = \frac{1}{8} + \frac{1}{2}\,\mathrm{sinc}^2\left(\frac{1}{8}\right)\cos\left(2\pi \frac{t}{T_0}\right)$$

$$P_y = \int_{-\infty}^{+\infty} |y(t)|^2\,dt = \int_{-\infty}^{+\infty} |Y(f)|^2\,df = \frac{1}{64} + 2\cdot\frac{1}{16}\,\mathrm{sinc}^2\left(\frac{1}{8}\right) =$$

$$= \frac{1}{64} + \frac{1}{8}\,\mathrm{sinc}^2\left(\frac{1}{8}\right)$$

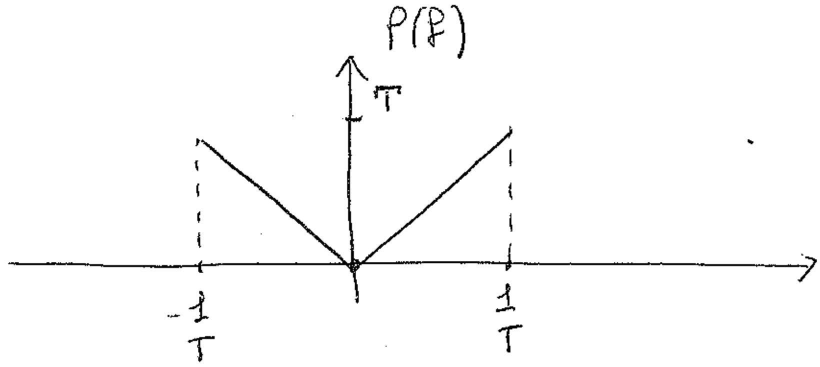
$$E_y = \infty$$

Esercizio 2

1) $E_s = 4 \cdot \frac{1}{2} \cdot E_p = 2 \cdot E_p$

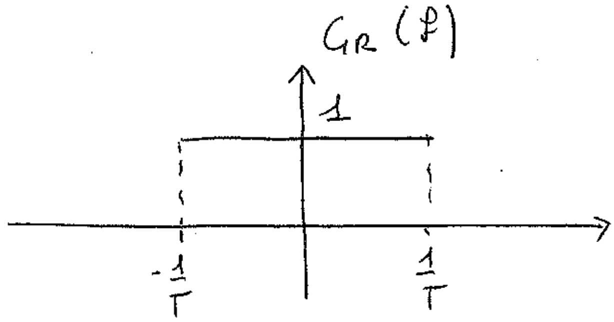
Dove E_p e' l'energia dell'impulso sagomatore

$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2 \, df = \frac{2T}{3}$$



$$E_s = \frac{4T}{3}$$

2) $P_n = \frac{N_0}{2} \int_{-\infty}^{+\infty} |G_R(f)|^2 \, df = \frac{N_0}{T}$



3) La DSP del segnale PAM trasmesso e' definita come

$$S_s(f) = \frac{1}{T} S_x(f) \, |P(f)|^2$$

La funzione di autocorrelazione dei simboli si ricava come segue:

$$R_x(m) = E\left\{ x_i \cdot x_{i+m} \right\} = \begin{cases} E\{x_i^2\} & m = 0 \\ E\{x_i\}^2 & m \neq 0 \end{cases}$$

essendo i simboli indipendenti

$$R_x(m) = \begin{cases} 2 & m = 0 \\ 1 & m \neq 0 \end{cases}$$

$$R_x(m) = \delta(m) + 1$$

Quindi $S_x(f)$ si ottiene trasformando la $R_x(m)$

$$S_x(f) = 1 + \frac{1}{T}\delta(f)$$

$$S_s(f) = \frac{1}{T}\left[1 + \frac{1}{T}\delta(f)\right]\left|T(fT)\,\mathrm{rect}\left(fT/2\right)\right|^2$$

4) $G(f) = P(f) \cdot G_R(f) = P(f)$

$G(f)$ puo' essere riscritto per convenienza nel modo seguente

$$G(f) = \left[T - T\left(1 - |f|T\right)\right] \mathrm{rect}\left(\frac{fT}{2}\right)$$

Antitrasformando si ottiene $g(t)$ che deve rispettare la condizione di Nyquist negli istanti di campionamento.

$$g(t) = 2\,\mathrm{sinc}\left(\frac{2t}{T}\right) - \mathrm{sinc}^2\left(\frac{t}{T}\right)$$

$$g(kT) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

5) $y_k = g(0)\, x_k + n_k = x_k + n_k$

$n_k \in \mathcal{N}(0, P_n)$

$$P_e = \frac{1}{2} Q\left(\sqrt{\frac{T}{4N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{9T}{4N_0}}\right)$$