Sol. TESTO 20/09/2007

$$X(f) = 2A\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) \otimes \frac{1}{2}\left[\delta(f-B)+\delta(f+B)\right] =$$

$$= A\left(1-\frac{|f-B|}{B}\right)\mathrm{rect}\left(\frac{f-B}{2B}\right) + A\left(1-\frac{|f+B|}{B}\right)\mathrm{rect}\left(\frac{f+B}{2B}\right)$$

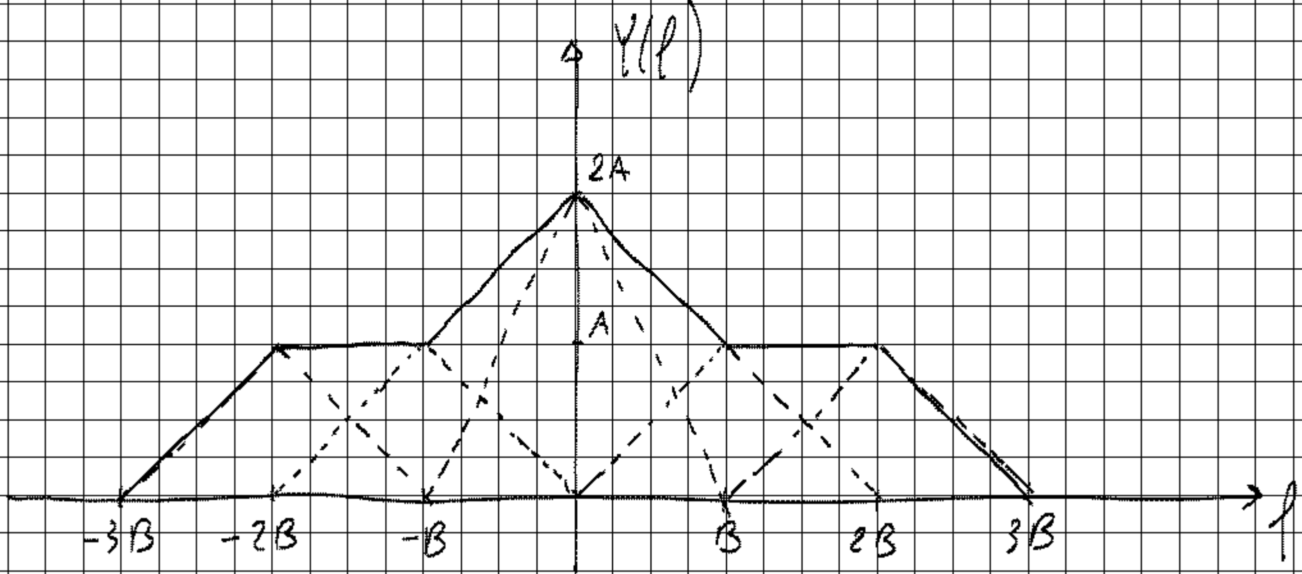
$$H(f) = \mathrm{rect}\left(\frac{f}{4B}\right)$$

$$P(f) = \mathrm{rect}\left(\frac{f}{B}\right)$$

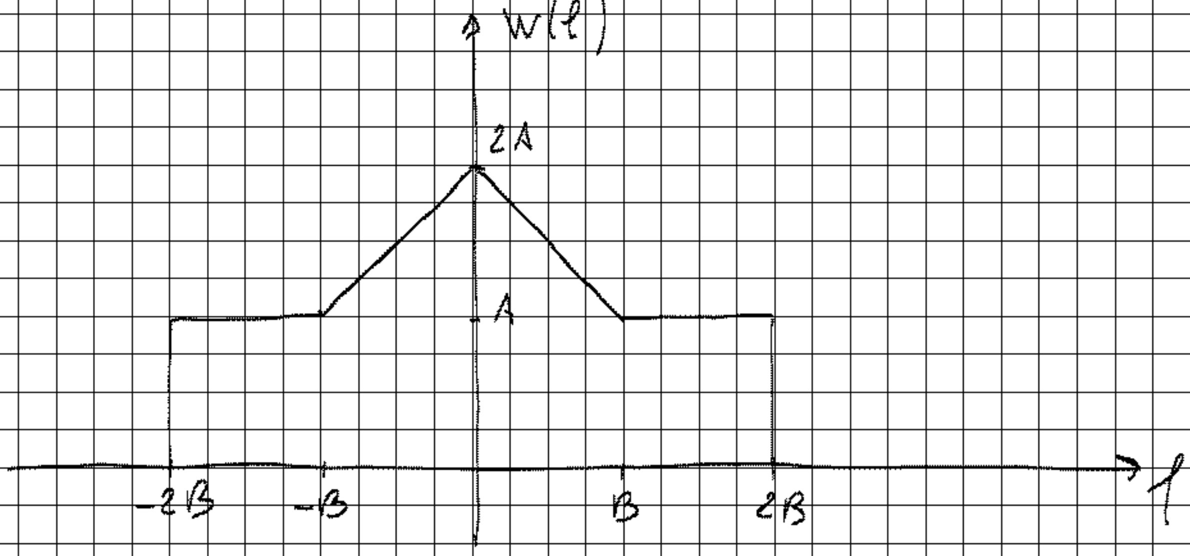
1) $v(t) = x(t)\cdot 2\cos(2\pi B t)$

$$V(f) = X(f)\otimes\left[\delta(f-B)+\delta(f+B)\right]$$

$$Y(f) = X(f) + V(f)$$



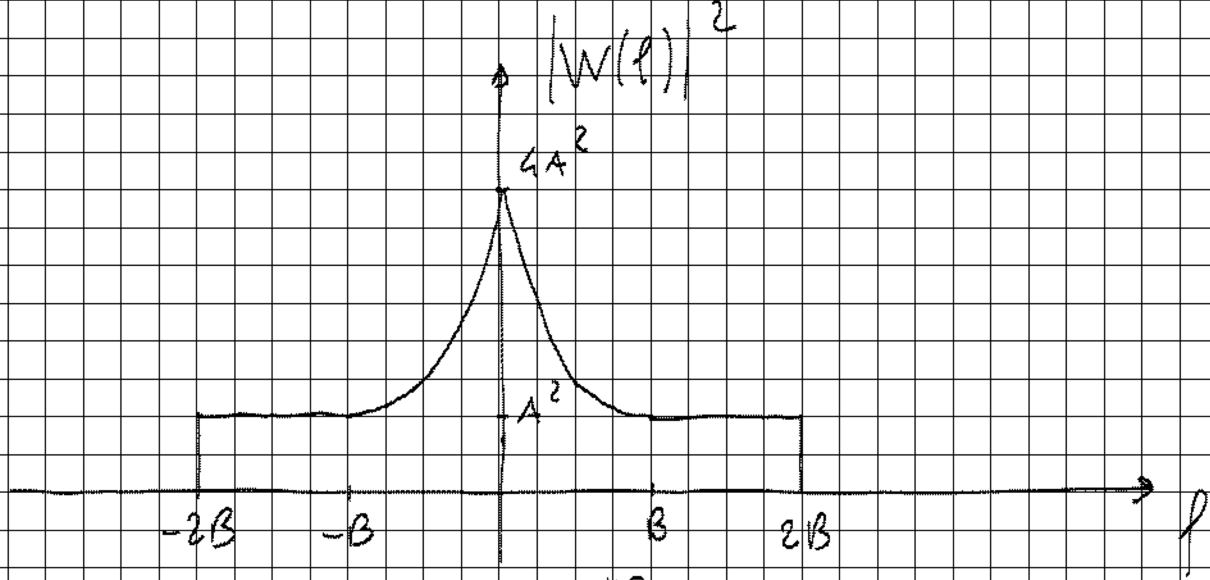
2) $W(f) = Y(f)\,H(f)$



$$W(f) = A\,\mathrm{rect}\left(\frac{f}{4B}\right) + A\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$w(t) = 4AB\,\mathrm{sinc}(4Bt) + AB\,\mathrm{sinc}^2(Bt)$$

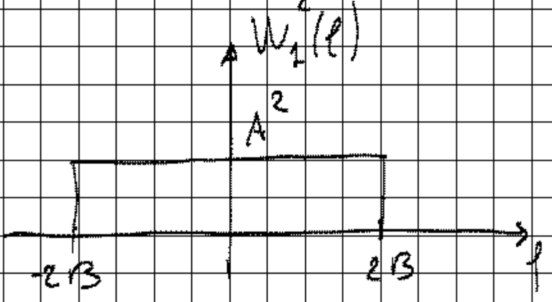
B) $$E_w = \int_{-\infty}^{+\infty} |w(t)|^2\,dt = \int_{-\infty}^{+\infty} |W(f)|^2\,df$$

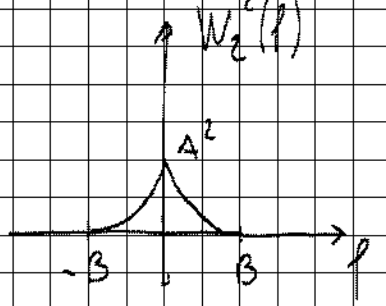


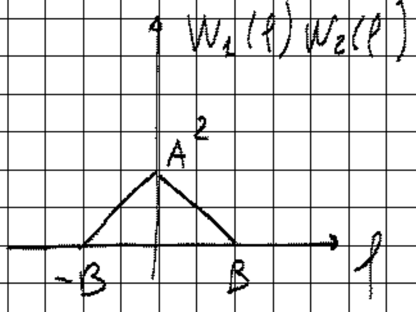
$$E_w = \int_{-\infty}^{+\infty} |W_1(f) + W_2(f)|^2\,df = \int_{-\infty}^{+\infty} (W_1(f) + W_2(f))^2\,df$$

$$W_1(f) = A\,\mathrm{rect}\left(\frac{f}{4B}\right), \quad W_2(f) = A\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$E_w = \int_{-\infty}^{+\infty} W_1(f)^2 \, df + \int_{-\infty}^{+\infty} |W_2(f)|^2 \, df + 2\int_{-\infty}^{+\infty} W_1(f) W_2(f) \, df$$





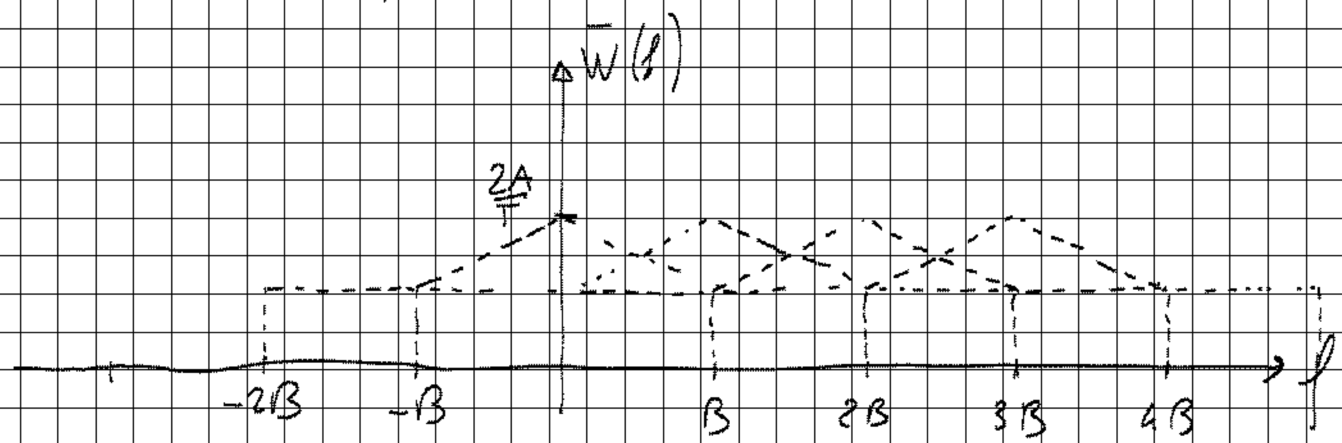


$$E_w = 4A^2B + \frac{2}{3}A^2B + 2A^2B = \frac{20}{3}A^2B$$

4) $w[n]$ non è campionata alla frequenza di Nyquist

$$T > \frac{1}{4B}$$

5) $$\bar{W}(f) = \frac{1}{T}\sum_n W\left(f - \frac{n}{T}\right)$$



$$\bar{W}(f) = \frac{1}{T}\left[\sum_n W_1\left(f - \frac{n}{T}\right) + \sum_n W_2\left(f - \frac{n}{T}\right)\right]$$

$$W(f) = W_1(f) + W_2(f)$$

$$W_1(f) = A \,\mathrm{rect}\left(\frac{f}{4B}\right)$$

$$W_2(f) = A\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$\bar{W}_1(f) = \frac{1}{T}\sum_n W_1\left(f - \frac{n}{T}\right) = \frac{4A}{T} = 4AB$$

$$\bar{W}_2(f) = \frac{1}{T}\sum_n W_2\left(f - \frac{n}{T}\right) = \frac{A}{T} = AB$$

$$\bar{W}(f) = 5AB$$

$$w(n) = 5AB\,\delta[n]$$

$$z(t) = \sum_n w[n]\,p(t - nT) = 5AB\,p(t) = 5AB^2 \operatorname{sinc}(Bt)$$

N.B. $w[n] = w(nT) = w\left(\frac{n}{B}\right) = 4AB \operatorname{sinc}(4n) + AB \operatorname{sinc}^2(n)$

$$= 5AB\,\delta[n]$$