Esame 10-06-2010 ⑫

Esercizio 1

$y(t) = \sum_n x(t - nT) \qquad x(t) = \text{rect}\left(\frac{2t}{3T}\right) = \text{rect}\left(\frac{t}{3T/2}\right)$

$y(t) = \sum_n \text{rect}\left(\frac{t - nT}{3T/2}\right)$

$Y_k = \frac{1}{T} X\left(\frac{k}{T}\right); \quad X(f) = \frac{3T}{2} \text{sinc}\left(\frac{3T}{2} f\right)$

$Y_k = \frac{1}{T} \cdot \frac{3T}{2} \text{sinc}\left(\frac{1}{T} \cdot \frac{3T}{2} k\right) = \frac{3}{2} \text{sinc}\left(\frac{3}{2} k\right)$

$P_y = \sum_k |Y_k|^2 = \sum_k \left|\frac{3}{2} \text{sinc}\left(\frac{3}{2}k\right)\right|^2 = \sum_k \frac{9}{4} \text{sinc}^2\left(\frac{3}{2}k\right)$

Esercizio 2

$r(t) = \sum_i a_i\, g_T(t - iT) \cos(2\pi f_0 t) - \sum_i b_i\, g_T(t - iT) \,\text{sen}(2\pi f_0 t) + w(t) =$

$= \sum_i \cos(\alpha_i)\, g_T(t - iT) \cos(2\pi f_0 t) - \sum_i \text{sen}(\alpha_i)\, g_T(t - iT)\, \text{sen}(2\pi f_0 t) + w(t) =$

$= \sum_i g_T(t - iT) \cos(2\pi f_0 t + \alpha_i) + w(t)$

$c_i = a_i + j b_i = \cos(\alpha_i) + j\, \text{sen}(\alpha_i) = e^{j\alpha_i} \in C_i\ [m_0, m_1, m_2, m_3]$

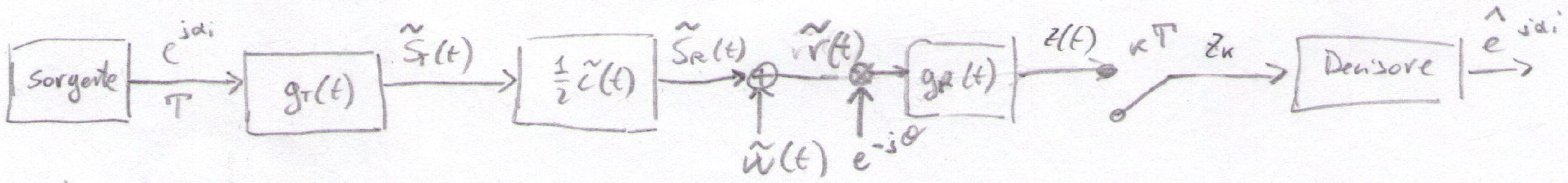
$\tilde{S}_T(t) = \sum_i c_i\, g_T(t - iT) = \sum_i e^{j\alpha_i} g_T(t - iT)$

$\bar{E}_T = \sum_{k=0}^{3} P_k E_k; \quad P_k = \frac{1}{4}; \quad E_k = \int_{-\infty}^{\infty} |S_T(t)|^2 dt = \int_{-\infty}^{\infty} |g_T(t) \cos(2\pi f_0 t + m_k)|^2 dt$

$E_k = \int_{-\infty}^{\infty} g_T^2(t) \cos^2(2\pi f_0 t + m_k)\, dt = \frac{1}{2}\int_{-\infty}^{\infty} g_T^2(t)\, dt + \frac{1}{2}\int_{-\infty}^{\infty} g_T^2(t) \cos(4\pi f_0 t + 2 m_k)\, dt =$

$= \frac{1}{2} E_{g_T} + \frac{1}{2} \underbrace{\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\frac{t + \frac{T}{2}}{T}\right)^2 \cos(4\pi f_0 t + 2 m_k)\, dt}_{\simeq 0} = \frac{1}{2} E_{g_T} \quad \forall\, m_k$

$\bar{E}_T = \sum_{k=0}^{3} \frac{1}{4} \cdot \frac{1}{2} E_{g_T} = \frac{1}{8} \cdot 4 E_{g_T} = \frac{1}{2} E_{g_T}$ ①



②

$g(t) = g_T(t) \otimes c(t) \otimes g_R(t) = g_{TC}(t) \otimes g_R(t)$

$= \left(\frac{t + T/2}{T}\right) \text{rect}\left(\frac{t}{T}\right) \otimes -A\left(\frac{t - T/2}{T}\right) \text{rect}\left(\frac{t}{T}\right) =$

$g(t) = \int_{-\infty}^{\infty} g_{TC}(\tau)\, g_R(t - \tau)\, d\tau \Rightarrow g(0) = \int_{-\infty}^{\infty} g_{TC}(\tau)\, g_R(-\tau)\, d\tau =$

$= \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\frac{\tau + T/2}{T}\right) \cdot -A\left(\frac{-\tau + \frac{T}{2}}{T}\right) d\tau = -A \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\frac{-\tau^2}{T^2} + \frac{1}{2}\right) d\tau = +\frac{A}{T^2}\int_{-\frac{T}{2}}^{\frac{T}{2}} \tau^2\, d\tau - \frac{A}{2}\int_{-\frac{T}{2}}^{\frac{T}{2}} 1\, d\tau =$

$= \frac{A}{T^2} \cdot \frac{T^3}{12} - \frac{A}{2} T = A\left(\frac{1 - 6T}{12}\right) \Rightarrow A = \frac{12}{1 - 6T}$

$$n_c(t) = w_c(t)\, 2\cos(2\pi f_0 t) \otimes g_R(t)$$

$$P_{n_c} = \int_{-\infty}^{\infty} S_{n_c}(f)\, df \;; \quad S_{n_c}(f) = S_{w_c}(f)\, |G_R(f)|^2$$

$$S_{w_c}(f) = \frac{S_{\tilde{w}}(f) + S_{\tilde{w}}(-f)}{4} \;; \quad S_{\tilde{w}}(f) = \begin{cases} 4 S_w(f+f_0) & f \geq f_0 \\ \emptyset & \text{altrove} \end{cases} = 2N_0 \operatorname{rect}\left(\frac{f}{B}\right)$$

$$S_{w_c}(f) = N_0 \operatorname{rect}\left(\frac{f}{B}\right) \;; \quad S_{n_c}(f) = N_0 \operatorname{rect}\left(\frac{f}{B}\right) |G_R(f)|^2$$

$$P_{n_c} = \int_{-\infty}^{\infty} N_0 \operatorname{rect}\left(\frac{f}{B}\right) |G_R(f)|^2\, df = N_0 \int_{-\frac{B}{2}}^{\frac{B}{2}} |G_R(f)|^2\, df = N_0 \int_{-\frac{B}{2}}^{\frac{B}{2}} |g_R(t)|^2\, dt = N_0 \int_{-\frac{B}{2}}^{\frac{B}{2}} \left| A\left(\frac{t-\frac{T}{2}}{T}\right) \operatorname{rect}\left(\frac{t}{T}\right) \right|^2 dt$$

$$= N_0 \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \left(\frac{t^2}{T^2} + \frac{1}{4} - \frac{t}{T}\right) dt = N_0 \frac{A^2}{T^2} \left[\frac{t^3}{3}\right]_{-\frac{T}{2}}^{\frac{T}{2}} + N_0 \frac{A^2}{4} \left[t\right]_{-\frac{T}{2}}^{\frac{T}{2}} - N_0 \frac{A^2}{T} \left[\frac{t^2}{2}\right]_{-\frac{T}{2}}^{\frac{T}{2}} =$$

$$= N_0 \frac{A^2}{T^2} \left(\frac{T^3}{24} + \frac{T^3}{24}\right) + N_0 \frac{A^2}{4} \left(\frac{T}{2} + \frac{T}{2}\right) - N_0 \frac{A^2}{T} \left(\frac{T^2}{8} + \frac{T^2}{8}\right) =$$

$$= \frac{N_0 A^2 T}{12} + \cancel{\frac{N_0 A^2 T}{4}} - \cancel{\frac{N_0 A^2 T}{4}} = \frac{N_0 A^2 T}{12} = P_{n_s} \quad 3$$

$$P(e) : z(t) = \sum_i e^{j\alpha_i} g(t - iT) + \tilde{n}(t) \;; \quad z_k = e^{j\alpha_k} + \tilde{n}_k \;; \quad \tilde{n}_k = \tilde{n}(t_k) \in \mathcal{N}(\emptyset, \sigma_{\tilde{n}}^2)$$

$$P(e) = 1 - P(c) \;; \quad P(c)\Big|_{z_k = e^{j\frac{\pi}{4}}} = P_r\{z_R > \emptyset\} \cdot P_r\{z_I > \emptyset\} = P_r\{\cos(\tfrac{\pi}{4}) > \emptyset\} \cdot P_r\{\operatorname{sen}(\tfrac{\pi}{4}) > \emptyset\}$$

$$P(c)\Big|_{z_k = e^{j\frac{\pi}{4}}} = 2 P_r\left\{\frac{\sqrt{2}}{2} > \emptyset\right\}$$