RELAZIONE TRA TCF e TSF

PERIODOCIZZAZIONE

$$y(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0) \qquad T_0 = \text{periodo di } y(t)$$

$$y(t) = y(t - kT_0) \qquad \text{periodico}$$

Dimostrazione

$$y(t-kT_0) = \sum_{n=-\infty}^{+\infty} x(t - kT_0 - nT_0) = \sum_{n=-\infty}^{+\infty} x[t-(k+n)T_0]$$

$$= (k+n = n') = \sum_{n'=-\infty}^{+\infty} x(t - n'T_0) = y(t)$$

$$y(t) \overset{TSF}{\Longleftrightarrow} Y_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right)$$

$$Y_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \sum_{k=-\infty}^{+\infty} x(t-kT_0)\, e^{-j2\pi n f_0 t}\, dt =$$

$$= \frac{1}{T_0} \sum_{k=-\infty}^{+\infty} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t-kT_0)\, e^{-j2\pi n f_0 t}\, dt = (t-kT_0 = \alpha)$$

$$= \frac{1}{T_0} \sum_{k=-\infty}^{+\infty} \int_{-\frac{T_0}{2}-kT_0}^{\frac{T_0}{2}-kT_0} x(\alpha)\, e^{-j2\pi n f_0 \alpha}\, d\alpha\; \underbrace{e^{-j2\pi n f_0 k T_0}}_{1}$$

$$= \frac{1}{T_0} \int_{-\infty}^{+\infty} x(\alpha)\, e^{-j2\pi n f_0 \alpha}\, d\alpha = \frac{1}{T_0} X\left(\frac{n}{T_0}\right)$$