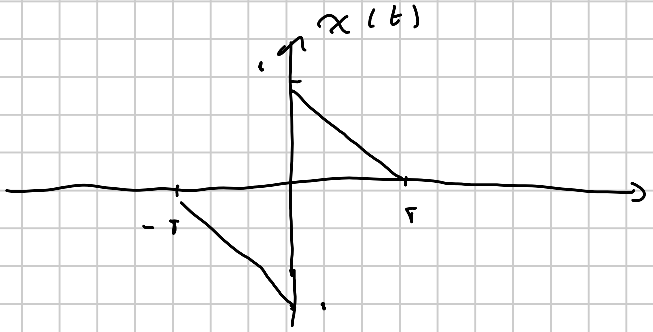
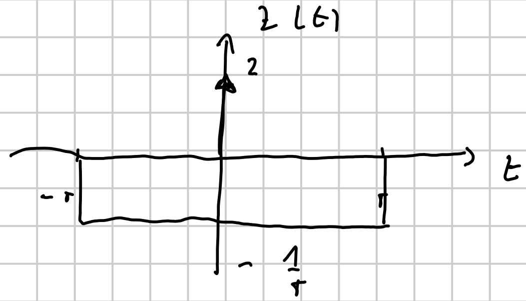
ESERCIZIO 1



$Y(f)$?

Y_k ?

$$y(t) = \sum_n x(t - 4Tn)$$



. TEO DERIVAZIONE

$$z(t) = \frac{d}{dt} x(t)$$

$$x(t) = \int_{-\infty}^{t} z(t)\, dt$$

$$z(t) = -\frac{1}{T} \operatorname{rect}\left(\frac{t}{2T}\right) + 2\delta(t)$$

$$Z(f) = -\frac{1}{\cancel{T}} 2\cancel{T} \operatorname{sinc}(f 2T) + 2$$

$$Z(0) = 0$$

$$Z(f) = j2\pi f X(f) \Rightarrow X(f) = \frac{Z(f)}{j2\pi f}$$

$$X(f) = \frac{2\left[1 - \operatorname{sinc}(f2T)\right]}{j2\pi f}$$

$$\boxed{T_0 = 4T}$$

$$Y(f) = \frac{1}{\cancel{2}4T} \sum_k \frac{\cancel{2}\left(1 - \operatorname{sinc}\left(\frac{k}{4T}2T\right)\right)}{j2\pi \frac{k}{4T}} \delta\left(f - \frac{k}{4T}\right)$$

$$Y_k = \frac{1 - \operatorname{sinc}\left(\frac{k}{2}\right)}{j\pi k}$$

ESERCIZIO 2

$$s(t) = \text{sinc}\left(2Bt - \frac{1}{2}\right) + \text{sinc}\left(2Bt + \frac{1}{2}\right)$$

$$x(t) = s(t)\sin(2\pi Bt) \qquad E_x, P_x ?$$

$$y(t) = \sum_k s\left(t - \frac{2k}{B}\right) \qquad E_y, P_y ?$$

$$s(t) = \text{sinc}\Big(2B\big(t - \underbrace{\tfrac{1}{4B}}_{t_0}\big)\Big) + \text{sinc}\Big(2B\big(t + \underbrace{\tfrac{1}{4B}}_{t_0}\big)\Big)$$

$$S(f) = \frac{1}{2B}\,\text{rect}\left(\frac{f}{2B}\right)e^{-j2\pi f t_0} + \frac{1}{2B}\,\text{rect}\left(\frac{f}{2B}\right)e^{+j2\pi f t_0} =$$

$$= \frac{1}{B}\,\text{rect}\left(\frac{f}{2B}\right)\left(\frac{e^{-j2\pi f t_0} + e^{+j2\pi f t_0}}{2}\right) =$$

$$= \frac{1}{B}\,\text{rect}\,\frac{f}{2B}\,\cos(2\pi f t_0)$$

$$X(f) = \frac{1}{2j}\left[S(f - f_0) - S(f + f_0)\right] =$$

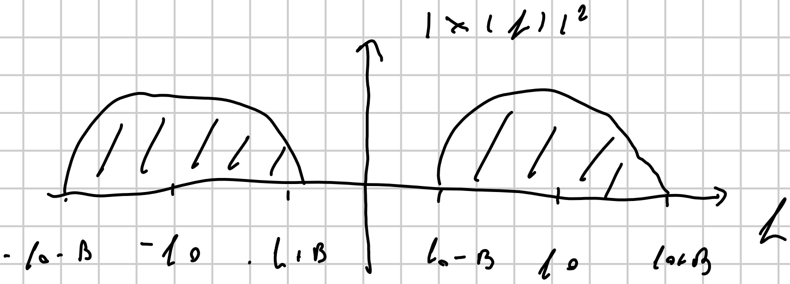
$$= \frac{1}{2jB}\left[\text{rect}\left(\frac{f - f_0}{2B}\right)\cos\left(\pi\frac{(f - f_0)}{2B}\right) + \right.$$

$$\left. - \text{rect}\left(\frac{f + f_0}{2B}\right)\cos\left(\pi\frac{(f + f_0)}{2B}\right)\right] =$$

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2\,dt = \int_{-\infty}^{+\infty} |X(f)|^2\,df =$$

$$|X(f)|^2 = \frac{1}{4B^2}\left[\text{rect}\left(\frac{f - f_0}{2B}\right)\cos^2\left(\pi\frac{(f - f_0)}{2B}\right) + \right.$$

$$\left. + \text{rect}\left(\frac{f + f_0}{2B}\right)\cos^2\left(\pi\frac{(f + f_0)}{2B}\right)\right]$$



$$E_x = \frac{2}{4B^2}\int_{f_0-B}^{f_0+B} \cos^2\left(\frac{\pi (f-f_0)}{2B}\right) df =$$

$$f' = f - f_0$$

$$= \frac{2}{4B^2}\int_{-B}^{B} \cos^2\left(\frac{\pi f'}{2B}\right) df' =$$

$$= \frac{2}{4B^2}\int_{-B}^{B} \left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi f'}{B}\right)\right] df' = \frac{1}{2B^2} \qquad B = \frac{1}{2B}$$

$$P_x = 0$$

$$y(t) = \sum_k x\left(t - \frac{2k}{B}\right) \qquad T_0 = \frac{2}{B}$$

$$E_y = \infty$$

$$P_y = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |y(t)|^2 dt = \sum_k |Y_k|^2$$

$$Y_k = \frac{1}{T_0} S\left(\frac{k}{T_0}\right) = \frac{B}{2} S\left(k\frac{B}{2}\right) =$$

$$= \frac{B}{2}\frac{1}{B} \operatorname{rect}\left(\frac{kB/2}{2B}\right) \cos\left(\pi\frac{B/2}{2B}\right) =$$

$$= \frac{1}{2}\operatorname{rect}\left(\frac{k}{4}\right) \cos\left(\frac{k\pi}{4}\right)$$

$$Y_k = \begin{cases} \frac{1}{2} & k=0 \\ \frac{\sqrt{2}}{4} & k=\pm 1 \\ 0 & \text{altrove} \end{cases} \qquad P_y = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$$

ESERCIZIO 3 — AM CON FASE

$$y(t) = x(t)\cos(2\pi f_0 t + \varphi)$$

$$x(t) = AB\,\mathrm{sinc}^2(Bt)$$

$$Y(f) = A\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) \otimes \frac{1}{2}\left[e^{j\varphi}\delta(f-f_0) + e^{-j\varphi}\delta(f+f_0)\right]$$

$$= \frac{A}{2}e^{j\varphi}\left(1 - \frac{|f-f_0|}{B}\right)\mathrm{rect}\left(\frac{f-f_0}{2B}\right) +$$

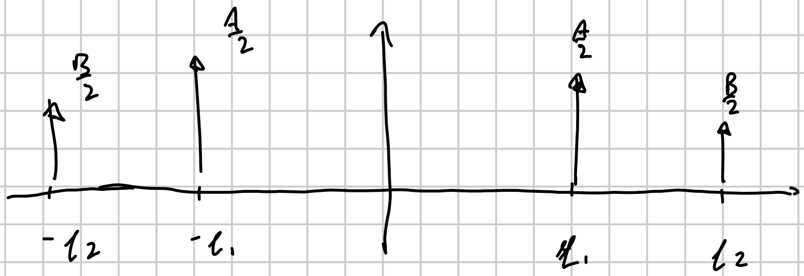
$$+ \frac{A}{2}e^{-j\varphi}\left(1 - \frac{|f+f_0|}{B}\right)\mathrm{rect}\left(\frac{f+f_0}{2B}\right)$$

ESERCIZIO 4

$$x(t) = A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)$$

$$f_1/f_2 = k \in \mathbb{R} \text{ (non intero)}$$

$$X(f) = \frac{A}{2}\left[\delta(f-f_1) + \delta(f+f_1)\right] + \frac{B}{2}\left[\delta(f-f_2) + \delta(f+f_2)\right]$$



SISTEMI

ESERCIZIO 1

$$y(t) = \int_{\tau}^{t} x(\alpha)\, d\alpha$$

1) LINEARITÀ ?
2) STAZIONARIETÀ ?
3) STABILITÀ BIBO ?
4) ISTANTANEITÀ ?

1) $x(t) = a\, x_1(t) + b\, x_2(t)$

$$y(t) = T[x(t)] = a\, T[x_1(t)] + b\, T[x_2(t)]$$

$$y(t) = \int_{\tau}^{t} [a\, x_1(t) + b\, x_2(t)]\, dt =$$

$$= a \int_{\tau}^{t} x_1(t)\, dt + b \int_{\tau}^{t} x_2(t)\, dt =$$

$$= a\, T[x_1(t)] + b\, T[x_2(t)]$$

LINEARE

2) $y(t - t_0) = T[x(t - t_0)]$

$$\int_{\tau}^{t} x(\alpha - t_0)\, d\alpha = \int_{\tau - t_0}^{t - t_0} x(\beta)\, d\beta =$$

$$= \underbrace{\int_{\tau - t_0}^{\tau} x(\beta)\, d\beta}_{\neq} + \underbrace{\int_{\tau}^{t - t_0} x(\beta)\, d\beta}_{y(t - t_0)} =$$

NO STAZIONARIO

3) STABILITA' BIBO

$|x(t)| \le M \qquad |y(t)| \le K$

$x(\alpha) = M$

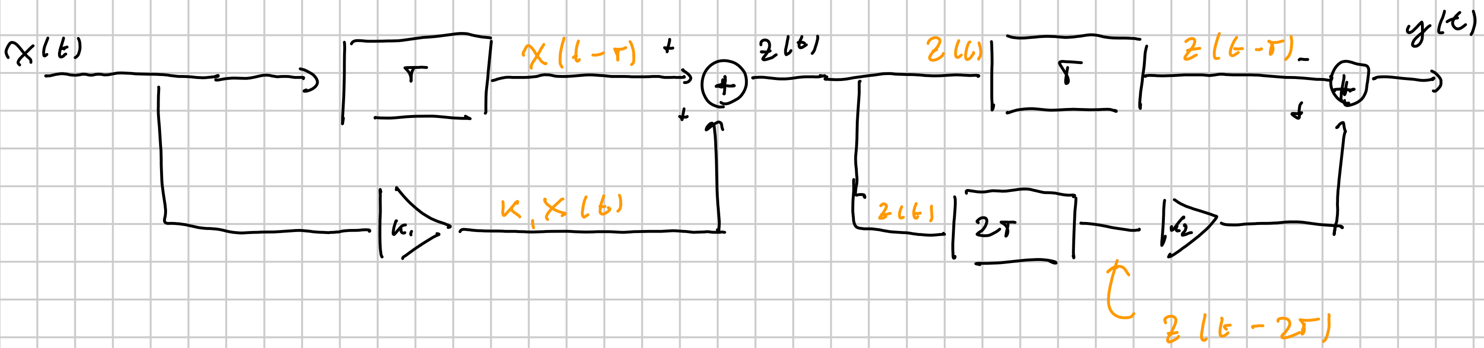
$$\int_{\tau}^{t} x(\alpha)\, d\alpha = \int_{\tau}^{t} M \, d\alpha = M(t-\tau) = y(t)$$

NO STABILE

4) ISTANTANEITA'

$y(t) = \int_{\tau}^{t} x(\alpha)\, d\alpha$ E' CON MEMORIA

ESERCIZIO 2



$z(t) = x(t-\tau) + k_1 x(t)$

$y(t) = -z(t-\tau) + k_2 z(t-2\tau)$

$y(t) = -\left(x(t-2\tau) + k_1 x(t-\tau)\right) + k_2\left(x(t-3\tau) + k_1 x(t-2\tau)\right) =$

$= -k_1 x(t-2\tau) + x(t-2\tau)(k_1 k_2 - 1) + k_2 x(t-3\tau)$

LINEARITA'

$x(t) = a\, x_1(t) + b\, x_2(t)$

$$y(t) = -k_1\left(a x_1(t-T) + b x_2(t-T)\right) +$$

$$+\left(a x_1(t-2T) + b x_2(t-2T)\right)(k_1 k_2 - 1) +$$

$$+ k_2\left(a x_1(t-3T)\right) + b x_2(t-3T) = \dots$$

2) STAZIONARIETÀ

$$x(t-t_0) \Rightarrow -k_1 x(t-t_0-T) + x(t-t_0-2T)(k_1 k_2 - 1) +$$

$$+ k_2 x(t-t_0-3T) = y(t-t_0)$$

STAZIONARIO

3) MEMORIA

$y(t)$ dipende da $x(t-T)$, $x(t-2T)$ $x(t-3T)$

4) CAUSALITÀ sì

5) STABILITÀ

$|x(t)| \le M \quad \forall t$

$$|y(t)| = \left| -k_1 x(t-T) + (k_2 k_1 - 1)x(t-2T) + k_2 x(t-3T) \right| \le$$

$$\le |k_1| M + |k_1 k_2 - 1| M + |k_2| M \le +\infty$$

STABILE

$h(t)$ risp. impulsiva

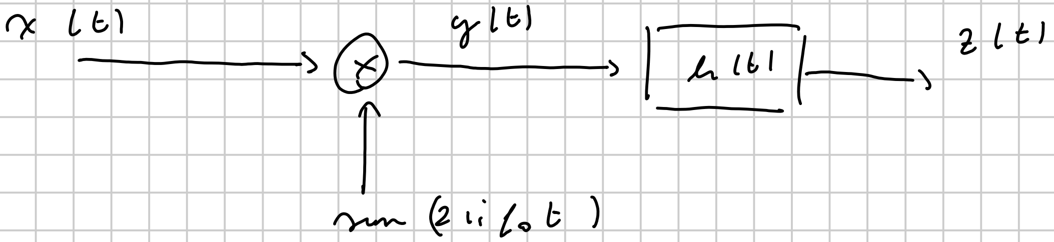
$H(f)$ risp. in frequenza

$x(t) = \delta(t)$

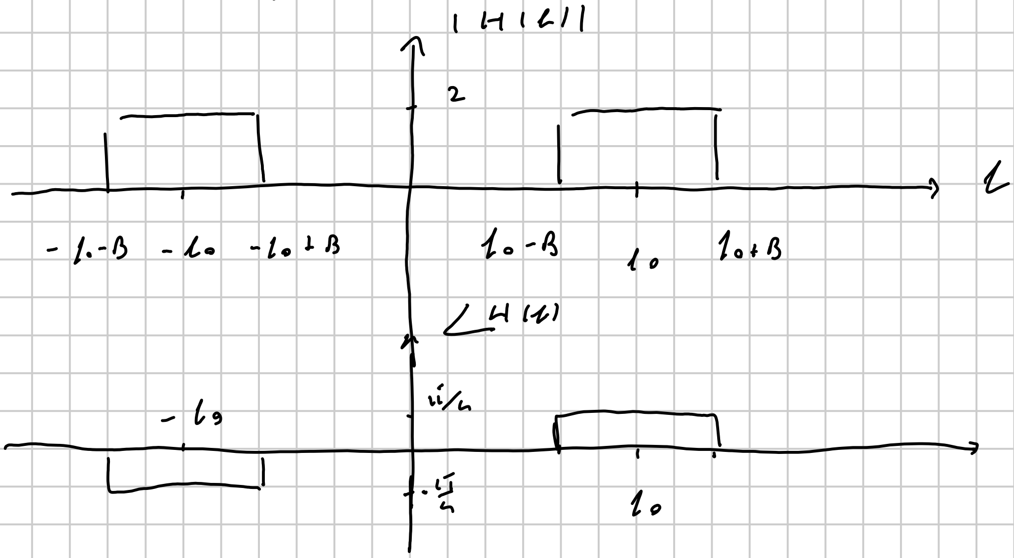
$$y(t) = h(t) = -k_1 \delta(t-T) + (k_2 k_1 - 1)\delta(t-2T) + k_2 \delta(t-3T)$$

$$H(f) = -k_1 e^{-j 2\pi f T} + (k_2 k_1 - 1) e^{-j 4\pi f T} + k_2 e^{-j 6\pi f T}$$

ESERCIZIO 3



$$X(f) = A\left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

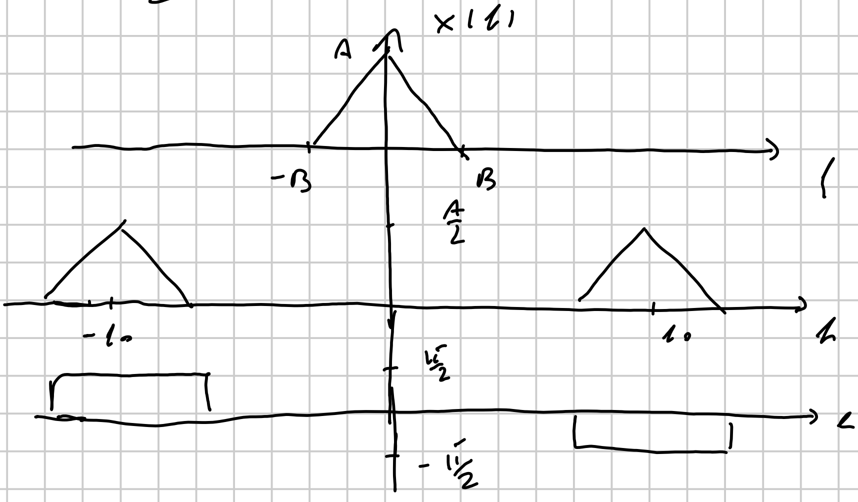


$z(t)$? $\qquad E_z$?

$$y(t) = x(t) \sin(2\pi f_0 t)$$

$$Y(f) = X(f) \otimes \frac{1}{2j}\left[\delta(f-f_0) - \delta(f+f_0)\right] =$$

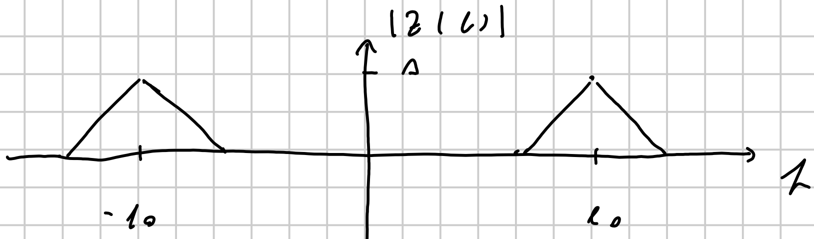
$$= \frac{e^{-j\frac{\pi}{2}}}{2} X(f-f_0) + \frac{e^{j\frac{\pi}{2}}}{2} X(f+f_0)$$

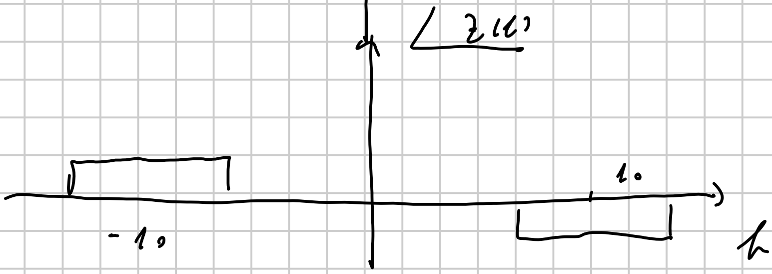


$$Z(f) = Y(f)\, H(f)$$

$$|Z(f)| = |Y(f)|\, |H(f)|$$

$$\angle Z(f) = \angle Y(f) + \angle H(f)$$





$$Z(f) = A\left(1 - \frac{|f - f_0|}{B}\right) \mathrm{rect}\left(\frac{f - f_0}{2B}\right) e^{-j\frac{\pi}{4}} +$$

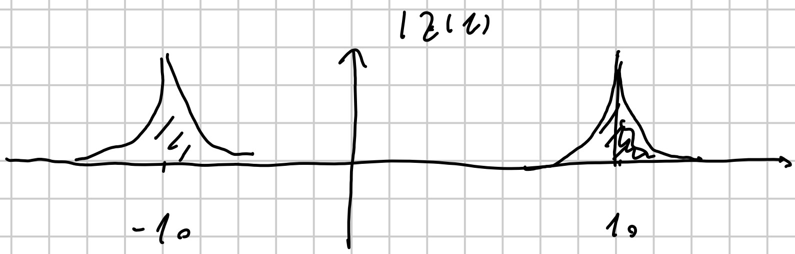
$$+ A\left(1 - \frac{|f + f_0|}{B}\right) \mathrm{rect}\left(\frac{f + f_0}{2B}\right) e^{j\frac{\pi}{4}} =$$

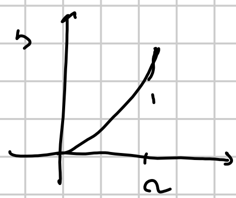
$$= A\left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right) \otimes \left[\delta(f - f_0) e^{-j\frac{\pi}{4}} + \delta(f + f_0) e^{j\frac{\pi}{4}}\right]$$

$$z(t) = AB\, \mathrm{sinc}^2(tB) \left[e^{+j\left(2\pi f_0 t - \frac{\pi}{4}\right)} + e^{-j\left(2\pi f_0 t - \frac{\pi}{4}\right)}\right] =$$

$$= 2AB\, \mathrm{sinc}^2(tB) \cos\left(2\pi f_0 t - \frac{\pi}{4}\right)$$

$$E_z = \int_{-\infty}^{+\infty} |z(t)|^2\, dt = \int_{-\infty}^{+\infty} |Z(f)|^2\, df$$





$$\int_0^a y(t)\, dt = \frac{ab}{3}$$

$$E_z = 4\int_{t_0-B}^{t_0} |z(t)|^2\, dt = 4\,\frac{A^2 B}{3}$$

ESERCIZIO 4

$$x_i(t) \longrightarrow \boxed{h(t)} \longrightarrow y_i(t)$$

$h(t) = \mathrm{sinc}\left(\frac{t}{T}\right)\mathrm{sinc}\left(\frac{t}{2T}\right)$

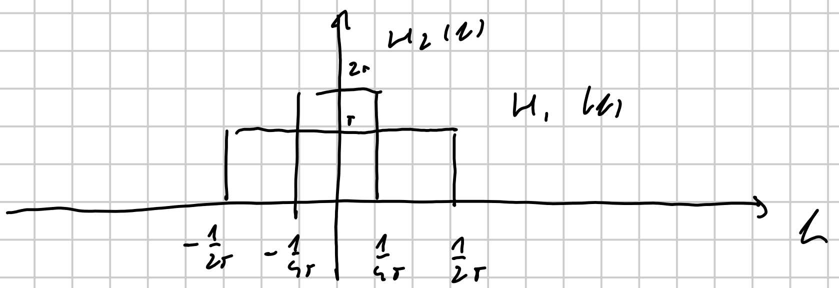
$x_1(t) = e^{-t/T}\, u(t)$

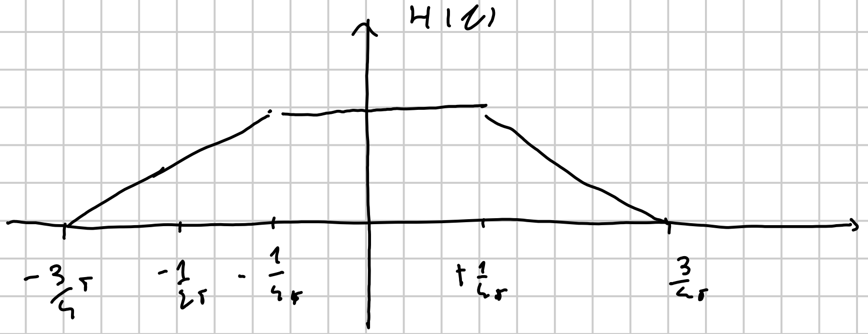
$x_2(t) = \mathrm{sinc}\left(\frac{t-T/2}{3T}\right) + \mathrm{sinc}\left(\frac{t+T/2}{3T}\right)$

$x_3(t) = \mathrm{sinc}^2\left(\frac{t}{T}\right)$

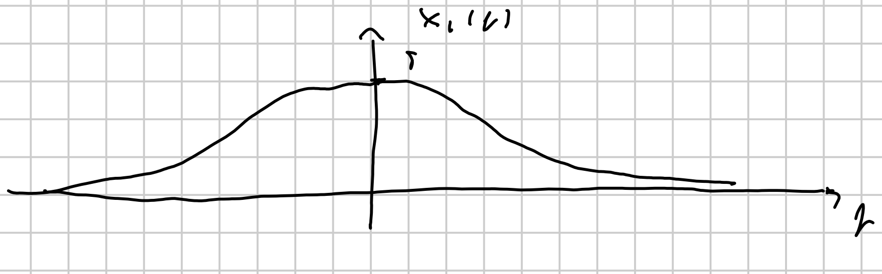
$$H(f) = \underbrace{T\,\mathrm{rect}\left(\frac{f}{1/T}\right)}_{H_1(f)} \otimes \underbrace{2T\,\mathrm{rect}\left(\frac{f}{1/2T}\right)}_{H_2(f)} =$$

$$= H_1(f) \otimes H_2(f)$$





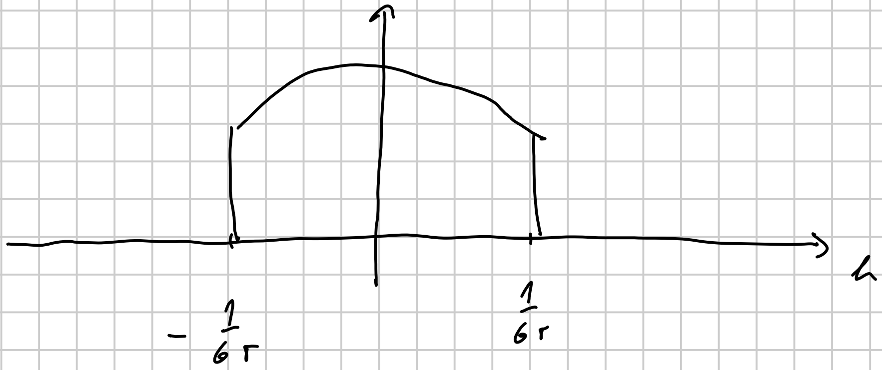
$$X_1(f) = \frac{T}{1+j2\pi f T} \Rightarrow |X_1(f)| = \frac{T}{1+4\pi^2 f^2 T^2}$$



y_1 distorta in ampiezza

$$X_2(f) = 3T \operatorname{rect}\left(\frac{f}{1/3T}\right) e^{-j2\pi f T/2} + 3T \operatorname{rect}\left(\frac{f}{1/3T}\right) e^{j2\pi f T/2} =$$

$$= 6T \operatorname{rect}\left(\frac{f}{1/3T}\right) \cos(\pi f T)$$



NO DISTORSIONE

$$E_{Y_2} = \int_{-\infty}^{+\infty} \left| 6T^2 \operatorname{rect}\left(\frac{f}{1/3T}\right) \cos(\pi f T)^2 \right|^2 df =$$

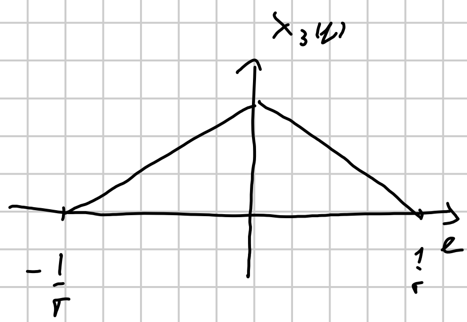
$$= 6T^2 \int_{-\frac{1}{6T}}^{\frac{1}{6T}} \cos^2(\pi f T)\, df = 6T^2 \int_{-\frac{1}{6T}}^{\frac{1}{6T}} \left[\frac{1}{2} + \frac{1}{2}\cos(2\pi f T)\right] df =$$

$$= 6T^2 \left[\frac{1}{6T} + \frac{1}{2} \sin(2\pi f t) \Big|_{-\frac{1}{6T}}^{\frac{1}{6T}} \right] =$$

$$= 6T^2 \left[\frac{1}{6T} + \frac{\sqrt{3}}{2}\right]$$

$$P_{Y_2} = 0$$

$$X_3(f) = T\left(1 - \frac{|f|}{1/T}\right) \operatorname{rect}\left(\frac{f}{2/T}\right)$$



$|Y_3(f)| = |X_3(f)|\,|H(f)|$ è distorto in un mezzo