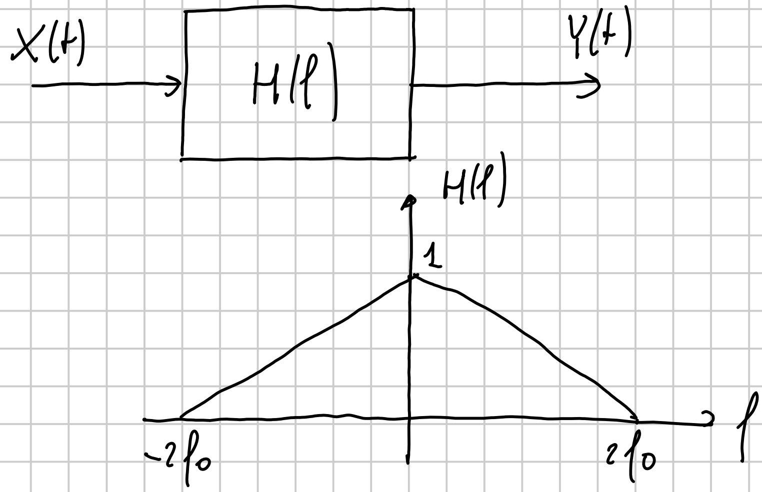
Es. PROCESSI ALEATORI PARAMETRICI

$$X(t) = A\cos(2\pi f_0 t) + B\sin(2\pi f_0 t)$$

$$A, B \in \mathcal{N}(0, \sigma^2) \quad , \quad \sigma^2 = 4 \qquad \text{indipendenti}$$

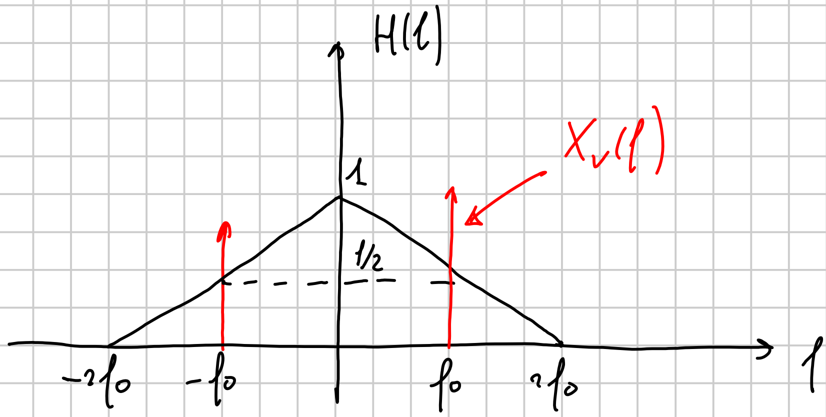


- Ricavare la $f_Y(y;t)$

Soluzione:

$$x_r(t) \Rightarrow X_r(f) = \frac{A}{2}\left(\delta(f-f_0) + \delta(f+f_0)\right) + \frac{B}{2j}\left(\delta(f-f_0) - \delta(f+f_0)\right)$$

$$Y_r(f) = X_r(f)\,H(f) =$$



$$Y_r(f) = \frac{1}{2}X_r(f) \Rightarrow y_r(t) = \frac{1}{2}x_r(t)$$

$$Y(t) = \frac{1}{2}X(t) = \frac{A}{2}\cos(2\pi f_0 t) + \frac{B}{2}\sin(2\pi f_0 t)$$

$$Y = Y(\bar{t}) = \frac{A}{2}\cos(2\pi f_0 \bar{t}) + \frac{B}{2}\sin(2\pi f_0 \bar{t})$$

$$= K_1 A + K_2 B$$

$$K_1 = \frac{1}{2}\cos(2\pi f_0 \bar{t}) \quad , \quad K_2 = \frac{1}{2}\sin(2\pi f_0 \bar{t})$$

$$Y \in \mathcal{N}(\eta_Y, \sigma_Y^2)$$

$$f_Y(y) = f_Y(y;t) = \frac{1}{\sqrt{2\pi\sigma_Y^2}} e^{-\frac{(y-\eta_Y)^2}{2\sigma_Y^2}}$$

$$\eta_Y = E[K_1 A + K_2 B] = K_1 \underbrace{E[A]}_{=0} + K_2 \underbrace{E[B]}_{=0} = 0$$

$$\sigma_Y^2 = E[Y^2] - \eta_Y^2 = E[Y^2] = E[(K_1 A + K_2 B)^2] =$$

$$= K_1^2 E[A^2] + K_2^2 E[B^2] + 2K_1K_2 \underbrace{E[AB]}_{=0} =$$

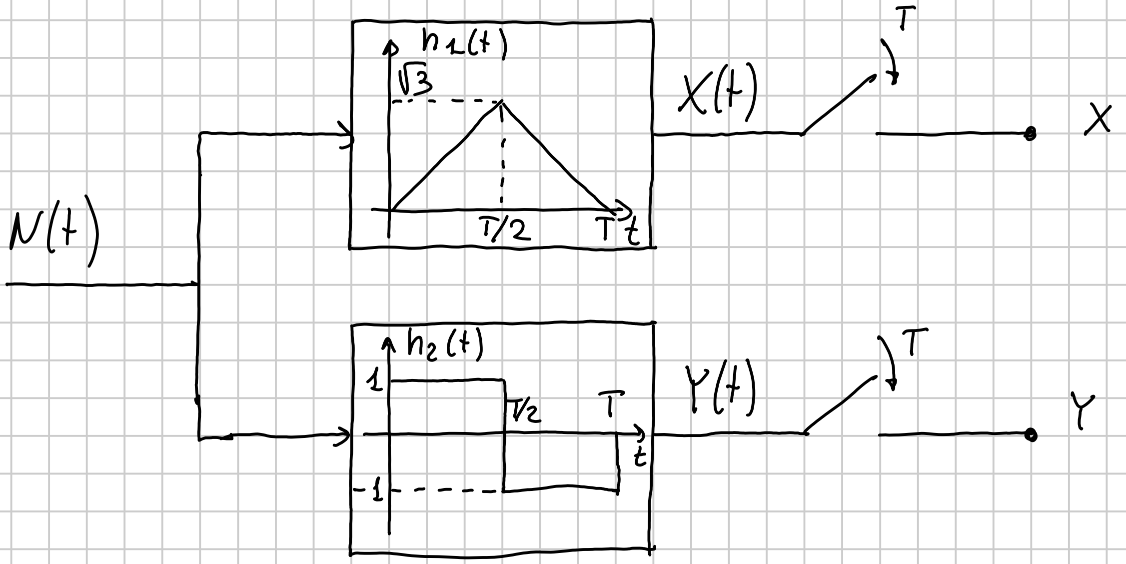
$$= K_1^2 \sigma^2 + K_2^2 \sigma^2 = (K_1^2 + K_2^2)\sigma^2$$

$$K_1^2 + K_2^2 = \frac{1}{4}\cos^2(2\pi f_0 t) + \frac{1}{4}\sin^2(2\pi f_0 t) = \frac{1}{4}$$

$$\sigma_Y^2 = \frac{1}{4} \cdot 4 = 1$$

$$Y \in \mathcal{N}(0,1) \quad (\text{STANDARD})$$

ES. PROCESSI ALEATORI NON - PARAMETRICI



$N(t)$ Gaussiano con $S_n(f) = \frac{N_0}{2}$

Calcolare:

- $P\{X > Y\}$
- $E[XY]$

Soluzione:

$$X = X(T) = \left[N(t) \otimes h_1(t)\right]_{t=T} = \int_{-\infty}^{+\infty} N(\tau)\, h_1(T-\tau)\, d\tau$$

$$Y = Y(T) = \int_{-\infty}^{+\infty} N(\tau)\, h_2(T-\tau)\, d\tau$$

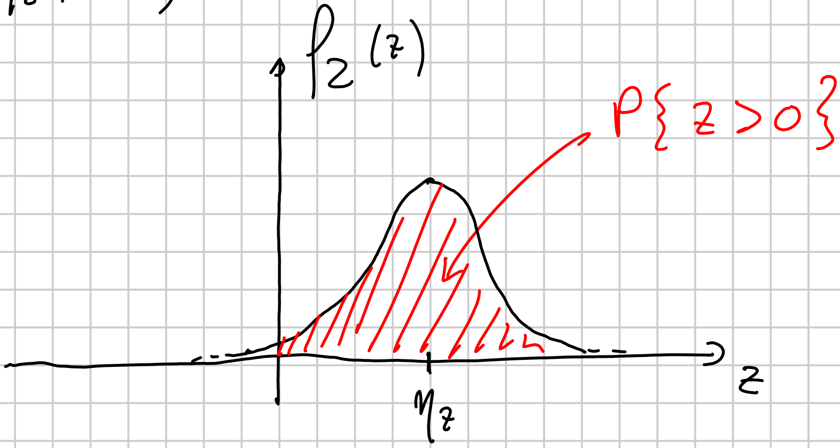
$$X, Y \in \mathcal{N}$$

$$X \in \mathcal{N}(\eta_x, \sigma_x^2) \quad , \quad Y \in \mathcal{N}(\eta_Y, \sigma_Y^2)$$

$$P\{X > Y\}$$

$$Z = X - Y \quad \Rightarrow \quad P\{X > Y\} = P\{Z > 0\}$$

$$Z \in \mathcal{N}(\eta_z, \sigma_z^2)$$



$$\eta_Z = E[X - Y] = E\left[\int_{-\infty}^{+\infty} N(\tau)\, h_1(T-\tau)\, d\tau \; + \right.$$

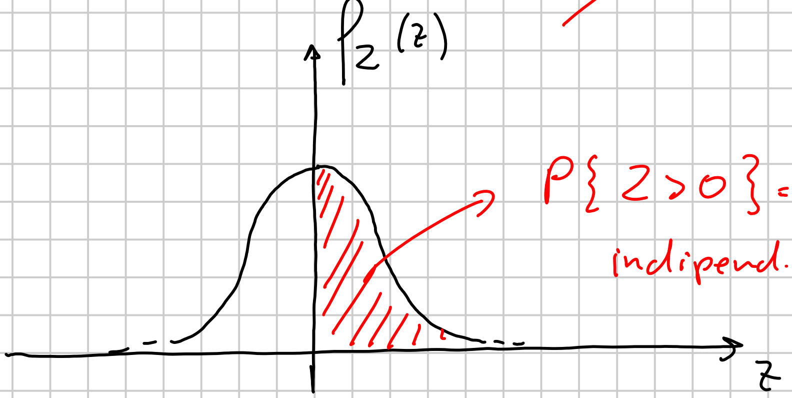
$$\left. - \int_{-\infty}^{+\infty} N(\tau)\, h_2(T-\tau)\, d\tau \right]$$

$$= E\left[\int_{-\infty}^{+\infty} N(\tau) \left[h_1(T-\tau) - h_2(T-\tau) \right] d\tau \right] =$$

$$= \int_{-\infty}^{+\infty} \underbrace{E[N(\tau)]}_{=0} \left[h_1(T-\tau) - h_2(T-\tau) \right] d\tau = 0$$

$$E[N(\tau)] = 0$$

$$W(t) = N(t) + A \quad \Rightarrow \quad \cancel{S_w(f) = \frac{N_0}{2}}$$



$$P\{Z>0\} = P\{X>Y\} = \frac{1}{2}$$

indipend. da σ_z^2

2) $E[XY] = E\left[\int_{-\infty}^{+\infty} N(\tau)\, h_1(T-\tau)\, d\tau \int_{-\infty}^{+\infty} N(\alpha)\, h_2(T-\alpha)\, d\alpha \right]$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E[N(\tau) N(\alpha)]\, h_1(T-\tau)\, h_2(T-\alpha)\, d\tau\, d\alpha$$

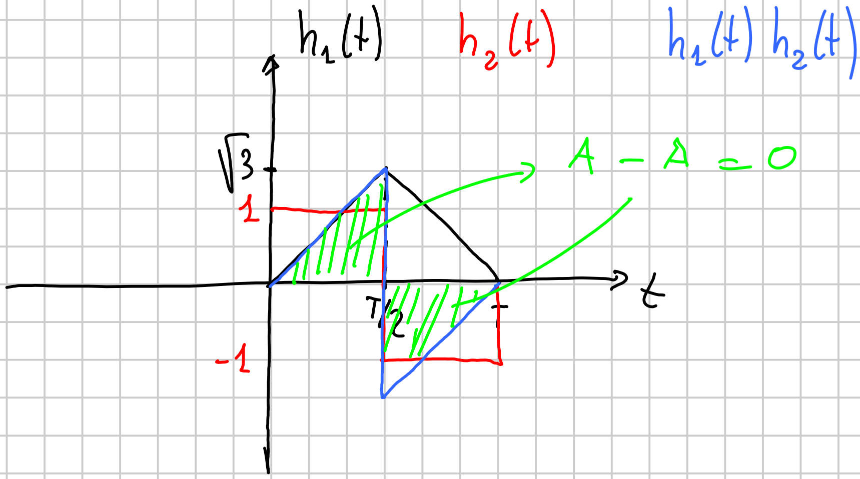
$$E[N(\tau) N(\alpha)] = R_N(\beta) = \frac{N_0}{2} \delta(\beta) = \frac{N_0}{2} \delta(\tau-\alpha)$$

$$\beta = \tau - \alpha$$

$$E[XY] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{N_0}{2} \delta(\tau-\alpha)\, h_1(T-\tau)\, h_2(T-\alpha)\, d\tau\, d\alpha$$

$$= \frac{N_0}{2} \int_{-\infty}^{+\infty} h_1(T-\tau)\, h_2(T-\tau)\, d\tau \qquad (T-\tau = t)$$

$$= \frac{N_0}{2} \int_{-\infty}^{+\infty} h_1(t)\, h_2(t)\, dt = 0$$



X e Y incorrelate

$$E[(X-\eta_X)(Y-\eta_Y)] = E[XY] = 0$$

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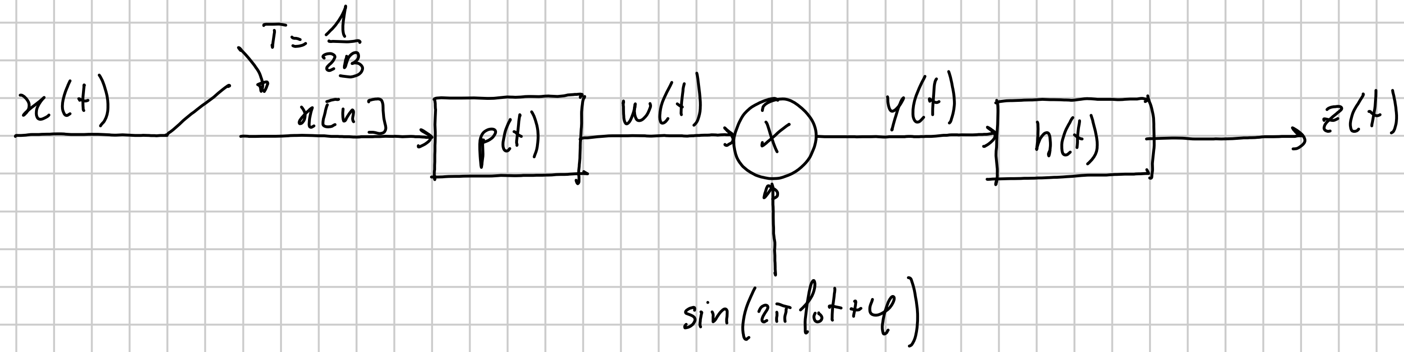


$x(t) = B\,\mathrm{sinc}^2(Bt)$

$h(t) = 2B\,\mathrm{sinc}(2Bt)$

$p(t) = 2B\,\mathrm{sinc}(2Bt)\cos(4\pi Bt)$

Si calcoli

1) $z(t)$
2) E_z, P_z
3) φ tale che E_z sia max

Soluzione:

1) $$X(f) = \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$\bar{X}(f) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X\left(f - \frac{k}{T}\right) = 2B\sum_{k=-\infty}^{+\infty} X(f - k2B)$$

$$P(f) = \mathrm{rect}\left(\frac{f}{2B}\right) \otimes \left[\frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)\right] =$$

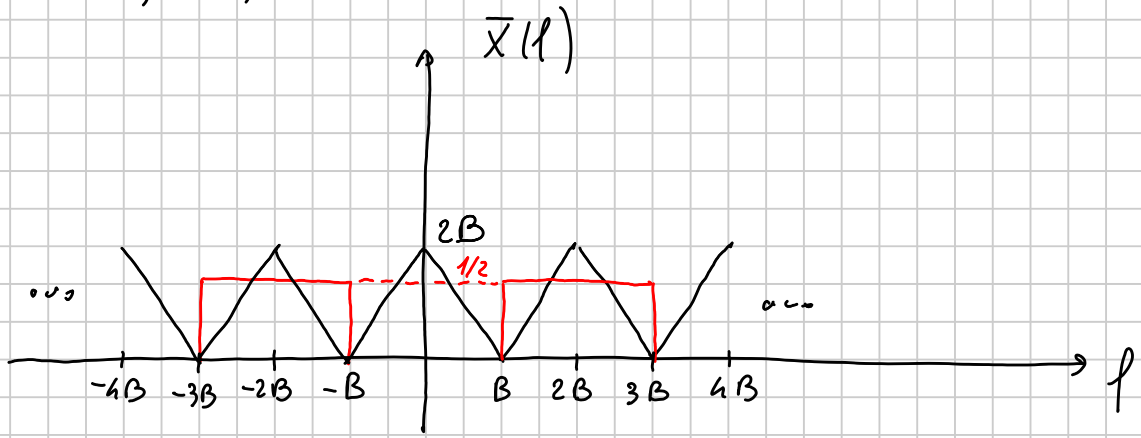
$$= \mathrm{rect}\left(\frac{f-f_0}{2B}\right) + \mathrm{rect}\left(\frac{f+f_0}{2B}\right)$$

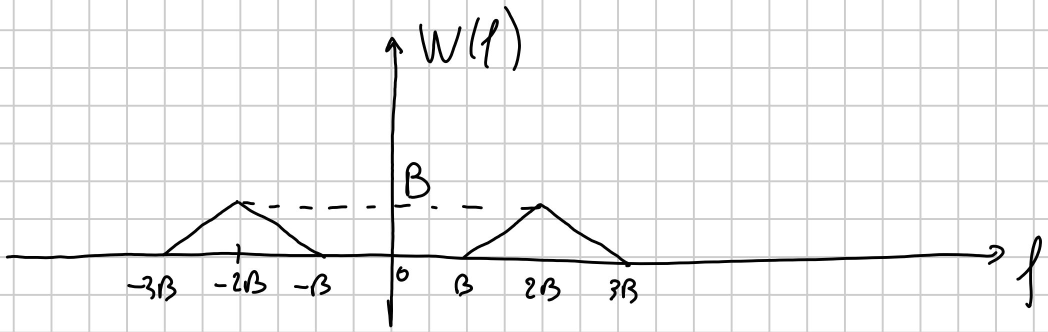
$f_0 = 2B$

$$= \frac{1}{2}\mathrm{rect}\left(\frac{f-2B}{2B}\right) + \frac{1}{2}\mathrm{rect}\left(\frac{f+2B}{2B}\right)$$

$$H(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$W(f) = \bar{X}(f)\, P(f)$$





$$W(f) = B\left(1 - \frac{|f-2B|}{B}\right) \mathrm{rect}\left(\frac{f-2B}{2B}\right) + B\left(1 - \frac{|f+2B|}{B}\right) \mathrm{rect}\left(\frac{f+2B}{2B}\right)$$

$$c(t) = \sin(2\pi f_0 t + \varphi) = \cos(2\pi f_0 t + \varphi') \quad , \varphi' = \varphi - \frac{\pi}{2}$$

$$C(f) = \frac{e^{j\varphi'}}{2}\delta(f - f_0) + \frac{e^{-j\varphi'}}{2}\delta(f + f_0)$$

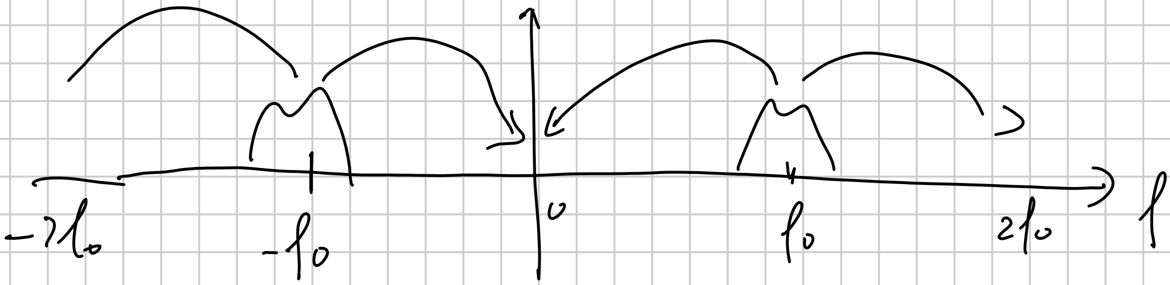
$f_0 = 2B$

$$= \frac{e^{j\varphi'}}{2}\delta(f - 2B) + \frac{e^{-j\varphi'}}{2}\delta(f + 2B)$$

$$Y(f) = W(f) \otimes C(f) = W(f - 2B)\frac{e^{j\varphi'}}{2} + W(f + 2B)\frac{e^{-j\varphi'}}{2}$$

$$= B\frac{e^{j\varphi'}}{2}\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) + \mathrm{comp}\ (2f_0)$$

$$+ \frac{B e^{-j\varphi'}}{2} \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right) + \text{comp}\left(-2f_0\right)$$



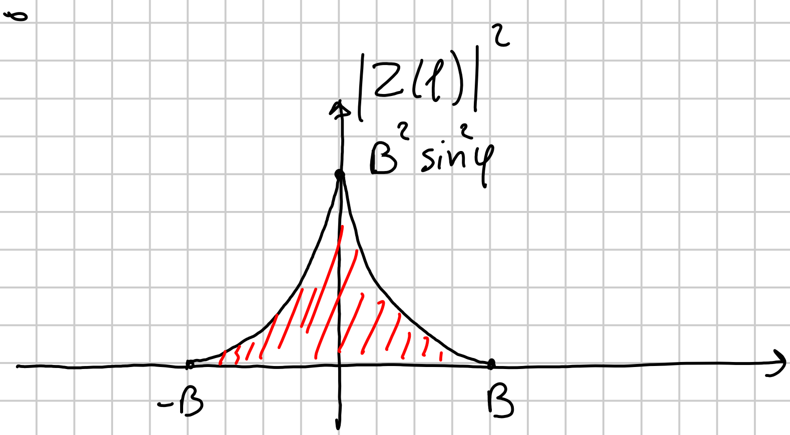
$$Z(f) = \frac{B}{2} \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right) \left(e^{j\varphi'} + e^{-j\varphi'}\right)$$

$$= B \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right) \cos \varphi'$$

$$= B \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right) \sin \varphi$$

$$z(t) = B^2 \,\mathrm{sinc}^2(Bt) \sin \varphi$$

2) $$E_z = \int_{-\infty}^{+\infty} |Z(f)|^2 \, df = \frac{2}{3} B^3 \sin^2 \varphi \Rightarrow P_z = 0$$



3) $$E_z = E_z^{(\max)} \Rightarrow \varphi = \frac{\pi}{2} + k\pi$$

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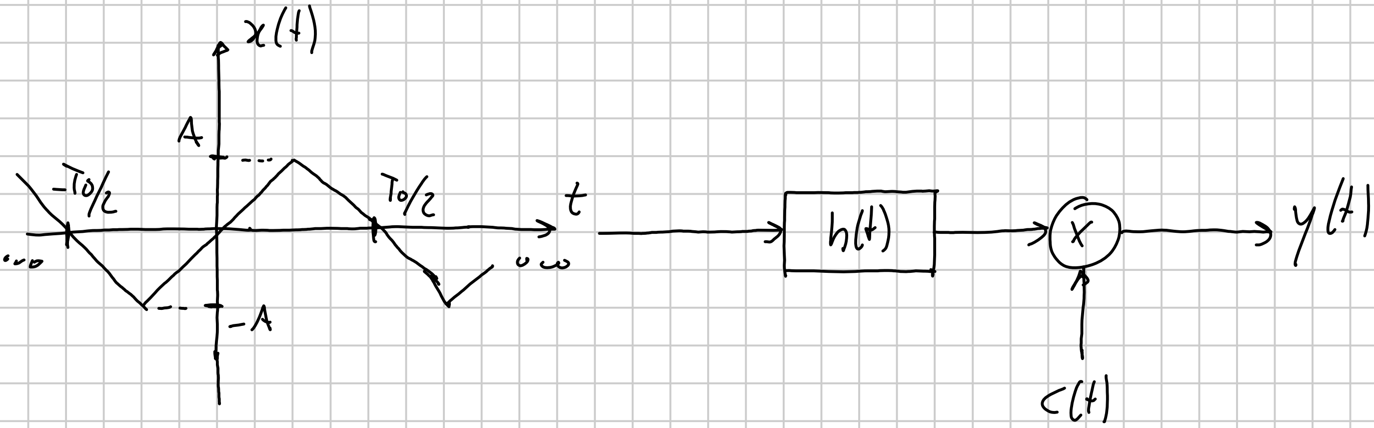


$$h(t) = \frac{3}{T_0}\,\mathrm{sinc}\left(\frac{3t}{T_0}\right)$$

$$c(t) = \sin\left(2\pi \frac{t}{T_0}\right)$$

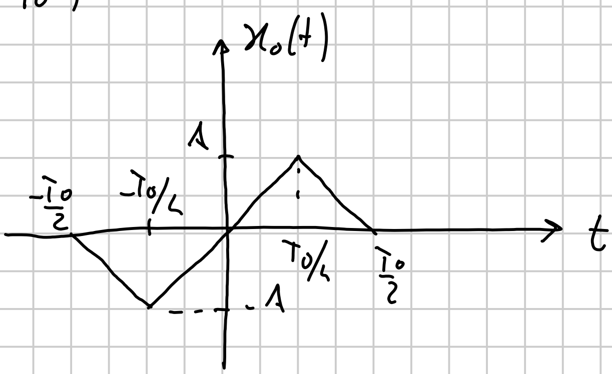
Si calcoli

1) $X(f)$
2) $y(t)$
3) E_y, P_y

Soluzione

1) $$X(f) = \sum_{k=-\infty}^{+\infty} X_k\, \delta\left(f - \frac{k}{T_0}\right)$$

$$X_k = \frac{1}{T_0} X_0\left(\frac{k}{T_0}\right) \quad , \quad X_0(f) = TCF\left[x_0(t)\right]$$



$$x(t) = \sum_{k=-\infty}^{+\infty} x_0(t - kT_0)$$

$$x_0(t) = A\left(1 - \frac{|t - T_0/4|}{T_0/4}\right) \mathrm{rect}\left(\frac{t - T_0/4}{T_0/2}\right) +$$

$$- A\left(1 - \frac{|t + T_0/4|}{T_0/4}\right) \mathrm{rect}\left(\frac{t + T_0/4}{T_0/2}\right)$$

$$X_0(f) = A\frac{T_0}{4} \mathrm{sinc}^2\left(\frac{T_0}{4} f\right) e^{-j 2\pi f \frac{T_0}{4}} +$$

$$- A\frac{T_0}{4} \mathrm{sinc}^2\left(\frac{T_0}{4} f\right) e^{+j 2\pi f \frac{T_0}{4}}$$

$$= A\frac{T_0}{4} \mathrm{sinc}^2\left(\frac{T_0}{4} f\right) (-2j) \left(\frac{e^{j 2\pi f \frac{T_0}{4}} - e^{-j 2\pi f \frac{T_0}{4}}}{2j}\right)$$

$$= -j\frac{A T_0}{2} \mathrm{sinc}^2\left(\frac{T_0 f}{4}\right) \sin\left(\frac{\pi f T_0}{2}\right)$$

$$X_k = \frac{1}{T_0} X_0\left(\frac{k}{T_0}\right) = \frac{1}{T_0}\left(-j\frac{A T_0}{2}\right) \mathrm{sinc}^2\left(\frac{T_0}{4}\frac{k}{T_0}\right) \sin\left(\frac{\pi T_0}{2}\frac{k}{T_0}\right)$$

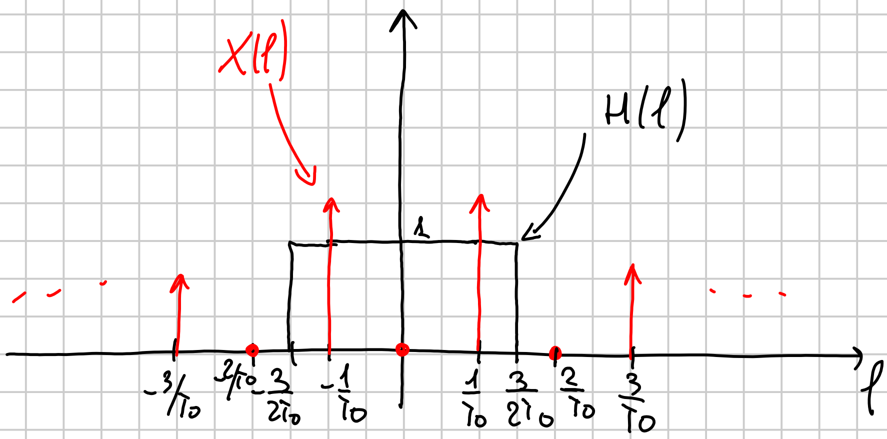
$$= -j\frac{A}{2} \mathrm{sinc}^2\left(\frac{k}{4}\right) \sin\left(k\frac{\pi}{2}\right)$$

$$X(f) = \sum_{k=-\infty}^{+\infty} \left(-j\frac{A}{2}\right) \mathrm{sinc}^2\left(\frac{k}{4}\right) \sin\left(k\frac{\pi}{2}\right) \delta\left(f - \frac{k}{T_0}\right)$$

2) $w(t) = x(t) \otimes h(t)$

$$W(f) = X(f)\, H(f)$$

$$H(f) = \mathrm{rect}\left(\frac{f}{3/T_0}\right)$$



$$W(f) = -j\frac{A}{2}\,\text{sinc}^2\left(\frac{1}{4}\right)\delta\left(f-\frac{1}{T_0}\right) + j\frac{A}{2}\,\text{sinc}^2\left(\frac{1}{4}\right)\delta\left(f+\frac{1}{T_0}\right)$$

$$Y(f) = W(f) \otimes C(f)$$

$$C(f) = \frac{1}{2j}\delta\left(f-\frac{1}{T_0}\right) - \frac{1}{2j}\delta\left(f+\frac{1}{T_0}\right)$$

$$Y(f) = -\frac{A}{4}\,\text{sinc}^2\left(\frac{1}{4}\right)\delta\left(f-\frac{2}{T_0}\right) + \frac{A}{4}\,\text{sinc}^2\left(\frac{1}{4}\right)\delta(f) +$$

$$+\frac{A}{4}\,\text{sinc}^2\left(\frac{1}{4}\right)\delta(f) - \frac{A}{4}\,\text{sinc}^2\left(\frac{1}{4}\right)\delta\left(f+\frac{2}{T_0}\right)$$

$$= \frac{A}{4}\,\text{sinc}^2\left(\frac{1}{4}\right)\left[2\delta(f) - \left(\delta\left(f-\frac{2}{T_0}\right) + \delta\left(f+\frac{2}{T_0}\right)\right)\right]$$

$$y(t) = \frac{A}{4}\,\text{sinc}^2\left(\frac{1}{4}\right)\left[2 - 2\cos\left(4\pi\frac{t}{T_0}\right)\right] =$$

$$= \frac{A}{2}\,\text{sinc}^2\left(\frac{1}{4}\right)\left[1 - \cos\left(\frac{4\pi t}{T_0}\right)\right]$$

3) $$P_y = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \frac{A^2}{4}\,\text{sinc}^4\left(\frac{1}{4}\right)\left[1 - \cos\left(\frac{4\pi t}{T_0}\right)\right]^2 dt =$$

$$= \frac{A^2}{4T_0}\,\text{sinc}^4\left(\frac{1}{4}\right)\left[T_0 + \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}}\left(\frac{1}{2} + \underbrace{\frac{1}{2}\cos\left(\frac{8\pi t}{T_0}\right)}_{0}\right)dt + \right.$$

$$\left. -\underbrace{2\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}}\cos\left(\frac{4\pi t}{T_0}\right)dt}_{=0}\right]$$

$$E_y = \infty$$

$$= \frac{A^2}{4T_0}\,\text{sinc}^4\left(\frac{1}{4}\right)\left[T_0 + \frac{T_0}{2}\right] = \frac{A^2}{4\cancel{T_0}}\,\text{sinc}^4\left(\frac{1}{4}\right)\frac{3}{2}\cancel{T_0}$$