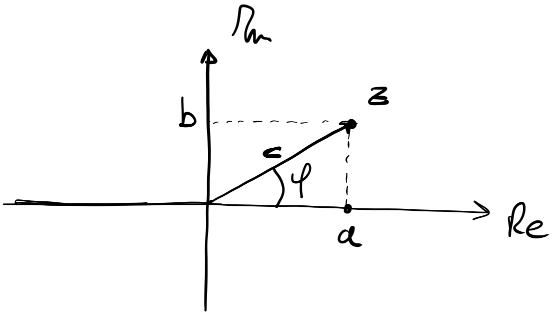
NUMERI COMPLESSI

$$z = \underbrace{a}_{\text{parte reale}} + i\,\underbrace{b}_{\text{parte immaginaria}}$$

rappresentazione cartesiana



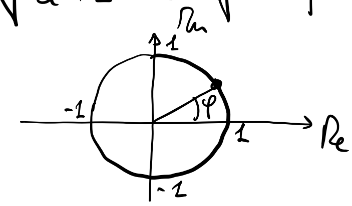
$$\begin{cases} c = \sqrt{a^2 + b^2} & \text{modulo} \\ \varphi = tg^{-1} \dfrac{b}{a} & \text{fase} \end{cases}$$

rappresentazione polare

$$z = \underbrace{c}_{\text{modulo}}\, e^{\overbrace{j\varphi}^{\text{fase}}}$$

$$e^{j\varphi} \triangleq \underbrace{\cos\varphi}_{a} + j\,\underbrace{\sin\varphi}_{b}$$

$$|e^{j\varphi}| = 1 = \sqrt{a^2 + b^2} = \sqrt{\cos^2\varphi + \sin^2\varphi}$$



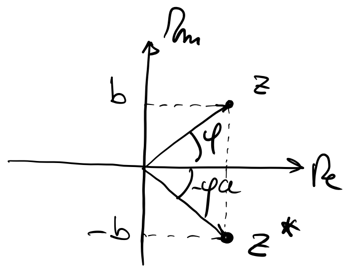
$$z = c\,e^{j\varphi} = \underbrace{c\cos\varphi}_{a} + j\,\underbrace{c\sin\varphi}_{b} \qquad j, i = \sqrt{-1}$$

$$\begin{cases} a = c\cos\varphi \\ b = c\sin\varphi \end{cases}$$ trasf da pol. a cart.

$$\begin{cases} c = \sqrt{a^2+b^2} \\ \varphi = tg^{-1}\dfrac{b}{a} \end{cases}$$ trasf da cart. a pol.

$$a = \mathrm{Re}\{z\} = \frac{z + z^*}{2} = \frac{a + \cancel{jb} + a - \cancel{jb}}{2}$$

$$z^* = a - jb$$



$$b = \mathrm{Im}\{z\} = \frac{z - z^*}{j2} = \frac{\cancel{a} + jb - (\cancel{a} - jb)}{2j}$$

Operazioni algebriche

·) somma/sottrazione

$$z = z_1 \pm z_2 \qquad z_1, z_2 \in \mathbb{C}$$

$$z_1 = a_1 + j\,b_1$$
$$z_2 = a_2 + j\,b_2$$

$$z = a_1 + j\,b_1 \pm a_2 + j\,b_2 = a_1 \pm a_2 + j\,(b_1 \pm b_2)$$

·) prodotto

$$Z = z_1 \cdot z_2 = (a_1 + j b_1)(a_2 + j b_2) = a_1 a_2 + j a_1 b_2 + j a_2 b_1 - b_1 b_2$$

$$= a_1 a_2 - b_1 b_2 + j(a_1 b_2 + a_2 b_1)$$

$z_1 = a_1 + j b_1$

$z_2 = a_2 + j b_2$

$z_1 = c_1 e^{j\varphi_1}$

$z_2 = c_2 e^{j\varphi_2}$

$$z = c_1 e^{j\varphi_1} c_2 e^{j\varphi_2}$$

$$= c_1 c_2 e^{j(\varphi_1 + \varphi_2)}$$

·) rapporto

$$Z = \frac{z_1}{z_2}$$

$z_1 = a_1 + j b_1$

$z_2 = a_2 + j b_2$

$$z = \frac{a_1 + j b_1}{a_2 + j b_2} \cdot \frac{a_2 - j b_2}{a_2 - j b_2}$$

$$= \frac{a_1 a_2 + b_1 b_2 + j(a_2 b_1 - a_1 b_2)}{a_2^2 \cancel{- j a_2 b_2} \cancel{+ j a_2 b_2} + b_2^2}$$

$$|z|^2 = z \cdot z^* = c e^{j\varphi} \cdot c e^{-j\varphi} = c^2$$

$z_1 = c_1 e^{j\varphi_1}$

$z_2 = c_2 e^{j\varphi_2}$

$$Z = \frac{c_1 e^{j\varphi_1}}{c_2 e^{j\varphi_2}} = \frac{c_1}{c_2} e^{j(\varphi_1 - \varphi_2)}$$

$z(t) \qquad t \in \mathbb{R} \qquad z \in \mathbb{C}$

$$z(t) = a(t) + j\, b(t)$$

$$= c(t)\, e^{j\varphi(t)}$$

$$\varphi = tg^{-1} \frac{b}{a} \qquad \in \quad [0, 2\pi)$$

$$\int_{K_1}^{K_2} z(t)\, dt = \int_{K_1}^{K_2} a(t)\, dt + j \int_{K_2}^{K_2} b(t)\, dt$$

$$\frac{d}{dt} z(t) = \frac{d}{dt} a(t) + j \frac{d}{dt} b(t)$$

SEGNALI

<u>segnali deterministici</u>

- segnali reali
- per ogni istante temporale si conosce il valore del segnale
- spesso sono rappresentati con funzioni analitiche
  - sin, cos, log., polinomi, ecc. ecc.

segnali aleatori

- es. segnale rumore
- caratterizzazione statistica

$x(t)$ ← variabile ind.

↑ variabile dip

	tempo continuo	tempo discreto
ampiezza continua	segnali analogici	sequenze
ampiezza discreta	segnali quantizzati	segnali numerici

Segnali analogici

$x(t)$

→ potenza istantanea

$$P_x(t) \triangleq |x(t)|^2$$

↳ $x \in \mathbb{C}$

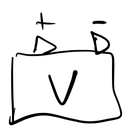
quando $x \in \mathbb{R} \Rightarrow P_x(t) = x^2(t)$

→ energia

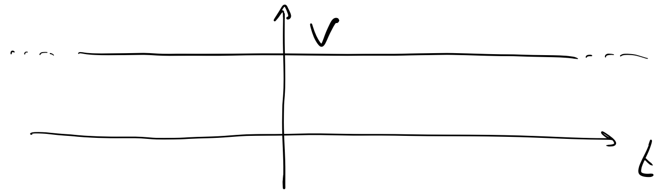
$$E_x \triangleq \int_{-\infty}^{+\infty} P_x(t)\,dt = \int_{-\infty}^{+\infty} |x(t)|^2\,dt$$

⇒ energia
- → finita (segnali fisici)
- → infinita (segnali ideali)

es segnale ad energia infinita



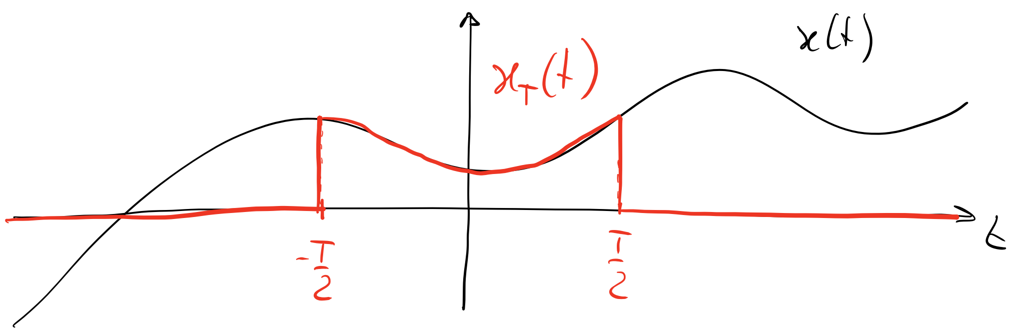
$$x(t) = V \qquad \forall t$$



$$E_x = \int_{-\infty}^{+\infty} V^2 \, dt = \infty$$

·) versione troncata di un segnale

$$x(t) \longrightarrow x_T(t) \triangleq \begin{cases} x(t) & -\frac{T}{2} \le t \le \frac{T}{2} \\ 0 & \text{altrove} \end{cases}$$



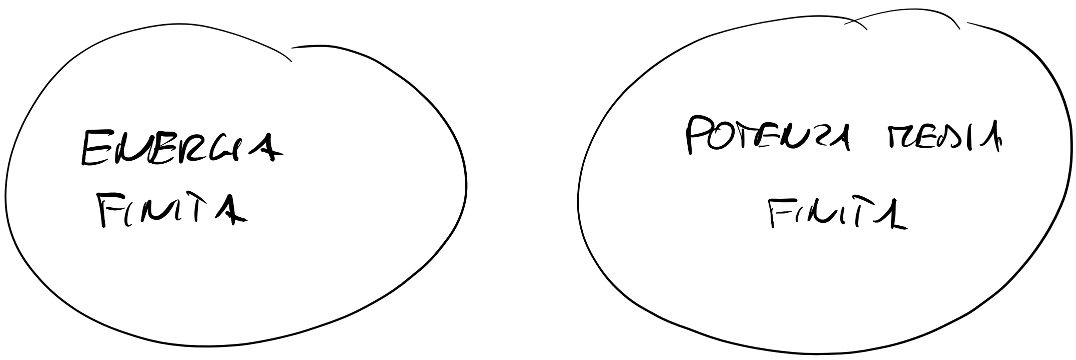
·) Potenza media

$$P_{x_T} \triangleq \frac{E_{x_T}}{T}$$

$$P_x \triangleq \lim_{T \to \infty} \frac{E_{x_T}}{T} = \boxed{\lim_{T \to \infty} \frac{1}{T} \overbrace{\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 \, dt}^{E_{x_T}}}$$

$x(t) = V$

$$P_x \triangleq \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} V^2 \, dt = \lim_{T \to \infty} \frac{1}{\cancel{T}} V^2 \cdot \cancel{T} = V^2$$



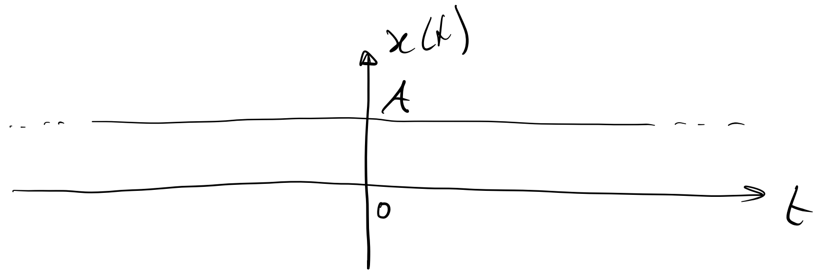
•) se $x(t)$ ha $E_x < \infty \Rightarrow P_x = 0$

•) se $x(t)$ ha $P_x = \underset{\neq 0}{K} < \infty \Rightarrow E_x = \infty$

Segnali tipici

- Costante

$$x(t) = A \qquad \forall t$$



·) Energia

$$E_x = \int_{-\infty}^{+\infty} A^2 \, dt = \infty$$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \, dt = A^2$$

$$x_{eff} = |A|$$

$$x_m = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A \, dt = \lim_{T \to \infty} \frac{1}{\cancel{T}} A \cancel{T} = A$$

·) Sinusoide

$$x(t) = \underbrace{A}_{\text{ampiezza}} \cos\left(2\pi \underbrace{f_0}_{\text{frequenza}} t + \underbrace{\varphi}_{\text{fase}}\right)$$

·) Energia

$$E_x = \int_{-\infty}^{+\infty} \left| A\cos(2\pi f_0 t + \varphi) \right|^2 dt$$

$$= A^2 \int_{-\infty}^{+\infty} \cos^2(2\pi f_0 t + \varphi)\, dt$$

$$= A^2 \int_{-\infty}^{+\infty} \left[\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0 t + 2\varphi) \right] dt$$

$$= A^2 \frac{1}{2} \underbrace{\int_{-\infty}^{+\infty} 1 \cdot dt}_{\infty} + \frac{A^2}{2} \int_{-\infty}^{+\infty} \cos(4\pi f_0 t + 2\varphi)\, dt$$

$$\infty \quad + \frac{A^2}{2} \frac{1}{4\pi f_0} \sin(\cdot) \Big|_{-\infty}^{+\infty}$$

$$= \infty$$

·) Potenza media

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A^2 \cos^2(2\pi f_0 t + \varphi)\, dt$$

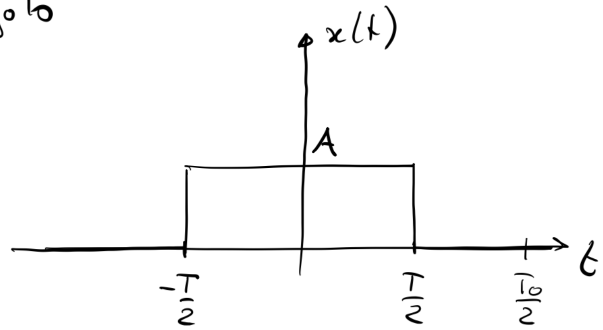
$$= \lim_{T \to \infty} \frac{1}{T} \left[\frac{A^2}{2} T + \frac{A^2}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(4\pi f_0 t + 2\varphi)\, dt \right]$$

$$= \lim_{T \to \infty} \frac{A^2 \cancel{T}}{\cancel{T} 2} + \lim_{T \to \infty} \frac{K}{T} \qquad -\frac{A^2}{2}\frac{1}{4\pi f_0} < K \leq \frac{A^2}{2}\frac{1}{4\pi f_0}$$

$$\lim_{T \to \infty} \frac{K}{T} \to 0$$

$$= \frac{A^2}{2}$$

·) rettangolo



$$x(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} A & -\frac{T}{2} \le t \le \frac{T}{2} \\ 0 & \text{altrove} \end{cases}$$

·) Energia

$$E_x = \int_{-\infty}^{+\infty} A^2\,\mathrm{rect}^2\left(\frac{t}{T}\right) dt = A^2 \int_{-\infty}^{+\infty} \mathrm{rect}\left(\frac{t}{T}\right) dt$$

$$= A^2 \int_{-\frac{T}{2}}^{\frac{T}{2}} 1\,dt = A^2 T < \infty$$

segnale di classe energia finita

$$P_x = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} A^2\,\mathrm{rect}^2\left(\frac{t}{T}\right) dt$$

$T_0 > T$

$$= \lim_{T_0 \to \infty} \frac{1}{T_0} A^2 T = \lim_{T_0 \to \infty} \frac{A^2 T}{T_0} = 0$$