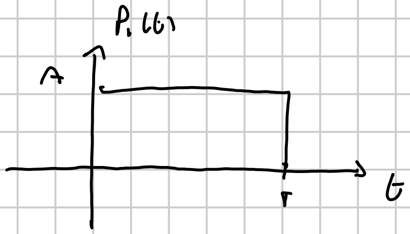
ESERCIZIO 1

$$s(t) = \sum_n x[n]\, p(t - nT)$$

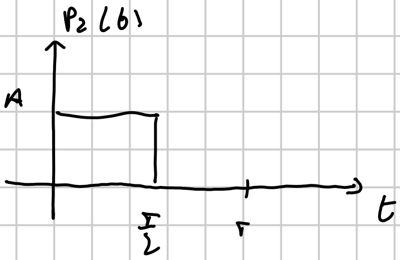
$x[n] \in A_s = \{0, 1\}$ SEGNALAZIONE ON-OFF

Calcolare $S_s(f)$ e P_s nel caso in cui

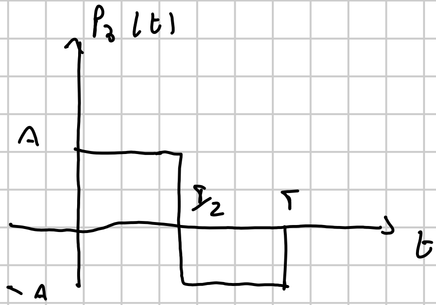
1) $p(t) = \begin{cases} A & 0 \le t \le T \\ 0 & \text{altrove} \end{cases}$



2) $p(t) = \begin{cases} A & 0 \le t \le \frac{T}{2} \\ 0 & \text{altrove} \end{cases}$



3) $p(t) = \begin{cases} A & 0 \le t \le \frac{T}{2} \\ -A & \frac{T}{2} \le t \le T \\ 0 & \text{altrove} \end{cases}$



$$S_s(f) = \frac{1}{T} S_x(f)\, |P(f)|^2$$

$$S_x(f) = \text{TFS}\left[R_x[m] \right]$$

$$R_x[m] = E\{x[i+m]\, x[i]\} = \begin{cases} E\{x^2[i]\} = \frac{1}{2} & m = 0 \\ E\{x^2[i]\} = \frac{1}{4} & m \neq 0 \end{cases}$$

$$R_x[m] = \frac{1}{4} + \frac{1}{4}\delta[m]$$

$$S_x(f) = \frac{1}{4} + \sum_k \frac{1}{4T}\delta\left(f - \frac{k}{T}\right)$$

$$S_s(f) = \frac{1}{T}\left(\frac{1}{4} + \sum_k \frac{1}{4T}\delta\left(f - \frac{k}{T}\right)\right) A^2 T^2 \operatorname{sinc}^2(fT) =$$

$$= \frac{A^2 T}{4} \operatorname{sinc}^2(fT) + A^2 T \sum_k \frac{1}{4T} \delta\left(f - \frac{k}{T}\right) \operatorname{sinc}^2\left(\frac{k}{T} T\right) =$$

$$= \frac{A^2 T}{4} \operatorname{sinc}^2(fT) + \frac{A^2}{4} \delta(f)$$

$$P_s = \frac{A^2}{4} + \int_{-\infty}^{+\infty} \frac{A^2 T}{4} \operatorname{sinc}^2(fT)\, df = \frac{A^2}{4} + \frac{A^2 T}{4} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\frac{1}{T} \operatorname{rect}\left(\frac{t}{T}\right)\right)^2 dt =$$

$$= \frac{A^2}{4} + \frac{A^2}{4} = \frac{A^2}{2}$$

2)

$$|P_2(f)|^2 = \frac{A^2 T^2}{4} \operatorname{sinc}^2\left(f \frac{T}{2}\right)$$

$$S_s(f) = \frac{1}{T} \left(\frac{1}{4} + \frac{1}{4T} \sum_k \delta\left(f - \frac{k}{T}\right) \right) \frac{A^2 T^2}{4} \operatorname{sinc}^2\left(f \frac{T}{2}\right) =$$

$$= \frac{A^2 T}{16} \operatorname{sinc}^2\left(f \frac{T}{2}\right) + \frac{A^2}{16} \sum_k \operatorname{sinc}^2\left(\frac{k}{\cancel{T}} \frac{\cancel{T}}{2}\right) \delta\left(f - \frac{k}{T}\right)$$

$$P_s = \int_{-\infty}^{+\infty} S_s(f)\, df = \frac{A^2 T}{4} \int_{-\infty}^{+\infty} \operatorname{sinc}^2\left(f \frac{T}{2}\right) df + \frac{A^2}{16} \sum_k \operatorname{sinc}^2\left(\frac{k}{2}\right) =$$

$$= \frac{A^2}{8} + \frac{A^2}{8} = \frac{A^2}{4}$$

3)

$$|P_3(f)|^2 = A^2 T^2 \operatorname{sinc}^2\left(f \frac{T}{2}\right) \sin^2\left(\pi f \frac{T}{2}\right)$$

$$S_s(f) = A^2 T \left(\frac{1}{4} + \frac{1}{4T} \sum_k \delta\left(f - \frac{k}{T}\right) \right) \operatorname{sinc}^2\left(f \frac{T}{2}\right) \sin^2\left(\pi f \frac{T}{2}\right) =$$

$$= \frac{A^2 T}{4} \operatorname{sinc}^2\left(f \frac{T}{2}\right) \sin^2\left(\pi f \frac{T}{2}\right) + \frac{A^2}{4} \sum_T \operatorname{sinc}^2\left(\frac{k}{2}\right) \sin^2\left(\frac{\pi k}{2}\right) \delta\left(f - \frac{k}{T}\right)$$

$$P_s = \int_{-\infty}^{+\infty} S_s(f)\, df = \frac{A^2}{4} + \frac{A^2}{4} \sum_k \operatorname{sinc}^2\left(\frac{k}{2}\right) \sin^2\left(\frac{k\pi}{2}\right) = \frac{A^2}{4} + \frac{A^2}{4} = \frac{A^2}{2}$$