ESERCIZIO

Sia data la trasformazione ingresso-uscita di un sistema monodimensionale

$$y(t) = \int_T^t x(\alpha)\, d\alpha$$

Dire se il sistema gode delle seguenti proprietà:

1) linearità
2) stazionarietà
3) stabilità
4) istantaneità

Soluzione

1) Linearità

$$x(t) = a\, x_1(t) + b\, x_2(t)$$

$$y(t) = a\, T[x_1(t)] + b\, T[x_2(t)]$$

$$y(t) = \int_T^t [a\, x_1(\alpha) + b\, x_2(\alpha)]\, d\alpha =$$

$$= a \int_T^t x_1(\alpha)\, d\alpha + b \int_T^t x_2(\alpha)\, d\alpha =$$

$$= a\, T[x_1(t)] + b\, T[x_2(t)]$$ (LINEARE)

2) Stazionarietà

$$y(t-t_0) = T[x(t-t_0)]$$

$$\int_T^t x(\alpha - t_0)\, d\alpha = (\alpha - t_0 = \beta) =$$

$$= \int_{T-t_0}^{t-t_0} x(\beta)\, d\beta = \int_{T-t_0}^{T} x(\beta)\, d\beta + \int_{T}^{t-t_0} x(\beta)\, d\beta$$

$$= y(t-t_0) + \int_{T-t_0}^{T} x(\beta)\, d\beta \neq y(t-t_0)$$

(NON STAZIONARIO)

3) STABILITÀ

$$|x(t)| \leq M \Rightarrow |y(t)| \leq K$$

Dimostriamo l'instabilità. Basta trovare una funzione $x(t)$: $|x(t)| \leq M$ per cui $|y(t)| = \infty$ per un qualunque "t".

$$|y(t)| = \left| \int_T^t x(\alpha)\, d\alpha \right|$$

scelgo $x(\alpha) = M$

$$|y(t)| = \left| \int_T^t M \, d\alpha \right| = M(t-T)$$

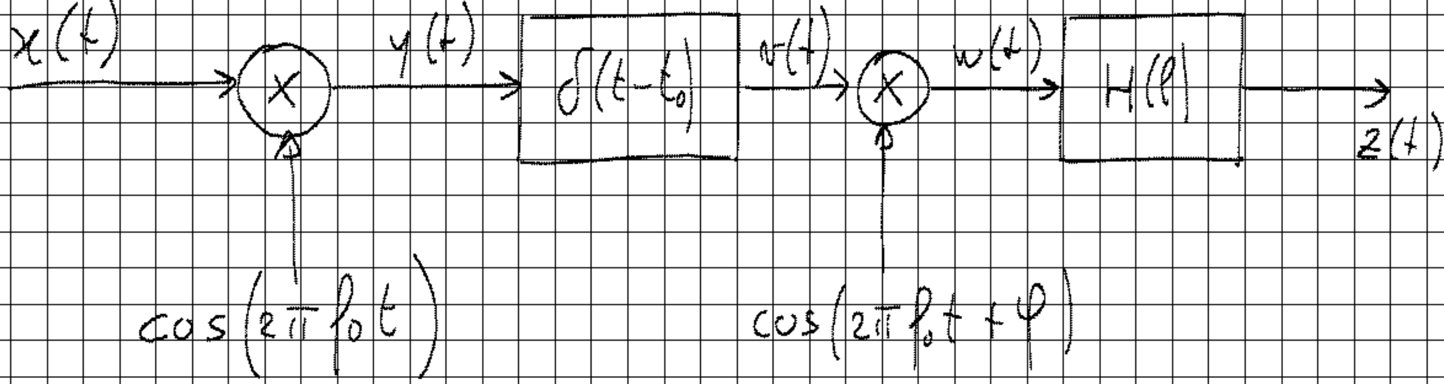
(INSTABILE)

4) ISTANTANEITÀ

$$y(t) = \int_T^t x(\alpha)\, d\alpha$$

(CON MEMORIA)

ESERCIZIO – SISTEMA DI COM. WIRELESS



$$X(f) = \left[1 + \cos\left(\frac{\pi f}{B}\right)\right] \mathrm{rect}\left(\frac{f}{2B}\right)$$

$H(f)$ passa basso ideale di banda B

Calcolare:

1) $Y(f)$
2) $W(f)$
3) $z(t)$
4) Determinare il valore di φ affinché $z(t)$ abbia energia massima

Soluzione

1) $$Y(f) = \frac{1}{2}\left[X(f-f_0) + X(f+f_0)\right] =$$

$$= \frac{1}{2}\left[1 + \cos\left(\frac{\pi(f-f_0)}{B}\right)\right] \mathrm{rect}\left(\frac{f-f_0}{2B}\right) +$$

$$+ \frac{1}{2}\left[1 + \cos\left(\frac{\pi(f+f_0)}{B}\right)\right] \mathrm{rect}\left(\frac{f+f_0}{2B}\right)$$

2) $$Y'(f) = Y(f)\, e^{-j2\pi f t_0}$$

$$W(f) = Y'(f) \otimes \frac{1}{2}\left[\delta(f-f_0)e^{j\varphi} + \delta(f+f_0)e^{-j\varphi}\right]$$

$$= \frac{1}{4} e^{-j\left[2\pi(f-f_0)t_0 - \varphi\right]} \left[1 + \cos\left(\frac{\pi(f-2f_0)}{B}\right)\right] \mathrm{rect}\left(\frac{f-2f_0}{2B}\right) +$$

$$+ \frac{1}{4} e^{-j\left[2\pi(f+f_0)t_0 - \varphi\right]} \left[1 + \cos\left(\frac{\pi f}{B}\right)\right] \mathrm{rect}\left(\frac{f}{2B}\right) +$$

$$+ \frac{1}{4} e^{-j\left[2\pi(f-f_0)t_0 + \varphi\right]} \left[1 + \cos\left(\frac{\pi f}{B}\right)\right] \mathrm{rect}\left(\frac{f}{2B}\right) +$$

$$+ \frac{1}{4} e^{-j\left[2\pi(f+f_0)t_0 + \varphi\right]} \left[1 + \cos\left(\frac{\pi(f+2f_0)}{B}\right)\right] \mathrm{rect}\left(\frac{f+2f_0}{2B}\right)$$

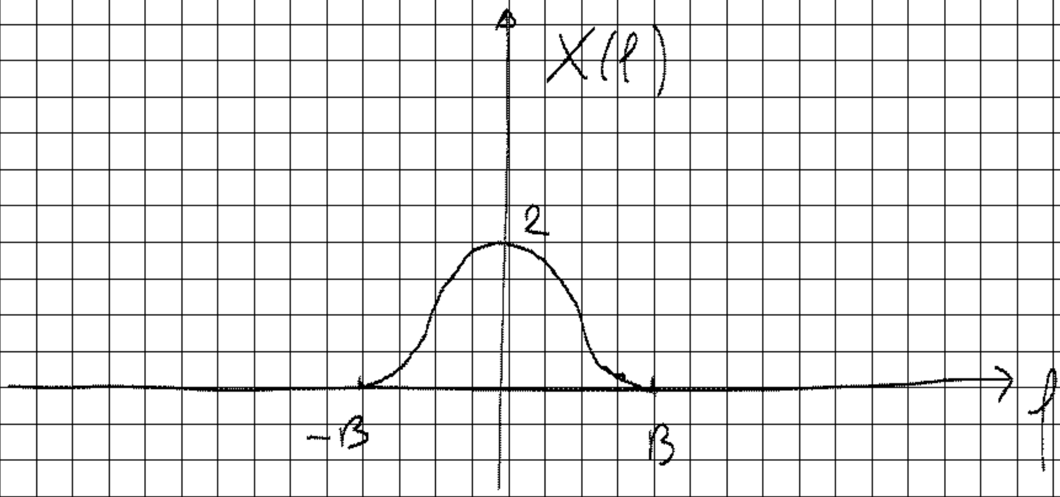
$$= \frac{1}{4} W_{2f_0}(f) + \frac{1}{4} W_{-2f_0}(f) +$$

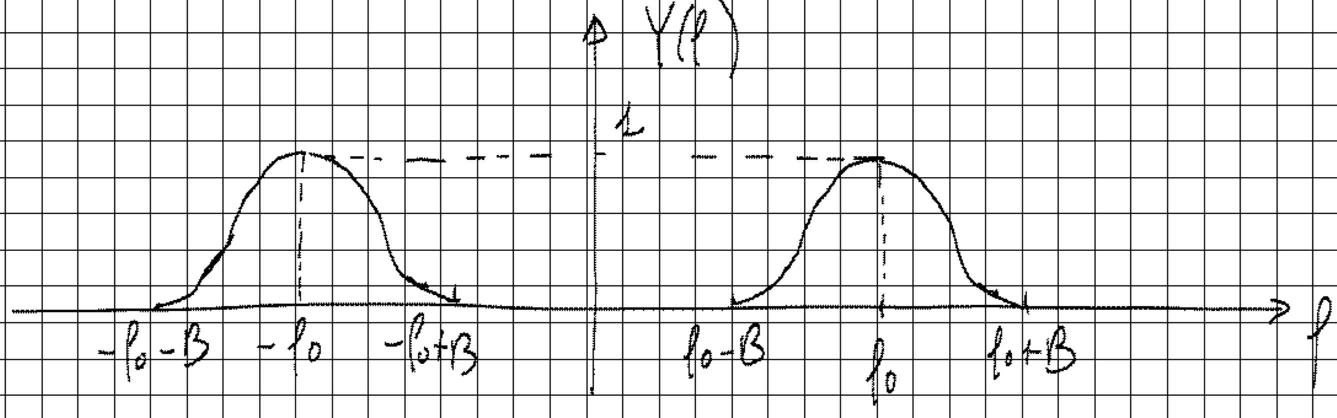
$$+ \frac{1}{4} e^{-j2\pi f t_0} \cdot \left[e^{-j(2\pi f_0 t_0 + \varphi)} + e^{j(2\pi f_0 t_0 + \varphi)} \right] \left[1 + \cos\left(\frac{\pi f}{2B}\right)\right] \mathrm{rect}\left(\frac{f}{2B}\right)$$

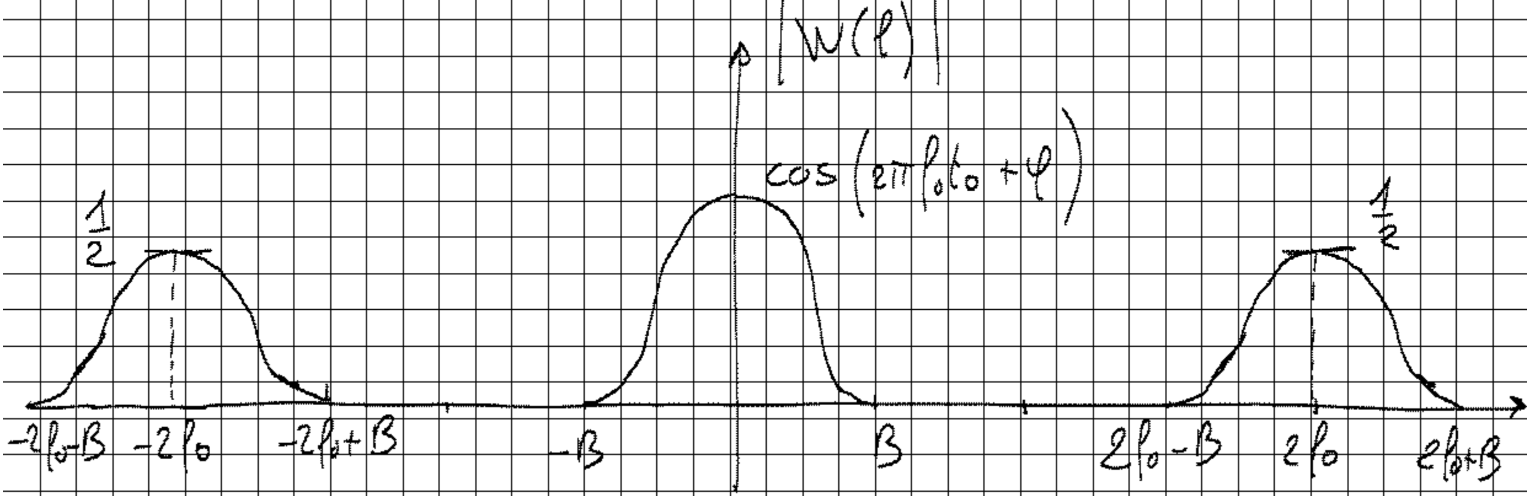
$$= \frac{1}{4}\left[W_{2f_0}(f) + W_{-2f_0}(f)\right] + \frac{1}{2} e^{-j2\pi f t_0} \cos(2\pi f_0 t_0 + \varphi)\left[1 + \cos\left(\frac{\pi f}{2B}\right)\right] \mathrm{rect}\left(\frac{f}{2B}\right)$$

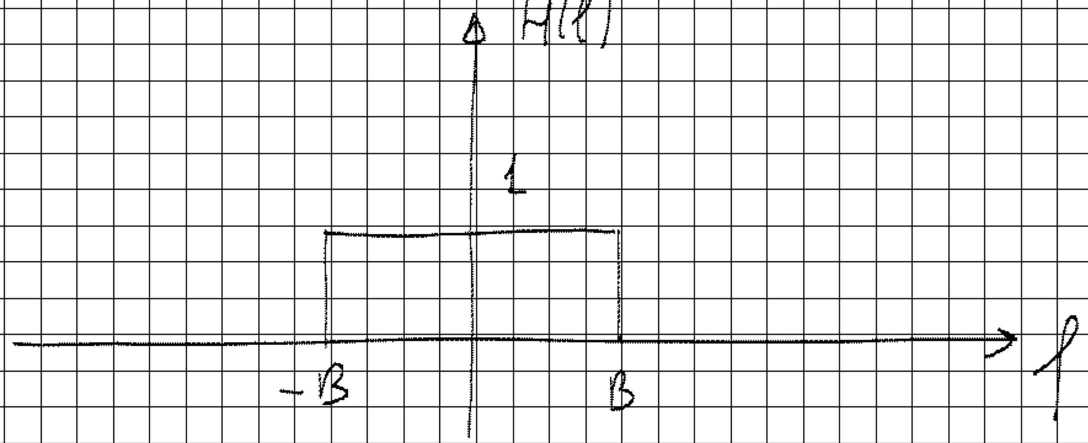
$$W_{2f_0}(f) = \frac{1}{4} e^{-j\left[2\pi(f-f_0)t_0 - \varphi\right]} \left[1 + \cos\left(\frac{\pi(f-2f_0)}{B}\right)\right] \mathrm{rect}\left(\frac{f-2f_0}{2B}\right)$$

$$W_{-2f_0}(f) = \frac{1}{4} e^{-j\left[2\pi(f+f_0)t_0 + \varphi\right]} \left[1 + \cos\left(\frac{\pi(f+2f_0)}{B}\right)\right] \mathrm{rect}\left(\frac{f+2f_0}{2B}\right)$$









3) $$Z(f) = \frac{1}{2} e^{-j2\pi f t_0} \cos(2\pi f_0 t_0 + \varphi)\left[1+\cos\left(\frac{\pi f}{2B}\right)\right] \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$= \frac{1}{2} \cos(2\pi f_0 t_0 + \varphi) \left[1+\cos\left(\frac{\pi f}{2B}\right)\right] \mathrm{rect}\,\frac{f}{2B} \cdot e^{-j2\pi f t_0}$$

$$z(t) = \frac{1}{2}\cos(2\pi f_0 t_0 + \varphi)\, x(t-t_0)$$

$z(t)$ è una replica ritardata ed attenuata del segnale sorgente $x(t)$

$$x(t) \overset{TCF}{\Longleftrightarrow} X(f)$$

$$X(f) = \mathrm{rect}\left(\frac{f}{2B}\right) + \cos\left(\frac{\pi f}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$x(t) = 2B\,\mathrm{sinc}(2Bt) + B\,\mathrm{sinc}(2Bt) \otimes \left[\delta\left(t-\frac{1}{2B}\right)+\delta\left(t+\frac{1}{2B}\right)\right]$$

$$x(t) = 2B\,\mathrm{sinc}(2Bt) + B\,\mathrm{sinc}\left[2B\left(t-\frac{1}{2B}\right) + 2B\left(t+\frac{1}{2B}\right)\right]$$

4) $$E_z = \int_{-\infty}^{+\infty} |z(t)|^2 \, dt =$$

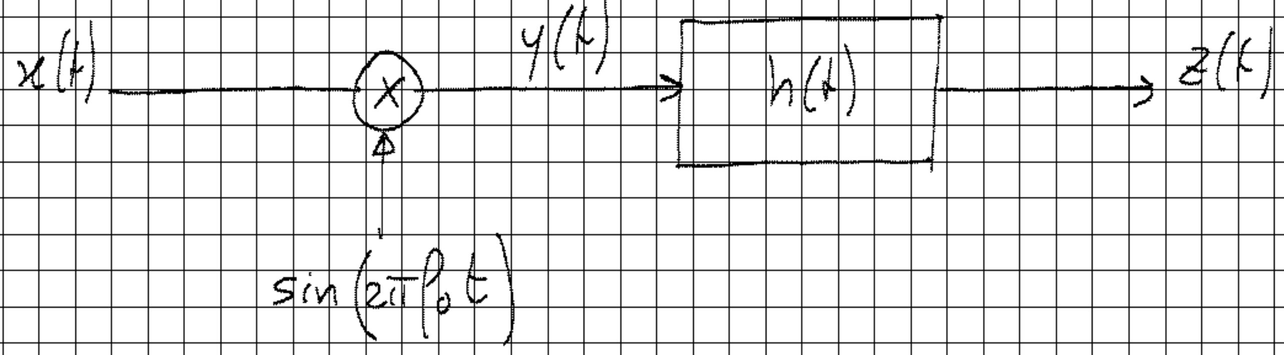
$$= \frac{1}{4} \left|\cos(2\pi f_0 t_0 + \varphi)\right|^2 \int_{-\infty}^{+\infty} |x(t-t_0)|^2 \, dt =$$

$$= \frac{1}{4} \left|\cos(2\pi f_0 t_0 + \varphi)\right|^2 \int_{-\infty}^{+\infty} |x(t')|^2 \, dt' =$$

$$= \frac{1}{4} \left|\cos(2\pi f_0 t_0 + \varphi)\right|^2 E_x$$

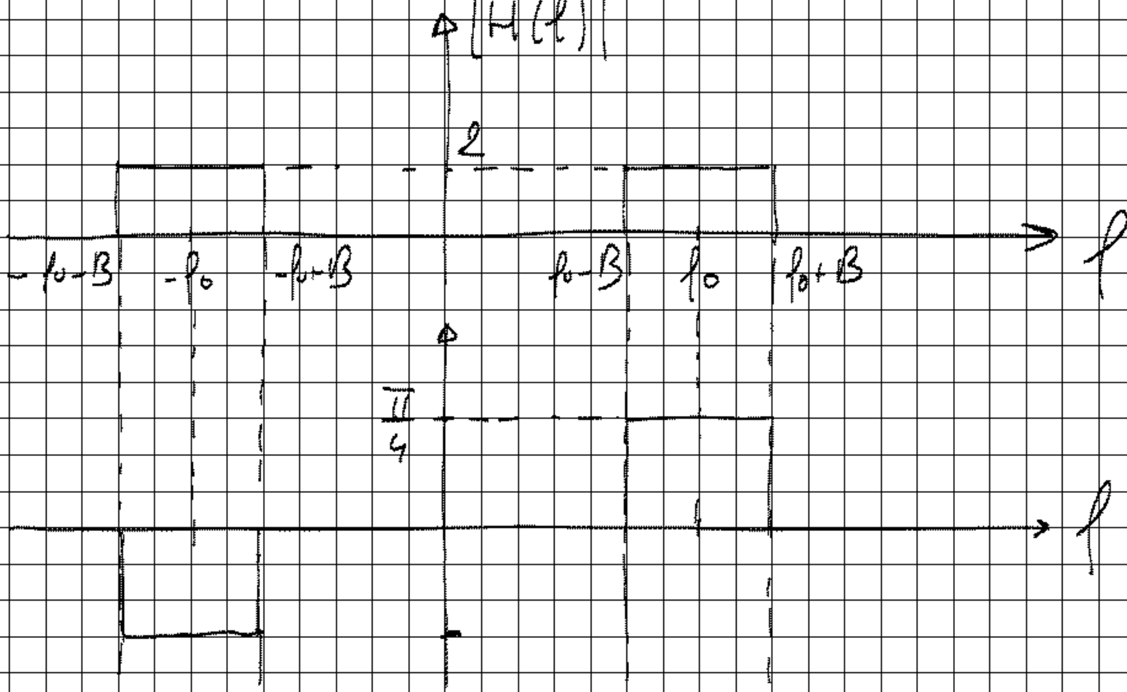
$E_{z_{max}} = \frac{1}{4} E_x$ quando $\varphi = -2\pi f_0 t_0 + k\pi$

ESERCIZIO – MODULAZIONE E FILTRAGGIO



Sia $X(f) = A\left(1 - \frac{|f|}{B}\right) \text{rect}\left(\frac{f}{2B}\right)$

e $H(f)$ definita attraverso il suo grafico



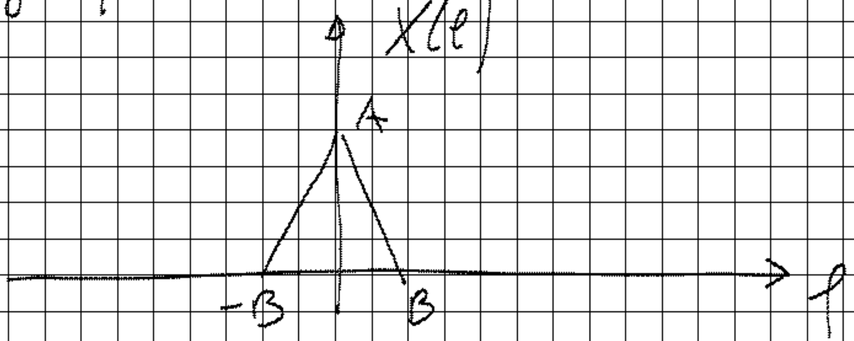
Calcolare:

1) Espressione analitica della $z(t)$

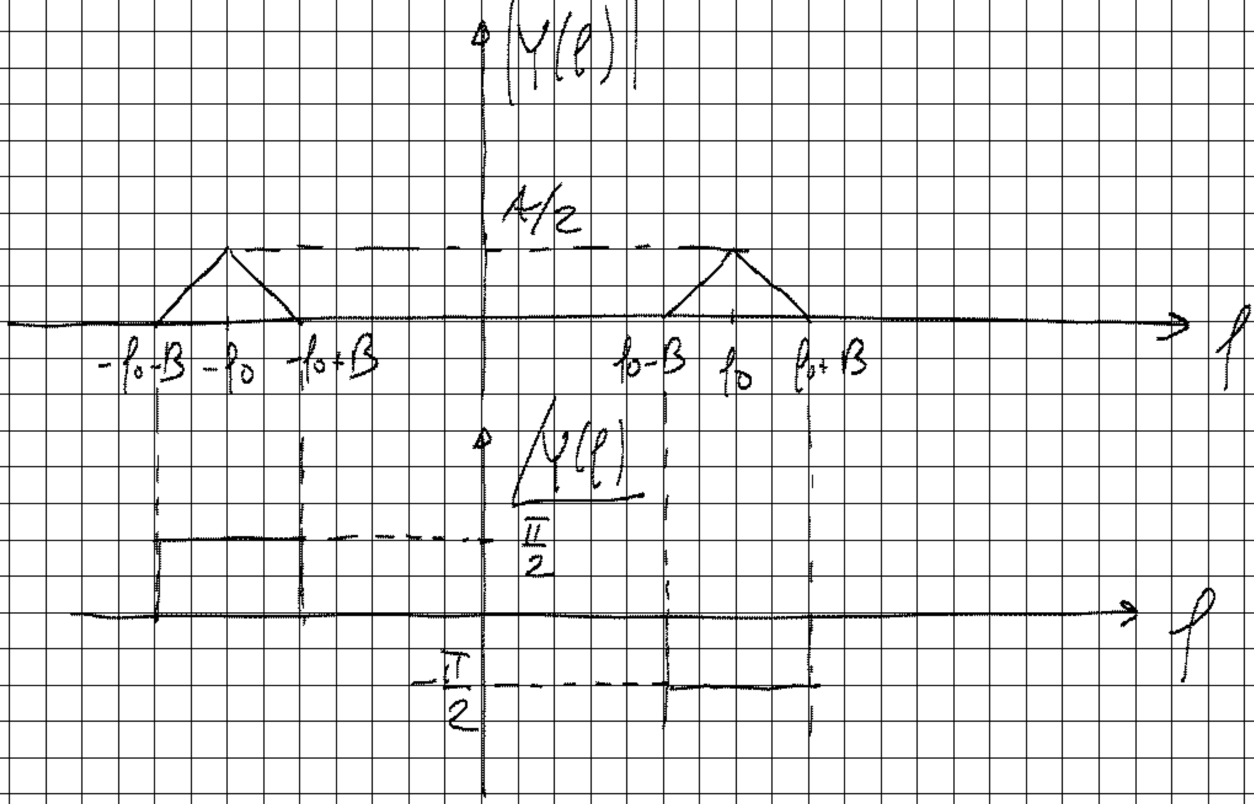
2) Energia della $z(t)$

Soluzione

Per via grafica



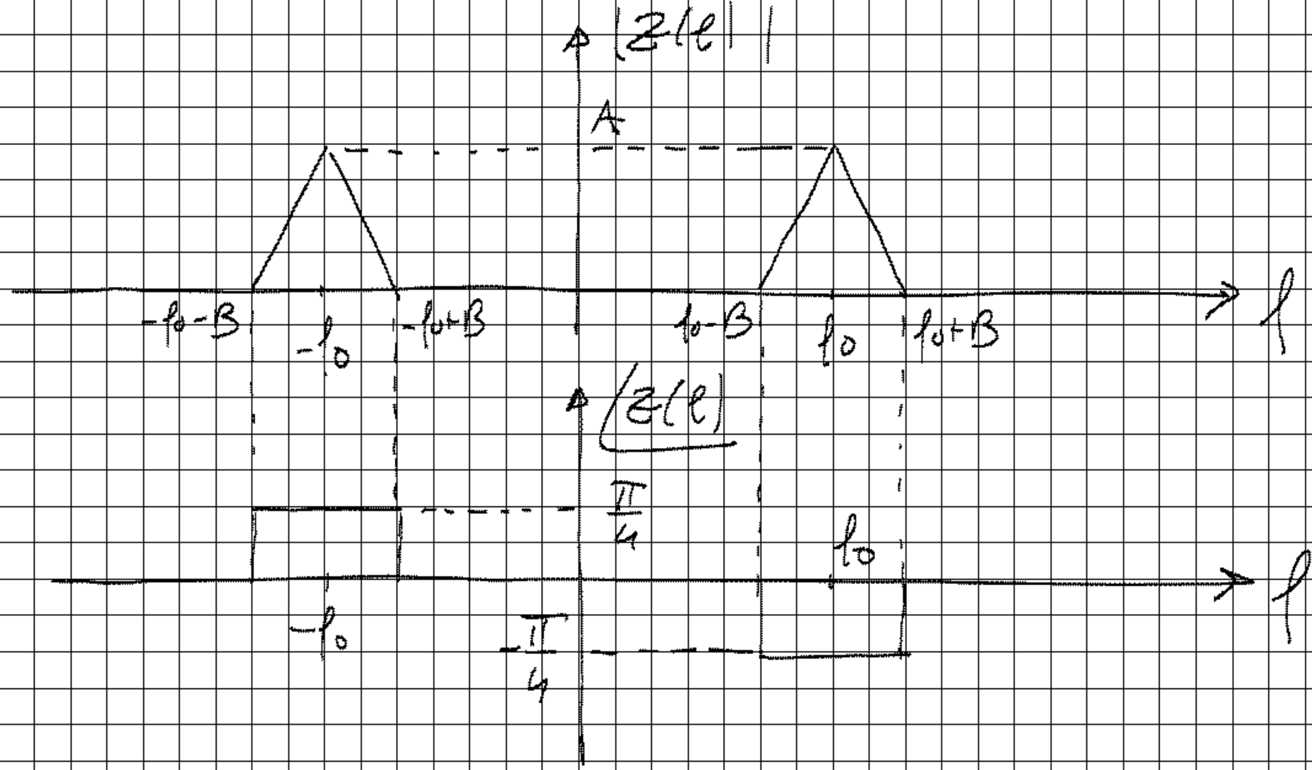
$$Y(f) = \frac{e^{-j\frac{\pi}{2}}}{2} X(f-f_0) + \frac{e^{j\frac{\pi}{2}}}{2} X(f+f_0)$$



$$Z(f) = Y(f)\, H(f)$$

$$|Z(f)| = |Y(f)|\, |H(f)|$$

$$\angle Z(f) = \angle Y(f) + \angle H(f)$$



$$Z(f) = A\left(1 - \frac{|f+f_0|}{B}\right) \mathrm{rect}\left(\frac{f+f_0}{2B}\right) e^{j\frac{\pi}{4}} +$$

$$+ A\left(1 - \frac{|f-f_0|}{B}\right) \mathrm{rect}\left(\frac{f-f_0}{2B}\right) e^{-j\frac{\pi}{4}} =$$

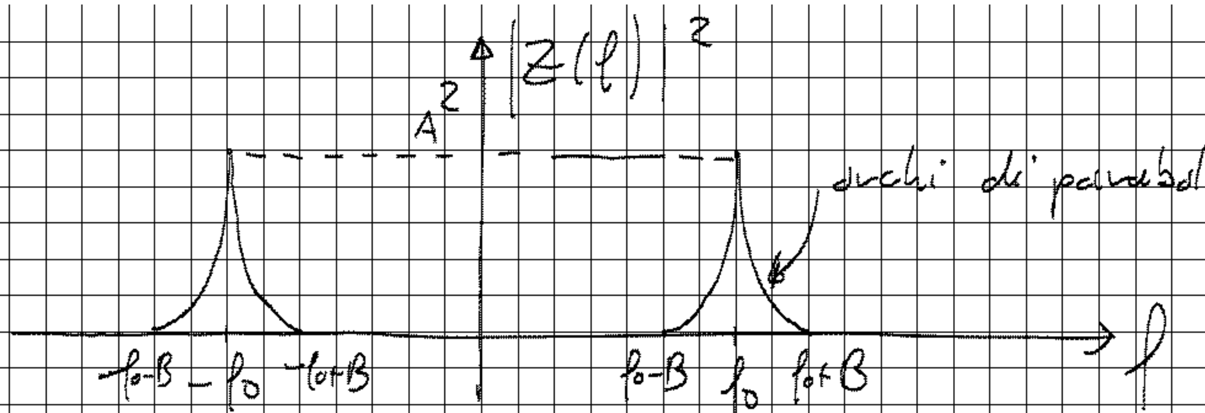
$$= A\left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right) \otimes \left[e^{-j\frac{\pi}{4}} \delta(f-f_0) + e^{j\frac{\pi}{4}} \delta(f+f_0) \right]$$

$$z(t) = B \,\mathrm{sinc}^2(Bt) \cdot \left[e^{j\left(2\pi f_0 t - \frac{\pi}{4}\right)} + e^{-j\left(2\pi f_0 t - \frac{\pi}{4}\right)} \right] =$$

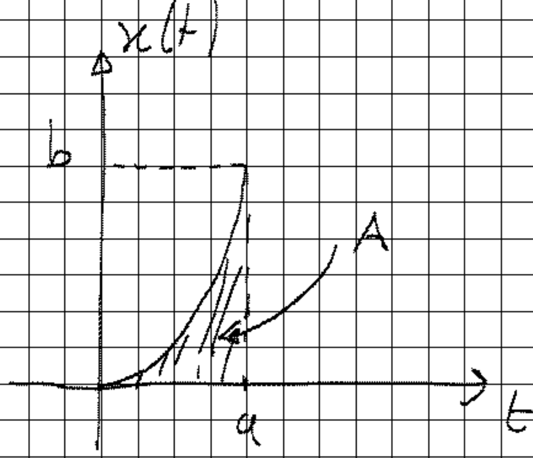
$$= 2B \,\mathrm{sinc}^2(Bt) \cos\left(2\pi f_0 t - \frac{\pi}{4}\right)$$

Calcolo dell'energia

$$E_z = \int_{-\infty}^{+\infty} |z(t)|^2 dt = \int_{-\infty}^{+\infty} |Z(f)|^2 df$$



Calcolo dell'area sottesa da un arco di parabola.



$$A = \int_0^a x(t)\,dt = \frac{ab}{3} \quad (\text{area del rettangolo}/3)$$

Dimostrazione:

$$A = \int_0^a \frac{b}{a^2} t^2\,dt = \frac{b}{a^2}\int_0^a t^2\,dt = \frac{b}{a^2}\left.\frac{t^3}{3}\right|_0^a = \frac{b}{a^2}\frac{a^3}{3} = \frac{ab}{3}$$

$$E_z = 4 \cdot \frac{B \cdot A^2}{3} = \frac{4}{3}A^2 B$$

ESERCIZIO — DISTORSIONI LINEARI

$$x_i(t) \longrightarrow \boxed{h(t)} \longrightarrow y_i(t)$$

) $h(t) = \mathrm{sinc}\left(\frac{t}{T}\right)\mathrm{sinc}\left(\frac{t}{2T}\right)$

) $x_1(t) = e^{-\frac{t}{T}}u(t)$

) $x_2(t) = \mathrm{sinc}\left(\frac{t-T/2}{3T}\right) + \mathrm{sinc}\left(\frac{t+T/2}{3T}\right)$

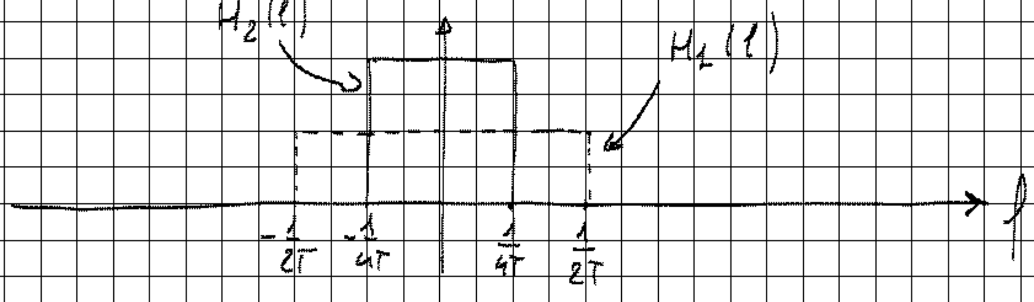
) $x_3(t) = \mathrm{sinc}^2\left(\frac{t}{T}\right)$

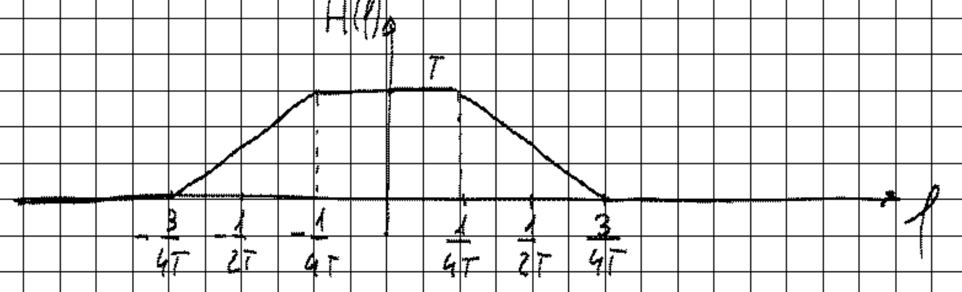
1) Dire se e quali uscite sono distorte e che tipo di distorsione hanno subito
2) Calcolare energia e potenza di $y_2(t)$

Soluzione:

1) Calcolo di $y_1(t)$

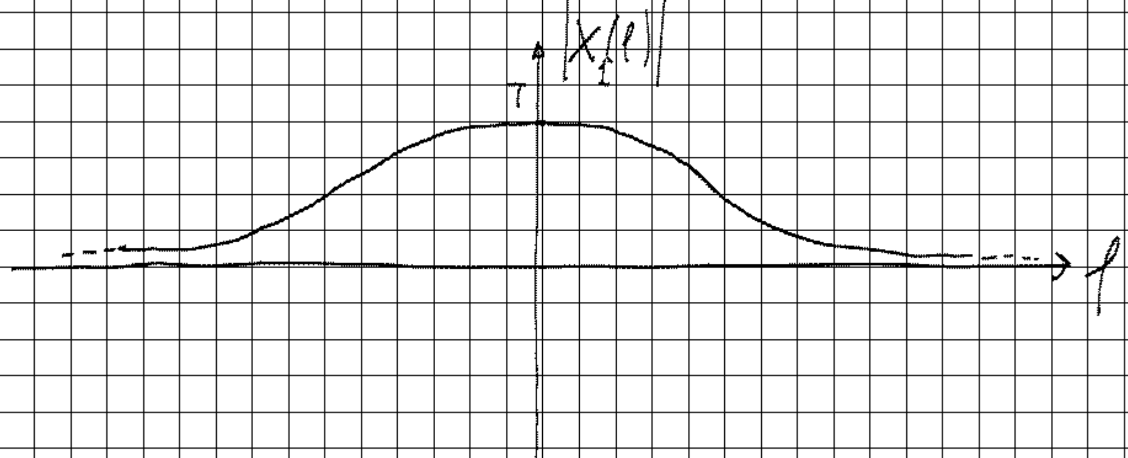
$$H(f) = \underbrace{T\,\mathrm{rect}\left(\frac{f}{1/T}\right)}_{H_1(f)} \otimes \underbrace{2T\,\mathrm{rect}\left(\frac{f}{\frac{1}{2T}}\right)}_{H_2(f)}$$





N.B. $H(f)$ non può provocare distorsioni di fase, ma solo distorsioni di ampiezza

$$X_1(f) = \frac{T}{1 + j2\pi f T} \quad , \quad |X_1(f)| = \frac{T}{\sqrt{1 + 4\pi^2 f^2 T^2}}$$



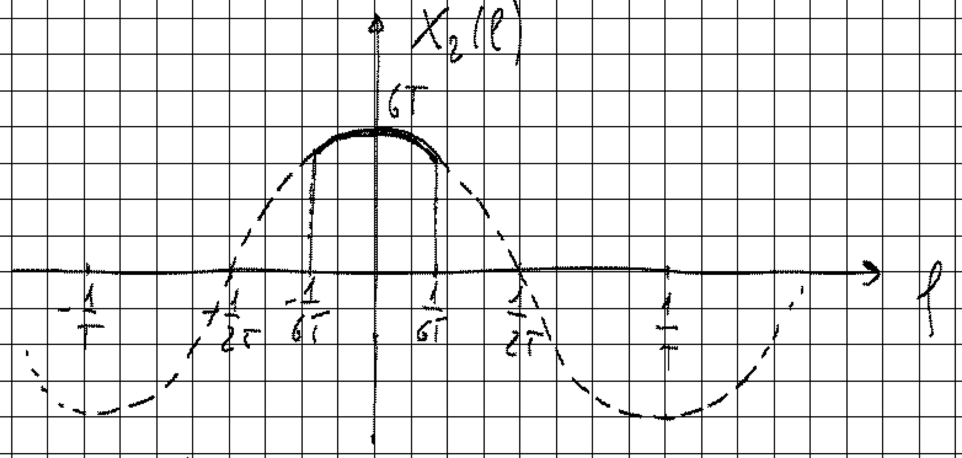
$|Y_1(f)| = |X_1(f)|\,|H(f)|$ è distorto in ampiezza

$\angle Y_1(f) = \angle X(f) + 0$ non è distorto in fase

Calcolo di $y_2(t)$

$$X_2(f) = 3T\,\mathrm{rect}\left(\frac{f}{\frac{1}{3T}}\right) e^{-j2\pi f \frac{T}{2}} + 3T\,\mathrm{rect}\left(\frac{f}{\frac{1}{3T}}\right) e^{j2\pi f \frac{T}{2}} =$$

$$= 6T\,\mathrm{rect}\left(\frac{f}{\frac{1}{3T}}\right) \cos(\pi f T)$$



$Y_2(f) = 4T^2 X_2(f)$ È una replica fedele (non distorto)

$$E_{y_2} = \int_{-\infty}^{+\infty} \left| 4T^2 \cdot 6T \, \mathrm{rect}\left(\frac{f}{\frac{1}{3T}}\right) \cos(\pi f T) \right|^2 df =$$

$$= \left(24T^3\right)^2 \int_{-\frac{1}{6T}}^{\frac{1}{6T}} \cos^2(\pi f T)\, df = \left(24T^3\right)^2 \left[\frac{1}{6T} + \int_{-\frac{1}{6T}}^{\frac{1}{6T}} \cos(2\pi f T)\, df\right] =$$

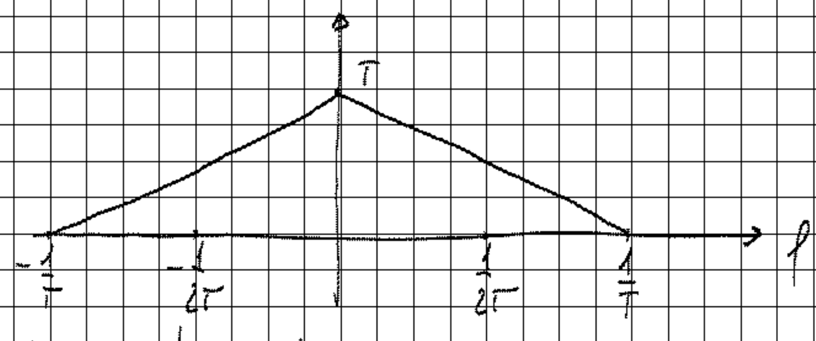
$$= \left(24T^3\right)^2 \left[\frac{1}{6T} + \sin(2\pi f T)\Big|_{-\frac{1}{6T}}^{\frac{1}{6T}}\right] = \left(24T^3\right)^2 \left[\frac{1}{6T} + \sin\left(\frac{\pi}{3}\right) - \sin\left(-\frac{\pi}{3}\right)\right] =$$

$$= \left(24T^3\right)^2 \left[\frac{1}{6T} + \sqrt{3}\right]$$

$P_{y_2} = 0$

Calcolo di $y_3(t)$

$$X_3(f) = T\left(1 - \frac{|f|}{\frac{1}{T}}\right) \mathrm{rect}\left(\frac{f}{\frac{2}{T}}\right)$$

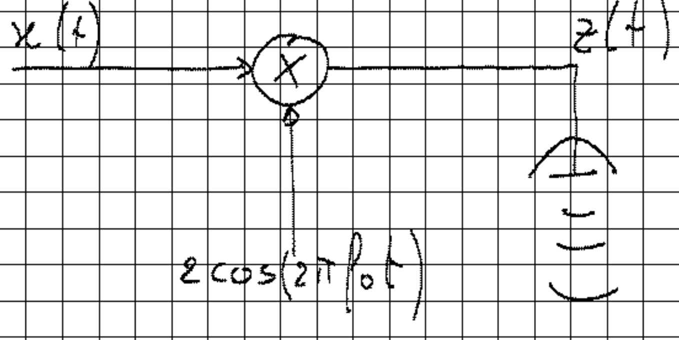


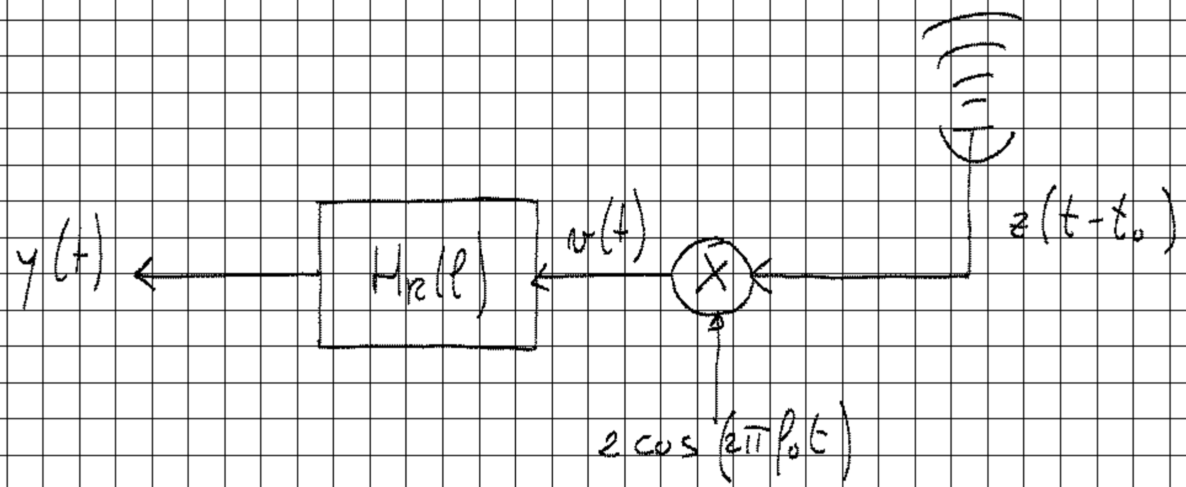
$|Y(f)| = |X(f)|\,|H(f)|$, $\angle Y(f) = \angle X(f)$

è distorto in ampiezza non è distorto in fase

ESERCIZIO - SISTEMA DI TRASMISSIONE RADIO CON MODULAZIONE "DUAL SIDE BAND" (DSB)

$$x(t) = AB\,\mathrm{sinc}^2(Bt)$$





1) Calcolare $Z(f)$ e disegnarne il grafico

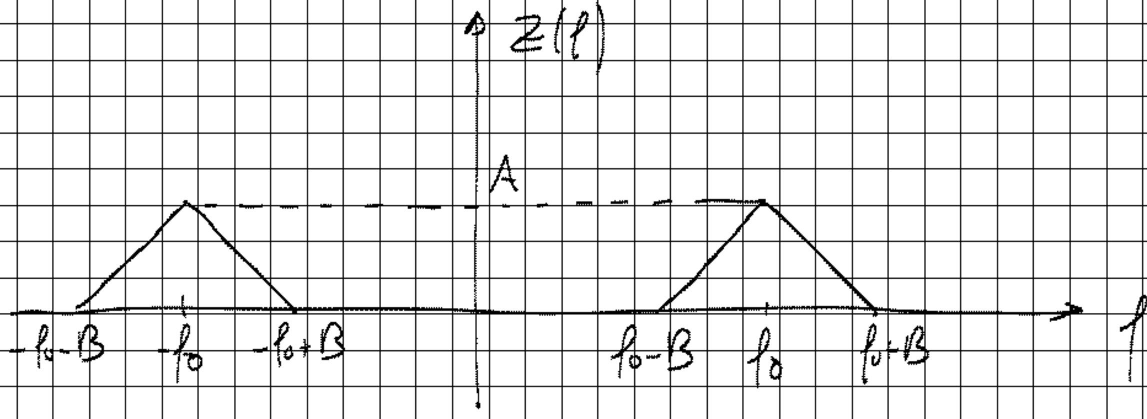
2) Calcolare $y(t)$ e dire se il sistema di trasmissione (visto come un macrosistema lineare e stazionario) introduce distorsione

Soluzione

$$X(f) = A\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$Z(f) = X(f) \otimes \left[\delta(f-f_0) + \delta(f+f_0)\right] = X(f-f_0) + X(f+f_0)$$

$$= A\left(1-\frac{|f-f_0|}{B}\right)\mathrm{rect}\left(\frac{f-f_0}{2B}\right)+A\left(1-\frac{|f+f_0|}{B}\right)\mathrm{rect}\left(\frac{f+f_0}{2B}\right)$$



$$w(t) = z(t-t_0)$$

$$W(f) = Z(f)\,e^{-j2\pi f t_0}$$

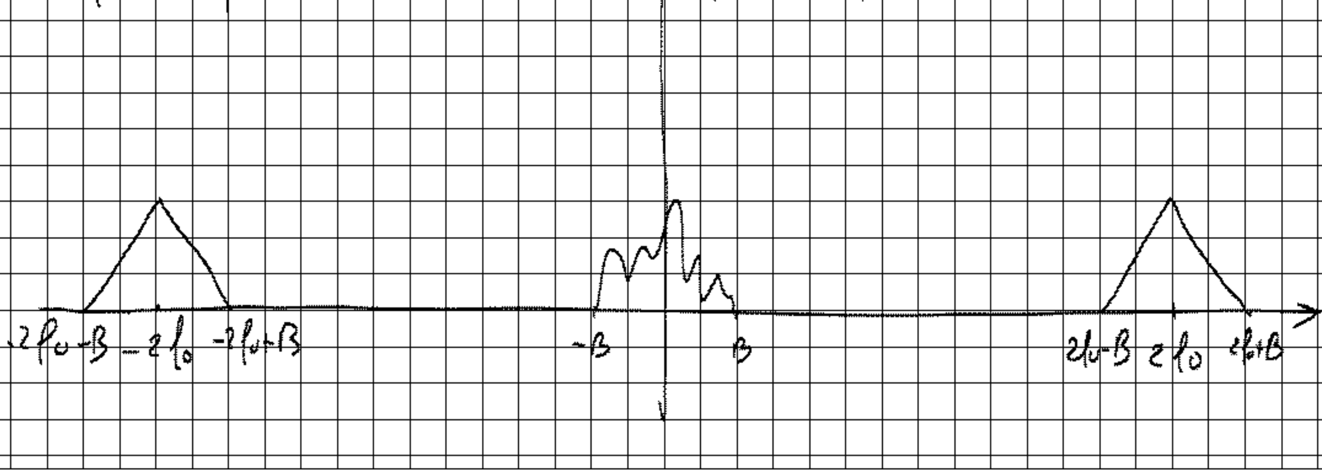
$$V(f) = Z(f)\,e^{-j2\pi f t_0} \otimes \left[\delta(f-f_0)+\delta(f+f_0)\right] =$$

$$= Z(f-f_0)\,e^{-j2\pi(f-f_0)t_0} + Z(f+f_0)\,e^{-j2\pi(f+f_0)t_0} =$$

$$= \left[X(f-2f_0)+X(f)\right]e^{-j2\pi(f-f_0)t_0} +$$

$$+ \left[X(f)+X(f+2f_0)\right]e^{-j2\pi(f+f_0)t_0} =$$

Grafico qualitativo



$$Y(f) = X(f)\left[e^{-j2\pi(f-f_0)t_0} + e^{-j2\pi(f+f_0)t_0}\right] =$$

$$= e^{j2\pi f t_0} X(f)\left[e^{j2\pi f_0 t_0} + e^{-j2\pi f_0 t_0}\right] =$$

$$= 2e^{j2\pi f t_0} X(f)\cos(2\pi f_0 t_0)$$

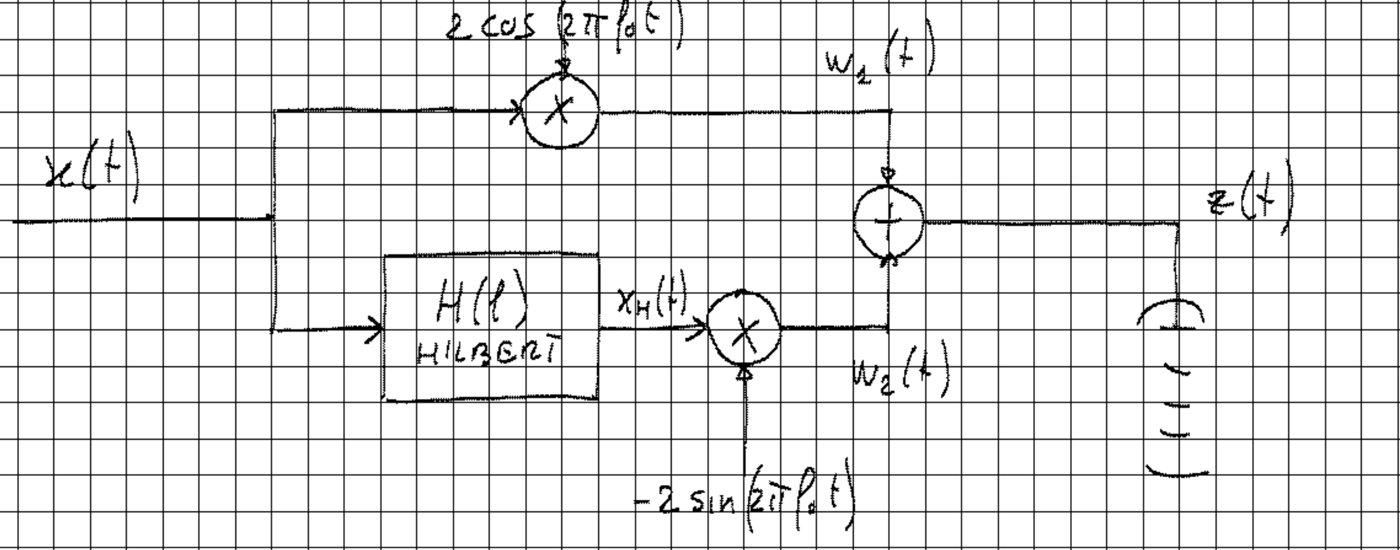
$$y(t) = 2\cos(\varphi_0)\, x(t-t_0) \quad , \quad \varphi_0 = 2\pi f_0 t_0$$

$y(t)$ e' una replica fedele

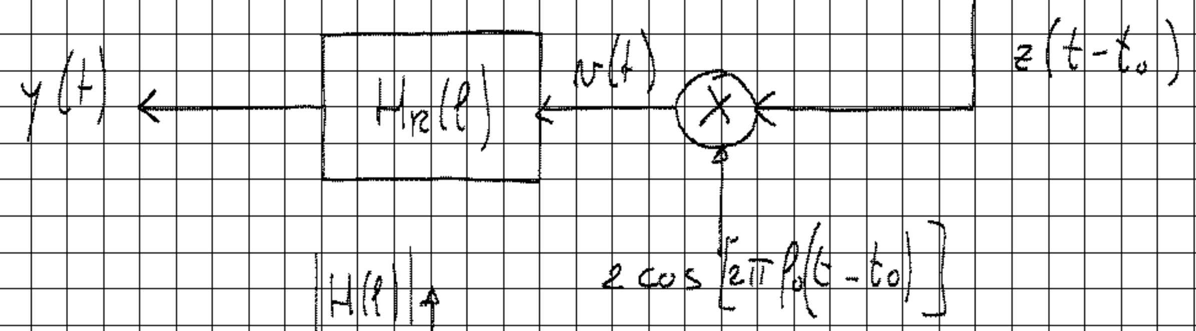
N.B. attenzione a $\cos(\varphi_0)$ poiche' rappresenta una attenuazione. Tale attenuazione deriva dal mezzo di propagazione dell'onda e.m.

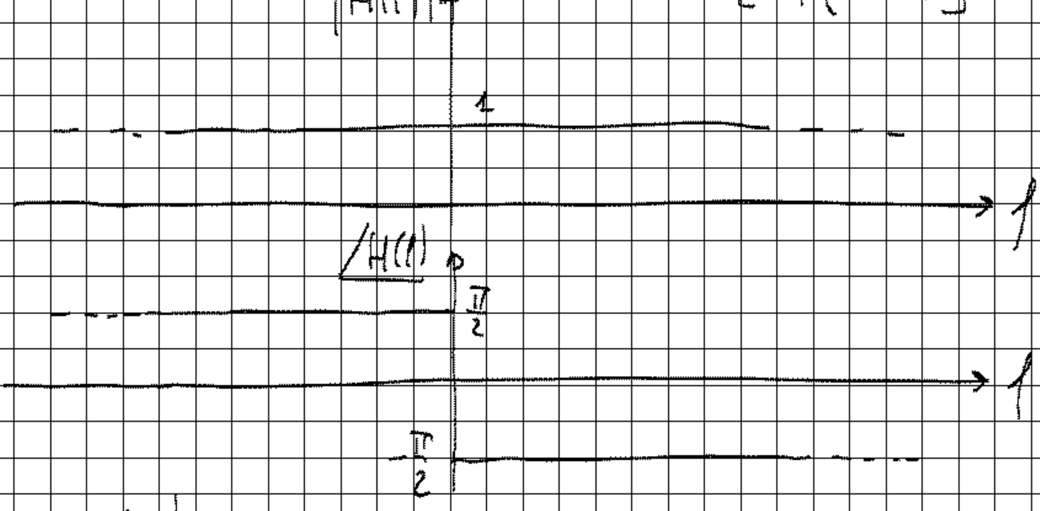
ESERCIZIO – MODULAZIONE SSB

Sia $x(t) = AB\,\mathrm{sinc}(Bt)$ in ingresso al sistema



$H_R(f)$ = passabasso ideale di banda B





1) Calcolare $Z(f)$

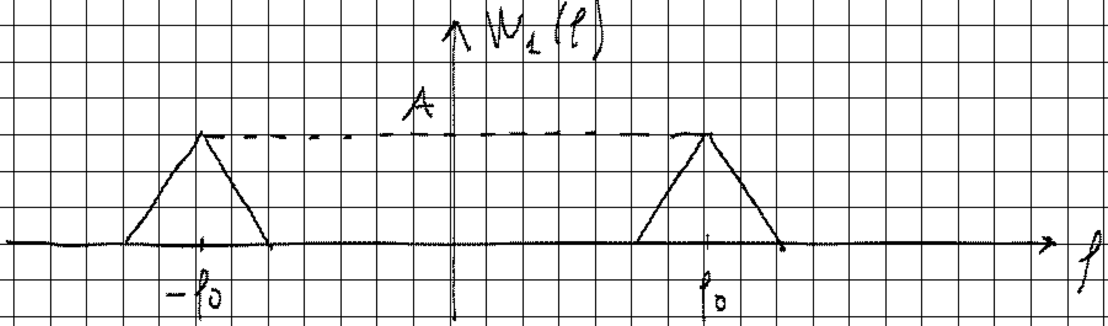
2) Calcolare $y(t)$ e dire se è una replica fedele di $x(t)$

Soluzione

$$z(t) = w_1(t) + w_2(t)$$

$$w_1(t) = 2\,x(t)\cos(2\pi f_0 t)$$

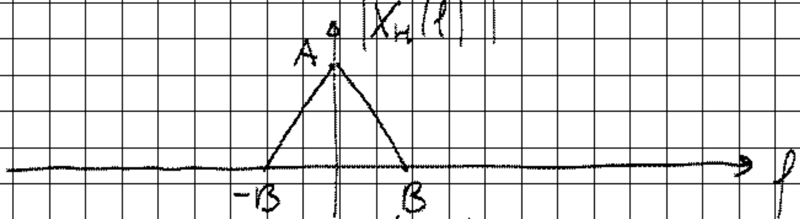
$$W_1(f) = X(f-f_0) + X(f+f_0)$$

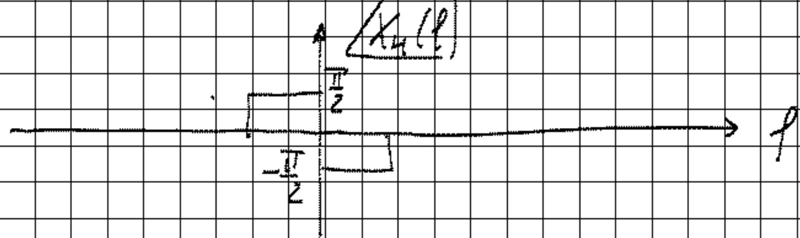


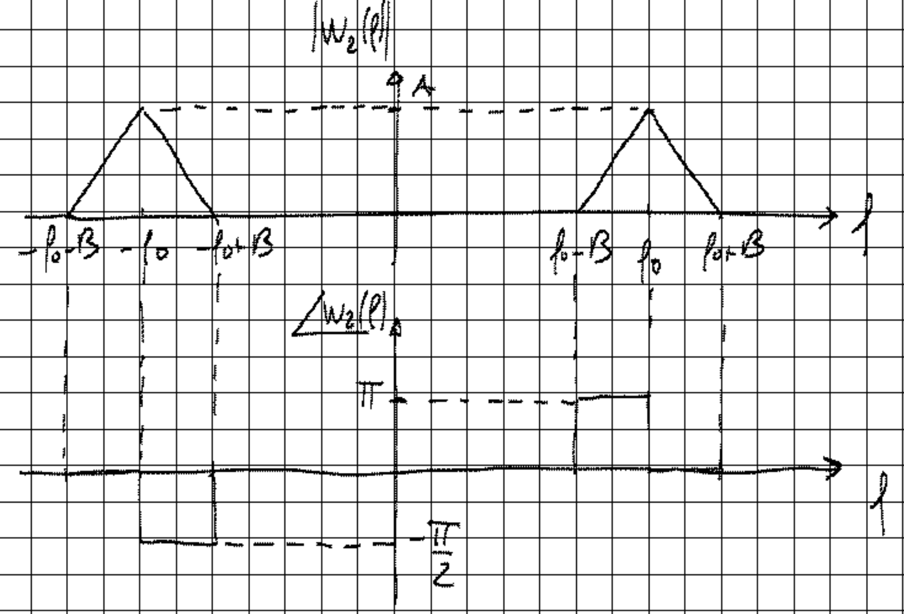
$$w_2(t) = -2\,x_H(t)\sin(2\pi f_0 t)$$

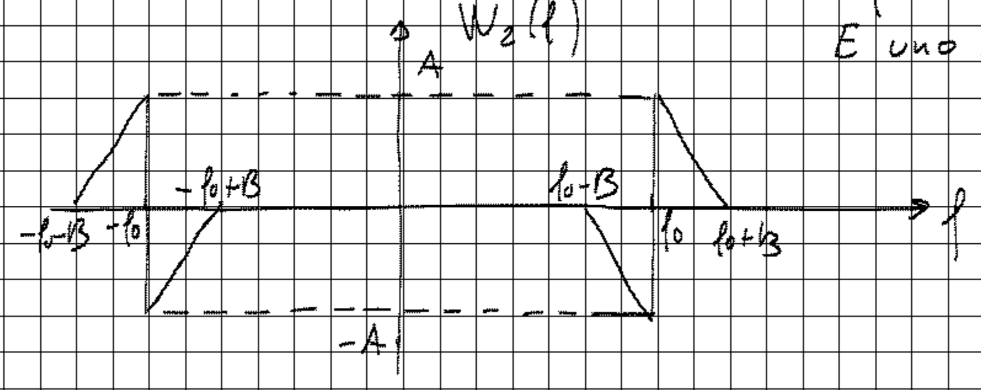
$$W_2(f) = -\left[X_H(f+f_0)\,e^{j\frac{\pi}{2}} + X_H(f-f_0)\,e^{-j\frac{\pi}{2}}\right] =$$

$$= X_H(f-f_0)\,e^{j\frac{\pi}{2}} + X_H(f+f_0)\,e^{-j\frac{\pi}{2}}$$



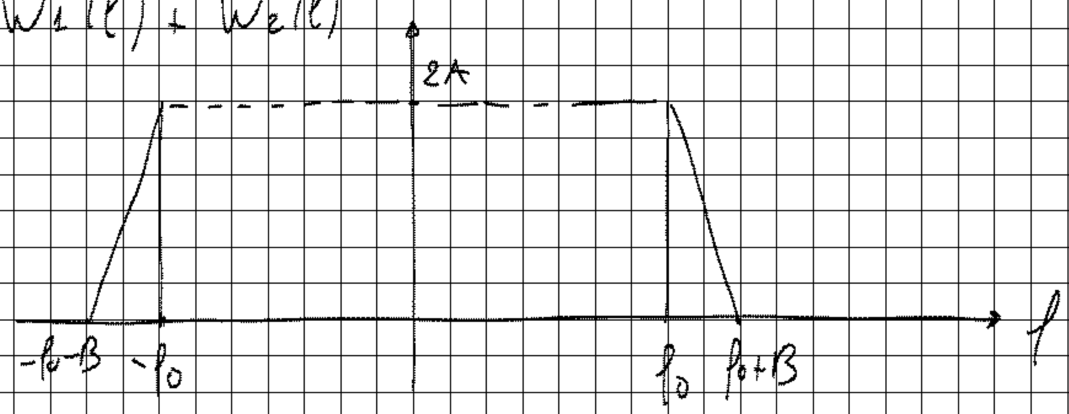






È uno spettro reale

$$Z(f) = W_1(f) + W_2(f)$$



$$Z(f) = 2A\left(1 - \frac{|f-f_0|}{B}\right) \mathrm{rect}\left(\frac{f - f_0 - \frac{B}{2}}{B}\right) + $$
$$+ 2A\left(1 - \frac{|f+f_0|}{B}\right) \mathrm{rect}\left(\frac{f + f_0 + \frac{B}{2}}{B}\right)$$

N.B. la banda del segnale trasmesso si è dimezzata rispetto alla modulazione DSB

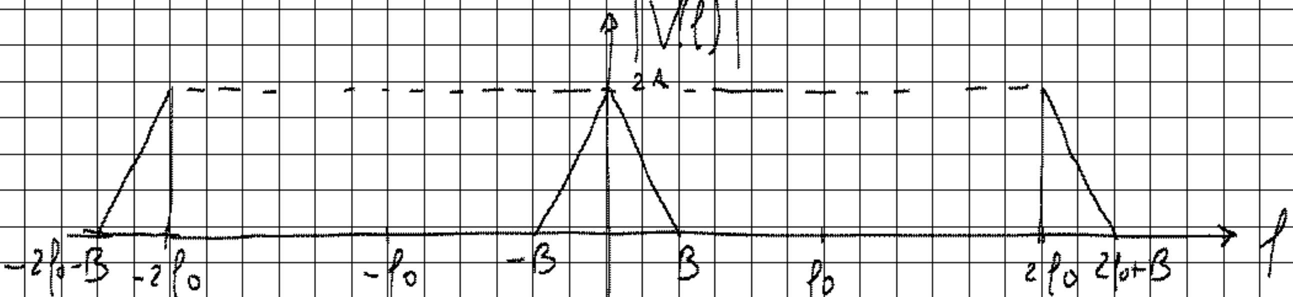
2) $v(t) = 2z(t-t_0)\cos\left[2\pi f_0(t-t_0)\right] =$

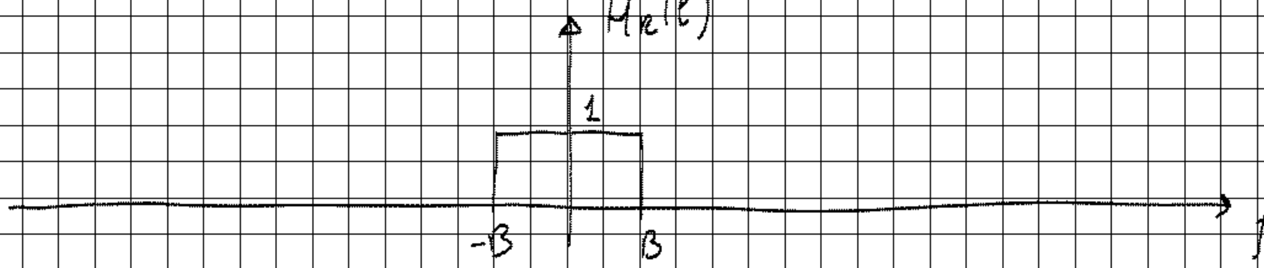
$= 2z(t)\cos(2\pi f_0 t) \otimes \delta(t-t_0)$

$$V(f) = \left[Z(f-f_0) + Z(f+f_0)\right] e^{-j2\pi f t_0} =$$

$$= \left[A\left(1-\frac{|f-2f_0|}{B}\right) rect\left(\frac{f-2f_0-B/2}{B}\right) + A\left(1-\frac{|f+2f_0|}{B}\right) rect\left(\frac{f+2f_0+B/2}{B}\right) + A\left(1-\frac{|f|}{B}\right) rect\left(\frac{f-\frac{B}{2}}{B}\right) + A\left(1-\frac{|f|}{B}\right) rect\left(\frac{f+\frac{B}{2}}{B}\right)\right] e^{-j2\pi f t_0}$$

$$= \left[V_{2f_0}(f) + V_{-2f_0}(f) + A\left(1-\frac{|f|}{B}\right) rect\left(\frac{f}{2B}\right)\right] e^{-j2\pi f t_0}$$



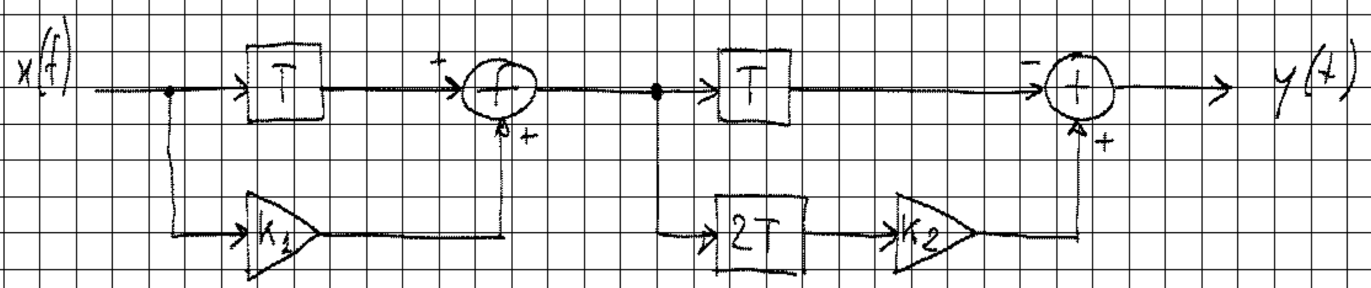


N.B. $\angle H_R(f) = 0$

$$Y(f) = 2A\left(1-\frac{|f|}{B}\right) rect\left(\frac{f}{2B}\right) e^{-j2\pi f t_0}$$

$y(t) = 2x(t-t_0)$ è ovviamente una replica fedele

ESERCIZIO - STUDIO DI UN SISTEMA



·) Dire se il sistema e'

·) Stazionario

·) Lineare

·) Senza memoria

·) Causale

·) Stabile BIBO

·) Calcolare inoltre la risposta impulsiva e in frequenza

Soluzione

Calcoliamo l'uscita $y(t)$ in funzione dell'ingresso $x(t)$

Definiamo $z(t)$ l'uscita del primo sommatore

$$z(t) = x(t-T) + k_1 x(t)$$

$$y(t) = -z(t-T) + k_2 z(t-2T)$$

$$y(t) = -x(t-2T) - k_1 x(t-T) + k_2 x(t-3T) + k_2 k_1 x(t-2T)$$

$$= -k_1 x(t-T) + (k_2 k_1 - 1) x(t-2T) + k_2 x(t-3T)$$

1) Stazionarieta'

$$x(t-t_0) \longrightarrow \boxed{T[\cdot]} \longrightarrow y'(t) = y(t-t_0)$$

$$z'(t) = x(t-t_0-T) + k_1 x(t-t_0)$$

$$y'(t) = -z'(t-T) + k_2 z(t-2T)$$

$$y'(t) = -x(t-t_0-2T) - K_1 x(t-t_0-T) + K_2 x(t-t_0-3T) +$$
$$+ K_1 K_2 \, x(t-t_0-2T) =$$
$$= -K_1 x(t-t_0-T) + (K_1K_2 - 1) x(t-t_0-2T) + K_2 x(t-t_0-3T)$$
$$= y(t-t_0)$$

È STAZIONARIO

2) LINEARITÀ

$$x(t) = a x_1(t) + b x_2(t) \longrightarrow \boxed{T[\cdot]} \longrightarrow y(t) = a y_1(t) + b y_2(t)$$

$$z(t) = a\, x_1(t-T) + a K_1 x_1(t) + b x_2(t-T) + b K_1 x_2(t)$$
$$y(t) = -\underline{a x_1(t-2T)} - \underline{a K_1 x_1(t-T)} - b x_2(t-2T) +$$
$$- b K_1 x_2(t-T) + \underline{a K_2 x_1(t-3T)} + \underline{a K_1 K_2 x_1(t-2T)} +$$
$$+ b K_2 x_2(t-3T) + b K_1 K_2 x_2(t-2T) =$$
$$= a\left[-K_1 x_1(t-T) + (K_1K_2 - 1) x_1(t-2T) + K_2 x_1(t-3T)\right] +$$
$$+ b\left[-K_1 x_2(t-T) + (K_1K_2 - 1) x_2(t-2T) + K_2 x_2(t-3T)\right]$$
$$= a\, y_1(t) + b\, y_2(t)$$

IL SISTEMA È LINEARE

3) MEMORIA

L'uscita dipende dall'ingresso ad istanti precedenti per cui IL SISTEMA È CON MEMORIA

4) CAUSALITÀ

L'uscita non dipende dall'ingresso ad istanti futuri per cui IL SISTEMA È CAUSALE

5) STABILITÀ BIBO

$$|x(t)| < M \quad \forall t$$

$$|y(t)| = \left|-K_1 x(t-T) + (K_2K_1 - 1) x(t-2T) + K_2 x(t-3T)\right|$$

$$\leq K_1 \left| x(t-T) \right| + \left| K_2 K_1 - 1 \right| \cdot \left| x(t-2T) \right| + |K_2| \cdot \left| x(t-3T) \right|$$

$$\leq K_1 \cdot M + |K_2 K_1 - 1| \cdot M + |K_2| \cdot M = N < \infty$$

IL SISTEMA È STABILE BIBO

.) Il sistema è lineare e stazionario per cui ha senso parlare di risposta impulsiva ed in frequenza

$$x(t) = \delta(t) \longrightarrow \boxed{SLS} \longrightarrow y(t) = h(t)$$

$$h(t) = y(t) \Big|_{x(t) = \delta(t)}$$

$$h(t) = -K_1 \, \delta(t-T) + (K_1 K_2 - 1)\, \delta(t-2T) + K_2 \, \delta(t-3T)$$

$$H(f) = -K_1 e^{-j2\pi f T} + (K_1 K_2 - 1) e^{-j4\pi f T} + K_2 e^{-j6\pi f T}$$