Soluzioni dei compiti # 3

(1a) $L(t) = p + t(q-p)$
$$= (1,2,3) + t((2,3,5) - (1,2,3))$$
$$= (1,2,3) + t(1,1,0)$$

$2 = 5-3$

(1b) $L(t) = (0,1,1)$?

$1+t = 0 \implies$ $t = -1$

$2+t = 1$? $2+(-1) = 1$ ✓

$3+2t = 1$? $3+2(-1) = 1$ ✗ ✓

$\therefore (0,1,1)$ non c' su questa linea.

(2a) $L(t) = p + tv = (1,1,1) + t(1,2,3)$

(2b) $1 + 3t = 0 \implies t = -1/3$

$L(-1/3) = (1 - 1/3, 1 - 2/3, 0) = (2/3, 1/3, 0)$

(3) $L(t) = (1,0,1) + t((0,1,0) - (1,0,1))$
$$= (1,0,1) + t(-1,1,-1) = (x(t), y(t), z(t))$$

$\therefore L(t) \cap \{x - y + z = 0\}$

$\implies x(t) + y(t) + z(t) = 0$

$\therefore \underbrace{(1-t)}_{x(t)} + \underbrace{t}_{y(t)} + \underbrace{(1-t)}_{z(t)} = 2 - t = 0$

$\implies t = 2$

$L(2) = (-1, 2, -1)$

①

(4) $\underline{X}(s,t) = p + s(q-p) + t(r-p)$

$= (6,6,0) + s((8,4,0)-(6,6,0))$
$\qquad + t((9,3,1) - (6,6,0))$

$= (6,6,0) + s(2,-2,0) + t(3,-3,1)$

$= (x(s,t), y(s,t), z(s,t))$

$\therefore \quad x(s,t) + y(s,t) = (6+2s+3t) + (6-2s-3t)$

$\qquad\qquad = 12$

$\therefore \quad \{\underline{X}(s,t) \mid (s,t) \in \mathbb{R}^2\} = \{(x,y,z) \in \mathbb{R}^3 \mid x+y=12\}$

(5b) $\left.\begin{array}{l} A - B + C = D \\ -A + B - C = D \\ A - C = D \end{array}\right\} \longrightarrow \left(\begin{array}{ccc|c} 1 & -1 & 1 & D \\ -1 & 1 & -1 & D \\ 1 & 0 & -1 & D \end{array}\right) \Big\} \text{ lezioni 1}$

$\left(\begin{array}{ccc|c} 1 & -1 & 1 & D \\ -1 & 1 & -1 & D \\ 1 & 0 & -1 & D \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} +1 & -1 & 1 & D \\ 0 & 0 & 0 & 2D \\ 1 & 0 & -1 & D \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 1 & D \\ 0 & 1 & -2 & \mathbf{0} \\ 0 & 0 & 0 & 2D \end{array}\right) \quad \therefore \boxed{D = 0}$

$B - 2C = 0 \Rightarrow B = 2C$

$A - B + C = A - C = 0 \Rightarrow A = C$

$(A, B, C) = C(1, -2, 1)$

$\underline{\text{Scelta} \quad C = 1} \qquad x - 2y + z = 0$

(5b) $\left.\begin{array}{l} A + B = D \\ A + B + C = D \\ 4A - 2B = D \end{array}\right\} \quad \left(\begin{array}{ccc|c} 1 & 1 & 0 & D \\ 1 & 1 & 1 & D \\ 4 & -2 & 0 & D \end{array}\right)$

$\longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & D \\ 0 & 0 & 1 & 0 \\ 0 & -6 & 0 & -3D \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & D \\ 0 & -6 & 0 & -3D \\ 0 & 0 & 1 & 0 \end{array}\right)$

②

⑤b) ∴ $C = 0$

$B = \frac{1}{2} D$

$A + \frac{1}{2} D \cancel{=} D \implies A = \frac{1}{2} D$

$D = 0 \implies (A, B, C) = 0$

Scelta $D = 1$

$\frac{1}{2} x + \frac{1}{2} y = 1$.

$(4, 2, -1) : \left(\frac{1}{2}\right)(4) + \left(\frac{1}{2}\right)(-2) + (0)(1) = 1$

$\implies (4, 2, -1)$ è su questo piano.

⑥

$a + b = (2, 5)$

$a - b$

$a = (3, 3)$

$(-1, 2) = b$

$(0, 0)$

$a - b = (3, 3) - (-1, 2) = (4, 1)$

$a + 2b = (-1, 5)$

$(-3, 3)$

$(2, 2)$

a

$2b$

$(0, 0)$

$a = (-3, 3)$
$2b = (2, 2)$
$\overline{a + 2b = (-1, 5)}$

$(-3, 3) = a$

$a - 3b = (-6, 0)$

$(0, 0)$

$(-3, -3) = -3b$

$a = (-3, 3)$
$-3b = (-3, -3)$
$\overline{a - 3b = (-6, 0)}$