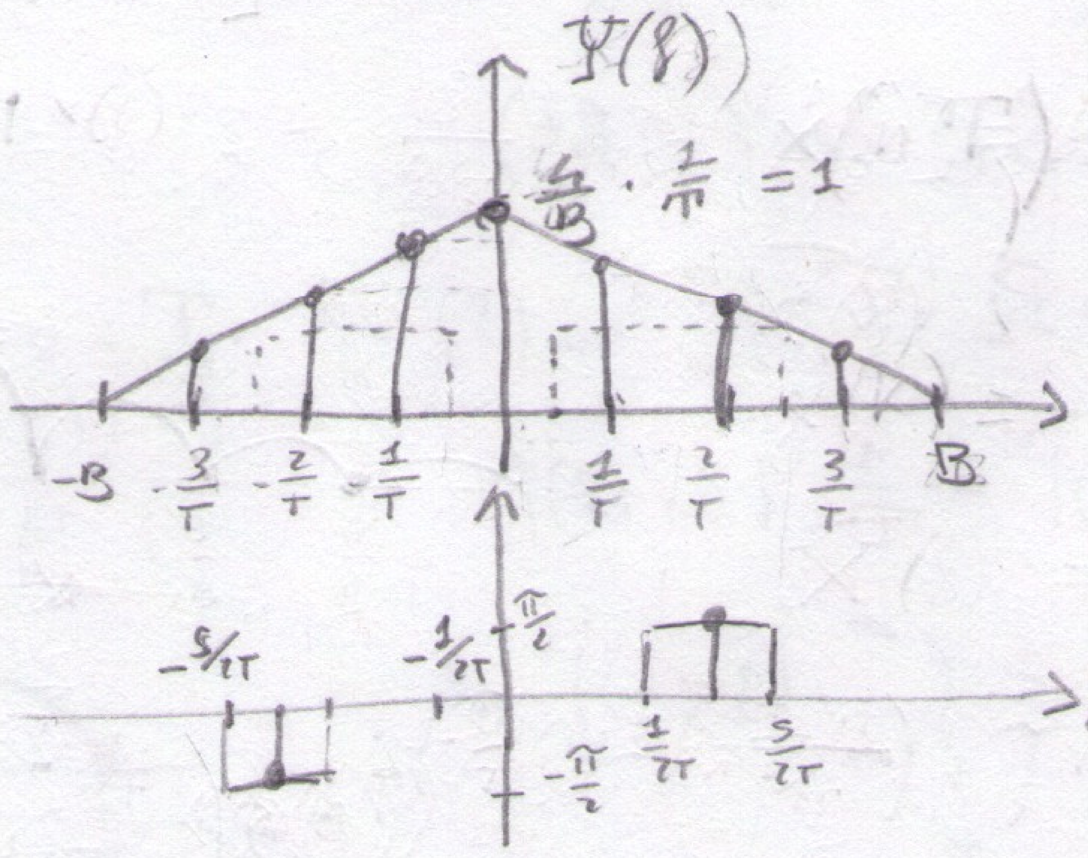
Esercizio 1

$x(t)=\sum_n x_0(t-nT)$; $x_0(t)=4\,\text{sinc}^2(Bt)$ $\quad T=\frac{4}{B}$ segnale periodicizzato

$X_0(f)=\frac{4}{B}\left(1-\frac{|f|}{B}\right)\text{rect}\left(\frac{f}{2B}\right)$

$T=\frac{4}{B} \Rightarrow \frac{1}{T}=\frac{B}{4}$



$Y(f)=\frac{1}{2}\delta\left(f+\frac{2}{T}\right)e^{-j\frac{\pi}{2}}+\frac{3}{4}\delta\left(f+\frac{1}{T}\right)+\frac{3}{4}\delta\left(f-\frac{1}{T}\right)+\frac{1}{2}\delta\left(f-\frac{2}{T}\right)e^{j\frac{\pi}{2}}$

$y(t)=\frac{-1\cdot 2j}{2}\left[\frac{e^{j2\pi\frac{2}{T}t}-e^{-j2\pi\frac{2}{T}t}}{2j}\right]+\frac{3\cdot 2}{4}\left[\frac{e^{j2\pi\frac{1}{T}t}+e^{-j2\pi\frac{1}{T}t}}{2}\right]=$

$=\frac{-2\cdot -j}{2}\,\text{sen}\left(2\pi\frac{t}{T}\right)+\frac{3}{2}\cos\left(2\pi\frac{t}{T}\right)=\frac{4}{B}\,\text{sen}\left(4\pi\frac{t}{T}\right)+\frac{3}{2}\cos\left(2\pi\frac{t}{T}\right)$ (1)

Esercizio 2

$E_{ST}=\sum_{m=0}^{1}E_k P_k=E_0P_0+E_1P_1$

$E_0=\int_{-\infty}^{\infty}|s_k(t)|^2dt=\int_{-\infty}^{\infty}|m_0\,g_T(t)|^2dt=\int_{-\infty}^{\infty}(-e)^2|g_T(t)|^2dt=e^2\int_{-\infty}^{\infty}|G_T(f)|^2df=$

$=e^2\int_{-\frac{1}{T}}^{\frac{1}{T}}\left(T^2|f|e^{-|f|}\right)^2df=e^2\int_{-\frac{1}{T}}^{\frac{1}{T}}T^4f^2e^{-2|f|}df=e^2T^4\int_{-\frac{1}{T}}^{\frac{1}{T}}f^2e^{-2|f|}df=2e^2T^4\int_0^{\frac{1}{T}}f^2e^{-2f}df=$

$=2e^2T^4\left[\frac{e^{-2f}}{(-2)^2}(-2f-1)\right]_0^{\frac{1}{T}}=2e^2T^4\left(\frac{e^{-\frac{2}{T}}}{4}\left(-\frac{2}{T}-1\right)+\frac{1}{4}\right)=\frac{e^{2-\frac{2}{T}}}{2}T^4\left(-\frac{2}{T}-1\right)+\frac{e^2T^4}{2}=$

$=-e^{2-\frac{2}{T}}T^3-\frac{e^{2-\frac{2}{T}}}{2}T^4+\frac{e^2T^4}{2}$

$E_1=\int_{-\infty}^{\infty}|s_k(t)|^2dt=\int_{-\infty}^{\infty}|m_1\,g_T(t)|^2dt=\int_{-\infty}^{\infty}(2e)^2|g_T(t)|^2dt=4e^2\int_{-\infty}^{\infty}|G_T(f)|^2df=$

$=2\cdot 4e^2T^4\left[\frac{e^{-\frac{2}{T}}}{4}\left(-\frac{2}{T}-1\right)+\frac{1}{4}\right]=2e^2T^4\left(e^{-\frac{2}{T}}\cdot-\frac{2}{T}-e^{-\frac{2}{T}}+\frac{1}{4}\right)=$

$=2e^2T^4\left(-\frac{2e^{-2/T}}{T}-e^{-2/T}+\frac{1}{4}\right)$

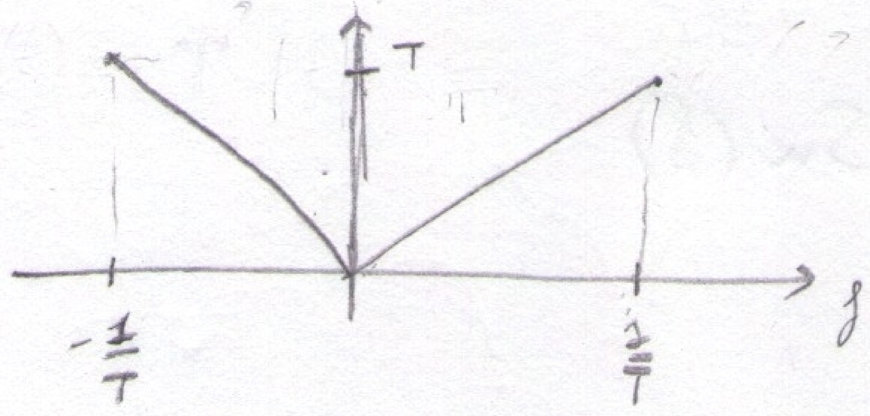
$E_{ST}=\frac{-e^{2-\frac{2}{T}}}{2}T^3-\frac{e^{2-\frac{2}{T}}}{4}T^4+\frac{e^2T^4}{4}-\frac{2e^{2-\frac{2}{T}}T^4}{T}-e^{2-\frac{2}{T}}T^4+\frac{e^2T^4}{4}=$

$=\cancel{\frac{-2e^{2-\frac{2}{T}}T^4-e^{2-\frac{2}{T}}T^5+e^2T^5-6e^{2-\frac{2}{T}}T^4-4e^{2-\frac{2}{T}}T^5+e^2T^5}{4T}}=$

$=-2e^{2-\frac{2}{T}}T^3-\frac{5}{4}e^{2-\frac{2}{T}}T^4+\frac{1}{2}e^2T^4$ (1) sbagliato...

$$g(t) \triangleq g_{TC}(t) \otimes g_R(t) \rightleftarrows G(f) = G_{TC}(f)\, G_R(f)$$

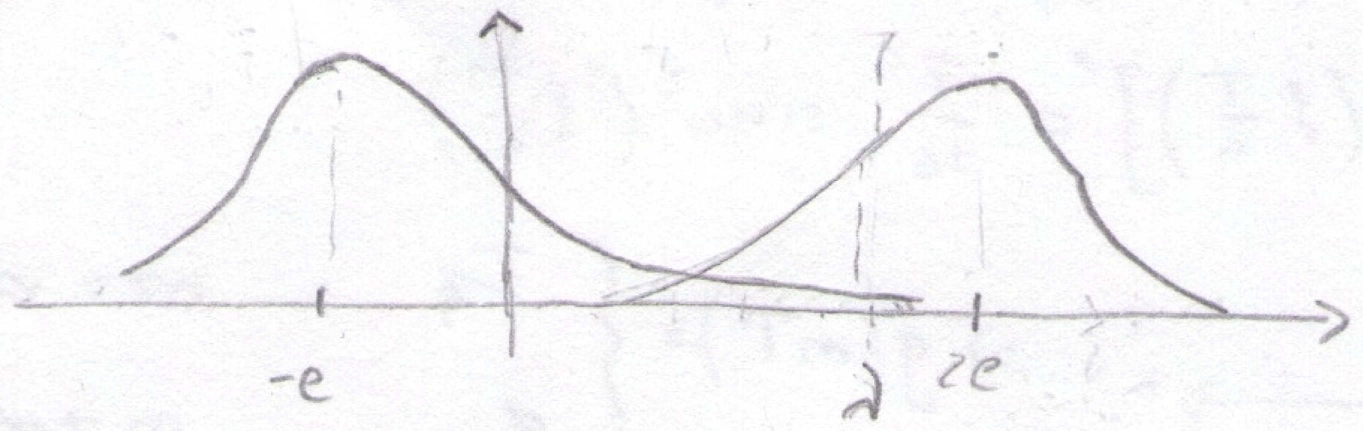
$$G(f) = T^2 |f|\, e^{-|f|}\, \mathrm{rect}\left(\frac{f}{2/T}\right) \cdot e^{|f|}\, \mathrm{rect}\left(\frac{f}{2/T}\right) = T^2 |f| \cdot \mathrm{rect}\left(\frac{f}{2/T}\right)$$



$$\sum_n G\left(f - \frac{n}{T}\right) = T \quad \text{ok} \quad \Rightarrow \text{ok anche nel tempo}$$

$$x_k = a_k + n_k \qquad n(t) \triangleq w(t) \otimes g_R(t) \; ; \; n_k \triangleq n(kT) \in \mathcal{N}(0, \sigma_n^2)$$

$$x_k\big|_{k=0} = -e + n_k \in \mathcal{N}(-e, \sigma_n^2) \; ; \; x_k\big|_{k=1} = 2e + n_k = \in \mathcal{N}(2e, \sigma_n^2)$$



$$P_r\left[\hat{-e} \mid 2e\right] = P_r\left[x_k \le \lambda \mid 2e\right] = 1 - Q\left(\frac{\frac{3e}{2} - 2e}{\sigma_n}\right) = 1 - Q\left(\frac{-e}{2\sigma_n}\right) = Q\left(\frac{e}{2\sigma_n}\right)$$

$$P_r\left[\hat{2e} \mid -e\right] = P_r\left[x_k > \lambda \mid -e\right] = Q\left(\frac{\frac{3e}{2} + e}{\sigma_n}\right) = Q\left(\frac{5e}{2\sigma_n}\right)$$

$$\sigma_n^2 = P_n = \int_{-\infty}^{\infty} S_w(f) |G_R(f)|^2 df = \frac{N_0}{2} \int_{-\infty}^{\infty} |G_R(f)|^2 df = \frac{N_0}{2}\left(2T^4\left[\frac{e^{-\frac{2}{T}}}{4}\left(-\frac{2}{T} - 1\right) + \frac{1}{4}\right]\right) =$$

$$= N_0 T^4 \left[\left(-\frac{1}{2}\frac{e^{-2/T}}{T} - \frac{e^{-\frac{2}{T}}}{4}\right) + \frac{1}{4}\right] = -\frac{N_0 T^3 e^{-2/T}}{2} - \frac{N_0 T^4 e^{-2/T}}{4} + \frac{N_0 T^4}{4}$$

$$P_r(e) = \frac{1}{2} Q\left(\frac{e}{2\sigma_n}\right) + \frac{1}{2} Q\left(\frac{5e}{2\sigma_n}\right) \quad (2)$$

$$\lambda_{opt} = 0{,}5 \quad (3)$$