Esercizio # 1 compitino 01/06/2010

Al ricevitore di Fig. 1 e' applicato il segnale

$$r(t) = \sum_i a_i g_T(t - iT) \cos(2\pi f_0 t) + w(t)$$

$w(t)$ Gaussiano ~~bianco~~ a valor medio nullo con $S_w(f)$ in Fig2.

$G_T(f) = \text{rect}\left(\frac{fT}{2}\right)$

$a_i \in A = [-1, +1]$ indipendenti ed equiprobabili.

$G_R(f) = T\left(1 - |f|T\right) \text{rect}\left(\frac{fT}{2}\right)$

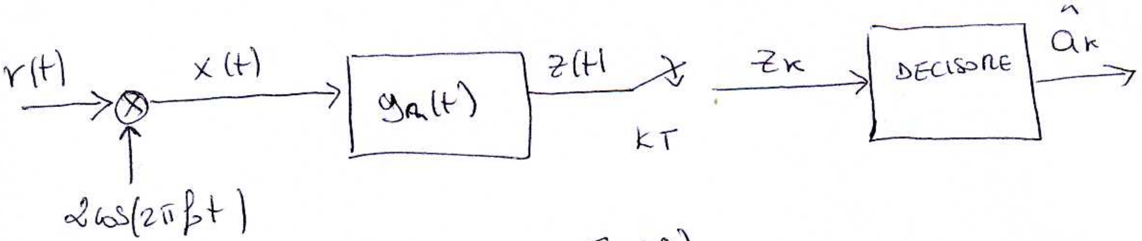
Si determini:

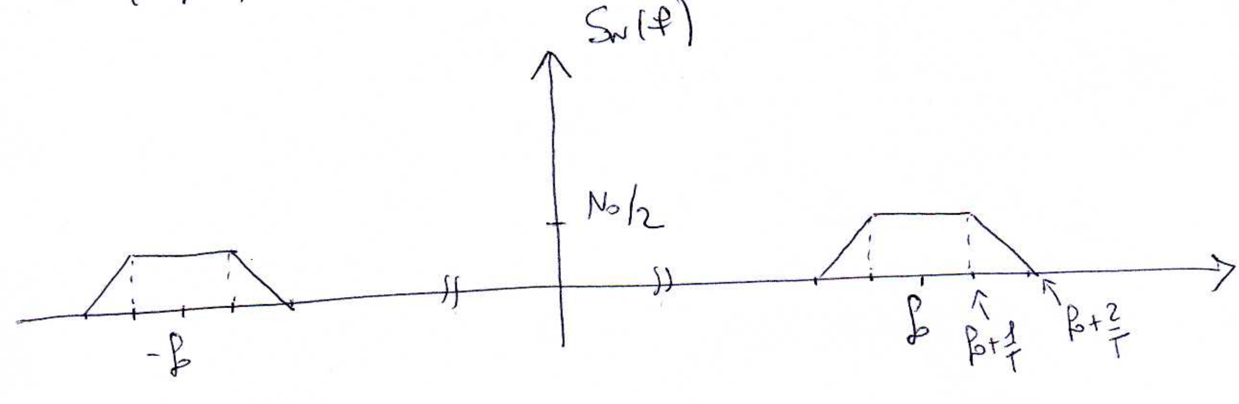
1) Energia media per simbolo ricevuto

2) DSP del rumore all'uscita di $g_R(t)$

3) Risposta impulsiva del sistema $g(t) = g_T(t) \otimes g_R(t)$

4) La probabilità di errore se $\lambda = 1/4$





1) $E_S = E\left\{ \int_0^T s^2(t)\,dt \right\}$ $\qquad s(t) = \sum_i a_i\, g_T(t-iT)$

$E\{a_i a_{i+m}\} = R_a(m) = \delta[m]$

$$E_S = \int_{-\infty}^{+\infty} s^2(t)\,dt = \int_{-\infty}^{+\infty} G_T^2(f)\,df = \int_{-\frac{1}{T}}^{1/T} 1\,df = \frac{2}{T}$$

3) $x(t) = r(t) \cdot 2\cos(2\pi f_0 t) =$

$= \sum_i a_i\, g_T(t-iT)\cos(2\pi f_0 t)\cdot 2\cos(2\pi f_0 t) + w(t)\, 2\cos(2\pi f_0 t) =$

$= \sum_i a_i\, g_T(t-iT)\left(1 - \cos(4\pi f_0 t)\right) +$

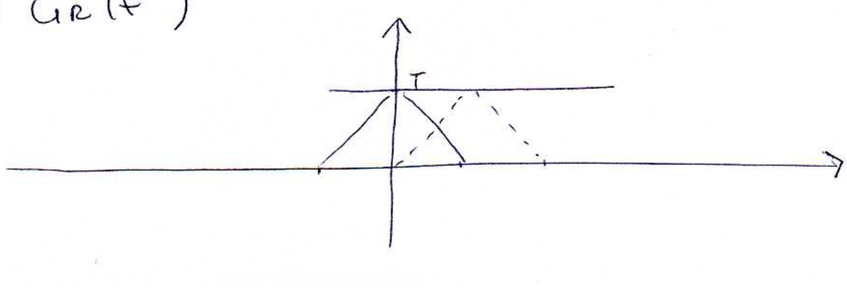
$w_c(t)\left(1+\cos(4\pi f_0 t)\right) + w_s(t)\sin(4\pi f_0 t)$

$z(t) = \sum_i a_i\, g(t-iT) + w_c(t) \otimes g_R(t)$

$g(t) = g_T(t) \otimes g_R(t)$

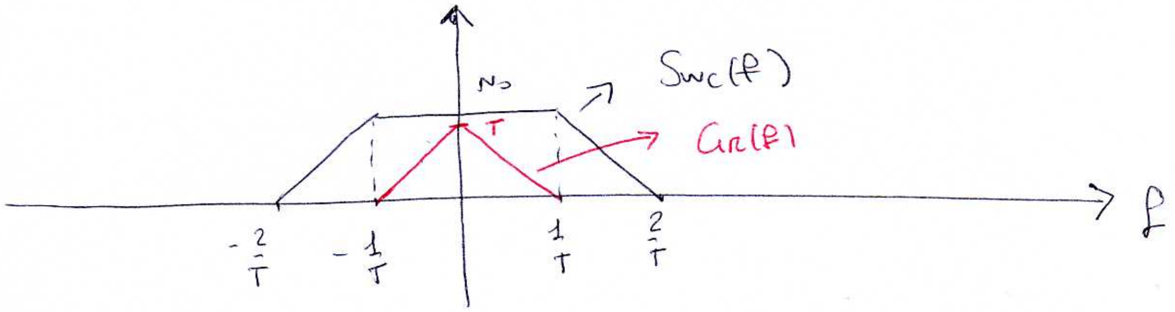
$G(f) = G_T(f)\, G_R(f) = T\, G_R(f)$

$\sum_k G\left(f - \frac{k}{T}\right) = T$



2) $n_c(t) = w_c(t) \otimes g_R(t)$

$$S_{n_c}(f) = S_{w_c}(f)\,|G_R(f)|^2 = N_0\,|G_R(f)|^2$$

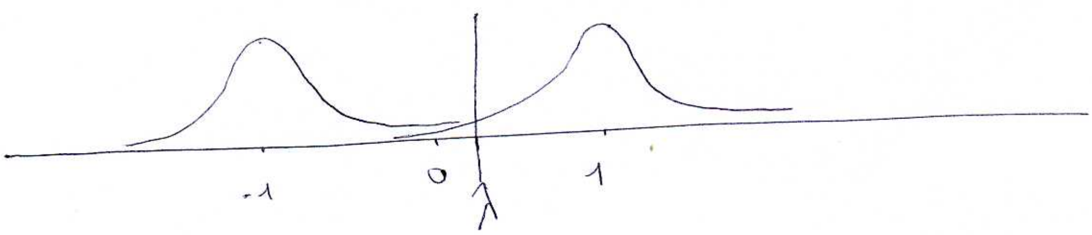


$$\sigma_n^2 = N_0 2 \int_0^{1/t} G_n(f)^2\, df = 2N_0 \cdot \frac{1}{T} \cdot T^2 \cdot \frac{1}{3} = \frac{2 N_0 T}{3}$$

4) $z_k = a_k + n_k$

$z_k|_{a_k=1} \in \mathcal{N}(1, \sigma_n^2)$

$z_k|_{a_k=-1} \in \mathcal{N}(-1, \sigma_n^2)$



$$P(e) = \frac{1}{2} \Pr\{1 + n_k < \lambda | a_k = 1\} + \frac{1}{2} \Pr\{-1 + n_k > \lambda \,|\, a_k = -1\}$$

$$\Pr\{1 + n_k < \lambda \,|\, a_k = 1\} = Q\left(\frac{1 - 1/4}{\sigma_n}\right) = Q\left(\frac{3}{4\sigma_n}\right)$$

$$\Pr\{-1 + n_k > \lambda \,|\, a_k = -1\} = Q\left(\frac{1/4 + 1}{\sigma_n}\right) = Q\left(\frac{5}{4\sigma_n}\right)$$