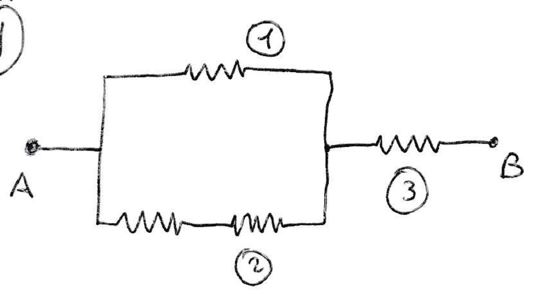
ES.

(1)



$$P_F^{2} = P\{\text{ramo } ② \text{ funziona}\} = (1-0,3)\cdot(1-0,15) = 0,595$$

$$P_G^{2} = P\{\text{ramo } ② \text{ è guasto}\} = 1-0,595 = 0,405$$

$$P_G^{1//2} = P_G^{1}\cdot P_G^{2} = 0,25\cdot 0,405 = 0,10125$$

$$P_F^{1//2} = 1 - P_G^{1//2} = 0,89875$$

$$P_F^{AB} = P_F^{3}\cdot P_F^{1//2} = (1-0,2)\cdot 0,89875 = 0,719$$

ES.

(2) $F_X(x) = P\{X \leq x\} = \left(1 - e^{-\frac{x}{\lambda}}\right)u(x)$

a) $P\{X > 15 \text{ minuti}\} = \frac{1}{e}$

$$1 - F_X(15) = \frac{1}{e}$$

$$1 - \left(1 - e^{-\frac{15}{\lambda}}\right) = \frac{1}{e}$$

$$e^{-\frac{15}{\lambda}} = e^{-1}$$

$$-\frac{15}{\lambda} = -1$$

$$\boxed{\lambda = 15 \text{ minuti}}$$

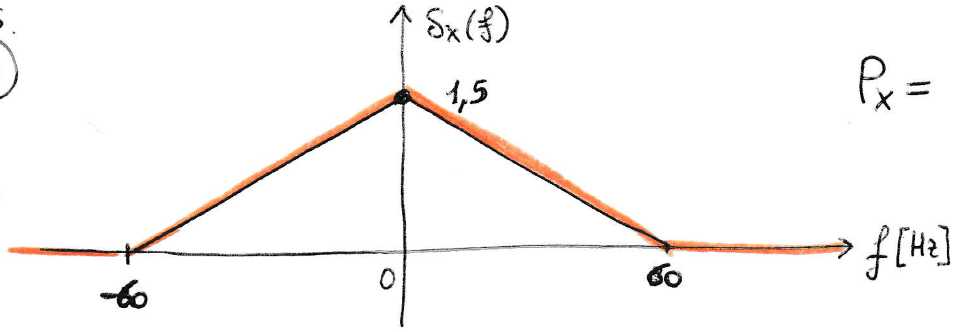
$$P\{X \leq x_0\} = F_X(x_0) = 0,1 = \frac{1}{10}$$

$$1 - e^{-\frac{x_0}{15}} = \frac{1}{10} \rightarrow e^{-\frac{x_0}{15}} = \frac{9}{10} \rightarrow -\frac{x_0}{15} = \ln\left(\frac{9}{10}\right)$$

$$x_0 = 15\cdot \ln\left(\frac{10}{9}\right) \simeq 1,58 \text{ minuti} \simeq 95 \text{ secondi}$$

ES. 3

a)

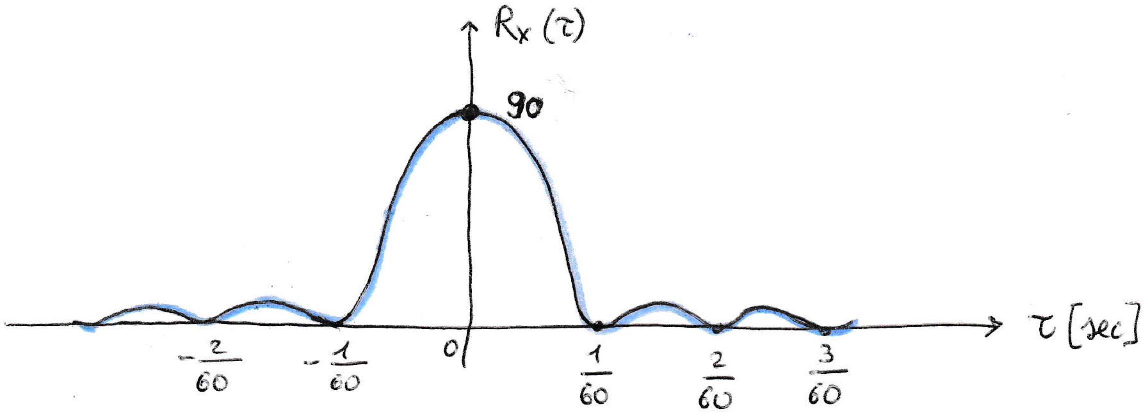


$$P_x = \frac{120 \cdot S_x(0)}{2} = 90$$

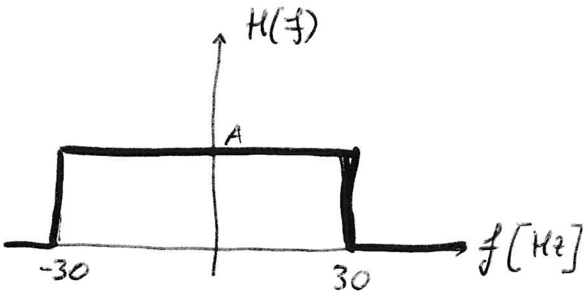
$$\downarrow$$

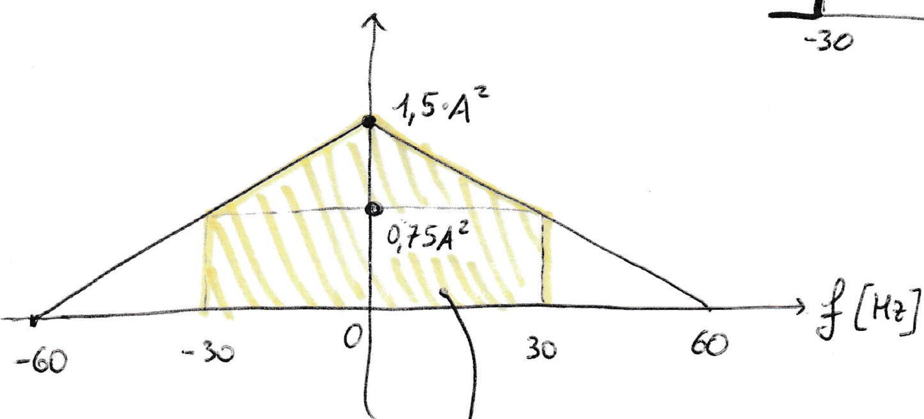
$$S_x(0) = 1{,}5$$

b) $R_x(\tau) \rightleftharpoons S_x(f) \leadsto R_x(\tau) = 90 \cdot \mathrm{sinc}^2(60\,\tau)$



c) $S_y(f) = S_x(f) \cdot |H(f)|^2$





$$P_y = 2 \cdot \underbrace{\frac{(1{,}5A^2 + 0{,}75A^2) \cdot 30}{2}}_{\text{area del trapezio}} = 67{,}5 \cdot A^2$$

ESERCIZIO 4

a) $x(t) = e^{-\frac{t}{T}} u(t)$

Calcoliamo la trasformata di Fourier:

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-\frac{t}{T}}\, e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-\left(\frac{1}{T}+j2\pi f\right)t}\, dt$$

Tenuto conto che:

$$\int_{0}^{\infty} e^{-at}\, dt = -\frac{1}{a}\, e^{-at}\Big|_{0}^{\infty} = -\frac{1}{a}(0-1) = \frac{1}{a}$$

$$= \frac{1}{\frac{1}{T}+j2\pi f} = \frac{T}{1+j2\pi T f}$$

b) $$X(f) = \frac{T}{1 + j\, f/f_T} \qquad f_T = \frac{1}{2\pi T}$$

$$|X(f)|^2 = \frac{T^2}{1 + \left(f/f_T\right)^2}$$

$$\boxed{\left. |X(f)|^2 \right|_{dB} = 10 \log_{10} \frac{|X(f)|^2}{|X(0)|^2}}$$

$$\left. |X(f)|^2 \right|_{dB} = -3\,dB \longrightarrow \frac{|X(f)|^2}{|X(0)|^2} = \frac{1}{2}$$

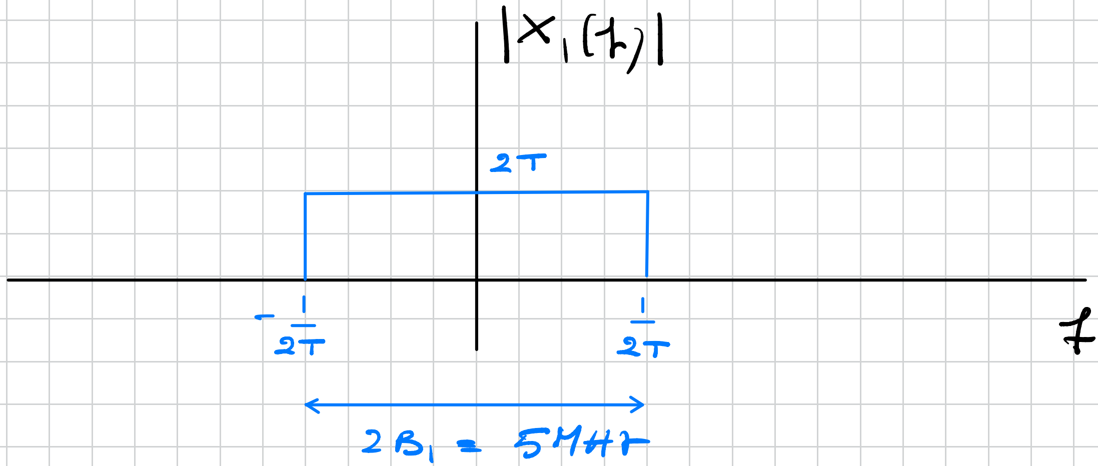
$$\frac{|X(f)|^2}{|X(0)|^2} = \left. \frac{1}{1 + \left(f/f_T\right)^2} \right|_{\boxed{f = f_T}} = \frac{1}{2}$$

Quindi $B_{-3dB} = f_T = \frac{1}{2\pi T} = \frac{1}{2\pi}\ kHz \cong 160\ Hz$

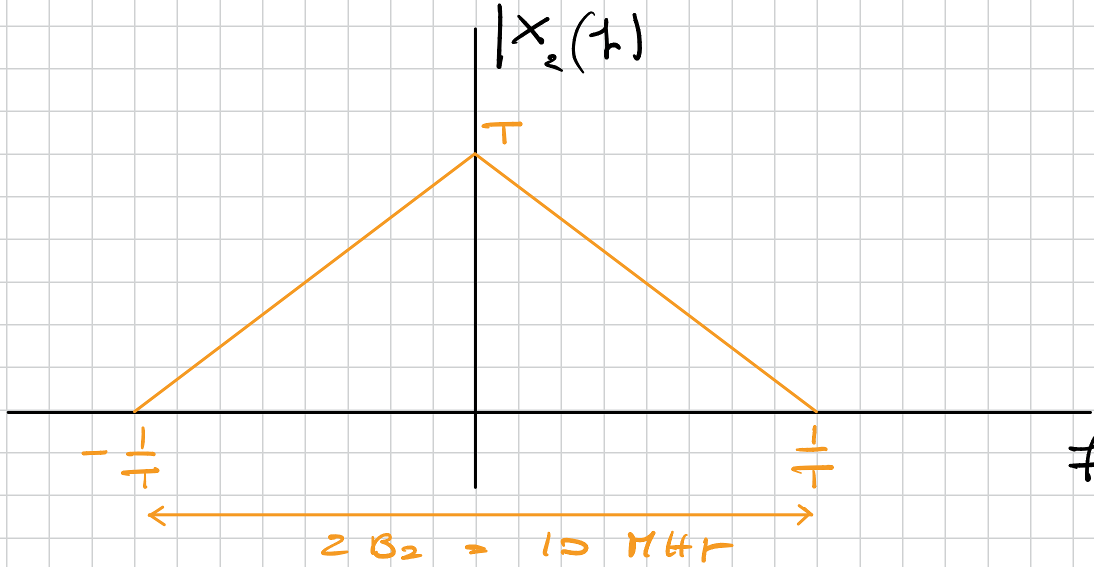
ESERCIZIO 5

a)

$$x_1(t) = \mathrm{sinc}\left(\frac{t}{2T}\right) \rightleftharpoons X_1(f) = 2T\,\mathrm{rect}(2fT)$$



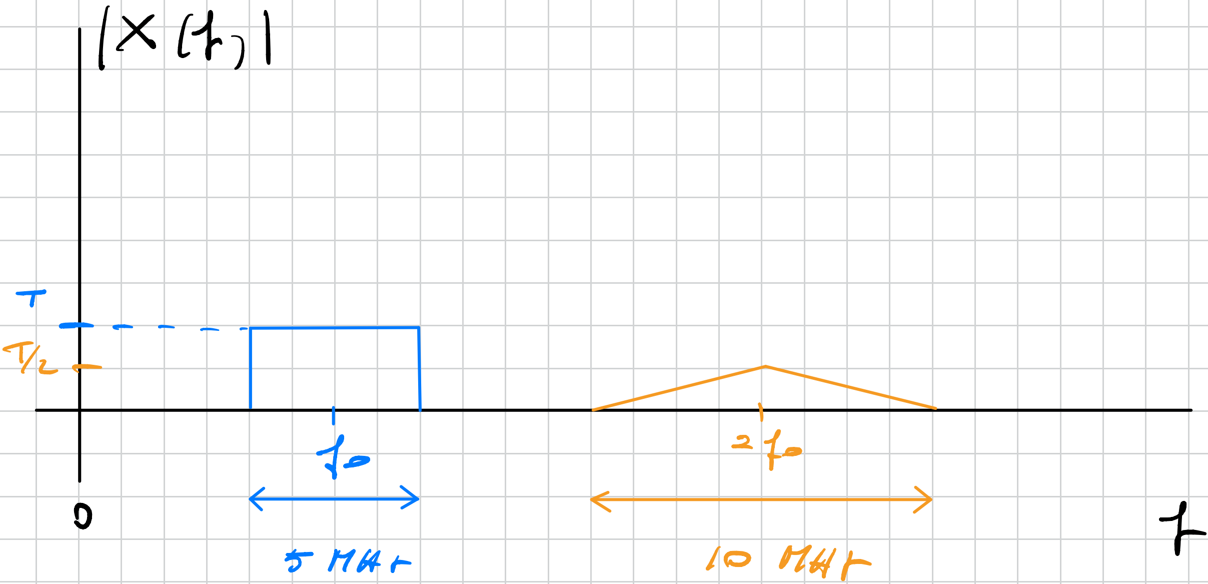
$$x_2(t) = \mathrm{sinc}^2\left(\frac{t}{T}\right) \rightleftharpoons X_2(f) = T^2\,\mathrm{rect}(fT) \otimes \mathrm{rect}(fT)$$



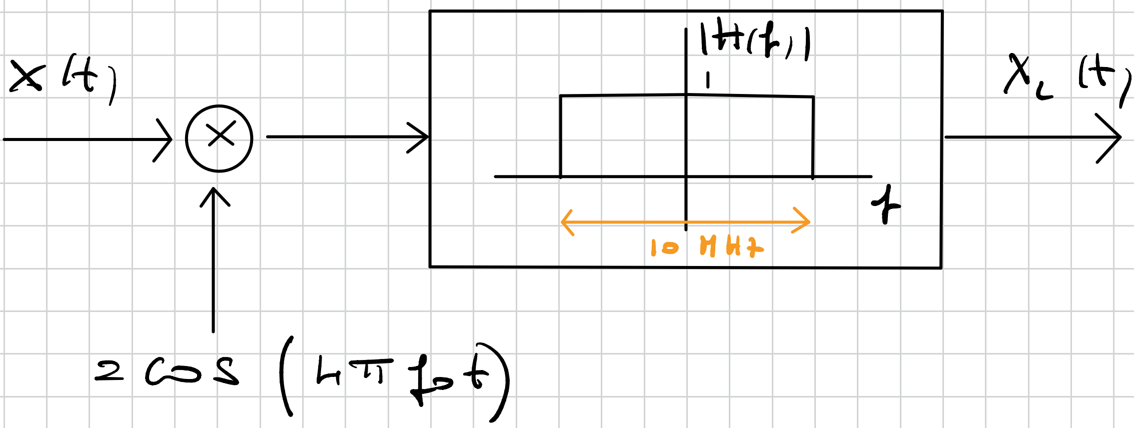
Tenuto di conto che:

$$X(f) = \frac{X_1(f+f_0) + X_1(f-f_0)}{2}$$

$$+ \frac{X_2(f+2f_0) + X_2(f-2f_0)}{2}$$



b) Per recuperare il segnale $x_2(t)$ devo demodularlo con un cos( ) alla frequenza $2f_0$ e filtrarlo con un filtro passa-basso di banda 5 MHz



ESERCIZIO 6

La matrice controllo di parità è:

$$\underline{\underline{H}} = [\underline{\underline{P}}^T, \underline{\underline{I}}_3] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

La sindrome di parità è:

$$\boxed{\underline{s} = \underline{y}\, \underline{\underline{H}}^T = [0\ 1\ 0]}$$

Siccome $\underline{s} = \underline{y}\, \underline{\underline{H}}^T = \underline{e}\, \underline{\underline{H}}^T$ si trova

$$\boxed{\underline{e} = [0\ 0\ 0\ 0\ 1\ 0]}$$

da cui segue $\underline{y} + \underline{e} = [0\ 1\ 1\ 1\ 1\ 0]$

Esercizio 7

a) La densità spettrale di potenza è

$$S_s(f) = \frac{1}{T_s} S_a(f) \, |G_r(f)|^2$$

dove

$$S_a(f) = \sum_m R_a(m) \, e^{-j 2\pi f T_s}$$

$$R_a(m) = \begin{cases} \sigma_a^2 + \eta_a^2 & m = 0 \\ \eta_a^2 & m \neq 0 \end{cases}$$

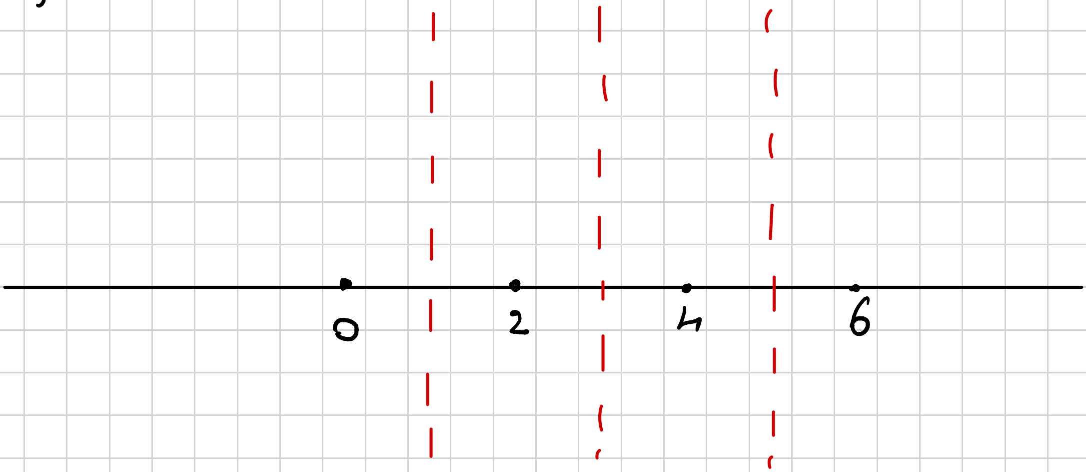
Calcoliamo η_a e σ_a^2:

$$\eta_a = E\{a_i\} = \frac{1}{4}(0 + 2 + 4 + 6) = 3$$

$$\sigma_a^2 = E\{a_i^2\} - \eta_a^2 = \frac{1}{4}\left(0^2 + 2^2 + 4^2 + 6^2\right) - 3^2$$

$$= \frac{56}{4} - 9 = 5$$

b)



La probabilità di errore è la stessa di una 4-PAM convenzionale dal momento che dipende da $Q\left(\frac{h-4}{\sigma}\right)$

Quindi:

$$P(e) = \frac{1}{4}\left(2\,P(e \mid c_k = 2) + 2\,P(e \mid c_k = 0)\right)$$

$$= \frac{1}{2}\left(2\,Q\left(\frac{1}{\sqrt{4}}\right) + Q\left(\frac{1}{\sqrt{4}}\right)\right) = \frac{3}{2}\,Q\left(\frac{1}{\sqrt{4}}\right)$$

ESERCIZIO 8

a) Tenuto conto che $B = \frac{1+\alpha}{T_S} = \frac{1+\alpha}{r \log_2 M} R_b$

$$\boxed{\eta = \frac{R_b}{B} = \frac{r \log_2 M}{1+\alpha} = \frac{5}{4} \qquad R_b = \eta B = 5\ Mbit/s}$$

Il tempo necessario per trasmettere il file è

$$\boxed{T_{FILE} = \frac{S_{FILE}}{R_b} = 0.2\ s}$$

b) La probabilità di errore in ingresso al codificatore è

$$P^{BIT}(e) = Q\left(\frac{1}{\bar{\Gamma}_n}\right) = Q\left(\sqrt{E_s/N_0}\right) = Q\left(\sqrt{\frac{E_b}{N_0} r \log_2 M}\right)$$

Se $\frac{E_b}{N_0} = 6\ dB = 4$ si ottiene $P^{BIT}(e) = Q\left(\sqrt{6}\right) \cong 6 \cdot 10^{-3}$