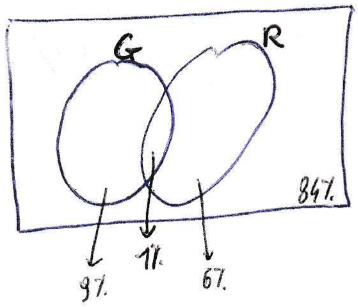
1



$G = \{\text{rubinetto gocciola}\}$ $\qquad P\{\bar{G}\cap\bar{R}\} = ?$

$P(\bar{G}) = 0.9 \quad P\{G\} = 0.1$

$R = \{\text{rubinetto rovinato}\}$

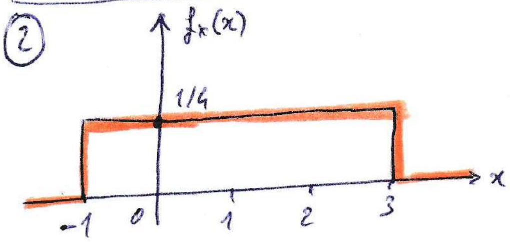
$P\{R\} = 0.07 \quad P\{\bar{R}\} = 0.93$

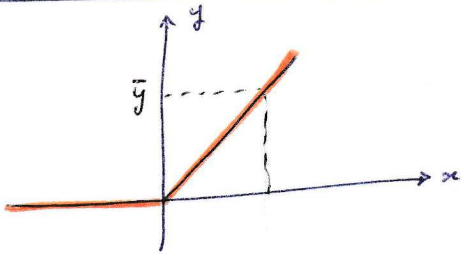
$P\{G\cap R\} = 0.01$

$$P\{\bar{G}\cap\bar{R}\} = P\{\overline{G\cup R}\} = 1 - P\{G\cup R\} = 0,84$$

$$P\{G\cup R\} = P\{G\} + P\{R\} - P\{G\cap R\} = 0,1 + 0,07 - 0,01 = 0,16$$

2



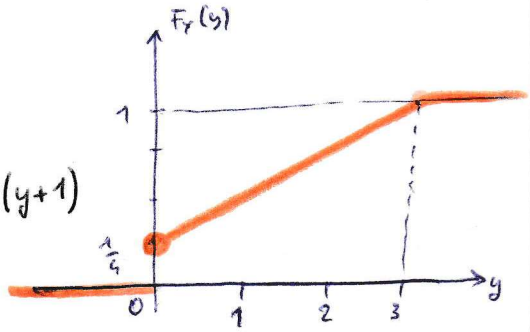


$y<0: \quad F_Y(y) = 0$

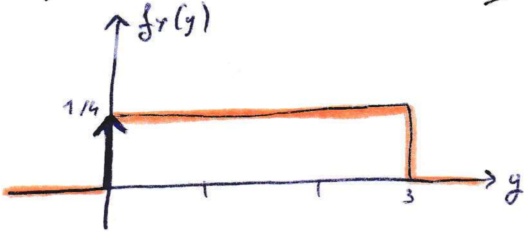
$y=0: \quad F_Y(0) = \int_{-\infty}^{0} f_x(x)\,dx = \int_{-1}^{0} \frac{1}{4}\,dx = \frac{1}{4}$

$0<y<3: \quad F_Y(y) = \int_{-\infty}^{y} f_x(x)\,dx = \int_{-1}^{y} \frac{1}{4}\,dx = \left.\frac{x}{4}\right|_{-1}^{y} = \frac{1}{4}(y+1)$

$y \geq 3: \quad F_Y(y) = 1$



b) $$\frac{1}{4}\left[\delta(y) + \mathrm{rect}\left(\frac{y-\frac{3}{2}}{3}\right)\right] = f_Y(y)$$



c) $$\eta_Y = \int_{-\infty}^{+\infty} y\, f_Y(y)\, dy = \int_{-\infty}^{+\infty} \frac{y}{4}\,\delta(y)\, dy + \frac{1}{4}\int_{-\infty}^{+\infty} y\cdot \mathrm{rect}\left(\frac{y-3/2}{3}\right) dy =$$

$$\int_a^b f(x)\,\delta(x-x_0) = \begin{cases} f(x_0), & x_0 \in (a,b) \\ 0, & x_0 \notin (a,b) \end{cases} \qquad = 0 + \frac{1}{4}\int_0^3 y\, dy = \frac{1}{4}\, \frac{y^2}{2}\Big|_0^3 = \frac{1}{4}\cdot\frac{9}{2} = \frac{9}{8}$$

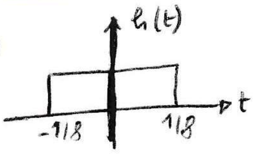
---

(3) $$X(t) \longrightarrow \boxed{h(t)} \longrightarrow Y(t)$$

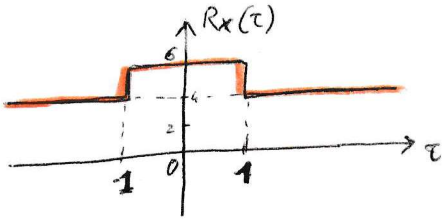
$\eta_X(t) = \eta_X = 2$

$C_X(t_1, t_2) = 2\cdot \mathrm{rect}\left(\frac{t_1 - t_2}{2}\right) = 2\,\mathrm{rect}(\tau/2) = C_X(\tau)$

$h(t) = \mathrm{rect}(4t)$



a) $\left.\begin{array}{l} X(t)\ \text{SSL} \\ h(t)\ \text{LTI} \end{array}\right\}\ Y(t)\ \text{SSL}$



b) $R_X(\tau) = C_X(\tau) + \eta_X^2 = 4 + 2\,\mathrm{rect}(\tau/2)$

c) $\eta_Y(t) = \eta_Y = \eta_X\cdot H(0) = \eta_X \int_{-\infty}^{+\infty} h(t)\, dt = 2\cdot\frac{1}{4}\cdot 1 = \frac{1}{2}$

d) $S_Y(f) = S_X(f)\cdot |H(f)|^2$

$S_X(f) \rightleftharpoons R_X(\tau) \Rightarrow S_X(f) = 4\,\delta(f) + 2\cdot 2\,\mathrm{sinc}(2f) = 4\left[\delta(f) + \mathrm{sinc}(2f)\right]$

$h(t) \rightleftharpoons H(f) \Rightarrow H(f) = \frac{1}{4}\cdot \mathrm{sinc}(f/4)$

$S_Y(f) = \left[\delta(f) + \mathrm{sinc}(2f)\right]\cdot \mathrm{sinc}^2(f/4)/4$

ESERCIZIO 4

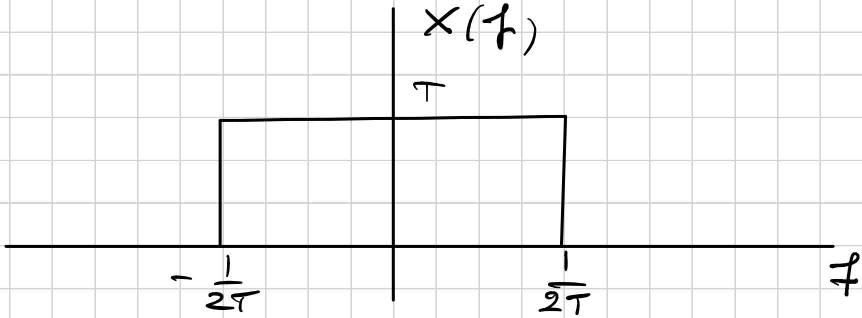
Vedi soluzione Esempio 4.16 libro del Prof. Luise. (PAG. 200)

ESERCIZIO 5

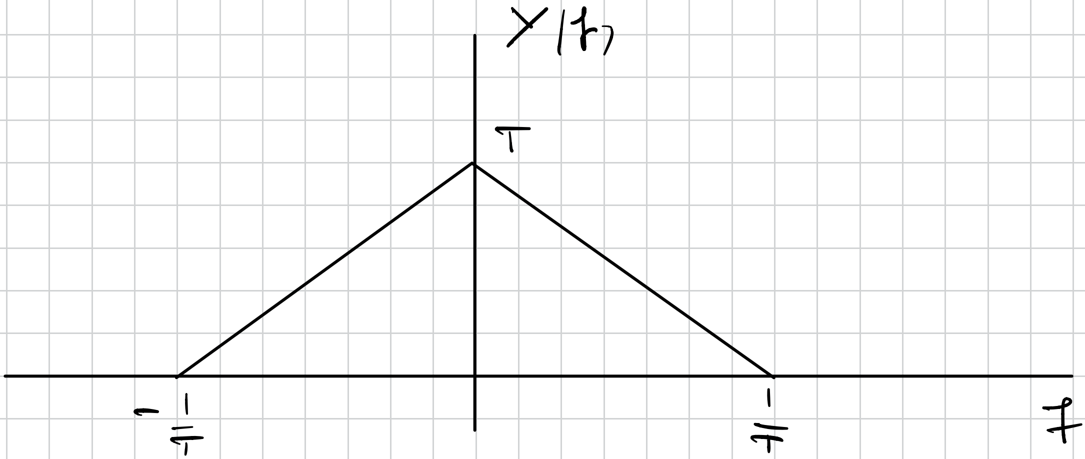
$$x(t) = \mathrm{sinc}\left(\frac{t}{T}\right) \qquad T = 1\,\mu s \qquad \frac{1}{T} = 1\,MHz$$

$$y(t) = x^2(t) = \mathrm{sinc}^2\left(\frac{t}{T}\right)$$

$$\boxed{\mathrm{sinc}\left(\frac{t}{T}\right) \rightleftarrows T\,\mathrm{rect}(fT)}$$

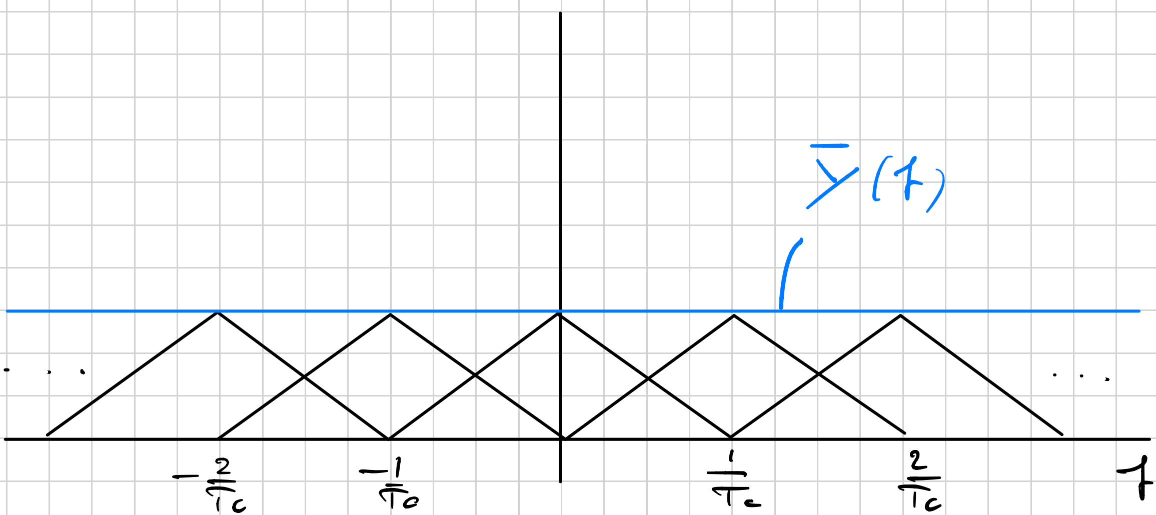


$$Y(f) = X(f) \otimes X(f)$$



$$\bar{Y}(f) = \sum_n y(nT_c)\, e^{-j2\pi f n T_c} = \frac{1}{T_c} \sum_k Y\left(f - \frac{k}{T_c}\right)$$

Tenuto conto che $\frac{1}{T_c} = 1\,\text{MHz} = \frac{1}{T}$



ESERCIZIO 6

$$\underline{\underline{H}} = \left[\underline{\underline{P}}^T, \underline{\underline{I}}_3 \right] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$\underline{e}\ \underline{\underline{H}}^T$	$\underline{s}$
tutti errori di peso 1	
0 0 0 0 0 1	0 0 1
0 0 0 0 1 0	0 1 0
0 0 0 1 0 0	1 0 0
0 0 1 0 0 0	0 1 1
0 1 0 0 0 0	1 0 1
1 0 0 0 0 0	1 1 0

Siccome a ciascun errore di peso 1 corrisponde una sola sindrome il codice li può correggere tutti.

Esercizio 7

a)
$$\eta_a = E\{a_i\} = 1$$

$$\sigma_a^2 = E\{a_i^2\} - \eta_a^2 = 2 - (1)^2 = 1$$

La densità spettrale di potenza del segnale è:

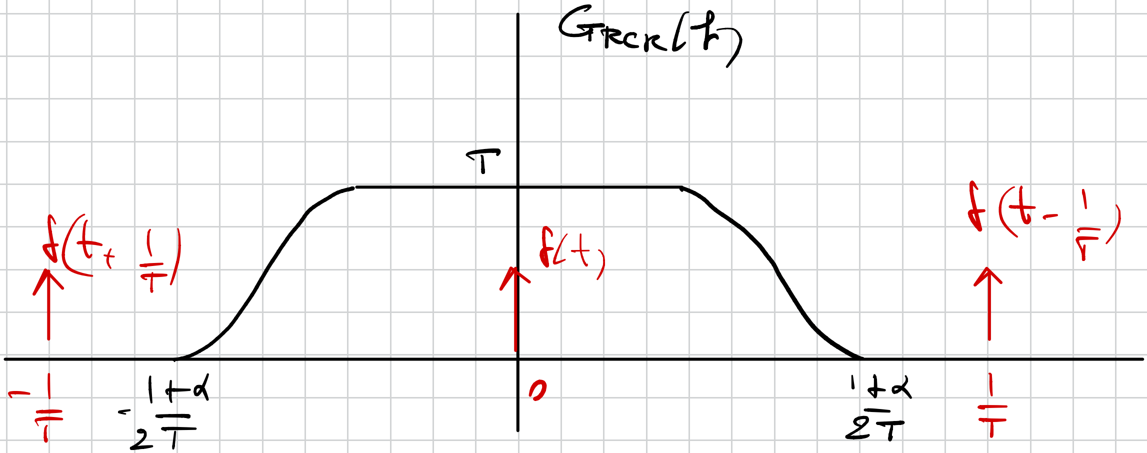
$$S_s(f) = \frac{1}{T} S_a(f) |G_T(f)|^2 = \frac{1}{T} S_a(f) G_{RCR}(f)$$

dove

$$S_a(f) = \frac{1}{T}\left(\sigma_a^2 + \frac{\eta_a^2}{T} \sum_l \delta\left(f - \frac{l}{T}\right)\right) G_{RCR}(f)$$

$$= \frac{\sigma_a^2}{T} G_{RCR}(f) + \frac{\eta_a^2}{T^2} \sum_l G_{RCR}\left(\frac{l}{T}\right) \delta\left(f - \frac{l}{T}\right)$$

Tenuto di conto che:



si ottiene:

$$S_a(f) = \frac{\sigma_a^2}{T} G_{RCR}(f) + \frac{\eta_a^2}{T^2} G_{RCR}(0)\, \delta(f)$$

$$= \frac{\sigma_a^2}{T} G_{RCR}(f) + \frac{\eta_a^2}{T} \delta(f)$$

La potenza del segnale e'

$$P_s = \int_{-\infty}^{\infty} S_a(f)\, df$$

$$= \frac{\sigma_a^2}{T} \int_{-\infty}^{\infty} G_{RCR}(f)\, df + \frac{\eta_a^2}{T} \int_{-\infty}^{\infty} \delta(f)\, df$$

$$= \frac{\sigma_a^2}{T} + \frac{\eta_a^2}{T} = \frac{2}{T}$$

L'energia del segnale è

$$E_s = P_s T = 2$$

b) La probabilità di errore è

$$P(e) = Q\left(\frac{1}{\sigma}\right) = Q\left(\sqrt{\frac{2}{N_0}}\right)$$

Tenuto conto che $\frac{E_s}{N_0} = \frac{2}{N_0}$ si ottiene

$$P(e) = Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

In una 2-PAM convenzionale si ha

$$P(e) = Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

La perdita è pari a 3 dB

Esercizio 8

a)

$$R_b = \frac{10\ \text{Mbit}}{0.1\ s} = 100\ \text{Mbit/s}$$

$$B = \frac{1+\alpha}{T_s} = \frac{1+\alpha}{r \log_2 M} R_b$$

$$= \frac{1.2}{\frac{3}{4}\log_2 16} R_b = 40\ \text{Mbit/s}$$

$$\eta_{sp} = \frac{R_b}{B} = \frac{5}{2}\ \text{bit/s/Hz}$$

b)

$$R_b = \frac{10\ \text{Mbit}}{0.05\ s} = 200\ \text{Mbit/s}$$

Raddoppiando R_b la banda rimane costante se l'ordine della mappa è tale che $\log_2 M = 8$ anziché $\log_2 M = 4$. Devo utilizzare 256-QAM.