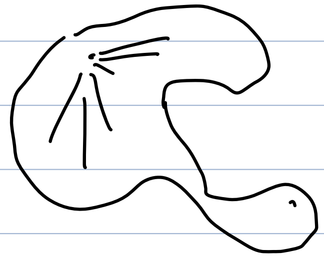
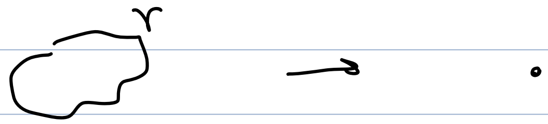
$$f(x,y) = \frac{y}{\sqrt{1+2xy}}\, i + \frac{x}{\sqrt{1+2xy}}\, j \qquad \Omega = \{ 1+2xy > 0 \}$$

Ω STELLATO RISPETTO A x_0 SE $\forall x$ $[x_0, x] \subseteq \Omega$



Ω SEMPLICEMENTE CONNESSO

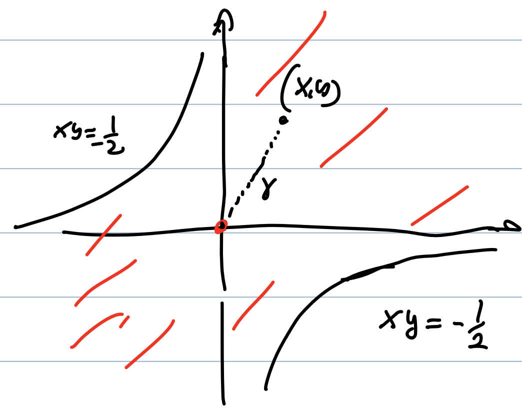
$\forall \gamma \in C^1$ $\gamma(a) = \gamma(b)$ TRASFORMAZIONE CONTINUA



$\gamma(t) \subseteq \Omega$

$$\Omega = \left\{ (x,y) : xy > -\frac{1}{2} \right\}$$

È STELLATO RISPETTO A $(x_0, y_0) = (0,0)$



$$f(x,y) = \frac{y}{\sqrt{1+2xy}}\, i + \frac{x}{\sqrt{1+2xy}}\, j$$

?

SE $= \partial_x U\, i + \partial_y U\, j$

V

$$\partial_y(\partial_x U) = \partial_x(\partial_y U) \qquad \partial_y\left(\frac{y}{\sqrt{1+2xy}}\right) = \partial_x\left(\frac{x}{\sqrt{1+2xy}}\right)$$

$$\partial_y\left(y\,(1+2xy)^{-1/2}\right) = 1\,(1+2xy)^{-1/2} + y\left(-\frac{1}{2}\right)(1+2xy)^{-3/2} \cdot 2x$$

$$= \frac{1}{\sqrt{1+2xy}} - \frac{xy}{\sqrt{(1+2xy)^3}} = \frac{1+2xy-xy}{\sqrt{(1+2xy)^3}} = \frac{1-xy}{\sqrt{(1+2xy)^3}}$$

$$\partial_x\left(x\,(1+2xy)^{-1/2}\right) = (1+2xy)^{-1/2} + x\left(-\frac{1}{2}\right)(1+2xy)^{-3/2}\, 2y$$

$$U(x,y) = \int_\gamma f \cdot dx \qquad \gamma(a) = (x_0, y_0) \qquad \gamma(b) = (x,y) \qquad [a,b] = [0,1]$$

(1)

$$\gamma(t) = (tx, ty) \qquad t \in [0,1]$$

$$\int_\gamma f \cdot dx = \int_0^1 \underbrace{\frac{tyx}{\sqrt{1+2t^2xy}} + \frac{txy}{\sqrt{1+2t^2xy}}}_{f_1(\gamma(t))\gamma_1'(t) + f_2(\gamma(t))\gamma_2'(t)} dt = \int_0^1 \frac{2txy}{\sqrt{1+2xyt^2}} dt$$

$$= \frac{1}{2}\int_0^1 \frac{4xyt}{\sqrt{1+2xyt^2}} dt$$

$$1 + 2xyt^2 = u$$

$$du = 2xy \cdot 2t\,dt = 4xyt\,dt$$

$$= \frac{1}{2}\int_1^{1+2xy} \frac{du}{\sqrt{u}} = \sqrt{u}\Big|_1^{1+2xy} = \sqrt{1+2xy} - \sqrt{1}$$

---

(2)

$$\partial_x U = \frac{y}{\sqrt{1+2xy}} \qquad \frac{d}{\partial x}\left(\sqrt{1+2xy}\right) = \frac{\partial}{\partial x}(1+2xy)^{1/2}$$

$$= \frac{1}{2}(1+2xy)^{-1/2} \cdot 2y$$

$$\boxed{U(x,y) = \sqrt{1+2xy} + \varphi(y)}$$

$$\partial_x U(x,y) = \frac{y}{\sqrt{1+2yx}} + \partial_x \cancel{\varphi(y)}$$

$$\partial_y U(x,y) = \frac{x}{\sqrt{1+2xy}} = \frac{x}{\sqrt{1+2xy}} + \varphi'(y) \Longrightarrow \varphi'(y) = 0$$

$$\varphi(y) = c$$

$$\boxed{U(x,y) = \sqrt{1+2xy} + c}$$

$$\omega = \frac{y}{\sqrt{1+2xy}} dx + \frac{x}{\sqrt{1+2xy}} dy$$

ω È ESATTA

$$f(x,y) = \frac{y}{\sqrt{1+2xy}} i + \frac{x}{\sqrt{1+2xy}} j$$

CERCATO POTENZIALE U

$\omega = dU$ | $f = \nabla U$

CONTROLLARE ω CHIUSA | CONTROLLATO CHE $\mathrm{rot}\, f = 0$

CICLOIDE

$$\gamma(t) = \big(t - \sin(t),\ 1 - \cos(t)\big) \qquad t \in [0, 2\pi]$$

$$\int_\gamma f \cdot dx = \int_0^{2\pi} f(\gamma(t)) \cdot \gamma'(t)\, dt = \int_0^{2\pi} \frac{(1-\cos(t))(1-\cos t)}{\sqrt{1 + 2\,(t - \sin(t))(1-\cos(t))}} + \dots$$

$$f_1(t)\gamma_1'(t) \qquad + f_2(\gamma(t))\gamma_2'(t)$$

$$\int_\gamma f \cdot dx = U(\gamma(2\pi)) - U(\gamma(0))$$

$$f(x,y,z) = 2yz\, i + 2z(x+3y)\, j + [y(2x+3y) + 2z]\, k$$

$\mathbb{R}^3$ $\quad \mathrm{rot}\, f = 0 \qquad \mathrm{rot}\, f = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ f_1 & f_2 & f_3 \end{vmatrix} = \nabla \times f$

$(\mathrm{rot}\, f)_i = \varepsilon_{ijk} \partial_j f_k$ SOMMATO SU j e k

ε_{ijk} TENSORE TOTALMENTE ANTISIMMETRICO $\qquad \varepsilon_{ijk} = 0$ SE DUE INDICI SONO UGUALI

$\varepsilon_{123} = \varepsilon_{312} = \varepsilon_{231} = 1$

$\varepsilon_{213} = \varepsilon_{321} = \varepsilon_{132} = -1$

$$(u \wedge v)_i = (u \times v)_i = \varepsilon_{ijk} u_j v_k$$

$$\partial_y f_3 - \partial_z f_2 = \partial_y \big(y(2x+3y)\big) - \partial_z \big(2z(x+3y)\big)$$

$$= 2x + 6y - (2x + 6y) = 0$$

$$\partial_z f_1 - \partial_x f_3 = \partial_z (2yz) - \partial_x \big(y(2x+3y) + 2z \big)$$

$$= 2y - 2y = 0$$

$$\partial_x f_2 - \partial_y f_1 = \partial_x \left(2z(x+3y)\right) - \partial_y (2yz)$$

$$= 2z - 2z = 0$$

$$\operatorname{rot} f = 0$$

$\exists\, U$ t.q $f = \nabla U$

$$\partial_x U = f_1 = 2yz \implies U(x,y,z) = 2xyz + \varphi(y,z)$$