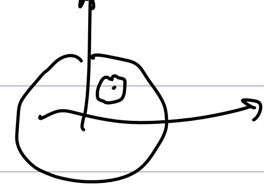
$B(0,1) \subset \mathbb{R}^n$ è aperta



$x^0 \in B(0,1)$ $\quad \|x^0\| < 1$

$R < 1 - \|x^0\|$

$x \in B(x^0, R) \Rightarrow \|x\| < 1$

$\|x\| = \|x - x^0 + x^0\| \le \|x - x^0\| + \|x^0\| < R + \|x^0\| < 1 - \|x^0\| + \|x^0\| \le 1$

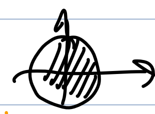
$f \in C([a,b]) \Rightarrow m, M \quad \min_{[a,b]} f = m \quad \max_{[a,b]} f = M$

$f : \mathbb{R}^n \to \mathbb{R}$ continua $\quad Q = [a_1,b_1] \times [a_2,b_2] \times \dots \times [a_n,b_n]$

$= \prod_{i=1}^{n} [a_i, b_i]$

$f \in C(Q) \Rightarrow \exists \min_Q f \quad \max_Q f \qquad Q_0 = [0,1]^n$

$B = \{x : \|x\| \le 1\} = \{x \in \mathbb{R}^n : \sum_{i=1}^{n} x_i^2 \le 1\}$



TEO WEIERSTRASS

$f : A \to \mathbb{R}$ $\quad A \subseteq \mathbb{R}^n$ $\quad A$ chiuso e limitato

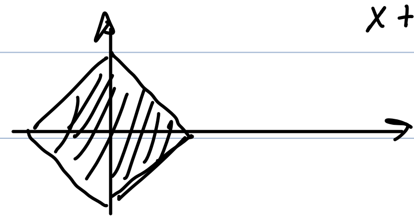
compatto

$\Rightarrow \quad m = \min_A f \quad M = \max_A f$

A limitato $\overset{def}{\iff} \exists R > 0 \quad A \subseteq B(0,R)$

$B' = \{\underset{\in \mathbb{R}^n}{x} : \underbrace{\sum_{i}^{n} |x_i|}_{\|x\|_1} \le 1\}$ $\qquad n = 2 \quad B' = \{(x,y) \in \mathbb{R}^2 \quad |x| + |y| \le 1\}$

$x + y \le 1$



$\{x : \|x\|_1 \le 1\}$

$B'' = \{x \in \mathbb{R}^n : \underbrace{\max_{i=1,..n} |x_i|}_{\|x\|_\infty} \le 1\}$

REGIONE sottoinsieme di $\mathbb{R}^n$ $A \neq \emptyset$ aperto connesso

$R = A \cup Z \qquad \emptyset \subseteq Z \subseteq \partial A$

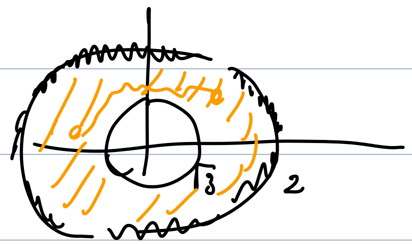
$R = \{ x \in \mathbb{R}^2 : \sqrt{4 - x^2 - y^2} < 1 \}$

$4 - x^2 - y^2 \geq 0 \qquad x^2 + y^2 \leq 4 \qquad \overline{B(0,2)}$

$4 - x^2 - y^2 < 1$

$x^2 + y^2 > 3 \qquad 3 < x^2 + y^2 \leq 4$

$\sqrt{3} < \|x\| \leq 2$



$f : \underset{\mathbb{R}^n}{\underset{\cap}{A}} \to \mathbb{R} \qquad x^0 \in A \qquad f$ è continua in x^0

$\forall \varepsilon > 0 \quad \exists \delta > 0 : \underset{\mathbb{R}^n}{\|x - x_0\|} < \delta \qquad \underset{\mathbb{R}}{|f(x) - f(x_0)|} < \varepsilon$

f, g continue in $x_0 \Rightarrow$ $f+g$, $f \cdot g$, $\frac{f}{g}$ $g(x_0) \neq 0$ continue

DIM $\quad |f(x) + g(x) - (f(x_0) + g(x_0))| < \varepsilon$

$|f(x) - f(x_0) + g(x) - g(x_0)| \leq \underset{\varepsilon/2}{\underset{\wedge}{|f(x) - f(x_0)|}} + \underset{\varepsilon/2}{\underset{\wedge}{|g(x) - g(x_0)|}}$

$\|x - x_0\| < \delta_1 \Rightarrow \qquad \|x - x_0\| < \delta_2$

$\delta = \min\{\delta_1, \delta_2\} \Rightarrow |f(x) + g(x) - (f(x_0) + g(x_0))| < \varepsilon \qquad |x - x_0| < \delta$

$f(x_1, \dots, x_n) = \sin(x_2)$ continua

$f(x, y) = \sin(x) + \cos(y)$

$f(x_1, x_2) = 2x_1 + 5x_2 \qquad x^0 = (3, 1)$

$|f(x) - f(x^0)| = |2(x_1 - 3) + 5(x_2 - 1)| \leq 2\, \underset{\wedge}{|x_1 - 3|} + 5\, \underset{\wedge}{|x_2 - 1|}$

$\boxed{|x_i| \leq \|x\|}$ $\qquad \leq 2\|x - x^0\| + 5\|x - x^0\|$

$$\boxed{|\omega_i| \le \|\omega\|} \qquad \le 2\|x-x^0\| \ldots$$

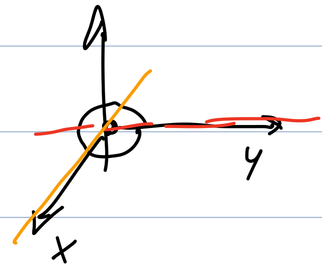
PROIEZIONI

$$\le 7\|x-x^0\|$$

$$f(x,y) = \frac{x^4+y^4}{x^2+y^2} \qquad \mathbb{R}^2 \setminus \{0\} \qquad f \in C(\mathbb{R}^2\setminus\{0\})$$

$$\tilde{f}(x,y) = \begin{cases} \dfrac{x^4+y^4}{x^2+y^2} & \mathbb{R}^2\setminus\{0\} \\ \boxed{0} & x=0 \end{cases} \qquad \tilde{f} \in C(\mathbb{R}^2)$$

$$0 \overset{?}{=} \lim_{(x,y)\to(0,0)} \tilde{f}(x,y)$$



$$\lim_{x\to x^0} f(x) = L \qquad x \in \mathbb{R}^n$$

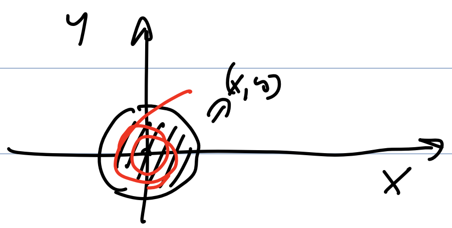
$$\forall \varepsilon>0 \quad \exists \delta>0 : \quad x \in \mathrm{Dom} f \ \ \underset{x\neq x^0}{\|x-x^0\|<\delta} \qquad |f(x)-L|<\varepsilon$$

$$\tilde{f}(0,y) = \frac{0+y^4}{0+y^2} = y^2 \xrightarrow{y\to 0} 0 \qquad \tilde{f}(x,0) = \frac{x^4+0}{x^2+0} = x^2 \xrightarrow{x\to 0} 0$$

$$0 \le \frac{x^4+y^4}{x^2+y^2} \le \frac{x^4+2x^2y^2+y^4}{x^2+y^2} = \frac{(x^2+y^2)^2}{x^2+y^2} = x^2+y^2 = \|x\|^2$$

$$0 \le \tilde{f}(x,y) \le x^2+y^2$$

$$|\tilde{f}(x,y)| < \varepsilon \qquad x^2+y^2 < \varepsilon \qquad \|(x,y)\| < \sqrt{\varepsilon} = \delta$$



$$\Rightarrow \quad |\tilde{f}(x,y)| < \varepsilon$$

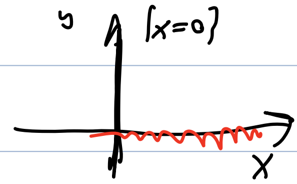
$$x = \rho\cos\theta \qquad |\tilde{f}(x,y)| = |\tilde{f}(\rho\cos\theta, \rho\sin\theta)| < \varepsilon$$

$$y = \rho\sin\theta$$

$$\frac{\rho^4\cos^4\theta + \rho^4\sin^4\theta}{\rho^2\cos^2\theta + \rho^2\sin^2\theta} = \frac{\rho^4(\cos^4\theta+\sin^4\theta)}{\rho^2 \ \cos^2\theta+\sin^2\theta} = \rho^2(\cos^4\theta+\sin^4\theta)$$

$$\le 2\rho^2$$

$$|f(\rho\cos\theta, \rho\sin\theta)| \leq 2\rho^2$$



- $f(x,y) = \dfrac{x^3+y^2}{x^2+y^2} \qquad (x,y) \neq (0,0)$

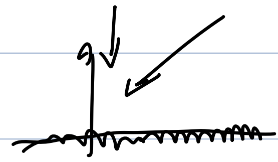
$$\lim_{(x,y)\to(0,0)} f(x,y) = N.E.$$

$$f(0,y) = \frac{0+y^2}{0+y^2} = 1 \xrightarrow{y\to 0} 1$$

$\not\parallel$

$$f(x,0) = \frac{x^3+0}{x^2+0} = x \xrightarrow{x\to 0} 0$$

$$f(x,y) = \begin{cases} x\, e^{\frac{x}{y}} & y \neq 0 \\ 0 & y = 0 \end{cases}$$



$f(0,y) = 0$

$y = mx \qquad m \neq 0$

$$\lim_{(x,y)\to(0,0)} f(x,y) = 0$$

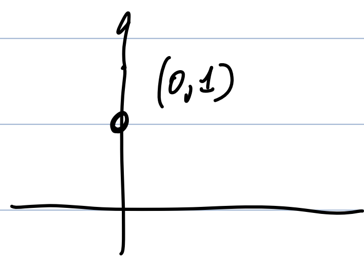
$$f(x,mx) = x\, e^{\frac{x}{mx}} = x\, e^{\frac{1}{m}} \xrightarrow[x\to 0]{} 0$$

$$f(x,x^2) = x\, e^{\frac{x}{x^2}} = x\, e^{\frac{1}{x}} \xrightarrow[x\to 0^+]{} +\infty$$

$x > 0$

$$\boxed{\frac{\cos\theta}{\sin\theta}}$$

$$f(\rho\cos\theta, \rho\sin\theta) = \underset{\substack{\downarrow \\ 0}}{\rho}\ \underset{\text{limitato}}{\cos\theta}\ e^{\frac{\cos\theta}{\sin\theta}}$$



$$\lim_{(x,y)\to(0,1)} \frac{x \ln y}{\sqrt{x^2+(y-1)^2}} = 0$$

$x^0 = (0,1)$

$x = \rho\cos\theta$

$y = 1+\rho\sin\theta$

$\xrightarrow{\rho\to 0} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$x_1^0 + \rho\cos\theta = x_1$

$x_2^0 + \rho\sin\theta = x_2$

$$\frac{\rho\cos\theta \ln(1+\rho\sin\theta)}{\sqrt{\rho^2\cos^2\theta + \rho^2\sin^2\theta}} = \frac{\cancel{\rho}\cos\theta \ln(1+\rho\sin\theta)}{\cancel{\rho}}$$

$$\lim_{t \to 0} \frac{\ln(1+t)}{t} = 1 \qquad \left| \frac{\ln(1+t)}{t} \right| \leq 2 \qquad |t| < \delta$$

$$t = \rho \sin\theta$$

$$\Downarrow$$

$$|\ln(1+t)| \leq 2|t|$$

$$|\ln(1+\rho\sin\theta)| \leq 2\rho|\sin\theta| \leq 2\rho$$

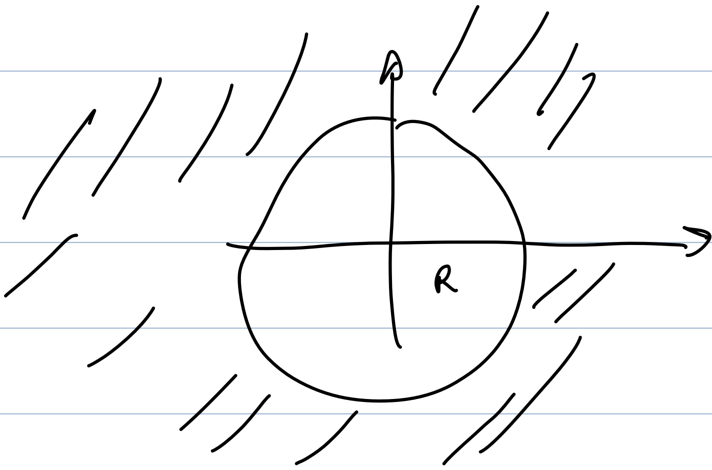
$$|\cos\theta \ln(1+\rho\sin\theta)| \leq 2\rho \xrightarrow{\rho\to 0} 0$$

$$\lim_{x \to +\infty} f(x) = L \qquad \lim_{x \to -\infty} f(x) = L$$

$x \in \mathbb{R}^n$

$$\lim_{x \to \infty} f(x) = L \qquad \lim_{\|x\| \to +\infty} f(x) = L$$

$$\forall \varepsilon > 0 \quad \exists R > 0 \,, \forall \; \|x\| > R \quad |f(x) - L| < \varepsilon$$



$$\lim_{x \to \infty} e^{-\frac{1}{|x_1| + |x_2|}} = 0$$

$$\lim_{\|x\| \to +\infty} e^{-\frac{1}{|x_1| + |x_2|}} = 0$$

$f : \mathbb{R}^n \to \mathbb{R}$

$$\lim_{x \to \infty} f(x) = +\infty$$

$$\forall M \in \mathbb{R} \quad \exists R > 0 \;:\; \forall x : \|x\| > R \quad f(x) > M$$

$$\lim_{\substack{x \to \infty \\ \|x\| \to +\infty}} \frac{x_1^4 + x_2^4}{x_1^2 + x_2^2} = +\infty \; ?$$

$x_1 = \rho \cos\theta$

$x_2 = \rho \sin\theta$

$$\lim_{\rho \to +\infty} \frac{\rho^4(\cos^4\theta + \sin^4\theta)}{\rho^2} = \lim_{\rho \to +\infty} \rho^2 (\cos^4\theta + \sin^4\theta)$$

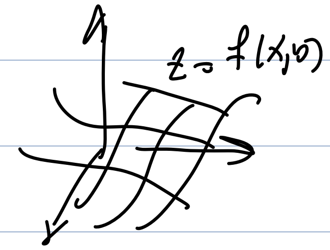
$$\cos^4\theta + \sin^4\theta = (\cos^2\theta + \sin^2\theta)^2 - 2 \sin^2\theta \cos^2\theta$$

$$= 1 - \frac{1}{2} 4 \sin^2\theta \cos^2\theta$$

$$= 1 - \frac{1}{2} (\sin(2\theta))^2 \geq \frac{1}{2}$$

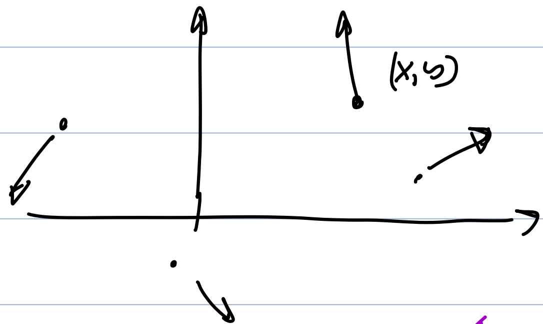
$$\rho^4(\cos^4\theta + \sin^4\theta) \geq \frac{\rho^3}{2} \xrightarrow[\rho \to +\infty]{} +\infty$$

$f(x,y) = z$



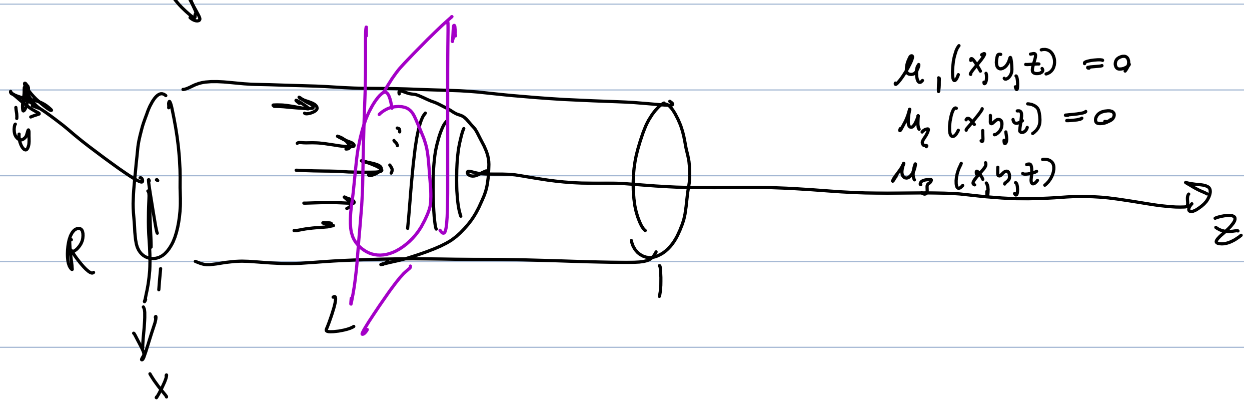
$$L_c = \{(x,y) : f(x,y) = c\}$$

$f_1(x,y)$, $f_2(x,y)$ $\qquad \mathbb{R}^2 \to \mathbb{R}^2$



$f_1(x,y) = -y$

$f_2(x,y) = x$



$u_1(x,y,z) = 0$

$u_2(x,y,z) = 0$

$u_3(x,y,z)$

$$u_3 = \frac{G}{4\mu}(R^2 - r^2) \qquad G = \frac{\Delta p}{L}$$

$$u_M = \frac{1}{\pi R^2}\int_{\text{SEZIONE}} u_3$$

$$\int_Q f(x,y)\,dx\,dy$$

$$\int_B f(x,y)\,dx\,dy$$