$$\frac{\partial f}{\partial x_i}(x^0) = \lim_{t\to 0} \frac{f(x^0 + te_i) - f(x^0)}{t} \qquad \frac{\partial f}{\partial v}(x^0) = \lim_{t\to 0} \frac{f(x^0 + tv) - f(x^0)}{t}$$

$$f(x) = \begin{cases} \frac{x_1 x_2}{\|x\|} & x \neq 0 \\ 0 & x = 0 \end{cases} \qquad \frac{\partial f}{\partial x_1}(x) = \frac{x_2 \|x\| - x_1 x_2 \frac{x_1}{\|x\|}}{\|x\|^2} \quad x \neq 0$$

$$\frac{\partial}{\partial x_1}\|x\| = \frac{\partial}{\partial x_1}\left(\sqrt{x_1^2 + x_2^2}\right) = \frac{1}{\cancel{2}} \frac{\cancel{2}x_1}{\sqrt{x_1^2 + x_2^2}}$$

$$\frac{\partial f}{\partial x_1}(0,0) = \lim_{t \to 0} \frac{f(t,0) - 0}{t} = 0 \qquad \frac{\partial f}{\partial x_2}(0,0) = 0$$

$$f(x_1, x_2) = \frac{\rho \cos\theta \, \rho \sin\theta}{\rho} = \rho \cos\theta \sin\theta \xrightarrow{\rho \to 0} 0$$

$$f(x) = \begin{cases} \frac{x_1 x_2}{\|x\|^2} & x \neq 0 \\ 0 & x = 0 \end{cases} \qquad f(t,0) = 0 \qquad \partial_{x_1} f(0,0) = \partial_{x_2} f(0,0) = 0$$

$$f(x) = \frac{\cancel{\rho^2} \cos\theta \sin\theta}{\cancel{\rho^2}} \quad \not\xrightarrow{\rho \to 0} \quad NE$$

$$f(x) = \begin{cases} \left[\dfrac{x^2 y}{x^4+y^2}\right]^2 & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$\boxed{\begin{array}{l} f \text{ non continue in } (0,0) \\ \text{ma } \forall v \neq 0 \\ \dfrac{\partial f}{\partial v}(0,0) = 0 \end{array}}$$

$$f(x, \lambda x^2) = \left[\frac{x^2 \lambda x^2}{x^4 + \lambda^2 x^4}\right]^2 = \frac{\lambda^2}{(1+\lambda^2)^2}$$

$$\frac{\partial f}{\partial v}(0,0) = \lim_{t \to 0} \frac{f(0+tv) - f(0)}{t}$$

$$\lim_{x \to 0} f(x, \lambda x^2) = \frac{\lambda^2}{(1+\lambda^2)^2}$$

$$v = (v_1, v_2)$$

$$\frac{\partial f}{\partial v}(0,0) = \lim_{t \to 0} \frac{f(t v_1, t v_2) - f(0,0)}{t} = \lim_{t \to 0} \left[\frac{t^2 v_1^2 \, t v_2}{t^4 v_1^4 + t^2 v_2^2}\right]^2 \frac{1}{t}$$

$$= \lim_{t \to 0} \frac{1}{t} \frac{t^6}{t^4} \left(\frac{v_1^2 v_2}{v_2^2 + t^2 v_1^4}\right)^2 \xrightarrow{t \to 0} 0$$

$f : (a,b) \to \mathbb{R}$ derivabile in x_0 x_0 pt di max, min locale

$x_0 \in]a,b[\Rightarrow f'(x_0) = 0$

$\not\Leftarrow$

$f : A \to \mathbb{R}$ $f(x^0) = \max_A f$ $x^0 \in \mathring{A} \iff \exists \varepsilon > 0 : B(x^0, \varepsilon) \subset A$

$A \subset \mathbb{R}^n$

$\partial_{x_1} f(x^0)$, $\partial_{x_2} f(x^0)$ ... $\partial_{x_n} f(x^0)$ ESISTONO

$\Rightarrow \partial_i f(x^0) = 0$

DIM $F(t) = f(x^0 + t e_i)$ $t = 0$ $F(t) = f(x^0)$

$|t| < \varepsilon$ x^0 pt di max $f(x^0) \geq f(x)$

$F(t) \leq F(0) \Rightarrow F'(0) = 0 = \dfrac{\partial f}{\partial x_i}(x^0)$

$$f(x,y) = x^3 \qquad (x,y) = (0,0) \qquad \partial_x f(0,0) = \partial_y f(0,0) = 0$$

$$\partial_x f(x,y) = 3x^2$$

$$\partial_y f(x,y) = 0$$

$$f(x,y) = x^2 - y^2 \qquad \nabla f(0,0) = \left(\partial_x f(0,0), \partial_y f(0,0)\right) = (0,0)$$

$$\partial_x f(x,y) = 2x$$

$$\partial_y f(x,y) = -2y$$

$f: \mathbb{R} \to \mathbb{R}$

$$f(x) = \underbrace{f(x_0) + f'(x_0)(x-x_0)}_{\text{retta tangente in } (x_0, f(x_0))} + \omega(x)$$

$$\lim_{x \to x_0} \frac{\omega(x)}{x - x_0} = 0 \qquad f \text{ DERIVABILE IN } x_0$$

$f: \mathbb{R}^2 \to \mathbb{R}$

$$f(x) = \underbrace{f(x^0) + \left(\nabla f(x^0), x - x^0\right)}_{\text{PIANO TANGENTE A } (x^0, f(x^0))} + \omega(x)$$

$$\lim_{x \to x^0} \frac{\omega(x)}{\|x - x^0\|} = 0$$

ESISTENZA DI $\nabla f(x^0)$ NON BASTA

$f: A \to \mathbb{R}$, $A \subseteq \mathbb{R}^n$ È DIFFERENZIABILE IN $x^0 \in \mathring{A}$

$\exists\ \varphi \in \mathcal{L}(\mathbb{R}^n, \mathbb{R})$ t.c.

$$\to \quad f(x) = f(x^0) + \varphi(x - x^0) + \omega(x) \qquad \lim_{x \to x^0} \frac{\omega(x)}{\|x - x^0\|} = 0$$

φ IL DIFFERENZIALE IN x^0

$$\varphi \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}) \qquad \varphi(x) = (a, x) = \sum_{i=1}^n a_i x_i$$

$$\boxed{a = \nabla f(x^0)}$$

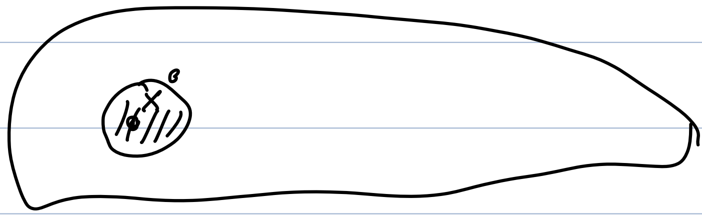
TEO DEL DIFFERENZIALE TOTALE $\qquad$ 270

$f: A \to \mathbb{R}$ $\quad A \subseteq \mathbb{R}^n$ $\qquad x^0 \in \mathring{A}$ $\qquad \partial_i f(x)$ ESISTE IN $B(x^0; r)$

$i = 1, \dots, n$

$\partial_i f$ È CONTINUA IN x^0

$\Rightarrow$ f È DIFFERENZIABILE IN x^0 $\qquad \varphi(x) = (\nabla f(x^0), x)$



DEF. f DIFFERENZIABILE in $x^0 \in \mathbb{R}^n$ $n \geq 1$

$$f(x) = f(x^0) + \varphi(x-x^0) + \omega(x) \qquad \lim_{x \to x^0} \frac{\omega(x)}{\|x-x^0\|} = 0$$

$\varphi \in \mathcal{L}(\mathbb{R}^n, \mathbb{R})$ $\qquad df(x_0)(x-x^0)$

φ è il differenziale $\varphi = df(x^0)$

f DIFFERENZIABILE in x^0 $\Rightarrow$ f continua in x^0

$$f(x) = f(x^0) + \varphi(x-x^0) + \underset{\substack{\downarrow \; x \to x^0 \\ 0}}{\omega(x)}$$

$$\varphi(x-x^0) = \sum_{i=1}^{n} a_i (x_i - x_i^0)$$

$$|\varphi(x-x^0)| = \left|\sum a_i(x_i - x_i^0)\right| = |(a, x-x^0)| \overset{CS}{\leq} \|a\| \, \|x-x^0\| \xrightarrow{x \to x^0} 0$$

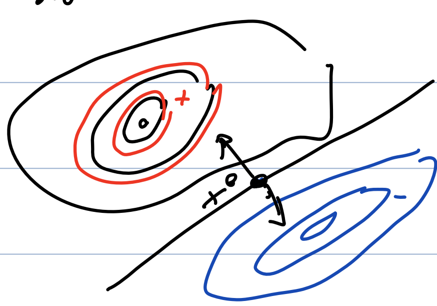
f DIFFERENZIABILE in x^0 $\Rightarrow$ $\frac{\partial f}{\partial v}(x^0) = (v, \nabla f(x^0))$

$v \neq 0$

$$\frac{\partial f}{\partial 0}(x^0) = \lim_{t \to 0} \frac{f(x^0 + t0) - f(x^0)}{t} = 0$$

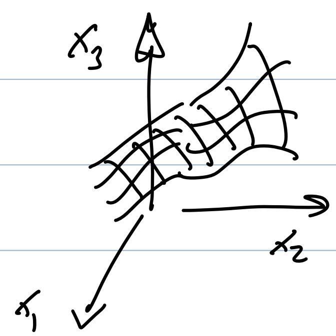
OSS $\|v\| = 1$ $\qquad \left|\frac{\partial f}{\partial v}(x^0)\right| = |(v, \nabla f(x^0))| \leq \|v\| \, \|\nabla f(x^0)\|$

$$-\|\nabla f(x^0)\| \leq \frac{\partial f}{\partial v}(x^0) \leq \|\nabla f(x^0)\|$$



$f(x^0) = 0$

$n = 2$ $\qquad f(x_1, x_2) = x_3$



$g(x) = f(x^0) + (\nabla f(x^0), x - x^0)$

PIANO PASSANTE PER $(x^0, f(x^0))$

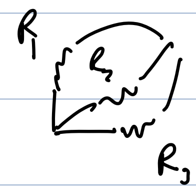
$$G_f = \{ (x_1, x_2, x_3) : x_3 = f(x^0) + (\nabla f(x^0), x - x^0) \} \approx \mathbb{R}^2$$
$$\cap \; \mathbb{R}^3$$

$$f(x_1, \dots, x_n) = x_{n+1}$$

$$G_f = \{ (x_1, \dots, x_n, x_{n+1}) \subseteq \mathbb{R}^{n+1} : x_{n+1} = f(x^0) + (\nabla f(x^0), x - x^0) \}$$
$$\approx \mathbb{R}^n$$

CIRCUITO R_1, R_2, R_3 PARALLELO

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \mathbb{R}^3 \to \mathbb{R}$$



$$R = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \qquad \frac{\partial R}{\partial R_1}$$

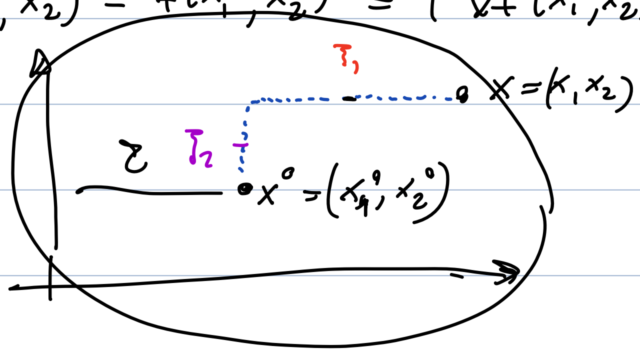
TEO LAGRANGE $f \in C([a,b])$ f deriv. in $]a,b[$

$$\exists\, \xi \in]a,b[\; : \; f(b) - f(a) = f'(\xi)(b-a)$$

$$\lim_{x \to x^0} \frac{\omega(x)}{\|x - x^0\|} = 0 \iff \omega(x) = o(\|x - x^0\|)$$

$$f(x_1, x_2) = f(x_1^0, x_2^0) + (\nabla f(x_1^0, x_2^0), (x_1 - x_1^0, x_2 - x_2^0)) + \omega(x)$$

$$f(x_1, x_2) - f(x_1^0, x_2^0) = (\nabla f(x_1^0, x_2^0), (x_1 - x_1^0, x_2 - x_2^0)) + \omega(x)$$



$$f(x) - f(x^0) = f(x_1, x_2) - f(x_1^0, x_2) + f(x_1^0, x_2) - f(x_1^0, x_2^0)$$

$$= \partial_{x_1} f(\xi_1, x_2)(x_1 - x_1^0) + \frac{\partial f}{\partial x_2}(x_1^0, \xi_2)(x_2 - x_2^0)$$

$$f(x) - f(x^0) - (\nabla f(x_0), (x - x^0)) \overset{?}{=} \omega(x) \qquad \omega(x) = o(\|x - x^0\|)$$

$$f(x) - f(x^0) - (\nabla f(x_0), (x - x^0)) = \left(\frac{\partial}{\partial x_1} f(\xi_1, x_2) - \frac{\partial}{\partial x_1} f(x_1^0, x_2^0)\right)(x_1 - x_1^0)$$
$$+ \left(\frac{\partial}{\partial x_2} f(x_1^0, \xi_2) - \frac{\partial}{\partial x_2} f(x_1^0, x_2^0)\right)(x_2 - x_2^0)$$

$$= A\,(x_1 - x_1^0) + B(x_2 - x_2^0)$$

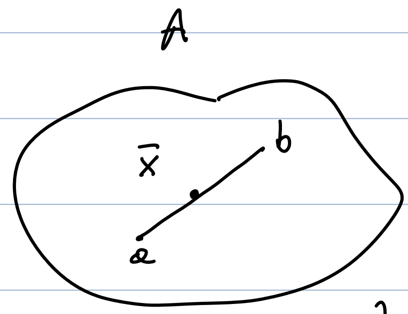
$$\left| f(x) - f(x^0) - (\nabla f(x^0), x - x^0) \right| \leq \left| A(x_1 - x_1^0) + B(x_2 - x_2^0) \right|$$

$$\leq |A|\,|x_1 - x_1^0| + |B|\,|x_2 - x_2^0|$$
$$\leq (|A| + |B|)\,(|x_1 - x_1^0| + |x_2 - x_2^0|)$$
$$\leq (|A| + |B|)\,\|x - x^0\|$$

$$\frac{|\omega(x)|}{\|x - x^0\|} \leq \frac{(|A| + |B|)\cancel{\|x - x^0\|}}{\cancel{\|x - x^0\|}} = |A| + |B| \to 0 \qquad \begin{array}{l} x \to x^0 \\ \xi_1 \to x_1^0 \\ x_2 \to x_2^0 \end{array}$$

TEO LAGRANGE

$f : A \to \mathbb{R}^1$, $A \subseteq \mathbb{R}^n$



$$S[a,b] = \{ x \in \mathbb{R}^n : x = a + t(b-a) \quad t \in [0,1] \} \subseteq A$$

$f \in C(S[a,b])$ $\qquad f$ differenziabile in $S[a,b] \setminus \{a,b\}$

$$\Rightarrow \quad \exists \ \bar{x} \in \overset{\circ}{S}[a,b] \qquad f(b) - f(a) = (\nabla f(\bar{x}), b-a)$$

Dim

$$F(t) = f(a + t(b-a)) \qquad t \in [0,1]$$

$F \in C([0,1])$ $\qquad F'$ ESISTE IN $]0,1[$ $\qquad a, b \in \mathbb{R}^n$

$$\frac{d}{dt}\left(f(a + t(b-a)) \right) = \frac{d}{dt} f(\gamma(t)) \qquad \gamma(t) = a + t(b-a)$$

$$\frac{d}{dt} f(\gamma(t)) \overset{\text{def}}{=} \sum_{i=1}^{n} \frac{\partial}{\partial x_i} f(\gamma(t)) \frac{d \gamma_i(t)}{dt} = (\nabla f(\gamma(t)), \dot{\gamma}(t))$$

$$\frac{d}{dt} F(t) = (\nabla f(a + t(b-a)), b-a) = \sum \frac{\partial}{\partial x_i} f(a + t(b-a)), \frac{d}{dt} a_i + t(b_i - a_i)$$

$n = 1 \qquad f'(\gamma(t))\gamma'(t)$

$$F(1) - F(0) = F'(\xi)(1-0)$$

$$f(b) - f(a) = (\nabla f(a + \xi(b-a)), b-a)$$

$$a + \xi(b-a) = \bar{x} \in \overset{\circ}{S}[a,b]$$