

$$P_F^2 = P\{\text{resistor ② funziona}\} = (1 - 0,3) \cdot (1 - 0,15) = 0,595$$

$$P_G^2 = P\{\text{resistor ② è guasto}\} = 1 - 0,595 = 0,405$$

$$P_G^{1/2} = P_G^1 \cdot P_G^2 = 0,25 \cdot 0,405 = 0,10125$$

$$P_F^{1/2} = 1 - P_G^{1/2} = 0,89875$$

$$P_F^{AB} = P_F^3 \cdot P_F^{1/2} = (1 - 0,2) \cdot 0,89875 = 0,719$$

ES. ②

$$F_X(x) = P\{X \leq x\} = (1 - e^{-\frac{x}{\lambda}}) u(x)$$

$$a) P\{X > 15 \text{ minuti}\} = \frac{1}{e}$$

$$1 - F_X(15) = \frac{1}{e}$$

$$1 - (1 - e^{-\frac{15}{\lambda}}) = \frac{1}{e}$$

$$e^{-\frac{15}{\lambda}} = e^{-1}$$

$$-\frac{15}{\lambda} = -1$$

$$\boxed{\lambda = 15 \text{ minuti}}$$

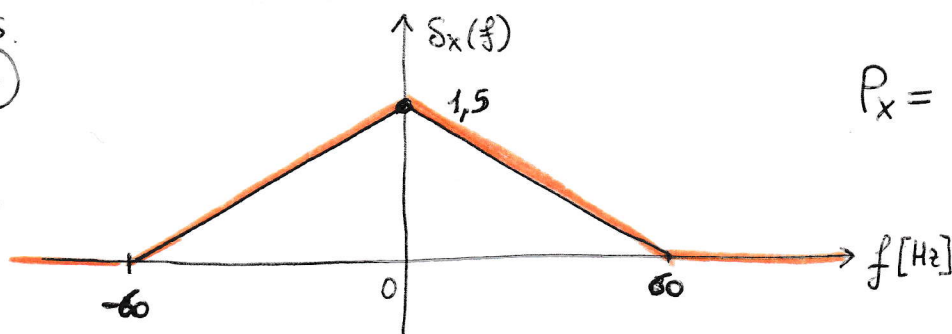
$$P\{X \leq x_0\} = F_X(x_0) = 0,1 = \frac{1}{10}$$

$$1 - e^{-\frac{x_0}{15}} = \frac{1}{10} \rightarrow e^{-\frac{x_0}{15}} = \frac{9}{10} \rightarrow -\frac{x_0}{15} = \ln\left(\frac{9}{10}\right)$$

$$x_0 = 15 \cdot \ln\left(\frac{10}{9}\right) \simeq 1,58 \text{ minuti} \simeq 95 \text{ secondi}$$

ES.  
3

a)

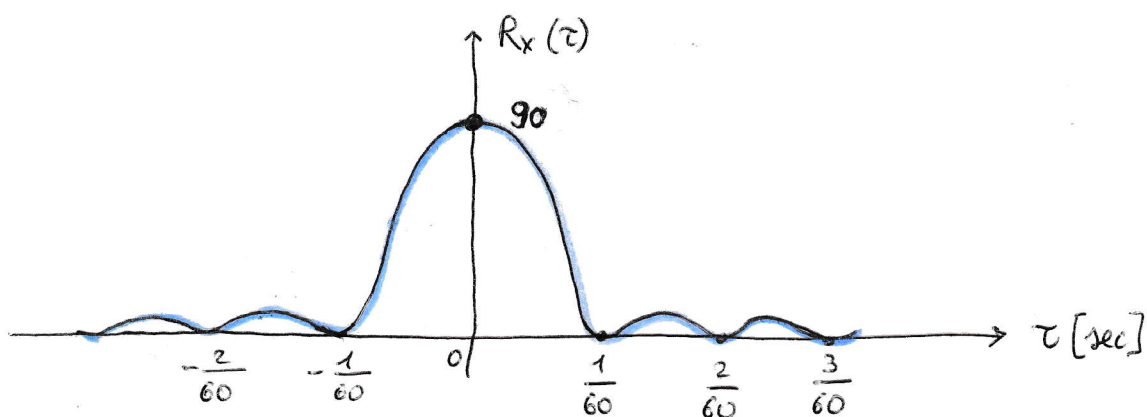


$$P_x = \frac{120 \cdot S_x(0)}{2} = 90$$

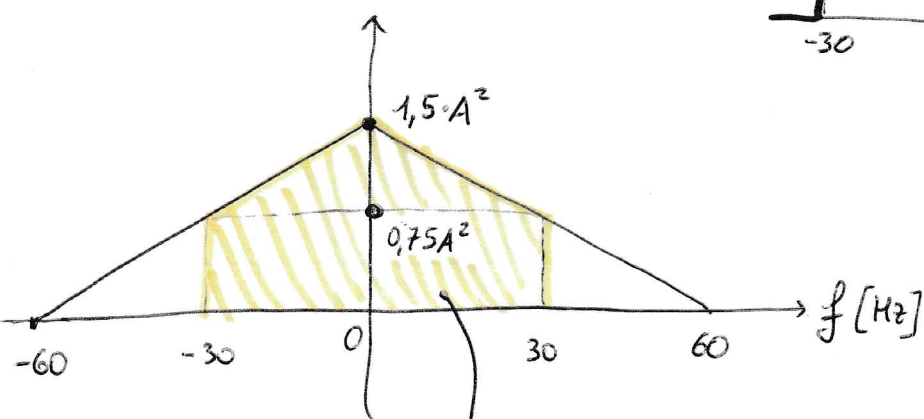
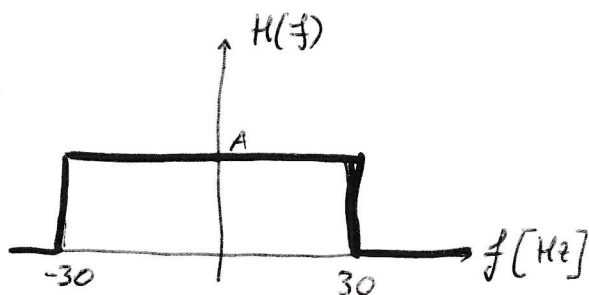
$$\downarrow$$

$$S_x(0) = 1,5$$

b)  $R_x(\tau) \Leftrightarrow S_x(f) \leadsto R_x(\tau) = 90 \cdot \text{sinc}^2(60\tau)$



c)  $S_y(f) = S_x(f) \cdot |H(f)|^2$



$$P_y = 2 \cdot \frac{(1,5A^2 + 0,75A^2) \cdot 30}{2} = 67,5 \cdot A^2$$

↓  
area del trapecio

## ESERCIZIO 4

a)  $x(t) = e^{-\frac{t}{T}} u(t)$

Calcoliamo la trasformata di Fourier:

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \\ &= \int_0^{\infty} e^{-\frac{t}{T}} e^{-j2\pi ft} dt \\ &= \int_0^{\infty} e^{-\left(\frac{1}{T} + j2\pi f\right)t} dt \end{aligned}$$

Tanto noto che:

$$\int_0^{\infty} e^{-at} dt = -\frac{1}{a} e^{-at} \Big|_0^{\infty} = -\frac{1}{a} (0 - 1) = \frac{1}{a}$$

$$= \frac{1}{\frac{1}{T} + j2\pi f} = \frac{T}{1 + j2\pi T f}$$

$$b) \quad X(f) = \frac{T}{1 + j f / f_T}$$

$$f_T = \frac{1}{2\pi T}$$

$$|X(f)|^2 = \frac{T^2}{1 + \left(\frac{f}{f_T}\right)^2}$$

$$\left|X(f)\right|_{dB}^2 = 10 \log_{10} \frac{|X(f)|^2}{|X(0)|^2}$$

$$\left|X(f)\right|_{dB}^2 = -3 \text{ dB} \longrightarrow \frac{|X(f)|^2}{|X(0)|^2} = \frac{1}{2}$$

$$\frac{|X(f)|^2}{|X(0)|^2} = \frac{1}{1 + \left(\frac{f}{f_T}\right)^2} = \frac{1}{2}$$

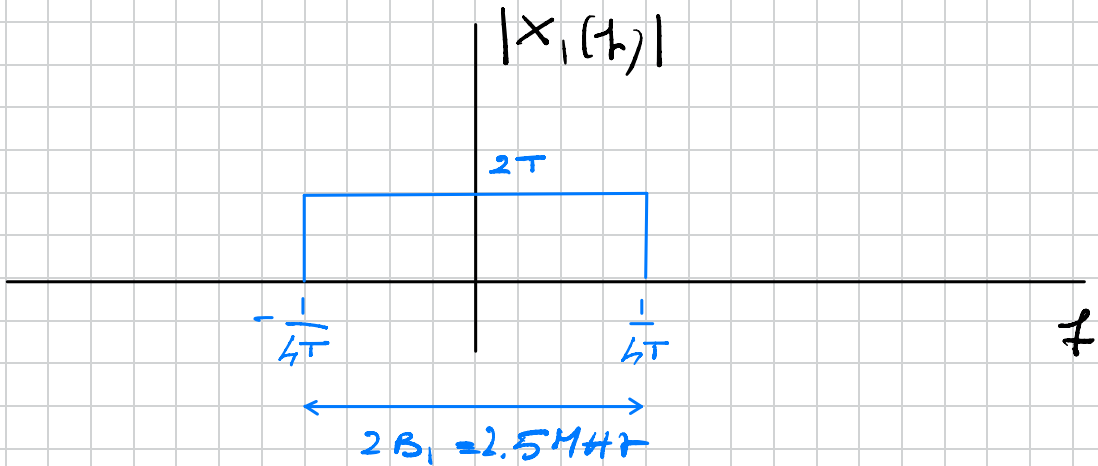
$$f = f_T$$

$$\text{Quindi } B_{-3\text{dB}} = f_T = \frac{1}{2\pi T} = \frac{1}{2\pi} \text{ kHz} \approx 160 \text{ Hz}$$

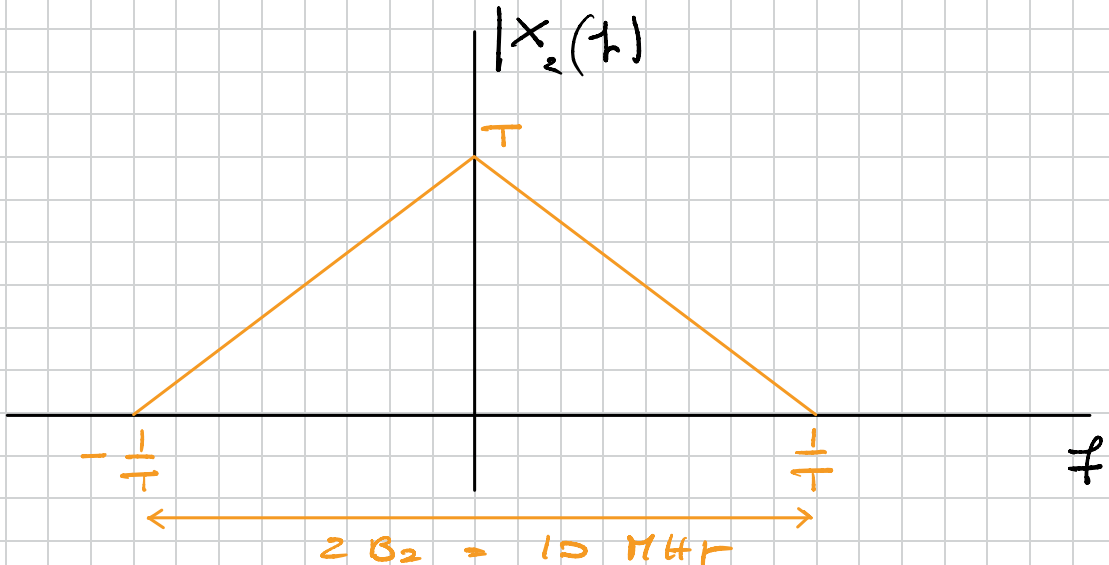
## Exercício 5

a)

$$x_1(t) = \text{sinc}\left(\frac{t}{2T}\right) \iff X_1(f) = 2T \text{ rect}(fT)$$

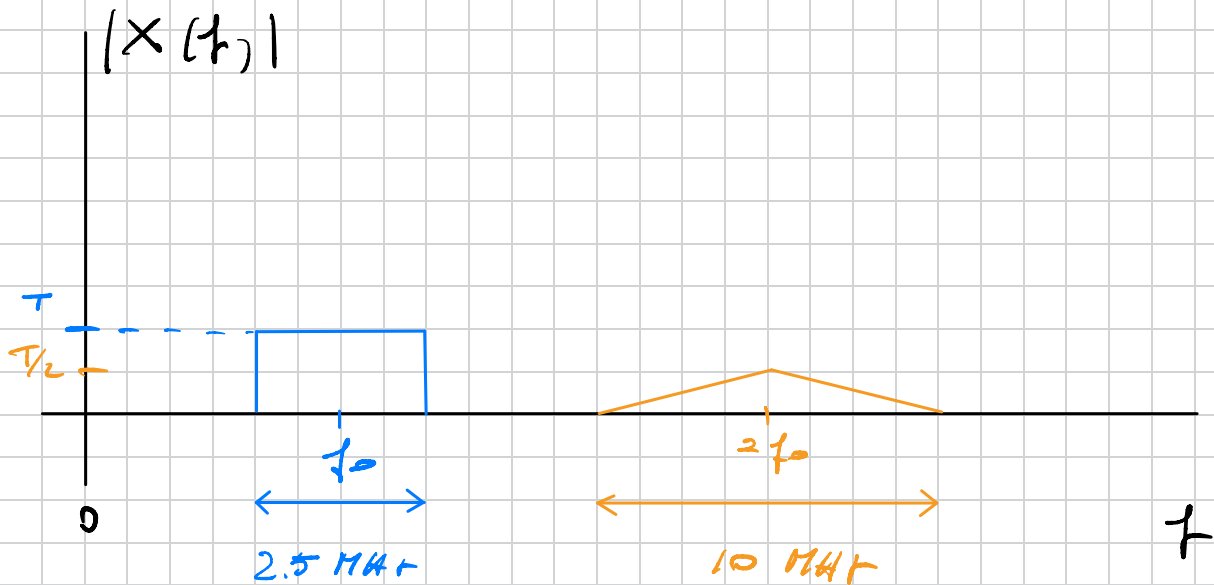


$$x_2(t) = \text{sinc}^2\left(\frac{t}{T}\right) \iff X_2(f) = T^2 \text{ rect}(fT) \otimes \text{rect}(fT)$$

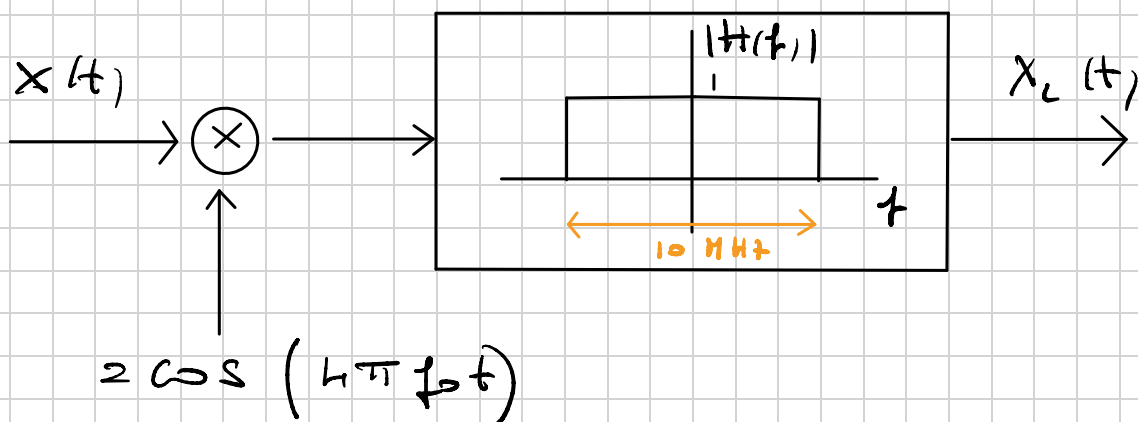


Tenuto di conto che :

$$X(f) = \frac{X_1(f + f_0) + X_1(f - f_0)}{2} + \frac{X_2(f + 2f_0) + X_2(f - 2f_0)}{2}$$



b) Per recuperare il segnale  $x_2(t)$ , devo demodularlo con un  $\cos(\cdot)$  alla frequenza  $2f_0$  e filtrarlo con un filtro passa-banda di banda  $5\text{ MHz}$



## ESERCIZIO 6

La matrice controllo di parità è:

$$\underline{H} = \left[ \underline{P}^T, \underline{H}_3 \right] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

La sindrome di parità:

$$\underline{s} = \underline{y} \underline{H}^T = [0 \ 1 \ 0]$$

Siccome  $\underline{s} = \underline{y} \underline{H}^T = \underline{e} \underline{H}^T$  si trova

$$\underline{e} = [0 \ 0 \ 0 \ 0 \ 1 \ 0]$$

da cui segue  $\underline{y} + \underline{e} = [0 \ 1 \ 1 \ 1 \ 1 \ 0]$



## Esercizio 7

a) La densità spettrale di potenza è

$$S_S(f) = \frac{1}{T_S} S_x(f) |G_T(f)|^2$$

dove

$$S_x(f) = \sum_m R_x(m) e^{-j2\pi f T_S m}$$

$$R_x(m) = \begin{cases} \sigma_x^2 + \gamma_x^2 & m=0 \\ \gamma_x^2 & m \neq 0 \end{cases}$$

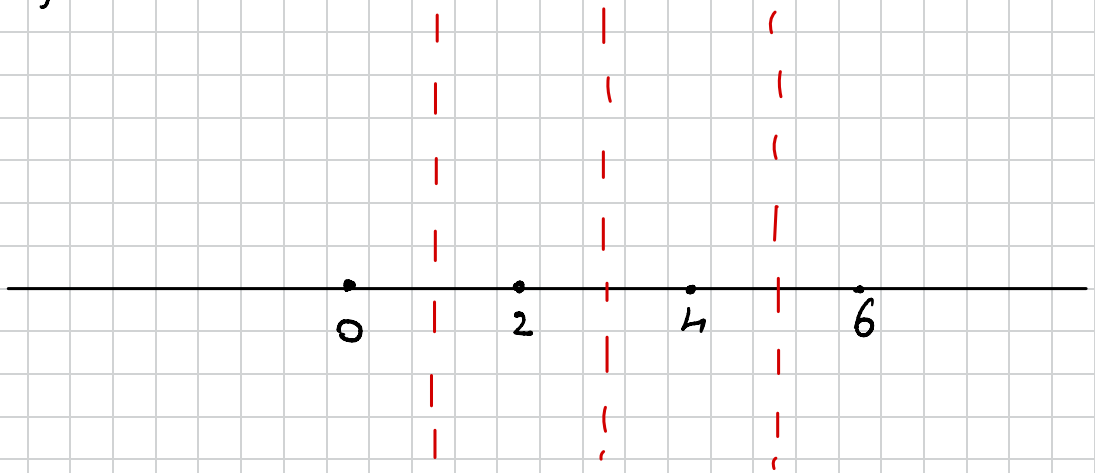
Calcoliamo  $\gamma_x$  e  $\sigma_x^2$ :

$$\gamma_x = E\{x_i\} = \frac{1}{4} (0 + 2 + 4 + 6) = 3$$

$$\sigma_x^2 = E\{x_i^2\} - \gamma_x^2 = \frac{1}{4} (0^2 + 2^2 + 4^2 + 6^2) - 3^2$$

$$= \frac{56}{4} - 9 = 5$$

b)



la probabilità di errore e la storia di una  
 $k$ -PAM (anche HMM) dal momento che  
dipende da  $Q\left(\frac{k-1}{\sigma}\right)$

Quindi:

$$\begin{aligned} P(e) &= \frac{1}{L} \left( 2 P(e | c_k = 2) + 2 P(e | c_k = 0) \right) \\ &= \frac{1}{2} \left( 2 Q\left(\frac{1}{\sigma_k}\right) + Q\left(\frac{1}{\sigma_k}\right) \right) = \frac{3}{2} Q\left(\frac{1}{\sigma_k}\right) \end{aligned}$$

## ESERCIZIO 8

a) Tenuto conto che  $B = \frac{1+\alpha}{T_S} = \frac{1+\alpha}{r \log_2 D} R_b$

$$\eta = \frac{R_b}{B} = \frac{r \log_2 D}{1+\alpha} = \frac{5}{4}$$

$$R_b = \eta B = 5 \text{ Mbit/s}$$

Il tempo necessario per trasmettere il file è

$$T_{\text{FILE}} = \frac{S_{\text{FILE}}}{R_b} = 0.2 \text{ s}$$

b) La probabilità di errore in ingresso al codificatore è

$$P_{\text{e}}^{\text{RIS}} = Q\left(\frac{1}{\sqrt{\gamma_n}}\right) = Q\left(\sqrt{\frac{E_s}{N_0}}\right) = Q\left(\sqrt{\frac{E_b r \log_2 D}{N_0}}\right)$$

Se  $\frac{E_b}{N_0} = 6 \text{ dB} = 4 \text{ m} \Rightarrow$  avere  $P_{\text{e}}^{\text{RIS}} = Q(\sqrt{6}) \approx 6 \cdot 10^{-3}$