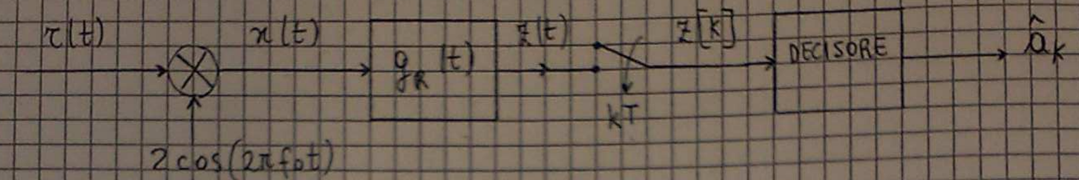
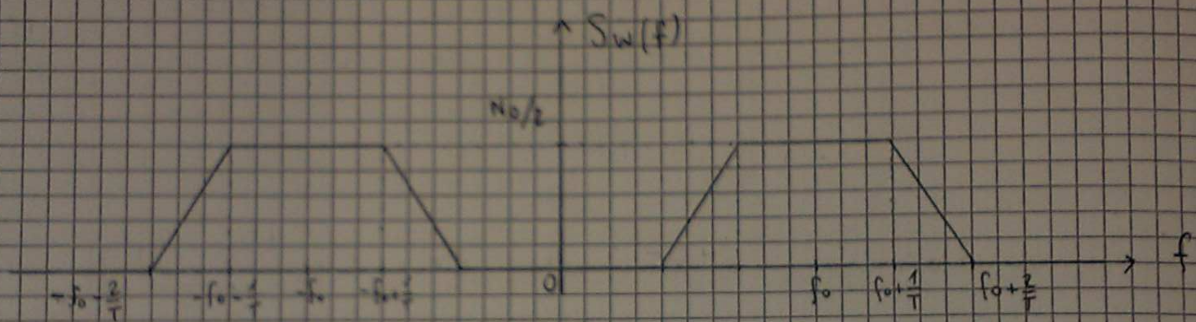
REAL PSK

Compito del 28/05/2007

es. 3) Il segnale $r(t) = \sum_i g_T(t - iT) \cdot \cos(2\pi f_0 t + a_i) + w(t)$ viene applicato al seguente sistema:



dove $w(t)$ è un rumore gaussiano bianco con valor medio $\eta_w = 0$ e densità spettrale di potenza $S_w(f)$:



Si ha $TCF[g_T(t)] = G_T(f) = \cos^2\left(\pi f \frac{T}{2}\right) \text{rect}\left(f \frac{T}{2}\right)$.

I simboli $a_i \in \left\{0, \frac{\pi}{2}\right\}$ sono equiprobabili ed indipendenti;

$G_R(f) = T \cdot \text{rect}\left(f \cdot \frac{T}{2}\right)$.

Il decisore ha la seguente strategia:

$$\hat{a}_k = \begin{cases} 0 & \text{se } z[k] \geq 1/4 \\ \pi/2 & \text{se } z[k] < 1/4 \end{cases}$$

Determinare:

1) energia media per simbolo ricevuta;

2) $S_{\tilde{w}}(f)$, $S_{w_c}(f)$, $S_{w_s}(f)$

3) $g(t) = g_T(t) \otimes g_R(t)$

4) BER.

Calcolo l'energia media per simbolo ricevuta

$$\overline{E}_R = \frac{1}{2} E_r\Big|_{\theta_i=0} + \frac{1}{2} E_r\Big|_{\theta_i=\frac{\pi}{2}}$$

$$E_r\Big|_{\theta_i=0} = \int_{-\infty}^{+\infty} \left(g_r(t) \cdot \cos(2\pi f_0 t)\right)^2 dt =$$

$$= \int_{-\infty}^{+\infty} g_r^2(t) \cdot \left(\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0 t) \right) dt =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} g_r^2(t)\, dt + \frac{1}{2}\int_{-\infty}^{+\infty} g_r^2(t) \cos(4\pi f_0 t)\, dt =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} g_r^2(t)\, dt \underset{\text{Parseval}}{=} \frac{1}{2}\int_{-\infty}^{+\infty} |G_r(f)|^2 df = \frac{1}{2}\int_{-\infty}^{+\infty} \left| \frac{1}{T} \cdot G_N(f,1) \right|^2 df$$

ovvero $E_r\Big|_{\theta_i=0} = E_r\Big|_{\theta_i=\frac{\pi}{2}} = \overline{E}_R = \frac{1}{2}\int_{-\infty}^{+\infty} \cos^4\left(\pi f \frac{T}{2}\right) \mathrm{rect}\left(f\frac{T}{2}\right) df$

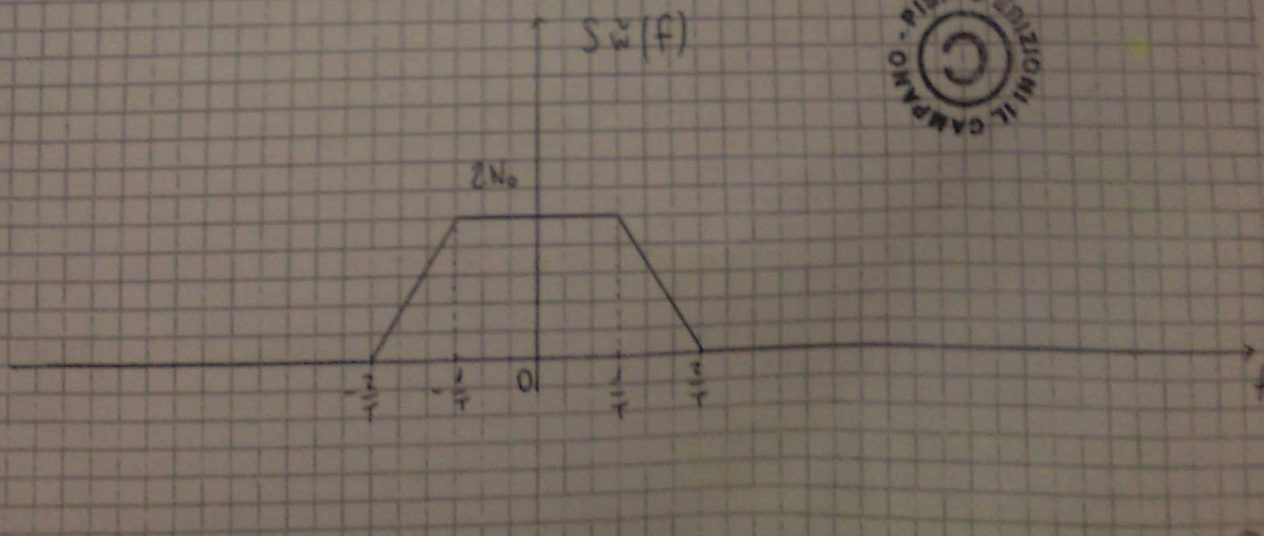
$$\Rightarrow \overline{E}_R = \frac{1}{2}\int_{-1/T}^{1/T} \left(\frac{1}{2} + \frac{1}{2}\cos^2(\pi f T) \right) df = \frac{1}{2T} + \frac{1}{4}\int_{-1/T}^{1/T} \cos^2(\pi f T)\, df$$

$$= \frac{1}{2T} + \frac{1}{4}\int_{-1/T}^{1/T} \left(\frac{1}{2} + \frac{1}{2}\cos(2\pi f T) \right) df = \frac{1}{2T} + \frac{1}{4T} + \underbrace{\frac{1}{8}\int_{-1/T}^{1/T} \cos(2\pi f T)\, df}_{\text{integrale nullo}}$$

$$\Rightarrow \overline{E}_R = \frac{3}{4T} \qquad (1)$$

Densità spettrale di potenza dell'inviluppo complesso $\tilde{w}(t)$:

$$S_{\tilde{w}}(f) = \begin{cases} 4\, S_w(f+f_0) & \forall f \geq -f_0 \\ 0 & \text{altrove} \end{cases}$$



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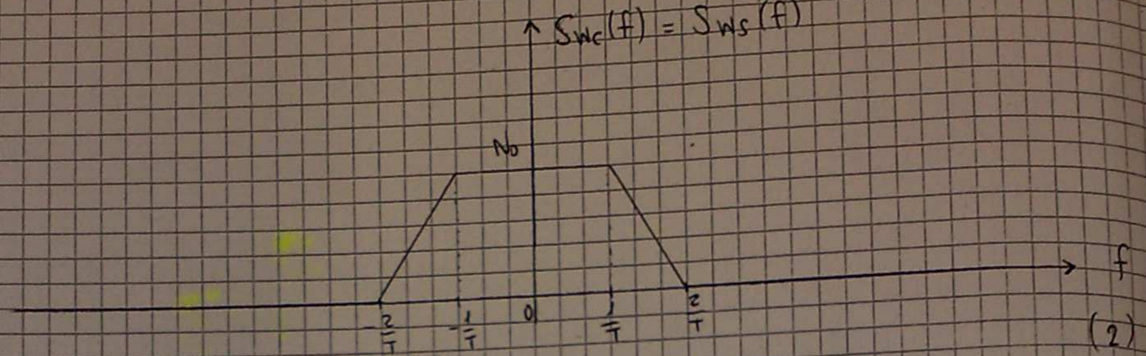
Ora:

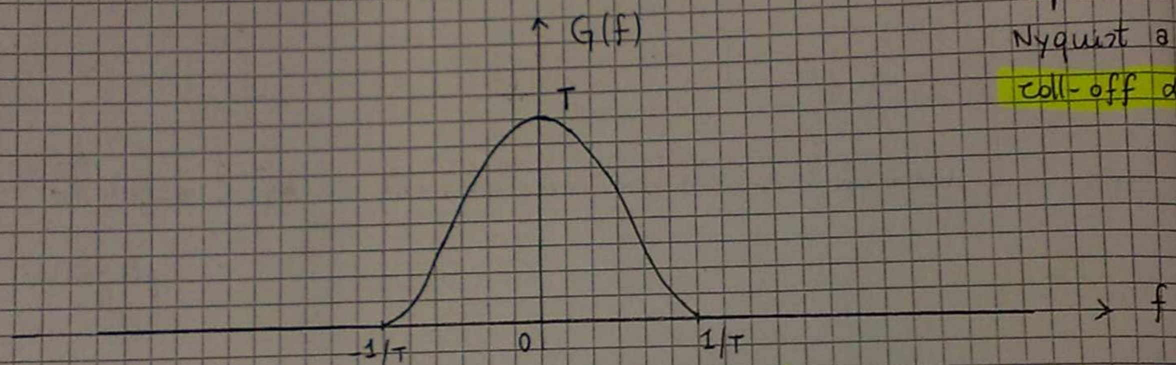
$$S_{w_c}(f) = S_{w_s}(f) = \frac{S_{\tilde{w}}(f) + S_{\tilde{w}}(-f)}{4}$$



(2)

$g(t) = g_T(t) \otimes g_R(t)$, in frequenza:

$$G(f) = G_T(f)\, G_R(f) = T \cdot \cos^2\left(\pi f \frac{T}{2}\right) \cdot \mathrm{rect}\left(\frac{f}{2/T}\right) = \underbrace{G_N(f, 1)}_{\text{impulso di Nyquist a roll-off } \alpha = 1}$$



Quindi $g(t) = TCF^{-1}[G(f)] = TCF^{-1}\left[\left(\frac{T}{2} + \frac{T}{2}\cos(\pi f T)\right) \mathrm{rect}\left(\frac{f}{2/T}\right)\right] =$

$$= \left[\frac{T}{2}\delta(t) + \frac{T}{4}\left(\delta\left(t - \frac{T}{2}\right) + \delta\left(t + \frac{T}{2}\right)\right)\right] \otimes \frac{2}{T}\,\mathrm{sinc}\left(\frac{2}{T}t\right) =$$

$$= \mathrm{sinc}\left(\frac{2}{T}t\right) + \frac{1}{2}\left[\mathrm{sinc}\left(\frac{2}{T}\left(t - \frac{T}{2}\right)\right) + \mathrm{sinc}\left(\frac{2}{T}\left(t + \frac{T}{2}\right)\right)\right] \quad (3$$

Per calcolare la BER bisogna intanto determinare $z(t)$:

$$n(t) = r(t) \cdot 2\cos(2\pi f_0 t) = 2 \cdot \cos(2\pi f_0 t) \cdot \sum_i g_T(t - iT)\cos(2\pi f_0 t + \vartheta$$

$$+ \underbrace{2 \cdot \cos(2\pi f_0 t) \cdot w(t)}_{\text{rumore statisticamente uguale a } w(t)} = \sum_i \left[g_T(t - iT) \cdot 2\cos(2\pi f_0 t + \vartheta_i)\cos(2\pi f_0 t)\right] + w$$

Per le formule di Werner:

$$2\cos(\alpha)\cos(\beta) = \cos(\alpha + \beta) + \cos(\alpha - \beta)$$

$\Rightarrow x(t) = \sum_i \left(g_T(t-iT) \cdot \left[\cos(4\pi f_0 t + a_i) + \cos(a_i) \right] \right) + w(t)$

Visto che $g_R(t)$ è un filtro passa-basso, posso considerare solo le componenti in banda-base di $x(t)$:

$$x(t)\Big|_{\text{in B.B.}} = \sum_i \left(g_T(t-iT) \cdot \cos(a_i) \right) + w(t)$$

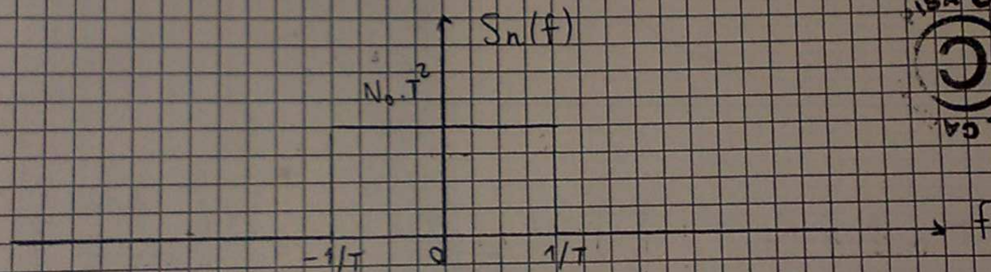
Ora sia $b_i = \cos(a_i) \Rightarrow b_i = \begin{cases} 1 & \text{se } a_i = 0 \\ 0 & \text{se } a_i = \pi/2 \end{cases}$

$$\Rightarrow x(t)\Big|_{\text{in B.B.}} = \sum_i b_i \cdot g_T(t-iT) + w(t) \Rightarrow$$

$$\Rightarrow z(t) = x(t) \otimes g_R(t) = \sum_i b_i \cdot g_T(t-iT) + n(t)$$

dove $n(t) = g_R(t) \otimes w_c(t)$ la cui densità spettrale di potenza $S_n(f)$ è la seguente:

$$S_n(f) = S_{w_c}(f) \cdot |G_R(f)|^2 = N_0 \cdot T^2 \cdot \text{rect}\left(\frac{f}{2/T}\right)$$



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Il fatto che $g(t)$ è di Nyquist ($G(f) = G_N(f, 1)$) implica che $ISI = 0$

$$z[k] = b_k + n[k] \in \mathcal{N}(b_k, \sigma^2)$$

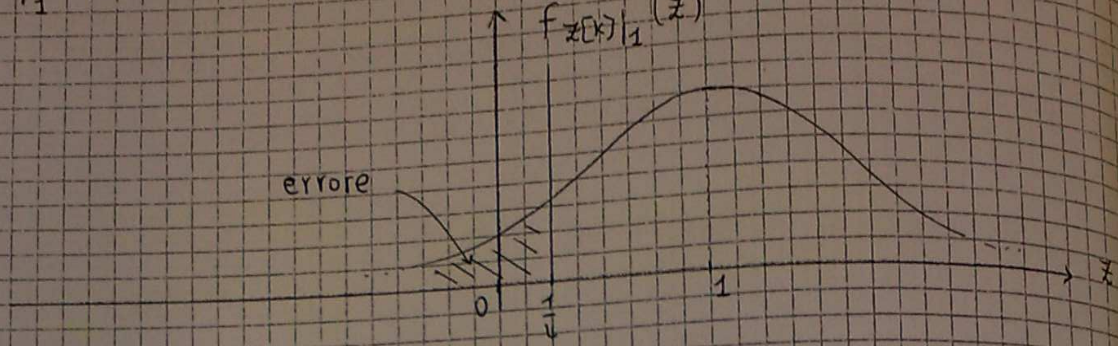
$$\sigma^2 = \int_{-\infty}^{+\infty} S_n(f)\, df = 2 N_0 T$$

La strategia di decisione diventa:

$$\hat{a}_k = \begin{cases} 0 & \text{se } z[k] \geq 1/4 \\ \pi/2 & \text{se } z[k] < 1/4 \end{cases} \Leftrightarrow \hat{b}_k = \begin{cases} 1 & \text{se } z[k] \geq 1/4 \\ 0 & \text{se } z[k] < 1/4 \end{cases}$$

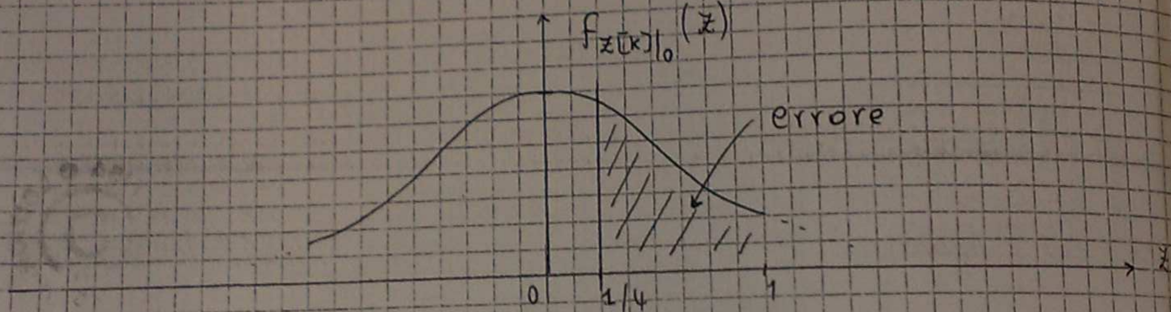
$$B.E.R. = P(e) = \frac{1}{2} P(e|1) + \frac{1}{2} P(e|0)$$

$$z[k]\big|_1 = 1 + n[k] \in N(1, \sigma^2)$$



$$P(e|1) = Q\left(\frac{3/4}{\sigma}\right) = Q\left(\frac{3/4}{\sqrt{2N_0T}}\right)$$

Analogamente $z[k]\big|_0 = n[k] \in N(0, \sigma^2) \Rightarrow P(e|0) = Q\left(\frac{1/4}{\sigma}\right)$



Pertanto $B.E.R. = \frac{1}{2} \cdot \left[Q\left(\frac{3/4}{\sqrt{2N_0T}}\right) + Q\left(\frac{1/2}{\sqrt{2N_0T}}\right) \right]$ (4)