TEO WEIERSTRASS $f: A \to \mathbb{R}$ A chiuso + limitato

$\Rightarrow \exists \; m = \min_A f \quad M = \max_A f$

TEO ZERI

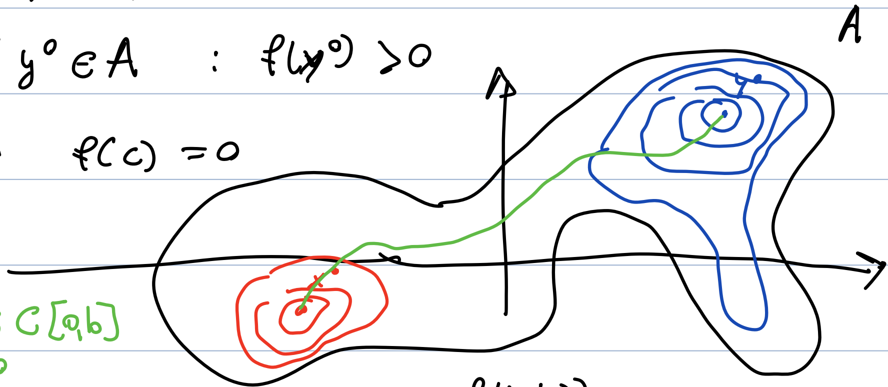
$f: A \to \mathbb{R}$ A connesso (per archi) $f \in C(A)$

$A \subseteq \mathbb{R}^n$

$\exists \; x^0 \in A \; : \; f(x^0) < 0$

$\exists \; y^0 \in A \; : \; f(y^0) > 0$

$\Rightarrow \exists \; c \in A \; : \; f(c) = 0$



$\gamma: [a,b] \to \mathbb{R}^n \quad \gamma \in C[a,b]$

$\gamma(a) = x^0 \quad \gamma(b) = y^0$

$F(t) = f(\gamma(t)) \quad F(a) = f(x^0) < 0$

$F \in C([a,b]) \to \mathbb{R} \quad \Rightarrow \exists \bar{t} \; : F(\bar{t}) = 0 \quad F(b) = f(y^0) > 0$

LAGRANGE

$f \in C[a,b]$, $f'\ ]a,b[$, $\exists c \in]a,b[\ : f'(c) = \frac{f(b)-f(a)}{b-a}$

$f : [a,b] \to \mathbb{R}$

LAGRANGE NON VALE $f : \mathbb{R} \to \mathbb{R}^m \quad m > 1$

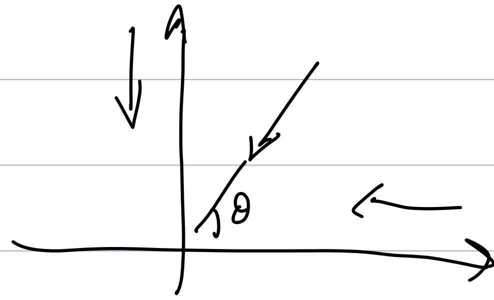
$f : [a,b] \to \mathbb{R}^2 \qquad f(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix} \qquad f'(t) = \begin{pmatrix} f_1'(t) \\ f_2'(t) \end{pmatrix}$

$f(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} \qquad t \in [0, 2\pi] \qquad f(0) = f(2\pi)$

?? $\exists \bar{t} \in]0, 2\pi[\ : f'(\bar{t}) = \frac{f(2\pi) - f(0)}{2\pi} = 0$

$f'(t) = \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} \qquad |f'(t)|^2 = \sin^2(t) + \cos^2(t) = 1 \neq 0$

(1) $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$



$f \in C(\mathbb{R}^2 \setminus \{(0,0)\})$

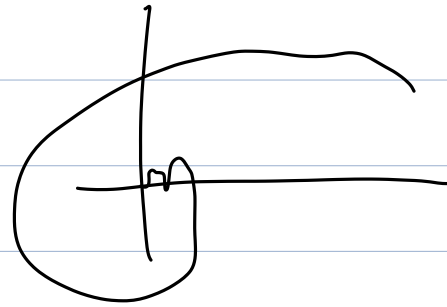
$f(0,y) = 0 \quad \forall y \neq 0$

$f(x,0) = 0 \quad \forall x \neq 0$

$f(x,y) = \dfrac{\rho \cos\theta \, \rho \sin\theta}{\rho^2} = \cos\theta \sin\theta$

(2) $f(x,y) = \varphi(x) + \psi(y) \qquad \psi, \varphi \in C(\mathbb{R}) \Rightarrow f \in C(\mathbb{R}^2)$

$$f(x,y) = \begin{cases} x \cdot \sin\left(\frac{1}{y}\right) & y \neq 0 \\ 0 & y = 0 \end{cases}$$

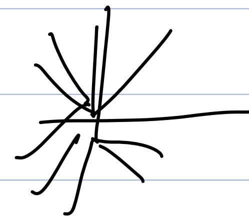


f è continua in $(0,0)$?

$$f(x,y) = \underset{\substack{\downarrow \\ 0}}{\rho} \underset{\substack{\downarrow \\ \text{LIMITATO}}}{\cos\theta} \underset{\text{LIMITATO}}{\sin\left(\frac{1}{\rho \sin\theta}\right)} \xrightarrow{\rho \to 0} 0$$

$$\lim_{(x,y) \to (x^0,y^0)} f(x,y) \neq \lim_{y \to y^0} \lim_{x \to x^0} f(x,y)$$

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$



$y = mx$ $\qquad\qquad \forall\, m \neq 0$

$$f(x,mx) = \frac{x^2\, mx}{x^4 + m^2x^2} = \frac{\cancel{x^2}}{\cancel{x^2}} \cdot \frac{mx}{m^2+x^2} \xrightarrow{x \to 0} 0$$

$$f(x,x^2) = \frac{x^4}{x^4+x^4} = \frac{1}{2}$$

$y(x) = x^{m/n}$ $\qquad$ m, n primi tra loro

$$f(x, x^{m/n}) \xrightarrow[x \to 0]{} 0$$

$$f(x,y) = \begin{cases} \dfrac{e^{-1/x^2}\, y}{e^{-2/x^2}+y^2} & (x,y) \neq (0,0) \\ e & (x,y) = (0,0) \end{cases}$$

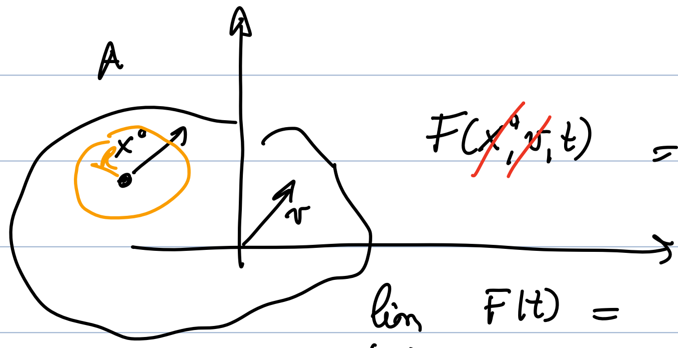
$$f(x, e^{-1/x^2}) \to \frac{1}{2}$$

$f : [a,b] \to \mathbb{R}$

$$f'(t) = \lim_{t \to t_0} \frac{f(t) - f(t_0)}{t - t_0} \qquad t \in]a,b[\qquad f'_+(a) \qquad f'_-(b)$$

$$\frac{f(x) - f(y)}{x - y}$$ NON HA SENSO $\qquad x \in \mathbb{R}^n \qquad f : A \to \mathbb{R}$, $A \subseteq \mathbb{R}^n$

$\| tv \| < R$

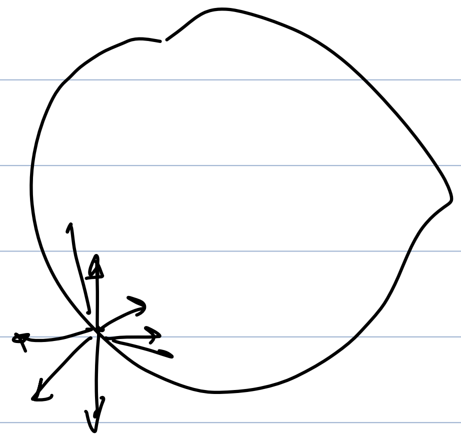


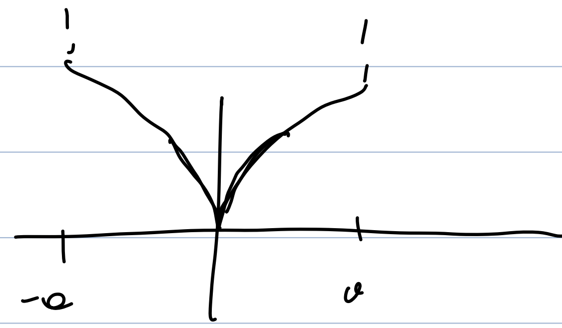
$$F(\cancel{x^0}, \cancel{v}, t) = \frac{f(x^0 + (tv)) - f(x^0)}{t} \qquad t \in \mathbb{R} \quad v \in \mathbb{R}^n$$

$$\lim_{t \to 0} F(t) = \frac{\partial f}{\partial v}(x^0)$$ DERIVATA DIREZIONALE DI f in x^0

se x^0 è interno

$\exists\, B(x^0, R) \subset A \qquad t : \ |t|\,\|v\| < R$





$$\{ (x,y) \in \mathbb{R}^2 : |x| < a \quad y > \sqrt{|x|} \}$$

$$v = e_i = (0, \ldots 0, \overset{i}{1}, 0 \ldots 0) \qquad \frac{\partial f}{\partial v}(x^0) = \frac{\partial f}{\partial x_i}(x^0)$$

$$\frac{\partial f}{\partial x_i}(x^0) = f_{x_i}(x^0) = D_i f(x^0)$$ DERIVATA PARZIALE DI f RISPETTO A x_i

nabla

$$\nabla f(x^0) = \left(\frac{\partial f}{\partial x_1}(x^0), \frac{\partial f}{\partial x_2}(x^0), \ldots, \frac{\partial f}{\partial x_n}(x^0) \right)$$

GRADIENTE DI f

$$f(x,y) = x^2 \sin(y)$$

$$\frac{\partial f}{\partial x}(x,y) = 2x \sin(y)$$

$$\frac{\partial f}{\partial y}(x,y) = x^2 \cos(y)$$

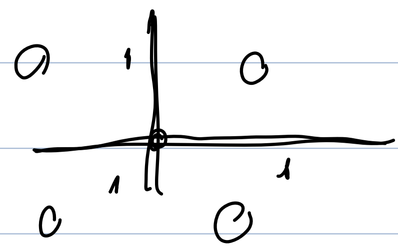
$$f(x,y,z) = e^{xyz}$$

$$\frac{\partial f}{\partial x}(x,y,z) = e^{xyz} \frac{\partial}{\partial x}(xyz)$$

$$= e^{xyz}\, yz$$

$f: \mathbb{R} \to \mathbb{R}$ $\quad$ f DERIV. IN x_0 $\Rightarrow$ f continua in x_0

$$f(x,y) = \begin{cases} 1 & xy = 0 \\ 0 & xy \neq 0 \end{cases}$$



$$\frac{\partial f}{\partial x}(0,0) = 0$$

$$\frac{\partial f}{\partial y}(0,0) = 0$$

$\mathbb{R}^2$
$\cup$

(1) $B = \left(B(0,2) \setminus \left([-1,1] \times \{0\}\right)\right) \cup \left(]-1,1[\times \{3\}\right)$

$\overset{\circ}{B}$ $\quad \overline{B}$ $\quad \partial B$

(2) $\lim\limits_{(x,y) \to (0,0)} \dfrac{x^2 - x^3 + y^2 - y^3}{x^2+y^2}$

(4) $\lim\limits_{(x,y,z) \to (0,0,0)} \dfrac{xy + yz^2 + xz^2}{x^2+y^2+z^4}$

(3) $\lim\limits_{(x,y) \to (0,0)} \dfrac{x-y}{x+y}$