Soluzione Ese 2

1) $E_b = \frac{1}{2} E_{b0} + \frac{1}{2} E_{b1}$

$E_{b0} = 0$

$E_{b1} = E_p = 1$

$E_b = \frac{1}{2}$

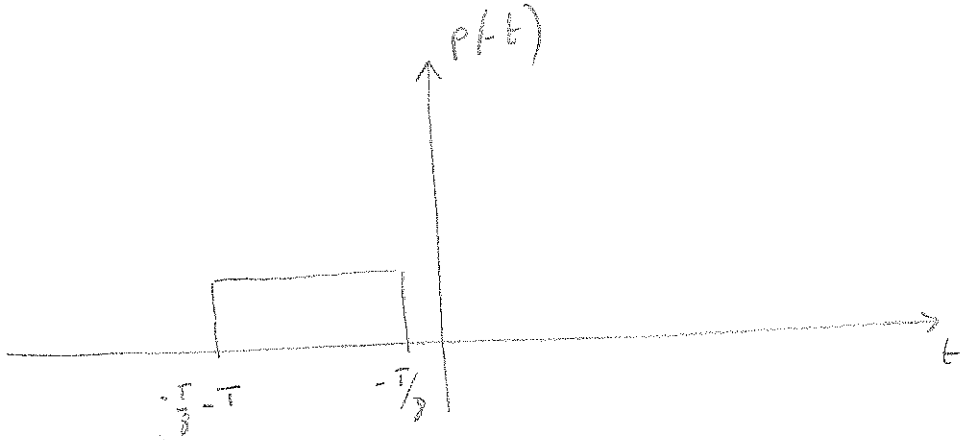
2) filtro adattato

$r(t) = A\, p(t_0 - t)$

$p(t) = \sqrt{\frac{1}{T}}\ \text{rect}\left(\frac{t - (T/8 + T/2)}{T}\right) = \sqrt{\frac{1}{T}}\ \text{rect}\left(\frac{t - 5T/8}{T}\right)$

$t_0 = \frac{T}{8} + T = \frac{9T}{8}$

$r(t) = A\, p\left(\frac{9T}{8} - t\right)$



3) $\sigma_{nu}^2 = \frac{N_0}{2} \int_{-\infty}^{+\infty} r^2(t)\, dt = A^2 \frac{N_0}{2}$.

a) $C(f) = 1$

$$H(f) = P(f) \cdot R(f)$$

$$P(f) = \sqrt{\frac{1}{T}} \cdot T \; \text{sinc}(fT) \; e^{-j2\pi f 5T/8}$$

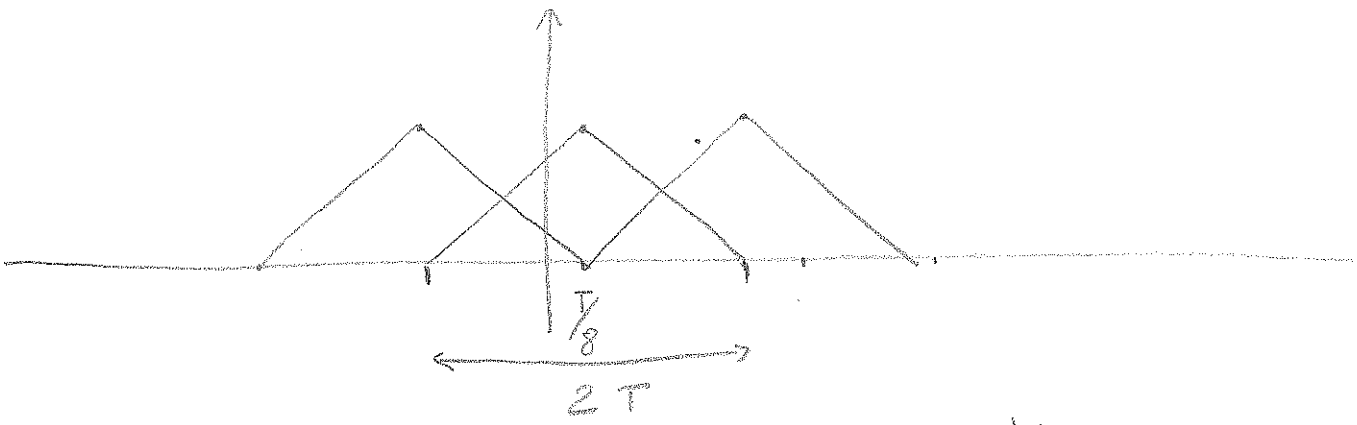
$$R(f) = A\, P(f)\, e^{j2\pi f 9T/8} = A\sqrt{T}\, \text{sinc}(fT)\, e^{j2\pi f T/2}$$

$$H(f) = A\, T\, \text{sinc}^2(fT)\, e^{-j2\pi f T/8}$$

$$h(t) = A\left(1 - \frac{|t - T/8|}{T}\right) \text{rect}\left(\frac{t - T/8}{2T}\right)$$

Il segnale prima del campionatore è

$$y(t) = A \sum_k x[k]\, h(t - kT) + n_u(t)$$



L'istante di campionamento ottimo è quindi

$$t_k = \frac{T}{8} + kT$$

5) $y_k = A\,\acute{x}[k] + n_0^k$

$$P_e\left(x[k]=0\right) = Q\left(\frac{1/4}{\sqrt{A^2 N_0/2}}\right) = Q\left(\sqrt{\frac{1}{8A^2 N_0}}\right)$$

$$P_e\left(x[k]=1\right) = Q\left(\sqrt{\frac{(A^2-1/4)^2}{A^2 N_0/2}}\right)$$

La BER è minima quando $P_e\left(x[k]=0\right) = P_e\left(x[k]=1\right)$

$$\frac{1}{8A^2 N_0} = \frac{2(A-1/4)^2}{A^2 N_0}$$

$$\frac{1}{8} = 2A^2 + \frac{1}{8} - A$$

$$A(2A-1) = 0 \quad \Rightarrow \quad A = \frac{1}{2}$$