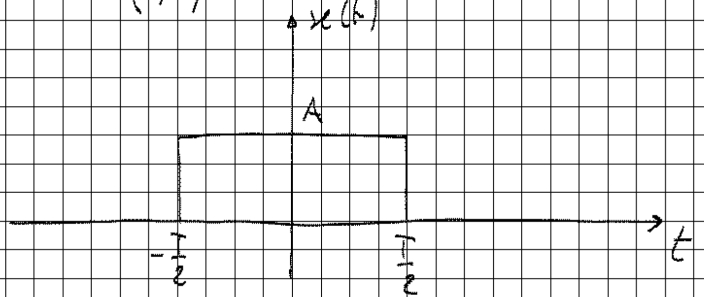
ESERCIZIO - TCF DI UN RETTANGOLO

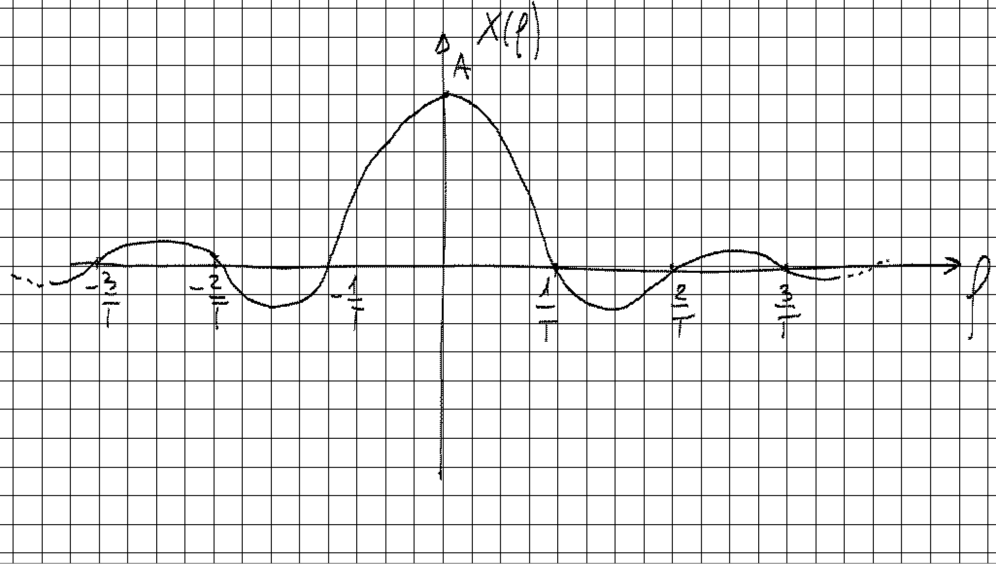
$$x(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$$



$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt = A\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-j2\pi f t}\, dt =$$

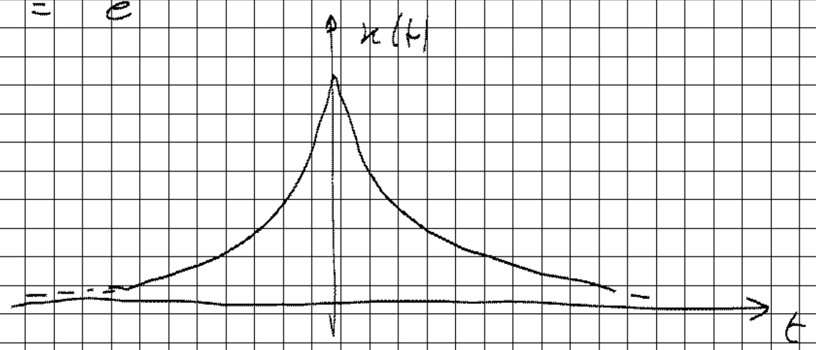
$$= \frac{A}{-j2\pi f}\, e^{-j2\pi f t}\Big|_{-\frac{T}{2}}^{\frac{T}{2}} = \frac{A}{j2\pi f}\left(e^{j\pi f T} - e^{-j\pi f T}\right) =$$

$$= \frac{A}{\pi f}\frac{T}{T}\sin(\pi f T) = AT\,\mathrm{sinc}(fT)$$



ESERCIZIO – TCF DI UNA ESPONENZIALE BIL.

$$x(t) = e^{-|t|}$$



$$X(f) = \int_{-\infty}^{+\infty} e^{-|t|} e^{-j2\pi f t} \, dt =$$

$$= \int_{-\infty}^{0} e^{(1-j2\pi f)t} \, dt + \int_{0}^{+\infty} e^{-(1+j2\pi f)t} \, dt =$$

$$= \frac{1}{1-j2\pi f} e^{(1-j2\pi f)t} \Big|_{-\infty}^{0} - \frac{1}{1+j2\pi f} e^{-(1+j2\pi f)t} \Big|_{0}^{+\infty} =$$

$$= \frac{1}{1-j2\pi f} + \frac{1}{1+j2\pi f} = \frac{1+j2\pi f + 1 - j2\pi f}{1+4\pi^2 f^2} = \frac{2}{1+4\pi^2 f^2}$$

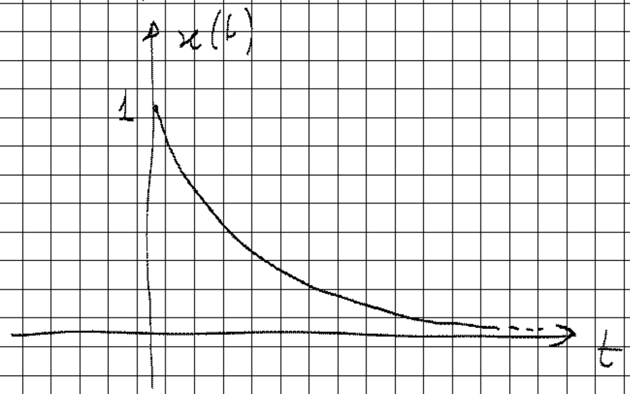
N.B. $x(t)$ reale e pari

$\Updownarrow$

$X(f)$ reale e pari

ESERCIZIO - TCF DI UNA ESPONENZIALE UNIL.

$$x(t) = e^{-t} u(t)$$



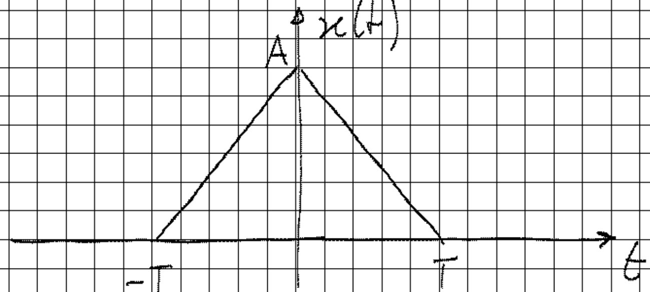
$$X(f) = \int_{-\infty}^{+\infty} e^{-t} u(t)\, e^{-j2\pi f t}\, dt =$$

$$= \int_{0}^{+\infty} e^{-t} e^{-j2\pi f t}\, dt = \int_{0}^{+\infty} e^{-(1+j2\pi f)t}\, dt =$$

$$= -\frac{1}{1+j2\pi f}\, e^{-(1+j2\pi f)t}\Big|_{0}^{+\infty} = \frac{1}{1+j2\pi f}$$

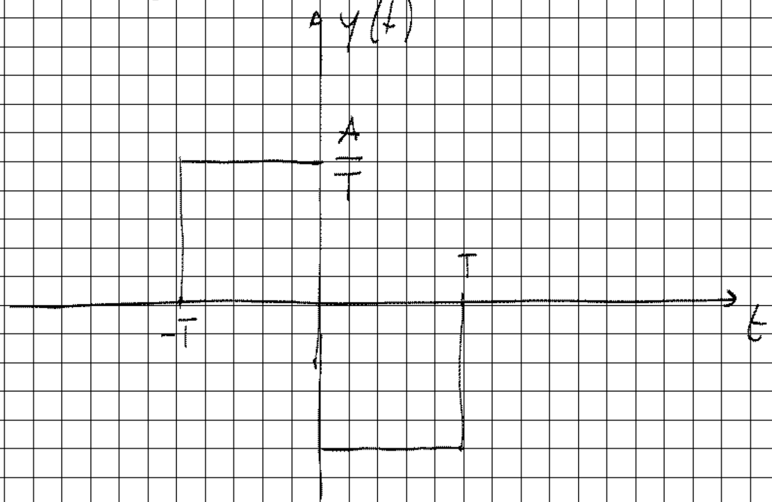
ESERCIZIO – TCF DI UN TRIANGOLO

$$x(t) = A\left(1 - \frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right)$$



Soluzione

$$y(t) = \frac{A}{T}\,\mathrm{rect}\left(\frac{t+T/2}{T}\right) - \frac{A}{T}\,\mathrm{rect}\left(\frac{t-T/2}{T}\right)$$



$$x(t) = \int_{-\infty}^{t} y(\alpha)\, d\alpha$$

$$X(f) = \frac{Y(f)}{j2\pi f}$$

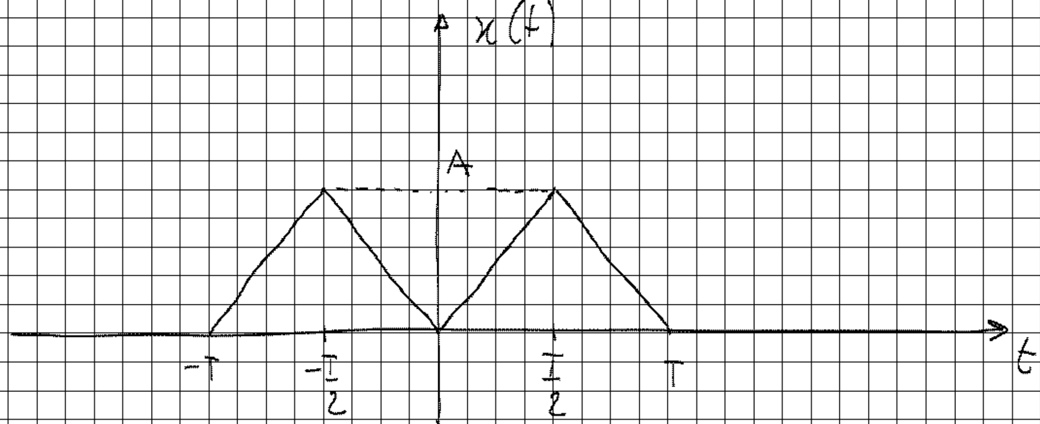
$$Y(f) = \frac{A}{T} T \,\mathrm{sinc}(Tf)\, e^{j2\pi f \frac{T}{2}} - \frac{A}{T} T \,\mathrm{sinc}(Tf)\, e^{-j2\pi f \frac{T}{2}}$$

$$= A \,\mathrm{sinc}(Tf) \left[e^{j\pi f T} - e^{-j\pi f T} \right] = j2A \,\mathrm{sinc}(Tf) \sin(\pi T f)$$

$$X(f) = \frac{j2A \,\mathrm{sinc}(Tf) \sin(\pi T f)}{j2\pi f} = A \,\mathrm{sinc}(Tf) \frac{\sin(\pi T f)}{\pi f} =$$

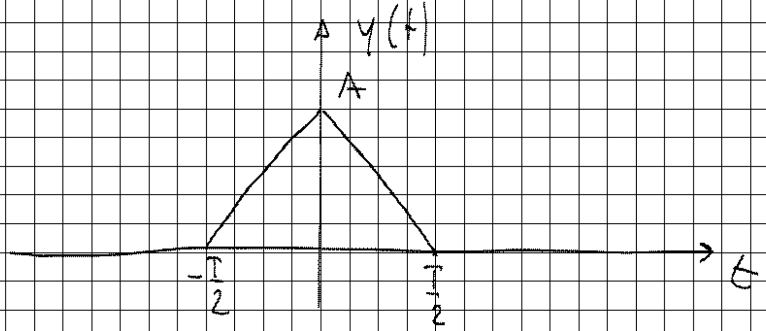
$$= AT \,\mathrm{sinc}^2(Tf)$$ spettro reale e pari

ESERCIZIO - TCF DI DUE TRIANGOLI



Soluzione

$$y(t) = A\left(1 - \frac{|t|}{T/2}\right) \mathrm{rect}\left(\frac{t}{T}\right)$$



$$x(t) = y\left(t - \frac{T}{2}\right) + y\left(t + \frac{T}{2}\right)$$

$$X(f) = Y(f)\, e^{-j2\pi f \frac{T}{2}} + Y(f)\, e^{j2\pi f \frac{T}{2}} =$$

$$= Y(f)\left[e^{j\pi f T} + e^{-j\pi f T}\right] = 2Y(f)\cos(\pi f T)$$

$$Y(f) = AT' \operatorname{sinc}^2(T' f) \qquad T' = \frac{T}{2}$$

$$= A\frac{T}{2} \operatorname{sinc}^2\left(\frac{T}{2} f\right)$$

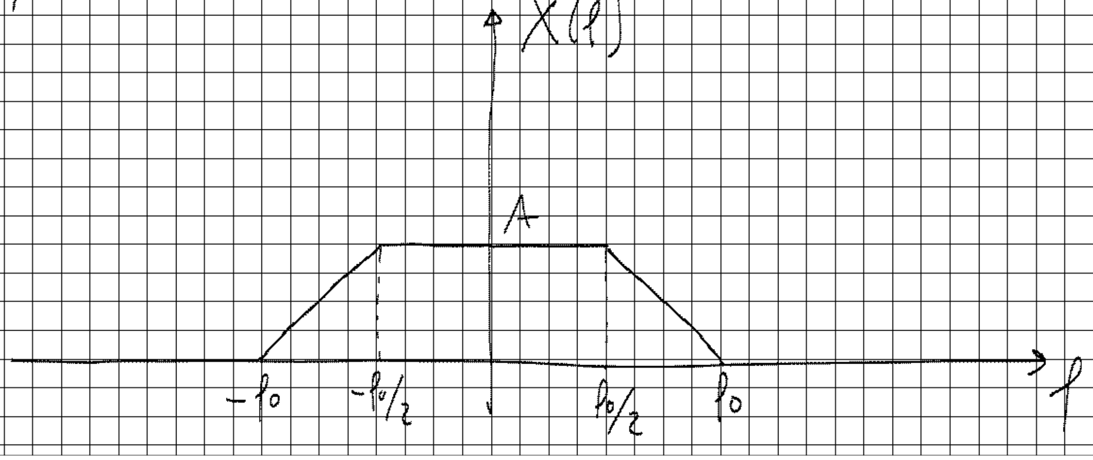
$$X(f) = AT \operatorname{sinc}^2\left(\frac{T}{2} f\right) \cos(\pi f T)$$

spettro reale e pari

NB. Da notare la dualità con il teorema della modulazione

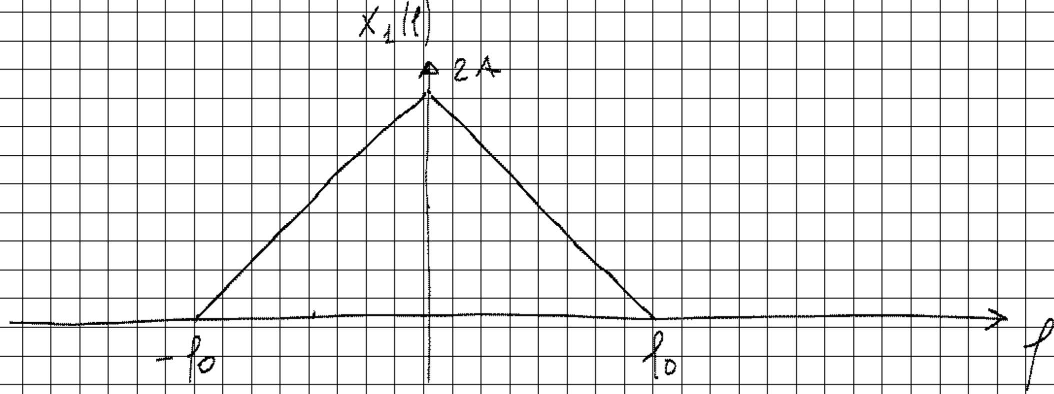
ESERCIZIO

Calcolare la ATCF (FT^{-1}) del seguente spettro:



Soluzione I

$$X(f) = X_1(f) - X_2(f)$$



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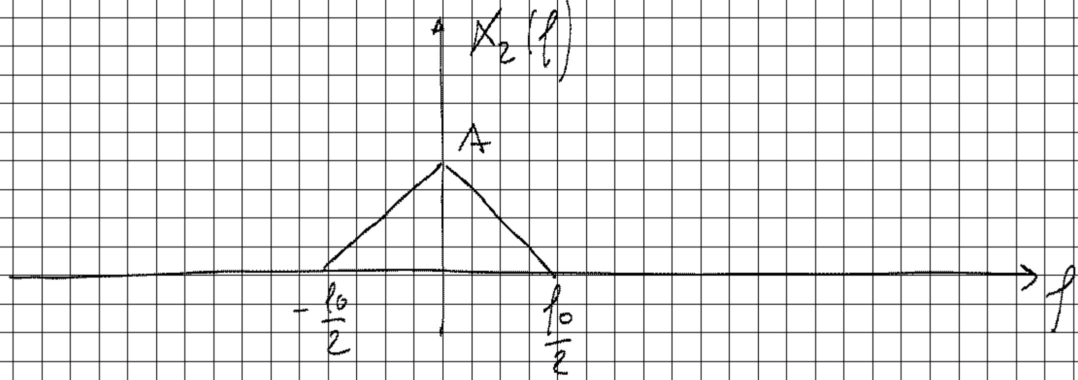


$$X_1(f) = 2A\left(1 - \frac{|f|}{f_0}\right) \mathrm{rect}\left(\frac{f}{2f_0}\right)$$

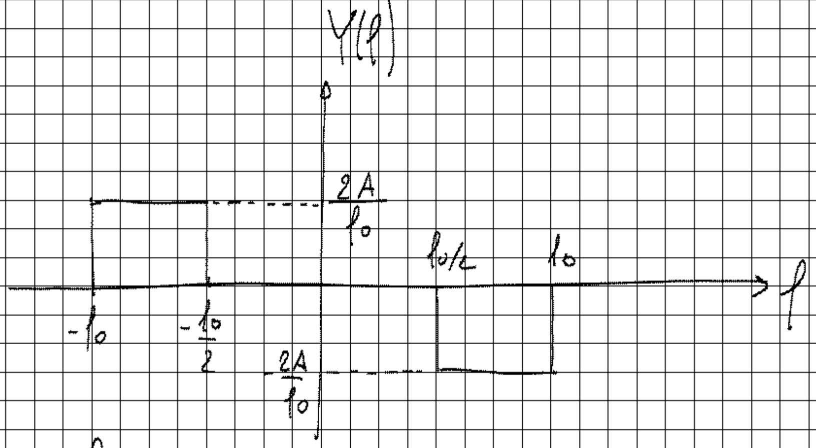
$$X_2(f) = A\left(1 - \frac{|f|}{f_0/2}\right) \mathrm{rect}\left(\frac{f}{f_0}\right)$$

$$x_1(t) = 2Af_0 \,\mathrm{sinc}^2(f_0 t)$$

$$x_2(t) = A\frac{f_0}{2} \,\mathrm{sinc}^2\left(\frac{f_0}{2} t\right)$$

$$x(t) = Af_0\left[2\,\mathrm{sinc}^2(f_0 t) - \frac{1}{2}\,\mathrm{sinc}^2\left(\frac{f_0}{2} t\right)\right]$$

Soluzione II



$$X(f) = \int_{-\infty}^{f} Y(\alpha)\, d\alpha$$

$$x(t) = -\frac{y(t)}{j2\pi t}$$

$$Y(f) = \frac{2A}{f_0}\left[\mathrm{rect}\left(\frac{f+\frac{3}{4}f_0}{f_0/2}\right) - \mathrm{rect}\left(\frac{f-\frac{3}{4}f_0}{f_0/2}\right)\right]$$

$$y(t) = \frac{2A}{f_0}\,\frac{f_0}{2}\,\mathrm{sinc}\left(\frac{f_0}{2}t\right) e^{-j2\pi\frac{3}{4}f_0 t} +$$

$$-\frac{2A}{f_0}\,\frac{f_0}{2}\,\mathrm{sinc}\left(\frac{f_0 t}{2}\right) e^{j2\pi\frac{3}{4}f_0 t} =$$

$$= -A\,\mathrm{sinc}\left(\frac{f_0}{2}t\right)\left[e^{j\frac{3}{2}\pi f_0 t} - e^{-j\frac{3}{2}\pi f_0 t}\right] =$$

$$= -2jA\,\mathrm{sinc}\left(\frac{f_0}{2}t\right)\sin\left(\frac{3}{2}\pi f_0 t\right)$$

$$x(t) = \frac{Y(t)}{-j2\pi t} = \frac{-2jA}{-j2\pi t}\,\text{sinc}\left(\frac{f_0}{2}t\right)\sin\left(\frac{3}{2}\pi f_0 t\right) =$$

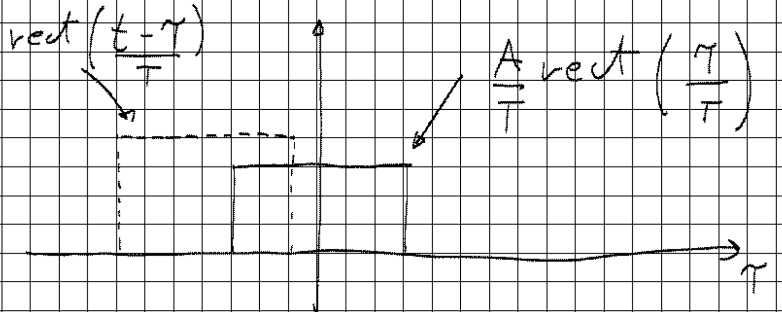
$$= A\,\text{sinc}\left(\frac{f_0}{2}t\right)\frac{\sin\left(\frac{3}{2}\pi f_0 t\right)}{\frac{3}{2}f_0\pi t}\cdot\frac{3}{2}f_0 =$$

$$= \frac{3}{2}Af_0\,\text{sinc}\left(\frac{f_0}{2}t\right)\text{sinc}\left(\frac{3}{2}f_0 t\right)$$

TCF DEL TRIANGOLO (Prodotto di Convoluzione)

$$x(t) = A\left(1-\frac{|t|}{T}\right)\text{rect}\left(\frac{t}{T}\right)$$

$$x(t) = \frac{A}{T}\text{rect}\left(\frac{t}{T}\right)\otimes\text{rect}\left(\frac{t}{T}\right) = \int_{-\infty}^{+\infty}\frac{A}{T}\text{rect}\left(\frac{\tau}{T}\right)\text{rect}\left(\frac{t-\tau}{T}\right)d\tau$$



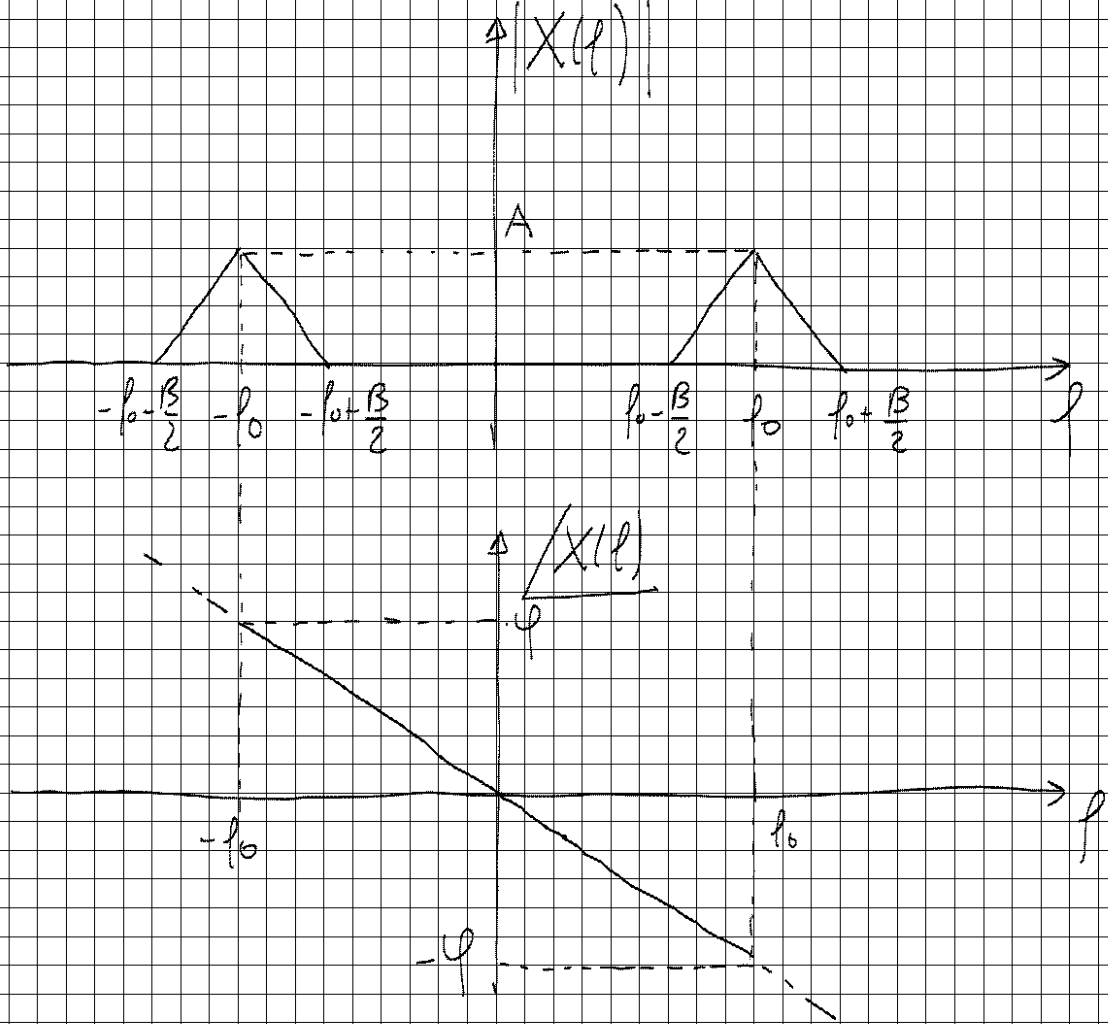
$$x(t) = x_1(t)\otimes x_2(t)$$

$$x_1(t) = \frac{A}{T}\text{rect}\left(\frac{t}{T}\right)\overset{TCF}{\Longleftrightarrow}X_1(f) = A\,\text{sinc}(Tf)$$

$$x_2(t) = \text{rect}\left(\frac{t}{T}\right)\overset{TCF}{\Longleftrightarrow}X_2(f) = T\,\text{sinc}(Tf)$$

$$X(f) = X_1(f)X_2(f) = AT\,\text{sinc}^2(Tf)$$

ESERCIZIO – ANTITRASFORMATA

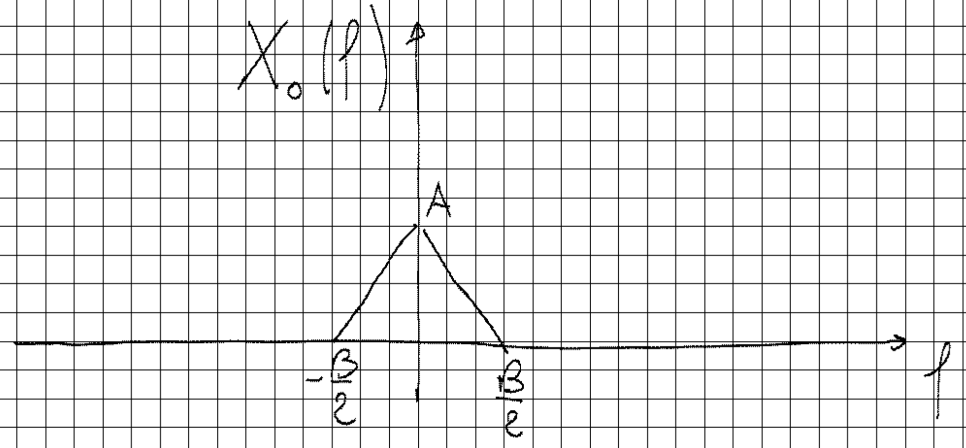


Calcolare $x(t)$

Soluzione

$$X(f) = |X(f)| \cdot e^{-j\angle X(f)}$$

·) Scrivo prima l'espressione analitica dello spettro di ampiezza $|X(f)|$



$$|X(f)| = X_0(f-f_0) + X(f+f_0)$$

$$X_0(f) = A\left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right)$$

·) $\angle X(f)$

$$\angle X(f) = af + b$$

$$\left.\begin{array}{l} b = 0 \\ a = \dfrac{-\varphi}{f_0} \end{array}\right\} \Rightarrow \angle X(f) = -\frac{\varphi}{f_0} f$$

$$X(f) = \left[X_0(f-f_0) + X_0(f+f_0) \right] e^{-j\frac{\varphi}{f_0} f}$$

$$= \underbrace{\left[X_0(f-f_0) + X_0(f+f_0) \right]}_{A(f)} e^{-j2\pi f t_0} \quad , \quad t_0 = \frac{\varphi}{2\pi f_0}$$

$$x(t) = a(t-t_0) \quad , \quad a(t) \overset{TCF}{\Longleftrightarrow} A(f)$$

$$a(t) = x_0(t)\, e^{j2\pi f_0 t} + x_0(t)\, e^{-j2\pi f_0 t} =$$

$$= 2x_0(t) \left[\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} \right] =$$

$$= 2x_0(t) \cos(2\pi f_0 t)$$

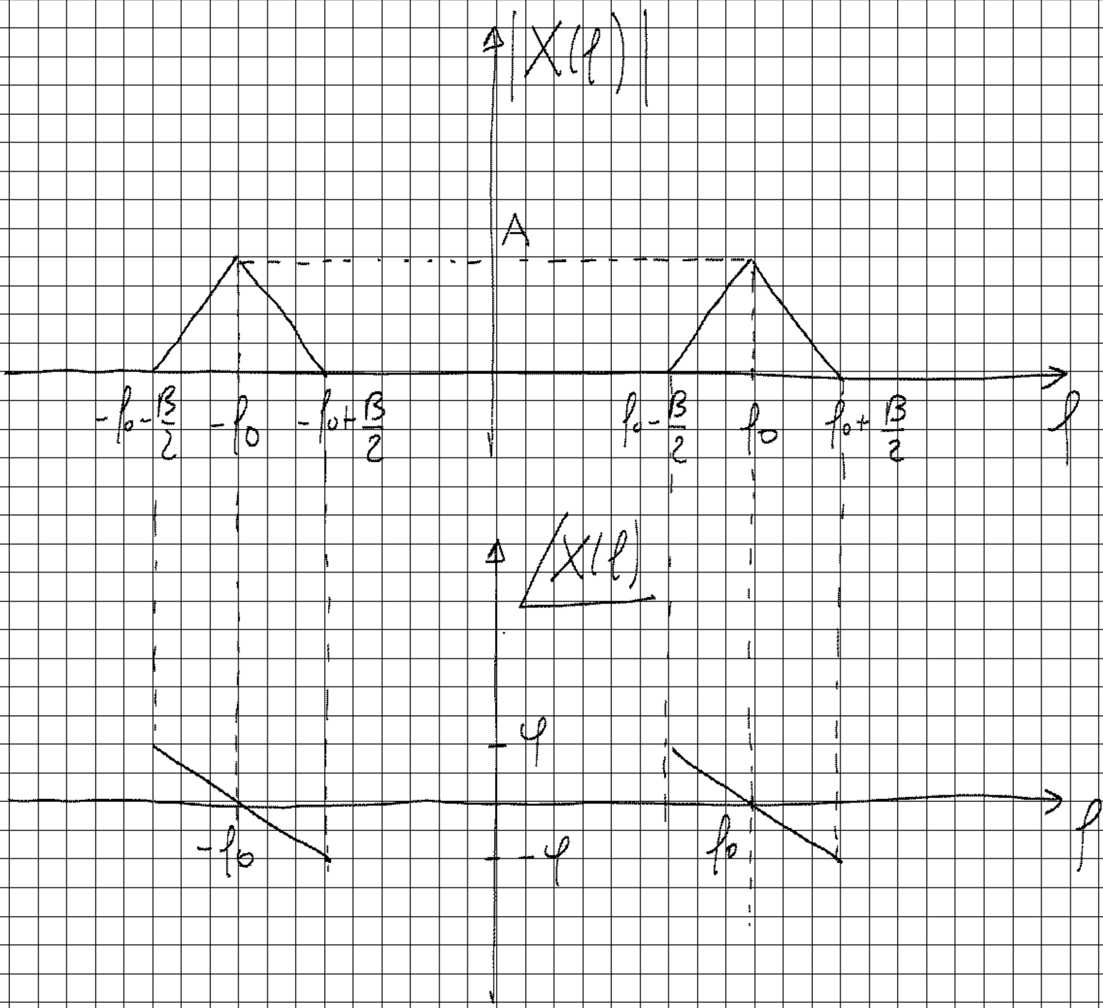
$$x(t) = 2x_0(t-t_0) \cos\left[2\pi f_0 (t-t_0) \right]$$

$$x_0(t) = \frac{AB}{2} \operatorname{sinc}^2\left(\frac{B}{2} t \right)$$

$$x(t) = AB \operatorname{sinc}^2\left[\frac{B}{2}(t-t_0) \right] \cos\left[2\pi f_0 (t-t_0) \right]$$

$$t_0 = \frac{\varphi}{2\pi f_0}$$

ESERCIZIO - ANTITRASFORMATA (VARIANTE)

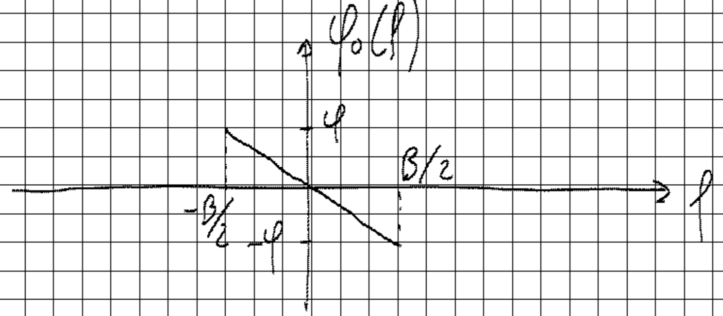


Calcolare $x(t)$

$$X_0(f) = A\left(1 - \frac{|f|}{B/2}\right) \mathrm{rect}\left(\frac{f}{B}\right)$$

$$X(f) = X_0(f-f_0)\, e^{j\varphi_1(f)} + X(f+f_0)\, e^{j\varphi_2(f)}$$

$$\angle X(f) = \varphi_1(f) + \varphi_2(f) = \varphi_0(f-f_0) + \varphi_0(f+f_0)$$



$$\varphi_0(f) = af + b$$

$$\left.\begin{array}{l} b = 0 \\ a = -\dfrac{\varphi}{B/2} = -\dfrac{2\varphi}{B} \end{array}\right\} \varphi_0(f) = -\frac{2\varphi}{B} f$$

$$X(f) = X_0(f-f_0)\, e^{j\varphi_0(f-f_0)} + X_0(f+f_0)\, e^{j\varphi_0(f+f_0)} =$$

$$= X_0(f-f_0)\, e^{-j\frac{2\varphi}{B}(f-f_0)} + X_0(f+f_0)\, e^{-j\frac{2\varphi}{B}(f+f_0)} =$$

$$= X_0(f-f_0)\, e^{-j2\pi(f-f_0)t_0} + X_0(f+f_0)\, e^{-j2\pi(f+f_0)t_0}$$

$$t_0 = \frac{\varphi}{\pi B}$$

$$Y(f) = X_0(f)\, e^{-j2\pi f t_0}$$

$$X(f) = Y(f-f_0) + Y(f+f_0)$$

$$x(t) = y(t)\, e^{+j2\pi f_0 t} + y(t)\, e^{-j2\pi f_0 t} =$$

$$= 2y(t) \cos(2\pi f_0 t)$$

$$y(t) = x_0(t - t_0)$$

$$x_0(t) = \frac{AB}{2} \operatorname{sinc}^2\left(\frac{B}{2} t\right)$$

$$x(t) = 2 \frac{AB}{2} \operatorname{sinc}^2\left[\frac{B}{2}(t - t_0)\right] \cos(2\pi f_0 t)$$

$$t_0 = \frac{\varphi}{\pi B}$$

N.B. la differenza tra le soluzioni delle due varianti

- nel primo caso il segnale modulato e' ritardato di una quantita' pari a t_0
- nel secondo caso solo il segnale modulante e' ritardato di una quantita' pari a t_0