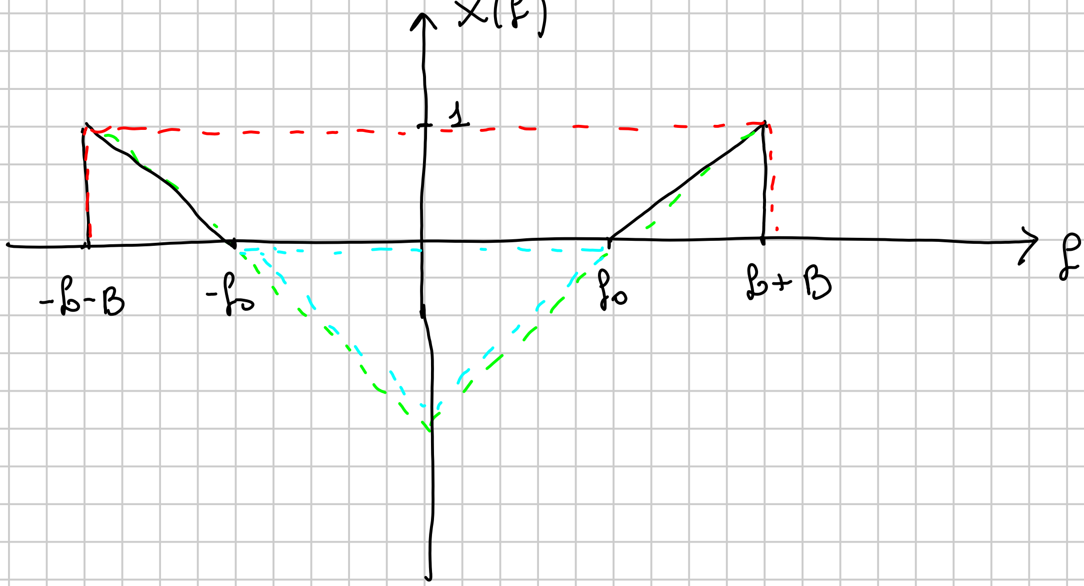
Compitino 2003

Esercizio 1

Si calcoli la TCF^{-1} dello spettro in figura.

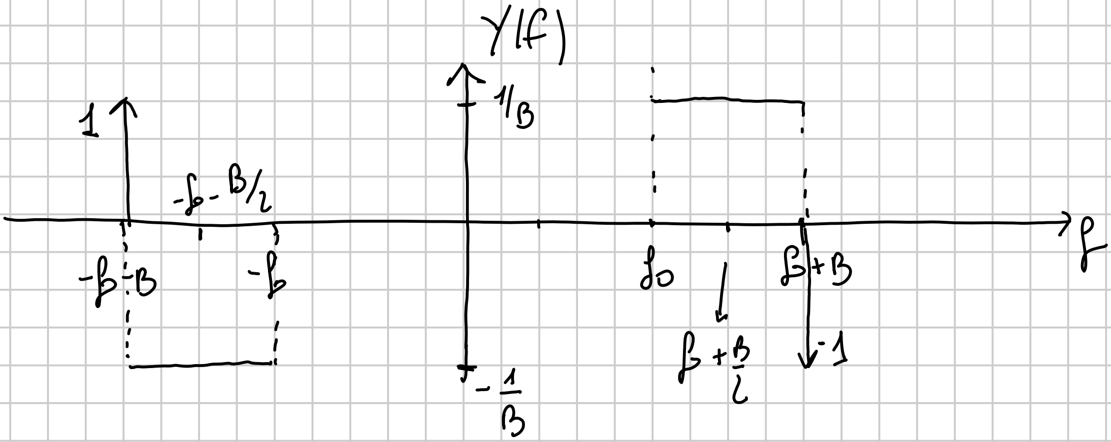


Teorema della derivata

$$Y(f) = \frac{\partial}{\partial f} X(f) \quad \rightleftarrows \quad y(t) = -j2\pi t\, x(t)$$

Se $\quad y(t)\big|_{t=0} = 0 \quad$ allora $\quad x(t) = -\dfrac{y(t)}{j2\pi t}$

$$y(0) = 0 \qquad \int_{-\infty}^{+\infty} Y(f)\, df = 0$$



$$Y(f) = \delta\left(f + f_0 + B\right) - \delta\left(f - (f_0 + B)\right) -$$

$$- \frac{1}{B} \operatorname{rect}\left(\frac{f + (f_0 + B/2)}{B}\right) + \frac{1}{B} \operatorname{rect}\left(\frac{f - (f_0 + B/2)}{B}\right)$$

$$y(t) = -2j \sin\left(2\pi (f_0 + B) t\right) - \operatorname{sinc}(tB)\, e^{-j 2\pi t (f_0 + B/2)} + \operatorname{sinc}(tB)\, e^{+j 2\pi t (f_0 + B/2)} =$$

$$= -2j \sin\left(2\pi t (f_0 + B)\right) + \operatorname{sinc}(tB) \left(e^{+j 2\pi t (f_0 + B/2)} - e^{-j 2\pi t (f_0 + B/2)} \right) =$$

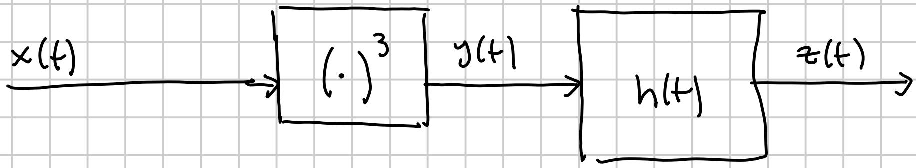
$$= -2j \sin\left(2\pi t (f_0 + B)\right) + 2j \operatorname{sinc}(tB) \cdot \sin\left(2\pi t \left(f_0 + \frac{B}{2}\right)\right)$$

$$x(t) = -\frac{y(t)}{2j\pi t} =$$

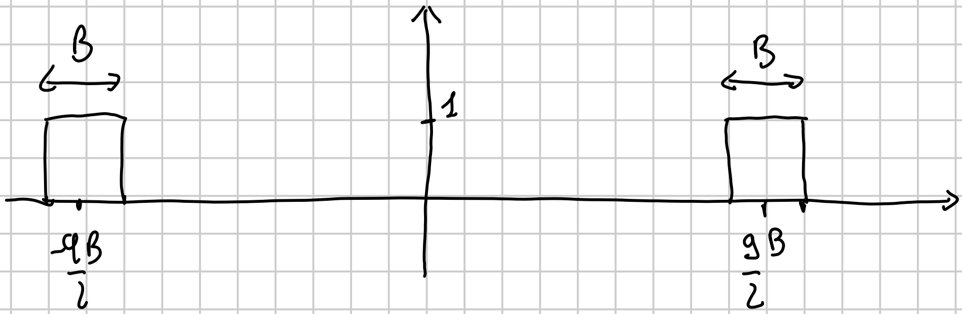
$$= \frac{\sin\left(2\pi t (f_0 + B)\right)}{\pi t} - \operatorname{sinc}(tB) \frac{\sin\left(2\pi t (f_0 + B/2)\right)}{\pi t} =$$

$$= 2(f_0 + B) \operatorname{sinc}\left(2t (f_0 + B)\right) - 2\left(f_0 + \frac{B}{2}\right) \operatorname{sinc}(tB) \cdot \operatorname{sinc}\left(2t (f_0 + B/2)\right)$$

Esercizio 2



$$x(t) = A\cos(3\pi B t + \theta)$$



- Calcolare $Y(f)$ e $y(t)$ e rappresentare i grafici di modulo e fase

- Determinare l'espressione di $z(t)$

$$y(t) = x^3(t) = A^3 \cos^3(3\pi B t + \theta) =$$

$$= A^3 \cos(3\pi B t + \theta) \cdot \cos^2(3\pi B t + \theta) =$$

$$= A^3 \cos(3\pi B t + \theta) \left(\frac{1}{2} + \frac{1}{2}\cos(6\pi B t + 2\theta) \right) =$$

$$= \frac{A^3}{2}\cos(3\pi B t + \theta) + \frac{A^3}{2}\cos(3\pi B t + \theta)\cos(6\pi B t + 2\theta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$= \frac{A^3}{2}\cos(3\pi Bt+\theta) + \frac{A^3}{2}\left(\frac{1}{2}\cos(9\pi Bt+3\theta) + \frac{1}{2}\cos(3\pi Bt+\theta)\right)$$

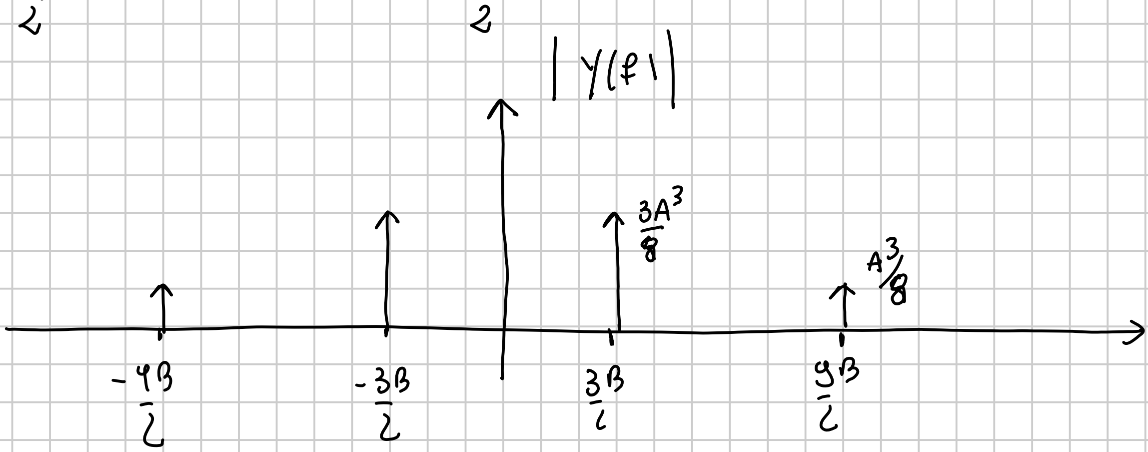
$$= \cos(3\pi Bt+\theta)\left(\frac{A^3}{2}+\frac{A^3}{4}\right) + \frac{A^3}{4}\cos(9\pi Bt+3\theta) =$$

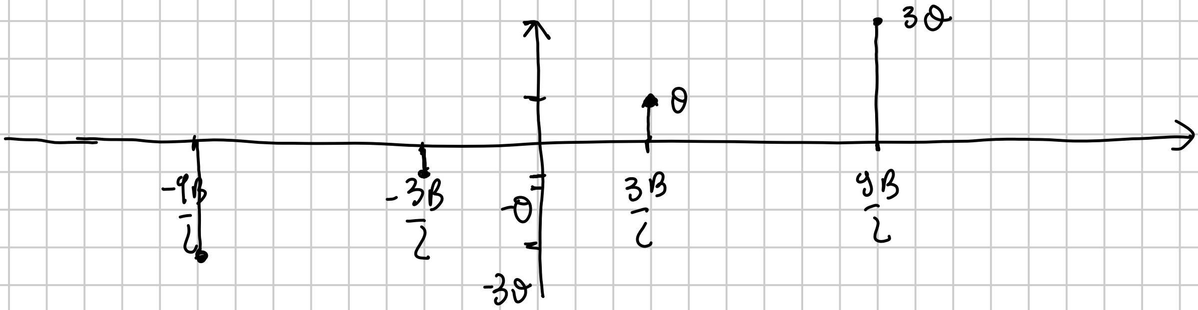
$$= \cos(\underbrace{3\pi Bt+\theta}_{2\pi\frac{3B}{2}t+\theta})\,\frac{3A^3}{4} + \frac{A^3}{4}\cos(\underbrace{9\pi Bt+3\theta}_{2\pi\frac{9B}{2}t+3\theta})$$

$$Y(f) = \frac{3A^3}{4}\left(\frac{\delta(f-f_{01})e^{i\theta} + \delta(f+f_{01})e^{-i\theta}}{2}\right) +$$

$$\frac{A^3}{4}\left(\frac{\delta(f-f_{02})e^{i3\theta} + \delta(f-f_{02})e^{-i3\theta}}{2}\right)$$

$$f_{01} = \frac{3B}{2} \qquad f_{02} = \frac{9B}{2}$$



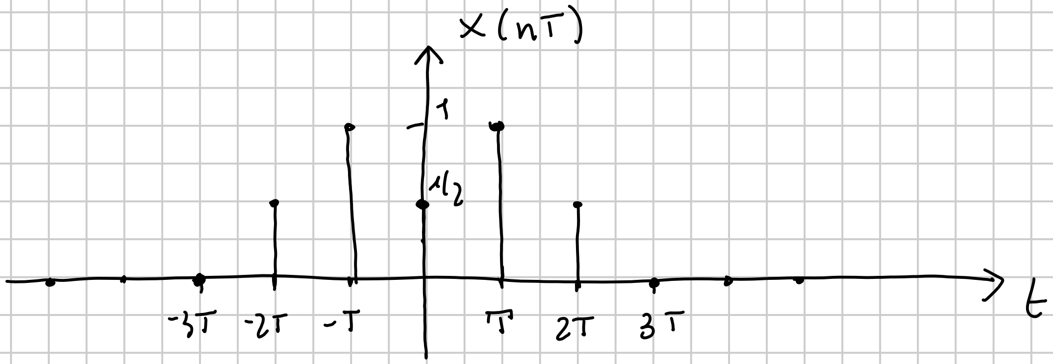


$$Z(f) = \frac{A^3}{8}\left(\delta\left(f - \frac{9B}{2}\right)e^{j3\theta} + \delta\left(f + \frac{9B}{2}\right)e^{-j3\theta}\right)$$

$$z(t) = \frac{A^3}{4}\cos\left(2\pi \frac{9B}{2} t + 3\theta\right)$$

Esercizio 3

Si calcoli la Trasformata Discreta di Fourier (TDF), $\overline{X}(f)$ della sequenza in figura.



$$x[n] = \frac{1}{2}\delta[n] + 1\,\delta[n-1] + 1\,\delta[n+1] + \frac{1}{2}\delta[n-2] + \frac{1}{2}\delta[n+2]$$

Teorema del ritardo

$$\overline{X}(f) = \frac{1}{2} + e^{-j2\pi fT} + e^{+j2\pi fT} + \frac{1}{2}e^{-j2\pi f\cdot 2T} + \frac{1}{2}e^{j2\pi f 2T}$$

$$= \frac{1}{2} + 2\cos(2\pi f T) + \cos(2\pi f 2T)$$

COMPITINO 19- APRILE 2010

Esercizio 1

SIA DATO IL SISTEMA

$$y(t) = |x(t)| + \int_a^t x(\alpha)\, d\alpha$$

DIRE SE IL SISTEMA È:

- LINEARE, STAZIONARIO, CON MEMORIA, STABILE, CAUSALE

- LINEARE

$$x(t) = a x_1(t) + b x_2(t)$$

$$T[x(t)] = |a x_1(t) + b x_2(t)| + \int_a^t (a x_1(t) + b x_2(t))\, dt \neq$$

$$\neq a|x_1(t)| + b|x_2(t)| + \int_a^t a x_1(t)\, dt + \int_a^t b x_2(t)\, dt = a y_1(t) + b y_2(t)$$

NON È LINEARE

- STAZIONARIO

$x(t-t_0) \Rightarrow y(t-t_0)$

$\alpha - t_0 = \alpha'$

$$T[x(t-t_0)] = |x(t-t_0)| + \int_b^t x(\alpha - t_0)\, d\alpha =$$

$$= |x(t-t_0)| + \int_{b-t_0}^{t-t_0} x(\alpha')\, d\alpha'$$

$$y(t-t_0) = |x(t-t_0)| + \int_b^{t-t_0} x(\alpha)\, d\alpha$$

NON È STAZIONARIO

- MEMORIA

Il sistema ha memoria perché l'uscita dipende da valori precedenti a t.

- CAUSALE

Se $b < t$

Il sistema è causale perché dipende solo da istanti precedenti a t.

- STABILITÀ

$|x(t)| < M$ per ogni t

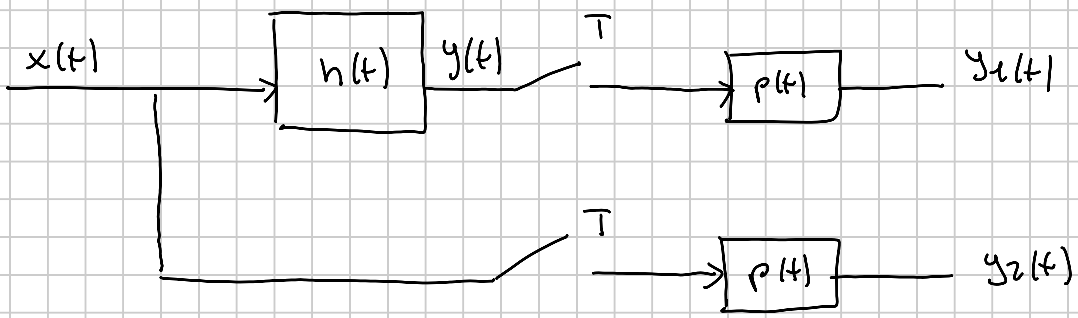
$$y(t) = |x(t)| + \int_b^t x(\alpha)\,d\alpha$$

$$|y(t)| = \left| \, |x(t)| + \int_b^t x(\alpha)\,d\alpha \right| \leq |x(t)| + \left| \int_b^t x(\alpha)\,d\alpha \right| \leq$$

$$|x(t)| + \int_b^t |x(\alpha)|\,d\alpha \leq M + \int_b^t M\,d\alpha \leq N \quad \text{per } t < \infty$$

$M + (t-b)M = K \quad t < \infty$ NON STABILE

Esercizio 2



$x(t) = 2AB \operatorname{sinc}^2(2Bt)$

$h(t) = 2B \operatorname{sinc}(2Bt)$

$p(t)$ interpolatore cardinale di banda B

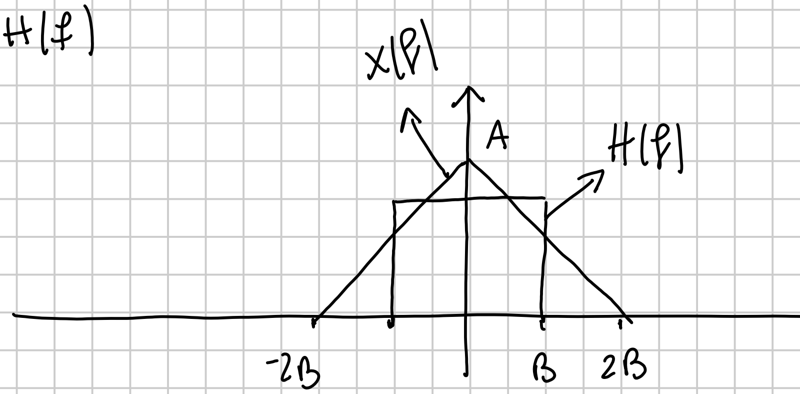
$T = \frac{1}{2B}$

- Calcolare l'espressione analitica di $y_1(t)$ e $y_2(t)$
- Calcolare l'energia di $y_1(t)$ e $y_2(t)$
- Calcolare la potenza di $y_1(t)$ e $y_2(t)$

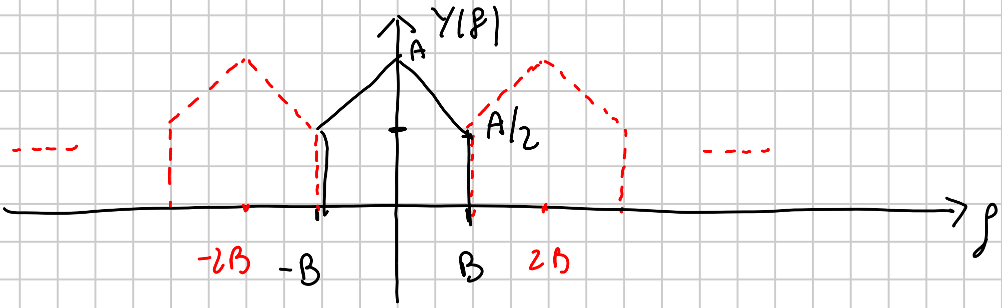
$$X(f) = A\left(1 - \frac{|f|}{2B}\right) \text{rect}\left(\frac{f}{4B}\right)$$

$$H(f) = \text{rect}\left(\frac{f}{2B}\right)$$

$$Y(f) = X(f)\,H(f)$$



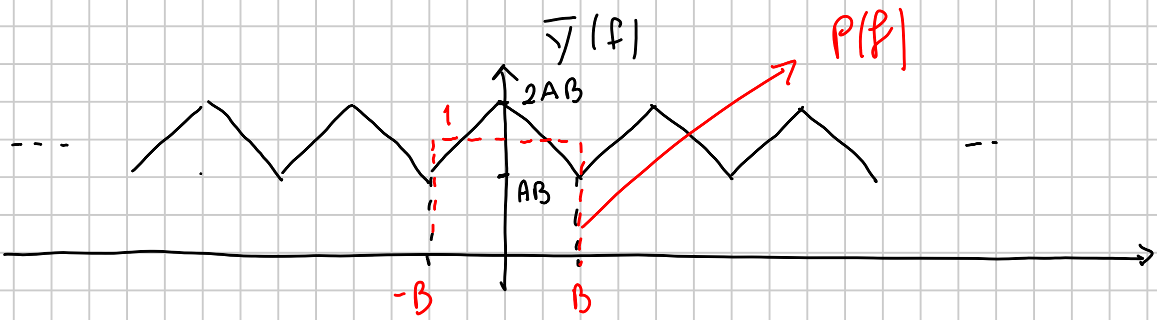
$$\frac{A}{2B} \cdot f \Big|_{f=\frac{B}{2}} = \frac{A}{2}$$

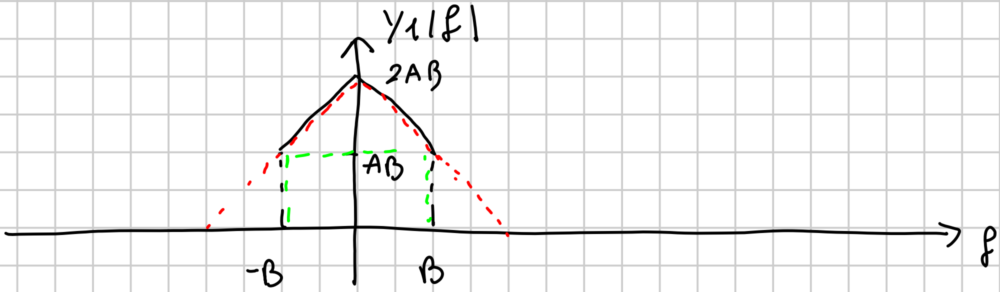


$$T = \frac{1}{2B}$$

$$\bar{Y}(f) = \frac{1}{T} \sum_n Y\left(f - \frac{n}{T}\right) =$$

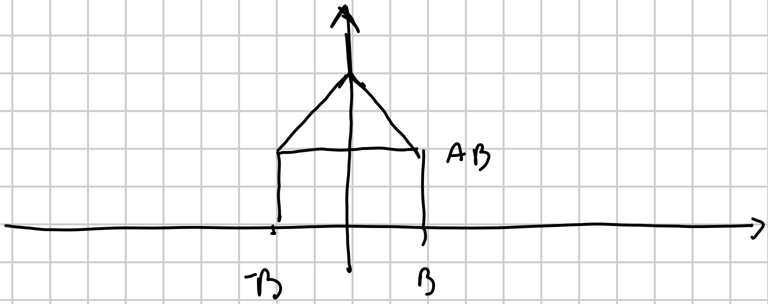
$$= 2B \sum_n Y(f - n2B)$$





$$Y_1(f) = 2AB\left(1 - \frac{|f|}{2B}\right) \mathrm{rect}\left(\frac{f}{4B}\right) \cdot \mathrm{rect}\left(\frac{f}{2B}\right) =$$

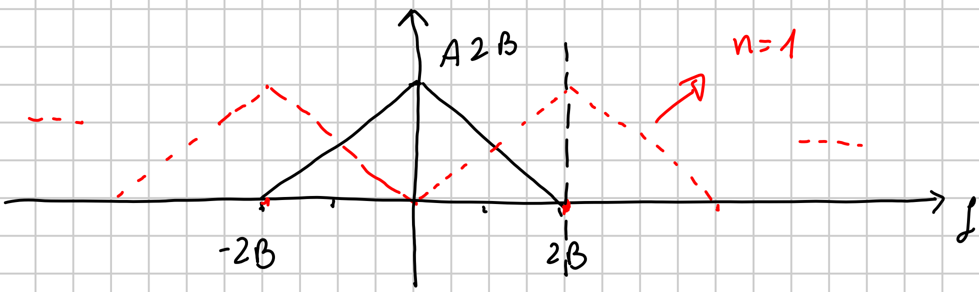
$$= 2AB\left(1 - \frac{|f|}{2B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

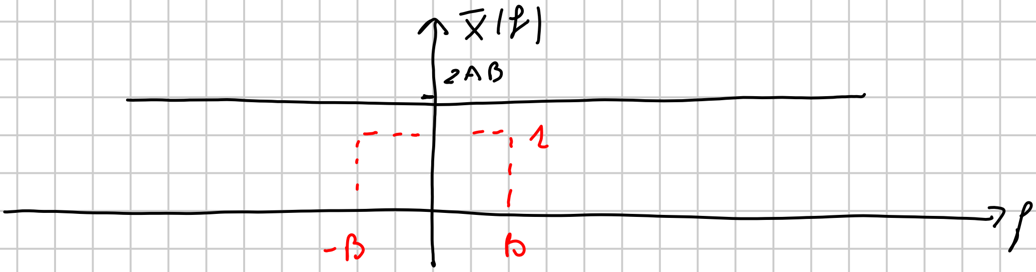


$$Y_1(f) = AB\,\mathrm{rect}\left(\frac{f}{2B}\right) + AB\left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$y_1(t) = A2B^2\,\mathrm{sinc}(t\,2B) + AB^2\,\mathrm{sinc}^2(t\,B)$$

$$\bar{X}(f) = 2B \sum_n X(f - 2Bn)$$





$$Y_2(f) = \overline{X}(f)\, P(f) \qquad P(f) = \text{rect}\left(\frac{f}{2B}\right)$$

$$Y_2(f) = 2AB\, \text{rect}\left(\frac{f}{2B}\right)$$

$$y_2(t) = 4AB^2\, \text{sinc}\left(t\, 2B\right)$$

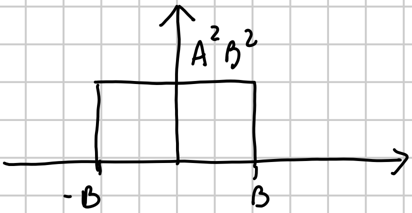
2)
$$Y_1(f) = AB\, \text{rect}\left(\frac{f}{2B}\right) + AB\left(1 - \frac{|f|}{B}\right) \text{rect}\left(\frac{f}{2B}\right)$$

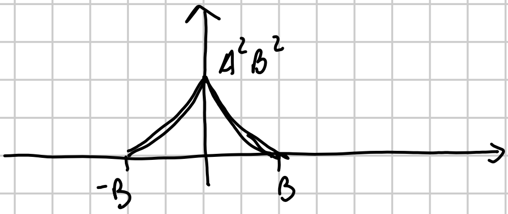
$$Y_2(f) = 2AB\, \text{rect}\left(\frac{f}{2B}\right)$$

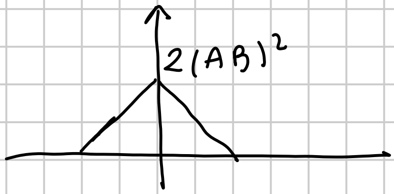
$$E_{y_1} = \int_{-\infty}^{+\infty} |Y_1(f)|^2\, df = \int_{-\infty}^{+\infty} (Y_1(f))^2\, df =$$

$$= \int_{-B}^{B} (AB)^2\, df + 2\int_{0}^{B} (AB)^2 \left(1 - \frac{f}{B}\right)^2 df +$$

$$\int_{-B}^{B} 2(AB)^2 \left(1 - \frac{|f|}{B}\right) df$$







$$\int_{-B}^{B} A^2B^2\, df = 2B\cdot A^2B^2$$

$$2\int_{0}^{B} A^2B^2\left(1-\frac{f}{B}\right)^2 df = 2A^2B^2\int_{0}^{B}\left(1+\frac{f^2}{B^2}-\frac{2f}{B}\right)df =$$

$$= 2A^2B^2\left[f+\frac{f^3}{3B^2}-\frac{\cancel{2}f^2}{\cancel{2}B}\right]_0^B = 2A^2B^2\left[\cancel{B}+\frac{B}{3}-\cancel{B}\right] =$$

$$= 2A^2B^2\left[\cdot\frac{B}{3}\right] = \frac{2}{3}A^2B^3$$

$$4\int_{0}^{B} A^2B^2\left(1-\frac{f}{B}\right)df = 4A^2B^2\int_{0}^{B}\left(1-\frac{f}{B}\right)df =$$

$$= 4A^2B^2\left[f-\frac{f^2}{2B}\right]_0^B = 4A^2B^2\left[B-\frac{B}{2}\right] = 2A^2B^3$$

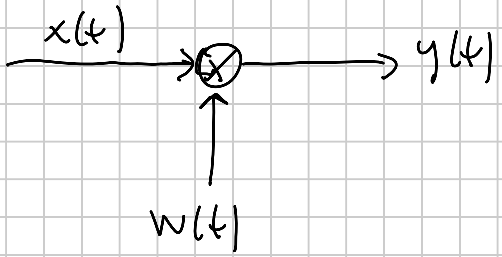
$$E_{Y_1} = 2A^2B^3+\frac{2}{3}A^2B^3+2A^2B^3 = \frac{14}{3}A^2B^3$$

$$E_{y2} = \int_{-\infty}^{+\infty} |Y_2(f)|^2 \, df = \int_{-B}^{B} 4A^2B^2 \, df = 4A^2B^2 \cdot 2B = 8A^2B^3$$

$y_1(t)$ e $y_2(t)$ sono segnali ad energia finita e quindi hanno potenza nulla

$$P_{y1} = P_{y2} = 0$$

Esercizio 3)



$$x(t) = \sum_{n=-\infty}^{+\infty} \mathrm{rect}\left(\frac{t-2Tn}{T/2}\right)$$

$$w(t) = \cos\left(\frac{\pi t}{T}\right)$$

Calcolare

X_n, $X(f)$

Y_n, $Y(f)$

Energia e potenza di $y(t)$

$$x_0(t) = \mathrm{rect}\left(\frac{t}{T/2}\right)$$

$$x(t) = \sum_n x_0(t - n2T) \qquad T_0 = 2T$$

$$w(t) = \cos\left(\frac{\pi t}{T}\right) = \cos\left(2\pi t \cdot \frac{1}{2T}\right) \qquad T_0 = 2T$$

$y(t) = x(t) \cdot w(t)$ sarà un segnale periodo $T_0 = 2T$

$$y(t) = \sum_n y_0(t - nT_0)$$

$$Y_n = \frac{1}{T_0} Y_0\left(\frac{n}{T_0}\right)$$

$$X_n = \frac{1}{T_0} X_0\left(\frac{n}{T_0}\right)$$

$$X_0(f) = \frac{T}{2} \,\mathrm{sinc}\left(f \frac{T}{2}\right)$$

$$X_n = \frac{T}{2} \cdot \frac{1}{2T} \,\mathrm{sinc}\left(\frac{T}{2} \cdot \frac{n}{2T}\right) = \frac{1}{4} \,\mathrm{sinc}\left(\frac{n}{4}\right)$$

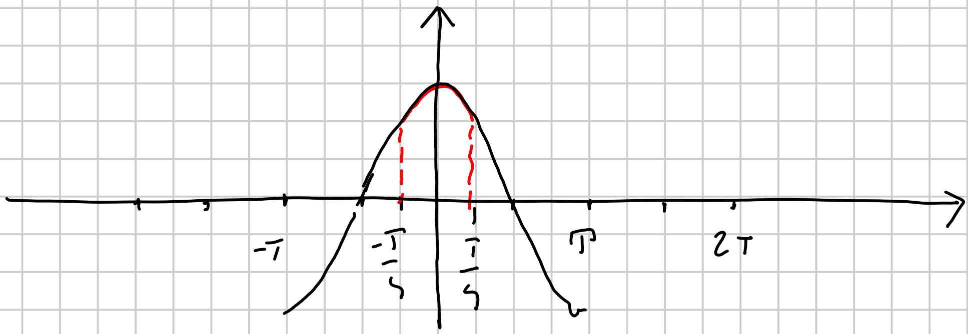
$$X(f) = \sum_{n=-\infty}^{+\infty} X_n \,\delta\left(f - \frac{n}{2T}\right)$$

$$x_0(t) = \mathrm{rect}\left(t/_{T/2}\right)$$

$$w_0(t) = \cos\left(\frac{\pi t}{T}\right) \cdot \mathrm{rect}\left(\frac{t}{T_0}\right) = \cos\left(\frac{\pi t}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right)$$

$$y_0(t) = x_0(t) \cdot w_0(t) = \cos\left(\frac{\pi t}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right) \mathrm{rect}\left(\frac{t\,2}{T}\right) =$$

$$= \cos\left(\frac{\pi t}{T}\right) \mathrm{rect}\left(\frac{2t}{T}\right)$$



$$Y_0(f) = \frac{T}{2} \operatorname{sinc}\left(f\frac{T}{2}\right) \otimes \left[\frac{1}{2}\,\delta\left(f - \frac{1}{2T}\right) + \frac{1}{2}\,\delta\left(f + \frac{1}{2T}\right)\right] =$$

$$= \frac{T}{4} \operatorname{sinc}\left(\frac{T}{2}\left(f - \frac{1}{2T}\right)\right) + \frac{T}{4} \operatorname{sinc}\left(\frac{T}{2}\left(f + \frac{1}{2T}\right)\right) =$$

$$= \frac{T}{4} \operatorname{sinc}\left(f\frac{T}{2} - \frac{1}{4}\right) + \frac{T}{4} \operatorname{sinc}\left(f\frac{T}{2} + \frac{1}{4}\right)$$

$$Y_n = \frac{1}{T_0}\, Y_0\left(\frac{n}{T_0}\right) = \frac{1}{2T}\, Y_0\left(\frac{n}{2T}\right)$$

$$Y_n = \frac{T}{4} \cdot \frac{1}{2T} \operatorname{sinc}\left(\frac{T}{2} \cdot \frac{n}{2T} - \frac{1}{4}\right) + \frac{T}{4} \cdot \frac{1}{2T} \operatorname{sinc}\left(\frac{T}{2}\frac{n}{2T} + \frac{1}{4}\right)$$

$$= \frac{1}{8} \operatorname{sinc}\left(\frac{n-1}{4}\right) + \frac{1}{8} \operatorname{sinc}\left(\frac{n+1}{4}\right)$$

$$Y(f) = \sum_{n=-\infty}^{+\infty} Y_n \, \delta\left(f - \frac{n}{2T}\right)$$

$$E_y = \infty$$

$$P_y = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} |y(t)|^2 dt = \sum_{n=-\infty}^{+\infty} |Y_n|^2$$

$$P_y = \frac{1}{2T} \int_{-T}^{T} |y(t)|^2 dt = \frac{1}{2T} \int_{-\frac{T}{4}}^{T/4} \cdot \cos^2\left(\frac{\pi t}{T}\right) dt =$$

$$= \frac{1}{2T} \int_{-\frac{T}{4}}^{T/4} \left(\frac{1}{2} + \frac{1}{2} \cos\left(\frac{2\pi t}{T}\right)\right) dt =$$

$$= \frac{1}{2T} \int_{-\frac{T}{4}}^{T/4} \frac{1}{2} dt + \frac{1}{2T} \int_{-\frac{T}{4}}^{T/4} \frac{1}{2} \cos\left(\frac{2\pi t}{T}\right) dt =$$

$$\frac{1}{4T} \cdot \frac{T}{2} + \frac{1}{4T} \sin\left(\frac{2\pi t}{T}\right) \cdot \frac{T}{2\pi} \Bigg|_{-T/4}^{T/4} = \frac{1}{8} + \frac{1}{8\pi} \sin\left(\frac{2\pi t}{T}\right) \Bigg|_{-T/4}^{T/4} =$$

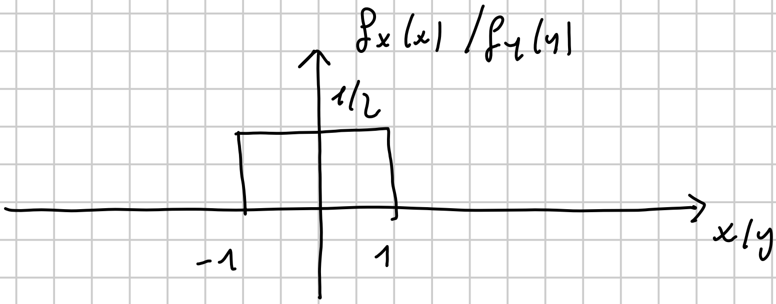
$= \frac{1}{8} + \frac{1}{4\pi}$

Esercizio 7.11

Sono assegnate 2 v.A indipendenti

$X \in U(-1,1)$

$Y \in U(-1,1)$



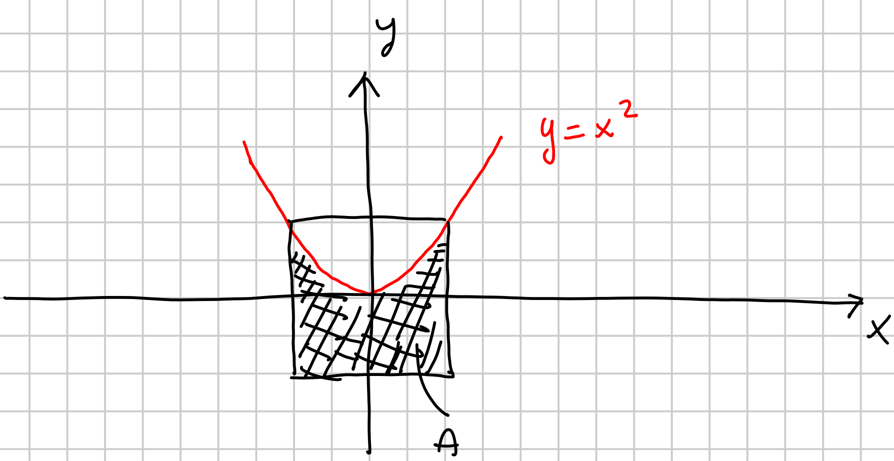
Calcolare la probabilità che l'equazione di secondo grado in α, abbia radici reali

$$\alpha^2 + 2X\alpha + Y = 0$$

$\Delta = b^2 - 4ac = 4X^2 - 4Y \geq 0$ affinché abbia radici reali

$X^2 - Y \geq 0$

$Y \leq X^2$



$Pr\{ Y \leq X^2 \}$

$$f_{xy} = f_x(x)\, f_y(y)$$

$$\Pr\{ y \leq x^2 \} = \iint_A f_{xy}\, dx\, dy = \frac{1}{2} \cdot \frac{1}{2} \iint_A 1\, dx\, dy =$$

$$= \frac{1}{4} \cdot \left(\text{Area del dominio} \right)$$

$$\int_{-1}^{+1} \int_{-1}^{x^2} \frac{1}{4}\, dy\, d\alpha = \frac{1}{4} \cdot \int_{-1}^{1} y \Big|_{-1}^{x^2} dx$$

$$= \frac{1}{4} \int_{-1}^{1} \left(x^2 + 1 \right) dx = 2\, \frac{1}{4} \int_0^1 \left(x^2 + 1 \right) dx =$$

$$= \frac{1}{2} \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{2} \left[\frac{1}{3} + 1 \right] = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$$