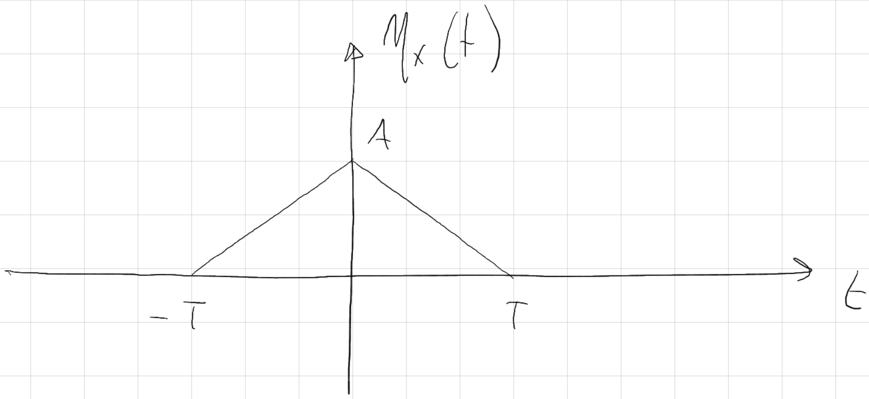
Soluzione Compitino 31/05/2019 FILA A

Es #1

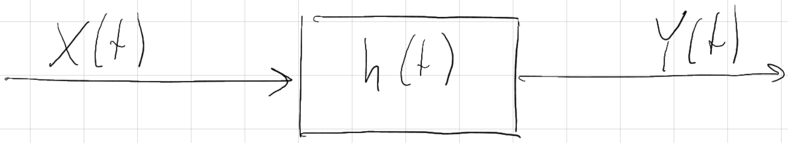
a) $$\eta_X(t) = E\left[X(t)\right] = \int_{-\infty}^{+\infty} A \,\mathrm{rect}\left(\frac{t-t_0}{T}\right) \frac{1}{T} \mathrm{rect}\left(\frac{t_0}{T}\right) dt_0$$

$$= \frac{A}{T} \int_{-\infty}^{+\infty} \mathrm{rect}\left(\frac{t_0}{T}\right) \mathrm{rect}\left(\frac{t-t_0}{T}\right) dt_0 = \frac{A}{T} \mathrm{rect}\left(\frac{t}{T}\right) \otimes \mathrm{rect}\left(\frac{t}{T}\right)$$

$$= \frac{A}{T} T \left(1 - \frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right) = A\left(1 - \frac{|t|}{T}\right) \mathrm{rect}\left(\frac{t}{2T}\right)$$

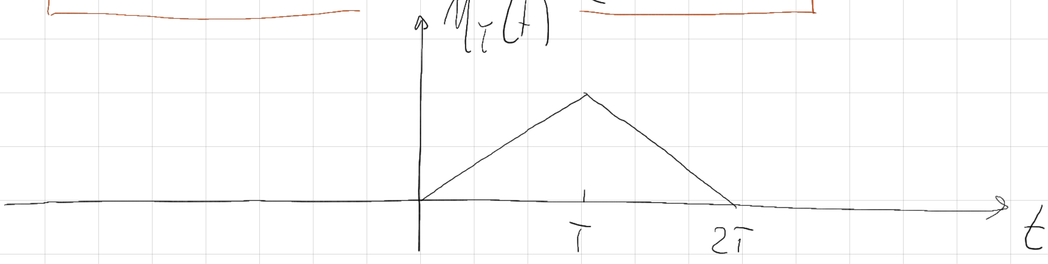


b)



$$E\left[Y(t)\right] = \eta_Y(t) = \eta_X(t) \otimes h(t) = \frac{1}{A}\eta_X(t-T)$$

$$= \boxed{\left(1 - \frac{|t-T|}{T}\right) \mathrm{rect}\left(\frac{t-T}{2T}\right)}$$



Es #2

a) $E_s = E[x^2]\, E_p = \frac{22}{3} \cdot \frac{T}{2} = \boxed{\frac{11}{3} T'}$

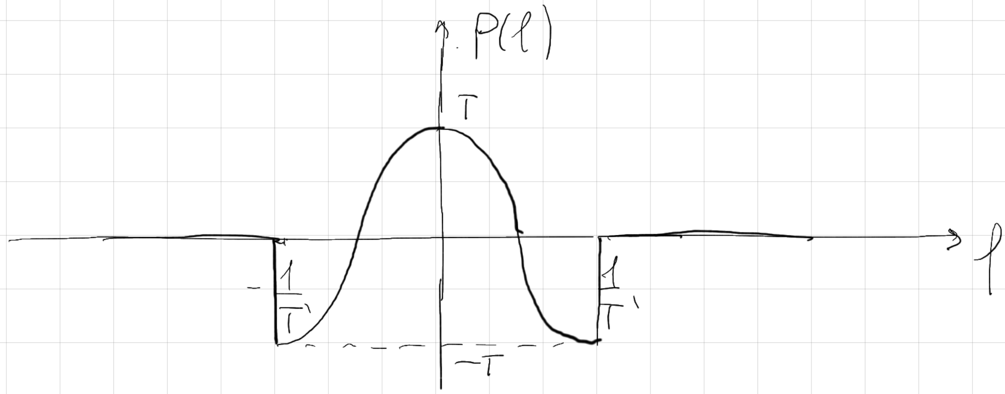
$$E[x^2] = \frac{1}{3}(-2)^2 + \frac{2}{3}(3)^2 = \frac{4}{3} + 6 = \frac{22}{3}$$

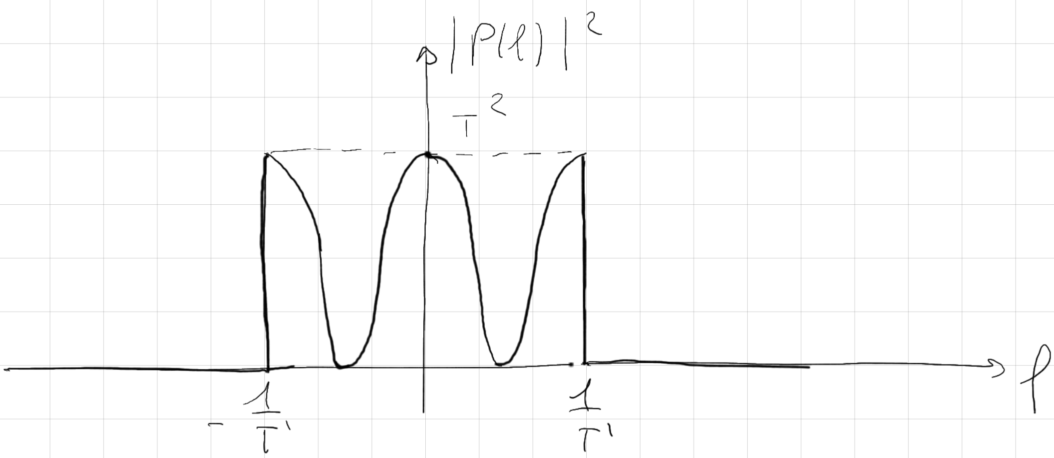
$$p(t) = \operatorname{sinc}\left(\frac{2t}{T'} - 1\right) + \operatorname{sinc}\left(\frac{2t}{T'} + 1\right)$$

$$= \operatorname{sinc}\left[\frac{2}{T'}\left(t - \frac{T'}{2}\right)\right] + \operatorname{sinc}\left[\frac{2}{T'}\left(t + \frac{T'}{2}\right)\right]$$

$$P(f) = \frac{T'}{2} \operatorname{rect}\left(f\frac{T'}{2}\right)\left[e^{-j\pi f T'} + e^{j\pi f T'}\right]$$

$$= T' \operatorname{rect}\left(f\frac{T'}{2}\right) \cos(\pi f T')$$





$$E_p = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T'^2 \cos^2(\pi f T')\, df = \frac{T'}{2}$$

b) $S_s(f) = \frac{1}{T} S_x(f) \, |P(f)|^2$

$S_x(f) = TFS\left[R_x[m]\right]$

$R_x[m] = C_x[m] + \eta_x^2$

$\eta_x = E[x] = \frac{1}{3}(-2) + \frac{2}{3}(3) = -\frac{2}{3} + \frac{6}{3} = \frac{4}{3}$

$$C_x[m] = \begin{cases} \sigma_x^2 & m = 0 \\ 0 & m \neq 0 \end{cases}$$

$$\sigma_x^2 = E\left[(x - \eta_x)^2\right] = \frac{1}{3}\left(-2 - \frac{4}{3}\right)^2 + \frac{2}{3}\left(3 - \frac{4}{3}\right)^2$$

$$= \frac{1}{3} \cdot \left(-\frac{10}{3}\right)^2 + \frac{2}{3}\left(\frac{5}{3}\right)^2 = \frac{100}{9} + \frac{50}{9} = \frac{50}{3}$$

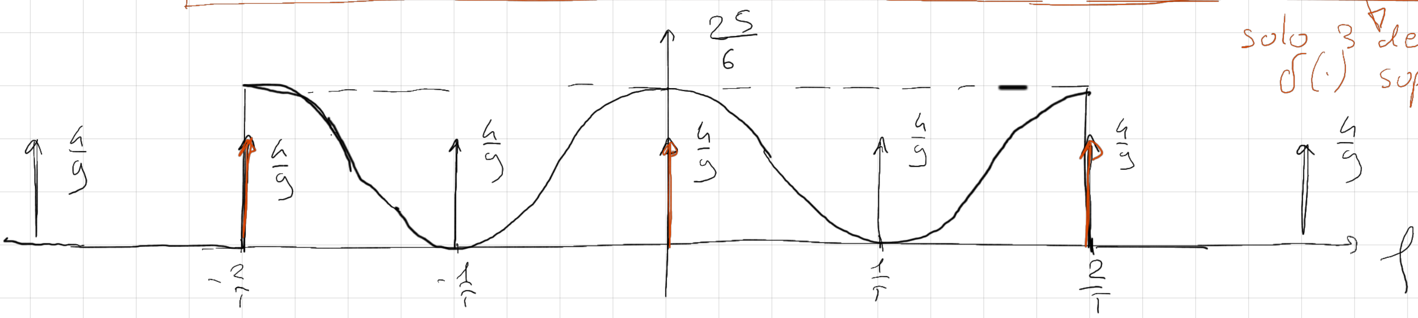
$R_x[m] = \frac{50}{3} \delta[m] + \frac{16}{9}$

$S_x(f) = \frac{50}{3} + \frac{16}{9} \frac{1}{T} \sum_n \delta\left(f - \frac{n}{T}\right)$

$$S_s(f) = \frac{1}{T}\left(\frac{50}{3} + \frac{16}{9T} \sum_n \delta\left(f - \frac{n}{T}\right)\right) T^2 \, \mathrm{rect}\left(\frac{fT}{2}\right) \cos^2(\pi f T)$$

$$= \boxed{\frac{25}{6} T \, \mathrm{rect}\left(\frac{fT}{4}\right) \cos^2\left(\frac{\pi f T}{2}\right) + \frac{4}{9}\left[\delta(f) + \delta\left(f - \frac{2}{T}\right) + \delta\left(f + \frac{2}{T}\right)\right]}$$

solo 3 delle $\delta(\cdot)$ sopravvivono



c) $P_{n_u} = \int_{-\infty}^{+\infty} S_n(f)\, |H_R(f)|^2\, df = \frac{N_0}{2} E_{H_R}$

$E_{H_R} = E_p = \frac{T'}{2}$

$P_{n_u} = \frac{N_0}{2} \frac{T'}{2} = \frac{N_0 T'}{4} = \frac{N_0 T}{8}$

d) $h(t) = p(t) \otimes h_R(t) = p(t) \otimes p(t)$

$H(f) = |P(f)|^2 = T'^2 \, \text{rect}\left(\frac{fT'}{2}\right) \cos^2(\pi f T')$

$\frac{T^2}{8} \text{rect}\left(\frac{fT}{4}\right) + \frac{T^2}{8} \text{rect}\left(\frac{fT}{4}\right) \cos(\pi f T)$

$h(t) = \frac{T^2}{8} \frac{4}{T} \text{sinc}\left(\frac{4t}{T}\right) + \frac{T^2}{8} \frac{4}{T} \text{sinc}\left(\frac{4t}{T}\right) \left[\frac{1}{2} \delta\left(t - \frac{T}{2}\right) + \frac{1}{2} \delta\left(t + \frac{T}{2}\right)\right]$

$= \frac{T}{2} \text{sinc}\left(\frac{4t}{T}\right) + \frac{T}{4} \text{sinc}\left[\frac{4}{T}\left(t - \frac{T}{2}\right)\right] + \frac{T}{4} \text{sinc}\left[\frac{4}{T}\left(t + \frac{T}{2}\right)\right]$

$= \frac{T}{2} \text{sinc}\left(\frac{4t}{T}\right) + \frac{T}{4} \text{sinc}\left(\frac{4t}{T} - 2\right) + \frac{T}{4} \text{sinc}\left(\frac{4t}{T} + 2\right)$

$h(nT) = \frac{T}{2} \text{sinc}(4n) + \frac{T}{4} \text{sinc}(4n - 2) + \frac{T}{4} \text{sinc}(4n + 2)$

$= \frac{T}{2} \delta[n]$ $\Leftarrow$ cond. di Nyquist verificata nel tempo $\Rightarrow$ ASSENZA DI ISI

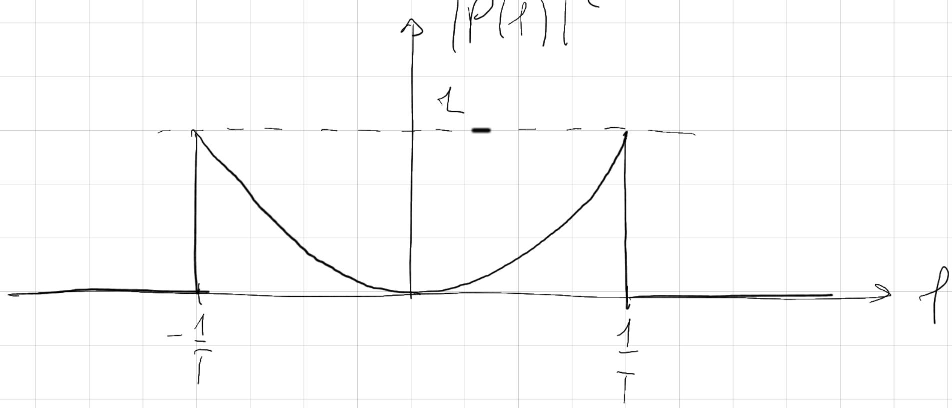
e) $\boxed{P_E(b) = \frac{1}{3} Q\left(\frac{T}{\sqrt{\frac{N_0 T}{8}}}\right) + \frac{2}{3} Q\left(\frac{\frac{3}{2}T}{\sqrt{\frac{N_0 T}{8}}}\right)}$

Es #3

a) $E_s = \frac{1}{2}\left[E[x_c^2] + E[x_s^2]\right] E_p = \frac{1}{2}\cdot(7+3)\,\frac{2}{3T} = \boxed{\frac{10}{3T}}$

$$E[x_c^2] = \frac{1}{4}(-1)^2 + \frac{3}{4}(3)^2 = \frac{1}{4} + \frac{27}{4} = \frac{28}{4} = 7$$

$$E[x_s^2] = \frac{1}{3}(-1)^2 + \frac{2}{3}(2)^2 = \frac{1}{3} + \frac{8}{3} = 3$$



$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2 \, df = \frac{2}{3T}$$

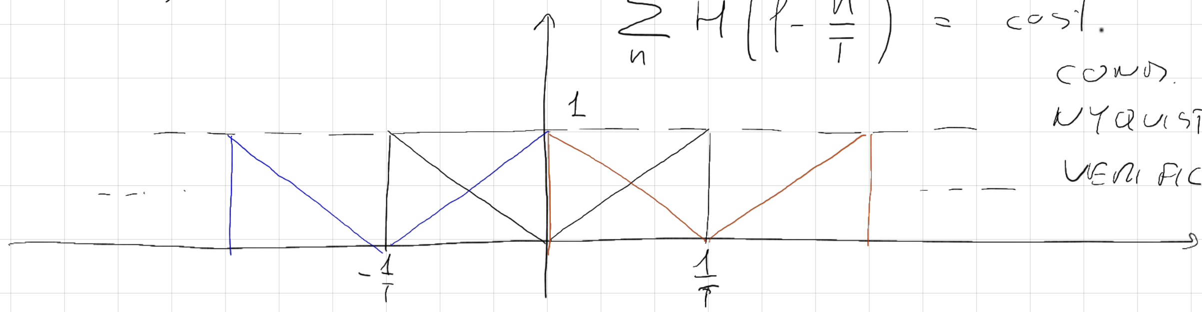
b) $P_{n_{u_c}} = P_{n_{u_s}} = P_{n_u} = N_0 E_{H_R} = N_0 \cdot \frac{2}{T} = \frac{2N_0}{T}$

c) verifichiamo la cond. di Nyquist

$$H(f) = P(f) H_R(f) = P(f)$$

$$\sum_n H\left(f - \frac{n}{T}\right) = \text{cost.}$$

COND. DI NYQUIST VERIFICATA



$$H(f) = \text{rect}\left(\frac{fT}{2}\right) - \left(1 - \frac{|f|}{1/T}\right)\text{rect}\left(\frac{f}{2/T}\right)$$

$$h(t) = \frac{2}{T}\,\text{sinc}\left(\frac{2t}{T}\right) - \frac{1}{T}\,\text{sinc}^2\left(\frac{t}{T}\right)$$

$$h(0) = \frac{2}{T} - \frac{1}{T} = \frac{1}{T}$$

$$P_E(a) = P_E^c(b)\left(1 - P_E^s(b)\right) + P_E^s(b)\left(1 - P_E^c(b)\right) + P_E^c(b)\,P_E^s(b)$$

$$P_E^c(b) = \frac{1}{4}\,Q\left(\frac{\frac{1}{T}}{\sqrt{\frac{2N_0}{T}}}\right) + \frac{3}{4}\,Q\left(\frac{\frac{3}{T}}{\sqrt{\frac{2N_0}{T}}}\right)$$

$$P_E^s(b) = \frac{1}{3}\,Q\left(\frac{\frac{1}{T}}{\sqrt{\frac{2N_0}{T}}}\right) + \frac{2}{3}\,Q\left(\frac{\frac{2}{T}}{\sqrt{\frac{2N_0}{T}}}\right)$$