$f(x,y) = \sqrt{1-x^2-y^2}$ $\qquad 0 \le x^2+y^2 \le 1$ $\qquad f$ concava

$v^T Hf(x,y)\, v \le 0 \qquad \forall\, x,y \in B(0,1) \qquad \forall\, v \in \mathbb{R}^2$

$\partial_x f = -x\,(1-x^2-y^2)^{-1/2}$ $\qquad f(x,y) = (1-x^2-y^2)^{1/2}$

$\partial_y f = -y\,(1-x^2-y^2)^{-1/2}$

$$\begin{pmatrix}\partial_{xx}f & \partial_{xy}f \\ \partial_{xy}f & \partial_{yy}f\end{pmatrix} = \begin{pmatrix} -(\quad)^{-1/2} - x\left(-\cancel{\tfrac{1}{2}}\right)(\quad)^{-3/2}(-\cancel{2}x), & -x\left(-\cancel{\tfrac{1}{2}}\right)(\quad)^{-3/2}(-\cancel{2}y) \\ \square, & -(\quad)^{-1/2} - y\left(-\cancel{\tfrac{1}{2}}\right)(\quad)^{-3/2}(-\cancel{2}y) \end{pmatrix}$$

$$= \begin{pmatrix} -\left(\dfrac{1}{\sqrt{(\quad)}} + \dfrac{x^2}{\sqrt{(\quad)^3}}\right), & -\dfrac{xy}{\sqrt{(\quad)^3}} \\ \square & -\left(\dfrac{1}{\sqrt{\quad}} + \dfrac{y^2}{\sqrt{(\quad)^3}}\right) \end{pmatrix}$$

$\mathrm{Tr}\, Hf(x,y) < 0 \quad \Rightarrow \quad \lambda_1 + \lambda_2 < 0$

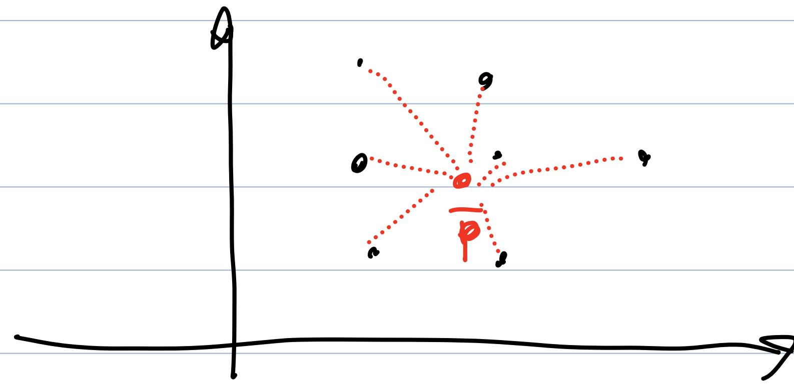
$$\begin{pmatrix} -\dfrac{1-\cancel{x^2}-y^2+\cancel{x^2}}{\sqrt{(\quad)^3}} & , & \dfrac{-xy}{\sqrt{(\quad)^3}} \\ \square & , & -\dfrac{1-x^2-\cancel{y^2}+\cancel{y^2}}{\sqrt{(\quad)^3}} \end{pmatrix}$$

$$\frac{(1-y^2)(1-x^2)}{(\quad)^3} - \frac{x^2y^2}{(\quad)^3} = \frac{1-y^2-x^2+\cancel{x^2y^2}-\cancel{x^2y^2}}{(1-x^2-y^2)^3}$$

$$= \frac{1}{(1-x^2-y^2)^2} > 0 \qquad \det Hf > 0 \qquad \lambda_1\lambda_2 > 0$$

$$\Rightarrow \quad \lambda_1, \lambda_2 < 0$$

$$p_i = (x_i, y_i) \qquad i = 1 \ldots n$$



$$f(p) = \sum_{i=1}^{n} (x_i - x_p)^2 + (y_i - y_p)^2 = \sum_{i=1}^{n} \|p - p_i\|^2$$

$$f(\bar{p}) \leq f(p) \qquad \forall\, p \in \mathbb{R}^2$$

$$\frac{\partial f}{\partial x_p}(p) = -2 \sum_{i=1}^{n} (x_i - x_p) = 0 \qquad \frac{\partial f}{\partial y_p} = -2 \sum_{i=1}^{n} (y_i - y_p) = 0$$

$$\nabla f = 0 \qquad \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_p \qquad \sum_{i=1}^{n} y_i = \sum_{i=1}^{n} y_p$$

$$\sum_{i=1}^{n} x_i = n\, x_p \qquad \sum_{i=1}^{n} y_i = n\, y_p$$

$$\Rightarrow \quad x_p = \frac{1}{n} \sum_{i=1}^{n} x_i = \bar{x} \qquad y_p = \frac{1}{n} \sum_{i=1}^{n} y_i = \bar{y}$$

MEDIA ARITMETICA

$$Hf = \begin{pmatrix} \frac{\partial^2 f}{\partial x_p^2} & \frac{\partial^2 f}{\partial x_p \partial y_p} \\ \square & \frac{\partial^2 f}{\partial y_p^2} \end{pmatrix} = \begin{pmatrix} 2n & 0 \\ 0 & 2n \end{pmatrix}$$

(x_i, y_i) DATI

x_i non tutte uguali

$y = \beta x + \alpha$

$y_i = \beta x_i + \alpha + \varepsilon_i$

$$E(\alpha, \beta) = \sum_{i=1}^{n} \varepsilon_i^2$$

METODO DEI MINIMI QUADRATI

REGRESSIONE (FIT) LINEARE)

$$= \sum (\beta x_i + \alpha - y_i)^2$$

$$\frac{\partial E}{\partial \beta} = 2 \sum (\beta x_i + \alpha - y_i) x_i \qquad \frac{\partial E}{\partial \alpha} = 2 \sum (\beta x_i + \alpha - y_i)$$

$$\bar{x} = \frac{1}{n} \sum x_i \qquad \bar{y} = \frac{1}{n} \sum y_i \qquad \bar{q} = \frac{\sum x_i^2}{n} \qquad \bar{p} = \frac{\sum x_i y_i}{n}$$

$$\sum \beta x_i^2 + \sum \cancel{\alpha} x_i - \sum x_i y_i = 0 \qquad \sum \beta x_i + \sum \alpha - \sum y_i = 0$$

$$\beta \frac{\sum x_i^2}{n} + \alpha \frac{\sum x_i}{n} = \frac{\sum x_i y_i}{n} \qquad \beta \frac{\sum x_i}{n} + \frac{n\alpha}{n} = \frac{\sum y_i}{n}$$

$$\beta \bar{q} + \alpha \bar{x} = \bar{p} \qquad \beta \bar{x} + \alpha = \bar{y}$$

$$\begin{cases} \bar{q}\beta + \bar{x}\alpha = \bar{p} \\ \bar{x}\beta + \alpha = \bar{y} \end{cases} \qquad A = \begin{pmatrix} \bar{q} & \bar{x} \\ \bar{x} & 1 \end{pmatrix} = q - \bar{x}^2 > 0$$

$$0 \leq \sigma_x^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$ VARIANZA DI x_i

$$\sigma_x^2 = 0 \iff x_i - \bar{x} = 0 \Rightarrow x_i = \bar{x} \quad \forall i$$

$$= \frac{1}{n} \sum \left(x_i^2 - 2\bar{x} x_i + \bar{x}^2 \right) = \frac{1}{n} \sum x_i^2 - 2 \frac{1}{n} \bar{x} \sum x_i + \frac{\bar{x}^2 \sum 1}{n})$$

$$= \frac{\sum x_i^2}{n} - 2\bar{x}^2 + \bar{x}^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 = \bar{q} - \bar{x}^2 > 0$$

$$\beta = \frac{\bar{p} - \bar{x}\bar{y}}{\bar{q} - \bar{x}^2} = \frac{\sigma_{xy}}{\sigma_x^2} \qquad \alpha = \bar{y} - \beta \bar{x}$$

$q - \bar{x}$ $\quad \sigma_x$

$$\sigma_{xy} = \frac{1}{n} \sum (x_i - \bar{x})(y_i - \bar{y})$$

COVARIANZA DI x_i e y_i

(1) $$= \bar{p} - \bar{x}\bar{y}$$

(2) $$HE(\beta,\alpha) = 2n \begin{pmatrix} \bar{q} & \bar{x} \\ \bar{x} & 1 \end{pmatrix}$$

DEFINITA POSITIV