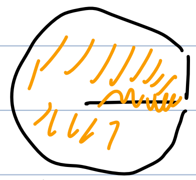
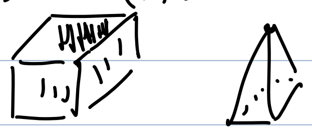
PORZIONE DI SUPERFICIE REGOLARE IN $\mathbb{R}^3$

$T \subseteq \mathbb{R}^2$ $\quad$ T Regione (chiuso $\overline{\mathring{T}} = T$, misurabile) limitato



$\varphi : T_u \to \mathbb{R}^3_x$

$T_{(u,v)} \to \mathbb{R}^3_{(x,y,z)}$



(1) $\varphi \in C^1$

(2) φ iniettiva

(3) $\forall\, u \in \mathring{T}$ $\quad$ rank $\frac{\partial \varphi}{\partial u} = 2$ $\quad$ rank $(J\phi)$

$\varphi : T \to \varphi(T)$ è bijettiva

$\varphi(T)$ SOSTEGNO

$$\varphi = (\varphi_1, \varphi_2, \varphi_3)$$

PORZIONE DI SUPERFICIE CARTESIANA $\quad f \in C^1(T)$

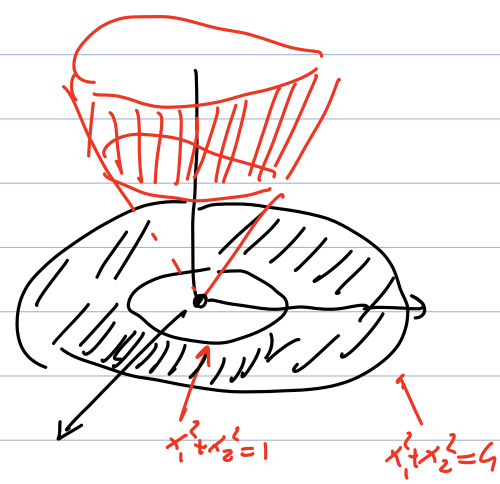
$$(u,v) \longrightarrow \underbrace{u}_{\varphi_1}\, i + \underbrace{v}_{\varphi_2}\, j + \underbrace{f(u,v)}_{\varphi_3}\, k$$

$$J\varphi = \begin{pmatrix} \partial_u \varphi_1 & \partial_v \varphi_1 \\ \partial_u \varphi_2 & \partial_v \varphi_2 \\ \partial_u \varphi_3 & \partial_v \varphi_3 \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} \\ \begin{matrix} \frac{\partial f}{\partial u} & \frac{\partial f}{\partial v} \end{matrix} \end{pmatrix}$$

rank $J\varphi(u,v) = 2$ $\quad \forall (u,v) \in T$

$$T = \{ \cancel{x} : 1 \le \|x\| \le 2 \}$$

$$(x_1, x_2) \longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ \sqrt{x_1^2 + x_2^2} \end{pmatrix} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \partial_{x_1} \sqrt{x_1^2 + x_2^2} & \partial_{x_2} \sqrt{x_1^2 + x_2^2} \end{pmatrix}$$

$$\frac{\partial}{\partial x_1} \sqrt{x_1^2 + x_2^2} = \frac{x_1}{\sqrt{x_1^2 + x_2^2}}$$

$$\sigma : [0, \pi] \times [0, 2\pi] \longrightarrow \mathbb{R}^3 \qquad \varphi \in [0, \pi] \qquad \theta \in [0, 2\pi]$$

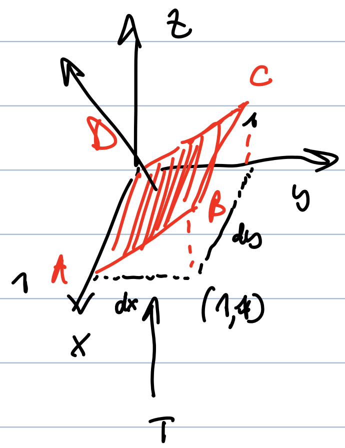
$$\sigma(\varphi,\theta) = \begin{cases} R\sin(\varphi)\cos(\theta) \\ R\sin(\varphi)\sin(\theta) \\ R\cos(\varphi) \end{cases} \qquad T = [0,\pi] \times [0,2\pi]$$

SEMI - SFERA SUPERIORE CENTRATA IN $(0,0,0)$ E DI RAGGIO R

$$\{ x_1, x_2 : x_1^2 + x_2^2 \leq R^2 \} = T'$$

$$\sigma : T' \to \mathbb{R}^3 \qquad \sigma(x) = \begin{pmatrix} x_1 \\ x_2 \\ \sqrt{R^2 - x_1^2 - x_2^2} \end{pmatrix}$$

$(x,y) \in [0,1] \times [0,1]$



$\varphi = (x,y) \longrightarrow (x,y,y)$ CARTESIANA

$\text{Area}(ABCD) = 1 \cdot \sqrt{2} = \sqrt{2}$

$$\boxed{\int_T \left\| \frac{\partial\varphi}{\partial u} \wedge \frac{\partial\varphi}{\partial v} \right\| du\,dv := A(\varphi)} \qquad \varphi \in C^1(T)$$

$$n = \frac{\partial\varphi}{\partial u} \wedge \frac{\partial\varphi}{\partial v} \quad \text{vettore normale} \qquad \boxed{n \wedge \frac{\partial\varphi}{\partial u} = 0} \quad \boxed{n \wedge \frac{\partial\varphi}{\partial v} = 0}$$

$$\Pi(u,v) = \varphi(u_0,v_0) + \frac{\partial\varphi}{\partial u}(u_0,v_0)(u-u_0) + \frac{\partial\varphi}{\partial v}(u_0,v_0)(v-v_0)$$

PIANO TG ALLA SUPERFICIE IN (u_0,v_0)

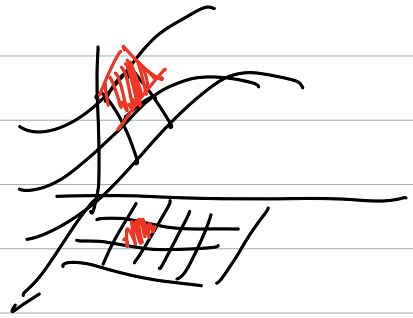
$$\frac{\partial\varphi}{\partial x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \frac{\partial\varphi}{\partial y} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{vmatrix} = (0\cdot 1 - 0\cdot 1)\, i - (1 - 0)\, j + (1\cdot 1 - 0)\, k$$

$$= 0\, i - 1 j + 1 \cdot k = n$$

$$\|n\| = \sqrt{0^2 + (-1)^2 + (1)^3} = \sqrt{2}$$

$$\int_T \sqrt{2}\, dx\,dy = \int_0^1 dx \int_0^1 dy\, \sqrt{2} = \sqrt{2}$$



$$T = \{ x_1^2 + x_2^2 \leq R^2 \}$$

$$\varphi(x_1, x_2) = \begin{pmatrix} x_1 \\ x_2 \\ \sqrt{R^2 - x_1^2 - x_2^2} \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ \frac{\partial}{\partial x_1} (R^2 - x_1^2 - x_2^2)^{1/2} \end{pmatrix} = \frac{\partial \varphi}{\partial x_1} = \begin{pmatrix} 1 \\ 0 \\ \frac{1}{2} (R^2 - x_1^2 - x_2^2)(-2x_1) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -\frac{x_1}{\sqrt{R^2 - x_1^2 - x_2^2}} \end{pmatrix}$$

$$\frac{\partial \varphi}{\partial x_2} = \begin{pmatrix} 0 \\ 1 \\ -\frac{x_2}{\sqrt{R^2 - x_1^2 - x_2^2}} \end{pmatrix}$$

$$n = \frac{\partial \varphi}{\partial x_1} \wedge \frac{\partial \varphi}{\partial x_2} = \begin{pmatrix} i & j & k \\ 1 & 0 & -\frac{x_1}{\sqrt{R^2 - x_1^2 - x_2^2}} \\ 0 & 1 & -\frac{x_2}{\sqrt{R^2 - x_1^2 - x_2^2}} \end{pmatrix} \begin{array}{l} \\ = \partial_1 f \\ = \partial_2 f \end{array}$$

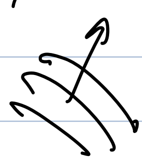
$$n = -f_{x_1} i + f_{x_2} j + k$$

$$= \left[\frac{x_1}{\sqrt{R^2 - x_1^2 - x_2^2}} \right] i - \left[-\frac{x_2}{\sqrt{R^2 - x_1^2 - x_2^2}} \right] j + 1 \quad k$$

SE SUPERFICIE È CARTESIANA

$(x_1, x_2, f(x_1, x_2))$

$$\begin{pmatrix} i & j & k \\ 1 & 0 & \partial_{x_1} f \\ 0 & 1 & \partial_{x_2} f \end{pmatrix} = (\quad) + 1 k$$

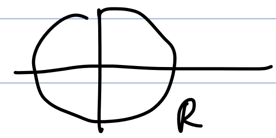


$n_3 > 0$

$$\|n\| = \sqrt{\frac{x_1^2}{\ldots} + \frac{x_2^2}{\ldots} + 1} = \sqrt{\cancel{x_1^2} + \cancel{x_2^2} + R^2 - \cancel{x_1^2} - \cancel{x_2^2}}$$

$$\int \frac{}{R^2-x_1^2-x_2^2} \qquad \frac{}{R^2-x_1^2-x_2^2} \qquad \int \frac{}{R^2-x_1^2-x_2^2}$$

$$= \frac{R}{\sqrt{R^2-(x_1^2+x_2^2)}}$$

$T =$ 

$$\int_T \frac{R}{\sqrt{R^2-(x_1^2+x_2^2)}}\, dx_1 dx_2$$

$$R \int_0^R d\rho \int_0^{2\pi} d\theta \frac{\rho}{\sqrt{R^2-\rho^2}} = 2\pi R \int_0^R \rho \left(R^2-\rho^2\right)^{-1/2} d\rho$$

$$= -\pi R \int_0^R -2\rho \left(R^2-\rho^2\right)^{-1/2} d\rho$$

$$= -\pi R \left. \frac{1}{1-\frac{1}{2}} \left(R^2-\rho^2\right)^{1-\frac{1}{2}} \right|_0^R = -2\pi R \left. \left(R^2-\rho^2\right)^{1/2} \right|_0^R$$

$$= 2\pi R^2$$

SE σ CARTESIANA $\quad \sigma(x_1, x_2) = \begin{pmatrix} x_1 \\ x_2 \\ f(x_1, x_2) \end{pmatrix} \quad (x_1, x_2) \in T$

$$A(\sigma) = \int_T \sqrt{1 + (f_{x_1})^2 + (f_{x_2})^2}\, dx_1 dx_2$$

$$\begin{cases} R \sin(\varphi)\cos(\theta) \\ R \sin(\varphi)\sin(\theta) \\ R\cos(\varphi) \end{cases} \qquad \varphi \in \left[0, \tfrac{\pi}{2}\right] \qquad \theta \in [0, 2\pi]$$

$$n(\varphi, \theta) = \begin{pmatrix} i & j & k \\ R\cos(\varphi)\cos(\theta) & R\cos(\varphi)\sin(\theta) & -R\sin(\varphi) \\ -R\sin(\varphi)\sin(\theta) & R\sin(\varphi)\cos(\theta) & 0 \end{pmatrix}$$

$$\left(0 + R^2 \sin^2(\varphi)\cos(\theta)\right) i - \left(-R^2\sin^2(\varphi)\sin\theta\right) j + R^2\left(\cos^2(\theta)\sin(\varphi)\cos(\varphi) + \sin^2(\theta)\sin(\varphi)\cos(\varphi)\right) k$$

$$= R^2\left[\sin^2(\varphi)\cos(\theta)\, i + \sin^2(\varphi)\sin(\theta)\, j + \sin(\varphi)\cos(\varphi)\, k\right]$$

$$= R^2 \sqrt{\sin^4(\varphi)\cos^2(\theta) + \sin^4(\varphi)\sin^2(\theta) + \sin^2(\varphi)\cos^2(\varphi)}$$

$$= R^2 \sqrt{\sin^4(\varphi) + \sin^2(\varphi)\cos^2(\varphi)}$$

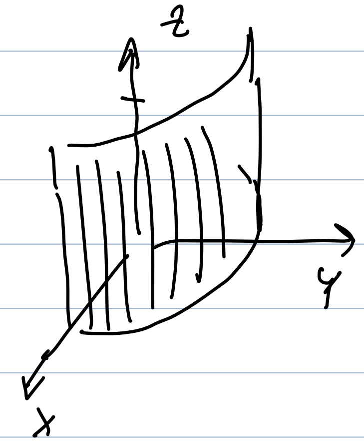
$$= R^2 \sqrt{\sin^2(\varphi)\,\cancel{[\sin^2(\varphi) + \cos^2(\varphi)]}} = R^2 \sin(\varphi)$$

$$= \int_0^{\pi/2} d\varphi \int_0^{2\pi} d\theta \; R^2 \sin\varphi = 2\pi R^2 \int_0^{\pi/2} \sin\varphi \, d\varphi =$$

$$= 2\pi R^2 \left(-\cos(\varphi)\Big|_0^{\pi/2} \right) = 2\pi R^2$$

$u_1 \in [0,\pi] \qquad u_2 \in [0,1] \qquad T = [0,\pi] \times [0,1]$

$$\sigma(u_1,u_2) = \begin{pmatrix} \cos(u_1) \\ \sin(u_1) \\ u_2 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$\sigma \in C^1 \qquad \sigma$ iniettiva

$$\left(\frac{\partial \sigma}{\partial u_1} \,\Big|\, \frac{\partial \sigma}{\partial u_2} \right)$$

$$\begin{pmatrix} -\sin(u_1) & 0 \\ \cos(u_1) & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \partial_1 \sigma_1 & \partial_2 \sigma_1 \\ \partial_1 \sigma_2 & \partial_2 \sigma_2 \\ \partial_1 \sigma_3 & \partial_2 \sigma_3 \end{pmatrix}$$

$$W_\sigma = (\det M_1)^2 + (\det M_2)^2 + (\det M_3)^2 > 0$$

$$M_1 = \begin{pmatrix} \partial_1 \sigma_2 & \partial_2 \sigma_2 \\ \partial_1 \sigma_3 & \partial_2 \sigma_3 \end{pmatrix} \qquad M_2 = \begin{pmatrix} \partial_1 \sigma_1 & \partial_2 \sigma_1 \\ \partial_1 \sigma_3 & \partial_2 \sigma_3 \end{pmatrix} \qquad M_3 = \begin{pmatrix} \partial_1 \sigma_1 & \partial_2 \sigma_1 \\ \partial_1 \sigma_2 & \partial_2 \sigma_2 \end{pmatrix}$$

$$\sqrt{W_\sigma} \overset{!}{=} \left\| \frac{\partial \sigma}{\partial u_1} \times \frac{\partial \sigma}{\partial u_2} \right\| \qquad \int_T \sqrt{W_\sigma}\, du\, dv = A(\sigma)$$

$$[J\sigma]^T [J\sigma] = \begin{vmatrix} E & F \\ F & G \end{vmatrix} \qquad \det \begin{vmatrix} E & F \\ F & G \end{vmatrix} = W_\sigma$$

$$EG - F^2 = W_\sigma$$

$$A(\sigma) = \int \sqrt{EG - F^2}\, du\, dv$$

$$n = \begin{pmatrix} i & j & k \\ \sin(u_1) & \cos(u_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \cos(u_1)\, i + \sin(u_1)\, j + 0\, k$$

$$\|n\| = \sqrt{\cos^2(u_1) + \sin^2(u_1)} = 1$$

$$\int_T \|n\|\, du_1\, du_2 = \int_T 1\, du_1\, du_2 = \int_0^1 \int_0^{\pi} du_1\, du_2 = \pi$$

$$\vec{a}, \vec{b} \in \mathbb{R}^3 \qquad \vec{a} \wedge \vec{b} \neq 0 \qquad \vec{c} \in \mathbb{R}^3$$

EQ CARTESIANA DEL PIANO PASSANTE PER $\vec{c}$ e PARALLELO AD $\vec{a}$ e $\vec{b}$

$u, v \in \mathbb{R}$

$$\boxed{\alpha x + \beta y + \gamma z = \delta} \Leftarrow \vec{x} = \vec{a} u + \vec{b} v + \vec{c}$$

$$\vec{x} - \vec{c} = u \vec{a} + v \vec{b}$$

$a \wedge b$ ortogonale al piano

$$a \wedge (\vec{x} - \vec{c}) = v\, \vec{a} \wedge \vec{b}$$

$$(a \wedge b) \cdot (\vec{x} - \vec{c}) = \underbrace{\boxed{(\vec{a} \wedge \vec{b}) \cdot \vec{a}}}_{0}\, u + \underbrace{\boxed{(\vec{a} \wedge \vec{b}) \cdot \vec{b}}}_{0}\, u$$