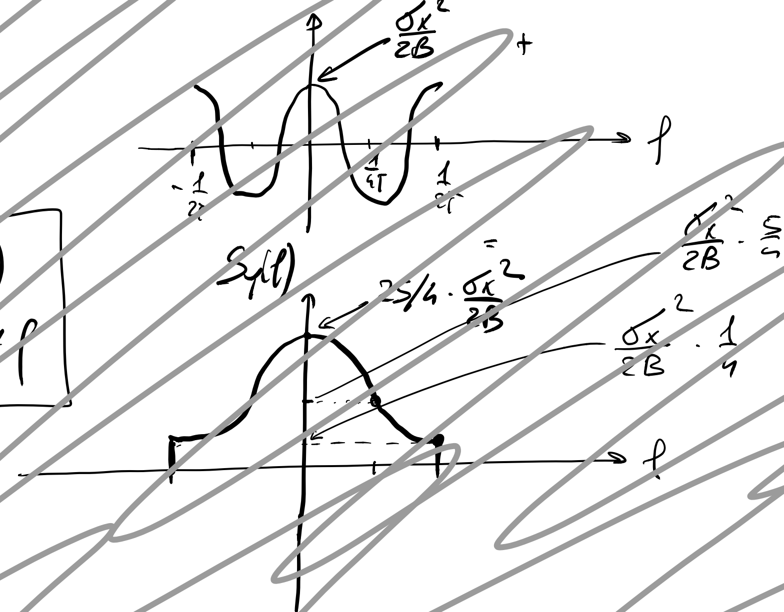
~~$S_Y(f) = S_Y(-f)$~~

~~$S_Y(f) \geq 0 \quad \forall f$~~



Il grafico di $R_Y(\tau)$ lo evitiamo (complicato)

ESERCIZIO – 13/01/2020

$C_X(\tau)$ di un processo $X(t)$

$$C_X(\tau) = A\, e^{-\alpha|\tau|} \cos(2\pi f_0 \tau)$$

$$f_X(x;t) \in \mathcal{U}[0, 10]$$

$$\Rightarrow S_X(f) = ?$$

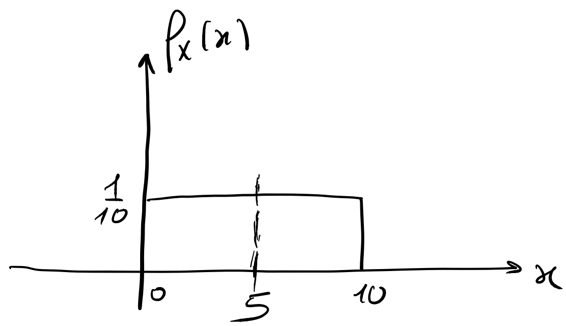
$$S_X(f) = TCF\left[R_X(\tau) \right]$$

$$R_X(\tau) = C_X(\tau) + \eta_X^2(\tau)$$

$$\eta_X(t) = \int_{-\infty}^{+\infty} x\, f_X(x;t)\, dx$$

$f_X(x;t)$ non dipende da "t" poiché è distribuito tra 0 e 10 indipendentemente dal tempo.

$$\eta_X = \int_{-\infty}^{+\infty} x \, f_X(x) \, dx = \int_{-\infty}^{+\infty} x \, \frac{1}{10} \, \mathrm{rect}\left(\frac{x-5}{10}\right) dx$$



$$= \frac{1}{10} \int_{0}^{10} x \, dx = \frac{1}{10} \left. \frac{x^2}{2} \right|_0^{10} = \frac{1}{20}(100-0) = \frac{100}{20} = 5$$

$$R_X(\tau) = \underbrace{A \, e^{-\alpha|\tau|} \cos(2\pi f_0 \tau)} + 25$$

$$S_X(f) = \underbrace{TCF\left[A \, e^{-\alpha|\tau|} \cos(2\pi f_0 \tau)\right]} + 25\,\delta(f)$$

$$S_X'(f) = \frac{A}{2}\left[S_{X_0}(f-f_0) + S_{X_0}(f+f_0)\right]$$

$$S_{X_0}(f) = TCF\left[e^{-\alpha|\tau|}\right]$$

$$S_{x_0}(f) = \int_{-\infty}^{+\infty} e^{-\alpha|\tau|} e^{-j2\pi f\tau} d\tau$$

$$= \int_{-\infty}^{0} e^{\alpha\tau} e^{-j2\pi f\tau} d\tau + \int_{0}^{+\infty} e^{-\alpha\tau} e^{-j2\pi f\tau} d\tau$$

$$= \int_{-\infty}^{0} e^{(\alpha - j2\pi f)\tau} d\tau + \int_{0}^{+\infty} e^{-(\alpha + j2\pi f)\tau} d\tau$$

$$= \frac{1}{\alpha - j2\pi f} + \frac{1}{\alpha + j2\pi f} = \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$$

$$\Rightarrow S_x'(f) = \frac{A}{\cancel{2}}\left[\frac{\cancel{2}\alpha}{\alpha^2 + 4\pi^2 (f-f_0)^2} + \frac{2\alpha}{\alpha^2 + 4\pi^2 (f+f_0)^2}\right]$$

$$= \alpha A \left[\frac{1}{\alpha^2 + 4\pi^2 (f-f_0)^2} + \frac{1}{\alpha^2 + 4\pi^2 (f+f_0)^2}\right]$$

$$\Rightarrow S_x(f) = \alpha A \left[\frac{1}{\alpha^2 + 4\pi^2 (f-f_0)^2} + \frac{1}{\alpha^2 + 4\pi^2 (f+f_0)^2}\right] + 2S\,\delta(f)$$

~~ESERCIZIO – 17/07/2018~~

~~$N(t)$ Gaussiano bianco di banda B con potenza $N_0 B$~~

~~$N(t) \rightarrow$ SLS $h(t)$ $\rightarrow X(t)$, $t=0$ $\rightarrow X = X(t)\big|_{t=0} = X(0)$~~

~~$h(t) = e^{-2t} u(t)$~~