Soluzioni pac 1 compito 4

Problema 1

$i \times j = k$
$j \times k = i$
$k \times i = j$

(a) $(1,0,0) \times [(1,1,0) \times (0,1,1)]$

$(1,1,0) \times (0,1,1) = (1,-1,1)$

$(i+j) \times (j+k) = i \times j + i \times k + j \times j + j \times k$

$= k - j + i = (1,-1,1)$

$(1,0,0) \times (1,-1,1) = i \times (i - j + k)$

$= (i \times i) - (i \times j) + (i \times k)$

$= (0,-1,-1)$

(b) $[(1,0,0) \times (1,1,0)] \times (0,1,1)$

$(1,0,0) \times (1,1,0) = i \times (i + j)$

$= (i \times i) + (i \times j) = k$

$(0,0,1) \times (0,1,1) = k \times (j + k)$

$= (k \times k) + k \times j = -i = (-1,0,0)$

Problema 2

$\cos\theta = \dfrac{(u,v)}{|u|\,|v|}$ $\qquad (u,v) = ((1,1,1),(2,1,t))$

$= 3 + t$

$|u| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$ $\qquad |v| = \sqrt{2^2+1^2+t^2} = \sqrt{5+t^2}$

$\cos(\pi/6) = 1/2$

$\Rightarrow \dfrac{1}{2} = \dfrac{3+t}{\sqrt{3}\sqrt{5+t^2}} \Rightarrow \sqrt{3}\sqrt{5+t^2} = 6 + 2t$

$\Rightarrow (3)(5+t^2) = 36 + 24t + 4t^2$

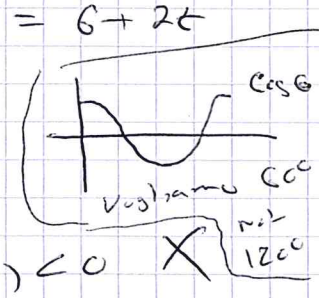
$\Rightarrow 3t^2 + 15 = 36 + 24t + 4t^2$

$\Rightarrow t^2 + 24t + 21 = 0$

$\Rightarrow t = -12 - \sqrt{123}$ $\qquad (u,v) < 0$

$t = \sqrt{123} - 12$ $\qquad (u,v) > 0$

Soluzione



Problema 3

(a) $\langle (1,-1,0), (1,1,1) \rangle = 0 \Rightarrow u \perp w$

$\langle (0,1,-1), (1,1,1) \rangle = 0 \Rightarrow v \perp w$

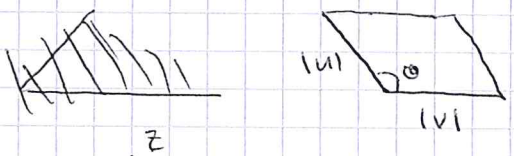
(b) $|u| = \sqrt{1^2+(-1)^2+0^2} = \sqrt{2}$

$|v| = \sqrt{0^2+(1)^2+(-1)^2} = \sqrt{2}$

(c) $\cos\theta = \dfrac{(u,v)}{|u||v|} = \dfrac{(1)(0)+(-1)(1)+(0)(1)}{\sqrt{2}\sqrt{2}}$

$= -1/2 \Rightarrow \theta = 120^\circ \text{ o } 2\pi/3$

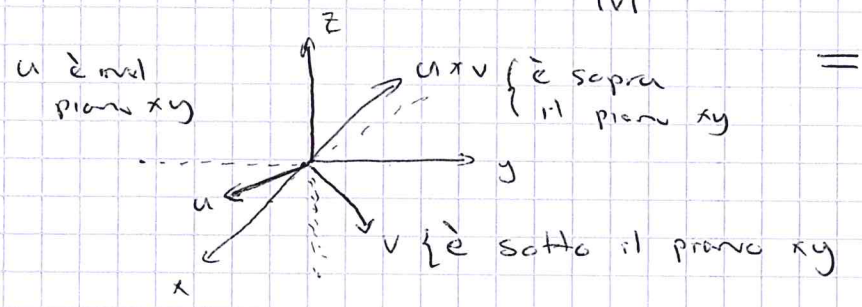
(d)



$\text{Area} = |u||v|\sin\theta$

$= \sqrt{2}\sqrt{2}\sin(2\pi/3)$

$= (2)\dfrac{\sqrt{3}}{2} = \sqrt{3}$



$|u \times v| = \sqrt{3}$

$u \times v \perp$ il piano generato di u e v

$u \times v$ è sopra il piano xy

$\Rightarrow \boxed{u \times v = (1,1,1)}$

(e) $u \times v = (i-j) \times (j-k)$

$= i \times j - i \times k - j \times j + j \times k$

$= k + j + i = (1,1,1)$

$(1,1,1) \perp u, v$

$|(1,1,1)| = \sqrt{3}$

$(1,1,1)$ sopra il piano xy

Problema 4

$$L(t) = (1,0,1) + t((0,1,0) - (1,0,1))$$
$$= (1,0,1) + t(-1,1,-1)$$

$$\Rightarrow x(t) = 1-t,\ y(t) = t,\ z(t) = 1-t$$

$$\Rightarrow 2 = x(t) + 2y(t) + 3z(t)$$

$$2 = 1 - t + 2t + 3(1-t)$$
$$= 4 - 2t \Rightarrow -2t = -2 \Rightarrow \boxed{t = 1}$$

$$L(1) = (0,1,0)$$

Problema 5

(a) $\underline{X}(s,t) = (1,2,3) + s((3,1,2) - (1,2,3)) + t((2,3,1) - (1,2,3))$

(b) Substituim i punts: $(1,2,3)$, $(3,1,2)$, $(3,3,1)$ nell'equació

$$Ax + By + Cz = D$$

$$\left.\begin{array}{l} A + 2B + 3C = D \\ 3A + B + 2C = D \\ 2A + 3B + C = D \end{array}\right\} \longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & D \\ 3 & 1 & 2 & D \\ 2 & 3 & 1 & D \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & D \\ 0 & -5 & -7 & -2D \\ 0 & -1 & -5 & -D \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & D \\ 0 & 1 & 5 & D \\ 0 & 5 & 7 & 2D \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & D \\ 0 & 1 & 5 & D \\ 0 & 0 & -18 & -3D \end{array}\right) \quad \therefore -18C = -3D$$

Per semplicitat, sia $D = 6$

$$\Rightarrow C = 1, \quad B + 5C = 6 \Rightarrow B = 1$$
$$A + 2B + 3C = 6 \Rightarrow A = 1$$

$x + y + z = 6 \longrightarrow \boxed{\frac{1}{6}x + \frac{1}{6}y + \frac{1}{6}z = 1}$

Problema 6

$\pi: x+y+2z=3$, $N=(1,1,2)$; $\pi': x+2y+3z=5$, $N'=(1,2,3)$

$N \times N' = (-1,-1,1)$

Sea $x=1$ $\quad 1+y+2z=3$
$1+2y+3z=5$

$y+2z=2 \Rightarrow y+2z=2$
$2y+3z=4 \quad\quad -z=0 \Rightarrow z=0 \Rightarrow y=2$

$(1,2,0) \in \pi \cap \pi'$

$L(t) = (1,2,0) + t(-1,-1,1)$

Problema 7

(a) $u-v = (4,4,1)-(1,4,5) = (3,0,4)$

distancia $= |u-v| = \sqrt{3^2+0^2+4^2} = 5$

(b) $\frac{u}{|u|} = \frac{(4,4,1)}{\sqrt{16^2+16^2+1}}$

$\frac{v}{|v|} = \frac{(1,4,5)}{\sqrt{1^2+4^2+5^2}}$

Problema 6

$(y,x) = ((1,1,1),(1,2,2))$
$= 1+2+2 = 5$

$(x,x) = 1^2+2^2+2^2 = 9$

$\frac{(y,x)}{(x,x)} x = \frac{5}{9}(1,2,2)$