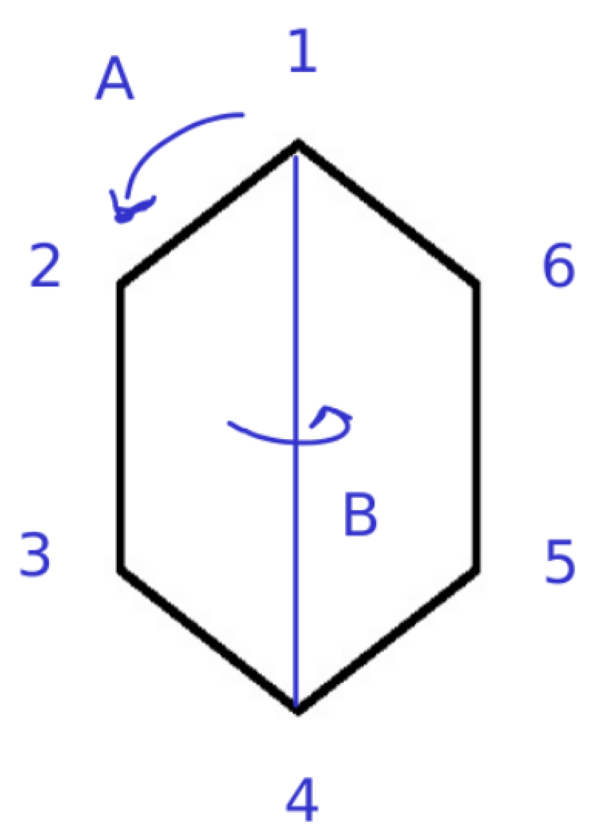
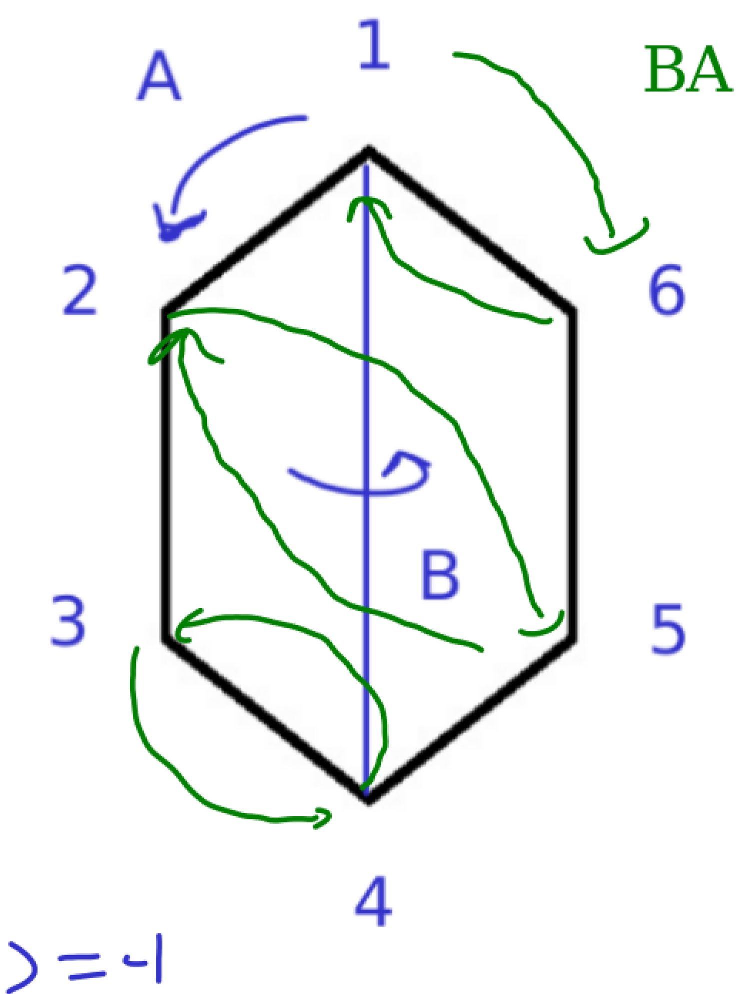
591AA 21/22 – ELENCO DEI PROBLEMI 13

Problema 1. Trova le matrici A e B che corrispondono alle permutazioni dell'esagono mostrato sotto



Problema 2. Trova il prodotto BA delle matrici dal problema 1. Verifica che $(BA)^2$ sia la matrice identità (uno sulla diagonale, zero altrove)

Problema 1 & 2:



$$A = \begin{array}{c} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \\ \begin{array}{cccccc} e_2 & e_3 & e_4 & e_5 & e_6 & e_1 \end{array} \end{array}$$

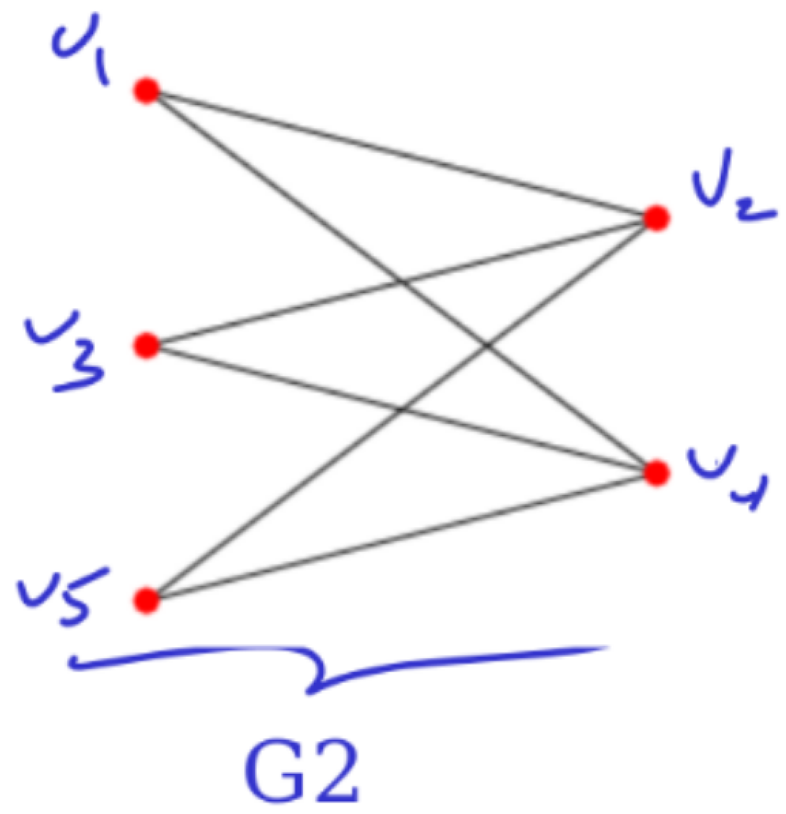
$A(1) = 2$, $A(2) = 3$, $A(3) = 4$
$A(4) = 5$, $A(5) = 6$, $A(6) = 1$

$$B = \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \\ \begin{array}{cccccc} e_1 & e_6 & e_5 & e_4 & e_3 & e_2 \end{array} \end{array}$$

$B(1) = 1$, $B(2) = 6$, $B(3) = 5$
$B(4) = 4$, $B(5) = 3$, $B(6) = 2$

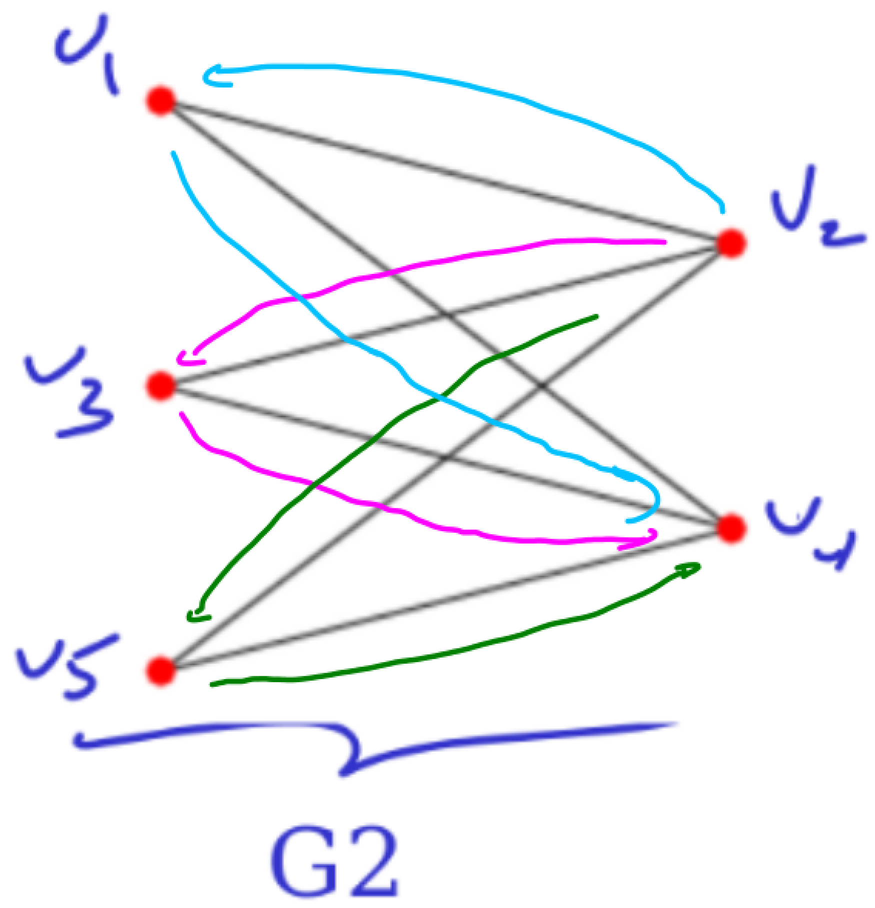
$$BA = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Problema 3. Trova la matrice del grafico G2 mostrato di seguito.



Problema 4. Quanti precorsi di lunghezza 2 ci sono da v_2 a v_4 nel grafico G2?

Problema 3 & 4



$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 & 2 \end{pmatrix}$$

Problema 5. Trova una fattorizzazione della matrice

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

usando l'eliminazione gaussiana

Problema 5

$$A = \begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}. \xrightarrow[R_2=R_2-4R_1]{} \begin{bmatrix}1&2&3\\0&-3&-6\\7&8&9\end{bmatrix}$$

$$\xrightarrow[R_3=R_3-7R_1]{} \begin{bmatrix}1&2&3\\0&-3&-6\\0&-6&-12\end{bmatrix} \xrightarrow[R_3=R_3-2R_2]{} \begin{bmatrix}1&2&3\\0&-3&-6\\0&0&0\end{bmatrix} = A'$$

$$E_3 E_2 E_1 A = A' \Longrightarrow A = \underbrace{E_1' E_2' E_3'}_{\begin{bmatrix}1&0&0\\4&1&0\\7&2&1\end{bmatrix}} A'$$

m_1': $R_2 = R_2 + 4R_1$

$$E_1' = \begin{pmatrix}1&0&0\\4&1&0\\0&0&1\end{pmatrix}$$

m_2': $R_3 = R_3 + 7R_1$ $\qquad E_2' = \begin{pmatrix}1&0&0\\0&1&0\\7&0&1\end{pmatrix}$

m_3': $R_3 = R_3 + 2R_2$ $\qquad E_3' = \begin{pmatrix}1&0&0\\0&1&0\\0&2&1\end{pmatrix}$

$$E_1' E_2' E_3' = \begin{pmatrix}1&0&0\\4&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\7&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&2&1\end{pmatrix}$$

$$= \begin{pmatrix}1&0&0\\4&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\7&2&1\end{pmatrix} = \begin{pmatrix}1&0&0\\4&1&0\\7&2&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\4&1&0\\7&2&1\end{pmatrix}\begin{pmatrix}1&2&3\\0&-3&-6\\0&0&0\end{pmatrix} = \begin{pmatrix}1&2&3\\4&5&6\\7&8&9\end{pmatrix}$$