Soluzione I Compitino 2017 - Fila A

Es 3

$$X(f) = X_0(f) \otimes \left[\delta(f-f_0) + \delta(f+f_0)\right]$$

$$X_0(f) = |X_0(f)| \cdot e^{j \angle X_0(f)}$$

$$|X_0(f)| = 2\,\mathrm{rect}\left(\frac{f}{2B}\right) - \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

$$\angle X_0(f) = 2\pi f t_0 \quad , \quad t_0 = \frac{1}{2B}$$

$$X_0(f) = \left[2\,\mathrm{rect}\left(\frac{f}{2B}\right) - \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)\right] e^{-j2\pi f(-t_0)}$$

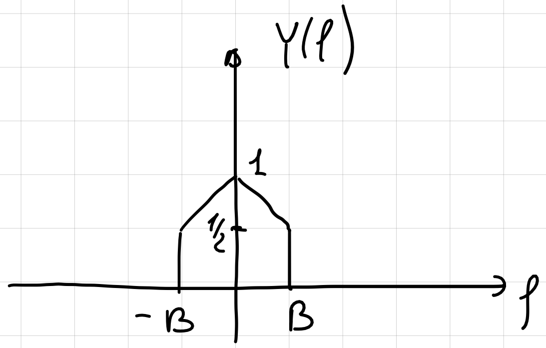
$$x(t) = 2\,x_0(t)\cos(2\pi f_0 t)$$

$$x_0(t) = 4B\,\mathrm{sinc}\left[2B(t+t_0)\right] - B\,\mathrm{sinc}^2\left[B(t+t_0)\right]$$

Es. 4

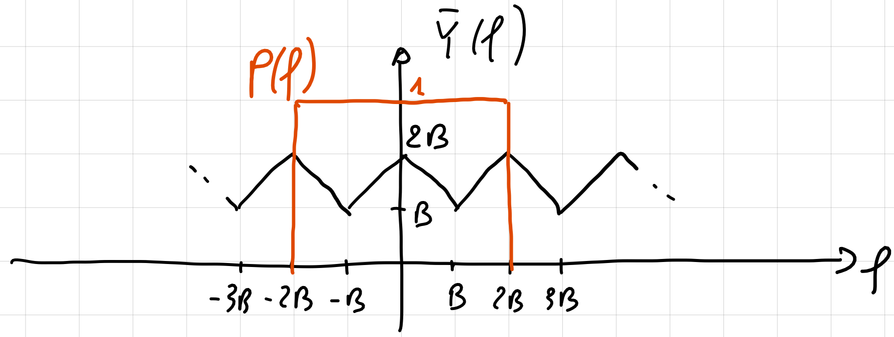
$$X(f) = \left(1 - \frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{4B}\right) \quad , \quad H(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$Y(f) = \left(1 - \frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) = \frac{1}{2}\mathrm{rect}\left(\frac{f}{2B}\right) + \frac{1}{2}\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$



1) $y(t) = B\,\mathrm{sinc}(2Bt) + \frac{B}{2}\mathrm{sinc}^2(Bt)$

2) $\bar{Y}(f) = 2B \sum_{n=-\infty}^{+\infty} Y(f - n2B) = 2B \sum_{n=-\infty}^{+\infty} \left(1 - \frac{|f - n2B|}{2B}\right) \mathrm{rect}\left(\frac{f - n2B}{2B}\right)$



$$Z(f) = \bar{Y}(f)\,P(f) = 2B\,\mathrm{rect}\left(\frac{f}{4B}\right) - 2B\left(1 - \frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{4B}\right)$$

$$+ 2B\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

3) $z(t) = 8B^2\,\mathrm{sinc}(4Bt) - 4B^2\,\mathrm{sinc}^2(2Bt) + 2B^2\,\mathrm{sinc}^2(Bt)$

4) $E_z = \int_{-\infty}^{+\infty} |z(t)|^2 \, dt = \int_{-\infty}^{+\infty} |Z(f)|^2 \, df = \int_{-2B}^{2B} Z^2(f) \, df =$

$$= 4\int_0^B Z^2(f)\,df = 4\int_0^B (2B - f)^2\,df = 4\int_0^B 4B^2\,df + 4\int_0^B f^2\,df$$

$$- 16B\int_0^B f\,df = 16B^3 + \frac{4}{3}B^3 - 8B^3 = \frac{28}{3}B^3$$

$P_z = 0$

Fila B

Es 3

$$X(f) = X_0(f) \otimes \left[\delta(f-f_0) + \delta(f+f_0)\right]$$

$$X_0(f) = |X_0(f)| \cdot e^{j \angle X_0(f)}$$

$$|X_0(f)| = \mathrm{rect}\left(\frac{f}{2B}\right) + \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) - \left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right)$$

$$\angle X_0(f) = 2\pi f t_0 \quad , \quad t_0 = \frac{1}{4B}$$

$$X_0(f) = \mathrm{rect}\left(\frac{f}{2B}\right) + \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) - \left(1 - \frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right) e^{-j2\pi f(-t_0)}$$

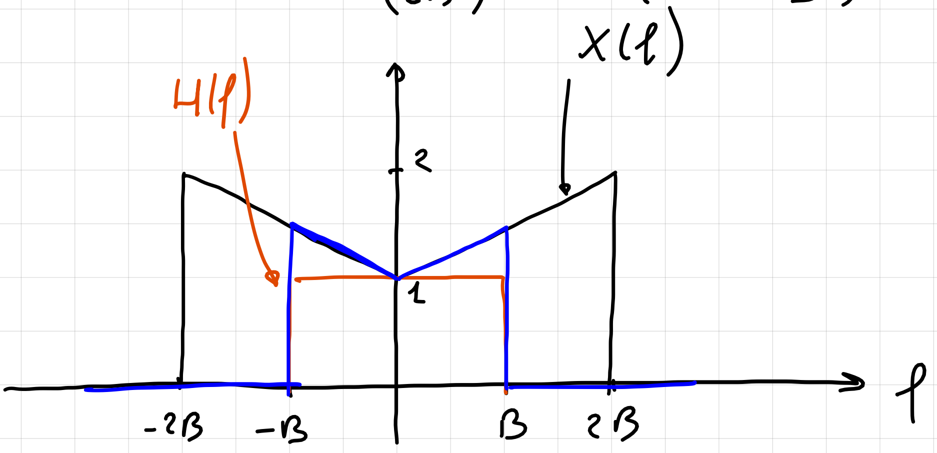
$$x(t) = 2\, x_0(t) \cos(2\pi f_0 t)$$

$$x_0(t) = 2B\,\mathrm{sinc}\left[2B(t+t_0)\right] + B\,\mathrm{sinc}^2\left[B(t+t_0)\right] - \frac{B}{2}\,\mathrm{sinc}^2\left[\frac{B}{2}(t+t_0)\right]$$

Es. 4

$$X(f) = 2\,\mathrm{rect}\left(\frac{f}{4B}\right) - \left(1 - \frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{4B}\right) \quad , \quad H(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$

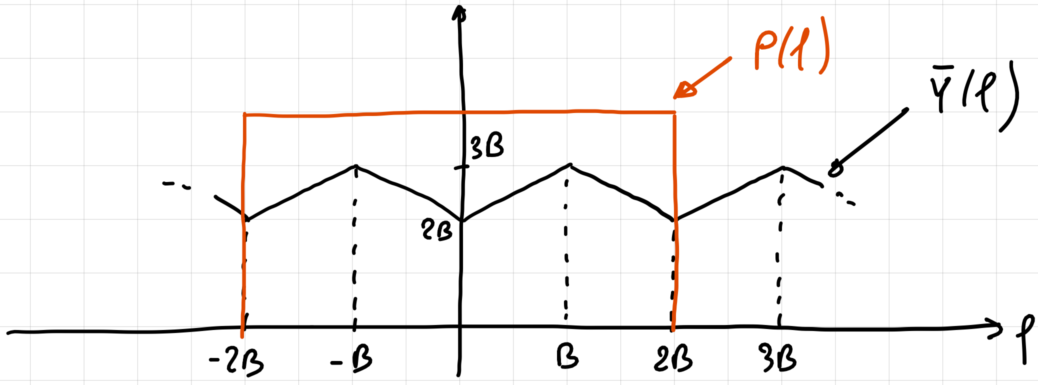
$$Y(f) = X(f)\,H(f) = \frac{3}{2}\,\mathrm{rect}\left(\frac{f}{2B}\right) - \frac{1}{2}\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$



1) $y(t) = 3B\,\mathrm{sinc}(2Bt) - \frac{B}{2}\,\mathrm{sinc}^2(Bt)$

2) $\bar{Y}(f) = 2B \sum_{n=-\infty}^{+\infty} Y(f - n2B)$

$$= 2B \sum_n \frac{3}{2}\,\mathrm{rect}\left(\frac{f - n2B}{2B}\right) - \frac{1}{2}\left(1 - \frac{|f - n2B|}{B}\right)\mathrm{rect}\left(\frac{f - n2B}{2B}\right)$$



$P(f) = \mathrm{rect}\left(\frac{f}{4B}\right)$

$$Z(f) = \bar{Y}(f)\,P(f) = 2B\,\mathrm{rect}\left(\frac{f}{4B}\right) + 2B\left(1 - \frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{4B}\right) +$$

$$- 2B\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$

3) $z(t) = 8B^2\,\mathrm{sinc}(4Bt) + 4B^2\,\mathrm{sinc}^2(2Bt) - 2B^2\,\mathrm{sinc}^2(Bt)$

4) $E_z = \int_{-\infty}^{+\infty} |x(t)|^2\,dt = \int_{-\infty}^{+\infty} |Z(f)|^2\,df = 4\int_0^B (2B + f)^2\,df$

$$= 4\int_0^B 4B^2\,df + 4\int_0^B f^2\,df + 16B\int_0^B f\,df$$

$$= 16B^3 + \frac{4}{3}B^3 + 8B^3 = \frac{76}{3}B^3 \quad \Rightarrow \quad P_z = 0$$

FILA C

Es 3

$$X(f) = X_0(f) \otimes \left[\delta(f-f_0) + \delta(f+f_0)\right]$$

$$X_0(f) = |X_0(f)| \cdot e^{j\angle X_0(f)}$$

$$|X_0(f)| = 2\,\mathrm{rect}\left(\frac{f}{2B}\right) - 2\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) + \left(1-\frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right)$$

$$\angle X_0(f) = 2\pi f t_0 \quad , \quad t_0 = \frac{1}{2B}$$

$$X_0(f) = \left[2\,\mathrm{rect}\left(\frac{f}{2B}\right) - 2\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) + \left(1-\frac{|f|}{B/2}\right)\mathrm{rect}\left(\frac{f}{B}\right)\right] e^{-j2\pi f(-t_0)}$$

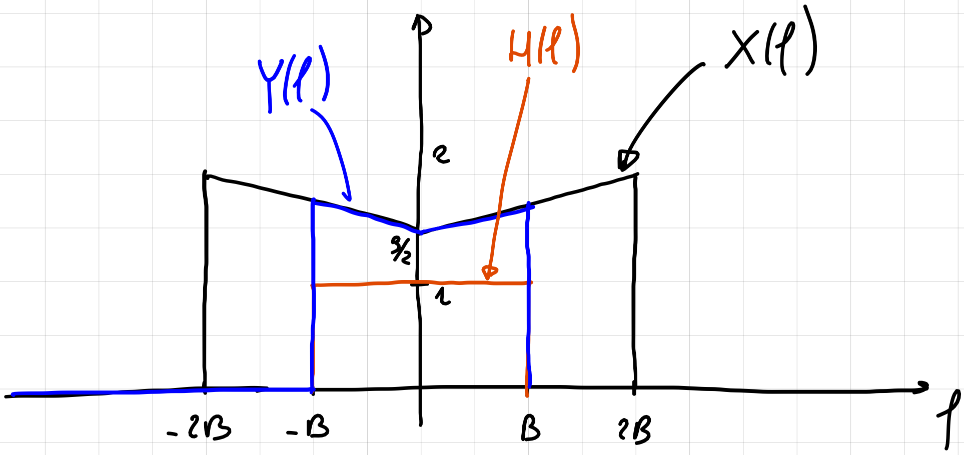
$$x(t) = 2\,x_0(t)\cos(2\pi f_0 t)$$

$$x_0(t) = 4B\,\mathrm{sinc}\left[2B(t+t_0)\right] - 2B\,\mathrm{sinc}^2\left[B(t+t_0)\right] + \frac{B}{2}\,\mathrm{sinc}^2\left[\frac{B}{2}(t+t_0)\right]$$

Es. 4

$$X(f) = 2\,\mathrm{rect}\left(\frac{f}{4B}\right) - \frac{1}{2}\left(1-\frac{|f|}{2B}\right)\mathrm{rect}\left(\frac{f}{4B}\right) \quad , \quad H(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$Y(f) = X(f)\,H(f) = \frac{7}{4}\,\mathrm{rect}\left(\frac{f}{2B}\right) - \frac{1}{4}\left(1-\frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$$



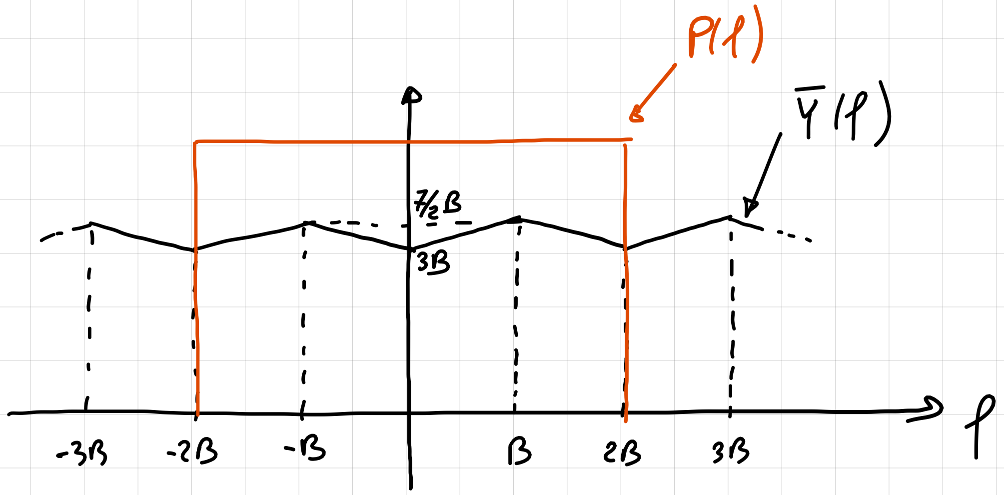
1) $y(t) = \frac{7}{2} B \,\mathrm{sinc}(2Bt) - \frac{B}{4} \mathrm{sinc}^2(Bt)$

2) $\bar{Y}(f) = 2B \sum_{n=-\infty}^{+\infty} Y(f - n2B)$

$$= 2B \sum_n \frac{7}{4} \mathrm{rect}\left(\frac{f-n2B}{2B}\right) - \frac{1}{4}\left(1 - \frac{|f-n2B|}{B}\right) \mathrm{rect}\left(\frac{f-n2B}{2B}\right)$$

$P(f) = \mathrm{rect}\left(\frac{f}{4B}\right)$

$Z(f) = \bar{Y}(f)\, P(f) = 3B\, \mathrm{rect}\left(\frac{f}{4B}\right) + B\left(1 - \frac{|f|}{2B}\right) \mathrm{rect}\left(\frac{f}{4B}\right) - B\left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$



3) $z(t) = 12 B^2 \mathrm{sinc}(4Bt) + 2B^2 \mathrm{sinc}^2(2Bt) - B^2 \mathrm{sinc}^2(Bt)$

4) $E_z = \int_{-\infty}^{+\infty} |z(t)|^2 dt = \int_{-\infty}^{+\infty} |Z(f)|^2 df = 4 \int_0^B \left(3B + \frac{f}{2}\right)^2 df$

$$= 4 \int_0^B 9B^2 df + 4 \int_0^B \frac{f^2}{4} df + 4 \int_0^B 3Bf\, df$$

$$= 36 B^3 + \frac{B^3}{3} + 6B^3 = \frac{127}{3} B^3 \Rightarrow P_z = 0$$