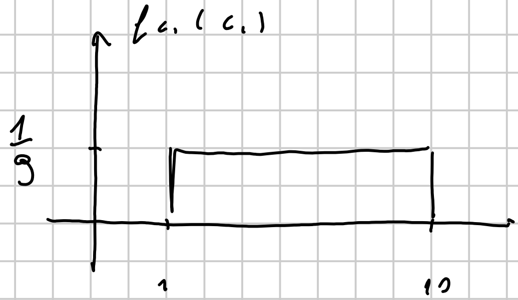
COMPITINO II 2013 FILA A

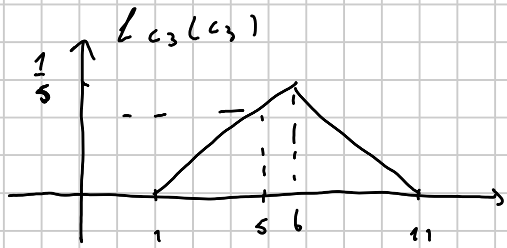
ESERCIZIO 1

C_1



C_2 $$f_{c_2}(c_2) = \frac{1}{5} e^{-\frac{c_2}{5}} u(c_2)$$

C_3



$A = \{$ STAMPARE ALMENO 100 PAGINE IDENTICHE $\}$

$$Pr\{A\} = \frac{1}{3} Pr\{A \mid \text{scelgo stampante } 1\} + \frac{1}{3} Pr\{A \mid \text{scelgo } 2\} + \frac{1}{3} Pr\{A \mid \text{scelgo } 3\}$$

$$Pr\{A \mid \text{scelgo } i\} = Pr\{C_i \leq 5 \text{ mlt/pag}\}$$

$$Pr\{A \mid \text{scelgo } 1\} = \frac{1}{9} \cdot 4 = \frac{4}{9}$$

$$Pr\{A \mid \text{scelgo } 2\} = \int_0^5 \frac{1}{5} e^{-\frac{c_2}{5}} dc_2 = \frac{1}{5}(-5) e^{-c_2/5}\Big|_0^5 =$$

$$= \left(1 - \frac{1}{e}\right)$$

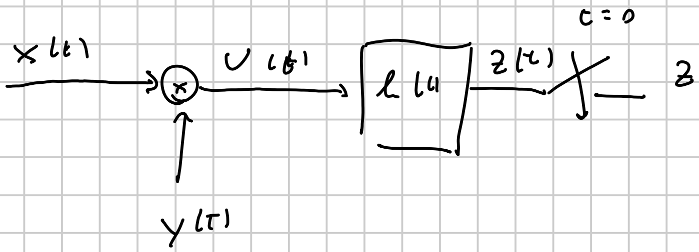
$Pr\{A \mid \text{Scelgo } 3\} = \frac{8}{25}$

$Pr\{A\} = \frac{4}{27} + \frac{8}{75} + \frac{1}{3}\left(1 - \frac{1}{e}\right)$

ESERCIZIO 2

$x(t) = \frac{A}{T} \operatorname{sinc}\left(\frac{t}{T}\right) \cos(2\pi f_0 t)$

$y(t) = \cos(2\pi f_0 t + \theta)$



$A \in \mathcal{U}(0, 1)$

$\theta \in \mathcal{U}(-\pi, \pi)$

$U(t) = x(t)\, y(t) = \frac{A}{T} \operatorname{sinc}\left(\frac{t}{T}\right) \cos(2\pi f_0 t) \cos(2\pi f_0 t + \theta) =$

$= \frac{1}{2T} A \operatorname{sinc}\left(\frac{t}{T}\right) \left[\cos(4\pi f_0 t + \theta)\ \cos\theta\right]$

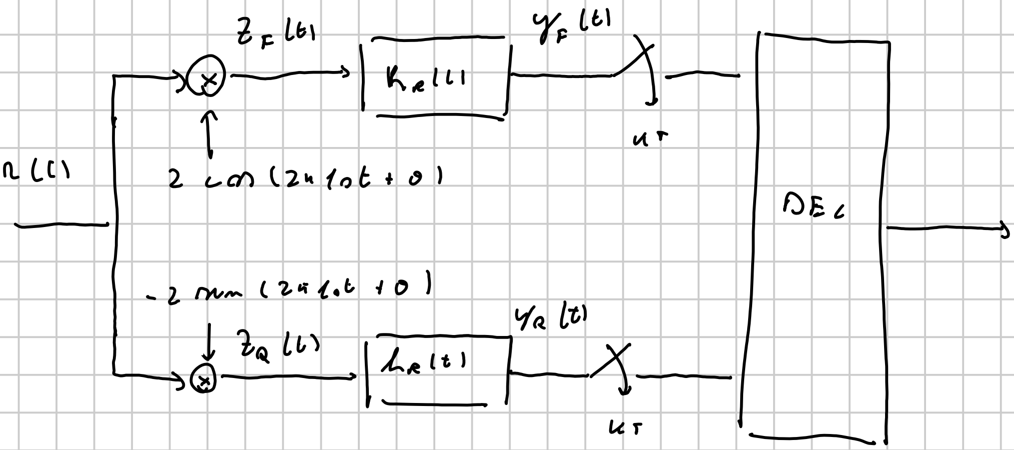
$z(t) = \frac{1}{2} A \operatorname{sinc}\left(\frac{t}{T}\right) \cos(\theta)$

$z(0) = z = \frac{1}{2} A \cos(\theta)$

$E\{z\} = E\left\{\frac{1}{2} A \cos(\theta)\right\} = \frac{1}{2} E\{A\} \underset{\overset{\shortparallel}{0}}{E\{\cos\theta\}} = 0$

$E\{z^2\} = \frac{1}{4} \underset{\overset{\shortparallel}{\frac{1}{3}}}{E\{A^2\}} \underset{\overset{\shortparallel}{\pi}}{E\{\cos^2\theta\}} = \frac{1}{4} \frac{\pi}{3} = \sigma_z^2$

ESERCIZIO 3



$$E_s = E\left\{\int_{-\infty}^{+\infty} s_i^2(t)\,dt\right\}$$

$$s_i(t) = x_c(i)\,p(t-iT)\cos(2\pi f_0 t) - x_s(i)\,p(t-iT)\sin(2\pi f_0 t)$$

$$E_s = E\left\{\int_{-\infty}^{+\infty}\Big(x_c^2(i)\,p^2(t-iT)\cos^2(2\pi f_0 t) + x_s(i)\,p^2(t-iT)\sin^2(2\pi f_0 t) +\right.$$

$$\left. - 2\,x_c(i)\,x_s(i)\,p^2(t-iT)\cos(2\pi f_0 t)\sin(2\pi f_0 t)\Big)dt\right\} =$$

$$= E\{x_c^2(i)\}\int_{-\infty}^{+\infty} p^2(t-iT)\left(\frac{1}{2}+\frac{1}{2}\cos(4\pi f_0 t)\right)dt +$$

$$+ E\{x_s^2(i)\}\int_{-\infty}^{+\infty} p^2(t-iT)\left(\frac{1}{2}-\frac{1}{2}\cos(4\pi f_0 t)\right)dt +$$

$$- E\{x_c(i)\,x_s(i)\}\int_{-\infty} p^2(t-iT)\,\frac{1}{2}\sin(4\pi f_0 t)\,dt =$$

$$= \frac{1}{2}E\{x_c^2(i)\}E_p + \frac{1}{2}E\{x_s^2(i)\}E_p =$$

$$= \frac{1}{2}\cdot\frac{3}{2}E_p + \frac{1}{2}E_p = \frac{5}{2}E_p = \frac{5}{2}B$$

$$E_p = \int_{-\infty}^{+\infty} |P(f)|^2 \, df = B$$

$$r(t) = s(t) \otimes c(t) + n(t) =$$

$$= s(t - t_0) + n(t) =$$

$$= \sum_n x_c(n)\, p(t - nT - t_0) \cos(2\pi f_0 t \underbrace{- 2\pi f_0 t_0}_{\alpha = -2\pi f_0 t_0}) +$$

$$+ \sum_n x_s(n)\, p(t - nT - t_0) \sin(2\pi f_0 t - 2\pi f_0 t_0) + n(t)$$

RAMO IN FASE

$$z_F(t) = z_{Fs}(t) + z_{Fn}(t)$$

$z_{Fn}(t) \rightarrow$ componente di rumore

$z_{Fs}(t) \rightarrow$ componente di segnale

$$z_{Fs}(t) = \sum_n x_c(n)\, p(t - nT - t_0) \cos(2\pi f_0 t + \alpha)\, 2\cos(2\pi f_0 t + \theta) +$$

$$+ \sum_n x_s(n)\, p(t - nT - t_0) \sin(2\pi f_0 t + \alpha)\, 2\cos(2\pi f_0 t + \theta) =$$

$$= \sum_n x_c(n)\, p(t - nT - t_0) \left(\cos(\alpha - \theta) + \cos(4\pi f_0 t + \alpha + \theta) \right)$$

$$- \sum_n x_s(n)\, p(t - nT - t_0) \left(\sin(\alpha - \theta) + \sin(4\pi f_0 t + \alpha + \theta) \right)$$

$$y_F(t) = y_{Fs}(t) + y_{Fn}(t)$$

$$y_{Fs}(t) = \sum_n x_c(n)\, g(t - nT) \cos(\alpha - \theta) +$$

$$- \sum_n x_s(n)\, g[t - nT]\, \sin(\alpha - \theta)$$

$$g(t) = p(t) \otimes c(t) \otimes h_R(t)$$

$$G(f) = P(f)\, C(f)\, H_R(f) = P^2(f)$$

$$y_{Fn}(t) = n_{nF}(t)$$

$$P_{nnF} = N_0 \int_{-\infty}^{+\infty} |H_R(f)|^2\, df = N_0 \int_{-\infty}^{+\infty} P^2(f)\, df = N_0 B$$

RAMO IN QUADRATURA

$$z_{Rs}(t) = - \sum_n x_c(n)\, p(t - nT - t_0) \cos(2\pi f_0 t + \alpha)\, 2 \sin(2\pi f_0 t + \theta) +$$

$$+ \sum_n x_s(n)\, p(t - nT - t_0) \sin(2\pi f_0 t + \alpha)\, 2 \sin(2\pi f_0 t + \theta) =$$

$$= \sum_n x_c(n)\, p(t - nT - t_0) \sin(4\pi f_0 t + \alpha + \theta) \sin(\alpha - \theta) +$$

$$+ \sum x_s(n)\, p(t - nT - t_0) \left(\cos(\alpha - \theta) - \cos(4\pi f_0 t + \alpha + \theta) \right)$$

$$y_{Rs}(t) = \sum_n x_s(n)\, h(t - nT) \cos(\alpha - \theta) +$$

$$+ \sum_n x_c(n)\, h(t - nT) \sin(\alpha - \theta)$$

ASSENZA DI CROSS-TALK

$$\alpha - \theta = 2\pi k$$

$$\theta = \alpha + 2k\pi = -2\pi f_0 t_0 + 2k\pi$$

$P_{max} = N_0 B$

Verifica assenza di ISI

$G(f) = P^2(f)$

$$\sum_k G\left(f - \frac{k}{T}\right) = 1 \Rightarrow h(n) = \frac{1}{T}$$

$$P_E^{(QAM)}(b) = P_E^{(F)}(b) + P_E^{(Q)}(b)$$

P. err. ramo in quadratura

Probabilità di errore ramo in fase

$$P_E^{(F)} = \frac{1}{2} Q\left(\frac{1/T}{\sqrt{N_0 B}}\right) + \frac{1}{2} Q\left(\frac{2/T}{\sqrt{N_0 B}}\right)$$

$$P_E^{Q} = Q\left(\frac{1/T}{\sqrt{N_0 B}}\right)$$

ESERCIZIO 4

1) NO perché $S_x(f)$ avrebbe componenti < 0

2) No perché non ha il massimo in 0

3) SI rispetta tutte le proprietà