

Corso di Laurea in Ingegneria Informatica

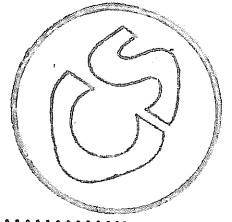
Prova Scritta di Elettrotecnica

(12 cred.: 1, 3, 4, 5; 9 cred.: 1, 2 or 5, 3, 6; 6 cred.: 2, 5, 6)

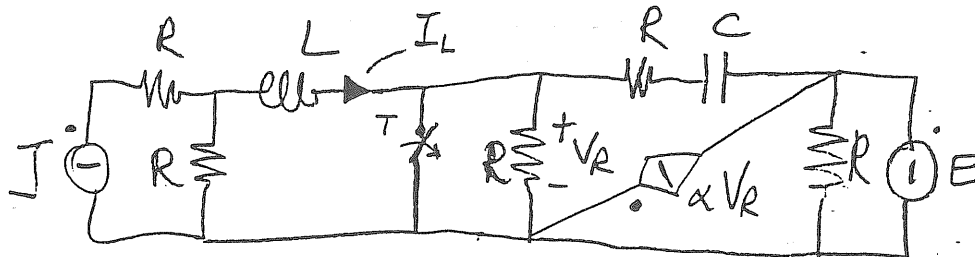
Pisa, ~~12 luglio~~ 2002

23/05/04

Allievo .....



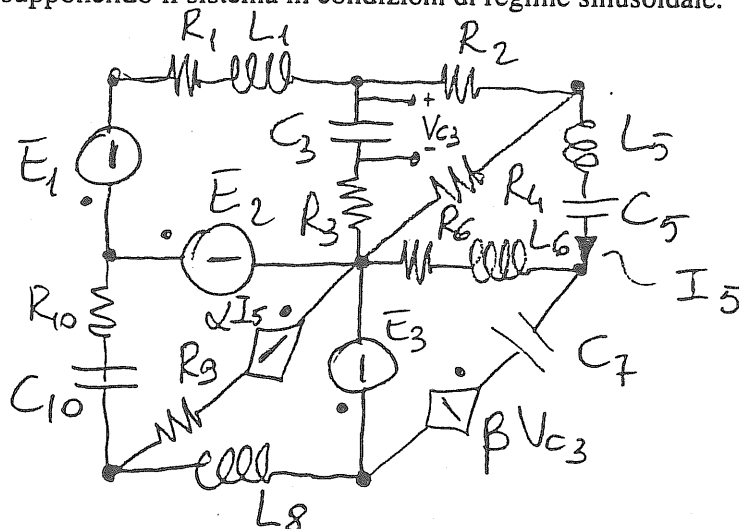
1. Il circuito di figura è in condizione di regime per  $t < 0$ . Determinare l'evoluzione temporale della corrente  $I_L(t)$  per  $t \geq 0$ .



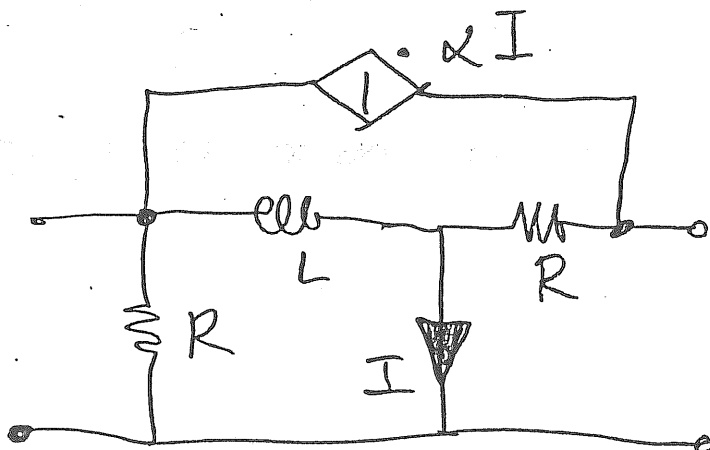
$R = 10 \Omega$ ;  
 $L = 10 \text{ mH}$ ;  
 $C = 200 \mu\text{F}$ ;  
 $\alpha = 2$ ;  
 $J = 10 \text{ A}$ ;  
 $E = 100 \text{ V}$ ;

Il tasto T si apre all'istante  $t = 0$ .

2. Per il circuito di figura scrivere un sistema di equazioni di equilibrio con il metodo delle tensioni nodali, supponendo il sistema in condizioni di regime sinusoidale.

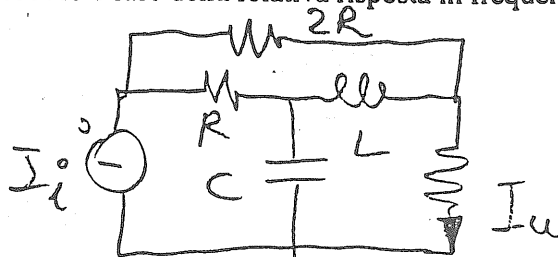
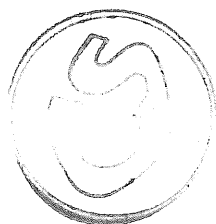


3. Per il doppio bipolo rappresentato in figura, determinare la matrice dei parametri T alla pulsazione  $\omega = 1000 \text{ rad/sec}$ .



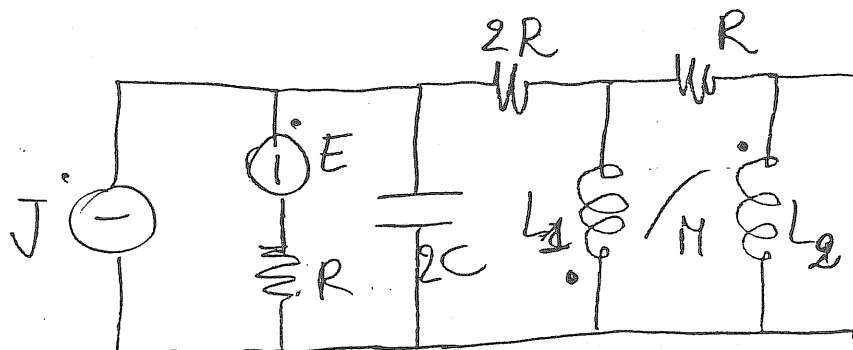
$R = 10 \Omega$ ;  
 $L = 20 \text{ mH}$ ;  
 $\alpha = 2$ ;

4. Per la rete di figura determinare la funzione di trasferimento  $I_u/I_i$  e tracciarne i diagrammi di Bode di modulo e fase della relativa risposta in frequenza.



$$\begin{aligned} R &= 20 \\ L &= 10 \text{ mH} \\ \alpha &= 0.5 \\ C &= 100 \mu\text{F} \end{aligned}$$

5. Considerando in condizioni di regime periodico la rete di figura determinare l'energia elettromagnetica media immagazzinata nel condensatore C.



$$\begin{aligned} R &= 10 \Omega; \\ L_1 &= L_2 = 10 \text{ mH}; \\ M &= 10 \text{ mH}; \\ C &= 100 \mu\text{F}; \\ \alpha &= 2; \\ J &= 10 \text{ A}; \end{aligned}$$

$$E = 10\sqrt{2} \sin(1000t + \pi/4) \text{ V};$$

6. Un generatore trifase simmetrico con tensioni di fase 220 V e frequenza 50 Hz, alimenta una macchina asincrona tramite un trasformatore con fase primarie e secondarie a stella. Determinare le perdite nel ferro del trasformatore e la potenza trasferita all'asse dell'asincrono quando funziona con scorrimento  $s = 0.7$ .

#### TRASFORMATORE

Prova a vuoto:

$$V_{10} = 380 \text{ V}; \quad I_{10} = 2 \text{ A} \quad P_{10} = 205 \text{ W}$$

Prova in corto circuito:

$$V_{1cc} = 300 \text{ V}; \quad I_{1cc} = 10 \text{ A} \quad P_{1cc} = 3 \text{ kW}$$

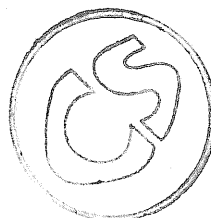
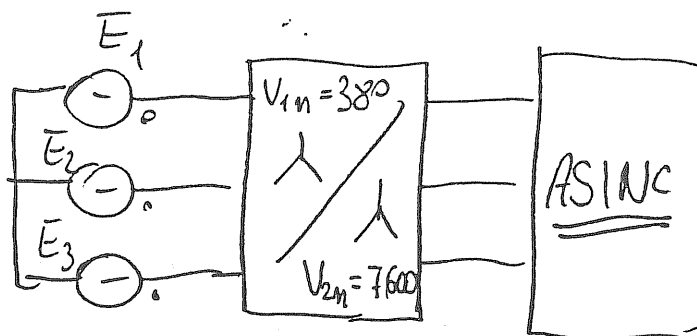
#### MACCHINA ASINCRONA

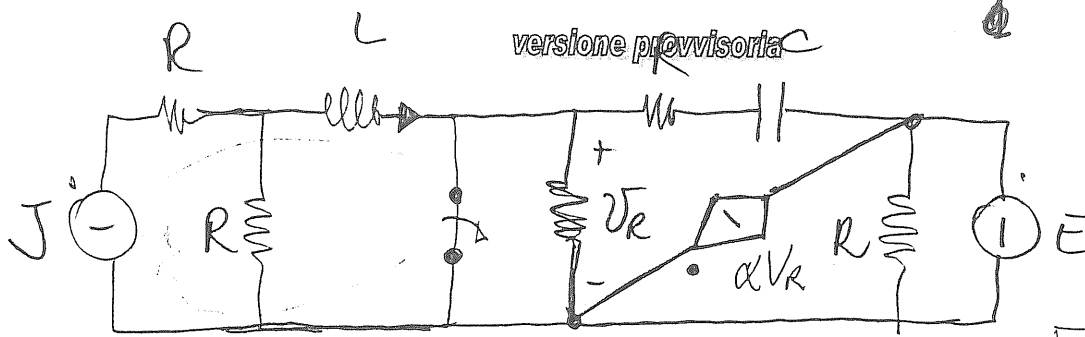
Prova a vuoto:

$$V_{10} = 7600 \text{ V}; \quad I_{10} = 20 \text{ A} \quad P_{10} = 41.2 \text{ kW}$$

Prova in corto circuito:

$$V_{1cc} = 60 \text{ V}; \quad I_{1cc} = 10 \text{ A} \quad P_{1cc} = 675 \text{ W}; \quad R_{1s} = 1.1 \Omega; \quad X_{1s} = 1.3 \Omega; \quad K = 0.5; \quad (E_1 = kE_2)$$

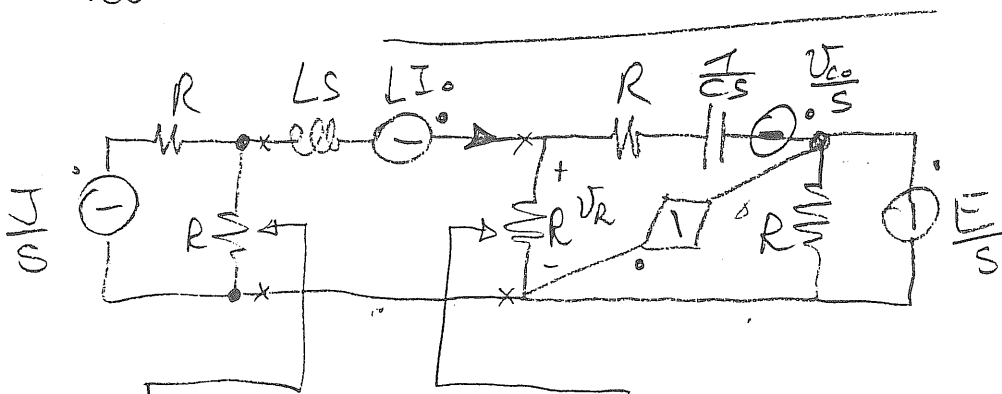




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Per  $t < 0$   $I_{OL} = J$   
 $V_{Co} = E$

$R = 10$   
 $L = 10 \text{ mH}$   
 $C = 200 \mu\text{F}$   
 $J = 10 \text{ A}$   
 $E = 100 \text{ V}$   
 $\alpha = 2$

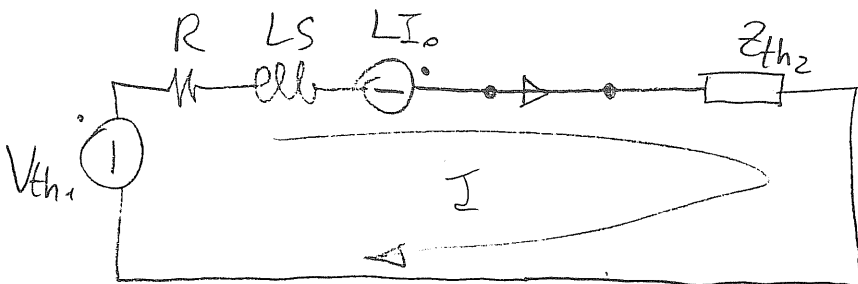
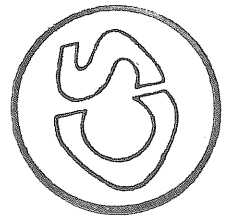


$$V_{th1} = \frac{RJ}{s}$$

$$Z_{th1} = R$$

$$V_{th2} = 0$$

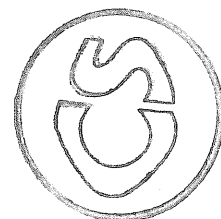
$$Z_{th2} = \frac{R(RCs + 1)}{2RCs + 1}$$



$$I = \frac{V_{th1} + LI_0}{R + sL + Z_{th2}} = \frac{RJ + LI_0 s}{s \left[ R + sL + \frac{R(RCs + 1)}{2RCs + 1} \right]}$$

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$$I(s) = \frac{(RJ + LI_0 s)(2RCs + 1)}{s \left[ \underset{1}{2R^2Cs} + \underset{1}{2RLCs^2} + \underset{1}{R^2Cs} + \underset{1}{R} + \underset{1}{R} + \underset{1}{Ls} \right]}$$



$$I(s) = \frac{2RC LI_0}{2RLC} \frac{\left(s + \frac{RJ}{LI_0}\right) \left(s + \frac{1}{2RC}\right)}{s \left(s^2 + \left(\frac{3R^2C + L}{2RLC}\right)s + \frac{2R}{2RLC}\right)}$$

$$I(s) = I_0 \frac{(s + 1000)(s + 250)}{s(s + 1330.4)(s + 353.6)}$$

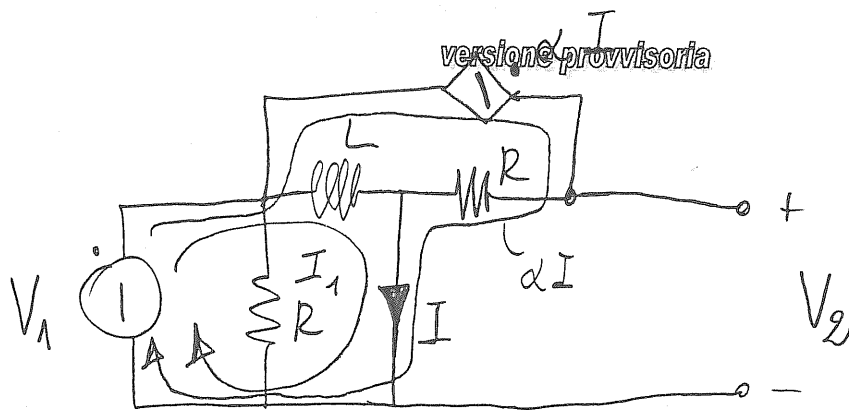
$$I(s) = I_0 \left[ \frac{A}{s} + \frac{B}{s + 1330.4} + \frac{C}{s + 353.6} \right]$$

$$A = I(s) \cdot s \Big|_{s=0} = 0.5$$

$$B = I(s) \cdot (s + 1330.4) \Big|_{s=-1330.4} = 0.3106$$

$$C = I(s) \cdot (s + 353.6) \Big|_{s=-353.6} = 0.1894$$

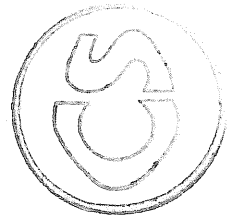
$$i(t) = 10 \cdot \left[ 0.5 + 0.3106 \cdot e^{-1330.4t} + 0.1894 \cdot e^{-353.6t} \right] u(t)$$



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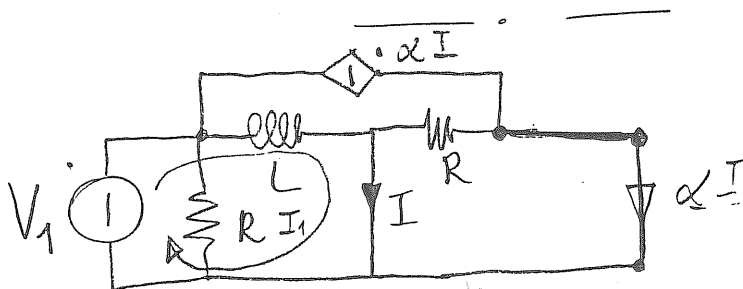


$$V_1 = j\omega L I_1 \Rightarrow I_1 = \frac{V_1}{j\omega L}$$

$$I = I_1 + \alpha I \Rightarrow I(1 - \alpha) = I_1 \Rightarrow I = \frac{I_1}{1 - \alpha}$$

$$V_2 = R \alpha I = \frac{R \alpha}{1 - \alpha} I_1 = \frac{\alpha R}{1 - \alpha} \cdot \frac{V_1}{j\omega L}$$

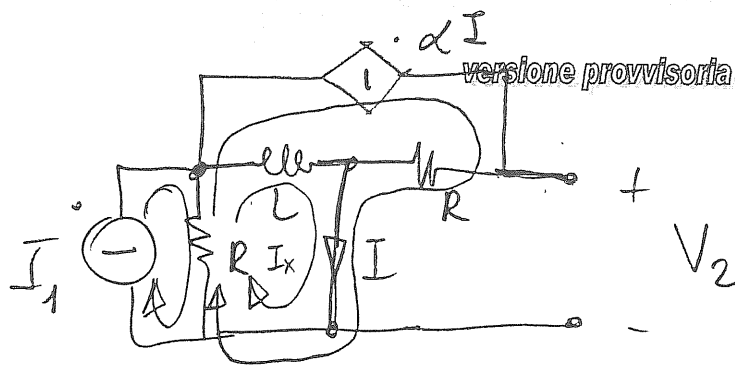
$$\frac{1}{A} = \frac{V_2}{V_1} = \left( \frac{\alpha R}{1 - \alpha} \cdot \frac{1}{j\omega L} \right) =$$



$$I_1 = \frac{V_1}{j\omega L} ; I = I_1$$

$$-I_2 = \alpha I = \alpha I_1 = \frac{\alpha V_1}{j\omega L}$$

$$\frac{1}{B} = \frac{(-I_2)}{V_1} = \frac{\alpha}{j\omega L} =$$



(4)

(2)

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$$(R + j\omega L) I_x - R I_1 + R \alpha I = 0$$

$$I = I_x + \alpha I \Rightarrow I = \frac{I_x}{1 - \alpha}$$

$$V_2 = R \alpha I = \frac{R \alpha}{1 - \alpha} I_x$$

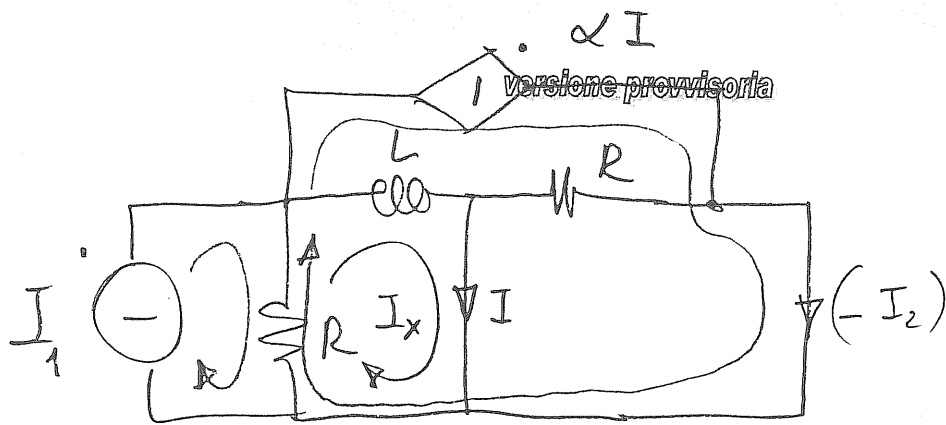
$$\left( R + j\omega L + \frac{R \alpha}{1 - \alpha} \right) I_x = R I_1$$

$$I_x = \frac{R(1 - \alpha)}{(R + j\omega L)(1 - \alpha) + \alpha R} I_1 = \frac{R(1 - \alpha)}{R + j\omega L - \cancel{R} j\omega L \alpha + \cancel{\alpha R}} I_1$$

$$I_x = I_1 \frac{R(1 - \alpha)}{R + j\omega L(1 - \alpha)}$$

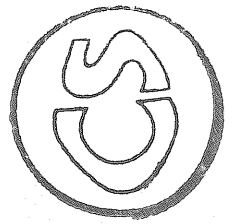
$$V_2 = \frac{R \alpha}{1 - \alpha} \cdot \frac{R(1 - \alpha)}{R + j\omega L(1 - \alpha)} I_1$$

$$\frac{1}{C} = \frac{V_2}{I_1} = \frac{\alpha R^2}{R + j\omega L(1 - \alpha)} =$$



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$$\begin{cases} (R + j\omega L) I_x + R \alpha I - R I_1 = 0 \\ I = I_x \\ -I_2 = \alpha I = \alpha I_x \end{cases}$$

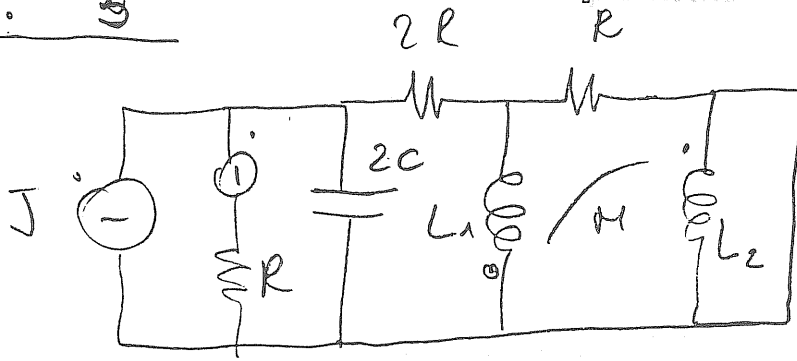
$$(R + j\omega L + \alpha R) I_x = R I_1 \Rightarrow I_x = \frac{R}{R(1 + \alpha) + j\omega L} I_1$$

$$-I_2 = \frac{\alpha R}{R(1 + \alpha) + j\omega L} I_1$$

$$\frac{1}{D} = \frac{-I_2}{I_1} = \frac{\alpha R}{R(1 + \alpha) + j\omega L} =$$

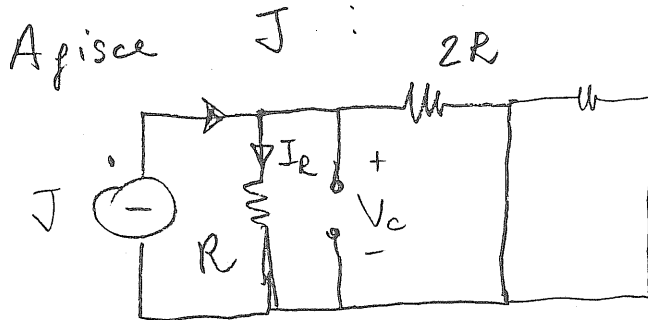
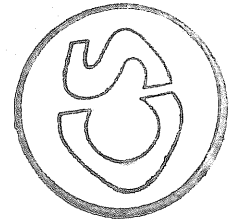
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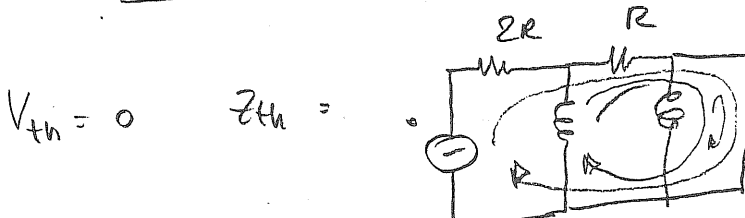
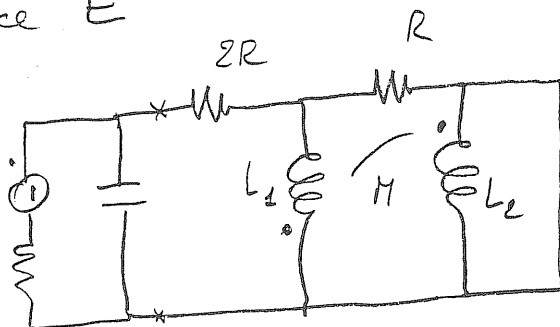
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$$\left. \begin{aligned} V_c &= R I_R \\ I_R &= \frac{J \cdot 2R}{3R} = \frac{2}{3} J \end{aligned} \right\} V_c = \frac{2}{3} R J$$

$$W' = \frac{1}{2} C V_c^2 =$$

Agisce E



$$\begin{cases} 0 = (j\omega L_1 + R) I_1 + R I_p - j\omega M I_2 \\ 0 = j\omega L_2 I_2 - j\omega M I_1 \end{cases}$$

$$I_2 = \frac{M}{L_2} I_1$$

$$0 = \left[ (R + j\omega L_1) - \frac{j\omega M^2}{L_2} \right] I_1 + R I_p \Rightarrow I_1 = \frac{R I_p}{(R + j\omega L_1) - \frac{j\omega M^2}{L_2}}$$



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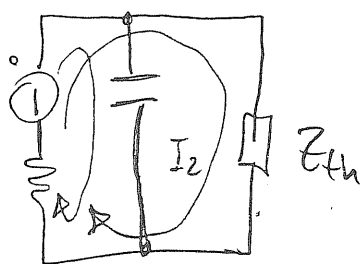
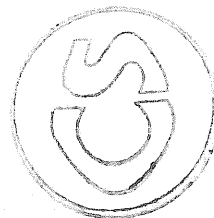
$$V_p = 3R I_p + \frac{R^2 I_p}{R + j\omega L_1 - j\frac{\omega M^2}{L_2}}$$

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$$Z_{th} = 3R + \frac{R^2}{R + j\omega L_1 - j\frac{\omega M^2}{L_2}} =$$



$$\begin{cases} \dot{E} = (R + Z_{th}) \dot{I}_2 + R \dot{I}_1 \\ \dot{E} = (R + \frac{1}{j\omega C}) \dot{I}_1 + R \dot{I}_2 \end{cases}$$

$$\Rightarrow \dot{I}_1 = 0.1552 + j0.0172$$

$$\dot{I}_2 = -0.07 + j0.62$$

$$\dot{V}_{cs} = \bar{Z}_{th} \cdot \dot{I}_2 = -2.76 + j24 \text{ V}$$



$$V_{cseff} \approx 25 \text{ V}$$

$$\bar{W}_c'' = \frac{1}{2} C V_{cseff}^2 = 0.0312$$

$$\bar{W} = \bar{W}_c' + \bar{W}_c'' = 0.2534 \text{ J}$$

Corso di Laurea in Ingegneria Informatica

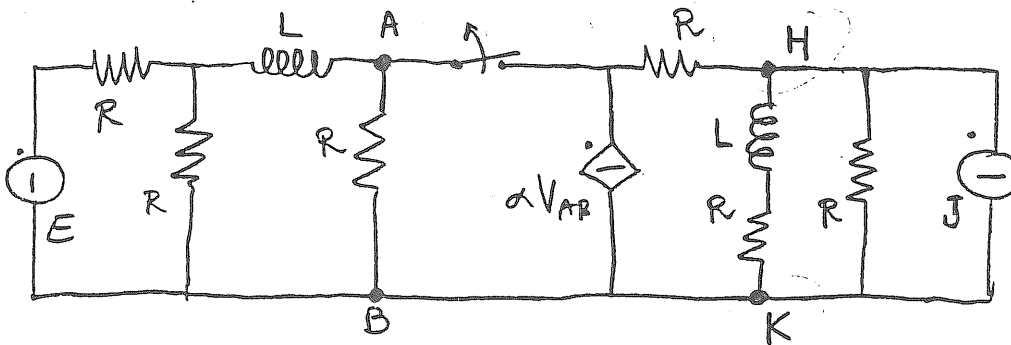
Prova Scritta di Elettrotecnica

(12 cred.: 1, 3, 4, 5; 9 cred.: 1, 2 or 5, 3, 6; 6 cred.: 2, 5, 6)

Pisa, 10 luglio 2004

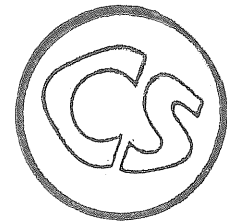
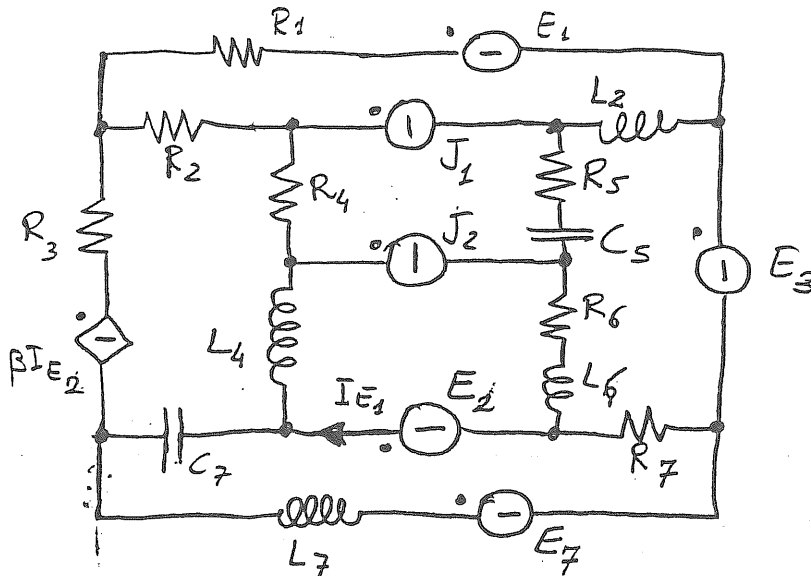
Allievo .....

1. Il circuito di figura è in condizione di regime per  $t < 0$ . Determinare l'evoluzione temporale della tensione  $V_{HK}(t)$  a seguito dell'apertura del tasto che avviene all'istante  $t=0$ .

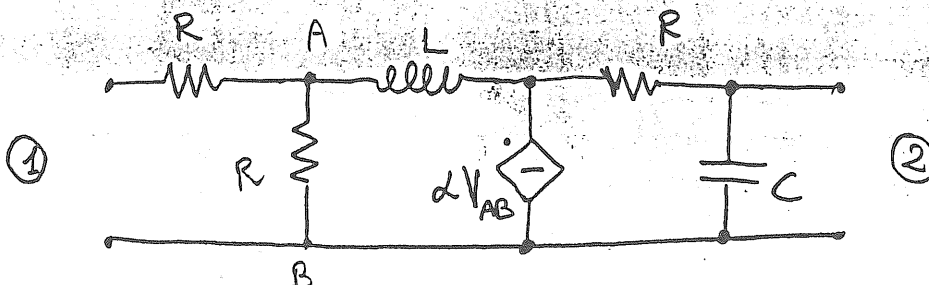


$R = 10 \Omega$ ;  
 $L = 10 \text{ mH}$ ;  
 $\alpha = 2$ ;  
 $J = 10 \text{ A}$ ;  
 $E = 100 \text{ V}$ ;

2. Per il circuito di figura scrivere un sistema di equazioni di equilibrio con il metodo delle correnti di maglia, supponendo il sistema in condizioni di regime sinusoidale.

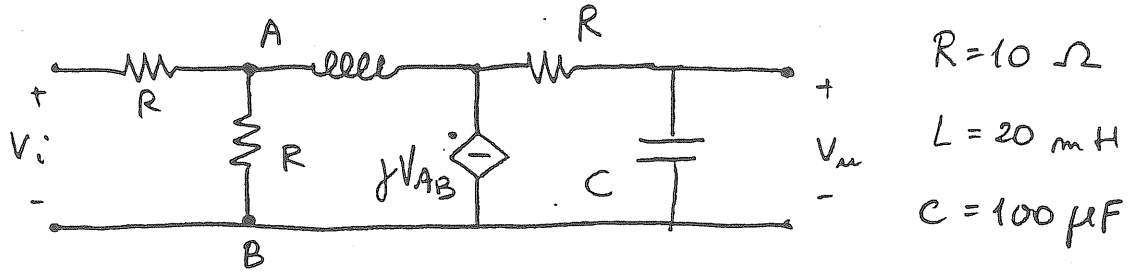


3. Per il doppio bipolo rappresentato in figura, determinare la matrice dei parametri  $h$  alla pulsazione  $\omega = 1000 \text{ rad/sec}$ .

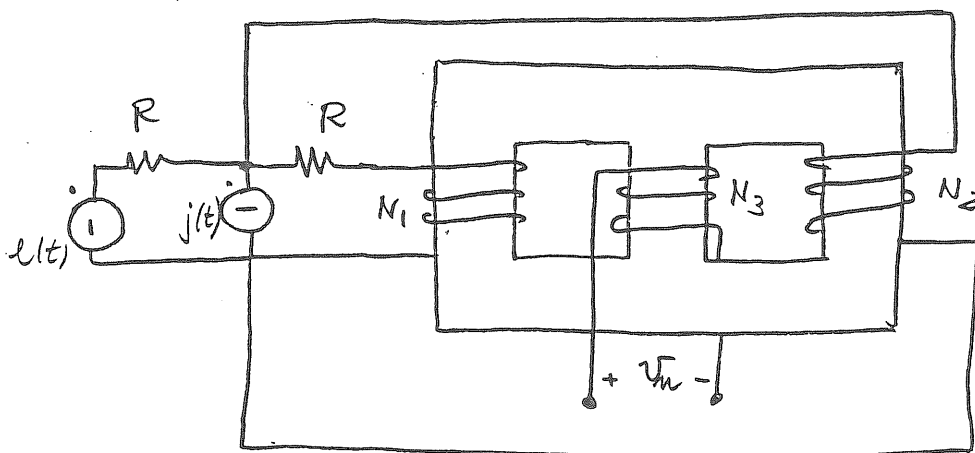


$R = 10 \Omega$ ;  
 $L = 20 \text{ mH}$ ;  
 $\alpha = 3$ ;  
 $C = 100 \mu\text{F}$

4. Per la rete di figura determinare la funzione di trasferimento  $V_u/V_i$ , studiare la stabilità al variare del parametro  $\gamma$  e tracciarne i diagrammi di Bode di modulo e fase della relativa risposta in frequenza.



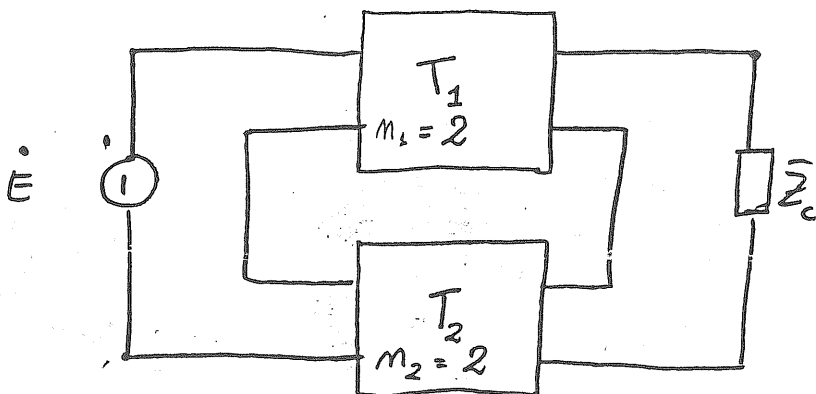
5. Considerando in condizioni di regime la rete di figura determinare la tensione  $V_u(t)$  ai morsetti dell'avvolgimento composto da  $N_3$  spire.



$\mu = 2000$ ;  $R = 10 \Omega$   
 $S = 9 \text{ cm}^2$ ,  $l = 5 \text{ cm}$   
 $u(t) = 100 \sin 314 t$   
 $i(t) = 2 \cos(314 t + \frac{\pi}{6})$   
 $N_1 = 50$ ;  $N_2 = 100$   
 $N_3 = 150$

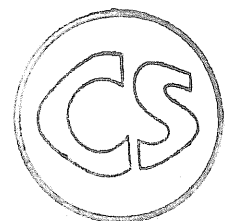
6. Con riferimento al sistema in figura, trovare la potenza attiva sul carico e determinare come si ripartisce tale potenza tra i due trasformatori.

TRASFORMATORE 1	TRASFORMATORE 2
Prova a vuoto:	Prova a vuoto:
$V_{10} = 400 \text{ V}$ ; $I_{10} = 2.5 \text{ A}$ $P_{10} = 250 \text{ W}$	$V_{10} = 420 \text{ V}$ ; $I_{10} = 3.5 \text{ A}$ $P_{10} = 300 \text{ W}$
Prova in corto circuito:	Prova in corto circuito:
$V_{1cc} = 150 \text{ V}$ ; $I_{1cc} = 18 \text{ A}$ $P_{1cc} = 2200 \text{ W}$	$V_{1cc} = 110 \text{ V}$ ; $I_{1cc} = 14 \text{ A}$ $P_{1cc} = 1100 \text{ W}$ ;



$Z_c = 10 + j4$

$E = 400 \text{ V}$

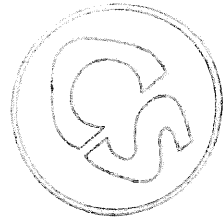
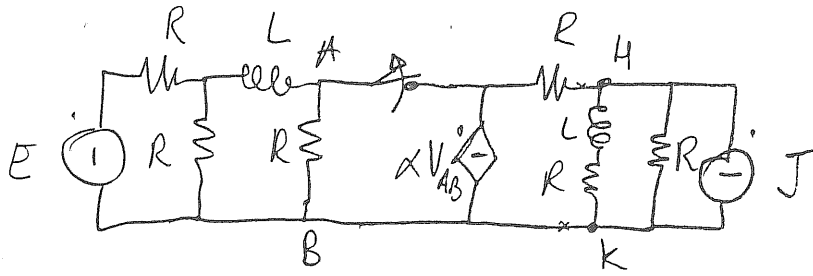


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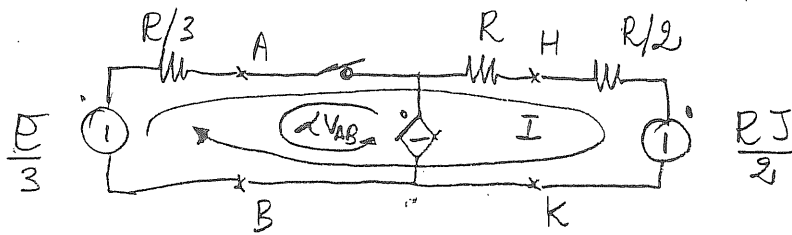
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①

Per  $t < \infty$ 

Appl. Thevenin tra AB e H-K



$$E_{th} = \frac{E}{\frac{3}{R} \cdot \frac{R}{2}} = \frac{E}{3}$$

$$R_{th} = \frac{R}{3}$$

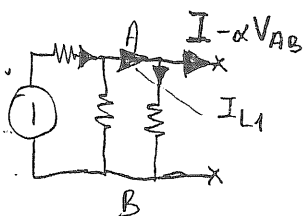
$$\left\{ \begin{aligned} \frac{E}{3} - \frac{RJ}{2} &= \left[ \frac{R}{3} + R + \frac{R}{2} \right] I - \frac{R}{3} \alpha V_{AB} \end{aligned} \right.$$

$$\left\{ \begin{aligned} V_{AB} &= \frac{E}{3} - \frac{R}{3} I + \frac{R}{3} \alpha V_{AB} \Rightarrow V_{AB} \left( 1 - \frac{\alpha R}{3} \right) = \frac{E}{3} - \frac{R}{3} I \Rightarrow V_{AB} = \frac{1}{2} \left( \frac{E - RI}{3 - \alpha} \right) \end{aligned} \right.$$

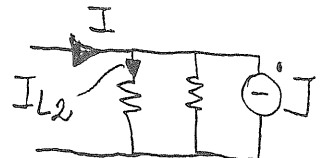
$$\frac{E}{3} - \frac{RJ}{2} = \frac{11}{6} R I - \frac{\alpha R E}{3} \left( \frac{1}{3 - \alpha R} \right) + \frac{\alpha R^2 I}{3(3 - \alpha R)}$$

$$\frac{E}{3} - \frac{RJ}{2} + \frac{\alpha R}{3} \left( \frac{E}{3 - \alpha R} \right) = I \left[ \frac{11}{6} R + \frac{\alpha R^2}{3(3 - \alpha R)} \right]$$

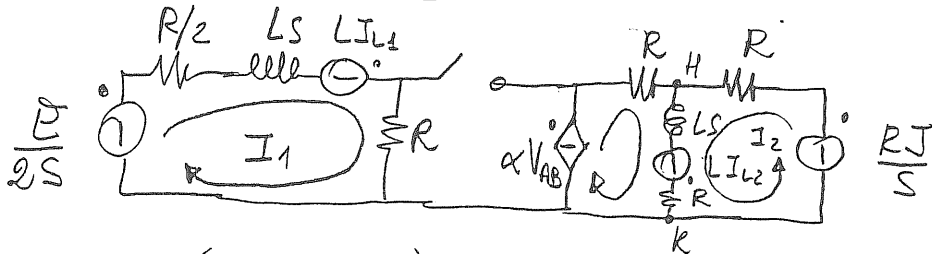
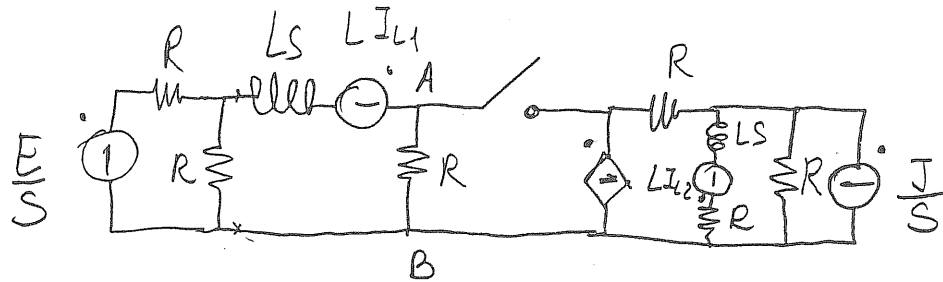
$$I = \frac{\frac{E}{3} - \frac{RJ}{2} + \frac{\alpha R E}{3(3 - \alpha R)}}{\frac{11R}{6} + \frac{\alpha R^2}{3(3 - \alpha R)}} = -3.8776 \text{ A} ; V_{AB} = -8.1633 \text{ V}$$



$$I_{L1} = (I - \alpha V_{AB}) + \frac{V_{AB}}{R} = 11.63 \text{ A}$$



$$I_{L2} = \frac{I}{2} + \frac{J}{2} = 3.0612 \text{ A}$$



$$I_1 = \frac{\left(\frac{E}{2S} - LI_{L1}\right)}{\frac{3}{2}R + LS} \Rightarrow V_{AB} = RI_1 = \frac{R(E - 2LI_{L1}S)}{S(3R + 2LS)}$$

$$\frac{RJ}{S} + LI_{L2} = (2R + LS)I_2 + (R + LS)\alpha \frac{R(E - 2LI_{L1}S)}{S(3R + 2LS)}$$

$$I_2 = \frac{\frac{RJ + LI_{L2}S}{S} - \frac{\alpha R(R + LS)(E - 2LI_{L1}S)}{S(3R + 2LS)}}{(2R + LS)}$$

$$I_2 = \frac{(RJ + LI_{L2}S)(3R + 2LS) - \alpha R(R + LS)(E - 2LI_{L1}S)}{S(2R + LS)(3R + 2LS)}$$

$$V_{HK} = \frac{RJ}{S} - RI_2 = \frac{RJ(2R + LS)(3R + 2LS) - R(RJ + LI_{L2}S)(3R + 2LS) + \alpha R^2(R + LS)(E - 2LI_{L1}S)}{S(2R + LS)(3R + 2LS)}$$

$$V_{HK} = \frac{-0.4514S^2 - 224.5S + 23 \times 10^4}{S(20 + 10^3S)(30 + 2 \times 10^3S)} = \frac{-0.4514}{10^3 \cdot 2 \cdot 10^3} \cdot \frac{(S + 1.0045 \times 10^3)(S - 507.21)}{S(S + 2000)(S + 1500)} = -2.25 \times 10^3$$

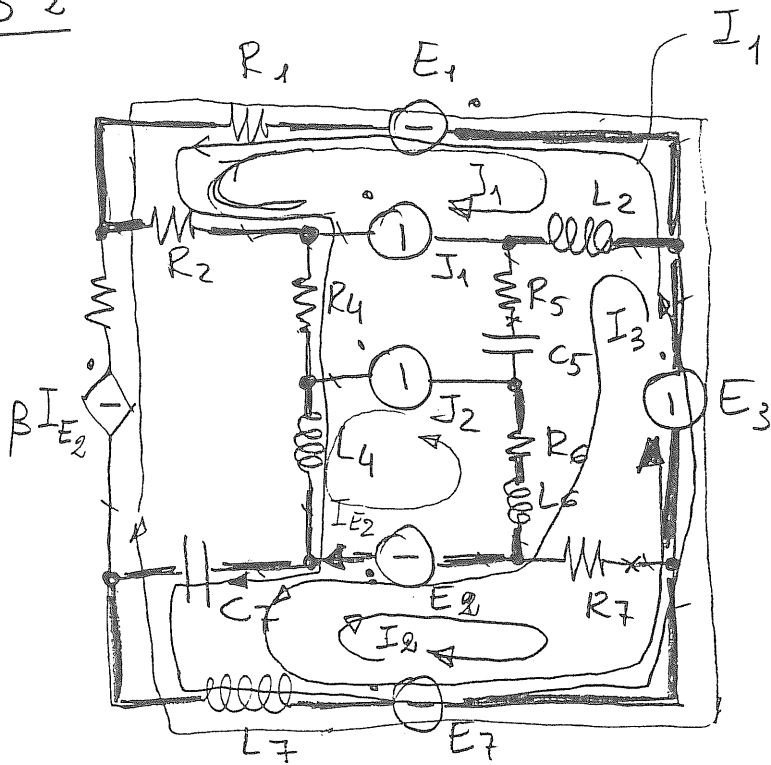
$$V_{HK} = K \left[ \frac{A}{S} + \frac{B}{S + 2000} + \frac{C}{S + 1500} \right]$$

$$A = V_{HK} \cdot S \Big|_{S=0} = -0.1698; \quad B = V_{HK}(S + 2000) \Big|_{S=-2000} = 2.4859; \quad C = V_{HK}(S + 1500) \Big|_{S=-1500} = -1.3261$$

$$V_{HK} = K \left[ A + B e^{-2000t} + C e^{-1500t} \right] u(t)$$

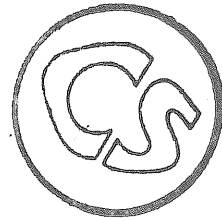
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$$M_{eq} = n - (n-1) - M_{pc}$$

$$15 - 9 - 3 = \underline{\underline{3}}_{eq}$$



$$-\dot{E}_1 - \dot{E}_7 + \dot{E}_3 = \left( R_1 + R_2 + R_4 + j\omega L_4 + \frac{1}{j\omega C_7} + j\omega L_7 \right) \dot{I}_1 - (R_1 + R_2) \dot{I}_1 - (R_1 + j\omega L_7) \beta \dot{I}_{E2} + j\omega L_4 \dot{I}_2 - \left( j\omega L_7 + \frac{1}{j\omega C_7} \right) \dot{I}_2 + \left( j\omega L_7 + \frac{1}{j\omega C_7} \right) \dot{I}_3$$

$$-\dot{E}_2 + \dot{E}_7 = \left( R_7 + j\omega L_7 + \frac{1}{j\omega C_7} \right) \dot{I}_2 + \left( j\omega L_7 + \frac{1}{j\omega C_7} \right) \dot{I}_3 - \left( j\omega L_7 + \frac{1}{j\omega C_7} \right) \dot{I}_1 + j\omega L_7 \beta \dot{I}_{E2}$$

$$\dot{E}_2 + \dot{E}_3 - \dot{E}_7 = \left( j\omega L_2 + R_5 + \frac{1}{j\omega C_5} + R_6 + j\omega L_6 + \frac{1}{j\omega C_7} + j\omega L_7 \right) \dot{I}_3 + \left( j\omega L_7 + \frac{1}{j\omega C_7} \right) \dot{I}_2 + \left( j\omega L_7 + \frac{1}{j\omega C_7} \right) \dot{I}_1 + j\omega L_7 \beta \dot{I}_{E2} - (R_6 + j\omega L_6) \dot{I}_2 + j\omega L_2 \dot{I}_1$$

$$\dot{I}_{E2} = \dot{I}_3 - \dot{I}_2 - \dot{I}_1$$

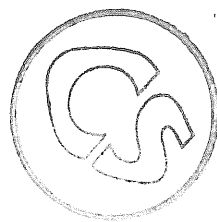
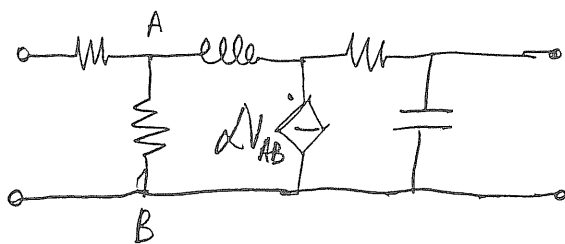

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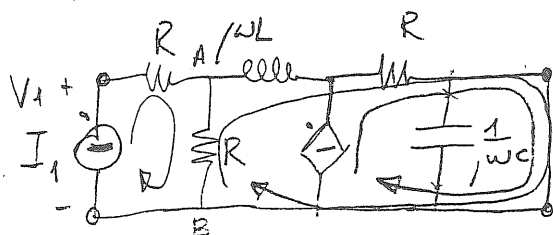
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①



$$\begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases} \quad \begin{aligned} h_{11} &= \left. \frac{V_1}{I_1} \right|_{V_2=0}; & h_{21} &= \left. \frac{I_2}{I_1} \right|_{V_2=0} \\ h_{12} &= \left. \frac{V_1}{V_2} \right|_{I_1=0}; & h_{22} &= \left. \frac{I_2}{V_2} \right|_{I_1=0} \end{aligned}$$

 $h_{11}$ 

$$\begin{cases} 0 = (2R + j\omega L) I_x + R \alpha V_{AB} - R I_1 \\ V_{AB} = R I_1 - R I_x \end{cases}$$

$$0 = (2R + j\omega L) I_x + \alpha R^2 I_1 - \alpha R^2 I_x - R I_1$$

$$I_x = I_1 \frac{R(\alpha R - 1)}{[R(2 - \alpha R) + j\omega L]}$$

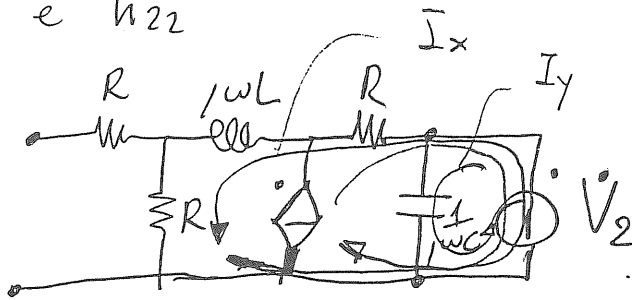
$$V_1 = 2R I_1 + R I_x = \left\{ 2R - \frac{R^2(\alpha R - 1)}{[R(2 - \alpha R) + j\omega L]} \right\} I_1$$

$$\boxed{h_{11}} = \frac{V_1}{I_1} = \frac{2R^2 - \frac{3}{2}\alpha R^3 + 2j\omega L R - \alpha R^3 + R^2}{[R(2 - \alpha R) + j\omega L]} = \boxed{30.30 + j0.736} \, \Omega$$

$$\dot{I}_2 = -I_x - \alpha V_{AB} = -\dot{I}_1 \left[ \frac{R(\alpha R - 1)}{[R(2 - \alpha R) + j\omega L]} \right] - \alpha R \dot{I}_1 + \frac{\alpha R (R(\alpha R - 1))}{(R(2 - \alpha R) + j\omega L)} \dot{I}_1$$

$$\boxed{h_{21}} = \frac{\dot{I}_2}{\dot{I}_1} = \boxed{-59.88 - j2.134}$$

$h_{12}$  e  $h_{22}$



$$\begin{cases} \dot{V}_2 = (2R + j\omega L) \dot{I}_x - \alpha \dot{V}_{AB} R \\ \dot{V}_2 = \frac{\dot{I}_y}{j\omega C} \Rightarrow \dot{I}_y = \frac{\dot{V}_2}{j\omega C} \\ \dot{V}_{AB} = R \dot{I}_x \end{cases}$$

$$\dot{V}_2 = (2R + j\omega L - \alpha R^2) \dot{I}_x \Rightarrow \dot{I}_x = \frac{\dot{V}_2}{2R + j\omega L - \alpha R^2}$$

$$\dot{V}_1 = \dot{V}_{AB} = R \dot{I}_x = \frac{R \dot{V}_2}{R(2 - \alpha R) + j\omega L}$$

$$\boxed{h_{12}} = \frac{\dot{V}_1}{\dot{V}_2} = \frac{R}{R(2 - \alpha R) + j\omega L} = \boxed{-0.0355 - j0.0025}$$

$$\dot{I}_2 = \dot{I}_x + \dot{I}_y - \alpha \dot{V}_{AB} = \frac{\dot{V}_2}{R(2 - \alpha R) + j\omega L} + \frac{\dot{V}_2}{j\omega C} - \frac{\alpha R \dot{V}_2}{R(2 - \alpha R) + j\omega L}$$

$$\boxed{h_{22}} = \frac{\dot{I}_2}{\dot{V}_2} = \frac{1 - \alpha R}{R(2 - \alpha R) + j\omega L} + \frac{1}{j\omega C} = \boxed{0.103 - j8.9926} \Omega^{-1}$$

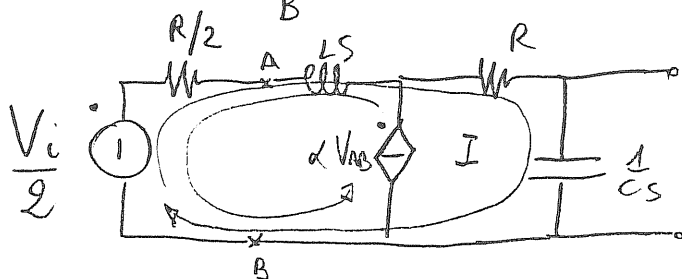
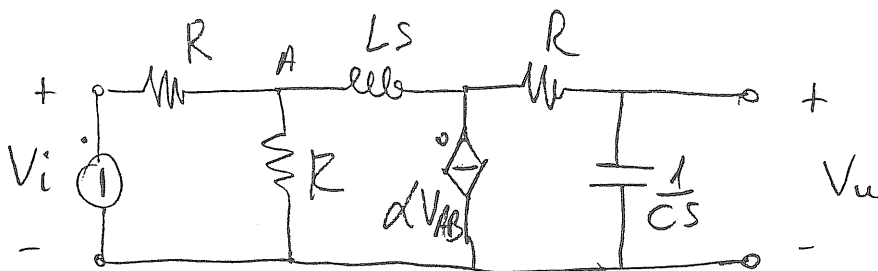


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6S 4

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①



$$\begin{cases} \frac{V_i}{2} = \left( \frac{3R}{2} + LS + \frac{1}{CS} \right) I - \alpha V_{AB} \left( \frac{R}{2} + LS \right) \\ V_{AB} = \frac{V_i}{2} - \frac{R}{2} I + \frac{R}{2} \alpha V_{AB} \Rightarrow V_{AB} \left( \frac{2-\alpha R}{2} \right) = \frac{V_i - RI}{2} \end{cases}$$

$$V_{AB} = \frac{V_i - RI}{2 - \alpha R} = \frac{V_i}{2 - \alpha R} - \frac{RI}{2 - \alpha R}$$

$$I \left[ \frac{1}{2} + \frac{\alpha(R+2LS)}{2(2-\alpha R)} \right] = \left[ \frac{3RCS + 2LCS^2 + 2}{2CS} + \frac{\alpha R}{2 - \alpha R} \cdot \left( \frac{R + 2LS}{2} \right) \right] I$$

$$\left[ \frac{2 - \alpha R + \alpha R + 2\alpha LS}{2(2 - \alpha R)} \right] = \left[ \frac{(2LCS^2 + 3RCS + 2)(2 - \alpha R) + \alpha R(R + 2LS)CS}{2CS(2 - \alpha R)} \right] I$$

$$I = V_i \frac{(2 + 2\alpha LS)CS}{(2LCS^2 + 3RCS + 2)(2 - \alpha R) + \alpha RCS(R + 2LS)}$$

$$\frac{V_u}{V_i} = \frac{I}{CS} = \cancel{V_i} \frac{(2 + 2\alpha LS)CS}{CS \left[ (2LCS^2 + 3RCS + 2)(2 - \alpha R) + \alpha RCS(R + 2LS) \right]}$$

$$4LCS^2 + 6RCS + 4 - 2\alpha RLCS^2 - \frac{2}{3}\alpha R^2CS - 2\alpha R + \alpha R^2CS + 2\alpha RLCS^2$$

$$\left[ 4LCS^2 + 2RC(3 - \alpha R)S + 2(2 - \alpha R) \right]$$

65 u

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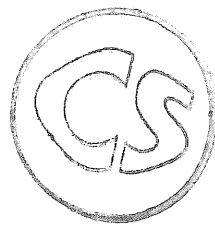
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(2)

Discussione stabilità:

$$3 - \alpha R \geq 0 \Rightarrow \alpha \leq \frac{3}{R} = 0.3$$

$$2 - \alpha R \geq 0 \Rightarrow \alpha \leq \frac{2}{R} = 0.2$$



Per  $\alpha > 0.2$  instabile

Per  $\alpha = 0.2$  marginalm. stabile

Per  $\alpha < 0.2$  sint. stabile

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$$\alpha = 0 \quad W = \frac{2}{4LCs^2 + 6RCs + 4} \Rightarrow \frac{1}{24} \cdot \frac{1}{1 + \frac{3RC}{42}s + LCs^2}$$

$$W(\omega) = \frac{1}{2} \cdot \frac{1}{1 + \frac{3}{2}RC \cdot j\omega + LC\omega^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}; \quad \zeta = \frac{3}{4}RC\sqrt{LC} \quad \left[ \quad W(\omega) = \frac{1}{2} \cdot \frac{1}{1 + 2\zeta \frac{\omega}{\omega_0} + \frac{\omega^2}{\omega_0^2}} \right]$$

$$W(\omega) = \frac{1}{2} \cdot \frac{1}{\left(1 - \frac{j\omega}{p_1}\right)\left(1 - \frac{j\omega}{p_2}\right)}$$

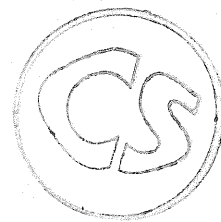
$$p_1 = -375 + j599.48$$

$$p_2 = -375 - j599.48$$

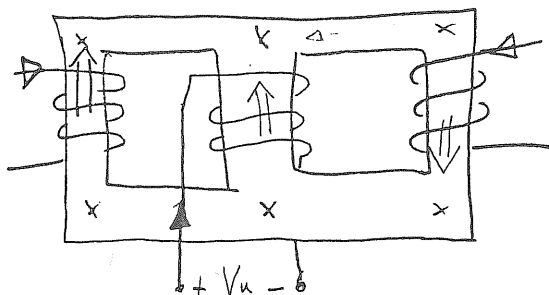
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OS 5

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(1)



$$R_{V1} = [3R // R] + 3R = \frac{15R}{4}$$

$$R_{V2} = R_{V1} = \frac{15R}{4}$$

$$R = \frac{l}{\mu S} = 2.21 \times 10^4 \text{ H}^{-1}$$

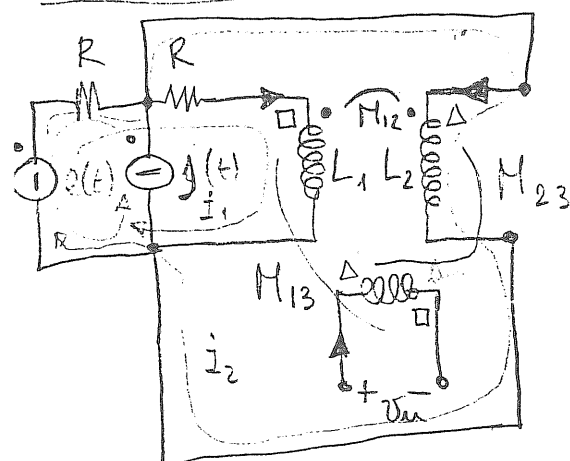
$$L_1 = \frac{N_1^2}{R_{V1}} \approx 30 \text{ mH}$$

$$L_2 = \frac{N_2^2}{R_{V2}} \approx 120.6 \text{ mH}$$

$$M_{12} = \frac{N_1 N_2}{\frac{15R}{4}} \cdot \frac{3R}{4R} = 15.1 \text{ mH} \oplus$$

$$M_{13} = \frac{N_1 N_3}{\frac{45R}{4}} \cdot \frac{3R}{4R} = 67.9 \text{ mH} \ominus$$

$$M_{23} = \frac{N_2 N_3}{\frac{45R}{4}} \cdot \frac{3R}{4R} = 135.7 \text{ mH} \oplus$$



$$E = 100 \text{ V}$$

j

$$J = 2 e^{j(\frac{\pi}{6} + \frac{\pi}{2})} = -1 + j1.73 \text{ A}$$

$$E = (2R + j\omega L_1) I_1 - RJ + j\omega M_{12} I_2 + RI_2 \Rightarrow I_1 = 3.80 - j0.428 \text{ A}$$

$$E = (R + j\omega L_2) I_2 - RJ + j\omega M_{12} I_1 + RI_1 \Rightarrow I_2 = 0.41 - j1.2 \text{ A}$$

$$V_u = \frac{2}{3} \cdot \frac{E}{2} - j\omega M_{13} I_1 + j\omega M_{23} I_2 = 42.38 - j63.40 \text{ V}$$

$$V_u(t) = 72.26 \sin(314t - 0.98) \text{ V}$$

$$\phi_1 = \frac{N_1 I_1}{R_{V1}} \Rightarrow \phi_{13} = \phi_1 \cdot \frac{3}{4}$$

$$\phi_2 = \frac{N_2 I_2}{R_{V2}} \Rightarrow \phi_{23} = \phi_2 \cdot \frac{3}{4}$$

VERIFICA CON IL FLUSSO

$$\phi_3 = \phi_{23} - \phi_{13} = -0.0013 - j0.0009 \text{ Wb}$$

$$V_u = \frac{2}{3} \cdot \frac{E}{2} - j\omega M_{13} I_1 + j\omega M_{23} I_2 = 42.38 - j63.40 \text{ V}$$

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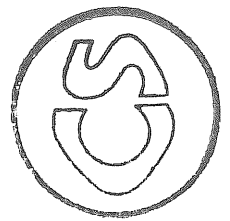
Es. 6

(Supprimi: trascurare l'impedenza di magnetizzazione  $\bar{Z}_0$ )

Calcolo delle impedenze  $\bar{Z}_0$  e  $\bar{Z}_{cc}$

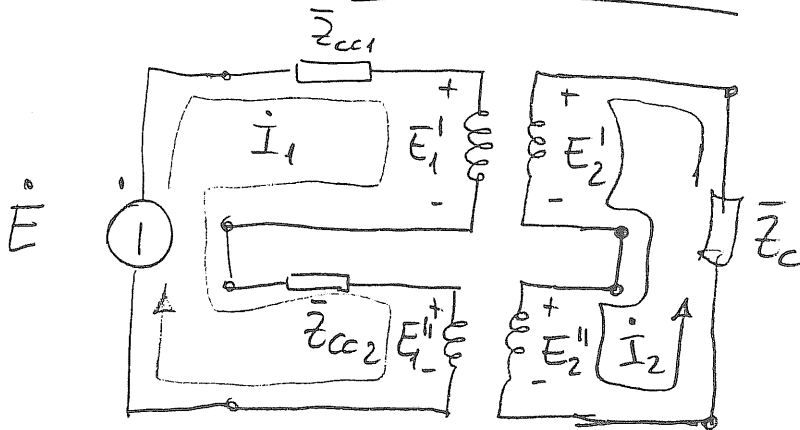
(T1)  $R_{01} = \frac{V_{10}^2}{P_{10}} ; |Y_0| = \frac{I_{10}}{V_{10}} ; B_{10} = \sqrt{Y_{10}^2 - G_{10}^2} =$

$X_{10} = \frac{1}{B_{10}} ; \bar{Z}_{01} = \frac{R_{10} + jX_{10}}{R_{10} + jX_{10}} = 40 + j155 \Omega$



$\begin{cases} P_{1cc} = V_{1cc} I_{1cc} \cos \varphi \\ |Z_{1cc}| = \frac{V_{1cc}}{I_{1cc}} \end{cases} \Rightarrow \cos \varphi = \frac{P_{1cc}}{V_{1cc} I_{1cc}} = 0.8148$   
 $\bar{Z}_{cc1} = |Z_{1cc}| [\cos \varphi + j \sin \varphi] = 6.8 + j4.8 \Omega$

(T2)  $\bar{Z}_{02} = 24.5 + j117.5 \Omega ; \bar{Z}_{cc2} = 5.61 + j5.49 \Omega$



$$\begin{cases} 1 & \dot{E} = (\bar{Z}_{cc1} + \bar{Z}_{cc2}) \dot{I}_1 + \dot{E}_1' + \dot{E}_1'' \\ 2 & 0 = \dot{E}_2' + \dot{E}_2'' + \bar{Z}_c \dot{I}_2 \\ 3 & \dot{E}_1' = M_1 \dot{E}_2' \\ 4 & \dot{E}_1'' = M_2 \dot{E}_2'' \\ 5 & \dot{I}_1' = -\frac{1}{M_1} \dot{I}_2' \\ 6 & \dot{I}_1'' = -\frac{1}{M_2} \dot{I}_2'' \end{cases}$$

$M_1 = M_2 = 2 = M$

dalla 2, sostituendo la 3 e la 4:

$\dot{E}_1' + \dot{E}_1'' = -M \bar{Z}_c \dot{I}_2$

sostit. in 1:  $\Rightarrow \dot{E} = (\bar{Z}_{cc1} + \bar{Z}_{cc2}) \dot{I}_1 - M \bar{Z}_c \dot{I}_2$

tenendo conto della 5 e 6:  $\dot{E} = -(\bar{Z}_{cc1} + \bar{Z}_{cc2}) \frac{\dot{I}_2}{M} - M \bar{Z}_c \dot{I}_2 \Rightarrow$   
 $\dot{I}_2 = \frac{-M \dot{E}}{\bar{Z}_{cc1} + \bar{Z}_{cc2} + M^2 \bar{Z}_c} = -12.2 + j6.12 A$

$\bar{Z} = Z_T^2 = 106 + j0.711 kVA \cdot P = 186 kVA$