29/05/2013

$$X(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right) + B\,\mathrm{rect}\left(\frac{t-T/2}{T}\right)$$

A, B sono V.A. indip.

$$A \in \mathcal{U}[0,1] \quad , \quad B \in \mathcal{U}[0,2]$$

$$h(t) = \frac{1}{T}\,\mathrm{rect}\left(\frac{t}{T}\right)$$

·) Calcolare la $f_Z(z)$

$$Y(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right) \otimes \frac{1}{T}\,\mathrm{rect}\left(\frac{t}{T}\right) + \underbrace{B\,\mathrm{rect}\left(\frac{t-T/2}{T}\right)} \otimes \frac{1}{T}\,\mathrm{rect}\left(\frac{t}{T}\right)$$

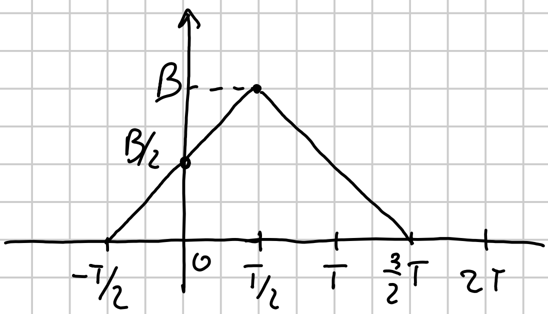
$$= A\,\frac{1}{\cancel{T}} \cdot \cancel{T}\left(1 - \frac{|t|}{T}\right)\mathrm{rect}\left(\frac{t}{2T}\right) +$$

$$+ B \cdot \frac{1}{\cancel{T}} \cdot \cancel{T}\left(1 - \frac{|t-T/2|}{T}\right)\mathrm{rect}\left(\frac{t-T/2}{2T}\right)$$

$$\left[B\,\mathrm{rect}\left(\frac{t}{T}\right) \otimes \delta(t - T/2)\right] \otimes \frac{1}{T}\,\mathrm{rect}\left(\frac{t}{T}\right)$$

$$\left[B\,\mathrm{rect}\left(\frac{t}{T}\right) \otimes \frac{1}{T}\,\mathrm{rect}\left(\frac{t}{T}\right)\right] \otimes \delta(t - T/2)$$

$$Z = Y(0) = A + \frac{B}{2}$$

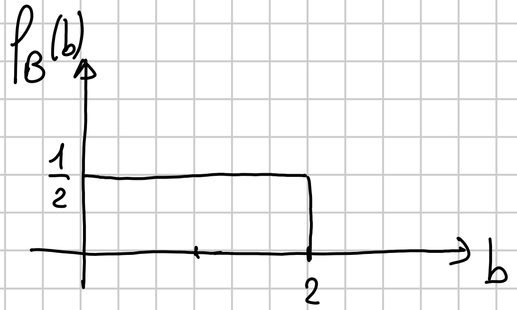


$$Z = A + \frac{B}{2}$$

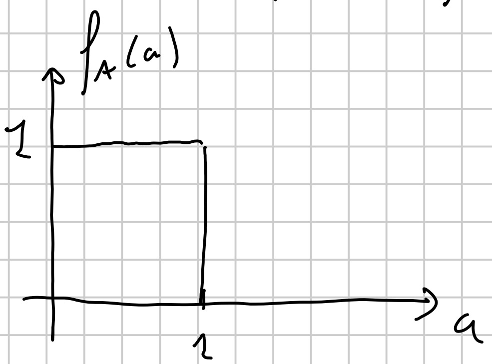
$$Z = X + Y$$, X, Y sono v.A. indipendenti

$$f_Z(z) = f_X(z) \otimes f_Y(z)$$

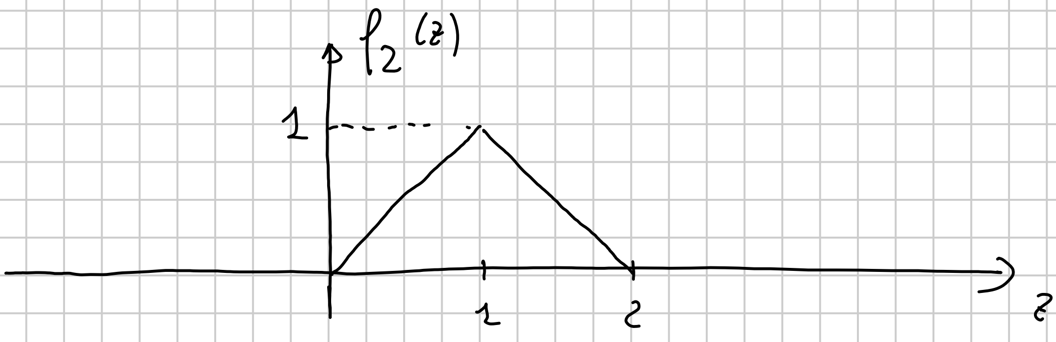
$$C = \frac{B}{2} \Rightarrow f_C(c) = \frac{f_B(b)}{|g'(b)|}\Bigg|_{b=g^{-1}(c)} = \frac{\cancel{\frac{1}{2}}\, rect\left(\frac{b-1}{2}\right)}{\cancel{\frac{1}{2}}}\Bigg|_{b=g^{-1}(c)}$$



$$f_C(c) = rect\left(\frac{2c-1}{2}\right) = rect\left(\frac{c-1/2}{1}\right) = f_A(a)$$



$$f_Z(z) = rect\left(\frac{z-1/2}{1}\right) \otimes rect\left(\frac{z-1/2}{1}\right) = \left(1 - \frac{|z-1|}{1}\right) rect\left(\frac{z-1}{2}\right)$$

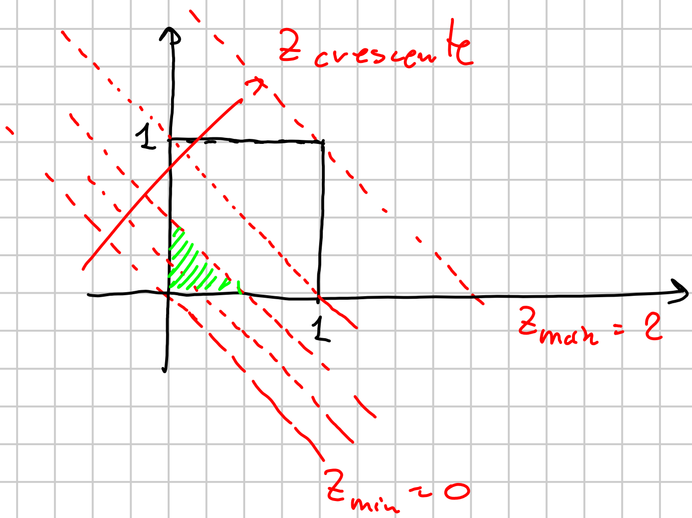


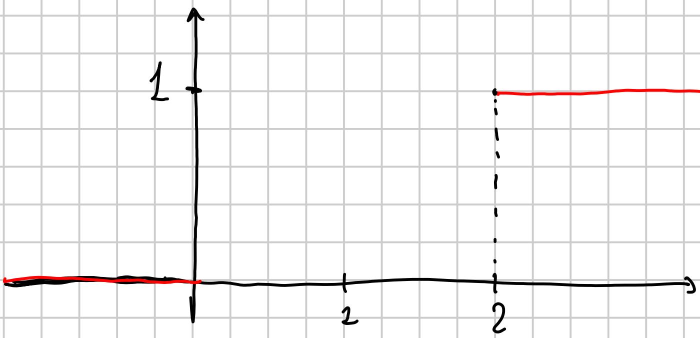
SOL. ALTERNATIVA

$$Z = A + \frac{B}{2}$$

$$F_Z(z) = P\{Z \le z\} = P\left\{A + \frac{B}{2} \le z\right\}$$

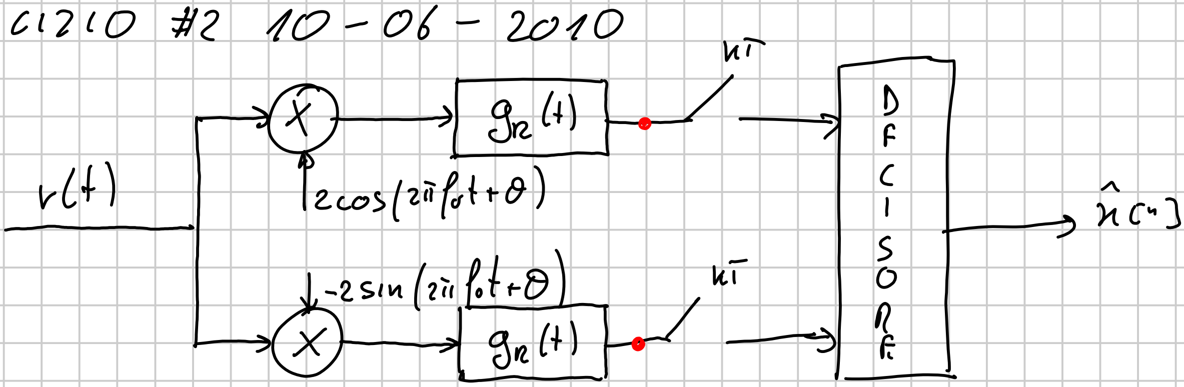
$$= P\{A + C \le z\}$$



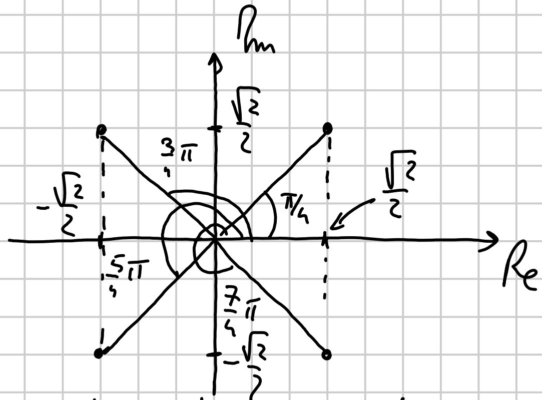


$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

ESERCIZIO #2 10-06-2010



$f_0 >> B$



4 - QPSK

4 - QAM

$$a_i \in A_s^{(c)} \left\{ \pm \frac{\sqrt{2}}{2} \right\}$$

$$b_i \in A_s^{(s)} \left\{ \pm \frac{\sqrt{2}}{2} \right\}$$

simboli ind. ed equiprob.

$$r(t) = \sum_i a_i \, g_T(t-iT) \cos(2\pi f_0 t) - \sum_i b_i \, g_T(t-iT) \sin(2\pi f_0 t) + w(t)$$

$w(t)$ Gaussiano e bianco nella banda del segnale

Ip:

1) $g_T(t) = \left(\frac{t + T/2}{T}\right) \mathrm{rect}\left(\frac{t}{T}\right)$

2) $c(t)$ e' ideale

3) $g_R(t) = -A \left(\frac{t - T/2}{T}\right) \mathrm{rect}\left(\frac{t}{T}\right)$

Domande

1) $E_S = E\left[\int_{-\infty}^{+\infty} s_i^2(t) \, dt \right]$

$$s_i(t) = a_i \, g_T(t-iT) \cos(2\pi f_0 t) - b_i \, g_T(t-iT) \sin(2\pi f_0 t)$$

$$E_s = E\Big[\int_{-\infty}^{+\infty} a_i^2 \, g_T^2(t-iT) \cos^2(2\pi f_0 t) +$$

$$+ b_i^2 \, g_T^2(t-iT) \sin^2(2\pi f_0 t) +$$

$$- 2 \, a_i b_i \, g_T^2(t-iT) \cos(2\pi f_0 t) \sin(2\pi f_0 t) \, dt \Big]$$

$$= E\left[a_i^2\right] \int_{-\infty}^{+\infty} g_T^2(t-iT)\left[\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0 t)\right] dt \; +$$

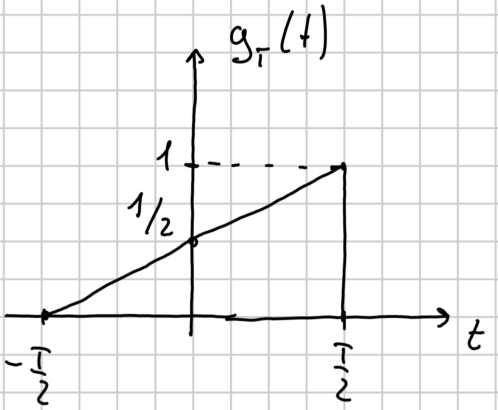
$$+ \; E\left[b_i^2\right] \int_{-\infty}^{+\infty} g_T^2(t-iT)\left[\frac{1}{2} - \frac{1}{2}\cos(4\pi f_0 t)\right] dt \; +$$

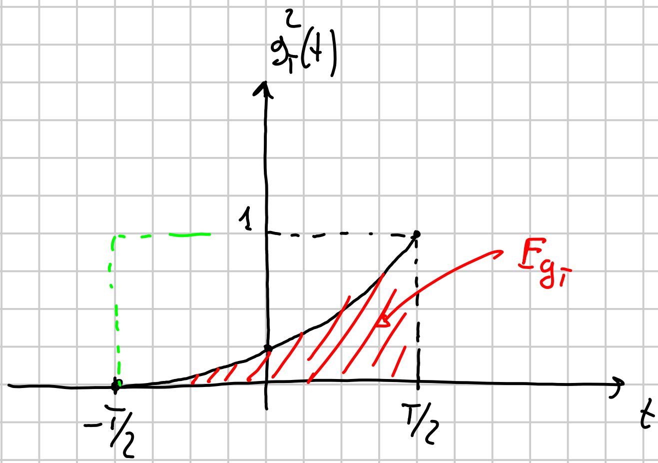
$$- \; 2\underbrace{E\left[a_i \cdot b_i\right]}_{=0} \int_{-\infty}^{+\infty} g_T^2(t-iT)\, \frac{1}{2}\sin(4\pi f_0 t)\, dt$$

$$E_S \simeq \frac{1}{2}\cdot\frac{1}{2} E_{g_T} + \frac{1}{2}\cdot\frac{1}{2} E_{g_T} = \frac{E_{g_T}}{2} = \boxed{\frac{T}{6}}$$

$$E\left[a_i \cdot b_i\right] = \underbrace{E\left[a_i\right]}_{=0} \underbrace{E\left[b_i\right]}_{=0} = 0$$

$$E_{g_T} = \int_{-\infty}^{+\infty} g_T^2(t)\, dt = \frac{T}{3}$$



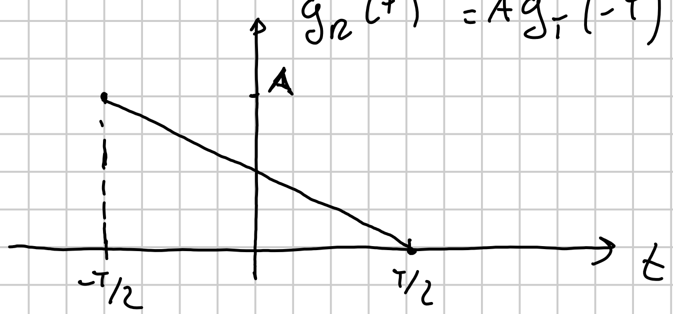


2) $$h(t) = g_T(t) \otimes c(t) \otimes g_R(t) = g_T(t) \otimes g_R(t)$$

$$g_R(t) = A\left(\frac{T/2 - t}{T}\right) \mathrm{rect}\left(\frac{t}{T}\right)$$

$$g_R(t) = A\, g_T(-t)$$

F.A. !!



$$h(0) = 1$$

$$h(t)\Big|_{t=0} = \int_{-\infty}^{+\infty} g_T(\tau)\, g_R(t-\tau)\, d\tau \Big|_{t=0} =$$

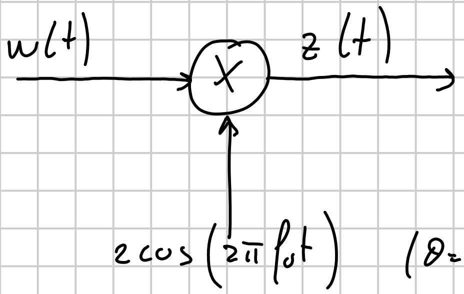
$$= A \int_{-\infty}^{+\infty} g_T(\tau)\, g_T(\tau - t)\, d\tau \Big|_{t=0} = A \int_{-\infty}^{+\infty} g_T^2(\tau)\, d\tau =$$

$$= A\, E_{g_T} = A \frac{T}{3} = 1 \Rightarrow \boxed{A = \frac{3}{T}}$$

3) $P_{n_M}^{(c)}$, $P_{n_M}^{(s)}$ nei casi $\theta = 0$, $\theta = \frac{\pi}{12}$

$$w(t) = w_c(t) \cos(2\pi f_0 t) - w_s(t) \sin(2\pi f_0 t)$$

$$S_{w_c}(f) = S_{w_s}(f) = N_0$$



$(\theta = 0)$

$$z_c(t) = \left[w_c(t) \cos(2\pi f_0 t) - w_s(t) \sin(2\pi f_0 t) \right] \cdot 2 \cdot \cos(2\pi f_0 t)$$

$$= w_c(t) \left[1 + \cos(4\pi f_0 t) \right] - w_s(t) \sin(4\pi f_0 t)$$

$$n_c(t) = z_c(t) \otimes g_R(t)$$

$$S_{n_c}(f) = N_0 \left| G_R(f) \right|^2$$

$$P_{n_c} = N_0 E_{g_R} = N_0 A^2 E_{g_T} = N_0 \frac{9}{T^2} \cdot \frac{T}{3} = \frac{3N_0}{T}$$

$$z_s(t) = \left[w_c(t)\cos(2\pi f_0 t) - w_s(t)\sin(2\pi f_0 t)\right]\left[-2\sin(2\pi f_0 t)\right] =$$

$$= -w_c(t)\sin(4\pi f_0 t) + w_s(t)\left[1 - \cos(4\pi f_0 t)\right]$$

$$n_s(t) = z_s(t) \otimes g_R(t)$$

$$S_{n_s}(f) = N_0 |G_R(f)|^2 \Rightarrow P_{n_s} = \frac{3N_0}{T}$$

CASO $\theta = \frac{\pi}{12}$

$$z_c(t) = \left[w_c(t)\cos(2\pi f_0 t) - w_s(t)\sin(2\pi f_0 t)\right] 2\cos(2\pi f_0 t + \theta)$$

$$= w_c(t)\left[\cos(4\pi f_0 t + \theta) + \cos\theta\right] +$$

$$- w_s(t)\left[\sin(4\pi f_0 t + \theta) + \sin(-\theta)\right]$$

$$\Rightarrow n_c(t) = z_c(t) \otimes g_R(t)$$

$$n_c(t) = \left[w_c(t)\cos\theta + w_s(t)\sin\theta\right] \otimes g_R(t)$$

$$= \cos\theta \, w_c(t) \otimes g_R(t) + \sin\theta \, w_s(t) \otimes g_R(t)$$

$$S_{n_c}(f) = \cos^2\theta \, N_0 |G_R(f)|^2 + \sin^2\theta \, N_0 |G_R(f)|^2$$

$$= N_0 |G_R(f)|^2 \Rightarrow P_{n_c} = \frac{3N_0}{T}$$

sul ramo in quadratura è lo stesso ...

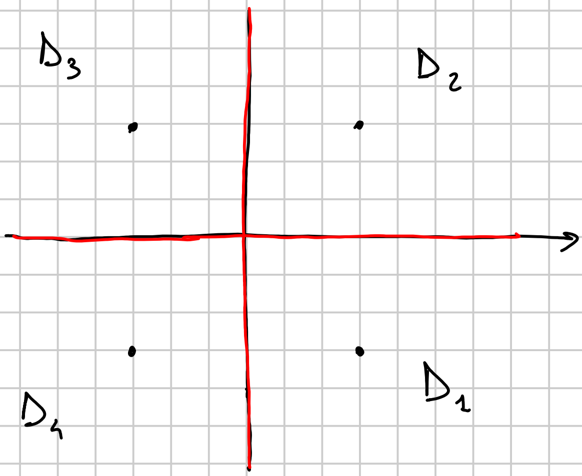
$$P_{n_s} = \frac{3N_0}{T}$$

Si deduce che la presenza di una fase nell' O.L. non cambia le statistiche di rumore.

4) $P_E^{QAM}(M)$ nelle ipotesi

·) $\theta = 0$

·) strategia di decisione sia la seguente



·) Assicuriamoci che:

·) ASSENZA DI CROSS-TALK

·) ASSENZA DI ISI

·) F.A. (GIA' NOTO)

$\Rightarrow$ 4-QAM $\Rightarrow$ DOPPIA 2-PAM

$$\Rightarrow P_E^{QAM}(4) \simeq 2\, P_E^{PAM}(2)$$

$$P_E^{QAM}(4) = P_E^{PAM}(M_c)\left(1 - P_E^{PAM}(M_s)\right) + P_E^{PAM}(M_s)\left(1 - P_E^{PAM}(M_c)\right) + $$
$$+ P_E^{PAM}(M_c)\, P_E^{PAM}(M_s) \qquad P_E \ll 1$$

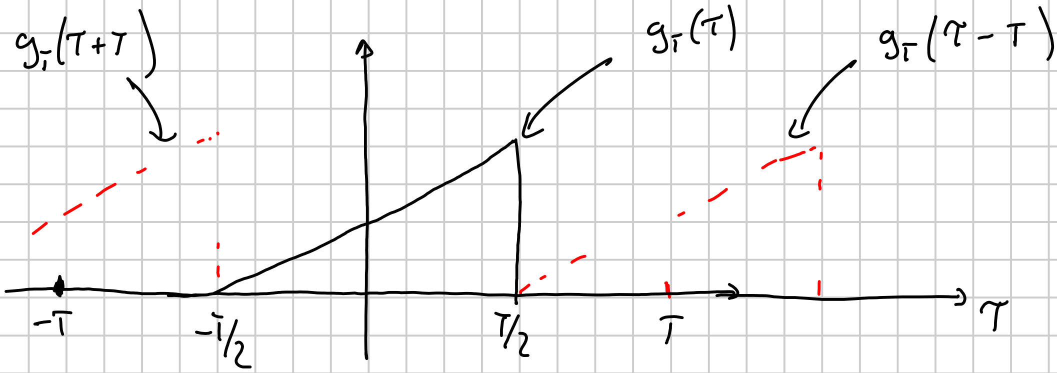
$\theta = 0 \Rightarrow$ ASSENZA DI CROSS-TALK

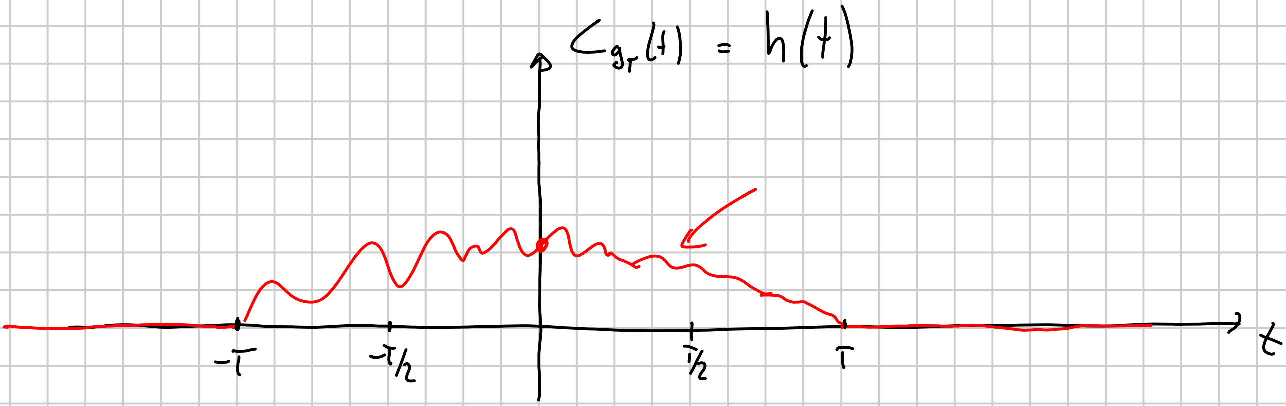
$\Rightarrow$ ASSENZA DI ISI

$$g_T(t) = \left(\frac{t + T/2}{T}\right) \mathrm{rect}\left(\frac{t}{T}\right)$$

$$h(t) = g_T(t) \otimes g_R(t) = A\, g_T(t) \otimes g_T(-t) =$$

$$= A\, C_{g_T}(t)$$





$$h[n] = h(nT) = \begin{cases} h(0) & n = 0 \\ 0 & \text{altrove} \end{cases} \Rightarrow \text{ASSENZA DI ISI}$$

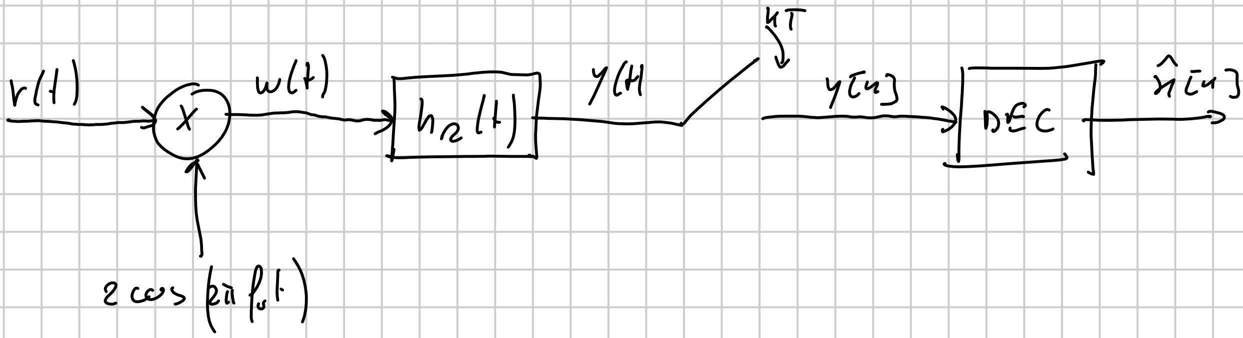
$$P_E^{PAM}(2) = P_E^{PAM}(b) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{SNR}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{T}{6N_0}}\right)$$

$$SNR_c = \frac{h^2(0) \cdot E\left[x^2[n]\right]}{\sigma_{n_c}^2} = SNR_s = \frac{1 \cdot 1/2}{\frac{3N_0}{T}} = \frac{T}{6N_0}$$

$$P_E^{QAM}(M) \simeq \mathrm{erfc}\left(\sqrt{\frac{T}{6N_0}}\right)$$

$$P_E^{QAM}(n) = 2 \cdot \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{T}{6N_0}}\right) \underbrace{\left(1 - \operatorname{erfc}\left(\sqrt{\frac{T}{6N_0}}\right)\right)}_{\simeq 1} + \frac{1}{4} \operatorname{erfc}^2\left(\sqrt{\frac{T}{6N_0}}\right)$$

ES 2 07/02/2013



$$r(t) = s(t) + n(t)$$

$$s(t) = \sum_k x[k]\, p(t - kT) \cos(2\pi f_0 t - \pi/3)$$

$x[k] \in A_s = \{-2, 1\}$ ind. equip.

$$p(t) = 2B \operatorname{sinc}(2Bt) + B \operatorname{sinc}\left(2B\left(t - \frac{1}{2B}\right)\right) + B \operatorname{sinc}\left(2B\left(t + \frac{1}{2B}\right)\right), \quad f_0 \gg B, \quad T = \frac{1}{B}$$

$c(t)$ ideale

$n(t)$ è bianco in banda con DSP $\frac{N_0}{2}$

$h_R(t)$ è passa-basso ideale di banda B

$\lambda = 0$

1) $E_s = E\left[\int_{-\infty}^{+\infty} x^2[n]\, p^2(t-nT) \cos^2(2\pi f_0 t - \pi/3)\, dt\right]$

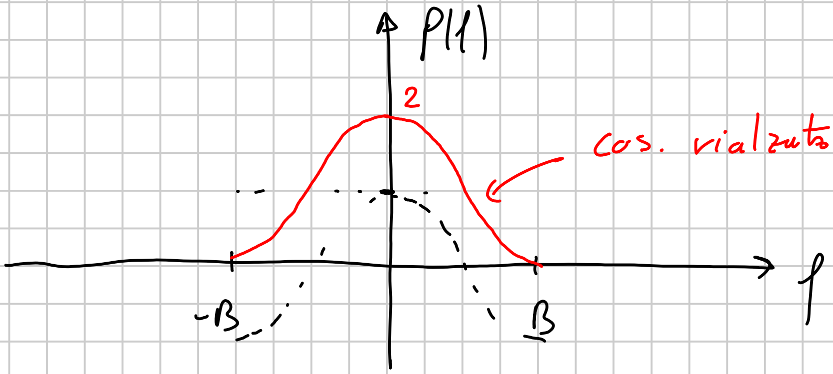
$$= E\left[x^2[n]\right] \int_{-\infty}^{+\infty} p^2(t-nT) \cos^2(2\pi f_0 t - \pi/3)\, dt$$

$$\simeq \left(\frac{1}{2}\cdot 4 + \frac{1}{2}\cdot 1\right) \frac{1}{2} E_p = \frac{1}{2}\cdot 3B = \frac{3}{2}B$$

$E_p = ?$

$$P(f) = \mathrm{rect}\left(\frac{f}{2B}\right) + \frac{1}{2}\mathrm{rect}\left(\frac{f}{2B}\right)\left[e^{-j\frac{2\pi f}{2B}} + e^{j2\pi f/2B}\right]$$

$$= \mathrm{rect}\left(\frac{f}{2B}\right)\left[1 + \cos\left(2\pi f/2B\right)\right]$$



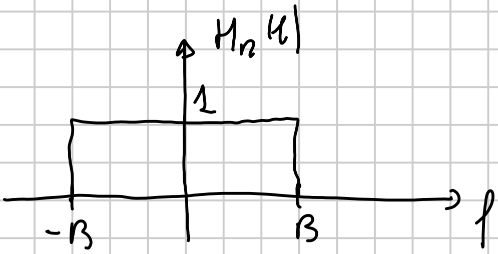
$$E_p = \int_{-\infty}^{+\infty} P^2(f)\, df$$

$$P^2(f) = \left[1 + \cos\left(2\pi f/2B\right)\right]^2 \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$= \left[1 + \left(\frac{1}{2} + \frac{1}{2}\cos\left(2\pi f/B\right)\right) + 2\cos\left(2\pi f/2B\right)\right] \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$E_p = \int_{-\infty}^{+\infty} P^2(f)\, df = 2B + B = 3B$$

2) $$P_{n_u} = \int_{-\infty}^{+\infty} N_0 \, |H_R(f)|^2 \, df = N_0 \, E_{h_R} = 2B N_0$$



3) $P_E(b)$

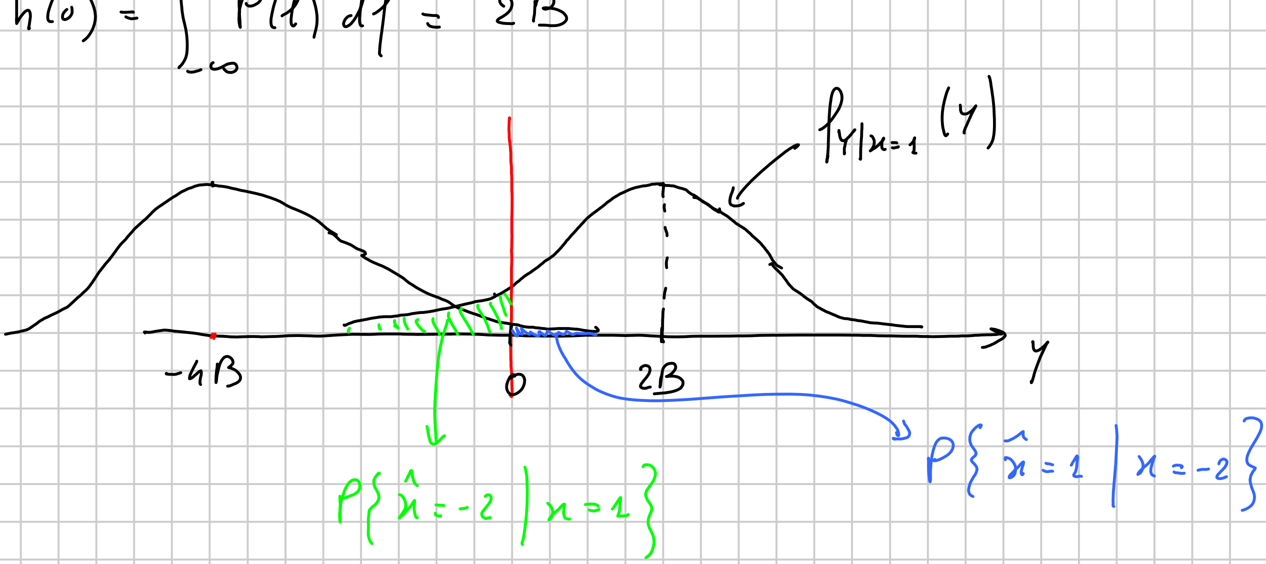
$H(f) = P(f)\, H_R(f) = P(f)$ $\Rightarrow$ coseno rialzato $\Rightarrow$ ASSENZA DI ISI

·) NON HO F.A.

$$P_E(2) = P\{\hat{x} = -2 \mid x = 1\} \underbrace{P\{x=1\}}_{1/2} + P\{\hat{x} = 1 \mid x = -2\} \underbrace{P\{x=-2\}}_{1/2}$$

$$y \mid x=1 \;=\; h(0)\cdot 1 + n_u \;=\; 2B + n_u$$

$$h(0) = \int_{-\infty}^{+\infty} P(f)\, df = 2B$$



$$P\{\hat{x}=-2 \mid x=1\} = Q\left(\frac{2B}{\sqrt{2BN_0}}\right) = Q\left(\sqrt{\frac{2B}{N_0}}\right)$$

$$P\{\hat{x}=1 \mid x=-2\} = Q\left(\frac{4B}{\sqrt{2BN_0}}\right) = Q\left(\sqrt{\frac{8B}{N_0}}\right)$$

$$P_E(b) = \frac{1}{2}Q\left(\sqrt{\frac{2B}{N_0}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{8B}{N_0}}\right)$$