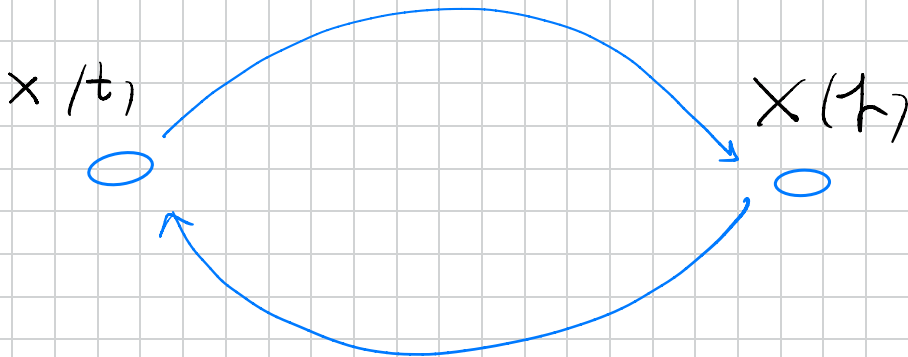


TRASFORMATA CONTINUA DI FOURIER

TCTF



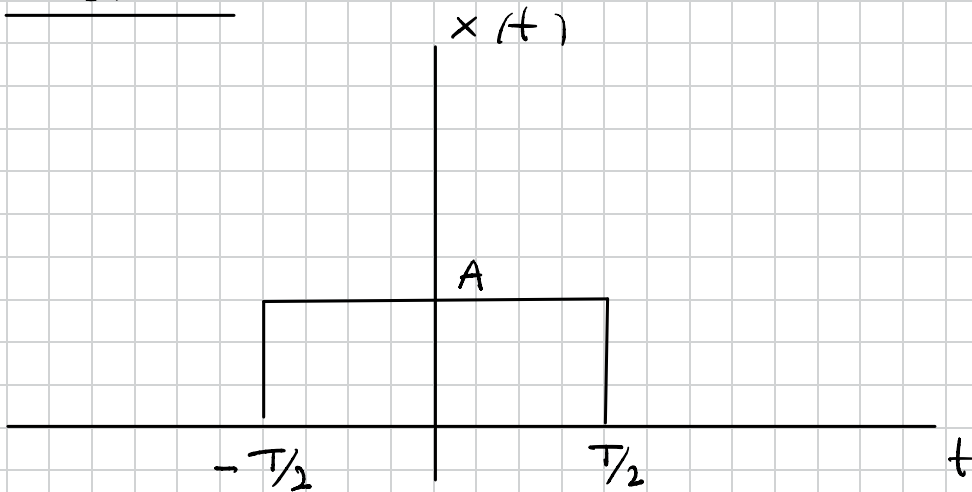
ATCF

$$X(f) \triangleq \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \quad \underline{\text{Analisi}}$$

$$x(t) \triangleq \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \quad \underline{\text{Sintesi}}$$

$$\boxed{x(t) \Leftrightarrow X(f)}$$

ESEMPIO



$$x(t) = \text{rect}\left(\frac{t}{T}\right) \Rightarrow X(f),$$

$$X(f) \triangleq \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

$$= \int_{-T/2}^{T/2} A e^{-j2\pi f t} dt$$

$$= A \frac{1}{-j2\pi f} e^{-j2\pi f t} \bigg|_{-T/2}^{T/2}$$

$$= \frac{A}{-j2\pi f} \left(e^{-j2\pi f \frac{T}{2}} - e^{+j2\pi f \frac{T}{2}} \right)$$

$$= \frac{A}{-j2\pi f} \underbrace{\left(e^{-j\pi f T} - e^{j\pi f T} \right)}_{-2j \sin(\pi f T)}$$

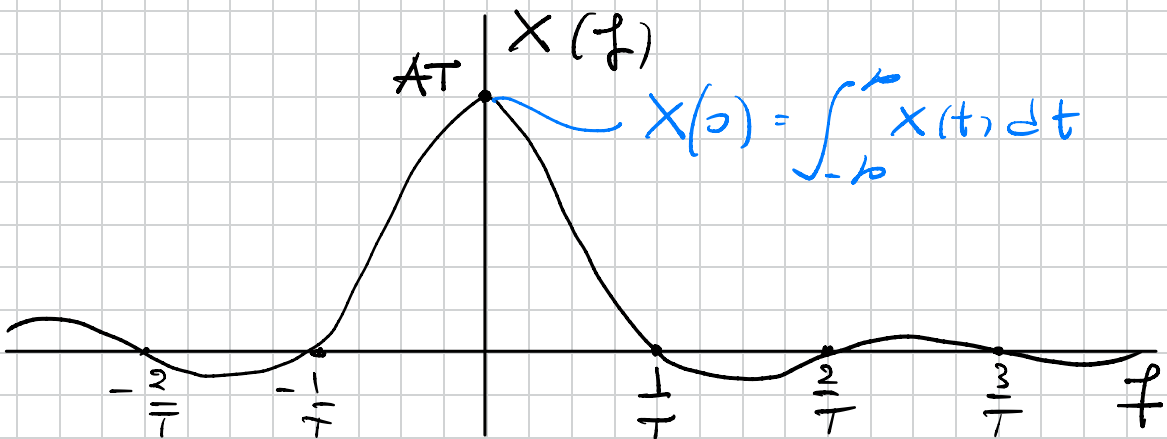
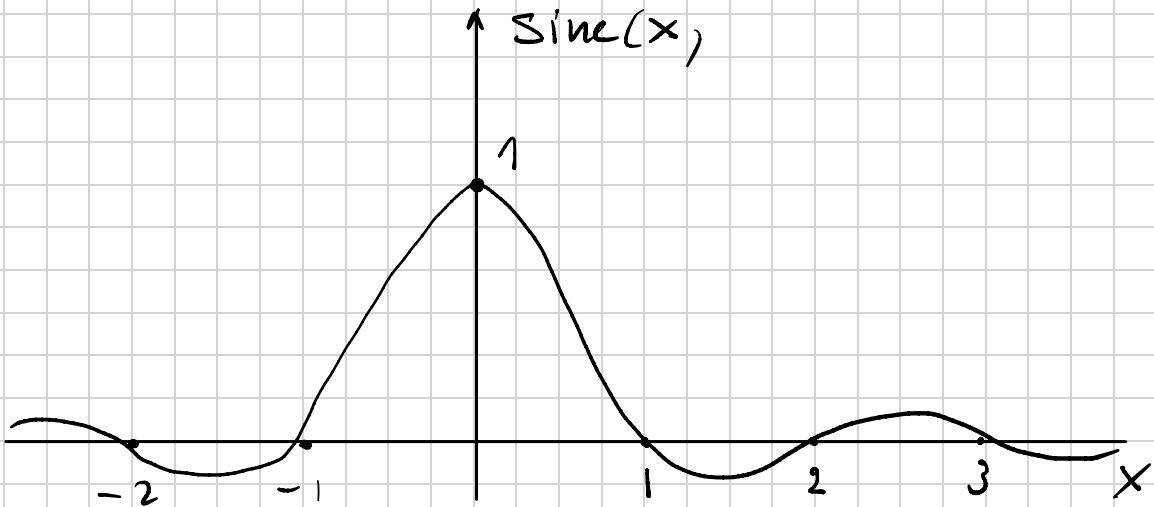
$$\begin{array}{l} e^{j\vartheta} + e^{-j\vartheta} = 2 \cos \vartheta \\ e^{j\vartheta} - e^{-j\vartheta} = 2j \sin \vartheta \end{array}$$

$$= \frac{A T}{\pi f T} \sin(\pi f T)$$

$$\text{sinc}(x) \triangleq \frac{\sin(\pi x)}{\pi x} \quad \text{sinc}(0) = 1$$

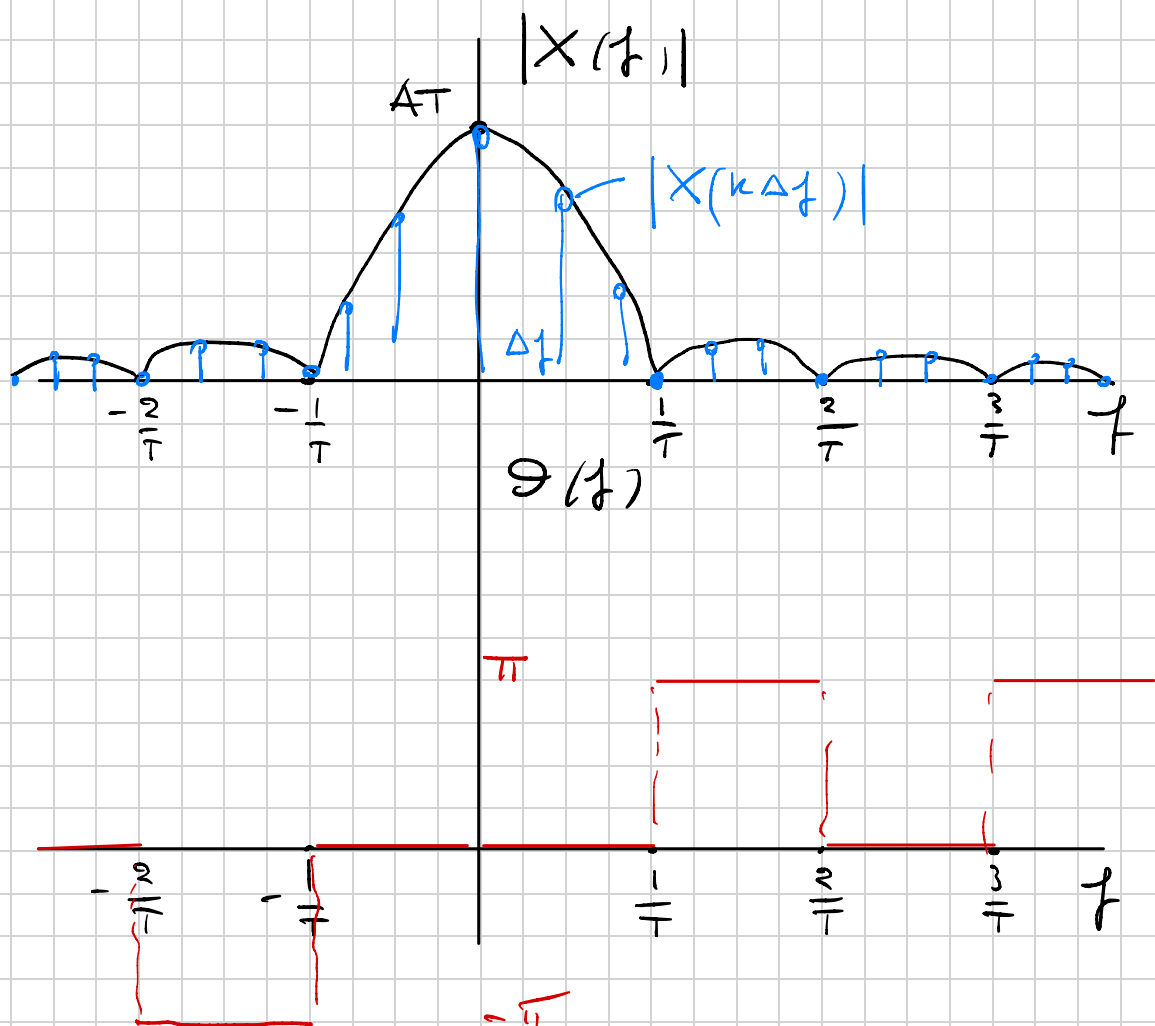
$$= A T \operatorname{sinc}\left(\frac{f}{T}\right)$$

$$A \operatorname{rect}\left(\frac{t}{T}\right) \Rightarrow A T \operatorname{sinc}\left(\frac{f}{T}\right)$$



$$X(f) = R(f) + j I(f) \neq \phi$$

$$= |X(f)| e^{j \theta(f)}$$



$$x(t) \in \mathbb{R}$$

$$x^*(t) = \int_{-\infty}^{\infty} x^*(t) e^{+j2\pi ft} dt$$

$$= \int_{-\infty}^{\infty} x(t) e^{j2\pi ft} dt$$

$$= x(-t)$$

$$x^*(-t) = x(t)$$

Simmetria
Hermitiana

$$|x(-t)| e^{-j\vartheta(-t)} = |x(t)| e^{j\vartheta(t)}$$

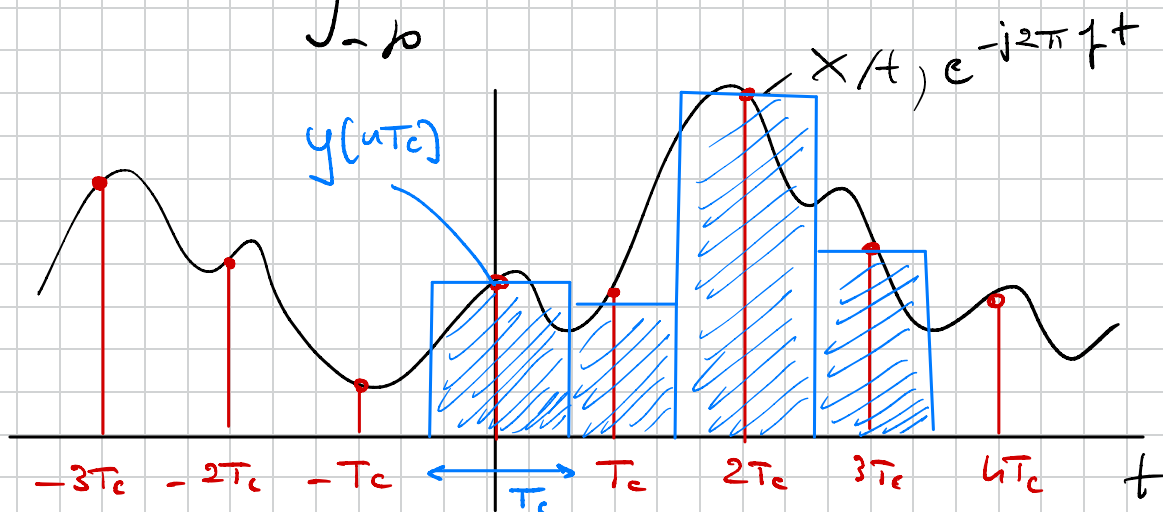
$$|x(-t)| = |x(t)|$$

$$-\vartheta(-t) = \vartheta(t)$$

Amplitude: pari

Fase: dispari

$$X(f) = \int_{-b}^b x(t) e^{-j2\pi f t} dt$$



$$\approx \sum_n \underbrace{x(nT_c) e^{-j2\pi f nT_c}}_{y(nT_c) T_c} T_c$$

$$X(k\Delta f) = \sum_n x(nT_c) e^{-j2\pi k n \Delta f T_c} T_c$$

$$\Delta f T_c = \frac{1}{N T_c}$$

FFT

Complexity: $N \log_2 N$

$$\begin{bmatrix} \left\{ \right. \\ X(k \Delta f) \\ \left. \right\} \end{bmatrix} = \begin{bmatrix} \text{---} \\ e^{-j 2\pi k_n \Delta f T_c} \\ \text{---} \end{bmatrix} \begin{bmatrix} \left\{ \right. \\ X(n T_c) \\ \left. \right\} \end{bmatrix}$$

Note how the matrix looks

$$\begin{matrix} N \times 1 & & N \times N & N \times 1 \\ \underline{X} & = & T_c \underline{F} & \underline{X} \end{matrix}$$

$$\left(\underline{F} \right)_{k,n} = e^{-j 2\pi k_n \Delta f T_c}$$

Complexity: $N \cdot N$

ie $N = N$ allora N^2