Esame 13 - 11 - 08

Esercizio 1

$$y(t) = -x(t) + 2x(t-T) - x(t-2T)$$

$$= x(t) \otimes \{ -\delta(t) + 2\delta(t-T) - \delta(t-2T) \}$$

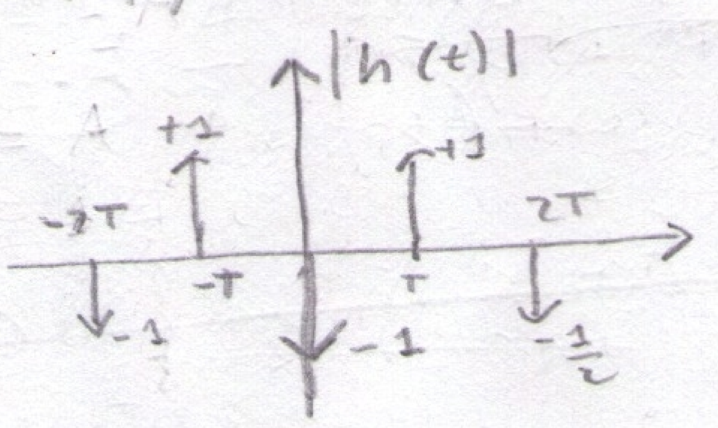
$$h(t) = -\delta(t) + 2\delta(t-T) - \delta(t-2T)$$ (1)

$$H(f) = -1 + 2e^{+j2\pi Tf} - e^{+j2\pi 2Tf}$$

$$= -1 + 2\cos(2\pi Tf) + j2\,\mathrm{sen}(2\pi Tf) - \cos(4\pi Tf) - j\,\mathrm{sen}(4\pi Tf)$$

$$\mathrm{Re}\{H(f)\} = -1 + 2\cos(2\pi Tf) - \cos(4\pi Tf) =$$

$$\mathrm{Re}\{h(t)\} = -\delta(t) + \delta(t-T) + \delta(t+T) - \tfrac{1}{2}\delta(t-2T) - \tfrac{1}{2}\delta(t+2T)$$



(1)

$$y(t) = -A\cos(2\pi f_0 t + \theta) + 2A\cos(2\pi f_0[t-T] + \theta) - A\cos(2\pi f_0[t-2T] + \theta) =$$

$$= -A\cos\left(2\pi \frac{21}{4T} t + \theta\right) + 2A\cos\left(2\pi \frac{21}{4T} t - 2\pi \frac{21}{4T} T + \theta\right) - A\cos\left(2\pi \frac{21}{4T} t - 2\pi \frac{21}{4T} \cdot 2T + \theta\right) =$$

$$= -A\cos\left(2\pi \frac{21}{4T} t + \theta\right) + 2A\cos\left(2\pi \frac{21}{4T} t - \frac{21\pi + 2\theta}{2}\right) - A\cos\left(2\pi \frac{21}{4T} t - 21\pi + \theta\right)$$ (2)

Esercizio 2

$n(t) = [w(t) \cdot 2\cos(2\pi f_0 t)] \otimes p(t) \quad ; \quad \tilde{n}_c(t) = \tilde{w}_c(t) \otimes p(t)$

$S_{\tilde{n}_c}(f) = S_{\tilde{w}_c}(f) |P(f)|^2$

$R_w(\tau) = m(\tau) \cos(2\pi f_0 \tau) = 2N_0 B \,\mathrm{sinc}(2\tau B) \cos(2\pi f_0 \tau) \rightleftharpoons S_w(f)$

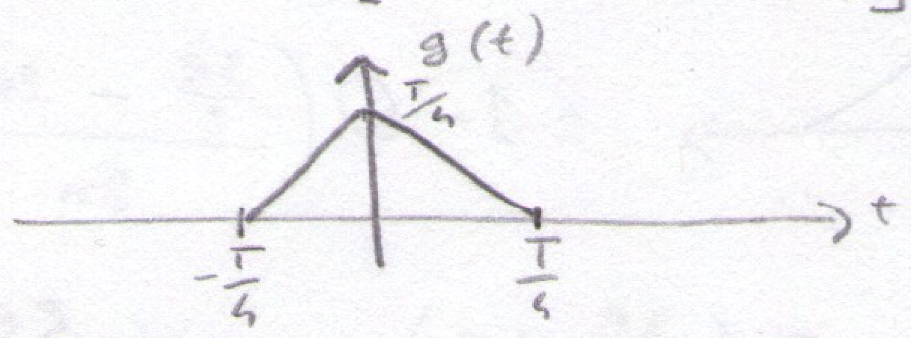
$S_w(f) = \frac{N_0}{2} \mathrm{rect}\left(\frac{f-f_0}{2B}\right) + \frac{N_0}{2} \mathrm{rect}\left(\frac{f+f_0}{2B}\right)$

$S_{\tilde{w}}(f) = \begin{cases} 4 S_w(f+f_0) & f \geq -f_0 \\ \emptyset & \text{altrove} \end{cases} = 2N_0 \,\mathrm{rect}\left(\frac{f}{2B}\right) \quad ; \quad S_{\tilde{w}_c}(f) = \frac{S_{\tilde{w}}(f) + S_{\tilde{w}}(-f)}{4} = N_0 \,\mathrm{rect}\left(\frac{f}{2B}\right)$

$S_{\tilde{n}_c}(f) = N_0 \,\mathrm{rect}\left(\frac{f}{2B}\right) \left|\frac{T}{4} \mathrm{sinc}\left(\frac{T}{4} f\right)\right|^2 = N_0 \,\mathrm{rect}\left(\frac{f}{2B}\right) \cdot \frac{T^2}{16} \mathrm{sinc}^2\left(\frac{T}{4} f\right) = \frac{N_0 T^2}{16} \mathrm{sinc}^2\left(\frac{T}{4} f\right)$ 1

$g(t) \triangleq p(t) \otimes p(t) \Rightarrow G(f) = P^2(f) = \left[\frac{T}{4} \mathrm{sinc}\left(f \frac{T}{4}\right)\right]^2 = \frac{T^2}{16} \mathrm{sinc}^2\left(f \frac{T}{4}\right)$

$g(t) = \frac{T}{4}\left(1 - \frac{t}{T/4}\right) \mathrm{rect}\left(\frac{t}{T/2}\right)$



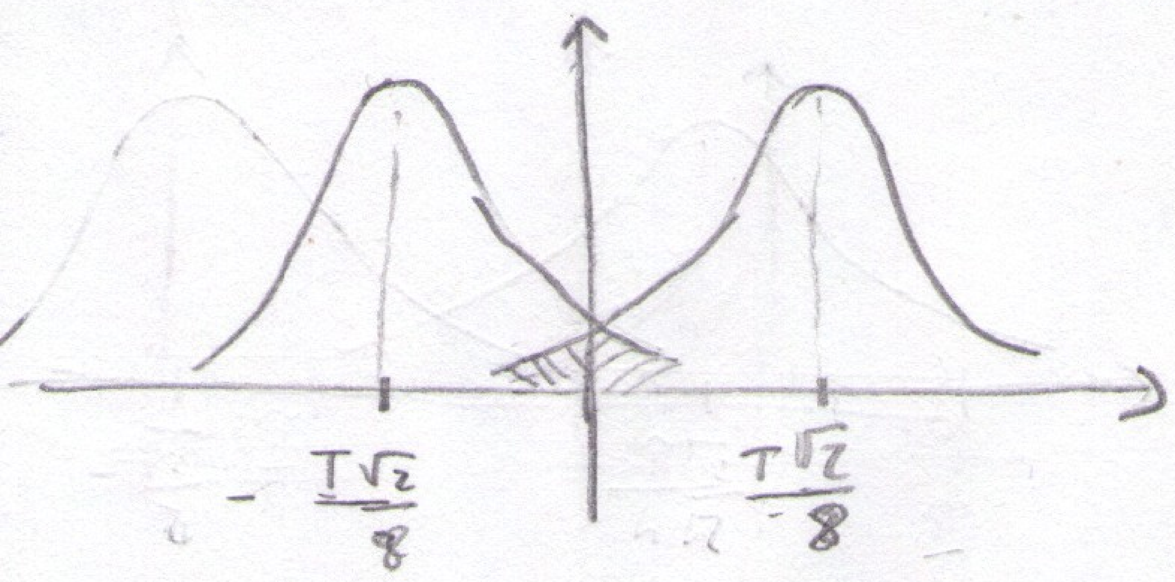
$\Rightarrow g(mT) = \begin{cases} T/4 & m = \emptyset \\ \emptyset & m \neq \emptyset \end{cases}$ ok 2

$r(t) = \mathrm{Re}\{\tilde{r}(t) e^{j2\pi f_0 t}\} = \mathrm{Re}\{e^{j\theta_k} e^{j2\pi f_0 t}\}$

$\tilde{r}(t) = e^{j\theta_k} \quad ; \quad y(t) \triangleq \mathrm{Re}\{\tilde{r}(t)\} \Rightarrow \cos(\theta_k)$

$z(t) = \cos(\theta_k) \otimes p(t) + \tilde{n}_c(t) \quad ; \quad z_k = \cos(\theta_k)\, p(\emptyset) + n_{k_c} \quad ; \quad n_{k_c} = \tilde{n}_c(kT) \in \mathcal{N}(\emptyset, \sigma_{n_c}^2)$

$z_k = \frac{T}{4}\cos(\theta_k) + n_{k_c} \quad \in \mathcal{N}(\emptyset, \sigma_{n_c}^2)$



$P_r\left[\widehat{\tfrac{T}{4}\cos\left(\tfrac{\pi}{4}\right)} \,\middle|\, \tfrac{T}{4}\cos\left(-\tfrac{3\pi}{4}\right)\right] = P_r\left[\hat{z}_k\Big|_{k=\frac{\pi}{4}} > \emptyset \,\middle|\, \tfrac{T}{4}\cos\left(-\tfrac{3\pi}{4}\right)\right] =$

$= Q\left(\frac{\emptyset + \frac{T\sqrt{2}}{8}}{\sigma_{n_c}}\right) = Q\left(\frac{T\sqrt{2}}{8\sigma_{n_c}}\right)$

$P_r\left[\widehat{\tfrac{T}{4}\cos\left(-\tfrac{3\pi}{4}\right)} \,\middle|\, \tfrac{T}{4}\cos\left(\tfrac{\pi}{4}\right)\right] = 1 - Q\left(\frac{\emptyset - \frac{T\sqrt{2}}{8}}{\sigma_{n_c}}\right) = Q\left(\frac{T\sqrt{2}}{8\sigma_{n_c}}\right)$

$P_r(e) = \frac{1}{2} Q\left(\frac{T\sqrt{2}}{8\sigma_{n_c}}\right) + \frac{1}{2} Q\left(\frac{T\sqrt{2}}{8\sigma_{n_c}}\right) = Q\left(\frac{T\sqrt{2}}{8\sigma_{n_c}}\right)$

$n(kT)\Big|_{kT=\emptyset} = \frac{N_0 T^2}{16} \Rightarrow \sigma_{n_c}^2 = P_{n_c} = \frac{N_0 T^2}{16} \Rightarrow \sigma_n = \sqrt{\frac{N_0 T^2}{16}} = \frac{\sqrt{N_0}\, T}{4}$

$P_r(e) = Q\left(\frac{T\sqrt{2}}{\cancel{2}8 \cdot \frac{\sqrt{N_0}\, T}{4_1}}\right) = Q\left(\frac{\sqrt{2}}{2\sqrt{N_0}}\right) = Q\left(\sqrt{\frac{2}{4N_0}}\right)$ 3