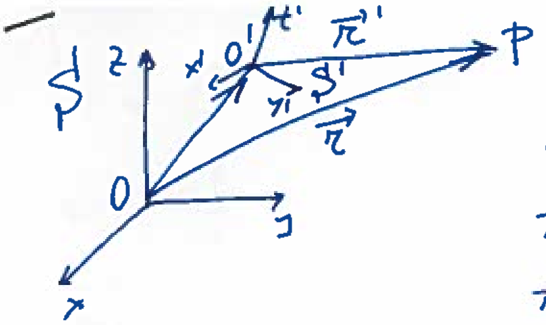

$$\vec{r} = \vec{OO'} + \vec{r}' \qquad (*)$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r}' = x'\hat{i}' + y'\hat{j}' + z'\hat{k}'$$

$$\vec{OO'} = x_{o'}\hat{i} + y_{o'}\hat{j} + z_{o'}\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$ vel di P in S

$$\vec{v}' = \frac{d\vec{r}'}{dt} = \frac{dx'}{dt}\hat{i}' + \frac{dy'}{dt}\hat{j}' + \frac{dz'}{dt}\hat{k}'$$ vel di P in S'

$$\vec{v}_{o'} = \frac{dx_{o'}}{dt}\hat{i} + \frac{dy_{o'}}{dt}\hat{j} + \frac{dz_{o'}}{dt}\hat{k}$$ vel di O' rispetto a S

deriviamo (*)

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{OO'}}{dt} + \frac{d\vec{r}'}{dt} =$$

$$= \frac{d}{dt}\left(x_{o'}\hat{i} + y_{o'}\hat{j} + z_{o'}\hat{k}\right) + \frac{d}{dt}\left(x'\hat{i}' + y'\hat{j}' + z'\hat{k}'\right) =$$

$$= \frac{dx_{o'}}{dt}\hat{i} + \frac{dy_{o'}}{dt}\hat{j} + \frac{dz_{o'}}{dt}\hat{k} + \frac{dx'}{dt}\hat{i}' + \frac{dy'}{dt}\hat{j}' + \frac{dz'}{dt}\hat{k}' +$$

$$+ x'\frac{d}{dt}\hat{i}' + y'\frac{d}{dt}\hat{j}' + z'\frac{d}{dt}\hat{k}' =$$

$$= \vec{v}_{o'} + \vec{v}' + x'\frac{d}{dt}\hat{i}' + y'\frac{d}{dt}\hat{j}' + z'\frac{d}{dt}\hat{k}'$$

Si dimostra che:

$$\frac{d}{dt}\hat{i}' = \vec{\omega}\times\hat{i}' \qquad \frac{d\hat{j}'}{dt} = \vec{\omega}\times\hat{j}' \qquad \frac{d\hat{k}'}{dt} = \vec{\omega}\times\hat{k}' \qquad (o)$$

$$x'\frac{d}{dt}\hat{i}' + y'\frac{d}{dt}\hat{j}' + z'\frac{d}{dt}\hat{k}' = x'\vec{\omega}\times\hat{i}' + y'\vec{\omega}\times\hat{j}' + k'\vec{\omega}\times\hat{k}' =$$

$$= \vec{\omega}\times\left(x'\hat{i}' + y'\hat{j}' + z'\hat{k}'\right) = \vec{\omega}\times\vec{r}'$$

Quindi: $\vec{v} = \vec{v}_{o'} + \vec{v}' + \vec{\omega}\times\vec{r}'$, $\frac{d\vec{r}'}{dt} = \vec{v}' + \vec{\omega}\times\vec{r}'$

Si definisce la velocità di trascinamento:

$$\vec{v}_T = \vec{v} - \vec{v}' = \vec{v}_{o'} + \vec{\omega}\times\vec{r}'$$

è la velocità rispetto a S di un punto P^* solidale a S' che all'istante t coincide con P. (ovvero sarebbe la velocità di P rispetto a S se P fosse fermo rispetto al sistema S').

Quindi:

$$\vec{v} = \vec{v}' + \vec{v}_T$$

Studiamo a considerare le accelerazioni

$$\vec{a} = \frac{d^2x}{dt^2}\hat{\imath} + \frac{d^2y}{dt^2}\hat{\jmath} + \frac{d^2z}{dt^2}\hat{k} \quad \text{in } S$$

$$\vec{a}' = \frac{d^2x'}{dt^2}\hat{\imath}' + \frac{d^2y'}{dt^2}\hat{\jmath}' + \frac{d^2z'}{dt^2}\hat{k}' \quad \text{in } S'$$

$$\vec{a}_{O'} = \frac{d\vec{v}_{O'}}{dt}$$

Deriviamo: $\vec{v} = \vec{v}_{O'} + \vec{v}' + \vec{\omega}\times\vec{r}'$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}_{O'}}{dt} + \frac{d\vec{v}'}{dt} + \frac{d\vec{\omega}}{dt}\times\vec{r}' + \vec{\omega}\times\frac{d\vec{r}'}{dt}$$

Come prima:

$$\frac{d\vec{v}'}{dt} = \frac{d}{dt}\left(\frac{dx'}{dt}\hat{\imath}' + \frac{dy'}{dt}\hat{\jmath}' + \frac{dz'}{dt}\hat{k}'\right) =$$

$$= \left(\frac{d^2x'}{dt^2}\hat{\imath}' + \frac{d^2y'}{dt^2}\hat{\jmath}' + \frac{d^2z'}{dt^2}\hat{k}'\right) + \frac{dx'}{dt}\frac{d\hat{\imath}'}{dt} + \frac{dy'}{dt}\frac{d\hat{\jmath}'}{dt} + \frac{dz'}{dt}\frac{d\hat{k}'}{dt} =$$

$$= \vec{a}' + \frac{dx'}{dt}\vec{\omega}\times\hat{\imath}' + \frac{dy'}{dt}\vec{\omega}\times\hat{\jmath}' + \frac{dz'}{dt}\vec{\omega}\times\hat{k}' =$$

$$= \vec{a}' + \vec{\omega}\times\frac{dx'}{dt}\hat{\imath}' + \vec{\omega}\times\frac{dy'}{dt}\hat{\jmath}' + \vec{\omega}\times\frac{dz'}{dt}\hat{k}' =$$

$$= \vec{a}' + \vec{\omega}\times\vec{v}'$$

$$\vec{\omega}\times\frac{d\vec{r}'}{dt} = \vec{\omega}\times(\vec{v}' + \vec{\omega}\times\vec{r}') = \vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}')$$

$$\vec{a} = \frac{d\vec{v}_{O'}}{dt} + \vec{a}' + \vec{\omega}\times\vec{v}' + \frac{d\vec{\omega}}{dt}\times\vec{r}' + \vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}') =$$

$$= \vec{a}' + \vec{a}_{O'} + \vec{\omega}\times(\vec{\omega}\times\vec{r}') + \frac{d\vec{\omega}}{dt}\times\vec{r}' + 2\vec{\omega}\times\vec{v}' \qquad (\blacklozenge)$$

L'accelerazione di trascinamento:

$$\vec{a}_T = \vec{a}_{O'} + \vec{\omega}\times(\vec{\omega}\times\vec{r}') + \frac{d\vec{\omega}}{dt}\times\vec{r}'$$

è l'accelerazione in S di un punto P^* fisso in S' che coincide con P all'istante t. Quindi si trova imponendo $\vec{a}'=0$ e $\vec{v}'=0$ nell'espressione ($\blacklozenge$).

L'acc. $2\vec{\omega}\times\vec{v}' = \vec{a}_{coriolis}$

$$\vec{a} = \vec{a}' + \vec{a}_T + \vec{a}_{coriolis}$$

Dimostriamo (0)

$$\frac{d}{dt}\hat{\imath}' = \vec{\omega} \times \hat{\imath}'$$
$$\frac{d}{dt}\hat{\jmath}' = \vec{\omega} \times \hat{\jmath}'$$
$$\frac{d}{dt}\hat{k}' = \vec{\omega} \times \hat{k}'$$

$$\frac{d\hat{\imath}'}{dt} = \cancel{\left(\frac{d\hat{\imath}'}{dt}\cdot \hat{\imath}'\right)\hat{\imath}'} + \left(\frac{d\hat{\imath}'}{dt}\cdot \hat{\jmath}'\right)\hat{\jmath}' + \left(\frac{d\hat{\imath}'}{dt}\cdot \hat{k}'\right)\hat{k}'$$

$$\frac{d\,\hat{\imath}'^2}{dt} = 0 \qquad \frac{d\hat{\imath}'}{dt}\cdot \hat{\imath}' + \hat{\imath}'\cdot\frac{d\hat{\imath}'}{dt} = 0 \qquad 2\,\frac{d\hat{\imath}'}{dt}\cdot \hat{\imath}' = 0$$

$$\begin{cases} \dfrac{d\hat{\imath}'}{dt} = \left(\dfrac{d\hat{\imath}'}{dt}\cdot\hat{\jmath}'\right)\hat{\jmath}' + \left(\dfrac{d\hat{\imath}'}{dt}\cdot\hat{k}'\right)\hat{k}' \\ \dfrac{d\hat{\jmath}'}{dt} = \left(\dfrac{d\hat{\jmath}'}{dt}\cdot\hat{\imath}'\right)\hat{\imath}' + \left(\dfrac{d\hat{\jmath}'}{dt}\cdot\hat{k}'\right)\hat{k}' \\ \dfrac{d\hat{k}'}{dt} = \left(\dfrac{d\hat{k}'}{dt}\cdot\hat{\imath}'\right)\hat{\imath}' + \left(\dfrac{d\hat{k}'}{dt}\cdot\hat{\jmath}'\right)\hat{\jmath}' \end{cases}$$ e analogamente

le 6 componenti tra ( ) non sono indip. perché:

$$\hat{\imath}'\cdot\hat{\jmath}' = 0 \Rightarrow \frac{d\hat{\imath}'}{dt}\cdot\hat{\jmath}' = -\frac{d\hat{\jmath}'}{dt}\cdot\hat{\imath}'$$
$$\hat{\imath}'\cdot\hat{k}' = 0 \Rightarrow \frac{d\hat{\imath}'}{dt}\cdot\hat{k}' = -\frac{d\hat{k}'}{dt}\cdot\hat{\imath}'$$
$$\hat{\jmath}'\cdot\hat{k}' = 0 \Rightarrow \frac{d\hat{\jmath}'}{dt}\cdot\hat{k}' = -\frac{d\hat{k}'}{dt}\cdot\hat{\jmath}'$$

Ci sono solo 3 componenti indipendenti

Definiamo il vettore $\vec{\omega} \equiv (\omega_x, \omega_y, \omega_z)$

$$\omega_x = \frac{d\hat{\jmath}'}{dt}\cdot\hat{k}'$$
$$\omega_y = \frac{d\hat{k}'}{dt}\cdot\hat{\imath}'$$
$$\omega_z = \frac{d\hat{\imath}'}{dt}\cdot\hat{\jmath}'$$

$$\frac{d\hat{\imath}'}{dt} = \left(\frac{d\hat{\imath}'}{dt}\cdot\hat{\jmath}'\right)\hat{\jmath}' - \left(\frac{d\hat{k}'}{dt}\cdot\hat{\imath}'\right)\hat{k}' = \omega_z\hat{\jmath}' - \omega_y\hat{k}' = \vec{\omega}\times\hat{\imath}' = \begin{vmatrix} \hat{\imath}' & \hat{\jmath}' & \hat{k}' \\ \omega_x & \omega_y & \omega_z \\ 1 & 0 & 0 \end{vmatrix}$$

analogamente

$$\frac{d\hat{\jmath}'}{dt} = \left(-\frac{d\hat{\imath}'}{dt}\cdot\hat{\jmath}\right)\hat{\imath}' + \left(\frac{d\hat{\jmath}'}{dt}\cdot\hat{k}'\right)\hat{k}' = -\omega_z\hat{\imath}' + \omega_x\hat{k}' = \vec{\omega}\times\hat{\jmath}' = \begin{vmatrix} \hat{\imath}' & \hat{\jmath}' & \hat{k}' \\ \omega_x & \omega_y & \omega_z \\ 0 & 1 & 0 \end{vmatrix}$$

$$\frac{d\hat{k}'}{dt} = \left(\frac{d\hat{k}'}{dt}\cdot\hat{\imath}'\right)\hat{\imath}' - \left(\frac{d\hat{\jmath}'}{dt}\cdot\hat{k}'\right)\hat{\jmath}' = \omega_y\hat{\imath}' - \omega_x\hat{\jmath}' = \vec{\omega}\times\hat{k}' = \begin{vmatrix} \hat{\imath}' & \hat{\jmath}' & \hat{k}' \\ \omega_x & \omega_y & \omega_z \end{vmatrix}$$