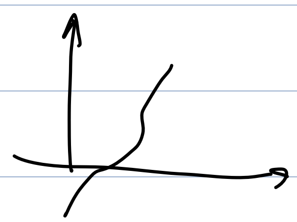
ANALISI 2

$f: \mathbb{R}^n \to \mathbb{R}^m \qquad n \geq 2 \qquad m \geq 1$

$x \in \mathbb{R}^n \qquad x = (x_1, \dots, x_n) \qquad (x,y,z) \in \mathbb{R}^3$

$f: \mathbb{R} \to \mathbb{R}^m \qquad f(t) = \begin{pmatrix} f_1(t) \\ \vdots \\ f_m(t) \end{pmatrix}$ curva

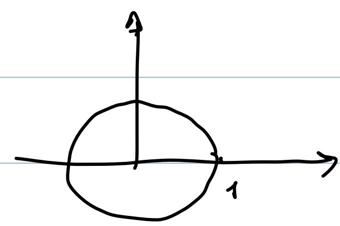


$\frac{d}{dt} f(t) = \begin{pmatrix} f_1'(t) \\ \vdots \\ f_m'(t) \end{pmatrix} \qquad \int_a^b f(t)\,dt = \begin{pmatrix} \int_a^b f_1(t)\,dt \\ \vdots \\ \int_a^b f_m(t)\,dt \end{pmatrix}$

$f: [a,b] \to \mathbb{R}^m$ curva continua $f \in C^0([a,b])$

$f([a,b])$ È IL SOSTEGNO DELLA CURVA

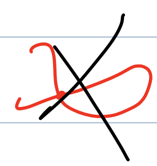
$f(t) \doteq [0,2\pi] \to \mathbb{R}^2 \qquad f(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$



$f(t): [0, 4\pi) \to \mathbb{R}^2 \qquad f(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$

$f: [a,b] \to \mathbb{R}^m$ è curva di classe $C^k \qquad f \in C^k([a,b])$

f è curva semplice



i) $f(a) \neq f(b)$ $\quad f$ è iniettiva

ii) $f(a) = f(b)$ $\quad f$ è chiusa $\quad f$ iniettiva su $[a,b[$

$f(x_1, \dots, x_n) \in \mathbb{R}$

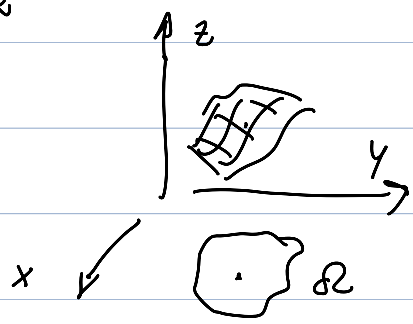
$f: [a,b] \to \mathbb{R} \qquad G_f = \{ (x,y) \in \mathbb{R}^2 : y = f(x) \}$

$f: \Omega \to \mathbb{R}, \quad \Omega \subseteq \mathbb{R}^n \qquad G_f = \{ (x,y) \in \Omega \times \mathbb{R} \quad y = f(x) \}$

$\Omega \subset \mathbb{R}^2$

$f : \Omega \to \mathbb{R}$ $\qquad z = f(x,y)$

$\qquad x_3 = f(x_1, x_2)$ $\qquad \mathbb{R}^{n+1}$



$x \to T(t, x_1, x_2, x_3)$

$x \mapsto \|x\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2} = (x, x)$

$x \mapsto (a, x) \qquad a \in \mathbb{R}^n$

$= \sum_{i=1}^{n} a_i x_i = x_i a_i$ NOTAZIONE DI EINSTEIN

$f : \mathbb{R}^n \to \mathbb{R}$ $\qquad \max f \qquad \sup f \qquad$ f limitata

$f(x) < f(y)$ $\qquad \max f = M \qquad M \in \mathbb{R}$

$x < y \; \cancel{x, y \in \mathbb{R}^n}$ $\qquad f(x) \leq M \quad \forall x \in \mathbb{R}^2$

$\exists \bar{x} \in \mathbb{R}^n \; f(\bar{x}) = M$

f limitata $\quad \exists K : \; |f(x)| \leq K \quad \forall x \in \mathbb{R}^n$

$\lim_{x \to x_0} f(x) = L \qquad f :]a, b[\to \mathbb{R}$

$\forall \varepsilon > 0 \quad \exists \delta > 0 : \quad \forall x \in]a, b[\quad \underset{x \neq x_0}{|x - x_0| < \delta} \quad |f(x) - L| < \varepsilon$

$\lim_{x \to x^0} f(x) = L \qquad f : \mathbb{R}^n \to \mathbb{R}$

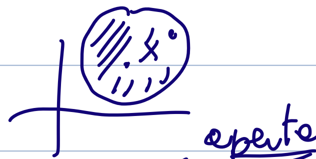
$x^0 = (x_1^0, \dots, x_n^0)$

$\forall \varepsilon > 0 \quad \exists \delta > 0 : \quad \forall x \in \mathbb{R}^n \quad \underset{x \neq x^0}{\|x - x^0\| < \delta} \quad |f(x) - L| < \varepsilon$

$|x - x_0| < \delta \qquad x, x_0 \in \mathbb{R}$

$x_0-\delta \quad x_0 \quad x_0+\delta$

$\|x - x^0\| < \delta \qquad x, x^0 \in \mathbb{R}^2$



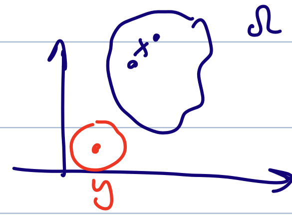
$B(x^0, R) = \{x \in \mathbb{R}^n : \|x - x^0\| < R\}$ palla ~~centrata~~ aperta centrata in x^0 e raggio $R > 0$

$\|x - y\| = d(x, y)$

$\overline{B(x^0, R)} = \{x \in \mathbb{R}^n : \|x - x^0\| \leq R\}$ palla chiusa

$S^{n-1}(x^0, R) = \{x \in \mathbb{R}^n : \|x - x^0\| = R\}$ superficie sferica

(DEF PUNTO INTERNO)



$x^0 \in \Omega \subseteq \mathbb{R}^n$ $\quad x^0$ è INTERNO

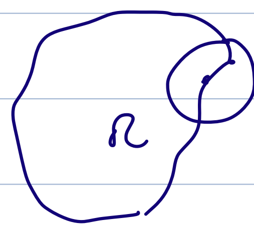
SE $\exists R > 0 \quad B(x^0, R) \subset \Omega$

DEF PUNTO ESTERNO

$y \in \mathbb{R}^n$ è ESTERNO a $\Omega \subseteq \mathbb{R}$

$\exists R > 0 \qquad B(y, R) \cap \Omega = \emptyset$

PUNTO DI FRONTIERA



$z \in \mathbb{R}^n$ È DI FRONTIERA PER Ω

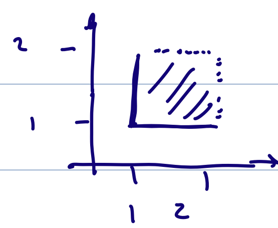
$\forall R > 0 \qquad B(z, R) \cap \Omega \neq \emptyset$

$\qquad\qquad\quad B(z, R) \cap (\mathbb{R}^n \setminus \Omega) \neq \emptyset$

$\Omega \subset \mathbb{R}^n$ è aperto se tutti i suoi punti sono interni

$\partial\Omega$ È LA FRONTIERA DI Ω

<u>ES</u> $\quad B(0,1) \subseteq \mathbb{R}^n$ È APERTO



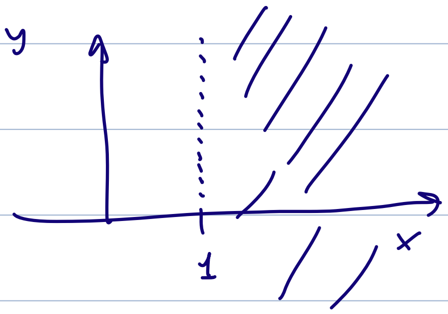
$\mathring{\Omega}$ PARTE INTERNA DI Ω

$\partial\Omega$

$\overline{\Omega}$ CHIUSURA DI $\Omega \qquad \Omega \cup \partial\Omega$

$\mathring{\Omega} \subseteq \Omega \subseteq \overline{\Omega}$

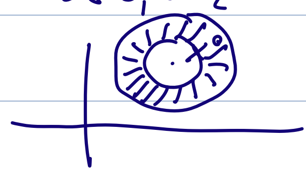
$\Omega = \{ (x,y) \subseteq \mathbb{R}^2 \quad x > 1 \}$ SEMI SPAZIO



$\overline{\Omega} = \{ (x,y) \subseteq \mathbb{R}^2 : x \geq 1 \}$ $\quad \mathbb{R}^2 \setminus \Omega = \{ (x,y) \subseteq \mathbb{R}^2 : x \leq 1 \}$

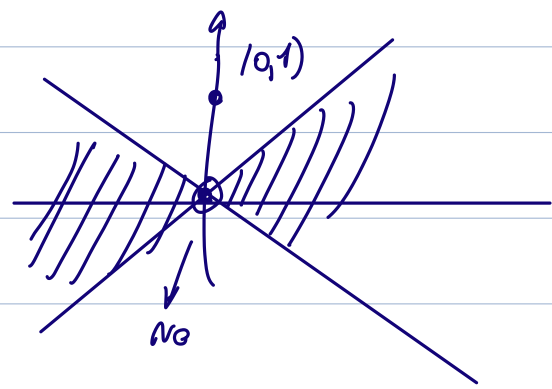
$\Omega = \{ x \in \mathbb{R}^n : r_1 < \| x - x^0 \| < r_2 \}$ $\quad 0 < r_1 < r_2$

CORONA CIRCOLARE $n = 2$



$\Omega = \{ (x,y) \subseteq \mathbb{R}^2 : \frac{y^2}{x^2} \leq 1$ oppure $x^2 + (y-1)^2 \leq 0 \}$

$y^2 \leq x^2 \qquad |y| \leq |x|$



$\bar{x} = (0,1) \in \Omega$ È ISOLATO

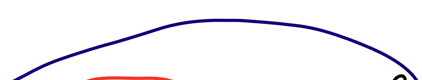
$\exists \; R > 0 \qquad B(\bar{x}, R) \cap \Omega = \{ \bar{x} \}$

PUNTO DI ACCUMULAZIONE PER Ω

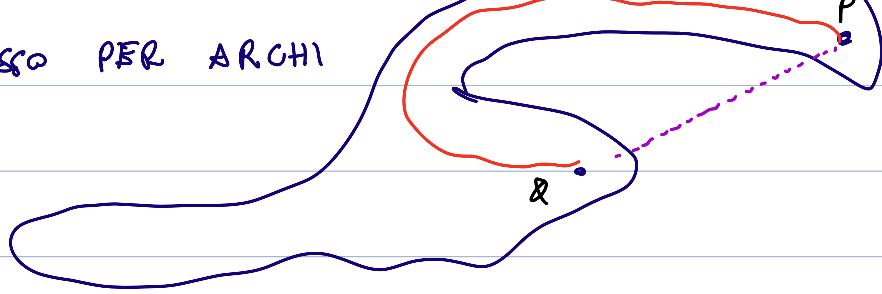
$y \in \mathbb{R}^n$ È DI ACCUMULAZIONE $\forall R > 0 \quad B(y, R) \cap \Omega \neq \emptyset$

$\overline{ab} = S[a,b] = \{ x \in \mathbb{R}^n \quad x = a + \underbrace{t(b-a)}_{(1-t)a + tb} \quad t \in [0,1] \}$

SEGMENTO CHIUSO DI ESTREMI $a, b \in \mathbb{R}^n$



Ω CONNESSO PER ARCHI



$\forall\, P, Q \in \Omega$

$\exists\ \gamma : [0,1] \to \mathbb{R}^n \qquad \gamma(0) = P \qquad \gamma(t) \subseteq \Omega$

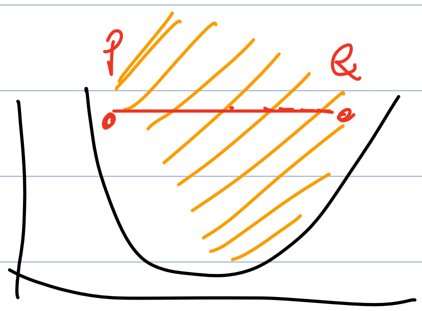
$\gamma(1) = Q$

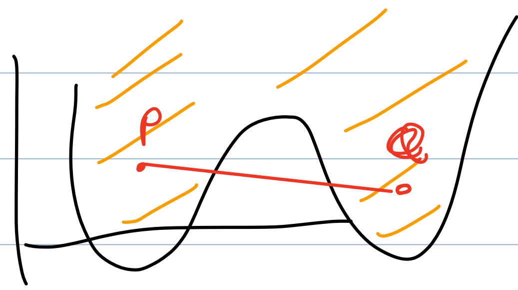
Ω CONVESSO

$\forall\, P, Q \in \Omega \qquad S[P,Q] = \overline{PQ} \subseteq \Omega$

$f : \mathbb{R} \to \mathbb{R}$ CONVESSA $\iff$ $Epi(f) = \{ (x,y) \in \mathbb{R}^2 \ , \ y > f(x) \} \subseteq \mathbb{R}^2$

$Epi(f)$ CONVESSO





$f(x) = \sqrt{1 - \|x\|^2} \qquad x \in \mathbb{R}^n \qquad f : \mathbb{R}^n \to \mathbb{R}^+$

$Dom(f) = \{ x \in \mathbb{R}^n : \ 1 - \|x\|^2 \geq 0 \}$

$\|x\|^2 \leq 1 \qquad \|x\| \leq 1 \iff \overline{B(0,1)}$

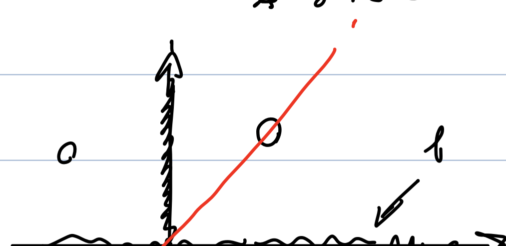
$f(x) = \sqrt{1 - x^2 - y^2} \qquad z = \sqrt{1 - x^2 - y^2} \qquad z \geq 0$

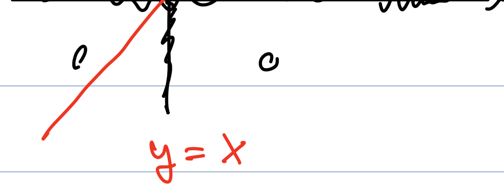
$z^2 = 1 - x^2 - y^2$

$x^2 + y^2 + z^2 = 1$

$f : \mathbb{R}^2 \to \mathbb{R}$

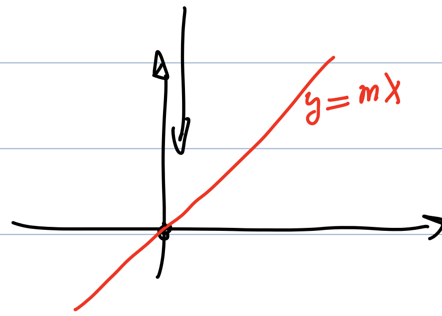
$f(x,y) = \begin{cases} 1 & xy = 0 \\ 0 & xy \neq 0 \end{cases}$





$$f(0,y) - f(0,0) = 1 - 1 = 0$$

$$f(x,0) - f(0,0) = 1 - 1 = 0$$



$$f(x,y) = (y - x^2)(y - 3x^2)$$

$$f(0,0) = 0$$

$$f(0,y) = (y-0)(y-0) = y^2$$

$$f(x,0) = (0 - x^2)(0 - 3x^2) = 3x^4$$

$$f(x,mx) = (mx - x^2)(mx - 3x^2) = 3x^4 - 3mx^3 - mx^3 + m^2x^2$$

$$= 3x^4 - 4mx^3 + m^2x^2$$

$$f'(x) = 12x^3 - 12mx^2 + 2m^2x \qquad f'(0) = 0$$

$$f''(x) = 36x^2 - 2 \cdot 12mx + 2m^2 \qquad f''(0) = 2m^2 > 0$$

$$f(a, 2a^2) = (2a^2 - a^2)(2a^2 - 3a^2) = a^2(-1)a^2 = -a^2 < 0$$

$\Rightarrow$ $(0,0)$ non è pt. di minimo