$$\int_a^b f(x)\,dx \qquad f:[a,b)\to\mathbb{R}$$

$$f(x,y) \qquad f:[a,b]\times[c,d]\to\mathbb{R} \qquad f \text{ continua}$$

$$\phi(x) = \int_c^d f(x,y)\,dy$$

TEOREMA $f\in C([a,b]\times[c,d]) \implies \phi\in C([a,b])$

$$|\phi(x)-\phi(x_0)| = \left|\int_c^d (f(x,y)-f(x_0,y))\,dy\right| \overset{?}{<} \varepsilon \quad \text{SE } |x-x_0|<\delta$$

$$\leq \int_c^d |f(x,y)-f(x_0,y)|\,dy < \int_c^d \varepsilon\,dy < \varepsilon(d-c)$$

f continua in $(x_0,\bar y)$

$\forall\, \varepsilon>0 \quad \exists\, \delta>0 \quad : \sqrt{|x-x_0|^2+|y-\bar y|^2}<\delta \implies |f(x,y)-f(x_0,\bar y)|<\varepsilon$

$\delta = \delta(\varepsilon, \cancel{x_0,\bar y})$

$f\in C([a,b]\times[c,d]) \implies f$ è uniformemente continua

$$\lim_{x\to x_0}\int_c^d f(x,y)\,dy = \int_c^d \lim_{x\to x_0} f(x,y)\,dy$$

$f: A\times B\to\mathbb{R}$, $A\subseteq\mathbb{R}^n$, $B\subseteq\mathbb{R}$ $\qquad f(x,y)=f(x_1,\dots,x_n,y) \qquad f\in C(\bar A\times\bar B)$

A, B limitati $\quad B\supset[c,d]$

$$\implies \phi(x)=\phi(x_1,\dots,x_n) = \int_c^d f(x_1,\dots,x_n,y)\,dy \text{ è continua in } \bar A.$$

TEO $f\in C^1(\bar A\times\bar B)$

$$\phi(x) = \int_c^d f(x_1,\dots x_n,y)\,dy \qquad \frac{\partial}{\partial x_i}\phi(x)$$

$$\implies \frac{\partial\phi}{\partial x_i}(x) = \int_c^d \frac{\partial f}{\partial x_i}(x_1,\dots,x_n,y)\,dy$$

DIM $\phi(x) = \int^d f(x,y)\,dy$

$$\frac{\phi(x+h)-\phi(x)}{h} = \int_c^d \frac{f(x+h,y)\,dy - \int_c^d f(x,y)dy}{h} = \int_c^d \frac{f(x+h,y)-f(x,y)}{h}\,dy$$

$$= \int_c^d \partial_x f(x+\xi h, y)\,dy \qquad (\text{LAGRANGE})$$

$$\frac{\phi(x+h)-\phi(x)}{h} - \int_c^d \frac{\partial f}{\partial x}(x,y)\,dy$$

$$= \int_c^d \partial_x f(x+\xi h,y) - \partial_x f(x,y)\,dy$$

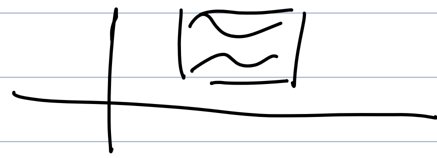
$\downarrow$ 0 $h \to 0$ unif in $[c,d]$

TEOREMA

$f \in C^1(\bar{A}\times\bar{B})$

$\alpha(x), \beta(x) \in \bar{B} \qquad \forall x \in \bar{A}$

$$\phi(x) = \int_{\alpha(x)}^{\beta(x)} f(x,y)\,dy$$



$$\boxed{\begin{aligned}&\phi(x) = \int_{\alpha(x)}^{\beta(x)} f(y)\,dy \qquad F'(x) = f(x) \qquad \text{ANALISI 1}\\ &\phi(x) = F(\beta(x)) - F(\alpha(x))\\ &\phi'(x) = F'(\beta(x))\beta' - F'(\alpha(x))\alpha'(x) = f(\beta(x))\beta'(x) - f(\alpha(x))\alpha'(x)\end{aligned}}$$

$$\phi'(x) = \int_{\alpha(x)}^{\beta(x)} \frac{\partial f}{\partial x}(x,y)\,dy + f(\beta(x))\beta'(x) - f(\alpha(x))\alpha'(x)$$

$$f(x) = \begin{cases} \frac{\sin(x)}{x} & x>0 \\ 1 & x=0 \end{cases} \qquad f \in C([0,+\infty))$$

$$\int_0^{+\infty} \frac{\sin(x)}{x}\,dx \quad \text{ESISTE FINITO} \qquad \int_0^{+\infty} \left|\frac{\sin(x)}{x}\right| dx = +\infty$$

$$F(x) = \int_0^x \frac{\sin(t)}{t}\,dt \qquad F'(x) = \frac{\sin(x)}{x}$$

$$\int_0^b \frac{\sin(x)}{x}\,dx = Si(b) - Si(0)$$

$$+\infty \qquad -\lambda x \qquad \lambda > 0$$

$$\int_0 \frac{\sin(x)}{x} e \quad dx$$

$$\phi(\lambda) = \int_0^{+\infty} f(\lambda, x)\, dx$$

$$\phi_M(\lambda) = \left(\int_0^M \frac{\sin(x)}{x} e^{-\lambda x} dx\right)' = \int_0^M \frac{\partial}{\partial\lambda}\left(\frac{\sin(x)}{x} e^{-\lambda x}\right) dx$$

$$\phi'_M(\lambda) = \int_0^M - \cancel{x} \underbrace{\frac{\sin(x)}{\cancel{x}}}_{f'} \underbrace{e^{-\lambda x}}_{g} dx$$

$$= \underbrace{\cos(x)}_{f} \underbrace{e^{-\lambda x}}_{g} \Big|_0^M - \int_0^M \underbrace{\cos(x)}_{f} \underbrace{e^{-\lambda x}(-\lambda)}_{g'(x)} dx$$

$$= \cos(x) e^{-\lambda x} \Big|_0^M + \lambda \int_0^M \underbrace{\cos(x)}_{f'} \underbrace{e^{-\lambda x}}_{g} dx$$

$$= \cos(x) e^{-\lambda x} \Big|_0^M + \lambda \left[\sin(x) e^{-\lambda x} \Big|_0^M - \int_0^M \sin(x) e^{-\lambda x} (-\lambda)\, dx \right]$$

$$= \cos(x) e^{-\lambda x} + \lambda \sin(x) e^{-\lambda x} \Big|_0^M \ast \lambda^2 \int_0^M - \sin(x) e^{-\lambda x} dx$$

$$\phi'_M(\lambda) = \cos(x) e^{-\lambda x} + \lambda \sin(x) e^{-\lambda x} \Big|_0^M - \lambda^2 \phi'_M(\lambda)$$

$$(1+\lambda^2)\, \phi'_M(\lambda) = \cos(x) e^{-\lambda x} + \lambda \sin(x) e^{-\lambda x} \Big|_0^M$$

$$= \cos(M) e^{-\lambda M} - 1 + \lambda \sin(M) e^{-\lambda M} - 0$$

$$\xrightarrow[M \to +\infty]{} = -1$$

$$\phi'(\lambda) = -\frac{1}{1+\lambda^2} \qquad \phi(\lambda) = c - \operatorname{arctg}(\lambda)$$

$$\phi(\lambda) = \int_0^{+\infty} \underbrace{\frac{\sin(x)}{x}}_{\text{LIMITATO}} \underbrace{e^{-\lambda x}}_{\downarrow\, 0} dx = c - \operatorname{arctg}(\lambda)$$

$$\lim_{\lambda \to +\infty} \phi(\lambda) = 0 \implies \lim_{\lambda \to +\infty} c - \operatorname{arctg}(\lambda) = c - \frac{\pi}{2} = 0$$

$$c = \frac{\pi}{2}$$

$$\phi(\lambda) = \frac{\pi}{2} - \operatorname{arctg}(\lambda) = \int_0^{+\infty} \frac{\sin(x)}{x} e^{-\lambda x} dx$$

$$\downarrow \lambda \to 0 \qquad\qquad \downarrow \lambda \to 0$$

$$\int^{+\infty}$$

$$\frac{\pi}{2} = \int_0 \frac{\sin(x)}{x}\,dx$$

$$\lim_{\lambda \to +\infty} \phi(\lambda) = \lim_{\lambda \to +\infty} \int_0^{+\infty} \frac{\sin(x)}{x} e^{-\lambda x} = \int_0^{\infty} \lim_{\lambda \to +\infty} \frac{\sin(x)}{x} e^{-\lambda x}\,dx$$

$$= \int_0^{+\infty} 0\,dx$$

$$\int_0^{+\infty} \frac{\sin(x)}{x} e^{-\lambda x} = \boxed{\int_0^{n} \frac{\sin(x)}{x} e^{-\lambda x}} + \boxed{\int_n^{+\infty} \frac{\sin(x)}{x} e^{-\lambda x}}$$

$$\left| \int_n^{+\infty} \frac{\sin(x)}{x} e^{-\lambda x} \right| < \int_n^{+\infty} \left|\frac{\sin(x)}{x}\right| e^{-\lambda x} \leq \int_n^{+\infty} e^{-\lambda x}\,dx$$

$$\lim_{b \to +\infty} \int_n^{b} e^{-\lambda x}\,dx = \lim_{b \to +\infty} \left. \frac{e^{-\lambda x}}{-\lambda} \right|_n^b$$

$$= \lim_{b \to +\infty} \frac{e^{-\lambda b} - e^{-\lambda n}}{-\lambda} = \frac{e^{-\lambda n}}{\lambda} \xrightarrow[n \to +\infty]{} 0$$

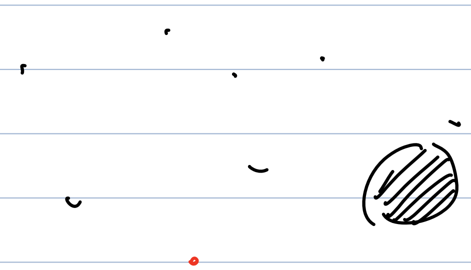
FISSO n GRANDE $\left| \int_n^{+\infty} \frac{\sin(x)}{x} e^{-\lambda x} \right| < \frac{\varepsilon}{2}$

$\left| \frac{\sin(x)}{x} e^{-\lambda x} \right| < \frac{\varepsilon}{2n}$ λ GRANDE

Δ LAPLACIANO

$$\Delta \Phi = 4\pi G \rho$$

$$\Delta \phi = \sum_{i=1}^{3} \frac{\partial^2 \phi}{\partial x_i^2} = \partial_{11}^2 \phi + \partial_{22}^2 \phi + \partial_{33}^2 \phi$$



$$F = -\nabla \phi$$

funzione armonica

$$\boxed{\Delta f = f_{xx} + f_{yy} = 0}$$ EQ LAPLACE

$$f(x,y) = e^x \sin(y)$$

$f_x = e^x \sin(y)$ $\quad f_{xx} = e^x \sin(y)$

$f_y = e^x \cos(y)$ $\quad f_{yy} = e^x(-\sin(y))$

$f(x,y,z) = x^2+y^2+z^2$ $\quad\quad$ $\Delta f = f_{xx} + f_{yy} + f_{zz} = 2+2+2 = 6$

$f_x = 2x$

$f_{xx} = 2$

$f(x,y) = \ln(x^2+y^2)$ $\quad\quad$ $(x,y) \neq (0,0)$

$\Delta f = f_{xx} + f_{yy} = 0$ $\quad\quad$ $f_x = \frac{1}{x^2+y^2} 2x$

$$f_{xx} = \frac{2(x^2+y^2) - 2x \cdot 2x}{(x^2+y^2)^2} = \frac{2x^2+2y^2-4x^2}{(x^2+y^2)^2}$$

$$f_{yy} = \frac{2x^2-2y^2}{(x^2+y^2)^2} \quad\quad = \frac{2y^2-2x^2}{(x^2+y^2)^2}$$

$\nabla \cdot \vec{E} = 0$ $\quad$ divergenza di E uguale a 0

$\operatorname{div} \vec{E} = 0$

$\nabla f = \operatorname{grad} f$

$$\nabla \cdot \vec{E} = \sum_{i=1}^{3} \frac{\partial}{\partial x_i} E_i \quad\quad \vec{E} \in \mathbb{R}^3 \Rightarrow \operatorname{div} \vec{E} \in \mathbb{R}$$

$$E = (x-y,\ y+z,\ z-x)$$

$$\operatorname{div} E = \nabla \cdot E = \frac{\partial}{\partial x}(x-y) + \frac{\partial}{\partial y}(y+z) + \frac{\partial}{\partial z}(z-x)$$

$$= 1 + 1 + 1 = 3$$

$$\left\langle \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right), (E_1, E_2, E_3) \right\rangle = \frac{\partial}{\partial x_1} E_1 + \frac{\partial}{\partial x_2} E_2 + \frac{\partial}{\partial x_3} E_3$$

$$\nabla^2 f = \Delta f = \nabla \cdot (\nabla f) = \left\langle \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right), \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3}\right) \right\rangle$$

$$= \frac{\partial}{\partial x_1}\frac{\partial f}{\partial x_1} + \frac{\partial}{\partial x_2}\frac{\partial f}{\partial x_2} + \frac{\partial}{\partial x_3}\frac{\partial f}{\partial x_3} = \Delta f$$

$$\nabla \times \vec{B} = \operatorname{rot} \vec{B} = \operatorname{curl} \vec{B} \qquad \text{ROTORE DI } B$$

$$\parallel$$

$$\vec{B} \in \mathbb{R}^3 \qquad \vec{\nabla} \times \vec{B} \in \mathbb{R}^3$$

$$\det \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$= i\left(\frac{\partial}{\partial y} B_3 - \frac{\partial}{\partial z} B_2\right) - j\left(\frac{\partial}{\partial x} B_3 - \frac{\partial}{\partial z} B_1\right) + k\left(\frac{\partial}{\partial x} B_2 - \frac{\partial}{\partial y} B_1\right)$$

$$f(x) = g(\|x\|) \qquad \begin{array}{l} g : \mathbb{R} \to \mathbb{R} \\ f : \mathbb{R}^n \to \mathbb{R} \end{array} \qquad \|x\| = \left(\sum_{k=1}^{n} x_k^2\right)^{1/2}$$

$$\frac{\partial}{\partial x_i} \|x\| = \frac{1}{2}\left(\sum x_k^2\right)^{-1/2} \cdot \frac{\partial}{\partial x_i}\left(\sum x_k^2\right)$$

$$= \frac{2}{2}\left(\sum x_k^2\right)^{-1/2} \cdot \delta_{ik} x_k$$

$$= \frac{x_i}{\|x\|}$$

$$\frac{\partial}{\partial x_i} g(\|x\|) = g'(\|x\|) \frac{\partial}{\partial x_i} \|x\|$$

$$= g'(\|x\|) \frac{x_i}{\|x\|}$$

$$\Delta f = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2} = \sum_{i=1}^{n} \frac{\partial^2 g}{\partial x_i^2} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_i} g(\|x\|)$$

$$\frac{\partial}{\partial x_i}\left(g'(\|x\|) \frac{x_i}{\|x\|}\right) = g''(\|x\|) \frac{\partial}{\partial x_i}\|x\| \frac{x_i}{\|x\|} + g'(\|x\|) \frac{\partial}{\partial x_i}\left(\frac{x_i}{\|x\|}\right)$$

$$= g''(\|x\|) \frac{x_i x_i}{\|x\|^2} + g'(\|x\|) \left[\frac{1\,\|x\| - x_i \frac{x_i}{\|x\|}}{\|x\|^2}\right]$$

$$= g''(\|x\|) \frac{x_i^2}{\|x\|^2} + g'(\|x\|) \frac{\|x\| - \frac{x_i^2}{\|x\|}}{\|x\|^2}$$

$$= g''(\|x\|) \frac{x_i^2}{\|x\|^2} + g'(\|x\|) \left[\frac{1}{\|x\|} - \frac{x_i^2}{\|x\|^3}\right]$$

$$\Delta g = g''(\|x\|) \sum \frac{x_i^2}{\|x\|^2} + g'(\|x\|) \sum\left(\frac{1}{\|x\|} - \frac{x_i^2}{\|x\|^3}\right)$$

$$= g''(\|x\|) \frac{\cancel{\|x\|^2}}{\cancel{\|x\|^2}} + g'(\|x\|) \left(\frac{n}{\|x\|} - \frac{\cancel{\|x\|^2}}{\|x\|^3}\right)$$

$$= g''(\|x\|) + g'(\|x\|)\left(\frac{n-1}{\|x\|}\right)$$

$$\Delta \omega(\|x\|) = 0 \qquad \omega(\|x\|) = \omega(\rho) \qquad \rho = \|x\|$$

$$\omega''(\rho) + (n-1)\frac{\omega'(\rho)}{\rho} = 0 \qquad \omega'(\rho) = f(\rho)$$

$$f'(\rho) = -(n-1)\frac{f(\rho)}{\rho} \qquad \text{EQ VARIABILI SEPARABILI}$$

$n = 2 \qquad n = 3$

$$f(x,y,z) = x^3\sqrt{y^2+z^2} \qquad df(2,3,4)$$

$\exists R > 0$

$f \in C^1(\mathbb{R}^3 \setminus \{0\}) \qquad x^0 = (2,3,4) \qquad B(x^0, R)$

f è C^1 in $B(x^0, R)$

$$df(2,3,4),(x,y,z) = \nabla f(2,3,4)(x,y,z)$$

$$\partial_x f\big|_{(2,3,4)} = 3x^2\sqrt{y^2+z^2}\Big|_{(2,3,4)} = 3\cdot 2^2\sqrt{3^2+4^2} = 3\cdot 4\sqrt{25} = 3\cdot 4\cdot 5$$

$$\partial_y f = x^3 (y^2+z^2)^{-1/2}\,\frac{1}{2}\cdot 2y \qquad \partial_y f\big|_{(2,3,4)} = 2^3\,\frac{1}{5}\,3$$

$$\partial_z f = x^3 (y^2+z^2)^{-1/2}\cdot\frac{1}{2}\cdot 2z \qquad \partial_z f = \frac{2^3}{5}\,4$$

$$df(2,3,4)(x,y,z) = 3\cdot 4\cdot 5x + \frac{2^3\cdot 3}{5}y + \frac{2^3\cdot 4}{5}z$$

$$(1,98)^3\sqrt{(3,01)^2+(3,97)^2} = f(1,98,\ 3,01,\ 3,97) = f(2,3,4) + df_{(2,3,4)}(\ ) + o(\ )$$

$1,98 \simeq 2 \qquad 3,01 \simeq 3 \qquad 3,97 \simeq 4$

$$f(x) = \boxed{f(x^0) + (\nabla f(x^0), x-x^0)} + \omega(x) \qquad x = (1,98,\ 3,01,\ 3,97)$$

$x^0 = (2,3,4)$

$$f(x) = f(x^0) + \varphi(x - x^0) + \omega(x)$$

φ è il differenziale φ è lineare

$$= df(x^0)(x - x^0)$$

$$\varphi(x+y) = \varphi(x) + \varphi(y)$$

$$= \varphi[x - x^0]$$

$$\varphi(\lambda x) = \lambda \varphi(x)$$

SE f è diff. in x^0 $\Rightarrow$ $\varphi(x) = (\nabla f(x^0), x)$