- Calcola la trasformata di Laplace di $x(t) = (4t - 3\cos(5t))e^{-2t}$

$$x(t) = 4te^{-2t} + 3\cos(5t)e^{-2t}$$

$$X(s) = \mathcal{L}\{4te^{-2t}\} + \mathcal{L}\{3\cos(5t)e^{-2t}\}$$

$$= 4\mathcal{L}\{te^{-2t}\} + 3\mathcal{L}\{\cos(5t)e^{-2t}\}$$

$$= 4\cdot\frac{1}{(s+2)^2} + 3\cdot\frac{s+2}{(s+2)^2+5^2}$$

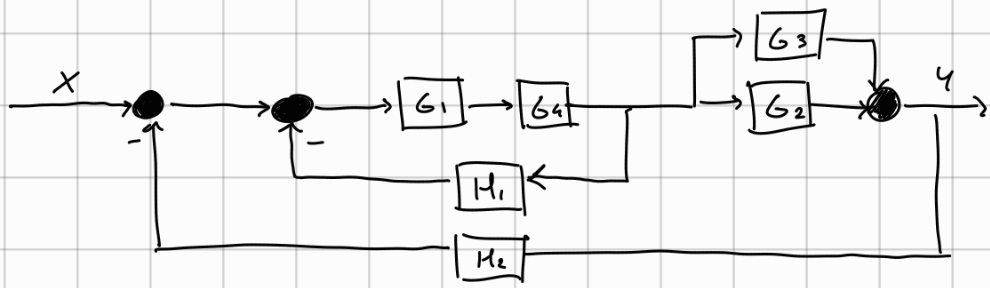
Dalla tabella

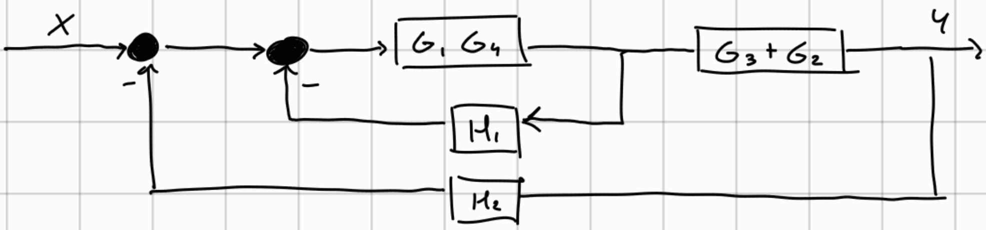
- Calcola l'anti trasformata di Laplace di $G(s) = 2 + \frac{3}{2s^3+3s^2+s}$

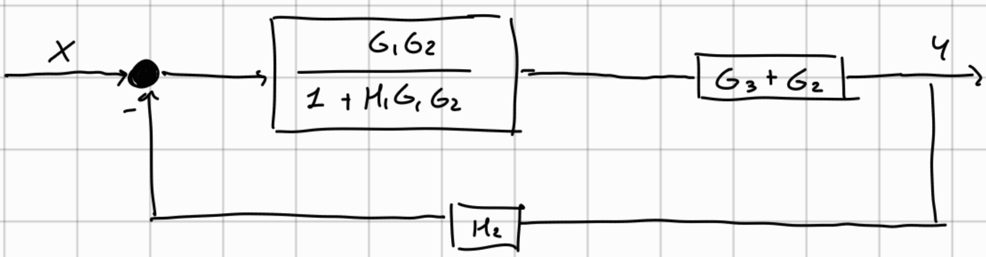
$$x(t) = 2\delta(t) + \mathcal{L}^{-1}\left\{\frac{3/2}{s^3+\frac{3}{2}s^2+\frac{s}{2}}\right\} = 2\delta(t) + \mathcal{L}^{-1}\left\{\frac{3/2}{s(s+\frac{1}{2})(s+1)}\right\}$$

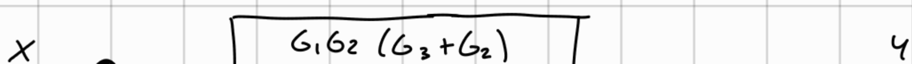
$$= 2\delta(t) + \mathcal{L}^{-1}\left\{\frac{3}{s} + \frac{-3}{s+\frac{1}{2}} + \frac{1}{s+1}\right\} = 2\delta(t) + 3 - 6e^{-\frac{1}{2}t} + 3e^{-t}$$

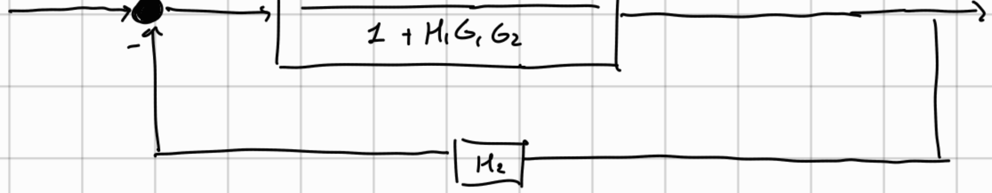
- Calcola la funzione di trasferimento equivalente per:











$$\frac{Y(s)}{X(s)} = \frac{\dfrac{G_1 G_2 (G_2 + G_3)}{1 + H_1 G_1 G_2}}{1 + \dfrac{G_1 G_2 (G_2 + G_3) H_2}{1 + H_1 G_1 G_2}}$$

- Scrivi la funzione di trasferimento di un sistema del 2° ordine la cui risposta a $1(t)$ è caratterizzata da:

1) Guadagno statico $G(0) = 5$
2) Coefficiente di smorzamento $\xi = 0.5$
3) Tempo di assestamento $t_s = 3s$
4) Nessuno zero

$$G(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n} s + 1}$$

2,4) $\xi = 0.5 \Rightarrow G(s) = \dfrac{1}{\frac{s^2}{\omega_n^2} + \frac{1}{\omega_n} s + 1}$

1) $G(0) = 5 \Rightarrow G(s) = 5 \cdot \dfrac{1}{\frac{s^2}{\omega_n^2} + \frac{1}{\omega_n} s + 1}$

3) $t_s \approx -\dfrac{1}{\xi \omega_n} \ln(0.05) \Rightarrow \omega_n = \dfrac{2}{3} \ln(0.05) = 2 \Rightarrow G(s) = \dfrac{5}{\frac{s^2}{4} + \frac{1}{2} s + 1}$

- Traccia la risposta a $1(t)$ per $G(s) = \dfrac{20(3 + 0.1s)(s^2 + 10s + 160)}{(2s + 10)(0.1s + 5)(s^2 + 2s + 400)}$

I poli sono: $s = -5 \quad s = -50 \quad s = -1 \pm \sqrt{399}\, i$

L'ultimo è il più lento, per cui $\hat{G}(s) \approx \dfrac{K}{s^2 + 2s + 400}$

Sapendo che i $G(0)$ devono essere uguali:

$G(0) = \dfrac{12}{25} \Rightarrow \hat{G}(0) = \dfrac{K}{400} = \dfrac{12}{25} \Rightarrow K = 192$

Dunque $G(s) \approx \tilde{G}(s) = \dfrac{192}{s^2+2s+400} = \dfrac{12}{25}\,\dfrac{1}{\frac{s^2}{400}+\frac{s}{200}+1} = \dfrac{12}{25}\,\dfrac{1}{\frac{s^2}{\omega_n^2}+\frac{2\xi}{\omega_n}s+1}$

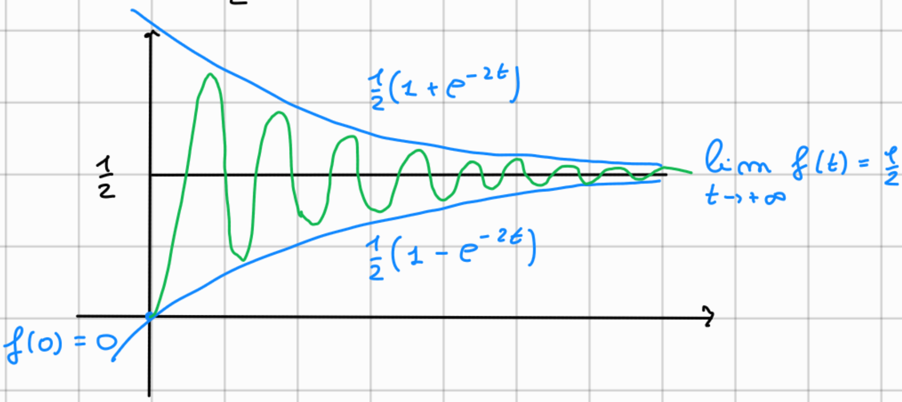
dove $\omega_n^2 = 400 \Rightarrow \omega_n = 20$

$\dfrac{2\xi}{\omega_n} = \dfrac{1}{200} \Rightarrow \dfrac{2\xi}{20} = \dfrac{1}{200} \quad \xi = \dfrac{20}{400} = \dfrac{1}{20}$

Allora $y(t)$ di $G(s)$ per $1(t)$ equivale a

$$y(t) = \frac{12}{25}\cdot\left[1 - \frac{1}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t}\, \sin\left(\omega_n\sqrt{1-\xi^2}\,t + \arccos(\xi)\right)\right]$$

$$\approx \frac{1}{2}\cdot\left[1 - e^{-2t}\sin\left(20t + \frac{\pi}{2}\right)\right]$$



- Per il sistema descritto dalla $G(s)$ sopra, calcolare:
  1) Il valore finale per $u(t) = 1(t)$
  2) Il tempo di assestamento
  3) Il periodo delle oscillazioni

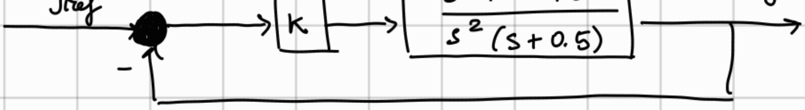
1) $FV = \lim_{s\to 0} s\cdot\frac{1}{s}G(s) = G(0) = \frac{12}{15}$

2) $t_s \approx -\dfrac{1}{\xi\omega_n}\ln(0.05) = -\dfrac{1}{\frac{1}{20}\cdot 20}\ln(0{,}05) = 3s$

3) $T_P = \dfrac{2\pi}{\omega_n\sqrt{1-\xi^2}} \approx \dfrac{2\pi}{\omega_n} = \dfrac{2\pi}{20} = \dfrac{\pi}{10} = 0.314s$

- Determinare il range di stabilità del seguente sistema:

$s^2+2s+64$



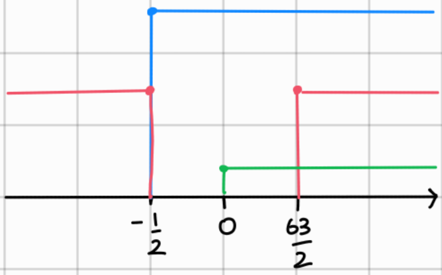
$$G(s) = \frac{K\,\frac{s^2+2s+64}{s^2(s+0.5)}}{1+K\,\frac{s^2+2s+64}{s^2(s+0.5)}} = \frac{K(s^2+2s+64)}{s^2(s+0.5)+K(s^2+2s+64)} = \frac{\text{Non ci interessa}}{s^3+s^2\left(\frac{1}{2}+K\right)+s\cdot 2K+64K}$$

s^3	1	$2K$
s^2	$\frac{1}{2}+K$	$64K$
s^1	$\frac{2K^2-63K}{K+\frac{1}{2}}$	
s^0	$64K$	

$\frac{1}{2}+K > 0$

$\frac{2K^2-63K}{\frac{1}{2}+K} > 0$

$64K > 0$



$K > \frac{63}{2}$

- Tracciare i diagrammi di Bode per il sistema sopra

$$\frac{s^2+2s+64}{s^2(s+0.5)} = \frac{1}{s}\cdot\frac{1}{s}\cdot\frac{1}{2s+1}\cdot\frac{s^2+2\cdot\overbrace{\frac{1}{8}}^{\xi}\cdot\overbrace{8}^{\omega_0}s+8^2}{8^2}\cdot 2\cdot 8^2$$