Esame 16-09-2010

Esercizio 2

1) $\bar{E}_T = P_{s_T}\cdot T$; $P_{s_T} = \int_{-\infty}^{\infty} S_T(f)\,df$; $S_T(f) = \frac{E\{a_i^2\}}{T}|G_T(f)|^2$

$E\{a_i^2\} = \sum_{k=0}^{1} m_k^2\, p_k = a_0^2 p_0 + a_1^2 p_1 = 0^2\cdot\frac{1}{2} + 2^2\cdot\frac{1}{2} = 2$

$S_{s_T}(f) = \frac{2}{T} G_T^2(f) = \frac{2}{T}T^2(1-|fT|)^2\, rect\left(\frac{f}{\frac{2}{T}}\right) = 2T(1-|fT|)^2\, rect\left(\frac{f}{\frac{2}{T}}\right)$

$P_{s_T} = \int_{-\infty}^{\infty} S_{s_T}(f)\,df = \int_{-\infty}^{\infty} 2T(1-|fT|)^2 rect\left(\frac{f}{\frac{2}{T}}\right) df = 2T\int_{-\frac{1}{T}}^{\frac{1}{T}}(1-|fT|)^2 df \overset{pari}{=} 2T\cdot 2\int_0^{\frac{1}{T}}(1-fT)^2 df =$

$= 4T\int_0^{\frac{1}{T}}(1+f^2T^2-2fT)\,df = 4T\cdot\frac{1}{T} + 4T\cdot T^2\left[\frac{f^3}{3}\right]_0^{\frac{1}{T}} - 4T\cdot 2T\left[\frac{f^2}{2}\right]_0^{\frac{1}{T}} = 4 + 4T^3\frac{1}{3T^3} - 8T^2\frac{1}{2T^2} =$

$= 4+\frac{4}{3}-4 = \frac{4}{3}$

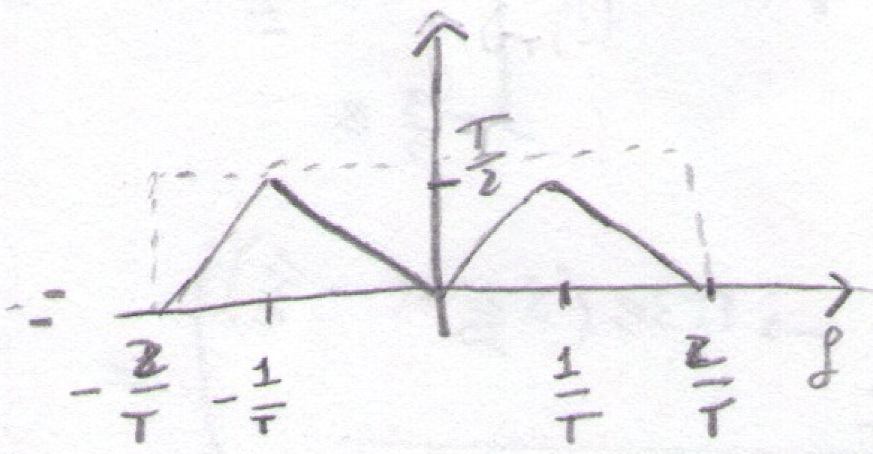
$\bar{E}_T = \frac{4}{3}T$

2) $x(t) = r(t)\otimes g_R(t) = \left[\sum_i a_i\, g_T(t-iT)\cos(2\pi f_0 t) + w(t)\right]\otimes g_R(t) =$

$= \sum_i a_i\, g_T(t-iT)\cos(2\pi f_0 t)\otimes g_R(t) + w(t)\otimes g_R(t) =$

$= \sum_i a_i\, g(t-iT) + \underbrace{w(t)\otimes g_R(t)}_{n(t)}$

$g(t-iT) = g_T(t-iT)\cos(2\pi f_0 t)\otimes g_R(t) \rightleftharpoons \left[\frac{1}{2}G_T(f-f_0)+\frac{1}{2}G_T(f+f_0)\right] G_R(f) =$



$= \frac{1}{2}G_T(f-f_0)+\frac{1}{2}G_T(f+f_0) = G(f)$

Soddisfa Nyquist? $G\left(f+\frac{k}{T}\right) = T \quad \forall k \Rightarrow g(m) = \begin{cases} K & m=0 \\ 0 & m\neq 0\end{cases}$

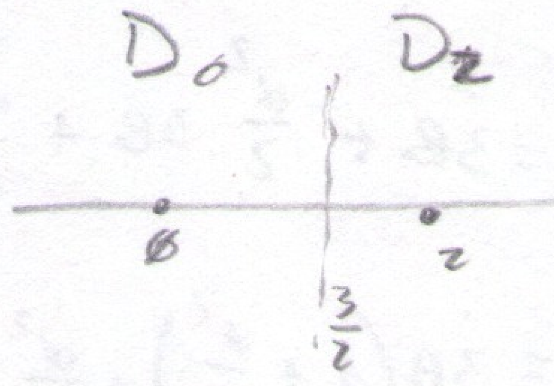
$g(0) = \int_{-\infty}^{\infty} G(f)\,df = \frac{2}{T}\cdot\frac{T}{2} = 1 \Rightarrow K=1$

$x_k = x(kT) = \sum_i a_i\, g(kT-iT) + n_k = \sum_i a_i\, g[(k-i)T] + n_k$

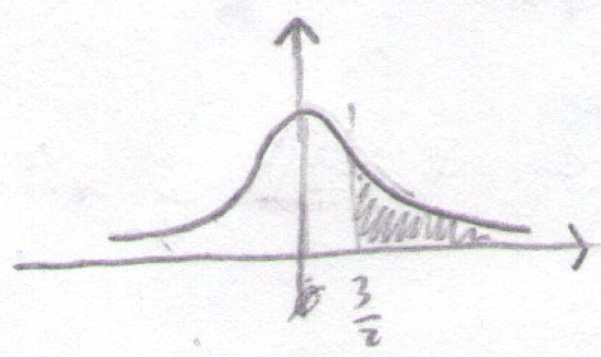
per ogni $(k-i)\neq 0$ per Nyquist $g[(k-i)T]=0 \Rightarrow$ sopravvive solo $k-i=0 \Rightarrow k=i$

$x_k = a_k\, g(0) + n_k = a_k + n_k$

$P_r(e) = P_r(\hat{2}|0)P(0) + P_r[\hat{0}|2]P(2) = \frac{1}{2}\left[P_r(\hat{2}|0)\; P_r(\hat{0}|2)\right]$

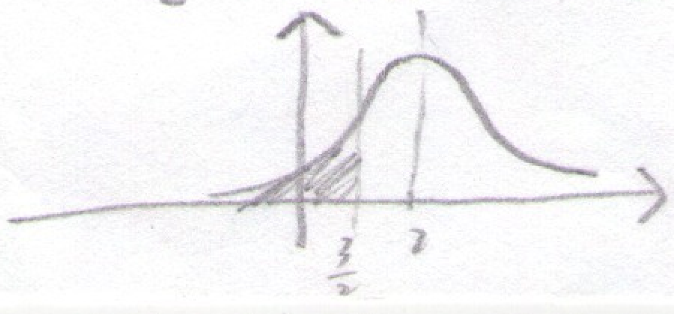


$P_r[\hat{2}|0] = P_r[x_k>\lambda|0]$, $x_k = 0+n_k = n_k \in N(0,\sigma_n^2)$



$Q\left(\frac{\frac{3}{2}-0}{\sigma_n}\right) = Q\left(\frac{3}{2\sigma_n}\right)$

$P_r[\hat{0}|2] = P_r[x_k\le\lambda|2]$; $x_k = 2+n_k \in N(2,\sigma_n^2)$



$1-Q\left(\frac{\frac{3}{2}-2}{\sigma_n}\right) = 1-Q\left(-\frac{1}{2\sigma_n}\right) = Q\left(\frac{1}{2\sigma_n}\right)$

$$\sigma_n^2 = P_n = \int_{-\infty}^{\infty} S_n(f)\,df \quad ; \quad S_n(f) = S_w(f)\,|G(f)|^2 = \frac{N_0}{2}|G_R(f)|^2$$

$$P_n = \int_{-\infty}^{\infty} S_n(f)\,df = \int_{-\infty}^{\infty} \frac{N_0}{2}|G_R(f)|^2\,df = \frac{N_0}{2}\cdot 2\cdot\frac{2}{T} = \frac{2N_0}{T} = \sigma_n^2$$

$$Pr[\hat{2}|\emptyset] = Q\left(\frac{3}{2\sigma_n}\right) = Q\left(\frac{3}{2\sqrt{\frac{2N_0}{T}}}\right) = Q\left(\sqrt{\frac{9T}{8N_0}}\right)$$

$$Pr[\hat{\emptyset}|2] = Q\left(\frac{1}{2\sigma_n}\right) = Q\left(\sqrt{\frac{T}{8N_0}}\right)$$

$$P(e) = \frac{1}{2}\left[Q\left(\sqrt{\frac{3T}{4N_0}}\right) + Q\left(\sqrt{\frac{T}{8N_0}}\right)\right]$$

• Esercizio 1

$$\xrightarrow{x(t)} \boxed{h(t)} \xrightarrow{y(t)}$$

$$X(f) = 2\left(1-\frac{|f|}{3B}\right)rect\left(\frac{f}{6B}\right) - \left(1-\frac{|f|}{2B}\right)rect\left(\frac{f}{4B}\right)$$

$$x(t) = 2\cdot 3B\, sinc^2(t3B) - 2B\, sinc^2(t2B) = 6B\, sinc^2(3Bt) - 2B\, sinc^2(2Bt)$$

$$Y(f) = H(f)X(f) = H(f) = \left(1+\alpha\cos\left[\frac{2\pi f}{f_0}\right]\right) e^{-j2\pi\frac{f}{B}}\, rect\left(\frac{f}{3B}\right)$$

$$Y(f) \rightleftharpoons y(t) = \int_{-\infty}^{\infty} Y(f)\, e^{j2\pi f t}\,df = \int_{-\frac{3}{2}B}^{\frac{3}{2}B} \left\{1+\alpha\cos\left[\frac{2\pi f}{f_0}\right]\right\} e^{\left[-j2\pi\frac{f}{B} + j2\pi f t\right]}\,df =$$

$$= \left[\frac{1}{j2\pi[t-\frac{1}{B}]} e^{j2\pi f[t-\frac{1}{B}]}\right]_{-\frac{3}{2}B}^{\frac{3}{2}B} + \frac{\alpha}{2}\int_{-\frac{3}{2}B}^{\frac{3}{2}B} \left(e^{j2\pi\frac{f}{f_0}} + e^{-j2\pi\frac{f}{f_0}}\right) e^{j2\pi f[t-\frac{1}{B}]}\,df =$$

$$= \frac{1}{j2\pi(t-\frac{1}{B})}\left(\frac{e^{j3\pi B(t-\frac{1}{B})} - e^{-j3\pi B(t-\frac{1}{B})}}{2j}\right) + \frac{\alpha}{2}\left[\frac{1}{j2\pi(\pm\frac{1}{f_0}+t-\frac{1}{B})} e^{j2\pi f(\pm\frac{1}{f_0}+t-\frac{1}{B})}\right]_{-\frac{3}{2}B}^{\frac{3}{2}B} =$$

$$= \frac{3B}{\pi(t-\frac{1}{B})}\,\frac{sen[3\pi B(t-\frac{1}{B})]}{3B} + \frac{\alpha}{2\pi(\pm\frac{1}{f_0}+t-\frac{1}{B})}\left(\frac{e^{j\pi 3B(\pm\frac{1}{f_0}+t-\frac{1}{B})} - e^{-j\pi 3B(\pm\frac{1}{f_0}+t-\frac{1}{B})}}{2j}\right) =$$

$$= 3B\, sinc\left[3B\left(t-\frac{1}{B}\right)\right] + \frac{\alpha\, 3B}{2\pi(\pm\frac{1}{f_0}+t-\frac{1}{B})}\,\frac{sen\left[3B\pi\left(\pm\frac{1}{f_0}+t-\frac{1}{B}\right)\right]}{3B} =$$

$$= 3B\, sinc\left[3B\left(t-\frac{1}{B}\right)\right] + \frac{\alpha 3B}{2}\, sinc\left[3B\left(\frac{1}{f_0}+t-\frac{1}{B}\right)\right] + \frac{\alpha 3B}{2}\, sinc\left[3B\left(-\frac{1}{f_0}+t-\frac{1}{B}\right)\right]$$

$$E_y = \int_{-\infty}^{\infty}|y(t)|^2\,dt = \int_{-\infty}^{\infty}|Y(f)|^2\,df = \int_{-\frac{3}{2}B}^{\frac{3B}{2}}\left[1+\alpha^2\cos^2\left(\frac{2\pi f}{f_0}\right) + 2\alpha\cos\left(\frac{2\pi f}{f_0}\right)\right]df =$$

$$= 3B + \frac{\alpha^2}{2}\cdot 3B + \frac{\alpha^2}{2}\int_{-\frac{3}{2}B}^{\frac{3}{2}B} \frac{e^{j4\pi f\frac{1}{f_0}} + e^{-j4\pi f\frac{1}{f_0}}}{2}\,df + 2\alpha\int_{-\frac{3}{2}B}^{\frac{3}{2}B}\frac{e^{j2\pi f\frac{1}{f_0}} + e^{-j2\pi f\frac{1}{f_0}}}{2}\,df =$$

$$= 3B\left(1+\frac{\alpha^2}{2}\right) + \frac{\alpha^2}{4}\frac{1}{j4\pi\frac{1}{f_0}}\left[e^{j4\pi f\cdot\frac{1}{f_0}}\right]_{-\frac{3}{2}B}^{\frac{3}{2}B} + \frac{\alpha^2}{4}\frac{1}{-j4\pi\frac{1}{f_0}}\left[e^{-j4\pi f\frac{1}{f_0}}\right]_{-\frac{3}{2}B}^{\frac{3}{2}B} + \alpha\frac{1}{j2\pi\frac{1}{f_0}}\left[e^{j2\pi f\cdot\frac{1}{f_0}}\right]_{-\frac{3}{2}B}^{\frac{3}{2}B} + \alpha\frac{1}{-j2\pi\frac{1}{f_0}}\left[e^{-j2\pi f\cdot\frac{1}{f_0}}\right]_{-\frac{3}{2}B}^{\frac{3}{2}B} =$$

$$= 3B\left(1+\frac{\alpha^2}{2}\right) + \frac{\alpha^2}{j16\pi\frac{1}{f_0}}\left[\cancel{e^{j4\pi\frac{3}{2}B\cdot\frac{1}{f_0}}} - \cancel{e^{-j4\pi\frac{3}{2}B\frac{1}{f_0}}} - \cancel{e^{j4\pi\frac{3}{2}B\frac{1}{f_0}}} + \cancel{e^{-j4\pi\frac{3}{2}B\frac{1}{f_0}}}\right] + \frac{\alpha}{j2\pi\frac{1}{f_0}}[\emptyset] =$$

$$= 3B\left(1+\frac{\alpha^2}{2}\right)$$