

TEORIE DI TCT

- LINEARITA'
- RITARDO
- DUALITA'
- CAMBIAMENTO DI SCALA
- TRASLATAZIONE

LINEARITA'

$$X_1(t) \Rightarrow X_1(\tau)$$

$$X_2(t) \Rightarrow X_2(\tau)$$

$$\alpha_1 X_1(t) + \alpha_2 X_2(t) \Rightarrow \alpha_1 X_1(\tau) + \alpha_2 X_2(\tau)$$

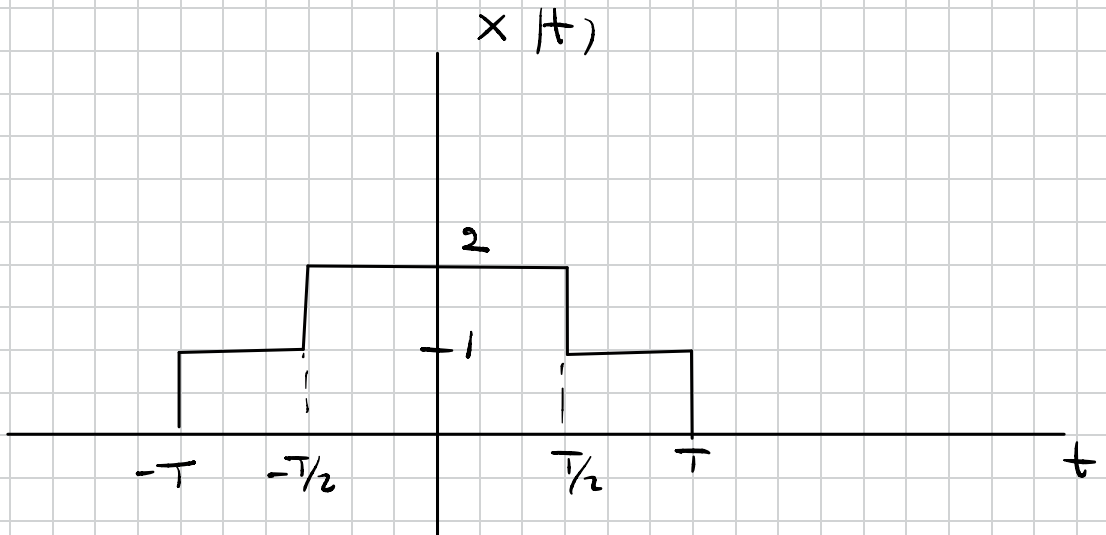
$$X(f) = \int_{-b}^b X(t) e^{-j2\pi f t} dt$$

$$\int_{-b}^b (\alpha_1 X_1(t) + \alpha_2 X_2(t)) e^{-j2\pi f t} dt$$

$$= \underbrace{\alpha_1 \int_{-b}^b X_1(t) e^{-j2\pi f t} dt}_{X_1(f)} + \underbrace{\alpha_2 \int_{-b}^b X_2(t) e^{-j2\pi f t} dt}_{X_2(f)}$$

$$= \alpha_1 X_1(f) + \alpha_2 X_2(f)$$

Esercizio



$$x(t) = x_1(t) + x_2(t)$$

$$x_1(t) = \text{rect}\left(\frac{t}{2T}\right) \Leftrightarrow 2T \text{sinc}\left(2fT\right)$$

$$x_2(t) = \text{rect}\left(\frac{t}{T}\right) \Leftrightarrow T \text{sinc}(fT)$$

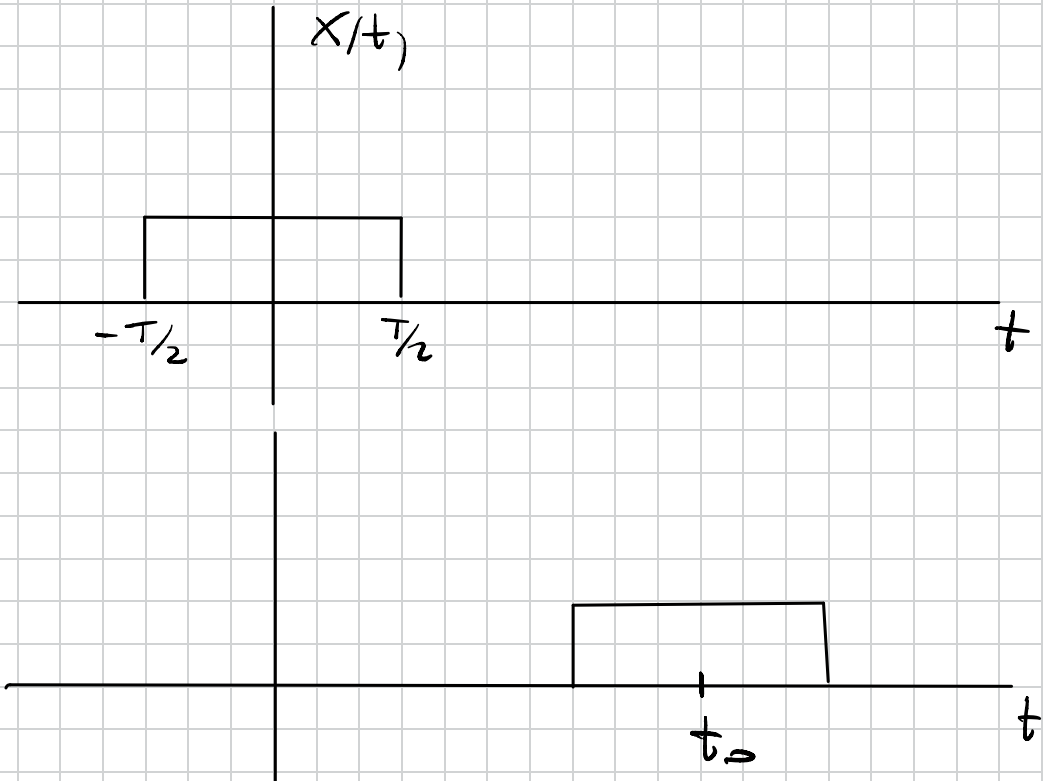
$$x(f) = x_1(f) + x_2(f)$$

$$= 2T \text{sinc}(2fT) + T \text{sinc}(fT)$$

TEOREMA RITARDO

$$x(t) \Rightarrow X(f)$$

$$x(t - t_0) \Rightarrow X(f) e^{-j2\pi f t_0}$$



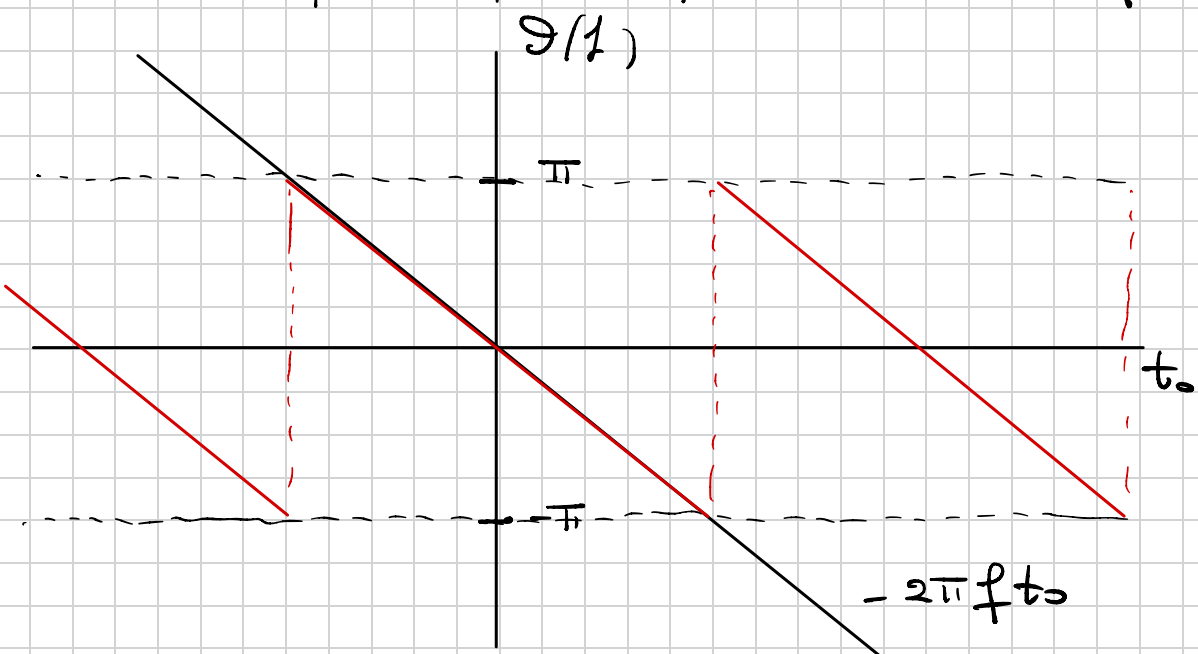
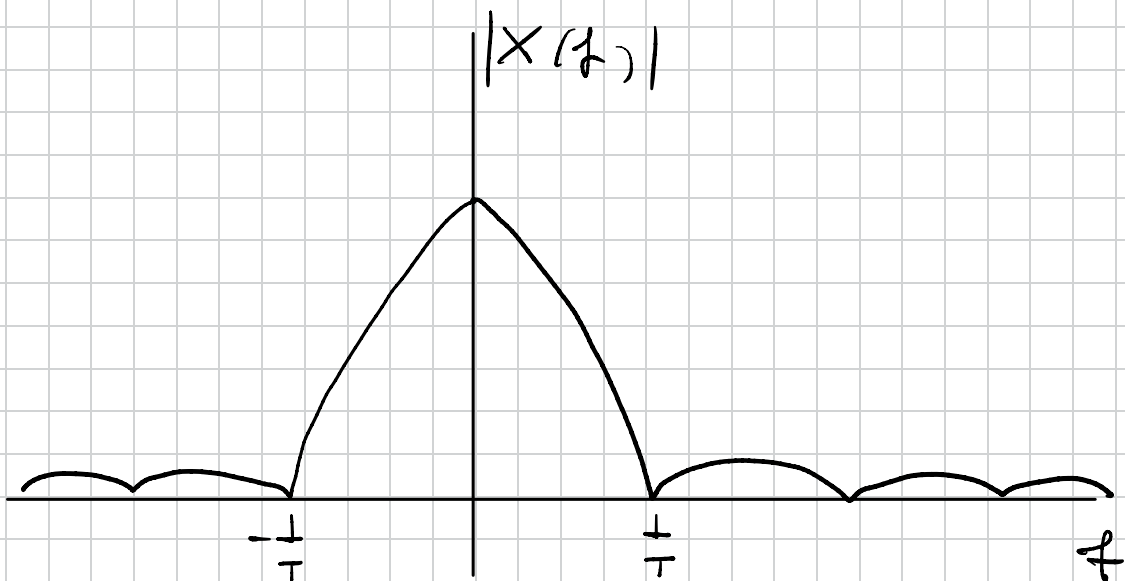
$$\int_{-b}^b X(t-t_0) e^{-j2\pi f t} dt$$

$$\begin{aligned} t-t_0 &= \alpha \\ &= \int_{-b}^b X(\alpha) e^{-j2\pi f(\alpha+t_0)} d\alpha \end{aligned}$$

$$= \int_{-b}^b X(\alpha) e^{-j2\pi f \alpha} e^{-j2\pi f t_0} d\alpha$$

$$= e^{-j2\pi f t_0} \underbrace{\int_{-b}^b X(\alpha) e^{-j2\pi f \alpha} d\alpha}_{X(f)}$$

$$= X(f) e^{-j2\pi f t_0}$$



TEOREMA DUPLA

$$X(t) \Leftrightarrow X(f)$$

$$X(t) \Leftrightarrow X(-f)$$

$$\text{rect}\left(\frac{t}{T}\right) \Leftrightarrow T \text{sinc}(fT)$$

$$\overset{B}{\cancel{T}} \text{sinc}\left(t \overset{B}{\cancel{T}}\right) \Leftrightarrow \text{rect}\left(\frac{f}{\overset{B}{\cancel{T}}}\right)$$

demo.

$$X(f) = \int_{-b}^b X(t) e^{-j2\pi f t} dt$$

$$\begin{aligned} X(t) &= \int_{-b}^b X(f) e^{-j2\pi f t} df \\ &= \int_{-b}^b X(-f) e^{j2\pi f t} df \end{aligned}$$

THEOREM PRODUCTIONS

$$X(t) \Leftrightarrow X(f)$$

$$X(t) \cos(2\pi f_0 t) \Leftrightarrow \frac{X(f-f_0) + X(f+f_0)}{2}$$

demo

$$\int_{-b}^b X(t) \cos(2\pi f_0 t) e^{-j2\pi f t} dt$$

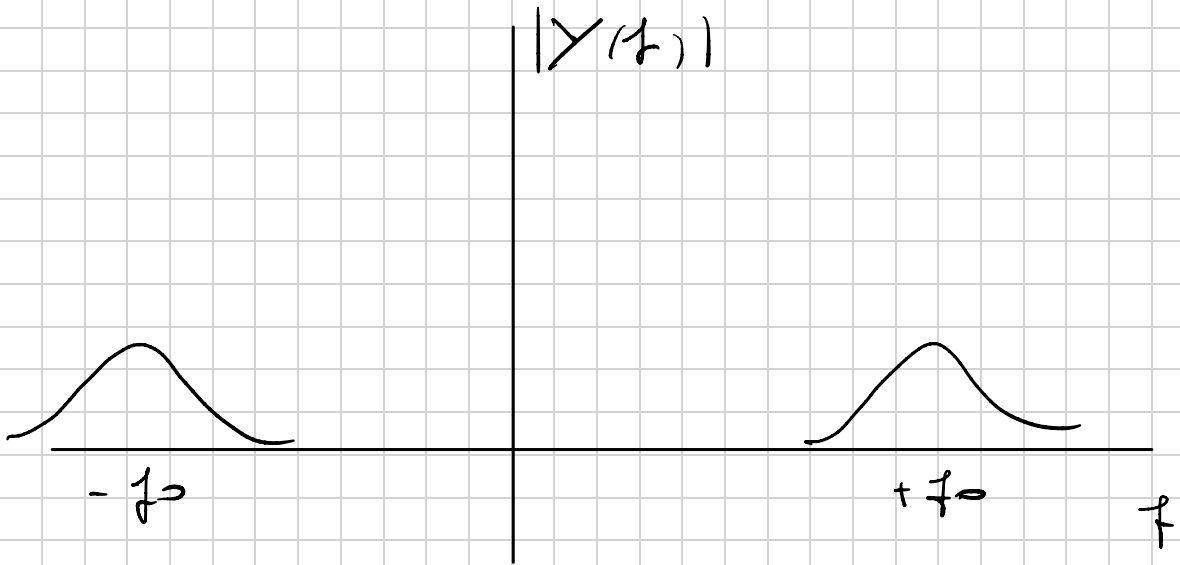
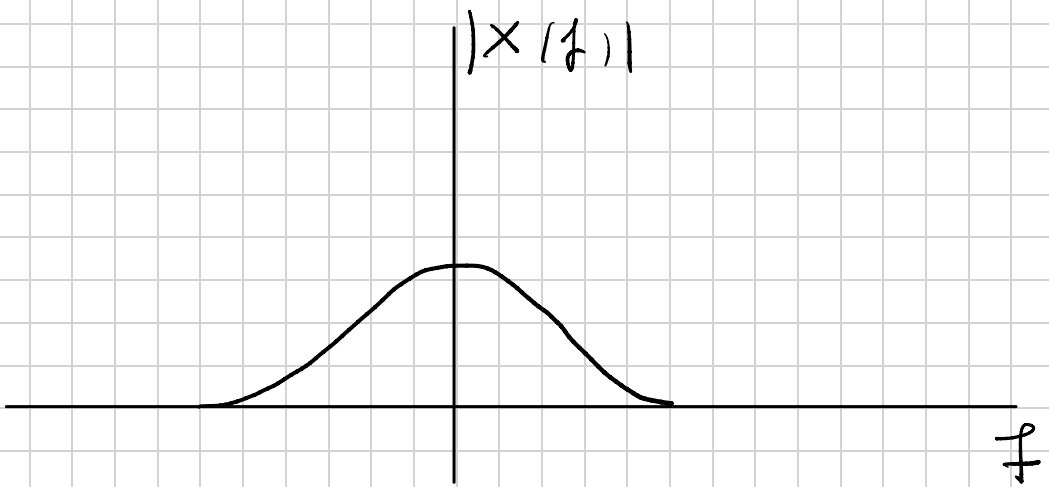
$$= \int_{-b}^b X(t) \left(\frac{e^{j2\pi f t} + e^{-j2\pi f t}}{2} \right) e^{-j2\pi f t} dt$$

$X(f-f_0)$

$$= \frac{1}{2} \left(\int_{-b}^b X(t) e^{-j2\pi(f-f_0)t} dt + \right.$$

$$\left. \int_{-b}^b X(t) e^{-j2\pi(f+f_0)t} dt \right)$$

$X(f+f_0)$



$$y(t) = x(t) \cos(2\pi f_0 t)$$