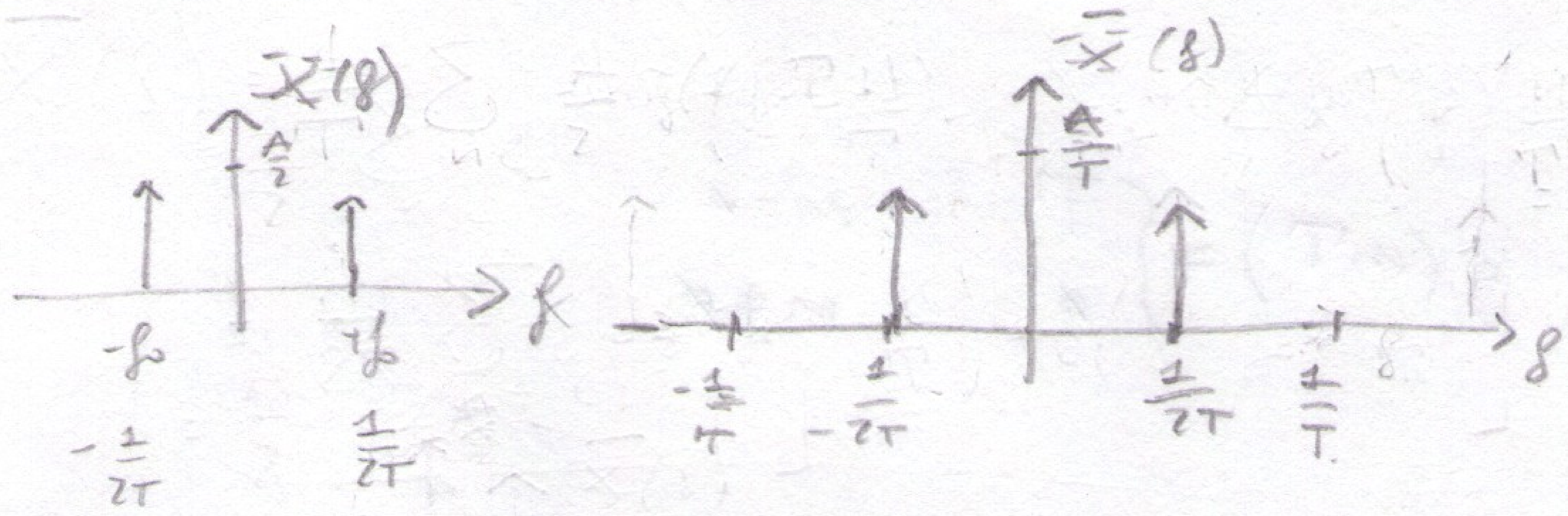
2:30 Esame 22/07/2010



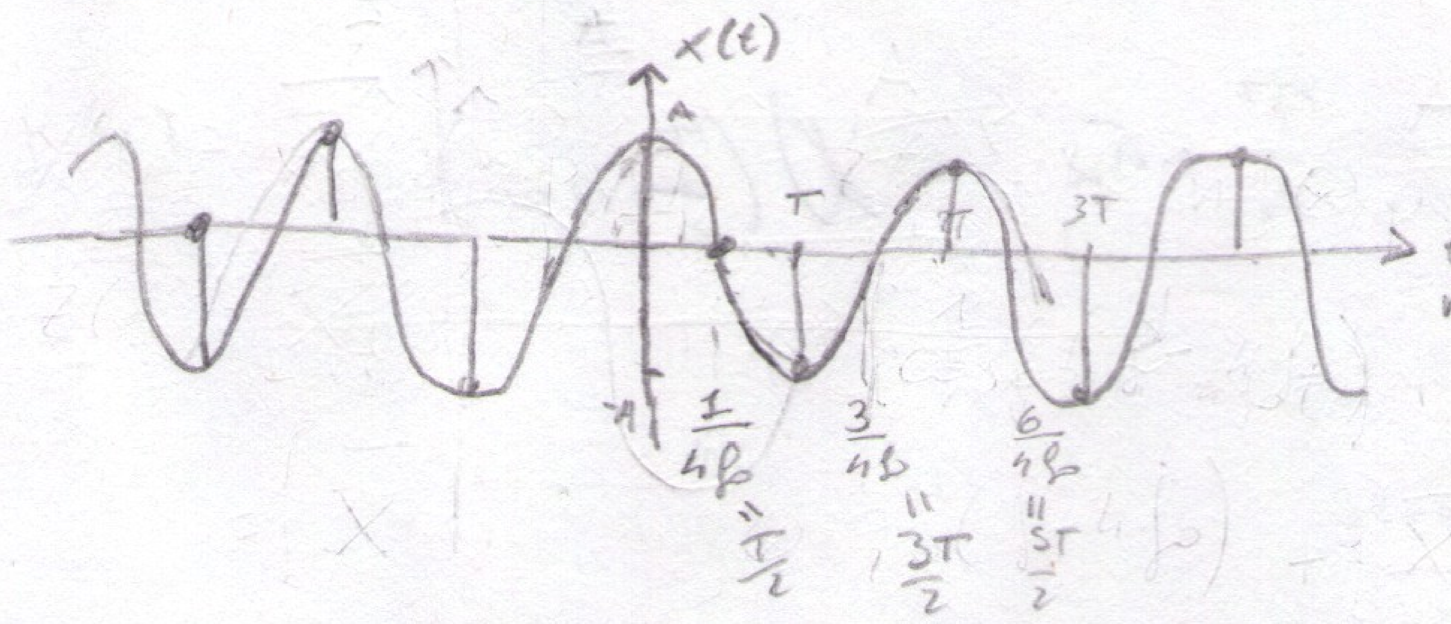
Esercizio 1

$$\frac{x(t)}{X(f)} \xrightarrow{T} \frac{x[n]}{\bar{X}(f)} \qquad \bar{X}(f) = \frac{1}{T}\sum_n X\left(f-\frac{n}{T}\right)$$

$$x(t) = A\cos(2\pi f_0 t) \rightleftharpoons X(f) = \frac{A}{2}\delta(f-f_0) + \frac{A}{2}\delta(f+f_0)$$




(1)




Esercizio 2

$$E_T = \sum_{k=0}^{1} P_k E_k \;;\; P_k = \frac{1}{2} \;;\; E_k = \int_{-\infty}^{\infty} |s_T(t)|^2\,dt$$

$$s_T(t) = m_k\, g_T(t)\,\mathrm{sen}^2(2\pi f_0 t+\theta) = m_k\, g_T(t)\left[\frac{1}{2}-\frac{1}{2}\cos(4\pi f_0 t+2\theta)\right]$$

$$E_k = \int_{-\infty}^{\infty} \left| m_k\, g_T(t)\frac{1}{2} - m_k\, g_T(t)\frac{1}{2}\cos(4\pi f_0 t+2\theta)\right|^2 dt =$$

$$= \int_{-\infty}^{\infty}\left[m_k^2 g_T^2(t)\frac{1}{4} + m_k^2 g_T^2(t)\frac{1}{4}\cos^2(4\pi f_0 t+2\theta) - 2m_k^2 g_T^2(t)\frac{1}{4}\cos(4\pi f_0 t+2\theta)\right]dt =$$

$$= \frac{1}{4}m_k^2 E_{g_T} + \frac{1}{4}m_k^2\int_{-\infty}^{\infty}\frac{1}{2}g_T^2(t)\,dt + \frac{1}{4}m_k^2\underbrace{\int_{-\infty}^{\infty}\frac{1}{2}g_T^2(t)\cos(8\pi f_0 t+4\theta)\,dt}_{\simeq 0} +$$

$$-\frac{1}{2}m_k^2\underbrace{\int_{-\infty}^{\infty} g_T^2(t)\cos(4\pi f_0 t+2\theta)\,dt}_{\simeq 0} =$$

$$= \frac{1}{4}m_k^2 E_{g_T} + \frac{1}{8}m_k^2 E_{g_T} = \frac{3}{8}m_k^2 E_{g_T}$$

$$E_T = P_0 E_0 + P_1 E_1 = \frac{1}{2}E_{g_T}\frac{3}{8} + \frac{1}{2}E_{g_T}\frac{3}{8} = \frac{3}{8}E_{g_T} \quad (1)$$

$$n(t) = w(t)\,2\cos(2\pi f_0 t)\otimes g_R(t)$$

$$\tilde{n}_c(t) = \tilde{w}_c(t)\otimes g_R(t) \;;\; P_{\tilde{n}_c} = \int_{-\infty}^{\infty} S_{\tilde{n}_c}(f)\,df \;;\; S_{\tilde{n}_c}(f) = S_{\tilde{w}_c}(f)\,|G_R(f)|^2$$

$$S_{\tilde{w}}(f) = \begin{cases} 4S_w(f+f_0) & f \geq -f_0 \\ 0 & \text{altrove}\end{cases} = 2N_0\,\mathrm{tri}\left(\frac{f}{2B}\right) \;;\; S_{\tilde{w}_c} = \frac{S_{\tilde{w}}(f)+S_{\tilde{w}}(-f)}{4} = N_0\,\mathrm{tri}\left(\frac{f}{2B}\right)$$

$$P_{\tilde{n}_c} = \int_{-\infty}^{\infty} N_0\,\mathrm{tri}\left(\frac{f}{2B}\right)|G_R(f)|^2\,df = N_0 T\int_{-\frac{1}{2T}}^{\frac{1}{2T}}\mathrm{tri}\left(\frac{f}{2B}\right)df = N_0 T\int_{-\frac{1}{2T}}^{\frac{1}{2T}}\mathrm{tri}\left(\frac{f}{1/T}\right)df = N_0 \quad (2)$$

$$g(t) = g_{Tc}(t)\otimes g_R(t) \Rightarrow G(f) = G_{Tc}(f)\,G_R(f) = G_R^2(f) = T\,\mathrm{rect}\left(\frac{f}{1/T}\right)$$

$$r(t) = \underbrace{\sum_i a_i\, g_T(t-iT)}_{S_T(t)} \operatorname{sen}^2(2\pi f_0 t+\theta) + w(t) =$$

$$= S_T(t)\cdot \operatorname{Re}\left\{ j\operatorname{sen}(2\pi f_0 t+\theta)\, e^{j\theta}\, e^{j2\pi f_0 t}\right\} = \operatorname{Re}\left\{S_T(t)\, j\operatorname{sen}(2\pi f_0 t+\theta)\, e^{j\theta} e^{j2\pi f_0 t}\right\}$$

$$\tilde r(t) = S_T(t)\, j\operatorname{sen}(2\pi f_0 t+\theta)\, e^{j\theta} = S_T(t)\, j\operatorname{sen}(2\pi f_0 t+\theta)\left[\cos(\theta) - j\operatorname{sen}(\theta)\right] =$$

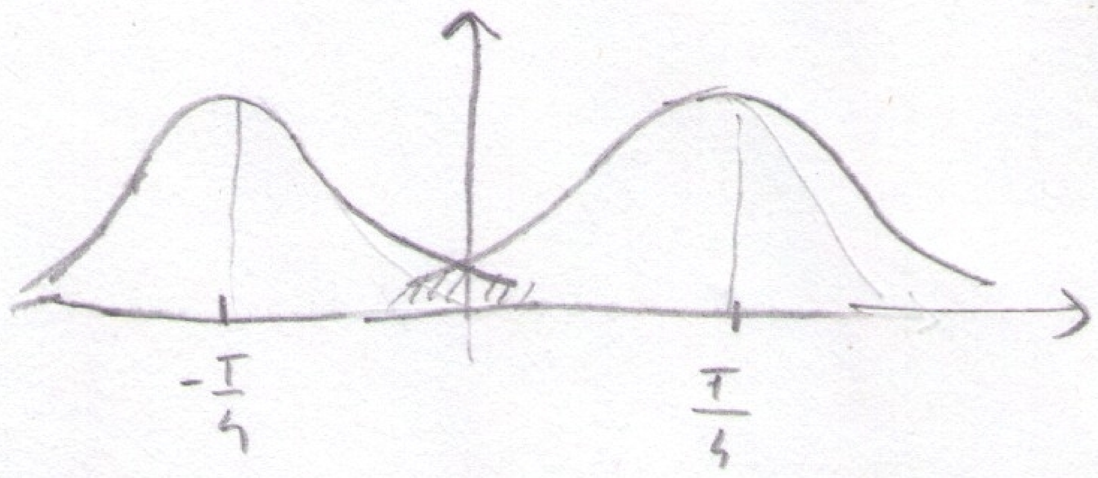
$$= S_T(t)\, j\operatorname{sen}(2\pi f_0 t+\theta)\cos(\theta) + S_T(t)\operatorname{sen}(2\pi f_0 t+\theta)\operatorname{sen}(\theta) + n(t)$$

$$\tilde y(t) = \frac{1}{2} S_T(t)\operatorname{sen}(2\pi f_0 t+\theta) + n(t)$$

$$z(t) = \sum_i \frac{1}{2} a_i\, g(t-iT)\operatorname{sen}(2\pi f_0 t+\theta) + n(t) \quad ; \quad g(mT) = \begin{cases} T & m=0 \\ 0 & m\neq 0\end{cases}$$

$$z_k = a_k \frac{T}{2}\operatorname{sen}(\theta) + \tilde n_k \cong a_k \frac{T}{4} + \tilde n_k$$

$$z_k\Big|_{a_k=-1} = -\frac{T}{4} + \tilde n_k \in \mathcal{N}\left(-\frac{T}{4}, \delta_{\tilde n}^2\right) \quad ; \quad z_k\Big|_{a_k=+1} = \frac{T}{4} + \tilde n_k \in \mathcal{N}\left(\frac{T}{4}, \delta_{\tilde n}\right)$$




$$P(e)\Big|_{a_k=-1} = P(e)\Big|_{a_k=+1} =$$

$$P(e) = Q\left(\frac{0+\frac{T}{4}}{\delta_{\tilde n}}\right) = Q\left(\frac{T}{4\,\delta_{\tilde n}}\right) = Q\left(\sqrt{\frac{T^2}{16 N_0}}\right)$$

$$\delta_{\tilde n} = \sqrt{\delta_{\tilde n}^2} = \sqrt{N_0} \quad ; \quad P(e) = BER = Q\left(\sqrt{\frac{T^2}{16 N_0}}\right)$$