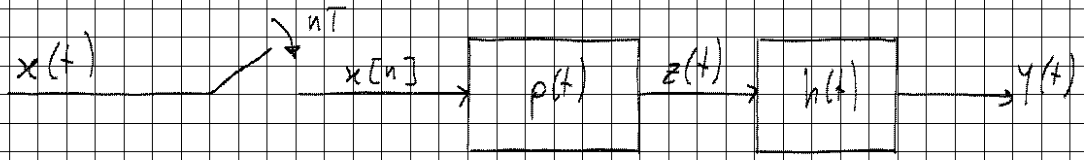
# ESERCIZIO — CAMPIONAMENTO E RICOSTRUZIONE




.) $x(t) = \text{sinc}\left(\frac{t}{T}\right)$

.) $p(t)$ = interpolatore a mantenimento

.) $H(f) = \frac{\pi f T}{\sin(\pi f T)} \, \text{rect}(fT)$

→ Calcolare $y(t)$

Soluzione:

$$x[n] = x(nT) = \text{sinc}^2\left(\frac{nT}{T}\right) = \text{sinc}^2(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$x[n] = \delta[n]$$

$$z(t) = \sum_n x[n] \, p(t - nT) = p(t)$$

$$y(t) = p(t) \otimes h(t)$$

$$Y(f) = P(f) \, H(f)$$

$$p(t) = \text{rect}\left(\frac{t - T/2}{T}\right) \overset{TCF}{\Longleftrightarrow} P(f) = T \, \text{sinc}(Tf) \, e^{-j\pi f T}$$

$$Y(f) = T \, \frac{\sin(\pi f T)}{\pi f T} \, e^{-j\pi f T} \cdot \frac{\pi f T}{\sin(\pi f T)} \, \text{rect}(fT) =$$

$$= T e^{-j\pi f T} \, \text{rect}(fT)$$

$$y(t) = \operatorname{sinc}\left(\frac{t - T/2}{T}\right)$$

N.B. attenzione: sembra un sistema che produce una replica fedele. In realtà è solo un caso. Basta provare a sostituire $x(t) = \operatorname{sinc}^2\left(\frac{t}{T}\right)$ per dimostrare che non lo è.

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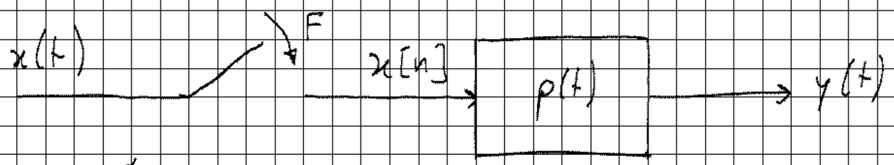


$$x(t) = e^{-\frac{t}{\alpha}} u(t)$$

$p(t)$ interpolatore a mantenimento

$$F = \frac{1}{T} = \frac{10}{\alpha}$$

Calcolare $E_x$ e $E_y$

Soluzione

$$E_x = \int_{-\infty}^{+\infty} e^{-\frac{2t}{\alpha}} u(t)\, dt = \int_0^{+\infty} e^{-\frac{2t}{\alpha}} dt = -\frac{\alpha}{2} e^{-\frac{2t}{\alpha}} \Big|_0^{\infty} =$$

$$= \frac{\alpha}{2}$$

$$y(t) = \sum_n x[n]\, \operatorname{rect}\left(\frac{t - T/2 - nT}{T}\right)$$

$$E_y = \int_{-\infty}^{+\infty} \sum_n x^2[n]\, \operatorname{rect}\left(\frac{t - T/2 - nT}{T}\right) dt =$$

$$= \sum_n x^2[n] \int_{-\infty}^{+\infty} \mathrm{rect}\left(\frac{t - T/2 - nT}{T}\right) dt =$$

$$= T\sum_n x^2[n] = T\sum_{n=0}^{\infty} e^{-\frac{2}{\alpha}nT} = T\sum_{n=0}^{\infty} a^n \quad , \quad a = e^{-\frac{2T}{\alpha}} < 1$$

$$= T\,\frac{1}{1-a} = \frac{T}{1-e^{-\frac{2T}{\alpha}}} = \frac{\frac{\alpha}{10}}{1-e^{-\frac{1}{5}}} = \frac{1}{5\left(1-e^{-\frac{1}{5}}\right)}\, E_x$$

$$\simeq 1{,}1\, E_x$$

Era da aspettarsi che $E_y > E_x$