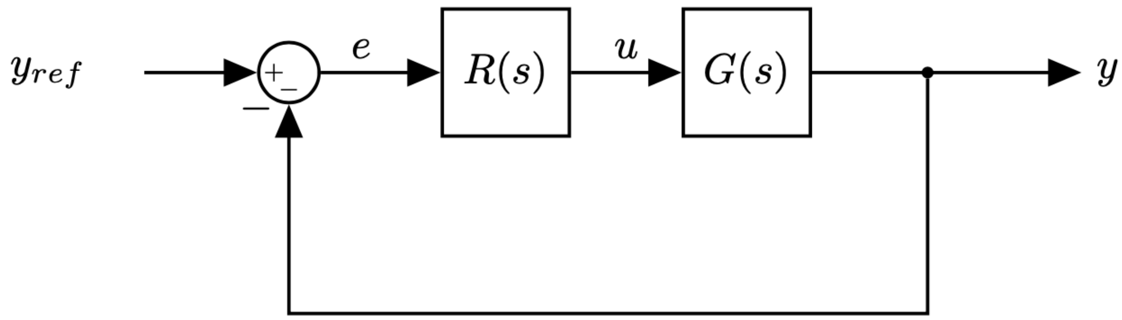
Main types of loops and transfer functions

In [ ]:
```
#| default_exp loops_and_tfs
```

The classical viewpoint

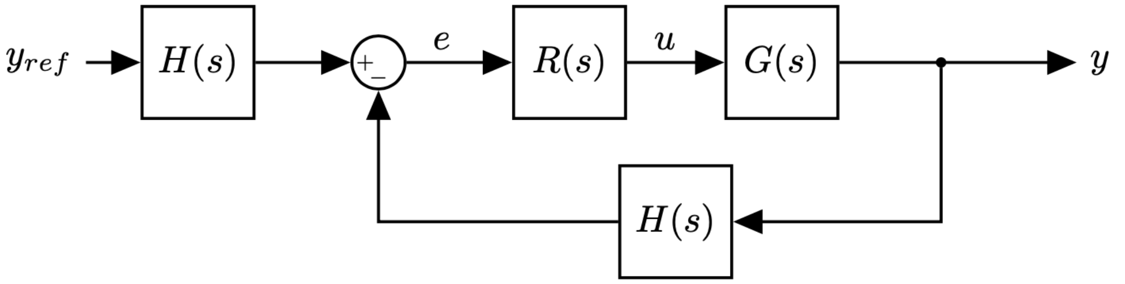
Standard "servo" or tracking configuration of classical feedback control:

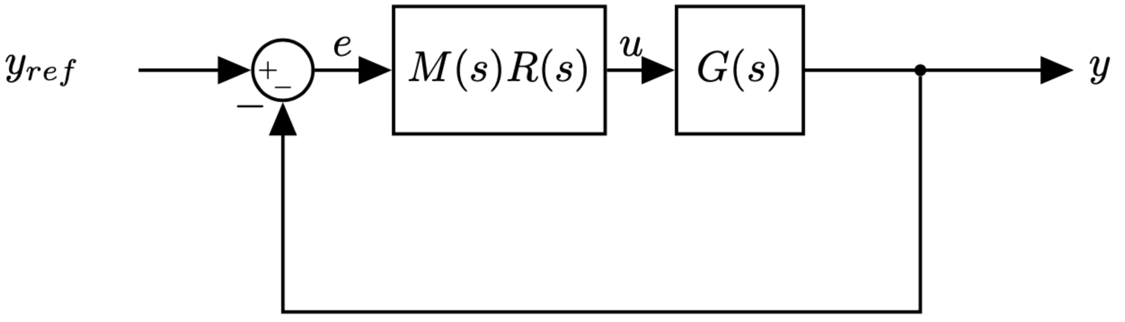


- $R(s)$: Controller/Compensator
- $G(s)$: Plant
- $Y_{ref}(s)$: input (reference)
- $e(s)$: error
- $U(s)$: control signal

- This is the "standard control loop"

Alternatives

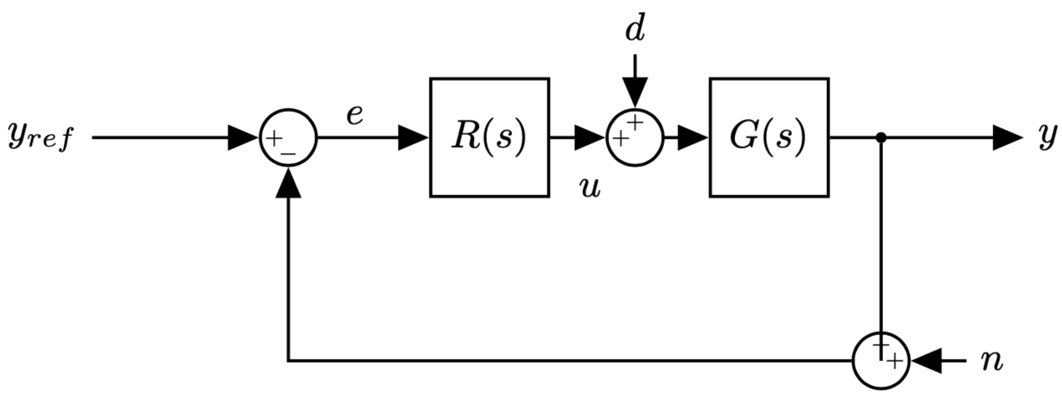




- We can always reshape them into the standard form.

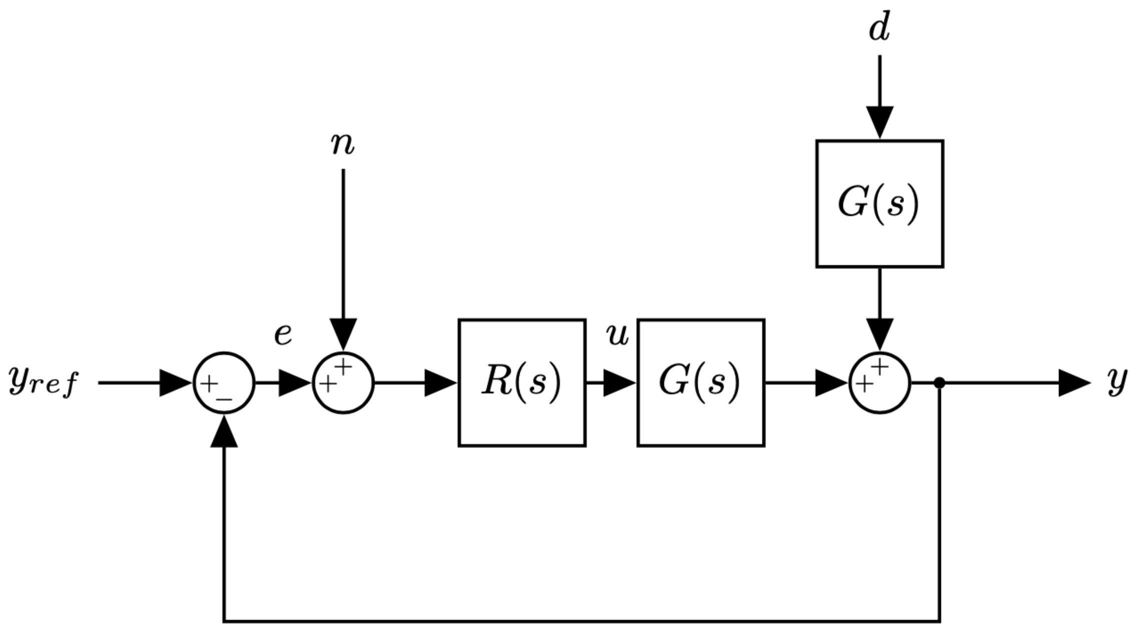
Disturbances

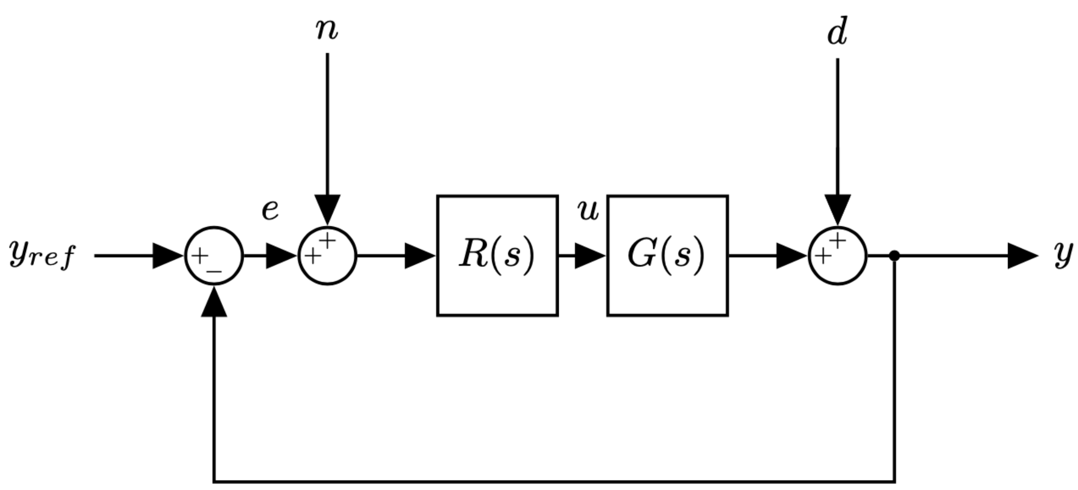
More in general:



- Load disturbance: assumed to act on the process input (but can enter in many different ways)
- Measurement disturbance
- The process is a system with three inputs (control signal, load disturbance and measurement noise) and one output, the measured signal.

Or equivalently





- The feedback loop is influenced by three signals: y_{ref}, d, and n.
- There are three interesting outputs: e, u, y

Comments

- Attenuation of load disturbances is often a primary goal for control
- Load disturbances are typically dominated by low frequencies (slow varying).
  - Example: car cruise control: disturbance is gravity and changes with the slope of the road
- Measurement noise corrupts the information about the process vari- able that the sensors delivers
- Measurement noise is typically higher frequency (average typically zero)
- Sometimes sensors have dynamics: often very accurate values are provided by slow sensors

In a typical control design problem we would choose the compensator $R(s)$ such as:

- the closed-loop system is stable
- the loop gain $R(s)G(s)$ has large magnitude at frequencies (low frequencies typically) where the power of the reference input r (and the plant disturbance d) is concentrated
- the loop gain has small magnitude at frequencies (high frequency typically) where the power of the measurement noise n is concentrated).

Note To obtain all the previous requirements it is convenient to state the close loop stability in terms of the open loop loop gain. This is provided by the *Nyquist stability criterion* as we will see later.

To understand the second and third requirements more, we can write the transfer function between any input-output pair

$$E(s) = \frac{1}{1+RG}Y_{ref}(s), \qquad Y(s) = \frac{1}{1+RG}D(s)$$

$$Y(s) = \frac{RG}{1+RG}Y_{ref}(s), \qquad Y(s) = \frac{RG}{1+RG}N(s)$$

$$U(s) = \frac{R}{1+RG}Y_{ref}(s)$$

- Some transfer functions are the same
- All transfer functions have $1+RG$ at the denominator: we only study stability once
- Transfer functions on the left column give the response of process variable to the set point
- The transfer function $\frac{1}{(1+RG)}$ tells how the process variable reacts to load disturbances
- The transfer function $\frac{RG}{(1+RG)}$ gives the response of the output signal to measurement noise.

- The transfer function $L(s) = R(s)G(s)$ is called *loop gain* or *loop transfer function*.

- **Sensitivity function**

$$S(s) = \frac{1}{1+RG}$$

- Describes how feedback influences the disturbances
- If $|RG|$ is large at frequencies where the power of the disturbance is concentrated, then $|S|$ is small and the effect of the disturbance on the output is attenuated
- Lower values of $|S|$ means higher attenuation of the external disturbance.
- Typically, plant disturbances are low frequency, and one would like $|RG|$ to be large at low frequency

Note

- The same transfer function also relates e and y_{ref}.
- If we want y to track r with good accuracy we want a small response of the error signal to the driving signal (*tracking accuracy*).

- $|S|$ should be small (or equivalently $|RG|$ large) at frequencies where the power of y_{ref} is concentrated.
- In most applications the reference signal is slowly varying and this means that we need $|RG|$ to be large at low frequencies

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- **Complementary sensitivity**

$$T(s) = \frac{RG}{1+RG}$$

- Maps the noise input n to the output y

- Noise rejection defines high frequency specifications
- $S+T=1$

- Note that T is also the transfer function from y_{ref} to y.

- If $|RG|$ is small at frequencies where the noise n is concentrated then $|T|$ will be small and the effect of the noise on the output is minimised.
- Measurement noise tend to occur at high frequency and this means that typically we would like $|RG|$ to be small at high frequency
- This constraint does not conflict with the low-frequency constraints for the disturbance d and the reference y_{ref}.

Control design task

- Given a plant G, we need to design a compensator R such that:
- The loop gain magnitude |RG| is large at low frequencies (to track the reference and reject disturbance)
- The loop gain magnitude |RG| rolls off (steeply decreases) to low values at high frequencies (to reject measurement noise)
- The stability of the system must be guaranteed

Final Note:

- Remember that we can have additional requirements on:
  - steady state response. This is typically specified in terms of response to known signals (e.g., step response error less than desired threshold (e.g. <3%), etc)
  - transient response (e.g. settling time, raise time, maximum overshoot - see also notebook `05_system_response.ipynb`
