ANALISI ENERGETICA DI SEGNALI APERIODICI

Correlazione

$$C_{xy}(\tau) \triangleq \int_{-\infty}^{+\infty} x(t)\, y^*(t-\tau)\, dt$$

Autocorrelazione

$$C_x(\tau) \triangleq \int_{-\infty}^{+\infty} x(t)\, x^*(t-\tau)\, dt$$

Proprietà della autocorrelazione

1) $C_x(0) = \int_{-\infty}^{+\infty} |x(t)|^2\, dt = E_x$

2) $C_x^*(\tau) = C_x(-\tau)$

Dimostrazione

$$C_x(-\tau) = \int_{-\infty}^{+\infty} x(t)\, x^*[t-(-\tau)]\, dt =$$

$$= \int_{-\infty}^{+\infty} x(t)\, x^*(t+\tau)\, dt = (t' = t+\tau) =$$

$$= \int_{-\infty}^{+\infty} x(t'-\tau)\, x^*(t')\, dt' = C_x^*(\tau)$$

TCF DELLA AUTOCORRELAZIONE

$$C_x(\tau) \overset{TCF}{\Longleftrightarrow} |X(f)|^2 = S_x(f)$$

Dimostrazione

$$S_x(f) = \int_{-\infty}^{+\infty} C_x(\tau)\, e^{-j2\pi f\tau}\, d\tau =$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x(t)\, x^*(t-\tau)\, dt\, e^{-j2\pi f\tau}\, d\tau =$$

$$= \int_{-\infty}^{+\infty} x(t) \int_{-\infty}^{+\infty} x^*(t-\tau)\, e^{-j2\pi f\tau}\, d\tau\, dt = (\alpha = t-\tau)$$

$$= \int_{-\infty}^{+\infty} x(t) \int_{-\infty}^{+\infty} x^*(\alpha)\, e^{-j2\pi f(t-\alpha)}\, d\alpha\, dt =$$

$$= \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi ft}\, dt \int_{-\infty}^{+\infty} x^*(\alpha)\, e^{j2\pi f\alpha}\, d\alpha =$$

$$= X(f) \left[\int_{-\infty}^{+\infty} x(\alpha)\, e^{-j2\pi f\alpha}\, d\alpha \right]^* = X(f)\, X^*(f) = |X(f)|^2$$

Proprietà

$$C_x(\tau) = FT^{-1}\left[S_x(f) \right]$$

$$C_x(\tau) = \int_{-\infty}^{+\infty} S_x(f)\, e^{j2\pi f\tau}\, df =$$

$$= \int_{-\infty}^{+\infty} |X(f)|^2 e^{j2\pi f\tau}\, df$$

$$C_x(0) = \int_{-\infty}^{+\infty} |X(f)|^2 df = E_x$$

$S_x(f) = |X(f)|^2$ = DENSITÀ SPETTRALE DI ENERGIA

NB Per calcolare l'energia di un segnale aperiodico si possono prendere due strade

I) $C_x(0) = \int_{-\infty}^{+\infty} |x(t)|^2 dt$

II) $\int_{-\infty}^{+\infty} |X(f)|^2 df$

TEOREMA DI PARSEVAL

$$\int_{-\infty}^{+\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{+\infty} X(f)\, Y^*(f)\, df$$

Dimostrazione

$$\int_{-\infty}^{+\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} X(f)\, e^{j2\pi ft}\, df\, y^*(t)\, dt$$

$$= \int_{-\infty}^{+\infty} X(f) \int_{-\infty}^{+\infty} y^*(t)\, e^{j2\pi ft}\, dt\, df = \int_{-\infty} X(f)\, Y^*(f)\, df$$

Esempio

Calcolare l'energia del segnale

$$x(t) = A \operatorname{sinc}(Bt)$$

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2\, dt = A^2 \int_{-\infty}^{+\infty} \operatorname{sinc}^2(Bt)\, dt$$

$$X(f) = \frac{A}{B} \operatorname{rect}\left(\frac{f}{B}\right)$$

$$E_x = \int_{-\infty}^{+\infty} |X(f)|^2\, df = \frac{A^2}{B^2} \int_{-\infty}^{+\infty} \operatorname{rect}\left(\frac{f}{B}\right) df = \frac{A^2}{B}$$

CORRELAZIONE E CONVOLUZIONE

$$C_{xy}(\tau) = x(\tau) \otimes y^*(-\tau)$$

Dimostrazione

$$x(\tau) \otimes y^*(-\tau) = \int_{-\infty}^{+\infty} x(t)\, y^*[-(\tau - t)]\, dt$$

$$= \int_{-\infty}^{+\infty} x(t)\, y^*(t-\tau)\, dt = C_{xy}(\tau)$$