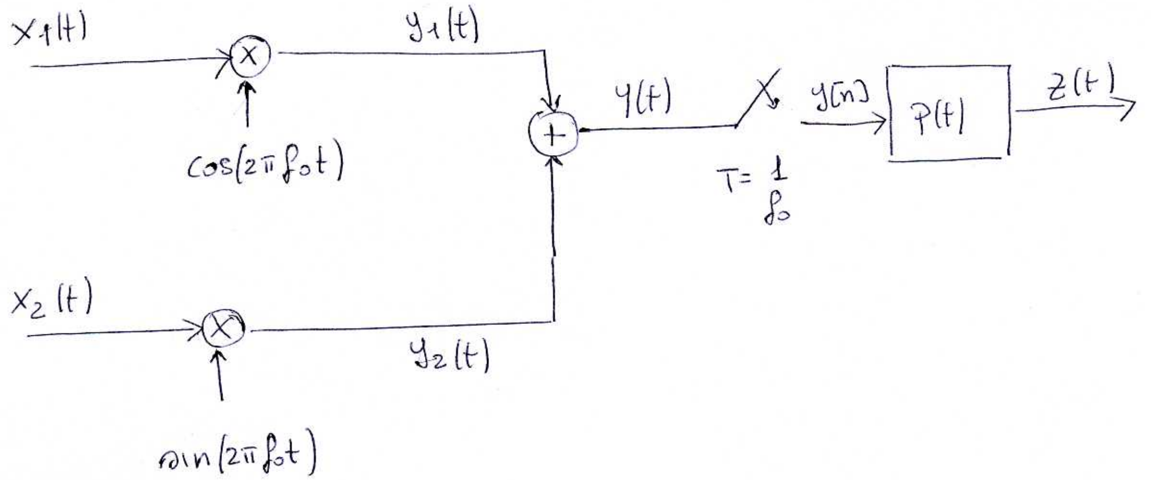
Esercizio # 1 COMPITO 19/07/2012



$$x_1(t) = 2B \operatorname{sinc}(2Bt) \rightleftharpoons X_1(f) = \operatorname{rect}\left(\frac{f}{2B}\right)$$

$$x_2(t) = B \operatorname{sinc}^2(Bt) \rightleftharpoons X_2(f) = \left(1 - \frac{|f|}{B}\right) \operatorname{rect}\left(\frac{f}{2B}\right)$$

1) $y_1(t) = x_1(t) \cdot \cos(2\pi f_0 t)$

$$Y_1(f) = X_1(f) \otimes \left[\frac{\delta(f-f_0) + \delta(f+f_0)}{2}\right]$$

$y_2(t) = x_2(t) \cdot \sin(2\pi f_0 t)$

$$Y_2(f) = X_2(f) \otimes \left[\frac{\delta(f-f_0) - \delta(f+f_0)}{2j}\right]$$

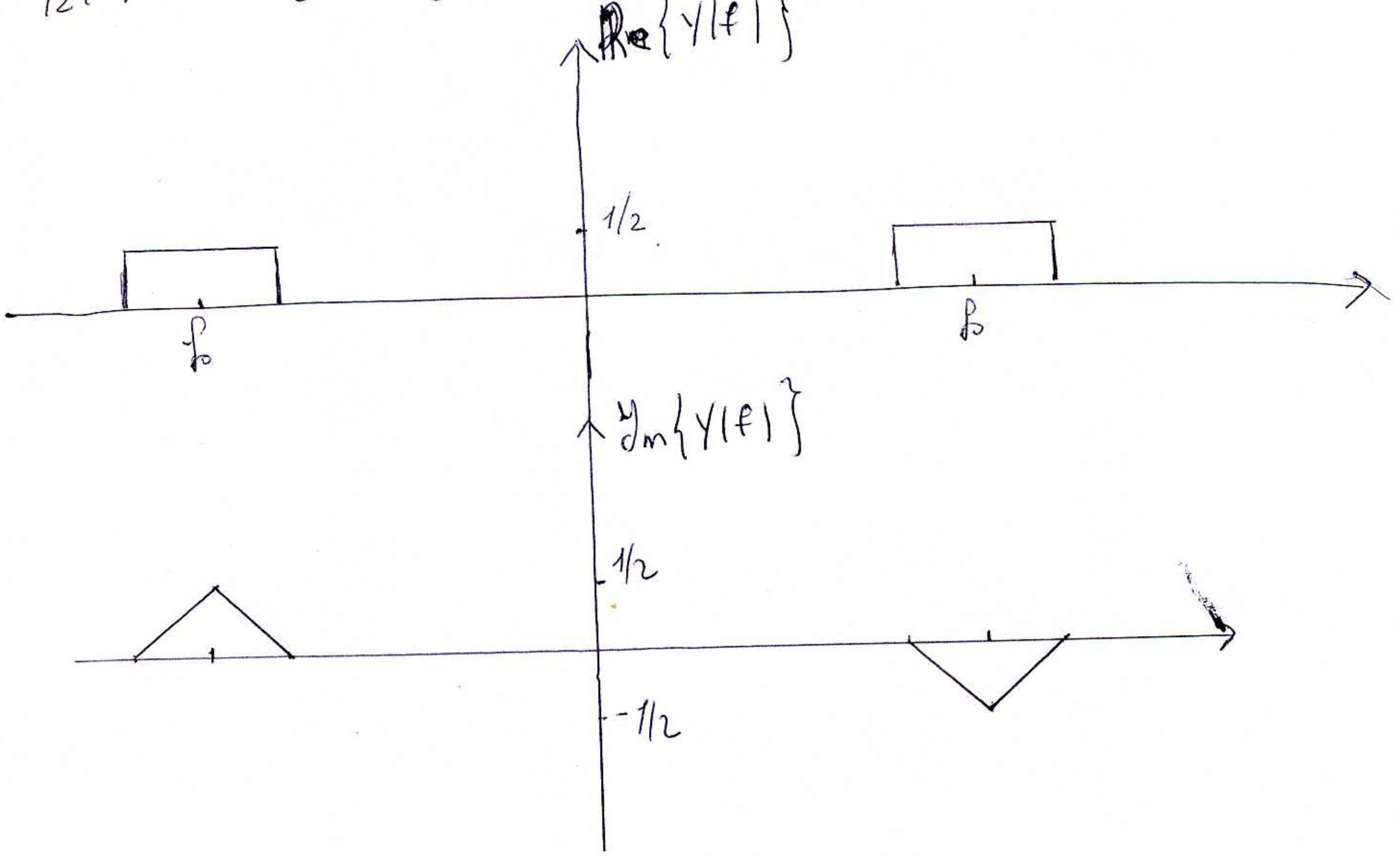
$$Y_1(f) = \frac{1}{2}\left[\text{rect}\left(\frac{f-f_0}{2B}\right) + \text{rect}\left(\frac{f+f_0}{2B}\right)\right]$$

$$Y_2(f) = \frac{j}{2}\left[\left(1-\frac{|f+f_0|}{B}\right)\text{rect}\left(\frac{f+f_0}{2B}\right) - \left(1-\frac{|f-f_0|}{B}\right)\cdot\text{rect}\left(\frac{f-f_0}{2B}\right)\right]$$

$$Y(f) = Y_1(f) + Y_2(f)$$

$$Y_1(f) = \Re\{Y(f)\}$$

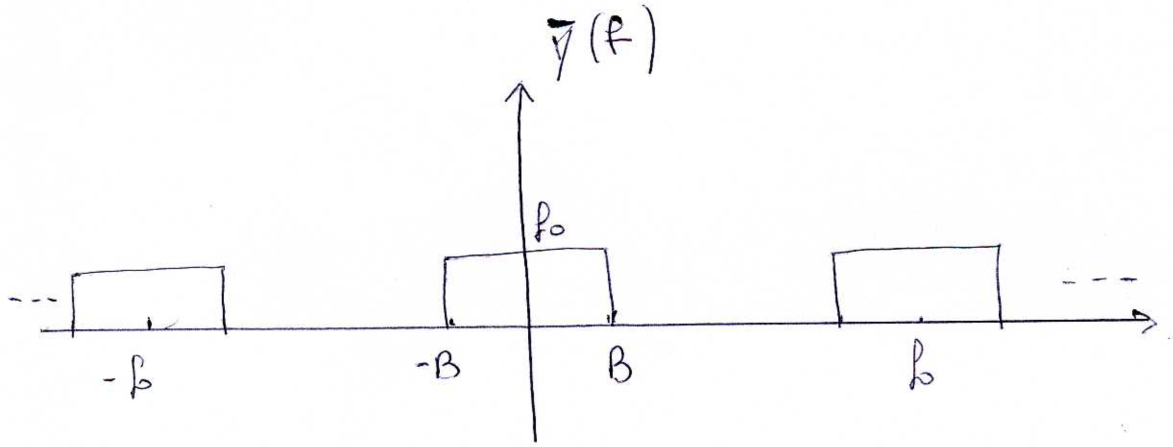
$$Y_2(f) = \Im\{Y(f)\}$$



$$|Y(f)| = \sqrt{Y_1^2(f) + Y_2^2(f)} \qquad \angle Y(f) = \operatorname{atan}\left(\frac{Y_2(f)}{Y_1(f)}\right)$$

2) $$y[n] \rightleftharpoons \bar{Y}(f) = \frac{1}{T}\sum_k Y\left(f - \frac{k}{T}\right) =$$

$$= f_0 \sum_k Y(f - kf_0)$$

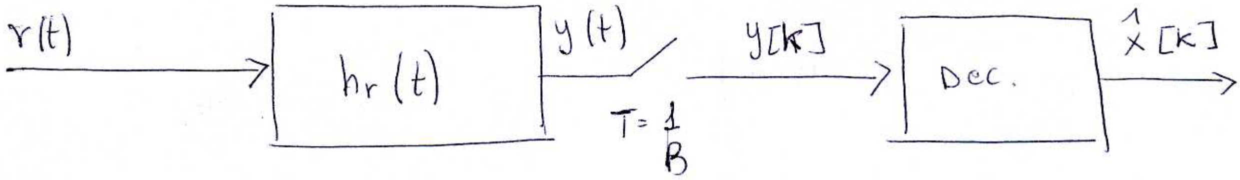


$$\mathrm{Im}\{\bar{Y}(f)\} = 0$$

$$z(t) = f_0 \,\mathrm{sinc}(Bt) \qquad \Rightarrow \qquad E_z = f_0$$

$$Z(f) = \frac{f_0}{B}\,\mathrm{rect}\left(\frac{f}{B}\right) \qquad P_z = 0$$

Esercizio # 2 compito 19/07/2012



$$s(t) = \sum_k x[k]\, p(t - kT)$$

$x[k] \in \{-2; +3\}$ indipendenti ed equiprobabili.

$p(t) = 2B \operatorname{sinc}(2Bt)$

$c(t) = B \operatorname{sinc}^2(Bt)$

$S_N(f) = \frac{N_0}{2}$

$h_r(t) = 2B \operatorname{sinc}(2Bt)$

1) $E_s = \frac{E_{s1}}{2} + \frac{E_{s2}}{2}$

$E_{s1} = \int_{-\infty}^{+\infty} s_1^2(t)\, dt \qquad s_1(t) = -2 \cdot p(t)$

$E_{s2} = \int_{-\infty}^{+\infty} s_2^2(t)\, dt \qquad s_2(t) = 3 \cdot p(t)$

$$E_p = \int_{-\infty}^{+\infty} p^2(t)\, dt = \int_{-\infty}^{+\infty} P^2(f)\, df = 2B$$

$$P(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$E_s = \frac{1}{2} \cdot 4 \cdot E_p + \frac{1}{2} \cdot 9 \cdot E_p = 13B$$

2) $y(t) = r(t) \otimes h_r(t) =$

$$= \sum_k x[k]\, h(t-kT) + w(t)$$

$$h(t) = p(t) \otimes c(t) \otimes h_r(t)$$

$$w(t) = n(t) \otimes h_r(t) \qquad S_N(f) = \frac{N_0}{2}$$

$$P_{nu} = \int_{-\infty}^{+\infty} S_w(f)\, df = N_0 B$$

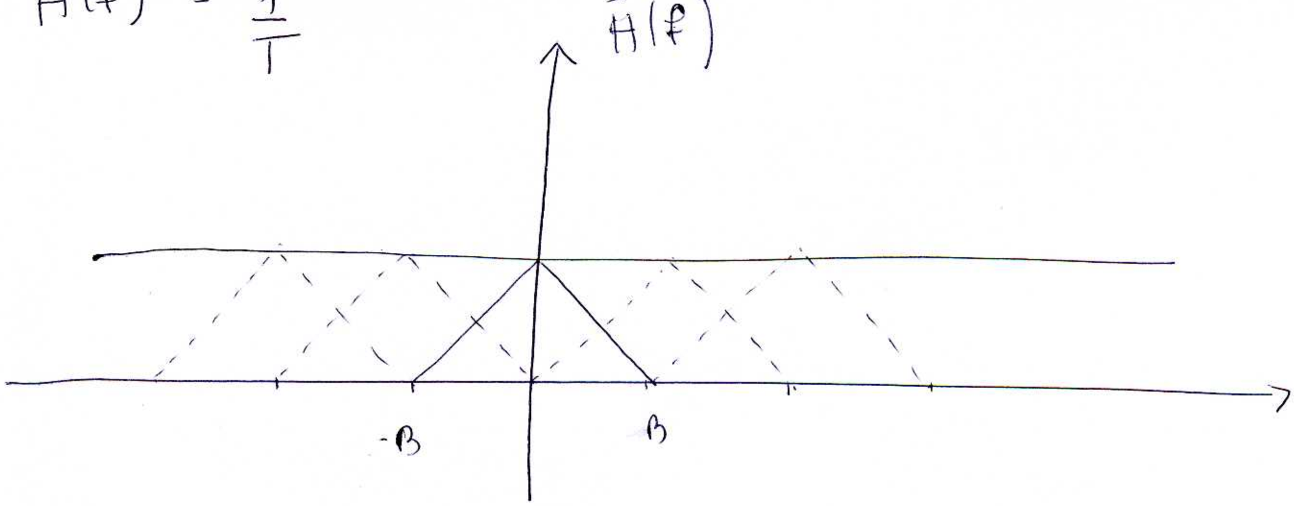
3) Prima di calcolare la $P_E(b)$ è necessario verificare se è valida la [redacted] condizione di Nyquist.

$$h(t) = p(t) \otimes c(t) \otimes h_r(t)$$

$$H(f) = P(f) \cdot C(f) \cdot H_r(f) = \mathrm{rect}\left(\frac{f}{2B}\right) \cdot \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right) \cdot \mathrm{rect}\left(\frac{f}{2B}\right)$$

$$H(f) = \left(1 - \frac{|f|}{B}\right) \mathrm{rect}\left(\frac{f}{2B}\right)$$

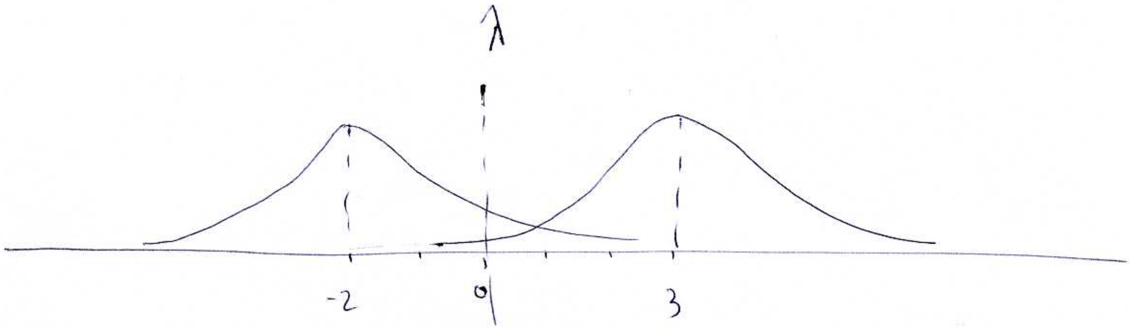
$$\bar{H}(f) = \frac{1}{T}$$



La condizione di Nyquist all'istante di campionamento è verificata.

$$y[k] = h(0) \cdot x[k] + w[k] = x[k] + w[k]$$

$w[k] \in \mathcal{N}(0, \sigma_{nu})$



$$P_E(b) = \frac{1}{2} \Pr\{\hat{x}[k] = -2 \mid x[k] = 3\} + \frac{1}{2} \Pr\{\hat{x}[k] = 3 \mid x[k] = -2\} =$$

$$= \frac{1}{2} \cdot Q\left(\frac{3}{\sqrt{N_0 T}}\right) + \frac{1}{2} \cdot Q\left(\frac{2}{\sqrt{N_0 T}}\right)$$