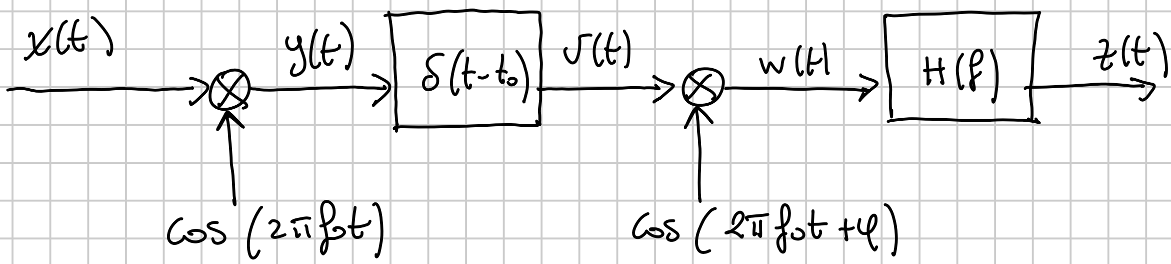
Esercizio #1 SISTEMA WIRELESS



$$X(f) = \left[1 + \cos\left(\frac{\pi f}{B}\right)\right] \mathrm{rect}\left(\frac{f}{2B}\right)$$

$H(f)$ passa basso ideale di banda B

$$H(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$$

Calcolare:

1) $Y(f)$
2) $W(f)$
3) $Z(f)$ e $z(t)$
4) Determinare il valore di φ per cui $z(t)$ ha energia massima

(1) $y(t) = x(t) \cdot \cos(2\pi f_0 t)$ MODULAZIONE

$$Y(f) = X(f) \otimes \left[\frac{\delta(f-f_0) + \delta(f+f_0)}{2}\right]$$

$$Y(f) = \frac{X(f-f_0)}{2} + \frac{X(f+f_0)}{2}$$

2) $v(t) = y(t) \otimes \delta(t-t_0) = y(t-t_0)$ TEOREMA DEL RITARDO

$$V(f) = Y(f)\, e^{-j2\pi f t_0} =$$

$$= \frac{1}{2}\left[X(f-f_0) + X(f+f_0)\right] e^{-j2\pi f t_0}$$

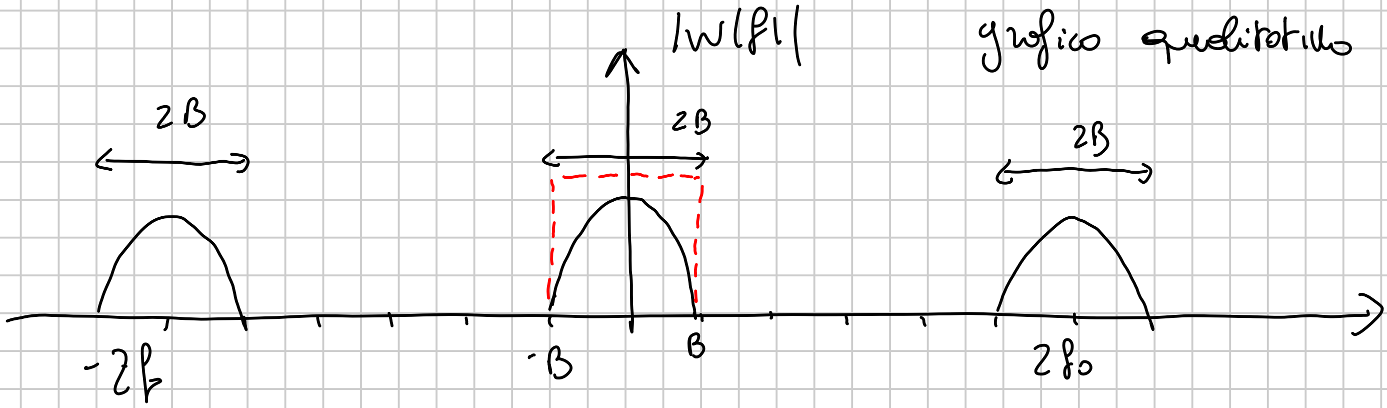
3) $w(t) = v(t) \cdot \cos(2\pi f_0 t + \varphi)$

$$W(f) = V(f) \otimes \left[\frac{\delta(f-f_0)\, e^{i\varphi} + \delta(f+f_0)\, e^{-i\varphi}}{2}\right] =$$

$$= \frac{1}{2} V(f-f_0)\, e^{i\varphi} + \frac{1}{2} V(f+f_0)\, e^{-i\varphi}$$

$$V(f-f_0) = \frac{1}{2} e^{-j2\pi t_0 (f-f_0)} \left[X(f-2f_0) + X(f)\right]$$

$$V(f+f_0) = \frac{1}{2} e^{-j2\pi t_0 (f+f_0)} \left[X(f) + X(f+2f_0)\right]$$



$$Z(f) = W(f) \cdot H(f) =$$

$$= \frac{1}{4} e^{-i 2\pi t_0 (f - f_0)} e^{i\varphi} X(f) + \frac{1}{4} e^{-i 2\pi t_0 (f + f_0)} e^{-i\varphi} X(f) =$$

$$= \frac{1}{4} X(f)\, e^{-i 2\pi t_0 f} \left[e^{i(2\pi f_0 t_0 + \varphi)} + e^{-i(2\pi f_0 t_0 + \varphi)} \right] =$$

$$= \frac{1}{2} X(f) \cos(2\pi f_0 t_0 + \varphi)\, e^{-i 2\pi t_0 f} \qquad -B \leq f \leq B$$

$$z(t) = \frac{1}{2} \cos(2\pi f_0 t_0 + \varphi)\, x(t - t_0)$$

5) $$E_z = \int_{-\infty}^{+\infty} z^2(t)\, dt = \int_{-\infty}^{+\infty} |Z(f)|^2\, df =$$

$$= \int_{-\infty}^{+\infty} \frac{1}{4} |X(f)|^2 \cos^2(2\pi f_0 t_0 + \varphi)\, df = \frac{1}{4} \cos^2(2\pi f_0 t_0 + \varphi) \cdot E_x$$

$$E_z \big|_{max} = \frac{1}{4} E_x$$

$$\cos^2(2\pi f t_0 + \varphi) = 1 \qquad 2\pi f t_0 + \varphi = k\pi$$

$$\varphi = k\pi - 2\pi f t_0$$

Esercizio #2 MODULAZIONE E FILTRAGGIO

$$x(t) \rightarrow \bigotimes \xrightarrow{y(t)} \boxed{h(t)} \rightarrow z(t)$$

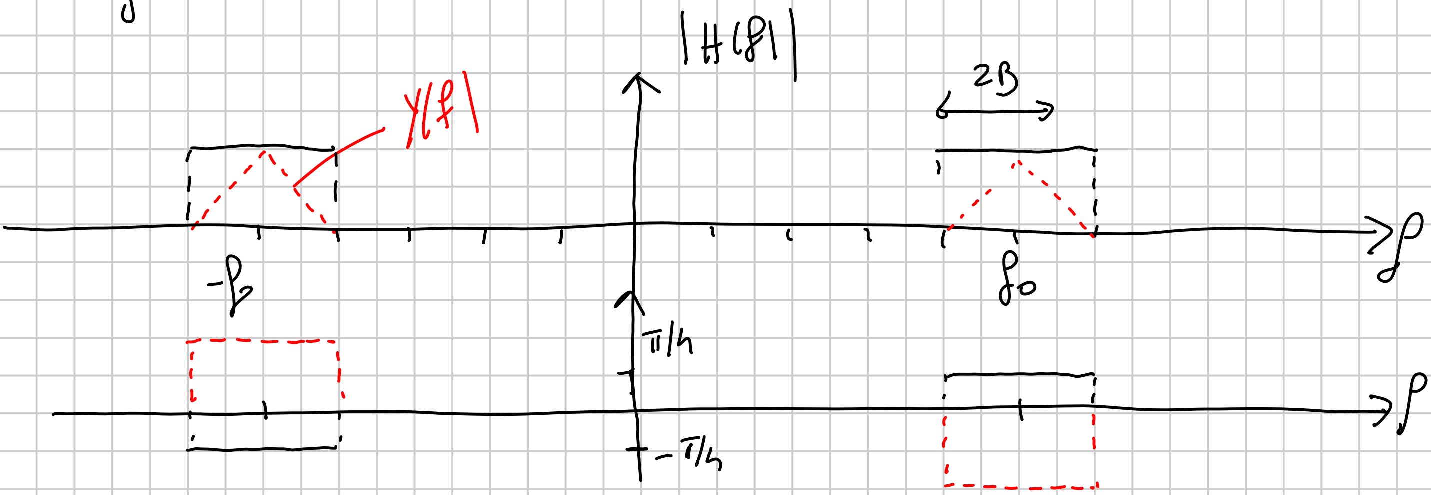
$$\uparrow \sin(2\pi f t)$$

$$X(f) = A\left(1 - \frac{|f|}{B}\right) \text{rect}\left(\frac{f}{2B}\right)$$

calcolare:

1) $z(t)$

2) Energia e Potenza di $z(t)$



$$j = e^{+i\pi/2} \qquad -j = e^{-i\pi/2}$$

(1) $y(t) = x(t)\,\sin(2\pi f_0 t)$

$$Y(f) = X(f) \otimes \left[\frac{\delta(f-f_0) - \delta(f+f_0)}{2j}\right] = \frac{1}{2j} X(f-f_0) - \frac{1}{2j} X(f+f_0)$$

$$Z(f) = Y(f)\cdot H(f) = \frac{1}{2j} X(f-f_0)\, e^{j\pi/4} - \frac{1}{2j} X(f+f_0)\, e^{-j\pi/4} =$$

$$= X(f) \otimes \left[\frac{1}{2}\delta(f-f_0)\, e^{-j\pi/2}\cdot e^{j\pi/4} + \frac{1}{2}\delta(f+f_0)\, e^{j\pi/2}\, e^{-j\pi/4}\right]$$

$$= X(f) \otimes \left[\frac{1}{2}\delta(f-f_0)\, e^{-j\pi/4} + \frac{1}{2}\delta(f+f_0)\, e^{j\pi/4}\right]$$

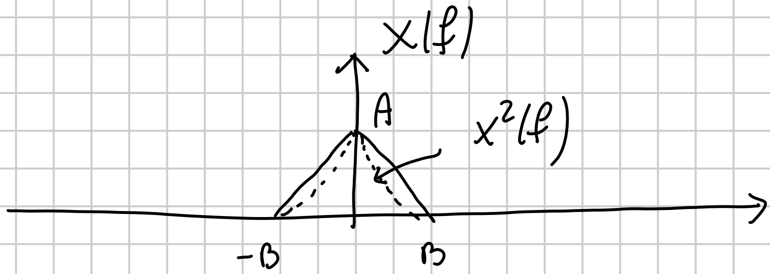
$$z(t) = x(t)\cdot\cos(2\pi f_0 t - \pi/4)$$

Energia finita e potenza nulla

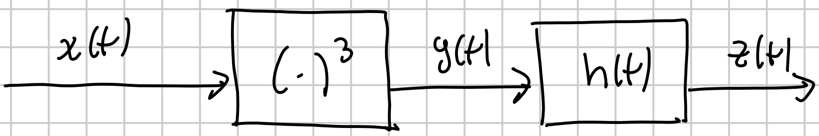
(2) $$E_z = \int_{-\infty}^{+\infty} |Z(f)|^2\, df = \int_{-\infty}^{+\infty} z^2(t)\, dt = \int_{-\infty}^{+\infty} x^2(t)\cdot\cos^2(2\pi f_0 t - \pi/4)\, dt =$$

$$= \int_{-\infty}^{+\infty} x^2(t)\cdot\left(\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0 t - \pi/2)\right) dt =$$

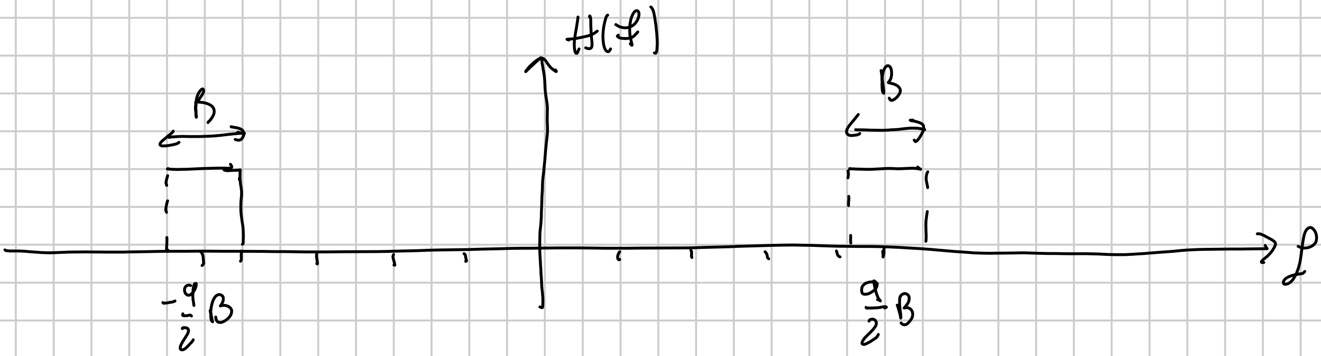
$$= \int_{-\infty}^{+\infty} x^2(t)\cdot\frac{1}{2} = \frac{1}{2}\int_{-\infty}^{+\infty} x^2(t)\, dt = \frac{1}{2}\cdot 2\frac{A^2 B}{3} = \frac{A^2 B}{3}$$



Esercizio # 3



$x(t) = A \cos(3\pi Bt + \theta)$



1) Calcolare $Y(f)$ e $y(t)$ e rappresentare i grafici di modulo e fase

2) Determinare $z(t)$

(1) $$y(t) = x^3(t) = A^3 \cos^3(3\pi Bt+\theta) =$$

$$= A^3 \cos(3\pi Bt+\theta) \cdot \cos^2(3\pi Bt+\theta) =$$

$$= A^3 \cos(3\pi Bt+\theta) \cdot \left(\frac{1}{2} + \frac{1}{2}\cos(6\pi Bt + 2\theta)\right)$$

$$= \frac{A^3}{2}\cos\left(3\pi Bt+\theta\right) + \frac{A^3}{2}\cos\left(3\pi Bt+\theta\right)\cdot\cos\left(6\pi Bt+2\theta\right)$$

$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$= \frac{A^3}{2}\cos\left(3\pi Bt+\theta\right) + \frac{A^3}{2}\left(\frac{1}{2}\cos\left(9\pi Bt+3\theta\right) + \frac{1}{2}\cos\left(3\pi Bt+\theta\right)\right) =$$

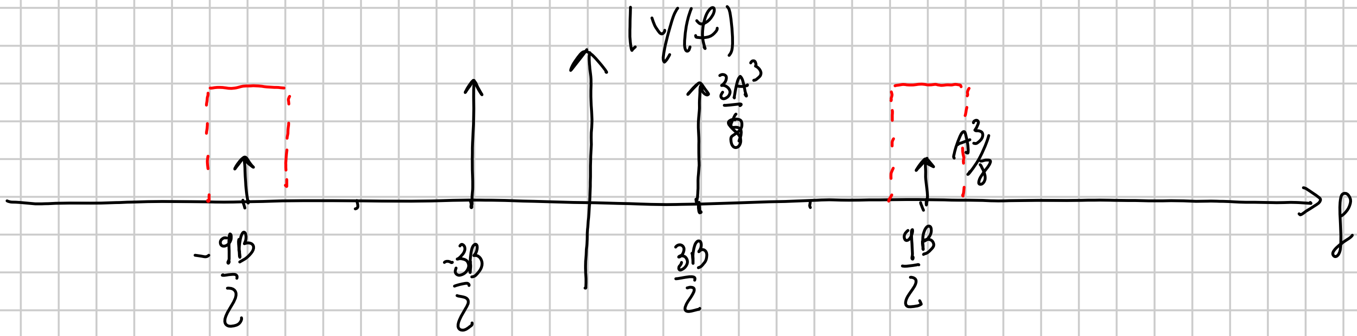
$$= \underbrace{\cos\left(3\pi Bt+\theta\right)}_{2\pi \underbrace{\frac{3B}{2}}_{f_1} t+\theta}\underbrace{\left(\frac{A^3}{2}+\frac{A^3}{4}\right)}_{\frac{3A^3}{4}} + \frac{A^3}{4}\underbrace{\cos\left(9\pi Bt+3\theta\right)}_{2\pi \underbrace{\frac{9B}{2}}_{f_2} t+3\theta}$$

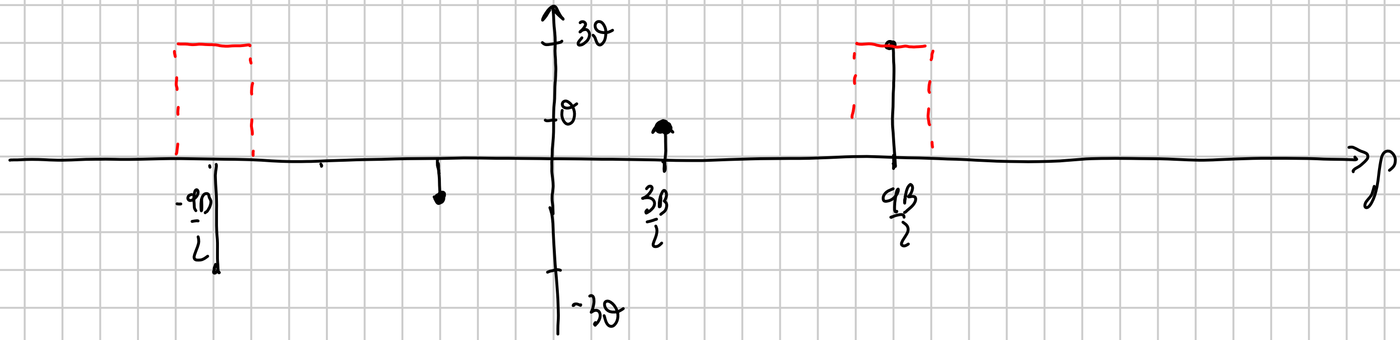
$$Y(f) = \frac{3A^3}{4}\left(\frac{\delta(f-f_1)e^{i\theta} + \delta(f+f_1)e^{-i\theta}}{2}\right) +$$

$$\frac{A^3}{4}\left(\frac{\delta(f-f_2)e^{i3\theta} + \delta(f+f_2)e^{-i3\theta}}{2}\right)$$

$$f_1 = \frac{3B}{2}$$

$$f_2 = \frac{9B}{2}$$





$$z(f) = \frac{A^3}{8}\left(\delta\left(f - \frac{9B}{2}\right) e^{j3\theta} + \delta\left(f + \frac{9B}{2}\right) e^{-j3\theta} \right)$$

$$z(t) = \frac{A^3}{4} \cos\left(2\pi \frac{9B}{2} t + 3\theta \right)$$

Esercizio — TRASFORMAZIONE DI V.A.

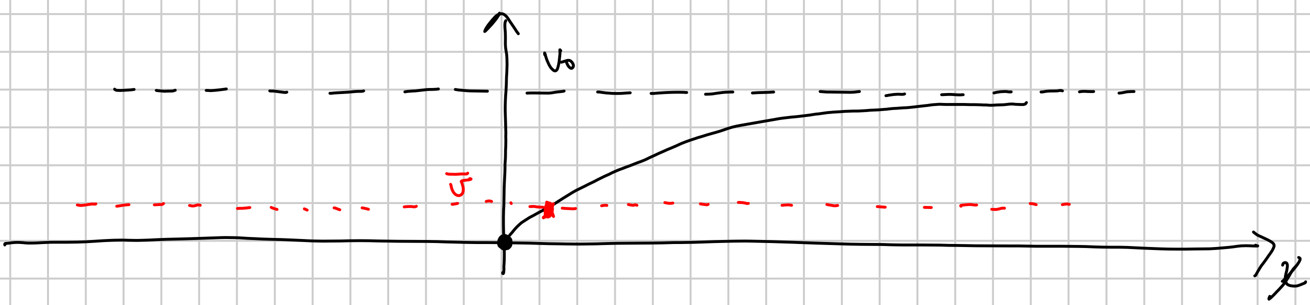
X V.A. con ddp esponenziale

$$f_X(x) = \frac{1}{2\alpha} e^{-x/2\alpha} u(x)$$

Determinare la ddp della variabile V ottenuta:

$$v(x) = V_0 \left[1 - e^{-x/\alpha} \right] u(x) \qquad v = g(x)$$

Disegnare la funzione



fissato un certo valore di $\bar{v}$ devo trovare le soluzione dell'equazione

$$\bar{v} = g(x_i)$$

·) per $v < 0$ e $v \geq V_0$ non ci sono soluzione

$$\Downarrow$$

$$f_V(v) = 0 \quad \text{per } v < 0 \text{ e } v \geq V_0$$

·) $0 \leq v < V_0$ esiste 1 soluzione

$$V_0 \left(1 - e^{-x_1/\alpha}\right) = V$$

$$\left(1 - e^{-x_1/\alpha}\right) = \frac{V}{V_0}$$

$$e^{-x_1/\alpha} = -\frac{V}{V_0} + 1$$

$$-\frac{x_1}{\alpha} = \ln\left(1 - \frac{V}{V_0}\right)$$

$$x_1 = -\alpha \ln\left(1 - \frac{V}{V_0}\right)$$

TEOREMA FONDAMENTALE

$$f_V(v) = \frac{f_X(x_1)}{|v'(x_1)|}$$

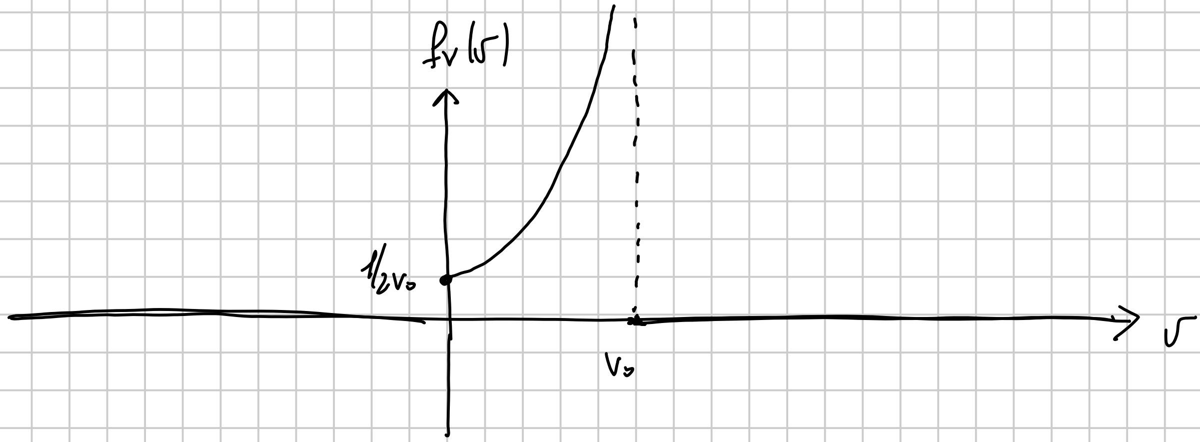
$$v'(x) = \frac{d}{dx} v(x) = -V_0 \left(-\frac{1}{\alpha}\right) e^{-x/\alpha} \qquad x \geq 0$$

$$v'(x_1) = \frac{V_0}{\alpha} e^{+\frac{\cancel{\alpha}}{\cancel{\alpha}} \ln\left(1 - \frac{V}{V_0}\right)} = \frac{V_0}{\alpha}\left(1 - \frac{V}{V_0}\right)$$

$$f_X(x_1) = \frac{1}{2\alpha} e^{-x/2\alpha} \Big|_{x = x_1} = \frac{1}{2\alpha} e^{+\frac{1}{2} \ln\left(1 - \frac{V}{V_0}\right)} =$$

$$= \frac{1}{2\alpha} e^{\ln\left(1 - \frac{V}{V_0}\right)^{1/2}} = \frac{1}{2\alpha}\sqrt{1 - \frac{V}{V_0}}$$

$$f_V(v) = \frac{\frac{1}{2\alpha}\sqrt{1-\frac{v}{V_0}}}{\frac{V_0}{\alpha}\left(1-\frac{v}{V_0}\right)} = \frac{1}{2V_0}\frac{1}{\sqrt{1-\frac{v}{V_0}}} \qquad v \in [0, V_0)$$



Esercizio #2

$$X \in \mathcal{N}(0,1)$$

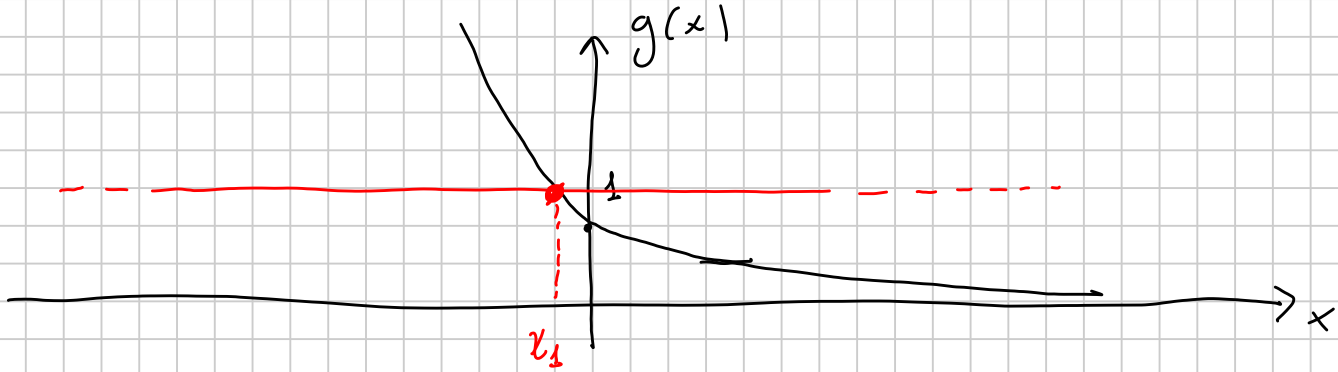
Determinare la ddp di Y e il suo valor medio

$$Y = e^{-X} \qquad \text{LOGNORMALE}$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

(1) $$f_Y(y) = \sum_{i=1}^{n} \frac{f_X(x_i)}{|g'(x_i)|}$$

$$g(x) = e^{-x}$$



$y \leq 0$ non esistono soluzioni $\Rightarrow f_Y(y) = 0$ per $y \leq 0$

$y > 0$ devo trovare $x_1 = g(x_1)$

$$y = e^{-x_1}$$

$$\ln(y) = -x_1$$

$$x_1 = -\ln(y)$$

ⓐ $$f_x(x_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\ln^2(y)}{2}}$$

ⓑ $$g(x) = e^{-x}$$

$$g'(x) = -e^{-x}$$

$$g'(x_1) = -e^{+\ln(y)} = -y$$

$$f_y(y) = \frac{1}{\sqrt{2\pi} \cdot y} e^{-\frac{\ln^2(y)}{2}}$$

(2) $$\eta_Y = \int_{-\infty}^{+\infty} y \, f_Y(y) \, dy$$

$$\eta_Y = \int_{-\infty}^{+\infty} g(x) \, f_X(x) \, dx$$

$$\eta_Y = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \, e^{-\left(\frac{x^2}{2}+x\right)} \, dx \qquad \underbrace{(ax+b)^2}_{y^2}$$

$$\frac{x^2}{2}+x = \frac{1}{2}\left[(x+1)^2-1\right]$$

$$\eta_Y = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \, e^{-\frac{1}{2}(x+1)^2} \, e^{+\frac{1}{2}} \, dx = e^{+1/2} \overset{=1}{\underbrace{\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \, e^{-\frac{1}{2}(x+1)^2} \, dx}_{\mathcal{N}(-1,1)}} =$$

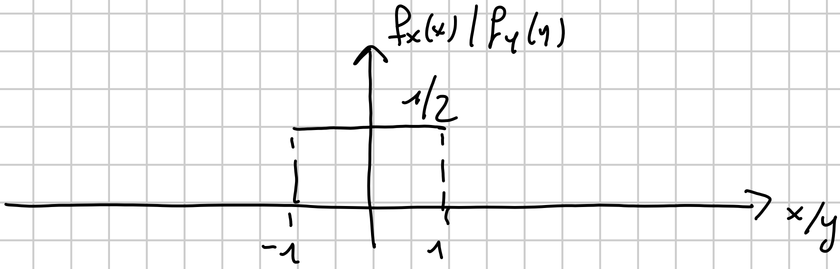
$$= e^{1/2}$$

Esercizio SISTEMI DI VARIABILI

Siano X e Y V.A. indipendenti

$$X \in U(-1,1)$$

$$Y \in U(-1,1)$$



Calcolare la probabilità che l'equazione di secondo grado

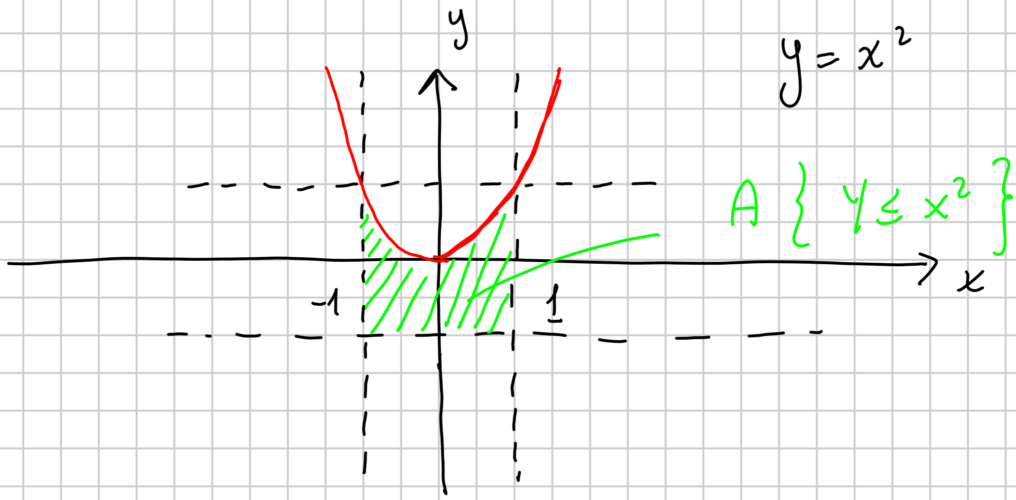
$$\alpha^2 + 2X\alpha + Y = 0$$ abbia radici reali.

$$\Delta \triangleq b^2 - 4ac = 4X^2 - 4Y \geq 0$$

$$\Pr\{4X^2 - 4Y \geq 0\} = ?$$

$$X^2 - Y \geq 0$$

$$Y \leq X^2$$



$$\Pr\{y \leq x^2\} = \iint_A f_{xy}(x,y)\, dx\, dy$$

$$f_{xy}(x,y) = f_x(x) \cdot f_y(y) = \frac{1}{4} \operatorname{rect}\left(\frac{x}{2}\right) \operatorname{rect}\left(\frac{y}{2}\right)$$

$$\Pr\{y \leq x^2\} = \frac{1}{4} \iint_A 1\, dx\, dy$$

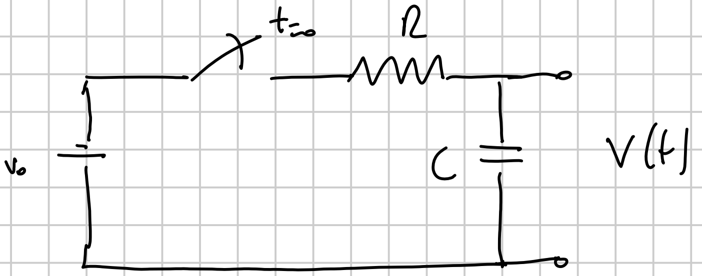
Area del dominio A

$$= \frac{1}{4} \int_{-1}^{1} \int_{-1}^{x^2} dy\, dx = \frac{1}{4} \int_{-1}^{1} y \Big|_{-1}^{x^2} dx =$$

$$= \frac{1}{4} \int_{-1}^{1} (x^2+1)\, dx = 2 \cdot \frac{1}{4} \int_{0}^{1} (x^2+1)\, dx =$$

$$= \frac{1}{2} \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{2} \left[\frac{1}{3} + 1 \right] = \frac{2}{3}$$

Esercizio 2 SISTEMI DI V.A.



R e C sono 2 variabili aleatorie indipendenti con ddp uniforme

A t=0 e' applicato al circuito una tensione $V_0 = 1V$.

Calcolare la probabilità che la tensione ai capi del condensatore all'istante $t_0 = 1ms$ sia inferiore a $\left(1 - \frac{1}{e}\right)$

SOLUZIONE

La tensione ai capi del condensatore è:

$$v(t) = V_0 \left(1 - e^{-t/RC}\right) u(t)$$

$$V(t_0) = V_0 \left(1 - e^{-t_0/RC}\right)$$

$$V_0 \left(1 - e^{-t_0/RC}\right) \leq 1 - \frac{1}{e}$$

evento di cui devo calcolare la probabilità

$$\Pr\left\{ V_0 \left(1 - e^{-t_0/RC}\right) \leq 1 - \frac{1}{e} \right\} = \iint_A f_{RC}(r,c)\, dr\, dc$$

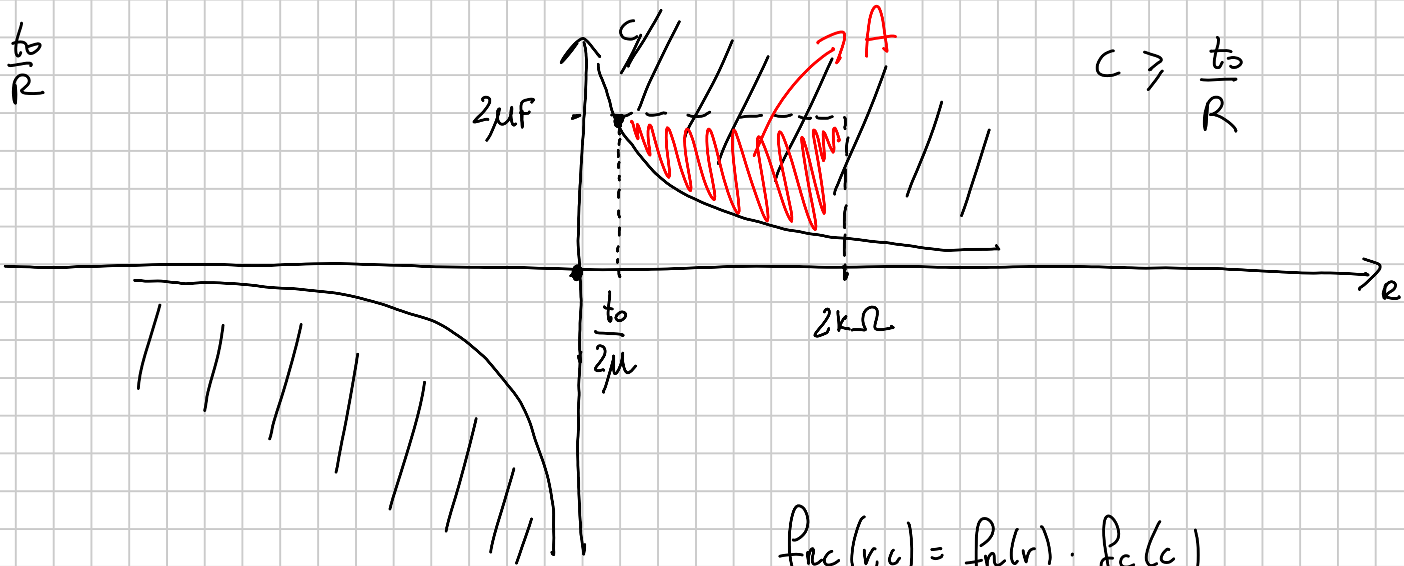
$V_0 = 1V$

$$1 - e^{-t_0/RC} \leq 1 - e^{-1}$$

$$\frac{t_0}{RC} \leq 1 \quad \Rightarrow \quad RC \geq t_0$$

$$A = \left\{ RC \geq t_0 \right\}$$

$C = \frac{t_0}{R}$



$$f_{RC}(r,c) = f_R(r) \cdot f_C(c)$$

$$Pr(A) = \int_{r=\frac{t_0}{2\mu}}^{2k} \int_{c=\frac{t_0}{r}}^{2\mu} f_{RC}(r,c)\, dr\, dc =$$

$$= \int_{\frac{t_0}{2\mu}}^{2k} \int_{\frac{t_0}{r}}^{2\mu} \frac{1}{2000} \frac{1}{2 \cdot 10^{-6}} \, dr\, dc =$$

$$= \frac{1}{4 \cdot 10^{-3}} \int_{\frac{t_0}{2\mu}}^{2k} \left[c \right]_{\frac{t_0}{r}}^{2\mu} dr =$$

$$= \frac{1}{4\, 10^{-3}} \int_{\frac{t_0}{2\mu}}^{2k} \left[2\, 10^{-6} - \frac{t_0}{r} \right] dr = \frac{1}{4\, 10^{3}} \left[2\, 10^{-6} r - t_0 \ln(r) \right]_{\frac{t_0}{2\mu}}^{2k} =$$

$$= \frac{3}{4} - \frac{1}{4}\left(\ln\left(2000\right) - \ln\left(\frac{1}{2}10^3\right)\right)$$

ESERCIZIO PROCESSI

Sia $x(t) = A \cdot \cos(2\pi f_0 t + \Theta)$

A V.A. esponenziale con valor medio η

Θ $U(-\pi, \pi)$

A e Θ sono indipendenti

Calcolare $\eta_x(t)$ e $P_x(t)$ e dire se è SSL

SOLUZIONE

$$f_A(a) = \frac{1}{\eta} e^{-a/\eta} u(a)$$

$$f_\Theta(\theta) = \frac{1}{2\pi} \text{rect}\left(\frac{\theta}{2\pi}\right)$$

(1) $\eta_x(t) = E\{x(t)\} = E\{A \cdot \cos(2\pi f_0 t + \Theta)\} =$

$= E\{A\} \cdot E\{\cos(2\pi f_0 t + \Theta)\}$

$$= \eta \cdot E\{\cos(2\pi f_0 t+\Theta)\} =$$

$$= \eta \int_{-\infty}^{+\infty} g(\theta)\cdot f_\Theta(\theta)\, d\theta = \eta \cdot \int_{-\infty}^{+\infty} \cos(2\pi f_0 t+\theta)\cdot \frac{1}{2\pi}\, \mathrm{rect}\left(\frac{\theta}{2\pi}\right) d\theta$$

$$= \frac{\eta}{2\pi}\int_{-\pi}^{\pi}\cos(2\pi f_0 t+\theta)\,d\theta = \frac{\eta}{2\pi}\Big[\sin(2\pi f_0 t+\theta)\Big]_{-\pi}^{\pi} =$$

$$= \frac{\eta}{2\pi}\Big[\sin(2\pi f_0 t+\pi) - \sin(2\pi f_0 t-\pi)\Big] = 0$$

$$\eta_x(t)=0$$

(2) $P_x(t) = E\{x^2(t)\} = E\{A^2\cdot\cos^2(2\pi f_0 t+\Theta)\} =$

$$= E\left\{A^2\left(\frac{1}{2}+\frac{1}{2}\cos(4\pi f_0 t+2\Theta)\right)\right\} =$$

$$= E\left\{\frac{A^2}{2}\right\} + E\left\{\frac{A^2}{2}\cos(4\pi f_0 t+2\Theta)\right\} =$$

$$= \eta^2 + E\left\{\frac{A^2}{2}\right\}\cdot E\{\cos(4\pi f_0 t+2\Theta)\} = \eta^2$$

$$P_x(t) = P_x = \eta^2$$

$x(t)$ è SSL se $\underline{\eta_x(t) = \eta_x}$ e $\underline{\underline{R_x(t_1,t_2) = R_x(t_2-t_1)}}$

$$\begin{cases} x(t_1) = A\cos(2\pi f t_1 + \Theta) \\ x(t_2) = A\cos(2\pi f t_2 + \Theta) \end{cases}$$

$$R_x(t_1,t_2) = E\{x(t_1)\cdot x(t_2)\} =$$

$$= E\{A\cos(2\pi f t_1 + \Theta)\cdot A\cos(2\pi f t_2 + \Theta)\} =$$

$$= E\{A^2\cdot\cos(2\pi f t_1 + \Theta)\cdot\cos(2\pi f t_2 + \Theta)\} =$$

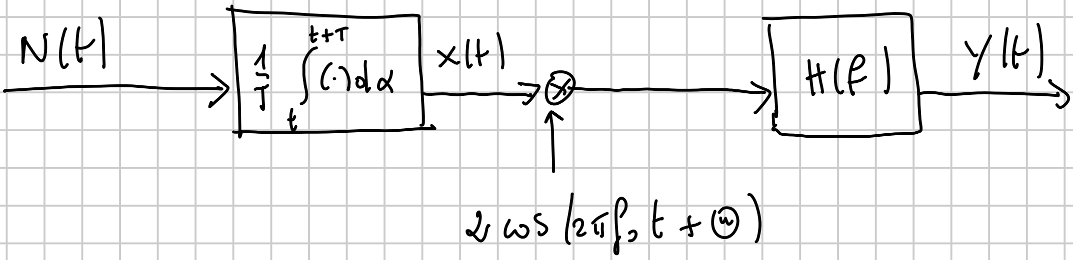
$$\cos\alpha\cdot\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$$

$$= E\{A^2\}\cdot E\left\{\frac{1}{2}\cos\left(2\pi f(t_1+t_2)+2\Theta\right) + \frac{1}{2}\cos\left(2\pi f(t_1-t_2)\right)\right\} =$$

$$= 2\eta^2\cdot\left[E\left\{\frac{1}{2}\cos\left(2\pi f(t_1+t_2)+2\Theta\right)\right\} + E\left\{\frac{1}{2}\cos\left(2\pi f(t_1-t_2)\right)\right\}\right] =$$

$$= \cancel{8}\eta^2 \cdot \frac{1}{\cancel{8}} \cos\left(2\pi f_0 (t_1 - t_2)\right) = \eta^2 \cos\left(2\pi f_0 \tau\right) \qquad \tau = t_1 - t_2$$

Esercizio Processi #2



$N(t)$ processo gaussiano bianco SSL $S_N(f) = \xi$

Θ V.A. indipendente da $N(t)$ $U(-\pi, \pi)$

Calcolare la DSP del processo $Y(t)$ $S_Y(f)$

SOLUZIONE

Poiché l'integratore è un sistema lineare e stazionario

$X(t)$ sarà un processo Gaussiano e SSL

$$\eta_X(t) = \eta_N \cdot H_1(0) = 0 \qquad \text{perché } \eta_N = 0$$

$H_1(f)$ è la risposta in frequenza dell'integratore

$$R_X(\tau) = R_N(\tau) \otimes h_1(\tau) \otimes h_1(-\tau)$$

$$S_X(f) = S_N(f) \left| H_1(f) \right|^2$$

$$h_1(t) = \frac{1}{T} \, rect\left(\frac{t - T/2}{T}\right)$$

$$H_1(f) = sinc(fT)\, e^{-j 2\pi f T/2} \Rightarrow |H_1(f)|^2 = sinc^2(fT)$$

$$R_x(\tau) = \xi\, \delta(\tau) \otimes h_1(\tau) \otimes h_1(-\tau) = \xi\, \delta(\tau) \otimes R_h(\tau) = \xi\, R_h(\tau) =$$

$$= \xi \frac{1}{T}\left(1 - \frac{|\tau|}{T}\right) rect\left(\frac{\tau}{2T}\right)$$

$$s(t) = 2\cos(2\pi f_0 t + \Theta) \qquad \Theta \in U(-\pi, \pi)$$

Dall' esercizio precedente $\Rightarrow$ $\eta_s = 0$

$$R_s(\tau) = 2\cos(2\pi f_0 \tau)$$

$z(t) = x(t) \cdot s(t)$ Calcoliamo valor medio e funzione di autocorrelazione

$$\eta_z(t) = E\{z(t)\} = 2E\{x(t) \cdot \cos(2\pi f_0 t + \Theta)\} =$$

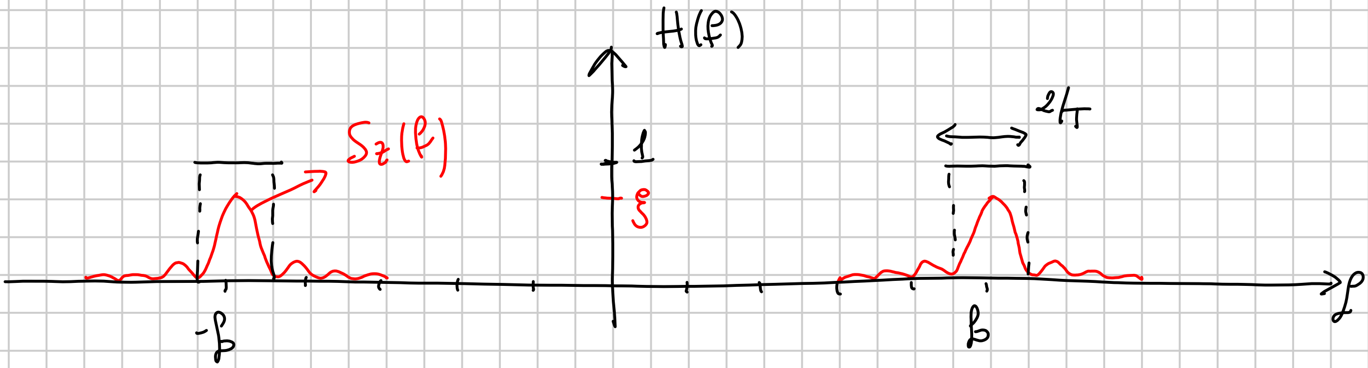
$$= 2E\{x(t)\} \cdot E\{\cos(2\pi f_0 t + \Theta)\} = 0$$

$$R_z(t_1,t_2) = R_z(t, t-\tau) = E\{z(t)\, z(t-\tau)\} =$$

$$= 4\, E\{x(t)\, x(t-\tau) \cos(2\pi f_0 t + \Theta) \cos(2\pi f_0 (t-\tau) + \Theta)\} =$$

$$= 4\, E\{x(t)\, x(t-\tau)\} \cdot E\{\cos(2\pi f_0 t + \Theta) \cos(2\pi f_0 (t-\tau) + \Theta)\} =$$

$$= R_x(\tau) \cdot R_s(\tau) = 2 R_x(\tau) \cdot \cos(2\pi f_0 \tau)$$



$$S_y(f) = S_z(f)\, |H(f)|^2$$

$$S_z(f) = S_x(f - f_0) + S_x(f + f_0)$$

$$S_x(f) = \xi\, \mathrm{sinc}^2(fT)$$

Se $f_0 \gg \frac{1}{T}$ $\operatorname{sinc}^2((f+f_0)T)$ e $\operatorname{sinc}^2((f-f_0)T)$ si possono considerare ISOLATE, quindi:

$$S_Y(f) \simeq \xi \left[\operatorname{sinc}^2((f+f_0)T) \operatorname{rect}\left(\frac{(f+f_0)T}{2}\right) + \operatorname{sinc}^2((f-f_0)T) \operatorname{rect}\left(\frac{(f-f_0)T}{2}\right) \right]$$

È POSSIBILE RICAVARE LA DDP DI 1° ORDINE DI $Y(t)$, $f_Y(y;t)$?

Non possiamo sapere se $Y(t)$ è Gaussiano, però CONDIZIONATAMENTE AD UN VALORE DI Θ, la ddp di 1° ordine di $Y(t)$, $f_{Y|\Theta}(y|\theta;t)$ è Gaussiana.

Infatti se fissiamo un valore di Θ $S(t)$ diventa deterministico e $X(t)$ sarà quindi un processo Gaussiano e poiché $H(f)$ è un sistema lineare e stazionario, lo sarà anche $Y(t)$.

Conoscendo quindi la ddp di 1° ordine di $Y(t)$ condizionata a Θ e la ddp di Θ, possiamo ricavare la ddp congiunta di $Y(t)$ e Θ:

$$f_{Y\Theta}(y,\theta;t) = f_{Y|\Theta}(y|\theta;t) \cdot f_\Theta(\theta)$$

Adesso è possibile ricavare la ddp marginale dalla congiunta

$$f_Y(y;t) = \int_{-\infty}^{+\infty} f_{Y\Theta}(y,\theta;t)\, d\theta$$