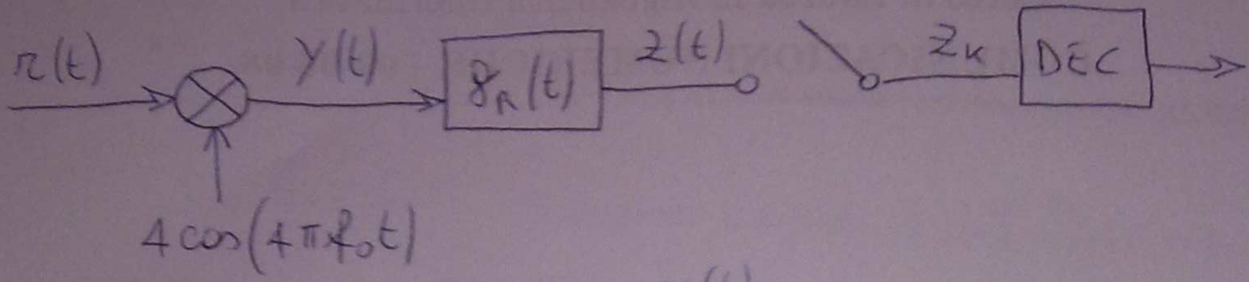
COMPITO 3/6/08



$$r(t)=\overbrace{\sum_i a_i\, g_T(t-iT)\cos^2(2\pi f_0 t+\theta)}^{x(t)}+w(t)$$

$f_0 \gg 1/T$, $\theta=-\pi/4$, a_i ind. e equiprob $\in A=[\pm 1]$

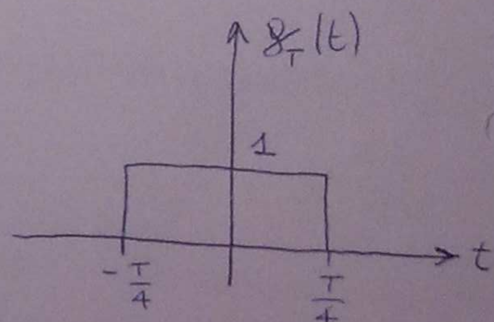
$w(t)$ è Gaussiano a media nulla con $S_w(f)=\dfrac{N_0}{2}$

$$g_T(t)=g_R(t)=\mathrm{rect}\left(\frac{t}{T/2}\right)$$

1. Energia trasmessa media per simbolo

$$x(t)=\underbrace{\frac{1}{2}\sum_i a_i\, g_T(t-iT)}_{x_1(t)}+\underbrace{\frac{1}{2}\sum_i a_i\, g_T(t-iT)\cos(4\pi f_0 t+2\theta)}_{x_2(t)}$$

$$E_{x_1}=\frac{1}{4}\int_{-\infty}^{+\infty} g_T^2(t)\,dt=\frac{1}{4}\cdot\frac{T}{2}=\frac{T}{8}$$



$$E_{x_2}=\frac{1}{4}\int_{-\infty}^{+\infty} g_T^2(t)\cos^2(4\pi f_0 t+2\theta)\,dt=$$

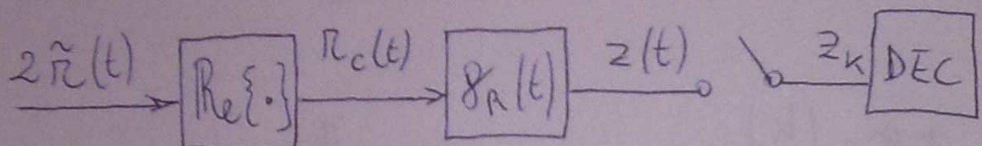
$$=\frac{1}{8}\int_{-\infty}^{+\infty} g_T^2(t)\,dt+\underbrace{\frac{1}{8}\int_{-\infty}^{+\infty} g_T^2(t)\cos(8\pi f_0 t+4\theta)\,dt}_{=0}=$$

$$=\frac{1}{8}\,\frac{T}{2}=\frac{T}{16}$$

$$\Rightarrow E_s=E_{x_1}+E_{x_2}=\frac{3T}{16}$$

(2) 2. La potenza media della componente di rumore all'uscita di $g_R(t)$

Schema equivalente in banda base riferito alla frequenza $2f_0$:



$\tilde{r}(t) = \tilde{x}(t) + \tilde{w}(t)$ con $\tilde{w}(t) = w_c(t) + j\, w_s(t)$

$\tilde{x}(t) = \sum_i a_i\, g_T(t - iT)\, e^{j2\theta}$

Nel dominio della frequenza, si ricorda:

$$\tilde{X}(f) = \begin{cases} 2X(f + 2f_0) & f \geq -2f_0 \\ 0 & f < -2f_0 \end{cases}$$

$$S_{\tilde{w}}(f) = \begin{cases} 4S_w(f + 2f_0) & f \geq -2f_0 \\ 0 & f < -2f_0 \end{cases}$$

$r_c(t) = 2\sum_i a_i\, g_T(t - iT)\cos(2\theta) + 2w_c(t)$

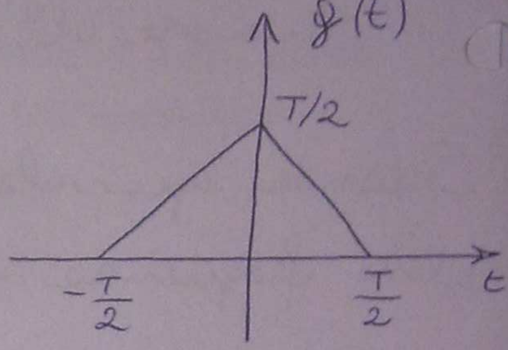
$n_c(t) = 2w_c(t) \otimes g_R(t)$

$S_{n_c}(f) = 2N_0\, |G_R(f)|^2$

$$\Rightarrow \sigma_{n_c}^2 = \int_{-\infty}^{+\infty} 2N_0\, |G_R(f)|^2\, df = 2N_0 \int_{-\infty}^{+\infty} g_R^2(t)\, dt =$$

$$= 2N_0 \frac{T}{2} = N_0 T$$

$$z(t) = 2 \sum_i a_i\, g(t - iT) \cos(2\theta) + n_c(t)$$



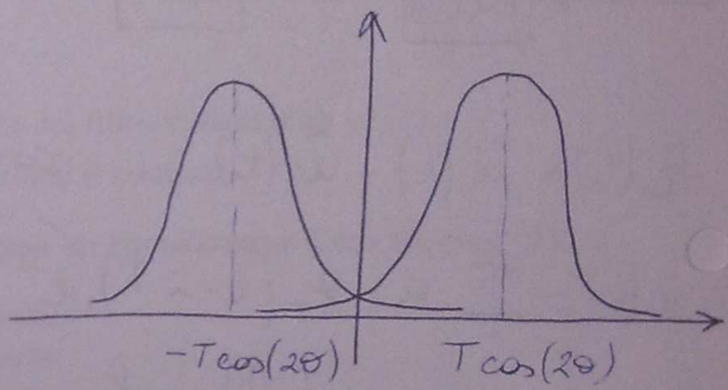
$$z(k) = 2 \frac{T}{2} a_k \cos(2\theta) + n_c(k)$$

$$n_c(k) \in \mathcal{N}(0, \sigma_{nc}^2)$$

con $\sigma_{nc}^2 = N_0 T$

$$z|1 = T\cos(2\theta) + n_c(k)$$

$$z|-1 = -T\cos(2\theta) + n_c(k)$$



$$P(e) = Q\left(\frac{T\cos 2\theta}{\sqrt{N_0 T}}\right)\Bigg|_{\theta = -\frac{\pi}{4}} = Q(0) = \frac{1}{2}$$