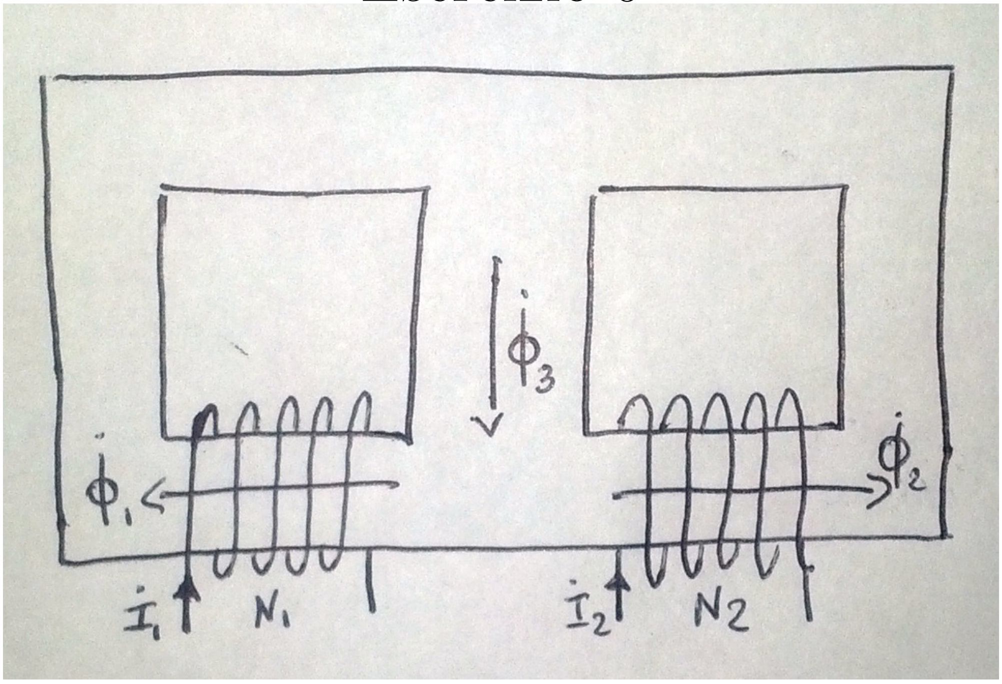
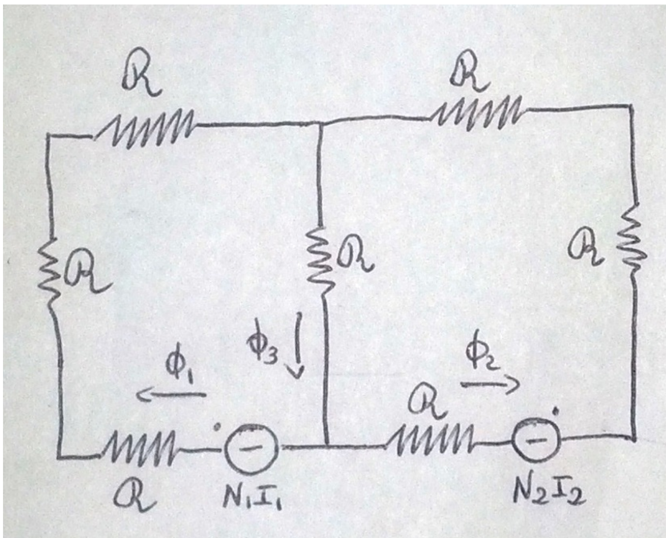
Esercizio 0



$$R = \frac{l}{S\mu_0\mu_r} = \frac{10 \cdot 10^{-2}}{10 \cdot 10^{-4} \cdot 4\pi \cdot 10^{-7} \cdot 10^3} = 79557,47H^{-1}$$

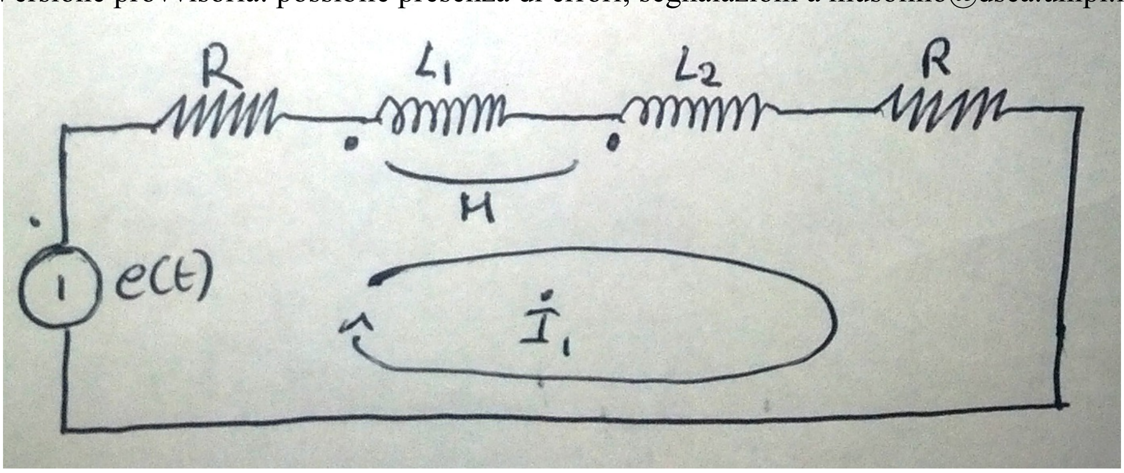


$$R_{v1} = R_{v2} = 3R + \frac{3}{4}R = \frac{15}{4}R = 298.340,51H^{-1}$$

$$L_1 = \frac{N_1^2}{R_{v1}} = 33,5mH$$

$$L_2 = \frac{N_2^2}{R_{v2}} = 75,4mH$$

$$M = \frac{N_1N_2}{4R_{v1}} = 12.6mH$$



$$\dot{E} = (R + j\omega(L_1 + L_2 + 2M))\dot{I}_1 \rightarrow \dot{I}_1 = 0.76 - j0.29$$

$$\begin{cases} \dot{\phi}_3 &= \dot{\phi}_2 + \dot{\phi}_1 \\ N_1\dot{I}_1 &= 3R\dot{\phi}_1 + R\dot{\phi}_3 \\ N_2\dot{I}_1 &= 3R\dot{\phi}_2 + R\dot{\phi}_3 \end{cases}$$

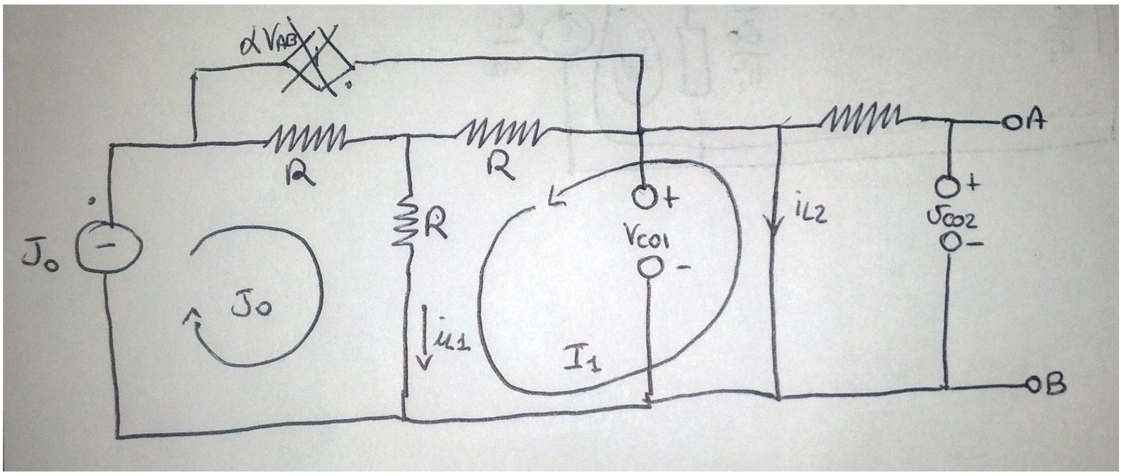
$$\dot{\phi}_3 = 0,48 \cdot 10^{-3} - 0,18 \cdot 10^{-3}j$$

$$\dot{V}_3 = j\omega N_3\dot{\phi}_3 = 13.50 + j35.81 = 38.27e^{j1.21}V_{eff}$$

$$V_3(t) = 38.27\cos(500t + 1.21)V$$

Esercizio 1

Condizioni Iniziali



$V_{AB} = V_{C02} = 0 \qquad$ per $t < 0$

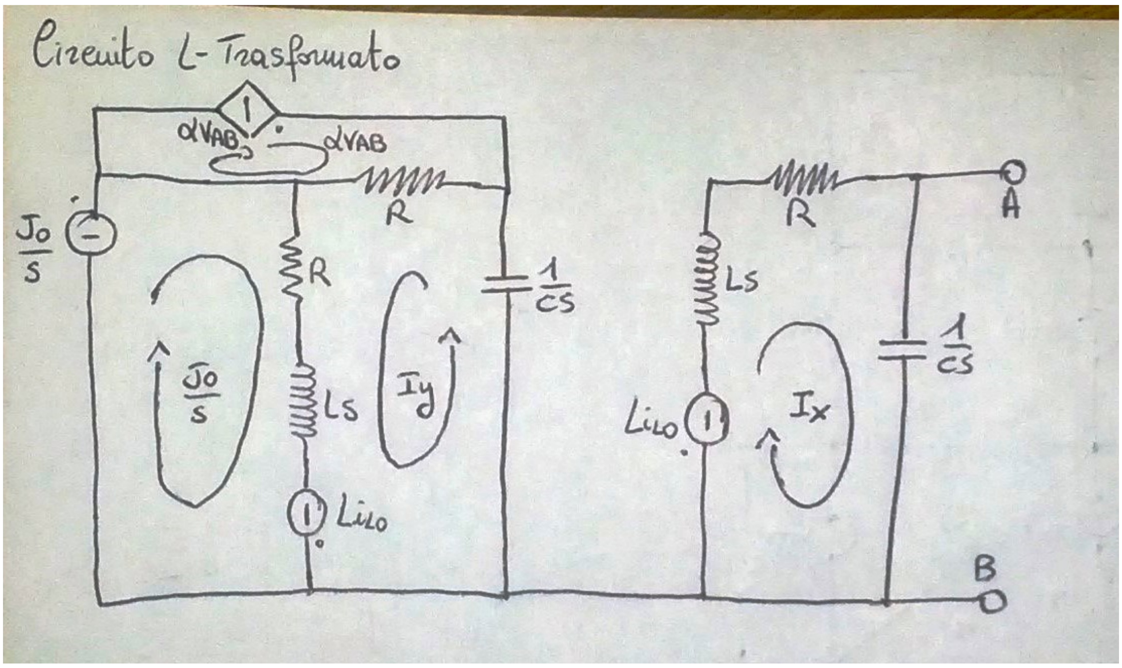
$$0 = (2R)I_1 + RJ_0 => I_1 = -\frac{J_0}{2}$$

$$i_{L1} = J_0 - \frac{J_0}{2} = \frac{J_0}{2} \quad i_{L2} = \frac{J_0}{2} = -I_1 i_{L0} = i_{L2} = i_{L1} = \frac{J_0}{2}$$

$$V_{C01} = 0$$

poiche' vi e' un corto circuito in parallelo alle armature

Circuito L-Trasformato



$$-Li_{L0} = (Ls + \frac{1}{Cs} + R)I_X$$

$$V_{AB} = \frac{1}{Cs}I_X = \frac{1}{Cs}\frac{-Li_{L0}Cs}{LCs^2 + RCs + 1} = -\frac{Li_{L0}}{Lcs^2 + Rcs + 1}$$

$$Li_{L0} = (2R + Ls + \frac{1}{Cs})I_y + (R + Ls)\frac{J_0}{s} + \frac{\alpha RLi_{L0}}{LCs^2 + RCs + 1}$$

$$\downarrow$$

$$I_y = -\frac{CJ_0(CL^2s^3 + 3RLCs^2 - s(\alpha RL + 2R^2C - 1) + 2R)}{2(LCs^2 + 2RCs + 1)(LCs^2 + RCs + 1)}$$

$$V_L(s) = -Li_{L0} + Ls[\frac{J_0}{s} - \frac{CJ_0(CL^2s^3 + 3RLCs^2 - s(\alpha RL + 2R^2C - 1) + 2R)}{2(LCs^2 + 2RCs + 1)(LCs^2 + RCs + 1)}]$$

$$\downarrow$$

$$V_L(s) = \frac{LJ_0(LCs^2(\alpha R + 1) + RCs + 1)}{2(LCs^2 + 2RCs + 1)(LCs^2 + RCs + 1)}$$

$$\downarrow$$

$$V_L(s) = \frac{4000000(2s^2 + 125s + 2500000)}{(s^2 + 1000s + 20000000)(s^2 + 2000s + 20000000)}$$

$$\downarrow$$

$$V_L(s) = \frac{7500s}{s^2 + 1000s + 20000000} - \frac{2500(3s - 200)}{s^2 + 2000s + 20000000}$$

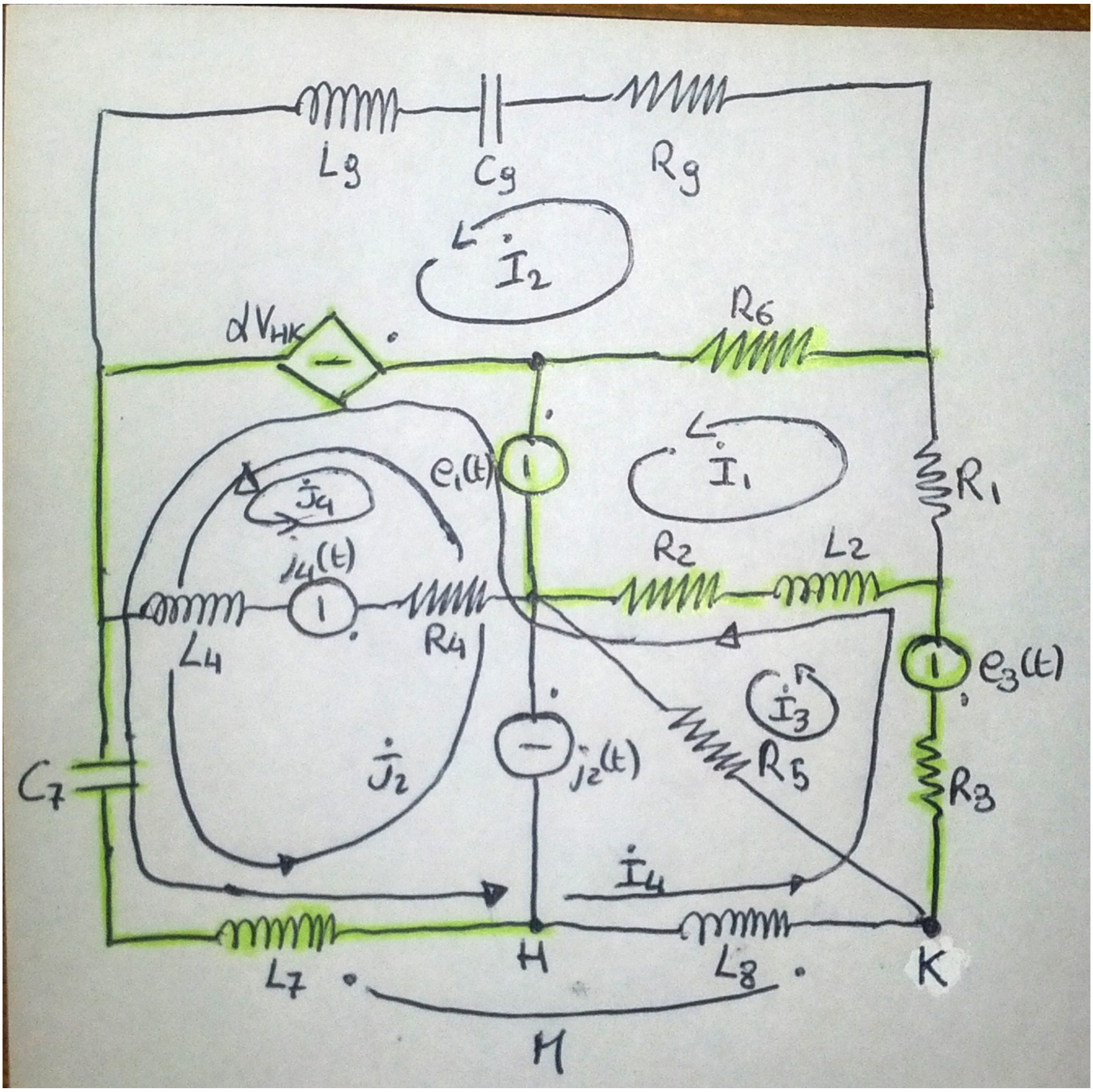
$$V_L(s) = \frac{A}{s + 500 + j4444.1} + \frac{A^*}{s + 500 - j4444.1} + \frac{B}{s + 1000 + j4358.9} + \frac{B^*}{s + 1000 - j4358.9}$$

$$A = 3750 - j421.91 \cong 3773.66e^{-j0,11} = M_1e^{j\phi_1}$$

$$B = -3750 + j917.66 \cong 3860.65e^{j2,90} = M_2e^{j\phi_2}$$

$$V_L(t) = [2M_1e^{-500t}cos(4444.1t - \phi_1) + 2M_2e^{-1000t}cos(4358.9t - \phi_2)]u(t) =$$
$$= [7547.32e^{-500t}cos(4444.1t + 0,11) + 7721.3e^{-1000t}cos(4358.9t - 2,87)]u(t)V$$

Esercizio 2



$N = 7; R = 12; N_{corde} = R - N + 1 \rightarrow 6$

- Per le correnti di maglia: $N_{eq} = R - N + 1 - N_{GenCorrente} \rightarrow 4$
- Per le tensioni nodali: $N_{eq} = N - 1 - N_{GenTensione} \rightarrow 4$ (Devo trasformare in Norton $e_3(t)$)

Metodo delle correnti di maglia.

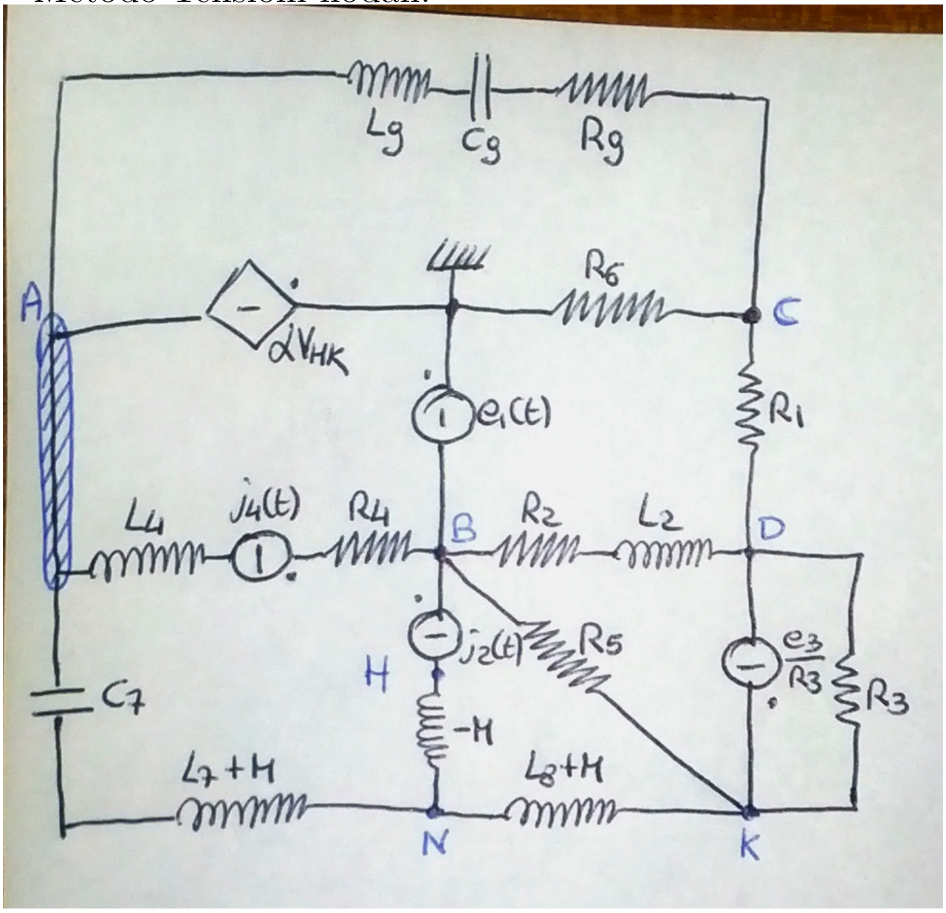
Associo a $j_4(t), j_2(t), e_1(t), e_3(t)$ rispettivamente i fasori $\dot{J}_4, \dot{J}_2, \dot{E}_1, \dot{E}_3$.

Sistema equazioni:

$$
\begin{cases}
\begin{aligned}
-\dot{E}_1 = \;& (R_6 + R_2 + j\omega L_2 + R_1)\dot{I}_1 - R_6\dot{I}_2 - (R_2 + j\omega L_2)\dot{I}_3 + \\
& -(R_2 + j\omega L_2)\dot{I}_4 \\
\alpha\dot{V}_{HK} = \;& -R_6\dot{I}_1 + (R_6 + R_9 + \tfrac{1}{j\omega C_9} + j\omega L_9)\dot{I}_2 \\
-\dot{E}_3 = \;& -(R_2 + j\omega L_2)\dot{I}_1 + (R_2 + j\omega L_2 + R_3 + R_5)\dot{I}_3 + \\
& +(R_2 + j\omega L_2 + R_3)\dot{I}_4 \\
-\dot{E}_3 - \alpha\dot{V}_{HK} + \dot{E}_1 = \;& -(R_2 + j\omega L_2)\dot{I}_1 + (R_2 + j\omega L_2 + R_3)\dot{I}_3 + \\
& +(R_2 + j\omega L_2 + \tfrac{1}{j\omega C_7} + j\omega L_8 + j\omega L_7)\dot{I}_4 \\
& +(\tfrac{1}{j\omega C_7} + j\omega L_7)\dot{J}_2 + 2j\omega M\dot{I}_4 + j\omega M\dot{J}_2
\end{aligned}
\end{cases}
$$

$$\dot{V}_{HK} = j\omega L_8\dot{I}_4 + j\omega M\dot{I}_4 + j\omega M\dot{J}_2 \qquad \text{(Equazione Generatore controllato)}$$

Metodo Tensioni nodali:



Applichiamo la trasformazione a T della mutua, ricordando che il nodo H corrisponde alla giuntura tra $-M$ e $j_2(t)$, chiameremo quindi il nuovo nodo N. Inoltre trasformiamo il generatore di tensione $e_3(t)$ nel suo equivalente Norton.

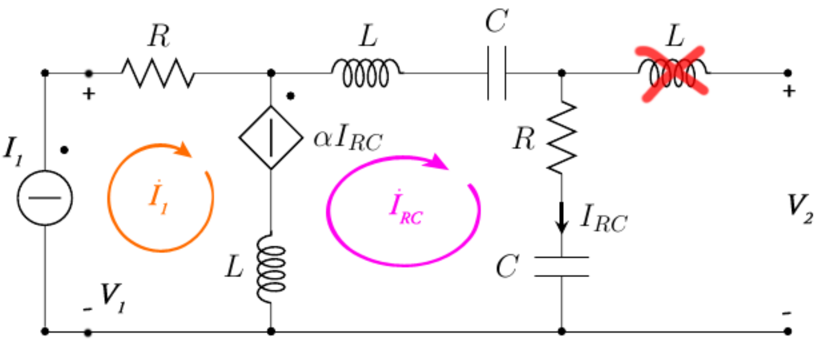
Con la scelta del nodo di riferimento in figura abbiamo: $\dot{V}_A = -\alpha \dot{V}_{HK}$ e $\dot{V}_B = -\dot{E}_1$

$$\begin{cases} 0 = & -\frac{1}{R_9 + j\omega L_9 + \frac{1}{j\omega C_9}}\dot{V}_A + (\frac{1}{R_6} + \frac{1}{R_1} + \frac{1}{R_9 + j\omega L_9 + \frac{1}{j\omega L_6}})\dot{V}_C - \frac{1}{R_1}\dot{V}_D \\ -\frac{\dot{E}_3}{R_3} = & -\frac{1}{R_2 + j\omega L_2}\dot{V}_B - \frac{1}{R_1}\dot{V}_C + (\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_2 + j\omega L_2})\dot{V}_D - \frac{1}{R_3}\dot{V}_K \\ -\dot{J}_2 = & -\frac{1}{j\omega(L_7+M) + \frac{1}{j\omega C_7}}\dot{V}_A + \frac{1}{j\omega M}\dot{V}_B + (\frac{1}{\frac{1}{j\omega C_7} + j\omega(L_7+M)} + \frac{1}{j\omega(L_8+M)} - \frac{1}{j\omega M})\dot{V}_N + \\ & -\frac{1}{j\omega(L_8+M)}\dot{V}_K \\ \frac{\dot{E}_3}{R_3} = & -\frac{1}{R_5}\dot{V}_B - \frac{1}{R_3}\dot{V}_D - \frac{1}{j\omega(L_8+M)}\dot{V}_N + (\frac{1}{R_3} + \frac{1}{R_5} + \frac{1}{j\omega(L_8+M)})\dot{V}_K \end{cases}$$

Equazione Generatore controllato per le tensioni nodali

$$\dot{V}_{HK} = -\dot{J}_2(-j\omega M) + \dot{V}_N - \dot{V}_K$$

Esercizio 3



Generatore di corrente a sinistra, circuito aperto a destra!

$$\alpha \dot{I}_{RC} = (2j\omega L + \frac{2}{j\omega C} + R)\dot{I}_{RC} - j\omega L \dot{I}_1$$

$$\dot{I}_{RC} = \frac{j\omega L}{2j\omega L + \frac{2}{j\omega C} + R - \alpha}\dot{I}_1 = \bar{H}\dot{I}_1 = (-0.014 + j0.169)\dot{I}_1$$

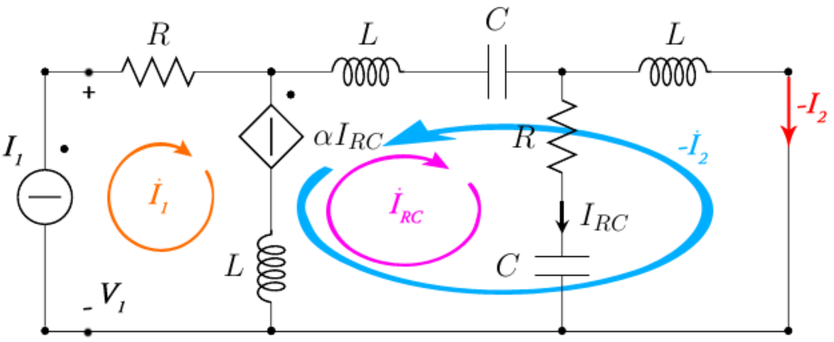
$$\dot{V}_2 = (R + \frac{1}{j\omega C})\bar{H}\dot{I}_1 = (0.97 + j8.59)\dot{I}_1$$

$$\boxed{\frac{1}{C} = \left.\frac{\dot{V}_2}{\dot{I}_1}\right|_{-\dot{I}_2=0} = (R + \frac{1}{j\omega C})\bar{H} = 0.97 + j8.59\Omega}$$

$$\dot{V}_1 = R\dot{I}_1 + \alpha\bar{H}\dot{I}_1 + j\omega L(\dot{I}_1 - \bar{H}\dot{I}_1)$$

$$\boxed{\frac{1}{A} = \left.\frac{\dot{V}_2}{\dot{V}_1}\right|_{-\dot{I}_2=0} = \frac{(R + \frac{1}{j\omega C})\bar{H}\dot{I}_1}{[R + \alpha\bar{H} + j\omega L(1 - \bar{H})]\dot{I}_1} = \frac{(R + \frac{1}{j\omega C})\bar{H}}{R + \alpha\bar{H} + j\omega L(1 - \bar{H})} = 0.046 + j0.160}$$

Generatore di corrente a sinistra, cortocircuito a destra!



$$\begin{cases} \alpha \dot{I}_{RC} = (2j\omega L + \frac{2}{j\omega C} + R)\dot{I}_{RC} - j\omega L \dot{I}_1 + (2j\omega L + \frac{1}{j\omega C})(-\dot{I}_2) \\ \alpha \dot{I}_{RC} = (2j\omega L + \frac{1}{j\omega C})\dot{I}_{RC} - j\omega L \dot{I}_1 + (3j\omega L + \frac{1}{j\omega C})(-\dot{I}_2) \end{cases}$$

Sottraendo membro a membro.

$$0 = (\frac{1}{j\omega C} + R)\dot{I}_{RC} - j\omega L(-\dot{I}_2)$$

$$\dot{I}_{RC} = \frac{j\omega L}{R + \frac{1}{j\omega C}}(-\dot{I}_2)$$

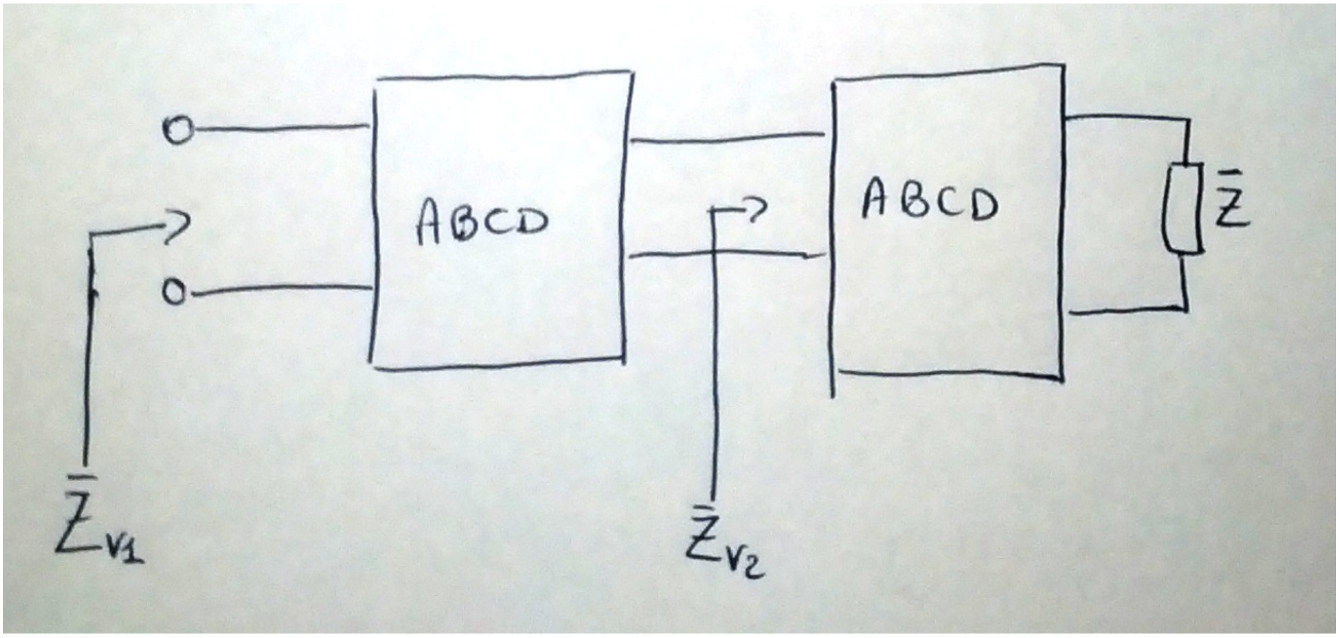
$$\begin{cases} \dot{I}_{RC} = \bar{K}\dot{I}_1 = (-0.013 + j0.093)\dot{I}_1 \\ (-\dot{I}_2) = \bar{H}\dot{I}_1 = (0.597 - j0.037)\dot{I}_1 \end{cases}$$

$$\boxed{\frac{1}{D} = -\frac{\dot{I}_2}{\dot{I}_1}\bigg|_{\dot{V}_2=0} = \bar{H} = 0.597 - j0.037}$$

$$\dot{V}_1 = R\dot{I}_1 - \alpha \dot{I}_{RC} + j\omega L(\dot{I}_1 + (-\dot{I}_2) - \dot{I}_{RC}) = R\dot{I}_1 - \alpha\bar{K}\dot{I}_1 + j\omega L\dot{I}_1 + j\omega L\bar{H}\dot{I}_1 - j\omega L\bar{K}\dot{I}_1$$

$$\boxed{\frac{1}{B} = -\frac{\dot{I}_2}{\dot{V}_1}\bigg|_{\dot{V}_2=0} = \frac{\bar{H}}{R - \alpha\bar{K} + j\omega L + j\omega L\bar{H} - j\omega L\bar{K}} = 0.011 - j0.003S}$$

Calcolo delle Z



$$Z_{V2} = \frac{\dot{V}_1}{\dot{I}_1} = \frac{A\dot{V}_2 + B(-\dot{I}_2)}{C\dot{V}_2 + D(-\dot{I}_2)} = \frac{AZ(-\dot{I}_2) + B(-\dot{I}_2)}{CZ(-\dot{I}_2) + D(-\dot{I}_2)} = \frac{AZ + B}{CZ + D} = 52.96 - j5.99\Omega$$

$$Z_{V1} = \frac{\dot{V}_1}{\dot{I}_1} = \frac{AZ_{V2} + B}{CZ_{V2} + D} = 57.77 + j6.86\Omega$$

Esercizio 4

$$G_m = \frac{P_{10}}{V_{10}^2} = 1,25 \cdot 10^{-3} S$$

$$R_m = \frac{1}{G_m} = 802,22 \Omega$$

$$|Y_m| = \frac{I_{10}\sqrt{3}}{V_{10}} = 6.84 \cdot 10^{-3}$$

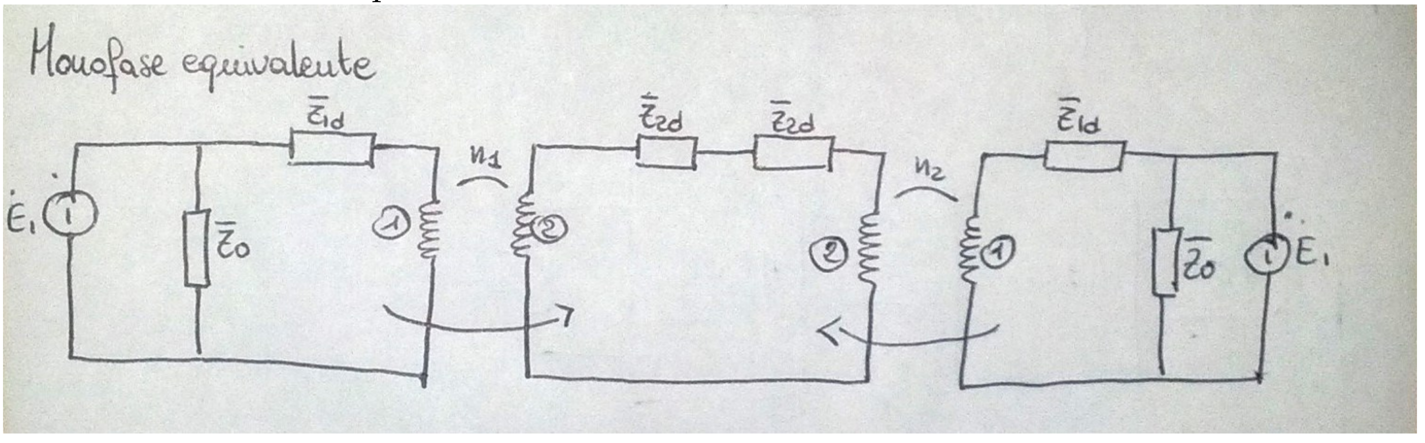
$$B_m = \sqrt{Y_m^2 - G_m^2} = 6.72 \cdot 10^{-3} S$$

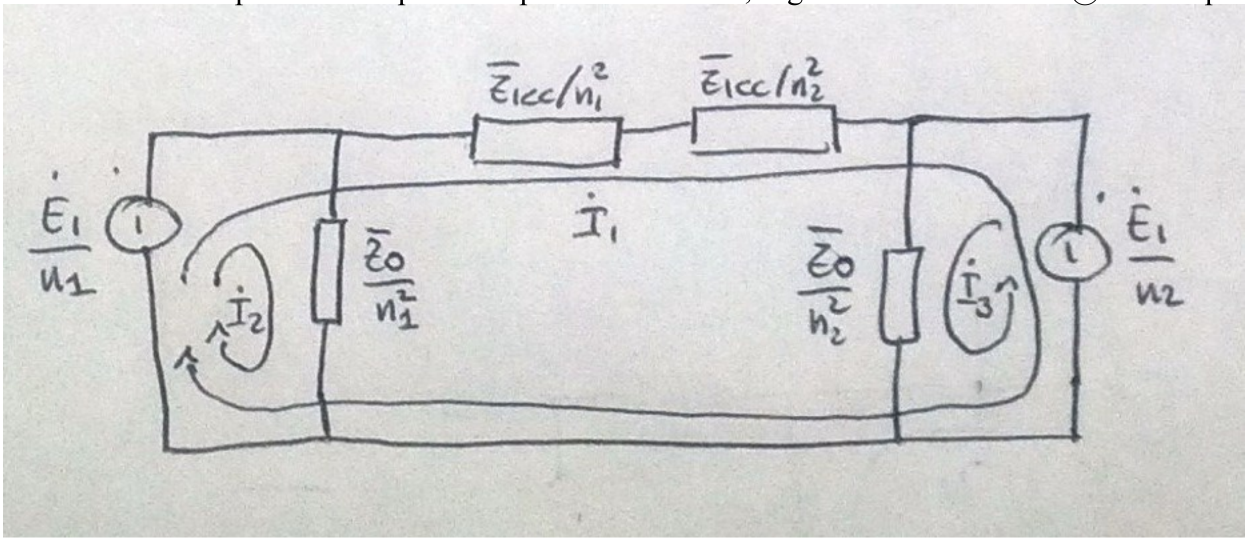
$$Z_0 = \frac{1}{G_m - jB_m} = 26.75 + j143.83 \Omega$$

$$cos\varphi_{cc} = \frac{P_{cc}\sqrt{3}}{V_{cc} I_{cc}} = 0.352 \qquad |Z_{1cc}| = 0.768$$

$$Z_{cc} = |Z_{cc}|(cos\varphi_{cc} + j\sqrt{1 - cos^2\varphi_{cc}}) = Z_{1d} + n^2 Z_{2d} = 0,27 + j0.719 \Omega$$

Riscrivo il monofase equivalente.





$\frac{E_1}{n_1} - \frac{E_1}{n_2} = I_1(\frac{Z_{1cc}}{n_1^2} + \frac{Z_{1cc}}{n_2^2})$

$$\dot{I}_1 = \frac{\dot{E}_1(\frac{1}{n_1} - \frac{1}{n_2})}{Z_{1cc}(\frac{1}{n_1^2} + \frac{1}{n_2^2})} = \frac{\dot{E}_1}{Z_{1cc}} \frac{(n_2 - n_1)n_1 n_2}{n_1^2 + n_2^2}$$

$$P_{fe1} = (\frac{E_1}{n_1})^2 3n_1^2 G_m \quad P_{fe2} = (\frac{E_1}{n_2})^2 3n_2^2 G_m \quad P_{cu1} = 3\frac{R_{cc}}{n_1^2}|\dot{I}_1|^2 \quad P_{cu2} = 3\frac{R_{cc}}{n_2^2}|\dot{I}_1|^2$$

$Con\ n_1 = 0,1$ e $n_2 = 0,11$
$\rightarrow P_{fe1} = 182.5W \ \ P_{fe2} = 181.5W$
$Con\ n_1 = 0,1$ e $n_2 = 0,15$
$\rightarrow P_{fe1} = 182.5W \ \ P_{fe2} = 181.58W$
In genere la potenza totale negli avvolgimenti di rame delle due macchine vale:

$$P_{cu} = 3R_{1cc}(\frac{1}{n_1^2} + \frac{1}{n_2^2})\frac{E_1^2}{|Z_{1cc}|^2}\frac{(n_2 - n_1)^2 n_1 n_2}{(n_2^2 + n_1^2)^2} = \frac{3R_{1cc}E_1^2}{|Z_{1cc}|^2}\frac{(n_2 - n_1)^2}{n_1 n_2(n_1^2 + n_2^2)}$$

Che, in funzione dei rapporti spira, ~~de~~cresce all'aumentare della differenza di questi ultimi.