

# HW #1

## Problem 1

(a)  $(1, 1, 2, 3) + (1, 2, 3, 5) = (2, 3, 5, 8)$

(b)  $2(1, 2, 4, 8) + (1, 3, 7, 15) = (2, 4, 8, 16) + (1, 3, 7, 15)$   
 $= (3, 7, 15, 31)$

## Problem 2

(a)  $3(x, y, z, t) + (1, 4, 16, 25) = (4, 16, 4, 16)$

$$\Rightarrow 3(x, y, z, t) = (4, 16, 4, 16) - (1, 4, 16, 25)$$

$$= (3, 12, -12, -9)$$

$$\Rightarrow (x, y, z, t) = \frac{1}{3}(3, 12, -12, -9) = (1, 4, -4, -3)$$

(b)  $2(x, y, z, t) \neq (-1, 2, 2, -1) = (1, 2, 2, 1)$

$$\Rightarrow 2(x, y, z, t) = (1, 2, 2, 1) - (-1, 2, 2, -1)$$

$$= (2, 0, 0, 2)$$

$$\Rightarrow (x, y, z, t) = \frac{1}{2}(2, 0, 0, 2) = (1, 0, 0, 1)$$

## Problem 3 :

s	f	g	f+g	f-2g
Napoli	3	1	4	1
Roma	2	2	4	-2
Pisa	1	3	4	-5

## Problem 4

(a)  $(1+i) + (2+3i) = 3+4i$

(b)  $(1+i)(2+3i) = (1)(2) + (1)(3i) + (i)(2) + (i)(3i)$   
 $= 2 + 3i + 2i - 3 = -1 + 5i$

(c)  $\frac{1+i}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{(1)(2) + (1)(-3i) + (i)(2) + (i)(-3i)}{4+9}$   
 $= \frac{(2+3) + (-3+2)i}{13} = \frac{5-i}{13}$

# Problema 5

HW #7

$$\textcircled{a} \quad \begin{cases} 3x + 2y = 1 \\ 2x + 3y = -1 \end{cases} \xrightarrow{R_2 \rightarrow -\frac{2}{3}R_1 + R_2} \begin{cases} 3x + 2y = 1 \\ (3 - \frac{4}{3})y = -1 - \frac{2}{3} \end{cases}$$

$$\Rightarrow \begin{cases} 3x + 2y = 1 \\ \frac{5}{3}y = -\frac{5}{3} \end{cases} \Rightarrow y = -1, \quad 3x + 2y = 1$$

$$\Rightarrow y = -1, \quad x = 1$$

$$\textcircled{b} \quad \begin{cases} 2x - y = 3 \\ x + 2y = 4 \end{cases} \xrightarrow{R_1 \leftrightarrow R_2} \begin{cases} x + 2y = 4 \\ 2x - y = 3 \end{cases}$$

$$\xrightarrow{R_2 = R_2 - 2R_1} \begin{cases} x + 2y = 4 \\ -5y = -5 \end{cases} \Rightarrow y = 1, \quad x + 2y = 4$$

$$y = 1, \quad x = 2$$

$$\textcircled{c} \quad \begin{cases} x + y + z = 1 \\ 2x + 3y + 4z = 5 \\ 3x + 4y + 4z = 1 \end{cases} \xrightarrow{\begin{matrix} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{matrix}} \begin{cases} x + y + z = 1 \\ y + 2z = 3 \\ y + z = -2 \end{cases}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{cases} x + y + z = 1 \\ y + z = -2 \\ y + 2z = 3 \end{cases} \xrightarrow{R_3 = R_3 - R_2} \begin{cases} x + y + z = 1 \\ y + z = -2 \\ z = 5 \end{cases}$$

$$\Rightarrow z = 5, \quad y + z = -2 \Rightarrow y = -7$$

$$x + y + z = 1 \Rightarrow x - 7 + 5 = 1 \Rightarrow x = 3$$

$$(x, y, z) = (3, -7, 5)$$

# Problema 6

$$\textcircled{a} \quad \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1-2 \\ 2-1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\textcircled{b} \quad \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 4-6 \\ 2-2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

$$\textcircled{c} \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 1+3 & 2+5 \\ 1+3 & 2+5 \end{pmatrix} = \begin{pmatrix} 4 & 7 \\ 4 & 7 \end{pmatrix}$$

$$\textcircled{d} \quad \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 8 & 16 \end{pmatrix} = \begin{pmatrix} 2+8 & 4+16 \\ 2-8 & 4-16 \end{pmatrix} = \begin{pmatrix} 10 & 20 \\ -6 & -12 \end{pmatrix}$$

### Problema 7

$$(a) \quad \det \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} = (1)(5) - (2)(3) = -1$$

$$(b) \quad \det \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = (2)(2) - (3)(1) = 1$$

### Problema 8

$$\det \begin{pmatrix} 1-x & 3 \\ -1 & 5-x \end{pmatrix} = (1-x)(5-x) + 3$$

$$= 5 - 6x + x^2 + 3 = x^2 - 6x + 8$$

$$= (x-2)(x-4)$$

$$(x-2)(x-4) = 0 \implies x=2, x=4$$