SOLUZIONE ES. 1 08/06/2012

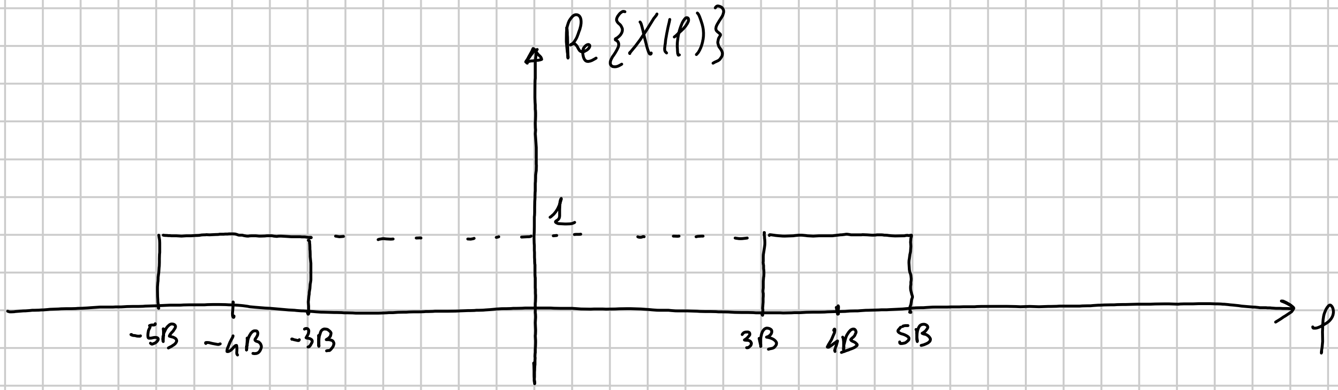
1) $x(t) = x_1(t)\cos(2\pi f_0 t) - x_2(t)\sin(2\pi f_0 t)$

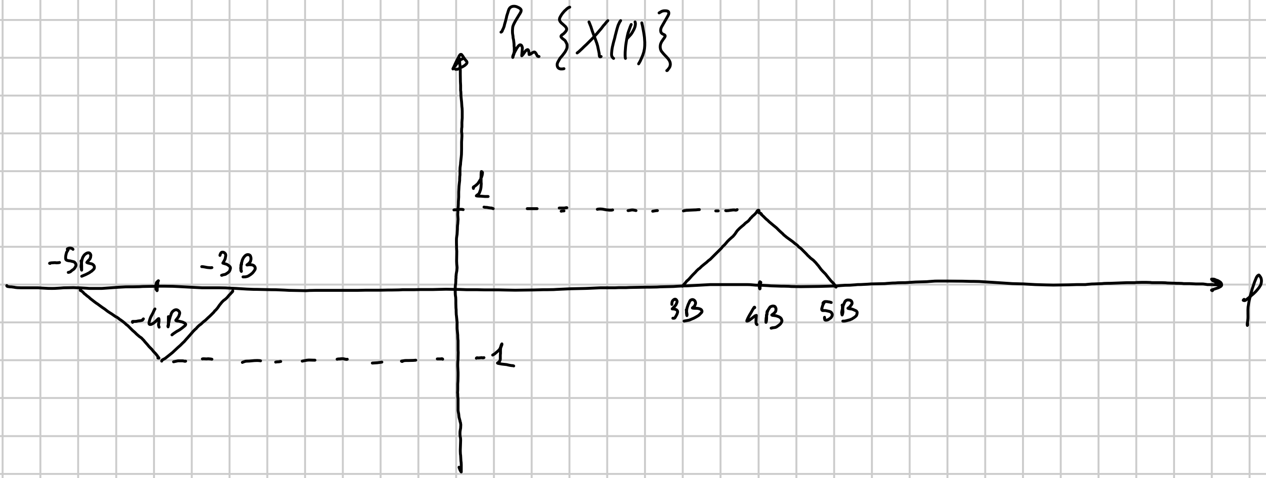
$$X(f) = X_1(f) \otimes \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right] +$$

$$- X_2(f) \otimes \frac{1}{2j}\left[\delta(f-f_0) - \delta(f+f_0)\right] =$$

$$= \frac{1}{2}\left[X_1(f-f_0) + X_1(f+f_0)\right] + j\frac{1}{2}\left[X_2(f-f_0) - X_2(f+f_0)\right]$$

$X_1(f) = \mathrm{rect}\left(\frac{f}{2B}\right)$, $X_2(f) = \left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right)$

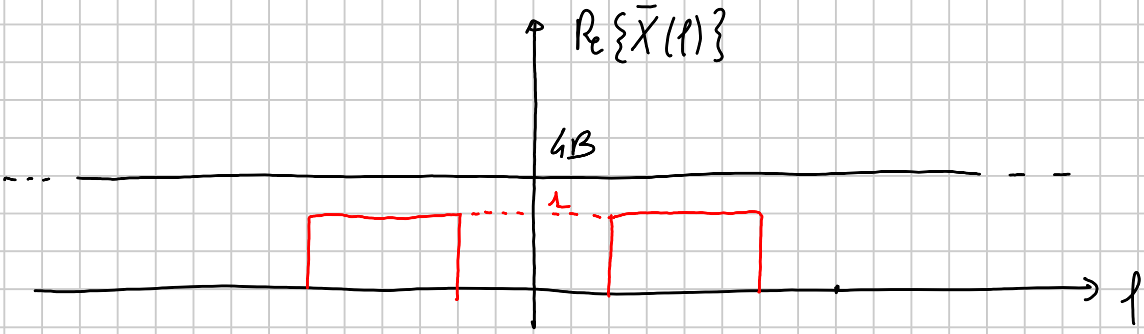


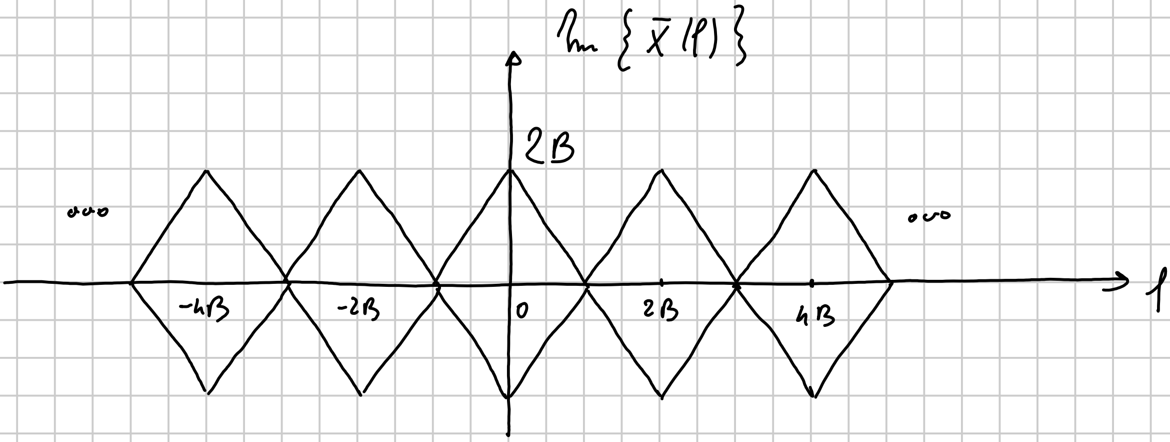


2) $x[n] = x(nT)$

$$\bar{X}(f) = 2B \sum_{n=-\infty}^{+\infty} X(f - n2B) =$$

$$2B \sum_{n=-\infty}^{+\infty} \mathrm{Re}\{X(f-n2B)\} + j\,2B \sum_{n=-\infty}^{+\infty} \mathrm{Im}\{X(f-n2B)\}$$





$$Y(f) = H_{BP}(f)\,\bar{X}(f) = H_{BP}(f)\,Re\{\bar{X}(f)\} + j\,H_{BP}(f)\,\underbrace{Im\{\bar{X}(f)\}}_{=0}$$

$$= 4B\left[rect\left(\frac{f-2B}{2B}\right) + rect\left(\frac{f+2B}{2B}\right)\right]$$

$$w(t) = y(t)\cos(2\pi f_0 t)$$

$$W_1(f) = \frac{1}{2}Y(f-2B) + \frac{1}{2}Y(f+2B) =$$

$$= 4B\,rect\left(\frac{f}{2B}\right) + \text{comp. reale a } \pm 2f_1 = \pm 4B$$

$$Z_1(f) = W_1(f)\,H_{LP}(f) = W_1(f)\,rect\left(\frac{f}{2B}\right) = 4B\,rect\left(\frac{f}{2B}\right)$$

$$z_1(t) = 8B^2\,sinc(2Bt)$$

$$W_2(f) = -\frac{1}{2j}Y(f-2B) + \frac{1}{2j}Y(f+2B) =$$

$$= -\frac{1}{2j} 4B \, \mathrm{rect}\left(\frac{f}{2B}\right) + \frac{1}{2j} 4B \, \mathrm{rect}\left(\frac{f}{2B}\right) + \text{comp a } \pm 2f_1 = \pm 4B$$

$$Z_2(f) = W_2(f) H_{LP}(f) = 0 \Rightarrow z_2(t) = 0$$

3) $E_{z_1} = 16B^2 \cdot 2B = 32B^3 \Rightarrow P_{z_1} = 0$

$E_{z_2} = 0 \Rightarrow P_{z_2} = 0$

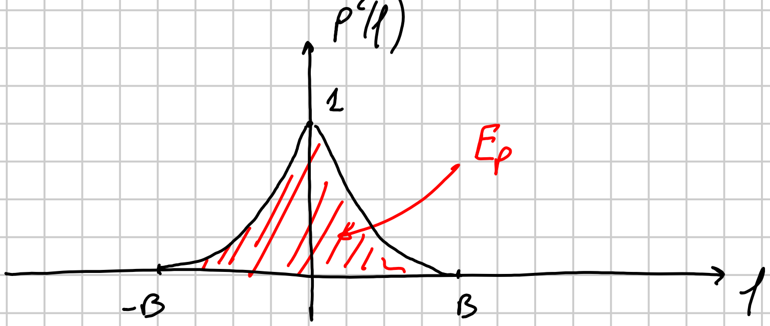
SOLUZIONE ES. 2 08/06/2012

1) $$E_s = E\left[\int_{-\infty}^{+\infty} x^2[n] \, p^2(t) \, dt\right] = \int_{-\infty}^{+\infty} E[x^2[n]] \, p^2(t) \, dt =$$

$$= E[x^2[n]] \, E_p$$

$$E[x^2[n]] = \frac{1}{2}(-1)^2 + \frac{1}{2}(2)^2 = \frac{1}{2} + 2 = \frac{5}{2}$$

$$E_p = \int_{-\infty}^{+\infty} p^2(t) \, dt = \int_{-\infty}^{+\infty} P^2(f) \, df = \frac{2}{3} B$$



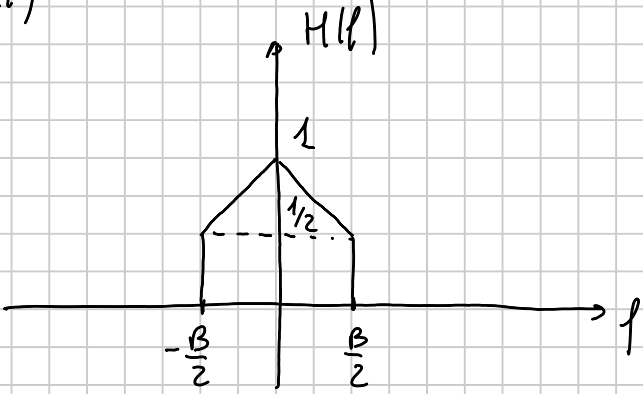
$$E_s = \frac{5}{2} \cdot \frac{2}{3} B = \frac{5}{3} B$$

2) $$S_{n_u}(f) = S_n(f) \, |H_R(f)|^2 = \frac{N_0}{2} \mathrm{rect}\left(\frac{f}{B}\right)$$

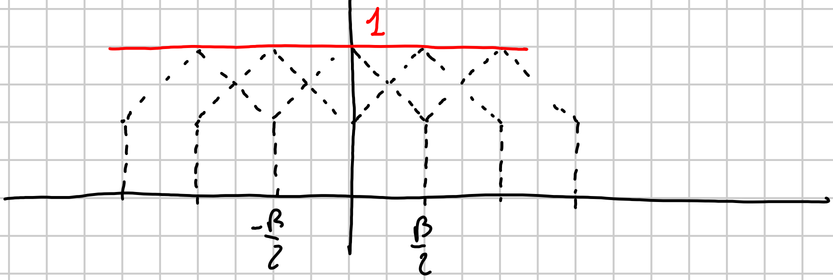
$$P_{n_u} = \int_{-\infty}^{+\infty} S_{n_u}(f) \, df = \frac{N_0 B}{2}$$

3) $\displaystyle\sum_{n=-\infty}^{+\infty} H\left(f-\frac{n}{T}\right) = K$

$H(f) = P(f)\, H_R(f)$



$\displaystyle\sum_n H\left(f-n\frac{B}{2}\right) = 1$



Cond. di Nyquist verificata

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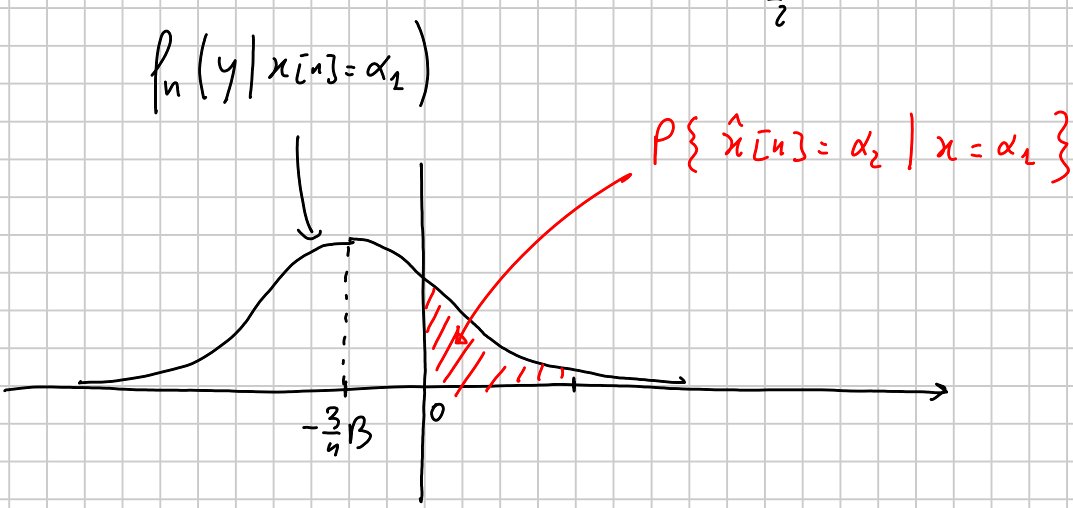
4) $\displaystyle y[n] = x[n]\, h(0) = x[n]\int_{-\infty}^{+\infty} H(f)\, df = \frac{3}{4}B\, x[n]$

$$P_E(2) = P_E(b) = P\{\hat{x}[n]=\alpha_1 \mid x[n]=\alpha_2\}\, \overset{=\frac{1}{2}}{P\{\alpha_2\}} +$$

$$+\, P\{\hat{x}[n]=\alpha_2 \mid x[n]=\alpha_1\}\, \underset{\frac{1}{2}}{\underset{\parallel}{P\{\alpha_1\}}}$$

$\alpha_1 = -1$

$\alpha_2 = 2$



$$P\{\hat{x}[n]=\alpha_2 \mid x[n]=\alpha_1\} = Q\left(\frac{\frac{3}{4}B}{\sqrt{\frac{N_0 B}{2}}}\right) = Q\left(\sqrt{\frac{9}{8}\frac{B}{N_0}}\right)$$

$$P\{\hat{x}[n] = \alpha_1 \mid x[n] = \alpha_1\} = Q\left(\frac{\frac{3}{2}B}{\sqrt{\frac{N_0 B}{2}}}\right) = Q\left(\sqrt{\frac{9}{2}\frac{B}{N_0}}\right)$$

$$P_E(b) = \frac{1}{2} Q\left(\sqrt{\frac{9}{8}\frac{B}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{9}{2}\frac{B}{N_0}}\right)$$