Promemoria sui Numeri Complessi

$z = a + jb$

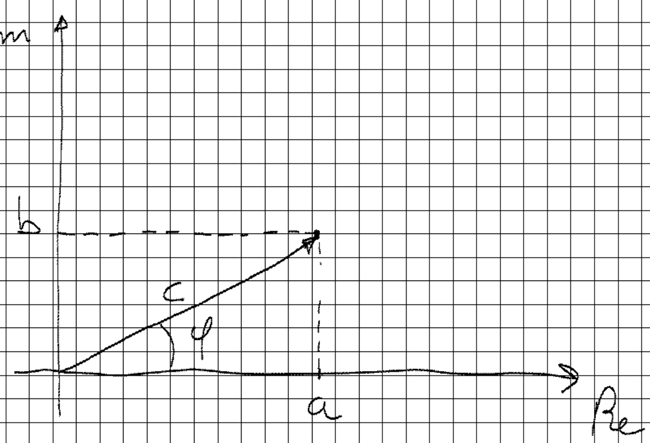
$a = \mathrm{Re}\{z\}$ = parte reale

$b = \mathrm{Im}\{z\}$ parte immaginaria

$z = c\, e^{j\varphi}$

c = modulo

φ = fase



$c = \sqrt{a^2 + b^2}$

$\varphi = \mathrm{arctg}\left[\dfrac{b}{a}\right]$

$a = c \cos\varphi$

$b = c \sin\varphi$

$z = a + jb = c\cos\varphi + j\, c\sin\varphi = c\, e^{j\varphi}$

$\left(e^{j\varphi} = \cos\varphi + j\sin\varphi\right)$

$\mathrm{Re}\{z\} = a = \dfrac{1}{2}\left[z + z^*\right]$

$\mathrm{Im}\{z\} = b = \dfrac{1}{2}\left[z - z^*\right]$

Operazioni Algebriche

$\cdot)\ z_1 = a_1 + j b_1 = c_1 e^{j\varphi_1}$

$\cdot)\ z_2 = a_2 + j b_2 = c_2 e^{j\varphi_2}$

$\cdot)\ z = z_1 \pm z_2 = (a_1 \pm a_2) + j(b_1 \pm b_2)$

$$\cdot)\ z = z_1 \cdot z_2 = (a_1 a_2 - b_1 b_2) + j(a_1 b_2 + a_2 b_1) = \\ = c_1 c_2 e^{j(\varphi_1 + \varphi_2)}$$

$$|z|^2 = z z^* = c e^{j\varphi} \cdot c e^{-j\varphi} = c^2 e^{j(\varphi - \varphi)} = c^2$$

$$z^2 = z \cdot z = c e^{j\varphi} \cdot c e^{j\varphi} = c^2 e^{j2\varphi} = \\ = (a^2 - b^2) + j2ab$$

$$\cdot)\ z = \frac{z_1}{z_2} = z_1 z_2^{-1} = \frac{c_1 e^{j\varphi_1}}{c_2 e^{j\varphi_2}} = \frac{c_1}{c_2} e^{j(\varphi_1 - \varphi_2)}$$

Funzioni complesse di variabile reale

$z(t)\ ,\quad z \in \mathbb{C}\ ,\quad t \in \mathbb{R}$

$$z(t) = a(t) + j b(t) = c(t) e^{j\varphi(t)}$$

$$\int_a^b z(t)\, dt = \int_a^b a(t)\, dt + j \int_a^b b(t)\, dt$$

$$\frac{d}{dt}\left[z(t)\right] = \frac{d}{dt}\left[a(t)\right] + j \frac{d}{dt}\left[b(t)\right]$$