$f(x,y) = (x-y)(x^2+y^2-1)$

$f(x,y) = xy$ $\qquad x^2 - xy + y^2 - 1 = 0 = G(x,y)$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\qquad \frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$

$(x, y)\ A\begin{pmatrix} x \\ y \end{pmatrix} = v^T A v$ $\qquad v = \begin{pmatrix} x \\ y \end{pmatrix}$

$= \langle Av, v \rangle$

$(x\ y)\ A\begin{pmatrix} x \\ y \end{pmatrix} = 1$

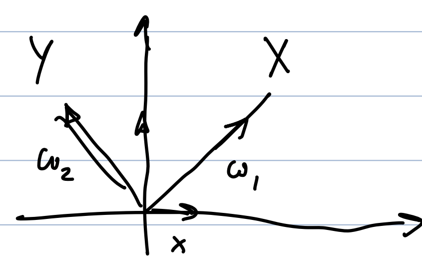
$A = \begin{pmatrix} 1 & -1/2 \\ -\frac{1}{2} & 1 \end{pmatrix}$ $\qquad (x, y)\begin{pmatrix} 1 & -1/2 \\ 1/2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

$\det(A - \lambda I) = 0$ $\qquad \begin{vmatrix} 1-\lambda & -1/2 \\ -1/2 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - \left(\frac{1}{4}\right) = \lambda^2 - 2\lambda + 1 - \frac{1}{4} = 0$

$\lambda^2 - 2\lambda + \frac{3}{4} = 0$ $\qquad \lambda = 1 \pm \sqrt{1 - \frac{3}{4}} = 1 \pm \sqrt{\frac{1}{4}} = 1 \pm \frac{1}{2}$ $\begin{matrix} 3/2 \\ 1/2 \end{matrix}$

$A\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} x \\ y \end{pmatrix}$ $\qquad A\begin{pmatrix} x \\ y \end{pmatrix} = \lambda_1 \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{cases} x - \frac{1}{2}y = \frac{1}{2}x \\ -\frac{1}{2}x + y = y \end{cases}$ $\qquad \frac{x}{2} = \frac{y}{2}$




$A\omega_1 = \frac{1}{2}\omega_1$

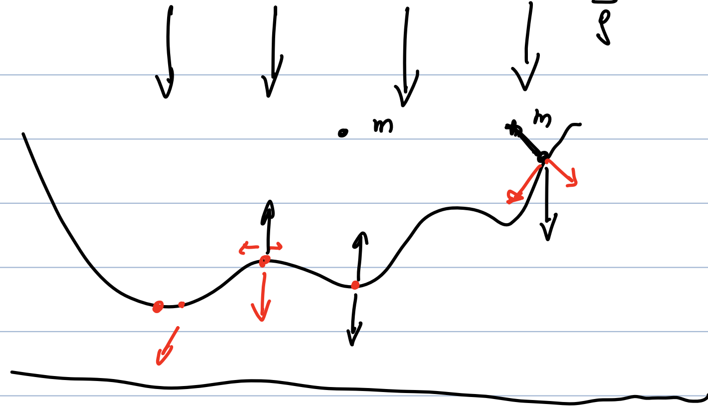
$A\omega_2 = \frac{3}{2}\omega_2$

$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{3}{2} \end{pmatrix}$

$\mathcal{L}(x,y,\lambda) = f(x,y) - \lambda G(x,y)$ $\qquad \nabla \mathcal{L} = 0$ per trovare pt. eventuali max e min vincolati

$\begin{cases} \frac{\partial \mathcal{L}}{\partial x} = 0 \\ \frac{\partial \mathcal{L}}{\partial y} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \end{cases}$ $\qquad (x_0, y_0, \lambda_0)$ $\qquad \nabla f(x_0, y_0) = \lambda_0 \nabla G(x_0, y_0)$

PUNTO MATERIALE $\quad F = \nabla V$



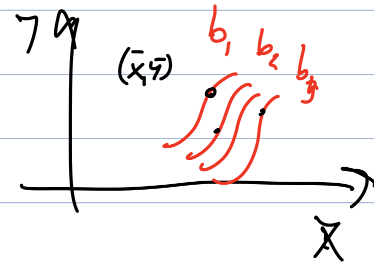
$G(x,y)=0$

$f(\bar{x}(b),\bar{y}(b))$

$\parallel$

$$f(\bar{x},\bar{y}) = \max_{G(x,y)=b} f(x,y) = M(b) \qquad b \in \,]b_1, b_2[$$

$$\begin{cases} f_x(\bar{x}(b),\bar{y}(b)) = \bar{\lambda}(b)\, G_x(\bar{x}(b),\bar{y}(b)) \\ f_y(\bar{x}(b),\bar{y}(b)) = \bar{\lambda}(b)\, G_y(\bar{x}(b),\bar{y}(b)) \\ G(\bar{x}(b),\bar{y}(b)) = b \end{cases} \qquad \nabla G \neq 0$$




$$\frac{d}{db} G(\bar{x}(b),\bar{y}(b)) = G_x \frac{d}{db}\bar{x}(b) + G_y \frac{d}{db}\bar{y}(b) = 1$$

$$\bar{\lambda} G_x \frac{d}{db}\bar{x} + \bar{\lambda} G_y \frac{d}{db}\bar{y} = \bar{\lambda}$$

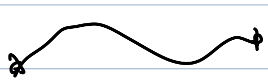
$$\frac{d}{db} M(b) = f_x(\bar{x}(b),\bar{y}(b)) \frac{d\bar{x}}{db} + f_y(\bar{x}(b),\bar{y}(b)) \frac{d\bar{y}(b)}{db} = \bar{\lambda}$$

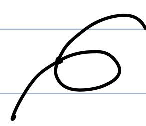
PREZZO OMBRA

CURVE $\quad \varphi : [a,b] \to \mathbb{R}^n \quad$ continua $\qquad \varphi|_{[a,b[}$ iniettiva

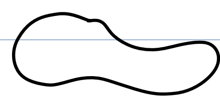
curva semplice

$\varphi(a) = \varphi(b)$ curva chiusa





SOSTEGNO DI $\varphi$ è $\mathrm{Im}\,\varphi$



$$\varphi(t) = \begin{pmatrix} \varphi_1(t) \\ \vdots \\ \varphi_n(t) \end{pmatrix} \in C[a,b] \iff \varphi_i \in C[a,b] \qquad i = 1 \dots n$$

$\varphi, \psi \in C^{1,r}$ $\qquad \gamma \in C^1$

$\varphi, \psi$ EQUIVALENTI $\qquad \exists\ \gamma : [a,b] \to [c,d]$ invertibile

$\gamma^{-1}$ continua $\qquad \gamma' \neq 0$

$\psi(\gamma(t)) = \varphi(t)$

$\varphi : [a,b] \to \mathbb{R}^n \qquad \tilde{\varphi}(t) = \varphi(a+b-t) \qquad t \in [a,b]$

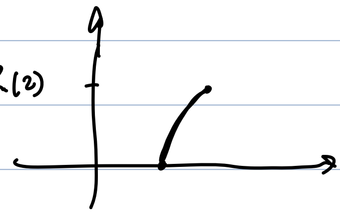
(1) $\varphi(t) = (at+b,\ ct+d) \qquad a \neq 0 \qquad t \in \mathbb{R}$

$$\begin{cases} x(t) = at+b \\ y(t) = ct+d \end{cases} \qquad t = \frac{x-b}{a}$$

$$y = c\left(\frac{x-b}{a}\right) + d = \frac{c}{a}x - \frac{bc}{a} + d$$

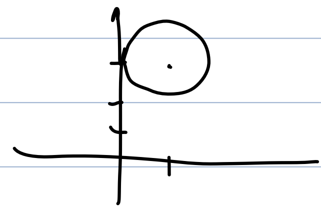
(2) $\varphi(t) = (t, f(t)) \qquad f \in C^1$

$\varphi(t) = (t, \ln(t)) \qquad t \in [1,2]$




(3) $(x(t), y(t)) = (1+\cos(t),\ 3+\sin(t)) \qquad t \in [0,2\pi] \qquad t \in [0,4\pi]$

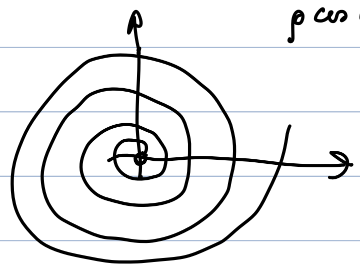
$(x(t),\ y(t))$



$$(\underbrace{x(t)-1}_{=\cos(t)})^2 + (\underbrace{y(t)-3}_{=\sin(t)})^2 = 1$$

(4) $(x(t), y(t)) = (\underbrace{t\cos(t)}_{=\rho\cos\theta},\ \underbrace{t\sin(t)}_{=\rho\sin\theta}) \qquad t \in [0,+\infty[$

$$\boxed{\begin{matrix}\rho = t \\ \theta = t\end{matrix}}$$

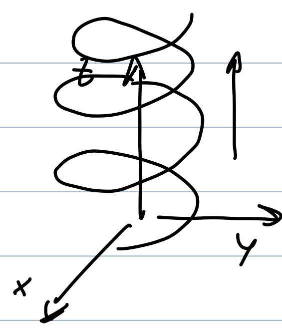


$$x^2(t) + y^2(t) = t^2(\cos^2(t) + \sin^2(t)) = t^2$$

SPIRALE DI ARCHIMEDE

(5) $\varphi(t) = (\cos(t), \sin(t), t)$

$(x(t),\ y(t),\ z(t))$




$(\cos[(t-1)^3],\ \sin[(t-1)^3])$

$-\sin((t-1)^3)\cdot 3(t-1)^2,\ \cos(\ )\,3(t-1)^2$

$\sin^2(\ )\ 9(t-1)^4 + \cos^2(\ )\ 9(t-1)^4$

$9(t-1)^4 = \|\varphi'\|^2$

CURVA REGOLARE $\quad \varphi \in C^1$

$\|\varphi'(t)\| \neq 0$

$$\varphi(t) = \begin{pmatrix}\varphi_1(t) \\ \vdots \\ \varphi_n(t)\end{pmatrix} \qquad \varphi'(t) = \begin{pmatrix}\varphi_1'(t) \\ \varphi_2'(t) \\ \vdots \\ \varphi_n'(t)\end{pmatrix}$$

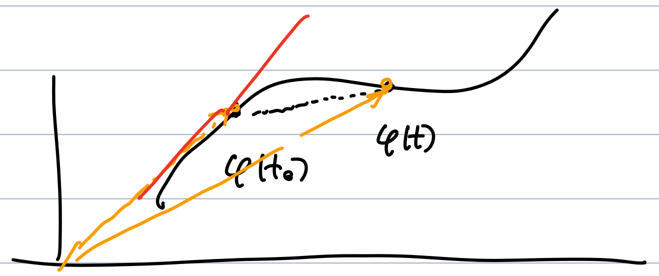
$$[\varphi_1'(t)]^2 + [\varphi_2'(t)]^2 + \dots + [\varphi_n'(t)]^2 \neq 0$$

CURVA REGOLARE A TRATTI

$[a,b] \qquad [a,t_1]\quad [t_1,t_2]\quad \dots\quad ,[t_{n-1},b]$

$I_1 \quad I_2$

t.c. $\varphi|_{I_j}$ è REGOLARE

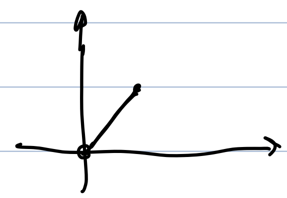


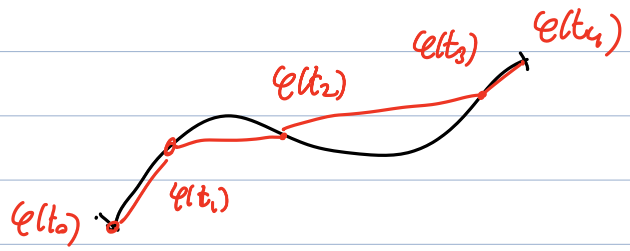

$$\frac{\varphi(t)-\varphi(t_0)}{t-t_0} \xrightarrow{t\to t_0} \varphi'(t_0)$$

$\varphi(t_0)+\varphi'(t_0)(t-t_0)$ curva

retta $t_g$ in $\varphi(t_0) \in \mathbb{R}^n$

$\varphi(t)=(t,t) \qquad t\in[0,1]$






$\varphi:[a,b]\to\mathbb{R}^n$

$\mathcal{P} = \{t_0=a<t_1<t_2<\dots<t_n=b\}$

$$L(\mathcal{P}) = \|\varphi(t_1)-\varphi(t_0)\| + \|\varphi(t_2)-\varphi(t_1)\| + \|\varphi(t_3)-\varphi(t_2)\| + \|\varphi(t_4)-\varphi(t_3)\|$$
$$= \sum_{k=1}^{n} \|\varphi(t_k)-\varphi(t_{k-1})\| \geq 0$$

$\varphi$ È RETTIFICABILE $\qquad \sup_{\mathcal{P}} L(\mathcal{P}) < +\infty$

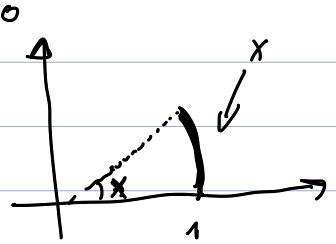
$$L(\varphi) = \sup_{\mathcal{P}} L(\mathcal{P}) = \int_a^b \|\varphi'(t)\|\,dt$$

$(\cos(t),\sin(t)) = \varphi(t) \qquad t\in[0,2\pi] \qquad \varphi'(t) = (-\sin(t),\cos(t))$

$t\in[0,4\pi]$

$\|\varphi'(t)\| = \sqrt{\sin^2(t)+\cos^2(t)} = 1$

$$\int_0^{2\pi} 1\,dt = 2\pi$$




$t\in[0,x] \qquad \int_0^x 1 = x$

$(\cos(t),\sin(t))$

$\varphi(t) = (R\cos(t), R\sin(t)) \qquad t\in[0,2\pi]$

$$\int_0^{2\pi} \sqrt{R^2\sin^2(t)+R^2\cos^2(t)}\,dt = \int_0^{2\pi} \sqrt{R^2} = 2\pi R$$

$$\sum_{k=1}^{n} \|\varphi(t_k)-\varphi(t_{k-1})\| \le \int_a^b \|\varphi'(t)\|\,dt \qquad \forall \text{ scelte } \overset{a}{\overset{\shortparallel}{t_0}} < t_1 \cdots < t_n = b$$

$$\sum \| \qquad \| \ge \int_a^b \|\varphi'(t)\|\,dt - \varepsilon$$

$f : [a,b] \to \mathbb{R}$ $\qquad \left| \int_a^b f(x)\,dx \right| \le \int_a^b |f(x)|\,dx$

$f \in C([a,b])$

$\vec{f} : [a,b] \to \mathbb{R}^n$ $\qquad \int_a^b \vec{f}(x)\,dx = \begin{pmatrix} \int_a^b f_1(x)\,dx \\ \vdots \\ \int_a^b f_n(x)\,dx \end{pmatrix}$

$\vec{f} \in C[a,b]$

$$\star \quad \left\| \int_a^b \vec{f}(x)\,dx \right\| \le \int_a^b \|\vec{f}(x)\|\,dx$$

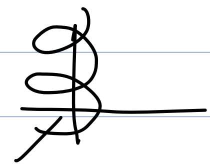
$$\varphi_i(t_k) - \varphi_i(t_{k-1}) = \int_{t_{k-1}}^{t_k} \varphi_i'(t)\,dt$$

$$\|\varphi(t_k) - \varphi(t_{k-1})\| = \left\| \int_{t_{k-1}}^{t_k} \varphi'(t)\,dt \right\|$$

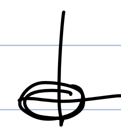
$$\sum \|\varphi(t_k) - \varphi(t_{k-1})\| \le \sum \int_{t_{k-1}}^{t_k} \|\varphi'(t)\|\,dt = \int_a^b \|\varphi'(t)\|\,dt$$

$$\begin{cases} x(t) = R\cos(t) \\ y(t) = R\sin(t) \\ z(t) = pt \end{cases} \qquad R > 0 \qquad t \in [0, 2\pi]$$

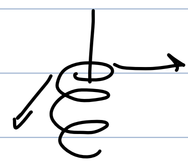
$p > 0$



$p = 0$



$p < 0$



$$\int_0^{2\pi} \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$$

$$= \int_0^{2\pi} \sqrt{R^2\sin^2(t) + R^2\cos^2(t) + p^2}\,dt = \int_0^{2\pi} \sqrt{R^2+p^2} = 2\pi\sqrt{R^2+p^2}$$

SPIRALE $\qquad \rho = f(\theta)$

$x(\theta) = f(\theta)\cos(\theta)$ $\qquad f(\theta) = \theta$ $\qquad \theta \in [0, 2\pi]$

$y(\theta) = f(\theta)\sin(\theta)$

$$\int_0^{2\pi} \|\varphi'(\theta)\|\,d\theta = \int_0^{2\pi} \sqrt{f^2(\theta) + [f'(\theta)]^2}\,d\theta = \int_0^{2\pi} \sqrt{1+\theta^2}\,d\theta$$

$\theta = \sinh(t)$

$d\theta = \cosh(t)$

$x'(\theta) = f'(\theta)\cos(\theta) - f(\theta)\sin(\theta)$

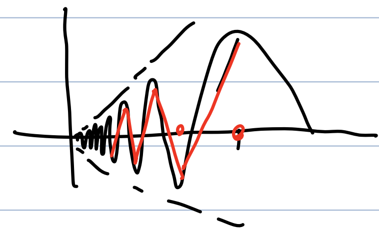
$y'(\theta) = f'(\theta)\sin(\theta) + f(\theta)\cos(\theta)$

$$[x'(\theta)]^2 + [y'(\theta)]^2 = [f'(\theta)]^2 \cos^2\theta + f^2(\theta)\sin^2(\theta) - 2f(\theta)f'(\theta)\cancel{\sin\theta\cos\theta}$$

$$+ [f'(\theta)]^2 \sin^2(\theta) + f^2(\theta)\cos^2(\theta) + 2ff'\cancel{\sin\theta\cos\theta}$$

$$= [f'(\theta)]^2 + f^2(\theta)$$

$$f(t) = \begin{cases} t\sin\frac{1}{t} & t>0 \\ 0 & t=0 \end{cases}$$




$f \in C([0,+\infty[)$

$f \in C^1(]0,+\infty[)$

$f'(0)$ N.E.

$\varphi(t) = (t, f(t)) \qquad t \in [0, \frac{2}{\pi}]$

NON RETTIFICABILE