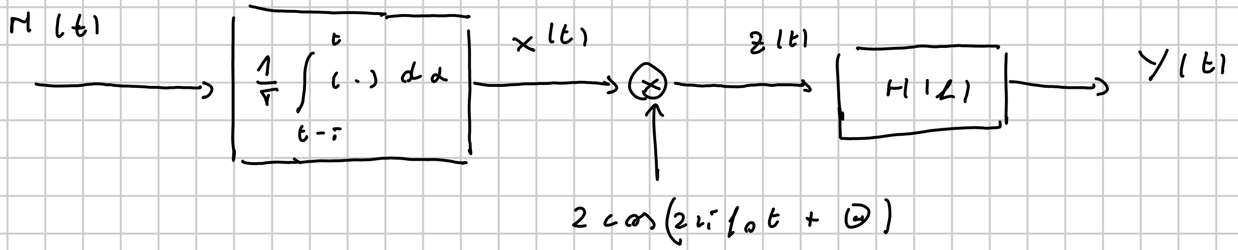
# ESERCIZIO 11




$n(t)$ è un processo SSL con $S_n(f) = \zeta$

(Gaussiano bianco)

$\Theta$ È UNA v.a. INDIP. DA $n(t)$ $\mathcal{U}[-\pi, \pi]$


$S_y(f)$

$X(t)$ è Gaussiano e SSL

$$\eta_x = \eta_n H(0) = 0$$

$$R_x(\tau) = R_n(\tau) \otimes h_1(\tau) \otimes h_1(-\tau)$$

$$\Updownarrow$$

$$S_x(f) = S_n(f) \, |H_1(f)|^2$$

$$h_1(t) = \frac{1}{T} \, \mathrm{rect}\left(\frac{t - T/2}{T}\right)$$

$$\Updownarrow$$

$$H_1(f) = \mathrm{sinc}(fT) \, e^{-j2\pi f \frac{T}{2}} \Rightarrow |H_1(f)|^2 = \mathrm{sinc}^2(fT)$$

$$R_x(\tau) = \underbrace{\zeta \, \delta(\tau)}_{R_n(\tau)} \otimes \underbrace{h_1(\tau) \otimes h_1(-\tau)}_{R_h(\tau)} = \zeta \, R_h(\tau) =$$

$S_n(f) = \zeta$

$$= \mathcal{F}\ \frac{1}{T}\left(1 - \frac{|\tau|}{T}\right) \mathrm{rect}\left(\frac{\tau}{2T}\right) = R_x(\tau)$$

$$s(t) = 2\cos(2\pi f_0 t + \Theta) \qquad \Theta \in \mathcal{U}(-\pi, \pi)$$

$$\eta_s = 0$$

$$R_s(\tau) = 2\cos(2\pi f_0 \tau)$$

$$z(t) = x(t) \cdot s(t)$$

$$\eta_z(t) = E\{z(t)\} = 2\, E\{x(t)\cos(2\pi f_0 t + \Theta)\} \overset{\text{(IND)}}{=}$$

$$= 2\, E\{x(t)\}\, E\{\cos(2\pi f_0 t + \Theta)\} = 0$$

$$R_z(t_1, t_2) = R_z(t, t-\tau) = E\{z(t)\, z(t-\tau)\} =$$

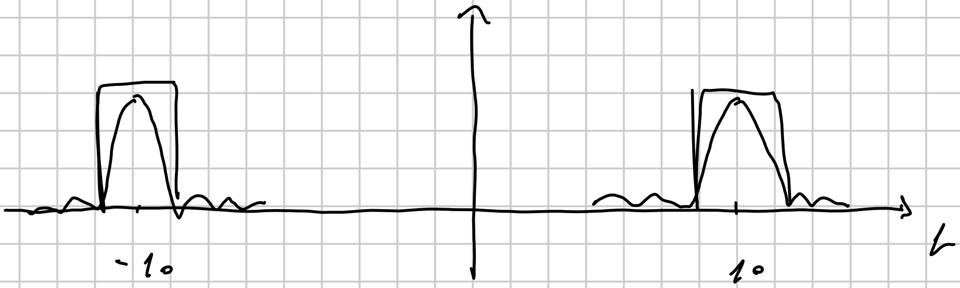
$$= 4\, E\{x(t)\, x(t-\tau) \cos(2\pi f_0 t + \Theta)\cos(2\pi f_0(t-\tau) + \Theta)\} \overset{\text{IND}}{=}$$

$$= 4\, E\{x(t)\, x(t-\tau)\}\, E\{\cos(2\pi f_0 t + \Theta)\cos(2\pi f_0(t-\tau) + \Theta)\} =$$

$$= 4\, R_x(\tau)\, R_s(\tau) = 8\, R_x(\tau)\cos(2\pi f_0 \tau)$$

$$S_y(f) = S_z(f)\, |H(f)|^2$$

$$S_z(f) = 4\, \mathcal{F}\left\{ \mathrm{sinc}^2((f+f_0)T) + \mathrm{sinc}^2((f-f_0)T) \right\}$$



LE 2 REPLICHE CENTRATE A $\pm f_0$ POSSONO CONSIDERARSI DISGIUNTE PERCHÈ $\boxed{f_0 \gg \frac{1}{T}}$

$$S_Y(f) \cong \frac{1}{2}\left[ \mathrm{sinc}^2((f+f_0)T)\,\mathrm{rect}\left(\frac{f+f_0}{2/T}\right) + \mathrm{sinc}^2((f-f_0)T)\,\mathrm{rect}\left(\frac{f-f_0}{2/T}\right)\right]$$

È POSSIBILE RICAVARE LA ddp DEL 1° ORDINE DI $Y(t)$, $f_Y(y;t)$ ?

CONDIZIONATAMENTE A $\Theta$, IL PROCESSO $Y(t)$ È GAUSSIANO QUINDI $f_{Y|\Theta}(y|\theta;t)$ È LA D.D.P. DI UNA GAUSSIANA.

$f_Y(y|\theta;t)$ è la ddp di una gaussiana

$f_\Theta(\theta)$ È NOTA $\Theta \sim \mathcal{U}[-\pi,\pi]$

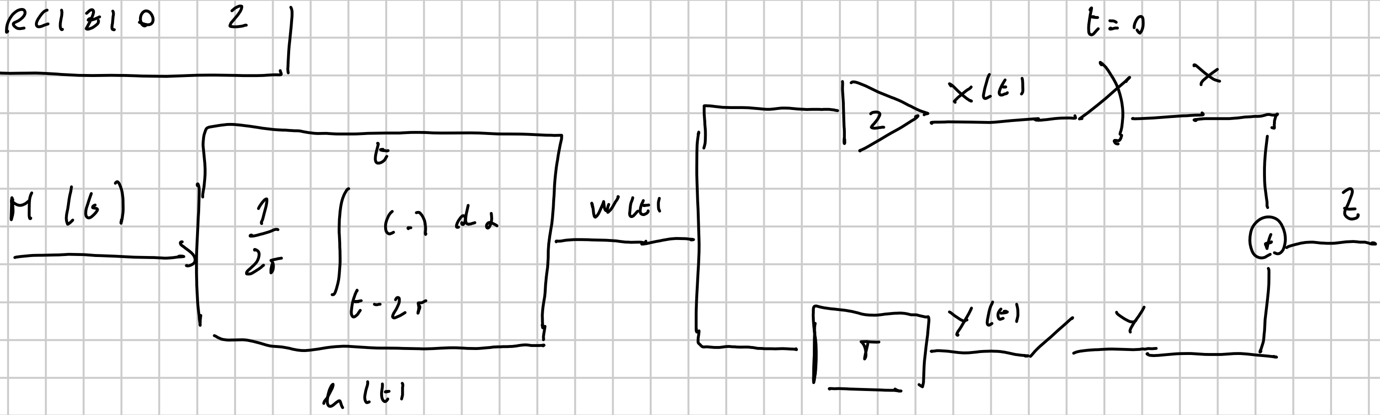
POSSO CALCOLARE LA d.d.p. CONGIUNTA

$$f_{Y\Theta}(y,\theta;t) = f_{Y|\Theta}(y|\theta;t)\cdot f_\Theta(\theta)$$

E DALLA CONGIUNTA POSSO RICAVARE LA MARGINALE

$$f_Y(y;t) = \int_{-\infty}^{+\infty} f_{Y\Theta}(y,\theta;t)\,d\theta$$

# ESERCIZIO 2




$M(t)$ GAUSSIANO BIANCO (SSL) $S_M(f) = 2T$

$h(t) = \frac{1}{2T}\,\mathrm{rect}\left(\frac{t-T}{2T}\right)$

$X(t) = 2\,W(t) \Rightarrow X(0) = X = 2W(0)$

$Y(t) = W(t-T) \Rightarrow Y(0) = Y = W(-T)$

$Z = X + Y$

$f_Z(z)$

X e Y SONO V. A. CONG. GAUSSIANE

$Z = X + Y \qquad (\eta_Z, \sigma_Z^2)$

$\eta_W(t) = \underset{\downarrow\, 0}{\eta_M(t)} \cdot H(0) = 0$

$R_W(\tau) = \left(1 - \frac{|\tau|}{2T}\right)\mathrm{rect}\left(\frac{\tau}{4T}\right)$

$$E\{z\} = E\{X + Y\} = E\{X\} + E\{Y\} = 0$$

$$\sigma_z^2 = E\{(z - \eta_z)^2\} = E\{z^2\} =$$

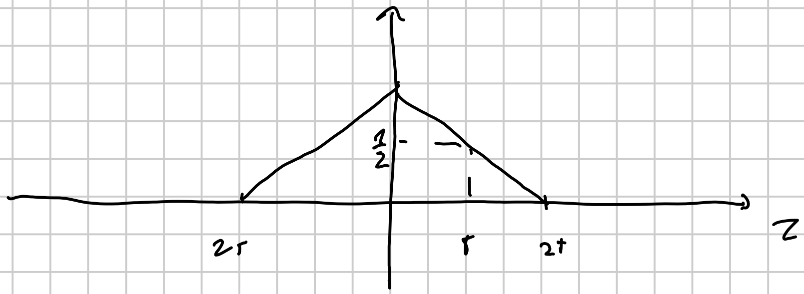
$$= E\{(2w(0) + w(-\tau))^2\} =$$

$$= E\{4w^2(0) + w^2(-\tau) + 4w(0)w(-\tau)\} =$$

$$= 4E\{w^2(0)\} + E\{w^2(-\tau)\} + 4E\{w(0)w(-\tau)\} =$$

$$= 4R_w(0) + R_w(0) + 4R_w(\tau) =$$

$$= 4 + 1 + 4 \cdot \frac{1}{2} = 7$$




$\eta_z = 0$ $\qquad$ $\sigma_z^2 = 7$

$$f_z(z) = \frac{1}{\sqrt{2\pi 7}} e^{-\frac{z^2}{2 \cdot 7}} = \frac{1}{\sqrt{14\pi}} e^{-\frac{z^2}{14}}$$

# ESERCIZIO 3

$x(t)$ PROCESSO ALEATORIO GAUSSIANO SSL

$$R_x(\tau) = \delta(\tau) + 1$$

$$|H(f)| = \sqrt{\left(1 - \frac{|f|}{2B}\right) \text{rect}\left(\frac{f}{4B}\right)}$$

$$\xrightarrow{x(t)} \boxed{H(f)} \longrightarrow y(t)$$

$R_y(\tau)$ $P_y$ ?

$$S_x(f) = \mathcal{F}^{-1}\{R_x(\tau)\} = 1 + \delta(f)$$

$$R_y(\tau) = R_x(\tau) \otimes h(\tau) \otimes h(-\tau)$$

$$\Updownarrow$$

$$S_y(f) = S_x(f)\,|H(f)|^2 = (1 + \delta(f))\left(1 - \frac{|f|}{2B}\right) \text{rect}\left(\frac{f}{4B}\right)$$

$$P_y = \int_{-\infty}^{+\infty} S_y(f) = 1 + \frac{4B \cdot 1}{2} = 1 + 2B$$

$$R_y(\tau) = \frac{1}{\tau} \text{sinc}^2(2B\tau) + 1$$

$Y(t_1)$ e $Y(t_2)$ v.a estratte da $Y(t)$

SOTTO QUALI CONDIZIONI DI $t_1$ e $t_2$ $Y(t_1) = y_1$ E $Y(t_2) = y_2$ SONO INDIPENDENTI?

$$E^2\{Y\} = \eta_Y^2 = \lim_{\tau \to \infty} R_Y(\tau) = 1$$

$$\Downarrow$$

$$\boxed{\eta_Y = 1}$$

$$C_{Y_1 Y_2} = E\{(Y_1 - \eta_{Y_1})(Y_2 - \eta_{Y_2})\} =$$

$$= R_Y(\tau) - \eta_{Y_1}\eta_{Y_2} = \frac{1}{T} \mathrm{sinc}^2(2B\tau) + \cancel{1} - \cancel{1} =$$

$$= 2B \,\mathrm{sinc}^2(2B\tau) \stackrel{?}{=} 0$$

$$\tau = \frac{k}{2B} \qquad \tau \neq 0$$

$$t_2 = t_1 + \frac{k}{2B}$$

INCORRELATE $\Rightarrow$ INDIPENDENTI