ESERCITAZIONE 29-02-2012

TRASFORMATA SERIE DI FOURIER

Esercizio #1

$$x(t) = A\cos(2\pi f_0 t + \varphi) + B\sin(4\pi f_0 t)$$

-) calcolare lo spettro di $x(t)$

-) Disegnare spettro di ampiezza e fase.

$x(t)$ reale e periodico $\rightleftarrows$ X_n con simmetria Hermitiana.

EQUAZIONE DI SINTESI

$$x(t) = \frac{A}{2}\left[e^{j2\pi f_0 t}e^{j\varphi} + e^{-j2\pi f_0 t}e^{-j\varphi}\right] + \frac{B}{2j}\left[e^{j4\pi f_0 t} - e^{-j4\pi f_0 t}\right] =$$

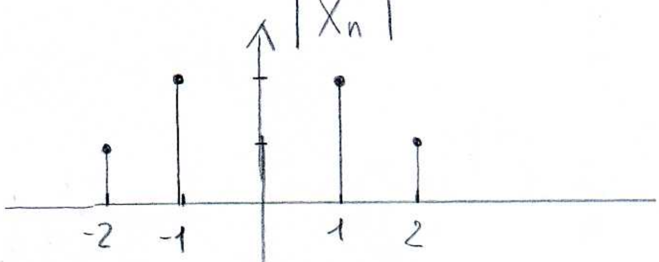
$$= \frac{A}{2}e^{j\varphi}e^{j2\pi f_0 t} + \frac{A}{2}e^{-j\varphi}e^{-j2\pi f_0 t} + \frac{B}{2}e^{-j\pi/2}e^{j4\pi f_0 t} - \frac{B}{2}e^{-j\pi/2}e^{-j4\pi f_0 t} =$$

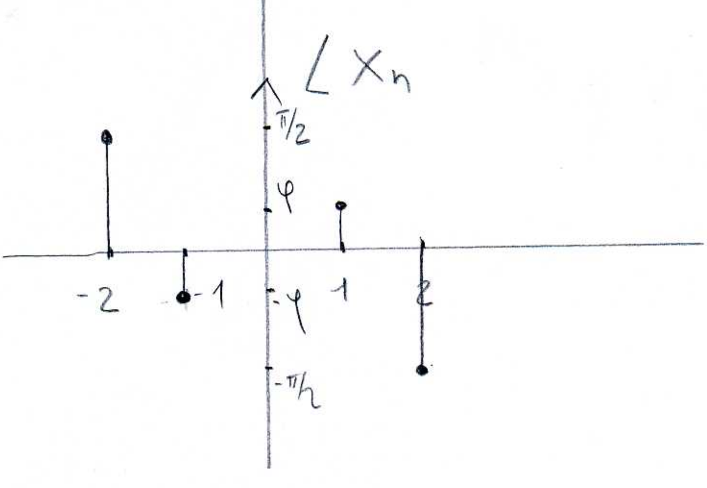
$$X_1 = \frac{A}{2}e^{j\varphi}$$

$$X_{-1} = \frac{A}{2}e^{-j\varphi}$$

$$X_2 = \frac{B}{2}e^{-j\pi/2}$$

$$X_{-2} = \frac{B}{2}e^{j\pi/2}$$





Esercizio #2

$$z(t) = A \cos(2\pi f_0 t + \varphi) \cdot \cos(4\pi f_0 t)$$

$z(t)$ reale e periodico $\rightleftarrows$ Z_n con simmetria Hermitiana

EQUAZIONE DI SINTESI

$$z(t) = \frac{A}{4}\left[e^{j2\pi f_0 t + j\varphi} + e^{-j2\pi f_0 t - j\varphi}\right] \cdot \left[e^{j4\pi f_0 t} + e^{-j4\pi f_0 t}\right] =$$

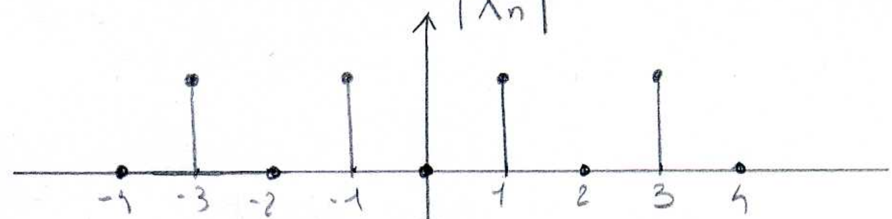
$$= \frac{A}{4}\left[e^{j6\pi f_0 t + j\varphi} + e^{-j2\pi f_0 t + j\varphi} + e^{j2\pi f_0 t - j\varphi} + e^{-j6\pi f_0 t - j\varphi}\right]$$

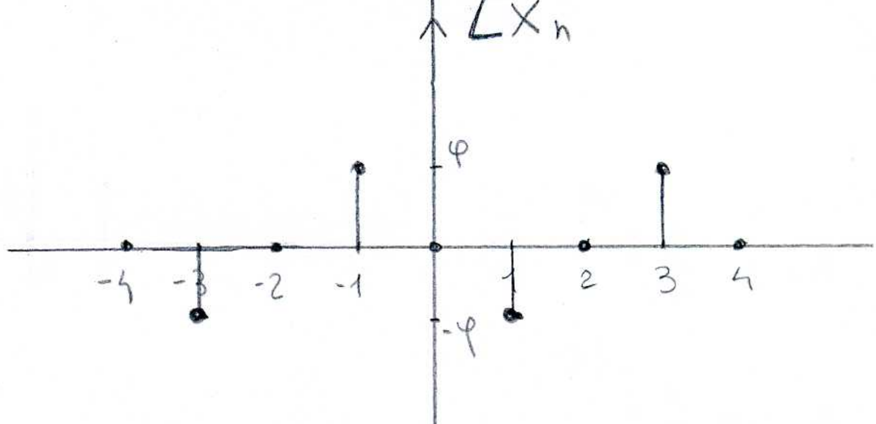
$$X_1 = \frac{A}{4} e^{-j\varphi}$$

$$X_{-1} = \frac{A}{4} e^{j\varphi}$$

$$X_3 = \frac{A}{4} e^{j\varphi}$$

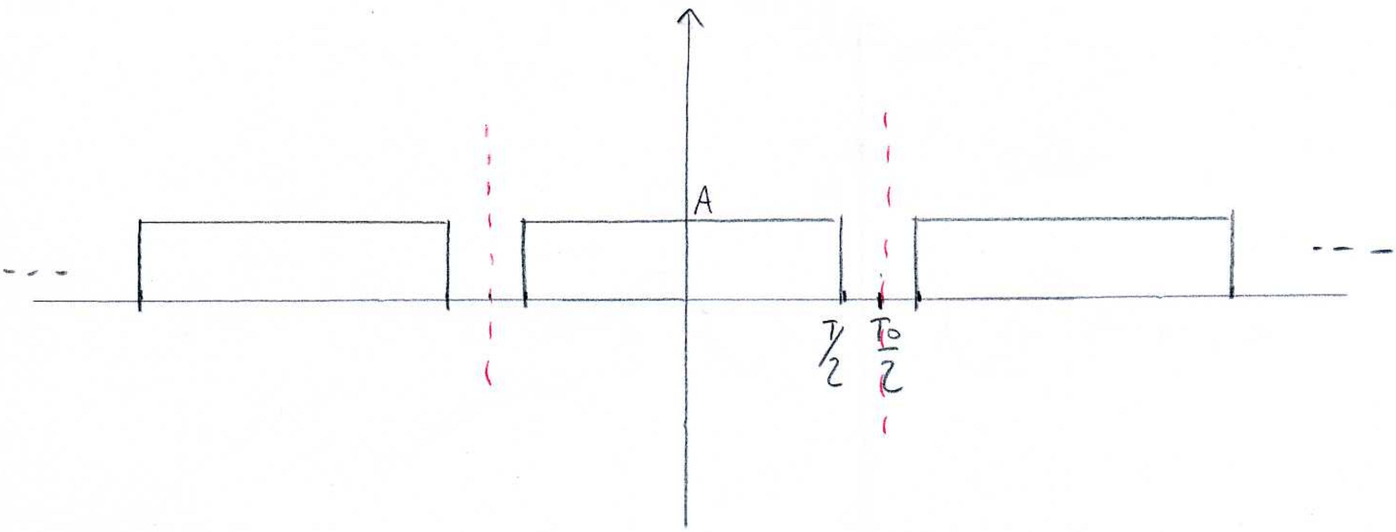
$$X_{-3} = \frac{A}{4} e^{-j\varphi}$$





Esercizio #3

TRENO DI IMPULSI RETTANGOLARI



$x_0(t) = A \, \mathrm{rect}\left(\frac{t}{T}\right)$ $x(t) = A \sum_{k=-\infty}^{+\infty} \mathrm{rect}\left(\frac{t-kT_0}{T}\right)$

~~Se~~ $x(t)$ reale periodico e pari $\rightleftharpoons$ $\mathcal{Im}\{X_n\} = 0$; $X_n = X_{-n}$

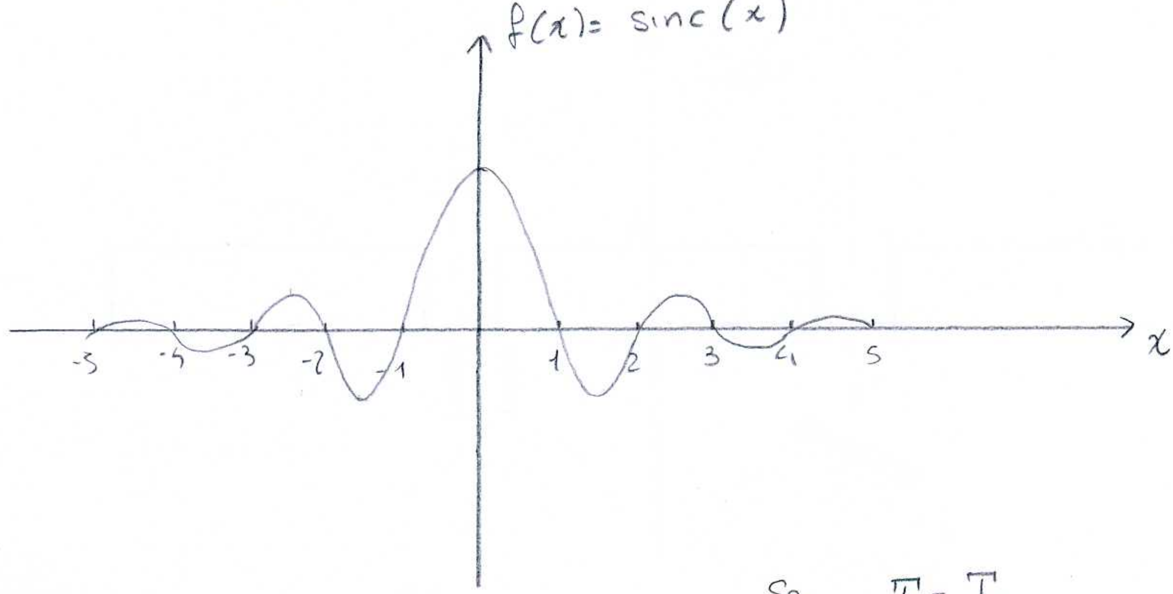
EQUAZIONE DI ANALISI

$$X_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \, e^{-j2\pi n f_0 t} \, dt = \frac{1}{T_0} \int_{-T/2}^{T/2} A \, e^{-j2\pi n f_0 t} \, dt =$$

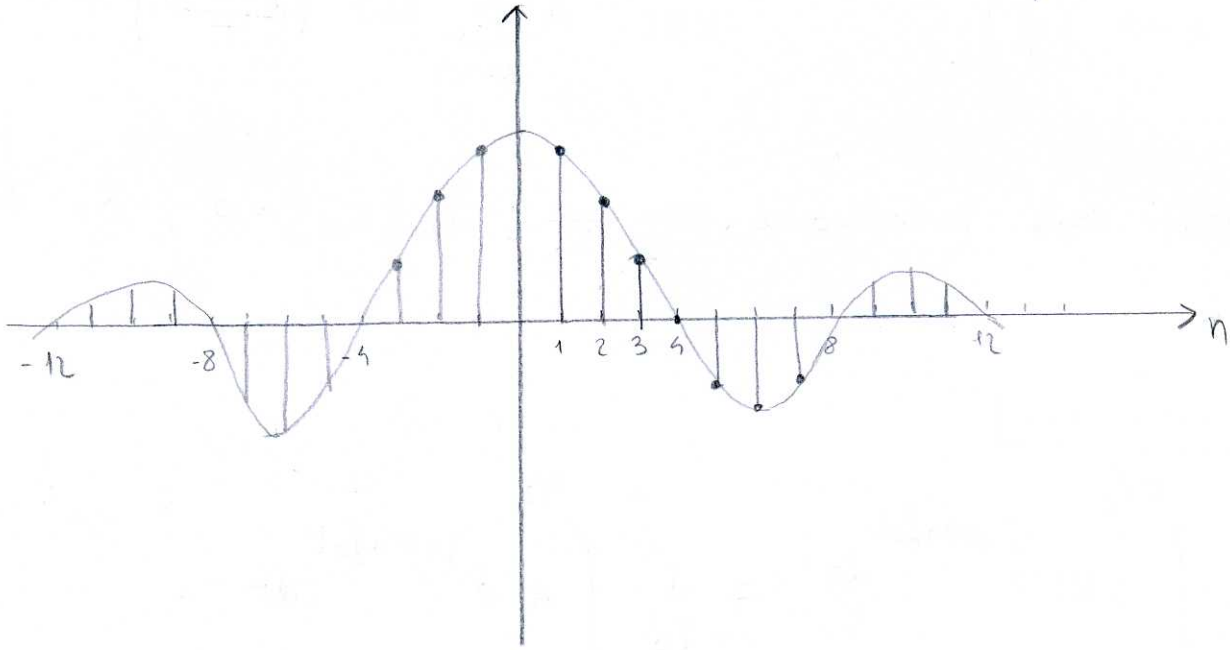
$$= \frac{A}{T_0} \left(- \frac{e^{-j2\pi f_0 n t}}{j2\pi n f_0} \right)_{-T/2}^{T/2} = - \frac{A}{j2\pi n} \left(e^{-j2\pi f_0 n \frac{T}{2}} - e^{j2\pi n f_0 \frac{T}{2}} \right) =$$

$$= \frac{A}{\pi n} \left(\frac{e^{j\pi n f_0 T} - e^{-j\pi n f_0 T}}{2j} \right) = \frac{A}{\pi n} \sin\left(\pi n f_0 T\right) =$$

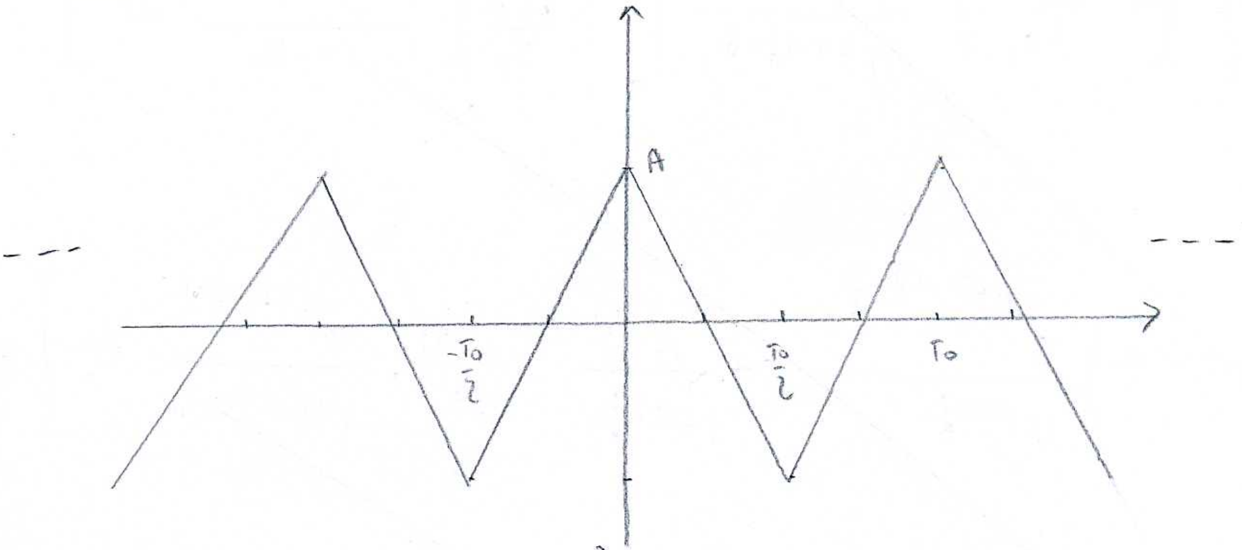
$$X_n = A f_0 T \cdot \frac{\sin(n \pi f_0 T)}{n \pi f_0 T} = A f_0 T \cdot \operatorname{sinc}(n f_0 T)$$



Se $T = \frac{T_0}{4}$



Esercizio #4



$$x_0(t) = \left(A - \frac{4A}{T_0}|t|\right) \text{rect}\left(\frac{t}{T_0}\right)$$

$$x(t) = -x(t - T_0/2)$$

$$x(t) = \sum_{k=-\infty}^{+\infty} x_0(t - kT_0)$$

$x(t)$ periodico, reale, pari e alternativo $\rightleftarrows$ $X_n = X_{-n}$; $\Im m\{X_n\} = 0$

$X_n = 0$ per n pari

La TSF per segnali alternativi è

$$X_n = \begin{cases} 0 & \text{per n pari} \\ \frac{2}{T_0}\int_0^{T_0/2} x(t)\, e^{-j2\pi n f_0 t}\, dt & \text{per n dispari} \end{cases}$$

$$X_n = \frac{2}{T_0}\int_0^{T_0/2} \left(A - \frac{4A}{T_0}t\right) e^{-j2\pi n f_0 t}\, dt =$$

$$X_n = \frac{2}{T_0}\left[\left(A - \frac{4At}{T_0}\right)\frac{e^{-j2\pi n f t}}{-j2\pi n f_0}\Bigg|_0^{T_0/2} + \int_0^{T_0/2}\frac{4A}{T_0}\,\frac{e^{-j2\pi n f t}}{-j2\pi n f_0}\,dt\right] =$$

$$= -\frac{1}{j\pi n}\left[(A-2A)\,e^{-j\pi n} - A + \frac{4A}{T_0}\,\frac{e^{-j2\pi n f t}}{-j2\pi n f_0}\Bigg|_0^{T_0/2}\right] =$$

$$= -\frac{A}{j\pi n}\left[e^{-j\pi n} - 2e^{-j\pi n} - 1 - \frac{2}{j\pi n}\left(e^{-j\pi n} - 1\right)\right] =$$

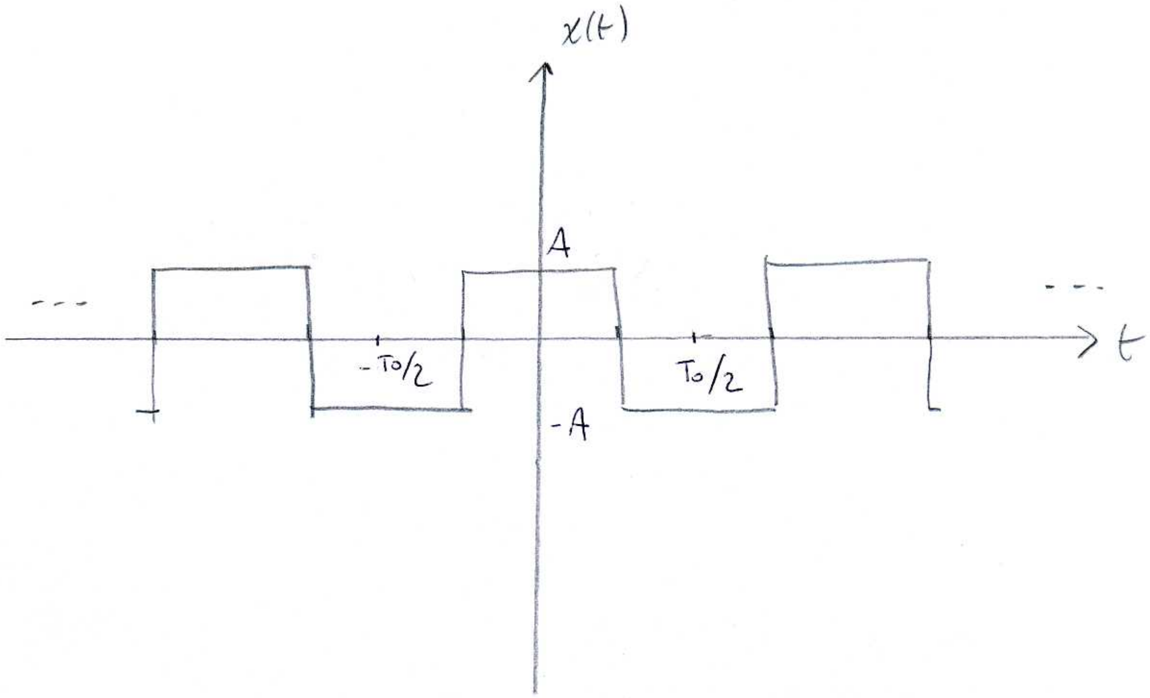
$$= -\frac{A}{j\pi n}\left[-e^{-j\pi n} - 1 - \frac{2}{j\pi n}\left(e^{-j\pi n} - 1\right)\right] =$$

$$= \frac{A}{j\pi n}\left(e^{-j\pi n} + 1\right) - \frac{2A}{\pi^2 n^2}\left(e^{-j\pi n} - 1\right)$$

per n dispari $e^{-j\pi n} = -1$

$$X_n = \frac{4A}{\pi^2 n^2}$$

Esercizio 5



$x(t)$ pari, reale, periodico e determinato $\rightleftarrows$ $X_n = X_{-n}$; $Im\{X_n\} = 0$

$X_n = 0$ per n pari

$$X_k = \frac{2}{T_0} \int_0^{T_0/2} x(t)\, e^{-j2\pi n f_0 t}\, dt =$$

$$= \frac{2}{T_0} \int_0^{T_0/4} A\, e^{-j2\pi n f_0 t}\, dt - \frac{2}{T_0} \int_{T_0/4}^{T_0/2} A\, e^{-j2\pi n f_0 t}\, dt =$$

$$= \frac{2A}{T_0} \left[\left. \frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0} \right|_0^{T_0/4} - \left. \frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0} \right|_{T_0/4}^{T_0/2} \right] =$$

$$= \frac{\cancel{2}A}{-j\cancel{2}\pi n \cancel{f_0}\cancel{T_0}} \left[e^{-j\cancel{2}\pi n \cancel{f_0}\frac{\cancel{T_0}}{\cancel{4}\,2}} - 1 - \left(e^{-j\cancel{2}\pi n \cancel{f_0}\frac{\cancel{T_0}}{\cancel{2}}} - e^{-j\cancel{2}\pi n \cancel{f_0}\frac{\cancel{T_0}}{\cancel{4}\,2}} \right) \right] =$$

$$= \frac{jA}{\pi n} \left[e^{-j\frac{\pi n}{2}} - 1 - e^{-j\pi n} + e^{-j\pi n/2} \right] =$$

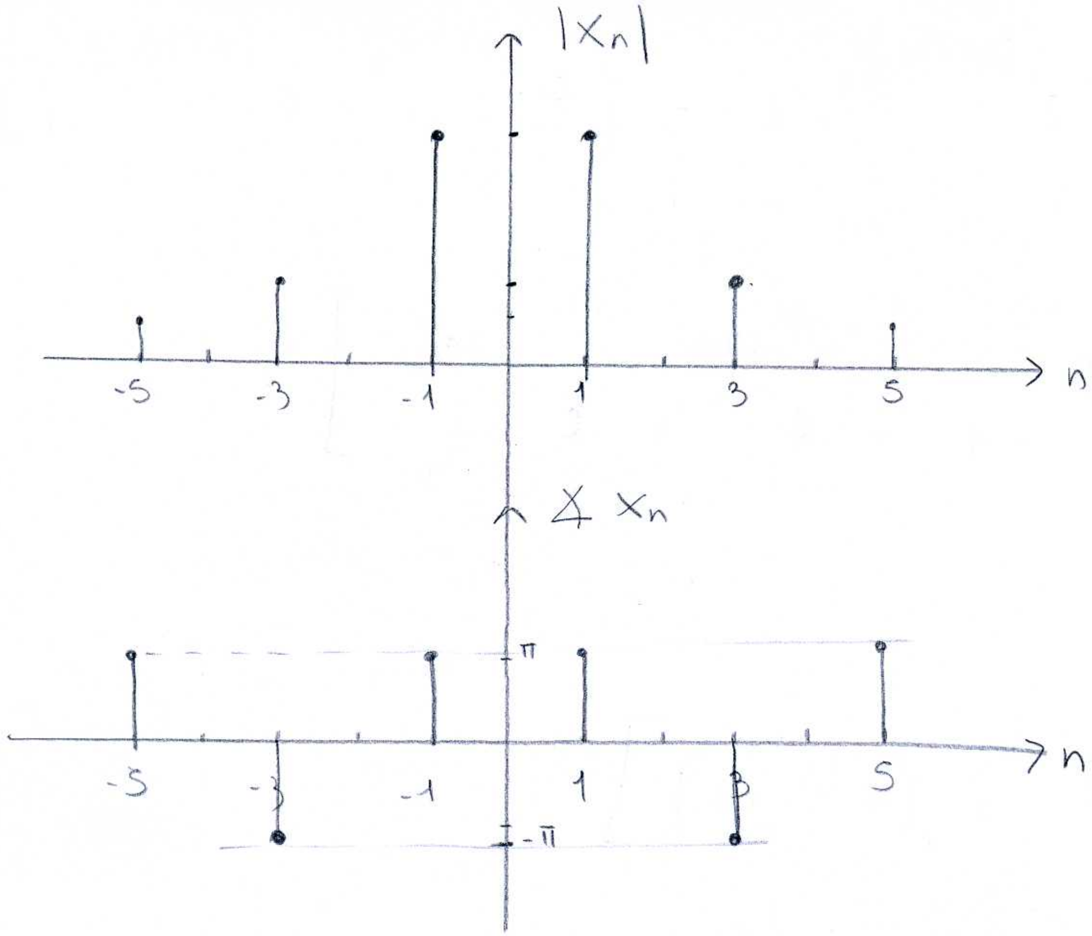
$$= -\frac{jA}{\pi n} \left[-2e^{-j\frac{\pi n}{2}} + \left(e^{-j\pi n} + 1 \right) \right]$$

per n dispari $e^{-j\pi n} = -1$

$$X_n = +\frac{2A}{\pi n} j(-j)^n = \frac{2A}{\pi n} (-1)^{\frac{n-1}{2}}$$

$$X_1 = \frac{2A}{\pi} \qquad X_3 = -\frac{2A}{3\pi} \qquad X_5 = \frac{2A}{5\pi}$$

$$X_{-1} = \frac{2A}{\pi} \qquad X_{-3} = -\frac{2A}{3\pi} \qquad X_{-5} = \frac{2A}{5\pi}$$



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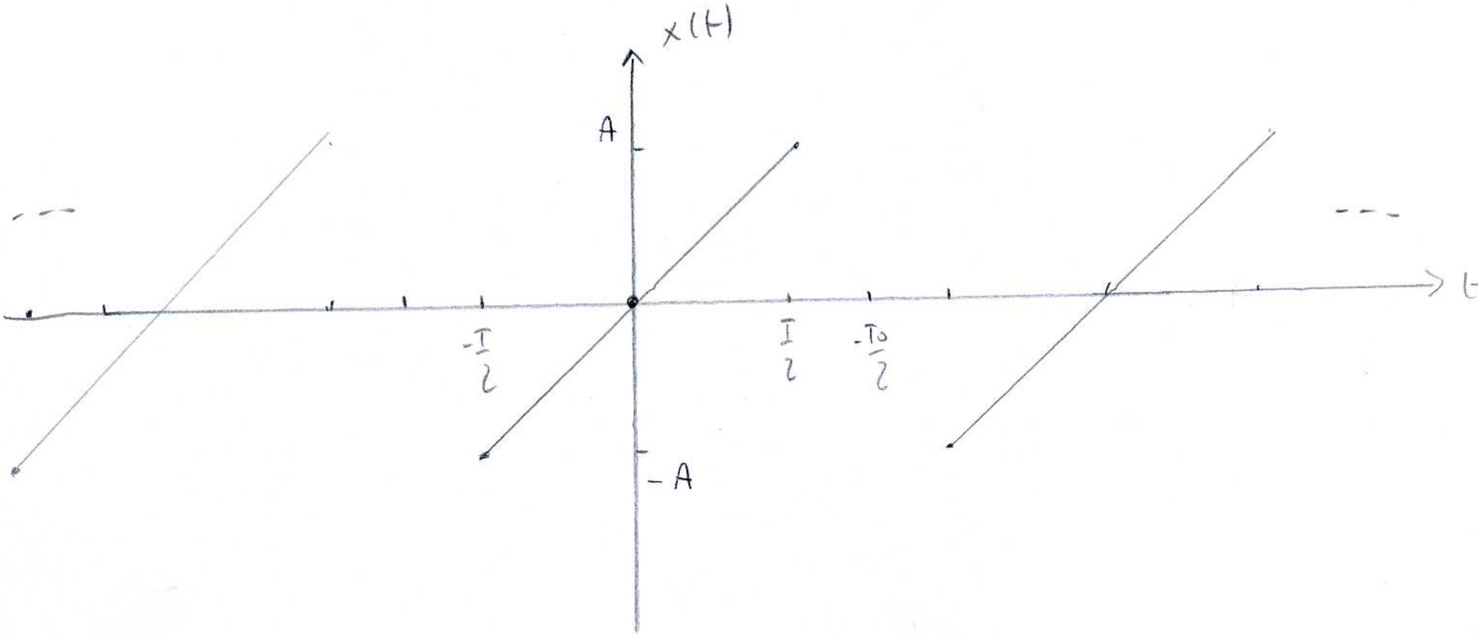
Esercizio #6



a) Determinare l'espressione di $x(t)$ in un periodo

b) calcolarne lo spettro.

a) $x_0(t) = \frac{2A}{T} t \, \mathrm{rect}\left(\frac{t}{T}\right)$

b) $x(t)$ reale, periodico e dispari $\rightleftarrows$ $\mathrm{Re}\{X_n\} = 0$; $X_n = -X_{-n}$

$$X_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \, e^{-j2\pi n f_0 t} \, dt = \frac{1}{T_0} \int_{-\frac{T}{2}}^{T/2} \frac{2A}{T} t \, e^{-j2\pi n f_0 t} \, dt =$$

$$= \frac{2A}{T T_0} \left[\left. \frac{t \, e^{-j2\pi n f_0 t}}{-j2\pi n f_0} \right|_{-\frac{T_0}{2}}^{T_0/2} - \int_{-\frac{T_0}{2}}^{T_0/2} \frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0} \, dt \right] =$$

$$= \frac{8A}{T f_0} \cdot \frac{1}{-j8\pi n f_0} \left[\frac{T}{2} \left(e^{-j2\pi n f_0 \frac{T}{2}} + e^{j2\pi n f_0 \frac{T}{2}} \right) + \left. \frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0} \right|_{-\frac{T}{2}}^{T/2} \right] =$$

$$= +\frac{jA\cancel{T}}{\pi n \cancel{T}} \cos(\pi n f_0 T) - \frac{jA}{\pi^2 n^2 f_0 T} \sin(\pi n f_0 T) =$$

$$= j\frac{A}{\pi n} \left[\cos(n\pi f_0 T) - \frac{\sin(\pi n f_0 T)}{\pi n f_0 T} \right] =$$

$$= j\frac{A}{\pi n} \left[\cos(\pi n f_0 T) - \mathrm{sinc}(n f_0 T) \right]$$