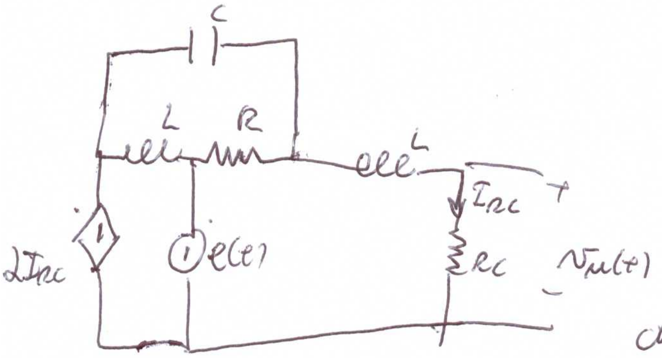
# Esercizi svolti di elettrotecnica

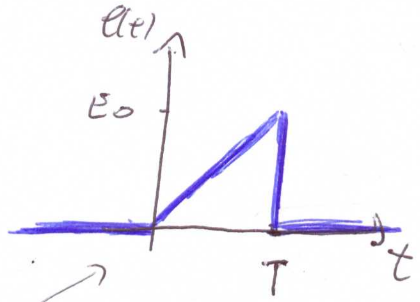
***Federico Balestri***

Appunti del 2016-12-17

## Licenza

Exe: 29/6/12




Il circuito (senza tratti) viene sollecitato da questa forma d'onda.

Le condizioni iniziali sono sempre $\emptyset$, perché se non agisce $e(t)$ non agisce nient'altro nel circuito.

$R = 5\,\Omega$ $\alpha = 3$ $v_u(t) = ?$

$L = 2\,mH$ $E_0 = 10\,V$

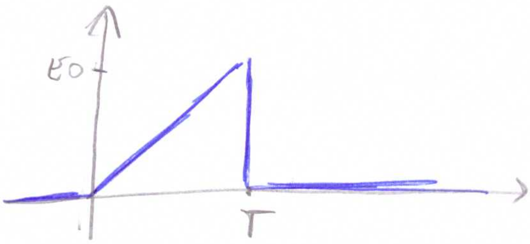
$C = 30\,\mu F$ $T = 10\,ms$

$t<0$ il circuito è senza alimentazione

$0 \le t \le T$ $\frac{E_0}{T} t\, u(t) = e$

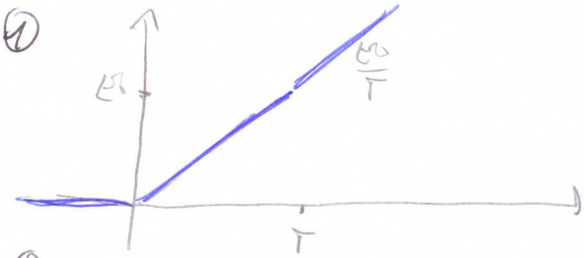
$t>T$ il circuito è senza alimentazione

Essendo il circuito lineare e tempoinvariante, posso applicare la sovrapposizione degli effetti e il circuito darà le stesse risposte (a meno di un fattore di traslazione temporale)




= ① + ② + ③

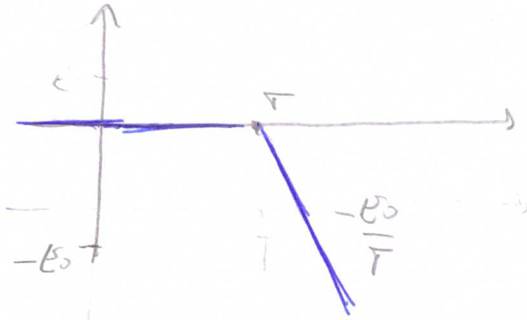
①



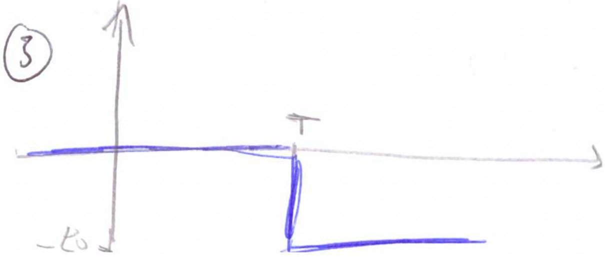

$$e_0(t) = \frac{E_0}{T} t\, u(t) - \frac{E_0}{T}(t-T)u(t-T) - E_0 u(t-T)$$

Posso far agire singolarmente le tre componenti con le stesse condizioni iniziali (poste a $\emptyset$)

②

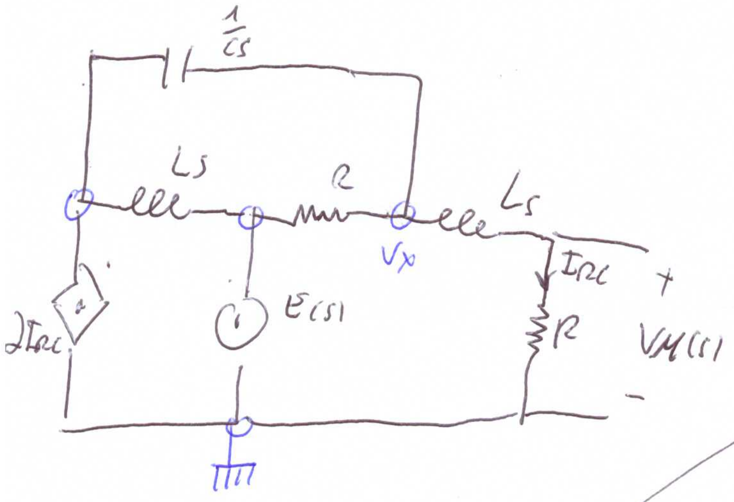



③



$$E_1(s) = \frac{E_0}{T}\frac{1}{s^2}$$ ← qui trasformata con Laplace

(ADN)




Con tensioni di nodo:

$$\begin{cases} 0 = V_x\left(\frac{1}{R} + Cs + \frac{1}{R+Ls}\right) - Cs\,\alpha I_{RL} - \frac{1}{R}E_1(s) \\ I_{RL} = \frac{V_x}{R+Ls} \end{cases}$$

$$V_M = I_{RL} \cdot R$$

$$(R+Ls)I_{RL}\left(\frac{1}{R} + Cs + \frac{1}{R+Ls}\right) - Cs\,\alpha I_{RL} - \frac{1}{R}E_1(s) = 0$$

$$0 = \cancel{(R+Ls)}\left(\frac{R+Ls+CsR(R+Ls)+R}{\cancel{R}\cancel{(R+Ls)}} - \frac{RCs\alpha}{\cancel{R}}\right)I_{RL} - \frac{1}{\cancel{R}}E_1(s)$$

$$I_{RL}\left(2R + Ls + R^2Cs + RLsCs - \alpha RCs\right) - E_1(s) = 0$$

$$I_{RL} = \frac{E_1(s)}{RLCs + (L+R^2C-\alpha RC)s + 2R} = \frac{E_1(s)}{RLC\left[s^2 + \left(\frac{1}{RC} + \frac{R}{L} - \frac{\alpha}{L}\right)s + \frac{2}{LC}\right]} \; *$$

* Da qui sostituisco la $E_1(s) = \frac{E_0}{T}\frac{1}{s^2}$

$$V_M^{(1)}(s) = \frac{1}{LC}\frac{E_0}{T}\frac{1}{s^2\left[s^2 + \left(\frac{1}{RC} + \frac{R}{L} - \frac{\alpha}{R}\right)s + \frac{2}{RC}\right]} = \frac{1}{LC}\frac{E_0}{T}\left[\frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+s_1} + \frac{D}{s+s_2}\right]$$

~~$E_1(s) =$~~

* Sostituisco $E_1(s) = \frac{E_0}{\cancel{T}}\frac{1}{s} = E_3(s)$

$$V_M^{(3)}(s) = \frac{1}{LC}E_0\frac{1}{s\left[s^2 + \left(\frac{1}{RC} + \frac{R}{L} - \frac{\alpha}{R}\right)s + \frac{2}{RC}\right]}$$

$$V_M^{(1)}(s) \Rightarrow V_M^{(1)}(t) = At\,u(t) + B\,u(t) + Ce^{-s_1t}u(t) + De^{-s_1t}u(t)$$

$$V_M^{(3)}(s) \Rightarrow V_M^{(3)}(t) = K\,u(t) + He^{-s_1t}u(t) + Me^{-s_1t}$$

$$V_M(t) = V_M^{(1)}(t) - V_M^{(1)}(t-T) - V_M^{(3)}(t-T)$$

$V_M^{(1)}$ e $V_M^{(2)}$ sono la stessa equazione a meno di una traslazione temporale $t-T$

Exe :

$$[s+(\sigma+j\omega)]\,[s+(\sigma-j\omega)]\,(\dots)\,(\dots)$$

In caso di radici complesse coniugate anche A e B sono complessi coniugati.

$$\lim_{s\to -(\sigma+j\omega)} (s+(\sigma+j\omega))\frac{N}{D}=A \qquad \lim_{s\to -(\sigma-j\omega)} (s+(\sigma-j\omega))\frac{N}{D}=B$$

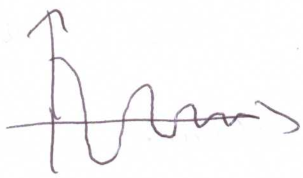
$$\frac{A}{s+(\sigma+j\omega)}+\frac{B}{s+(\sigma-j\omega)}+\frac{\quad}{\quad}\dots$$

$$A=K+jH=\sqrt{K^2+H^2}\,e^{j\alpha}$$

$$B=K-jH=\sqrt{K^2+H^2}\,e^{-j\alpha}$$

$$A e^{-(\sigma+j\omega)t}\mu(t)+B e^{-(\sigma-j\omega)t}\mu(t)=M e^{j\alpha}e^{-\sigma t}e^{j\omega t}\mu(t)+M e^{-j\alpha}e^{-\sigma t}e^{-j\omega t}\mu(t)=$$

$$=2Me^{-\sigma t}\mu(t)\left[\frac{e^{j(\omega t-\alpha)}+e^{-j(\omega t-\alpha)}}{2}\right]=2Me^{-\sigma t}\cos(\omega t-\alpha)\,\mu(t)$$



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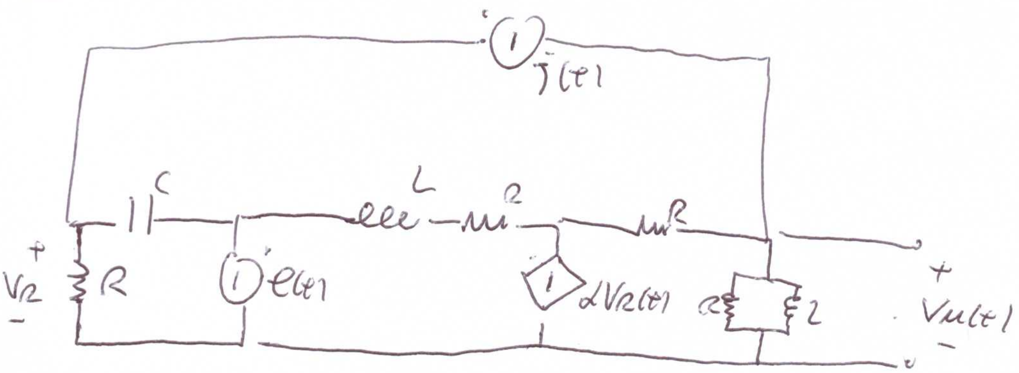
Exe : 1/1/16




$j(t)=100\,\text{sen}(500t)$

$e(t)$

$R=10\,\Omega$

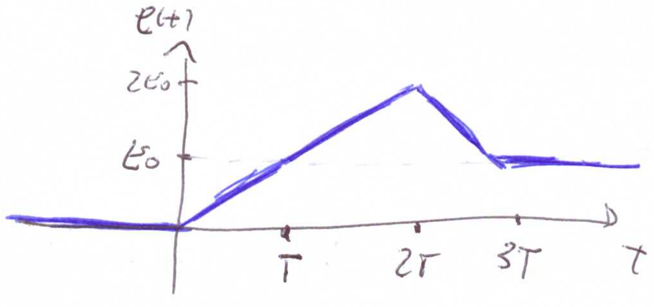
$C=100\,\mu F$

$L=10\,mH$

$T=10\,ms$

$E_0=20\,V$

$V_u(t)=?$

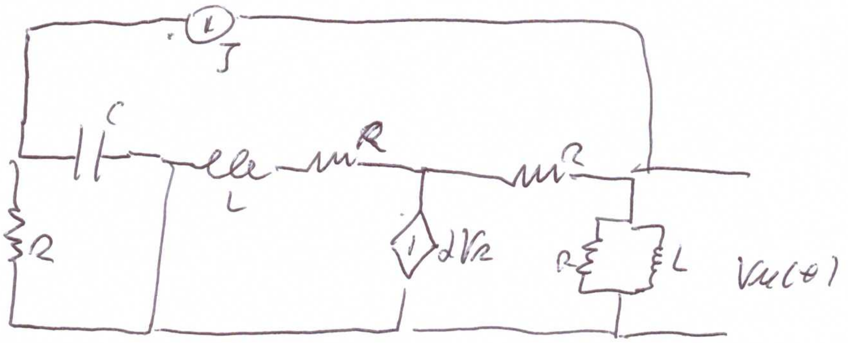


AEN

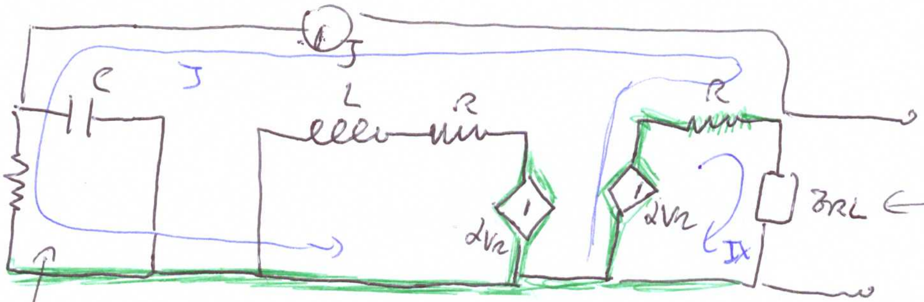
Circuito lineare → sovrapposizione degli effetti: posso far agire $j(t)$ e $e(t)$ separatamente

Le condizioni iniziali per $e(t)$ sono nulle, perché facendo agire i due generatori separatamente è come se l'altro non fosse mai esistito, per cui per $t<0$ non ci sono correnti. (se agisce $e(t)$)

Faccio agire J:




Lo disegno in maniera migliore:




$$Z_{RL} = \frac{R\, j\omega L}{R + j\omega L}$$

$$Z_{RC} = \frac{R \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

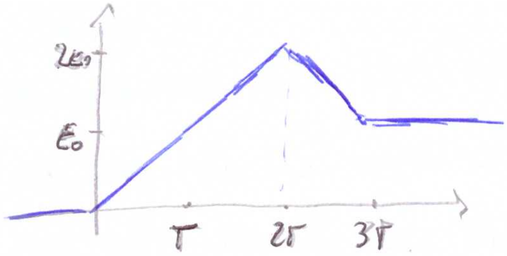
Con correnti di maglia: ALBERO

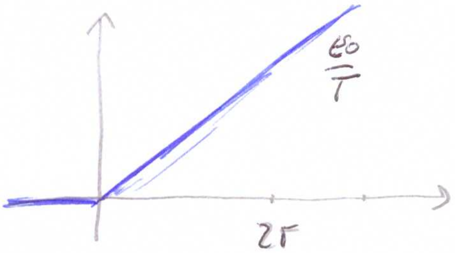
$$\begin{cases} \alpha V_R = (R + Z_{RL}) I_x + RJ \\ \alpha \bar{Z}_{RC} \bar{J} = (R + Z_{RC}) I_x + RJ \end{cases}$$

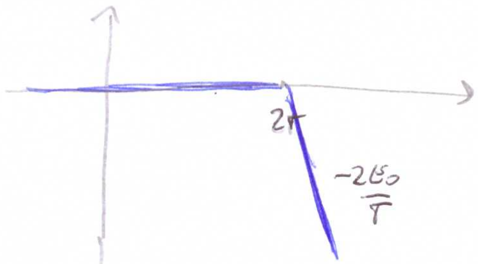
$$V_u^J = \frac{\alpha Z_{RC} - R}{R + \bar{Z}_{RL}} \bar{Z}_{RL} J = a + jb$$

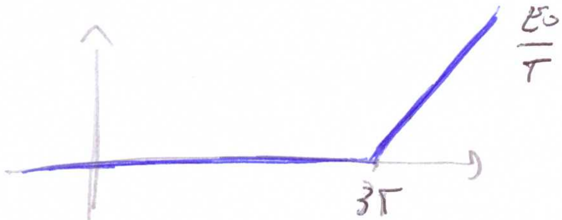
$$V_u^J = \sqrt{a^2+b^2}\, \text{sen}\left(\omega t - tg^{-1}\frac{b}{a} + \pi\right) \quad \text{se } a<0$$

Facciamo agire solo $e(t)$

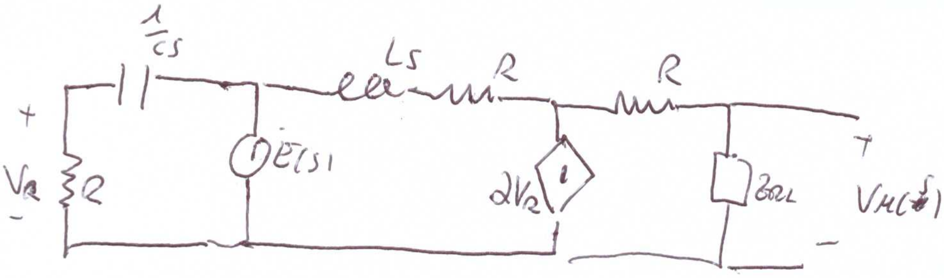



$$e(t) = \frac{E_0}{T} t\,u(t) - \frac{2E_0}{T}(t-2T)\,u(t-2T) + \frac{E_0}{T}(t-3T)\,u(t-3T)$$




$V_R$ si trova con partitore di tensione da $E(s)$:

$$V_R = \frac{E(s)}{R+\frac{1}{Cs}} R = \frac{E(s)R}{\frac{RCs+1}{Cs}} = \frac{Cs\,E(s)R}{RCs+1}$$

$V_u$ si trova con partitore di tensione da $2V_R$

$$V_u(s) = \frac{2V_R}{Z_{RL}+R} Z_{RL} = \frac{\frac{RLs}{R+Ls}}{\frac{R+Ls}{RLs}+R} 2V_R = \frac{\frac{RLs}{R+Ls}}{\frac{R^2+2RLs+RLs}{R+Ls}} 2V_R = \frac{2s}{2Ls+R}\, 2\left(\frac{E(s)}{RCs+1} RCs\right)$$

$$\frac{E_0}{T}\frac{1}{s^2} \cdot \frac{2RLCs^2}{(2Ls+R)(RCs+1)} = \frac{E_0}{T}\frac{2RLC}{2RLC\left(s+\frac{R}{2L}\right)\left(s+\frac{1}{L}\right)} \Rightarrow V_u^{(1)} = \frac{E_0}{T} 2\left[\frac{A}{s+\frac{R}{2L}} + \frac{B}{s+\frac{1}{RC}}\right]$$

AFN

Dalla $V_u^{(1)}$ posso trovare le altre per traslazione:

$$V_u(t) = V_u^{T}(t) + V_u^{(1)}(t)\mu(t) + 2V_u^{(1)}(t-2\tau)\mu(t-2\tau) + V_u^{(1)}(t-3\tau)\mu(t-3\tau) =$$

$$= \sqrt{a^2+b^2}\,\text{sen}\left(500t + tg^{-1}\left(\frac{b}{a}\right) + \overset{\text{se } a<0}{\pi}\right) + \frac{E_0}{T}\alpha\left(Ae^{-\frac{R}{2L}t} + Be^{-\frac{1}{RC}t}\right)\mu(t) - \frac{2E_0}{T}\alpha\left(Ae^{-\frac{R}{2L}(t-2\tau)} + Be^{-\frac{1}{RC}(t-2\tau)}\right)\mu(t-2\tau) + \frac{E_0}{T}\alpha\left(Ae^{-\frac{R}{2L}(t-3\tau)} + Be^{-\frac{1}{RC}(t-3\tau)}\right)\mu(t-3\tau)$$

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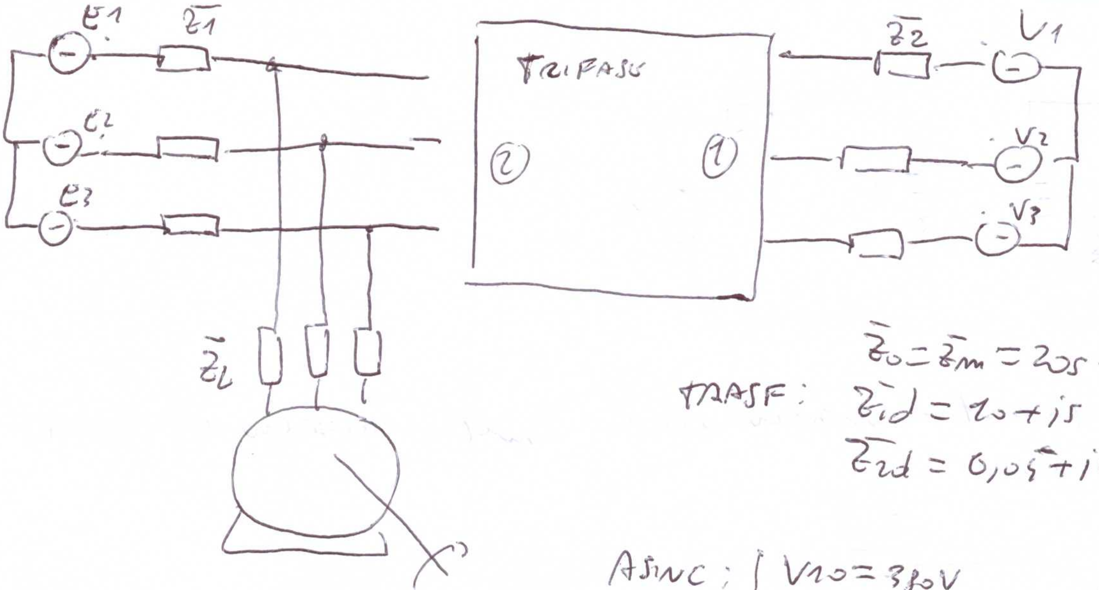
Exe:




$P_{fe}^{T} = ?$

$P_{mecc}^{A} = ?$

TRASF: $\bar{Z}_0 = \bar{Z}_m = 205 + j150$

$\bar{Z}_{1d} = 10 + j5$

$\bar{Z}_{2d} = 0{,}05 + j0{,}02 \quad n = 2$

ASINC: $\begin{cases} V_{10} = 380\,V \\ I_{10} = 2\,A \\ P_0 = 700\,W \end{cases}$ $\begin{cases} V_{1cc} = 150\,V \\ I_{1cc} = 8\,A \\ P_{1cc} = 1500\,W \end{cases}$

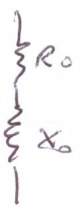
$k = 0{,}5 \qquad s = 0{,}75$

$G_m^A = \frac{P_{10}}{V_{10}^2}$

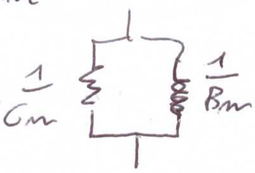
$Y_m^A = \frac{V_{10}}{\sqrt{3} I_{10}}$

$B_m^A = \sqrt{Y_m^2 - G_m^2}$

$\bar{Z}_m^A = \frac{1}{G_m - jB_m} = R_0 + jX_0$



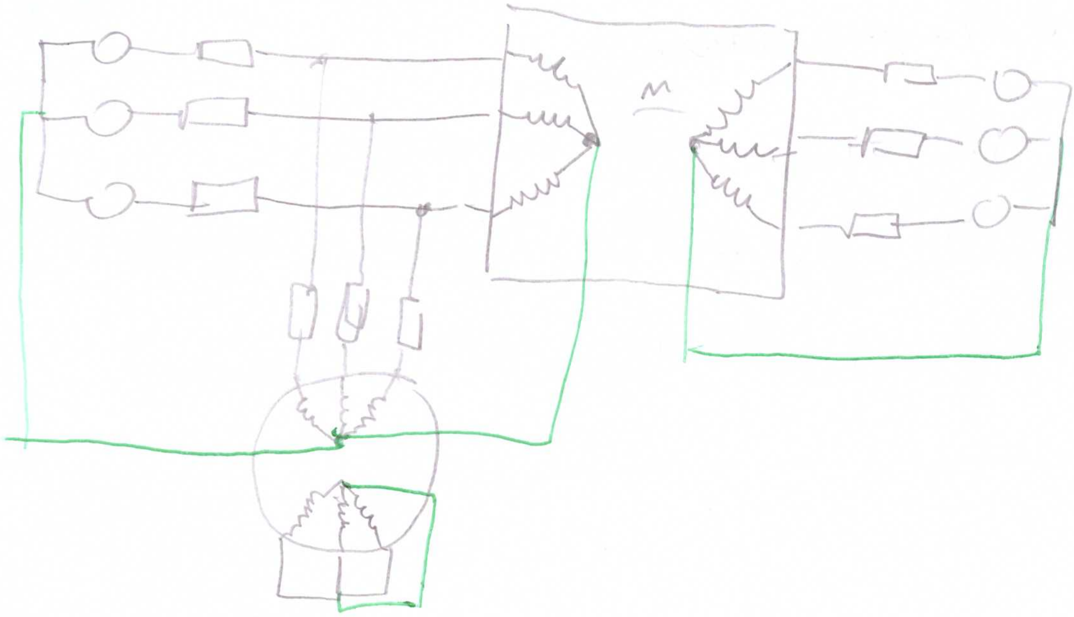

$R_0 \neq G_m$

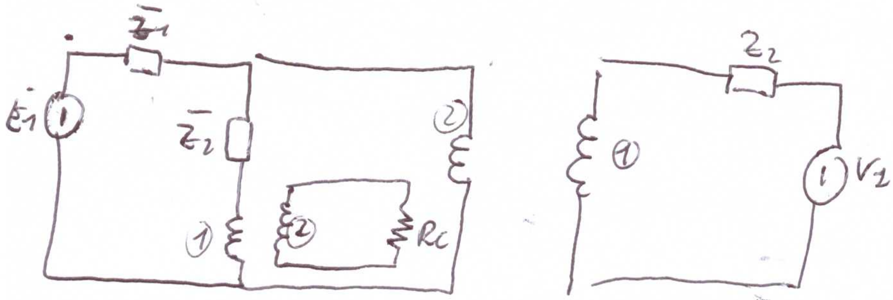



$Z_{1cc} = \frac{V_{1cc}}{\sqrt{3} I_{1cc}}$ $\qquad \cos\varphi = \frac{P_{1cc}}{\sqrt{3} V_{1cc} \cdot I_{1cc}}$

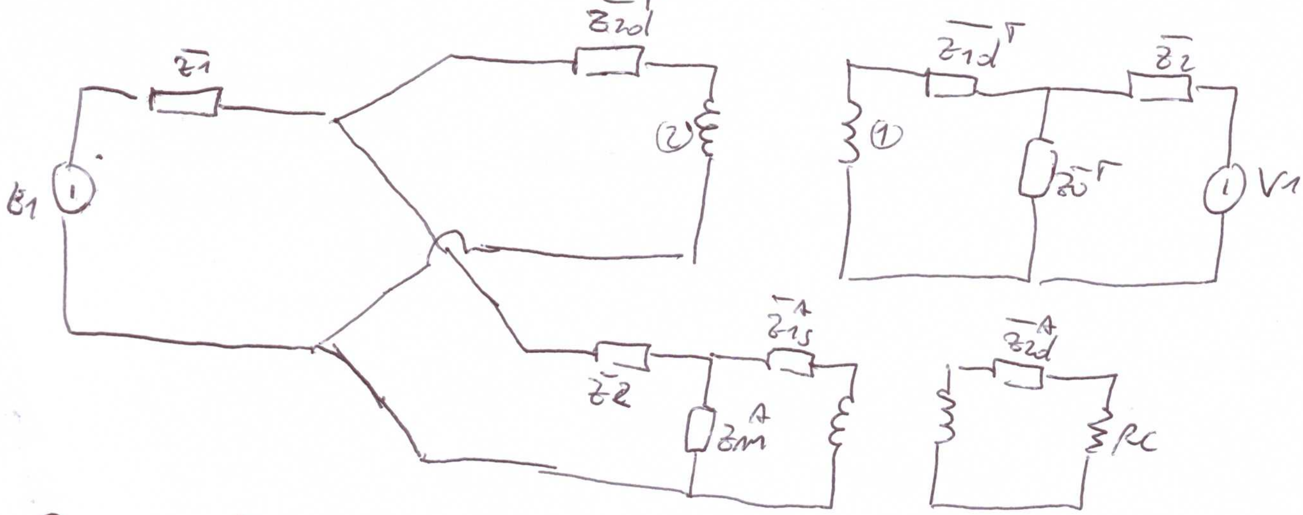
$\bar{Z}_{1cc} = Z_{1cc}(\cos\varphi + j\sqrt{1-\cos^2\varphi})$

Costruisco il circuito equivalente considerando i trasformatori ideali e collegando i centri stella.

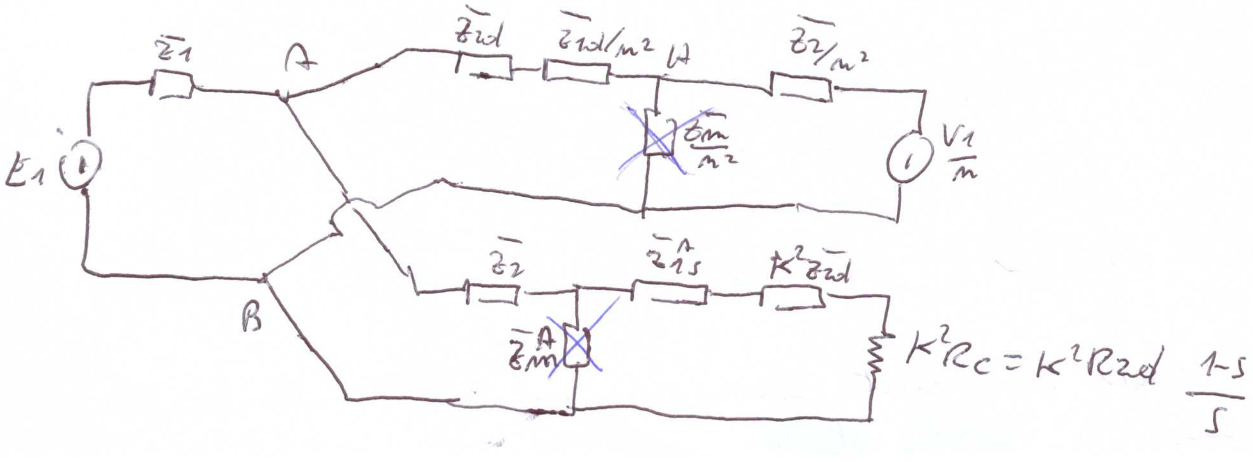





Mi ricordo che i trasformatori non sono ideali. Inoltre ridisegno il circuito in maniera più chiara




Sposto $I^{\Gamma} \to II^{\Gamma}$ e $I^{\Delta} \leftarrow II^{\Delta}$ perché devo calcolare $I_{Z_0}^{\Gamma}$ e $I_{R_c}$



A6N

Siccome $\frac{Z_m^T}{n^2}$ e $Z_m^A$ sono molto grandi rispetto alle impedenze a loro in parallelo, $\boxed{\text{le posso eliminare}}$. X

Con Millman:

$$V_{AB} = \frac{\frac{E_1}{Z_1} + \frac{V_1}{n}\cdot\frac{1}{Z_2/n^2 + Z_{2d}^T + Z_{1d}^T/n^2}}{\frac{1}{Z_1} + \frac{1}{\frac{Z_{2d}}{n^2} + Z_{2d}^T + \frac{Z_{1d}^T}{n^2}} + \frac{1}{Z_1 + Z_{1cc}^A + k^2 R_C}}$$

$$I_{RC} = \frac{V_{AB}}{Z_1 + Z_{1cc}^A + k^2 R_C} \Rightarrow P_m = 3k^2 Re\left|\frac{I}{k^2 R_C}\right|^2$$ ← non sono sicuro della formula

$V_{thk} \Rightarrow$ $$I_T = \frac{V_{AB} - \frac{V_1}{n}}{Z_{2d}^T + \frac{Z_{1d}^T}{n^2} + \frac{Z_2}{n^2}} \rightarrow V_{thk} = \frac{V_1}{n} + \frac{Z_2}{n^2} + I_T$$

$$\cancel{\frac{Z_m^T}{n^2}} \parallel \frac{Z_0}{n^2} \Rightarrow y = \frac{n^2}{Z_0} = \frac{n^2}{R_0 + jX_0} = \frac{n^2}{R_0^2 + X_0^2} R_0 - j(\dots)$$

$G$

$P_{Fe}^T = 3\,G\,V_{thk}^2$ oppure $P_{Fe}^T = \frac{R_0}{n^2}\,3\left|\frac{V_{thk}}{\frac{Z_0}{n^2}}\right|^2$

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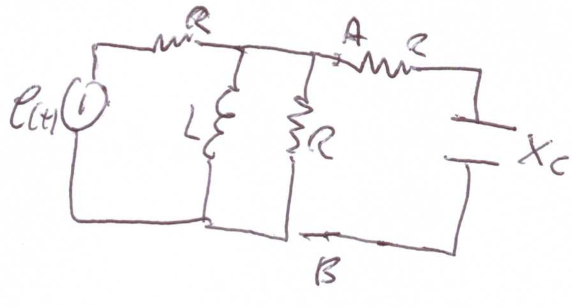
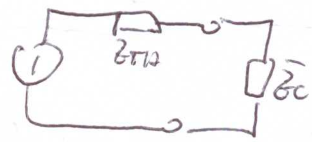
Exe: Determinare il carico affinché assorba la massima potenza




Per il teorema del massimo trasferimento di potenza devo riportarmi a una configurazione del tipo




Quindi basta applicare Thevenin a sinistra dei morsetti AB

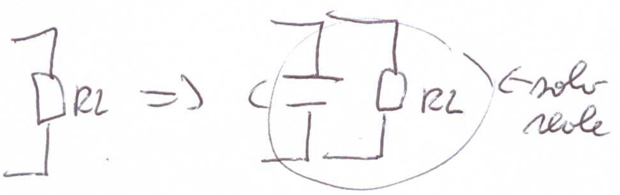
Mi interessa solo la $Z_{TH}$

$$\bar{Z}_{TH} = \frac{\frac{R}{2} j\omega L}{\frac{R}{2} + j\omega L} = \frac{j\omega L R}{R + 2j\omega L} = \frac{\omega^2 L^2 R^2 + j\omega L R^2}{R^2 + 4\omega^2 L^2}$$

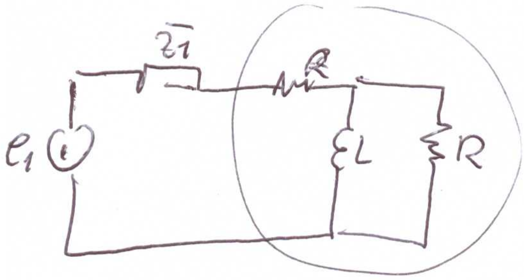
$$R_{TH} = \frac{\omega^2 L^2 R}{R^2 + 4\omega^2 L^2} = R_C$$

$$X_{TH} = \frac{\omega L R^2}{R^2 + 4\omega^2 L^2} = -X_C$$

Exe: RIFASAMENTO




Rifasare ~~[illegible]~~ significa aggiungere a un'impedenza ohmico induttiva una capacità in parallelo in modo che il tutto sia ~~solo~~ reale.




Trovare capacità per rifasare il carico

$$\bar{Y} = j\omega C + \frac{1}{R + j\omega L} = j\omega C + \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2}$$

$$\omega C = \frac{\omega L}{R^2 + \omega^2 L^2}$$

Nota bene che $R_{eq}\Big|_{\text{rifasamento}} = \dfrac{R^2 + \omega^2 L^2}{R} \neq R$

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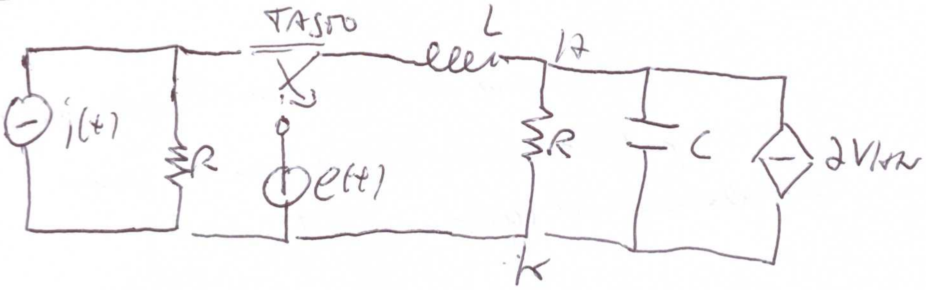
Exe: 25/02/11



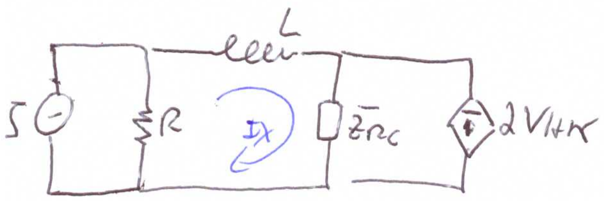

$j(t) = 5\cos(\omega_0 t + \frac{\pi}{8})$

$e(t) = 50\,V$

Al tempo $t=0$ il tasto si abbassa e il generatore di tensione inizia a lavorare

Le condizioni iniziali ci sono solo per J, perché prima di $t=0$ agisce solo il generatore di corrente

AHN




$$\bar{Z}_{RC} = \frac{R \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$0 = I_x (R + j\omega L + \bar{Z}_{RC}) + 2V_{HK} \bar{Z}_{RC} - RJ$$

$$V_{HK} = \bar{Z}_{RC}(I_x + 2V_{HK}) \Rightarrow V_{HK} = \frac{\bar{Z}_{RC} \cdot I_x}{1 - 2\bar{Z}_{RC}}$$

Mettiamo che $I_x = -3 + j8\,A$ $\quad V_{HK} = 15 - j40\,V$

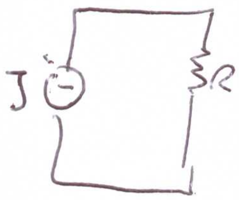
$$i_x(t) = \sqrt{9+64} \cos\left(500t + \pi - tg^{-1}\left(\frac{8}{3}\right)\right)$$

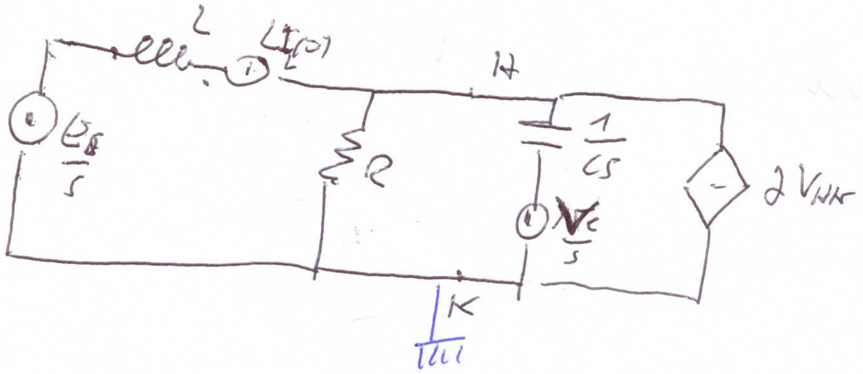
$$v_{HK}(t) = \sqrt{15^2 + 40^2} \cos\left(500t - tg^{-1}\left(\frac{40}{15}\right)\right)$$

$$i_x(0^-) = \sqrt{9+64} \cos\left(\pi - tg^{-1}\left(\frac{8}{3}\right)\right) \qquad v_{HK}(0^-) = \sqrt{15^2+40^2} \cos\left(-tg^{-1}\left(\frac{40}{15}\right)\right)$$

$i_L(0^-)$ $\quad$ condizioni iniziali $\quad$ $v_C(0^-)$

Calcolo transitorio:




Con tensioni di nodo:

$$V_{HK} \Rightarrow 2V_{HK} = V_{HK}\left(Cs + \frac{1}{R} + \frac{1}{Ls}\right) - Cs \frac{V_C(0)}{s} - \left(\frac{E_0}{s} + LI_L(0)\right)\frac{1}{Ls}$$

$$2V_{HK} = V_{HK} \frac{RLCs^2 + Ls + R}{RLs} - CV_C(0) - \frac{E_0}{Ls^2} - \frac{I_L(0)}{s}$$

$$V_{HK}\left(\frac{RLCs^2 + Ls + R}{RLs} - 2\right) = \frac{LCV_C(0)s^2 + E_0 + LsI_L(0)}{Ls^2}$$

$$V_{HK} = \frac{RLCV_C(0)s^2 + RE_0 + RLI_L(0)}{s\left(RLCs^2 + L(1-2R)s + R\right)}$$

$$V_{1tK} = \frac{1}{LC}\,\frac{LC\,v_C(0)\,s^2 + L\,i_L(0) + E_0}{s\left(s^2 + \frac{1}{RC}(1-\alpha R)s + \frac{1}{LC}\right)}$$

Da qui lo riporto in funzione di $t$

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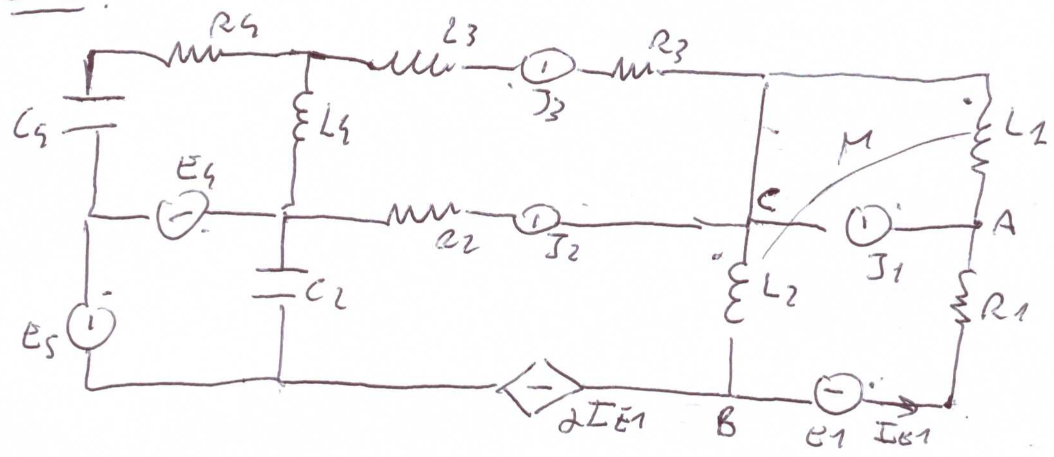
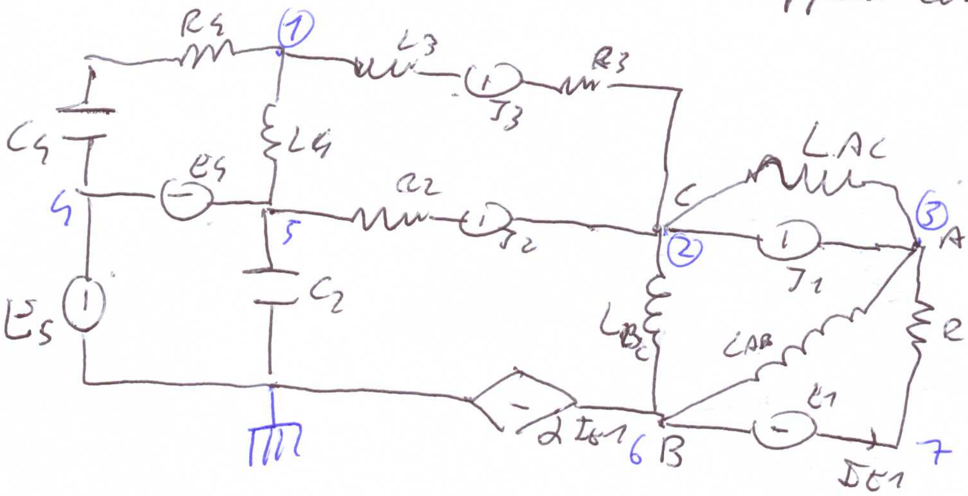
**Exe:** risolvere con tensioni di nodo



Sostituiamo gli induttori mutuamente accoppiati con equivalente a triangolo



$$J_3 = V_1\left(\frac{1}{j\omega L_3} + \frac{1}{R_4 + \frac{1}{j\omega C_4}}\right) + 0V_2 - 0V_3 - \frac{1}{R_4 + \frac{1}{j\omega C_4}} - \frac{1}{j\omega L_5}V_5 + 0V_6$$

~~$J_2 =$~~

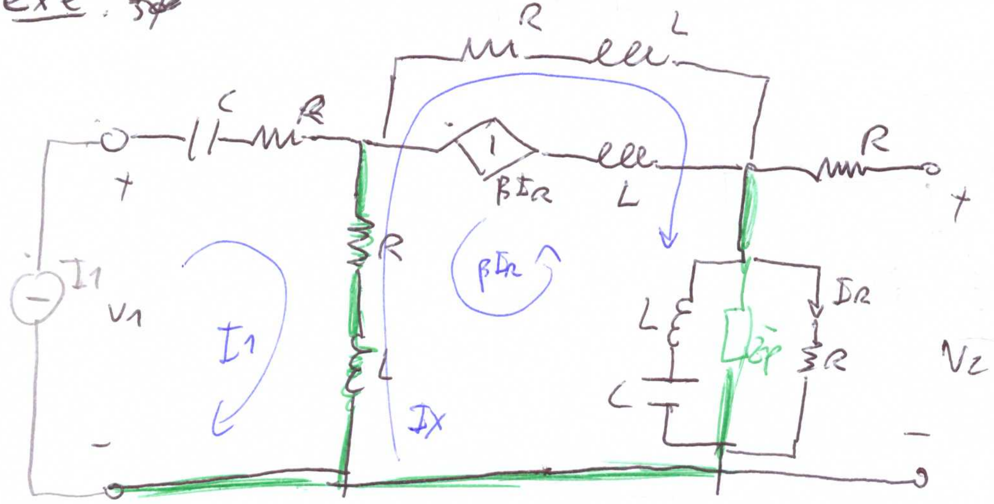
$$J_2 - J_1 - J_3 = 0V_1 + V_2\left(\frac{1}{j\omega L_{AC}} + \frac{1}{j\omega L_{BC}}\right) - V_3\,\frac{1}{j\omega L_{AC}} + 0V_5 + 0V_5 - \frac{1}{j\omega L_{BC}}V_6$$

$$J_1 = 0V_1 - \frac{1}{j\omega L_{AC}}V_2 + V_3\left(\frac{1}{j\omega L_{AC}} + \frac{1}{j\omega L_{AB}} + \frac{1}{R_1}\right) - \frac{1}{j\omega L_{AB}}V_6 - \frac{1}{R_1}(e_1 + V_6)$$ supernodo 7

$$J_{e1} = \frac{(e_1 + V_6) - V_3}{R_1}$$

Exe : 3

AJN




Calcolare i parametri ABCD

Determino prima $\frac{1}{C}$ aggiungendo un generatore di corrente a sinistra e poi usando il metodo delle ~~...~~ correnti di maglia

ALBERO NON OTTIMALE (ho messo nell'albero la porta dove scorre $I_R$)

$$Z_p = \frac{R\left(j\omega L + \frac{1}{j\omega C}\right)}{R + j\omega L + \frac{1}{j\omega C}}$$

$$0 = I_x(2R + 2j\omega L + Z_p) - I_1(R + j\omega L) - \beta I_R(R + j\omega L + Z_p)$$

$$I_R = \frac{(I_x - \beta I_R)\left(j\omega L + \frac{1}{j\omega C}\right)}{R + j\omega L + \frac{1}{j\omega C}} = I_x \frac{\frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}}{1 + \beta \frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}} = \bar{K} I_x$$

$$I_x = \frac{R + j\omega L}{2R + 2j\omega L + Z_p - \bar{K}(R + j\omega L + Z_p)} I_1 = \bar{H} I_1$$

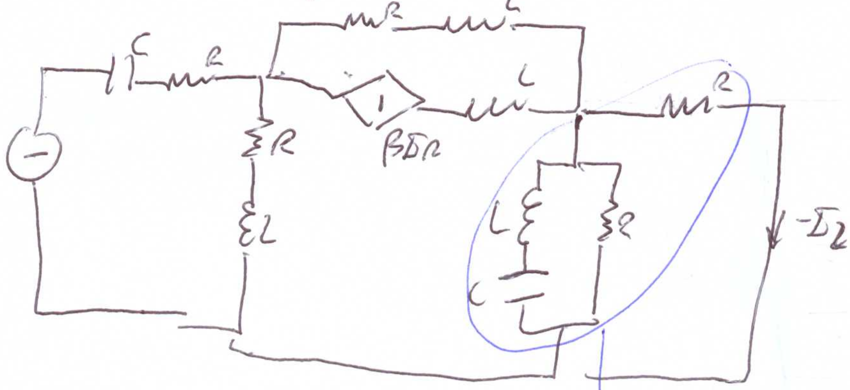
$$V_2 = R I_R = R \bar{K} I_x = R \cdot \bar{K} \bar{H} I_1$$

$$\boxed{\frac{1}{C} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = R \bar{H} \bar{K}}$$

$$V_1 = \left(R + \frac{1}{j\omega C}\right) I_1 + (R + j\omega L) \overset{I_x}{\bar{H} I_1} + R \bar{H} \bar{K} I_1 = \left[R + \frac{1}{j\omega C} + (R + j\omega L)\bar{H} + R \bar{H} \bar{K}\right] I_1$$

$$\boxed{\frac{1}{A} = \frac{V_2}{V_1} = \frac{R \bar{H} \bar{K}}{R + \frac{1}{j\omega C} + (R + j\omega L)\bar{H} + R \bar{H} \bar{K}}}$$

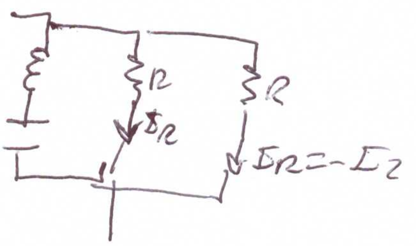
Per calcolare gli altri due parametri: cortocircuito la porta 2 e metto un generatore di corrente alla porta 1




$$Z_p^{(2)} = Z_p // R = \frac{\frac{R}{2}\left(j\omega L + \frac{1}{j\omega C}\right)}{\frac{R}{2} + \left(j\omega L + \frac{1}{j\omega C}\right)}$$

Sostituendo la $Z_p^{(2)}$ ottengo esattamente lo stesso circuito di prima con l'unica differenza $Z_p^{(2)}$ al posto di $Z_p$

$I_R = -I_2$




$V_1 = \left(R + \frac{1}{j\omega C}\right) I_1 + (R + j\omega L) H_1 I_1 + R H_1 K_1 I_1$

$\frac{1}{D} = \ldots$

$\frac{1}{B} = \ldots$