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| **MATHEMATICS DEPARTMENT**  **Year 11 Specialist – 2016**  **Test Number 6: Resource Free**  **Trigonometric Identities and Complex Numbers** |

**Name: \_\_\_\_\_\_SOLUTIONS\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Teacher: DDA**

**Marks: 52**

**Time Allowed: 45 minutes**

**Weight: 7%  
Instructions:** Show your working where appropriate remembering you must show working for questions worth more than 2 marks. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Part A

1 sec (45°) =

A 

B 

C 

D 

E 

[1 mark]

2 cos  =

A −2

B 

C 

D 

E  [1 mark]

3 sec =

A  −sec 

B −cos 

C  cos 

D  sec 

E  sec  [1 mark]

4 cos (270° − α) =

A   sin (α)

B −sin (α)

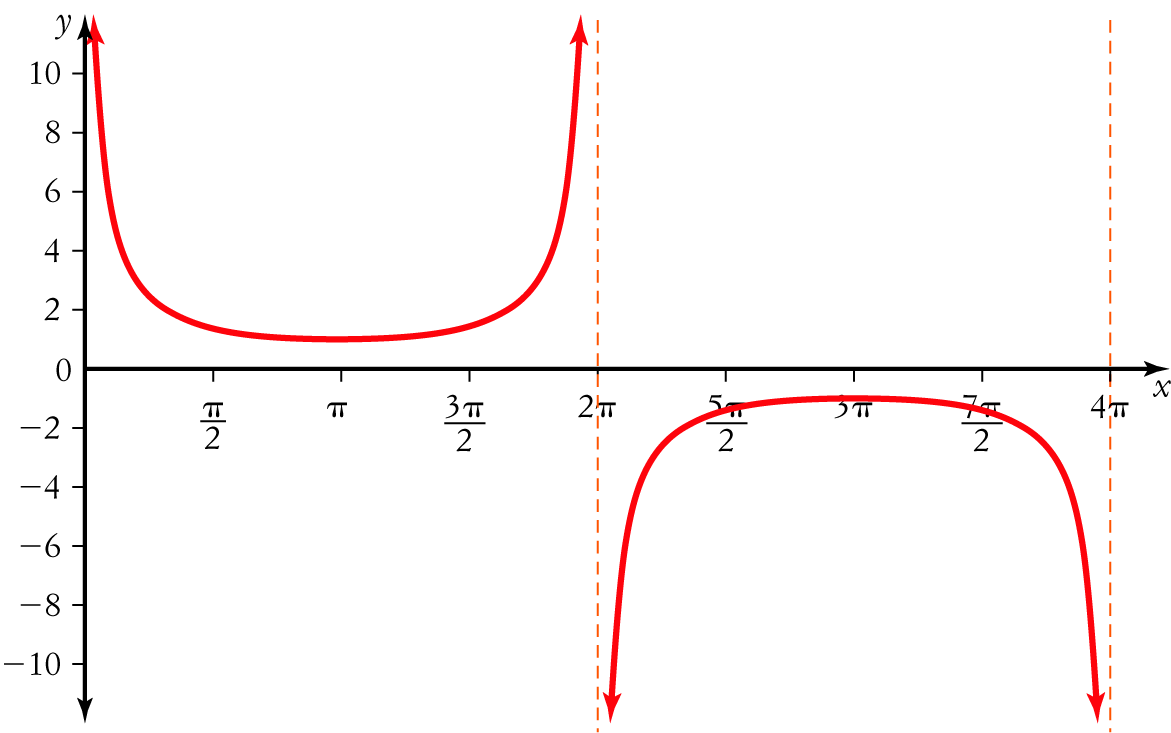
C   cos (α)

D −cos (α)

E   cos (270°)

[1 mark]

5 The graph shown below could have the equation:



A  y = 2 cosec (x)

B y = 2 sec (x − 1)

C y =  cosec (x)

D y =  sin (x)

E   y = cosec  [1 mark]

6 The value of  is:

A  9 − 16i

B   25

C 5

D 9 + 16i

E −7

[1 mark]

7 The roots of x2 − 4x + 5 = 0 are:

A   x = 2 ± i

B (x − 2 − i)(x − 2 + i)

C  x = −2 ± i

D  x = −4 ± i

E  x = 4 ± i

[1 mark]

8 If  then the value of v2 is

A  2

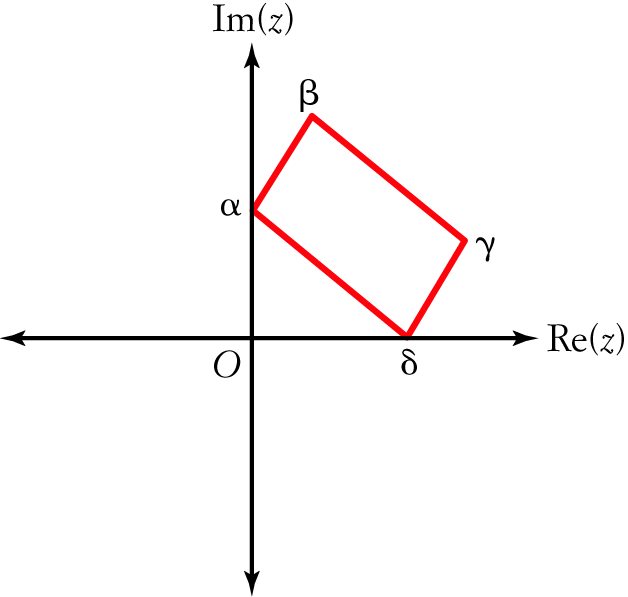
B i

C 2i

D 4i

E   4

[1 mark]

9 

In the diagram, the complex numbers α, β, γ, δ form a parallelogram, such that α and δ lie on the axes. The point β is described by:

A   α + δ + γ

B  α + δ − γ

C α − δ − γ

D α − δ + γ

E −α − δ − γ

[1 mark]

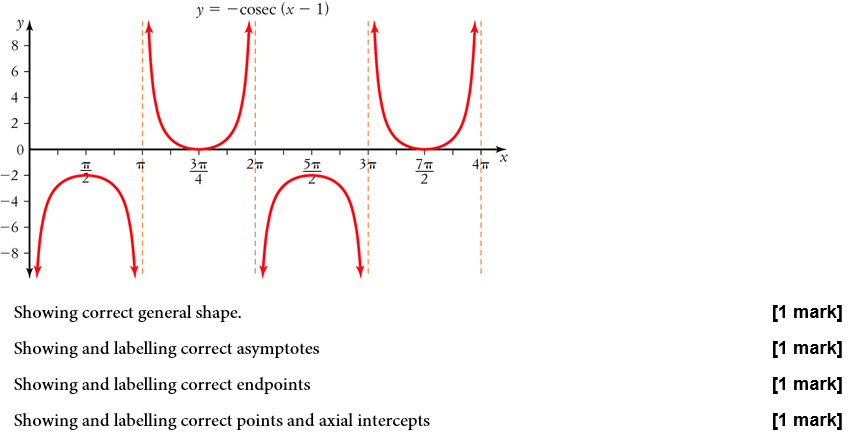
Part B

9 short answer questions

35 marks

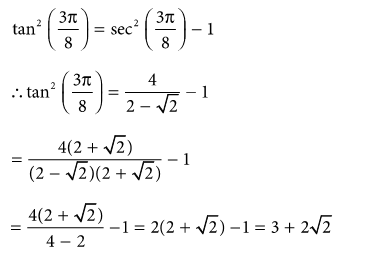
Show your working where appropriate.

10 Sketch the graph of y = −cosec (x) − 1 for 0 ≤ x ≤ 4π.



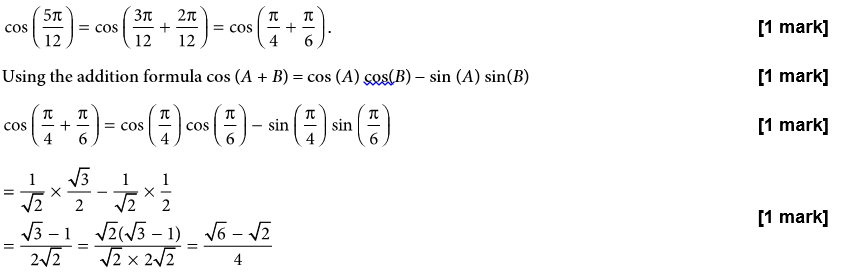
[4 marks]

11 Using the identity 1 + tan2 (θ) = sec2 (θ) evaluate  given that .



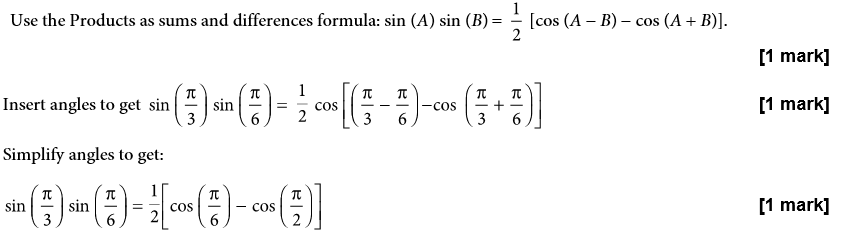
[3 marks]

12 Using a suitable compound angle formula evaluate cos.



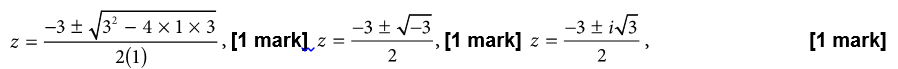
[4 marks]

13 Express sin sin  as a sum.

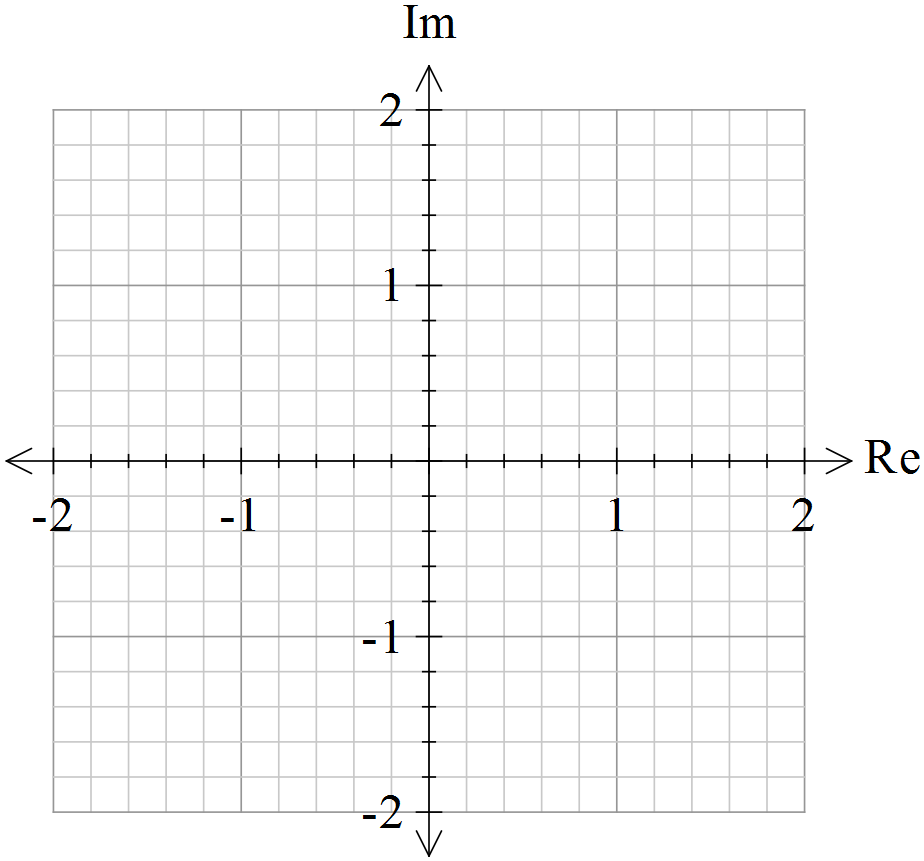


[3 marks]

14 a  Solve z2 + 3z + 3 = 0 over the complex plane.



b Sketch the solutions to z2 + 3z + 3 = 0 on an Argand diagram, labelling them z1 and z2.





[2 marks]

c  What geometric property do the complex roots of quadratic equations with real coefficients display?



[1 mark]

15 If (a + bi)2 = 5 + 12i, where a, b ∈ R. Find all possible values of a and b.

a2 − b2 + 2abi = 5 + 12i [1 mark]

Equating real and imaginary parts: a2 − b2 = 5, 2ab = 12 ⇒ ab = 6 [1 mark]

[1 mark]

[2 marks]

16 If z = 6 + 8i find in simplest form:

a   [1 mark]

 or z−1 = 0.06 − 0.08i [1 mark]

b  [1 mark]

 [1 mark]

c  [1 mark]

 [1 mark]

17 If w = x + yi, where x, y ∈ R, show that:

a 

 [1 mark]

 [1 mark]

b 

 [1 mark]

 [1 mark]

Part C

2 analysis questions

8 marks

Show your working where appropriate.

18 Rewrite the expression sin (2x) cosec (x) − sec (x) cos (2x) in terms of cos (x) only. (Simplify as much as possible.)

sin (2x) cosec (x) − sec (x) cos (2x) = 2 sin (x) cos (x) × [2 cos2 (x) − 1]

[1 mark]

 [1 mark]

 [1 mark]

19 Prove that cot2 (2θ) = .

RHS = 

 [2 marks]

 [2 mark]



= LHS [1 mark]

Alternative methods may be used.

[5 marks]