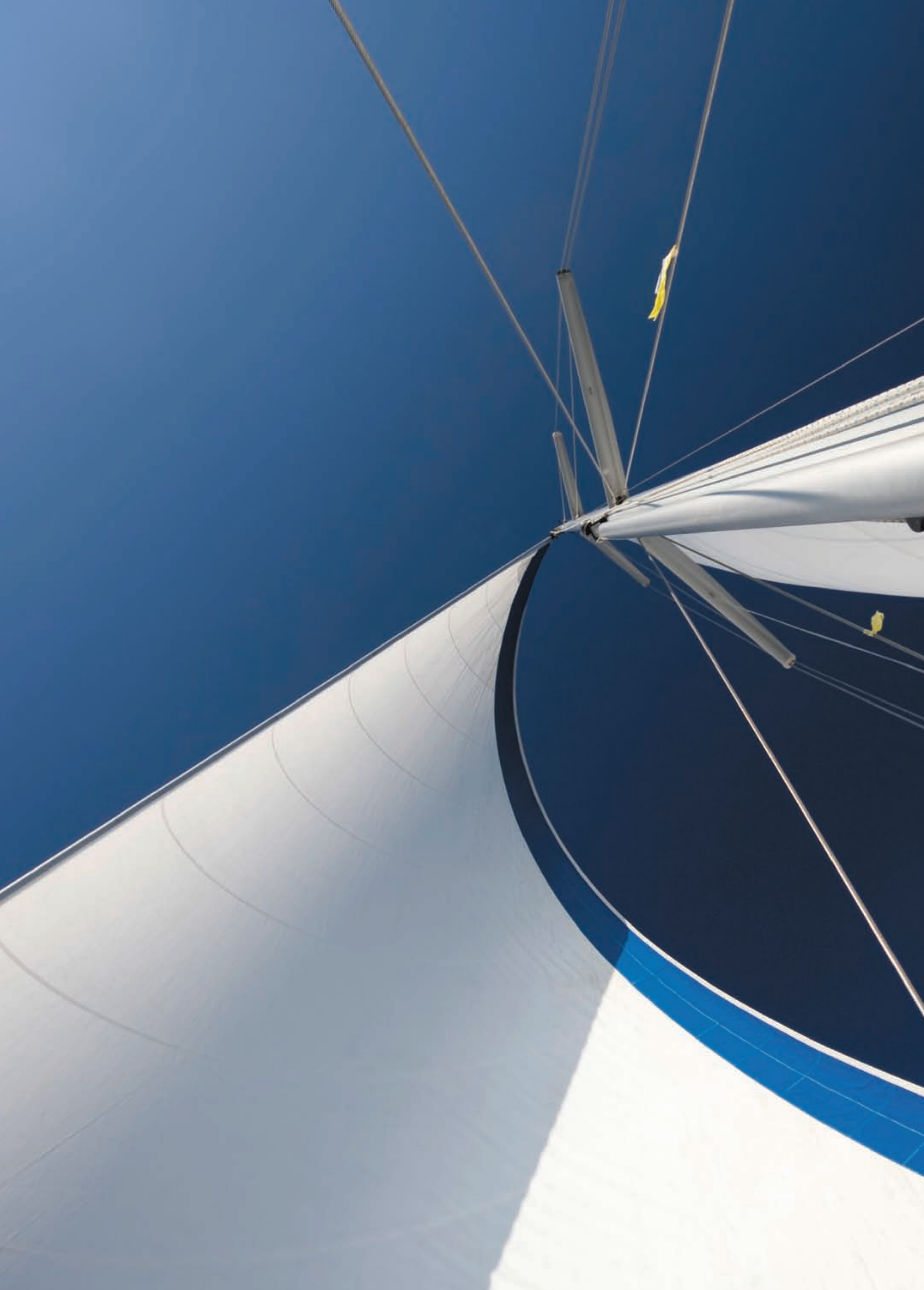
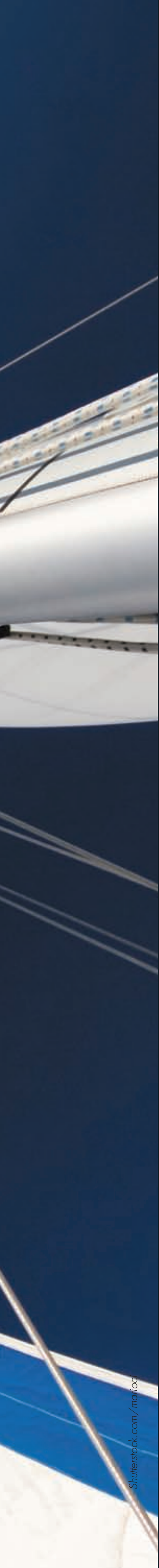




10

TERMINOLOGY

adjacent
angle of depression
angle of elevation
area of a triangle
compass bearing
cosine rule
Heron's formula
hypotenuse
included angle
non-included angle
non-right-angled triangle
opposite
right-angled triangle
sine rule
true bearing

Prior learning

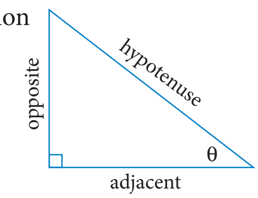
TRIGONOMETRY

APPLICATIONS OF TRIGONOMETRY

APPLICATIONS OF TRIGONOMETRY

- **review the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle (ACMGM034)**
- **determine the area of a triangle given two sides and an included angle by using the rule Area = $\frac{1}{2}ab \sin C$, or given three sides by using Heron's rule, and solve related practical problems (ACMGM035)**
- **solve problems involving non-right-angled triangles using the sine rule (ambiguous case excluded) and the cosine rule (ACMGM036)**
- **solve practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of bearings in navigation (ACMGM037).**

10.01 FINDING AN UNKNOWN SIDE



Trigonometry has many real-life applications, some of which are construction and navigation. It allows us to measure unknown lengths and angles that may be too difficult to measure directly, for example, measuring the height of a building.

When we are working with **right-angled triangles** we need to be able to name the sides of the triangle. The convention is to look at the given angle, θ (named 'theta'), then label the sides according to their position in relation to θ, as the **hypotenuse**, **adjacent** side or **opposite** side.

There are three **trigonometric functions,** each defined by a specific ratio, which can be used to find the lengths of unknown sides and the size of unknown angles in right-angled triangles.

IMPORTANT

For angle θ in the diagram above:

Function	Abbreviation	Ratio	Initials
sine	sin	$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$	SOH
cosine	cos	$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$	CAH
tangent	tan	$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$	TOA

Some students memorise the initials of these ratios, SOH-CAH-TOA (pronounced 'so car toe ah'), while others use a phrase such as '**S**ome **O**ld **H**ens **C**an **A**lways **H**ide **T**heir **O**ld **A**ge' to remember them. Find your own mnemonic (memory aid) to remember this.

A trigonometric ratio can be used to calculate the length of an unknown side in a right-angled triangle if one angle and one side are known. Select the ratio that links the angle to both the unknown side and the known side.

9780170250252

IMPORTANT

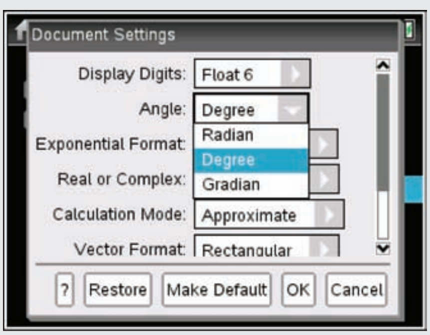
Before you begin your calculations you must have your calculator set to 'degree' mode. Your CAS will then identify that you are working in degrees and you will not have to use the degrees symbol when working with angles.

TI-Nspire CAS

On your home screen select 5: Settings.

From the menu select: Document Settings.

Open the menu within the Angle option. Highlight Degree using the ▼ key. Press [enter]. Press [tab] until [Make Default] is highlighted then press [enter].



9780170250252

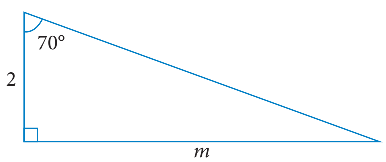
ClassPad

Using the $\sqrt{\alpha}$ Main application, check **Deg** is showing on the bottom toolbar. If the mode is either **Rad** or **Gra**, tap until **Deg** appears.

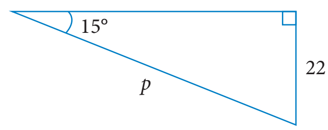
Example 1

Calculate the value of the pronumerals, correct to two decimal places.

a

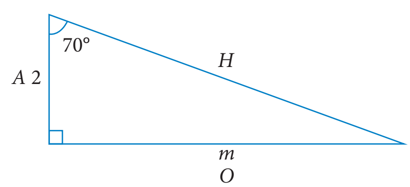


b



Solution

a Draw and label the sides of the triangle.



Select and write the appropriate ratio. If you have values on the opposite and the adjacent side, the ratio will be tangent.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute known values into the equation.

$\theta = 70°$, adjacent $= 2$

$$\tan(70°) = \frac{m}{2}$$

Solve the equation.

$$2 \times \tan(70°) = \frac{m}{\cancel{2}} \times \cancel{2}$$

$$m = 2\tan(70°)$$

$$m = 5.49$$

9780170250252

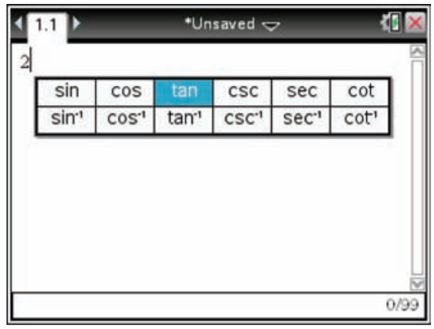
TI-Nspire CAS

Press 2.

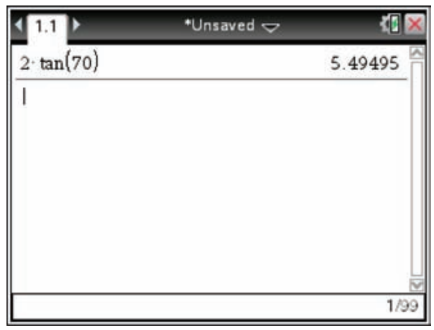
Press trig.

Select ‘tan’.

Key in 70. Press ctrl enter.



The side opposite the angle, represented by m, equals approximately 5.49 units.



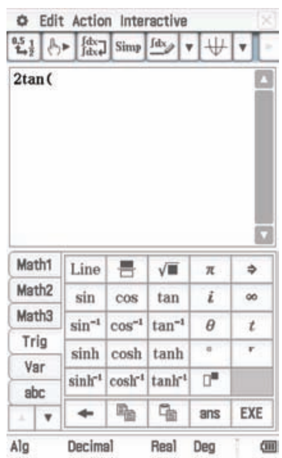
ClassPad

Use the Main $\sqrt{\alpha}$ Application in **Decimal** mode.

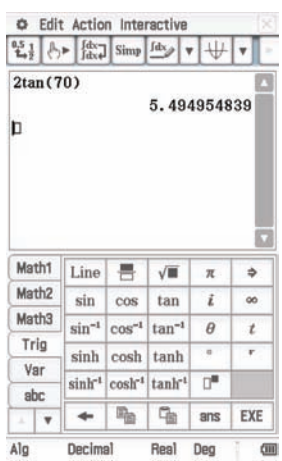
Press 2.

Press Keyboard.

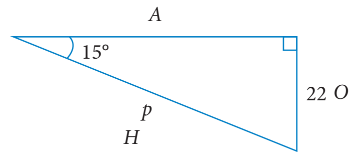
Tap Trig and tan.



Type 70). Press EXE.



b Draw and label the sides of the triangle.



Select and write the appropriate ratio. If you have values on the opposite and the hypotenuse, the ratio will be sin.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

Substitute known values into the equation.

$\theta = 15°$, opposite = 22, hypotenuse = p

$$\sin(15°) = \frac{22}{p}$$

Solve the equation.

Multiply both sides by p.

$$p \times \sin(15°) = \frac{22}{\cancel{p}} \times \cancel{p}$$

$$p \sin(15°) = 22$$

Divide both sides by sin (15°).

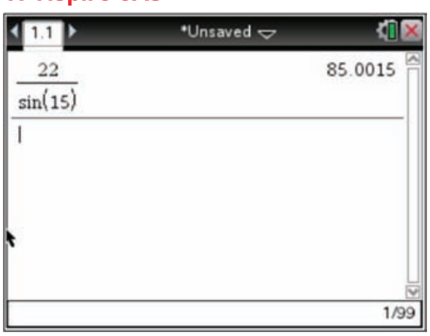
$$\frac{p \sin(15°)}{\sin(15°)} = \frac{22}{\sin(15°)}$$

$$p = \frac{22}{\sin(15°)}$$

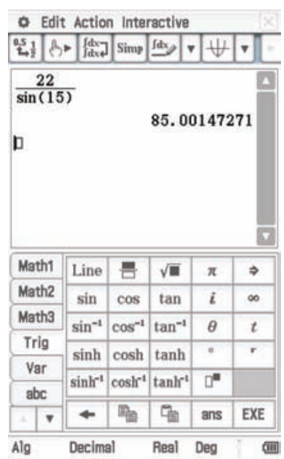
$$p = 85$$

The hypotenuse, represented by p, equals approximately 85 units.

TI-Nspire CAS

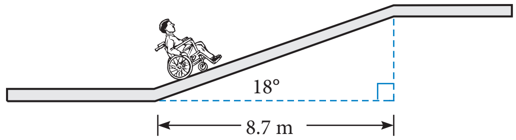


ClassPad



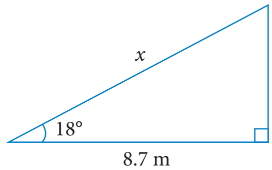
Example 2

A wheelchair ramp is inclined at 18° to the horizontal. How long is the ramp if it links two levels 8.7 m apart horizontally? Round your answer correct to one decimal place.

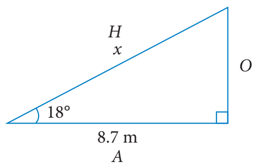


Solution

Redraw the triangle using the given values. Represent the length of the ramp by x.



Label the sides of the triangle.



Select and write the appropriate ratio. If you have values on the adjacent side and the hypotenuse, the ratio will be cosine.

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Substitute the known values into the equation.

$\theta = 18°$, adjacent $= 8.7$

$$\cos(18°) = \frac{8.7}{x}$$

Solve the equation.

Multiply both sides by x.

$$x\cos(18°) = \frac{8.7}{\cancel{x}} \times \cancel{x}$$

Divide both sides by $\cos(18°)$.

$$\frac{x\cos(18°)}{\cos(18°)} = \frac{8.7}{\cos(18°)}$$

$$x = \frac{8.7}{\cos(18°)}$$

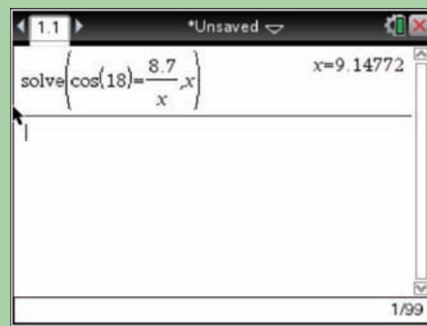
Evaluate and round correct to one decimal place.

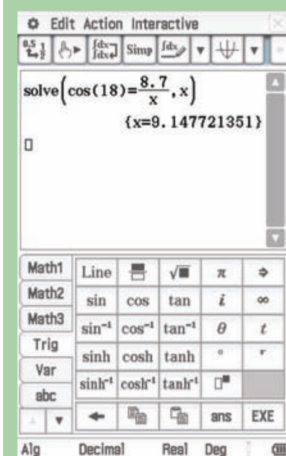
$$x = 9.1477...$$
$$\approx 9.1$$

Write the answer.

The length of the ramp is 9.1 metres.

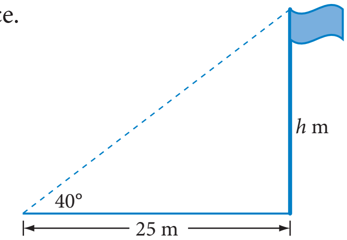
A CAS can be used to solve for an unknown side length.





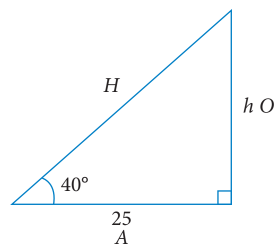
Example 3

Find the height, h m, of this flagpole correct to one decimal place.



Solution

Redraw the triangle using the given values.
Represent the height of the flagpole by h.
Label the sides of the triangle.



Select and write the appropriate ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute the known values into the equation and solve for h.

$$\tan(40°) = \frac{h}{25}$$

$$h = 25 \tan(40°)$$
$$= 20.9774...$$
$$\approx 20.98$$

Write the answer including units.

The height of the flagpole is 20.98 m.

WS Identifying the correct trigonometric ratio

EXERCISE 10.01 Finding an unknown side

Concepts and techniques

1 Evaluate each of these expressions correct to two decimal places.

a $7 \tan(28°)$

b $\dfrac{25}{\sin(10°)}$

c $\dfrac{7}{\tan(63°)}$

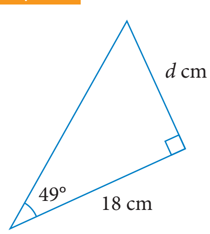
d $8 \cos(12°)$

e $\dfrac{11.2}{\tan(77°)}$

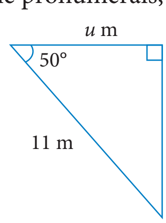
f $7 \sin(25°)$

2 Example 1a Calculate the value of the pronumerals, correct to two decimal places.

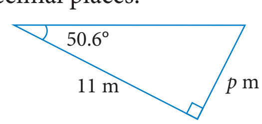
a



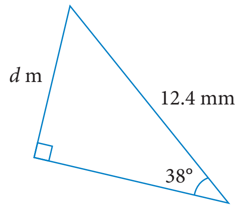
b

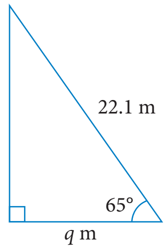


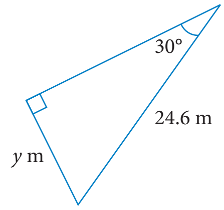
c



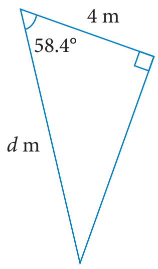
9780170250252

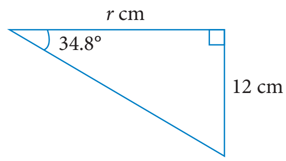
d


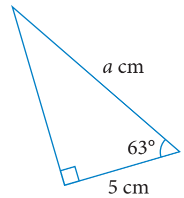
e


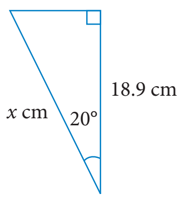
f


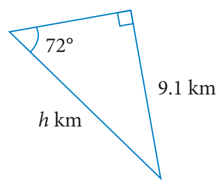
3 Example 1b Find the value of the pronumerals, correct to two decimal places.

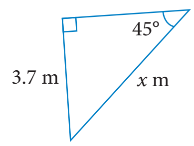
a


b


c


d


e


f


4 The expression used to find the value of x in the following triangle is:

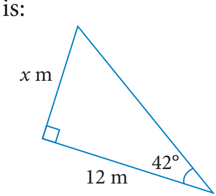
A $12 \cos(42°)$

B $12 \tan(42°)$

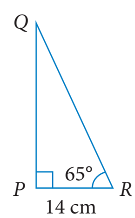
C $\dfrac{12}{\tan(42°)}$

D $\dfrac{12}{\cos(42°)}$

E $12 \sin(42°)$



5 A triangle PQR has the following dimensions: $\angle P = 90°$, $\angle R = 65°$ and $PR = 14$ cm. Find the length of side PQ, correct to one decimal place.



6 In ΔHIJ, $\angle J = 90°$, $\angle I = 26°$ and $HJ = 5.1$ m. Find the length of HI, correct to one decimal place.

7 Which of the following is not equal to the unknown side in the following triangle?

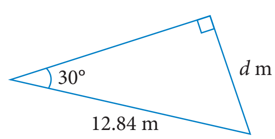
A $12.84 \cos(60°)$

B $12.84 \tan(60°)$

C $\dfrac{12.84}{2}$

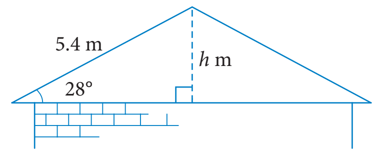
D 6.42

E $12.81 \sin(30°)$

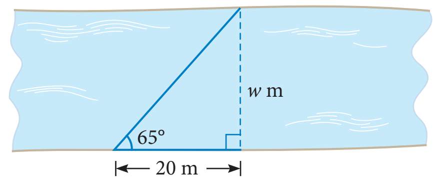


Reasoning and communication

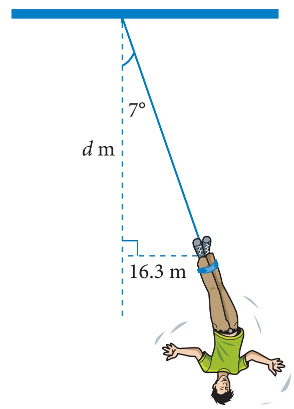
8 Example 2 The pitch of a roof is 28°. If the peak of the roof is 5.4 m from the gutter, how much higher is the peak than the gutter? Give your answer correct to one decimal place.



9 Example 3 Find the width, w metres, of the river shown in the diagram, correct to one decimal place.



10 A bungy jumper leaps off a bridge at an angle of 7° to the vertical. If he is swinging 16.3 m off-centre, calculate the vertical distance that he has dropped correct to one decimal place.



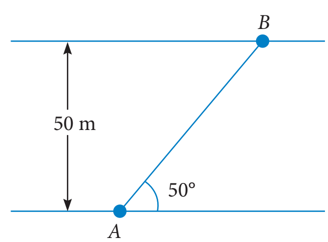
11 A rectangular field of width 38 m has a diagonal path that makes an angle of 40° with the 38 m side. Find the length of the path correct to the nearest 0.1 m.

12 A ladder leaning against a building makes an angle of 22° with the building. If the foot of the ladder is 2.8 m from the building, calculate, correct to two decimal places,

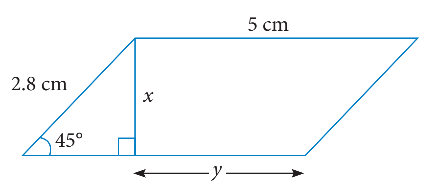
a the length of the ladder.

b the height it reaches above the ground.

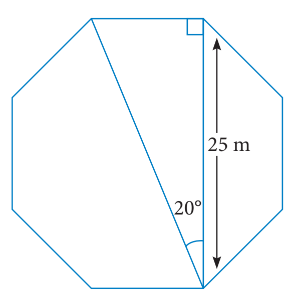
13 A swimmer swims across a river 50 m wide. She starts at point A and finishes at point B. What is the actual distance that she swam (to the nearest metre)?



14 In the parallelogram on the right calculate the values of x and y correct to one decimal place.



15 A right-angled triangle is drawn inside a regular octagon as shown on the right. Calculate the perimeter of the octagon correct to one decimal place.



9780170250252

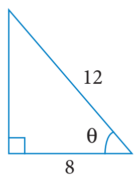
10.02 FINDING AN UNKNOWN ANGLE

Trigonometric ratios can also be used to find unknown angles in right-angled triangles if two side lengths are known. Select the appropriate function linking the unknown angle with the two known sides. Solve to find the angle by using the inverse trigonometric function.

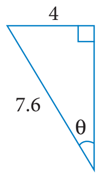
Example 4

Calculate the values of the unknown angles, correct to the nearest degree.

a

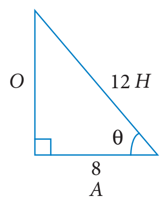


b



Solution

a Draw and label the sides of the triangle.



Select and write the appropriate ratio. $\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

Substitute the known values into the equation. $\cos(\theta) = \dfrac{8}{12}$

Solve the equation by using $\cos^{-1}$ to evaluate θ. $\theta = \cos^{-1}\left(\dfrac{8}{12}\right)$

$\theta = 48.1897...$

$\approx 48°$

The inverse of a trigonometric function is represented in the following way.

The inverse of sine = $\sin^{-1}$.

The inverse of cosine = $\cos^{-1}$.

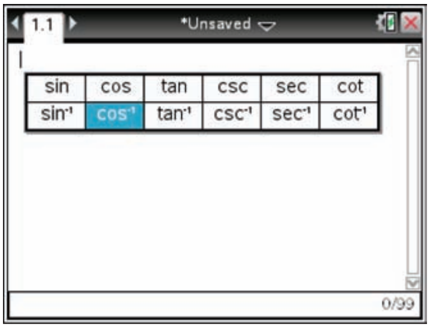
The inverse of tangent = $\tan^{-1}$.

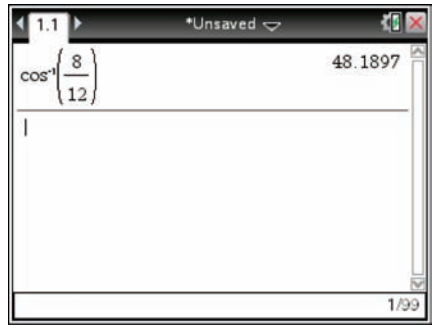
TI-Nspire CAS

Press [trig] and select $\cos^{-1}$.

Key in $\left(\dfrac{8}{12}\right)$. Press [ctrl] [enter].

Calculate and round to the nearest degree.





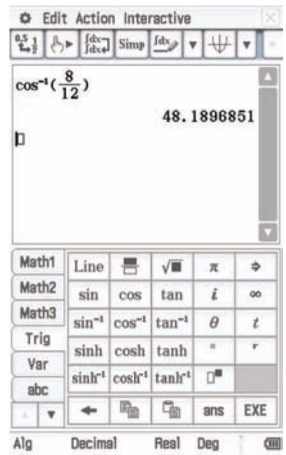
ClassPad

Use the Main $\sqrt{\alpha}$

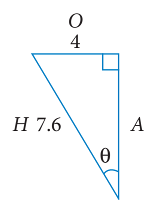
Application in **Decimal** mode.

Press [Keyboard] then tap [Trig] and [$\cos^{-1}$].

Key in $\left(\frac{8}{12}\right)$. Press [EXE].



b Draw and label the sides of the triangle.



Select and write the appropriate ratio.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

Substitute known values into the equation.

$$\sin(\theta) = \frac{4}{7.6}$$

Solve the equation by using $\sin^{-1}$ to evaluate θ.

$$\theta = \sin^{-1}\left(\frac{4}{7.6}\right)$$

$$\theta = 31.7569...$$

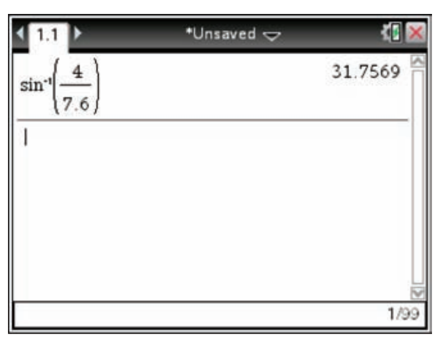
$$\approx 32°$$

TI-Nspire CAS

Press [trig] and select $\sin^{-1}$.

Key in $\left(\frac{4}{7.6}\right)$. Press [ctrl] [enter].

Calculate and round to the nearest degree.



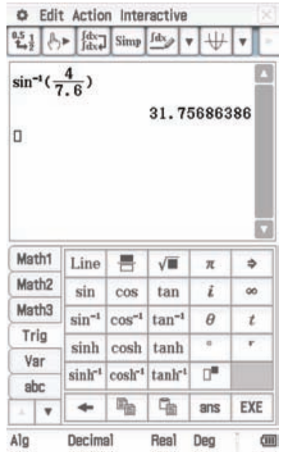
ClassPad

Use the Main $\sqrt{\alpha}$ Application in **Decimal** mode.

Press [Keyboard] then tap [Trig] and [$\sin^{-1}$].

Key in $\left(\frac{4}{7.6}\right)$.

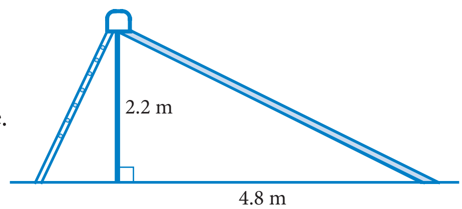
Press [EXE]. Calculate and round to the nearest degree.



Example 5

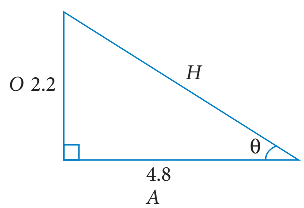
A playground slide is made up of a ladder and a metal slide. The top of the metal slide is 2.2 m from the ground and the bottom of the slide is 4.8 m from a point directly under the top of the slide.

What angle does the slide make with the ground, correct to the nearest degree?



Solution

Draw and label the sides of the triangle.



Select and write the appropriate ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute the known values into the equation.

$$\tan(\theta) = \frac{2.2}{4.8}$$

Solve the equation by using $\tan^{-1}$ to evaluate θ.

$$\theta = \tan^{-1}\left(\frac{2.2}{4.8}\right)$$

Calculate and write the answer, remembering to round to the nearest degree.

$$\theta = 24.6236\ldots$$
$$\approx 25^\circ$$

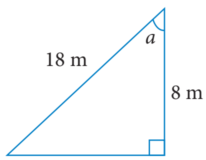
The metal slide makes an angle of 25° with the ground.

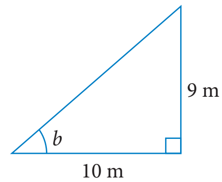
EXERCISE 10.02 Finding an unknown angle

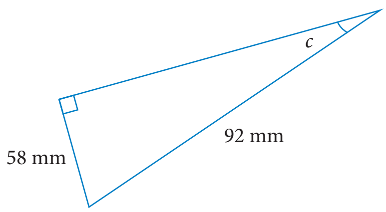
WS
Calculating lengths and angles

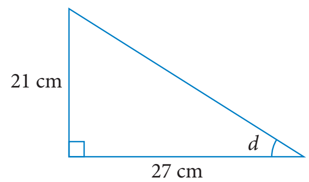
Concepts and techniques

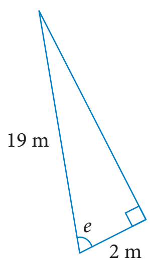
1 Example 4 Find the value of the pronumeral in each of the following, correct to the nearest degree.

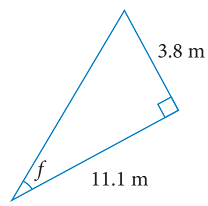
a


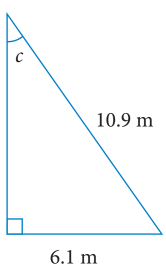
b


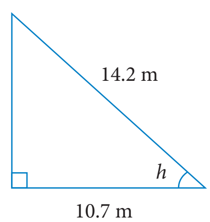
c


d


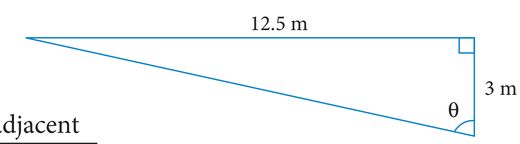
e


f


g


h


2 To calculate the value of θ in the following triangle, which trigonometric ratio would you use?



A $\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}}$

B $\sin(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

C $\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

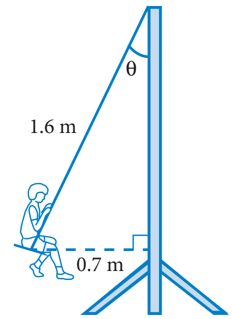
D $\tan(\theta) = \dfrac{\text{adjacent}}{\text{opposite}}$

E $\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$

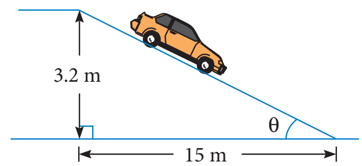
9780170250252

Reasoning and communication

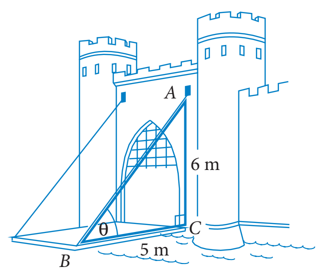
3 Example 5 Caitlin is on a swing of length 1.6 m. When she is 0.7 m away from the swing's frame, what angle does the swing make with the vertical? Give your answer to the nearest degree.



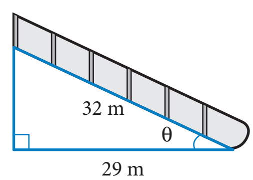
4 Find the angle of inclination, θ, of this ramp in a multistorey car park, correct to the nearest degree.



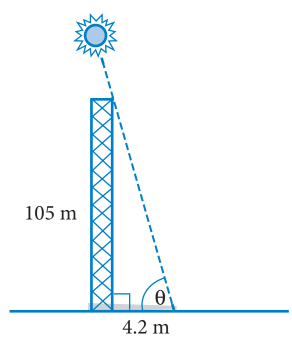
5 The drawbridge in front of this castle is 5 m long and is pulled by chains from 6 m above ground level. What angle do the chains make with the ground, to the nearest degree?



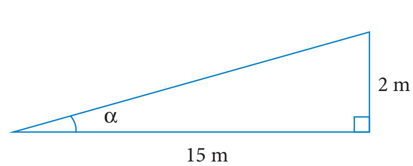
6 An escalator ramp has a length of 32 m and covers a horizontal distance of 29 m. What is its angle of inclination, θ, to the nearest degree?



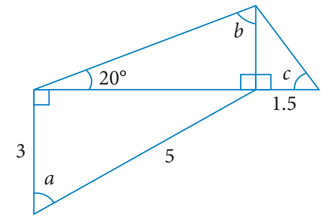
7 What angle do the Sun's rays make with the horizontal when a tower that is 105 m tall has a shadow of 4.2 m? Give your answer correct to the nearest degree.



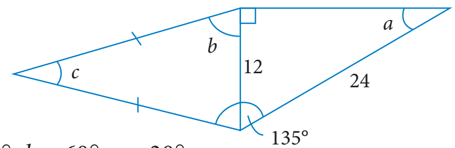
8 A hill has a gradient of $\frac{2}{15}$, which means that it rises 2 m for every 15 m horizontally. Find the angle that the hill makes with the horizontal, correct to the nearest degree.



9 In the diagram below, find the values of a, b, and c to the nearest degree.



10 An isosceles triangle is placed next to a right-angled triangle as shown in the diagram. The magnitudes of angles a, b, and c to the nearest degree are:



A $a = 30°, b = 30°, c = 30°$

B $a = 30°, b = 60°, c = 30°$

C $a = 30°, b = 75°, c = 30°$

D $a = 60°, b = 75°, c = 30°$

E $a = 30°, b = 30°, c = 60°$

INVESTIGATION Investigating the trigonometric ratios

a i Use your calculator to find the ratios of the angles in this table (correct to four decimal places).

θ	1°	10°	30°	45°	60°	80°	89°
sin (θ)							
cos (θ)							
tan (θ)							

ii Look at the sine and cosine ratios for 30° and 60°. Why do we get these values?

iii What happens to the sine and cosine ratios as θ increases in size from 0° to 90°?

iv What happens to the tan ratio as θ increases in size from 0° to 90°?

b i Using your ruler and a protractor accurately draw a right-angled triangle that has one angle measuring 30°.

ii Use your protractor to measure the size of the third angle.

iii Is there only one possible triangle with the angle measurements described? Explain your reasoning.

iv Measure the sides of your triangle using your ruler and then calculate the sine, cosine and tangent ratios for 30°. Compare them with those in the completed table from question **a** part **i**.

v Repeat part **iv** for the third angle of the triangle (60°).

c i Using your ruler and a protractor accurately draw a right-angled triangle that has one angle measuring 45°.

ii Use your protractor to measure the size of the third angle.

iii Is there only one possible triangle with the angle measurements described? Explain your reasoning.

iv Measure the sides of your triangle using your ruler and then calculate the sine, cosine and tangent ratios for 45°. Compare them with those in the completed table from question **a** part **i**.

9780170250252

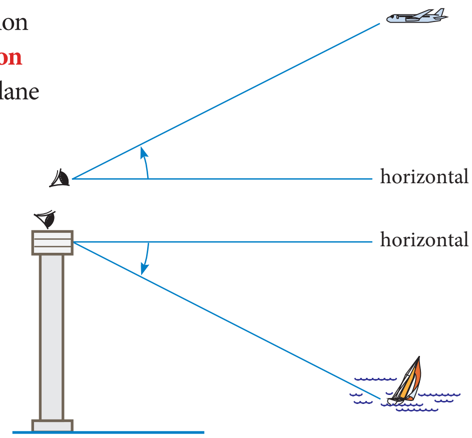
10.03 ANGLES OF ELEVATION AND DEPRESSION

One application of trigonometry is the use and calculation of angles of elevation and depression. **Angles of elevation** and **angles of depression** are taken from a horizontal plane or reference line as shown on the right.

The angle of elevation is the angle looking up.

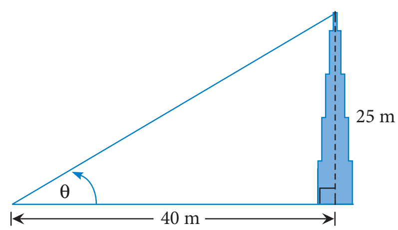
The angle of depression is the angle looking down.

Remember that the angle is measured from the horizontal. To distinguish between the two types of angles you could remember that when we feel 'elevated' things are 'looking up'. When we feel 'depressed' things are 'looking down'.



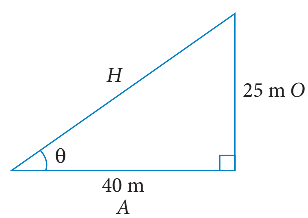
Example 6

Imran stands 40 m from the base of a 25 m tower. What is the angle of elevation, to the nearest degree, of the tower from Imran?



Solution

Draw and label the sides of the triangle.



Select and write the appropriate ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute the known values into the equation.

$$\tan(\theta) = \frac{25}{40}$$

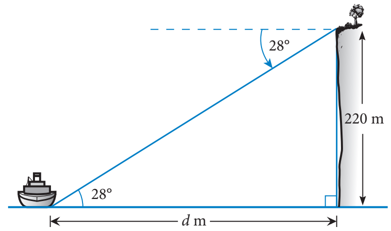
Solve the equation and write the answer, remembering to round to the nearest whole degree.

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{25}{40}\right)\\ &= 32.0053\ldots\\ &\approx 32^\circ\end{aligned}$$

The angle of elevation is 32°.

Example 7

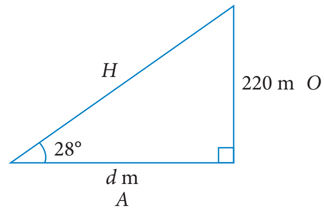
The angle of depression of a ship from the top of a 220 m vertical cliff is 28°. To the nearest metre, how far is the ship from the base of the cliff?



Solution

Draw and label the sides of the triangle.

The angle of depression of the ship from the cliff top is 28°, so the angle of elevation from the ship to the cliff top must also be 28°.



Select and write the appropriate ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute the known values into the equation.

$$\tan(28°) = \frac{220}{d}$$

Solve the equation and write the answer, remembering to round to the nearest whole metre.

$$d \times \tan(28°) = \frac{220}{\cancel{d}} \times \cancel{d}$$

$$d\tan(28°) = 220$$

$$\frac{d\tan(28°)}{\tan(28°)} = \frac{220}{\tan(28°)}$$

$$d = \frac{220}{\tan(28°)}$$

$$= 413.76...$$

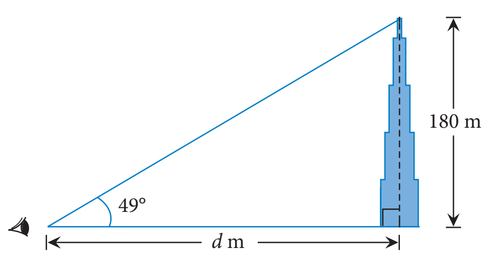
$$\approx 414 \text{ m}$$

The ship is 414 m from the base of the cliff.

EXERCISE 10.03 Angles of elevation and depression

Reasoning and communication

1 Example 6 From street level, Sonja sees the top of a 180 m tower at an angle of elevation of 49°. How far, to the nearest metre, is she from the tower?

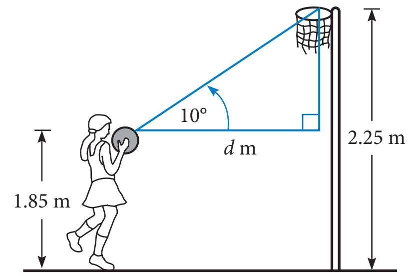


9780170250252

2 Jaz is shooting for goal towards a netball ring at an angle of 10°. Find, correct to two decimal places:

a the distance of the ball from the ring

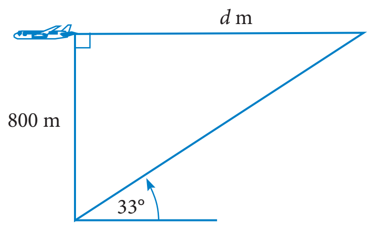
b the distance, d, from Jaz to the goalpost.



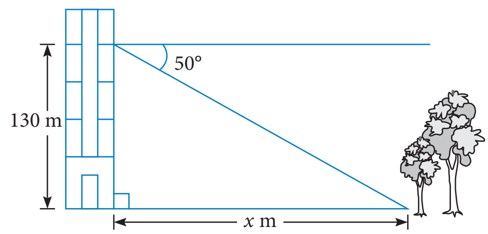
3 Ben observed a plane at an angle of elevation of 33°. It flew at a constant height of 800 m until it was directly above him.

a How far was the plane from Ben when he first saw it (correct to two decimal places)?

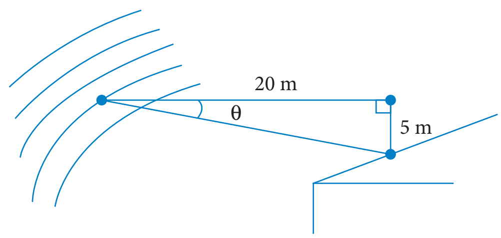
b What was the distance, d, travelled by the plane (correct to two decimal places)?



4 Example 7 From her apartment, 130 m above ground level, Maddy sights the park at an angle of depression of 50°. How far, to the nearest metre, is the park from the base of Maddy's building?



5 In a concert hall, Liam's seat is 20 m from the stage and 5 m above it. What is his angle of depression to the stage, correct to the nearest degree?



6 Alexis is on a swing of length 1.4 m. She pulls back 0.6 m from the frame to commence swinging. Which of the following is the expression that will give you the angle that the swing makes with the vertical?

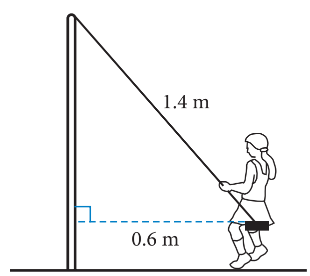
A $\cos^{-1}\left(\dfrac{0.6}{1.4}\right)$

B $\sin^{-1}\left(\dfrac{1.4}{0.6}\right)$

C $\tan^{-1}\left(\dfrac{0.6}{1.4}\right)$

D $\cos^{-1}\left(\dfrac{1.4}{0.6}\right)$

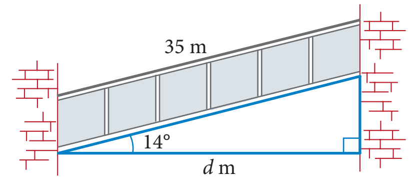
E $\sin^{-1}\left(\dfrac{0.6}{1.4}\right)$



7 A pedestrian walkway, linking two buildings, is 35 m long and is inclined at 14° to the horizontal. Calculate, correct to two decimal places:

a the distance between the two buildings

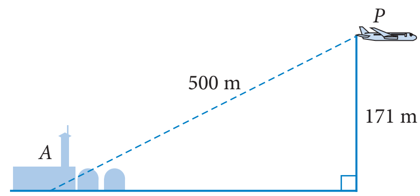
b the vertical rise of the skyway.



8 A plane, *P*, leaves an airport, *A*, and flies 500 m to reach a height of 171 m.

a What is the angle of elevation of the plane from the airport (to the nearest degree)?

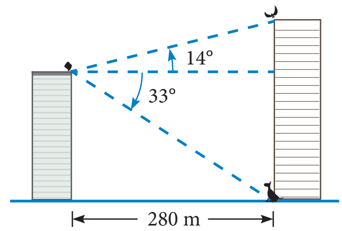
b What is the horizontal distance between the plane and the airport (correct to two decimal places)?



9 Amien stands on the roof of his apartment block and sees a bird on the roof of a neighbouring office building at an angle of elevation of 14°. At the same time, he sees a dog at the bottom of the office building at an angle of depression of 33°. If the two buildings are 280 m apart:

a how far, correct to two decimal places, is the bird from Amien?

b how far, correct to two decimal places, is the bird from the dog?

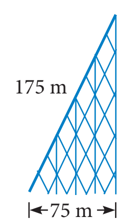


10 A 2.5 m totem pole casts a shadow of 2 m.

a What is the length of the shadow of a 6.4 m tree at the same time, correct to two decimal places?

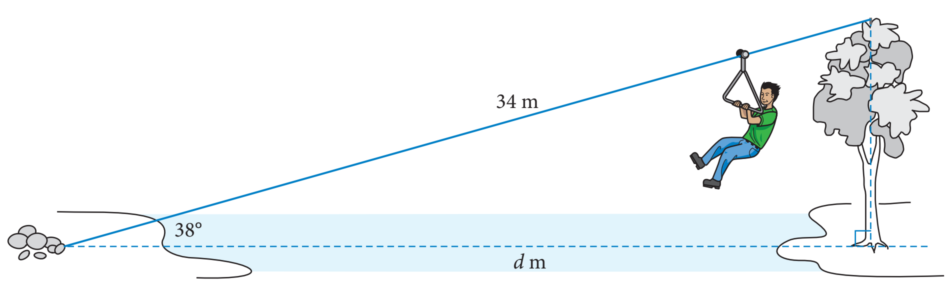
b What is the angle of elevation of the Sun, correct to the nearest degree?

11 A section of a rollercoaster is shown in the diagram. What is the angle of elevation of this section to the nearest degree?



12 A flying fox is constructed between the top of a tree and a large rock. It has a length of 34 m and an angle of elevation of 38°. Which of the following is the (horizontal) distance between the tree and the rock, correct to one decimal place?

A 20.9 m B 26.0 m C 26.6 m D 26.7 m E 26.8 m



9780170250252

INVESTIGATION Using a clinometer to measure heights

You will need: A clinometer, a trundle wheel, a metre ruler or height chart and a calculator. Choose a tall object outside, such as a tree, a flagpole, a football goalpost, a tower or a building. You will use a clinometer and a trundle wheel to measure the object's height.

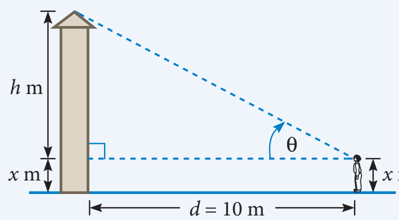
a Copy the following table.

Distance, d	Angle of elevation, θ	Height, $h = d \tan(\theta)$	Revised height, $h + x$
10 m			
15 m			
20 m			
30 m			
50 m			

b Use the trundle wheel to measure 10 m from the base of the object and stand at that point.

c Use the clinometer to view and measure the angle of elevation, θ, to the top of the object. Write this value in the 'Angle of elevation, θ' column in the table.

d $\tan(\theta) = \frac{h}{d}$, so $h = d\tan(\theta)$. Calculate h correct to two decimal places. Write that value in the 'Height, $h = d\tan(\theta)$' column of the table.



e Measure x, the height of the observer's eye above the ground in metres, correct to two decimal places.

f Calculate $h + x$ and write the answer in the 'Revised height, $h + x$' column of the table. This is the height of the object.

g Repeat this experiment for other values of d, at different distances from the base of your object, using the values given in the table, or some of your own.

h How do your calculated results compare with each other? They are all meant to be the same, but why might the answers be different?

i How could we come up with a single accurate value for the height of the object, using these results?

j Use the clinometer to measure and calculate the heights of other objects around your school.

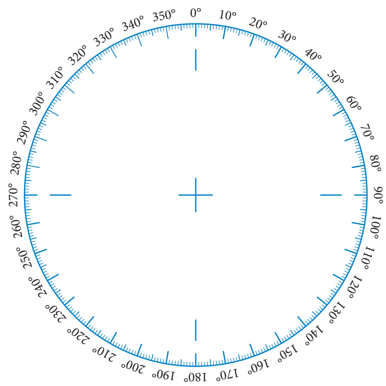
10.04 BEARINGS

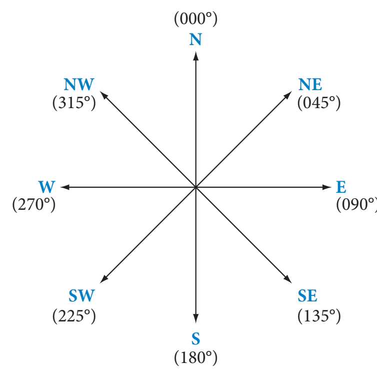
Bearings use angles to show the direction of one location from a given point. Bearings or directions are found by connecting the two points with a straight line on a horizontal plane.

There are two types of bearings: **true bearings** and **compass bearings**.

True bearings

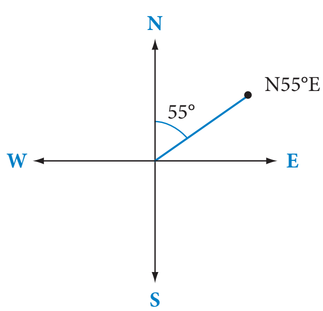
True bearings (also known as three-figure bearings) are angles measured in a clockwise direction from north, which is at 000°T. The diagram below allows you to see how one full revolution of a circle (360°) compares to the compass rose.

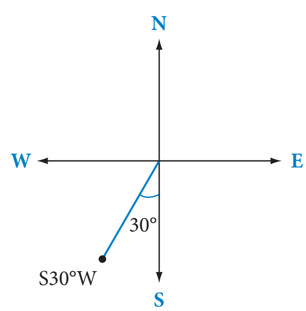




Compass bearings

Compass bearings differ to true bearings although they are still referenced to a compass rose. Compass bearings are referenced from either the north or the south; and are measured in either an east or west direction.



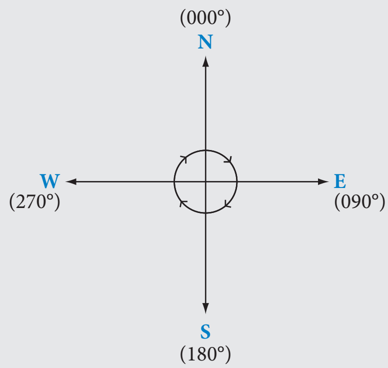


In the diagram above, you can see how the bearing is taken from either north or south. With compass bearings, north (N) or south (S) is written first. The angle, which is between 0° and 90°, is written next and this is followed by the reference to east (E) or west (W).

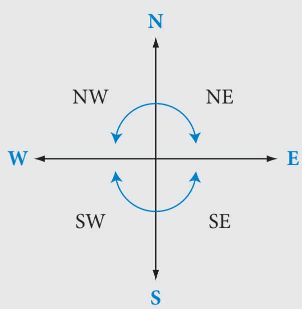
9780170250252

IMPORTANT

True bearings always start from north and go in a clockwise direction. True bearings will be between 000°T and 360°T (with north being at both 000°T and 360°T).



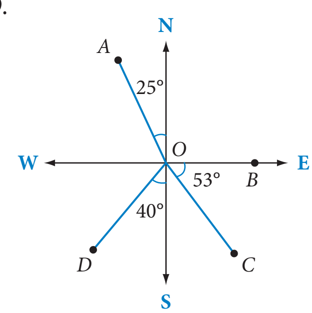
Compass bearings depend on which quadrant is being referenced. Compass bearings are referenced from either north or south.



Example 8

a Write the true bearings of locations A, B, C and D from point O.

b Write the compass bearings of locations A, B, C and D from point O.



Solution

a True bearings are measured clockwise from north.

The first bearing from north is B. B is due east, so it is 90° clockwise from north.	The bearing of B is 090°T.
C is 53° past east.	The bearing of C is $(90° + 53°)$T = 143°T.
D is 40° past south (180°).	The bearing of D is $(180° + 40°)$T = 220°T.
A is 25° before north (360°).	The bearing of A is $(360° - 25°)$T = 335°T.

b Compass bearings depend on which quadrant the point is in. Point B is on east.

B = east

C is in the SE quadrant and forms an angle of 37° with south.

$90° - 53° = 37°$
The bearing of C is S 37° E.

D is in the SW quadrant and forms an angle of 40° with south.

The bearing of D is S 40° W.

A is in the NW quadrant and forms an angle of 25° with north.

The bearing of A is N 25° W.

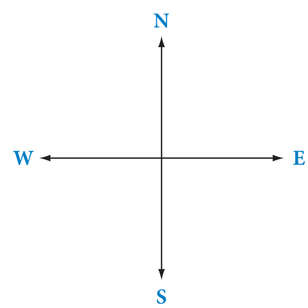
Example 9

Sketch each of the following bearings on a compass rose:

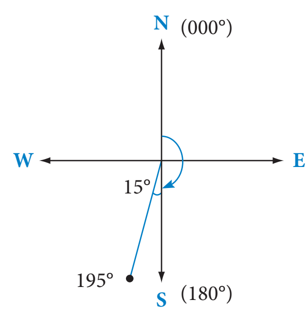
a 195°T. **b** N 25° W.

Solution

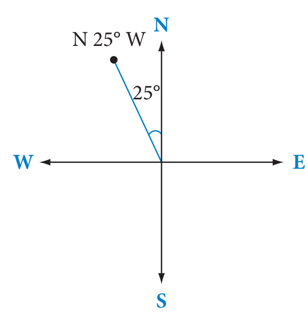
Draw a compass rose.



a 195° is 15° greater than 180°, therefore draw the bearing at 15° past south.



b This compass bearing is in the NW quadrant. It is 25° from north.



9780170250252

EXERCISE 10.04 Bearings

Bearings

Identifying bearings

Concepts and techniques

1 Write the true bearings of each of the following.

a south b north-east c west

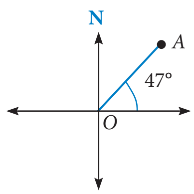
d south-west e north-west f east

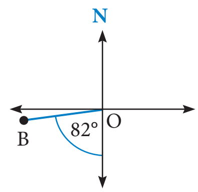
2 In which quadrant (NE, SE, SW, NW) will each of these bearings lie?

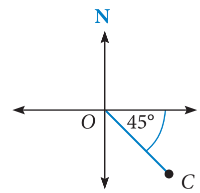
a 260° b 145° c 073°

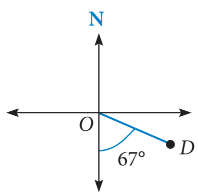
d 102° e 340° f 237°

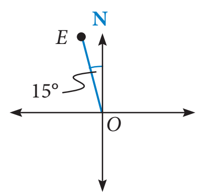
3 Example 8 Write the true bearing of each of the following points from *O*.

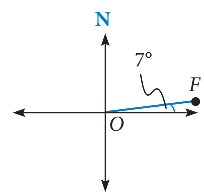
a


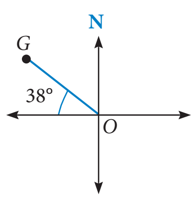
b


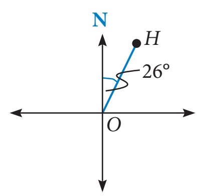
c


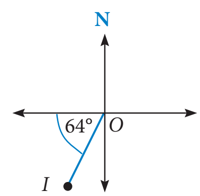
d


e


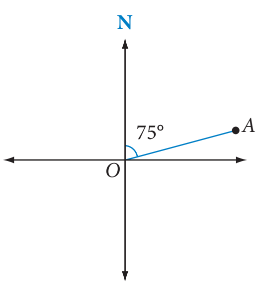
f


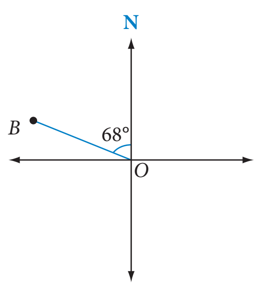
g


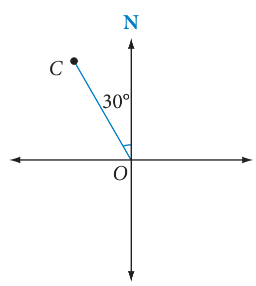
h


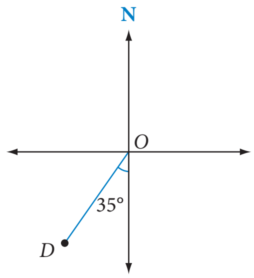
i


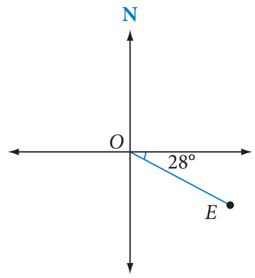
4 Write the compass bearing of each of the following points from *O*.

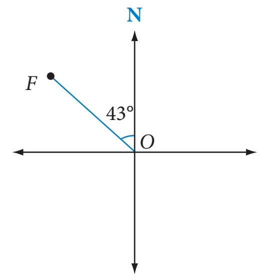
a


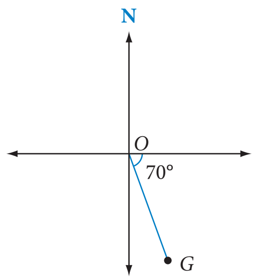
b


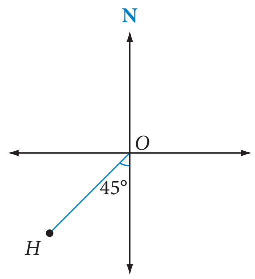
c


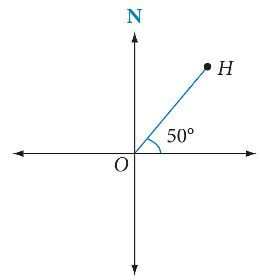
d


e


f


g


h


i


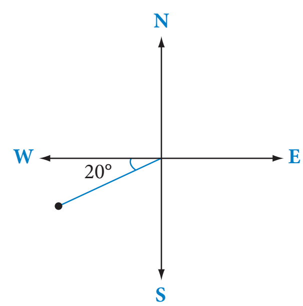
5 147° is in which quadrant?

A NW B NE C SW D SE E NS

6 The compass bearing displayed at right is:

A S 70° W B S 20° W C W 70° S

D N 20° W E S 250° W



7 Example 8 Sketch each of the following bearings on a compass rose.

a 352°T b N 40° W

c 166°T d 078°T

e S 22° W f 303°T

g 147°T h 270°T

i N 10° E j S 70° E

k 065°T l S 58° W

10.05 BEARING APPLICATIONS

For problems involving bearings, take note where the bearing is taken from. This is the point where we draw the centre of our compass rose and from where we take the bearing.

It is important to always work from an accurate diagram and to have north pointing up. To find the lengths and angles in problems involving bearings, you will need to apply all of your trigonometry skills.

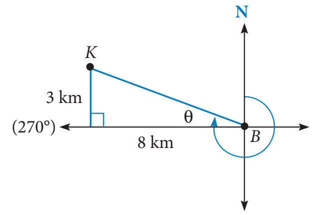
9780170250252

Example 10

Kerr's Creek, K, is 8 km west and 3 km north of Bobtown, B.

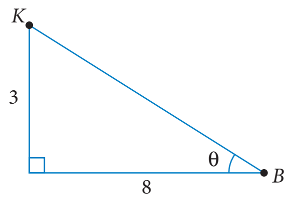
a Find the true bearing, correct to the nearest degree, of Kerr's Creek from Bobtown.

b Find the direct distance between Kerr's Creek and Bobtown correct to one decimal place.



Solution

a Find the right-angled triangle in the diagram and draw it, including all of the relevant information.



Select and write the appropriate ratio.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{3}{8}$$

Calculate θ.

$$\theta = \tan^{-1}\left(\frac{3}{8}\right)$$
$$= 20.5560\ldots$$
$$\approx 21°$$

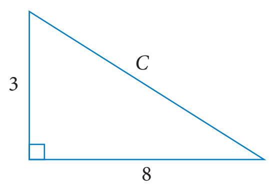
The true bearing is taken from North in a clockwise direction. Add θ to 270° (the angle between north and west).

$$21° + 270° = 291°$$

Write the answer.

The true bearing of Kerr's Creek from Bobtown is 291°T.

b The direct distance can be found using Pythagoras' theorem.

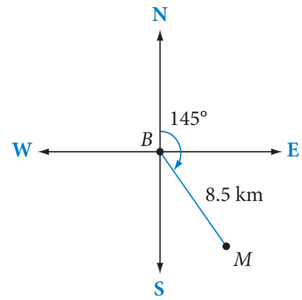


$$a^2 + b^2 = C^2$$
$$3^2 + 8^2 = C^2$$
$$9 + 64 = C^2$$
$$73 = C^2$$
$$C = \sqrt{73}$$
$$= 8.5440\ldots$$
$$\approx 8.5$$

The direct distance between Kerr's Creek and Bobtown is 8.5 km.

Example 11

Matthew walked for 8.5 km on a true bearing of 145° from base camp. His position is shown by M on the diagram on the right.



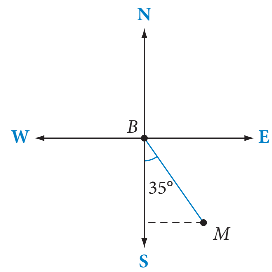
a How far south is he from base camp (correct to one decimal place)?

b How far east is he from base camp (correct to one decimal place)?

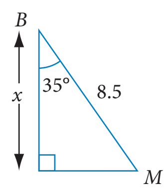
c What is the true bearing of base camp from Matthew?

Solution

a Find the right-angled triangle in the diagram.



Draw it, including all of the relevant information.



Select and write the appropriate ratio.

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Substitute the values into the equation and solve for the unknown length.

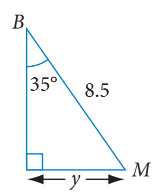
$$\cos(35°) = \frac{x}{8.5}$$

$$x = 8.5 \cos(35°)$$
$$= 6.9627...$$
$$\approx 7.0 \text{ km}$$

Write the answer.

Matthew is 7.0 km south of base camp.

b Using the same right-angled triangle, calculate the distance east (y) using the sine ratio.



You could use either Pythagoras or the tan ratio to find the length of y, but it is always better to use the values given in the original question.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin(35°) = \frac{y}{8.5}$$

$$y = 8.5 \sin(35°)$$
$$= 4.8754...$$
$$\approx 4.9 \text{ km}$$

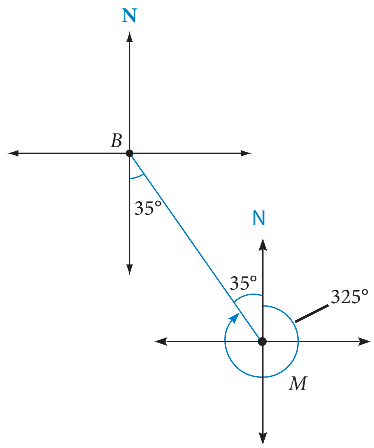
Write the answer.

Matthew is 4.9 km east of base camp.

9780170250252

c To find the true bearing of the base from Matthew we need to sketch his journey using two compass roses.

Using alternate angles, it can be seen that the angle formed between north and Matthew's journey is 35°.



The true bearing must go in a clockwise direction from north.

$360° - 35° = 325°$

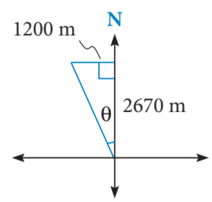
Write the answer.

The bearing of the base camp from Matthew is 325°T.

EXERCISE 10.05 Bearing applications

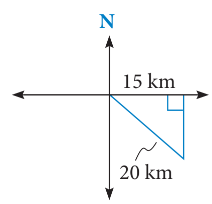
Reasoning and communication

1 Example 10 Selena walks 2670 m north, then turns and walks 1200 m west, as shown in the diagram.



  a Find the value of θ, to the nearest degree.
  b Calculate Selena's true bearing from her starting point.
  c How far is Selena from her starting point (to the nearest 10 m)?

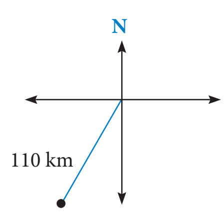
2 A ship sails in a south-easterly direction for 20 km until it is 15 km east of its starting point.



  a What is the bearing of the starting position from the ship's current position?
  b How far south has the boat sailed, to the nearest km?

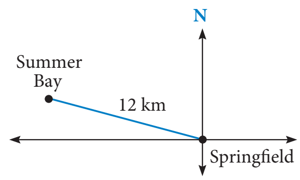
3 A triangular orienteering course starts at Alpha and passes through the checkpoints Bravo and Charlie before finishing back at Alpha. Bravo is 8.5 km due east of Alpha and Charlie is 10.5 km due south of Bravo.
  a Draw a diagram showing this information.
  b Calculate the length of the circuit to the nearest 0.1 km.
  c Find the compass bearing of Charlie from Alpha to the nearest degree.
  d Find the compass bearing of Alpha from Charlie to the nearest degree.

4 Example 11 Amy drives on a true bearing of 210° for 110 km. Find:



  a how far west Amy is from her starting point, correct to the nearest kilometre
  b how far south Amy is from her starting point, correct to the nearest kilometre
  c the true bearing of Amy's starting point from her finishing point.

5 Springfield and Summer Bay are 12 km apart and the compass bearing of Summer Bay from Springfield is N 65° W. How far is Summer Bay west of Springfield, correct to two decimal places?



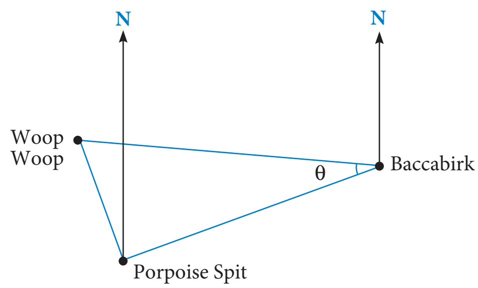
6 A hot air balloon travelled 15 km due east but drifted 1.8 km south. What is its true bearing from its starting point, to the nearest degree?

7 Two cars leave from the same place: the first heads due east, the other on a true bearing of 165°. After travelling 8 km, the first car is due north of the second car. How far has the second car travelled, to the nearest kilometre?

8 The town of Woop Woop is 22 km from the town of Porpoise Spit, on a true bearing of 340°. The town of Baccabirk is 48 km from Porpoise Spit, on a true bearing of 070°.

a Find θ to the nearest degree.

b Find the true bearing of Woop Woop from Baccabirk. (*Hint*: Use alternate angles.)

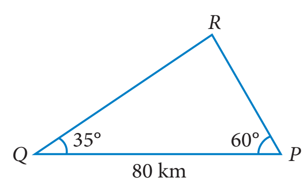


9 Farmer Fred needs to complete the fencing for his triangular pig enclosure. The enclosure currently has two fences. From point O, one fence line is on a bearing of N 30° E for 200 m and the other is on a bearing of S 60° E for 400 m. Find the length of fencing (to the nearest metre) required to complete the pig enclosure.

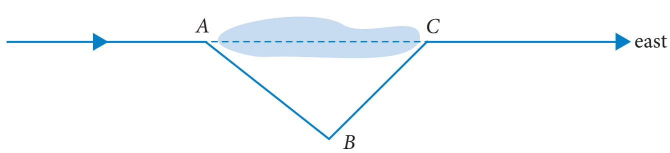
10 Petersville (P) is 80 km due east of Queensfield (Q). Rossmore (R) is on a true bearing of 055° from Queensfield and 300° from Petersville as shown in the diagram.

a What is the true bearing of Queensfield from Petersville?

b What is the true bearing of Queensfield from Rossmore?

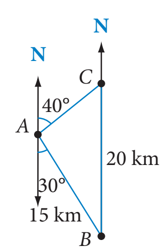


11 A surveyor travelling due east needs to make a detour on a true bearing of 128° to avoid a flooded area. After detouring 8 km, he turns and travels NE until he rejoins the original line of travel.



Copy the diagram above and mark on it all the information given and then find the size of each angle in $\triangle ABC$.

12 Emily is going on a three-day bushwalking adventure. She starts at camp site A and walks for 15 km on a compass bearing of S 30° E to camp site B. On the second day, she walks 20 km due north to campsite C. On the third day, Emily plans to return to campsite A. She knows campsite C is on a compass bearing of N 40° E from campsite A.
What compass bearing does Emily need to travel on in order to return to camp site A?



9780170250252

10.06 THE SINE RULE: FINDING AN UNKNOWN SIDE

Pythagoras' theorem and trigonometric ratios can be used when working with right-angled triangles. When working with **non-right-angled triangles**, the **sine rule** can be used to calculate side lengths and angles. We will consider finding unknown side lengths first.

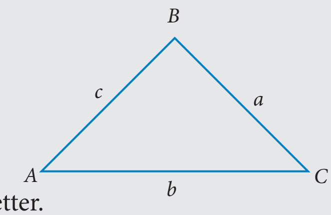
IMPORTANT

The sine rule for the triangle ABC shown on the right is

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}.$$

A, B and C represent the angles and a, b and c represent the side lengths.

The angle is opposite the side length represented by the same letter.



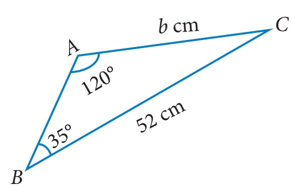
The sine rule can be used if given a non-right-angled triangle and either of the following conditions exist:

– one side and two angles are given

– two sides and a **non-included angle** are given.

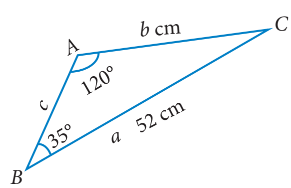
Example 12

Calculate the side length b, correct to two decimal places.



Solution

Label the sides of the triangle using a, b, and c; ensuring that the sides are opposite the angles represented by the same letter.



Select and write the appropriate section of the sine rule.

$$\frac{b}{\sin(B)} = \frac{a}{\sin(A)}$$

Substitute the known values into the equation.

$$\frac{b}{\sin(35°)} = \frac{52}{\sin(120°)}$$

Solve the equation.

$$\sin(35°)\times\frac{b}{\sin(35°)}=\frac{52\sin(35°)}{\sin(120°)}$$

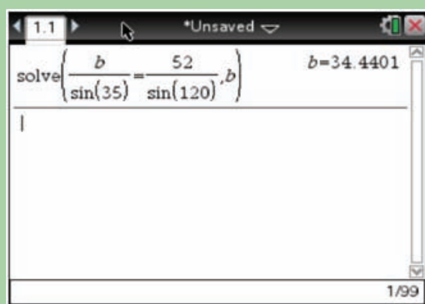
$$b=\frac{52\sin(35°)}{\sin(120°)}$$

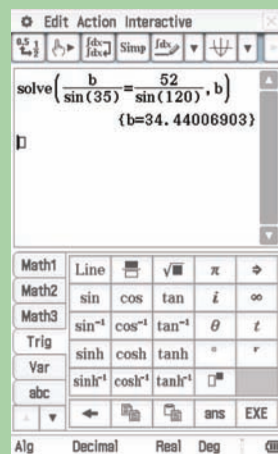
$$=34.440\ldots$$

Write the answer with units.

$$\approx 34.44 \text{ cm}$$

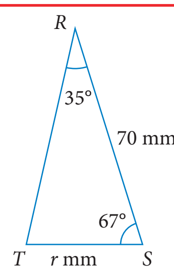
A CAS can be used to solve for the unknown side.





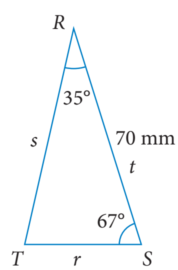
Example 13

In $\triangle RST$, $\angle R = 35°$, $\angle S = 67°$, and $t = 70$ mm. Find r, correct to one decimal place.



Solution

Label the sides of the triangle.



Note that letters other than a, b and c can be used. It is best to match the letters used to label the side lengths to the letters used to label the angles.

To solve for r, angle T is needed since its corresponding side is given.

$$T = 180° - 35° - 70°$$
$$= 75°$$

9780170250252

Write the sine rule using the appropriate pronumerals.

$$\frac{r}{\sin(R)} = \frac{t}{\sin(T)}$$

Substitute the known angles and side length into the rule.

$$\frac{r}{\sin(35°)} = \frac{70}{\sin(75°)}$$

Solve the equation.

$$\sin 35° \times \frac{r}{\sin 35°} = \frac{70 \sin 35°}{\sin 75°}$$

$$r = \frac{70 \sin 35°}{\sin 75°}$$

$$= 41.5667...$$

Write the answer with the correct units.

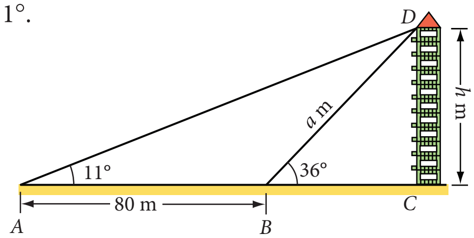
$$\approx 41.57 \text{ mm}$$

Example 14

Kasun observes a tower at an angle of elevation of 11°. Walking 80 m towards the tower, he finds that the angle of elevation increases to 36°.

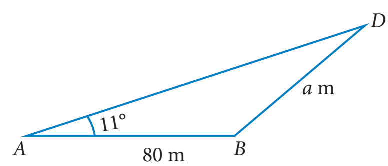
a Show that $a = \frac{80 \sin(11°)}{\sin(25°)}$.

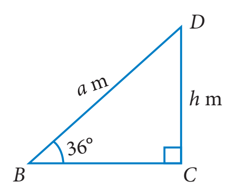
b Hence evaluate the height, h metres, of the tower correct to two decimal places.

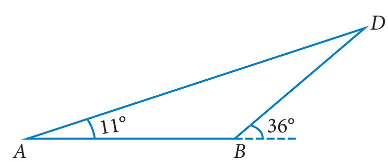


Solution

a Redraw the two triangles separately.





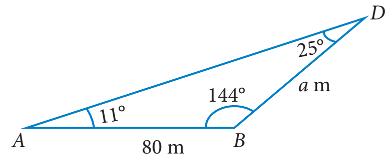


The non-right-angled triangle has more information, therefore use it first. With the given information we can find $\angle ABD$ and $\angle ADB$.

$$\angle ABD = 180° - 36° = 144°$$

$$\angle ADB = 180° - 11° - 144° = 25°$$

Label the non-right-angled triangle.



Show that $a = \dfrac{80\sin(11°)}{\sin(25°)}$ using the sine rule.

$$\frac{a}{\sin(A)} = \frac{d}{\sin(D)}$$

$$\frac{a}{\sin(11°)} = \frac{80}{\sin(25°)}$$

$$\sin(11°) \times \frac{a}{\sin(11°)} = \frac{80\sin(11°)}{\sin(25°)}$$

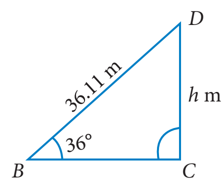
$$a = \frac{80\sin(11°)}{\sin(25°)} \text{ as required.}$$

b Evaluate BD from part **a**.

$$BD = \frac{80\sin(11°)}{\sin(25°)}$$

$$= 36.11940...$$

Now that length BD is known, it is possible to work with the right-angled triangle to find h.



IMPORTANT

Use the unrounded answer 36.11940… in the calculations rather than rounding to a smaller number of decimal places so that rounding errors do not occur.

Use the sine ratio to solve for h.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin(36°) = \frac{h}{36.11940\ldots}$$

$$h = 36.11940\ldots \sin(36°)$$

$$= 21.2304\ldots$$

$$\approx 21.23$$

Write the answer.

The height of the tower is 21.23 m.

9780170250252

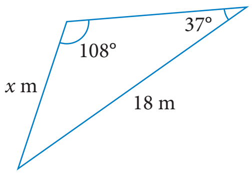
EXERCISE 10.06 The sine rule: finding an unknown side

The sine rule – Finding lengths of sides

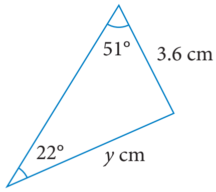
Concepts and techniques

1 Example 12 Find the value of the pronumeral, correct to two decimal places.

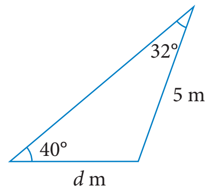
a



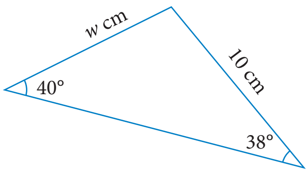
b



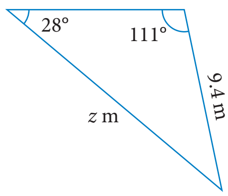
c



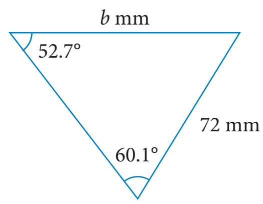
d



e



f



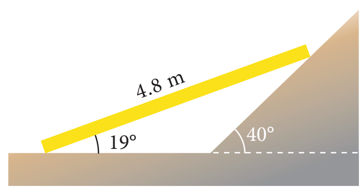
2 Example 13 In $\triangle MTV$, $\angle M = 71°$, $\angle T = 46°$ and $TV = 18.3$ cm. Calculate the length of MV, correct to one decimal place.

3 In $\triangle JKL$, $\angle J = 25°$, $\angle K = 87°$ and $KL = 7.8$ m. The length of JL, correct to one decimal place, is:

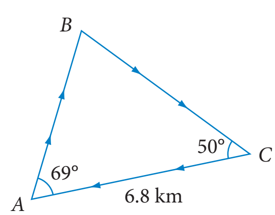
A 3.3 m B 16.7 m C 17.1 m D 18.4 m E 148.8 m

Reasoning and communication

4 A 4.8 m plank is placed between the ground and a ramp that is inclined at 40° to the ground.
If the plank is inclined at 19° to the ground, how far up the ramp is the other end of the plank (give your answer correct to two decimal places)?



5 Which of the following expressions will give you the total distance of the orienteering course outlined on the right?



A $\dfrac{6.8\sin(69°)}{\sin(61°)} + \dfrac{6.8\sin(50°)}{\sin(61°)} + 6.8$

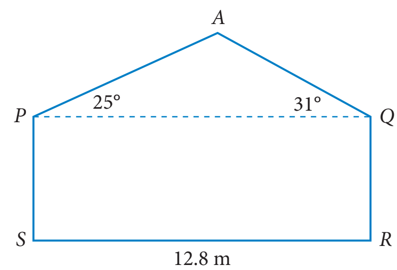
B $\dfrac{6.8\sin(61°)}{\sin(69°)} + \dfrac{6.8\sin(61°)}{\sin(50°)} + 6.8$

C $\dfrac{\sin(69°)}{6.8\sin(61°)} + \dfrac{\sin(50°)}{6.8\sin(61°)} + 6.8$

D $\dfrac{6.8\sin(69°)}{\sin(50°)} + \dfrac{6.8\sin(61°)}{\sin(69°)} + 6.8$

E $\dfrac{\sin(50°)}{6.8\sin(69°)} + \dfrac{\sin(69°)}{6.8\sin(61°)} + 6.8$

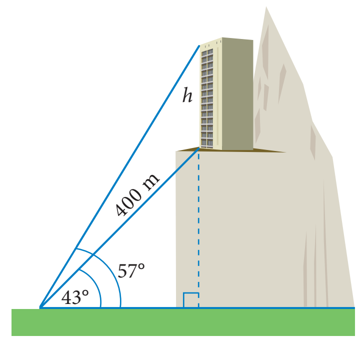
6 A possum walks across the two sides, PA and AQ, of the roof of this house. Find the total distance travelled across the roof, correct to one decimal place.



7 Example 14 Rhys observes the top and bottom of a mountain resort to be at angles of elevation of 57° and 43° respectively. He is 400 m from the bottom of the resort.

a Copy this diagram and find the sizes of all angles.

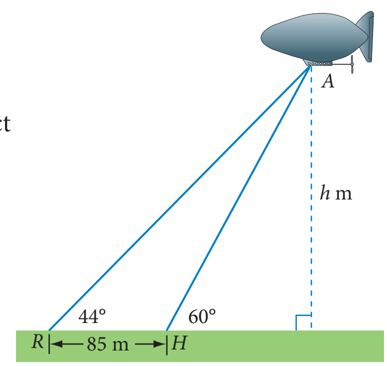
b Find the height of the resort, h, correct to two decimal places.



8 Hayley (H) and Rebecca (R), standing 85 m apart, observe an airship (A) at angles of elevation of 60° and 44° respectively.

a Find the distance between Hayley and the airship, correct to two decimal places.

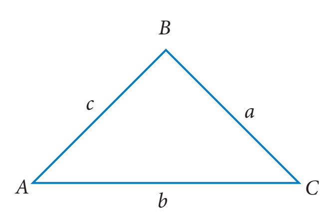
b Calculate the height of the airship, h metres, correct to two decimal places.



10.07 THE SINE RULE: FINDING AN UNKNOWN ANGLE

The sine rule, in the form as shown below, can also be used to find unknown angles in non-right-angled triangles. This form of the rule keeps the unknown value in the numerator, making the equation easier to solve.

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$



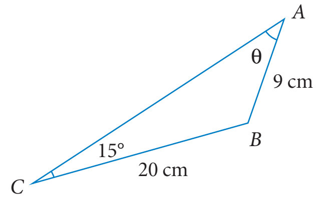
where A, B and C are the angles and a, b and c are the side lengths.

It doesn't matter which form of the sine rule you begin with as long as you label your triangle carefully and substitute the values correctly.

9780170250252

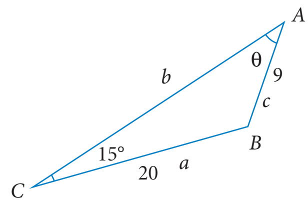
Example 15

Find the size of the angle θ, correct to the nearest degree.



Solution

Label the sides of the triangle.



Since $\theta = A$, we need to use the corresponding side a, and angle C and side c are also known.

$$\frac{\sin(A)}{a} = \frac{\sin(C)}{c}$$

Substitute in the known values.

$$\frac{\sin(\theta)}{20} = \frac{\sin(15°)}{9}$$

Solve the equation for sin (θ).

$$20 \times \frac{\sin(\theta)}{20} = \frac{20\sin(15°)}{9}$$

$$\sin(\theta) = \frac{20\sin(15°)}{9}$$

Use the inverse sine function to calculate the size of the angle. Calculate and write your answer, rounding to the nearest whole degree.

$$\theta = \sin^{-1}\left[\frac{20\sin(15°)}{9}\right]$$

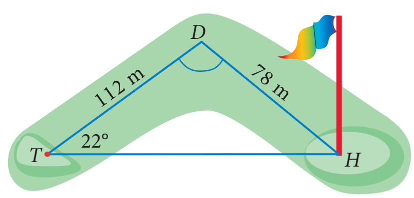
$$= 35.1103...$$

$$\approx 35°$$

Example 16

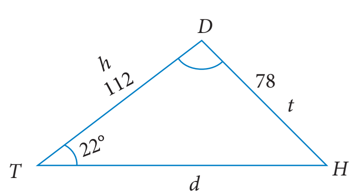
The 18th hole at a golf course has a tee at T, a hole at H and a dogleg at D.

If $TD = 112$ m, $DH = 78$ m and $\angle HTD = 22°$, find $\angle TDH$ correct to the nearest degree.



Solution

Label the sides of the triangle.



For the purpose of using the sine rule:

$\angle HTD = T$

$\angle TDH = D$

$\angle DHT = H$

Since d is not known, D cannot be calculated using the sine rule. However, H can be calculated and this can be used to find D.	$\dfrac{\sin(H)}{h} = \dfrac{\sin(T)}{t}$
Substitute the known values into the sine rule to calculate H.	$\dfrac{\sin(H)}{112} = \dfrac{\sin(22°)}{78}$ $\sin(H) = \dfrac{122\sin(22°)}{78}$ $H = \sin^{-1}\left[\dfrac{122\sin(22°)}{78}\right]$ $\approx 35.87°$
Since all the internal angles of a triangle must add to 180°, D can now be calculated.	$D + T + H = 180°$ $D + 22° + 35.87° = 180°$ $D = 180° - 22° - 35.87°$ $= 122.13°$
Write your answer, rounding to the nearest whole degree.	$\angle TDH = 122°$

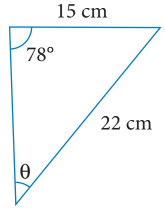
The sine rule – Finding angles

EXERCISE 10.07 The sine rule: finding an unknown angle

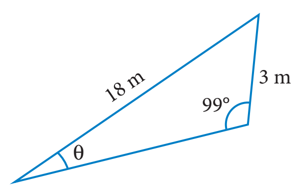
Concepts and techniques

1 Example 15 Find θ in each triangle. Give your answer correct to the nearest degree.

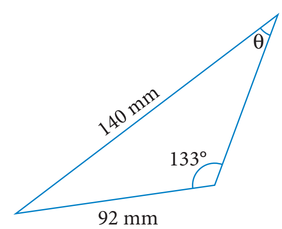
a



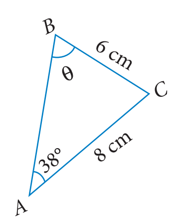
b



c



2 The expression for finding θ in the triangle at right is:



A $\dfrac{8\sin(38°)}{6} \times \dfrac{8\sin(38°)}{6}$

B $\sin^{-1}\left[\dfrac{6\sin(38°)}{8}\right]$

C $\sin^{-1}\left[\dfrac{8\sin(38°)}{6}\right]$

D $\dfrac{8\sin[\sin(38°)]}{6}$

E $\dfrac{6\sin(38°)}{8}$

9780170250252

3 Triangle PQR has sides $PQ = 15$ mm, $QR = 14.7$ mm and $\angle PRQ = 62°$. The values of $\angle QPR$ and $\angle PQR$ respectively are:

A 59°, 59° **B** 60°, 58° **C** 60°, 30° **D** 62°, 56° **E** 30°, 60°

4 In $\triangle RCN$, $NR = 7.8$ m, $NC = 4.5$ m and $\angle C = 120°$.

a Calculate the size of $\angle R$, to the nearest degree.

b What is the size of $\angle N$, to the nearest degree?

c What type of triangle is $\triangle RCN$?

d What is the length of CR correct to two decimal places?

5 Example 16 A non-right-angled triangle PQR has $\angle RPQ = 72°$, side $PR = 26.1$ m and side $QR = 32.8$ m.

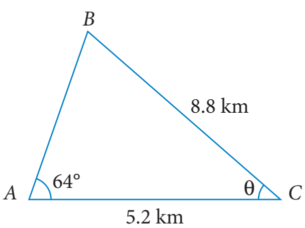
a Draw a diagram showing the information given.

b Find the value of $\angle PQR$, correct to the nearest degree.

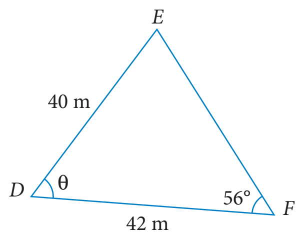
c Hence find the value of $\angle PRQ$, correct to the nearest degree.

6 Find θ in each triangle. Give your answer correct to the nearest degree.

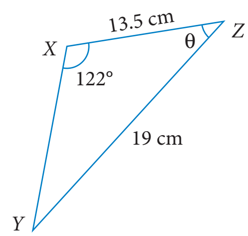
a



b



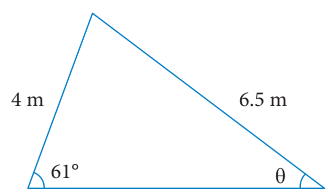
c



7 A surveyor positions three checkpoints M, N and P around a lake at a picnic ground. $MN = 728$ m, $MP = 638$ m and $\angle N = 57°$. Find the size of $\angle M$, correct to the nearest degree.

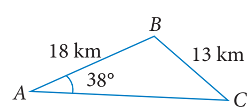
A 47° **B** 50° **C** 73° **D** 76° **E** 107°

8 The gable end of a roof is shown here. Find the value of θ to the nearest degree.

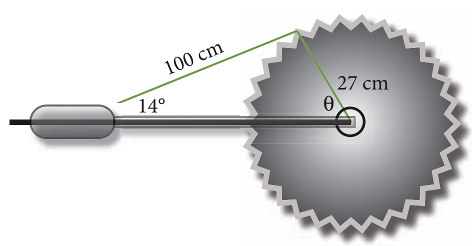


Reasoning and communication

9 Deanna canoes down a river from point A to point B, then on to point C. If $AB = 18$ km, $BC = 13$ km, and $\angle CAB = 38°$, calculate $\angle ABC$ to the nearest degree.

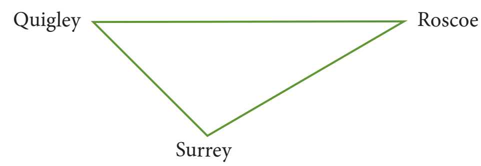


10 The crank and connecting rod of an engine are 27 cm and 100 cm long respectively. What angle does the crank make with the horizontal when the angle made by the connecting rod is 14°? Answer to the nearest degree.



11 Litza is taking a scenic helicopter flight over Melbourne. She leaves Melbourne and flies 20 km due south over the bay. The helicopter then turns on an angle of 120° to fly back over land. When Litza realises that she is running out of petrol she flies 36 km back to the airport where she started. Determine the angle (to the nearest degree) that Litza must turn to return to the airport.

12 Surrey is 21 km on a bearing of 140° from Quigley. Roscoe is 31 km from Surrey and due east of Quigley. Find the bearing of Surrey from Roscoe to the nearest degree.



10.08 THE COSINE RULE

The **cosine rule** can be used to calculate unknown side lengths and angles in non-right-angled triangles when there is not enough information to use the sine rule.

IMPORTANT

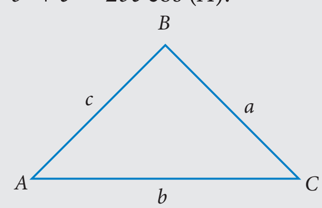
The cosine rule for the triangle ABC can be written as $a^2 = b^2 + c^2 - 2bc \cos(A)$.

It can also be written in other forms:

$b^2 = a^2 + c^2 - 2ac \cos(B)$

or

$c^2 = a^2 + b^2 - 2ab \cos(C)$.



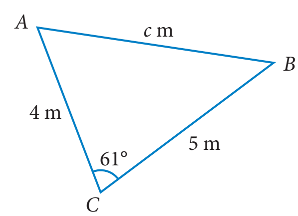
The cosine rule can be used if given a non-right-angled triangle under either of the following conditions exist:

– two side lengths and an **included angle** are given

– three side lengths are given.

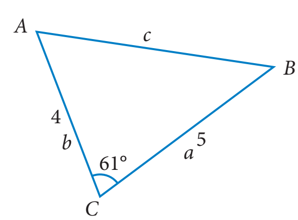
Example 17

Find the value of the pronumeral in the following triangle, correct to two decimal places.



Solution

Label the sides of the triangle using a, b, and c; ensuring that the sides are opposite the angles represented by the same letter.



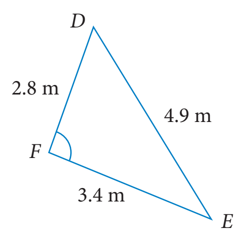
Write the appropriate variation of the cosine rule.	$c^2 = a^2 + b^2 - 2ab\cos(C)$
Substitute the known values into the equation.	$c^2 = 5^2 + 4^2 - 2 \times 5 \times 4 \times \cos(61°)$
Calculate.	$c^2 = 25 + 16 - 40\cos(61°)$ $= 21.6076...$
Find c by taking the square root of both sides.	$c = \sqrt{21.6076...}$ $= 4.6483...$
Write your answer, rounding to two decimal places and including units.	$c \approx 4.65$ m

If given all three side lengths and no angles, the cosine rule can be transposed in order to calculate the size of an angle.

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc} \quad \text{or} \quad \cos(B) = \frac{a^2 + c^2 - b^2}{2ac} \quad \text{or} \quad \cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

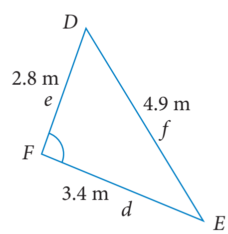
Example 18

Triangle DEF has $DE = 4.9$ m, $EF = 3.4$ m and $DF = 2.8$ m. Find angle F, correct to the nearest degree.



Solution

Label the sides of the triangle.



Note that letters other than a, b and c can be used. It is essential to match the letters used to label the side lengths to the letters used to label the opposite angles.

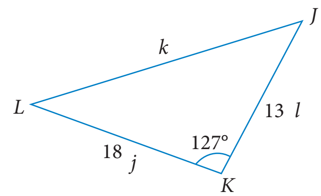
Write the appropriate variation of the cosine rule.	$\cos(F) = \frac{d^2 + e^2 - f^2}{2de}$
Substitute the known values into the equation and solve for cos (F).	$\cos(F) = \frac{3.4^2 + 2.8^2 - 4.9^2}{2 \times 3.4 \times 2.8}$ $= -\frac{4.61}{19.04}$
Solve for F and write your answer. Remember to round to the nearest whole degree.	$F = \cos^{-1}\left(-\frac{4.61}{19.04}\right)$ $= 104.0118...$ $\approx 104°$

Example 19

Kelly, Jessica and Lena are testing their new walkie-talkies. Kelly is 18 m from Lena and 13 m from Jessica. The angle formed between Lena, Kelly and Jessica is 127°. Calculate the distance between Lena and Jessica, correct to the nearest metre.

Solution

Label the sides and angles of the triangle. The distance between (L) Lena and (J) Jessica is represented by k as it is the side opposite K (Kelly).



Write the cosine rule. $k^2 = j^2 + l^2 - 2jl\cos(K)$

Substitute known values into the equation. $k^2 = 18^2 + 13^2 - 2 \times 18 \times 13 \times \cos(127°)$

Calculate k^2. $k^2 = 324 + 169 - 468 \times \cos(127°)$

$= 774.6494...$

Find k by taking the square root of both sides. $k = \sqrt{774.6494...}$

$= 27.8325...$

≈ 27.83

Write the answer. The distance between Lena and Jessica is 28 metres.

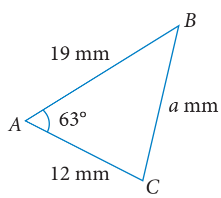
The cosine rule – Angles and sides

EXERCISE 10.08 The cosine rule

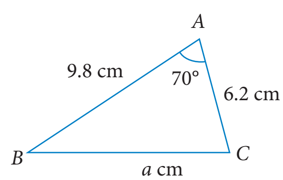
Concepts and techniques

1 Example 17 Find the length of the unknown side in each triangle, correct to two decimal places.

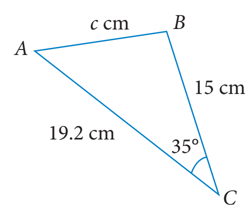
a



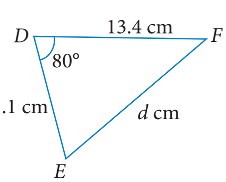
b



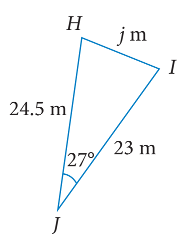
c



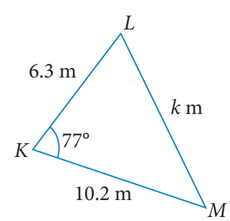
d



e



f



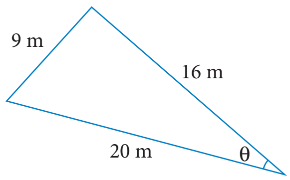
9780170250252

2 In $\triangle ABC$, $\angle B = 38°$, $AB = 12$ m and $BC = 17.5$ m. The length of AC to the nearest metre is:

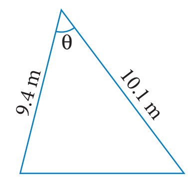
A 9 m **B** 10 m **C** 11 m **D** 14 m **E** 119 m

3 Example 18 Find θ, to the nearest degree, in each triangle.

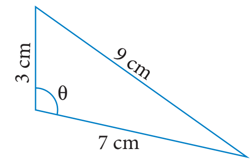
a



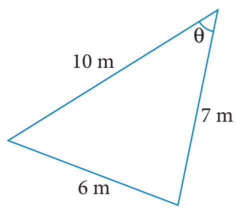
b



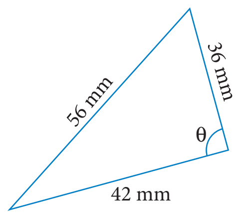
c



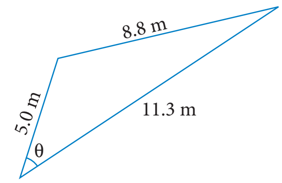
d



e



f



4 In $\triangle GST$, $GS = ST = 8.4$ cm and $GT = 12.7$ cm.

a Calculate the size of $\angle S$, to the nearest degree.

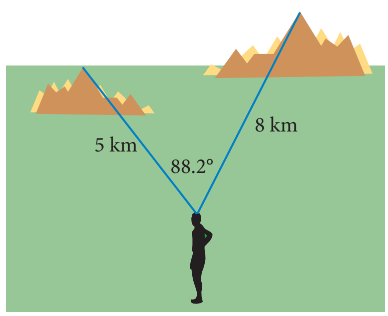
b Calculate the size of $\angle G$, to the nearest degree.

c What type of triangle is $\triangle GST$?

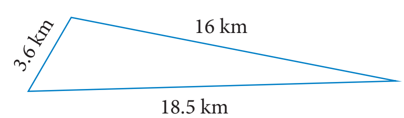
d What is the size of $\angle T$?

Reasoning and communication

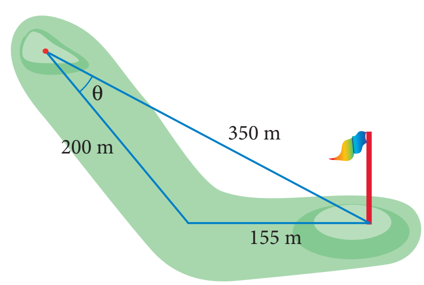
5 Example 19 Angus observes the peaks of two hills from a plain. One is at a distance of 5 km and the other is at a distance of 8 km. If the angle between the lines of sight is 88.2°, calculate the distance between the peaks, correct to the nearest kilometre.



6 A triathlon has a triangular circuit with three legs: swimming 3.6 km, cycling 18.5 km and running 16 km. What is the size of the angle between the cycling and running legs? Answer correct to the nearest degree.



7 A golf hole is 350 m in a straight line from the tee. A ball is driven 200 m from the tee but it is off-line. The ball lands 155 m to the right of the hole. By what angle, to the nearest degree, is the shot off-line?

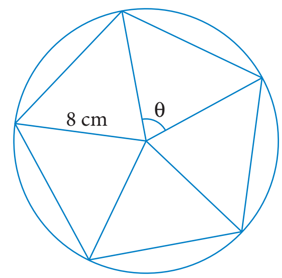


8 A regular pentagon is inscribed in a circle of radius 8 cm.

a How many degrees in a revolution?

b What is the size of angle θ?

c Calculate the perimeter of the pentagon, giving the answer correct to two decimal places.

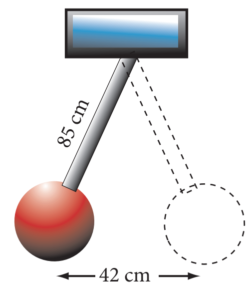


9 The two legs of a ladder are 2.1 m long and stand on the ground 0.9 m apart. Calculate, correct to the nearest degree:

a the angle between the legs

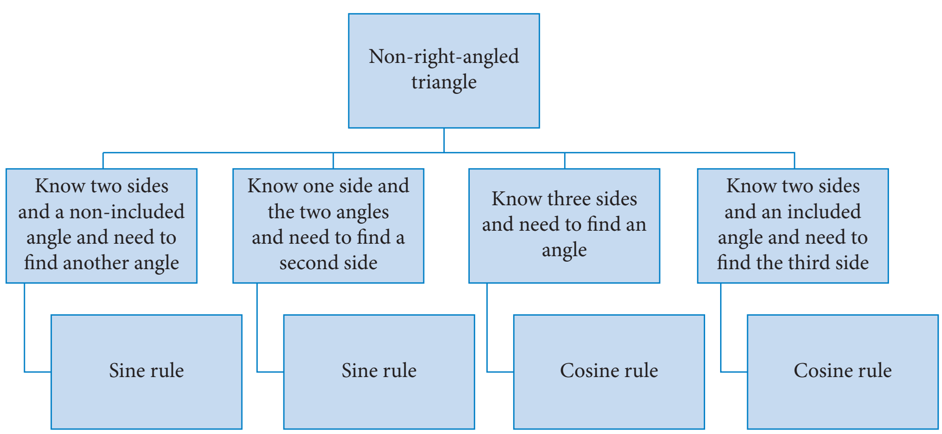
b the angle that each leg makes with the ground.

10 A pendulum of length 85 cm swings through a horizontal distance of 42 cm. What is the angle swept by the pendulum? Answer correct to the nearest degree.



10.09 MIXED APPLICATIONS INVOLVING NON-RIGHT-ANGLED TRIANGLES

When working with non-right-angled triangles either the sine rule or cosine rule can be used to calculate an unknown angle or side length. Each rule has its own purpose and it is important to be familiar with what each rule is used for. The following flow chart will assist you in choosing the appropriate rule.



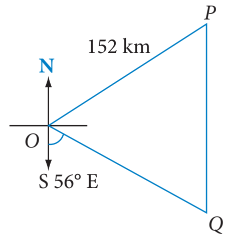
9780170250252

Example 20

Two ships, P and Q, leave port O at the same time. Ship P sails on a course NE, while ship Q sails on a compass bearing of S 56° E. After P sails 152 km, it is due north of Q. Calculate, correct to two decimal places:

a the distance between P and Q

b how far Q has sailed.

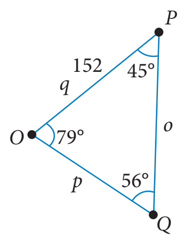


Solution

a Redraw the triangle and calculate the internal angles.

$\angle POQ = 180° - 45° - 56°$
$= 79°$
$\angle OPQ = 45°$
$\angle OQP = 56°$

Label angles O, P and Q, and side lengths o, p and q.



Use the sine rule to calculate the distance between P and Q (labelled as o on the triangle).

$$\frac{o}{\sin(O)} = \frac{q}{\sin(Q)}$$

$$\frac{o}{\sin(79°)} = \frac{152}{\sin(56°)}$$

$$o = \frac{152\sin(79°)}{\sin(56°)}$$

$$= 179.9765...$$

$$\approx 180$$

Write your answer, rounding to the nearest whole kilometre, including units.

The distance between P and Q is 180 km.

b Use the sine rule to calculate the distance that Q has travelled (labelled as p on the triangle).

$$\frac{p}{\sin(P)} = \frac{q}{\sin(Q)}$$

$$\frac{p}{\sin(45°)} = \frac{152}{\sin(56°)}$$

$$p = \frac{152\sin(45°)}{\sin(56°)}$$

$$= 129.6445...$$

$$\approx 130$$

Write your answer, rounding to the nearest whole kilometre, including units.

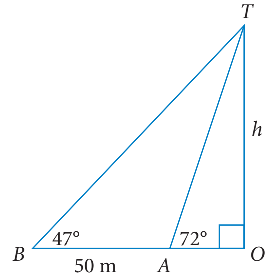
Q has sailed 130 km.

Example 21

The angle of elevation of a tower from point A is 72°. From point B, 50 m further away from the tower than A, the angle of elevation is 47°.

a Find the length of AT correct to two decimal places.

b Find the height h of the tower correct to two decimal places.

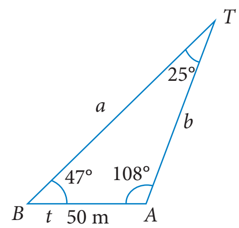


Solution

a To find AT, use $\triangle BAT$ as there is not enough information to use $\triangle BOT$. $\angle BAT$ and $\angle BTA$ can also be found.

$$\begin{aligned}\angle BAT &= 180° - 72° \\ &= 108° \\ \angle BTA &= 180° - 108° - 47° \\ &= 25°\end{aligned}$$

Draw and label the diagram carefully.



Solve for AT using the sine rule. Let $AT = b$.

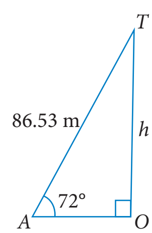
$$\frac{b}{\sin(47°)} = \frac{50}{\sin(25°)}$$

$$\begin{aligned}b &= \frac{50\sin(47°)}{\sin(25°)} \\ &= 86.5265... \\ &\approx 86.53\end{aligned}$$

Write your answer, rounding to two decimal places, and include the unit.

The length of AT is 86.53 m.

b To find h, we use $\triangle AOT$ with the length of AT found in part **a**.



Use the sine ratio to solve for h.

$$\sin(\theta) = \frac{O}{H}$$

$$\sin(72°) = \frac{h}{86.53}$$

$$\begin{aligned}h &= 86.53\sin(72°) \\ &= 82.2949... \\ &\approx 82.29\end{aligned}$$

Write your answer, rounding to two decimal places, and include the unit.

The height of the tower is 82.29 m.

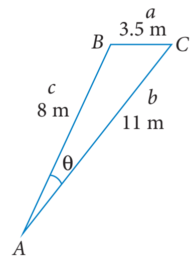
9780170250252

Example 22

Football posts are 3.5 m apart. If a footballer is standing 8 m from one post and 11 m from the other, find the angle within which the ball must be kicked to score a goal, correct to the nearest degree.

Solution

Sketch a diagram illustrating the above situation.



As three side lengths are known and an angle needs to be calculated, the cosine rule is used.

Select and write the appropriate variation of the cosine rule.

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Substitute in the known values and solve.

$$\cos(A) = \frac{11^2 + 8^2 - 3.5^2}{2 \times 11 \times 8}$$

$$= \frac{172.75}{176}$$

$$A = \cos^{-1}\left(\frac{172.75}{176}\right)$$

$$= 11.0279...$$

$$\approx 11°$$

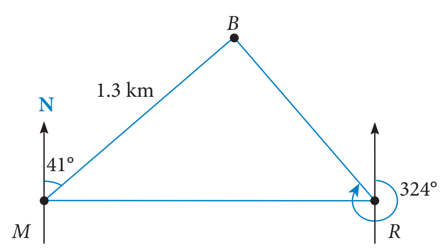
Write your answer, remembering to round to the nearest whole degree.

The angle within which the ball must be kicked is 11°.

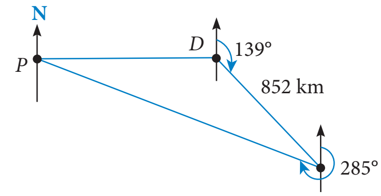
EXERCISE 10.09 Mixed applications involving non-right-angled triangles

Reasoning and communication

1 Example 20 A boat is sinking 1.3 km out to sea from a marina. Its bearing is 041° from the marina and 324° from a rescue boat. The rescue boat is due east of the marina. How far, correct to 2 decimal places, is the rescue boat from the sinking boat?



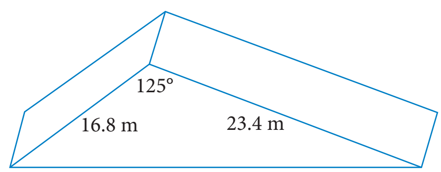
2 A plane flies from Dubbo on a bearing of 139° for 852 km, then turns and flies on a bearing of 285° until it is due west of Dubbo. How far from Dubbo is the plane, to the nearest km?



3 Example 21 The angle of elevation of the top of a flagpole from a point a certain distance away from its base is 20°. After walking 80 m towards the flagpole, the angle of elevation is 75°. Find the height of the flagpole, correct to the nearest metre.

4 Example 22 A triangular field ABC has sides $AB = 85$ m and $AC = 50$ m. If B is on a bearing of 065° from A and C is on a bearing of 166° from A, find the length of BC, correct to the nearest metre.

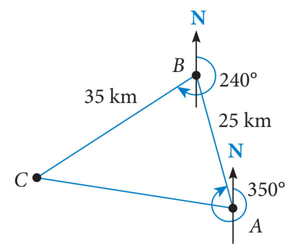
5 A triangular roof is 16.8 m up to its peak, then 23.4 m on the other side with a 125° angle at the peak as shown. Find the horizontal length of the roof correct to one decimal place.



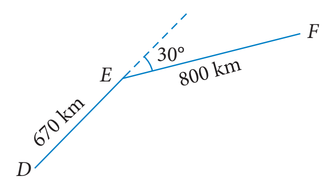
6 Claire just returned from a three-day cycling trip. On the first day she left point A on a true bearing of 350° and cycled for 25 km to point B. On the second day she cycled for 35 km on a true bearing of 240° to reach point C. On the third day she returned to point A but she didn't record the distance or bearing.

a What distance did she travel on the third day (correct to two decimal places)?

b What is the true bearing of point A from point C (to the nearest degree)?



7 A plane flies from Dunstan to Emmett, a distance of 670 km. It then turns 30° and flies another 800 km to Frankston. Calculate the distance between Dunstan and Frankston, correct to the nearest kilometre.



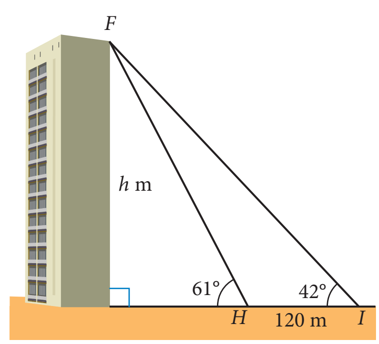
8 At point I, Justine observes the top, F, of an office tower at an angle of elevation of 42°. She walks 120 m towards the tower to H, where the angle of elevation increases to 61°.

a Explain why $\angle IFH = 19°$.

b Show that $FH = \dfrac{120\sin(42°)}{\sin(19°)}$.

c Hence show that $h = \dfrac{120\sin(42°)\sin(61°)}{\sin(19°)}$.

d Evaluate h to the nearest metre.



9780170250252

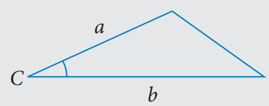
10.10 AREA OF A TRIANGLE: TRIGONOMETRY

The **area of a triangle** can be found using the formula $A = \frac{1}{2}bh$, where b is the base length and h is the perpendicular height. There is another formula that can be used to calculate the area of a triangle if given the length of two sides and the angle between them.

IMPORTANT

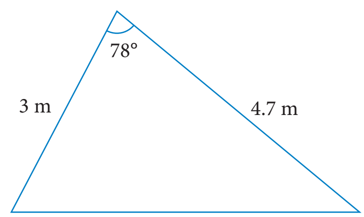
When two side lengths and the included angle are known:

$$A_{\text{triangle}} = \frac{1}{2}ab\sin(C)$$



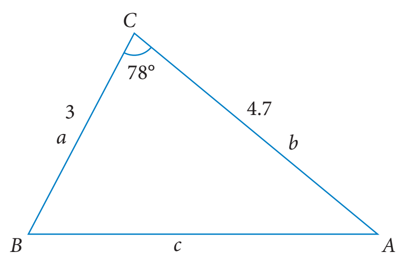
Example 23

Calculate the area of the following triangle correct to two decimal places.



Solution

Label the side lengths a, b and c and the corresponding angles A, B and C.



Identify the values of a, b and C.

$a = 3$
$b = 4.7$
$C = 78°$

Substitute the known values into the formula.

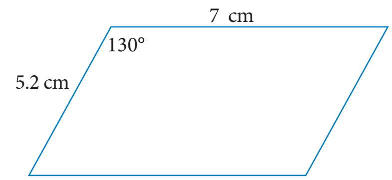
$$A = \frac{1}{2}ab\sin(C)$$
$$= \frac{1}{2} \times 3 \times 4.7 \times \sin(78°)$$

Evaluate, round correct to two decimal places and write your answer.

$A = 6.8959\ldots$
$\approx 6.90 \text{ m}^2$

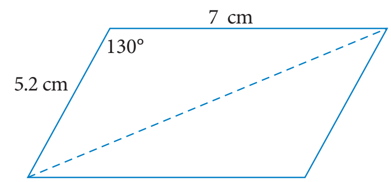
Example 24

A parallelogram has sides of length 7 cm and 5.2 cm, and an included angle of 130°. Calculate its area, correct to one decimal place.



Solution

The question specifies that the angle is included, so the appropriate formula to calculate half the area of the parallelogram is $A = \frac{1}{2}\ ab \sin(C)$.



Identify the values of a and b: that is, the side lengths. Identify the value of the included angle.

$a = 7$ cm, $b = 5.2$ cm
$C = 130°$

Substitute the known values into the formula.

$$\begin{aligned} A &= \frac{1}{2} ab \sin(C) \\ &= \frac{1}{2} \times 7 \times 5.2 \times \sin(130°) \\ &= 13.94 \text{ cm}^2 \end{aligned}$$

Remember that a parallelogram is constructed of two non-right-angled triangles. Multiply your answer by 2, round correct to one decimal place and write your answer.

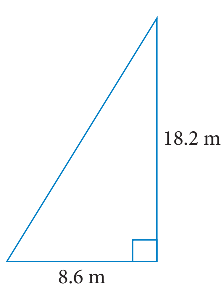
$$\begin{aligned} \text{Area of parallelogram} &= 2 \times 13.94 \text{ cm}^2 \\ &= 27.9 \text{ cm}^2 \end{aligned}$$

EXERCISE 10.10 Area of a triangle: trigonometry

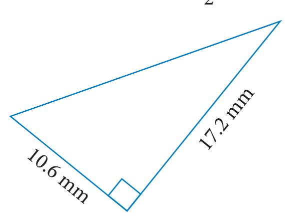
Concepts and techniques

1 Calculate the area of the following triangles using $A = \frac{1}{2}bh$.

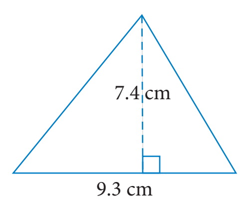
a



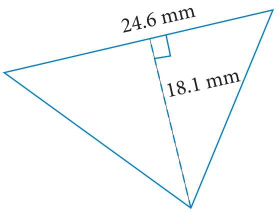
b

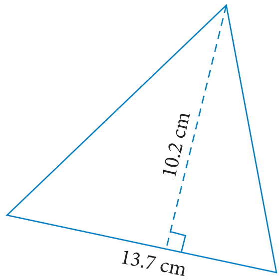


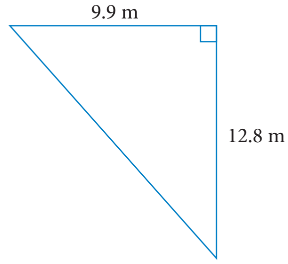
c



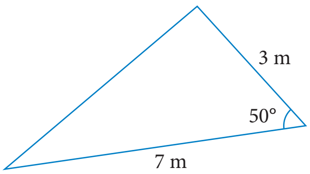
9780170250252

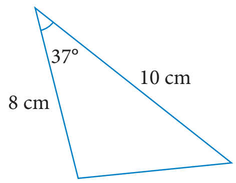
d


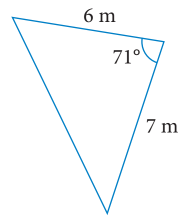
e


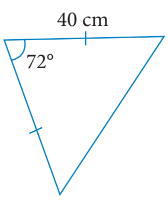
f


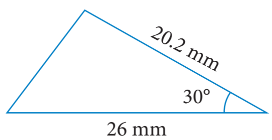
2 Example 23 Calculate the area of the following triangles using $A = \frac{1}{2}ab \sin(C)$, correct to two decimal places.

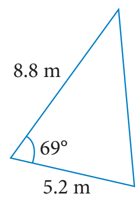
a


b


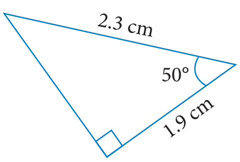
c


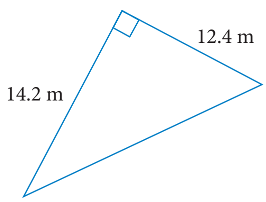
d


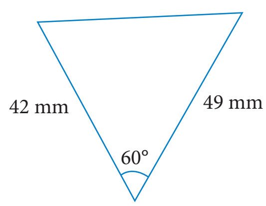
e


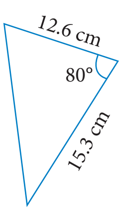
f


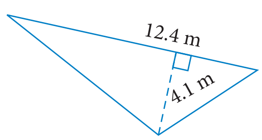
3 Calculate the area of the following triangles correct to one decimal place.

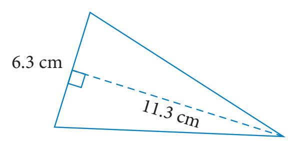
a


b


c


d


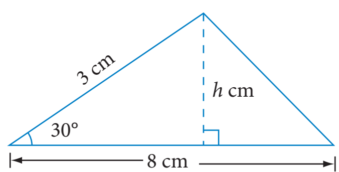
e


f


4 Calculate the area of this triangle:

a using $A = \frac{1}{2}ab \sin(C)$.

b using $A = \frac{1}{2}bh$ (find h using trigonometry first).



5 Calculate, correct to two decimal places, the area of an equilateral triangle with side lengths of 7 cm:

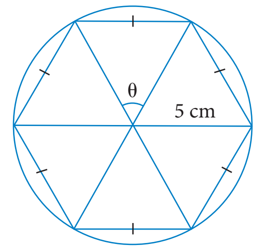
a using the formula $A = \frac{1}{2}ab \sin(C)$.

b using the formula $A = \frac{1}{2}bh$ (after first finding the value of h).

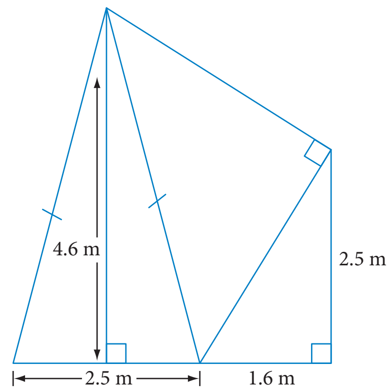
Reasoning and communication

6 Example 24 A regular hexagon is inscribed in a circle of radius 5 cm. The area of the hexagon is:

A 10.83 cm^2 B 54.13 cm^2
C 64.95 cm^2 D 75.00 cm^2
E 78.54 cm^2



7 Jim has designed the following garden bed as a feature in his backyard. He needs to fill the garden bed with top soil. Calculate the total area that requires top soil.

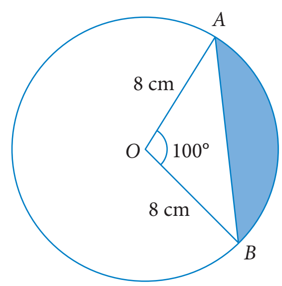


8 O is the centre of a circle with radius 8 cm shown at right.

a Calculate, correct to two decimal places, the area of sector OAB using the formula $A = \frac{\theta}{360}\pi r^2$.

b Calculate the area of ΔOAB, correct to two decimal places.

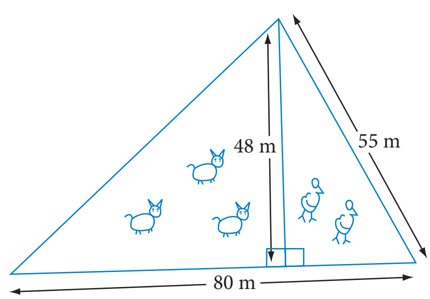
c Hence, find the area of the shaded segment.



9 Farmer Mac has divided his farm into two sections as illustrated. On one side he has chickens and on the other he has pigs.

a Calculate the area of each section, correct to two decimal places.

b Hence calculate the total area of the farm, correct to the nearest square metre.



9780170250252

10.11 AREA OF A TRIANGLE: HERON'S FORMULA

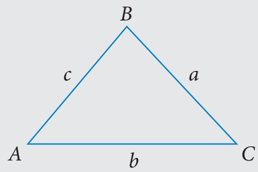
The area of a triangle can also be found using **Heron's formula**. Heron's formula is used when all three side lengths are known.

IMPORTANT

Heron's formula:

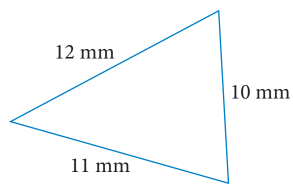
$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

where $s=\frac{a+b+c}{2}$ (the semi-perimeter).



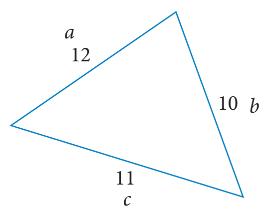
Example 25

Calculate the area, correct to two decimal places, of the following triangle.



Solution

Label the sides of the triangle a, b and c.



To calculate the semi-perimeter, write the formula for the semi-perimeter and substitute in the known values.

$$\begin{aligned} s &= \frac{a+b+c}{2} \\ &= \frac{12+10+11}{2} \\ &= 16.5 \end{aligned}$$

To calculate the area, write Heron's formula and substitute in the known side lengths and semi-perimeter.

Evaluate.

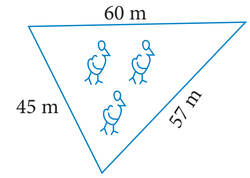
Round to two decimal places and write the answer with the appropriate unit.

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{16.5(16.5-12)(16.5-10)(16.5-11)} \\ &= \sqrt{2654.4375} \\ &= 51.5212\ldots \end{aligned}$$

$$\text{Area} \approx 51.52\ \text{mm}^2$$

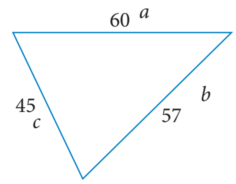
Example 26

Local law states that owners require a certain amount of area for each chicken kept on a property. Mike has the following space allocated for keeping chickens. Calculate this area to the nearest square metre.



Solution

Label the sides of the triangle a, b and c.



Write the formula for the semi-perimeter and substitute in the known values.

$$s=\frac{a+b+c}{2}$$
$$=\frac{60+57+45}{2}$$
$$=81$$

Substitute the known side lengths and semi-perimeter into Heron's formula.

$$A=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{81(81-60)(81-57)(81-45)}$$

Evaluate.

$$=\sqrt{1\,469\,664}$$
$$=1212.2969...$$

Round to two decimal places and write the answer with the appropriate unit.

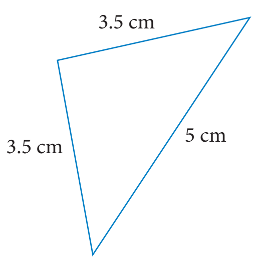
$$\text{Area} \approx 1212.30\text{ m}^2$$

EXERCISE 10.11 Area of a triangle: Heron's formula

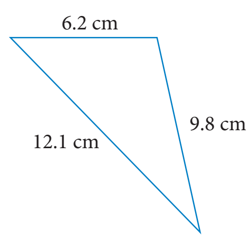
Concepts and techniques

1 Example 1 Calculate the area of the following triangles, correct to two decimal places.

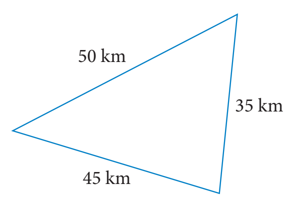
a



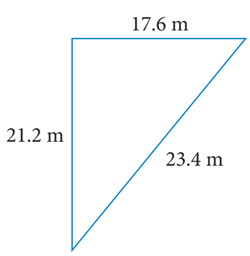
b



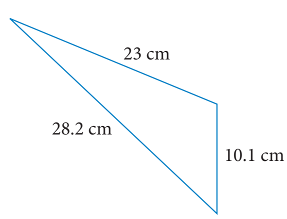
c



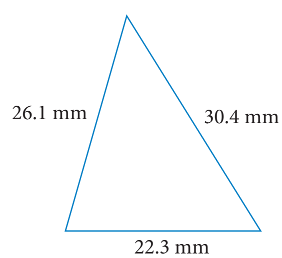
d



e



f



9780170250252

2 In triangle ABC, $AB = 12$ cm, $AC = 7$ cm and $BC = 9$ cm. To calculate the area, which of the following equations would be used?

A $A=\sqrt{14(14-12)(14-7)(14-9)}$

B $A=\sqrt{28(28-12)(28-7)(28-9)}$

C $A=\dfrac{12+7+9}{2}$

D $A=\dfrac{1}{2}\times 12\times 7$

E $A=\dfrac{1}{2}\times 12\times 7\times 9$

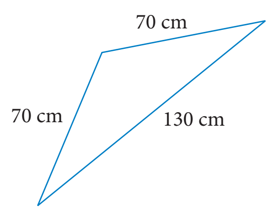
3 Find the area (correct to two decimal places) of an equilateral triangle with a perimeter of 18 cm using:

a Heron's formula

b $A=\dfrac{1}{2}ab\sin(C)$

Reasoning and communication

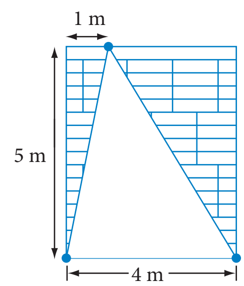
4 Theo is designing a television cabinet to fit in the corner of his lounge room. He needs to know the area of the top of the cabinet so he knows how much paint is required. Calculate the area of the top of the cabinet to the nearest cm^2.



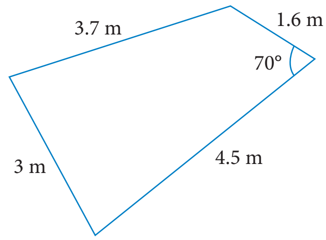
5 A shade sail is to be tied to three posts on a rectangular patio, as shown in the diagram on the right.

a Calculate the dimensions of this shade sail , correct to two decimal places.

b Calculate the area of material required to make the shade sail (to the nearest square metre).



6 Eric has designed a fishpond in the shape of a quadrilateral for a local park, as shown in the diagram below. Calculate the total area of the fishpond, correct to the nearest square metre.



10 CHAPTER SUMMARY
APPLICATIONS OF TRIGONOMETRY

- The **trigonometric ratios** allow us to solve unknown side lengths and unknown angles in right-angled triangles.
- We use the mnemonic (memory aid) **SOH-CAH-TOA** to remember the trigonometric ratios:

Ratio	Meaning	Initials
sine	$\sin(\theta)=\frac{\text{opposite}}{\text{hypotenuse}}$	SOH
cosine	$\cos(\theta)=\frac{\text{adjacent}}{\text{hypotenuse}}$	CAH
tangent	$\tan(\theta)=\frac{\text{opposite}}{\text{adjacent}}$	TOA

- **Angles of elevation** are taken from the horizontal looking up.
- **Angles of depression** are taken from the horizontal looking down.
- There are two types of **bearings.**
  - **True bearings** are taken from north (0°) in a clockwise direction. True bearings will always be between 0° and 360° and are generally written with three digits (e.g. 060°)
  - **Compass bearings** depend on which quadrant the bearing is in. Compass bearings either go from north or south and in an east or west direction. Compass bearings will always be between 0° and 90°. An example of a compass bearing is N 60° E.
- The **sine rule**: $\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$ is used to find an unknown side or angle in a non-right-angled triangle under the following conditions:
  - two sides and a non-included angle are known
  - one side and two angles are known.
- The **cosine rule**: $a^2 = b^2 + c^2 - 2bc\cos(A)$ is used to find an unknown side.
- A variation of the cosine rule,

  $$\cos(A)=\frac{b^2+c^2-a^2}{2bc}$$

  is used to find an unknown angle in a non-right-angled triangle under the following conditions:
  - two sides and an included angle are known
  - three sides are known.
- There are three different formulas which can be used to calculate the **area of a triangle**:
  - $A=\frac{1}{2}bh$
  - $A=\frac{1}{2}ab\sin(C)$
  - $A=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\frac{a+b+c}{2}$ (the semi-perimeter)

9780170250252

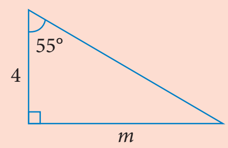
10

CHAPTER REVIEW

APPLICATIONS OF TRIGONOMETRY

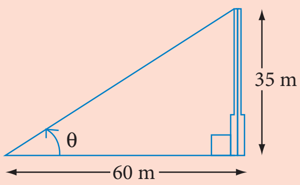
Multiple choice

1 Example 1 The value of the pronumeral, correct to two decimal places, in the diagram on the right is:



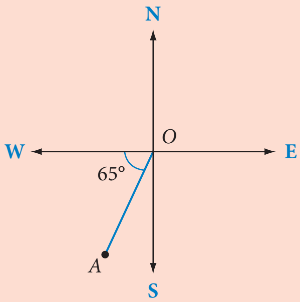
A 2.29 B 2.80 C 3.28
D 5.71 E 6.97

2 Example 6 Kevin stands 60 m from the base of a 35 m tower. The angle of elevation, to the nearest degree, of the tower from Kevin is:



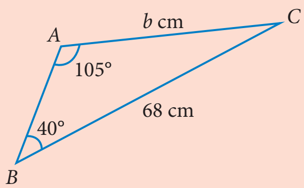
A 30° B 31° C 36°
D 54° E 60°

3 Example 8 Respectively, the true bearing and compass bearing of A from O is:



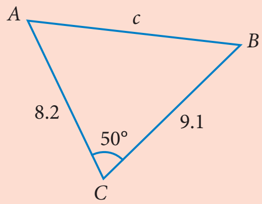
A 215°, S 65° W
B 065°, W 65° S
C 205°, W 65° S
D 215°, S 25° W
E 205°, S 25° W

4 Example 12 The value of b, correct to two decimal places, is:



A 42.22 cm
B 43.63 cm
C 45.25 cm
D 67.88 cm
E 102.18 cm

5 Example 17 Which of the following expressions is equal to c?



A $8.2^2 + 9.1^2$

B $\sqrt{8.2^2 + 9.1^2 - 2 \times 8.2 \times 9.1 \cos(50°)}$

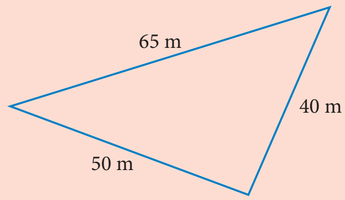
C $8.2^2 + 9.1^2 - 2 \times 8.2 \times 9.1 \cos(50°)$

D $\sqrt{8.2^2 + 9.1^2}$

E $\dfrac{8.2 \sin(50°)}{9.1}$

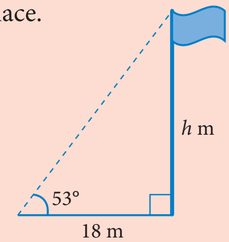
6 Example 25 The area of the triangle (correct to two decimal places) is:

A 77.50 m^2 B 155.00 m^2 C 999.51 m^2
D 1000.00 m^2 E 1300.00 m^2

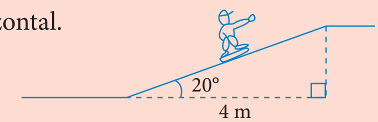


Short answer

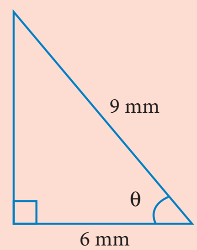
7 Example 3 Find the height, *h* m, of this flagpole correct to one decimal place.



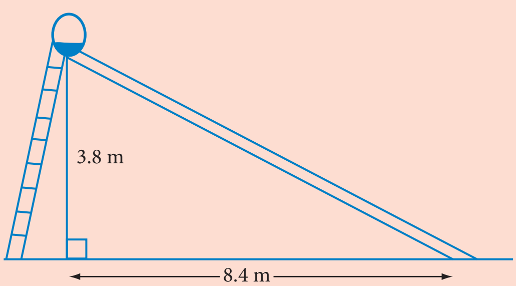
8 Example 2 A skateboard ramp is inclined at 20° to the horizontal. How long is the ramp if it links two levels that are 4 m apart? Give your answer correct to one decimal place.



9 Example 4 Find the unknown angle, correct to the nearest degree.

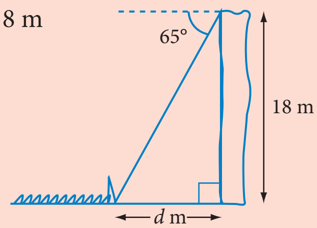


10 Example 5 A pool slide is made up of a ladder and a plastic slide. The top of the plastic slide is 3.8 m from the ground and the bottom of the slide is 8.4 m from a point directly under the top of the slide. To the nearest degree, what angle does the slide make with the ground?



 9780170250252

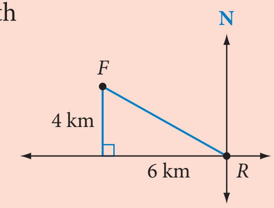
11 Example 7 The angle of depression of a buoy from the top of a 18 m vertical cliff is 65°. To the nearest metre, how far is the buoy from the base of the cliff?



12 Example 10 Flannigan's Primary School, F, is 6 km west and 4 km north of Riverside Mall, R.

a Find the true bearing, correct to the nearest degree, of Flannigan's Primary School from Riverside Mall.

b Find the direct distance between Flannigan's Primary School and Riverside Mall, correct to one decimal place.

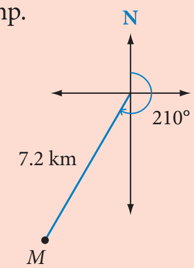


13 Example 11 Max walked for 7.2 km on a true bearing of 210° from base camp. His position is shown by M on the diagram on the right.

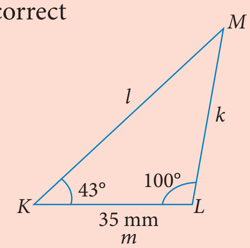
a How far south is he from base camp (correct to one decimal place)?

b How far west is he from base camp (correct to one decimal place)?

c What is the true bearing of base camp from Max?



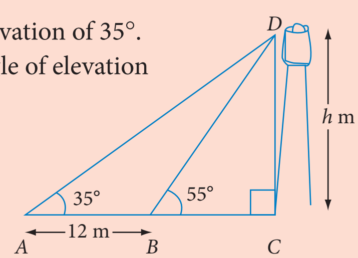
14 Example 13 In $\triangle KLM$, $\angle K = 43°$, $\angle L = 100°$, and $m = 35$ mm. Find k, correct to one decimal place.



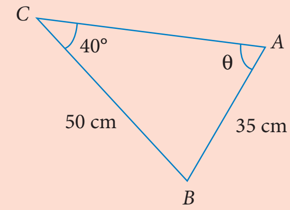
15 Example 14 Monique observes a lighthouse at an angle of elevation of 35°. Walking 12 m towards the lighthouse, she finds that the angle of elevation increases to 55°.

a Calculate the distance between B and D, correct to two decimal places.

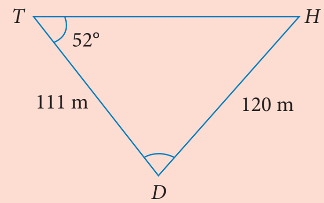
b Hence evaluate the height, h metres, of the lighthouse, correct to two decimal places.



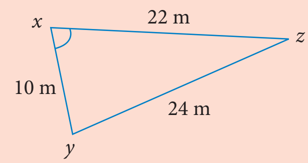
16 Example 15 Find the value of θ, correct to the nearest degree, in the following triangle.



17 Example 16 The 12th hole at a golf course has a tee at T, a hole at H and a dogleg at D. If $TD = 111$ m, $DH = 120$ m and $\angle HTD = 52°$, find $\angle TDH$ correct to the nearest degree.



18 Example 18 Triangle XYZ has $XY = 10$ m, $YZ = 24$ m and $XZ = 22$ m. Find angle X, correct to the nearest degree.

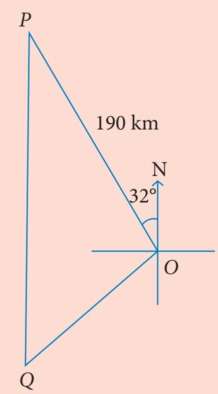


19 Example 19 James, Hannah and Caitlin are playing hide-and-seek. James is 12 m from Hannah and 19 m from Caitlin. The angle formed between Hannah, James and Caitlin is 105°. Calculate the distance between Hannah and Caitlin, correct to the nearest metre.

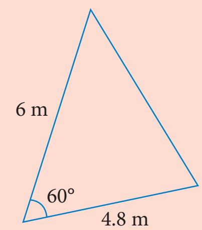
20 Example 20 Two ships, P and Q, leave port O at the same time. Ship P sails on a course N 32° W, while ship Q sails SW. After P sails 190 km, it is due north of Q. Calculate, correct to two decimal places:

a the distance between P and Q.

b how far Q has sailed.

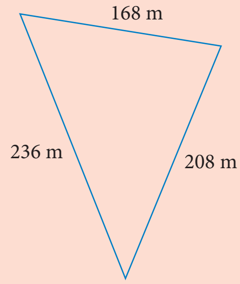


21 Example 24 Calculate the area of the following triangle using $A = \frac{1}{2}ab\sin(C)$, correct to two decimal places.



9780170250252

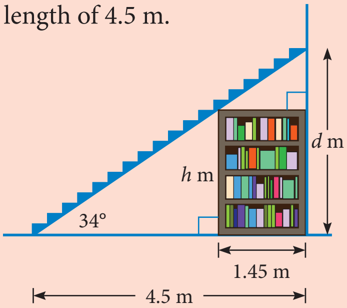
22 Example 27 Michelle has measured the dimensions of a triangular dam on her property, as shown in the diagram on the right. Calculate the area of the dam, correct to the nearest square metre.



Application

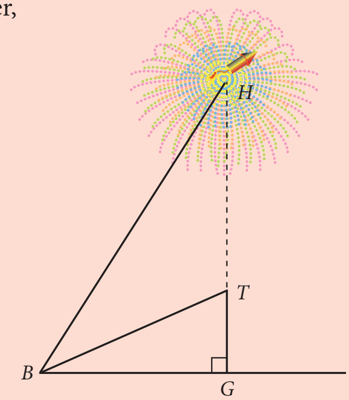
23 A staircase is inclined at an angle of 34° and has a horizontal length of 4.5 m. A bookcase, 1.45 m long, is placed under the stairs. Find, correct to the nearest metre,

- a the vertical height, h, of the tallest bookcase that will fit under the stairs
- b the distance, d, of the top step of the staircase from the ground
- c the length from the bottom of the staircase to the top.



24 Boun (B) sees fireworks being launched from the top of a tower, TG, at an angle of elevation of 38°. When the fireworks are at their highest point, H, 800 m above the tower, the angle of elevation is 80°.

- a Copy the diagram and mark on it all the information given.
- b Show that $\angle BTH = 128°$.
- c Calculate the distance between Boun and the fireworks at H, correct to the nearest metre.
- d Calculate the distance between Boun and the top, T, of the tower, correct to the nearest metre.
- e Calculate the height of the tower, correct to the nearest metre.



Practice quiz