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| **MATHEMATICS DEPARTMENT**  **Year 11 Specialist – 2016**  **Test Number 5: Resource Rich**  **Real Numbers and Proof and Matrix Arithmetic** |

**Name: \_\_\_\_\_\_\_Solutions\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Teacher: DDA**

**Marks: 44**

**Time Allowed: 45 minutes**

**Weight: 7%  
Instructions:** You are permitted 1 A4 page of notes and your calculators. Show your working where appropriate remembering you must show working for questions worth more than 2 marks. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Part A

7 multiple-choice questions

1 mark each: 7 marks

Circle the correct answer.

1 , where a, b ∈ **Ζ** cannot be:

A an irrational number

B an integer

C a transcendental number

D an odd number

E a rational number

[1 mark]

2 28 − 1 is:

A a perfect number.

B a Mersenne prime.

C a deficient number.

D an even number.

E   an abundant number.

[1 mark]

3 If n is divisible 3, then a counterexample to show that n2 - 1 is divisible by 4 or 5 or both is a false statement is when n is equal to:

A   3

B   6

C  9

D 12

E  15

[1 mark]

4 To prove a statement A, assume A is false. Then show that  ⇒ B, where B and  cannot both be true. Hence  is false so A is true. This is the method used for which one of the following proofs?

A direct

B mathematical induction

C contraposition

D counterexample

E contradiction

[1 mark]

5 If A is a 3 × 4 matrix and B is a 2 × 3 matrix, then the number of elements in matrix BA would be:

A  2

B  3

C  4

D 8

E  12

[1 mark]

6 If A =  and B =  which one of the following could not be found?

A    AB

B   A + B

C  B2

D  A−1

E − B

[1 mark]

7 If  then the elements m and n, respectively, will have the values:

A  2, 

B   2, 

C  8, 6

D , 2

E  1, 4

[1 mark]

Part B

8 short answer questions

27 marks

Show your working where appropriate.

8 State whether the following sets are closed under addition. If they are not closed give an example to show that they are not.

a {−2, 0, 2}

Not closed. Example: 2+2 = 4

b R

Closed.

[3 marks]

9 Convert  to a fraction in the form , a, b ∈ **Ζ**+ where a and b have no common factors other than 1.

Let

[3 marks]

10 Use a proof by contraposition to show that if 5n + 1 is odd, then n is even, where n ∈ **Ζ**.

RTP: 5n+1 odd n is even, n

Proof: Consider the contrapositive statement: n is odd, n5n+1 even [1 mark]

If n is odd, n

Then n = 2k + 1 for some k ∈ Z.

5n + 1 = 5(2k + 1) + 1

= 10k + 5 + 1 [2 marks]

= 10k + 6

= 2(5k + 3), which is even, so 5n + 1 is even.

Therefore the contraposition statement is true.

Hence by contrapositon if 5n + 1 is odd, then n is even, where n ∈ Z. QED

[1 mark]

11 Show by mathematical induction that: ,   
where n ∈ **Z+**.

For n =1, LHS = ==

RHS =  = so the proposition is true for n = 1.

[1 mark]

Assume it is true for n = k. Then 

[1 mark]

So 

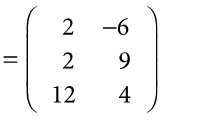
[1 mark]

so  [1 mark]

Thus the proposition is true for n = k + 1.

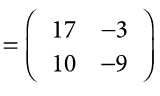
By the principle of mathematical induction  for   
 n ∈ N. QED [1 mark]

12 Find each of the following:

a 



b 



c 

[3 marks]

13 A = . Work out the inverse of A.

Using M−1 = 

 [1 mark]

so the inverse is  [2 marks]

[3 marks]

14 Using A =  and B = , find C such that 2A−1 + C = B.



so  [1 mark]

2A−1 + C = B

so C = B − 2A−1 [1 mark]

= 

= 

=  [1 mark] [3 marks altogether]

15 Using A =  and B = , find matrix D such that BD = A.



so [1 mark]

BD = A

so B−1BD = B−1A

so D =  [1 mark]

= 

=  [1 mark] [3 marks altogether]

Part C

2 analysis questions

10 marks

Show your working where appropriate.

16 An airline company has a fleet of 5 planes of type A, 8 planes of type B, 4 planes of type C and 10 planes   
of type D.

Each plane type has three seat categories: Economy, Business Class and First Class. The table below shows the number of seats in each seating category for these planes.

|  |  |  |  |
| --- | --- | --- | --- |
| Class | Economy | Business | First |
| A | 300 | 60 | 40 |
| B | 150 | 50 | 20 |
| C | 120 | 40 | 0 |
| D | 100 | 0 | 0 |

a  Write down two matrices whose product shows the total number of seats in each class.

 [1 mark] and  [1 mark]

b Find the number seats in each class.

  =  [1 mark]

Economy 4180, Business 860, First 360 [1 mark]

On a particular day each aircraft made a single flight, 10% of the Economy seats were vacant, 20% of the Business seats were vacant and 30% of First class seats were vacant.

c  Write down a matrix whose product with the matrix in part b can be used to find the total number of vacant seats on that day, and find this number.

 [1 mark]



There were 698 vacant seats.

There were 115 vacant seats on this particular day. [1 mark]

17 Using the idea of a multiplicative inverse of a matrix, and clearly showing that use, solve this system of equations.



[4 marks]



End of questions.