



# Year 11 Specialist Mathematics

MELVILLE  
SENIOR HIGH SCHOOL

Semester 1, March 2021

## Test 2: Vectors

Weighting: 7%

[Australian Curriculum Reference Numbers: 1.2.1-1.2.9, 1.2.12, 1.2.14]

Total Time: 50min

Total Marks = 50 52

Student Name: SOLUTIONS

Teacher: \_\_\_\_\_

### TO BE PROVIDED BY THE STUDENT

Standard Items: Pens, pencils, eraser, sharpener, correction tape/fluid, highlighters, ruler.

Special Items:

- Drawing instruments, templates
- A maximum of three CAS calculators satisfying the conditions set by the Curriculum Council  
*for use in the Calculator Allowed section only*

### TO BE PROVIDED TO THE STUDENT

- A formula sheet will be provided

### INSTRUCTIONS TO STUDENTS:

You are required to attempt ALL questions.

Write answers in the spaces provided beneath each question.

Marks are shown with the questions.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.

It is recommended that students do not use a pencil, except in diagrams

**Part A – Calculator Free (25 minutes)** /25

**Part B – Calculator Assumed (25 minutes)** /~~25~~ 27

**Final mark** /~~50~~ 52

**Question 1**

Two forces are given by  $\mathbf{F}_1 = -3\mathbf{i} + 5\mathbf{j}$  N and  $\mathbf{F}_2 = 2\mathbf{i} - \mathbf{j}$  N. Determine

a)  $\mathbf{F}_1 - \mathbf{F}_2$ .

(1 mark)

$$\begin{pmatrix} -3 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 \\ 6 \end{pmatrix} \checkmark$$

b)  $5\mathbf{F}_1 + 10\mathbf{F}_2$ .

(2 marks)

$$= \begin{pmatrix} -15 \\ 25 \end{pmatrix} + \begin{pmatrix} 20 \\ -10 \end{pmatrix} = \begin{pmatrix} 5 \\ 15 \end{pmatrix} \checkmark$$

c)  $|\mathbf{F}_1|$ .

$$= \sqrt{(-3)^2 + (5)^2} = \sqrt{9+25} = \sqrt{34} \checkmark$$

(2 marks)

**Question 2**

4 marks

Consider the vectors  $\mathbf{p} = \begin{pmatrix} -7 \\ 8 \end{pmatrix}$ ,  $\mathbf{q} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$  and  $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ .

Given that  $\mathbf{p} = \lambda\mathbf{q} + \mu\mathbf{r}$ , determine the value of  $\lambda$  and the value of  $\mu$ .

$$\begin{pmatrix} -7 \\ 8 \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \end{pmatrix} \checkmark$$

$$\therefore 3\lambda + \mu = -7 \quad ① \checkmark$$

$$-4\lambda - 2\mu = 8 \quad ② \checkmark$$

$$① \times 2: \cancel{-6\lambda} \quad 6\lambda + 2\mu = -14 \quad ③ \checkmark$$

$$\begin{matrix} ② + ③ \\ \hline \end{matrix}$$

$$\Rightarrow 2\lambda = -6$$

$$\lambda = -3$$

$$\lambda = -3 \checkmark$$

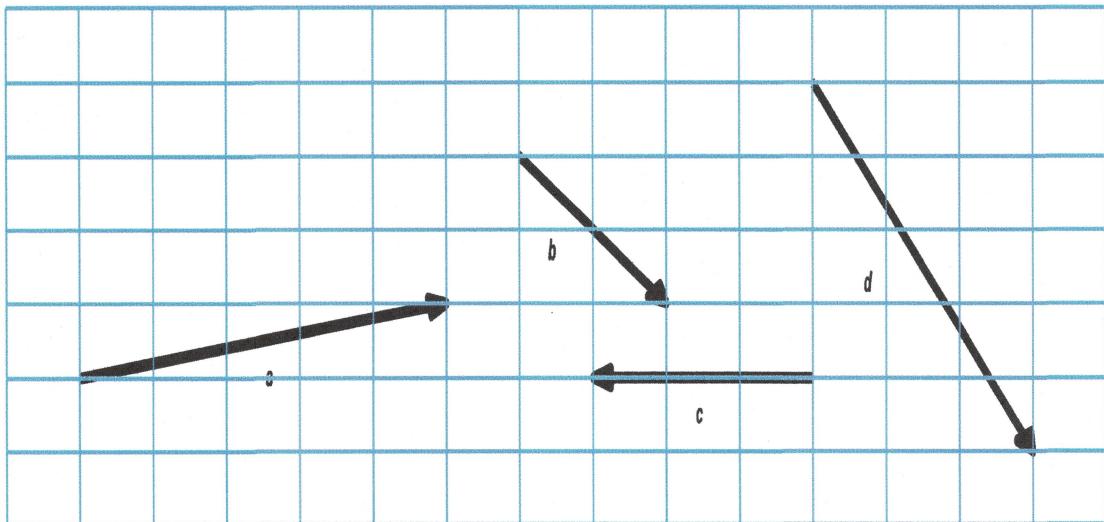
$$\text{sub into } ① \quad 3(-3) + \mu = 7$$

$$\mu = 2 \checkmark$$

$$\mu = 2$$

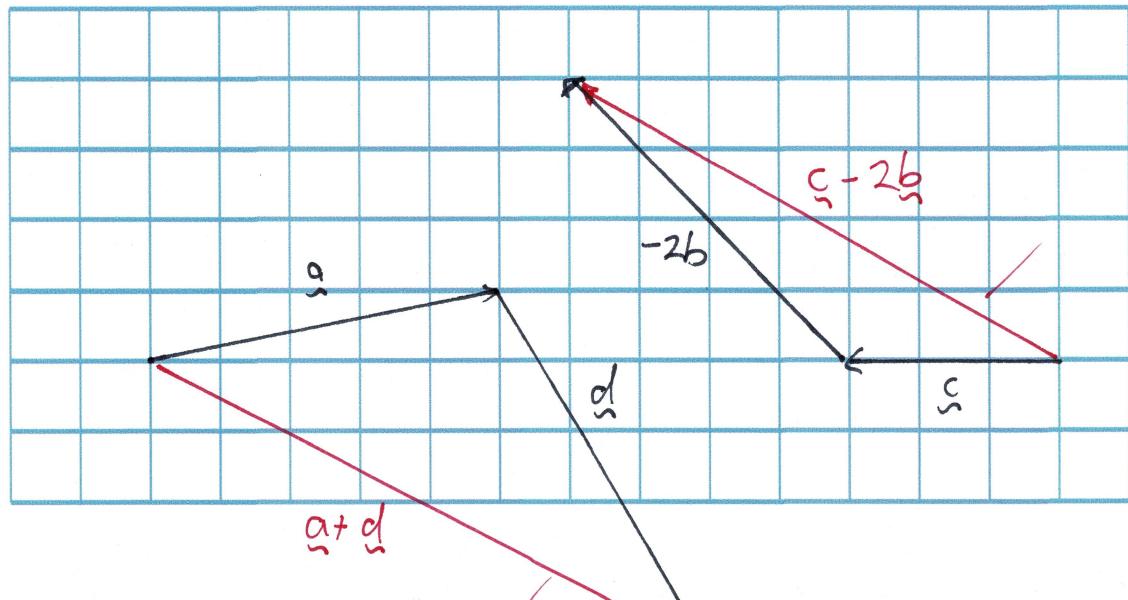
**Question 3**

Use the vectors below and the grid provided to draw each of the following vectors:



a)  $\underline{a} + \underline{d}$

b)  $\underline{c} - 2\underline{b}$

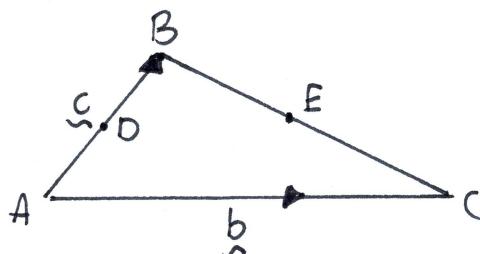


Must have arrows

## Question 4

In triangle  $ABC$ ,  $\overrightarrow{AB} = c$ ,  $\overrightarrow{AC} = b$ .  $D$  is the midpoint of  $AB$  and  $E$  is the midpoint of  $CB$ .

- a) Draw a diagram to illustrate all the given information. (2 marks)



/ triangle with vectors  
/ midpoints  
-1 neatness

- b) Express each of the following in terms of  $b$  and/or  $c$ . (4 marks)

(i)  $\overrightarrow{AD}$  (1)

$$\frac{1}{2} \underline{c} /$$

(ii)  $\overrightarrow{CD}$  (1)

$$= -\underline{b} + \frac{1}{2} \underline{c} /$$

(iii)  $\overrightarrow{AE}$  (2)

$$\begin{aligned} & \underline{c} + \frac{1}{2} \underline{BC} \\ &= \underline{c} + \frac{1}{2} (-\underline{c} + \underline{b}) \\ &= \frac{1}{2} \underline{c} + \frac{1}{2} \underline{b} / \end{aligned}$$

$$\begin{aligned} \overline{BC} &= \overline{BA} + \overline{AB} \\ &= -\underline{c} + \underline{b} / \end{aligned}$$

**Question 5**

Given  $\mathbf{a} = 12\mathbf{i} + 5\mathbf{j}$ ,  $\mathbf{b} = (x+1)\mathbf{i} + 2\mathbf{j}$ ,  $\mathbf{c} = -5\mathbf{i} + (y-3)\mathbf{j}$ . Find:

- a) The value of  $x$  if  $\mathbf{a} + \mathbf{b} = 7\mathbf{i} + 7\mathbf{j}$  (1 mark)

$$12 + (x+1) = 7$$

$$x = -6$$

- c) The value(s) of  $y$  if  $\mathbf{a}$  and  $\mathbf{c}$  have the same magnitude. (4 marks)

$$|\mathbf{a}| = 13$$

$$\therefore \sqrt{(-5)^2 + (y-3)^2} = 13$$

$$25 + y^2 - 6y + 9 = 169$$

$$y^2 - 6y - 135 = 0$$

$$(y+9)(y-15) = 0$$

$$y = -9 \text{ or } 15$$

$$169 - 9 = 160$$

$$160 - 25 = 135$$

OR  
 $\mathbf{a} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$      $\mathbf{c} = \begin{pmatrix} -5 \\ y-3 \end{pmatrix}$

$\therefore y-3$  must equal  $-12$  or  $12$  to have same magnitude.

$$\text{ie. } y = -9 \text{ or } 15$$

- d) A unit vector (in component form) and in the opposite direction for  $\mathbf{a}$ . (3 marks)

$$|\mathbf{a}| = 13$$

$$\therefore -\hat{\mathbf{a}} = \frac{1}{13} \begin{pmatrix} -12 \\ -5 \end{pmatrix}$$

$$= -\frac{12}{13} \mathbf{i} - \frac{5}{13} \mathbf{j}$$

**End of Section One**

**Additional working space**

Question \_\_\_\_\_



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## Part B: Calculator Assumed Section

Time Allowed: 25 minutes

[Australian Curriculum Reference Numbers: 1.2.1-1.2.9, 1.2.12, 1.2.14]

Marks =

25 / 27

Student Name:

SOLUTIONS

Teacher:

### INSTRUCTIONS TO STUDENTS:

- You are allowed a CAS calculator
- You are not allowed any notes
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1. Given  $\mathbf{a} = (5, 125^\circ)$ ,  $\mathbf{b} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$  and  $\mathbf{c} = 6\mathbf{i} - \mathbf{j}$ , determine:  
 (Answers 2 decimal places)

a)  $2\mathbf{a} - 3\mathbf{b}$ , in component form.

$$\begin{bmatrix} 0.26 \\ 26.19 \end{bmatrix} = 0.26\mathbf{i} + 26.19\mathbf{j} \quad /$$

b)  $4(\mathbf{c} - 2\mathbf{a})$ , in polar form, with a positive angle.

$$= \begin{pmatrix} 59.63 \\ -38.07^\circ \end{pmatrix} = \begin{bmatrix} 59.63 \\ <321.93^\circ \end{bmatrix} \quad /$$

c) Vector  $\mathbf{d}$  that has a magnitude of 5 and is in the same direction of  $\mathbf{a} + \mathbf{b} + \mathbf{c}$ , in terms of  $\mathbf{i}$  and  $\mathbf{j}$ .

$$\underline{\mathbf{a} + \mathbf{b} + \mathbf{c}} = 1.13\mathbf{i} - 2.90\mathbf{j} \quad /$$

$$\text{unit vector}(\underline{\mathbf{a} + \mathbf{b} + \mathbf{c}}) = 0.36\mathbf{i} - 0.93\mathbf{j} \quad /$$

$$\underline{\mathbf{d}} = 5(0.36\mathbf{i} - 0.93\mathbf{j})$$

$$= 1.82\mathbf{i} - 4.66\mathbf{j} \quad /$$

[1, 2, 3 = 6 Marks]

use exact values to

Calculator Allowed (25 minutes)

2. If  $\mathbf{a} = -6\mathbf{i} + 9\mathbf{j}$  and  $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j}$ , show that the vector in the direction of  $\mathbf{a}$  that has one third the magnitude of  $\mathbf{b}$  is  $-\frac{2}{3}\mathbf{i} + \mathbf{j}$ .

$$|\mathbf{a}| = \sqrt{117} = 3\sqrt{13}$$

$$\hat{\mathbf{a}} = \frac{1}{3\sqrt{13}} \begin{pmatrix} -6 \\ 9 \end{pmatrix} = \begin{pmatrix} -2 \\ \sqrt{13} \\ 3 \end{pmatrix}$$

$$|\mathbf{b}| = \sqrt{13}$$

$$\therefore \text{magnitude} = \frac{\sqrt{13}}{3}$$

$$\text{Vector} = \frac{\cancel{\sqrt{13}}}{3} \begin{pmatrix} -2 \\ \cancel{\sqrt{13}} \\ 3 \\ \cancel{\sqrt{13}} \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} \\ 1 \\ \frac{3}{3} \end{pmatrix} = -\frac{2}{3}\mathbf{i} + \mathbf{j}$$

5  
[★ marks]

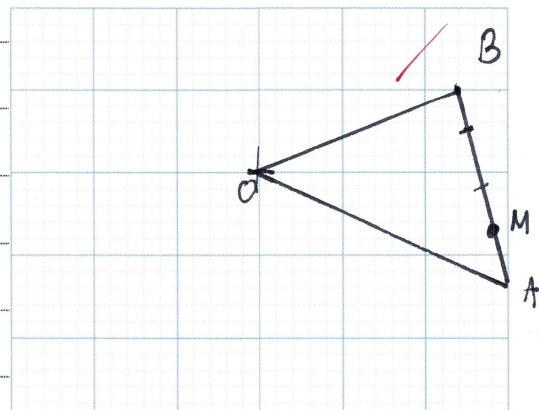
3. Points A and B have position vectors  $|5\mathbf{i} - 7\mathbf{j}$  and  $|2\mathbf{i} + 5\mathbf{j}$  respectively. Find the position vector of the point that divides AB internally in the ratio 1:3.

$$\overline{OM} = \overline{OA} + \frac{1}{4} \overline{AB}$$

$$\overline{AB} = \overline{AO} + \overline{OB} = \begin{pmatrix} -15 \\ 7 \end{pmatrix} + \begin{pmatrix} 12 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 12 \end{pmatrix}$$

$$\therefore \overline{OM} = \begin{pmatrix} 15 \\ -7 \end{pmatrix} + \frac{1}{4} \begin{pmatrix} -3 \\ 12 \end{pmatrix}$$

$$= \begin{pmatrix} 15 \\ -7 \end{pmatrix} + \begin{pmatrix} -\frac{3}{4} \\ 3 \end{pmatrix} = \begin{pmatrix} 14.25 \\ -4 \end{pmatrix}$$



\* Various answers - mark accordingly \*

[5 marks]

4. A plane can maintain a speed of 280 km/h in still air. A wind is blowing at 42 km/h from a bearing of 160°.

- a) If the plane flies on a bearing of 036° and is then blown off course by the wind, in what direction (to the nearest degree) does the plane now fly and how far will it travel in 6.5 hours?

$$|r| = \sqrt{280^2 + 42^2 - 2(280)(42) \cos 124}$$

$$|r| = 305.477 \text{ km/h} \checkmark$$

$$\text{Distance} = 6.5 \times 305.477$$

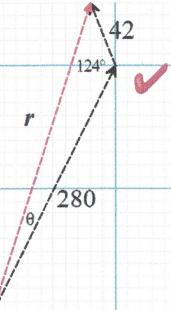
$$\text{Distance} = 1985.6 \text{ km} \checkmark$$

$$\frac{\sin \theta}{42} = \frac{\sin 124}{305.477}$$

$$\theta = 6.55^\circ \checkmark$$

$$\therefore \text{Bearing of } 029^\circ \checkmark$$

*The plane will travel 1985.6 km on a bearing of 029°*



- b) If the plane wishes to fly on a bearing of 220° to an airport 420km away, how long will the journey take (to the nearest minute) and in what direction should the aircraft be pointing, to the nearest degree?

$$\frac{\sin \alpha}{42} = \frac{\sin 120}{280} \checkmark$$

$$\alpha = 7.464 \checkmark$$

$$\therefore \theta = 52.536 \checkmark$$

$$\therefore \text{Bearing of } 180 + 32.536^\circ = 213^\circ \checkmark$$

$$|r| = \sqrt{280^2 + 42^2 - 2(280)(42) \cos 52.536}$$

$$r = 256.63 \text{ km/h} \checkmark$$

$$t = \frac{\text{distance}}{\text{speed}} = 1.6366 \text{ hours} = 1\text{hr } 38\text{min} \checkmark$$

*The trip will take 1hr 38 min on a bearing of 213°*

[5, 6 = 11 marks]

**Calculator Allowed (25 minutes)**

\*\*\*Extra space for working out\*\*\*