BOOK OVERVIEW

Year 11 Maths Methods for the Australian Curriculum

Chapter 1: Equations and polynomials		
For Teachers	**For Students**	
Nominal time: 3 weeks	**You should know:**	**You should be able to:**
Curriculum topics and statements **Functions and graphs** Lines and linear relationships • examine examples of direct proportion and linearly related variables (ACMMM002) (see sections 1.03, 1.04, 1.09) • recognise features of the graph of $y = mx + c$, including its linear nature, its intercepts and its slope or gradient (ACMMM003) (see section 1.04) • solve linear equations (ACMMM005) (see sections 1.03, 1.05, 1.06) Review of quadratic relationships • examine examples of quadratically related variables (ACMMM006) (see sections 1.01, 1.02, 1.07) • solve quadratic equations using the quadratic formula and by completing the square (ACMMM008) (see sections 1.02, 1.07, 1.08) • find the equation of a quadratic given sufficient information (ACMMM009) (see section 1.06) • find turning points and zeros of quadratics and understand the role of the discriminant (ACMMM010) (see section 1.08) Inverse proportion • examine examples of inverse proportion (ACMMM012) (see section 1.09)	• The meanings of the terms *binomial, coefficient, constant, cubic, direct proportion, discriminant, factor theorem, formula, highest common factor, inverse proportion, leading term, linear, monic, null factor law, polynomial, quadratic, quartic, relationship, remainder theorem, root, simultaneous equations, subject, trinomial* and *variable* • The meanings of the inequality symbols $<, >, \leq, \geq$, the proportion symbol $\propto$ and the discriminant symbol Δ • The distributive law $a(b + c) = ab + ac$ • The perfect square expansions $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$ • The difference of two squares $(a + b)(a - b) = a^2 - b^2$ • The standard quadratic expression $ax^2 + bx + c$ • The slope–intercept form of a straight line $y = mx + c$ • The equation $y = kx$ of simple direct proportion • The equation $y = \frac{k}{x}$ of simple inverse proportion • The null factor law: if $ab = 0$ then $a = 0$, $b = 0$ or $a = b = 0$ • The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	• Identify linear, quadratic and cubic equations • Simplify expressions using like terms • Expand brackets including binomial and cubic expressions • Factorise expressions using highest common factors or grouping and standard expressions • Factorise trinomial expressions using at least one of the cross, decomposition or fraction methods • Solve linear equations • Solve direct proportion problems • Solve inverse proportion problems • Recognise the graphs of variables in direct or inverse proportion • Transpose a formula to make a particular variable the subject • Use simple formulas • Solve simultaneous equations in two unknowns using graphical, elimination or substitution methods • Solve quadratic equations using factorisation, completion of the square or the quadratic formula as appropriate • Use the discriminant to determine the nature of the roots of a quadratic equation • Identify the leading term, coefficients and degree of a polynomial

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<table>
<tr><th colspan="3">Chapter 1: Equations and polynomials</th></tr>
<tr><th>For Teachers</th><th colspan="2">For Students</th></tr>
<tr><th>Nominal time: 3 weeks</th><th>You should know:</th><th>You should be able to:</th></tr>
<tr><td>Powers and polynomials
• recognise features of the graphs of $y = x^n$ for $n \in \mathbf{N}$, $n = -1$ and $n = \frac{1}{2}$, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM014) AC (see section 1.04)
• identify the coefficients and the degree of a polynomial (ACMMM015) AC (see sections 1.10–1.12)
• expand quadratic and cubic polynomials from factors (ACMMM016) AC (see sections 1.01, 1.10–1.12)
• factorise cubic polynomials in cases where a linear factor is easily obtained (ACMMM018) AC (see section 1.11)
• solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained (ACMMM019) AC (see section 1.12)
Functions
• use function notation, domain and range, independent and dependent variables (ACMMM023) AC (see section 1.04)
• understand the concept of the graph of a function (ACMMM024) AC (see section 1.04, 1.05)</td><td>• The discriminant $\Delta = b^2 - 4ac$
• The standard form of a polynomial
• The remainder theorem for real polynomials: if $P(x)$ is divided by $x - a$ then the remainder is $P(a)$
• The factor theorem for real polynomials: $x - a$ is a factor of $p(x)$ if and only if $P(a) = 0$
• The perfect cube expansions $(a + b)^3 = a^3 + 3a^2 + 3ab^2 + b^3$ and $(a - b)^3 = a^3 - 3a^2 + 3ab^2 - b^3$</td><td>• Use the factor theorem to factorise simple polynomial expressions
• Solve simple polynomial equations
• Recognise and use perfect cube expansions
• Apply reasoning skills in algebra
• Interpret algebraic solutions to problems and comment on the reasonableness of solutions
• Express arguments, logic and strategies used in solving problems using algebra in written or verbal form</td></tr>
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Chapter 2: Sets and basic probability		
For Teachers	**For Students**	
Nominal time: 2 weeks	**You should know:**	**You should be able to:**
Curriculum topic and statements	• The meanings of the terms *binomial coefficient, binomial expansion, binomial theorem, cardinality, certain, combination, complement, disjoint, element, elementary outcomes, event, expected frequency, impossible, intersection, mutually exclusive, null set, Pascal's triangle, probability, proper subset, random, relative frequency, sample space, set, subset, trial, uniform probability, distribution, union, universal set, Venn diagram, well-defined* • The meanings of the symbols $\{\}$, $\varnothing$, $\cup$, $\in$, $\notin$, :, $\subset$,$\subseteq$,$\cup$,$\cap$,A', $\overline{A}$, $\lvert A \rvert$, $n(A)$, $\boldsymbol{N}$, $\boldsymbol{Z}$, $\boldsymbol{J}$, $\boldsymbol{Q}$, $\boldsymbol{R}$, $\boldsymbol{Z}^+$, $\boldsymbol{W}$, $\boldsymbol{R}\backslash 0$, $\boldsymbol{R}\backslash\{0\}$, !, $\begin{pmatrix} n \\ r \end{pmatrix}$, ${}^nC_r$, S • The meaning of equal sets • The binomial expansion of $(x = y)^n$ • The formula for the number combinations of r objects from n distinct objects: $\begin{pmatrix} n \\ r \end{pmatrix} = {}^nC_r = \frac{n!}{r!(n-r)!}$ $= \frac{n \times (n-1)(n-2)\ldots(n-r+1)}{r!}$ • The definition of the probability $P(A)$ of an event A in a subset of a sample space S as a number such that: $0 \leq P(A)$, $P(S) = 1$, $P(\varnothing) = 0$ and the probability of a union of disjoint events is the sum of the probabilities of the events • The formula $P(E) = \frac{n(E)}{n(S)}$ for the probability in a finite sample space of a uniform probability distribution • The formula $P(\overline{A}) = 1 - P(A)$	• Recognise and use set notation • Recognise whether sets are mutually exclusive or intersecting • Recognise and construct subsets or proper subsets of a given set • Find the intersections, unions and complements of given sets • Find the cardinality of a finite set • Construct and use Venn diagrams • Construct and use sample spaces • Use the binomial theorem to expand expressions or find individual terms • Use the formula for $\begin{pmatrix} n \\ r \end{pmatrix}$ to find the coefficients of terms in the binomial expansion of $(x = y)^n$ • Find the probabilities of simple events in finite sample spaces • Use the formula for $\begin{pmatrix} n \\ r \end{pmatrix}$ to find probabilities • Find expected frequencies • Find relative frequencies • Use relative frequencies as estimates of probability • Use the formula for the probability of a union of sets to find probabilities • Use the formula for the probability of the union or intersection of mutually exclusive sets to find probabilities • Solve simple probability problems • Apply reasoning skills in probability • Interpret solutions to probability problems and comment on the reasonableness of solutions
Counting and probability Combinations • understand the notion of a combination as an ordered set of r objects taken from a set of n distinct objects (ACMMM044) (see section 2.03) • use the notation $\begin{pmatrix} n \\ r \end{pmatrix}$ and the formula $\begin{pmatrix} n \\ r \end{pmatrix} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects (ACMMM045) (see section 2.03) • expand $(x + y)^n$ for small positive integers n (ACMMM046) (see section 2.02) • recognise the numbers $\begin{pmatrix} n \\ r \end{pmatrix}$ as binomial coefficients, (as coefficients in the expansion of $(x + y)^n$) (ACMMM047) (see section 2.02) • use Pascal's triangle and its properties (ACMMM048) (see section 2.02) Language of events and sets • review the concepts and language of outcomes, sample spaces and events as sets of outcomes (ACMMM049) (see section 2.04) • use set language and notation for events, including $\overline{A}$ (or A') for the complement of an event A, $A \cap B$ for the intersection of events A and B, and $A \cup B$ for the union, and recognise mutually exclusive events (ACMMM050) (see sections 2.01, 2.04) • use everyday occurrences to illustrate set descriptions and representations of events, and set operations (ACMMM051) (see sections 2.01, 2.04)		

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Chapter 2: Sets and basic probability		
For Teachers	**For Students**	
Nominal time: 2 weeks	**You should know:**	**You should be able to:**
Review of the fundamentals of probability • review probability as a measure of 'the likelihood of occurrence' of an event (ACMMM052) (see section 2.05) • review the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty (ACMMM053) (see sections 2.05–2.07) • review the rules: $P(\bar{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (ACMMM054) (see sections 2.06–2.08) • use relative frequencies obtained from data as point estimates of probabilities (ACMMM055) (see section 2.08)	• The formula for expected frequency of an event A in n trials: $P(A) \times n$ • The formula for relative frequency: $\text{relative frequency} = \dfrac{\text{frequency of event}}{\text{total frequency}}$ • The formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for the probabilities of combined events • The formulas $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$ for the probabilities of mutually exclusive events	• Express arguments, logic and strategies used in solving probability problems in written or verbal form

Chapter 3: Trigonometry		
For Teachers	**For Students**	
Nominal time: 4 weeks	**You should know:**	**You should be able to:**
Curriculum topic and statements **Trigonometric functions** Cosine and sine rules • review sine, cosine and tangent as ratios of side lengths in right-angled triangles (ACMMM028) AC (see section 3.01) • understand the unit circle definition of cos (θ), sin (θ) and tan (θ) and periodicity using degrees (ACMMM029) AC (see section 3.02) • examine the relationship between the angle of inclination of a line and the gradient of that line (ACMMM030) AC (see section 3.06) • establish and use the sine and cosine rules and the formula $Area = \frac{1}{2}bc\sin(A)$ for the area of a triangle (ACMMM031) AC (see sections 3.03, 3.05, 3.06) Circular measure and radian measure • define and use radian measure and understand its relationship with degree measure (ACMMM032) AC (see sections 3.07, 3.08) • calculate lengths of arcs and areas of sectors in circles (ACMMM033) AC (see section 3.08) • recognise the exact values of sin (θ), cos (θ) and tan (θ) at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (ACMMM035) AC (see sections 3.02, 3.07)	• The meanings of the terms *adjacent, angle of depression, angle of elevation, arc, bearings, circular measure, cosine, cosine rule, hypotenuse, included angle, opposite, radian, sector, segment, sine, sine rule, tangent* • The meanings of the angle symbols °, ′ and ″ • The conversions 60′ = 1° and 60″ = 1° • The trigonometric ratios in terms of the sides of a right-angled triangle: $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$ and $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$ • The exact trigonometric ratios of 0°, 30°, 45°, 60° and 90° • The definitions of trigonometric ratios of any magnitude in terms of the unit circle: $\sin(\theta) = y$, $\cos(\theta) = x$ and $\tan(\theta) = \frac{y}{x}$ • The ASTC (CAST) diagram for the signs of angles of any magnitude • The formula for the area of a triangle : $A = \frac{1}{2}ab\sin(C)$ • The sine rule $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ or $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$ • The cosine rule $c^2 = a^2 + b^2 - 2ab\cos(C)$ or $\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$ • The definition of one radian as the angle at the centre of a circle subtended by an arc of length equal to the radius of the circle • The conversion between radians and degrees: $1^c = 180°$ • The formula for the length of an arc: $l = r\theta$ • The formula for the area of a sector: $A = \frac{1}{2}r^2\theta$	• Convert angles between degrees, minutes and seconds and degrees • Use trigonometric ratios to find unknown sides or angles in right-angled triangles • Convert between bearings expressed in terms of compass points and three-figure bearings • Solve problems involving angles of depression or elevation and bearings using right-angled triangles • Use exact ratios to find exact solutions of right-angled triangles where appropriate • Use the ASTC (CAST) diagram to find the signs and values of trigonometric ratios of angles of any magnitude in terms of the values of trigonometric ratios of acute angles • Find the exact values of trigonometric ratios of angles of any magnitude where appropriate • Use the formula to find the area of triangle given two sides and the enclosed angle • Use the sine rule to find unknown sides or angles in triangles where appropriate • Use the cosine rule to find unknown sides or angles in triangles where appropriate • Solve problems involving right-angled triangles using appropriate methods • Convert angles between degrees and radians • Find the lengths of arc and the areas of sectors and segments of circles • Apply reasoning skills in trigonometric problems • Interpret solutions to trigonometric problems and comment on the reasonableness of solutions • Express arguments, logic and strategies used in solving trigonometric problems in written or verbal form

Chapter 4: Functions and graphs		
For Teachers	**For Students**	
Nominal time: 3 weeks	**You should know:**	**You should be able to:**
Curriculum topic and statements **Functions and graphs** Lines and linear relationships • determine the coordinates of the midpoint of two points (ACMMM001) (see section 4.03) • recognise features of the graph of $y = mx + c$, including its linear nature, its intercepts and its slope or gradient (ACMMM003) (see sections 4.03, 4.04) • find the equation of a straight line given sufficient information; parallel and perpendicular lines (ACMMM004) (see section 4.04) Review of quadratic relationships • examine examples of quadratically related variables (ACMMM006) (see section 4.05) • recognise features of the graphs of $y = x^2$, $y = a(x - b)^2$ and $y = a(x - b)(x - c)$, including their parabolic nature, turning points, axes of symmetry and intercepts (ACMMM007) (see sections 4.05, 4.07) • find turning points and zeros of quadratics and understand the role of the discriminant (ACMMM010) (see sections 4.05, 4.06) • recognise features of the graph of the general quadratic $y = ax^2 + bx + c$ (ACMMM011) (see sections 4.05, 4.06) Inverse proportion • examine examples of inverse proportion (ACMMM012) (see section 4.07) • recognise features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x - b}$, including their hyperbolic shapes, and their asymptotes (ACMMM013) (see section 4.07)	• The meanings of the terms *asymptote*, *axis of symmetry*, *codomain*, *concave*, *dependent variable*, *dilation*, *discontinuous*, *discriminant*, *domain*, *even*, *function*, *gradient*, *hyperbola*, *independent variable*, *intercept*, *linear function*, *midpoint*, *odd*, *parabola*, *point of inflection*, *quadratic function*, *range*, *reflection*, *relation*, *translation*, *turning point*, *zeros* • The meaning of the symbols Δ and ∞ • The definition of a relation as a set of ordered pairs or mapping • The definition of a function as a relation where each value of the domain is in only one ordered pair • The vertical line test • The equations of translations, dilations and reflections in the x or y axes of given functions • The formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ for the distance between two points • The formula $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ for the midpoint of two given points • The formulas $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$ and $m = \tan(\theta)$ for the gradient of a straight line • The conditions $m_1 = m_2$ and $m_1 m_2 = -1$ for straight lines to be parallel or perpendicular respectively • The equations $y = mx + c$, $y - y_1 = m(x - x_1)$, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ and $\frac{x}{a} + \frac{y}{b} = 1$ for the equation of a straight line (linear function) • The standard form of the equation of a straight line	• Identify the domain, codomain and range of a relation or function • Use the vertical line test to determine if a relation is a function • Identify a function as continuous or discontinuous • Recognise odd and even functions • Recognise the equations of translations, dilations and reflections in the x or y axes of given functions • Sketch translations, dilations and reflections in the x or y axes of given functions • Find intercepts of given functions • Find the distance between two points • Find the midpoint of two points • Solve problems involving midpoint and distance • Find the gradient of a straight line from given points or its inclination • Find the inclination of a straight line from its gradient • Determine whether lines are parallel, perpendicular or neither • Find the equations of straight lines using appropriate methods • Express the equation of a straight line in appropriate forms • Determine the gradient and intercepts of a straight line from its equation • Solve problems involving gradients and straight lines • Determine the y-intercept, zeros, axis of symmetry and turning point of a quadratic function • Sketch the graph of a quadratic function • Find the discriminant of a quadratic function

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Chapter 4: Functions and graphs		
For Teachers	**For Students**	
Nominal time: 3 weeks	**You should know:**	**You should be able to:**
Powers and polynomials • recognise features of the graphs of $y = x^n$ for $n \in \mathbf{N}$, $n = -1$ and $n = \frac{1}{2}$, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM014) (see sections 4.03–4.07) • recognise features of the graphs of $y = x^3$, $y = a(x - b)^3 + c$ and $y = k(x - a)(x - b)(x - c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM017) (see section 4.07) Graphs of relations • recognise features of the graphs of $x^2 + y^2 = r^2$ and of $(x - a)^2 + (y - b)^2 = r^2$, including their circular shapes, their centres and their radii (ACMMM020) (see section 4.08) • recognise features of the graph of $y^2 = x$ including its parabolic shape and its axis of symmetry (ACMMM021) (see section 4.08) Functions • understand the concept of a function as a mapping between sets, and as a rule or a formula that defines one variable quantity in terms of another (ACMMM022) (see section 4.01) • use function notation, domain and range, independent and dependent variables (ACMMM023) (see section 4.01) • understand the concept of the graph of a function (ACMMM024) (see sections 4.01, 4.02) • examine translations and the graphs of $y = f(x) + a$ and $y = f(x + b)$ (ACMMM025) (see section 4.02) • examine dilations and the graphs of $y = cf(x)$ and $y = f(kx)$ (ACMMM026) (see section 4.02) • recognise the distinction between functions and relations, and the vertical line test (ACMMM027) (see section 4.01)	• The equation of a quadratic function in the forms $y = ax^2 + bx + c$ and $y = d(x - h)^2 + k$ • The formulas $x = -\frac{b}{2a}$ and $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ for the axis of symmetry and turning point of a quadratic function • The parabolic shape of the graph of a quadratic function • The formula $\Delta = b^2 - 4ac$ for the discriminant of a quadratic function • The vertical and horizontal asymptotes of hyperbolas of the form $y = \frac{1}{x}$ or $y = \frac{1}{x - a}$ • The shapes of functions of the form $y = x^n$ for $n \in \mathbf{N}^+$ • Cubic functions of the forms $y = x^3$, $y = a(x - b)^3$ and $y = k(x - a)(x - b)(x - c)$ • The equation $(x - a)^2 + (x - b)^2 = r^2$ of a circle • The equation $y^2 = ax$ of simple parabolas with horizontal axes of symmetry	• Use the discriminant to determine the nature of the roots of a quadratic equation • Sketch the graphs of hyperbolas of the form $y = \frac{1}{x}$ or $y = \frac{1}{x - a}$ and simple dilations or translations • Sketch the graphs of functions of the form $y = x^n$ for $n \in \mathrm{N}^+$ • Sketch cubic functions of the forms $y = x^3$, $y = a(x - b)^3$ and $y = k(x - a)(x - b)(x - c)$ • Sketch a circle from its equation and determine its centre and radius • Find the equation of a circle given its centre and radius • Recognise the equation of a parabola with a horizontal axis of symmetry • Sketch the equation of a simple relation $y^2 = ax$ or simple translation • Solve problems involving functions and graphs • Apply reasoning skills in problems involving functions and graphs • Interpret solutions to problems involving functions and graphs and comment on the reasonableness of solutions • Express arguments, logic and strategies used in solving problems involving functions and graphs in written or verbal form

Chapter 5: Conditional probability		
For Teachers	**For Students**	
Nominal time: 2 weeks	**You should know:**	**You should be able to:**
Curriculum topic and statements **Counting and probability** Conditional probability and independence • understand the notion of a conditional probability and recognise and use language that indicates conditionality (ACMMM056) (see sections 5.01–5.04, 5.06) • use the notation $P(A\|B)$ and the formula $P(A \cap B) = P(A\|B)P(B)$ (ACMMM057) (see sections 5.04, 5.06, 5.07) • understand the notion of independence of an event A from an event B, as defined by $P(A\|B) = P(A)$ (ACMMM058) (see sections 5.05–5.07) • establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B, and recognise the symmetry of independence (ACMMM059) (see sections 5.05–5.07) • use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events (ACMMM060) (see section 5.07)	• The meanings of the terms *combination, complement, conditional probability, dependent events, independent events, intersection, mutually exclusive, tree diagram, two-way table, union, Venn diagram* • The definition of the conditional probability $P(A\|B)$ of event A given event B • The formulas $P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$ and $P(A \cap B) = P(A\|B)P(B)$ for conditional probability • The definition of independent events as events A and B for which $P(A\|B) = P(A)$ or $P(B\|A) = P(B)$ • The equivalence of the statements $P(A\|B) = P(A)$, $P(B\|A) = P(B)$ and $P(A \cap B) = P(A) \times P(B)$ • The definition of dependent events as those which are not independent: $P(A\|B) \neq P(A)$, $P(B\|A) \neq P(B)$ and $P(A \cap B) \neq P(A) \times P(B)$ • The formula for the number combinations of r objects from n distinct objects: $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$ $= \dfrac{n \times (n-1)(n-2)...(n-r+1)}{r!}$ • The formula $P(\bar{A}) = 1 - P(A)$ • The formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for the probabilities of combined events • The formulas $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$ for the probabilities of mutually exclusive events	• Construct and use tree diagrams, two-way tables and Venn diagrams to find probabilities involving conditions • Find conditional probabilities • Use formulas to calculate probabilities involving conditional probability • Solve problems involving conditional probability • Solve problems involving dependent and independent events • Use estimates of probability to determine whether events are likely to be dependent or independent • Apply reasoning skills in problems involving conditional probability, dependent and independent events • Interpret solutions to problems involving conditional probability, dependent and independent events and comment on the reasonableness of solutions • Express arguments, logic and strategies used in solving problems involving conditional probability, dependent and independent events in written or verbal form

Chapter 6: Trigonometric functions and graphs

For Teachers	For Students	
Nominal time: 4 weeks	**You should know:**	**You should be able to:**
Curriculum topic and statements **Trigonometric functions** Trigonometric functions • understand the unit circle definition of cos (θ), sin (θ) and tan (θ) and periodicity using radians (ACMMM034) (see section 6.01) • recognise the exact values of sin (θ), cos (θ) and tan (θ) at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (ACMMM035) (see section 6.05) • recognise the graphs of $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$ on extended domains (ACMMM036) (see sections 6.01–6.04) • examine amplitude changes and the graphs of $y = a \sin(x)$ and $y = a \cos(x)$ (ACMMM037) (see sections 6.02–6.04) • examine period changes and the graphs of $y = \sin(bx)$, $y = \cos(bx)$ and $y = \tan(bx)$ (ACMMM038) (see section 6.03) • examine phase changes and the graphs of $y = \sin(x + c)$, $y = \cos(x + c)$ and $y = \tan(x + c)$ (ACMMM039) (see section 6.04) • the relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$ (ACMMM040) (see section 6.04) • prove and apply the angle sum and difference identities (ACMMM041) (see section 6.05) • identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems (ACMMM042) (see section 6.07) • solve equations involving trigonometric functions using technology, and algebraically in simple cases (ACMMM043) (see section 6.06)	• The meanings of the terms *amplitude, circular measure, cycle, period, periodic, phase, radian, sinusoidal, trigonometric functions, trigonometric identities* • The definition of a periodic function as one for which has a value t such that $f(x) = f(x + t)$ for all x. • The shapes, amplitudes and periods of the graphs of the trigonometric functions $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$ • The amplitude a, period $\frac{2\pi}{b}$ and vertical translation c of the function $y = a \sin(bx) + c$ • The amplitude a, period $\frac{2\pi}{b}$ and vertical translation c of the function $y = a \cos(bx) + c$ • The period $\frac{\pi}{b}$ and vertical translation c of the function $y = a \tan(bx) + c$ • The relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$ • The trigonometric identities $\tan(x) = \frac{\sin(x)}{\cos(x)}$, $\sin^2(x) + \cos^2(x) = 1$, $\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$ and $\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$	• Sketch the basic trigonometric functions $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$ over specified domains • Find the amplitudes of simple sinusoidal functions • Find the periods, translations, dilations and phase shifts of simple trigonometric functions • Sketch simple trigonometric functions using their amplitudes, periods, translations, dilations and phase shifts or by using significant points • Model period phenomena using the sinusoidal functions • $y = a \sin(bx) + c$ or $y = a \cos(bx) + c$ • Solve simple trigonometric equations over specified domains using graphical or algebraic means as appropriate • Solve problems involving periodic phenomena or trigonometric equations • Apply reasoning skills in problems involving trigonometric functions • Interpret solutions to problems involving trigonometric functions and comment on the reasonableness of solutions • Express arguments, logic and strategies used in solving problems involving trigonometric functions in written or verbal form

Chapter 7: Rates of change		
For Teachers	For Students	
Nominal time: 2 weeks	**You should know:**	**You should be able to:**
Curriculum topic and statements **Introduction to differential calculus** Rates of change • interpret the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the average rate of change of a function f (ACMMM077) (see sections 7.01–7.04) • use the Leibniz notation δx and δy for changes or increments in the variables x and y (ACMMM078) (see section 7.05) • use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $y = f(x)$ (ACMMM079) (see sections 7.05, 7.06) • interpret the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y = f(x)$ (ACMMM080) (see sections 7.05, 7.06)	• The meanings of the terms *acceleration, approximate rate of change, average rate of change, average speed, chord, dependent variable, displacement, gradient of a graph, independent variable, instantaneous, instantaneous rate of change, rate of change, secant, secant from below, secant from above, secant around a point, small change, speed, tangent, velocity* • The meanings of the symbols Δ and δ • The formula $\frac{\Delta y}{\Delta x}$ for the rate of change of x with respect to y and its value m for a straight line • The formula $\frac{y_2 - y_1}{x_2 - x_1}$ for the average rate of change • The formula $\frac{\Delta x}{\Delta t}$ for the average rate of change of x over the period Δt • The definitions of position, displacement, velocity and acceleration • The instantaneous rate of change of a function as the slope of the tangent to the curve • The meanings of the terms *secant from below a point, secant above a point*, and *secant around a point* • The approximation $\frac{\delta y}{\delta x}$ for the instantaneous rate of change of a function	• Find the rate of change of one variable with respect to another • Find the average rate of change of one variable with respect to another • Find the average rate of change of a variable with respect to time • Find average velocity and average acceleration of an object • Approximate the instantaneous rate of change of a function at a point using a tangent to the curve • Find the approximate rate of change of a function at a point using $\frac{\delta y}{\delta x}$ for secants below, around or above the point • Use approximations of the rates of change of a function to suggest a function for the rate of change of the original function • Apply reasoning skills in problems involving rates of change • Interpret solutions to problems involving rates of change and comment on the reasonableness of solutions • Express arguments, logic and strategies used in solving problems involving rates of change in written or verbal form

Chapter 8: Arithmetic and geometric sequences		
For Teachers	**For Students**	
Nominal time: 4 weeks	**You should know:**	**You should be able to:**
Curriculum topic and statements **Arithmetic and geometric sequences and series** Arithmetic sequences • recognise and use the recursive definition of an arithmetic sequence: $d = t_{n+1} - t_n$ (ACMMM068) (see sections 8.01, 8.02) • use the formula $t_n = t_1 + (n - 1)d$ for the general term of an arithmetic sequence and recognise its linear nature (ACMMM069) (see section 8.03) • use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest (ACMMM070) (see sections 8.03, 8.04) • establish and use the formula for the sum of the first n terms of an arithmetic sequence (ACMMM071) (see section 8.04) Geometric sequences • recognise and use the recursive definition of a geometric sequence: $t_{n+1} = rt_n$ (ACMMM072) (see sections 8.01, 8.05) • use the formula $t_n = ar^{n-1}$ for the general term of a geometric sequence and recognise its exponential nature (ACMMM073) (see section 8.06) • understand the limiting behaviour as $n \to \infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r (ACMMM074) (see section 8.06) • establish and use the formula $S_n = \frac{t_1(r^n - 1)}{r - 1}$ for the sum of the first n terms of a geometric sequence (ACMMM075) (see section 8.07) • use geometric sequences in contexts involving geometric growth or decay, such as compound interest (ACMMM076) (see section 8.07) • [Optional] establish and use the formula for the limiting sum of a geometric sequence (see section 8.08)	• The meanings of the terms *arithmetic progression, arithmetic sequence, common difference, common ratio, general term, geometric progression, geometric sequence, limiting sum, partial sum of a sequence, recursive, sequence, series, sum to infinity, term* • The definition of an arithmetic sequence • The formula $d = t_{n+1} - t_n$ for the common difference of an arithmetic sequence • The formulas $t_n = t_1 + (n - 1)d$ and $t_{n+1} = t_n + d$ or $t_n = a + (n - 1)d$ and $t_{n+1} = a + d$ for the terms of an arithmetic sequence • The definition of a series • The formulas $S_n = \frac{n}{2}[2t_1 + (n - 1)d]$ and $S_n = \frac{n}{2}[t_1 + t_n]$ or $S_n = \frac{n}{2}[2a + (n - 1)d]$ and $S_n = \frac{n}{2}[a + l]$ for the sum of an arithmetic sequence to n terms • The definition of a geometric sequence • The formula $r = \frac{t_n}{t_{n-1}}$ for the common ratio of an geometric sequence • The formula $t_n = ar^{n-1}$ or $t_n = t_1r^{n-1}$ for the terms of an geometric sequence • The condition $-1 < r < 1$ for the terms of a geometric sequence to converge (to 0) • The formulas $S_n = \frac{a(r^n - 1)}{r - 1}$ and $S_n = \frac{a(1 - r^n)}{1 - r}$ or $S_n = \frac{t_1(r^n - 1)}{r - 1}$ and $S_n = \frac{t_1(1 - r^n)}{1 - r}$ for the sum of an geometric sequence to n terms • [Optional] The condition $-1 < r < 1$ for a geometric sequence to converge and its limiting sum $S_\infty = \frac{a}{1 - r}$	• Find the next terms of simple sequences • Find recursive expressions for the terms of simple sequences • Find expressions for the general terms of simple sequences • Recognise an arithmetic sequence • Find the terms of arithmetic sequences • Find the sum to n terms of an arithmetic sequence (series) • Solve problems involving arithmetic sequences • Recognise a geometric sequence • Find the terms of geometric sequences • Describe the behaviour of a geometric sequence as $n \to \infty$ • Find the sum to n terms of a geometric sequence (series) • Solve problems involving geometric sequences • [Optional] Find the sum to infinity of a geometric series • Apply reasoning skills in problems involving arithmetic and geometric sequences • Interpret solutions to problems involving arithmetic and geometric sequences and comment on the reasonableness of solutions • Express arguments, logic and strategies used in solving problems involving arithmetic and geometric sequences in written or verbal form

Chapter 9: Derivatives

For Teachers

Nominal time: 3 weeks

Curriculum topic and statements

Introduction to differential calculus

The concept of the derivative

- examine the behaviour of the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit (ACMMM081)
(see sections 9.01, 9.02)
- define the derivative as $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$ (ACMMM082)
(see sections 9.02, 9.03)
- use the Leibniz notation for the derivative: $\dfrac{dy}{dx} = \lim\limits_{\delta x \to 0} \dfrac{\delta y}{\delta x}$ and the correspondence $\dfrac{dy}{dx} = f'(x)$ where $y = f(x)$ (ACMMM083)
(see section 9.03)
- interpret the derivative as the instantaneous rate of change (ACMMM084)
(see sections 9.02, 9.05)
- interpret the derivative as the slope or gradient of a tangent line of the graph of $y = f(x)$ (ACMMM085)
(see sections 9.02, 9.06)

Computation of derivatives

- estimate numerically the value of a derivative, for simple power functions (ACMMM086)
(see sections 9.02, 9.03)
- examine examples of variable rates of change of non-linear functions (ACMMM087)
(see sections 9.05, 9.07)
- establish the formula $\dfrac{d}{dx}x^n = nx^{n-1}$ for positive integers n by expanding $(x+h)^n$ or by factorising $(x+h)^n - x^n$ (ACMMM088)
(see section 9.04)

Applications of derivatives

- find instantaneous rates of change (ACMMM092)
(see section 9.05)
- find the slope of a tangent and the equation of the tangent (ACMMM093)
(see section 9.06)

For Students

You should know:

- The meanings of the terms *acceleration, average rate of change, chord, derivative, differentiation, displacement, distance, first principles, gradient, instantaneous, limit, marginal, secant, tangent, velocity*
- The symbols $\lim\limits_{x \to c}$, $f'(x)$, $\dfrac{dy}{dx}$ and $\dfrac{dy}{dx}\Big|_{x=a}$
- The formula for the gradient of a secant
- The definition of the derivative of a function
- The formula for the derivative of x^n
- The formula for the derivative of kx^n
- The equivalence of the derivative, the instantaneous rate of change of a function, the slope of the tangent and the gradient of a graph
- The definitions of position, displacement, velocity and acceleration
- The definitions of velocity and acceleration in terms of derivatives

You should be able to:

- Find the values of simple limits by approximation of by using algebra
- Estimate the derivative of a function using numerical methods
- Find the derivative of simple functions from first principles
- Find the derivatives of functions of the form $y = x^n$ or $y = kx^n$
- Find the derivative of a function at a point
- Find the slopes of the curves and the instantaneous rates of change of functions of the form $y = x^n$ or $y = kx^n$
- Find the slopes and equations of tangents to functions of the form $y = x^n$ or $y = kx^n$
- Solve problems involving simple derivatives
- Apply reasoning skills in problems involving simple derivatives
- Interpret solutions to problems involving simple derivatives and comment on the reasonableness of solutions
- Express arguments, logic and strategies used in solving problems involving simple derivatives in written or verbal form

Chapter 10: Properties of derivatives		
For Teachers	**For Students**	
Nominal time: 2 weeks	**You should know:**	**You should be able to:**
Curriculum topic and statements **Introduction to differential calculus** Properties of derivatives • understand the concept of the derivative as a function (ACMMM089) (see section 10.01) • recognise and use linearity properties of the derivative (ACMMM090) (see sections 10.02, 10.03) • calculate derivatives of polynomials and other linear combinations of power functions (ACMMM091) (see section 10.04) Applications of derivatives • find instantaneous rates of change (ACMMM092) (see section 10.06) • find the slope of a tangent and the equation of the tangent (ACMMM093) (see section 10.05) Anti-derivatives • calculate anti-derivatives of polynomial functions and apply to solving simple problems involving motion in a straight line (ACMMM097) (see section 10.07)	• The meanings of the terms *anti-derivative*, *anti-differentiation*, *average rate of change*, *decreasing*, *derivative*, *differentiation*, *gradient function*, *increasing*, *instantaneous rate of change*, *integration*, *integral*, *linear product*, *linear combination*, *marginal cost*, *primitive*, *stationary*, *tangent* • The notation for the derivative • The gradient function as the derivative of a function • The relationship between the derivative and whether the function is increasing or decreasing • The rules $\frac{d}{dx}k = 0$, $\frac{d}{dx}[kf(x)] = kf'(x)$, $\frac{d}{dx}x^n = x^{n-1}$, $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$ and $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$ • The rule $\frac{d}{dx}[af(x) + bg(x)] = af'(x) + bg'(x)$ for the derivative of a linear combination of functions • The derivative is both the instantaneous rate of change of a function and the slope of the tangent • The anti-derivative of a function $f(x)$ as a function $F(x)$ whose derivative is equal to the given function: $F'(x) = f(x)$ • The rule for the anti-derivative of ax^n: if $f'(x) = ax^n$ then $f(x) = \frac{x^{n+1}}{n+1}$ for an integer $n \geq 0$ • If $F'(x) = f(x)$, then all anti-derivatives of $f(x)$ are of the form $F(x) + c$	• Sketch the gradient function from a sketch of the graph of a simple function and vice versa • Find the derivatives of simple functions including polynomials • Find the slopes and equations of tangents to polynomial functions • Find the instantaneous rates of change of polynomial functions • Solve problems involving the derivatives of polynomial functions • Find the anti-derivatives of simple functions including polynomials • Solve problems involving the anti-derivatives of simple functions • Apply reasoning skills in problems involving derivatives and anti-derivatives of polynomials • Interpret solutions to problems involving derivatives and anti-derivatives of polynomials and comment on the reasonableness of solutions • Express arguments, logic and strategies used in solving problems involving derivatives and anti-derivatives of polynomials in written or verbal form

Chapter 11: Exponential functions		
For Teachers	**For Students**	
Nominal time: 3 weeks	**You should know:**	**You should be able to:**
Curriculum topic and statements	• The meanings of the terms *base*, *decay function*, *exponent*, *exponential equation*, *exponential function*, *growth factor*, *growth function*, *half-life*, *horizontal asymptote*, *index*, *indicial equation*, *initial value*, *power*, *radical*, *radiocarbon dating*, *scientific notation*, *significant figures*, *transformation* • The index laws for and extensions to integers and rational numbers: $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$, $(ab)^n = a^n b^n$, $a^0 = 1\ (a \neq 0)$, $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$ $(p, q \in \mathbf{Z}, q > 0)$ • Scientific notation, accuracy and significant figures • The general form $f(x) = Aa^{x+b} + c$ of an exponential function • The conditions $a > 0$ and $0 < a < 1$ for exponential functions to be characterised as growth or decay functions respectively • The general shape of the graph of a simple exponential function of the form $f(x) = a^x$ • The dilation and translation effects of the variables A, b and c on the shape of the graph of the simple exponential function	• Simplify expressions involving indices • Change numbers between scientific notation and normal notation • State the accuracy of numbers to the number of significant figures and round to a specified number of significant figures • Recognise and use key characteristics of exponential functions • Sketch the graphs of simple exponential functions and functions of the form $f(x) = Aa^{x+b} + c$ using transformations or key features of the function, including the locations of asymptotes • Use exponential functions to model physical phenomena • Find exact or approximate solutions to exponential equations as appropriate • Solve problems involving exponential functions or equations • Apply reasoning skills in problems involving exponential functions or equations • Interpret solutions to problems involving exponential functions or equations and comment on the reasonableness of solutions • Express arguments, logic and strategies used in solving problems involving exponential functions or equations in written or verbal form
Exponential functions Indices and the index laws • review indices (including fractional indices) and the index laws (ACMMM061) AC (see section 11.01) • use radicals and convert to and from fractional indices (ACMMM062) AC (see sections 11.01, 11.02) • understand and use scientific notation and significant figures (ACMMM063) AC (see section 11.03) Exponential functions • establish and use the algebraic properties of exponential functions (ACMMM064) AC (see section 11.04) • recognise the qualitative features of the graph of $y = a^x$ $(a > 0)$ including asymptotes, and of its translations $y = a^x + b$ and $y = a^{x+c}$ (ACMMM065) AC (see sections 11.05, 11.06) • identify contexts suitable for modelling by exponential functions and use them to solve practical problems (ACMMM066) AC (see section 11.08) • solve equations involving exponential functions using technology, and algebraically in simple cases (ACMMM067) AC (see section 11.07)		

Chapter 12: Applications of derivatives		
For Teachers	**For Students**	
Nominal time: 4 weeks	**You should know:**	**You should be able to:**
Curriculum topic and statements **Introduction to differential calculus** Applications of derivatives • find instantaneous rates of change (ACMMM092) (see section 12.01) • construct and interpret position–time graphs, with velocity as the slope of the tangent (ACMMM094) (see section 12.02) • sketch curves associated with simple polynomials; find stationary points, and local and global maxima and minima; and examine behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM095) (see sections 12.04–12.06) • solve optimisation problems arising in a variety of contexts involving simple polynomials on finite interval domains (ACMMM096) (see section 12.07) Anti-derivatives • calculate anti-derivatives of polynomial functions and apply to solving simple problems involving motion in a straight line (ACMMM097) (see section 12.03)	• The meanings of the terms *acceleration*, *average rate of change*, *displacement*, *dominant term*, *flow rate*, *global maximum*, *global minimum*, *gradient function*, *greatest value*, *instantaneous rate of change*, *least value*, *local maximum*, *local minimum*, *optimisation*, *point of horizontal inflection*, *stationary point*, *velocity* • The derivative is both the instantaneous rate of change of a function and the slope of the tangent • The characteristics of displacement velocity and acceleration in terms of derivatives and anti-derivatives • The relationships between displacement–time, velocity time and acceleration time graphs • The relationship between the signs of the derivative and the nature of stationary points • The relationship between the dominant term and the behaviour of a function as $x \to \pm\infty$	• Find the instantaneous rates of change of polynomial functions • Solve problems involving the instantaneous rates of change of polynomial functions • Use derivatives and anti-derivatives to find the displacements, velocities and accelerations of particles moving in a straight line • Sketch graphs involving motion in a straight line • Solve problems involving particles moving in a straight line • Find the stationary points of polynomial functions • Determine the nature of the stationary points of polynomials • Use the y-intercept, zeros, dominant term, increasing and decreasing nature and stationary points of a polynomial appropriately to sketch its graph • Find local and global maxima and minima of polynomial functions using algebraic or numerical means as appropriate • Find the greatest and least values of polynomial functions on specified intervals • Model simple optimisation problems using polynomials • Solve simple optimisation problems • Apply reasoning skills in problems involving instantaneous rates of change, motion in a straight line, local and global maxima and minima, greatest and least values on and interval or optimisation • Interpret solutions to problems involving instantaneous rates of change, motion in a straight line, local and global maxima and minima, greatest and least values on and interval or optimisation and comment on the reasonableness of solutions • Express arguments, logic and strategies used in solving problems involving instantaneous rates of change, motion in a straight line, local and global maxima and minima, greatest and least values on and interval or optimisation in written or verbal form