MATHEMATICS METHODS UNIT 2 **TEST 4**

RESOURCE FREE **NAME:** ____________________

TIME ALLOWED: 20 MIN

Show all your working clearly. *Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.*

QUESTION 1

(2,3,3 marks)

Simplify each of the following leaving answers with positive indices:

(a) $\dfrac{(6a^3b^2)^2}{(-3a^2b^3)^3} = \dfrac{36a^6b^4}{-27a^6b^9}$

$= -\dfrac{4}{3b^5}$

(b) $\dfrac{6x^4y^{-5}}{2x^2y^{-3}} \div \dfrac{2x^2}{(2x^3)^2} = \dfrac{3x^2y^{-2}}{1} \times \dfrac{4x^6}{2x^2}$

$= \dfrac{12x^8y^{-2}}{2x^2}$

$= \dfrac{6x^6}{y^2}$

(c) $\dfrac{7^{(x+2)} - 49}{7^x - 1} = \dfrac{7^x \times 7^2 - 7^2}{7^x - 1}$

$= \dfrac{7^2(7^x - 1)}{(7^x - 1)}$

$= \underline{49}$

8

QUESTION 2 **(2,2 marks)**

Simplify:

(a) $25^{\frac{3}{2}} = (5^2)^{\frac{3}{2}}$

$= 5^3 = 125$ ✓✓

(b) $(4^{-1})^{-0.5} = 4^{0.5}$

$= \sqrt{4} = \pm 2$ ✓✓

QUESTION 3 **(3 marks)**

The eighth term and twelfth term of an arithmetic sequence are 24 and 40 respectively. Find the general rule for this sequence.

$T_8 = 24 \quad T_{12} = 40$

$T_{12} = T_8 + 4d$

$40 = 24 + 4d$

$16 = 4d$

$4 = d$

$T_8 = T_1 + 7d$

$24 = T_1 + 7 \times 4$

$T_1 = -4$

$T_n = -4 + (n-1)4$

$T_n = -4 + 4n - 4$

$\underline{T_n = 4n - 8}$ ✓✓✓

QUESTION 4 **(3,3 marks)**

Solve each of the following:

(a) $5^x \times 25^{(x+1)} = 0.04$

$5^x \times 5^{2(x+1)} = \frac{1}{25}$

$5^x \times 5^{2x+2} = 5^{-2}$

$5^{3x+2} = 5^{-2}$

$3x + 2 = -2$

$3x = -4 \qquad x = \frac{-4}{3}$ ✓✓✓

(b) $\sqrt{50x} - \sqrt{18x} = \sqrt{2}$

$5\sqrt{2x} - 3\sqrt{2x} = \sqrt{2}$

$2\sqrt{2x} = \sqrt{2}$

$4(2x) = 2$

$8x = 2$

$x = \frac{1}{4}$ ✓✓✓

TOTAL MARKS: 21

MATHEMATICS METHODS UNIT 2 **TEST 4**

RESOURCE RICH **NAME:** ______________________

TIME ALLOWED: 40 MINUTES

Show all your working clearly. *Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.*

QUESTION 5

(2,2,2,2 marks)

An arithmetic progression has first term -20 and common difference 3.

(a) Find the general rule for this sequence and use it to find the twentieth term.

$T_n = -20 + (n-1)3$ $\quad T_{20} = -20 \times 19 \times 3$

$= 3n - 23$ ✓ $\quad = 37$ ✓

(b) Use the formula for the sum of an arithmetic progression to find the sum of the first 20 terms.

$S_{20} = \frac{20}{2}(2(-20) + 19 \times 3)$ or $S_{20} = \frac{20}{2}(-20 + 37)$

$= 170$ ✓✓ $\quad = 170$

(c) Find the recursive rule for this sequence and use this to find the first **positive term** in the sequence.

$\left.\begin{array}{l} T_{n+1} = T_n + 3 \\ T_1 = -20 \end{array}\right\}$ ✓ $\quad T_8 = 1$ ✓

(d) Use your calculator to find n so that the **sum** of the first n terms is positive for the first time.

$T_{15} = 15 \quad \therefore n = 15$ ✓✓

QUESTION 6

(2,2,2 marks)

A geometric sequence is described by the rule $T_n = 5 \times 3^n$ where $n = 1,2,3,\ldots$

(a) Find the first three terms of the sequence.

$T_1 = 5 \times 3$ $\quad T_2 = 5 \times 3^2$ $\quad T_3 = 5 \times 3^3$

$= 15$ $\quad = 45$ $\quad = 135$ ✓✓

(b) State the recursive rule for this sequence. $\quad r = \frac{45}{15} = 3$

$T_{n+1} = T_n \times 3$ ✓

$T_1 = 15$ ✓

(c) How many terms less than 1000 are there in this sequence?

4 terms ✓✓

QUESTION 7 **(1,1,2,1 marks)**

Scientists were trying to increase the population of a rare species of fish so they placed 50 fish in a small lake and monitored the population monthly. They discovered that the population of fish, P, increased according to the rule:

$P = 50 \times 1.15^t$, where t was the time in months since the first fish was placed in the lake.

(a) What was the population of the fish after 18 months?

618·77 ~ 619 fish ✓

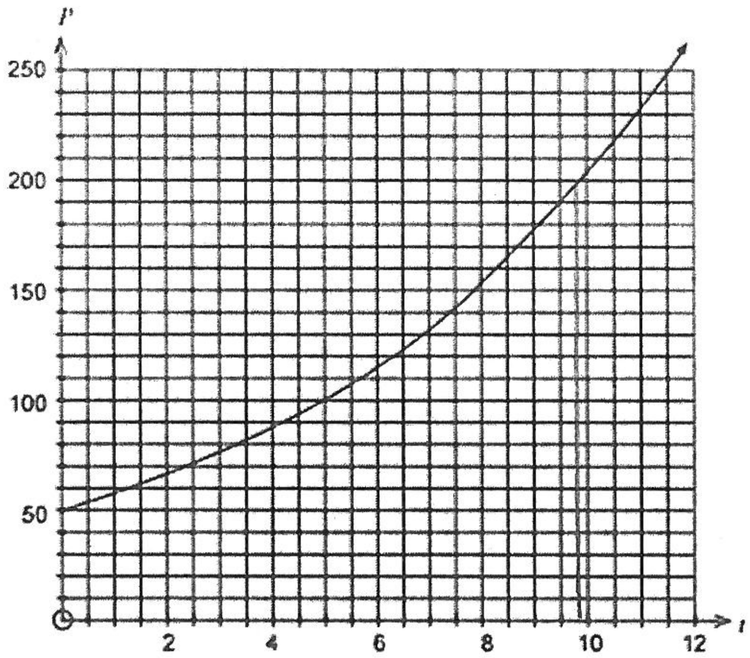
(b) By what percentage did the population of the fish increase each month?

15% ✓

(c) Complete the recursive rule representing the growth of the population of the fish.

$P_{n+1} = P_n \times 1.15$ ✓ $P_0 = 50$ ✓

A graph of the fish population is shown below. Use the graph to determine during which month the population of the fish first exceeded 200. Clearly indicate all working on the graph and state your solution.



∴ in 10th month ✓

QUESTION 8

(1,2,2 marks)

A rubber ball is dropped from a height of 300cm onto a horizontal surface. Each time it hits the ground it will bounce upwards to a height that is 60% of the height of the previous bounce. Find

(a) the height reached on the first bounce,

$T_1 = 300 \times 0{\cdot}6$

$= 180\text{cm}$ ✓

(b) the height reached on the fifth bounce,

$T_5 = 300 \times 0{\cdot}6^5$

$= 23{\cdot}33\text{cm}$ ✓✓

(c) the total vertical distance travelled by the ball before it comes to rest on the ground.

$T_1 = 300 \times 0{\cdot}6$

$S_\infty = \frac{a}{1-r} = \frac{300 \times 0{\cdot}6}{1-0{\cdot}6} = 450\text{m}$ ✓

$\therefore$ Total Distance

$= 300 + 2 \times S_\infty$

$= 300 + 900$

$= 1200\text{cm}$

$= 12\text{m}$ ✓

QUESTION 9

(2,2 marks)

The sum to infinity of a general series with first term 10 is 40.

(a) Determine the common ratio of this series.

$T_1 = 10 \qquad S_\infty = 40$

$S_\infty = \frac{a}{1-r}$

$40 = \frac{10}{1-r}$

$1 - r = 0{,}25$

$r = 0{\cdot}75$ ✓✓

(b) Find the recursive rule of the sequence of this series.

$T_{n+1} = T_n \times 0{\cdot}75$

$T_1 = 10$ ✓✓

QUESTION 10 **(4,3,1,3 marks)**

On 1st January 2014 Mrs McKernan starts a savings plan by investing \$1000 into an account that guarantees 7% interest per annum provided she commits to invest a further \$1000 on the 1st January every year. The plan will finish on 1st January 2029.

Date	Value of deposit made on				Total Value
	1st Jan 2014	1st Jan 2015	1st Jan 2016	1st Jan 2017	
1st Jan 2014	\$1000 T_0				\$ 1000
1st Jan 2015	$\$1000 \times 1.07$ T_1	\$1000			\$ 2070
1st Jan 2016	$\$1000 \times 1.07^2$ T_2	$\$1000 \times 1.07$	\$1000		\$ 3214.90
1st Jan 2017	$\$1000 \times$ T_3	$\$1000 \times 1.07^2$ T_2	$\$1000 \times 1.07$ T_1	\$1000 T_0	\$ 4439.94 ✓✓✓✓

(a) Complete the last row and last column table for this situation.

(b) Determine T_1 and the common ratio, and use this to write a recursive rule for the value of T_n the deposit after n years.

$T_1 = 1000 \times 1.07$ ✓ $r = 1.07$ ✓

$T_n = T_{n-1} \times 1.07$ ✓

(c) What is the value of n on 1st January 2029?

$n = 15.$ ✓

(d) Determine how much will be in the account following the deposit of \$1000 made on 1st January 2029, to the nearest 10 cents.

$T_1 = 1000 \times 1.07$

Amt in Account $= \dfrac{1000 \times 1.07(1.07^{15} - 1)}{1.07 - 1} + 1000$

$= \$26888.05 + 1000$

$= \$27888.05$

$\sim \$27888.10$ nearest 10 cents ✓✓✓

OR

$T_1 = 1000$

$S_{16} = \dfrac{1000(1.07^{16} - 1)}{1.07 - 1}$

$= \$27888.10$

TOTAL MARKS: 39