School Name
Mathematics Test 2017

Year 10 **Geometric Reasoning** Non Calculator

Skills and Knowledge Assessed:

- Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)
- Formulate proofs involving congruent triangles and angle properties (ACMMG243)

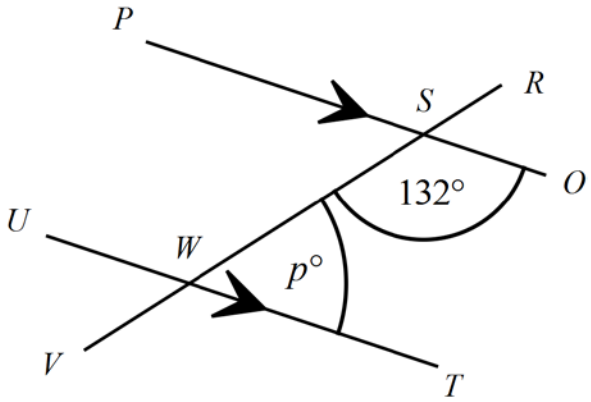
Name_____

Extended Answer Test

Answers should be supported by relevant mathematical reasoning and/or calculations
Full marks may not be awarded for answers with no reasoning.
Complete any diagrams and write all working and answers in the spaces provided on this test paper.

Marks

1. (a) Find the value of p. **2**



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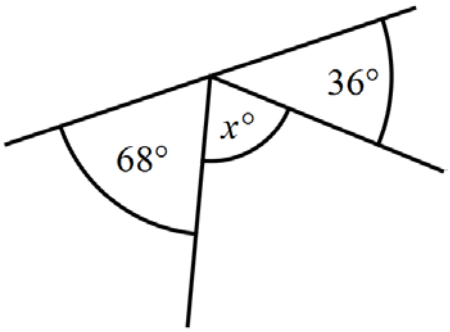
(b) Find the value of x. **2**



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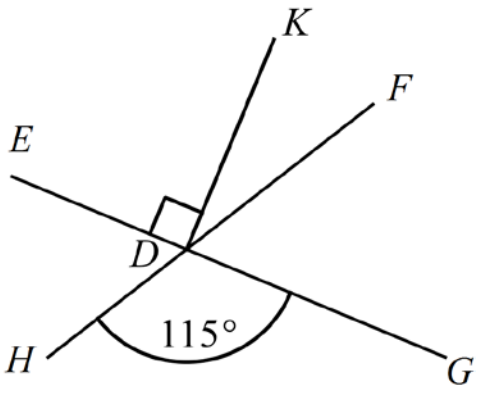
(c) Find the size of $\angle FDK$. **2**



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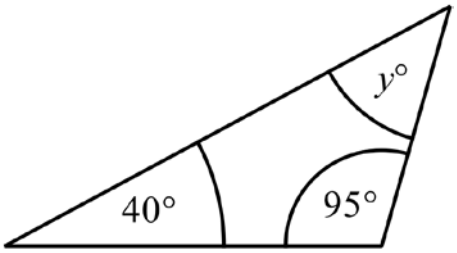
2. (a) Find the value of y. **2**



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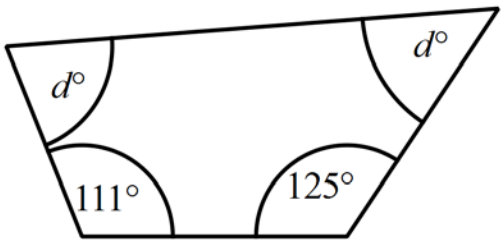
(b) Find the value of *d*. **2**



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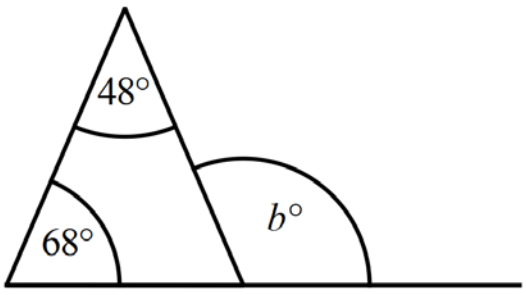
(c) Find the value of *b*. **2**



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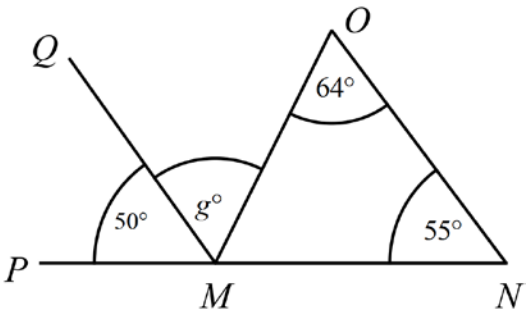
Marks

3. (a) Find the value of g. **3**



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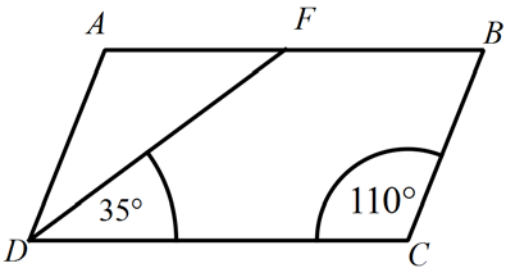
(b) $ABCD$ is a parallelogram. **3**
F is a point on AB such that $\angle FDC = 35°$.
Find the size of $\angle$ ADF.



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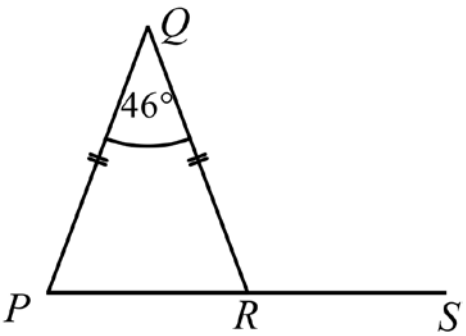
(c) PQR is an isosceles triangle. PR is produced to S. **3**
Find the size of $\angle QRS$.



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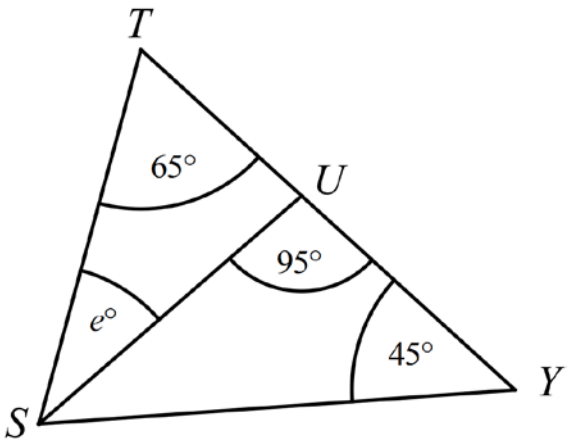
Marks

4. (a) In ΔSTY $\angle T = 65°$ and $\angle Y = 45°$. **3**
U is a point on TY such that $\angle SUY = 95°$.
Find the value of e.



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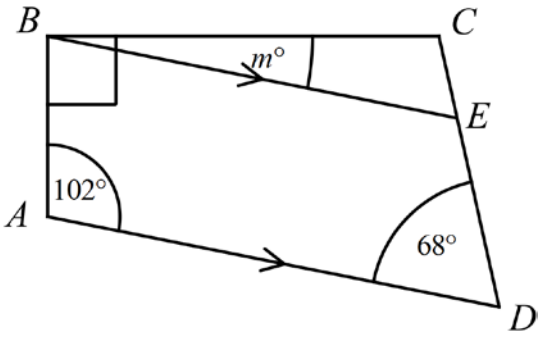
(b) In the diagram, $BC \perp BA$ and $BE \parallel AD$. **3**
Find the value of m.



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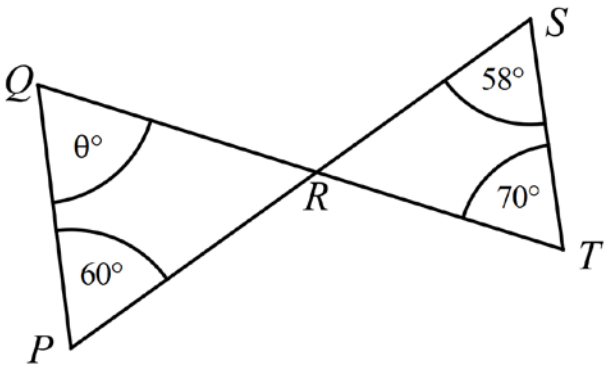
(c) Find the value of θ. **3**



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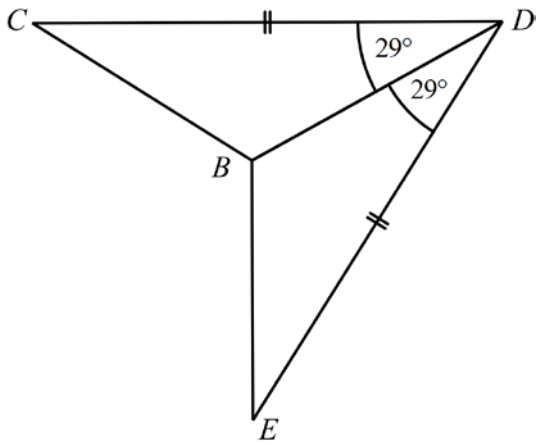
Marks

5. (a) Prove that $\Delta BCD \equiv \Delta BED$. **3**



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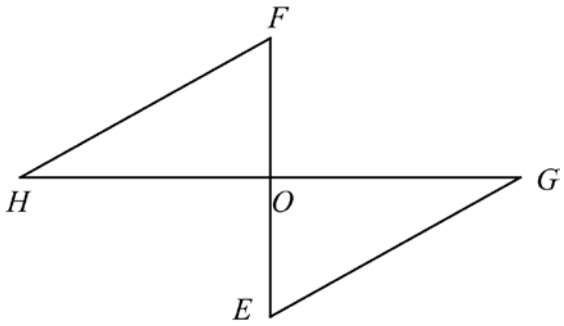
Marks

(b) *EF* and *GH* are two line segments which bisect one another at right angles at *O*
FH and *EG* are joined to create ΔFHO and ΔEGO.



(i) Prove that $\Delta FHO \equiv \Delta EGO$. **2**

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(ii) *FG* and *HE* are joined to form the quadrilateral *FGEH*.
List two other pairs of congruent triangles apart from the pair in part (i). **2**

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(iii) What type of quadrilateral is *FGEH*? Give reasons for your choice. **2**

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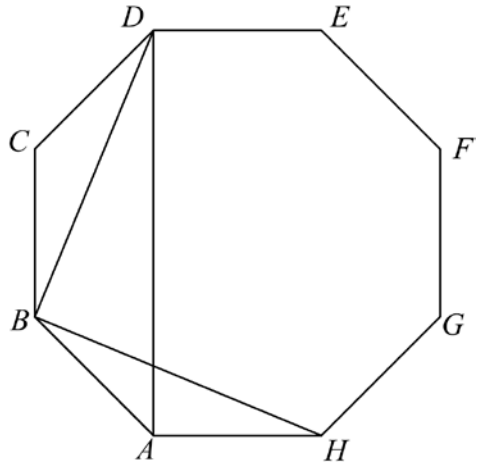
Marks

(c) The diagram below shows a regular octagon with three diameters draw inside it.



(i) Show that $\angle BCD = 135°$. **2**

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(ii) Show that $\angle DBA = 112.5°$. **2**

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(iii) Show that $\angle DBH = 90°$. **2**

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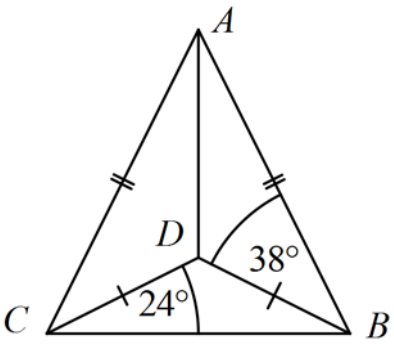
Marks

6. (a) ΔABC is an isosceles triangle with $AB = AC$. **3**
D is a point on the interior of ΔABC such that $DC = DB$.
$\angle DCB = 24°$ and $\angle ABD = 38°$
Find the size of $\angle ADB$.



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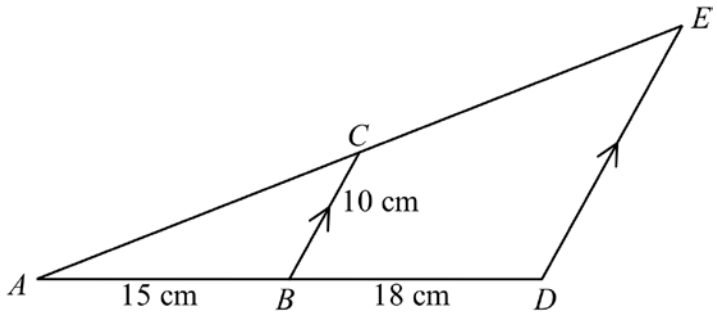
Marks

(b) In the diagram below $BC \parallel DE$, $AB = 15$ cm, $BD = 18$ cm and $BC = 10$ cm.



(i) Prove that $\Delta ACB \,|||\, \Delta$ AED. **3**

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(ii) Find the length of ED. **2**

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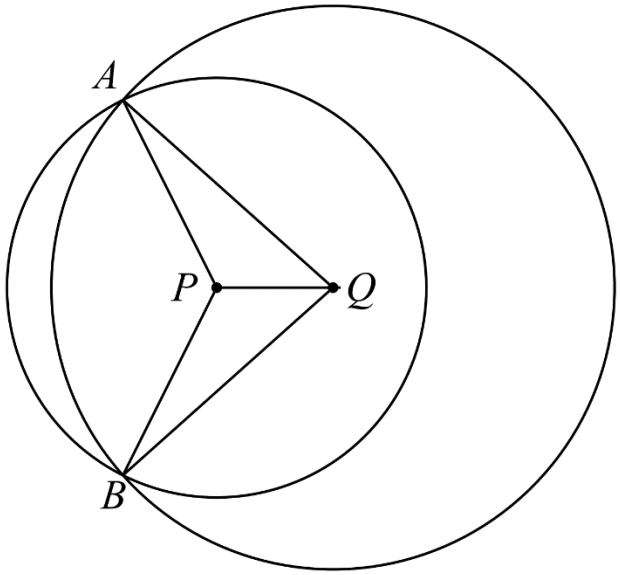
Marks

(c) In the diagram below, P is the centre of the smaller circle and Q is the centre of the larger circle.
A and B are the points of intersection of the two circles.



(i) Prove that $\Delta AQP \equiv \Delta BQP$ and hence that $\angle PAQ = \angle PBQ$. **3**

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(ii) What name best describes the type of quadrilateral for $AQBP$?
Give reasons for your choice.

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School Name
Mathematics Test 2017

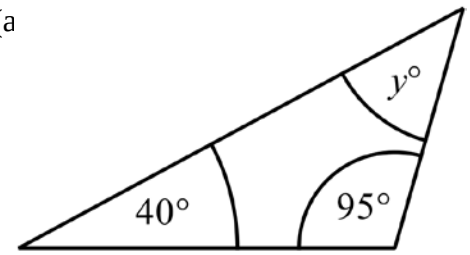
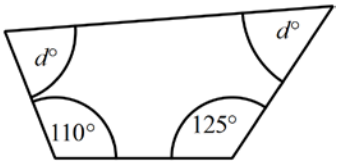
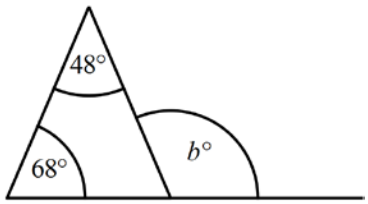
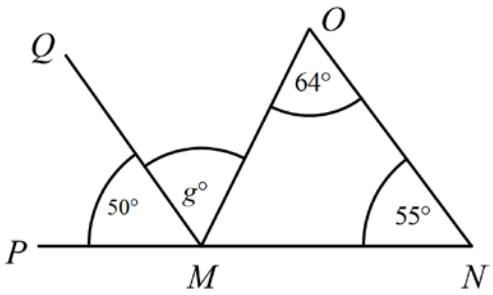
Year 10

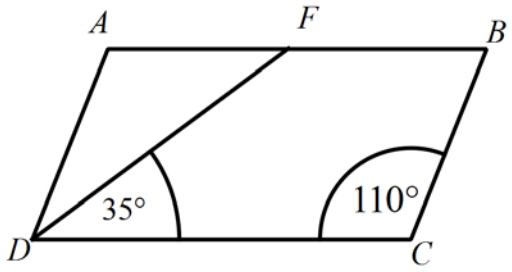
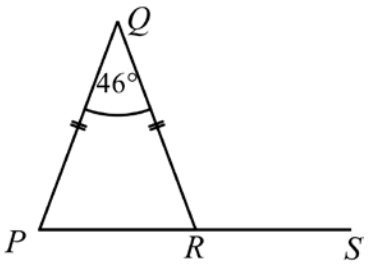
Geometric Reasoning

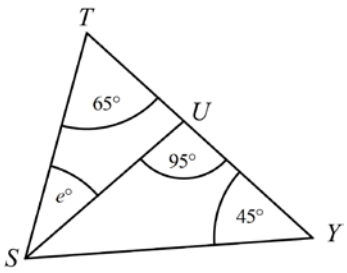
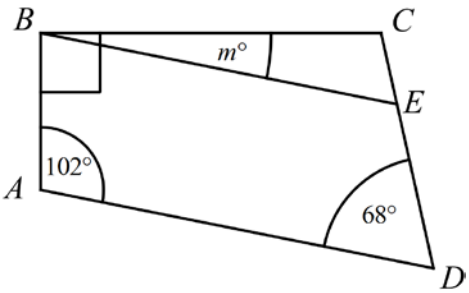
Calculator Allowed
Longer Answer Section

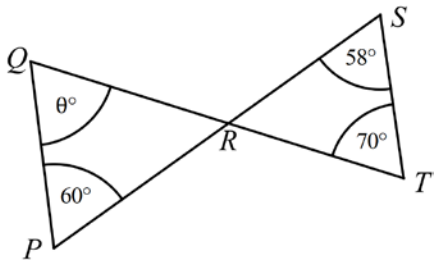
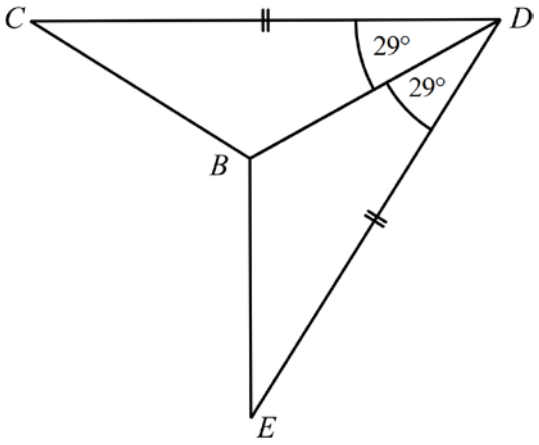
ANSWERS

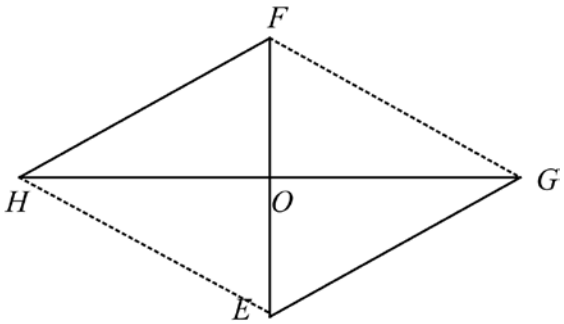
Question	Answer	Marks
1.	(a) $p = 180 - 132 = 48°$ (cointerior angles on parallel lines)	**2 marks for correct answer with reason.** **1 mark if answer or reason is wrong or not provided.**
	(b) $x + 36 + 68 = 180$ (angles on straight line) $x = 180 - (68 + 36)$ $x = 76°$	**2 marks for correct answer with reason.** **1 mark if answer or reason is wrong or not provided.**
	(c) $\angle EDF = \angle HDG = 115°$ (vertically opp angles) $\angle FDK = 115° - 90°$ (adjacent angles) $\angle FDK = 25°$	**2 marks for correct answer with reason.** **1 mark if answer or reason is wrong or not provided.**

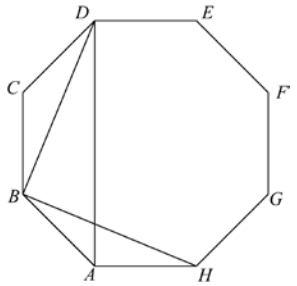
Question	Answer	Marks
2.	(a  $y + 40 + 95 = 180$ (angle sum Δ) $y = 180 - 135$ $y = 45$	**2 marks for correct answer with reason.** **1 mark if answer or reason is wrong or not provided.**
	(b)  $d + d + 125 + 111 = 360$ (angle sum quadrilateral) $2d = 360 - 236$ $2d = 124$ $d = \frac{124}{2} = 62$	**2 marks for correct answer with reason.** **1 mark if answer or reason is wrong or not provided**
	(c) $b = 48 + 68$ (exterior angle Δ) $b = 116$  	**2 marks for correct answer with reasons.** **1 mark if answer is calculated incorrectly or if reasoning is wrong or not provided**
3.	(a) $\angle PMO = 64 + 55$ $= 119°$ (ext angle of Δ) $\angle QMO =$ g` $= 119 - 50$ $g = 69$ (adjacent angles)  	**3 marks for correct answer with all reasons.** **2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided** **1 mark for an answer that shows some correct reasoning.**

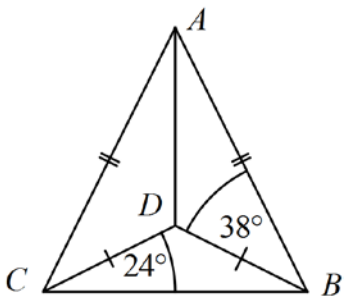
Question	Answer	Marks
	(b) $\angle AFD = 35°$ (alt angle on $\parallel$ lines) $\angle DAF = 110°$ (opp angle of $\parallel gram$) $\angle ADF = 180 - (35 + 110)$ (angle sum of Δ) $\angle ADF = 35°$ 	**3 marks for correct answer with all reasons.** **2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided** **1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided**
	(c) $\angle PRQ \times 2 + 46 = 180$ (angle sum isosceles Δ) $\angle PRQ = \frac{180 - 46}{2} = \frac{134}{2} = 67$ $\angle QRS = q = 180 - 67$ $= 113°$ (angles on st line) 	**3 marks for correct answer with all reasons.** **2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided** **1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided**

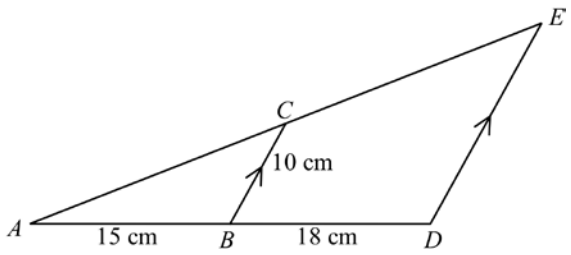
Question	Answer	Marks
4.	(a) $\angle TSY = 180 - 65 - 45$ $= 70$ (angle sum Δ) $\angle USY = 180 - 95 - 45$ $= 40$ (angle sum Δ) $\angle TSU = x° = 70° - 40°$ $= 30$ (adjacent angles) 	**3 marks for correct answer with all reasons.** **2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided** **1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided**
	(b) $\angle BCD = \angle BCE = 360 - (90 + 102 + 68)$ $= 360 - 260 = 100$ (angles sum quad) $\angle BEC = 68$ (alternate $\angle$ on $\parallel$ lines) $\angle CBE = m = 180 - (68 + 100)$ $= 12$ ($\angle$ sum Δ) 	**3 marks for correct answer with all reasons.** **2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided** **1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided**

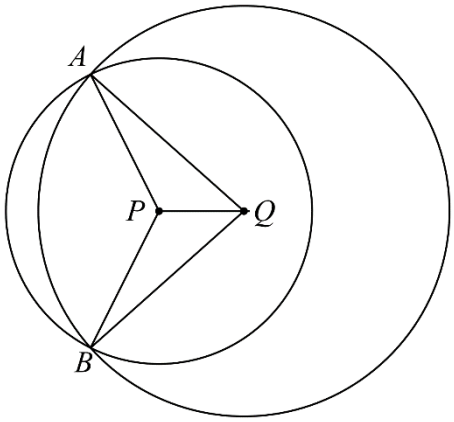
Question	Answer	Marks
	(c) $\angle SRT = 180 - (58 + 70)$ $= 52°$ (angles sum Δ) $\angle QRP = 52$ (vert opp $\angle$) and PQR $= \theta° = 180 - (60 + 52)$ $\theta = 68$ ($\angle$ sum Δ)  	**3 marks for correct answer with all reasons.** **2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided** **1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided**
5.	(a) In ΔBCD and ΔBED $CD = ED$ (given) $\angle BDC = \angle BDE = 29°$ (given) BD is common $\therefore \Delta BCD \equiv \Delta BED$ (SAS)  	**3 marks for correct conclusion with right congruence test and all steps of reasoning given.** **2 marks for correct conclusion with one step of reasoning incorrect or not provided or if conclusion uses wrong test** **1 mark if at least one correct statement needed for the proof is provided**

Question	Answer	Marks
	(b)  (i) In ΔFHO and ΔEGO $FO = EO$ (FE bisected at O) $\angle HOF = \angle GOE = 90°$ (Vert opp $\angle$) $HO = GO$ (GH bisected at O) $\therefore \Delta FHO \equiv \Delta EGO\,(SAS)$ (ii) Other pairs include: ΔFHO and ΔFGO ΔFGO and ΔEGO ΔHOE and ΔGOE ΔHOE and ΔFGO ΔHFG and ΔHEG (iii) $FGEH$ is a rhombus, as the diagonals bisect at right angles. (Or mention the congruent triangles giving all sides equal)	**(i)** **2 marks for correct conclusion with right congruence test and all steps of reasoning given.** **1 mark for correct conclusion with one step of reasoning incorrect or not provided or if conclusion uses wrong test** **(ii)** **1 mark each for any two pairs** **(iii)** **1 mark for correct name and 1 mark for any valid reason**

Question	Answer	Marks
	(c)  (i) angle sum of polygon $= (n-2) \times 180$ angle sum of octagon $= (8-2) \times 180$ $= 6 \times 180 = 1080°$ angle in a regular octagon $= \frac{1080}{8} = 135°$ (ii) $\angle CDB = \angle CBD$ (*base* $\angle$ isos Δ) $2 \times \angle CBD + 135 = 180$ ($\angle$ sum Δ) $2 \times \angle CBD = 180 - 135 = 45°$ $\angle CBD = \frac{45}{2} = 22.5°$ $\angle DBA + \angle CBD = 135°$ (adjacent $\angle$) $\angle DBA = 135° - 22.5° = 112.5°$ (iii) $\angle CBD = 22.5°$ (from part (*ii*)) $\angle ABH = 22.5°$ (similarly to (*ii*)) $\angle DBA = 112.5°$ (from part (*ii*)) $\angle DBH + \angle ABH = \angle DBA$ (adjacent angles) $\angle DBH + 22.5° = 112.5°$ $\angle DBH = 112.5° - 22.5° = 90°$	**(i)** **2 marks for correct conclusion with right reasoning given.** **1 mark for a minor error in reasoning.** **ii)** **2 marks for correct conclusion with right reasoning given.** **1 mark for a minor error in reasoning.** **iii)** **2 marks for correct conclusion with right reasoning given.** **1 mark for a minor error in reasoning.**

Question	Answer	Marks
6.	(a)  $AB = AC$ (given) $DB = DC$ (given) AD is common $\therefore \Delta ABD \equiv \Delta ACD\ (SSS)$ (Formal proof of Congruence not needed, but should be quoted) $\therefore \angle ADB = \angle ADC$ (corres angles in cong Δ) $\angle DBC = \angle DCB = 24°$ (Base $\angle$ isos Δ) $\angle CDB = 180 - 24 \times 2 = 132$ $\angle ADB = \frac{360 - 132}{2} = 114$ (angles at a point) **Alternative** $\angle DBC = \angle DCB = 24°$ (Base $\angle$ isos Δ) $\therefore \angle ABC = 62°$ (adjacent angles) $\angle ACB = 62°$ (Base $\angle$ isos Δ) $\angle CAB = 180 - 2 \times 62 = 56°$ ($\angle$ sum Δ) $\angle DAB = \angle DAC = 28°$ (equal adjacent angles) and $ADB = 180 - 38 - 28 = 114°$ ($\angle$ sum Δ)	**3 marks for correct answer with all reasons.** **2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided** **1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided**

Question	Answer	Marks
	(b)  (i) In ΔACB and ΔAED $\angle CAB = \angle EAD$ (common $\angle le$) $\angle ACB = \angle AED$ (corr $\angle$ on $\parallel$ lines) $\angle ABC = \angle A\Delta$ (corr $\angle$ on $\parallel$ lines) $\therefore \Delta ACB \,\|\|\| \Delta AED$ (corr $\angle$ are equal) (ii) $\frac{ED}{CB} = \frac{AD}{AB}$ (corr sides in same ratio) $\frac{ED}{10} = \frac{15+18}{15}$ $ED = \frac{33}{15} \times 10 = 22$ cm	**(i)** **3 marks for correct conclusion with right similarity test and all steps of reasoning given.** **2 marks for correct conclusion with one step of reasoning incorrect or not provided or if congruence uses wrong test or conclusion is incorrect or not provided** **1 mark if at least two correct statements needed for the proof are provided** **(ii)** **2 marks if answer is calculated correctly with reasoning/working provided** **1 mark if answer is calculated incorrectly with some correct working/reasoning**

Question	Answer	Marks
	(c) (i) in ΔAQP and ΔBQP $AP = BP$ (equal radii of circle centre P) $AQ = BQ$ (equal radii of circle centre Q) PQ is common $\therefore \Delta AQP \equiv \Delta BQP$ (SSS) $\therefore \quad \angle PAQ = \angle PBQ$ (corresp $\angle$ in cong Δ 's)  (ii) Quadrilateral $AQBP$ is a kite. Reason : There are two pairs of adjacent sides equal formed by the radii of the two circles.	**(i)** **3 marks for correct conclusion with right congruence test and all steps of reasoning given.** **2 marks for correct conclusion with one step of reasoning incorrect or not provided or if conclusion uses wrong test** **1 mark if at least one correct statement needed for the proof is provided** **(ii)** **1 mark each for name of quadrilateral and any valid reason which quotes a property unique to a kite.**