MINDARIE
SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS NOW

MATHEMATICS: SPECIALIST 3 & 4

SEMESTER 2 2016

TEST 6

Resource Free

Reading Time: 2 minutes
Time Allowed: 1~~7~~6 minutes

Total Marks: 1~~7~~6

1. [3, ~~5~~4 marks]

A particle moves in a straight line and, at time t, the displacement from a fixed origin is x, while v denotes the velocity $\frac{dx}{dt}$.

(a) If $v = 3t^2 - 4$, and $x = 0$ when $t = 0$, find the value of t for which x next has the value 0.

$x = t^3 - 4t + c$

$c = 0$

$x = t^3 - 4t$ ✓

$0 = t(t^2 - 4)$ ✓

$\therefore t = 0$ or ± 2

So $t = 2$ sec ✓

(b) If the particle moves in the region $x < 2$, and $v = (x-2)^2$, find the acceleration when $v = 4$.

$\frac{dv}{dx} = 2(x-2)$

$a = v\frac{dv}{dx} = 2(x-2)^3$ ✓

$a = 2(-2)^3$

$= -16$ m/s^2 ✓

$4 = (x-2)^2$ ✓

$\pm 2 = x - 2$

$x = 0$ or $\cancel{4}$ ✓

2. [2, 3, 2, 2 marks]

(a) A researcher was constructing a 95% confidence interval for the average length of time it took for subjects to complete a puzzle. Would increasing the number of subjects increase or decrease the width of the confidence interval? Justify your answer.

Decrease as error $= \frac{\sigma}{\sqrt{n}}$.

as n increases, error (& hence CI) will decrease.

(b) Given that a 92% confidence interval corresponds to a z-score of 1.75, determine a 92% confidence interval for the average length of a piece of string given that 100 pieces of string had a mean length of 30 cm with a standard deviation of 16 cm.

$E = 1.75 \times \frac{16}{\sqrt{100}}$

$= 1.75 \times 1.6$

$= 2.8$ 1.6 1.2

So 27.2 to 32.8

(c) The length of time between phone calls at a call centre is exponentially distributed with both a mean and a standard deviation of 5 minutes. The length of time between ~~100~~ 64 phone calls is recorded. Let $\bar{T}$ be the average time between these phone calls. Describe the distribution that a large number of samples of $\bar{T}$ would follow.

Normal distribution with mean $= 5$

& sd $= \frac{5}{\sqrt{64}}$

$= 5/8$ min

(d) When a fair, standard ten-sided die is rolled, then the outcome forms a uniform probability distribution, with a mean of 5.5 and a standard deviation of 2.8723 (to 4 d.p.)

In an experiment, n fair, standard, ten-sided dice are rolled and the mean for the rolls is calculated. The experiment is repeated 100 times, and the mean and standard deviation of the means is calculated.

Andrew, Kelly and Patsy each performed the above procedure. One used 10 dice, another 30 and another 50 (that is, $n = 10$ or $n = 30$ or $n = 50$).

Andrew's results were:	mean = 5.35 and	standard deviation = 0.8954
Kelly's results were:	mean = 5.43 and	standard deviation = 0.3871
Patsy's results were:	mean = 5.48 and	standard deviation = 0.5794

Who was the person who was *most likely* to have rolled 50 dice (ie $n = 50$)? Justify your answer.

Kelly as her s.d. is smallest.

sd. of means $= \frac{\sigma}{\sqrt{n}}$

So as $n \uparrow$, $\sigma_{\bar{x}} \downarrow$.

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Resource Assumed

Reading Time: 3 minutes
Time Allowed: 26 minutes

Total Marks: 26

3. [3, 2 marks]

(a) A particle moves with Simple Harmonic Motion. If its acceleration at distance s metres from the mean position is c, prove that the period of the motion is $2\pi\sqrt{\frac{s}{c}}$.

$\ddot{x} = -k^2 x$

when $x = +s$, $\ddot{x} = -c$ ✓
(or $x = -s$, $\ddot{x} = +c$)

$-c = -k^2 s$

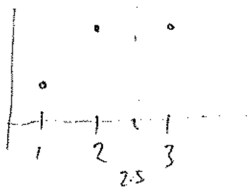
$\therefore k^2 = \frac{c}{s}$ $\quad k = \sqrt{\frac{c}{s}}$ ✓

$T = \frac{2\pi}{k}$

$= \frac{2\pi}{\sqrt{\frac{c}{s}}}$

$= 2\pi\sqrt{\frac{s}{c}}$ ✓

(b) At the end of three successive seconds the distances of a particle moving with Simple Harmonic Motion as measured from its mean position are 1, 5 and 5 metres. Find the period of the motion.



max @ 2.5 ✓ $\Rightarrow T = 10$ sec. ✓

4. [1, 3, 3, 2 marks]

A major bank was concerned that customers who rang the customer service phone line were waiting too long to be served. A random sample of 90 phone calls found that the wait time was normally distributed with a mean of 8.3 minutes and a standard deviation of 2.1 minutes.

(a) What is the probability that a customer had to wait for between 7.5 and 12.5 minutes to be served?

$P(7.5 < t < 12.5) = 0.6256.$ ✓

(b) Construct a 95% confidence interval for the actual mean wait time to be served.

$Z_{95} = 1.96$ ✓

$E = 1.96 \times \frac{2.1}{\sqrt{90}}$

≈ 0.43 min.

$\therefore 7.87 < \bar{t} < 8.73$ min ✓ ✓

(c) How many phone calls should the bank sample in order to be 99% confident that the mean wait time was within 30 seconds of the actual mean wait time?

$Z_{99} = 2.5758$ ✓

$E < 0.5$

$2.5758 \times \frac{2.1}{\sqrt{n}} < 0.5$

$n > 117.04$ ✓

$\therefore$ 118 calls ✓

(d) The bank manager states that the average waiting time for customers who rang the customer service phone line was less than 8 minutes. Does the data support his comment? Justify your answer.

At a 95% confidence ✓ 8 min does within conf. Int.

$\therefore$ data supports comment. ✓

5. [3, 3, 2, 3 marks]

A zoologist studying the rare Bigaitch Scrubrat captures a random sample of 50 Bigaitch Scrubrats. She determines that the mean weight of these Scrubrats was 4.25 kg with a standard deviation of 0.85 kg.

(a) Calculate a 97.5% confidence interval for the true mean weight of Bigaitch Scrubrats.

$Z_{97.5} = 2.2414$ ✓

$E = 2.2414 \times \frac{0.85}{\sqrt{50}}$

$= 0.27$

$\therefore$ 3.98 ✓ to 4.52 kg ✓

Twelve years later the zoologist does a follow-up research project and captures another sample of 50 Bigaitch Scrubrats.

(b) Determine the probability that these scrubrats have a mean weight in excess of 4.3 kg. State the distribution being used with all parameters.

$\bar{X} \sim N\left(4.25, \left(\frac{0.85}{\sqrt{50}}\right)^2\right)$ ✓ ✓

$P(\bar{X} > 4.3) = 0.3387$ ✓

The zoologist discovers that these scrubrats have a mean of 4.55 kg with a standard deviation of 0.93 kg.

(c) State, with reason, what the zoologist could conclude about this second sample of Bigaitch Scrubrats.

4.55 kg is not within the interval. ✓

At a 97.5% confidence level, the weights are significantly different ✓

(d) How big a sample of scrubrats would the zoologist need to take if she wanted a 90% confidence interval of the true mean to have a total width of 0.2 kg?

$Z_{90} = 1.645$ ✓

$E = 0.1$ ✓

$0.1 < 1.645 \times \frac{0.93}{\sqrt{n}}$

$\therefore n > 235.2$ ✓

$\therefore 236$ ✓