Calculator Assumed
Applications of Differentiation – Mixed Problems

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 3 = 5 marks]

(a) The gradient function of $f(x)$ is given by $f'(x) = -ax^2 + 2$. If $f(x)$ has a stationary point at (2, 10), find the value of *a*.

(b) If the gradient function is given by $\frac{dy}{dx} = kx - 2$, determine the value of *k* if the function passes through the origin and the point (-1, 3.5)

Question Two: **[7 marks]**

Determine all the maximum and minimum turning points of $f(x) = \frac{x^3}{3} - ax$, giving your answers in terms of a.

Question Three: [1, 2, 2, 1, 2 = 8 marks]

The revenue obtained from producing and selling x items of a product, $\$R$ is given by $R(x) = (x+5)^2 + 4$.

(a) Calculate the revenue for producing and selling 10 items.

(b) Use the revenue function given to calculate the revenue for producing and selling the 11th item.

(c) Calculate the average revenue per item when 11 items are produced and sold.

(d) Determine an expression for marginal revenue.

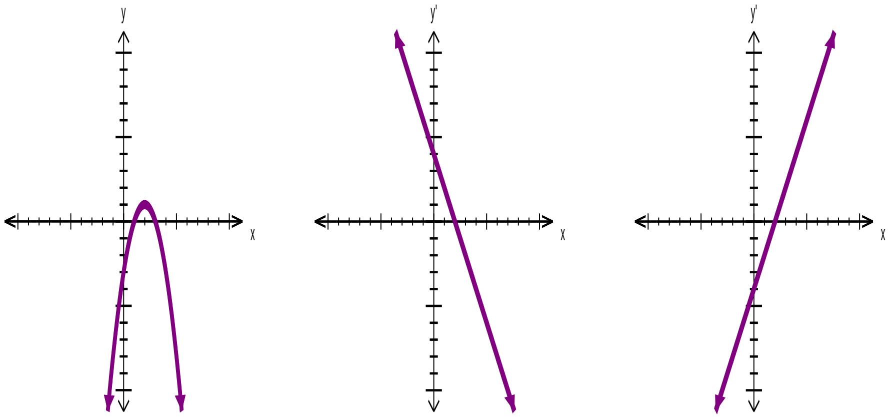
(e) Hence calculate $R'(10)$ and interpret your answer.

Question Four: [2, 3 = 5 marks]

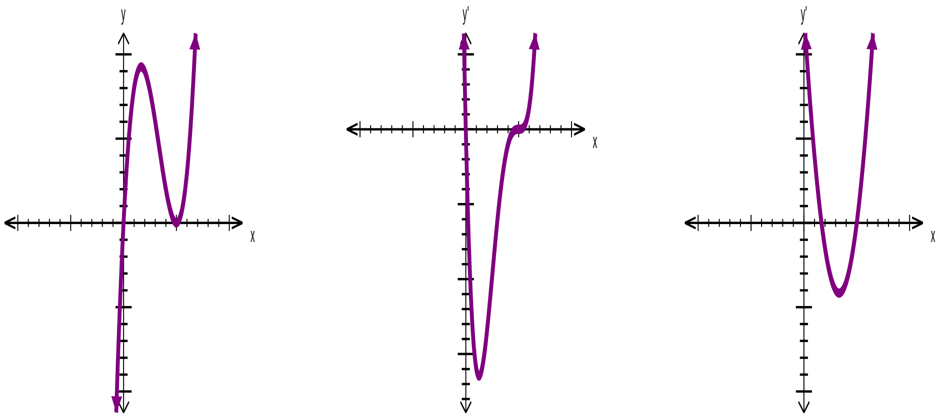
In each of the situations below, a function graph is given and then two possibilities for the gradient function graph.

Circle the correct gradient function to match the function graph and explain your reasoning.

(a)



(b)



Question Five: **[5, 2, 3 = 10 marks]**

The displacement of a body moving in linear motion is given by $x = 2t^3 + at^2 + bt + c$ metres where t is time in seconds.

The initial velocity is -4 m/s and the initial displacement from the origin is 5 m. The object changes direction when t = 2.

(a) Determine the values of a, b and c.

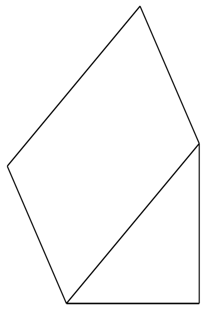
(b) Determine when the body again returns to its initial position.

(c) Calculate the distance travelled by the body in the first 4 seconds.

Question Six: [2, 3, 5 = 10 marks]

The frame of a right triangular prism is to be constructed using wire. The hypotenuse is 1 cm more than the length of the base, x cm. The length of the prism is twice the height, h cm.

(a) Label all sides of the diagram provided below.



(b) The length of wire to be used to construct the prism is 154 cm long. Use this information to write a constraint for h in terms of x.

(c) Use calculus methods to determine the dimensions of the prism which achieve maximum volume.

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SOLUTIONS
Calculator Assumed
Applications of Differentiation – Mixed Problems

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Question One: [2, 3 = 5 marks]

(a) The gradient function of $f(x)$ is given by $f'(x) = -ax^2 + 2$. If $f(x)$ has a stationary point at (2, 10), find the value of a.

$$-a(2)^2 + 2 = 0 \checkmark$$
$$-4a + 2 = 0$$
$$-4a = -2$$
$$a = \frac{1}{2} \checkmark$$

(b) If the gradient function is given by $\frac{dy}{dx} = kx - 2$, determine the value of k if the function passes through the origin and the point (-1, 3.5)

$$y = \frac{kx^2}{2} - 2x + c \checkmark$$
$$0 = \frac{k(0)^2}{2} - 2(0) + c$$
$$\therefore c = 0 \checkmark$$
$$3.5 = \frac{k(-1)^2}{2} - 2(-1)$$
$$3.5 = \frac{k}{2} + 2$$
$$1.5 = \frac{k}{2}$$
$$k = 3 \checkmark$$

Question Two: [7 marks]

Determine all the maximum and minimum turning points of $f(x)=\dfrac{x^3}{3}-ax$, giving your answers in terms of a.

$f'(x)=x^2-a$ ✓
$x^2-a=0$ ✓
$x^2=a$
$x=\pm\sqrt{a}$ ✓
$f''(x)=2x$
$f''(-\sqrt{a})=-2\sqrt{a}<0 \therefore \text{max}$ ✓
$f''(\sqrt{a})=2\sqrt{a}>0 \therefore \text{min}$ ✓

$$f(\sqrt{a})=-\frac{2a^{\frac{3}{2}}}{3}$$

$$f(-\sqrt{a})=\frac{2a^{\frac{3}{2}}}{3}$$

$$\text{min } \left(\sqrt{a},\frac{-2a^{\frac{3}{2}}}{3}\right)$$ ✓

$$\text{max } \left(-\sqrt{a},\frac{2a^{\frac{3}{2}}}{3}\right)$$ ✓

Question Three: [1, 2, 2, 1, 2 = 8 marks]

The revenue obtained from producing and selling x items of a product, $\$R$ is given by $R(x)=(x+5)^2+4$.

(a) Calculate the revenue for producing and selling 10 items.

$R(10)=(15)^2+4=\$229$ ✓

(b) Use the revenue function given to calculate the revenue for producing and selling the 11th item.

$R(11)-R(10)=260-229=\$31$ ✓
✓

(c) Calculate the average revenue per item when 11 items are produced and sold.

$$\frac{R(11)-R(0)}{11-0}=\frac{260-29}{11}=\$21$$ ✓
✓

(d) Determine an expression for marginal revenue.

$R'(x)=2x+10$ ✓

(e) Hence calculate $R'(10)$ and interpret your answer.

$R'(10)=2\times10+10=30$ ✓

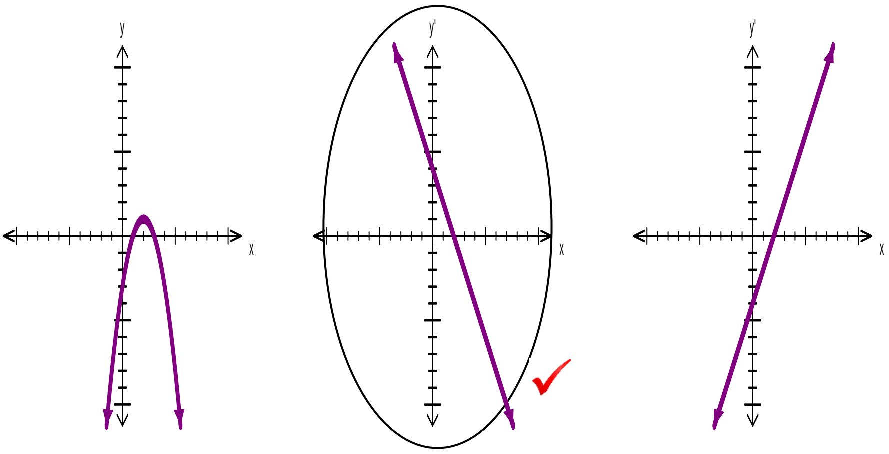
The marginal or approximate extra revenue for producing and selling the 11th item is \$30. ✓

Question Four: [2, 3 = 5 marks]

In each of the situations below, a function graph is given and then two possibilities for the gradient function graph.

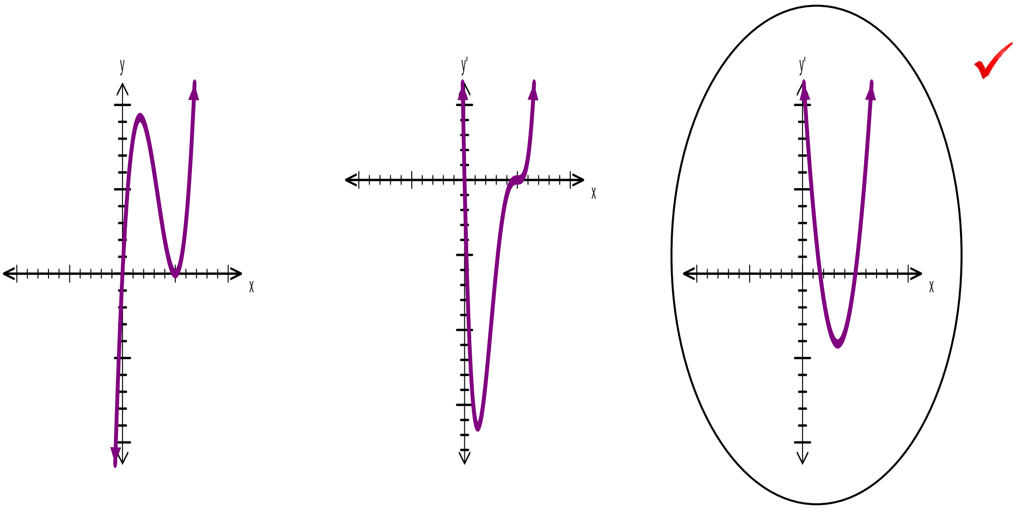
Circle the correct gradient function to match the function graph and explain your reasoning.

(a)



The parabola first has a positive gradient and hence the gradient function graph must first be above the x-axis. The gradient is horizontal momentarily, hence passing through the x-axis on the gradient graph, and then the gradient is negative and must be below the x-axis on the gradient graph.

(b)



The original function is a cubic and so the gradient function will be a quadratic. The two stationary points on the original function are x-intercepts on the gradient graph and the positions above and below the x-axis on the gradient graph match the gradients on the original.

Question Five: **[5, 2, 3 = 10 marks]**

The displacement of a body moving in linear motion is given by $x = 2t^3 + at^2 + bt + c$ metres where t is time in seconds.

The initial velocity is -4 m/s and the initial displacement from the origin is 5 m. The object changes direction when t = 2.

(a) Determine the values of a, b and c.

$c = 5$ ✓

$\frac{dx}{dt} = 6t^2 + 2at + b$ ✓

$6(0)^2 + 2a(0) + b = -4$

$b = -4$ ✓

$6(2)^2 + 2a(2) - 4 = 0$ ✓

$4a = -20$

$a = -5$ ✓

(b) Determine when the body again returns to its initial position.

$2t^3 - 5t^2 - 4t + 5 = 5$ ✓

$t = 3.14s$

✓

(c) Calculate the distance travelled by the body in the first 4 seconds.

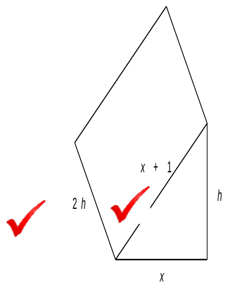
$D = 5 + 7 + 7 + 37 = 56m$ ✓

✓ ✓

Question Six: [2, 3, 5 = 10 marks]

The frame of a right triangular prism is to be constructed using wire. The hypotenuse is 1 cm more than the length of the base, x cm. The length of the prism is twice the height, h cm.

(a) Label all sides of the diagram provided below.



(b) The length of wire to be used to construct the prism is 154 cm long. Use this information to write a constraint for h in terms of x.

$$2x + 2(x+1) + 2h + 3(2h) = 154$$
$$2x + 2x + 2 + 2h + 6h = 154$$
$$8h = 152 - 4x$$
$$h = 19 - 0.5x$$

(c) Use calculus methods to determine the dimensions of the prism which achieve maximum volume.

$$V = 0.5x \times h \times 2h$$
$$V = x(19 - 0.5x)^2$$
$$\frac{dV}{dx} = 0.25(3x^2 - 152x + 1444)$$
$$0.25(3x^2 - 152x + 1444) = 0$$
$$x = 38,\ 12.67$$
$$x = 12.67cm \text{ max}$$
$$h = 19 - 0.5(12.67) = 12.665cm$$
$$L = 2 \times 12.665 = 25.33cm$$