Full Name: Mrs Griffin

Mathematics Applications
YEAR 12

Investigation 3 – Finance

Semester 2 2017

Take Home Section

Time allowed: One week

Marks Available: No marks are allocated toward this section.

Materials required: Writing implements, correction fluid/tape or eraser, ruler, Scientific or CAS calculator

Instructions:

1. Write your answers in the spaces provided in this Question/Answer Booklet.
2. **Show all your working clearly in preparation for the Validation Test.** Your working should always be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Remember in the Validation Test, incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.
3. CAS calculators will be permitted to be used during the validation test.
4. No notes will be allowed.

This investigation examines the concept of interest.

Definition of terms

Define each of the following terms, in relation to interest, to clarify your own understanding:

Principal: The amount of money you borrow or invest.

Interest: Calculated as a percentage of the principal and added to the principal (flat rate interest)

Reducible Interest: Interest is calculated on the amount you owe. Interest will decrease over time.

Repayment: The regular payment that must be made

Per annum: each year.

Quarterly: four time periods per year

Monthly: occurring once a month - twelve per year.

Owing: amount of money yet to be paid.

Opening Balance: the amount of money owing at the start of a specified time period.

Closing Balance: the amount of money remaining at the end of a specified time period

Loan: the amount of money that is borrowed.

Compound: ie compound interest → getting interest added at regular periods and then having the next interest instalment calculated ie get interest on your interest.

Interest Free: not having any extra charges added to the principal during a time period.

Final Payment: the amount of the last payment required to "clear" the loan ie no outstanding balance.

Reducible Interest

When repayments are made on the money owing, the interest can be of the reducible type. The interest rate stays the same, but the amount of interest paid will be reduced, because repayments are being made. A housing loan is an example of a reducible loan.

Example 1 – Interest per annum

A loan of $15 000 is needed for a new car. The interest rate is 12% per annum and added yearly. Repayments of $3000 are made each year. How much is owed after 2 years?

Time	Principal	Interest $\frac{12}{100}$	Principal + Interest	Repayment	Amount owing
1st year	15 000	15 000 x 0.12 = 1 800	15 000 + 1 800 = 16 800	3000	16 800 – 3000 = $13 800
2nd year	13 800	13 800 x 0.12 = 1 653	13 800 + 1 653 = 15 456	3000	15 456 – 3000 = $12 456

After 2 years $12 456 is owed

Example 2 – Interest per quarter

quarter (see table)

A loan of $15 000 is needed for a new car. The interest rate is 3% per quarter and added quarterly. Repayments of $750 are made each year. How much is owed after 2 years?

Time	Principal	Interest $\frac{12}{100} \div 4$	Principal + Interest	Repayment	Amount owing
1st quarter	15 000	15 000 x 0.03 = 450	15 000 + 450 = 15 450	750	15 450 – 750 = $14 700
2nd quarter	14 700	14 700 x 0.03 = 441	14 700 + 441 = 15 141	750	15 141 – 750 = $14 391
3rd quarter	14391	431.73	14 822.73	750	14 072.73
4th quarter	14 072.73	422.18	14 494.91	750	13 744.91
5th quarter	13 744.91	412.35	14 157.26	750	13 407.26
6th quarter	13 407.26	402.22	13 809.48	750	13 059.48

7th quarter	13 059.48	391.78	13 451.26	750	12 701.26
8th quarter	12 701.26	381.04	13 082.30	750	12 332.30

After 2 years $12 332.30 is owed

Both of the examples used the same Principal and time period.

Describe the difference in the repayment and the interest rate.

Repayments are made more often, so that
* the interest that is calculated is calculated
on a <u>lower</u> balance.

Total repayment over 1 year is the same amount
* (750 × 4 = 3000)

Interest p.a is the same
3% per quarter = (3 × 4)% p.a.

Which example gives the best loan for the borrower? Why?

2nd loan

- more frequent repayments
- interest calculated on lower balance

2nd loan has reducible interest
not a flat rate

1. Emma borrowed $10 000 to buy her car. Interest is charged on the opening balance each month at a rate of 9% per annum. Emma repays $ 1000 each month (except for the final payment). The final payment cannot exceed the regular payments. The table below shows Emma's account over the life of the loan.

What is the interest rate used in the table?

9% p.a = $\frac{9}{12}$ % per month ⟹ Rate = 0.75 % per month

Why is this rate different to the annual rate?

Interest is reducible and calculated monthly.

Month	Opening Balance	Interest	Repayment	Closing Balance
1	$10 000.00	$75	$1 000.00	$9 075.00
2	$9 075.00	$68.06	$1 000.00	$8 143.06
3	$8 143.06	$61.07	$1 000.00	$7 204.14
4	$7 204.14	$54.03	$1 000.00	$6 258.17
5	$6 258.17	$46.94	$1 000.00	$5 305.10
6	$5 305.10	$39.79	$1 000.00	$4 344.89
7	$4 344.89	$32.59	$1 000.00	$3 377.48
8	$3377.48	$25.33	$1000	2402.81
9	$2402.81	$18.02	$1000	1420.83
10	$1420.83	$10.66	$1000	431.49
11	$431.49	$3.24	$434.73	0.00
12				
13				

a. Complete the table above to find how long Emma takes to repay the loan. State the amount of the final payment.

Emma takes 11 months to repay the loan.
Final payment = $434.73

b. How much interest would Emma have paid for the loan? Show clearly how you obtained your answer.

Interest = $434.73

(total of interest column)

2. Dan borrows $5 000 from the bank to purchase a car. The annual rate is advertised at 8% per annum for his personal loan.

Calculate the monthly interest rate.

$\frac{8}{12}$ % pa = $\frac{2}{3}$ % per month

Dan repays $ 200 at the end of each month. Interest is calculated monthly. The following table shows the progress of his loan on a monthly basis.

Month	Opening Balance	Interest for the month	Repayment	Amount owing at end of month
1	$5 000.00	$33.33	200	$4833.33
2	$4 833.33	**a** 32.22	200	**b** 4665.55
3	4665.55	$31.10	200	$4496.66
4	$4496.66	$29.98	200	$4326.64
5	$4326.64	$28.84	200	$4155.48
6	$4155.48	$27.70	200	$3983.18
7	$3983.18	$26.55	200	$3809.74
8	$3809.74	$25.40	200	$3635.14
9	$3635.14	$24.23	200	$3459.37
10	$3459.37	$23.06	200	$3282.43
11	$3282.43	$21.88	200	$3104.32
12	$3104.32	$20.70	200	$2925.01
13	$2925.01	$19.50	200	$2744.51
14	$2744.51	$18.30	200	$2562.81
15	$2562.81	$17.09	200	$2379.90
16	$2379.90	$15.87	200	$2195.76
17	$2195.76	$14.64	200	$2010.40
18	$2010.40	$13.40	200	$1823.80
19	$1823.80	$12.16	200	$1635.96
20	$1635.96	$10.91	200	$1446.87
21	$1446.87	$9.65	200	$1256.51
22	$1256.51	$8.38	200	$1064.89
23	$1064.89	$7.10	200	$871.99
24	$871.99	$5.81	200	$677.80
25	$677.80	$4.52	200	$482.32
26	$482.32	$3.22	200	$285.54
27	$285.54	$1.90	200	87.44
28	**c** 87.44	$0.58	**d** 88.02	0

Determine the value of

a: $32·22

b: $4665·55

c: $87·44

d: $88·02

How much did Dan actually pay for the car? (including interest)

Pay = 27 × 200 + 88·02
= $5488·02

Calculate the total amount of interest that Dan paid.

Interest = 5488·02 − 5000
= $488·02

If the bank compounded interest on the daily balance, rather than charged simple interest on the monthly balance, would the total amount of interest have been less, more, or the same as above? Explain.

More as interest is being charged more often, and repayment frequency not changing

Interest $488·02 vs. $489·80

(Can check with equivalent effective rate → 8·3% pa $i_e = \left(1+\frac{i}{n}\right)^n - 1$

Dan could have borrowed the $5000 from his family on an 'interest free' basis, provided he pays it back in 2 years.

Assuming Dan makes equal monthly payments, calculate the minimum he would need to pay each month.

Payment = $5000 ÷ 24 (2 × 12 months)
= $208·33

But would need to pay 208·34 to ensure $5000 is fully repaid

End of Take Home Section of the Investigation