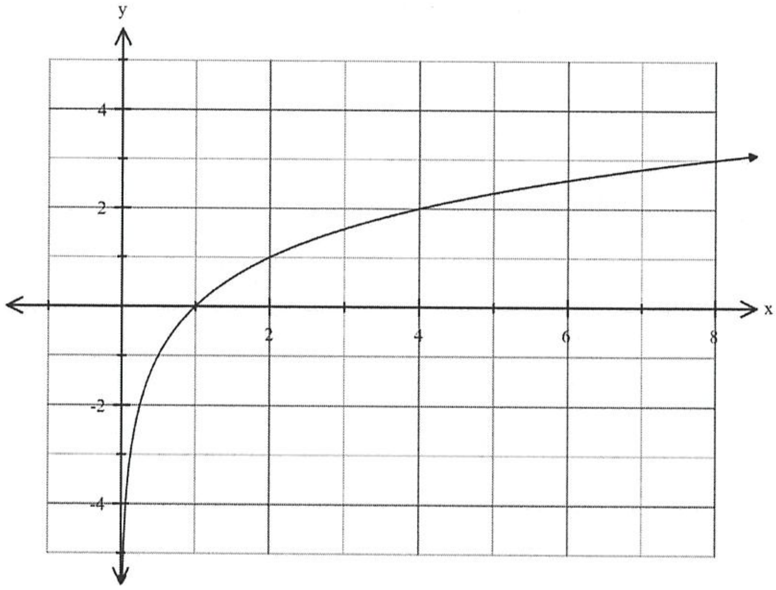
Methods Unit 4 Practice Test
(Calculator Free) Name____________________

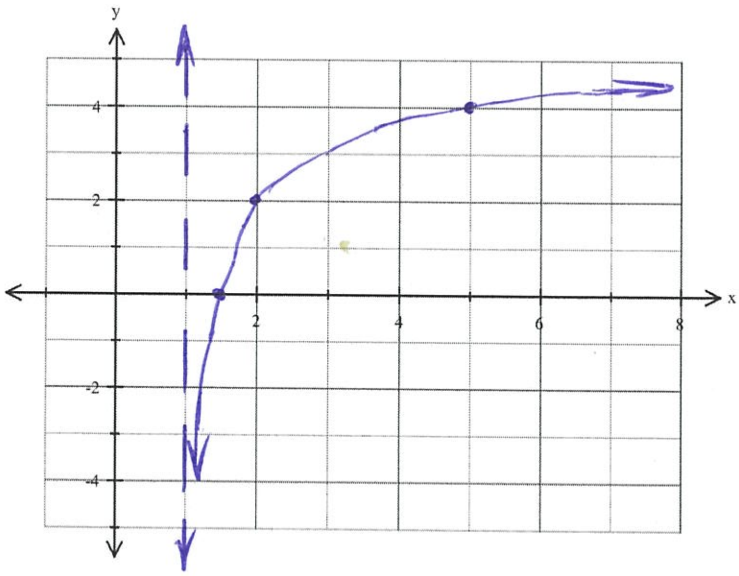
Time: 22 minutes Marks: 22

1. [2, 1 marks]

The graph below shows $y = \log_a(x)$ for $x \leq 0$.



a) Sketch $y = \log_a(x - 1) + 2$, clearly indicating any intercepts and/or asymptotes.



b) Determine a, giving the reasoning behind your answer.

$\log_a 4 = 2$

$\therefore a^2 = 4$

$a = 2$

2. [1, 2, 2 marks]

If log 2 = x and log 3 = y write in terms of x and y:

a) log 1.5 $= \log \frac{3}{2}$

$= \log 3 - \log 2$

$= y - x$

b) log 72 $= \log 2^3 . 3^2$

$= 3\log 2 + 2\log 3$

$= 3x + 2y$

c) log 1200 $= \log 2^2 . 3 . 100$

$= 2\log 2 + \log 3 + \log 100$

$= 2x + y + 2$

3. [2 marks]

Write as a single logarithm:

$$2\log_3(x) - \tfrac{1}{2}\log_3(y) + 2$$

$= \log_3 x^2 - \log_3 \sqrt{y} + \log_3 9$

$= \log \frac{9x^2}{\sqrt{y}}$

4. [4 marks]

An object moves along a straight line such that its displacement, x metres, from the origin at time t seconds, is x = 2t + ln (2t – 1).

Find its velocity and acceleration of the object at t = 1 second.

$v = \frac{dx}{dt} = 2 + \frac{2}{2t-1}$

If $t = 1$, $v = 4$ m/sec

$a = \frac{dv}{dt} = -\frac{4}{(2t-1)^2}$

If $t = 1$, $a = -4$ m/sec^2

5. [4 marks]

Show that $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\cos 2x}{\sin 2x}$ dx = 0.5 (ln 2 - ln$\sqrt{3}$)

$$= \left[\frac{1}{2}\ln(\sin(2x))\right]_{\pi/6}^{\pi/4}$$

$$= \frac{1}{2}\ln 1 - \frac{1}{2}\ln\left(\frac{\sqrt{3}}{2}\right)$$

$$= -\frac{1}{2}\left(\ln\sqrt{3} - \ln 2\right)$$

$$= \frac{1}{2}\left(\ln 2 - \ln\sqrt{3}\right)$$

6. [1, 3 marks]

Given that y = $\log_3(x + 3)$,

a) Determine the vertical asymptote.

$x = -3$

b) Determine the x- and y-intercepts.

If $y=0$ $\log_3(x+3) = 0$

$x+3 = 3^0$

$x = -2 \Rightarrow (-2, 0)$

If $x=0$ $y = \log_3(3)$

$= 1 \Rightarrow (0, 1)$

MINDARIE SENIOR COLLEGE

Methods Unit 4 Practice Test
(Calculator Assumed) Name___________________

Time: 43 minutes Marks: 43

7. [2, 4 marks]

The temperature in a freezer once turned on is determined by the function where $T = 30 - 15e^{0.055t}$, where T is measured in degrees Celsius and t is the time in minutes after the freezer is turned on.

a) By how many degrees (correct to 2 decimal places) does the temperature drop in the first 5 minutes?

$15 - 10.25 = 4.75°C$

b) Demonstrating the use of logarithms, find how long will it will take to reach 0^oC (correct to the nearest minute).

$30 - 15e^{0.055t} = 0$

$e^{0.055t} = 2$

$\ln e^{0.055t} = \ln 2$

$0.055t = \ln 2$

$t = \ln 2 / 0.055$

$= 13$ min.

8. [5 marks]

A continuous random variable X has a p.d.f. such that

$$f(x) = \begin{cases} \frac{3(a+x-0.5x^2)}{8} & 0 \le x \le 2 \\ 0 & for\ all\ other\ values\ of\ x \end{cases}$$

Determine a.

$\int_0^2 \frac{3a + 3x - 1.5x^2}{8}\, dx = 1$

$\Rightarrow \frac{1}{8}\left[3ax + \frac{3x^2}{2} - \frac{x^3}{2}\right]_0^2 = 1$

$\Rightarrow \frac{1}{8}(6a + 6 - 4) = 1$

$6a + 2 = 8$

$6a = 6$

$a = 1$

9. [4 marks]

Find exactly the equation of the tangent to the curve y = ln $(\frac{4}{x+1})$ at the point where x = 1.

$$\frac{dy}{dx} = -\frac{1}{x+1}$$

If $x=1$ $\frac{dy}{dx} = -\frac{1}{2}$

$y = \ln 2$

$\therefore y = \frac{1}{2}x + \frac{1}{2} + \ln 2$.

10. [3, 3 marks]

a) Find algebraically the intersection of y = $\frac{3}{x}$ and y = 4 – x.

$4 - x = 3/x$

$4x - x^2 = 3$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3$ or 1

ie. $(1,3)$ or $(3,1)$

b) By calculus methods, determine exactly the area between these curves.

$$\int_1^3 4 - x - \frac{3}{x} \, dx$$

$$= \left[4x - \frac{x^2}{2} - 3\ln(x)\right]_1^3$$

$$= (7.5 - 3\ln 3) - (3.5 - 0)$$

$$= 4 - 3\ln 3$$

11. [2, 1, 2 marks]

A continuous random variable X has a pdf such that f(x) = 0.8 – 0.15x defined over interval [1, a]

a) Find a.

$$\int_1^a 0.8 - 0.15x \, dx = 1$$

Solving from Classpad

$a = 3$

Determine:

b) P(1.2 ≤ X ≤ 2.5)

0.67925

c) P(X ≤ 2 | X ≥ 1.5)

$$\frac{0.5}{0.69375}$$

$= 0.7207$

12. [2, 2, 2, 2 marks]

Let X be the time (in minutes) it takes for a student to swim 200 metres, where

$$f(x) = \begin{cases} \frac{13-2x}{8} & for\ 2 \le x \le 3 \\ 0 & elsewhere \end{cases}$$

a) Show that this is a probability density function.

$$\int_2^3 \frac{13-2x}{8}\,dx$$

$$= \left[\frac{13x}{8} - \frac{x^2}{8}\right]_2^3$$

$$= \frac{30}{8} - \frac{22}{8}$$

$$= 1 \qquad \therefore \text{It is a p.d.f.}$$

b) Determine the mean time.

$$E[x] = \int_2^3 x\left(\frac{13-2x}{8}\right)dx$$

$$= 2.48 \text{ min}$$

c) Determine the variance.

$$VAR[x] = \int_2^3 \left(\frac{13-2x}{8}\right)(x-2.48)^2\,dx$$

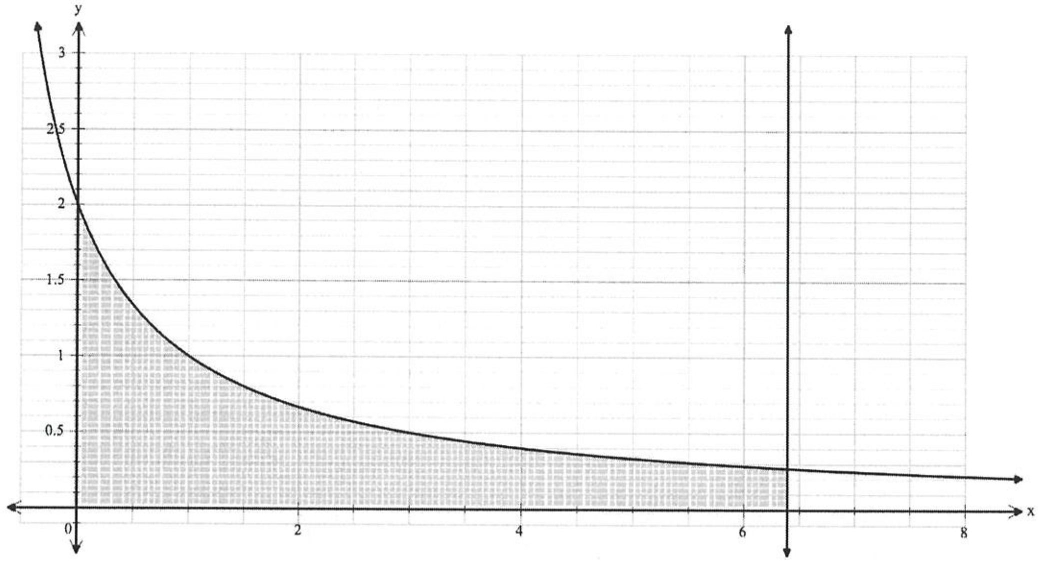
$$= 0.0829 \text{ min}$$

d) State the mean time and variance in seconds.

$E[x] = 148.75$ sec

$VAR[x] = 298.44$ sec.

13. [5 marks]



The area between y = $\frac{2}{x+1}$, the x-axis, the y-axis and x = k is 4 square units. Determine k exactly, showing all reasoning.

$$\int_0^k \frac{2}{x+1}\,dx = 4$$
$$\left[2\ln(x+1)\right]_0^k = 4$$
$$2\ln(k+1) - 2\ln 1 = 4$$
$$2\ln(k+1) = 4$$
$$\ln(k+1) = 2$$
$$k+1 = e^2$$
$$\therefore\ k = e^2 - 1$$

14. [4 marks]

If there are x octaves between a note of f_1 Hertz (Hz) and one of f_2 Hz then x = $\frac{1}{log2} \cdot log\frac{f_2}{f_1}$

If there is a frequency range of 3 octaves between f_1 and f_2 where f_1 = 100 Hz, show that f_2 = 800 Hz.

$$\frac{1}{\log 2} \cdot \log\left(\frac{f_2}{100}\right) = 3$$
$$\log\left(\frac{f_2}{100}\right) = 3\log 2$$
$$= \log 8$$
$$\therefore\ \frac{f_2}{100} = 8$$
$$f_2 = 800\ Hz$$