Calculator Assumed
Arithmetic and Geometric Sequences
Mixed Applications 3

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [4, 3 = 7 marks]

(a) Determine the first term and the tenth term of the arithmetic sequence with $T_2 = 21$ $S_3 = 63$ and $S_4 = 106$.

(b) Determine the possible values for x if the following terms represent the first three terms in a geometric sequence: $x+5,\ 8x,\ 20x+12$

Question Two: [1, 2, 2, 2, 2 = 9 marks]

The value of a car bought in 2007 depreciates by 7% each year. A Mitsubishi Lancer was bought was for $24 000 in 2007.

(a) What was the value of the car in 2008?

(b) Write a general rule which defines the value of the Lancer each year after 2007.

(c) When will the Lancer first be worth less than $15000?

(d) How much value has the car lost in total by 2018?

(e) Can this model of depreciation be used indefinitely? Explain your answer.

Question Three: [2, 1, 1, 2, 3 = 9 marks]

Little Lucy saves some of her pocket money each week and puts it into her piggy bank. By the end of the 10th week she has $65 saved and by the end of the 26th week she has $145 saved. She always saves the same amount.

(a) How much is she putting away into savings each week?

(b) How much did she have in her piggy bank in the first week?

(c) How much will she have saved in total by the end of the year?

(d) If Little Lucy is saving up for something costing $500, when will she have enough money?

(e) If instead Little Lucy had been putting away double the amount she is currently saving, how much time would she save on reaching her goal of $500?

Question Four: [1, 2, 2, 2 = 7 marks]

A ball is dropped from a height of 3 m. On each bounce back up the ball reaches a height 80% of its previous height.

(a) Determine the height reached on the first bounce.

(b) Write a recursive rule defining the height of each bounce after the first bounce.

(c) Calculate the total vertical distance travelled by the ball after 10 bounces.

(d) If the ball bounces indefinitely, calculate the total vertical distance travelled.

Question Five: **[4 marks]**

Martin is on a diet. He currently weighs 95 kg. In the first week of the diet he loses 2 kg. In the second week he loses 1.7 kg and in the third week 1.445 kg. If his weight loss continues in this way, how much can he expect to weigh at the end of his weight loss journey?

Question Six: **[4 marks]**

Over the next 10 years, the average rate of inflation is expected to be 2.8% p.a.

The Jones family currently spend $450 a week on groceries for their family and estimate that they will need to increase their weekly spending by $12 each year to cover inflation.

Are they correct? Show full mathematical reasoning to support your answer.

Question Seven: [5 marks]

The fourth term of a geometric sequence is 3125 and the sum of the 5^{th}, 6^{th} and 7^{th} terms is 76171.875.

Determine a recursive rule for this geometric sequence.

education equals

SOLUTIONS
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Question One: **[4, 3 = 7 marks]**

(a) Determine the first term and the tenth term of the arithmetic sequence with $T_2 = 21$ $S_3 = 63$ and $S_4 = 106$.

$T_4 = 106 - 63 = 43$ ✓
$21 + 2d = 43$
$2d = 22$
$d = 11$ ✓
$a = 21 - 11 = 10$ ✓
$T_{10} = 10 + 11(10 - 1) = 109$ ✓

(b) Determine the possible values for x if the following terms represent the first three terms in a geometric sequence: $x+5,\ 8x,\ 20x+12$

$$\frac{20x+12}{8x} = \frac{8x}{x+5}$$ ✓

$x = 3,\ x = \dfrac{-5}{11}$

✓ ✓

Question Two: [1, 2, 2, 2, 2 = 9 marks]

The value of a car bought in 2007 depreciates by 7% each year. A Mitsubishi Lancer was bought was for \$24 000 in 2007.

(a) What was the value of the car in 2008?

$24000 \times 0.93 = \$22320$ ✓

(b) Write a general rule which defines the value of the Lancer each year after 2007.

$V_n = 24000 \times 0.93^n$

✓ ✓

(c) When will the Lancer first be worth less than \$15000?

$V_n < 15000$ ✓

$n = 7$

2014 ✓

(d) How much value has the car lost in total by 2018?

$V_{11} = 10802.49$ ✓

$24000 - 10802.49 = \$13197.51$ ✓

(e) Can this model of depreciation be used indefinitely? Explain your answer.

✓ No. Eventually the value of the car will have a value of \$0 using this model, and the car will retain some value, even if only for its parts.

✓

Question Three: [2, 1, 1, 2, 3 = 9 marks]

Little Lucy saves some of her pocket money each week and puts it into her piggy bank. By the end of the 10th week she has $65 saved and by the end of the 26th week she has $145 saved. She always saves the same amount.

(a) How much is she putting away into savings each week?

$65+16d=145$ ✓

$16d=80$

$d=5$

$\$5\ a\ week$ ✓

(b) How much did she have in her piggy bank in the first week?

$a+9\times 5=65$

$a=20$

$\$20$ ✓

(c) How much will she have saved in total by the end of the year?

$T_{52}=\$275$ ✓

(d) If Little Lucy is saving up for something costing $500, when will she have enough money?

$T_n=500$ ✓

$n=97$

$After\ 97\ weeks$ ✓

(e) If instead Little Lucy had been putting away double the amount she is currently saving, how much time would she save on reaching her goal of $500?

$20+10(n-1)=500$ ✓

$n=49$ ✓

$Save\ 48\ weeks$ ✓

Question Four: [1, 2, 2, 2 = 7 marks]

A ball is dropped from a height of 3 m. On each bounce back up the ball reaches a height 80% of its previous height.

(a) Determine the height reached on the first bounce.

$3 \times 0.8 = 2.4m$ ✓

(b) Write a recursive rule defining the height of each bounce after the first bounce.

$H_n = 0.8 \times H_{n-1};\ H_1 = 2.4$
✓ ✓

(c) Calculate the total vertical distance travelled by the ball after 10 bounces.

$S_{10} \times 2 + 3 = 10.71 \times 2 + 3 = 24.42m$
✓ ✓

(d) If the ball bounces indefinitely, calculate the total vertical distance travelled.

$S_{\infty} \times 2 + 3 = 12 \times 2 + 3 = 27m$
✓ ✓

Question Five: [4 marks]

Martin is on a diet. He currently weighs 95 kg. In the first week of the diet he loses 2 kg. In the second week he loses 1.7 kg and in the third week 1.445 kg. If his weight loss continues in this way, how much can he expect to weigh at the end of his weight loss journey?

$T_n = 2 \times 0.85^{n-1}$ ✓✓

$S_\infty = 13.33$ ✓

$95 - 13.33 = 81.67kg$ ✓

Question Six: [4 marks]

Over the next 10 years, the average rate of inflation is expected to be 2.8% p.a.

The Jones family currently spend \$450 a week on groceries for their family and estimate that they will need to increase their weekly spending by \$12 each year to cover inflation.

Are they correct? Show full mathematical reasoning to support your answer.

$T_{10} = 450 \times 1.028^{10-1} = \$576.97$ ✓✓

$450 + 9 \times 12 = \$546$ ✓

No they are not correct. In the second year they will be short \$0.60 and in subsequent years they will fall short each week by increasing amounts.

✓

Question Seven: [5 marks]

The fourth term of a geometric sequence is 3125 and the sum of the 5th, 6th and 7th terms is 76171.875.

Determine a recursive rule for this geometric sequence.

$a\times r^{4-1}=3125$ ✓

$a\times r^{4}+a\times r^{5}+a\times r^{6}=76171.875$ ✓✓

$a=200 \quad r=2.5$ ✓

$T_n=2.5\times T_{n-1};\ T_1=200$ ✓