Solutions

2021 Calc free (20 mins)

Question 1 (3,3)

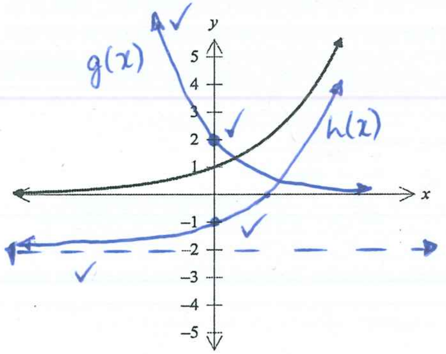
Simplify, where necessary indices to be expressed in terms of positive powers.

b) $\frac{2^{14}+2^{16}}{\sqrt[3]{8^{15}}} = \frac{2^{14}(1+2^2)}{(2^3)^{\frac{1}{3}\times 15}} = \frac{2^{14}(5)}{2^{15}} = \frac{5}{2}$ ✓

c) $\sqrt{100g^3h^4} \div (16g^2h^8)^{\frac{1}{4}} = \frac{10g^{3/2}h^2}{2g^{1/2}h^2} = 5g$ ✓

Question 3 (4,3)

The graph of $f(x) = a^x$ is drawn below.



a) **Sketch** on the axes above the following functions. Clearly label each one.

(i) $h(x) = a^x - 2$ (2)

(ii) $g(x) = 2a^{-x}$ must show to be steeper. (2)

b) Given $k(x) = -\left(\frac{a^x}{a^2}\right)$, state the transformations that must occur for $f(x)$ to become $k(x)$.

$f(x) \rightarrow k(x)$

$a^x \rightarrow -a^{x-2}$ ∴ translate 2 units right ✓

reflects about the x-axis. ✓

Question 4 (3,3,4)

Solve

a) $5x^{\frac{4}{3}} + 5x^{\frac{4}{3}} = 160$

$10x^{4/3} = 160$

$x^{4/3} = 16$ ✓

$(x^{4/3})^{3/4} = 16^{3/4}$ ✓

$x = (\pm 2)^3$

$= \pm 8$ ✓

b) $\frac{25^{x+3}}{125^{2-x}} = 5$

$\frac{5^{2(x+3)}}{5^{3(2-x)}} = 5^1$ ✓

$5^{2x+6-6+3x} = 5^1$ ✓

$5x = 1$

$x = \frac{1}{5}$ ✓

1. Consider the sequence: 15 , ____ , ____ , 405 , ...

a. Given that these terms follow an arithmetic sequence, determine the value of T_2. [3]

$T_4 = 405 = 15 + 3d$ ✓

$390 = 3d$

$d = 130$ ✓

$T_2 = 15 + 130$

$= 145$ ✓

b. Given that these terms follow a geometric sequence, determine the value of T_3. [3]

$T_4 = 405 = 15r^3$ ✓

$27 = r^3$

$r = 3$ ✓

$T_3 = \frac{405}{3}$

$= 135.$ ✓

2. Determine all the possible values of x given that $4, x, (x^2-3)$ are consecutive terms of a geometric sequence. [4]

$\frac{x}{4} = \frac{x^2-3}{x}$

$x^2 = 4x^2 - 12$

$12 = 3x^2$

$4 = x^2$

∴ $x = \pm 2$

Calc Assumed (30 mins)

Question 5 (1,4,3)

The school mixed netball team of 12, is to be chosen from 8 males and 12 females who are trialing for the team.

a) How many different teams are possible if there are no restrictions? $^{20}C_{12} = 125\,970$

b) **Show how** to determine the probability that the team selected will comprise of (do not actually calculate)

(i) 6 males and 6 females? (2)

$$\frac{^{8}C_{6} \times {}^{12}C_{6}}{125\,970}$$ ✓ ✓

(ii) there must be more women than men, but at least 4 men? (2)

$$\frac{^{8}C_{4} \times {}^{12}C_{8} + {}^{8}C_{5} \times {}^{12}C_{7}}{125\,970}$$

Tony, a male, and Angelica, a female, are two of the players trialing.

c) Determine the probability that Tony is not selected, Angelica is and there are exactly 5 males selected? (Answer to 3 sig figs)

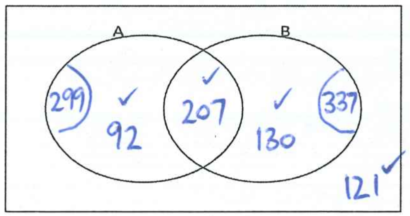
$$\frac{^{1}C_{0} \times {}^{7}C_{5} \times {}^{1}C_{1} \times {}^{11}C_{6}}{125\,970} = 0.0770$$

Question 6 (4,4)

A mobile phone retailer classified recent sales of 550 phones by the age of customer; whether they were aged under 30 or aged "30 and above" and whether they had bought the phone outright, or bought on a plan.

Of the sales, 337 were bought on a plan, 299 were from customers aged under 30,and 121 were from customers "30 and over" who had bought their phone outright.

a) Based on the information above complete the Venn diagram below where Event A occurs if the customer was aged under 30 ; event B occurs if the phone was bought on a plan.



$U = 550$ (A) = < 30 (B) = Plan.

b) A recent sale is selected at random from those recorded above. Determine the following probabilities: (leave answers in fraction form)

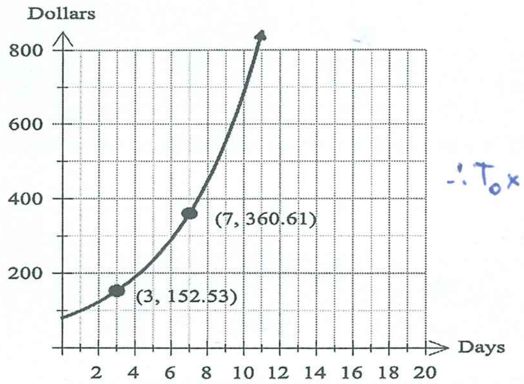
(i) $P(\bar{A})$ $\frac{130+121}{550} = \frac{251}{550}$ ✓ (1)

(ii) $P(\bar{A} \cup B)$ $\frac{251+207}{550} = \frac{458}{550}$ ✓ (1)

(iii) $P(A|\bar{B})$ $\frac{92}{92+121} = \frac{92}{213}$ ✓ ✓ (2)

Question 7 (2, 2)

A person invested in one Bitecoin and plotted the value in dollars D against the time t in days since the purchase of the coin. Two of the points on the graph are shown below.



a) What price did the investor pay for this coin?

$152.53 \times r^{7-3} = 360.61$

$r^4 = \frac{360.61}{152.53}$

$r = 1.24$

$\therefore T_0 \times r^3 = 152.53$

$T_0 = \$80$

b) On what day will this investment first exceed a value of $876?

$80 \times 1.24^t = 876$ ✓

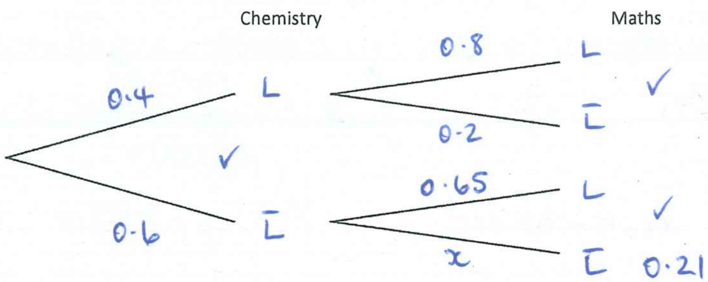
$t = 11.12$

$\therefore$ Day 12. ✓

Question 8 (4,1,3)

It was noted that on the days Llyod had Chemistry, he would be late to this class 40% of the time. His Maths teacher then noticed that if maths was next and he was late to chemistry then he would be late to maths 80% of the time. She also noted that he was on time to both classes 21% of the time.

a) Complete the tree diagram below to show the probabilities of Llyod's punctuality regarding his attendance to his maths class following his chemistry class



$\therefore 0.6x = 0.21$

$x = 0.35$ ✓

b) Determine the probability that Llyod is on time for his maths class.

$0.4 \times 0.2 + 0.21 = 0.29$ ✓

c) Given Llyod is late for Maths what is the probability he is on time for Chemistry? (Answer to 2 sig figs)

$$\frac{0.6 \times 0.65}{0.4 \times 0.8 + 0.6 \times 0.65} = 0.5493 \approx 0.55$$ ✓

Question 9 (6,3)

Events A and B occur at random, and it is known that P(A) = 0.2 and P(A ∪ B) = 0.68.

a) Determine P(B) when

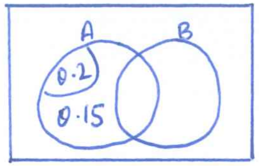
(i) A and B are mutually exclusive. (1)

$P(A) + P(B) = P(A \cup B)$

$0.2 + P(B) = 0.68$

$P(B) = 0.48$

(ii) P (A ∩ B') = 0.15 (2)



$0.15 + P(B) = P(A \cup B)$

$P(B) = 0.68 - 0.15$

$= 0.53$

(iii) A and B are independent. (3)

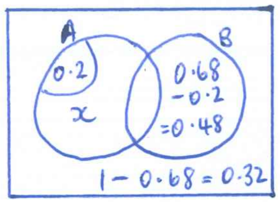
$P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

$0.68 = 0.2 + P(B) - 0.2\,P(B)$

$0.48 = 0.8\,P(B)$

$P(B) = 0.6$

b) Determine P(A ∩ B) if P(A | B') = 0.36



$P(A) = 0.2$

$\therefore P(\bar{A}) = 0.8$

$P(A|B') = \dfrac{x}{x + 0.32}$

$0.36(x + 0.32) = x$

$x = 0.18$

$P(A \cap B) = 1 - (0.18 + 0.48 + 0.32)$

$= 0.02.$

3. A geometric sequence with $T_2 = 43.75$ has a sum to infinity of 400. Determine all possible values of T_1 for this sequence. [3]

$T_2 = ar = 43.75$

$S_\infty = \dfrac{a}{1-r} = 400$

Solve simultaneous equations

$a = 50$ or $a = 350$

$\therefore T_1 = 50$ $T_1 = 350$

4. A sequence is defined by $T_{n+1} = T_n - 3.55,\ T_1 = 835$.

a. Determine T_{150}. [1]

$T_{150} = 306.05$

b. Determine the sum of the first 35 terms, S_{35}. [1]

$S_{35} = 27\,112.75$

c. Determine the value of n that will maximise S_n and state the corresponding value of S_n. Explain why this value of S_n is the maximum. [3]

$n = 236$

$S_n = 98\,618.5$

From T_{237} onwards, the values are negative

∴ the sum of the terms will start to decrease.

END OF ASSESSMENT