Solutions and marking key for Test 1 for concurrent Unit 3 and Unit 4 program

Section One – calculator-free section **(30 marks)**

Question 1 (3.1.6, 3.1.15) **(8 marks)**

(a) Given $z=\sqrt{3}+i$ evaluate z^6 giving the answer in Cartesian form. (2 marks)

Given $z=\sqrt{3}+i$ evaluate z^6

$$z^6=\left(\sqrt{3}+i\right)^6=\sqrt{3}^6+6.\left(\sqrt{3}\right)^5.(i)^1+15.\left(\sqrt{3}\right)^4.(i)^2+20.\left(\sqrt{3}\right)^3.(i)^3+15.\left(\sqrt{3}\right)^2.(i)^4+6.\left(\sqrt{3}\right)^1.(i)^5+(i)^6$$

$$=27-135+45-1+\left(54\sqrt{3}-60\sqrt{3}+6\sqrt{3}\right)i$$

$$=-64$$

OR $z=2cis\left(\frac{\pi}{6}\right)\Rightarrow z^6=64cis(\pi)=-64$

Specific behaviours	Mark	Item
Expands the Cartesian form of z^6	1	simple
Simplifies correctly	1	simple
Or		
Expresses z^6 in polar form	1	simple
Expresses the answer in Cartesian form	1	simple

(b) (4 marks)

Given $Z_1=cis\left(\frac{\pi}{3}\right)$ and $Z_2=cis\left(\frac{\pi}{4}\right)$ evaluate the following in exact Cartesian form:

(i) $\overline{Z_1}$ (ii) iZ_2 (iii) $cis\left(\frac{\pi}{12}\right)$

Given $Z_1=cis\left(\frac{\pi}{3}\right)$ and $Z_2=cis\left(\frac{\pi}{4}\right)$ evaluate the following in exact Cartesian form:

(i) $\overline{Z_1}=\frac{1-\sqrt{3}i}{2}$ (ii) $iZ_2=\frac{-\sqrt{2}+\sqrt{2}i}{2}$

(iii) $cis\left(\frac{\pi}{12}\right)=\frac{Z_1}{Z_2}=\frac{1+\sqrt{3}i}{2}\times\frac{2}{\sqrt{2}+\sqrt{2}i}=\left(\frac{\left(\sqrt{2}+\sqrt{6}\right)+\left(\sqrt{6}-\sqrt{2}\right)i}{4}\right)$

Specific behaviours	Mark	Item
Writes the Cartesian form of $\overline{Z_1}$ correctly	1	simple
Writes the Cartesian form of iZ_2 correctly	1	simple
Expresses polar term for $\overline{Z_3}$ in Cartesian form	1	complex
Simplifies the Cartesian form correctly	1	complex

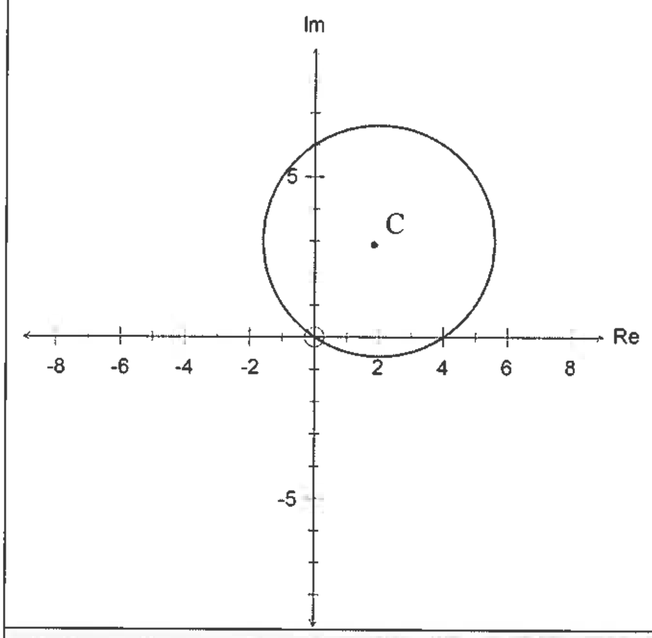
(c) Solve $x^2 - 6x + 13 = 0$ for $x \in \text{Im}$ in exact form. (2 marks)

Solve $x^2 - 6x + 12 = 0$ $\Leftrightarrow \quad x^2 - 6x + 9 = -3$ $\Leftrightarrow \quad (x-3)^2 = -3$ $\Leftrightarrow \quad (x-3)^2 = 3i^2$ $\Leftrightarrow \quad x = 3 \pm \sqrt{3}\,i$	Or Solve $x^2 - 6x + 12 = 0$ $\Leftrightarrow \quad a = 1, b = -6$ and $c = 12$ $\Leftrightarrow \quad x = \dfrac{6 \pm \sqrt{36 - 48}}{2}$ $\Leftrightarrow \quad x = \dfrac{6 \pm \sqrt{-12}}{2} = 3 \pm \sqrt{3}i$

Specific behaviours	Mark	Item
Completes the square correctly	1	simple
Solves the equation using the exact form	1	simple
Or		
Uses the quadratic formula	1	simple
Simplifies the expressions to the correct exact form	1	simple

Question 2 (1.1.7) **(6 marks)**

(a) Sketch the set of points defined by $|z - (2 + 3i)| = \sqrt{13}$.



Specific behaviours	Mark	Item
Draws a circle	1	simple
Has the correct centre *(2 + 3i)*	1	simple
Has the correct radius	1	simple
Circumference passes through *(0, 0) (4 + 0i)* and *(0 + 6i)*	3	simple

Section Two – calculator-assumed section **(20 marks)**

Question 5 (3.1.7) **(10 marks)**

(a) Expand and simplify the expression $F(\theta)=(\cos\theta+i\sin\theta)^5$. (2 marks)

(b) Hence, express the $\mathrm{Re}(F)$ in terms of $\cos\theta$. (3 marks)

(c) Use $\mathrm{Re}(F)$ to solve the equation $16x^5-20x^3+5x-1=0$ and express the solutions in trigonometric form. (5 marks)

Question 6 (3.1.7) **(10 marks)**

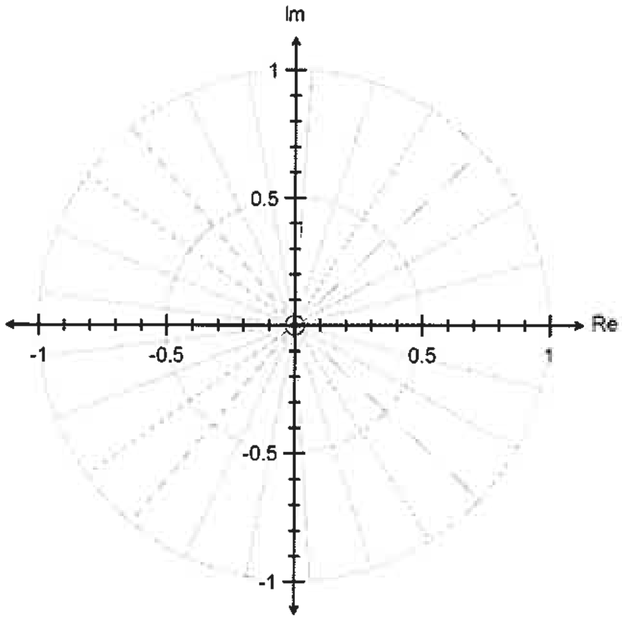
Given $z=\cos\theta+i\sin\theta$:

(a) Express $\dfrac{\left(z-\frac{1}{z}\right)}{i\left(z+\frac{1}{z}\right)}$ in trigonometric form. (4 marks)

(b) Show $z^2+\dfrac{1}{z^2}=2\cos 2\theta$ and hence prove $\cos 2\theta=2\cos^2\theta-1$. (6 marks)

Question 3 **(3.1.11, 3.1.12)** **(8 marks)**

Determine and locate all solutions in the Argand plane to the equation $z^5 = 1$.



Question 4 **(3.1.13, 3.1.15)** **(8 marks)**

Given $H(z) = z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8$

(a) Evaluate $H(i)$, $H(-i)$ and $H(2)$. (3 marks)

(b) Hence, find all roots of the equation $z^5 - 2z^4 + 5z^3 - 10z^2 + 4z - 8 = 0$. (5 marks)