SCOTCH
COLLEGE

12 Mathematics Methods 2022

Test 2 – Integration and Applications

Section 1: Calculator-Free

Time allowed: 20 minutes **Maximum marks: 20**

Name: Solutions.

Teacher: Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No Calculators and no notes are permitted.

Question 1 [6 marks]

Evaluate or determine the following definite or indefinite integrals.

a) $\int_0^2 (4x+6)(x^2+3x)^2\,dx$ [3]

$$= \left[\frac{2\cancel{(4x+6)}(x^2+3x)^3}{\cancel{(2x+3)}(3)}\right]_0^2$$

$$= \frac{2}{3}(x^2+3x)^3\Big]_0^2$$

$$= \frac{2000}{3} - 0$$

$$= \frac{2000}{3}$$

b) $\int(\sqrt{x}+\frac{2}{x^2})\,dx$[3]

$$\int x^{1/2} + 2x^{-2}\,dx$$

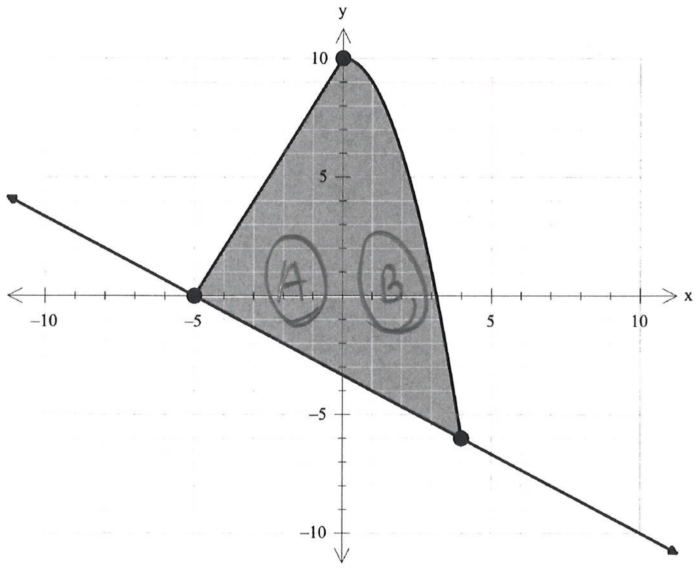
$$\frac{2x^{3/2}}{3} - 2x^{-1} + C.$$

Question 2 [6 marks]

Below is a graph with parts of three functions graphed. They are;

$$y = -x^2 + 10$$
$$y = 2x + 10$$
$$3y = -2x - 10$$

$y = -\frac{2}{3}x - \frac{10}{3}$



Showing use of calculus, determine the exact area of the shaded region.

gint $(0, \frac{-10}{3})$.

$\therefore$

Area A $\Delta = \frac{1}{2} \times 5 \times (10 + \frac{10}{3})$

$= \frac{1}{2} \times 5 \times \frac{40}{3}$

$= \frac{100}{3}$

Area B $= \int_0^4 -x^2 + 10 - (-\frac{2}{3}x - \frac{10}{3})\, dx$

$= \int_0^4 -x^2 + \frac{2}{3}x + \frac{40}{3}\, dx$

$= \left[\frac{-x^3}{3} + \frac{x^2}{3} + \frac{40}{3}x\right]_0^4$

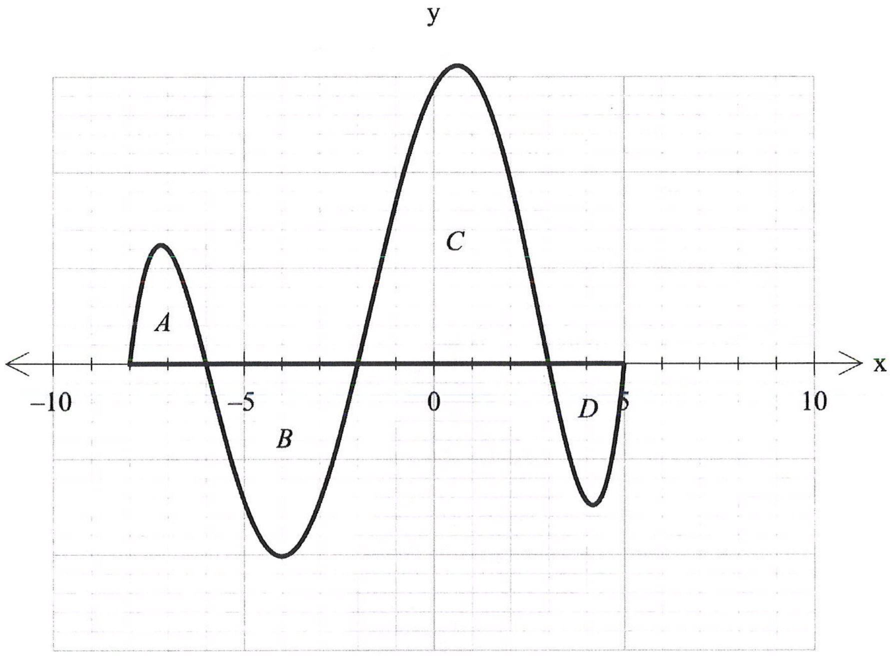
$= \left(-\frac{64}{3} + \frac{16}{3} + \frac{160}{3}\right) - (0) \Rightarrow \frac{112}{3}$

$\therefore$ T.A $= \frac{100}{3} + \frac{112}{3}$

$= \frac{212}{3}$ units^2

$(70.\dot{6})$.

Question 3 (8 marks)



Above is part of the graph of $y = f(x)$ which has x-intercepts at $-8, -6, -2, 3, 5$.
The areas of various parts of the graph are;

A = 4 square units B = 15 square units C = 23 square units D = 6 square units

Using this information, determine the following;

a) The area between $f(x)$ and the x-axis between 5 and -6. [2]

$6+23+15 = 44\,u^2$

b) $\int_{-8}^{3} f(x).\,dx$ [1]

$23-15+4$

$= 12$

c) $\int_{-2}^{5} 2f(x) - 5.\,dx$ [3]

$2\int_{-2}^{5} f(x) - \int_{-2}^{5} 5.\,dx$

$2\times(23-6) - 35$

$= -1$

d) If $\int_{-2}^{0} f(x).\,dx = 8$ determine the value of $\int_{-5}^{0} f(-x)\,dx$ [2]

$\int_{-5}^{0} f(-x).\,dx \equiv \int_{0}^{5} f(x).\,dx$

$= -6 + (23-8)$

$= 9$

-----END OF SECTION ONE-----

SCOTCH
COLLEGE

12 Mathematics Methods 2022

Test 2 – Integration and Applications

Section 1: Calculator-Assumed

Time allowed: 25 minutes **Maximum marks: 25**

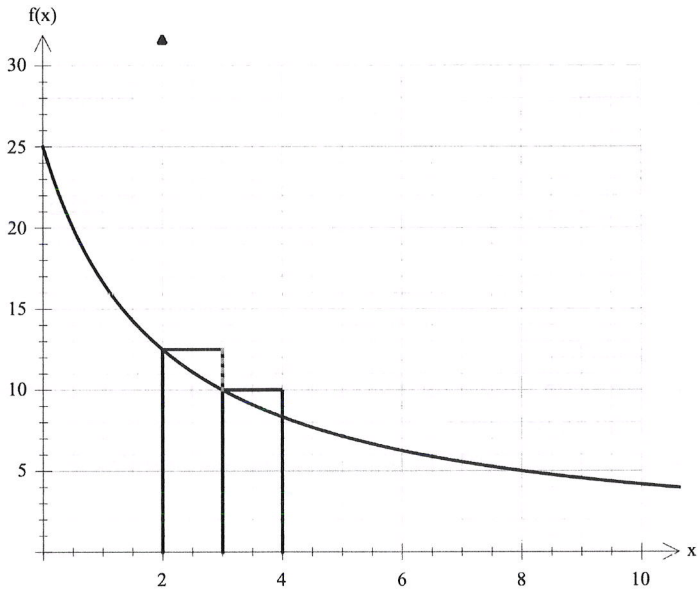
Name:______________________________ **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- Calculators and 1 page (2 sides) of personal notes are permitted.

Question 4 (5 marks)

Below is the graph of $f(x) = \frac{50}{x+2}$ with two circumscribed rectangles drawn.



The value of $\int_2^6 f(x).dx$ is exactly $50\ln 2$.

a) Using the 2 rectangles drawn, and two more of the same width, determine an overestimate for $\int_2^6 f(x)\,dx$. Give your answer correct to 2 decimal places. [2]

$f(2) = 12.5$ $f(6) = 6.25.$

$f(3) = 10$

$f(4) = \frac{25}{3}$

$f(5) = \frac{50}{7}$

$E = 1 \times \left(12.5 + 10 + \frac{25}{3} + \frac{50}{7}\right)$

$= 37.98$

Using 4 rectangles of the same width, an underestimate for $\int_2^6 f(x)\,dx$ is 31.73 correct to 2 decimal places.

b) By using your answer from part a) and the information given give a decimal estimate for $200\ln 2$. [3]

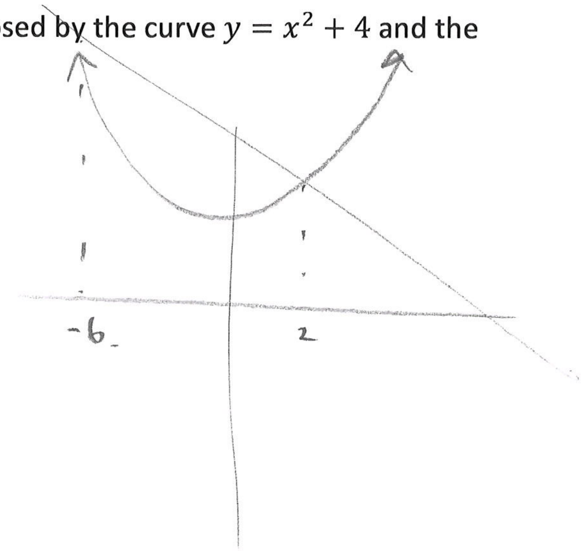
$50\ln 2 \approx \frac{31.73 + 37.98}{2}$

≈ 34.855

$\therefore 200\ln 2 \approx 139.42.$

Question 5 (4 marks)

Determine, showing the use of calculus, the area enclosed by the curve $y = x^2 + 4$ and the line $y = 16 - 4x$.



$x^2 + 4 = 16 - 4x$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$\int_{-6}^{2} 16 - 4x - (x^2+4)\, dx$

$\int_{-6}^{2} -x^2 - 4x + 12 \, dx$

$\left[\frac{-x^3}{3} - 2x^2 + 12x \right]_{-6}^{2}$

$= \frac{256}{3}$ $(85.\dot{3})$ units^2

Question 6 (8 marks)

A particle has acceleration $a(t) = 2t - 8$ and has initial velocity of jm/s and the change in displacement during the first 2 seconds is $\frac{-160}{3}$m

a) Show that $j = -20$ [3]

$v(t) = t^2 - 8t + j$

$\int_0^2 t^2 - 8t + j \, dt = -\frac{160}{3}$

$\left[\frac{t^3}{3} - 4t^2 + jt\right]_0^2 = -\frac{160}{3}$

$\frac{8}{3} - 16 + 2j = -\frac{160}{3} \Rightarrow j = -20$

b) Determine the average velocity over the first 5 seconds. [2]

Δ in disp $= \int_0^5 t^2 - 8t - 20 \, dt$

$= \frac{-475}{3}$m $(-158.\dot{3})$

$\therefore$ Ave$_v$ $= \frac{-475}{15}$ $(-31.\dot{6})$ m/s.

c) Determine the acceleration 1 second after the particle is at rest. [2]

$v(t) = (t-10)(t+2)$

$\therefore t = 10$

$\therefore a(11) = 2(11) - 8$

$= 14$ m/s^2

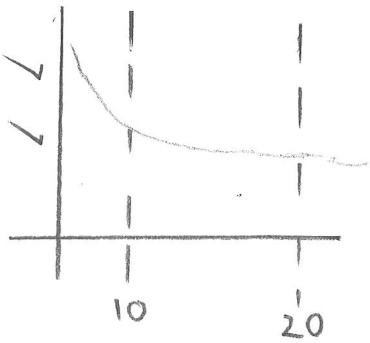
ping units once

Question 7 (9 marks)

a) By first drawing a diagram of the situation, determine the area enclosed between the x-axis, the lines $x = 10$ and $x = 20$ and the curve $f(x) = \frac{100}{x^2}$. [4]

$$\int_{10}^{20} \frac{100}{x^2} \, dx$$

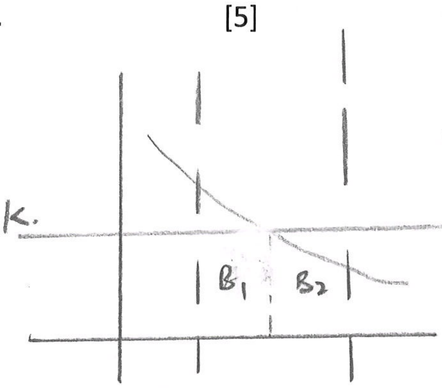
$$= 5$$



b) The line $y = k$ cuts this area into two different size areas, one above the line (Area A) and one below the line (Area B). Area B is $(8\sqrt{5} - 13)$ units^2. By showing use of integration involving Area B and $f(x)$, Determine the value of k. [5]
Hint: $0.25 < k < 1$

POI $\quad k = \frac{100}{x^2}$

$$x = \sqrt{\frac{100}{k}}$$



Rectangle B_1

$$\underbrace{\left(\sqrt{\frac{100}{k}} - 10\right) \times k}_{\text{Rectangle } B_1} + \int_{\sqrt{\frac{100}{k}}}^{20} \frac{100}{x^2} \, dx = 8\sqrt{5} - 13$$

$$10\sqrt{k} - 10k + \left[\frac{-100}{x}\right]_{\sqrt{\frac{100}{k}}}^{20} = 8\sqrt{5} - 13$$

$k = \frac{4}{5}$ (0.8) (ignore other answer).

-----END OF SECTION 2-----