MATHEMATICS:SPECIALIST 1 & 2 2017

TEST 5

Calculator Free

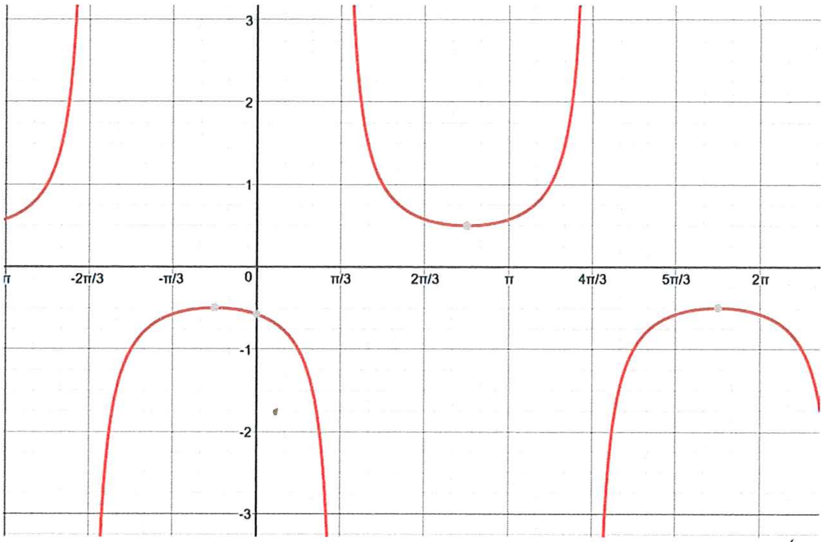
Reading time: 2 minutes

Time Allowed: 35 minutes

Total Marks: 28

1. **[4 marks]**

Find the equation of the function graphed below:



$y = \frac{1}{2}\operatorname{cosec}\left(x - \frac{\pi}{3}\right)$

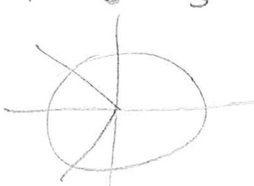
OR

$y = \frac{1}{2}\sec\left(x + \frac{\pi}{6}\right)$

2. **[5 marks]**

Solve the following equation $\sqrt{2}\sec(x + \frac{\pi}{3}) = -2,$ $0 \le x \le 2\pi$

$\frac{\pi}{3} \le x + \frac{\pi}{3} \le \frac{7\pi}{3}$



$\sec\left(x + \frac{\pi}{3}\right) = -\frac{2}{\sqrt{2}}$

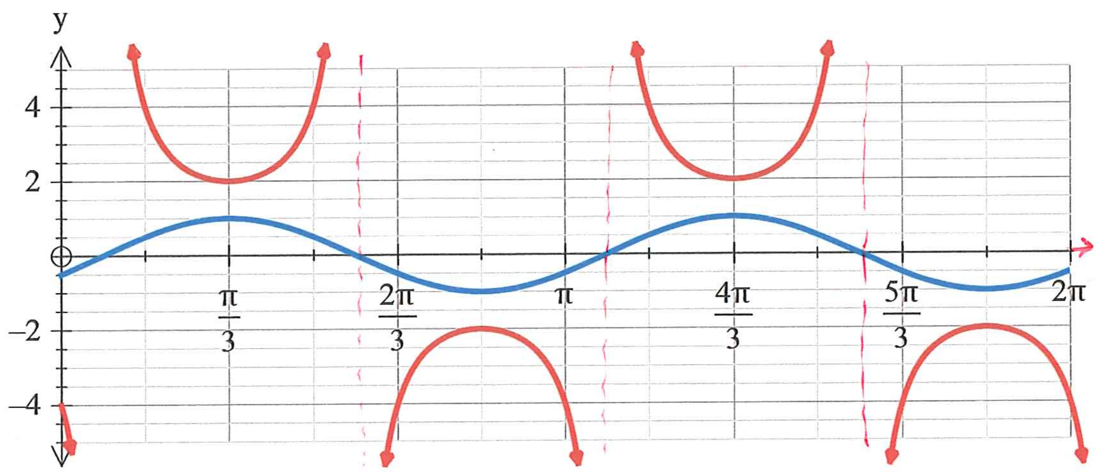
$\cos\left(x + \frac{\pi}{3}\right) = -\frac{1}{\sqrt{2}}$

$x + \frac{\pi}{3} = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$

$x = \frac{5\pi}{12}$ or $\frac{11\pi}{12}$

3. **[3, 3 = 6 marks]**

(a) Draw a sketch of $y = \cos 2(x - \frac{\pi}{3})$ $\quad 0 \le x \le 2\pi$



✓ period
✓ phase shift.
✓ domain.

✓ asymptotes
✓ TP's
✓ shape

(b) Hence draw a sketch (on the same set of axis as part 'a') of $y = 2 \sec 2(x - \frac{\pi}{3})$ $\quad 0 \le x \le 2\pi$

4. **[4 marks]**

Prove the identity $\cot A + \tan A = \sec A \operatorname{cosec} A$.

$$\text{LHS} = \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A} \checkmark$$

$$= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A} \checkmark$$

$$= \frac{1}{\sin A \cos A} \checkmark$$

$$= \sec A \operatorname{cosec} A \checkmark$$

$$= \text{RHS}.$$

5. **[2, 4 – 6 marks]**

(a) Show how to express $5.\overline{25}$ as a rational number. (2 marks)

$x = 5.\overline{25}$

$100x = 525.2525\ldots$

$x = 5.2525\ldots$ ✓

$99x = 520$

$x = \frac{520}{99}$ ✓ which is rational.

(b) Prove by contradiction that $\sqrt[3]{4}$ is an irrational number. (4 marks)

Assume that cube root of 4 is rational

so that $\sqrt[3]{4} = \frac{a}{b}$, where a, b integers with no common factors ✓

$4 = \frac{a^3}{b^3}$

$a^3 = 4b^3 = 2(2b^3)$ ✓

Hence a must be even $\Rightarrow a = 2n, n \in \mathbb{Z}$

$4b^3 = (2n)^3$

$4b^3 = 8n^3$ ✓

$b^3 = 2n^3 \Rightarrow b$ must be even

But if a & b both even, this contradicts that they have no common factors

Hence cube root of 4 is irrational

✓

6. **[2, 2 = 4 marks]**

Find **all** the solutions to the following equations for x in radians

(a) $\sin x = \frac{\sqrt{3}}{2}$

$$x = \begin{cases} \frac{\pi}{3} + 2\pi n \\ \frac{2\pi}{3} + 2\pi n \end{cases}, \quad n \in \mathbb{Z}$$

(b) $\cot x = \frac{1}{\sqrt{3}}$

$\tan x = \sqrt{3}$

$$x = \frac{\pi}{3} + \pi n, \quad n \in \mathbb{Z}$$

Eastern Goldfields College

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Total Marks: 22

7. **[1, 3, 1 = 5 marks]**

For the sequence **4, 13, 22, 31, . . .,**

(a) Find an expression (in simplest form) for the general term T_n, the nth term, of this sequence.

$T_n = 4 + (n-1)9$
$= 9n - 5$ ✓

(b) Prove that the sum of any two consecutive terms of this sequence is always odd.

Let T_n, T_{n+1} be consecutive terms of sequence

$T_n + T_{n+1} = 9n - 5 + 9(n+1) - 5$
$= 9n - 5 + 9n + 9 - 5$ ✓✓
$= 18n - 1$

Since $18n$ is even
$18n - 1$ must be odd. ✓

(c) Use a counter example to disprove that "The sum of any three terms of this sequence is always even."

$4 + 13 + 22 = 39$ which is odd ✓

8. **[3, 3, 2, 3 = 11 marks]**

The motion of a small body moving along a straight track was recorded by a video camera for 20 seconds. An analysis of the motion showed the distance, x cm, of the body from a fixed point O on its path t seconds after recording began was given by $x(t) = 3cos\frac{\pi t}{4} - 4sin\frac{\pi t}{4}$.

(a) The distance can also be given by $x(t) = a\sin(\frac{\pi t}{4} + b)$, where a and b are real constants.

Determine the values of a and b.

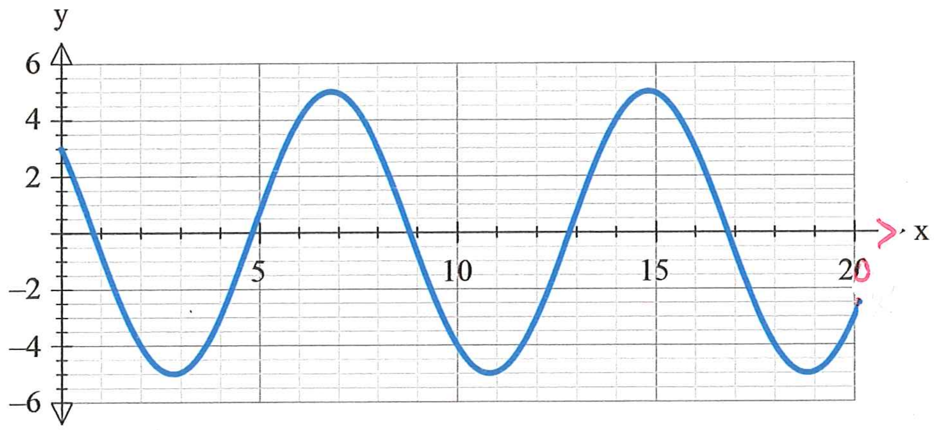
$a\sin\frac{\pi t}{4}\cos b + a\cos\frac{\pi t}{4}\sin b = 3\cos\frac{\pi t}{4} - 4\sin\frac{\pi t}{4}$

$a\cos b = -4 \qquad a\sin b = 3$ ✓

$a = -5 \qquad b = -0.6435$

✓ ✓

(b) Graph $y = x(t)$ on the axes below for $0 \leq t \leq 20$



✓ TP's in correct posn
✓ x intercepts
✓ domain

(c) State the period and the amplitude of the graph of $y = x(t)$

$P = \frac{2\pi}{\frac{\pi}{4}} = 8$ seconds ✓

$a = 5$ cm ✓

(d) Determine the percentage of the first 20 seconds that the body was at least four cm away from the point O.

$7.6387 - 6 = 1.6387$ ✓

$\frac{5 \times 1.6387}{20} \times 100 \approx 41\%$ ✓

9. **[3, 3 = 6 marks]**

(a) Use an appropriate product-to-sum identity to show that (sin 105°) × (sin 15°) = 0.25.

$$
\begin{aligned}
LHS &= \sin 105 \times \sin 15 \\
&= \tfrac{1}{2}(\cos 90 - \cos 120) \\
&= \tfrac{1}{2}(0 - (-\tfrac{1}{2})) \\
&= \tfrac{1}{4} \\
&= 0.25. \\
&= RHS
\end{aligned}
$$

(b) Use an appropriate product-to-sum identity to show that the equation below has two solutions in the domain $0° \le x \le 180°$.

$$\cos(x + 15°)\cos(x - 15°) = \frac{\sqrt{3}}{4}$$

$$\tfrac{1}{2}\left(\cos(x+15+x-15) + \cos(x+15-x+15)\right)$$

$$= \tfrac{1}{2}(\cos 2x + \cos 30)$$

$$\frac{\sqrt{3}}{4} = \tfrac{1}{2}\cos 2x + \tfrac{1}{2}\frac{\sqrt{3}}{2}$$

$$0 = \tfrac{1}{2}\cos 2x$$

$$0 = \cos 2x \qquad 0 \le 2x \le 360$$

$$2x = 90 \text{ or } 270$$

$$x = 45° \text{ or } 135° \quad \text{ie } 2 \text{ sol}^{\text{ns}}$$