Name: ANN

Esperance SHS

Year 12 MATHEMATICS METHODS

TEST 3 2017

Calculator Free

Total Marks: 33

Reading: 2 minutes Time Allowed: 35 minutes

Question 1 **(9 marks)**

(a) Solve $\log_x 64 = 3$. (1 mark)

$x^3 = 64$

$x = 4$

(b) Rearrange the equation $11^{2+3x} = 33$ for x. (2 marks)

$\log_{11} 33 = 2 + 3x$

$x = \frac{1}{3}(\log_{11} 33 - 2)$

$\log_{11} 33 = \log_{11} 3 + \log_{11} 11$

OR $= \log_{11} 3 + 1$

$= \frac{1}{3}(\log_{11} 33 - 1)$

(c) Solve $2(\log_3 x + 1) = \log_3 25$. (3 marks)

$\log_3 x + 1 = \log_3 25^{\frac{1}{2}}$

$\log_3 x + \log_3 3 = \log_3 5$

$3x = 5$

$x = 5/3$

(d) If $a = \log_5 4$ and $b = \log_5 8$, express the following in terms of a and b:

(i) $\log_5 32$ (1 mark)

$= \log_5 8 + \log_5 4$

$= a + b$

(ii) $\log_5 400$ (2 marks)

$= \log_5 (25 \times 16)$

$= \log_5 25 + \log_5 16$

$= 2 + 2a$

OR $\log_5 25 + \log_5 8 + \log_5 2$

$= 2 + b + \frac{1}{2} a$

OR $2b - a + 2$

Question 2 **(11 marks)**

Differentiate the following, simplifying where possible.

(a) $y = \dfrac{\log_e(4-x^3)}{12}$ (2 marks)

$$\frac{dy}{dx} = \frac{1}{12}\left(\frac{-3x^2}{4-x^3}\right)$$

$$= \frac{-x^2}{16-4x^3}$$

(b) $y = \dfrac{x}{\ln x}$ (3 marks)

$$\frac{dy}{dx} = \frac{\ln x \times 1 - x \times \frac{1}{x}}{(\ln x)^2}$$

$$= \frac{\ln x - 1}{(\ln x)^2}$$

c) $y = \ln\left(\dfrac{\sqrt{x+1}}{1-x}\right)$ (4 marks)

$$= \ln(x+1)^{\frac{1}{2}} - \ln(1-x)$$

$$= \frac{1}{2}\ln(x+1) - \ln(1-x)$$

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{x+1} - \frac{-1}{1-x}$$

$$= \frac{1}{2(x+1)} + \frac{1}{1-x}$$

d) $y = \log_4(1+x^3)^2$ (2 marks)

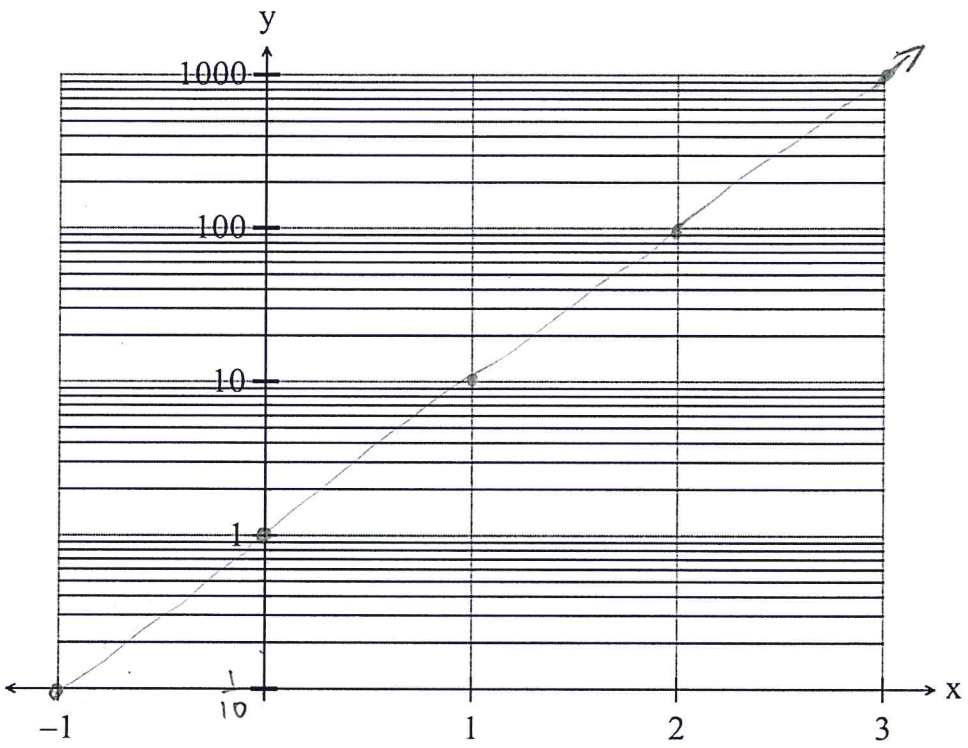
$$= \frac{2\ln(1+x^3)}{\ln 4}$$

$$\frac{dy}{dx} = \frac{2}{\ln 4}\left(\frac{3x^2}{1+x^3}\right)$$

$$= \frac{6x^2}{\ln 4(1+x^3)}$$

Question 3 **(5 marks)**

(a) Sketch $y = 10^x$ on the following semi-log paper. (2)



(b) (i) Find the derivative of $ln\sqrt{x^2-5}$ (2)

$y = \frac{1}{2} \ln(x^2-5)$

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{2x}{x^2-5}\right)$

$= \frac{x}{x^2-5}$

(ii) hence determine $\int \frac{x}{x^2-5}\,dx$ *for* $x^2-5>0$ (1)

$= \ln\sqrt{x^2-5} + C$

Question 4 **(8 marks)**

(a) Evaluate the following

$$\int_2^3 \left(x^2+x+1+\frac{1}{x}\right)dx$$ (3)

$$= \left[\frac{x^3}{3}+\frac{x^2}{2}+x+\ln x\right]_2^3$$

$$= \left(9+\frac{9}{2}+3+\ln 3\right)-\left(\frac{8}{3}+2+2+\ln 2\right)$$

$$= \frac{59}{6}+\ln\left(\frac{3}{2}\right)$$

(b) Determine [3]

$$\int \frac{x+1}{3x^2+6x+3}dx$$

$$= \frac{1}{6}\int \frac{6x+6}{3x^2+6x+3}dx$$

$$= \frac{1}{6}\ln(3x^2+6x+3)+C$$

(c) *Hence*, or otherwise evaluate :

$$\int_0^1 \frac{x+1}{3x^2+6x+3}dx$$

exactly. [2]

$$= \frac{1}{6}\left[\ln(3+6+3)-\ln(3)\right]$$

$$= \frac{1}{6}\left[\ln(4)\right]$$

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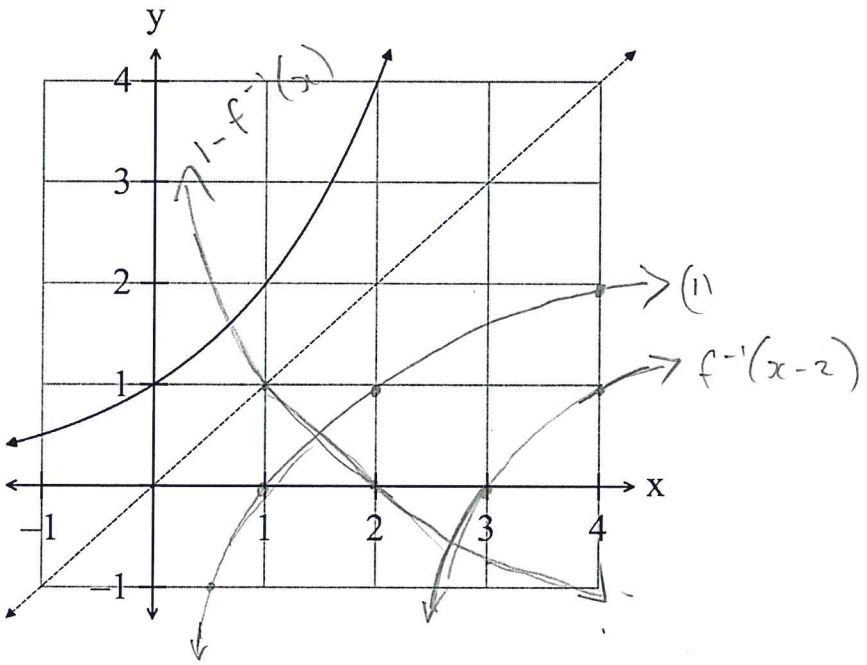
Resource Rich

Total Marks: 23
Reading: 2 minutes Time Allowed: 25 minutes

Question 5 **(7 marks)**

(a) The graph of $f(x) = 2^x$ is shown on the set of axes below,



Sketch on the same set of axes

(i) $y = f^{-1}(x)$, the inverse of the function $f(x) = 2^x$. (2)

ie $\log_2 x$

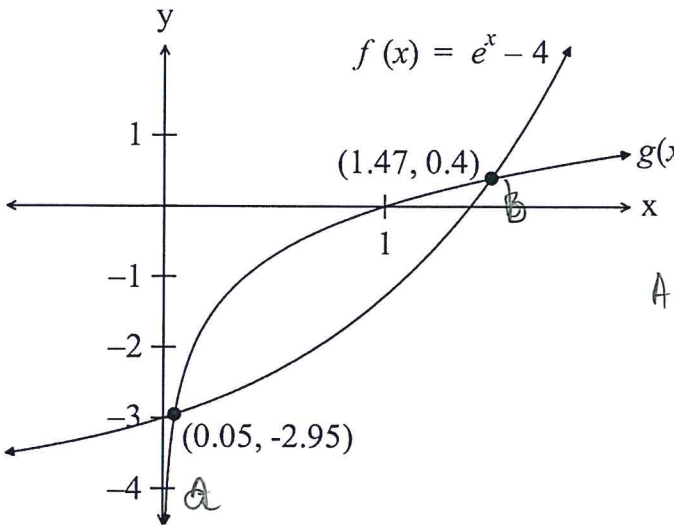
(ii) $y = f^{-1}(x-2)$ (3)

(iii) $y = 1 - f^{-1}(x)$. (2)

Question 6 **(7 marks)**

(a) Use your calculator to find the area enclosed between the two functions $f(x)=e^x-4$ and $g(x)=ln(x)$ as shown in the diagram below.

The points of intersection are shown.



$A = \int_a^b [g(x) - f(x)]\,dx$

$= 1.678 \text{ unit}^2$

–1 for not writing some sort of equation.

(3)

(b) A small colony of quolls live in hummock grasslands on the sand plains not far from Port Hedland. The population of this colony was studied in 2002. The population can be modelled by the equation $P(t)=22(ln(t+3))$ where t is in years starting in 2002.

(i) What was the population in 2002? (2)

$P(0) = 24.69$

i.e. 24

(ii) In what year will the population reach 100? (2)

$100 = 22[\ln(t+3)]$

$t \simeq 91.2$ years

i.e. 2093

accept 2094?

EXAM SOLNS

D.D.

Question 7 **(9 marks)**

(a) Simplify each of the following by expressing each as a single logarithmic term.

(i) $2\log c + \log\left(d^{\frac{1}{2}}\right) - \log e$ [2]

$= \log\left(\frac{c^2 \cdot d^{\frac{1}{2}}}{e}\right)$

(ii) $2\log_3 4 - \log_3 4 + 1$ [2]

$= \log_3 16 - \log_3 4 + \log_3 3$

$= \log_3\left(\frac{16 \times 3}{4}\right) = \log_3 12$

(b) If $\log_b a = c$, $\log_b d = j$, $\log_b f = g$ then find in terms of c, g and j.

(i) $\log_b(ad)$ [1]

$= \log_b a + \log_b d$

$= c + j$

(ii) $\log_b(\sqrt{fd})$ [2]

$= \frac{1}{2}\log_b fd$

$= \frac{1}{2}[\log_b f + \log_b d]$

$= \frac{1}{2}(g + j)$

(iii) $3\log_b\left(\frac{af}{\sqrt{d}}\right)$ [2]

$= 3 \cdot \left(\log_b a + \log_b f - \log_b d^{\frac{1}{2}}\right)$

$= 3 \cdot \left(c + g - \frac{1}{2}j\right)$