Baldivis Secondary College

Question/Answer Booklet

MATHEMATICS APPLICATIONS UNIT 1
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
    *   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    *   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed 65% (98 Marks)

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(4 marks)**

A student travelling to Singapore and Malaysia for a holiday bought 250 Australian Dollars worth of Singapore Dollars and 150 Australian Dollars worth of Malaysian Ringgit.

The exchange rates are shown below.

Exchange rates	Singapore Dollar	Malaysian Ringgit
One Australian Dollar	1.14	2.95

(a) How many Singapore Dollars did the student buy? (1 mark)

$$250 \times 1.14 = 285 \text{ SD}$$

(b) How many Malaysian Ringgit did the student buy? (1 mark)

$$150 \times 2.95 = 442.50 \text{ MR}$$

On their return to Australia the student exchanged their remaining 67 Singapore Dollars and 82 Malaysian Ringgits back into Australian dollars.

(c) How many Australian Dollars did the student receive? (2 marks)

$$67 \div 1.14 = 58.77$$

$$82 \div 2.95 = 27.80$$

$$58.77 + 27.80 = \$86.57$$

Question 9   **(8 marks)**

A young person qualifies for youth allowance of $272.80 per fortnight, so long as they do not earn more than $415 before tax in that time.

In any fortnight that they do earn more than $415, their allowance will be reduced by 50 cents in the dollar for earnings over $415 and up to $498, and reduced by 60 cents in the dollar for earnings over $498.

(a)   Sam has a part time job for eight hours every Saturday and six hours every Sunday that pays $22.25 per hour.

       (i)   Calculate her fortnightly earnings.   (2 marks)

$$22.25 \times (8 + 6) \times 2 = \$623.00$$

       (ii)   Calculate her fortnightly youth allowance. Justify your answer.   (3 marks)

$$623.00 - 498.00 = 125.00$$
$$125.00 \times 0.6 = 75.00$$

$$498.00 - 415.00 = 83.00$$
$$83.00 \times 0.5 = 41.50$$

$$272.80 - 75.00 - 41.50 = \$156.30$$

(b)   Sam has been offered an extra six hours work on a Friday but a friend has told her that she will not qualify for youth allowance if she takes these additional hours. Show that her friend is correct.   (3 marks)

$$22.25 \times (8 + 6 + 6) \times 2 = 890.00$$

$$890.00 - 498.00 = 392.00$$
$$392.00 \times 0.6 = 235.20$$

$$235.20 + 41.50 = \$276.70$$

Hence no YA as it is reduced by $276.70 – more than the actual allowance of $272.80.

Question 10 **(8 marks)**

(a) At a supermarket, a certain brand of coffee is sold in four different sizes: 125g for $2.95, 200g for $4.70, 375g for $8.70 and 500g for $11.95. On the price tag of the 200g size is displayed the information $2.35 per 100g of coffee. Calculate the price per 100g for the other three sizes and hence state the most economical size to buy if price is the only factor to consider. (3 marks)

> 2.95 ÷ 1.25 = $2.36
>
> 8.70 ÷ 3.75 = $2.32
>
> 11.95 ÷ 5 = $2.39
>
> The 375g is best buy.

(b) At a certain cinema, the price of a movie ticket is $15.50.

(i) The cinema sells a book of 20 movie tickets with a discount of 12% off the regular price. What is the price of the book of 20 tickets? (2 marks)

> 20 × 15.50 = 310
>
> 310 × 0.88 = $272.80

(ii) The price of a movie ticket is to be increased by 8%. What is the new price, correct to the nearest 5c? (1 mark)

> 15.50 × 1.08 = 16.74
>
> Price is $16.75 to nearest 5c.

(c) A sales person received a percentage commission of the value of sales each month. Following sales of $7960 in one month their commission amounted to $99.50. What was the percentage rate of commission? (2 marks)

> 99.50 ÷ 7960 = 0.0125
>
> 0.0125 × 100 = 1.25%

See next page

Question 11   **(7 marks)**

A school sportsground is 225 m long, 68 m wide and has semicircular ends.
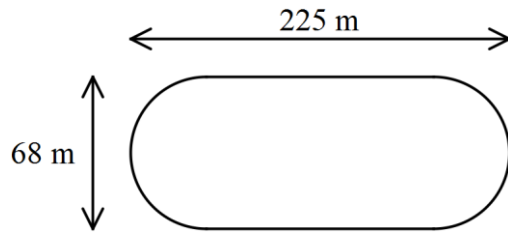
(a)   Determine the area of the sports ground.   (3 marks)

$$225 - 68 = 157$$
$$68 \times 157 = 10676$$

$$68 \div 2 = 34$$
$$\pi \times 34^2 = 3632$$

$$10676 + 3632 = 14308 \text{ m}^2$$

(b)   If the sportsground is to be fenced around its perimeter at a cost of $56 per metre, determine the total cost of the fencing, giving your answer to the nearest thousand dollars.   (4 marks)

$$2 \times \pi \times 34 = 213.6$$

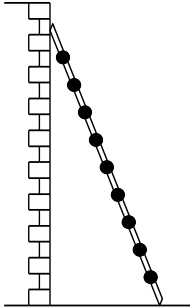$$213.6 + 157 + 157 = 527.6$$

$$527.6 \times 56 = 29545.6$$
$$\approx \$30\ 000$$

See next page

Question 12 **(9 marks)**

(a) A 2.7 m long ladder leans against a vertical wall as shown. The top of the ladder is 2.4 m above the horizontal ground.

(i) Determine how far the base of the ladder is from the wall, rounded to two decimal places. (2 marks)

$$2.7^2 - 2.4^2 = 1.53$$

$$\sqrt{1.53} = 1.24 \text{ m}$$

(ii) If the top of the ladder slips 30 cm down the wall, calculate how far the base of the ladder slips away from the wall. (4 marks)
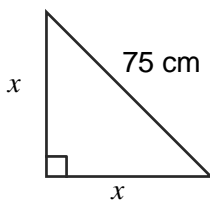
$$2.4 - 0.3 = 2.1$$

$$2.7^2 - 2.1^2 = 2.88$$

$$\sqrt{2.88} = 1.70$$

$$1.70 - 1.24 = 0.46 \text{ m (or 46 cm)}$$

(b) A right-angled triangle has a hypotenuse of length 75 cm and two other sides, both of length x, as shown in the diagram. Determine x. (3 marks)

75 cm

x

x

$$x^2 + x^2 = 75^2$$
$$2x^2 = 5625$$
$$x^2 = 2812.5$$
$$x = 53 \text{ cm}$$

See next page

Question 13   (9 marks)

The profit, P, made by a small fast-food outlet in a week is calculated using the formula
$P = S - W - C$ where S is the sales total, W is the part-time wages bill and C stands for other costs. The business employs 8 part-time staff who each work 12 hours per week for a wage of $9.20 per hour.

(a)   Determine the part-time wages bill for a week.   (1 mark)

$$8 \times 12 \times 9.20 = \$883.20$$

(b)   In a week when sales of $3200 are made and other costs amount to $1440,

    (i)   What profit is made?   (1 mark)

$$3200 - 883.20 - 1440 = \$876.80$$

    (ii)   State this profit as a percentage of the sales total.   (1 mark)

$$\frac{876.80}{3200} \times 100\% = 27.4\%$$

The manager of the business wants to make some changes in order to improve the weekly profit.

(c)   One choice is to reduce part-time staff wages by 5%. What effect would this have on the profit made in (b) (i) above?   (2 marks)

$$883.20 \times \frac{5}{100} = 44.16$$
Hence increase profit by $44.16

(d)   An alternative is to increase prices. If other costs remain at $1440 and no change is made to the part-time wages, by what percentage do sales of $3200 need to be increased to achieve the same increase in profit as in (c) above?   (2 marks)

$$\frac{44.16}{3200} \times 100\% = 1.38\%$$
Hence increase prices by 1.38%

(e)   The manager's wages are based on a commission of the total weekly sales. Explain how each of the changes made in (c) and (d) would affect the manager's wages.   (2 marks)
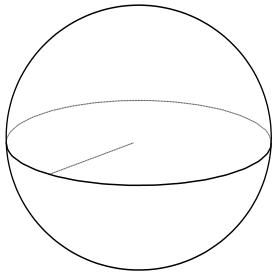
Reducing part-time wages will have no effect on manager's wage.

Increasing prices will increase the manager's wage, so long as higher prices do not lead to a decrease in sales.

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Question 14 **(8 marks)**

The sketch below shows a solid wooden sphere with a radius of 7.5 cm.

(a) Calculate

    (i) the volume of the sphere. (2 marks)

$$V = \frac{4(7.5)^3}{3}$$
$$= 1767 \text{ cm}^3$$

    (ii) the total surface area of the sphere. (2 mark)

$$A = 4\pi(7.5)^2$$
$$= 707 \text{ cm}^2$$

(b) The wooden sphere is sliced in half to create two hemispheres. Calculate

    (i) the volume of one of the hemispheres. (1 mark)

$$1767 \div 2 = 884 \text{ cm}^3$$

    (ii) the total surface area of one of the hemispheres. (3 marks)

$$A = 707 \div 2 + \pi(7.5)^2$$
$$= 353.5 + 176.7$$
$$= 530 \text{ cm}^2$$

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Question 15 **(9 marks)**

Four investors held shares in three companies as shown in the matrix below.

$$\begin{array}{c} \\ \text{Samway} \\ \text{Transpac} \\ \text{Urbon} \end{array} \begin{array}{cccc} \text{Ann} & \text{Bo} & \text{Cal} & \text{Dee} \\ \begin{bmatrix} 250 & 100 & 150 & 200 \\ 150 & 200 & 200 & 200 \\ 200 & 100 & 200 & 150 \end{bmatrix} \end{array}$$

Initially, the values of one share in Samway, Transpac and Urbon were $3.30, $5.50 and $8.40 respectively.

(a)   Determine which investor held the least number of shares and state this number. (1 mark)

Bo – 400 shares.

(b)   Represent the initial values of the shares in a 1 by 3 matrix.   (1 mark)

$$\begin{bmatrix} 3.3 & 5.5 & 8.4 \end{bmatrix}$$

(c)   Using matrix multiplication and your calculator, or otherwise, determine the initial values of each investor's share portfolio.   (3 marks)

Ann: $3330

Bo: $2270

Cal: $3275

Dee: $3020

(d)   After six months, the values of one share in Samway, Transpac and Urbon had changed to $4.10, $6.10 and $6.80 respectively. Determine the new values of each investor's share portfolio.   (2 marks)

Ann: $3300

Bo: $2310

Cal: $3195

Dee: $3060

(e)   Which investor made the largest loss in value over the six month period, and how much was this loss?   (2 marks)

Cal – made a loss of $80.

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Question 16   **(10 marks)**

A sculpture was sold for $22 000 by an art gallery.

(a) The sculpture could be bought on terms in which a premium of 8% was added to the purchase price and then the total amount repaid to the gallery in 12 equal monthly repayments. How much would each repayment be?   (2 marks)

$$22000 \times 1.08 = 23760$$
$$23760 \div 12 = \$1980 \text{ per month}$$

(b) The sculpture was expected to increase in value at an annual rate of 9%. What is the expected value of the sculpture three years after it was purchased?   (2 marks)

$$22000 \times 1.09^3 = \$28490.64$$
$$\approx \$28\ 500$$

(c) An amount of $22 000 can also be invested using two different saving schemes:

Scheme A, in which compound interest of 5.4% pa is paid monthly; and

Scheme B, in which simple interest of 5.9% pa is paid annually.

(i) Which of these schemes would pay the most interest over three years on the principal of $22 000? Justify your answer with clear working.   (4 marks)

Scheme A

$$F = 22000\left(1 + \frac{5.4}{100 \times 12}\right)^{12 \times 3}$$
$$= 25859.53$$
$$I = 22000 - 25859.53$$
$$= \$3\ 859.53$$

Scheme B

$$I = \frac{22000 \times 5.9 \times 3}{100}$$
$$= \$3\ 894$$

Scheme B would pay the most interest, by $34.47

(ii) Would the best scheme in part (c) (i) always be the best scheme? Explain your answer.   (2 marks)

No. In the short term, scheme B will be better, but in the long term, scheme A will be better due to the compounding of interest.

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Question 17   **(9 marks)**

A rectangular prism measuring 30cm by 40 cm by 8cm has a circular hole bored through its center from one side to the other as shown below.
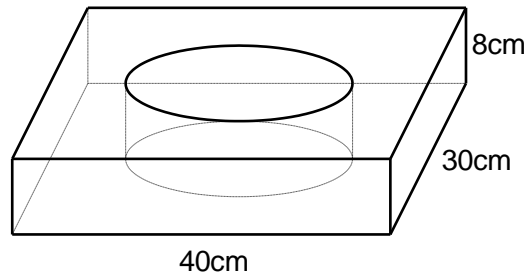
(a)   What was the volume of the rectangular prism before the circular hole was bored?

                                                                                  (1 mark)

$$V = 8 \times 30 \times 40 = 9600 \text{ cm}^3$$

After the circular hole was bored, the volume of the remaining prism was half of the original volume.

(b)   Show with calculations that the radius of the circular hole must be 13.82cm, correct to 2 decimal places.   (3 marks)

$$9600 \div 2 = 4800$$
$$4800 = \pi \times r^2 \times 8$$
$$\therefore r = 13.81976598 \approx 13.82 \text{ cm to 2 decimal places}$$
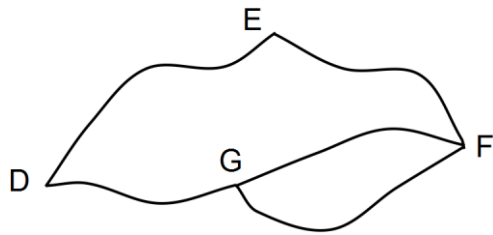
(c)   Determine the total surface area of the solid that remains after the circular hole is bored as shown above, correct to the nearest cm^2.   (5 marks)

$$\text{SA of whole rectangular prism } = 2(8 \times 30 + 30 \times 40 + 40 \times 8) = 3520$$
$$\text{Area of two circles removed } = \pi \times 13.82^2 \times 2 = 1200$$
$$\text{Area of cylinder wall added } = 2 \times \pi \times 13.82 \times 8 = 695$$
$$\text{TSA } = 3520 - 1200 + 695 = 3015 \text{ cm}^2$$

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Question 18 **(9 marks)**

(a)   The sketch map below shows the roads linking four country towns. Display the number of direct paths between the towns in a matrix. (3 marks)
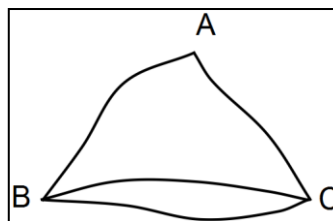

$$\begin{array}{c c} & \begin{array}{c c c c} D & E & F & G \end{array} \\ \begin{array}{c} D \\ E \\ F \\ G \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

(b)   The matrix T below shows the number of direct paths between another three towns.

$$\begin{array}{c c} & \begin{array}{c c c} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \end{array}$$

(i)   Sketch a diagram to show a possible layout of roads linking these towns. (2 marks)

(ii)   Use your calculator to determine the square of matrix T. (2 marks)

$$T^2 = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 1 \\ 2 & 1 & 5 \end{bmatrix}$$

(iii) How many routes, travelling along just **one** path, could a motorist take if they wanted to start at C and finish at A? (1 mark)
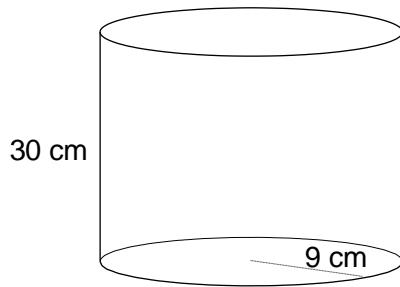
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(iv)   How many routes, travelling along **two** paths, could a motorist take if they wanted to start and finish at B? (1 mark)

5

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Question 19   **(8 marks)**

The diagram below shows a solid cylinder moulded from plastic with a radius of 9 cm and a height of 30 cm.

(a)   Calculate

    (i)   the total surface area of this cylinder.   (2 marks)

$$A = 2\pi(9)(30) + 2\pi(9)^2$$
$$= 2205 \text{ cm}^2$$

    (ii)   the volume of this cylinder.   (1 mark)

$$V = \pi(9)^2(30)$$
$$= 7634 \text{ cm}^3$$

(b)   A similar cylinder has dimensions three times the size of the one shown above.

    (i)   Determine the surface area scale factor.   (1 mark)

$$3^2 = 9$$

    (ii)   Calculate the total surface area of this cylinder.   (1 mark)

$$2205 \times 9 = 19845 \text{ cm}^2$$

(c)   Another similar cylinder has a volume of 25 765 cm³. Determine the height of this cylinder.
      (3 marks)

Volume scale factor: $\dfrac{25765}{7634} = 3.375$

Length scale factor: $\sqrt[3]{3.375} = 1.5$

Height: $30 \times 1.5 = 45$ cm

End of questions

Additional working space

Question number: _____