MATHEMATICS APPLICATIONS
YEAR 12

Investigation 1 – Sequences

Semester 1 2018

Time allowed: 45 minutes

Marks Available: 40 marks

Materials required: Writing implements, correction fluid/tape or eraser, ruler,
Scientific or CAS calculator

Instructions:

1. Write your answers in the spaces provided in this Question/Answer Booklet.
2. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

The Fibonacci sequence is the sequence of numbers 1, 1, 2, 3, 5, 8, ... where each number (or term) is the sum of two previous numbers.

The first term is denoted F_1, the second F_2, the third F_3 and so on.

1. [9 marks]

For the Fibonacci sequence, determine the value of:

a) the next 6 terms in the sequence (ie. $F_7, F_8, \ldots, F_{11}, F_{12}$) [2]

13, 21, 34, 55, 89, 144 ✓✓

b) $F_3 \times F_1 - (F_2)^2 =$ [1]

$2 \times 1 - 1^2 = 1$ ✓

c) $F_4 \times F_2 - (F_3)^2 =$ [1]

$3 \times 1 - 2^2 = -1$ ✓

d) $F_5 \times F_3 - (F_4)^2 =$ [1]

$5 \times 2 - 3^2 = 1$ ✓

e) $F_6 \times F_4 - (F_5)^2 =$ [1]

$8 \times 3 - 5^2 = -1$ ✓

f) $F_{11} \times F_9 - (F_{10})^2 =$ [1]

$89 \times 34 - 55^2 = 1$ ✓

g) Evaluate $F_n \times F_{n-2} - (F_{n-1})^2$ [2]

(i) when n is odd

$F_n \times F_{n-2} - (F_{n-1})^2 = 1$ ✓

(ii) when n is even.

$F_n \times F_{n-2} - (F_{n-1})^2 = -1$ ✓

2. [9 marks]

A closely related sequence, called the Carbonacci sequence is 2, 2, 4, 8, 32, 256,... The first term is denoted C_1, the second C_2, the third term C_3 and so on.

a) Describe how the Carbonacci sequence is formed: [2]

multiplying two consequtive terms to get the next

b) Complete the given table: [2]

2^1	2	2^6	64
2^2	4	2^7	128
2^3	8	2^8	256
2^4	16	2^9	512
2^5	32	2^{10}	1024

c) Describe any relationship you can see between the Carbonacci numbers and the Fibonacci numbers. [3]

Carbonacci sequence is $2^{Fib.}$ ✓ accurate representation

ie $C_2 = 2^{F_2}$

$C_3 = 2^{F_3}$ ✓ examples ✓ basic idea

and so on

d) Give an expression for C_n, the general term of the Carbonacci sequence, **in terms of the Fibonacci numbers**. [2]

$C_n = 2^{F_n}$ ✓ expression

✓ correct notation

3. [7 marks]

a) Using the table generated in question (2b) for powers of 2, what pattern can you see for the last digits of each number? [2]

e.g. If the power is a multiple 4 the number ends in a ...

If multiple of 4 ends in a 6 ✓

If it leaves a remainder of 1 it ends in a 2

✓ 2 ✓ 4

✓ 3 ✓ 8 ✓ idea

b) **Using your pattern** predict what the last digit for 2^{47} would be. Show all working to explain how you used the pattern discovered in part (3a) [2]

2^{47} $47 \div 4 = 11 \text{ r } 3$ ✓

ends in an 8 ✓

c) Now **use your pattern** to predict what the last digit of C_{11} would be. Show all working. [3]

$C_{11} = 2^{F_{11}}$

$= 2^{89}$ ✓

$89 \div 4 = 22 \text{ r } 1$ ✓

ends in a 2 ✓

4. [13 marks]

A new sequence, the Lucas sequence is the set of numbers that is obtained in a similar way to the Fibonacci numbers. Each new term is obtained by adding the previous 2 terms. However in the Lucas sequence, the first term, denoted L_1, is 1 and the second term, L_2 is 3.

a) Write down the first 10 terms of the Lucas sequence. [3]

1, 3, 4, 7, 11, 18, 29, 47, 76, 123 ✓3 all correct

−1 for one arithmetic error, −2 for two, 0 after

b) Determine the value of the following and continue the next five lines of the pattern: [8]

(i) $F_1 + F_3 =$ 3

(ii) $F_2 + F_4 =$ 4

(iii) $F_3 + F_5 =$ 7

(iii) $F_4 + F_6 = 11$

(iv) $F_5 + F_7 = 18$

(v) $F_6 + F_8 = 29$

(vi) $F_7 + F_9 = 47$

(vii) $F_8 + F_{10} = 76$

c) Using the pattern in part (4b), describe the relationship between the terms in the Fibonacci sequence and the terms in the Lucas sequence. [2]

$L_2 = F_1 + F_3$

$L_3 = F_2 + F_4$ ✓ idea

$L_n = F_{n-1} + F_{n+1}$ ✓ rule.

End of Investigation