HOLY CROSS

W-16

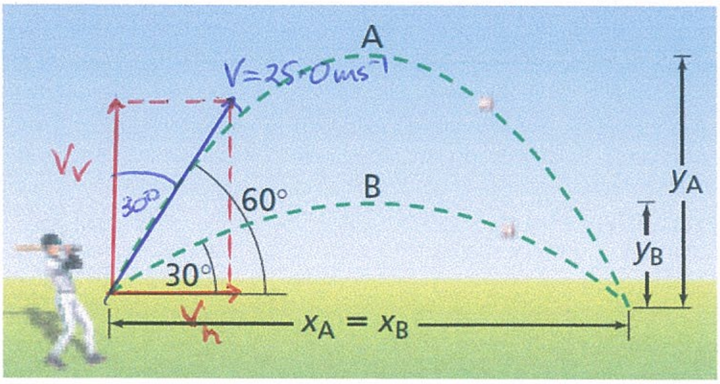
12 PHYSICS ATAR
TEST 4 - MOTION

NAME: SOLUTIONS

MARK: $\frac{\;}{56}$

Instructions:
Assume negligible air resistance unless the question explicitly asks you to consider for real situations or qualitative arguments.

1. Two baseballs in the figure below were hit with the same initial speed of 25.0 ms^{-1}.



(a) For trajectory A, determine the magnitude of the:

(i) horizontal component of the initial velocity. (1 mark)

$V_h = 25.0 \cos 60.0^\circ = \underline{12.5 \text{ ms}^{-1}}$ (1)

(ii) vertical component of the initial velocity. (1 mark)

$V_v = 25.0 \cos 30.0^\circ = \underline{21.6 \text{ ms}^{-1}}$ (1)

(b) For trajectory B, determine the magnitude of the:

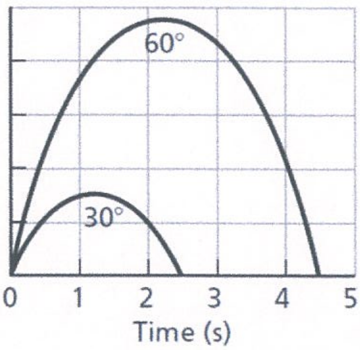
(i) horizontal acceleration. (1 mark)

0 ms^{-2} (1)

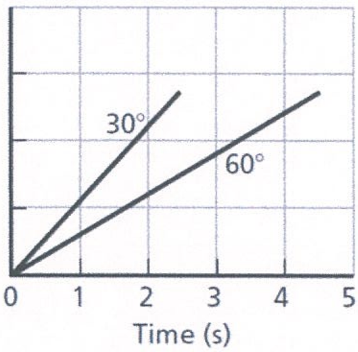
(ii) vertical acceleration (1 mark)

9.80 ms^{-2} (1)

(c) For both trajectories, which graph shows:



Graph 1



Graph 2

(i) the horizontal distance travelled versus time. (1 mark)

graph 2 (1)

(ii) the vertical distance travelled versus time. (1 mark)

graph 1 (1)

(iii) Using the equations of motion, calculate the time of flight of trajectory A (projected at 60°). (2 marks)

$\downarrow$ +ve

$v = ?$
$u = -21.6\,ms^{-1}$
$a = 9.80\,ms^{-2}$
$t = ?$
$s = 0\,m$

$s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad 0 = -21.6t + \frac{1}{2}(9.80)t^2$ (1)

$\Rightarrow 21.6\cancel{t} = 4.90t^{\cancel{2}}$

$\Rightarrow \quad t = \underline{4.41\,s}$ (1)

(iv) Calculate the maximum height reached in trajectory A (projected at 60°). (2 marks)

Consider movement to the highest point.

$\downarrow$ +ve

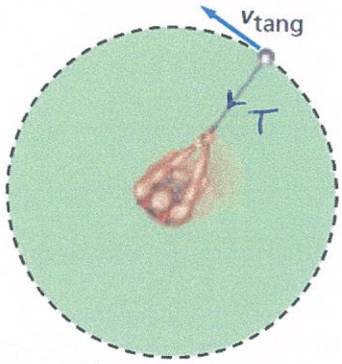
$v = 0\,ms^{-1}$
$u = -21.6\,ms^{-1}$
$a = 9.80\,ms^{-2}$
$t = ?$
$s = ?$

$v^2 = u^2 + 2as$

$\Rightarrow \quad 0 = (-21.6)^2 + 2(9.80)s$ (1)

$\Rightarrow \quad \underline{s = 23.8\,m}$ (1)

2. An athlete whirls a 7.00 kg hammer of radius 1.80 m from the axis of rotation in a horizontal circle as shown right.



(a) If the hammer makes one revolution in 1.00 s:

(i) calculate the centripetal acceleration of the hammer? (3 marks)

$$a_c = \frac{v^2}{r}$$
$$= \frac{4\pi^2 r}{T^2} \quad \left(\text{since } v = \frac{2\pi r}{T}\right) \quad (1)$$
$$= \frac{4\pi^2(1.80)}{(1.00)^2} \quad (1)$$
$$= \underline{71.1\ ms^{-2} \text{ towards the centre}} \quad (1)$$

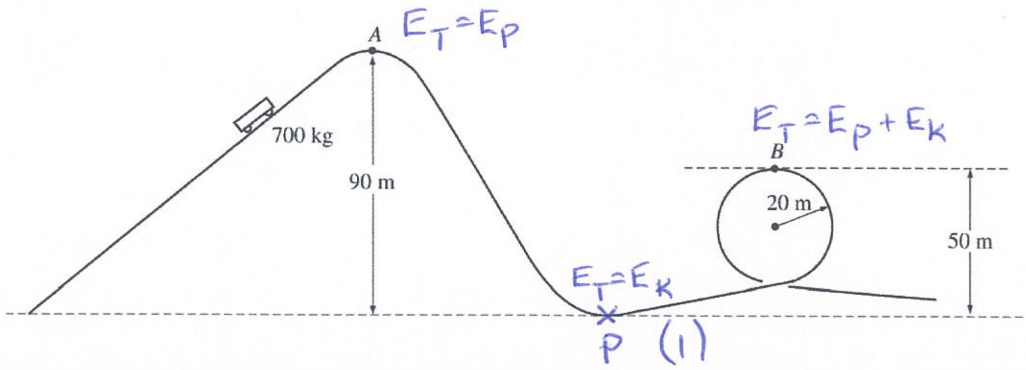
(ii) state the direction of the centripetal acceleration? (1 mark)

towards the centre (1)

(b) Determine the tension in the chain? (2 marks)

$$T = F_c$$
$$= \frac{mv^2}{r}$$
$$= \frac{(7.00)(71.1)}{1.00}$$
$$= \underline{498\ N \text{ towards the centre}}.$$

3.



A roller coaster at an amusement park lifts a car of mass M kg to point A at a height of 90.0 m above the lowest point on the track, as shown above.

The car starts from rest at point A, rolls with negligible friction down the incline and follows the track around a loop of radius 20.0 m.

Point B, the highest point on the loop, is at a height of 50.0 m above the lowest point on the track.

(a) (i) Indicate on the figure the point P at which the maximum speed of the car is attained. (1 mark)

See diagram

(ii) Calculate the value of this maximum speed. (3 marks)

$E_T = E_p\ (\text{top}) = E_K\ (\text{bottom})$

$\Rightarrow \quad \cancel{m}gh = \frac{1}{2}\cancel{m}v^2 \qquad (1)$

$\Rightarrow \quad v = \sqrt{2gh}$

$= \sqrt{2(9.80)(90.0)} \qquad (1)$

$= \underline{42.0\ \text{ms}^{-1}} \qquad (1)$

(b) Calculate the speed of the car at point B. (3 marks)

$E_T = E_p\ (\text{at A}) = E_p + E_K\ (\text{at B})$

$\Rightarrow \quad \cancel{m}gh_A = \cancel{m}gh_B + \frac{1}{2}\cancel{m}v^2 \qquad (1)$

$\Rightarrow \quad (9.80)(90.0) = (9.80)(50.0) + \frac{1}{2}v^2 \qquad (1)$

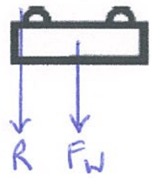
$\Rightarrow \quad 392 = \frac{1}{2}v^2$

$\Rightarrow \quad \underline{v = 28.0\ \text{ms}^{-1}} \qquad (1)$

(c) (i) On the figure of the car below, draw and label vectors to represent the forces acting on the car when it is upside down at point B. (3 marks)

[1 mark off for extra forces]

$\Sigma F = F_c \downarrow$



(3)

(ii) Calculate the magnitude of all the forces identified in (c) (i), given that M is 7.00 x 10^2 kg. (3 marks)

$$F_W = mg$$
$$= (7.00 \times 10^2)(9.80)$$
$$= \underline{6.86 \times 10^3 \text{ N}} \quad (1)$$

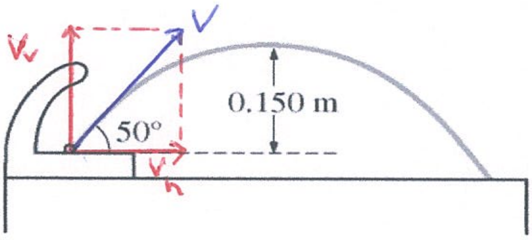
$$\Sigma F = F_c = R + F_W$$
$$\Rightarrow \quad R = F_c - F_W$$
$$= \frac{mv^2}{r} - mg \quad (1)$$
$$= (7.00 \times 10^2)\left[\frac{(28.0)^2}{(20.0)} - 9.80\right]$$
$$= \underline{2.06 \times 10^4 \text{ N}} \quad (1)$$

(d) Assume that friction is not negligible. How could the loop be modified to maintain the same speed at the top of the loop as found in (b)?

Justify your answer. (2 marks)

- Decrease the height of B. (1)
- Energy is lost due to friction, so E_T decreases. To achieve same E_K, E_P must decrease. (1)

4.



A drinking fountain projects water at an initial angle of 50.0° above the horizontal. The water reaches a maximum height of 0.150 m above the point of exit.

(a) Calculate the speed at which the water leaves the fountain. (4 marks)

Consider vertical movement to the top. ↓+ve

$v = 0\ ms^{-1}$ (1)

$u = -V\cos 40.0°\ ms^{-1}$

$a = 9.80\ ms^{-2}$

$t = ?$

$s = -0.150\ m$

$v^2 = u^2 + 2as$

$\Rightarrow 0 = (-V\cos 40.0)^2 + 2(9.80)(-0.150)$ (1)

$\Rightarrow V^2 = 5.01$ (1)

$\Rightarrow \underline{V = 2.24\ ms^{-1}}$ (1)

(b) The radius of the fountain's exit hole is 4.00 x 10^{-3} m. Calculate the rate of volume flow of the water. (2 marks)

$V = A \times l$ and $v = \frac{l}{t}$

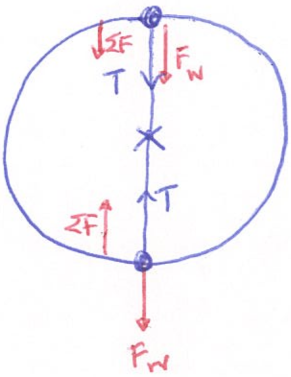
Rate of volume flow $= \frac{V}{t} = \frac{A \times l}{t}$ (1)

$= \pi r^2 v$

$= \pi (4.00 \times 10^{-3})^2 (2.24)$

$= \underline{1.13 \times 10^{4}\ m^3 s^{-1}}$ (1)

5. A ball on a light string moves in a vertical circle. Analyse and describe the motion of this system considering the effects of gravity and tension using free-body diagrams. Is the system in uniform circular motion? Explain. (5 marks)



At top: $\Sigma F = F_c = T + F_W$ (1)

At bottom: $\Sigma F = F_c = T - F_W$ (1)

This implies that F_c (top) > F_c (bottom) (1)

$\Rightarrow$ v (top) > v (bottom) (1)

$\therefore$ Not uniform circular motion (1)

6. A car is travelling with speed v around a curve of radius r (shown left).

(a) Determine an equation for the angle θ at which a road should be banked so that no friction is required. Draw forces on the free body diagram to derive the equation. (4 marks)

VERTICALLY $\Sigma F_v = 0$

$\Rightarrow R\cos\theta = F_W$

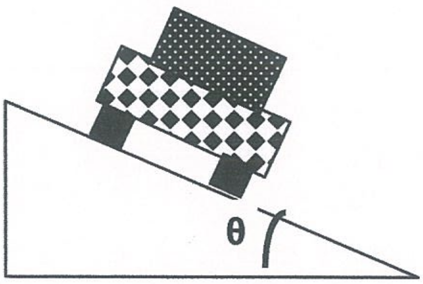
$\Rightarrow R\cos\theta = mg$ ① (1)

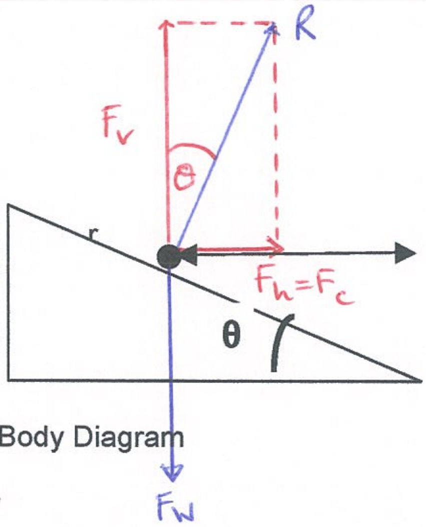
HORIZONTALLY $R\sin\theta = F_c$

$\Rightarrow R\sin\theta = \frac{mv^2}{r}$ ② (1)

$\frac{②}{①} \Rightarrow \frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{r} \times \frac{1}{mg}$ (1)

$\Rightarrow \tan\theta = \frac{v^2}{rg}$ (1)





Free Body Diagram

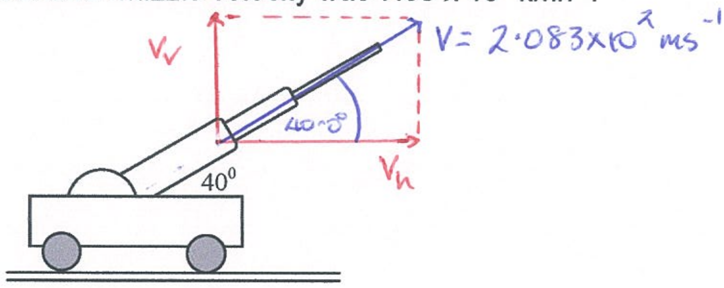
(b) Calculate the angle for a road that has a curve of radius 50.0 m with a design speed of 50 kmh^{-1}. (2 marks)

$$\tan\theta = \frac{v^2}{gr}$$

$$= \frac{(13{\cdot}89)^2}{(9{\cdot}80)(50{\cdot}0)} \quad (1)$$

$$\Rightarrow \quad \underline{\theta = 21{\cdot}5^\circ} \quad (1)$$

7. Big Bertha was the name of a huge cannon used by the Germans in the First World War to fire across the Channel to England. This gun was mounted on a train and could be moved to different positions. The shell's muzzle velocity was 7.50 x 10^2 kmh^{-1}.



(a) What would be the maximum height that the shell reached above the ground? (2 marks)

VERTICALLY ↓+ve Consider movement to the top.

$v = 0\ ms^{-1}$

$u = -2{\cdot}083 \times 10^2 \cos 50{\cdot}0^\circ\ ms^{-1}$

$a = 9{\cdot}80\ ms^{-2}$

$t = ?$

$s = ?$

$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad 0 = (-2{\cdot}083 \times 10^2 \cos 50{\cdot}0^\circ)^2 + 2(9{\cdot}80)s \quad (1)$$

$$\Rightarrow \quad s = -9{\cdot}15 \times 10^2\ m$$

$\therefore$ $\underline{\text{Height} = 9{\cdot}15 \times 10^2\ m \text{ above the ground.}}$ (1)

(b) What is the range of the gun if the angle of elevation is 40.0°? (2 marks)

VERTICALLY Consider the whole motion. ↓+ve

$v = ?$

$u = -208.3 \cos 50.0^\circ \text{ ms}^{-1}$

$a = 9.80 \text{ ms}^{-2}$

$t = ?$

$s = 0 \text{ m}$

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow 0 = (-208.3 \cos 50.0^\circ)t + \frac{1}{2}(9.80)t^2$$

$$\Rightarrow t = 27.33 \text{ s}. \qquad (1)$$

HORIZONTALLY

$$v_h = \frac{s_h}{t}$$

$$\Rightarrow s_h = (208.3 \cos 40.0^\circ)(27.33)$$

$$= \underline{4.36 \times 10^3 \text{ m}}. \qquad (1)$$

(c) What would be the gun's maximum range when its angle of elevation can be increased or decreased? (2 marks)

Max range at $\theta = 45.0^\circ$. ↓+ve

VERTICALLY

$v = ?$

$u = -208.3 \cos 45.0^\circ \text{ ms}^{-1}$

$a = 9.80 \text{ ms}^{-2}$

$t = ?$

$s = 0 \text{ m}$

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow 0 = (-208.3 \cos 45.0^\circ)t + \frac{1}{2}(9.80)t^2$$

$$\Rightarrow t = 30.06 \text{ s}. \qquad (1)$$

HORIZONTALLY

$$v_h = \frac{s_h}{t}$$

$$\Rightarrow s_h = (208.3 \cos 45.0^\circ)(30.06)$$

$$= \underline{4.43 \times 10^3 \text{ m}}. \qquad (1)$$

(d) What would be the velocity and angle of the shell 5.00 seconds after firing? Assume the angle of elevation remains at 40.0°. (2 marks)

HORIZONTALLY

$$V_h = 208.3 \cos 40.0^\circ$$
$$= 1.60 \times 10^2 \ ms^{-1}$$

VERTICALLY $\downarrow$ +ve

$v = ?$

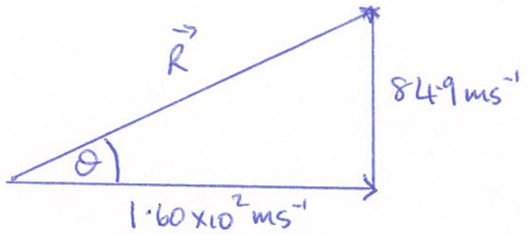
$u = -208.3 \cos 50.0^\circ \ ms^{-1}$

$a = 9.80 \ ms^{-2}$

$t = 5.00 s$

$s = ?$

$$v = u + at$$
$$= -208.3 \cos 50.0^\circ + (9.80)(5.00)$$
$$= -84.9 \ ms^{-1}$$



$$\vec{R} = \sqrt{(1.60 \times 10^2)^2 + (84.9)^2}$$
$$= 181 \ ms^{-1} \qquad (1)$$

$$\tan\theta = \frac{84.9}{1.60 \times 10^2}$$
$$\Rightarrow \theta = 28.0^\circ \qquad (1)$$

$\therefore$ velocity is $181 \ ms^{-1}$ at 28.0° to the horizontal