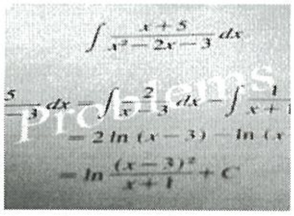
Rossmoyne SHS	METHODS 12 Mathematics SPOT TEST 2, 2019 Week 5 Term 1 Time allowed: 20 minutes No Calculator – 20 mks	

Name : Solutions

1. [9 mks]

Determine each of the following

(a) $\int_3^x \frac{d}{dt}(1-t^2)dt = \int_3^x (-2t)\,dt = \left[-t^2+c\right]_3^x$

$= \left[-x^2+c\right]-\left[-9+c\right]$

$= 9-x^2$

(b) $\frac{d}{dx}\int_5^x \sqrt{t.(1-t)^7}\,dx = \sqrt{x(1-x)^7}$

(c) $-\int_5^0 2\pi\,dx = -2\pi(0-5)$

$= 10\pi$

(d) $\frac{d}{dx}\int_0^2 \frac{t^2+1}{t-1}dt = 0$

(e) $\frac{d}{dx}\int_{-1}^{x^2} \frac{1+u}{u}\,du = f(x^2).\,2x$

$= \left(\frac{1+x^2}{x^2}\right).2x$

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$\int_2^5 = \int_2^3 + \int_3^5$

2. [3 mks]

If $\int_2^5 f(x)dx = 6$ and $\int_2^3 f(x)dx = 1$ find $\int_3^5 -f(x)dx$

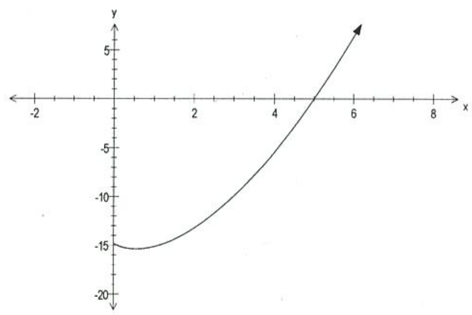
$$-\int_3^5 f(x)\,dx = -\left[\int_2^5 f(x)\,dx - \int_2^3 f(x)\,dx\right] \checkmark\checkmark$$
$$= -[6-1]$$
$$= -5 \checkmark$$

3. [2,2 mks]

Consider the curve defined by the equation

$$y = f(x) = \int_5^x (4\sqrt{t}-3)\,dt \qquad \text{where } x \geq 0$$

The graph of $y = f(x)$ is shown below



Determine

(a) $f'(4)$

$$f'(x) = \frac{d}{dx}\int_5^x (4\sqrt{t}-3)\,dt$$
$$= 4\sqrt{x} - 3 \checkmark$$
$$f'(4) = 5 \checkmark$$

(b) the x-coordinate of the stationary point on the curve

$$f'(x) = 0$$
$$\Rightarrow 4\sqrt{x} - 3 = 0 \checkmark$$
$$\sqrt{x} = \frac{3}{4}$$
$$x = \frac{9}{16} \checkmark$$

4. [4 mks]

If $\int_{-5}^{5} f(x)\,dx = 10$ find

$$\int_{-4}^{6} f(-x+1)dx$$

$f(-x+1)$

$x \to -x \quad f(x+1)$

$x \to x-1 \quad f(x)$

$$\overset{x \to -x}{=} \int_{-6}^{4} f(x+1)\,dx \checkmark\checkmark$$

$$\overset{x \to x-1}{=} \int_{-5}^{5} f(x)\,dx \checkmark$$

$$= 10 \checkmark$$

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