Manjimup S.H.S Year 11 2015
Mathematical Methods Investigation 1
In-Class validation

HERON'S RULE

Name: ANSWERS.

Score: .... out of 35 .... %

Total marks: 35 Time permitted: 45 minutes

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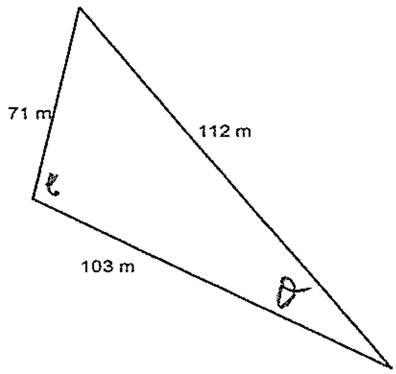
1. Use Heron's formula to calculate the area, to 2 decimal places, of this scalene triangle shown.



Show working.

$$S = \frac{71+112+103}{2} = 143$$

$$\Rightarrow A = \sqrt{143(143-71)(143-112)(143-103)}$$

$$= 3573.10\,m^2$$

[3 marks]

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2. Use trigonometrical rules to find the area of this triangle and to verify your solution to question 1. Show clearly the mathematics you used.

$$\cos\theta = \frac{103^2+112^2-71^2}{2(103)(112)} \Rightarrow A = \frac{(103)(112)\sin 38.28}{2}$$

$$\Rightarrow \theta = 38.28^\circ$$

$$= 3573.\cancel{3}\,\cancel{m}^2$$

[3 marks]

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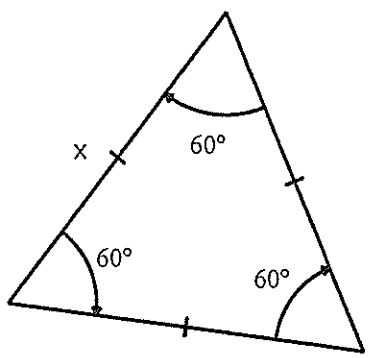
3. This equilateral triangle has the *same area* as the triangle from question one. Determine the length of the side lengths, accurate to 2 decimal places.

Show working.



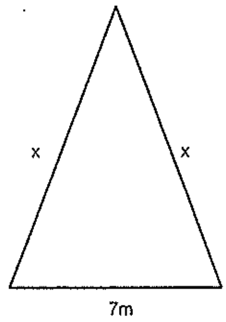
$$3573.10 = \frac{1}{2}x^2 \sin 60^\circ$$

$$\Rightarrow x^2 = 8251.72$$

$$\Rightarrow x = 90.84m$$

[3 marks]

4. a) Show working to demonstrate that the semi-perimeter, s, of this triangle can be expressed as s = x + 3.5



$P = 2x + 7$

$s = \frac{2x+7}{2}$

$s = x + 3.5$ as required.

b) Using Heron's rule, write a simplified algebraic expression for the area of this triangle in terms of x.

$A = \sqrt{(x+3.5)(x+3.5-x)(x+3.5-x)(x+3.5-7)}$

$= \sqrt{(x+3.5)(3.5)^2(x-3.5)} = 3.5\sqrt{(x^2-12.25)}$

c) Determine the value of x, when the area of the triangle is 25.18 m^2.
Indicate your method of solution.

solve $25.18 = \sqrt{(x+3.5)(3.5)^2(x-3.5)}$

$x = 8.0004$

$x \approx 8m$

[1,3,2 = 6 marks]

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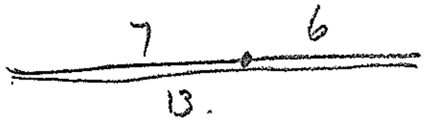
5. a) Use Heron's rule to find the area of a triangle with sides 6cm, 7cm and 13 cm.

$s = 13$

$A = \sqrt{13(13-6)(13-7)(13-3)} = 0\ cm^2$

b) Explain why this solution occurs, using a diagram to assist you.

It has no area because the lengths do not form a triangle, merely a straight line



[1,2 = 3 marks]

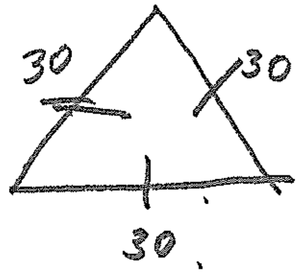
6. a) Find the maximum area possible for a triangle with a perimeter of 90 cm.
Show clearly how you did this and use a sketch to illustrate your solution.

Use equilateral triangle. $\Rightarrow$ $s = 45$

$a = b = c = 30$

$$A = \sqrt{45(15)(15)(15)}$$

$$= 389.7\text{cm}^2$$



b) The rule for the maximum area of a triangle with a semi-perimeter of s is equal to $\sqrt{\frac{s^4}{27}}$

Show working to verify this rule is correct for the triangle in question 6a.

If $a = 30$, $s = 45$

$$\Rightarrow \sqrt{\frac{45^4}{27}} = 389.7\text{cm}^2$$

[3,2 = 5 marks]

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7. Determine the dimensions of a triangle with an area of 1 hectare (10 000 m^2) that has the smallest perimeter. Show how you did this.

Equilateral triangle.

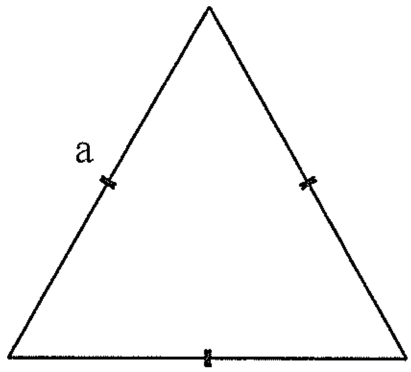
Solve $\sqrt{\frac{s^4}{27}} = 10000$ $\Rightarrow s = 227.95\text{m}$

$\Rightarrow$ lengths of sides $= 151.97\text{m}$.

[3 marks]

An equilateral triangle with lengths of 151.97m.

8. a) Given the following equilateral triangle, find an expression for s, in terms of a, the side length



s = $\frac{3a}{2}$

b) Substitute this into Heron's rule to find an unsimplified expression for area in terms of a.

A = $\sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$

c) Show algebraic working to demonstrate that A is given by the expression $A=\sqrt{\frac{3a^4}{16}}$.

You may use your Casio to assist with this if you wish but show each step.

$A=\sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)^3}$

$=\sqrt{\frac{3a}{2}\cdot\frac{a^3}{8}} = \sqrt{\frac{3a^4}{16}}$ as required

d) Investigate what happens to the area of an equilateral triangle when the side lengths are doubled. Show working and conclusions below.

$a=1 \rightarrow A=\sqrt{\frac{3}{16}}=\frac{\sqrt{3}}{4}$

$a=2 \rightarrow A=\sqrt{\frac{3.16}{16}}=\sqrt{3}$

$\Rightarrow$ Area is 4 times as large

[1,2,3,2 = 8 marks]

THE END