SCOTCH COLLEGE

12 Mathematics Methods 2020

Test 1 – Differentiation and Logarithms

Section 1: Calculator-free

Time allowed: 25 minutes **Maximum marks: 26**

Name:_____ **Teacher:** Foster | Giese

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No calculators or personal notes are permitted.

Question 1 **[1, 1, 3, 3 = 8 marks]**

Solve the following equations.

(a) $\log_{10} x = -2$

(b) $\log_x x^2 = x$

(c) $2^{x+1} = 3^{x-1}$

(d) $e^{2x} = e^x + 6e^0$

Question 2 **[3 marks]**

$\log_2 7 \approx 2.8$ and $\log_2 3 \approx 1.6$. Calculate the approximate value of $\log_2 24 - \log_2 14$.

Question 3 **[3, 3 = 6 marks]**

Differentiate the following (do not simplify your answers).

(a) $f(x) = \frac{3(x^4-10)^5}{x^2}$

(b) $y = (2 + x^2)\sqrt{x} + \frac{3}{x^3}$

Question 4 **[4 marks]**

Consider the quadratic function $y = ax^2 + bx + 5$. This function has a tangent that is $y = 4x + 6$ at the point $(1,10)$. Find the values of a and b.

Question 5 **[5 marks]**

The cost, $\$C$, to construct a water tank in the shape of cylinder with a height of h m and a radius of r m is given by the formula $C = 120(2\pi rh + 2\pi r^2)$. The cost of constructing a water tank with a height of 10m and radius of 5m is approximately \$56 550.

Use the incremental formula to calculate the approximate cost of a water tank with a height of 10m and a radius of $(5 + \frac{1}{\pi})$m.

END OF SECTION 1

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12 Mathematics Methods 2020

Test 1 – Differentiation and Logarithms

Section 2: Calculator-assumed

Time allowed: 20 minutes **Maximum marks: 19**

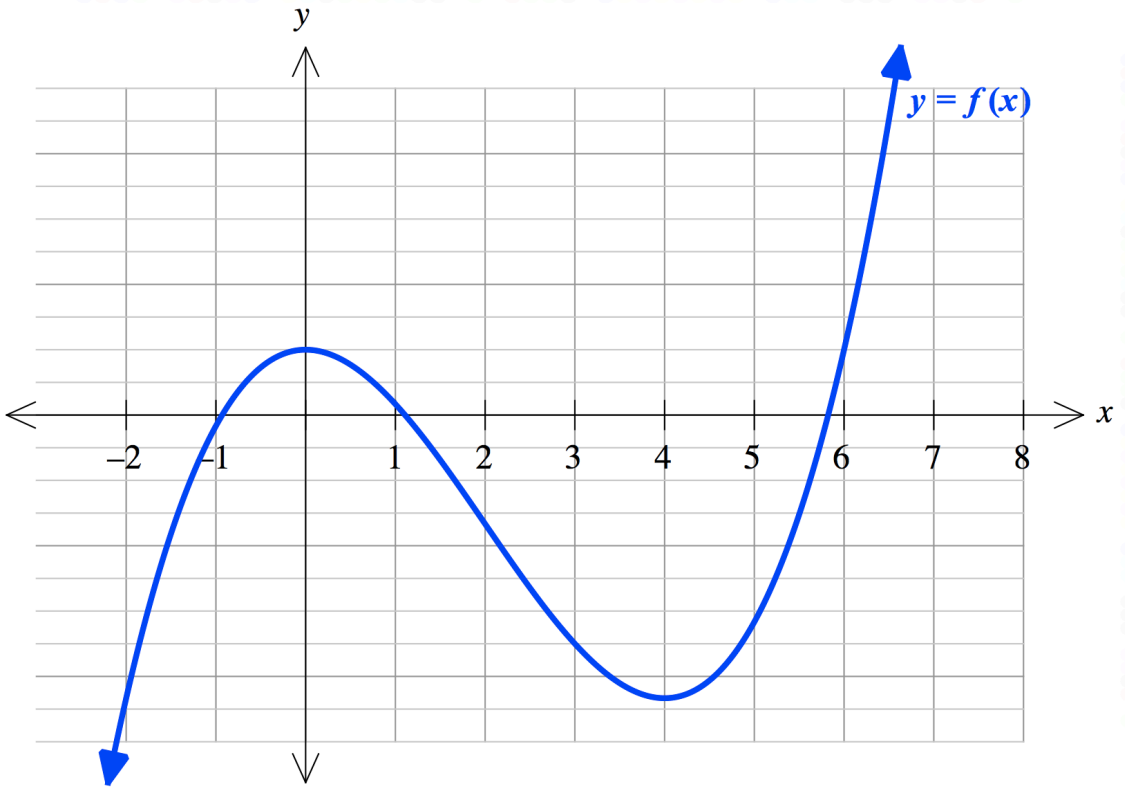
Name:_____ **Teacher:** Foster | Giese

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- Calculators and 1xA4 double-sided page of personal notes are permitted.

Question 6 **[3 marks]**

A graph of the function $y = f(x)$ is given below.
Sketch the graphs of the functions $y = f'(x)$ and $y = f''(x)$ on the number plane below.



Question 7 **[1, 3, 3, 2 = 9 marks]**

A particle is initially at rest before it moves in a straight line. Its displacement, x mm, from the origin after t seconds can be described by the following equation.

$$x = \frac{t^3}{3} - 4.5t^2 + 8t + 22, \quad 0 \leq t \leq 12$$

(a) What is the initial displacement of the particle?

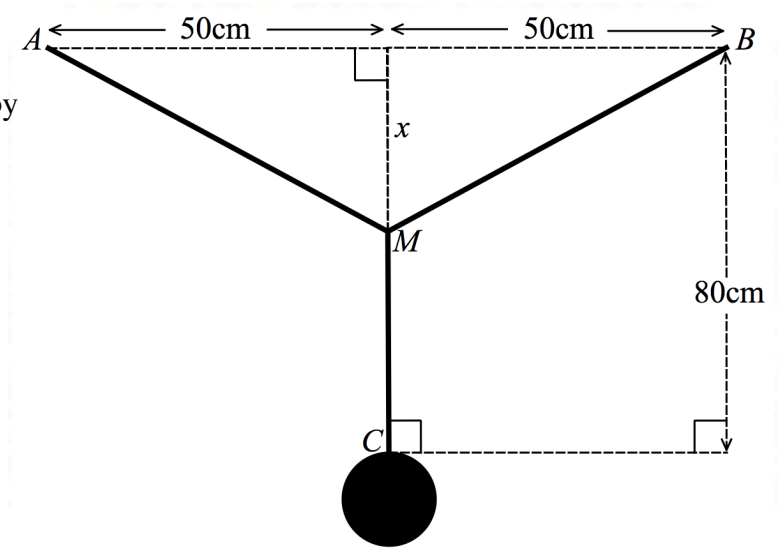
(b) Use calculus to show that the particle is at rest twice in the first 12 seconds.

(c) Is the particle travelling faster the first time it returns to the origin or the second time?

(d) What is the maximum distance that this particle is from the origin?

Question 8 **[2, 5 = 7 marks]**

Two sparrows are flying level with each other 1m apart and are each carrying one end of a piece of string. One end of a second piece of string is tied at M to the string carried by the birds while the other end is attached to a hook on the surface of a small coconut. The coconut is 80cm lower than the sparrows.



(a) Show that the total length of all the string, L cm, is given by $L = 2\sqrt{50^2 + x^2} - x + 80$.

(b) Using calculus techniques, show that there is a minimum length of string that can be achieved and justify it is a minimum. Determine the length of both pieces of string when this occurs.

END OF TEST