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Calculator Assumed
Applications of Differentiation – Rectilinear Motion

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 1, 1 = 3 marks]

(a) If the displacement of an object is modeled by a quadratic function, what type of function will model the velocity of the object?

(b) If the velocity of an object is modeled by a quadratic function, what type of function will model the displacement of the object?

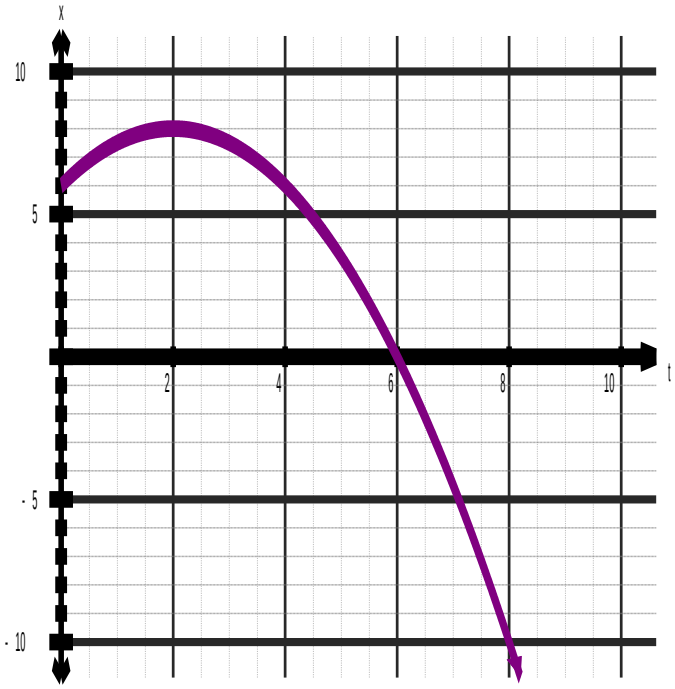
(c) If the velocity of an object is modeled by a quadratic function, what type of function will model the rate of change of the velocity?

Question Two: [3 marks]

The velocity, in m/s , of a particle is modeled by $v = -2t - 5$. Determine the function which models the displacement of the particle if at 5 seconds the displacement is -48 m.

Question Three: [2, 1, 1, 3, 2 = 9 marks]

The displacement of a particle moving in linear motion is graphed below, where t is time in seconds, and x is displacement in metres.



(a) What is the initial displacement of the particle? Interpret your answer.

(b) Determine when $v(t) = 0$.

(c) When does the particle pass the origin?

(d) What is the distance travelled by the particle in the first 8 seconds?

(e) Determine the approximate speed of the particle at $t = 6$.

Question Four: [1, 3, 3, 2, 1, 2, 3 = 14 marks]

A particle moves in linear motion and its displacement, d m, from the origin is given by $d = t^3 - 13t^2 + 50t - 56$, where t is time in seconds.

(a) Determine the displacement of the particle after 3 seconds.

(b) When will the particle be at a distance of 1 m from the origin?

(c) Determine, using calculus methods, when the particle changes direction.

(d) Determine when the particle is at the origin.

(e) Calculate the velocity of the particle at $t = 5$ seconds.

(f) Calculate the total distance travelled by the particle in the 7^{th} second.

(g) Calculate when the speed of the particle is 5m/s.

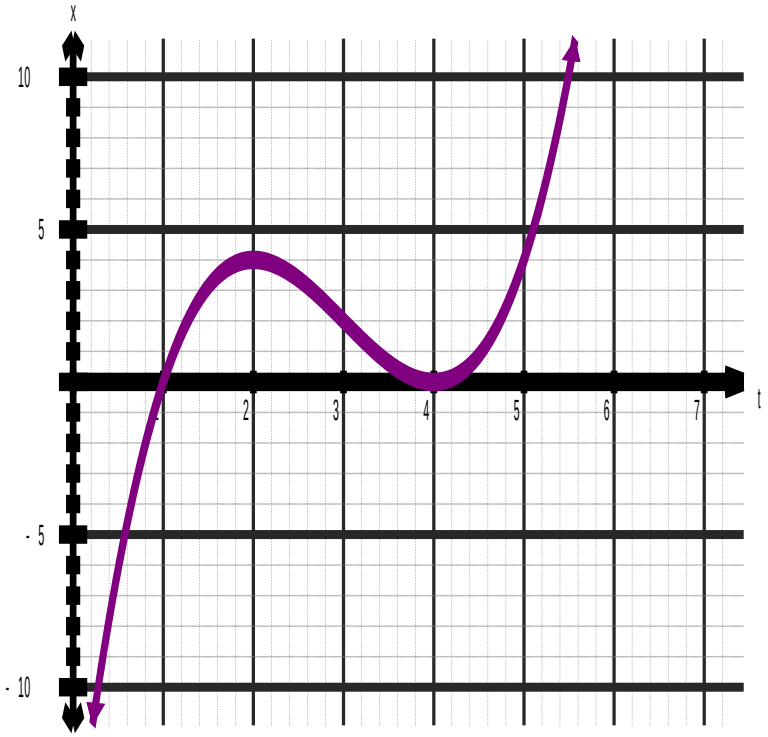
Question Five: **[5 marks]**

The displacement of an object moving in linear motion is given by $x = at^3 + bt$, where x is the displacement of the object in metres from the origin and t is the time in seconds.

If $\frac{dx}{dt} = 4$ when initially at the origin and $x(3) = -42$, find the values of a and b.

Question Six: **[4 marks]**

Sketch, on the same set of axes, a possible graph of the instantaneous rate of change of displacement if the displacement of an object moving in linear motion is graphed below.



Question Seven: [1, 2, 1, 3 = 7 marks]

The velocity, in m/s, of a particle moving in linear motion is monitored and its velocity at various times is given below.

t	0	1	2	3	4
v	5	3	-3	-13	-27

(a) Determine the initial velocity of the particle.

(b) Determine when the particle has a speed of 3 m/s.

(c) During which second does the particle change direction?

(d) Determine the equation of the function which models the velocity of this particle.

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SOLUTIONS
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Applications of Differentiation – Rectilinear Motion

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Question One: [1, 1, 1 = 3 marks]

(a) If the displacement of an object is modeled by a quadratic function, what type of function will model the velocity of the object?

linear ✓

(b) If the velocity of an object is modeled by a quadratic function, what type of function will model the displacement of the object?

cubic ✓

(c) If the velocity of an object is modeled by a quadratic function, what type of function will model the rate of change of the velocity?

linear ✓

Question Two: [3 marks]

The velocity, in m/s , of a particle is modeled by $v = -2t - 5$. Determine the function which models the displacement of the particle if at 5 seconds the displacement is -48 m.

$x = \frac{-2t^2}{2} - 5t + c$ ✓

$x = -t^2 - 5t + c$

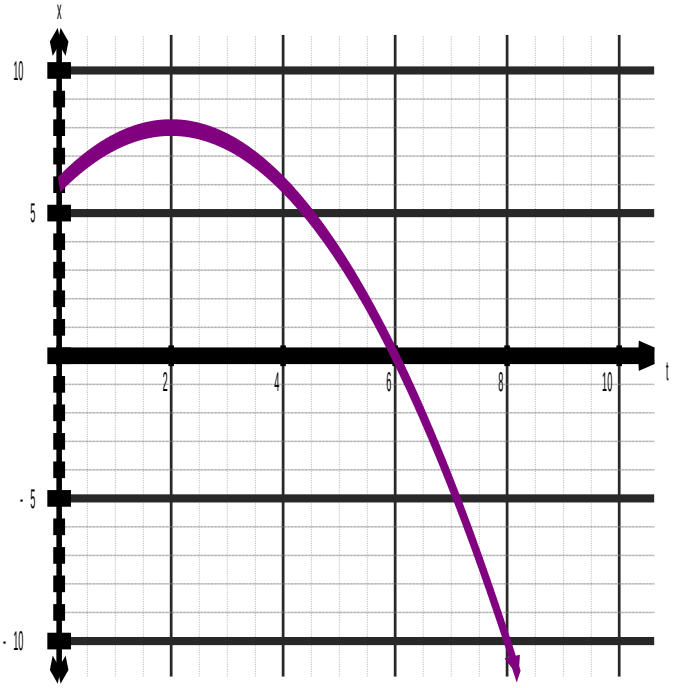
$-48 = -(5)^2 - 5(5) + c$ ✓

$c = 2$

$x = -t^2 - 5t + 2$ ✓

Question Three: **[2, 1, 1, 3, 2 = 9 marks]**

The displacement of a particle moving in linear motion is graphed below, where t is time in seconds, and x is displacement in metres.



(a) What is the initial displacement of the particle? Interpret your answer.

$\approx 6m$ The particle is initially 6m from the origin. ✓ ✓

(b) Determine when $v(t) = 0$.

$t = 2$ ✓

(c) When does the particle pass the origin?

$t = 6$ ✓

(d) What is the distance travelled by the particle in the first 8 seconds?

$D = 2 + 8 + 10 = 18m$ ✓ ✓ ✓

(e) Determine the approximate speed of the particle at $t = 6$.

$\approx 4m/s$ ✓ ✓

Question Four: **[1, 3, 3, 2, 1, 2, 3 = 14 marks]**

A particle moves in linear motion and its displacement, d m, from the origin is given by $d = t^3 - 13t^2 + 50t - 56$, where t is time in seconds.

(a) Determine the displacement of the particle after 3 seconds.

$d = 4m$ ✓

(b) When will the particle be at a distance of 1 m from the origin?

$t = 1.91s,\ t = 2.11s,\ t = 3.83s, t = 4.16s,\ t = 6.93s,\ t = 7.06s$

✓ ✓ ✓

(c) Determine, using calculus methods, when the particle changes direction.

$\frac{dd}{dt} = 3t^2 - 26t + 50 = 0$ ✓

$t = 2.88s,\ t = 5.79s$

✓ ✓

(d) Determine when the particle is at the origin.

$d = 0$ ✓

$t = 2s,\ t = 4s,\ t = 7s$

✓

(e) Calculate the velocity of the particle at t = 5 seconds.

$$\frac{dd}{dt} = 3t^2 - 26t + 50$$
$$3(5)^2 - 26(5) + 50 = -5m/s$$ ✓

(f) Calculate the total distance travelled by the particle in the 7th second.

$$d(7) - d(6) = 0 - -8 = 8m$$
✓ ✓

(g) Calculate when the speed of the particle is 5m/s.

$$5 = 3t^2 - 26t + 50$$
$$t = 2.39s,\ 6.28s$$ ✓
$$-5 = 3t^2 - 26t + 50$$ ✓
$$t = 5s,\ 3.67s$$
✓

Question Five: **[5 marks]**

The displacement of an object moving in linear motion is given by $x = at^3 + bt$, where x is the displacement of the object in metres from the origin and t is the time in seconds.

If $\frac{dx}{dt} = 4$ when initially at the origin and $x(3) = -42$, find the values of a and b.

$\frac{dx}{dt} = 3at^2 + b$ ✓

$3a(0)^2 + b = 4$ ✓

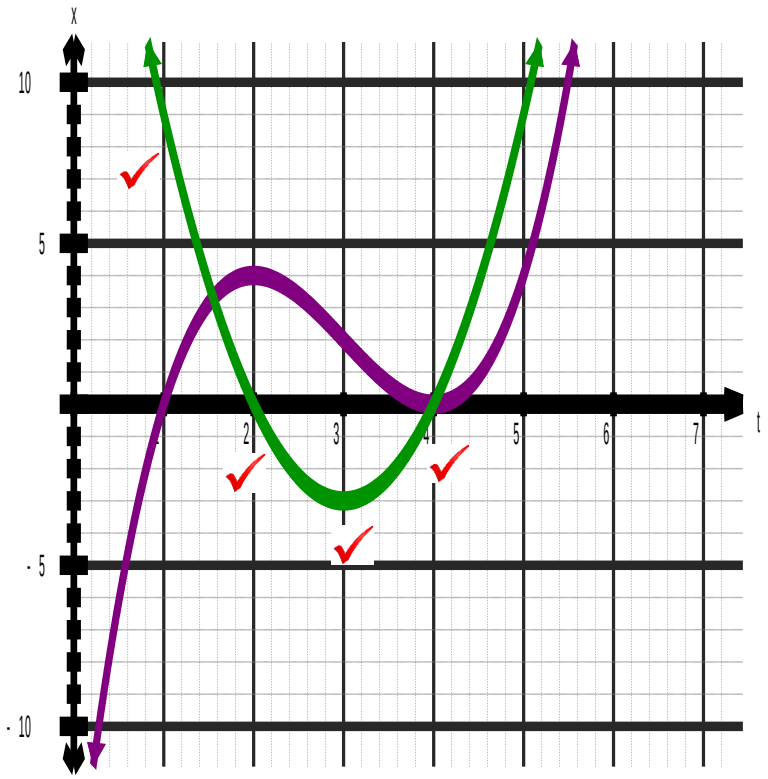
$b = 4$ ✓

$-42 = a(3)^3 + 4(3)$ ✓

$a = -2$ ✓

Question Six: **[4 marks]**

Sketch, on the same set of axes, a possible graph of the instantaneous rate of change of displacement if the displacement of an object moving in linear motion is graphed below.



Question Seven: [1, 2, 1, 3 = 7 marks]

The velocity, in m/s, of a particle moving in linear motion is monitored and its velocity at various times is given below.

t	0	1	2	3	4
v	5	3	-3	-13	-27

(a) Determine the initial velocity of the particle.

$5m/s$ ✓

(b) Determine when the particle has a speed of 3 m/s.

$t = 1s,\ t = 2s$
✓ ✓

(c) During which second does the particle change direction?

During the 2nd second. ✓

(d) Determine the equation of the function which models the velocity of this particle.

$v = -2t^2 + 5$ ✓
✓✓