

Mathematics Methods Unit 4 2019  
Investigation 3: Pressure and Balloons  
Validation Exercise

Name: Marking Key

Marks: \_\_\_\_ / 40 Time allowed 55 minutes.

Mobile phones must be switched off and stored in bags.  
The mark for this section will constitute 100% of the total investigation mark. Notes will not be allowed in this section, however calculators will be allowed.  
Answer the questions in the spaces provided.

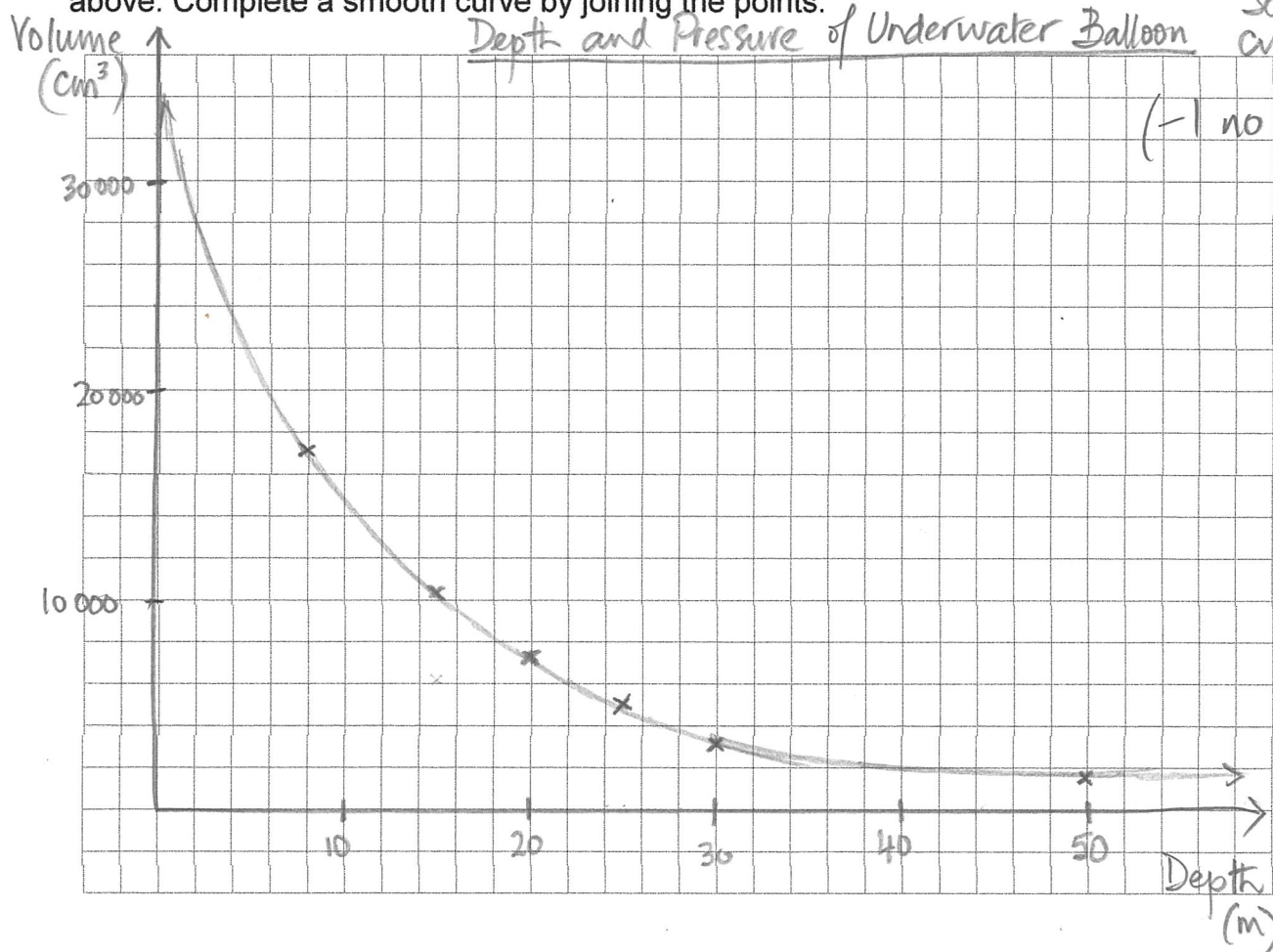


In a deep-sea diving experiment, a balloon is inflated while submerged at 30 metres depth below the surface of the water. The Volume of the balloon (in  $\text{cm}^3$ ) was found to increase as the depth (in metres) got less. The table below gives information on how the volume of a balloon changes at various depths.

$d$ (m)	25	15	8	50	20	30
$V$ ( $\text{cm}^3$ )	5 213.2	10 498.1	17 136.3	905.9	7 397.9	3 673.7

1. [ 5 marks]

On the graph grid below, choosing the scale of your axes carefully, plot the points above. Complete a smooth curve by joining the points.



2. <sup>12</sup>~~9~~<sup>14</sup> = 11 marks]

- a) Verify the nature of the relationship between depth and the volume of the balloon by considering a number of options that produce curved graphs. You should show and compare the results you have found to verify that you have found the best option.

✓ Quadratic regression gives  $r^2 = 0.9936139$  ✓  
✓ Cubic " "  $r^2 = 0.999945$  ✓  
✓ Quartic " "  $r^2 = 0.99999998$  ✓  
✓ Exponential =  $r^2 = 1$  ✓

so exponential is best ✓  
with  $y = 30000 \times e^{-0.07x}$  ✓

Only the above regressions are explored (ie not linear) as the points are not in a straight line ✓

- b) Were the other options considered like or very different from the preferred option? Why should this be?

The others were very close too ✓  
This is maybe because the domain values are (depth)  
so close to each other. ✓  
(or similar)

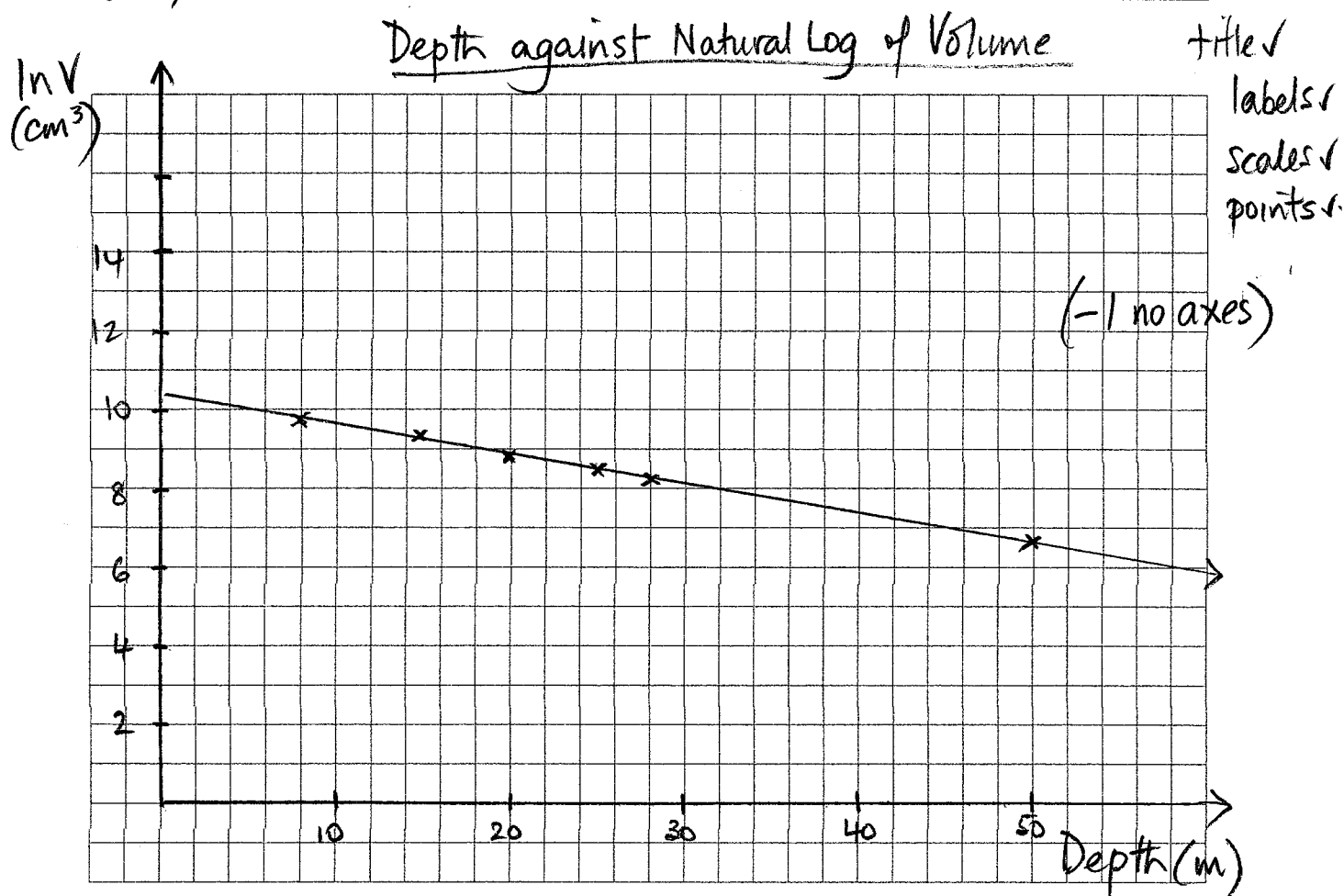
3, 5, 6 2 19.  
3. [8, 7, 5, 1, 2, 1] = 24 marks

The perfect match or correlation between two variables is verified by a straight line graph. By applying a particular function to one of the variables show that this possible and verifiable.

- a) Complete the table below and b) redraw your graph accordingly.

$d(m)$	8	15	20	25	30	50
$\ln V (cm^3)$	9.75	9.26	8.91	8.56	8.21	6.81

✓✓



- c) Use your graph to verify the original function (to two decimal places) of the relationship between the depth and the volume of the underwater balloon.

Gradient of line =  $\frac{6.81 - 9.75}{42} \checkmark = -0.07 \checkmark$

Y-intercept is approx. 10.31  $\checkmark$  based on  $6.81 = 50x - 0.07 + y$   
 $\Rightarrow y = 10.31$

$V_0 = e^{10.31} \approx 30031$  (0dp)  $\checkmark$  so  $V \approx 30031 e^{-0.07d} \checkmark$

- d) Comment on any possible reason you may not have the same exact answer as in Question 2.

- The difficulty of plotting exact points on the graph ✓  
means the result is more approximate than accurate.  
or Limited by small size of grid or rounding  
or other considerations

- e) Comment on the Volume of the balloon at the surface. Why might this not be theoretically possible?

$$V = 30000 \times e^{-0.07 \times 0} \checkmark$$
$$\approx 30000$$

Theoretically the exponential function has an asymptote at  $V=0$  ✓

- f) When would the volume of the balloon be zero?

Never, as this is also an asymptote ✓

4. [ 2 marks]

How might this information be important for any diver?

The lungs are like balloons - their volume will increase as the diver goes down. ✓

He/she needs to let their breath out when ascending ✓

Comparison needs to be to lungs, not to oxygen tanks.