Full Name: SOLUTIONS

MATHEMATICS APPLICATIONS

Test 6 – Time Series and Finance

Chapters 6 and 7

Semester 2 2016

Calculator Assumed

Time allowed

Working time for this section: 50 minutes
Marks available: 47 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (3 marks)
Renee wants to earn $1000 interest in 2 years. She invests in an account earning 9.8% p.a. simple interest.
How much would she need to invest? Answer to the nearest one hundred dollars.

$$1000 = \frac{P \times 9.8 \times 2}{100} \checkmark$$

$$P = 5102 \checkmark$$

$\therefore$ invest $5100 ✓

2. (4 marks)
Michael purchases office furniture valued at $3700 on terms of $370 deposit and 52 fortnightly installments of $71.56.

a. How much interest does he pay in total? [3]

$52 \times 71.56 = 3721.12$ ✓

$3721.12 - 3700 + 370 = 391.12$ ✓

$\therefore$ Interest paid is $391.12 ✓

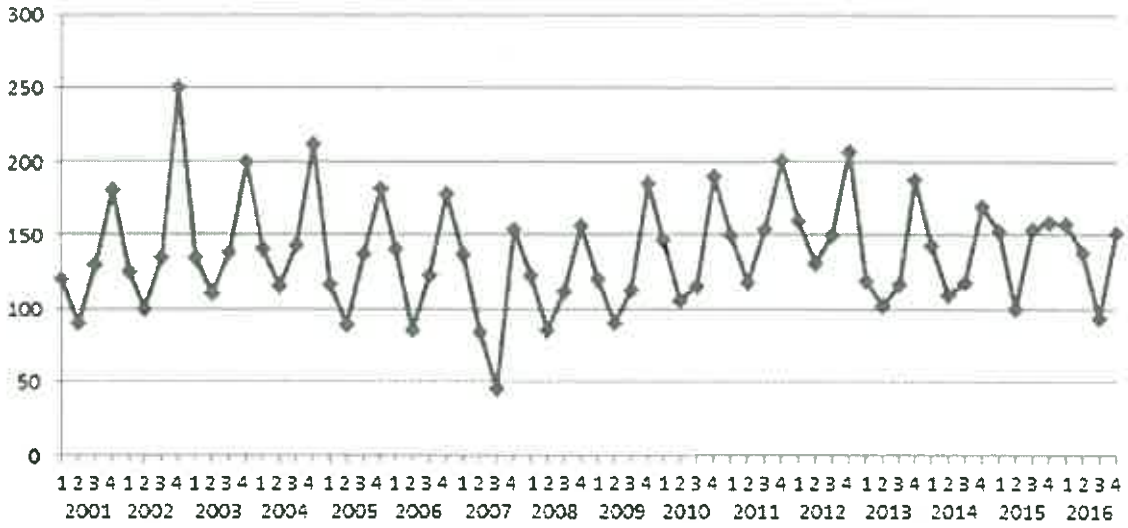
b. Calculate the annual flat rate of interest charged. Write your answer as a percentage correct to two decimal places. [2]

$$391.12 = \frac{3330 \times R \times 2}{100} \checkmark$$

$R = 5.87\%$ p.a. ✓

3. (7 marks)

The graph below shows the sales of vehicles in a city for the various quarters between 2001 and 2016 inclusive.



a. What moving average would you apply to this data? [1]

period is 4 quarters

∴ 4pt CMA ✓

b. What is the purpose of applying a moving average to this data? [1]

a moving average smooths the data ✓

c. Describe any long term fluctuations (cycles) in this time series. [2]

Appears to be a cycle every 4 years

Peaks go up for 4 years, down 4 years, etc ✓½

d. Describe any unusual fluctuations in this time series. [3]

2002 Q4 is unusually high ✓

2007 Q3 is down but is usually above Q2. ✓

2016 Q3 is down but is usually above Q2 ✓

* a couple of other minor inconsistencies - no marks.

4. (3 marks)

Bradley invests $8000 in an account paying 7.8% p.a. compounded quarterly.

a. Write a recursive expression that can be used to find the value of his investment at the end of 3 years. [2]

$T_{n+1} = \left(1 + \frac{7.8}{400}\right) T_n$, $T_0 = 8000$ ✓ ✓

$T_{n+1} = \left(1 + \frac{1.95}{100}\right) T_n$, $T_0 = 8000$

$T_{n+1} = 1.0195 T_n$, $T_0 = 8000$

} options

b. Find the value of his investment at the end of 3 years. [1]

3 years $\Rightarrow n = 12$

$T_{12} = 10086.41$

$\therefore$ \$10086.41 ✓

5. (4 marks)

Simon wants to loan money for a kitchen renovation. He has been given two options:

i. Loan A – 11.3% p.a. compounded quarterly

ii. Loan B – 11.1% p.a. compounded monthly

Which is the better ~~investment~~ loan and why?

$e = \left(1 + \frac{i}{n}\right)^n - 1$

Loan A $e_A = \left(1 + \frac{0.113}{4}\right)^4 - 1$

$= 0.11788 \Rightarrow 11.79\%$ ✓

Loan B $e_B = \left(1 + \frac{0.111}{12}\right)^{12} - 1$

$= 0.11682 \Rightarrow 11.68\%$ ✓

$\therefore$ Loan B ✓ is the better option because it is lower and will save him interest ✓

6. (5 marks)

The accountant of a company calculated the following seasonal components based on quarterly sales over a number of years of gathering data.

Quarter	Seasonal Component (%)	Seasonal Component (Decimal)
January	70	0.7
April	130	1.3
July	110	1.1
October	90 ✓	0.9

a. Complete the table for October and giving justification for your decision below. [2]

Seasonal components must add to 400 in this case. ✓

The following table shows the sales for 2015.

Quarter	Sales ($)
January	15 000
April	18 000
July	12 000
October	19 000

÷ 0.7 = 21428 (½)
÷ 1.3 = 13846 (½)
÷ 1.1 = 10909 (½)
÷ 0.9 = 21111 (½)

Each quarter had a different sales team leader. A bonus is paid to that sales team leader whose sales figures (in seasonally adjusted terms) were the best.

b. Showing justification, which sales leader should receive the bonus? [3]

Based on calculations above, the January sales leader should receive the bonus ✓

7. (12 marks)

A table of export data from Australia is given below.

Time Period (t)	Year	Quarter	Value of exports ($million)	4pt CMA	Yearly mean	% of the mean
1	2012	1	A		191.75	104.3
2		2	180			93.9
3		3	192	192.375		100.1
4		4	195	194.25		101.7
5	2013	1	205	195.875	196.5	104.3
6		2	190	196.375		96.7
7		3	195	198.375		99.2
8		4	196	B		99.7
9	2014	1	220	200.25	C	109.3
10		2	190	200.75		94.4
11		3	195			96.9
12		4	200			99.4

a. Determine the values of A, B and C. [3]

$192.375 \times 4 = \frac{A}{2} + 180 + 192 + 195 + \frac{205}{2}$

$A = 200$ ✓

OR $191.75 = \frac{A + 180 + 192 + 195}{4}$

$B = \left(\frac{190}{2} + 195 + 196 + 200 + \frac{190}{2}\right) \div 4$

$= 200.25$ ✓

$C = \frac{220 + 190 + 195 + 200}{4}$

$= 201.25$ ✓

The following table shows the seasonal indices.

Year	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
2012	104.3	93.9	100.1	101.7
2013	104.3	96.7	99.2	99.7
2014	109.3	94.4	96.9	99.4
Seasonal Index (%)	106.0	D	E	100.3

b. Determine the values of D and E. [2]

D 95% ✓

E 98.7% ✓

c. Determine the deseasonalised values for:

i. Quarter 1, 2012 [1]

$\frac{200}{1.06} = 188.68$ ✓

ii. Quarter 3, 2013 [1]

$\frac{195}{0.987} = 197.57$ ✓

A regression line is fitted to the deseasonalised values and its equation is $y = 1.04x + 189.73$

d. Is the export value increasing or decreasing with time? Justify your answer with reference to the regression line. [2]

increasing with time. Gradient is positive (1.04)

e. What is the potential value of the exports for Quarter 1, 2015? [3]

$y = 1.04 \times 13 + 189.73$

$= 203.25$ ✓

$203.25 \times 1.06 = 215.445$ ✓

Potential value is $215.45 ✓

8. (9 marks)

$100 000 is invested at a rate of 4.75% per annum with interest compounding monthly.

a. Calculate the monthly interest rate. [1]

$\frac{4.75}{12} = 0.396\%$ ✓

b. Calculate the value of the investment after 10 years and the total interest earned. [3]

Using Financial

$r = 4.75$

$n = 120$

Future \$160650.72

Interest \$60650.72 ✓✓

✓ evidence of reasoned working

Using Sequence

$r = 0.00396$

$n = 120$

Future \$160682.72

Interest \$60682.72

$r = 4.75 \div 1200$

$n = 120$

Future \$160650.72

Interest \$60650.72

c. Find the time taken for the investment to double its initial value. [2]

$n = 176$ (using Financial or Sequence with $4.75 \div 1200$)

$\therefore$ 176 months (14 yrs 8 mths) ✓✓

→ same with rounded rate

d. If the interest rate per annum were to be doubled, determine the time taken for the initial investment to double. Assume that the interest is compounded monthly. [3]

88 months ✓

Change r to 9.5% ✓

$200000 = 100000\left(1 + \frac{9.5}{1200}\right)^{12t}$ ✓ working/routine

If rate is written down then assume working on calc (Full marks) ✓

End of Test

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You may use this space to extend or re-attempt an answer to a question or questions and should you do so then number the question(s) attempted and cross out any previous unwanted working.