education equals

Topic: Standard scores and Normal Distribution

Time: 45 mins   Marks: /45 marks

No Calculator Allowed

Question One: [2, 2, 2, 2 : 8 marks]

Complete the table below. Each line should contain a test score, the mean and standard deviation for that set of scores and the test score written as a standard score (z-score).

Score	z-score	Mean	Standard Deviation
a) 10		8	1
b) 6		12	2
c)	2	15	2.5
d) 10	- 0.8		3

Question Two: [2, 3, 4: 9 marks]

X is a normally distributed random variable with a mean of 21 and a standard deviation of 3.

a)   What is the probability that X is between 18 and 24?

b)   What is the probability that X is less than 15?

c)   What is the probability that X is between 18 and 30?

Question Three: [3, 3, 1: 7 marks]

A particular mobile phone company has batteries which they claim last for 5 years. In actual fact their battery life span is normally distributed with a mean of 7 years and standard deviation of 1.5 years.

a)   Calculate the probability that a randomly selected mobile phone battery lasts between 2.5 and 4 years?

b)   Sage is lucky enough to have been using the same mobile phone battery for the past 11.5 years. What is the probability of it lasting longer than 11.5 years?

Another company's mobile phone batteries have a life span which is also normally distributed. The life span of these batteries has a standard deviation of 3 years.

c)   If 68% of their mobile phone batteries last for between 5 and 11 years, what is the mean life span of this company's mobile phone batteries?

Question Four: [2, 2, 4: 8 marks]

Anthony is about to eat chips from a 175g packet of chips. Before he eats the chips he pours them out onto a scale and notices that in fact the weight of the chips was actually 183g, not 175g as stated on the packet.

After some research into the matter, Anthony discovers that the true weight of the chips is normally distributed with a mean weight of 180g.

A representative from the manufacturer of these chips claims that the majority of their chip packets will weigh between 171g and 189g. With only 0.3% falling outside of this range.

a)   Calculate the standard deviation of the weight of these packets of chips.

b)   What is the probability that Anthony's next packet of chips weighs more than his last packet?

A different chip company also sells chips in 175g packets with the weights of the packets being normally distributed and having the same mean as the first company. This second company is happy with 95% of the weights of their chips being between 171g and 189g.

c)   Calculate the probability that a randomly selected packet of chips, from this second company, weighs between 175.5g and 184.5g.

Question 5: [4, 2: 6 marks]

Nicole's marks for her first semester exams in English, Maths and Chemistry were 65, 80 and 57.5 respectively. The mean and standard deviation for English were $\mu = 70 \; and \; \sigma = 2.5$, for Maths $\mu = 70 \; and \; \sigma = 5$, and for Chemistry $\mu = 50 \; and \; \sigma = 3$.

a)   Use z-scores to compare her exam results and determine which was her best result.

b)   What percentage of the class scored less than Nicole in the English exam?

Question 6: [1, 2: 3 marks]

People with a z-score greater than 3 on an IQ test are considered to be geniuses. If the scores on a particular IQ test have a mean of 100 and a standard deviation of 12 points;

a)   What is the cut off score for a genius?

b)   What percentage of the population are geniuses?

Question 7: [2, 2: 4 marks]

People wanting to drive a truck have to take a test and they are given a score. 95% of the population who took this test scored between 40 and 100 points.

a)   Calculate the mean and standard deviation of the truck license test.

A mark of 55 and above is considered a pass for the truck license test.

b)   What percentage of people fail this test?

Question One: [2, 2, 2, 2: 8 marks]

Complete the table below. Each line should contain a test score, the mean and standard deviation for that set of scores and the test score written as a standard score (z-score).
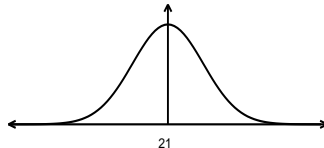
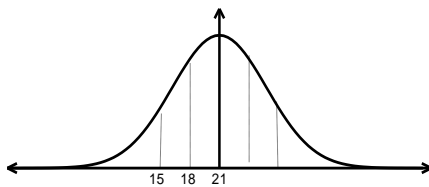| Score	z-score	Mean	Standard Deviation
a) 10	$\frac{10-8}{1} = 2$ ✔ ✔	8	1
b) 6	$\frac{6-12}{2} = -3$ ✔ ✔	12	2
c) 20	$\frac{x-15}{2.5} = 2$ ✔ $x - 1.5 = 5$ ✔	15	2.5
d) 10	✔ $\frac{10-\bar{x}}{1} = -0.8$ ✔	12.4	3

Question Two: [2, 3, 4: 9 marks]

X is a normally distributed random variable with a mean of 21 and a standard deviation of 3.

a)   What is the probability that X is between 18 and 24?

$$P(18 \leq X \leq 24) \approx 0.66$$

✔
✔

b)   What is the probability that X is less than 15?

$$P(X \leq 15) = 0.5 - \frac{0.95}{2}$$ ✔

$$= 0.5 - 0.475$$ ✔

$$= 0.025$$ ✔

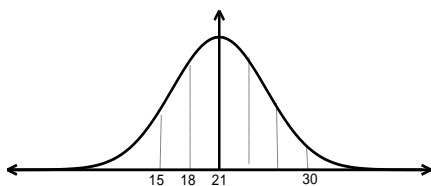c)   What is the probability that X is between 18 and 30?

$$P(18 \leq X \leq 30) = \frac{0.95}{2} + \frac{0.997}{2}$$ ✔ ✔

$$= 0.475 + 0.4985$$ ✔

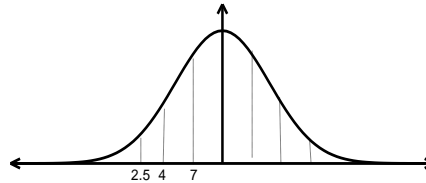$$= 0.9735$$ ✔

Question Three: [3, 3, 1: 7 marks]

A particular mobile phone company has batteries which they claim last for 5 years. In actual fact their battery life span is normally distributed with a mean of 7 years and standard deviation of 1.5 years.

a) Calculate the probability that a randomly selected mobile phone battery lasts between 2.5 and 4 years?

$P(-3 \le Z \le -2) = 0.4985$ ✓

$= -0.475$ ✓

$= 0.0235$ ✓

b) Sage is lucky enough to have been using the same mobile phone battery for the past 11.5 years. What is the probability of it lasting longer than 11.5 years?
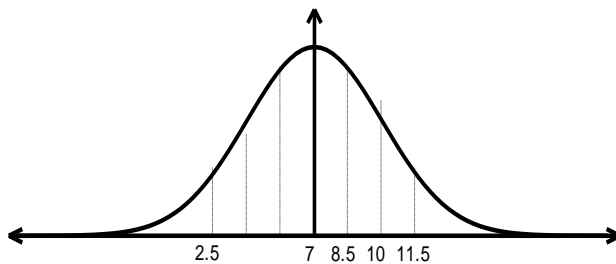
$B \sim N(7, 1.5^2)$

$7 + 3 \times 1.5 = 11.5$

$7 - 3 \times 1.5 = 2.5$

$P(2.5 \le X \le 11.5) = 99.7\%$ ✓

$1 - 0.997 = 0.003$

$0.003 \div 2 = 0.0015$ ✓
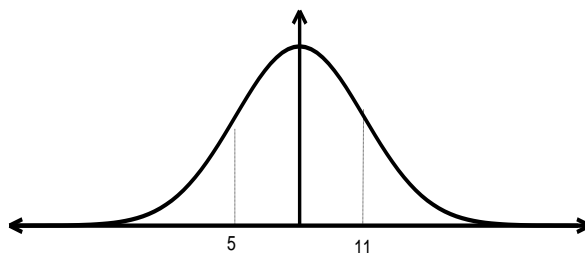
$P(X > 11.5) = 0.0015$

$= 0.15\%$ ✓

Another company's mobile phone batteries have a life span which is also normally distributed. The life span of these batteries has a standard deviation of 3 years.

c) If 68% of their mobile phone batteries last for between 5 and 11 years, what is the mean life span of this company's mobile phone batteries?

$\bar{x} = 8$ ✓

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Question Four: [2, 2, 4: 8 marks]

Anthony is about to eat chips from a 175g packet of chips. Before he eats the chips he pours them out onto a scale and notices that in fact the weight of the chips was actually 183g, not 175g as stated on the packet.

After some research into the matter, Anthony discovers that the true weight of the chips is normally distributed with a mean weight if 180g.

A representative from the manufacturer of these chips claims that the majority of their chip packets will weigh between 171 g and 189 g. With only 0.3% falling outside of this range.

a)   Calculate the standard deviation of the weight of these packets of chips.
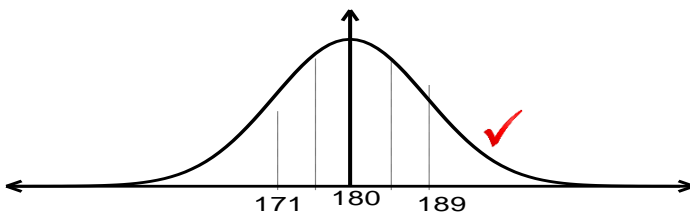
Standard deviation = 3g   ✓   ✓
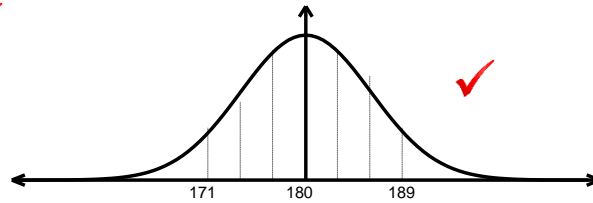
b)   What is the probability that Anthony's next packet of chips weighs more than his last packet?

$P(c \geq 183) = 0.5 - 0.34$ ✓

$= 0.16$ ✓

A different chip company also sells chips in 175g packets with the weights of the packets being normally distributed and having the same mean as the first company. This second company is happy with 95% of the weights of their chips being between 171g and 189g.

c)   Calculate the probability that a randomly selected packet of chips, from this second company, weighs between 175.5g and 184.5g.

Standard deviation = 4.5g   ✓ ✓

$P(175.5 \leq C \leq 184.5) = 0.68$

68%   ✓

Question 5: [4, 2: 6 marks]

Nicole's marks for her first semester exams in English, Maths and Chemistry were 65, 80 and 57.5 respectively. The mean and standard deviation for English were $\mu = 70$ and $\sigma = 2.5$, for Maths $\mu = 70$ and $\sigma = 5$, and for Chemistry $\mu = 50$ and $\sigma = 3$.

a)   Use z-scores to compare her exam results and determine which was her best result.

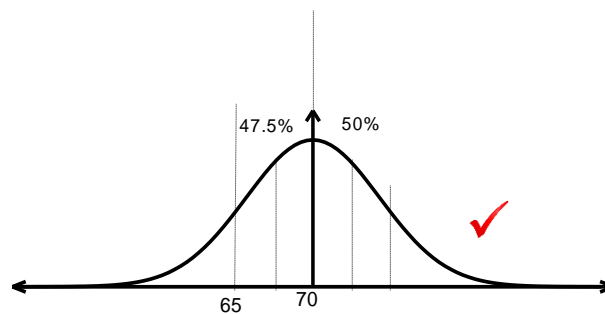$$Z_{English} = \frac{65 - 70}{2.5} = -2 \qquad \checkmark$$

$$Z_{Maths} = \frac{80 - 70}{5} = 2 \qquad \checkmark$$

$$Z_{Chemistry} = \frac{57.5 - 50}{3} = 2.5 \qquad \checkmark$$

$$\therefore Chemistry\ was\ her\ best\ result \qquad \checkmark$$

b)   What percentage of the class scored less than Nicole in the English exam?
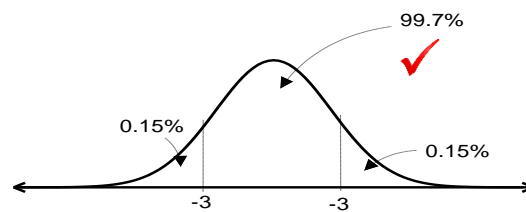
2.5 %

Question 6: [1, 2: 3 marks]

People with a z-score greater than 3 on an IQ test are considered to be geniuses. If the scores on a particular IQ tests have a mean of 100 and a standard deviation of 12 points;

a)   What is the cut off score for a genius?

$100 + 3 \times 12 = 136$ ✓

b)   What percentage of the population are geniuses?
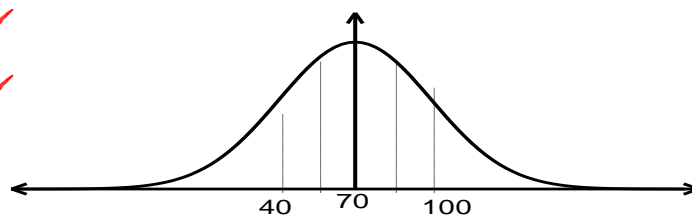
0.15% ✓

Question 7: [2, 2: 4 marks]

People wanting to drive a truck have to take a test and they are given a score. 95% of the population who took this test scored between 40 and 100 points.

a)   Calculate the mean and standard deviation of the truck license test.

$\bar{x} = 70$ ✓

$\sigma = 15$ ✓

A mark of 55 and above is considered a pass for the truck license test.

b)   What percentage of people fail this test?

$34\% + 50\% = 84\%\ of\ people$ ✓ ✓