education equals

Calculator Assumed
Arithmetic and Geometric Sequences
Mixed Applications 2

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 2 = 8 marks]

Edith is knitting a woolen scarf that will measure 40 cm wide by 2 m in length. On the first day she knits 2000cm^2. Each subsequent day Edith knits 25% less than the day before.

(a) How much of the scarf does Edith knit on the second day?

(b) On which day does she knit less than half of the amount she knitted on the first day?

(c) On which day has she finished knitting 75% of the total scarf?

(d) Will Edith ever finish knitting this scarf? Justify your answer with full mathematical reasoning.

Question Two: [2, 3, 2, 2, 2 =11 marks]

Homer and Marge started saving for their daughter Lisa's college education from the day she was born, 9th May 1990.

On the day she was born they deposited $1000 in a bank account that has earned interest at 5% p.a. ever since, compounded yearly.

Each year on Lisa's birthday, Marge and Homer deposit an extra $200 in her account.

(a) How much money was in this account after Lisa turned 1 year old?

(b) Write a recursive rule which defines the amount in the account each year, the day after Lisa's birthday.

(c) How much is in the account on 9th May 1999, just before Marge and Homer deposit the $200?

(d) Marge and Homer decide to withdraw all the money from the account on the day after Lisa's 18th birthday so that she can use the money for university. How much will Lisa receive?

(e) If Marge and Homer had chosen an account which compounded the 5% p.a. monthly rather than annually, what effect would this have had on the amount Lisa received when she turned 18? (No calculations are required in your answer).

Question Three: [2, 2, 2, 2, 1, 2 = 11 marks]

Aoife is training for a half marathon. Her goal is to be able to run the full 21 km in 100 minutes.

On her first run she completes the half marathon in 150 minutes. On her second run it takes her 148 minutes to complete the 21 km, and on the third run it takes her 146 minutes. Her run times continue in this way.

(a) How long will it take her to run the half marathon on her 10^{th} training session?

(b) How long will it take her to run the half marathon on her nth training session?

(c) When will she be able to run the half marathon in less than 2 hours?

(d) How long has she spent training during her first seven runs?

(e) If her training continues in this way, when will she achieve her goal?

(f) Aoife wonders if her training can reap these sort of results indefinitely. Explain whether Aoife can keep achieving these results.

Question Four: [1, 2, 3, 3, 1, 2, 2, 1 =15 marks]

Mr and Mrs Chapman borrowed $500 000 to buy a house on the 20th January 2010. The bank charges them interest at 6.2% p.a. compounded monthly. Each month they make a repayment on their loan of $3500.

(a) State the monthly interest rate.

(b) How much do they owe at the end of the first month, after their monthly repayment?

(c) Write a recursive rule which defines the balance on their loan at the end of each month.

(d) When will the Chapman's have paid off half of their home loan? State the year and month.

(e) How long will it take to pay back the entire loan if the interest rate and repayments remain constant?

(f) State two strategies the Chapman's could use to pay off their loan sooner.

(g) Calculate the total value of the repayments they have made to pay off their loan.

(h) Hence calculate the interest the total amount of interest they have paid.

education equals

SOLUTIONS
Calculator Assumed
Arithmetic and Geometric Sequences
Mixed Applications 2

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 2 = 8 marks]

Edith is knitting a woolen scarf that will measure 40 cm wide by 2 m in length. On the first day she knits 2000cm². Each subsequent day Edith knits 25% less than the day before.

(a) How much of the scarf does Edith knit on the second day?

$2000 \times 0.75 = 1500cm^2$ ✓
✓

(b) On which day does she knit less than half of the amount she knitted on the first day?

$1000 > 2000 \times 0.75^{n-1}$ ✓
$Day\ 4$
✓

(c) On which day has she finished knitting 75% of the total scarf?

$A = 200 \times 40 = 8000cm^2$
$S_n = 6000$ ✓
$n = 5$
$Day\ 5$ ✓

(d) Will Edith ever finish knitting this scarf? Justify your answer with full mathematical reasoning.

$S_\infty = \dfrac{2000}{1-0.75} = 8000$ ✓

At this rate Edith will need to knit for an infinite amount of time to finish her scarf, so realistically she will come very close to finishing the scarf after a month.
✓

Question Two: **[2, 3, 2, 2, 2 =11 marks]**

Homer and Marge started saving for their daughter Lisa's college education from the day she was born, 9th May 1990.

On the day she was born they deposited $1000 in a bank account that has earned interest at 5% p.a. ever since, compounded yearly.

Each year on Lisa's birthday, Marge and Homer deposit an extra $200 in her account.

(a) How much money was in this account after Lisa turned 1 year old?

$1000 \times 1.05 + 200 = \$1250$ ✓ ✓ ✓

(b) Write a recursive rule which defines the amount in the account each year, the day after Lisa's birthday.

$T_n = 1.05 \times T_{n-1} + 200;\ T_0 = 1000$ ✓ ✓ ✓

(c) How much is in the account on 9th May 1999, just before Marge and Homer deposit the $200?

$T_9 - 200 = \$3556.64$ ✓ ✓

(d) Marge and Homer decide to withdraw all the money from the account on the day after Lisa's 18th birthday so that she can use the money for university. How much will Lisa receive?

$T_{18} = \$8033.10$ ✓ ✓

(e) If Marge and Homer had chosen an account which compounded the 5% p.a. monthly rather than annually, what effect would this have had on the amount Lisa received when she turned 18? (No calculations are required in your answer).

✓ Monthly compounding of the same annual interest rate would result in a larger overall amount. The more often interest is compounded, the more quickly it accrues. ✓

Question Three: [2, 2, 2, 2, 1, 2 = 11 marks]

Aoife is training for a half marathon. Her goal is to be able to run the full 21 km in 100 minutes.

On her first run she completes the half marathon in 150 minutes. On her second run it takes her 148 minutes to complete the 21 km, and on the third run it takes her 146 minutes. Her run times continue in this way.

(a) How long will it take her to run the half marathon on her 10th training session?

$T_{10} = 150 - 2(10-1) = 132$ minutes

(b) How long will it take her to run the half marathon on her nth training session?

$T_n = 150 - 2(n-1)$

(c) When will she be able to run the half marathon in less than 2 hours?

$150 - 2(n-1) < 120$

$n = 17$

By the 17th run

(d) How long has she spent training during her first seven runs?

$S_7 = 1008$ minutes

(e) If her training continues in this way, when will she achieve her goal?

$150 - 2(n-1) = 100$

$n = 26$

After 26 runs

(f) Aoife wonders if her training can reap these sort of results indefinitely. Explain whether Aoife can keep achieving these results.

Aoife cannot continue in this way indefinitely as this will lead to her being able to run the half marathon in only a few minutes, or a negative amount of time, both of which are not possible.

Question Four: [1, 2, 3, 3, 1, 2, 2, 1 =15 marks]

Mr and Mrs Chapman borrowed $500 000 to buy a house on the 20th January 2010. The bank charges them interest at 6.2% p.a. compounded monthly. Each month they make a repayment on their loan of $3500.

(a) State the monthly interest rate.

$6.2 \div 12 = 0.5167\%$ ✓

(b) How much do they owe at the end of the first month, after their monthly repayment?

$500000 \times 1.0051666667 - 3500 = \$499083.33$ ✓

✓

(c) Write a recursive rule which defines the balance on their loan at the end of each month.

$A_n = 1.005167 \times A_{n-1} - 3500;\ A_0 = 500000$

✓ ✓ ✓

(d) When will the Chapman's have paid off half of their home loan? State the year and month.

$A_n < 250000$ ✓

$n = 171$

April 2024

✓ ✓

(e) How long will it take to pay back the entire loan if the interest rate and repayments remain constant?

$A_n = 0$

$n = 260$ ✓

August 2031

(f) State two strategies the Chapman's could use to pay off their loan sooner.

They could increase their monthly repayments and/or they could make weekly or fortnightly repayments to ensure more repayments are made each year.

(g) Calculate the total value of the repayments they have made to pay off their loan.

$= 259 \times 3500 + 3431.71$

$= \$909931.71$

(h) Hence calculate the interest the total amount of interest they have paid.

$\$909931.71 - 500000 = \$409931.71$