WILLETTON SENIOR HIGH SCHOOL

YEAR 12 MATHEMATICS METHODS

TEST 2 2023

Section 1: Calculator Free

Student Name: Solutions

Circle your teacher's name.

Miss Ahern	**Mr Galbraith**
Mrs Gatland	**Mrs Sun**

Mark: ______ / 28

-1 overall if use wrong variable in questions eg x instead of t

Time: 25 mins

For section 1 of this test:

No notes.

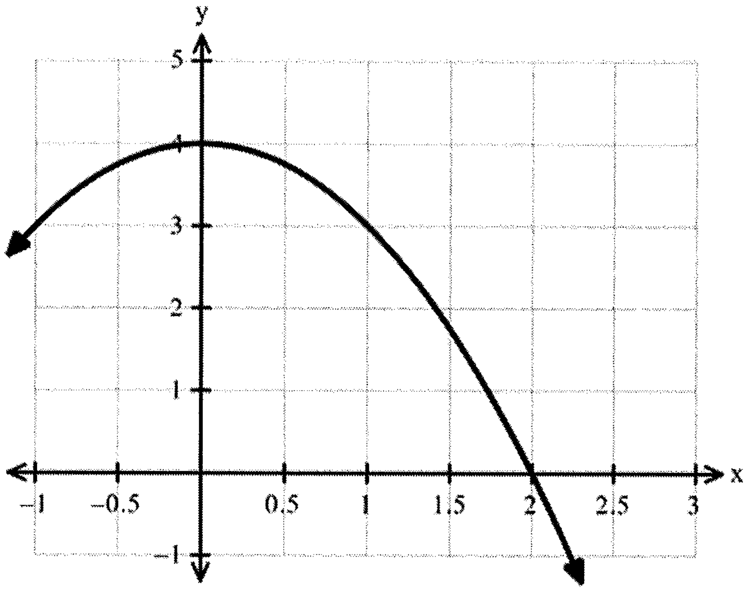
No calculators.

Formula sheet as provided.

Show working.

QUESTION ONE (5 MARKS)

A graph of y = f(x) with a table of values of the function for given values of x, is shown below.



x	0	0.5	1	1.5	2
y	4	$\frac{15}{4}$	3	$\frac{7}{4}$	0

a) Find the approximate area under the curve y = f(x) between x = 0 and x = 2 using the areas of four circumscribed rectangles of equal width.

[2]

$$\text{Area} = 0.5\left(4 + \frac{15}{4} + 3 + \frac{7}{4}\right) \checkmark$$
$$= 0.5\left(\frac{28}{4} + \frac{22}{4}\right)$$
$$= 0.5 \times \frac{50}{4}$$
$$= \frac{25}{4} \checkmark$$

b) Find the approximate area under the curve y = f(x) between x = 0 and x = 2 using the areas of four inscribed rectangles of equal width.

[2]

$$\text{Area} = 0.5\left(\frac{15}{4} + 3 + \frac{7}{4} + 0\right) \checkmark$$
$$= 0.5\left(\frac{34}{4}\right)$$
$$= \frac{17}{4} \checkmark$$

c) Using your answers from parts 'a' and 'b', find an improved estimate for the area under the curve y = f(x).

[1]

$$\text{Area} \approx \frac{\frac{25}{4} + \frac{17}{4}}{2} = \frac{42}{4} \times \frac{1}{2} = \frac{21}{4} \checkmark$$

See next page

QUESTION TWO (5 MARKS)

Determine the following:

a) $\frac{d}{dx}[x^2 e^{\sin(x)}]$ $= x^2 e^{\sin(x)} \times \cos(x) + e^{\sin(x)} \times 2x$ ✓ ✓

$= x^2 e^{\sin(x)} \times \cos(x) + 2x e^{\sin(x)}$

[2]

OR

$x e^{\sin(x)}(x\cos(x) + 2)$

b) $\int(3e^{2x} - cos2x)dx$ $= \frac{3}{2}e^{2x} - \frac{1}{2}\sin(2x) + C$ ✓ ✓

[2]

-1 missing constant

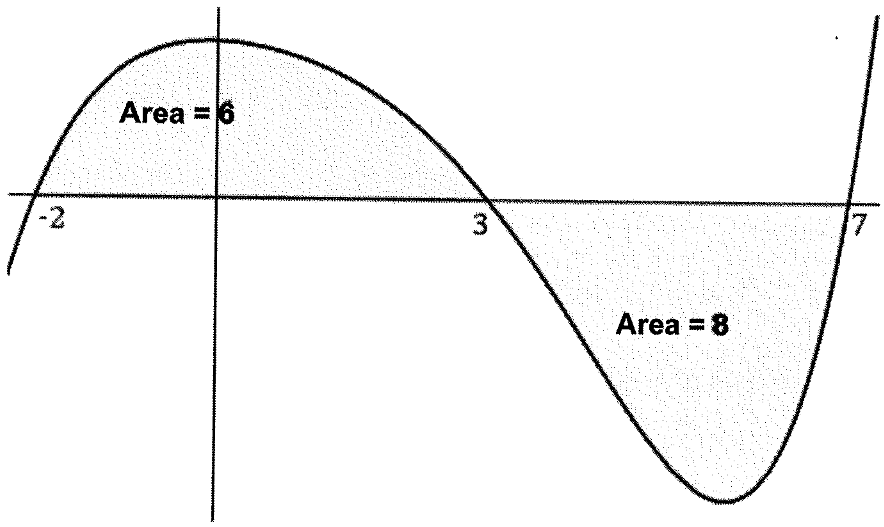
c) $\frac{d}{dx}(\int_{\frac{\pi}{2}}^{x} \sin(t)\, dt)$ $= \sin(x)$ ✓

[1]

QUESTION THREE (5 MARKS)

The function $f(x)$ is shown below.

The areas enclosed between the graph, the x-axis and the lines $x = -2$ and $x = 7$ are marked in the appropriate regions.



Determine:

a) $\int_{-2}^{7} f(x)dx$. $6 - 8 = -2$ ✓ [1]

b) the area enclosed between the graph of $f(x)$ and the x axis. $6 + 8 = 14$ ✓ [1]

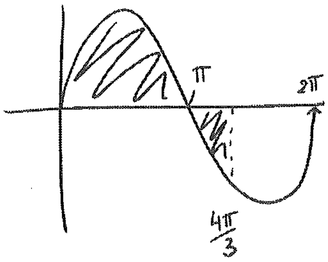
c) $\int_{3}^{7}(-f(x))dx$ $-(-8) = 8$ ✓ [1]

d) $\int_{3}^{-2} f(x)dx$ $-\int_{-2}^{3} f(x)dx = -6$ ✓ [1]

e) $\int_{-2}^{7} |f(x)|\, dx$ $6 + 8 = 14$ ✓ [1]

QUESTION FOUR (4 MARKS)

Find the area between the graph of y = sin(x) and the x axis between x = 0 and x = $\frac{4\pi}{3}$.



$$\text{Area} = \int_0^{\pi} \sin(x)\,dx - \int_{\pi}^{\frac{4\pi}{3}} \sin(x)\,dx \checkmark$$

$$= \Big[-\cos(x)\Big]_0^{\pi} - \Big[-\cos(x)\Big]_{\pi}^{\frac{4\pi}{3}} \checkmark$$

$$= [1-(-1)] - [\tfrac{1}{2}-(1)] \checkmark$$

$$= 2-(-\tfrac{1}{2})$$

$$= 2\tfrac{1}{2} \text{ units}^2 \checkmark$$

QUESTION FIVE (3 MARKS)

A particle is moving in a straight line with acceleration a m/s^2 at time t seconds with equation $a(t) = 40e^{0.4t}$

Find the total change in velocity between 0 and 5 seconds as an exact value.

$\therefore$ total change in velocity

$$\int_0^5 40e^{0.4t}\,dt \checkmark$$

$$= \left[\frac{40}{0.4}e^{0.4t}\right]_0^5 \checkmark$$

$$= \left[100e^{0.4t}\right]_0^5$$

$$= 100e^2 - 100 \checkmark$$

OR

$$100(e^2-1)$$

See next page

QUESTION SIX (6 MARKS)

a) Find $\frac{dy}{dx}$ given that $y = 3x\ sin\ 2x$ [2]

$$\frac{dy}{dx} = 3x \times 2\cos 2x + 3\sin 2x$$
$$= 6x\cos 2x + 3\sin 2x$$
✓ ✓

b) Use your answer from part (a) to determine $\int x \cos 2x\ dx$. [4]

$$\frac{d}{dx}(3x\sin 2x) = 6x\cos 2x + 3\sin 2x$$

$$\int \frac{d}{dx}(3x\sin 2x)dx = \int(6x\cos 2x + 3\sin 2x)dx$$ ✓

$$\int \frac{d}{dx}(3x\sin 2x)dx = \int(6x\cos 2x)dx + \int(3\sin 2x)dx$$

$$3x\sin 2x + c_1 = 6\int(x\cos 2x)dx - \frac{3}{2}\cos 2x + c_2$$
✓ ✓

$$6\int(x\cos 2x)dx = 3x\sin 2x + \frac{3}{2}\cos 2x + c$$

$$\int x\cos 2x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$$ ✓

END OF SECTION

WILLETTON SENIOR HIGH SCHOOL

YEAR 12 MATHEMATICS METHODS

TEST 2 2023

Section 2: Calculator Allowed

Student Name: ______________________________

Circle your teacher's name.

Miss Ahern	Mr Galbraith
Mrs Gatland	Mrs Sun

Mark: __________ / 25

Time: 25 mins

For this test:

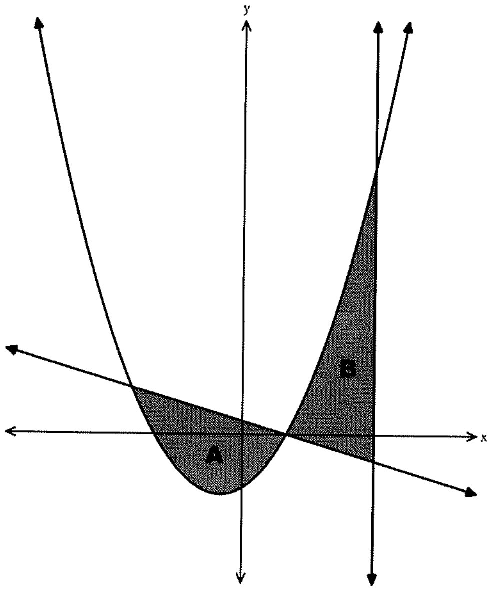
Scientific calculators and Classpads are allowed

One A4 single side of notes is allowed

Show any working in the spaces provided

QUESTION SEVEN (5 MARKS)

The graph below shows two shaded areas.
A is the area bounded by the curve $y = 0.5x^2 + x - 4$ and the line $y = -0.5x + 1$.
B is the area bounded between the curve $y = 0.5x^2 + x - 4$, the line $y = -0.5x + 1$ and the line $x = p$.



a) Given that the area of **B** is $\frac{116}{3}$ units^2, determine the value of p. [3]

$0.5x^2 + x - 4 = -0.5x + 1 \quad x = -5$ OR $x = 2$ ✓

$\int_2^p 0.5x^2 + x - 4 - (-0.5x + 1)\,dx = \frac{116}{3}$ ✓

$\int_2^p 0.5x^2 + 1.5x - 5\,dx = \frac{116}{3}$

$p = 6$ ✓

b) Determine the exact area shaded in total. [2]

$A = \int_{-5}^{2} -0.5x + 1 - (0.5x^2 + x - 4)\,dx + \frac{116}{3}$ ✓

$= \frac{343}{12} + \frac{116}{3}$

$= \frac{269}{4}$ units^2 OR 67.25 units^2 ✓

QUESTION EIGHT (7 MARKS)

An initially full water tank is emptied in 125 minutes at a rate of $V'(t) = 0.02t^2 - 2.1t - 50$ litres per minute.

a) Determine the flow rate of the water leaving the tank at $t = 15$ mins. [2]

$V'(15) = 0.02(15)^2 - 2.1(15) - 50$ ✓

$= -77 L/min$ ✓ OR $77L/min$ leaving tank

b) Determine the capacity of the water tank to the nearest litre. [2]

$V = \int_0^{125} |0.02t^2 - 2.1t - 50| \, dt$ ✓

$= 9635L$ ✓

c) Determine the time, to the nearest 0.1 minute, that it will take to reduce the volume of water in the tank to a fifth of its capacity. [3]

$9635 \div 5 = 1927L$ leave tank is $7708L$ ✓

$-7708 = \int_0^m 0.02t^2 - 2.1t - 50 \, dt$ ✓

$m = 84.6$ mins ✓ to reduce to a fifth capacity

QUESTION NINE (4 MARKS)

The number of bacteria (N) in a certain culture at time t weeks is modelled by the function $N = N_0 e^{kt}$, where when $t = 2$, $N = 101$ and when $t = 4, N = 203$.
Calculate the rate at which the bacteria is increasing when the number of bacteria in the culture reaches 500.

$101 = N_0 e^{2k}$

$203 = N_0 e^{4k}$ ✓

$\frac{203}{101} = e^{2k}$

$k = 0.34904$ ✓

$\frac{dN}{dt} = kN$

$= 0.349 \times 500$ ✓

$= 174.5$ bacteria/week

See next page

QUESTION TEN (4 MARKS)

When building a house, an optional feature is to have a heating system installed within the concrete slab. This system comprises of tubing installed within the concrete slab and water runs though the tubing to heat the concrete during winter.

The number of litres/ minute of water flowing through the tubing over t minutes can be modelled by the rule; $\frac{dV}{dt} = 2[\cos\left(\frac{\pi t}{3}\right) + \sin\left(\frac{\pi t}{9}\right) + 3]$. Determine the amount of water that runs through the pipes during the time period for one cycle.

[Hint: Consider the graph of the function to determine the time period for one cycle first]

looking at graph peaks at 5.854 and 23.854 ✓

∴ period is 18 mins ✓

$V = \int_0^{18} 2[\cos(\frac{\pi t}{3}) + \sin(\frac{\pi t}{9}) + 3]dt$ OR $V = \int_{5.854}^{23.854} 2[\cos(\frac{\pi t}{3}) + \sin(\frac{\pi t}{9}) + 3]dt$ ✓

$= 108L$ ✓

QUESTION ELEVEN (5 MARKS)

Given that $F(x) = \int_1^x f(t)\,dt$, with $F(2) = 11$ and $F''(x) = 6x$, find $f(t)$

$F'(x) = f(x) \Rightarrow F''(x) = f'(x)$

$\Rightarrow f'(x) = 6x$ ✓

$f(x) = 3x^2 + c$ ✓

$F(2) = \int_1^2 f(t)\,dt = 11$

$\therefore \int_1^2 (3t^2 + c)dt = 11$ ✓

$[t^3 + ct]_1^2 = 11$ ✓

$8 + 2c - 1 + c = 11 \Rightarrow c = 4$

$f(t) = 3t^2 + 4$ ✓

END OF SECTION