Name SOLUTIONS

Calculator Assumed **Notes Permitted**

Marks available: 21 **Time allowed: 25 minutes**

1. [3 marks]

Using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or otherwise, determine the exact solutions to $2x^2 - 2x + 25 = 0$.

$0.5 \pm \dfrac{\sqrt{-196}}{4}$ ✓ $= \dfrac{1}{2} \pm 3.5i$ ✓✓

2. [5 marks]

If w = 5 – 3**i** and c = 2 + **i** determine exactly in the form a + bi

a) c^2

$3 + 4i$ ✓

b) $\overline{w}\,\overline{c}$

$(5-3i)(2+i)$ ✓

$13 - i$ ✓

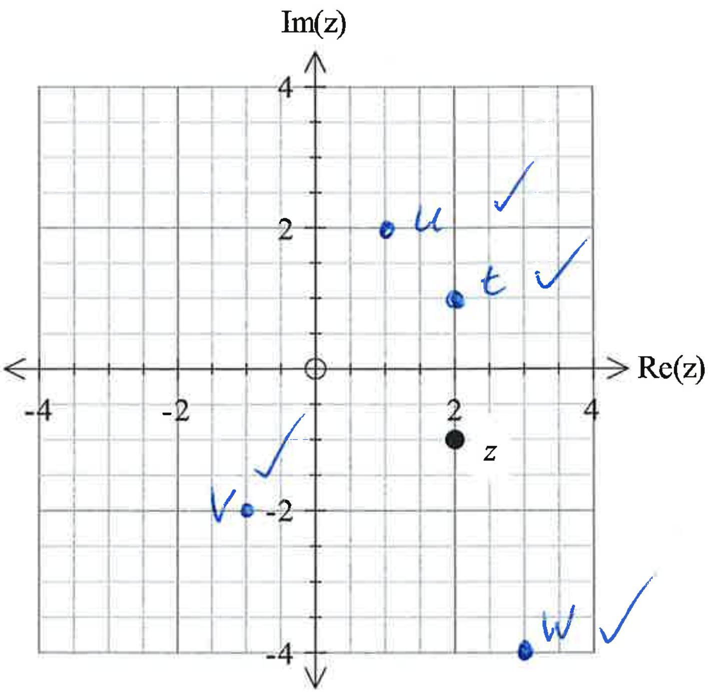
c) $\dfrac{w}{c}$

$\dfrac{5-3i}{2+i} \times \dfrac{2-i}{2-i}$ ✓

$= \dfrac{7}{5} - \dfrac{11}{5}i$ ✓

3. [4 marks]

The Argand diagram below shows the complex number z. On the same diagram **plot and label** the four complex numbers given by $t = \overline{z}$, u = **i** z , v = $\mathbf{i}^3 z$, w = z^2.



4. **[4 marks]**

Prove by contradiction that $\dfrac{\sqrt{3}-\sqrt{5}}{4}$ is irrational.

Assume $\dfrac{\sqrt{3}-\sqrt{5}}{4} = a$ where a is rational ✓

$\sqrt{3}-\sqrt{5} = 4a$

$(\sqrt{3}-\sqrt{5})^2 = 16a^2$

$3+5-2\sqrt{15} = 16a^2$

$\sqrt{15} = 4-8a^2$ ✓

If a is rational, $4-8a^2$ must also be rational ✓
But $\sqrt{15}$ is irrational which contradicts our assumption that a is rational. Hence $\dfrac{\sqrt{3}-\sqrt{5}}{4}$ is irrational ✓

5. **[5 marks: 1, 4]**

The variables k and m are both positive integers such that $m^2+3=2k$.

i. Explain why m must always be odd.

$2k$ is even, hence m^2+3 is even
Since 3 is odd m^2 is odd
So m must be odd. ✓

ii. Using the fact that an odd integer can be written in the form $2n+1$, or otherwise, prove that k is always the sum of three square numbers.

$2k = m^2+3$

$= (2n+1)^2+3$ ✓

$= 4n^2+4n+4$

$k = 2n^2+2n+2$ ✓

$= n^2+n^2+2n+1+1$

$= n^2+(n+1)^2+1^2$ ✓✓