

Name \_\_\_\_\_

Time: 18 minutes

Marks: 18

All reasoning must be shown to be awarded full marks.

1. [3, 3 marks]

- a) Based on Bureau of Meteorology records, there is a 20% chance that it will rain on any given day in November. A Methods class wishes to simulate the daily rainfall in November, 2018 and determine how many days it will rain. Describe how they could do this and explain your reasoning.

Using  $\text{RAND}(1, 5)$  or  $\text{RANDLIST}(30, 1, 5)$  ✓  
 GENERATE 30 RANDOM NOS → FOR EACH DAY IN NOVEMBER.  
 LET 1, 2, 3, 4 BE FINE / 5 BE RAIN. ✓  
 REPEAT SEVERAL TIMES + FIND  $\bar{x}$ . ✓  
 (NOTE: COULD USE  $\text{RANDBIN}(1, 0.2)$ )

- b) Year 12 male students are typically between 165 and 195 cm in height, and normally distributed. A Methods class wish to simulate a set of 10 Year 12 boys' heights. Describe a method they could use to do this and comment on their simulation.

Using  $\text{RANDNORM}(5, 180, 10)$  ✓  
 OR  $\text{INT}(\text{RANDNORM}(\quad))$  ✓  
 TO GENERATE 10 RANDOM NOS WITH  $\mu = 180$ ,  $\sigma_x = 5$  ✓  
 THIS IS TOO SMALL A SAMPLE. NEED TO DO GREATER  $n$ , OR REPEAT SEVERAL TIMES. ✓

2. [2 marks]

The same Methods class of 15 students determine that because there are 6 girls in the class the sample proportion of girls doing Math Methods is  $\hat{p} = 0.4$   
 Discuss this proposition in terms of the population proportion  $p$ .

ONE SAMPLE IS NOT APPROPRIATE TO APPROXIMATE A NORMAL DISTRIBUTION. THEY WOULD NEED TO COLLECT MORE SAMPLES. ✓  
 (NEED AT LEAST  $n \cdot p > 10$  AND  $n(1-p) > 20$ .) ✓

3. [2, 3, 2 marks]

In an exam, the final marks are found to be normally distributed with a mean of 55% and a standard deviation of 15%. Based on the **68%, 95%, 99.7% rule**

a) Approximately what percentage of students scored above 40%?

$$P(X > 40) = P(Z > -1) = 84\%$$

b) If 10 000 students sat the exam, approximately how many scored above 85%?

$$P(X > 85) = P(Z > 2) = 0.025$$
$$0.025 \times 10000 = 250$$

c) If Greg had a standardized score of -1.1, what was his actual score?

$$\frac{x - 55}{15} = -1.1$$
$$x - 55 = -16.5$$
$$x = 38.5$$

4. [2, 1 marks]

The kakapo or night parrot, also called owl parrot, is a species of large, flightless, nocturnal, ground-dwelling parrot of the family Strigopoidea, endemic to New Zealand. To ascertain how many are left in the wild, 10 are caught and tagged. At a later date, another 25 are caught and 3 of them are found to be tagged.

a) Estimate the population of kakapo in the wild.

$$\frac{10}{P} = \frac{3}{25}$$
$$3P = 250$$
$$P \approx 83 \text{ kakapo}$$

b) Outline one factor that may make this estimate of the population unreliable.

- Too much time between capture/recapture
- Different region
- any reasonable response

Name \_\_\_\_\_

Time: 36 minutes

Marks: 36

All reasoning must be shown to be awarded full marks.

5. [2, 2, 2, 2 marks]

For a certain type of laptop, the length of time required to fully recharge the battery is normally distributed with a mean of 100 minutes and a standard deviation of 15 minutes.

Determine the probability that, for a randomly selected laptop, the battery will take

a) more than 2 hours to recharge?

$$X \sim N(100, 15^2) \quad \checkmark$$

$$P(X > 120) = 0.0912 \quad \checkmark$$

b) less than 2 hours, given it takes more than 1 hour?

$$\frac{0.9090}{0.9082} = 0.9085 \quad \checkmark$$

The manufacturer has determined that 2% of batteries will need replacement because they take too long to fully recharge, and recall these batteries for replacement.

c) What is the minimum time a recalled battery will take to recharge?

$$P(X < k) = 0.98 \quad \checkmark$$

$$k = 130.8 \text{ min.} \quad \checkmark$$

Five laptops are selected at random.

d) Determine the probability that more than one will require more than 2 hours to recharge.

$$Y \sim \text{Bin}(5, 0.0912) \quad \checkmark$$

$$P(Y \geq 2) = 0.0690 \quad \checkmark$$

6. [1, 2, 2, 2, 1, 1 marks]

The management at a Conference Centre was concerned about the quality of the free pens that it provided to those attending. A staff member tested a random sample of 150 pens and found that 21 of them fail to write.

(a) If  $p$  is the true proportion of pens that fail to write and  $\hat{p}$  is the corresponding sample proportion, use the above sample to determine

(i)  $\hat{p}$ .  $\hat{p} = \frac{21}{150} = 0.14$  ✓

(ii) the approximate margin of error for a 95% confidence interval for  $p$ .

$$1.960 \sqrt{\frac{0.14(0.86)}{150}} = 0.0555$$
 ✓ ✓

(iii) an approximate 95% confidence interval for  $p$ .

$$0.0845 < p < 0.1955$$
 ✓ ✓

(b) The stationery company that supplies pens to the conference centre claim that no more than 1 in 10 pens fail to write. Use your previous working to comment on the validity of this claim.

0.1 lies within this conf. interval, so it is valid. ✓ ✓

(c) How does the margin of error change in (a) (ii) if

(i) the quality of the pens had been better?

decreases ✓

(ii) the required level of confidence decreased?

decreases ✓



7. [5, 2 marks]

An AFL club wishes to change the Club jumper, but will survey their members first. The category of member and the sex of the members are indicated below:

	Male	Female
Child (0 – 15 years)	3 259	3 345
Adult	21 456	20 159
Concession	10 308	10 287

As the Club Secretary, you are asked to survey members by selecting a sample to determine whether this is acceptable proposition to them.

a) Describe how you would do this.

Total = 68814 ✓  
For a sample of 800, select ✓

	m	F
Child	24	24
Adult	156	146
Concess.	75	75

by using stratified sampling. ✓  
(must give one example) ✓  
Use alphabetic lists on ✓  
numbering each group ✓

b) What type of bias could be present in your survey if

i) You post a questionnaire and reply-paid envelope to your selected sample.

if. Some people may not reply. ✓

ii) You stand at the gate of the stadium as the game starts and question the members as they arrive.

if. Assumes that all members are attending ✓

8. [5 marks]

In survey of  $n$  people, regarding the time taken to speak to someone at Centrelink over the phone, it was found that 152 people felt the wait time was too high.

The confidence interval calculated for this was  $0.701 < p < 0.819$ . Determine  $n$ , thus find the confidence level (correct to the nearest integer).

$$\begin{aligned}\hat{p} &= 0.76 \quad \checkmark \\ \frac{152}{n} &= 0.76 \quad \checkmark \\ n &= 200. \quad \checkmark \\ k \sqrt{\frac{(0.76)(0.24)}{200}} &= 0.059 \quad \checkmark \\ k &= 1.953 \quad \checkmark \\ 95\% \text{ confidence interval} &\quad \checkmark\end{aligned}$$

9. [2, 2, 3 marks]

Precision Parts makes components for machinery. They find that, using laser technology to measure the mass of each component, the mass of parts being produced are normally distributed with a mean of 125 grams and a standard deviation of 0.25 grams.

- a) They produce 1 000 components an hour. A component is rejected if it is outside the limits of 124.5 and 125.5 grams. How many are rejected?

$$\begin{aligned}P(124.5 < X < 125.5) &= 0.9545 \quad \checkmark \\ 0.9545 \times 100 &= 954 \text{ (or } 955) \quad \checkmark \\ \text{So 46 are rejected.} &\quad \checkmark\end{aligned}$$

- b) Consider 3 components selected at random. What is the probability at least one is rejected?

$$\begin{aligned}Y &\sim \text{BIN}(3, 0.0455) \quad \checkmark \\ P(X \geq 1) &= 0.2077 \quad \checkmark\end{aligned}$$

- c) It is considered that too many components are rejected. The machine is recalibrated such that no more than 20 components will be rejected in one hour. Given the mean remains unchanged, what is the standard deviation?

$$\begin{aligned}P(-k < X < k) &= 0.98 \quad \checkmark \\ k &= 2.326 \quad \checkmark \\ \text{If } \frac{125.5 - 125}{\sigma} &= 2.326 \quad \checkmark \\ \sigma &= 0.215 \text{ grams.} \quad \checkmark\end{aligned}$$