WILLETTON SENIOR HIGH SCHOOL

MATHEMATICS METHODS – UNIT TWO

TEST TWO 2022

SECTION ONE: Calculator Free

STUDENT NAME: ....Draft 2 Solutions........

TOTAL MARKS: .... / 36

TIME ALLOWED: 30 mins

CIRCLE YOUR TEACHER'S NAME:

Mrs Kalotay	Ms Leow	Mr Riemer	Ms Mack
Ms Thompson	Ms Smirke	Mrs Gatland	

- Formulae sheet supplied.
- No calculators allowed.
- If a question is worth more than 2 marks, sufficient working must be shown to justify your answer, in order to receive full marks.

Question 1 [1,2,3,2,2=10 marks]

The graph of the quadratic function $y = -2(x+1)^2 + 8$

(a) Determine the line of symmetry. [1]

$x = -1$ ✓

(b) Determine the nature and coordinate of turning point. [2]

maximum ✓

$(-1, 8)$ ✓

must be in coordinates.

take away 1 mark per part (b) & (c)

(c) Determine the coordinates of the intercepts. [3]

$x = 0$

$y = -2(1)^2 + 8$

$y = 6$

$(0,6)$ ✓

$y = 0$

$0 = -2(x+1)^2 + 8$

$4 = (x+1)^2$

$x + 1 = 2$ $\quad$ $x + 1 = -2$

$x = 1$ $\quad$ $x = -3$

$(1, 0)$ ✓

$(-3, 0)$ ✓

(d) If the function passes through the point $(d, -10)$. Determine value(s) of d. [2]

$y = -10$

$-10 = -2(x+1)^2 + 8$

$2(x+1)^2 = 18$

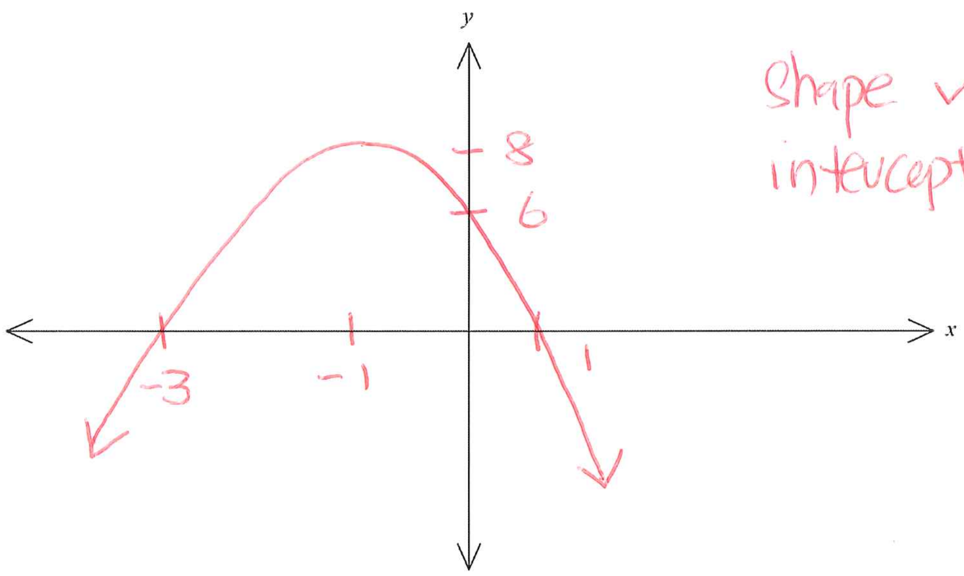
$(x+1)^2 = 9$

$x + 1 = 3$ $\quad$ $x + 1 = -3$

$x = 2$ $\quad$ $x = -4$

$d = 2$ or -4 ✓ ✓

(e) Sketch the quadratic function in the given grid with all notable features. [2]



Shape ✓

intercepts ✓

Question 2 [3,3=6 marks]

(a) Expand and simplify $(2x+1)^4$. [3]

$(2x)^4 + 4(2x)^3 + 6(2x)^2 + 4(2x) + 1$ ✓

$= 16x^4 + 32x^3 + 24x^2 + 8x + 1$ ✓✓ -1 per error

(b) State the coefficient of the x^3 term when $\left(2x - \frac{3}{x}\right)^5$ is expanded. [3]

✓ $5(2x)^4\left(-\frac{3}{x}\right)^1$ ✓

$= 5(16x^4)\left(-\frac{3}{x}\right)$

$= -240x^3$

coefficient $= -240$ ✓

Question 3 [3 marks]

Given that $2x^2 - 4px + 2p^2 - 2p + 5 = 0$ has no real roots, determine the greatest integer value of p. [3]

$a = 2$

$b = -4p$

$c = 2p^2 - 2p + 5$

no real roots

$b^2 - 4ac < 0$

$(-4p)^2 - 4(2)(2p^2 - 2p + 5) < 0$ ✓

$16p^2 - 16p^2 + 16p - 40 < 0$

$16p < 40$

$p < \frac{40}{16}$

$p < 2.5$ ✓

greatest integer of $p = 2$ ✓

Question 4 [3 marks]

Determine the radius and the coordinates of the centre of the circle with equation: [3]

$$x^2 + y^2 + 6x - 4y - 3 = 0$$

$x^2 + 6x + \left(\frac{6}{2}\right)^2 + y^2 - 4y + \left(\frac{4}{2}\right)^2 = 3 + \left(\frac{6}{2}\right)^2 + \left(\frac{4}{2}\right)^2$

$(x+3)^2 + (y-2)^2 = 3 + 9 + 4$

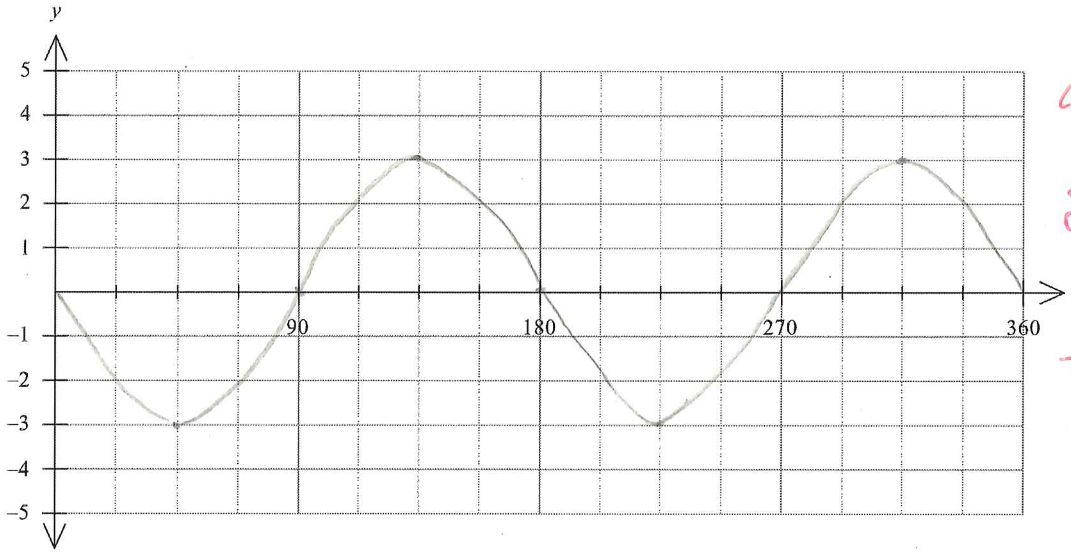
$(x+3)^2 + (y-2)^2 = 16$ ✓

centre $= (-3, 2)$ ✓ radius $= 4$ ✓

Question 5 [3,3=6 marks]

Sketch the following functions on the axes given.

(a) $y = 3\sin(2x + \pi)$ for $0 \leq x \leq 360°$ [3]

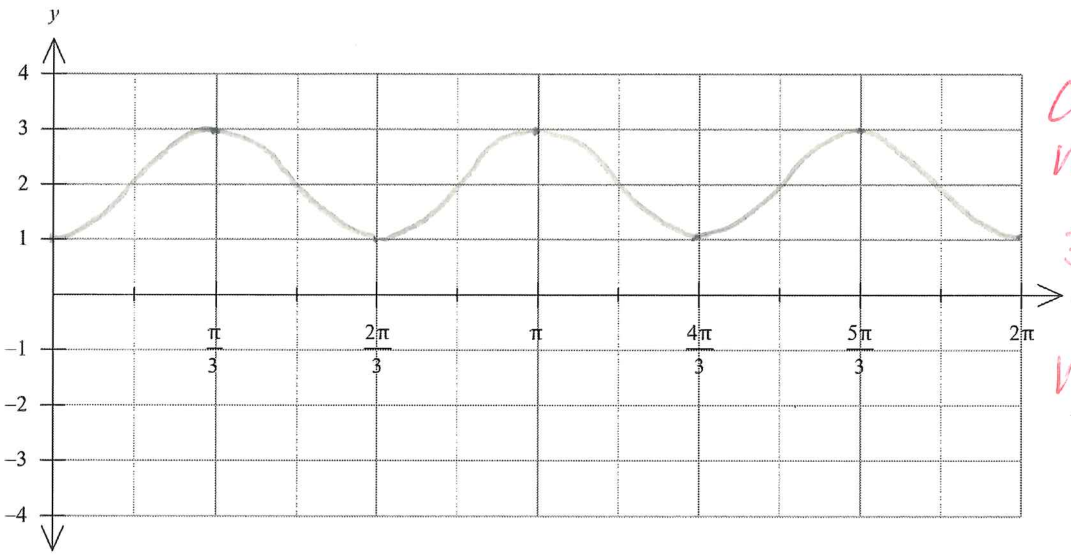


amplitude = 3 ✓

2 cycles ✓

translate 90° left ✓

(b) $y = -\cos(3x) + 2$ for $0 \leq x \leq 2\pi$ [3]



Correct max & min ✓

3 cycles ✓

vertical reflection ✓

Question 6 [3 marks]

Consider the function $y = -\cos\left(2x + \frac{\pi}{2}\right) + 3$. Determine the coordinates which will give the maximum value of this function for the domain $0 \leq x \leq \pi$.

need to find min value as it is a vertical reflection.

min value → $(\pi, -1)$ ✓

$-\frac{\pi}{2}$ $\left(\frac{\pi}{2}, -1\right)$

$\div 2$ $\left(\frac{\pi}{4}, -1\right)$

$\left(\frac{\pi}{4}, 1\right)$ $x -1$

$\left(\frac{\pi}{4}, 4\right)$ $+3$

✓ ✓

Question 7 [5 marks]

Determine all the intercepts of the function $f(x) = 2x^3 - 11x^2 + 2x + 15$. [5]

let $x = -1$

$f(-1) = 2(-1) - 11(1) + 2(-1) + 15$

$= -2 - 11 - 2 + 15$

$= 0$

Hence $(x+1)$ is a factor ✓ Finding 1st factor

$$\begin{array}{r} 2x^2 - 13x + 15 \\ x+1 \overline{) 2x^3 - 11x^2 + 2x + 15} \\ 2x^3 + 2x^2 \\ \hline -13x^2 + 2x \\ -13x^2 - 13x \\ \hline 15x + 15 \\ 15x + 15 \\ \hline 0 \end{array}$$ ✓

Finding quadratic factor

$2x^2 - 13x + 15 = (2x-3)(x-5)$ ✓

completely factorise

intercepts

$$\left.\begin{array}{l} y = 15 \\ x = -1 \\ x = \frac{3}{2} \\ x = 5 \end{array}\right\}$$ ✓✓

-1 per missing intercept

End of Calc Free

WILLETTON SENIOR HIGH SCHOOL

MATHEMATICS METHODS – UNIT TWO

TEST TWO 2022

SECTION TWO: Calculator Assumed

STUDENT NAME: ....

TOTAL MARKS: .... / 21

TIME ALLOWED: 20 mins

CIRCLE YOUR TEACHER'S NAME:

Mrs Kalotay	Ms Leow	Mr Riemer	Ms Mack
Ms Thompson	Ms Smirke	Mrs Gatland	

- Formulae sheet supplied.
- One page of A4 notes allowed.
- Classpads and scientific calculators are allowed.
- If a question is worth more than 2 marks, sufficient working must be shown to justify your answer, in order to receive full marks.

Question 8 [1,3=4 marks]

Mya is choosing which five subjects she wants to study in Year 11. If there are eight humanities subjects and six science subjects on offer at her school:

(a) Determine the total numbers of choices Mya can make in selecting her five subjects. [1]

$^{14}C_5 = 2002$ ✓

(b) If Mya has to choose either three from the humanities and two from the sciences, or two from the humanities and three from the sciences, determine the number of choices Mya has. [3]

$^{8}C_3 \times {}^{6}C_2$ ✓ $+$ $^{8}C_2 \times {}^{6}C_3$ ✓

$= 840 + 560$

$= 1400$ ✓

Question 9 [2,2=4 marks]

For a given light source, the intensity (l) in lumens, of light is inversely proportionate to the square of the distance (d) in metres from the light source.

(a) If the intensity is 20 lumens when observed from a distance of 5 metres, determine a formula relating the intensity and distance. [2]

$l = \frac{k}{d^2}$

$20 = \frac{k}{5^2}$

$k = 500$ ✓

$l = \frac{500}{d^2}$ ✓

(b) Hence, determine the distance of an observer from the given light source for an intensity of 40 lumens. [2]

$40 = \frac{500}{d^2}$ ✓

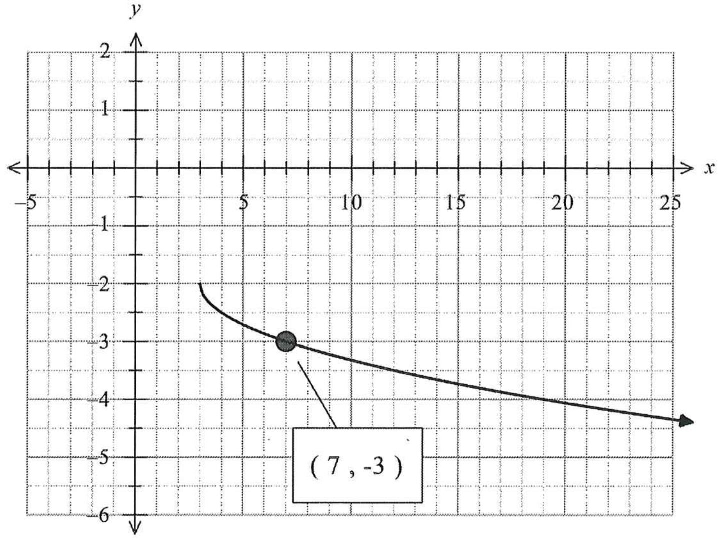
$d^2 = \frac{500}{40}$

$d = 3.536$ m ✓

Question 10 [3,2,3=8 marks]

Determine the equations of the following graphs.

(a) [3]



$y = a\sqrt{x-3} - 2$ ✓ ✓

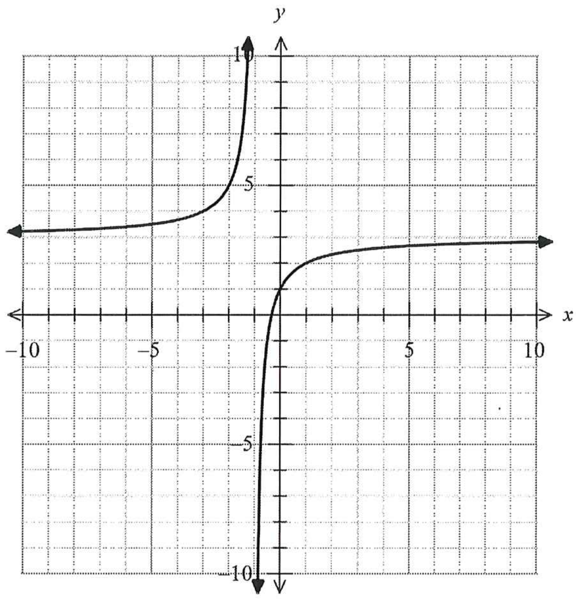
sub $(7, -3)$

$a = -\frac{1}{2}$

$y = -\frac{1}{2}\sqrt{x-3} - 2$

✓

(b) [2]



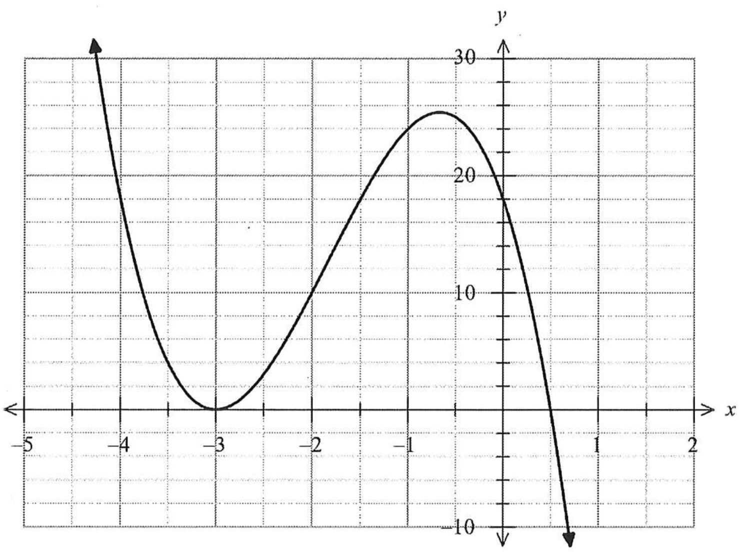
$y = \frac{a}{x+1} + 3$ ✓

sub $(0, 1)$

$a = -2$

$y = \frac{-2}{x+1} + 3$ ✓

(c) [3]



$y = a(x+3)^2(2x-1)$

✓ ✓

sub $(0, 18)$

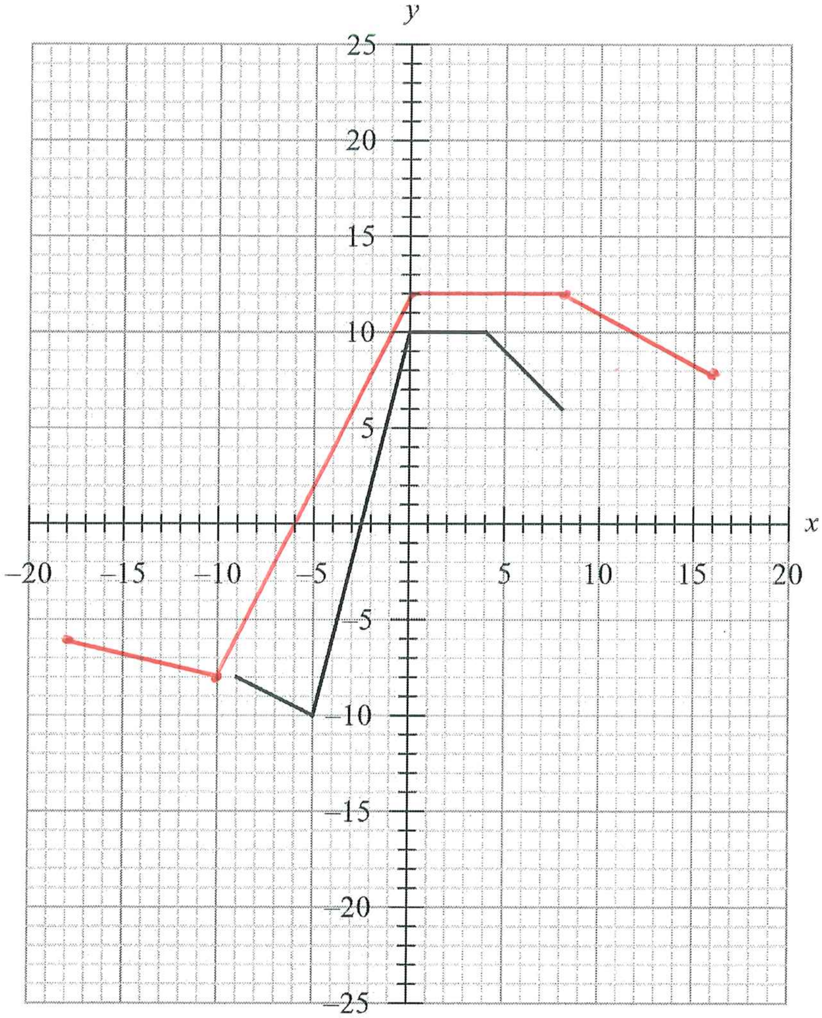
$a = -2$

$y = -2(x+3)^2(2x-1)$

✓

Question 11 [2,2,1=5 marks]

The graph of $y = f(x)$ has been drawn in the diagram below.



(a) Describe the transformation required for $y = f(x)$ to become $y = f\left(\frac{1}{2}x\right) + 2$. [2]

dilate horizontally with scale factor 2 ✓

vertically translate up 2 units ✓

(b) Sketch the transformed graph of $y = f\left(\frac{1}{2}x\right) + 2$ in the diagram above. ~~[2]~~ 3

all vertices correct and joined.

-1 per error

(c) State the domain for $y = f\left(\frac{1}{2}x\right) + 2$. [1]

$-18 \leq x \leq 16$ ✓

End of Calc Assumed