Mathematics Department

11 Maths Methods Test 6 Even

Introduction to Differentiation

Name

Section 1 – Resource Free – Students can have the formula sheet

Marks : ~~30~~ ~~29~~ 31 **Time:** ~~32~~ ~~30~~ 33 **minutes (maximum)**

1: [2,2,2,2 = ~~8~~ 13 marks]

Differentiate each of the following with respect to 'x'.

a) $y = x^5 + 4x^2 - 8x + 3$

$\frac{dy}{dx} = 5x^4 + 8x - 8$ ✓✓

b) $g(x) = \frac{3}{10x^5}$ $= \frac{3}{10} x^{-5}$

$g'(x) = \frac{-15}{10} x^{-6}$

$g'(x) = \frac{-3}{2x^6}$ ✓✓

c) $f(x) = (8 - x)(5x^2 + 7)$

$f(x) = 40x^2 + 56 - 5x^3 - 7x$

$f'(x) = 80x - 15x^2 - 7$ ✓✓

d) $y = (3 - 5x)^2$

$y = 9 - 30x + 25x^2$

$\frac{dy}{dx} = -30 + 50x$ ✓✓

e) $y = \frac{8x^3 - 7}{x}$

$y = 8x^2 - 7x^{-1}$

$\frac{dy}{dx} = 16x + \frac{7}{x^2}$ ✓✓✓

f) $y = \sqrt[5]{x^2}$

$y = x^{2/5}$

$y' = \frac{2}{5x^{3/5}}$ ✓✓

~~positive~~

−1 no positive

2: [2 marks]

State one function whose derivative is **2x – 1**.

$F(x) = x^2 - x + 7$

✓✓

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3. [4 marks]

$-2 \quad -1$

Find the <u>equation of the tangent line</u> to the curve $\mathbf{y = 2x^3 - x^2 + 6}$ at the point **(-1,3)**. ✓

$$\frac{dy}{dx} = 6x^2 - 2x$$ ✓

$$\left.\frac{dy}{dx}\right|_{x=-1} = 6 + 2 = 8$$ ✓

$$\therefore \quad y = 8x + c$$

$$3 = -8 + c$$ ✓

$$c = 11$$

$\therefore$ Eqn Tangent $y = 8x + 11.$ ✓

4. [~~5~~ 6 marks]

Given $y = (2x + 1)^2$ and $u = 2x + 1$, find:

$y = 4x^2 + 4x + 1$

a) $\frac{dy}{dx} = 8x + 4$ ✓✓

b) $\frac{du}{dx} = 2$ ✓

c) y in terms of u. $y = u^2$ ✓

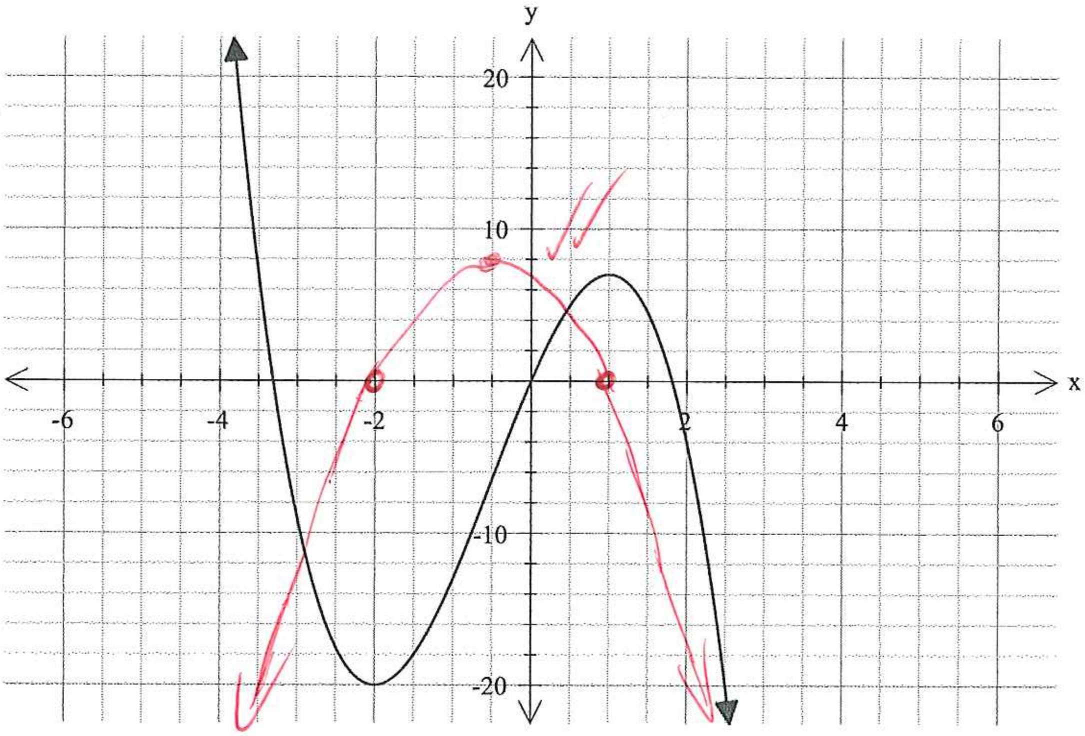
d) $\frac{dy}{du}$ in terms of x $\quad \frac{dy}{du} = 2u$ ✓

ie $\frac{dy}{du} = 2(2x+1)$ ✓

$\boxed{10}$

5. [5 marks] 6

Below is a graph of y = f(x)



a) State the value(s) of x for which:

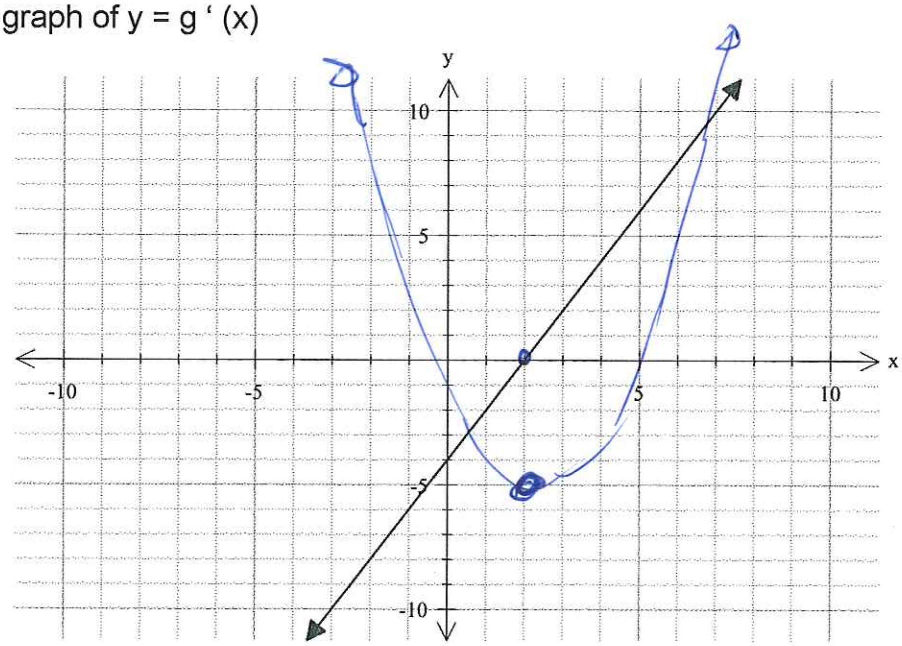
i) f ' (x) < 0 $x \leq -2$ or $x \geq 1$

ii) f ' (x) = 0 $x = -2$ or $x = 1$

iii) f ' (x) > 0 $-2 \leq x < 1$

b) On the grid above, draw a possible graph of y = f ' (x)

6. [2 marks]
Below is a graph of y = g ' (x)



On the grid above, draw a possible graph of y = g(x)

Mathematics Department

11 Maths Methods Test 6 Odd

Introduction to Differentiation

Name

Section 2 – Resource Rich – calculators, formula sheet and 1 page of notes
Marks : 25 **Time: 27 minutes (minimum)**

1. [5 marks] 7

a) Given the function $f(x) = 5x^3$, complete the following table:

Point P	Point Q	Gradient of PQ (3 decimal places)
(2, 40)	(3, 135)	$\frac{135-40}{3-2} = \frac{95}{1} = 95$
(2, 40)	(2.1, 46.305)	$\frac{6.305}{0.1} = 63.05$
(2, 40)	(2.01, 40.60)	$= 60.301$
(2, 40)	(2.001, 40.06003)	$= 60.030$

b) Find the value of the gradient to the curve $f(x) = 5x^3$, at the point (2, 40)

$f'(x) = 15x^2$
$f'(2) = 60$

2. **[4 marks]**
Given $f(x) = 6x - 3$, find:

a) $f(x + h) = 6(x+h) - 3 = 6x + 6h - 3$

b) Show that $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = 6$

$$\lim_{h \to 0} \frac{6x+6h-3-(6x-3)}{h}$$
$$= \lim_{h \to 0} \frac{6x+6h-3-6x+3}{h}$$
$$= \lim_{h \to 0} \frac{6h}{h}$$
$$= 6$$

3. [4 marks]
Given that $f(t) = 4x^3 - 2x$, find:

a) f ' (t)

b) Hence, or otherwise, find when f ' (t) = 1

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3. [4 marks]
Given that $f(t) = 4x^3 - 2x$, find:

a) $f'(t)$ $= 12t^2 - 2$ ✓

b) Hence, or otherwise, find when $f'(t) = 1$ ✓

ie $12t^2 - 2 = 1$

$12t^2 = 3$

$t^2 = \frac{1}{4}$

$t = \pm\frac{1}{2}.$ ✓✓

4. [5 marks]
Find the point(s) of intersection between the curves $y = x^2 + 2x + 3$ and $y = x^2 - x + 6$. Hence, find the equation of the tangent(s) to the curve $y = x^2 - x + 6$ at the point(s) of intersection.

Solve $x^2 + 2x + 3 = x^2 - x + 6$

$x = 1$ ✓

If $y = x^2 - x + 6$

$\frac{dy}{dx} = 2x - 1$ ✓

$\left.\frac{dy}{dx}\right|_{x=1} = 1$

when $x = 1$, $y = 6$ ✓ $\quad 6 = \cancel{1} + c$

$c = 5$ ✓✓

$\therefore y = x + 5$

take of 2 if no justification

5. [5 marks]

$y = 2x + 1$ is a tangent to the curve $y = a\cancel{x^2}x^3 + bx$, at the point (1, 3). Find the values of a and b.

$m = 2$ when $x = 1$ $\quad \frac{dy}{dx} = \cancel{2}3ax^2 + b$ ✓

ie $2 = \cancel{2}3a + b$ ✓✓

using (1, 3) $\quad$ and $\quad 3 = a + b$ ✓

$-1 = 2a$

$a = -\frac{1}{2}$ ✓

$b = \cancel{2}3\frac{1}{2}$ ✓

$\boxed{14}$