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Calculator Free
Arithmetic Sequences

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 3 = 7 marks]

Define each of the following sequences recursively and state T_6 :

(a)   10, 8, 6, 4 ...

(b)   90, 100, 110, 120 ...

(c) $\dfrac{1}{8}, \dfrac{1}{4}, \dfrac{3}{8}, \dfrac{1}{2} ...$

Question Two: [1, 2, 2 = 5 marks]

Consider the sequence $U_n = -3 + U_{n-1}$; $U_2 = 5$

(a)   Calculate U_3

(b)   Calculate U_1

(c)   Determine the general term of the sequence.

Question Three: **[2, 2, 1, 2, 2 = 9 marks]**

The first term of an arithmetic sequence is 2 and the 5th term is 14.

    (a)   Determine the common difference of this sequence.

    (b)   Hence or otherwise define this sequence recursively.

    (c)   Calculate T_{10}

    (d)   Calculate S_{10}

    (e)   Determine when the sequence first has a value greater than 100.

Question Four: **[2 marks]**

Show that the sequence $T_n = 10 - 3(n-1)$ can be written as $T_n = -3n + 13$

Question Five: **[4 marks]**

Determine when the value of the sequence $T_{n+1} = T_n + 10$; $T_1 = 5$ and of the sequence $T_n = 290 - 5(n-1)$ are equivalent.

Question Six: **[4, 2 = 6 marks]**

The 12th term of an arithmetic sequence is 57 and the 21st term is 84.

(a)   Determine the value of the first term and the common difference of this sequence.

(b)   Hence calculate S_5

Question Seven: [2, 2, 2 = 6 marks]

The first three terms of an arithmetic sequence are $x+2$, $3x+7$, $5x+12$

    (a)   Determine the common difference of this sequence.

    (b)   Determine an expression for T_7

    (c)   Determine an expression for S_{10}, simplifying your answer.

Question Eight: [3, 3 = 6 marks]

The first three terms of an arithmetic series are $S_1 = 18$, $S_2 = 32$, $S_3 = 42$

    (a)   Determine the first three terms of the sequence.

    (b)   Hence determine when the sequence will have a value of -34.

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Question One: **[2, 2, 3 = 7 marks]**

Define each of the following sequences recursively and state T_6 :

(a)   10, 8, 6, 4 ...

$T_n = T_{n-1} - 2; T_1 = 10$ ✓

$T_6 = 0$ ✓

(b)   90, 100, 110, 120 ...

$T_n = T_{n-1} + 10; T_1 = 90$ ✓

$T_6 = 140$ ✓

(c) $\dfrac{1}{8}, \dfrac{1}{4}, \dfrac{3}{8}, \dfrac{1}{2} ...$

✓

$T_n = T_{n-1} + \dfrac{1}{8}; T_1 = \dfrac{1}{8}$ ✓

$T_6 = \dfrac{6}{8} = \dfrac{3}{4}$ ✓

Question Two: **[1, 2, 2 = 5 marks]**

Consider the sequence $U_n = -3 + U_{n-1}$; $U_2 = 5$

(a)   Calculate U_3 $U_3 = -3 + 5 = 2$ ✓

(b)   Calculate U_1

$5 = -3 + U_1$ ✓

$U_1 = 8$ ✓

(c)   Determine the general term of the sequence.

$U_n = 8 - 3(n-1)$

✓   ✓

Question Three: [2, 2, 1, 2, 2 = 9 marks]

The first term of an arithmetic sequence is 2 and the 5th term is 14.

(a)   Determine the common difference of this sequence.

$$2 + 4d = 14 \checkmark$$
$$4d = 12$$
$$d = 3 \checkmark$$

(b)   Hence or otherwise define this sequence recursively.

$$T_n = T_{n-1} + 3; \ T_1 = 2 \checkmark$$
$$\checkmark$$

(c)   Calculate T_{10}

$$T_n = 2 + 3(n-1)$$
$$T_{10} = 2 + 3(10-1) = 29 \quad \checkmark$$

(d)   Calculate S_{10}

$$\checkmark$$
$$S_{10} = \frac{10}{2}\left(2 \times 2 + 3(10-1)\right) = 155 \checkmark$$

(e)   Determine when the sequence first has a value greater than 100.

$$100 = 2 + 3(n-1) \checkmark$$
$$98 = 3(n-1)$$
$$32\frac{2}{3} = n - 1$$
$$33\frac{2}{3} = n$$
$$\therefore T_{34} \quad \checkmark$$

Question Four: [2 marks]

Show that the sequence $T_n = 10 - 3(n-1)$ can be written as $T_n = -3n + 13$

$$T_n = 10 - 3n + 3 = 13 - 3n$$
$$\checkmark \qquad \checkmark$$

Question Five: [4 marks]

Determine when the value of the sequence $T_{n+1} = T_n + 10$; $T_1 = 5$ and of the sequence $T_n = 290 - 5(n-1)$ are equivalent.

$5 + 10(n-1) = 290 - 5(n-1)$ ✓

$5 + 10n - 10 = 290 - 5n + 5$ ✓

$10n + 5n = 290 + 5 + 5$

$15n = 300$ ✓

$n = 20$ ✓

Question Six: [4, 2 = 6 marks]

The 12th term of an arithmetic sequence is 57 and the 21st term is 84.

(a)   Determine the value of the first term and the common difference of this sequence.

$57 + 9d = 84$ ✓

$9d = 27$

$d = 3$ ✓

$57 = a + 3(12-1)$ ✓

$57 = a + 33$

$a = 24$ ✓

(b)   Hence calculate S_5

$S_5 = \dfrac{5}{2}\left(48 + 3(5-1)\right) = 2.5 \times 60 = 150$ ✓ ✓

Question Seven: **[2, 2, 2 = 6 marks]**

The first three terms of an arithmetic sequence are $x+2$, $3x+7$, $5x+12$

(a)   Determine the common difference of this sequence.

$5x+12-(3x+7) = 5x+12-3x-7 = 2x+5$ ✓

✓

(b)   Determine an expression for T_7

$T_7 = x+2+(2x+5)(7-1)$ ✓
$= x+2+12x+30$
$= 13x+32$ ✓

(c)   Determine an expression for S_{10}, simplifying your answer.

✓

$S_{10} = 5[2(x+2)+(2x+5)(10-1)]$
$= 5[2x+4+18x+45]$
$= 5[20x+49]$
$= 100x+245$ ✓

Question Eight: **[3, 3 = 6 marks]**

The first three terms of an arithmetic series are $S_1 = 18$, $S_2 = 32$, $S_3 = 42$

(a)   Determine the first three terms of the sequence.

$T_1 = 18$ ✓
$T_2 = 32-18 = 14$ ✓
$T_3 = 42-32 = 10$ ✓

(b)   Hence determine when the sequence will have a value of -34.

$-34 = 18-4(n-1)$ ✓
$-52 = -4(n-1)$ ✓
$13 = n-1$
$n = 14$ ✓

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