Mathematics Methods Unit 3&4

Test 5 – Normal Distribution

Baldivis Secondary College

Name: _____ **Total Marks:**_____

Task type:	**Response**	
Time allowed for this task:	60 minutes, in-class, under test conditions Calculator Free 30 minutes	28 marks
	Calculator-assumed 30 minutes	30 marks
Materials required:	Calculator with CAS capability (to be provided by the student)	
Standard items:	Pens (blue/black preferred) , pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters	
Special items:	Drawing instruments, templates, notes on one unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations	
Marks available:	**58 marks**	

Task weighting: 7%

Section One - Calculator Free

Question 1 **[6 marks]**

For a set of data values that are normally distributed, approximately 68% of the values will lie within one standard deviation of the mean, approximately 95% of the values will lie within two standard deviations of the mean and approximately 99.7% of the values will lie within three standard deviations of the mean.

If the heights of a large group of male senior school students are normally distributed with a mean μ = 160 cm and standard deviation σ = 14 cm, use the above information to answer the following questions:

(a) A statistician says that almost all of the students have heights in the range 118 cm to 202 cm. Comment on her statement.

(2 marks)

Since 118cm and 202cm are within 3 standard deviations of the mean ≈ 99.7% will be within this range.

(b) Approximately what percentage of the students has a height less than 146 cm?

(2 marks)

50 − 34 = 16

≈ 16%

(c) Approximately 2.5% of the students are taller than what height?

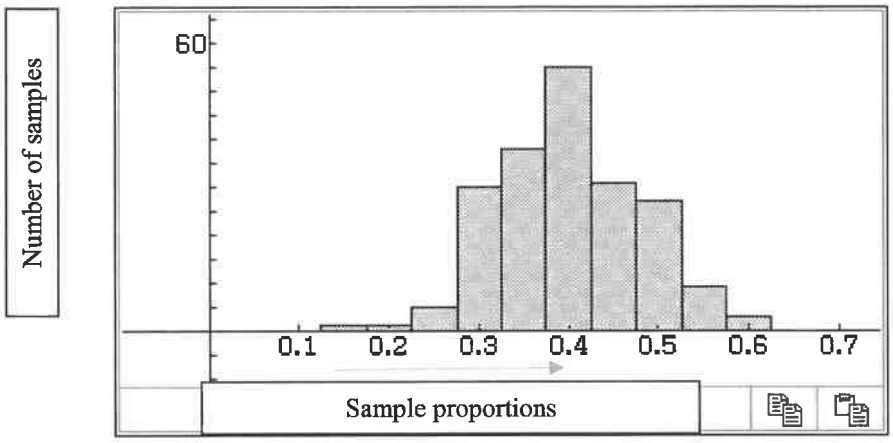
(2 marks)

2 standard deviations 28cm

160 + 28 = 188cm

Question 2 **[4 marks]**

(a) A number of sample proportions ($\hat{p}$) each of the same size (50) gave rise to the graph shown below.



(i) Approximately, how many samples were involved (1 mark)

≈ 200

allow 190 - 210.

(ii) Estimate p , the population proportion. (1 marks)

≈ 0.4

(b) A local radio station carries out regular polls of its listeners on items of current interest. In one such poll, listeners were asked to telephone the station and just answer yes or no to the following question:

Should the AFL introduce the red card send-off rule into all games?

The poll was carried out between 7:00 am and 9:00 am one morning.

Give two problems associated with this method of sampling and suggest why each problem might cause misleading conclusions to be drawn. (2 marks)

List two possibilities

- Survey restricted to listeners of one station, not representative of the whole population
- Self selecting model indicates bias.
- Time may exclude groups of people.
- Access to telephone presumed.
- People could respond more than once
- Nature means football fans may be more likely to respond.

Question 3 **[10 marks]**

The heights of fairy penguins in a particular geographic location are normally distributed with a mean height of 32 cm and a standard deviation of 1.5 cm.

Use the 68%, 95% and 99.7% rule to calculate each of the following.

a) Determine the probability that a randomly selected fairy penguin is taller than 30.5 cm. (2 Mark)

$P(x>30.5) = P(z>-1)$
$= 0.34 + 0.5$
$= 0.84$

b) Determine the probability of a randomly selected fairy penguin being shorter than 30.5 cm if it is known that they are in the 0.5 quantile. (2 Mark)

$\frac{P(X<30.5)}{P(X<32)} = \frac{0.16}{0.5} = 0.32$

c) In a sample of 2000 penguins, how many would you expect to be taller than 35cm? (2 Mark)

$P(x>35) = P(Z>2)$
$= 2.5\%$
$2000 \times 0.025 = 50$

d) What is the maximum height of the shortest 2.5% of penguins in this location? (2 Mark)

$32 + (-1.5) \times 2 = 29\text{cm}$

e) In a different geographic location the mean height of the fairy penguins found there is 33 cm. If 97.5% of he penguins are shorter than 38cm, and their heights are also normally distributed, what is the standard deviation for this population? (2 Mark)

$\frac{38-33}{\sigma} = 2 \Rightarrow \frac{5}{2} = \sigma$
$\sigma = 2.5$

Question 4 **[8 marks]**

a) The following table indicates Ben's maths test scores and the relevant class statistics.

Ben's maths Test scores

	Ben's score	Class mean	Class standard deviation
Test 1	54	60	8
Test 2	40	48	12
Test 3	63	72	10

Rank the tests from best to worst for Ben's performance relative to the class performance. Show working to justify your ranking.

(4 marks)

Test 1 $\frac{54-60}{8} = -0.75$ ✓

Test 2 $\frac{40-48}{12} = -\frac{8}{12} = 0.\dot{6}$ ✓

Test 3 $\frac{63-72}{10} = \frac{-9}{10} = -0.9$ ✓

Test 2, Test 1, Test 3. ✓

b) On another test, Ben scored 55% and had a standard score of -0.5 and his friend Mike scored 70% and his standard score was 1.

Calculate the mean and standard deviation for this test.

(4 marks)

$\frac{55-\mu}{\sigma} = -0.5$ $\frac{70-\mu}{\sigma} = 1$ ✓

$55-\mu = -0.5(70-\mu)$

$90 = 1.5\mu$

$\mu = 60$ ✓

$\frac{70-60}{\sigma} = 1$ ✓

$\sigma = 10$ ✓

Section Two – Resources Allowed **Name:**_____

Question 5 **[5 marks]**

The height of sunflowers is normally distributed with a mean of 1.6m and a standard deviation of 8cm.

a) What percentage of sunflowers are between 155cm and 170cm? (2 marks)

$\mu = 160 \quad \sigma = 8$

$P(155 < x < 170) = 0.6284$

$= 62.84\%$

b) What percentage of sunflowers are below a 150cm? (1 mark)

$P(x < 150) = 0.1056$

$= 10.6\%$

c) In a field of 2000 sunflowers how many will taller 175cm? (2 marks)

$P(x > 175) = 0.0304$

$0.0304 \times 2000 = 60.79$

≈ 61

Question 6 **[3 marks]**

A machine produces components whose weights are normally distributed. The intention is for the machine to be calibrated to produce components with a mean of 250g with only 1% of the components having a weight less than 245g. Determine the standard deviation of the calibrated machine.

$P(x < 245) = P\left(z < \frac{245-250}{\sigma}\right) = 0.01$

$\frac{245-250}{\sigma} = -2.326348$

$\sigma = 2.1492$

$= 2.15g$

Question 8 **[4 marks]**

A survey of 115 randomly selected teachers found that 15 of them were aged over 60.
A second survey of 1200 teachers found that 94 were aged over 60.

a) Considering the first survey, what is the sample proportion of teachers aged over 60?

(1 mark)

$\frac{15}{115} = 0.1304$

b) Considering the second survey, what is the sample proportion of teachers aged over 60?

(1 mark)

$\frac{94}{1200} = 0.0783$

c) Which sample proportion is likely to be the better estimate of the population proportion? Why?

(2 marks)

Second larger sample size.

Question 9 **[4 marks]**

The probability of an adult being left handed is 15%. A random sample of 400 adults was taken from the population.

a) State the standard deviation, σ, of $\hat{p}$, which is equal to the proportion of adults who are left handed in any sample of size 400.

(2 marks)

$\sqrt{\frac{0.15 \times 0.85}{400}} = 0.01785$ ✓ ✓

b) Using a normal approximation, determine the probability that in any random sample of 400 adults between 50 and 70 will be left handed.

(2 marks)

$P(\frac{50}{400} < X < \frac{70}{400}) = 0.8386$ ✓ ✓

Question 7 **[10 marks]**

Motorists travelling on the South-West Highway between Manjimup and Bridgetown had their speeds monitored over recent weeks. The speeds are normally distributed with a mean of 106 km/h and a standard deviation of 6 km/h. The speed limit is 110 km/h.

a) What percentage of motorists are speeding? (2 marks)

$X \sim N(106, 6^2)$

$P(x > 110) = 0.2525 \approx 25.25\%$

b) Find the lowest speed at which the fastest 5% of motorists are travelling. (2 marks)

$P(x > k) = 0.05$

$k = 115.9 \text{ km/h}$.

In order to reduce speeding the police have issued a warning that any motorist doing between 112 km/h and 115 km/h will be stopped and cautioned, whilst those doing over 115 km/h will be stopped and fined $380.

c) Determine the probability that a motorist was fined if they had to stop. (1 marks)

$$\frac{P(x > 115)}{P(x > 112)} = \frac{0.0668}{0.1587} = 0.4211$$

One afternoon the police counted 820 motorists who went through the speed trap.

d) Determine the expected total dollar amount of the fines they issued. (1 marks)

$820 \times 0.0668 = 54.78$

$54 \times 380 = \$20520$.

The police campaign was successful so that all motorists have reduced their speeds by 5%.

e) Find the probability that the next five motorists chosen would not be stopped. (4 marks)

$Y = 0.95X \quad Y \sim N(100.7, 5.7^2)$

$P(Y > 112) = 0.02371$.

$T = B(5, 0.02371) \quad P(T = 0) = 0.8869$

10

Question 7 **[4 marks]**

The proportion of working adults who miss breakfast on week days is estimated to be 40%. Michael and Jenny take a random sample of 250 adults.
Michael obtained his sample by selecting the first 250 workers he met coming off the train at Elizabeth Quay from 8 am on a Monday morning.

a) Discuss briefly **two** possible sources of bias in Michaels's sample.

(2 marks)

Poss answers
- Location - only one location
- Time - only people on a mon morning.
- Selection - first 250 - not random from all workers

1 mark for a correct source with explanation

Jenny suggests that a better sampling scheme is to obtain a random sample of 250 voters and contact them by telephone.

b) Outline **one** source of bias in Jenny's sampling scheme.

(1 mark)

Poss Sol - only those with telephone numbers will be selected
- Not everyone will answer when called

c) Which of Michael's or Jenny's sampling scheme is better? Provide a reason for your answer

(1 mark)

Jenny better selects randomly from whole population

- states Jenny with reason.