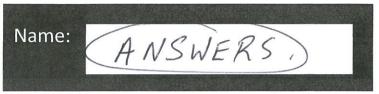
Manjimup S.H.S Year 12 2016 Mathematical Methods Investigation 2 Out of Class Section

PROBABILITY/STATS



DATE: Handout:

Thursday April 27th 2016

Due:

Thursday May 4th 2016

Test day:

Friday May 5th 2016

- The Out of Class Investigation is designed for you to learn the essentials needed for the In-Class validation.
- This is the "Take Home" part of the Investigation. It does not count towards your mark for this investigation.
- You will need your Casio Classpad.

This investigation covers three main ideas:

- The Standard Deviation/Variance of a distribution.
- The concept of Binomial Probability
- Mathematical Expectation.

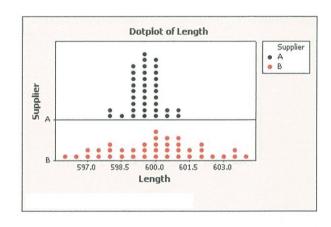
Standard Deviation/Variance Part One:

The variance and standard deviation are measures of how widely spread out or dispersed a distribution is.

Important note:

Standard deviation =
$$\sqrt{\text{var}\,iance}$$

1. Consider the dot plot to the right comparing the lengths of metal piping from two suppliers. The suppliers are paid to produce 600 mm length pipes for a company.



Without doing any calculations, explain which supplier has the highest variance and standard deviation of the length of pipe provided. Give reasons for your answer.

Supplier B has a higher variance and standard deviation because it is more spread out.

2. The definition of variance is the average of the square deviations from the mean. i.e $\sigma_x^2 = \frac{\sum (x-\overline{x})}{2}$

You will need to calculate the deviation from the mean of each score, square them, add them and divide by the number of scores. Use this table to calculate the variance and standard deviation of these scores by a netball side in the first quarter. Check your answer by using your calculator.

Mean of data
$$\bar{x} = 10$$
.

Goals	Deviation from the	Square deviations
	mean $x-x$	$(x-\overline{x})^2$
7	~ 3	9
8	-2	4
10	0	O
11	Ì	1
14	4	30
	Total	30.

Calculations:

$$6x^2 = \frac{30}{5} = 6.0$$

Variance; $6 \cdot 0$

Standard deviation (2 dp)

3. Here is the result of using the Statistics Aplet on your Casio.

Explain in words what is represented by

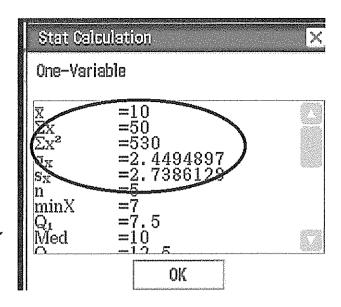
a)
$$\bar{x} = \text{Mean of all scores}$$

b)
$$\sum x = Sum \ of all scoren$$

b)
$$\sum x = Sum \text{ of all scoren.}$$

c) $\sigma_x = Standard deviation.$

Please note that the term sx represents the standard deviation of a sample and is calculated slightly differently than $\sigma_{_{\mathrm{r}}}$.



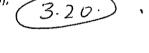
Use the table method to practice calculating the variance and standard deviation of the number of residents in each house in a street. $\{1,1,2,2,5,10\}$. Check your answer using your calculator afterwards.

		-
Residents	Deviation from	Square
	mean	deviations
ſ	-2.5	6.25
	-2.5	6.25
2	-1.5	2-25
2	-1.5	2-25
5	1.5	2.25
10.	6.5	42-25
	Total	64.5

Calculations,

$$6x^{2} = \frac{61.5}{6} = 10.25$$
.
 $6x = \sqrt{10.25} = 3.20$.
Variance; 10.25

Standard deviation (2 d.p);



5. An alternative rule for variance is $\sigma_x^2 = \frac{\sum x^2}{x^2} - (\bar{x})^2$. In words this means the mean of the square scores taken away from the mean squared. Repeat the calculations from the table above using this rule to verify the answer is the same.

Residents	Square of the
	scores
	1
1	l
2	4
2	4
5	25
10	100
Total	135.

$$6x^{2} = \frac{135}{6} - 3.5^{2}$$
$$= 10.25$$

Is it the same?
$$y.e.5$$

Use your calculator for these questions:

To get an A grade a student must get at least 1.5 standard deviations above the mean. Looking at the final scores below determine which students will get an A grade.

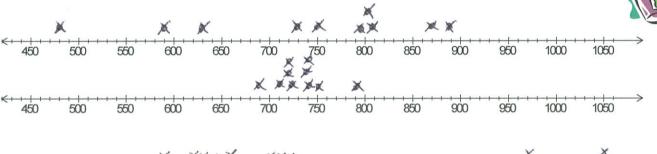
Student	Α	В	С	D	Е	F	G	Н		J	K
Mark	51	53	56	57	60	63	70	78	(86)	63	(89)

$$\overline{x} = 66$$

7. A wealthy overseas investor is looking to set up a vineyard somewhere in the South-West. The investor has 3 potential sites to choose from and to help her choose she looks at the rainfall for each site for the last 10 years. The rainfall in millimeters rounded to the nearest 10mm over the last 10 years is shown.

Site A	480	730	810	750	810	870	590	630	890	800	
Site B	790	710	720	720	740	720	690	750	740	740	
Site C	660	700	620	710	630	640	720	1050	970	590	

a) Plot the dot frequency diagram for each site.





- b) Calculate the following statistics for each Site
- Explain what the standard deviation of each site indicates for the wine investor.

This meanines the variability of the rainfull for each site over the last ten years

	Mean	Standard
		Deviation
Site A	736	124.5
Site B	732	25 - 6.
Site C	729	147.

Which site would you choose if you were the wine investor? Justify your answer.

has the most consistent Site B 7 rain full

What have you learned about standard deviation/variance?

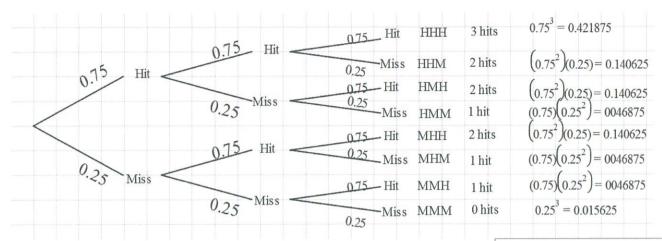
they are meanues of variability (or consistency).

Bernoulli Trials (the Binomial Probability Distribution) Part Two:

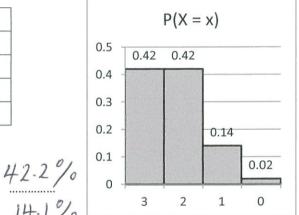
Consider this scenario: A basketball player has a 75% (¾) success rate for foul shooting. He is fouled outside the 3 point line (giving him 3 free shots) with 0.1 seconds to go and his side is 1 point behind.

What are his team's chances of winning after he completes his 3 shots?

Consider this diagram and accompanying table;



Х	Calculations	P(X=x)
3	0.75 ³	0.421875 = 42.2%
2	3 x 0.75 ² x 0.25	0.421875 = 42.2%
1	3 x 0.75 x 0.25 ²	0.140625 = 14.1%
0	0.25 ³	0.015625 = 1.6%



42-2+42-2=84.4%.

Use this information above to answer the following.

d)

- What is the chance of getting all 3 free shots? a)
- What is the chance of getting 1 out of 3 shots in? b)
- What is the chance that the player's team won after he completes his 3 free shots? c)

Explain why the calculation for 2 shots in was
$$3 \times 0.75^2 \times 0.25 = 0.421875$$
.

There are 3 ways of getting 2 shots in $\Rightarrow 3 \times p(hit) \times p(hit) \times p(miss)$.

Imagine another player, with a 60% free shot success rate, was fouled instead. Recalculate the probabilities in the e) table below.

Х	Calculations	P(X=x)
3	0.60	0-216 = 21-6/0
2	3x'0.62 x0.4	0.432 = 43.2%
1	3x 0.6x 0.42	0.288 = 28.89
0	0.43	0.064= 6.4%

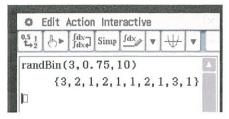
f) What is now the chance the team wins? Was it reduced by much?

win) =
$$p(3hih) + 2p(2hih) = 64.8\%$$
 compared to 84.4%
 $p(win) = p(3hih) + 2p(2hih) = 64.8\%$ compared to 84.4%

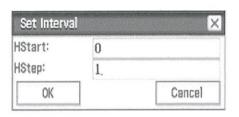
- You calculated the theoretical chances of the team winning in the previous questions for those two players but the 10. world is random and the real world won't always follow the theory exactly. In this question we will run 100 trials, using your calculator to see how closely the experiment can match the theory.
 - Place the expression randbin(3.0.75.10) on your calculator and press a) execute. (randbin can be found in the catalogue of your calculator).

Explain what each of the numbers in randbin(3,0.75,10) represents.

- Now we will run the trial 100 times, not 10 times! b)
 - type in randbin(3,0.75,100)
 - save this list into List 1 (see to the right).
 - Go to statistics and check to see if your numbers are in List1.
 - Draw a histogram of your data by using SetGraph.
 - Make your graph start at 0 and step up in ones.
 - Use the trace button to move across your graph and check the total for each column.



randBin(3, 0, 75, 100) {2,3,2,2,2,3,3,2,2, ans⇒list1 {2,3,2,2,2,3,3,2,2,



For extra help at home go the Classpad Help Series on the internet, and then go to Australian Senior Courses M34 and Random Sample from Binomial Distribution.

Place your results in the table below and calculate the probabilities, c)

٥,	Security and Security	I results on ca		* R	enult will vary.
Х	0	1	2	3	100.9.
Frequency	0	15	44	41	
Relative frequency	0%	15%	44 /0	41%	

☼ Zoom Analysis Calc ◆ Trace The NOT ▼ c=1

Place experimental data and theoretical data from previous question d) into the following tables and comment on how similar or different the results are.

х	0	1	2	3
Theoretical probability	4 6 %	14.1%	42.2%	42.2%
Relative frequency	.0%	15%	44 %	41%

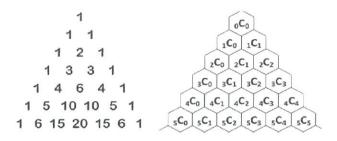
Theoretical and Experimental data closely match in this case

A Bernoulli sequence/ the binomial probability distribution occur under the following conditions.

- in each trial there are two outcomes only, usually
- designated success or failure. the probability of success and failure from one trial to another stays the same
- The trials are independent of each other, what happens on one doesn't affect the chances of the other.

Shortcut for theoretical calculation:

The formula for any individual probability calculated from the Binomial probability distribution uses the combination rule but Pascal's triangle can be used for small values.



The rule for Binomial probability:

if X is a binomial random variable then the probability of x successes in n trials is

$$P(X = x) = {}^{n}C_{x}p^{x}q^{n-x}.$$

For instance in question 9, n = 3 (shots), p = 0.75, q = 0.25 and x was the number of Hits, so P(2 hits) = ${}^{3}C_{2}(0.75)^{2}(0.25)^{1}$ = 0.421875.

- 11. Consider this situation: In June in Pemberton there is a 65% chance of rain on any particular day. A family is going to spend a 4 day holiday in Pemberton in the July holidays.
 - Calculate the probability distribution for the number of rainy days (R) out of the 4.

r	Calculations	P(R=r)
0	0.354	0.015
1	4C, (0.35) (0.65)	0.111
2	4(2 (0.35) (0.65)	0.311
3	+ (3 (0.35)' (0.65)'	0.384.
4	0.654	0.179



Model this situation on the calculator for the family and record the experimental relative frequency distribution below.

Х	0	1	2	3	4
Frequency	2	12	45	35	.6.
Relative frequency	0.02	0.12	0.45	0.35	0.06

Compare the theoretical to the experimental results below.

Х	0	1	2	3	4
Theory	0.015	0.111	0-311	0-384	0.179
Relative frequency	0.02	0.12	0.45	0.35	0.06
	Close	Clae	14% diff	Close	12% dift

The family will be happy if it only rains on 2 days or less. Give the theoretical probability and experimental probability the family will be happy.

THEORY:
$$p(r \le 2) = 0.437$$
 | $Experiment: p(r \le 2) = 0.59$
= 43.7%

12. Records show that 42% of young drivers pass their driving test on the first go. From a class of 9 students calculate

Part Three: Mathematical Expectation

The mathematical expectation of a probability distribution is the expected result in the long run. The formal definition is $E(x) = \sum x \cdot P(x)$ which means the sum of the products of the probability of an event and the result for each event.

For example: Would it be worth playing this game with a spinner if each spin costs me \$10? Note: prize of \$1 = 10 - 1\$ loss of \$9

Answer:

	A	В	С	D	
X	-\$9.50	-\$9	-\$8	+\$40	
p(x)	0.5	0.25	0.125	0.125	Total
x.P(x)	-\$4.75	-\$2.25	-\$1	\$5	-\$3



Answer: $-\$9.50 \times 0.5 + -\$9 \times 0.25 + -\$8 \times 0.125 + \$40 \times 0.125 = -\$3$

I would expect to lose \$3 per game, therefore no, it would not be worth playing this game.

13. An amateur sports club decides to play a gambling game which costs \$5 to play.

You throw a die and if one comes up you win \$1, if two comes up you win \$2, three appears therefore \$3, four appears you win \$4, five appears the player wins \$6 and if a six comes up you win \$12.

How much would the club expect to win each game? Show working.

hrow	1	2	3	4	5	6	
X	-\$4	-\$3	-\$2	-\$1	+\$1	+\$7	
P(x)	1/6	1	1/6	1/6	1/6	1/1	Total
.P(x)	70	- \$0.50	-\$0.33	-30.16	\$+0.16	6.	-\$0.3

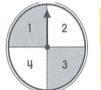
Experted loss per gare = [50.33.]

14. Counter Products.

This game is played these spinners. The numbers are multiplied.

a) Show all outcomes in the table below.

	1	2	3	4	5
1	1	2:	3	4	5
2	2.	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20





Counter products

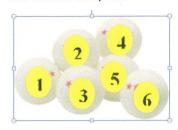
You pay \$2 to play.

You spin two spinners and multiply the results. If a product of 16 comes up your payout is \$30 If you get a product of 2 or 9 your payout is \$2.

b) Calculate t	he expected los	s of the players p	per game. Show working be	lpw.
Zeml Ovacone.	龙 428	0	-\$2.	-
P(X=x)	1 20	3 20	20	Total.
XeP(x=x)	\$1.40]	0	-\$1.60	-\$0.20
	Expe	it to	lae \$0.20	per gane

15. Game #3

This game is a Lotto Game. It is called Lucky Six.



For the purposes of this game make your lucky numbers 1, 2 and 3.

LUCKY SIX.

You pay \$8 to play.

Numbers 1 to 6 are placed in a barrel. You choose 3 numbers beforehand. Three of the 6 numbers are drawn out of the

barrel. These are called the Lucky numbers.

If your 3 numbers match with the 3 drawn out your payout is \$60

If two of your lucky numbers match with 2 of the numbers drawn out your payout is \$7.

اهر List out all 20 ways of getting three numbers. It has been started for you.

4/									
1/1/2/3	(1,2,4)	(1,2,5)	(1,2,6)	(1,3,4)	(13,5)	(1,3,6)	1,4,5	1,4,6	1,5,6
(2.3.4)	(235)	(2,3,6)	2.4.5	24.6		345	3.4.6	3,5,6	4.5,6
				1 1 2 1		1 1			

- b) Find the chances of winning each prize. $p(Match_3) = \frac{1}{20} p(Match_3^2)$
- c) Explain why the total sample space is 6C_3 . $\simeq 20 \ \text{//}$
- Choosing 3 from 6.
 d) Explain why the chance of Matching 3 is $\frac{{}^3C_3.{}^3C_0}{{}^6C_3}$. Choose 3 from 3 = $\frac{1}{20}$
- Explain why the chance of Matching 2 is $\frac{{}^{3}C_{2} \cdot {}^{3}C_{1}}{{}^{6}C_{3}} = \frac{9}{20}$ Choose 2 from 3 winning n of Choose 1 from 3 other nos

f) Use a probability distribution table to determine the expected loss for the player for each gam · Match 3. Match 2. Wather.

Experted loss = $52 \times \frac{1}{20} + (-1) \times \frac{9}{10} + -8 \times \frac{10}{20}$ = |-\$1.85 per gare.

16. Show working to prove that the player will expect to lose \$0.75 for each game played in Lucky Ten.

Use combinations to help!

$$P(Match 3) = {}^{3}C_{3}{}^{7}C_{0} = \frac{1}{120}$$

$$P(Match 2) = \frac{3(2^{7}(1))}{\frac{10}{3}} = \frac{21}{120}$$

$$p(lose) = \frac{120}{120} - \frac{22}{120} = \frac{98}{120}$$

LUCKY TEN.

You pay \$5 to play.

Numbers 1 to 10 are placed in a barrel.

You choose 3 numbers beforehand.

Three of the 10 numbers are drawn out of the barrel. These are called the Lucky numbers.

If your 3 numbers match with the 3 drawn out your payout is \$300

If two of your lucky numbers match with 2 of the numbers drawn out your payout is \$10.

Expertation: $\frac{1}{120} \times (3295) + \frac{21}{120} \times (35) + \frac{99}{120} \times (-35) = (-$0.75)$ gare

17. The Roulette Wheel

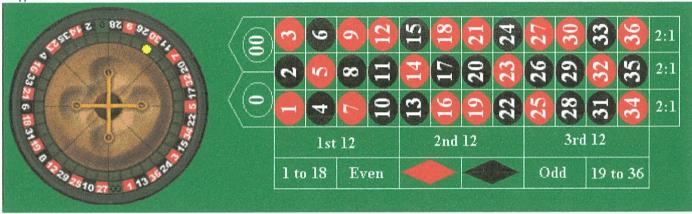
According to <u>Richard Epstein</u>, roulette is the oldest casino game still in operation. It's invention has been variously attributed to <u>Blaise Pascal</u>, the Italian mathematician Don Pasquale, and several others. In any event, the roulette wheel was first introduced into Paris in 1765. Here are the characteristics of the wheel:

The (American) roulette wheel has 38 slots numbered 00, 0, and 1-36.

- 1. Slots 0, 00 are green;
- 2. Slots 1, 3, 5, 7, 9, 12, 14, 16, 18, 19, 21, 23, 25, 27, 30, 32, 34, 36 are red;
- 3. Slots 2, 4, 6, 8, 10, 11, 13, 15, 17, 20, 22, 24, 26, 28, 29, 31, 33, 35 are black.

Except for 0 and 00, the slots on the wheel alternate between red and black. The strange order of the numbers on the wheel is intended so that high and low numbers, as well as odd and even numbers, tend to alternate.

A typical American roulette wheel and table



The roulette <u>experiment</u> is very simple. The wheel is spun and then a small ball is rolled in a groove, in the opposite direction as the motion of the wheel. Eventually the ball falls into one of the slots. Naturally, we assume mathematically that the wheel is fair, so that the random variable *X*

There are many ways to bet on Roulette. Assume it costs \$1 to play each time.

a) Single number bet: match the number get \$36 back i.e win \$35. Calculate the expected loss or gain for a player.

$$p(\text{win}) = \frac{1}{38}$$
 Expect = $\frac{1}{38} \times (\$35) + \frac{37}{38} \times (-1) = [-\$0.0526]$
 $p(\text{lose}) = \frac{37}{38}$

b) Three number bet; bet on a vertical row of 3 numbers. Pays out \$12 if you win. Calculate expectation.

A French Roulette wheel only has 37 numbers, it only has one zero, not two like the American Casinos.

c) Calculate the expected loss or gain for playing a single bet number game for \$1 on a French roulette wheel.

$$P(win) = \frac{1}{37}$$

 $P(lose) = \frac{36}{37}$ Expect = $\frac{1}{37} \times ($35) + \frac{36}{37} \times (-1) = [-$0.027/game]$

d) Comment on which country's casino is likely to make the most money; the American or French. Justify your answer.

28. The Game of Craps!

Craps is an American gambling game based on throwing two dice and adding the result.

One bet you ca make in craps is a "Field bet" which pays \$2 if a total of 3,4,9,10 or 11 comes up and \$3 if a total of 2 or 12 comes up. All other numbers pay nothing and the game costs \$1 to play.

Prove the game loses the gambler more than 5 cents a game! Show all working.

Summarise what you now know about:

1. Standard deviation and Variance.

2. The Binomial probability distribution.

3. Expectation.

Strange Definitions

Aardvark

A vark that thinks it's tough!

A vark that thinks it's tough!

A very popular make of car

Abdomen Anglei

Men with beer bellies! Someone who is good at maths

Abigail Alarm Clock

Strong wind heard in a monastery! Something to scare the living daylight into you

Absent Minded Announce

Oh! I seem to have forgotten this one! One sixteenth of a pound

Absolute Antelope

The best musical instrument in the World, ever! When two ants run off to get married

Abundance Anti-body
Lots of dancing cakes Uncle's fat wife
Abyss Anti-freeze

An abbot's wife A female relative from Greenland

Accidental

When you fall and knock your teeth out on the way to

the dentist

Acorn Appear

An oak in a nutshell Something you fish off

Actor Apricots

A man who tries to be everything but himself Beds for baby monkeys

Adult

A person who has stopped growing at both ends and has

started growing in the middle

Archaeologist

An old gorilla girlfriend

A person who's career lies in ruins

Advertising

Makes you think you've always wanted something that

you've never heard of before

Area Code

Apex

A cold that hits one part of the country at a time

ABDICATE: To give up all hope of ever having a flat stomach.

ACCOUNT: A Countess's husband.

ACRE: Literally means the amount of land plowable in one day. So in my case it would be four feet by four feet.

ADULT: A person who has stopped growing at both ends and is now growing in the middle. **AMNESIA:** condition that enables a woman who has gone through labor to have more babies.

ANTIQUE: An item your grandparents bought, your parents got rid of, and you're buying again.

ARBITRATOR (ar'-bi-tray'-ter): A cook that leaves Arby's to work at Burger King.

ATHEISM is a non-prophet organization. -- George Carlin

AVOIDABLE (uh-voy'-duh-buhl'): What a bullfighter tries to do.

BALDERDASH: A rapidly receding hairline.

BARIUM: What we do to most people when they die.

BATHROOM: A room used by the entire family, believed by all except Mom to be self-cleaning.

BEAUTY PARLOR: Places where women curl up and dye.

BOSS: Someone who is early when you are late and late when you are early.

CANNIBAL: Someone who is fed up with people. **CANTALOUPE:** Gotta get married in a church.

CAR SICKNESS: The feeling you get when the car payment is due.

CATALOGS: Rails used to build cow fences.

CHICKENS: The only animals you eat before they are born and after they are dead.

CLASSIC: A book which people praise, but do not read. **CLOTHES DRYER:** An appliance designed to eat socks.

COFFEE: A person who is coughed upon.

COLLEGE: The four year period when parents are permitted access to the telephone.

COMMITTEE: A body that keeps minutes and wastes hours.

COMPROMISE: The art of dividing a cake in such a way that everybody believes he got the biggest piece.

CONFERENCE: The confusion of one man multiplied by the number present.

CONTROL (kon'-trol): A short, ugly inmate.

COURTESY: The art of yawning with your mouth closed.

DERANGE: Where de buffalo roam.

DICTIONARY: A place where success comes before work.

DIPLOMAT: A person who tells you to go to hell in such a way that you actually look forward to the trip.

DIVORCE: Future tense of marriage.

DOCTOR: A person who kills your ills by pills, and kills you with his bills. **DUMBWAITER:** One who asks if the kids would care to order dessert.

DUST: Mud with the juice squeezed out.

EGOTIST: Someone who is usually me-deep in conversation.

EMERGENCY NUMBERS: Police station, Fire Department and Places that deliver.

Etc: A sign to make others believe that you know more than you actually do.\

ETERNITY: The last two minutes of a football game.

EYEDROPPER (i'-drop-ur): A clumsy ophthalmologist.

EXPERIENCE: The name men give to their mistakes.

FABLE: A story told by a teenager arriving home after curfew.

FAMILY PLANNING: the art of spacing your children the proper distance apart to keep you on the edge of financial disaster.

FANCY RESTAURANT: One that serves cold soup on purpose.

FATHER: A banker provided by nature.

FEEDBACK: The inevitable result when the baby doesn't appreciate the strained carrots.

FLABBERGASTED: Appalled over how much weight you have gained.

FULL NAME: What you call your child when you're mad at him/her.

GOSSIP: A person who will never tell a lie if the truth will do more damage.

GRANDMOTHER: A baby-sitter who doesn't hang around the refrigerator.

GRANDPARENTS: The people who think your children are wonderful even though they're sure you're not raising them right.

GROCERY LIST:. What you spend half an hour writing, then forget to take with you to the store.

GUM: Adhesive for the hair.

HAIR DRESSER: Someone who is able to create a style you will never be able to duplicate again.

See "Magician."

HANDKERCHIEF: Cold Storage.

HEARSAY: What toddlers do when anyone mutters a dirty word.

HEROES (hee-rhos'): What a guy in a boat does.

HINDSIGHT: What one experiences from changing too many diapers.

HORS D'OEUVRES: A sandwich cut into 20 pieces.

IMPREGNABLE: A woman whose memory of labor is still vivid.

INDEPENDENT: How we want our children to be as long as they do everything we say.

INFLATION: Cutting money in half without damaging the paper.

KISSING: A means of getting two people so close together that they can't see anything wrong with

each other

MINIMUM: Minidad's wife