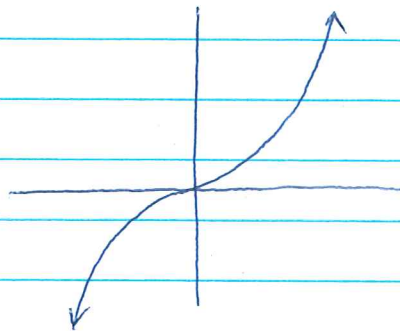


PART A - EPW.

i. (a)



$$\begin{aligned}
 (b) \quad \tanh(x) &= \frac{\sinh(x)}{\cosh(x)} \\
 &= \frac{\frac{1}{2}(e^x - e^{-x})}{\frac{1}{2}(e^x + e^{-x})} \\
 &= \frac{e^x - e^{-x}}{e^x + e^{-x}}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad (i) \quad \cosh(0) &= \frac{1}{2}(1+1) \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \lim_{x \rightarrow \infty} \frac{2}{e^x + e^{-x}} &= \lim_{x \rightarrow \infty} \frac{2}{e^x + \frac{1}{e^x}} \\
 &= 0.
 \end{aligned}$$

$$\begin{aligned}
 2 \quad (a) \quad (i) \quad \sinh(-x) &= \frac{1}{2}(e^{-x} - e^x) \\
 &= -\frac{1}{2}(e^x - e^{-x}) \\
 &= -\sinh(x)
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \sinh 2x &= 2 \sinh x \cosh x \\
 &= 2 \cdot \frac{1}{2}(e^x + e^{-x}) \cdot \frac{1}{2}(e^x - e^{-x}) \\
 &= \frac{1}{2}(e^x + e^{-x})(e^x - e^{-x}) \\
 &= \frac{1}{2}(e^{2x} - e^{-2x}) \\
 &= \sinh 2x
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad \cosh^2 x - \sinh^2 x &= \left(\frac{1}{2}(e^x + e^{-x})\right)^2 - \left(\frac{1}{2}(e^x - e^{-x})\right)^2 \\
 &= \frac{e^{2x} + 2 + e^{-2x}}{4} - \left(\frac{e^{2x} - 2 + e^{-2x}}{4}\right) \\
 &= \frac{4}{4} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 2 \quad (b) \quad \cosh 2x &= \frac{1}{2}(e^{2x} + e^{-2x}) \\
 &= 2 \sinh^2 x + 1
 \end{aligned}$$

$$\begin{aligned}
 \sinh^2 x &= \frac{e^{2x} - 2 + e^{-2x}}{4} \\
 &= \frac{e^{2x} + e^{-2x}}{4} - \frac{1}{2}
 \end{aligned}$$

$$2 \sinh^2 x = \frac{e^{2x} + e^{-2x}}{2} - 1$$

$$\begin{aligned}
 (c) \quad (i) \quad (\cosh x + \sinh x) &= \frac{1}{2}(e^x + e^{-x}) + \frac{1}{2}(e^x - e^{-x}) \\
 &= e^x
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad (\cosh x + \sinh x)^n &= e^{xn}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad (a) \quad (i) \quad \frac{d}{dx}(\sinh x) &= \frac{d}{dx}\left(\frac{1}{2}(e^x - e^{-x})\right) \\
 &= \frac{1}{2}(e^x + e^{-x}) \\
 &= \cosh x
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \frac{d}{dx}(\cosh x) &= \frac{d}{dx}\left(\frac{1}{2}(e^x + e^{-x})\right) \\
 &= \frac{1}{2}(e^x - e^{-x}) \\
 &= \sinh(x)
 \end{aligned}$$

$$(b) \quad (i) \quad \frac{d}{dx}(\sinh^2 x)$$

$$\begin{aligned}
 &= \frac{d}{dx}(\sinh x \cdot \sinh x) \\
 &= \cosh x \cdot \sinh x + \sinh x \cdot \cosh x \\
 &= 2 \cosh x \cdot \sinh x \\
 &= \sinh 2x
 \end{aligned}$$

$$(ii) \quad \frac{d}{dx}(\cosh(3x))$$

$$= 3 \sinh 3x$$

$$(iii) \quad \frac{d}{dx} \ln(\sinh x)$$

$$\begin{aligned}
 &= \frac{\cosh x}{\sinh x} \\
 &= \frac{1}{\tanh x}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad (a) \quad (i) \quad \cos^2 x + \sin^2 x &= 1 \\
 (\cosh x)^2 + (\sinh x)^2 &= 1 \\
 \cosh^2 x - \sinh^2 x &= 1
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \cos 2x &= 1 - 2 \sin^2 x \\
 \cosh 2x &= 1 - 2(\sinh x)^2 \\
 &= 1 + 2 \sinh^2 x
 \end{aligned}$$