SOLUTIONS

Time limit = 40 minutes.

Name: _____   Score: _____ / 34

Calculators allowed
Access to approved Sample Mathematics Specialist formulae sheet is permitted.
Answers should be in 4 decimal places.

Q1. [3,3,2]   [8 marks]
The standard deviation of the durability of Performance Racing tyres is 410 kilometres. Racing experts plan to estimate μ, the mean lifetime of these tyres, using the mean lifetime of a random sample of the tyres.

(a) The experts would like to be 95% confident that the mean lifetime of tyres in the sample is within 50 kilometres of μ. How large a sample should they take?

$$50 = 1.96 \left(\frac{410}{\sqrt{n}} \right) \quad \sqcap$$

$$n = 258.309184 \quad \sqcap$$

They should take a sample of at least 259 tyres. √

(b) Suppose that a random sample of 80 tyres is taken, and the mean lifetime of these tyres is 1245 kilometres. Based on this sample, determine a 90% confidence interval for μ.

The 90% interval is $1245 \pm 1.645 \left(\dfrac{410}{\sqrt{80}} \right)$ √

$= (1169.6, 1320.4) \quad \sqcap$

$1169.6009 < \mu < 1320.3991$ √

C-Level	0.90
σ	410
$\bar{x}$	1245
n	80

Lower	0.90	⊓ ⊓
Upper	410	
$\bar{x}$	1245	
n	80	

(c) The manufacturer claims that the mean lifetime of Performance Racing tyres is at least 1250 kilometres. Does the sample in part (b) provide a strong reason to doubt this claim? Justify your answer.

No, 1250 lies within the 90% confidence interval. We should generally doubt the claim only if 1250 lay outside the confidence interval – and a 90% interval is a narrower interval than others that could be used (say 95%, 99%) to test the claim. 1250 would lie well within these intervals.   ⊓   ⊓

Q2. [2,2] [4 marks]

A machine is designed to produce ball bearings. The quality control officer sampled 400 items and calculated a mean radius of 16 mm and standard deviation of 0.2 mm.

(a) Calculate the 99% confidence interval for the population mean of the radii of the ball bearings,

$$16 - 2.576 \times \frac{0.2}{\sqrt{400}} \leq \mu \leq 16 + 2.576 \times \frac{0.2}{\sqrt{400}} \checkmark$$

$$15.974 \leq \mu \leq 16.026$$

$$15.9742 \leq \mu \leq 16.0258 \checkmark$$

(b) The company was asked to make ball bearings with radii accurate to within ± 0.05mm with a 95% confidence. What size sample must be taken to satisfy this request?

$$n = \left(\frac{1.96 \times 0.2}{0.05} \right)^2 = 61.46 \; 56$$

So $n = 62 \checkmark$

Q3. [2,2,1] [5 marks]

A survey was conducted via the internet. It asked participants who logged onto a football website; "what fine should a football player receive for major indiscretion?" The statistics of the population resulted in a mean of $4212 and a standard deviation of $1004.

Using a sample of 1256 respondents, the website supervisor calculated the 95% confidence interval to be $4156.47 to $4267.53.

(a) Is the confidence level correct? Explain.

$$4212 - \left(1.96 \times \frac{1004}{\sqrt{1256}} \right) = 4267.53 \qquad 4267.5248$$

$$4212 + \left(1.96 \times \frac{1004}{\sqrt{1256}} \right) = 4156.47 \qquad 4156.4752$$

$\therefore$ Yes it is $\checkmark$

(b) Does this sample represent the view of the population? Explain.

No, only those that use that website

(Or   Yes, mean is within interval)

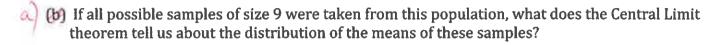(c) Describe the effect on the length of confidence interval if the confidence level were to be increased to 99%

The confidence interval gets wider $\checkmark$

Q4. [1,3,3] [7 marks]

If the numbers 1 to 100 form the population, then the mean and standard deviation of the population are 50.5 and 28.87 respectively.

(a) Show clearly why the mean of the population is 50.5.

$$\frac{1+2+3+...+100}{100} = \frac{5050}{100} = 50.5$$

(b) If all possible samples of size 9 were taken from this population, what does the Central Limit theorem tell us about the distribution of the means of these samples?

$$X \approx N\left(50.5, \left(\frac{28.87}{\sqrt{9}}\right)^2\right)$$

Or   1. Normally distributed

2. Mean = population mean

3. Standard deviation = $\dfrac{28.87}{\sqrt{9}} = \dfrac{28.87}{3}$

(c) The diagram below shows 3 lists of 9 randomly generated counting numbers from 1 to 100 generated by the calculator

▼   Edit   Calc   SetGraph

	List 1	List 2	List 3
1	49	15	75
2	3	20	70
3	89	45	72
4	76	7	19
5	72	16	74
6	45	6	81
7	36	48	83
8	43	46	27
9	86	8	11
10			
11			
Calc=	randList (9,1,100)		
Deg Auto Decimal			

Given that the 95% confidence interval for the population mean is $31.64 \leq \mu \leq 69.36$
Indicate whether these lists above are representative samples, giving your reasons.

$\bar{x}_1 = 55.4$ Yes

$\bar{x}_3 = 23.4$ No

$\bar{x}_3 = 56.9$ Yes

Q5. [2,2] [4 marks]

(a) A company wishes to estimate the average age of its employees. From past information it was found the standard deviation was 3.5 years. A sample of 60 employees is selected and the mean is calculated as 32.5 years. Find the 95% confidence interval of the population mean.

The z score for the 95% confidence interval is 1.96

$$\bar{x} - z\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z\frac{\sigma}{\sqrt{n}}$$

$$32.5 - 1.96 \times \frac{3.5}{\sqrt{60}} \le \mu \le 32.5 + 1.96 \times \frac{3.5}{\sqrt{60}}$$

$$31.6144 \le \mu \le 33.3856$$

ie. With a 95% confidence, the average age of employees lies between 26.607 and 28.113 years.

(b) How large a sample should the company use to be 95% sure that the sample is within 1 year of the sample mean of 32.5 years,

$$n = \left(\frac{z\sigma}{w}\right)^2$$

$$n = \left(\frac{1.96 \times 3.5}{1}\right)^2 \approx 47.0596$$

The required sample size is 48 employees.

Q6. [1,2,2,2] [7 marks]

On the basis of the results obtained from a random sample of 60 bags produced by a mill, the 95% confidence interval for the mean weight of flour in a bag is found to be (3.1kg, 4.6kg).

(a) Find the value σ, the standard deviation of the normal population from which the sample is drawn.

$$\bar{x} - 1.96 \leq x \leq \; \leq \mu \leq \bar{x} + 1.96 \; \frac{\sigma}{\sqrt{65}}$$

$$\bar{x} - 1.96 \; \frac{\sigma}{\sqrt{60}} = 3.1$$

$$\bar{x} + 1.96 \; \frac{\sigma}{\sqrt{60}} = 4.6 \quad \checkmark$$

solve it simultaneously

$$\left(0 + 2\left(\frac{1.96}{60}\right) \sigma = 1.5 \right) \quad \sigma = 2.964 \quad \overset{2.964017867}{\checkmark}$$

(b) Find the value of $\bar{x}$, the mean weight of the sample.

$$\bar{x} - 1.96 \; \frac{2.964}{\sqrt{60}} = 3.1$$

$$\bar{x} = 3.850 \quad \checkmark$$

(c) Calculate the 99% confidence interval for the mean weight of flour in a bag.

$k = 2.576$ for 99% confidence interval

$$2.8644 \leq x \leq 4.8356 \quad \checkmark$$

$$2.864 \leq x \leq 4.836$$

(d) Using the sample mean from (a) as the best estimate for the population mean, what is the probability that the sample mean of a larger sample of 200 bags is less than 3.8kg.

$$\bar{x} = 3.85$$

$$\bar{\sigma} = \frac{2.9640}{\sqrt{200}} = 0.2095864499$$

$$X \sim N\left(3.85, \; 0.2095864499^2\right) \quad \checkmark$$

$$P(x < 3.8) = 0.4057 \; (4dp) \quad \checkmark$$