



**Calculator Free
Applications of Differentiation**

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [7 marks]

Consider the function $A = x^3 - \frac{3x^2}{2} - 18x + 10$.

Determine the maximum value of A , justifying your answer with full mathematical reasoning.

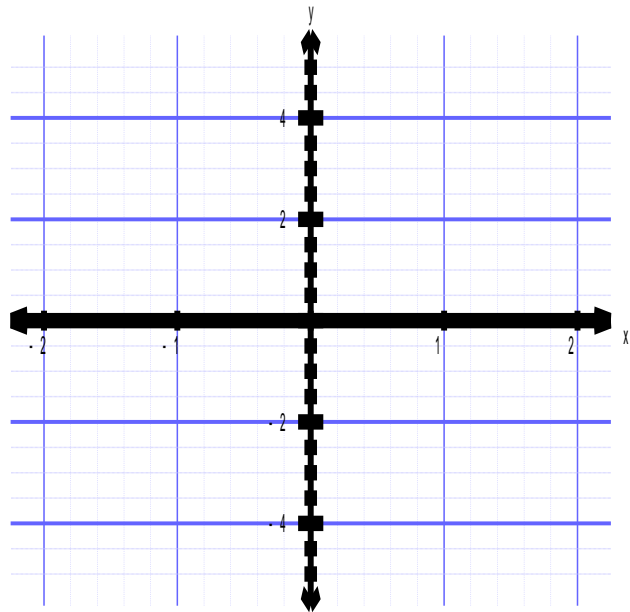
Question Two: [10 marks]

Determine the global maximum and minimum of the function $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2 + 1$

over the domain $-1 \leq x \leq 1$.

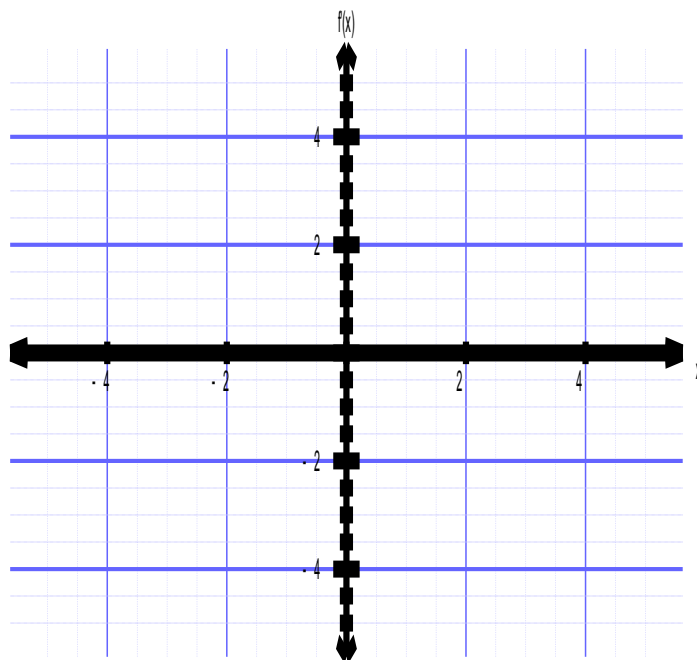
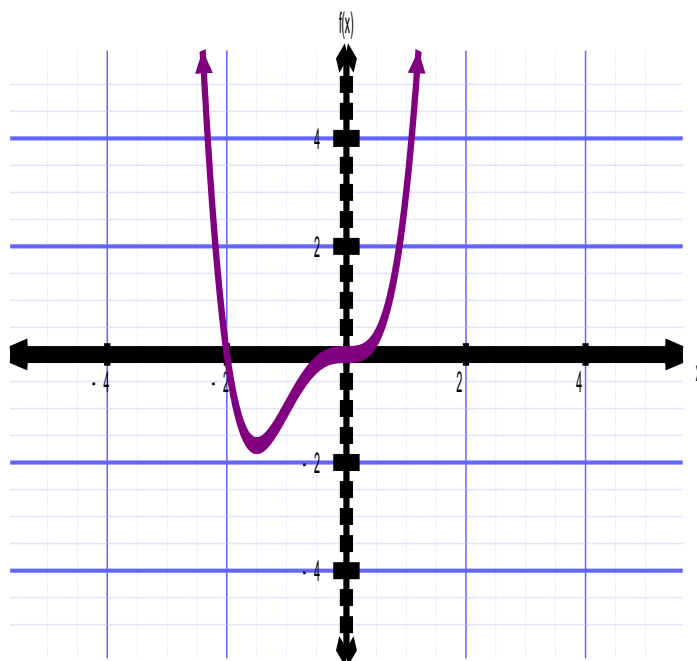
Question Three: [10 marks]

Sketch a graph of the function $y = 4x^3 - 6x^2$ by finding all intercepts and all stationary points and their nature.

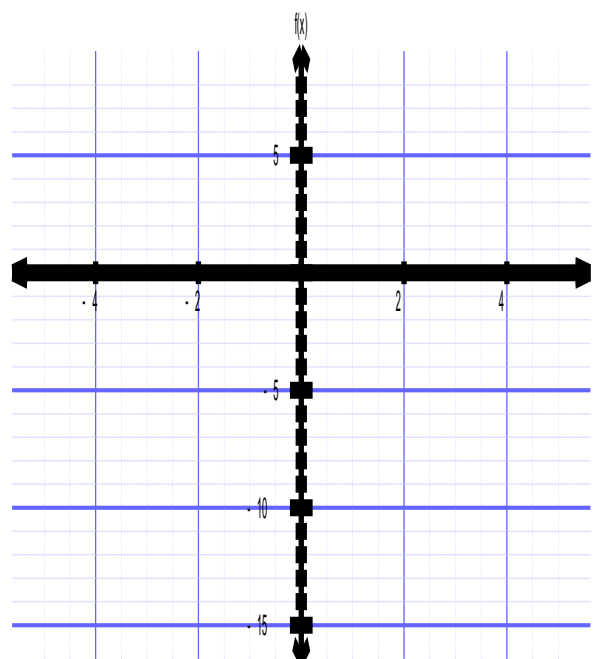
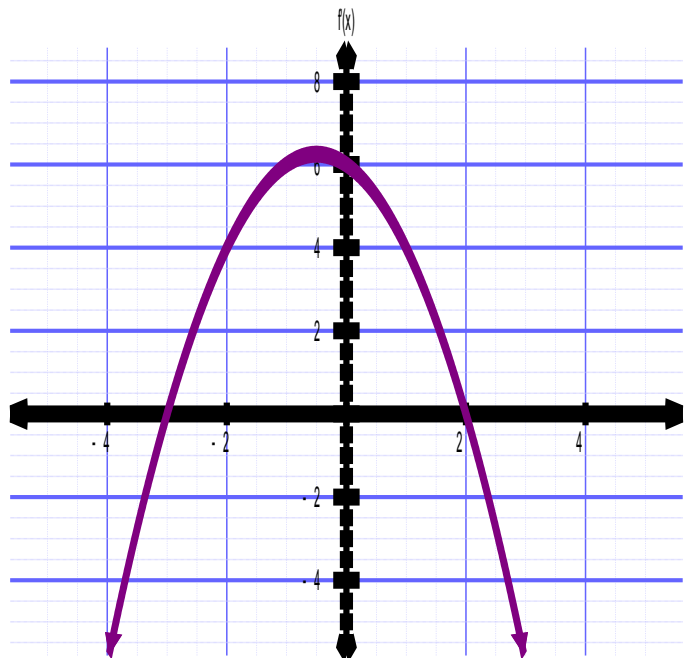


Question Four: [5, 5 = 10 marks]

- (a) Sketch a possible gradient function, $f'(x)$, using the graph of $f(x)$ below.

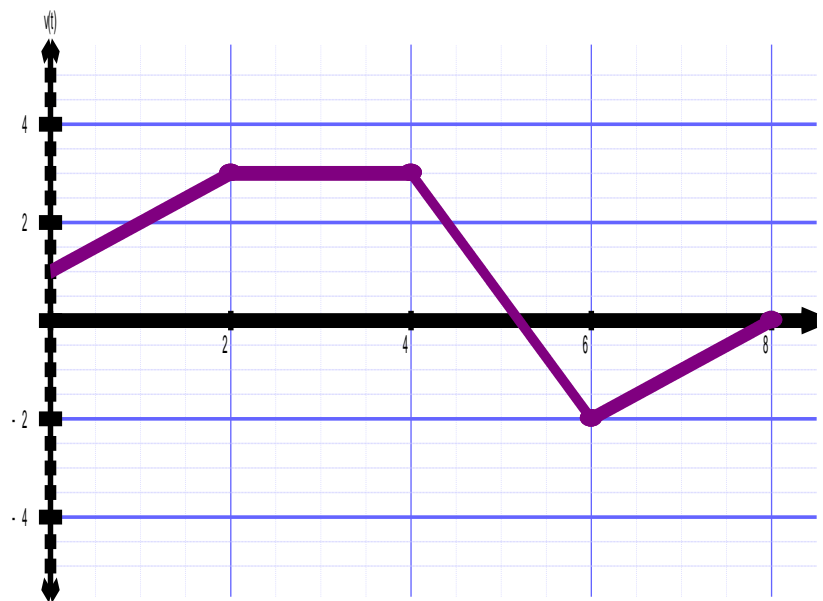


- (b) Sketch a possible function, $f(x)$, using the graph of $f'(x)$ below.



Question Five: [1, 1, 2, 2, 2 = 8 marks]

The graph below shows the velocity-time graph of a moving object, where time is in seconds and velocity is in m/s.



- (a) Determine the speed of the object when $t = 1$.
- (b) Determine the velocity of the object when $t = 7$.
- (c) Determine the speed of the object when $t = 6$.
- (d) Determine an expression for the rate of change of displacement with respect to time for this object in the first 2 seconds.
- (e) Determine an expression for the rate of change of velocity with respect to time for this object between $t = 6$ and $t = 8$.



SOLUTIONS
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Question One: [7 marks]

Consider the function $A = x^3 - \frac{3x^2}{2} - 18x + 10$.

Determine the maximum value of A , justifying your answer with full mathematical reasoning.

$$\frac{dA}{dx} = 3x^2 - 3x - 18 \quad \checkmark$$

$$3x^2 - 3x - 18 = 0 \quad \checkmark$$

$$3(x^2 - x - 6) = 0$$

$$3(x-3)(x+2) = 0 \quad \checkmark$$

$$x = 3, x = -2 \quad \checkmark$$

$$\frac{d^2A}{dx^2} = 6x - 3 \quad \checkmark$$

$$x = 3 \quad \frac{d^2A}{dx^2} = 15 > 0 \quad \therefore \text{min}$$

$$x = -2 \quad \frac{d^2A}{dx^2} = -15 < 0 \quad \therefore \text{max} \quad \checkmark$$

$$A(-2) = (-2)^3 - \frac{3(-2)^2}{2} - 18(-2) + 10$$

$$= -8 - 6 + 36 + 10$$

$$= 32 \quad \checkmark$$

Question Two: [10 marks]

Determine the global maximum and minimum of the function $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2 + 1$

over the domain $-1 \leq x \leq 1$.

$$f(-1) = \frac{1}{4} + \frac{1}{3} - 1 + 1 = \frac{7}{12} \quad \checkmark$$

$$f(1) = \frac{1}{4} - \frac{1}{3} - 1 + 1 = \frac{-1}{12} \quad \checkmark$$

$$f'(x) = x^3 - x^2 - 2x \quad \checkmark$$

$$x^3 - x^2 - 2x = 0 \quad \checkmark$$

$$x(x^2 - x - 2) = 0 \quad \checkmark$$

$$x(x-2)(x+1) = 0 \quad \checkmark$$

$$x = 0, x = 2, x = -1 \quad \checkmark$$

$$f''(x) = 3x^2 - 2x - 2 \quad \checkmark$$

$$f''(-1) = 3 > 0 \therefore \text{min} \quad \checkmark$$

$$f''(0) = -2 < 0 \therefore \text{max} \quad \checkmark$$

$$f(0) = 1 \quad \checkmark$$

Global max is 1

$$\text{Global min is } \frac{-1}{12} \quad \checkmark$$

Question Three: [10 marks]

Sketch a graph of the function $y = 4x^3 - 6x^2$ by finding all intercepts and all stationary points and their nature.

$$x = 0 \quad y = 0$$

$$4x^3 - 6x^2 = 0$$

$$2x^2(2x - 3) = 0$$

$$x = 0 \quad x = 1.5$$

$$(0, 0) \quad (1.5, 0) \quad \checkmark \checkmark$$

$$\frac{dy}{dx} = 12x^2 - 12x \quad \checkmark$$

$$12x^2 - 12x = 0$$

$$12x(x - 1) = 0 \quad \checkmark$$

$$x = 0, \quad x = 1 \quad \checkmark$$

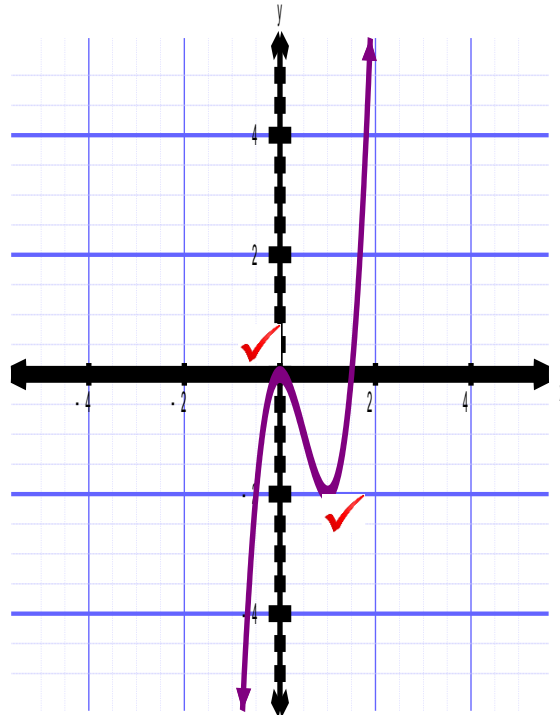
$$\frac{d^2y}{dx^2} = 24x - 12 \quad \checkmark$$

$$x = 0 \quad \frac{d^2y}{dx^2} = -12 < 0 \therefore \text{max}$$

$$x = 1 \quad \frac{d^2y}{dx^2} = 12 > 0 \therefore \text{min} \quad \checkmark$$

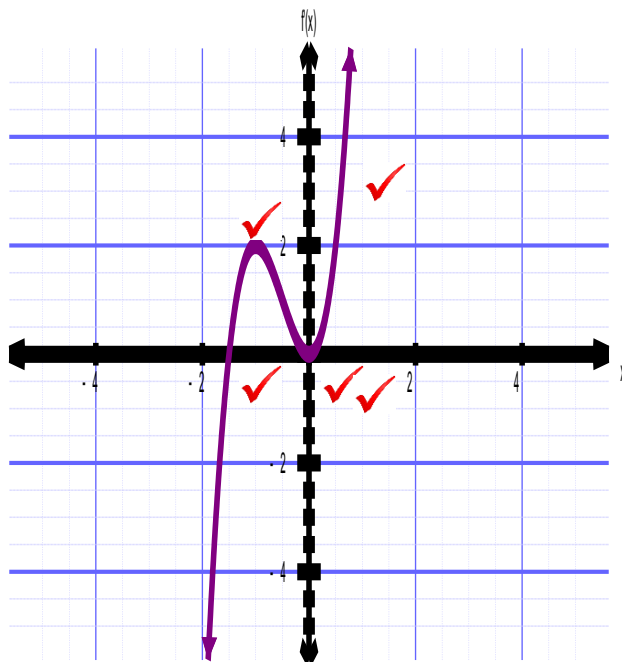
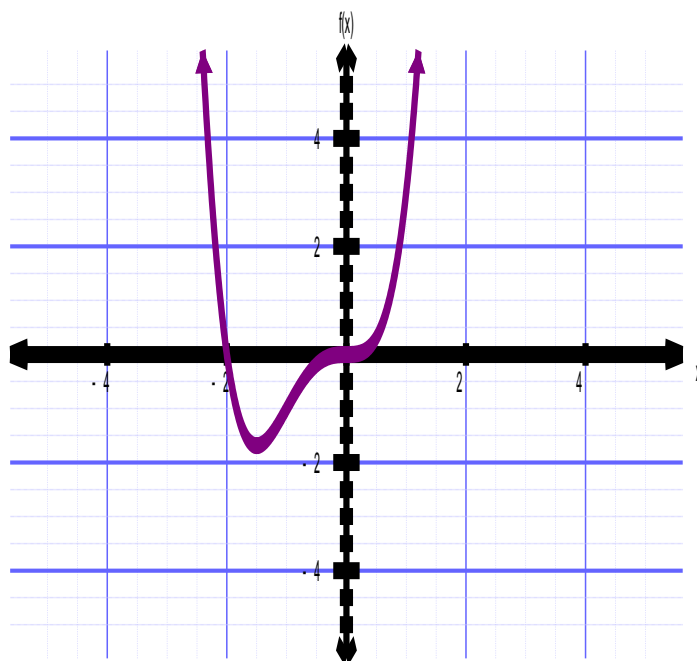
$$x = 1 \quad y = -2$$

$$(1, -2) \quad \checkmark$$

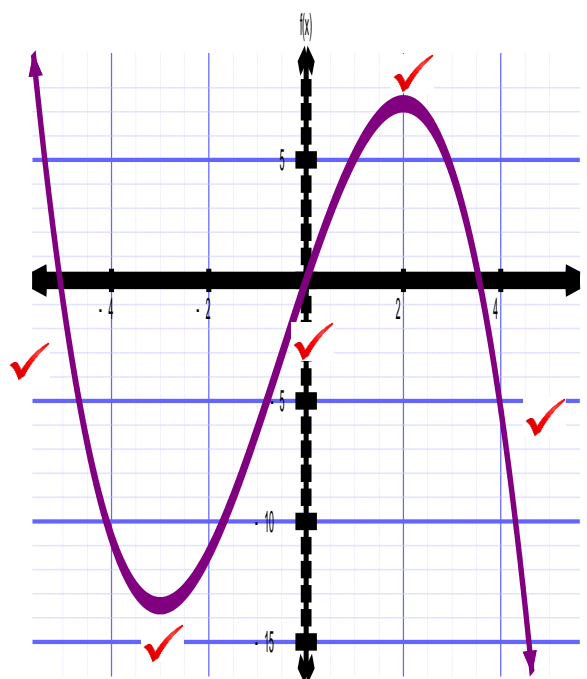
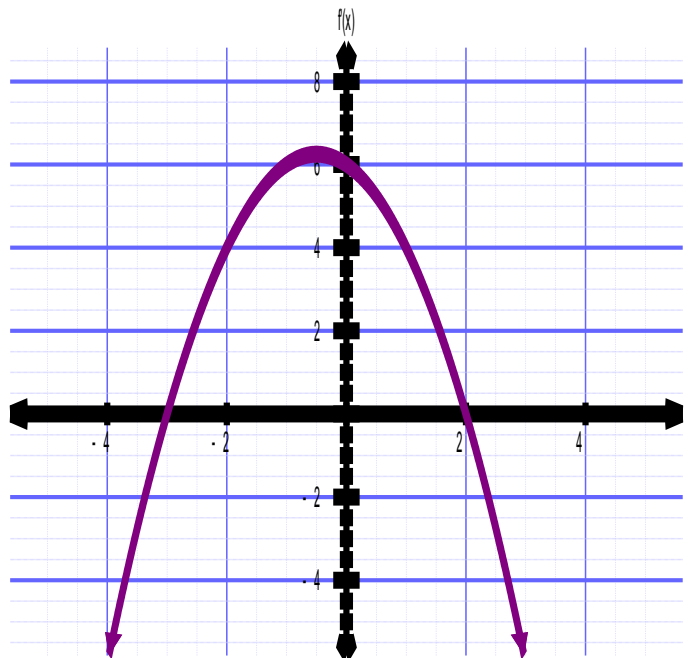


Question Four: [5, 5 = 10 marks]

- (a) Sketch a possible gradient function, $f'(x)$, using the graph of $f(x)$ below.

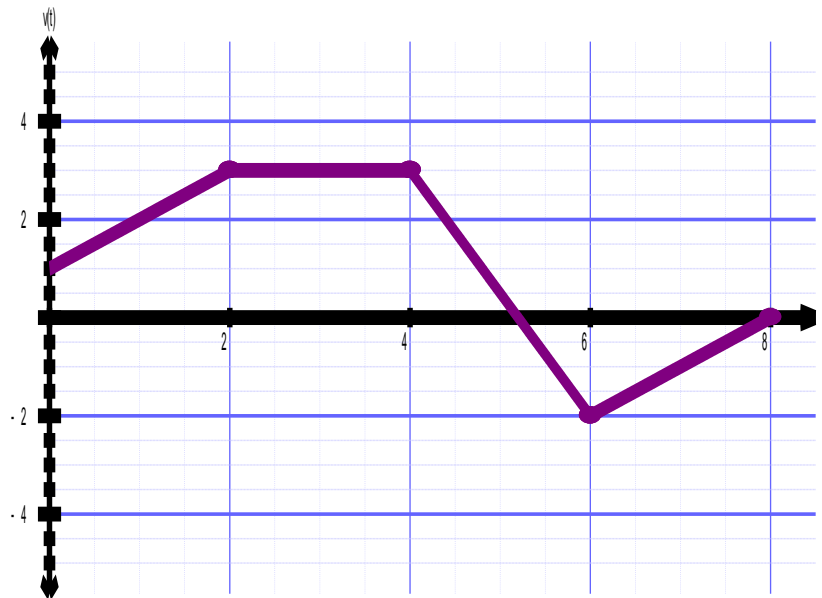


- (b) Sketch a possible function, $f(x)$, using the graph of $f'(x)$ below.



Question Five: [1, 1, 2, 2, 2 = 8 marks]

The graph below shows the velocity-time graph of a moving object, where time is in seconds and velocity is in m/s.



- (a) Determine the speed of the object when $t = 1$.

2 m/s ✓

- (b) Determine the velocity of the object when $t = 7$.

-1 m/s ✓

- (c) Determine the speed of the object when $t = 6$.

2 m/s ✓✓

- (d) Determine an expression for the rate of change of displacement with respect to time for this object in the first 2 seconds.

$\frac{dx}{dt} = t + 1$ ✓✓

- (e) Determine an expression for the rate of change of velocity with respect to time for this object between $t = 6$ and $t = 8$.

$\frac{dv}{dt} = 1$ ✓✓