Mathematics Methods – ATAR Year 11 Unit 1

Test 2

Conditions:
Time for the task: Up to 55 minutes, in-class, under test conditions

Materials required:

Section One: Calculator-free	Standard writing equipment
Section Two: Calculator-assumed	Calculator (to be provided by the student)
Other materials allowed:	Drawing templates, 1 page of notes in Section Two
Marks available:	**62 marks**
	Section One: Calculator-free (30 marks)
	Section Two: Calculator-assumed (32 marks)
Task weighting:	**10%**

Section One: Calculator-free (30 marks)

Suggested time: 25 minutes

Question 1 **(2, 2=4 marks)**

Given that the following graph shows a quadratic function,

a) determine its equation in the form $y=a(x-b)^2+c$.

$y = a(x-2)^2 - 5$

$(0,3) \rightarrow 3 = a(0-2)^2 - 5 \rightarrow a=2$ ✓

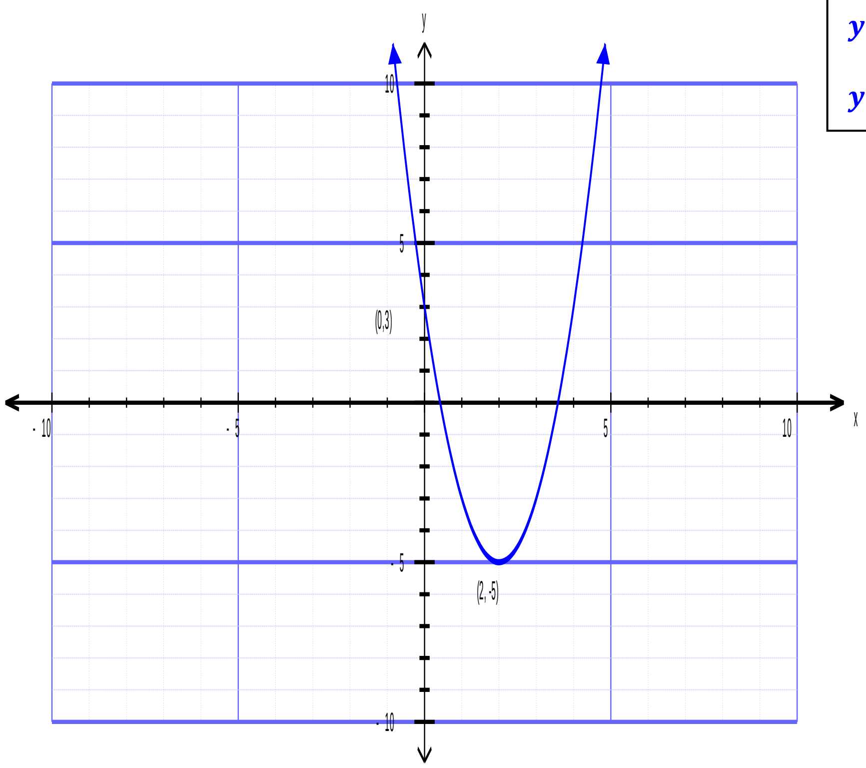
$y = 2(x-2)^2 - 5$ ✓

b) expand the brackets in your answer to a) to rewrite the equation of the graph in the form $y= px^2+qx+r$

$y = 2(x-2)^2 - 5$

$y = 2(x^2 - 4x + 4) - 5$ ✓

$y = 2x^2 - 8x + 3$ ✓



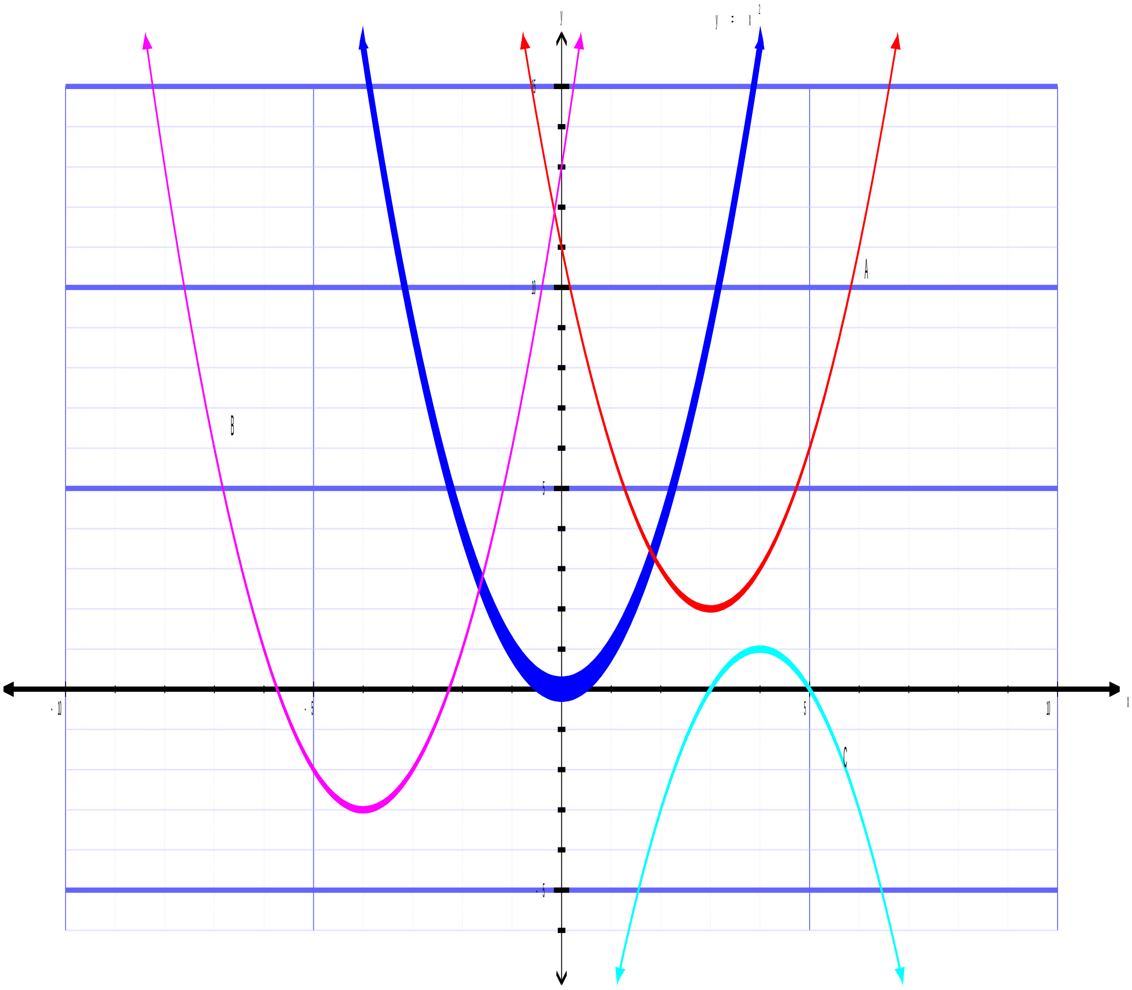
Question 2 **(1,1,1=3 marks)**

In the graph below the curves are all of the same shape as the curve shown bold. State the equation of each of the other graphs given that the curve shown bold is defined by $y = x^2$.

Equation of curve A: $y = (x-3)^2 + 2$ ✓

Equation of curve B: $y = (x+4)^2 - 3$ ✓

Equation of curve C: $y = -(x-4)^2 + 1$ ✓



Question 3 **(2, 1, 2,1= 6 marks)**

For the quadratic function with equation $y = 2(x+3)(x-5)$ determine

(a) the coordinates of the points where the graph crosses the x-axis.

(-3,0) and (5,0)

✓ ✓

(b) the equation of the line of symmetry.

x = (-3+5)/2 , i.e. x=1 ✓

(c) the coordinates of the maximum/minimum turning point, stating which of these it is.

(1, y(1)) = (1, -32). It's a minimum turning point (*a* =2 >0)

✓ ✓

(d) the coordinates of the point where the curve cuts the y-axis.

x = 0 y=2(3)(-5)=-30 →(0, -30) ✓

Question 4 **(3 marks)**

Express the quadratic function y = x^2+10x+29 in the form $y = a(x-b)^2 + c$ and hence determine the coordinates and nature of the turning point.

$y = (x+5)^2 + 4$ ✓

Turning point (-5, 4) ✓

It's a minimum turning point since a=1 >0 ✓

Question 5 **(3 marks)**

Solve the quadratic equation $6x^2$ +13x + 6 =0 by factorising or by using the quadratic formula.

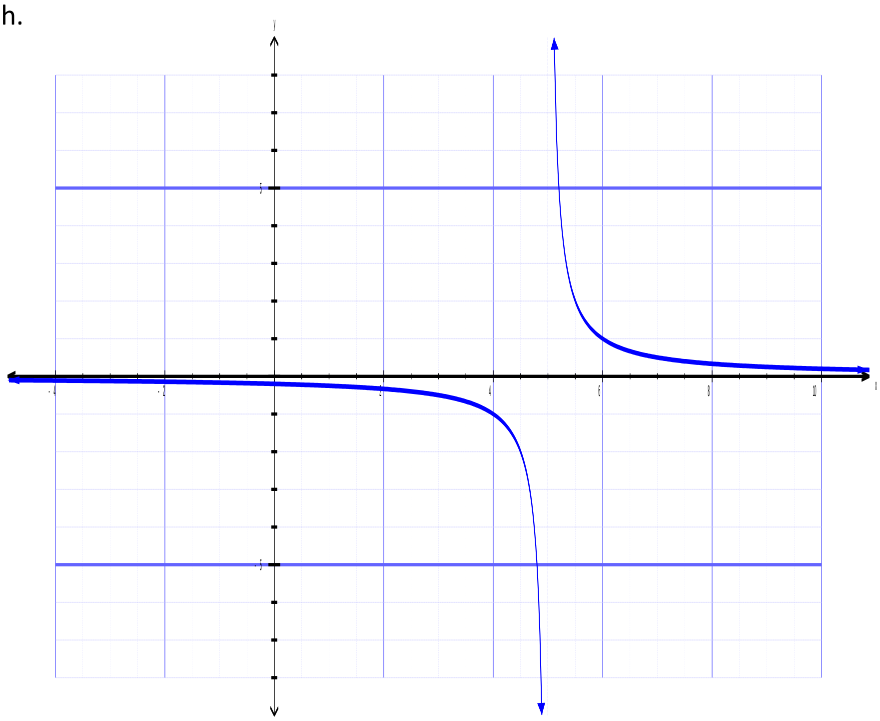
$\mathbf{6x^2 + 13x + 6 = 6x^2 + 9x + 4x + 6 = 3x(2x+3) + 2(2x+3) = (3x+2)(2x+3)}$

$\mathbf{6x^2 + 13x + 6 = 0} \rightarrow \mathbf{(3x+2)(2x+3) = 0}$ ✓✓

$x = \frac{-2}{3}$ or $x = \frac{-3}{2}$ ✓

Question 6 **(1 marks)**

The graph below has an equation in the form $y = \frac{1}{x+c}$. Find the value of c and hence find the equation of the graph.



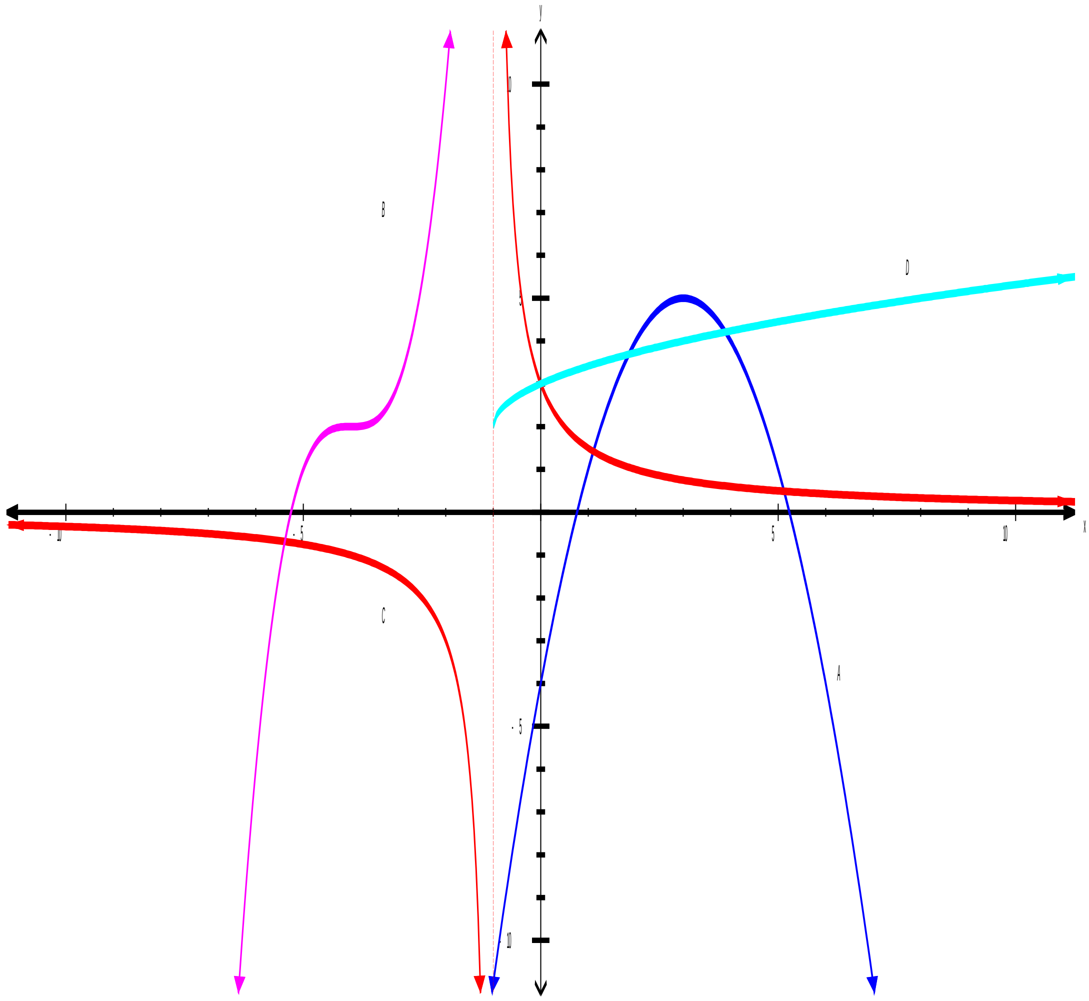
For the graph of $y = \frac{1}{x+c}$, the vertical asymptote is x= -c.

From the graph vertical asymptote is x=5. Therefore, c = -5 and

the equation of the graph is $y = \frac{1}{x-5}$ ✓

Question 7 **(4 marks)**

Match the graphs with the appropriate equation from the list below.



Graph A: $y = -(x-3)^2 + 5$ ✓

Graph B: $y = \frac{3}{x+1}$ ✓

Graph C: $y = (x+4)^3 + 2$✓

Graph D: $y = \sqrt{x+1} + 2$ ✓

$y = x^3 - 5$
$y = (x-5)^2 + 3$
$y = -(x-3)^2 + 5$
$y = 5(x-3)^2$
$y = \dfrac{3}{x+1}$
$y = (x+4)^3 + 2$
$y = \dfrac{1}{x+3}$
$y = \sqrt{x+1} + 2$
$y = \sqrt{x-1} + 2$

Question 8 **(2, 2, 2=6 marks)**

Describe how the graph of each of the functions in list A can be transformed into the graphs of the corresponding functions in list B.

A	B	description
$y=(x-1)^3$	$y=(x+4)^3-10$	Translate horizontally 5 units to the left and vertically 10 units down✓✓.
$y=\frac{1}{x+1}$	$y=\frac{-1}{x+3}$	Reflect about the x-axis then translate horizontally 2 units to the left.✓✓
$y=\sqrt{x+2}$	$y=\sqrt{x-1}+5$	Translate horizontally 3 units to the right then vertically 5 units up.✓✓

Section Two: Calculator-assumed (32 marks)

Suggested time: 30 minutes

Question 9 **(1, 2,2=5 marks)**

Given that $x^3 + x^2 - 22x - 40 = (x+2)(x^2+bx+c)$,

a) Determine the value of c by inspection.

2c = -40. Therefore c = -20 ✓

b) With your answer in a) in place of c, expand $(x+2)(x^2+bx+c)$ and hence determine b.

$(x+2)(x^2+bx-20) = x^3 + (b+2)x^2 + (2b-20)x - 40$ ✓

→ b+2= 1 and 2b-20=-22 → b= -1 ✓

c) Factorise $x^3 + x^2 - 22x - 40$.

$x^3 + x^2 - 22x - 40 = (x+2)(x^2 - x - 20) = (x+2)(x-5)(x+4)$

✓ ✓

Question 10 **(2, 2 =4 marks)**

For each of the following situations write an equation that connects the variables:

a) The variable x varies inversely as y, and when x=0.2, y=5.

$y = \frac{k}{x}$

$5 = \frac{k}{0.2} \rightarrow k = 1 \rightarrow y = \frac{1}{x}$ ✓✓

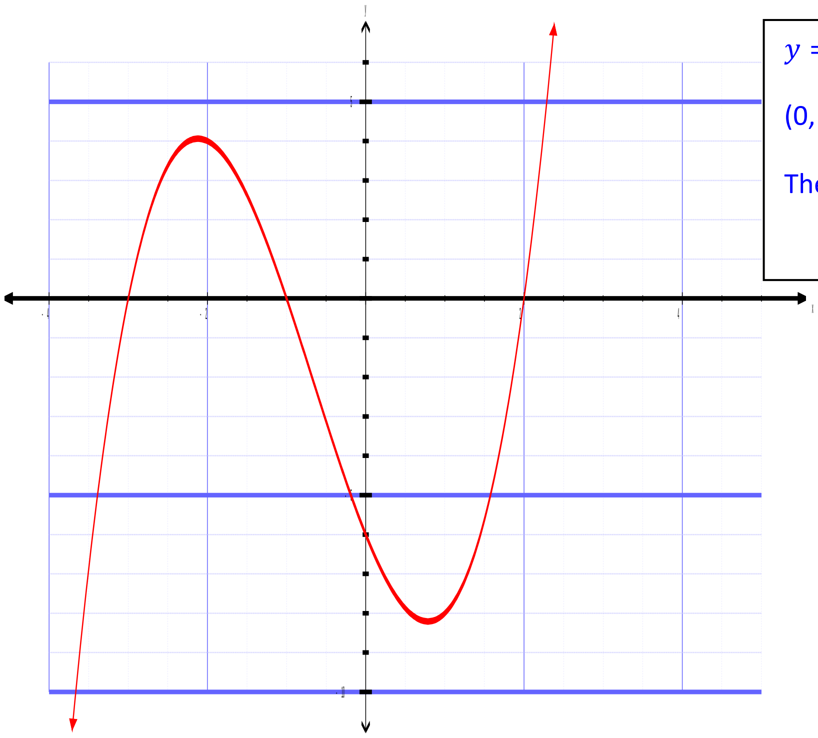
b) The volume (V) of a certain gas is inversely proportional to the surrounding pressure (P). From observations, it is known that when 10 units of pressure are exerted, 2 units of volume result.

$V = \frac{k}{P}$

$2 = \frac{k}{10} \rightarrow k = 20 \rightarrow V = \frac{20}{P}$ ✓✓

Question 11 **(3 marks)**

Given that the following graph represents a cubic function, determine the equation of the function.



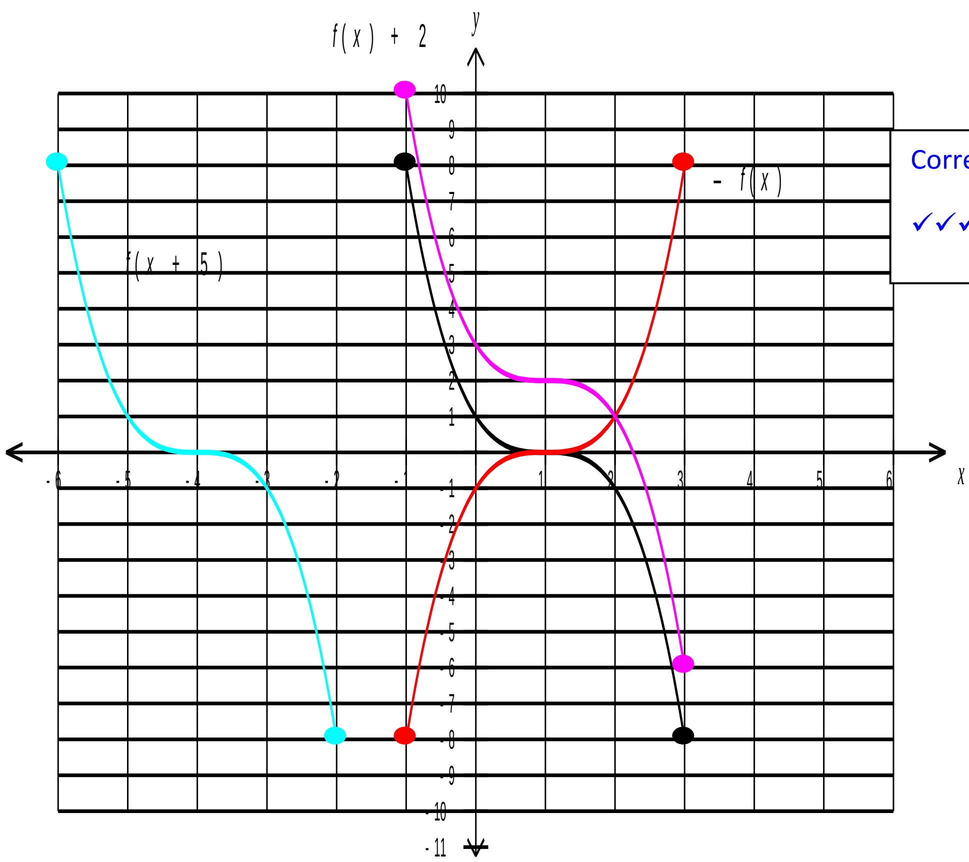
$y = a(x+3)(x+1)(x-2)$ ✓

(0, -6)→-6=a(3)(1)(-2) → $a = 1$ ✓

Therefore $y = (x+3)(x+1)(x-2)$ ✓

Question 12 **(3 marks)**

The grid shows a sketch of a function $y = f(x)$. Sketch the functions $y = -f(x)$, $y = f(x) + 2$ and $y = f(x+5)$ clearly labelling each of these.



Correct graphs and labels.

✓✓✓

Question 13 **(2,2,1= 5 marks)**

A cubic function f is defined by $f(x) = 2x^3 - 3x^2 + px + 4$. Given that $f(-1) = 0$, determine

a) the value of p.

$0 = 2(-1)^3 - 3(-1)^2 + p(-1) + 4$ ✓

$0 = -p - 1 \rightarrow p = -1$ ✓

b) using your answer in a) in place of p, factorise f(x) and hence determine the coordinates of the points where the graph of the function cuts the x-axis.

$f(x) = 2x^3 - 3x^2 - x + 4 = (x+1)(2x^2 + bx + 4)$ (since f(-1)=0) ✓

$f(x) = (x+1)(2x^2 - 5x + 4)$ ✓

Note that $2x^2 - 5x + 4$ has a negative discriminant.

$\therefore$ The graph cuts the x-axis at (-1, 0) only. ✓

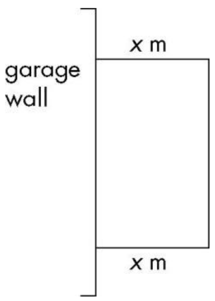
c) the coordinates of the point where the graph of the function cuts the y-axis.

When x=0 $f(0) = 2(0)^3 - 3(0)^2 - 0 + 4 = 4.$

The graph cuts the y-axis at (0, 4). ✓

Question 14 **(1, 1, 2, 1,1= 6 marks)**

Johanna wants to construct a dog pen in her backyard. She is going to use one of the walls of her garage and some cyclone wire and posts to construct the rectangular pen. Johanna has 16 m of wire available to make the other three sides of the pen.



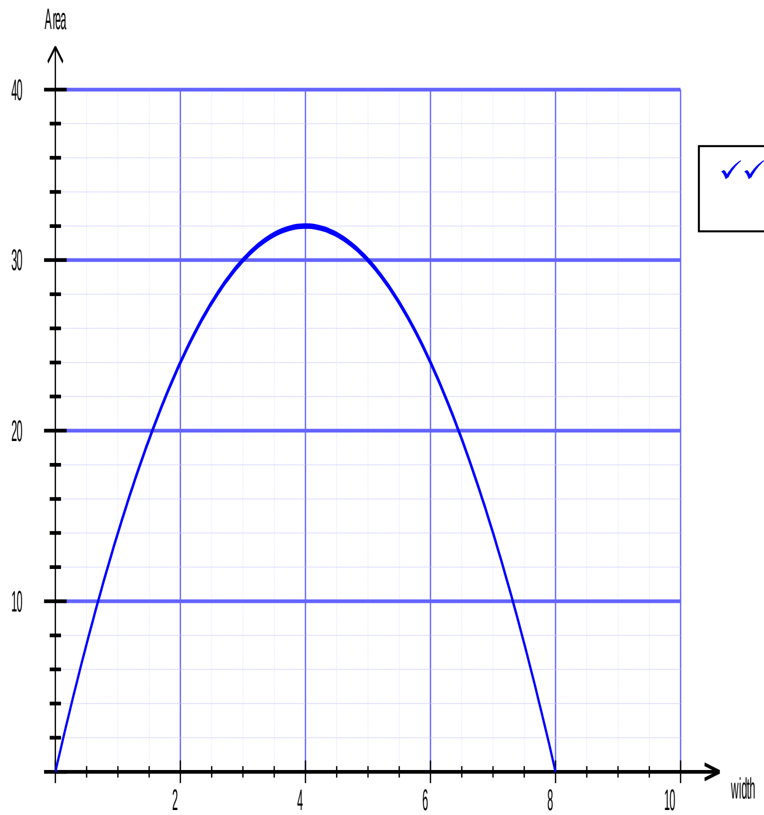
(a) If the width of the dog pen is to be x m, find an expression (in terms of x) for the length of the dog pen.

$16 - 2x$ m ✓

(b) Show that the area of the dog pen, A m^2, is given by the expression $A = -2x^2 + 16x$.

$A = x(16 - 2x) = 16x - 2x^2 = -2x^2 + 16x$ ✓

(c) Sketch the graph of the area of the dog pen, A m^2, against the width of the dog pen, x m. (Show width on the horizontal axis.)



✓✓

(d) If the dog pen is to be as large as possible, find its dimensions.

Width = 4m, length 16-2(4) = 8m ✓

(e) Find the area of the largest rectangular dog pen that Johanna can construct.

32 m^2 ✓

Question 15 **(1 mark)**

Calculate the discriminant of the quadratic equation $x^2 + 5x + 7 = 0$ and hence determine whether the graph of the function $y = x^2 + 5x + 7$ crosses the x-axis or not.

$$D = 5^2 - 4(1)(7) = -3$$

Since the discriminant is negative, the graph of the function doesn't cross the x-axis. ✓

Question 16 **(1, 1 = 2 marks)**

Find the equation of a circle with centre (3, -2) and radius 5, giving your answer in the form

a) $(x - p)^2 + (y - q)^2 = c$

$(x - 3)^2 + (y + 2)^2 = 25$✓

b) $x^2 + y^2 + dx + ey = c$

$x^2 - 6x + 9 + y^2 + 4y + 4 = 25$

Or $x^2 + y^2 - 6x + 4y = 12$✓

Question 17 **(1, 2 = 3 marks)**

Write the vertical and horizontal asymptotes of each of the following equations.

a) $y = \frac{4}{x-8}$

Vertical asymptote $x = 8$

Horizontal asymptote $y = 0$ ✓

b) $y = \frac{1}{2x+5} + 3$

Vertical asymptote $x = \frac{-5}{2}$ ✓

Horizontal asymptote $y = 3$ ✓