12 Mathematics Methods 2022

Test 3 – Calculus of trig, exponential and log functions

Section 2: Calculator-assumed

Time allowed: 25 minutes **Maximum marks: 26**

Name:_____ **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- Calculators and 1xA4 double-sided page of personal notes are permitted.

Question 5 **[3 marks]**

If $y = 3\sin 2x + 2\ \cos 2x$, determine the value of k for which $ky = \frac{d^2y}{dx^2}$

Question 6 **[4 marks]**

Determine the values of a and b where $f(x) = ax\ln(bx)$, $f(1) = 12$ and $f'(1) = 16$.

Question 7 **[2, 3 = 5 marks]**

Let $g(x)$ be a function such that $g(-14) = g(16) = -72,\ g(-2) = g(4) = 0, g(1) = 3$ and $g'(1) = 0$.

If $g'(x) > 0\ for -14 \leq x < 1$ and $g'(x) < 0\ for\ 1 < x \leq 16$, evaluate:

a) $\int_1^{16} g'(x)dx$

b) What is the area bounded by the graph of $g'(x)$ and the x-axis between $x = -14$ and $x = 16$? Justify your answer.

Question 8 **[2, 2, 2 = 6 marks]**

V is the volume of water left in a 350Litre container, t minutes after a leak occurs. The rate of change of V can be modelled by $\frac{dV}{dt} = kV$. After 3 minutes, 43 litres of water were lost.

a) Write an equation to model the amount of water left in the tank after t mins given that the container was full when the leak started.

b) To the nearest litre, how much water was lost in the first 20mins?

c) Determine the rate at which water is leaving the tank, when the tank has 10Litres of water remaining.

Question 9 **[2, 3, 3 = 8 marks]**

An AFL match starts at 2:40pm.

The rate at which spectators enter a stadium is given by $S'(t) = \frac{9000e^{-0.05t}}{(1+4e^{-0.05t})^2}$ spectators per minute, where t is the time from 1pm.

a) At what rate were spectators entering the stadium when the match started?

b) When, to the nearest minute, were spectators entering at the fastest rate and what was this rate?

c) If 5000 spectators were already in the stadium at 1pm, how many spectators were in the stadium when the match starts?

END OF TEST