Full Name: SOLUTIONS

MATHEMATICS APPLICATIONS

Test 6 – Finance

Chapter 8

Semester 2 2016

Calculator Assumed

Time allowed

Working time for this section:	55 minutes
Marks available:	50 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items:	pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters
Special items:	drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (5 marks)
Zac took out a loan for $28 000. He makes monthly repayments of $500 and interest is calculated at a rate of 6.2% p.a., adjusted monthly.

a. Write a difference equation that gives the amount still owing on the loan after n months. [2]

$$T_{n+1} = \left(1 + \frac{6.2}{1200}\right) T_n - 500 \quad , \quad T_0 = 28000$$

✓ ✓

b. How much does Zac owe after 4 years? [1]

$8699.07

c. How much interest has Zac paid in this time? [1]

paid in $24000 in payments

$8699.07 - 4000 = \underline{\$4699.07}$

diff between P and PMT

interest

d. At this rate, how long until Zac pays off the loan? [1]

I 6.2
PV 28000
PMT -500
FV 0
P/Y 12
C/Y 12

N = 66.28

∴ 67 months. ✓

(5 years 7 months)

2. (4 marks)

Kayte takes out a loan for $10 000 at 10.75% p.a. interest, adjusted quarterly.

a. How many years until the loan is repaid if Kayte repays $607.99 per quarter? [1]

22 quarters

∴ 5 years 6 months ✓

b. If Kayte needs to pay off the loan in 3 years, find the quarterly repayment required. [1]

$985.97 ✓

c. How much interest would Kayte save by repaying according to part **b** rather than part **a**? [2]

985.97 × 12 = $11831.64

607.99 × 22 = $13375.78 ✓

$1544.14 saving ✓

3. (8 marks)

May & Sydney operate an investment account for their children's education with an initial deposit of $5000. They make regular monthly deposits of $2000 at the end of each month for 10 years. The table below shows the first few months of the account. Assume that the account pays a fixed rate of interest of 6% per annum. The monthly growth is calculated on the opening monthly balance and credited to the account at the end of each month.

Month	Opening Balance ($)	Growth for the month ($)	Monthly Deposit ($)	Balance at the end of the month ($)
1	5000	25	2000	7025
2	7025	35.125	2000	9060.125
3	9060.125	45.30625	2000	11105.43 ✓ $\frac{1}{2}$

a. Complete the table above to find the account balance at the end of the third month. [2]

b. Write a recursive rule to determine the account balance at the end of each month. [2]

$$T_{n+1} = \left(1+\frac{6}{1200}\right)T_n + 2000 \quad , \quad T_1 = 5000$$
✓ ✓

c. Determine the account balance at the end of the 10-year period. [2]

$n = 120$ ✓

$336855.68 ✓

d. Find the total interest earned after 10 years. [2]

$120 \times 2000 = 240000$

opening $= 5000$

Interest $= 336855.68 - 245000$ ✓

$= \$91855.68$ ✓

4. (9 marks)

Julie buys a car with a purchase price of $16 000. However, she has been told to expect the car to depreciate in value. The value of the car after *n* years can be determined by using the recursive rule.

$$T_{n+1} = 0.86T_n, \quad T_0 = 16000$$

a. Complete the table below to show the value of the car at the end of each year, to the nearest dollar. [2]

without rounding

1

n	0	1	2	3
Value of car after *n* years ($)	16 000	13760	11834	10177

✓ 1/2

b. Use the information above to determine the rate of depreciation of Julie's car per year. [1]

14% per annum ✓

c. Determine a rule for the *n*th term of the sequence of values found in part (a). [2]

$T_n = 16000 \times 0.86^n$

✓ ✓

d. Determine the value of Julie's car after eight years, correct to the nearest dollar. [2]

$T_8 = 16000 \times 0.86^8$

$= \$4787.49$ ✓

$\therefore \$4787$ ✓

e. Julie decides that she will sell her car at the end of the year in which its value drops to half of the purchase price. After how many years should she sell her car? [2]

$8000 = 16000 \times 0.86^n$

$n = 4.596$ ✓

$\therefore$ at the end of fifth year ✓

5. (11 marks)
Thomas has borrowed $20 000 from a bank at a reducible interest rate of 15% per annum with interest accrued and repayments made monthly. Standard repayments are set at $500 per month.

The table below shows the progress of the loan for the first six months. All values have been rounded to the nearest cent.

Month	**Amount owing at beginning of month**	**Interest for the month**	**Repayment**	**Amount owing at end of month**
1	20 000.00	250.00	500.00	19 750.00
2	19 750.00	246.88	500.00	19 496.88
3	19 496.88	243.71	500.00	19 240.59
4	19 240.59	240.51	500.00	18 981.09
5	18 981.09	237.26	500.00	18 718.36
6	18 718.36	***A***	500.00	***B***

a. What is the monthly interest rate? [1]

$\frac{15}{12} = 1.25\%$ per month

b. Determine the values of ***A*** and ***B***. [2]

$A = 233.98$

$B = 18452.34$

c. Determine the length of time it will take Thomas to pay off the loan. [1]

56 months

d. Determine the total amount Thomas pays over the duration of the loan. [3]

amount owing at end of 55th month \$394.39

interest for 56th month \$4.93

total paid $(55 \times 500) + (394.39 + 4.93)$

$= \$27899.32$

e. The bank suggests that Thomas need only make repayments of \$250 per month. Describe how this would affect the length of time and total amount he pays over the duration of the loan. [2]

He would never pay off the loan, because the repayment = interest, hence principal never decreases

✓reasoning

f. After listening to advice, Thomas decides that he wants to pay off the loan completely in two years, making equal payments each month over that time. Determine the amount of each repayment he will need to make in order to make this happen (correct to the nearest cent). [2]

Financial mode

N 24
I 15
PV 20000
FV 0
P/Y 12
C/Y 12

Payment \$969.73

6. (13 marks)

a. Alex is about to retire and is planning to take an annuity from his pension fund. He sets up the pension fund on his 65th birthday with $500 000 and he estimates the fund can generate a growth rate of 6% per year. He plans to start withdrawing an annuity of $40 000 starting on his following birthday.

i. Write a recurrence relation to calculate the total amount in the fund directly after each withdrawal. [3]

$T_{n+1} = 1.06T_n - 40000$ ✓ ✓ , $T_0 = 500000$ ✓

ii. For how many years will Alex be able to receive his annuity of $40 000? [2]

23 years $\sqrt{2}$

(time as a decimal is one mark)

iii. Assuming that all other conditions are the same, explain what would happen if Alex decided to withdraw $30 000 per year instead of $40 000 per year. [2]

6% of 500 000 = $30 000 ✓

∴ principal will not decrease ✓

b. Abbey sets up her pension fund on July 1 2016 with a principal of $850 000. The fund guarantees an annual growth rate of 7.5% compounded monthly and she plans to take an annuity of $75 000 each year on July 1, starting in 2017.

i. Calculate the balance in the fund after the annuity is withdrawn in July 2020. [2]

n = 4 (July 2020) ✓

N 4
I 7.5
PV 850000
PMT -75000
P/Y 1
C/Y 12

Balance $809 531.47 ✓

The investment fund revised its annual interest rate to 9% compounded monthly on July 1 2020 guaranteed for the period to July 2025 and Abbey continued withdrawing $75 000 as usual.

ii. Calculate the balance in the fund after a withdrawal is made on July 1 2025. [2]

initial balance 809531.47 ✓

2021 -> N = 1

Balance when N = 5

New Balance $815197.73 ✓

iii. Calculate, to the nearest $100, the maximum amount Abbey could withdraw annually, starting in 2020, without decreasing her balance. [2]

initial 809531.47
future 809531.47

Abbey could withdraw $75939.63 ✓

$75900.00 ✓

End of Test

Additional working space

Question number: ______