

Topic: Simultaneous Equations and Applications

Time: 45 mins Marks: /45 marks

Question One: [4, 4, 3, 4: 15 marks]

a) Use the method of elimination to solve the simultaneous equations

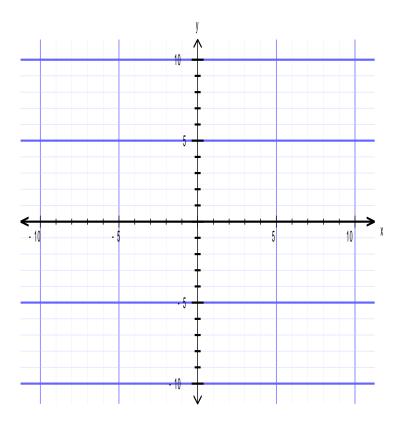
$$2x - 4y = 24$$
 and $6x - 20y = 0$

b) Use the method of substitution to solve the simultaneous equations

$$\frac{2y}{5} = 4x - 6$$
 and $13 = 3x - 2y$

c) Use graphical methods to solve the simultaneous equations

$$2x + 3y = 12$$
 and $y = -4x + 4$



d) Write down the pair of simultaneous equations represented by this matrix equation $\begin{bmatrix} 8 & 6 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -16 \end{bmatrix}$

and solve the simultaneous equations.

Question Two: [2, 4: 6 marks]

Ben and Holly each buy tickets for themselves and their families for the elf concert. Ben buys two child's tickets and four adult tickets and he pays \$24. Holly buys three children's tickets and three adult tickets and she spends \$21.

a) Write two equations in terms of, *c*, the price of a child's ticket and, *a*, the price of an adult ticket, for what each Ben and Holly spent.

b) Solve your equations to determine the cost of each type of ticket.

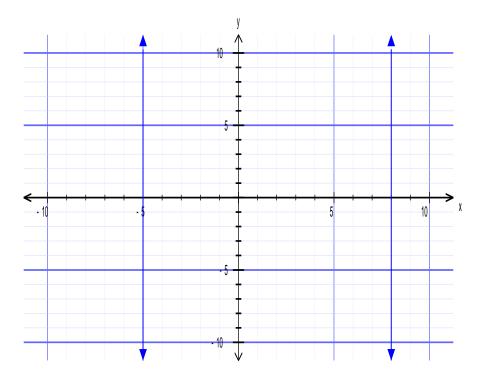
Question Three: [4, 3: 7 marks]

a) For the equations mx + my = p and y = 4x + m show that m(m + 5x) = 0 when p = 0.

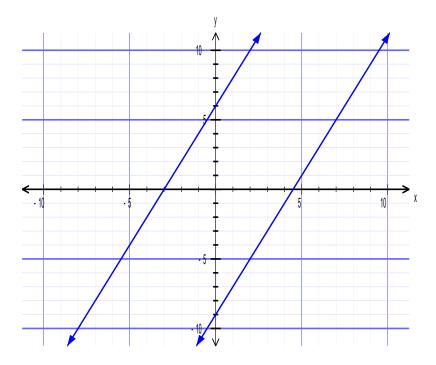
b) If m = -1 and p = 0 solve for x and y.

Question Four: [4, 5: 9 marks]

a) These two parallel lines form two sides of a rectangle with an area of $52units^2$. If one of the vertices of the rectangle is at (-5,2) determine the possible coordinates of the other three.



b) These two parallel lines form two sides of a rectangle with one of the vertices of the rectangle being at (0,6).



- i) Determine the equation of the line which forms a third side of this rectangle and includes the given vertex.
- ii) Hence or otherwise determine the coordinates of a second vertex of the rectangle.

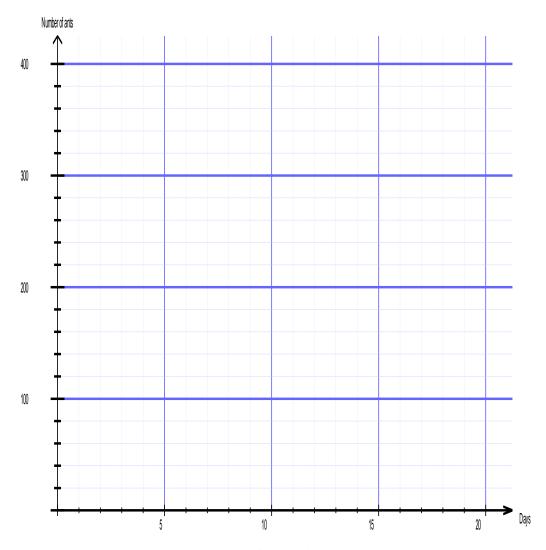
Question Five: [4, 2, 2: 8 marks]

Alisa has an ant farm and she notes that the population of ants can be modeled by the linear equation A = 50 + 5d with d being the number of days since she got the ant farm.

Alisa's brother had an ant farm of his own long before Alisa got hers so he already had 350 ants when Alisa got her farm.

Alisa's brother's ant farm contracts a disease on the day which Alisa got her ant farm. Alisa's brother's ants no longer reproduce but instead 15 ants are dying a day.

a) Draw two equations on the graph below for the number of ants which Alisa and her brother have.



- b) How many ants do they each have after 10 days?
- c) Hence determine after how many days Alisa and her brother have the same number of ants.



Topic: Simultaneous Equations and Applications SOLUTIONS

Time: 45 mins Marks: /45 marks

Question One: [4, 4, 3, 4: 15 marks]

a) Use the method of elimination to solve the simultaneous equations

$$2x - 4y = 24$$
 and $6x - 20y = 0$

$$2x - 4y = 24 \times 3 \qquad \checkmark$$

$$6x - 20y = 0$$

$$6x - 12y = 72(1)$$

$$6x - 20y = 0$$
 (2)

$$(2) - (1)$$

$$-8y = -72$$

$$y = 9$$

$$6x - 180 = 0$$

$$6x = 180$$

$$x = 30$$

b) Use the method of substitution to solve the simultaneous equations

$$\frac{2y}{5} = 4x - 6$$
 and $13 = 3x - 2y$

$$2y = 20x - 30$$

$$2y = 20x - 30$$
 $y = 10x - 15$

$$13 = 3x - 2(10x - 15) \checkmark$$

$$13 = 3x - 20x + 30$$

$$-17 = -17x$$

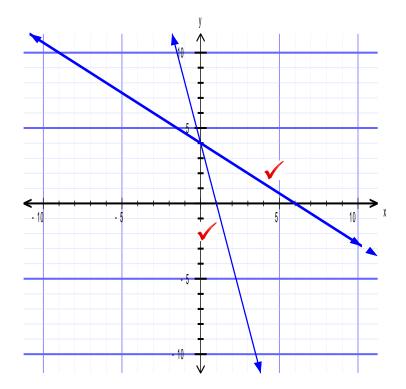
$$1 = x$$

$$y = 10 - 15$$

$$y = -5$$

c) Use graphical methods to solve the simultaneous equations

$$2x + 3y = 12$$
 and $y = -4x + 4$



 $solution = (0,4) \checkmark$

d) Write down the pair of simultaneous equations represented by this matrix equation $\begin{bmatrix} 8 & 6 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -16 \end{bmatrix}$

and solve the simultaneous equations.

$$8x + 6y = 12(1)$$

$$\checkmark$$

$$-4x + 2y = -16 \times 2$$

$$-8x + 4y = -32$$
 (2)

$$(1) + (2)$$

$$10y = -20$$

$$y = -2$$

sub into (1)
$$8x - 12 = 12$$

$$8x = 24$$

$$x = 3$$

Question Two: [2, 4: 6 marks]

Ben and Holly each buy tickets for themselves and their families for the elf concert. Ben buys two child's tickets and four adult tickets and he pays \$24. Holly buys three children's tickets and three adult tickets and she spends \$21.

a) Write two equations in terms of, *c*, the price of a child's ticket and, *a*, the price of an adult ticket, for what each Ben and Holly spent.

$$2c + 4a = 24 \qquad \checkmark$$
$$3c + 3a = 21 \qquad \checkmark$$

b) Solve your equations to determine the cost of each type of ticket.

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2c = 4a = 24 \times 3
3c + 3a = 21 \times 2
6c + 12a = 72
6c + 6a = 42
6a = 30
a = 5 adult's ticket costs $5
2c + 20 = 24
2c = 4
c = 2 child's ticket costs $2
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Question Three: [4, 3: 7 marks]

a) For the equations mx + my = p and y = 4x + m show that m(m + 5x) = 0, when p = 0.

$$mx + m(4x + m) = p \qquad \checkmark$$

$$mx + 4mx + m^2 = p \qquad \checkmark$$

$$5mx + m^2 = p \qquad \checkmark$$

$$5mx + m^2 = 0$$

$$m(5x+m)=0 \qquad \checkmark$$

b) If m = -1 and p = 0 solve for x and y.

$$-1(5x-1)=0 \qquad \checkmark$$

$$-5x + 1 = 0$$

$$x = \frac{1}{5} \quad \checkmark$$

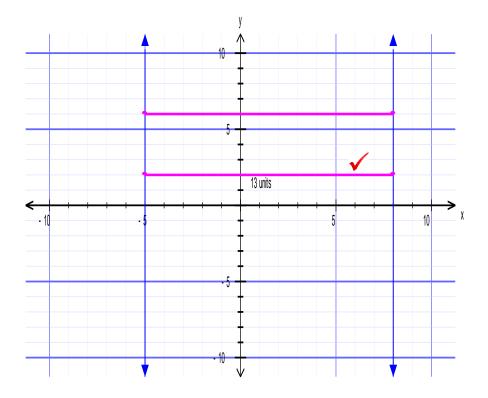
$$y = 4 \times \frac{1}{5} - 1$$

$$y = \frac{4}{5} - \frac{5}{5}$$

$$y = -\frac{1}{5} \checkmark$$

Question Four: [4, 5: 9 marks]

a) These two parallel lines form two sides of a rectangle with an area of $52units^2$. If one of the vertices of the rectangle is at (-5,2) determine the possible coordinates of the other three.

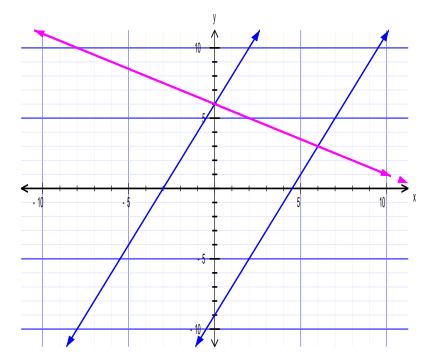


13 units long, ∴ 4 units wide



Other possible coordinates are (8,2) (8,6) and (-5,6)

b) These two parallel lines form two sides of a rectangle with one of the vertices of the rectangle being at (0,6).



i) Determine the equation of the line which forms a third side of this rectangle and includes the given vertex.

$$m_1 = 2$$
 $m_{perpendicular} = -\frac{1}{2}$ point (0,6)
$$y = -\frac{1}{2}x + 6$$

ii) Hence or otherwise determine the coordinates of a second vertex of the rectangle.

See graph \rightarrow Second vertex = (6,3) \checkmark

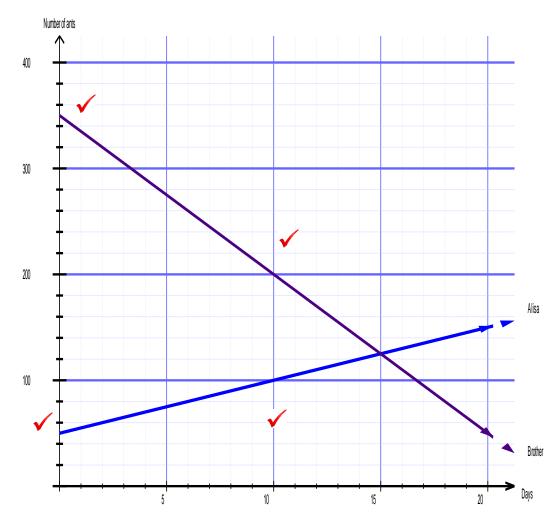
Question Five: [4, 2, 2: 8 marks]

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Alisa's brother had an ant farm of his own long before Alisa got hers so he already had 350 ants when Alisa got her farm.

Alisa's brother's ant farm contracts a disease on the day which Alisa got her ant farm. Alisa's brother's ants no longer reproduce but instead 15 ants are dying a day.

a) Draw two equations on the graph below for the number of ants which Alisa and her brother have.



b) How many ants do they each have after 10 days?

Alisa has 100 ants and her brother has 200 ants.



c) Hence determine after how many days Alisa and her brother have the same number of ants.

