Name: SOLUTIONS Date: _____

Specialist Unit 1

Test 2, 2015

Topics – Vectors, Relative displacement and velocity

	49
	=___ %

Total Time: *60 minutes*

Total Reading: *5 minutes*

Total Working: *55 minutes*

Weighting: *5% of the year.*

*This test comprises of **TWO sections**. The **first section** is **calculator free** where no calculators of any kind are to be used. The **second section** is **calculator assumed** where a CAS calculator may be used. All questions must be answered in both sections. **Answers should be rounded appropriately**. All working should be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.*

No pens, pencils, highlights etc. may be used during reading time. This time is to be used to read through the assessment and check that you understand what is being asked of you. You may speak with the teacher/supervisor during this time (by putting up your hand and waiting patiently for them to approach you) but you may only ask clarification questions and not how to solve the problems. After reading time has ended, you may not ask any more questions.

SECTION 1: CALCULATOR FREE

Time: *25 minutes*

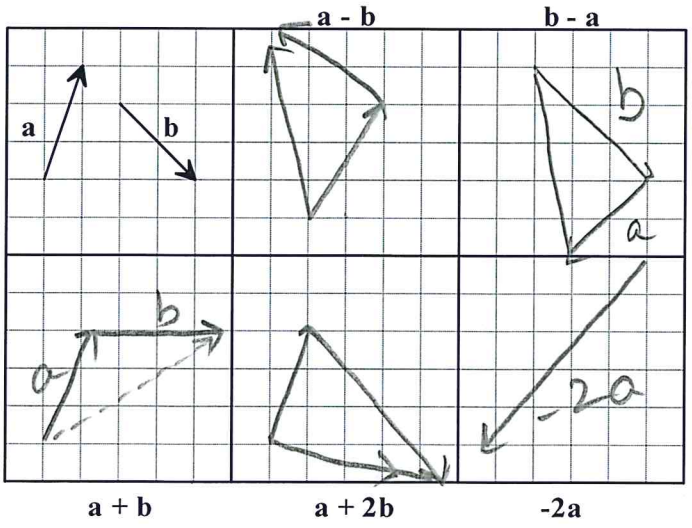
Reading: *2 minutes*

Working: *23minutes*

Marks for Section 1: *23*

Equipment Allowed: *Nil*

1. [5 marks: 1, 1, 1, 1, 1]
Complete this grid by drawing the resultant of the vector operations.



2. [2 marks]

Given that $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j}$ and $\mathbf{b} = \mathbf{i} + k\mathbf{j}$, find k if $|a - b| = \sqrt{85}$

$a - b = (3i - 5j) - (i + kj) = 2i + (-5 - k)j$

$|a - b| = \sqrt{85}$

$\therefore \sqrt{2^2 + (-5-k)^2} = \sqrt{85}$

$4 + (5+k)^2 = 85$

$(5+k)^2 = 85 - 4$

$(5+k)^2 = 81$

$(5+k)^2 = (9)^2$

$5 + k = \pm 9$

Hence $k = -14$

$k = 4$

3. [3 marks]

Seven people, including 3 friends, are seated in a row at random. In how many ways can the three friends be seated together?

$= 5! \times 3!$

$= 120 \times 6$

$= 720$

∴ There are 720 ways in which the three friends can be seated together.

4 [4 marks]

Find n if:

$9 \times {}^{n+1}P_3 = 10 \times {}^{n}P_3$

$\frac{9 \times (n+1)!}{(n+1-3)!} = \frac{10 \times n!}{(n-3)!}$

$\frac{9(n+1)!}{(n-2)!} = \frac{10\, n(n-1)(n-2)(n-3)!}{(n-3)!}$

$\frac{9(n+1)n(n-1)(n-2)!}{(n-2)!} = 10n(n-1)(n-2)$

$$9(n+1)n(n-1) = 10n(n-1)(n-2)$$

$$9(n+1) = 10(n-2)$$

$$9n+9 = 10n-20$$

$$9n-10n = -20-9$$

$$-n = -29$$

$$\therefore n = 29$$

5. [4 Marks]

Use the vectors (3, 2), (–2, 5) and (7, 3) to demonstrate that the scalar product is distributive over vector addition (i.e. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$).

$$L.H.S = a \cdot (b+c)$$

$$= (3,2) \cdot [(-2,5) + (7,3)]$$

$$= (3,2)\ (5,8)$$

$$= 15 + 16$$

$$= 31$$

$$R.H.S = a \cdot b + a \cdot c$$

$$= (3,2) \cdot (-2,5) + (3,2) \cdot (7,3)$$

$$= -6 + 10 + 21 + 6$$

$$= 31$$

$$\therefore L.H.S = R.HS$$

QED

6.

[5 Marks]

Given **a** = (–3, 5), **b** = (2, 7), **c** = (1, 9), **d** = (8, –2) and **e** = (–4, –2), find:

a **e + e + e + e**

$= 4e$

$= 4(-4,-2)$

$= (-16,-8)$

b 5e

$= 5(-4,-2)$

$= (-20,-10)$

c 6d – 4b

$= 6(8,-2) - 4(2,7)$

$= (48-8), (-12-28)$

$= (40,-40)$

d 3(2c – a)

$= 3(2(1,9) - (-3,5))$

$= 6(1,9) - 3(-3,5)$

$= (6,54) - (-9,15) = (15,39)$

e 8d – 12e

$8(8,-2) - 12(-4,-2)$

$(64,-16) - (-48,-24)$

$(112, 40)$

~ END OF TEST SECTION 1 ~

Name: _____ Date: _____

SECTION 2: CALCULATOR ASSUMED

Time:	*30 minutes*	**Marks for Section 2:**	*26*
Reading:	*3 minutes*	**Equipment Allowed:**	*½ page notes (A4 one side), CAS calculator*
Working:	*26 minutes*		

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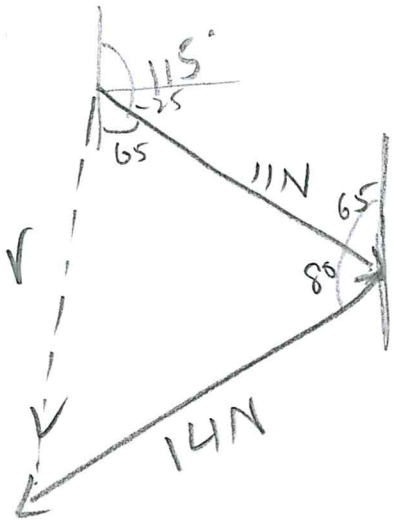
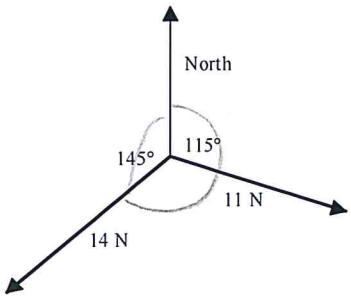
7. [5 marks]
Find the magnitude and direction of the resultant of this pair of forces. Give your answer correct to two decimal places.





$r^2 = 11^2 + 14^2 - 2 \times 14 \times 11 \times \cos 80$

$r^2 = 263.52$

$r = 16.23\ N$

$\frac{\sin\theta}{14} = \frac{\sin(80)}{16.23}$

$\sin\theta = \frac{14 \times \sin(80)}{16.23}$

$\sin\theta = 0.8493$

$\theta = \sin^{-1}(0.8493)$

$\theta = 58.14°$

8. **[3 marks]**

A large circular table at a restaurant has 9 people from an office seated at random around it.
What is the probability of Mary being seated between David and Peter?

No. of ways of seating $= 8! = 40320$

Consider Mary, David & Peter as group with Mary in the middle.

2 possible orders with Mary in the middle.

So the no. of ways of seating is $6! \times 2$

$= 1440$

So probability $= \frac{1440}{40320} = \frac{1}{28} \approx 3.6\%$

9. **[7 marks: 2, 2, 3]**

Given that $\mathbf{a} = -3\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{c} = 3\mathbf{i} - 2\mathbf{j}$ find

a) a unit vector in the same direction as **b**.

$|b| = \sqrt{2^2+1^2} = \sqrt{5}$ ✓

unit vector $= \frac{1}{\sqrt{5}}(2i+j)$ ✓

b) a vector in the same direction as **b** but equal in magnitude to **a**.

$|a| = \sqrt{3^2+4^2} = \sqrt{25} = 5$ ✓

vector $= \frac{5}{\sqrt{5}}(2i+j)$ ✓

c) a vector in the same direction as the resultant of **a, b** and **c** but equal in magnitude to **a.**

$a+b+c = -3i+4j+2i+j+3i-2j$

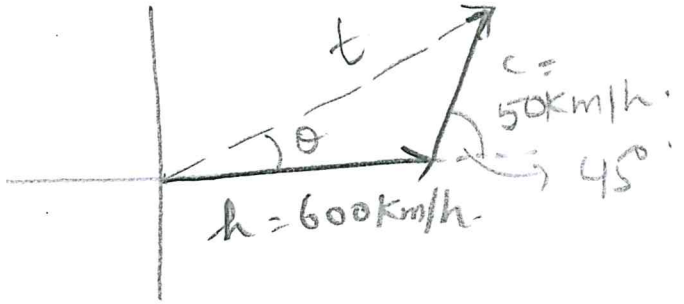
$= 2i+3j$

$|a+b+c| = \sqrt{2^2+3^2} = \sqrt{13}$ ✓

vector $= \frac{5}{\sqrt{8}}(2i+3j)$

10. [5 marks]

An aircraft, heading due east in still air at an airspeed of 600 km/h^{-1}. It encounters a wind blowing towards the north east at 50 km/h^{-1}. Calculate the aircraft's direction and ground speed.



$t = h + c$

$= 600i + [50\cos(45)i + 50\sin(45)j]$

$= 635.35i + 35.35j$

$t = \sqrt{(635.35)^2 + (35.35)^2}$

$= 636.33$

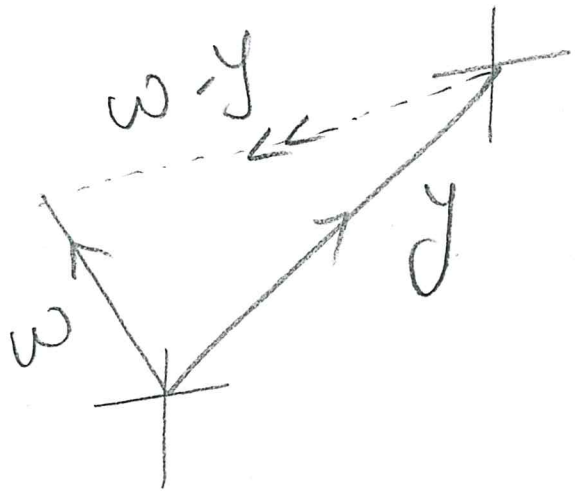
$u = \tan^{-1}\left(\frac{35.35}{635.35}\right) \approx 3.2°$

The aircraft is flying at about 636.3 km/h in the direction N 86.8° E.

11. [6 marks: 3, 3]

A yacht is moving with velocity $2\mathbf{i}+5\mathbf{j}$ kmh^{-1} relative to a stationary observer on the shore. A sailor on board the yacht measures the wind as blowing with velocity $-3\mathbf{i}-2\mathbf{j}$ kmh^{-1}.

a) Sketch a clearly labelled velocity vector diagram that shows the relationship between the velocity of the yacht relative to the shore, the velocity of the wind relative to the shore and the velocity of wind relative to the yacht.



b) Find the velocity of the wind with respect to the stationary observer on the shore.

$$w = y + (w - y)$$
$$= (2i + 5j) + (-3i - 2j)$$
$$= -i + 3j$$

~ END OF TEST SECTION 2 ~