MINDARIE SENIOR COLLEGE

Methods Unit 3 Test 1, 2018
(Calculator Free)

Time: 20 minutes Marks: 20

1. [2, 2, 2 marks]

Determine $\frac{dy}{dx}$ for each of the following but **do not simplify**:

a) y = (2x - 3)(1 - 4x)^3

$$\frac{dy}{dx} = 2(1-4x)^3 + (2x-3).3(1-4x)^2(-4)$$

✓ ✓

b) y = $\frac{x^2}{3x-2}$

$$\frac{dy}{dx} = \frac{2x(3x-2) - x^2.3}{(3x-2)^2}$$

✓ ✓

c) y = $\frac{2}{(1-2x)^2}$

$$= 2(1-2x)^{-2}$$ ✓

$$\frac{dy}{dx} = -4(1-2x)^{-3}.(-2)$$ ✓

2. [2, 2 marks]

Determine the following integrals, expressing in their simplest form:

a) $\int 4(2x-1)^5 \ dx$

$$= \frac{4(2x-1)^6}{6.2} + c$$ ✓

$$= \frac{(2x-1)^6}{3} + c$$ ✓

b) $\int (x-1)(2x+5) \ dx$

$$= \int 2x^2 + 3x - 5 \ dx$$ ✓

$$= \frac{2x^3}{3} + \frac{3x^2}{2} - 5x + c$$ ✓

3. [1, 4, 2 marks]

For the curve y = $\frac{x^2}{x-2}$

a) State the equation of the vertical asymptote.

$x = 2$ ✓

b) Show that this curve has two stationary points, and state the coordinates.

$y = \frac{2x(x-2) - x^2}{(x-2)^2}$ ✓

$= \frac{x^2 - 4x}{(x-2)^2} = 0$ ✓

$\therefore x^2 - 4x = 0$

$x(x-4) = 0$ ✓

$x = 0, \text{ or } x = 4$

i.e. $(0,0)$ or $(4,8)$ ✓

c) Given that $\frac{d^2y}{dx^2} = \frac{8}{(x-2)^3}$ determine the local minimum.

If $x = 0$ $\frac{d^2y}{dx^2}$ is $-$ve $\Rightarrow$ local max. ✓

If $x = 4$ $\frac{d^2y}{dx^2}$ is $+$ve $\Rightarrow$ local min

$\therefore (4,8)$ is the local min ✓

4. [3 marks]

Given that y = $\frac{x-1}{x-2}$, use the incremental formula to determine the approximate change in y, as x increases from 4 to 4.01.

$\frac{dy}{dx} = \frac{(x-2) - (x-1)}{(x-2)^2}$ ✓

if $x = 4$ $\frac{dy}{dx} = -\frac{1}{(4-2)^2}$

$= -0.25$ ✓

$\delta y = -0.25(0.01)$

$= -0.0025.$ ✓

Methods Unit 3 Test 1, 2018
(Calculator Assumed)

Time: 40 minutes   Marks: 40

5. [2, 2, 4, 3 marks]

The velocity of a particle is given by $v(t) = t^2 - 2t - 8$ m/sec. Initially, the particle has a displacement of 2 m.

a) When is the particle stationary?

$t^2 - 2t - 8 = 0$ ✓
$(t-4)(t+2) = 0$
ie $t = 4$ sec. ✓

b) When is the acceleration zero?

$a(t) = 2t - 2 = 0$ ✓
$t = 1$ sec. ✓

c) Determine the displacement when the particle is stationary.

$x(t) = \int t^2 - 2t - 8 \, dt.$
$= \frac{t^3}{3} - t^2 - 8t + c.$ ✓
if $t=0$, $c=2$ ✓
$x(t) = t^3/3 - t^2 - 8t + 2.$ ✓
$x(4) = 24\,2/3$ m. ✓

d) How far did the particle travel in the first 6 seconds?

If $t=0$, $x(0) = 2$ m
if $t=4$, $x(4) = -24\,2/3$ m ✓✓
if $t=6$, $x(6) = -10$

dist $= 26\,2/3 + 14\,2/3$
$= 41\,1/3$ m. ✓

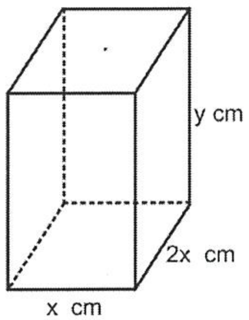
6. [6 marks]

The equation of the tangent to the curve $y = (ax - 3)^3 - 2$ at the point (1, 6) is $y = bx + c$.
Determine a, b and c.

If $x = 1$, $y = 6$
$(a-3)^2 - 2 = 6$
Solving $a = 5$
$\frac{dy}{dx} = 15(5x-3)^2$
If $x = 1$ $\frac{dy}{dx} = 60$
$\therefore c = -54$
thus $a = 5$, $b = 60$, $c = -54$

7. [2, 7 marks]

A wire frame is constructed as indicated in the diagram below. 48 cm of wire is required to make it.



a) Show that $y = 12 - 3x$.

$4x + 8x + 4y = 48.$
$12x + 4y = 48$
$\therefore y = 12 - 3x.$

b) Determine the dimensions of the box and its volume, such that the volume is at a maximum.

$V = x \cdot 2x(12 - 3x)$
$= 24x^2 - 6x^3$
$\frac{dV}{dx} = 48x - 18x^2 = 0$
$6x(8 - 3x) = 0$
$\therefore x = 2\frac{2}{3}$ cm.
$\frac{dV^2}{dx^2} = 48 - 36x$
If $x = 2\frac{2}{3}$ $\frac{d^2V}{dx^2}$ is $-$ve, thus Volume is at a maximum.
thus sides are $2\frac{2}{3}$ cm, $5\frac{1}{3}$ cm and 4 cm.
giving a volume of $56\frac{8}{9}$ cm^3

8. [3, 3, 3 marks]

Consider the curve $y = x^3 - 6x^2$

a) Use calculus to determine the point of infection of the curve.

$\frac{dy}{dx} = 3x^2 - 12x$ ✓

$\frac{d^2y}{dx^2} = 6x - 12 = 0$ ✓

$\therefore x = 2$

ie. $(2, -16)$ ✓

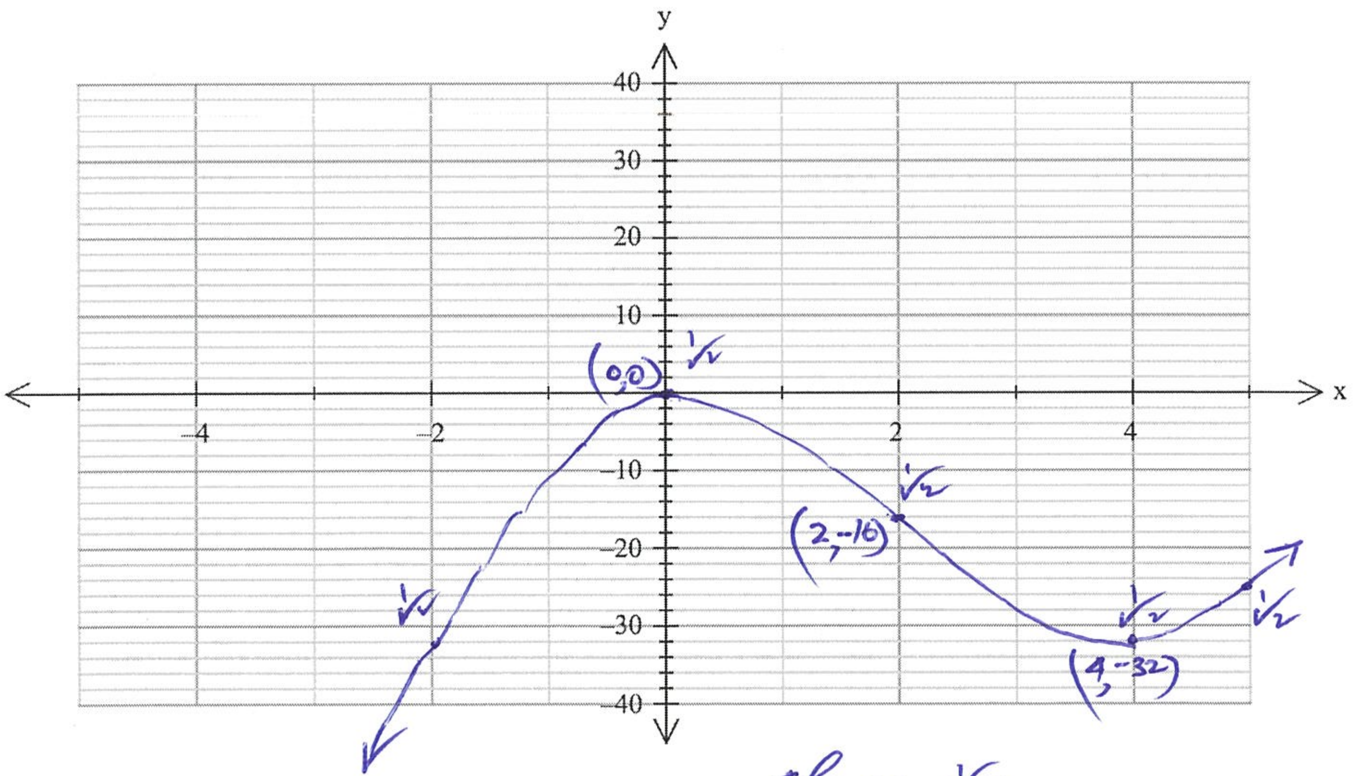
b) State the interval(s) for which the curve is concave down. Show reasoning.

If $x = 1$, $\frac{d^2y}{dx^2} = -6$ ✓

If $x = 3$, $\frac{d^2y}{dx^2} = 6$ ✓

Concave down for $x < 2$ ✓

c) Sketch the curve below, clearly labelling all critical points with coordinates.



shape ½ ½

9. [5 marks]

Given that $f'(x) = \frac{4}{(2x-1)^2}$ and that $f(1) = 4$, determine $f(0)$. Show all reasoning.

$$f(x) = \int 4(2x-1)^{-2}\,dx$$

$$= \frac{4(2x-1)^{-1}}{-2} + c \quad ✓$$

$$= -2/_{2x-1} + c. \quad ✓$$

$$f(1) = 4, \quad \therefore c = 6 \quad ✓$$

$$\therefore f(x) = -2/_{2x-1} + 6. ✓$$

$$\Rightarrow f(0) = 8 \quad ✓$$