MINDARIE
SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS NOW

MATHEMATICS: SPECIALIST 1 & 2

SEMESTER 2 2015

TEST 4

Resource Free

Time Allowed: 25 minutes

Total Marks: 24

1. [1, 2, 2 marks]

Given the matrices $A = \begin{bmatrix} -4 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -5 \\ 3 & 6 \end{bmatrix}$, and I is the identity matrix, calculate

(a) A – B $= \begin{bmatrix} -6 & 7 \\ -3 & -5 \end{bmatrix}$ ✓

(b) BA $= \begin{bmatrix} 2 & -5 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -8 & -1 \\ -12 & 12 \end{bmatrix}$ ✓✓ (−1 /error)

(c) BA + 2B – 3I $= \begin{bmatrix} -8 & -1 \\ -12 & 12 \end{bmatrix} + \begin{bmatrix} 4 & -10 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} -7 & -11 \\ -6 & 21 \end{bmatrix}$ ✓✓ (−1 /error)

2. [2 marks]

Prove that $\tan 105° = \frac{1+\sqrt{3}}{1-\sqrt{3}}$

$\tan 105° = \tan(60° + 45°)$ ✓

$= \dfrac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$

$= \dfrac{\sqrt{3} + 1}{1 - \sqrt{3}.1}$ ✓

$= \dfrac{1+\sqrt{3}}{1-\sqrt{3}}$

3. [1, 4, 5 marks]

(a) Find the value(s) of k for which $\begin{bmatrix} k & 1 \\ 4 & 3 \end{bmatrix}$ is singular.

$3k - 4 = 0 \quad k = 4/3$ ✓

(b) (i) Determine the inverse of the $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$.

$\frac{1}{2}\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ ✓

(ii) Using matrices, solve the simultaneous equations:

$$2x + y = \cancel{3}4$$
$$4x + 3y = 6$$

sets up matrices ✓

$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

premultiplies by inverse ✓

$\frac{1}{2}\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

solutions ✓

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$\therefore x = 3 \quad y = -2$

(c) Given that A = $\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 6 & -21 \\ -4 & 15 \end{bmatrix}$

(i) determine AB $= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ ✓

(ii) hence, solve

$$6x - 21y = 6$$
$$-4x + 15y = -2$$

Sets up matrices ✓

$\begin{bmatrix} 6 & -21 \\ -4 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

premultiplies by "inverse". ✓

$\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 6 & -21 \\ -4 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 6 \end{bmatrix}$

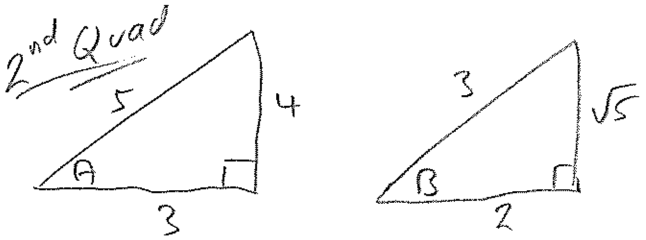
✓ $\begin{bmatrix} 2x \\ 3y \end{bmatrix} = \begin{bmatrix} 16 \\ 6 \end{bmatrix} \quad \therefore \quad x = 8, \; y = 2$ ✓

4. [3, 4 marks]

(a) Given $\cos\theta = \frac{2}{\sqrt{5}}$ for $0 \le \theta \le \frac{\pi}{2}$ determine the exact value of $\cos 2\theta$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$= 2\left(\frac{2}{\sqrt{5}}\right)^2 - 1$$
$$= 2 \cdot \frac{4}{5} - 1$$
$$= 8/5 - 1$$
$$= 3/5$$

(b) Given that A is an obtuse angle with $\sin A = \frac{4}{5}$ and B is an acute angle with $\cos B = \frac{2}{3}$ determine exactly the value for $\sin(A - B)$.



$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$
$$= \left(\frac{4}{5}\right)\cdot\left(\frac{2}{3}\right) - \left(\frac{-3}{5}\right)\cdot\left(\frac{\sqrt{5}}{3}\right)$$
$$= \frac{8}{15} + \frac{3\sqrt{5}}{15}$$
$$= \frac{8 + 3\sqrt{5}}{15}$$

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Resource Assumed

Time Allowed: 30 minutes   Total Marks: 29

5. [2, 2, 2, 3 marks]

Given that A, B, C, and X are all square matrices of the same order, and that all necessary inverse matrices exist, then re-arrange the following equations to make X the subject (i.e. X = ...)

(a) A + BX = C

$-A$ ✓ $BX = C - A$

Premult. B^{-1} ✓ $B^{-1}BX = B^{-1}(C-A)$

$X = B^{-1}(C-A)$

(b) XAB = C

$XABB^{-1} = CB^{-1}$ Post mult. B^{-1} ✓

$XA = CB^{-1}$

$XAA^{-1} = CB^{-1}A^{-1}$ Post mult. A^{-1} ✓

$X = CB^{-1}A^{-1}$

(c) C(X – B) = A

Premult C^{-1} ✓ $C^{-1}C(X-B) = C^{-1}A$

$X - B = C^{-1}A$

$+B$ ✓ $X = C^{-1}A + B$

(d) BX = A + CX

$BX - CX = A$ $-CX$ ✓

$(B-C)X = A$ $(B-C)X$ ✓

$(B-C)^{-1}(B-C)X = (B-C)^{-1}A$ Premult. $(B-C)^{-1}$ ✓

$X = (B-C)^{-1}A$

6. [4 marks]

Rewrite 3 cos θ + 5 sin θ in the form R cos (θ – α), where α is an acute angle in degrees.

$3\cos\theta + 5\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$\left.\begin{matrix} 3 = R\cos\alpha \\ 5 = R\sin\alpha \end{matrix}\right\}$ ✓

$3^2 + 5^2 = R^2$

$34 = R^2$

$R = \sqrt{34}$ ✓

$\frac{5}{3} = \frac{R\sin\alpha}{R\cos\alpha}$

$5/3 = \tan\alpha$

$\alpha = 59°$ ✓

$\sqrt{34}\cos(\theta - 59°)$ ✓

7. [2, 2, 2, 2 marks]

A company displays motor vehicles for sale in two different showrooms. Matrix P shows the number of vehicles for sale in each showroom. Matrix Q shows the petrol and oil requirements (in litres) for the sedans and 4-wheel drive vehicles. Matrix R gives the cost per litre for petrol and oil.

$$P = \begin{array}{c c} & \begin{array}{cc} \text{Sdn} & \text{4WD} \end{array} \\ \begin{array}{c} \text{Showroom A} \\ \text{Showroom B} \end{array} & \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \end{array}, \quad Q = \begin{array}{c c} & \begin{array}{cc} \text{Pet} & \text{Oil} \end{array} \\ \begin{array}{c} \text{Sdn} \\ \text{4WD} \end{array} & \begin{bmatrix} 45 & 4 \\ 60 & 6 \end{bmatrix} \end{array} \text{ and } R = \begin{array}{c c} & \$ \\ \begin{array}{c} \text{Pet} \\ \text{Oil} \end{array} & \begin{bmatrix} 1.40 \\ 2.50 \end{bmatrix} \end{array}$$

(a) Give a matrix S which shows the total (combined) cost of petrol and oil products for each type of vehicle.

$S = QR$

$$= \begin{array}{c c} \$ & \\ \begin{bmatrix} 73 \\ 99 \end{bmatrix} & \begin{array}{c} \text{sdn} \\ \text{4wd} \end{array} \end{array}$$

✓✓

(b) Show how S can be used, with one of the given matrices, to obtain a matrix M listing the total cost of petrol and oil products for each showroom. Hence find the matrix M.

$M = PS$

$$= \begin{array}{c c} \$ & \\ \begin{bmatrix} 417 \\ 391 \end{bmatrix} & \begin{array}{c} A \\ B \end{array} \end{array}$$

✓✓

(c) Give a matrix T which shows the separate petrol and oil requirements for each showroom.

$T = PQ.$

$$= \begin{array}{c c} & \begin{array}{cc} P & O \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 255 & 24 \\ 240 & 22 \end{bmatrix} \end{array}$$

✓✓

(d) Show how T can be used, with one of the given matrices, to obtain the matrix M in (b).

$$TR = \begin{bmatrix} 417 \\ 391 \end{bmatrix}$$

✓✓

8. [4, 4 marks]

Prove the following:

(a) $\frac{\cos\theta}{1+\sin\theta} + \tan\theta = \frac{1}{\cos\theta}$

$$LHS = \frac{\cos\theta}{1+\sin\theta} + \tan\theta$$

$$= \frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta} \checkmark$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} + \frac{\sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta + \sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)} \checkmark$$

$$= \frac{\cancel{1+\sin\theta}}{\cos\theta\cancel{(1+\sin\theta)}} \checkmark \quad \sin^2 + \cos^2 = 1$$

$$= \frac{1}{\cos\theta} \checkmark$$

$$= RHS$$

(b) $\cos 3A = \cos A.(1 - 4\sin^2 A)$

$$LHS = \cos 3A$$

$$= \cos(2A + A) \checkmark$$

$$= \cos 2A \cos A - \sin 2A \sin A$$

$$= (1 - 2\sin^2 A)\cos A - 2\sin A \cos A \sin A \checkmark \checkmark$$

$$= (1 - 2\sin^2 A)\cos A - 2\sin^2 A \cos A$$

$$= (1 - 2\sin^2 A - 2\sin^2 A)\cos A$$

$$= (1 - 4\sin^2 A)\cos A \checkmark$$

$$= RHS$$