BALDIVIS SECONDARY COLLEGE

APPLICATIONS - Unit 3 & 4

2022 Test 2- Sequences

Baldivis
Secondary College

Student Name Answers Teacher Name Yes

Time allowed for this task: 55 minutes, in-class, test conditions.

Section 1: 20 minutes + 2 minutes reading time
Section 2: 30 minutes + 3 minutes reading time

Materials required:

Section 1 Calculator free section (19 marks)
Standard writing equipment
SCSA Formula Sheet

Section2 Calculator assumed section (31 marks)
Calculator (to be supplied by the student)
SCSA formula Sheet
One page A4 (double sided) hand written notes

Other materials allowed: Drawing templates

Marks available: **50 marks**

Task Weighting: **6%**

Section 1: Resource Free **19 marks**

Question 1 (11 marks: 4, 4, 3)

a) A geometric sequence has $T_3 = 4$ and $T_6 = 32$

I. Determine the recursive rule.

$T_{n+1} = 2T_n \quad T_1 = 1$ ✓ ✓

II. By determining the **explicit** rule, calculate the 5th term

$T_n = 1 \times 2^n$ ✓

$T_5 = 1 \times 2^5$
$= 32$ ✓

b) An arithmetic sequence has $T_3 = -5$ and $T_6 = 4$

I. Determine the recursive rule.

$T_{n+1} = T_n + 3, \quad T_1 = -11$ ✓ ✓

II. By determining the **explicit** rule, calculate the 5th term

$T_n = -11 + 3n$ ✓

$T_5 = 1$ ✓

c) For the following sequence determine the recursive rule and T_7

T_1	T_2	T_3	T_4	T_5
4	-8	16	-32	64

$\times -2$ $\times -2$

✓ ✓

$T_{n+1} = -2T_n, \quad T_1 = 4$

$T_7 = 256$ ✓

Question 2 (6 marks: 2, 2, 2)

Ryan is attempting to collect a set of 300 football cards. In the first month he collects 50 cards, and in each following month he collects 20 new cards.

a) Find the first term and common difference

$T_1 = 50$ ✓ common diff $+20$ ✓

b) How many cards will he collect in 6 months.

T_1	T_2	T_3	T_4	T_5	T_6
50	70	90	110	130	150

✓ 150 cards ✓

c) How long will it take him to collect all 300 cards.

$50 + 20n = 300$

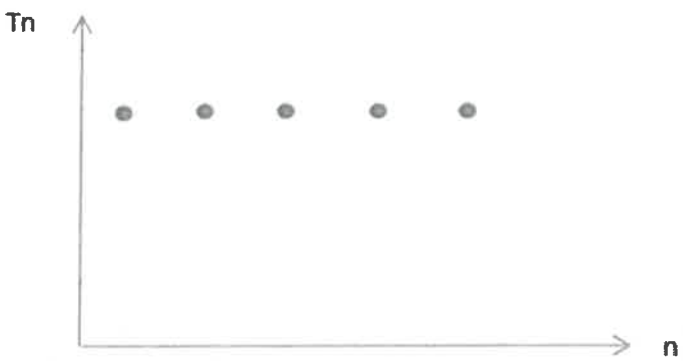
$20n = 250$

$n = 13.5$ ✓

14 months ✓

Question 3: (2 marks)

This graph could be thought of as a special Geometric progression or a special Arithmetic progression



a) If this is a GP, state the common ratio.

1 ✓

b) If this is a AP state the common difference.

0 ✓

<u>End of Resource Free</u>

Section 2: Resource Allowed

31 marks

Question 4: (3 marks: 1, 2)

A sequence has a recursive formula given by:

$$T_{n+1} = 0.6T_n, \quad T_1 = 150$$

a) Determine the first five terms of the sequence.

1	2	3	4	5
150	90	54	32.4	19.44

✓ - all correct

b) What percentage increase or decrease occurs with each successive term?

decrease ✓ 40% ✓

Question 5: (4 marks: 2,2)

A new road 75km long is being laid. At the end of Stage 1, 35km of road has been laid.
It took 45 days to complete Stage 1.
For Stage 2, covering the remaining 40km, an extra 600m of new road is completed each day.

Let $t(n)$ be the length of road completed at the start of day n in Stage 2.

a) Write a recursive equation for the length of completed road at the start of day n.

km $t_{n+1} = t_n + 0.6$ ✓ $t_1 = 35$ km ✓

or

m $t_{n+1} = t_n + 600$ $t_1 = 3500$ m

b) Find how long it would take for the entire road to be laid.

$T_{68} = 75.2$ ✓

67 days ✓

Question 6: (5 marks: 1,1,2,1)

A house is valued each year over the course of 5 years

Year	1	2	3	4	5	6
	\$450000	\$477000	\$505620	\$535957.20	\$568114.63	\$602201.51

a) Show that the house follows a geometric sequence

$\frac{477000}{450000} = 1.06$ $\frac{505620}{477000} = 1.06$ ✓

each term is found by ×1.06 by previous term hence geometric

b) Find the annual rate of increase as a percentage

6% ✓

c) Write a general rule for the terms in the sequence

✓ ✓

$T_n = 450000 \times 1.06^{n-1}$

d) Find the value of the house in year 30

$T_{30} = 450000 \times 1.06^{29}$

$= \$2438274.56$ ✓

Question 7: (4 marks: 2,2)

Consider the sequence 1, *x*, 25.

a) Use an appropriate method to find the value of *x* if the sequence is an arithmetic sequence.

$1 + n = x$ $25 - x = n$ ✓

solve simultaneously $x = 13$ $n = 12$

$x = 13$ ✓

b) Use an appropriate method to find the value of *x* if the sequence is a geometric sequence where all the terms are positive.

~~18n=x~~ $\frac{x}{1} = \frac{25}{x}$ ✓

$x^2 = 25$

$x = 5$ or -5

$x = 5$ ✓

Question 8: (8 marks: 3,2,2,1)

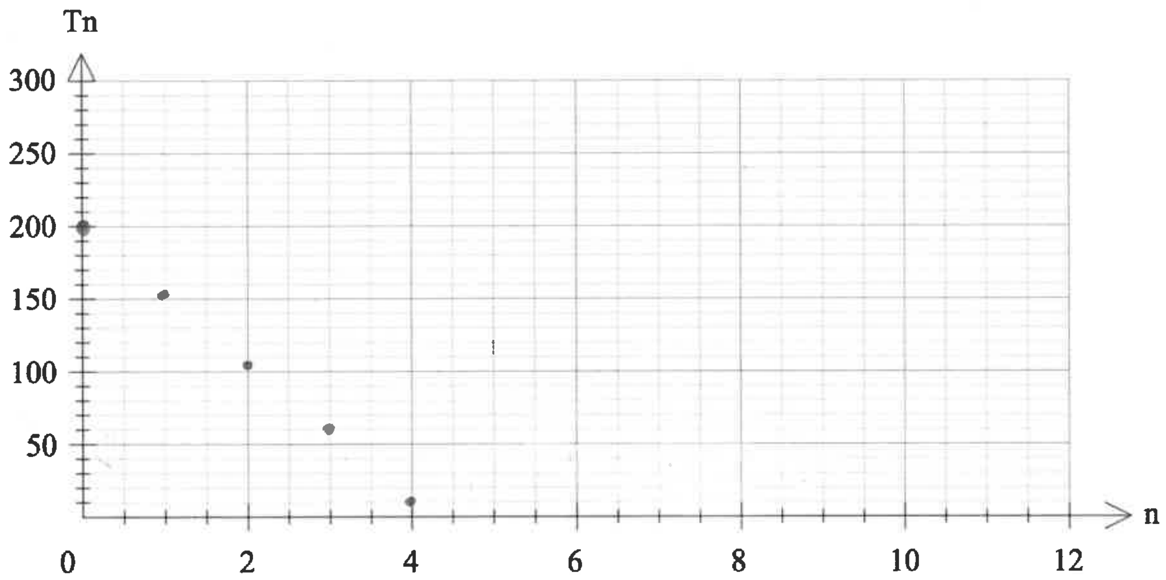
On a private property in Rosa Brook, the owner releases a population of 200 marron into her dam. She expects that the marron population will grow at a rate of 20% per year and she plans to capture 50 to eat each year.

a) Write a first order linear recurrence relation to model this situation

$T_{n+1} = 1.02T_n - 50$ ✓ ✓

$T_0 = 200$ ✓

b) Plot the terms of the sequence on the axes below



c) Describe what is expected to happen to the population of marron over time

Decrease to zero

d) How many marrons should the owner harvest each year to achieve a 'steady state' situation?

$200 \times 0.2 = 40$

40 marron.

Question 9: (7 marks: 2, 3, 2)

A fish farmer initially stocked a tank with 50 small fish. At the end of each month, the farmer caught some of the largest fish and sold them before adding more, smaller fish to the tank. The number of fish in the tank at the start of the n^{th} month is given by F, where

$F_{n+1} = 0.7F_n + 120,\ F_1 = 50$

(a) Use the recurrence relation to state

(i) the number of smaller fish added to the tank each month.

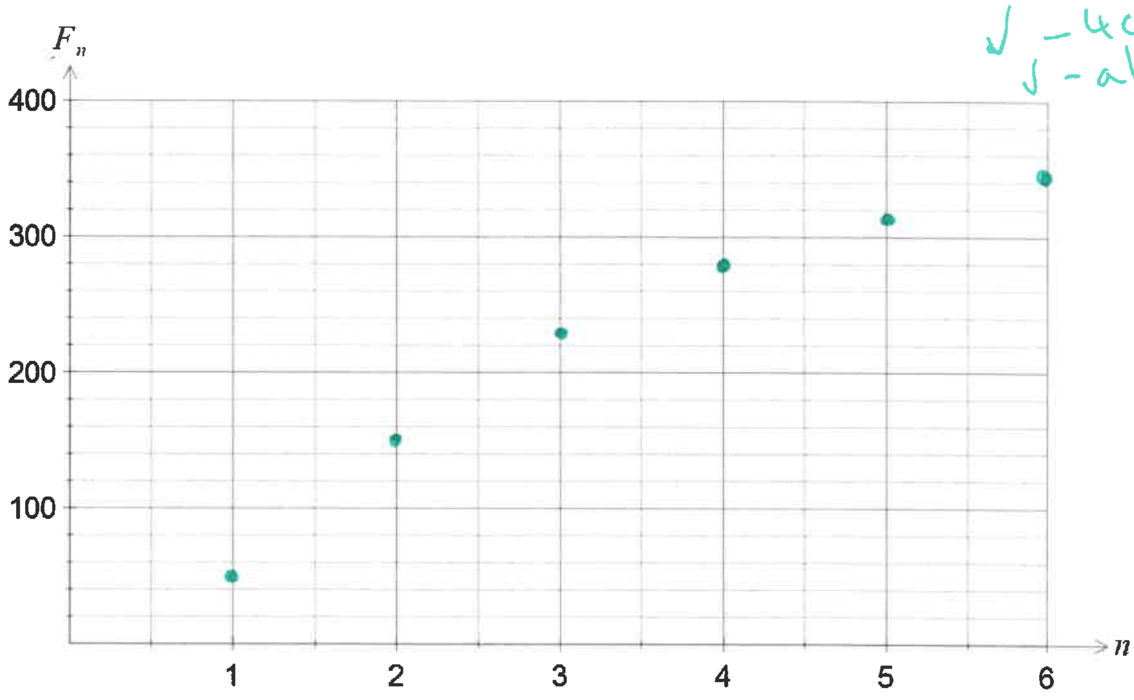
120 ✓

(ii) the percentage of the fish caught and sold each month.

30% ✓

(b) Graph F_n on the axes below for $1 \le n \le 6$.

✓ - 2 correct
✓ - 4 correct
✓ - all correct.



(c) Assuming this model continues, comment on how the number of fish in the tank changes over the next few years.

✓ increase
✓ 400 steadystate.

steady state of 400

~~There will be 400 fish.~~

Fish numbers will ~~approach~~ increase to 400 ✓

End of Resource Allowed