WILLETTON SENIOR HIGH SCHOOL

YEAR 12 MATHEMATICS METHODS

TEST 1 2023

Section 1: Calculator Free

Student Name: SOLUTIONS

Circle your teacher's name.

Miss Ahern	Mr Galbraith
Mrs Gatland	Mrs Sun

Mark: ________ / 26

Time: 25 mins

For section 1 of this test:

No notes.

No calculators.

Formula sheet as provided.

Show working.

QUESTION ONE (2, 3, 3 = 8 MARKS)

a) Find the second derivative of $y = \frac{1}{2x}$, simplifying your answer

,[2]

$$y' = \frac{-2}{4x^2} = \frac{-1}{2x^2}$$ ✓

$$y'' = \frac{4x}{4x^4} = \frac{1}{x^3}$$ ✓

b) Differentiate $m = 2n^4(3n-2)^3$ with respect to n, writing your answer in factorised form.

[3]

$$\frac{dm}{dn} = 2n^4 \times 3(3n-2)^2 \times 3 + (3n-2)^3 \times 8n^3$$ ✓ ✓

$$= 18n^4(3n-2)^2 + (3n-2)^3 \times 8n^3$$

$$= 2n^3(3n-2)^2\left[9n + 4(3n-2)\right]$$

$$= 2n^3(3n-2)^2(21n-8)$$ ✓

c) If $f(x) = \frac{2x}{x-1}$ and $g(x) = \sqrt{x}$ determine $(f \circ g)'(x)$, simplifying your answer.

[3]

$$(f\circ g)'(x) = f'(g(x)) \times g'(x)$$

$$= \frac{2(\sqrt{x}-1) - 2\sqrt{x}}{(\sqrt{x}-1)^2} \times \frac{1}{2\sqrt{x}}$$ ✓ ✓

$$= \frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$$ ✓

OR

$$f[g(x)] = \frac{2\sqrt{x}}{\sqrt{x}-1}$$

$$\frac{d}{dx} f[g(x)]$$

$$= \frac{(\sqrt{x}-1) \times x^{-\frac{1}{2}} - 2\sqrt{x}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{(\sqrt{x}-1)^2}$$ ✓

$$= \frac{1 - x^{-\frac{1}{2}} - 1}{(\sqrt{x}-1)^2}$$ ✓

$$= \frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$$ ✓

QUESTION TWO (2, 2 = 4 MARKS)

a) Find the antiderivative of $(3x+7)^{\frac{4}{3}}$

$$\frac{1}{7}(3x+7)^{\frac{7}{3}} + c$$ ✓✓ [2]

b) Determine $\int \frac{8x-16}{\sqrt{x^2-4x+7}}\,dx$

$$4\int \frac{2x-4}{\sqrt{x^2-4x+7}}\,dx$$ ✓ [2]

$$= 8\sqrt{x^2-4x+7} + c$$ ✓

QUESTION THREE (3 MARKS)

Determine $f(x)\ if\ f'(x) = \frac{2}{(4-2x)^2}$ and $f(1) = 2$

$$f(x) = \frac{1}{4-2x} + c$$ ✓

$$f(1) = \frac{1}{2} + c = 2 \Rightarrow c = \frac{3}{2}$$ ✓

$$\therefore f(x) = \frac{1}{4-2x} + \frac{3}{2}$$ ✓

QUESTION FOUR (3, 2 = 5 MARKS)

An athlete who has been running at a steady speed of 5 m/s, decides to accelerate for a period of 6 seconds. During this 6 second period the acceleration increases at a constant rate from 0 m/s^2 to 3 m/s^2

a) At what speed is the athlete running at the end of the acceleration period?

[3]

$a(t) = 0.5t$ ✓

$v(t) = \frac{1}{4}t^2 + c_1, \quad v(0) = c_1 = 5$

$\therefore v(t) = \frac{1}{4}t^2 + 5$ ✓

$v(6) = \frac{1}{4} \times 6^2 + 5 = 14$

speed is 14 m/s ✓

b) How far does the athlete travel during the acceleration period?

[2]

$x(t) = \frac{t^3}{12} + 5t + C_2, \quad x(0) = C_2 = 0$

$\therefore x(t) = \frac{t^3}{12} + 5t$ ✓

$x(6) = \frac{6 \times 36}{12} + 5 \times 6 = 18 + 30 = 48$

athlete travels 48m ✓

OR

$\left(x(6) - x(0) = (48 + C_2) - (C_2) = 48\text{m} \right)$ ✓

QUESTION FIVE (6 MARKS)

For the graph of $y = f(x) = 7 + 4x - x^3 - \frac{1}{4}x^4$, determine the location of any points of inflection.

$f'(x) = 4 - 3x^2 - x^3$

$f''(x) = -6x - 3x^2$ ✓

$-6x - 3x^2 = 0 \Rightarrow -3x(2+x) = 0$

$\Rightarrow x = 0$ or $x = -2$ ✓

$f(0) = 7$ and $f(-2) = 3$

For $(0,7)$ ✓: $f'(0) > 0$ and $f''(0) = 0$

$\therefore$ (oblique) point of inflection at $(0,7)$ ✓

For $(-2,3)$ ✓: $f'(-2) = 0$ and $f''(-2^-) < 0$

and $f''(-2^+) > 0$

$\therefore$ (horizontal) point of inflection

at $(-2,3)$ ✓

marks: finding $f''(x)$ ✓

solving $f''(x) = 0$ ✓

finding $(0,7)$ ✓ and giving reason

to show a point of inflection ✓

finding $(-2,3)$ ✓ and giving reason

to show a point of inflection ✓

END OF SECTION 1

WILLETTON SENIOR HIGH SCHOOL

YEAR 12 MATHEMATICS METHODS

TEST 1 2023

Section 2: Calculator Assumed

Student Name: SOLUTIONS

Circle your teacher's name.

Miss Ahern	Mr Galbraith
Mrs Gatland	Mrs Sun

Mark: ________ / 24

Time: 25 mins

For section 2 of this test:

Calculators allowed.

One A4 single side of notes allowed.

Show working.

Formula sheet as provided.

QUESTION SIX (3, 2 = 5 MARKS)

A particle moves along a straight line. Its displacement, x cm from a fixed-point O on the line at time t seconds, is given by $x = \frac{10t}{t^2+1}$.

a) Determine the velocity of the particle when the acceleration is zero.

[3]

Solve $\frac{d^2x}{dt^2} = 0 \Rightarrow t = 0$ or $t = \pm\sqrt{3}$. Reject $t = -\sqrt{3}$

$\therefore t = 0$ or $\sqrt{3}$ ✓

$v(0) = \frac{dx}{dt}\Big|_{t=0} = 10$ cm/s ✓

$v(\sqrt{3}) = \frac{dx}{dt}\Big|_{t=\sqrt{3}} = -\frac{5}{4}$ cm/s ✓

b) At what time(s) is the speed of the particle increasing?

[2]

Speed increasing when $v > 0$ and $a > 0$, or $v < 0$ and $a < 0$.

(From 'graphs' of v and a or using 'solve')

$1 < t < \sqrt{3}$ ✓

QUESTION SEVEN (2,2 = 4 MARKS)

The acceleration 'a' in cm/s^2 of a particle P moving in a straight line from a fixed point O at time t seconds is given by a = 12 – 6t. The initial velocity of P is equal to -9 cm/s when it is 4 cm to the right of O.

a) At what time(s) does the particle experience maximum velocity?

[2]

$V = 12t - 3t^2 - 9$ ✓

From 'graph' of v or

$\therefore$ maximum velocity at $t = 2$ seconds ✓

$\frac{dV}{dt} = 12 - 6t$

$\frac{dV}{dt} = 0$ when $t = 2$

for $t < 2$: $a > 0$

for $t > 2$: $a < 0$

b) When is the position of the particle to the left of the fixed point O?

[2]

$x = 6t^2 - t^3 - 9t + 4$ ✓

Particle left of O when $x < 0$

(Solving $6t^2 - t^3 - 9t + 4 < 0$)

i.e. $t > 4$ ✓

QUESTION EIGHT (3 MARKS)

The side, x, of a cube is measured with 3% error. Estimate, with the aid of the increments formula, the approximate percentage error in the surface area of the cube.

$A = 6x^2$, $\frac{\delta x}{x} = 3\%$ ✓

$\frac{\delta A}{\delta x} \approx \frac{dA}{dx} \Rightarrow \frac{\delta A}{A} \approx \frac{\frac{dA}{dx} \times \delta x}{A} = \frac{12x \times 0.03x}{6x^2}$ ✓

$= 0.06$

Approximate percentage error is 6% ✓

QUESTION NINE (2,1,3 = 6 MARKS)

A television company has 1000 subscribers who are paying \$5 per month. The company can get 100 more subscribers for each \$0.10 decrease in the monthly fee.

a) If x represents the monthly fee, show that the number of subscribers(N) in terms of x is $-1000x + 6000$. [2]

$N = 1000 + 100\left(\frac{5-x}{0.1}\right)$ ✓ $= 1000 + 1000(5-x)$ ✓

$= 1000 + 5000 - 1000x = -1000x + 6000$

b) Find the revenue(R) in terms of x. [1]

$R = x(-1000x + 6000) = -1000x^2 + 6000x$ ✓

c) Find the monthly fee that will yield the maximum revenue and state this maximum revenue. [3]

From graph of R, maximum turning point is $(3, 9000)$ ✓

A monthly fee of \$3 ✓ will produce a maximum revenue of \$9000 ✓

OR) $\frac{dR}{dx} = -2000x + 6000$, $\frac{dR}{dx} = 0$ when $x = 3$

$\frac{d^2R}{dx^2} = -2000 < 0 \Rightarrow$ Max. TP at $x = 3$

(or : for $x < 3$, $\frac{dR}{dx} > 0$ and for $x > 3$, $\frac{dR}{dx} < 0$)

QUESTION TEN (4, 2 = 6 MARKS)

A cubic function f has the rule $f(x) = ax^3 + bx^2 + \frac{150}{7}x$. The graph of this cubic function has a stationary point at $x = 1$ and a point of inflection at $x = 3$.

a) Find the values of a and b exactly. [4]

$f'(x) = 3ax^2 + 2bx + \frac{150}{7}$

$\therefore f'(1) = 3a + 2b + \frac{150}{7} = 0 \quad \ldots (1)$ ✓

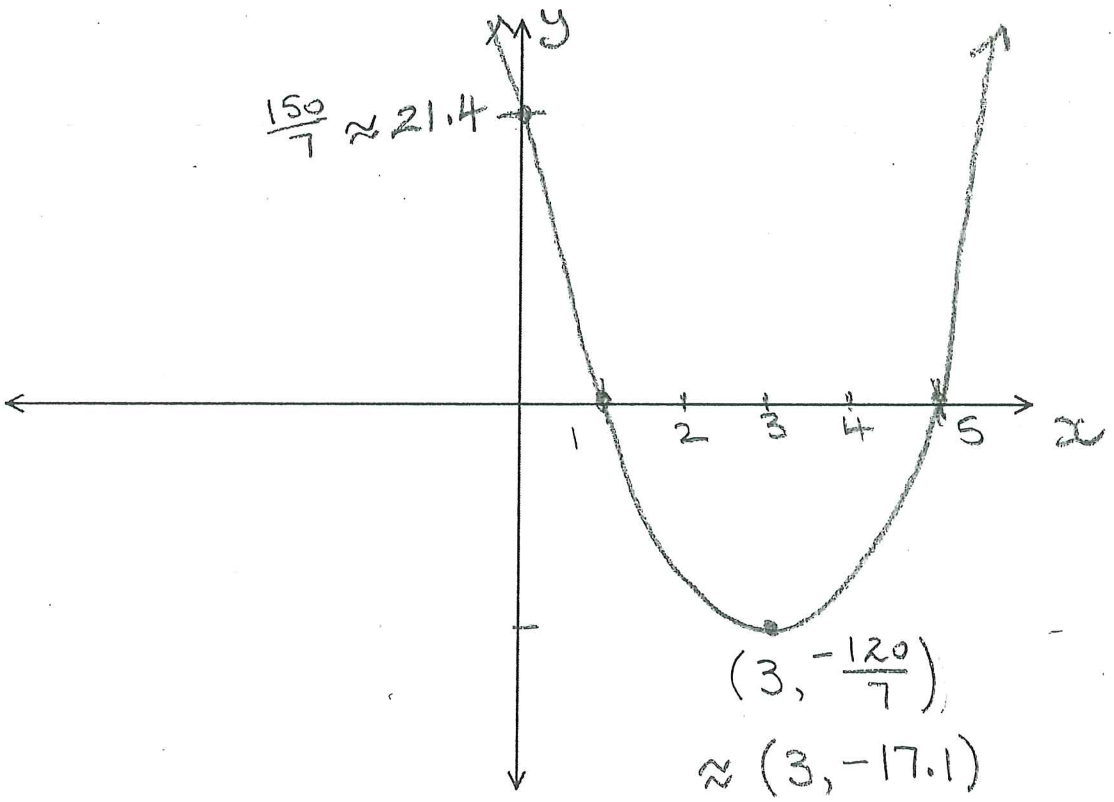
$f''(x) = 6ax + 2b$

$\therefore f''(3) = 18a + 2b = 0 \quad \ldots (2)$ ✓

Solving (1) and (2) simultaneously gives

$a = \frac{10}{7}$ ✓, $b = \frac{-90}{7}$ ✓

b) Sketch the graph of $y = f'(x)$ below. [2]



intercepts ✓

T.P ✓

END OF SECTION 2