WILLETTON SENIOR HIGH SCHOOL

MATHEMATICS METHODS – UNIT TWO

TEST FOUR 2021

SECTION ONE: Calculator Free

STUDENT NAME: **……**

TOTAL MARKS: **…….. / 37**

TIME ALLOWED: **30 mins**

CIRCLE YOUR TEACHER'S NAME:

Mrs Kalotay Ms Leow Mr Riemer Mrs Scoles

Ms Thompson Ms Tsen Mr Whiteley

- Formulae sheet supplied.
- No calculators allowed.
- If a question is worth more than 2 marks, sufficient working must be shown to justify your answer, in order to receive full marks.

1. Consider the sequence: 15 , _____ , _____ , 405 , …

   a. Given that these terms follow an arithmetic sequence, determine the value of T_2. [3]

   b. Given that these terms follow a geometric sequence, determine the value of T_3. [3]

2. Determine all the possible values of x given that 4, x, $(x^2 - 3)$ are consecutive terms of a geometric sequence. [4]

3. Determine the derivative of the following functions.
   a. $f(x) = 3x^4 - 5$ [1]

   b. $f(x) = 4\pi^2 - \sqrt{x}$ [2]

   c. $f(x) = \frac{5x^2-4}{x^3}$ [2]

4. Determine the derivative of $y = x^3 - 12x + 3$ using first principles. [4]

5. Find the equation of the tangent to the curve $y = x^2 + 4x - 1$ at the point where $x = 1$. [4]

6. Determine the anti-derivative for each of the following functions.
   a. $3x^4 - 6$ [1]

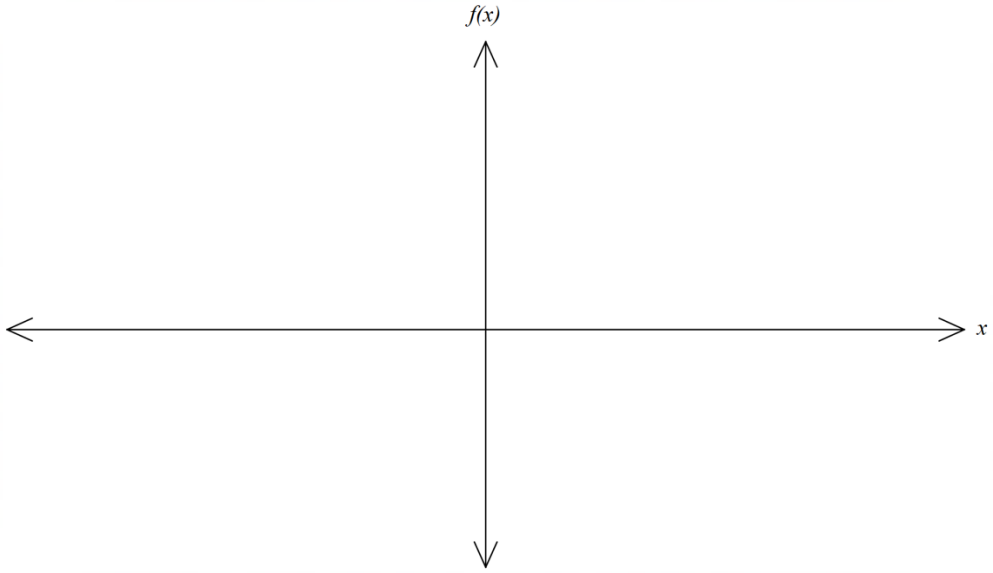
   b. $2x(3x - 1)^2$ [2]

7. A function is defined by $f(x) = x^4 - 8x^2 + 7$. After factorisation, $f(x) = (x^2 - 7)(x^2 - 1)$.

   a. Given that $\left(\sqrt{7}, 0\right)$ and $\left(-\sqrt{7}, 0\right)$ are the coordinates of the x-intercepts of $f(x)$, determine the coordinates of the remaining x-intercepts. [2]

   b. Use calculus techniques to determine the nature and location of any stationary points. [6]

   c. In the space below, draw a sketch of the function, including the location of any intercepts, stationary points and indicate the behaviour as $x \to \pm\infty$. [3]



WILLETTON SENIOR HIGH SCHOOL

MATHEMATICS METHODS – UNIT TWO

TEST FOUR 2021

SECTION TWO: Calculator Assumed

STUDENT NAME: **……**

TOTAL MARKS: **…….. / 25**

TIME ALLOWED: **20 mins**

CIRCLE YOUR TEACHER'S NAME:

Mrs Kalotay Ms Leow Mr Riemer Mrs Scoles

Ms Thompson Ms Tsen Mr Whiteley

- Formulae sheet supplied.
- One page of A4 notes allowed.
- Classpads and scientific calculators are allowed.
- If a question is worth more than 2 marks, sufficient working must be shown to justify your answer, in order to receive full marks.

8. A fatal disease is caused by a particular bacteria increasing in such a way that the number of bacteria present after t hours is given by $N(t) = 5t^3 + 4t^2 + 300$. Determine:

   a. The number of bacteria present initially. [1]

   b. The average rate of increase of bacteria in the first 10 hours. [2]

   c. The expression for the instantaneous rate of change of bacteria. [1]

   d. The rate of change of the number of bacteria at:

      i. t=150 minutes. [1]

      ii. t=1 day. [1]

9. A geometric sequence with $T_2 = 43.75$ has a sum to infinity of 400. Determine all possible values of T_1 for this sequence. [3]

10. A sequence is defined by $T_{n+1} = T_n - 3.55,\ T_1 = 835$.

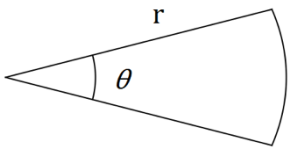
    a. Determine T_{150}. [1]

    b. Determine the sum of the first 35 terms, S_{35}. [1]

    c. Determine the value of n that will maximise S_{n} and state the corresponding value of S_{n}. Explain why this value of S_{n} is the maximum. [3]

11. The gradient of the tangent of a graph is given by $m = x^2 + x + k$, where k is a constant. Given that $f(0) = -2$ and $f(-1) = 0$, determine the equation of the graph. [3]

12. The perimeter of a sector of a circle, of radius r cm and angle θ radians, is 60 cm.



a. Show that $\theta = \frac{60}{r} - 2$. [2]

b. Hence, show that the area of the sector is given by $30r - r^2$. [2]

c. Use calculus to determine the maximum area of the sector and state the corresponding values of r and θ. [4]