GREENWOOD COLLEGE	**Greenwood College** **Year 12 Applications** **Test 2 2020** **Resource-Free**

Name....... Marking key .... (60)

No calculators nor notes allowed.
18 mark total.

Formula sheet allowed.
20 minute time limit

Question 1 [4 marks: 2, 2]

A number sequence is an arithmetic progression. The third term is 13 $\overset{a+2d}{}$ and the seventh term is 33.

$$a+6d$$

2 (a) Determine the common difference.

$$6d - 2d = 33 - 13$$
$$4d = 20$$
$$d = 5 \checkmark\checkmark$$

2 (b) Find the first term.

$$T_1 = T_3 - 2d$$
$$= 13 - 2 \times 5$$
$$= 3 \checkmark\checkmark$$

Question 2 [5 marks: 3, 2]

A geometric sequence has a common ratio of -3. The fifth term in the sequence is 405.

2 (a) Find the first term in the sequence

$$T_5 = a \times r^4 \checkmark$$
$$405 = a \times (-3)^4 \checkmark$$
$$405 = 81a$$
$$a = \frac{405}{81} = 5 \checkmark$$

2 (b) Write down the recurrence relation that describes this sequence.

$$T_n = -3T_n, \quad T_1 = 5$$
$$\checkmark \qquad\qquad \checkmark$$

9

Question 3

[6 marks: 2, 2, 2]

Below are four consecutive numbers of an arithmetic sequence. The middle two numbers are missing

$$...... , 24, \underset{d}{\underline{\;\;\;\;\;}} \overset{43}{,} \underset{d}{\underline{\;\;\;\;\;}} \overset{62}{,} \underset{d}{\underline{\;\;\;\;\;}} 81, ...$$

(a) What are the two missing numbers?

$$3d = 81 - 24$$
$$d = 19$$

(b) If the first term T_1 is 24, find the general rule for T_n.

$$T_n = a + (n-1)d$$
$$= 24 + (n-1) \times 19$$

(c) Now suppose the kth term T_k is 24. Find the general rule for T_n.

$$24 = a + (n-1)19$$

Question 4

[3 marks]

The number sequence $4000, $4120, $4240, $4360,... represents the current value of Mary's investment at the end of each year.

What type of interest has been used by the financial institution to calculate the interest? Explain your choice.

Simple

Constant difference of

$120

9

<table>
<tr><td>GREENWOOD G COLLEGE</td><td>Greenwood College
Year 12 Applications
Test 2 2020
Resource-Allowed</td></tr>
</table>

Name...

Formula sheet, one A4 page notes and calculators allowed.

~~3/~~ mark total.   35 minute time limit
42

Question 5

[7 marks: 2, 2, 2, 1]

Anthony wants to purchase a home and needs to borrow $400 000. A bank agrees to lend him the money at an interest rate of 7.2% per annum with interested paid on a monthly basis. Anthony can afford to make monthly repayments of $4200.

2 (a) Write a recursive rule that will enable you to determine the amount Anthony will owe after n months.

$$a_{n+1} = a_n + \frac{0.072 a_n}{12} - 4200 \qquad a_0 = 400\,000$$

$$= 1.006\, a_n - 4200$$

2 (b) Calculate the total interest that Anthony will pay on this loan.

$$141 \times 4200 + (4200 - 1511.05)$$
$$- 400000$$

$$\$194\,888.95 \checkmark$$

2 (c) After 50 repayments have been made, how much will Anthony still owe the bank, to the nearest dollar?

$$a_{50} = \$295405 \checkmark\checkmark$$

1 (d) How many months will it take for Anthony to repay this loan? $142 \checkmark$

$$a_{140} = \$6831.92$$
$$a_{141} = \$2672.91$$
$$a_{142} = -\$1511.05$$

7

Question 6 [11 marks: 2, 3, 2, 2, 2]

Helen borrows $15000 at a flat rate of 6% per annum. The loan is to be repaid over 3 years.

2 (a) Find the total interest paid on this loan.

$$SI = P \times R \times T$$
$$= 15000 \times 0.06 \times 3$$
$$= \$2700 \checkmark\checkmark$$

3 (b) Calculate the monthly repayment.

$$MR = \frac{15000 + 2700}{36} \checkmark$$
$$= \$491.67 \checkmark$$

2 (c) Write a explicit formula to show the how much Helen owes after the nth month.

$$T_n = (15000 + 2700) - 491.67n$$
$$17700$$

(d) After making 10 monthly repayments (using part b), Helen is able to increase her repayments to $600 per month. We can assume no penalty for early repayment. Still owes $12783.30

$$(15000 + 2700) - 491.67 \times 10$$

2 (i) How long will it take her to pay off the loan?

Solve $12783.30 - 600n = 0$

$$n = 22 \rightarrow 10 + 22$$
$$32 \text{ month} \checkmark\checkmark$$

2 (ii) What is the amount of the her last repayment?

$$\$12783.30 - 600 \times 21$$
$$= \$183.30 \checkmark\checkmark$$

11

Question 7 [6 marks: 3, 3]

Mary plans to travel overseas in October 2023 so she decides to save for her trip by making monthly deposits of $500 into an investment account. This account accrues interest at the rate of 5% p.a. compounded monthly. Mary opens this account on the 1st November 2020 with a deposit of $500 with all future deposits being made on the first day of the month.

3 (a) Write a recursive formula to show the balance at the beginning of each month.

$$a_{n+1} = a_n + \frac{0.05}{12}a_n + 500 \;,\; a_0 = 500$$

3 (b) How much will be in this account on the 30th September 2023?

$$a_{35} = 19376.67$$
$$a_{36} = 19957.40$$
$$a_{37} = 20540.56$$

$$1 \text{ Nov } 2020 \rightarrow 30 \text{ Sep } 2023$$
$$n = 36 - 1$$
$$= 35$$

$$Bal = a_{35} - 500$$
$$= 18876.78 \;\checkmark$$

6

Question 8

[13 marks: 1, 5, 1, 2, 1, 1, 2]

Anne borrowed $30000 for an overseas holiday. She agreed to repay the loan by making monthly repayments of $600.

Month	Balance at start of the month ($)	Interest earned during the month ($)	Monthly repayments ($)	Balance at the end of the month ($)
1	30 000	315	600	29 715
2	29 715	312.01	600	
3	A	B	600	C
"	"	"	"	"
"	"	"	"	"
36	18 020.35	D	600	E

(a) What is the monthly interest rate on this loan?

$$\frac{315}{30000} \times 100\% = 1.05\% \checkmark$$

(b) Determine the values of A to E.

A = 29427.00 ✓

B = 308.98 ✓

C = 29135.99 ✓

D = 189.21 ✓

E = 17609.56 ✓

(c) How much is still owing after 12 months?

$26 375.42 ✓

(d) Write a recursive rule which will determine the amount owing at the end of each month?

$$a_{n+1} = a_n + 0.0105 a_n - 600, \quad a_0 = 30000 \checkmark$$

(e) How long will it take to repay the loan?

$$a_{71} = \$161.32$$

$$a_{72} = -\$436.98$$

72 months ✓

10

1 (f)   What is the amount of the final repayment made by Anne?

$$600 - 436.98 = \$163.02$$

2 (g)   What was the cost of this loan?

$$\left(71 \times 600 + 600 - 436.98\right) - 30000$$
$$= \$12763.02$$

Question 9   **[5 marks: 3, 2]**

3 (a)   A sequence defined by the recurrence relation $a_{n+1} = ka_n + 6$ where $a_1 = -4$. If the terms of this sequence approach a limiting value of 15, find the value of k.

$$15 = 15k + 6$$
$$15k = 9$$
$$k = \frac{9}{15}$$
$$= 0.6$$

2 (b)   Consider the sequence defined by the recurrence relation $b_{n+1} = 0.7b_n + 3,\ where\ b_1 = 1$. Find the value of this limit.

$$x = 0.7x + 3$$
$$0.3x = 3$$
$$x = 10$$

8