LAKELAND SHS MATHEMATICS
MATHEMATICS METHODS UNIT 2
TEST 2 – PART A

WEEK 5, T3 - 2015	
Score (x / 22)	%

NAME: MARKING KEY

Time allowed: 20 min

CALCULATOR FREE AND NOTE FREE SECTION

1. [1, 2 = 3 marks]

a) List the first 6 terms of the sequence defined by the difference equation, $T_{n+1} = \frac{1}{2}T_n$, $T_1 = 64$.

64, 32, 16, 8, 4, 2, ...

b) State the general equation for this sequence.

$T_n = 64(0.5)^{n-1}$

2. The 5th term of an arithmetic sequence is 41 and the 11th term is 83. [2, 1, 2 = 5 marks]

a) Find the arithmetic difference d of the sequence

$d = \frac{83-41}{11-5}$ ✓

$= \frac{42}{6}$

$= 7$ ✓

b) Find the first term a of the sequence

$T_5 = 41$

$a + 4 \times 7 = 41$

$a = 13$ ✓

c) Find the sum of the first twelve terms of the sequence

$S_{12} = \frac{12}{2}(13 \times 2 + 11 \times 7)$ ✓

$= 6(26 + 77)$

$= 6 \times 103$

$= 618$ ✓

3. [3, 4 = 7 marks]

A sequence is given by $\mathbf{T}_{n+2} = 2\mathbf{T}_{n+1} - \mathbf{T}_n$ where $\mathbf{T}_1 = 5$ and $\mathbf{T}_2 = -1$

(a) Write down the values of T_3 , T_4 and T_5

$T_1 = 5$

$T_2 = -1$

$T_3 = 2 \times -1 - 5$
$= -7$ ✓

$T_4 = 2 \times -7 - (-1)$ ✓
$= -13$

$T_5 = 2 \times -13 - (-7)$ ✓
$= -19$

(b) The above sequence can be rewritten in the form $\mathbf{T}_n = \boldsymbol{pn} + \boldsymbol{k}$ where p and k are constants and n is the term number. What are the values of **p** and **k?**

$5, -1, -7, -13, -19, \ldots$ is an AP.

$a = 5 \quad d = -6$ ✓

$T_n = 5 - 6(n-1)$ ✓
$= -6n + 11$

$p = -6$ ✓

$k = 11$ ✓

4. In the following sequence 8, –8, –24, ... [1, 2, 2, 2 = 7 marks]

a) state the type of sequence

Arithmetic

b) define the sequence with a recursive definition

$T_{n+1} = T_n - 16 , \ T_1 = 8$

c) find T_{11}

$T_{11} = 8 - 16 \times 10$
$= -152$

d) find S_6

$S_6 = 3 \times (2 \times 8 - 16 \times 5)$
$= 3 \times (16 - 80)$
$= 3 \times -64$
$= -192$

Question 10 [2,2,2 = 6 marks]

A stalagmite is an icicle being formed by calcite dropping from the roof of a cave to form a cone on the floor. A particular cave has a height of 8 meters. A stalagmite in this cave started forming at the beginning of 2003. In that year it grew 30 cm. If the stalagmite grows at the rate of 95% of the previous year's growth each year from that year on, then find:

$T_1 = 30$ 2003 $r = 0.95$

a) the increase in height during 2008

$T_n = ar^{n-1}$ T_6 is 2008.

$T_6 = 30(0.95)^5$

$= 23.2$ cm

b) the height of the stalagmite at the end of 2020

End of 2020 is 2020 − 2003 + 1.

$= n = 18$

$S_{18} = \dfrac{a(1-r^n)}{1-r} = \dfrac{30(1-0.95^{18})}{0.05} = 362$ cm

c) whether the stalagmite will ever reach the roof of the cave (state and prove your answer).

$S_\infty = \dfrac{a}{1-r}$

$= \dfrac{30}{0.05}$

$= 600$ cm

∴ The stalagmite will never reach the 8m high cave roof.

Question 11 [2, 2, 1, 2 = 7 marks]

Bailey takes out a loan of $20 000. Interest of 18% p.a. calculated monthly (i.e. 1.5% per month) of the outstanding balance is added at the end of each month, then Bailey makes his monthly repayment of $450 .

(a) State a recursive formula which shows Bailey's end of month balance .

$R = 18 \div 12 = 1.5\%$ $T_{n+1} = \cancel{0.985}\ 1.015 \times T_n - 450$

a_{n+1} Type a_0

Using that recursive formula and Sequence mode on your CAS, find:

(b) the amount he still owes immediately after the $450 repayment at the end of month 10

$18394.59

(c) How long he takes to pay off the loan ?

74 MONTHS

d) If he wished to take ten years to pay off the loan ,what would be the size of his regular monthly repayment?

120 MONTHS

THIS WILL TAKE A LOT OF TRIAL AN ADJUSTMENT IN: SEQUENCE MODE ON CAS.

LEAVE OUT!

LAKELAND SHS MATHEMATICS

MATHEMATICS METHODS UNIT 2

TEST 2 – PART B

WEEK 5, T3 - 2015	
Score (x / 30)	%

NAME: MARKING KEY

Time allowed: 30 min

CALCULATOR AND NOTES ASSUMED

5. [1, 1, 1, 2 = 5 marks]

In the general term of the sequence defined by $T_n = 30(0.85)^{n-1}$ find:

a) T_4

$T_4 = 30(0.85)^3$

$= 18.4$ (1 d.p.)

b) The common ratio

0.85

c) S_5

$$S_5 = \frac{30(1-0.85^5)}{0.15}$$

$= 111.3$ (1 d.p.)

d) S_∞

$$S_\infty = \frac{30}{0.15}$$

$= 200$

6. [1, 2, 3, 2 =8 marks]

Due to a sterilisation experiment on a farm, the number of rabbits breeding will be 400 in the first year of the experiment, 360 in the second year, 324 in the third year and so on.

Find the:

a) the percentage by which the number of rabbits breeding is decreasing each year

$r = \frac{360}{400} = 0.9$ 10% decrease

b) recursive definition for the number of rabbits

$T_{n+1} = T_n \times 0.9, \quad T_1 = 400$

c) number of rabbits breeding in the sixth year of the experiment

$T_6 = 400(0.9)^5$

$= 236$ (nearest whole)

d) number of years into the experiment before the number of rabbits breeding drops below 30.

$30 = 400(0.9)^{n-1}$

$n = 25.5848$ SOLVED WITH CAS

26 years.

7. [2, 2, 2, 2, 2 = 10 marks]

Harry rolls a soccer ball on a flat, constant surface of 60 metres in length. In the first second the soccer ball covers 5 metres. In next second, due to friction, the soccer ball only covers 4.50 metres or 90% of the distance covered in the previous second, subsequently this pattern continues.

a) State a recursive definition that describes the sequence of distance travelled by the soccer ball in each second.

$T_{n+1} = T_n \times 0.9, \quad T_1 = 5$

b) Write a general formula for the distance travelled in the n[th] second.

$T_n = 5(0.9)^{n-1}$

c) Calculate (to the nearest centimetre) the distance travelled **in** the 15th second.

$T_{15} = 5(0.9)^{14}$

$= 1.14\,m$

114 cm

d) During what second does the soccer ball travel over 20 metres from the start?

$S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{5(1-0.9^n)}{0.1}$

$50(1-0.9^n) = 20$

Solve with CAS

$n = 4.848$

In the 5th second.

e) Determine whether the soccer ball will reach the end of the 60 metre track (prove your answer).

$S_\infty = \frac{5}{0.1}$

$= 50$ metres

No it will not reach 60m.

8. [1, 3, 3 = 7 marks]

Michelle agrees to pay an interest free loan of \$4380 in a number of instalments. Each instalment is \$30 more than the previous instalment and the first instalment is \$200.

a) Write down the first three instalments.

200, 230, 260 ✓

b) Use the formula for the sum of an AP to show that $4380 = 15n^2 + 185n$ where n is the number of instalments.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$a = 200,\ d = 30,\ S_n = 4380$

$4380 = \frac{n}{2}[2 \times 200 + 30(n-1)]$ ✓

$= \frac{n}{2}(400 + 30n - 30)$

$= \frac{n}{2}(370 + 30n)$ ✓

$= 185n + 15n^2$ ✓

c) Use a suitable method to determine the number of instalments required to pay off the loan completely. (Write down which method you used)

$15n^2 + 185n = 4380$ ✓

SOLVE BY CAS

$x = 12$ and $x = -24.3$ ✓

12 installments. ✓