

# Mathematics Methods Unit 4

## Test 6

### Logarithmic Functions, CRV, Normal Distributions and Sampling



Name: Solutions

Time allowed: 20 minutes

Marks: /20

#### Section One: Calculator Free

Show working out where appropriate.

##### Question 1

[2, 2 = 4 marks]

Evaluate each of the following:

a)  $\log_6 27 + \log_6 40 - \log_6 5 = \log_6 \left( \frac{27 \times 40}{5} \right)$   
 $= \log_6 (3^3 \times 2^3)$  working  
 $= 3 \log_6 (6)$   
 $= 3$  ✓

b)  $\frac{\log_2 16}{\log_2 32} = \frac{\log_2 (2^4)}{\log_2 (2^5)}$  ✓  
 $= \frac{4}{5}$  ✓

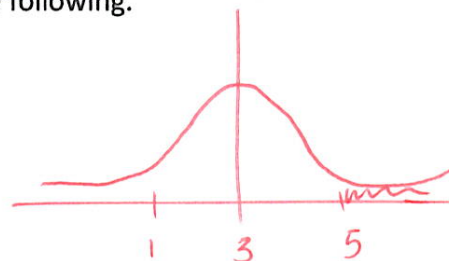
##### Question 3

[1, 2, 3 = 6 marks]

Use the 68%, 95%, 99.7% rule to give approximate answers to the following.

a) If  $\mu = 3$  and  $\sigma = 2$ , find

i)  $P(X > 5) = 0.16$  ✓



ii)  $P(X < 1 | X < 5) = \frac{0.16}{0.84}$  ✓  
 $= \frac{4}{21}$  ✓

b) If  $X = 3$  has a z score of -2 and  $X = 8$  has a z score of 2, find  $\mu$  and  $\sigma$

$$z = \frac{X - \mu}{\sigma}$$
$$-2 = \frac{3 - \mu}{\sigma}$$
$$-2\sigma = 3 - \mu$$
$$-2\sigma + \mu = 3$$

$$2 = \frac{8 - \mu}{\sigma}$$
$$2\sigma = 8 - \mu$$
$$2\sigma + \mu = 8$$

$$\begin{array}{r} -2\sigma + \mu = 3 \\ 2\sigma + \mu = 8 \\ \hline 2\mu = 11 \\ \mu = 5.5 \end{array}$$
 ✓

$$2\sigma = 8 - 5.5$$
$$2\sigma = 2.5$$
$$\sigma = 1.25$$
 ✓

✓ working

**Question 3****[2 marks]**

Sarah was employed to complete a study of a local wren's population in the Binningup area prior to the building of the new Desalination Plant. During her first visit to the area she captured and tagged 200 local wrens. On her second visit she captured 75 wrens of which 20 had been tagged previously. **Estimate the number of wrens in the area.**

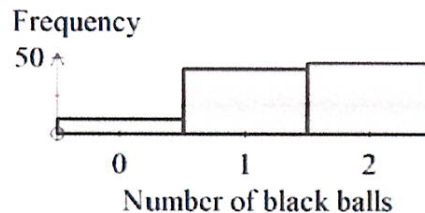
$$\frac{x}{200} = \frac{75}{20} \quad \checkmark$$

$$x = 750 \text{ wrens} \quad \checkmark$$

**Question 4****[2, 3, 3, = 8 marks]**

A barrel contains a large number of black and white balls, such that the ratio of black to white balls is 8:2.

The graph below shows the results of a simulation of an experiment in which two balls are randomly drawn from the barrel, the number of black balls noted and then the balls are replaced, for a total of 100 times.



- a) Comment on the distribution shown above.

Distribution is binomial  $\checkmark$  & because  $p$  is large (0.8) the distribution is negatively skewed.  $\checkmark$

- b) Determine the probability that when two balls are randomly drawn from the barrel, both balls are the same colour.

$X$  is number of black balls:

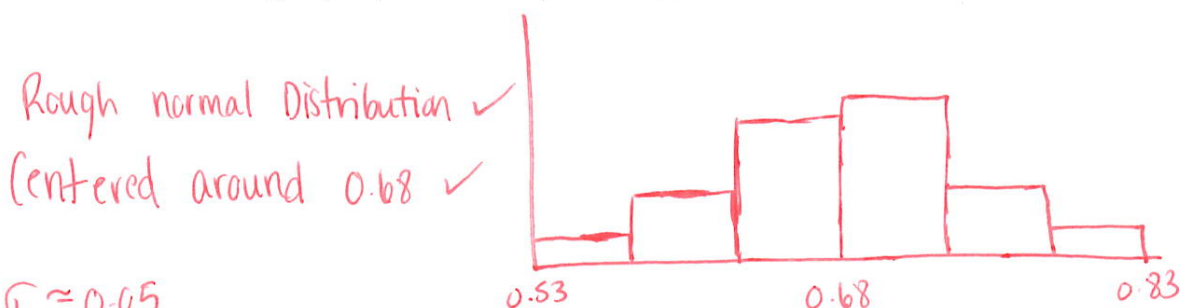
$$P(X=0) = 0.2^2 = 0.04 \quad \checkmark$$

$$P(X=2) = 0.8^2 = 0.64 \quad \checkmark$$

$$P(\text{Same colour}) = 0.04 + 0.64 = 0.68 \quad \checkmark$$

The same simulation is repeated another 100 times, and the proportion of draws in which both balls are the same colour is noted for each simulation.

- c) Sketch a frequency histogram to illustrate the likely distribution of these proportions, noting any key features of your sketch.

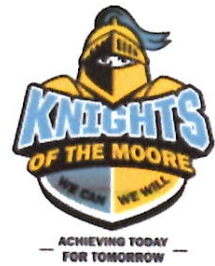


Most proportions fall between 0.53 & 0.83  $\checkmark$   
(or draw bars representing  $\pm 3\sigma$ )

## Mathematics Methods Unit 4

### Test 6

#### Logarithmic Functions, CRV, Normal Distributions and Sampling



Name: Solutions

Time allowed: 35 minutes

Marks: /30

#### Section Two: Calculator Assumed

Show working out where appropriate.

1 side of A4 notes allowed

Calculator allowed

#### Question 5

[1, 3, 3 = 7 marks]

In a random sample of 275 passenger cars in a city, it was found that 92 had a manual transmission.

- a) Estimate the percentage of all passenger cars in the city with a manual transmission.

$$\frac{92}{275} = 0.3345 \approx 33.5\% \quad \checkmark$$

- b) Calculate a 80% confidence interval for the estimate in (a), and briefly explain the meaning of the 80% level.

$$2 \text{ score for } 80\% \text{ CI} = 1.282 \quad \checkmark$$

$$0.335 \pm 1.282 \sqrt{\frac{0.335 \times (1 - 0.335)}{275}} \quad \checkmark$$

$$(0.298, 0.371) \quad \checkmark$$

or use E-activity

- c) Assuming the estimate in (a) is correct, what size sample of passenger cars, to the nearest 10, would have to be taken in order for the width of a 95% confidence interval to be no more than 0.1?

$$Z_{95} = 1.96$$

$$p = \hat{p} = 0.3345$$

$$e = 0.1 \div 2 \\ = 0.05 \quad \checkmark$$

E-activity

$$n = ?$$

$$z = 1.96$$

$$p = 92/275$$

$$e = 0.05$$

$$n \approx 340 \text{ cars} \quad \checkmark$$

or other working



### Question 6

[1, 2, 4 = 7 marks]

- a) Briefly define a random sample

A random sample is one in which every member of the population has an equal chance of being selected. ✓

- b) A random sample of Year 12 students was undertaken from which the 90% confidence interval for the proportion of students who planned to take a gap year after finishing Year 12 was determined to be (0.034, 0.252).

- i) Explain why the reliability of this interval estimate is not immediately evident.

Sample size is not stated. ✓  
Unless size is large, distribution is unlikely to be normal leading to an unreliable interval. ✓

- ii) Determine the corresponding 95% confidence interval based on this survey.

$$p = \frac{0.034 + 0.252}{2} = 0.143 \quad \checkmark$$

$$E = 0.252 - 0.143 = 0.109$$

$$E_{\text{new}} = \frac{1.96}{1.645} \times E = \frac{1.96}{1.645} \times 0.109 = 0.130 \quad \checkmark$$

$$0.143 \pm 0.130 = (0.013, 0.273) \quad \checkmark$$

### Question 7

[1, 1, 4, 1, 2 = 8 marks]

A random sample of three items is selected from a batch of 15 items which contains five defective.

- a) What is  $p$ , the proportion of defectives in the batch?

$$p = \frac{5}{15} = \frac{1}{3} \quad \checkmark$$

- b) What are the possible values of the sample proportion  $\hat{p}$  of defective in the sample?

$$0, \frac{1}{3}, \frac{2}{3}, 1 \quad \checkmark$$

- c) Construct a probability distribution table which summarises the sampling distribution of the sample proportion of defectives in the sample.

Number of defective items in a sample	0	1	2	3
$P$	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$P(P=p)$	$\frac{\binom{5}{0}\binom{10}{3}}{\binom{15}{3}} = \frac{120}{455}$	$\frac{\binom{5}{1}\binom{10}{2}}{\binom{15}{3}} = \frac{225}{455}$	$\frac{\binom{5}{2}\binom{10}{1}}{\binom{15}{3}} = \frac{100}{455}$	$\frac{\binom{5}{3}\binom{10}{0}}{\binom{15}{3}} = \frac{16}{455}$

✓ ✓ ✓ ✓

Q7 cont next page

- d) Use your sampling distribution to determine the probability that the proportion of defectives is less than 0.4.

$$\frac{120 + 225}{455} = \frac{345}{455} \checkmark$$

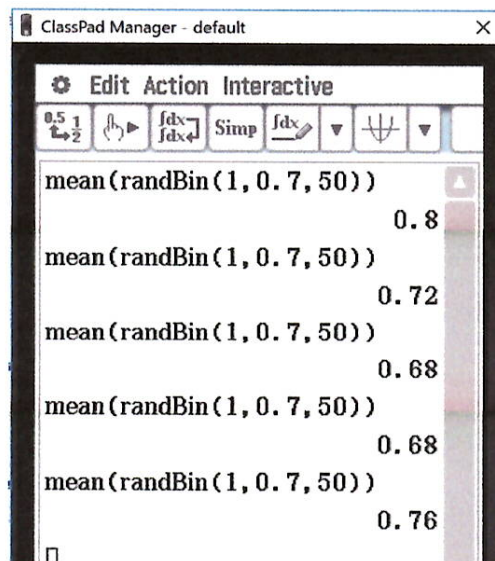
- e) Find  $P(\hat{P} < 0.4 | \hat{P} > 0)$

$$\frac{225}{455} \div \frac{335}{455} = \frac{225}{335} \checkmark$$

### Question 8

[4, 1, 4 = 9 marks]

The probability of occurrence of a given property is  $p = 0.7$ . The simulation of five samples, each of size 50, was conducted, and the mean of each sample is given below.



- a) Calculate a 90% confidence interval for the first sample.

$$\begin{aligned} \text{Lower} &: 0.706953 \checkmark \\ \text{Upper} &: 0.893047 \checkmark \\ \hat{p} &: 0.8 \checkmark \\ n &: 50 \checkmark \end{aligned}$$

- b) Hence comment on the position of  $p$  in this confidence interval.

$p$  is just within this confidence interval  $\checkmark$

- c) Repeat part (a) and (b) for the fourth simulation.

$$\begin{aligned} \text{Lower} &: 0.5714895 \checkmark \\ \text{Upper} &: 0.7885105 \checkmark \\ \hat{p} &: 0.68 \checkmark \\ n &: 50 \checkmark \end{aligned}$$

$p$  is well within this confidence interval  $\checkmark$