Year 11 Mathematics Specialist

Test 4 - Trigonometry

MELVILLE
SENIOR HIGH SCHOOL

Student Name : _____

Part One – Resource Free

Part One contains 6 questions worth 39 marks

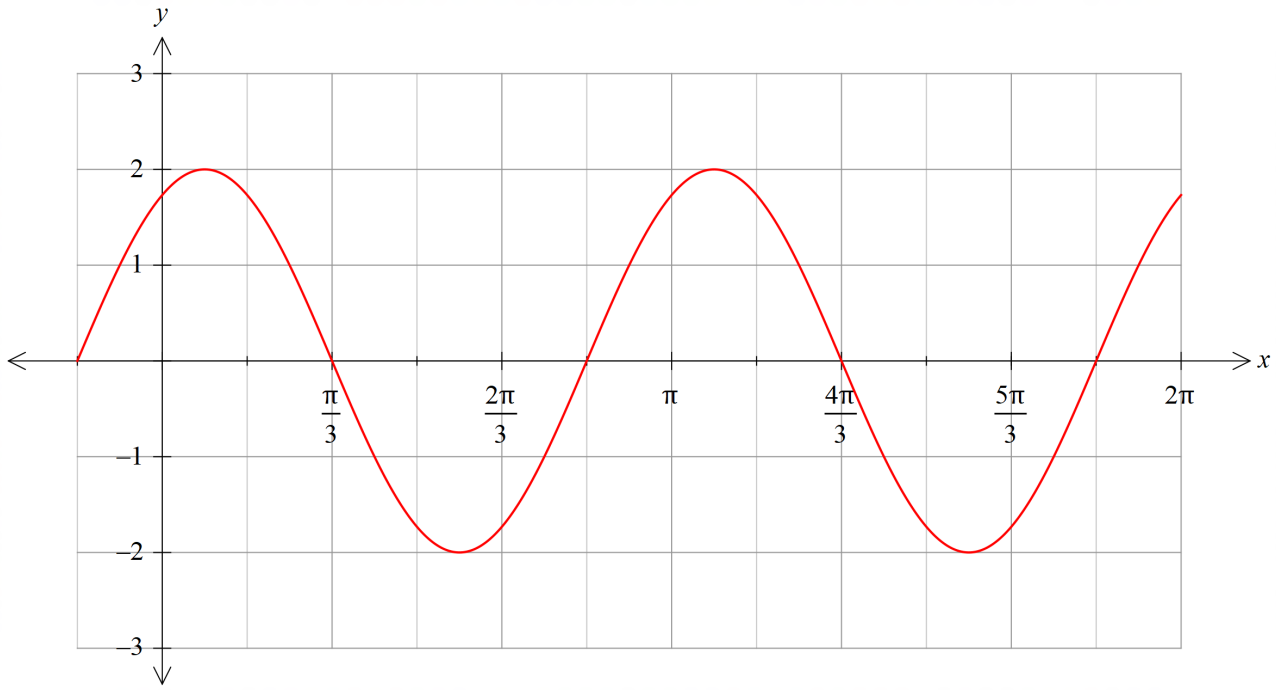
Time Allowed : 40 minutes

INSTRUCTIONS TO STUDENTS:
You are required to attempt ALL questions,
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.

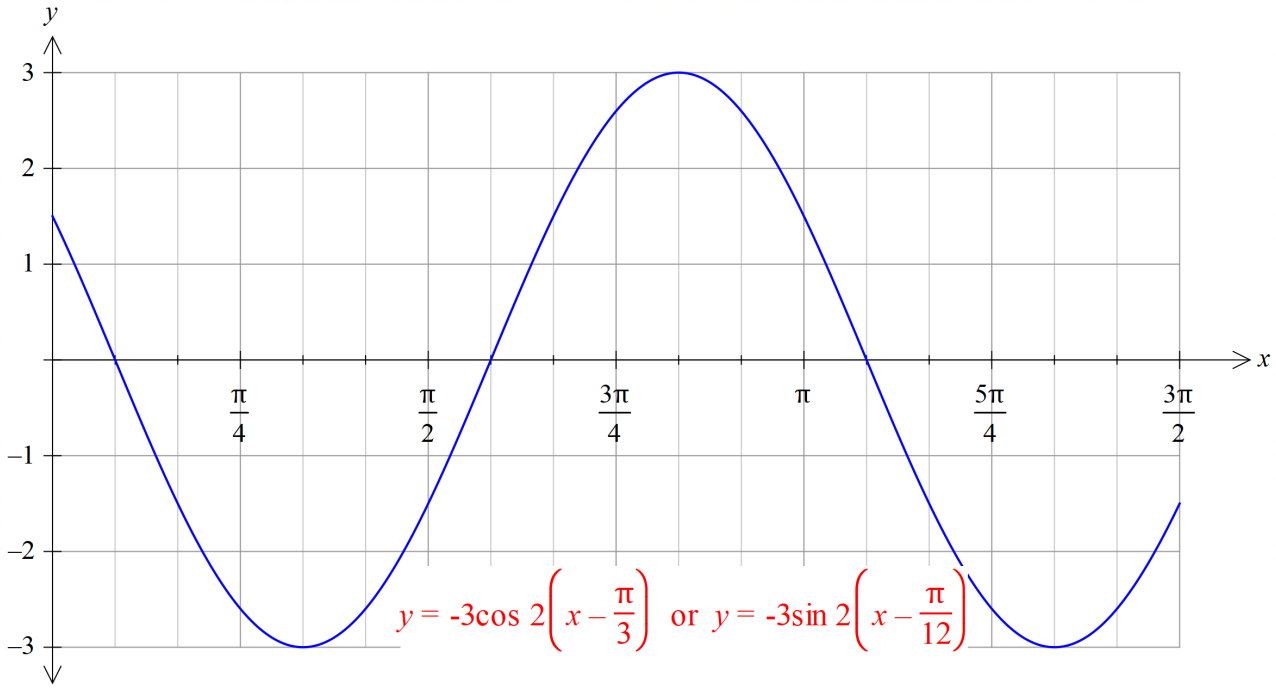
Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be answered for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.

Question 1 **3, 3 - 6 marks**

a) Sketch the function $y = 2\sin\left(2x + \frac{\pi}{3}\right)$ on the set of axes below



b) Determine the equation of this graph.



$y = -3\cos 2\left(x - \frac{\pi}{3}\right)$ or $y = -3\sin 2\left(x - \frac{\pi}{12}\right)$

Question 2 **3, 4 - 7 marks**

Solve the following equations:

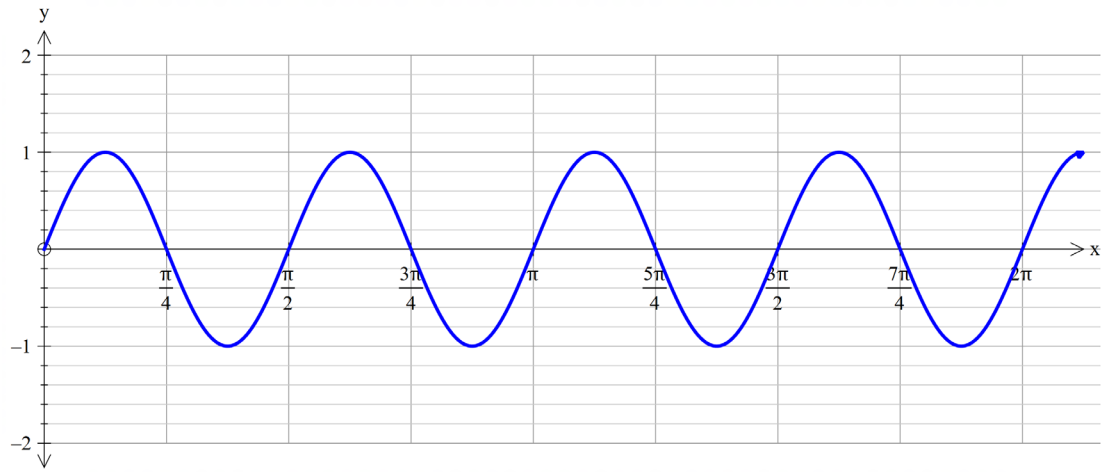
a) $\sin 4x = 0$ for $0 \le x \le 2\pi$.

$\sin 4x = 0 \qquad 0 \le 4x \le 8\pi$

$4x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi, 8\pi$

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi$

Identifies correct RA in first 0 to 2pi
Extend to 8pi
Divides by 4 to have 8 answers



b) $\cos 2x - \cos x = 0$ for $0 \le x \le \pi$.

$2\cos^2 x - 1 - \cos x = 0$ (replaces $\cos 2x$ with $2\cos^2 x - 1$)

$(2\cos x + 1)(\cos x - 1) = 0$ (correctly factorises quadratic)

$\cos x = -\frac{1}{2}$ or $\cos x = 1$

$x = \frac{2\pi}{3} \qquad x = 0$ (identifies 2 correct solutions)

Question 3 **3, 2 - 5 marks**

a) Show $\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$.

$$RHS = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$

$= \sqrt{2}\left[\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right]$ (correctly expands)

$= \sqrt{2}[\sin x . \frac{1}{\sqrt{2}} + \cos x . \frac{1}{\sqrt{2}}]$ (correctly substitutes exact values)

$= \sin x + \cos x$ (correctly simplifies)

b) Hence solve $\sin x + \cos x = \sqrt{2}$ for $0 \le x \le \pi$.

$$\sqrt{2}\sin\left(x + \frac{\pi}{4}\right) = \sqrt{2}$$

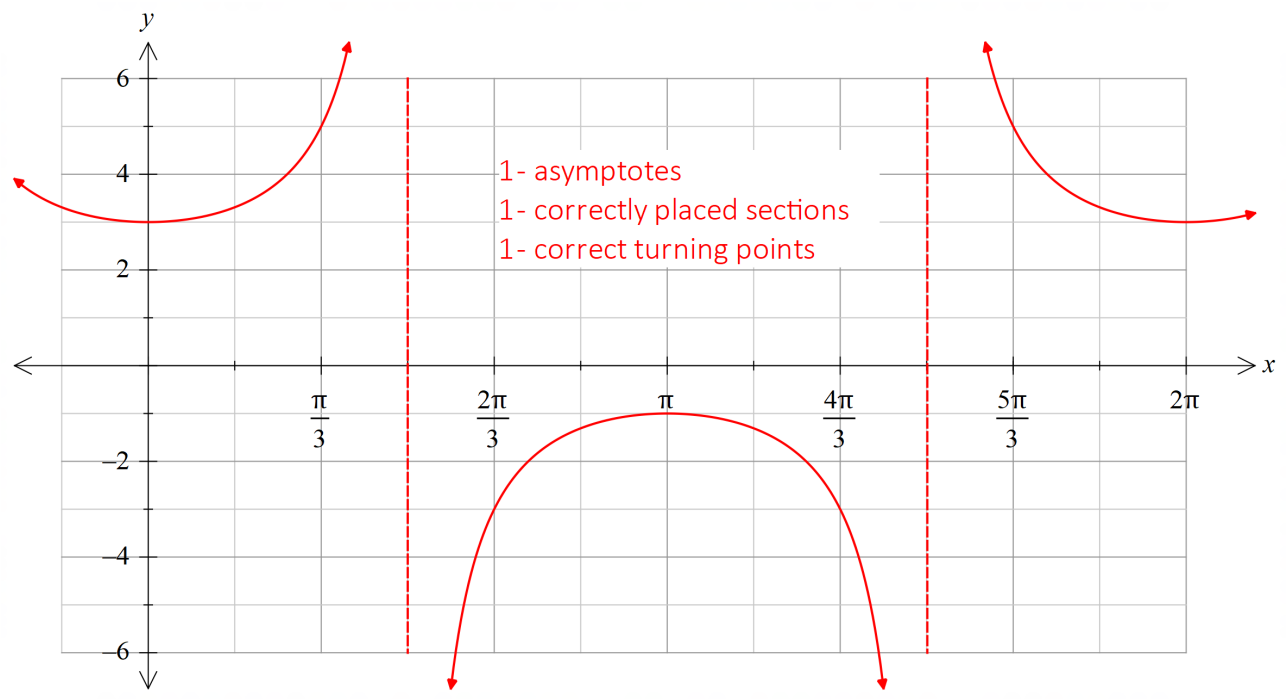
$$\sin\left(x + \frac{\pi}{4}\right) = 1$$

$$x + \frac{\pi}{4} = \frac{\pi}{2}$$

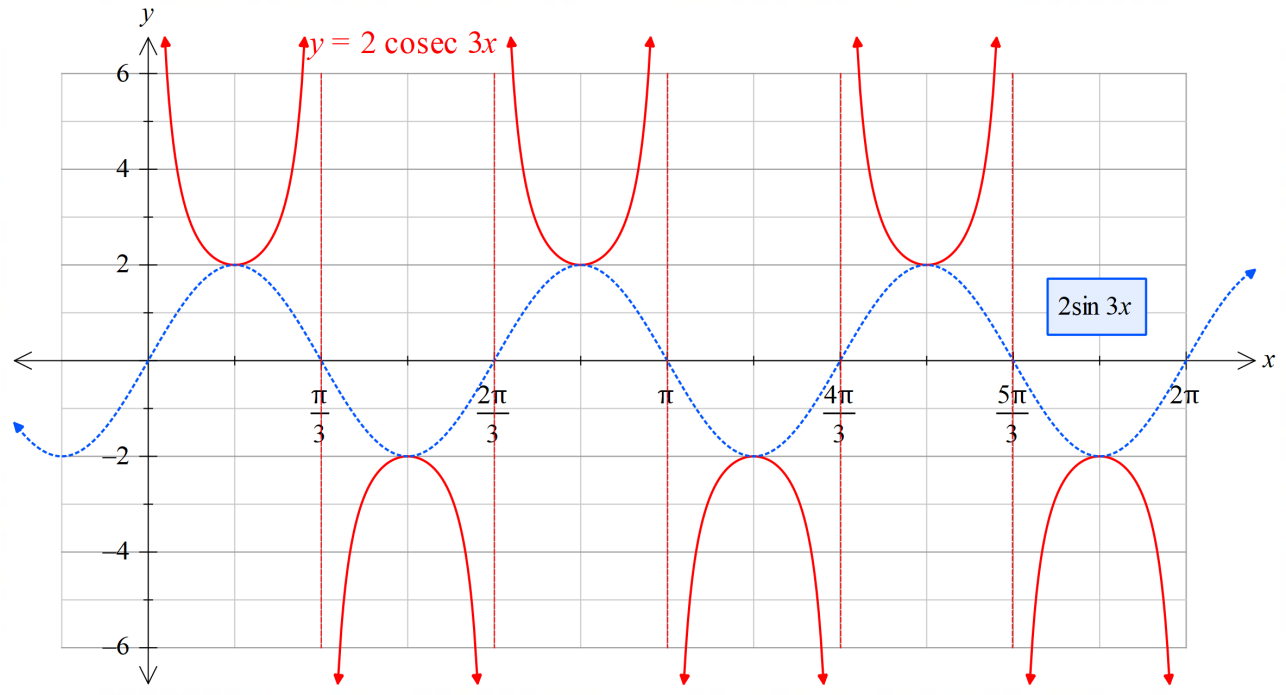
$$x = \frac{\pi}{4}$$

Question 4 **3,3 - 6 marks**

a) On the axes provided, sketch the graph of $y = 2\sec x + 1$.



b) Determine the equation of the graph below.



Question 5 **4 marks**

Simplify $\dfrac{\cos\left(\dfrac{\pi}{2}-\theta\right)}{\operatorname{cosec}(\pi+\theta)}$.

$$\left(\cos\frac{\pi}{2}\cos\theta+\sin\frac{\pi}{2}\sin\theta\right)\div\left(\frac{1}{(\sin\pi\cos\theta+\cos\pi\sin\theta)}\right)$$
$$=(0\cos\theta+1\sin\theta)\times(0\sin\theta+(-1)\sin\theta)$$
$$=\sin\theta\times(-\sin\theta)$$
$$=-\sin^2\theta$$

Correct expansions
Recognises reciprocal of cosec as sin and multiplies
Substitutes in exact values
Simplifies

Question 6 **3 marks**

a) Prove that $\sec\theta-\tan\theta\sin\theta=\dfrac{1}{\sec\theta}$.

$$LHS=\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}.\sin\theta$$
$$=\frac{1-\sin^2\theta}{\cos\theta}$$
$$=\frac{\cos^2\theta}{\cos\theta}$$
$$=\cos\theta$$
$$=\frac{1}{\sec\theta}$$

Replaces with sin & cos
Combines to make single fraction
Simplifies and recognises reciprocal

Question 7 **4, 3, 2 - 9 marks**

a) Find the exact value of tan 75°.

$$\tan(30°+45°)=\frac{\tan30°+\tan45°}{1-\tan30°\tan45°}$$

$$=\frac{\frac{1}{\sqrt{3}}+1}{1-\left(\frac{1}{\sqrt{3}}\right)(1)}$$ correctly subs

$$=\frac{1+\sqrt{3}}{\sqrt{3}}\div\frac{\sqrt{3}-1}{\sqrt{3}}$$ combines to single fractions

$$=\frac{1+\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$ rationalises result

$$=\frac{3+2\sqrt{3}+1}{3-1}$$

$$=\frac{4+2\sqrt{3}}{2}$$ simplifies

$$=2+\sqrt{3}$$

b) If $\cos x = \frac{12}{13}$ and $0 \le x \le \frac{\pi}{2}$, find the exact value of $\sin\left(\frac{x}{2}\right)$.

$$\cos x = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

$$\frac{12}{13} = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

$$2\sin^2\left(\frac{x}{2}\right) = \frac{1}{13}$$

$$\sin^2\left(\frac{x}{2}\right) = \frac{1}{26}$$

$$\sin\left(\frac{x}{2}\right) = \frac{1}{\sqrt{26}}$$

$$= \frac{\sqrt{26}}{26}$$

c) Give the exact value of $\left(\sin 22.5° + \cos 22.5°\right)^2$

$$\sin^2 22.5° + \cos^2 22.5° + 2\sin 22.5° \cos 22.5°$$

$$= 1 + \sin 45°$$

$$= 1 + \frac{\sqrt{2}}{2}$$

$$= \frac{2 + \sqrt{2}}{2}$$

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Test 4 - Trigonometry

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Student Name : _____

Part Two – Resource Allowed

Part Two contains 3 questions worth 21 marks

Time Allowed : 15 minutes

TO BE PROVIDED BY THE STUDENT

A maximum of one A4 page of notes, one sided.
Standard Items : Pens, pencils, eraser, sharpener, correction tape/fluid, highlighters, ruler.

Special Items: Drawing instruments, templates.
A maximum of three CAS calculators satisfying the conditions set by the SCSA.

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Question 1 **3, 4 - 7 marks**

Find all the solutions to the equations:

a) $2\cos 3x+\sqrt{3}=0.$

$$\cos 3x=-\frac{\sqrt{3}}{2}$$

$$3x=\frac{5\pi}{6},\frac{7\pi}{6}+2\pi n, n\in\mathbb{Z}$$

$$x=\frac{5\pi}{18},\frac{7\pi}{18}+\frac{2\pi n}{3}, n\in\mathbb{Z}$$

$$\therefore x=\begin{cases}\frac{5\pi}{18}+\frac{2\pi n}{3}\\ \frac{7\pi}{18}+\frac{2\pi n}{3}\end{cases}, n\in\mathbb{Z}$$

$$\therefore x=\begin{cases}\pm\frac{5\pi}{18}+\frac{2\pi n}{3}\end{cases}, n\in\mathbb{Z}$$

b) $2\sin^2(2\theta)+7\sin(2\theta)-4=0.$

$(2\sin 2\theta-1)(\sin 2\theta+4)=0$ *factorises*

$\sin 2\theta=-4$ or $\sin 2\theta=\frac{1}{2}$

no such θ

$$2\theta=\frac{\pi}{6},\frac{5\pi}{6},+2\pi n, n\in\mathbb{Z}$$

$$\theta=\frac{\pi}{12},\frac{5\pi}{12}+\pi n, n\in\mathbb{Z}$$

$$\theta=\begin{cases}\frac{\pi}{12}+\pi n\\ \frac{5\pi}{12}+\pi n\end{cases}, n\in\mathbb{Z}$$

Question 2 **3, 2, 2 - 7 marks**

a) Express $y = 3\sin x - \cos x$ in the form $y = R\sin(x-\alpha)$, where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$.

$$R^2 = 3^2 + (-1)^2$$
$$R = \sqrt{10}$$

$$R\sin(x-\alpha) = R\sin x\cos\alpha - R\cos x\sin\alpha$$
$$= 3\sin x - \cos x$$

$$R\cos\alpha = 3 \qquad R\sin\alpha = 1$$
$$\cos\alpha = \frac{3}{R} \qquad \sin\alpha = \frac{1}{R}$$

$$\tan\alpha = \frac{1}{3}$$
$$\alpha = 0.32$$

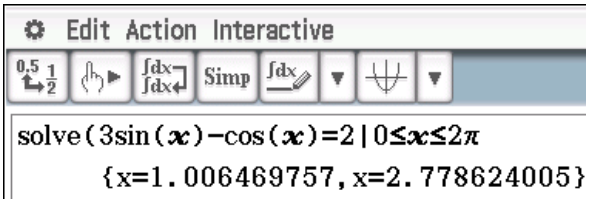
$$y = 3\sin x - \cos x = \sqrt{10}\sin(x-0.32)$$

b) Find the minimum value of this expression and state the smallest positive value of x for which this minimum value occurs.

$\sin(x-0.32)$ has a minimum value of -1,
$\therefore \sqrt{10}\sin(x-0.32)$ has a minimum value of $-\sqrt{10}$

$$\sin(x-0.32) = -1$$
$$x - 0.32 = \frac{3\pi}{2}$$
$$x = 5.03$$

c) Solve the equation $3\sin x - \cos x = 2$ for $0 \le x \le 2\pi$.



$x = 0.006, 2.779$

Question 3 **1, 2, 1, 3 - 7 marks**

The height, h metres, of a rider above the ground on a large Ferris wheel, at time t minutes after it starts moving can be determined by the equation $h(t) = -68\cos\left(\frac{\pi t}{15}\right) + 70$.

a) At what height do riders get into the seats on the wheel?

$h(0) = -68\cos\left(\frac{\pi(0)}{15}\right) + 70 = 2$

At a height of 2m above the ground.

b) How long does the wheel take for one revolution?

Period $= \frac{2\pi}{\frac{\pi}{15}} = 30$ minutes

c) What is the maximum height reached by a rider on the wheel?

Maximum height

$-68 \times (-1) + 70 = 138$ m

d) A rider completes 1 revolution on the wheel.
For how many minutes is the rider more than 100 metres above ground?

20.318 – 9.682
= 10.636 minutes
= -10 minutes 38 seconds