Eastern Goldfields College

MATHEMATICS: SPECIALIST 1 & 2
SEMESTER 2 2016

TEST 4
Resource Free

Time Allowed: 25 minutes

Total Marks: 24

1. [1, 2, 2 marks]

Given the matrices $A = \begin{bmatrix} -4 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -5 \\ 3 & 6 \end{bmatrix}$, and I is the identity matrix, calculate

(a) A – B

$$\begin{bmatrix} -6 & 7 \\ -3 & -5 \end{bmatrix} \checkmark$$

(b) BA

$$\begin{bmatrix} 2 & -5 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -1 \\ -12 & 12 \end{bmatrix} \checkmark\checkmark \text{ (-1 per error)}$$

(c) BA + 2B – 3I

$$= \begin{bmatrix} -8 & -1 \\ -12 & 12 \end{bmatrix} + \begin{bmatrix} 4 & -10 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -11 \\ -6 & 21 \end{bmatrix} \text{ (-1/error)} \checkmark\checkmark$$

2. [2 marks]

Prove that $\tan 105° = \frac{1+\sqrt{3}}{1-\sqrt{3}}$

$$LHS = \tan 105° = \tan(45+60) \checkmark$$

$$= \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \checkmark$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$= RHS$$

3. [1, 4, 5 marks]

(a) Find the value(s) of k for which $\begin{bmatrix} k & 1 \\ 4 & 3 \end{bmatrix}$ is singular.

$$3k - 4 = 0 \qquad k = \frac{4}{3}$$

(b) (i) Determine the inverse of the $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$.

$$\frac{1}{2}\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

(ii) Using matrices, solve the simultaneous equations:

$$2x + y = 4$$
$$4x + 3y = 6$$

✓ set up matrices

✓ premult. by inverse

✓ solution ✓ $\frac{1}{2}$

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \quad \therefore x = 3 \; y = -2$$

(c) Given that $A = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -21 \\ -4 & 15 \end{bmatrix}$

(i) determine AB

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

(ii) hence, solve

$$6x - 21y = 6$$
$$-4x + 15y = -2$$

✓ sets up matrices

✓ premult by inverse

$$\begin{bmatrix} 6 & -21 \\ -4 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 6 & -21 \\ -4 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 6 \end{bmatrix}$$

$$\checkmark \begin{bmatrix} 2x \\ 3y \end{bmatrix} = \begin{bmatrix} 16 \\ 6 \end{bmatrix} \qquad x = 8,\; y = 2 \checkmark$$

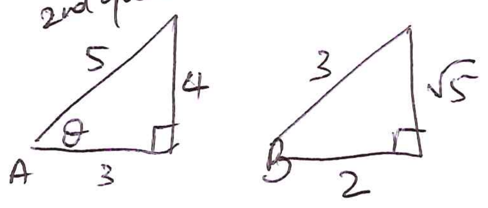
4. [3, 4 marks]

(a) Given $\cos\theta = \frac{2}{\sqrt{5}}$ for $0 \le \theta \le \frac{\pi}{2}$ determine the exact value of $\cos 2\theta$

$$\cos 2\theta = 2\cos^2\theta - 1 \checkmark$$
$$= 2\left(\frac{2}{\sqrt{5}}\right)^2 - 1 \checkmark$$
$$= 2 \cdot \frac{4}{5} - 1$$
$$= \frac{3}{5} \checkmark$$

(b) Given that A is an obtuse angle with $\sin A = \frac{4}{5}$ and B is an acute angle with $\cos B = \frac{2}{3}$ determine exactly the value for $\sin(A - B)$.

2nd quad



✓

$$\sin(A-B) = \sin A\cos B - \cos A \sin B$$
$$= \left(\frac{4}{5}\right)\left(\frac{2}{3}\right) - \left(\frac{-3}{5}\right)\left(\frac{\sqrt{5}}{3}\right) \checkmark$$
$$= \frac{8}{15} + \frac{3\sqrt{5}}{15}$$
$$= \frac{8 + 3\sqrt{5}}{15}. \checkmark$$

MATHEMATICS: SPECIALIST 1 & 2
SEMESTER 2 2016

TEST 4

Calculator Assumed

Eastern Goldfields College

Time Allowed: 30 minutes

Total Marks: 29

5. [2, 2, 2, 3 marks]

Given that A, B, C, and X are all square matrices of the same order, and that all necessary inverse matrices exist, then re-arrange the following equations to make X the subject (i.e. X = ...)

(a) A + BX = C

$$BX = C - A \quad \checkmark -A$$
$$B^{-1}BX = B^{-1}(C-A) \quad \checkmark \text{premult } B^{-1}$$
$$X = B^{-1}(C-A)$$

(b) XAB = C

$$XABB^{-1} = CB^{-1} \quad \checkmark \text{post mult } B^{-1}$$
$$XA = CB^{-1}$$
$$XAA^{-1} = CB^{-1}A^{-1} \quad \checkmark \text{post mult } A^{-1}$$
$$X = CB^{-1}A^{-1}$$

(c) C(X – B) = A

$$C^{-1}C(X-B) = C^{-1}A \quad \checkmark \text{premult } C^{-1}$$
$$X - B = C^{-1}A \quad \checkmark +B$$
$$X = C^{-1}A + B$$

(d) BX = A + CX

$$BX - CX = A \quad -CX \checkmark$$
$$(B-C)X = A \quad (B-C)X \checkmark$$
$$(B-C)^{-1}(B-C)X = (B-C)^{-1}A \quad \text{pre mult } (B-C)^{-1}$$
$$X = (B-C)^{-1}A$$

6. [4 marks]

Rewrite 3 cos θ + 5 sin θ in the form R cos (θ – α), where α is an acute angle in degrees.

$$3\cos\theta + 5\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

$$\left.\begin{aligned} 3 &= R\cos\alpha \\ 5 &= R\sin\alpha \end{aligned}\right\} \checkmark \qquad 3^2 + 5^2 = R^2, \quad \sqrt{34} = R \checkmark$$

$$\frac{5}{3} = \tan\alpha$$
$$\alpha = 59^\circ \checkmark \qquad \sqrt{34}\cos(\theta - 59^\circ) \checkmark$$

7. [2, 2, 2, 2 marks]

A company displays motor vehicles for sale in two different showrooms. Matrix P shows the number of vehicles for sale in each showroom. Matrix Q shows the petrol and oil requirements (in litres) for the sedans and 4-wheel drive vehicles. Matrix R gives the cost per litre for petrol and oil.

$$P = \begin{array}{c} \\ \text{Showroom A} \\ \text{Showroom B} \end{array} \begin{array}{c} \text{Sdn} \quad \text{4WD} \\ \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \end{array}, \quad Q = \begin{array}{c} \\ \text{Sdn} \\ \text{4WD} \end{array} \begin{array}{c} \text{Pet} \quad \text{Oil} \\ \begin{bmatrix} 45 & 4 \\ 60 & 6 \end{bmatrix} \end{array} \quad \text{and} \quad R = \begin{array}{c} \\ \text{Pet} \\ \text{Oil} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 1.40 \\ 2.50 \end{bmatrix} \end{array}$$

(a) Give a matrix S which shows the total (combined) cost of petrol and oil products for each type of vehicle.

$S = QR$

$$= \begin{array}{c} \$ \\ \begin{bmatrix} 73 \\ 99 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{sdn} \\ \text{4wd} \end{array}$$

(b) Show how S can be used, with one of the given matrices, to obtain a matrix M listing the total cost of petrol and oil products for each showroom. Hence find the matrix M.

$M = PS$

$$= \begin{array}{c} \$ \\ \begin{bmatrix} 417 \\ 391 \end{bmatrix} \end{array} \begin{array}{l} \\ A \\ B \end{array}$$

(c) Give a matrix T which shows the separate petrol and oil requirements for each showroom.

$T = PQ$

$$= \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} P \quad\; O \\ \begin{bmatrix} 255 & 24 \\ 240 & 22 \end{bmatrix} \end{array}$$

(d) Show how T can be used, with one of the given matrices, to obtain the matrix M in (b).

$$TR = \begin{bmatrix} 417 \\ 391 \end{bmatrix}$$

8. [4, 4 marks]

Prove the following:

(a) $\frac{\cos\theta}{1+\sin\theta} + \tan\theta = \frac{1}{\cos\theta}$

$$LHS = \frac{\cos\theta}{1+\sin\theta} + \tan\theta$$

$$= \frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} + \frac{\sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta + \sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cancel{1+\sin\theta}}{\cos\theta\cancel{(1+\sin\theta)}} \qquad \sin^2 + \cos^2 = 1$$

$$= \frac{1}{\cos\theta}$$

$= RHS$

(b) $\cos 3A = \cos A.(1 - 4\sin^2 A)$

$LHS = \cos 3A$

$= \cos(2A + A)$

$= \cos 2A \cos A - \sin 2A \sin A$

$= (1 - 2\sin^2 A)\cos A - 2\sin A \cos A \sin A$

$= (1 - 2\sin^2 A)\cos A - 2\sin^2 A \cos A$

$= (1 - 2\sin^2 A - 2\sin^2 A)\cos A$

$= (1 - 4\sin^2 A)\cos A$

$= RHS$