SHENTON COLLEGE

# SHENTON COLLEGE

**Examination Semester One 2019**
**Question/Answer Booklet**

# MATHEMATICS SPECIALIST UNIT 3

## Section One
## (Calculator-free)

Your name _____

## Time allowed for this section

Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

## Material required/recommended for this section

**To be provided by the supervisor**
Question/answer booklet for Section One.
Formula sheet.

**To be provided by the candidate**
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this examination

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Student Score |
|---|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 52 | |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | |
| | | | **Total** | 150 | |

**Section One: Calculator-free** **35% (52 Marks)**

This section has **eight (8)** questions**.** Answer **all** questions**.** Write your answers in the spaces provided**.**

Working time: 50 minutes.

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**Question 1** **(6 marks)**

(a) Determine the modulus and argument of $\frac{3}{1-i}$. (3 marks)

(b) Determine $z^2$ in the form $a+bi$, where $a, b \in \mathbb{R}$, when $z = 4\cos\left(\frac{\pi}{6}\right) + 4i\sin\left(\frac{\pi}{6}\right)$. (3 marks)

**Question 2** **(4 marks)**

The equations of three planes are shown below.

$$x - y + 3z = 11$$
$$x + 2y - 2z = 0$$
$$x - y + z = 9$$

(a) Determine the coordinates of the point of intersection of the planes. (3 marks)

(b) Determine the distance of the point of intersection of the planes from the origin. (1 mark)

**Question 3** **(6 marks)**

(a) State whether the planes with equations $2x - y + z = 2$ and $x + 3y + 2z = 1$ are perpendicular. Justify your answer. (2 marks)

(b) Determine the Cartesian equation of the plane that passes through the three points with position vectors shown below. (4 marks)

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \qquad \mathbf{c} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

**Question 4** **(6 marks)**

Functions $f$ and $g$ are defined over their natural domains by $f(x) = \sqrt{8-x}$ and $g(x) = 3 + \frac{4}{\sqrt{x}}$.

(a) State the domain of

(i) $g(x)$. (1 mark)

(ii) $g^{-1}(x)$. (2 marks)

(b) Determine $f \circ g(x)$ and the natural domain of this composite function. (3 marks)

**Question 5** **(7 marks)**

Four functions are defined as

$$f(x) = x^2 + 4x - 5, \qquad g(x) = 3x^2 + 2x - 1, \qquad h(x) = x + 5, \qquad k(x) = x - 1$$

Determine the equations of all asymptotes of the following graphs.

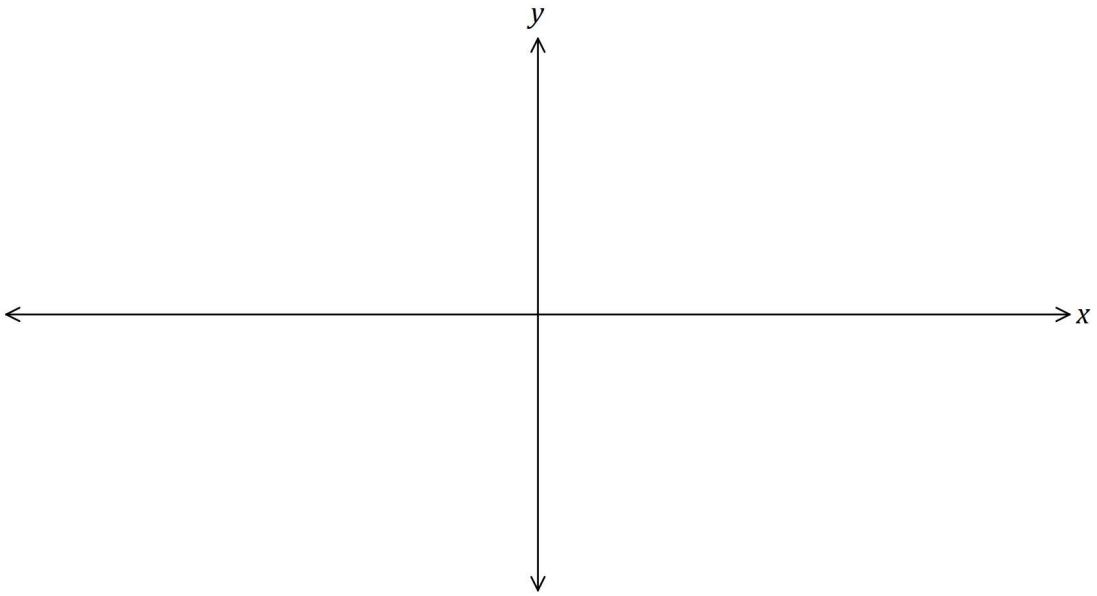
(a) $y = \dfrac{h(x)}{f(x)}$. (2 marks)

(b) $y = \dfrac{f(x)}{g(x)}$. (2 marks)

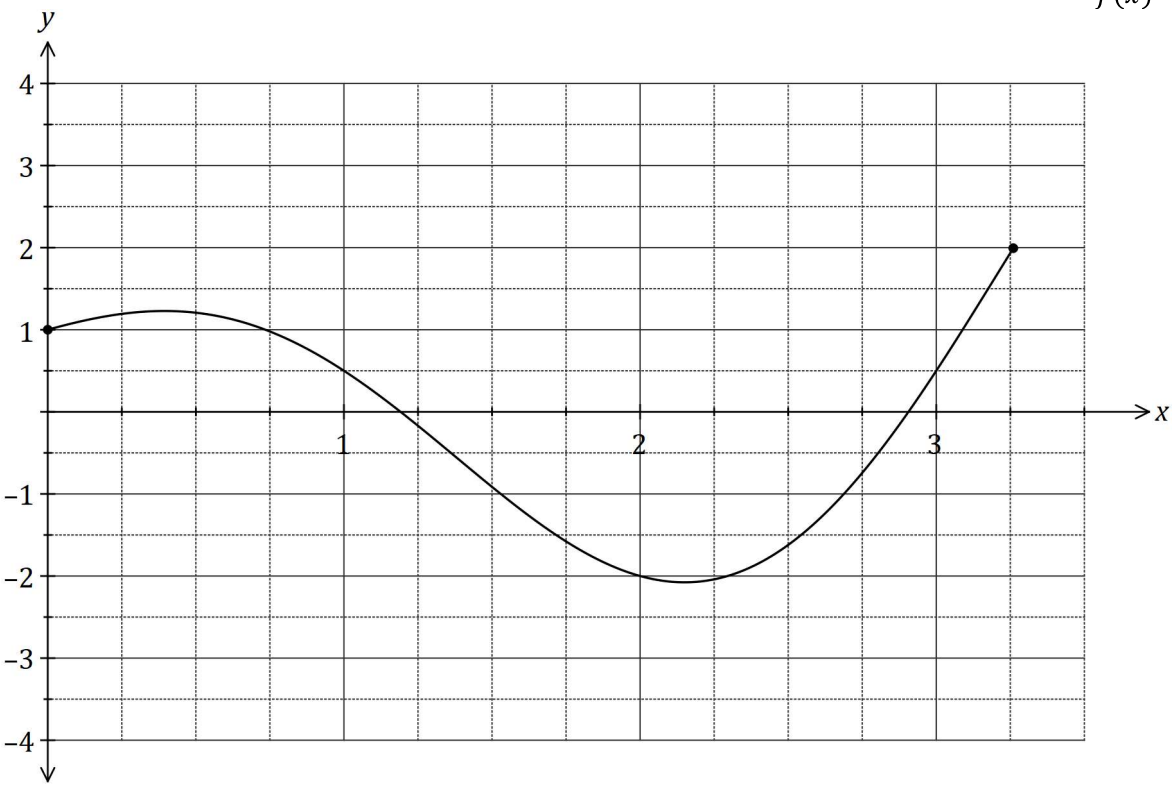
(c) $y = \dfrac{g(x)}{k(x)}$. (3 marks)

**Question 6** **(7 marks)**

(a) On the axes below, sketch the graph of $y = \dfrac{2x}{|x|}$. (3 marks)

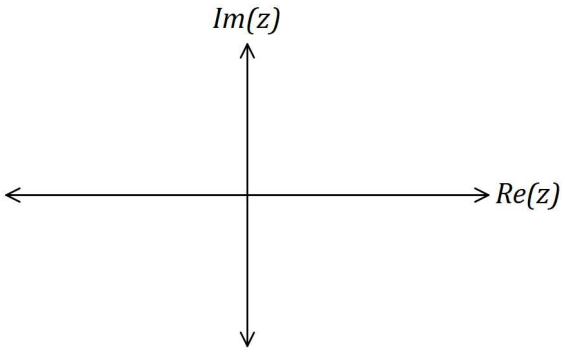



(b) The graph of $y = f(x)$ is shown below. On the same axes draw the graph of $y = \dfrac{1}{f(x)}$. (4 ma



**Question 7** **(8 marks)**

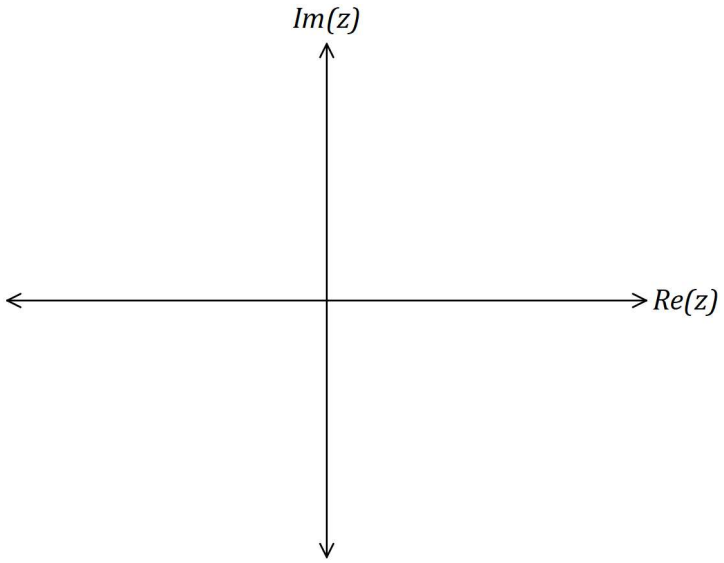
(a) Sketch the locus of points $z$ in the complex plane determined by $\arg(z + 3i) = \frac{3\pi}{4}$. (3 marks)




(b) Another locus of points $z$ in the complex plane is determined by $z\bar{z} + z + \bar{z} = 8$.

(i) Show that this locus can also be defined in the form $|z - w| = k$, clearly showing the value of constant $w$ and the value of constant $k$. (3 marks)

(ii) Sketch the locus on the axes below. (2 marks)



**Question 8** **(8 marks)**

Let $z = x + yi$ and $z^2 = a + bi$ where $a, b, x, y \in \mathbb{R}$.

(a) Show that $\sqrt{a^2 + b^2} + a = 2x^2$. (4 marks)

(b) By solving the equation $z^4 - 16z^2 + 100 = 0$ for $z^2$ or otherwise, determine the roots of the equation in Cartesian form. (4 marks)