Name: ....

Resource Free	/25	%
Resource Rich	/35	%
Total	/60	%

Mathematics Methods, Year 12, 2017

Test 2 – Further differentiation and applications, Integrals

25 minutes working time.

Calculator Free Section (no notes, no calculators) SCSA Formula sheet allowed

1. (5 marks)

Determine each of the following. Express your answers with positive indices.

(a) $\int (2x+4)\,dx$ (1)

$= x^2 + 4x + c$ ✓

(b) $\int \frac{3}{(2t-1)^2}\,dt = \int 3(2t-1)^{-2}$ (2)

$= \frac{3(2t-1)^{-1}}{-1 \times 2}$ ✓

$= -\frac{3}{2(2t-1)} + c$ ✓

(c) $\int \frac{e^{2x}+e^{-3x}}{e^{\cancel{2}x}}\,dx$ (2)

$= \int e^{x} + e^{-4x}$ ✓

$= e^{x} - \frac{1}{4e^{4x}}$ ✓

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2. (4 marks)

Determine the value of each of the following **exactly**

(a) $\int_0^{-3} (x-1)\,dx$ $= \left[\frac{x^2}{2} - x\right]_0^{-3}$ ✓ (2)

$= \left[\frac{(-3)^2}{2} - -3\right]$

$= 7.5$ ✓

(b) $\int_0^1 e^{2x} + e^{x+1} dx$ $= \left[\frac{e^{2x}}{2} + e^{x+1}\right]_0^1$ ✓ (2)

$= \left[\frac{e^2}{2} + e^2\right] - \left[\frac{e^0}{2} + e^1\right]$

$= \frac{3}{2}e^2 - e - \frac{1}{2}$ ✓

3 (4 marks)

Find each of the following

(a) $\frac{d}{dx}\int_0^x \sqrt{t^2-3}\,dt$ $= \sqrt{x^2-3}$ ✓ (1)

(b) $\frac{d}{dt}\int_{-3}^t \frac{1}{(x-2)^4}\,dx$ $= \frac{1}{(t-2)^4}$ ✓ (1)

(c) $\frac{d}{dx}\int_{-3}^{2x} (a^2+4)\,da$ $= \left((2x)^2 + 4\right) \times 2$ (2) ✓

$= 8(x^2+1)$ ✓

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4. (5 marks)

State the exact value of

(a) $\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n$ $= e^2$ ✓ (1)

Consider the function $f(x) = e^{-2x}x^3$

(b Show clearly that $f'(x) = x^2e^{-2x}(3-2x)$ (2)

$e^{-2x} \times 3x^2 + x^3 \times -2e^{-2x}$ ✓

$x^2e^{-2x}(3-2x)$ ✓

Hence, or otherwise;

(c) Determine the exact co-ordinates of the curve $f(x) = e^{-2x}x^3$ where the gradient is zero. (2)

$x^2e^{-2x}(3-2x) = 0$

$x = 0$ or $\frac{3}{2}$ ✓

$\therefore (0,0)$ and $\left(\frac{3}{2}\ \ \frac{27e^{-3}}{8}\right)$ ✓

5. (6 marks)

Find the derivative of each of the following

(a) $y = \sin 2x + cos\frac{x}{2}$ (2)

$dy/dx = 2\cos 2x - \frac{1}{2}\sin\frac{x}{2}$ ✓ ✓

(b) $y = sin^2t + 3cos2t + \pi$ (2)

$\frac{dy}{dx} = 2(\sin t)(\cos t) - 6\sin 2t.$ ✓ ✓

(c) $y = (3x+2)\cos(x^2)$ (3)

$\frac{dy}{dx} = (3x+2) \times -2x\sin x^2 + \cos(x^2) \times 3$ ✓ ✓

$= 3\cos(x^2) - 2x(3x+2)\sin(x^2)$ ✓

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Mathematics Methods, Year 12, 2017

Test 2 – Further differentiation and applications, Integrals

Resource Rich	/35	%

35 minutes working time. Calculator Assumed Section (notes allowed), SCSA Formula sheet and calculators allowed

1 (9marks)

The number of white rhinos in Africa has been decreasing at a rate proportional to the number present since 1993. At the beginning of 1993 there were 440 white rhinos in Africa.

That is $\frac{dW}{dt} = -kW$ where k is the constant of proportionality and t is the number of years since 1993.

(a) Show clearly that $W = W_o e^{-kt}$ satisfies the above equation (2)

If $W = W_0 e^{-kt}$ ✓, then $\frac{dW}{dt} = -kW_0 e^{-kt}$
$= -ky$ ✓

By the beginning of 2000 there were only 356 white rhinos in Africa.

(b) Determine

(i) The value of W_o (1)

$W_0 = 440$ ✓

(ii) The value of the constant of proportionality, correct to three decimal places. (2)

✓ $356 = 440e^{-k7}$ ✓ $\therefore k = 0.030$.

Hence, or otherwise,

(c) Determine the expected number of white rhinos at the beginning of 2010. (2)

✓ $W = 440e^{-0.03(17)}$ ✓ $= 264$.

(d) During which year the number of white rhinos will first fall below 300. (2)

$300 = 440e^{-0.03t}$ ✓

$t = 12.77$ $\therefore 1993 + 12.77 = 2005$ ✓

2. (10 marks)
A particle is moving in rectilinear motion with acceleration a at any time t, $in\ ms^{-2}$, given as $a = 6t - 1$

Initially the particle is at the origin with a velocity of $-2ms^{-1}$

Determine:

(a) The velocity of the particle at any time t. (2)

$v = \int 6t - 1 \, dt = 3t^2 - t + c$ ✓

when $t = 0$ $v = -2$

$\therefore -2 = 3(0)^2 - 0 + c$

$c = -2$

$\therefore v = 3t^2 - t - 2$ ✓

(b) When the particle is again at the origin. (3)

$x = \int 3t^2 - t - 2 \, dt$ ✓

$= t^3 - \frac{t^2}{2} - 2t + c$

when $t = 0$ $x = 0$

$\therefore c = 0$

$\therefore x(t) = t^3 - \frac{t^2}{2} - 2t$ ✓

$0 = t^3 - \frac{t^2}{2} - 2t$

$t = 0 + 1.69s$.

$\therefore 1.69s$ ✓

(c) The minimum velocity of the particle (2)

$v = 3t^2 - t - 2$

$\frac{dv}{dt} = 6t - 1$

$0 = 6t - 1$

$t = 1/6$ ✓

$v(\frac{1}{6}) = 3(\frac{1}{6})^2 - \frac{1}{6} - 2$ ✓

$= -2.08 \, ms^{-1}$

(d) The total distance travelled by the particle in the first three seconds. (3)

Total distance =

$\int_0^3 |3t^2 - t - 2| \, dt$ ✓✓

$= 19.5m.$ ✓

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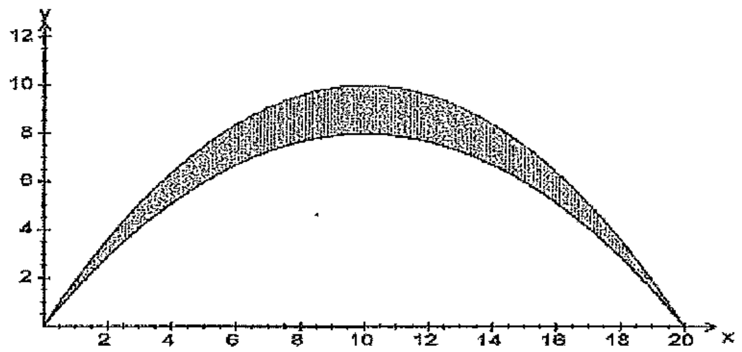
3. (10 Marks)

The roof of a stage is constructed using the plans drawn below. The distances are in metres and the horizontal axis represents the floor of the stage. The two parabolas shown represent supports.



The equations of the supports are given by:

$$f(x) = -0.1x(x-20) \, and \; g(x) = -0.08x(x-20)$$

Determine:

(a) The distance between the supports at the highest point above the stage. (2)

$f(x)$ max at $(10,10)$; $g(x)$ max at $(10,8)$.

$\therefore$ Distance between supports $= 10-8 = 2m.$

(b) The distance between the supports 4 metres from the right hand side of the stage. (2)

$f(16) = 6.4 \quad g(16) = 5.12$

$\therefore 6.4 - 5.12 = 1.28m.$

A tight wire is to be connected from the origin to the lower support at a point where $x = 8$.

(c) Determine the equation of this tight wire. (2)

$g(8) = 7.68 \quad \therefore y = \frac{7.68}{8}x = 0.96x$

(d) State the integral which would be used to determine the area between the supports above the stage (i.e. the shaded region) (2)

$\int_0^{20} -0.1x(x-20) - -0.08x(x-20)$

$= -0.02x(x-20)$

Hence, or otherwise;

(e) Determine the area between the supports, correct to two decimal places. (2)

$\int_0^{20} -0.02x(x-20) = 26.67$ units^2

$\frac{}{10}$

4. (6 marks)

The rate of change of cost, in dollars of producing x tonnes of fertiliser is such that:

$$\frac{dC}{dx} = 30x - 30\sqrt{x}$$

Determine:

(a) The rate at which cost is changing at the instant when 4 tonnes of fertiliser are being produced. (2)

$$\left.\frac{dC}{dx}\right|_{x=4} = 30(4) - 30\sqrt{4} = \$60$$

(b) The extra cost involved in producing 16 tonnes instead of 9 tonnes of fertiliser. (2)

$$\int_9^{16} 30x - 30\sqrt{x}\,dx = \$1885$$

(c) An integral which displays the actual cost of producing the 25^{th} tonne of fertiliser. (Do not evaluate) (2)

$$\int_{24}^{25} 30x - 30\sqrt{x}\,dx$$

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