SCOTCH COLLEGE

12 Mathematics Methods 2023

Test 2 – Integration and Applications

Section 1: Calculator-Free

Time allowed: 20 minutes **Maximum marks: 20**

Name:_____ **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No Calculators and no notes are permitted.

Question 1 [3 marks]

Find y in terms of x given that $\frac{dy}{dx} = 3x^2 - 4$ and $y = 5$ when $x = -3$.

Question 2 [6 marks]

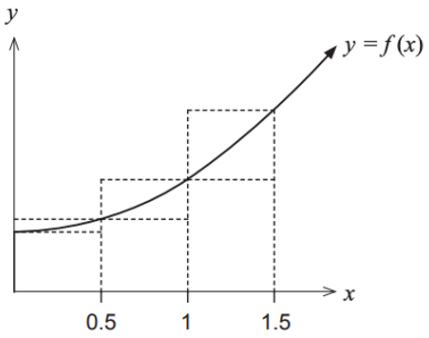
Evaluate the following definite integrals.

a) $\int_1^4 (\frac{2}{\sqrt{x}} + 1)dx$ **[3]**

b) $\int_0^3 7(2-x)^3 dx$ **[3]**

Question 3 [5 marks]

Consider below the function of $f(x)$ and its table of values at various points.



$\boldsymbol{x}$	0	0.5	1	1.5
$\boldsymbol{f(x)}$	15	18	22	27

a) Using the rectangles shown in the diagram above, show that: **[3]**

$$27.5 < \int_0^{1.5} f(x)\,dx < 33.5$$

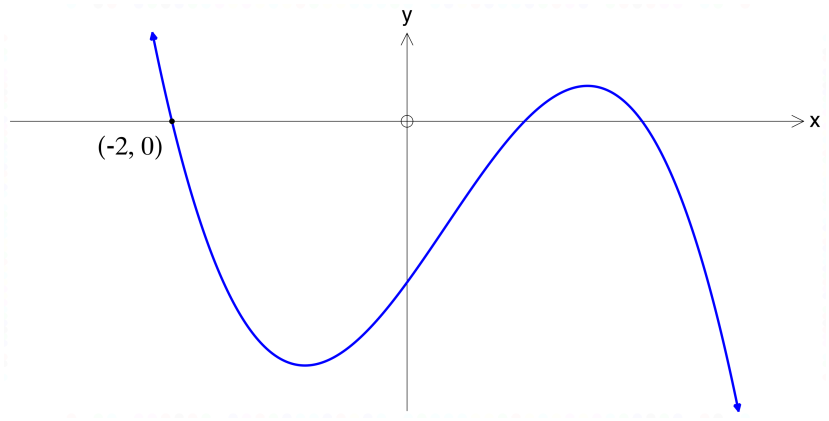
The process used above is repeated using rectangles of **half the width** to obtain the following result;

$$a < \int_0^{1.5} f(x)\,dx < b$$

b) Without calculating, suggest appropriate values of a and b. **[2]**

Question 4 [6 marks]

Some of the features of the graph of $y = -3x^3 + 3x^2 + 12x - 12$ are shown below.



a) Determine the other two roots of the graph.

b) Showing use of Calculus, determine the total area enclosed by the graph and the x-axis.

-----END OF SECTION ONE-----