GREENWOOD COLLEGE

Methods 11 Test 2 2018
Quadratic and Exponential Functions
Total Marks: 60   Time Allowed: 60 minutes
Name: _____

SECTION A – Resource Free
40 minutes – 43 marks

ALL working must be shown for full marks.

1.   **[3, 1, 1 = 5 marks]**

A parabola has the equation $y = (x + 6)(10 - x)$.

a) Find the coordinates of the x and y intercepts of the parabola.

$(-6, 0)$ $(10, 0)$ $(0, 60)$
✓   ✓   ✓

b) Find the equation of the line of symmetry

$\dfrac{-6+10}{2} = 2$ $x = 2$ ✓

c) Find the coordinates of the turning point of the parabola and state the nature of the turning point.

$y = (2+6)(10-2)$
$\quad = 64$

$(2, 64)$ ✓

2.   **[8 marks]**

Complete the following table with the appropriate information.

– 2.25

	x-intercept	y-intercepts	Domain	Range
$y = x^2 + 9x + 14$	$(-7, 0)$ ✓ $(-2, 0)$	$(0, 14)$ ✓	$\{x : x \in R\}$ ✓	$\{y : y \ge \quad y \in R\}$ ✓
$y = \dfrac{1}{x+5} - 3$	$(-\frac{14}{3}, 0)$ ✓	$0, -\frac{14}{5})$ ✓	$\{x : x \neq -5, x \in R\}$ ✓	$\{y : y \neq -3, y \in R\}$ ✓

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3. **[3, 3, 3, 7 = 16 marks]**

Solve the following exponential equations:

a) $3^{2x-1} \times 9^x = 243$

$3^{2x-1} \times 3^{2x} = 3^5$ ✓ base 3

$2x-1+2x = 5$ ✓ index laws

$4x = 6$

$x = \frac{3}{2}$ ✓ Solve

b) $5^{-x} = 0.04$

$5^{-x} = \frac{4}{100}$

$5^{-x} = \frac{1}{25}$ ✓ decimal to fraction

$5^{-x} = 5^{-2}$ ✓ base 5

$x = 2$ ✓ solve

c) $\sqrt{(2x-3)^3} = 8$

$(2x-3)^3 = 64$ ✓

$2x-3 = 4$ ✓

$2x = 7$

$x = \frac{7}{2}$ ✓

d) $4 \times 2^{2x} - 34 \times 2^x + 16 = 0$ $\quad let\ p = 2^x$

$4p^2 - 34p + 16 = 0$ ✓ replace P

$2(2p^2 - 17p + 8) = 0$ ✓ factorize 2

$2(2p^2 - p - 16p + 8) = 0$

$2(p(2p-1) - 8(2p-1)) = 0$

$2(p-8)(2p-1) = 0$ ✓ factorize

$p = 8$ or $\frac{1}{2}$ ✓ Solve p

$2^x = 8$ or $\frac{1}{2}$

$2^x = 2^3$ or 2^{-1} ✓ replace p with 2^x

$x = 3$ or -1 ✓ Solve x

4. **[2, 2 = 4 marks]**

a) Find the value(s) for k for which $x^2 - 10x + k = 0$ has exactly one solution.

$b^2 - 4ac = 0$

$100 - 4k = 0$ ✓

$k = 25$ ✓

c) Find the value(s) for k for which $kx^2 + 6x - 2 = 0$ has no real solutions.

$b^2 - 4ac < 0$

$36 + 8k < 0$ ✓

$36 < -8k$

$\frac{36}{-8} > k$ ✓

5.   [3, 3 = 6 marks]

A parabola has the equation $y = f(x)$ where $f(x) = k(x + a)^2 + 16$ where a is a constant.

   a) Find a and k if the parabola has a turning point at (-2, 16) and $f(0) = -4$

$$y = k(x+2)^2 + 16 \checkmark \qquad a = 2 \checkmark$$
$$-4 = 4k + 16 \qquad\qquad k = -5 \checkmark$$
$$k = -5$$

   c) Find a and k if $f(3) = f(-5) = 0$

$$LoS = \frac{-b}{2a} \qquad\qquad 0 = k(x+1)^2 + 16$$
$$= -\frac{3+-5}{2} \qquad\qquad 0 = k(3+1)^2 + 16 \checkmark$$
$$= +1 \qquad\qquad\qquad -16 = 16k$$
$$\qquad\qquad\qquad\qquad k = -1 \checkmark$$
$$a = +1 \checkmark$$

6.   [1, 1, 1, 1 = 4 marks]

Consider the graph of y = f(x) shown.
Find the image of the point (-3, 9) under
the following transformations:

   a) y = f(x+3)

$$(-6, 9) \checkmark$$

   b) y = f(-x)

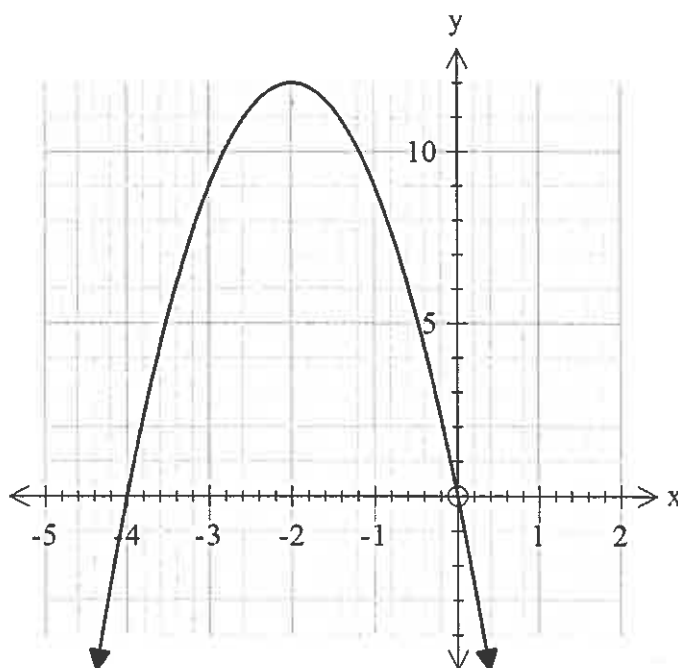$$(3, 9) \checkmark$$

   c) y = f(2x) + 1

$$\left(-\frac{3}{2}, 10\right) \checkmark$$
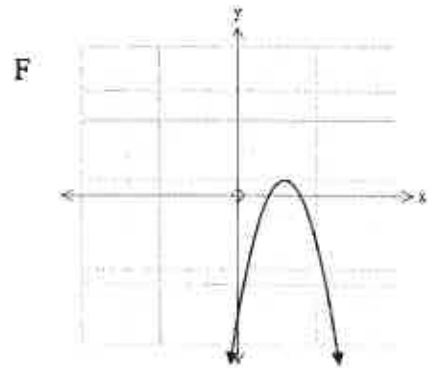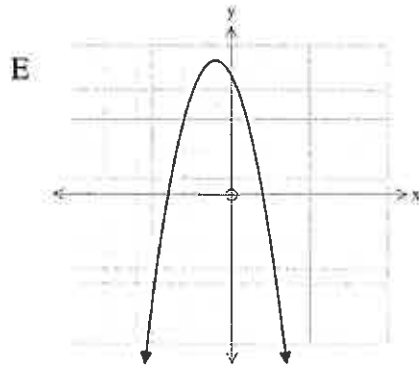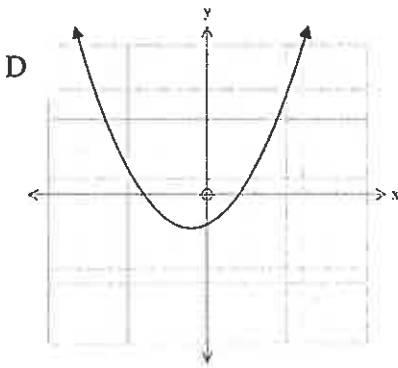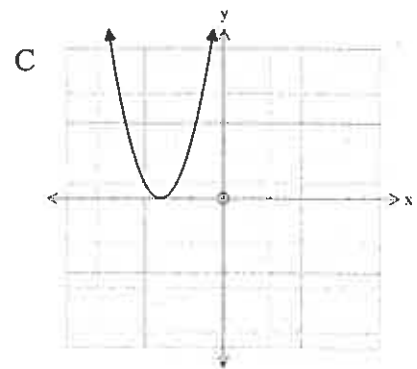
   d) y = -2f(x)

$$(-3, -18) \checkmark$$

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7.   [6 marks]

Match the graphs with the equations.

A

B

C

D

E

F
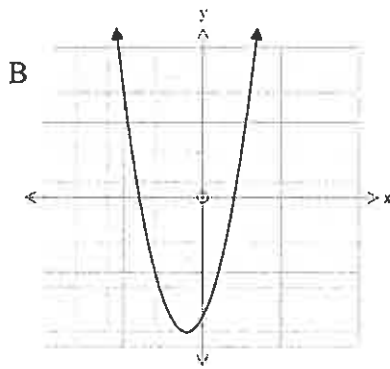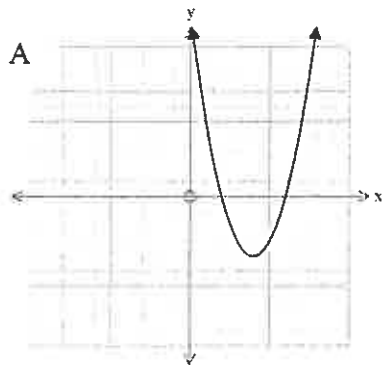
$y = a(x-b)(x+a)$ Graph ___D___ ✓ $y = (x-a)(x-b)$ Graph ___A___ ✓

$y = (x+a)^2$ Graph ___C___ ✓ $y = x^2 - bx - c$ Graph ___B___ ✓

$y = -x^2 + bx + c$ Graph ___E___ ✓ $y = -(x-a)^2 + b$ Graph ___F___ ✓

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8. [2, 2, 2 = 6 marks]

The graph of a reciprocal function has asymptotes with equation $x = -2$ and $y = 4$.

a) Write two possible equations for this function.

$$y = \frac{1}{x+2} + 4 \quad \checkmark \qquad y = \frac{2}{x+2} + 4 \quad \checkmark$$

b) Write the equation of this function if it has a y-intercept at $(0, 5)$.

$$y = \frac{a}{x+2} + 4 \qquad 5 = \frac{a}{x+2} + 4 \qquad a = 2 \; \checkmark$$

$$1 = \frac{a}{2+2} \qquad\qquad y = \frac{2}{x+2} + 4 \; \checkmark$$

c) Write the equation of this function if it has a x-intercept at $(-3, 0)$.

$$0 = \frac{a}{-3+2} + 4 \qquad\qquad y = \frac{4}{x+2} + 4 \; \checkmark$$

$$-4 = \frac{a}{-1} \; \checkmark$$

$$a = 4$$

9. [5 marks]

The sides of a right triangle are $(x+1)$ cm, $(x+3)$ cm and $(x+5)$ cm.
Find the length of each side. TO EARN FULL MARKS YOU MUST SHOWING ALGEBRAIC WORKING.

$$(x+5)^2 = (x+1)^2 + (x+3)^2 \qquad \checkmark$$

$$x^2 + 10x + 25 = x^2 + 2x + 1 + x^2 + 6x + 9$$

$$x^2 - 2x - 15 = 0 \; \checkmark$$

$$(x-5)(x+3) = 0 \; \checkmark$$

$$x = 5 \text{ or } -3 \; \checkmark$$

can't be -3 as   6cm 8cm 10cm ✓
can't have a
negative length

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