Year 11 Methods

Test 2A

Topic 1: Trigonometry and Radian Measure

Test 1 Non-Calculator

Name:

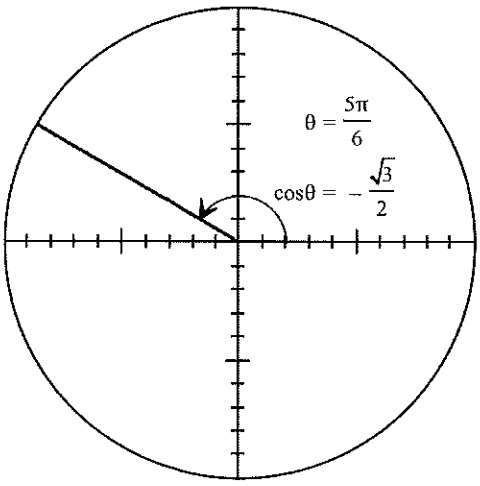
Instructions to candidates

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

Time Allocation: 20 minutes

Total Marks: 23

Question 1 (1,4marks)



The diagram below shows a unit circle with angle $\theta = \frac{5\pi}{6}$ and $\cos\theta = -\frac{\sqrt{3}}{2}$ as marked.

(a) State the value of θ in degrees.

(b) State exact values for $\sin\theta$ and $\tan\theta$.

Question 2 [2, 2 marks]

Solve each of the following over the given domain:

a) $\sin\theta = -\frac{1}{2} \quad ; 0^\circ \le \theta \le 360^\circ$

b) $\cos\theta = \frac{1}{2} \quad ; -\pi \le \theta \le \pi$

Question 3 (5 marks)

(a) Convert the following to radians:

(i) 145º (1 mark)

(b) Convert the following to degrees:

(ii) $\frac{7\pi}{8}$ (1 mark)

Give the **exact** values for each of the following.

(a) $\tan 135^\circ$ (b) Cos 60 (1, 1 mark)

(b) $\sin 45^\circ \times \tan 60^\circ$ (1 mark)

Question 4 (2,3,2,2 marks)

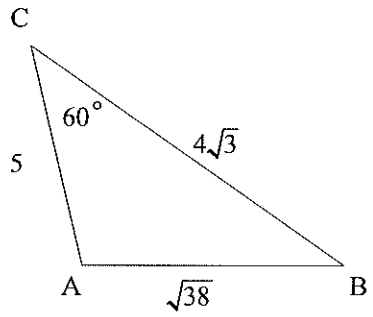
(a) Find the exact length of the arc formed by an angle of $\frac{\pi}{3}$ in a circle of radius 12 cm. (2 marks)

(b)

Find the radius of a circle if the area of the sector is $\frac{3\pi}{4}\ cm^2$ and the angle subtended at the centre of the circle is $\frac{\pi}{6}$. (3 marks)

(c) The circumference of a circle is 10 m. Find the length of an arc that is cut off by an angle of $\frac{\pi}{9}$ subtended at the centre of the circle, correct to two decimal places. (2 marks)

(a) Find the area of ΔABC. (2 marks)



Mathematics Methods Unit 1

Name: _____

Test 2A

Topic 1: Trigonometry and Radian Measure

Section Two:

Calculator-assumed

Time allowed for this section: 25 minutes

Total marks for this section: 23 marks

Materials required/recommended for this section:

SCSA Formula Sheet

Notes on one sheet (both sides) of A4 paper

Up to three approved calculators

Instructions to candidates

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

Question 5 (2, 2 marks)

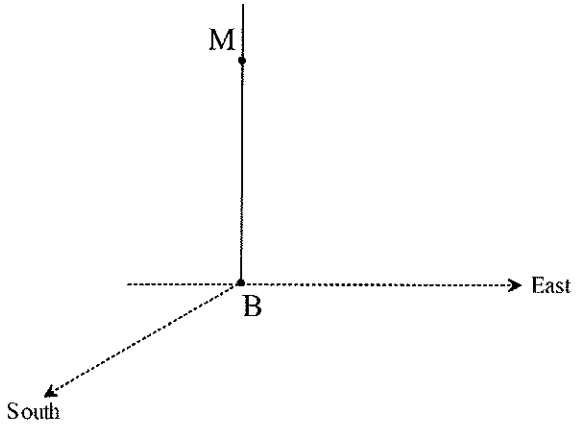
(a) A plane flies on a bearing of 60^0 for 325 km from Point X and then flies a further 590 km on a bearing of 174^0 to Point Y.

(i) How far from X, to the nearest km, is the plane now? (2 marks)

(ii) What is the bearing of the plane from Point X, to the nearest degree? (2 marks)

QUESTION 6 (5, 2 marks)

A wireless mast is supported by two wires MP and MQ, each attached to it at the point M, 12m from the base A of the mast. P,Q, and B are all on level ground, Q being due East and P due South of the mast. If MP and MQ are respectively 15m and 18m long:

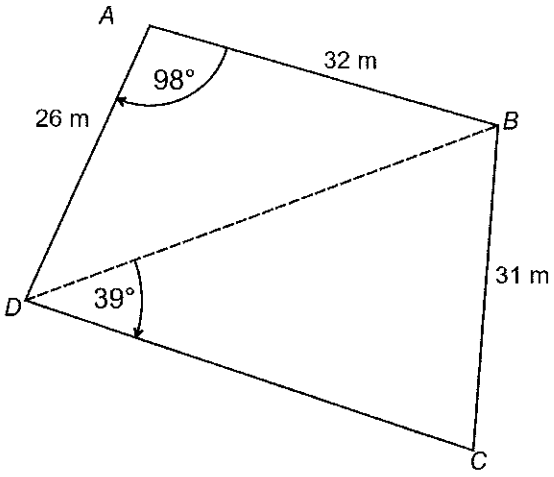


(a) Find the distance between P and Q

(b) Find the angle QMP between the wires

Question 7 (2, 2 marks)

Use quadrilateral ABCD to:



(a) Determine the length of DB, correct to two decimal places. (2 marks)

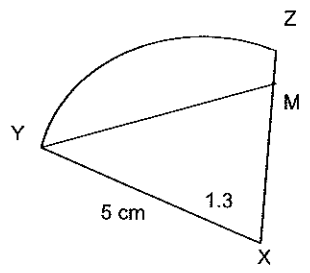
(b) Determine the size of ∠BCD. (2 marks)

Question 7(2,3,3 marks)

The figure shown is such that XY = YM = 5 cm. The angle at the centre of the sector is 1.3 radians.

Calculate:

(a) the area of the sector XYZ. (2 marks)



(b) the length XM.

(3 marks)

(c) the area of YMZ.

(3 marks)

Year 11 Methods

Test 2A

Topic 1: Trigonometry and Radian Measure
Test 1 Non-Calculator

Name: Solutions

Instructions to candidates

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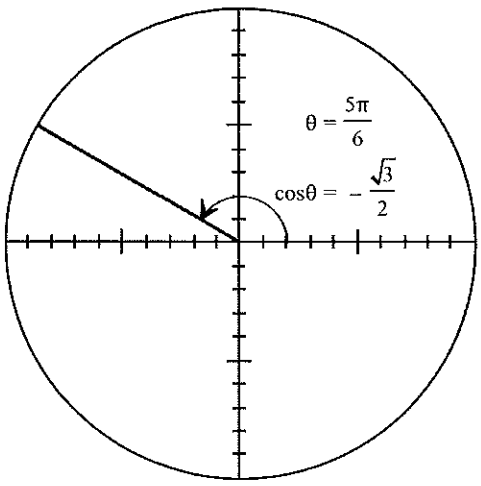
Time Allocation: 20 minutes

Total Marks: 23

Question 1 (1,4marks)

The diagram below shows a unit circle with angle $\theta = \frac{5\pi}{6}$ and $\cos\theta = -\frac{\sqrt{3}}{2}$ as marked.



(a) State the value of θ in degrees.

$\frac{5\pi}{6} \times \frac{180^{30}}{\pi} = 150^\circ$ ✓

(b) State exact values for $\sin\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$ ✓

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = 1 - \left(\frac{\sqrt{3}}{2}\right)^2$ ✓

$= 1 - \frac{3}{4}$ ✓

$= \frac{1}{4}$

$\sin\theta = \pm\sqrt{\frac{1}{4}} \Rightarrow \frac{1}{2}$

$\sin\theta = \frac{1}{2}$ ✓

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{1}{2} \times -\frac{2}{\sqrt{3}}$

$= -\frac{1}{\sqrt{3}}$ ✓

Question 2 [2, 2 marks]

Solve each of the following over the given domain:

a) $\sin\theta = -\frac{1}{2}$; $0^\circ \le \theta \le 360^\circ$

Sin is in 3 or 4th Quad ✓

$\sin\theta = 30^\circ$

$\theta = 210^\circ$ or 330° ✓

b) $\cos\theta = \frac{1}{2}$; $-\pi \le \theta \le \pi$

Cos is positive in 1st or 4th ✓

$\cos\theta = \frac{1}{2} = \cos 60^\circ$

$\theta = 60^\circ$ or -60° ✓

Question 3 (5 marks)

(a) Convert the following to radians:

(i) 145º $\quad 145 \times \frac{\pi}{180} = \frac{29\pi}{36}$ ✓ (1 mark)

(b) Convert the following to degrees:

(ii) $\frac{7\pi}{8}$ $\quad \frac{7\pi}{8} \times \frac{180^{45}}{\pi} = 157.5^\circ$ (1 mark)

Give the **exact** values for each of the following.

(a) $\tan 135^\circ$ (b) Cos 60 (1, 1 mark)

$-\tan 45^\circ = -1$ ✓ $\quad \cos 60^\circ = \frac{1}{2}$ ✓

(b) $\sin 45^\circ \times \tan 60^\circ$ (1 mark)

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{1}$

$= \frac{\sqrt{3}}{\sqrt{2}}$

Question 4 (2,3,2,2 marks)

(a) Find the exact length of the arc formed by an angle of $\frac{\pi}{3}$ in a circle of radius 12 cm. (2 marks)

$l = r\theta$

$= 12 \times \frac{\pi}{3}$ ✓

$= 4\pi$ cm ✓

(b)

Find the radius of a circle if the area of the sector is $\frac{3\pi}{4}\ cm^2$ and the angle subtended at the centre of the circle is $\frac{\pi}{6}$. (3 marks)

$A = \frac{1}{2} r^2 \theta$

$\frac{3\pi}{4} = \frac{1}{2} r^2 \times \frac{\pi}{6}$ ✓

$\frac{3\pi}{4} \times \frac{2}{1} \times \frac{6}{\pi} = r^2$

$9 = r^2$ ✓

$3 = r$ ✓ $r = 3$ cm

(c) The circumference of a circle is 10 m. Find the length of an arc that is cut off by an angle of $\frac{\pi}{9}$ subtended at the centre of the circle, correct to two decimal places. (2 marks)

$2\pi r = 10$

$r = \frac{10^5}{2\pi}$

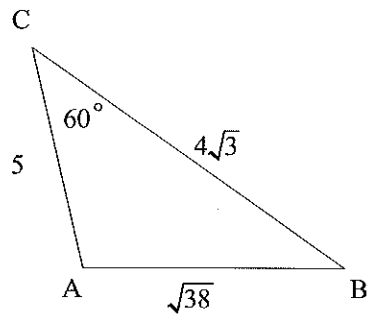
$r = \frac{5}{\pi}$ m ✓

$l = r\theta$

$= \frac{5}{\pi} \times \frac{\pi}{9}$ ✓

$l = \frac{5}{9} \Rightarrow 0.56$ m

(a) Find the area of ΔABC. (2 marks)



Area $= \frac{1}{2}ab\sin\theta$

$= \frac{1}{2} \times 5 \times 4\sqrt{3} \times \sin 60$

$= \frac{1}{2} \times 5 \times 4\sqrt{3} \times \frac{\sqrt{3}}{2}$ ✓

$= 5 \times 3$

$= 15$ units ✓

Mathematics Methods Unit 1

Name: _____

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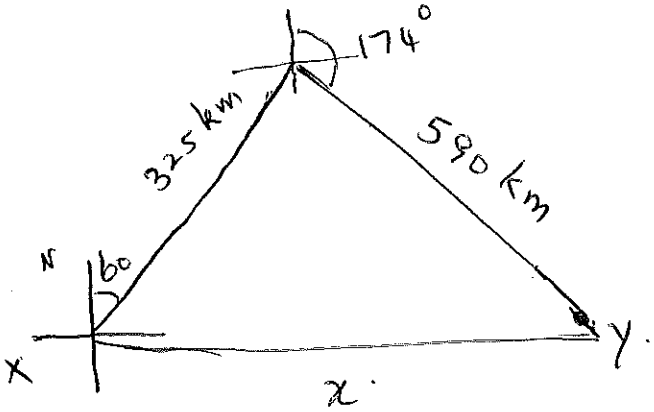
Question 5 (2, 2 marks)

(a) A plane flies on a bearing of 60^0 for 325 km from Point X and then flies a further 590 km on a bearing of 174^0 to Point Y.

(i) How far from X, to the nearest km, is the plane now? (2 marks)



$x^2 = 325^2 + 590^2 - 2 \times 325 \times 590 \cos 66^\circ$

$x = \sqrt{297741.4974}$

$x = 545.6569$

$= 546$ km

(ii) What is the bearing of the plane from Point X, to the nearest degree? (3 marks)

$$\frac{590}{\sin\theta} = \frac{545.7}{\sin 66^\circ}$$ ✓

$$\sin\theta = \frac{590 \times \sin 66^\circ}{545.7}$$

$$\sin^{-1} = 0.9877$$ ✓

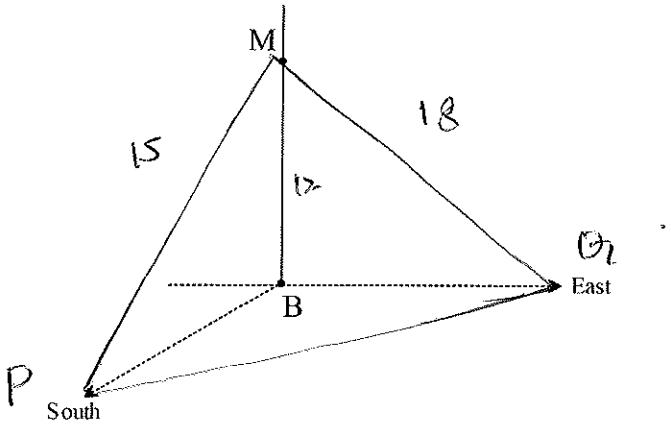
$$= 81.03$$

Bearing $60 + 81$

$= 141^\circ$ ✓

QUESTION 6 (5, 2 marks)

A wireless mast is supported by two wires MP and MQ, each attached to it at the point M, 12m from the base A of the mast. P,Q, and B are all on level ground, Q being due East and P due South of the mast. If MP and MQ are respectively 15m and 18m long:



(a) Find the distance between P and Q

$$PB = 15^2 - 12^2$$ ✓

$$= 9\text{ m}$$ ✓

$$QB = 18^2 - 12^2$$ ✓

$$= \sqrt{180}\text{ m}$$ ✓

$$PQ = 9^2 + (\sqrt{180})^2$$ ✓

$$= \sqrt{261}\text{ m}$$

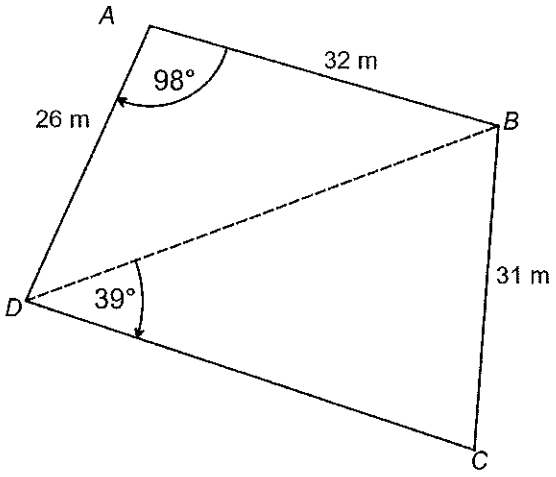
(b) Find the angle QMP between the wires

$$\cos\theta = \frac{15^2 + 18^2 - (\sqrt{261})^2}{2 \times 15 \times 18}$$

$$= 57.8^\circ$$

Question 7 (2, 2 marks)

Use quadrilateral ABCD to:



(a) Determine the length of DB, correct to two decimal places. (2 marks)

$$DB = 26^2 + 32^2 - 26 \times 2 \times 32 \times \cos 98$$

$$= \sqrt{193158}$$

$$= 43.95 \text{ m}$$

(b) Determine the size of ∠BCD. (2 marks)

$$\frac{31}{\sin 39^\circ} = \frac{43.95}{\sin\theta}$$

$$\sin\theta = \frac{43.95}{31} \times \sin 39^\circ$$

$$\theta = 63.15^\circ \text{ or } 116.85^\circ \quad 63^\circ \text{ or } 117^\circ$$

Question 7(2,3,3 marks)

The figure shown is such that XY = YM = 5 cm. The angle at the centre of the sector is 1.3 radians.

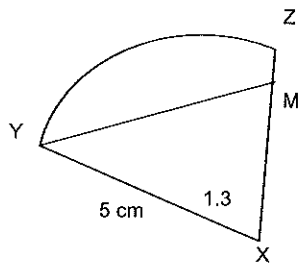
Calculate:

(a) the area of the sector XYZ. (2 marks)

$\frac{1}{2} r^2 \theta$ ✓

$= \frac{1}{2} \times 5 \times 5 \times 1.3$

$= 16.25 \text{ cm}$ ✓



(b) the length XM. (3 marks)

$XM = 5^2 + 5^2 - 2 \times 25 \times \cos(\pi - 2.6)$ ✓

$= 50 - 50 \cos 0.54$ ✓

$= 2.67 \text{ cm}$

✓

(c) the area of YMZ. (3 marks)

Area of YMZ = Area of Sector − Area of triangle ✓

$= 16.25 - \left(\frac{1}{2} \times 5 \times 5 \times \sin 0.54\right)$

$= 9.82 \text{ cm}^2$

✓