Eastern Goldfields College

Student Name MARKING KEY

Eastern Goldfields College

Mathematics Methods 2015

Test 5– Calculator Free

Working Time: 30 minutes

Total Marks: 40 marks

1. [2 marks – 1, 1]

The table shows the temperature of a liquid over a period of time.

Time (minutes)	0	5	10	15	20	25
Temperature (°C)	58	44	32	25	21	19

Determine the average rate of change of temperature of the liquid

(i) over the first ten minutes.

$\frac{32-58}{10} = -2.6$°C/min ✓

(ii) between 15 and 20 minutes.

$\frac{21-25}{5} = -0.8$°C/min ✓

2. [3 marks – 1, 2]

For each of the following, recognise and state the function being differentiated.

(a) $\lim\limits_{h\to 0}\frac{(x+h)^2-2(x+h)-(x^2-2x)}{h}$

$f(x) = x^2 - 2x$ ✓

(b) $\lim\limits_{h\to 0}\frac{3x^2+6xh+3h^2-3x^2}{h}$

$f(x) = 3x^2$ ✓✓

3. [3 marks]

Differentiate from first principles $y = 5x^2$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{5(x+h)^2 - 5x^2}{h} \checkmark$$

$$= \lim_{h \to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 - \cancel{5x^2}}{h} \checkmark$$

$$= \lim_{h \to 0} \frac{5\cancel{h}(2x+h)}{\cancel{h}}$$

$$= 10x \checkmark$$

4. [9 marks – 1, 1, 2, 2, 3]

Determine the gradient function for each of the following functions:

(a) $f(x) = -x^4$

$f'(x) = -4x^3$ ✓

(b) $h(x) = 3x^2 + 6x - 5$

$h'(x) = 6x + 6$ ✓

(c) $g(x) = 2x(3x-5) = 6x^2 - 10x$ ✓

$g'(x) = 12x - 10$ ✓

(d) $y = (x-1)^2 = x^2 - 2x + 1$ ✓

$\frac{dy}{dx} = 2x - 2$ ✓

(e) $f(x) = \frac{3x^3 - 9x^4}{6x^2} = \frac{x}{2} - \frac{3x^2}{2}$ ✓

$f'(x) = \frac{1}{2} - 3x$ ✓ ✓

5. [11 marks – 3, 4, 4]

(a) Determine the gradient of the function $f(x) = -x^3 - 5x$ at the point (2, -18).

$f'(x) = -3x^2 - 5$ ✓

$f'(2) = -3.4 - 5$

$= -17$ ✓

(b) Determine the equation of the tangent to the curve $g(x) = 2x(x-1)$ at $x = -2$.

$y = 12$ ✓

$= 2x^2 - 2x$

$g'(x) = 4x - 2$ ✓

$g'(-2) = -10.$ ✓

$y = -10x + c$

$12 = 20 + c$

$-8 = c$ ✓

$\therefore y = -10x - 8$

(c) Determine the coordinates of the point(s) on the curve $y = -2x^3$ where the gradient is -24.

$\frac{dy}{dx} = -6x^2$

$-6x^2 = -24$ ✓

$x^2 = 4$

$x = \pm 2$ ✓

$(2, -16)$ $(-2, 16)$

✓ ✓

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Test 5– Calculator Assumed

Working Time: 30 minutes

Total Marks: 31 marks

8. [7 marks – 2, 2, 1, 2]

An arithmetic sequence has an 9th term of 267 and a 14th term of 237.

(i) The sequence is defined by the rule $T_n = a + (n-1)d$. Determine the values of a and d.

$237 - 267 = -30$
$5d = -30$
$d = -6$ ✓
$267 = a + 8(-6)$
$315 = a$ ✓

(ii) Write a recursive rule for this sequence.

$T_{n+1} = T_n - 6, \quad T_1 = 315$ ✓ ✓

(iii) Calculate T_{50}.

$T_{50} = 21$ ✓

(iv) If $T_1 + T_2 + ... + T_n = 0$, determine the value of n.

$\frac{n}{2}(2a - 6(n-1)) = 0$ ✓
$n = 0, \; n = 106$ ✓

9. [10 marks – 2, 1, 2, 2, 3]

(a) Determine x if the terms 12, x, 27 form part of a geometric sequence.

$$\frac{x}{12} = \frac{27}{x}$$

$$x^2 = 12 \times 27$$

$$x = \pm 18 \quad ✓✓$$

(b) A team of workers is using a pile driver to drive wooden poles 4 metres long into the ground. The first hit of the pile driver drives a pole 50 cm into the ground. The second hit drives the pole another 40 cm into the ground. The third hit drives the pole another 32 cm into the ground and successive distances driven by the pile driver form a geometric sequence.

(i) How much further will the fourth hit drive the pole into the ground?

$$40 \div 50 = 0.8$$

$$32 \times 0.8 = 25.6\text{cm} \quad ✓$$

(ii) Determine the total distance the wooden pole has been driven into the ground after 12 hits of the pile driver.

$$S_{12} = \frac{50(1-0.8^{12})}{1-0.8} = 232.82\text{cm}$$

✓ ✓

(iii) If the workers continued in this way for some time, what length of the wooden pole will always be left above the ground? Justify your answer.

$$S_\infty = \frac{50}{1-0.8} = 250 \quad ✓$$

$$400 - 250 = 150 \text{ cm} \quad ✓$$

(c) A geometric series has a first term of 80 and a sum to infinity of 800. Determine the minimum number of terms of this sequence required so that their sum exceeds 700.

$$800 = 80 \div (1-r) \Rightarrow r = 0.9 \; ✓$$

$$\frac{50(1-0.9^n)}{1-0.9} \geq 700 \Rightarrow n \overset{=}{\cancel{\geq}} 20.$$

✓ ✓

6. [6 marks – 2, 2, 2]

The first three terms of an arithmetic sequence are $x+2,\ 3x+7,\ 5x+12$

(a) Determine the common difference of this sequence.

$3x+7-(x+2)$ ✓

$= 2x+5.$ ✓

(b) Determine an expression for T_7

$T_7 = x+2+6(2x+5)$ ✓

$= x+2+12x+30$

$= 13x+32$ ✓

$x+2+6(2.5)$

(c) Determine an expression for S_{10}, simplifying your answer.

$S_{10} = 5(2(x+2)+(2x+5)9)$ ✓

$= 5(2x+4+18x+45)$ ✓

$= 100x+245$

7. [6 marks]

The function $f(x)=ax^2+bx+c$ passes through the point (0, -4) and has a gradient of -15 at the point (-2, 10).

Determine the values of a, b and c.

$f'(0) = -4 = c$ ✓

$f(x) = ax^2+bx-4.$

$f'(x) = 2ax+b$ ✓

$-15 = -4a+b$ ✓ ①

$14 = 4a-2b$

$-1 = -b$

$1 = b$ ✓

$4 = a$ ✓

$10 = 4a-2b-4$

✓ $14 = 4a-2b$ ②

$14 = 4a \times 4/3$

$14 - 4/3 = 4a$

$40/3$ $38/3$

$\times 6/3$

$33.33/12$ ✓

10. [6 marks]

Peta starts her first job in an interior architecture firm. She is offered the following two pay packages.

Option A: Starting salary of $45 000 with a $1500 increase each year.

Option B: Starting salary of $40 000 with a 5% increase each year.

If she plans to stay at the firm for 10 years, which pay package should she choose? Explain your answer with mathematical reasoning.

$A_{10} = 45000 + 1500(9) = \$58500$ ✓

$S_{10} = \$517500$ ✓

$B_{10} = 40000 \times 1.05^{9} = \$62053$ ✓

$S_{10} = \$503115.70$ ✓

Although 10th yr greater option B, the total amt earned over ten yrs is greater with option ~~B~~ A. ✓ ✓

11. [8 marks – 3, 1, 4]

An infectious disease epidemic in a third world country records rapid growth in the number of new cases reported until an overseas aid agency is able to offer large scale medical assistance and the number of new cases reported decreases according to a constant ratio.

This geometric pattern is maintained until the epidemic is wiped out and the aid agency is able to leave.

Month	Jan 12	Feb 12	Mar 12	Apr 12	May 12	Jun 12	Jul 12	Aug 12
No of New Cases	454	3815	207312	39366	26244	17496	11664	7776

(a) Which month corresponds to the first term of the geometric sequence? April 2012 ✓
What is the common ratio? $\frac{2}{3}$ ✓✓

211581

(b) How many new cases were reported in September 2012?

5184 ✓

(c) How many new cases in total were reported for the time period January 2012 to June 2013 inclusive?

$S_{15} = \frac{39366\left(1 - \frac{2}{3}^{15}\right)}{1 - \frac{2}{3}} = 117828$ ✓ ✓

Total = 329409 ✓✓