Name _____

Section 1 – Resource Free – Students can have the formula sheet

Marks : ~~29~~ 34   Time: ~~30~~ 35 minutes (maximum)

1.   [7 marks]

Simplify the following (expressing with positive indices):

(a) $\dfrac{8 \times 2^{-7} \times 3^5}{9 \times 2^{-5} \times 81}$

$= \dfrac{2^3\, 3^5\, 2^5}{3^2\, 2^7\, 3^4}$ ✓

$= \dfrac{2}{3}$ ✓

(b) $\dfrac{7a^{-5}b^2}{(-2a^3b)^2} \div \dfrac{21a^{-3}b^2}{4a^{-1}b}$

$= \dfrac{7b^2}{2^2 a^6 a^5 b^2} \times \dfrac{2^2 ba^3}{a.\,21.\,b^2}$ ✓✓

$= \dfrac{7}{3a^9 b}$ ✓

(c) $\dfrac{\left(\sqrt[3]{x^2}\right)}{\left(\sqrt{3}x^{\frac{5}{6}}y^{-1}\right)^2}$

$= \dfrac{x^{2/3}}{3\, x^{5/3}\, y^{-2}}$ ✓

$= \dfrac{y^2}{3x}$ ✓

2.   [~~5~~ 7 marks]

Solve the following equations

a) $9^{1-x} = 27^{x-2}.3^{x+2}$

$3^{2(1-x)} = 3^{3(x-2)}.3^{x+2}$

$2-2x = 3x-6 + x+2$

$4 = 4x$

$x = 1$ ✓✓✓

b) $2x + y = 10$ and $25^{3x+y} = 625^2$

$5^{2(3x+y)} = (5^3)^2$

$6x+2y = 6$

$2x+y = 10$ ✓✓✓

$2x = -14$

$x = -7$

$y = 24$

$\boxed{14}$

3. [4 marks] 7

a) Find the exact value of sin (-15°)

$$= \sin(30-45)$$
$$= \sin 30 \cos 45 - \cos 30 \sin 45$$
$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$
$$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

✓
✓
✓
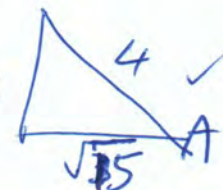
b) Given that $\sin A = \frac{1}{4}$ and $\tan B = \frac{3}{4}$, where A is acute and $180° < B < 270°$

Find:

a) $\cos B = \frac{\sqrt{15}}{4} - \frac{4}{5}$ ✓

b) $\cos(A-B)$

$$= \cos A \cos B + \sin A \sin B$$
$$= \frac{\sqrt{15}}{4} \cdot \left(-\frac{4}{5}\right) + \frac{1}{4}\left(\frac{-3}{5}\right)$$
$$= -\frac{4\sqrt{15}}{20} + \frac{3}{20}$$
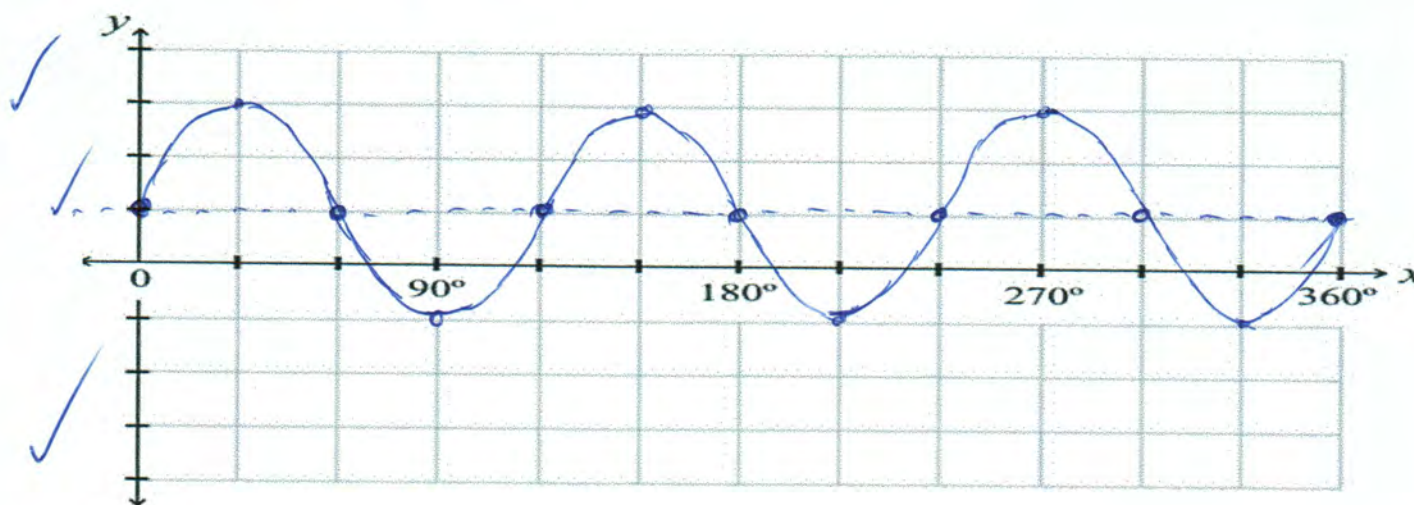$$= \frac{-3 - 4\sqrt{15}}{20}$$

✓✓

4. [5 marks]
Graph $y = 2\cos 3x + 1$ $0° \leq x \leq 360°$

✓ Period = 120°

✓
✓
✓

11

5. **[5 marks]**

Prove the **identities**:

a) $\sin x \tan x + \cos x = \dfrac{1}{\cos x}$

$$LHS = \sin x \frac{\sin x}{\cos x} + \cos x$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x}$$

✓✓✓

b) Prove $\sin(x + 2n\pi) = \sin x$

$$LHS = \sin x \cos 2n\pi + \cos x \sin n\pi$$

$$= \sin x \cdot 1 + \cos x \cdot 0$$

$$= \sin x$$

✓✓

6. **[5 marks]**

If $t_{10} = 100$ and $t_{15} = 175$, find the first term, the common difference and hence the nth term for the arithmetic sequence.

$$5d = 75$$
$$d = 15 \quad ✓✓$$

$$100 = a + 9d$$
$$100 = a + 135$$
$$a = -35 \quad //$$

$$T_n = -35 + (n-1)$$

$\boxed{9}$

Name

Section 2 – Resource Rich – calculators, formula sheet and 1 page of notes
Marks : 31

Time: 30 minutes (minimum)

7. **[1, 1, 1, 2, 2 = 7 marks]**

Drilling tests show that in sinking a well, the distance drilled each hour decreases by 10%. A depth of 20 metres is drilled in the first hour.

(a) Find how much is drilled in the second and third hours.

$$G.P. \quad a = 20 \quad r = 0.9$$
$$a_2 = 18m \quad a_3 = 16.2m$$

[1]

(b) Explain why the distances drilled each hour will form a geometric progression.

Common ratio $r = 0.9$

[1]

(c) Find the distance drilled in the 10th hour, correct to the nearest centimetre.

$$a_{10} = 7.748m = 7.75m$$

[1]

(d) How long will it take to drill a depth of 100 metres? (answer to the nearest minute)

$$S_n = 100$$
$$a = 20$$
$$r = 0.9$$

$$n = 6.58h$$
$$6 \text{ hours } 35 \text{ mins}$$

[2]

(e) The contractors estimate that the gas they are looking for is somewhere between 250 and 300m below ground? When would they expect to first find the gas? Some justification is required for this question.

Never reach since $S_\infty = 200m$

[2]

$\boxed{7}$

8.   [2, 2, 2, 2 = 8 marks]

Amy is left a sum of $ 460 000 in a will and considers investing the money in a bank account paying 6.25% **pa** interest compounded annually. She wants to withdraw $ 40 000 per year for living expenses.

a)   Write a **recursive set** of equations for this information.

$$T_{n+1} = T_n \times 1.0625 - 40000 \qquad T_0 = 460000$$

✓ ✓

b)   For how many years will she be able to withdraw this amount from the account?

20 years

runs out during 21st year

c)   What is the total amount of interest she would receive before her investment ran out of money?

$$21 \times 40000 - 2954$$
$$= 837046$$

Interest = $377046

✓✓

d)   If she wanted her money to last about 40 years, how much could she take out each year? (Some justification is needed for your answer.)

Trial and error

$ 31 500

✓✓

9   [3 marks]

A hiker sets out on a 100km hike. She walks 36 km on the first day and $\frac{2}{3}$ that distance on the second. Every day thereafter she walks $\frac{2}{3}$ of the distance she walked on the day before.
Will the hiker cover the distance of 100 km to complete the walk and if so, on what day will she complete the task?   GP $a = 36$ $S_n = 100$ ✓

$$r = \frac{2}{3}$$

$$n = 6.419$$

hence during 7th day.

✓
✓
10

10. **[4 marks]**

   a) Insert four evenly spaced numbers between 8 and 36.

   $\underset{a_1}{8}$ $\underline{13 \cdot 6}$ $\underline{19 \cdot 2}$ $\underline{24 \cdot 8}$ $\underline{30 \cdot 4}$ $\underset{a_6}{36}$ $d = 5 \cdot 6$ ✓/

   b) Insert three numbers between 4 and 2500 so that they are in geometric progression.

   $\underset{a}{4}$ $\underline{20}$ $\underline{100}$ $\underline{500}$ $\underset{a_5}{2500}$ $r = 5$ ✓✓

11. **[4 marks]**

   Find the sum of all powers of 2 between 500 and 50000.

   $2, 4, 8$ $r = 2$

   $n = 9$ $a_9 = 512$ ✓/

   $n = 15$ $a_{15} = 32768$ ✓

   Let $a_1 = 512$ $S_7 = 65024$ ✓

   $n = 7$ ✓

   $r = 2$ ✓

12. **[2 marks]**

   Solve $3 - 2\cos(3x + \frac{\pi}{3}) = 1$ $\pi \le x \le 3\pi$ 3 sf

   $x = 4 \cdot 54$ or $6 \cdot 63$ or $8 \cdot 73$ ✓✓

   $\boxed{10}$