

SCOTCH
COLLEGE



12 Mathematics Methods 2023

Test 2 – Integration and Applications

Section 2: Calculator-Assumed

Time allowed: 25 minutes

Maximum marks: 25

Name: Solutions

Teacher: Foster | Kelly

Instructions:

- Show all working clearly.
 - Sufficient detail must be shown for marks to be awarded for reasoning.
 - A formula sheet will be provided.
 - Calculators and 1 A4 page (2 sides) of personal notes are permitted.
- .

Question 5 [7 marks]

The velocity in metres per second of an object that travels on a straight line is given by;

$$v(t) = \frac{8t(t^2 - 6t + 4)}{3} \text{ for } 0 \leq t \leq 5$$

Calculate the:

- a) time(s) that the object is at rest

[2]

$$v(t) = 0; \quad t = 0, 0.764 \text{ secs}$$

- b) acceleration of the object at $t = 3$

[1]

$$a(3) = v'(3) = -13\frac{1}{3} \text{ m/s}^2$$

- c) change in displacement of the object during the third second

[2]

$$\int_2^3 v(t) dt = -31\frac{1}{3} \text{ m}$$

- d) total distance travelled by the object

[2]

$$\int_0^5 |v(t)| dt = 118.59 \text{ m}$$

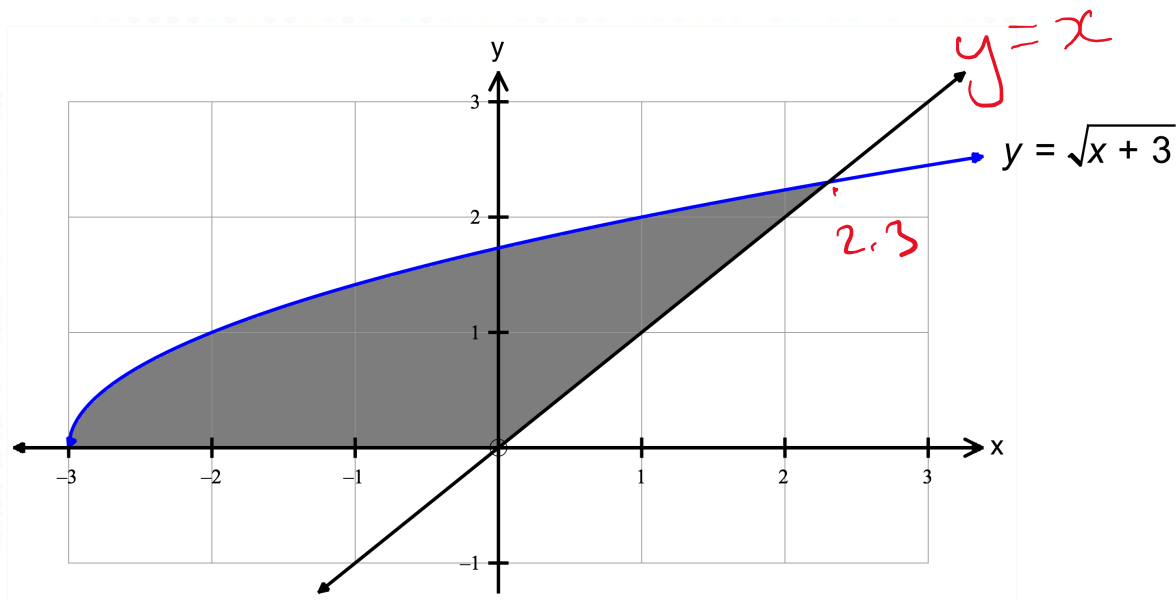
unit penalties
throughout question

-1

Question 6 [3 marks]

Show the use of integral(s) to determine the shaded area below.

Give your answer to two decimal places.



meet when, $x = \sqrt{x+3}$
 $x = \frac{\sqrt{13}}{2} + \frac{1}{2}$

Shaded area = $\int_{-3}^0 \sqrt{x+3} dx + \int_0^{\frac{\sqrt{13}}{2} + \frac{1}{2}} (\sqrt{x+3} - x) dx$ ✓ FT

= 5.49 units² ✓ FT

✓

Question 7 [5 marks]

The birth rate, in thousands, for a population of insects observed over a 10-year period from the start of 2010 is given by:

$$b(t) = 8 + 3t, 0 \leq t \leq 10$$

- a) At what rate is the population of insects changing at 5 years?

[1]

$$b(5) = 23 \text{ thousand insects / year.}$$

- b) How many insects are born over the 10-year period modelled by the function $b(t)$?

[2]

$$\int_0^{10} b(t) dt = 230\,000$$

- c) At what time does the total number of insects born reach 50 000?

Give your answer to the nearest month.

[2]

$$\int_0^x 8 + 3t dt = 50$$

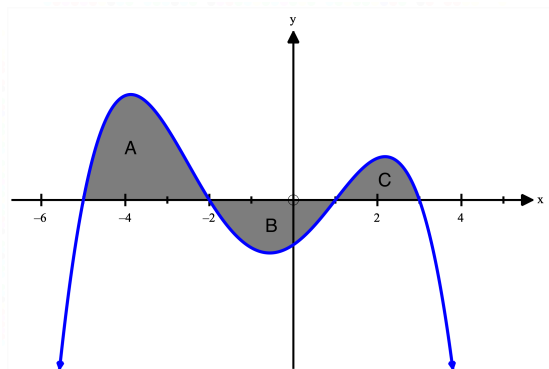
$$x = 3.6929$$

\therefore during Aug 2013

unit penalty (-1)

Question 8 [6 marks]

The graph of $y = f(x)$ as well as the area of each region enclosed by the curve and the x - axis is shown in the table below.

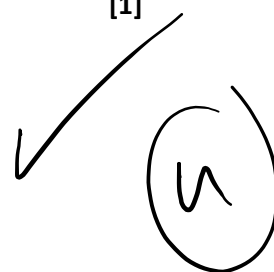


Region	A	B	C
Area of region	12	15	7

- a) Determine the area enclosed between the graph of $y = f(x)$ and the x - axis, from $x = -5$ to $x = 3$.

[1]

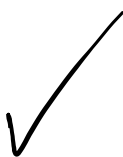
$$12 + 15 + 7 = 34 \text{ units}^2$$



- b) Determine the value of:

$\int_{-2}^3 f(x) dx$ [1]

$$-15 + 7 = -8$$



ii) $\int_{-2}^{-5} \frac{f(x)}{4} dx$ [2]

$$= -\frac{1}{4} (12) = -3$$

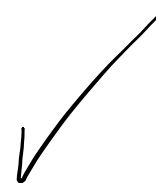


i) $\int_{-5}^1 3 + f(x) dx$

[2]

$$= [3x]_{-5}^1 + [12 - 15]$$

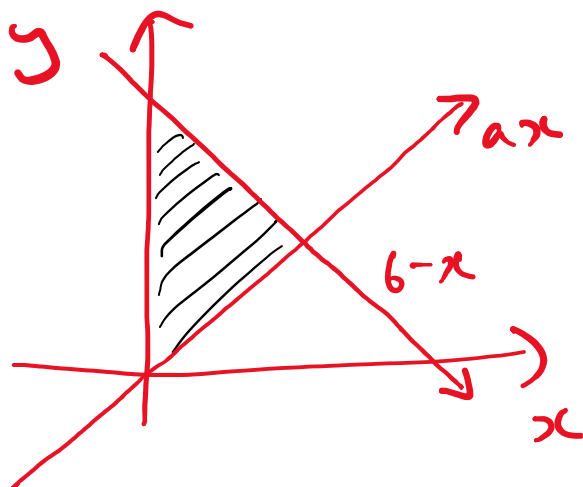
$$= (3 + 15) - 3 = 15$$



Question 9 [4 marks]

The area enclosed by the lines $y = ax$, $y = 6 - x$ and the positive y -axis is k units² where a is greater than zero.

By first writing an integral, determine the value of a terms of k .



$$ax = 6 - x$$

$$x(a+1) = 6$$

$$x = \frac{6}{a+1}$$

✓

solve

$$k = \int_0^{\frac{6}{a+1}} (6-x) - ax \, dx, a$$

✓ ✓

$$a = \frac{18}{k} - 1$$

✓