Name:................................

| Resource Free | /24 | % |
|---|---|---|
| Resource Rich | /36 | % |
| Total | /60 | % |

# Mathematics Methods, Year 12, 2018

Test 2 – Differentiation and Integration

24 minutes working time.

Calculator Free Section (no notes, no calculators) SCSA Formula sheet allowed

1. (5 marks)

Determine each of the following. Express your answers with positive indices.

(a) $\int \frac{1}{2}x^3\,dx \quad = \frac{x^4}{8}$ ✓ (1)

(b) $\int \frac{3}{\sqrt{(2-3x)}}\,dt \quad = 3(2-3x)^{-1/2}$ (2)

$$= \frac{3(2-3x)^{1/2}}{\frac{1}{2}\times -3} \checkmark$$

$$= -2\sqrt{2-3x} \checkmark$$

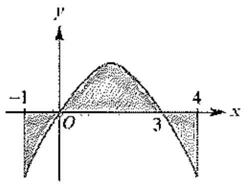
(c) $\int 8cos4x - sin4x\,dx$ (2)

$$= \frac{8\sin 4x}{4} - -\frac{\cos 4x}{4} \checkmark$$

$$= 2\sin 4x + \frac{\cos 4x}{4}. \checkmark$$

2. (4 marks)

The graph of $y = x(3-x)$ is shown below.




(a) Evaluate $\int_{-1}^{4} x(3-x)dx$ (2)

$= \int_{-1}^{4} 3x - x^2 \, dx$

$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_{-1}^{4}$ ✓

$= \left[\frac{3(4)^2}{2} - \frac{4^3}{3}\right] - \left[\frac{3(-1)^2}{2} - \frac{(-1)^3}{3}\right]$ ✓

$= \left[24 - \frac{64}{3}\right] - \left[\frac{3}{2} + \frac{1}{3}\right] = 5/6 \text{ units}^2$

(b) Find the exact area of the shaded region in the figure. (2)

$= \int_{0}^{3} 3x - x^2 - 2\int_{3}^{4} 3x - x^2 \, dx.$

$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 - 2\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_3^4$ ✓

$= \left[\frac{3(3)^2}{2} - \frac{3^3}{3}\right] - 2\left[\left(\frac{3(4)^2}{2} - \frac{4^3}{3}\right) - \left(\frac{3(3)^2}{2} - \frac{3^3}{3}\right)\right]$

$= \left[\frac{27}{2} - 9\right] - 2\left[\left(24 - \frac{64}{3}\right) - \left(\frac{27}{2} - 9\right)\right]$

$= 4\frac{1}{2} - 2\left[-\frac{11}{6}\right]$

$= 8\frac{1}{6} \text{ units}^2.$ ✓

3 (4 marks)

Find each of the following

(a) $\frac{d}{dx}\int_0^x \sqrt{t^2-3}\,dt$ $= \sqrt{x^2-3}$. ✓ (1)

(b) $\frac{d}{dx}\int_x^4 (t^3-1)\,dt$ $= -(x^3-1)$ ✓ (1)

(c) $\frac{d}{dx}\int_{-3}^{4x} \left(\frac{5+t}{t-4}\right) dt$ $= \left(\frac{5+4x}{4x-4}\right) \cdot 4$ ✓ ✓ (2)

4. (5 marks)

Given that $y = x^2e^x$, prove that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2e^x(1+x) = 0$

$$y = x^2 e^x$$

$$\frac{dy}{dx} = 2x e^x + x^2 e^x \checkmark$$

$$\frac{d^2y}{dx} = \left(2e^x + 2xe^x\right) + \left(2xe^x + x^2e^x\right) \checkmark$$

$$= 4xe^x + x^2e^x + 2e^x. \checkmark$$

$$\checkmark$$

$$LHS = \left(4xe^x + x^2e^x + 2e^x\right) - \left(2xe^x + x^2e^x\right) - 2e^x(1+x)$$

$$= 4xe^x + x^2e^x + 2e^x - 2xe^x - x^2e^x - 2e^x - 2xe^x \checkmark$$

$$= 0$$

$$= RHS.$$

5. (6 marks)

Find the derivative of each of the following; you do not need to simplify your answer.

(a) $y = cos\frac{\pi}{4}$ (1)

$dy/dx = 0$. ✓

(b) $y = \cos(1-2x)^3$ (2)

$$\frac{dy}{dx} = -\sin(1-2x)^3 \times 3(1-2x)^2 \times -2.$$ ✓ ✓

(c) $y = cos2x\sin(1+3x)$ (3)

$$\frac{dy}{dx} = -2\sin 2x \sin(1+3x) + \cos 2x \cos(1+3x) \times 3.$$ ✓ ✓ ✓

Name: ....................................

# Mathematics Methods, Year 12, 2017
## Test 2 – Differentiation and Integration

36 minutes working time. Calculator Assumed Section (notes allowed), SCSA Formula sheet and calculators allowed

| Resource Rich | /36 | % |
|---|---|---|

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1 (10marks)

Biologists analysing the spread of feral animals in outback Western Australia note that the rate of growth of the population of a species of European Wild rabbit (*Oryctolagus cuniculus*) is proportional to its population at any time. The approximate number of rabbits at the start of 2000 was 850 000 and the continuous percentage growth rate is 2.5% per year.

(a) Determine the equation relating the population ($P$) of the rabbits to the time ($t$) years after the beginning of 2000. (2)

$P = 850000\, e^{0.025t}$ ✓✓

(b) Find the population of the rabbits at the beginning of 2016, to the nearest 100 rabbits. (2)

$P = 850000\, e^{0.025(16)}$ ✓ $= 1268050.993$

$= 1268100$ ✓

(c) What is the instantaneous rate of growth of the number of rabbits at the start of 2016? (2)

$\frac{dP}{dt} = 0.025P$ ✓ $= 0.025 \times 1268050.993$

$= 31701$ ✓

(d) In order to control the rapid growth of the feral rabbits, the *myoxama* virus was introduced at the beginning of 2016. The population decay after the virus was introduced is modelled by $\frac{dP}{dt} = -0.185P$, where $P$ is the population after the beginning of 2016. (4)

In what year will the rabbit population be reduced to less than 250000.

$250000 = 1268050\, e^{-0.185t}$ ✓

$t = 8.78$ ✓

During 2024 the population of rabbits will fall to below 250000. ✓✓

2. (9 marks)

A particle is moving in rectilinear motion such that velocity, *m/min* at any time, *t* minutes , is given by:

$$v(t) = 4t^3 - 12t^2 + 6t + 4$$

If the particle begins 4 metres to the left of the origin, determine:

(a) When the particle stops. (2)

$4t^3 - 12t^2 + 6t + 4 = 0$ ✓

✓ $t = 1.37$ min and 2min.

(b) The displacement at any time *t*. (2)

$x = \int v(t)\ dt$

$= \int 4t^3 - 12t^2 + 6t + 4\ dt$

$= t^4 - 4t^3 + 3t^2 + 4t + c$. ✓

When $t = 0$, $x = -4$.

$\therefore\ c = -4$.

$x = t^4 - 4t^3 + 3t^2 + 4t - 4$. ✓

(c) The acceleration as the particle passes through the origin. (3)

$a = \frac{d}{dx} v(t)$.

✓ $= 12t^2 - 24t + 6$

✓ $x = 0$, when $t = 1$ and 2

$A(1) = -6$ m/min$^2$ ✓

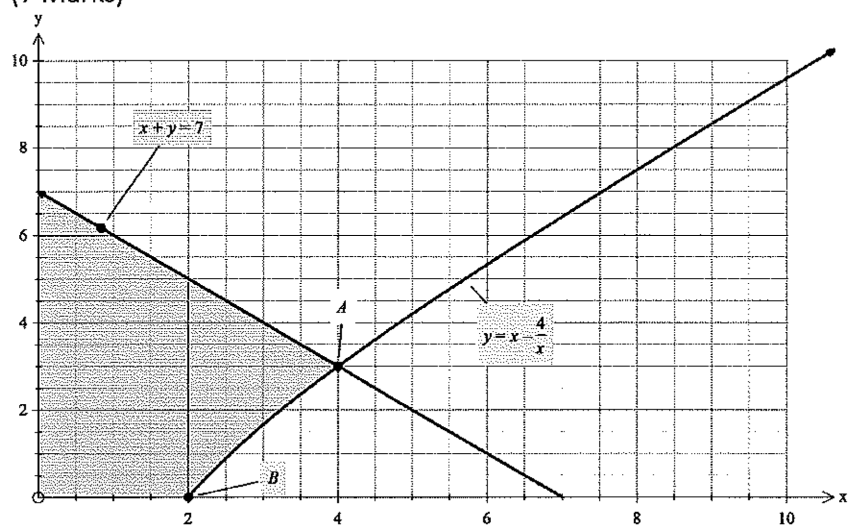
$A(2) = 6$ m/min$^2$

(d) The total distance travelled by the particle in the first two minutes. (2)

Total Distance $= \int_0^2 |4t^3 - 12t^2 + 6t + 4|\ dt$. ✓

✓ $= 4.7$ m.

3. (7 Marks)




The shaded area is bounded by the x and y axes, the line $x + y = 7$ and the curve with equation $y = x - \frac{4}{x}$

(a) Determine the co-ordinates of A and B (3)

A $\left.\begin{cases} x+7=7 \\ y = x - \frac{4}{x} \end{cases}\right|_{x,y} = (4,3)$ ✓✓

B. $x - \frac{4}{x} = 0$

$x^2 = 4$

$x = \pm 2.$

$(2,0)$ ✓

(b) State an expression, which when evaluated will determine the area of the shaded region. Give the area correct to two decimal places. (4)

$= \int_0^4 7 - x \, dx - \int_2^4 x - \frac{4}{x} \, dx.$ ✓ ✓

$= 20 - 3.227$ ✓

$= 16.77$ units$^2$. ✓

4. (4marks)

The rate of change in temperature °$C$ in a steel bar at any time $t$ minutes after an initial reading is given as:

$$\frac{dT}{dt} = 10 + \frac{300}{(t+1)^3} \quad where\ t > 0$$

The temperature of the bar one minute after the initial reading in 80°C.

Determine:

(a) The change in the temperature of the bar during the second minute. (2)

$$\int_1^2 10 + \frac{300}{(t+1)^3}\, dt = 30.83°C.$$

✓ ✓

(b) The temperature of the bar at any time $t$. ✓ (2)

$$T = \int 10 + \frac{300}{(t+1)^3} = 10x - \frac{150}{(x+1)^2} + c.$$

when $T = 80°C$ $t = 1$.

$$\therefore 80 = 10x - \frac{150}{(x+1)^2} + c.$$

$$c = 107.5.$$

✓

$$\therefore T = 10x - \frac{150}{(x+1)^2} + 107.5.$$

5. 6 marks

A particle, initially at rest at a displacement of 1 metre from the origin 0, moves in a straight line so that its velocity $v\ metres/\sec after\ t\ seconds$ is given by the equation

$v = 3\cos t - \sin t$

a. If after $time\ t$, the $displacement\ is\ x\ metres$ and the $acceleration\ is\ a\ metre/sec^2$, show that $x + a = 0$ (4)

$x = \int v(t)\, dt.$

$= 3\sin t + \cos t + C.$

when $t=0$, $x=1$

$1 = 3\sin 0 + \cos 0 + C$

$C=0$

$\therefore x = 3\sin t + \cos t.$ ✓✓

$a = \frac{dv}{dt}.$

$= -3\sin t - \cos t.$ ✓

$LHS = -3\sin t - \cos t + 3\sin t + \cos t.$ ✓

$= 0$

$= RHS.$

b. Find the maximum acceleration of the particle. (2)

$a = -3\sin t - \cos t.$

$\frac{da}{dt} = -3\cos t + \sin t.$

let $\frac{da}{dt} = 0$

$\therefore \sin t = 3\cos t.$

$\frac{\sin t}{\cos t} = 3.$

$\tan t = 3.$

$t = \tan^{-1} 3$

$= 1.249.$ ✓

$a(1.249) = 3.162\ m/sec^2$ ✓