Student Name:______________________

Eastern Goldfields College

Mathematics Applications U3&4 2017

Investigation 2

Working Time: 55 minutes | CALCULATORS ARE ALLOWED | Total Marks: 50

COMPETITION

In this investigation, networks and matrices will be used to represent competition between teams and players. Use the letters provided to represent the nodes.

Question 1 (10 marks: 2, 2, 2, 2, 2)

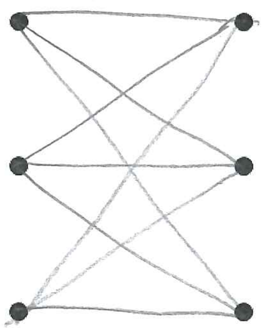
Team A has three players:
- Kyle (K),
- Rod (R) and
- Ahmed (A)

Team B has three players:
- Stefan (S),
- John (J) and
- Nigel (N)

Each person in Team A plays a match against each person in Team B.

These nine matches can be represented by a bipartite graph in which the members of Team A form one set of nodes and the members of Team B form another set of nodes. There is an edge drawn from each member of Team A to each member of Team B.

(a) Draw the bipartitie graph described above. [2]



(b) The adjacency matrix for this non-directed network is given below. [2]
Describe what the elements of the adjacency matrix represent.

$$\begin{array}{c} \\ K \\ R \\ A \end{array}\begin{array}{c} \begin{array}{ccc} S & J & N \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

every element = '1' ⇒ every player plays each opposition player exactly once.

or the 1's rep all edges from 1 set to other.

(c) Team C has four players: Lou (L); Nat (N); Trish (T); and Ali (A).

Team D has four players: Di (D); Indi (I); Jane (J); and Pat (P)

Each person in Team C plays a match against each person in Team D.

Represent the matches between the two teams as

(i) A bipartite graph [2]

✓ labeled ✓ complete

(ii) An adjacency matrix [2]

$$\begin{array}{c} \\ L \\ N \\ T \\ A \end{array}\begin{array}{c} \begin{array}{cccc} D & I & J & P \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

✓ labelled. ✓ all elements = 1.

(d) For two teams of n players in each team where each person in the first team plays a match against each player in the second team, describe the adjacency matrix. [2]

- square matrix with dimensions $n \times n$
- all elements = 1

Question 2 (12 marks: 1, 1, 2, 2, 4, 2)

An extra person has joined Teams C and D.

Team C now has five players: Lou (L), Nat (N), Trish (T), Ali (A) and Barb (B).

Team D now has five players: Di (D), Indi (I), Jane (J), Pat (P) and Kath (K).

Each person from each team plays three matches against three players from the other team and these are represented by a ✓ in the table below.

TEAM C

TEAM D

	L	N	T	A	B
D	✓	✓	✓		
I			✓	✓	✓
J	✓			✓	✓
P		✓	✓		✓
K	✓	✓		✓	

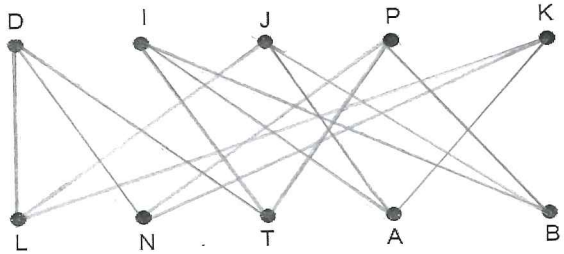
(a) Name the people that Jane plays. [1]

Lou, Ali + Barb. ✓ R1 ✓

(b) How many matches altogether? [1]

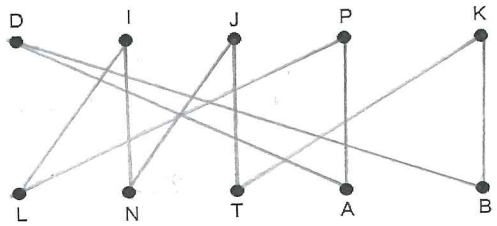
15 ✓

(c) Use a bipartite graph to represent the matches played between the two teams. [2]



✓ ✓

(d) The complement of this graph is the graph where the edges all represent the matches that were NOT played between the two players at each node. Draw the complement described. [2]



✓ ✓

(e) Determine the adjacency matrix for: [4]

(i) the played matches in part (c)

Both matrices ✓

	L	N	T	A	B
D	1	1	1	0	0
I	0	0	1	1	1
J	1	0	0	1	1
P	0	1	1	0	1
K	1	1	0	1	0

✓

(ii) the matches which were not played in part (d)

	L	N	T	A	B
D	0	0	0	1	1
I	1	1	0	0	0
J	0	1	1	0	0
P	1	0	0	1	0
K	0	0	1	0	1

✓

✓ Both match bipartite graphs. F.T.

(f) Describe the matrix formed by adding the two matrices from part (e). [2]

✓ 5 × 5 matrix where all elements = 1 ✓

Question 3 (8 marks: 1, 1, 1, 1, 3, 1)
Three more players have joined the A and B teams. The new teams are as follows:

Team A: Kyle (K), Rod (R), Ahmed (A), Barry (B), Dong (D) and Terry (T)

Team B: Stefan (S), John (J), Nigel (N), Fraser (F), Chas (C) and Peter (P)

Each person in Team A is scheduled to play each person in Team B. Some matches have already been played. The adjacency matrix of the complement of the graph for all matches played is provided below.

$$\begin{array}{c} \\ S \\ J \\ N \\ F \\ C \\ P \end{array}\begin{array}{c} \begin{array}{cccccc} K & R & A & B & D & T \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(a) On the graph represented by this matrix, how many edges will connect Chas to Team A players? [1]

4 ✓

(b) How many edges in total will there be on the graph represented by the matrix given? [1]

19 ✓

(c) How can you use the elements of the matrix to determine the answer to part (b)? [1]

Add all elements. ✓

or count all the 1's

(d) Which players has John already played? [1]

Rod, Barry + Terry. ✓R/W

(e) Which player has not played any matches? Justify your answer. [3]

✓Stefan ∵ matrix = compliment of games played. ∴ all elements = 1 ⇒ no game played. ✓✓

Stephen has all 1's in his row.

or ✓(1) ⇒ game.

(f) Which node on the bipartite graph of matches played will be unconnected from the network? [1]

S or Stefan ✓

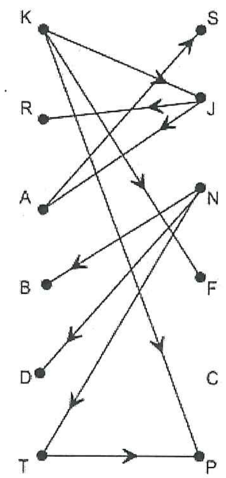
For Questions 4 and 5, winning and losing matches will be considered and the graphs will be digraphs.

Question 4 (9 marks: 2, 1, 1, 3, 2)
The digraph below represents the matches played between members of Team A and members of Team B. No connection indicates a match has not been played and a win is represented by ?. →

Example F ⟶ G indicates F beats G and G loses to F

Team A: Kyle (K), Rod (R), Ahmed (A), Barry (B), Dong (D) and Terry (T)

Team B: Stefan (S), John (J), Nigel (N), Fraser (F), Chas (C) and Peter (P)



Must state: Team A → Team B or {K, R, A, B, D, T} {S, J, N, F, C, P}

✓ identifies nodes in 2 groups

✓ explains connection

(a) The graph is bipartite. Explain. [2]

- No player on same team plays another player in their own team
- 2 sets of nodes. ⇒ 1 ∃ team each set of nodes.
- Each team represents i.e. Team A (K, R, A, B, D, T) = set 1 & (S, J, N, F, C, P) = set 2.

(b) Which player has not played any matches? [1]

C or Chas ✓

(c) How many matches have been played? [1]

10 ✓

	In	Out
L	1	3
N	1	3
T	2	2
A	2	2
D	4	0

(d) Which players have:

i. won every match they have played? [1]

Kyle + Nigel ✓R/W

ii. lost every match they have played? [2]

Stefan Rod Barry Dony Peter Fraser

✓=4 ✓✓=6.

(e) Describe two of the features of the adjacency matrix for this network. [2]

- 12x12 matrix 10 1's ✓ = 1 feature ✓ = 2nd feature
- any element = 0 ⇒'s loss or no game
- Not symmetrical ∵ directed (win/loses)
- any element = 1 ⇒'s win for person in that row and loss for player in that col
- leading diagonal all 0's as no-one plays a game against themselves ✓

Question 5 (11 marks: 1, 1, 2, 2, 2, 1, 2)

Team E has five players: Lily (L), Nora (N), Tricia (T), Alison (A) and Deirdre (D).

They need to determine a playing order so they play each other. The results of these matches are given in the table below.

	L	N	T	A	D
L beats	-	✓	✓	X	✓
N beats	X	-	✓	✓	✓
T beats	X	X	-	✓	✓
A beats	✓	X	X	-	✓
D beats	X	X	X	X	-

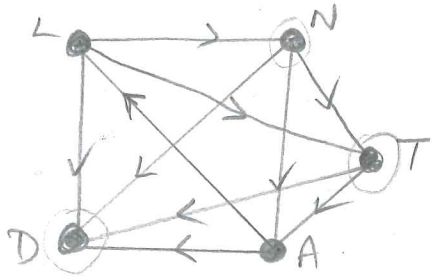
(a) Who wins the most matches? Lily + Nora. [1] ✓R/W

(b) Who loses the most matches? Deirdre [1] ✓

It is NOT symmetrical across the leading diagonal ∵ One person must win a game. i.e./eg. L v N · L=1 N=0. ∵ L won N lost. ✓ compliant to each other

OK

(c) Use a digraph to represent this information. [2]



✓ correct nodes + connections

✓ correct direction

(d) Is the digraph drawn in part (c) bipartite? Justify your conclusion. [2]

No ∵ all players played each other ×1. ✓ complete

To be bipartite there must be 2 distinct groups/sets of players, where each player with 1 set cannot play another since all played each other ≠ bipartite ✓ reason

(e) Determine the adjacency matrix for the digraph. [2]

	L	N	T	A	D
L	0	1	1	0	1
N	0	0	1	1	1
T	0	0	0	1	1
A	1	0	0	0	1
D	0	0	0	0	0

✓ 5×5 matrix labeled.

✓ all elements.

(f) Explain the nature of the elements along the leading diagonal. [1]

All Ø ∵ cannot play yourself ✓ in relation to the Q n/scenario

(g) Describe and explain the symmetry of the adjacency matrix about the leading diagonal. [2]

Is symmetric across leading diagonal wrt games played. ✓✓

e.g. $E_{NT} = 1$ & $E_{TN} = 0$

⇒ N played T + N won (1) & T lost (0)

ie. Where there is a 1 there is a corresponding 0 in the symmetrical [illegible]

~ END OF INVESTIGATION ~