Mathematics Department

11 Maths Methods Test 7 Odd

Applications of Differentiation, Exponential Functions and Antidifferentiation

Name

Section 1 – Resource Free – Students can have the formula sheet
Marks : 18 **Time: 18 minutes (maximum)**

1: [2, 2, 3, 3 = 10 marks]

Find the anti-derivative of the following:

a) $\frac{dy}{dx} = 3x^3 + 4x$

$y = \frac{3x^4}{4} + 2x^2 + c$

✓✓

b) $\frac{dy}{dx} = \frac{4}{x^2}$ $= 4x^{-2}$

$y = -4x^{-1} = \frac{-4}{x} + c$ ✓✓

c) $\frac{dy}{dx} = (8x^2+1)(4x-3)$

$\frac{dy}{dx} = \cancel{3}2x^3 - 24x^2 + 4x - 3$

$y = 8x^4 - 8x^3 + 2x^2 - 3x + c$

✓✓✓ lose one no c

d) $\frac{dy}{dx} = \frac{9x^3 - 8x^4}{x^2}$ $= 9x - 8x^2$

$y = \frac{9x^2}{2} - \frac{8x^3}{3} + c$

✓✓✓

2. (4 marks)

Find y as a function of x given $\frac{dy}{dx} = 4x^3 + 15x^2 - 14x$ and $y = 1$ when $x = 1$

$y = x^4 + 5x^3 - 7x^2 + c$ ✓✓

Subst $(1,1)$ $1 = 1 + 5 - 7 + c$ ✓

$c = 2$

$\therefore y = x^4 + 5x^3 - 7x^2 + 2$ ✓

3. (4 marks)

$\$C$ is the cost of producing x units of a product. If $C'(x) = 200 - 4x$ find the extra cost incurred by producing 40 units rather than 20.

$C(x) = 200x - 2x^2$

$C(20) = 4000 - 800 = 3200$

$C(40) = 8000 - 3200 = 4800$

$\therefore$ Extra cost is $\$1600$

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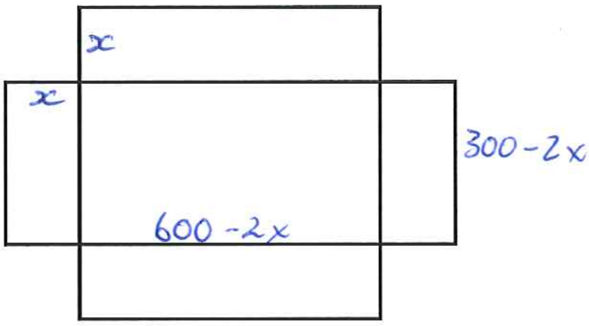
Section 2 – Resource Rich – calculators, formula sheet and 1 page of notes

Marks : ~~**39**~~ 40 **Time: 40 minutes (minimum)**

1. [2, 4 = 6 marks]

A piece of heavy cardboard, measuring 600cm x 300cm, is to be made into an open box. This is to be done by removing a square from each corner and folding up the resulting flaps to form the box.

a) If x is the length of the side of the square cut out of the box, find an expression for the volume of the box in the form $V = ax^3 + bx^2 + cx + d$



$V = x(600-2x)(300-2x)$ ✓

$V = 4x^3 - 1800x^2 + 180000x$ ✓

b) Use calculus techniques to find the maximum possible volume of this box (to the nearest litre).

$\frac{dV}{dx} = 12x^2 - 3600x + 18000$ ✓

Max $\frac{dV}{dx} = 0$

ie $x = 63.4$, or 236.6 ✓

$\therefore x = 63.4$

$V = 5196152\ cm^3$ ✓

$V = 5196\ L$ ✓

$\boxed{6}$

2. [1, 3, 4, 1, 1 = 10 marks]

The displacement s (in metres) at time *t* (in seconds) of a particle moving in a horizontal straight line is given by

$$s(t) = (t - 3)(2t + 3)(t - 6).$$

Find:

a) the initial displacement of the particle

$S(0) = (-3)(3)(-6) = 54m$

b) Use calculus to determine when the particle changes direction.

Change when $s'(t) = 0$

$s(t) = 2x^3 - 15x^2 + 9x + 54$

$s'(t) = 6t^2 - 30t + 9$

$t = 0.32$ or $t = 4.68$

c) The total distance travelled in the first 5 seconds (to the nearest metre).

$S(0) = 54$] 1.41

$S(0.32) = 55.41$] 82.82

$S(4.68) = -27.41$] 1.41

$S(5) = -26$

Total distance = 85.64

$\approx$ 86 m

d) The velocity of the particle when t = 5 $s'(5) = 9$ m/s.

e) The average speed of the particle over the first five seconds.

Av Speed: $\frac{85.64}{5}$

$= 17.13$ m/s

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3. **[3, 1, 2 = 6 marks]**
Palaeontologists establish the age of artefacts by the process of carbon dating. They use the fact that the half-life of Carbon C_{14} is about 5600 years and that C_{14} decays at an exponential rate. [Half life is the time it takes to go from 100% to 50% of the original amount]

a) Find an equation, in the form $P = a\,r^t$, to represent the percentage of C_{14} remaining, with respect to time. (give r accurate to 6 decimal places)

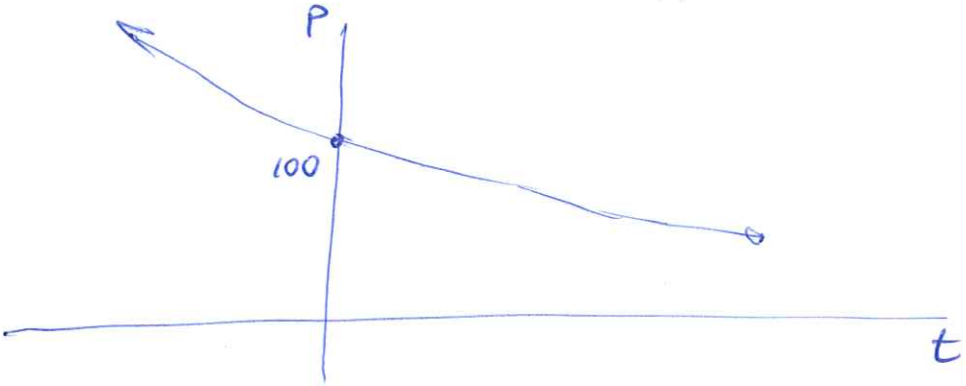
$P = 100\,r^t$

$50 = 100\,r^{5600}$

$r = 0.999876$

$\therefore\ P = 100\,(0.999876)^t$

b) Do a sketch to represent the amount of C_{14} remaining over time.



After testing an artefact, they found that the mass of C_{14} still present was 12% of the original amount.

c) What (to the nearest 1000 years) was the age of the artefact?

$12 = 100\,(0.999876)^t$

$t = 17000$ years old.

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4. [1, 1, 2, 2, 2, 2 = 10 marks]

The population of swans nesting in Carine Glades was recorded at the beginning of September every year, starting in 2010. It is found that the population satisfies the function:

$S(t) = 800 + 395t - 2.5t^2$ (t is the number of years, after 2010).

a) How many swans were counted in 2010?

$S(0) = 800$

b) How many were counted in 2012? $S(2) = 1580$

c) What was the average rate of change in the population over these two years?

$Av = \frac{780}{2} = 390$ swans/yr

d) What was the instantaneous rate of change when t = 2?

$\frac{ds}{dt} = 395 - 5x$

$\left.\frac{ds}{dx}\right|_{x=2} = 385$ swans/yr

e) After how many years will the population be a maximum?

$\frac{ds}{dt} = 0$ when $x = 79$

f) If there are no unforeseen disturbances to the area, for how many years will there be swans at Carine Glades?

$\cancel{x} = 160$ years

5. **[3, 2, 1, 2 = 8 marks]**

The melting point of gold is 1062°C. After being poured into an ingot block, the gold is placed into a cool room which is kept at a constant temperature of 30°C.

After a minute the temperature of the ingot is 900°C and decreasing exponentially.

a) Write a formula to find the temperature of the ingot at any time t.

$T = 1032\, r^{t} + 30$

$900 = 1032\, r + 30$

$r = 0.843$

$T = 1032\,(0.843)^{t} + 30$

b) When will the temperature of the ingot get down to 40°C (the temperature that they will be able to pick it up by hand and move it)?

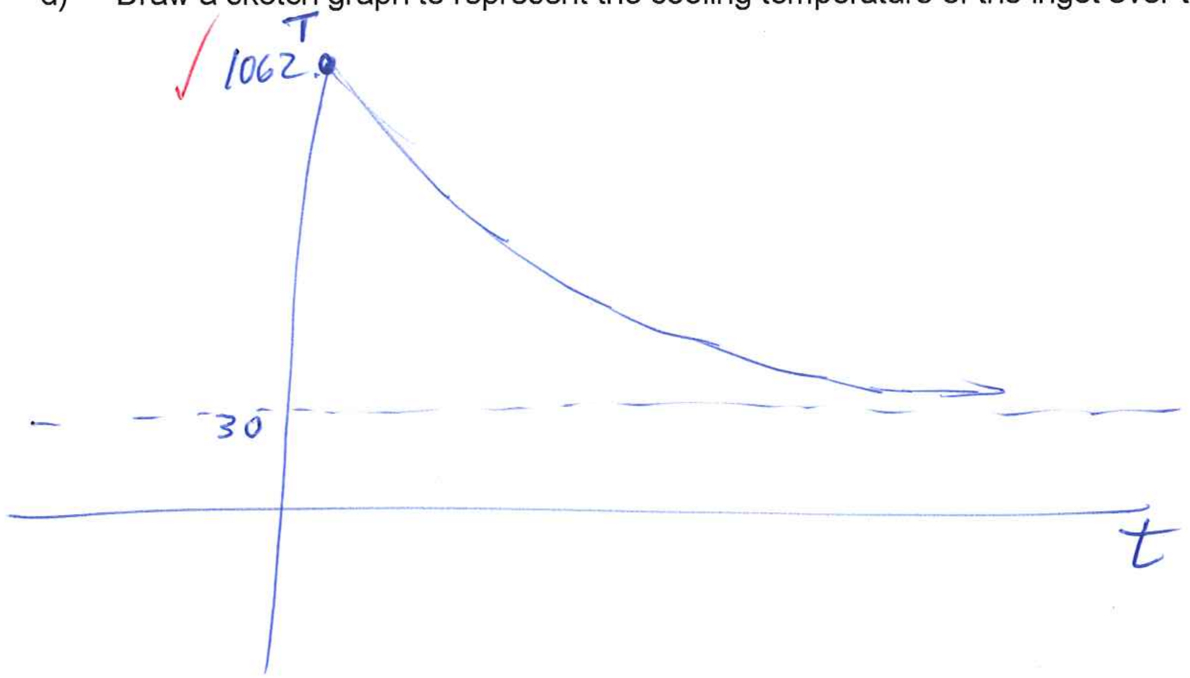
$40 = 1032\,(0.843)^{t} + 30$

$x = 27.14$ minutes

c) If left in the room, when will the temperature of the ingot reach 28°C?

never, cannot do it

d) Draw a sketch graph to represent the cooling temperature of the ingot over time.



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