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CORPUS CHRISTI COLLEGE
SEQUERE DOMINUM

12 ATAR Physics

Gravity & Satellites Test 2016

Student name: Soln

1. An asteroid of mass 2.55 x 10^{22} kg and diameter of 772 km orbits the sun with a speed of 17.9 kms^{-1}.

a. Calculate the gravitational field strength at any point on the surface of the asteroid? **[4 marks]**

$$g = \frac{GM}{r^2} = \frac{(6.67\times10^{-11})(2.55\times10^{22})}{(3.86\times10^{5})^2}$$

$$= \frac{1.70\times10^{12}}{1.49\times10^{11}} = 11.4\ Nkg^{-1}$$

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b. Calculate the orbital radius of the asteroid. **[3 marks]**

$$F_c = F_g \quad \therefore \quad \frac{Mv^2}{r} = \frac{GMm}{r^2}$$

$$\therefore r = \frac{GM}{v^2} = \frac{(6.67\times10^{-11})(1.99\times10^{30})}{(17.9\times10^{3})^2}$$

$$\therefore r = 4.16\times10^{4}\ m.$$

3

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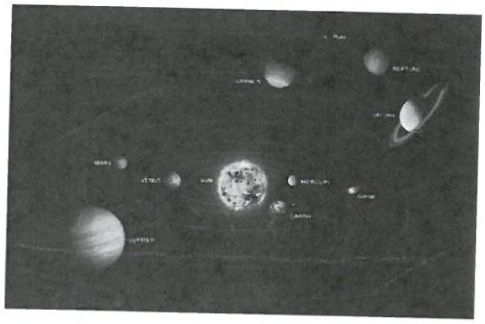
c. With what force does the asteroid attract the sun? **[4 marks]**

$$F = \frac{GM_S M_A}{r^2} = \frac{(6.67\times10^{-11})(1.99\times10^{30})(2.55\times10^{22})}{(4.16\times10^{4})^2}$$

(4)

$$= \frac{3.39\times10^{42}}{1.73\times10^{23}} = 1.96\times10^{19}\ \text{N}$$ TOWARDS ASTEROID.

2. The solar system consists of a number of planets in approximately circular orbits around the sun. The quotient, r^3/T^2, for each planet has the same value.



a. Show, by using algebraic manipulation of the equations learned in class, that the relationship r^3/T^2, is a constant value. **[4 marks]**

$$F_g = F_c \quad \therefore \quad \frac{GM_1 m_2}{r^2} = \frac{m_2 4\pi^2 r}{T^2}$$

$$\therefore \quad \frac{T^2}{(r^2)(r)} = \frac{4\pi^2}{GM_1}$$

(4)

or $$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$ WHERE M IS CENTRAL MASS.

$$= \text{Const}$$

(8)

b. What is the numerical value of this constant, (r^3/T^2)? **[3 marks]**

$$\frac{r^3}{T^2} = \frac{(6.67\times10^{-11})(1.99\times10^{30})}{(4)(3.14)^2}$$ ✓

$$= \frac{1.327\times10^{20}}{39.44}$$

$$= \underline{3.37\times10^{18}\ m^3 s^{-2}}$$ ✓ ✓ (3)

c. Mercury takes 88 days to orbit the sun, while Venus takes 225 days. Calculate the maximum distance that could ever exist between Mercury and Venus. (If you did not calculate a value in Part (b), use a value of 4×10^{18}) **[5 marks]**

MERCURY:

$r^3 = (3.37\times10^{18})T^2 = (3.37\times10^{18})(88\times24\times3600)^2$ ✓

$= 1.95\times10^{32}$ $\therefore$ $\underline{r_m = 5.80\times10^{10}\ m}$ ✓

VENUS:

$r^3 = (3.37\times10^{18})T^2 = (3.37\times10^{18})(225\times24\times3600)^2$

$= 1.27\times10^{33}$ $\therefore$ $\underline{r_v = 1.08\times10^{11}\ m}$ ✓

$\therefore$ MAX DIST $= r_m + r_v$ ✓

$= \underline{1.66\times10^{11}\ m}$ ✓ (5)

NOTE: WITH 4×10^{18} MAX DIST $= 7.21\times10^{11}\ m$.

(8)

3. Just after lift-off a space shuttle rocket is accelerating vertically upwards. An astronaut inside states that she feels heavier.

a. Explain, in terms of the forces acting on her, why she feels heavier.

[3 marks]

- NORMAL REACTION FROM SEAT = mg. ✓
- FORCE OF ACCEL = ma. ✓
- TOTAL FORCE ACTING = mg + ma ✓
  ∴ FEELS HEAVIER.

3

b. As the shuttle continues to accelerate vertically upwards, **at the same rate**, she notices she feels her weight decreasing. Explain why. **[3 marks]**

- $g = \frac{GM}{r^2}$ ✓
- AS $r \uparrow$, $g \downarrow$. ✓
- mg + ma WILL DECREASE. ✓
  ∴ WEIGHT DECREASING.

3

c. The space shuttle launches from Cape Canaveral, Florida, USA. This is the location on mainland USA, closest to the equator. Explain how this might assist with the launch.

[3 marks]

- EQUATOR HAS GREATEST ROTATIONAL SPEED. ✓
- LAUNCHING IN DIR^ OF SPIN WILL ADD EARTH'S ROTATIONAL SPEED TO LAUNCH VEHICLE SAVING FUEL COST. ✓
  (OR) INC MAX WEIGHT LAUNCHED.

3

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4. Callisto is the largest moon orbiting Jupiter. Callisto takes 16 days to complete each orbit, at a distance of 1.88×10^9 metres from the centre of Jupiter. Use this data to calculate the mass of Jupiter. **[4 marks]**

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2} \quad \therefore \quad M = \frac{(r^3)(4\pi^2)}{(T^2)\,G}$$ ✓ (4)

$$\therefore \quad m = \frac{(1.88\times10^9)^3(39.4)}{(16\times24\times3600)^2(6.67\times10^{-11})} = \frac{2.62\times10^{29}}{128}$$ ✓

$$\therefore \quad M_J = 2.05\times10^{27}\ \text{Kg}$$ ✓

5. The planet Mercury has a radius of 1.30×10^6 m, a mass of 3.30×10^{23} kg and its day is 58.65 earth days. A 25 kg satellite is positioned into a geostationary orbit. How high above the surface of Mercury is the satellite orbiting? **[6 marks]**

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2} \quad \therefore \quad r^3 = \frac{(T^2)\,GM}{4\pi^2}$$ ✓ (6)

$$\therefore \quad r^3 = \frac{(58.65\times24\times3600)^2(6.67\times10^{-11})(3.3\times10^{23})}{(39.4)}$$ ✓ ✓

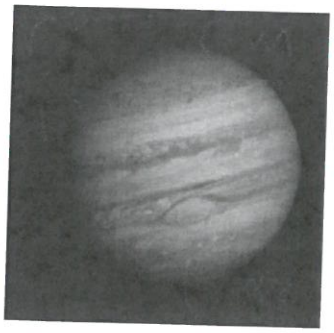
$$r^3 = \frac{5.65\times10^{26}}{39.4} \quad \therefore \quad r = 2.43\times10^8\ \text{m}$$ ✓

$$\therefore \quad \text{HEIGHT} = (2.43\times10^8) - (1.3\times10^6)$$ ✓

$$= 2.42\times10^8\ \text{m}$$ ✓

(10)

6. The table shown below gives astronomical data for a planet currently orbiting the sun.



Mass: 1.90 X 10^{27} kg (317.9 Earths)
Radius (equatorial): 71 492 km
Mean density: 1.33 g cm^3
Distance from Sun: 778 330 000 km
Rotational period: 0.4135 days
Orbital Period: 4332.71 days
Escape velocity: 59.56 kms^{-1}
Apparent magnitude: -2.70
Surface temperature: -121°C (cloud)
Atmospheric composition: hydrogen (90%), helium (10%)

a. What is the least massive planet in the solar system? **[1 mark]**

MERCURY. ✓ (1)

b. Calculate the escape velocity of the planet Jupiter. **[4 marks]**

$$V^2 = \frac{2GM}{r} \quad \checkmark$$

$$= \frac{2(6.67\times10^{-11})(1.90\times10^{27})}{(71492\times10^{3})} \quad \checkmark$$

$$= \frac{2.5346\times10^{17}}{(71492\times10^{3})}$$

$$= 3.55\times10^{9} \quad \checkmark$$

$$\therefore V = 59.5\ kms^{-1} \quad \checkmark$$

(4)

(5)