EASTERN GOLDFIELDS COLLEGE

MATHEMATICS APPLICATIONS U1 - 2016

Investigation 2

Take home notes and calculator Assumed Time: 60 min

Part A - _____ / 2 marks Part B - _____ /58 marks **TOTAL** _____ **/ 60**

ALPHABET CODES

- MATRICES -

Name: MARKING KEY

1. **Encode** "MORE AMMO URGENT", following these steps: **(12 marks)**

a) Write the message in numbers. (1 marks)

[ ]

13	15	18	5	27	1	13	13	15	27	21	18	7	5	14	20

✓

b) Write these as a series of matrices. (4 marks)

$\begin{bmatrix} 13 & 5 \\ 18 & 5 \end{bmatrix}$ ✓ $\begin{bmatrix} 27 & 1 \\ 13 & 13 \end{bmatrix}$ ✓ $\begin{bmatrix} 15 & 27 \\ 21 & 18 \end{bmatrix}$ ✓ $\begin{bmatrix} 7 & 5 \\ 14 & 20 \end{bmatrix}$ ✓

c) Multiply these matrices by the encoding matrix, which is $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ (4 marks)

$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 13 & 15 \\ 18 & 5 \end{bmatrix} = \begin{bmatrix} 44 & 35 \\ 31 & 20 \end{bmatrix}$ ✓ $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 27 & 1 \\ 13 & 13 \end{bmatrix} = \begin{bmatrix} 67 & 15 \\ 40 & 14 \end{bmatrix}$ ✓

$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 15 & 27 \\ 21 & 18 \end{bmatrix} = \begin{bmatrix} 51 & 72 \\ 36 & 45 \end{bmatrix}$ ✓ $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 7 & 5 \\ 14 & 20 \end{bmatrix} = \begin{bmatrix} 28 & 30 \\ 21 & 25 \end{bmatrix}$ ✓

Complete the table below by following these steps: (3 marks)

d) Write the resulting code.
e) Rewrite the code after subtracting 27 from the values where necessary.
f) Change back into letters, ready to send.

d)	44	35	31	20	67	15	40	14	51	72	36	45	28	30	21	25
e)	17	8	4	20	13	15	13	14	24	18	9	18	1	3	21	25
f)	Q	H	D	T	m	O	m	N	X	R	I	R	A	C	U	Y

✓ ✓ ✓

-1 errors.
ft a → f.

2. **Decode** the following message, using the **encoding** matrix $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ **(15 marks)**

a) Covert to numbers (1 mark)

R	X	H	E	S	R	K	-	R	P	P	X	Q	E	E	V
18	24	8	5	19	18	11	27	18	16	16	24	17	5	5	22

✓

b) What is the **decoding** matrix **to be** used? (2 marks)

$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ ✓ a → d swap
b & c x -1 ✓

c) Apply the decoding matrix. (8 marks)

$$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 18 & 24 \\ 8 & 5 \end{bmatrix} = \begin{bmatrix} 28 & 43 \\ -38 & -62 \end{bmatrix}$$ ✓✓

$$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 19 & 18 \\ 11 & 27 \end{bmatrix} = \begin{bmatrix} 27 & 9 \\ -35 & 0 \end{bmatrix}$$ ✓✓

$$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 18 & 16 \\ 16 & 24 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ -22 & 0 \end{bmatrix}$$ ✓✓

$$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 17 & 5 \\ 5 & 22 \end{bmatrix} = \begin{bmatrix} 29 & -12 \\ -41 & 29 \end{bmatrix}$$ ✓✓

(4 marks)

d)	28	43	-38	-62	27	9	-35	0	20	8	-22	0	29	-12	-41	29	✓
e)	1	16	16	19	27	9	19	27	20	8	5	27	2	15	13	2	✓✓
f)	A	P	P	S	–	I	S	–	T	H	E	–	B	O	m	B	✓

-1 errors ft errors from d) → f)

3. Messages can also be sent in code by **adding** matrices together. Consider the following message: **(7 marks)**

ONE BEER NOW

a) Assign a number to each letter. (1 mark)

O N E – B E E R – N O W
15 14 5 27 2 5 5 18 27 14 15 23 ✓

b) Set up 2 x 2 matrices for these. (2 marks)

$\begin{bmatrix} 15 & 14 \\ 5 & 27 \end{bmatrix}$ $\begin{bmatrix} 2 & 5 \\ 5 & 18 \end{bmatrix}$ ✓ $\begin{bmatrix} 27 & 14 \\ 15 & 23 \end{bmatrix}$ ✓

c) Use the **encoding** 2 x 2 matrix $\begin{bmatrix} 2 & 7 \\ 13 & 5 \end{bmatrix}$ to encode the message. (2 marks)

$\begin{bmatrix} 15 & 14 \\ 5 & 27 \end{bmatrix} + \begin{bmatrix} 2 & 7 \\ 13 & 5 \end{bmatrix} = \begin{bmatrix} 17 & 21 \\ 18 & 32 \end{bmatrix}$

$\begin{bmatrix} 2 & 5 \\ 5 & 18 \end{bmatrix} + \begin{bmatrix} 2 & 7 \\ 13 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ 18 & 23 \end{bmatrix}$ ✓

$\begin{bmatrix} 27 & 14 \\ 15 & 23 \end{bmatrix} + \begin{bmatrix} 2 & 7 \\ 13 & 5 \end{bmatrix} = \begin{bmatrix} 29 & 21 \\ 28 & 28 \end{bmatrix}$ ✓

d) Reassign letters to complete the message. Remember to take 27 from those numbers greater than 27. (2 marks)

17 21 18 32 — 4 12 18 23 — 29 21 28 28 ✓

Q U R E — D L R W — B U A A ✓

4. To decode the message a **decoding** matrix is needed. The decoding matrix is $\begin{bmatrix} -2 & -7 \\ -13 & -5 \end{bmatrix}$

Use this decoding matrix to decode the message below. **(8 marks)**

V O N Y U G V Y

22 15 14 25 21 7 22 25 ✓

$\begin{bmatrix} 22 & 15 \\ 14 & 25 \end{bmatrix} + \begin{bmatrix} -2 & -7 \\ -13 & -5 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 1 & 20 \end{bmatrix}$ ✓✓

$\begin{bmatrix} 21 & 7 \\ 22 & 25 \end{bmatrix} + \begin{bmatrix} -2 & -7 \\ -13 & -5 \end{bmatrix} = \begin{bmatrix} 19 & 0 \\ 9 & 20 \end{bmatrix}$ ✓✓

20 8 1 20 19 ? 9 20 ✓

T H A T S ? I T ✓

27

5. Using your calculator, save these matrices and perform the following calculations or state if they cannot be determined. **(6 marks)**

$A = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix}$ $C = \begin{bmatrix} 4 & -2 & 6 \\ 1 & 0 & 7 \end{bmatrix}$ $D = \begin{bmatrix} 4 & 6 \\ -3 & 2 \end{bmatrix}$

a) A + D

$\begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -3 & 7 \end{bmatrix}$ ✓✓ −1 ∃ error

b) 3A - D

$3A = 3 \times \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 0 & 15 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -12 \\ 3 & 13 \end{bmatrix}$ ✓✓ −1 ∃ error

c) C - 2B

C 2×3 B 3×1 ≠

cannot be determined ✓

∵ rows ≠ columns ≠ } ✓

6. When multiplying two matrices in the method you have used means that:

Two matrices can be multiplied together provided the number of columns in the first matrix equals the number of rows in the second matrix

Knowing this and using the matrices from question 5, state if the following calculations can be determined or not. If they can be determined, use your calculator to calculate the answer. **(10 marks)**

a) A x B

$A_{2\times 2} \neq B_{3\times 2}$

cannot be determined. ✓

∵ A columns ≠ B rows ✓

b) A x D

$A_{2\times 2} = D_{2\times 2}$ ✓

$\begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 4 & 6 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 2 \\ -15 & 10 \end{bmatrix}$ ✓✓

−1 ∃ error

c) C x 2B

$C_{2\times 3} \times B_{3\times 1} = 2\times 1$ ✓

$$2B = 2 \times \begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & 6 \\ 1 & 0 & 7 \end{bmatrix} \times \begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 68 \\ 40 \end{bmatrix}$$ ✓✓

d) B^2

$B_{3\times 1} \times B_{3\times 1}$ ✓

$\neq$ ✓

cannot be determined

rows $\neq$ columns

$$\begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix} \times \begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix}$$

e) $A^2 - D^2$

$A_{2\times 2} \times A_{2\times 2}$ ✓

$$A^2 = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -12 \\ 0 & 25 \end{bmatrix}$$

$D_{2\times 2} \times D_{2\times 2}$ ✓

$$D^2 = \begin{bmatrix} 4 & 6 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 4 & 6 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 36 \\ -18 & -14 \end{bmatrix}$$

$$A^2 - D^2 = \begin{bmatrix} 1 & -12 \\ 0 & 25 \end{bmatrix} - \begin{bmatrix} -2 & 36 \\ -18 & -14 \end{bmatrix} = \begin{bmatrix} 3 & -48 \\ 18 & 39 \end{bmatrix}$$ ✓✓

END OF PART B

-1 ∄ error.