MINDARIE
SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS

MATHEMATICS: SPECIALIST 1 & 2

SEMESTER 1 2019
TEST 1

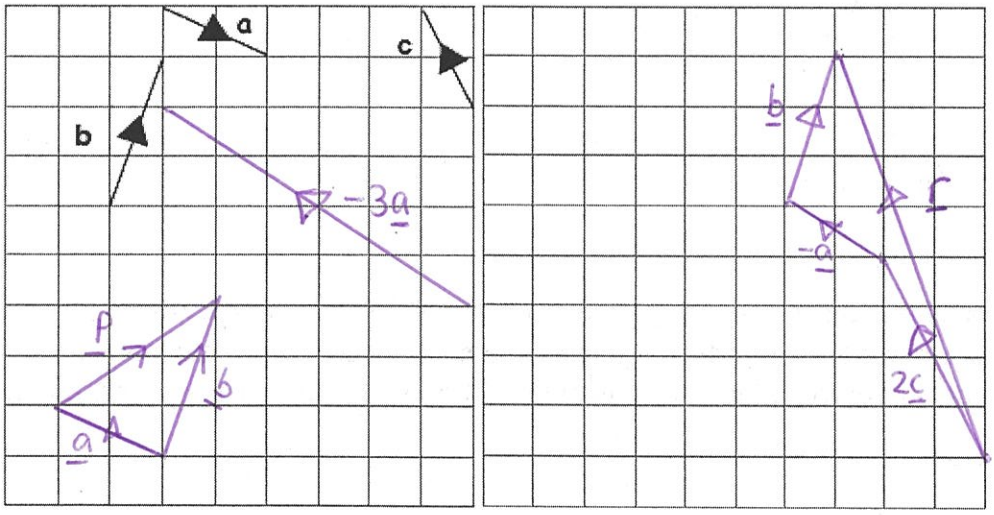
Name_____

Calculator Free

Time allowed: 15 mins

Total marks: 13

1. [4 marks:1, 1, 2]
Vectors **a** , **b** and **c** are as shown below.



On the grid, sketch and label the vectors p, q and r where:

p=a+b ✓

q=-3a ✓

r=2c-a+b ✓✓

2. [6 marks: 2, 4]

(a) Evaluate ${}^6C_4 + {}^7C_4$

$$\frac{6!}{(6-4)!4!} + \frac{7!}{(7-4)!4!}$$

$$= \frac{6!}{2!4!} + \frac{7!}{3!4!} \quad \checkmark$$

$$= \frac{6\times5}{2} + \frac{7\times6\times5}{6} = 15 + 35 = \underline{\underline{50}} \quad \checkmark$$

(b) Show that:

$${}^nC_{r+1} = \frac{n-r}{r+1} \times {}^nC_r$$

$$= \frac{n-r}{r+1} \times \frac{n!}{(n-r)!\,r!} \quad \checkmark$$

$$= \frac{(n-r)\,n!}{(r+1)(n-r)!\,r!} \quad \checkmark$$

$$= \frac{n!}{(r+1)!\,(n-r-1)!} \quad \checkmark$$

$$= \frac{n!}{(n-(r+1))!\,(r+1)!} = {}^nC_{r+1} \quad \checkmark$$

3. [3 marks]

A bag contains lettered tiles consisting of: 3 E's, 4 R's, 6 T's, 5 Z's and 2 A's.
Tilly selects tiles at random from the bag and does not replace them. How many tiles need to be drawn out to guarantee that Tilly has at least 4 of one letter? Justify your answer.

E	R	T	Z	A	
III	III	III	III	II	= 14 tiles ✓

Next tile will be R, T or Z which will mean 4 of the same letter so 15 tiles are needed. ✓✓

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Calculator Assumed

Time allowed: 45 mins

Total marks: 43

4. [8 marks: 2, 2, 2, 2]

How many 5 letter words can be formed using the letters from the word ABSOLUTE if:

a) letters cannot be repeated.

$8P_5 = 6720$ ✓ ✓

b) the word must start with B.

$B \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} = 840$ ✓ ✓

c) the word must start and end with O and L in either order.

$\underline{2} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{1} = 240$ ✓ ✓

d) the word must start and end with a vowel.

$\underline{4} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} = 1440$ ✓ ✓

5. [4 marks]

A 4 digit code is to be made from the numbers 1, 2, 3, 4, 5, 6 and 7 or the numbers 3, 4, 5, 6, 7 and 8. How many different codes are possible?

$7P4 + 6P4 - 5P4$ ✓

$= 840 + 360 - 120$

$= 1080$ ✓

6. [5 marks: 2, 4]

a) How many ways are there of arranging the 8 letters of the word SYLLABUS?

$\frac{8!}{2!\,2!} = 10080$ ✓

b) Find the number of different arrangements of 4 letters that can be made using the letters of the word SYLLABUS.

S Y L A B U
S L

All different letters: $6 \times 5 \times 4 \times 3 = 360$ ✓

2 S's + ?? : $6 \times 5 \times 4 = 120$ ✓

SS__ S_S_ S__S
-SS- -S-S --SS

2 L's + ?? $6 \times 5 \times 4 = 120$

2 S's + 2 L's 6 ✓

total = 606 ✓

7. [5 marks: 1, 2, 2]

A committee of five members is to be chosen from a group of 5 teachers and 3 senior school students.

a) How many different committees could be formed?

$^8C_5 = 56$ ✓

b) How many committees would contain 3 teachers and 2 students?

$^5C_3 \times {}^3C_2 = 30$
✓ ✓

c) How many committees contain at least 2 students? ⇒ 2 or 3 students.

$^5C_3 \times {}^3C_2 + {}^5C_2 \times {}^3C_3$ ✓

$= 30 + 10$

$= 40.$ ✓

8. [4 marks]

How many integers between 1 and 500 inclusive are divisible by 4 or 7?

$500 \div 4 = 125$
$500 \div 7 = 71.4...$ ✓✓ $500 \div 28 = 17.8...$

$n(\text{div. by 4} \cup \text{div. by 7}) = n(\text{div. by 4}) + n(\text{div. by 7}) - n(\text{div by 4 and 7})$

$= 125 + 71 - 17$ ✓

$= 179$ ✓

9. [6 marks: 1, 2, 3]

The 4 members of the band U2, (Bono, The Edge, Larry and Adam) their manager (Guy) and their tour manager (Willie) arrange themselves in line for a photograph.
How many arrangements are there in which:

a) Bono is first

B x 5!

= 120 ✓

b) The Edge and Bono are together

Only 5 options

✓ 5! x 2 (as E + B can swap places)

= 240 ✓

c) The Edge and Bono are together and Guy and Willie are together

only 4 options

4! x 2 x 2 ✓✓

= 96 ✓

10. [2 marks]

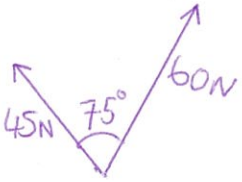
There are 20 000 students studying at a University, with at least one student from each of the 196 countries in the world. What is the largest number of students certain to be from the same country?

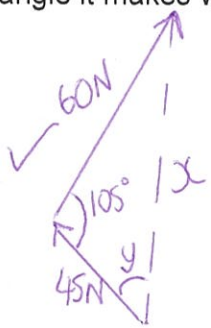
20 000 ÷ 196 ✓ = 102.0408...

103 students ✓

11. [5 marks]

Two vectors have magnitudes of 45N and 60N and the angle between their directions is 75°. Sketch a diagram showing these forces and calculate the magnitude of the resultant force and the angle it makes with the smaller force.





$x^2 = 45^2 + 60^2 - 2 \times 45 \times 60 \times \cos 105$ ✓✓

$x = 83.8\,N$

Need angle y:

$\frac{60}{\sin y} = \frac{x}{\sin 105}$ ✓

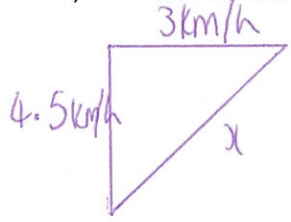
$y = 43.8°$ ✓

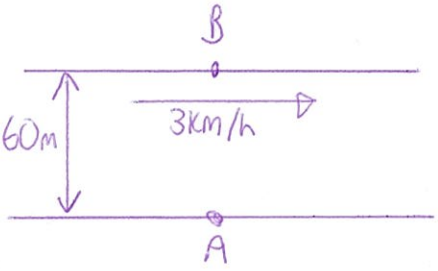
12. [4 marks: 2, 2]

Jess wants to take her boat on a river from point A on one bank to point B directly opposite on the other bank. The river is 60m wide and flows at 3km/h parallel to the bank. Jess can maintain a rowing speed of 4.5km/h in still water.

If she starts at A and directs her boat towards B when she starts rowing:

a) What would her resultant velocity be?



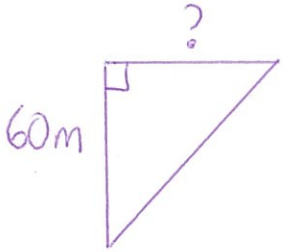


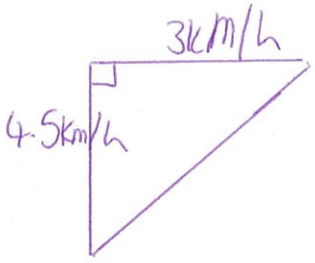
✓

$x = \sqrt{4.5^2 + 3^2}$ ✓

$= 5.4$ km/h

b) How far away from point B would she end up?





Using similar triangles:

$60 \div 4.5 = 13\frac{1}{3}$ ✓

$3 \times 13\frac{1}{3} = 40\,m$ ✓

Jess would be 40m away from B.