12 Mathematics Methods 2021

Test 1 – Differentiation and Logarithms

Section 1: Calculator-free

Time allowed: 25 minutes **Maximum marks: 25**

Name:______________________________ **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No calculators or personal notes are permitted.

Question 1 **[2, 2 = 4 marks]**

Calculate the following;

a) $\log 1000\sqrt{10}$ b) $\log_{81} 3$

Question 2 **[2, 3 = 5 marks]**

Differentiate the following (do not simplify your answers).

a) $y = \frac{3x^2-5x}{6x-5}$

b) $f(x) = (8-x)(7x^2+4x)^3$

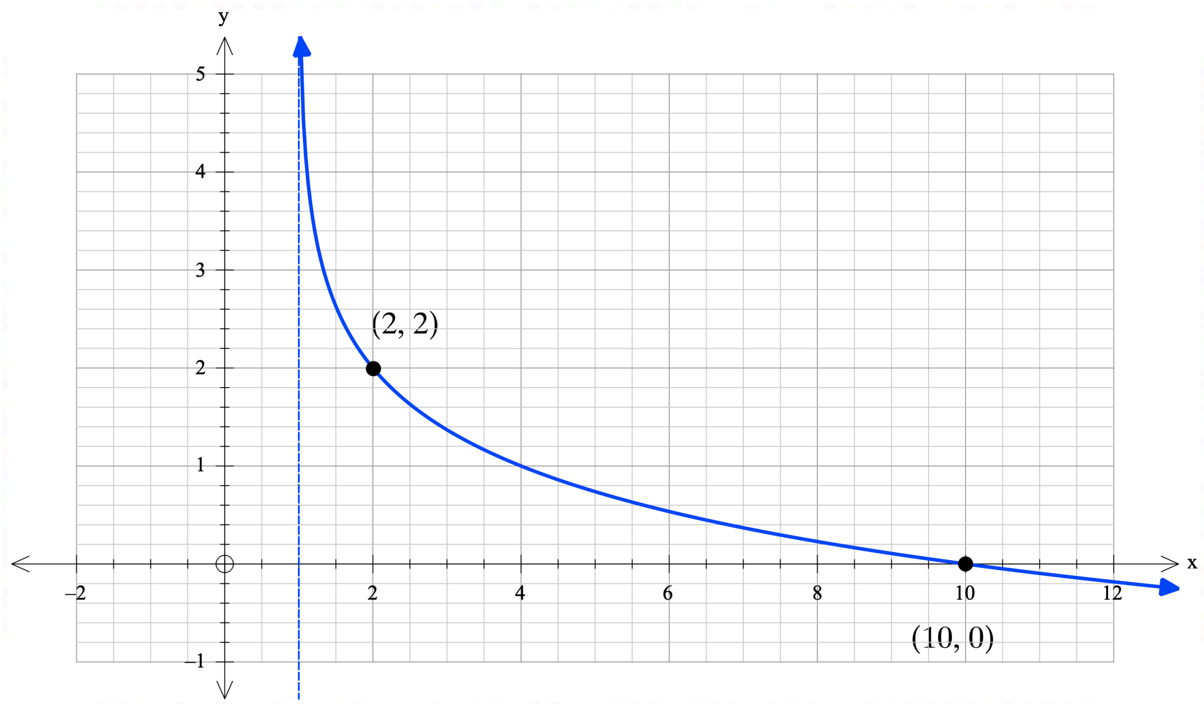
Question 3 **[5 marks]**

Determine the coordinates of any points on the function $y = -\frac{6}{(x-4)}$ whose tangents are parallel to the line $3x - 2y = 6$

Question 4 **[4 marks]**

The graph of $y = -\log_b(x + c) + d$ is drawn below.

If there is a vertical asymptote at $x = 1$, determine the values of b, c and d.



Question 5 **[3, 4 = 7 marks]**

Solve each of the following equations for x;

a) $16^{\,x+1} = (\sqrt{8})^{\,6x-2}$

b) $12(2^x) = 7 + \frac{10}{2^x}$ (giving answer in form $a + \log_2 b$)

END OF SECTION 1

12 Mathematics Methods 2021

Test 1 – Differentiation and Logarithms

Section 2: Calculator-assumed

Time allowed: 20 minutes **Maximum marks: 20**

Name:______________________________ **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
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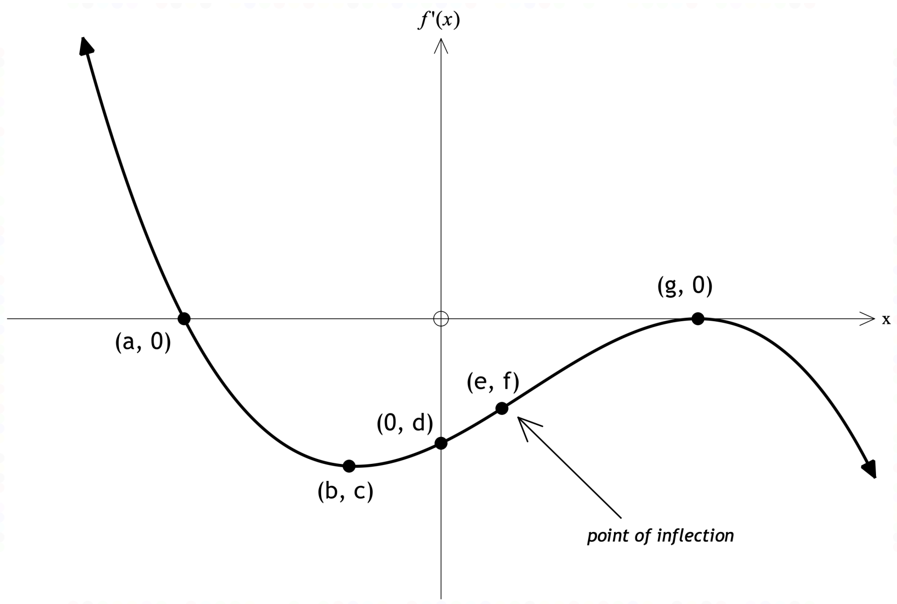
- A formula sheet will be provided.
- Calculators and 1xA4 double-sided page of personal notes are permitted.

Question 6 **[4 marks]**

Use the *increments formula* to determine the percentage change in the radius of a sphere when its surface area decreases by 3%.

Question 7 **[2, 2 = 4 marks]**

The graph of the **derivative**, $f'(x)$, is drawn below.



a) On the graph above, sketch a possible graph of the second derivative $f''(x)$.

b) Determine the x values of any stationary points, and their nature, on $f(x)$.

Question 8 **[2, 3 = 5 marks]**

A closed cylindrical can, with base radius r and height h, has a volume of $250\pi \text{ cm}^3$.

a) Show that the total area, $A \text{ cm}^2$, of metal required to make the can is given by $A = 2\pi r^2 + \frac{500\pi}{r}$

b) If the material for the curved side of the can costs $\$0.001$ per cm^2 and the material for each of the circular ends costs $\$0.003$ per cm^2, determine;

i. The area of material used to minimise cost.

ii. The minimum cost to produce a can.

Question 9 **[2, 2, 3 = 7 marks]**

Two particles, P and Q, both travel along the same straight line.

Their displacements, s metres, after t seconds ($t \geq 0$) from a fixed-point O on the line are given by;

$s_P = 3t^3 - 81t + 5$

$s_Q = -2(t-1)(t-4)$

a) Calculate the initial distance between the particles.

b) At what time(s), does particle P change direction?

c) At $t = 4$, is particle Q speeding-up or slowing-down? Justify your answer.

END OF TEST