Name. Solutions

Mathematics Specialist Unit Test 6

Matrices and Complex Numbers

Mark:_____/54

Section 1: Non-Calculator
Time Allocation: 20 mins Marks: ____/19

1 [6 marks: 2, 2, 2]

Determine the transformation matrix representing each of the following:

a) reflection about the y axis

$$\begin{bmatrix} & 0 \\ & 1 \end{bmatrix} \checkmark$$

b) dilation parallel to the x axis scale factor 4

$$\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \checkmark\checkmark$$

c) reflection about the line y = x tan(30^0)

$$\begin{bmatrix} & \sin 60^\circ \\ & -\cos 60^\circ \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \checkmark\checkmark$$

2. [3 marks]

Solve the following equation, leaving any non-real solutions in the form a + bi

$$4x^2 + 4x + 2 = 0$$

$$\frac{-4 \pm \sqrt{16 - 4\times4\times2}}{2\times4} \checkmark$$

$$= \frac{-4 \pm \sqrt{-16}}{8} \checkmark$$

$$= \frac{-4 \pm 4i}{8} = -\frac{1}{2} \pm \frac{1}{2}i \checkmark$$

3. [4 marks]

Express the following as a product of two linear factors:

$$x^2 - 2x + 10$$

$\frac{2 \pm \sqrt{4-4\times1\times10}}{2}$

$= \frac{1 \pm \sqrt{-36}}{2}$ ✓

$= 1 \pm 3i$ ✓

$(x-(1+3i))(x-(1-3i))$ ✓

$= (x-1-3i)(x-1+3i)$ ✓

3/28

4. [6 marks1,1,2,2]

The transformation T is represented by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

a) Describe in words the transformation T.

Refection about the x-axis ✓

b) The transformationis applied to the line with the equation y=x. Find the equation of the resulting line

$y = -x$

c) The point A is mapped to the point with the coordinates (k, k+1) under transformation T. find the coordinates of point A. Justify your answer.

Since T is a reflection about the x-axis ✓

$A' = (k, -k+1)$ ✓

y becomes -ve

d) The transformation T is combined with the transformation represented by the matrix **M**. All the entries in **M** are positive. The effects of the combined transformation is represented by the matrix $\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$.

Find the matrix **M**. show clearly your reasoning.

$$M \times \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \checkmark$$

but all entries need to be positive

$$M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \checkmark$$

one mark for

two marks for

END OF SECTION 1

Name: Solutions Teacher's Name: ____________________

Mathematics Specialist Unit 2

Test 6 2018

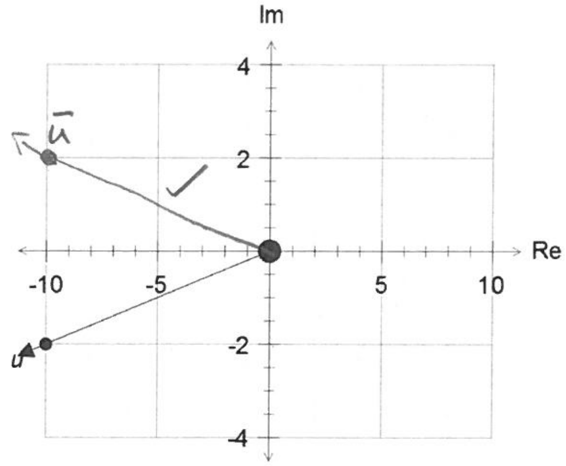
Section 2: Calculator Assumed

Instructions: *Show all working clearly, in sufficient detail to allow your answers to be checked and for part marks to be awarded for reasoning. Calculators are permitted.*

Time Allocation: 40 mins Total Marks: /35

1. [3 marks: 1, 1, 1]

The complex number u is shown on the Argand diagram to the right.



a) Express u in the form $a \pm bi$.

$-10-2i$ ✓

b) State the equation of $\bar{u}$ in the form $a \pm bi$

$-10+2i$ ✓

c) Show $\bar{u}$ on the Argand diagram.

on diagram

2. [8 marks: 3, 2, 3]

Given that w = 3 - 2i and z = 4 + 3i, determine:

$w+z = 7+i$

a) $(w+z)^2$

$(7+i)(7+i)$ ✓

$49+14i+i^2$ ✓

$48+14i$ ✓

b) $\overline{wz}$

$wz = (3-2i)(4+3i)$ ✓

$= 12+9i-8i+6$

$= 18+i$ ✓

c) $w + \dfrac{\overline{w}}{z}$

$3-2i + \dfrac{3+2i}{4+3i} \times \dfrac{4-3i}{4-3i}$ ✓

$= 3-2i + \dfrac{12-9i+8i+6}{16+9}$

$= 3-2i + \dfrac{18-i}{25}$ ✓

$= \dfrac{93}{25} - \dfrac{51}{25}i$ ✓

3. [4 marks]

Solve the equation to find Z. (Hint: Let $Z = x + yi$)

$$Z + 2\overline{Z} = (3 - i)(1 + 2i)$$

Let $Z = x + yi$

$x + yi + 2x - 2yi = 3 + 6i - i - 2i^2$

$3x - yi = 5 + 5i$

$\therefore\ 3x = 5 \qquad -y = 5$

$x = \frac{5}{3} \qquad y = -5$

4. [5 marks: 2, 2, 1]

The transformations S and F are represented by $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ respectively.

a) Find the single matrix that represents the successive transformations S then F.

$FS = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$

b) Find a single matrix that will reverse the sequence of transformations in part a).

$\begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}$ or $\frac{1}{2}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$

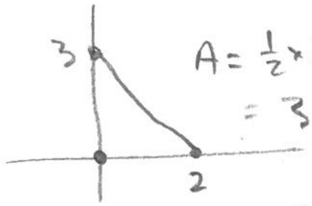
c) Hence, or otherwise, find the point whose image is (8, 4) under the sequence of transformations in part a).

$= \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 8 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} -4 \\ -4 \end{bmatrix}$

5. [6 marks]

The triangle with vertices O (0, 0), A (2, 0) and B (0, 3) is mapped to the triangle O'A'B' by the transformation represented by the matrix $\begin{bmatrix} 4 & x \\ x & y \end{bmatrix}$. The area of triangle O'A'B' is 36 cm^2 and A' has coordinates (8, 0). Find the value of x and y.



$A = \frac{1}{2} \times 2 \times 3$

$= 3 \text{ cm}^2$ ✓

$\begin{bmatrix} 4 & x \\ x & y \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 8 \\ 2x \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$ ✓

$\therefore x = 0$

$\det = \frac{\text{new Area}}{\text{old Area}}$

$= \frac{36}{3} = 12$ ✓

$\det = 4y - x^2 \qquad \therefore 4y - x^2 = 12$ ✓

$4y = 12$

$y = 3$

$\therefore x = 0$ ✓ and $y = 3$ ✓

6. [4 marks: 3, 1]

a) All points on the line $y = x + 1$ are rotated 45° clockwise about the origin. Find the equation of the image line.

$45°$ clockwise $= 315°$ anticlockwise

$\begin{bmatrix} \cos 315 & -\sin 315 \\ \sin 315 & \cos 315 \end{bmatrix} \begin{bmatrix} x \\ x+1 \end{bmatrix}$ ✓

$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} x \\ x+1 \end{bmatrix} = \begin{bmatrix} \frac{2x+1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ ✓

$y = \frac{1}{\sqrt{2}}$ ✓

b) Find the single transformation matrix that will reverse the transformation described in part a).

$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ ✓

a
28 7. [5 marks: 2,3]

a) Find the matrix representation of a combination of linear transformations that map the points (1,0) to (-2,0) and (2,1) to (-3,1).

$$M\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}-2&-3\\0&1\end{bmatrix}$$ ✓

$$M=\begin{bmatrix}-2&-3\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}^{-1}$$

$$=\begin{bmatrix}-2&1\\0&1\end{bmatrix}$$ ✓

b) Show that it is not possible to find a combination of linear transformations that will map (1,0) to (3,0), (2,1) to (4,1) and (3, 1) to (5,1).

$$M_1\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}3&4\\0&1\end{bmatrix}$$

$$M_1=\begin{bmatrix}3&4\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}^{-1}=\begin{bmatrix}3&-2\\0&1\end{bmatrix}$$ ✓

$$M\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}5\\1\end{bmatrix}$$ ✓

$$\begin{bmatrix}3&-2\\0&1\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}7\\1\end{bmatrix}$$

$$\begin{bmatrix}7\\1\end{bmatrix}\neq\begin{bmatrix}5\\1\end{bmatrix}$$ ✓

$\therefore$ It's not possible

or

$$M_2\begin{bmatrix}1&3\\0&1\end{bmatrix}=\begin{bmatrix}3&5\\0&1\end{bmatrix}$$

$$M_2=\begin{bmatrix}3&-4\\0&1\end{bmatrix}$$ ✓

~~not the same~~

$M_1\neq M_2$ ✓

$\therefore$ not possible

END OF SECTION 2