Full Name: SOLUTIONS

MATHEMATICS APPLICATIONS

Test 7 – Directed Graphs and Networks

Chapter 9

Semester 2 2018

Calculator Free

Time allowed

Working time for this section: 30 minutes
Marks available: 30 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

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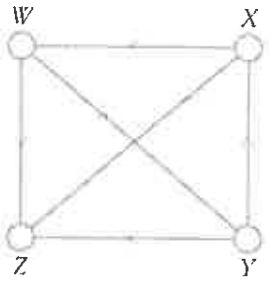
1. (5 marks)
Given the digraph shown on the right,



a. Construct an adjacency matrix (M) [2]

From \ To	W	X	Y	Z
W	0	0	1	1
X	1	0	0	1
Y	0	1	0	1
Z	0	0	0	0

(½ each row)

b. How many two-stage pathways exist? [1]

5 two stage pathways ✓

c. How many three-stage pathways exist? List one such pathway. [2]

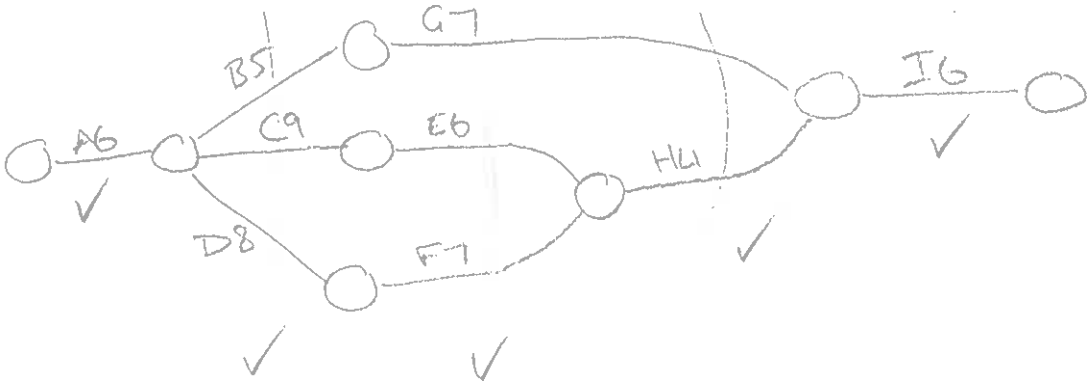
3 three-stage pathways (1)

W → Y → X → W
Y → X → W → Z
Y → X → W → Y

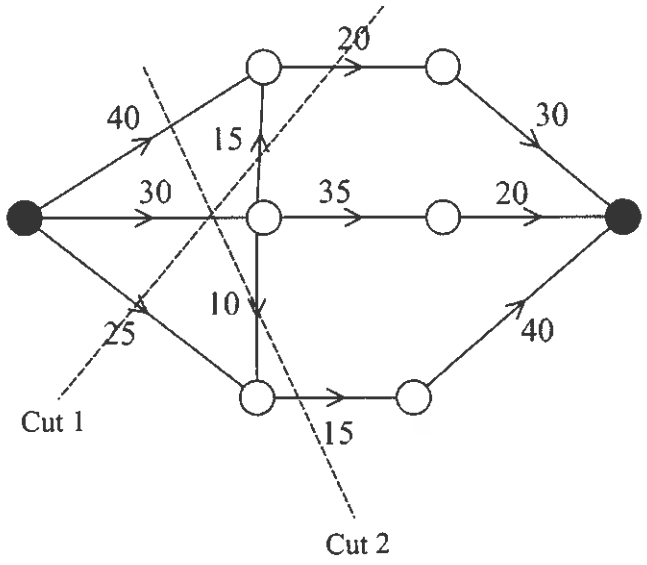
one of these (1)

2. (5 marks)
Draw the network for the activity table below.

Activity	Time (hours)	Immediate predecessor/s
A	6	None
B	5	*A*
C	9	*A*
D	8	*A*
E	6	*C*
F	7	*D*
G	7	*B*
H	4	*E,F*
I	6	*G,H*



3. (7 marks)
The flow capacity, in cars per minute, for a series of one-way roads is shown in the directed graph below.



a. Determine:

i. the flow capacity of cut 1 [1]

$20 + 30 + 25 = 75$ ✓

ii. the flow capacity of cut 2 [1]

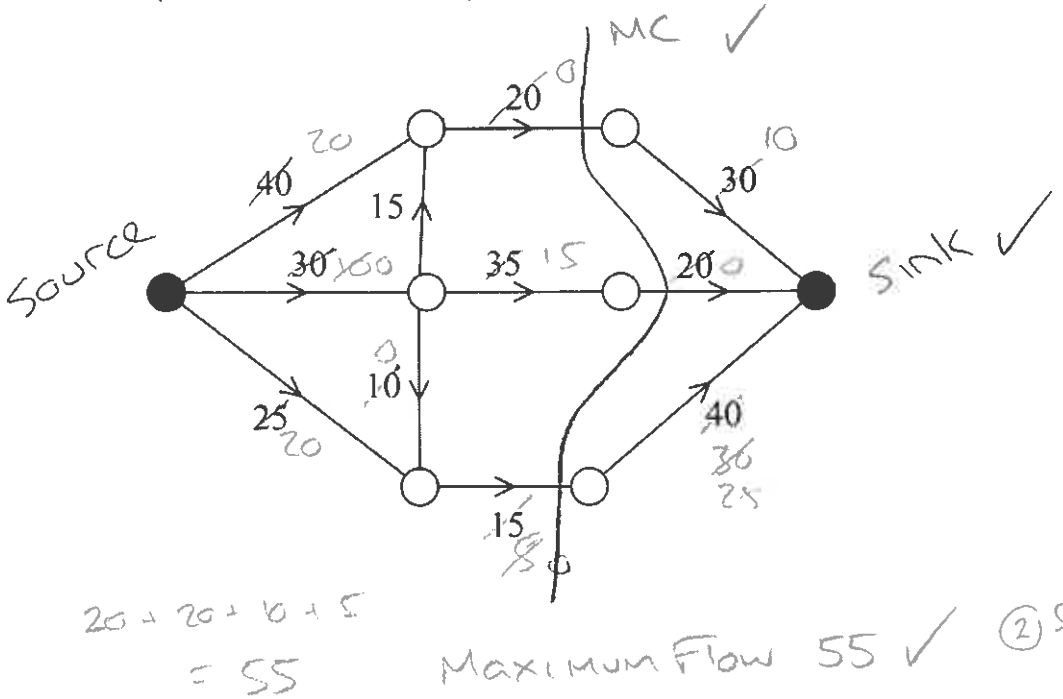
$40 + 30 + 15 = 85$ ✓

b. On the digraph below,

i. identify the source and the sink by labelling them [1]

ii. determine the maximum flow in cars per minute from source to sink [3] working!

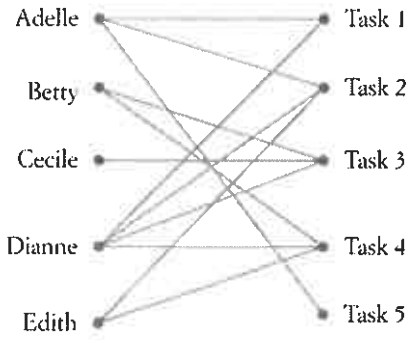
iii. identify the minimum cut that equals the maximum flow [1]



$20 + 20 + 10 + 5$
$= 55$

Maximum Flow 55 ✓ (2) for demonstrated working.

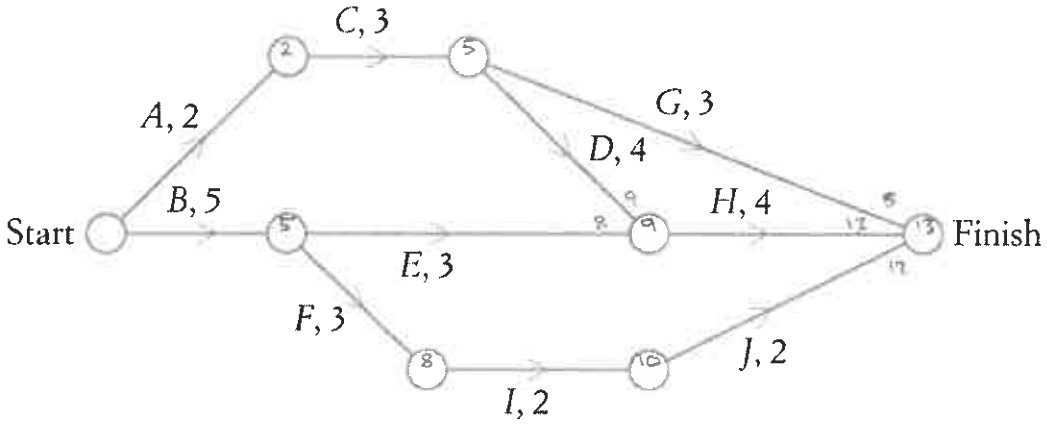
4. (3 marks)
Determine the allocation of the five tasks for Adelle, Betty, Cecile, Dianne and Edith if each person must complete a different task.



Adelle – Task 5
Betty – Task 4
Cecile – Task 3
Dianne – Task 1
Edith – Task 2

T1 – Dianne
T2 – Edith
T3 – Cecile
T4 – Betty
T5 – Adelle

5. (10 marks)
A couple decides to remove an old shed from their block and build a new garage. The 10 activities that need to be completed for this project are shown in the network below where activity times are in weeks.



a. Find the minimum time required to complete the project. [2]

13 weeks ✓ (1) working.

b. Find the float time for activity *E*. [1]

1 week ✓

c. Determine the impact on the project completion time if:

i. activity *C* is delayed by 1 week [1]

Extends project by 1 week ✓
(ACDH is CP)

ii. activity *G* is delayed by 2 weeks [1]

No impact ✓
(Float goes from 5 to 3)

iii. activity *F* is delayed by 2 weeks [1]

Extends project by 1 week ✓
(Also changes CP)

iv. activity *B* is delayed by 3 weeks. [1]

Extends project by 2 weeks ✓
(Also changes CP)

The owners are prepared to pay an additional cost to reduce the completion time. The cost of reducing time in each activity is $1500 per week; however, each activity may only be reduced by 1 week.

d. Determine the cost of reducing the project completion time by the maximum number of weeks. [3]

Reduce CP activities by 1 week each ✓

Reduce B, E, F and I by 1 week each ✓

8 x 1500 = $12000 ✓

End of Test

Additional working space

Question number: _____

Full Name: SOLUTIONS

MATHEMATICS APPLICATIONS

Test 7 – Directed Graphs and Networks

Chapter 9

Semester 2 2018

Calculator Assumed

Time allowed

Working time for this section: 20 minutes
Marks available: 19 marks

Material required/recommended for this section

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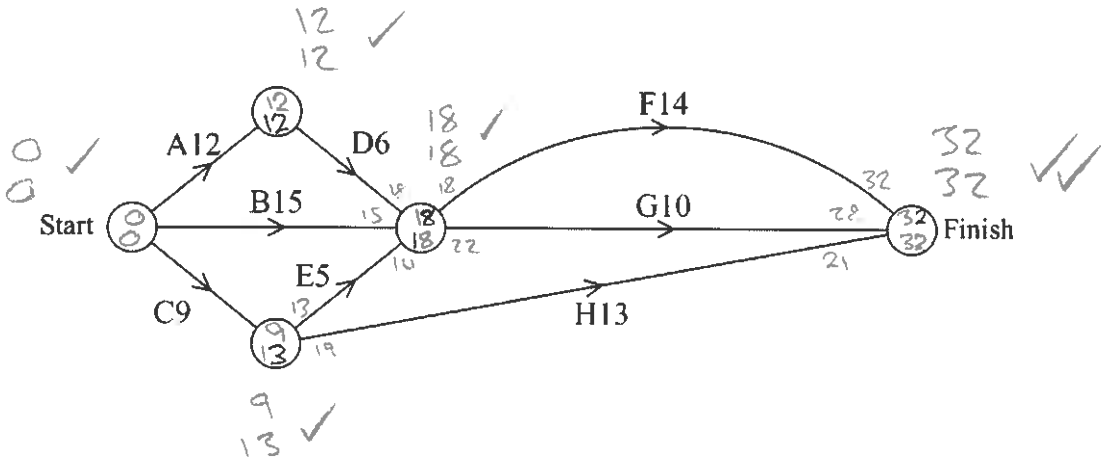
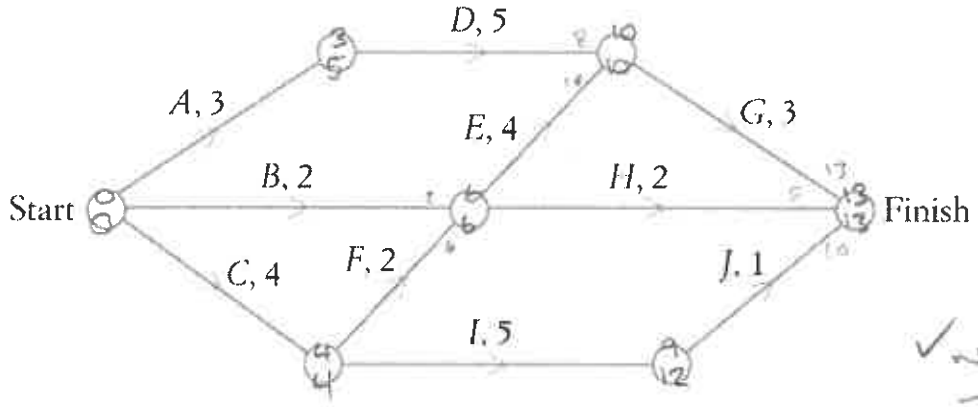
1. (6 marks)
Calculate the earliest starting times and the latest starting times for each activity in the network below. Activity times shown are in hours.



2. (3 marks)
Determine the critical path and the project completion time for the following network. Activity times shown are in days.



MCT 13 days ✓

CP C → F → E → G ✓

3. (10 marks)
A furniture store must arrange deliveries to four customers. The store contracts its deliveries to four different drivers. The distance in kilometres that each driver will need to travel to complete the deliveries is shown in the table below.

	Driver 1	Driver 2	Driver 3	Driver 4
Customer *A*	17	22	17	19
Customer *B*	19	20	20	17
Customer *C*	21	24	23	18
Customer *D*	21	18	20	19

The deliveries will be allocated in order to minimise the total distance that must be travelled. The Hungarian algorithm is to be used to find this minimum value.

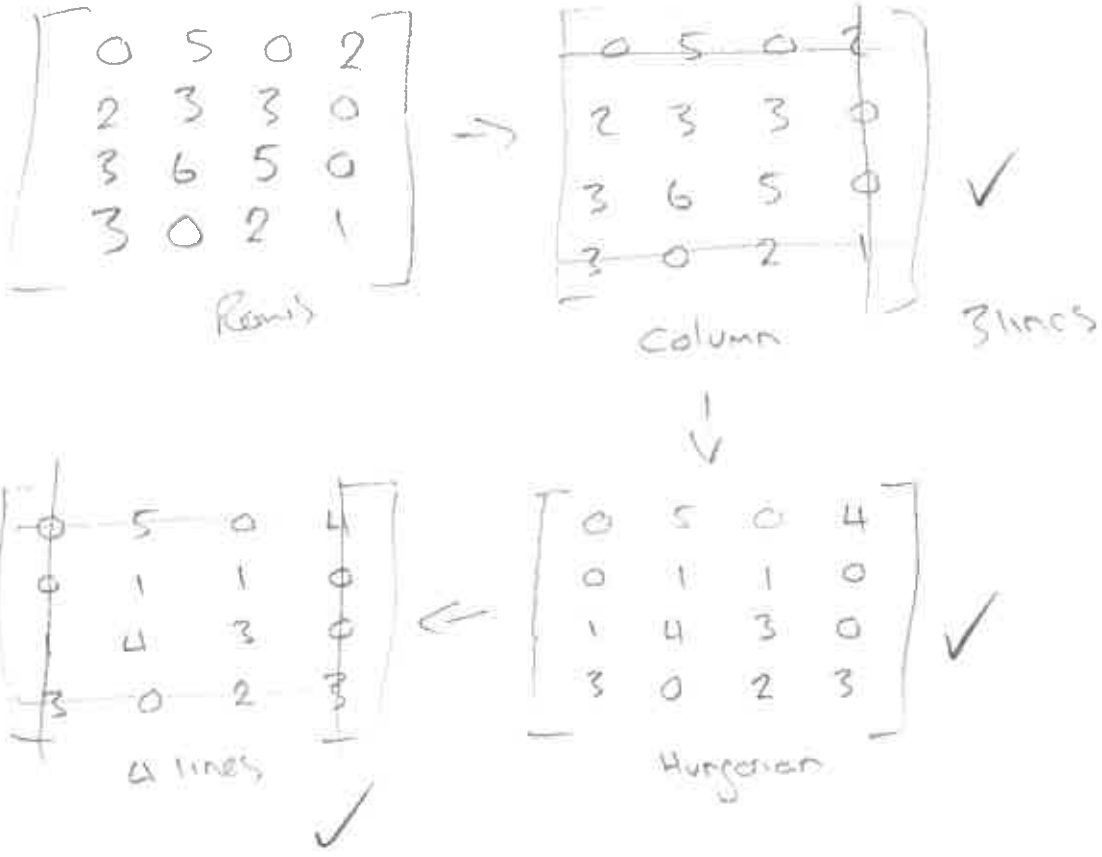
a. Step 1 of the Hungarian algorithm is to subtract the minimum entry in each row from each element in the row.

Complete step 1 for Customer *C* by writing the missing values in the table below. [2]

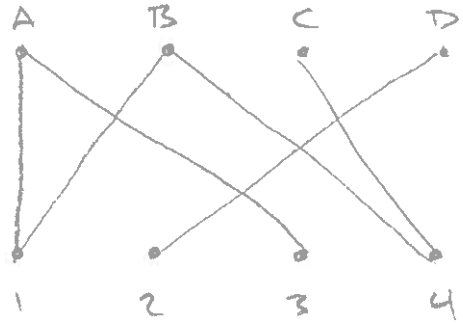
	Driver 1	Driver 2	Driver 3	Driver 4
Customer *A*	0	5	0	2
Customer *B*	2	3	3	0
Customer *C*	3	6	5	0
Customer *D*	3	0	2	1

(½ each)

b. Complete the Hungarian algorithm so that an allocation can be made. [3]



c. Draw the bipartite graph for the final matrix. [2]



✓½

d. Determine the allocation of drivers to customers. [2]

A3 B1 C4 D2

(½ each)

e. Find the total distance travelled by the drivers. [1]

17 + 19 + 18 + 18 = 72

72 km ✓

End of Test

Additional working space

Question number: _____