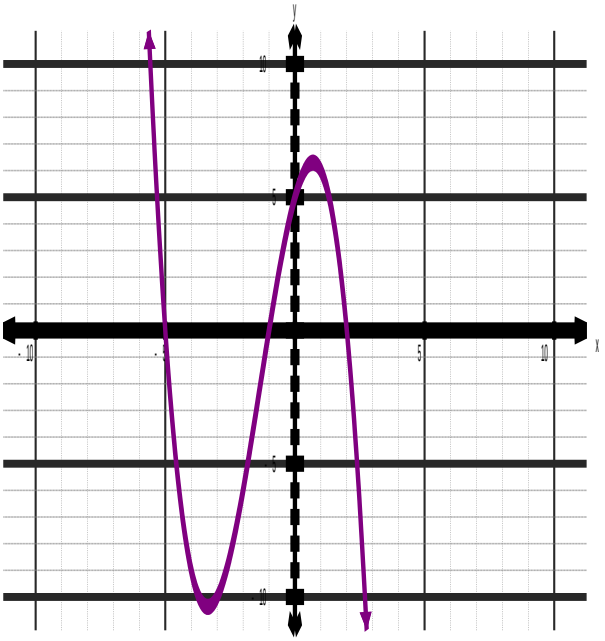
education equals

Calculator Assumed
Applications of Differentiation – Curve Sketching

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 2, 2, 2, 2 = 9 marks]

Examine the graph drawn below.



(a) State the y-intercept of the function.

(b) For what values of x is the gradient zero?

(c) For what values of x is the gradient negative?

(d) State the global maximum and the local minimum over the domain $-5 \leq x < -1$

(e) On the same set of axes, sketch a possible graph of $\frac{dy}{dx}$.

Question Two: [6 marks]

A curve has equation $f(x) = -(x+2)(x-4)$. Determine the equations of the tangents to the curve at the x – intercepts.

Question Three: [6 marks]

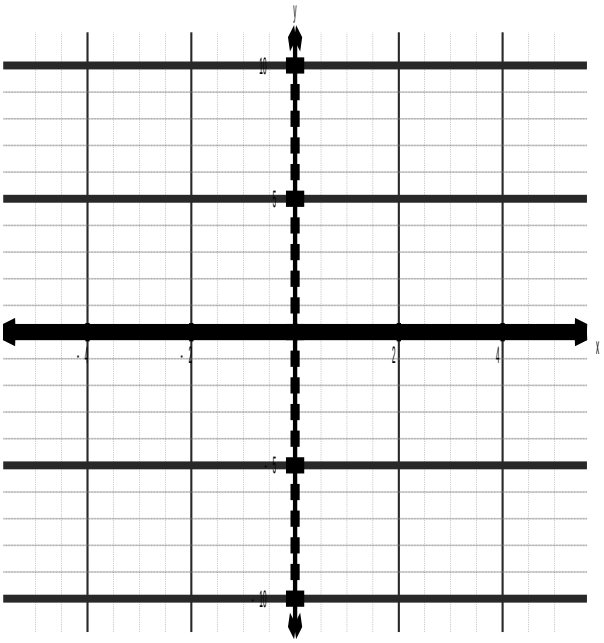
On the axes below, sketch the function with the following features:

$f(0) = 1$

$f(-1) = f(5) = 0$

$f'(-1) = f'(3) = 0$

$f'(x) > 0$ for $-1 < x < 2$



Question Four: [6 marks]

Show, using calculus methods, that the function $g(x) = (x-2)^2(x+1)^3$ has stationary points at $x = -1, x = 0.8, x = 2$.

Hence state the nature of each stationary point.

Question Five: [5 marks]

The equation of the tangent to the curve $f(x) = 2x^2 - 6x + k$ at $x = 1$ is $y = mx - 3$.

Determine the value of m and k.

Question Six: [8 marks]

The graph of the function $y = 2x^3 - ax^2 + bx + c$ has a y – intercept at (0, 10) and only one stationary point located at $x = \frac{1}{2}$.

Determine, using calculus methods, the values of a, b and c.

Question Seven: [5 marks]

The function $y = \frac{1}{3}x^3 - ax^2 + bx + 4$ has two stationary points, one at $x = -2$ and the other at $x = 3$.

Determine the values of *a* and *b* showing all working.

SOLUTIONS
Calculator Assumed
Applications of Differentiation – Curve Sketching

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Total Marks: 45
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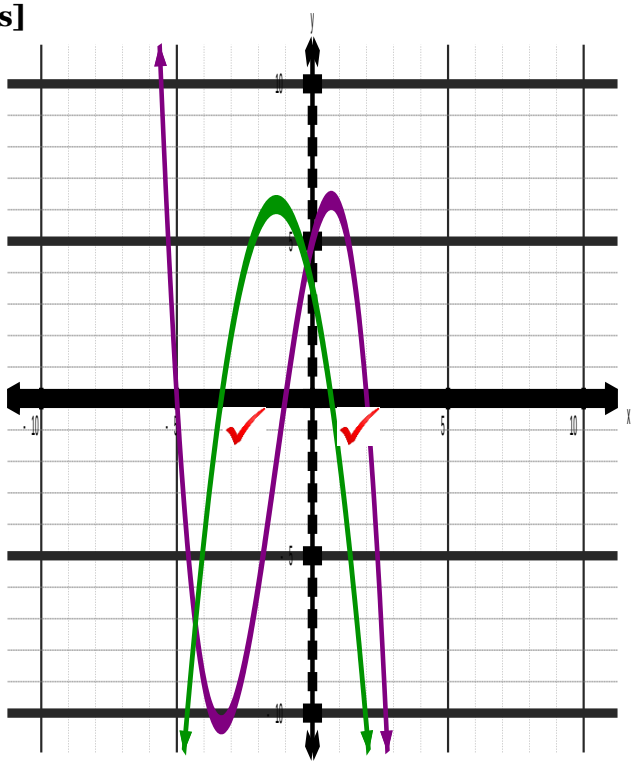
Question One: **[1, 2, 2, 2, 2 = 9 marks]**

Examine the graph drawn below.



(a) State the y-intercept of the function.

$(0,5)$ ✓

(b) For what values of x is the gradient zero?

$x \approx -3.5,\ x \approx 0.5$
✓ ✓

(c) For what values of x is the gradient negative?

$x < -3.5,\ x > 0.5$
✓ ✓

(d) State the global maximum and the local minimum over the domain $-5 \le x < -1$

local min : -10.2 ✓
global max : 0 ✓

(e) On the same set of axes, sketch a possible graph of $\frac{dy}{dx}$.

Question Two: [6 marks]

A curve has equation $f(x) = -(x+2)(x-4)$. Determine the equations of the tangents to the curve at the x – intercepts.

$(-2,0)\ (4,0)$ ✓

$f(x) = -x^2 + 2x + 8$

$f'(x) = -2x + 2$ ✓

$f'(-2) = 6$ ✓

$f'(4) = -6$ ✓

$y = 6x + c$

$0 = 6(-2) + c$

$y = 6x + 12$ ✓

$y = -6x + c$

$0 = -6(4) + c$

$y = -6x + 24$ ✓

Question Three: [6 marks]

On the axes below, sketch the function with the following features:

$f(0) = 1$

$f(-1) = f(5) = 0$

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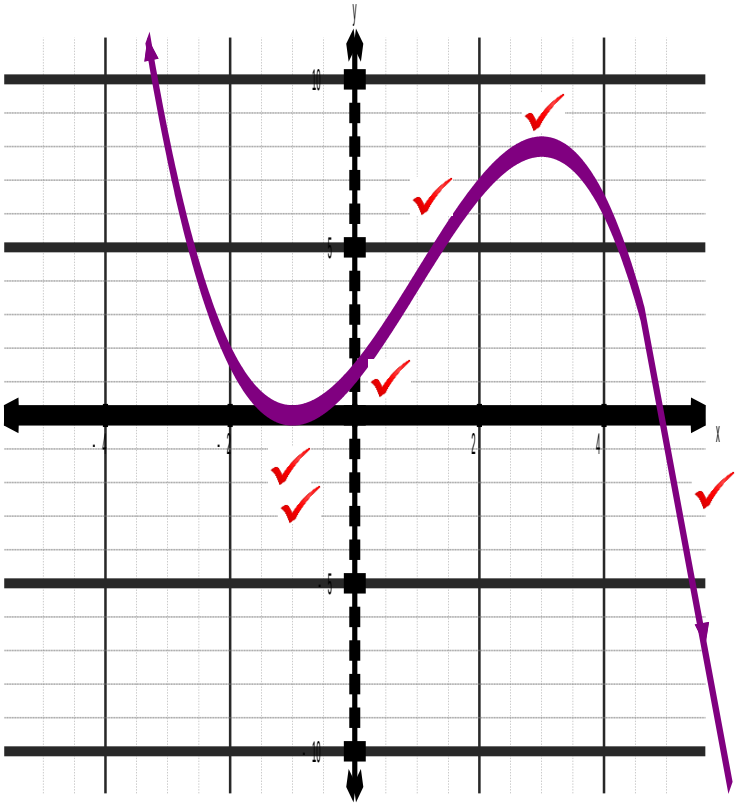
$f'(x) > 0$ for $-1 < x < 2$



Question Four: [6 marks]

Show, using calculus methods, that the function $g(x) = (x-2)^2(x+1)^3$ has stationary points at $x = -1, x = 0.8, x = 2$.

Hence state the nature of each stationary point.

$g(x) = x^5 - x^4 - 5x^3 + x^2 + 8x + 4$ ✓

$g'(x) = 5x^4 - 4x^3 - 15x^2 + 2x + 8$

$5x^4 - 4x^3 - 15x^2 + 2x + 8 = 0$ ✓

$x = -1, 0.8, 2$

$f''(x) = 20x^3 - 12x^2 - 30x + 2$ ✓

$f''(-1) = 0$ horizontal point of inflection ✓

$f''(0.8) = -19.44$ max ✓

$f''(2) = 54$ min ✓

Question Five: [5 marks]

The equation of the tangent to the curve $f(x) = 2x^2 - 6x + k$ at $x = 1$ is $y = mx - 3$.

Determine the value of m and k.

$f'(x) = 4x - 6$ ✓

$f'(1) = -2$ ✓

$m = -2$

$y = -2x - 3$

$y = -2(1) - 3$ ✓

$y = -5$ ✓

$-5 = 2(1)^2 - 6(1) + k$

$-1 = k$ ✓

Question Six: [8 marks]

The graph of the function $y = 2x^3 - ax^2 + bx + c$ has a y – intercept at (0, 10) and only one stationary point located at $x = \frac{1}{2}$.

Determine, using calculus methods, the values of a, b and c.

$c = 10$ ✓

$\frac{dy}{dx} = 6x^2 - 2ax + b$ ✓

$6(\frac{1}{2})^2 - 2a(\frac{1}{2}) + b = 0$

$-a + b = -1.5$ ✓

$\frac{d^2y}{dx^2} = 12x - 2a$ ✓

$x = \frac{1}{2}\ \frac{d^2y}{dx^2} = 0$ ✓

$12(\frac{1}{2}) - 2a = 0$ ✓

$-2a = -6$

$a = 3$ ✓

$-1 \times 3 + b = -1.5$

$b = 1.5$ ✓

Question Seven: [5 marks]

The function $y = \frac{1}{3}x^3 - ax^2 + bx + 4$ has two stationary points, one at $x = -2$ and the other at $x = 3$.

Determine the values of a and b showing all working.

$\frac{dy}{dx} = x^2 - 2ax + b$ ✓

$(-2)^2 - 2a(-2) + b = 0$

$4a + b = -4$ ✓

$(3)^2 - 2a(3) + b = 0$

$-6a + b = -9$ ✓

$a = 0.5$ ✓

$b = -6$ ✓