Name:

SOLUTIONS

Date: _____



Mathematics Methods Unit 1&2

Test 7, 2016

Chapters 8

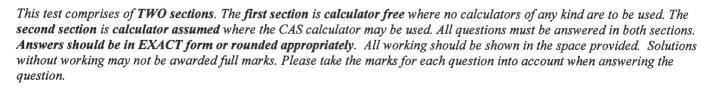
Topic – Sequences

Total Time:

60 minutes

Weighting:

6% of the year.



SECTION 1: CALCULATOR FREE

Time: 21 minutes

Equipment Allowed: SCSA Formula sheets

Marks for Section 1: 21 marks

1. [4 marks: 2, 2]

Write down the first four terms of the sequence defined by:

a)
$$T_n = T_{n-1} - 5$$
 and $T_1 = -1$

$$T_1 = -1$$
 $T_2 = -1 - 5 = -6$
 $T_3 = -6 - 5 = -11$
 $T_4 = -11 - 5 = -16$

b)
$$T_n = 3T_{n-1} - 2$$
 and $T_1 = 2$

$$T_1 = 2$$

 $T_2 = 3(2) - 2 = 4$
 $T_3 = 3(4) - 2 = 10$
 $T_4 = 3(10) - 2 = 28$





2. [5 marks: 3, 2]

An arithmetic sequence is described by the rule: $T_n = 200 - 5n$, where n = 1, 2, 3, 4, 5...

a) Find the first three terms of the sequence

$$N=1$$
 , $T_1 = 200 - 5(1) = 195$
 $N=2$, $T_2 = 200 - 5(2) = 190$
 $N=3$, $T_3 = 200 - 5(3) = 185$ V

b) State the recursive rule for this sequence

$$T_n = T_{n-1} - 5$$
 $T_1 = 195$

3. [4 marks]

If $t_8 = 24$ and $t_{12} = 40$, find the first term and the common difference, given the terms are from an arithmetic progression. $t_0 = a + (n-1)d$

$$t_8 = 24$$
 $t_{12} = 40$
 $24 = a + (8-1)d$
 $40 = a + (12-1)d$
 $40 = a + 11d$

FIR term = 4

[3 marks: 1, 1, 1]

The number of feral cats in a National park was initially 3500 but decreasing at an annual rate of 7.5%. Write a rule describing the number of cats after t years.

- The number of bacteria in a culture after t minutes is described by the rule $B = 50(1.12)^t$
- i) Describe the percentage change in the number of bacteria each minute.



Find how many bacteria were present initially.



[5 marks: 2, 2, 1]

Consider the sequence: 50, 42, 34, 26, ...

Determine:

a) The recursive rule for the sequence

$$T_n = T_{n-1} - 8$$
 V
 $T_n = 50$ V

b) The general rule for any term in the sequence

$$T_n = a + (n-1)d$$

 $T_n = 50 + (n-1)(-8)$
= 50 - 8n + 8
= 58 - 8n

c) Show using the general rule fom b) that -30 is a term in the sequence

$$-30 = 58 - 80$$

 $-88 = -80$
 $11 = 0$

Name:			
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SECTION 2: CALCULATOR ASSUMED

Time: 39 minutes

Equipment Allowed: Curriculum Council Formula sheets, CAS calculator, 1 page of notes (A4 one side)

3 each

a = 15

r=45 =3

Marks for Section 2: 36 marks

6. 6 marks: 2, 2, 2]

A geometric sequence is described by the rule $T_n = 5 \times 3^n$ where n = 1, 2, 3, 4, 5...

a) Find the first four terms of the sequence

$$N=1$$
, $T_1 = 5 \times 3^1 = 15$
 $N=2$, $T_2 = 5 \times 3^2 = 45$
 $N=3$, $T_3 = 5 \times 3^3 = 135$
 $N=4$, $T_4 = 5 \times 3^4 = 405$

b) How many terms less than (a) are there in this sequence?

c) Find the sum of the first 11 terms, show your method

$$S_n = \frac{\alpha(r^n - 1)}{r - 1}$$

$$S_n = \frac{15(3^n - 1)}{3 - 1}$$

$$= 1328595$$

7. [4 marks: 2, 2]



Given the sequence 6, 10, 14, 18, 22, ...

a) Determine the sum of the first twenty terms

$$S_n = {}^{n}2(2a + (n-1)d)$$

 $S_{20} = {}^{20}2(2(6) + (20-1)(4))$
 $= 880$

b) Does the number 624 belong in this sequence?

$$T_n = a + (n-1)d$$

 $624 = 6 + (n-1)4$
 $624 = 6 + 4n - 4$
 $622 = 4n$
 $155.5 = n$ No, 624 does not belong
in the sequence.

- [4 marks: 2, 2]
- a) Find the number of terms in the following sequence:

b) Which term in the following sequence is the first that is greater than 1000 000:

$$10000000 < 9 \times (\frac{3}{2})^{n-1}$$
 (Solve)

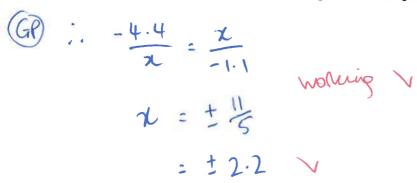
: The 30th term 15the 15T greater shan 1000 000

9. [6 marks: 4, 2]

a) The first term of a geometric sequence is 2, and the fifth term is 162. $t_n = \alpha \times r^{n-1}$ Find the sum of the first 10 terms.

$$t_1 = 2$$
 $t_5 = 162$
 $2 = a \times r^{\circ}_{out} \cdot 162 = a \times r^{4} \quad (solve)$
 $a = 2$ $S_{10} = a(r^{\circ}-1)$ OR $S_{10} = 2((-3)^{\circ}-1)$
 $r = \pm 3$ $= 2(3^{\circ}-1)$ $= -29524$

b) If -1.1, x, -4.4 are consecutive terms of a geometric sequence, find x.



10. [3 marks]

A hiker sets out on a 100km hike. She walks 36 km on the first day and $\frac{2}{3}$ that distance on the second. Every day thereafter she walks $\frac{2}{3}$ of the distance she walked on the day before.

Will the hiker cover the distance of 100 km to complete the walk and if so, on what day will she complete the task?

$$\alpha = 36$$
 $\Gamma = \frac{2}{3}$
Sn = 100

:.
$$Sn = \alpha(r^n-1)$$

$$100 = 36(2/3^n-1) \quad (some)$$

$$\frac{2}{3}$$

36,20,13.33

11. [6 marks: 2, 1, 1, 2]

Drilling tests show that in sinking a well, the distance drilled each hour decreases by 10%. A depth of 20 metres is drilled in the first hour.

a) Find how much is drilled in the second and third hours.

$$t_1 = 20$$

 $t_2 = 20 \times 0.9^2 = 18 \text{ m}$
 $t_3 = 20 \times 0.9^2 = 16.2 \text{ m}$

b) Explain why the distances drilled each hour will form a geometric progression.

c) Find the distance drilled in the 10th hour, correct to the nearest centimetre.

$$t_{10} = 20 \times 0.9^{9} = 7.7484$$

d) How long will it take to drill a depth of 100 metres? (answer to the nearest minute)

$$S_n = 100$$

 $S_n = a(r^n - 1)$
 $r - 1$
 $100 = 20(0.9^n - 1) v (solve)$
 $0.9 - 1$
 $n = 6.57881$ hovs.
 $6 h 35 muic$

Yr 1 Rent \$300 / week 2 Rent \$320 / week

12. [7 marks: 2, 2, 3]

Kaya rents her house out at \$300 per week.

Each year she increases the weekly rent by \$20. a) Find a recursive rule for the weekly rent after n years.

$$T_n = T_{n-1} + 20$$
 $T_1 = 300$

How long will it take for the weekly rent to double?

How many years would it take for the total rent to be collect to exceed \$200 000?

$$S_n = \frac{1}{2}(2\alpha + (n-1)d)$$

 $200000 = \frac{1}{2}(2(300) + (n-1)(1040)) \times (50)$
 10^{-1} (9) By the end of the 10^{-1} year.