Calculator Free
The Derivative and Rates of Change

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: **[1, 2, 2, 2, 2 = 9 marks]**

For each of the following, recognise and state the function being differentiated .

(a) $\lim_{h\to 0}\dfrac{(x+h)^2-2(x+h)-(x^2-2x)}{h}$

(b) $\lim_{h\to 0}\dfrac{3x^2+6xh+3h^2-3x^2}{h}$

(c) $\lim_{h\to 0}\dfrac{(x+h)^3-4x-4h-x^3+4x}{h}$

(d) $\lim_{h\to 0}\dfrac{-2xh-x^2-h^2+x^2}{h}$

(e) $\lim_{h\to 0}\dfrac{10xh+5h^2}{h}$

Question Two: **[3, 3, 3 =9 marks]**

Evaluate each of the following:

(a) $\lim\limits_{h \to 0} \dfrac{(1+h)^2 - 1^2}{h}$

(b) $\lim\limits_{h \to 0} \dfrac{-(2+h)^3 + 8}{h}$

(c) $\lim\limits_{h \to 0} \dfrac{\sqrt{16+h} - 4}{h}$

Question Three: [2, 2, 2 = 6 marks]

Consider the function $f(x) = -2x + 3$.

(a) Determine the instantaneous rate of change of the function when $x = 3$.

(b) Calculate the average rate of change for the function between $x = 3$ and $x = 5$.

(c) Explain why your answer to part (a) and part (b) are the same.

Question Four: [1, 3, 2, 2 = 8 marks]

The cost of a new piece of technology over the next decade is modeled by the function $C(x) = -0.5x^2 + 1200$, where C is the cost in dollars and x is the number of days since the technology was introduced.

The company who invented the technology is interested in the change in cost between day 10 and day 11.

(a) For this time period, state ∂x.

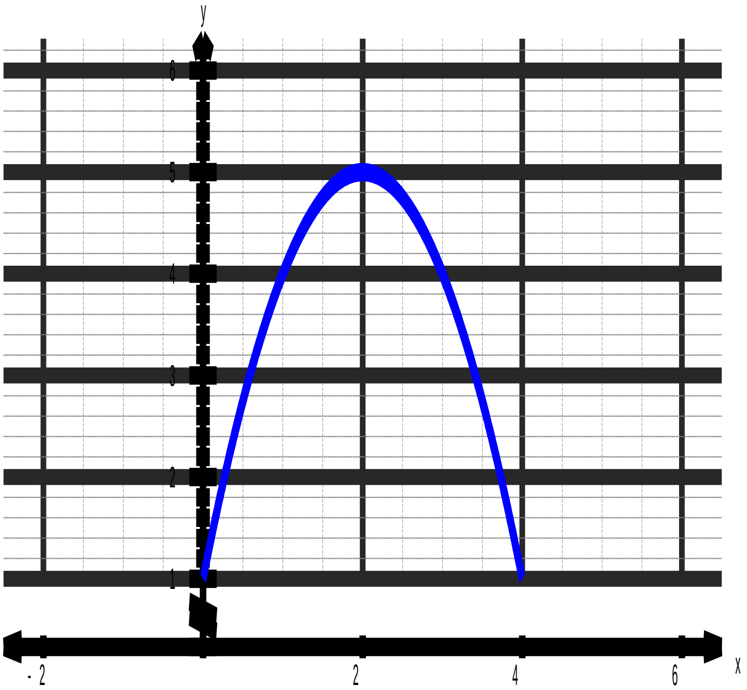
(b) Hence evaluate ∂C.

(c) Evaluate the exact difference in costs between day 10 and day 11.

(d) Explain why your answers to part (b) and (c) are different.

Question Five: **[5 marks]**

Sketch, accurately, the tangent to the curve $f(x) = -(x-2)^2 + 5$ at $x = 3$.



Question Six: [3 marks]

The equation of the tangent to the curve $f(x) = ax^2$ is $y = -4x - 2$ when $x = -1$.

Determine the value of a.

Question Seven: [5 marks]

Differentiate $y = x^2 - 3x$ using first principles.

SOLUTIONS
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Question One: **[1, 2, 2, 2, 2 = 9 marks]**

For each of the following, recognise and state the function being differentiated .

(a) $\lim_{h\to 0}\dfrac{(x+h)^2-2(x+h)-(x^2-2x)}{h}$ $f(x)=x^2-2x$ ✓

(b) $\lim_{h\to 0}\dfrac{3x^2+6xh+3h^2-3x^2}{h}$ $f(x)=3x^2$ ✓✓

(c) $\lim_{h\to 0}\dfrac{(x+h)^3-4x-4h-x^3+4x}{h}$ $f(x)=x^3-4x$ ✓✓

(d) $\lim_{h\to 0}\dfrac{-2xh-x^2-h^2+x^2}{h}$ $f(x)=-x^2$ ✓✓

(e) $\lim_{h\to 0}\dfrac{10xh+5h^2}{h}$ $f(x)=5x^2$ ✓✓

Question Two: **[3, 3, 3 = 9 marks]**

Evaluate each of the following:

(a) $\lim_{h \to 0} \frac{(1+h)^2 - 1^2}{h}$

$f(x) = x^2$ ✓
$f'(x) = 2x$ ✓
$f'(1) = 2$ ✓

(b) $\lim_{h \to 0} \frac{-(2+h)^3 + 8}{h}$

$f(x) = -x^3$ ✓
$f'(x) = -3x^2$ ✓
$f'(2) = -12$ ✓

(c) $\lim_{h \to 0} \frac{\sqrt{16+h} - 4}{h}$

$f(x) = \sqrt{x}$ ✓
$f'(x) = 0.5x^{-0.5}$ ✓
$f'(16) = \frac{1}{8}$ ✓

Question Three: [2, 2, 2 = 6 marks]

Consider the function $f(x) = -2x + 3$.

(a) Determine the instantaneous rate of change of the function when $x = 3$.

$f'(x) = -2$

$f'(3) = -2$

(b) Calculate the average rate of change for the function between $x = 3$ and $x = 5$.

$$\frac{f(5) - f(3)}{5 - 3} = \frac{-7 - -3}{2} = -2$$

(c) Explain why your answer to part (a) and part (b) are the same.

Because the function is linear and therefore the gradient is constant at all points.

Question Four: [1, 3, 2, 2 = 8 marks]

The cost of a new piece of technology over the next decade is modeled by the function $C(x) = -0.5x^2 + 1200$, where C is the cost in dollars and x is the number of days since the technology was introduced.

The company who invented the technology is interested in the change in cost between day 10 and day 11.

(a) For this time period, state ∂x.

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(b) Hence evaluate ∂C.

$$\frac{\partial C}{\partial x} = -x$$

$\partial C = -(10) \times \partial x$

$\partial C = -10$

(c) Evaluate the exact difference in costs between day 10 and day 11.

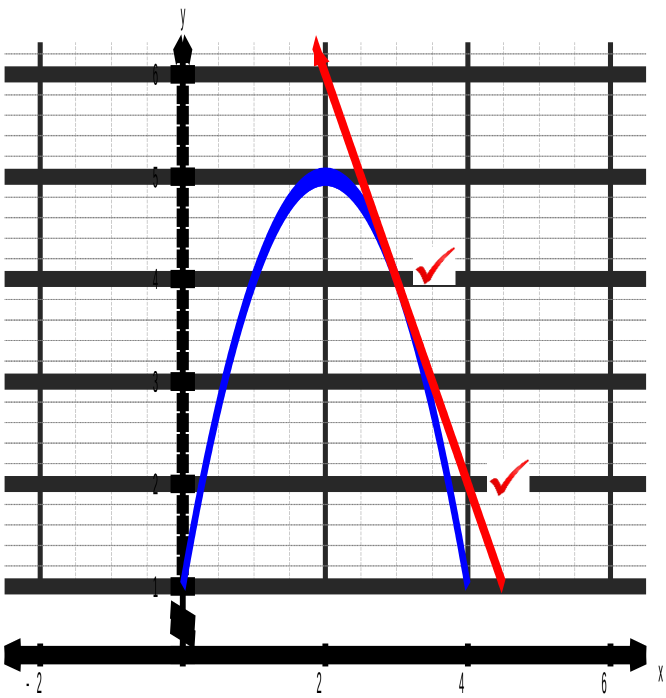
$C(11) - C(10) = 1139.5 - 1150 = -10.5$

(d) Explain why your answers to part (b) and (c) are different.

Part (b) gives the approximate change based on the change in gradient at that point. It is an approximation, while part (c) is an exact change.

Question Five: [5 marks]

Sketch, accurately, the tangent to the curve $f(x) = -(x-2)^2 + 5$ at $x = 3$.



$f(x) = -(x^2 - 4x + 4) + 5$

$f(x) = -x^2 + 4x + 1$

$f'(x) = -2x + 4$

$f'(3) = -2$

$f(3) = 4$

$y = -2x + 10$

Question Six: [3 marks]

The equation of the tangent to the curve $f(x) = ax^2$ is $y = -4x - 2$ when $x = -1$.

Determine the value of *a*.

$f'(x) = 2ax$

$2a(-1) = -4$

$a = 2$

Question Seven: [5 marks]

Differentiate $y = x^2 - 3x$ using first principles.

$$\lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 3)}{h}$$

$$= 2x - 3$$