GREENWOOD COLLEGE
Mathematics Methods Units 3 & 4
Test 3 Integration 2019

Name Marking Key Mark ____/17

All electronic devices must be switched off and in bags.
Access to Formulae Sheet allowed. No notes.
No calculators allowed in this section. Time limit 15 minutes.

1. **[1, 1 = 2 marks]**

For a positive function $a(x)$, $\int_2^5 a(x)dx = 10$ and $\int_5^9 a(x)dx = 12$.
Determine:

a) $\int_5^5 a(x)dx$

$= 0$ ✓

b) $\int_2^9 a(x)dx$

$= 22$ ✓

2. **[4 marks]**

Use the Fundamental Theorem of Calculus to find the area between the x-axis and $y = \sqrt{x}$ from $x = 1$ to $x = 9$

$\int_1^9 \sqrt{x}\,dx = \left[\frac{2}{3}x^{3/2}\right]_1^9$ antiderivative ✓

$= \frac{2}{3} \times 27 - \frac{2}{3}$ substitutions ✓✓

$= 18 - \frac{2}{3}$ simplification ✓

$= 17\frac{1}{3}$ units^2

3. **[2,2,2 = 6 marks]**

Integrate with respect to :

(Missing +c = −1 mark total)

a) $\cos x \sin^2 x$

$= \frac{\sin^3 x}{3} + c$ ✓ ✓

b) $2e^{2x} - e^{-3x}$

$= e^{2x} + \frac{e^{-3x}}{3} + c$ ✓ ✓

c) $4\sqrt{5-x}$

integration of bracket ✓
readjustments ✓

$y' = 4(5-x)^{1/2}$

$y = \frac{4(5-x)^{3/2}}{3/2 \times -1} = \frac{-8(5-x)^{3/2}}{3} + c$ or $\frac{-8}{3}\sqrt{(5-x)^3} + c$

4. **[3 marks]**

Determine $\frac{d}{dx}(x\cos x)$ and hence find $\int x\sin x\,dx$.

$\frac{d}{dx}\, x\cos x$

$= \cos x - x\sin x$ (derivative ✓)

$\therefore \int \cos x\,dx - \int x\sin x\,dx = x\cos x + c$ (reverse the differentiation ✓)

$\therefore \int x\sin x\,dx = \int \cos x\,dx - x\cos x + k.$ $(k=-c)$

or $\sin x - x\cos x + k$ (rearrange to make $\int x\sin x\,dx$ the subject ✓)

5. **[2,2 = 4 marks]**

Determine $\frac{dy}{dx}$ for the following:

a) $y = \int_1^x \frac{1}{\sqrt{1+t^2}}\,dt$

$= \frac{1}{\sqrt{1+x^2}}$ ✓

by F.T.C.

(no C = ✓)

b) $y = \int_a^x \frac{d}{dx}(4t^3)\,dt$

$= \int_a^x 4t^3\,dt$ ($\frac{d}{dx}$ has no effect)

$= x^4 - a^4$ (substitution ✓)

$= 4x^3$ (derivative ✓

END OF SECTION

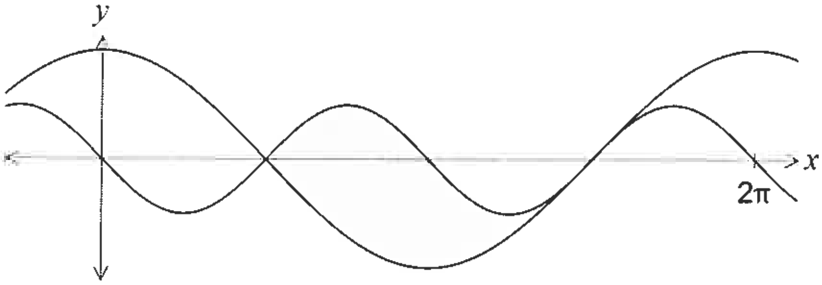
GREENWOOD COLLEGE
Mathematics Methods Units 3 & 4
Test 3 Integration 2019

Name Marking Key Mark /~~38~~ 36

All electronic devices must be switched off and in bags.
Access to Formulae Sheet and one sheet of A4 notes allowed. Use of approved calculators is assumed in this section.
Time limit 40 minutes.

6. **[~~5~~ 4 marks]**

The shaded region on the graph below is enclosed by the curves $y = -\sin(2x)$ and $y = 2\cos x$.



Show that the area of the region is 4 square units.

Intersections are at (1.5708, 0) and (4.712389, 0)

(intersections ✓✓)

$$\text{Area} = \int_{1.5708}^{4.712389} -\sin 2x - 2\cos x \, dx$$

(correct integral ✓)

$$= \left[\frac{\cos 2x}{2} - 2\sin x \right]_{1.5708}^{4.712389}$$

(order of functions correct or | | ✓)

$$= 4$$

7. **[2,2,6,2 = 12 marks]**

Callum is happily paddling his kayak on the Greenwood College Outdoor Education camp until his craft hits a rock. Water begins to leak in at the rate of $R_1(t) = 5 - 5e^{-0.2t}$ litres per minute, where t is the time in minutes.
Callum starts to bail the water out of his kayak , removing at a rate of $R_2(t) = 6 - 6e^{-0.1t}$ litres per minute.

a) After 2 minutes, at what rate is the water:

i) leaking into the kayak

1.64839977 L/min
✓

ii) being bailed from the kayak

1.087615482 L/min
✓

b) Is the amount of water in the kayak increasing or decreasing after 2 minutes ? Explain your answer.

increasing ✓
because water is coming in at a higher rate than being bailed out
(reason ✓)

c) Evaluate the following integrals, and interpret their meaning:

i) $\int_0^3 R_1(t)dt$

= 3.720290902
✓
(integral)

This is the amount of water in Litres leaked into Kayak in first 3 minutes
✓
(explanation)

ii) $\int_2^5 R_2(t)dt$

= 5.267994398
(integral ✓)

This is amount of water in litres bailed out of the kayak between 2 minutes and 5 minutes
(explanation ✓)

iii) $\int_0^8 [R_1(t) - R_2(t)]dt$

= 5.087675103
(integral ✓)

Amount of water in the kayak in litres 8 minutes after hitting the rock
(explanation ✓)

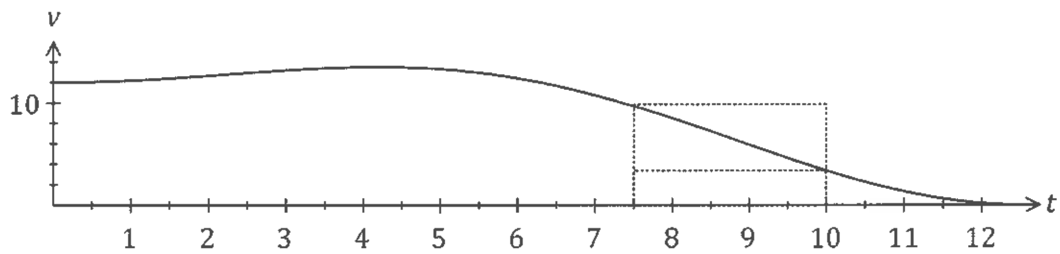
d) How much water is in the kayak after 10 minutes ?

$\int_0^{10} (5 - 5e^{-0.2t} - 6 + e^{-0.1t})\,dt$
= 6.31061561l Litres

(integral ✓)
(answer ✓)

8. **[2,5,1 = 8 marks]**

The speed, in metres per second, of a car approaching a stop sign is shown in the graph below and can be modelled by the equation $v(t) = 6(1 + \cos(0.25t) + \sin^2(0.25t))$, where t represents the time in seconds.



The area under the curve for any time interval represents the distance travelled by the car.

(a) Complete the table below, rounding to two decimal places.

t	0	2.5	5	7.5	10
$v(t)$	12.00	12.92	13.30	9.66	3.34

values ✓
rounding ✓

(b) Complete the following table and hence estimate the distance travelled by the car during the first ten seconds by calculating the mean of the sums of the inscribed areas and the circumscribed areas, using four rectangles of width 2.5 seconds.
(The rectangles for the 7.5 to 10 second interval are shown on the graph.)

Interval	0 – 2.5	2.5 – 5	5 – 7.5	7.5 – 10
Inscribed area	30.0	32.3	24.15	8.35
Circumscribed area	32.3	33.25	33.25	24.15

Inscribed total = 94.8

Circumscribed total = 122.95

Estimate = $\frac{94.8 + 122.95}{2}$

$\simeq$ 108.875 m

(values 1st col ✓
values 2nd col. ✓
values 3rd. col. ✓
sums ✓
estimate that rounds to 109 ✓)

(c) Suggest one change to the above procedure to improve the accuracy of the estimate.

Use larger number of or thinner rectangles.

(valid suggestion ✓)

9. **[5 marks]**

The area bounded by the curve $y = e^{2-x}$ and the lines $y = 0$, $x = 1$ and $x = k$ is exactly $e - 1$ square units. Determine the value of the constant k, given that $k > 1$.

$$\int_1^k e^{2-x}\,dx$$ integral ✓

$$= -e^{2-k} - (-e^1)$$ antiderivative ✓

$$= e - e^{2-k}$$ & substitution ✓

Therefore $e - e^{2-k} = e - 1$ equation ✓

$$\Rightarrow e^{2-k} = 1.$$

$$\Rightarrow k = 2$$ solves for k ✓

10. [2,~~3~~ 2,3 = ~~8~~ 7 marks]

A particle is initially at the origin and moving to the right at 5 cms^{-1}. It accelerates with time according to the rule $a(t) = 4 - 2t$ cms^{-2}.

a) Determine the velocity function of the particle.

$v(t) = 4t - t^2 + c$ (antiderivative ✓)

At $t = 0$ $v(t) = 5$ so $c = 5$

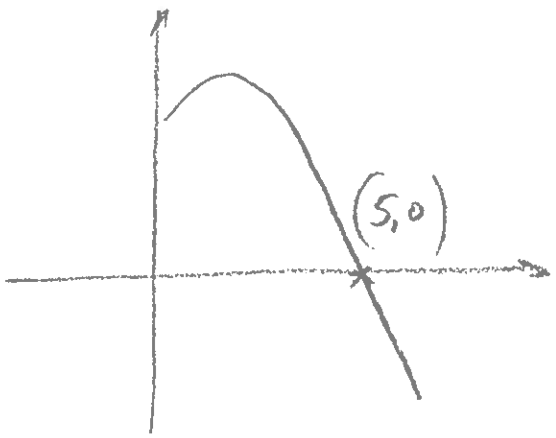
$\Rightarrow v(t) = 4t - t^2 + 5$ (evaluates c ✓)

b) For the first 6 seconds of motion, determine the displacement of the particle.

$\int_0^6 (4t - t^2 + 5)\,dt$ integral ✓

$= 30$ cm correct value ✓

c) For the first 6 seconds of motion, determine the total distance travelled.



Distance $= \int_0^5 v(t)\,dt - \int_5^6 v(t)\,dt$

or $\int_0^6 |v(t)|\,dt$ correct integrals ✓✓

$= 33\tfrac{1}{3} - (-3\tfrac{1}{3})$

$= 36\tfrac{2}{3}$ cm correct value ✓

END OF PAPER