Yr 12 METHODS TEST 1 2017

DIFFERENTIATION, APPLICATIONS AND EXPONENTIALS

Time: 20 minutes **Total: 19 marks**

Student Name: ___________________ **Teacher: ____________**

Instructions: ***Show all working clearly.***
Sufficient detail must be shown for marks to be awarded for reasoning.
NO CALCULATOR AND NO PERSONAL NOTES ALLOWED

Question 1. (8 marks)

a) Differentiate the following. No need to simplify your answer.

i) $y = x^2 e^{5x+1}$ [2]

ii) $\frac{3x^3-5x}{x+3}$ [2]

b) The derivative of $y = (2x+1)(4x^2+2x)^4$ can be written in the form; [4]

$$\frac{dy}{dx} = 4(4x^2+2x)^3(ax^2+bx+2)$$

Determine the values of a and b.

Question 2. (4 marks)

A cylinder with constant height (h) has a volume modelled by $V = \pi r^2 h$. Using the incremental formula, determine the percentage change in radius (r) if the volume is increased by 3%.

Question 3. (7 marks)

Solve for x in each of the following;

a) $\log_2 x = 4$ [1]

b) $\log_x 4 = \frac{1}{2}$ [2]

c) $\log_2 x + \log_2(x-7) = 3$ [4]

Yr 12 METHODS TEST 1 2017

DIFFERENTIATION, APPLICATIONS AND EXPONENTIALS

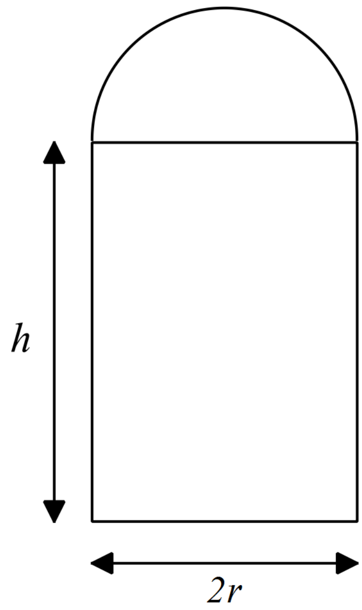
Time: 35 minutes **Total: 35 marks**

Student Name: __________________ **Teacher: ____________**

Instructions: ***Show all working clearly.***
Sufficient detail must be shown for marks to be awarded for reasoning.
CALCULATOR AND 1 PAGE PERSONAL NOTES ALLOWED

Question 4. **(12 marks)**

The diagram shows an arched church wooden window frame, to be made from 10m of timber.



Note: All lines drawn are part of the frame

a) Show that $h = \frac{10-4r-\pi r}{2}$

[2]

b) Show that the area of the window is $A = 10r - r^2\left(4 + \frac{\pi}{2}\right)$ [3]

Hence, or otherwise,

c) show that the **exact** value of r that maximises the area is $r = \frac{10}{8+\pi}$

[4]

d) Suppose the radius (r) is increased by 10cm. Find the approximate change, using calculus methods, in the height of the window if the 10m of timber restriction still applies. [3]

Question 5. **(10 marks)**

A particle is travelling in a straight line such that its displacement in metres (d) at any time in seconds (t) is such that;

$d = \frac{t^3 - 3t^2 - 9t + 6}{3}$, $0 \leq t \leq 6$ where a positive (d) value represents the particle to the right of a particular point A.

a) Is the particle initially to the right or the left of point A? [1]

b) Showing the use of calculus, determine the velocity of the particle at $t = 2$. [2]

c) Determine the distance travelled by the particle in the first 4 seconds. [3]

d) What is the maximum speed the particle obtains over the given domain? [2]

e) How long does the particle spend to the right of point A in the given domain? [2]

Question 6. (6 marks)

Iodine-131 is present in radioactive water from the nuclear power industry.

It has a half-life of eight days. This means that every eight days, one half of the iodine-131 decays to a form that is not radioactive.

This decay can be represented by the equation $N = N_0 e^{kt}$,

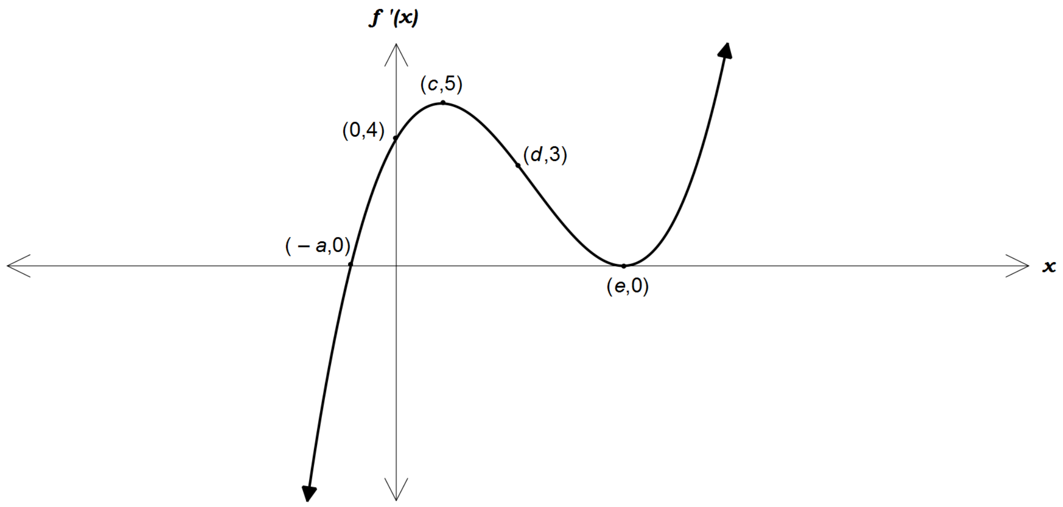
Where $N =$ amount of iodine-131 present after t days, and
$N_0 =$ amount of iodine-131 present initially.

a) Showing use of logarithms, determine the value of k correct to three decimal places. [3]

b) If 125 milligrams of iodine-131 are considered to be safe, how many full days will it take for 88 grams of iodine-131 to decay to a safe amount? [3]

Question 7. (7 marks)

The graph below shows the **derivative** $f'(x)$ of a function.



a) Use the graph of $f'(x)$ to state the x-values of all stationary points of the original function (and their nature) and the x-values of the points of inflection. [5]

b) Given that the original function $f(x)$ passes through the point $(d, 10)$, write an equation (**in terms of *d***) for the line that is tangential to the function at $(d, 10)$. [3]