Show how to use the graph to find:

a the speed of the car after 12 s

b the speed gain between 4.0 s and 6.2 s.

13.5 Solving problems using algebra

Algebraic analysis of motion under constant acceleration makes use of the *suvat* equations, for which the following symbols apply:

- distance interval = s (s is not a particular point on a line)
- initial speed = u
- final speed = v
- acceleration = a
- time interval = t (t is not an instantaneous time)

The equations for straight-line motion with constant acceleration are:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2 \qquad s = vt - \frac{1}{2}at^2$$

$$s = \frac{u+v}{2}t$$

$$v^2 = u^2 + 2as$$

Each equation connects four variables:

- Knowing three variables, a fourth can be deduced
- With four known variables, the fifth can be deduced.

WORKED EXAMPLE

An arrow is launched vertically with a speed of 40 m s^{-1}.

a Calculate the time taken to reach the highest point.

b Determine the maximum height to which the arrow rises.

- Take air friction to be negligible.
- Assume acceleration due to gravity = 9.80 m s^{-2}.

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ANSWER

a The acceleration due to gravity is uniform (constant) and negative with respect to the change of speed as the arrow rises:

$s = ?\ \ u = 40\ \mathrm{m\,s^{-1}}\ \ v = 0\ \mathrm{m\,s^{-1}}\ \ a = {}^{-}9.80\ \mathrm{m\,s^{-2}}\ \ t = 4.1\ \mathrm{s}$

$$v = u + at$$

$$\Rightarrow t = \frac{v-u}{a}$$

$$\Rightarrow t = \frac{0\ \mathrm{m\,s^{-1}} - 40\ \mathrm{m\,s^{-1}}}{{}^{-}9.80\ \mathrm{m\,s^{-2}}}$$

$$\Rightarrow t = \frac{{}^{-}40\ \mathrm{m\,s^{-1}}}{{}^{-}9.80\ \mathrm{m\,s^{-2}}}$$

$$\Rightarrow t = 4.1\ \mathrm{s}\ (4.08\ \mathrm{s})$$

b $s = ?\ \ u = 40\ \mathrm{m\,s^{-1}}\ \ v = 0\ \mathrm{m\,s^{-1}}\ \ a = {}^{-}9.80\ \mathrm{m\,s^{-2}}\ \ t = 4.08\ \mathrm{s}$

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow s = 40\ \mathrm{m\,s^{-1}} \times 4.08\ \mathrm{s} + \frac{1}{2}({}^{-}9.80\ \mathrm{m\,s^{-2}}) \times (4.08\ \mathrm{s})^2$$

$$\Rightarrow s = 82\ \mathrm{m}\ (81.6\ \mathrm{m})$$

Alternative 1: Note that $v = 0\,\mathrm{m\,s^{-1}}$ at the top simplifies the calculation:

$s = ?\ \ u = 40\ \mathrm{m\,s^{-1}}\ \ v = 0\ \mathrm{m\,s^{-1}}\ \ a = {}^{-}9.80\ \mathrm{m\,s^{-2}}\ \ t = 4.08\ \mathrm{s}$

$$s = vt - \frac{1}{2}at^2$$

$$\Rightarrow s = 0\ \mathrm{m\,s^{-2}} - \frac{1}{2}({}^{-}9.80\ \mathrm{m\,s^{-2}}) \times (4.08\ \mathrm{s})^2$$

$$\Rightarrow s = 82\ \mathrm{m}\ (81.6\ \mathrm{m})$$

Alternative 2:

$s = ?\ \ u = 40\ \mathrm{m\,s^{-1}}\ \ v = 0\ \mathrm{m\,s^{-1}}\ \ a = {}^{-}9.80\ \mathrm{m\,s^{-2}}\ \ t = 4.08\ \mathrm{s}$

$$s = \frac{(u+v)}{2}t$$

$$\Rightarrow s = \frac{(40\ \mathrm{m\,s^{-1}} + 0\ \mathrm{m\,s^{-1}})}{2} \times 4.08\ \mathrm{s}$$

$$\Rightarrow s = 82\ \mathrm{m}\ (81.6\ \mathrm{m})$$

Alternative 3:

$s = ?\ \ u = 40\ \mathrm{m\,s^{-1}}\ \ v = 0\ \mathrm{m\,s^{-1}}\ \ a = {}^{-}9.80\ \mathrm{m\,s^{-2}}\ \ t = 4.08\ \mathrm{s}$

$$v^2 = u^2 + 2as$$

$$\Rightarrow s = \frac{v^2 - u^2}{2a}$$

$$\Rightarrow s = \frac{(0\ \mathrm{m\,s^{-1}})^2 - (40\ \mathrm{m\,s^{-1}})^2}{2 \times ({}^{-}9.80\ \mathrm{m\,s^{-2}})}$$

$$\Rightarrow s = 82\ \mathrm{m}\ (81.6\ \mathrm{m})$$

Thus, once the fourth variable has been deduced (in this case, t), a variety of solution strategies can be used to deduce the fifth variable (in this case, s). With experience, it is possible to select the simplest of the possibilities more often.

QUESTIONS

1 Complete the following table:

s (m)	u (m s^{-1})	v (m s^{-1})	a (m s^{-2})	t (s)
35	10	5.0		
	15	35	6.0	
		0	$^{-}10$	8.0
36			10	12
96	12			5.0

2 An aeroplane travelling at 40 m s^{-1} accelerates uniformly to 60 m s^{-1} in 10 s.

a Calculate the acceleration of the aeroplane.

b Determine the speed of the aeroplane after:

i 4.0 s

ii 10 s.

c Calculate the distance travelled by the aeroplane while it was accelerating.

3 A rock is dropped from a height and reaches a speed of 300 m s^{-1} just before it lands. Find the height from which it was dropped.

- Ignore any air friction.
- Assume acceleration due to gravity = 9.80 m s^{-2}.

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4 A jet takes 0.95 km to reach its take-off speed of $123\,\mathrm{km\,h^{-1}}$.

a Calculate the time taken for the jet to reach its take-off speed.

b Determine the acceleration of the jet.

c There is a 2.1 km safety zone at the end of the runway. The pilot needs to abort the take-off when travelling at $84\,\mathrm{km\,h^{-1}}$. Find the minimum average acceleration needed in order for the jet to come to a stop within the safety zone.

5 Given the maximum deceleration of a car is $8.0\,\mathrm{m\,s^{-2}}$ and the reaction time for the driver is 1.0 s, compare the stopping distances for the car from these initial speeds.

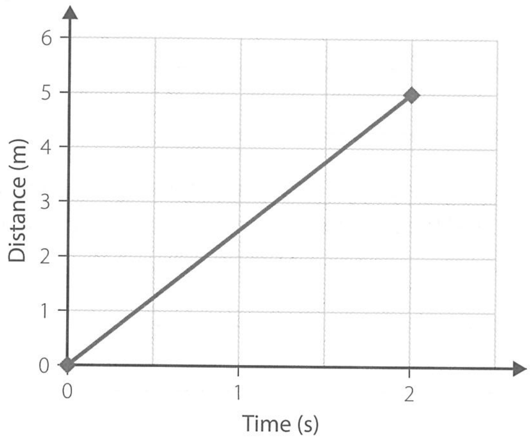
a $90\,\mathrm{km\,h^{-1}}$ ($25\,\mathrm{m\,s^{-1}}$)

b $45\,\mathrm{km\,h^{-1}}$

EVALUATION

1 The graph shows the position of a particle while moving along a straight line.

FIGURE 13.5.1



The speed of the particle at 1.5 s is closest to:

A $2.5\,m\,s^{-1}$.

B $4.0\,m\,s^{-1}$.

C $5.0\,m\,s^{-1}$.

D $10\,m\,s^{-1}$.

2 Velocity is measured as:

A distance divided by time.

B time rate of change of distance.

C time rate of change of displacement.

D the time it takes to move from one position to another.

3 It takes Van an hour to travel the first 80 km of a trip and an hour and a half for the remaining 135 km. What is Van's average speed for the journey?

A $80\,km\,h^{-1}$

B $85\,km\,h^{-1}$

C $86\,km\,h^{-1}$

D $88\,km\,h^{-1}$

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4 The instantaneous speed at a particular time is deduced from a distance–time graph by finding:

A the difference in height of the graph from start to finish and dividing by time.

B the difference in height of the graph from start to finish.

C the gradient of the graph at that particular time.

D the area under the graph.

5 The graph in Figure 13.5.2 shows how the speed of a car changes over time.

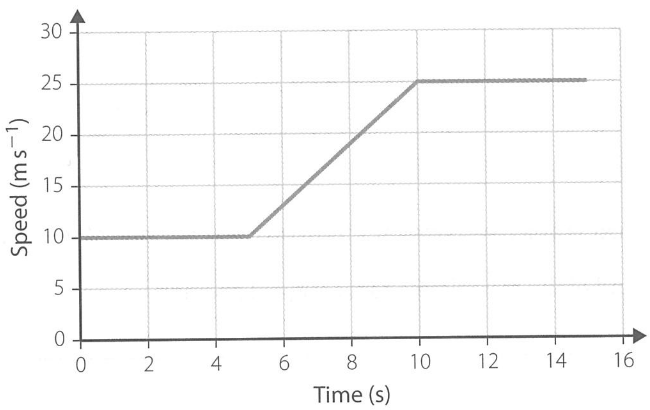

FIGURE 13.5.2

The distance travelled by the car between 4.0 s and 10 s is:

A 25 m.

B 125 m.

C 150 m.

D 250 m.

6 The speed of a cyclist changes from 4.0 $m\,s^{-1}$ to 10 $m\,s^{-1}$ in 3.0 s. What is the average acceleration of the cyclist?

A 2.0 $m\,s^{-2}$.

B 3.0 $m\,s^{-2}$.

C 6.0 $m\,s^{-2}$.

D 10 $m\,s^{-2}$.

7 The area under an acceleration–time graph shows:

A the change in acceleration.

B the distance travelled.

C the change in speed.

D the average speed.

8 A car travelling at 16 $m\,s^{-1}$ accelerates uniformly at 3.0 $m\,s^{-2}$ for 4.0 s. The speed of the car after 4.0 s is:

A 19 $m\,s^{-1}$.

B 20 $m\,s^{-1}$.

C 24 $m\,s^{-1}$.

D 28 $m\,s^{-1}$.

9 How far will a car that accelerates uniformly from $12\,m\,s^{-1}$ to $20\,m\,s^{-1}$ in 4.0 s travel?

A 48 m

B 64 m

C 88 m

D 112 m

10 A car accelerates uniformly from $12\,m\,s^{-1}$ to $20\,m\,s^{-1}$ in 4.0 s. How far will it travel in that time?

A 48 m

B 64 m

C 88 m

D 112 m

11 A car is 65 km from a town. After 90 minutes the car is at a new position, 15 km on the other side of the town.

a Calculate the displacement of the car at the end of the trip relative to its original position.

____

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b Calculate the average speed of the car in $m\,s^{-1}$.

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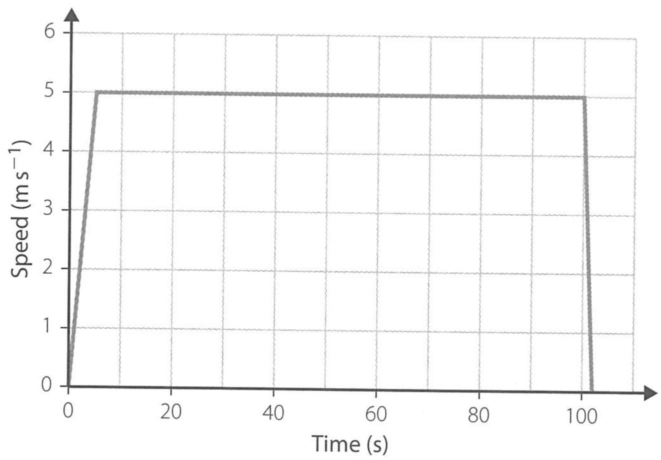
12 The speed–time graph in Figure 13.5.3 represents a cyclist's journey.

FIGURE 13.5.3



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a Find the maximum speed attained by the cyclist.

b While braking, did the cyclist decelerate at a rate greater than or less than their initial acceleration? Explain.

c Calculate the acceleration of the cyclist in the last 2.0 s of the ride.

d Calculate the distance travelled by the cyclist.

e Calculate the average speed of the cyclist in the first 45 s.

13 A ball is propelled vertically upwards from a 40 m high cliff at a speed of $8.0\,\mathrm{m\,s^{-1}}$.

- Ignore air resistance.
- Assume $g = 9.8\,\mathrm{m\,s^{-2}}$.

a Find the maximum height to which the ball rises.

b Calculate the speed at which the ball hits the ground below the cliff.

c Find the time that the ball spends in the air.

14 A car is travelling at $99\,\mathrm{km\,h^{-1}}$ when the driver notices a cow on the road. It takes 1.4 s for the driver to begin braking, and a further 2.4 s to stop just in front of the cow. Find the braking distance. Show all working.

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14 Forces

LEARNING

Summary

- Forces are applied to objects and can affect their motion. Forces can cause objects to speed up, slow down, change direction or change speed and direction simultaneously.
- Forces are external actions applied by one object, A (*agent*), on another object, B (*receiver*). This is written in symbol form as:

  $\vec{F}$(by A on B)
- The interaction between objects is mutual.
  - Object A exerts a force on object B: $\vec{F}$(by A on B)
  - Object B exerts a force on A: $\vec{F}$(by B on A).
- Forces are vectors. They can be represented as arrows:
  - length – proportional to the magnitude of the force
  - direction – points from tail to head in the direction of the force.
- Like all vectors, forces can be:
  - added and subtracted geometrically
  - multiplied by a scalar, including ($^-1$), which reverses the direction.
- There are two types of forces:
  - contact force – force acting when two objects appear to be touching
  - non-contact force – force acting when two objects are clearly separated.
- Electrostatic force is the force applied by an electric charge on another electric charge:

TABLE 14.0.1 Charges of the same sign repel; charges of different signs attract

	⊕	⊖
⊕	Repel	Attract
⊖	Attract	Repel

- Magnetic force is the force applied by one magnetic pole on another magnetic pole:
  - same poles repel
  - opposite poles attract.

TABLE 14.0.2 Magnetic poles of the same sign repel; magnetic poles of different signs attract

	NORTH POLE	SOUTH POLE
NORTH POLE	Repel	Attract
SOUTH POLE	Attract	Repel