WILLETTON SENIOR HIGH SCHOOL

MATHEMATICS METHODS – UNIT TWO

TEST FOUR 2022

SECTION ONE: Calculator Free

STUDENT NAME: Solutions

TOTAL MARKS: .... / 51

TIME ALLOWED: 35 mins

CIRCLE YOUR TEACHER'S NAME:

Mrs Gatland	Mrs Kalotay	Ms Leow	Ms Mack
Mr Riemer	Ms Smirke	Ms Thompson	

- Formulae sheet supplied.
- No calculators allowed.
- If a question is worth more than 2 marks, sufficient working must be shown to justify your answer, in order to receive full marks.

QUESTION 1 [2, 2 = 4 marks]

State the next three terms for each of the sequences below:

a. $T_n = -2T_{n-1}$, $T_1 = -2$

$T_2 = -2 \times -2 = 4$

$T_3 = -2 \times 4 = -8$

$T_4 = -2 \times -8 = 16$

$4, -8, 16$ ✓✓

b. $T_{n+1} = T_n + 2n$, $T_1 = -5$

$T_2 = -5 + 2(1) = -3$

$T_3 = -3 + 2(2) = 1$

$T_4 = 1 + 2(3) = 7$

$-3, 1, 7$ ✓✓

* -1/error

QUESTION 2 [1, 2, 3 =6 marks]

Determine the gradient function for each of the following.

a. $y = 2x^5 - 4$

$y' = 10x^4$ ✓

b. $y = \frac{5x^3+4x^2}{x}$

$y = 5x^2 + 4x$ ✓

$y' = 10x + 4$ ✓

c. $y = (x+3)^3$

$y = 1x^3 + 3 . x^2 . 3^1 + 3 . x . 3^2 + 1 . 3^3$ ✓

$y = x^3 + 9x^2 + 27x + 27$ ✓

$y' = 3x^2 + 18x + 27$ ✓

QUESTION 3 [2 marks]

Write the recursive formula for the following sequence: $\frac{1}{2}, \frac{5}{4}, \frac{4}{2}, \frac{11}{4}, \ldots$

$3/4 \quad 3/4 \quad 3/4$

$T_{n+1} = T_n + \frac{3}{4}$ ✓ $T_1 = \frac{1}{2}$ ✓

QUESTION 4 [4 marks]

The gradient of a curve is given by $\frac{dy}{dx} = a + 3x$, where a is a constant. Given the curve has a stationary point at $(2, 5)$, determine its equation.

$y' = a + 3x$

$y = ax + \frac{3x^2}{2} + c$

$y' = 0$ at $x = 2$

$0 = a + 3(2)$

$-6 = a$

$y = -6x + \frac{3}{2}x^2 + c$ at $(2,5)$

$5 = -6(2) + \frac{3}{2}(2^2) + c$

$5 = -12 + 6 + c$

$11 = c$

$y = \frac{3x^2}{2} - 6x + 11$

QUESTION 5 [1, 3 = 4 marks]

The general term of a sequence is given by $T_n = 4n + 8$. Calculate:

a. T_5

$T_5 = 4.5 + 8$

$= 28$

b. Which term of the sequence is the first to exceed 217?

$217 = 4n + 8$

$209 = 4n$

$52\frac{1}{4} = n$

$\therefore T_{53}$

QUESTION 6 [3, 1 = 4 marks]

A research department finds that the revenue produced by pricing an item at $\$p$ is related by the equation $R = -3p^2 + 45p$.

a. Calculate $\frac{dR}{dp}$ when:

p = 4

p = 8

$R' = -6p + 45$

$= -6(4) + 45$

$= 21$

$= -6(8) + 45$

$= -3$

b. Should the research team recommend increasing or decreasing the price from $8?

Decrease

QUESTION 7 [4, 2 = 7 marks]

For a geometric sequence; $T_1 = x - 2, \quad T_2 = x + 1, \quad T_3 = x + 5$

Determine:

a. The first three terms.

$\frac{x+1}{x-2} = \frac{x+5}{x+1}$

$(x+1)^2 = (x+5)(x-2)$

$x^2 + 2x + 1 = x^2 + 3x - 10$

$x = 11$

$T_1 = 11 - 2 = 9$

$T_2 = 11 + 1 = 12$

$T_3 = 11 + 5 = 16$

b. The general rule of the sequence.

$T_n = 9\left(\frac{4}{3}\right)^{n-1}$

QUESTION 8 [3, 3, 1, 4 = 11 marks]

For the function $f(x) = 5 - 2x^2$,

a. Find an expression for $f(2 + h)$.

$= 5 - 2(2+h)^2$ ✓

$= 5 - 2(4 + 4h + h^2)$

$= 5 - 8 - 8h - 2h^2$ ✓

$= -2h^2 - 8h - 3$ ✓

b. Show, using first principles, that the average rate of change of $f(x) = 5 - 2x^2$ from $x = 2$ to $x = 2 + h$ is $-2h - 8$.

$\frac{f(2+h) - f(2)}{h} = \frac{-2h^2 - 8h - 3 - (-3)}{h}$ ✓

$= \frac{-2h^2 - 8h}{h}$ ✓

$= \frac{\cancel{h}(-2h - 8)}{\cancel{h}}$ ✓

$= -2h - 8$

c. Hence, find the gradient of the tangent to the curve $f(x) = 5 - 2x^2$ at $x = 2$, showing how you use your answer from part (b) . * 2

as $h \to 0$; $-8 - 2(0) = -8$ ✓

*must show $h = 0$

d. Determine the equation of the tangent to the curve $f(x) = 5 - 2x^2$ at $x = 2$.

$m = -8$

& $(2, -3)$ ✓

$y = -8x + c$ ✓

$-3 = -8(2) + c$

$13 = c$ ✓

$\therefore y = -8x + 13$ ✓

QUESTION 9 [6, 3 = 9 marks]

a. Using calculus techniques, determine the stationary points and their nature, for the function $y = (x-1)^2(x+2)$.

$y = x^3 - 3x + 2$ ✓

$y' = 3x^2 - 3$ ✓

$y' = 0 \quad 3(x^2 - 1) = 0$

$3(x-1)(x+1) = 0$

$x = \pm 1$ ✓

$x = 1$

0	1	2
\	—	/

$\therefore (1, 0)$ minimum ✓

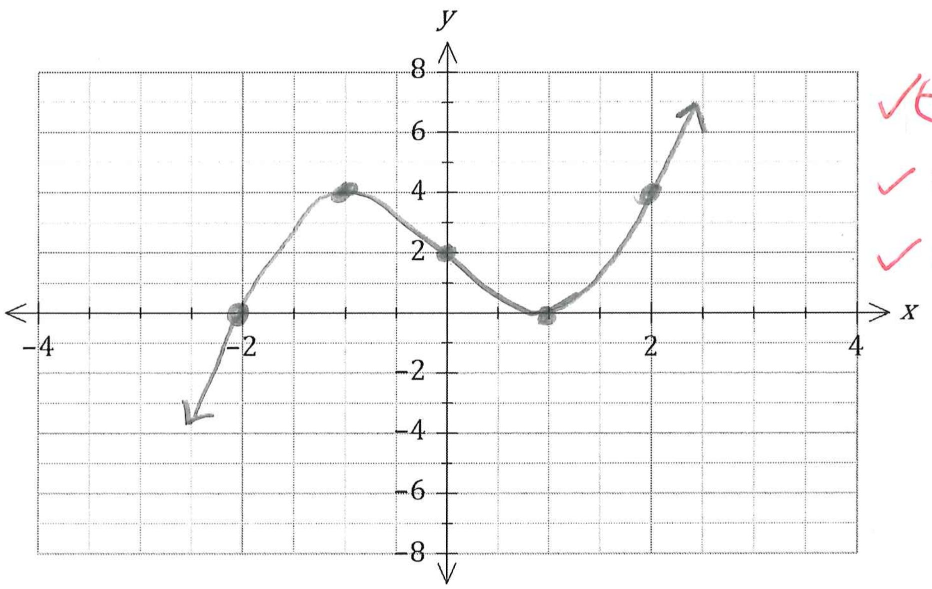
test ✓

$x = -1$

−2	−1	0
/	—	\

$\therefore (-1, 4)$ maximum ✓

b. Sketch the graph of the function $y = (x-1)^2(x+2)$ showing clearly the x and y intercepts, stationary points and indicate the behaviour of the graph as $x \to +\infty$ and $x \to -\infty$



✓ (−1, 4) & (1, 0)

✓ x/y intercept

✓ curve

END OF SECTION

WILLETTON SENIOR HIGH SCHOOL

MATHEMATICS METHODS – UNIT TWO

TEST FOUR 2022

SECTION TWO: Calculator Assumed

STUDENT NAME: Solutions

TOTAL MARKS: .... / 14

TIME ALLOWED: 15 mins

CIRCLE YOUR TEACHER'S NAME:

Mrs Gatland	Mrs Kalotay	Ms Leow	Ms Mack
Mr Riemer	Ms Smirke	Ms Thompson	

- Formulae sheet supplied.
- Calculators/Classpads allowed.
- 1A4 page of notes ONE SIDE only
- If a question is worth more than 2 marks, sufficient working must be shown to justify your answer, in order to receive full marks.

QUESTION 10 [4, 1 = 5 marks]

To manufacture x items costs a company $\$(40x + 15000)$. If the company has set a sale price of $\$(150 - 0.02x)$ per item, calculate:

a. The number of items that should be produced to provide a maximum profit.

Prof = Rev − Cost

$P = (150 - 0.02x)x - (40x + 15000)$ ✓

$= -0.02x^2 + 110x - 15000$ ✓

$P' = -0.04x + 110$ ✓

$P' = 0$; $x = 2750$ ✓(classpad)
items

b. The price per item to achieve this profit.

$\$150 - 0.02(2750) = \$95$ ✓

QUESTION 11 [4 marks]

The sum to infinity of a geometric sequence is equal to 25, while the first two terms of this sequence add up to 9. Find the value(s) of T_1 and r which satisfy these conditions.

$25 = \frac{a}{1-r}$ ① ✓

$a + ar = 9$ ② ✓

simult. eqns.

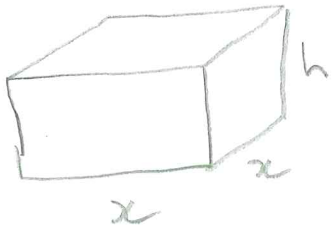
$a = 5$ $r = 0.8$ ✓

$a = 45$ $r = -0.8$ ✓

* must have both a & r correct for 1 mark

QUESTION 12 [5 marks]

An open, $500m^3$ rectangular storage tank, with a square base, is to be constructed. Using calculus methods, calculate the area of sheet metal required for the construction, if the area of metal used is to be minimized.



$\text{Vol} = 500m^3$

$500 = x^2 h$

$h = \frac{500}{x^2}$ ✓

$SA = x^2 + 4xh$

$= x^2 + 4x\left(\frac{500}{x^2}\right)$

$SA = x^2 + \frac{2000}{x}$ ✓

$SA' = 2x - \frac{2000}{x^2}$ ✓

$SA' = 0$, $2x - \frac{2000}{x^2} = 0$

$x = 10m$ ✓

$h = 5m$.

$SA = 10^2 + \frac{2000}{10}$

$= 300m^2$ ✓

END OF PAPER