Name SOLUTIONS

Mathematics Specialist Unit 2

**Calculator Free** **No Notes**

**Marks available: 29** **Time allowed: 30 minutes**

**1. [7 marks: 2, 2, 3]**

Solve for x.

i. $x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$x = \pm 2i$ ✓✓

ii. $x^2 + x + 4 = 0$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(4)}}{2(1)}$

$= \dfrac{-1 \pm \sqrt{-15}}{2}$

$= -\frac{1}{2} \pm \frac{\sqrt{15}}{2} i$

✓✓

iii. $x^3 + 9x = 0$

$x(x^2 + 9) = 0$

$\Rightarrow x = 0$ ✓

$x^2 + 9 = 0$

$x^2 = -9$

$x = \pm 3i$

✓✓

**2. [3 marks: 1, 2]**

For the complex number $z = 8 - 3\mathbf{i}$, find

i. Im(z)

$-3$ ✓

ii. $|z|$

$\sqrt{8^2 + (-3)^2} = \sqrt{73}$

✓ ✓

**3. [3 marks]**

The solutions to a quadratic equation are $x = -2 + \mathbf{i}$ and $x = -2 - \mathbf{i}$. Determine the original quadratic equation.

$(x - (-2 + i))(x - (-2 - i)) = 0$

$(x + 2 - i)(x + 2 + i) = 0$ ✓

$x^2 + 2x - ix + 2x + 4 - 2i + ix + 2i - i^2 = 0$ ✓

$x^2 + 4x + 5 = 0$ ✓

**4.** **[2 marks]**

Prove that the product of a complex number and its conjugate is a real number.

Let $z = a+bi \Rightarrow \bar{z} = a - bi$

$z.\bar{z} = (a+bi)(a-bi)$ ✓

$= a^2 + abi - abi + b^2$

$= a^2 + b^2$ which is a real number ✓

**5.** **[9 marks]**

Let the complex numbers $z_1 = 2 + k\,i$ and $z_2 = -5 + 12i$, where $k$ is a real number.

Determine all values of $k$ if:

i. $[\boldsymbol{Im}(z_1)]^2 = \boldsymbol{Re}(z_2)$

$k^2 = -5$ ✓

$k = \pm\sqrt{5}i$

but $k$ is real $\therefore$ no values of $k$ ✓

ii. $z_1^2 = z_2$

$(2+ki)(2+ki) = -5+12i$

$4 - k^2 + 4ki = -5 + 12i$ ✓

Equating reals $4 - k^2 = -5$

$k = \pm 3$ ✓

Equat Im $4k = 12 \Rightarrow k = 3$ ✓

$\therefore k = 3$ ✓

iii. $\dfrac{6Z_1}{z_2} = -i$

$\dfrac{6(2+ki)}{-5+12i} = -i$

$6(2+ki) = -i(-5+12i)$ ✓

$12 + 6ki = 5i + 12$ ✓

$6ki = 5i$

$6k = 5$

$k = \frac{5}{6}$ ✓

6. **[5 marks]**

Use mathematical induction to prove that the sum of the first n multiples of 5 is given by

$$5 + 10 + 15 + \ldots\ldots + 5n = \frac{5n(n+1)}{2}.$$

Check $n = 1$

$$5(1) = \frac{5(1)(1+1)}{2}$$

$$5 = 5 \checkmark$$

Assume works for $n = k$

$$\Rightarrow 5 + 10 + 15 + \ldots + 5k = \frac{5k(k+1)}{2} \checkmark$$

Consider case for $n = k+1$

$$5 + 10 + 15 + \ldots + 5k + 5(k+1) = \frac{5k(k+1)}{2} + 5(k+1) \checkmark$$

$$= \frac{5k^2 + 5k}{2} + 5k + 5$$

$$= \frac{5k^2 + 5k + 10k + 10}{2}$$

$$= \frac{5(k^2 + 3k + 2)}{2}$$

$$= \frac{5(k+1)(k+2)}{2}$$

$$= \frac{5(k+1)(k+1+1)}{2} \checkmark\checkmark$$

$\therefore$ Proven by M.I.