Esperance Senior High School
Year 11 Specialist Mathematics

INVESTIGATION 1 2016

Total Marks 33 **Minutes: 40** **Name:**

Part 1 **(10 Marks)**

a) The expression $a+b$ contains two terms and hence is called a binomial. Complete the following, writing the expanded version in powers of a and powers of b (gathering like terms). The first 3 have been done for you and the 4th has been partially completed. (5 Marks)

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$

$(a+b)^3 = (a+b)(a+b)(a+b) + = (a+b)(a^2+2ab+b^2) =$ $a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 =$ $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^5 =$ $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

b) What do you notice in the powers of a? (1 Mark)

decrease by 1 each term

c) What do you notice in the powers of b? (1 Mark)

increase by 1 each term

d) Looking at the patterns found above and ignoring the coefficients (numbers in front of the variables), complete the following general rule for the expansion of $(a+b)^n$ (2 Marks)

$a^n b^0 + a^{n-1}b^1 + a^{n-2}b^2 + \ldots + a^1 b^{n-1} + a^0 b^n$

(ie the first 3 terms and the last 2 terms)

e) How many terms are there in the expansion of $(a+b)^n$ (1 Mark)

$n+1$

Part 2 **(11 Marks)**

The method for determining the coefficients can be found if they are written in a triangular array.

a) Copy and complete this triangular array. (2 Marks)

With $(a+b)^n$

$n=0$ 1

$n=1$ 1 1

$n=2$ 1 2 1

$n=3$ 1 3 3 1

$n=4$ 1 4 6 4 1

$n=5$ 1 5 10 10 5 1

b) How do the numbers is the above array relate to the coefficients of the terms when you expanded the binomials in Question 1 part A? (1 Mark)

the same

c) This Triangular array is called Pascal's Triangle. Although it allows us to find the coefficients it is time consuming and a little painful. To find a particular row of coefficients we need a preceding row. (4 Marks)

Write down the expansion of

i) $(a+b)^6$ $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

ii) $(a+b)^7$ $a^7 + 7a^2b + 21a^3b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$

d) Is there a better way? Complete the following (2 Marks)

$n=0$ ${}^0C_0$

$n=1$ ${}^1C_1$ ${}^1C_1$

$n=2$ ${}^2C_0$ ${}^2C_1$ ${}^2C_2$

$n=3$ ${}^3C_0$ ${}^3C_1$ ${}^3C_2$ ${}^3C_3$

$n=4$ ${}^4C_0$ ${}^4C_1$ ${}^4C_2$ ${}^4C_3$ ${}^4C_4$

$n=5$ ${}^5C_0$ ${}^5C_1$ ${}^5C_2$ ${}^5C_3$ ${}^5C_4$ ${}^5C_5$

e) In General (2 Marks)

$(a+b)^n =$ ${}^nC_0 a^n$ + ${}^nC_1 a^{n-1} b$ + ${}^nC_2 a^{n-2} b^2$ + ... + ${}^nC_n b^n$

Part 3 **(12 Marks)**

From your observations in Part A and Part B

a) Expand $(1+2x)^4$ (Hint: let $a=1$ and $b=2x$) (3 Marks)

$= {}^4C_0 1^4 + {}^4C_1 1^3(2x) + {}^4C_2 1^2(2x)^2 + {}^4C_3 1^1(2x)^3 + {}^4C_4(2x)^4$

$= 1 + 4(2x) + 6(2x)^2 + 4(2x)^3 + 1(2x)^4$

$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$

b) Expand $(2m-3n)^3$ (4 Marks)

$= {}^3C_0(2m)^3 + {}^3C_1(2m)^2(-3n) + {}^3C_2(2m)(-3n)^2 + {}^3C_3(-3n)^3$

$= 8m^3 + 3(2m)^2(-3n) + 3(2m)(-3n)^2 + 1(-3n)^3$

$= 8m^3 - 36m^2 + 54mn^2 - 27n^3.$

c) Write down the first 3 terms of $(3-2x)^{10}$ (2 Marks)

$= {}^{10}C_0 3^{10} + {}^{10}C_1 3^9(-2x) + {}^{10}C_2 3^8(-2x)^2$

$= 59049 - 393660x + 1180980x^2.$

d) Consider the expansion of $(p+q)^{10}$

i) How many terms are there? 11 (1 Mark)

ii) Find the middle term. (2 Marks)

${}^{10}C_5$ $252p^5q^5$