Practice Test 1

Calculator Free Section

Name: SOLUTIONS **Score:** ______ / 18

No calculators or notes are to be used.
Mathematics Specialist Formula Sheet is permitted
Time limit = 20 minutes

1. (4 marks)

Given that z is a complex number solve $\frac{z-6}{z+1} = 5z-3$, giving the solution(s) exactly in the form $a + bi$.

$$z - 6 = (5z-3)(z+1)$$ ✓ Multiply to remove denominator

$$z - 6 = 5z^2 + 2z - 3$$

$$5z^2 + z + 3 = 0$$ ✓ quadratic

$$z = \frac{-1 \pm \sqrt{1^2 - 4 \times 5 \times 3}}{2 \times 5}$$ ✓ solve for z

$$= -\frac{1}{10} \pm \frac{\sqrt{59}}{10} i$$ ✓ solution

2. (3 marks)

Rewrite $\{z: |z-(2+4i)| = |z-1|\}$ as a cartesian equation.

$(x-2)^2+(y-4)^2 = (x-1)^2+y^2$ ✓ circles

$\cancel{x^2}-4x+4+\cancel{y^2}-8y+16 = \cancel{x^2}-2x+1+\cancel{y^2}$ ✓ working

$-2x-8y = -19$

$\therefore 2x+8y=19$ or $y = -\frac{x}{4}+\frac{19}{8}$ ✓ solution

3. (3,3 = 6 marks)

a) Given $z = a-3i$ and $z + c\bar{z} = 12+6i$, determine a and c.

$a-3i+c(a+3i) = 12+6i$

$a(c+1)=12$ and $-3i+3ci = 6i$ ✓ equating

$c=3$ ✓ solution

$a(3+1)=12$

$a=3$ ✓ solution

b) Given $w = b\text{cis}\frac{5\pi}{6}$ and $|(\bar{w})^3| = 0.001$, determine b and give the principal argument of w^3.

$w = 0.1\text{cis}\left(\frac{5\pi}{6}\right)$ so $b=0.1$ ✓

$w^3 = (0.1)^3\text{cis}\left(\frac{5\pi}{6}\times 3\right)$

$= 0.001\text{cis}\left(\frac{5\pi}{2}\right)$ ✓ DM expansion

$\therefore$ principal argument is $\frac{\pi}{2}$ ✓ solution

4. (5 marks)

Use De Moivre's Theorem to find all the roots of $z^4 = -8 + 8\sqrt{3}i$. Give your answers exactly in polar form, with r >0 and $-\pi < \vartheta \leq \pi$.

$$-8 + 8\sqrt{3}\,i = 16\,\text{cis}\left(\frac{2\pi}{3}\right) ✓$$

$$\left[16\,\text{cis}\left(\frac{2\pi}{3}\right)\right]^{1/4} = 2\,\text{cis}\left(\frac{2\pi}{12}\right) = 2\,\text{cis}\left(\frac{\pi}{6}\right)$$

$\therefore$ roots in form $z = 2\,\text{cis}\left(\frac{\pi}{6} + \frac{2k\pi}{4}\right)$

$$z_0 = 2\,\text{cis}\left(\frac{\pi}{6}\right) ✓$$

$$z_1 = 2\,\text{cis}\left(\frac{\pi}{6} + \frac{\pi}{2}\right) = 2\,\text{cis}\left(\frac{4\pi}{6}\right) = 2\,\text{cis}\left(\frac{2\pi}{3}\right) ✓$$

$$z_2 = 2\,\text{cis}\left(\frac{\pi}{6} + \pi\right) = 2\,\text{cis}\left(\frac{7\pi}{6}\right) = 2\,\text{cis}\left(-\frac{5\pi}{6}\right) ✓$$

$$z_3 = 2\,\text{cis}\left(\frac{\pi}{6} + \frac{3\pi}{2}\right) = 2\,\text{cis}\left(\frac{10\pi}{6}\right) = 2\,\text{cis}\left(-\frac{\pi}{3}\right) ✓$$

End of Section One

Practice Test 1

Calculator Assumed Section

Name: ________________________ **Score: _____ / 29**

Calculators allowed and one double-sided page of A4 notes.
Mathematics Specialist Formula Sheet is permitted
Time limit = 30 minutes

5. (2,2,2,2 marks)

Suppose that z is a complex number with modulus r and argument θ. Express in terms of r and θ the modulus and argument of each of the four complex numbers z_1, z_2, z_3, and z_4 where

(i) $z_1 = z^2$

mod r^2 ✓

arg 2θ ✓

(ii) $z_2 = 2z$

mod $2r$ ✓

arg θ ✓

(iii) $z_3 = z^{-1}$

mod $\frac{1}{r}$ ✓

arg $-\theta$ ✓

(iv) $z_4 = -iz$

mod r ✓

arg $\theta - \frac{\pi}{2}$ ✓

6. (2,2,2 marks)

The complex numbers of z and w are such that:

$z = 3\text{cis}(\ -\frac{\pi}{6}\ )$ and $w = \text{cis}(\ \frac{3\pi}{4}\ )$

Determine the following.

a) $z + w$ (in a + bi form)

$$3\text{cis}\left(\frac{-\pi}{6}\right) + \text{cis}\left(\frac{3\pi}{4}\right)$$

$$= \frac{3\sqrt{3}}{2} + \frac{3i}{2} + \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

$$= \left(\frac{3\sqrt{3}-\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}-3}{2}\right)i$$

✓ ✓

b) wz (in cis form)

$$3\text{ cis}\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$$

$$= 3\text{cis}\left(\frac{7\pi}{12}\right)$$

✓ ✓

c) $\frac{1}{z}$ (in cis form and a + bi form)

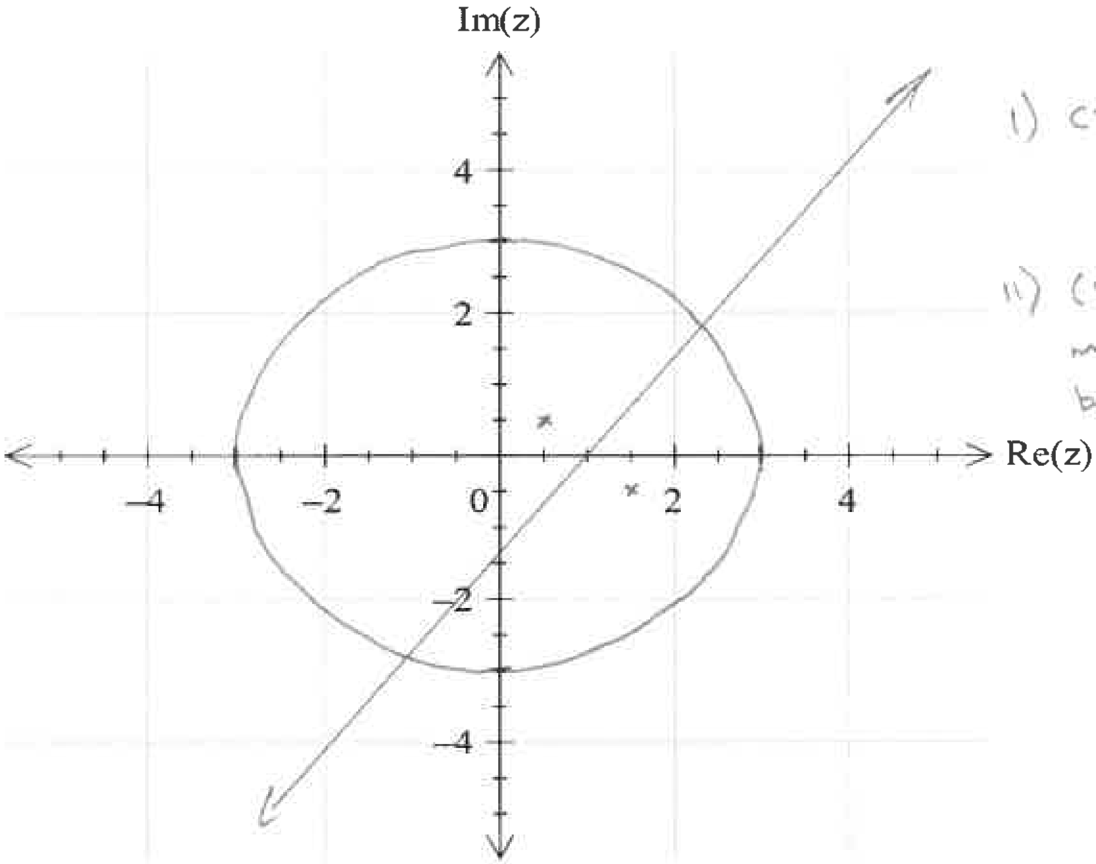
$$\frac{1}{3\text{cis}\left(\frac{\pi}{6}\right)} = \frac{1}{3}\text{cis}\left(\frac{-\pi}{6}\right) \checkmark$$

$$= \frac{\sqrt{3}}{6} - \frac{i}{6} \checkmark$$

7. (5,5 marks)

a) On the Argand Diagram below, sketch and label both of the sets of points specified:

i) $z\bar{z} = 9$ **ii)** $|z - 1 - i| = |z - 3 + i|$



i) circle shape ✓
radius 3 ✓

ii) (1, i) and (3, -i) ✓
midpoint ✓
bisector ✓

b) Determine to four decimal places the largest possible argument, $-\pi < \theta \le \pi$, of the complex number that satisfies $\{z : z\bar{z} = 9\} \cap \{z : |z - 1 - i| = |z - 3 + i|\}$

$(x-1)^2 + (y-1)^2 = (x-3)^2 + (y+1)^2$ ✓

$x^2 + y^2 = 9$ ✓

Simultaneous

$x = \frac{\sqrt{14}}{2} + 1, \quad y = \frac{\sqrt{14}}{2} - 1$ ✓

$\tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right)$

✓

$\frac{\frac{\sqrt{14}}{2} - 1}{\frac{\sqrt{14}}{2} + 1}$ $\therefore \theta = 0.2945$ radians ✓

$(16.87°)$

8. (4,1 marks)

a) Use de Moivre's theorem to show that $\cos 4x = \cos^4 x + \sin^4 x - 6\cos^2 x \sin^2 x$

$\text{cis}\, 4x = (\cos x + i\sin x)^4$ ✓

$= \cos^4 x + 4\cos^3 x\, i\sin x - 6\cos^2 x \sin^2 x - 4\cos x\, i\sin^3 x + \sin^4 x$ ✓

Equate real ✓ $\cos 4x = \cos^4 x - 6\cos^2 x \sin^2 x + \sin^4 x$ ✓

b) Use this result to show that $\cos 4x = 1 - 8\cos^2 x \sin^2 x$

$\cos^4 x - 6\cos^2 x \sin^2 x + \sin^4 x$

$= (\cos^2 x + \sin^2 x)^2 - 2\cos^2 x \sin^2 x - 6\cos^2 x \sin^2 x$ ✓

$= 1 - 8\cos^2 x \sin^2 x$

End of Test