

Mathematics Department

Course: ATMAA

Topic Title: Graph and Networks

Test 3



Student Name: Solutions.

Date: _____

Special Instructions: Calculator Allowed

Time Allowed: 20 mins

1 page of A4 notes and Formula Sheet Allowed

Marks: / 56

Question 1.

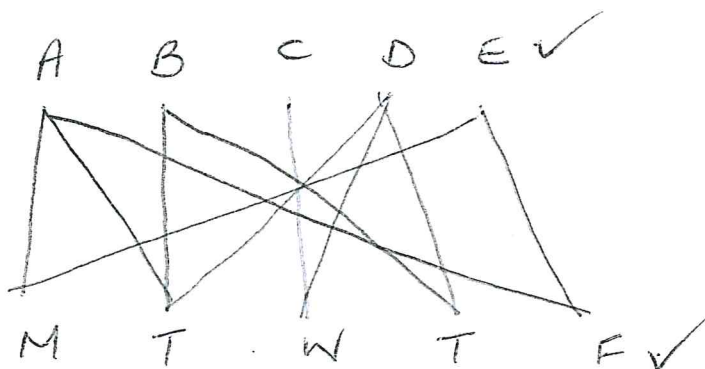
(4 marks)

Draw a bipartite graph to represent the below information.

Student services' is manned by five volunteers from Monday to Friday.

One volunteer is required per day.

- Abbie is available Monday, Tuesday and Friday.
- Bob is available Tuesday and Thursday.
- Cathy is available Wednesday.
- Dennis is available Tuesday, Wednesday and Thursday.
- Esther is available Monday and Friday.



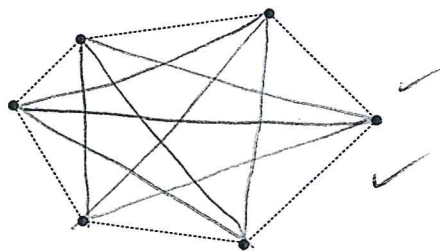
Question 2.

(2, 2: 4 marks)

The diagram shows a graph with six vertices and six edges

a) Explain what the term "complete graph" means:

All vertices are connected to all other vertices by an edge.

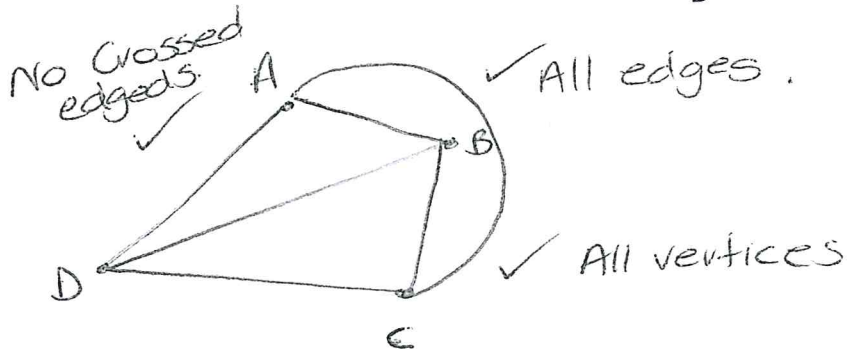
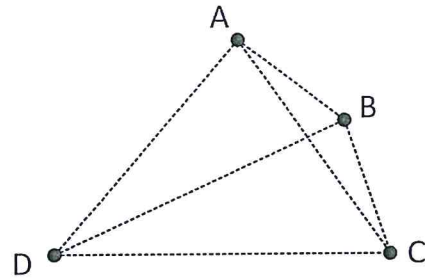


b) Add edges to the graph so that the graph becomes a complete graph.

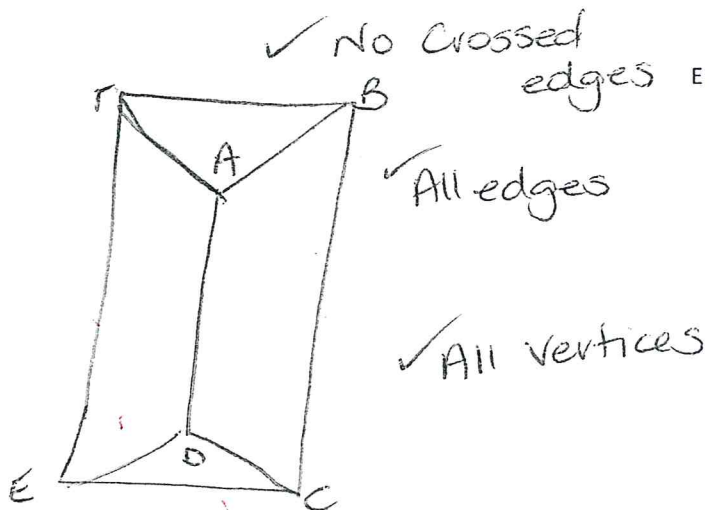
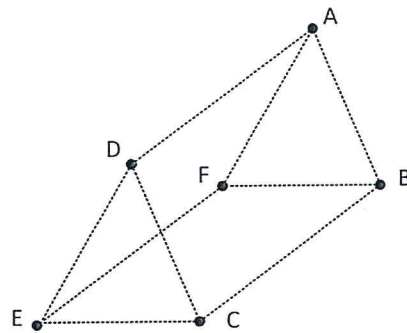
Question 3.

(3, 3: 6 marks)

- a) ABCD is a triangular pyramid with $\triangle BCD$ as its base.
Represent the pyramid ABCD as a planar graph.



- b) ABCDEF is a triangular prism with BCEF as its base.
Represent the prism ABCDEF as a planar graph.



$$V = 6$$

$$E = 9$$

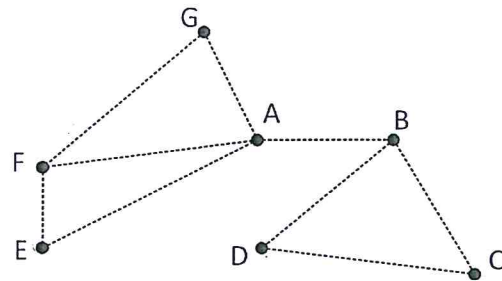
Question 4.

(2, 2, 2, 2, 2, 4: 16 marks)

The accompanying diagram shows a graph.

- a) Determine with reasons if the graph is simple.

All vertices are connected, no loops or multiple edges ✓



- b) Determine with reasons if the edge AB is a bridge.

Yes AB is a bridge, it connects two graphs/networks. ✓

- c) This graph does not have a Hamiltonian path. Add an extra edge to this graph so that a Hamiltonian path may be formed. State this path.

$D \rightarrow C \rightarrow B \rightarrow A \rightarrow E \rightarrow F \rightarrow G.$ (7).
No repeated vertices.

- d) Verify that Euler's rule works for this graph.

$$V + F = E + 2 \quad \checkmark$$

$$7 + 4 = 9 + 2. \quad \checkmark$$

$$11 = 11$$

e) Identify the semi-Eulerian trail in this graph.

* Starts & Finishes odd degrees.
* Uses every edge.

$F \rightarrow G \rightarrow A \rightarrow E \rightarrow F \rightarrow A \rightarrow B \rightarrow C \rightarrow D \rightarrow B$

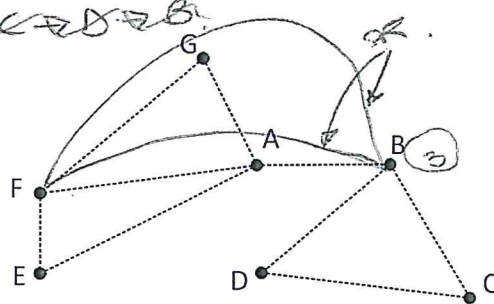
f) Add an additional edge to the graph so that the Eulerian circuit may be formed.

i) Explain the reason for your choice.

$F \rightarrow G \rightarrow A \rightarrow E \rightarrow F \rightarrow A \rightarrow B \rightarrow C \rightarrow D \rightarrow B$

F & B ~~are~~ have odd degrees. Adding an additional edge to the two vertices creates ~~all~~ all vertices have even degrees.

(3)



ii) State this circuit

$B \rightarrow C \rightarrow D \rightarrow B \rightarrow A \rightarrow G \rightarrow F \rightarrow E \rightarrow A \rightarrow F \rightarrow B$

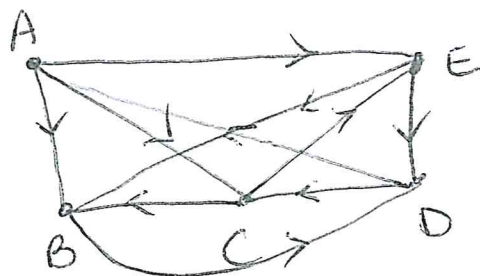
Question 5.

(4, 3, 3, 4, 3: 17 marks)

Teams A, B, C, D and E are in Group 1 of a soccer competition where they play each other exactly once. The results of the games are:

- A defeated B, C and E
- B defeated D
- C defeated B and E
- D defeated A and C
- E defeated B and D

a) Draw a digraph that represents the results of these matches where a directed edge points from the winning teams to the losing teams.



✓ ✓ All vertices (5)
All edges (6).
Directed.

✓ ✓

b) The digraph in (a) can be expressed as an adjacency matrix \mathbf{M} . The element "1" is to indicate a win and the element "0" to indicate a loss.

Complete the matrix \mathbf{M}

	A	B	C	D	E
A	0	1	1	0	1
B	0	0	0	1	0
C	0	1	0	0	1
D	1	0	1	0	0
E	0	1	0	1	0

-1/2 for each incorrect.

c) The elements in M^2 represents the number of "two-stage wins" between teams. An incomplete M^2 is shown below. The element in row 1 column 2 indicates that A has 2 "two-stage wins" against :

A beat E who beat B and

A beat C who beat B.

Complete the matrix M^2 below.

$$M^2 = \begin{matrix} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 0 & 2 & 0 & 2 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 2 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix} \quad \begin{matrix} \checkmark \\ \checkmark \\ \checkmark \\ \checkmark \\ \checkmark \end{matrix}$$

d) Calculate $M + M^2$. Then multiply your answer with the column matrix $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

The resulting matrix gives the total number of direct win and "two-stage wins" for each team. Use this answer to rank the teams.

$$\begin{bmatrix} 0 & 3 & 1 & 2 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 2 & 1 \\ 1 & 2 & 2 & 0 & 2 \\ 1 & 1 & 1 & 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 5 \\ 7 \\ 5 \end{bmatrix} \quad \checkmark \checkmark$$

$$\begin{aligned} A &= 8 \\ D &= 7 \\ C &= 5 \\ E &= 5 \\ B &= 3 \end{aligned} \quad \checkmark$$

e) Calculate $M + M^2 + M^3$. Then multiply your answer with $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Use this answer to rank the teams.

$$\begin{bmatrix} 2 & 4 & 3 & 5 & 2 \\ 1 & 2 & 2 & 1 & 2 \\ 2 & 2 & 2 & 3 & 1 \\ 1 & 5 & 2 & 4 & 3 \\ 2 & 3 & 3 & 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \checkmark \checkmark$$

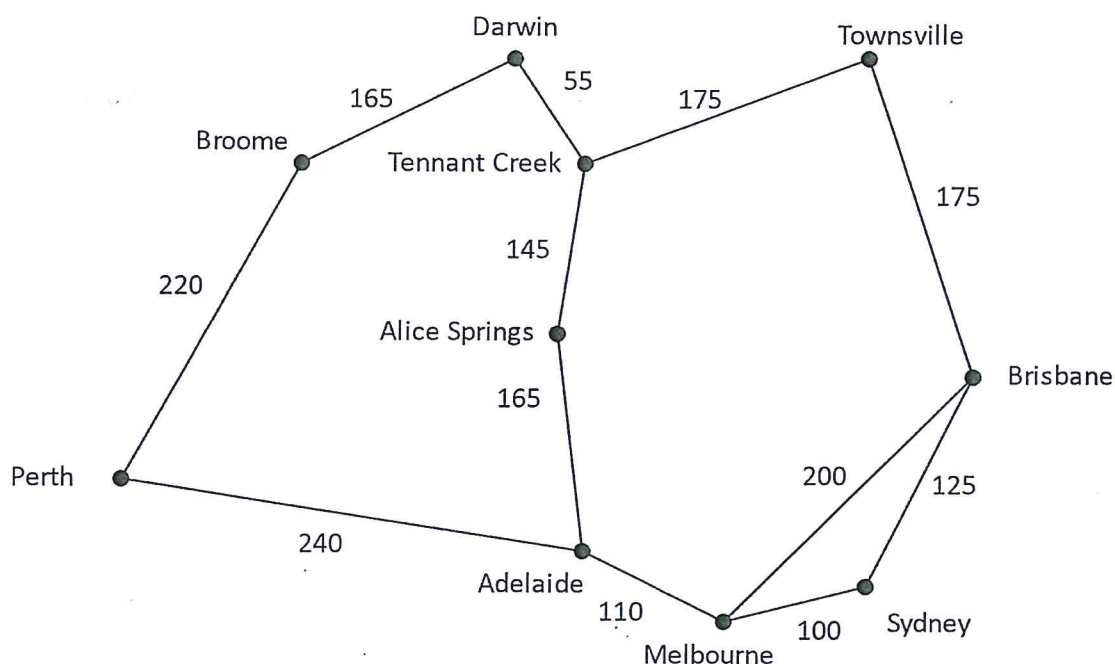
$$\begin{bmatrix} 16 \\ 8 \\ 10 \\ 15 \\ 12 \end{bmatrix}$$

$$\begin{aligned} A &= 16 \\ D &= 15 \\ E &= 12 \\ C &= 10 \\ B &= 8 \end{aligned} \quad \checkmark$$

Question 6.

(5, 6: 11 marks)

When John retired, he drove around mainland Australia with his wife Janet in their Winnebago. The diagram below shows the amount of diesel John and Janet used for each part of their journey. The average price of diesel was \$2.00 per Litre.



- a) State the route from Sydney to Broome for which the cost of diesel usage is a minimum and state the diesel cost for this route.

$$S \rightarrow M \rightarrow A \rightarrow AS \rightarrow TK \rightarrow D \rightarrow B$$

$$100 + 110 + 165 + 145 + 55 + 165 = 740$$

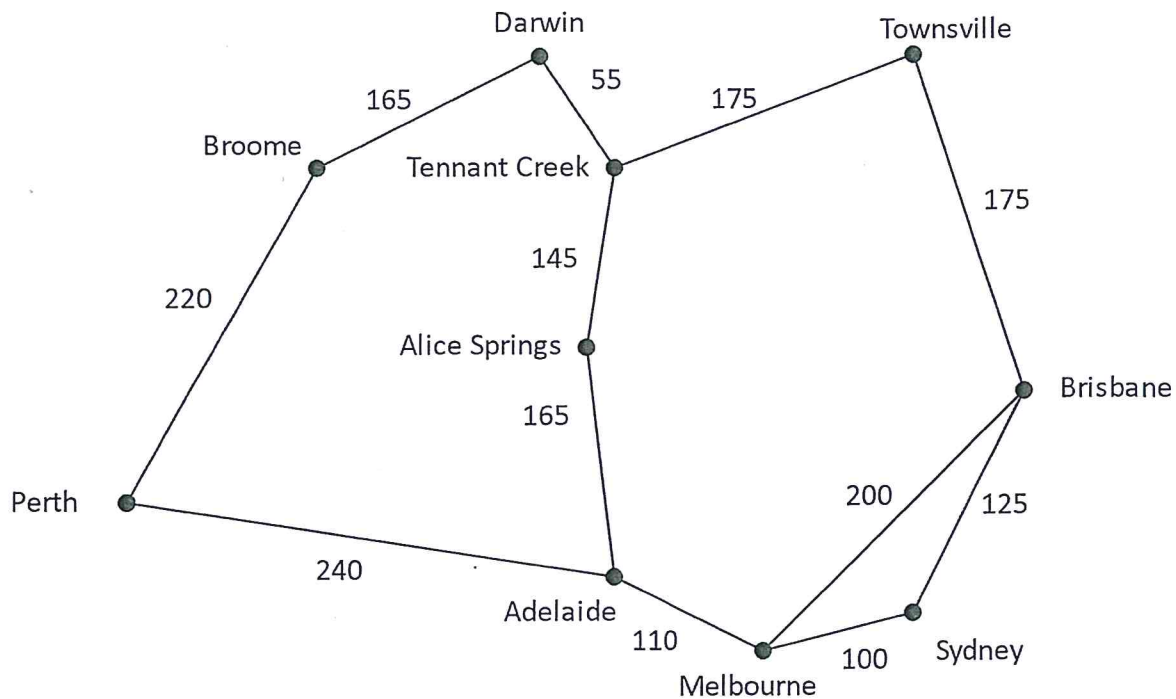
Syd \rightarrow mel - Ade \rightarrow Perth - Broome. ✓✓✓

= 670 litres of diesel used. ✓

$$670 \times \$2.00 = \$1340.$$

✓

- b) From Sydney, John and Janet visited each of the towns listed at least once.
John and Janet complete their journey in Sydney.



- a) State the route for which the cost of diesel usage is a minimum and state the diesel cost for this route.

(12)

Sydney - Bris - Towns - Tennant Creek - Alice Springs
- Tennant Creek - Darwin - Broome - Perth -
Adelaide - Melbourne - Sydney ✓✓✓✓✓

Total diesel = 1655 litres. ✓

$$1655 \times \$2 = \$3310 \quad \checkmark$$

1 visiting each town ✓

S, M, A, P, B, D, TC, AS, TC, T, Bris, S. → correct.