1. Determine the gradient function of the curve f(x) = $x^3 - x$, using first principles.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{(x+h)^3 - (x+h) - (x^3 - x)}{h}$$
$$= \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 - x - h - x^3 + x}{h}$$
$$= \lim_{h \to 0} \frac{3hx^2 + 3h^2x + h^3 - h}{h}$$
$$= \lim_{h \to 0} 3x^2 + 3hx + h^2 - 1$$
$$= 3x^2 - 1$$

2. Determine the derivative of the following functions.

a. $y = 3x^3 + 2x^4 - 4x^5 + \pi$

$$\frac{dy}{dx} = 9x^2 + 16x^3 - 20x^4$$

b. $f(x) = (3x + 7)^2 = 9x^2 + 42x + 49$

$$f'(x) = 18x + 42$$

c. $f(x) = x^7 + 2x^6 - 3x^5 + 4x^4 - 5x^3 + 6x^2 - 7x + 8$

$$f'(x) = 7x^6 + 12x^5 - 15x^4 + 16x^3 - 15x^2 + 12x - 7$$

3. Find the antiderivatives of the following.

a. $\int 2x^3 - 5x^2 + 2 \, dx = \frac{2x^4}{4} - \frac{5x^3}{3} + 2x + c$

$$= \frac{x^4}{2} - \frac{5x^3}{3} + 2x + c$$

b. $5t^4 - t^2 + 3 = \frac{5t^5}{5} - \frac{t^3}{3} + 3t + c = t^5 - \frac{t^3}{3} + 3t + c$

4. Determine using Calculus, the coordinates on the curve $y = 2x^3 - 3x^2 + 4$ where the gradient is zero.

$$\frac{dy}{dx} = 6x^2 - 6x$$
$$6x^2 - 6x = 0$$
$$6x(x-1) = 0 \quad \therefore x = 0 \text{ or } x = 1 \quad \therefore$$

Points are $(0, 4)$, $(1, 3)$

5. Determine the following indefinite integrals

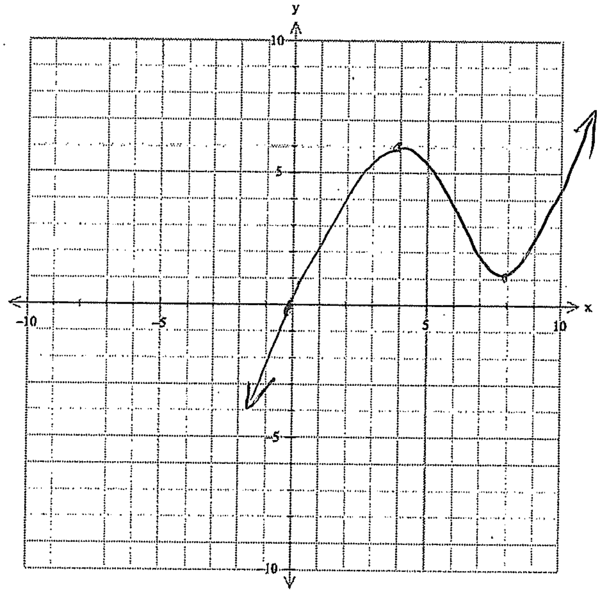
a. $\int(4x + 3x^2 - 6x + 3)dx$ $\quad \frac{4x^2}{2} + x^3 - \frac{6x^2}{2} + 3x + c = 2x^2 + x^3 - 3x^2 + 3x + c$

b. $\int(2p^3 - 2p^4 + 2p)dp$ $\quad = 2\frac{p^4}{4} - \frac{2p^5}{5} + \frac{2p^2}{2} + c$

$$= \frac{p^4}{2} - \frac{2p^5}{5} + p^2 + c$$

6. Sketch the function $y = f(x)$ given that:

- $f(x)$ is continuous for all values of x
- $f(x)$ cuts the x–axis only at the origin
- $f'(x) = 0$ only at $x = 4$ and $x = 8$
- The gradient is negative only for $4 < x < 8$
- $f(x) > 0$ only for $x > 0$



7. Consider the curve f(x) = 2x³ + 3x² + 4. Find the equation(s) of all lines tangent to f(x) and parallel to y = 12x – 1.

$f'(x) = 6x^2 + 6x$

$6x^2 + 6x = 12$

$6(x+2)(x-1) = 0$

$\therefore x = -2$ or $x = 1$

if $x = -2$ $f(-2) = 0$

if $x = 1$ $f(1) = 9$

$M = 12$

$y = 12x + c$

$(-2, 0)$

$0 = 12(-2) + c$

$c = 24$

$\therefore y = 12x + 24$

$(1, 9)$

$9 = 12(1) + c$

$c = -3$

$\therefore y = 12x - 3$

8. Given that the gradient function of a curve is $\frac{dy}{dx} = x^2 - \frac{4x}{3} + 2$, determine the equation of the curve if it passes through the point (3, -1)

$y = \int x^2 - \frac{4x}{3} + 2\, dx$

$= \frac{x^3}{3} - \frac{4x^2}{6} + 2x + c$

$= \frac{x^3}{3} - \frac{2x^2}{3} + 2x + c$

Sub $(3, -1)$

$-1 = \frac{3^3}{3} - \frac{2(3)^2}{3} + 2(3) + c$

$-1 = 9 - 6 + 6 + c$

$-1 = 9 + c$

$c = -10$

$\therefore y = \frac{x^3}{3} - \frac{2x^2}{3} + 2x - 10$

9. The position of an electron in a magnetic field changes with time. The rule which states its position, y millimetres from a fixed point, as a function of time, t seconds is; y = 16t – t³.

a. State its position when the magnetic field is first turned on. If $t = 0$ $y = 0$ mm

b. State its position when t = 1 If $t = 1$ $y = 15$ mm

c. Determine the maximum position of the electron from the fixed position and the time it reaches the maximum position.

$\frac{dy}{dt} = 16 - 3t^2$

$16 - 3t^2 = 0$

$3t^2 = 16$

$t^2 = \frac{16}{3}$

$t = \sqrt{\frac{16}{3}}$ or $-\sqrt{\frac{16}{3}}$ ← reject

$\therefore t = \frac{4}{\sqrt{3}}$ s $= 2.309$ s

$y = 16 \times \frac{4}{\sqrt{3}} - \left(\frac{4}{\sqrt{3}}\right)^3$

$= \frac{64}{\sqrt{3}} - \frac{64}{3\sqrt{3}}$ mm

$= 24.63$ mm

10. A company manufactures lasers, receives \$25 in revenue per laser. The cost of making x lasers is given by c(x) = $x^3 - 35x^2 - 795x + 70$.

a. Write an equation for the profit made from selling x lasers.

$$P = R - C = 25x - (x^3 - 35x^2 - 795x + 70)$$
$$= 25x - x^3 + 35x^2 + 795x - 70$$
$$= \$(-x^3 + 35x^2 + 820x - 70)$$

b. How many lasers should be made and sold in order to maximise profit?

$$\frac{dP}{dx} = -3x^2 + 70x + 820$$

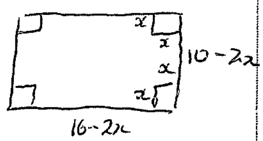
$$\frac{dP}{dx} = 0 \quad \text{if} \quad x = -8.67 \text{ or } 31.9$$

reject

x	30	31.9	31
$\frac{dP}{dx}$	+ve	0	-ve
	/	—	\

∴ Max if no of lasers is 32.

11. An open box is to be made from a rectangular sheet of card measuring 16cm by 10cm, by cutting four equal squares from the corners and folding up the flaps. Find the length of the sides of the square that makes the volume as large as possible and find this largest volume.



$$V = x(16-2x)(10-2x)$$
$$= x(160 - 32x - 20x + 4x^2)$$
$$= 160x - 52x^2 + 4x^3$$

$$\frac{dV}{dx} = 160 - 104x + 12x^2$$

$$\frac{dV}{dx} = 0 \quad \text{if } x = 2 \text{ or } x = 6.\dot{6}$$

reject as impossible

x	1	2	3
$\frac{dV}{dx}$	+ve	0	-ve
	/	—	\

∴ Max

$$\therefore V = 2(16 - 2(2))(10 - 2(2))$$
$$= 144 \text{cm}^3$$

12. Organisers of the 2007 Slam-it festival know that if they sell tickets at \$150 each they will sell 5000 tickets. for every 50c drop in the price, the number of tickets sold increases by 50. It costs the organisers \$250000 per day to run the festival

a. Write an expression that represents the price of the tickets, if the number of tickets sold is given by (5000 + 50x), where x is the number of 50c increases.

$$\text{Price} = 150 - 0.5x$$

b. Hence, show that R(x) = $750000 + 5000x - 25x^2$

$$R = (150 - 0.5x)(5000 + 50x)$$
$$= 750\,000 + 7500x - 2500x - 25x^2$$
$$= 750\,000 + 5000x - 25x^2$$

c. Show that the profit per day from the concert in terms of x is given by; P(x) = $500000 + 5000x - 25x^2$

$$\text{Profit} = 750\,000 + 5000x - 25x^2 - 250\,000$$
$$= 500\,000 + 5000x - 25x^2$$

d. Using differentiation, find the maximum profit the organisers make and at what price should the tickets be sold at to achieve this profit?

$$P'(x) = 5000 - 50x$$
$$0 = 5000 - 50x$$
$$x = 100$$

$$\text{Price} = 150 - 0.5(100)$$
$$= \$100$$

$$\text{Profit} = 500000 + 5000(100) - 25(100)^2$$
$$= \$750\,000$$