



**Calculator Assumed
Applications of Differentiation -
Optimisation**

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [5 marks]

Verify, using calculus techniques, that a maximum revenue of \$36 000 is obtained by this revenue function: $R = 20000 + 800x - 10x^2$.

Question Two: [2, 2, 4, 2 = 10 marks]

In the corner of his garden, Tony wants to section off an area of grass for his rabbits to roam around. He has 12 metres of fencing he wants to use in addition to the brick wall that encloses his backyard on all sides. Tony wants to create the maximum area possible within his constraints. Let L be the length of his enclosure in metres and W the width of the enclosure in metres.

- (a) Explain why $L = 12 - W$.
- (b) Show that the area of the enclosure can be given by $A = 12W - W^2$.
- (c) Calculate, using calculus techniques, the maximum area of the enclosure.
- (d) Hence determine the dimensions of the enclosure.

Question Three: [3, 7 = 10 marks]

A piece of wire 50 cm long will be cut into two pieces. One piece will be bent to create a circle and the other piece will be bent to form a square.

- (a) Show that the side length of the square, x cm, is given by:

$$x = 12.5 - \frac{\pi r}{2}$$

- (b) Determine, using calculus techniques, the length of each piece of wire such that the sum of the areas of the circle and the square are a minimum.

Question Four: [1, 2, 1, 1, 2, 4 = 11 marks]

A grower is ready to pick his figs ready for market. On average each tree has 25 figs ready for picking and he can sell them each for \$1.50.

For each day he waits to pick the figs, he thinks there will be an extra two figs that are ripe and ready for picking, but he will lose \$0.02 per fig each day he waits.

- (a) What is his revenue per tree if he picks and sells all the figs today?

- (b) What is his revenue per tree if he picks and sells all the figs tomorrow?

- (c) How many figs will be ready to pick if he waits x days from today?

- (d) What will be the price per fig if he waits x days from today to pick them?

- (e) Write an expression representing the grower's total revenue if he waits x days to pick them.
- (f) Hence use calculus techniques to determine how many days the grower should wait to pick his figs in order to achieve maximum revenue.

The wood used for the top and front of the cabinet will cost \$50 per square metre and the wood used for all the other sides will cost \$30 per square metre.

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SOLUTIONS
Calculator Assumed
Applications of Differentiation -
Optimisation

Time: 45 minutes

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Question One: [5 marks]

Verify, using calculus techniques, that a maximum revenue of \$36 000 is obtained by this revenue function: $R = 20000 + 800x - 10x^2$.

$$\frac{dR}{dx} = 800 - 20x \quad \checkmark$$

$$800 - 20x = 0 \quad \checkmark$$

$$x = 40 \quad \checkmark$$

$$\frac{d^2R}{dx^2} = -20 < 0 \therefore \text{max} \quad \checkmark$$

$$R(40) = 36000 \quad \checkmark$$

Question Two: [2, 2, 4, 2 = 10 marks]

In the corner of his garden, Tony wants to section off an area of grass for his rabbits to roam around. He has 12 metres of fencing he wants to use in addition to the brick wall that encloses his backyard on all sides. Tony wants to create the maximum area possible within his constraints. Let L be the length of his enclosure in metres and W the width of the enclosure in metres.

- (a) Explain why $L = 12 - W$.

Tony will use his brick wall for two sides, leaving the 12m of fencing for the other two sides of the enclosure. Together the length and width add to 12m, so the length can be calculated by subtracting the width from the available 12m.

- (b) Show that the area of the enclosure can be given by $A = 12W - W^2$.

$$A = W \times L$$

$$A = W \times (12 - W)$$

$$A = 12W - W^2$$

- (c) Calculate, using calculus techniques, the maximum area of the enclosure.

$$A = 12W - W^2$$

$$A' = 12 - 2W$$

$$12 - 2W = 0$$

$$W = 6$$

$$A = 6 \times (12 - 6) = 36m^2$$

- (d) Hence determine the dimensions of the enclosure.

$$L = 6m$$

$$W = 6m$$

Question Three: [3, 7 = 10 marks]

A piece of wire 50 cm long will be cut into two pieces. One piece will be bent to create a circle and the other piece will be bent to form a square.

- (a) Show that the side length of the square, x cm, is given by:

$$x = 12.5 - \frac{\pi r}{2}$$

$$4x + 2\pi r = 50$$

$$4x = 50 - 2\pi r$$

$$x = \frac{50 - 2\pi r}{4} = 12.5 - \frac{\pi r}{2}$$

- (b) Determine, using calculus techniques, the length of each piece of wire such that the sum of the areas of the circle and the square are a minimum.

$$A = x^2 + \pi r^2$$

$$A = \left(12.5 - \frac{\pi r}{2}\right)^2 + \pi r^2$$

$$A = 156.25 - 12.5\pi r + \frac{\pi^2 r^2}{4} + \pi r^2$$

$$\frac{dA}{dr} = -12.5\pi + \pi^2 r + 2\pi r$$

$$-12.5\pi + \pi^2 r + 2\pi r = 0$$

$$r = 2.7$$

$$\text{Circle: } 2\pi(2.7) = 16.96\text{cm}$$

$$\text{Square: } 50 - 16.96 = 33.04\text{cm}$$

Question Four: [1, 2, 1, 1, 2, 4 = 11 marks]

A grower is ready to pick his figs ready for market. On average each tree has 25 figs ready for picking and he can sell them each for \$1.50.

For each day he waits to pick the figs, he thinks there will be an extra two figs that are ripe and ready for picking, but he will lose \$0.02 per fig each day he waits.

- (a) What is his revenue per tree if he picks and sells all the figs today?

$$25 \times 1.50 = \$37.50 \quad \checkmark$$

- (b) What is his revenue per tree if he picks and sells all the figs tomorrow?

$$27 \times 1.48 = \$39.96 \quad \checkmark \checkmark$$

- (c) How many figs will be ready to pick if he waits x days from today?

$$25 + 2x \quad \checkmark$$

- (d) What will be the price per fig if he waits x days from today to pick them?

$$1.50 - 0.02x \quad \checkmark$$

- (e) Write an expression representing the grower's total revenue if he waits x days to pick them.

$$R(x) = (25 + 2x)(1.50 - 0.02x)$$



- (f) Hence use calculus techniques to determine how many days the grower should wait to pick his figs in order to achieve maximum revenue.

$$R(x) = (25 + 2x)(1.50 - 0.02x)$$

$$R(x) = -0.04x^2 + 2.5x + 37.5$$

$$R'(x) = -0.08x + 2.5$$



$$-0.08x + 2.5 = 0$$



$$x = 31.25$$



$$\therefore 31 \text{ days}$$



Question Five: [4, 5 = 9 marks]

Paul is making a TV cabinet in the shape of a rectangular prism. To fit his living space he decides it has to be 80 cm in width. The total volume of the cabinet will be 1.824 m^3 .

The wood used for the top and front of the cabinet will cost \$50 per square metre and the wood used for all the other sides will cost \$30 per square metre.

- (a) Determine an expression for the total cost of the cabinet in terms of x only, where x is the length of the cabinet in metres.

$$V = x \times H \times 0.8 \quad \checkmark$$

$$0.8xH = 1.824$$

$$H = \frac{2.28}{x} \quad \checkmark$$

$$C = 50\left(x \times \frac{2.28}{x} + 0.8 \times x\right) + 30\left(1.6 \times \frac{2.28}{x} + x \times \frac{2.28}{x} + 0.8 \times x\right) \quad \checkmark \quad \checkmark$$

- (b) Hence use calculus techniques to determine the dimensions of the cabinet which will minimise Paul's costs.

$$C'(x) = \frac{0.04(1600x^2 - 2736)}{x^2} \quad \checkmark$$

$$\frac{0.04(1600x^2 - 2736)}{x^2} = 0 \quad \checkmark$$

$$x = -1.31, 1.31 \quad \checkmark$$

$$L = 1.31\text{m} \quad \checkmark$$

$$H = 1.74\text{m} \quad \checkmark$$

$$W = 0.8\text{m}$$