Full Name: _SOLUTIONS_

MATHEMATICS
APPLICATIONS

Test 6 – Finance

Chapter 7 & 8

Semester 2 2018

Calculator Assumed

Time allowed
Working time for this section: 55 minutes
Marks available: 60 marks

Material required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper,
                    and up to three calculators satisfying the conditions set by the Curriculum
                    Council for this course.

Important note to candidates
No other items may be used in this section of the examination. It is **your** responsibility to ensure
that you do not have any unauthorised notes or other items of a non-personal nature in the
examination room. If you have any unauthorised material with you, hand it to the supervisor **before**
reading any further.

1. **(8 marks)**

   Consider the recursive formula $V_{n+1} = 1.072V_n$, $V_0 = 6800$. This recursive formula can be used to calculate the value of an investment compounded annually for t years.

   a. What amount is being invested? [1]

   $6800 ✓

   b. What annual interest rate is being applied? [1]

   7.2% p.a. ✓

   c. What will be the value of this investment after 8 years? [1]

   $11 859.52 ✓

   d. How much interest is earned after 8 years? [1]

   11859.52 − 6800 = $5059.52 ✓

   e. How much interest is earned during the 8th year? [1]

   $796.54 ✓ (8 to 8)

   Consider the terms of this investment being compounded monthly instead of compounded annually.

   f. What will be the value of the investment after 8 years under the new terms? [1]

   $12075.78 ✓

   g. How much interest is earned during the 8th year under the new terms? [1]

   (85 to 96)

   $836.48 ✓

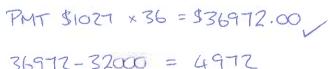   h. What is the effective annual rate of interest under the new terms? [1]

   7.44% p.a. (7.44242)

2. (7 marks)

Elyse borrows $32 000 at a rate of 9.63% per annum compounded monthly. The loan is to be fully repaid with equal monthly instalments over 3 years.

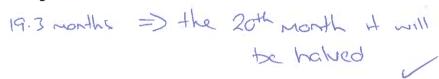a. Calculate the total amount of interest that will be paid on this loan. [2]

PMT $1027 × 36 = $36972.00 ✓

36972 - 32000 = 4972

∴ Interest $4972 ✓

b. How long does it take to reduce the loan to half of the amount borrowed. [1]

19.3 months ⟹ the 20th month it will be halved ✓

c. If the repayment amount on this loan is halved, does the duration of the loan double. Justify your response mathematically. [1]

halving payment results in 86.7 months. This is more than double, so no, it does not double the duration ✓

d. What monthly repayment amount will halve the duration of the loan? [1]

$1916.38 ✓

e. How much will be saved if the duration of the loan is halved by increasing monthly repayments? [2]

34494.83 - 32000 = 2494.83

4972 - 2494.83 = 2477.17 ✓

∴ $2477.17 saved ✓

3. (12 marks)

In order to purchase a car, Ellen needs to borrow $8500 and plans to repay the loan by making a payment at the end of each year. Interest on this loan is charged at a rate of 8.5% per annum compounded annually.

The table below summarises Ellen's repayments and outstanding balance.

Years (n)	Amount owing at the start of the year ($)	Amount owing at the end of the year ($)
1	8500	7622.50
2	7622.50	A
3	A	5637.40
4	5637.40	4516.58
5	4516.58	3300.49
6	3300.49	B
7	B	
8		

a. Calculate the amount of the repayment Ellen made at the end of the first year. [2]

$$8500 \times \frac{8.5}{100} \times - PMT = 7622.50 \checkmark$$

$$PMT = \$1600 \checkmark$$

b. What portion of the principal is repaid by this first repayment? [1]

$$8500 - 7622.50 = \$877.50 \checkmark$$

c. If Ellen repays the same amount at the end of each year for the first seven years determine the missing table values **A** and **B**. [2]

$$A = \$6670.41 \checkmark$$

$$B = \$1981.03 \checkmark$$

d. Ellen makes one final repayment at the end of the eighth year and repays the loan.
    Find the amount of this final repayment.   [1]

$596.11   ✓   table, first negative
                add to $1600

e. Calculate the total interest paid on this loan.   [2]

$3296.11   ✓₂   if incorrect but
                  reasonable working
                  then one mark.

f. Write a recursive formula for A_n, the amount owed at the end of *n* years. [2]

$$A_n = (1.085) A_{n-1} - 1600 \quad ✓$$

$$A_0 = 8500 \quad ✓$$

g. Show how to use this recursive formula to find the value of **B**.   [2]

$$B = A_6$$

$$A_6 = 1.085 A_5 - 1600$$

$$= 1.085 (3300.49) - 1600 \quad ✓$$

$$= 1981.03$$

hence $B = \$1981.03$ ✓

4. (15 marks)

The table below shows the mortgage details on Laoghaire's housing loan from May 2011 to May 2012.

Year	Month	Balance at start of month	Interest	Repayment	Balance at end of month
2011	May	100 000.00	575.00	3000.00	97 575.00
	Jun	97 575.00	561.06	3000.00	95 136.06
	Jul	95 136.06	547.03	3000.00	92 683.09
	Aug	92 683.09	532.93	3000.00	90 216.02
	Sep	90 216.02	518.74	3000.00	87 734.76
	Oct	87 734.76	504.47	3000.00	85 239.23
	Nov	85 239.23	468.82	3000.00	82 708.05
	Dec	82 708.05	454.89	3000.00	80 162.94
2012	Jan	80 162.94	440.90	3000.00	77 603.84
	Feb	77 603.84	426.82	3000.00	75 030.66
	Mar	75 030.66	412.67	3000.00	72 443.33
	Apr	72 443.33	398.44	3000.00	A
	May	B	C	3000.00	D

a. Examine the table and determine which month the monthly interest rate changed. [1]

Big drop in interest in November 2011 ✓

b. Calculate the new annual interest rate. Show working. [2]

$$\frac{468.82}{85239.23} \times 1200 = 6.6\% \ p.a.$$ ✓ ✓

c. Write a recursive formula for B_n, the balance owed at the end of n months after the interest rate changed. [2]

$$B_n = 1.0055 B_{n-1} - 3000$$ ✓

$$B_0 = 85239.23$$
or
$$B_1 = 82708.05$$
either ✓

d. Complete the table by finding the entries for A, B, C and D. [4]

$A = 69841.77$ ✓

$B = 69841.77$ ✓

$C = 384.13$ ✓

$D = 67225.90$ ✓

* no penalty for no $. not in table and these are table values.

e. When does the amount that Laoghaire owes fall below $50 000 for the first time? [1]

December 2012 ✓

f. Write a recursive formula for A_t, the monthly opening balance of this mortgage account after t months before the interest rate was changed. [2]

$$A_t = 1.00575 A_{t-1} - 3000$$ ✓

$$A_0 = 100000$$ ✓

* can have A_0 or A_1 (no difference)

g. **Show** how to use the recursive formula found in **f.** to calculate the balance at the start of August 2011. [2]

August 2011 $(t=4)$

$$A_4 = 1.00575 A_3 - 3000$$ ✓

$$= 1.00575 (95136.06) - 3000$$

$$= 92 683.09$$ ✓ ∴ balance $92683.09

h. Calculate the total interest paid for this loan. [1]

$10941.19 ✓

5. (12 marks)

Adam deposits $100 into an account at the end of every month for 4 years. The account earns interest at a rate of 9% per annum compounded monthly.

a. Find the total amount of Adam's deposits. [1]

$$\$4800 \quad \checkmark$$

b. Write a recursive rule that gives the amount A_n in Adam's account after the nth month. [2]

$$A_{n+1} = \left(1 + \frac{0.09}{12}\right) A_n + 100 \quad \checkmark$$

$$A_1 = 100 \quad \checkmark$$

c. What amount will be in Adam's account after the 4 years? [1]

$$\$5752.07 \quad \checkmark$$

d. How much interest did the investment earn? [1]

$$\$952.07 \quad \checkmark$$

Bianca deposits $100 into an account at the start of every month for 4 years. The account earns interest at a rate of 9% per annum compounded monthly.

e. Find the total amount of Bianca's deposits. [1]

$$\$4800 \quad \checkmark$$

f. Write a recursive rule that gives the amount A_n in Bianca's account after the nth month. [2]

$$B_{n+1} = \left(1 + \frac{0.09}{12}\right)(B_n + 100) \quad \checkmark$$

$$B_1 = 100.75 \quad \checkmark$$

g.   What amount will be in Bianca's account after the 4 years?   [1]

$5795.21   ✓

h.   How much interest did the investment earn?   [1]

$995.21   ✓

i.   State the difference, if any,
     i.   In the amount invested by Adam and Bianca.   [1]

amounts the same   ✓

     ii. In the amount of interest earned.   [1]

Bianca earned $43.14 more   ✓

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6. (8 marks)
    a.   A sum of $400 000 is invested in a perpetuity at an interest rate of 8.1% per annum.
         i.   Find the monthly payment that the perpetuity provides.   [1]

         Q = PE
         Q = 400 000 × 8.1 ÷ 100
            = $32400   ✓   ↓

         ii. After 2 years of monthly payments what is the balance of the
              perpetuity account?   [1]

         $400 000 ✓

    b.   Another perpetuity pays $1900 monthly when the interest rate is 5% per annum. If the
         interest rate increases to 7% per annum how much will the perpetuity pay monthly?   [2]

         @ 5%   Principal is $38000   ✓

         @ 7%, Payment $2660   ✓

c. Murtagh has $1 000 000 in his superannuation account and would like to receive an annuity of $10 000 per month. It is invested at a rate of 6% per annum compounded monthly.

   i. How long will his savings last?   [1]

138.97 months

138 months or 11½ years ✓ either

   ii. If he would like this annuity to last for 10 years, what would be the value of his monthly payment?   [1]

$ 11102.05 when 120 months ✓

End of Test

Additional working space

Question number: _____