

High School Mathematics Test 2015

Year 10

Non Right Triangle Trigonometry

Calculator Allowed

Skills and Knowledge Assessed:

- Apply Pythagoras' theorem and trigonometry to solving three - dimensional problems in right-angled triangles (ACMMG276)
- Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies (ACMMG274)
- Solve simple trigonometric equations (ACMMG275)
- Establish the sine, cosine and area rules for any triangle and solve related problems (ACMMG273)

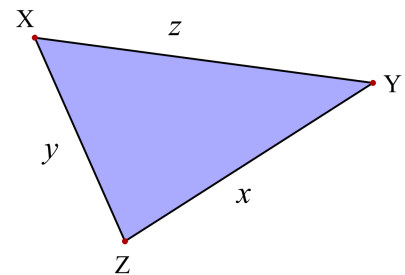
Name _____

Section 1 Short Answer Section

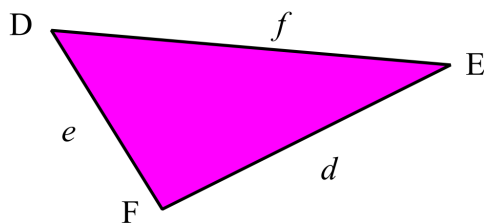
Write all working and answers in the spaces provided on this test paper.

1. Use the sine rule to write an equation which could be used to find the value of y in $\triangle XYZ$?

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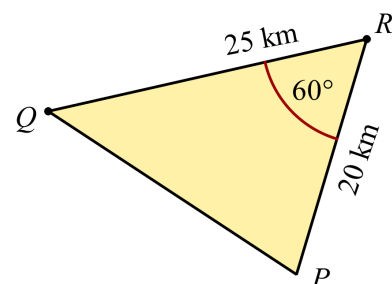
2. Write a statement of the cosine rule that could be used to find the value of f in $\triangle DEF$?



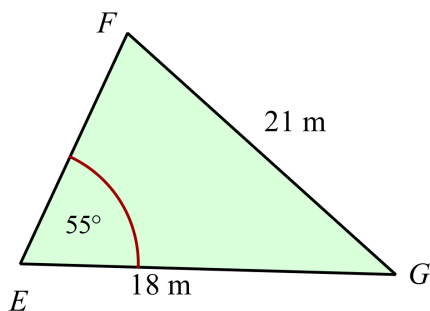
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3. Use the cosine rule to find the length of PQ correct to 1 decimal place.

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4. Use the sine rule to find the size of $\angle EFG$.



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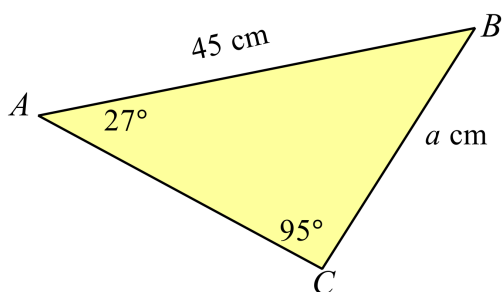
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5. If $\sin \theta = 0.5$ and $0 \leq \theta \leq 180^\circ$ find all the possible values of θ .

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6. Find the value of a , correct to the nearest mm.



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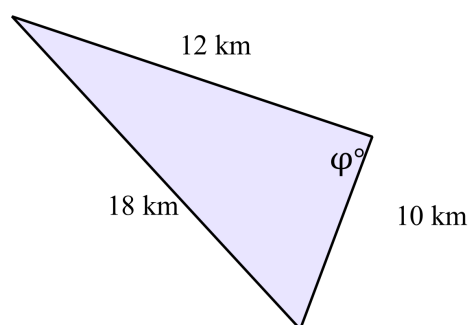
7. What is the value of φ ?
Answer to the nearest degree.

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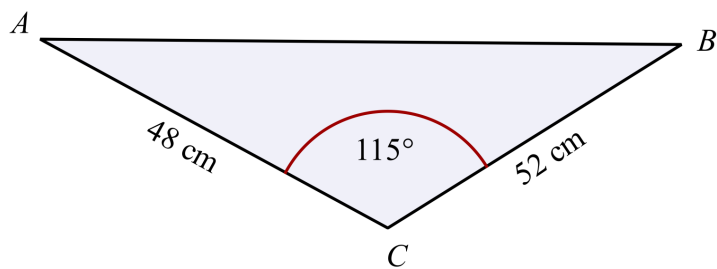
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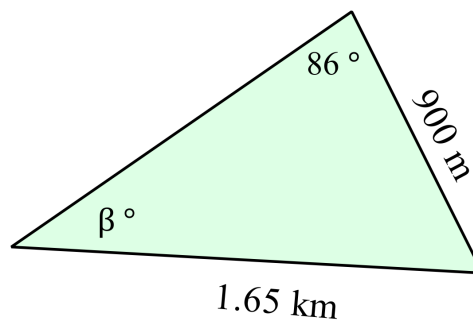
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8. Find the area of $\triangle ABC$.
Answer to the nearest cm^2 .

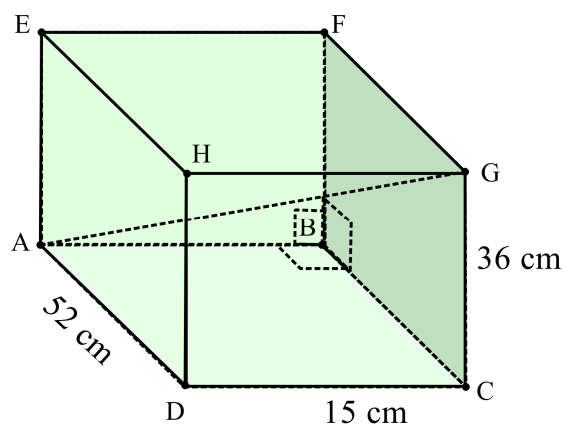


9. Find the value of β .
Answer to the nearest degree.

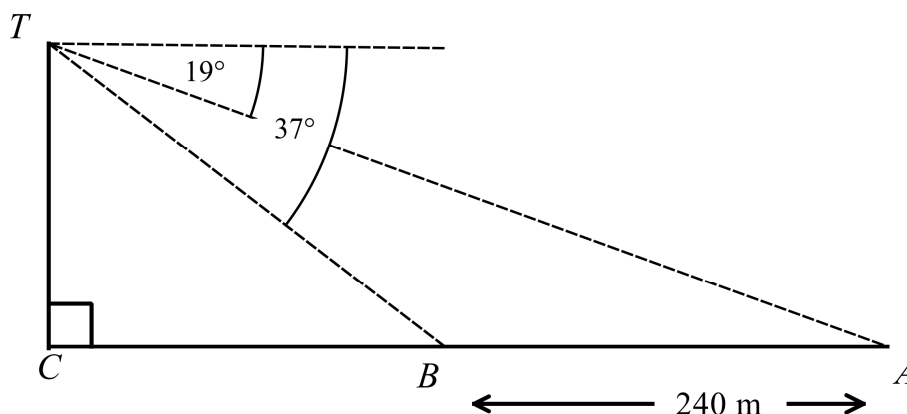


10. Find all possible values of α between 0° and 360° such that $\tan \alpha = \frac{4}{5}$.
Answer to the nearest degree.

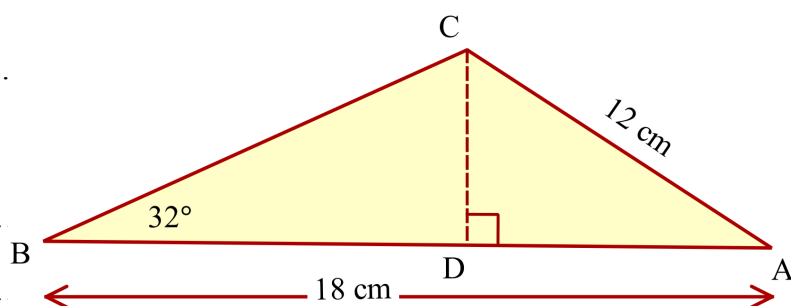
11. What is the length of the interval AG in the rectangular prism shown?



12. From the top of a tower (T) the angles of depression of two points A and B are 19° and 37° respectively. If A and B are 240 m apart in a direct line to the base of the tower, calculate the distance BT , correct to the nearest metre.



13. Find the length of CD correct to the nearest mm.



14. Two planes leave airports M and N at the same time, both flying to a third airport O . The plane leaving M flies on a bearing of 114° and the plane from N flies on a bearing of 055° . M is $1\,125$ km due north of N . How far is O from M ?

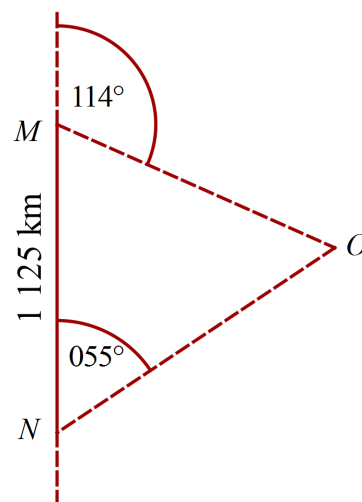
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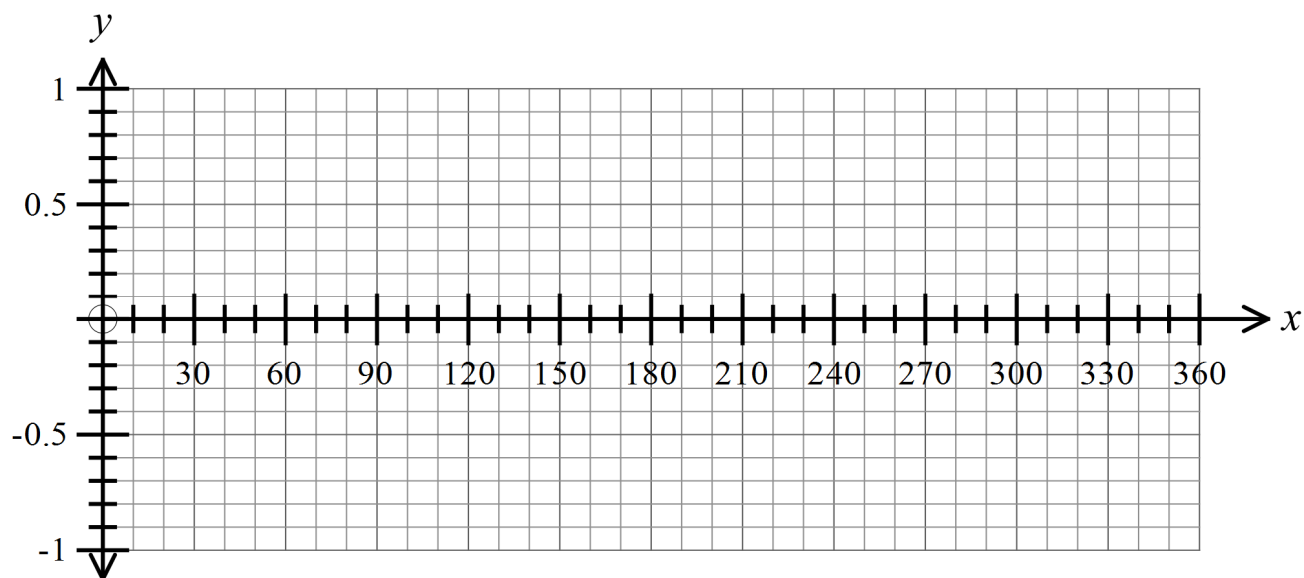
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15. Use the table below to sketch $y = \sin x^\circ$ for $0 \leq x \leq 360$.

x	0	30	60	90	120	150	180	210	240	270	300	330	360
$\sin x$	0.00	0.50	0.87	1.00	0.87	0.50	0.00	-0.50	-0.87	-1.00	-0.87	-0.50	0.00



High School Mathematics Test 2015

Year
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Non Right Triangle Trigonometry

Calculator Allowed

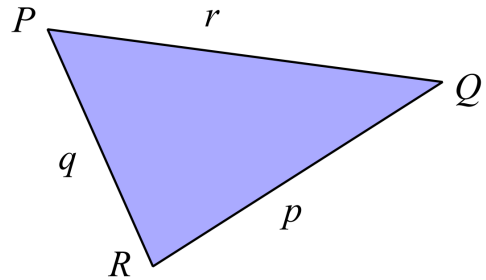
Name _____

Section 2 Multiple Choice Section

Mark all your answers on the accompanying multiple choice answer sheet, not on this test paper. You may do any working out on this test paper. Calculators are allowed for this section.

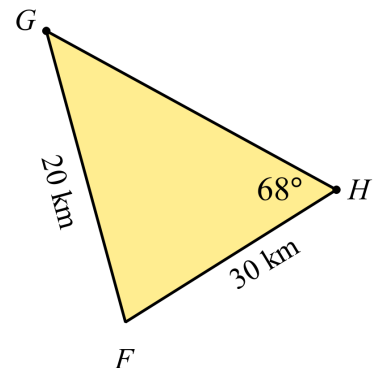
1. Which statement of the cosine rule could be used to find $\angle R$ in $\triangle PQR$?

- A. $\cos R = \frac{p^2 + q^2 - r^2}{2pq}$
- B. $\cos R = \frac{r^2 + q^2 - p^2}{2rq}$
- C. $\cos R = \frac{p^2 + r^2 - q^2}{2pr}$
- D. $\cos R = \frac{p^2 + q^2 - r^2}{2qr}$



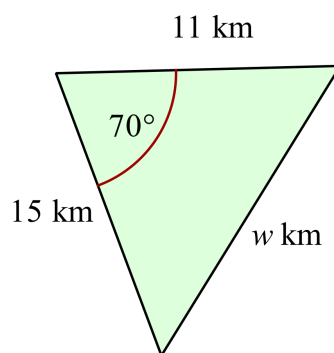
2. Which calculation using the sine rule could be used to find the size of $\angle G$?

- A. $\frac{\sin G}{20} = \frac{\sin 68^\circ}{30}$
- B. $\frac{\sin G}{68} = \frac{\sin 30^\circ}{20}$
- C. $\frac{\sin G}{30} = \frac{\sin 20^\circ}{68}$
- D. $\frac{\sin G}{30} = \frac{\sin 68^\circ}{20}$



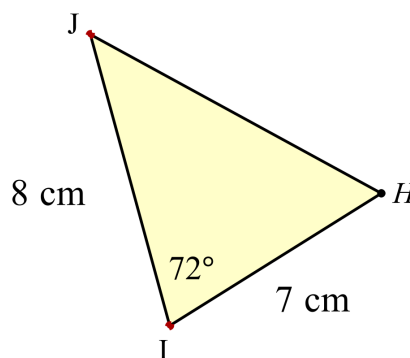
3. Use the cosine rule to find the value of w , correct to 1 decimal place.

- A. 8.0
B. 15.3
C. 17.0
D. 21.4



4. Use the area formula to find the area of $\triangle HIJ$, correct to the nearest 10^{th} of a square cm.

- A. 19.2 cm^2
B. 26.6 cm^2
C. 53.3 cm^2
D. 63.5 cm^2

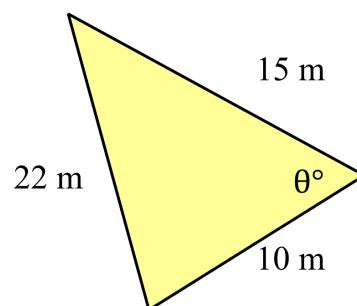


5. Solve $2 \cos x = 1$ for $0^\circ \leq x^\circ \leq 360^\circ$.

- A. $x = 60^\circ$ or 120° .
B. $x = 60^\circ$ or 300° .
C. $x = 120^\circ$ or 300° .
D. $x = 240^\circ$ or 300° .

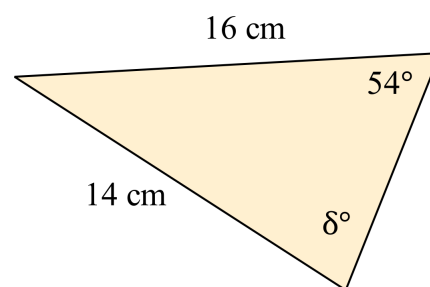
6. Calculate the value of θ correct to the nearest degree.

- A. 23°
B. 35°
C. 122°
D. 157°



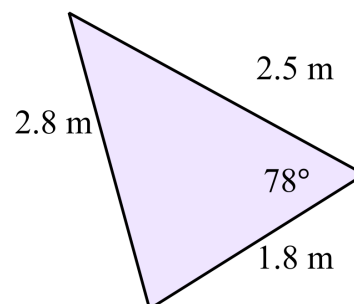
7. Find the value of δ , to the nearest degree.

- A. 11°
- B. 13°
- C. 45°
- D. 68°



8. Find the area of the triangle shown correct to 1 decimal place.

- A. 2.2 m^2
- B. 2.5 m^2
- C. 3.4 m^2
- D. 4.4 m^2

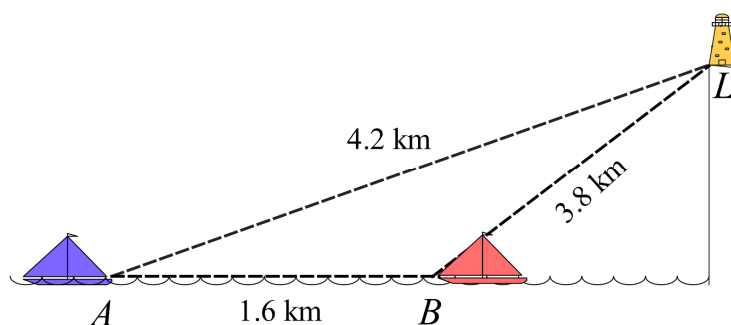


9. The direct distance from ship A to the lighthouse L is 4.2 km and from ship B to the lighthouse is 3.8 km.

The two ships are in a straight line to the lighthouse and are 1.6 km apart.

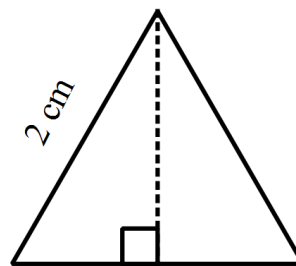
What is the angle of elevation of the lighthouse from ship A?

- A. 22°
- B. 45°
- C. 65°
- D. 93°



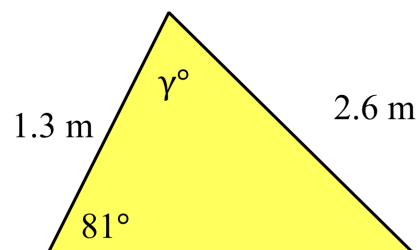
10. An equilateral triangle is shown with the perpendicular bisector of one side drawn in. Use the equilateral triangle to find the exact value of $\cos 30^\circ$.

- A. $\frac{1}{2}$
B. $\frac{1}{\sqrt{3}}$
C. $\frac{\sqrt{3}}{2}$
D. $\sqrt{3}$



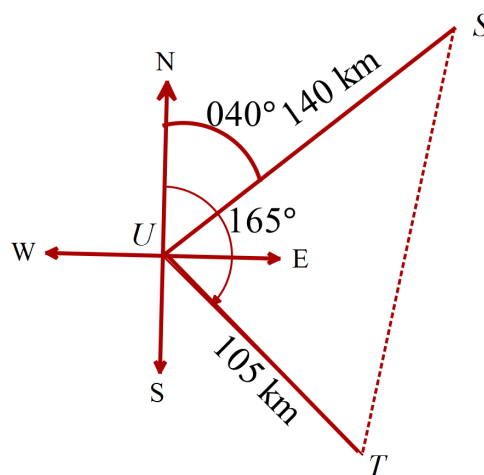
11. Find the value of γ correct to the nearest degree.

- A. 30°
B. 69°
C. 111°
D. 150°

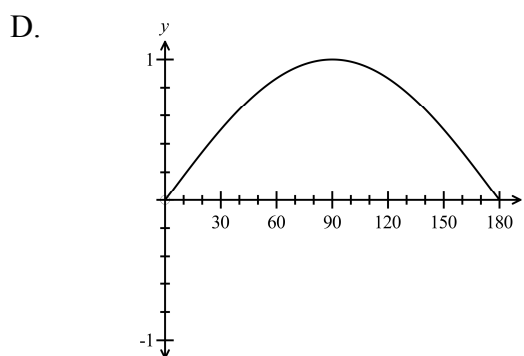
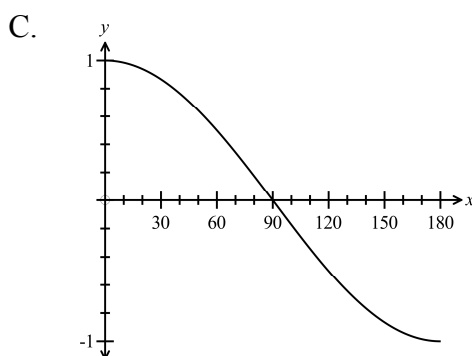
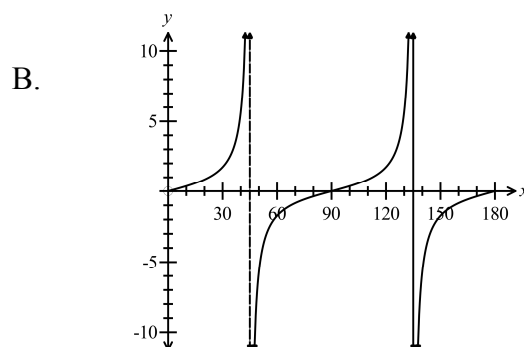
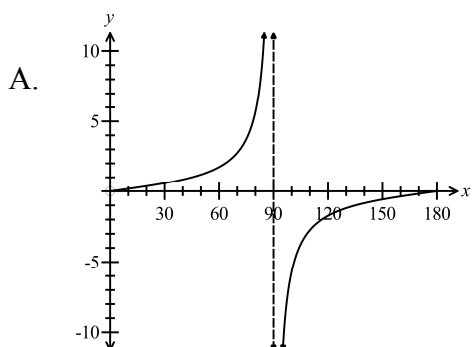


12. Ship S is 140 km on a bearing of 040° from Port U .
Ship T is 105 km on a bearing of 165° from Port U .
Calculate the distance ST , correct to the nearest 10^{th} of a kilometre.

- A. 117.3 km
B. 197.6 km
C. 217.9 km
D. 242.9 km



13. Which graph shows a sketch of $y = \tan x$ for $0^\circ \leq x \leq 180^\circ$.

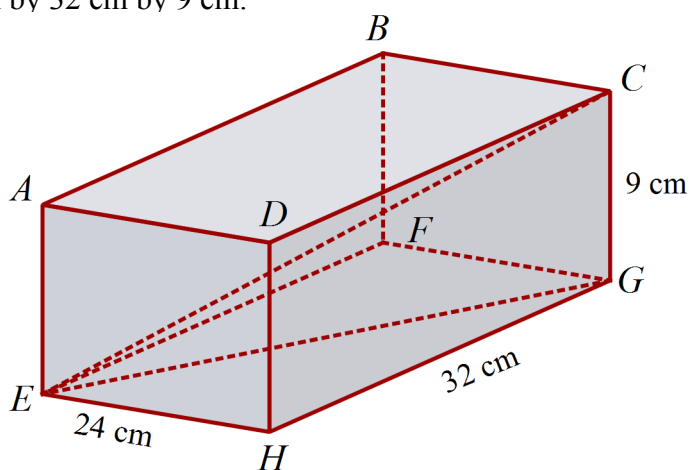


14. If $0 \leq x \leq 360$; for what values of x is $\cos x^\circ = 0$?

- A. $x = 0$ and $x = 180$
- B. $x = 0$ and $x = 270$
- C. $x = 90$ and $x = 180$
- D. $x = 90$ and $x = 270$

15. The rectangular prism shown measures 24 cm by 32 cm by 9 cm.
What is the size of $\angle CEG$?

- A. 13°
- B. 34°
- C. 56°
- D. 76°



High School Mathematics Test 2015

Multiple Choice Answer Sheet

Non Right Triangle Trigonometry

Name _____

Completely fill the response oval representing the most correct answer.

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|-----|---|-----------------------|---|-----------------------|---|-----------------------|---|-----------------------|
| 1. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 2. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 3. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 4. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 5. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 6. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 7. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 8. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 9. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 10. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 11. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 12. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 13. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 14. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 15. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |

High School Mathematics Test 2015

Year 10

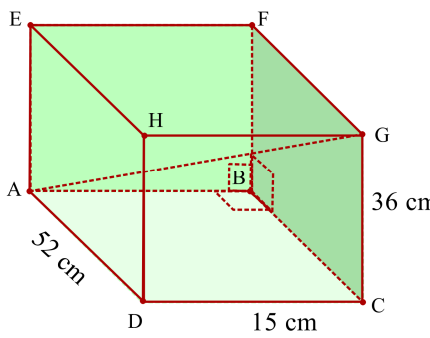
Non Right Triangle Trigonometry

Calculator

Section 1 Short Answer Section

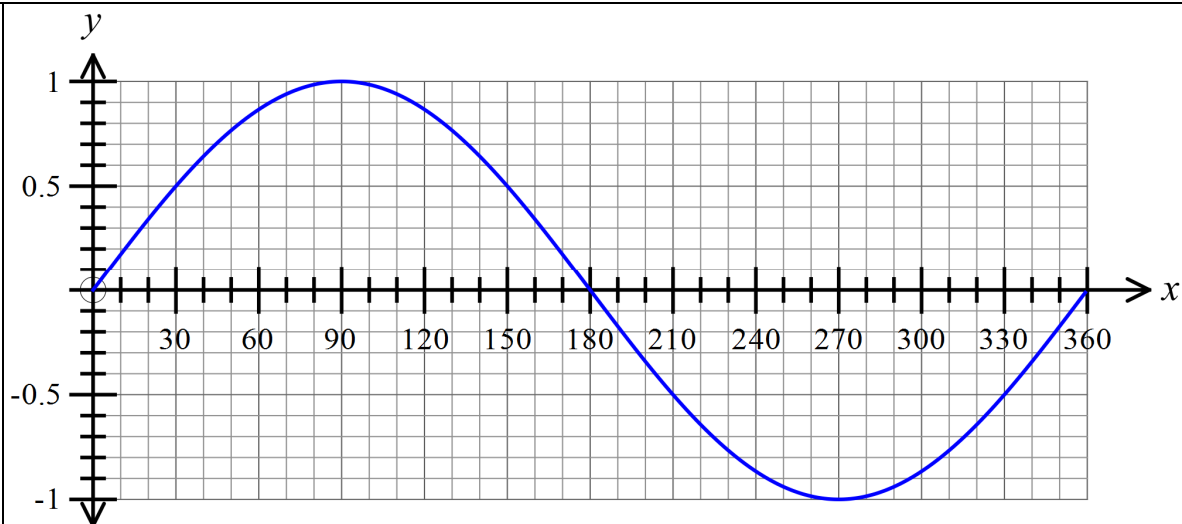
ANSWERS

No.	WORKING	ANSWER
1.	$\frac{y}{\sin Y} = \frac{x}{\sin X} = \frac{z}{\sin Z} \qquad \frac{y}{\sin Y} = \frac{z}{\sin Z}$ $\frac{y}{\sin Y} = \frac{x}{\sin X} \qquad y = \frac{z(\sin Y)}{\sin Z} \qquad y = \frac{x(\sin Y)}{\sin X}$	Any of the equations shown at left.
2.	$f^2 = e^2 + d^2 - 2ed \cos F$	$f^2 = e^2 + d^2 - 2ed \cos F$
3.	$PQ^2 = 25^2 + 20^2 - 2 \times 25 \times 20 \times \cos 60$ $= 625 + 400 - 500$ $= 525$ $PQ = \sqrt{525} = 22.91287$ $= 22.9 \text{ km (1 dec place)}$	22.9 km
4.	$\frac{\sin F}{18} = \frac{\sin 55^\circ}{21}$ $\sin F = \frac{18 \times \sin 55^\circ}{21}$ $= 0.702130$ $F = \sin^{-1} 0.702130$ $= 44.598171$ $= 45^\circ \text{ (nearest degree)}$	45°
5.	$\sin \theta = 0.5$ $\theta = \sin^{-1} (0.5)$ $\theta = 30^\circ \text{ and since } \sin \text{ is } +ve \text{ in } 2nd \text{ quadrant}$ $\theta = 180 - 30 = 150^\circ$	$\theta = 30^\circ \text{ or } 150^\circ.$
6.	$\frac{a}{\sin 27} = \frac{45}{\sin 95}$ $a = \frac{\sin 27 \times 45}{\sin 95}$ $= 20.507610$ $= 20.5 \text{ cm (nearest mm)}$	20.5 cm or 205 mm

7.	$\cos \varphi = \frac{12^2 + 10^2 - 18^2}{2 \times 10 \times 12}$ $= -\frac{80}{240}$ $\varphi = \cos^{-1} \left(-\frac{80}{240} \right)$ $= 109.4712$ $= 109^\circ$	109°
8.	$\text{Area} = \frac{1}{2} \times 48 \times 52 \times \sin 115^\circ$ $= 1131.072$ $= 1131 \text{ cm}^2 \text{ (nearest cm}^2\text{)}$	1131 cm^2
9.	$\frac{\sin \beta}{900} = \frac{\sin 86^\circ}{1650}$ $\sin \beta = \frac{900 \times \sin 86^\circ}{1650}$ $= 0.544126$ $\beta = \sin^{-1} 0.544126$ $= 32.96495$ $= 33^\circ \text{ (nearest degree)}$	33°
10.	$\tan \alpha = \frac{4}{5}$ $\alpha = \tan^{-1} \left(\frac{4}{5} \right)$ $= 38.6598$ $= 39^\circ$ <p>\tan is also +ve in 3rd quadrant, so $\alpha = 180 + 39$ $\alpha = 219^\circ$</p>	39° and 219°
11.	<p>In $\triangle ADC$</p> $AC^2 = 52^2 + 15^2$ $= 2929$ $AC = \sqrt{2929}$ $AG^2 = \sqrt{2929}^2 + 36^2$ $= 4225$ $AG = \sqrt{4225} = 65 \text{ cm}$ 	65 cm

12.	$\angle BTA = 37 - 19 = 18^\circ$ $\angle TAB = 19^\circ$ (alternate angles) $\frac{BT}{\sin 19^\circ} = \frac{240}{\sin 18^\circ}$ $BT = \frac{\sin 18^\circ}{\sin 19^\circ} \times 240$ $= 252.85456$ $= 253 \text{ m}$	253 m
13.	<p>In $\triangle ABC$</p> $\frac{\sin C}{18} = \frac{\sin 32^\circ}{12}$ $\sin C = \frac{18 \times \sin 32^\circ}{12}$ $= 0.79488$ $C = \sin^{-1}(0.79488)$ $= 52.6438$ $= 53^\circ$ or since C is obtuse $C = 180 - 53 = 127^\circ$ <p>In $\triangle ACB$</p> $\angle A = 180 - 127 - 32 = 21^\circ$ <p>In $\triangle ACD$</p> $\sin A = \frac{CD}{12}$ $CD = 12 \times \sin 21$ $= 4.3004$ $= 4.3 \text{ cm}$	4.3 cm
14.	<p>In $\triangle MNO$</p> $\angle NMO = 180 - 114 = 66^\circ$ $\angle MON = 180 - (66 + 55) = 59^\circ$ $\frac{MO}{\sin 55^\circ} = \frac{1125}{\sin 59^\circ}$ $MO = \frac{1125 \times \sin 55^\circ}{\sin 59^\circ}$ $= 1075.10639879$ $= 1\,075 \text{ km}$	1 075 km

15.



High School Mathematics Test 2015

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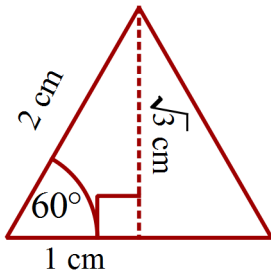
Non Right Triangle Trigonometry

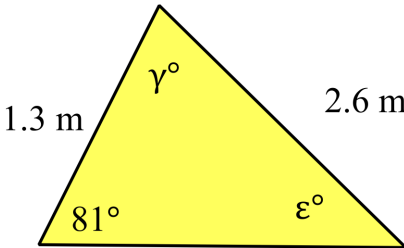
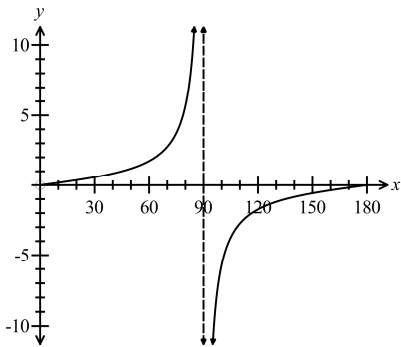
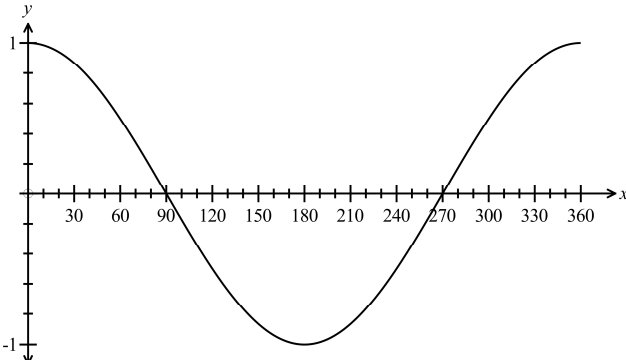
Calculator Allowed

Section 2 Multiple Choice Section

ANSWERS

No.	WORKING	ANSWER
1.	$\cos R = \frac{p^2 + q^2 - r^2}{2pq}$	A
2.	$\frac{\sin G}{30} = \frac{\sin 68^\circ}{20}$	D
3.	$w^2 = 11^2 + 15^2 - 2 \times 11 \times 15 \times \cos 70$ $= 346 - 112.866$ $= 233.1334$ $w = \sqrt{233.1334}$ $= 15.2687$ $= 15.3 \text{ (1 dec place)}$	B
4.	$\text{Area} = \frac{1}{2} a b \sin C$ $= \frac{1}{2} h j \sin I$ $= \frac{1}{2} \times 8 \times 7 \times \sin 72^\circ$ $= 26.629582456264300019260301334623$ $= 26.6 \text{ cm}^2 \text{ (nearest 10th cm}^2\text{)}$	B
5.	$2 \cos x^\circ = 1$ $\cos x^\circ = \frac{1}{2}$ $x^\circ = \cos^{-1} \left(\frac{1}{2} \right)$ Acute value $x = 60^\circ$ for $0 \leq x \leq 360$. ASTC cos is +ve in 1st and 4th quadrants $x = 60^\circ \text{ or } 360 - 60^\circ = 300^\circ$	B

6.	$\cos \theta = \frac{15^2 + 10^2 - 22^2}{2 \times 10 \times 15}$ $= -\frac{159}{300}$ $\theta = \cos^{-1} \left(-\frac{159}{300} \right)$ $= 122^\circ$	C
7.	$\frac{\sin \delta}{16} = \frac{\sin 54^\circ}{14}$ $\sin \delta = \frac{16 \times \sin 54^\circ}{14}$ $= 0.92459$ $\delta = \sin^{-1} 0.92459$ $= 67.606739$ $= 68^\circ \text{ (nearest degree)}$	D
8.	$\text{Area} = \frac{1}{2} a b \sin C$ $= \frac{1}{2} \times 2.5 \times 1.8 \times \sin 78^\circ$ $= 2.2008321016510626853392751827066$ $= 2.2 \text{ m}^2 \text{ (1 decimal place)}$	A
9.	$\cos \theta = \frac{1.6^2 + 4.2^2 - 3.8^2}{2 \times 1.6 \times 4.2}$ $= \frac{5.76}{13.44}$ $\theta = \cos^{-1} \left(\frac{5.76}{13.44} \right)$ $= 64.6230$ $= 65^\circ$	C
10.	$\sin 60^\circ = \frac{O}{H} = \frac{\sqrt{3}}{2}$ 	C

11.	<p>First find ε.</p> $\frac{\sin \varepsilon}{1.3} = \frac{\sin 81^\circ}{2.6}$ $\sin \varepsilon = \frac{1.3 \times \sin 81^\circ}{2.6}$ $= 0.49384$ $\varepsilon = \sin^{-1}(0.49384)$ $= 29.5935$ $= 30^\circ$ $\gamma = 180 - (81 + 30)$ $= 180 - 111$ $= 69^\circ$		B
12.	$\angle SUT = 165^\circ - 40^\circ = 125^\circ$ $ST^2 = 140^2 + 105^2 - 2 \times 140 \times 105 \times \cos 125^\circ$ $= 30625 + 16863.14722$ $ST^2 = 47488.147229$ $ST = 217.917$ $= 217.9 \text{ km (nearest 10th)}$	C	
13.		A	
14.	<p>Using a graph $x = 90$ and $x = 270$.</p> 	D	

15.	<p>Using Pythagoras Theorem</p> $EG^2 = 24^2 + 32^2$ $= 1600$ $EG = \sqrt{1600} = 40$ $\tan \angle CEG = \frac{CG}{EG} = \frac{9}{40}$ $\angle CEG = \tan^{-1} \left(\frac{9}{40} \right)$ $= 12.68038$ $= 13^\circ \text{ (nearest degree)}$	A
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High School Mathematics Test 2015

Multiple Choice Answer Sheet Non Right Triangle Trigonometry

Name ANSWERS

Completely fill the response oval representing the most correct answer.

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|-----|---|----------------------------------|---|----------------------------------|---|----------------------------------|---|----------------------------------|
| 1. | A | <input checked="" type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 2. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input checked="" type="radio"/> |
| 3. | A | <input type="radio"/> | B | <input checked="" type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 4. | A | <input type="radio"/> | B | <input checked="" type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 5. | A | <input type="radio"/> | B | <input checked="" type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 6. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input checked="" type="radio"/> | D | <input type="radio"/> |
| 7. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input checked="" type="radio"/> |
| 8. | A | <input checked="" type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 9. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input checked="" type="radio"/> | D | <input type="radio"/> |
| 10. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input checked="" type="radio"/> | D | <input type="radio"/> |
| 11. | A | <input type="radio"/> | B | <input checked="" type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 12. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input checked="" type="radio"/> | D | <input type="radio"/> |
| 13. | A | <input checked="" type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
| 14. | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input checked="" type="radio"/> |
| 15. | A | <input checked="" type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |