Question 1 (2, 3 = 5 marks)

Find the derivative of the following functions, fully simplifying where possible

a) $f(x) = \sqrt{x}(2x^3 + 1)$

$f(x) = 2x^{\frac{7}{2}} + x^{\frac{1}{2}}$ ✓ OR $f'(x) = 6x^2(\sqrt{x}) + \frac{2x^3+1}{2\sqrt{x}}$ ✓

$f'(x) = 7x^{\frac{5}{2}} + \frac{1}{2\sqrt{x}}$

$= \frac{14x^3+1}{2\sqrt{x}}$ ✓

$= \frac{6x^2(2x) + 2x^3+1}{2\sqrt{x}}$

$= \frac{14x^3+1}{2\sqrt{x}}$ ✓

Answers: $f'(x) = \frac{14x^3+1}{2\sqrt{x}}$, $\frac{\sqrt{x}(14x^3+1)}{2x}$, $\frac{14\sqrt{x^7}+\sqrt{x}}{2x}$ $= 7x^{\frac{5}{2}} + \frac{1}{2\sqrt{x}}$

b) $y = \frac{6x}{(x^2-5x+6)^3}$

$y' = \frac{6(x^2-5x+6)^3 - 6x(3)(2x-5)(x^2-5x+6)^2}{(x^2-5x+6)^6}$ ✓

$= \frac{6(x^2-5x+6)^2[x^2-5x+6-3x(2x-5)]}{(x^2-5x+6)^6}$ ✓

$= \frac{6(x^2-5x+6-6x^2+15x)}{(x^2-5x+6)^4}$

$= \frac{6(-5x^2+10x+6)}{(x^2-5x+6)^4}$ ✓

OR $y = 6x(x^2-5x+6)^{-3}$ ✓

$y' = 6(x^2-5x+6)^{-3} + 6x(-3)(x^2-5x+6)^{-4}(2x-5)$

$= 6(x^2-5x+6)^{-4}[x^2-5x+6-3x(2x-5)]$ ✓

$= \frac{6(x^2-5x+6-6x^2+15x)}{(x^2-5x+6)^4}$

$= \frac{6(-5x^2+10x+6)}{(x^2-5x+6)^4}$ ✓

Answers $y' = \frac{6(-5x^2+10x+6)}{(x^2-5x+6)^4}$ OR $\frac{-6(5x^2-10x-6)}{(x^2-5x+6)^4}$

Question 2 (3 marks)

Given that $u = g(x) = 4x - 3$ and $f(u) = \frac{1}{u^2+7}$, determine $(f o g)'(x)$. Simplify your answer where possible.

$f(g(x)) = \frac{1}{(4x-3)^2+7}$ ✓

$= \frac{1}{16x^2-24x+16}$

$= \frac{1}{8(2x^2-3x+2)} = \frac{1}{8}(2x^2-3x+2)^{-1}$

$(f(g(x)))' = \frac{1}{8}(2x^2-3x+2)^{-2}(-1)(4x-3)$ ✓

$= \frac{-(4x-3)}{8(2x^2-3x+2)^2}$ ✓

Question 3 (1, 2 = 3 marks)

Determine the following;

a) The antiderivative of $\frac{3}{\sqrt{x}} - 4x^3 + \frac{2}{5x^3}$

$F(x) = 6\sqrt{x} - x^4 - \frac{1}{5x^2} + C$ ✓

-1 missing +C

b) $\int 10(6x+1)(6x^2+2x+1)^3 dx$

$5\int(12x+2)(6x^2+2x+1)^3 dx$ ✓

$F(x) = \frac{5(6x^2+2x+1)^4}{4} + C$ ✓

Question 4 (7 marks)

The position of a particle is described by the function $x(t) = -\frac{t^3}{3} + t^2 + 8t + 1$ for $t \geq 0$, where t is in seconds and $x(t)$ in cm. Determine the distance travelled by the particle when the acceleration reaches -8 m/s².

$v(t) = -t^2 + 2t + 8$ ✓

$a(t) = -2t + 2$ ✓

$-8 = -2t + 2$

$t = 5$ ✓

Start point: $0 = -t^2 + 2t + 8$

$0 = t^2 - 2t - 8$

$0 = (t-4)(t+2)$

$t = 4$ or $t = -2$ reject. ✓

distance

$x(0) = 1$

$x(4) = \frac{-64}{3} + 16 + 32 + 1$

$= \frac{-64}{3} + 49$

$= \frac{147-64}{3}$

$= \frac{83}{3}$ ✓ $= 27\frac{2}{3}$

$x(5) = \frac{-125}{3} + 25 + 40 + 1$

$= \frac{-125}{3} + 66$

$= \frac{198-125}{3}$

$= \frac{73}{3}$ ✓ $= 24\frac{1}{3}$

Distance covered $= \left(\frac{83}{3} - 1\right) + \left(\frac{83}{3} - \frac{73}{3}\right)$

$= \frac{80}{3} + \frac{10}{3}$

$= 30$ cm. ✓

Question 5 (6 marks)

A small moving body, W moves in a straight line with acceleration a m/s² at time t s given by the function $a = At + B$.

Initially, W had a displacement of 12 m from a fixed point of O and moves with a velocity of 3 m/s. Two seconds later, W has a displacement of 1.8 cm and a velocity of -2 cm/s.

Determine the value of the constants A and B.

$V = \int a\,dt$

$= \int (At + B)\,dt$

$V = \frac{At^2}{2} + Bt + C$

$V(0) = 3 = C$

$\therefore V = \frac{At^2}{2} + Bt + 3$ ✓

$x = \int \left(\frac{At^2}{2} + Bt + 3\right) dt$

$= \frac{At^3}{6} + \frac{Bt^2}{2} + 3t + C$

$x(0) = 12 = C$

$\therefore x = \frac{At^3}{6} + \frac{Bt^2}{2} + 3t + 12$ ✓

$x(2) = 1.8 = \frac{8A}{6} + \frac{4B}{2} + 6 + 12$

$-16.2 = \frac{4A}{3} + 2B$ ✓ $\rightarrow 2B = -16.2 - \frac{4A}{3}$

$V(2) = -2 = \frac{4A}{2} + 2B + 3$

$-5 = 2A + 2B$ ✓ $\rightarrow 2B = -5 - 2A$

$-5 - 2A = -16.2 - \frac{4A}{3}$

$\frac{-2A}{3} = -11.2$

$A = 16.8$ ✓

$2B = -5 - 2(16.8)$

$B = -19.3$ ✓

$\therefore A = 16.8,\ B = -19.3$

END OF SECTION

WILLETTON SENIOR HIGH SCHOOL

YEAR 12 MATHEMATICS METHODS

TEST 1 2022

Section 1: Calculator Free

Student Name: Solutions

Circle your teacher's name

Miss Ahern **Ms Arora** **Mrs Gatland**

Mrs Sun **Mrs Tay**

Mark: ________ **/ 24**

Time: **25 mins**

For this test:

Scientific calculators and Classpads are NOT allowed

Show any working in the spaces provided