Manjimup S.H.S Year 11 2015
Mathematical Methods Investigation 2
Out of Class Section

TRIG CURVES

Name: SOLUTIONS

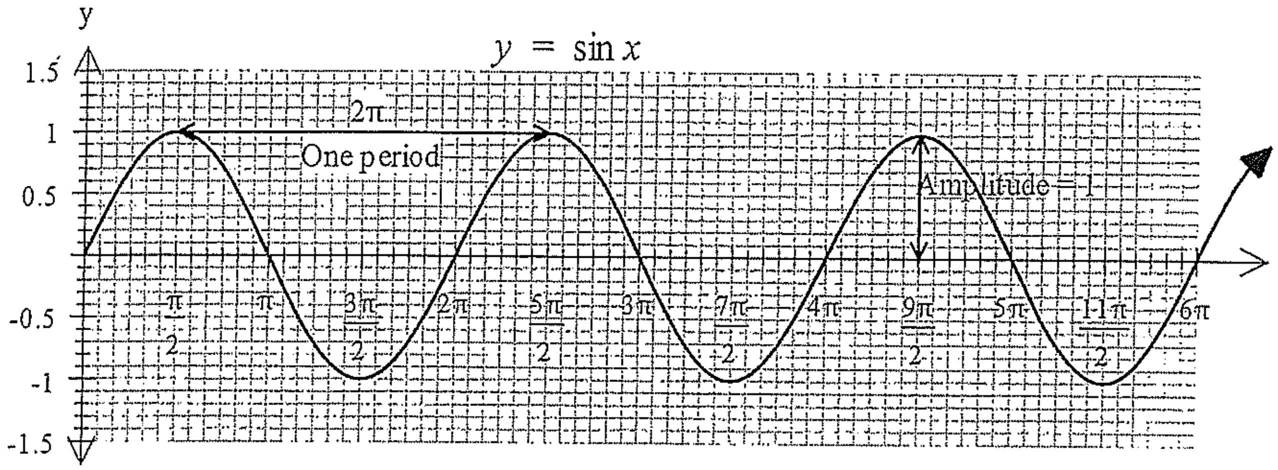
- *The Out of Class Investigation is designed for you to learn the essentials needed for the In-Class validation.*
- *This is the "Take Home" part of the Investigation. It does not count towards your mark for this investigation.*
- *You will need your Casio Classpad.*
- *You are permitted to take this section into the "In Class" validation test to assist you.*

Date of Validation: Tuesday 12th May.

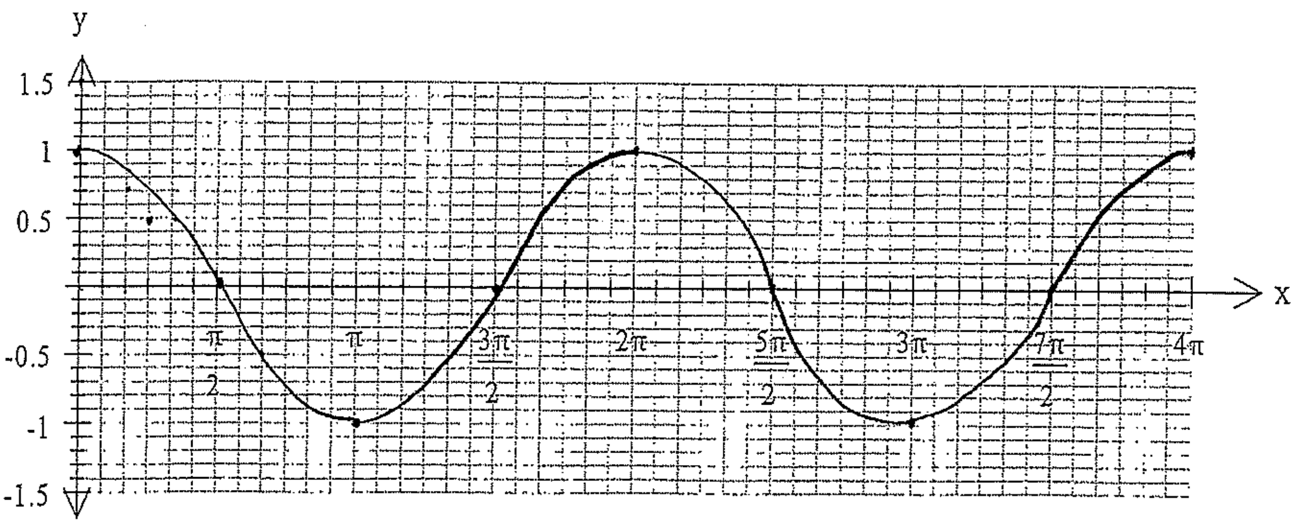
Shown below is a graph of y = sin x for the domain $0 \le x \le 6\pi$. It shows the curve as it goes through 3 cycles.

The period is the length of one cycle of the curve. It is often best to measure from maximum to maximum.
The amplitude of a curve is equal to *half* of the distance from the maximum value to the minimum value.

The sine curve below has a period of 2π and an amplitude of 1.



1. Using radians draw a graph of y = cos x between 0 and 4π. Use your calculator to help.

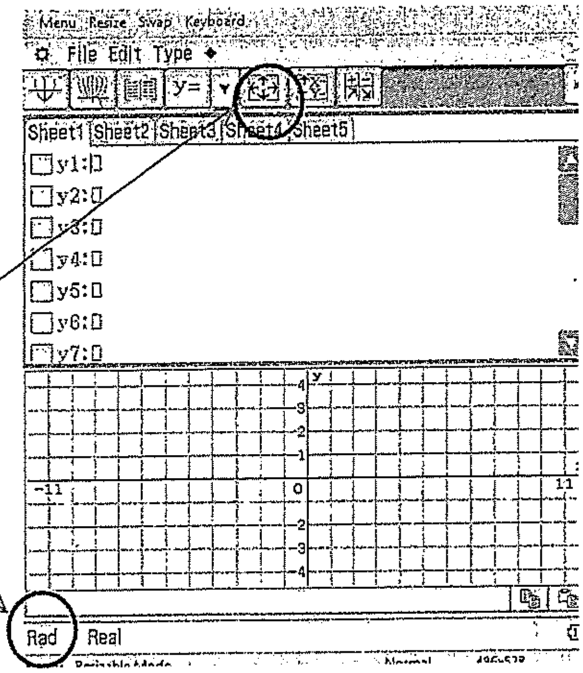


a) State the period of y = cos x and the amplitude of y = cos x. Period 2π Amp = 1.

b) Compare the differences and similarities of y = cos x and y = sin x.

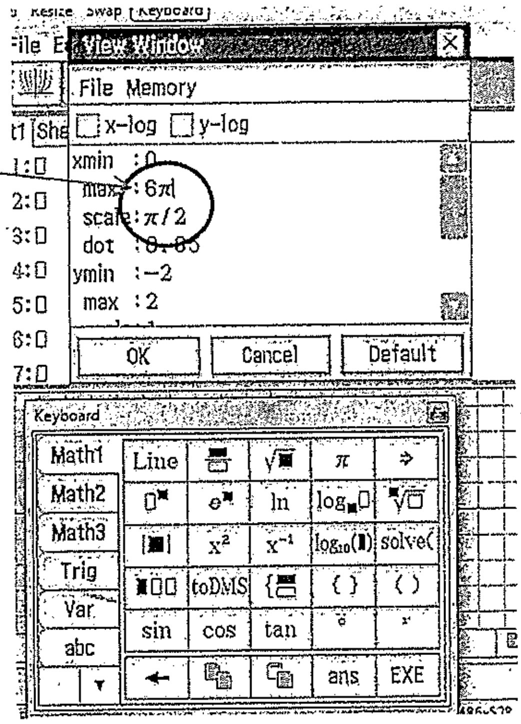
- Same basic curve
sine starts at 0, cosine at 1.

HOW TO SET UP YOUR CALCULATOR TO DRAW TRIG GRAPHS



1. IN GRAPH & TAB set your calculator to radian mode.

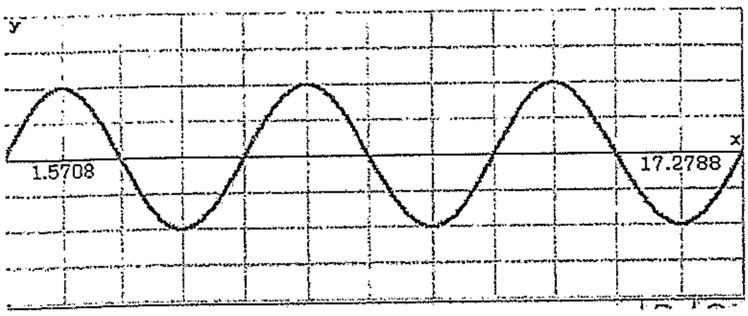
2. Go to View Window.



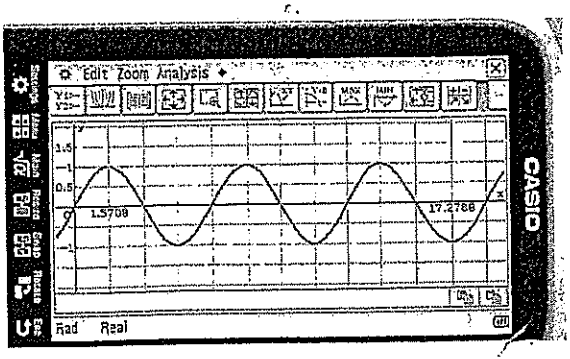
3. Set your x-axis from 0 to whatever value of π you need, using the π on the keyboard..

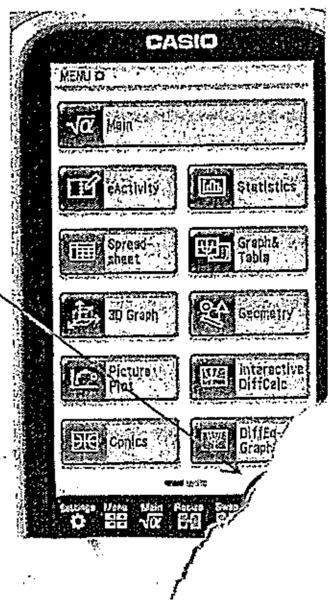
4. Set your scale as either $\dfrac{\pi}{2}$ (about 1.57) or $\dfrac{\pi}{6}$.

5. Your graph will appear as thus:

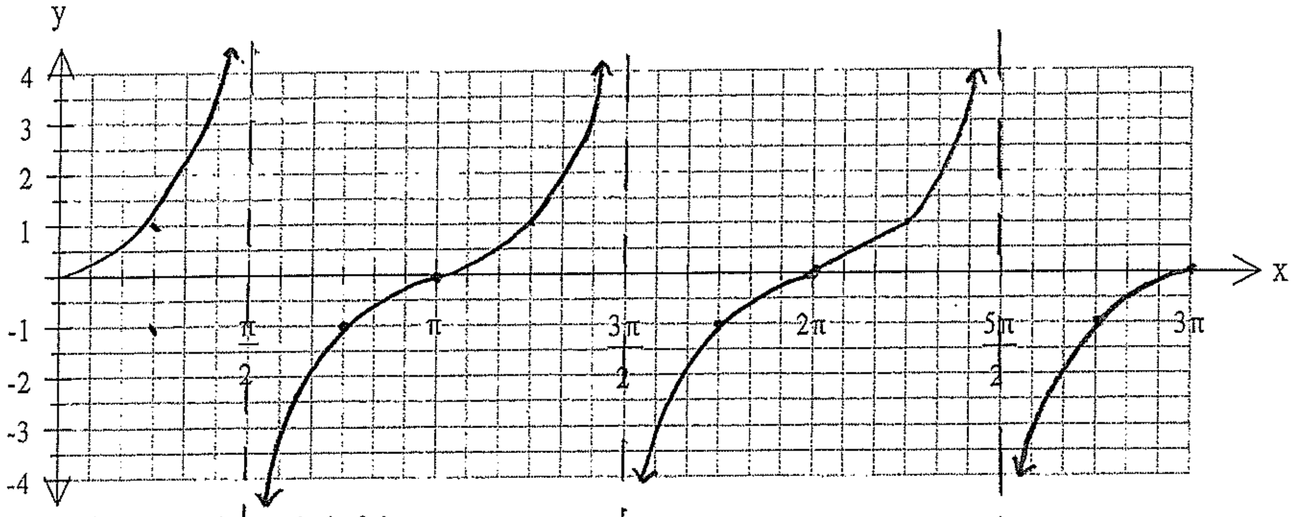


6. You may see the graph better if you rotate your screen to landscape view.





2. Plot the function y = tan x for the domain $0 \le x \le 3\pi$, indication the presence of vertical asymptotes.



a) State the period of the y = tan x curve. Period = π.

b) Discuss the amplitude of the y = tan x curve. Amplitude is infinite.

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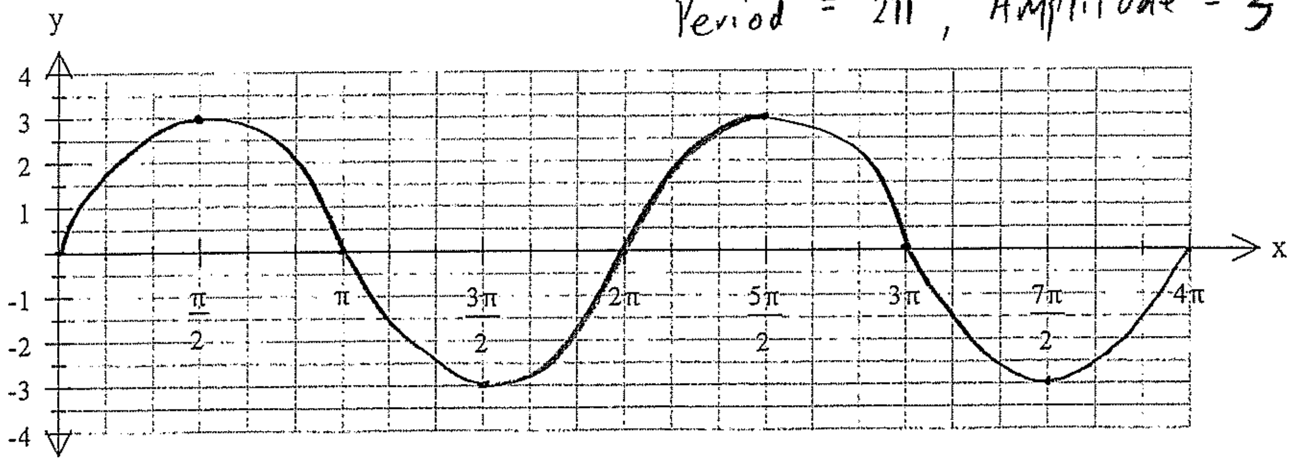
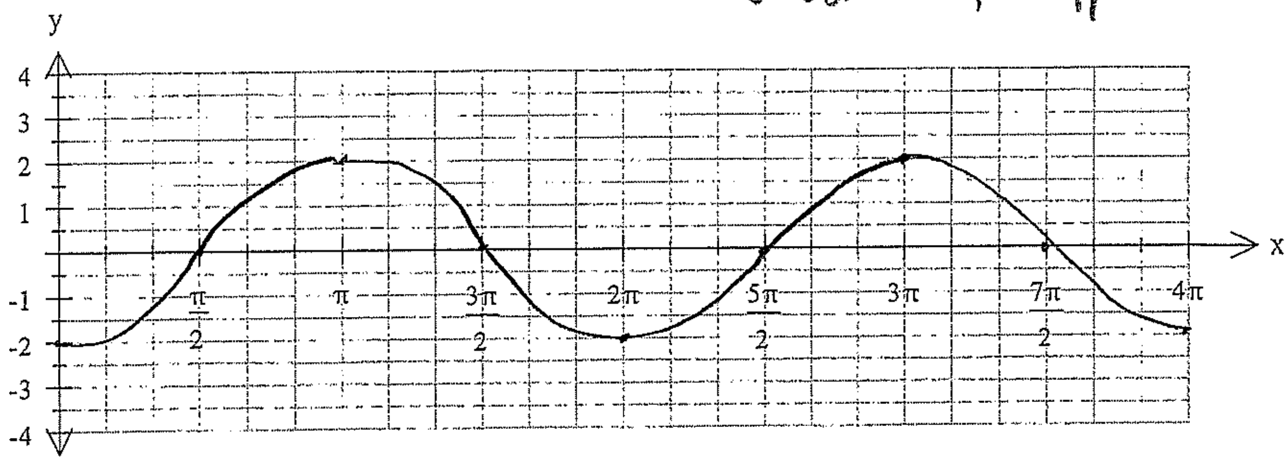
3. Plot the curve y = 3 sin x and y = -2 cos x on the axes below.

State the period and amplitude of each.

Period = 2π, Amplitude = 3.



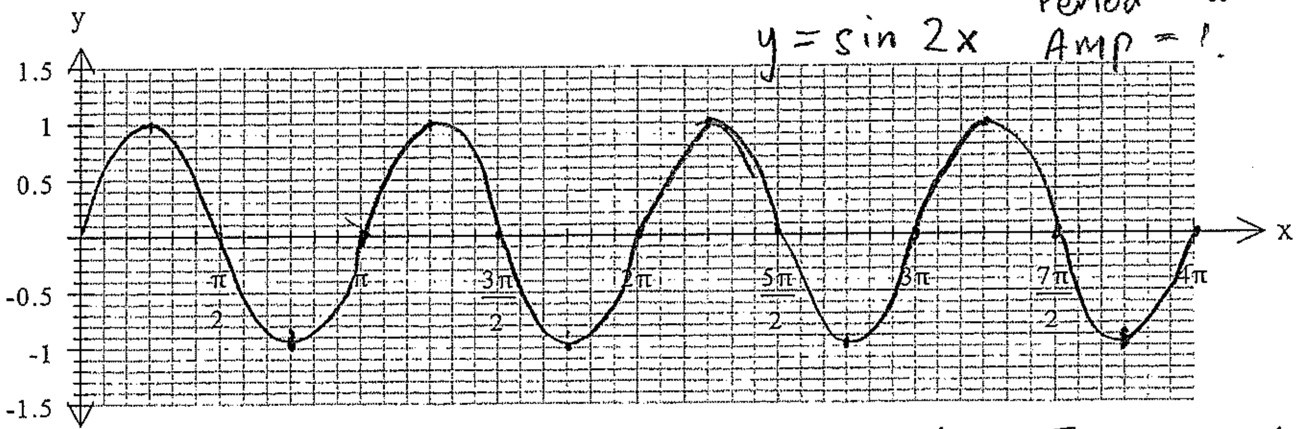
Period = 2π, Ampl = 2



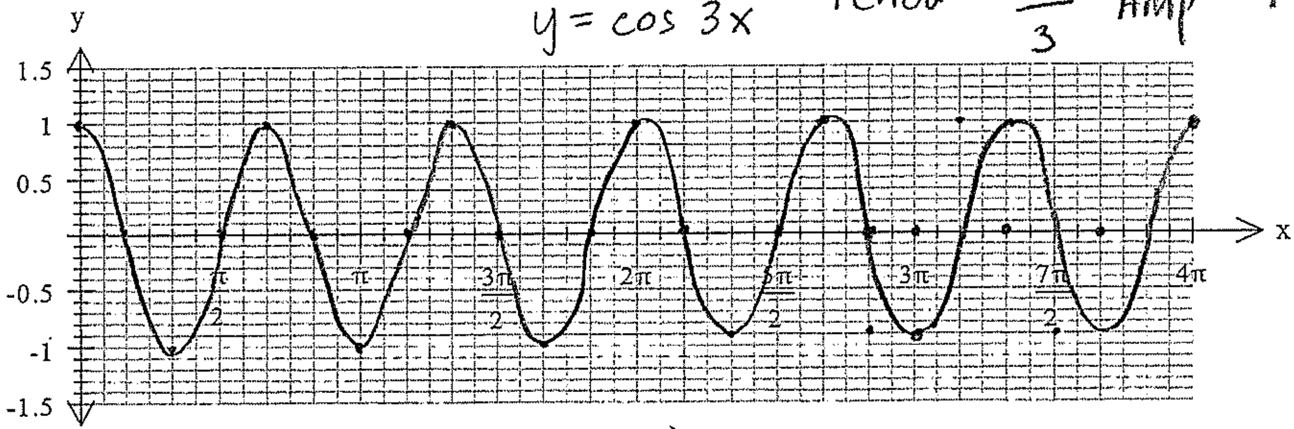
Discuss the transformations that have occurred.

4. Plot the functions y = sin 2x, y = cos 3x and y = sin $\left(\frac{x}{2}\right)$. State the amplitude and periods of each.

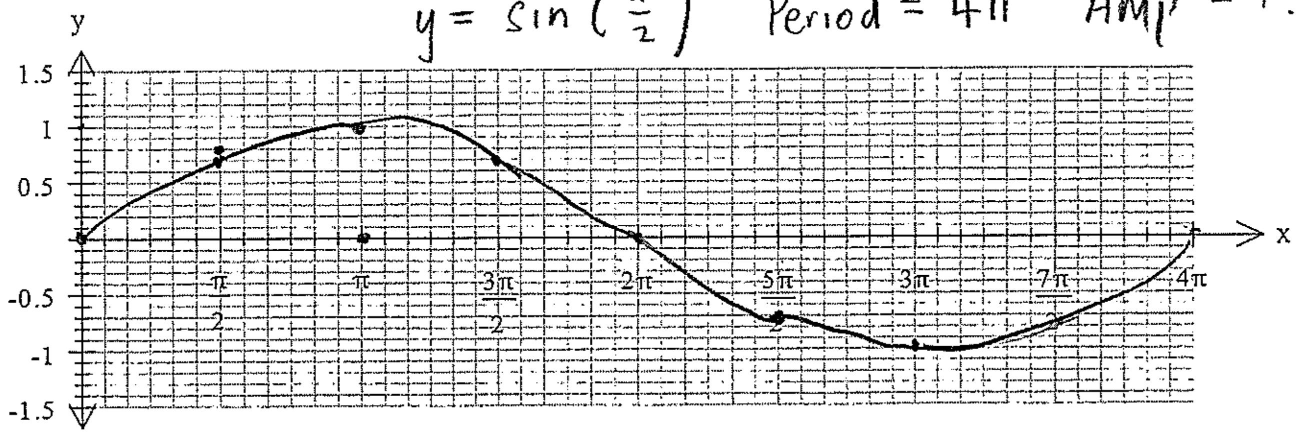
$y = \sin 2x$ Period $= \pi$ Amp $= 1$.



$y = \cos 3x$ Period $= \frac{2\pi}{3}$ Amp $= 1$.



$y = \sin\left(\frac{x}{2}\right)$ Period $= 4\pi$ Amp $= 1$.



5. State the effect of b in the rules y = sin bx and y = cos bx.

Change of Period.

6. State the period of the functions y = sin 4x, y = cos 0.2x and y = tan 2x.

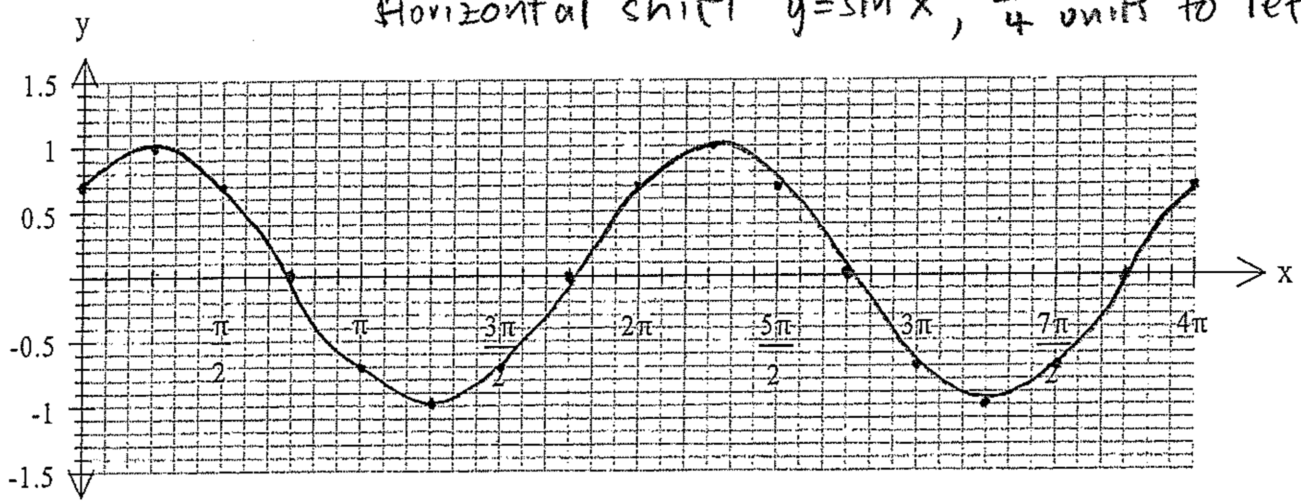
$y = \sin 4x$	$y = \cos 0.2x$	$y = \tan 2x$
$P = \frac{2\pi}{4} = \frac{\pi}{2}$	$P = \frac{2\pi}{0.2} = 10\pi$	$P = \frac{\pi}{2}$

7. State the rule for the period of y = sin bx and y = cos bx in terms of b: Period = $\frac{2\pi}{b}$.

8. State the rule for the period of the graph of y = tan bx in terms of b. Period = $\frac{\pi}{b}$

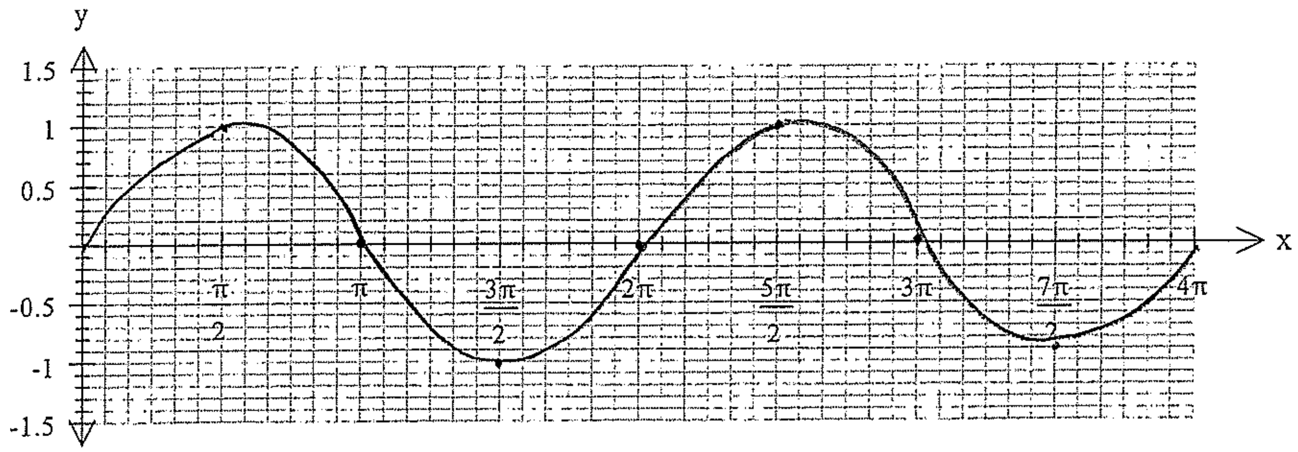
9. What is the effect on the sin x graph if it is transformed into $y = \sin(x + \frac{\pi}{4})$. Draw it below.

Horizontal shift $y = \sin x$, $\frac{\pi}{4}$ units to left.



10. What is the effect on the cos x graph if it transformed into the rule $y = \cos(x - \frac{\pi}{2})$? Shift $\frac{\pi}{2}$ units to the right

11. Draw the graph of $y = \cos(x - \frac{\pi}{2})$ below.



12. What is the $y = \cos(x - \frac{\pi}{2})$ graph the same as? $y = \sin x$

Explain why. The sinx graph is simply a cos x graph translated $\frac{\pi}{2}$ to the right.

13. What is the effect on a sine graph by transforming it into $y = \sin x + 3$. Graph it on your calculator and comment.
(Hint; What will the new mean position be?) $y = \sin x$ graph, translated 3 units up.

14. Describe what the graph of $y = 2\cos x - 11$ will look like. (Hint; What will the new mean position be?)
Vertical shift 11 units down, with an amplitude of 2.

15. Putting it all together: Explain the transformations that have occurred to $y = \cos x$ to alter it to
$y = -4\cos(2x - \pi) + 3$ ← up 3

effected bout x-axis

Amp=4 Period is now π

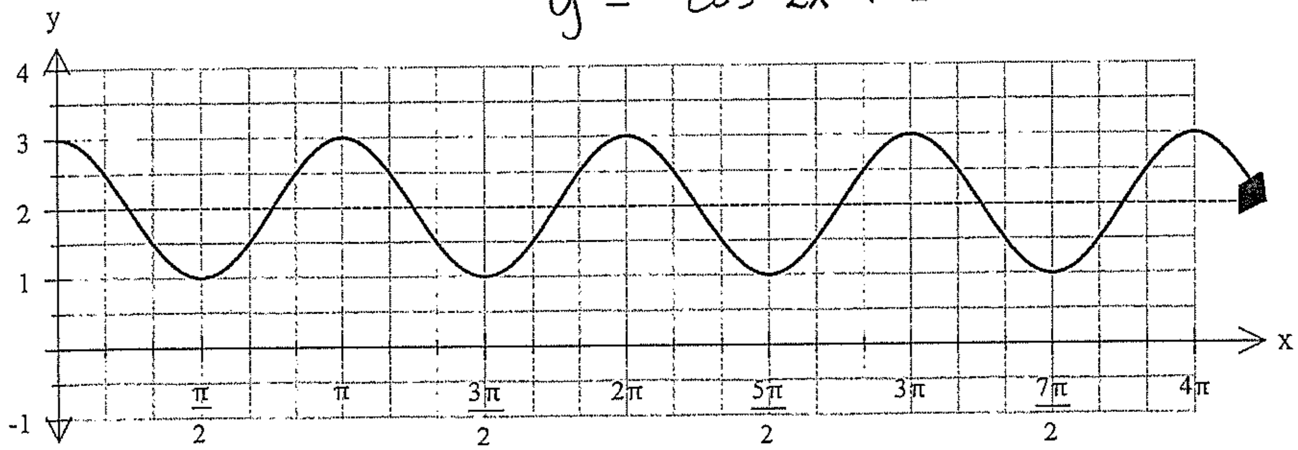
Translated π to the right

16. Using the ideas of transformations complete the table for the following trigonometry rules. Use your calculator if you are not sure.

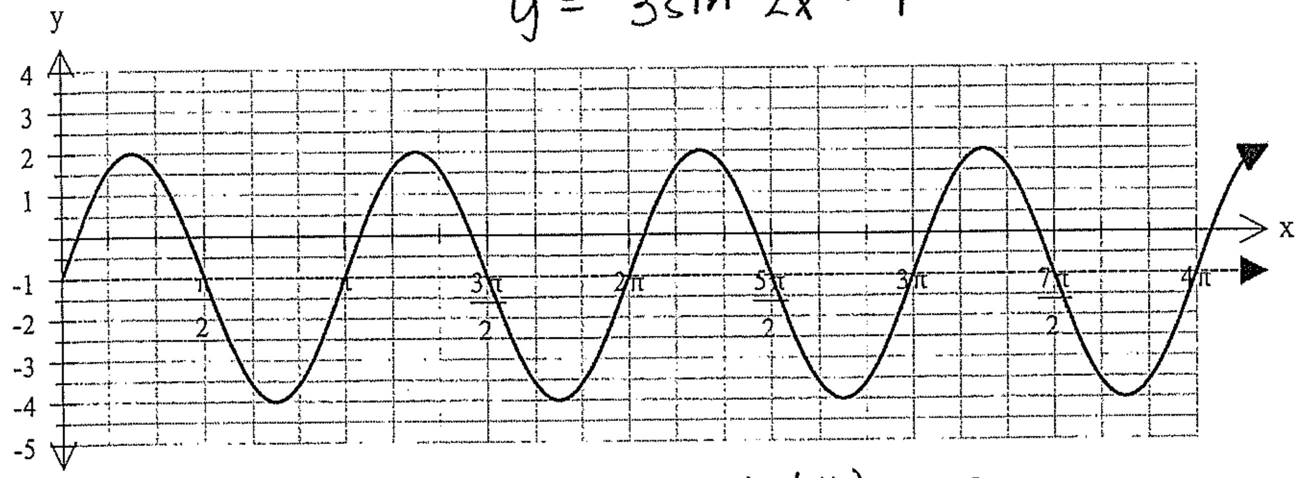
Rule	Period	Amplitude	Mean position	Maximum value	Minimum value	Horizontal shift
$y = 3 \sin 2x + 5$	π	2	$y = 5$	8	2	none
$y = -3 \cos 3x - 4$	$2\pi/3$	3	$y = -4$	1	-7	none
$y = 0.6 \sin (x - 2) + 1.2$	2π	0.6	$y = 1.2$	1.8	0.6	2 right
$y = -2 \sin (\pi x) - 1$	$2\pi/\pi = 2$	2	$y = -1$	1	-3	none
$y = \tan(0.5x + \frac{\pi}{2})$	$\frac{\pi}{0.5} = 2\pi$	Infinite	$y = 0$	∞	$-\infty$	$\frac{\pi}{2}$ to left

17. Find the rules for these graphs.

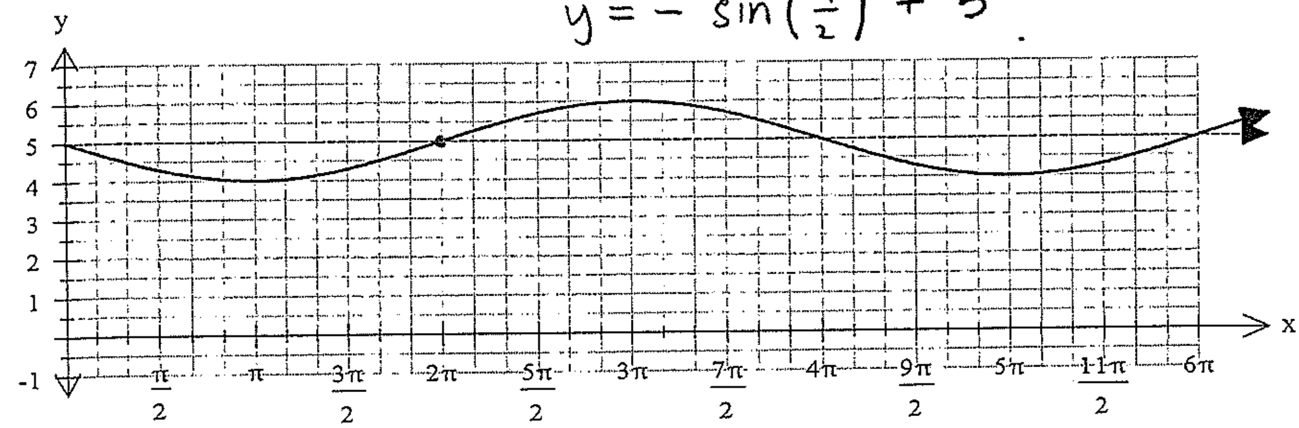
$y = \cos 2x + 2.$



$y = 3\sin 2x - 1$



$y = -\sin\left(\frac{x}{2}\right) + 5$



The following questions will show you some examples of how trigonometrical rules are used.

18. Use your calculator and new found knowledge to answer this question.

Tides in a particular harbour are such that the depth of water (in metres) in the harbour is given by:

$$D = 3\sin(\frac{\pi t}{12}) + 9$$

where t is the number of hours after midnight each day.

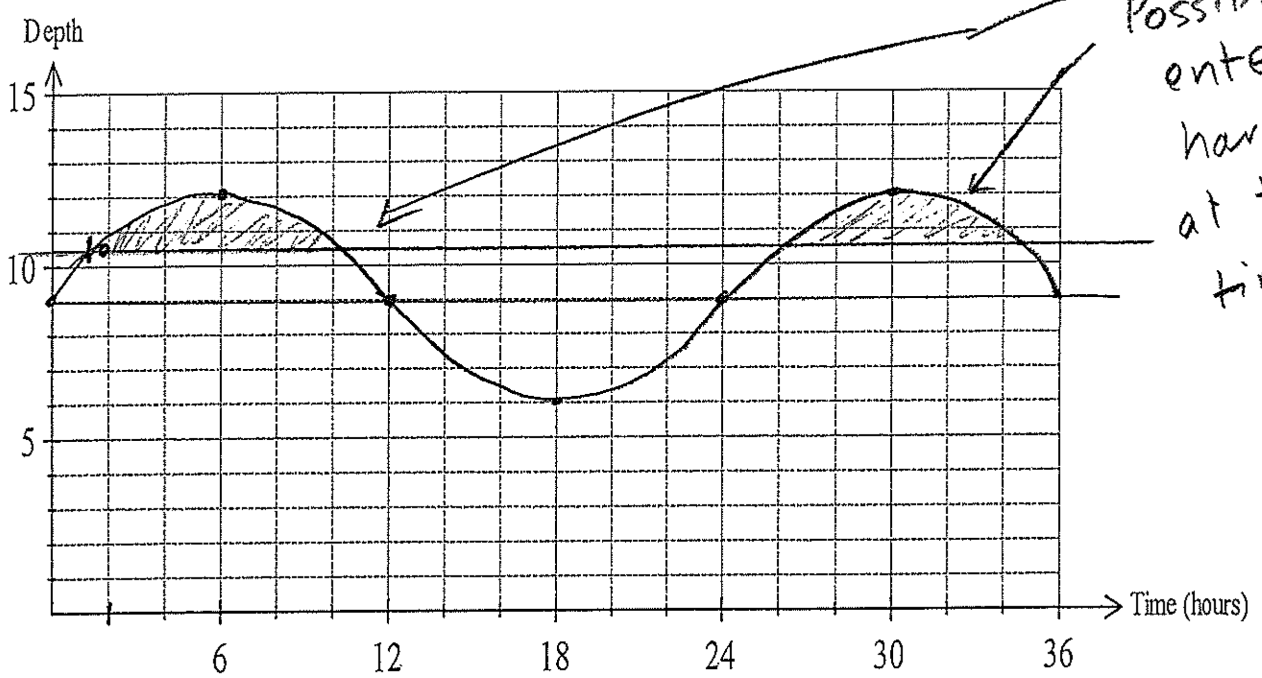
(a) At what time does low tide occur each day? midnight + 18 = 6 pm.

(b) What is the depth of water in the harbour at high tide? 12 metres.

(c) What is the depth of the water in the harbour at 3.00 pm? $t = 15 \Rightarrow D = 6.88m$.

(d) What is the maximum and minimum depth of the tide? Max 12m Min 6m.

(e) Sketch the graph of $D = 3\sin(\frac{\pi t}{12}) + 9$ for $0 \leq t \leq 36$



Possible to enter harbour at these times.

(f) A ship carrying a load has draught (i.e. the depth to which the ship sinks into the water when it is floating) of 10.5 metres.

(i) Show on your graph the times is it possible for the ship to enter the harbour?

(ii) Determine these times, for any day, correct to the nearest minute.

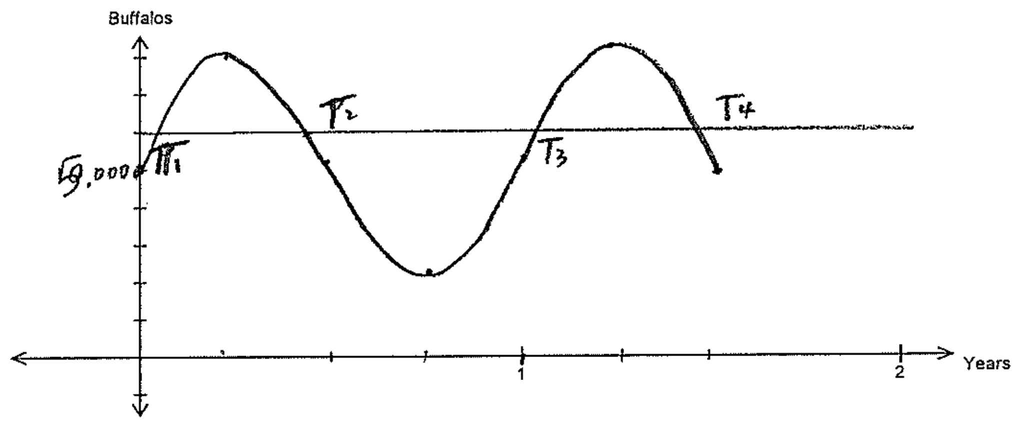
2am → 10am

Between 2am and 10am.

19. The population of a water buffalo herd in the Northern Territory is modelled by:

$P(t) = 5000 + 3000\sin(2\pi t)$ where t is the number of years since the first estimate was made.

(a) Sketch the graph showing the population for $0 \leq t \leq 2$



(b) Find the maximum herd size and when it occurs

Maximum = 8,000 after $\frac{1}{2}$ year.

(c) When is the herd size 6000?

T_1 = 0.054 years = 20 days

T_2 = 0.4459 years = 163 days.

T_3 = 1.054 years = 1 year and 20 days

T_4 = 1.4459 year = 1 year and 163 days

20. The ultraviolet radiation rate, $R(t)$, from the sun t hours after sunrise on February 14 is modelled by:

$$R(t) = 9.2 + 5.8\sin\left[\frac{\pi}{6}(t-2)\right], \quad 0 \le t \le 14$$

(a) What is the UV rate at sunrise (t = 0)?

$t = 0,\ R(0) = 4.1771$.

(b) Find the greatest and least UV rates

Max = 15, Min = 3.4.

(c) What is the period of the function?

$P = \dfrac{2\pi}{\frac{\pi}{6}} = 12$ hours.

(d) Find when the UV rate is 9.

$9 = R(t) \Rightarrow 1.93$ hours and 8.07 hours
116 minutes 484 min

(e) If sunrise is at 6:15 am, at what time does the maximum UV rate occur?

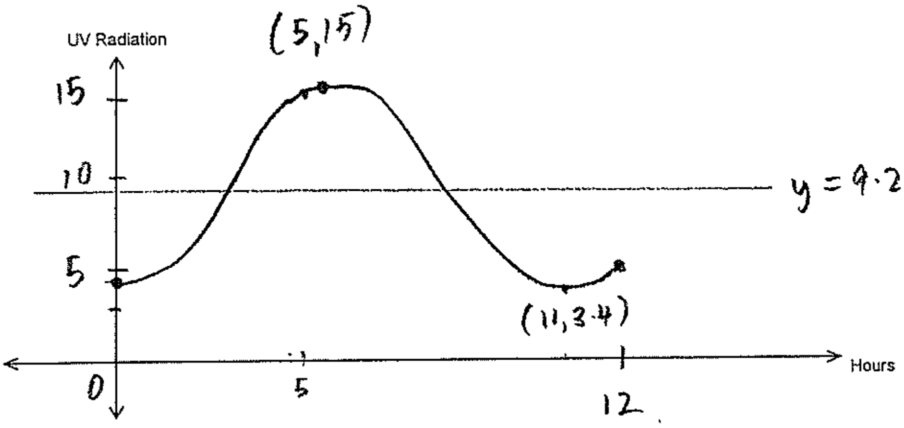
After 5 hours ie At 11.15am

(f) The most dangerous times for sunburn occur when the UV rate is 12 or above. If sunrise is at 6:15 am, during what times of the day is the UV radiation most dangerous?

Between 2.962 hours and 7.0378h
= 178min and 422m.
3h 58 min — 7h 2 min

9:13am ⟶ 1:17pm
Above 12.

(g) Sketch $R(t)$ against time for $0 \le t \le 14$



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