Name: _____

Differentiation and Applications

Extended Investigation: Roller Coaster Track Design

Mathematics Methods Unit 3

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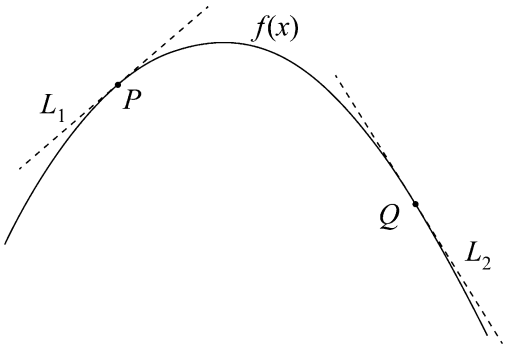
Part A (10 marks)

The first task is to design an *ascent* and *drop* for a section of roller coaster track.

You will connect two straight stretches of track, $y = L_1(x) = m_1 x + k_1$ and $y = L_2(x) = m_2 x + k_2$, with part of a parabola $y = f(x) = ax^2 + bx + c$, where x and $f(x)$ are measured in metres.

By studying photographs of roller coasters you decide to make the slope of the *ascent* 0.7 and the slope of the *drop* - 1.8.

For the track to be smooth there can't be abrupt changes in direction. To ensure smooth transitions between the linear segments and the parabola - L_1 and L_2 need to be tangents to the parabola *f(x)* at the transition points P and Q.



To simplify the equations for the functions involved, you decide to place the *origin* at *P*. Furthermore, the horizontal distance between *P* and *Q* is 16 metres.

Find the equations of the *three segments* of track indicated below for the given domains, showing all appropriate steps of logic:

$$L_1(x) = m_1 x + k_1 \quad \text{for } x \leq 0$$

$$f(x) = ax^2 + bx + c \quad \text{for } 0 \leq x \leq 16$$

$$L_2(x) = m_2 x + k_2 \quad \text{for } x \geq 16$$

Part B (38 marks)

After conducting some research on roller coaster specifications, your task is to design a roller coaster and specify the functions for your roller coaster track from the ***starting point*** (entrance on the left) to its ***finish point*** (exit on the right). The three segments of track from Part A must be included in your track.

You must determine the equations for each function used (minimum of 4) for the section on the left to join L_1 and the equations for each function used (minimum of 3) from L_2 to the exit point, ensuring a smooth transition between intersection points between each function or section of roller coaster track. You must show detailed working, equations, constraints, reasons for the choices etc.

You must carefully consider:

-length and height constraints
-using a range of functions in your design
-consider different techniques for generating the functions

- Use graphing software, draw a detailed, fully labelled, smooth graph showing what each section looks like. You must observe the constraints of each section and need to also produce a final graph showing all sections of the roller coaster track together.

- Include a discussion of any assumptions, limitations and improvements in the design of your roller coaster.

The report should include the following:

- **an outline of the problem and context**
- **the method required to find a solution, in terms of the mathematical model or strategy used**
- **the application of the mathematical model or strategy, including**
  - relevant data and/or information
  - mathematical calculations and results, using appropriate representations
  - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
- **the results and conclusions in the context of the problem.**

A bibliography and appendices, as appropriate, may be used.

The investigation report, excluding bibliography and appendices if used, must be a maximum of 15 A4 pages if written, or the equivalent. The maximum page limit is for single-sided A4 pages with minimum font size 10. Page reduction, such as 2 A4 pages reduced to fit on 1 A4 page, is not acceptable. Conclusions, interpretations and/or arguments that are required for the assessment must be presented in the report, and not in an appendix. Appendices are used only to support the report, and do not form part of the assessment decision.

Mark Allocations for Task

Part A

$L_1(x) = m_1 x + k_1$ for $x \leq 0$

$f(x) = ax^2 + bx + c$ for $0 \leq x \leq 16$

$L_2(x) = m_2 x + k_2$ for $x \geq 16$

1 mark for each variable and 1 mark for each domain **(Max 10 marks)**

Part B

Minimum of 4 functions for the section on the left to join L_1

1 mark for each equation and 1 mark for the domain **(Max 16 marks)**
2 marks for consideration of gradients at junctions.

Minimum of 3 equations from L_2 to the exit point

1 mark for each equation and 1 mark for the domain **(Max 12 marks)**
2 marks for consideration of gradients at junctions.

The report:

- **an outline of the problem and context** **(Max 2 marks)**
- **description of the method required to find a solution, in terms of the mathematical model or strategy used** **(Max 2 marks)**
- **the application of the mathematical model or strategy, including**
    - relevant data and/or information **(Max 4 marks)**
    - mathematical calculations and results, using appropriate representations (graphs etc) and notation
- **the results and conclusions in the context of the problem.** **(Max 2 marks)**
    - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results