Methods 11 Test 5 2018
Trigonometry
Total Marks: 60   Time Allowed: 60 minutes

Name: _Marking Key_

SECTION A – Resource Free   40 minutes – 37 marks
ALL working must be shown for full marks.

1.   **[3, 5 = 8 marks]** $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Solve the equations:

a) $\sin(2x) = \dfrac{\sqrt{2}}{2}$ for $0 \le x \le 2\pi$ $\therefore \ 0 \le 2x \le 4\pi$

$$2x = \frac{\pi}{4}, \ \frac{3\pi}{4}, \ \frac{9\pi}{4}, \ \frac{11\pi}{4} \checkmark$$

$$\therefore x = \frac{\pi}{8}, \ \frac{3\pi}{8}, \ \frac{9\pi}{8}, \ \frac{11\pi}{8} \checkmark$$

b) $\sqrt{3}\sin(3x) = -\cos(3x)$ *for* $0 \le x \le 2\pi$ $0 \le 3x \le 6\pi$

$$\frac{\sin(3x)}{\cos(3x)} = -\frac{1}{\sqrt{3}} \checkmark$$

$$\therefore \tan 3x = \frac{-1}{\sqrt{3}}$$

$$\therefore 3x = \frac{5\pi}{6}, \ \frac{11\pi}{6}, \ \frac{17\pi}{6}, \ \frac{23\pi}{6}, \ \frac{29\pi}{6}, \ \frac{35\pi}{6}$$

$$\therefore x = \frac{5\pi}{18}, \ \frac{11\pi}{18}, \ \frac{17\pi}{18}, \ \frac{23\pi}{18}, \ \frac{29\pi}{18}, \ \frac{35\pi}{18} \checkmark$$

2.   **[6 marks]**

Complete the following table.

Function	Period	Amplitude (where applicable)
$y = 5 \sin(3x°)$	$\frac{2\pi}{3}$ ✓	5 ✓
$y = -6 \cos(\frac{x}{4} + 30°)$	8π ✓	6 $\binom{\text{not}}{-6}$ ✓
$v = 7 \tan(2t + \pi)$	$\frac{\pi}{2}$ ✓	∞ or N/A ✓

3. [1, 2, 3 = 6 marks]

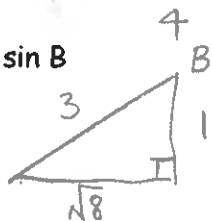Given that $\sin A = \frac{4}{5}$ and $\cos B = \frac{1}{3}$, where A and B are acute, find the exact value of:

a) cos A

$\therefore \cos A = \frac{3}{5}$ ✓

(triangle with hypotenuse 5, sides 3 and 4, vertex A)

b) sin B

so $\sin B = \frac{\sqrt{8}}{3}$ ✓✓

(triangle with hypotenuse 3, side 1, base $\sqrt{8}$, vertex B)
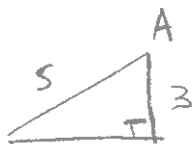
c) cos (A - B) $= \cos A \cos B + \sin A \sin B$

$= \frac{3}{5} \times \frac{1}{3} + \frac{4}{5} \times \frac{\sqrt{8}}{3}$ ✓

$= \frac{1}{5} + \frac{4\sqrt{18}}{15}$ ✓   or $\frac{3 + 4\sqrt{18}}{15}$ or $\frac{3 + 12\sqrt{2}}{15}$

or $\frac{1 + 4\sqrt{2}}{5}$

4. [5, 5 = 10 marks]

Use an appropriate trigonometry identity to find the exact value of:
(Simplify to a whole number denominator)

a)   tan 105°

$= \tan(60 + 45)$ ✓

$= \dfrac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$ ✓

$= \dfrac{\sqrt{3} + 1}{1 - \sqrt{3} \times 1}$ ✓✓

$= \dfrac{\sqrt{3} + 1}{1 - \sqrt{3}} \times \dfrac{1 + \sqrt{3}}{1 + \sqrt{3}}$

$= \dfrac{4 + 2\sqrt{3}}{1 - 3}$

$= -2 - \sqrt{3}$ ✓

b)   cos 75°

$= \cos(45° + 30°)$ ✓

$= \cos 45° \cos 30° - \sin 45° \sin 30°$ ✓

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$ ✓✓

$= \dfrac{\sqrt{3}}{2\sqrt{2}} - \dfrac{1}{2\sqrt{2}}$
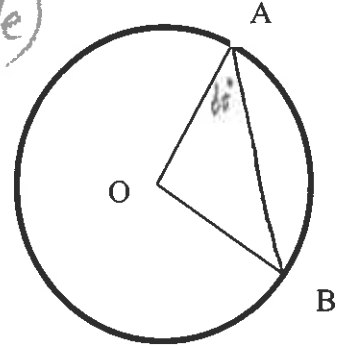
$= \dfrac{\sqrt{2}(\sqrt{3} - 1)}{4}$ ✓

$= \dfrac{\sqrt{6} - \sqrt{2}}{4}$

5. **[3, 2, 2 = 7 marks]**

In the circle of radius 5 cm with centre O drawn below, $\angle OAB = 60°$

a) Find the exact size of triangle OAB. Explain how you arrived at your answer.

$OA = OB = 5cm$ (radius of circle)

Since $\triangle OAB$ is isosceles $\angle OBA = 60°$

and $\angle AOB = 60°$

$\therefore$ equilateral ✓

so $AB = 5cm$ ✓

b) Find the exact area of the minor segment formed by the cord AB.

$Area = \frac{1}{2} \times 5^2 \left(\frac{\pi}{3} - \sin \frac{\pi}{3} \right)$ ✓

$= \frac{25}{2} \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) cm^2$ ✓

$= \frac{25}{2} \times \frac{2\pi - 3\sqrt{3}}{6}$

$= \frac{50\pi}{12} - \frac{75\sqrt{3}}{12}$ or $\frac{25\pi}{6} - \frac{25\sqrt{3}}{4}$

c) Find the exact perimeter of the minor segment formed by the cord AB.

$Arc\ AB = 5 \times \frac{\pi}{3}$

$= \frac{5\pi}{3} cm$ ✓

$Perimeter = 5 + \frac{5\pi}{3} cm$ ✓

Methods 11 Test 5 2018 (trigonometry)

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6.   [1, 1, 1, 1, 5 = 9 marks]

The body temperature θ (Celsius) of a reptile in summer at time t hours after midnight is given by

$\Theta = 15 - 5 \sin\left(\frac{\pi t}{12}\right)$

a) State the period for θ

$$\frac{2\pi}{\pi/12} = 24 \checkmark$$

b) What is the range of body temperature experienced by the reptile?

$$\text{Range} = 20 - 10 = 10°C \checkmark$$

c) Find the maximum body temperature of the reptile and state when this first occurs after midnight.

$20°C$ at 6 pm $\checkmark$

d) Find the minimum body temperature of the reptile and state when this first occurs after midnight.

$10°C$ at 6 am $\checkmark$

e) Use an algebraic method to find the first time (to the nearest minute) when the temperature of the reptile is 16° Celsius.

$\text{Solve } 16 = 15 - 5 \sin\left(\frac{\pi t}{12}\right) \checkmark$

$\Rightarrow -1 = 5 \sin\left(\frac{\pi t}{12}\right) \checkmark$

$\Rightarrow \sin\left(\frac{\pi t}{12}\right) = -\frac{1}{5} \checkmark$

$\Rightarrow t = 12.679 \checkmark$

So time is 12.46 pm to nearest minute $\checkmark$

7.   **[5, 5 = 10 marks]**

Solve for θ within the domain:

   a) $\cos(\theta + 30°) = \sin\theta$ $for\ 0 \le \theta \le 360°$

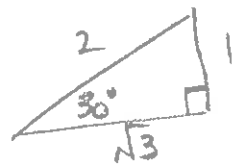$\Rightarrow$ $\cos\theta\cos 30° - \sin\theta\sin 30° = \sin\theta$ ✓

$\Rightarrow$ $\dfrac{\sqrt{3}\cos\theta}{2} - \dfrac{\sin\theta}{2} = \sin\theta$ ✓

$\Rightarrow$ $\dfrac{\sqrt{3}\cos\theta}{2} = \dfrac{3\sin\theta}{2}$ ✓

$\Rightarrow$ $\sqrt{3}\cos\theta = 3\sin\theta$

$\Rightarrow$ $1 = \dfrac{3\sin\theta}{\sqrt{3}\cos\theta}$

$\Rightarrow$ $\tan\theta = \dfrac{1}{\sqrt{3}}$ ✓ $\therefore \tan\theta = 30°, 210°$ ✓

   b) $\sin\left(\theta + \dfrac{\pi}{4}\right) = \sqrt{2}\cos\theta$ $for\ 0 \le \theta \le 2\pi$

$\Rightarrow$ $\sin\theta\cos\dfrac{\pi}{4} + \cos\theta\sin\dfrac{\pi}{4} = \sqrt{2}\cos\theta$ ✓

$\Rightarrow$ $\dfrac{\sin\theta}{\sqrt{2}} + \dfrac{\cos\theta}{\sqrt{2}} = \sqrt{2}\cos\theta$ ✓

$\Rightarrow$ $\sin\theta + \cos\theta = 2\cos\theta$ ✓

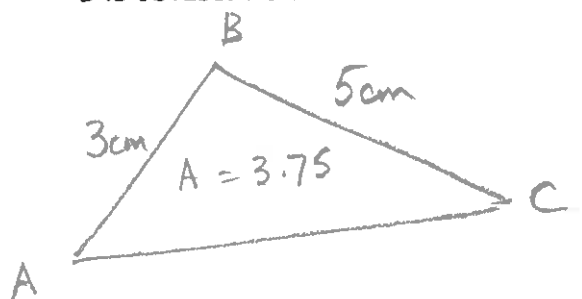$\Rightarrow$ $\sin\theta = \cos\theta$ ✓

$\Rightarrow$ $\tan\theta = 1$ ✓

$\Rightarrow$ $\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$ ✓

8. **[4 marks]**

If a triangle ABC has an area of 3.75 cm² and a=5cm and c=3cm, find the length of b if angle B is obtuse. YOU MUST SHOW ALL WORKINGS TO RECEIVE FULL MARKS!!!

$$3.75 = \frac{1}{2} \times 3 \times 5 \times \sin B \checkmark$$

$$\Rightarrow \quad \sin B = \frac{3.75}{7.5} = \frac{1}{2}$$

$$\Rightarrow \quad B = 30° \text{ or } 150°$$

obtuse, so 150° $\checkmark$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$= 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos 150° \checkmark$$

$$b = 7.744724793 \text{ cm} \checkmark$$