# Yr 12 METHODS TEST 1 2018

## DIFFERENTIATION, APPLICATIONS AND EXPONENTIALS

**Time: 30 minutes** **Total: 28 marks**

**Student Name:** Solutions **Teacher:** ____________

**Instructions:** ***Show all working clearly.***
***Sufficient detail must be shown for marks to be awarded for reasoning.***
**NO CALCULATOR AND NO PERSONAL NOTES ALLOWED**

Question 1. (9 marks)

a) Determine the tangent of the graph of $y = 2(3x^2 + 2)^3$ at the point (1,250) [4]

$y' = 6(3x^2+2)^2(6x)$

@1 $y' = 6(3(1)^2+2)^2(6)$

$= 900$

$\therefore y = 900x - 650$

b) Determine the coordinates of any stationary points on the function $y = \frac{x+7}{x-2} + x$ [5]

$y' = \frac{(x-2)(1) - (x+7)(1)}{(x-2)^2} + 1$

$0 = \frac{-9}{(x-2)^2} + 1$

$-(x-2)^2 = -9$

$(x-2)^2 = 9$

$x - 2 = \pm 3$

$x = 5$ or $-1$

$\therefore (5, 9)$

$\therefore (-1, -3)$

Question 2. (6 marks)

Given that $\log_9 5 = a$ and $\log_9 6 = b$, write the following in terms of $a$ and $b$.

a) $\log_9 25$ [1]

$2a.$

b) $\log_9 180$ [2]

$2b + a.$

c) $\log_9 18$ [3]

$\log_9(6 \times 3) = \log_9 6 + \log_9 3$

$= b + \frac{1}{2}.$

Question 3. (4 marks)

A sphere is has an initial volume of $\frac{32\pi}{3}$ cm$^3$.

Use the incremental formula to determine the change in radius if the volume of the sphere is increased by 3cm$^3$.

$$\frac{32\pi}{3} = \frac{4}{3}\pi r^3$$

$$8\pi = \pi r^3$$

$$8 = r^3$$

$$r = 2$$

$$\frac{\Delta V}{\Delta r} = 4\pi r^2$$

$$\Delta V = 4\pi r^2 \times \Delta r$$

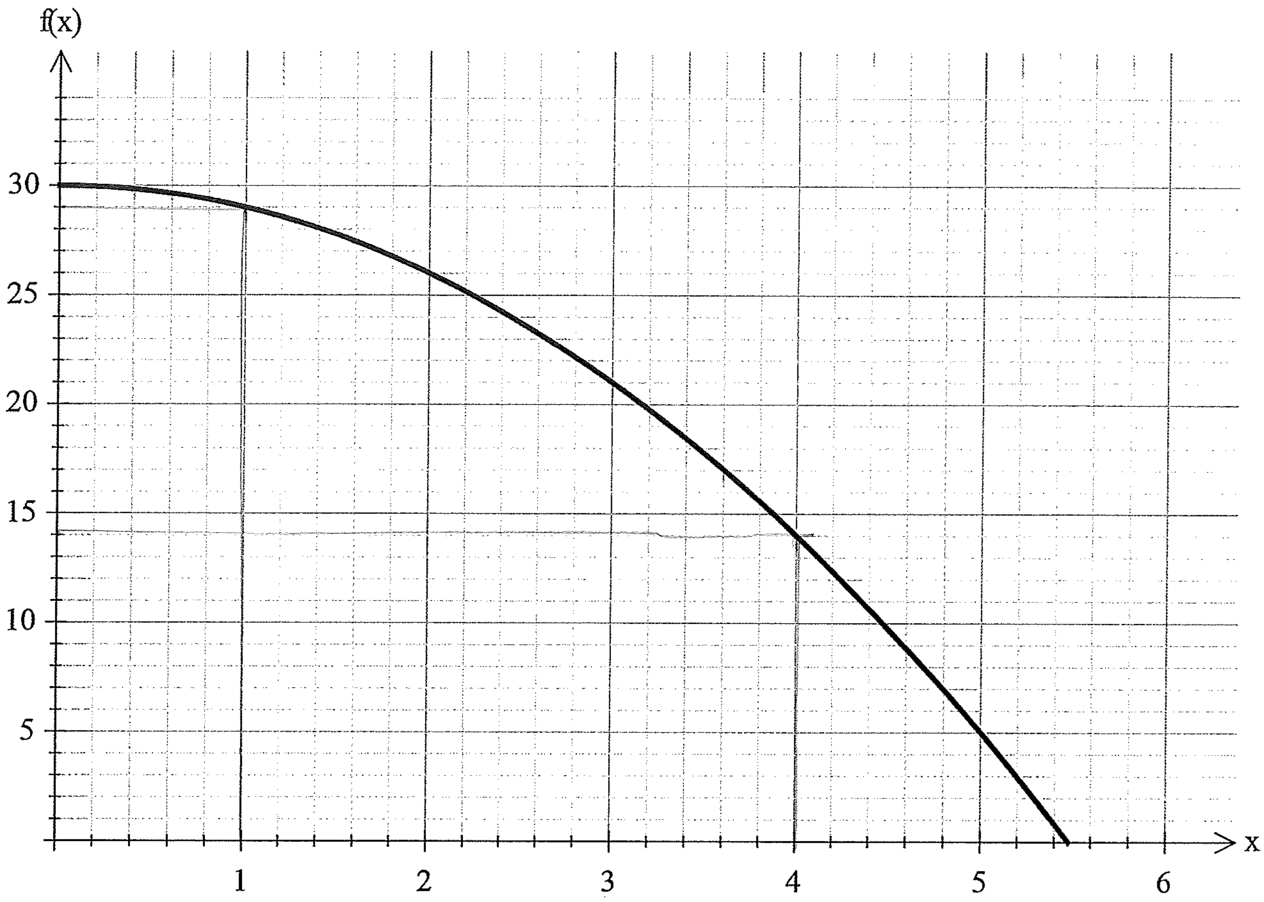
$$3 = 4\pi(2)^2 \times \Delta r$$

$$\Delta r = \frac{3}{16}$$

$$\frac{3}{16\pi}$$

Question 4. (8 marks)

Consider the graph below of $f(x) = -x^2 + 30 \quad 0 \le x \le \sqrt{30}$




Rectangles can be created by drawing a vertical line up from any $x$ value until that line hits the curve and then horizontally until it hits the $y$ axis.

a) Draw in two such rectangles. One using an $x$ value of 1 and the other using an $x$ value of 4. [1]

b) Calculate the area of each of these two rectangles. [2]

$1 \times 29 = 29\,u^2$

$4 \times 14 = 56\,u^2$

Question 4 (continued)

c) Use calculus to show determine the exact $x$ value would give the rectangle with the greatest area. [4]

$$A = x(-x^2 + 30)$$

$$= -x^3 + 30x$$

$$A' = -3x^2 + 30$$

$$3x^2 = 30$$

$$x^2 = \pm\sqrt{10}$$

$$\therefore \sqrt{10}$$

d) State the exact maximum area of this rectangle. [1]

$$A = -(\sqrt{10})^3 + 30\sqrt{10}$$

$$= -10\sqrt{10} + 30\sqrt{10}$$

$$= 20\sqrt{10} \text{ units}^2$$

# Yr 12 METHODS TEST 1 2018

## DIFFERENTIATION, APPLICATIONS AND EXPONENTIALS

**Time: 25 minutes** **Total: 25 marks**

**Student Name:** Solutions **Teacher:** ____________

**Instructions:** *Show all working clearly.*
*Sufficient detail must be shown for marks to be awarded for reasoning.*
**CALCULATOR AND 1 PAGE OF PERSONAL NOTES ALLOWED**

Question 4. (9 marks)

A small body is moving in a straight line with velocity $v(t) = 2t^2 - 19t + 30$ m/s, where $t$ is the time in seconds, since the body first passed through the origin.

a) Determine an expression for $x(t)$, the displacement of the body at time $t$. [2]

$$x(t) = \frac{2t^3}{3} - \frac{19t^2}{2} + 30t.$$

b) Show that the body is stationary twice and find the change in displacement of the body between these two moments. [4]

$$0 = 2t^2 - 19t + 30$$

$$0 = (2t-15)(t-2)$$

$$t = 7.5 \text{ or } 2.$$

$$x(4) = 82/3$$

$$x(7.5) = -\frac{225}{8}$$

$$\therefore \text{ change } \frac{1331}{24} \text{ m.}$$

55.46m.

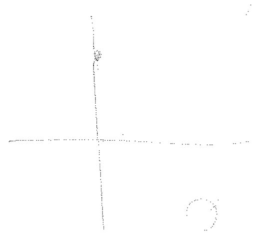
c) Determine the position of the body when it's velocity is a minimum. [3]

$$v'(t) = 4t - 19.$$

$$0 = 4t - 19$$

$$t = 19/4$$

$$x\left(\frac{19}{4}\right) = -\frac{19}{48} \text{ m}$$



Question 5. (8 marks)

A cylindrical oil drum, or radius $r$ m and height $h$ m, has circular ends constructed from material costing \$75 per square metre and sides constructed from material costing \$40 per square metre.

a) Determine an expression for the cost of construction $C$, in dollars. [1]

$C = 150\pi r^2 + 80\pi r h.$

b) If the oil drum must be constructed for \$250, show that the volume of the oil drum is given by, $V = \frac{25r - 15\pi r^3}{8}$ [3]

$V = \pi r^2 h.$

$250 = 150\pi r^2 + 80\pi r h.$

$\frac{250 - 150\pi r^2}{80\pi r} = h.$

$V = \pi r^{\cancel{2}} \left( \frac{250 - 150\pi r^2}{80\cancel{\pi}\cancel{r}} \right)$

$= \frac{25r - 1.5\pi r^3}{8}$

c) Use calculus methods to determine the dimensions that maximise the volume of the oil drum, and state this maximum volume. [4]

$V' = \frac{25}{8} - \frac{45}{8}\pi r^2$

$0 = \frac{25}{8} - \frac{45}{8}\pi r^2$

$r = 0.4205.$

$h = 1.577$

$V = 0.876\, m^3.$

Question 6. (8 marks)

A polynomial function $f(x) = ax^4 + bx^2 + c$, where $a$ , $b$ and $c$ are real constants, has the following features:

- $f(x) = 0$ **only** for $x = -2$ and $x = 2$
- $f'(x) = 0$ **only** for $x = -1, x = 0$ and $x = 1$
- $f'(x) > 0$ **only** for $-1 < x < 0$ and $x > 1$
- $f''(0) < 0$

a) At the point where the curve intersects the $y$ axis, is the graph concave up or concave down? Explain your answer. [2]

DOWN. As $f'(0) = 0$ $\therefore$ MAX
$f''(0) < 0$

b) Is $c$ positive or negative? Explain your answer. [2]

negative.
gradient negative at $x = -2$. ($x$ int $\therefore$ below $x$ axis)
doesn't get to $x$ axis again until $x = 2$

c) Sketch a possible graph of the function on the axes below. [4]