11AEMAM Test 3 2021

TIME ALLOCATION FOR THIS TEST

Section 1 – <u>No</u> Calculators Allowed

minutes reading time: 2 minutes
minutes working time: 20 minutes

Section 2 – Calculators allowed

minutes reading time: 3 minutes
minutes working time: 25

Section 1:	/
Section 2:	/
Total:	/
	%

Material required/recommended for this test

To be provided by the supervisor
Question/answer booklets for Sections One and Two.
SCSA 11AEMAM Formulae Sheet

To be provided by the candidate

Section One:

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special materials: *drawing instruments, templates, notes on a maximum of one unfolded sheet of A4 paper*

Section Two:

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special materials: *drawing instruments, templates, notes on a maximum of one unfolded sheet of A4 paper, notes on a maximum of one unfolded sheet of A4 paper, up to three approved calculators, CAS, graphics, or scientific.*

Important note to candidates

No other items may be taken into the test room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the test room. If you have any unauthorised material with you, hand it to the teacher **before** reading any further.

Section	Reading Time	Working time	Marks	Score
Resource free	**2 minutes**	**20**	**20**	%
Resource rich	**3 minutes**	**25**	**23**	%
Total	**5 minutes**	**45**	**43**	%

Calculator Free Section: 25 minutes

1. **[2 marks]**

Consider the function shown. Decide whether it is linear, quadratic or neither:

x	0	1	2	3	4	5
f(x)	-3	1	7	15	25	37

+4   +6   +8   +10   +12

+2   +2   +2   +2   ← second difference pattern
is 2, therefore f(x) is quadratic

2. **[4 marks: 2, 1, 1]**

For the function f(x) = (2 − x)(x + 4)

   i)   Solve when f(x) = 0

$$x = 2 \text{ or } -4$$

   ii)   What is the y-intercept of the function?

$$y \text{ intercept} = (0, 8)$$

   iii)   What is the axis of symmetry of the function?

$$\text{Axis of symmetry } x = -1$$

3. **[3 marks : 1, 1, 1]**

A quadratic function is given by $f(x) = (x + 1)^2 - 4$

For this function determine

a) The coordinates of the y-intercept

$$f(0) = (0+1)^2 - 4 \qquad y \text{ intercept } (0, -3)$$
$$= -3$$

b) The equation of the line of symmetry

$$x = -1$$

c) The coordinates of the turning point

$$\text{Turning point} = (-1, -4)$$

4. **[5 marks – 2, 3]**

A quadratic has equation $y = x^2 - 8x + 3$. Determine

i. The location and nature of the turning point.

$$\text{Line of symmetry} \qquad y = 4^2 - 8(4) + 3$$
$$x = \frac{-b}{2a} \qquad\qquad = 16 - 32 + 3$$
$$= \frac{8}{2(1)} \qquad\qquad = -13$$
$$= 4 \qquad \text{Min turning point} = (4, -13)$$

ii. the **exact values** of the zeros of the quadratic.

$$x = \frac{-(-8) \pm \sqrt{64 - 4(1)(3)}}{2} \qquad = \frac{8 \pm 2\sqrt{13}}{2}$$
$$= \frac{8 \pm \sqrt{64 - 12}}{2} \qquad = 4 \pm \sqrt{13}$$
$$= \frac{8 \pm \sqrt{52}}{2} \qquad = 4 + \sqrt{13} \text{ or } 4 - \sqrt{13}$$

5. [6 marks: 1, 2, 1, 2]

For the function $y = 2x^2 - 2x - 4$, determine:

a. The equation of the line of symmetry

$$\text{Line of symmetry} \quad x = \frac{-b}{2a}$$
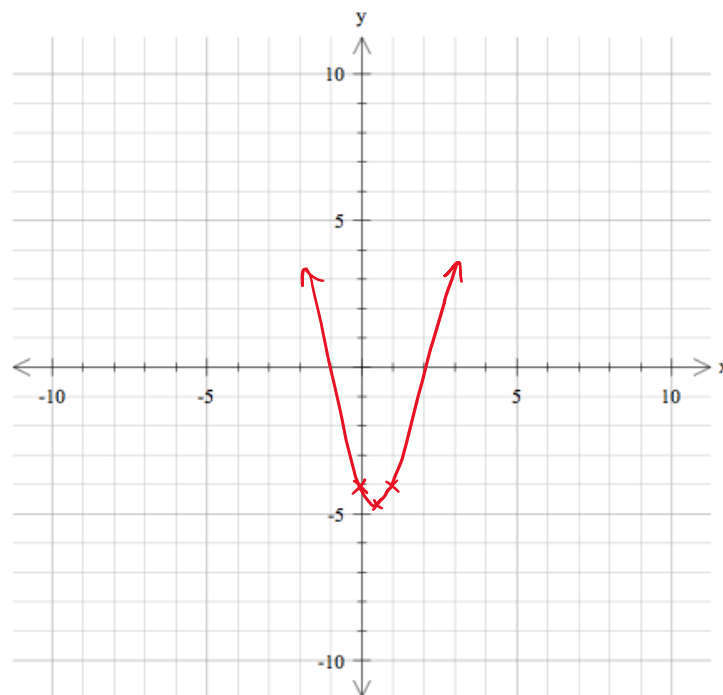$$= \frac{2}{4}$$
$$= \frac{1}{2}$$

$$x = \frac{1}{2}$$

b. The location and nature of the turning point

$$y = 2\left(\tfrac{1}{2}\right)^2 - 2\left(\tfrac{1}{2}\right) - 4$$
$$= 2\left(\tfrac{1}{4}\right) - 1 - 4$$
$$= \frac{2}{4} - \frac{4}{4} - \frac{16}{4}$$
$$= -\frac{18}{4}$$

$$\text{Min turning point} = \left(\tfrac{1}{2}, -\tfrac{9}{2}\right)$$

c. The y-intercept

$$y\text{-intercept} = (0, -4)$$

d. Sketch the function

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-4)}}{2(-2)}$$
$$= \frac{2 \pm \sqrt{4 + 32}}{4}$$
$$= \frac{2 \pm 6}{4}$$
$$= 1 \text{ or } 2$$

END OF CALCULATOR FREE SECTION

Calculator Allowed Section   Name: ………………………………
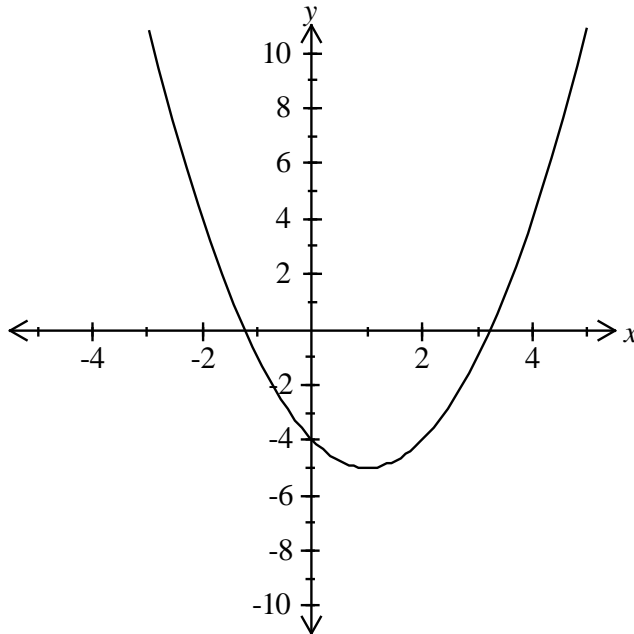
Reading time: 2 minutes

Working time: 25 minutes   Marks: 23

6.   **[5 marks: 1, 1, 1, 1, 1]**
          A student was asked to sketch the graph of $y = (x-1)^2 - 5$. The sketch drawn by the student is reproduced
          below and represents the temperature y°C of an object changing with time x (minutes).

          a)   Which values of x are valid (make sense) if x represents time?

                              $x \geq 0$

          b)   What was the initial temperature of the object?

                              $-4°C$

          c)   When did the body's temperature reach 0°?

                              3.24 min (2dp)

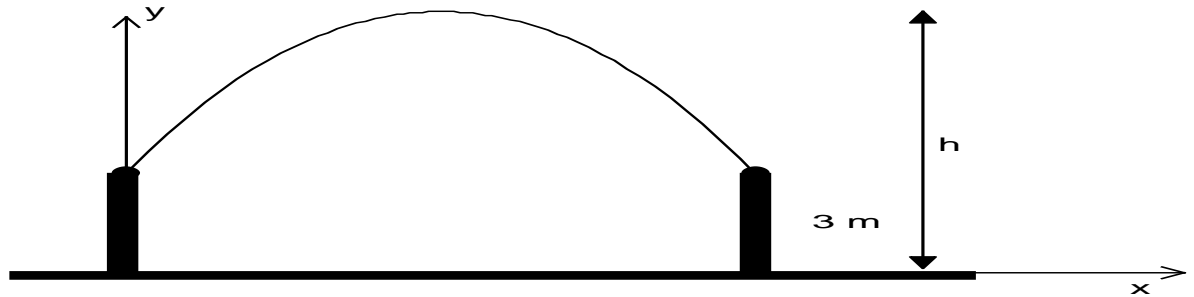   d)   What was the minimum temperature reached by the body?

                              $-5°C$

          e)   When was the minimum temperature reached?

                              1 minute

7. [10 marks: 2, 2, 3, 3]

A bridge over a freeway is in the form of a parabola, supported on both sides by a 3 metre high concrete pillar.

NOT DRAWN TO SCALE

The height above ground level for any value x measured from the base of the left hand pillar is given by

$$h = 3 + \frac{4}{15}x - \frac{8}{900}x^2$$

a) Calculate the height of the bridge 3 metres from the left hand pillar.

$x = 3$, $h = 3.72m$ *Graph function*
Table of Values

b) Calculate the width of the freeway from pillar to pillar.

When $h = 3$, $x = 0$ or 30 *Solve function*
Width of freeway = 30m

c) How far from the left hand pillar is the height of the bridge 4.5 metres?

$h = 4.5$ $x = 7.5$ or 22.5 *Solve function*
The height of the bridge at 4.5 m occurs 7.5m and 22.5m from the left hand pillar.

d) A train line is to be built in the centre of the freeway. What is the maximum height of the train, if it is to pass safely under the bridge with a two metre clearance? Assume that the train has a width of 2 metres and a flat roof.

Centre of the bridge is at 15m.
Train has width of 2m
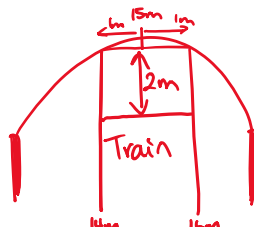
Height at 14m = 4.99m
Height at 16m = 4.99m
Train needs 2m clearance
Max Height of train = 4.99 − 2
= 2.99m

8. **[4 marks: 1, 1, 2]**

Solve the following quadratic equations by factorising or using the quadratic formula (in simplified exact form). Show your working out to receive full marks:

a. $(2x + 3)(x - 4) = 0$

$2x + 3 = 0 \qquad x - 4 = 0$
$2x = -3 \qquad x = 4$
$x = \frac{-3}{2}$

b. $x^2 - 6x - 16 = 0$

$(x - 8)(x + 2) = 0$
$x = 8 \text{ or } -2$

c. $3x^2 - 4x - 12 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{4 \pm \sqrt{(-4)^2 - 4(3)(-12)}}{2(3)}$

$= \dfrac{4 \pm \sqrt{16 + 144}}{6}$

$= \dfrac{4 \pm \sqrt{160}}{6}$

$= \dfrac{4 \pm 4\sqrt{10}}{6}$

$\dfrac{2 \pm 2\sqrt{10}}{3}$

$x = \dfrac{2 + 2\sqrt{10}}{3} \text{ or } \dfrac{2 - 2\sqrt{10}}{3}$

9. **[4 marks]**

Isabel throws a ball for her dog to catch. The path of the ball is parabolic and can be modelled by the equation $h = -x^2 + 4x + 1.5$ where h is the height in metres of the ball above the ground and x is the horizontal distance of the ball from Isabel.

If Isabel's dog is 4m away from her, how far does he have to jump to catch the ball?
Provide a sketch to illustrate your answer.

$h = -x^2 + 4x + 1.5$

1.5

4
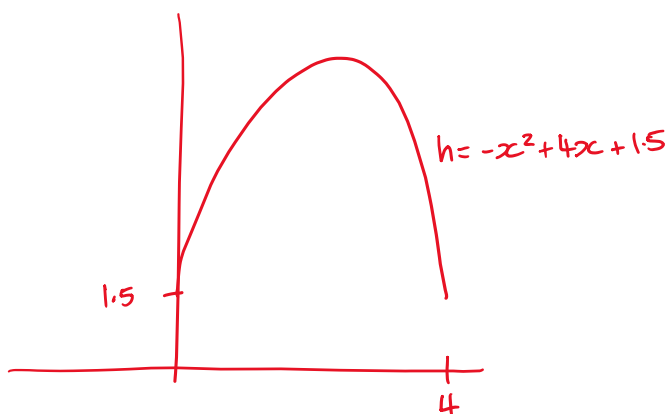
$x = 4$
$h = -(4)^2 + 4(4) + 1.5$
$= -16 + 16 + 1.5$
$= 1.5$

Isabel's dog will need to jump 1.5m to catch the ball.

THIS PAGE HAS BEEN LEFT BLANK INTENTIONALLY TO PROVIDE SPACE FOR WORKING IF REQUIRED

END OF TEST