



**Greenwood College
Year 12 Applications
Test 5 Finance 2019
Resource-Allowed**

Name..... Marking key 150

Formula sheet, one A4 page single-sided of notes and calculators allowed.

50 mark total.

55 minute time limit.

Question 1

(6 marks)

Mary was keen to compare interest rates offered by different banks, so she decided to construct a table showing the effective annual rates of interest (%). Part of this table is shown below.

↓

Compounding period	Rate of interest (p.a.)				
	4%	4.5%	5%	5.5%	6%
Quarterly	4.060	4.577	5.095	5.614	6.136
Monthly ← 12	4.074	4.594	5.116	5.641	6.168
Daily	4.081	4.602	5.127	5.654	6.183

- (a) Show how the value of 5.641% in the table was calculated. (3 marks)

$$\begin{aligned}
 ERI &= \left(1 + \frac{0.055}{12}\right)^{12} - 1 \checkmark \\
 &= 0.5641 \checkmark \\
 &= 5.641 \checkmark
 \end{aligned}$$

simple interest

- (b) Mary has \$15000 to invest. Using the effective annual rates of interest values only, determine the value of her investment at the end of 2 years if she has been offered a rate of 5% p.a. with interest compounded quarterly. (3 marks)

$$\begin{aligned}
 FV &= 15000 \checkmark + 15000 \times 0.05095 \times 2 \checkmark \\
 &= 15000 + 1528.50 \checkmark \\
 &= \$16528.50 \checkmark
 \end{aligned}$$

Question 2

13
(14 marks)

Jeff has borrowed \$16 000 from a bank at a reducible interest rate of 18% per annum with interest accrued and repayments made monthly. Standard repayments are set at \$500 per month.

The table below shows the progress of the loan for the first six months. All values have been rounded to the nearest cent.

Month	Amount owing at beginning of month	Interest for the month	Repayment	Amount owing at end of month
1	16 000.00	240.00	500.00	15 740.00
2	15 740.00	236.10	500.00	15 476.10
3	15 476.10	232.14	500.00	15 208.24
4	15 208.24	228.13	500.00	14 936.37
5	14 936.37	224.04	500.00	14 660.41
6	14 660.41	A	500.00	B

- (a) What is the monthly interest rate? (1 mark)

$$18 \div 12 = 1.5\% \text{ per month}$$

- (b) Determine the values of A and B.

Show the recursive formula used. (3 marks)

$$A = \$217.91$$

$$B = \$14\,380.32$$

$$T_{n+1} = T_n + 0.015T_n - 500$$

$$T_0 = 16000$$

- (c) Determine the length of time it will take Jeff to pay off the loan. (2 marks)

$$44 \text{ months}$$

- (d) Determine the total amount Jeff pays over the duration of the loan. (3 marks)

Amount owing at the end of the 43 month = \$454.08 ✓

Interest 44th month = \$6.81 ✓

$$\text{Total} = (43 \times 500) + 454.08 + 6.81 \\ = \$21960.89 \checkmark$$

- (e) The bank suggests that Jeff need only make repayments of \$240 per month. Describe how this would affect the length of time and total amount he pays over the duration of the loan. (2 marks)

He will never pay^{off} the loan. ✓
Using this suggestion the repayment would = the interest per month, so he only ever pays interest and the balance never decreases. ✓

- (f) After listening to advice, Jeff decides that he wants to pay off the loan completely in two years, making equal payments each month over that time. Determine the amount of each repayment he will need to make in order to make this happen? (2 marks)

Compound Interest

$$\begin{aligned} N &= 24 \\ I\% &= 18 \\ PV &= 16000 \rightarrow PMT = 798.7856 \\ FV &= 0 \\ P/Y &= 12 \\ C/Y &= 12 \end{aligned}$$

↓
798.79 ✓✓

Question 3

(7 marks)

Lynn is about to retire and is planning to take an annuity from her pension fund. She sets up the pension fund on her 65th birthday with \$500 000 and she estimates the fund can generate a growth rate of 6% per year. She plans to start withdrawing an annuity of \$40 000 starting on her following birthday.

- (a) Write a recurrence relation to calculate the total amount in the fund directly after each withdrawal. (3 marks)

$$T_{n+1} = 1.06 T_n - 40000, T_0 = 500000$$

\uparrow ✓ \uparrow ✓

- (b) For how many years will Lynn be able to receive her annuity of \$40 000?

(2 marks)

Number of years is 23 since in the 24th year there would not be enough money for a withdrawal of \$40 000. ✓✓

- (c) Assuming that all other conditions are the same, explain what would happen if Lynn decided to withdraw \$30 000 per year instead of \$40 000 per year?

Since 6% of \$500 000 is \$30 000 (2 marks) ✓
the principal would not decrease. ✓

Question 4

(6 marks)

John sets up his pension fund on July 1 2018 with a principal of \$850 000. The fund guarantees an annual growth rate of 7.5% compounded monthly and he plans to take an annuity of \$75 000 each year on July 1, starting in 2019.

- (a) Calculate the balance in the fund after the annuity is withdrawn in July 2022.

$$N = 4 \quad \rightarrow$$

(2 marks)

Compound Interest

$$N \quad 4$$

$$I\% \quad 7.5$$

$$PV \quad 850000 \rightarrow FV = -809531.47 \checkmark$$

$$PMT \quad -75000$$

$$P/Y \quad 1$$

$$C/Y \quad 12$$

The investment fund revised its annual interest rate to 9% compounded monthly on July 1 2022 guaranteed for the period to July 2027 and John continued withdrawing \$75 000 as usual.

- (b) Calculate the balance in the fund after a withdrawal is made on July 1 2027.

(2 marks)

$$\text{Initial balance} = \$809531.47 \checkmark$$

$$2023 \rightarrow N = 1$$

Balance in 2025

occurs when $N = 5$

$$\text{New balance} = \$815197.37 \checkmark \leftarrow$$

in July 2027

$$N \quad 5$$

$$I\% \quad 9$$

$$PV \quad 809531.47$$

$$PMT \quad -75000$$

$$FV \quad \checkmark$$

$$P/Y \quad 1$$

$$C/Y \quad 12$$

- (c) Calculate, to the nearest \$100, the maximum amount John could withdraw annually, starting in 2022, without decreasing his balance.

(2 marks)

$$\text{Initial balance} = \$809531.47$$

$$\text{Future value} = \$809531.47$$

Abbey could withdraw

$$\$75939.63 \checkmark \leftarrow$$

$$= \$75900 \checkmark$$

(nearest \$100)

$$N \quad 1$$

$$I\% \quad 9$$

$$PV \quad 809531.4722$$

$$PMT$$

$$FV \quad -809531.4722$$

$$P/Y \quad 1$$

$$C/Y \quad 12$$

Question 5

(3 marks)

Mia has \$30 000 to invest. If she chooses to invest this money in an account earning compound interest at the rate of 3.5% per annum, determine the time required for her to double her investment, if interest is paid monthly.

$$\text{Solve } 30000 \left(1 + \frac{0.035}{12}\right)^N = 60000 \checkmark \checkmark$$

$$\downarrow$$

$$N = 238 \text{ months } \checkmark$$

Question 6

(6 marks)

Numspa, a former high school student and now a successful business owner, wishes to set up a perpetuity of \$2000 per year to be paid to a deserving student from his school. The perpetuity is to be paid at the start of the year in one single payment.

- (a) A financial institution has agreed to maintain an account for this perpetuity paying a fixed rate of 5.2% p.a. compounded monthly.

Find the amount that Numspa needs to invest.

(3 marks)

$$\text{Solve } x \left(1 + \frac{0.052}{12}\right)^{12} - 2000 = x$$

$$x = \$37\,553.46 \checkmark$$

- (b) Numspa allows himself 4 years to accumulate the required investment in part (a) by making regular monthly payments into an account paying 4.7% p.a. compounded quarterly.

Determine the monthly payment to reach the required amount after 4 years if Numspa starts the account with an initial deposit of \$5000. (3 marks)

Compound Interest

N 16

I% 4.7

✓✓

PV -5000

PMT → -1955.90

FV 37553.46

✓
\$1955.90 per month

Question 7

6
(2 marks)

Roger borrowed \$250 000 at a rate of 6% per annum compounded monthly. He makes monthly repayments of \$3000 to repay the loan.

(a) How long will it take Roger to repay the loan?

(2 marks)

Compound Int
 $N \longrightarrow 108.0685744$
 $I\% 6$
 $PV 250000$
 $PMT -3000$
 $FV 0$
 $P/Y 12$
 $C/Y 12$

✓✓
 108 months to
 repay the loan.

After 2 years, Roger decreases his monthly payments to \$2500.

(b) Calculate by how many months the length of the loan is increased by this reduction in repayments.

(4 marks)

$N 24$
 $I\% 6$
 $PV 250000$
 $PMT -3000$
 $FV \longrightarrow -205494.0783$ ✓
 $P/Y 12$
 $C/Y 12$

$N \longrightarrow 106.1262321$
 $I\% 6$
 $PV 205494.0783$
 $PMT -2500$
 $FV 0$
 $P/Y 12$
 $C/Y 12$

Take 106 months
 to repay remainder
 of the loan

Increase
 $(24 + 106) - 108 = 22$ ✓
 months

Question 8

(3 marks)

Situation 1

For a reducing balance loan, the interest on the amount owed is added to the balance owing before the repayment is made.

Situation 2

For a reducing balance loan, a repayment is made before the interest on the amount owed is calculated.

Which situation would result in a person repaying less in the long run? Show mathematical working to justify your choice.

Situation 2. ✓

Sit 1

$$\begin{aligned} T_{n+1} &= T_n + I \times T_n - \text{Repay} \\ &= (1+I)T_n - \text{Repay} \end{aligned}$$

Sit 2

$$\begin{aligned} T_{n+1} &= (T_n - \text{Repay}) \\ &\quad + I \times (T_n - \text{Repay}) \\ &= (1+I)T_n \\ &\quad - (1+I)\text{Repay} \end{aligned}$$

T_{n+1} for Sit 2 is lower than T_{n+1} for Sit 1. ✓✓