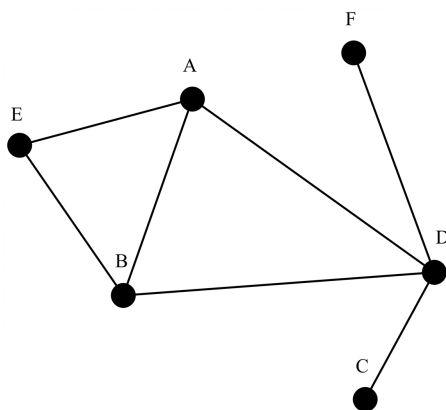Name: **SOLUTIONS**

Mark	/27

Time Allowed: 25 minutes

Resources Allowed: Formula sheet only.

Solutions without working may not be allocated full marks.

Question One   [8 marks]

a)   Redraw the network below so that it is planar. Write two closed trails each the length of 4 starting at vertex E.   (3 marks)
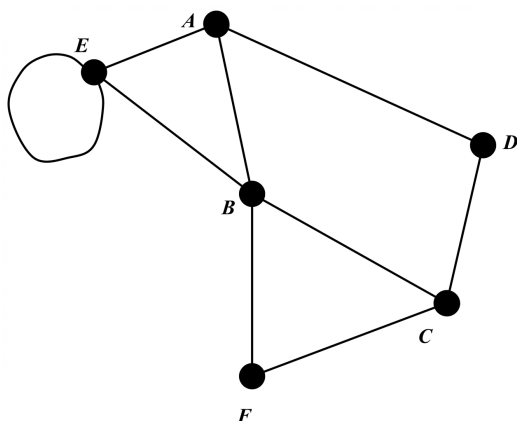

1 mark for all correct labels and edges and for planar (no edge is crossing any other edge).
Edges must touch the nodes.
Nodes are clearly and correctly defined

1 mark for writing close trail starting and finishing at E of length 4. E-A-D-B-E,
1 mark for writing close trail starting and finishing at E of length 4. E-B-D-A-E

No repeated edges

b)   Redraw the network M as a planar graph. Determine the adjacency matrix for the network.
                                                                 (5 marks)

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{bmatrix} A & B & C & D & E & F \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

1 mark for all correct labels and edges
Edges must touch the nodes.
Nodes are clearly defined

1 mark for planar (no edge is crossing any other edge).
1 mark for correct Matrix labels including "M="
1 mark for 3 correct rows/columns in the matrix (FT)
1 mark for all correct values (FT)

Question Two   [11 marks]

An employee has to be qualified with a certificate to operate some of the machines in a factory.

Employees in team 1; Abigail, Barak, Clyde, Dom, Eugene and Fran, have been assigned to work the same shift every day.

Each member has certificates to operate different machinery in the factory. In any shift all 5 machines have to be working at the same time.

TEAM MEMBERS	QUALIFICATIONS
Abigail	M1, M3, M4
Barak	M1, M2, M3, M4, M5
Clyde	M1, M2
Dom	M2, M3
Eugene	M2, M4
Fran	M1

a)   Draw a bipartite graph to represent the information about Team 1 and label it S_1.   (3 marks)

$S_1 =$
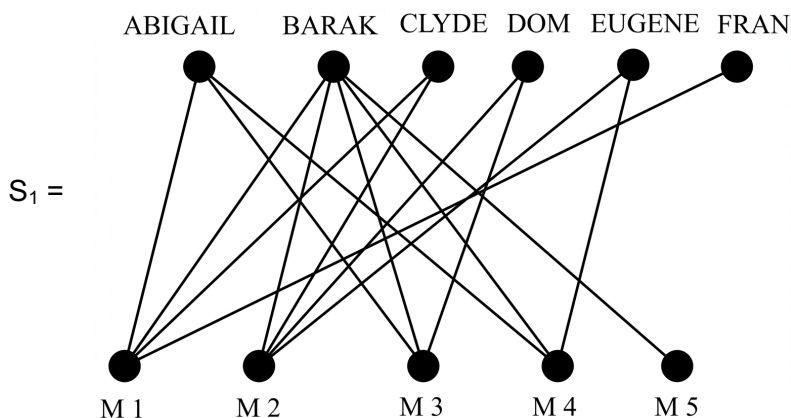
1 mark for all correct labels on each set of vertices
1 mark correctly labelling "$S_1 =$"

1 mark for correct Bi partite graph
Edges must touch the nodes.
Nodes are correctly and clearly defined.

b)   Based on the information about the team, identify the team member who can be chosen to help most of the members of the team during any shift. Justify your decision.   (2 marks)

Abigail is qualified to work on 3 machines.
She can replace Barak and Fran on Machine 1
She can replace Barak and Dom on Machine 2
She can replace Barak and Eugene on Machine 4
Every machine is allocated to every other member of the team so Abigail is most useful because she can any help 3 out of 5 people during a shift.

1 mark for choosing Abigail
1 mark for recognising 3 qualifications or saying that everyone else is allocated a machine.

Do not accept Barak as an answer because he is the only one who can operate M5 so he is effectively stuck.

c) The company wants Fran to add another certificate to her qualifications. Which machine would you recommend that Fran qualifies in? Justify your choice using the information from the bipartite graph or the table provided. (3 marks)

Fran should train and get a certificate in Machine M5. It is the only machine which has one person qualified to operate it; Barak. The rest of the machines have a back up operator.

> 1 mark for choosing Machine M5.
> 1 mark for recognising that Barak is the only one qualified for M5.
> 1 mark for saying that every machine has a back up operator.

d) Draw a subgraph S_2 that shows every member of the team and how every machine can be used during a Team 1 shift. (3 marks)

$S_2 =$
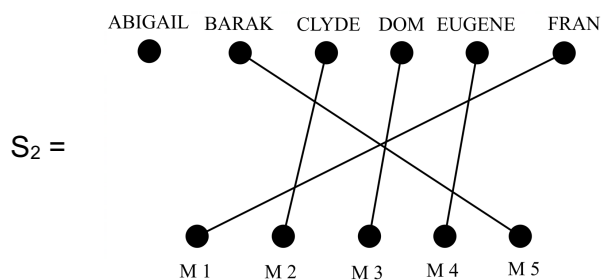
> 1 mark for correct labels in both vertices including "$S_2 =$"
> 1 mark for allocating that Barak to M5, Clyde to M2, Dom to M3, and Fran to M1.
> 1 mark for correct allocation of the rest as one sub graph. Of their answer in 2 (a) (Variations can happen see below). Edges must touch nodes, and nodes are clearly and correctly defined.

$S_2 =$
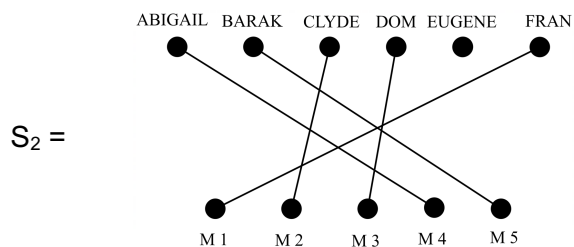
Question Three   **[8 marks]**

a)   A complete graph is denoted as K_n. Draw a graph denoted by K_5.   (2 marks)
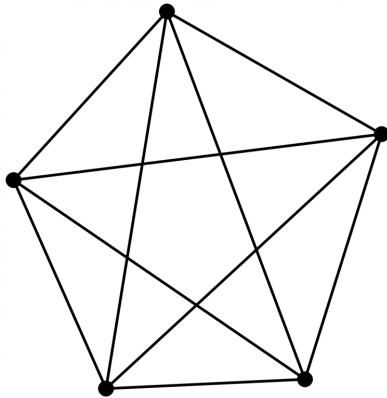
1 mark for complete graph.
Edges must touch the nodes.
Nodes are clearly defined

1 mark for graph with 5 nodes.

b)   Determine the total degree sum of a graph given by K_{12}.   (1 mark)

$$n = 12$$

$$n(n-1) = 12(12-1) = 132$$

1 mark for correct answer.

c)   What is the total number of edges in a graph denoted by K_9. Describe the graph accurately using terms such as Hamilitonian, Semi Hamilitonian, Eulerian, or Semi Eulerian graph? Justify your answer.   (5 marks)

$$n = 9$$

$$\frac{n(n-1)}{2} = \frac{9(9-1)}{2} = 36 \; edges$$

1 mark for choosing 9 nodes
1 mark for showing the working out.
1 mark for correct answer.
1 mark for saying the graph is Hamilitonian and saying that a every node can be reached without repeating nodes and that the walk will start and finish at the same node.
1 mark for saying the graph is Eulerian and saying that every node has even order of vertex.

END OF SECTION ONE