

Name \_\_\_\_\_


## Maths Specialist – Investigation 2016

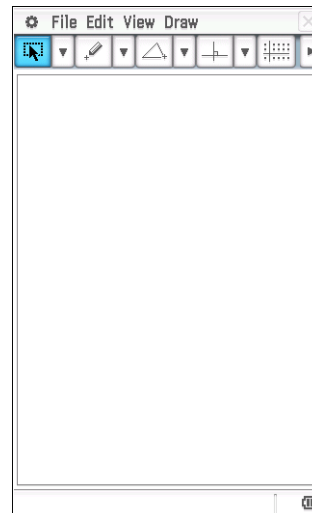
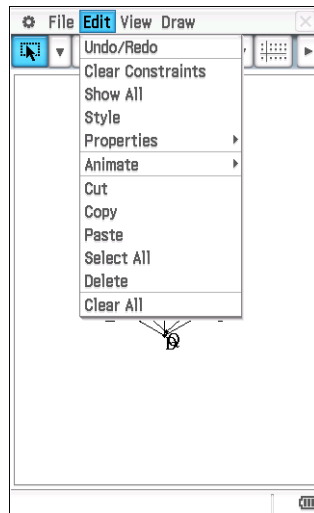
### Circle Geometry- Part One

Extended investigation

Part 1: Preparation activity


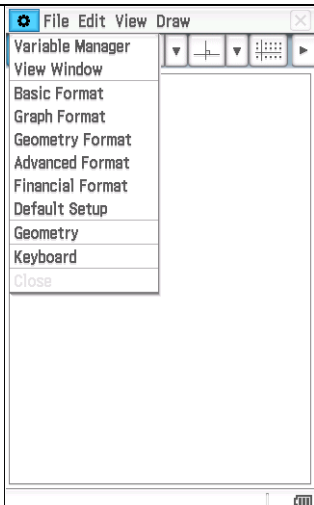
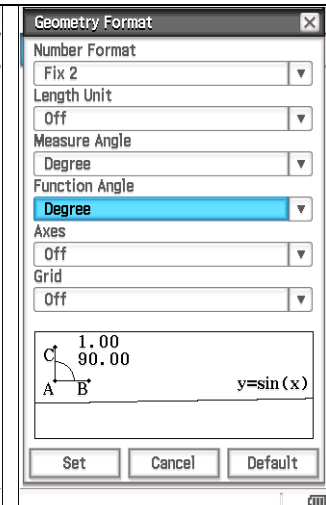
### Using the Geometry Application on the ClassPad

Tap  from the main menu to select the **Geometry** application.



To clear the screen, tap **Edit**, tap **Clear all**.

Apply the following CAS settings for the Geometry application.

<p>Tap </p> <p>Tap <b>Geometry Format</b></p> <p>Set <b>Number Format</b> to <b>Fix 2</b></p> <p>Set <b>Measure Angle</b> to <b>Degree</b></p> <p>Set <b>Function Angle</b> to <b>Degree</b></p> <p>Set <b>Axes</b> to <b>Off</b></p> <p>Tap <b>Set</b></p>	 
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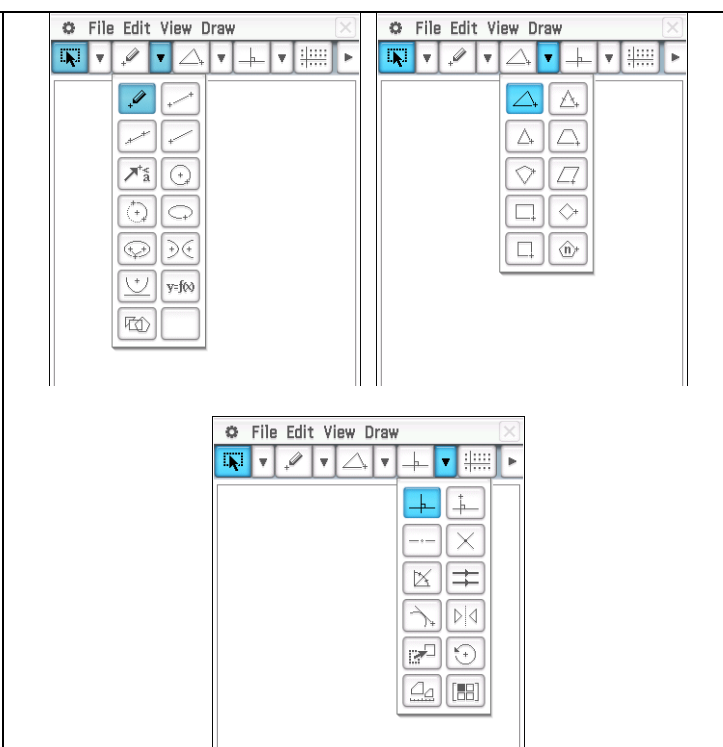
## Tools available in the Geometry application

By tapping on the down arrow, ▼, to the right of each of the drop down menus,





a number of tools become available.

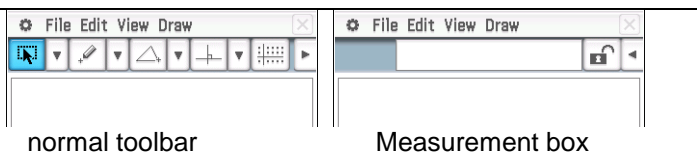
When a tool is referred to, the accompanying screen picture shows where the tool may be found.



## The Measurement Box

Tap  to display the Measurement Box.


Tap  to return to the normal toolbar.

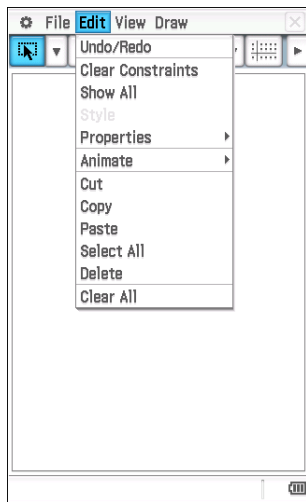


The **Measurement Box** can be used to perform operations such as:

- viewing measurements such as length, angle size
- to specify a measurement, e.g. set a length to represent a measurement of 12 cm or an angle to be of size 48°
- to name a point, line segment, angle, ...
- to determine whether a line is tangential to a circle
- to fix a line so that it is tangential to a circle

### NOTE:

- To **deselect an object** (or objects), tap  and then tap anywhere in space within the Geometry window.
- To undo tap **Edit**, tap **Undo / Redo**.



### Question 1

Write a definition for each of following.

**Central Angle:**

**Chord:**

**Cyclic quadrilateral:**

**Diameter:**

**Radius:**

**Major/Minor Arc:**

**Major/Minor Segment:**

**Alternate Segment:**



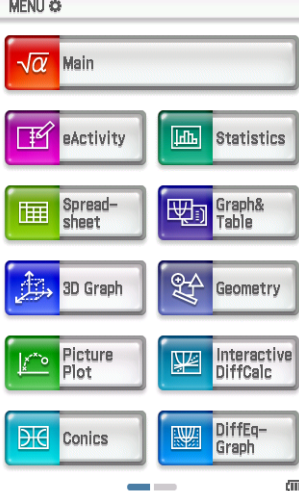
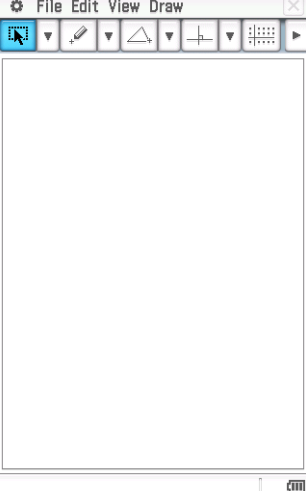

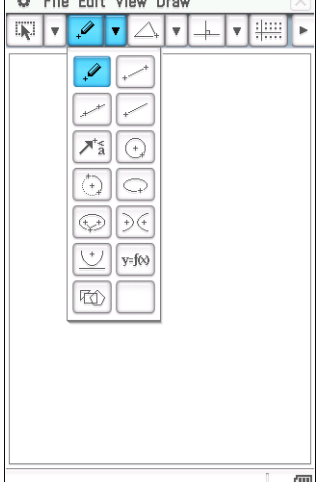


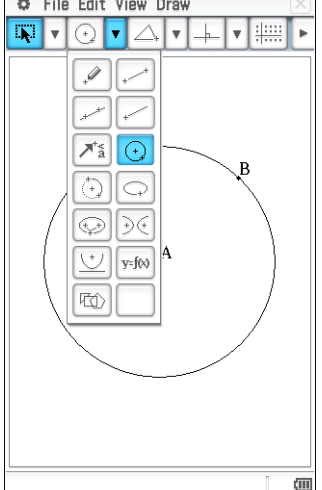
**Major/Minor Sector:**




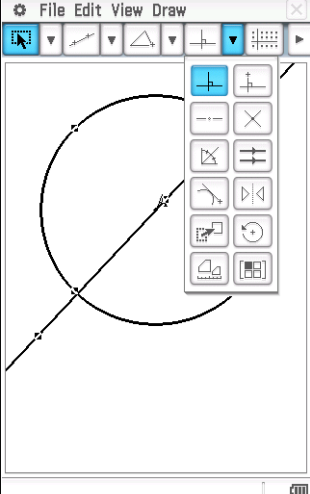



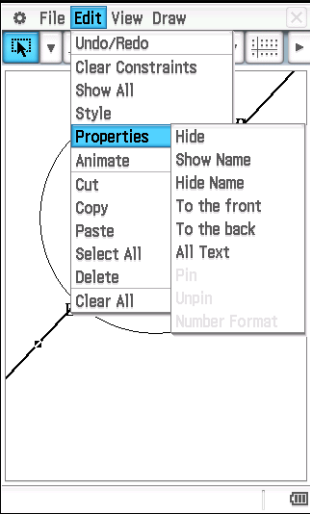





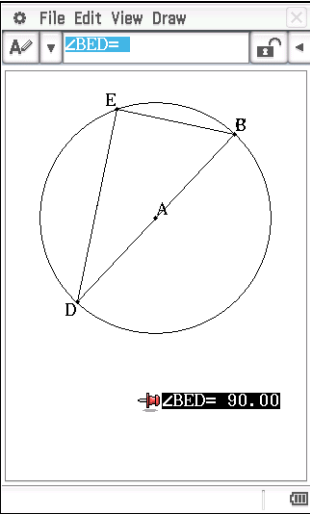
**Secant:**

**Semicircle:**

**Tangent:**

## Question 2

<p>Tap  .</p> <p>Tap  .</p> <p>Tap <b>File</b>, tap <b>New</b></p>		
<p>Draw a circle by tapping  and then tapping in two different places in the Geometry window.</p> <p>Tap <b>View</b>, tap <b>Zoom to Fit</b>.</p>		
<p>Draw a line passing through centre A and point B by tapping  , then tap A, tap B.</p> <p>Tap  .</p>		

<p>Select the circle and the line by tapping once on each of them.</p> <p>Choose  to locate point D, the point of intersection of the circle and the line.</p> <p>Draw diameter BD by tapping , then tap B, tap D.</p> <p>Tap .</p>	
<p>To hide the line passing through D, A and B, tap on any part of this line that lies outside the circle, tap <b>Edit</b>, tap <b>Properties</b>, tap <b>Hide</b>.</p> <p>Tap  and mark a point, E, on the circle.</p> <p>Tap  to draw two chords, BE and DE.</p> <p>You have now drawn triangle BDE where E is a point on the circle and DE is a diameter of the circle centre A,</p> <p>Tap .</p>	
<p>Tap  to display the Measurement Box.</p> <p>Measure the size of <math>\angle BED</math> by tapping once on BE and once on DE. The angle size of <math>\angle BED</math> will be displayed in the measurement box.</p> <p>Tap on the angle size of <math>\angle BED</math> displayed in the Measurement Box and drag it into the Geometry window.</p> <p>Name this angle BED by tapping  and using the  tab on the keyboard to type BED, press , press .</p> <p>Hide the keyboard.</p> <p>Tap in space.</p>	

Observe the size of  $\angle BED$  as point E moves around the circle by using one of the two methods below:

Method One

Tap E, tap the circle.

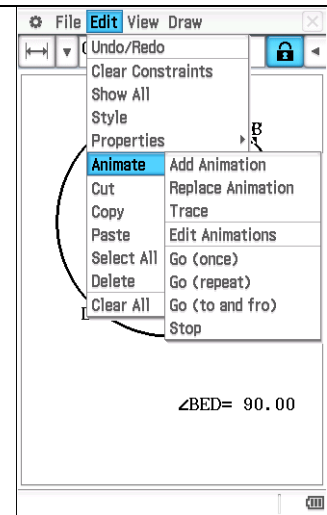
Tap **Edit**, tap **Animate**, tap **Add Animation**.

Tap **Edit**, tap **Animate**, tap **Go (once)**.

Method Two

Tap E, tap E a second time and drag it around the circle.

To save this file, tap **File**, tap **Save** and name the file Qn\_2.



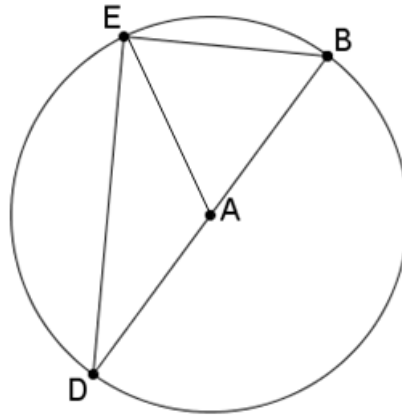
- (a) Why do you think  $\angle BED$  is referred to as the angle in the semicircle?
- (b) What do you notice about the size of  $\angle BED$  as point E moves around the circle?
- (c) Make a conjecture about the size of the angle in a semicircle.

**Conjecture:** The angle in a semicircle is ...

### Question 3

A proof of your conjecture about the angle in a semicircle has been started for you. Complete the proof. Remember that statements in the proof need to be justified.

#### Angle in a Semicircle Theorem



**Given:** Circle centre A, diameter BD. E is any point on the circle,  $\angle BED$  is an angle in the semicircle DBE.



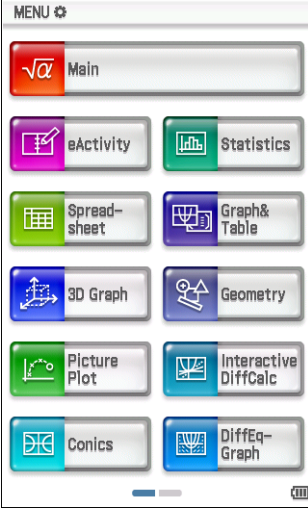
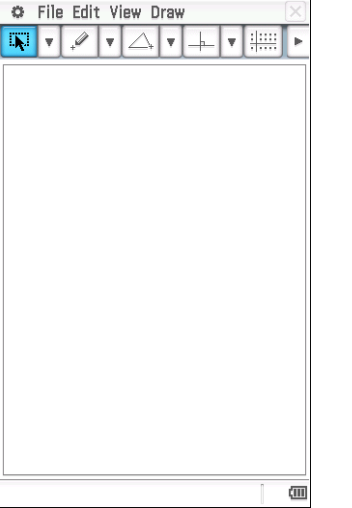

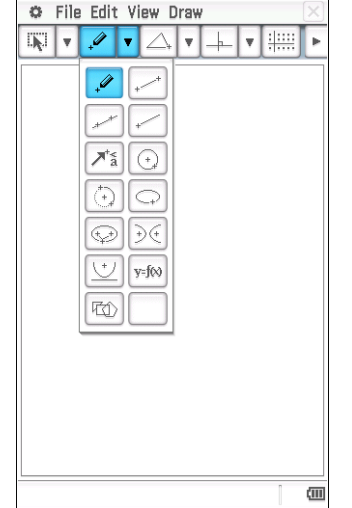




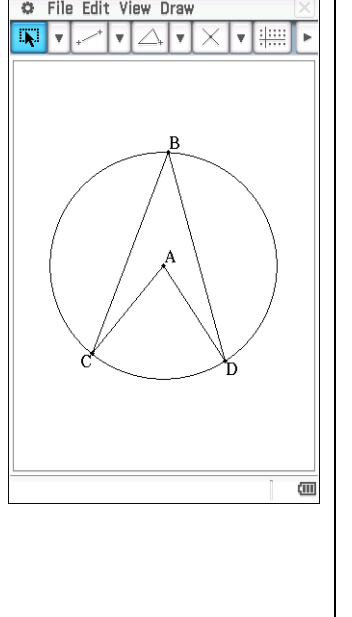
**To Prove:**









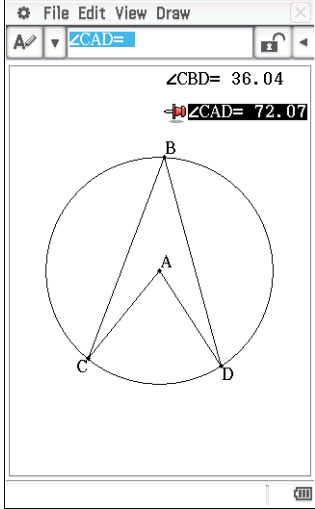
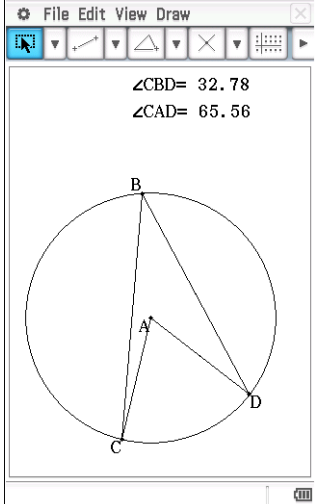
**Extension to the diagram:** Draw AE.

**Proof:**



## Question 4

<p>Tap  .</p> <p>Tap  .</p> <p>Tap <b>File</b>, tap <b>New</b></p>		
<p>Draw a circle by tapping  and then tapping in two different places in the Geometry window.</p> <p>Tap <b>View</b>, tap <b>Zoom to Fit</b>.</p>		
<p>Tap  and mark points, C and D, on the circle <b>as shown in the diagram</b>.</p> <p>Tap . Draw radii AC and AD; draw chords BC and BD.</p> <p>Your diagram should show that <math>\angle CBD</math> is subtended at the circumference by arc CD in the same segment as <math>\angle CAD</math> (see diagram). Recall, points can be moved by tapping on the point once and then tapping a second time to drag to a new position. Don't forget to tap in space to deselect before selecting a different item.</p> <p>Tap  .</p> <p>Tap  to display the Measurement Box.</p>		

<p>Display the size of <math>\angle CBD</math> by tapping BC and BD.</p> <p>Tap on the size of <math>\angle CBD</math> and drag it into the Geometry window.</p> <p>Name this angle CBD by tapping  and using the  tab on the keyboard to type CBD, press . Press .</p> <p>Tap in space.</p> <p>Display the size of <math>\angle CAD</math> by tapping AC and AD.</p> <p>Tap on the size of <math>\angle CAD</math> and drag it into the Geometry window.</p> <p>Name this angle CAD by tapping  and using the  tab on the keyboard to type CAD, press . Press .</p> <p>Tap in space.</p> <p>Hide the keyboard.</p>	
<p>Observe the size of angles <math>\angle CAD</math> and <math>\angle CBD</math> when points C and D respectively move on the circle.</p> <p>Tap C. Tap C a second time and drag it around the circumference such that <math>\angle CAD</math> and <math>\angle CBD</math> both remain in the same segment.</p> <p>Tap in space.</p> <p>Tap D. Tap D a second time and drag it around the circumference such that <math>\angle CAD</math> and <math>\angle CBD</math> both remain in the same segment.</p> <p>To save this file, tap <b>File</b>, tap <b>Save</b> and name the file Qn_4.</p>	

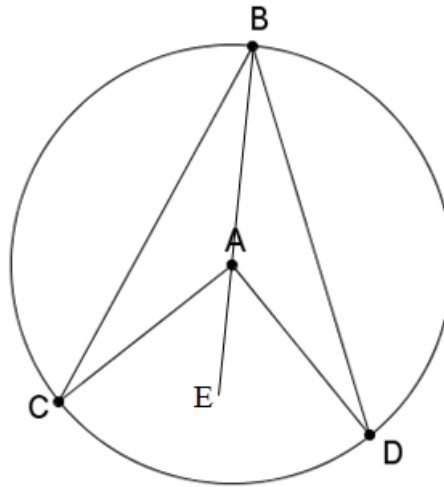
- By which arc are angles CBD and CAD both subtended?
- What do you notice about the size of the angle subtended at the centre of the circle,  $\angle CAD$ , and the size of the angle subtended at the circumference of the circle,  $\angle CBD$ ?
- Make a conjecture about the size of the angle at the centre subtended by an arc of a circle and the size of the angle at the circumference subtended by the same arc.

**Conjecture:** The size of the angle at the centre subtended by an arc of the circle is ...

### Question 5

A proof of your conjecture about the central angle has been started for you. Complete the proof. Remember that statements in the proof need to be justified.

#### Central Angle Theorem



**Given:** Circle centre A.  $\angle CAD$  is the angle subtended by arc CD at the centre and  $\angle CBD$  is the angle subtended by arc CD at the circumference

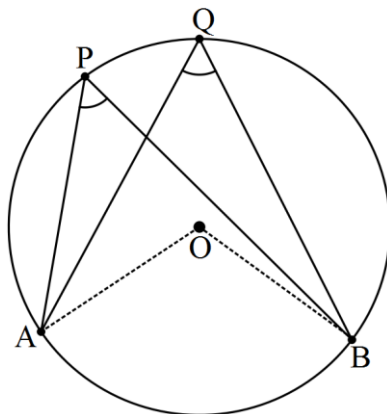
**To Prove:**

**Extension to the diagram:** Join BA and produce it to E.



Proof:	Statement	Reason
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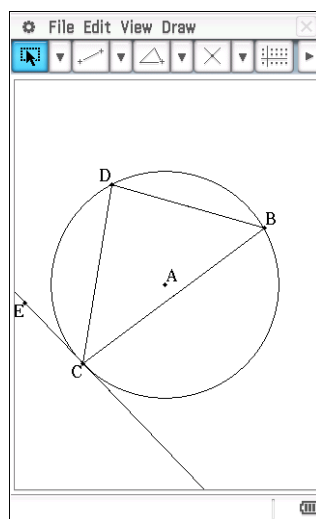
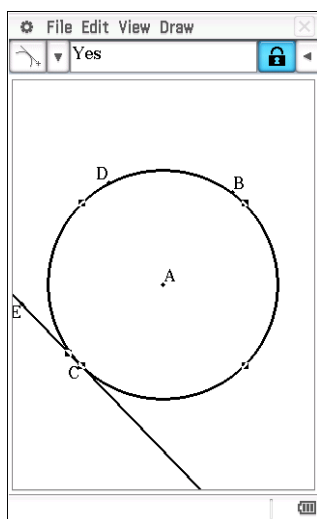
### Question 6

Prove that two angles at the circumference subtended by the same arc are equal, i.e.  $\angle APB = \angle AQB$ .



### Question 7

Use the Geometry application to draw a circle centre A and radius AB. Mark points C and D on the circle as shown in the diagram below. Draw EC tangential to the circle at C by drawing a line through C. Tap  to display the Measurement Box. Tap on the line, tap on the circle. If **No** is displayed, tap . EC is now tangential to the circle at C. Draw line segment EC and chords BD, CD and BC.  $\angle CBD$  is an angle in the alternate segment to  $\angle DCE$ .





Note: it will be necessary to draw line segment EC prior to measuring the size of  $\angle DCE$ .

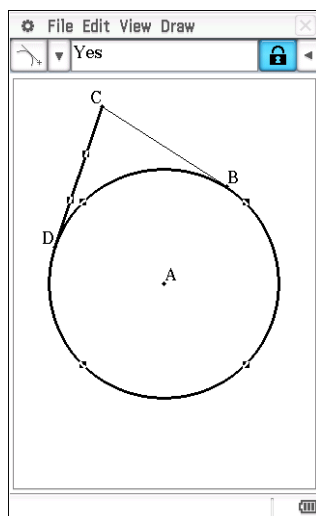
Save your file as Qn\_6.

- Whilst maintaining the location of  $\angle CBD$  in the alternate segment to  $\angle DCE$ , move point D on the circle. What do you notice about the size of  $\angle CBD$  and the size of  $\angle DCE$ ?
- Make a conjecture about the angles in the alternate segment.

**Conjecture:** An angle between a chord and a tangent is ...




### Question 8

Use the Geometry application to draw a circle centre A and radius AB. Position point C anywhere outside the circle and point D on the circle. Draw line segments CB and CD. Tap  to display the Measurement Box. Tap on CB, tap on the circle. If **No** is displayed, tap the tick, . CB is now a tangent to the circle at B. In a similar manner, make CD a tangent to the circle at D.



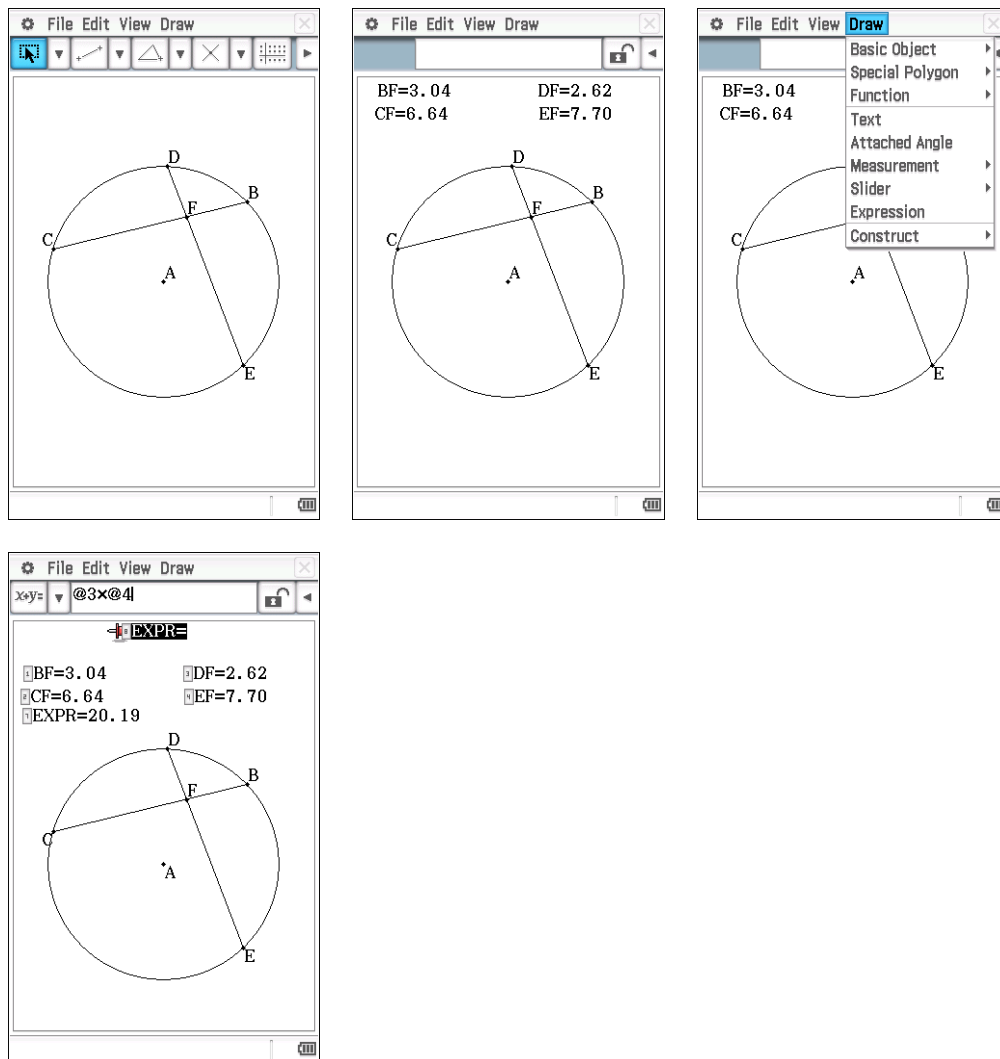
- (a) What do you notice about the lengths of tangents CB and CD as you change the location of point C?
- (b) Make a conjecture about the lengths of the tangents drawn from a point to a circle.



### Question 9

Use the Geometry application to draw a circle centre A and radius AB. Use  to draw intersecting chords BC and DE. Select BC and DE, tap  to determine F, the point of intersection of chords BC and DE. Tap  to display the Measurement Box.

Display the length of chord BF by tapping on B and on F. Tap on the size of BF and drag it into the Geometry window. Label this chord length BF.

In a similar manner, display the length of chords CF, DF and EF.

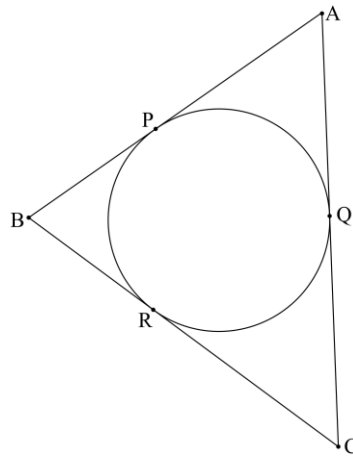


To calculate the product of BF and CF, tap Draw, tap Expression. Tap BF, press , tap CF, press . In a similar manner, display the product of DF and EF.

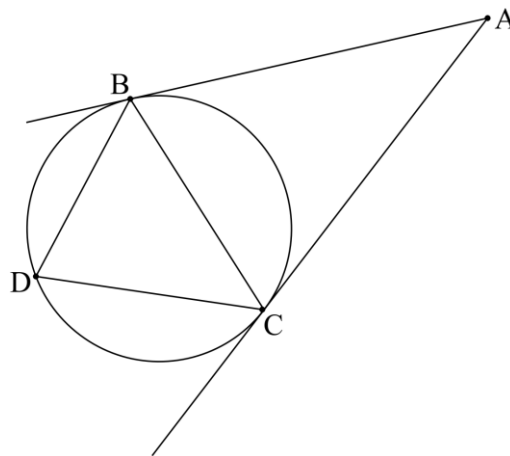
- What do you notice about  $BF \times CF$  and  $DF \times EF$  as you change the locations of C, D, E and F?
- Make a conjecture about the product of the lengths of the intervals on one chord and the product of the lengths of the intervals of an intersecting chord.

### Question 10

- (a) AB, AC and BC are tangents; AB = 15 cm; BC = 17 cm; BP = 9 cm. Find AC.



- (b) AB and AC are tangents;  $\angle BDC = 74^\circ$ . Find the size of  $\angle BAC$ .



- (c) PE is a tangent; AD is parallel to PC;  $\angle ADB = 55^\circ$ ,  $\angle BAC = 42^\circ$ . Find the size  $\angle EAD$  and  $\angle ABP$ .

