NAME

Esperance Senior High School

Mathematics Methods U3/4 Test 1 – 2018

Resource Free

Reading Time: 2 minutes
Working Time: 38 minutes
Marks: 38

Mathematics Methods Formula Sheet allowed

Question 1 **(10 marks)**

Differentiate the following with respect to x. Do not simplify. (Except negative indices)

(a) $f(x)=\dfrac{10}{x^5}$ (1 mark)

$f'(x) = \dfrac{50}{x^6}$

(b) $y=\dfrac{e^x}{2+\cos x}$ (3 marks)

$\dfrac{dy}{dx} = \dfrac{e^x(\cos(x)+\sin(x)+2)}{(\cos(x)+2)^2}$ ✓ ✓ ✓

(c) $g(x)=(8x-5)^3(x^2-3x)^4$ (3 marks)

$g'(x) = 3(8x-5)^2 \times 8 \times (x^2-3x)^4 + 4(x^2-3x)^3 \times (2x-3) \times (8x-5)^3$ ✓ ✓ ✓

(d) $y=\sin^3(2x+1)$. (3 marks)

$\dfrac{dy}{dx} = 3(\sin^2(2x+1)) \times 2\cos(2x+1)$ ✓ ✓ ✓

Question 2 **(4 marks)**

Using the chain rule find $\frac{dk}{dm}$ (in terms of m) given that $k = p^3 - 1 \text{ and } p = 3m^2 + 1$

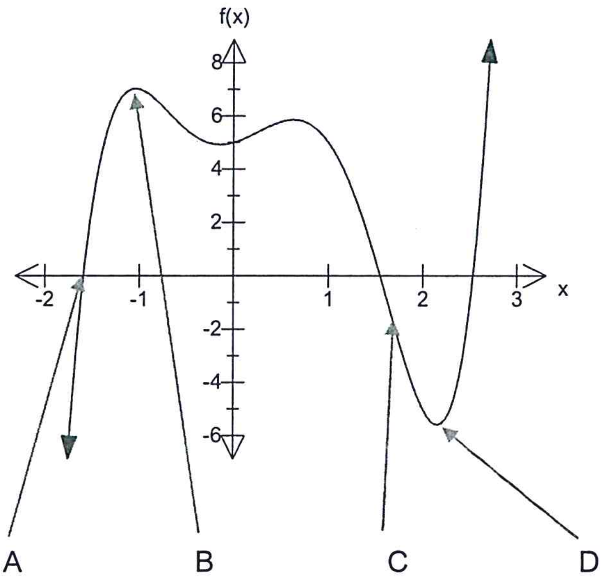
$\frac{dk}{dp} = 3p^2$ ✓ $\frac{dp}{dm} = 6m$ ✓ $\frac{dk}{dm} = \frac{dk}{dp} \times \frac{dp}{dm}$ ✓

$= 3p^2 \times 6m$

$= 3(3m^2+1)^2 \times 6m$ ✓

$= 18m(3m^2+1)^2$ ✓

Question 3 **(8 marks)**



At each indicated point on the diagram state whether it is a local maximum or minimum or a point of inflection or some other significant point. Also indicate what you know about $f'(x) \text{ and } f''(x)$ at each point.

A: root of $f(x)$ ✓

$f'(x) > 0$ ✓

$f''(x) < 0$

B: Local max of $f(x)$ ✓

$f'(x) = 0$ ✓

$f''(x) < 0$

C: Point of inflection ✓

$f'(x) < 0$ ✓

$f''(x) = 0.$

D: Local min ✓

$f'(x) = 0$ ✓

$f''(x) < 0$

Question 4 **(5 marks)**

The displacement of a body x metres from an origin at time t seconds is given by x = $t^3 - 3t^2 + 4, t \geq 0$. Find

(a) the initial velocity of the body (2 marks)

$$\frac{dx}{dt} = 3t^2 - 6t$$

(b) The acceleration of the body when its velocity is zero. (3 marks)

velocity zero if $3t^2 - 6t = 0$

$3t(t-2) = 0$

$t = 0$ or $t = 2$

$$\frac{d^2y}{dt^2} = 6t - 6$$

when $t = 0$ $\frac{d^2y}{dt^2} = -6$ $t = 2$ $\frac{d^2y}{dt^2} = 6 \, m/s^2$

Question 5 **(5 marks)**

If $y = \sqrt{x^3}$ find the approximate percentage change in y when x increases by 1%

$$y = x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}}$$

$$\delta y = \frac{dy}{dx} \delta x$$

$$\frac{\delta y}{y} = \frac{\frac{3}{2} x^{\frac{1}{2}}}{x^{\frac{3}{2}}} \times \delta x$$

$$= \frac{3}{2} \times \delta x$$

$$= \frac{3}{2} \times 0.01$$

$$= 0.015$$

$\therefore \delta y = 1.5\%$

Question 6 **(6 marks)**

Consider the function defined by $f(x)=\dfrac{x}{2}-\sqrt{x}$, $x\geq 0$.

(a) Determine the coordinates of the stationary point of $f(x)$. (3 marks)

$$f'(x) = \frac{1}{2} - \frac{1}{2\sqrt{x}} \checkmark$$

$$0 = \frac{1}{2} - \frac{1}{2\sqrt{x}}$$

$$x = 1 \checkmark$$

$$f(1) = \frac{1}{2} - \sqrt{1}$$

$$= -\frac{1}{2}$$

$\therefore$ Stationary at $\left(1, -\frac{1}{2}\right)$ ✓

(b) Use the second derivative test to determine the nature of the stationary point found in (a). (3 marks)

$$f''(x) = \frac{1}{4\sqrt{x^3}} \checkmark$$

$$f''(1) = \frac{1}{4}$$

$f''(1) > 0$ ✓ $\therefore$ Local minimum. ✓

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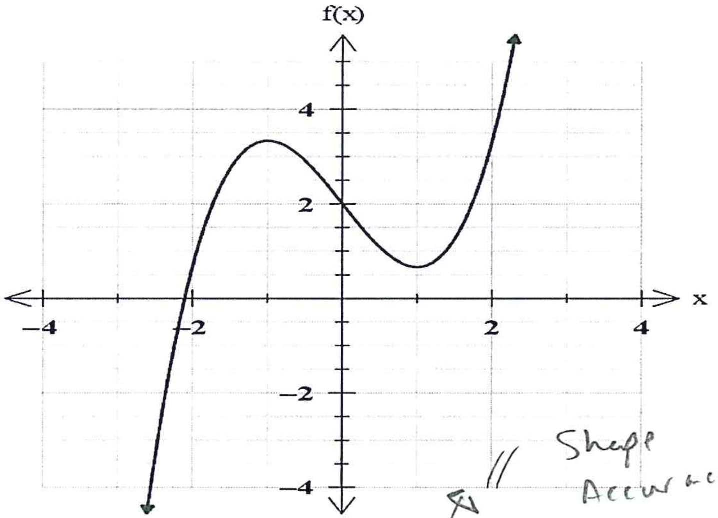
Resource Rich

Reading Time: 2 minutes
Working Time: 23 minutes
Marks: 23

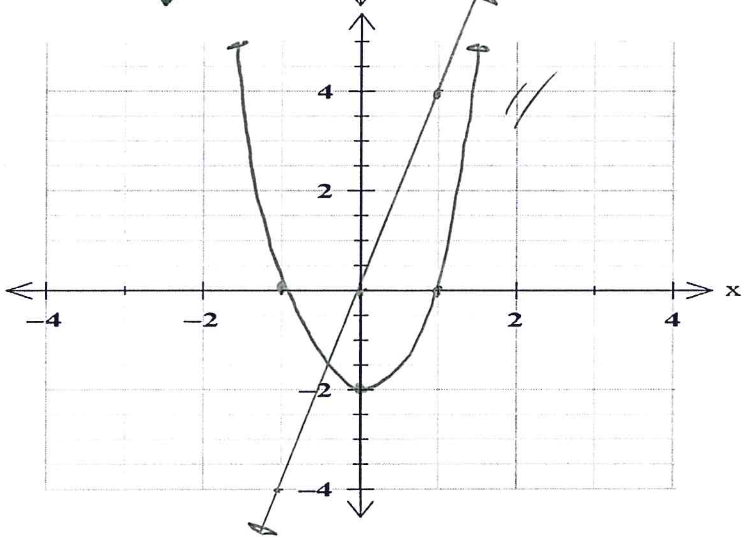
Mathematics Methods Formula Sheet allowed and 1 Page of notes allowed

Question 1 **(4 marks)**

On the one set of axes below sketch $f'(x)$ *and* $f''(x)$. Clearly indicate each graph.



Shape
Accuracy



Question 2 **(5 marks)**

The area of a segment with central angle θ in a circle of radius r is given by $A = \frac{r^2}{2}(\theta - \sin\theta)$. Use the increments formula to approximate the increase in area of a segment in a circle of radius 10 cm as the central angle increases from $\frac{\pi}{3}$ to $\frac{11\pi}{30}$.

$$A = \frac{10^2}{2}(\theta - \sin\theta)$$

$$= 50(\theta - \sin\theta)$$

$$\frac{dA}{d\theta} = 50(1 - \cos\theta)$$

when $\theta = \frac{\pi}{3}$ $\quad \frac{dA}{d\theta} = 50(1 - 0.5)$

$$= 25.$$

$$\delta\theta = \frac{11\pi}{30} - \frac{10\pi}{30}$$

$$= \frac{\pi}{30}$$

$$\delta A = \frac{dA}{d\theta} \times \delta\theta$$

$$= 25 \times \frac{\pi}{30}$$

$$= \frac{25\pi}{30}$$

$$= \frac{5\pi}{6} \text{ cm}^2$$

Question 3 **(9 marks)**

Amatil is planning a medium sized coke can with a volume of 250mL. Find the dimensions of the can to minimise the cost of manufacture (excluding the contents). NB $V=\pi r^2 h,\ \ SA=2\pi r(r+h),\ \ 1mL=1cm^3$

(a) Find an expression for h (1 mark)

$$250 = \pi r^2 h$$

$$h = \frac{250}{\pi r^2}$$

✓

(b) Show that the surface area $A=2\pi r^2+\frac{500}{r}$ (2 marks)

$$SA = 2\pi r\left(r+\frac{250}{\pi r^2}\right) ✓ \quad = 2\pi r^2 + \frac{500}{r} ✓$$

$$= 2\pi r^2 + \frac{500\pi r}{\pi r^2}$$

(c) Find an expression for A'(r) (2 marks)

$$A'(r) = 4\pi r - 500r^{-2} ✓$$

$$= 4\pi r - \frac{500}{r^2} ✓$$

(d) Using your calculator or otherwise, find and verify the values of r and h that minimises the cost of manufacture. (4 marks)

Local min / max when $A'(r)=0$

$$0 = 4\pi r - \frac{500}{r^2} ✓$$

$r = 3.4$ cm (1d.p)

$A'(3) < 0$ ✓

$A'(4) > 0$

$\therefore$ $r = 3.4$ gives minimum Area ✓

$h = 6.8$ cm (1d.p) ✓

Question 4 **(5 marks)**

A recent news report said that it took 34 months for the population of Australia to increase from 23 to 24 million people.

(a) Assuming that the rate of growth of the population can be modelled by the equation $\frac{dP}{dt} = kP$, where P is the population of Australia at time t months, determine the value of the constant k. (3 marks)

$P = P_0 e^{kt}$ ✓

$24 = 23 e^{k(34)}$ ✓

$k = 0.001252$ ✓

(b) Assuming the current rate of growth continues, how long will it take for the population to increase from 24 million to 25 million people? (2 marks)

$25 = 24 e^{0.001252 t}$ ✓

$t = 32.6$ months. ✓