Name: Soln.

Esperance SHS

Year 12 MATHEMATICS METHODS

TEST 4 2016

Calculator Free

Total Marks: 11

Reading: 1 minute Time Allowed: 11 minutes

Question 1 **(5 marks)**

Determine if each of the following is a Discrete Random variable. If not, give a reason.

a)

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$

NO ✓

$\Sigma P > 1$ ✓

b)

x	-3	-2	-1	0	1	2
$P(X = x)$	0.3	0.2	0.2	0.1	0.1	0.1

YES. ✓

c) $P(X = x) = \frac{x}{6}$ where X = 0,1,2,3

x	0	1	2	3
$P(x=x)$	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$

✓

$\Sigma P(x=x) = 1 \therefore$ YES ✓

Question 2 **(6 marks)**

The discrete random variable X has the probability distribution shown in the table below.

x	0	1	2	3
$P(X=x)$	$\frac{2a^2}{3}$	$\frac{1-3a}{3}$	$\frac{1+2a}{3}$	$\frac{4a^2}{3}$

Determine the value of the constant a.

$$\frac{2a^2}{3} + \frac{1-3a}{3} + \frac{1+2a}{3} + \frac{4a^2}{3} = 1$$ ✓

$$2a^2 + 1 - 3a + 1 + 2a + 4a^2 = 3$$ ✓

$$6a^2 - a - 1 = 0$$ ✓

$$(3a+1)(2a-1) = 0$$ ✓

$$a = -\frac{1}{3} \text{ or } \frac{1}{2}$$ ✓✓

Name: Ann

Esperance SHS
Year 12 MATHEMATICS METHODS
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Resource Rich

Total Marks: 29
Reading: 2 minutes Time Allowed: 30 minutes

Question 3 **(7 marks)**

It is known that 15% of Year 12 students in a large country study advanced mathematics.

A random sample of n students is selected from all Year 12's in this country, and the random variable X is the number of those in the sample who study advanced mathematics.

(a) Describe the distribution of X. (2 marks)

$X \sim Bin(n, 0.15)$

OR Binomial with n trials & $p = 0.15$

(b) If $n = 22$, determine the probability that

(i) three of the students in the sample study advanced mathematics. (1 mark)

$P(X = 3) = 0.2370$

(ii) more than three of the students in the sample study advanced mathematics. (1 mark)

$P(X \geq 4) = 0.4248$

(iii) none of the students in the sample study advanced mathematics. (1 mark)

$P(X = 0) = 0.0280$

(c) If ten random samples of 22 students are selected, determine the probability that at least one of these samples has no students who study advanced mathematics. (2 marks)

should be different letter but..?

$p = 0.0280 \quad n = 10$

$1 - P(X = 0)$

$= 0.2473$

Question 4 **(14 marks)**

(a) Determine the mean of a Bernoulli distribution with variance of 0.24. (3 marks)

$p(1-p) = 0.24$

$p = 0.4$ or 0.6.

$\therefore$ mean is 0.4 or 0.6.

(b) A Bernoulli trial, with probability of success p, is repeated n times. The resulting distribution of the number of successes has an expected value of 5.76 and a standard deviation of 1.92. Determine n and p. (4 marks)

$np = 5.76$

$npq = 1.92^2$

$\therefore np(1-p) = 1.92^2$

$\therefore 5.76(1-p) = 1.92^2$

$1-p = 0.64$

$\therefore p = 0.36$

$\therefore n = 16$.

(c) The probability that a student misses their bus to school is 0.2, and the probability that they miss the bus on any day is independent of whether they missed it on the previous day.

Over five consecutive weekdays, what is the probability that the student

(i) only misses the bus on Tuesday? (2 marks)

$0.2 \times 0.8^4 = 0.08192$

$5C_1 \times 0.2 \times 0.8^4$ 1 month.

(ii) misses the bus at least twice? (2 marks)

$n = 5 \quad p = 0.2 \qquad X \sim Bi(5, 0.2)$

$P(X \geq 2) = 0.2627$ ↑ not required.

(iii) misses the bus on Tuesday and on two other days? (3 marks)

$n = 4 \quad p = 0.2$

$P(X = 2) = 0.1536$. i.e. from 4 days must miss 2.

$P = 0.2 \times 0.1536$

$= 0.0307$.

Question 5 **(8 marks)**

The discrete random variable Y has the probability distribution shown in the table below.

y	-2	-1	0	1	2
$P(Y=y)$	0.4	0.2	0.1	0.1	0.2

(a) Determine $P(Y \geq 0 \mid Y \leq 1)$. (2 marks)

$= \frac{0.2}{0.8}$

$= \frac{1}{4}$

(b) Calculate

(i) $E(Y)$. (2 marks)

$-2\times0.4 + -1\times0.2 + 1\times0.1 + 2\times0.2$

$= -0.5$

(ii) $E(1-2Y)$. (1 mark)

$= 1 - 2(-0.5)$

$= 2$

(c) Calculate

(i) $Var(Y)$. (2 marks)

$= 2.45 \quad \left(\frac{49}{20}\right)$

(ii) $Var(1-2Y)$. (1 mark)

$= 2.45 \times 2^2$

$= 9.8$