GREENWOOD COLLEGE

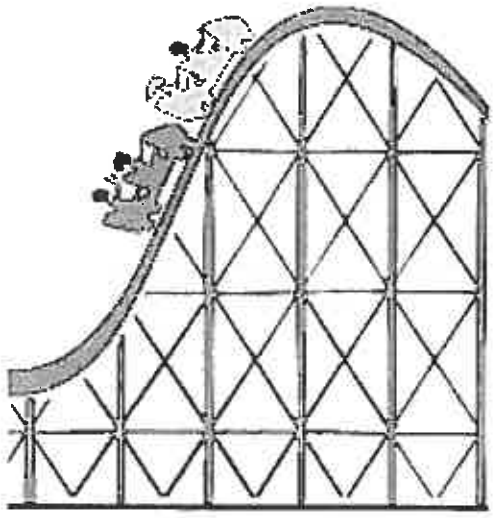
Mathematics Methods Unit 3 2019
Investigation 1: Roller Coaster Design

Name: Marking Key **Mark** ____ **/ 28**

The mark for this section will constitute 50% of the total investigation mark. Notes will not be allowed in this section, however calculators will be allowed.
Time allowed 40 minutes.

Part C

Answer the questions below in the spaces provided.



Question 1 **[15 marks]**

Your first task is to design an *ascent* and *drop* for the first section of roller coaster track.

You will connect two straight stretches of track, $y = L_1(x) = m_1 x + k_1$ and $y = L_2(x) = m_2 x + k_2$, with part of a parabola $y = f(x) = ax^2 + bx + c$, where x and $f(x)$ are measured in metres.

The slope of the *ascent* is 0.74 starting from the origin O and the slope of the *drop* is - 1.4.

Determine the equations of the *three segments* of track indicated below for the given domains, showing all appropriate steps of logic in the space below and on the next page:

$$L_1(x) = m_1 x + k_1 \text{ for } 0 \leq x \leq 9$$

$$f(x) = ax^2 + bx + c \text{ for } 9 < x < 15$$

$$L_2(x) = m_2 x + k_2 \text{ for } x \geq 15$$

Track ①

$y = 0.74x$ ✓

At $x = 9$ $y = 0.74 \times 9$

$= 6.66$ ✓

Track ② $f(x) = ax^2 + bx + c$

$\therefore f'(x) = 2ax + b$ ✓

$f'(9) = 2a(9) + b = 0.74$.

$f'(15) = 2a(15) + b = -1.4$

Simultaneous Equations based on gradient function at joining points

So $18a + b = 0.74$ ✓

$30a + b = -1.4$ ✓

Solve gives $a = \frac{-107}{600}$ (or $0.178\overline{3}$) ✓, $b = 3.95$ (or $\frac{79}{20}$) ✓

so $y = -\frac{107}{600}x^2 + 3.95x + C$

for $(9, 6.66)$ $6.66 = -\frac{107}{600}(9^2) + 3.95(9) + C$ ✓

$\Rightarrow C = -14.445$ ✓

so $y = -\frac{107}{600}x^2 + 3.95x - 14.445$ ✓

When $x = 15$, $y = 4.68$ ✓

Track 3 $y = -1.4x + C$ ✓

So $4.68 = -1.4 \times 15 + C$ ✓

$\Rightarrow C = 25.68$ ✓

so $y = -1.4x + 25.68$ ✓

Question 2 **[8 marks]**

Join the same two linear functions in Question 1 with a cubic function this time, given that the top point of the curve between the X-values of 9 and 15 is the point (11,8.2).

$f(x) = ax^3 + bx^2 + cx + d$

$\therefore f'(x) = 3ax^2 + 2bx + c$ ✓

points are $(9, 6.66)$, $(15, 4.68)$ and $(11, 8.2)$

So $0.74 = 3\times 9^2 a + 2\times 9b + c$ ✓

$-1.4 = 3\times 15^2 a + 30b + c$ ✓

$0 = 3\times 11^2 a + 22b + c$ ✓

ie. $\left.\begin{aligned} 243a + 18b + c &= 0.74 \\ 675a + 30b + c &= -1.4 \\ 363a + 22b + c &= 0 \end{aligned}\right\}$ Solve simultaneously

gives $a = \frac{1}{900}$, $b = \frac{-131}{600}$, $c = \frac{22}{5}$ ✓

$(0.00\dot{1})$ $(0.218\dot{3})$ (4.4)

Put into one point to find d. ✓

$6.66 = \frac{1}{900}\times 9^3 - \frac{131}{600}\times 9^2 + \frac{22}{5}\times 9 + d$

$\therefore d = -16.065$ ✓

(Using $(11, 8.2)$ $d = -15.26055556$)

So $y = \frac{1}{900}x^3 - \frac{131}{600}x^2 + \frac{22x}{5} - 16.065$ ✓

Question 3 **[2,1,1,1 = 5 marks]**

a) State two factors that reduce the speed of the roller coaster as it goes down the track?

friction ✓
air resistance ✓ (or gravity, when car going uphill.)

b) What force keeps the roller coaster on the track as it goes through a loop?

centrifugal force ✓

c) Under what circumstances might a negative ordinate (Y-axis) value be possible?

track goes underground (eg. in a tunnel) ✓

d) Where should the highest point of the track be?

at the start of the first descent ✓

END OF VALIDATION