Greenwood College
Year 12 Applications
Test 6 2019
Resource-Free

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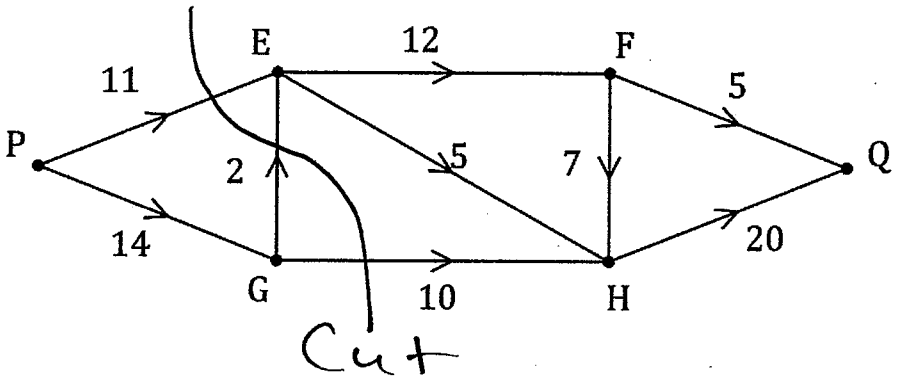
Name.....

No calculators nor notes allowed.
27 mark total.

Formula sheet allowed.
30 minute time limit.

Question 1 **(5 marks)**

The network shows a system of pipes with the maximum capacity for each pipe, in litres per second, shown on the edges.



(a) Determine the maximum flow through the system from P to Q. (3 marks)

PEFQ=5
PEFHQ=6
PGEHQ=2
PGHQ=10
Total 23L/s ✓

At least 3 paths with correct flow contribution. ✓
All paths with correct flow contribution. ✓

(b) Show the cut on the network that has capacity equal to the maximum flow. (2 marks)

See above. ✓✓

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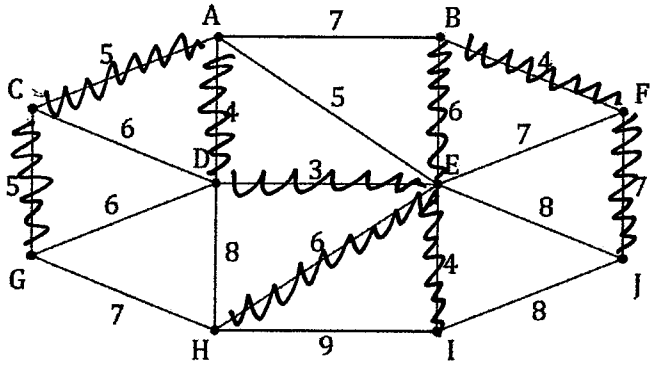
Question 2 **(5 marks)**

Cabling between ten distribution boards in a factory is to be upgraded to ensure the supply of electricity between all boards in an emergency. The upgrade costs between adjacent boards, in thousands of dollars, are shown on the edges in the weighted graph.



(a) Determine the minimum spanning tree for the graph, clearly showing it on the graph. (3 marks)

Tree. ✓
At least 7 correct edges. ✓
All correct edges. ✓

(b) Calculate the cost of upgrading the cabling that forms the minimum spanning tree. (2 marks)

Sum of edges = 44 ✓

Upgrade cost is $44 000. ✓

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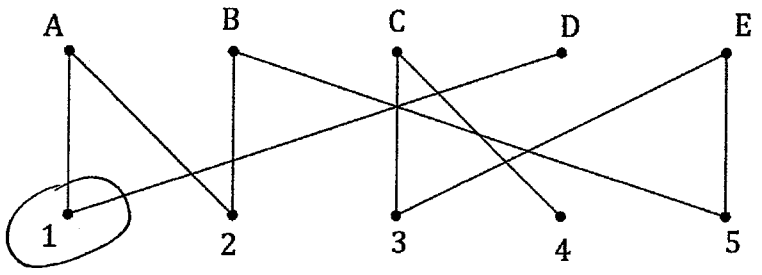
Question 3 **(7 marks)**

Five people, A, B, C, D and E are to be allocated to five tasks, $1, 2, 3, 4$ and 5. The bipartite graph below shows the tasks that each of the five people can carry out.



(a) Does the graph contain a bridge? Explain your answer. (2 marks)

Yes. ✓

If any of the edges in the graph were removed then the graph would no longer be connected. ✓

(b) Explain why the graph is not a complete bipartite graph. (1 mark)

All vertices from top set are not connected to all vertices in bottom set. ✓

(c) If person B is assigned to task 2, explain why a complete matching of people to tasks is not possible. (2 marks)

If B is assigned to task 2, then A must be assigned to task 1. ✓ However, task 1 is the only task that D can do and so either A or D cannot do a task.

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(d) Determine a complete matching of people to tasks. (2 marks)

A → 2 E → 3
B → 5
C → 4
D → 1

✓ for D1 and E3, rest wrong.

Question 4 **(10 marks)**

The following table shows the scores of four people, Peta, Qi, Ro and Sam after taking four tests in accounting (A), economics (E), finance (F) and law (L).

	Peta	Qi	Ro	Sam
A	11	15	10	14
E	11	16	13	12
F	12	13	14	11
L	10	12	11	12

Each of the four people are to be assigned to one of the four tests so that the total score is maximised. No-one can be assigned to more than one test.

(a) Explain why the Hungarian algorithm may be used to find the optimal assignment if each number in the table, n, is replaced by $16 - n$. (2 marks)

The Hungarian algorithm is used to find the minimum ✓ assignment. To maximise we must minimise the difference from the largest score (16). ✓

(b) Form a new table by replacing each number in the original table, n, with $16 - n$. (1 mark)

	Peta	Qi	Ro	Sam
A	5	1	6	2
E	5	0	3	4
F	4	3	2	5
L	6	4	5	4

(c) Use the Hungarian algorithm to determine how each of the people should be assigned to the four tests to maximise their total score, and state what this maximum score is. (5 marks)

Reduce rows.

	Peta	Qi	Ro	Sam
A	4	0	5	1
E	5	0	3	4
F	2	1	0	3
L	2	0	1	0

Reduce columns.

	Peta	Qi	Ro	Sam
A	2	0	5	1
E	3	0	3	4
F	0	1	0	3
L	0	0	1	0

Subtract 1 from uncovered and add 1 to those covered twice.

	Peta	Qi	Ro	Sam
A	1	0	4	0
E	2	0	2	3
F	0	2	0	3
L	0	1	1	0

	Peta	Qi	Ro	Sam
A				
E				
F				
L				

5

	Peta	Qi	Ro	Sam
A				
E				
F				
L				

Peta → L Ro → F ✓
Qi → E Sam → A

(d) A statistics test score (S) is recorded for each student. To apply the Hungarian algorithm again what has to be done to the matrix? (2 marks)

Another student's results ✓ are needed (add a column). of 0's ✓

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