education equals

Calculator Assumed
Topic: Mixed Trigonometry Applications

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [5 marks]

The area of an obtuse isosceles triangle with equal side lengths of 12 cm is $55.1552\,cm^2$.

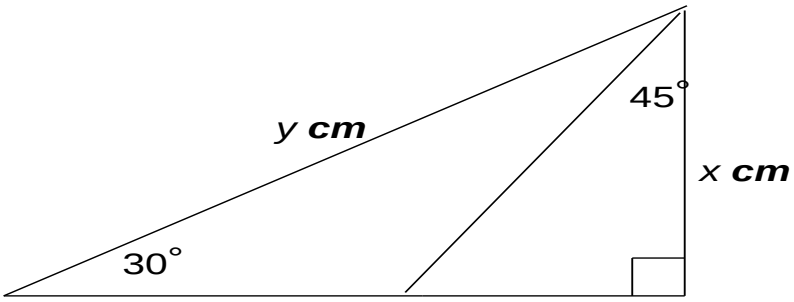
Determine the length of the third side.

Question Two: [3 marks]

Prove that $\tan(t-\frac{\pi}{4}) = \frac{\tan t - 1}{\tan t + 1}$

Question Three: [6 marks]

Show, with algebraic reasoning, that $y = 2x$



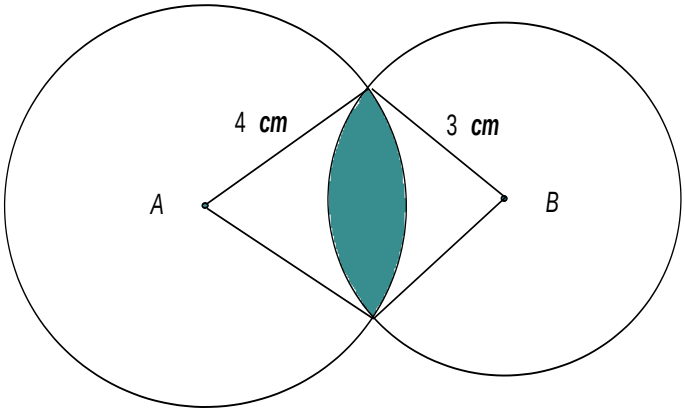
Question Four: [6 marks]

A and B are both acute angles such that $\sin A = \frac{5}{13}$ and $\tan B = \frac{3}{4}$.

Determine the exact value of $\cos(A - B)$

Question Five: [8 marks]

Calculate the area of the shaded region if it is known that the length of AB is 5 cm.



Question Six: [1, 6, 4 = 11 marks]

A cyclic quadrilateral, ABCD, has $\angle DAB = 100^\circ$, $AB = 10cm$, $AD = 9cm$ and $BC = 8cm$.
Calculate:

(a) the size of angle $\angle BCD$.

(b) the size of angle $\angle ADC$.

(c) the area of the quadrilateral.

Question Seven: [4, 2 = 6 marks]

A Ferris wheel has a diameter of 40m, with the centre 22m above the ground.
Customers riding the Ferris wheel, climb a few stairs and get on a carriage at the lowest point of the wheel.
The wheel makes one rotation every 120 seconds.

(a) Draw a sketch of one rotation of the Ferris wheel from when a customer steps onto the ride.

(b) Hence or otherwise find the cosine equation of the graph.

SOLUTIONS
Calculator Assumed
Topic: Mixed Trigonometry Applications

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [5 marks]

The area of an obtuse isosceles triangle with equal side lengths of 12 cm is $55.1552\,cm^2$.

Determine the length of the third side.

$55.1552 = \frac{1}{2} \times 12^2 \times \sin\theta$ ✓

$\theta = 50^\circ, 130^\circ$ ✓

$x^2 = 12^2 + 12^2 - 2 \times 12 \times 12 \times \cos 130^\circ$ ✓ ✓

$x = 21.75cm$ ✓

Question Two: [3 marks]

Prove that $\tan(t - \frac{\pi}{4}) = \frac{\tan t - 1}{\tan t + 1}$

$LHS : \tan(t - \frac{\pi}{4})$ ✓

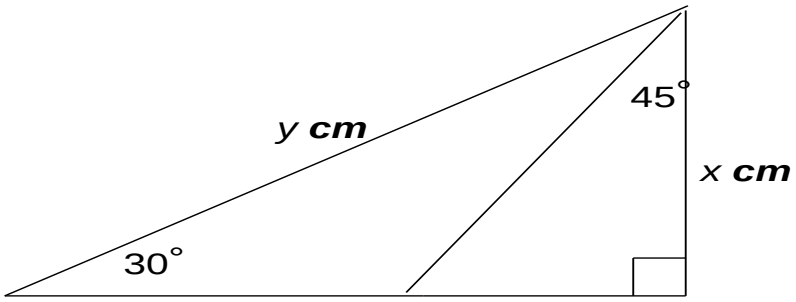
$= \frac{\tan t - \tan\frac{\pi}{4}}{1 + \tan t \tan\frac{\pi}{4}}$ ✓

$= \frac{\tan t - 1}{1 + \tan t} = RHS$

✓

Question Three: [6 marks]

Show, with algebraic reasoning, that $y = 2x$



$h = \sqrt{x^2 + x^2} = \sqrt{2}x$

$\frac{y}{\sin 135^\circ} = \frac{\sqrt{2}x}{\sin 30^\circ}$

$y = \frac{\sqrt{2}x \times \frac{1}{\sqrt{2}}}{\frac{1}{2}} = 2x$

Question Four: [6 marks]

A and B are both acute angles such that $\sin A = \frac{5}{13}$ and $\tan B = \frac{3}{4}$.

Determine the exact value of $\cos(A - B)$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

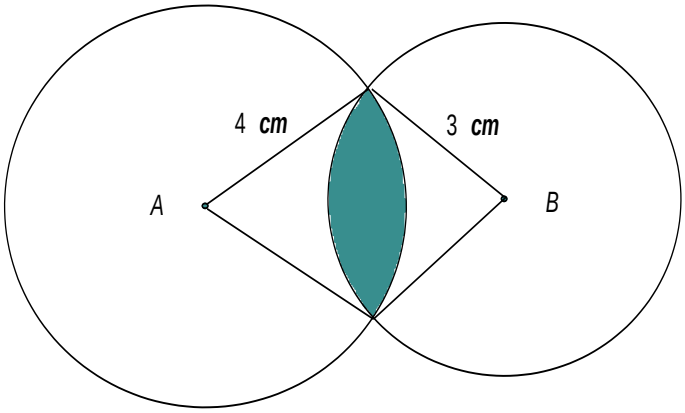
$= \frac{12}{13} \times \frac{4}{5} + \frac{5}{13} \times \frac{3}{5}$

$= \frac{48 + 15}{65}$

$= \frac{63}{65}$

Question Five: [8 marks]

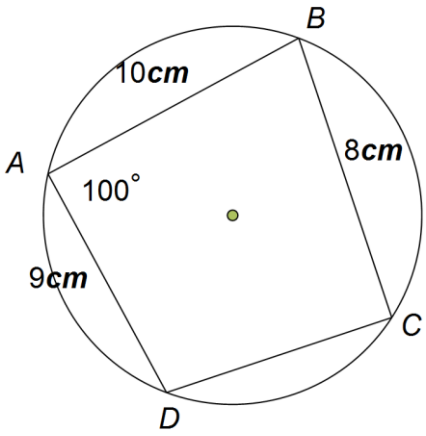
Calculate the area of the shaded region if it is known that the length of AB is 5 cm.



$$A = (\frac{1}{2} \times 4^2 \times 1.287 - \frac{1}{2} \times 4^2 \times \sin 1.287^R) + (\frac{1}{2} \times 3^2 \times 1.855 - \frac{1}{2} \times 3^2 \times \sin 1.855^R)$$
$$= 34.82 cm^2$$

Question Six: [1, 6, 4 = 11 marks]

A cyclic quadrilateral, ABCD, has $\angle DAB = 100^\circ$, $AB = 10cm$, $AD = 9cm$ and $BC = 8cm$.
Calculate:



(a) the size of angle $\angle BCD$.

80° ✓

(b) the size of angle $\angle ADC$.

$x^2 = 10^2 + 9^2 - 2\times 10\times 9\times \cos 100^\circ$ ✓

$x = 14.57cm$ ✓

$\frac{\sin\theta}{10} = \frac{\sin 100}{14.57}$ ✓

$\theta = 42.53^\circ$ ✓

$\frac{\sin\alpha}{8} = \frac{\sin 80}{14.57}$ ✓

$\alpha = 32.73^\circ$

$\theta + \alpha = 75.26^\circ$ ✓

(c) the area of the quadrilateral.

$A = \frac{1}{2}\times 10\times 9\times \sin 100 + \frac{1}{2}\times 8\times 13.64\times \sin 80$ ✓✓✓

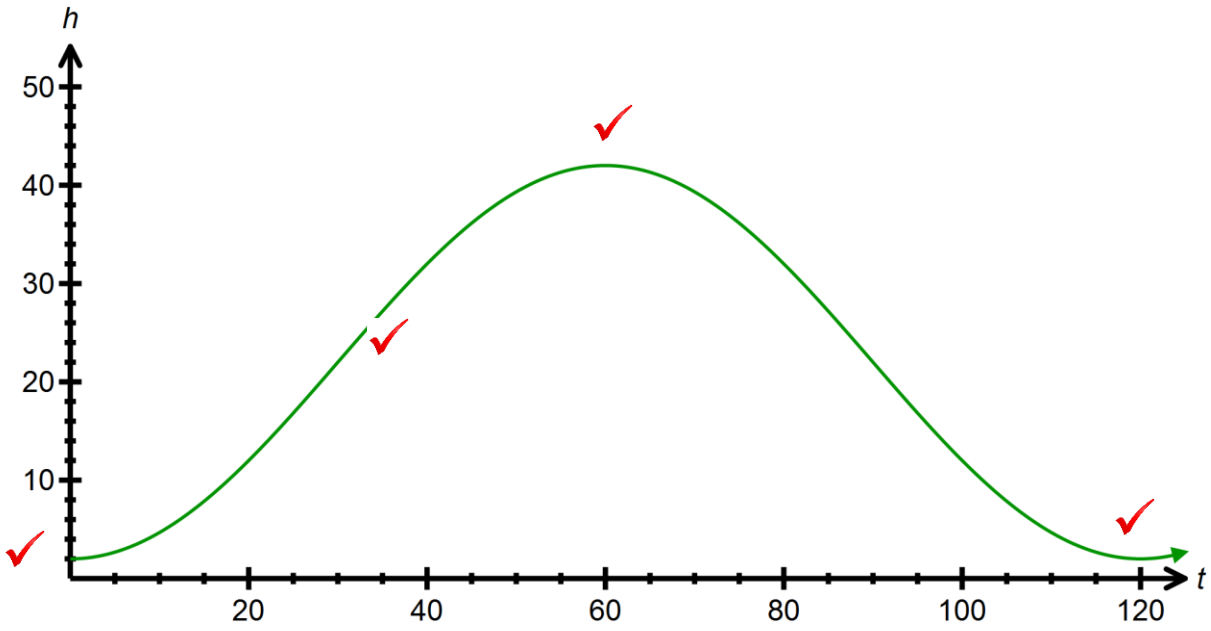
$= 98.05cm^2$

✓

Question Seven: [4, 2 = 6 marks]

A Ferris wheel has a diameter of 40m, with the centre 22m above the ground.
Customers riding the Ferris wheel, climb a few stairs and get on a carriage at the lowest point of the wheel.
The wheel makes one rotation every 120 seconds.

(a) Draw a sketch of one rotation of the Ferris wheel from when a customer steps onto the ride.



(b) Hence or otherwise find the cosine equation of the graph.

$$120 = \frac{2\pi}{k}$$

$$k = \frac{\pi}{60}$$

$$H = -20\cos\left(\frac{\pi t}{60}\right) + 22$$