



Yr 12 METHODS TEST 3 2018

**CALCULUS INVOLVING
EXPONENTIALS, LOGARITHMS
AND TRIGONOMETRIC FUNCTIONS**

Time: 25 minutes

Total: 24 marks

Student Name: _____

Teacher: _____

Instructions: *Show all working clearly.*

Sufficient detail must be shown for marks to be awarded for reasoning.

NO CALCULATOR AND NO PERSONAL NOTES ALLOWED

Question 1. (7 marks)

Determine an exact value for each of the following

a) The gradient of $y = \frac{x^2}{e^x + 1}$ at the point $(2, \frac{4}{e^2 + 1})$ [4]

b) $\int_0^4 \frac{4x+6}{x^2+3x+1} dx$ [3]

Question 2. (7 marks)

a) i) Determine $\frac{dy}{dx}$ if $y = \ln \frac{x^2+3}{x-8}$ [2]

ii) given $\frac{dy}{dx}$ can be written in the form $\frac{ax^2+bx+c}{(x^2+3)(x-8)}$, determine the values of a, b and c . [3]

b) Differentiate $y = [\ln(3x - 4)]^4$, do not simplify your answer [2]

Question 3. (6 marks)

A straight tunnel is going to be hollowed out of a large hill. The height, $h(x)$, of the tunnel at any distance, x metres, from the left hand edge can be modelled by the relationship;

$$h(x) = 6\sin(3x) \quad , \quad 0 \leq x \leq k$$

- a) If the x axis represents ground level, determine the value of k in radians. [1]

- b) Showing use of integration, determine the area of the tunnel's opening. [3]

- c) It is planned that 3 identical tunnels are to be cut from the same hill. If all 3 tunnels are 450m long, determine the amount of earth that will need to be removed to create the tunnels. [2]

Question 4. (4 marks)

Determine the exact value of $\int_0^{\frac{\pi}{3}} \sin x \cos x \, dx$

-----END OF SECTION A-----



Yr 12 METHODS TEST 3 2018

**CALCULUS INVOLVING
EXPONENTIALS, LOGARITHMS
AND TRIGONOMETRIC FUNCTIONS**

Time: 30 minutes

Total: 26 marks

Student Name: _____

Teacher: _____

Instructions: *Show all working clearly.*

Sufficient detail must be shown for marks to be awarded for reasoning.

CALCULATOR AND 1 PAGE OF PERSONAL NOTES ALLOWED

Question 5. (8 marks)

A group of biologists has decided that colonies of a native Australian animal are in danger if their populations are less than 1000. One such colony had a population of 2300 at the start of 2011. The population was growing continuously such that $\frac{dP}{dt} = 0.065P$, where P is the number of animals in the colony t years after the start of 2011.

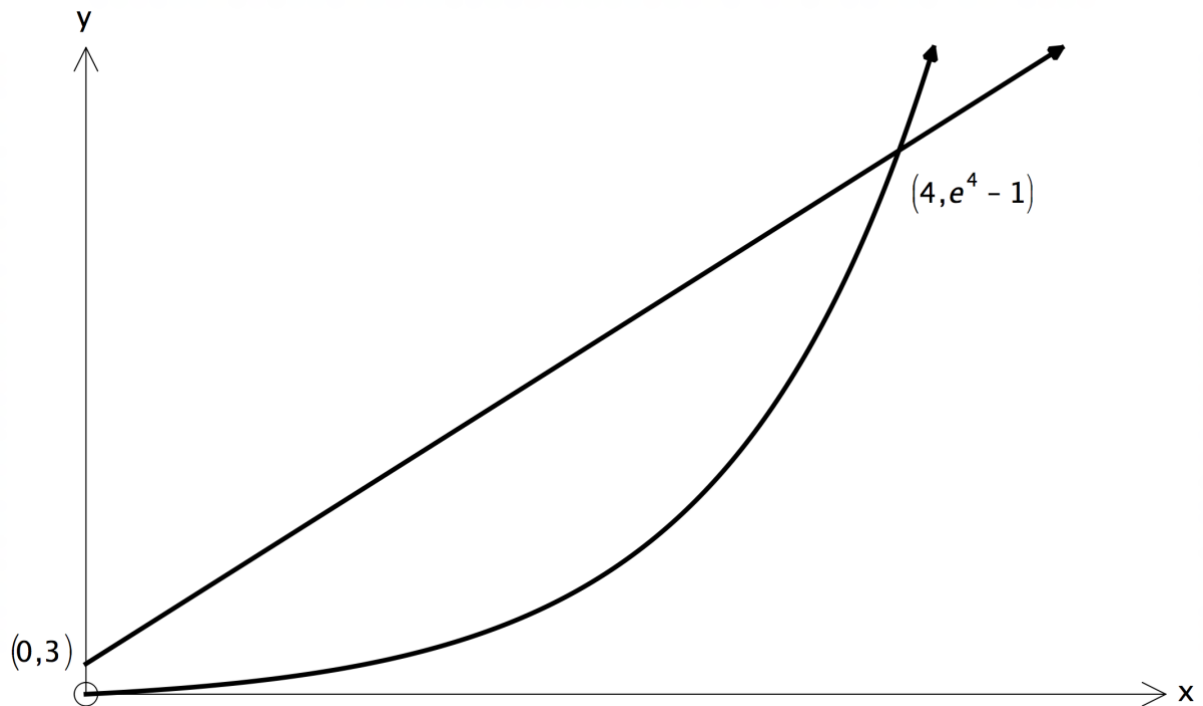
- a) Determine, to the nearest 10 animals, the population of the colony at the start of 2014. [2]

- b) Determine the rate of change of the colony's population when $t = 2.5$ [2]

- c) At the beginning of 2017 a disease caused the colony's population to decrease continuously at the rate of 8.25% of the population per year. If this rate continues, when will the colony become 'in danger'? Give your answer to the nearest month. [4]

Question 6. (8 marks)

Consider the graph of two functions below. The curve can be described by $g(x) = e^x - 1$.



- a) Show that the gradient of the line is $\frac{e^4}{4} - 1$. [2]

- b) Write an integral that could be used to determine the area enclosed by the two graphs and the y axis. [3]
Note: there is no need to calculate this integral.

- c) A vertical line is added to the graph with equation $x = k$, given $0 \leq k \leq 4$. If the area bounded by this line and the two graphs is 8 square units, determine the value of k . [3]

Question 7. (10 marks)

A particle travels in a straight line back and forth past a fixed origin such that its displacement in metres at time t seconds is given by;

$$x(t) = 2 + 3\sin 2t$$

a) What is the maximum distance the particle gets from the origin? [1]

b) Determine how many times the particle crosses the origin in the first 2π seconds. [2]

c) Showing use of calculus; [4]

i) determine the velocity of the particle at $t = 10$.

ii) determine the distance travelled by the particle in the first 80 seconds.

d) if $\int_0^\pi x(t)dt = k$, determine $\int_0^{10\pi} [x(t) + 3]dt$ in terms of k . [3]