

Name:

SOLUTIONS



Year 12 ATAR Applications

Test 2, 2023

<div><div>58</div><div>57</div></div> <div>= %</div>

Topics: Geometric Progressions First Order Sequences and Graph Theory

Total Time: 50 minutes

Weighting: 11% of the year.

This test comprises of **TWO sections**. The **first section** is **calculator free** where no calculators of any kind are to be used. The **second section** is **calculator assumed** where up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted. All questions must be answered in both sections. **Answers should be rounded appropriately**. All working should be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.

SECTION 1: CALCULATOR FREE

Time: 25 minutes	Marks for Section 1: 29
	Equipment Allowed: Formula Sheet

Please circle your teacher's surname

Cartwright

Hackett

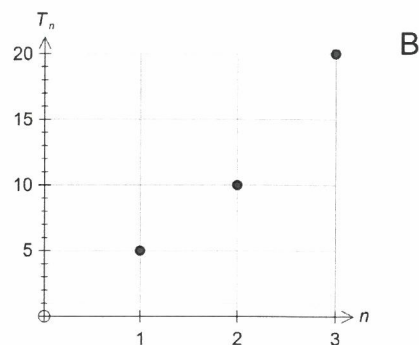
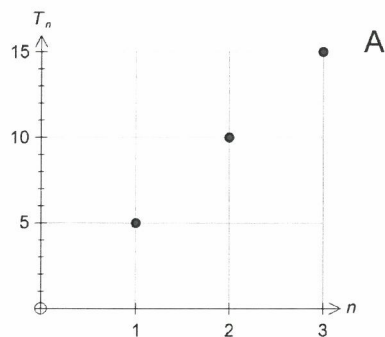
Jones

Kumar

Pope

Question 1 (2, 2, 1, 2 = 7 marks)

The graphs of A and B represent two sequences.



- (a) What type of sequence is B? Give a reason.

Geometric Sequence ✓
Common ratio, $r = \frac{10}{5} = \frac{20}{10} = 2$ ✓

- (b) State the recursive definition for B.

$T_{n+1} = 2T_n$ ✓, $T_1 = 5$ ✓

- (c) State true or false: $d_A > r_B$

d represents the common difference
 r represents the common ratio

True ✓

- (d) Circle the rule(s) for graph B.

☒ (i) $T_n = 5(2)^{n-1}$ ✓

(ii) $T_n = 2(5)^{n-1}$

☒ (iii) $T_n = 2.5(2)^n$ ✓

(iv) $n = 2.5(2)^{T_n}$

7

Question 2 (3 marks)

If a geometric sequence has $T_1 = 2$ and $T_5 = 162$ find the value of T_4 .

$$\begin{aligned}T_5 &= ar^4 \\162 &= 2 \times r^4 \\r^4 &= 81 \quad \checkmark \\r &= \pm 3\end{aligned}$$

$$\begin{aligned}\text{If } r &= 3 \\T_4 &= \frac{162}{3} \\&= 54 \quad \checkmark\end{aligned}$$

$$\begin{aligned}\text{If } r &= -3 \\T_4 &= \frac{162}{-3} \\&= -54 \quad \checkmark\end{aligned}$$

Question 3 (2 marks)

Discuss the long term behaviour of the sequence defined as:

$$b_{n+1} = 0.4b_n - 1, \quad b_1 = 3 \quad \text{for } n \geq 1$$

$$\text{let } b_{n+1} = b_n = x$$

$$x = 0.4x - 1 \quad \checkmark$$

$$0.6x = -1$$

$$x = \frac{-1}{0.6}$$

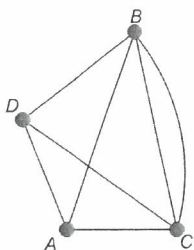
$$= \frac{-10}{6}$$

$$= -\frac{5}{3}$$

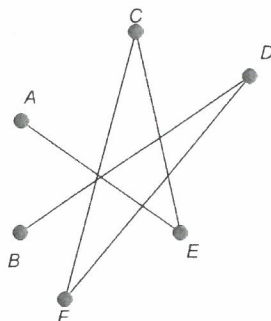
The sequence will approach $-\frac{5}{3}$ ✓

Question 4 (3 marks)

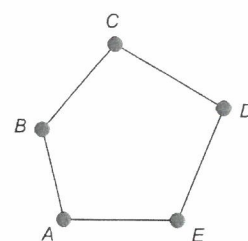
Graph 1



Graph 2



Graph 3



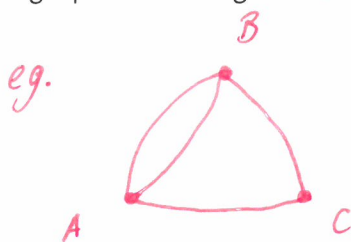
Complete the following table insert \checkmark (yes) or X (no) for the graphs shown.

	SIMPLE	COMPLETE	CONNECTED	PLANAR
Graph 1	X	\checkmark	\checkmark	X
Graph 2	\checkmark	X	\checkmark	X
Graph 3	\checkmark	X	\checkmark	\checkmark

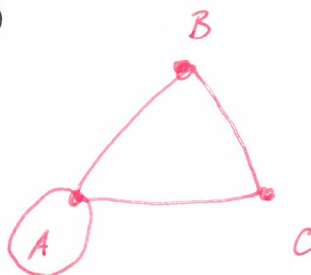
Question 5 (1, 2, 1, 2 = 6 marks)

Construct a graph that meets the following requirements. Clearly label the vertices.

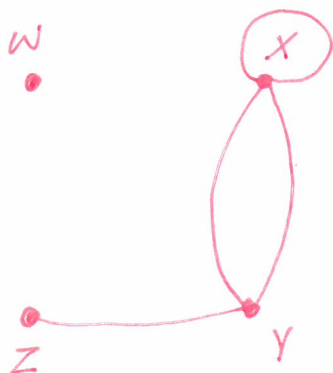
- (a) A graph with 4 edges and 3 vertices (A, B, C)



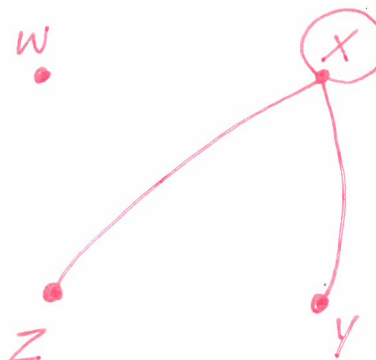
or



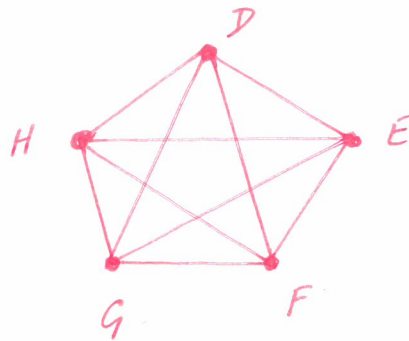
- (b) A graph with 4 vertices (W, X, Y, Z) where W is an isolate vertex, there is a loop at X, vertex X has an order of 4, vertices Y and Z are both odd vertices.



or

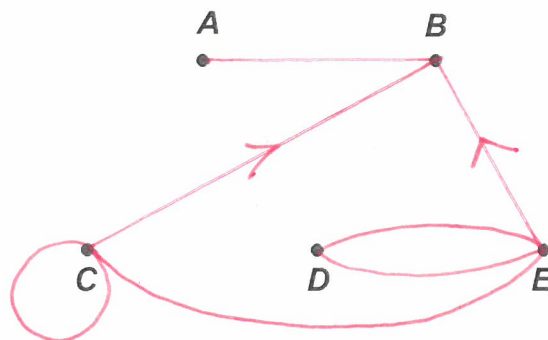


- (c) A complete graph with 5 vertices (D, E, F, G, H).



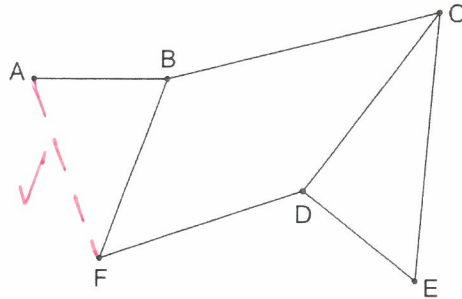
- (d) A graph that is described by the adjacency matrix:

		TO				
		A	B	C	D	E
FROM	A	0	1	0	0	0
	B	1	0	0	0	0
	C	0	1	1	0	1
	D	0	0	0	0	2
	E	0	1	1	2	0



Question 6 (1, 2, 2, 2, 1 = 8 marks)

This map shows the connecting passageways between Seaside Senior High School's main offices.



- (a) State the degree of vertex D.

3 ✓

- (b) State which of the edges of the network form(s) a bridge. Explain what a bridge is.

AB ✓

If AB is removed, A becomes isolated ✓
or disconnected

- (c) Is the graph Eulerian, semi-Eulerian or neither? Explain clearly.

Neither ✓

There are 4 odd vertices ✓

- (d) Is the graph semi-Hamiltonian, Hamiltonian or neither? Explain clearly.

Semi-Hamiltonian ✓

eg. E C D F B A - travel to all vertices ✓
- start/finish diff. vertices

- (f) Add one more edge to the network to make it Hamiltonian.

AF

END OF CALCULATOR FREE SECTION

Question 7 (2, 1, ¹2, 1, 1= ⁶7 marks)

A producer of large mining machinery uses the following equation to model its stock of bulldozers (B_n) where n is the number of years it has been producing bulldozers.

$$B_n = 1.1B_{n-1} - 250 \text{ where } B_0 = 2000$$

(a) Use the model to predict his bulldozer numbers after:

(i) one year.

$$B_1 = 1950 \quad \checkmark$$

(ii) ten years.

$$B_{10} = 1203.12 \\ \approx 1203 \quad \checkmark$$

The model shows that he sells 250 units each year.

(b) The before-sales production seems to be increasing by 10% each year. Why is this assumption misleading?

110% is less than sales \checkmark

(c) At this level of production, how long will the producer be able to maintain this level of sales? Your answer needs to be to the nearest year.

16 years \checkmark

It is considered sensible to maintain the stock level at 2000 units.

(d) How many units can be sold each year?

200 \checkmark

The stock level of large trucks can be modelled by a geometric sequence, with a common ratio of 0.9

(e) Stock levels are decreasing. Explain how this can be determined.

$r < 1 \quad \checkmark$

6

Question 8 (2, 2 = 4 marks)

A particular species of flightless bird lives only on Waddle Island. An introduced species infects the birds with a disease. The number, B_n , of infected birds on the island in week n obeys the equation,

$$B_{n+1} = 2B_n - 12, \text{ for } n = 1, 2, 3, \dots, \text{ where } B_1 = a.$$

- a) The number of infected birds on the island in week 4 is 44. How many birds were infected in the first week?

$$\begin{array}{llll} B_4 = 44 & 44 = 2B_3 - 12 & 28 = 2B_2 - 12 & 20 = 2B_1 - 12 \\ 56 = 2B_3 & 40 = 2B_2 & 32 = 2B_1 & \\ B_3 = 28 & B_2 = 20 & \underline{\underline{B_1 = 16}} & \end{array}$$

- b) Is the sequence generated by the rule for B_n arithmetic, geometric or neither?
Justify your answer.

Neither ✓

$$\begin{array}{l} \text{Not Geom.} \\ \left. \begin{array}{l} \frac{44}{28} = 1.57 \\ \frac{28}{20} = 1.4 \\ \frac{20}{16} = 1.25 \end{array} \right\} \text{no "r"} \end{array}$$

$$\begin{array}{l} \text{Not Arith} \\ \left. \begin{array}{l} 44 - 28 = 16 \\ 28 - 20 = 8 \\ 20 - 16 = 4 \end{array} \right\} \text{no "d"} \end{array}$$

④

Question 9 (2, 1, 1 = 4 marks)

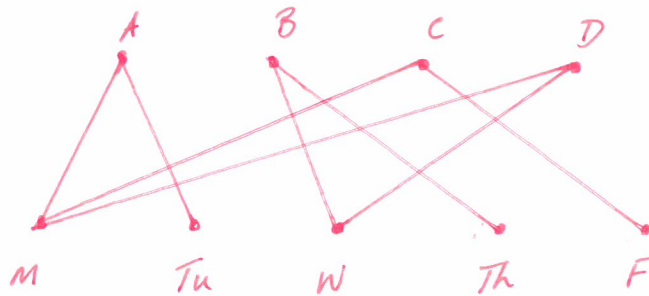
- (a) Draw a suitable graph to represent the following information.
On the five weekdays, Monday to Friday, four children; A, B, C and D, get homework help (HH).

Andrew gets HH on Mondays and Tuesdays.

Ben gets HH on Wednesdays and Thursdays.

Cate gets HH on Mondays and Fridays.

Dan gets HH on Mondays and Wednesdays.



- (b) A complete bipartite graph has nine vertices.

Determine:

- (i) the maximum number of edges.

$$4 \times 5 = 20 \text{ edges} \checkmark$$

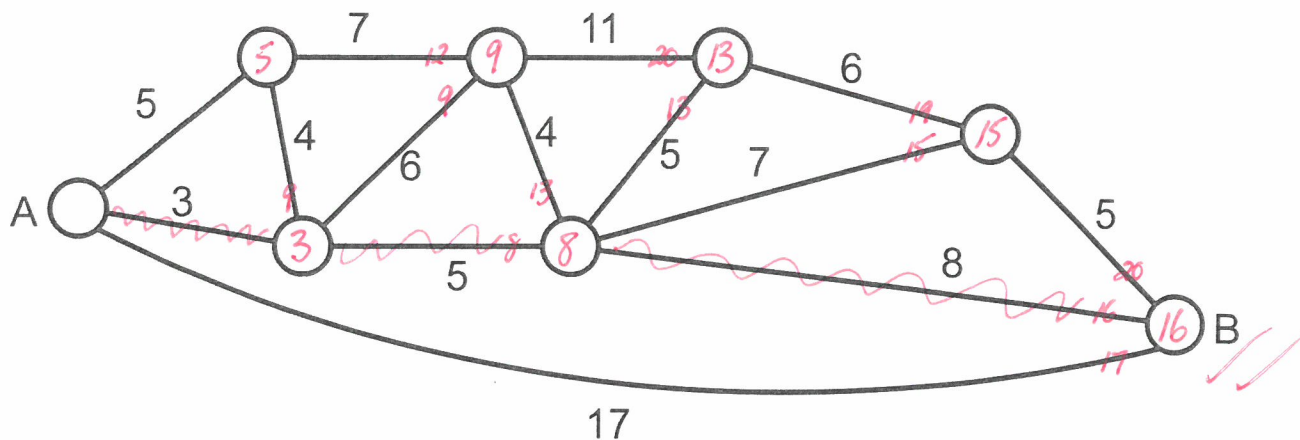


- (ii) the minimum number of edges.

$$8 \times 1 = 8 \text{ edges} \checkmark$$



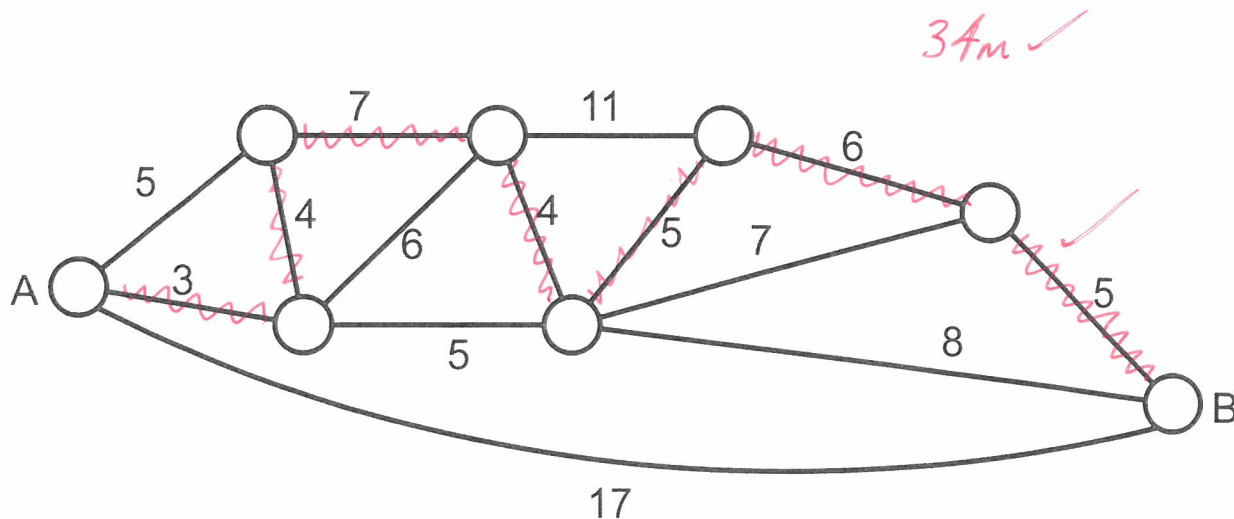
Question 10 (2, 1, 2, 1, = 6 marks)



- (a) (i) Determine the shortest path from A to B.
Show the path on the diagram. Lengths are in metres.
- (ii) State the length of the shortest path from A to B.

16m ✓

- (b) (i) Determine the shortest semi-Hamiltonian path from A to B, state its length and show it on the diagram below.



- (ii) State the length of the shortest Hamiltonian cycle, ABA.

$34m + 17m = 51m$ ✓

⑥

Question 11 (2, 2, 1, 3 = 8 marks)

Teams from various suburbs are in group in a basketball competition where they play each other exactly once. The teams are from Applecross (A), Brentwood (B), Carine (C), Dianella (D) and Ellenbrook (E).

The results of the games are:

- A defeated B, C and E
- B defeated D
- C defeated B and E
- D defeated A and C
- E defeated B and D.

- (a) Draw an adjacency matrix (M) to show this information. Use a "1" to indicate a win and a "0" to indicate a loss.

	A	B	C	D	E
A	0	1	1	0	1
B	0	0	0	1	0
C	0	1	0	0	1
D	1	0	1	0	0
E	0	1	0	1	0

- (b) Complete the matrix M^2 which is started for you below.

	A	B	C	D	E
A	0	2	0	2	1
B	1	0	1	0	0
C	0	1	0	2	0
D	0	2	1	0	2
E	1	0	1	1	0

④

- (c) What does the matrix M^2 represent in relation to the original statement at the top of this question?

M^2_{ij} means team "i" beat a team who beat team "j" ✓

- (d) The entry in row 1 column 2 of the above matrix M^2 (bolded), is interpreted as:

"Team A had 2 two stage wins against team B." ✓

These are described as:

"A beat C who beat B" and "A beat E who beat B." ✓ ✓

④

END OF CALCULATOR ASSUMED SECTION