SCOTCH
COLLEGE

12 Methods
Test 3
2019

Section 1

Calculus of Exponential, Logarithmic and Trigonometric Functions

Time: 20 minutes **Total: 20 marks**

Student Name: Solutions **Teacher: Foster / Giese / Reyhani**

Instructions: *Show all working clearly.*
Sufficient detail must be shown for marks to be awarded for reasoning.
NO CALCULATOR AND NO PERSONAL NOTES ALLOWED

Question 1. **(7 marks)**

Differentiate the following functions. Simplify fully where possible.

a) $\frac{d}{dx}\left(\ln\left(\frac{x}{2}\right)\right)^3$ **[3]**

$= 3\left[\ln\left(\frac{x}{2}\right)\right]^2 \times \frac{\frac{1}{2}}{\frac{x}{2}}$

$= \frac{3\left[\ln\left(\frac{x}{2}\right)\right]^2}{x}$

* 1 mark for $\ln\left(\frac{x}{2}\right)^2$

* 1 mark for $3\left(\frac{\left(\frac{1}{2}\right)}{\left(\frac{x}{2}\right)}\right)$

* 1 mark for fully simplifying

b) $\frac{d}{dx}\sin^4(\pi - x^3)$ **[2]**

$= 4(-3x^2)\cos(\pi - x^3)\sin^3(\pi - x^3)$

$= -12x^2\cos(\pi - x^3)\sin^3(\pi - x^3)$

* 1 mark for $4\sin^3(\pi - x^3)$ or $-3x^2\cos(\pi - x^3)$

* 1 mark for correct answer

c) $\frac{d}{dx} xe^{2x^5}$ **[2]**

$= x(10x^4 e^{2x^5}) + e^{2x^5}$

$= 10x^5 e^{2x^5} + e^{2x^5}$

Question 2. **(10 marks)**

Determine the following.

a) $\int \frac{24-12x}{x^2-4x+1}\,dx$ [2]

$= -6\ln|x^2-4x+1|+c$

* 1 mark for $\ln(x^2-4x+1)$
* 1 mark for correct answer

b) $\int \sin 3x \cos 3x\,dx$ [2]

$= \frac{\sin^2 3x}{6} + c$

OR

$-\frac{\cos^2 3x}{6} + c$

* 1 mark for $\sin^2 3x$ or $\cos^2 3x$
* 1 mark for correct answer

c) $\int_0^1 \frac{\cos 5x}{\sin 5x}\,dx$ [3]

$= \frac{1}{5}\left[\ln(\sin 5x)\right]_0^1$

$= \frac{1}{5}\left(\ln(\sin 5(1)) - \ln(\sin 5(0))\right)$

$= \frac{1}{5}\left(\ln(\sin 5) - \ln(\sin 0)\right)$

$\ln 0$ is undefined

d) $\int_{-1}^{0} \frac{e^{2x}+e^{4x}}{e^{3x}}\,dx$ [3]

$= \int_{-1}^{0} (e^{-x}+e^{x})\,dx$

$= \left[-e^{-x}+e^{x}\right]_{-1}^{0}$

$= (-e^{-0}+e^{0}) - (-e^{-(-1)}+e^{-1})$

$= (-1+1) - (\frac{1}{e} - e)$

$= e - \frac{1}{e}$

Question 3. **(3 marks)**

Find the equation of the tangent to the curve $y = \ln(3x^2)$ at the point $\left(\frac{e}{3}, \ln\left(\frac{e^2}{3}\right)\right)$.

$$\frac{dy}{dx} = \frac{6x}{3x^2}$$

$$= \frac{2}{x}$$

When $x = \frac{e}{3}$, $\frac{dy}{dx} = \frac{2}{\left(\frac{e}{3}\right)} = \frac{6}{e}$

Tangent:

$$\ln\left(\frac{e^2}{3}\right) = \frac{6}{e}\left(\frac{e}{3}\right) + c$$

$$c = \ln\left(\frac{e^2}{3}\right) - 2$$

$$\therefore y = \frac{6}{e}x + \ln\left(\frac{e^2}{3}\right) - 2$$

OR

$$y = \frac{6}{e}x - \ln 3$$

END OF SECTION

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Section 2

Calculus of Exponential,
Logarithmic and
Trigonometric Functions

Time: 25 minutes **Total: 25 marks**

Student Name: Solutions **Teacher: Foster / Giese / Reyhani**

Instructions: *Show all working clearly.*
Sufficient detail must be shown for marks to be awarded for reasoning.
CALCULATOR AND 1xA4 PAGE PERSONAL NOTES ALLOWED

Question 4. **(6 marks)**

The water level of the ocean (in metres) in Broome t hours after midnight on a particular day is given by the equation $w(t) = 9.5\sin(\frac{\pi}{6}(t+4)) + 9.9$.

a) State the initial water level at midnight on this particular day **[1]**

18.13 m

b) Find the water level at 10:45am **[1]**

When $t = 10.75$, $w(10.75) = 19.32$m

c) Find the minimum water level in Broome on this day **[1]**

0.4m

d) A tourist wants to visit a local wreck which is only visible when the water level is less than 2 metres. If the tourist is only free during the morning of this particular day, between what times (to the nearest minute) would he need to visit the wreck?

[3]

$3.88 < x < 6.12$

$\therefore$ Between 3:53am → 6:07am the wreck will be visible.

Question 5. **(9 marks)**

A doctor administers a 300mg dose of an antibiotic for a patient. The rate of change of the amount of antibiotics left in the patient's system after t hours is approximated by the following function.

$$\frac{dA}{dt} = -0.13A$$

The initial dose was administered at 8am.

a) Write an equation to represent the number of milligrams of antibiotics, A, left in the patient's system t hours later. **[2]**

$A = 300e^{-0.13t}$ ✓✓

b) Find the amount of antibiotic left in the patient's system at 1pm **[2]**

When $t=5$, $A \doteqdot 156.6\text{mg}$ ✓✓

c) Find the rate of change of the amount of antibiotics left in the patient's system at 10am **[2]**

When $t=2$, $\frac{dA}{dt} \doteqdot -30\text{mg/hr}$ ✓✓

d) The doctor knows that the antibiotics will only be effective if there is at least 60mg remaining in the patient's system. What is the latest time that he could administer another dose in order to make sure that the patient has an effective amount of the drug in their system at all times? **[3]**

$60 = 300e^{-0.13t}$ ✓

$t \doteqdot 12.38$ ✓

$12.38 + 8 = 20.38$

$(8:22\text{pm})$ ✓

Question 6. **(7 marks)**

The velocity, v, in ms^{-1} of a particle moving along a straight line is given by the following equation.

$$v = 2\cos\left(\frac{\pi t}{4}\right) + 0.3t$$

The particle has an initial displacement of -2.5 metres.

a) Determine the displacement of the particle after 4 seconds **[3]**

$x = \int v \, dt$

$x = \dfrac{8\sin\left(\frac{\pi t}{4}\right)}{\pi} + 0.15t^2 + c$

When $t=0$, $x = c = -2.5$

When $t=4$, $x = -0.1\,m$

b) Determine the total distance travelled by the particle during the first 10 seconds **[2]**

$d = \int_0^{10} |v| \, dt \doteqdot 20.23\,m$

c) Determine when the acceleration of the particle is zero during the first 10 seconds. **[2]**

$a = \frac{d}{dt} v = 0$

$t \doteqdot 0.24, 3.76, 8.24\,s$

Question 7. **[3 marks]**

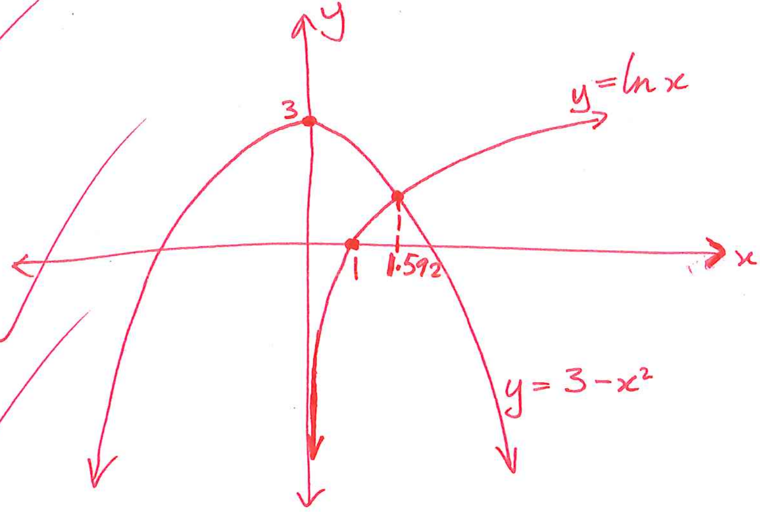
Consider the functions $f(x) = \ln x$ and $g(x) = 3 - x^2$. If the area enclosed by these two functions and the line $x = k$ is 4 units², find the possible value(s) of k.

$\ln x = 3 - x^2$

$x \doteqdot 1.592$

$\left| \int_{1.59}^{k} \ln x - (3 - x^2)\, dx \right| = 4$

$k \doteqdot 0.039, 2.915$



END OF SECTION