Name : MARKING KEY

Eastern Goldfields College
Year 12 MATHEMATICS METHODS

TEST 2 2017

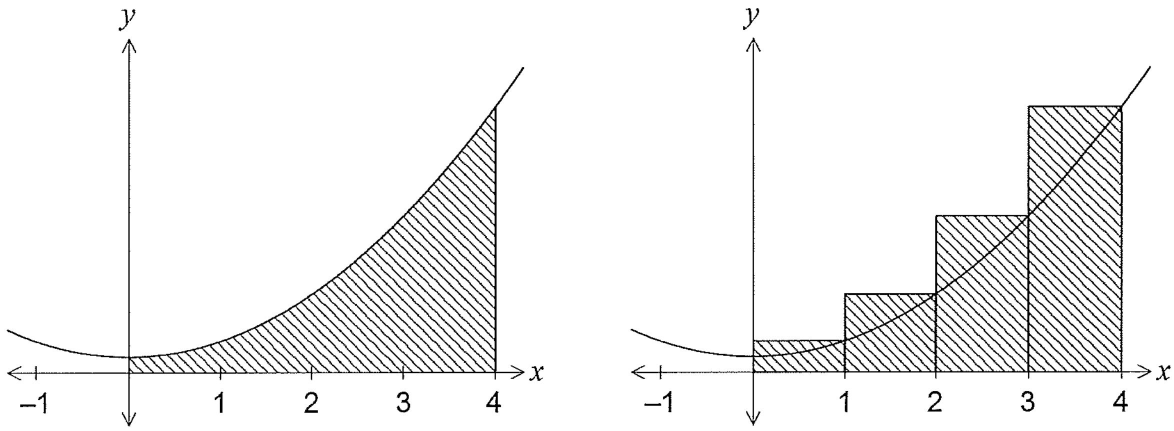
CALCULATOR FREE

Total Marks: 30

Reading: 2 minutes Time Allowed: 30 minutes

Question 1 [5 marks]

Part of the graph of $y = x^2 + 1$ is shown in the diagrams below.



An approximation for the area beneath the curve between $x = 0$ and $x = 4$ is made using rectangles as shown in the right-hand diagram. Determine the exact amount by which the approximate area exceeds the exact area.

$1\,(y(1) + y(2) + y(3) + y(4))$ ✓
$= 1\,(2 + 5 + 10 + 17)$
$= 34$ unit^2 ✓

$\int_0^4 x^2 + 1 \, dx$
$= \left[\frac{x^3}{3} + x\right]_0^4$ ✓
$= \frac{64}{3} + 4$
$= \frac{76}{3}$ ✓
$= 25\frac{1}{3}$ units^2.

$\therefore$ Approx exceeds exact by $8\frac{2}{3}$ units^2. ✓

Question 2 [2, 2, 2, 2 – 8 marks]

Find and simplify the following.

a) $\int (3x + \frac{5}{x^2})dx$

$= \frac{3x^2}{2} - \frac{5}{x} + c$

✓✓

b) $\int (3\sqrt{x})dx$

$= \frac{2}{\cancel{3}}\cancel{3}x^{3/2} + c$

$= 2x^{3/2} + c$

✓✓

c) $\int (4-3x)^5 dx$

$= \frac{(4-3x)^6}{-18} + c$

✓✓

d) $\int 6\cos(\frac{2x}{3})\, dx$

$= \frac{3}{\cancel{2}}\cancel{6}^{3} \sin\left(\frac{2x}{3}\right)$

$= 9\sin\left(\frac{2x}{3}\right) + c$

✓✓

Question 3 [3 marks]

Find as an exact value the definite integral $\int_0^4 2e^{0.5x} dx$

$= \int_0^4 2e^{\frac{1}{2}x}$ ✓

$= 2\left[2e^{\frac{1}{2}x}\right]_0^4$

$= 4e^2 - 4e$

✓ ✓

Question 4 [4 marks]

Find $\int (e^t - \frac{1}{e^t})^2 dt$

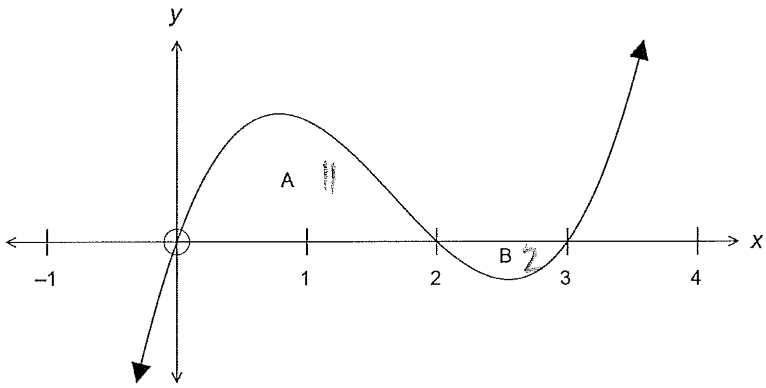
$= \int e^{2t} - 2 + \frac{1}{e^{2t}}\, dt$ ✓✓

$= \frac{e^{2t}}{2} - 2t - \frac{1}{2e^{2t}} + c$

✓✓

Question 5 [1, 1, 2 - 4 marks]

Part of the graph of $y = f(x)$ is shown below. The areas of regions A and B, bounded by the curve and the $x-$axis, are 11 and 2 square units respectively.



Evaluate:

(a) $\int_2^3 f(x)\,dx$ $= -2$ ✓

(b) $\int_0^3 f(x)\ dx$ $= 9$ ✓

(c) $\int_0^2 3f(x) - 2\ dx$ $= 3\int_0^2 f(x)\,dx - \int_0^2 2\,dx)$ ✓

$= 3.11 - 2x]_0^2$

$= 33 - 4 = 29$ ✓

Question 6 [2 marks]

Determine $\frac{d}{dx}\int_{-2}^{x} \frac{3}{\sqrt{5t-2}}\,dt$

$= \frac{3}{\sqrt{5x-2}}$ ✓✓

Question 7 [4 marks]

The curve of the function $g(x)$ has a stationary point at (4, –2) and a gradient function of $\frac{3x}{2} + m$ where m is a constant. Show that $g(2) = 1$.

$\frac{dy}{dx} = 0$ at $(4, -2)$

$0 = 6 + m$

$-6 = m$ ✓

$\frac{dy}{dx} = \frac{3x}{2} + m$

$y = \frac{3x^2}{4} + mx + c$

$= \frac{3x^2}{4} - 6x + c$

$-2 = 3.4 - 24 + c$

$10 = c$ ✓

$\therefore y = \frac{3x^2}{4} - 6x + 10$ ✓

$y(2) = 3 - 12 + 10$

$= 1$ ✓

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Total Marks: 36

Reading: 2 minutes Time Allowed: 36 minutes

Question 8 [2, 2, 2 - 6 marks]

The height of a tree is increasing at a rate of $\frac{dH}{dt}$ metres per month where $\frac{dH}{dt} = \frac{1}{50}(2t + \frac{t^2}{5})$

a) Find the increase in height of the tree in the 5th month.

$$✓ \int_4^5 \frac{1}{50}\left(2t + \frac{t^2}{5}\right) dt = 0.261m ✓$$

b) Find the average growth in the tree in the first 5 months.

$$✓ \frac{1}{5}\int_0^5 \frac{1}{50}\left(2t + \frac{t^2}{5}\right) dt = \frac{2}{15} cm ✓$$

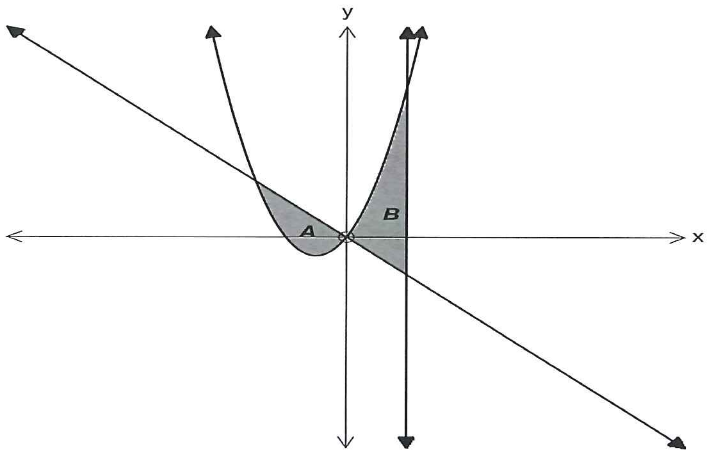
The initial height of the tree was 2 metres.

c) Find the height of the tree after 2 years.

$$\underbrace{2}_{✓} + \underbrace{\int_0^{24} \frac{1}{50}\left(2t + \frac{t^2}{5}\right) dt}_{✓} = 31.952m$$

Question 9 [1, 2, 2 - 5 marks]

The graph below shows two shaded areas. A is the area bounded by the curve $y = x(x+2)$ and the line $y = -x$. B is the area between the curve $y = x(x+2)$, the line $y = -x$ and the line $x = p$.



$x^2 + 2x = -x$
$x^2 + 3x = 0$
$x(x+3) = 0.$

(a) Determine the area of A.

$\int_{-3}^{0} -x - (x^2+2x)\,dx = 4.5 \text{ units}^2$ ✓

(b) Write down an integral that when evaluated will determine the area of B.

$x^2 + 2x = -p$

$\int_{0}^{p} x^2 + 2x + x \,dx$

$= \int_{0}^{p} (x^2 + 3x)\,dx$ ✓✓

(c) Find the value of p so that area B is 9 units, giving your answer correct to 3 decimal places.

$\int_{0}^{p} (x^2+3x)\,dx = 9$ ✓✓

$\therefore\ x = 2.033$ ✓

Question 10 [2, 2, 2 - 6 marks]

A particle P travels in a straight line. Its acceleration is given by $a = -4\pi^2\cos(2\pi t)$ ms^{-2}. The initial velocity of P is 2π ms^{-1}. Calculate

(a) the velocity of P when $t = 1.5$ seconds.

$\boxed{v = -2\pi\sin(2\pi t) + 2\pi}$

$v(1.5) = 2\pi\ ms^{-1}$ ✓✓

(b) the change in displacement in P in the first 1.5 seconds.

$\int_0^{1.5} -2\pi\sin(2\pi t) + 2\pi\ dt$ ✓ $= 7.42m$ ✓

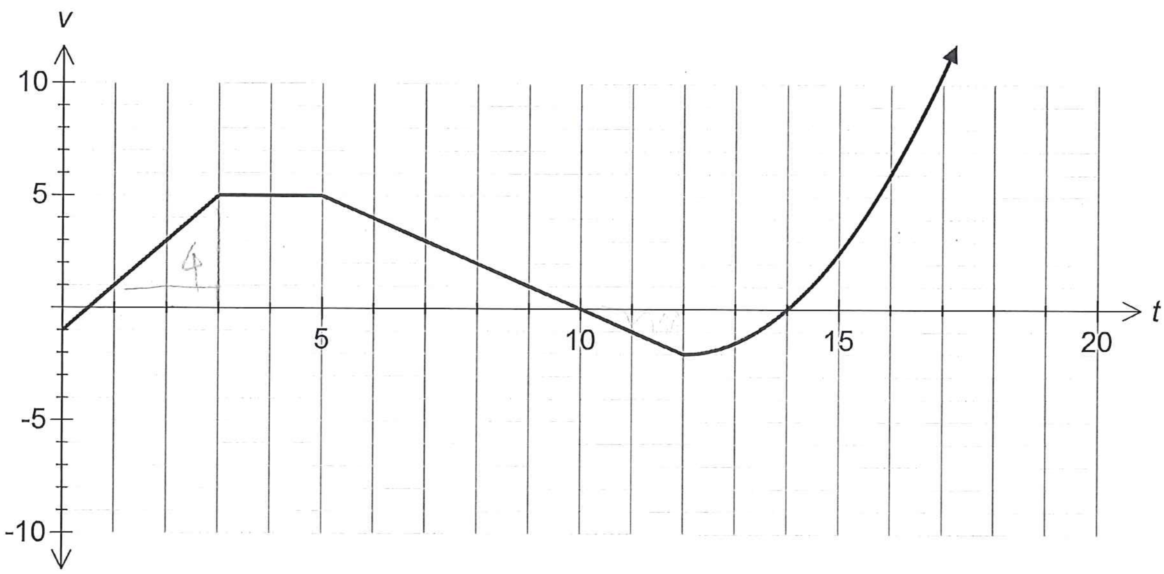
(c) the distance travelled by P in the first 1.5 seconds.

$\int_0^{1.5} \left| -2\pi\sin(2\pi t) + 2\pi \right| dt = 7.42m$

✓ ✓

Question 11 [1, 1, 2, 3, 2, 3, 2, 5 - 19 marks]

A particle moving in rectilinear motion has its velocity function graphed below, where t is time in seconds and v is in ms^{-1}.



(a) Determine the initial speed of the particle.

$1 \, ms^{-1}$

(b) Determine the acceleration of the particle during the 4th second.

$0 \, ms^{-2}$

(c) Calculate the displacement of the particle after 3 seconds.

6m

(d) Calculate the distance travelled by the particle in the first 12 seconds.

$10 + \frac{1}{2} + 6 + 12.5 + 2 = 31m$

(e) Determine when the particle has travelled a distance of 21 m since commencement.

6 sec. ✓✓

(f) State the times when the particle was at rest.

$\frac{1}{2}$s, 10s, 14s
✓ ✓ ✓

(g) When did the particle first return to the origin?

$t = 1$ sec ✓✓

(h) Calculate the distance travelled by the particle for $13 \le t \le 18$ if it is known that the velocity for $t \ge 12$ is given by $v(t) = at^2 + bt + c$.

✓ (12, −2) (14, 0) (16, 6)

$v(t) = \frac{1}{2}t^2 - 12t + 70$ ✓✓

$\int_{13}^{18} \left|\frac{1}{2}t^2 - 12t + 70\right| dt = 27.5$ m.
✓ ✓