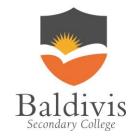
Name		

Maths Specialist - Investigation



Matrix Transformations

Investigation Part 1: Preparation activity

A point in the x-y plane can be expressed as a 2 \times 1 matrix: for example, the point (1, 2) can be written as $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

This point can then be transformed into another point by multiplication by a 2×2 matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Transformation matrix \times point = Transformed point

[NB: The transformation matrix is pre-multiplied by the point.]

Questions 1

Use the shape joined by the co-ordinates O(0,0), A(1,0), B(1,2) and C(0,2), to investigate the effect of 2x2 transformation matrices of these types:

Matrix	Calculation	Diagram	Effect – Shape, position, size
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Question 2				
Investigate the effect of a transformation by	y the following	matrices,	where k is a	constant.

$\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$	
$\begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$	
$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$	
$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$	

Question 3

a) Find the area of rectangle OABC

For each the matrices in question 1,

- b) find the absolute value of the determinant of the matrix.
- c) Find the area of O'A'B'C' the image of OABC under the transformation.

d) Find the value of $\frac{Area O'A'B'C'}{Area OABC}$

Are	ea OABC	1	T
Matrix	Determinant	Area O'A'B'C'	Area O'A'B'C' Area OABC

e) What do you notice?

By using the same transformation matrices above investigate the effect on a different shape. Can you predict the effect the transformation matrices will have? Can you predict the area of the image after it has been transformed the matrix?