12 Mathematics Methods 2021

Test 2 – Integration and Area

Section 1: Calculator-free

Time allowed: 20 minutes

Maximum marks: 20

Name: _____*Solutions*_____

Teacher: Foster | Kelly

Instructions:
- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No calculators or personal notes are permitted.

Question 1   (6 marks)

a) If $f'(x) = 3x^2 - 2$ and $f(-2) = 4$, determine $f(x)$.   [2]

$$f(x) = \frac{3x^3}{3} - 2x + C \quad \checkmark \quad \checkmark_{FT}$$

$$f(-2) = (-2)^3 - 2(-2) + c = 4 \ ; \ c = 8$$

$$\therefore \quad f(x) = x^3 - 2x + 8$$

b) Calculate $\int_0^4 \frac{-x}{\sqrt{x^2+9}} dx$ [4]

$$= -\frac{1}{2} \int_0^4 2x \left(x^2 + 9\right)^{-1/2} dx \quad \checkmark$$

$$= -\frac{1}{2} \left[\frac{\left(x^2 + 9\right)^{1/2}}{1/2} \right]_0^4 \quad \checkmark \quad = \left[-\sqrt{x^2+9} \right]_0^4$$

$$= \left(-\sqrt{25} \right) - \left(-\sqrt{9} \right) \quad \checkmark_{FT}$$
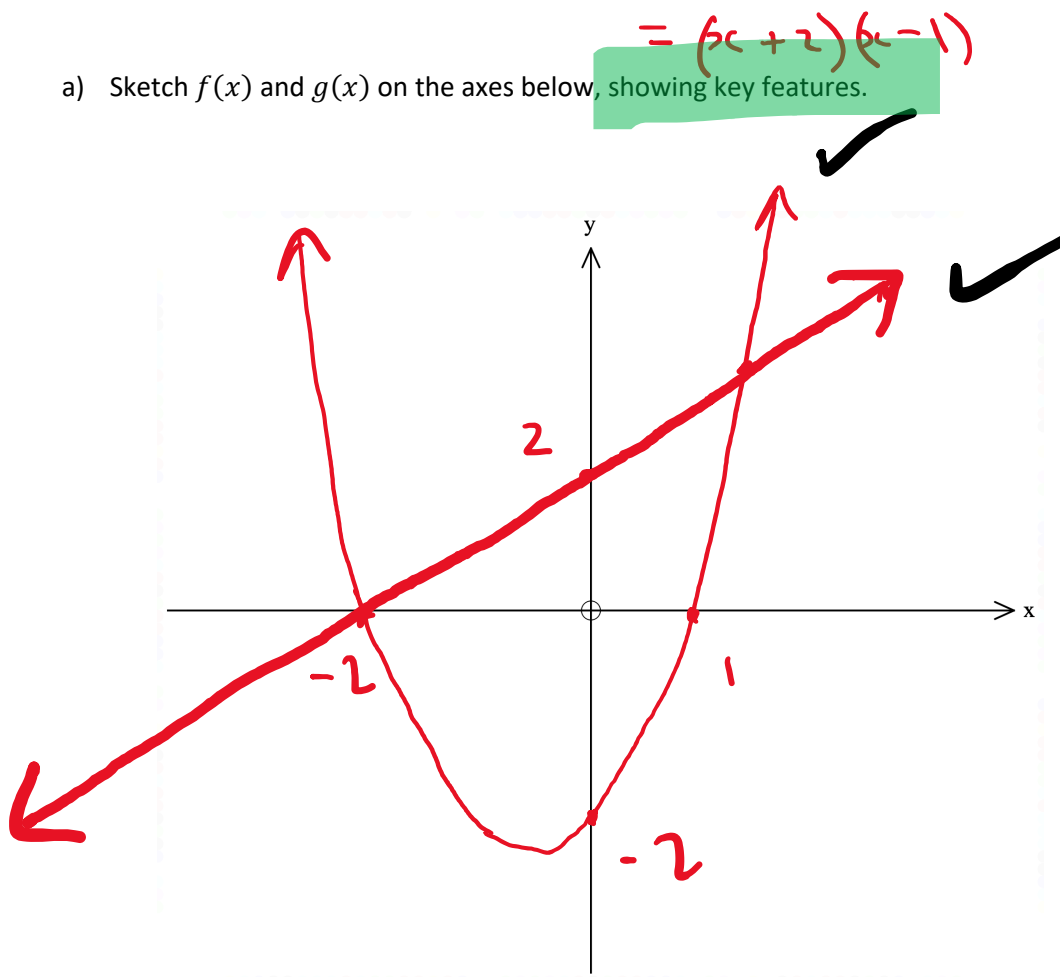
$$= -5 + 3 = -2 \quad \checkmark_{FT}$$

Question 2   (7 marks)

Consider the functions $f(x) = x + 2$ and $g(x) = x^2 + x - 2$

$$= (x+2)(x-1)$$

a)   Sketch $f(x)$ and $g(x)$ on the axes below, showing key features.   [2]

b)   State the x values of the points of intersection of $f(x)$ and $g(x)$.   [2]

$$x + 2 = x^2 + x - 2$$
$$0 = x^2 - 4$$

$$x = \pm 2$$

c)   Hence or otherwise, determine the area enclosed by $f(x)$ and $g(x)$.   [3]
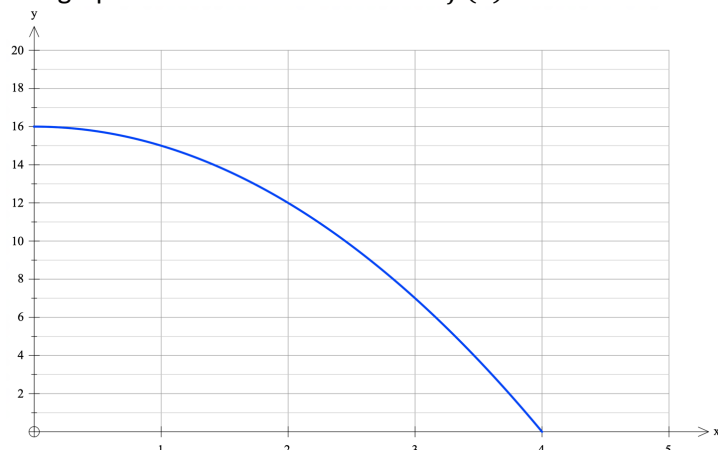
$$= \int_{-2}^{2} (x+2) - (x^2 + x - 2)\, dx = \int_{-2}^{2} -x^2 + 4\, dx$$

FT

$$= \left[\frac{-x^3}{3} + 4x \right]_{-2}^{2} = \left(\frac{-8}{3} + 8 \right) - \left(\frac{8}{3} - 8 \right) = 16 - \frac{16}{3}$$

FT

$$= 10\frac{2}{3} \text{ units}^2$$

FT

Question 3   **(7 marks)**

The graph below shows the function $f(x) = 16 - x^2$

a) An estimate for the area between the curve and the x −axis between $x = 0$ and $x = 4$ is required.   [4]

i)   Use 4 rectangles (each of width $1\ unit$) to find an overestimate for the area.

ii)   Use 4 rectangles (each of width $1\ unit$) to find an underestimate for the area.

$$= 16 \times 1 + 15 \times 1 + 12 \times 1 + 7 \times 1 \quad \checkmark$$

$$= 50 \text{ units}^2 \quad \checkmark \text{ FT}$$

$$= 15 + 12 + 7 + 0$$

$$= 34 \text{ units}^2 \quad \checkmark \text{ FT}$$

iii)   Determine the mean of the overestimate and underestimate.

$$= 42 \text{ units}^2$$

$\checkmark$ $\left(\begin{array}{l}\text{working to}\\\text{be shown}\\\text{if FT}\end{array}\right)$

b) Use calculus techniques to find the exact area.   [2]

$$\int_0^4 16 - x^2 \, dx = \left[16x - \frac{x^3}{3}\right]_0^4 = 64 - \frac{64}{3} \quad \checkmark$$

$$= 42\frac{2}{3} \text{ units}^2$$

c) If rectangles of $0.5\ units$ wide were used instead to estimate, (without calculations) suggest a possible over-estimate and under-estimate for the area.   [1]

over ;

$$42\frac{2}{3} < x < 50$$

under ;

$$34 < x < 42\frac{2}{3} \quad \checkmark$$

END OF SECTION

SCOTCH
COLLEGE

12 Mathematics Methods 2021

Test 2 – Integration and Area

Section 2: Calculator-assumed

Time allowed: 20 minutes   **Maximum marks: 20**

Name: ___Solutions___   **Teacher:** Foster | Kelly

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Question 4   (4 marks)

A train moves along a straight track with acceleration $\frac{t}{10} - 3 \ ms^{-2}$. If the initial velocity of the train is $45ms^{-1}$, determine the total distance the train travels in the first 2 minutes.

$$a = \frac{1}{10}t - 3$$

$$v = \frac{1}{10}\frac{t^2}{2} - 3t + c = \frac{t^2}{20} - 3t + 45$$

$$distance = \int_0^{120} |v(t)| \, dt = 12\,600 \, m$$

Question 5   (3 marks)

Find the total area enclosed by the graphs of $y_1 = x^3 - 4x$ and $y_2 = 3x + 6$.

$$= \int_{-2}^{-1} y_1 - y_2 \, dx + \int_{-1}^{3} y_2 - y_1 \, dx$$

$$= \qquad 0.75 \qquad\qquad + \quad 32$$

$$= \qquad 32.75 \ units^2$$

$(\checkmark\checkmark \text{ for no working})$

Question 6   **(4 marks)**

The marginal cost of producing x items is given by $y = 4.15 - 0.03x + 0.0012x^2$ $(0 < x \leq 80)$.

The initial costs are $215 before production.

a) Determine the cost of producing 60 items.   [2]

$$C_{(x)} = \int y \, dx$$
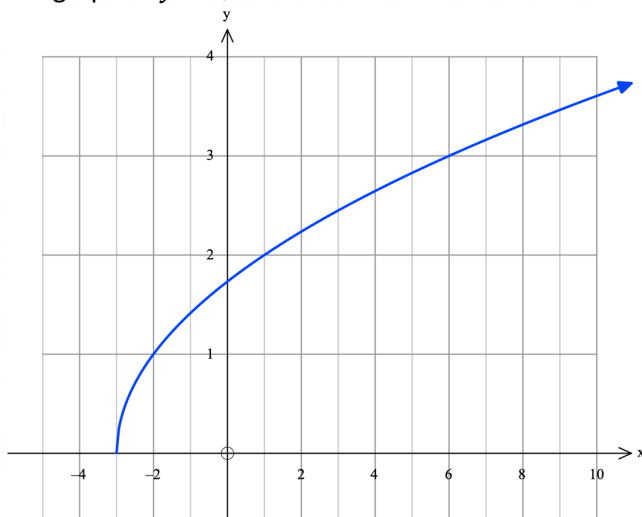
$$C(60) = \int_0^{60} y \, dx + 215 = \$496.40 \checkmark$$

b) Determine the difference in cost of producing 65 items rather than 60 items.   [2]

$$\int_{60}^{65} y \, dx = \$34.83 \qquad \checkmark\checkmark$$

Question 7   **(9 marks)**

The graph of $y = \sqrt{x+3}$ is drawn below.

a) Calculate the area of region A, given that A is enclosed between the curve, the $x-$ axis and the
    line $x = 10$.   [2]

$$\text{Area} = \int_{-3}^{10} y \, dx = 31.25 \text{ units}^2 \qquad \checkmark\checkmark$$

b) The line $x = k$ divides the region A into two regions B and C (where B lies left of $x = k$).

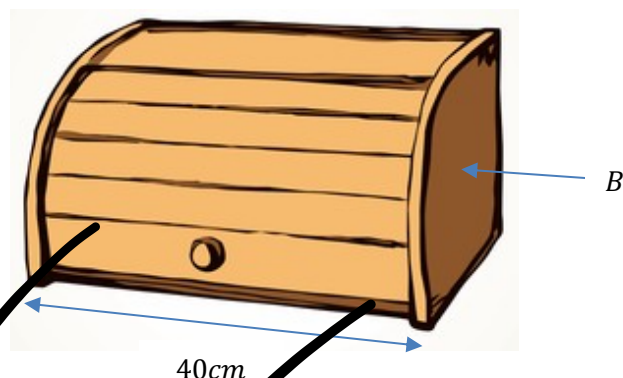Determine the value of k if the ratio of $B : C$ is $2 : 3$ [3]

$$\frac{2}{5} A = \int_{-3}^{k} y \, dx \quad \checkmark$$

$$k = 4.06$$

$\checkmark$

c)

B is used as a design for the cross-section of a $40cm$ long breadbin.

If each unit on the graph represents $7cm$, calculate the volume of the breadbin (rounding your answer to the nearest cm^3).

B

$\checkmark$ FT

$$\text{Volume} = \int_{-3}^{k} y \, dx \times 7^2 \times 40 \quad \checkmark$$

$40cm$

[4]

$$= 24\,999 \; cm^3 \quad \checkmark \; FT$$

END OF TEST