



Student Name _____

Eastern Goldfields College

Mathematics Methods Unit 1 2016

Test 1– Calculator Free Section

Working Time: ~~40~~ minutes

Total Marks: 45 marks

Calculators and notes are NOT permitted in this section.

Answer all of the following questions. Show all working to obtain full marks.

Question 1 (5 marks – 1, 2, 2)

Solve the following equations for x .

(a) $\frac{3x}{2} = \frac{6}{5}$.

$$15x = 12$$
$$x = \frac{12}{15} = \frac{4}{5} \checkmark$$

(b) $\frac{2x+3}{5} - \frac{x-2}{4} = 2$.

$$\frac{8x+12}{20} - \frac{5x-10}{20} = \frac{40}{20}$$
$$8x+12-5x+10=40 \checkmark$$
$$3x = 18$$
$$x = 6 \checkmark$$

(c) $4x^2 = 6x$.

$$4x^2 - 6x = 0$$
$$2x(2x-3) = 0 \checkmark$$
$$x = 0, \frac{3}{2} \checkmark$$

Question 2 (13 marks – 1, 1, 1, 1, 3, 3, 3)

(a)

- (i) What are the coordinates of the y -intercept of the graph of $y = (x-2)(x+3)$?

$$(0, -6) \checkmark$$

- (ii) State the number of roots of the graph of $y = 2x^2 + 1$.

none \checkmark

- (iii) What are the coordinates of the turning point of the graph of $y = 3(x+4)^2 + 2$?

$$(-4, 2) \checkmark$$

- (iv) What is the equation of the line of symmetry of the graph of $y = (x-3)(x-7)$?

$$x = 5 \checkmark$$

- (b) Determine the equation of the quadratic function that passes through the point $(0, -9)$ and has a turning point at $(-2, 3)$.

$$\checkmark -9 = a(0+2)^2 + 3$$

$$-9 = 4a + 3$$

$$-12 = 4a$$

$$3 = a \checkmark$$

$$y = -3(x+2)^2 + 3$$
$$(-3x^2 - 12x - 9)$$

(c) Solve the following equations.

(i) $x^2 + 4x + 6 = 2x^2 + 5x - 6$.

$$0 = x^2 + x - 12 \quad \checkmark$$

$$0 = (x+4)(x-3)$$

$$x = -4 \text{ or } 3 \quad \checkmark \quad \checkmark$$

(ii) $2(x-2)^2 = 100$.

$$(x-2)^2 = 50 \quad \checkmark$$

$$x-2 = \pm 5\sqrt{2}$$

$$x = 2 + 5\sqrt{2}, 2 - 5\sqrt{2}$$

\checkmark ~~www~~

Question 3 (8 marks – 1, 1, 1, 3, 2)

(a) A line has equation $3x - 2y = 12$. Determine

(i) the gradient of the line.

$$m = \frac{3}{2} \checkmark$$

(ii) the coordinates of the x -intercept of the line.

$$(4, 0) \checkmark$$

(iii) the coordinates of the y -intercept of the line.

$$(0, -6) \checkmark$$

(b) Determine the equation of the line that is perpendicular to the line in (a) and that passes through the point $(6, -1)$, giving your answer in the form $ax + by = c$, where a, b and c are integers.

$$m_{\perp} = -\frac{2}{3} \checkmark$$

$$y = -\frac{2}{3}x + c$$

$$-1 = -\frac{2}{3} \cdot 6 + c$$

$$3 = c \checkmark$$

$$\therefore y = -\frac{2}{3}x + 3$$

$$2x + 3y = 9 \checkmark$$

(c) The mid-point of $A(-4, 7)$ and B is $C(2, -2)$. Determine the coordinates of B .

the midpt of $B+C$

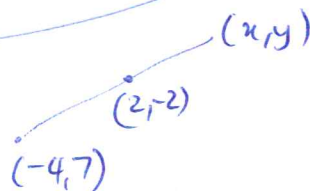
$$\frac{-4+x}{2} = 2$$

$$x = 8 \checkmark$$

$$\frac{7+y}{2} = -2$$

$$7+y = -4 \checkmark$$

$$y = -11 \checkmark$$



$$\left(\frac{8+2}{2}, \frac{-11-2}{2} \right) = (5, -6.5) \checkmark$$

Question 4 (3 marks – 2, 1)

- (a) Explain with reference to the discriminant in the quadratic formula, why the function $f(x) = 2x^2 - 12x + 18$ has only one root.

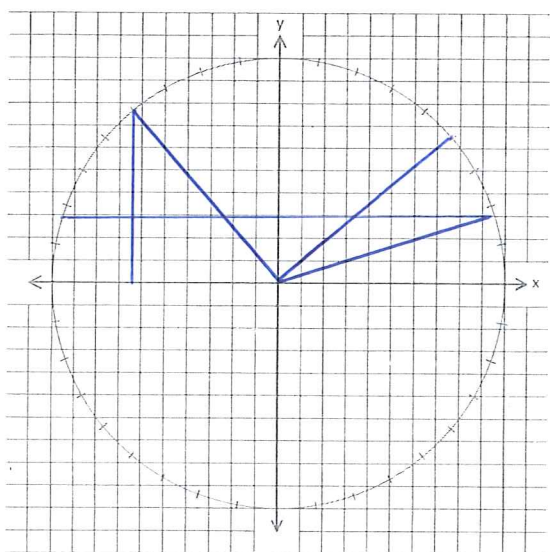
$$\begin{aligned}\Delta &= 12^2 - 4 \cdot 2 \cdot 18 \\ &= 144 - 144 \checkmark \\ &= 0 \\ &\therefore \text{only 1 root.} \checkmark\end{aligned}$$

- (b) From your answer in part(a), what can you say about the turning point of this function?

TP on x-axis \checkmark

Question 5 (4 marks – 1, 1, 2)

Use the unit circle below to answer the questions that follow. Give your answers to an appropriate degree of accuracy.



- (a) Determine the value of $\sin 40^\circ = 0.64 \checkmark$

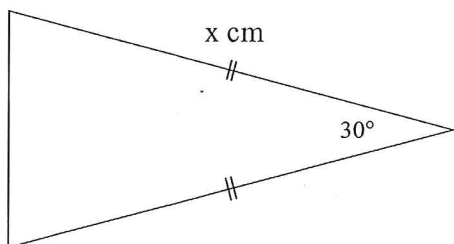
- (b) Determine the value of $\cos 130^\circ = -0.64 \checkmark$

- (c) Solve for x: $\sin x = 0.3$ for $0^\circ \leq x \leq 360^\circ$

$$x = 17^\circ, 163^\circ \checkmark$$

Question 6 (4 marks)

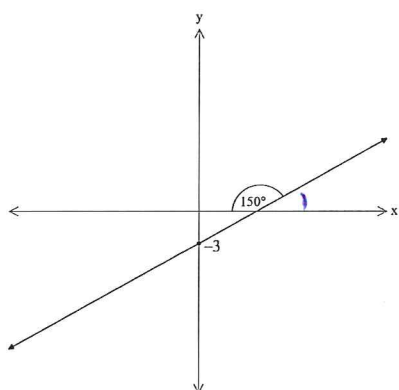
The triangle shown below has an area of 36 cm^2 , determine the value of x.



$$\begin{aligned}A &= \frac{1}{2} ab \sin C \\ 36 &= \frac{1}{2} x^2 \sin 30^\circ \checkmark \\ 36 &= \frac{1}{2} x^2 \cdot \frac{1}{2} \checkmark \\ 144 &= x^2 \checkmark \\ 12 &= x \checkmark\end{aligned}$$

Question 7 (2 marks)

Determine the equation of the linear function shown below. All values should be expressed in exact form.



$$\tan 30 = \frac{1}{\sqrt{3}}$$

$$y = \frac{1}{\sqrt{3}}x - 3$$

✓ rule.

Question 8 (2 marks – 1, 1)

State their exact value:

(a) $\cos 150^\circ = -\frac{\sqrt{3}}{2}$ ✓

(b) $\sin 135^\circ = \frac{1}{\sqrt{2}}$ ✓

Question 9 (4 marks)

Find the exact value of $\frac{\cos 60^\circ \cos 45^\circ}{\cos 30^\circ}$ expressed with a rational denominator.

$$\frac{\frac{1}{2} \cdot \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} = \frac{1}{2\sqrt{2}} \cdot \frac{2}{\sqrt{3}} = \frac{\sqrt{6}}{6}$$



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Mathematics Methods Unit 1 2016

Test 1– Calculator Assumed Section

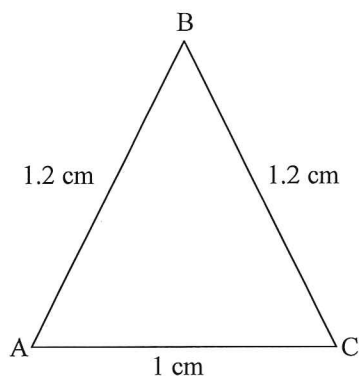
Working Time: 25 minutes

Total Marks: ²²25 marks

Question 10

⁹
(~~11~~ marks – 2, 2, ⁵7)

- (a) Consider the triangle below. The lengths of all sides of the triangle are given.



- (i) Determine the size of the angle BAC

$$\cos A = \frac{1.2^2 + 1^2 - 1.2^2}{2 \cdot 1.2 \cdot 1} \quad \checkmark$$

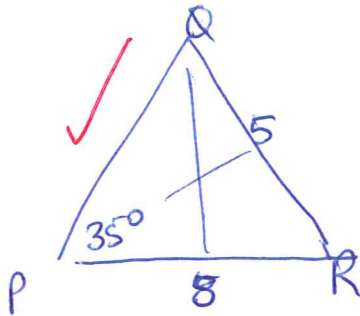
$$\underline{A = 65.4^\circ} \quad \checkmark$$

- (ii) Determine the area of the triangle ABC

$$\begin{aligned} A &= \frac{1}{2} \cdot 1.2 \cdot 1 \cdot \sin 65.4^\circ \quad \checkmark \\ &= 0.55 \text{ cm}^2 \quad \checkmark \end{aligned}$$

- (b) A second triangle, PQR , has $PR = 8$, $QR = 5$ and $\angle QPR = 35^\circ$.

Determine the length of PQ and the size of the angle PQR .



$$\frac{\sin Q}{8} = \frac{\sin 35}{5}$$

$$Q = 66.6^\circ \text{ or } 113.4^\circ$$

$$\angle QRP = 78.4^\circ$$

$$\frac{PQ}{\sin R} = \frac{5}{\sin 35}$$

$$PQ = 8.54$$

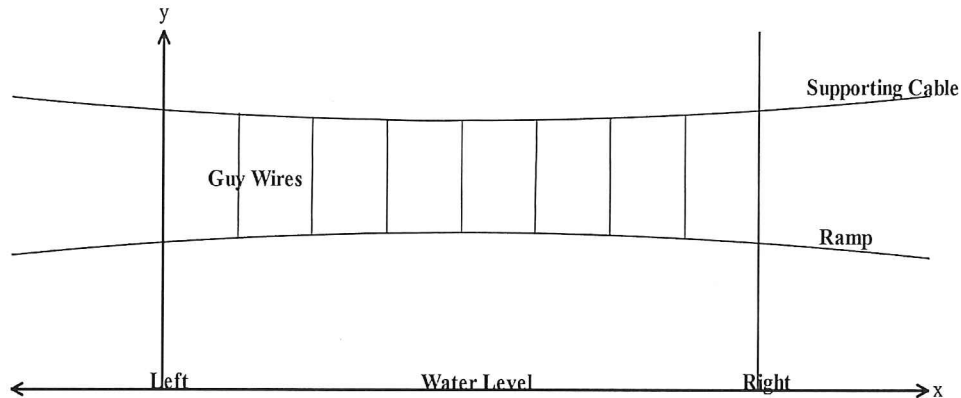
$$\angle QRP = 31.6^\circ$$

$$\frac{PQ}{\sin R} = \frac{5}{\sin 35}$$

$$RQ = 4.57$$

Question 11 (6 marks – 2, 2, 2)

A ramp walkway is to be built over a ravine. A river runs through the ravine. The walkway is to be attached to a supporting cable with guy wires as shown in the cross sectional diagram below. Both the ramp walkway and the supporting cable are in the shape of a quadratic function. The x -axis represents the water level, the left hand edge of the ravine aligns with the y -axis and the right hand edge of the ravine is also shown in the diagram. The guy wires are evenly spaced along the ramp.



The equation of the *ramp walkway* is given by the function $y = -0.001x^2 + 0.062x + 18.04$

The equation of the *supporting cable* is given by the function $y = 0.003x^2 - 0.186x + 25.18$

- (a) Find the maximum height of the ramp walkway above the water level.

19m. ✓✓

- (b) Determine the width of the ravine.

$x = 0 \Rightarrow \text{ramp } 18.04.$
Also 18.04 at 62m ✓✓

- (c) Find the distances from the left end that the supporting cable is exactly 24 metres above the water?

$x = 7.17\text{m and } 54.83\text{m}.$ ✓✓

Question 12 (3 marks – 2, 1)

The owner of a shop that sells computers calculates that his total weekly profit is given by the rule:

$$\text{Total profit in dollars} = mx - c,$$

where \$ m is the profit per computer sold, x is the number of computers sold in the week and \$ c is the fixed weekly cost of running the shop.

If he sells ten computers in a week his total profit is \$360.

If he only sells five computers in the week he makes a loss of \$190.

- (a) Calculate m and c .

$$\begin{aligned} m &= \frac{360 - (-190)}{10 - 5} \\ &= \frac{550}{5} \\ &= 110 \checkmark \end{aligned}$$

$$\begin{aligned} P &= 110x - c \\ 360 &= 110 \cdot 10 - c \\ 740 &= c \checkmark \end{aligned}$$

- (b) What is the least number of computers he can sell and still make a profit?

$$\begin{aligned} P &= 110x - 740 \\ 0 &= 110x - 740 \\ 6.73 &= x \\ x &= 7 \text{ computers } \checkmark \end{aligned}$$

Question 13 (4 marks – 2, 2)

Three consecutive positive integers are such that if you add the square of the smallest integer to the product of the other two integers the answer is 154. Determine the three integers.

(Note: An example of consecutive integers are 34, 35, 36 or -207, -206, -205)

- (a) Write an equation to describe this situation. Hint: Let the integers be x , $x+1$ and $x+2$.

$$x^2 + (x+1)(x+2) = 154 \checkmark \checkmark$$

$$\begin{aligned} x^2 + x^2 + 3x + 2 \\ 2x^2 + 3x + 2 \end{aligned}$$

- (b) Find the three integers.

$$\begin{aligned} x &= 8, -9.5 \checkmark \\ \therefore \text{Three integers } 8, 9, 10. \checkmark \end{aligned}$$