Mathematics Methods Unit 3&4
Test 1 Applications of differentiation
Name Solutions

Total time allowed: 55 minutes. **Total marks: 54 marks**
Section One: Calculator-free

Time allowed for this section: 28 minutes
Total marks for this section: 28 marks

Materials allowed for this section:
SCSA Formula Sheet (provided)

Instructions to candidates
Show all of your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

1. [1, 2, 3, 3 = 9 marks]
Differentiate the following with respect to x. Do not simplify. (Except to change all negative indices to positive)

(a) $f(x)=\frac{10}{x^5}$

$f(x) = 10x^{-5}$
$f'(x) = -50x^{-6}$
$= \frac{-50}{x^6}$ ✓

(b) $y=\frac{8}{(2x-3)^3}$

$y = 8(2x-3)^{-3}$ ✓
$y' = -24(2x-3)^{-4} \cdot 2$
$y' = \frac{-48}{(2x-3)^4}$ ✓

(c) $g(x)=(8x-5)^3(x^2-3x)^4$
$u \qquad\quad v$

$g'(x) = v\frac{du}{dx} + u\frac{dv}{dx}$

$= (x^2-3x)^4 \cdot 3(8x-5)^2 \cdot 8$ ✓ $+ (8x-5)^3 \cdot 4(x^2-3x)^3 \cdot (2x-3)$ ✓ ✓ brackets in chain rule.

(d) $h(x)=\frac{5x^2-6}{4x-2x^3} = \frac{u}{v}$

$h'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(4x-2x^3) \cdot 10x - (5x^2-6)(4-6x^2)}{(4x-2x^3)^2}$ ✓ ✓ ✓

2. [4 marks]

Using the chain rule find $\frac{dk}{dm}$ (in terms of m) given that $k = p^3 - 1$ *and* $p = 3m^2 + 1$

$k' = 3p^2$ $p' = 6m$

$$\frac{dk}{dm} = \frac{dk}{dp} \cdot \frac{dp}{dm}$$

$$= 3p^2 \cdot 6m$$

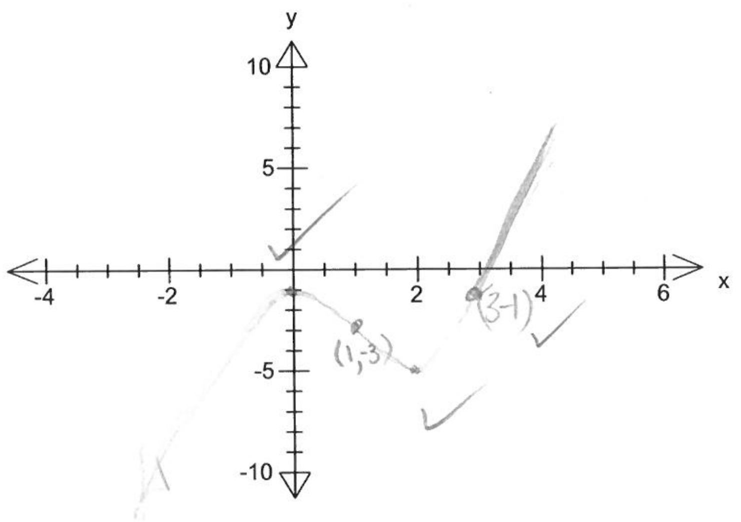
$$= 3(3m^2+1)^2 \cdot 6m$$

$$= 18m(3m^2+1)^2$$

3. [10 marks]

Locate and identify any stationary points and any points of inflection for the function $f(x) = x^3 - 3x^2 - 1$.

Using any additional information sketch the function on the given axes.



$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

Stat points at $f'(x) = 0$

$0 = 3x^2 - 6x$

$= 3x(x-2)$

$\therefore$ Stat points at

$x = 0$ & $x = 2$

$(0,-1)$ $(2,-5)$

Test nature stat pts

$f''(0) = -ve$ $\therefore$ concave down (max)

$f''(2) = +ve$ $\therefore$ concave up min

Inflection

$f''(x) = 0$

$6x - 6 = 0$

$6x = 6$

$x = 1$

Inflection at $(1,-3)$

4. [2, 3 = 5 marks]

The displacement of a body x metres from an origin at time t seconds is given by x = $t^3 - 3t^2 + 4, t \geq 0$. Find

(a) the initial velocity of the body

$\dot{x} = 3t^2 - 6t$ $\quad \dot{x} = 3(0)^2 - 6(0)$

At $t=0$ ~~[illegible]~~ $V = 0$

(b) The acceleration of the body when its velocity is zero.

~~$\dot{x}$~~

$0 = 3t^2 - 6t$

$= 3t(t-2)$

Velocity $= 0$ @ $t=0$ and $t=2$ sec

$\ddot{x} = 6t - 6$

at $t=0$ acceleration $= -6ms^{-2}$

at $t=2$ acceleration $= 6ms^{-2}$

$\therefore$ when velocity is 0 the accelration is $6ms^{-2}$

Mathematics Methods Unit 3&4
Test 1 Applications of differentiation
Name Solutions.

Total time allowed: 55 minutes. **Total marks: 54 marks**
Section One: Calculator Assumed

Time allowed for this section: 27 minutes
Total marks for this section: 26 marks

Materials allowed for this section:
SCSA Formula Sheet (provided)
Up to 3 SCSA Approved Calculators
Instructions to candidates
Show all of your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. [2, 1, 2, 2 = 7 marks]
Items being sold for $50 each have a production cost function
$C(x) = 10x + 0.04x^2 + 500$ dollars for producing x items.

(a) Find an expression for the profit, $P(x)$ corresponding to the manufacture and sale of x items.

$P(x) = R(x) - C(x)$ ✓
$= 50x - (10x + 0.04x^2 + 500)$
$= 50x - 10x - 0.04x^2 - 500 = 40x - 0.04x^2 - 500$ ✓

(b) Determine an expression for $P'(x)$

$P'(x) = 40 - 0.08x$ ✓

(c) Calculate $P'(100)$ and interpret this value.

$P'(100) = 40 - 0.08(100)$
$= 32$

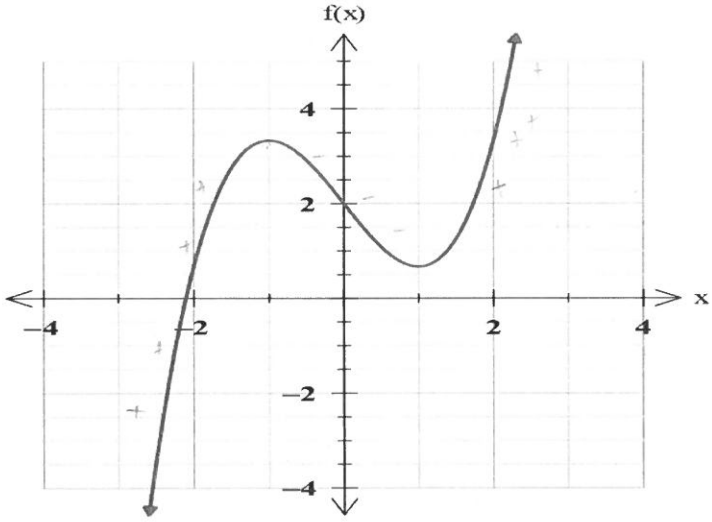
✓ The rate of increase in profit is $32 per item when 100 are produced

(d) Find the number of items manufactured and sold which will give the maximum profit.

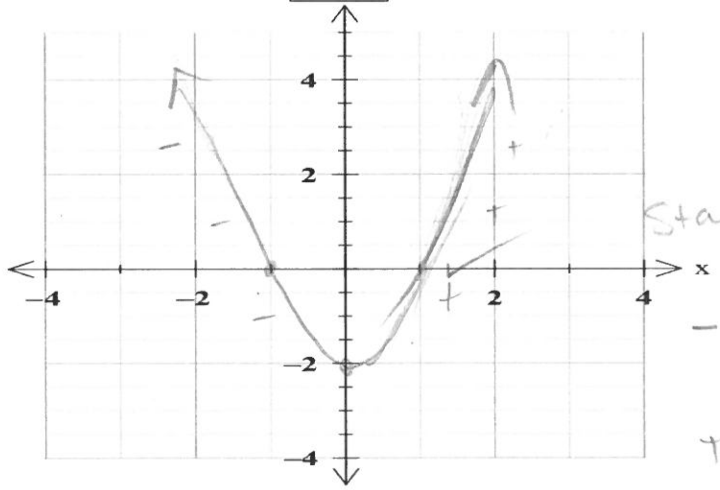
$0 = 40 - 0.08x$ ✓
$x = 500$ ✓

6. [4 marks]

On the axes below sketch $f'(x)$ *and* $f''(x)$. Clearly indicate each graph.

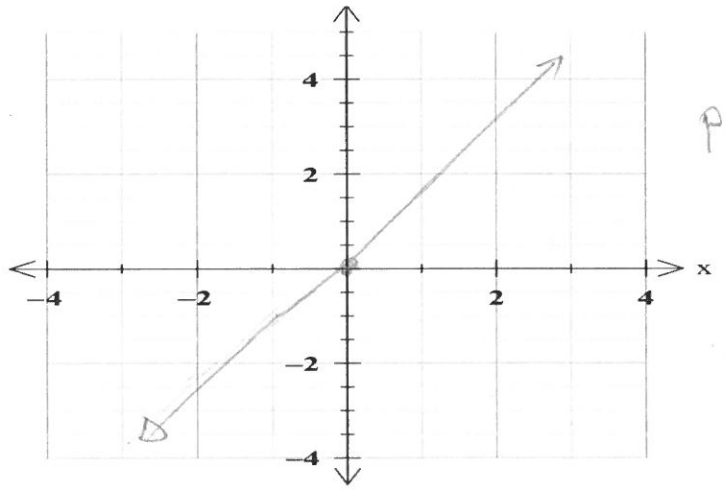


f ’(x)



Stat points → 0
−ve grad below x axis
+ve grad above x axis

f ’’(x)



point of inflection lines up with TP on $f'x$

(b) The approximate solution to f(x)=0 is −2.2

(Accept 2.1 to 2.3)

7. [2, 2, 4 ,3 = 11 marks]

An isosceles triangle has a perimeter of 60 metres. By letting the length of the congruent sides be x metres:

(a) show that the perpendicular height of the triangle is $\sqrt{60x-900}$ metres. (A diagram would be useful.)

$x^2 = h^2 + (\frac{1}{2}(60-2x))^2$

$x^2 = h^2 + (30-x)^2$

$x^2 - (30-x)^2 = h^2$

$\sqrt{x^2-(30-x)^2}$

$\sqrt{x^2-(900-60x+x^2)}$

$\sqrt{-900+60x}$

(b) Show that the area of the triangle $A(x)=(30-x)\sqrt{60x-900}\ m^2$

$\frac{1}{2} \times b \times h$

$A = \frac{1}{2}(60-2x)\cdot\sqrt{60x-900}$

$= (30-x)\sqrt{60x-900}$

(c) Show that A'(x) is $\frac{900-30x}{\sqrt{60x-900}} - \sqrt{60x-900}$

$A(x) = (30-x)(60x-900)^{\frac{1}{2}}$

$u \qquad v$

$A'(x) = (30-x)\ \frac{1}{2}(60x-900)^{-\frac{1}{2}}\cdot 60 + (60x-900)^{\frac{1}{2}}\cdot -1$

$= \frac{30\cancel{60}(30-x)}{\cancel{2}\sqrt{60x-900}} - \sqrt{60x-900}$

$= \frac{900-30x}{\sqrt{60x-900}} - \sqrt{60x-900}$

(d) Using your calculator or otherwise, find and verify the value of x that maximises the area.

Solve $0 = \frac{900-30x}{\sqrt{60x-900}} - \sqrt{60x-900}$ $\quad x = 20$

Check max: $\frac{d^2A}{dx^2}$ = at $x=20$ is negative

$\frac{d^2A}{dx^2} = -5.196$ $\therefore$ $x=20$ is a maximum

8. [4 marks]
The radius of a solid rubber wheel of an old wheelbarrow has decreased by 5% after years of wear. Use the method of small changes to approximate the percentage decrease in the volume of rubber in the wheel.

cylinder

$V = \pi r^2 \times h$

$\cancel{V = 2\pi r}$

$\frac{\delta V}{V} = \frac{dV}{dr} \cdot \frac{\delta r}{\delta V}$

$\frac{dV}{dr} = 2\pi r h$

$\frac{\delta r}{r} = \frac{5}{100}$

$\frac{\delta V}{V} = \frac{2\pi r h}{\pi r^2 h} \cdot \delta r$

$\frac{2}{r} \delta r$

$= 2 \frac{\delta r}{r}$

$= 2 \cdot \frac{5}{100}$

$= 0.1$

$= 10\%$ change in volume