MINDARIE SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS

MATHEMATICS: SPECIALIST 1 & 2

SEMESTER 2 2019
TEST 5

Name Marking Key.

Calculator Free

Time allowed: 32 mins

Total marks: 32

1. [2 marks]

Write the following recurring decimal as a fraction.

$x = 7.1465465465....$

$100x = 714.654654...$ ✓

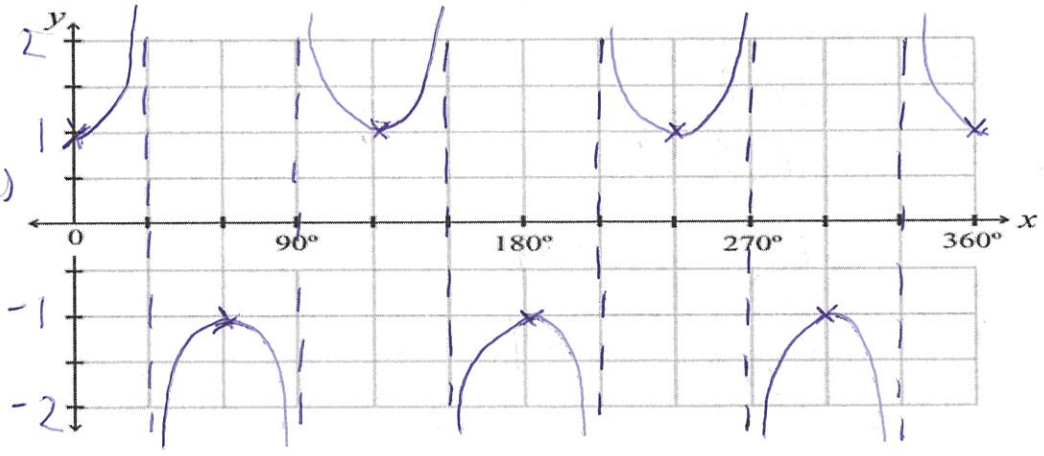
$100000x = 714654.654...$

$99900x = 713940$

$x = \frac{713940}{99900}$ ✓

2. [3 marks]

Sketch the graph of $y = sec3x$ for $0° \leq x \leq 360°$.



✓ shape
✓ asymptotes
✓ all correct

3. **[3 marks]**

Show that when any three odd numbers are added together, the result is an odd number.

$2m+1+2n+1+2p+1 = 2m+2n+2p+3$ ✓

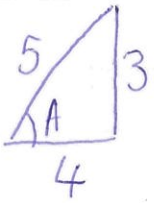
$= 2(m+n+p+1)+1$ ✓ factorising

$=$ even $+1$

$=$ odd number. ✓

4. **[1, 2, 2, 2 = 7 marks]**

Given that $cosA = -\frac{4}{5}$, where A is obtuse, find the exact value of:



$sinA = \frac{3}{5}$ ✓

$sin2A = 2\sin A\cos A$ ✓

$= 2 \times \frac{3}{5} \times -\frac{4}{5}$

$= -\frac{24}{25}$ ✓

$cos2A = \cos^2 A - \sin^2 A$ ✓

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$ ✓

$cot2A = \frac{\cos 2A}{\sin 2A}$ ✓

$= \frac{7/25}{-24/25} = -\frac{7}{24}$ ✓

5. **[2, 5 = 7 marks]**

Prove the following:

a) $cosec^2x = secx\ cosecx\ cotx$

$$RHS = \frac{1}{\cos x} \times \frac{1}{\sin x} \times \frac{\cos x}{\sin x} \checkmark$$

$$= \frac{\cos x}{\cos x \sin^2 x}$$

$$= \frac{1}{\sin^2 x} \checkmark$$

$$= cosec^2 x$$

$$= LHS.$$

b) $\frac{sinx+cosx}{cosxsinx} = sinx(1+tanx) + cosx(1+cotx)$

$$RHS = \sin x + \sin x \tan x + \cos x + \cos x \cot x \checkmark$$

$$= \sin x + \frac{\sin^2 x}{\cos x} + \cos x + \frac{\cos^2 x}{\sin x} \checkmark$$

$$= \frac{\sin^2 x \cos x}{\sin x \cos x} + \frac{\sin^3 x}{\sin x \cos x} + \frac{\cos^2 x \sin x}{\sin x \cos x} + \frac{\cos^3 x}{\sin x \cos x} \checkmark$$

$$= \frac{\sin^2 x(\cos x + \sin x) + \cos^2 x(\sin x + \cos x)}{\sin x \cos x} \checkmark$$

$$= \frac{(\cos x + \sin x)(\sin^2 x + \cos^2 x)}{\sin x \cos x} \checkmark$$

$$= \frac{\cos x + \sin x}{\sin x \cos x}$$

$$= LHS$$

6. **[3 marks]**

Show that:

$sin11xcos7x - sin8xcos4x = sin3xcos15x$

LHS $= \frac{1}{2}[\sin 18x + \sin 4x] - \frac{1}{2}[\sin 12x + \sin 4x]$ ✓

$= \frac{1}{2}[\sin 18x - \sin 12x]$ ✓

$= \cos\left(\frac{18+12}{2}\right)x \cdot \sin\left(\frac{18-12}{2}\right)x$ ✓

$= \cos 15x \sin 3x$

$= RHS$

7. **[4 marks]**

$y = \frac{1}{3}\sin 8(x-15) + 4$ ✓

Fully describe the transformations that take $y = sinx$ to $y = \frac{1}{3}\sin(8x - 120) + 4$.

Dilation parallel to y axis s.f $\frac{1}{3}$ ✓

Dilation parallel to x axis s.f $\frac{1}{8}$ ✓

Translation 15 units right, 4 units up. ✓

8. **[3 marks]**

Solve $secx = 2$, where x is in degrees.

$\cos x = \frac{1}{2}$ ✓

$x = 60°, 300°, \ldots$

$x_1 = 60 + 360k$ ✓ $\quad x_2 = 300 + 360k$ ✓

or

$-60 + 360k$

$k \in \mathbb{Z}$ ← $-\frac{1}{2}$ if this not written

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Calculator Assumed

Time allowed: 26 mins

Total marks: 26

9. [5 marks]

Prove, using contradiction that $\sqrt{5}$ is irrational.

Assume that $\sqrt{5}$ is rational and can be written as a fraction $\frac{a}{b}$ where $a, b \in \mathbb{Z}$ and $\frac{a}{b}$ is in its simplest form. ✓

$\sqrt{5} = \frac{a}{b}$

$5 = \frac{a^2}{b^2}$

$5b^2 = a^2$ ✓

a^2 is a multiple of 5
$\therefore$ a is a multiple of 5.

Let $a = 5n$ ✓

$5b^2 = (5n)^2$

$5b^2 = 25n^2$

$b^2 = 5n^2$ ✓

So b^2 is a multiple of 5
$\therefore$ b is a multiple of 5

✓ If a and b are both multiples of 5 then $\frac{a}{b}$ is not in its simplest form hence our assumption was false and $\sqrt{5}$ is irrational.

10. [3, 3 = 6 marks]

a) Write $3cosx - 5sinx$ in the form $rcos(x + \alpha)$.

$3\cos x - 5\sin x = r\cos x \cos\alpha - r\sin x \sin\alpha$

$r\cos\alpha = 3 \qquad r\sin\alpha = 5$

$r = \sqrt{3^2+5^2}$ ✓

$= \sqrt{34}$ ✓

$\alpha = \tan^{-1}(5/3)$

$= 59°$ ✓

$3\cos x - 5\sin x = \sqrt{34}\cos(x + 59°)$ ✓

b) Hence solve $3cosx - 5sinx + 2 = 0$ for $0° \leq x \leq 360°$.

$\sqrt{34}\cos(x + 59°) + 2 = 0$

$\cos(x + 59°) = -\frac{2}{\sqrt{34}}$ ✓

$x + 59 = 110.1°, 249.9°, ..$ ✓

$x = 51.1°, 190.9°$ ✓

11. [4 marks]

Solve $2sec^2x = 9tanx + 7$ for $0° \leq x \leq 360°$. Give answers to 2 decimal places.

$2(1+\tan^2 x) = 9\tan x + 7$

$2\tan^2 x - 9\tan x - 5 = 0$ ✓

let $y = \tan x$

$2y^2 - 9y - 5 = 0$

$(2y+1)(y-5) = 0$ ✓

$2\tan x + 1 = 0$ or $\tan x - 5 = 0$

$\tan x = -\frac{1}{2}$ $\tan x = 5$

$x = -26.6, 153.4, 333.4...$ $x = 78.7, 258.7...$

$x = 78.7°, 153.4°, 258.7°, 333.4°$ ✓✓

12. [3 marks]

Solve the following:

$\tan\left(2x - \frac{\pi}{6}\right) - 1 = 0$

$\tan\left(2x - \frac{\pi}{6}\right) = 1$

$2x - \frac{\pi}{6} = \frac{\pi}{4} + \pi k$ ✓

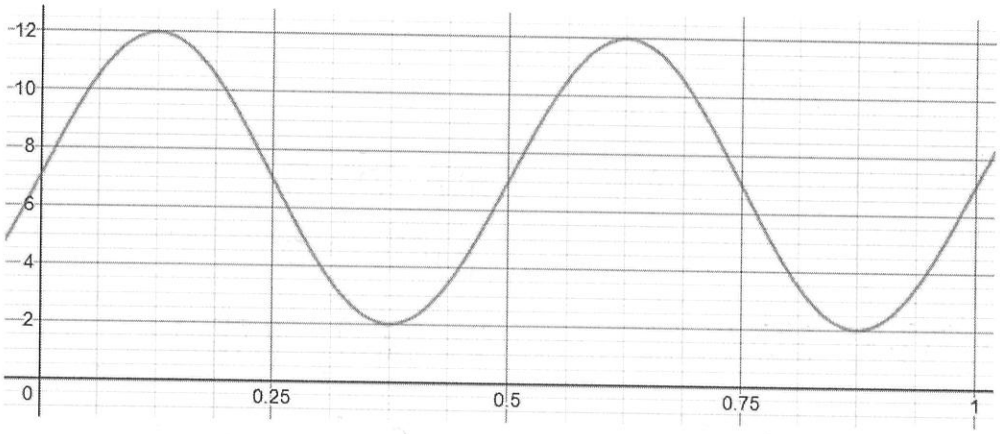
$2x = \frac{\pi}{4} + \frac{\pi}{6} + \pi k$

$= \frac{5\pi}{12} + \pi k$ ✓

$x = \frac{5\pi}{24} + \frac{\pi}{2}k \quad k \in \mathbb{Z}$ ✓

13. [1, 1, 3, 3 = 8 marks]

Juanita is lying on the beach watching the waves. She notices that the waves appear to roll up the beach at regular time intervals, and she is able to estimate the distance of the wave front from her toes over time. She scratches Cartesian axes in the sand and sketches the distance of the wave from her toes against time in minutes. She realises that the distance can be modelled by a sine curve $d = asinbt + c$, with time (t) in minutes along the horizontal axis and distance (d) in metres on the vertical axis.



a) State the maximum and minimum distances of the waves from her feet.

Max 12m Min 2m ✓

b) How many waves wash up on the beach each hour?

2 per minute → 120 per hour ✓

c) Find the values of a, b and c.

$a = 5$ ✓ $c = 7$ ✓

$b = \frac{2\pi}{0.5} = 4\pi$ ✓

Her beach umbrella is stuck into the sand 4.5m closer to the wave front than her toes and the waves are washing over its base.

d) Calculate the percentage of time for which the base of the umbrella is in the water.

$4.5 = 5\sin 4\pi t + 7$ ✓

$t = \frac{7}{24}, \frac{11}{24}$ ✓ base is under water between these two times.

$\frac{\frac{11}{24} - \frac{7}{24}}{0.5} = \frac{1}{3} \rightarrow 33\frac{1}{3}\%$ ✓