SCOTCH COLLEGE

12 Mathematics Methods 2023

Test 2 – Integration and Applications

Section 1: Calculator-Free

Time allowed: 20 minutes **Maximum marks: 20**

Name: Solutions **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No Calculators and no notes are permitted.

Question 1 **[3 marks]**

Find y in terms of x given that $\frac{dy}{dx} = 3x^2 - 4$ and $y = 5$ when $x = -3$.

$y = \int 3x^2 - 4\,dx = x^3 - 4x + c$ ✓

$5 = (-3)^3 - 4(-3) + c$ ✓ ; $c = 20$

$y = x^3 - 4x + 20$ ✓

Question 2 **[6 marks]**

Evaluate the following definite integrals.

a) $\int_1^4 (\frac{2}{\sqrt{x}} + 1)dx$ **[3]**

$\int_1^4 2x^{-1/2} + 1\,dx$

$= \left[\frac{2x^{1/2}}{1/2} + x\right]_1^4$

$= \left[4\sqrt{x} + x\right]_1^4$ ✓ ✓

$= [4 \cdot 2 + 4] - [4\sqrt{1} + 1]$

$= 12 - 5$

$= 7$ ✓FT

b) $\int_0^3 7(2-x)^3 dx$ **[3]**

$= \left[\frac{7(2-x)^4}{4(-1)}\right]_0^3$ ✓ ✓

$= -\frac{7}{4}\left[(-1)^4 - 2^4\right]$

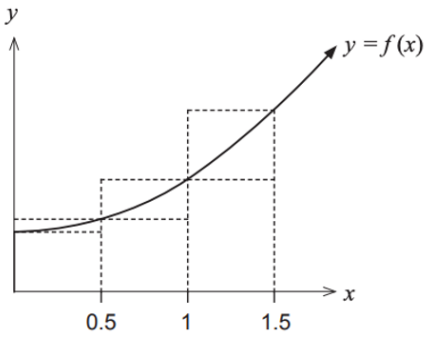
$= -\frac{7}{4}[1 - 16]$

$= \frac{-7 \cdot (-15)}{4}$

$= \frac{105}{4}$ ✓FT

Question 3 [5 marks]

Consider below the function of $f(x)$ and its table of values at various points.



x	0	0.5	1	1.5
$f(x)$	15	18	22	27

a) Using the rectangles shown in the diagram above, show that: **[3]**

$$27.5 < \int_0^{1.5} f(x)\,dx < 33.5$$

underestimate; $15(0.5) + 18(0.5) + 22(0.5) = 27.5$

overestimate; $18(0.5) + 22(0.5) + 27(0.5) = 33.5$

$\therefore \quad 27.5 < \int_0^{1.5} f(x)\,dx < 33.5$

The process used above is repeated using rectangles of **half the width** to obtain the following result;

$$a < \int_0^{1.5} f(x)\,dx < b$$

b) Without calculating, suggest appropriate values of a and b. **[2]**

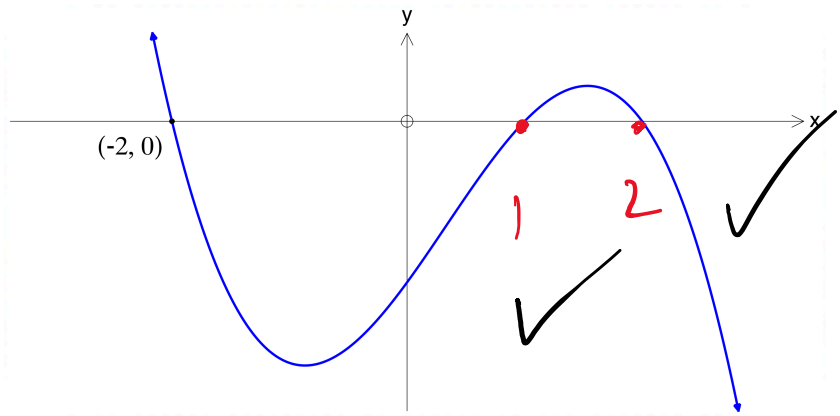
e.g. $29 < \int_0^{1.5} f(x)\,dx < 31$

$27.5 < a < 30.5$

$30.5 < b < 33.5$

Question 4 [6 marks]

Some of the features of the graph of $y = -3x^3 + 3x^2 + 12x - 12$ are shown below.



a) Determine the other two roots of the graph.

$$\begin{array}{r|rrrr} -2 & -3 & 3 & 12 & -12 \\ & \downarrow & 6 & -18 & 12 \\ \hline & -3 & 9 & -6 & 0 \end{array}$$

$$(x+2)(-3x^2+9x-6)=0$$

$$(x+2)(-3)(x^2-3x+2)=0$$

$$-3(x+2)(x-2)(x-1)=0$$

b) Showing use of Calculus, determine the total area enclosed by the graph and the x-axis.

$$\left|\int_{-2}^{1} y\,dx\right| + \int_{1}^{2} y\,dx \quad \checkmark FT$$

$$= \left[-\frac{3x^4}{4} + \frac{3x^3}{3} + \frac{12x^2}{2} - 12x\right]_{-2}^{1} + \left[-\frac{3x^4}{4} + x^3 + 6x^2 - 12x\right]_{1}^{2} \quad \checkmark FT$$

$$= \left[\left(-\frac{3}{4} + 1 + 6 - 12\right) - \left(-12 - 8 + 24 + 24\right)\right] + \left[\left(-12 + 8 + 24 - 24\right) - \left(-\frac{23}{4}\right)\right] \quad \checkmark FT$$

$$= \left[\left(-\frac{3}{4} - \frac{20}{4}\right) - (28)\right] + \left[-4 - \left(-\frac{23}{4}\right)\right]$$

$$= \frac{135}{4} + \frac{7}{4} = \frac{142}{4} = \frac{71}{2} \text{ units}^2 \quad \checkmark FT$$

-----END OF SECTION ONE-----