Manjimup S.H.S Year 11 2015
Mathematical Methods Investigation 1
Out of Class Section

HERON'S RULE

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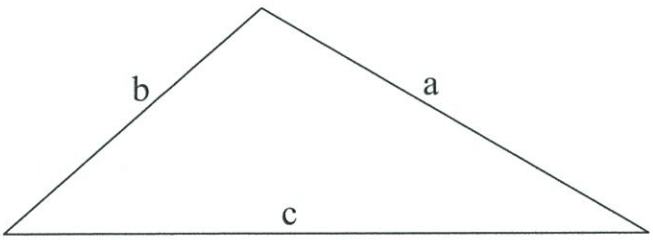
- *The Out of Class Investigation is designed for you to learn the essentials needed for the In-Class validation.*
- *This is the "Take Home" part of the Investigation. It does not count towards your mark for this investigation.*
- *You will need your Casio Classpad or a spreadsheet.*
- *You are permitted to take this section into the "In Class" validation test to assist you.*

The rule you have used to calculate the area of a triangle is A = ½bh where b is the length of the base and h is the perpendicular height.

However outside the school classroom people rarely have the base and the perpendicular height. They are more likely to be given the lengths of the 3 sides of the triangle. In 60 AD the mathematician Heron of Alexandria published a book with a rule for calculating the area of a triangle and ever since then it has been known as Heron's rule. It seems however that Archimedes knew the rule over 300 years before Heron!!!



$$s = \frac{a+b+c}{2}$$

s is called the semi-perimeter.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

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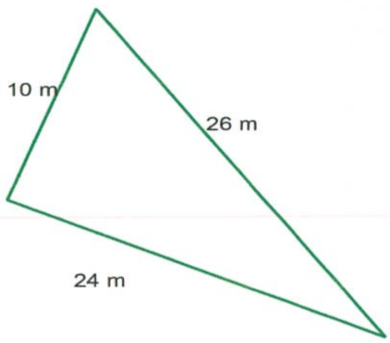
1. Use Heron's formula to calculate the area of this scalene triangle shown.

$$s = \frac{60}{2} = 30$$



$$A = \sqrt{30(30-10)(30-24)(30-26)}$$

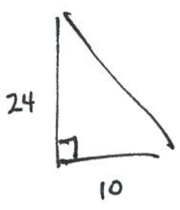
$$= \underline{120 m^2}.$$

2. Use Pythagoras' rule to prove that this triangle is right angled.

$10^2 + 24^2 = 676 \Rightarrow a^2 + b^2 = c^2$

$26^2 = 676 \Rightarrow$ Pythagorean triple $\Rightarrow$ Right-angled Δ.

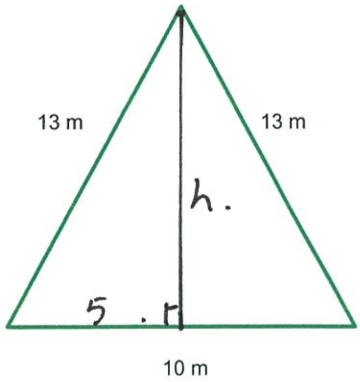
3. Use this result to show your result the area calculated from from Heron's Rule is correct.



$A = \frac{1}{2} bh$

$= \frac{1}{2}(10)(24) = 120 m^2 \hat{=}$ Heron's rule result

4. Use Heron's rule to determine the area of this isosceles triangle, with a perimeter of 26 cm.



$S = \frac{36}{2} = 18$

$A = \sqrt{18(18-13)^2(18-10)}$

$= 60m^2$

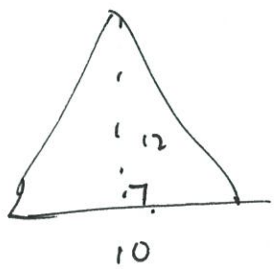
5. Find the perpendicular height of the triangle in question 4. Show all working.

$h^2 = 13^2 - 5^2$

$h = 12m.$

6. Use another method to determine the area of this triangle to confirm your solution in question 4 is correct.

Method : $A = \frac{1}{2} bh$



$= \frac{1}{2}(10)(12)$

$= 60m^2$

= Q4 answer.

7. Consider the following lengths of the sides of triangles each with a perimeter of 30cm (semi-perimeter of 15).

Find the area of each using Heron's rule.

Impossible?

Isosceles triangles?

Eauilateral?

Scalene?

Maximum Area?

Heron's rule on NumSolve?

(8,10,12) $A = 25.10cm^2$ (7,9,14) $A = 26.83 cm^2$

(6,11,13) $A = 32.86cm^2$ (10,10,10) $A = 43.30 cm^2$

(9,9,12) $A = 40.25cm^2$ (8,8,14) $A = 27.11cm^2$

(3,10,17) No real solution (7,7,16) No real solution

(4,12,14) $A = 22.25 cm^2$ (5,11,14) $A = 24.49cm^2$

Try some other triangles of your own with a fixed perimeter of 30 cm.
Show your solutions below.

8. Discuss what you found.

- Which triangles are impossible? Why? What happens to Heron's rule with these impossible triangles? What relationship must exist between a, b and c for the numbers to form a triangle?

Impossible triangles $\rightarrow$ $a + b < c$. $\frac{a \quad b}{c}$

They cannot form triangles

For possible triangle $a+b \cancel{k} > c$

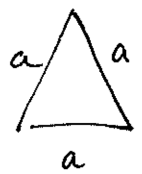
Heron's Law yields a ~~m~~ "complex sol^n^" or "no real answe" or $\sqrt{-ve}$ n°.

- What type of triangles create the largest area?

Equilateral triangles.

- Can you find a rule for the area of the largest triangle in terms of the semi-perimeter, s? Justify your answer using algebraic steps.

A_{max} = ...???????

If  $\Rightarrow s = \frac{3a}{2}$ $\Rightarrow a = \frac{2s}{3}$

So Area $= \sqrt{s(s-a)(s-a)(s-a)}$

$= \sqrt{s\left(s - \frac{2s}{3}\right)^3}$

$= \sqrt{s\left(\frac{s}{3}\right)^3} = \sqrt{\frac{s^4}{27}}$.

$A_{max} = \sqrt{\frac{s^4}{27}}$

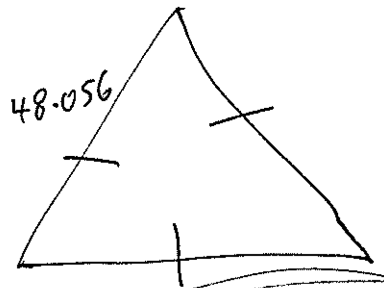
- What are the sides of a triangle with the smallest perimeter possible with a maximum area of 1000 m^2? Explain how you found this. What procedure did you use?

$A_{max} = 1000$.

Solve $1000 = \sqrt{\frac{s^4}{27}}$

$\Rightarrow s = 72.084$

$\Rightarrow a = 48.056m$.



$A_{max} = 1000\ m^2$.

A 48.056m equilateral triangle.