Chapter 4 **Fission and fusion**

Teacher Notes

Module 4.2 **Chain reactions and nuclear reactors**

Evaluation and Analysis 4.2.1 **Data analysis:** Critical mass   Total marks 20

Imagine that a particular sample of fissile material is manufactured only in 1 cm cubes. Each cubic centimetre of this material generates 6×10^6 neutrons each second; all of which are capable of going on to cause further fission should they meet another fissile atom. It is a property of this material that each square centimetre of surface area allows up to 3×10^6 of these neutrons to exit the cube every second, therefore these neutrons will not cause further fission. Hence, the quantity of material (in cm^3), and the surface area together determine whether a sample of this material is subcritical, critical or supercritical.

- A critical mass is defined as one in which a chain reaction can be sustained.
- A subcritical mass is defined as one in which a chain reaction cannot be sustained.
- A supercritical mass is defined as one in which a chain reaction will increase in rate.

Questions

1 What is the net number of neutrons escaping from a single (1 cm^3) cube?

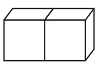
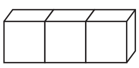
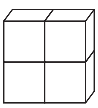
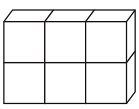
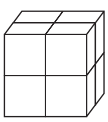
Total surface area $= 6 \times 1\ cm^2 = 6\ cm^2$

No. of neutrons able to escape $= 6 \times 3 \times 10^6 = 18 \times 10^6$ (1 mark)

2 A chain reaction is sustained if there are fewer atoms escaping than there are atoms being produced. Is a single cube able to sustain a chain reaction?

The number of neutrons able to escape $= 18 \times 10^6$*, which is more than the number produced* $= 6 \times 10^6$*, so a chain reaction is not sustainable.* (2 marks)

3 When two or more cubes are joined together to make a cuboid, the following data were obtained. Complete the following table, using your calculations of surface area and the information given above. (5 marks)

Number of cubes used	Total volume (cm^3)	Total surface area (cm^2)	No. of neutrons produced ($\times 10^6$)	No. of neutrons escaping ($\times 10^6$)	Critical?
2 	2	10	12	30	No
3 	3	14	18	42	No
4 	4	16	24	48	No
6 	6	22	36	66	No
8 	8	24	48	72	No

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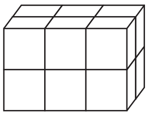
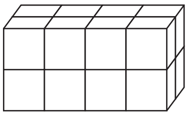
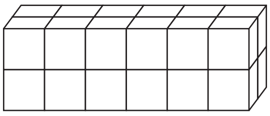
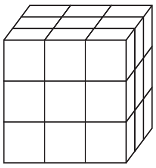
Number of cubes used	Total volume (cm^3)	Total surface area (cm^2)	No. of neutrons produced ($\times 10^6$)	No. of neutrons escaping ($\times 10^6$)	Critical?
12 	12	32	72	96	No
16 	16	40	96	120	No
24 	24	56	14432e	168	No
27 	27	54	162	162	Yes
30 (3 × 1 × 10)	30	86	180	258	No
36 (3 × 2 × 6)	36	72	216	216	Yes

4 Which of the combinations shown in the table would be 'critical'?

The 27 cm^3 cube, and the 36 cm^3 arrangement (since the rate of fission is sustained). (2 marks)

5 Are any of the combinations shown in the table adequate as the final configuration of a nuclear fission bomb?

No, there is no arrangement that has an excess of neutrons to increase the rate of fission. (1 mark)

6 Look just at the arrangements that have a surface area of 4 cm^3 at one end. What difference always occurs between the number of neutrons produced and the number escaping?

A pattern emerges in the table showing there is always a deficit of 24×10^6 neutrons between those produced and those escaped for every arrangement that has one side with an area of 4 cm^2. (1 mark)

7 With the help of calculations, explain why it is impossible for a rectangular prism of cross-sectional area 4 cm^2 and length *n* cm to sustain a chain reaction?

The number of neutrons produced $= \textit{volume} \times 6 \times 10^6$

$= (4 \times n) \times 6 \times 10^6$

$= 24 \times 10^6 \times n$

The number of neutrons able to escape $= \textit{surface area} \times 3 \times 10^6$

$= (8n + 8) \times 3 \times 10^6$

$= (n \times 24 \times 10^6) + (24 \times 10^6)$

$= 24 \times 10^6 \times (n + 1)$

The number of neutrons produced is always less than the number that can escape. (2 marks)

8 Write a formula for the number of neutrons produced in an *n* cm × *n* cm cube every second.

Neutrons produced $= \textit{volume} \times 6 \times 10^6$

$= n \times n \times n \times 6 \times 10^6$

$= 6 \times 10^6 \times n^3$ (2 marks)

9 Write a formula for the number of neutrons escaping from an n cm × n cm cube every second.

Neutrons escaping = *surface area* $\times 3 \times 10^6$

$= 6 \times n^2 \times 3 \times 10^6$

$= 18 \times 10^6 \times n^2$ (1 mark)

10 Show that the smallest cube that will form a supercritical mass is 3 cm × 3 cm × 3 cm. Use calculations incorporating your answers to questions **8** and **9** to support your answer.

To be critical: No. neutrons produced = No. neutrons escaping (1 mark)

$6 \times 10^6 \times n^3 = 18 \times 10^6 \times n^2$ (for a cube as calculated above)

Therefore: $6n^3 = 18n^2$

$6n^3 - 18n^2 = 0$

$6n^2(n - 3) = 0$

$n = 0$ or $n = 3$ (1 mark)

$n = 0$ means no neutrons produced or escaped, so $n = 3$ is the minimum size for a chain reaction to be sustained (controlled). Thus $n > 3$ will result in a supercritical mass. (1 mark)

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