MINDARIE SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS NOW

MATHEMATICS METHODS 1 & 2

Investigation 2

Polynomial Division

In-Class Validation **Total marks: 45** **Time allowed: 50 mins**

NAME _____

Question 1: [2,2 marks]

Using the Remainder Theorem, find the remainder when:

a) $P(x) = x^3 + 5x^2 - 2x - 2$ is divided by $(x + 1)$

$P(-1) = (-1)^3 + 5(-1)^2 - 2(-1) - 2$ ✓
$= -1 + 5 + 2 - 2$
$= 4$ ✓

b) $P(x) = x^3 - 3x^2 + x - 10$ is divided by $(x - 4)$

$P(4) = 4^3 - 3(4)^2 + 4 - 10$ ✓
$= 64 - 48 + 4 - 10$
$= 10$ ✓

Question 2: [3 marks]

A polynomial is given by $f(x) = 2x^3 + ax^2 + 6x + 3$ where a is a constant.
When $f(x)$ is divided by $(x - 2)$ there is a remainder of 7.
Find the value of a.

$f(2)$: $2(2)^3 + a(2)^2 + 6(2) + 3 = 7$ ✓
$16 + 4a + 12 + 3 = 7$
$4a + 31 = 7$ ✓
$4a = -24$
$a = -6$ ✓

Question 3: [2,2 marks]

Using the Factor Theorem, show that:

a) $(x - 1)$ is a factor of $P(x) = x^3 + x^2 + 3x - 5$

$P(1) = 1^3 + 1^2 + 3(1) - 5$ ✓

$= 1 + 1 + 3 - 5$

$= 0$ ✓

$\therefore (x-1)$ is a factor of $P(x)$

b) $(x + 2)$ is a factor of $P(x) = x^3 - 4x^2 - 7x + 10$

$P(-2) = (-2)^3 - 4(-2)^2 - 7(-2) + 10$ ✓

$= -8 - 16 + 14 + 10$

$= 0$ ✓

$\therefore (x+2)$ is a factor of $P(x)$

Question 4: [4,1 marks]

a) Use the factor theorem to fully factorise $P(x) = x^3 + 6x^2 - 13x - 42$.

$P(3) = 0$

$P(-2) = 0$

$P(-7) = 0$

✓✓ for working

$P(x) = (x+7)(x+2)(x-3)$ ✓✓ for factorised form.

b) Hence solve the equation $x^3 + 6x^2 - 13x - 42 = 0$.

$x = -7$

$x = -2$ ✓

$x = 3$

f.t from above if necessary.

Question 5: [3 marks]

Given that $(x - 3)$ is a factor of $P(x) = ax^3 + 2ax^2 + 3ax - 54$, find the value of a.

$P(3)$: $a(3)^3 + 2a(3)^2 + 3a(3) - 54 = 0$ ✓

$27a + 18a + 9a - 54 = 0$

$54a - 54 = 0$ ✓

$54a = 54$

$a = 1$ ✓

Question 6: [5,5 marks]

Complete the following divisions using one of the methods from parts D, E and F of the take home component.

a) $x^3 + 4x^2 - 7x - 10 \div (x + 5)$ $= x^2 - x - 2$ ✓

$= (x - 2)(x + 1)$ ✓

	x^2	$-x$	-2
x	x^3	$-x^2$	$-2x$
$+5$	$5x^2$	$-5x$	-10

✓✓✓ for method

b) $2x^3 + 5x^2 - x - 6 \div (x - 1)$ $= 2x^2 + 7x + 6$ ✓

$= (2x + 3)(x + 2)$ ✓

	$2x^2$	$+7x$	$+6$
x	$2x^3$	$+7x^2$	$+6x$
-1	$-2x^2$	$-7x$	-6

✓✓✓

Question 7: [5,5 marks]

Solve the following equations by factorising. You should only use the Factor Theorem to find the first factor.

a) $x^3 - 4x^2 - 3x + 18 = 0$

$(x+2)(x-3)^2 = 0$ ✓ for finding first factor

✓✓ for division

$x = -2$ or $x = 3$ ✓ for solutions.

b) $2x^3 + 5x^2 - x - 6 = 0$

$(2x+3)(x+2)(x-1) = 0$ ✓ for first factor

✓✓ for division

$x = -\frac{3}{2}$ $x = -2$ $x = 1$ ✓ for solutions.

Question 8: [6 marks]

Find the coordinates of the points where $2x^4 - 9x^3 - x^2 + 18x + 8$ crosses the x axis.

✓ $(2x+1)(x+1)(x-2)(x-4) = 0$

✓ $x = -\frac{1}{2}$ $x = -1$ $x = 2$ $x = 4$

✓✓✓ for method

- allow using factor theorem to find all 4