Test 1A Solutions

Wednesday, 6 March 2019 1:34 PM

Churchlands Mathematics Department 2019
Year 11 Mathematics Specialist
Test 1A:

CHURCHLANDS SENIOR HIGH SCHOOL

INSTRUCTIONS:
No Calculator
Notes not allowed
Full working must be shown for all questions (or parts) worth more than 2 marks.
Marks will be deducted for rounding and unit errors.

Name:_______________________ **Time:55 minutes** **Total**_______ **/ 57**

Question 1 **[1, 1 1 = 3 marks]**

Simplify, evaluating where possible, the following:

a) $\frac{10!}{7!\,5!}$

$$\frac{\cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8}}{\cancel{5} \times \cancel{4} \times 3 \times \cancel{2}}$$

$= 6$ ✓

b) $\frac{n!}{(n-2)!}$

$n(n-1)$ ✓

c) $\binom{10}{2}$

$$\frac{10!}{8!\,2!} = \frac{10 \times 9}{2}$$

$= 45$ ✓

Question 2 **[1, 1, 1 = 3 marks]**

For the statement "If the weather is nice, then I will take a walk" write its:

a) Inverse

If the weather is not nice, I will not take a walk ✓

b) Contrapositive

If I won't take a walk, the weather isn't nice ✓

c) Converse

If I take a walk, the weather is nice ✓

/6

Question 3 **[2, 2, 3, 3 = 10 marks]**

The digits 0, 1, 2, 3, 4, 5, 6 are to be arranged without repetition to form a 4 digit number which cannot ~~form~~ start with 0.

a) How many different arrangements are possible

$6 \times 6 \times 5 \times 4 = 720$ ✓✓

b) How many arrangements are greater than 3000

$4 \times 6 \times 5 \times 4 = 480$ ✓✓

c) How many arrangements are even

end with 0 : $6 \times 5 \times 4 \times 1 = 120$ ✓

with 2, 4, 6 : $5 \times 5 \times 4 \times 3 = 300$ ✓

$\therefore$ 420 ✓

d) How many arrangement are greater than 3000 or even

$n(>3000 \cap \text{even}) = 2 \times 5 \times 4 \times 4 + 2 \times 5 \times 4 \times 3$

$= 160 + 120$

$= 280$ ✓

$480 + 420 - 280 = 620$ ✓

✓

inclusion - exclusion

/10

Question 4 **[1, 1, 2, 2 = 6 marks]**

A box contains crayons of different colours. There are 5 blue, 3 green, and 2 black. How many crayons must be drawn to ensure that

a) There is 1 blue crayon

$5 + 1 = 6$ ✓

b) There are 2 crayons of each colour

$5 + 3 + 2 = 10$ ✓

c) There is a blue or a black crayon

$3 + 1 = 4$ ✓✓

d) There are more blue than black crayons

$3 + 2 + 3 = 8$ ✓✓

Question 5 **[2, 2, 2 = 6 marks]**

A committee of 10 is to be chosen from 7 farmers, 8 grocers and 5 distributers.
Determine the number of ways of selecting the committee if it has to contain:
[YOU MAY LEAVE YOUR ANSWERS AS FACTORIALS OR IN THE FORM $^{n}C_{r}$]

a) Exactly 3 farmers and 4 grocers

$^{7}C_{3} \times {}^{8}C_{4} \times {}^{5}C_{3}$ ✓✓

b) Exactly 2 distributers

$^{5}C_{2} \times {}^{15}C_{8}$ ✓✓

c) More farmers and grocers (combined) than distributers

$\binom{5}{0}\binom{15}{10} + \binom{5}{1}\binom{15}{9} + \binom{5}{2}\binom{15}{8} + \binom{5}{3}\binom{15}{7} + \binom{5}{4}\binom{15}{6}$

✓✓

/12

Question 6 **[6 marks]**

How many numbers less than 2000 are divisible by 3, 8, or 10

$\text{Int}\left[\frac{2000}{3}\right] = 666$ $\quad \text{Int}\left[\frac{2000}{8}\right] = 250$ $\quad \text{Int}\left[\frac{2000}{10}\right] = 200$

$\text{Int}\left[\frac{2000}{24}\right] = 83$ $\quad \text{Int}\left[\frac{2000}{30}\right] = 66$ $\quad \text{Int}\left[\frac{2000}{40}\right] = 50$

$\text{Int}\left[\frac{2000}{120}\right] = 16$

✓ 4 correct
✓ 5 correct
✓ 6 correct
✓ 7 correct

$(666 + 250 + 200) - (83 + 66 + 50) + 16$ ✓ inc.-exc.

$= 1116 - 199 + 16$

$= 933$

✓

/6

Question 7 **[2, 2, 2, 3 = 9 marks]**

How many 'words' can be made from the letters of the word CONTINUOUS, if

[YOU MAY LEAVE YOUR ANSWERS AS FACTORIALS OR IN THE FORM ^nC_r]

C N T S   O I U
N   O U

a) All ten letters are to be used

$$\frac{10!}{2!2!2!}$$ ✓✓

b) All ten letters are to be used and the letters C and S must be next to each other

$$\frac{9!2!}{2!2!2!} = \frac{9!}{2!2!}$$ ✓✓

c) Five letters are to be used and the word cannot contain more than one of each letter (i.e. no letter is to be repeated)

$$^7C_2 \times 5! \quad ✓✓ \quad \left[\text{or } ^7C_5 \times 5! = {}^7P_5 = \frac{7!}{2!}\right]$$ accept any

d) Five of the ten letters are to be used and the 'word' must contain exactly two vowels

$$\binom{4}{3}\binom{3}{2}\times 5! + \binom{3}{1}\binom{3}{2}\times\frac{5!}{2!} + \binom{3}{1}(2)\left(\frac{5!}{2!2!}\right) + \binom{4}{3}(2)\left(\frac{5!}{2!}\right)$$

✓ 2 correct
✓ 3 correct
✓ 4 correct

all diff. 1440 | repeat Ns, diff vowels 540 | repeat Ns, repeat vowels 180 | diff const, repeat vowels 480

Question 8 **[4 marks]**

Decide if the following statement is true or false:

" Given any 100 consecutive whole numbers, one can choose any 15 numbers so that at least two of the chosen numbers have a difference between them that is always divisible by 7 "

Justify your answer.

True ✓

For the difference to be divisible by 7, it has to have the same modulo 7. i.e. must be in the same form of either

$7n, 7n+1, 7n+2, \dots, 7n+6$ ✓

There are 100 pigeons [integers] and 7 pigeonholes [$7n, 7n+1, \dots, 7n+6$]

Int $\left[\frac{100}{7}\right] = 14 < 15$. ✓ So by the pigeonhole principle, at least 2 of the chosen integers will be the same modulo 7 (i.e. diff. div. by 7) ✓

/13

Question 9 **[4 marks]**

Show, using proof by contradiction, that if $a^2 - 2a + 7$ is even, then a is odd , a is an integer

Assume that $a^2 - 2a + 7$ is even and a is even ✓assume opp.

Then we can write $a = 2n$, $n \in \mathbb{Z}$

SO, $a^2 - 2a + 7 = (2n)^2 - 2(2n) + 7$ ✓ sub.

$= 4n^2 - 4n + 7$

$= 2(2n^2 - 2n + 3) + 1$ ✓simplify and show contradiction

$= 2m + 1$, $m \in \mathbb{Z}$ which is odd (contradiction!)

$\therefore$ a must be odd ✓conclude

$\Rightarrow$ If $a^2 - 2a + 7$ is even, then a is odd □

Question 10 **[6 marks]**

For each of the following statements, state if it is true or fase.

If it is false, provide a counter example

a) If the total exterior angle of a polygon is equal to 360°, then its total interior angle is also equal to 360°

✓ False , triangles have total ext 360° and int 180° ✓

b) ${}^{n}C_3 > {}^{n}C_2 \quad \forall\, n \geq 3$

False ✓, ${}^{3}C_3 = 1$, ${}^{3}C_2 = 3$, $1 \not> 3$ ✓

c) ${}^{n}C_0 > {}^{n}C_1$ for any integer $n \geq 1$

False ✓, ${}^{1}C_0 = {}^{1}C_1 = 1$ ✓

/10