High School
Mathematics Test 2014

Year 10

Geometric Reasoning

<div style="background:black">Calculator Allowed</div>

Name_____

Skills and Knowledge Assessed:

- Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)
- Formulate proofs involving congruent triangles and angle properties (ACMMG243)

Extended Answer Test.

Answers should be supported by relevant mathematical reasoning and/or calculations
Marks will not be awarded for answers with no reasoning..
Complete any diagrams and write all working and answers in the spaces provided on this test paper.

Marks

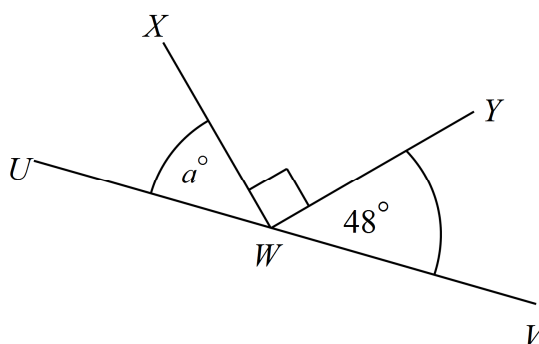1. (a)   Find the value of a.   **1**
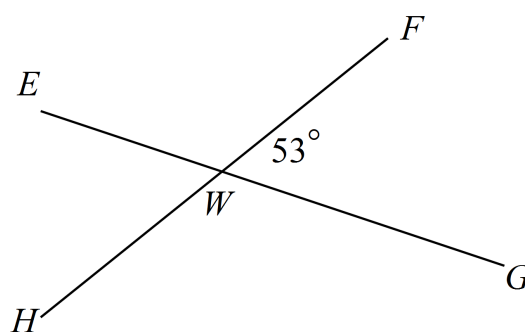
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(b)   Find the size of $\angle EWH$.   **1**

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Marks

(c) Find the value of b.   1

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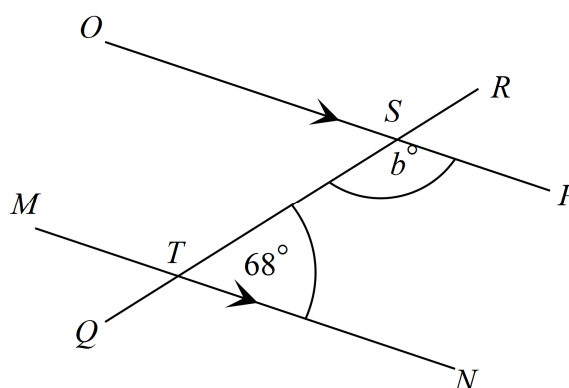
2. (a) *KL*, *MN* and *QR* are straight lines   2
which intersect at *P*.
Find the value of *s*.

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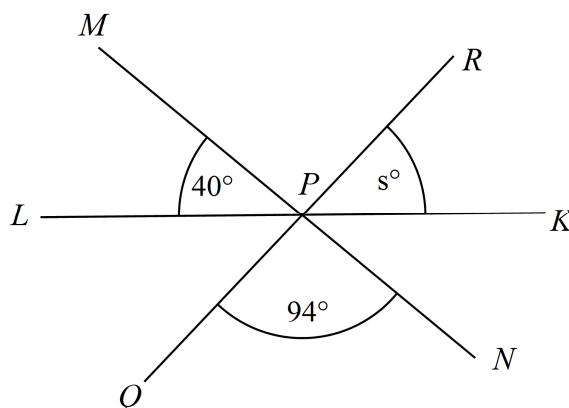
(b) *BA, CA, DA* and *EA* intersect at *A*.   2
∠*CAD* is a right angle.   Find the value of *k*.

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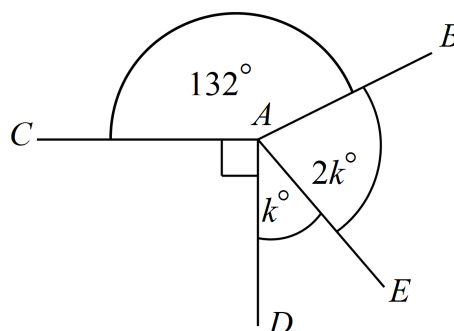
(c) *AB* || *CD*   2
HJ, IK and *AB* intersect at F.

Find the value of *x*.

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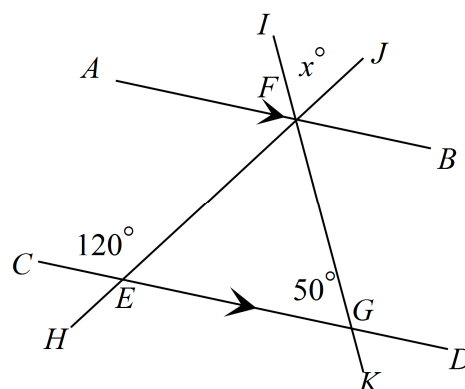
Marks

3. (a) $\triangle$ *GHI* is an isosceles triangle.   **2**
   GI is produced to *J*.
   Find the value of *y*.

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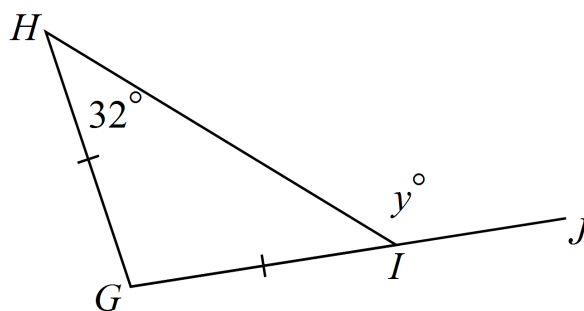
(b) *ABCD* is a rectangle.   **2**
  E is a point on the diagonal *BD*.
  Find the size of ∠*BEA*.

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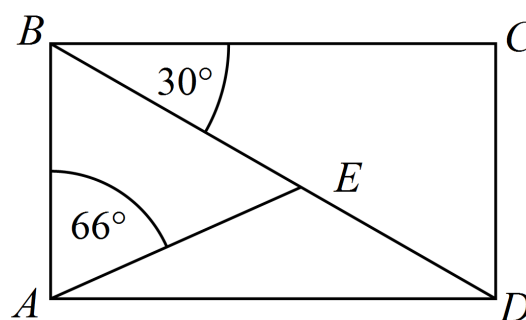
(c) *PQRS* is a kite whose diagonals intersect at *T*. **2**
  Find the size of ∠*TSP*.

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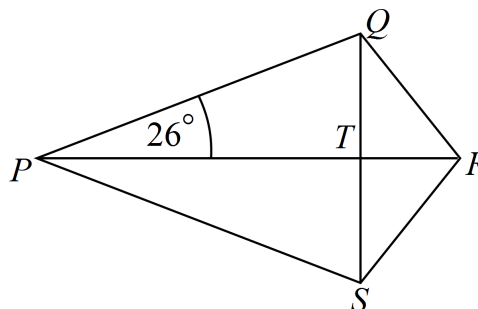
4. (a) Prove that $\triangle ABC \equiv \triangle FED$.   **3**

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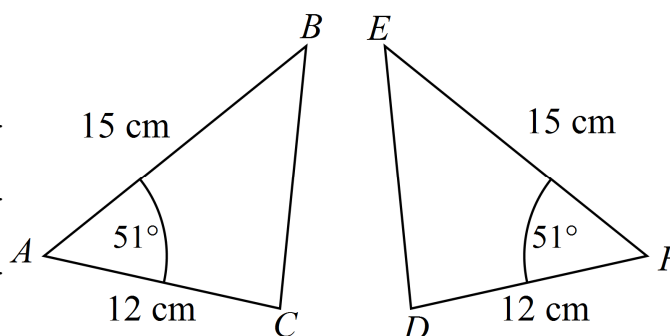
Marks

(b)   *VW || UX* and *VU || WX*.
Prove that $\Delta\,UVX\,\equiv\,\Delta\,WXV$.

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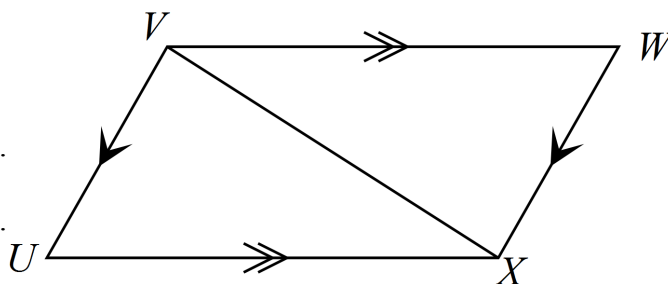
5. (a)   *KL || ON*.
Prove that $\Delta\,MLK\,|||\,\Delta\,MON$.

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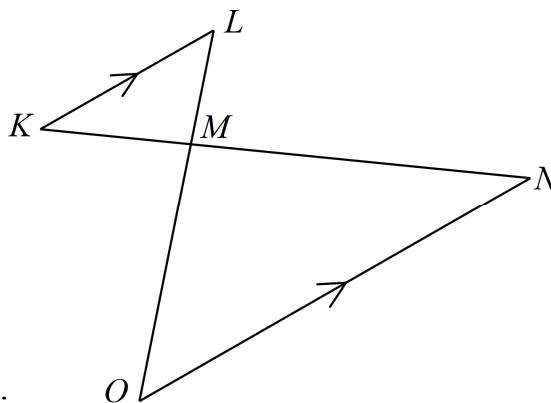
(b)   i)   A vertical tree EF casts a shadow FD which measures 12 m.
At the same time a vertical fence post BC casts a shadow CA which
measures 3.6 m.
Prove that the triangles DEF and ABC are similar.

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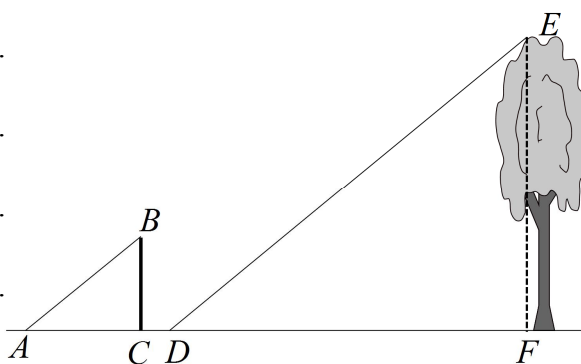
Marks

ii) If the fence post BC is 2.4 metres high, what is the height of the tree EF?   **1**

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6. a) The circles are concentric with O as the centre.   **3**
PQ =SR.
Prove that $\triangle OPQ \equiv \triangle OSR.$

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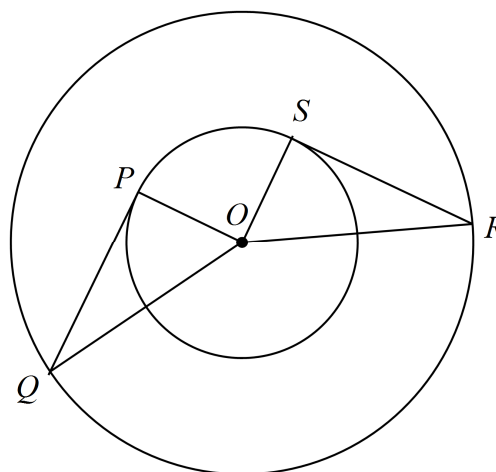
b) i) In $\triangle$ABC, *D* is the midpoint of *AB* and *E* is the midpoint of *BC*.   **2**

Prove that $\triangle ABC \;|||\; \triangle DBE.$

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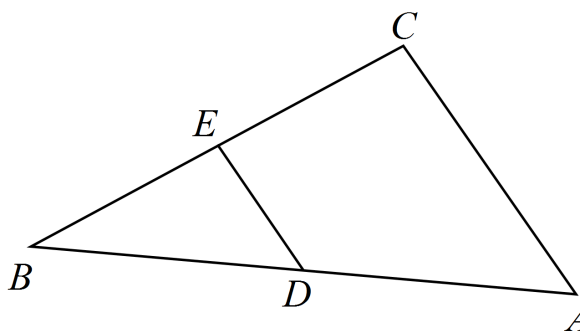
Marks

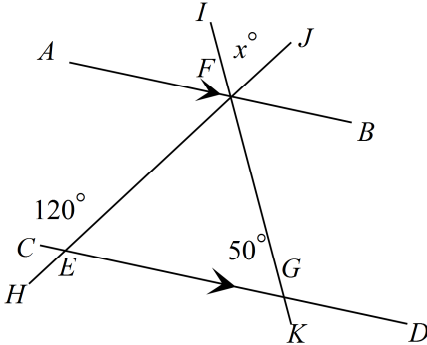ii) DF is drawn parallel to BC. Prove that $\triangle BED \equiv \triangle DFA$.   **3**
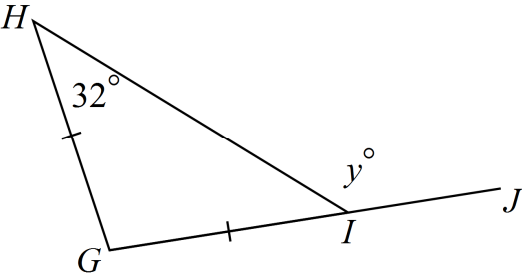
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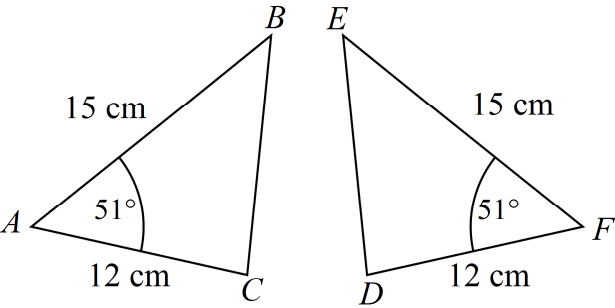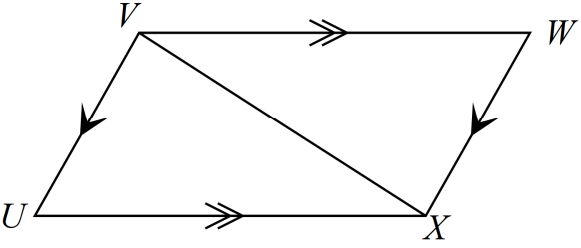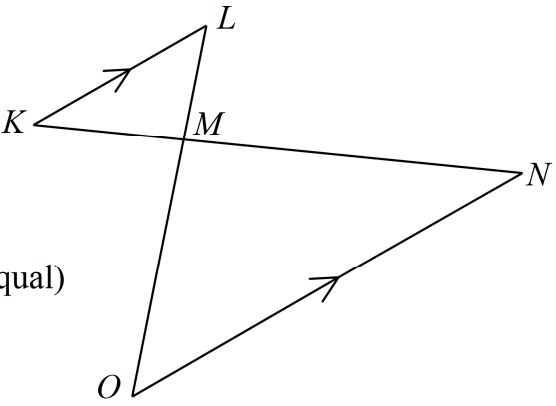…………………………………………

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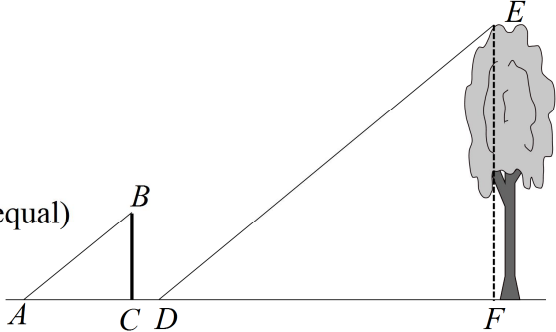
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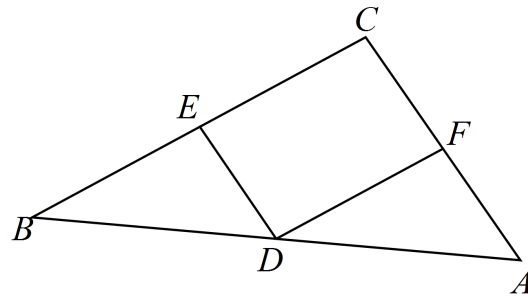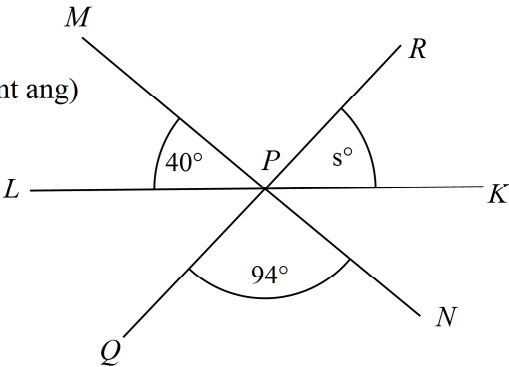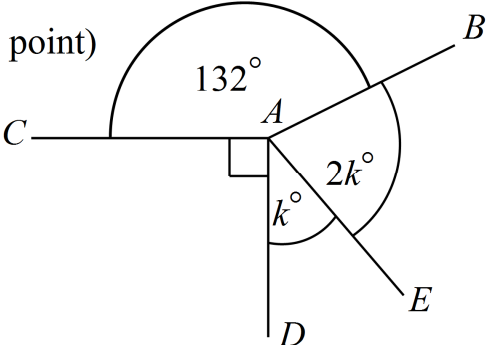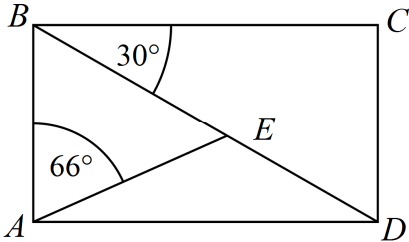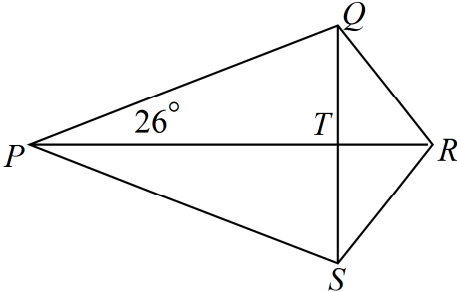
End of Test

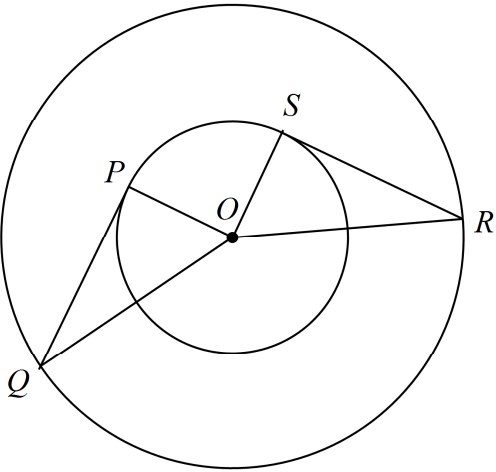# High School
Mathematics Test 2014
Geometric Reasoning
ANSWERS
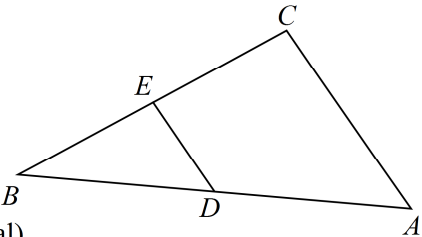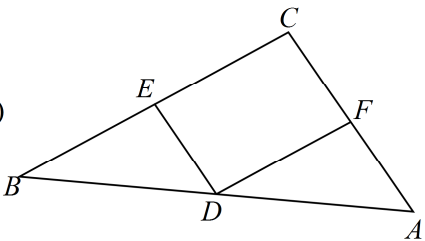
Section 3		
	Working and Answers	Marks
1. a)	$a + 90 + 48 = 180$ (Supplementary angles) $\therefore a = 180 - 138$ $a = 42^{\circ}$ 	1 mark for answer with reasons
b)	 $\angle EWH = \angle FWG$ (vertically opposite angles) $\therefore \angle EWH = 53^{\circ}$	1 mark for answer with reasons
c)	$\angle TSP + \angle STN = 180^{\circ}$ (cointerior/allied angles) $\therefore b + 68 = 180^{\circ}$ $b = 180 - 68 = 112^{\circ}$ 	1 mark for answer with reasons

2. a)	$\angle MPR = \angle QPN = 94^\circ$ (vert opp angles) $\angle LPM + \angle MPR + \angle RPK = 180^\circ$ (supplement ang) $40 + 94 + s = 180^\circ$ $s = 180 - 134$ $s = 46^\circ$	2 marks for correct answer with reasoning. 1 mark if an error in otherwise correct solution.
b)	$90 + 132 + 2k + k = 360^\circ$ (angles at a point) $222 + 3k = 360^\circ$ $3k = 360 - 222$ $3k = 138$ $k = \dfrac{138}{3} = 46^\circ$	2 marks for correct answer with reasoning. 1 mark if an error in otherwise correct solution.
c)	$\angle AFE + \angle CEF = 180^\circ$ (cointerior angles on ‖ lines) $\therefore \angle AFE = 60^\circ$ $\angle BFG = \angle FGE = 50^\circ$ (alt angles on ‖ lines) $\angle AFB + \angle EFG + \angle BFG = 180^\circ$ (supplementary ang) $60 + \angle EFG + 50 = 180$ $\therefore \angle EFG = 180 - 110 = 70^\circ$ $\angle IFJ = \angle EFG$ (vert opp ang) $\therefore x = 70^\circ$	2 marks for correct answer with reasoning. 1 mark if an error in otherwise correct solution.
3. a)	$\angle GIH = 32^\circ$ (base ang of isos $\triangle$) $\angle HIJ = 180 - 32 = 148^\circ$ (supp angles) $y = 148^\circ$	2 marks for correct answer with reasoning. 1 mark if an error in otherwise correct solution.

b)	$\angle ABE + 30 = 90$ (angles in rectangle are $90°$) $\angle ABE = 60°$ $\angle BEA + 66 + 60 = 180°$ (ang sum of $\triangle ABE$) $\angle BEA = 180 - 126 = 54°$	2 marks for correct answer with reasoning. 1 mark if an error in otherwise correct solution.
c)	$\angle PTQ = 90°$ (diagonals of kite meet at $90°$) $\angle PQT + 90 + 26 = 180$ (ang sum $\triangle PTQ$) $\angle PQT = 180 - 116 = 64°$ $\angle TSP = \angle PQT = 64°$ (symmetry of kite)	2 marks for correct answer with reasoning. 1 mark if an error in otherwise correct solution.
4. a)	In $\triangle ABC$ and $\triangle FED$ $AB = EF = 15$ cm (given) $AC = FD = 12$ cm (given) $\angle BAC = \angle EFD = 51°$ (given) $\therefore \triangle ABC \equiv \triangle FED$ (SAS)	3 marks for correct & complete proof. 2 if a reason or statement is incorrect or missing 1 if one correct statement is made toward proof.

b)	In $\triangle UVX$ and $\triangle WXV$ $\angle UVX = \angle WXV$ (alt ang on ‖ lines) $\angle UXV = \angle WVX$ (alt ang on ‖ lines) VX is common $\therefore \triangle UVX \equiv \triangle WXV$ (*AAS*)	3 marks for correct & complete proof. 2 if a reason or statement is incorrect or missing 1 if one correct statement is made toward proof.
5. a)	In $\triangle MON$ and $\triangle MLK$ $\angle O = \angle L$ (alt ang on ‖ lines) $\angle N = \angle K$ (alt ang on ‖ lines) $\angle OMN = \angle LMK$ (vert opp angles) $\therefore \triangle MON \parallel\parallel\parallel MLK$ (all corr angles equal)	3 marks for correct & complete proof. 2 if a reason or statement is incorrect or missing 1 if one correct statement is made toward proof.
b)	i) In $\triangle ABC$ and $\triangle DEF$ $\angle BAC = \angle EDF$ (corr ang on ‖ lines) $\angle BCA = \angle EFD$ (corr ang on ‖ lines) $\angle ABC = \angle DEF$ (ang sum of triangles) $\therefore \triangle ABC \parallel\parallel\parallel \triangle DEF$ (all corresp angles equal)	2 marks for correct answer with reasoning. 1 mark if an error in otherwise correct solution

b)	ii) $\dfrac{EF}{BC} = \dfrac{FD}{CA}$ (Ratios of corresponding sides are equal) $\dfrac{EF}{2.4} = \dfrac{12}{3.6}$ $EF = 2.4 \times \dfrac{12}{3.6}$ $= 8$ the height is 8 m.		1 mark for answer and working.
6. a)	 In ΔOPQ and ΔOSR $PQ = SR$ (given) $PO = SO$ (equal radii of circle PS) $OQ = OR$ (equal radii of circle QR) $\therefore \Delta OPQ \equiv \Delta OSR$ (SSS)		3 marks for correct & complete proof. 2 if a reason or statement is incorrect or missing 1 if one correct statement is made toward proof.
b)	In ΔABC and ΔDBE $\dfrac{AB}{DB} = \dfrac{2}{1}$ (D is midpoint of AB) $\angle B$ is common $\dfrac{CB}{EB} = \dfrac{2}{1}$ (E is midpoint of CB) $\therefore \Delta ABC \,|||\, \Delta DBE$ (2 pairs of sides in the same ratio and included angle equal) 		2 marks for correct answer with reasoning. 1 mark if an error in otherwise correct solution

b)	ii) In $\triangle BED$ and $\triangle DFA$ $\angle EBD = \angle FDA$ (Corr Ang on ∥ lines BE and DF) $\angle EDB = \angle FAD$ (Corr Ang in Similar $\triangle$) $BD = DA$ (D is midpoint of AB) $\therefore \triangle BED \equiv \triangle DFA$ (AAS)	3 marks for correct & complete proof. 2 if a reason or statement is incorrect or missing 1 if one correct statement is made toward proof.