Full Name: SOLUTIONS

MATHEMATICS

Specialist Units 3 & 4

Test 6 – Point and Interval Estimates and The Central Limit Theorem

Semester 2 2019

Calculator Assumed Only

Time allowed for this section

Working time for this section: 45 minutes
Marks available: 46 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (6 marks: 3, 3)

The time T in minutes that a particular flight arrives later than its scheduled time is uniformly distributed with $-25 \leq T \leq 45$. The population mean is $\mu(T) = 14$ and the population variance is $\sigma^2(T) = 975$

A sample of 40 arrival times is taken and the sample mean $\bar{T}$ is calculated.

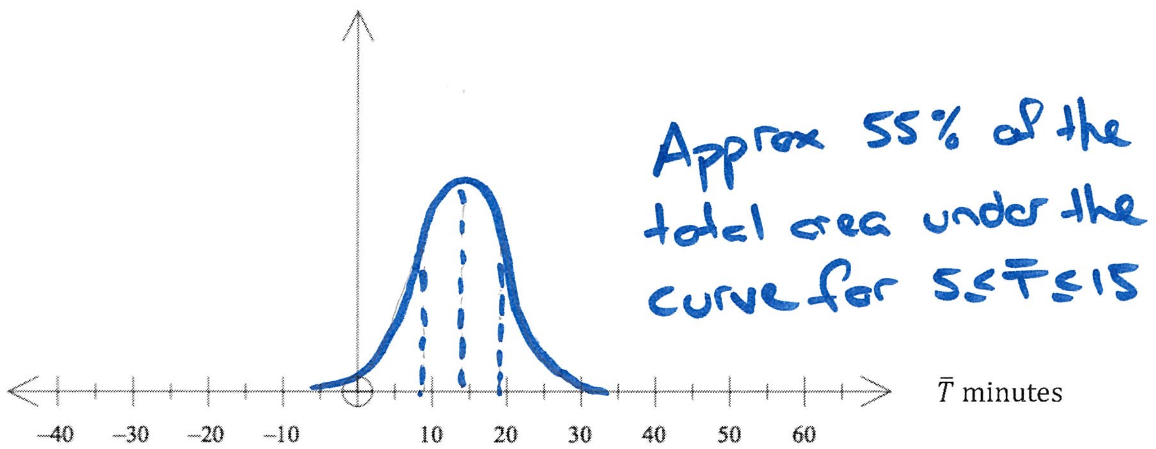
a) Determine $P(5 \leq \bar{T} \leq 15)$ correct to 2 decimal places.

$\bar{T} \sim N\left(14, \frac{975}{40}\right) \Rightarrow \sigma(\bar{T}) = 4.94$

$P(5 \leq \bar{T} \leq 15) = 0.55$ (2 d.p.)

b) If a large number of samples, each with 40 arrival times, is taken, sketch the likely distribution of the sample mean $\bar{T}$ below.

In the diagram, indicate or refer to the calculation from part a).



2. (7 marks: 1, 2, 2, 2)
On the basis of results obtained from a random sample of 81 bags produced by a mill, the 95% confidence interval for the mean weight of flour in a bag is found to be (514.56g, 520.44g).

a) Find the value of $\bar{x}$, the mean weight of the sample.

$$\bar{x} = \frac{514.56 + 520.44}{2}$$

$$= 517.5\text{g}$$

b) Find the value of σ, the standard deviation of the normal population from which the sample is drawn.

$$520.44 - 517.5 = 1.96 \frac{\sigma}{\sqrt{81}}$$

$$\sigma = 13.5\text{g}$$

c) Calculate the 99% confidence interval for the mean weight of flour in a bag.

$$517.5 \pm 2.576 \times \frac{13.5}{\sqrt{81}} = (513.636, 521.364)$$

d) Using the sample mean from a) as the best estimate for the population mean, what is the probability that the sample mean of a larger sample of 225 bags is less than 516g?

$$X \sim N\left(517.5, \frac{13.5^2}{225}\right)$$

$$P(X < 516) = 0.0478$$

3. (8 marks: 2, 2, 2, 2)
A random sample of 45 households in a suburb was selected as part of a study on electricity consumption, and the number of kilowatt hours (kWh) was recorded for each household. The mean consumption was found to be 415 kWh. In a very large study the previous year, it was found that the standard deviation of the consumption was 64 kWh.

a) Calculate a 98% confidence interval for the mean electricity consumption of households in this suburb.

$$415 \pm 2.326 \times \frac{64}{\sqrt{45}} = (392.8, 437.2)$$

b) An electricity supplier claimed that the mean consumption of the households in the suburb was 435 kWh. Does the sample above provide a strong reason to doubt his claim? Justify your answer.

No, 435 lies within the 98% C.I and so there is no reason to doubt the claim.

c) Thirty six (36) similar studies are planned for the suburb.

i. Determine the number of households that should be sampled in each of these studies to be 95% confident that the mean electricity consumption of households is within 15kWh of the true value.

$$n = \left(\frac{1.96 \times 64}{15}\right)^2$$

$$= 69.9 \qquad \therefore \text{Sample 70 households}$$

ii. How many of the 95% confidence intervals from these additional studies are not expected to contain the true mean? Justify your answer.

$$0.05 \times 36 = 1.8$$

Two of the 36 studies as we expect 95% of the intervals to contain the true mean.

4. (8 marks: 3, 4, 1)

Raphael drives to work each day from Monday to Friday. The mean time he takes to drive to work is μ minutes with a standard deviation of 2 minutes.

a) The average time he takes to drive to work over a total of 40 work days is 25 minutes with a standard deviation of 2 minutes. Determine a 99% confidence interval for μ minutes (correct to one decimal place).

$$25 \pm 2.576 \times \frac{2}{\sqrt{40}}$$

$$24.2 \leq \mu \leq 25.8 \text{ minutes}$$

b) The average time he takes to drive to work over a total of *n* work days is *k* minutes. A 99% confidence interval for μ for this sample is $23.80 \leq \mu \leq 25.20$. Find *k* and *n*.

$$k = \frac{23.8 + 25.2}{2} = 24.5 \text{ min.}$$

$$\text{Error} = 24.5 - 23.8 = 0.7$$

$$\text{Hence } 2.576 \times \frac{2}{\sqrt{n}} = 0.7$$

$$n = 54$$

c) Raphael calculates 10 sets of 90% confidence intervals to estimate μ. How many of these confidence intervals are expected to contain μ?

$$\text{Expected number} = 10 \times 0.9$$

$$= 9$$

5. (16 marks: 3, 3, 3, 3, 4)
The volume of water used by Backyard Landscaping to top up an ornamental pool has been observed to be normally distributed with mean $\mu = 215$ litres and standard deviation $\sigma = 20$ litres.

The ornamental pool is topped up 50 times. Determine the probability that the:

a) sample mean volume will be between 212 and 218 litres.

let $\bar{w}$ be the sample mean

$N(215, \sigma_{\bar{w}}^2) \Rightarrow \sigma_{\bar{w}} = 2.12$

$P(212 < \bar{w} < 218) = 0.843$

b) total volume of water used is less than 10.9 kilolitres.

$\bar{w} = \frac{10900}{50} = 218$ litres

$P(\bar{w} < 218) = 0.921$

Water is a scarce commodity and accuracy is required. The pool is topped up 50 times and the sample mean obtained is denoted by $\bar{W}$.

c) If it is required that $P(a \le \bar{W} \le b) = 0.99$, then determine the values of a and b, each correct to 0.1 litres.

$N(215, \sigma_{\bar{w}}^2) \Rightarrow \sigma_{\bar{w}} = 2.12$

$P(-k < z < k) = 0.99 \Rightarrow k = 2.5758$

$215 - 2.5758 \times 2.12 < \bar{w} < 215 + 2.5758 \times 2.12$

$a = 209.539$

$b = 222.461$

d) If the probability for the mean amount of water used differs from μ by less than five litres is 96%, find n, the number of waterings that need to be measured.

$\sigma_{\bar{w}} = \frac{20}{\sqrt{n}}$ $P(-k < z < k) = 0.96$

$\Rightarrow k = 2.0537$

$2.0537 \times \frac{20}{\sqrt{n}} < 5 \Rightarrow n > 67.48$

$\therefore$ n is 68 waterings

A rival company called Better Landscaping takes over the watering of the ornamental pool. Over 36 consecutive days, it was observed that the Better Landscaping company used a total of 8.01 kilolitres. The standard deviation for the 36 days was also 20 litres.

A representative from the Backyard Landscaping states that 'Better Landscaping are using significantly more water than we did when we were filling this pool. They are wasting water'.

e) Perform the calculations necessary to comment on this claim.

* μ_w = pop mean for Better Landscaping.

$\frac{8018}{36} = 222.5$ litres $N\left(222.5, \frac{15^2}{36}\right)$

$\sigma_{\bar{w}} = 2.5$

95% CI $217.6 < \mu_w < 227.4$

99% CI $216.05 < \mu_w < 228.95$

Backyard Landscapings mean is within both CI's $\therefore$ the claim is not vindicated

End of Test

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You may use this space to extend or re-attempt an answer to a question or questions and should you do so then number the question(s) attempted and cross out any previous unwanted working.