# BALDIVIS SECONDARY COLLEGE

## APPLICATIONS - Unit 3 & 4

## 2020 Test 3 – Networks

*Marking Guide*

Student Name _____   Teacher Name _____

**Time allowed for this task:** 55 minutes, in-class, test conditions.

Section 1: 28 minutes + 2 minutes reading time
Section 2: 22 minutes + 2 minutes reading time

| | | |
|---|---|---|
| **Materials required:** | **Section 1 Resource free section** | **(28 marks)** |
| | Standard writing equipment | |
| | SCSA Formula Sheet | |
| | | |
| | Section 2 Calculator assumed section | (22 marks) |
| | Calculator (to be supplied by the student) | |
| | SCSA formula Sheet | |
| | One page A4 (single sided) hand written notes | |

**Marks available:** 50 marks

**Task Weighting:** 7%

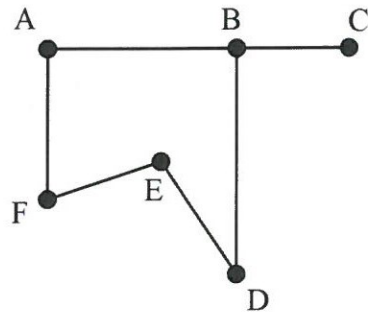**Section 1: Resource Free**                                    **[28 marks]**

**Question 1**                                          **[3,2,2,1,2 – 10 marks]**

Consider the network below:



(a) State the number of faces, vertices and edges.

$F = 2$  ✓
$V = 6$  ✓
$E = 6$  ✓

(b) State the degree of each vertex.

$A = 2$          $D = 2$
$B = 3$          $E = 2$          ✓✓    – 1 for each error
$C = 1$          $F = 2$

(c) Is the graph simple? Justify your answer.

Yes  –   no loops, multiple edges, direction
                    or weightings  ✓
✓

(d) Name an edge that is a bridge for the network.

BC  ✓

(e) Is the graph Eulerian, Semi-Eulerian or neither? Justify your answer.

Semi- Eulerian  ✓
Exactly 2 odd vertices  ✓

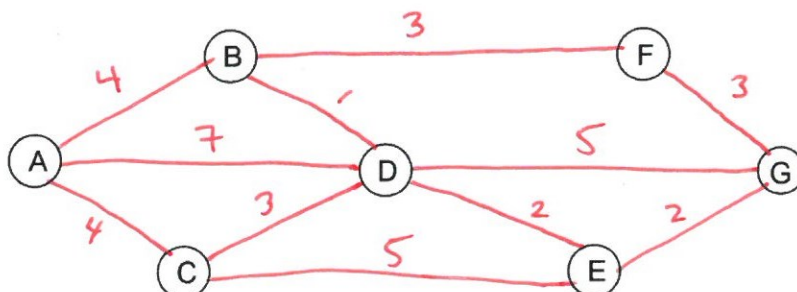**Question 2**                                    [3,2,2,2 - 9 marks]

The table below shows the distances, in kilometres, along direct footpaths between the checkpoints A, B, C, D, E, F and G at a national park.

|   | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| A | 4 | 4 | 7 | - | - | - |
| B |   | - | 1 | - | 3 | - |
| C |   |   | 3 | 5 | - |   |
| D |   |   |   | 2 | - | 5 |
| E |   |   |   |   | - | 2 |
| F |   |   |   |   |   | 3 |

(a) Complete the graph below to show this information.

✓ - edges
✓✓ - weighting

- 1 for each error



* must include distances on edges

A park worker is at Checkpoint D and needs to visit all the other checkpoints just once and then return to D.

(b)   The worker leaves Checkpoint D, walks to Checkpoint A and then visits the rest of the checkpoints just once before returning to D. List the other checkpoints visited, in order, and state the total length of this route.

✓   D - A - B - F - G - E - C - D    or   D - A - C - E - G - F - B - D

✓   Total length = 25 km

(c)   Is the trail in (b) a Hamiltonian path or cycle? Explain your answer.

Hamiltonian cycle ✓
visits all vertices just once, starting & finishing at same vertex ✓

(d)   A shorter cycle, starting and finishing at D, is possible for the worker. List the order the checkpoints should be visited to achieve the smallest total length and state this length.

✓   D - C - A - B - F - G - E - D  (or reverse)

✓   shortest = 21 km

**Question 3**                                    [3,3,3 - 9 marks]

Over the years, a regional authority has found that the presence of a supermarket, a bank, a service station and a hotel was a good indicator of a country town's well-being.

Below is shown a section of the table containing this information for some of the country towns.

| | | Service provided | | | |
|---|---|---|---|---|---|
| | | Bank (B) | Supermarket (M) | Service Station (S) | Hotel (H) |
| Towns | Katanning | Yes | Yes | Yes | Yes |
| | Collie | No | Yes | Yes | No |
| | Moora | No | No | Yes | Yes |

(a)    Display the information in the table as a bipartite graph.

/ – correct edges for Kat

/ – correct edges for Collie

/ – correct edges for Moora.
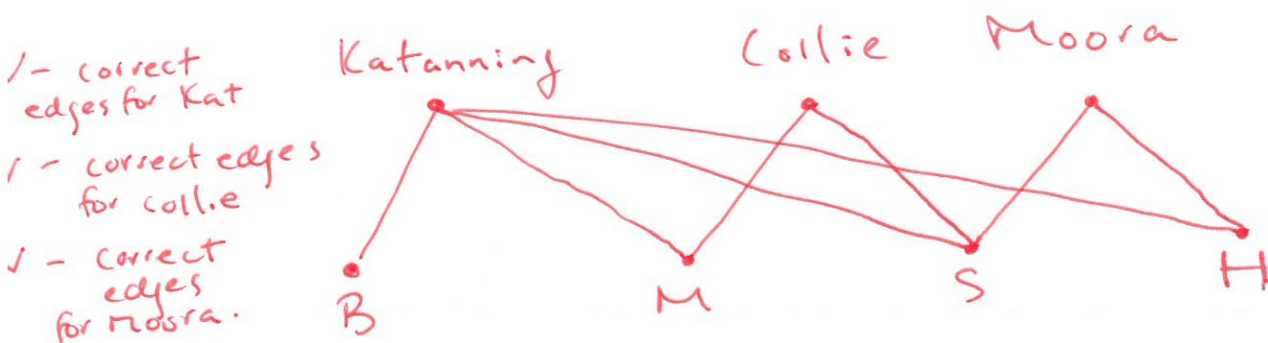


(b)    Looking at this bipartite graph of the three towns, an officer at the regional authority commented that the graph was planar.

(i)    When is a graph planar?

✓   when it can be drawn without any edges crossing

(ii)    Draw the bipartite graph again, showing that it is planar.

✓ – connects all towns with their respective services

✓ – draws network without any edges crossing.
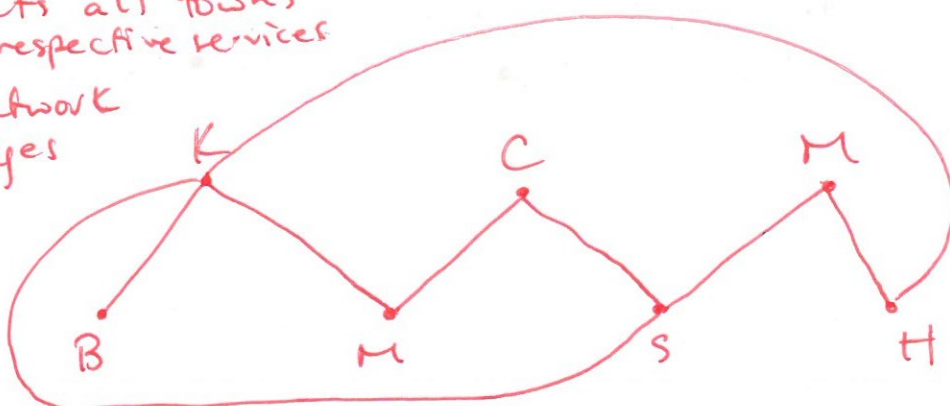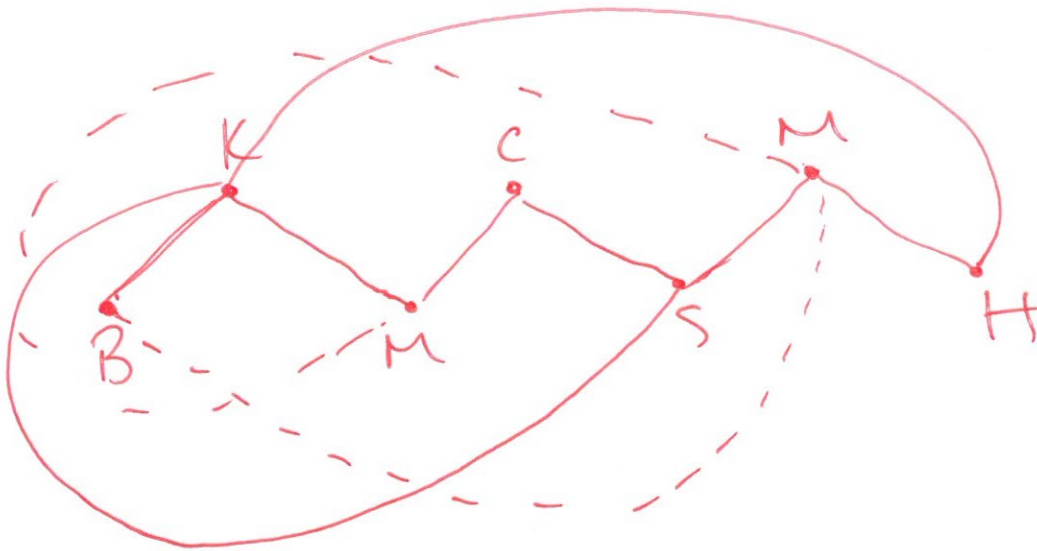
(c)     The officer also commented that if Moora had one more of the service providers, the
        bipartite graph would no longer be planar.

        (i)     To which of the service providers was the officer referring?

        ✓          Bank  or  Supermarket

        (ii)    Draw the bipartite graph again showing why with this additional piece of
                information displayed on the graph, it is no longer a planar graph?



✓ – draws a graph showing edge from Moora
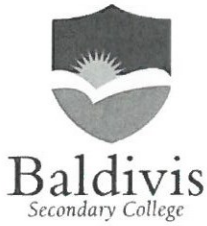      to bank or supermarket

✓ – describes (visually or with words) that
      you couldn't connect Moora to these
      services without edges crossing.

**End of Section 1**

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**Section 2 Calculator assumed section    (22 marks)**
Calculator (to be supplied by the student)
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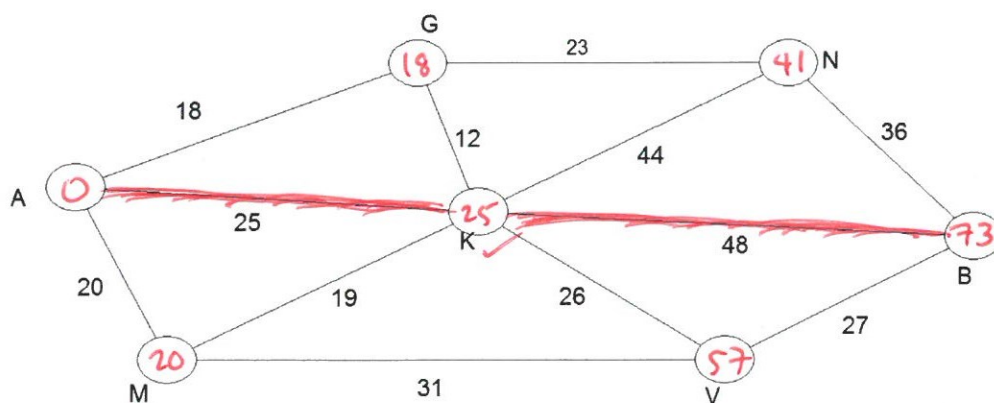One page A4 (single sided) hand written notes

**Marks available:**     **50 marks**

**Task Weighting:**     7%

**Question 4**                                                    **[3,3– 6 marks]**

The network below shows the distances in km of each road connecting seven towns.
Having just passed her driving test Yusuke decides to cruise around this network.

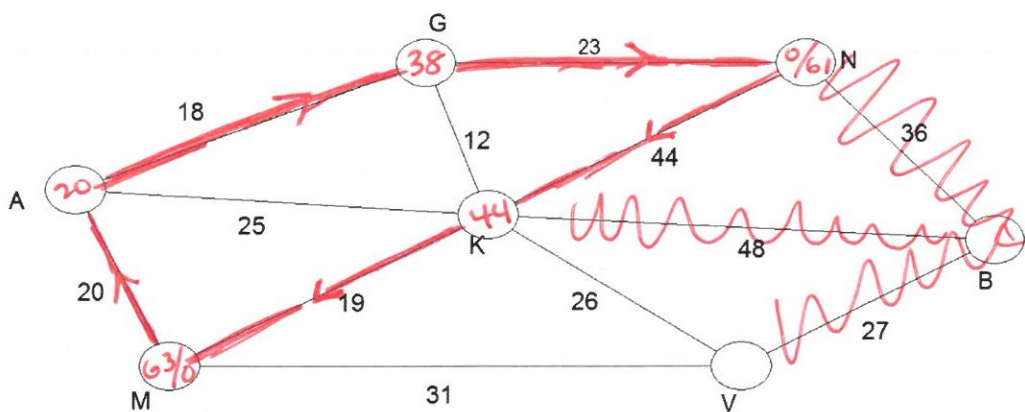(a)    Find the shortest path from A to B and state the
distance.



AKB ✓            distance – 73km ✓            All bubbles filled
                                              correctly ✓

(b)    Vanessa is at town N wants to travel to town M and back to N. All roads through  B
       are blocked and no road may be travelled more than once. Find the length of the
       shortest path showing all your working?



NKM = 44 + 19 = 63 km ✓

MAGN = 20 + 18 + 23 = 61 km ✓

length of shortest path = 63 + 61

= 124 km ✓

**Question 5**                                                    **[3,2,2,1– 8 marks]**

Adele, Bob, Chen and Daisy all have work in the same office and have their own email addresses. Adele and Bob both know each other's email addresses and Adele and Chen both know Daisy's email address. Bob also knows Chen's email address and Daisy knows Bob's email address.

(a)    Construct a digraph to show the above information, ignoring the fact that everyone knows their own email address.

✓ – vertices
✓ – edges
✓ – directions



or  equivalent
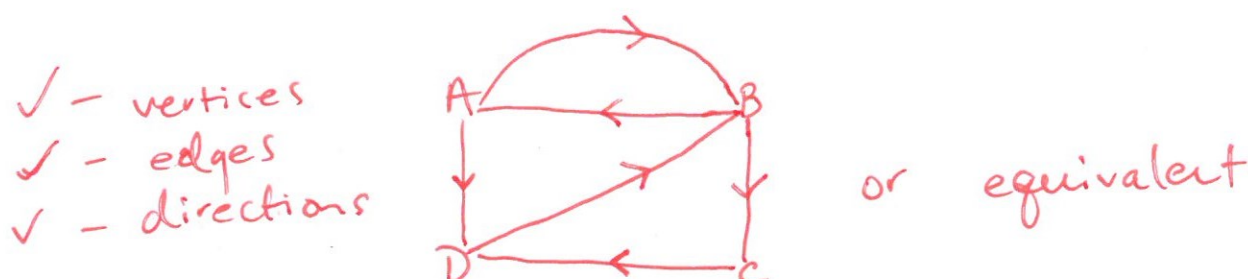
(b)    Construct an adjacency matrix, $A$, for the digraph in (a)

$$
\begin{array}{c}
 & \text{To} \\
\text{From} &
\begin{array}{c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} &
\left[\begin{array}{cccc}
0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0
\end{array}\right]
\end{array}
\end{array}
$$

✓ – constructs 4×4 matrix with correct labelling and 12 correct entries

✓ – above with all entries correct

(c)    Calculate the matrix $B$, where $B = A^2 + A$.

$$
B = \begin{bmatrix}
1 & 2 & 1 & 1 \\
1 & 1 & 1 & 2 \\
0 & 1 & 0 & 1 \\
1 & 1 & 1 & 0
\end{bmatrix}
$$

✓✓

(d)    Use matrix $B$ to identify which of the workers, if any, are unable to email the other three workers either directly or by asking one other worker to forward their email.
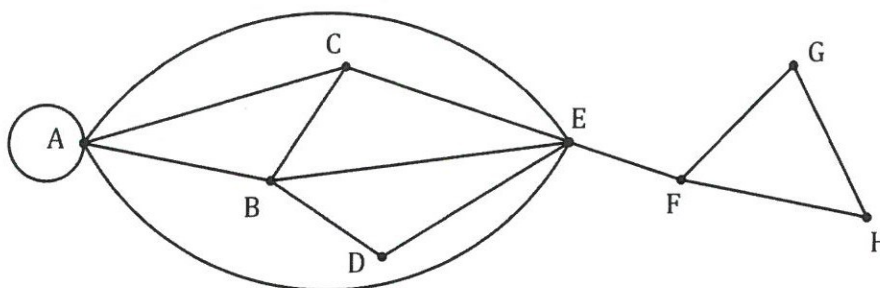
Chen cannot email Adele as $B_{31}$ is zero.

✓    Chen can ask Daisy to forward emails to Bob to forward to Adele.

**Question 6** [2,2,1,3 - 8 marks]

The graph below represents 14 canals that meet at locations $A$ to $H$.



(a) A canal enthusiast noticed that the graph contained a semi-Eulerian trail. State the two properties of a walk that make a semi-Eulerian trail.

✓ Trail must be open (start & finish @ different vertices)
✓ and includes every edge once.

(b) Verify that this graph is planar using Euler's rule

$V = 8$
$E = 14$ ✓
$f = 8$
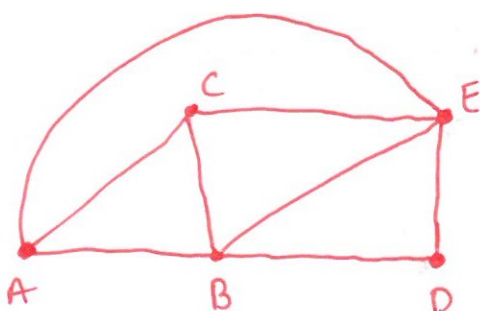
$V + f - e = 2$ ✓
$8 + 8 - 14 = 2$

(c) Suggest a suitable starting point for the canal enthusiast to begin a semi-Eulerian trail and indicate where they will finish the trail.

Start at C, finish at F ✓
or vice versa.

(d) Draw a subgraph of the above graph that is simple, connected, has no bridges and has 8 edges.



- removes RHS bridge ✓
- removes loop and one ✓ of the AE edges
- subgraph with 8 edges ✓