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Calculator Free
Geometric Sequences

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 3 = 7 marks]

Define each of the following sequences recursively and state T_6:

(a) 10, 20, 40, 80 ...

(b) 1000, 500, 250, 125 ...

(c) $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27} \ldots$

Question Two: [1, 2, 2 = 5 marks]

Consider the sequence $U_{n+1} = \frac{U_{n-1}}{-2}$; $U_2 = 50$

(a) Calculate U_3

(b) Calculate U_1

(c) Determine the general term of the sequence.

Question Three: [2, 2, 1, 2, 2 = 9 marks]

The first term of a geometric sequence is 6 and the 4th term is 48.

(a) Determine the common ratio of this sequence.

(b) Hence or otherwise define this sequence recursively.

(c) Calculate T_5

(d) Calculate S_4

(e) Determine when the sequence first has a value greater than 300.

Question Four: [2 marks]

Show that the sequence $A_n = 100 \times 0.5^{n-1}$ can be written as $A_n = 200 \times \left(\frac{1}{2}\right)^n$

Question Five: **[4 marks]**

Determine when the value of the sequence $T_{n+1} = 2T_n$; $T_1 = 16$ and of the sequence $T_n = 256 \times \frac{1}{2^{n-1}}$ are equivalent.

Question Six: **[5 marks]**

The 4th term of a geometric sequence is 5000 and the 7th term is 5 000 000.

Determine the value of the first term and the common ratio of this sequence.

Question Seven: [5, 1, 2 = 8 marks]

The first three terms of a geometric sequence are $x-2,\ x+1,\ x+5$

(a) Determine the common ratio of this sequence.

(b) Determine the value of the first term.

(c) Hence determine the values of a and b in the rule defining this sequence: $T_n = a \times b^n$

Question Eight: [3, 2 = 5 marks]

The first three terms of a geometric series are $S_1 = 3$, $S_2 = -3$, $S_3 = 9$

(a) Determine the first three terms of the sequence.

(b) Explain whether T_{45} will be positive or negative.

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Question One: [2, 2, 3 = 7 marks]

Define each of the following sequences recursively and state T_6:

(a) 10, 20, 40, 80 ...

$T_n = 2 \times T_{n-1}; T_1 = 10$ ✓
$T_6 = 320$ ✓

(b) 1000, 500, 250, 125 ...

$T_n = 0.5 \times T_{n-1}; T_1 = 1000$ ✓
$T_6 = 31.25$ ✓

(c) $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27} \ldots$

$T_n = \frac{2}{3} \times T_{n-1}; T_1 = 1$ ✓
$T_6 = \frac{32}{243}$ ✓✓

Question Two: [1, 2, 2 = 5 marks]

Consider the sequence $U_{n+1} = \frac{U_{n-1}}{-2}$; $U_2 = 50$

(a) Calculate U_3 $U_3 = \frac{50}{-2} = -25$ ✓

(b) Calculate U_1 $50 = \frac{U_1}{-2}$ ✓
$U_1 = -100$ ✓

(c) Determine the general term of the sequence.

$U_n = -100 \times (-\frac{1}{2})^{(n-1)}$
✓ ✓

Question Three: [2, 2, 1, 2, 2 = 9 marks]

The first term of a geometric sequence is 6 and the 4th term is 48.

(a) Determine the common ratio of this sequence.

$6 \times r^3 = 48$ ✓

$r^3 = 8$

$r = 2$ ✓

(b) Hence or otherwise define this sequence recursively.

$T_n = 2 \times T_{n-1}; T_1 = 6$ ✓

(c) Calculate T_5 ✓

$T_5 = 6 \times 2^{(5-1)}$

$= 6 \times 16 = 96$ ✓

(d) Calculate S_4

$S_4 = \frac{6(2^4 - 1)}{2 - 1} = 6 \times 15 = 90$ ✓ ✓

(e) Determine when the sequence first has a value greater than 300.

$300 = 6 \times 2^{(n-1)}$ ✓

$50 = 2^{(n-1)}$

$50 < 2^{(7-1)}$

$n = 7$ ✓

Question Four: [2 marks]

Show that the sequence $A_n = 100 \times 0.5^{n-1}$ can be written as $A_n = 200 \times \left(\frac{1}{2}\right)^n$

$A_n = 100 \times 0.5^n \times 0.5^{-1}$ ✓

$= 100 \times 2 \times 0.5^n$

$= 200 \times \left(\frac{1}{2}\right)^n$ ✓

Question Five: **[4 marks]**

Determine when the value of the sequence $T_{n+1} = 2T_n$; $T_1 = 16$ and of the sequence $T_n = 256 \times \frac{1}{2^{n-1}}$ are equivalent.

$16 \times 2^{n-1} = 256 \times 2^{(-n+1)}$ ✓

$\frac{2^{(n-1)}}{2^{(-n+1)}} = \frac{256}{16}$ ✓

$2^{2n-2} = 16$

$2^{2n-2} = 2^4$ ✓

$2n - 2 = 4$

$2n = 6$

$n = 3$ ✓

Question Six: **[5 marks]**

The 4th term of a geometric sequence is 5000 and the 7th term is 5 000 000.

Determine the value of the first term and the common ratio of this sequence.

$5000 \times r^3 = 5000000$ ✓

$r^3 = 1000$ ✓

$r = 10$ ✓

$5000 = a \times 10^3$ ✓

$5000 = a \times 1000$

$a = 5$ ✓

Question Seven: [5, 1, 2 = 8 marks]

The first three terms of a geometric sequence are $x-2,\ x+1,\ x+5$

(a) Determine the common ratio of this sequence.

$$\frac{x+1}{x-2}=\frac{x+5}{x+1}$$
$$(x+1)(x+1)=(x+5)(x-2)$$
$$x^2+2x+1=x^2+3x-10$$
$$-x=-11$$
$$x=11$$
$$r=\frac{11+1}{11-2}=\frac{12}{9}=\frac{4}{3}$$

(b) Determine the value of the first term.

$$a=11-2=9$$

(c) Hence determine the values of a and b in the rule defining this sequence: $T_n=a\times b^n$

$$b=\frac{4}{3}$$
$$a=\frac{27}{4}=6\frac{3}{4}$$

Question Eight: [3, 2 = 5 marks]

The first three terms of a geometric series are $S_1=3$, $S_2=-3$, $S_3=9$

(a) Determine the first three terms of the sequence.

$$T_1=3$$
$$T_2=-3-3=-6$$
$$T_3=9--3=12$$

(b) Explain whether T_{45} will be positive or negative.

Positive, the odd terms are positive and the even are negative.