

**SCOTCH  
COLLEGE**



## **12 Mathematics Methods 2019**

### **Test 2 – Integration and Area**

#### **Section 1: Calculator-free**

**Time allowed: 20 minutes**

**Maximum marks: 17**

**Name:** \_\_\_\_\_ **Teacher:** Foster | Giese | Reyhani

#### **Instructions:**

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No calculators or personal notes are permitted.

**Question 1 (9 marks)**

- a)** Perform the following indefinite integral, leaving your answer in simplest form.

[3]

$$\int \frac{1}{x^2} + 4x^5 dx$$

- b)** Determine the following definite integrals.

**i)**  $\int_0^2 (2x - 3)^3 dx$

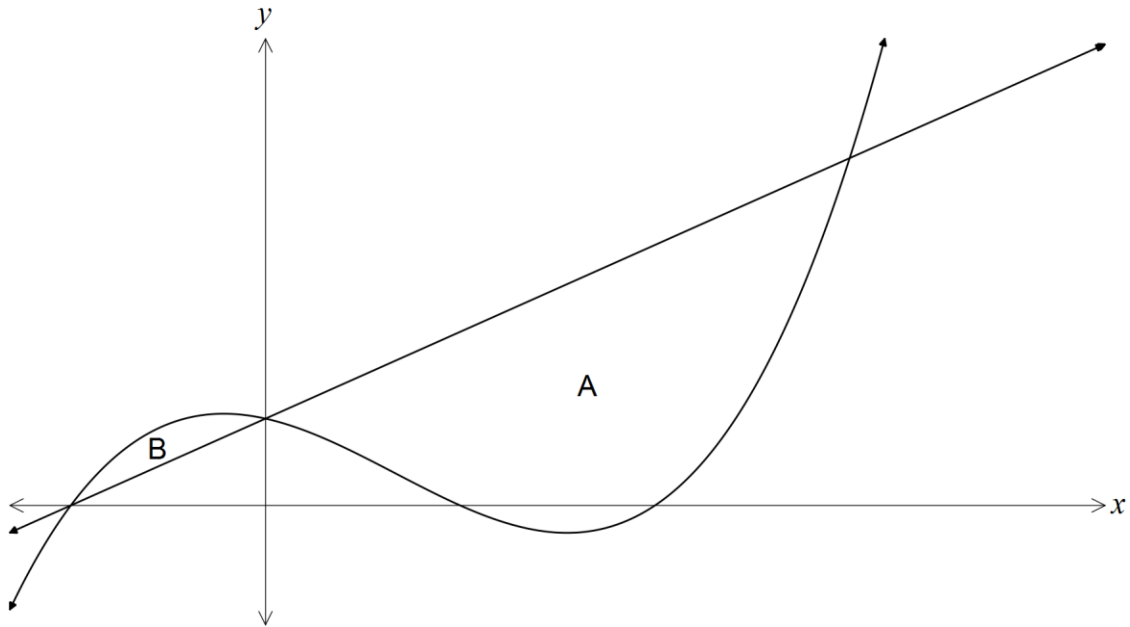
[3]

**ii)**  $\int_0^1 \frac{6x}{\sqrt{3x^2 + 1}} dx$

[3]

**Question 2 (8 marks)**

The graphs of  $y = 2x + 2$  and  $y = f(x)$  are shown below, where  $f(x) = x^3 - 2x^2 - x + 2$ .



- a) Use calculus techniques to show and justify that a point of inflection exists on the graph of  $y = f(x)$  when  $x = \frac{2}{3}$  [3]
- b) Two regions are trapped between the linear and cubic functions, marked A and B on the diagram. Show that the difference in the areas of these two regions is  $10\frac{2}{3}$  square units. [5]

**Extra working space**

**END OF SECTION**

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**12 Mathematics Methods 2019**

**Test 2 – Integration and Area**

**Section 2: Calculator-assumed**

**Time allowed: 25 minutes**

**Maximum marks: 23**

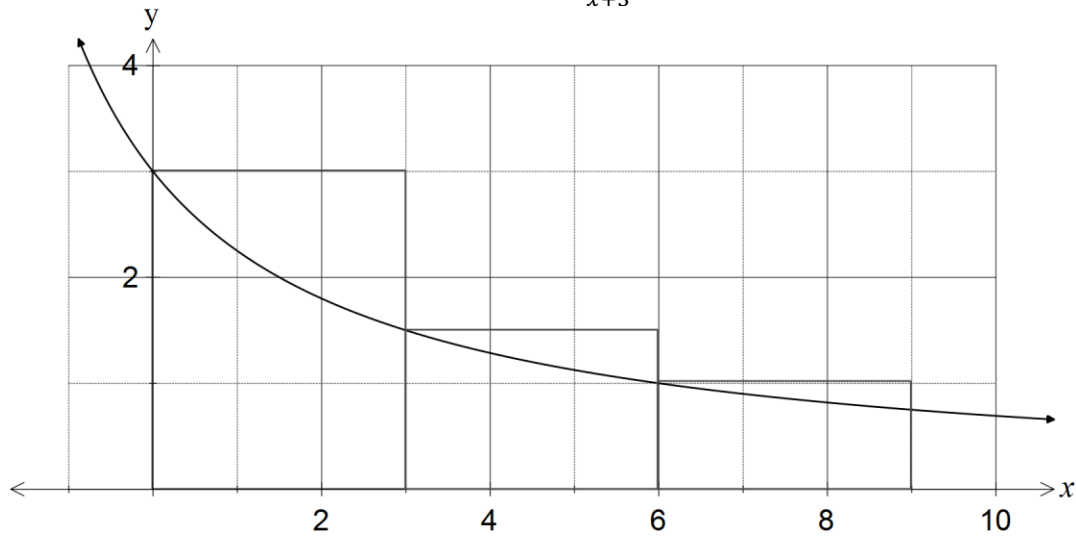
**Name:** \_\_\_\_\_ **Teacher:** Foster | Giese | Reyhani

**Instructions:**

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- Calculators and 1A4 double-sided page of personal notes are permitted.

**Question 3 (6 marks)**

The graph below shows the function  $f(x) = \frac{9}{x+3}$



An estimate for the area under the curve between  $x = 0$  and  $x = 9$  is required.

- a) Three circumscribed rectangles are shown on the diagram. Use these rectangles to calculate an over-estimate for the area. [2]
- b) Use three inscribed rectangles to calculate an under-estimate for the area. [2]
- c) Use your over- and under- estimates to calculate a better estimate for the area under the curve between  $x = 0$  and  $x = 9$ . [1]
- d) The exact area is  $18\log_e 2$ . Calculate the error in the best estimate above as a percentage of the exact area. [1]





**Question 4 (5 marks)**

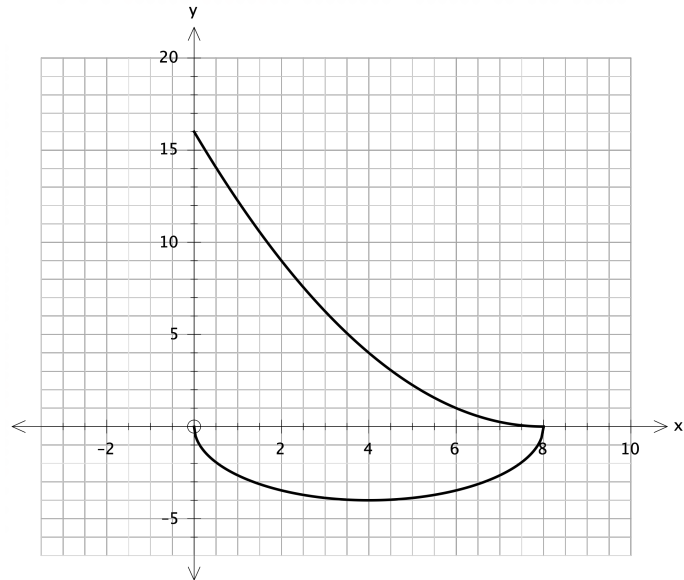
A motor vehicle slows down from an initial velocity of  $25 \text{ ms}^{-1}$  until it is stationary. During this interval, its acceleration  $t$  seconds after the brakes were applied is given by;

$$a(t) = \frac{t}{2} - 5 \text{ ms}^{-2}$$

- a) Determine the velocity of the vehicle after four seconds. [3]
- b) Calculate the distance travelled by the vehicle in the first two seconds after the brakes were applied. [2]

**Question 5 (5 marks)**

The graph below shows parts of the relationships  $y = 0.25(x - 8)^2$  and  $(x - 4)^2 + y^2 = 16$  that are being used to model a new fin for a surfboard. The picture below shows a cross-section of the fin. Both the  $x$  and  $y$  axis have the scale 1 unit = 2cm.



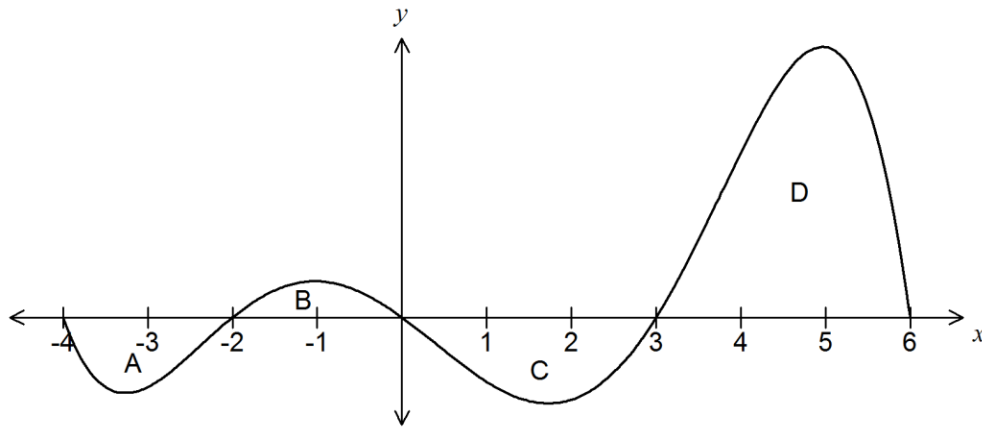
The fin is to be 1.5cm thick.

Determine the exact volume of a prototype of this fin.

**Question 6 (7 marks)**

The graph of the function  $y = f(x)$  is shown below for  $-4 \leq x \leq 6$ .

The area of each region enclosed by the curve and the  $x$ -axis is shown in the table below the graph.



Region	A	B	C	D
Area of region	5	3	11	25

- a) Determine the area enclosed between the graph of  $y = f(x)$  and the  $x$ -axis, from  $x = -4$  to  $x = 6$ . [1]

- b) Determine the value of

i)  $\int_{-2}^6 f(x) dx$  [1]      ii)  $\int_{-2}^3 3f(x) dx$  [2]

iii)  $\int_0^6 6 - f(x) dx$  [3]

**Extra working space**

**END OF TEST**