MINDARIE SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS NOW

MATHEMATICS: SPECIALIST 1 & 2

SEMESTER 2 2017

TEST 4

Resource Free

Reading Time: 2 minutes
Time Allowed: 19 minutes

Total Marks: 19

1. [2, 2, 2, 3 marks]

If $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 3 & -2 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

determine each of the following. If any cannot be determined, state this clearly and explain why.

(a) $2A - C = \begin{bmatrix} 3 & -4 & 5 \\ -1 & -1 & 6 \end{bmatrix}$ ✓✓

(b) CB = DNE ✓ – non-conformable. ✓

$(2\times3) \times (2\times2)$

(c) $BD = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$ ✓✓

(d) Matrix E such that $BE = D$.

$B^{-1}BE = B^{-1}D$ Pre mult. by B^{-1} ✓

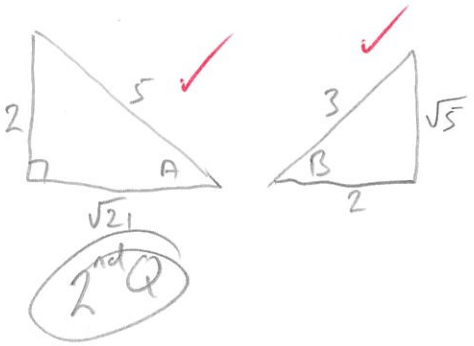
$E = \frac{-1}{9}\begin{bmatrix} -2 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ B^{-1} ✓

$= \frac{-1}{9}\begin{bmatrix} -5 \\ -3 \end{bmatrix}$ answer ✓

$= \begin{bmatrix} \frac{5}{9} \\ \frac{1}{3} \end{bmatrix}$

2. [5 marks]

Given that A is an obtuse angle with $\sin A = \frac{2}{5}$ and B is an acute angle with $\cos B = \frac{2}{3}$ determine exactly the value for $\cos(A - B)$.



2nd Q

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ ✓

$= -\frac{\sqrt{21}}{5} \cdot \frac{2}{3} + \frac{2}{5} \cdot \frac{\sqrt{5}}{3}$ ✓

$= \frac{-2\sqrt{21}}{15} + \frac{2\sqrt{5}}{15}$ ✓

$= \frac{2(\sqrt{5} - \sqrt{21})}{15}$

3. [1, 4 marks]

(a) Determine $\begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 0 \\ 0 & 4 & 2 \end{bmatrix} \begin{bmatrix} 4 & 6 & -8 \\ -1 & 1 & 2 \\ 2 & -2 & 1 \end{bmatrix}$ (A, B)

$= \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$ ✓

(b) Hence, solve the simultaneous equations (C)

$$\begin{aligned} 4x + 6y - 8z &= -2 \\ -x + y + 2z &= 8 \\ 2x - 2y + z &= -6 \end{aligned}$$

$BX = C$

$ABX = AC$

$10X = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 0 \\ 0 & 4 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 8 \\ -6 \end{bmatrix}$

$= \begin{bmatrix} -10 \\ 30 \\ 20 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

Sets up as matrices ✓

Pre multb. by A ✓

Finds AC ✓

solutions ✓

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Resource Assumed

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Time Allowed: 31 minutes

Total Marks: 31

4. [4 marks]

Some matrices have a special property called "anti-commutativity" where $AB = -BA$. These matrices are called Pauli-spin matrices and are useful in the study of electron spin in quantum mechanics.

Given that $P = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $Q = \begin{bmatrix} a & a \\ b & -1 \end{bmatrix}$ are anti-commutative, find a and b.

$PQ = \begin{bmatrix} a-b & a+1 \\ 2a-b & 2a+1 \end{bmatrix}$ ✓ $\quad -QP = \begin{bmatrix} -3a & +2a \\ b-2 & -1+b \end{bmatrix}$ ✓

$a+1 = 2a$

$1 = a$ ✓

$3 = -1+b$

$b = 4$ ✓

5. [4 marks]

Rewrite $2\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$, where α is an acute angle in degrees.

$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$2 = R\cos\alpha$ ✓

$1 = R\sin\alpha$

$R^2 = 5$

$R = \sqrt{5}$ ✓

$\tan\alpha = \frac{1}{2}$

$\alpha = 26.6°$ ✓

$\sqrt{5}\sin(\theta + 26.6°)$ ✓

6. [2, 1, 2, 1 marks]

Burger Barn's three locations sell hamburgers, fries, and soft drinks. Barn I sells 600 burgers, 600 orders of fries, and 700 soft drinks each day. Barn II sells 1500 burgers a day and Barn III sells 1150. Soft drink sales number 900 each day at Barn II and 825 a day at Barn III. Barn II sells 950 and Barn III sells 800 orders of fries per day.

(a) Display the daily sales figures in a 3×3 matrix.

labelled

$$\begin{matrix} & B & F & D \\ I \\ II \\ III \end{matrix}\begin{bmatrix} 600 & 600 & 700 \\ 1500 & 950 & 900 \\ 1150 & 800 & 825 \end{bmatrix}$$

M x ✓ labels ✓

$$\begin{matrix} & I & II & III \\ B \\ F \\ D \end{matrix}\begin{bmatrix} 600 & 1500 & 1150 \\ 600 & 950 & 800 \\ 700 & 900 & 825 \end{bmatrix}$$

Burgers cost \$2.50 each, fries \$1.20 per order, and soft drink costs \$0.80 each.

(b) Display the prices in matrix form.

$$\begin{matrix} B \\ F \\ D \end{matrix}\begin{bmatrix} 2.5 \\ 1.2 \\ 0.8 \end{bmatrix}$$ ✓

$$\begin{matrix} B & F & D \end{matrix}\begin{bmatrix} 2.5 & 1.2 & 0.8 \end{bmatrix}$$

(c) Use matrix multiplication to obtain a matrix that gives the daily revenue at each of the three locations.

-1/error

$$\begin{bmatrix} 600 & 600 & 700 \\ 1500 & 950 & 900 \\ 1150 & 800 & 825 \end{bmatrix}\begin{bmatrix} 2.5 \\ 1.2 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2780 \\ 5610 \\ 4495 \end{bmatrix}$$ ✓ ✓

$$\begin{bmatrix} 2.5 & 1.2 & 0.8 \end{bmatrix}\begin{bmatrix} 600 & 1500 & 1150 \\ 600 & 950 & 800 \\ 700 & 900 & 825 \end{bmatrix} = \begin{bmatrix} 2780 & 5610 & 4495 \end{bmatrix}$$

(d) ~~What~~ Show a matrix that could be used to determine the total daily revenue from all three locations?
You do NOT need to calculate the revenue.

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$ ✓ $$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

7. [3, 3 marks]

Given the matrix $A = \begin{bmatrix} 2x & 1 \\ 2 & x \end{bmatrix}$, find all values of x for which

(a) $A^2 = \begin{bmatrix} 38 & -9 \\ -18 & 11 \end{bmatrix}$,

$$= \begin{bmatrix} 4x^2+2 & 3x \\ 6x & x^2+2 \end{bmatrix}$$ ✓✓

$\therefore x = -3$ ✓

(b) A is singular. $2x^2 - 2 = 0$ ✓

$x = \pm 1$ ✓✓

8. [5, 6 marks]

Prove the following:

(a) $\frac{1-\cos 2\theta}{\sin 2\theta} = \tan\theta$

$$LHS = \frac{1-\cos 2\theta}{\sin 2\theta} \quad \checkmark$$

$$= \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta} \quad \checkmark$$

$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta} \quad \checkmark$$

$$= \frac{\sin\theta}{\cos\theta} \quad \checkmark$$

$$= \tan\theta \quad \checkmark$$

$$= RHS$$

(b) $2\sin A\cos\left(\frac{\pi}{6}+A\right) = \frac{\sqrt{3}\sin 2A+\cos 2A-1}{2}$

$$LHS = 2\sin A\cos\left(\tfrac{\pi}{6}+A\right)$$

$$= 2\sin A\left[\cos\tfrac{\pi}{6}\cos A - \sin\tfrac{\pi}{6}\sin A\right] \quad \checkmark \text{ Expands}$$

$$= 2\sin A\left[\tfrac{\sqrt{3}}{2}\cos A - \tfrac{1}{2}\sin A\right] \quad \checkmark \text{ substitutes}$$

$$= \frac{\sqrt{3}}{2}\cdot 2\sin A\cos A - \frac{1}{2}\cdot 2\sin^2 A.$$

$$= \frac{\sqrt{3}}{2}\sin 2A - \frac{1}{2}(1-\cos 2A) \quad \checkmark \quad \checkmark$$

$$= \frac{\sqrt{3}\sin 2A - 1 + \cos 2A}{2} \quad \checkmark$$

$$= RHS$$