Mathematics Methods Unit 4
Investigation 2: DRV Validation

Name: Solutions

Time allowed: 35 minutes | Calculator Assumed | Marks: /31

Prolonged exposure to loud music can result in hearing loss. The two major characteristics of sound are intensity and frequency (pitch). We are only considering intensity of sound, I, which is measured in watts/m ^{2}.

The sound intensity level, L, is a logarithmic measure given as

$$L = 10 \log\left(\frac{I}{I_0}\right) \text{ and measured in decibels (dB)}$$

$$I_0 = 10^{-12} \text{ watts/m}^2.$$

The reference intensity of sound, I_0, that all other intensities are compared to is 10^{-12} watts/m^2 because this is the weakest intensity of sound that can be detected by the human ear.

Question 1 **[2 marks]**

Determine the sound intensity level of normal piano practice when the intensity of the sound of the music is 10^{-5} watts/m ^{2}.

$L = 10\log\left(\frac{10^{-5}}{10^{-12}}\right)$
$= 10\log(10^{7})$
$= 70\log(10)$
$L = 70$ dB ✓

✓ working

Question 2 **[4, 3 = 7 marks]**

The maximum sound intensity level of an orchestra playing is 110 dB.

a) The sound intensity level of a live performance of a rock band can reach 135 dB. How many times more intense is the sound of the music of a live rock band than the music of an orchestra? (4 marks)

$135 = 10\log\left(\frac{I}{10^{-12}}\right)$
$10^{13.5} = \frac{I}{10^{-12}}$
$I = 10^{1.5}$ ✓

$110 = 10\log\left(\frac{I}{10^{-12}}\right)$
$10^{11} = \frac{I}{10^{-12}}$
$I = 10^{-1}$ ✓

$\frac{10^{1.5}}{10^{-1}} = 10^{2.5}$ ✓

∴ Live rock band is roughly 316 times more intense than an orchestra

b) The sound intensity level of chamber music in a small auditorium is around 90 dB. How many times less intense is the sound of the chamber music than the music of an orchestra? (3 marks)

$90 = 10\log\left(\frac{I}{10^{-12}}\right)$

$10^9 = \frac{I}{10^{-12}}$

$I = 10^{-3}$ ✓

$\frac{10^{-3}}{10^{-1}} = 10^{-2}$ ✓

Chamber music has 0.01 the intensity of an orchestra. ✓

Question 3 **[1, 1 = 2 marks]**

a) Given the range of the sound intensity levels, *L*, of the following musical instruments, which <u>two</u> instruments have the potential to do the most damage to the human ear? (1 mark)

	L (dB)
Violin	84-102
Cello	82-93
Oboe	90-94
Flute	85-110
Piccolo	95-112
Clarinet	92-102
French horn	90-105
Trombone	85-114
Timpani and bass drum rolls	107

✓

(b) What other factors need to be considered? (1 mark)

Duration of sound be played, proximity and other instrument surrounding them.

✓ (reasonable response)

Question 4 **(2, 3, 1, 1, 1, 2, 2 = 12 marks)**

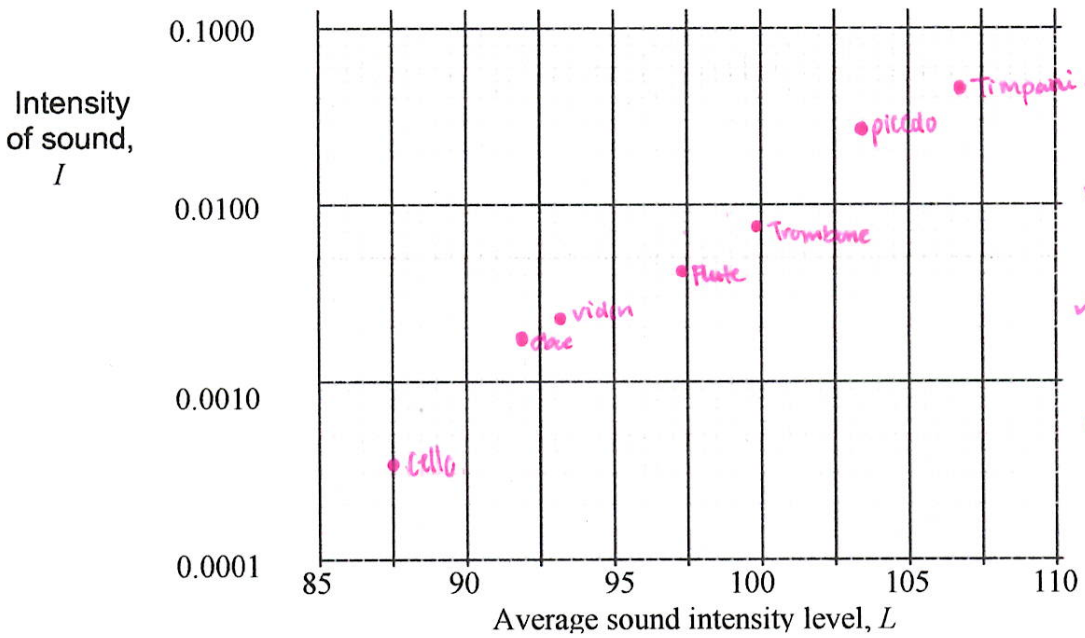
The table below shows the average sound intensity levels (L) and the intensity of the sound (I) of some instruments of a symphony orchestra. The ratios $\left(\frac{I}{I_0}\right)$ are also given for each instrument.

(a) Determine the missing values (i) – (iv) (2 marks)

musical instrument	average sound intensity level, L (dB)	$\left(\frac{I}{I_0}\right)$	intensity of sound, I (watts/m ^{2})
Violin	93	1 995 262 315	0.0019953
Cello	87.5	(i)	(iii)
Oboe	92	1 584 893 192	0.0015849
Flute	97.5	5 623 413 252	0.0056234
Piccolo	103.5	(ii)	(iv)
Trombone	99.5	8 912 509 301	0.0089125
Timpani and bass drum rolls	107	50 118 723 360	0.0501187

i) 562,341,325
ii) 22,387,211,390 ✓
iii) 0.0005623
iv) 0.0223872 ✓

(b) Plot the intensity of sound (I watts/m ^{2}) against the average sound intensity level (L dB) for any four of the musical instruments listed in the table above. (3 marks)



✓ plots 2 points correctly
✓ plots 3 points correctly
✓ plots 4 points correctly

Question 4 (continued

(c) Identify the relationship between the points you have plotted. (1 mark)

Points are almost collinear (linear relationship). ✓

(d) Using the graph or otherwise, determine the average sound intensity level of the sound from a musical instrument that has an intensity of sound of 0.1 watts/m ^{2}.

(1 mark)

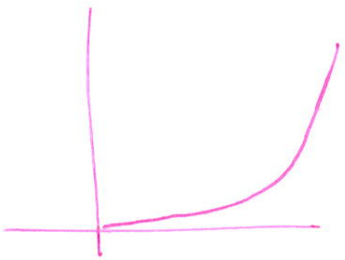
$\simeq$ 110 dB ✓

(e) Explain why a semi – logarithm grid was useful for the data graphed. (1 mark)

Semi-log graph is useful as the numbers have a large range of values even though the numbers are small. ✓

(f) (i) What shape would the $L - I$ graph take? Explain your decision. (2 marks)

The L-I graph would be exponential. ✓



Rough calculator plot

or →

$L = 10 \log\left(\frac{I}{I_0}\right)$

$\frac{L}{10} = \log\left(\frac{I}{I_0}\right)$

$10^{\frac{L}{10}} = \frac{I}{I_0}$

$I_0 10^{\frac{L}{10}} = I$

which is exponential

✓ (for graph or working)

(ii) Using your calculator or any other method, determine the equation that best fits the $L - I$ data. (2 marks)

Exp Reg $y = a \cdot e^{\wedge}(b \cdot x)$ – CP

a = 9.522 E-13

b = 0.2288644

r = 0.9553606

r^2 = 0.9127139

MSe = 0.280371 ✓

$\therefore y = (9.522 \times 10^{-13}) \times e^{0.2289x}$ ✓

Noise exposure

It is said that an increase of 3 decibels in sound intensity level will double the intensity of the sound.

Question 5 **[4, 4 = 8 marks]**

(a) By finding an expression for *I* at a sound intensity level of 80 dB and *I* at a sound intensity level of 83 dB, show why increasing the sound intensity level by 3 dB doubles the value of *I*. (4 marks)

$80 = 10\log\left(\frac{I}{I_0}\right)$ $\quad$ $83 = 10\log\left(\frac{I}{I_0}\right)$

$10^8 = \frac{I}{10^{-12}}$ $\quad$ $10^{8.3} = \frac{I}{10^{-12}}$

$I_{80} = 10^{-4}$ ✓ $\quad$ $I_{83} = 10^{-3.7}$ ✓

$\frac{10^{-3.7}}{10^{-4}} = 10^{0.3}$

$= 2$ ✓

∴ The intensity of the 83 dB sound is 2x that at 80 dB. ✓

(b) Show how this is the case at any sound intensity level. (4 marks)

$x = 10\log\left(\frac{I}{I_0}\right)$ $\quad$ $x+3 = 10\log\left(\frac{I}{I_0}\right)$

$10^{\frac{x}{10}} = \frac{I}{10^{-12}}$ $\quad$ $10^{\frac{x+3}{10}} = \frac{I}{10^{-12}}$

$10^{\frac{x}{10}} \times 10^{-12} = I_x$ ✓ $\quad$ $10^{\frac{x+3}{10}} \times 10^{-12} = I_{x+3}$ ✓

$\frac{10^{\frac{x+3}{10}} \times 10^{-12}}{10^{\frac{x}{10}} \times 10^{-12}} = 10^{\frac{x+3}{10} - \frac{x}{10}}$

$= 10^{\frac{3}{10}}$

$= 10^{0.3}$

$= 2$ ✓

∴ The rule works @ any level of x dB. ✓