

Full Name: SOLUTIONS



MATHEMATICS

Methods Units 1 & 2

Test 2 – Linear and Quadratic Functions, Polynomials

Chapters 4, 5, 6 and 7

Semester 1 2019

Section One - Calculator Free

Time allowed for this section

Working time for this section: 20 minutes

Marks available: 20 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (6 marks)

Suggest one possible equation each for the lines L1 and L2 if:

a) L1 and L2 are parallel to $x + 2y = 0$. $\Rightarrow y = -\frac{1}{2}x$ [2]

$$L1 = -\frac{1}{2}x + 3 \quad \checkmark$$

$$L2 = -\frac{1}{2}x - 2 \quad \checkmark \quad \text{same gradient}$$

b) L1 and L2 meet at the point with coordinates (0,4) and are perpendicular to each other. [2]

$$L1 = 2x + 4 \quad \checkmark$$

$$L2 = -\frac{1}{2}x + 4 \quad \checkmark$$

same y-intercept
and $m_1 m_2 = -1$

c) L1 and L2 do not intersect. [2]

$$L1 = 3x \quad \checkmark$$

$$L2 = 3x + 1 \quad \checkmark$$

same gradient

2. (4 marks)

A parabola has equation $y = k(x - a)(x - b)$ where k , a and b are constants with $a < b$. Find k , a and b if the parabola has an x-intercept at $(-3, 0)$, a turning point at $(1, 32)$ and a y-intercept at $(0, 30)$.

$$y = k(x - 1)^2 + 32$$

sub $(-3, 0)$

$$0 = k(16) + 32$$

$$-32 = k \times 16$$

$$k = -2$$

$$\therefore y = -2(x - 1)^2 + 32 \quad \checkmark$$

$$= -2(x^2 - 2x + 1) + 32$$

$$= -2x^2 + 4x + 30$$

$$= -2(x^2 - 2x - 15)$$

$$= -2(x + 3)(x - 5)$$

$$\therefore k = -2, a = -3, b = 5$$

$\checkmark \quad \checkmark \quad \checkmark$

3. (7 marks)

A curve has equation $y = 2x^3 - x^2 - 2x + 1$.

a) Find the coordinates of the x-intercepts of this curve.

[4]

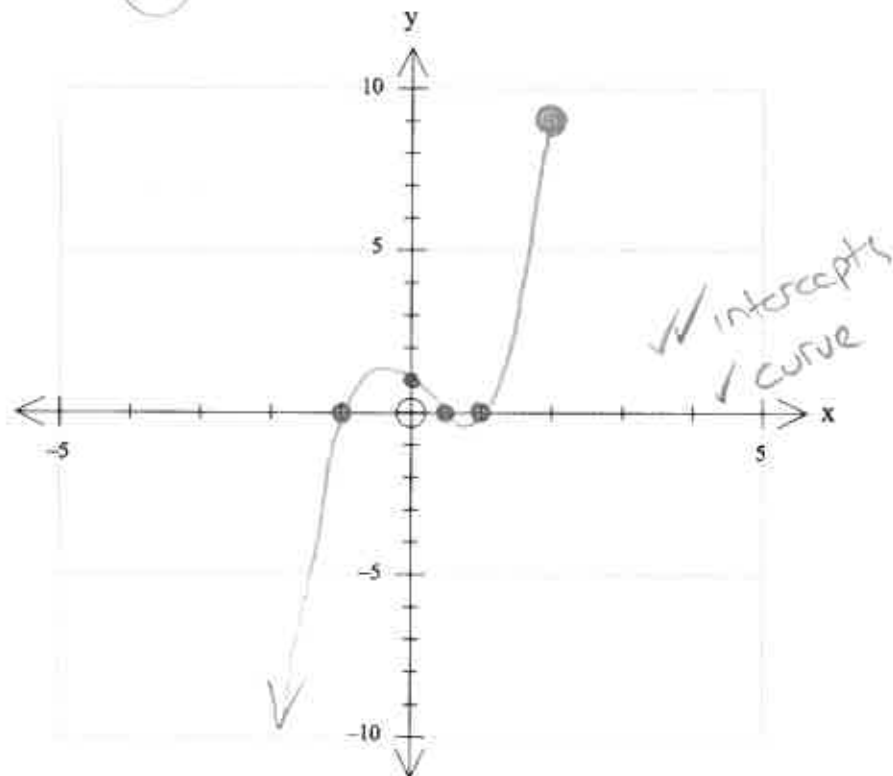
$$\begin{array}{r|rrrr}
 x=1 & 2 & -1 & -2 & 1 \\
 \checkmark & & 2 & 1 & -1 \\
 \hline
 & 2 & 1 & -1 & 0 \checkmark
 \end{array}$$

$$\begin{aligned}
 y &= (x-1)(2x^2+x-1) \\
 &= (x-1)(x+1)(2x-1) \quad \checkmark
 \end{aligned}$$

x-intercepts $(1,0)$, $(-1,0)$ and $(\frac{1}{2},0)$ ✓

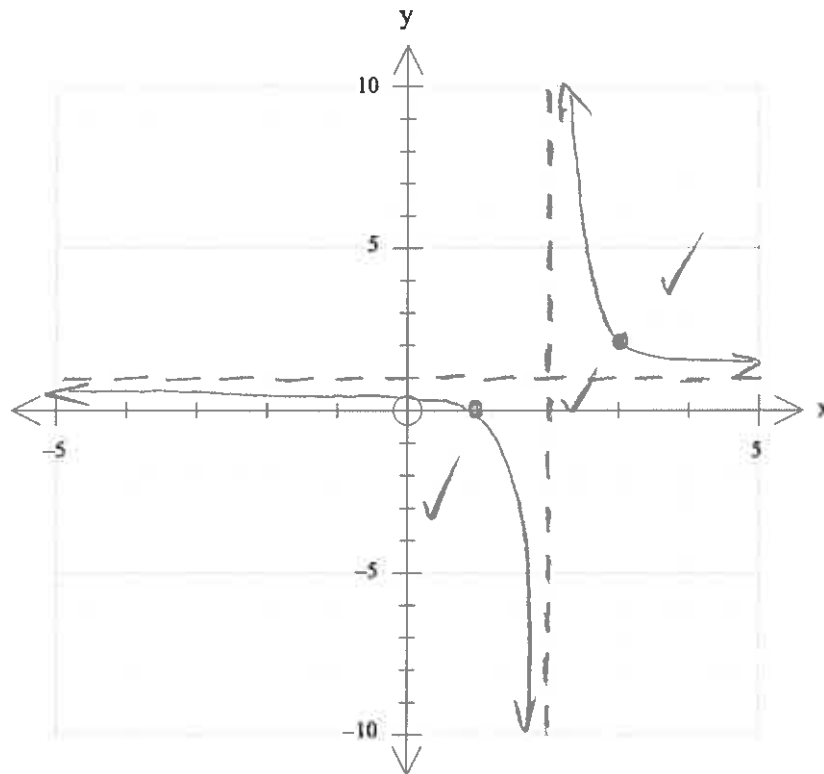
b) Sketch this curve for $-1.5 \leq x \leq 2$.

[3]



4. (3 marks)

Sketch the following function $y = \frac{1}{x-2} + 1$, clearly showing all asymptotes and labelling at least two points on the curve.



End of Section One

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Section Two - Calculator Assumed

Time allowed for this section

Working time for this section: 30 minutes

Marks available: 32 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

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1. (10 marks)

Bill, a plumber, charges a call-out fee of \$100 plus \$80 per half hour or part thereof. Ian, another plumber, does not charge a call-out fee but charges \$180 per hour or part thereof.

a) How much will Bill charge for a job that is estimated to take exactly 4 hours? [2]

$$100 + 160 \times 4 = \$740$$

b) How much will Ian charge for a job that is estimated to take exactly 4 hours? [1]

$$4 \times 180 = \$720$$

c) Determine which plumber will be cheaper to employ if a job is estimated to take 3 hours and 20 minutes. Justify your answer. [3]

$$3.5 \text{ h}$$

$$\text{Bill } 100 + 160 \times 3.5 = \$660$$

$$\text{Ian } 180 \times 3.5 = \$630$$

\therefore Ian is cheaper

d) Under what conditions will it be cheaper to employ Bill? Justify your answer. [4]

$$100 + 160x = 180x$$

$$100 = 20x$$

$$x = 5$$

\therefore Bill is cheaper for jobs longer than 5 hours or part thereof.

2. (12 marks)

For the cubic defined by $y = x^3 + 4x^2 - 3x - 7$

a) determine any stationary points and their nature [4]

From graph

Max @ $(-3, 11)$ ✓Min @ $(\frac{1}{3}, -7.52)$ ✓

b) state the zeros of the function [3]

 $x = -4.32, -1.12, 1.44$
✓ ✓ ✓

c) find where the cubic changes concavity [1]

inflection $(-1\frac{1}{3}, 1.74)$ ✓

d) describe the curve over its natural domain [4]

- an increasing function (positive a)
- positive gradient then negative then positive
- as $x \rightarrow -\infty, y \rightarrow -\infty$; as $x \rightarrow \infty, y \rightarrow \infty$
- other valid comments.

3. (3 marks)

If the function defined $g(x) = px^2 - 5x - 3$ passes through the points $(1, -6)$ and $(q, 4)$, find the possible values of p and q .

$$-6 = p - 8$$

$$p = 2 \quad \checkmark$$

$$4 = 2q^2 - 5q - 3$$

$$0 = 2q^2 - 5q - 7$$

$$q = -1 \text{ or } 3.5$$

✓

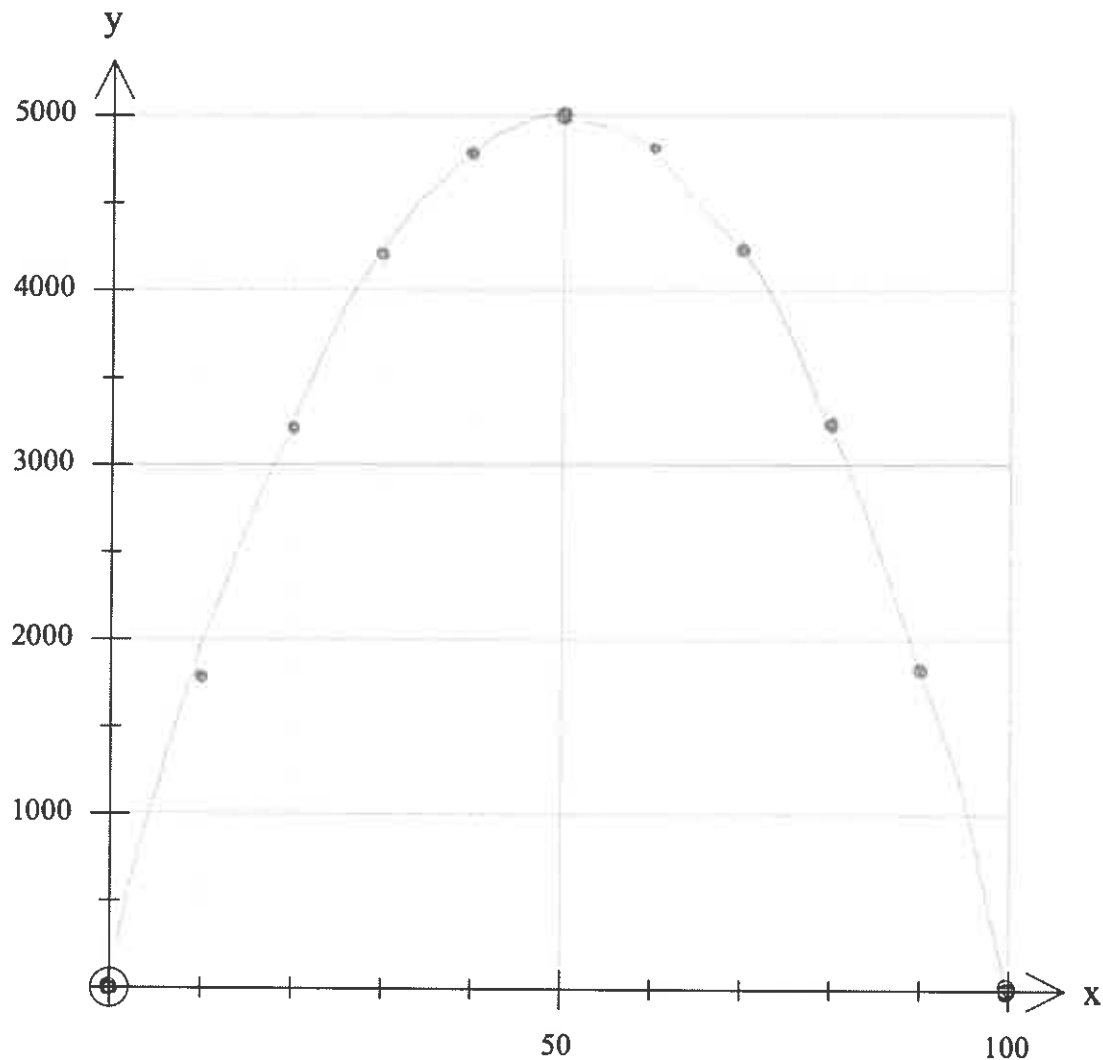
✓

4. (7 marks)

Gemma owns a hobby farm and needs to create a fenced-up area for her sheep using the back wall of her shed as one of the sides of the fenced-up area. She has 200 metres of fencing available. From what she could recall from her mathematics class when she was a student, to maximise the fenced-up area, she would need to maximise the function $A(x) = x(200 - 2x)$ where x is the width of the fenced-up area.

a) On the axes provided below sketch $A(x) = x(200 - 2x)$.

[2]



b) Find the coordinates of the turning point of function $A(x)$.

[1]

(50, 5000)

- c) Find the maximum possible area that can be fenced and the dimensions of that fenced-up area. [2]

Max Area 5000 m^2 ✓

DIMENSIONS 50 by 100 ✓

- d) Find the possible dimensions of the fenced-up area if its area is 3200 m^2 . [2]

$$3200 = x(200 - x)$$

$$0 = -x^2 + 200x - 3200$$

$$x = 20 \text{ or } 80 \quad \checkmark$$

End of Test

∴ either 20 by 160
or 80 by 40 ✓