Name: .....

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|---|---|---|
| **Calculator Free** | **/20** | **%** |
| **Calculator Allowed** | **/33** | **%** |
| **Total** | **/53** | **%** |

Mathematics Methods, Year 12, 2018
Test 3 – Further differentiation and applications, Integrals, Discrete Random Variables.
20 minutes working time.
Calculator Free Section (no notes, no calculators) SCSA Formula sheet allowed

**1. [3 marks]**

A probability distribution of a certain random variable $X$ is given by:

$P(X = x) = \frac{x}{10}$ ,where $x = 1, 2, 3, 4$.

Show that a probability distribution is formed.

$\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = \frac{10}{10} = 1$ ✓✓

∴ Probability distribution formed. ✓

**2. [2 marks]**

Give two reasons why the following cannot be a probability distribution.

| $x$ | 2 | 3 | 5 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.1 | 0.4 |

1. $x = 2$ occurs twice - not allowed ✓

2. probabilities add to $1.1 > 1$. ✓

4 1

**3. [3 marks: 1, 3]**

A data scientist tracked how many cups of coffee she drank every day at work over the course of a year. She used the data to build a probability distribution where the random variable ***x*** represents the number of cups of coffee she drank on a given day. Here is the partially completed distribution:

| ***x*** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| P(X = ***x***) | 0.2 | 0.25 | k | 0.15 | 0.1 |

a) Find k. $1 - 0.7 = 0.3 \quad \therefore k = 3$ ✓

b) Find the expected value from the given table.

$E(x) = 1 \times 0.2 + 2 \times 0.25 + 3 \times 0.3 + 4 \times 0.15 + 5 \times 0.1$ ✓

$= 0.2 + 0.5 + 0.9 + 0.6 + 0.5$

$= 2.7$ ✓✓

**4. [4 marks: 1, 3]**

Each of the following represent discrete probability functions. Determine the value of $k$ for each.

(a) $P(x) = \frac{1}{k};\ x = 1,2,3,...12$ $k = 12$ ✓

(b) $P(x) = \frac{x}{2k-1};\ x = 1,2,3,4,5,6$

$\frac{1+2+3+4+5+6}{2k-1} = 1$ ✓

$21 = 2k - 1$

$2k = 22$ ✓✓

$k = 11$

**5.** 3 [2 marks]

Determine the integral of $\int_0^4 e^{-3x}\,dx$

Leave your answer with positive indices.

$\int_0^4 e^{-3x}\,dx = \left[-\frac{e^{-3x}}{3}\right]_0^4 = -\frac{e^{-12}}{3} - \left(-\frac{e^0}{3}\right)$ ✓ ✓

$= -\frac{3}{e^{12}} + \frac{1}{3}$ ✓

**6.** **[4 marks: 3, 1]**

If $\frac{dA}{dt} = 6t^2 - 4e^{2t}$, and A = 3 when t = 0, find

a) A in terms of t

$A = 2t^3 - \frac{4e^{2t}}{2} + c$ ✓

$= 2t^3 - 2e^{2t} + c$

$3 = 0 - 2e^0 + c$

$c = 5$ ✓ $\therefore A = 2t^3 - 2e^{2t} + 5$ ✓

b) the exact value of A, when t = 1.

$A = 2(1)^3 - 2e^2 + 5$

$= 7 - 2e^2$ ✓

Name: .....

| Calculator Allowed | /13 | % |
|---|---|---|

Mathematics Methods, Year 12, 2018

Test 3 – Further differentiation and applications, Integrals, Discrete Random Variables.

35 minutes working time. Calculator Assumed Section (notes allowed), SCSA Formula sheet and calculators allowed

**1.** **[4 marks: 1, 3]**

Nick takes a free throw for basketball $n$ times. The probability, $p$, of Nick scoring is constant and all free throws are independent.

Let X be the number of times Nick scores a free throw in the $n$ attempts.

The mean of X is 32 and the standard deviation is 4.

a) State the distribution of X.

Binomial ✓

b) Determine $n$ and $p$.

$\text{mean of } X = n \times p = 32$

$\text{sd of } X = \sqrt{n \times p \times (1-p)} = 4$

$np(1-p) = 16$

$32(1-p) = 16$ ✓

$1-p = 0.5$

$p = 0.5$ ✓

$n \times 0.5 = 32$

$n = 64$ ✓

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**2. [~~11~~ marks - 1, 2, 2, 2, 1, 3]**

The probability that a new drug on the market will be effective against a particular disease is 0.8. Fifteen people suffering from this disease are given the drug.
Determine the probability that the drug is effective for:

a) 9 people

$$P(X=9) = {}^{15}C_9 \times (0.8)^9 \times (0.2)^6 = 0.0430$$ ✓

b) 8 or 10 people

$$P(X=8) + P(X=10) = 0.0138 + 0.1032 = 0.1170$$ ✓ ✓

c) More than 5 but less than 13 people

$$P(5 < X < 13) = P(6 \leq X \leq 12) = 0.6019$$ ✓ ✓

d) At least 8 people

$$P(X \geq 8) = 0.9958$$ ✓

e) At most 10 people

$$P(X \leq 10) = 0.1642$$ ✓

f) Less than 10 people given that it was effective for more than 7 people

$$P(X < 10 \mid X > 7) = \frac{P(8 \leq X \leq 9)}{P(X \geq 8)}$$ ✓

$$= \frac{0.0568}{0.9958}$$ ✓

$$= 0.0571$$ ✓

**3. [7 marks: 3, 4]**

In the following table, $x$ is a score in a game and P(X) is the probability of getting that score. The expected mean of the discrete probability distribution is 2.8.

a) Find the values of m and n.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| P(X = $x$) | 0.2 | m | 0.3 | n | 0.1 |

$0.2 + m + 0.3 + n + 0.1 = 1$

$m + n = 0.4$ $\quad m = 0.4 - n$

$1(0.2) + 2(m) + 3(0.3) + 4(n) + 5(0.1) = 2.8$

$0.2 + 2m + 0.9 + 4n + 0.5 = 2.8$ ✓

$2m + 4n + 1.6 = 2.8$

$2m + 4n = 1.2$

$m + 2n = 0.6$ $\quad m = 0.6 - 2n$

$0.4 - n = 0.6 - 2n$

$n = 0.2$ $\quad m = 0.2$ ✓✓

b) Calculate the standard deviation of the scores.

$Var = 0.2 \times (1-2.8)^2 + 0.2 \times (2-2.8)^2 + 0.3 \times (3-2.8)^2 + 0.2 \times (4-2.8)^2 + 0.1 \times (5-2.8)^2$ ✓

$= 0.648 + 0.128 + 0.012 + 0.288 + 0.484$

$= 1.56$ ✓

$SD = \sqrt{1.56} = 1.249$ ✓✓

**4. [7 marks: 2, 2, 3]**

The probability distribution of x where random variable, X is the sum of the uppermost numbers when two fair die are rolled is tabulated below.

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P(X=x) | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

Find:

a) $P(X > 6) = \frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{21}{36} = \frac{7}{12}$ ✓ ✓

b) $P(X < 10) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36} = \frac{30}{36} = \frac{5}{6}$ ✓ ✓

c) $P(X < 10 \mid X > 6)$ $\frac{P(6<X<10)}{P(X>6)} = \frac{\frac{20}{36}}{\frac{21}{36}} = \frac{20}{21}$ ✓ ✓ ✓

**5. [5 marks]**

Given that $y = 3t$, $t = e^{2p}$ and $p = 3x+2$, find $\frac{dy}{dx}$. and $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dp} \times \frac{dp}{dx}$$

$$= 3 \times 2e^{2p} \times 3 \quad \checkmark\checkmark$$

$$= 18e^{2p} \quad \checkmark$$

$$= 18e^{2(3x+2)}$$

$$= 18e^{6x+4} \quad \checkmark$$

$$\frac{d^2y}{dx^2} = 18e^{6x+4} \times 6$$

$$= 108e^{6x+4} \quad \checkmark$$