BALLAJURA COMMUNITY COLLEGE

Mathematics Applications
Unit 3 Year 12
Test 3: Calculator Free

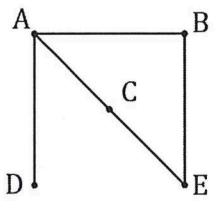
$\overline{25}$	$\overline{55}$

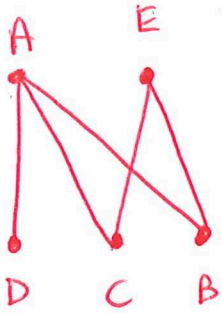
Working Time: 20 minutes
No notes or calculators permitted

Student Name: Solutions

1. (3, 3, 3: 9 marks)

a) The following bipartite graph shows the subjects studied by three students. Redraw the graph to clearly show the two sets of vertices and hence state which vertices represent the subjects studied.



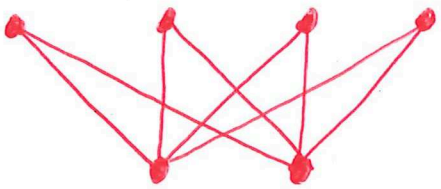


✓ Vertices
✓ Edges

✓ Subjects are A and E

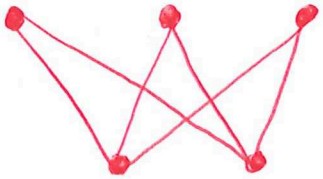
b) The complete bipartite graph denoted by $K_{p,q}$ has p vertices in one set and q vertices in the other set.

(i) Draw $K_{4,2}$ and state whether the graph is Eulerian, semi-Eulerian or neither. Justify your choice.



✓ Graph
✓ Eulerian
✓ All even vertices is a closed trail

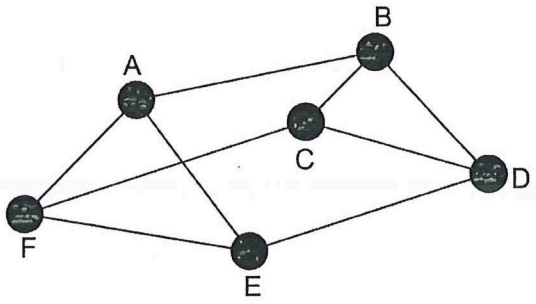
(ii) Draw $K_{3,2}$ and state whether the graph is Hamiltonian, semi-Hamiltonian or neither. Justify your choice.



✓ Graph
✓ Semi-Hamiltonian
✓ Visit all vertices with an open path

2. (1, 1, 1, 1: 4 marks)
For the graph shown below;

a) Determine the degree of each vertex



3 ✓

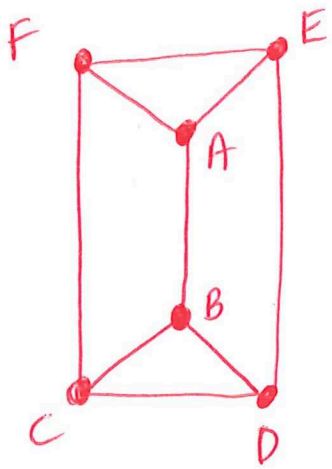
b) State the number of edges

9 ✓

c) State the number of faces

5 ✓

d) If the graph is planar, redraw the graph with no intersecting edges

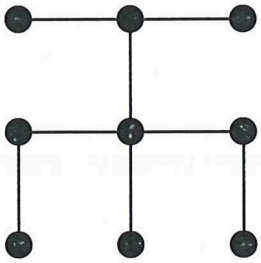


✓ graph needs to be labelled

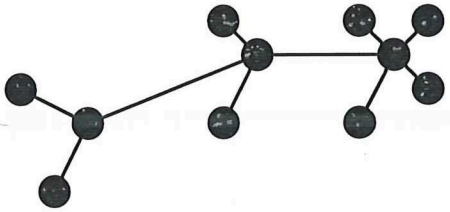
3. (2, 1, 1, 2: 6 marks)
A tree graph is a planar graph with no cycles. It is an undirected graph in which any two vertices are connected by exactly one path. The following questions all refer to tree graphs.

a) For the following trees state the number of vertices and the number of edges.

I.



II.



V=9 E=8 ✓

V=11 E=10 ✓

b) Draw a semi-Eulerian tree with 4 vertices and 3 edges.

✓

c) Is it possible to draw a tree with 4 vertices and 5 edges?

No ✓

d) Use Euler's formula to justify why tree graphs, with n vertices, are all planar.

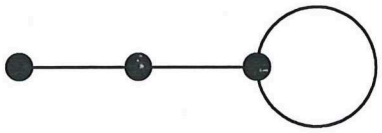
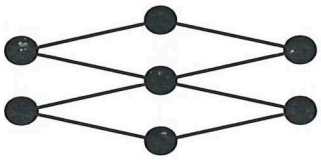
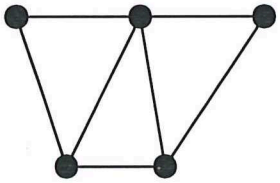
vertices = n
edges = $n-1$
faces = 1

✓

$f+v-e=2$
$1+n-(n-1)=2$.
$2=2$.

✓

4. (6 marks)
Classify the following graphs as Eulerian (E) or semi-Eulerian (SE), Hamiltonian (H) or semi Hamiltonian (SH) or no classification (N).

Graph	Classifications
	SE ✓ SH ✓
	E ✓ SH ✓
	SE ✓ H ✓

BALLAJURA COMMUNITY COLLEGE

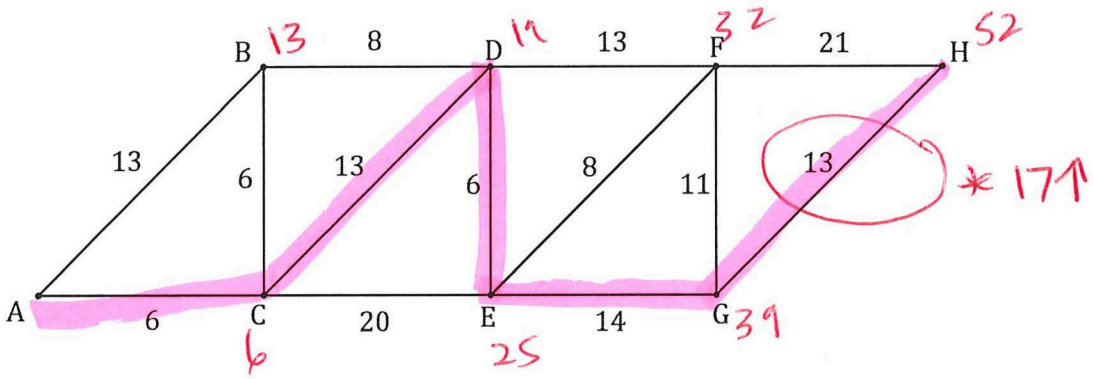
Mathematics Applications
Unit 3 Year 12 2023
Test 3: Calculator Assumed

27

Working Time: 30 minutes
Calculators and ½ page of notes permitted

Student Name: _____

5. (3, 3, 2: 8 marks)
The vertices in the graph below represent city landmarks and the weights on the edges are the times, in minutes, to travel between adjacent landmarks.



a) Determine the shortest time to travel from *A* to *H*, stating the route used.

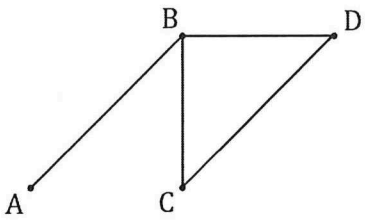
52 mins ACDEGH.

✓ path + time
✓ correct path
✓ correct time.

b) The travel times from *A* to *B* and from *G* to *H* both increase by 4 minutes. Explain how these changes affect your answer to part a above.

A to B – no effect ✓
G to H – time changes to 53 mins ✓
route change to ACDFH ✓

c) Construct the adjacency matrix *M* for the subgraph below;



$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

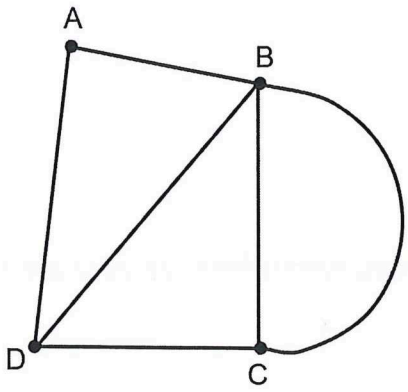
✓✓
(–1 per error
labelled
graph)

6. (1, 1, 1, 2, 2, 1: 8 marks)
Two road network systems are shown below. Network One is in table form and Network Two is in graph form.

Network One

	A	B	C	D
A	0	2	0	1
B	1	1	0	1
C	0	0	0	0
D	0	2	1	0

Network Two



a) Based on the values in the table, explain why Network One is a directed graph.

It is not symmetrical either side of the leading diagonal. ✓ B to A = 1 ✓ A to B = 2

b) Explain why Network Two can be classified as a planar graph.

$f = 4$
$e = 6$
$v = 4$

$f + v - e = 2$
$4 + 4 - 6 = 2$ ✓

(0 not edges crossing)

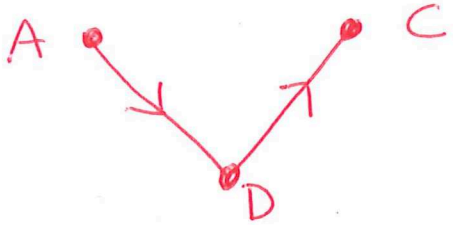
c) Use Network Two to describe a closed walk of length 5

A B C D B A ✓

d) When the adjacency matrix for Network One is squared the value in Row 1, Column 2 is 4. Explain the meaning of this value.

There are 4 2 step walks ✓ from A to B ✓
(① R1 x C2 = 4)

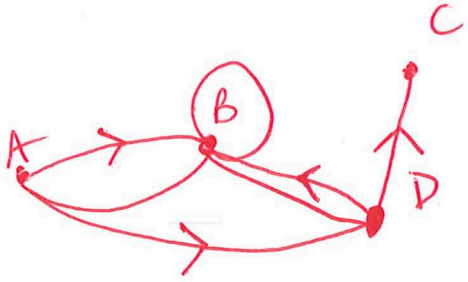
e) Draw a subgraph of Network One containing only the vertices A, C and D.



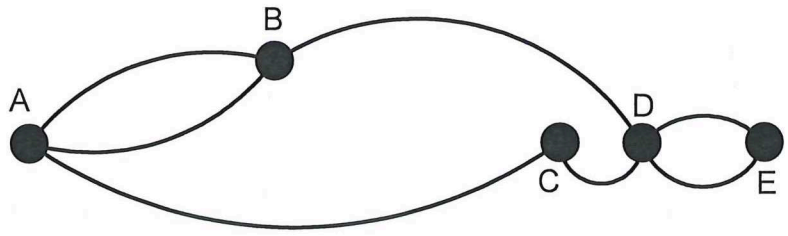
✓ graph
✓ arcs

f) Determine where a bridge exists in Network One

CD ✓



7. (3 marks)
Consider the following undirected graph.



There are 5 walks from A to A of length 2. These walks are: A-B-A via the top path, A-B-A via lower path, A-B-A as a loop in one direction, A-B-A as a loop in the other direction and A-C-A.

Complete the table below to show how many walks of length 2 there are between vertex E and the other vertices in the graph.

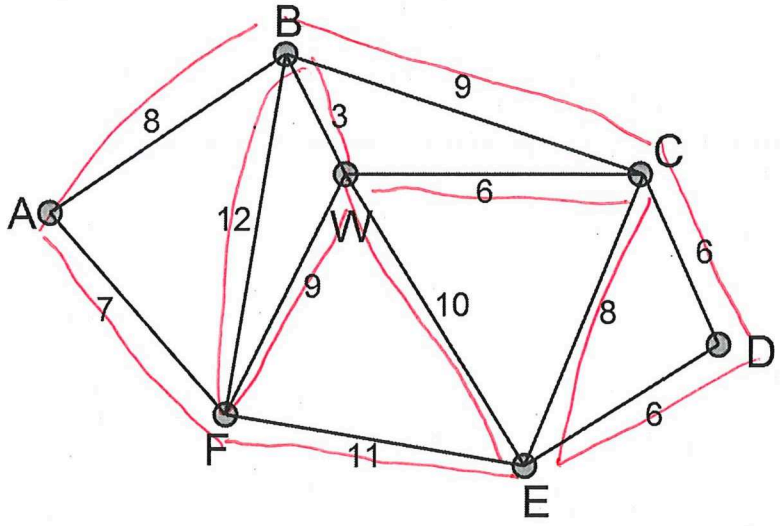
To

From	A	B	C	D	E
A	5	0	0	3	0
B	0	5	3	0	2
C	0	3	2	0	2
D	3	0	0	6	0
E	0	2	2	0	4

$$\begin{bmatrix} 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}^2$$

✓ (2 entries correct)
✓ (adjacency matrix for working)
✓ All entries correct

8. (1, 2, 2: 5 marks)
A clothing company owns a warehouse and 6 outlet stores.
The network below shows the location of the warehouse, W.
The warehouse supplies clothing to six outlet stores, A, B, C, D, E, F.
The numbers along the edges indicates the kilometres of connecting roads.



a) A delivery van is at store B. It must make a delivery to store E before returning to the warehouse. Determine the minimum distance travelled on this journey.

(BWEW) 23km ✓

b) A salesman wants to leave the warehouse and visit every store once before returning to the warehouse. Describe a route he could take and state the mathematical term used to describe this route.

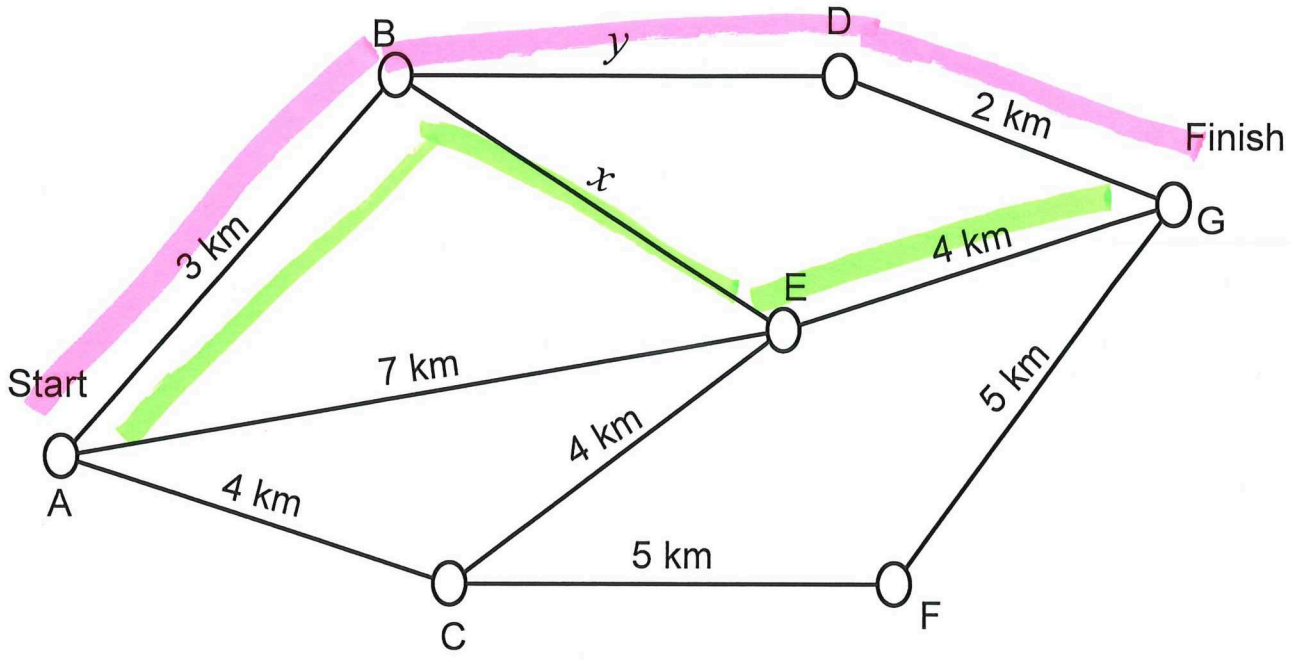
WBAFEDCW ✓ Hamiltonian cycle ✓
(-1 Hamiltonian graph, Hamiltonian closed path)

c) The owner of the company wants to check for competitors along the streets of this network describe a route she could take and state the mathematical term used to describe this route.

(W)BFWEFABCDEC(W) ✓
(13 vertices)

Eulerian Trail ✓
(-1 Eulerian graph. Eulerian Closed Trail)

9. (3 marks)



If the shortest path from start to finish is 9 km, state all possible values for x and y and the possible shortest path.

ABDG $y = 4$ ✓ and $x > 2$ ✓

ABEG $x = 2$ ✓ and $y > 4$