

Full Name: SOLUTIONS



MATHEMATICS METHODS

Test 1 – Trigonometry

Chapters 11, 12 and 13

Semester 1 2019

Section One - Calculator Free

Time allowed for this section

Working time for this section: 20 minutes

Marks available: 22 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (2 marks)

Convert the following to radians, giving exact values.

a) 45°

$$45 \times \frac{\pi}{180} = \frac{\pi}{4} \quad \checkmark$$

b) 300°

$$300 \times \frac{\pi}{180} = \frac{5\pi}{3} \quad \checkmark$$

2. (2 marks)

Convert the following radians to degrees.

a) $\frac{\pi}{6}$

$$\frac{\pi}{6} \times \frac{180}{\pi} = 30^\circ \quad \checkmark$$

b) $\frac{5\pi}{4}$

$$\frac{5\pi}{4} \times \frac{180}{\pi} = 225^\circ \quad \checkmark$$

3. (3 marks)

State which quadrant each of the following angles are in.

a) -285° 1st \checkmark b) $\frac{7\pi}{6}$ 3rd \checkmark c) $-\frac{13\pi}{4}$ 2nd \checkmark

4. (4 marks)

Use the unit circle shown below to determine each of the following, giving your answers in terms of either a, b, c or d.

a) $\sin 23^\circ$

b ✓

b) $\cos 51^\circ$

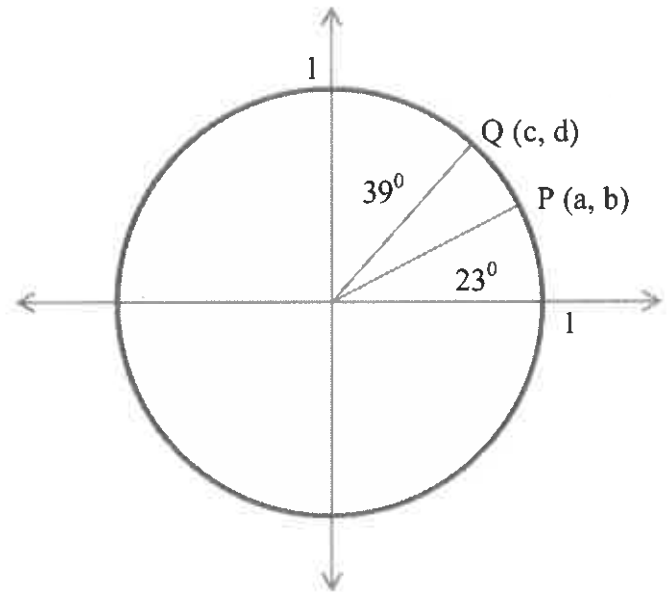
c ✓

c) $\cos 157^\circ$

-a ✓

d) $\sin 203^\circ$

-b ✓

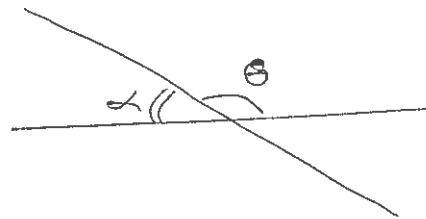


5. (2 marks)

For the line $3x + 3y = 12$ determine the angle of inclination the line makes with the x-axis.

(4, 0)

(0, 4)



$$\tan \alpha = -1$$

$$\alpha = 45^\circ$$

$$\therefore \theta = 135^\circ$$

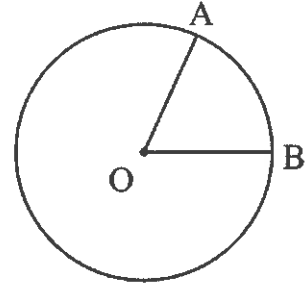
Angle of inclination is 135°

6. (9 marks)

In the circle of radius 5 cm with centre O drawn below, $\angle OAB = 30^\circ$

a) Find the exact dimensions of triangle OAB. Show your reasoning for each dimension. [4]

$$\begin{aligned}
 OA = OB &= 5 && \text{(radii)} \quad \checkmark \\
 \angle A = \angle B &= 30^\circ && \text{(isosceles)} \quad \checkmark \\
 \angle O &= 120^\circ && \text{(angle sum } \Delta) \quad \checkmark
 \end{aligned}$$



$$\begin{aligned}
 AB^2 &= 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 120 \\
 &= 50 + 25 \quad \checkmark \\
 &= 75
 \end{aligned}$$

$$\begin{aligned}
 AB &= \sqrt{75} \\
 &= 5\sqrt{3} \quad \text{either} \quad \checkmark
 \end{aligned}$$

b) Find the exact area of the minor segment formed by the chord AB. [3]

$$\text{Segment} = A_{\text{sector}} - A_{\text{triangle}}$$

$$= \frac{120}{360} \times \pi \times 5^2 \quad \checkmark - \frac{1}{2} \times 5 \times 5 \times \sin 120 \quad \checkmark$$

$$= \frac{25\pi}{3} - \frac{25\sqrt{3}}{2} \quad \checkmark$$

- c) Find the exact perimeter of the minor segment formed by the chord AB.

[2]

$$\text{Arc} = \frac{120}{360} \times 2 \times \pi \times 5$$

$$= \frac{10\pi}{3} \quad \checkmark$$

$$\text{Perimeter} = \frac{10\pi}{3} + 5\sqrt{3} \quad \checkmark$$

End of Section One

Full Name: SOLUTIONS



MATHEMATICS METHODS

Test 1 – Trigonometry

Chapters 11, 12 and 13

Semester 1 2019

Section Two - Calculator Assumed

Time allowed for this section

Working time for this section: 30 minutes

Marks available: 31 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

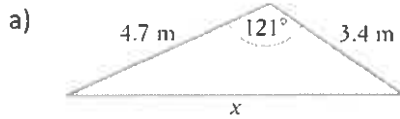
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

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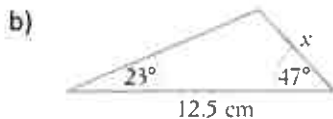
1. (6 marks)

Evaluate x correct to one decimal place.

[3]

$$x^2 = 4.7^2 + 3.4^2 - 2(4.7)(3.4) \cos 121 \quad \checkmark$$

$$x = 7.1 \text{ m} \quad \checkmark$$



[3]

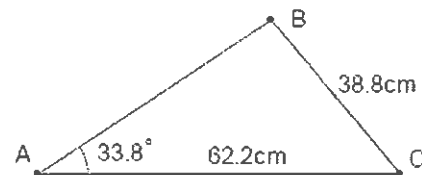
$$\frac{x}{\sin 23} = \frac{12.5}{\sin 110} \quad \checkmark$$

$$x = 5.2 \text{ cm} \quad \checkmark$$

2. (3 marks)

Calculate the smallest possible length of AB in the triangle shown below.

(The triangle is not drawn to scale).



$$38.8^2 = 62.2^2 + x^2 - 2x(62.2) \cos 33.8 \quad \checkmark$$

$$0 = x^2 - 103.37x + 2363.4$$

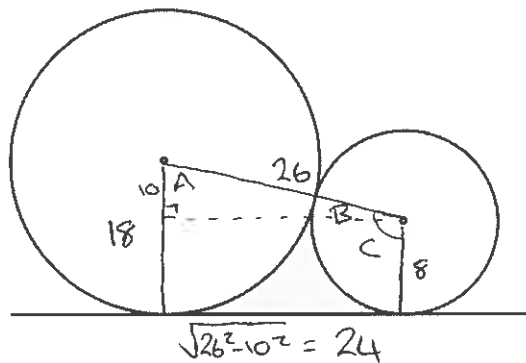
$$x = \underline{\underline{34.13}} \quad \text{OR} \quad x = 69.24 \quad \checkmark$$

Smallest length of AB is 34.13 cm \checkmark

* must show justification.

3. (9 marks)

Two circles, one of radius 8cm and the other of radius 18cm, with a common tangent, touch each other as shown in the diagram.



a) Calculate the perimeter of the shaded region.

[5]

$$\cos A = \frac{10}{26} \quad \sin B = \frac{10}{26} \quad C = 1.966$$

$$A = 1.176 \quad \checkmark \quad B = 0.395 \quad \checkmark$$

$$\text{arc}_1 = 18 \times 1.176$$

$$= 21.168 \quad \checkmark$$

$$\text{arc}_2 = 8 \times 1.966$$

$$= 15.725 \quad \checkmark$$

$$P = 24 + 21.168 + 15.725$$

$$= 60.893 \quad \checkmark$$

b) Calculate the area of the shaded region.

[4]

$$A_{\text{trap}} = (18+8) \times 24 \div 2$$

$$= 312 \quad \checkmark$$

$$\text{Sector A} = 0.5 \times 18^2 \times 1.176$$

$$= 190.512 \quad \checkmark$$

$$\text{Sector B} = 0.5 \times 8^2 \times 1.966$$

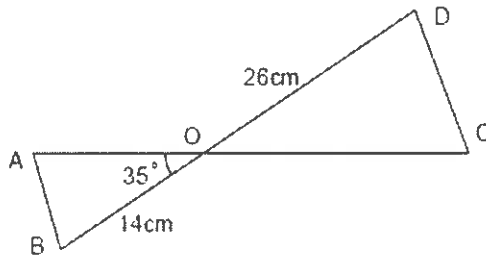
$$= 62.912 \quad \checkmark$$

$$A_{\text{shaded}} = 58.576 \text{ cm}^2$$

$$\checkmark$$

4. (3 marks)

In the diagram below (not to scale), the line AC intersects the line BD at O. The angle $\text{AOB} = 35^\circ$, and the lengths $\text{OB} = 14\text{cm}$, $\text{OD} = 26\text{cm}$ and $\text{AC} = 30\text{cm}$. If the area of triangle ODC is twice that of triangle OAB, determine the length OA.



$$\text{AC} = 30$$

$$\text{if } \text{AO} = x$$

$$\text{then } \text{OC} = 30 - x$$

✓

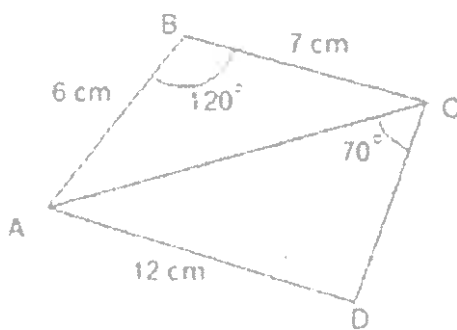
$$2 \left(\frac{1}{2} \times x \times 14 \times \sin 35 \right) = \frac{1}{2} \times (30 - x) \times 26 \times \sin 35 \quad \checkmark$$

$$27x \sin 35 = 390 \sin 35$$

$$x = 14.4 \text{ cm} \quad \checkmark$$

5. (4 marks)

In the quadrilateral ABCD, $\text{AB} = 6\text{cm}$, $\text{BC} = 7\text{cm}$, $\text{AD} = 12\text{cm}$, and $\text{ABC} = 120^\circ$, angle $\text{ACD} = 70^\circ$. Calculate the size of angle ADC. Give your answer to 2 decimals.



$$\text{AC}^2 = 6^2 + 7^2 - 2(6)(7)\cos 120$$

$$\text{AC} = \sqrt{127} \approx 11.27 \quad \checkmark$$

$$\frac{\sin 70}{12} = \frac{\sin D}{\sqrt{127}} \quad \checkmark$$

$$\angle \text{ADC} = 61.94^\circ \text{ or } 118.06^\circ$$

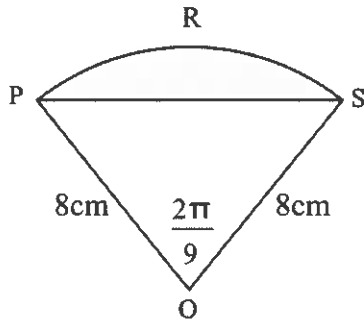
✓

either
answer

6. (3 marks)

The diagram shows a sector of a circle with centre O. The radius of the circle is 8 cm. PRS is an arc of the circle. PS is a chord of the circle. Angle POS = $\frac{2\pi}{9}$ radians

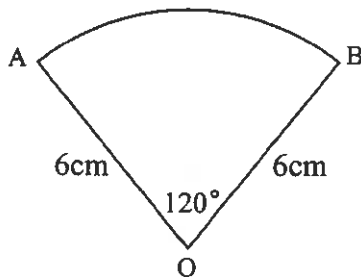
Calculate the area of the shaded segment. Give your answer correct to 3 significant figures.



$$\begin{aligned}
 A &= \frac{1}{2} r^2 (\theta - \sin \theta) \checkmark \\
 &= \frac{1}{2} \times 8^2 \times \left(\frac{2\pi}{9} - \sin \frac{2\pi}{9} \right) \checkmark \\
 &= 1.77 \text{ cm}^2 \checkmark
 \end{aligned}$$

7. (3 marks)

The diagram shows a sector of a circle with centre O. The radius of the circle is 6 cm and angle AOB = 120°. Find the arc length AB in terms of π .



$$120^\circ = \frac{2\pi}{3} \checkmark$$

$$\begin{aligned}
 \text{Arc} &= r\theta \\
 &= 6 \times \frac{2\pi}{3} \checkmark \\
 &= 4\pi \checkmark
 \end{aligned}$$