West Coast Collaborative
Specialist Mathematics Units 3 & 4
Test 2 2017
Calculator Free Section

Name: Marking Key

Score: _____ / 21

Polynomials & Functions   Section 1

All electronic devices must be switched off and in student bags.

Section 1 is worth approximately 40% of your final test mark.
No calculators or notes are to be used.
Access to approved Mathematics Specialist formulae sheet is permitted.
Simplify answers where possible.   **Time limit = 22 minutes.**

Question 1. (4 marks)

If $f(x) = x^2 + 1$ and $g(x) = \frac{2}{x}$ find the natural domain and corresponding range for

(a) fog(x) $= \frac{4}{x^2} + 1$ (2)

$D_g = x \neq 0$, $R_g = y \neq 0 \longrightarrow \boxed{R_{fog} : y > 1}$ ✓

$\boxed{D_{fog} = R : x \neq 0}$ ✓

(b) gof(x) $= \frac{2}{x^2+1}$ (2)

$D_f = R$ $R_f = y \geq 1 \longrightarrow \boxed{R_{gof} \text{ is } 0 < y \leq 2}$ ✓

$\boxed{D_{gof} \text{ is } x : x \in R}$ ✓

Question 2. (4 marks)

Find an expression for the inverse function of h where $h(x) = 1 + \frac{1}{3+x}$ and state the domain and range of h^{-1}.

$y = 1 + \frac{1}{3+x}$

$\Rightarrow y - 1 = \frac{1}{3+x}$

$\Rightarrow x + 3 = \frac{1}{y-1}$ ✓

$\Rightarrow x = \frac{1}{y-1} - 3$

So $h^{-1}(x) = \frac{1}{x-1} - 3$ ✓

$D_h = x \neq 3$

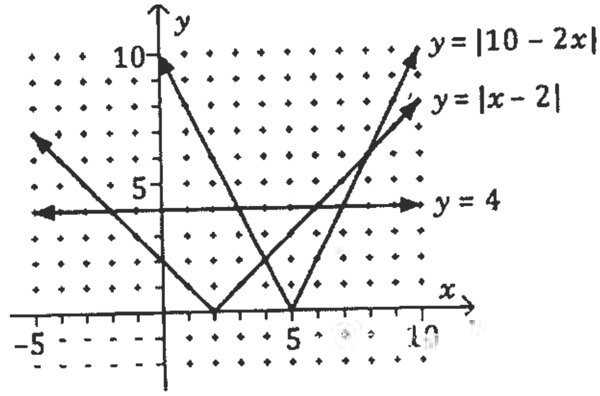
$R_h = y \neq 1$

So $D_{h^{-1}} = x : x \neq 1$

$R_{h^{-1}} = y : y \neq -3$

Question 3. (2 marks)

Use the diagram opposite to assist you to solve:



(a) $|x-2| = 4$

$x = -2, 6.$ ✓

(b) $|10-2x| = |x-2|$

$x = 4, 8$ ✓

Question 4. (4 marks)

Consider the nature of the graph of $y = \frac{x^2-3x-4}{x^3-2x^2-3x}$. Describe the effect on the y values of the graph as:

$y = \frac{(x-4)(x+1)}{x(x-3)(x+1)} = \frac{x-4}{x(x-3)}, x \neq -1$

(a) x tends to $+\infty$

$y \to 0$ ✓

(b) x tends to 3^+

$y \to -\infty$ ✓

$\frac{-ve}{+ve \times +ve}$

(c) x tends to -1^-

$y \to -5/4$ ✓

$\frac{(-1-4)}{(-1)(-1-3)} = \frac{-5}{4}.$

(d) x tends to 4^-

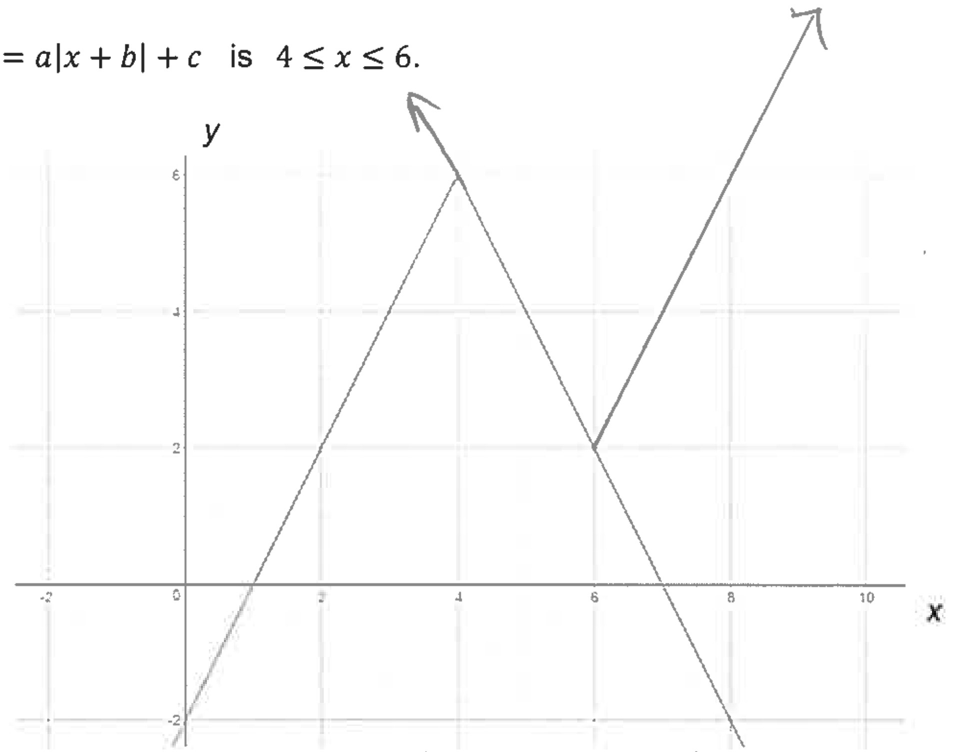
$y \to 0$ ✓

$x - 4 = 0$

Question 5. (3 marks)

The graph of $y = -2|x-4| + 6$ is shown below. Find a, b and c such that the solution to the equation

$-2|x-4| + 6 = a|x+b| + c$ is $4 \le x \le 6$.



Gradient = 2 so $a = 2$ ✓, vertex at $(6,2)$ so $b = -6$ ✓

$c = 2$ ✓

Question 6. (4 marks)

If $p(x) = 1 + \dfrac{2}{x}$ determine

(a) $p^{-1}(5)$ (2)

$y = 1 + \frac{2}{x}$

$\Rightarrow y - 1 = \frac{2}{x}$

$\Rightarrow x = \frac{2}{y-1}$

So $f(x)^{-1} = \frac{2}{x-1}$ ✓

$p^{-1}(5) = \frac{2}{5-1}$

$= \frac{1}{2}$ ✓

(b) The value(s) of x such that $p^{-1}(x) = p(x)$. (2)

$\frac{2}{x-1} = 1 + \frac{2}{x}$

$\Rightarrow \frac{2}{x-1} = \frac{x+2}{x}$ ✓

$\Rightarrow 2x = (x+2)(x-1)$

$\Rightarrow x^2 + x - 2 = 2x$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2$ or -1 ✓

or $p^{-1}(x) = p(x) = x$

So $1 + \frac{2}{x} = x$ ✓

$\Rightarrow x + 2 = x^2$

$\Rightarrow x^2 - x - 2 = 0$

So $x = 2$ or -1 ✓

End of Section One

Greenwood College | Balcatta Senior High School | Warwick Senior High School

West Coast Collaborative
Specialist Mathematics Units 3 & 4
Test 2 2017
Calculator Free Section

Name: Marking Key Score: _____ / 32

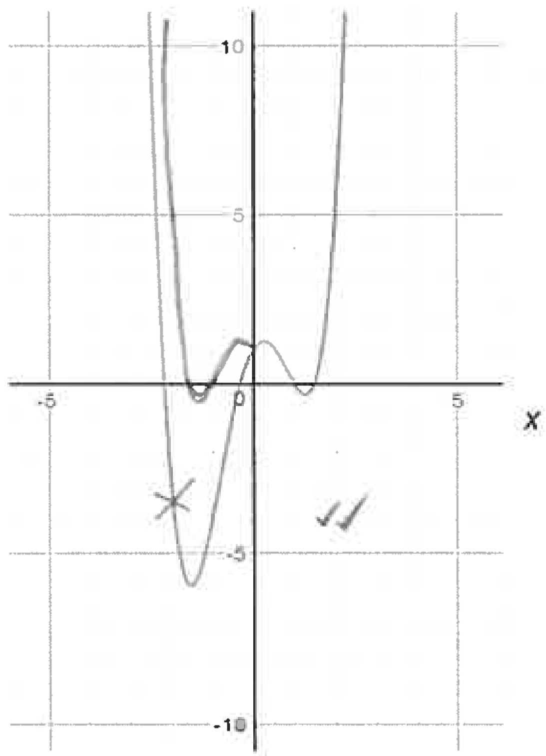
Polynomials & Functions Section 2

All electronic devices must be switched off and in student bags.

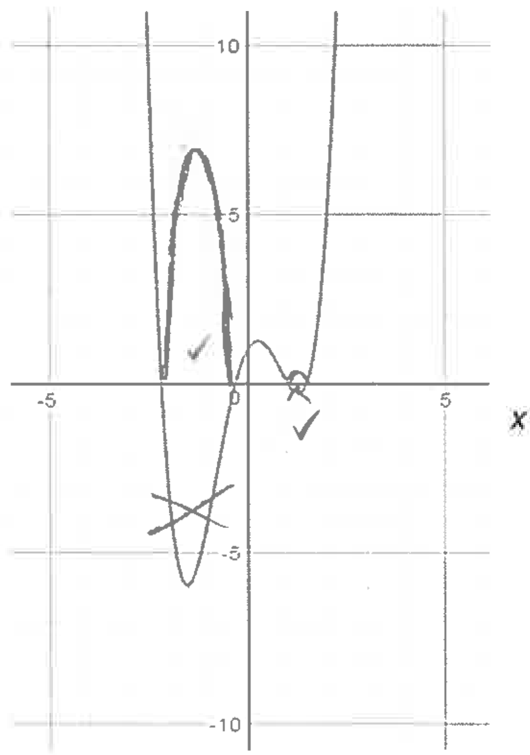
Section 1 is worth approximately 60% of your final test mark.
Calculators and 1 page of A4 notes, written on both sides, allowed.
Access to approved Mathematics Specialist formulae sheet is permitted.
Simplify answers where possible. **Time limit = 22 minutes.**

Question 7. (4 marks)
The graph of $y = p(x)$ is shown twice below. In diagram (a) show the graph of $y = p(|x|)$ while in diagram (b) show the graph of $y = |p(x)|$.

(a)



(b)



Question 8. (3 marks)

Find an expression for the inverse function of $f(x)$ where $f(x) = 4 + \frac{2}{2x-1}$ and state its domain and range.

$y = 4 + \frac{2}{2x-1}$

$\Rightarrow y - 4 = \frac{2}{2x-1}$

$\Rightarrow 2x - 1 = \frac{2}{y-4}$

$\Rightarrow 2x = \frac{2}{y-4} + 1$

$\Rightarrow 2x = \frac{2 + y - 4}{y-4}$

$\Rightarrow 2x = \frac{y-2}{y-4}$

$\Rightarrow x = \frac{y-2}{2(y-4)}$

$\Rightarrow f^{-1}(x) = \frac{x-2}{2(x-4)}$ ✓ or $x^{-1} = \frac{1}{x-4} + \frac{1}{2}$

$D_f : x \neq \frac{1}{2}$

$R_f : y \neq 4$

so $D_{f^{-1}} = x \in \mathbb{R} : x \neq 4$ ✓

$R_{f^{-1}} = y \in \mathbb{R} : y \neq \frac{1}{2}$ ✓

Question 9. (4 marks)

Given that $p(x) = x^2 - 4$ and $q(x) = \frac{1}{x}$ state the natural domain and the corresponding range of each of the following functions:

(a) $poq(x)$ $= \frac{1}{x^2} - 4$ (2)

Domain $= x \in R : x \neq 0$ ✓

Range $= y \in R : y > -4$ ✓

(b) $qop(x)$ $= \frac{1}{x^2-4}$ (2)

Domain $= x \in R : |x| \neq 2$ or $x \neq \pm 2$ ✓

Range $= y \in R :$ $y \leq -1/4 \cup y > 0$ ✓

Question 10. (6 marks)

Determine all of the asymptotes for each of the following functions:

(a) T(x) = $\frac{x^2+3x}{x-1}$ (2)

$$\begin{array}{r} x+4 \\ x-1 \overline{) x^2+3x+0} \\ \underline{x^2-x} \\ 4x- \\ \underline{4x-4} \\ 0 \end{array}$$

vertical assymptote at $x = 1$ ✓

oblique assymptote at $y = x + 4$ ✓

(b) U(x) = $\frac{x^2-9}{x(x+3)}$ $= \frac{\cancel{(x+3)}(x-3)}{x\cancel{(x+3)}}$ (2)

horizontal assymptote at $y = 1$ ✓

Vertical assymptote at $x = 0$ ✓

(c) V(x) = $\frac{x^2+1}{(2x-1)(x+2)}$ = (2)

horizontal assymptote at $y = 1/2$ ✓

vertical assymptotes at $x = 1/2$ and $x = -2$ ✓

Question 11. (4 marks)

Consider the graph of $y = \dfrac{x^2+x-2}{x^3-2x^2-x+2}$. $= \dfrac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x-2)(x+1)} = \dfrac{x+2}{(x-2)(x+1)}, x \neq 1$

(a) Locate any asymptotes. (2)

vertical assymptotes at $x = 2$ and $x = -1$

horizontal assymptote at $y = 0$

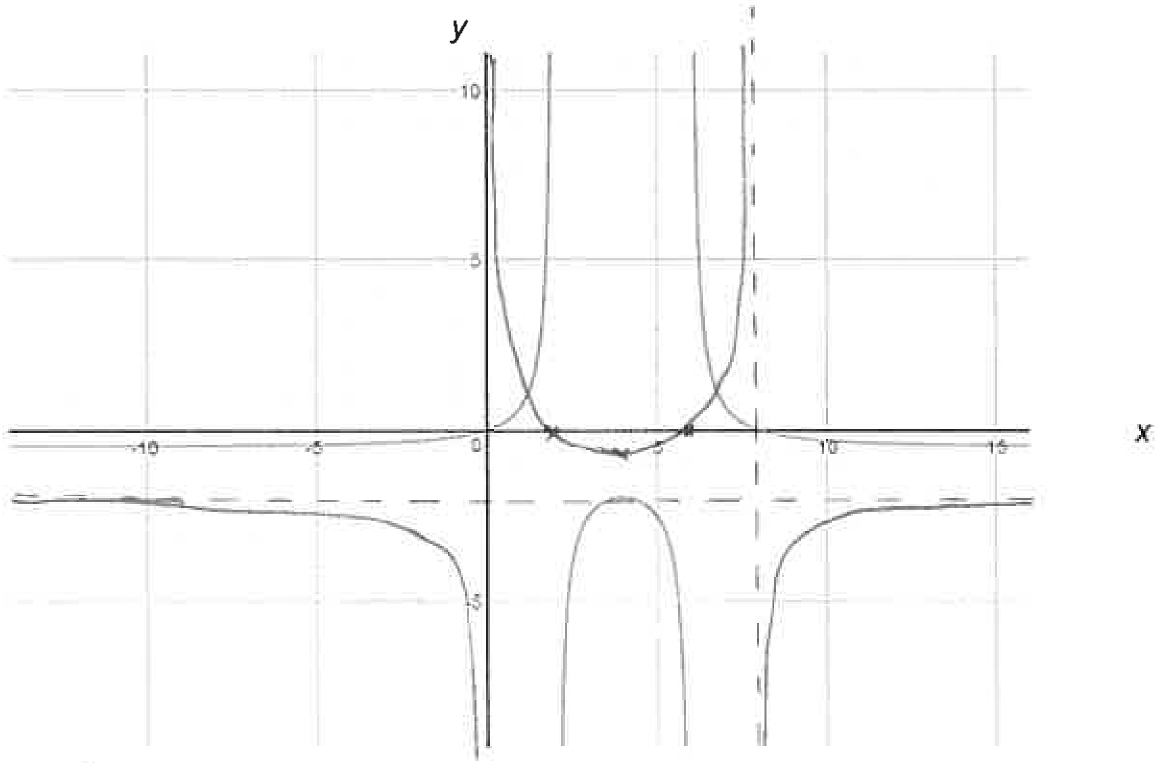
(b) Describe any other discontinuities. (2)

There is a discontinuity at $x = 1$ ✓

It is an open point ✓ at $(1, -1\frac{1}{2})$

Question 12. (5 marks)

The graph of $y = f(x)$ is shown below. It has asymptotes at $x = 2, x = 6, y = -\frac{1}{2}$. On the same set of axes draw the graph of $y = \dfrac{1}{f(x)}$, clearly showing any roots and asymptotes.



Asymptotes at $x = 0$, $x = 8$ and $y = -2$ ✓

T.P. at $(4, -\frac{1}{2})$ ✓

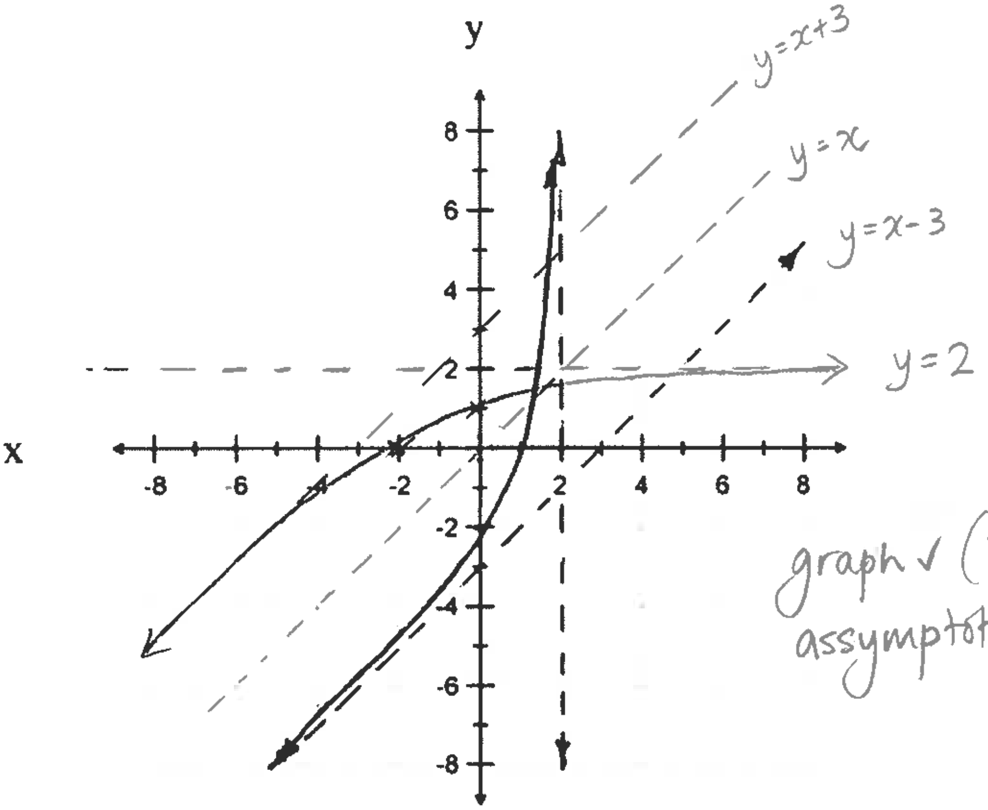
Graph $y = 1/f(x)$ intersect at points where $f(x) = 1$ ✓

Roots at $x = 2$ and 6 ✓

Symmetry of graph ✓

Question 13. (6 marks)

The graph of $g(x) = \dfrac{(x-4)(x-1)}{x-2}$, $x < 2$ is shown below:



graph ✓ (should pass through (-2,0) and (0,1))
assymptotes ✓

(a) Explain why g has an inverse function g^{-1} and find $g^{-1}(0)$. (2)

$g(x)$ is a one-to-one function ✓

$g(1) = 0$ so $g^{-1}(0) = 1$ ✓

(b) Sketch the graph of $y = g^{-1}(x)$ on the same set of axes above. Include any asymptotes in your sketch. (2)

(c) Solve for x , $g^{-1}(x) = 1.5$. (2)

$g^{-1}(x) = 1.5$

$g \circ g^{-1}(x) = g(1.5)$ ✓

$\Rightarrow \quad x = g(1.5)$

$\Rightarrow \quad x = 2.5$ ✓

Alternatively,

$y = 1.5$ for $g^{-1}(x)$

then $x = 1.5$ for $g(x)$ ✓

$g(1.5) = 2.5$ ✓

End of Test Paper