



Mathematics Applications Year 11
Test 2 ~ Financial Considerations and Matrices

Name Solutions & Marking Guide

Total Mark

Total
63

SCSA Objectives in this test may include:

- 1.1.6 use currency exchange rates to determine the cost in Australian dollars of purchasing a given amount of foreign currency, or the value of a given amount of foreign currency, when converted to Australian dollars.
- 1.1.7 calculate the dividend paid on a portfolio of shares given the percentage dividend or dividend paid for each share, and compare share values by calculating a price-to-earnings ratio.
- 1.2.4 use matrices for storing and displaying information that can be presented in rows and columns; for example, databases, links in social or road networks
- 1.2.5 recognise different types of matrices (row, column, square) and determine their size
- 1.2.5 recognise different types of matrices (zero, identity) and determine their size
- 1.2.6 perform matrix addition, subtraction, multiplication by a scalar, and matrix multiplication, including determining the power of a matrix using technology with matrix arithmetic capabilities when appropriate
- 1.2.7 use matrices, including matrix products and powers of matrices, to model and solve problems; for example, costing or pricing problems, squaring a matrix to determine the number of ways pairs of people in a communication network can communicate with each other via a third person

PART A - CALCULATOR FREE

Part A Mark

Total
24

- Instructions:
- Show all working in order for full marks to be awarded
 - Round answers to 2 decimal places unless otherwise stated
 - NO calculators are permitted in this section
 - SCSA Formula Sheet is permitted

1. State the order of each matrix below.

[2:1,1]

a) $\begin{bmatrix} 8 & 9 & 2 \\ 3 & 1 & 5 \end{bmatrix}$

Order 2x3 ✓

b) $\begin{bmatrix} 5 \\ 4 \\ 9 \\ 2 \end{bmatrix}$

Order 4x1 ✓

2. What is the order of the product matrix CD if $C = \begin{bmatrix} 3 & 8 & 7 \\ 1 & 6 & 7 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 7 \\ 3 & 6 \\ 9 & 5 \end{bmatrix}$? [1]

Order 2x2 ✓

2x3

3x2

3. What type of matrix has the same number of rows and columns?

[1]

Square ✓

4. To determine the new balance of an account after an increase of 3%, what number is the original balance multiplied by? [1]

1.03 ✓

5. To determine the value of a car after it has been depreciated (lowered in value) by 8%, what number is the original value multiplied by? [1]

0.92 ✓

6. Which situation below has the greatest discount (ie the most money)? Justify your answer. [4]

A 50% discount on goods normally priced at \$500.

B 40% discount on goods normally priced at \$600.

C 80% discount on goods normally priced at \$300.

A: \$250 ✓

B: $600 \div 5 \times 2$
= \$240 ✓

C: $300 \div 5 \times 4$
= \$240 ✓

∴ Situation A ✓ has the greatest discount.

7. State whether or not each of the following is possible, using the matrices below. Give reasons to justify your answers. [6:2,2,2]

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 5 & 7 \\ 87 & 32 & 47 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

a) AB Possible? No ✓ Reason rows ≠ columns ✓

b) AC Possible? Yes ✓ Reason rows = columns ✓

f) A + B Possible? Yes ✓ Reason same dimensions ✓

8. Show below a:

[2:1,1]

a) 2 x 2 identity matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

b) 3 x 3 identity matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

9. Perform the stated calculations with the given matrices.

[4:2,2]

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 8 \\ 6 & 10 \end{bmatrix}$$

$$C = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

a) AB

b) CB

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 8 \\ 6 & 10 \end{bmatrix}$$
$$= \begin{bmatrix} 5+12 & 8+20 \\ 15+24 & 24+40 \end{bmatrix} \checkmark$$

$$= \begin{bmatrix} 17 & 28 \\ 39 & 64 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \times \begin{bmatrix} 5 & 8 \\ 6 & 10 \end{bmatrix}$$
$$= \begin{bmatrix} 5a+6c & 8a+10c \\ 5b+6d & 8b+10d \end{bmatrix}$$

10. Given that Matrix Y has dimensions 3 by 5 and Matrix Z has dimensions 5 by 7:

[2:1,1]

a) State the dimensions of the product YZ.

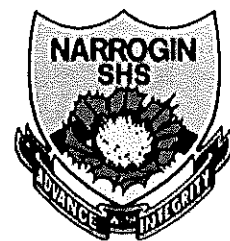
$$\underline{3 \times 7} \checkmark$$

$$\begin{array}{c} 3 \times 5 \times 5 \times 7 \\ \text{by} \\ \hline \end{array}$$

b) What can you say about the product ZY?

Not possible \checkmark

~ END OF TEST PART A ~



Mathematics Applications Year 11

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PART B - CALCULATOR ALLOWED

Part B Mark _____

39

- Instructions:
- Show all working in order for full marks to be awarded
 - Round answers to 2 decimal places unless otherwise stated
 - Classpads and scientific calculators are permitted
 - One double sided A4 page of notes and the SCSA Formula Sheet are permitted

8. A toy company has factories in Aytown, Beetown and Ceetown, all of which manufacture one particular type of doll and one particular type of robot. The matrix A below gives the production costs (in dollars) for Aytown. [6:2,2,2]

$$A = \begin{bmatrix} & \text{Doll} & \text{Robot} \\ \text{Material} & 4.25 & 5.60 \\ \text{Labour} & 3.50 & 3.75 \end{bmatrix}$$

In Beetown material costs are \$4.50 per doll and \$5.70 per robot, whilst labour costs are \$3.27 per doll and \$3.65 per robot. In Ceetown a doll has costs of \$4.30 and \$3.40 for materials and labour respectively, whilst a robot has costs of \$5.74 and \$3.70 for materials and labour respectively.

- a) Write down the production cost matrices for Beetown and Ceetown.

$$B = \begin{bmatrix} 4.50 & 5.70 \\ 3.27 & 3.65 \end{bmatrix} \checkmark$$

$$C = \begin{bmatrix} 4.30 & 5.74 \\ 3.40 & 3.70 \end{bmatrix} \checkmark$$

- b) Assuming each plant makes the same number of dolls and robots, write down the matrix representing the average production costs for the three factories.

$$\left[\begin{array}{cc} (4.25 + 4.50 + 4.30) \div 3 & (5.60 + 5.70 + 5.74) \div 3 \\ (3.50 + 3.27 + 3.40) \div 3 & (3.75 + 3.65 + 3.70) \div 3 \end{array} \right] = \begin{bmatrix} 4.35 & 5.68 \\ 3.39 & 3.70 \end{bmatrix}$$

- c) Suppose that labour costs are increased by 10% in Aytown, resulting in a new production cost matrix A' for Aytown. Find the matrix A'.

$$A' = 1.1 \times A = 1.1 \times \begin{bmatrix} 4.25 & 5.60 \\ 3.50 & 3.75 \end{bmatrix} = \begin{bmatrix} 4.675 & 6.16 \\ 3.85 & 4.125 \end{bmatrix}$$

6

9. The brokerage fees charged by a stockbroker are:

[2]

Share value	Up to \$15000	\$15000 and over
Brokerage	\$69.95	0.60% of the trade value

Determine the brokerage fee for 2000 shares @ \$7.60 per share.

$$2000 \times 7.60 = \$15200 \checkmark$$

$$\text{Fees will be } 0.6\% \text{ of } \$15200,$$

$$\text{ie } 0.6 \div 100 \times 15200 \checkmark$$

$$= \$91.20 \checkmark$$

10. Jason is paid an annual salary of \$68500. Determine Jason's income per:

[3:1,1,1]

a) week

b) fortnight

c) month

$$68500 \div 52$$

$$= \$1317.31 \checkmark$$

$$68500 \div 26$$

$$= \$2634.62 \checkmark$$

$$68500 \div 12$$

$$= \$5708.33 \checkmark$$

11. The table below shows the conversion rates for a number of international currencies. These were current as at March 16th, 2018.

[5:1,2,2]

	USD	EUR	GBP	INR	MYR	NZD	THB	SGD	JPY
1 AUD	0.77876	0.63276	0.55907	50.6021	3.05661	1.07345	24.3049	1.02400	82.5292

Use this information to answer the following questions.

a) 1 Australian dollar is equivalent to how many New Zealand dollars?

$$1.07345 \checkmark$$

b) Andrew wishes to purchase Euros for his trip overseas.
How many Euros will he be able to purchase for 3000 Australian dollars?

$$3000 \times 0.63276 \checkmark = 1898.28 \checkmark$$

c) After returning from an overseas holiday, Emma still has 2500 Japanese yen.
How much will this convert to when Emma trades her yen for Australian dollars?

$$2500 \div 82.5292 \checkmark = \$30.29 \checkmark$$

12. The table below shows the Youth Allowance rates for people aged between 16 and 24 who are studying full time. Students are able to earn up to \$415 per fortnight with no penalty, however if they earn over \$498 per fortnight, their fortnightly allowance is reduced by 60 cents in the dollar for every dollar they earn over \$498. [3]

Status	Fortnightly payment
Singles less than 18 years living at home	\$226.80
Single less than 18 years not living at home	\$414.40
Single older than 18 years living at home	\$272.80
Single older than 18 years not living at home	\$414.40
Single with children	\$542.90

Bradley is a 20 year old single student living away from home who has a part time job that pays him \$535 per fortnight. Determine the Youth Allowance that Bradley will be paid.

$$\begin{aligned}
 \text{Rate} &= \$414.40 \checkmark & 535 - 498 &= \$37 & \therefore \text{Allowance} &= 414.40 - 22.20 \\
 & & & & &= \$392.20 \checkmark \\
 & & 37 \times 0.6 &= \$22.20 \checkmark
 \end{aligned}$$

13. Xanthe purchases 8000 shares in Amity Confectionary at a cost of \$7.25 each. The company pays a dividend of 4.5% of the share price and a brokerage fee of 3% is paid to the stockbroker. [8:3,2,3]

Determine:

- a) the total cost of purchasing the shares (including the brokerage fee)

$$\begin{aligned}
 8000 \times 7.25 &= \$58000 \checkmark & 58000 + (3\% \text{ of } \$58000) &= 58000 + 1740 \checkmark \\
 & & &= \$59740 \checkmark
 \end{aligned}$$

- b) the total dividend paid

$$\begin{aligned}
 4.5 \div 100 \times 58000 &= \$2610 \checkmark
 \end{aligned}$$

- c) the total gain if Xanthe sells all 8000 shares at the end of one year @ \$9.05 per share.

$$\begin{aligned}
 \text{Cost} &= \$59740 & \text{Income} &= 2610 + (8000 \times 9.05) \\
 & & &= 2610 + 72400 \checkmark = \$75010 \checkmark \\
 \text{Profit} &= 75010 - 59740 = \$15270 \checkmark
 \end{aligned}$$

14. If $A = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 2 & 0 & 3 \end{bmatrix}$, find: [5:1,2,2]

a) $A - B$

b) $B + (A - C)$

c) $B - 2A + 3C$

$$\begin{bmatrix} 1 & 2 & 0 \\ -4 & 1 & -3 \\ 0 & 1 & -2 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 6 & -2 & 7 \\ -1 & 5 & 5 \\ 0 & -3 & 1 \end{bmatrix} \checkmark \checkmark$$

$$\begin{bmatrix} -2 & 3 & -11 \\ 19 & -4 & 7 \\ 5 & 0 & 10 \end{bmatrix} \checkmark \checkmark$$

15. Alice worked the following rates and hours.

[3]

Normal rate: 35 hours $\rightarrow 35$
 Time and a half: 4 hours $\rightarrow 6$
 Double time: 6 hours $\rightarrow 12$

If Alice earned \$1934.50, determine her normal rate of pay.

$$35 + (4 \times 1.5) + (6 \times 2) \checkmark$$

$$= \underline{53 \text{ equivalent hours}}$$

$$1934.50 \div 53 \checkmark$$

$$= \underline{\$36.50} \checkmark$$

16. The Sky High annual production will be held over 3 nights - Friday, Saturday and Sunday. Ticket prices are \$35 for Front Row Adult, \$25 for General Seating Adults, \$15 for Children and \$10 for those with Concession cards. The number of tickets sold for each night is given in the table below. [3]

State 2 matrices and show how matrix multiplication could be used to determine the total value of ticket sales for each of the three nights.

Matrix A: $\begin{bmatrix} 25 & 63 & 38 & 18 \\ 29 & 82 & 49 & 25 \\ 52 & 81 & 67 & 39 \end{bmatrix} \checkmark$

(must be transposed from original or no mark)

Matrix B: $\begin{bmatrix} 35 \\ 25 \\ 15 \\ 10 \end{bmatrix} \checkmark$

	Friday	Saturday	Sunday
Front Row Adult	25	29	52
General Adult	63	82	91
Children	38	49	67
Concession	18	25	39

$$\Rightarrow AB = \begin{bmatrix} 3200 \\ 4050 \\ 5240 \end{bmatrix} \Rightarrow \begin{array}{l} \text{Friday } \$3200 \\ \text{Saturday } \$4050 \\ \text{Sunday } \$5240 \end{array}$$

~ END OF TEST PART B ~

11

