School Name Mathematics Test 2017

Year 10

Congruence

Non-Calculator

Skills and Knowledge Assessed:

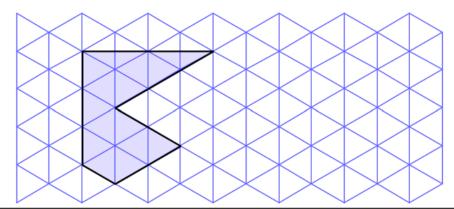
- Define congruence of plane shapes using transformations (ACMMG200)
- Develop the conditions for congruence of triangles (ACMMG201)
- Formulate proofs involving congruent triangles and angle properties (ACMMG243)

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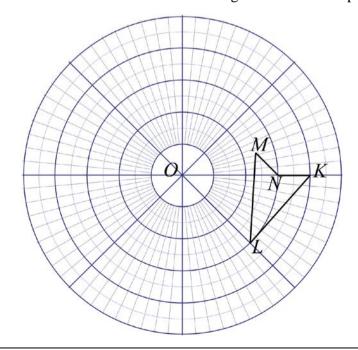
Section 1 Short Answer Section

Write all working and answers in the spaces provided on this test paper.

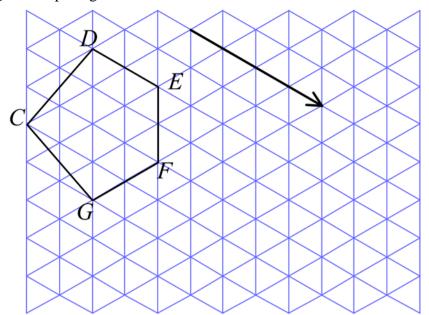
1. Use the grid to draw a shape which is congruent to the one shown.



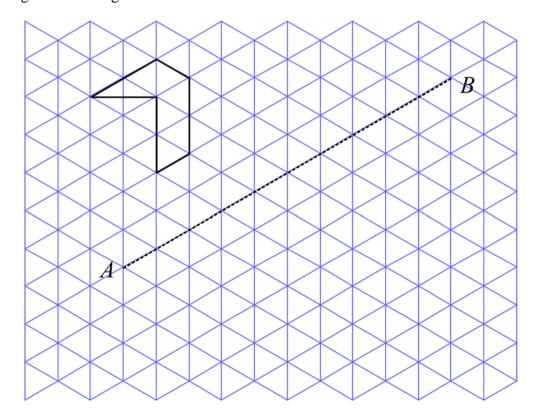
2. Draw the image of the kite *KLMN* when it is rotated through 160° about the point O.



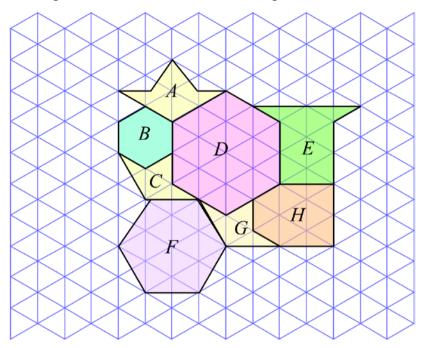
3. Draw the image of the pentagon *CDEFG* when it is translated 4 cm in the direction of the arrow.



4. Draw the image of the hexagon when it is reflected in the line *AB*.



5. Which two hexagons are congruent? (Write their letters in the space below.)



.....

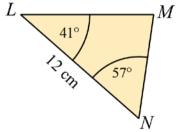
Explain what information is needed to use the triangle congruence test which is abbreviated as SSS.

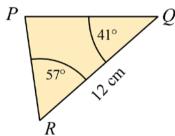
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7. Which of the congruence test (AAS, RHS, SAS or SSS) could be used to show that

 $\Delta LMN \equiv \Delta QPR$.

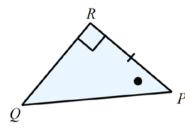
6.

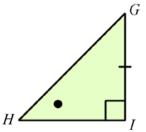




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8. Which two triangles below have enough information provided to show they are congruent to one another.

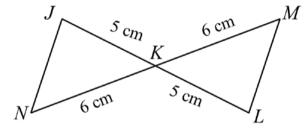




Explain your answer.

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9. Which of the congruence test (AAS, RHS, SAS or SSS) could be used to prove that $\Delta JKN \equiv \Delta LKM$?



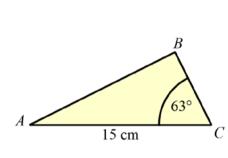
Explain your answer.

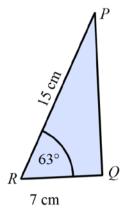
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10. In $\triangle ABC$ and $\triangle PQR$, AC = PR and $\angle ACB = \angle PRQ$.

What additional piece of information would need to be given, in order to prove that $\Delta ABC \equiv \Delta PQR$?

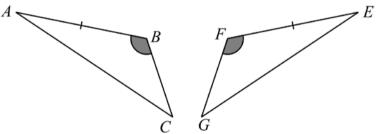
Explain your answer.





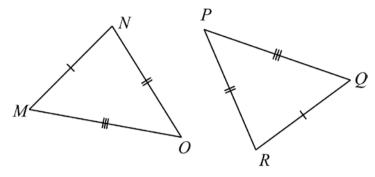
11. In the figure below, AB = EF and $\angle ABC = \angle EFG$.

Give one additional piece of information would allow you to show that Δ *ACB* \equiv Δ *DFE* and which congruence test would be used?



.....

12. In the triangles below, MN = QR, NO = PR and MO = PQ.



A proof that $\triangle MNO \equiv \triangle QRP$ has been started.

Complete the missing sections of the proof.

In \triangle MNO and \triangle QRP

$$MN = QR$$
 (given)

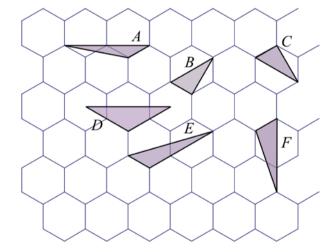
$$NO = PR$$
 (given)

$$\therefore \Delta \, MNO \, \equiv \, \Delta \, QRP \, \, \left(\, \underline{\hspace{1cm}} \, \right)$$

13. The diagram below is drawn on a grid of congruent hexagons.

It shows a number of triangles, labelled A to F.

Which pair of triangles are congruent and which test could be used to prove this?



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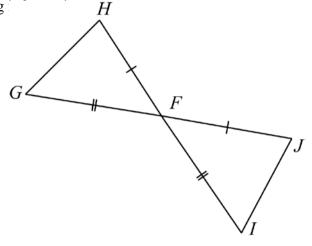
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14. In the diagram, GF = FI and HF = FJ.

HI and GJ are a straight line segments.

Complete the proof below, by filling in the missing

Prove that $\Delta FGH \equiv \Delta FIJ$.



15. In the diagram $\triangle PQR$ is isosceles, with QP = QR and QS is drawn perpendicular to PR.

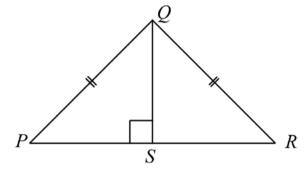
Complete the proof below, by writing in the reasons for each step.

Prove that $\triangle PSQ \equiv \triangle RSQ$. In $\triangle PQS$ and $\triangle RQS$ $\triangle PSQ = \angle RSQ = 90^{\circ}$ (.....)

$$PQ = RQ$$
 (.....)

 QS is

 $\therefore \Delta PQS \equiv \Delta RQS \ (.....)$



School Name Mathematics 2017

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Congruence

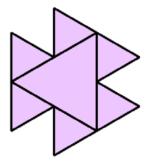
Calculator Allowed

Name____

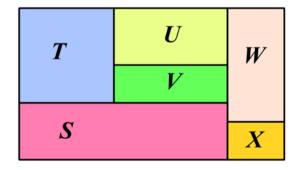
Section 2 Multiple Choice Section

Mark all your answers on the accompanying multiple choice answer sheet, not on this test paper. You may do any working out on this test paper. Calculators are allowed for this section.

- 1. How many congruent triangles are there in the shape below?
 - A. 4
 - B. 5
 - C. 6
 - D. 7



2. Which of the pairs of rectangles listed below are congruent?

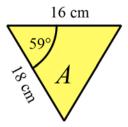


- A. S and W
- B. T and X
- C. U and V
- D. U and W

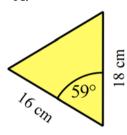
3. Triangle A has the measurements shown at right.

Which of the triangles below is congruent to Triangle A?

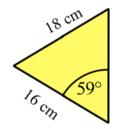
The diagrams are not to scale.



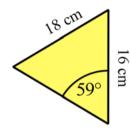
A.



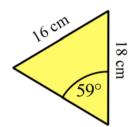
B.



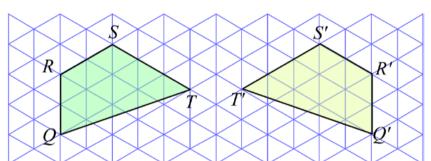
C.



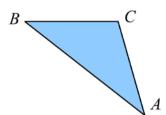
D.



- 4. What transformation could have been used to produce the congruent image Q'R'S'T'.
 - A. Reflection.
 - B. Rotation through 90°.
 - C. Rotation through 180°.
 - D. Translation.

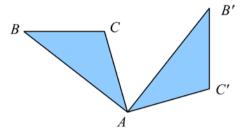


5. The triangle ABC is rotated through 270° in a clockwise direction about A.

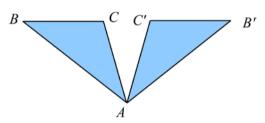


Which diagram shows the correct image?

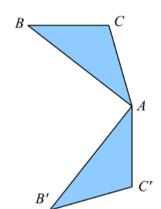
A.



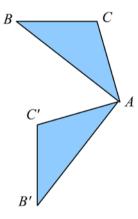
B.



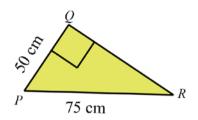
C.



D.

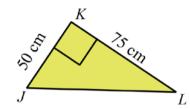


6. Which triangle is congruent to $\triangle PQR$?

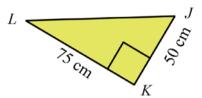


NOT TO SCALE

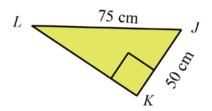
A.



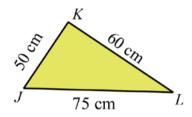
B.



C.

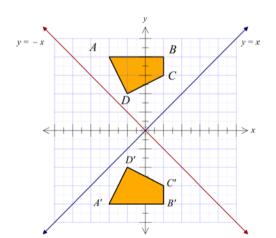


D.



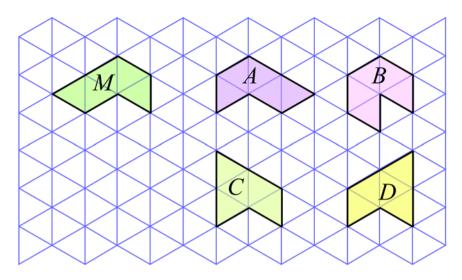
7. Figure ABCD is moved to an image A'B'C'D' by a single transformation.

What was the transformation?



- A. A clockwise rotation of 180° about the origin.
- B. A reflection in the *x* axis.
- C. A reflection in the line y = x.
- D. A reflection in the line y = -x

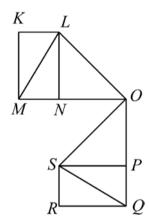
8. Which shape is congruent to shape *M*?



- A. Shape A B. Shape B C. Shape C D. Shape D
- 9. *KLNM* is a rectangle and *LNO* is a right isosceles triangle.

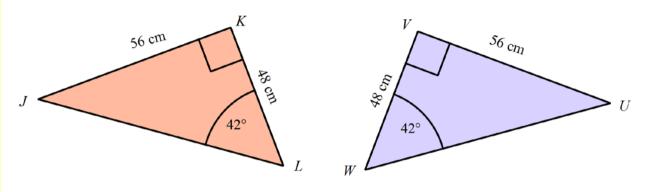
The trapezium *KLOM* is rotated through 90° in an anticlockwise direction about O.

Which of the following is a pair of congruent triangles?



- A. \triangle *KLM* and \triangle *SOP*
- B. $\triangle LON$ and $\triangle SPQ$
- C. \triangle *MLO* and \triangle *QSO*
- D. \triangle LMN and \triangle OSP

Which two congruence tests could be used to prove that $\Delta JKL \equiv \Delta UVW$? 10.



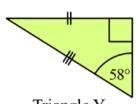
A. AAS or RHS AAS or SAS C. SAS or RHS B.

D. SAS or SSS

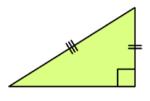
11. Which triangles are congruent?



Triangle X



Triangle Y



Triangle Z

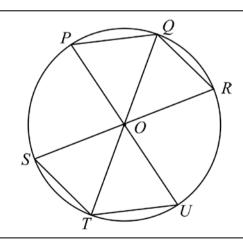
- All three triangles.
- B. Triangles X and Y.
- C. Triangles X and Z.
- Triangles Y and Z. D.
- 12. The circle shown has centre *O*.

PU, QT and RS are diameters of the circle.

Which pair of triangles are congruent?



- \triangle *POQ* and \triangle *SOT* B.
- \triangle QOR and \triangle TOU C.
- \triangle QOR and \triangle SOT D.

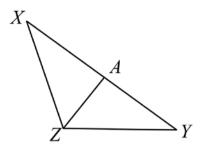


13. In $\triangle XYZ$, XZ = YZ and A is the midpoint of its longest side XY.

Without using any further properties of triangles, which of the congruence tests could be used to show

$$\Delta XZA \equiv \Delta YZA$$
?

- A. AAS
- B. RHS
- C. SAS
- D. SSS



NOT TO SCALE

14. In the figure below, $BC \parallel EF$.

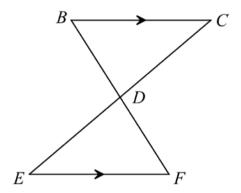
Which single additional piece of information would allow you to show that $\Delta BCD \equiv \Delta FED$?



B.
$$\angle BDC = \angle FED$$

C.
$$BD = FD$$

D.
$$CD = FD$$



15. PQRS is a parallelogram and T and U are the points on PQ and SR such that PT = SU.

One line has been left out of the proof below.

$$PT = SU$$
 (given)

$$\angle PTS = \angle UST$$
 (alternate angles on parallel lines)

$$\therefore \Delta PTS \equiv \Delta UST \text{ (SAS)}$$

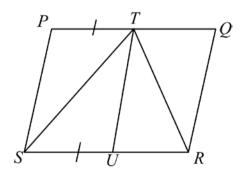
Which statement would complete the proof?

A.
$$SP = UT$$
 (opposite sides of a parallelogram)

B. ST is common

C.
$$\angle TSP = \angle STU$$
 (alternate angles on parallel lines)

D. PT = TQ (T is midpoint of PQ)



School Name Mathematics 2017

Year 10

1.

Congruence

Calculator Allowed

Name

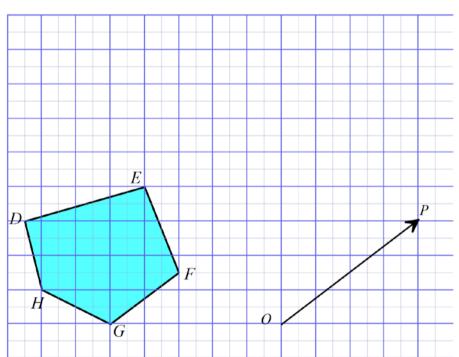
Section 3 Longer Answer Section

Write all working and answers in the spaces provided on this test paper.

Marks

2

(a) Translate the polygon *DEFGH* in the distance and direction of the ray *OP*.



(b) Label the image after the translation as D'E'F'G'H'.

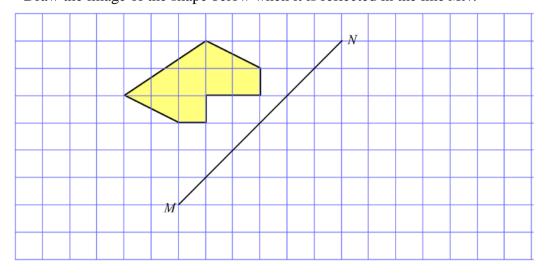
1

(c) What two properties are shared by the sides EF and E'F'?

2

2. Draw the image of the shape below when it is reflected in the line *MN*.

2



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| (c) | What could be said about the area of the shape and its image? | |
|-----|---|--|

1

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| Marks |
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3. (a) A triangle has three sides whose lengths are given below.

2

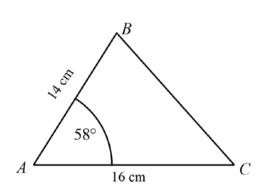
Use instruments to accurately draw the triangle. (Do not erase your construction lines.)

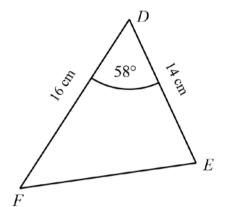
(b) A right triangle has a base which measures 10 cm, and a hypotenuse which is 14 cm in length.

Use instruments to accurately draw the triangle.

4. (a) Prove that $\triangle ABC \equiv \triangle DEF$.

3



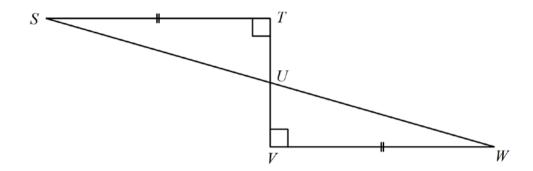


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(b) In the diagram below, ST = VW, $\angle STU = \angle UVW = 90^{\circ}$ and U is the midpoint of TV.

Prove that $\Delta STU \equiv \Delta WVU$.

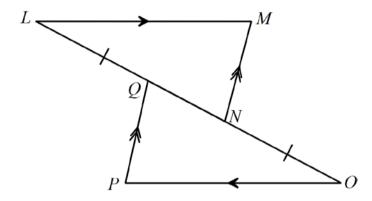


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(c) In the diagram below, $LM \parallel PO$, $MN \parallel PQ$ and LQ = NO.

3

Prove that $\triangle LMN \equiv \triangle OPQ$.



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School Name

Mathematics 2017

Multiple Choice Answer Sheet

Congruence

| | Completely | fill the re | sponse ova | l representing the most correct answer. |
|-----|----------------|-------------|--------------|---|
| 1. | A 🔾 | В | c O | D 🔾 |
| 2. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |
| 3. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |
| 4. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |
| 5. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |
| 6. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |
| 7. | $A \bigcirc$ | В | c 🔾 | D 🔾 |
| 8. | $A \ \bigcirc$ | В | c \bigcirc | D 🔾 |
| 9. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |
| 10. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |
| 11. | $A \bigcirc$ | В | c 🔾 | D 🔾 |
| 12. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |
| 13. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |
| 14. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |
| 15. | $A \bigcirc$ | В | c \bigcirc | D 🔾 |

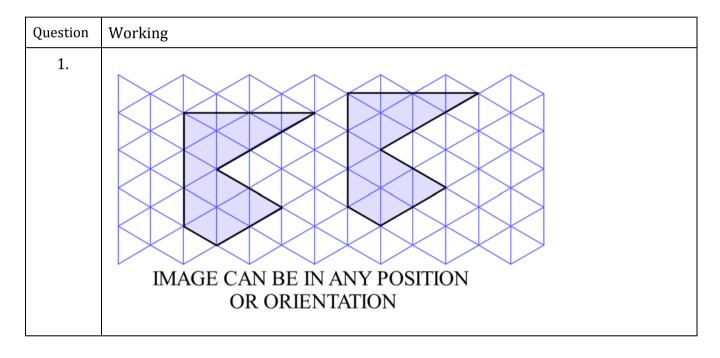
School Name Mathematics Test 2017

Year 10

Congruence

Non Calculator Section

ANSWERS



| Question | Working |
|----------|---------|
| 2. | |
| 3. | |

| Question | Working |
|----------|--|
| 4. | |
| 5. | C and G are the only congruent hexagons. |
| 6. | The three sides of one triangle are equal in length to the corresponding three sides of the second triangle. |
| 7. | The Test AAS could be used as there are a pair of angles which are equal and the side between them in each triangle is equal. |
| 8. | Triangles ABC and PQR are congruent, as the side which is equal is in the same corresponding position in both triangles relative to the equal angles so can use AAS. |
| 9. | The test SAS could be used, as there are a pair of corresponding sides equal, (given) and the angle between them is equal (vertically opposite angles) |
| 10. | We have one pair of equal angles and one pair of equal sides. In triangle PQR, the side RQ is given as 7cm giving SAS, so we need side BC = 7cm. |

| Question | Working | | | | |
|----------|---|--|--|--|--|
| 11. | We have a pair of equal angles and a pair of equal sides which form one arm of the an If we were given that $BC = FG$ we could use SAS. If we were given that $\angle A = \angle E$ we could use AAS. If we were given that $\angle C = \angle G$ we could use AAS. | | | | |
| 12. | In \triangle MNO and \triangle QRP | | | | |
| | MN = QR (given) | | | | |
| | NO = PR (given) | | | | |
| | MO = PQ (given) | | | | |
| | $\therefore \Delta MNO \equiv \Delta QRP (SSS)$ | | | | |
| 13. | Triangles B and C are congruent and any of the tests except RHS could be used to prove congruence. | | | | |
| 14. | | | | | |
| | In Δ FGH and Δ FIJ | | | | |
| | GF = FI. (given) | | | | |
| | $\angle GFH = \angle IFJ$ (vertically opposite angles) | | | | |
| | HF = FJ (given) | | | | |
| | $\Delta FGH \equiv \Delta FIJ (SAS)$ | | | | |

| Question | Working | | | |
|----------|--|--|--|--|
| 15. | In $\triangle PQS$ and $\triangle RQS$ $\angle PSQ = \angle RSQ = 90^{\circ}$ (equal right angles on straight line) | | | |
| | PQ = RQ (given) | | | |
| | QS is common | | | |
| | $\therefore \Delta PQS \equiv \Delta RQS (\mathbf{RHS})$ | | | |
| | | | | |

School Name Mathematics Test 2017

Congruence

Calculator Allowed Short Answer Section

Year 10

ANSWERS

| Question | | Answer |
|----------|--|--------|
| 1. | All of the smaller outer triangles are congruent, so there are 6 congruent triangles. | С |
| 2. | Rectangles U and W both measure 3 cm by 1.5 cm, so are congruent as all the angles are equal to 90° . | D |
| 3. | Triangle A has a pair of sides equal to those in triangle <i>KLM</i> and the angle included is equal. | A |
| 4. | A reflection in a vertical line would produce the image. | A |
| 5. | B C' A 270° B' | D |
| 6. | Triangle C is congruent as RHS could be used. | С |

| 7. | A reflection in the <i>x</i> axis. | В |
|-----|--|---|
| | y = -x A B C C C' B' | |
| 8. | Shape A is a reflection of shape M . | A |
| 9. | Δ <i>MLO</i> and Δ <i>QSO</i> as M rotates to Q and L rotates to S while O remains unchanged. | С |
| 10. | SAS using the right angle as the included angle. AAS using the two given angles and the corresponding side. | В |
| 11. | Triangle X Triangle Y Triangle Z After finding the missing angles in X and Y, they are congruent by AAS, but Y and Z are congruent by RHS, so all three are congruent. | A |
| 12. | There are two pairs of congruent triangles, both with equal sides due to equal radii and included angles due to vertically opposite angles. These are \triangle <i>QOR</i> and \triangle <i>SOT</i> and \triangle <i>POQ</i> and \triangle <i>TOU</i> Of these only \triangle <i>QOR</i> and \triangle <i>SOT</i> is listed. | D |
| 13. | XZ = YZ is given. A is the midpoint of its longest side XY so $AX = AY$. AZ is common. So there are three corresponding sides equal. SSS | D |

| 14. | $\angle B = \angle F$ (alternate angles on \parallel lines) $\angle C = \angle E$ (alternate angles on \parallel lines) $\angle BDC = \angle FDE$ (vert opp angles) So we need a pair of corresponding sides equal, $BD = FD$ is the only pair of the two listed which are in corresponding positions relative to the angles. | С |
|-----|--|---|
| 15. | To achieve SAS, we need a side which is the other arm of the given angle. <i>ST</i> being common gives this. | В |

School Name

Mathematics 2017

Multiple Choice Answer Sheet

Congruence

| | | | | Name | |
|-----|---|--------------|--------------|------|--|
| | Completely fill the response oval representing the most correct answer. | | | | |
| 1. | A 🔾 | В | c • | D 🔾 | |
| 2. | A 🔾 | В | c \bigcirc | D | |
| 3. | A • | В | c \bigcirc | D 🔾 | |
| 4. | A • | $B \bigcirc$ | c \bigcirc | D 🔾 | |
| 5. | A 🔾 | В | c \bigcirc | D | |
| 6. | $A \bigcirc$ | В | c | D 🔾 | |
| 7. | A 🔾 | В | c \bigcirc | D 🔾 | |
| 8. | A • | В | c \bigcirc | D 🔾 | |
| 9. | A 🔾 | В | C | D 🔾 | |
| 10. | A 🔾 | В | c \bigcirc | D 🔾 | |
| 11. | Α • | В | c \bigcirc | D 🔾 | |
| 12. | A 🔾 | В | c \bigcirc | D | |
| 13. | $A \bigcirc$ | В | c \bigcirc | D | |
| 14. | $A \bigcirc$ | В | C | D 🔾 | |
| 15. | $A \bigcirc$ | В | c \bigcirc | D 🔾 | |
| | | | | | |

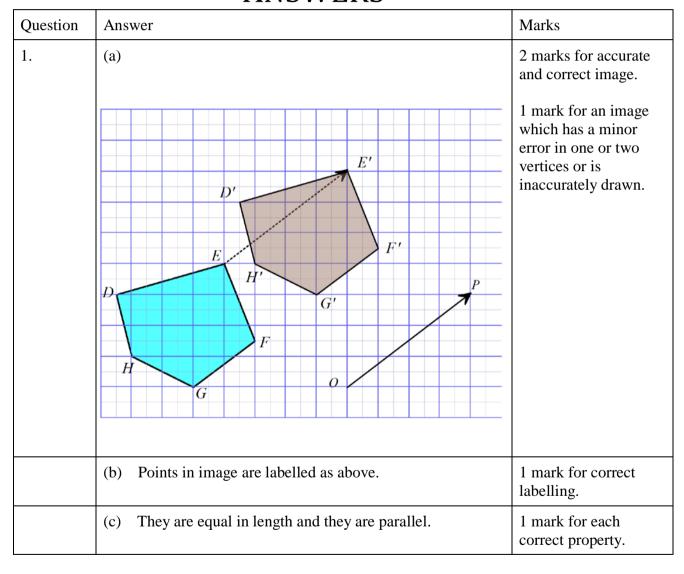
School Name Mathematics Test 2017

Congruence

Calculator Allowed Longer Answer Section

Year 10

ANSWERS



| Question | Answer | Marks | |
|----------|------------------------|--|--|
| 2. | | 2 marks for accurate and correct image. 1 mark for an image which has a minor error in one or two vertices or is inaccurately drawn. | |
| | (b) They are the same. | 1 mark for correct answer | |
| | (c) They are the same. | 1 mark for correct answer | |
| 3. | (a) | 2 marks for correctly drawn triangle in any orientation showing construction lines. 1 mark for correctly drawn triangle in any orientation without construction lines. 1 mark for slightly inaccurately drawn triangle in any orientation showing construction lines. | |

| Question | Answer | Marks |
|----------|--|--|
| | (c) | 2 marks for correctly drawn triangle in any orientation showing construction lines. 1 mark for correctly drawn triangle in any orientation without construction lines. 1 mark for slightly inaccurately drawn triangle in any orientation showing construction lines. |
| 4. | (a) In \triangle ABC and \triangle DEF AB = DE = 14 cm (given) \angle BAC = \angle EDF = 58° (given) AC = DF = 16 cm (given) \therefore \triangle ABC \equiv \triangle DEF (SAS) | 3 marks for complete proof with reasons 2 marks for complete proof except for a minor error 1 mark for partially correct proof (with incorrect reasons or lacking reasons) |
| | (b) In $\triangle STU$ and $\triangle WVU$. $\angle STU = \angle WVU = 90^{\circ}$ (given) $\angle TUS = \angle VUW$ (vert opp angles) $ST = VW$ (given) $\therefore \triangle STU \equiv \triangle WVU$ (AAS) Alternative: $TU = UV$ (U bisects TV) - therefore SAS as another proof | 3 marks for complete proof with reasons 2 marks for complete proof except for a minor error 1 mark for partially correct proof (with incorrect reasons or lacking reasons) |

| Question | Answer | Marks |
|----------|--|---|
| | (c) In $\triangle LMN$ and $\triangle OPQ$ $\angle MNL = \angle PQN$ (alt \angle on \parallel lines) | 3 marks for complete proof with reasons |
| | ∠ MLN = ∠ \overrightarrow{POQ} (alt ∠ on lines) \overrightarrow{QN} is common $\overrightarrow{LQ} = NO$ (given) ∴ $\overrightarrow{LQ} + \overrightarrow{QN} = NO + \overrightarrow{QN}$ ∴ $\overrightarrow{LN} = \overrightarrow{OQ}$ ∴ $\overrightarrow{\Delta} \overrightarrow{LMN} \equiv \overrightarrow{\Delta} \overrightarrow{OPQ} (AAS)$ | 2 marks for complete proof except for a minor error 1 mark for partially correct proof (with incorrect reasons or lacking reasons) |