Mathematics Methods

Investigation 1

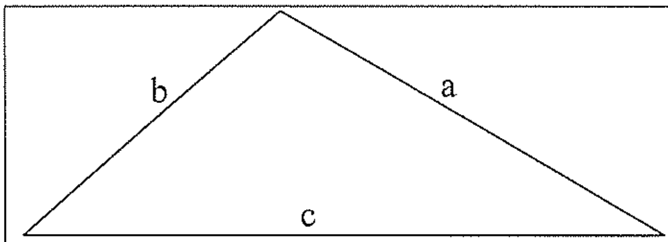
Heron's Rule

Take Home Section **Due:**

NAME Solutions

The rules you have used to calculate the area of a triangle are A = ½bh, where b is the length of the base and h is the perpendicular height, and A = $\frac{1}{2}abSinC$, where a and b are lengths of 2 sides and C is the angle between them.

However outside the school classroom people rarely have the base and the perpendicular height or an angle. They are more likely to be given the lengths of the 3 sides of the triangle. In 60 AD the mathematician Heron of Alexandria published a book with a rule for calculating the area of a triangle and ever since then it has been known as Heron's rule. It seems however that Archimedes knew the rule over 300 years before Heron!!!



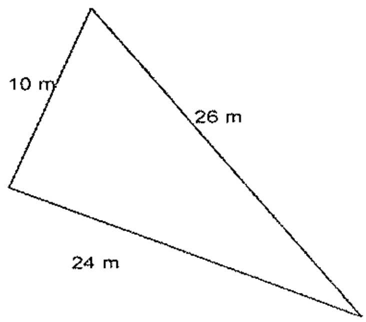
$$s = \frac{a+b+c}{2}$$

s is called the semi-perimeter.

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

Question 1:

a) Use Heron's formula to calculate the area of the scalene triangle shown below.



$S = \frac{10+26+24}{2} = 30.$

area $= \sqrt{30(30-10)(30-26)(30-24)}$

$= \sqrt{30 \times 20 \times 4 \times 6}$

$= \sqrt{14400}$

$= 120m^2$

b) Use Pythagoras' Theorem to prove that this triangle is right angled.

$10^2 + 24^2 = 676$

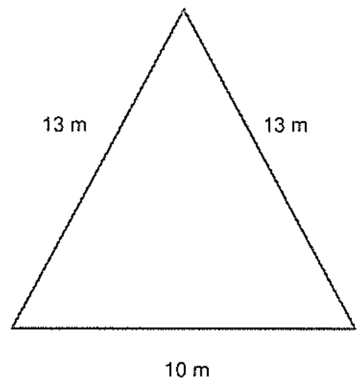
$\sqrt{676} = 26m$ as required.

c) Use this result to show the area calculated from Heron's Rule is correct.

$\frac{1}{2} \times 10 \times 24 = 120m^2$ $\therefore$ Heron's rule is correct.

Question 2:

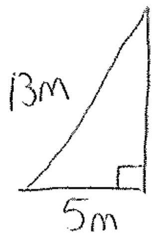
a) Use Heron's rule to determine the area of this isosceles triangle, with a perimeter of 36 cm.



$$S = \frac{13+13+10}{2}$$
$$= 18$$

$$\text{area} = \sqrt{18(18-13)(18-13)(18-10)}$$
$$= \sqrt{18 \times 5 \times 5 \times 8}$$
$$= \sqrt{3600}$$
$$= \underline{\underline{60m^2}}$$

b) Find the perpendicular height of the triangle in part (a). Show all working.



$$13^2 - 5^2 = 144$$
$$\sqrt{144} = \underline{\underline{12m}}$$

c) Use another method to determine the area of this triangle to confirm your solution to part a.

$\frac{1}{2} \times 10 \times 12 = 60m^2$ as required.

Question 3:

a) Consider the following lengths of the sides of triangles each with a perimeter of 30cm (semi-perimeter of 15).

Find the area of each using Heron's rule.

Impossible?

Isosceles triangles?

Equilateral?

Scalene?

Maximum Area?

Heron's rule on NumSolve?

(8,10,12)

39.7cm^2

(7,9,14)

26.8cm^2

(6,11,13)

32.9cm^2

(10,10,10)

43.3cm^2

(9,9,12)

40.2cm^2

(8,8,14)

27.1cm^2

(3,10,17)

no solution

(7,7,16)

no solution

(4,12,14)

22.2cm^2

(5,11,14)

24.5cm^2

Try some other triangles of your own with a fixed perimeter of 30 cm.
Show your solutions below.

b) Discuss what you found.

- Which triangles are impossible? Why? What happens to Heron's rule with these impossible triangles? What relationship must exist between a, b and c for the numbers to form a triangle?

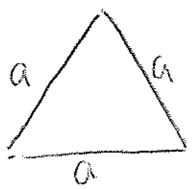
- $a + b > c$ means triangle is formed.

- Heron's rule doesn't work for $a + b < c$ as this results in have a negative number in the $\sqrt{\ }$

- What type of triangles create the largest area?

equilateral triangles

- Can you find a rule for the area of the largest triangle in terms of the semi-perimeter (s)? Justify your answer using algebraic steps.



$$S = \frac{a + a + a}{2}$$

$$= \frac{3a}{2}$$

$$a = \frac{2s}{3}$$

$$\sqrt{s\left(s - \frac{2s}{3}\right)\left(s - \frac{2s}{3}\right)\left(s - \frac{2s}{3}\right)}$$

$$= \sqrt{s\left(s - \frac{2s}{3}\right)^3}$$

$$= \sqrt{s\left(\frac{3s}{3} - \frac{2s}{3}\right)^3}$$

$$= \sqrt{s\left(s/3\right)^3}$$

$$= \sqrt{s \times \frac{s^3}{27}}$$

$$= \sqrt{\frac{s^4}{27}}$$

- What are the sides of a triangle with the smallest perimeter possible with a maximum area of 1000 m²? Explain how you found this. What procedure did you use?

$$1000 = \sqrt{\frac{s^4}{27}}$$

$$1\,000\,000 = \frac{s^4}{27}$$

$$27\,000\,000 = s^4$$

$$s = 72.08$$

$$a = \frac{2s}{3}$$

$$a = \frac{2 \times 72.08}{3}$$

$$a = 48.1\text{cm}$$

Each side is 48.1cm