Name   *Solutions* — Formula Sheet

Section 1 – Resource Free
Marks :   27   **Time: 30 minutes   (maximum)**

1.   **[6 marks]**

For the graph of $y = x^3 - x^2 - 4x + 4$, determine:

(a)   the coordinates of the point where the curve cuts the y-axis, $(0, 4)$ ✓

(b)   the coordinates of the points where the line cuts the x-axis (you may need to use the factor theorem to help you do this).

$$P(1) = 1 - 1 - 4 + 4 = 0 \quad (x-1)$$
$$P(-1) = -1 - 1 + 4 + 4$$
$$P(2) = 8 - 4 - 8 + 4 = 0 \quad (x-2)$$
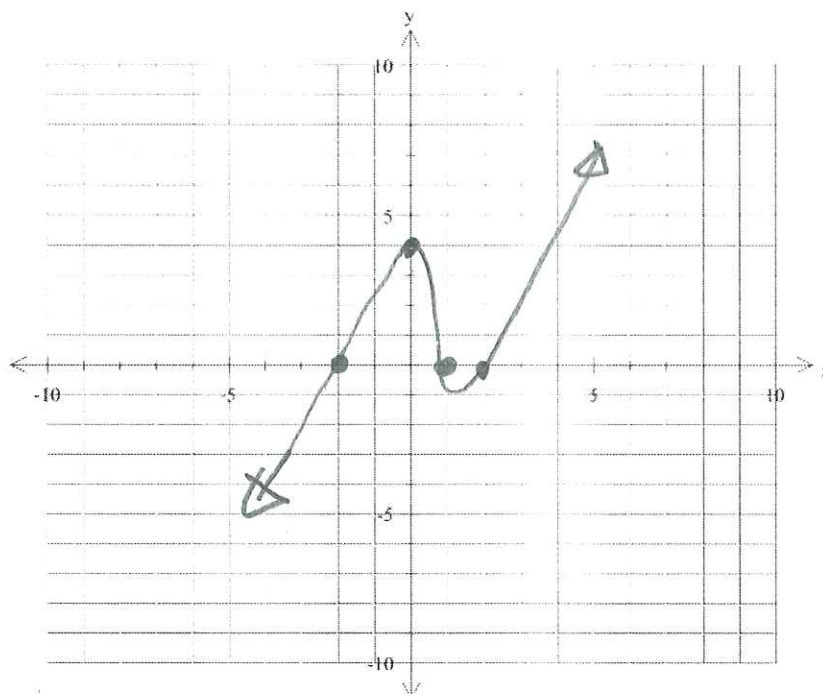$$\therefore \quad y = (x-1)(x-2)(x+2)$$

$(1, 0)$
$(2, 0)$
$(-2, 0)$ ✓✓✓✓

(c)   Draw a sketch of this graph.

✓✓

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2. [1, 1, 1 = 3 marks]

(a) Express $15°$ in radians leaving your answer as a multiple of π $\dfrac{15\pi}{180} = \dfrac{\pi}{12}$ ✓

(b) Express $\dfrac{5\pi}{4}$ radians in degrees $= 225°$ ✓

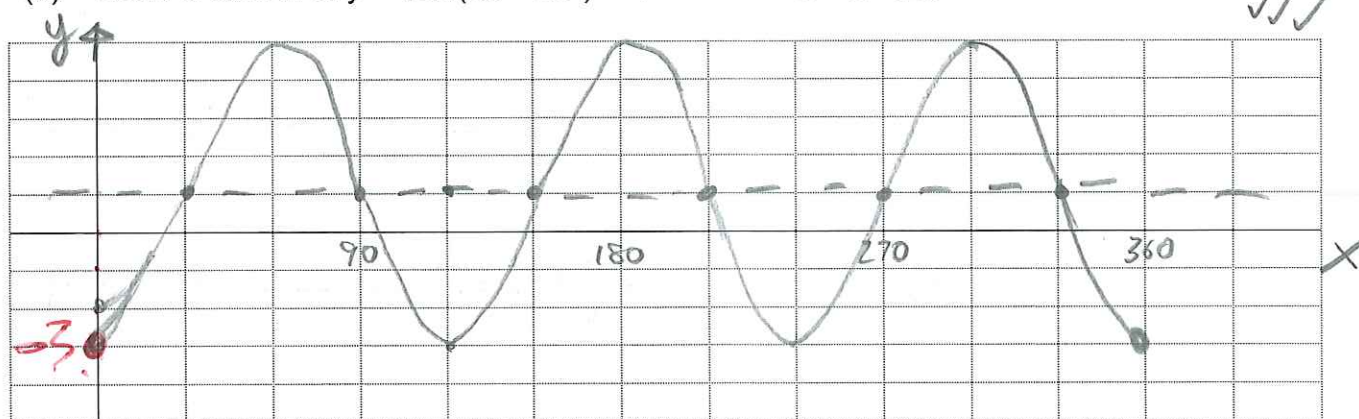(c) Find exactly, the exact value of $\cos\left(\dfrac{5\pi}{6}\right) = \cos 150° = -\dfrac{\sqrt{3}}{2}$ ✓✓

3. [5 marks]
Give the amplitude and period of:
(a) y = 4sin(3x − 90°) + 1

$Amp = 4$ ✓
$Period = 120°$ ✓

(b) Draw a sketch of y = 4sin(3x − 90°) + 1 $0 \le x \le 2\pi$ ~~360~~   ✓✓✓

4. [5 marks]

Given $P(A \cup B) = 0.72$, P(A) = 0.4 and P(B) = 0.5,

a) find $P(A \cap B))$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (2)
$0.72 = 0.4 + 0.5 - P(A \cap B)$
$P(A \cap B) = 0.18$ ✓✓

b) find P(A|B) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ (2)
$= \dfrac{0.18}{0.5}$
$= 0.36$ ✓✓

c) Are events A and B independent? (Justify your answer)   (1)
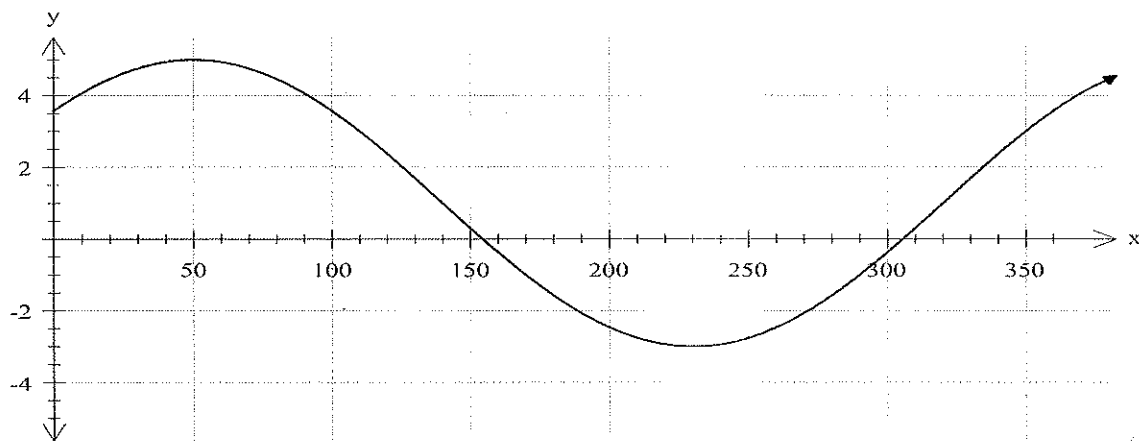
No since $P(A|B) \neq P(A)$ ✓

$\boxed{\dfrac{18}{}}$

5. **[4 marks]**
Give the equation of each of the following graphs:
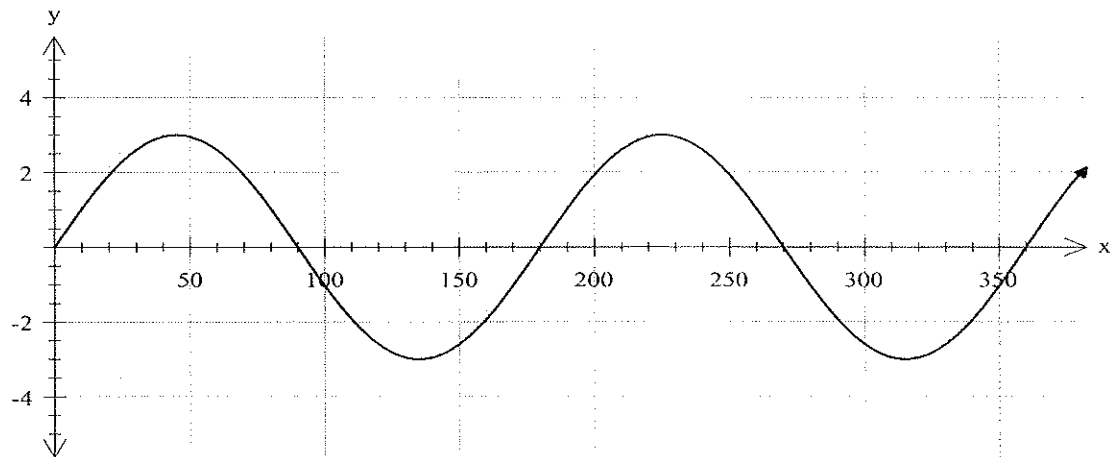
$$y = 3 \sin 2x \quad \checkmark\checkmark$$

$$y = 4 \cos(x - 50) + 1 \quad \checkmark\checkmark$$
$$\text{or} \quad y = 4 \sin(x + 40) + 1$$

6. **[4 marks]**

$(x - 2)$ and $(x + 1)$ are factors of $x^3 - 4x^2 + px + q$.
Find p and q and determine the third factor of the cubic expression

$$0 = 8 - 16 + p2 + q \qquad 2p + q = 8$$
$$0 = -1 - 4 + p + q \qquad -p + q = 5$$
$$3p = 3$$
$$p = 1 \quad \checkmark$$
$$q = 6 \quad \checkmark$$

$$\therefore x^3 - 4x^2 + x + 6 = (x-2)(x+1)(x-3) \quad \checkmark\checkmark$$

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Name _____

Section 2 – Resource Rich — 1 Page notes + Formula sheet

Marks : 31

Time: 30 minutes (minimum)

7. **[5 marks]**

Some Year 12 students purchased 30 items from two surf shops; High Tied and Surf Down.

The following table shows how many Tops, Shorts and Jeans were purchased from each shop.

	Tops	Shorts	Jeans
High Tied	4	9	4
Surf Down	3	8	2

Two items were chosen at random from the group of 30.

(a) Determine how many different ways there are of choosing the two items. [1]

$$\binom{30}{2}$$

Without evaluating answers, find the probability that:

(b) they were both from High Tied. [1]

$$P(\text{Both HT}) = \frac{\binom{17}{2}}{\binom{30}{2}} \qquad \frac{136}{435}$$

(c) one was from each shop. [1]

$$P(\text{Diff}) = \frac{\binom{17}{1}\binom{13}{1}}{\binom{30}{2}} \qquad 221$$

(d) ~~neither was a top from High Tied.~~ both same [1]

$$\frac{\binom{4}{0}\binom{26}{2}}{\binom{30}{2}} \qquad \frac{\overset{136}{\binom{13}{0}\binom{17}{2}} + \overset{78}{\binom{13}{2}\binom{17}{0}}}{\binom{30}{2}} = 214$$

(e) neither was a top and neither was from High Tied. [1]

$$\frac{\binom{10}{2}}{\binom{30}{2}} \qquad 45$$

[5]

7. **[5 marks]**

A deer farmer believes that the profit/loss of his farm may be affected by weather conditions. His computer is set up with appropriate software and the following table produced.

	Weather Favourable (F)	Weather Unfavourable ($\bar{F}$)	
Profit (P)	0.52	0.13	0.65
Loss ($\bar{P}$)	0.12	0.23	0.35
	0.64	0.36	1.00

(a) Complete the table.

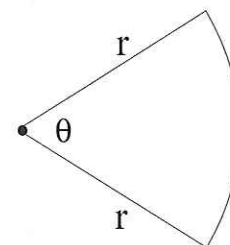(b) Find the probability that he makes a profit and the weather is unfavourable.

$$P(P \cap F) = 0.13$$

(c) Find the probability that the weather was favourable, given that he makes a loss.

$$P(F \mid \bar{P}) = \frac{0.12}{0.35} = \frac{12}{35}$$

$$\sim 0.34$$

8. **[7 marks]**

A sector of a circle is shown in the diagram opposite.

a) If r = 8cm and θ = 50°, find the perimeter of the sector (accurate to 1 decimal place).

$$P = \frac{50}{360} \times \pi \times 16 + 16$$

$$P = 23.0 \text{ cm}$$

b) If the radius is 9cm and the perimeter is 31.5 cm, find the size of θ in radians.

$$\text{arc} = 13.5$$

$$\theta = \frac{13.5}{9}$$

$$\theta = 1.5^r$$

c) Given that the area of the sector is 62cm² and θ = 1.4 radians. Find the radius of the sector (accurate to 2 significant figures).

$$A = \frac{1}{2} r^2 \theta$$

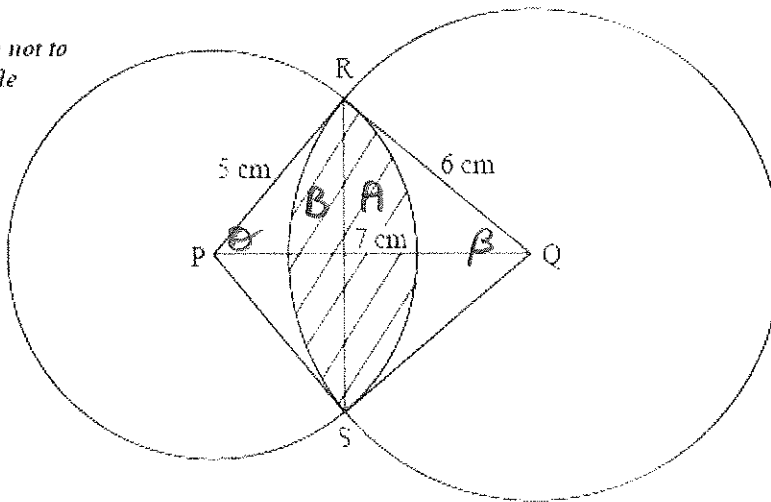$$r = 9.4 \text{ cm}$$

$$A = \frac{1}{2} r^2 (\theta - \sin\theta)$$

$$r = 9.5$$

$$\boxed{11}$$

9. **(7 marks)**

The diagram below shows a pair of intersecting circles with centres at P and Q with radii of 5 cm and 6 cm respectively. RS is the common chord of both circles and PQ is 7 cm.

diagram not to scale

Find the area of the shaded region.

Angles using cosine rule

$$\theta = 0.9969^r$$

$$\beta = 0.775^r$$

$$\text{Area}(B) = \frac{1}{2}r^2(\theta - \sin\theta)$$

$$= \frac{1}{2}6^2(2 \cdot 0.775 - \sin(1.5\ldots))$$

$$= 9.91$$

$$\text{Area}(A) = \frac{1}{2}5^2(2 \cdot 0997 - \sin(2 \times 0.997))$$

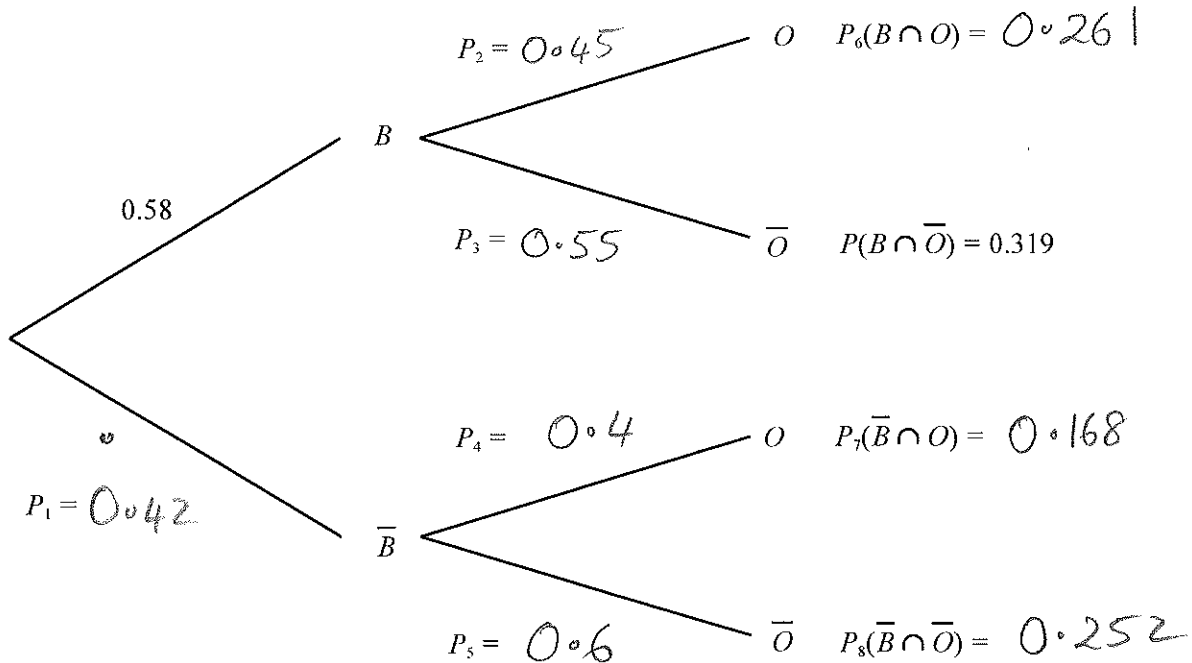$$= 13.52$$

$$\text{Total Area} = 23.43\ cm^2$$

10 **[8 marks]**

The clinical records of a large eye hospital indicate that

- 58% of patients are blue eyed (set B)
- 42.9% of patients belong to the blood group O (set O)
- 31.9% of patients are blue eyed and do not belong to blood group O

(a) Use this information to complete the probabilities P_1 to P_8 in the tree diagram below.

$P_2 = 0.45$ O $P_6(B \cap O) = 0.261$

B

0.58

$P_3 = 0.55$ $\overline{O}$ $P(B \cap \overline{O}) = 0.319$

$\frac{1}{2}$ each

$P_4 = 0.4$ O $P_7(\overline{B} \cap O) = 0.168$

$P_1 = 0.42$

$\overline{B}$

$P_5 = 0.6$ $\overline{O}$ $P_8(\overline{B} \cap \overline{O}) = 0.252$

(b) What is the probability that a randomly selected patient will

(i) belong to blood group O and have blue eyes?   (1 mark) ✓

0.261

(ii) have blue eyes or belong to blood group O? $0.58 + 0.168$ (1 mark)   ✓

$= 0.748$

(iii) not have blue eyes, given they do not belong to blood group O?   (2 marks)   ✓✓

$$P(\overline{B}|\overline{O}) = \frac{0.252}{0.571}$$

$$= 0.441$$

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