Student Name Marking Key

Eastern Goldfields College
Mathematics Applications U3&4 2018
Test 2 – Calculator Free Section

Working Time: 25 minutes **No Notes** **Total Marks: 26 marks**

Question 1 (6 marks: 2, 2, 2)

For the following sequences determine which are arithmetic, geometric or neither.

Provide a reason to support your answer.

a) 1, 2.5, 4, 5.5, …

Arithmetic ✓

Common difference = 1.5 ✓ must state common diff/ratio

$\frac{1}{2}$ if write add 1.5 X.

b) 5, -5, 5, -5, 5, -5, …

Geometric ✓

Common ratio = -1 ✓

c) 2, 1, 2, 1, 2, 1 …

Niether ✓ no common difference no common ratio ✓

Question 2 (11 marks: 3, 3, 3, 2)

a) A geometric sequence has $T_3 = 4$ and $T_6 = 32$. (3 marks: 2, 1)

i) Determine the recursive rule.

$32 = 4 \times r^3$

$8 = r^3$

$r = 2$

$T_1 = 1$ ✓

$T_{n+1} = 2T_n$ ✓

ii) Calculate the 5th term.

$T_5 = 16$ ✓

/9

b) An arithmetic sequence has $T_3 = -5$ and $T_6 = 4$. (3 marks: 2, 1)

i) Determine the recursive rule.

$4 = -5 + 3d$

$9 = 3d$

$d = 3$

$T_1 = -11$ ✓

$T_{n+1} = T_n + 3$ ✓

ii) Calculate the 5th term.

$T_5 = 1$ ✓

c) For the following sequence determine the recursive rule and T_7. (3 marks)

T_1	T_2	T_3	T_4	T_5
4	-8	16	-32	64

$r = \frac{-8}{4} = -2$ $\quad \frac{16}{-8} = -2$

✓ $T_1 = 4$

✓ $T_{n+1} = -2T_n$

✓ $T_7 = 256$

d) The Lucas sequence is defined by $L_{n+1} = L_n + L_{n-1}$ with $L_5 = 11$ and $L_6 = 18$. Determine the first 4 terms in the sequence. (2 marks)

$L_6 = L_5 + L_4$

$L_4 = L_6 - L_5$

$L_4 = 7$

$L_3 = 11 - 7 = 4$

$L_2 = 7 - 4 = 3$

1, 3, 4, 7 ✓ ✓

Question 3 (2 marks)
Renee bought a pair of dogs, and at the end of the first year decides to breed them. If the dogs have three puppies every year, find how many dogs Renee will own after 5 years.

$T_1 = 2$

$T_2 = 2 + 3 = 5$

$T_3 = 8$

$T_4 = 11$

$T_5 = 14$

$T_{n+1} = 3T_n + 2$
$T_5 = 3(5) + 2$
$T_5 = 17$ OR
0 marks

$T_n = a + (n-1)3$

$T_5 = 2 + (4)(3)$

$T_5 = 14$

process mark?

$T_n = a + (n-1)d$

$T_5 = 2 + (5)3$ ✓

$T_5 = 17$ 0 marks

2, 5, 8, 11, 14. ✓
5 years = 11 dogs X.

$3 \times 5 = 15$
$15 + 2 = 17$
17 dogs
0 marks

$T_1 = 2$
$T_{n+1} = T_n + 3$ ✓
$T_5 = 17$ X

Question 4 (5 marks: 2, 3)

The nth term of a sequence is given by the rule $T_n = 3^{n-1} + 5$

a) Find the first three terms in the sequence.

$T_1 = 3^0 + 5$ $\quad$ $T_2 = 3^1 + 5$ $\quad$ $T_3 = 3^2 + 5$

$= 6$ $\quad$ $= 8$ $\quad$ $= 14.$

✓ $\quad$ ✓

b) Find the first order recurrence relation that defines the sequence.

✓ $T_1 = 6.$ ft

$T_{n+1} = 3T_n - 10$

✓ ✓

If $T_1 = 5$ ✓ft

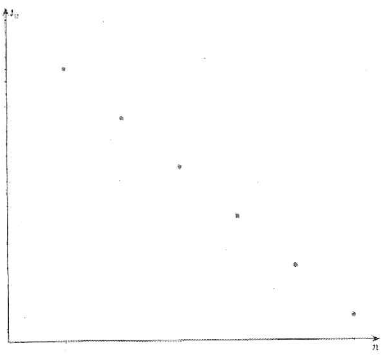
$T_{n+1} = 2T_n - 2$ ✓✓ft

only work for 1st 3 terms.

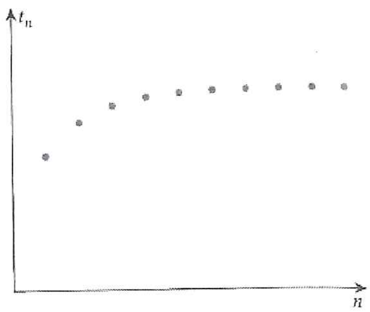
Question 5 (2 marks)

Match each of the following recursive rules with their respective graph.

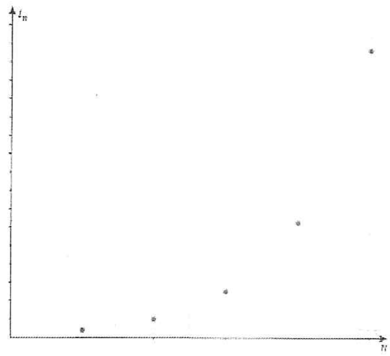
a) $T_{n+1} = T_n - 4;\ T_1 = 22$ $\quad$ b) $T_{n+1} = 2.5T_n;\ T_1 = 2$ $\quad$ c) $T_{n+1} = 0.5T_n + 3;\ T_1 = 4$



(a)



(c)



(b)

✓✓ all 3 correct

✓ 2 correct.

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Test 2 – Calculator Assumed Section

Working Time: 30 minutes **Notes Assumed** **Total Marks: 31 marks**

Question 1 [9 marks – 3, 3, 3]

The first and second terms of a sequence are 2 and 6 respectively.

(a) If these terms form part of a geometric sequence $\frac{6}{2} = 3 = r$

(i) list the next two terms,

$T_3 = 18$
$T_4 = 54$ ✓

(ii) state a recursive rule for the sequence.

$T_1 = 2$ ✓
$T_{n+1} = 3T_n$ ✓

(b) If the two terms form part of an arithmetic sequence, find $6-2 = 4 = d$

(i) the fifth term of the sequence,

$T_n = 2 + 4(n-1)$
$T_5 = 2 + 4(4)$
$= 18$

$T_5 = 18$ ✓

(ii) which term of the sequence is the first to exceed 100.

$100 > 2 + 4(n-1)$
$100 > 4n - 2$
$102 > 4n$
$25.5 > n$

$n = 26$ or T_{26} ✓✓

(c) If the recursive rule for the sequence is given by $T_n = T_{n-1} - T_{n-2}$, find

$T_3 = 6 - 2 = 4$

(i) T_4,

$T_4 = T_3 - T_2$
$= 4 - 6$

$T_4 = -2$ ✓

(ii) the smallest value of n, $n > 1$, for which $T_n = T_1$.

$2, 6, 4, -2, -6, -4, 2$ ✓ working

$n = 7$ ✓

Question 2 (7 marks – 1, 2, 2, 2)

Elsa is negotiating with her mother as to how much pocket money she will get. Elsa suggests starting with $50 in the first month and increasing this by $5 every month.

a) With this scheme, how much pocket money will Elsa receive 12 months from the start?

$105 ✓ R/W.

Elsa's mother says that increasing the amount by 5% each month is better for Elsa **in the long run**.

b) Use the table below to show how much pocket money Elsa will receive with her scheme and her mother's scheme for the first 5 months of the year.

	Month 1	Month 2	Month 3	Month 4	Month 5
Elsa's scheme	50	55	60	65	70 ✓
Mother's scheme	50	52.50	55.125	57.881	60.775 ✓ (3dp)

c) Is Elsa's mother correct? Justify your solution mathematically.

Yes ✓

	Elsa	Mum.
27 months	$180	$177.78 (2dp)
28 months	$185	$186.67

after 28 months, Elsa's mother scheme gives more. ✓

'In the long run' Elsa mum is correct

OR Elsa = AP = linear growth
Mother = GP = exponential increase

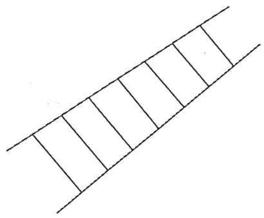
IF wrote No ⇒ XX (c)
then No ✓ FT in (d)

d) If the amount of pocket money Elsa will be paid is capped to a maximum of $120/month, does this effect which scheme is better? Justify your solution.

Yes. ✓ as Elsa will achive $120 in month 15 whereas she will have to wait until month 19 to achieve $120 by her mothers scheme ✓

Question 3 (5 marks – 2, 2, 1)

A ladder has 21 rungs and from the bottom to the top each rung is shorter than the one before it by a constant amount. The bottom rung is 400 mm long and the top rung is 320 mm.



a) How much shorter is each rung than the rung below it?

$400 - 320 = 20n$ ✓

$80 = 20n$

$n = 4$ mm. shorter ✓

$\frac{80}{21} = 3.8095$

3.81 (2dp) ✗ ✓ft only if working is shown

b) Give a recursive rule for calculating the length (T_n) of the nth rung from the rung below it. No working = XX

$T_1 = 400$ ✓ $\quad T_{n+1} = T_n - 4$ ✓

c) Give a non-recursive formula for calculating the length of any rung.

$T_n = 400 - 4(n-1)$ OR $T_n = 404 - 4n$ ✓

Question 4 **(6 marks: 3, 1, 2)**

A commercial fish farming operation has approximately 10,000 fish in one of its artificial lakes. It is thought that without any intervention this population will continue to increase at the rate of 1.5% per week.

However, intervention is planned that will see 200 fish harvested from the lake at the end of each week, the first harvest will be one week from now.

a) Give the first order linear recurrence relation for this sequence.

$T_0 = 10\,000$ ✓

$T_{n+1} = 1.015T_n - 200$ ✓ ✓

b) How many weeks does it take for the number of fish in the lake to fall below 8000?

$T_{31} = 8044.9$

$T_{32} = 7965.6$

32 weeks ✓

if used $T_1 = 10\,000$ then

b) Describe what happens to fish farming operation in the long term.

In the long run there will be no fish. ✓

@ week 93 = 22 fish

week 94 = 0 fish. ✓

Question 5 **(4 marks – 3, 1)**

The numbers 5, x and 49 are the first three terms of the sequence defined by the first order recurrence relation $T_{n+1} = rT_n + 1$; $T_1 = 5$

a) Find the values of r and x, given that $x > 0$.

$T_2 = r(5)+1$

$x = 5r+1$

$T_3 = r(x)+1$

$49 = r(x)+1$

Working

$x = 16$ ✓ f.t.

$r = 3$ ✓✓

So if r = ~~XX~~

x = f.t. & reasonable

75 + <49

b) Find the fourth term in the sequence.

$T_4 = 3(T_3)+1$

$= 3(49)+1$

$T_4 = 148$ ✓ f.t.

END OF TEST