

Non-Calculator Section (No calculator or notes, formula sheet provided)

Time: 19 minutes Marks: 19 marks

1. [2, 2 marks]

Determine if X in each of the following is a Discrete Random variable. Give a reason for your choice.

a)

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$

✓
No. Prob ADD TO > 1

b) $P(X=x) = \frac{x}{6}$ where $X = 0, 1, 2, 3$

x	0	1	2	3
$P(X=x)$	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$

✓
YES (SEE TABLE)

2. [2, 2 marks]

Find the derivative of each of the following, but do not simplify.

a) $y = \sin 3x + \cos^3 x$

$$\frac{dy}{dx} = 3 \cos 3x - 3 \cos^2 x \sin x$$

✓ ✓

b) $y = e^{2x} \cdot \sin(2x-1)$

$$\frac{dy}{dx} = 2e^{2x} \sin(2x-1) + 2e^{2x} \cos(2x-1)$$

✓ ✓

3. [2, 2 marks]

Find

a) $\int 6 \cos\left(\frac{2x}{3}\right) dx$

$$= 9 \sin\left(\frac{2x}{3}\right) + C$$

✓✓

b) $\int (\sin^2 x \cdot \cos x) dx$

$$= \frac{1}{3} \sin^3 x + C$$

✓✓

① for no +C.

4. [1, 2 marks]

Consider the Discrete Random Variable X defined by the table

x	2	3	11
P(X=x)	$\frac{1}{3}$	$\frac{1}{2}$	a

a) Find the value of a, expressing your answer as a fraction.

$$a = 1 - \frac{5}{6}$$

$$= \frac{1}{6}$$

✓

b) Determine E[X].

$$E[X] = \frac{2}{3} + \frac{3}{2} + \frac{11}{6}$$

$$= 4$$

✓✓

5. [4 marks]

Find an equation for the tangent line to the curve $y = 3 \sin(2x) - \cos(2x)$ at $x = \frac{\pi}{4}$

$$\frac{dy}{dx} = 6 \cos(2x) + 2 \sin(2x)$$

✓

If $x = \frac{\pi}{4}$ $\frac{dy}{dx} = 2$ ✓

$$y = 3$$

✓

$$y = 2x + 3 - \frac{\pi}{2}$$

✓

Calculator Section (calculator and notes allowed, formula sheet provided)

Time: 36 minutes Marks: 36 marks

6. [1, 1, 1 marks]

If for the Discrete Random Variable X , $E(X) = 3$ and $\text{Var}(X) = 4$ determine

- a) $E(2X + 1)$ b) $\text{Var}(X - 5)$ c) $\text{SD}(-3X)$

7 ✓

4 ✓

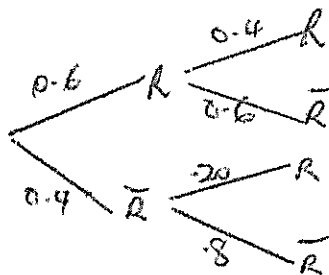
6 ✓

7. [4, 1, 2 marks]

Nick organised a camping weekend for 2 days in winter. The weather bureau says there is a 60% chance of rain on the first day. If it rains on the first day, there is a 40% chance of rain on the second day. If it doesn't rain on the first day, there is only a 20% chance of rain on the second day.

Let X be the number of days it rains on the weekend.

- a) Using a tree diagram or another method, determine the probability distribution for X .



x	0	1	2
$P(X=x)$	0.32	0.44	0.24

- b) Find the probability that it rains on at least one day.

$$P(x \geq 1) = 0.68 \quad \checkmark$$

- c) Find the probability that it rains on both days given it rains on at least one day.

$$P(x=2 | x \geq 1) = \frac{0.24}{0.68} \quad \checkmark$$

$$= \frac{6}{17} \text{ or } 0.3529 \quad \checkmark$$

8. [2, 2, 2, 2, 3 marks]

Laura sells smart phones. The table below shows the probability of a particular number of smart phones, X , being sold each month.

x	10	20	30	40	50
$P(X=x)$	0.18	0.35	0.25	0.12	0.10

- a) Determine the expected number of smart phone sales Laura makes each month.

$$E[X] = 1.8 + 7 + 7.5 + 4.8 + 5 \quad \checkmark$$

$$= 26.1 \text{ phones} \quad \checkmark$$

- b) Laura is paid a retainer of \$1500 a month and \$22 for each phone she sells. Find her expected monthly pay.

$$22(26.1) + 1500 = \$2074.20 \quad \checkmark \quad \checkmark$$

Laura takes delivery of 40 smart phones from her supplier. It is known that there is a 3% chance that a smart phone will have a defect.

- c) What is the probability that exactly one of the phones will have a defect?

$$X \sim \text{bin}(40, 0.03) \quad \checkmark$$

$$P(X=1) = 0.3658 \quad \checkmark$$

- d) Find the chance that more than one of the phones will have a defect.

$$P(X \geq 2) = 0.3385 \quad \checkmark$$

- e) What is the minimum number of smart phones that Laura should order so that the chance of no phones having a defect is less than 0.002?

$$P(X=0) = 0.97$$

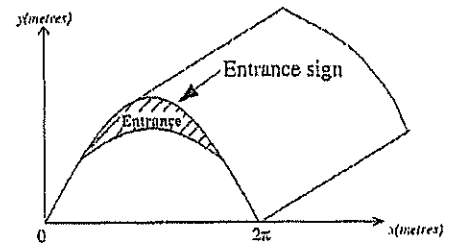
$$\text{If } (0.97)^n (0.03) < 0.002 \quad \checkmark$$

$$n > 204.03 \quad \checkmark$$

$$\text{ie } 205 \text{ phones.} \quad \checkmark$$

9. [2, 4 marks]

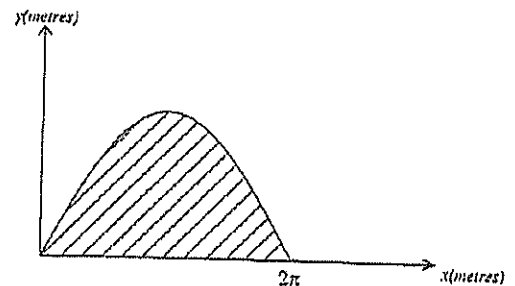
At the new Mindarie Funworld Theme Park there is a boat ride inside a tunnel. The entrance to the tunnel is shown in the diagram opposite. The shape of the tunnel itself is found from the rule $y = 2 \sin\left(\frac{x}{2}\right)$ whilst the lower edge of the sign is $y = \sin\left(\frac{x}{2}\right) + \frac{1}{2}$.



- a) Find the exact cross-sectional area of the whole tunnel entrance as shown in the second diagram.

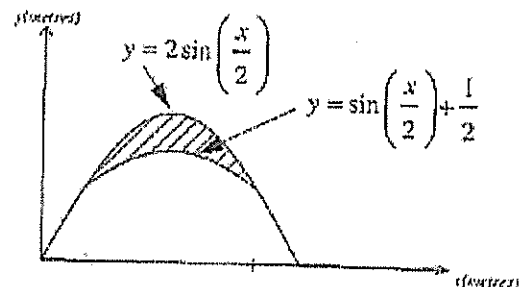
$$\int_0^{2\pi} 2 \sin\left(\frac{x}{2}\right) dx \quad \checkmark$$

$$= 8 \text{ m}^2 \quad \checkmark$$



- b) Accurate to 2 decimal places, find the cross sectional area of the entrance sign as shown in the second diagram.

Sufficient working must be shown to receive full marks.



$$2 \sin\left(\frac{x}{2}\right) = \sin\left(\frac{x}{2}\right) + \frac{1}{2}$$

$$\sin\left(\frac{x}{2}\right) = \frac{1}{2}$$

$$\frac{x}{2} = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

$$x = \frac{\pi}{3} \text{ or } \frac{5\pi}{3} \quad (\text{or } 5.236 \text{ or } 1.047)$$

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} \left[2 \sin\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) - \frac{1}{2} \right] dx \quad \checkmark$$

$$= 1.37 \text{ m}^2 \quad \checkmark$$

10. [3, 1, 1, 2 marks]

Given that a discrete random variable is binomially distributed such that $X \sim \text{bin}(n, 0.8)$, and has a standard deviation of 2, determine

a) the expected value $E[X]$

$$\begin{aligned}np(1-p) &= 2^2 \\(0.8)(0.2)p &= 4 \quad \checkmark \\0.16n &= 4 \\n &= 25 \quad \checkmark \\\therefore E[X] &= 20 \quad \checkmark\end{aligned}$$

Use your value of n to determine, correct to 4 decimal places,

b) $P(X = 20)$ $0.1960 \quad \checkmark$

c) $P(X \leq 15)$ $0.0173 \quad \checkmark$

d) $P(X > 18 \mid X < 22)$ $\frac{0.5460}{0.7660} \quad \checkmark$
 $= 0.7127 \quad \checkmark$

11. [2 marks]

Assume a student does a 20-question multiple choice test. Each question has 5 possible answers, only one of which is correct. In order to pass, she must achieve at least 12 correct answers.

She knows the answers to the first 5 questions, but guesses the rest. What is the probability she will pass the test?

$$\begin{aligned}X &\sim \text{bin}(15, 0.2) \quad \checkmark \\P(X \geq 7) &= 0.0181 \quad \checkmark\end{aligned}$$