SHENTON COLLEGE

Examination Semester One 2018
Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 3
Section One
(Calculator-free)

Your name _____

Time allowed for this section
Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section
To be provided by the supervisor
Question/answer booklet for Section One.
Formula sheet.

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

| Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
			Total	150	100

Section One: Calculator-free 35% (53 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (6 marks)

The graph of $y = f(x)$ is shown below, where f is defined by $f(x) = \dfrac{5}{1 + \sqrt{x - 3}}$.
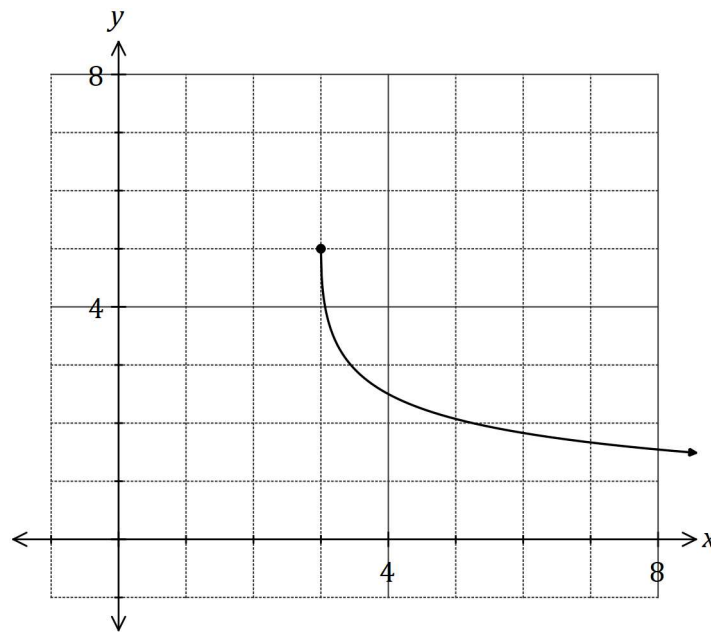
(a) Sketch the graph of $y = f^{-1}(x)$ on the same axes. (2 marks)

(b) Determine the defining rule for $y = f^{-1}(x)$ and state its domain. (4 marks)

Question 2   **(7 marks)**

Consider $f(z) = 5z^3 + 2z^2 + 10z + 4$, where z is a complex number.

(a)   Determine, with reasons, which of the following are factors of $f(z)$.

         (i) $z - 2$.   (2 marks)
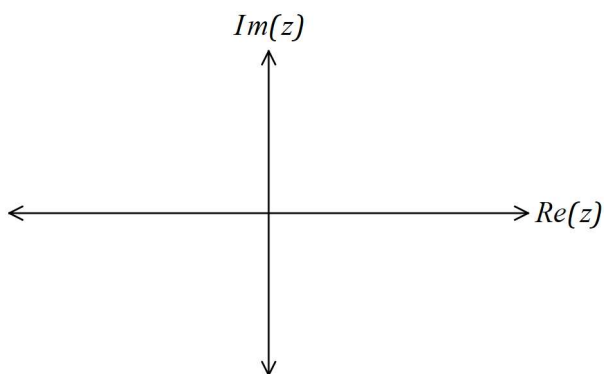
         (ii) $z - \sqrt{2}i$.   (2 marks)

(b)   Solve the equation $f(z) = 0$.   (3 marks)

Question 3 **(7 marks)**

(a) Locate the roots of the complex equation $z^5 - 1 = 0$ in the Argand plane below. (3 marks)

(b) State the sum of all the roots of the complex equation $z^5 - 1 = 0$. (1 mark)

(c) Let u be any 5th root of unity, where $\text{Im}\, u \neq 0$.

Show that $(1 + u)^2(1 + u^3) = 1 + u + u^4$. (3 marks)

Question 4 **(6 marks)**

(a)   Solve this system of equations. (3 marks)

$$x + y + 2z = 1$$
$$4x + y - z = 7$$
$$3x - y + z = 14$$

(b)   Determine the value of constant a so that the following system of equations does not have a unique solution and give a brief geometric interpretation of the system of equations with this value. (3 marks)

$$x + y + 2z = 1$$
$$4x + y - z = 7$$
$$ax - y + z = 14$$

Question 5 **(10 marks)**

The points A, B and C have position vectors $(2, 1, -1)$, $(-1, b, -2)$ and $(1, -1, 0)$ respectively.

(a)   Determine the vector equation for the line through A and C.   (2 marks)

(b)   Determine, in terms of b, the Cartesian equation of the plane containing A, B and C.

(5 marks)

(c)   The line with equation $\mathbf{r} = (-2, -2, 1) + \mu(p, -2, 7)$ is perpendicular to the plane containing A, B and C. Determine the values of the constants b and p.   (3 marks)
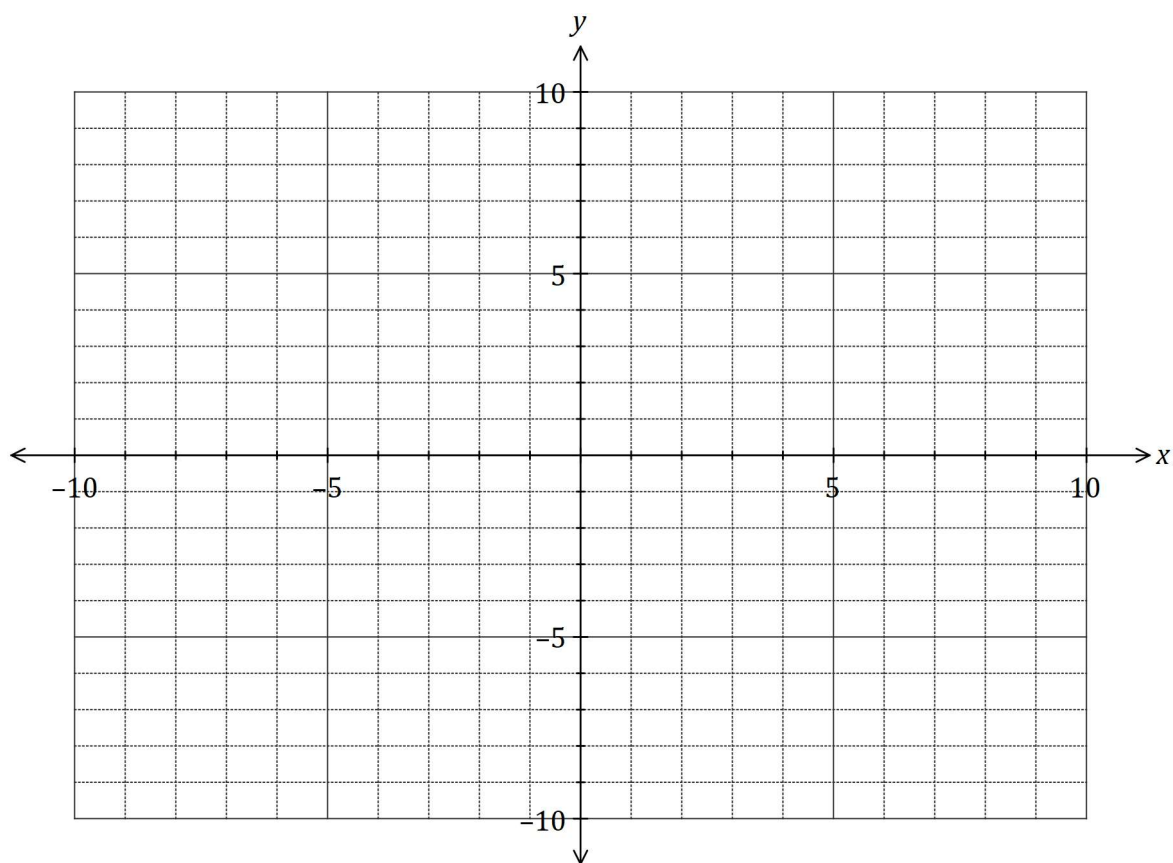
Question 6 (5 marks)

The complex numbers u and v satisfy the equations $u - v = 2i$ and $uv = 10$.

Solve the equations for u and v, giving your solution(s) in the form $x + yi$, where x and y are real.

Question 7 (6 marks)

The graph of $y = \dfrac{2x^2 - 18}{(x - 2)(x + 4)}$ has no stationary points. Sketch the graph.

Question 8   **(6 marks)**

A function is defined by $f(x) = \dfrac{5-x}{(5x+8)(2x-3)}$.

(a)   State the natural domain of $f(x)$.   (1 mark)

(b)   State the equations of all asymptotes of the graph of $y = x \cdot f(x)$.   (2 marks)

(c)   The graph of $y = \dfrac{1}{f(x)}$ has an asymptote with equation $y = ax + b$. Determine the values of the constants a and b.   (3 marks)