



Calculator Free
Pascal's Triangle, Binomial Expansion,
Combinations

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 1, 1, 1, 1, 2 = 7 marks]

There are four coloured pens on a desk: red, green, blue and pink.

How many ways can you choose:

- (a) One pen from the desk?
- (b) Two pens from the desk?
- (c) Three pens from the desk?
- (d) Four pens from the desk?
- (e) No pens from the desk?
- (f) Use the above results to provide the simplified expansion of $(m + y)^4$

- (e) Lucy is getting married and wants to choose three of her eight best friends to be her bridesmaids. How many choices does she have?

- (f) The Principal of a school has narrowed down the decision for Head Boy and Head Girl to 3 male students and 3 female students. How many choices does she have to form the Head prefect pair?

- (g) Little Lucas is staying overnight at his grandparents' house. He is allowed to pack 2 books and 1 toy car to take with him. If he has 9 books and 7 toy cars to choose from, how many does he have?

- (h) Mya is choosing which five subjects she wants to study in Year 11. She can either choose three from the humanities and two from the sciences, or three from the sciences and two from the humanities. If there are 8 humanities subject and 6 science subjects on offer at her school, how many choices does she have?

Question Three: [1, 2, 3, 1, 2 =9 marks]

Determine the value of the unknown in each of the following equations:

(a) $\binom{n}{4} = \frac{15!}{11! \times 4!}$

(b) ${}^8C_x = 56$

(c) $\binom{12}{3} = \frac{12!}{9! \times 3!} = y$

(d) $\binom{n}{n-1} = 24$

(e) ${}^9C_r = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}$

Question Four: [5 marks]

By using a suitable binomial expansion, calculate $(1.5)^5$.

Question Five: [7 marks]

Solve for n in the following equation showing full algebraic working.

$$\binom{n}{2} = \binom{2n}{1}; n \neq 0$$



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Question One: [1, 1, 1, 1, 1, 2 = 7 marks]

There are four coloured pens on a desk: red, green, blue and pink.

How many ways can you choose:

- (a) One pen from the desk?

$${}^4C_1 = 4 \quad \checkmark$$

- (b) Two pens from the desk?

$${}^4C_2 = 6 \quad \checkmark$$

- (c) Three pens from the desk?

$${}^4C_3 = 4 \quad \checkmark$$

- (d) Four pens from the desk?

$${}^4C_4 = 1 \quad \checkmark$$

- (e) No pens from the desk?

$${}^4C_0 = 1 \quad \checkmark$$

- (f) Use the above results to provide the simplified expansion of $(m + y)^4$

$$m^4 + 4m^3y + 6m^2y^2 + 4my^3 + y^4 \quad \checkmark \quad \checkmark$$

- (e) Lucy is getting married and wants to choose three of her eight best friends to be her bridesmaids. How many choices does she have?

$$\binom{8}{3} = 56 \text{ choices} \quad \checkmark$$

- (f) The Principal of a school has narrowed down the decision for Head Boy and Head Girl to 3 male students and 3 female students. How many choices does she have to form the Head prefect pair?

$$\binom{3}{1} \times \binom{3}{1} = 3 \times 3 = 9 \text{ choices}$$

$\checkmark \qquad \qquad \checkmark$

- (g) Little Lucas is staying overnight at his grandparents' house. He is allowed to pack 2 books and 1 toy car to take with him. If he has 9 books and 7 toy cars to choose from, how many does he have?

$$\binom{9}{2} \times \binom{7}{1} = 36 \times 7 = 252 \text{ choices}$$

$\checkmark \qquad \qquad \checkmark$

- (h) Mya is choosing which five subjects she wants to study in Year 11. She can either choose three from the humanities and two from the sciences, or three from the sciences and two from the humanities. If there are 8 humanities subject and 6 science subjects on offer at her school, how many choices does she have?

$$\binom{8}{3} \times \binom{6}{2} + \binom{8}{2} \times \binom{6}{3} = 56 \times 15 + 28 \times 20 = 840 + 560 = 1400 \text{ choices}$$

$\checkmark \qquad \checkmark \qquad \qquad \checkmark$

Question Three: [1, 2, 3, 1, 2 =9 marks]

Determine the value of the unknown in each of the following equations:

(a) $\binom{n}{4} = \frac{15!}{11! \times 4!}$ $n = 15$ ✓

(b) ${}^8C_x = 56$ $x = 3$ $x = 5$ ✓ ✓

(c) $\binom{12}{3} = \frac{12!}{9! \times 3!} = y$ $\frac{12 \times 11 \times 10}{3 \times 2 \times 1} = \frac{440}{2} = 220$ ✓ ✓

(d) $\binom{n}{n-1} = 24$ $n = 24$ ✓

(e) ${}^9C_r = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}$ $\frac{9!}{5!4!}$ ✓
 $r = 4$ ✓

Question Four: [5 marks]

By using a suitable binomial expansion, calculate $(1.5)^5$.

$$\begin{aligned} & \left(1 + \frac{1}{2}\right)^5 \checkmark \\ &= 1^5 + 5(1)^4\left(\frac{1}{2}\right) + 10(1)^3\left(\frac{1}{2}\right)^2 + 10(1)^2\left(\frac{1}{2}\right)^3 + 5(1)\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5 \checkmark \\ &= 1 + \frac{5}{2} + \frac{10}{4} + \frac{10}{8} + \frac{5}{16} + \frac{1}{32} \checkmark \\ &= \frac{32 + 80 + 80 + 40 + 10 + 1}{32} \checkmark \\ &= \frac{243}{32} \checkmark \end{aligned}$$

Question Five: [7 marks]

Solve for n in the following equation showing full algebraic working.

$$\binom{n}{2} = \binom{2n}{1}; n \neq 0$$

$$\frac{n!}{(n-2)!2!} = 2n$$

$$\frac{n(n-1)}{2} = 2n$$

$$n(n-1) = 4n$$

$$n^2 - n - 4n = 0$$

$$n^2 - 5n = 0$$

$$n(n-5) = 0$$

$$n = 5$$