



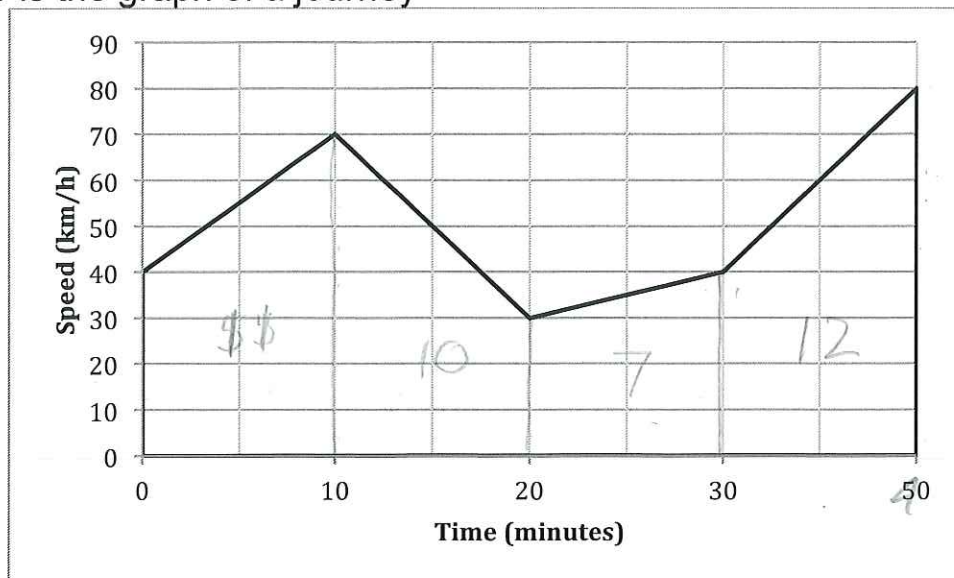
Margaret River Senior High School

Year 12 Investigation Mathematics Methods

Area Under A Curve – Validation Test

1. Below is the graph of a journey

[6 marks]



a) What is the area of each square in the grid and what does it represent?

$$10 \times 5 = 50 \text{ km/h min}$$

$$= 5000 \text{ m/h min}$$

$$= 833.3 \text{ m}$$

b) What is the area under the curve?

400 squares

or 2000

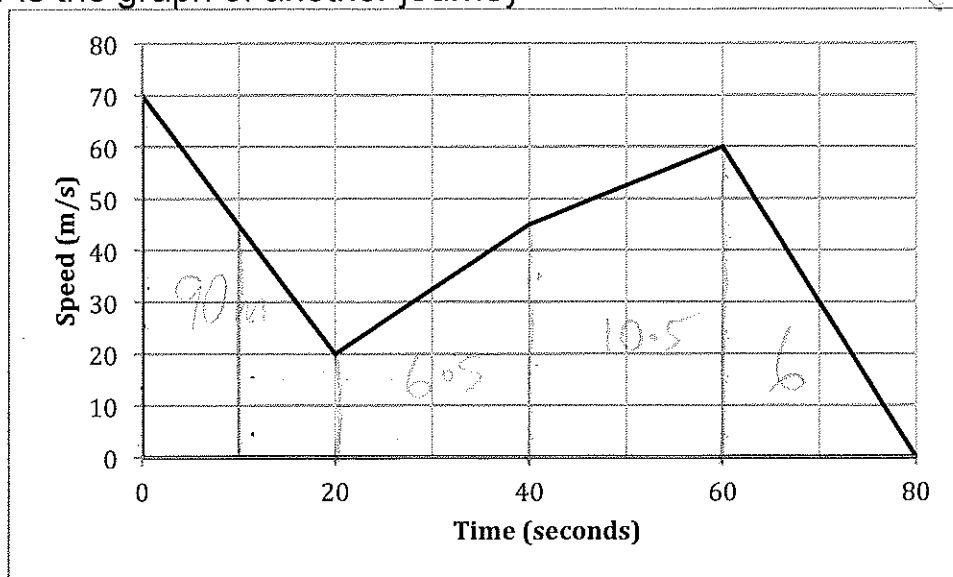
c) What is the significance of your answer above to the journey?

33333m travelled

$33\frac{1}{3} \text{ km.}$

2. Below is the graph of another journey

(8 marks)



How far did the person travel:

a) in the first 10 seconds

$$11.5 \div 2$$

$$50.25 \times 100 = 5025 \text{ m}$$



b) between 10 and 35 seconds

$$\begin{matrix} 6.5 & 3.25 \\ 6.5 & 6.5 \end{matrix}$$

$$9.75 \times 100 = 975 \text{ m}$$



c) during the first 50 seconds

$$32 \times 100 = 3200 \text{ m}$$



d) During what time period did the person have the greatest acceleration (positive or negative - explain your answer)

60 → 80 seconds



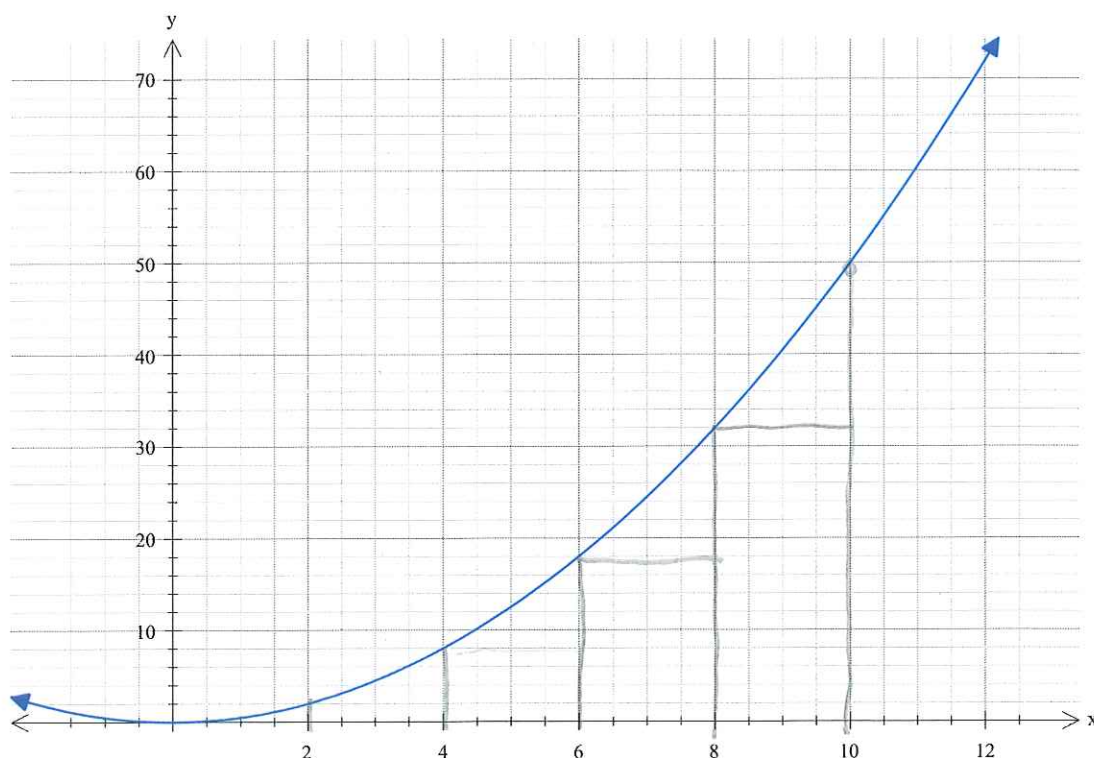
e) When did the person have the greatest velocity?

0 seconds



3. Below is a graph of $y = \frac{x^2}{2}$

[7 marks]



Approximate the area between the curve and the x-axis from 0 to 10, using intervals of 2, and then:

- a) find the sum of the left hand rectangles.

120 sq

		Area
0-2	0	0
2-4	2	4
4-6	8	16
6-8	18	36
8-10	32	64

- b) find the sum of the right hand rectangles

220 sq

2	4
8	16
18	36
32	64
50	100

- c) Find the approximate area between the curve and the x-axis from 0 to 10.

$$\frac{220 + 120}{2} = 170 \text{ sq}$$

- d) What would be the approximate area between the curve and the x-axis from 2 to 10?

$$\frac{120 + 216}{2} = 168 \text{ sq}$$

[11]

4. As part of this investigation we went on a bus trip and discovered (were reminded) that the area under a speed graph gave us the distance travelled.

Thinking about the relationship between speed and distance, answer the following questions.

A bus is travelling with a speed given by $v = 4t^3 + 6t^2 + 10$ m/s $0 \leq t \leq 100$ s

- a) Does the bus ever travel with a negative speed? Justify your answer.

No since whole graph above zero ✓

- b) How far would the bus travel during the 100s trip? ✓✓

$$s = t^4 + 2t^3 + 10t + c$$

$$s(0) = 0$$

$$s(100) = 10\,300\,100$$

102 001 Kms.

- c) How far would bus travel between $t = 40$ and $t = 100$ s

$$s(40) = 2688\,400$$

Distance 9931206 , kms ✓✓

99312600 ms.

5. Without counting squares, find the area under the curve $y = \frac{x^2}{2}$;

- a) between the interval $0 \leq x \leq 10$?

$$\int y = \frac{x^3}{6}$$

$$A(0) = 0$$

$$A(10) = 1000/6$$

$$166.7u^2$$

- b) between the interval $4 \leq x \leq 10$?

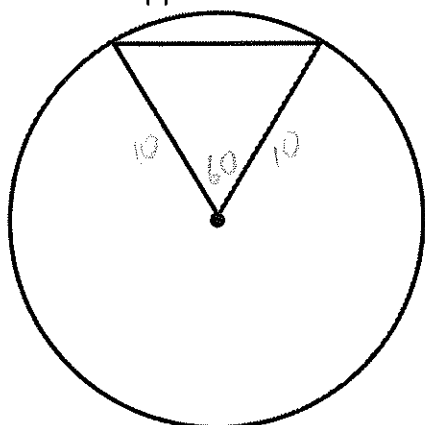
$$1000/6 - \frac{64}{6} = \frac{936}{6}$$

$$= 156.00$$

✓✓

6. A similar approach to the left hand rectangles and right hand rectangles can be used to find the approximate area of a circle.

- a) If we consider a circle of radius 10cm. We could break it up into 6 internal triangles and 6 external triangles. Find the area of each triangle and hence find an approximation for internal area of the circle.

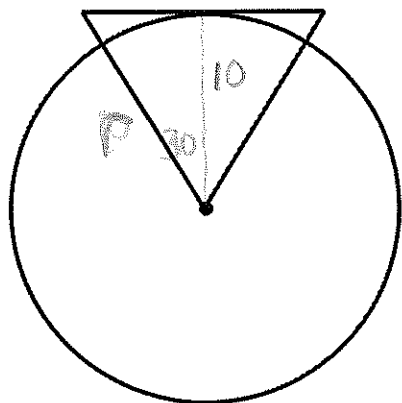


$$\text{Area} = 6 \times \frac{1}{2} \times 10^2 \times \sin 60$$

$$= 259.8 \text{ cm}^2$$

✓✓

- b) Now break it up into 6 external triangles. Find the area of each triangle and hence find an approximation for external area of the circle.



$$\cos 30 = \frac{10}{r}$$

$$r = 11.54$$

$$\text{Area} = 6 \times \frac{1}{2} \times 11.54^2 \times \sin 60$$

$$= 346.4 \text{ cm}^2$$

✓✓

- c) By averaging these results, find a good approximation to the area of the circle, radius 10cm.

$$303.01 \text{ cm}^2$$

✓

- d) How does this compare to the exact area of the circle?

$$A_1 = \pi \times 10^2$$

$$= 314 \text{ cm}^2$$

✓✓

It is close but smaller by
11 cm²

7. Repeat question 6 only this time use 12 triangles instead of 6.

[6 marks]

$$\begin{aligned}\text{Internal Area} &= 12 \times \frac{1}{2} \times 10^2 \times \sin 30 \\ &= 300 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{External Area} &= 12 \times \frac{1}{2} \times \left(\frac{10}{\cos 15} \right)^2 \times \sin 30 \\ &= 321.54\end{aligned}$$

$$\therefore \text{Average} = 310.8 \text{ cm}^2$$

this is much closer