

Broome SHS 2016
Year 12 Mathematics Methods
Test 6
Sampling and Confidence Intervals

Name:

Score:

out of

18

Non-Calculator Section (No calculator nor notes, formula sheet is provided)

Time: 20 minutes

Marks: 18 marks

/50

1. [3 marks]

Ornithologists erected a bird net in an area of a forest catching 240 birds. They tagged these birds and then released them. A couple of months later they erected their net in another area and caught 180 birds. Of these 30 were tagged. What would be a good prediction of the total number of birds in the forest?

$$\frac{30}{180} = \frac{240}{x}$$

$$x = \frac{240 \times 180}{30}$$

$$x = 1440 \text{ birds.}$$

✓

✓

✓

2. [4 marks]

In a school of 600 students, the following table shows the methods students get to school.

Method	Bus	Car	Walk	Other	
Frequency	302	200	66	32	600

A stratified random sample of 30 students is to be chosen so each group is represented in their correct proportion.

Determine how many of each group should be chosen for the sample.

$$\frac{30}{600} = \frac{1}{20}$$

$$\frac{1}{20} \times 302$$

$$= 15.1$$

15 bus

✓

$$\frac{1}{20} \times 200$$

$$= 10$$

10 Car

✓

$$\frac{1}{20} \times 66$$

$$= 3.3$$

3 walk

✓

$$\frac{1}{20} \times 32$$

$$= 1.6$$

2 others

✓

3. [1,1 = 2 marks]

The Mathsville Shire Council wishes to determine the opinion of its ratepayers on the new plans for the Euler St development. Identify possible sources of bias inherent in these surveys.

- a) The Council sets up a table in Euler St between 8am and 9am every day for 1 week and stops every 8th person who passes by and gets them to complete the opinion survey.

Same time everyday not random enough.

✓

- b) The Council selects 100 of their ratepayers randomly from their list and mails out the survey with a reply paid envelope for the return.

Self selecting respondents many may choose not to participate

✓

4. [1,3,2 = 4 marks]

A sample of 900 people found that 180 were able to twitch their nose without moving any other face muscles.

- a) What is the sample proportion (\hat{p}) for people able to twitch their nose?

$$\frac{180}{900} = \frac{1}{5}$$

✓

- b) If we were to take many such samples, what would you expect the standard deviation of the sample proportions to equal?

$$s = \sqrt{\frac{\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)}{900}}$$

$$= \sqrt{\frac{4}{25 \cdot 900}}$$

$$= \frac{2}{5 \cdot 30} = \frac{1}{75}$$

✓

✓

✓

- c) Given that 95.5% of the scores are within 2 standard deviations from the mean, what would be the 95.5% confidence interval for the sample proportion of people who could twitch their nose?

$$\frac{1}{5} - \frac{2}{75} \leq p \leq \frac{1}{5} + \frac{2}{75}$$

✓

$$\frac{13}{75} \leq p \leq \frac{17}{75}$$

✓

5. [1,1,3 = 5 marks]

To estimate the proportion of Perth's population that has been to an AFL match this year a sample of n people were surveyed. A confidence interval based on 2 standard deviations ($z = 2$), yielded the confidence interval: $0.12 \leq p \leq 0.28$.

a) State the margin of error.

$$\frac{0.28 - 0.12}{2} = 0.08.$$

b) State the sample mean \hat{p} .

$$\hat{p} = \frac{0.28 + 0.12}{2} = 0.2$$

c) Show working to demonstrate that there must have been 100 people in the sample.

$$0.08 = 2 \sqrt{\frac{(0.2)(0.8)}{n}}$$

$$0.04 = \sqrt{\frac{(0.2)(0.8)}{n}}$$

$$0.04 = \sqrt{\frac{0.16}{n}}$$

$$0.0016 = \frac{0.16}{n}$$

$$n = \frac{0.16}{0.0016} = 100 \text{ as required.}$$

Test 6
Sampling and Confidence Intervals

Name:

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out of

32

Calculator Section (Calculators and 1 page (A4) of notes permitted, formula is sheet provided)

Time: 35 minutes

Marks: 32 marks

6. [1,2,2,1 = 6 marks]

In a random sample of 50 people, 18 indicated that they had used public transport in the last year sometime.

a) State the sample proportion, \hat{p} . $\hat{p} = \frac{18}{50} = 0.36$ ✓

b) Determine the sample standard deviation.

$$s = \sqrt{\frac{(0.36)(0.64)}{50}} = 0.067882 \quad \checkmark \checkmark$$

c) Determine a 95% confidence interval for the population proportion p .

$$Z = 1.96 \quad E = 1.96(0.067882) = 0.133$$

$$\hat{p} \pm 0.133$$

$$0.227 < p < 0.493 \quad \checkmark \checkmark$$

d) Describe what happens to the confidence interval width if we increased our level of confidence to 99%.

$$99\% \quad 0.185 < p < 0.535$$

99% Confidence Interval is much wider than the 95% confidence interval. ✓

7. [3,3,2,2 = 10 marks]

It is known that 60% of Western Australian Public school teachers earn over \$80 000.

- a) A sample of 300 WA teachers is surveyed about their salary.
Find the probability that more than 0.57 of the teachers in the sample had a salary more than \$80 000.

$$s = \sqrt{\frac{0.57(0.43)}{300}} = 0.028583$$
$$\hat{p} \sim N(0.6, 0.028583^2)$$
$$P(\hat{p} > 0.57) = 0.853$$

- b) Give a range, using the 90% level of confidence.

$$p \pm 1.645(0.028583)$$
$$0.553 < p < 0.647$$

Use the 90% confidence interval to compare and discuss the following samples.

- c) Algebra SHS staff had 27 out of 43 staff with a salary over \$80 000.

$$\hat{p} = \frac{27}{43} = 0.627$$

Sample mean within above interval so proportion of ASHS is not significantly different.

- d) Geometry SHS staff random survey showed 46 out of 94 staff had a salary over \$80 000.

$$\hat{p} = \frac{46}{94} = 0.489$$

90% Confidence interval

$$0.405 < \hat{p} < 0.574$$

Mean outside interval and only a slight overlap between confidence intervals. Unlikely the Geom. SHS is from same dist.

8. [4 marks]

It is thought that about 68% of all Year 12 students have their driver's licence by the time they leave high school. How large a sample would be needed to establish this to within a margin of error of 5% at the 95% confidence level?

$$1.96 \quad 0.05 = 1.96 \sqrt{\frac{(0.68)(0.32)}{n}} \quad \checkmark \checkmark$$

$$n = 334.37 \quad \checkmark$$

Need a sample of 335 or larger. \checkmark

9. [2,2,3 = 7 marks]

A random sample of 75 people were asked "Do you prefer AFL to soccer".
From this the confidence interval $0.763 \leq x \leq 0.917$ was established at the x% level.

a) How many agreed with the question in the survey?

$$\hat{p} = \frac{0.763 + 0.917}{2} = 0.84 = 63 \text{ people} \quad \checkmark \checkmark$$

b) Find the standard deviation of the sample.

$$\sqrt{\frac{(0.84)(0.16)}{75}} = 0.042332 \quad \checkmark \checkmark$$

c) Determine the confidence level, x%, of the survey.

$$N(0.84, 0.042332^2) \quad \checkmark$$

$$P(0.763 < \hat{p} < 0.917) = 0.931 \quad \checkmark$$

$$\text{Confidence level} = 93.1\% \quad \checkmark$$

10. [3,2 = 5 marks]

The WA Health Department has calculated that 18% of Western Australians had a flu shot this year.

- a) A new sample of 150 was taken as to who had the flu shot this year and X = number of who had the flu shot was recorded. Give a range using the 95% confidence interval, within you would expect X to be in.

$$s = \sqrt{\frac{(0.18)(0.82)}{150}} \quad \checkmark$$
$$= 0.03137$$

$$1.96 \cdot$$
$$0.18 \pm 1.96(0.03137)$$

$$0.18 \pm 0.061 \quad \checkmark$$

$$0.119 < p < 0.241 \quad \checkmark$$

- b) If 8 surveys were taken, each of size 150, using the 95% confidence level determine the probability that less than seven of the samples would include the true value of $p = 0.18$.

$$T \sim \text{Bin}(8, 0.95) \quad \checkmark$$

$$P(T < 7) = 0.0572 \quad \checkmark$$