



Calculator Free
The Unit Circle, Radian Measure and
Trigonometric Equations

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 2 = 3 marks]

- (a) Explain what relationship radian measure represents.
- (b) Hence or otherwise explain what 1 radian looks like on a circle with radius 5 cm. Draw a diagram to assist your explanation.

Question Two: [1, 2, 1, 2 = 6 marks]

Express each of these angles as radians, in terms of π , leaving your answers as exact values.

- (a) 180° (c) 60°
- (b) 210° (d) 330°

Question Three: [1, 1, 2, 2 = 6 marks]

Express each of these angles in degrees, leaving answers in terms of π where necessary:

(a) 2π

(c) 2^R

(b) $\frac{\pi}{4}$

(d) $\frac{7\pi}{10}$

Question Four: [1, 1, 1, 1, 2, 2 = 8 marks]

Use the unit circle diagram below to approximate the value of:

(a) $\sin 115^\circ$

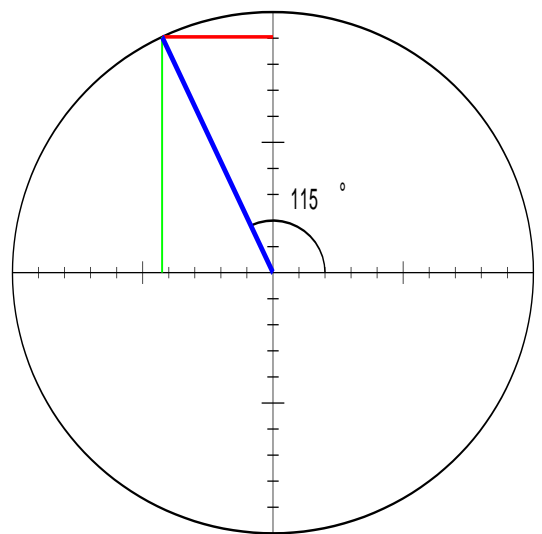
(b) $\cos 115^\circ$

(c) $\sin 65^\circ$

(d) $\cos 295^\circ$

(e) $\cos 25^\circ$

(f) $\tan 245^\circ$



Question Five: [1, 1, 1, 1, 1, 3 = 8 marks]

Express each of the following as a simplified exact value:

(a) $\sin 45^\circ$

(b) $\cos \frac{\pi}{6}$

(c) $\tan \frac{5\pi}{6}$

(d) $\cos \frac{7\pi}{4}$

(e) $\sin \frac{8\pi}{3}$

(f) $\sin \frac{7\pi}{6} + \tan \frac{\pi}{3}$

Question Six: [2, 2, 3, 3, 4 = 14 marks]

Solve each of the following trigonometric equations over the given domain.

(a) $\sin \theta = -\frac{1}{2}; 0^\circ \leq \theta \leq 360^\circ$

(b) $\cos \theta = \frac{1}{2}; -\pi \leq \theta \leq \pi$

(c) $3 \tan \theta = \sqrt{3}; 0 \leq \theta \leq 2\pi$

(d) $\cos 2\theta = -\frac{1}{\sqrt{2}}; 0 \leq \theta \leq 2\pi$

(e) $(\sin \theta)^2 = \frac{3}{4}; -\pi \leq \theta \leq 2\pi$



SOLUTIONS
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Question One: [1, 2 = 3 marks]

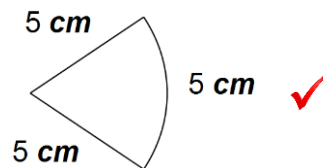
- (a) Explain what relationship radian measure represents.

The ratio between the arc length and the radius.



- (b) Hence or otherwise explain what 1 radian looks like on a circle with radius 5 cm. Draw a diagram to assist your explanation.

The sector drawn has radius 5cm and an arc length of 5 cm if the angle measure is 1 radian.



Question Two: [1, 2, 1, 2 = 6 marks]

Express each of these angles as radians, in terms of π , leaving your answers as exact values.

- (a) 180°

π ✓

- (c) 60°

$= \frac{\pi}{3}$ ✓

- (b) 210°

$= \frac{210}{180} \times \pi$ ✓
 $= \frac{7\pi}{6}$ ✓

- (d) 310°

$= \frac{310}{180} \times \pi$ ✓
 $= \frac{11\pi}{6}$ ✓

Question Three: [1, 1, 2, 2 = 6 marks]

Express each of these angles in degrees, leaving answers in terms of π where necessary:

(a) 2π

360° ✓

(c) 2^R

$$= \frac{2}{\pi} \times 180 \quad \checkmark$$

$$= \frac{360}{\pi} \quad \checkmark$$

(b) $\frac{\pi}{4}$

45° ✓

(d) $\frac{7\pi}{10}$

$$= \frac{7\pi}{10} \times \frac{180}{\pi} \quad \checkmark$$

$$= 126^\circ \quad \checkmark$$

Question Four: [1, 1, 1, 1, 2, 2 = 8 marks]

Use the unit circle diagram below to approximate the value of:

(a) $\sin 115^\circ$
 ≈ 0.9 ✓

(b) $\cos 115^\circ$
 ≈ -0.42 ✓

(c) $\sin 65^\circ$
 ≈ 0.9 ✓

(d) $\cos 295^\circ$
 ≈ 0.42 ✓

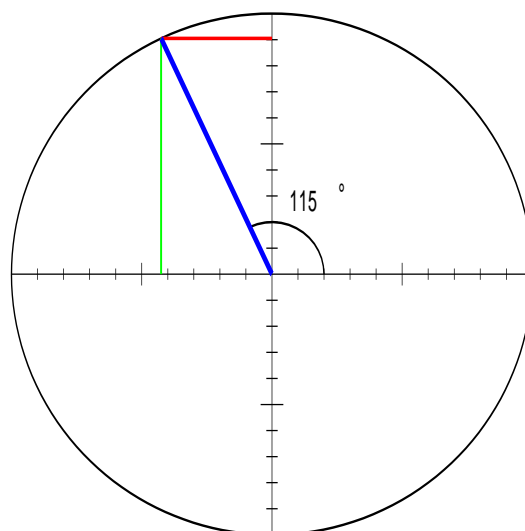
(e) $\cos 25^\circ$
 ≈ 0.9 ✓ ✓

(f) $\tan 245^\circ$

$$= \frac{\sin 245}{\cos 245} \quad \checkmark$$

$$= \frac{-0.9}{-0.42}$$

$$= \frac{90}{42} \quad \checkmark$$



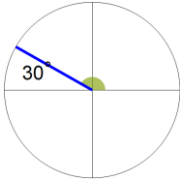
Question Five: [1, 1, 1, 1, 1, 3 = 8 marks]

Express each of the following as a simplified exact value:

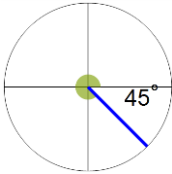
(a) $\sin 45^\circ$
 $= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ✓

(b) $\cos \frac{\pi}{6}$
 $= \frac{\sqrt{3}}{2}$ ✓

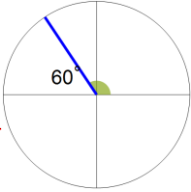
(c) $\tan \frac{5\pi}{6}$
 $= \frac{-1}{\sqrt{3}}$ ✓



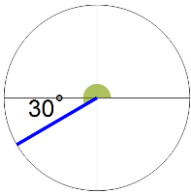
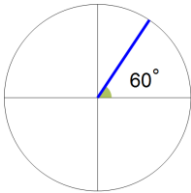
(d) $\cos \frac{7\pi}{4}$
 $= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ✓



(e) $\sin \frac{8\pi}{3}$
 $= \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ ✓



(f) $\sin \frac{7\pi}{6} + \tan \frac{\pi}{3}$
 $= \frac{-1}{2} + \sqrt{3}$ ✓
 $= \frac{2\sqrt{3}-1}{2}$ ✓

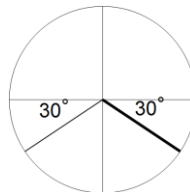



Question Six: [2, 2, 3, 3, 4 = 14 marks]

Solve each of the following trigonometric equations over the given domain.

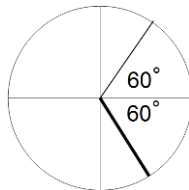
(a) $\sin \theta = -\frac{1}{2}; 0^\circ \leq \theta \leq 360^\circ$

$\theta = 210^\circ, 330^\circ$



(b) $\cos \theta = \frac{1}{2}; -\pi \leq \theta \leq \pi$

$\theta = -\frac{\pi}{3}, \frac{\pi}{3}$

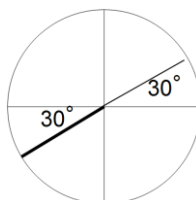


(c) $3 \tan \theta = \sqrt{3}; 0 \leq \theta \leq 2\pi$

$\tan \theta = \frac{\sqrt{3}}{3}$

$\tan \theta = \frac{1}{\sqrt{3}}$ ✓

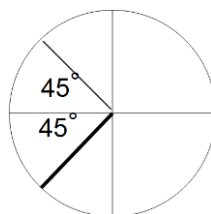
$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$



(d) $\cos 2\theta = -\frac{1}{\sqrt{2}}; 0 \leq \theta \leq 2\pi$

$2\theta = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$ ✓ ✓

$\theta = \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$ ✓



(e) $(\sin \theta)^2 = \frac{3}{4}; -\pi \leq \theta \leq 2\pi$

$\sin \theta = \pm \frac{\sqrt{3}}{2}$ ✓

$\theta = \pm \frac{\pi}{3}, \pm \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

