education equals

Calculator Free
Trigonometric Functions and Trigonometric Identities

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 3, 3 =8 marks]

Describe the transformations that have transformed $f(x)$ to $g(x)$ in the situation below.

(a) $f(x) = \sin x$ $g(x) = 3\sin(2x)$

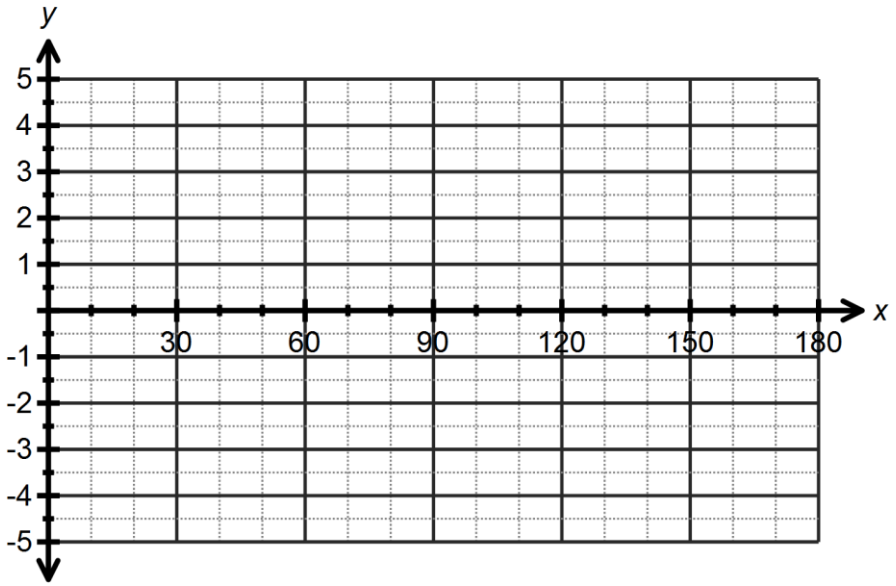
(b) $f(x) = -\cos x$ $g(x) = \cos(x - \frac{\pi}{4}) + 1$

(c) $f(x) = \tan x$ $g(x) = \tan(\frac{x}{2} + \frac{\pi}{6})$

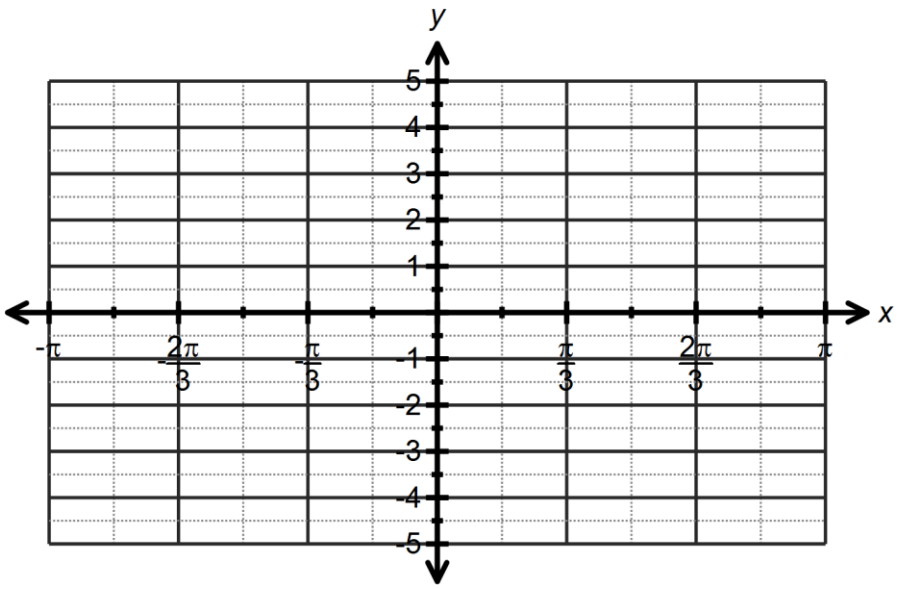
Question Two: [2, 3, 3, 4 = 12 marks]

Sketch each of the following functions on the axes below:

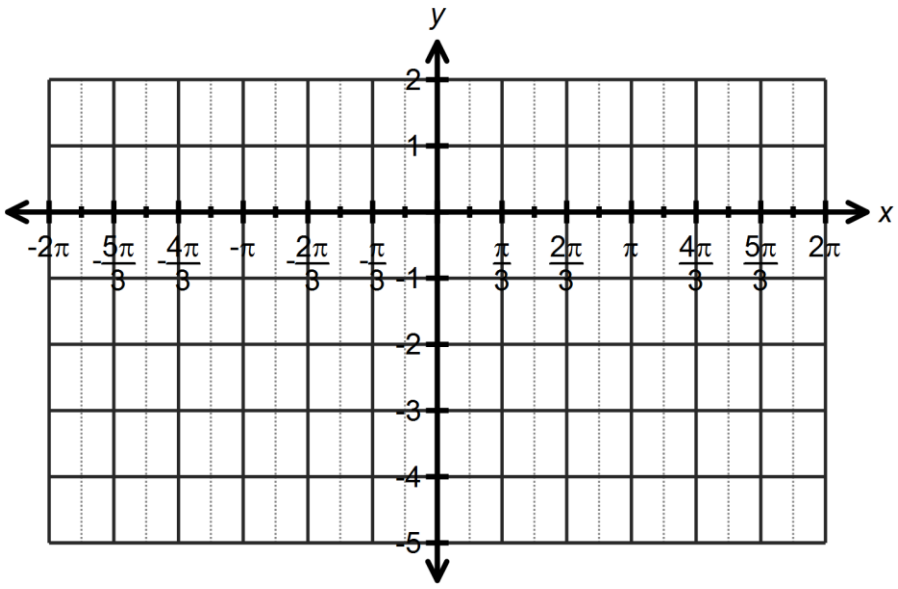
(a) $y = \tan 2x$



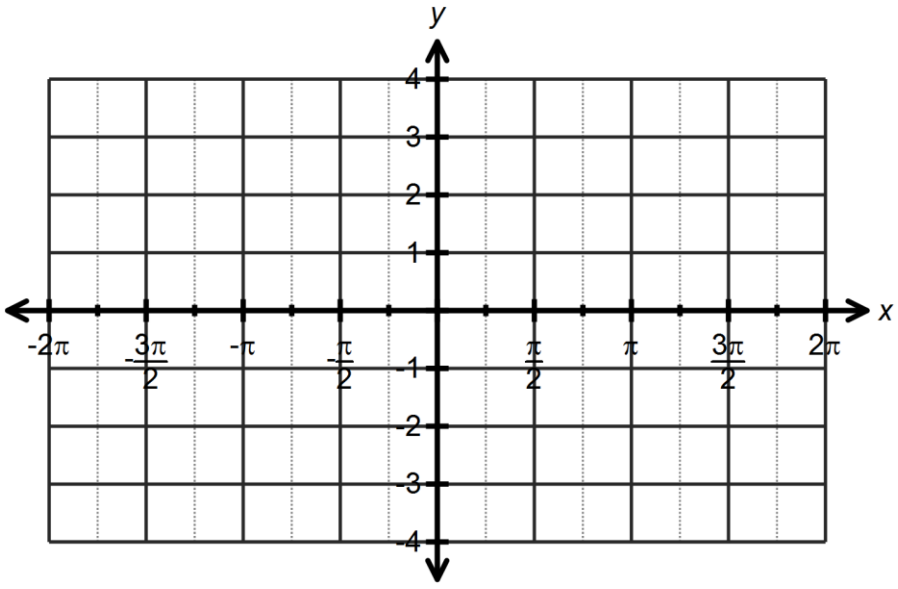
(b) $y = -3\sin 3x$



(c) $y = \cos(x + \frac{\pi}{6}) - 2$



(d) $y = -2\sin\left(\frac{x}{2}\right) + 1$



Question Three: [2, 2, 2 = 6 marks]

Calculate the values of the unknowns for each of the following:

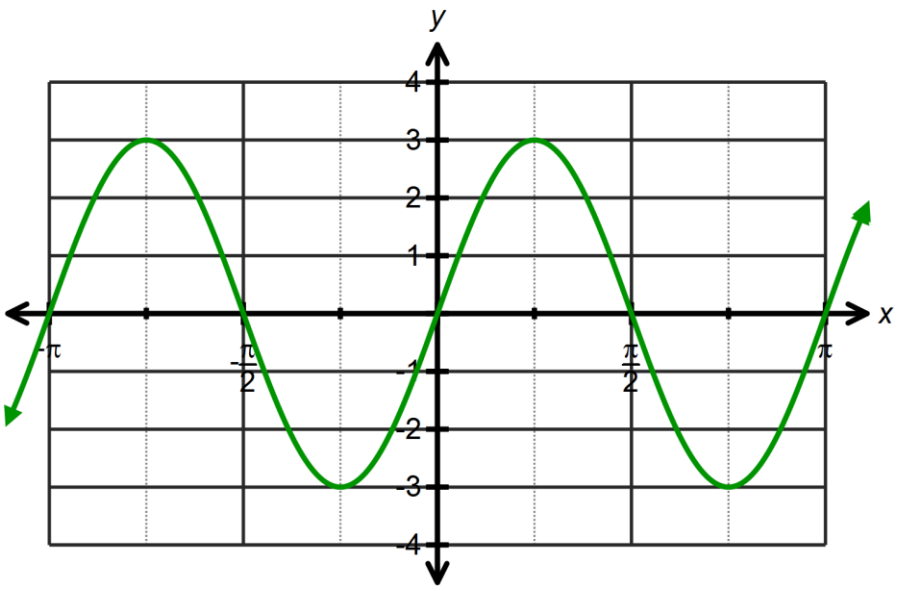
(a) $\sin 30^\circ = \cos\theta;\ 0^\circ \le \theta \le 90^\circ$

(b) $\cos\frac{\pi}{4} = \sin\theta;\ \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

(c) $-\sin\frac{\pi}{3} = \cos\theta\ ;\ 0 \le \theta \le \pi$

Question Four: [5 marks]

Determine the equation of the graph drawn below as both a sine and a cosine function.



Question Five: [3, 3, 4, 4 = 14 marks]

Use the angle sum or difference property to find the exact value for each of the following, simplifying all answers.

(a) $\sin 15^\circ$

(b) $\cos 165^\circ$

(c) $\sin(-75)^\circ$

(d) $\tan 345^\circ$

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Trigonometric Functions and Trigonometric Identities

Time: 45 minutes
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Question One: [2, 3, 3 =8 marks]

Describe the transformations that have transformed $f(x)$ to $g(x)$ in each situation below.

(a) $f(x) = \sin x$ $g(x) = 3\sin(2x)$

Horizontal dilation scale factor ½ ✓

Vertical dilation scale factor 3 ✓

(b) $f(x) = -\cos x$ $g(x) = \cos(x - \frac{\pi}{4}) + 1$

Translate $\frac{\pi}{4}$ units right ✓

Reflect about the $x-$axis ✓

Translate 1 unit up ✓

(c) $f(x) = \tan x$ $g(x) = \tan(\frac{x}{2} + \frac{\pi}{6})$

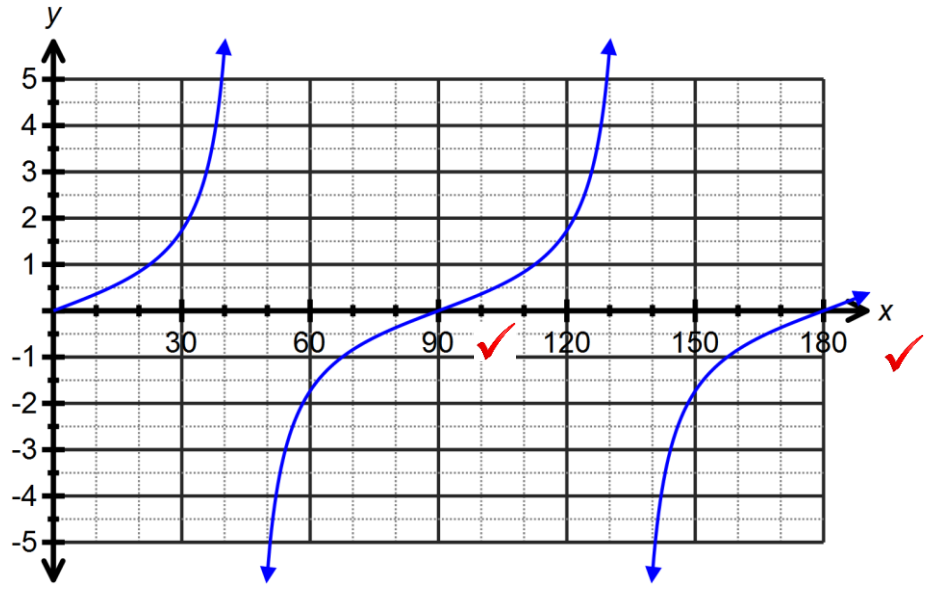
Horizontal translation $\frac{\pi}{3}$ left ✓

Horizontal dilation scale factor 2 ✓

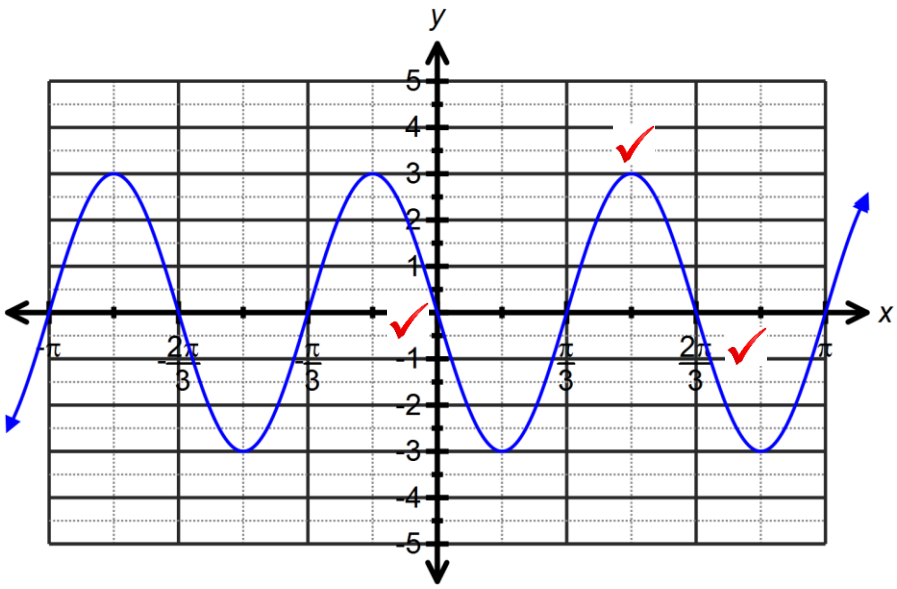
Question Two: [2, 3, 3, 4 = 12 marks]

Sketch each of the following functions on the axes below:

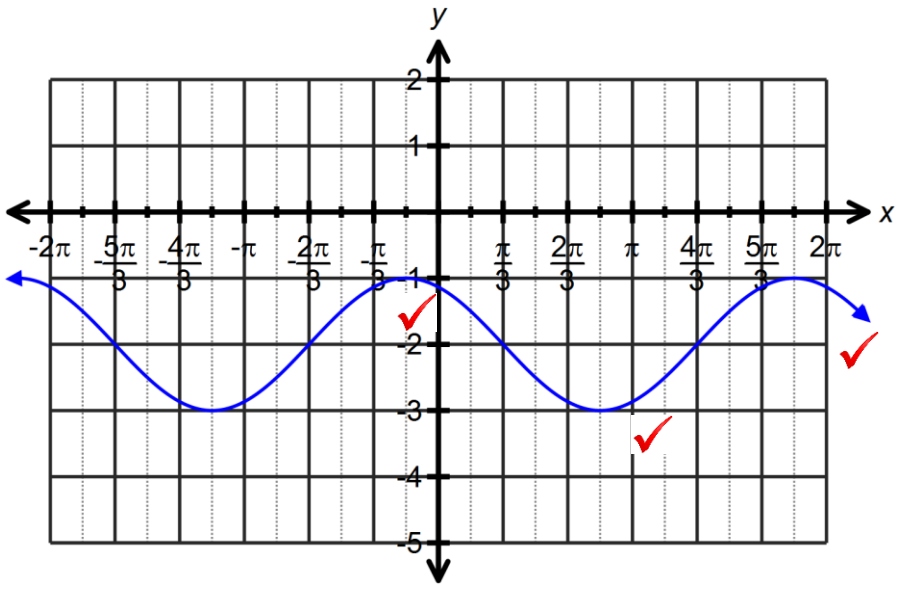
(a) $y = \tan 2x$



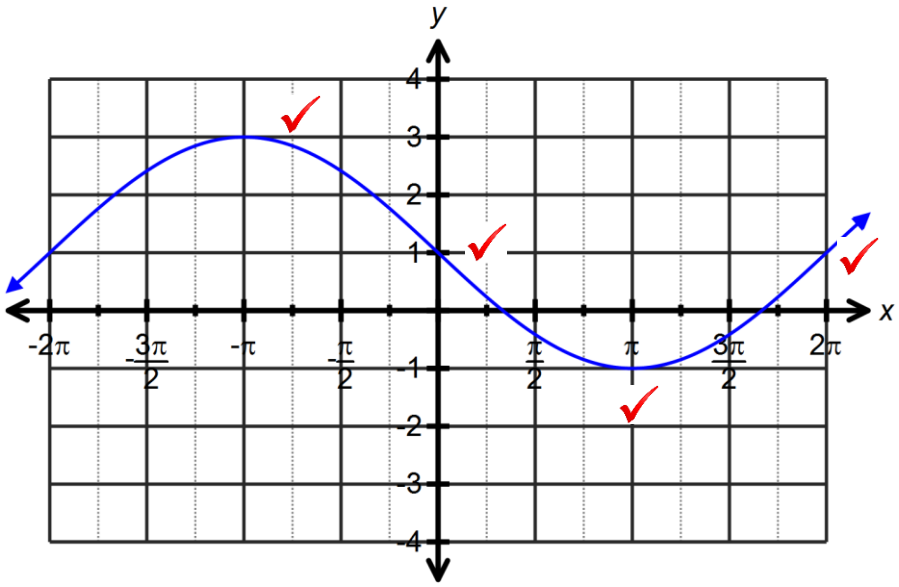
(b) $y = -3\sin 3x$



(c) $y = \cos(x + \frac{\pi}{6}) - 2$



(d) $y = -2\sin\left(\frac{x}{2}\right) + 1$



Question Three: [2, 2, 2 = 6 marks]

Calculate the values of the unknowns for each of the following:

(a) $\sin 30^\circ = \cos\theta;\ 0^\circ \le \theta \le 90^\circ$

$\theta = 60^\circ$ ✓ ✓

(b) $\cos\frac{\pi}{4} = \sin\theta;\ \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

$\theta = \frac{3\pi}{4}$ ✓ ✓

(c) $-\sin\frac{\pi}{3} = \cos\theta\ ;\ 0 \le \theta \le \pi$

✓ $\frac{-\sqrt{3}}{2} = \cos\theta$

$\theta = \frac{2\pi}{3}$ ✓

Question Four: [5 marks]

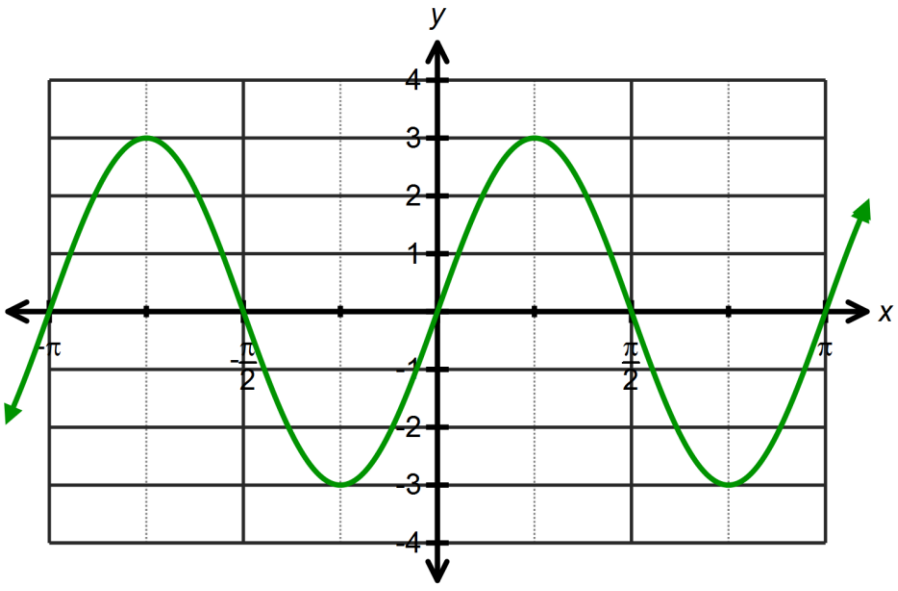
Determine the equation of the graph drawn below as both a sine and a cosine function.

✓ ✓

$y = 3\sin 2x$

$y = 3\cos 2(x - \frac{\pi}{4})$

✓ ✓ ✓



Question Five: [3, 3, 4, 4 = 14 marks]

Use the angle sum or difference property to find the exact value for each of the following, simplifying all answers.

(a) $\sin 15^\circ$

$= \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

(b) $\cos 165^\circ$

$= \cos(120^\circ + 45^\circ) = \cos 120^\circ \cos 45^\circ - \sin 120^\circ \sin 45^\circ$

$= \frac{-1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{2} - \sqrt{6}}{4}$

(c) $\sin(-75)^\circ$

$= -\sin(75^\circ) = -\sin(30^\circ + 45^\circ) = -(\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ)$

$= -(\frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2})$

$= \frac{-\sqrt{2} - \sqrt{6}}{4}$

(d) $\tan 345^\circ$

$= \tan(-15)^\circ = \tan(30^\circ - 45^\circ) = \frac{\tan 30^\circ - \tan 45^\circ}{1 + \tan 30^\circ \tan 45^\circ}$

$= \frac{\frac{1}{\sqrt{3}} - 1}{1 + \frac{1}{\sqrt{3}} \times 1} = \frac{\frac{1}{\sqrt{3}} - 1}{1 + \frac{1}{\sqrt{3}}}$