MINDARIE SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS

Methods Unit 4 Test 4, 2018

(Calculator Free) Name___________________

Time: 22 minutes Marks: 22

1. [2, 2 marks]

Determine $\frac{dy}{dx}$ for each (you do not need to simplify):

a) $y = ln\left[\frac{2x-1}{3x+1}\right]$

$y = \ln(2x - 1) - \ln(3x + 1)$

$\frac{dy}{dx} = \frac{2}{2x-1} - \frac{3}{3x+1}$

✓ for separating logs
✓for correct derivative

b) $y = x^2. \ln(\sin x)$

$\frac{dy}{dx} = 2x.\ln(\sin x) + x^2(\frac{\cos x}{\sin x})$

✓ for product rule
✓ for correct derivative

2. [2, 2 marks]

Determine the following:

a) $\int \frac{1}{3-2x} \quad dx$

$= - ½ \int \frac{-2}{3-2x} \quad dx$

$= - ½ \ln|3 – 2x| + c$

✓ for rewriting
✓ for correct integration

b) $\int \frac{\cos(2x)}{\sin(2x)} \quad dx$

$½ \int \frac{\cos(2x)}{\sin(2x)} \quad dx$

$= ½ \ln|\sin(2x)| + c$

✓ for rewriting
✓ for correct integration

3. [4 marks]

Solve $2^{x-1} = 3^{3x}$, leaving your answer in exact form.

$\log 2^{x-1} = \log 3^{3x}$
$(x - 1) \log 2 = (3x) \log 3$
$x \log 2 – \log 2 = 3x \log x$
$x \log 2 – 3x \log 3 = \log 2$
$x(\log 2 – 3 \log 3) = \log 2$

$$x = \frac{log2}{\log\frac{2}{27}}$$

$\boxed{\checkmark \text{ for log of both sides}}$
$\boxed{\checkmark \text{for rearranging}}$
$\boxed{\checkmark \text{ factorizing}}$
$\boxed{\checkmark \text{for solution}}$

4. [3, 2 marks]

The continuous random variable X is defined by the p.d.f.

$$f(x) = \begin{cases} \frac{q}{x} & for\ \ 1 \le x \le 3 \\ 0 & elsewhere \end{cases}$$

a) Determine the exact value of q.

$$\int_1^3 \frac{q}{x}\ dx\ =\ q[\ln(x)]_1^3 = 1$$

$$= q\ (\ln(3) – \ln(1)) = 1$$

$$q = \frac{1}{\ln 3}$$

$\boxed{\checkmark \text{ for integral equal to 1}}$
$\boxed{\checkmark \text{integration}}$
$\boxed{\checkmark \text{ solution}}$

b) Determine P(2 < x < 3)

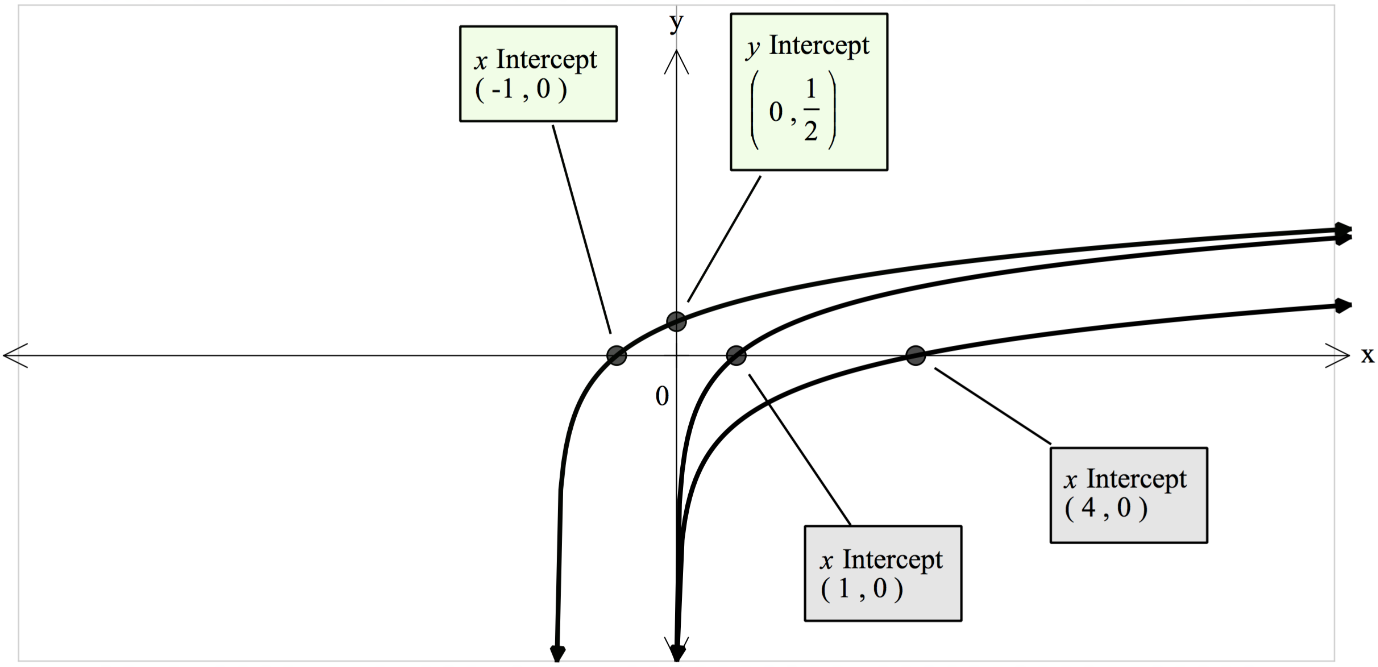
$$\int_2^3 \frac{1}{x.ln3}\ dx = \frac{ln3}{ln3} – \frac{ln2}{ln3}$$

$$= 1 - \frac{ln2}{ln3}$$

$\boxed{\checkmark \text{ for integral}}$
$\boxed{\checkmark \text{ solution}}$

5. [5 marks]

The diagram below shows $y = \log_a(x)$, $y = \log_a(x + b)$ and $y = \log_a(x) + c$.
Determine a, b and c.



Translates 2 units to left so b = 2

$\log_a 2 = \frac{1}{2}$
$\sqrt{a} = 2 \implies a = 4$

$\log_4 4 + c = 0$
$c = -1$

✓ for b = 2
✓ for substituting (0, ½)
✓ for a = 4
✓ for substituting (4, 0)
✓ for c = -1

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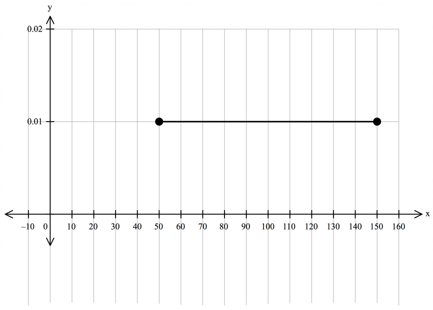
Time: 43 minutes Marks: 43

Show working in sufficient detail to support your answers. Incorrect answers given without supporting reasoning may not be allocated any marks.

6. [2, 3, 2, 2 marks]

The serving time, T seconds, for a customer at an ATM is a uniformly distributed random variable, where $50 \le T \le 150$.

a) Sketch this distribution function below, using appropriate scales on each axis.



✓ for k = 0.01
✓uniform distribution $50 \le T \le 150$

b) Find the expected value and standard deviation for this distribution.

E[X] = 100 sec

Var[X] = $\int_{50}^{150} 0.01(x-100)^2 \; dx$
= 833.33

SD[X] = 28.87

✓ for E[X]
✓for using integration to find variance
✓for standard deviation

c) Evaluate $P(T \ge 100 \mid T \le 120)$

$\frac{0.2}{0.7}$ = 0.286

✓ for values of 0.2 & 0.7
✓simplified answer

d) What is the probability that exactly 3 of the next 5 customers will require at least 2 minutes to be served?

Binomial distribution with n = 5 and p = 0.3
P(X = 3) = 0.1325

✓ for correctly identifying binomial and the parameters
✓ answer

7. [2, 3, 3, 3 marks]

The life (in years) of a light globe has a p.d.f. which can be modelled by:

$$f(x) = \begin{cases} \frac{4x}{3} & for\ 0 \leq x \leq 1 \\ \frac{4}{3x^5} & for\ x > 1 \end{cases}$$

a) Determine P(X < 1)

$$\int_0^1 \frac{4x}{3}\ dx\ = 0.6667$$

✓ for correctly identifying definite integral
✓ answer

b) Determine P(X < 3)

$$\frac{2}{3} + \int_1^3 \frac{4}{x^5}\ \ dx$$

= 0.9959

✓ for correctly identifying 2/3 +
✓ remaining integral
✓ answer

c) Determine the expected value for this distribution.

E[X] = $\int_0^1 x.\frac{4x}{3}\ dx + \int_1^\infty x.\frac{4}{3x^5}\ \ dx$

= 0.8889

✓✓ for correctly each part
✓ answer

d) If you had 1000 globes, how many would you expect to last longer than 3 years?

1 – 0.9959 = 0.0041 from part b

0.0041 x 1000 = 4 globes

✓✓ for correct probability
✓ answer as whole number

8. [2, 2, 4 marks]

a) Consider the shaded area shown between the graph of $y = e^x$, the y-axis and $y = 2$.

i) Determine the exact coordinates of point A.

$e^x = 2$
$x = \ln 2 \Longrightarrow (\ln 2, 2)$

$\boxed{\checkmark\checkmark \text{ for exact value of x}}$
$\boxed{\checkmark \text{ answer as a coordinate}}$

ii) Hence, or otherwise, determine the shaded area.

$\int_0^{ln2} 2 - e^x \ \ dx = 2\ln(2) - 1$ [or 0.386]

$\boxed{\checkmark\checkmark \text{ integral}}$
$\boxed{\checkmark \text{ answer}}$

b) If the area between $y = e^x$, the x-axis, the y-axis and $y = k$, where $k > 0$, is to be equal to 2 square units, determine the exact value of k.

$\int_0^k e^x \ \ dx = 2$

$[e^x]_0^k = 2$
$e^k - 1 = 2$
$e^k = 3$
$k = \ln 3$

$\boxed{\checkmark \text{ for integral}}$
$\boxed{\checkmark \text{for integration}}$
$\boxed{\checkmark \text{for equation}}$
$\boxed{\checkmark \text{ for value of k}}$

9. [1, 4 marks]

a) Determine f '(x), given f(x) = $\frac{\ln(x)}{x}$

$$f'(x) = \frac{\frac{1}{x}.x - \ln(x)}{x^2}$$

$$= \frac{1-\ln(x)}{x^2}$$

✓✓ for correct derivative

b) Hence, or otherwise, show that $\int_1^2 \frac{\ln(x)-1}{x^2} = \ln\left(\frac{1}{\sqrt{2}}\right)$

$\int_1^2 \frac{\ln(x)-1}{x^2}$ dx $= -\int_1^2 \frac{1-\ln(x)}{x^2}$ dx

$= -\left[\frac{\ln(x)}{x}\right]_1^2$

$= -\frac{1}{2}\ln(2)$

$= \ln\left(\frac{1}{\sqrt{2}}\right)$

✓ for use of part a)
✓for integrations
✓ substitution
✓showing result is as shown

10. [3, 1, 2, 2, 2 marks]

A continuous random variable X has a pdf such that $f(x) = 0.4e^{-0.4x}$ defined over interval $[0, \infty]$

a) Show that $P(X \leq k) = 1 - e^{-0.4k}$

$P(X \leq k) = \int_0^k 0.4e^{-0.4x}$

$= [-e^{-0.4x}]_o^k$

$= e^{-0.4k} + 1$ or $1 - e^{-0.4k}$

✓ for use of integral
✓for integration
✓showing result is as shown

Hence or otherwise determine:

b) $P(X \leq 5)$

$1 - e^{-2} = 0.8647$

✓ for use of part a)

c) $P(5 \leq X \leq 6)$

$0.9093 - 0.8647 = 0.0446$

✓ for correct values
✓ for correct answer

c) $P(X \leq 6 \mid X \geq 5)$

$\frac{0.0446}{0.1353} = 0.3296$

✓ for correct values
✓ for correct answer

d) the value of a, given that $P(X \leq a) = 0.2$

$1 - e^{-0.4a} = 0.2$

Thus $a = 0.56$

✓ for correct use of part a
✓ for value of a