



**MINDARIE**  
**SENIOR COLLEGE**

WHERE YOUR FUTURE BEGINS

**MATHEMATICS:**  
**SPECIALIST 1 & 2**

**SEMESTER 1      2018**  
**TEST 2**

Name \_\_\_\_\_

**Calculator Free**

Reading time: 2 mins  
Time allowed: 18 mins

Total marks: 17

1. [6 marks]

For the statement: The vehicle is a car then the vehicle has four wheels

Write down the converse, inverse and contrapositive statements and state whether they are true or false.

	Statement	True/False
Converse	If the vehicle has 4 wheels then it is a car. ✓	F ✓
Inverse	If the vehicle is not a car then it does not have 4 wheels. ✓	F ✓
Contrapositive	If the vehicle does not have 4 wheels then it is not a car. ✓	T ✓

2. [7 marks: 1, 2, 1, 3]

Three vectors are defined as  $\mathbf{a} = 3\mathbf{i} + 9\mathbf{j}$ ,  $\mathbf{b} = \mathbf{i} - 3\mathbf{j}$  and  $\mathbf{c} = -3\mathbf{i} - 4\mathbf{j}$ .

All answers to this question should be written in an exact form if they cannot be evaluated.

a) What is  $|\mathbf{a}|$ ?

$$\sqrt{3^2 + 9^2} = \sqrt{90} \quad \checkmark \text{ accept either.} \\ = 3\sqrt{10}$$

b) What is the unit vector  $\hat{\mathbf{b}}$ ?

$$\hat{\mathbf{b}} = \frac{\mathbf{b}}{|\mathbf{b}|} = \frac{\mathbf{i} - 3\mathbf{j}}{\sqrt{10}} \quad \begin{array}{l} \checkmark \text{ for } |\mathbf{b}| \\ \checkmark \text{ for } \hat{\mathbf{b}} \end{array}$$

c) Find a vector in the same direction as  $\mathbf{b}$  but is 7 units in length.

$$\frac{7}{\sqrt{10}}(\mathbf{i} - 3\mathbf{j}) \quad \checkmark$$

d) Find a vector that is the same direction as  $\mathbf{c}$  but is twice the length of  $\mathbf{b}$ .

$$\hat{\mathbf{c}} = \frac{1}{5}(-3\mathbf{i} - 4\mathbf{j}) \quad \checkmark$$

$$\frac{2\sqrt{10}}{5}(-3\mathbf{i} - 4\mathbf{j}) \quad \checkmark$$

3. [4 marks]

Prove the following statement using proof by contradiction.

The points A (2,3), B (-6,8) and C (-2,-3) are not collinear.

Assume that A, B and C are collinear. ✓

$$\vec{AB} = -2\hat{i} - 3\hat{j} - (-6\hat{i} + 8\hat{j}) = -8\hat{i} + 5\hat{j}$$

✓✓

$$\vec{AC} = -2\hat{i} - 3\hat{j} - 2\hat{i} - 3\hat{j} = -4\hat{i} - 6\hat{j}$$

$$(\vec{BC} = 6\hat{i} - 8\hat{j} - 2\hat{i} - 3\hat{j} = 4\hat{i} - 11\hat{j})$$

$\vec{AB}$  and  $\vec{AC}$  are not parallel, hence  
the points A, B, C cannot be collinear,  
and the original statement is true. ✓

\*Some of my students used gradients to prove  
this which I allowed. \*



**MINDARIE**  
**SENIOR COLLEGE**

WHERE YOUR FUTURE BEGINS

**MATHEMATICS:**  
**SPECIALIST 1 & 2**

**SEMESTER 1      2018**  
**TEST 2**

Name \_\_\_\_\_

**Calculator Assumed**

Reading time: 3 mins  
Time allowed: 40 mins

Total marks: 36

4. [3 marks]

A displacement of  $(3\mathbf{i}+2\mathbf{j})$  m is followed by a displacement of  $(7\mathbf{i}-6\mathbf{j})$  m. Find the magnitude and direction of the resultant displacement, giving your answers to 1 decimal place.

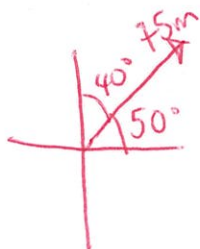
$$3\mathbf{i} + 2\mathbf{j} + 7\mathbf{i} - 6\mathbf{j} = 10\mathbf{i} - 4\mathbf{j}$$
$$\text{magnitude: } \sqrt{10^2 + 4^2} = \sqrt{116} = 10.8\text{m} \checkmark$$
$$\text{direction: } \tan^{-1}\left(\frac{4}{10}\right) = 21.8^\circ \checkmark$$



10.8m on a bearing of  $111.8^\circ$  ✓

5. [2 marks]

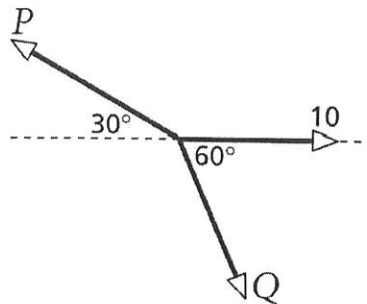
Damien walks 75m on a bearing of  $040^\circ$ . Taking  $\mathbf{i}$  and  $\mathbf{j}$  as the unit vectors in the directions east and north respectively, find his displacement. Give your answer in the form  $a\mathbf{i} + b\mathbf{j}$  where  $a$  and  $b$  are rounded to 1 decimal place.



$$75\cos 50^\circ \mathbf{i} + 75\sin 50^\circ \mathbf{j}$$
$$= 48.2\mathbf{i} + 57.5\mathbf{j} \checkmark$$

7. [5 marks]

The three forces shown in this diagram are in equilibrium. Find the values of P and Q.



$\perp$  direction:

$$-P \cos 30 + 10 + Q \cos 60 = 0 \quad \checkmark \checkmark$$

$\downarrow$  direction:

$$P \sin 30 - Q \sin 60 = 0 \quad \checkmark$$

Solve on classpad:

$$\begin{cases} Q \cos 60 - P \cos 30 = -10 \\ P \sin 30 = Q \sin 60 \end{cases}$$

$$P = 17.3 \text{ N} \quad \checkmark$$

$$Q = 10 \text{ N} \quad \checkmark$$

8. [3 marks]

Determine the coordinates of point B given that  $\mathbf{r}_A = 6\mathbf{i} - 4\mathbf{j}$  and  $\mathbf{r}_{AB} = -2\mathbf{i} + 4\mathbf{j}$ .

$$\mathbf{r}_{AB} = \mathbf{r}_A - \mathbf{r}_B$$

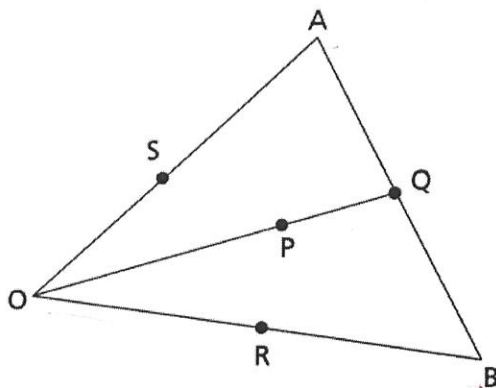
$$\begin{aligned} \mathbf{r}_B &= \mathbf{r}_A - \mathbf{r}_{AB} \quad \checkmark \\ &= 6\mathbf{i} - 4\mathbf{j} + 2\mathbf{i} - 4\mathbf{j} \\ &= 8\mathbf{i} - 8\mathbf{j} \quad \checkmark \end{aligned}$$

$$B = (8, -8) \quad \checkmark$$



6. [9 marks: 2, 2, 1, 1, 2]

A triangle OAB has midpoints on its sides Q, R and S as shown in the diagram below.



$\vec{OA} = \mathbf{a}$  and  $\vec{OB} = \mathbf{b}$ .

P is a point  $\frac{2}{3}$  of the way along Q.

a) Express  $\vec{AQ}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\vec{AB} = \mathbf{b} - \mathbf{a} \quad \vec{AQ} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$$

b) Express  $\vec{OQ}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ . Hence, or otherwise, express  $\vec{OP}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\begin{aligned} \vec{OQ} &= \vec{OA} + \vec{AQ} \\ &= \mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{OP} &= \frac{2}{3}\vec{OQ} \\ &= \frac{2}{3}\left(\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \\ &= \frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \end{aligned}$$

c) Express  $\vec{AR}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\vec{AR} = \vec{AO} + \vec{OR} = -\mathbf{a} + \frac{1}{2}\mathbf{b}$$

d) T is a point  $\frac{2}{3}$  along AR. Express  $\vec{AT}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\vec{AT} = \frac{2}{3}\vec{AR} = -\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$$

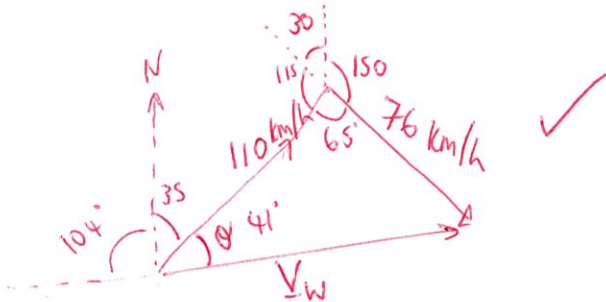
e) Using your answer to d), express  $\vec{OT}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ . What does this tell you about points P and T?

$$\begin{aligned} \vec{OT} &= \vec{OA} + \vec{AT} = \mathbf{a} - \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \\ &= \frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \end{aligned}$$

$\vec{OP}$  and  $\vec{OT}$  are the same, therefore P and T are in the same place.

9. [6 marks]

To a person travelling in a car at 110 km/h on a bearing of  $035^\circ$  the wind appears to come from a bearing of  $330^\circ$  at 76 km/h. Find the true velocity and direction of the wind.



$$|\underline{V}_w| = \sqrt{110^2 + 76^2 - 2 \times 110 \times 76 \times \cos 65^\circ} \quad \checkmark$$

$$|\underline{V}_w| = 104 \text{ km/h}$$

$$\text{Direction: } \frac{\sin \theta}{76} = \frac{\sin 65^\circ}{104} \quad \checkmark \quad \theta = 41^\circ \quad \checkmark$$

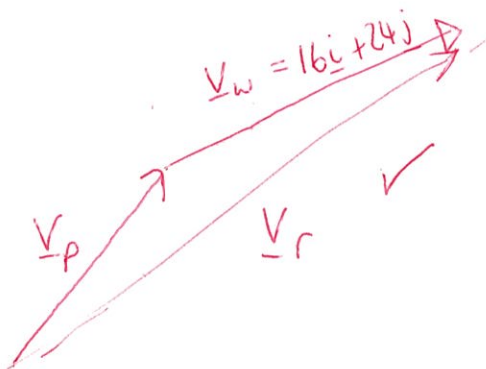
Velocity of wind is  $104 \text{ km/h}$  } on a bearing of  $076^\circ$   
or  
from  $256^\circ$

10. [8 marks]

A pilot needs to fly from Perth to Singapore which is a distance of 3900km on a bearing of  $315^\circ$ .

A Boeing 777 can fly at 892km/h in still air. On the particular day of the flight, there is a wind of  $(16\mathbf{i}+24\mathbf{j})$  km/h blowing.

Find the velocity vector (in the form  $a\mathbf{i}+b\mathbf{j}$ ) that the pilot should set and the time the flight will take.



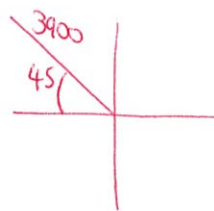
$$|V_p| = 892 \text{ km/h}$$

$$\therefore a^2 + b^2 = 892^2 \quad \checkmark$$

$$V_w = 16\mathbf{i} + 24\mathbf{j}$$

$$\begin{aligned} V_r &= 16\mathbf{i} + 24\mathbf{j} + a\mathbf{i} + b\mathbf{j} \\ &= (16+a)\mathbf{i} + (24+b)\mathbf{j} \quad \checkmark \end{aligned}$$

Distance needed to travel:



$$-3900\cos 45\mathbf{i} + 3900\sin 45\mathbf{j} \quad \checkmark$$

Use distance = speed  $\times$  time

$$-3900\cos 45\mathbf{i} + 3900\sin 45\mathbf{j} = t[(16+a)\mathbf{i} + (24+b)\mathbf{j}] \quad \checkmark$$

$$\left. \begin{aligned} -3900\cos 45 &= t(16+a) \\ 3900\sin 45 &= t(24+b) \\ a^2 + b^2 &= 892^2 \end{aligned} \right\} \begin{array}{l} \text{Solve} \\ \text{on} \\ \text{classpad.} \end{array} \quad \checkmark$$

$$a = -650.4$$

$$b = 610.4$$

$$t = 4.35 \text{ hours}$$

$$\text{Velocity vector: } -650.4\mathbf{i} + 610.4\mathbf{j} \quad \checkmark$$

$$\text{Time taken: } 4 \text{ hours } 21 \text{ mins} \quad \checkmark$$