SOLUTIONS

Specialist Unit 3&4 Test 6 2016
Calculator Section

Time limit = 40 minutes.

Name: ______________________________ **Score:** ___________ **/ 34**

Calculators allowed
Access to approved Sample Mathematics Specialist formulae sheet is permitted.
Answers should be in 4 decimal places.

Q1. [3,3,2] [8 marks]

The standard deviation of the durability of Performance Racing tyres is 410 kilometres. Racing experts plan to estimate μ, the mean lifetime of these tyres, using the mean lifetime of a random sample of the tyres.

(a) The experts would like to be 95% confident that the mean lifetime of tyres in the sample is within 50 kilometres of μ. How large a sample should they take?

$50 = 1.96\left(\frac{410}{\sqrt{n}}\right)$

n = 258.309184

They should take a sample of at least 259 tyres. ✓

(b) Suppose that a random sample of 80 tyres is taken, and the mean lifetime of these tyres is 1245 kilometres. Based on this sample, determine a 90% confidence interval for μ.

The 90% interval is $1245 \pm 1.645\left(\frac{410}{\sqrt{80}}\right)$ ✓

= (1169.6, 1320.4)

$1169.6009 < \mu < 1320.3991$ ✓

C-Level	0.90
σ	410
$\bar{x}$	1245
n	80

Lower	0.90
Upper	410
$\bar{x}$	1245
n	80

(c) The manufacturer claims that the mean lifetime of Performance Racing tyres is at least 1250 kilometres. Does the sample in part (b) provide a strong reason to doubt this claim? Justify your answer.

No, 1250 lies within the 90% confidence interval. We should generally doubt the claim only if 1250 lay outside the confidence interval – and a 90% interval is a narrower interval than others that could be used (say 95%, 99%) to test the claim.1250 would lie well within these intervals.

Q2. [2,2] [4 marks]

A machine is designed to produce ball bearings. The quality control officer sampled 400 items and calculated a mean radius of 16 mm and standard deviation of 0.2 mm.

(a) Calculate the 99% confidence interval for the population mean of the radii of the ball bearings,

$$16 - 2.576 \times \frac{0.2}{\sqrt{400}} \leq \mu \leq 16 + 2.576 \times \frac{0.2}{\sqrt{400}}$$ ✓

$$15.974 \leq \mu \leq 16.026$$

15.9742 ≤ μ ≤ 16.0258 ✓

(b) The company was asked to make ball bearings with radii accurate to within ±0.05mm with a 95% confidence. What size sample must be taken to satisfy this request?

$$n = \left(\frac{1.96 \times 0.2}{0.05}\right)^2 = 61.46\,56$$ ✓✓

So $n = 62$ ✓

Q3. [2,2,1] [5 marks]

A survey was conducted via the internet. It asked participants who logged onto a football website; "what fine should a football player receive for major indiscretion?" The statistics of the population resulted in a mean of $4212 and a standard deviation of $1004.

Using a sample of 1256 respondents, the website supervisor calculated the 95% confidence interval to be $4156.47 to $4267.53.

(a) Is the confidence level correct? Explain.

4267.5248

$$4212 - \left(1.96 \times \frac{1004}{\sqrt{1256}}\right) = 4267.53$$

✓ ∴ Yes it is ✓

$$4212 + \left(1.96 \times \frac{1004}{\sqrt{1256}}\right) = 4156.47$$

4156.4752

(b) Does this sample represent the view of the population? Explain.

No, only those that use that website ✓ ✓

(Or Yes, mean is within interval) ✓ ✓

(c) Describe the effect on the length of confidence interval if the confidence level were to be increased to 99%

The confidence interval gets wider ✓

Q4. [1,3,3] **[7 marks]**

If the numbers 1 to 100 form the population, then the mean and standard deviation of the population are 50.5 and 28.87 respectively.

(a) Show clearly why the mean of the population is 50.5.

$$\frac{1+2+3+\ldots+100}{100}=\frac{5050}{100}=50.5$$

a) (b) If all possible samples of size 9 were taken from this population, what does the Central Limit theorem tell us about the distribution of the means of these samples?

$$X \approx N\left(50.5, \left(\frac{28.87}{\sqrt{9}}\right)^2\right)$$

Or 1. Normally distributed

2. Mean = population mean

3. Standard deviation = $\dfrac{28.87}{\sqrt{9}}=\dfrac{28.87}{3}$

b) (c) The diagram below shows 3 lists of 9 randomly generated counting numbers from 1 to 100 generated by the calculator

▼ Edit Calc SetGraph

	List 1	List 2	List 3
1	49	15	75
2	3	20	70
3	89	45	72
4	76	7	19
5	72	16	74
6	45	6	81
7	36	48	83
8	43	46	27
9	86	8	11
10			
11			

Calc= randList (9,1,100)

Deg Auto Decimal

Given that the 95% confidence interval for the population mean is $31.64 \le \mu \le 69.36$

Indicate whether these lists above are representative samples, giving your reasons.

$\bar{x}_1 = 55.4$ Yes

$\bar{x}_3 = 23.4$ No

$\bar{x}_3 = 56.9$ Yes

Q5. [2,2] [4 marks]

(a) A company wishes to estimate the average age of its employees. From past information it was found the standard deviation was 3.5 years. A sample of 60 employees is selected and the mean is calculated as 32.5 years. Find the 95% confidence interval of the population mean.

The z score for the 95% confidence interval is 1.96

$$\bar{x} - z\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z\frac{\sigma}{\sqrt{n}}$$

$$32.5 - 1.96 \text{ x } \frac{3.5}{\sqrt{60}} \leq \mu \leq 32.5 + 1.96 \text{ x } \frac{3.5}{\sqrt{60}}$$

$$31.6144 \leq \mu \leq 33.3856$$

ie. With a 95% confidence, the average age of employees lies between 26.607 and 28.113 years.

(b) How large a sample should the company use to be 95% sure that the sample is within 1 year of the sample mean of 32.5 years,

$$n = \left(\frac{z\sigma}{w}\right)^2$$

$$n = \left(\frac{1.96 \times 3.5}{1}\right)^2 \approx 47.0596$$

The required sample size is 48 employees.

2-1

Q6. [~~1~~,~~2~~,2,2] [7 marks]

On the basis of the results obtained from a random sample of 60 bags produced by a mill, the 95% confidence interval for the mean weight of flour in a bag is found to be (3.1kg, 4.6kg).

(a) Find the value σ, the standard deviation of the normal population from which the sample is drawn.

$$\bar{x} - 1.96 \leq x \leq \leq \mu \leq \bar{x} + 1.96 \frac{\sigma}{\sqrt{65}}$$

$$\bar{x} - 1.96 \frac{\sigma}{\sqrt{60}} = 3.1$$

$$\bar{x} + 1.96 \frac{\sigma}{\sqrt{60}} = 4.6$$ ✓

solve it simultaneously $\left(0 + 2\left(\frac{1.96}{60}\right)\sigma = 1.5\right)$ $\sigma = 2.964$ ✓

2.964017867

(b) Find the value of $\bar{x}$, the mean weight of the sample.

$$\bar{x} - 1.96 \frac{2.964}{\sqrt{60}} = 3.1$$

$\bar{x} = 3.85\cancel{0}$ ✓

(c) Calculate the 99% confidence interval for the mean weight of flour in a bag.

k = 2.576 for 99% confidence interval

$2.8644 \leq x \leq 4.8356$ ✓ ✓

$2.864 \leq x \leq 4.836$

(d) Using the sample mean from (a) as the best estimate for the population mean, what is the probability that the sample mean of a larger sample of 200 bags is less than 3.8kg.

$\bar{x} = 3.85$

$\sigma = \frac{2.9640}{\sqrt{200}} = 0.209586449$

~~Find~~ $X \sim N(3.85, 0.2095864499^2)$ ✓

$P(X < 3.8) = 0.4057$ (4dp) ✓