Eastern Goldfields College

Maths Methods Unit 3 and 4
Test 3 2018

Calculator Free

Total Marks: 37 **Reading: 2 minutes** **Time Allowed: 40 minutes**

Maths Methods Formula Sheet may be used

Question 1 **(5 marks)**

(a) Simplify $\frac{3\log 100}{4\log 1000}$. $= \frac{\cancel{3}\times\cancel{2}}{2\cancel{4}\times\cancel{3}} = \frac{1}{2}$ ✓ (1 mark)

(b) Solve for x, where $\log_x 3 = \frac{1}{2}$. (2 marks)

$x^{\frac{1}{2}} = 3$ ✓

$x = 9$ ✓

(c) If $\log x = 0.313$, determine $\log\frac{1}{x^2}$. $= \log x^{-2}$ (2 marks)

$= -2\log x$ ✓

$= -0.626.$ ✓

Question 2 **(5 marks)**

Evaluate the following

(a) $\int \frac{3x}{5x^2-2}\,dx = \frac{3}{10}\int\frac{10x}{5x^2-2}\,dx$ ✓ (2 marks)

$= \frac{3}{10}\ln(5x^2-2) + c$ ✓

(b) $\int_0^{\frac{\pi}{k}} \frac{\sin(x)}{1+\cos(x)}\,dx$ where k is a constant. (3 marks)

$= -\ln(1+\cos x)\Big]_0^{\frac{\pi}{k}}$ ✓

$= -\ln\left(1+\cos\frac{\pi}{k}\right) - \left(-\ln(1+\cos 0)\right)$ ✓

$= -\ln\left(1+\cos\frac{\pi}{k}\right) + \ln 2$ ✓

Question 3 **(8 marks)**

Differentiate the following with respect to x, do not simplifying.

(a) $y = x^2 \ln(2x+3)$. (2 marks)

$$\frac{dy}{dx} = 2x \cdot \ln(2x+3) + x^2 \cdot \frac{2}{2x+3} \checkmark \checkmark$$

(b) $y = x \log_{10}(1+x)$ (3 marks)

$$= \frac{x \ln(1+x)}{\ln 10} \checkmark$$

$$\frac{dy}{dx} = \frac{1}{\ln 10}\left(\ln(1+x) \cdot 1 + x \cdot \frac{1}{1+x} \right) \checkmark \checkmark$$

(c) $y = \dfrac{\ln(2x-1)}{x}$ (3 marks)

$$\frac{dy}{dx} = \frac{x \cdot \frac{2}{2x-1} - \ln(2x-1) \times 1}{x^2} \checkmark \checkmark$$

✓ quotient

Question 4 **(4 marks)**

(a) Determine $\dfrac{d}{dx}[\ln(\cos^2 2x)]$ (2 marks)

$$= \frac{d}{dx}(2 \ln \cos 2x) \checkmark$$

$$= 2 \times \frac{-2 \sin 2x}{\cos 2x} \checkmark$$

$$= -4 \tan 2x$$

Hence,

(b) determine $\int \tan 2x \, dx$ (2 marks)

$$\int -4 \tan 2x \, dx = \ln(\cos^2 2x) + c \checkmark$$

$$\therefore \int \tan 2x \, dx = -\frac{1}{4} \ln(\cos^2 2x) + c \checkmark$$

Question 5 **(7 marks)**

A function is defined by $f(x) = \dfrac{2 + 2\ln x}{3x}$.

(a) State the natural domain of f. (1 mark)

$\{x \in \mathbb{R}, x > 0\}$

(b) Show that $f'(1) = 0$. (3 marks)

$$f'(x) = \frac{\frac{2}{x}(3x) - (2 + 2\ln x)3}{(3x)^2}$$

$$f'(1) = \frac{6-6}{3^2} = 0$$

✓ quotient
✓ $u'v$ and uv' expressions
✓ subst $x=1$, result $=0$

(c) Use the second derivative test to determine the nature of the stationary point of the function at $x = 1$. (3 marks)

$$f'(x) = -\frac{2\ln x}{3x^2}$$

$$f''(x) = -\left(\frac{\left(\frac{2}{x}\right)3x^2 - 2\ln x \cdot 6x}{(3x^2)^2}\right)$$

$$f''(1) = -\frac{6-0}{3^2} < 0 \quad \therefore \text{ pt is maximum}$$

✓ simplifies $f'(x)$ + diff with quotient rule.
✓ diff. correctly
✓ indicates + interprets sign of $f''(1)$

Question 6 **(3 marks)**

Find an exact solution for x if $7^{2x} = 5^{x-3}$.

$$7^{2x} = \frac{5^x}{5^3} \checkmark$$

$$5^3 = \frac{5^x}{7^{2x}} \checkmark$$

$$5^3 = \left(\frac{5}{49}\right)^x$$

$$\therefore \log_{\frac{5}{49}} 125 = x \checkmark$$

OR

$$2x\ln 7 = (x-3)\ln 5 \checkmark$$

$$2x\ln 7 = x\ln 5 - 3\ln 5$$

$$3\ln 5 = x(\ln 5 - 2\ln 7) \checkmark$$

$$\frac{3\ln 5}{\ln 5 - 2\ln 7} = x \checkmark$$

Question 7 **(5 marks)**

Let $y = \ln\sqrt{\dfrac{1+x^2}{1-x^3}}$.

(a) Rewrite y as the difference of two logarithms without the radical sign. (3 marks)

$$y = \tfrac{1}{2}\ln(1+x^2) - \tfrac{1}{2}\ln(1-x^3) \quad \checkmark \checkmark \checkmark$$

(b) Hence, find $\dfrac{dy}{dx}$. You do not need to simplify your answers. (2 marks)

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{2x}{1+x^2}\right) - \frac{1}{2}\left(\frac{-3x^2}{1-x^3}\right) \quad \checkmark \checkmark$$

Esperance SHS **Maths Methods Unit 3 / 4** **Test 3 2018**

Resource Rich

Reading: 2 minutes **Time Allowed: 24 minutes** **Total Marks: 22**

Maths Methods Formula Sheet, 1 page of notes and a CAS calculator may be used

Question 8 **(7 marks)**

(a) Given that $a = log_3 2$ and $b = log_3 5$, find in terms of a and b:

(i) $log_3 0.4$ (2 marks)

$= \log_3 \frac{2}{5}$ ✓

$= a - b.$ ✓

(ii) $log_3 30$ (2 marks)

$= \log_3 (2 \times 5 \times 3)$ ✓

$= \log_3 2 + \log_3 5 + \log_3 3$ ✓

(b) For each of the following, express p in terms of q.

(i) $log_e p = 2 log_e q$ (1 mark)

$p = q^2$ ✓

(ii) $\frac{e^{2p}}{3} = q$ (2 marks)

$e^{2p} = 3q.$

$\log_e 3q = 2p.$ ✓

$\ln 3q = 2p$

$p = \frac{\ln 3q}{2}$ ✓

Question 9 **(6 marks)**

The annual growth rate for an investment that is growing continuously is given by $r = \frac{1}{t}\ln\left(\frac{A}{P}\right)$ where P is the principal and A is the amount after t years. An investment of \$10 000 in Dell Computer stock in 2012 grew to \$31 800 in 2015.

(a) Assuming the investment grew continuously, what was the annual growth rate (to 4 decimal places)? (2 marks)

$$r = \frac{1}{3}\ln\left(\frac{31\,800}{10\,000}\right) = 0.3856. \quad //$$

(b) If Dell continues to grow at the same rate, what will the \$10 000 investment be worth in 2019? (2 marks)

$$0.3856 = \frac{1}{7}\left(\frac{A}{10\,000}\right) \qquad A = \$148\,706.50$$

//

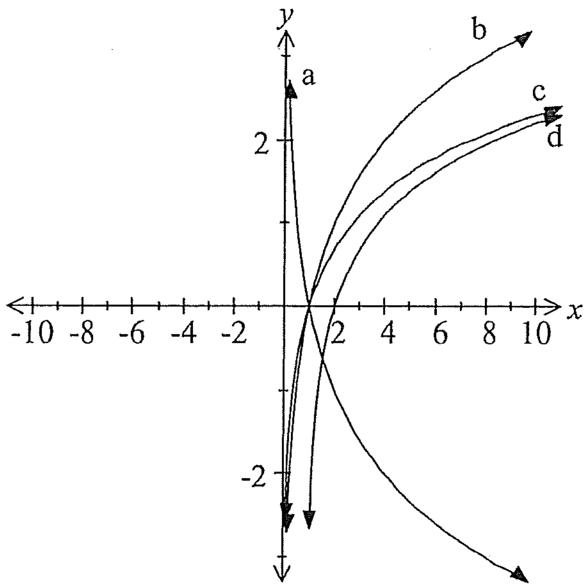
(c) Assuming the investment grew continuously at the same rate, how long will it take for the \$10 000 investment to grow to \$500 000? (2 marks)

$$0.3856 = \frac{1}{t}\left(\frac{500\,000}{10\,000}\right) \checkmark$$

$t = 10.1453$ years after 2012 ✓

Question 10 **(4 marks)**

Match the equation with the graph. (not all equations are used)



$y=\ln x$	c
$y=\log_{0.5} x$	a
$y=\log_2 x$	b
$y=\ln (x+1)$	
$y=\ln (x-1)$	d
$y=2 \ln x$	

Question 11 **(5 marks)**

The sound level L, in decibels (dB), for a single sound of pressure p, in millipascals (mPa), is calculated using the formula $L = 20\log\frac{p}{0.02}$, $p > 0$.

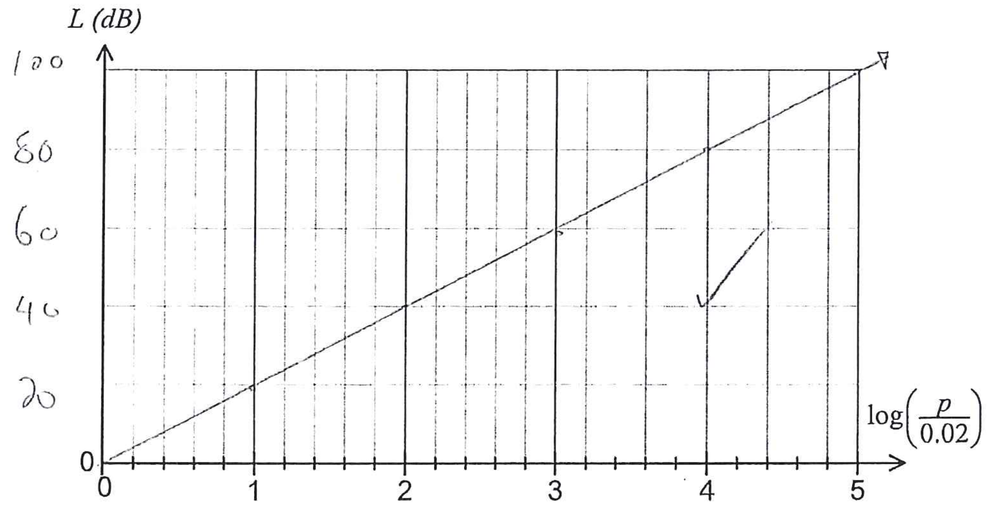
(a) Determine the sound level corresponding to a sound pressure of 0.02 mPa. (1 mark)

0 ✓

(b) Determine the sound pressure corresponding to a sound level of 80 dB. (2 marks)

$80 = 20\log\left(\frac{p}{0.02}\right)$ $p = 200$ mPa //

(c) Sketch the graph of the above function on the axes below with $\log\frac{p}{0.02}$ on the horizontal axis. Indicate the scale used on the vertical axis. (2 marks)



✓