Test 2 2016
Calculator Free Section

Name: ________________ Score: _____ / 31

Section 1 is worth 50% of your final test mark.
No calculators or notes are to be used.
Access to approved Sample Mathematics Specialist formulae sheet is permitted. **Time limit = 35 minutes.**

1. [5 marks: 2, 3]

The functions $f(x)$ and $g(x)$ are given by $f(x) = \sqrt{2x+4}$ and $g(x) = \frac{1}{2}x^2 - 4$.

a) Determine and simplify $f(g(x))$.

$fg(x) = \sqrt{2\left(\frac{1}{2}x^2 - 4\right) + 4}$ ✓

$= \sqrt{x^2 + 2}$ ✓

b) State the largest possible domain for $g(x)$ so that $f \circ g(x)$ is a function, and the range of $f \circ g(x)$.

$\xrightarrow{\mathbb{R}} \frac{1}{2}x^2 - 4 \xrightarrow{\geq -4} \quad \xrightarrow{\geq -2} \sqrt{2x+4}$

$\frac{1}{2}x^2 - 4 \geq -2$ ✓

$\frac{1}{2}x^2 \geq 2$

$x^2 \geq 4$

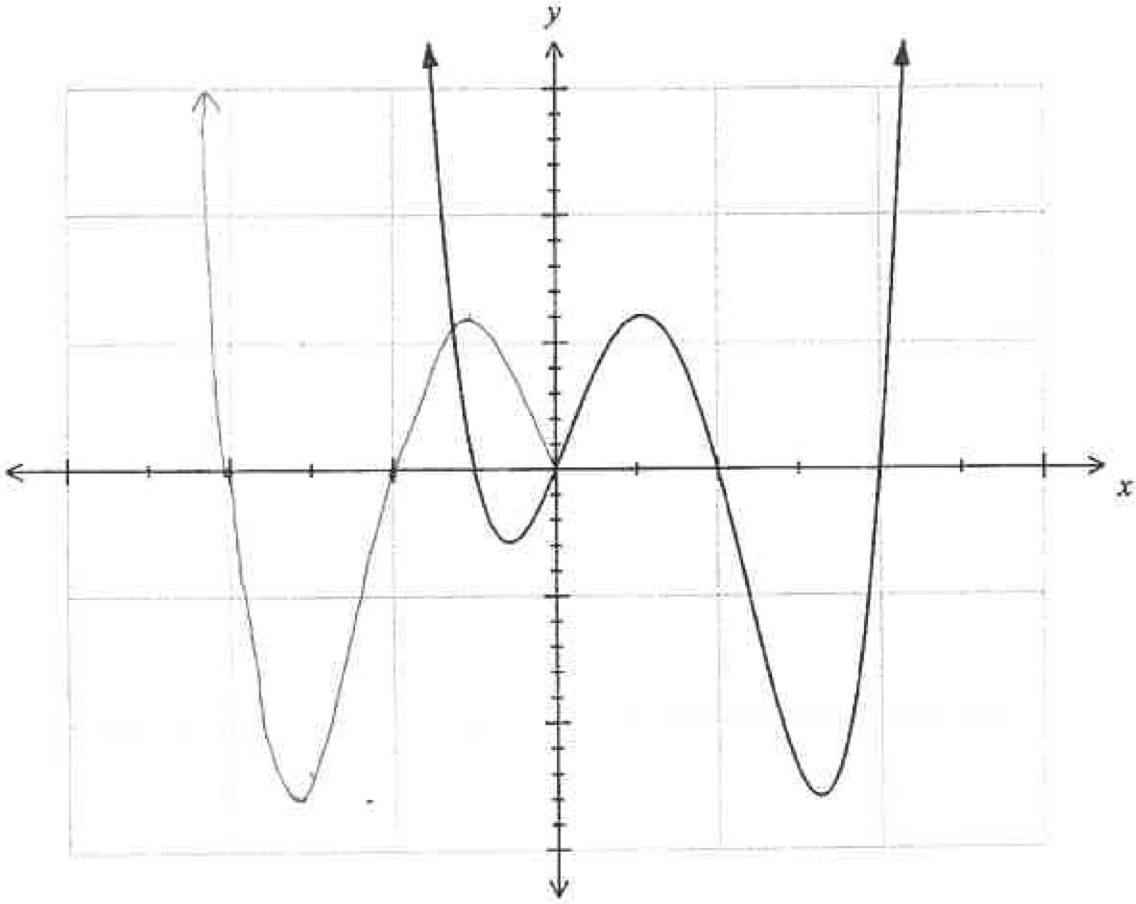
$x \leq -2 \quad x \geq 2$

$\therefore$ Domain $\{x \in \mathbb{R} : x \leq -2, x \geq 2\}$ ✓

Range $\{y \in \mathbb{R} : y \geq 0\}$ ✓

2. **[3 marks: 2, 1]**

The graph of $y = f(x)$ is shown.



a) Draw the graph of $y = f(|x|)$ on the same axes.

b) Briefly explain how to obtain the graph of $y = |f(x)|$ from $y = f(x)$.

Reflect any values of $f(x) < 0$ about the x axis.

3. [1, 2, 2, 4 = 9 marks]

Given $f(x) = (x-1)^2 - 5$ and $g(x) = \sqrt{1+x}$, determine:

a) f(0) $= -4$ ✓

b) fg(3) $g(3) = 2$ ✓

$f(2) = -4$

$\therefore fg(3) = -4$ ✓

c) Explain why fg(x) is a function for the natural domain of g(x) whereas gf(x) is not a function for the natural domain of f(x).

$\xrightarrow{\geq -1} \sqrt{1+x} \xrightarrow{\geq 0} \quad \xrightarrow{\mathbb{R}} (x-1)^2 - 5$ The range of $g(x)$ is a subset of the domain of $f(x)$ for the natural domain of $g(x)$. $\therefore fg(x)$ is a function. ✓

$\xrightarrow{\mathbb{R}} (x-1)^2 - 5 \xrightarrow{\geq -5} \quad \xrightarrow{\geq -1} \sqrt{1+x}$ The range of $f(x)$ is not a subset of the domain of $g(x)$ for the natural domain of $f(x)$. $\therefore gf(x)$ is not a function. ✓

d) Find the largest possible domain for f(x) so that the inverse of f(x) is a function. Using this domain for f(x), state the equation of $f^{-1}(x)$ and state the domain and range of $f^{-1}(x)$.

$\{x \in \mathbb{R} : x \geq 1\}$ ✓ or $\{x \in \mathbb{R} : x \leq 1\}$ ✓

$x = (y-1)^2 - 5$

$\sqrt{x+5} + 1 = y$

$\therefore f^{-1}(x) = \sqrt{x+5} + 1$ ✓

Domain $\{x \in \mathbb{R} : x \geq -5\}$ ✓

Range $\{y \in \mathbb{R} : y \geq 1\}$ ✓

$f^{-1}(x) = -\sqrt{x+5} + 1$ ✓

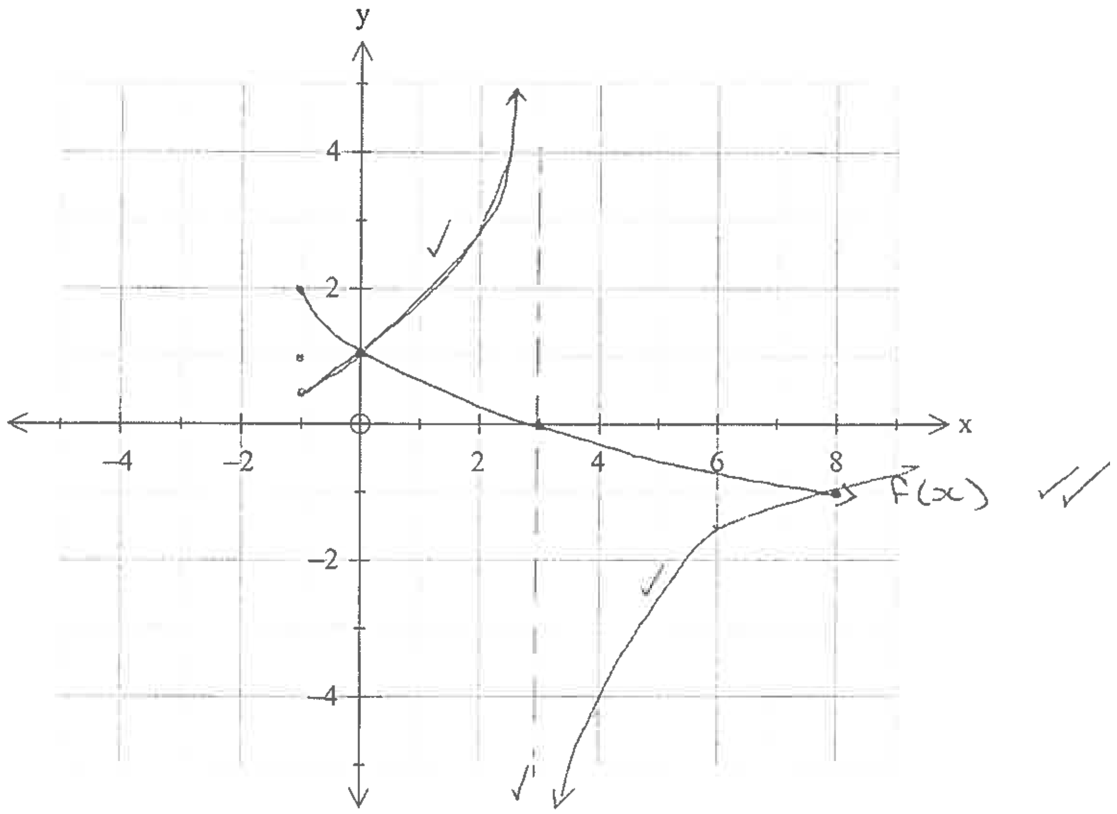
$\{x \in \mathbb{R} : x \geq -5\}$ ✓

$\{y \in \mathbb{R} : y \leq 1\}$ ✓

4. **[2, 3, 1 = 6 marks]**

a) Sketch the function $f(x) = 2 - \sqrt{x+1}$ on the axes below

b) Sketch the function $\frac{1}{f(x)}$ on the axes below



c) Use your sketch to state the domain of $\frac{1}{f(x)}$

$\{x \in \mathbb{R} : x \geq -1, x \neq 3\}$ ✓

5. [8 marks]

Sketch the following curve on the axes provided, showing any working out in the space provided. ~~Label any~~ Consider important features such as intercepts, asymptotes, turning points, . behaviour of the function approaching asymptotes and behaviour of the function for very large and very small values of x.

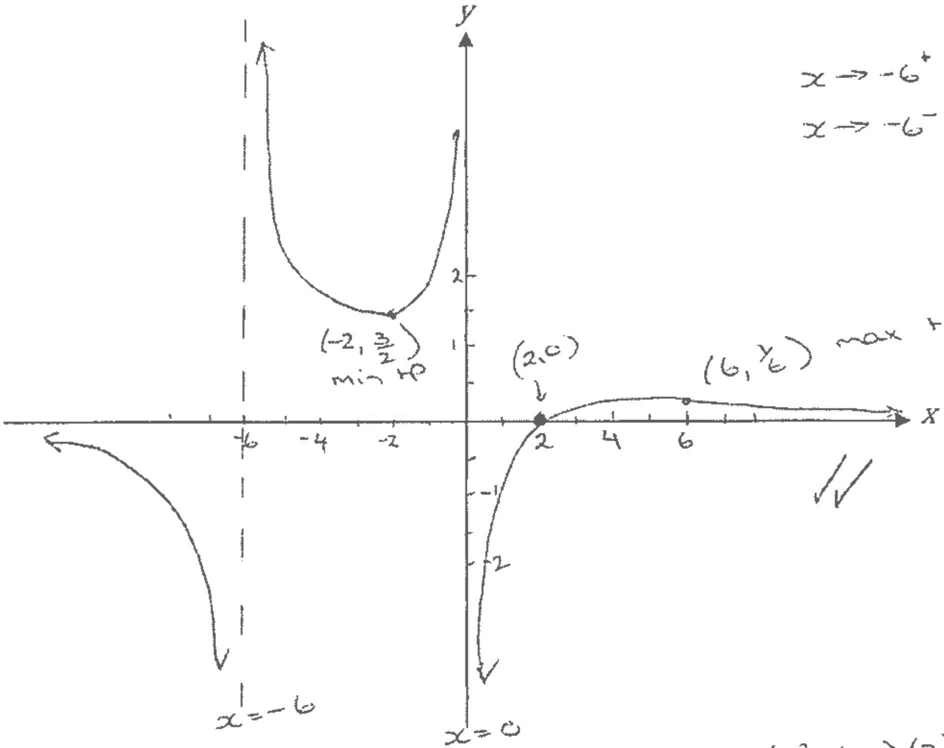
$$y = \frac{3(x-2)}{x(x+6)}$$

as $x \to 0^+$ $y \to -\infty$

$x \to 0^-$ $y \to +\infty$

$x \to -6^+$ $y \to \infty$

$x \to -6^-$ $y \to -\infty$



Roots $0 = 3(x-2)$

$x = 2$ $\therefore (2,0)$

y intercept: No y intercept

Vertical asymptotes: $x = 0$, $x = -6$ ✓

$$y = \frac{3x-6}{x^2+6x}$$

as $x \to +\infty$ $y \to \frac{3x}{x^2}$

$y \to \frac{3}{x}$

$y \to 0$ ✓

$x \to -\infty$ $y \to 0$

$$\frac{dy}{dx} = \frac{(x^2+6x)(3) - (3x-6)(2x+6)}{(x^2+6x)^2}$$ ✓

$$\frac{dy}{dx} = \frac{3x^2+18x - 6x^2 - 6x + 36}{(x^2+6x)^2}$$

$$0 = \frac{-3x^2+12x+36}{(x^2+6x)^2}$$

$0 = x^2 - 4x - 12$

$0 = (x-6)(x+2)$

$x = 6$ $x = -2$ ✓

$y = \frac{12}{72}$ $y = \frac{-12}{-8}$

$y = \frac{1}{6}$ $y = \frac{3}{2}$

$\therefore (6, \frac{1}{6})$ $(-2, \frac{3}{2})$ ✓

Test 2 2016
Calculator Section

Name: ______________________ Score: _____ / 28

Section 2 is worth 50% of your final test mark.
Calculators allowed and 1 page of A4 notes, writing on both sides.
Access to approved Sample Mathematics Specialist formulae sheet is permitted. **Time limit = 25 minutes.**

6. [6 marks]
Use an algebraic method to solve: $|3x+3| \geq |x-2|$

$x < -1$

$-(3x+3) \geq -(x-2)$ ✓

$-3x-3 \geq -x+2$

$-5 \geq 2x$

$-\frac{5}{2} \geq x$

$\therefore$ Solution $x \leq -\frac{5}{2}$ ✓

$-1 \leq x \leq 2$

$3x+3 \geq -(x-2)$ ✓

$3x+3 \geq -x+2$

$4x \geq -1$

$x \geq -\frac{1}{4}$

$\therefore$ Solution $-\frac{1}{4} \leq x \leq 2$ ✓

$x > 2$

$3x+3 \geq x-2$ ✓

$2x \geq -5$

$x \geq -\frac{5}{2}$

$\therefore$ Solution $x > 2$ ✓

$\therefore$ $x \leq -\frac{5}{2}$, $x \geq -\frac{1}{4}$.

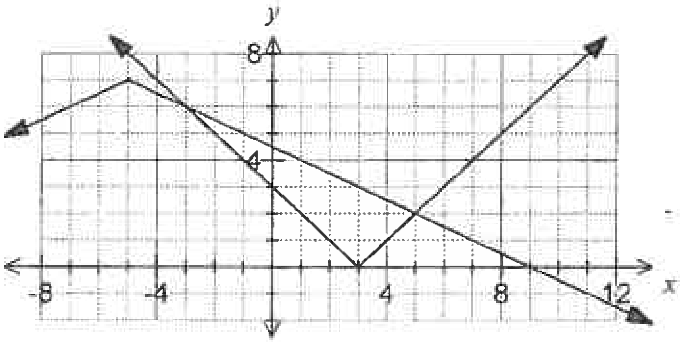
7. **[2, 2, 2, 3 = 9 marks]**

a) Solve $|7-3x| = c$, giving your solution(s) in terms of c.

$7 - 3x = c$ $\qquad\qquad$ $-7 + 3x = c$

$\frac{7-c}{3} = x$ ✓ $\qquad\qquad$ $x = \frac{c+7}{3}$ ✓

b) The graphs of $f(x) = |x-3|$ and $g(x) = 7 - \left|\frac{x}{2} + \frac{5}{2}\right|$ are shown below.



(i) Solve $g(x) = f(4)$. $\quad f(4) = 1$ ✓

$1 = 7 - \left|\frac{x}{2} + \frac{5}{2}\right|$

$x = 7, \quad x = -17$ ✓

(ii) Solve $|x-3| \geq 7 - \left|\frac{x}{2} + \frac{5}{2}\right|$

$x \leq -3, \qquad x \geq 5$

✓ ✓

(iii) Given that the solution to $|ax+b| = 7 - \left|\frac{x}{2} + \frac{5}{2}\right|$ is $-5 \leq x$, determine all possible values for a and b.

$(-5, 7) \quad (9, 0)$

$y = -\frac{7}{14}x + c$ ✓

$0 = -\frac{1}{2}(9) + c$

$\frac{9}{2} = c$

$y = -\frac{1}{2}x + \frac{9}{2}$ ✓

$|ax+b| = -\frac{1}{2}x + \frac{9}{2}$

$\therefore a = -\frac{1}{2}, \quad b = \frac{9}{2}$ or

$a = \frac{1}{2}, \quad b = -\frac{9}{2}$ ✓

8. **[6 marks]**

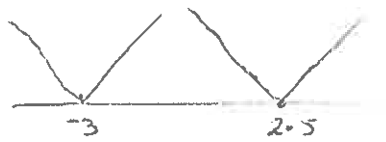
Rewrite $|2x - 5| + |x + 3| = y$ as a piecewise function.

$x < -3$

$-(2x-5) - (x+3) = y$

$-2x + 5 - x - 3 = y$

$-3x + 2 = y$



$-3 \leq x \leq \frac{5}{2}$

$-(2x-5) + (x+3) = y$

$-2x + 5 + x + 3 = y$

$-x + 8 = y$

$x > \frac{5}{2}$

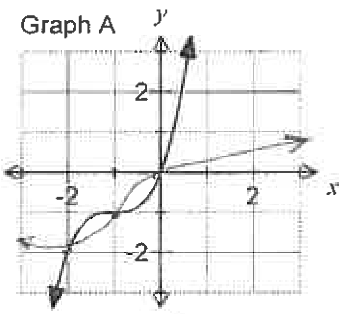
$2x - 5 + x + 3 = y$

$3x - 2 = y$

$$y = \begin{cases} -3x + 2, & x < -3 \\ -x + 8, & -3 \leq x \leq \frac{5}{2} \\ 3x - 2, & x > \frac{5}{2} \end{cases}$$

9. **[3, 2, 2 = 7 marks]**

a) For each graph below that shows a function, on the same axes sketch the inverse function. For those that do not show a function, clearly indicate which graph(s) and briefly give your reasoning in the space below the graph.



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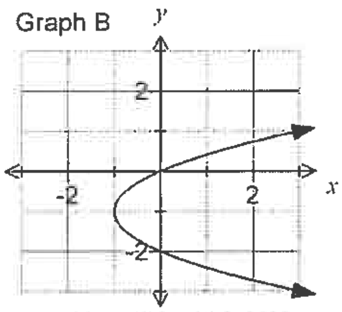
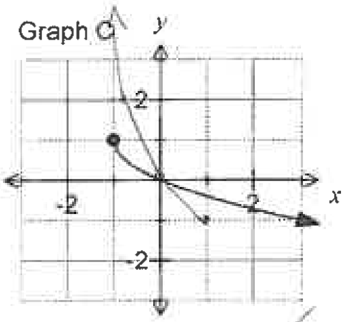


Not a function as one to many ✓



✓

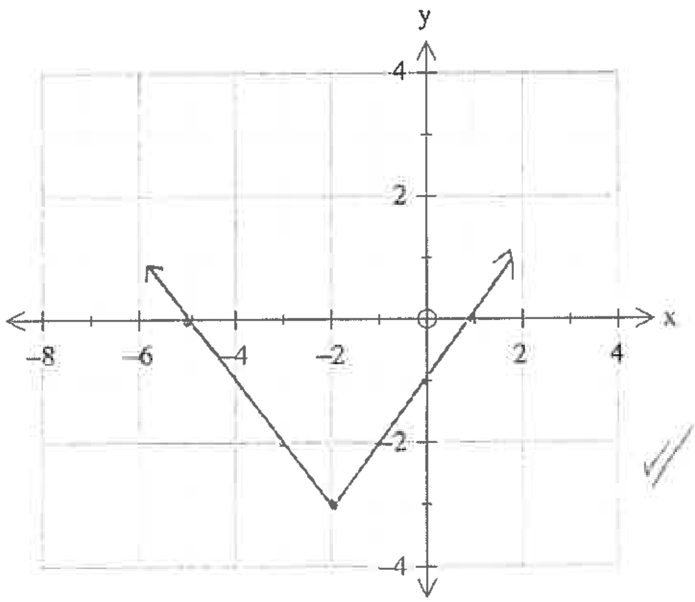
b) $f(x) = \sqrt{x} - 3$ and $g(x) = (x+2)^2$

i) Obtain an expression for $fg(x)$

$fg(x) = \sqrt{(x+2)^2} - 3$ ✓

$fg(x) = |x+2| - 3$ ✓

ii) Draw $fg(x)$ on the axes below



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