

Full Name: \_\_\_\_\_



## **MATHEMATICS**

### **Specialist Units 3 & 4**

#### **Test 4 – Integration**

#### **Chapters 13, 14 and ~~15~~**

**Semester 2 2017**

### **Section One – Calculator Free**

#### **Time allowed for this section**

Working time for this section: 35 minutes

Marks available: 35 marks

#### **Material required/recommended for this section**

##### **To be provided by the supervisor**

This Question/Answer booklet

Formula sheet

##### **To be provided by the candidate**

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

#### **Important note to candidates**

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (10 marks)

(a) Find  $\int \tan^2 4x dx$  using a suitable trigonometric formula. [2]

(b) Find  $\int (4x + 6)(2x + 1)^5 dx$  using a suitable substitution. [4]

(c) Find  $\int \frac{x+4}{(x+2)(x+1)} dx$  using partial fractions. [4]

2. (9 marks)

- (a) Determine the volume of the solid formed when the area in the first quadrant and enclosed by  $y = x^2$ , the line  $y = 3$  and the y-axis is rotated through one revolution about the y-axis. [4]

- (b) The area in the first quadrant enclosed by the curve  $y = \frac{1}{x^2}$ , the lines  $x = 1, x = k, k > 1$  and the x-axis is rotated  $360^\circ$  about the x-axis. If the volume of the solid generated is  $\frac{21\pi}{64}$  units<sup>3</sup> determine the value of the constant k. [5]

3. (8 marks)

(a) Determine  $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3 x \, dx$  using the substitution  $u = \sin x$ .

[4]

(b) Show that  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{\sin x + \cos x} \, dx = \frac{1}{2} \ln 2$

[4]

4. (8 marks)

(a) Use the substitution  $\tan\theta = x + 2$  to determine  $\int \frac{4}{x^2+4x+5} dx$

[4]

(b) Find  $\int (\sin 2x + \cos 2x) \cos 2x \, dx$

[4]

**End of Section One**

Full Name: \_\_\_\_\_



## **MATHEMATICS**

### **Specialist Units 3 & 4**

#### **Test 4 – Integration**

#### **Chapters 13, 14 and ~~15~~**

**Semester 2 2017**

### **Section Two – Calculator Assumed**

#### **Time allowed for this section**

Working time for this section: 20 minutes

Marks available: ~~20~~ marks  
10

#### **Material required/recommended for this section**

##### **To be provided by the supervisor**

This Question/Answer booklet

Formula sheet

##### **To be provided by the candidate**

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

#### **Important note to candidates**

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5. (4 marks)

The area enclosed by the x-axis, the lines  $x = 1$  and  $x = 4$  and the curve  $y = 1 + \sqrt{x}$  is rotated  $360^\circ$  about the x-axis. Calculate the volume of the solid generated to an accuracy of two decimal places.

6. (6 marks)

(a) Find  $\int \frac{5x^2 - 10x - 3}{(x+1)(x-1)^2} dx$  using partial fractions.

[3]

(b) Hence express  $\int_2^5 \frac{5x^2 - 10x - 3}{(x+1)(x-1)^2} dx$  as a single logarithm.

[3]