Narrogin Senior High School
Mathematics Department

Mathematics Applications Unit 2
Statistics ~ Normal Distribution

Name Solutions & Marking Guide Mark Total / 44

Section		Possible Marks	Marks Awarded
Part A:	Calculator free	20 marks	
Part B:	Calculator allowed	24 marks	
		Total Mark out of 44	

This test will include the following objectives from the SCSA course outline.

2.1.6 *use number deviations from the mean (standard scores) to describe deviations from the mean in normally distributed data sets*

2.1.7 *calculate quantiles for normally distributed data with known mean and standard deviation in practical situations*

2.1.8 *use the 68%, 95%, 99·7% rule for data one, two and three standard deviations from the mean in practical situations*

2.1.9 *calculate probabilities for normal distributions with known mean μ and standard deviation σ in practical situations*

Part A ~ Calculator Free 20 marks

1. A set of data is described below.

[4 marks:1,1,1,1]

N(80,25)

State: ?

a) the type of distribution

normal ✓

b) the mean

80 ✓

c) the standard deviation

5 ✓

d) the variance

25 ✓

4

2. Mangoes are graded as A, B or C grade, based on their diameters, which are normally distributed.

Diameter, x cm	$x > 12$	$8 \leq x \leq 12$	$x < 8$
Grade	A	B	C

The mean diameter is 10 cm, and 16% of mangoes are grade A.

[6 marks:2,2,2]

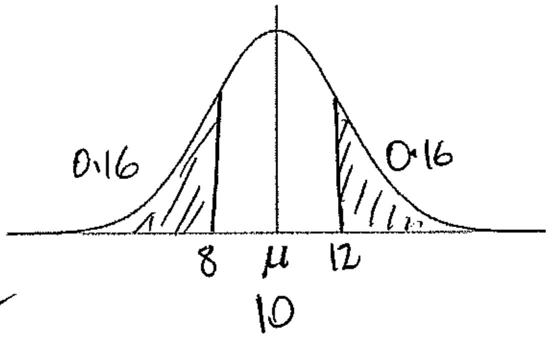
a) Determine the variance.

$100 - 16 - 16$

$= 100 - 32$

$= \underline{68\%}$ $\quad \therefore 8 = 10 - 1\sigma$ ✓

$\therefore \boxed{\sigma = 2} \Rightarrow \boxed{V = 4}$ ✓

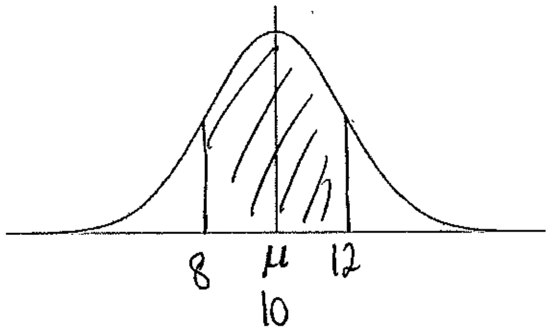


b) What percentage of mangoes are B grade?

B grade between 8 and 12 cm,

ie $\mu \pm 1\sigma$

$= 0.68$ ✓ $= \boxed{68\%}$ ✓



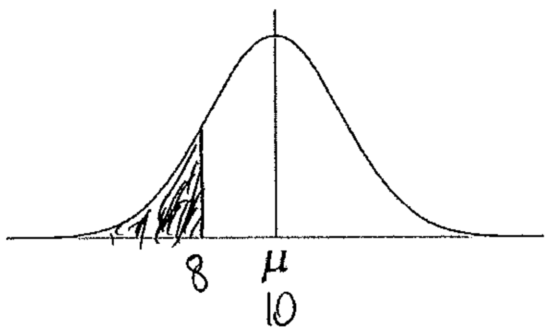
c) How many C grade mangoes would be in a picked lot of 500 mangoes?

$P(X < 8) = P(X > 12) = 0.16$

$n = 500$

500×0.16 ✓

$= \boxed{80 \text{ mangoes}}$ ✓



16
$\times 500$
$80\cancel{0}\cancel{0} \Rightarrow \div 100 = 80$

6

3. Plasma TVs have an average life span of 8 years with a standard deviation of 2 years. A replacement guarantee is in place for 2 years after purchase.

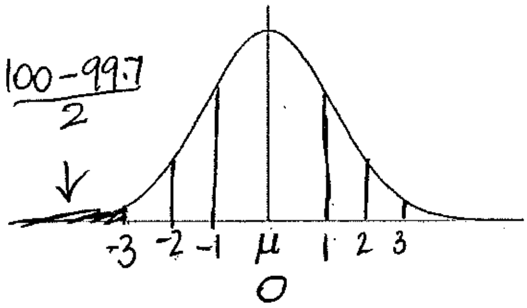
[6 marks:1,2,1,2]

a) What is the standard score for 2 years?

$$Z = \frac{x - \bar{x}}{\sigma} = \frac{2-8}{2} = \frac{-6}{2} \Rightarrow \boxed{Z = -3} \checkmark$$

b) What percentage would be expected to be replaced under the guarantee?

$$P = \frac{100 - 99.7}{2} \checkmark = \frac{0.3}{2} = \boxed{0.15\%} \checkmark$$



Phil's TV breaks down after 14 years.

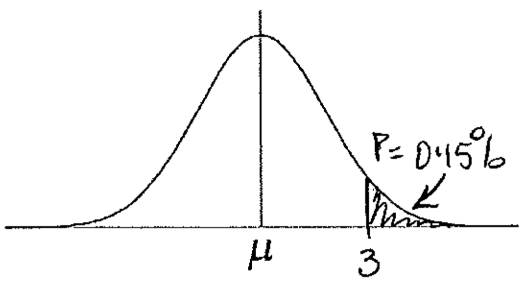
c) What is the standard score for that period?

$$Z = \frac{14-8}{2} = \frac{6}{2} = 3 \Rightarrow \boxed{Z = 3} \checkmark$$

d) How lucky/unlucky is Phil? Justify your answer mathematically.

* Phil is $\boxed{\text{very lucky}}$ ✓

* $\boxed{\text{Only } 0.15\%}$ of TV's lasted over 14 years. ✓



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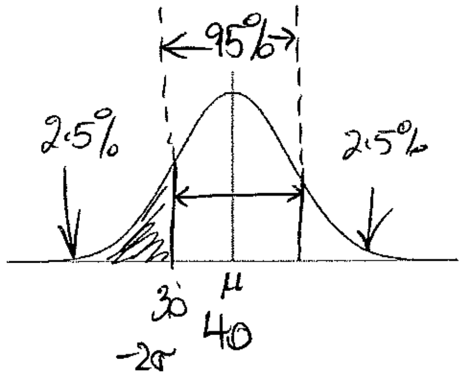
4. The ages of doctors in a city are normally distributed with a mean of 40 years and a standard deviation of 5 years. Use the "68, 95, 99·7" rule to determine the probability that a randomly selected doctor from the city:

[4 marks:1,1,1,1]

a) is less than 30

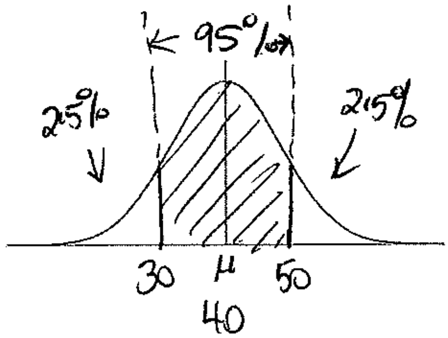


$40 - 30 = 10 \Rightarrow 10 \div 5 = 2$

$\therefore -2\sigma$ from the mean

$\boxed{2.5\% \text{ or } 0.025}$ ✓

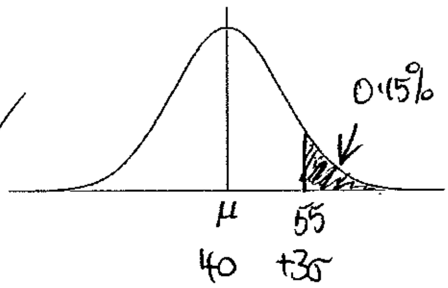
b) is between 30 and 50



2σ from the mean

$\therefore \boxed{95\% \text{ or } 0.95}$ ✓

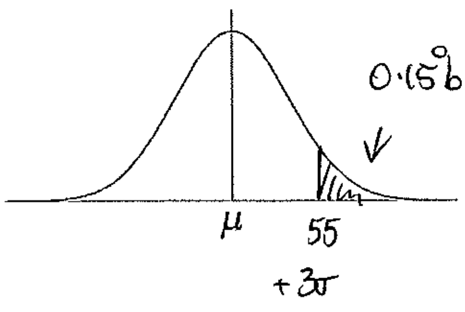
c) is more than 55



$55 = 40 + 3\sigma$

$\therefore P = \dfrac{100 - 99.7}{2} = \dfrac{0.3}{2} = \boxed{0.15\% \text{ or } 0.0015}$ ✓

d) is not more than 55



$100 - 0.15$

$= \boxed{99.85\% = 0.9985}$ ✓

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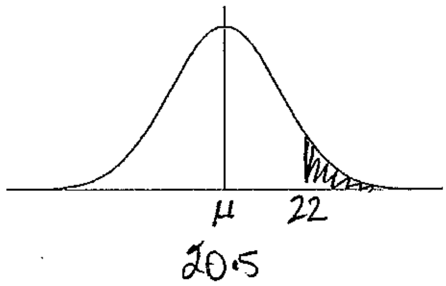
Part B ~ Calculator Allowed 29 marks

1. A variable X is normally distributed with mean 20·5 and standard deviation 4·3.
Find:

[5 marks: 1,2,2]

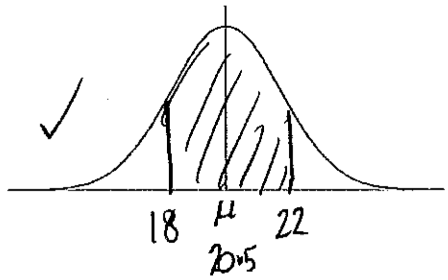
a) $P(X \geq 22)$

$\boxed{P = 0{\cdot}3636}$ ✓



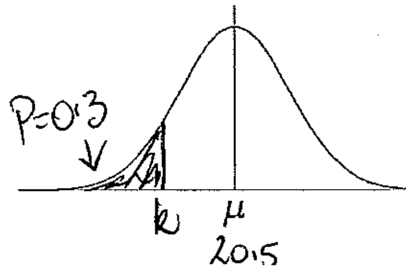
b) $P(18 \leq X \leq 22)$

$\boxed{P = 0{\cdot}3559}$ ✓

✓


c) k such that $P(X \leq k) = 0{\cdot}3$

$\boxed{k = 18{\cdot}25}$ ✓

✓


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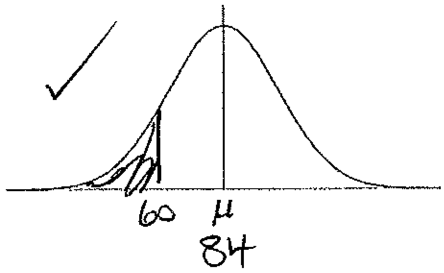
2. The weights of tomatoes sold by a market gardener were observed to be normally distributed with a mean of 84 g and a standard deviation of 12 g.

[8 marks: 2,1,2,3]

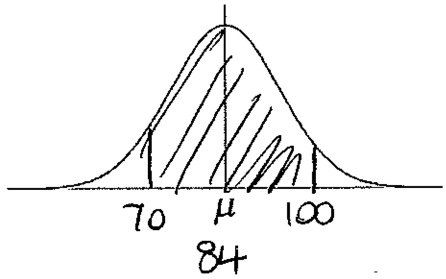
a) What proportion of tomatoes will weigh less than 60 g?

$\boxed{P = 0.0228}$



b) What is the probability that a randomly selected tomato will weigh between 70 and 100 g?

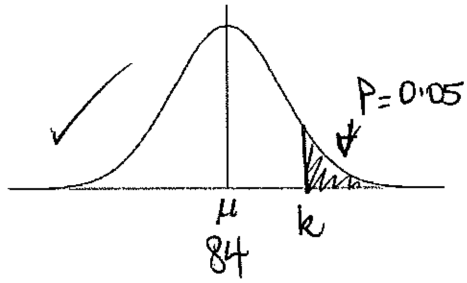
$\boxed{P = 0.7871}$



c) Above what weight will the heaviest 5% of tomatoes fall?

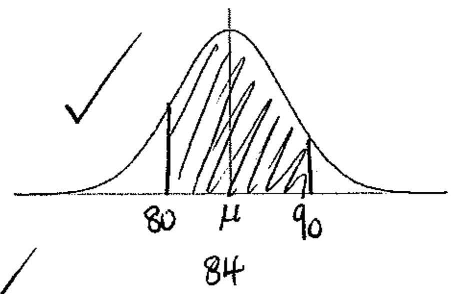
$P = 0.05$

$\boxed{k = 103.74g}$



d) Premium tomatoes, those with a weight between 80 g and 90 g, can be sold for more than other tomatoes. Estimate the number of premium tomatoes that would be found in a box of 288 randomly packed tomatoes.

$P(80 < X < 90) = \boxed{0.3220}$



$0.3220 \times 288 = 92.74 \approx \boxed{93 \text{ tomatoes}}$

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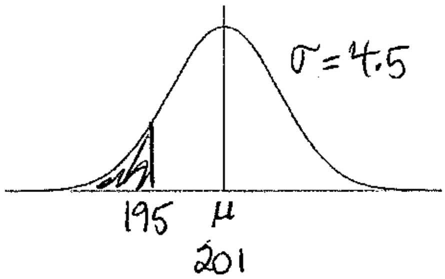
3. A machine is set to fill packets of potato chips with 200 g of chips. However, due to the inaccuracy of this type of machine the actual weights in packets are normally distributed with a mean of 201 g and a standard deviation of 4·5 g. A quality control measure used by the factory is to weigh each packet after filling and recycle any packet with less than 195 g.

[6 marks: 2,2,2]

a) What percentage of packets will be recycled?

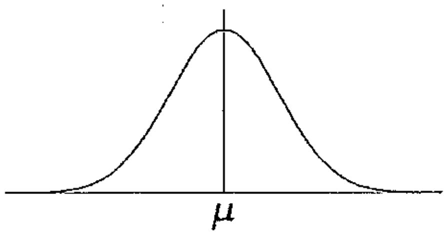
$P = 0{\cdot}0912 = \boxed{9{\cdot}12\%}$



b) If the factory produces 12000 packets per day how many will be recycled in one day?

$0{\cdot}0912 \times 12000$

$= 1094{\cdot}4 \approx \boxed{1094 \text{ packets}}$

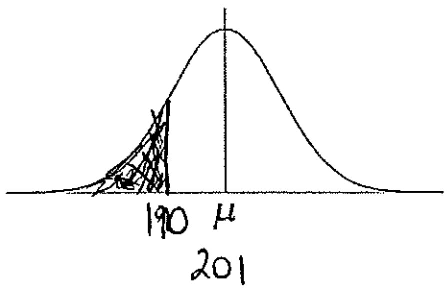


c) If a packet is selected from those destined for recycling what is the probability that its weight is less than 190 g?

$P(X<190) = 0{\cdot}0073$

$P(\text{recycled}) = 0{\cdot}0912$

$P = \dfrac{0{\cdot}0073}{0{\cdot}0912} = \boxed{0{\cdot}08}$



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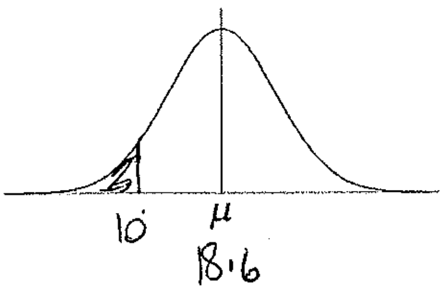
4. a) The maximum daily temperature at a city airport in January is known to follow a normal distribution with a mean of 18·6°C and a standard deviation of 4·5°C.

[10 marks: 1,2,2,2,1,2]

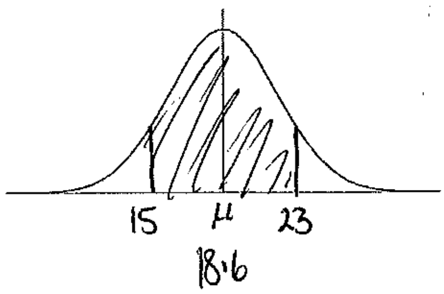
(i) What is the probability of a January day at the airport having a maximum temperature of less than 10°C?

$P(x < 10) = \boxed{0 \cdot 0280}$ ✓



(ii) How many days in January would you expect the airport to have a maximum temperature of between 15°C and 23°C?

$P(15 < x < 23) = \boxed{0 \cdot 6241}$ ✓

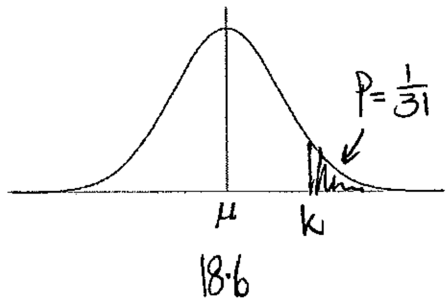


Number of days $= 0 \cdot 6241 \times 31 = 19 \cdot 3471$

$\approx \boxed{19 \text{ days}}$ ✓

(iii) Find the temperature at the airport for which only one day in January might be expected to be higher than.

$P(1 \text{ day}) = \frac{1}{31}$



$P(x > k) = \frac{1}{31}$ ✓

$\therefore \boxed{k = 26 \cdot 9°C}$ ✓

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