education equals

Calculator Free
Non-Linear Functions

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 2 = 8 marks]

$x^2 + y^2 = 25$ $\quad xy = -4$ $\quad y = x(x-4)^2$ $\quad \dfrac{3}{x-2} = y+5$ $\quad y^2 = 4x^2$

$y = 2-(x-4)^3$ $\quad y = \sqrt{3x-4}$ $\quad y = \dfrac{-2}{x+1}$ $\quad y = x(x-3)$

From the above list of functions and relations, state all those:

(a) representing a cubic function.

(b) representing polynomial functions.

(c) representing reciprocal functions.

(d) whose graphs have domains that do not exist for all real values.

Question Two: [1, 5, 2 = 8 marks]

(a) Show that $x-2$ is a factor of $f(x)=x^3+10x^2+8x-64$

(b) Hence factorise $f(x)$ and state the other two factors.

(c) Using your results from (a) and (b) determine the roots of the function $f(x)$.

Question Three: [2, 3, 1, 1 = 7 marks]

(a) Represent $f(x)=-2(x-1)^2-5$ in the form $f(x)=ax^2+bx+c$

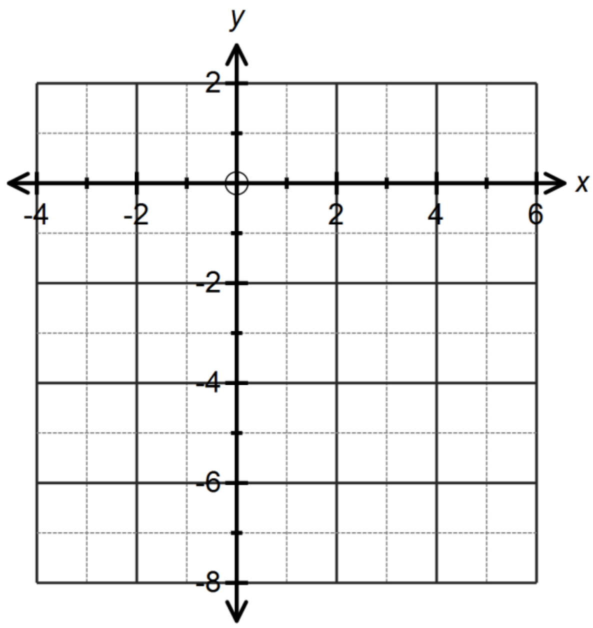
(b) Represent $g(x)=-x(x+4)(2x-3)$ in the form $g(x)=ax^3+bx^2+cx+d$

(c) State the degree of the polynomial given in part (b).

(d) State the coefficient of the x term in part (a).

Question Four: [2, 2 = 4 marks]

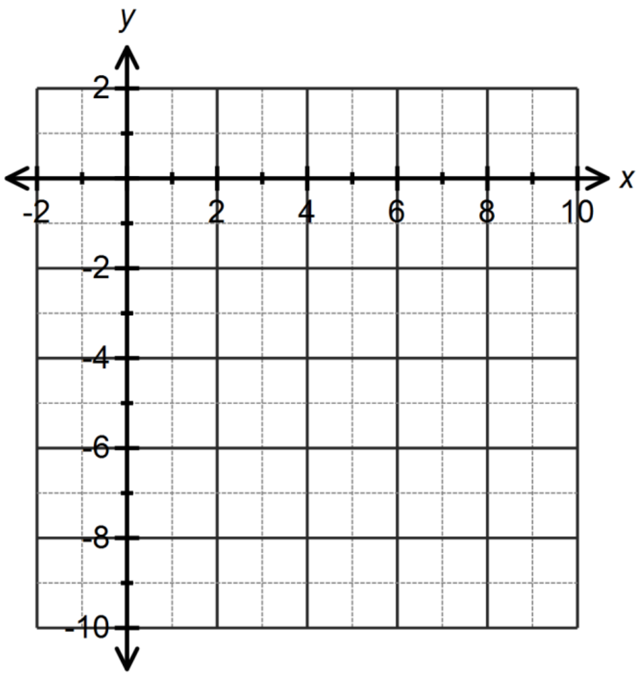
(a) Sketch the graph of $(x-2)^2+(y+3)^2=9$ on the axes below.



(b) Explain, using the vertical line test, why your graph does not represent a function.

Question Five: [2, 2, 2, 2 = 8 marks]

(a) Sketch the function $h(x) = -\sqrt{x-4} - 2$ on the axes below.



(b) Sketch the function $f(x) = \sqrt{x-4} - 2$ on the set of axes above.

(c) Write an equation which represents the combined relationship of the two graphs drawn.

(d) Does your equation in part (c) represent a function? Explain your answer.

Question Six: [2, 8 = 10 marks]

(a) Consider the function $f(x)=\dfrac{20}{x}$.

Over what domain would this function represent the following context: The number of hours required to lay bricks with x bricklayers.

(b) Consider the function $g(x)=\dfrac{-2}{x-1}+3$.

Determine:

(i) The equation of the horizontal asymptote.

(ii) The equation of the vertical asymptote.

(iii) The coordinates of the x and y intercepts.

(iv) The behaviour of y as $x \to \infty$

(v) The equation, in terms of x, of the function $h(x)=-g(x+5)-2$

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Question One: [2, 2, 2, 2 = 8 marks]

$x^2 + y^2 = 25$ $\quad xy = -4$ $\quad y = x(x-4)^2$ $\quad \dfrac{3}{x-2} = y+5$ $\quad y^2 = 4x^2$

$y = 2-(x-4)^3$ $\quad y = \sqrt{3x-4}$ $\quad y = \dfrac{-2}{x+1}$ $\quad y = x(x-3)$

From the above list of functions and relations, state all those:

(a) representing a cubic function.

$$y = x(x-4)^2 \checkmark$$
$$y = 2-(x-4)^3 \checkmark$$

(b) representing polynomial functions.

$$y = x(x-4)^2$$
$$y = 2-(x-4)^3$$
$$y = x(x-3)$$

✓ ✓

(c) representing reciprocal functions.

$$\frac{3}{x-2} = y+5 \checkmark$$
$$y = \frac{-2}{x+1} \checkmark$$
$$xy = -4$$

(d) whose graphs have domains that do not exist for all real values.

$$x^2 + y^2 = 25 \quad xy = -4 \checkmark$$
$$y = \sqrt{3x-4} \quad y = \frac{-2}{x+1} \checkmark$$

Question Two: [1, 5, 2 = 8 marks]

(a) Show that $x-2$ is a factor of $f(x)=x^3+10x^2+8x-64$

$$\begin{aligned} f(2) &= 2^3+10(2)^2+8(2)-64 \\ &= 8+40+16-64 \\ &= 0 \end{aligned}$$

(b) Hence factorise $f(x)$ and state the other two factors.

$$\begin{array}{r l} & x^2+12x+32 \\ x-2 \big) & x^3+10x^2+8x-64 \\ & \underline{x^3-2x^2} \\ & \quad 12x^2+8x \\ & \quad \underline{12x^2-24x} \\ & \qquad 32x-64 \\ & \qquad \underline{32x-64} \\ & \qquad\qquad 0 \end{array}$$

$$\begin{aligned} f(x) &= (x-2)(x^2+12x+32) \\ &= (x-2)(x+8)(x+4) \end{aligned}$$

The other two factors are $x+8$ and $x+4$

(c) Using your results from (a) and (b) determine the roots of the function $f(x)$.

$(2,0)$ $(-8,0)$ $(-4,0)$

Question Three: [2, 3, 1, 1 = 7 marks]

(a) Represent $f(x) = -2(x-1)^2 - 5$ in the form $f(x) = ax^2 + bx + c$

$$\begin{aligned} f(x) &= -2(x^2 - 2x + 1) - 5 \\ &= -2x^2 + 4x - 7 \end{aligned}$$

(b) Represent $g(x) = -x(x+4)(2x-3)$ in the form $g(x) = ax^3 + bx^2 + cx + d$

$$\begin{aligned} g(x) &= -x(2x^2 - 3x + 8x - 12) \\ &= -x(2x^2 + 5x - 12) \\ &= -2x^3 - 5x^2 + 12x \end{aligned}$$

(c) State the degree of the polynomial given in part (b).

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(d) State the coefficient of the x term in part (a).

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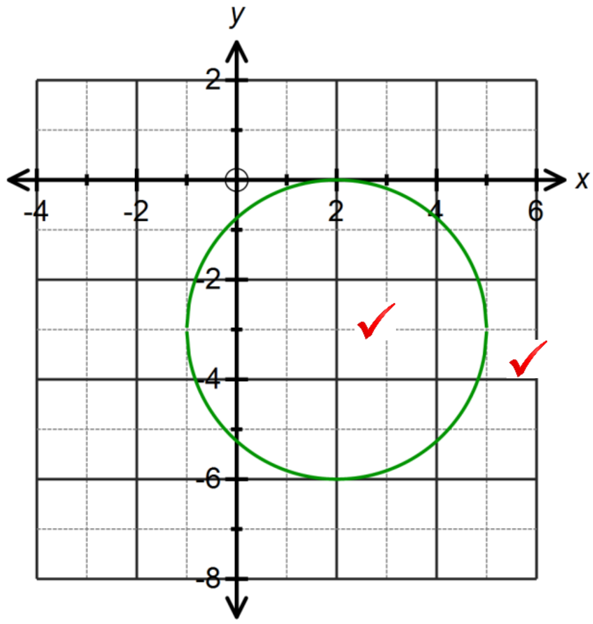
Question Four: [2, 2 = 4 marks]

(a) Sketch the graph of $(x-2)^2+(y+3)^2=9$ on the axes below.

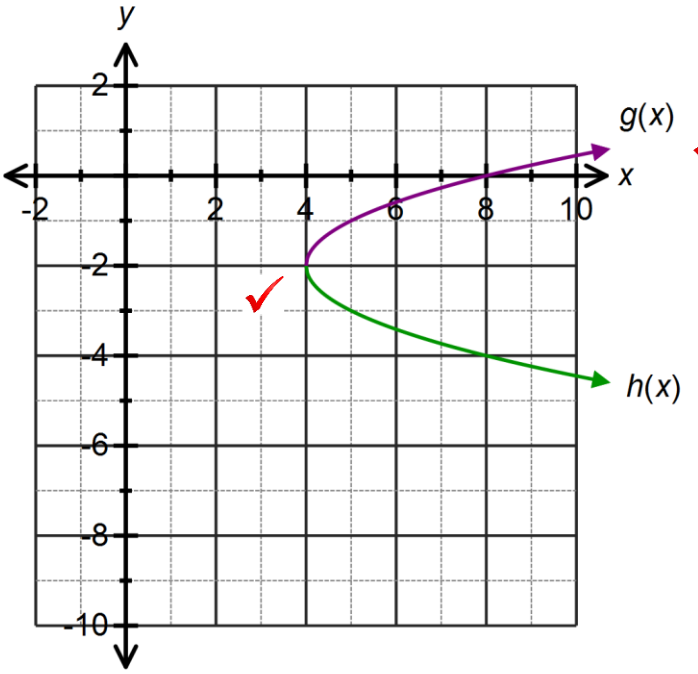


(b) Explain, using the vertical line test, why your graph does not represent a function.

On the interval $-1 < x < 5$, a vertical line through the graph shows that for every x value there are two y values and thus this is a 1 – to – many relationship and hence not a function.

Question Five: [2, 2, 2, 2 = 8 marks]

(a) Sketch the function $h(x) = -\sqrt{x-4} - 2$ on the axes below.



(b) Sketch the function $f(x) = \sqrt{x-4} - 2$ on the set of axes above.

(c) Write an equation which represents the combined relationship of the two graphs drawn.

$$x = (y+2)^2 + 4$$

(d) Does your equation in part (c) represent a function? Explain your answer.

No. The graph of this relationship, as shown above, fails the vertical line test and has more than one y value for each x value.

Question Six: [2, 8 = 10 marks]

(a) Consider the function $f(x) = \frac{20}{x}$.

Over what domain would this function represent the following context: The number of hours required to lay bricks with *x* bricklayers.

$$x : \{1, 2, 4, 5, 10, 20\}$$ ✓ ✓

(b) Consider the function $g(x) = \frac{-2}{x-1} + 3$.

Determine:

(i) The equation of the horizontal asymptote.

$$y = 3$$ ✓

(ii) The equation of the vertical asymptote.

$$x = 1$$ ✓

(iii) The coordinates of the *x* and *y* intercepts.

$$y = \frac{-2}{-1} + 3 = 5 \quad (0,5)$$ ✓

$$0 = \frac{-2}{x-1} + 3$$ ✓

$$-3(x-1) = -2$$

$$-3x = -5$$

$$x = \frac{5}{3} \qquad (\frac{5}{3}, 0)$$ ✓

(iv) The behaviour of *y* as $x \to \infty$

$$y \to 3$$ ✓

(v) The equation, in terms of *x*, of the function $h(x) = -g(x+5) - 2$

$$h(x) = -[\frac{-2}{x+5-1} + 3] - 2$$ ✓

$$= \frac{2}{x+4} - 5$$ ✓