MINDARIE
SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS

MATHEMATICS:
SPECIALIST 1 & 2

SEMESTER 1 2018
TEST 3

Name_____

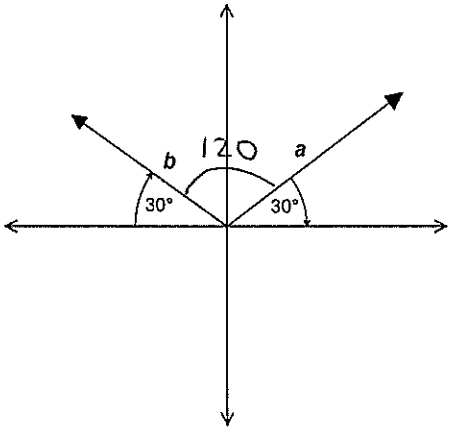
Calculator Free

Reading time: 2 mins
Time allowed: 23 mins

Total marks: 22

1. [5 marks: 2, 3]

Given the 2 vectors below ***a*** and ***b*** and $|\boldsymbol{a}| = 5$ and $|\boldsymbol{b}| = 6$



Determine the exact values of

(a) $\boldsymbol{a} \cdot \boldsymbol{a} = a^2$ ✓

$= 5^2$

$= 25$ ✓

(b) $\boldsymbol{b} \cdot \boldsymbol{a} = 6 \times 5 \times \cos 120$ ✓

$= 30 \times -\frac{1}{2}$ ✓

$= -15.$ ✓

2. [3 marks]

If **a** = 4**i** − 3**j** and **b** = -6**i** + 8**j**, find m if $\begin{pmatrix} 2 \\ m \end{pmatrix}$ is perpendicular to **b**.

$= 2\underline{i} + m\underline{j}$

$-6 \times 2 + 8 \times m = 0$ ✓

$-12 + 8m = 0$

$8m = 12$ ✓

$m = 12/8$

$m = 1.5$ ✓

3. [4 marks]

The vectors **a** and **b** are given by **a** = (5,12) and **b** = (2,-1).
Determine the **vector** projection of **a** on **b**.

$(\underline{a} \cdot \hat{\underline{b}})\hat{\underline{b}}$ $\qquad \hat{\underline{b}} = \frac{1}{\sqrt{5}}(2\underline{i} - \underline{j})$ ✓

$\|$

$\left[(5\underline{i} + 12\underline{j}) \cdot \frac{1}{\sqrt{5}}(2\underline{i} - \underline{j})\right] \frac{1}{\sqrt{5}}(2\underline{i} - \underline{j})$ ✓

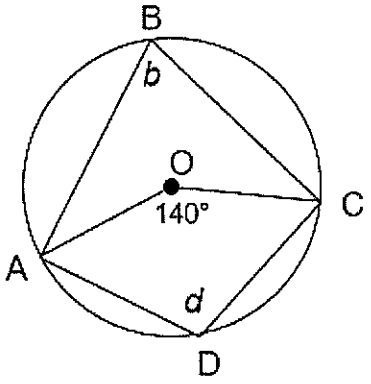
$5 \times 2 - 12$

$= \frac{-2}{\sqrt{5}} \times \frac{1}{\sqrt{5}}(2\underline{i} - \underline{j})$ ✓

$= \frac{-2}{5}(2\underline{i} - \underline{j})$ ✓

4. [6 marks: 3, 3]

(a) A circle centred at O has s∠AOC = 140°, as shown in the diagram. Determine the values of *b* and *d*. Justify your answers.



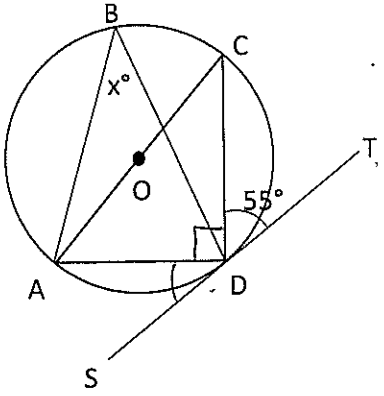
$b = 70^\circ$ ✓ – angle at centre is double angle at circumference.

$\angle AOC = 220^\circ$ ✓ – angle sum around a point.

$d = 110^\circ$ ✓ angle at centre is double angle at circumference.

– 1 no reasons.

(b) A circle centred at O has a tangent ST as shown in the diagram. Given that s∠CDT = 55°, determine the value of x. Justify your answer.



$\angle CDA = 90^\circ$ ✓ – angle in semi-circle

$\angle SDA = 35^\circ$ ✓ – angle sum on straight line.

$x = 35$ ✓ – alt. segment theorem.

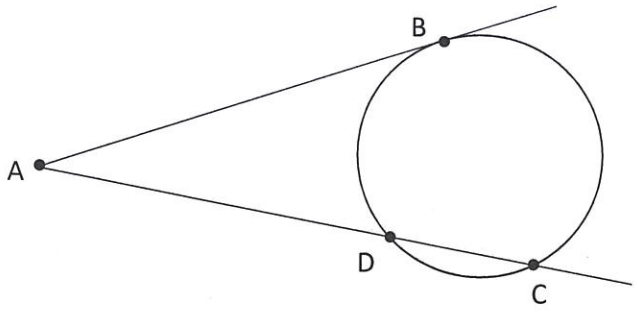
– 1 no reasons.

5. [4 marks: 1,3]

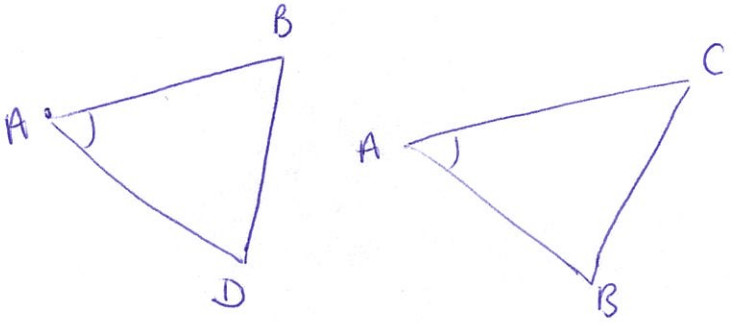
A line drawn from a point A forms a tangent to a circle at B. A secant from A cuts through the same circle at point C and D.

(a) State a relationship between the lengths of the line segments AB, AD and AC.

$AB^2 = AD \times AC$ ✓



(b) Hence prove that $\Delta ABD \sim \Delta ACB$.



$\angle BAD = \angle CAB$ - same angle ✓

$$\frac{AC}{AB} = \frac{\frac{AB^2}{AD}}{AB}$$

$$= \frac{AB^2}{ADAB}$$ ✓

$\frac{AC}{AD} = \frac{AB}{AD}$ - Sides in same ratio

$\therefore \Delta ABD \sim \Delta ACB$ (SAS) ✓

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Calculator Assumed

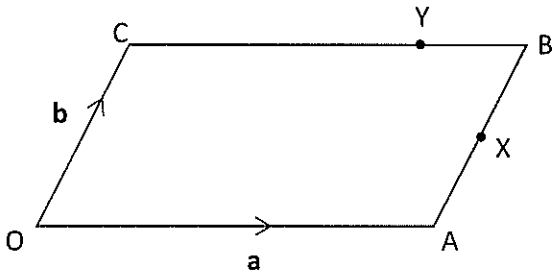
Reading time: 3 mins
Time allowed: 32 mins

Total marks: 30

6. [5 marks: 2, 3]

OABC is a parallelogram, X is the midpoint of AB and Y is such that $\overrightarrow{CY} = \frac{2}{3}\overrightarrow{CB}$.

Let $\overrightarrow{OA}$ = **a** and $\overrightarrow{OC}$ = **b**.



(a) Express $\overrightarrow{OX}$ and $\overrightarrow{OY}$ in terms of **a** and/or **b**.

$\overrightarrow{OX} = \underline{a} + \frac{1}{2}\underline{b}$ ✓

$\overrightarrow{OY} = \underline{b} + \frac{2}{3}\underline{a}$ ✓

(b) Show that $\overrightarrow{OX} \bullet \overrightarrow{OY} = \frac{4}{3}\mathbf{a} \bullet \mathbf{b} + 8$, given $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 2$.

$\overrightarrow{OX} \cdot \overrightarrow{OY} = \left(\underline{a} + \frac{1}{2}\underline{b}\right) \cdot \left(\underline{b} + \frac{2}{3}\underline{a}\right)$ ✓

$= \underline{a} \cdot \underline{b} + \frac{2}{3}a^2 + \frac{1}{2}b^2 + \frac{2}{6}\underline{a} \cdot \underline{b}$ ✓

$= \frac{4}{3}\underline{a} \cdot \underline{b} + \frac{2}{3}(9) + \frac{1}{2}(4)$ ✓

$= \frac{4}{3}\underline{a} \cdot \underline{b} + 8.$

7. [7 marks: 4, 3]

(a) Vectors **a** and **b** have the same magnitude and vectors **a** and **c** are perpendicular, where $\mathbf{a}=\begin{bmatrix} m \\ n \end{bmatrix}$, $\mathbf{b}=\begin{bmatrix} -4 \\ 6 \end{bmatrix}$ and $\mathbf{c}=\begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Determine the values of m and n.

$m^2+n^2 = 16+36$ ✓

$m^2+n^2 = 52$

$m \times 2 + n \times 3 = 0$

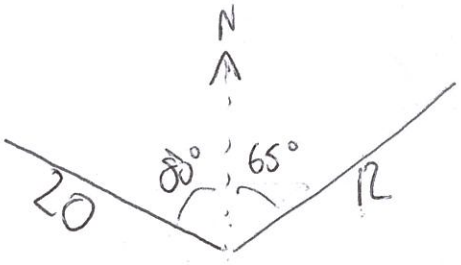
$2m + 3n = 0$ ✓

Solve on classpad

$m = -6$ ✓ or $m = 6$ ✓

$n = 4$ $\quad$ $n = -4$

(b) Determine the scalar projection of a velocity of 12 m/s on a bearing of 65° onto a velocity of 20 m/s on a bearing of 280°, giving your answer to three significant figures.



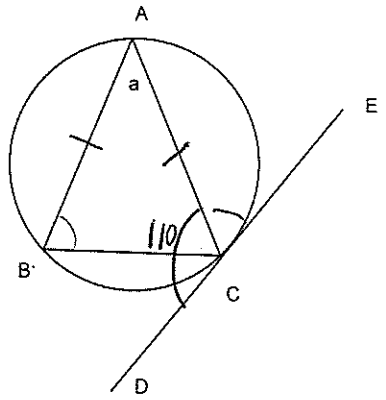
Scalar projection $= |\underline{a}|\cos\theta$

$= 12\cos 145$ ✓✓

$= -9.83$ ✓

8. [10 marks: 3, 4, 3]

(a) Determine the size of angle **a** in the diagram below.
A, B and C lie on a circle. DE is a tangent at C. AB = AC and ∠DCA = 110°



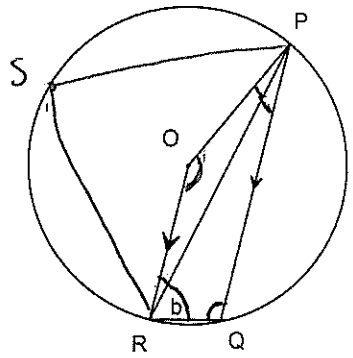
$\angle ECA = 70°$ – angle sum on a straight line.

✓ $\angle CBA = 70°$ – alt. segment

✓ $\angle DCB = 40°$ – base angles of isos. triangles same

$a = \angle BAC = 40°$ – alt. segment (or angle sum of tri.)
✓ ✓

(b) Determine the size of angle **b** in the diagram below.
P, Q and R lie on a circle. PQ is parallel to OR and ∠ORQ = 68°, ∠PRQ = b°,



$\angle ORQ = 68°$ then $\angle RQP = 112°$
✓ (supplementary between parallel lines)

$\angle ORP = 68 - b$ ✓
and $\angle OPR = 68 - b$ (isos. triangle)

$\angle RSP = 180 - 112$
$= 68°$ (opp. angles in cyclic quad.)
✓

$\angle ROP = 136°$ (angle at centre)

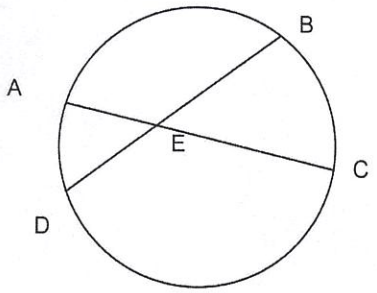
$\angle ORP = \frac{180 - 136}{2}$ (isos. triangle

$= 22°$

but $\angle ORP = 68 - b$ $\quad 68 - b = 22$ ✓

$\therefore b = 46°$

(c) In the circle shown (not to scale), chords AC and BD intersect at E.
If AE = $2x$, BE = $x-1$, CE = $2x-1$ and DE = $5x$, determine the length x.
Justify your answer.



$AE \times CE = BE \times DE$

$2x \times (2x-1) = (x-1) \times 5x$ ✓

$4x^2 - 2x = 5x^2 - 5x$

$0 = x^2 - 3x$ ✓

$0 = x(x-3)$

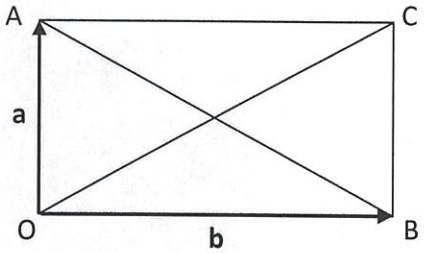
$x = 0 \quad x = 3$

↓ length can't be 0

✓ (must exclude $x=0$)

9. [4 marks]

Prove that if the diagonals of a rectangle are perpendicular then the rectangle is a square.



$\vec{OC} = \underline{a} + \underline{b}$ ✓

$\vec{AB} = \underline{b} - \underline{a}$

Diagonals perp. $\Rightarrow \vec{OC} \cdot \vec{AB} = 0$

$(\underline{a}+\underline{b}) \cdot (\underline{b}-\underline{a}) = 0$ ✓

$\underline{a}\cdot\underline{b} - a^2 + b^2 - \underline{a}\cdot\underline{b} = 0$

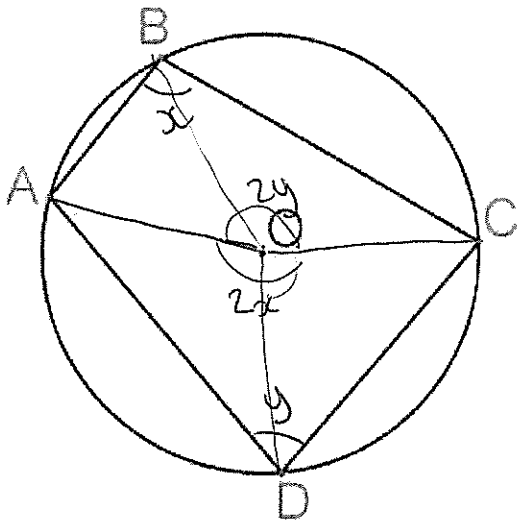
$b^2 - a^2 = 0$

$b^2 = a^2$ ✓

$|\underline{b}| = |\underline{a}|$ ∴ Rectangle is a square. ✓

10. [4 marks]

Prove that opposite angles in a cyclic quadrilateral are supplementary.
State reasons for each stage of your proof.



If $\angle ABC = x$ then

reflex $\angle AOC = 2x$ (angle at centre)

If $\angle ADC = y$ then $\angle AOC = 2y$ (angle at centre)

$2x + 2y = 360$ (angle sum around a point.)

✓ $2(x+y) = 360$

$\therefore x + y = 180$

and $\angle ADC + \angle ABC = 180$

Similarly, if $\angle DAB = w$ then reflex $\angle DOB = 2w$

$\angle DCB = z$ then $\angle DOB = 2z$ ✓

$2w + 2z = 360$

$2(w+z) = 360$

$w + z = 180$ ✓

and $\angle DAB + \angle DCB = 180$

Also accept splitting quadrilateral into 4 isosceles triangles.