Name : Ann

Esperance Senior High School
Year 12 MATHEMATICS METHODS
TEST 1 2017

CALCULATOR AND RESOURCE FREE

Total Marks: 27
Reading: 2 minutes **Time Allowed: 30 minutes**

1. [4 marks]

A function $y = f(x)$ is such that $y = ax^3 + bx^2 + 3x + 2$
The function has a maximum point at (1, 3).
Determine the values of a and b.

$\frac{dy}{dx} = 3ax^2 + 2bx + 3$

at $x = 1$ $\frac{dy}{dx} = 0$ & $y = 3$.

$\therefore 3 = a + b + 3 + 2$

i.e. $a + b = -2$

$\therefore a = -2 - b$

& $3a + 2b + 3 = 0$

$\therefore -6 - 3b + 2b + 3 = 0$

$\therefore b = -3$

$\therefore a = 1$

2. [4 marks]

If the radius of a sphere is measured with an error of at most 4%, estimate the percentage error in the volume of the sphere.

Find $\frac{\delta V}{V}$ $\frac{\delta r}{r} = 4\%$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\delta V \approx \frac{dV}{dr} \times \delta r$

$\frac{\delta V}{V} = \frac{4\pi r^2 \times \delta r}{\frac{4}{3}\pi r^3}$

$= 3 \times \frac{\delta r}{r}$

$= 12\%$

3. [12 marks]

Determine the gradient function $\frac{dy}{dx}$ for each of the following.

Leave your answers with positive indices, where necessary. Do not simplify.

(a) $y = ax^a$ (Where a is a positive constant) [1]

$\frac{dy}{dx} = a^2 x^{a-1}$

(b) $y = \frac{5x^3 + 4x^4 + \pi^2}{2x}$ [2]

$y = \frac{5}{2}x^2 + 2x^3 + \frac{\pi^2}{2x}$

$\frac{dy}{dx} = \frac{2x(15x^2 + 16x^3) - 2(5x^3 + 4x^4 + \pi^2)}{4x^2}$ OR $\therefore \frac{dy}{dx} = 5x + 6x^2 - \frac{\pi^2}{2x^2}$

(c) $y = (x^3 - 4x)^3(3x^2)$ [3]

$\frac{dy}{dx} = 3x^2 \cdot (3 \cdot (3x^2 - 4)(x^3 - 4x)^2 + 6x \cdot (x^3 - 4x)^3$

(d) $y = [3 + \cos(x/2)]^4$ [3]

$\frac{dy}{dx} = -4 \times \frac{1}{2} \sin(\frac{x}{2}) \cdot (3 + \cos(\frac{x}{2}))^3$

(e) $y = \frac{3x^2}{e^{5x} - 2}$ [3]

$\frac{dy}{dx} = \frac{(e^{5x} - 2) \cdot 6x - 3x^2(5e^{5x})}{(e^{5x} - 2)^2}$

4. [7 marks]

$$f(x) = -x^2 + 4x + 5$$

(a) Evaluate:

(i) $f(2)$ [1]

$= -4+8+5$

$= 9$

(ii) $f'(2)$ [1]

$f'(x) = -2x+4$

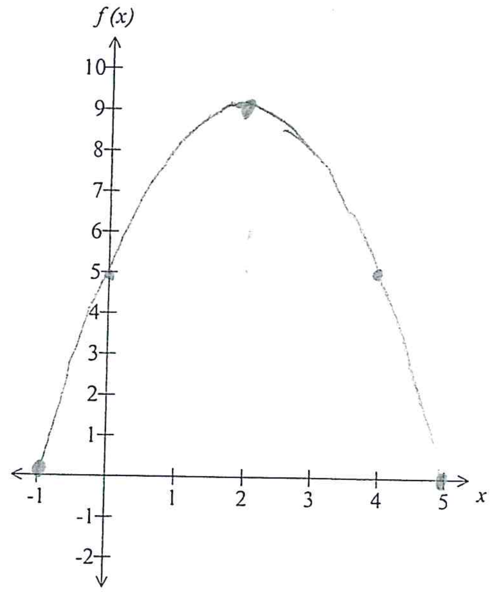
$f'(2) = 0$

(iii) $f''(2)$ [1]

$f''(x) = -2$

$\therefore f''(2) = -2$

(b) (i) Sketch the graph of $f(x)$ over the domain $-1 \le x \le 5$ on the axes provided. [1]



could use concave down

(ii) With reference to your sketch, explain the significance of each answer from part (a). [3]

Have a maximum (iii) turning point (ii) at the value of (2, 9) (i).

ie (ii) → stationary point

(iii) max.

(i) value.

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5. [4 marks]

A spherical balloon is being inflated by pumping gas into it at a rate of 5 m^3/minute. Determine the rate at which the diameter is increasing when the radius is 1 m. [4]

$V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 5$

$\frac{dV}{dr} = 4\pi r^2$ $\quad \frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = \frac{5}{4\pi r^2}$

when $r=1$

$\frac{dr}{dt} = \frac{5}{4\pi}$

$\therefore$ diameter is $\frac{10}{4\pi}$ m/m

6. [4 marks]

A population, y, increases according to the differential equation:

$$\frac{dy}{dt} = 0.04y \quad \text{where } t \text{ is the time, in years, after the start of 2000}$$

The population at the start of 2000 has size 1 000.

(a) State the equation for population, y, in terms of t. [1]

$y = 1000e^{0.04t}$

(b) State the population size when $t = 5$. [1]

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(c) Determine the doubling time for the population. [2]

$2 = e^{0.04t}$

$\therefore t = 17.33$ years

7. [11 marks]

The function $h(x) = \dfrac{2\pi}{3\sqrt{x}}(3x+2)$ has a global minimum value over the domain $0 < x \leq 2$.

(a) Complete the following table. [2]

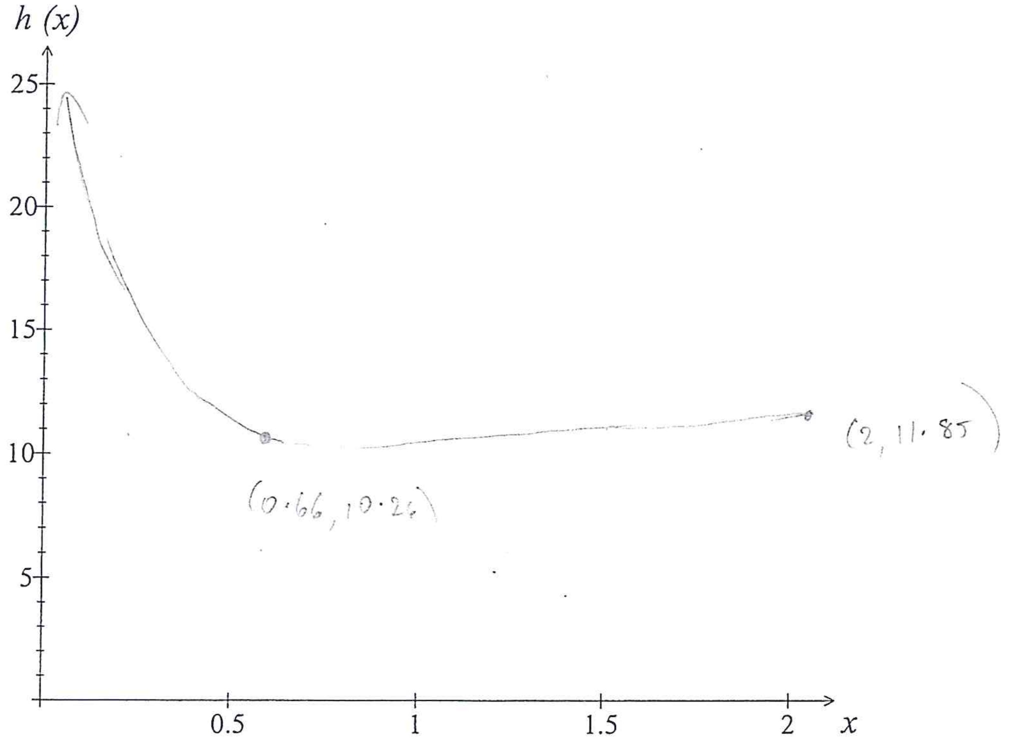
x	$h(x) = \dfrac{2\pi}{3\sqrt{x}}(3x+2)$
0.5	10.367
0.6	10.275
0.7	10.263
0.8	10.303

(b) Explain why the table shows that a local minimum exists in the domain. [1]

decreases from 0.5 → 0.7 but starts to increase

for 0.7 → 0.8

(c) Sketch the graph of $y = h(x)$ over the domain given on the axes provided, labelling clearly any important points. [2]



(d) State the minimum value of $h(x)$, correct to three decimal places, over the stated domain. [1]

10.260

Question 7 continued

(e) Use your calculator to determine the derivative $h'(x)$, giving your answer in fractional form, and with positive indices. [1]

$$h'(x) = \frac{3\pi x^{3/2} - 2\pi\sqrt{x}}{3x^2}$$

OR $$\frac{\pi \cdot (3x-2)}{3x^{3/2}}$$

(f) Explain how to use the derivative $h'(x)$ and your calculator to determine any stationary point of $h(x)$. State the x value of any stationary point. [2]

Solve $h'(x) = 0$

$x = \frac{2}{3}$.

(g) Use the "Sign Test", by completing the following table, to prove that the stationary point found in (f) is a local minimum. [1]

x	$x < \frac{2}{3}$	$x = \frac{2}{3}$	$x > \frac{2}{3}$
$h'(x)$	−ve	0	+ve

(h) State the nature of concavity of $h(x)$. [1]

concave up