education equals

Topic: Normal Distribution

Time: 45 mins   Marks: /45 marks

Calculator Assumed

Question One: [1, 1, 3, 2: 7 marks]

If $X \sim N(20, 9^2)$ calculate:

a) $P(X > 13)$

b) $P(15 \leq X \leq 30)$

c) $P(X < 28 | X > 9)$

d)   Find k such that $P(X < k) = 0.076$

Question Two: [2, 3, 2, 2: 9 marks]

The weights of local snapper sold at the fish market is normally distributed with a mean of 400g and a standard deviation of 3.2g.

The manager insists that it is unacceptable to sell a piece of snapper less than 395g.

a)   What proportion of their snapper is at an acceptable weight?

b)   Given that they weigh all of the fish before putting it out to sell and discard the unacceptable pieces of snapper, what is the probability that they sell a snapper which weighs at least 406g.

c)   Determine the weight exceeded by 7% of the snapper.

d)   In one day 125 pieces of snapper are sold. Estimate how many weigh more than 401g.

Question Three: [4 marks]

A normal distribution has a mean of 6 and standard deviation of 0.3. It is known that 220 scores fall short of 4.7. How many scores are in the distribution?

Question Four: [3, 3: 6 marks]

The prices of all properties sold in Western Australia on a particular day were very close to being normally distributed with 95% of the properties sold selling for between $400 000 and $850 000.

a)   Estimate the mean price for a property sold in Western Australia on that particular day. Show mathematical reasoning to justify your solution.

b)   Estimate the percentage of properties which sold for less than $520 000 on that day.

Question Five: [2, 2, 3, 3, 4: 14 marks]

Eloise likes to arrive at work before her boss in the morning. The time it takes her to get to work is normally distributed with a mean of 28 minutes and standard deviation of 6 minutes.

a)   If her boss arrives at work at 8:30am every day and Eloise leaves her house at 7:50am what is the probability of her arriving before her boss?

b)   With Eloise leaving her house at 7:50am, what is the probability of her arriving 5 minutes either side of her boss's arrival?

c)   If Eloise leaves her house at 8:00am, what is the probability of her not arriving between 8:20 and 8:40am?

d)   What is the latest time that Eloise should leave her house if she wants to cut the times she arrives after her boss to less than 1%?

e)   Eloise leaves her house on a particular day at 8:05am. Given that she arrives after her boss, what is the probability that she arrives before 8:40?

Question Six: [3, 2: 5 marks]

An egg farmer weighs all the eggs his chickens lay and categorises them according to weight for sale.

If the eggs weigh less than 50g it is considered a small egg.

If it weighs between 50g and 65g then it is considered medium sized.

Between 65g and 75g are large eggs and any egg weighing more than 75g is a jumbo egg.

The weights of the eggs are found to be normally distributed with a standard deviation of 3.5g.

a)   If 60% of the farmer's eggs are medium calculate the mean weight of an egg from his farm.

b)   What is the probability of a randomly selected egg being a jumbo egg?

Question One: [1, 1, 3, 2: 7 marks]

If $X \sim N(20, 9^2)$ calculate:

a) $P(X > 13)$

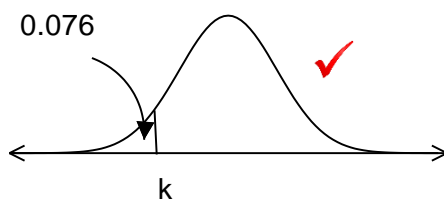 $= 0.7817$ ✓

b) $P(15 \leq X \leq 30)$

   $= 0.5775$ ✓

c) $P(X < 28 | X > 9)$

$= \dfrac{P(9 < X < 28)}{P(X < 9)}$ ✓

$= \dfrac{0.7022}{0.8892}$ ✓

$= 0.79 \,(2dp)$ ✓

d)   Find k such that $P(X < k) = 0.076$

0.076

k

✓

$k = 7.107 \,(2dp)$

Question Two: [2, 3, 2, 2: 9 marks]

The weights of local snapper sold at the fish market is normally distributed with a mean of 400g and a standard deviation of 3.2g.

The manager insists that it is unacceptable to sell a piece of snapper less than 395g.

a)   What proportion of their snapper is at an acceptable weight?

$$P(X > 395) = 0.9409$$

b)   Given that they weigh all of the fish before putting it out to sell and discard the unacceptable pieces of snapper, what is the probability that they sell a snapper which weighs at least 406g.

$$P(X > 406 \mid X > 395) = \frac{0.0304}{0.9409} = 0.032 \ (3dp)$$

c)   Determine the weight exceeded by 7% of the snapper.

$$P(X > k) = 0.07$$

$$k = 404.72g$$

d)   In one day 125 pieces of snapper are sold. Estimate how many weigh more than 401g.

$$P(X > 401) = 0.3773$$

$$0.3773 \times 125 = 47 \ pieces$$

Question Three: [4 marks]

A normal distribution has a mean of 6 and standard deviation of 0.3. It is known that 220 scores fall short of 4.7. How many scores are in the distribution?

$X \sim N(6, 0.3^2)$ ✓

$P(X < 4.7) = 0.0000073434$ ✓

$\dfrac{220}{x} = 0.0000073434$ ✓

$x = 29958875$ ✓

Question Four: [3, 3: 6 marks]

The prices of all properties sold in Western Australia on a particular day were very close to being normally distributed with 95% of the properties sold selling for between $400 000 and $850 000.

a) Estimate the mean price for a property sold in Western Australia on that particular day. Show mathematical reasoning to justify your solution.

$\mu + 2\sigma = 850\,000$ ✓ $\mu = 625\,000$ ✓

$\mu - 2\sigma = 400\,000$ ✓

b) Estimate the percentage of properties which sold for less than $520 000 on that day.

$\sigma = 112500$ ✓

$P(X < 520\,000) = 0.1753$ ✓

17.53% ✓

Question Five: [2, 2, 3, 3, 4: 14 marks]

Eloise likes to arrive at work before her boss in the morning. The time it takes her to get to work is normally distributed with a mean of 28 minutes and standard deviation of 6 minutes.

a) If her boss arrives at work at 8:30am every day and Eloise leaves her house at 7:50 what is the probability of her arriving before her boss?

$P(X < 40) = 0.9772$

b) With Eloise leaving her house at 7:50am, what is the probability of her arriving 5 minutes either side of her boss's arrival?

$P(35 < X < 45) = 0.1194$

c) If Eloise leaves her house at 8:00am, what is the probability of her not arriving between 8:20 and 8:40am?

$P(20 < X < 40) = 0.8860$

$1 - 0.8860 = 0.1140$

d) What is the latest time that Eloise should leave her house if she wants to cut the times she arrives after her boss to less than 1%?

$P(X > k) = 0.01$

$k = 41.96 \approx 42 \ mins$

$\therefore 7 \cdot 48 \ am$

e) Eloise leaves her house on a particular day at 8:05am. Given that she arrives after her boss, what is the probability that she arrives before 8:40?

$P(X < 35 | X > 25) = \frac{P(25<X<35)}{P(X>25)} \quad \frac{0.56979}{0.69146} = 0.824$

Question Six: [3, 2: 5 marks]

An egg farmer weighs all of the eggs his chickens lay and categorises them according to weight for sale.

If the eggs weigh less than 50g it is considered a small egg.

If it weighs between 50g and 65g then it is considered medium sized.

Between 65g and 75g are large eggs and any egg weighing more than 75g is a jumbo egg.

The weights of the eggs are found to be normally distributed with a standard deviation of 3.5g.

a) If 60% of the farmer's eggs are medium calculate the mean weight of an egg from his farm.

$$X \sim N(\mu, 3.5^2)$$

$$P(50 < X < 65) = 0.6$$

$$\mu = 64.113g$$

b) What is the probability of a randomly selected egg being a jumbo egg?

$$P(X > 75) = 0.000934$$