NAME: Answers

Response Item: Test 6

Networks and Decision Mathematics

Year 12 Applications Mathematics

Resource Free Assessment
Time: 55 minutes

Total:	/50
	%

Material required/recommended for this test

To be provided by the supervisor

Question/answer booklet and formula sheet.

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special materials: drawing instruments, templates.

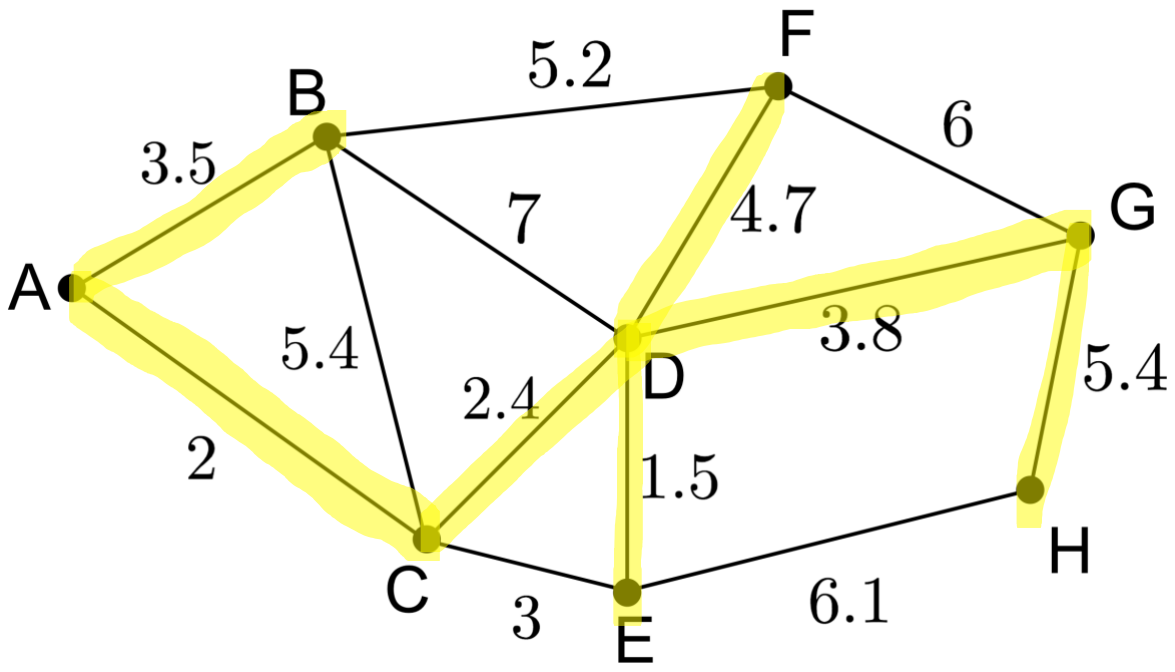
Important note to candidates

No other items may be taken into the test room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the test room. If you have any unauthorised material with you, hand it to the teacher **before** reading any further.

Although marks are not necessarily awarded for working, it is recommended that **enough working to justify** your responses is shown. Incorrect answers with no working will be awarded **zero** marks.

[9 marks – 3, 3, 3]

1. The network below shows the cost to seal dirt tracks (in thousands of dollars) between connected outback towns within a specific council's jurisdiction:



a) The council wishes to seal as many of the roads as they can. A minimal spanning tree is to be used to determine the shortest distance of road required to be sealed to connect all towns directly or indirectly:

i. Clearly identify on the network above the minimum spanning tree solution

(1) - uses at least 3 correct edges
(1) - " " " 6 " "
(1) - all correct

ii. Calculate the minimum cost

$3.5+2+2.4+1.5+4.7+3.8+5.4$ ✓ (sums edges)

$20+3.3=23.3$ ✓ (decimal ans)

$\therefore \$23300$ ✓ (correct conv.)

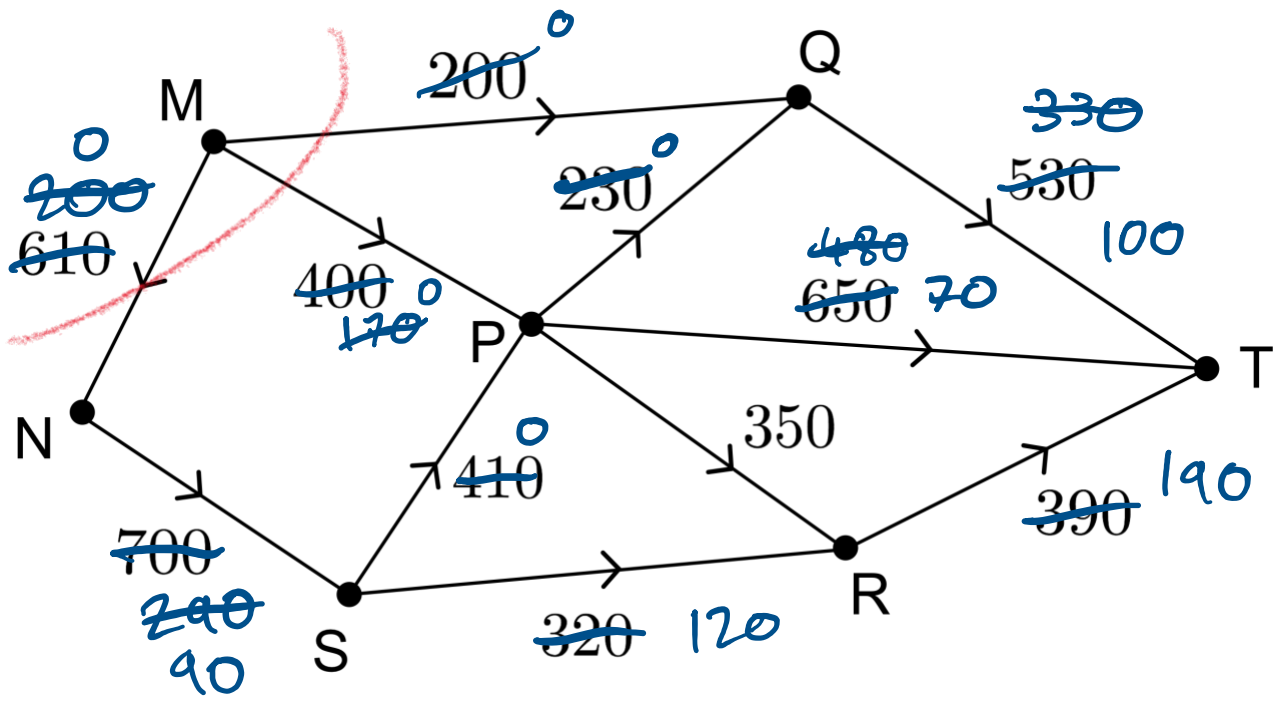
b) Due to annual flooding edge **AC** is not cost effective. <u>Explain</u> how this will change the solution to a):

AC will be replaced by BF ✓ (correct edge)

increasing cost by $3200 ✓ (increase in context) ✓ (calculates correct increase) (3.2)

[9 marks – 2, 4, 1, 1, 1]

2. The number of data packets a network of servers can send (and receive), in packets per second, is shown below:



a) Identify the source and the sink

Source = M ✓ Sink = T ✓ (only 1 mark if just 'M and T')

b) List each unique path with their flow rate and hence calculate the maximum flow for this network:

MQT = 200
MPQT = 230
MPT = 170
MNSPT = 410
MNSPRT = 200
Max Flow = 1210

① - 2+ paths
① - 4+ paths
① - 5 paths
① - correct max flow

c) Verify this maximum flow by drawing the minimum cut on the network above

as above ✓

d) One network connection (edge) is to be upgraded with the goal of increasing the max flow

i. Identify the edge to be upgraded to increase the max flow the most

MP ✓

ii. By how much should the connection's flow be increased in order to maximise flow without wasting resources

260 ✓ (if PR remains 350)

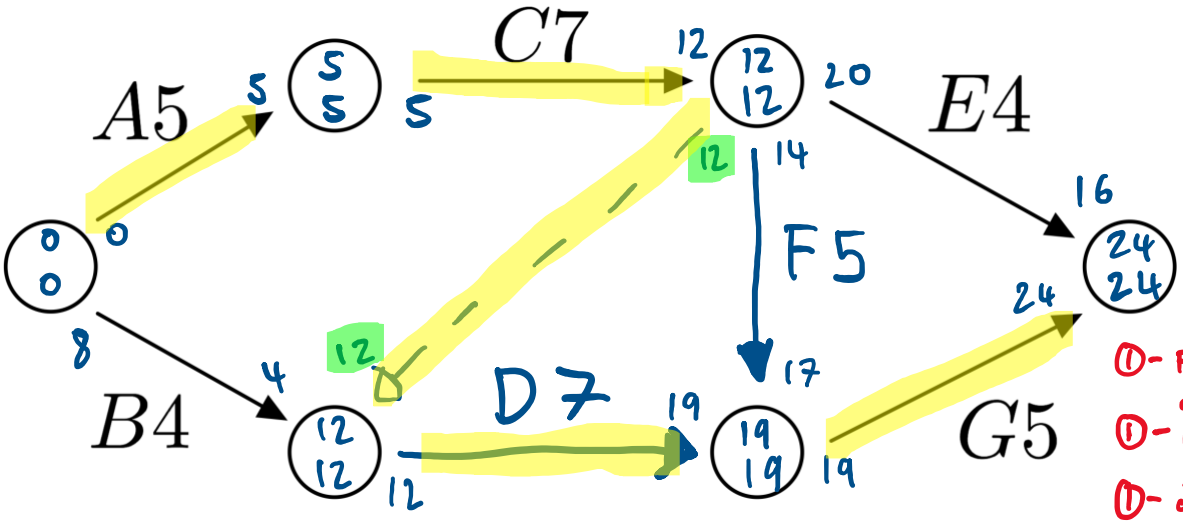
may have other correct answers if used different flow. Don't penalise if < 260

[15 marks – 3, 4, 3, 2, 3]

3. The tasks required to custom design and produce a specific type of prosthetic are shown in the table below along with the immediate predecessors for each activity:

Task	A	B	C	D	E	F	G
Time (hours)	5	4	7	7	4	5	5
Immediate Predecessors	–	–	A	B, C	C	C	D, F

a) Complete the project network below showing each task and its duration:



① - FS location correct
① - D7 location correct
① - dummy path correct

b) Determine the earliest start time and latest start time for all activities, showing this on the graph above, and list the critical path below:

ACDG ① - critical path
① - dummy path accounted for correctly
① - correct EST all (FT)
① - correct LST all

c) Calculate the slack for all tasks not on the critical path

F = 2 h ✓
B = 8 h ✓
E = 8 h ✓

(1 mark each)

d) Task F has been delayed by 3 hours due to an electrical fault, what effect, if any, will this have on the minimum completion time and the critical path?

✓ (delay 1 hour)

It will delay min. comp. time by 1 hour

and F will replace D on the critical path

✓ (F replaces D) (ACFG)

e) In an attempt to shorten the minimum completion time, resources are redirected from task E to tasks D and G. This causes task E to be delayed 4 hours while reducing the time taken for task D and G by 3 hours each.

Calculate the new minimum completion time to prove that this is a wise change:

① makes changes to D, G, E

① accounts for F correctly

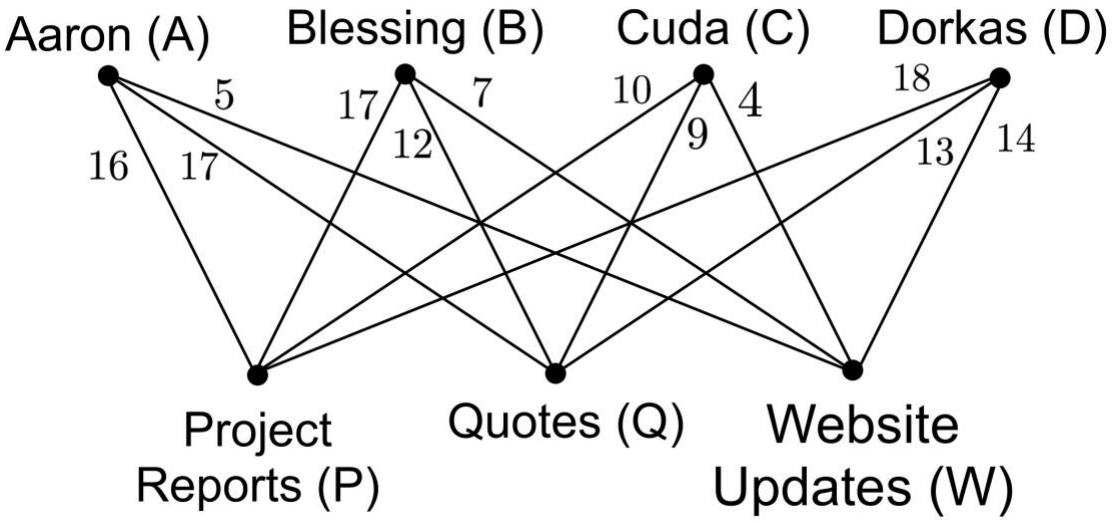
① correct new min comp. *

D finishes at 16 ∴ F now EST for G at 17. G then finishes at 19. However E delayed to 20. ∴ 4 hours better

*NO follow through if doesn't account for F

[9 marks – 1, 5, 3]

4. The bipartite graph below shows the time taken, in hours, by each employee to complete three tasks required to be completed weekly. Each employee was rotated through the tasks over a period of a few months and their average time taken was recorded. The company wants to minimise time time spent on these tasks so that the employees can more quickly be transferred to more lucrative projects.

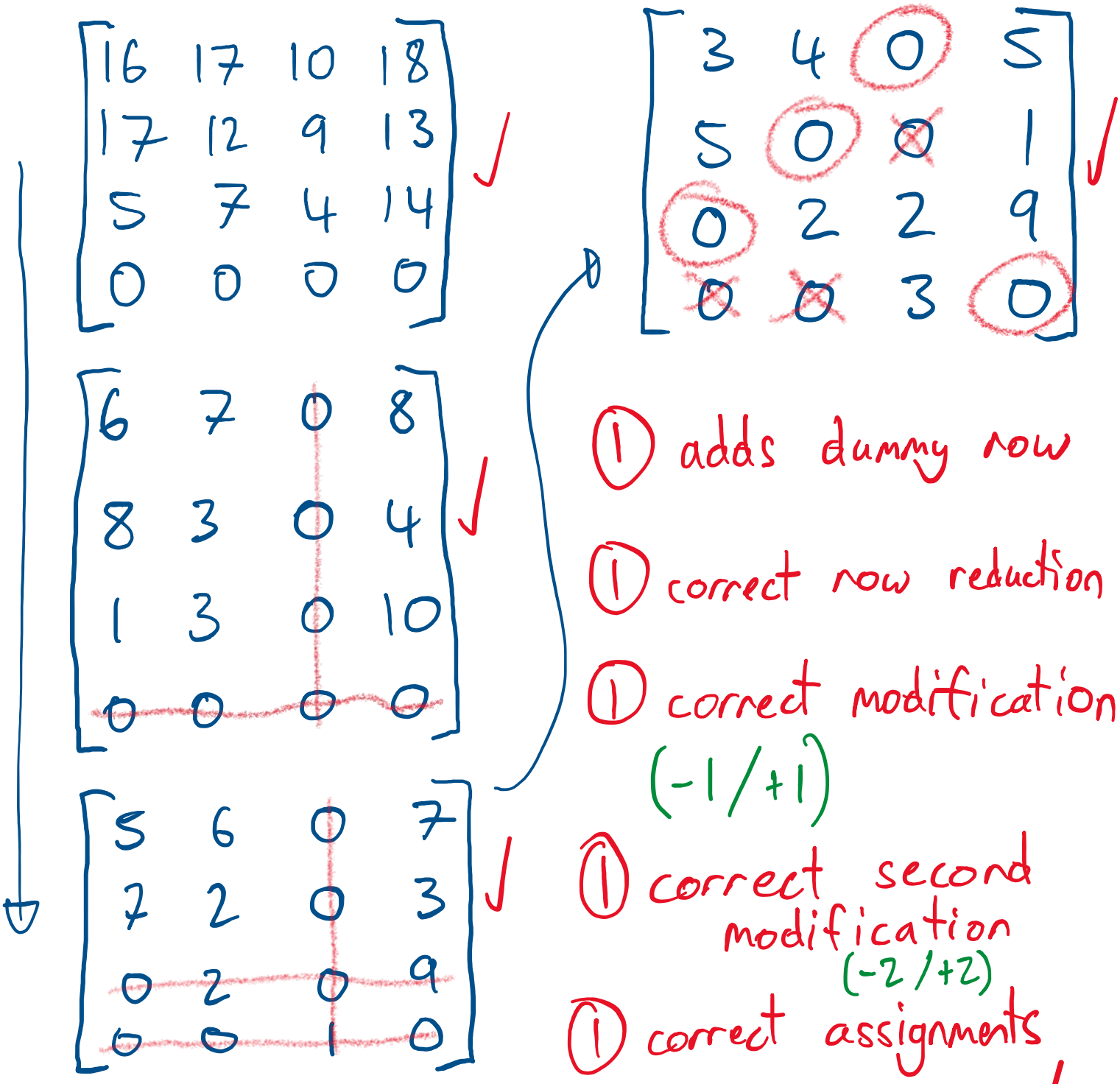


a) Complete the matrix below with the information from the graph above:

	A	B	C	D
P	16	17	10	18
Q	17	12	9	13
W	5	7	4	14

✓ (correct)

b) Convert the matrix into one that is appropriate for use with the Hungarian Algorithm and use this algorithm to allocate the employees to the tasks in order to minimise time spent on these tasks. Write the allocations in the table below:



Employee	Aaron	Blessing	Cuda	Dorkas
Task	W	Q	P	Unassigned

c) Before the employees began being rotated though the tasks they were originally assigned as follows: Aaron – not assigned; Blessing – Project Reports; Cuda – Website Updates; and Dorkas – Quotes.

Use Mathematical evidence to show how much time your allocation in **b)** will save compared to the original allocation:

BP CW DQ

Old Assigament: $17 + 4 + 13 = 34$ hours ✓

AW BQ CP

New Assignment: $5 + 12 + 10 = 27$ hours ✓

$\therefore$ 7 hours saved! ✓

(1) - calculates time for old ass. correctly

(1) - calculates given ass.

(1) - states difference in hours

[8 marks – 4, 2, 2]

5. The length of piping required to connect various troughs and taps on a farm directly are shown below (in hundreds of meters), the dashes indicate that a direct connection between those two locations is not feasible:

	A	B	C	D	E	F	G
A	-	15	19	27	32	-	
B	15	-	25	22	-	-	-
C	19	25	-	-	8	23	-
D	27	22	-	-	9	-	15
E	32		8	9		11	14
F	-	-	23		11	-	20
G		-	-	15	14	20	

① - evidence of Prim's or other method

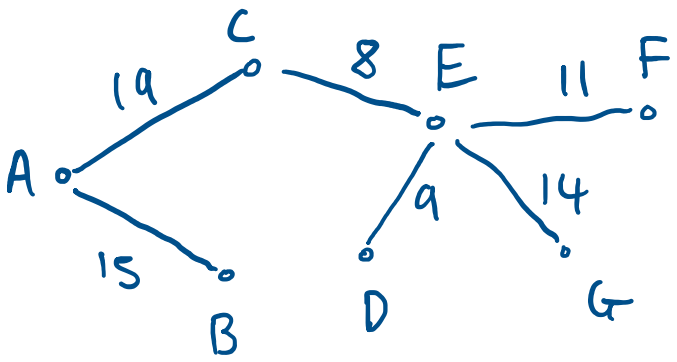
① - correct min. solution shown

a) Using Prim's algorithm or otherwise determine the minimum total length of pipes required and state this length below:

15 + 19 + 8 + 9 + 11 + 14 = 76 00 m

① adds values ① - ans in '00's of meters

b) Draw this network showing *only* the connections required to connect all locations with the minimum total length of pipe:



① - connections

① - values on edges

c) A new alternative connection between F and G has become feasible. Calculate the maximum length this connection can be (in whole meters) while still replacing one of the edges in your network in b):

Must be less than 14, ∴ 13 → 1300m

(also accept 1399m)

① - recognises must replace 14

① - converts answer to meters

End of Test