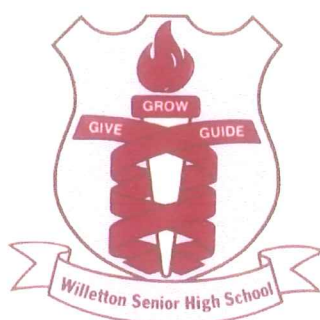


MATHEMATICS APPLICATIONS
YEAR 12 UNIT 3

TEST 1
BIVARIATE DATA and SEQUENCES

2023



PART A
CALCULATOR FREE

TIME: 30 mins

MARKS: 31 marks

STUDENT'S NAME:

Marking Guide

CIRCLE YOUR
TEACHER'S NAME:

Mr Ismail

Mrs Kalotay

Ms Mack

Mrs Smirke

Mrs Scoles

Ms Tsen

Mrs Scoles
TUTOR GROUP

Questions 1-3 are MULTIPLE CHOICE. Circle the correct answer.

[3 marks]

Question 1

In a scientific experiment, the explanatory variable was the amount of sleep (in hours) a new mother got per night during the first month following the birth of her baby.

The response variable would most likely have been:

- A the blood pressure of the baby
- B the amount of time (in hrs) spent by the mother reading
- ✓ ☒ C the mother's reaction time (in seconds) to a certain stimulus
- D the level of alertness of the baby

Question 2

Alen works as a personal trainer at the local gym. He wishes to analyse the relationship between the number of weekly training sessions and the weekly weight loss of his clients.

Which one of the following statements is correct?

- A When graphed, the number of weekly training sessions should be shown on the vertical axis as it is the response variable
- B When graphed, the weekly weight loss should be shown on the vertical axis as it is the explanatory variable
- C When graphed, the weekly weight loss should be shown on the horizontal axis as it is the response variable
- ✓ ☒ D When graphed, the number of weekly training sessions should be shown on the horizontal axis as it is the explanatory variable

Question 3

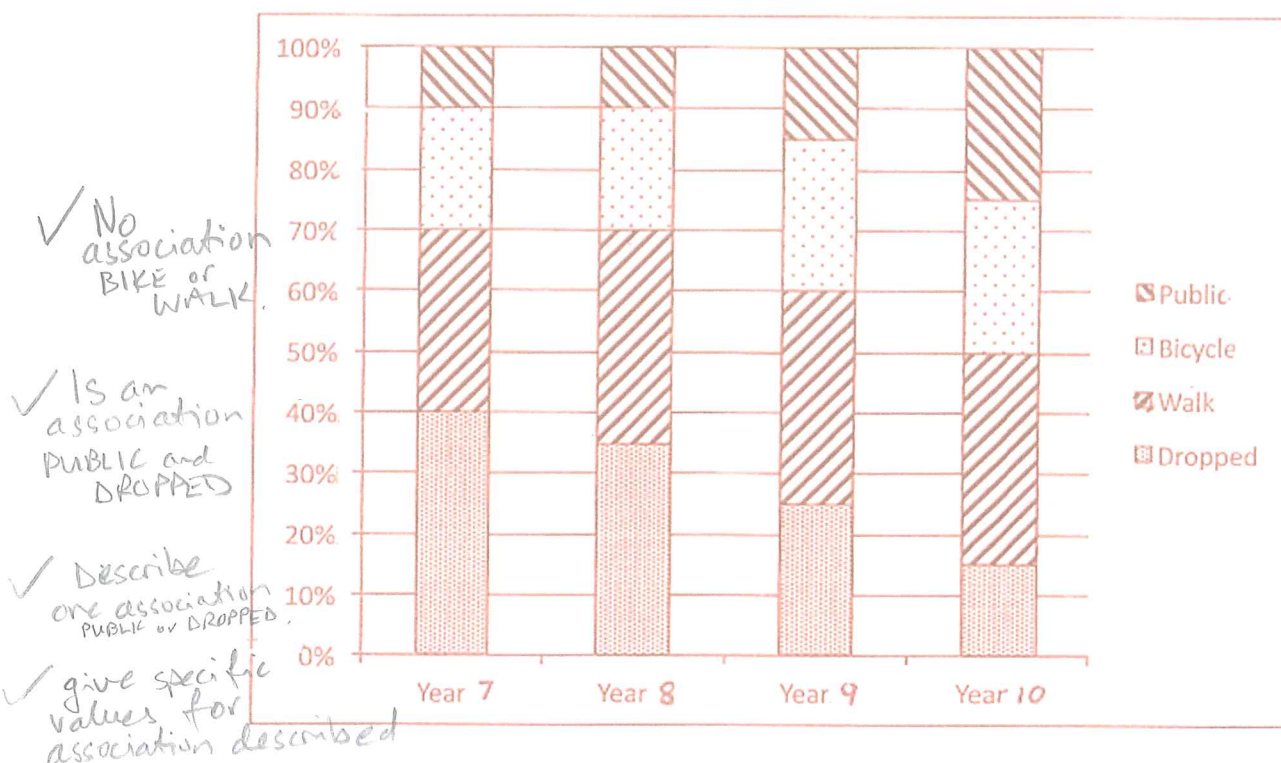
A pediatrician (doctor who provides medical care for children) investigated the relationship between the amount of time children aged two to five spend outdoors and the annual number of visits to his clinic. Which one of the following statements is not true?

- ✓ ☒ A It is impossible to identify the explanatory variable in this case
- B The annual number of visits to the pediatric clinic is the response variable
- C When graphed the amount of time spent outdoors should be shown on the horizontal axis
- D The annual number of visits to the pediatric clinic should be shown on the vertical axis

Question 4

[4 marks]

The Diagram below shows
the percentage of students in the listed school year groups
and their mode of travel to school.



Using the stacked 100% column graph above comment on whether there seems to be any association(s) between the variables, explaining your reasons and describe the association(s).

For riding a bike or walking there is no association with the year groups of students as proportions are similar

There is an association for taking public transport or being dropped off, as moving across the year groups the percentages change

For public transport, the older students are more likely to take it (positive or increasing association) percentages being 10, 10, 15 then 25% from yr7-10.

OR
For being dropped off the, as you move from yr7 to yr10 the proportion being dropped decreases, (negative association) as seen with percentages of 40, 35, 25, to 15%.

SEE NEXT PAGE

Question 5

[1 mark]

A study shows a strong correlation between house size and the life expectancy of the home owners. Suggest an underlying factor (confounding/lurking variable) that is more likely to be influencing the observed correlation.

- income ✓

- having children

(or any other reasonable answer)

* Note: must influence both variables.

Question 6

[2 marks]

A set of data giving the number of police traffic patrols on duty and the number of fatalities for the region was recorded and a correlation coefficient of $r = -0.8$ was found. Calculate the coefficient of determination and interpret its value in context.

$$r^2 = 0.64$$
 ✓

64% of the variation in fatalities can be explained by the variation in the number of police patrols ✓

Question 7

[3 marks]

The first few terms of three sequences are plotted on the graphs a), b) and c) below. Complete the table to match one of the six recursive rules to each graph.

(i) $T_{n+1} = T_n + 2, \quad T_1 = 1$

(iii) $T_{n+1} = T_n + 3, \quad T_1 = 1$

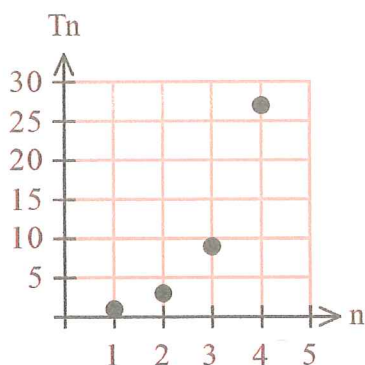
(v) $T_{n+1} = T_n - 4, \quad T_1 = 1$

(ii) $T_{n+1} = 2T_n, \quad T_1 = 1$

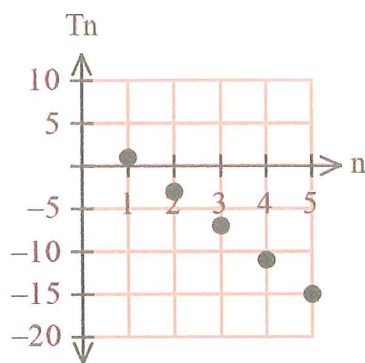
(iv) $T_{n+1} = 3T_n, \quad T_1 = 1$

(vi) $T_{n+1} = -4T_n, \quad T_1 = 1$

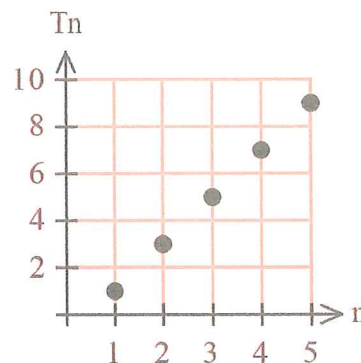
a)



b)



c)



Graph	a) ✓	b) ✓	c) ✓
Recursive rule	(iv) ✓	(v) ✓	(i) ✓

[5 marks]

Question 8

Determine the first four terms for the following sequences:

a) $T_n = 5T_{n-1} + 2, \quad T_1 = 1$ [2]

1, 7, 37, 187 ✓

b) $T_{n+1} = 2T_n - 5, \quad T_3 = -10$ [3]

1.25, -2.5, -10, -25 ✓

(if 1M F.T. do -10 as $T_1 \Rightarrow -10, -25, -55, -115$)

Question 9

[7 marks]

Given $T_2 = 6$ and $T_4 = 54$

Write the recursive rule and determine the third term for each of the following if:

a) The given terms are from an Arithmetic sequence. [3]

$T_4 - T_2 = 2d = 48$
 $d = 24$

$T_{n+1} = T_n + 24$ ✓ $T_1 = -18$ ✓

$T_3 = 30$ ✓

b) The given terms are from a Geometric sequence. [4]

$\frac{T_4}{T_2} = r^2 = 9$
 $r = \pm 3$ ✓

$T_{n+1} = 3T_n \quad T_1 = 2$

✓ recursive rules (with T_1)

$T_3 = 18$

✓ T_3

OR

$T_{n+1} = -3T_n \quad T_1 = -2$

$T_3 = -18$

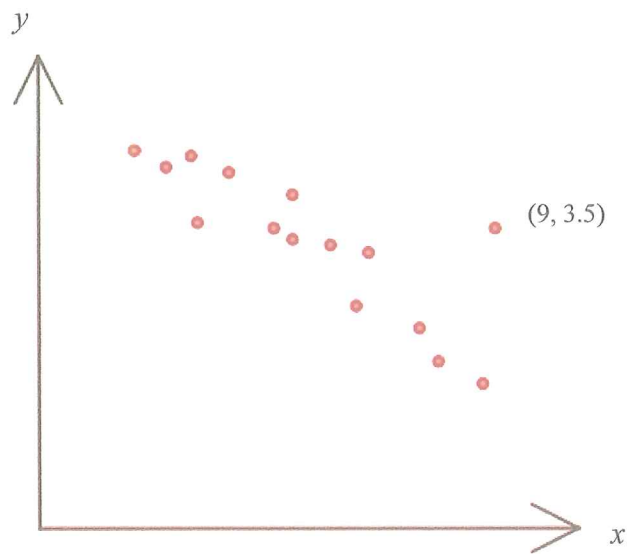
SEE NEXT PAGE

(-2M if they don't have second sequence and term with $r = -3$)

Question 10

[6 marks]

The following graph relates x and y .



The least squares regression line of y on x was fitted to the data.

The equation is given as: $\hat{y} = 11.5 - 1.2x$ and the coefficient of determination = 0.49

- a) Determine the correlation coefficient.

✓
- 0.7 ✓

[2]

- b) The point (9, 3.5) is referred to as an outlier ✓

[1]

- c) If the point (9, 3.5) was removed, state the effect this would have on the value of your response to part (a). Give a reason for your answer.

[2]

Value will be closer to -1 ✓ (value decreases)
as the relationship gets stronger ✓

- d) If $\bar{x} = 7$, calculate the value of \bar{y} .

[1]

$$\begin{aligned}\bar{y} &= \hat{y}(7) = 11.5 - 1.2(7) \\ &= 11.5 - 8.4 \\ &= 3.1 \quad \checkmark\end{aligned}$$

MATHEMATICS APPLICATIONS
YEAR 12 UNIT 3

TEST 1
BIVARIATE DATA and SEQUENCES

2023



PART B
CALCULATOR ASSUMED

TIME: 20 mins
MARKS: 23 marks

STUDENT'S NAME:

Marking Guide.

CIRCLE YOUR
TEACHER'S NAME:

Mr Ismail

Mrs Kalotay

Ms Mack

Mrs Smirke

Mrs Scoles

Ms Tsen

Mrs Scoles
TUTOR GROUP

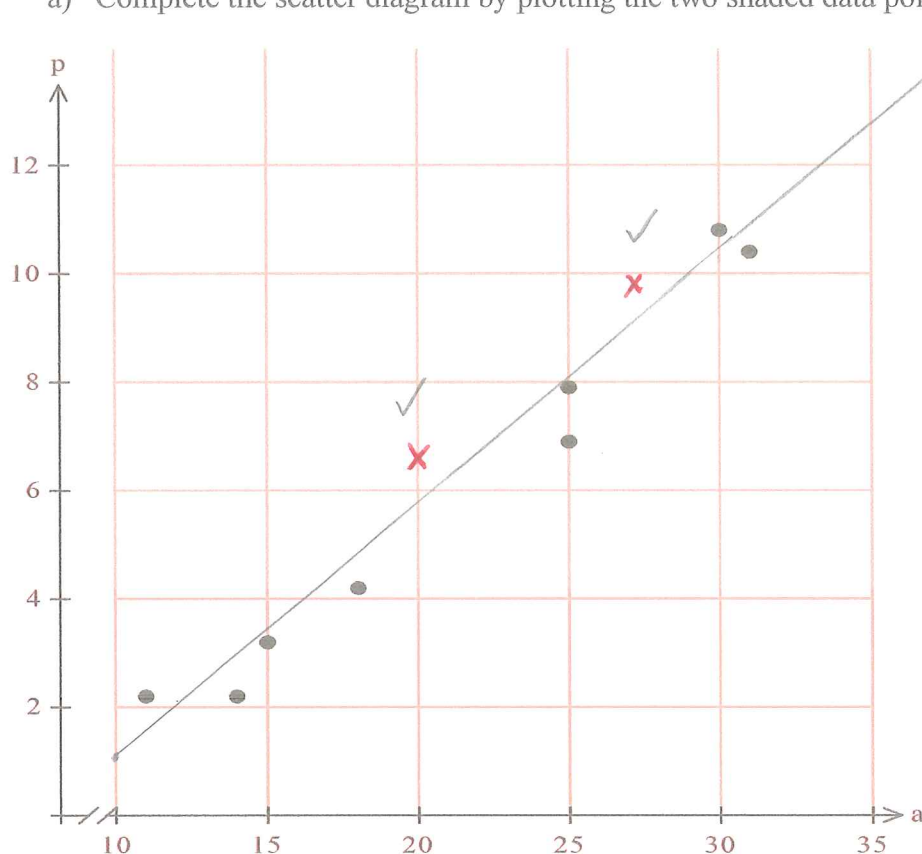
Question 11

[17 marks]

Data giving the annual advertising budgets (in \$1000) and the yearly profit increases (%) of ten companies are shown below.

Annual advertising budget (in \$1000) a	11	14	15	18	20	25	25	27	30	31
Yearly profit increase (%) p	2.2	2.2	3.2	4.2	6.5	6.9	7.9	9.8	10.8	10.4

- a) Complete the scatter diagram by plotting the two shaded data points in the table. [2]



- b) Determine the equation of the least squares regression line that models this data (giving values correct to three decimal places) and draw this line on the scatter diagram. [4]

$$\hat{p} = 0.466a - 3.646 \quad \checkmark \checkmark$$

(-1 rounding)
(-1 incorrect variables)

(10, 1.0)
(30, 10.3)

- c) Interpret the value of the gradient in the least squares line. [2]

The yearly profit increases by 0.47% for every \$1000 increase in the annual advertising budget.

✓ correct units
✓ correct statement

- d) What percentage of the variation of the yearly profit increase can be explained by the variation of the annual advertising budget? [1]

from CLASSPAD. $r^2 = 0.9566$
 $\therefore \sim 96\% \quad \checkmark$

Question 11 continued:

- e) Predict the yearly percentage profit increase if the annual advertising budget of a company was \$35 000 and comment on its reliability. [2]

$$\hat{p}(35) = 12.6\% \text{ (1dp)}$$

(or 12.7% using rounded eqn in part a)

Not reliable as an extrapolation.

- f) Is it possible to determine how much should be spent on the annual advertising budget for a company to achieve a 5% yearly profit increase?

If so, what is the value?

Not from eqn in a)

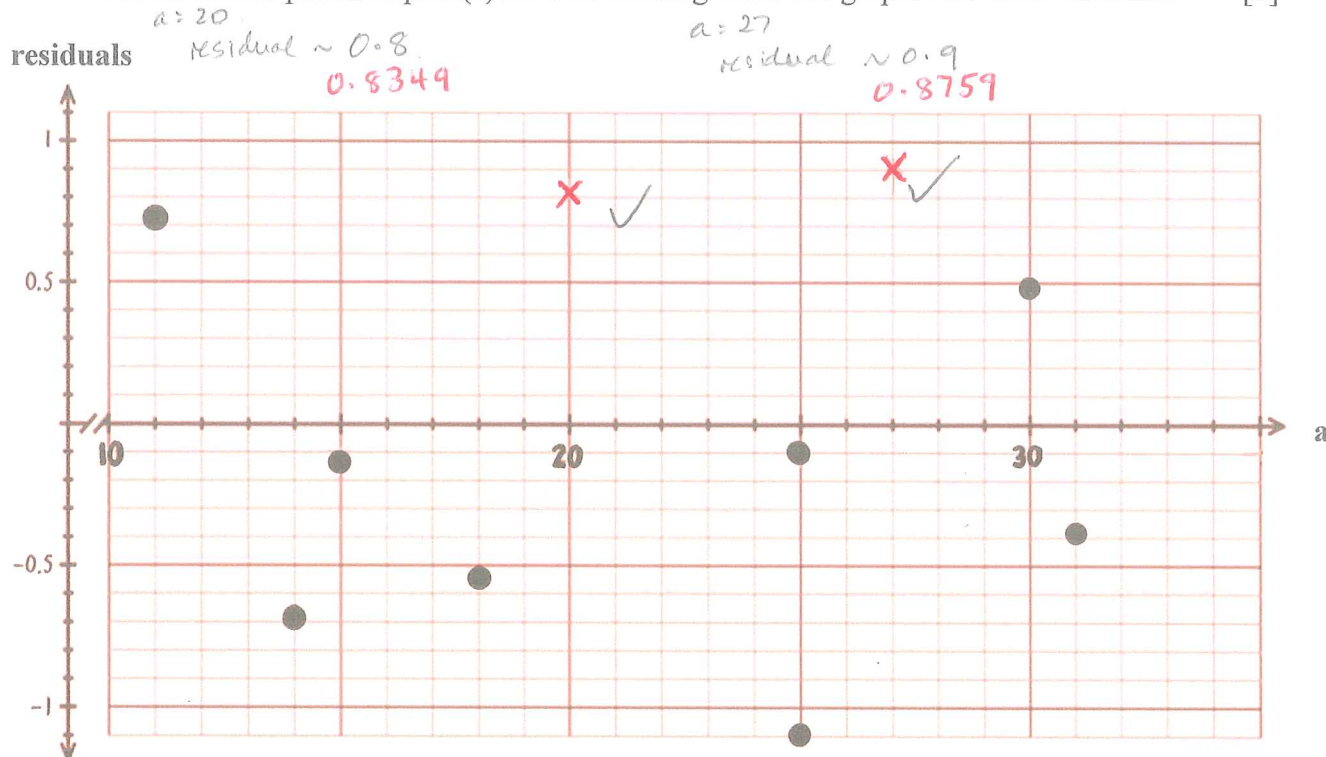
If not, why not and explain how you would determine it?

[2]

Least-squares line is only for predicting a response variable.
To predict advertising budget from given profit variable 'p' must be x variable on UASCPAS.
& 'a' must be y variable then find a new regression line.

- g) Below is the residual plot for the data in the table.

The same two points in part (a) are also missing from this graph. Add their residuals. [2]



- h) With reference to the residual plot, comment on whether a linear model is appropriate or not.

As the residual plot is random
a linear model is appropriate

[2]

[FT. if they didn't add 2 missing residuals and concluded linear not appropriate due to pattern in residuals]

SEE NEXT PAGE

Question 12

[3 marks]

Every year a new housing subdivision has 40 new houses completed. If there were initially 10 houses in the subdivision:

- a) Write a recursive rule to determine the number of houses in total in the subdivision each year. [2]

$$H_0 = 10$$

$$H_{n+1} = H_n + 40$$

$$H_1 = 50$$

can use any letter
for term.

- b) How many years will it be until there are at least 500 houses in the subdivision? [1]

$$13 \text{ yrs}$$

$$H_{12} = 490$$

$$H_{13} = 530$$

Question 13

[3 marks]

A couple sell a property for \$800 000 and decide to invest the money, at 4.5%p.a.
The plan is to have at least one million dollars before looking for another investment property.

- a) Write a recursive rule to determine the value of the investment at the end of each year. [2]

$$V_0 = 800\,000$$

$$V_{n+1} = 1.045 V_n$$

$$(or V_1 = 836\,000)$$

- b) How many years will it take for the investment to be worth more than one million? [1]

$$V_5 = 996\,945.55$$

$$1.0 \text{ E} + 6$$

$$V_6 = 1\,041\,808.10$$

$$1.0 \text{ E} + 6$$

$$\therefore 6 \text{ yrs.}$$