

Unit 1
Semester 1 2018
Mathematics Methods Test 3



Baldyvis
Secondary College

Name	
-------------	--

Full working out must be shown to get full marks.

Attempt all questions

Total Time: 60 minutes

- | | | |
|---|-------------------|-----------------|
| • Section 1 (Calculator Free): | 40 minutes | 39 marks |
| • Section 2 (Calculator Assisted): | 20 minutes | 17 marks |

Calculator Free

Question 1

[2, 2, 3 = 7 marks]

- a) Rewrite the equation $y = 2(x - 4)^2 + 7$ in the form of $y = ax^2 + bx + c$

$$\begin{aligned}
 &2(x - 4)^2 + 7 \\
 &2(x^2 - 4x - 4x + 16) + 7 \\
 &2x^2 - 16x + 32 + 7 \\
 &2x^2 - 16x + 39
 \end{aligned}$$

- b) For the graph $y = ax^2 + x - 1$ determine the value of a if the graph passes through the point $(-4, 11)$

$$\begin{aligned}
 11 &= a(-4)^2 + -4 - 1 \\
 11 &= 16a - 5 \\
 16 &= 16a \\
 a &= 1.
 \end{aligned}$$

- c) The graph of $y = x^2 + px + q$ has a turning point at $(6, -3)$. Determine the coordinates of the y -intercept of the parabola.

$$\text{line of symmetry} = x = \frac{-b}{2a}$$

$$6 = \frac{-p}{2(1)}$$

$$p = -12$$

$$y = x^2 - 12x + q$$

sub $6, -3$

$$-3 = (6)^2 - 12(6) + q$$

$$-3 = 36 - 72 + q$$

$$-3 = -36 + q$$

$$q = 33$$

Question 2

[1, 1, 1, 2, 2 = 7 marks]

- a) What are the coordinates of the y -intercept of the graph of $y = (x-2)(x+3)$?

$$y \text{ int} = (0, 6)$$

$$y = x^2 + 3x - 2x - 6$$

$$y = x^2 + x - 6$$

- b) State the number of roots of the graph of $y = 2x^2 + 1$.

Zero

- c) What are the coordinates of the turning point of the graph of $y = 3(x+4)^2 + 2$?

$$T.P = (-4, 2)$$

- d) What is the equation of the line of symmetry of the graph of $y = (x-3)(x-7)$?

$$x \text{ int } (3, 0) \text{ and } (7, 0)$$

$$\text{eq. of line of symmetry midpoint}$$

$$\circ (5, 0)$$

$$x = 5$$

- e) Determine the equation of the quadratic function that passes through the point $(0, -9)$ and has a turning point at $(-2, 3)$.

$$x = 0$$

$$y = -9$$

$$y = a(x-b)^2 + c$$

$$y = a(x+2)^2 + 3$$

$$-9 = a(0+2)^2 + 3$$

$$-9 = 4a + 3$$

$$4a = -12$$

$$a = -3$$

$$\therefore \text{Equation is}$$

$$y = -3(x+2)^2 + 3$$

Question 3

[2 Marks]

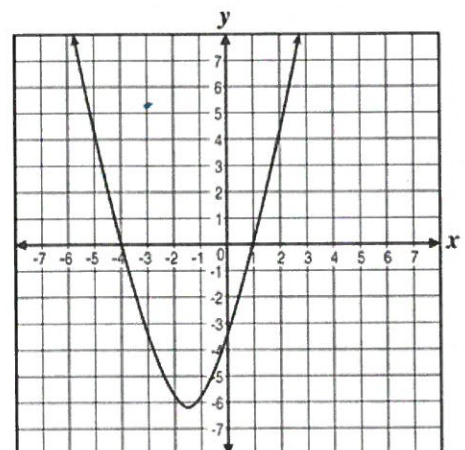
The graph of $y = x^2 + bx - 4$ is shown.
Determine the value of b

$$y = x^2 + bx - 4$$

$$(x-1)(x+4)$$

$$x^2 + 3x - 4$$

$$\therefore b = 3$$



Question 4**[6 marks : 2, 2, 2]**

Express the quadratic equation $y = x^2 - 8x + 15$ in the form:

a) $y = a(x - b)(x - c)$

$$y = (x - 5)(x - 3)$$

b) $y = a(x - b)^2 + c$

$$y = (x^2 - 8x + 16) + 15 - 16$$

perfect square

$$\therefore y = (x - 4)^2 - 1$$

c) Solve the equation $x^2 - 8x + 15 = 0$ using the quadratic formula

$$\begin{aligned} &= \frac{-8 \pm \sqrt{(-8)^2 - 4 \times 1 \times 15}}{2 \times 1} \\ &= \frac{8 \pm \sqrt{4}}{2} = \\ &\quad \swarrow \quad \searrow \\ &= \frac{8+2}{2} \quad = \frac{8-2}{2} \\ &= 5 \quad \quad = 3 \end{aligned}$$

Question 5**[2, 2, 6 = 10 marks]**

A quadratic function is given by $f(x) = (x - 2)^2 - 9$.

a) The function can also be written in the form $f(x) = x^2 + bx + c$.
Determine the values of b and c .

$$\begin{aligned} &(x - 2)^2 - 9 \\ &= x^2 - 4x + 4 - 9 \\ &= x^2 - 4x - 5 \\ \therefore &b = -4 \\ &c = -5 \end{aligned}$$

b) Solve the equation $f(x) = 0$.

$$\begin{aligned} &y = x^2 - 4x - 5 \\ &0 = (x - 5)(x + 1) \\ \therefore &x = 5 \\ &x = -1 \end{aligned}$$

c) For the graph of $y = (x - 2)^2 - 9$, state:

a. The coordinates of the turning point.

T.P (2, -9)

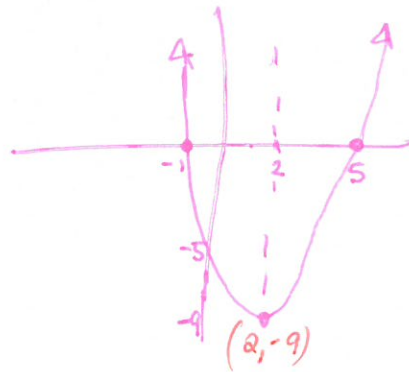
b. The equation of the line of symmetry.

line of symmetry $x=2$

c. The coordinates of all axes intercepts.

x int (5,0) (-1,0)
y int (0,-5)

d. Draw a sketch of the graph using the above information



Question 6

[7 marks : 1, 2, 2, 2]

For the equation $y = 2x^2 - 4x + 6$ determine;

a) the y-intercept

(0,6)

b) the line of symmetry

$$-\frac{b}{2a} = -\frac{(-4)}{4} = 1$$

c) the coordinates of the turning point

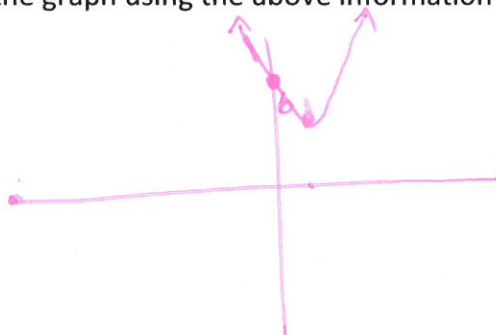
Turning point at $x=1$
 $y = 2(1)^2 - 4(1) + 6$
 $= 4$

d) Use the discriminant to determine the number of solutions.

$$\begin{aligned} &= \sqrt{b^2 - 4ac} \\ &= 4^2 - 4(2)(6) \\ &= 16 - 48 = -32 \end{aligned}$$

∴ No Roots

e) Draw a sketch of the graph using the above information



Mathematics Methods
Semester 1 2017
Calculator Assumed

Question 7

[8 marks]

Complete the table below.

Rule	y-intercept	roots	line of symmetry	turning point
$y = 15x^2 - 11x - 12$	0, -12	(0.6, 0) (1.34, 0)	$x = 0.36$	(0.36, -14.016)
$y = \frac{1}{2}(x-6)(x+2)$	0, -6	(6, 0) and (-2, 0)	$x = 2$	(2, -8)

$$y = a(x^2 - 4x - 12)$$

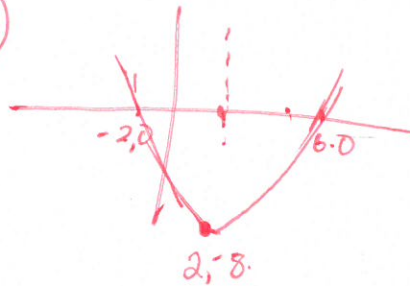
$$-8 = a(2^2 - 4(2) - 12)$$

$$-8 = a(-16)$$

$$a = \frac{1}{2}$$

Question 8

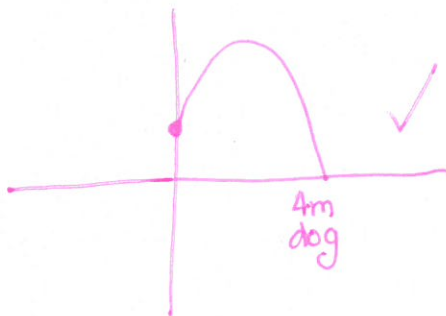
[3 marks]



Isabel throws a ball for her dog to catch. The path of the ball is parabolic and can be modeled by the equation $h = -x^2 + 4x + 1.5$ where h is the height in metres of the ball above the ground and x is the horizontal distance of the ball from Isabel.

If Isabel's dog is 4m away from her, how far does he have to jump to catch the ball?

Provide a sketch to illustrate your answer.



$$h = -x^2 + 4x + 1.5$$

Assuming 4m in front of Isabel

$$h = -4^2 + 4(4) + 1.5$$

$$= 1.5\text{m} \quad \checkmark$$

Dog has to jump 1.5m up.

✓

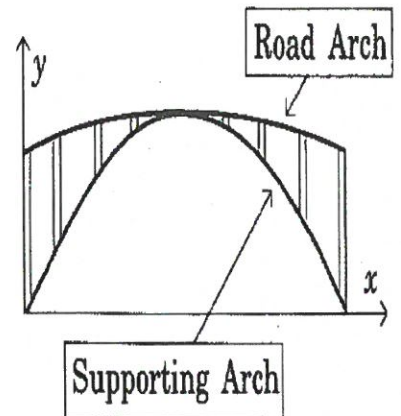
Question 9**[1 + 1 + 2 + 2 = 6 marks]**

A road bridge is to be constructed over a tidal river. The road arch is in the shape of a quadratic function, as is the supporting arch (see diagram).

With x and y axes as shown, the equations of each arch are as follows:

Road arch: $y = -\frac{x^2}{2250} + \frac{2x}{15} + 40$

Supporting arch: $y = \frac{2x}{3} - \frac{x^2}{450}$



[1 metre = 1 unit on each axis]

The x -axis is the mean water level with high and low tide levels being four metres either side of this mean.

a) Are the road arch and supporting arch, concave up or concave down?

Concave down ✓

b) Calculate the value of x at the mid-point of the bridge.

line of symmetry

$$x = \frac{-\frac{2}{15}}{2(-\frac{1}{2250})}$$

$$x = \frac{-b}{2a}$$

$$x = 150\text{m} \checkmark$$

c) Calculate the length of the vertical strut between the support arch and the road arch at a point one-quarter of the way along the bridge.

$$x=75 \quad \text{length strut} = \frac{(-75)^2}{2250} + \frac{2(75)}{15} + 40 = 40\text{m}$$

d) Calculate the maximum clearance between the water and bridge at:

i) low tide.

ii) high tide.

$$x=150 \quad y=60\text{m}$$

$$64\text{m}$$

$$56\text{m}$$