12 Mathematics Methods 2022

Test 1 – Differentiation and Logarithms

Section 1: Calculator-free

Time allowed: 20 minutes **Maximum marks: 20**

Name:_____ **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No calculators or personal notes are permitted.

Question 1 **[3 marks]**

By firstly considering the result of 5^3 , use the *increments formula* to estimate the result of 4.99^3.

Question 2 **[2, 2 = 4 marks]**

Solve the following equations exactly:

a) $9 \times 3^x - 20 = -3^x$

b) $\log_3(2x - 3) = 4$

Question 3 [2, 3 = 5 marks]

Differentiate the following with respect to x:

a) $\frac{-x^2}{2x+1}$

b) $(3x^3 - 4)(2 - x)^7$

Leaving your answer in form $(2 - x)^n(ax^k + bx^{k-1} + ..)$

Question 4 **[2, 2, 3 = 7 marks]**

The tangent to $y = x^3 + kx + 2$ at $x = 1$ is parallel to the line $y = 4x + 9$.

a) Determine the value of k

b)

i) Determine the equation of the tangent at $x = 1$.

ii) Determine the $x - values$ of any points where this tangent meets the curve again.

END OF SECTION 1

12 Mathematics Methods 2022

Test 1 – Differentiation and Logarithms

Section 2: Calculator-assumed

Time allowed: 25 minutes **Maximum marks: 25**

Name:_____ **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- Calculators and 1xA4 double-sided page of personal notes are permitted.

Question 5 **[1, 2, 2 = 5 marks]**

Let $p = \log_5 2$ and $q = \log_5 3$.

Write in terms of p and q:

a) $\log_5 6$

b) $\log_5 \frac{16}{27}$

c) $\log_5 2.4$

Question 6 **[1, 2, 2 = 5 marks]**

The weekly cost (in dollars) of producing x calculators is $C(x) = 40000 - 30x + 0.01x^2$

a) Determine the cost of producing 200 calculators.

b) Using calculus methods, determine the value of x that minimises production costs.

c) Each calculator is sold for $40.

Determine the value of x that maximises profit and what this maximum profit is.

Question 7 **[2, 2, 2, 2 = 8 marks]**

A particle P moves in a straight line with its position from a fixed point, O, given by: $s(t) = 8t - \frac{40}{(t+1)^2}$ cm, where t is the time in seconds and $t \geq 0$.

a) Determine functions for the:

i) velocity of the particle.

ii) acceleration of the particle.

b) Determine the average velocity during the first 5 seconds.

c) Explain why the particle is never stationary.

d) By considering both velocity and acceleration, describe what is happening to the particle at $t = 1$.

Question 8 **[1, 3, 3 = 7 marks]**

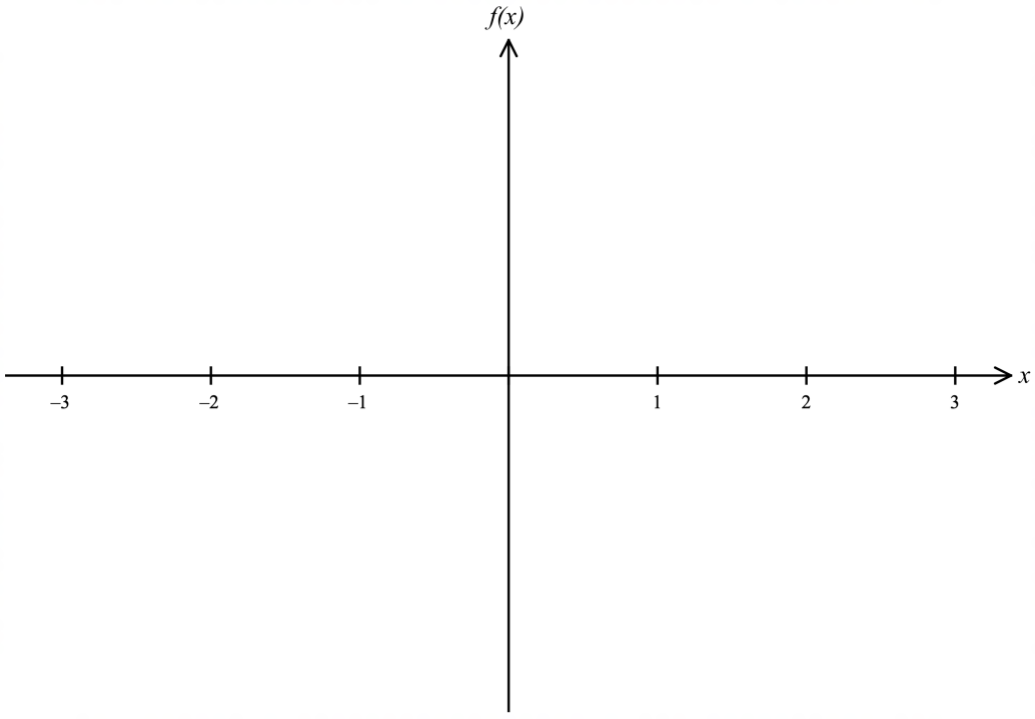
Properties of the function $f(x)$ are displayed in the table below.

x	-3	-2	-1	0	1	2
$f(x)$	$-$	0	0	$-$	0	$-$
$f'(x)$	$+$	$+$	$-$	$+$	0	$-$
$f''(x)$	$-$	$-$	0	$+$	$-$	$-$

a) Describe the concavity of the graph at $x = 0$. Justify your answer.

b) Fully describe all features of $f(x)$ at $x = 1$.

c) Sketch a possible graph of $f(x)$ below.



END OF TEST