MINDARIE SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS

Methods Unit 4 Test 4, 2018

(Calculator Free) Name___________________

Time: 22 minutes Marks: 22

Show working in sufficient detail to support your answers. Incorrect answers given without supporting reasoning may not be allocated any marks.

1. [2, 2 marks]

Determine $\frac{dy}{dx}$ for each (you do not need to simplify):

a) y = $ln\left[\frac{2x-1}{3x+1}\right]$

b) y = x^2. ln(sin x)

2. [2, 2 marks]

Determine the following:

a) $\int \frac{1}{3-2x} \quad dx$

b) $\int \frac{\cos(2x)}{\sin(2x)}$ dx

3. [4 marks]

Solve $2^{x-1} = 3^{3x}$, leaving your answer in exact form.

4. [3, 2 marks]

The continuous random variable X is defined by the p.d.f.

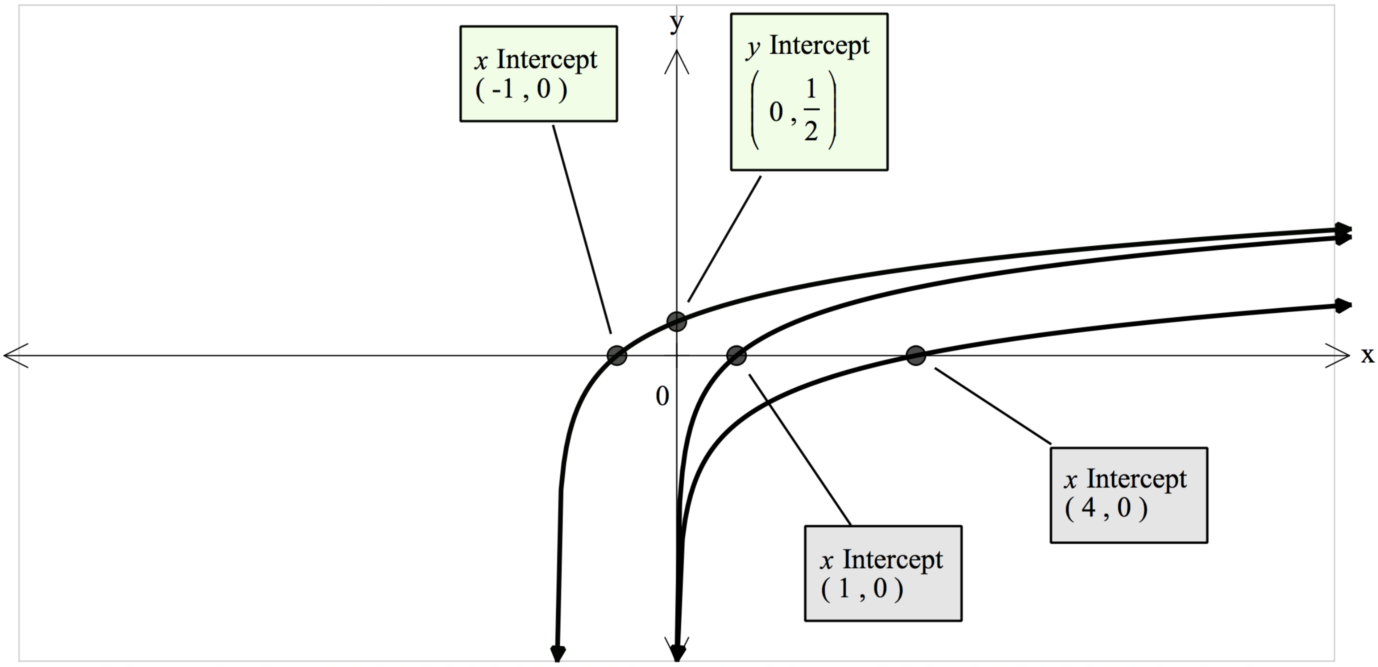
$$f(x) = \begin{cases} \frac{q}{x} & for\ \ 1 \le x \le 3 \\ 0 & elsewhere \end{cases}$$

a) Determine the exact value of *q*.

b) Determine $P(2 < x < 3)$

5. [5 marks]

The diagram below shows $y = \log_a(x)$, $y = \log_a(x + b)$ and $y = \log_a(x) + c$.
Determine a, b and c.



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Time: 43 minutes Marks: 43

Show working in sufficient detail to support your answers. Incorrect answers given without supporting reasoning may not be allocated any marks.

6. [2, 3, 2, 2 marks]

The serving time, T seconds, for a customer at an ATM is a uniformly distributed random variable, where $50 \leq T \leq 150$.

a) Sketch this distribution function below, using appropriate scales on each axis.

b) Find the expected value and standard deviation for this distribution.

c) Evaluate $P(T \geq 100 \mid T \leq 120)$

d) What is the probability that exactly 3 of the next 5 customers will require at least 2 minutes to be served?

7. [2, 3, 3, 3 marks]

The life (in years) of a light globe has a p.d.f. which can be modelled by:

$$f(x) = \begin{cases} \frac{4x}{3} & for\ 0 \le x \le 1 \\ \frac{4}{3x^5} & for\ x > 1 \end{cases}$$

a) Determine P(X < 1)

b) Determine P(X < 3)

c) Determine the expected value for this distribution.

d) If you had 1000 globes, how many would you expect to last longer than 3 years?

8. [2, 2, 4 marks]

a) Consider the shaded area shown between the graph of $y = e^x$, the y-axis and $y = 2$.

i) Determine the exact coordinates of point A.

ii) Hence, or otherwise, determine the shaded area.

b) If the area between $y = e^x$, the x-axis, the y-axis and $y = k$, where $k > 0$, is to be equal to 2 square units, determine the exact value of k.

9. [1, 4 marks]

a) Determine f '(x), given f(x) = $\frac{\ln(x)}{x}$

b) Hence, or otherwise, show that $\int_1^2 \frac{\ln(x)-1}{x^2} = \ln\left(\frac{1}{\sqrt{2}}\right)$

10. [3, 1, 2, 2, 2 marks]

A continuous random variable X has a pdf such that $f(x) = 0.4e^{-0.4x}$ defined over interval $[0, \infty]$

a) Show that $P(X \leq k) = 1 - e^{-0.4k}$

Hence or otherwise determine:

b) $P(X \leq 5)$

c) $P(5 \leq X \leq 6)$

c) $P(X \leq 6 \mid X \geq 5)$

d) the value of a, given that $P(X \leq a) = 0.2$