Full Name: SOLUTIONS

Ellenbrook
CHRISTIAN COLLEGE

Mathematics Methods
YEAR 11

Investigation 1 – Heron's Formula

Semester 1 2015

Time allowed: 70 minutes

Marks Available: 35 marks

Materials required: Writing implements, correction fluid/tape or eraser, ruler, Scientific or CAS calculator

Instructions:

1. Write your answers in the spaces provided in this Question/Answer Booklet.
2. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

1. (13 marks)

Heron's formula for calculating the area of a triangle is:

$$A = \sqrt{(s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

a) Use Heron's formula to calculate the area of a triangle with sides 5cm, 12cm and 13cm. [3]

$S = \frac{5+12+13}{2}$ ✓

$= 15$

$A = \sqrt{15(15-5)(15-12)(15-13)}$ ✓

$= 30\text{cm}^2$ ✓

b) Show that the triangle is in fact right angled and hence prove that your area calculation was correct. [2]

If right triangle then

$13^2 = 12^2 + 5^2$

$169 = 169$ ✓ Yes, right triangle

$A = \frac{1}{2} \times 5 \times 12$

$= 30\text{cm}^2$

$\therefore$ calculation correct

c) Explain why you cannot draw a triangle with sides 42cm, 20cm and 18cm. [2]

$42 > 20 + 18$

$\therefore$ sides 20 + 18 will not meet

d) Show what happens if you substitute the numbers from part (c) into Heron's formula. [2]

$S = \frac{42+20+18}{2}$

$= 40$

$A = \sqrt{40(40-42)(40-20)(40-18)}$

$= \sqrt{-35200}$

not possible; negative under root.

e) What relationship is necessary for each of *a*, *b* and *c* compared to *s* to form a triangle (in Heron's formula)? [1]

$a, b, c < S$

f) What are the dimensions of the equilateral triangle that has the same area as the triangle with side lengths 5cm, 12cm and 13cm from part (a). [3]

$30 = \sqrt{S(S-a)^3}$

$S = \frac{3a}{2}$

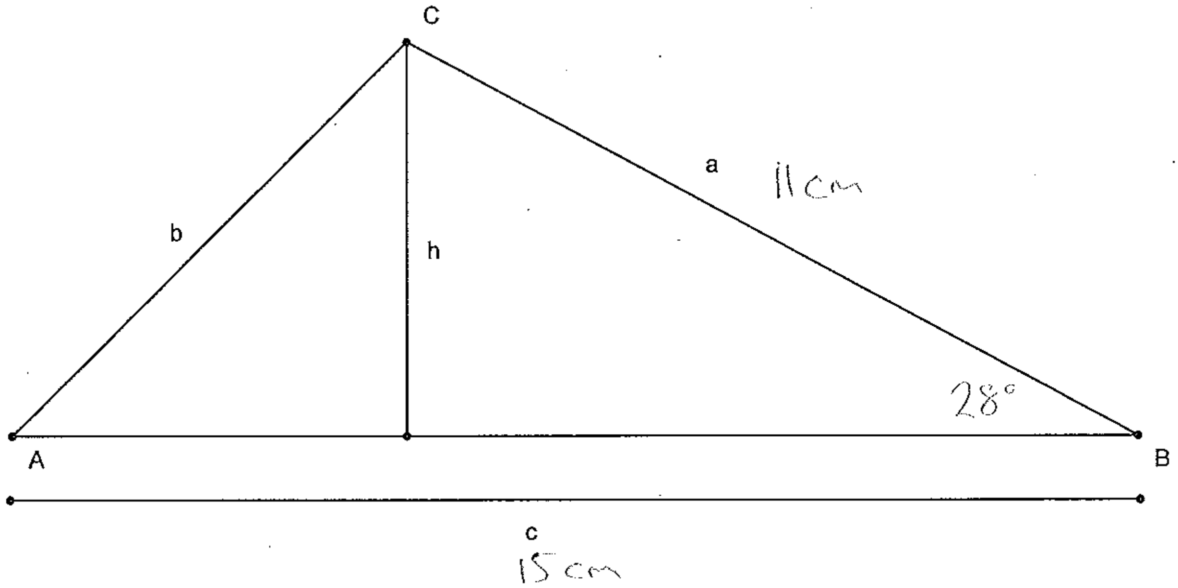
$30 = \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)^3}$

$900 = \frac{3a}{2} \times \frac{a^3}{8}$

$a^4 = 4800$

$a = 8.32\text{cm}$

2. (10 marks)
In the diagram below *a* (ie BC) is 11cm, *c* is 15cm and B or ∠ABC is 28°.



a) Use a ruler to measure the perpendicular height of this triangle and hence calculate its area. [2]

$h = 5.2\text{cm}$

$$A = \frac{1}{2} \times 15 \times 5.2$$
$$= 39\text{ cm}^2$$

b) Use trigonometry to check your answer in part (a). Clearly show your method. [3]

$$A = \frac{1}{2} \times 15 \times 11 \times \sin 28$$
$$= 38.73\text{ cm}^2$$

OR

$$A = \sqrt{16.7(5.7)(1.7)(9.31)}$$
$$= 38.8\text{cm}^2$$

$b^2 = 11^2 + 15^2 - 2(11)(15)\cos 28$

$b = 7.39\text{cm}$

$$s = \frac{7.39 + 11 + 15}{2}$$
$$= 16.7$$

c) For the ΔABC on the previous page, state:

i. how to calculate the area of the triangle in terms of *c* and *h* [1]

$$A = \frac{1}{2}ch$$

ii. how to calculate the perpendicular height using the sine ratio, in terms of *a* and *h* [1]

$$\left(\sin 28 = \frac{h}{11}\right)$$

$$h = a \sin B$$

iii. Use the information in part (i) and (ii) above to determine a formula for calculating the area of the triangle without using *h* in the equation. [2]

$$A = \frac{1}{2} ca \sin B$$

From the work you have just done it follows that $\Delta ABC = \dfrac{ab\sin C}{2} = \dfrac{ac\sin B}{2} = \dfrac{bc\sin A}{2}$

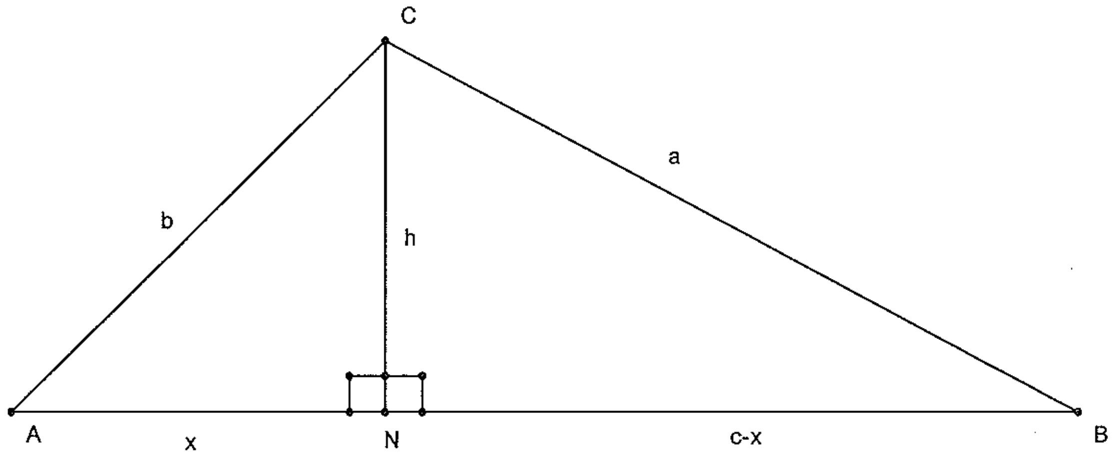
d) Explain how you can arrange the equation $\Delta ABC = \dfrac{ab\sin C}{2} = \dfrac{ac\sin B}{2}$ to show that $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$ [1]

$$\frac{\cancel{a}b\sin C}{\cancel{2}} = \frac{\cancel{a}c\sin B}{\cancel{2}}$$

$$\frac{\cancel{b}\sin C}{bc} = \frac{\cancel{c}\sin B}{bc}$$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

3. (12 marks)
Consider the triangle below



Complete the following:

a) In triangle ANC, use Pythagoras to find an equation for b^2 in terms of h and x. [1]

$$b^2 = x^2 + h^2$$

b) Rearrange to find an equation for h^2 in terms of b and x. [1]

$$h^2 = b^2 - x^2$$

c) In triangle BNC, use Pythagoras to find an equation for a^2 in terms of h, c and x. [1]

$$a^2 = h^2 + (c-x)^2$$

d) Rearrange to find an equation for h^2 in terms of a, c and x. [1]

$$h^2 = a^2 - (c-x)^2$$

e) Hence or otherwise, establish an equation for a^2 in terms of b, c, and x. [4]

$a^2 = h^2 + (c-x)^2$

$a^2 = b^2 - x^2 + (c-x)^2$ ✓½ simplify.

$a^2 = b^2 - x^2 + c^2 - 2cx + x^2$

$a^2 = b^2 + c^2 - 2cx$

f) Determine the algebraic value of cos A in triangle CAN [1]

$\cos A = \frac{x}{b}$

g) Rearrange your equation in part (f) to make x the subject. [1]

$x = b\cos A$

h) Substitute your answer into part (e) and identify which rule you have proven. [2]

$a^2 = b^2 + c^2 - 2bc\cos A$

cosine rule proven.

End of Investigation