SHENTON COLLEGE

Examination Semester One 2017
Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 3

Section One
(Calculator-free)

Your name ______________________________

Time allowed for this section

Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section One.
Formula sheet.

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	11	11	100	99	65
			Total	152	100

Section One: Calculator-free **35% (53 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

A sphere has its centre at R, which has position vector $2\mathbf{i} - \mathbf{k}$. If the diameter of the sphere is $\overrightarrow{PQ}$ and the position vector of Q is $8\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.

(a) Determine the position vector of P. (2 marks)

(b) Determine the vector equation of the sphere. (2 marks)

(c) The sphere intersects the y-axis where $y = a$. Determine the value(s) of the constant a. (2 marks)

Question 2 **(5 marks)**

A function is defined by $g(z) = 2z^4 - z^3 + 7z^2 - 4z - 4$.

(a) Show that $z = 1$ and $z = 2i$ are both zeros of $g(z)$. (2 marks)

(b) Determine all solutions to $g(z) = 0$. (3 marks)

Question 3 **(8 marks)**

Simplify the following into the form $x + iy$.

(a) $\dfrac{3}{2i} + 2i$. (2 marks)

(b) $\dfrac{1}{(2-i)^2}$. (3 marks)

(c) $\left(-\sqrt{2} + \sqrt{2}i\right)^6$. (3 marks)

Question 4 **(7 marks)**

The function f is defined by $f(x) = \frac{1}{1-x}$.

(a) Evaluate $f(f(-1))$. (1 mark)

(b) Determine and simplify an expression for $f \circ f(x)$. (2 marks)

(c) For $f \circ f(x)$, state the

(i) domain. (2 marks)

(ii) range. (2 marks)

Question 5 **(7 marks)**

(a) The equation $2z^2 + 3z + 5 = 0$ has roots of α and β. Determine the value of

(i) $\alpha + \beta$. (1 mark)

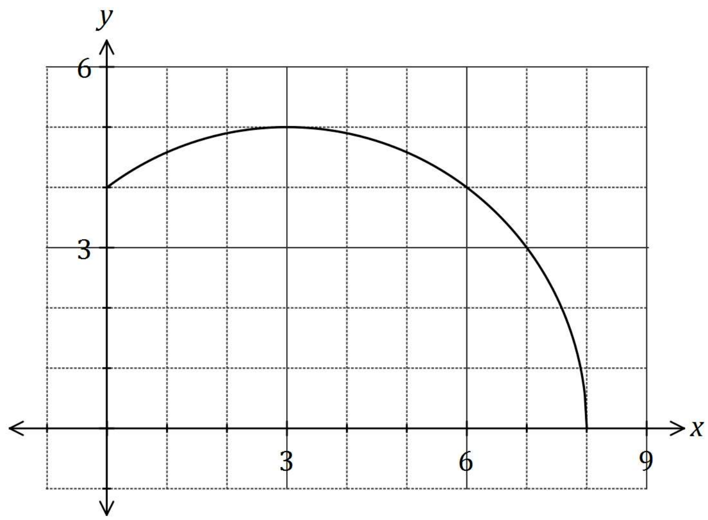
(ii) $\alpha\beta$. (1 mark)

(iii) $2\alpha^2 + 3\alpha + 5$. (1 mark)

(b) Determine the values of the real constants a and b if $z - 2 + i$ is a factor of $z^3 + az + b$. (4 marks)

Question 6 **(6 marks)**

Let $f(x) = \sqrt{16 + 6x - x^2},\ 0 \le x \le 8$. The graph of $y = f(x)$ is shown below.

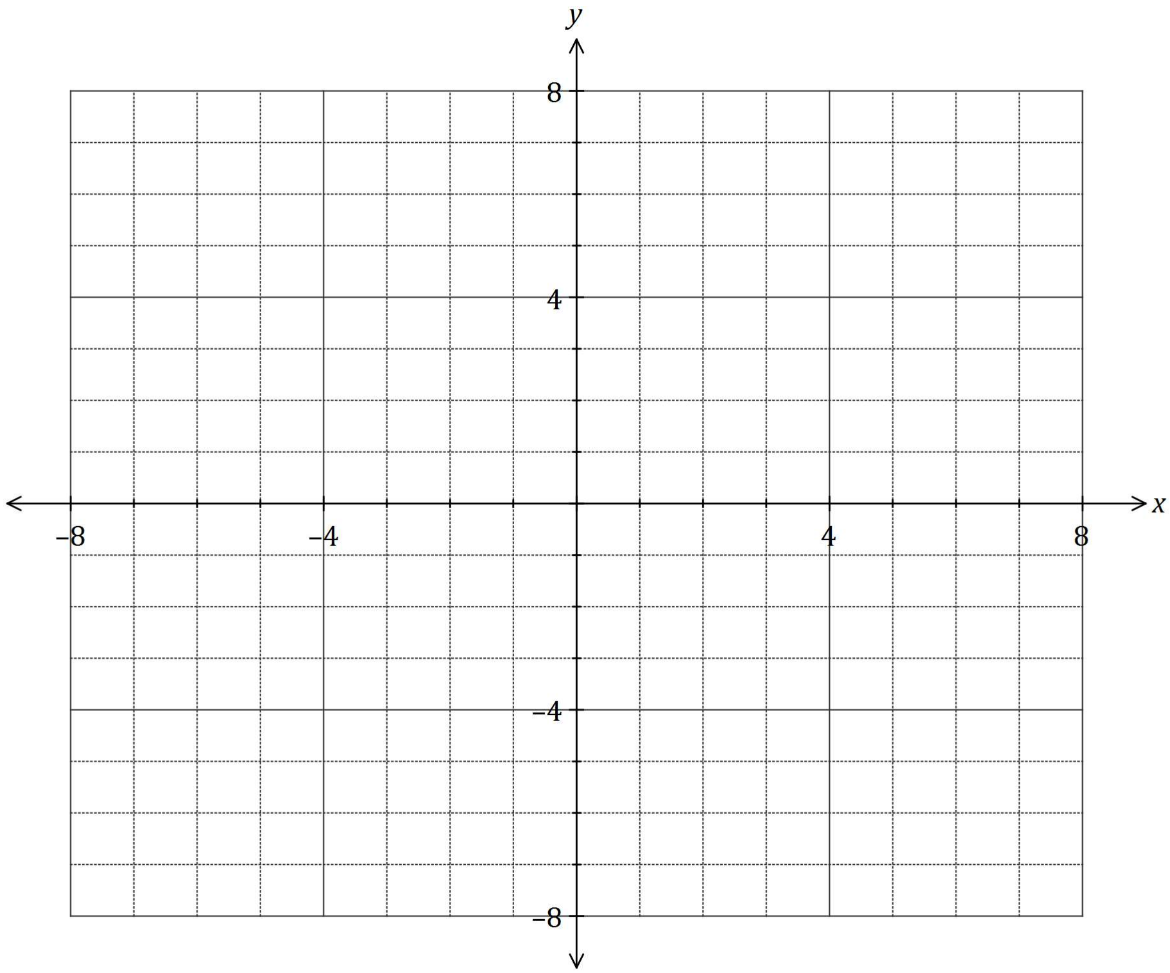


(a) In order that $y = f^{-1}(x)$ is a function, the domain of f must be restricted to $k \le x \le 8$. Explain why this restriction is necessary and state the minimum value of k. (2 marks)

(b) Using the restriction from (a), determine the inverse function of f and its domain. (4 marks)

Question 7 **(6 marks)**

On the axes below, draw the graph of $y = \frac{x^2}{x^2-2x-3}$, clearly showing key features and the behaviour of the curve near the asymptotes.



Question 8 **(8 marks)**

(a) Two of the solutions to the equation $z^n = 1, n \in \mathbb{Z}^+$, are $z = \text{cis}\frac{\pi}{2}$ and $z = \text{cis}\frac{\pi}{3}$.

(i) State another solution to the equation. (1 mark)

(ii) Determine, with reasons, the minimum value of n. (2 marks)

(ii) If $z = cis\frac{11\pi}{24}$ is <u>not</u> a solution to the equation then determine, with reasons, the maximum value of n. (2 marks)

(b) If $z = \text{cis}\frac{\pi}{4}$, determine the sum of the geometric series $1 + z + z^2 + z^3 +. \ldots + z^{24}$. Explain your answer. (3 marks)