Name____________________________

Maths Specialist - Investigation

Matrix Transformations

Investigation **Part 1: Preparation activity**

A point in the x-y plane can be expressed as a 2 × 1 matrix: for example, the point (1, 2) can be written as $\begin{bmatrix}1\\2\end{bmatrix}$.

This point can then be transformed into another point by multiplication by a 2 × 2matrix.

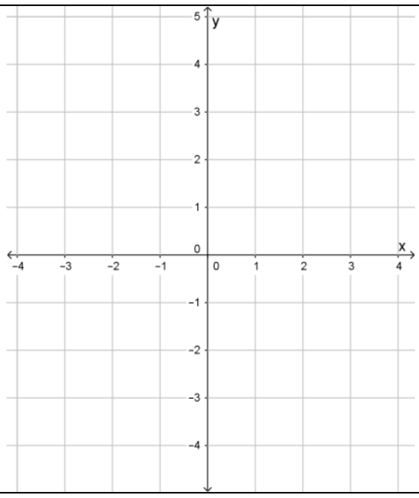
$$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} \times \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\\-2\end{bmatrix}$$

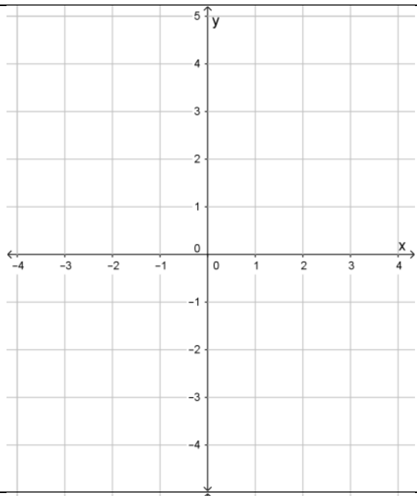
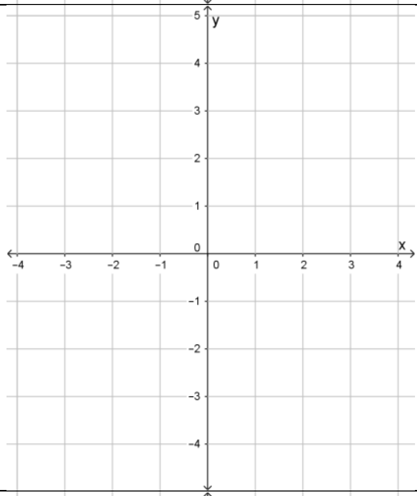
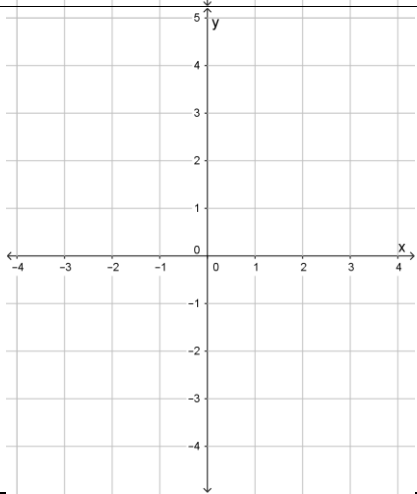
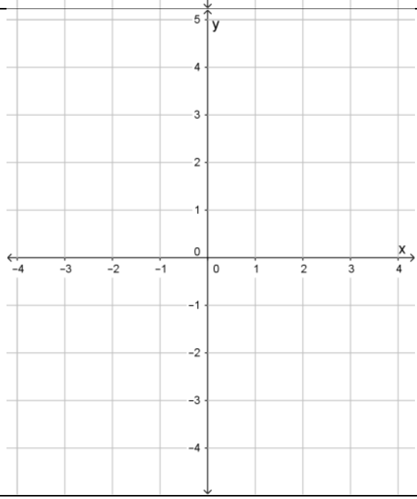
Transformation matrix × point = Transformed point

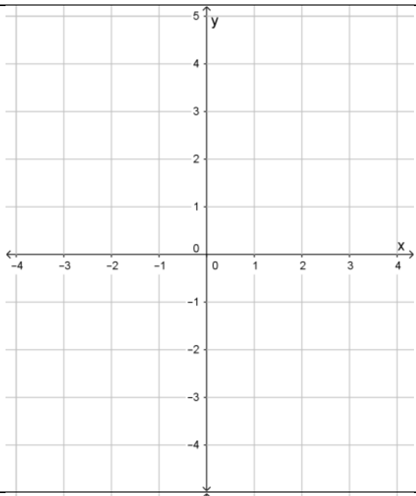
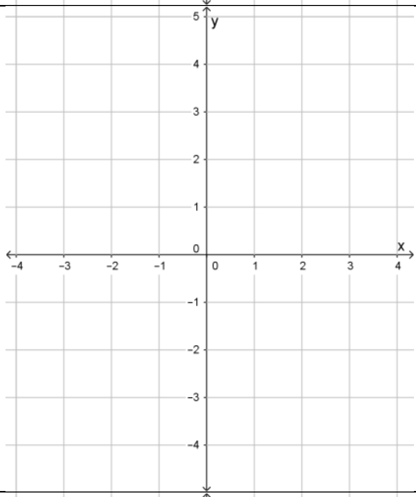
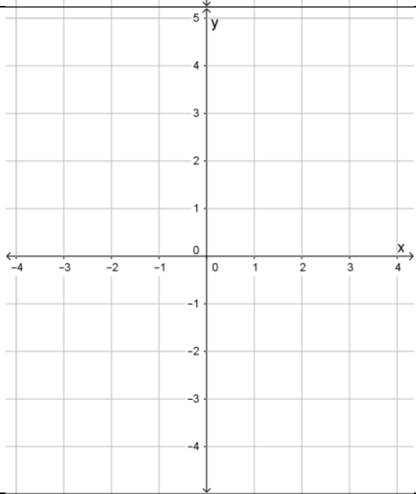
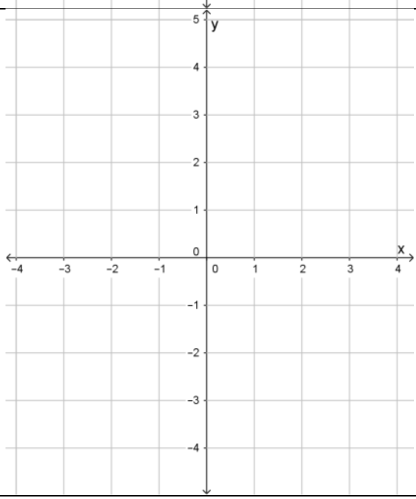
[NB: The transformation matrix is pre-multiplied by the point.]

Questions 1

Use the shape joined by the co-ordinates 0(0,0), A(1,0), B(1,2) and C(0,2), to investigate the effect of 2x2 transformation matrices of these types:

Matrix	Calculation	Diagram	Effect – Shape, position, size
$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$			

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$			
$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$			
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$			
$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$			

$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$			
$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$			
$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$			
$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$			

Question 2

Investigate the effect of a transformation by the following matrices, where k is a constant.

$\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$	
$\begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$	
$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$	
$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$	

Question 3

a) Find the area of rectangle OABC

For each the matrices in question 1,

b) find the absolute value of the determinant of the matrix.

c) Find the area of $\mathrm{O'A'B'C'}$ the image of OABC under the transformation.

d) Find the value of $\frac{Area\ \mathrm{O'A'B'C'}}{Area\ \mathrm{OABC}}$

Matrix	Determinant	$Area\ \mathrm{O'A'B'C'}$	$\frac{Area\ \mathrm{O'A'B'C'}}{Area\ \mathrm{OABC}}$

e) What do you notice?

By using the same transformation matrices above investigate the effect on a different shape.

Can you predict the effect the transformation matrices will have?

Can you predict the area of the image after it has been transformed the matrix?