Year 11 Mathematics Specialist

Test 1 – 5.5%

MELVILLE
SENIOR HIGH SCHOOL

Student Name : _____

Part One – Resource Free

Part One contains 10 questions worth 34 marks

Time Allowed : 35 minutes

INSTRUCTIONS TO STUDENTS:
You are required to attempt ALL questions,
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.

Question 1   1 mark

How many different debating teams of 3 people can be chosen from 6 people?

$$\binom{6}{3} = 20$$

Question 2   3 marks

Let $n \in \mathbb{Z}.$ Consider the statement : If $5n + 3$ is even, then n is odd.

a)   Write the inverse of this statement.

If 5n+3 is odd, then n is even

b)   Write the converse of this statement.

If n is odd, then 5n+3 is even

c)   Write the contrapositive of this statement.

If n is even, then 5n+3 is odd

Question 3   2 marks

How many four digit passwords can be formed using the digits using 2 letters followed by two numbers if :

a)   repetition is not allowed   (Do not evaluate)   26x25x10x9

b)   repetition is allowed?   (Do not evaluate)   26x26x10x10

Question 4   **2, 3 – 5 marks**

a)   Find n and r given $^nC_r = \dfrac{5!}{4!1!} + \dfrac{5!}{3!2!}$.
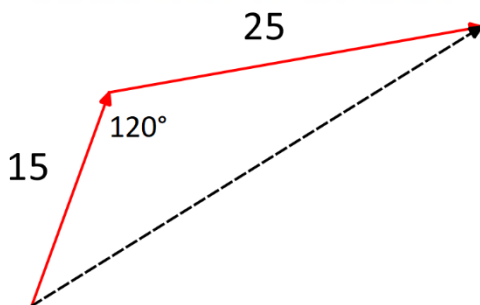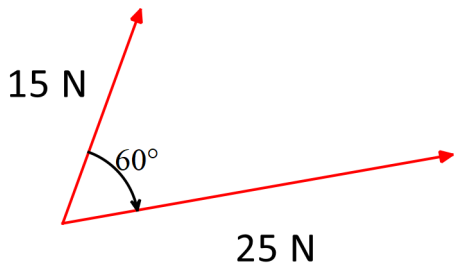
$$n=6, r=2 \text{ or } 4$$

b)   Simplify $\dfrac{1}{n!} + \dfrac{1}{(n+1)!}$

$$\dfrac{(n+1)!+n!}{n!(n+1)!}$$
$$=\dfrac{(n+1)n!+n!}{n!(n+1)!}$$
$$=\dfrac{n!(n+1+1)}{n!(n+1)!}$$
$$=\dfrac{n+2}{(n+1)!}$$

Question 5   4 marks

Two forces have magnitudes of 15 N and 25N and the angle between them is 60°.
a)   Draw a diagram to represent the situation
b)   Find the exact magnitude of the resultant.

$$r^2 = 15^2 + 25^2 - 2 \times 15 \times 25 \times \cos 120°$$
$$= 225 + 625 - 750 \times \left(-\frac{1}{2}\right)$$
$$= 1225$$
$$r = 35\ N$$

1 each diagram

1 1225 or substitution

1 35

Question 6   2 marks

A small child puts his hand into a container of 10 different lollies. Given that neither the container nor his hand is empty when he removes it, how many different handfuls of lollies could he have chosen?

$$2^{10} - 2 = 1022$$

Question 7   **2, 4 - 6 marks**

How many arrangements can be made from the letters of the word NEEDLE?

$$\frac{6!}{3!} = 120$$

How many 3-letter arrangements can be made from the letters of the word NEEDLE?

All different letters : $4 \times 3 \times 2 = 24$

2 E's & * : $\dfrac{3!}{2!} \times 3 = 9$

All E's : 1

Total : 34

Question 8   **3 marks**

Show that in any collection of 5 counting numbers, at least two will have the same remainder when divided by four.

When dividing by four, there are four possible remainders (0,1,2 or 3) which are the pigeonholes. If each of the numbers is a pigeon, with 5 pigeons and 4 pigeon holes, at least two of the numbers must have the same remainder.
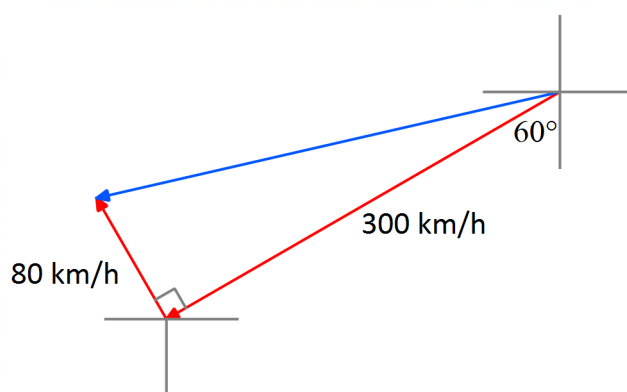
Given the set of numbers {1, 2, 3, …, 80}, how many of these are not divisible by 4 or 5 or 10?

An aircraft is flying on a bearing of 240° at 300 km/h. An 80 km/h wind is blowing from a bearing of 150°.

Draw a diagram to represent this situation, clearly indicating all relevant magnitudes and angles, to determine the final velocity of the aircraft, but do not evaluate.

1 indicate magnitudes and draws resultant

1 correct direction of wind

Indicates 90 degree angle

Student Name : _____

Part Two – Resource Allowed

Part Two contains 4 questions worth 22 marks

Time Allowed : 25 minutes

TO BE PROVIDED BY THE STUDENT

A maximum of one A4 page of notes, one sided.

Standard Items : Pens, pencils, eraser, sharpener, correction tape/fluid, highlighters, ruler.

Special Items: Drawing instruments, templates.

A maximum of three CAS calculators satisfying the conditions set by the SCSA.

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Question 1

a) How many ways can the letters of the word BEHAVIOUR be arranged?

$9! = 362880$

b) How many of these arrangements begin with the letter H followed by the letter A?

$HA7! = 5040$

c) How many of the arrangements have the vowels next to each other?

$5! \times 5! = 14400$

d) How many arrangements do not have A and H adjacent?

$9! - 2 \times 8!$
$= 362880 - 40320 \times 2$
$= 282240$

Question 2

Mrs Greenaway's book shelf contains 3 books by Alan Sadler and 6 other books. How many ways can the books be arranged if none of the Sadler books are placed next to teach other?
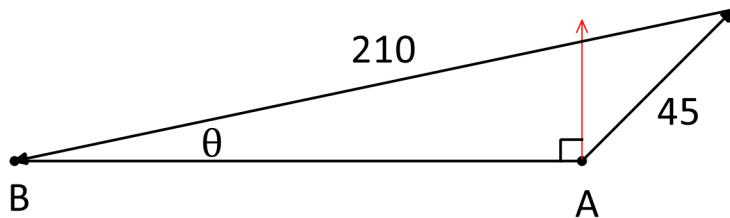
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$6! \times \binom{7}{3} \times 3! = 151200$

The air speed of a plane is 210 km/h. A steady wind is blowing from the south-west at 45 km/h.

a)   In what direction should the plane head to fly to B, which is due West of A?

$$\frac{\sin\theta}{45} = \frac{\sin135}{210}$$
$$\theta = 8.7°$$

Bearing $270 - 8.7 = 261°$

b)   How long will the flight take if B is 400 km away from A?

$$x^2 + 45^2 - 90x\cos135° = 210^2$$

By classpad $x = 175.76, -239.40$

Final velocity: 175.8km/h

$$\text{Time} = \frac{400}{175.8}$$
$$= 2.275 \text{ h}$$
$$= 2 \text{ hours 17 minutes}$$

Five boys and eight girls have nominated to participate in a mixed netball team of 7.
How many ways can this be done if:

a)   there are no restrictions

$$\binom{13}{7} = 1716$$

b)   there are four girls and three boys on the team

$$\binom{5}{3}\binom{8}{4} = 700$$

c)   there must be at least two boys on the team

$$\binom{5}{0}\binom{8}{7} + \binom{5}{1}\binom{8}{6} = 148$$

$$1716 - 148 = 1568$$

d)   Ben must be on the team

$$\binom{12}{6} = 924$$

e)   Francesca and Maxwell cannot be on the team together?

$$\binom{2}{2}\binom{11}{5} = 462$$

$$1716 - 462 = 1254$$