MATHEMATICS:SPECIALIST 3 & 4
SEMESTER 2016
TEST 4
Calculator Free

Eastern Goldfields College

Reading Time: 2 minutes
Time Allowed: 33 minutes

Total Marks: 31

1. [10 marks - 2, 3, 1, 2, 2]

Determine the following:

a) $\int 8\cos^3 x \sin x \, dx$

$= -\frac{8\cos^4 x}{4} + c$

$= -2\cos^4 x + c$

b) $\int 4\sin^3 x \, dx$

$= \int 4\sin^2 x \sin x \, dx$

$= 4\int (1-\cos^2 x)\sin x \, dx$

$= 4\int \sin x - \cos^2 x \sin x \, dx$

$= -4\cos x + \frac{4\cos^3 x}{3} + c$

e) $\int \sin x \, . \, e^{\cos x} dx$

$= -e^{\cos x} + c$

f) $\int \frac{2}{6x-3} dx$

$= \frac{1}{3}\ln|6x-3| + c$

g) $\int \frac{2x^2-3}{x^3} dx$

$= \int \frac{2}{x} - \frac{3}{x^3} dx$

$= 2\ln|x| + \frac{3}{2x^2} + C$

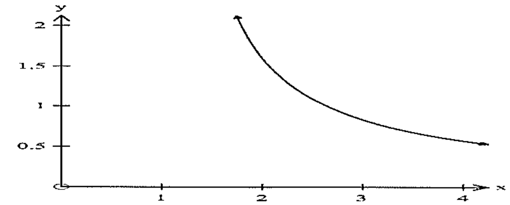
h) $\int \cos 3x \cos 2x - \sin 3x \sin 2x \, dx$

$= \int \cos 5x \, dx$

$= \frac{\sin 5x}{5} + c$

2. [8 marks – 4, 4]

Part of the graph of the function $f(x) = \frac{2x+4}{x^2+2x-3}$ is shown below



(a) Use partial fractions to show that $f(x) = \frac{1}{2(x+3)} + \frac{3}{2(x-1)}$

$f(x) = \frac{A}{x+3} + \frac{B}{x-1} = \frac{A(x-1) + B(x+3)}{(x+3)(x-1)}$

$A + B = 2$
$-A + 3B = 4$

$4B = 6$
$B = \frac{3}{2}, \; A = \frac{1}{2}$

$\therefore f(x) = \frac{1}{2(x+3)} + \frac{3}{2(x-1)}$

(b) Show that the area under the graph of $y = f(x)$ between $x = 2$ and $x = 3$ is $ln\left(\frac{4\sqrt{3}}{\sqrt{5}}\right)$

$\int_2^3 \left(\frac{1}{2(x+3)} + \frac{3}{2(x-1)}\right) dx$

$= \left[\frac{1}{2}\ln|x+3| + \frac{3}{2}\ln|x-1|\right]_2^3$

$= \frac{1}{2}\ln 6 + \frac{3}{2}\ln 2 - \frac{1}{2}\ln 5$

$= \frac{1}{2}\left(\ln\left(\frac{6 \times 8}{5}\right)\right)$

$= \frac{1}{2}\ln\frac{48}{5}$

$= \ln\left(\frac{48}{5}\right)^{\frac{1}{2}} = \ln\left(\frac{4\sqrt{3}}{\sqrt{5}}\right)$

3. [8 marks – 4, 4]

(a) Determine $\int_0^2 \frac{x}{\sqrt{x^2+1}}dx$

Let $u = x^2+1$

$\checkmark$ $\frac{du}{dx} = 2x$

$x=0,\ u=1$

$x=2,\ u=5$

$$\int_1^5 \frac{\cancel{x}}{u^{1/2}}\frac{du}{2\cancel{x}}$$

$$= \checkmark \ \frac{1}{2}\int_1^5 u^{-\frac{1}{2}}du$$

$$= \frac{1}{\cancel{2}}\left(2u^{1/2}\right)\Big]_1^5 \checkmark$$

$$= \sqrt{5} - 1 \quad \checkmark$$

OR otherwise

(b) Determine $\int(4cos^2x - 2sin^2x - 1)dx$

$$= \int 4\cos^2 x - 2(1-\cos^2 x) - 1\, dx \quad \checkmark$$

$$= \int 4\cos^2 x - 2 + 2\cos^2 x - 1\, dx$$

$$= \int 6\cos^2 x - 3\, dx$$

$$= 3\int 2\cos^2 x - 1\, dx \ \checkmark$$

$$= 3\int \cos 2x\, dx \quad \checkmark$$

$$= \frac{3}{2}\sin 2x + c \quad \checkmark$$

4. [5 marks]

Evaluate $\int_0^1 x\,(2x-1)^3\,dx$ using the substitution $u = 2x-1$

$x = \frac{u+1}{2}$

$\frac{du}{dx} = 2 \ \checkmark$

$\checkmark$ $x=0,\ u=-1$

$x=1,\ u=1$

$$= \int_{-1}^{1} x . u^3 \frac{dy}{2}$$

$$= \frac{1}{2}\int_{-1}^{1}\left(\frac{u+1}{2}\right)u^3\,du$$

$$\checkmark = \frac{1}{4}\int_{-1}^{1} u^4 + u^3\,du$$

$$\checkmark = \frac{1}{4}\left[\frac{u^5}{5} + \frac{u^4}{4}\right]_{-1}^{1}$$

$$= \frac{1}{4}\left(\frac{1}{5} + \cancel{\frac{1}{4}} - \left(-\frac{1}{5} + \cancel{\frac{1}{4}}\right)\right)$$

$$= \frac{1}{4}\cdot\frac{2}{5}$$

$$\checkmark = \frac{1}{10}$$

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MATHEMATICS:SPECIALIST 3 & 4
TEST 4 2016
Calculator Assumed

Reading Time: 2 minutes
Time Allowed: 23 minutes

Total Marks: 21

5. [6 marks - 2, 2, 2]

(a) An unknown function is such that its table of values are given below.

x	3	4	5	6	7	8	9	10	11	12	13
f(x)	13	13.8	14.1	14.6	15.3	15.7	15.9	16.5	17.1	18.6	20.1

Using the midpoint rule with 5 strips, determine the approximate area under the function between x = 3 and x = 13.

Sums $13.8+14.6+15.7+16.5+18.6=79.2$ ✓
multiplies by 2 to get 158.4 sq units ✓

(b) Determine the area under the curve of $f(x) = \frac{\sqrt{x}}{\ln x}$ between x = 2 and x = 10 using Simpson's rule with strips of width 0.5. Give your answer to 6 decimal places.

19.196212 ✓✓

(c) An unknown function is such that its table of values are given below.

x	1	1.5	2	2.5	3	...	10	10.5	11
f(x)	8.2	8.175	7.6	6.625	5.4	...	46	59.025	74.2

The formula for the trapezoidal rule is:

$$A \approx \frac{w}{2} \times \left[f(a_0) + 2\sum_{i=1}^{n-1} f(a_i) + f(a_n) \right]$$

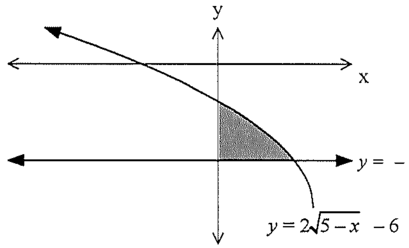
Given that $\sum(8.2 + 8.175 + 7.6 + \ldots + 74.2) = 333.2$ determine the approximate area under the function between x = 1 and x = 11 using the trapezoidal rule.

Calculates $f(a_0)+2\sum_{i=1}^{n-1} f(a_i)+f(a_n)=584$ ✓

Area = 146 sq units ✓

6. [8 marks - 4, 2, 2]

The graphs of $y = -a$ and $y = 2\sqrt{5-x} - 6$ are shown below:



(a) The expression for the shaded area (above) can be written in the form: $\int_0^b c \, dx$.

Determine expressions for b and c.

$2\sqrt{5-x}-6=-a$ ✓
$2\sqrt{5-x}=6-a$
$\sqrt{5-x}=\frac{6-a}{2}$
$5-x=\left(\frac{6-a}{2}\right)^2$ ✓
$x=5-\left(\frac{6-a}{2}\right)^2$

i.e. $b=5-\left(\frac{b-a}{2}\right)^2$ ✓

$c=2\sqrt{5-x}-6+a$ ✓

(b) Determine the values of a and b, if it is known that $2\sqrt{5-x} - 6 = -a$, when $x = 4$.

If $x=4$ $\quad b=5-\left(\frac{6-4}{2}\right)^2=4$ ✓
$a=6-2\sqrt{5-4}=4$ ✓

c) Determine the area of the shaded region to two decimal places. Working is not required.

$\int_0^4 2\sqrt{5-x}-2 \, dx = 5.57$ ✓✓

7. [7 marks – 3, 4}

(a) Find the exact volume of the solid of revolution formed when the line $3x + 4y = 36$ between the limits $y = 1$ and $y = 7$ is rotated about the y-axis.

$$3x + 4y = 36 \Rightarrow x = 12 - \frac{4}{3}y \checkmark$$

$$V = \pi \int_1^7 \left(12 - \frac{4}{3}y\right)^2 dy \checkmark$$

$$= \frac{896\pi}{3} \text{ cubic units} \checkmark$$

(b) When the same line, $3x + 4y = 36$, is rotated about the x-axis between the limits $x = 1$ and $x = a$, the volume of the solid of revolution formed is $\frac{489\pi}{2}$.
Determine the value of a.

$$3x + 4y = 36 \Rightarrow y = 9 - \frac{3}{4}x \checkmark$$

$$\checkmark \frac{489\pi}{2} = \pi \int_1^a \left(9 - \frac{3x}{4}\right)^2 dx$$

$$\frac{489\pi}{2} = \pi\left(\frac{4 \cdot \left(\frac{3a}{4} - 9\right)^3}{9} + \frac{3993}{16}\right)$$

$$a = 9 \checkmark\checkmark$$