

Full Name: SOLUTIONS



Ellenbrook
CHRISTIAN COLLEGE

MATHEMATICS METHODS

Test 1 – Trigonometry

Chapters 3 and 6

Semester 1 2015

Time allowed for this section

Working time for this section: 65 minutes

Marks available: 57 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

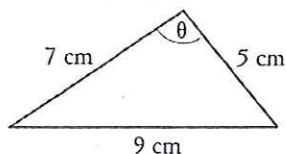
Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Multiple choice questions – select the best response

1. (1 mark)

Which statement is true?



A $\cos(\theta) = \frac{5^2 + 9^2 - 7^2}{2 \times 5 \times 9}$

B $\cos(\theta) = \frac{5^2 + 7^2 - 9^2}{5 \times 7}$

C $\cos(\theta) = \frac{9 + 7 - 5}{2 \times 9 \times 7}$

D $\cos(\theta) = \frac{5^2 + 7^2 - 9^2}{2 \times 5 \times 7}$

E $\cos(\theta) = \frac{9^2 + 7^2 - 5^2}{9 \times 7}$

2. (1 mark)

$\tan\left(\frac{4\pi}{3}\right) =$

A $\frac{1}{\sqrt{3}}$

B $-\sqrt{3}$

C $\sqrt{3}$

D $-\frac{1}{\sqrt{3}}$

E 1

3. (1 mark)

Which statement is not correct?

A $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

B $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

C $\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

D $\sin(x - y) = \sin(y)\cos(x) - \cos(y)\sin(x)$

E $\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

4. (1 mark)

Solve $\cos(2x) = -1$ for $0 \leq x \leq 2\pi$.

A $x = \frac{\pi}{2}, \frac{3\pi}{2}$

B $x = \left(\frac{\pi}{2}\right)$

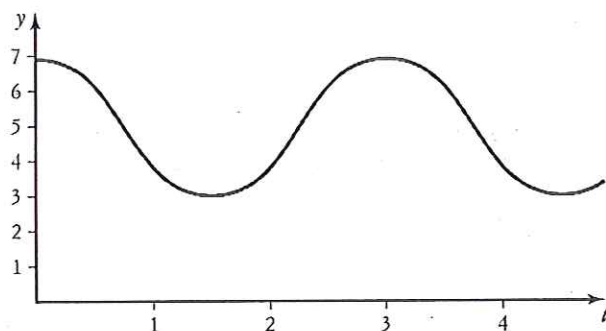
C $x = 0, \pi, 2\pi$

D $x = \left(\frac{3\pi}{2}\right)$

E $x = \pi, \left(\frac{3\pi}{2}\right)$

5. (1 mark)

Find the amplitude and period of the following graph.



A Amplitude 4, period 4

B Amplitude 4, period 3

C Amplitude 4, period $\frac{2\pi}{3}$

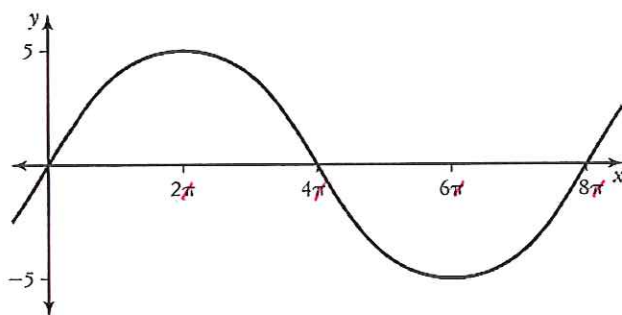
D Amplitude 2, period 3

E Amplitude 2, period $\frac{2\pi}{3}$

D

6. (1 mark)

The equation of the function below is:

A $y = 8 \sin(5x)$ B $y = 5 \sin(8x)$ C $y = 5 \sin\left(\frac{\pi x}{8}\right)$ D $y = 5 \sin\left(\frac{\pi x}{4}\right)$ E $y = 5 \sin(4\pi x)$

Mistake

remove π on x-axis

OR,

Bonus +1

 $y = 5 \sin\left(\frac{x}{4}\right)$

7. (1 mark)

Find the amplitude and period of the function $f(x) = 2 - 5 \cos(2\pi x)$.A Amplitude 2, period 2π B Amplitude 5, period π

C Amplitude -5, period 1

D Amplitude -5, period π

E Amplitude 5, period 1

8. (1 mark)

Find the exact value of $\sin(105^\circ)$.

A $\frac{\sqrt{3} - 1}{2\sqrt{2}}$

B $\frac{1 - \sqrt{3}}{2\sqrt{2}}$

C $\frac{\sqrt{3} + 1}{\sqrt{2}}$

D $\frac{\sqrt{3} - 1}{\sqrt{2}}$

E $\frac{\sqrt{3} + 1}{2\sqrt{2}}$



Short and Extended Answer Questions – show working for questions worth more than 2 marks

8. (2 marks)

Convert the following to radians, giving exact values.

a) 45°

$$45 \times \frac{\pi}{180} = \frac{\pi}{4}$$

b) 300°

$$300 \times \frac{\pi}{180} = \frac{5\pi}{3}$$

9. (2 marks)

Convert the following radians to degrees.

a) $\frac{\pi}{6}$

$$\frac{\pi}{6} \times \frac{180}{\pi} = 30^\circ$$

b) $\frac{5\pi}{4}$

$$\frac{5\pi}{4} \times \frac{180}{\pi} = 225^\circ$$

10. (3 marks)

State which quadrant each of the following angles are in.

a) -285°

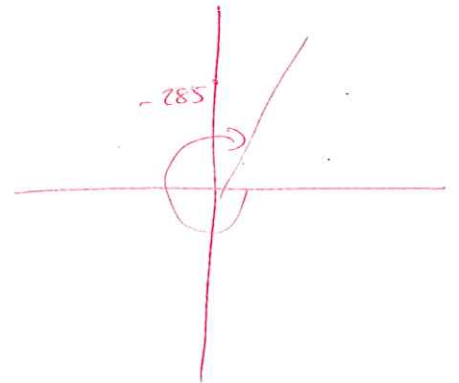
Q1

b) $\frac{7\pi}{6}$

Q3

c) $-\frac{13\pi}{4}$

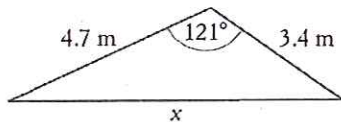
Q2



11. (6 marks)

Evaluate x correct to one decimal place.

a)

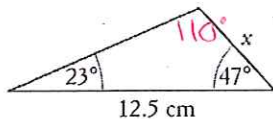


$$x^2 = 4.7^2 + 3.4^2 - 2 \times 4.7 \times 3.4 \cos 121^\circ$$

$$x = \underline{7.1 \text{ m}}$$

1 d.p.

b)



$$\frac{12.5}{\sin 110} = \frac{x}{\sin 23}$$

$$x = \underline{5.2 \text{ cm}}$$

1 d.p.

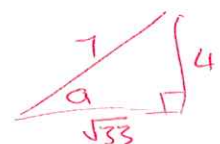
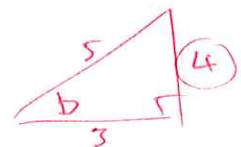
12. (5 marks)

If $\sin(a) = \frac{4}{7}$ and $\cos(b) = \frac{3}{5}$, find the exact value of $\cos(a+b)$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$= \frac{\sqrt{33}}{7} \times \frac{3}{5} - \frac{4}{7} \times \frac{4}{5}$$

$$= \frac{3\sqrt{33} - 16}{35}$$



13. (2 marks)

Using your CAS calculator, graph the functions $y = 2 \cos(x)$ and $y = x - 1$ on the same set of axes and graphically solve $2 \cos(x) = x - 1$ (you do not need to draw a graph).

$$(1.38, 0.38)$$

14. (2 marks)

Expand

a) $\tan(2a + b)$

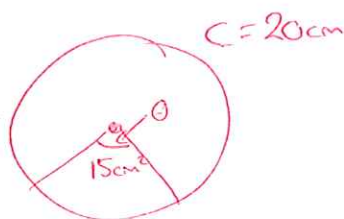
$$= \frac{\tan(2a) + \tan(b)}{1 - \tan(2a)\tan(b)}$$

b) $\cos(5a - 3b)$

$$= \cos(5a)\cos(3b) + \sin(5a)\sin(3b)$$

15. (5 marks)

The circumference of a circle is 20 cm. Find the angle subtended at the centre if it cuts off a sector with an area of 15 cm^2 .



$$20 = 2\pi r$$

$$r = \frac{10}{\pi}$$

$$15 = \frac{1}{2} \left(\frac{10}{\pi} \right)^2 \theta$$

$$15 = \frac{100}{2\pi^2} \times \theta$$

$$\theta = \frac{30\pi^2}{100} \quad (2.96 \text{ rad}) \quad \text{or} \quad 169.65^\circ$$

16. (3 marks)

Show that $\cos(2x) = 1 - 2\sin^2(x)$

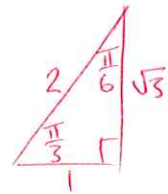
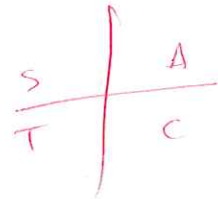
$$\begin{aligned}
 \cos(2x) &= \cos(x+x) \\
 &= \cos x \cos x - \sin x \sin x \\
 &= \cos^2 x - \sin^2 x \\
 &= 1 - \sin^2 x - \sin^2 x \\
 &= 1 - 2\sin^2 x \quad \text{QED}
 \end{aligned}$$

17. (6 marks)

Solve

a) $\cos(x) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$

$$x = \frac{\pi}{3} \text{ and } \frac{5\pi}{3}$$



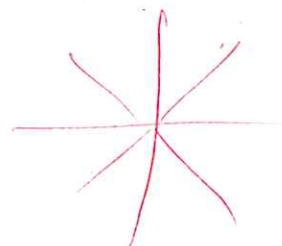
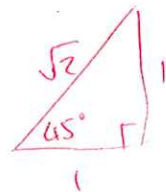
b) Solve $2\sin^2(\theta) - 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$.

$$2\sin^2(\theta) - 1 = 0$$

$$\sin^2(\theta) = \frac{1}{2}$$

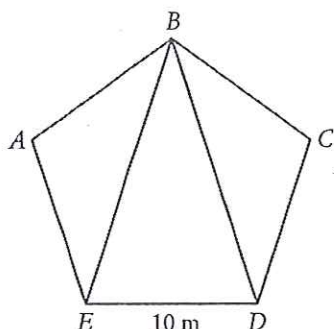
$$\sin(\theta) = \pm \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$



18. (13 marks)

The diagram below shows a regular pentagon with sides of 10 m.



- a) Use the sum of angles in a triangle to find the angle sum of any pentagon.

[1]

$$540^\circ$$

- b) Find the size of each angle inside a regular pentagon.

[1]

$$108^\circ$$

- c) Find the length of BE in the pentagon above.

[3]

$$\begin{aligned} BE &= \sqrt{10^2 + 10^2 - 2(10)(10)\cos 108^\circ} \\ &= \sqrt{200 - 200\cos 108^\circ} \\ &= 16.18 \text{ m} \end{aligned}$$

- d) Find the area of the pentagon.

[8]

$$\begin{aligned} A_{ABE} &= \frac{1}{2} \times 10 \times 10 \times \sin 108^\circ \quad (\Rightarrow 2 \text{ of these (BCD as well)}) \\ &= 47.55 \text{ m}^2 \quad (\text{Total } 95.11 \text{ m}^2) \end{aligned}$$

$$\begin{aligned} A_{BED} &= \frac{1}{2} \times 10 \times (\sqrt{16.18^2 - 5^2}) \\ &= 76.94 \text{ m}^2 \end{aligned}$$

$$\text{Total } 172.05 \text{ m}^2$$

More space available over page.

End of Test

