Baldivis
Secondary College

Mathematics Methods
2017

Unit 1
Unit 2

Student Information

Programme and Assessments
School Curriculum and Standards Authority Syllabus

Name: _____

Students Signature:_____

Parent/Carer Signature: _____

Welcome to Mathematics Applications. The following is a set of guidelines to help you succeed in this subject. You must ensure you bring the following items to each lesson; your workbook/s, 'Sadler' textbooks and other relevant texts, your scientific calculator / Classpad, pens, pencils, etc.

HOMEWORK

Homework will be given every lesson. There are two types of homework:

1. The first type includes finishing off a certain number of questions from the text or worksheet. If you are struggling with an exercise you should seek help as soon as you can from a friend or myself.
2. The second type of homework is study. Each night you should, at the very least, reread your day's work.

All students are expected to complete about 2 ½ - 3 hours home study per week spread over at least 5 days.

ASSESSMENT

Most assessment items for subjects in mathematics are conducted under test conditions. The approximate dates of these tests are indicated on the program. Students will be given notice of exact dates of tests as timing allows. Assessment will be cumulative throughout the course. It is very important that student achievement in the examinations reflect progressive marks awarded throughout the course.

IT IS VITAL THAT YOU ATTEND ON THE INDICATED DAY OF A MATHEMATICS ASSESSMENT.

Our experience indicates that students will not perform as well on an assessment if it is attempted at a later date.
Your Responsibilities; Students have a responsibility to attend regularly, participate in class activities and to perform assessment tasks. It must be emphasized that a student who is away on camp, excursion or school activity has the sole responsibility of following up with the teacher on any class work or missed assessment items.

Assessment Policy

Extensions

When the submission date for an assessment cannot be met due to circumstances beyond the student's control, parents or students may request extensions in advance of due dates. Reasons for seeking an extension include absences due to illness, and family emergencies. Pressure of assignments in other subjects or holidays taken during the school term are not sufficient reasons to warrant an extension. Students who are attending camps (such as an Outdoor Education Camp) are expected to meet deadlines that occur in their absence, or to negotiate revised submission dates with teachers before the camp. Students who are absent for an in-class assessment will complete the task at a later date at the discretion of the teacher. Students who fail to avail themselves of the opportunity to compete the task could be awarded an "E" grade. A "U" grade (Unfinished) will only be awarded when it is beyond the students control to complete the course requirements

Late work

Baldivis Secondary College aims to develop students' time management skills in order to prepare them for life after school. Students must plan their work to meet deadlines. Extending a deadline for an assignment reduces the time available for subsequent tasks and the problem compounds. Students who consistently meet deadlines are rewarded for their diligence through positive comments in the Attitude, Behaviour and Effort area of the report. Students who submit work late without an extension will have a "Late" recorded against their name, the parent will be contacted and a comment will appear on the student's report regarding their inability to meet deadlines. Although feedback will be provided on late work which is submitted within a reasonable time after the due date, **it may not be graded**. Extenuating circumstances are managed at the discretion of the Head of Curriculum and teachers.

Cheating

If cheating on an assessment task is established beyond reasonable doubt, the student will be penalized, including a loss of Good Standing. The teacher will be responsible for contacting the Deputy Principal (Student Services) regarding this. Students who cheat may be required to complete an alternative task or give a reduced mark.

Moderation

In Year 11 and 12, consistency between schools for a particular course is achieved through the moderation processes invoked by the School Curriculum and Standards Authority.

Program

Mathematics Applications 2017

Mathematics Applications is an ATAR course which focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering questions that involve analysing univariate and bivariate data, including time series data.

ASSESSMENT

Type of assessment	Weighting
Response Students respond using knowledge of mathematical facts, concepts and terminology, applying problem-solving skills and algorithms. Response tasks can include: tests, assignments, quizzes and observation checklists. Tests are administered under controlled and timed conditions.	40%
Investigation Students plan, research, conduct and communicate, the findings of an investigation/project. They can investigate problems identifying the underlying mathematics, or select, adapt and apply models and procedures to solve problems. This assessment type provides for the assessment of general inquiry skills, course-related knowledge and skills, and modelling skills. The 'Consumer Arithmetic' and 'Univariate Data' topics are recommended as suitable content areas for investigation. Evidence can include: observation and interview, written work or multimedia presentations.	20%
Examination Students apply mathematical understanding and skills to analyse, interpret and respond to questions and situations. Examinations provide for the assessment of conceptual understandings, knowledge of mathematical facts and terminology, problem-solving skills, and the use of algorithms. Examination questions can range from those of a routine nature, assessing lower level concepts, through to open-ended questions that require responses at the highest level of conceptual thinking. Students can be asked questions of an investigative nature for which they may need to communicate findings, generalise, or make and test conjectures. Typically conducted at the end of each semester and/or unit. In preparation for Unit 3 and Unit 4, the examination should reflect the examination design brief included in the ATAR Year 12 syllabus for this course. Where a combined assessment outline is implemented, the Semester 2 examination should assess content from both Unit 1 and Unit 2. However, the combined weighting of Semester 1 and Semester 2 should reflect the respective weightings of the course content as a whole.	40%

Assessment Outline

Style	Task	Weighting (%)	Due Date
Response (40%)	Test 1 – Trigonometry, Circular Measure, Radian Measure, Functions and Linear Relationships	6	Term 1 Week 5
	Test 2 – Quadratic Relationships, Powers, Polynomials and Inverse Proportion	7	Term 1 Week 9
	Test 3 – Trigonometry, Sets and Probability	7	Term2 Week 4
	Test 1 – Indices and Exponentials	6	Term 2 Week 9
	Test 2 – Sequences and Series	6	Term 3 Week 3
	Test 3 –Differentiation and Anit-derivatives	8	Term 3 Week 8
Investigation (20%)	Investigation 1 – Trigonometry	5	Term 1 Week 3 (Val'n Week 4)
	Investigation 2 – Counting Techniques and Pascal's Triangle	5	Term 2 Week 2 (Val'n Week 3)
	Investigation 1 – Exponential Functions	5	Term 2 Week 8
	Investigation 2 – Sequences	5	Term 3 Week 5
Examination (40%)	Semester 1 Examination **Unit 1**	15	Term 2 Week 5
	Semester 2 Examination **Unit 1 and Unit 2**	25	Term 3 Week 10

Baldivis Secondary College

Mathematics Methods Course Outline 2017

TEXT references; Mathematics Methods Unit 1, A J Sadler **MM 1**
Mathematics methods Unit 2, A J Sadler **MM 2**

Further texts ICE-EM Year 10/10A Books 1 & 2

WACE Revision Series Mathematics Methods
Year 11 Units 1 & 2 O.T. Lee **OTL**

WEBSITES School Curriculum and Standards Authority www.scsa.wa.edu.au
Classpad Calculators www.charliewatson.com
Connect Community – Mathematics Methods Units 1 & 2 http://connect.det.wa.edu.au

UNIT 1				
Week	**Content**	**Syllabus Reference**	**Text**	**Assessment**
Term 1 Week1,2	***Cosine and sine rules:*** • Review sine, cosine and tangent as ratios of side lengths in right-angled triangles • Understand the unit circle definition of $\cos\theta$, $\sin\theta$and $\tan\theta$and periodicity using degrees • Examine the relationship between the angle of inclination of a line and the gradient of that line • Establish and use the sine and cosine rules, including consideration of the ambiguous case, and the formula $Area$=0.5$bc\sin A$ for the area of a triangle.	1.2.1 1.2.2 1,2,3 1,2,4	**MM 1** Chapter 1	
Week 2, 3	***Circular measure and radian measure:*** • Define and use radian measure and understand its relationship with degree measure • Calculate lengths of arcs and areas of sectors and segments in circles.	1,2,5 1,2,6	**MM 1** Chapter 2	**Investigation 1** **Take home**
Week 4	***Functions:*** • Understand the concept of a function as a mapping between sets, and as a rule or a formula that defines one variable quantity in terms of another • Use function notation, determine domain and range, recognise independent and dependent variables • Understand the concept of the graph of a function	1,1,23 1,1,24 1,1,25	**MM 1** Chapter 3	**Investigation 1** **Validation**
Week 5	***Lines and linear relationships:*** • Determine the coordinates of the midpoint of two points • Determine an end-point given the other end point and the mid-point • Examine examples of direct proportion and linearly related variables	1,1,1 1,1,2 1,1,3	**MM 1** Chapter 4	

	• Recognise features of the graph of $y=mx+c$, including its linear nature, its intercepts and its slope or gradient • Determine the equation of a straight line given sufficient information; including parallel and perpendicular lines • Solve linear equations, including those with algebraic fractions and variables on both sides	1,1,4 1,1,5 1,1,6		**Test 1** **Chapters 1 - 4**
Week 6	***Review of quadratic relationships:*** • Examine examples of quadratically related variables • Recognise features of the graphs of $y=x^2$, $y=a(x-b)^2+c$, and $y=a(x-b)(x-c)$, including their parabolic nature, turning points, axes of symmetry and intercepts	1,1,7 1,1,8	**MM 1** Chapter 5	
Week 7	• Solve quadratic equations, including the use of the quadratic formula and completing the square • Determine the equation of a quadratic given sufficient information • Determine turning points and zeros of quadratics and understand the role of the discriminant • Recognise features of the graph of the general quadratic $y=ax^2+bx+c$.	1,1,9 1,1,10 1,1,11 1,1,12	**MM 1** Chapter 6	
Week8, 9	***Powers and polynomials:*** • Recognise features of the graphs of $y=x^n$ for $n\in\mathbf{N}$, $n=-1$ and $n=½$, including shape, and behaviour as $x\to\infty$ and $x\to-\infty$ • Identify the coefficients and the degree of a polynomial • Expand quadratic and cubic polynomials from factors • Recognise features and determine equations of the graphs of $y=x^3$, $y=a(x-b)^3+c$and $y=k(x-a)(x-b)(x-c)$, including shape, intercepts and behaviour as $x\to\infty$ and $x\to-\infty$ • Factorise cubic polynomials in cases where a linear factor is easily obtained • Solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained. • Examine translations and the graphs of $y=f(x)+a$and $y=f(x-b)$ • Examine dilations and the graphs of $y=cf(x)$ and $y=f(dx)$ • Recognise the distinction between functions and relations, and apply the vertical line test. ***Inverse proportion:*** • Examine examples of inverse proportion • Recognise features and determine equations of the graphs of $y=\frac{1}{x}$ and $y=\frac{a}{x-b}$, including their hyperbolic shapes, and their asymptotes. ***Graphs of relations:*** • Recognise features of the graphs of $x^2+y^2=r^2$ and $(x-a)^2+(y-b)^2=r^2$, including their circular shapes, their centres and their radii • Recognise features of the graph of $y^2=x$including its parabolic shape and its axis of symmetry.	1,1,15 1,1,16 1,1,17 1,1,18 1,1,19 1,1,20 1,1,26 1,1,27 1,1,28 1,1,13 1,1,14 1,1,21 1,1,22	**MM 1** Chapter 7	**Test 2** **Chapters 5-7**

Week 10,1	***Trigonometric functions:*** • Understand the unit circle definition of cosθ, sinθ and tanθ and periodicity using radians • Recognise the exact values of sinθ, cos$\theta$and tanθat integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ • Recognise the graphs of y=sin$x$,y=cos$x$, and y=tan$x$ on extended domains • Examine amplitude changes and the graphs of y=$a$sin$x$and y=$a$cos$x$ • Examine period changes and the graphs of y=sinbx, y=cosbx, and y=tan bx • Examine phase changes and the graphs of y=sin(x-c), y=cos(x-c) and y=tan (x-c) • Examine the relationships sin(x+$\frac{\pi}{2}$)=cos$x$ and cos(x−$\frac{\pi}{2}$)=sin$x$ • Prove and apply the angle sum and difference identities • Identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems • Solve equations involving trigonometric functions using technology, and algebraically in simple cases	1,2,7 1,2,8 1,2,9 1,2,10 1,2,11 1,2,12 1,2,13 1,2,14 1,2,15 1,2,16	**MM 1** Chapter 8	
Term 2 Week 2,3	***Language of events and sets:*** • Review the concepts and language of outcomes, sample spaces and events as sets of outcomes • Use set language and notation for events, including: a. A(or A') for the complement of an event A b. $A\cap B$ and $A\cup B$ for the intersection and union of events $A$and B, respectively c. A∩B∩C and A∪B∪C for the intersection and union of the three events A, B and C respectively d. recognise mutually exclusive events • Use everyday occurrences to illustrate set descriptions and representations of events, and set operations. ***Review of the fundamentals of probability:*** • Review probability as a measure of 'the likelihood of occurrence' of an event • Review the probability scale: 0≤$P(A)$≤1 for each event A, with $P(A)$=0 if $A$is an impossibility and $P(A)$=1 if $A$is a certainty • Review the rules: $P(\bar{A})$=1−$P(A)$ and $P(A\cup B)$=$P(A)$+$P(B)$−$P(A\cap B)$ • Use relative frequencies obtained from data as point estimates of probabilities. ***Conditional probability and independence:*** • Understand the notion of a conditional probability and recognise and use language that indicates conditionality • Use the notation $P(A\|B)$ and the formula $P(A\cap B)$=$P(A\|B)P(B)$ • Understand the notion of independence of an event $A$from an event B, as defined by $P(A\|B)$=$P(A)$ • Establish and use the formula $P(A\cap B)$=$P(A)P(B)$ for independent events $A$and B, and recognise the symmetry of independence • Use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events.	1,3,6 1,3,7 1,3,8 1,3,9 1,3,10 1,3,11 1,3,12 1,3,13 1,3,14 1,3,15 1,3,16 1,3,17	**MM 1** Chapter 9	**Investigation 2** **Take home** **Investigation 2** **Validation**

Week 4	***Combinations:*** • Understand the notion of a combination as an ordered set of r objects taken from a set of n distinct objects • Use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of $r$objects taken from a set of n distinct objects • Expand $(x+y)^n$ for small positive integers n • Recognise the numbers $\binom{n}{r}$ as binomial coefficients, (as coefficients in the expansion of $(x+y)^n$) • Use Pascal's triangle and its properties.	1,3,1 1,3,2 1,3,3 1,3,4 1,3,5	**MM 1** Chapter 10	**Test 3** **Chapters 8-10**
Week 5	**Semester 1 Examination**			
	UNIT 2			
Week 6, 7	***Indices and the index laws:*** • review indices (including fractional indices) and the index laws • use radicals and convert to and from fractional indices • understand and use scientific notation and significant figures.	2,1,1 2,1,2 2,1,3	**MM 2** Chapter 1	
Week 8	***Exponential functions:*** • establish and use the algebraic properties of exponential functions • recognise the qualitative features of the graph of $y=a^x$ ($a>0$) including asymptotes, and of its translations ($y=a^x+b$and $y=a^{x-c}$) • identify contexts suitable for modelling by exponential functions and use them to solve practical problems • solve equations involving exponential functions using technology, and algebraically in simple cases.	2,1,4 2,1,5 2,1,6 2,1,7	**MM 2** Chapter 2	**Investigation 1**
Week 9	***Arithmetic sequences:*** • recognise and use the recursive definition of an arithmetic sequence: $t_{n+1}=t_n+d$ • use the formula $t_n=t_1+(n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature • use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest • establish and use the formula for the sum of the first n terms of an arithmetic sequence.	2,2,1 2,2,2 2,2,3 2,2,4	**MM 2** Chapter 3/4	**Test 1** **Chapters 1 - 2**
Week 10	**Catch up**			

Semester 2

Week	Content	Syllabus Reference	Text	Assessment
Term 3 Week 1	***Arithmetic sequences:*** • recognise and use the recursive definition of an arithmetic sequence: $t_{n+1}=t_n+d$ • use the formula $t_n=t_1+(n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature • use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest • establish and use the formula for the sum of the first n terms of an arithmetic sequence.	2,2,1 2,2,2 2,2,3 2,2,4	**MM 2** Chapter 3/4	
Week 1, 2	***Geometric sequences:*** • recognise and use the recursive definition of a geometric sequence: $t_{n+1}=t_n r$ • use the formula $t_n= t_1 r_{n-1}$ for the general term of a geometric sequence and recognise its exponential nature • understand the limiting behaviour as $n\to\infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r • establish and use the formula $S_n= t_1 \frac{r^n-1}{r-1}$ for the sum of the first n terms of a geometric sequence • use geometric sequences in contexts involving geometric growth or decay, such as compound interest.	2,2,5 2,2,6 2,2,7 2,2,8 2,2,9	**MM 2** Chapter 3/4	
Week 3,4	***Introduction to Differentiation:*** ***Rates of change:*** • interpret the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the average rate of change of a function f • use the Leibniz notation δx and δy for changes or increments in the variables x and y • use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $y=f(x)$ • interpret the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y=f(x)$. ***The concept of the derivative:*** • examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h\to 0$ as an informal introduction to	2,3,1 2,3,2 2,3,3 2,3,4 2,3,5	**MM 2** Chapter 5	**Test 2** **Chapters 3-4**

	the concept of a limit • define the derivative $f'(x)$ as $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ • use the Leibniz notation for the derivative: $\frac{dy}{dx}=\lim_{\delta x\to 0}\frac{\delta y}{\delta x}$ and the correspondence $\frac{dy}{dx}=f'(x)$ where $y=f(x)$ • interpret the derivative as the instantaneous rate of change • interpret the derivative as the slope or gradient of a tangent line of the graph of $y=f(x)$.	2,3,6 2,3,7 2,3,8 2,3,9		
Week 5,6	***Applications of derivatives:*** • find instantaneous rates of change • find the slope of a tangent and the equation of the tangent • construct and interpret position-time graphs, with velocity as the slope of the tangent • recognise velocity as the first derivative of displacement with respect to time • sketch curves associated with simple polynomials; determine stationary points, and local and global maxima and minima; and examine behaviour as $x\to\infty$ and $x\to-\infty$ • solve optimisation problems arising in a variety of contexts involving polynomials on finite interval domains.	2,3,16 2,3,17 2,3,18 2,3,19 2,3,20 2,3,21	**MM 2** Chapter 6	**Investigation 2**
Week 7	***Anti-derivatives:*** • calculate anti-derivatives of polynomial functions	2,3,22	**MM 2** Chapter 7	
Week 8	• recognise velocity as the first derivative of displacement with respect to time • calculate anti-derivatives of polynomial functions	2,3,19 2,3,20	**MM 2** Chapter 8	**Test 3** **Chapters 5 - 8**
Week 9	**Revision**			
Week 10	**EXAM WEEK**			

Unit 1

Unit description

This unit begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of calculus. The basic trigonometric functions are then introduced. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of inferential statistics begins in this unit with a review of the fundamentals of probability and the introduction of the concepts of counting, conditional probability and independence. Access to technology to support the computational and graphical aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

understand the concepts and techniques in algebra, functions, graphs, trigonometric functions, counting and probability

solve problems using algebra, functions, graphs, trigonometric functions, counting and probability

apply reasoning skills in the context of algebra, functions, graphs, trigonometric functions, counting and probability

interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems

communicate their arguments and strategies when solving problems.

Unit content

This unit includes the knowledge, understandings and skills described below.

Topic 1.1: Functions and graphs (22 hours)

Lines and linear relationships

1.1.1 determine the coordinates of the mid-point between two points

1.1.2 determine an end-point given the other end-point and the mid-point

1.1.3 examine examples of direct proportion and linearly related variables

1.1.4 recognise features of the graph of $y = mx + c$, including its linear nature, its intercepts and its slope or gradient

1.1.5 determine the equation of a straight line given sufficient information; including parallel and perpendicular lines

1.1.6 solve linear equations, including those with algebraic fractions and variables on both sides

Quadratic relationships

1.1.7 examine examples of quadratically related variables

1.1.8 recognise features of the graphs of $y = x^2$, $y = a(x-b)^2 + c$, and $y = a(x-b)(x-c)$, including their parabolic nature, turning points, axes of symmetry and intercepts

1.1.9 solve quadratic equations, including the use of quadratic formula and completing the square

1.1.10 determine the equation of a quadratic given sufficient information

1.1.11 determine turning points and zeros of quadratics and understand the role of the discriminant

1.1.12 recognise features of the graph of the general quadratic $y = ax^2 + bx + c$

Inverse proportion

1.1.13 examine examples of inverse proportion

1.1.14 recognise features and determine equations of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x-b}$, including their hyperbolic shapes and their asymptotes.

Powers and polynomials

1.1.15 recognise features of the graphs of $y = x^n$ for $n \in \boldsymbol{N}, n = -1$ and $n = ½$, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$

1.1.16 identify the coefficients and the degree of a polynomial

1.1.17 expand quadratic and cubic polynomials from factors

1.1.18 recognise features and determine equations of the graphs of $y = x^3$, $y = a(x-b)^3 + c$ and $y = k(x-a)(x-b)(x-c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$

1.1.19 factorise cubic polynomials in cases where a linear factor is easily obtained

1.1.20 solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained

Graphs of relations

1.1.21 recognise features and determine equations of the graphs of $x^2 + y^2 = r^2$ and $(x-a)^2 + (y-b)^2 = r^2$, including their circular shapes, their centres and their radii

1.1.22 recognise features of the graph of $y^2 = x$, including its parabolic shape and its axis of symmetry

Functions

1.1.23 understand the concept of a function as a mapping between sets and as a rule or a formula that defines one variable quantity in terms of another

1.1.24 use function notation; determine domain and range; recognise independent and dependent variables

1.1.25 understand the concept of the graph of a function

1.1.26 examine translations and the graphs of $y = f(x) + a$ and $y = f(x-b)$

1.1.27 examine dilations and the graphs of $y = cf(x)$ and $y = f(dx)$

1.1.28 recognise the distinction between functions and relations and apply the vertical line test

Topic 1.2: Trigonometric functions (15 hours)

Cosine and sine rules

1.2.1 review sine, cosine and tangent as ratios of side lengths in right-angled triangles

1.2.2 understand the unit circle definition of $\cos\theta$, $\sin\theta$ and $\tan\theta$ and periodicity using degrees

1.2.3 examine the relationship between the angle of inclination of a line and the gradient of that line

1.2.4 establish and use the cosine and sine rules, including consideration of the ambiguous case and the formula $Area = \frac{1}{2}bc\ sin\ A$ for the area of a triangle

Circular measure and radian measure

1.2.5 define and use radian measure and understand its relationship with degree measure

1.2.6 calculate lengths of arcs and areas of sectors and segments in circles

Trigonometric functions

1.2.7 understand the unit circle definition of $\sin\theta$, $\cos\theta$ and $\tan\theta$ and periodicity using radians

1.2.8 recognise the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$

1.2.9 recognise the graphs of $y = \sin x, y = \cos x$, and $y = \tan x$ on extended domains

1.2.10 examine amplitude changes and the graphs of $y = a\sin x$ and $y = a\cos x$

1.2.11 examine period changes and the graphs of $y = \sin bx$, $y = \cos bx$ and $y = \tan\ bx$

1.2.12 examine phase changes and the graphs of $y = \sin(x - c)$, $y = \cos(x - c)$ and $y = \tan\ (x - c)$

1.2.13 examine the relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos x$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

1.2.14 prove and apply the angle sum and difference identities

1.2.15 identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems

1.2.16 solve equations involving trigonometric functions using technology, and algebraically in simple cases

Topic 1.3: Counting and probability (18 hours)

Combinations

1.3.1 understand the notion of a combination as a set of r objects taken from a set of n distinct objects

1.3.2 use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects

1.3.3 expand $(x + y)^n$ for small positive integers n

1.3.4 recognise the numbers $\binom{n}{r}$ as binomial coefficients (as coefficients in the expansion of $(x + y)^n$)

1.3.5 use Pascal's triangle and its properties

Language of events and sets

1.3.6 review the concepts and language of outcomes, sample spaces, and events, as sets of outcomes

1.3.7 use set language and notation for events, including:

a. $\bar{A}$ (or A') for the complement of an event A

b. $A \cap B$ and $A \cup B$ for the intersection and union of events A and B respectively

c. $A \cap B \cap C$ and $A \cup B \cup C$ for the intersection and union of the three events A, B and C respectively

d. recognise mutually exclusive events.

1.3.8 use everyday occurrences to illustrate set descriptions and representations of events and set operations

Review of the fundamentals of probability

1.3.9 review probability as a measure of 'the likelihood of occurrence' of an event

1.3.10 review the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty

1.3.11 review the rules: $P(\bar{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

1.3.12 use relative frequencies obtained from data as estimates of probabilities

Conditional probability and independence

1.3.13 understand the notion of a conditional probability and recognise and use language that indicates conditionality

1.3.14 use the notation $P(A|B)$ and the formula $P(A \cap B) = P(A|B)P(B)$

1.3.15 understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$

1.3.16 establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B, and recognise the symmetry of independence

1.3.17 use relative frequencies obtained from data as estimates of conditional probabilities and as indications of possible independence of events

Unit 2

Unit description

The algebra section of this unit focuses on exponentials. Their graphs are examined and their applications in a wide range of settings are explored. Arithmetic and geometric sequences are introduced and their applications are studied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically, by calculating difference quotients both geometrically as slopes of chords and tangents, and algebraically. Calculus is developed to study the derivatives of polynomial functions, with simple application of the derivative to curve sketching, the calculation of slopes and equations of tangents, the determination of instantaneous velocities and the solution of optimisation problems. The unit concludes with a brief consideration of anti-differentiation.

Learning outcomes

By the end of this unit, students:

understand the concepts and techniques used in algebra, sequences and series, functions, graphs and calculus

solve problems in algebra, sequences and series, functions, graphs and calculus

apply reasoning skills in algebra, sequences and series, functions, graphs and calculus

interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems

communicate arguments and strategies when solving problems.

Unit content

This unit builds on the content covered in Unit 1.

This unit includes the knowledge, understandings and skills described below.

Topic 2.1: Exponential functions (10 hours)

Indices and the index laws

2.1.1 review indices (including fractional and negative indices) and the index laws

2.1.2 use radicals and convert to and from fractional indices

2.1.3 understand and use scientific notation and significant figures

Exponential functions

2.1.4 establish and use the algebraic properties of exponential functions

2.1.5 recognise the qualitative features of the graph of $y = a^x$ $(a > 0)$, including asymptotes, and of its translations ($y = a^x + b$ and $y = a^{x-c}$)

2.1.6 identify contexts suitable for modelling by exponential functions and use them to solve practical problems

2.1.7 solve equations involving exponential functions using technology, and algebraically in simple cases

Topic 2.2: Arithmetic and geometric sequences and series (15 hours)

Arithmetic sequences

2.2.1 recognise and use the recursive definition of an arithmetic sequence: $t_{n+1} = t_n + d$

2.2.2 develop and use the formula $t_n = t_1 + (n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature

2.2.3 use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest

2.2.4 establish and use the formula for the sum of the first n terms of an arithmetic sequence

Geometric sequences

2.2.5 recognise and use the recursive definition of a geometric sequence: $t_{n+1} = t_n r$

2.2.6 develop and use the formula $t_n = t_1 r^{n-1}$ for the general term of a geometric sequence and recognise its exponential nature

2.2.7 understand the limiting behaviour as $n \to \infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r

2.2.8 establish and use the formula $S_n = t_1 \frac{r^n - 1}{r - 1}$ for the sum of the first n terms of a geometric sequence

2.2.9 use geometric sequences in contexts involving geometric growth or decay, such as compound interest

Topic 2.3: Introduction to differential calculus (30 hours)

Rates of change

2.3.1 interpret the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the average rate of change of a function f

2.3.2 use the Leibniz notation δx and δy for changes or increments in the variables x and y

2.3.3 use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $y = f(x)$

2.3.4 interpret the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y = f(x)$

The concept of the derivative

2.3.5 examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit

2.3.6 define the derivative $f'(x)$ as $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

2.3.7 use the Leibniz notation for the derivative: $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ and the correspondence

$\frac{dy}{dx} = f'(x)$ where $y = f(x)$

2.3.8 interpret the derivative as the instantaneous rate of change

2.3.9 interpret the derivative as the slope or gradient of a tangent line of the graph of $y = f(x)$

Computation of derivatives

2.3.10 estimate numerically the value of a derivative for simple power functions

2.3.11 examine examples of variable rates of change of non-linear functions

2.3.12 establish the formula $\frac{d}{dx}(x^n) = nx^{n-1}$ for non-negative integers n expanding $(x+h)^n$ or by factorising $(x+h)^n - x^n$

Properties of derivatives

2.3.13 understand the concept of the derivative as a function

2.3.14 identify and use linearity properties of the derivative

2.3.15 calculate derivatives of polynomials

Applications of derivatives

2.3.16 determine instantaneous rates of change

2.3.17 determine the slope of a tangent and the equation of the tangent

2.3.18 construct and interpret position-time graphs with velocity as the slope of the tangent

2.3.19 recognise velocity as the first derivative of displacement with respect to time

2.3.20 sketch curves associated with simple polynomials, determine stationary points, and local and global maxima and minima, and examine behaviour as $x \to \infty$ and $x \to -\infty$

2.3.21 solve optimisation problems arising in a variety of contexts involving polynomials on finite interval domains

Anti-derivatives

2.3.22 calculate anti-derivatives of polynomial functions