Question 1. (8 marks)

a) Differentiate the following. No need to simplify your answer.

i) $y = x^2 e^{5x+1}$ [2]

$y' = 2xe^{5x+1} + x^2(5e^{5x+1})$

ii) $\frac{3x^3-5x}{x+3}$ [2]

$$\frac{(x+3)(9x^2-5) - (3x^3-5x)(1)}{(x+3)^2}$$

b) The derivative of $y = (2x+1)(4x^2+2x)^4$ can be written in the form; [4]

$$\frac{dy}{dx} = 4(4x^2+2x)^3(ax^2+bx+2)$$

Determine the values of a and b.

$2(4x^2+2x)^4 + (2x+1)(8x+2)4(4x^2+2x)^3$

$(4x^2+2x)^3 + (8x^2+4x + 64x^2 + 48x + 8)$

$4(4x^2+2x)^3(18x^2+13x+2)$

$\therefore\ a = 18 \quad b = 13.$

Question 2. (4 marks)

A cylinder with constant height (h) has a volume modelled by $V = \pi r^2 h$. Using the incremental formula, determine the percentage change in radius (r) if the volume is increased by 3%.

$$\frac{\Delta V}{\Delta r} = 2\pi r h.$$

$$\frac{\Delta V}{V} = 0.03$$

$$\Delta V = 2\pi r h \cdot \Delta r$$

$$\frac{\Delta V}{V} = \frac{2\pi r h \cdot \Delta r}{\pi r^2 h.}$$

$$\frac{\Delta V}{V} = 2 \times \frac{\Delta r}{r}.$$

$$0.03 = 2 \times \frac{\Delta r}{r}$$

$$\therefore \frac{\Delta r}{r} = 0.015$$

1.5%

Question 3. (7 marks)

Solve for x in each of the following;

a) $\log_2 x = 4$ [1]

$x = 16$

b) $\log_x 4 = \frac{1}{2}$ [2]

$x = 16.$

c) $\log_2 x + \log_2(x - 7) = 3$ [4]

$\log_2 x(x-7) = \log_2 8$

$x(x-7) = 8$

$x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$

$\therefore x = 8 \quad x = -1.$

ignore.

Yr 12 METHODS TEST 1 2017

DIFFERENTIATION, APPLICATIONS
AND EXPONENTIALS

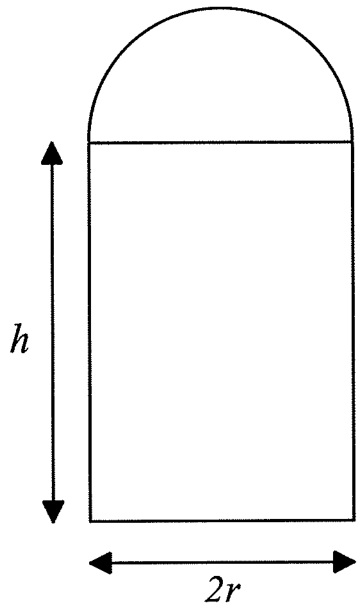
Time: 35 minutes **Total: 35 marks**

Student Name: Solutions. **Teacher:** ____________

Instructions: *Show all working clearly.*
Sufficient detail must be shown for marks to be awarded for reasoning.
CALCULATOR AND 1 PAGE PERSONAL NOTES ALLOWED

Question 4. (12 marks)

The diagram shows an arched church wooden window frame, to be made from 10m of timber.



a) Find an expression for h in terms of r. [2]

$10 = 4r + 2h + \pi r.$

$2h = 10 - 4r - \pi r$

$h = \frac{10 - 4r - \pi r}{2}.$

b) Show that the area of the window is $A = 10r - r^2\left(4 + \frac{\pi}{2}\right)$ [3]

$A = 2rh + \frac{1}{2}\pi r^2$

$= 2r\left(\frac{10 - 4r - \pi r}{2}\right) + \frac{1}{2}\pi r^2$

$= 10r - 4r^2 - \pi r^2 + \frac{1}{2}\pi r^2$

$= 10r - r^2\left(4 + \frac{\pi}{2}\right)$

Hence, or otherwise,

c) show that the **exact** value of r that maximises the area is $r = \frac{10}{8+\pi}$

[4]

$$A' = 10 - 2\left(4 + \frac{\pi}{2}\right)r$$

$$0 = 10 - (8 + \pi)r$$

$$\frac{10}{8+\pi} = r.$$

d) Suppose the radius (r) is increased by 10cm. Find the approximate change, using calculus methods, in the height of the window if the 10m of timber restriction still applies.

[3]

$\Delta r = 0.1$

$$\frac{\Delta h}{\Delta r} = -2 - \frac{\pi}{2}$$

$$\Delta h = \left(-2 - \frac{\pi}{2}\right)\Delta r$$

$$= \left(-2 - \frac{\pi}{2}\right)0.1$$

$$= -0.2 - \frac{\pi}{20}$$

$$= -0.357\text{m}$$ decrease

Question 5. (10 marks)

in metre in seconds.

A particle is travelling in a straight line such that its displacement (d) at any time (t) is such that;

3 -9 +6.

$d = \frac{t^3 - \cancel{6}t^2 \cancel{+9}x \cancel{+6}}{3}$, $0 \leq t \leq 6$ where a positive (d) value represents the particle to the right of a particular point A.

a) Is the particle initially to the right or the left of point A? [1]

right.

b) Showing the use of calculus, determine the velocity of the particle at $t = 2$. [2]

$v = t^2 - 2t - 3$

$v(2) = -3$ m/sec.

c) Determine the distance travelled by the particle in the first 4 seconds. [3]

$0 = t^2 - 2t - 3$

$0 = (t-3)(t+1)$

$t = 3$ turns around

$d(0) = 2$.

$d(3) = -7$.

$d(4) = -14/3$.

$\therefore 9 + 7/3 = \frac{34}{3}$ m

d) What is the maximum speed the particle obtains over the given domain? [2]

$v(0) = -3$

$v(3) = 0$

$v(6) = 21$.

$\therefore$ 21 m/sec.

e) How long does the particle spend to the right of point A? [2]

$d = 0$.

$t = -2.233$, 0.577 , 4.656.

$\therefore 0.577 + 1.344$

$= 1.921$ seconds.

Question 6. (6 marks)

Iodine-131 is present in radioactive water from the nuclear power industry.

It has a half-life of eight days. This means that every eight days, one half of the iodine-131 decays to a form that is not radioactive.

This decay can be represented by the equation $N = N_0 e^{kt}$,

Where $N =$ amount of iodine-131 present after t days, and
$N_0 =$ amount of iodine-131 present initially.

a) Determine the value of k correct to three decimal places. [3]

$$.5 = e^{8k}.$$

$$\ln .5 = 8k$$

$$\frac{\ln .5}{8} = k$$

$$-0.087 = k$$

↓

b) If 125 milligrams of iodine-131 are considered to be safe, how many full days will it take for 88 grams of iodine-131 to decay to a safe amount? [3]

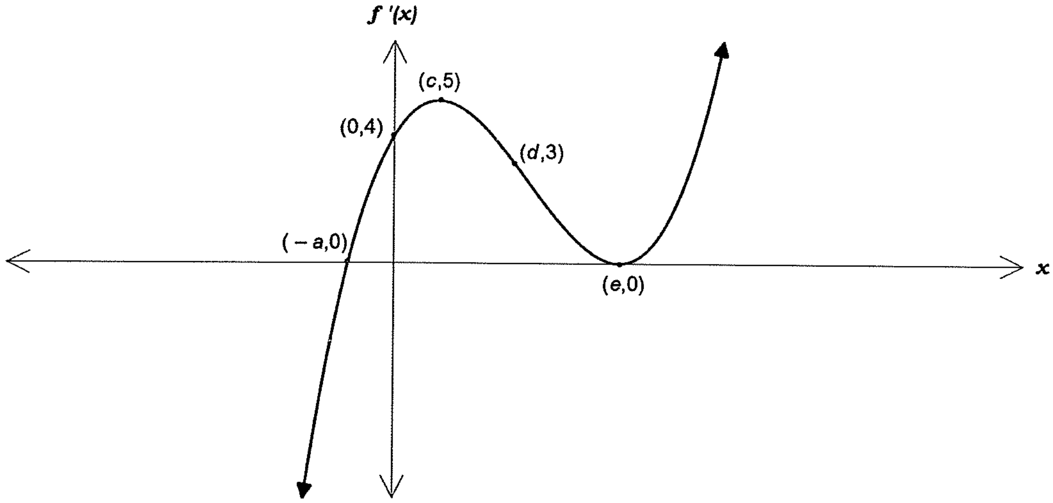
$$.125 = 88e^{-0.087t}.$$

$$t = 75.365$$

$\therefore$ 76 days.

Question 7. (7 marks)

The graph below shows the **derivative** $f'(x)$ of a function.



a) Use the graph of $f'(x)$ to state the x-values of all stationary points of the original function (and their nature) and the x-values of the points of inflection. [5]

$x = -a$ $x = e$ $x = c$

min. HPI VPI

b) Given that the original function $f(x)$ passes through the point $(d, 10)$, write an equation (**in terms of *d***) for the line that is tangential to the function at $(d, 10)$. [3]

$m = 3.$

$\therefore \ y = 3x + c$

using $(d, 10)$ $10 = 3d + c$

$10 - 3d = c$

$\therefore \ y = 3x + 10 - 3d.$