

2. [6 marks -2, 4]

(a) Evaluate

$${}^6C_2 + {}^7C_2$$

$$= \frac{6!}{4!2!} + \frac{7!}{5!2!}$$

$$= \frac{6 \times 5 \times 4!}{4! \times 2 \times 1} + \frac{7 \times 6 \times 5!}{5! \times 2 \times 1}$$

$$= 15 + 21 = 36$$

(b) Solve for x ${}^xP_3 = {}^{10}C_3$

$$\frac{x!}{(x-3)!} = \frac{10!}{3!7!}$$

$$\frac{x(x-1)(x-2)(\cancel{x-3})!}{(\cancel{x-3})!} = \frac{10 \times 9 \times 8 \times \cancel{7!}}{\cancel{7!}}$$

$$x(x-1)(x-2) = 120$$

$$x = 6$$

✓ expresses both in factorial term
✓ simplifies LHS
✓ simplifies RHS
✓ answer

3. [3 marks]

A bag is known to contain 1 white ball, 2 green balls, 3 blue balls, 4 yellow balls and 5 red balls.

Gillian selects balls at random from the bag.

Use the pigeonhole principle to determine how many balls have to be drawn to guarantee that Gillian has at least three of one colour. Justify your answer.

$$1 \text{ W } + 2 \text{ G } + 2 \text{ B } + 2 \text{ Y } + 1 \text{ R} = 10$$

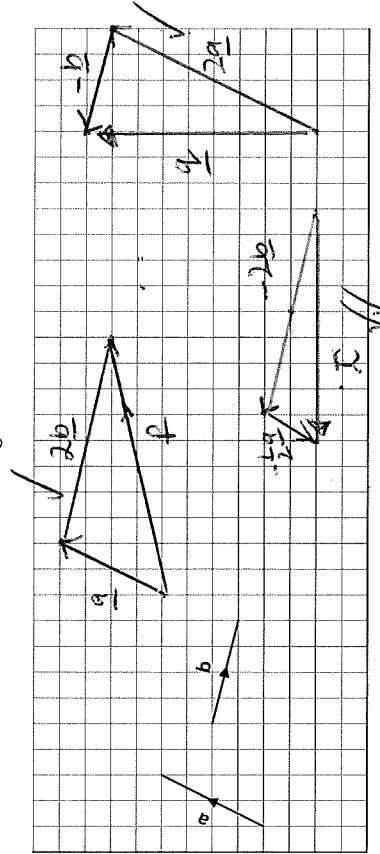
✓ reasoning
✓ answer

Reading Time: 2 minutes
Time Allowed: 15 minutes

Total Marks: 13

1. [4 marks -1, 1, 2]

Vectors **a** and **b** are as shown on the grid below.



-1 if no direction indicated.

On the grid above, sketch and label the vectors **p**, **q** and **r** where

$$\mathbf{p} = \mathbf{a} + 2\mathbf{b}$$

$$\mathbf{q} = 2\mathbf{a} - \mathbf{b}$$

$$\mathbf{r} = -2\mathbf{b} - 0.5\mathbf{a}$$



MATHEMATICS: SPECIALIST
SEMESTER 1 2018

TEST 1

Calculator Assumed

Reading Time: 3 minutes
Time Allowed: 47 minutes

Total Marks: 45

4. [8 marks - 2, 2, 2, 2]

4-digit numbers are made from the digits 1, 2, 3, 4, 5, (no repetition of digits allowed).

- (a) How many such 4-digit numbers can be made? ✓

$$\boxed{5} \boxed{4} \boxed{3} \boxed{2} = 120. \quad \checkmark$$

- (b) How many of those in (i) have a 3 as the first digit? ✓

$$\boxed{1} \boxed{4} \boxed{3} \boxed{2} = 24. \quad \checkmark$$

- (c) How many of those in (i) end with 13? ✓

$$\boxed{3} \boxed{2} \boxed{1} \boxed{1} = 6. \quad \checkmark$$

- (d) How many of those in (i) are even? ✓

$$\boxed{3} \boxed{4} \boxed{2} \boxed{2} = 48. \quad \checkmark$$

5. [3 marks]

How many digits greater than 900000 can be formed by arranging some or all of the following digits in a row? Digits may not be used more than once.

3, 1, 5, 4, 2, 9, 8

$$\begin{array}{|c|c|c|c|c|c|} \hline 1 & 6 & 5 & 4 & 3 & 2 \\ \hline \end{array} \quad \checkmark \quad \begin{array}{|c|c|c|c|c|c|} \hline 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ \hline \end{array} \quad \checkmark$$

if only 7 digit
1 mk only.

$$= 5760. \quad \checkmark$$

6. [6 marks - 1, 2, 3]

Find how many ways we can select a tuck shop roster of 6 mothers from a group of 18 mothers if

- (a) there are no restrictions

$$\binom{18}{6} = 18564 \quad \checkmark$$

- (b) 2 particular mothers insist on working together or not at all.

Both or neither

$$\checkmark \quad \binom{2}{2} \binom{16}{4} + \binom{2}{0} \binom{16}{6} \quad \checkmark$$

$$= 1820 + 8008$$

$$= 9828$$

- (c) 2 particular mothers refuse to work together.

neither or together

$$\checkmark \quad \binom{2}{0} \binom{16}{6}$$

or neither

$$+ \binom{1}{1} \binom{1}{0} \binom{16}{5} + \binom{1}{1} \binom{1}{0} \binom{16}{5}$$

$$\quad \quad \quad \binom{2}{1} \binom{16}{5}$$

$$= 16744. \quad \checkmark$$

7. [3 marks]

A student has 3 Mathematics books, 4 History books and 5 English books. In how many ways can he arrange them on a shelf if he keeps books on the same subject together.

$$3! \times 4! \times 5! = 103680$$

9. [6 marks- 2, 2, 2]

(a) How many different arrangements can be made from the letters of the word

BLACKBOARD?

$$\frac{10!}{2!2!} = 907200$$

(b) How many of these arrangements have the two A's together?

2 A's \Rightarrow 9 objects.

$$\frac{9!}{2!} = 181440$$

8. [4 marks]

Consider the set of integers between 1 and 1000 inclusive

Using the Inclusion-Exclusion Principle for Two Sets/Events

$$n(A \cup B) = n(A) + n(B) - n(A \cap B),$$

Determine how many of these integers are divisible by 3 or 5.

$$\begin{aligned} \text{Div by } 3 &= 333 \\ \text{Div by } 5 &= 200 \\ \text{Div by } 3+5 &= 66 \end{aligned}$$

$$\therefore \text{Div by } 3 \text{ or } 5 = 333 + 200 - 66 = 467$$

(c) How many of these arrangements begin with the two B's?

$$\frac{1}{B} \frac{1}{B} \frac{8!}{\text{rest}} = 20160$$

10. [3 marks]

From a class of 6 boys and 4 girls, 3 boys and 2 girls are to be selected for a debating team. How many speaking orders are there?

$$\binom{6}{3} \binom{4}{2} \times 5! = 14400$$

11. [5 marks]

Two vectors have magnitudes of 45 N and 60 N and the angle between their directions is 130° . Sketch a diagram to show their sum and use trigonometry to calculate the magnitude of the resultant and the angle it makes with the smaller vector.



$$|R|^2 = 45^2 + 60^2 - 2 \cdot 45 \cdot 60 \cdot \cos 50^\circ$$

$$|R| = 46.4 \text{ N} \quad \checkmark$$

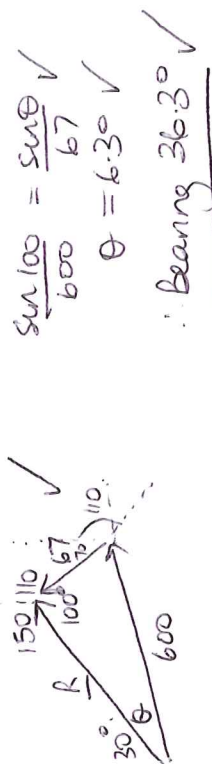
$$\frac{\sin \theta}{60} = \frac{\sin 50^\circ}{|R|}$$

$$\theta = 82^\circ \quad \checkmark$$

12. [7 marks - 4, 3]

A jet can maintain a speed of 600 km/h in still air. It needs to fly to a location 850 km away on a bearing of 030° but will encounter wind at 67 km/h blowing from 110° .

a) Determine what bearing the plane needs to take to reach the required location



b) Determine how long the journey would take to the nearest minute.

$$\frac{\sin 100^\circ}{600} = \frac{\sin (80 - 6.3^\circ)}{|R|}$$

$$|R| = 584.7 \quad \checkmark$$

$$\checkmark \frac{850}{584.7} = 1.45$$

$$= 1 \text{ hr } 27 \text{ mins} \quad \checkmark$$