YEAR 12 PHYSICS
ASSIGNMENT 2 - GRAVITATION

Name: SOLUTIONS **Mark:** $\overline{}$ **60**

1. Titan is the largest of Saturn's moons. Its orbital radius is 1.22×10^6 km. Use the Formulae and Data booklet and the data in the table below to determine the strength of Saturn's gravitational field where Titan orbits. Give your answer in Nkg^{-1} and ms^{-2}. (5 marks)

Planet	Mass (Earth masses)
Mercury	0.055
Venus	0.815
Earth	1.000
Mars	0.107
Jupiter	318
Saturn	95
Uranus	14.5
Neptune	17.2
Pluto	0.002

$$g = \frac{G M_s}{r^2}$$

(1)

$$= \frac{(6.67\times10^{-11})(95\times 5.97\times10^{24})}{(1.22\times10^{9})^2} \quad (1)$$

(1)

$$= 2.54\times10^{-2}\ ms^{-2} \quad (1)$$

2.54×10^{-2} Nkg^{-1} (1)

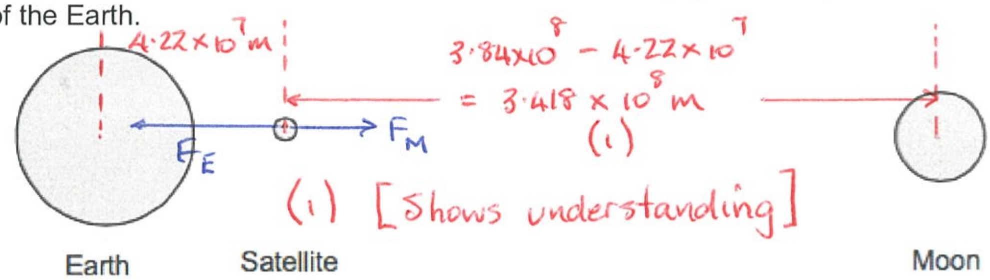
2.54×10^{-2} ms^{-2}

2. When a satellite is launched it is placed in an initial circular orbit around the Earth. Later some small jets on board the satellite will fire compressed gas for a set period of time to move it to the precise final circular orbit required. These gas jets point backward relative to the satellite's motion only and ***not*** toward or away from the Earth.

How can backward facing gas jets be used to raise the satellite to a higher final circular orbit? (4 marks)

- Gas jet increases the speed and E_K of the satellite. (1)
- This increase in E_K is converted into E_P. (1)
- Satellite rises to a higher orbit. (1)
- It slows down since $r \propto \frac{1}{v^2}$ (from $v^2 = \frac{GM_E}{r}$) (1)

3. A satellite orbits 4.22×10^7 m above the Earth's centre. At a certain point in its orbit around the Earth, the satellite and the Moon line up as shown in the diagram below. Show that in this position the influence of the Moon on the satellite is negligible, compared with the influence of the Earth. (7 marks)



$$F_E = \frac{GM_E m_S}{r^2}$$

$$= \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})\, m_S}{(4.22 \times 10^7)^2} \quad (1)$$

$$= 0.224\, m_S \quad (1)$$

$$F_M = \frac{GM_M M_S}{r^2}$$

$$= \frac{(6.67 \times 10^{-11})(7.35 \times 10^{22})\, m_S}{(3.418 \times 10^8)^2} \quad (1)$$

$$= 4.20 \times 10^{-5}\, m_S \quad (1)$$

$\therefore$ F_E is 5.33×10^3 times larger than F_M (1)

4. A GPS system uses the signals from four satellites to establish a position on the Earth's surface. The satellites have an orbital period of 12.0 hours but they are in different planes of orbit. Each satellite has an atomic clock that allows a signal to be emitted at prescribed intervals. The time difference between the four signals is used by the receiver to establish a position.

(a) By equating the relationship for centripetal force and gravitational force, show that the orbital velocity of each satellite is close to 3.90×10^3 ms^{-1}. Show all your workings. (5 marks)

$$F_g = F_c$$

$$\Rightarrow \frac{GM_E \cancel{M_s}}{r^2} = \frac{m_s v^2}{r} = \frac{4\pi^2 \cancel{m_s} r}{T^2} \quad (1)$$

$$\Rightarrow r^3 = \frac{GM_E T^2}{4\pi^2}$$

$$\Rightarrow r = \sqrt[3]{\frac{(6.67\times10^{-11})(5.97\times10^{24})(12.0\times3.60\times10^3)^2}{4\pi^2}} \quad (1)$$

$$= 2.66\times10^7 \text{ m} \quad (1)$$

$$V = \frac{2\pi r}{T}$$

$$= \frac{2\pi(2.66\times10^7)}{(12.0\times3.60\times10^3)} \quad (1)$$

$$= \underline{3.87\times10^3 \text{ ms}^{-1}} \quad (1)$$

(b) The manufacturers of the satellites deliberately build in a correction to the rate at which the clocks tick so that they run a little fast before they are put into orbit. Explain why they do this. (2 marks)

- Velocity is large. (1)
- Relativistic time dilation becomes significant to a very precise atomic clock. (1)

5. The Kepler NASA mission aims to search for planets orbiting stars in other solar systems. The star named Kepler-20 has been observed to have several planets orbiting it. Kepler 20 is 950 light-years from Earth.

Information about Kepler 20 and some of the planets orbiting it is summarised in the table below.

Astronomical object	Radius	Mass	Orbital period around Kepler 20
Star – Kepler 20	$0.944 \times \text{radius}_{SUN}$	$0.912 \times \text{mass}_{SUN}$	
Planet – Kepler 20b	$2.40 \times \text{radius}_{EARTH}$		290 days
Planet – Kepler 20e	$0.87 \times \text{radius}_{EARTH}$		6.1 days
Planet – Kepler 20f	$1.03 \times \text{radius}_{EARTH}$		19.6 days

(a) A light-year is an astronomical unit of distance. It is defined as the distance travelled by light in one year. Calculate the distance from Kepler 20 to Earth in kilometres.
(2 marks)

$$s = vt$$
$$= (950)(3.00 \times 10^8)(24.0)(60.0)(60.0)(365.25) \quad (1)$$
$$= 8.99 \times 10^{18} \text{ m}$$
$$= \underline{8.99 \times 10^{15} \text{ km}} \quad (1)$$

(b) Astronomers express the mass of Kepler 20 as $(0.912 \pm 0.035) \times \text{mass}_{SUN}$. Calculate the maximum value astronomers expect for the mass of Kepler 20. (2 marks)

Max value: $(0.912 + 0.035) m_{Sun}$ (1) $= (0.947)(1.99 \times 10^{30})$

$= \underline{1.88 \times 10^{30} \text{ kg}}$ (1)

(c) Calculate the orbital radius of Kepler 20e around Kepler 20. You should use the mass for Kepler 20 quoted in the table and assume the orbit is circular. (4 marks)

$$r^3 = \frac{GM_s T^2}{4\pi^2} \quad (1)$$

$$\Rightarrow r = \sqrt[3]{\frac{(6.67\times10^{-11})(0.912\times1.99\times10^{30})(6.10\times24\times60.0\times60.0)^2}{4\pi^2}} \quad (1)$$

(1)

$$= \underline{9.48\times10^9\ m} \quad (1)$$

(d) The mass of Kepler 20b is unknown but it has been speculated that it may have a density similar to that of Earth, 5520 kgm^{-3}. Calculate the surface gravity of Kepler 20b if its density is 5520 kgm^{-3}. (4 marks)

Reminder:

$$density = \frac{mass}{volume}$$

$$volume\ of\ a\ sphere = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(2.40\times6.37\times10^6)^3$$

$$= 1.50\times10^{22}\ m^3 \quad (1)$$

$$D = \frac{m}{V}$$

$$\Rightarrow m = DV$$

$$= (5520)(1.50\times10^{22})$$

$$= 8.28\times10^{25}\ kg \quad (1)$$

$$g = \frac{GM}{r^2}$$

$$= \frac{(6.67\times10^{-11})(8.28\times10^{25})}{(2.40\times6.38\times10^6)^2} \quad (1)$$

$$= \underline{23.6\ ms^{-2}} \quad (1)$$

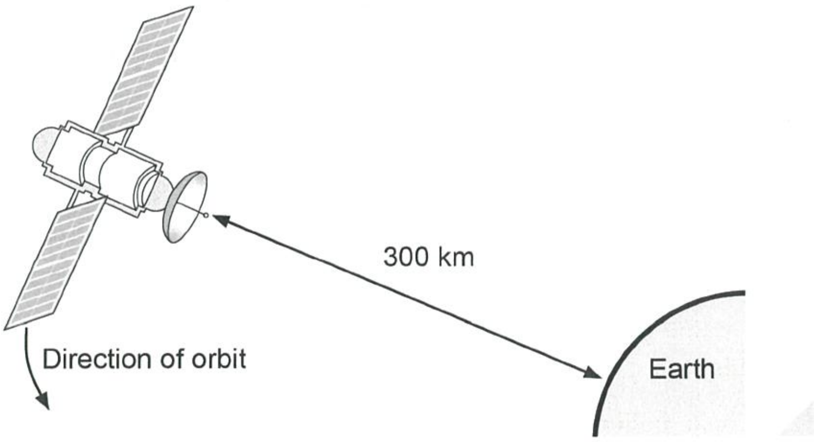
The Kepler mission is particularly concerned with finding planets that lie within the habitable zones of stars. A planet in a star's habitable zone receives the right amount of energy from the star to maintain liquid water on its surface, provided it also has an appropriate atmosphere.

(e) By comparing the Kepler 20 system and our own solar system, suggest which planet in the Kepler 20 system is most likely to lie in the habitable zone. Explain your answer. (3 marks)

- Kepler 20b (1)
- Given the star Kepler 20 is about the same size as our Sun (1)
  $\Rightarrow$ Choose a similar period of orbit for similar conditions. (1)

$\therefore$ Kepler 20 b

6. A 2.00 x 10^2 kg satellite is put into a low Earth orbit at an altitude of 3.00 x 10^2 km.



(a) Calculate the orbital speed of the satellite. (3 marks)

$$F_g = F_c$$

$$\Rightarrow \frac{G M_E \cancel{M_s}}{r^{\cancel{2}}} = \frac{\cancel{M_s} v^2}{\cancel{r}}$$

$$\Rightarrow v = \sqrt{\frac{G M_E}{r}} \quad (1)$$

$$= \sqrt{\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.37 \times 10^6 + 3.00 \times 10^5)}} \quad (1)$$

$$= \underline{7.73 \times 10^3 \text{ ms}^{-1}} \quad (1)$$

(b) Calculate the orbital period of the satellite. (2 marks)

$$v = \frac{2\pi r}{T}$$

$$\Rightarrow T = \frac{2\pi r}{v}$$

$$= \frac{2\pi(6.37\times10^6 + 3.00\times10^5)}{(7.73\times10^3)} \quad (1)$$

$$= \underline{5.42\times10^3\ s} \quad (1)$$

(c) Calculate the gravitational acceleration experienced by the satellite in orbit. (2 marks)

$$g = \frac{GM_E}{r^2}$$

$$= \frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.37\times10^6 + 3.00\times10^5)^2} \quad (1)$$

$$= \underline{8.95\ ms^{-2}} \quad (1)$$

[Could also use $a_c = \frac{v^2}{r}$]

An amateur astronomer on Earth measures the orbital period of the international space station in the night sky at 94.7 minutes.

(d) Calculate the altitude of the International Space Station based on an orbital period of 94.7 minutes. (3 marks)

$$r^3 = \frac{GM_E T^2}{4\pi^2}$$

$$\Rightarrow r = \sqrt[3]{\frac{(6.67\times10^{-11})(5.97\times10^{24})(94.7\times60.0)^2}{4\pi^2}} \quad (1)$$

$$= 6.88\times10^6\ m \quad (1)$$

$$r = r_E + h$$

$$\Rightarrow h = r - r_E$$

$$= 6.88\times10^6 - 6.37\times10^6$$

$$= \underline{5.10\times10^5\ m} \quad (1)$$

7. A group of students wanted to verify Kepler's laws of planetary motion. They chose to collect data on four moons of Jupiter.

(a) Complete the following table, using the data provided for the moons. (2 marks)

Moon	Orbital radius (r) (×10^6 m)	Orbital period (T) (×10^3 s)	r^3 (×10^{24})	T^2 (×10^9)
Metis	128	25.5	2.10	0.65
Adrastea	129	25.8	2.15	0.67
Amalthea	181	43.0	5.93	1.85
Thebe	222	58.3	10.9	3.40

[1 mark each]

(b) Plot the data from the table above onto the grid provided, demonstrating the relationship described by Kepler's laws of planetary motion. Draw the line of best fit. (4 marks)

(c) Using your graph, determine Kepler's constant (the ratio of r^3 to T^2). Express your answer in the appropriate significant figures. (3 marks)

$$\text{gradient} = \frac{\Delta r^3}{\Delta T^2}$$

Doesn't use data points (1)

$$= \frac{(10.8 - 0.0) \times 10^{24}}{(3.40 - 0.00) \times 10^9}$$

$$= \underline{3.18 \times 10^{15}\ m^3 s^{-2}}$$ (1) (3 sig. fig)

[Note: Units not given a mark]

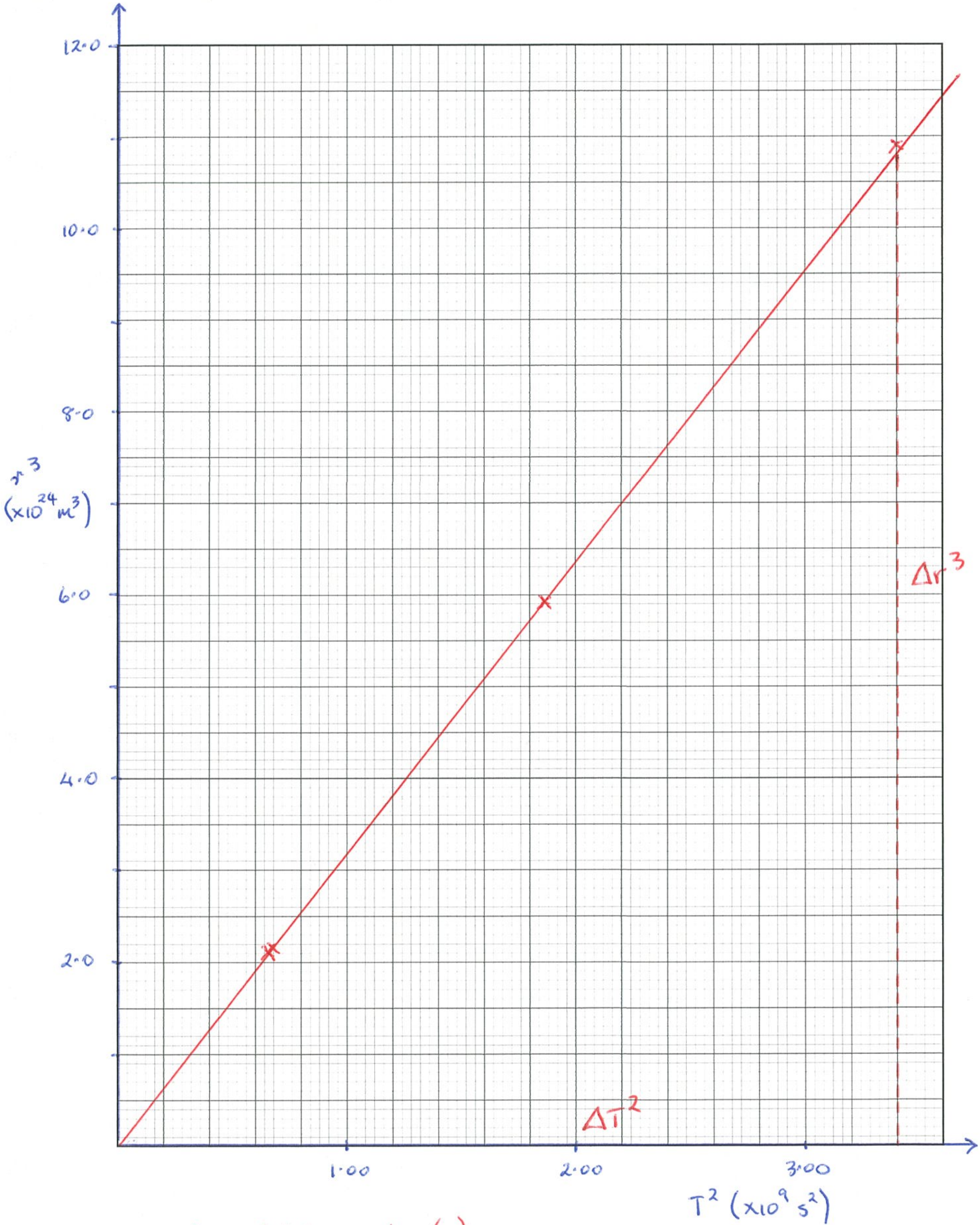
(d) Use your graph to determine the mass of Jupiter. (3 marks)

$$r^3 = \frac{G M_J T^2}{4\pi^2}$$

$$\Rightarrow M_J = \frac{r^3}{T^2} \cdot \frac{4\pi^2}{G}$$ (1)

$$= \frac{(3.18 \times 10^{15})\, 4\pi^2}{(6.67 \times 10^{-11})}$$ (1)

$$= \underline{1.88 \times 10^{27}\ kg}$$ (1)



Axes - labels + scales (1)
Correct units (1)
Accuracy (1)
Line of best fit (1)