Year 11 Mathematics Methods
Investigation 3
In Class Section

Name: ANSWERS

Score: ___ out of **50**

You may use your out of class section as notes for this test. CAS calculator permitted.

Part One: **[7 marks]**

Jordie draws the following diagrams (L shapes) that are made up of rectangles. The first diagram shows a shape that uses three rectangles.

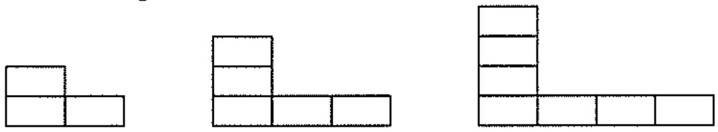


diagram 1 diagram 2 diagram 3

(a) Complete the table showing the number of rectangles needed for each shape.

Diagram (n)	1	2	3	4	5	6
Rectangles (L_n)	3	5	7	9	11	13

[1 mark]

successive

(b) Determine a ***recursive*** rule for the sequence of rectangles of the form

$L_{n+1} = $ $L_n + 2$, $L_1 = $ 3

[2 marks]

(c) Use the Recursive tab in ***Sequence*** and your rule from part b to determine L_{200}

$L_{200} = 401$.

[1 mark]

(d) Determine an Explicit rule for the number of rectangles needed in L_n.

$L_n = 2n + 1$

[1 mark]

(e) Which diagram number could you make if you had 200 rectangles? Show your method of solution.

$2n + 1 = 200$

$2n = 199$

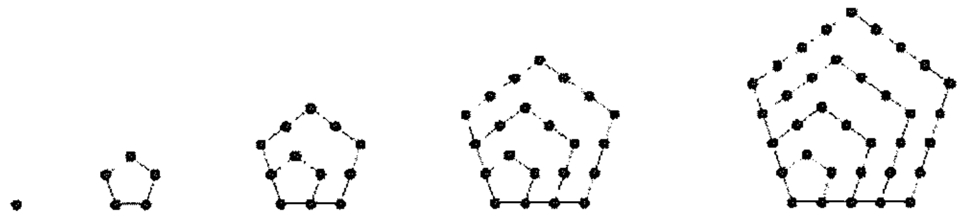
$n = 99.5$

[2 marks]

$\Rightarrow$ You could make diagram 99.

Part Two: [7 marks]

Tamara draws the first five pentagonal numbers as shown in the diagrams below.



1 **5** **12** **22** **35**

4 7 10 13

3 3 3

(a) What type of pattern do these numbers form?

Quadratic pattern.

[1 mark]

(b) Find a rule for the n^{th} pentagonal number P_n. Indicate your method of solution.

Sequence $(\{1, 5, 12, 22, 35\})$ $T_n = 1.5n^2 - 0.5n$.

or Quadfit

[2 marks]

(c) Use your rule from part b to determine which pentagonal number could be made if you had 5 925 dots?

solve $(5925 = 1.5n^2 - 0.5n)$

$n = 63.015$ $\Rightarrow$ You could make shape 63.

[2 marks]

(d) Complete the table below showing the rules for triangular, square and pentagonal numbers.

Type of number	n^{th} term rule.
Triangular number (*T*)	$T_n = 0.5n^2 + 0.5n$
Square number (*S*)	$S_n = n^2$
Pentagonal number (*P*)	$P_n = 1.5n^2 - 0.5n$.
Hexagonal number (*H*)	$H_n = 2n^2 - n$.

Use the three rules to predict a rule for hexagonal numbers (H). Place your predicted rule in the table above.

[1 marks]

(e) Using your rule, show working to verify that that the 5^{th} hexagonal number is 45.

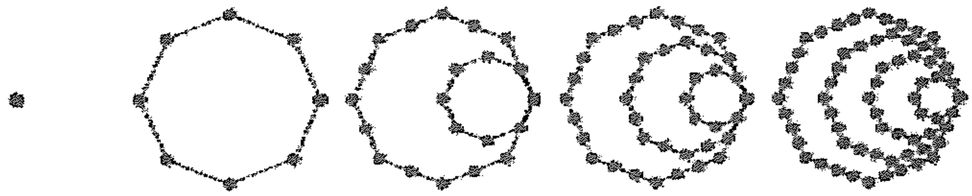
$H_5 = 2(5)^2 - 5$ ✓

$= 45$ ✓

[1 marks]

Part Three: **[8 marks]**

Claire draws the first five octagonal numbers (E_n) as shown in the diagrams.



Consider the table shown below.

n	1	2	3	4	5	6	7
E_n	1	8	21	40	65	96	133.
S_n	1	9	30	70	135	231	364

(a) Determine a rule for the n^{th} Octagonal number (E_n).

$$E_n = 3n^2 - 2n$$

[1 mark]

(b) Describe how the numbers in the S_n row are found and then determine a rule, in terms of n, for S_n.

S_n is found by summing all Octagonal numbers.

$$S_n = n^3 + 0.5n^2 - 0.5n .$$

[2 marks]

(c) Complete the missing entries in the table.

✓

[2 marks]

(d) Find S_{50}

$$S_{50} = 126\,225 .$$

[1 marks]

(f) Which is the smallest value for n for the sum of the Octagonal numbers S_n to first exceed 1 million?

solve $1\,000\,000 = n^3 + 0.5n^2 - 0.5n .$

[2 mark]

$n = 99.8$

$\Rightarrow n = 100$

Part Four: [8 marks]

The Fibonacci sequence begins as follows 1, 1, 2, 3, 5, 8, 13, 21, . . .

Ebonie defines this sequence recursively by the rule $F_n = F_{n-1} + F_{n-2}$, $F_1 = F_2 = 1$

(a) Use the Recursive tab in ***Sequence*** to enter this rule and complete the table below.

n	37	38	39	40
F_n	24 157 812	39 088 169	63245 986	102 334 155

[2 marks]

(b) The table below lists the first seven Fibonacci numbers and some values in a new sequence. This new sequence T_n is defined as

$T_n = 3F_n + 2F_{n-1}$ for $n \geq 2$

n	1	2	3	4	5	6	7
F_n	1	1	2	3	5	8	13
T_n	dne	5	8	13	21	34	55

Complete the table above.

[2 marks]

(c) You can write appropriate formula in ***Spreadsheet*** on the calculator to determine numbers in the sequences F_n and T_n.

Use ***Spreadsheet*** on the calculator to help you complete the table below.

[2 marks]

n	37	38	39	40
F_n	24 157 812	39 088 169	63 245 986	102 334 155
T_n	102 334 155	165 580 141	267 914 296	433 494 437

(d) Phoebe makes a conjecture that $T_n = F_{n+3}$ for $n \geq 2$

Test this conjecture for n = 37 and state whether it is true or false.
Show working and conclusion below.

$T_{37} = 102\ 334\ 155$

[2 marks]

$= F_{40} = F_{37+3}$

$\Rightarrow$ conjecture is true.

The glasses icon will help you see things a little more clearly!

Part Five: [10 marks]

Tariro borrows $40 000 to buy a car and is required to make repayments of $670 per month. The first three months of his loan details are shown below. Some amounts are missing.

Month	Amount owing	Interest	Payment	Balance
1	$40 000.00	$600.00	$670.00	39 930
2	39930	$598.95	$670.00	$39 858.95
3	$39 858.95	$ 597.88	$670.00	$ 39786.83.

(a) (i) Calculate the monthly interest rate that Tariro is paying?

$$\frac{600}{40\,000} = 0.015 = 1.5\%$$

(ii) What is the annual interest rate that Tariro is paying?

$$12 \times 1.5 = 18\% \text{ p.a.}$$

[2 marks]

(b) Complete the missing entries in the table above.

✓

[2 marks]

Nic makes the claim "He will have the loan paid off in less than 12 years"

(c) Check on Nic's statement. Is it correct? Explain your answer fully.

Using sequence $12 \times 12 = 144$ months

$a_{144} = + \$4845.24 \Rightarrow$ No, the loan has not been fully repaid after 12 years!

[2 marks]

(d) Tanisha has found that just before the last payment Tariro owed $471.05. Determine the amount of the last payment that must be made in order to reduce the loan to zero.

$$\$471.05 \times 1.015 = \$478.12$$

[1 mark]

(e) Calculate the total interest that Tariro has to pay over the life of the loan.

Repayments = 151 months × 670 + 478.12

= $101 648.12

[3 marks]

Interest paid = $101 648.12 − $40 000 = $61 648.12

Part Six: [4 marks]

The parents of a newborn, Harmony, are financially astute. They placed $780 in an account when she was born and then continue to deposit another $780 on her birthday until she turns 18. The account earns 4.85% p.a. compounding yearly.

(a) How much will be in Harmony's account on her 18th birthday? Describe/explain how you did this and what expression(s) you used on the calculator.

$a_{n+1} = a_n \times 1.0485 + 780 \quad a_0 = 780$

$a_{18} = \$23468.13$

[2 marks]

(b) If instead of 4.85% p.a. Harmony's parents could have found an account paying 5.25%, how much more money would she have by her 18th birthday? Show clearly how you found this answer.

$a_{n+1} = a_n \times 1.0525 + 780 \quad a_0 = 780$

$a_{18} = \$24421.39$

Extra $= \$24421.39 - \$23468.13$

[2 marks]

$= \$953.26$ extra

Part Seven: [6 marks]

To help banks and customers calculate repayments for different loans there is a formula. It is shown below.

$$A_o = \frac{Q\,[1 - (1+i)^{-n}]}{i}$$

Number of regular repayment needed

Amount borrowed

Regular repayment

Interest rate per payment period

Indicate clearly how you solved each of these questions.

(a) Find the regular monthly repayment needed to pay off a car loan of \$25 000 in seven years at an interest rate of 9.5% p.a.

$A_0 = 25\,000$

$Q = \ ?$

$i = 0.095/12$.

$n = 7 \times 12 = 84$

$$\text{Solve}\left(25\,000 = \frac{x\left[1-\left(1+\frac{0.095}{12}\right)^{-84}\right]}{0.095/12}\right.$$

$$x = \boxed{\$\,408.60\ /\ \text{month}}$$

[2 marks]

(b) A teacher borrows \$15 000 to go to a Maths conference in the USA. The interest rate is initially 6% p.a. and regular repayments of \$550 per month are made (with the exception of the last payment). The teacher is told the repayment time in months.

At the end of the 14th month, the lender informs the teacher that the annual rate of interest has increased to 12.75% p.a. Determine the new monthly repayment required in order to pay off the loan in the same amount of time.

Without changing, time would have been.

$$15000 = \frac{550\left(1-\left(1-\frac{0.06}{12}\right)^{-n}\right)}{(0.06/12)}$$

$\Rightarrow\ n = 29.39$ month

$\Rightarrow\ 30$ months

After 14th month, still owing.

$a_{n+1} = a_n \times \left(1+\frac{0.06}{12}\right) - 550, \quad a_0 = 15000$

$a_{14} = \$\,8129.49.$

Need to pay in 16 months.

$$\Rightarrow \text{solve}\left(8129.49 = \frac{Q\left(1-\left(1+\frac{0.1275}{12}\right)^{-16}\right.}{(0.1275/12)}\right.$$

$\Rightarrow\ Q = \$\,555.19.$

Repayment must be increased to $\boxed{\$\,555.19}$

[4 marks]

THE END