GREENWOOD COLLEGE

Methods 11 Test 1 2018
Quadratic and Linear Functions
Total Marks: 59 Time Allowed: 60 minutes

Name: _____

Resource Free
ALL working must be shown for full marks.

1. [4, 4, 3, 3 = 14 marks]

Solve the following:

a) $\frac{4}{4-x} = \frac{4}{x+2} + 1$

$\frac{4}{4-x} - \frac{4}{x+2} = 1$ ✓rearrange

$\frac{4(x+2)-4(4-x)}{(4-x)(x+2)} = 1$ ✓cross multiple

$4x+2 - 16 + 4x = -x^2 + 2x + 8$

$x^2 + 6x - 16 = 0$ ✓rearrange

$(x+8)(x-2)$

$x = -8$ or 2 ✓solve

b) $x^2 + 4x - 3 = 0$

OR

$x^2+4x+4-4-3=0$ ✓quadratic $a=1$ $b=4$ $c=-3$

$(x+2)^2 - 7 = 0$ ✓

$(x+2)^2 = 7$

$x+2 = \pm\sqrt{7}$ ✓

$x = \sqrt{7}-2$ ✓ or $-\sqrt{7}-2$

$x = \frac{-4 \pm \sqrt{16+12}}{2}$

$= \frac{-4 \pm \sqrt{28}}{2}$ ✓simplify

$= \frac{-4 \pm 2\sqrt{7}}{2}$ ✓simplify surd

$= -2+\sqrt{7}$ or $-2-\sqrt{7}$ ✓

c) $6x^2 - x - 2 = 0$

$6x^2 + 3x - 4x - 2 = 0$ ✓split

$3x(2x+1) - 2(2x+1) = 0$

$(3x-2)(2x+1) = 0$ ✓factorise

$x = \frac{2}{3}$ or $-\frac{1}{2}$ ✓solve

give ½ marks

d) solve by completing the square
$x^2 + 4x - 6 = 0$

$x^2+4x+4-6-4=0$

$(x+2)^2 - 10 = 0$ ✓complete square

$(x+2)^2 = 10$

$x+2 = \pm\sqrt{10}$ ✓

$x = +\sqrt{10}-2$ or $-\sqrt{10}-2$ ✓

2. [2, 2, 2, 2 = 8 marks]

Find the discriminant of each quadratic equation and state the number of real roots it has.

a) $x^2 - x - 4 = 0$ b^2-4ac

$a=1$, $b=-1$, $c=-4$

$1-(4\times1\times-4)$

$1+16$ ✓

$17 > 0$

2 real roots ✓

b) $2x^2 + 3x + 6 = 0$

$a=2$, $b=3$, $c=6$

$3^2-(4\times2\times6)$

$-39<0$ ✓

no real roots ✓

c) $x^2 + 6x + 9 = 0$

$a=1$, $b=6$, $c=9$

$6^2-(4\times1\times9)$

$36-36$ ✓

$0=0$

one real root ✓

d) $-2x^2 + 8x - 2 = 0$

$a=-2$, $b=8$, $c=-2$

$(8)^2-(4\times-2\times-2)$

$64-16$ ✓

$48>0$ ✓

2 real roots

3. [3, 2 = 5 marks]

The line with equation 7x + 5y = 70 intersects the x-axis and y-axis at A and B respectively.

a) Find the coordinates on the mid-point AB.

$5y = -7x + 70$

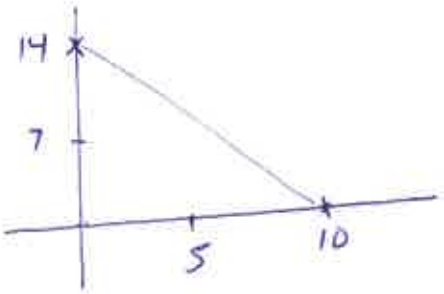
$y = -\frac{7}{5}x + 14$ ✓ rearrange

(0,14) (10,0)

✓ solve for intercepts

midpoint (5,7)

✓ find midpoint



b) Find the distance between A and B. Leave answer in exact form.

$d = \sqrt{14^2+10^2}$ ✓

$= \sqrt{196+100}$

$= \sqrt{296}$ ✓

4. [3, 2, 2 = 7 marks]

a) What is the equation of the line passing through the point (-5, 3) and is perpendicular to $2y = 10x - 6$?

$y = 5x - 3$

$m_1 = 5$ ✓

$m_2 = -\frac{1}{5}$

$y = -\frac{1}{5}x + c$

$3 = \frac{-5}{-5} + c$

$c = 2$ ✓

$y = -\frac{1}{5}x + 2$ ✓

b) Write the equation of the line that passes through the point (-2, 4) and is parallel to the equation $y = 3x + 2$.

$m_1 = m_2 = 3$

$y = 3x + c$

$4 = -6 + c$

$c = 10$

$y = 3x + 10$ ✓ ✓

c) A line passes through the point with coordinates (1, 2) and has a gradient of 2. If it passes through the point with coordinates (5, k) find the value of k.

$y = 2x + c$

$2 = 2 + c$

$c = 0$

$y = 2x$ ✓

$k = 2(5)$

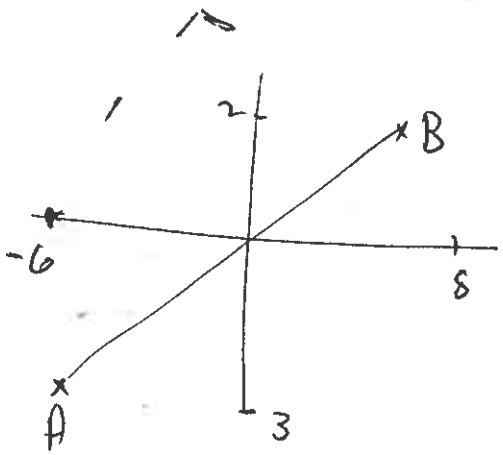
$k = 10$ ✓

5. **[6 marks]**

In the triangle ABC, A has co-ordiantes (-6, -3) while B has co-ordinates (k, 5) and C has co-ordinates (8, 2). Find the value of k if angle ABC is a right angle.

$m_{AB} = \frac{-3-5}{-6-k}$ ✓ $\quad m_{BC} = \frac{2-5}{8-k}$ ✓

$= \frac{-8}{-6-k}$ $\quad = \frac{-3}{8-k}$



$m_{AB} = \frac{-1}{m_{BC}}$

$\frac{-8}{-6-k} = \frac{8-k}{3}$ ✓

$-24 = -48 - 8k + 6k + k^2$ ✓

$k^2 + 2k - 24 = 0$ ✓

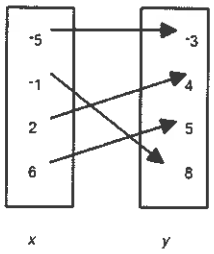
$(k-6)(k+4) = 0$

$k = 6$ or -4 ✓

6. **[2, 2, 2 = 6 marks]**

Define each of the following as a function, or a relation, giving a reason for your choice.

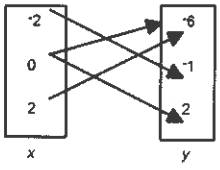
a)



one to one ✓

function ✓

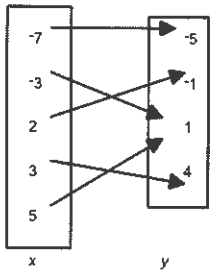
b)



one to many ✓

relation ✓

c)



many to one ✓

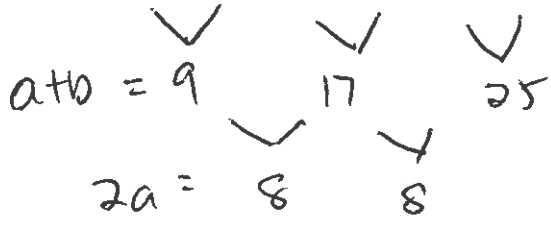
function ✓

7. [3, 5 = 8 marks]

Determine the equation for the following.

a)

x	0	1	2	3	4	5
y	-2	7	24	49	82	123



$a=4$

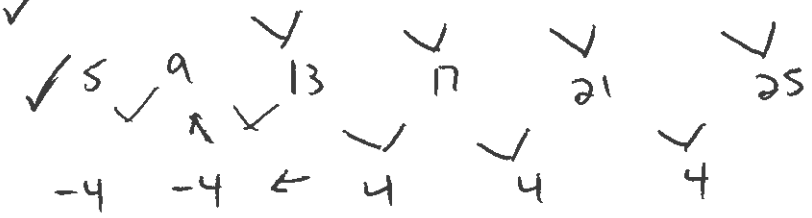
$c=-2$

$b=5$

$y = 4x^2 + 5x - 2$

b)

x	0	1	2	3	4	5	6	7
y	5	10	19	32	49	70	95	124



$a=2$

$c=5$

$b=3$

8. [1, 1, 1, 2 = 5 marks]

$y = 2x^2 + 3x + 5$

The value of a photocopier t years after purchase is given by $V = 9000 - 900t$ dollars.

a) Find V when $t = 4$.

$V = 9000 - (900 \times 4)$

$= \$5400$

b) Find t when V is 3600.

$3600 = 9000 - 900t$

$t = 6$ years

c) Find the orginial purchase price of the photocopier.

$9000

d) For what values of t is it reasonable to use this function? Give an explanation.

t has to be greater than or equal 0 $t \geq 0$ (1/2)

t has to be lyrs than or equal 10 $t \leq 10$ (1/2)

If older, value is negative (1/2)

Can be younger than 0 years (1/2)

Functions and graphs

Lines and Linear relationships

For points $P(x_1, y_1)$ and $Q(x_2, y_2)$

Mid-point of P and Q: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Gradient of the line through P and Q: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of the line through P with slope m: $y - y_1 = m(x - x_1)$

Parallel lines: $m_1 = m_2$

Perpendicular lines: $m_1 m_2 = -1$

General equation of a line: $ax + by + c = 0$ or $y = mx + c$

Quadratic relationships

For the general quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$

Completing the square: $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

Discriminant: $\Delta = b^2 - 4ac$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Graphs and Relations

Equation of a circle: $(x - a)^2 + (y - b)^2 = r^2$

where, (a, b) is the centre and r is the radius

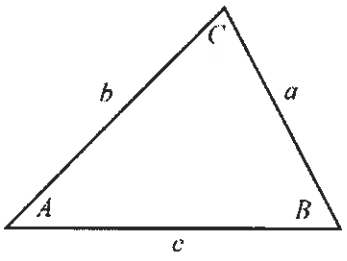
Trigonometric functions

Cosine and sine rules

For any triangle ABC with corresponding length of sides a, b, c

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



Area of Δ: $A = \frac{1}{2}ab\sin C$

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$