Mathematics Department

Course: ATMAA

Topic Title: Growth & Decay in Sequences

JTC

JOHN TONKIN COLLEGE

Test 1

Student Name: _____*Answers*_____

Date: _____

Special Instructions: Calculator Free

Formula Sheet Allowed

Time Allowed: 40 mins

Marks: / 35

Question 1. (2, 2, 1: 5 marks)

Consider the sequence: 18 25 32 39 46 . . . .

+7 +7 +7.

Find the :

a) recursive rule for this sequence.

$$T_{n+1} = T_n + 7 \quad , \quad T_1 = 18$$

✓ ✓

b) rule for the nth term of this sequence.

$$T_n = a + (n-1)d$$

$$= 18 + (n-1)7$$

$$= 18 + 7n - 7$$

$$T_n = 7n + 11 \checkmark$$

c) 50th term of this sequence. ✓

$$T_{50} = 7(50) + 11$$

$$= 361 \quad \checkmark$$

Question 2. (1, 1, 1: 3 marks)

Find the first 5 terms of the sequences given by the following recursive definitions.

a) $T_{n+1} = T_n - 3$, $T_1 = 78$

78, 75, 72, 69, 66 ✓

b) $T_n = 2T_{n-1} - 6$, $T_1 = 3$

$T_2 = 2(3) - 6$

$T_3 = 2(0) - 6$

$3, 0, -6, -18, -42$ $= -6$ $T_4 = 2(-6) - 6$ $= -18$

$T_5 = 2(-18) - 6$
$= -36 - 6$
$= -42$

c) $T_{n+1} = T_n + T_{n-1}$, $T_1 = 2$, $T_2 = 3$

$2, 3, 5, 8, 13$ ✓

Question 3.   (2, 2: 4 marks)

Consider the sequence: 5   15   45   135   405 ....

a) Find the recursive rule for this sequence. $\frac{15}{5} = 3$.

$T_{n+1} = 3T_n$ ✓ , $T_1 = 5$. ✓

b) Given the nth term is $T_n = a \times b^{n-1}$, find a and b.

$T_n = 5 \times 3^{n-1}$. $a = 5$ ✓
                            $b = 3$ ✓

Question 4.   (1, 1, 1: 3 marks)

For each of the following first order recurrence relations determine if they have a long term:
    (i) increasing
    (ii) decreasing or
    (iii) steady state solution

(a) $T_{n+1} = 0.2T_n + 2$, $T_1 = 10$ Steady State ✓

(b) $T_{n+1} = 3T_n + 1$, $T_1 = 2$ Increasing ✓

(c) $T_{n+1} = 2.2T_n - 3$, $T_1 = -1$ Decreasing ✓

Question 5. (5, 2: 7 marks)

The eighth term and twelfth term of an arithmetic sequence are 24 and 40 respectively.
a) Find the recursive rule for the sequence.

$$T_{n+1} = T_n + d \;,\; T_1 = a. \quad T_n = a + (n-1)d.$$
$$T_8 = a + (8-1)d.$$
$$\text{(1)} \quad 24 = a + 7d. \;\checkmark$$

$$T_8 = T_n + d$$

$$T_{n+1} = T_n + 4 \;,\; T_1 = -4. \;\checkmark \quad T_{12} = a + (12-1)d.$$
$$\text{(2)} \quad 40 = a + 11d.$$

Find $a \rightarrow 24 = a + 28.$
$$-4 = a \;\checkmark$$

$$\begin{array}{r} 40 = a + 11d \\ - \; 24 = a + 7d \\ \hline +16 = 4d \\ d = 4. \end{array} \;\checkmark$$

b) Find the fifth term

$$T_n = -4 + (n-1)4.$$
$$= -4 + 4n - 4.$$
$$T_n = 4n - 8$$

$$T_5 = 4(5) - 8$$
$$= 12. \;\checkmark \;\checkmark$$

Question 6. (2, 2: 4 marks)

The nth term of a geometric sequence is described by the rule $T(n) = 4 \times 2^n$, where $n = 1, 2, 3, 4, 5 \ldots$

a) Find the first 5 terms of the sequence.

$$T_1 = 4 \times 2 \qquad T_4 = 64$$
$$= 8 \qquad T_5 = 128 \;\checkmark$$
$$T_2 = 4 \times 4$$
$$= 16 \qquad \checkmark$$
$$T_3 = 4 \times 8$$
$$= 32.$$

b) State the recursive rule for this sequence.

$$T_{n+1} = 2T_n \;,\; T_1 = 8.$$
$$\checkmark \qquad \checkmark$$

Question 7.   (3, 3, 3: 9 marks)

A wedding photographer is quoting the following price for producing a wedding album for the newlyweds:

A fixed minimum cost of $150, with 80 photos in a hard-backed album. Further photos may also be added in lots of 10 photos at $0.70 per photo, up to a maximum of 200 photos.

He wants to set up a table below, showing:

- the type of album where T_1 is the basic album, with 80 photos at a cost of $150
- the number of photos in each of the possible album sizes
- the cost in dollars of each of the different albums.

a) Complete each of the blank cells of the table.

$0.70 \times 10 = \$7.00$ ✓

Type	T_1	T_2	T_3	T_4	T_5		T_n
Number of pictures	80	90	100	110	120.		200
$ cost of album	$150	157	164	171	178		234

✓

$$\left(\begin{array}{l} 200 = 10n + 70. \\ 130 = 10n. \\ 13 = n. \end{array} \right)$$ ✓

b) Write a rule that will calculate the number of pictures in album type $= T_n$.

$C_{13} = 143 + 7(13)$
$143 + 91$.

Therefore $T_n = a + (n-1)d$ ✓
$= 80 + 10n - 10$ ✓
$T_n = 10n + 70.$ ✓

✓

c) Write a rule that will calculate the cost of album type $= C_n.$

$C_n = a + (n-1)d$ ✓
$= 150 + 7n - 7$ ✓
$C_n = 143 + 7n$

✓