SHENTON COLLEGE

Examination Semester One 2017
Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two
(Calculator-assumed)

Your name ____

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section Two.
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	11	11	100	99	65
			Total	152	100

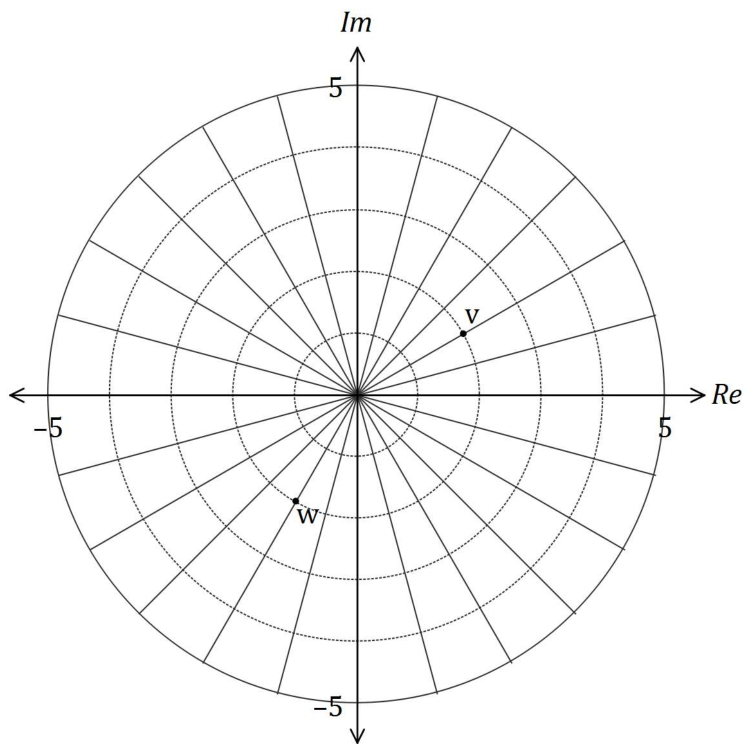
Section Two: Calculator-assumed 65% (99 Marks)

This section has **eleven (11)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

The complex numbers v and w are shown on the Argand diagram below.



On the diagram, clearly mark the complex numbers

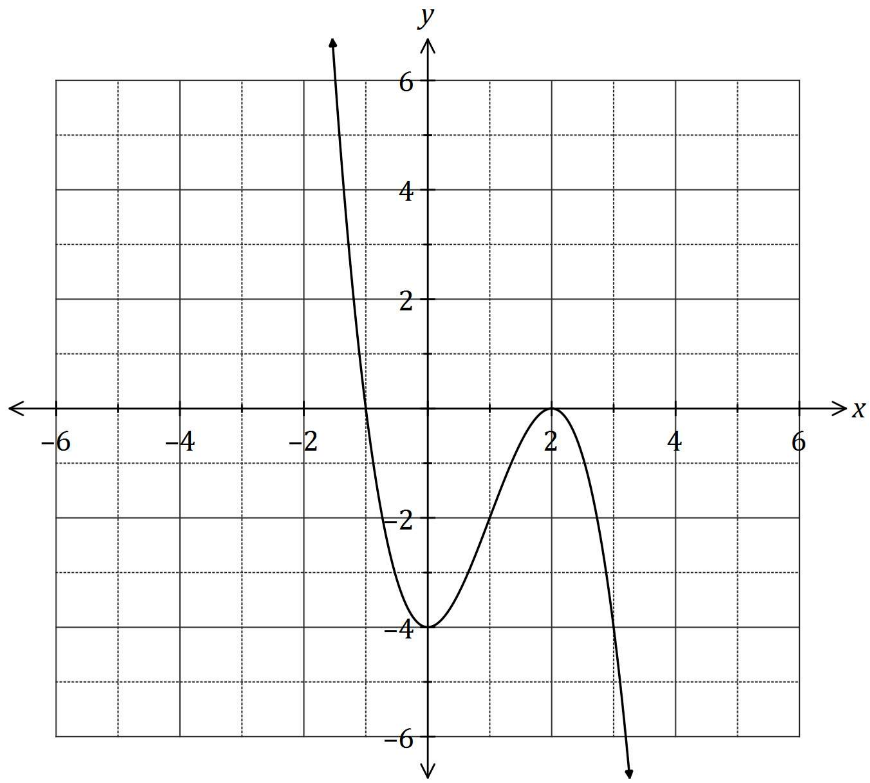
(a) $z_1 = vw$. (2 marks)

(b) $z_2 = \dfrac{v}{w}$. (2 marks)

(c) $z_3 = v - iw$. (2 marks)

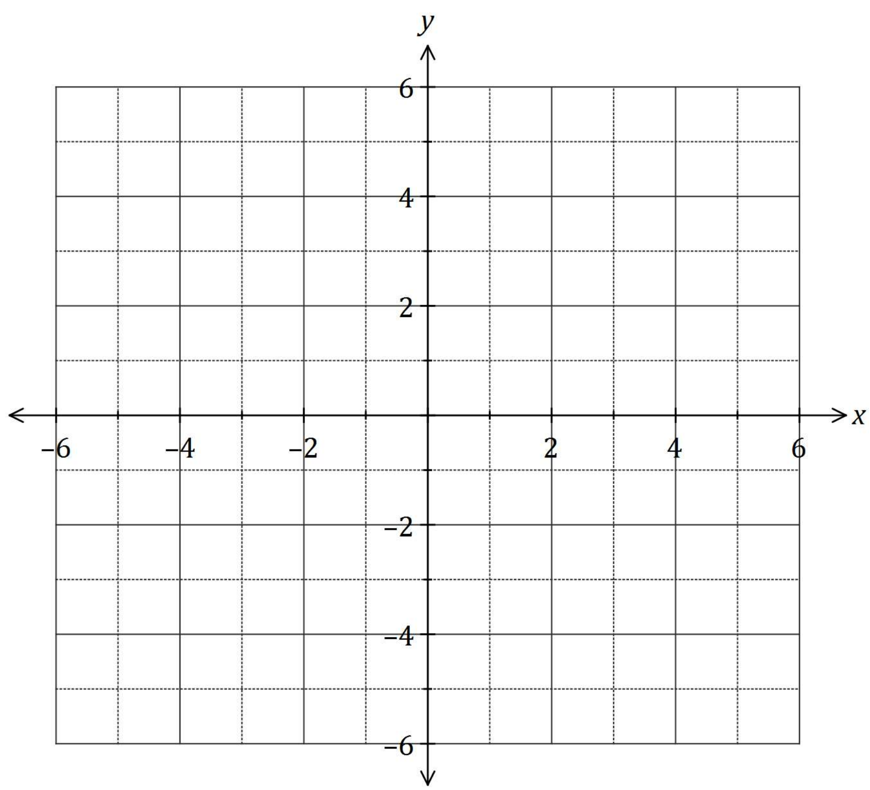
Question 10 **(8 marks)**

The graph of $y = f(x)$ is drawn below.

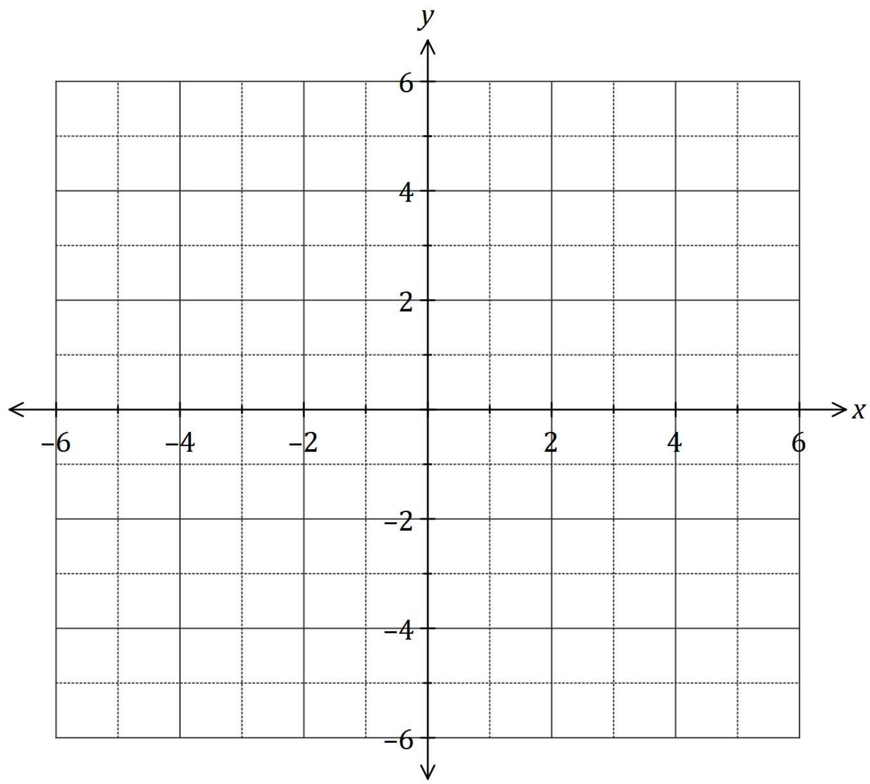


On the axes provided, sketch the graphs of

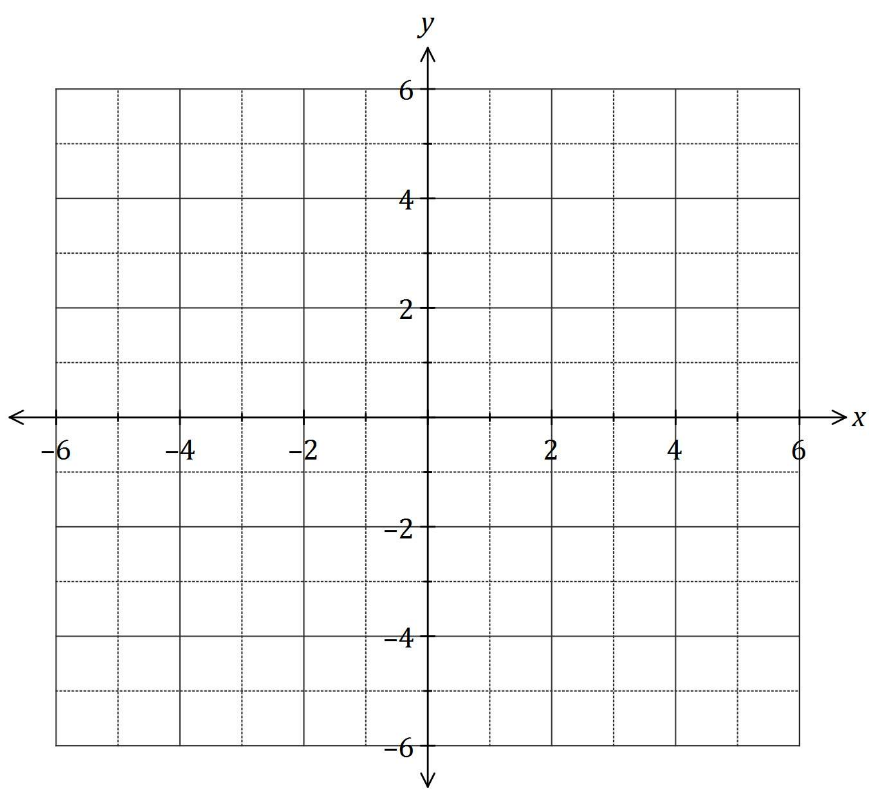
(a) $y = f(|x|)$. (2 marks)



(b) $y = f^{-1}(x)$. (2 marks)



(c) $y = \frac{1}{f(x)}$. (4 marks)



Question 11 **(9 marks)**

The position vectors of particles A and B are $\mathbf{r_A} = \begin{pmatrix} 15 - t \\ 4 - 3t \end{pmatrix}$ and $\mathbf{r_B} = \begin{pmatrix} 3t - 8 \\ 5 - t^2 \end{pmatrix}$, where t is the time in seconds, $t \geq 0$, and distances are measured in metres.

(a) Determine the speed of particle B when $t = 2$. (2 marks)

(b) Determine the Cartesian equation for the path of particle A, including any appropriate restrictions on the domain. (3 marks)

(c) Determine where the paths of the particles cross and whether the particles meet. Justify your answer. (4 marks)

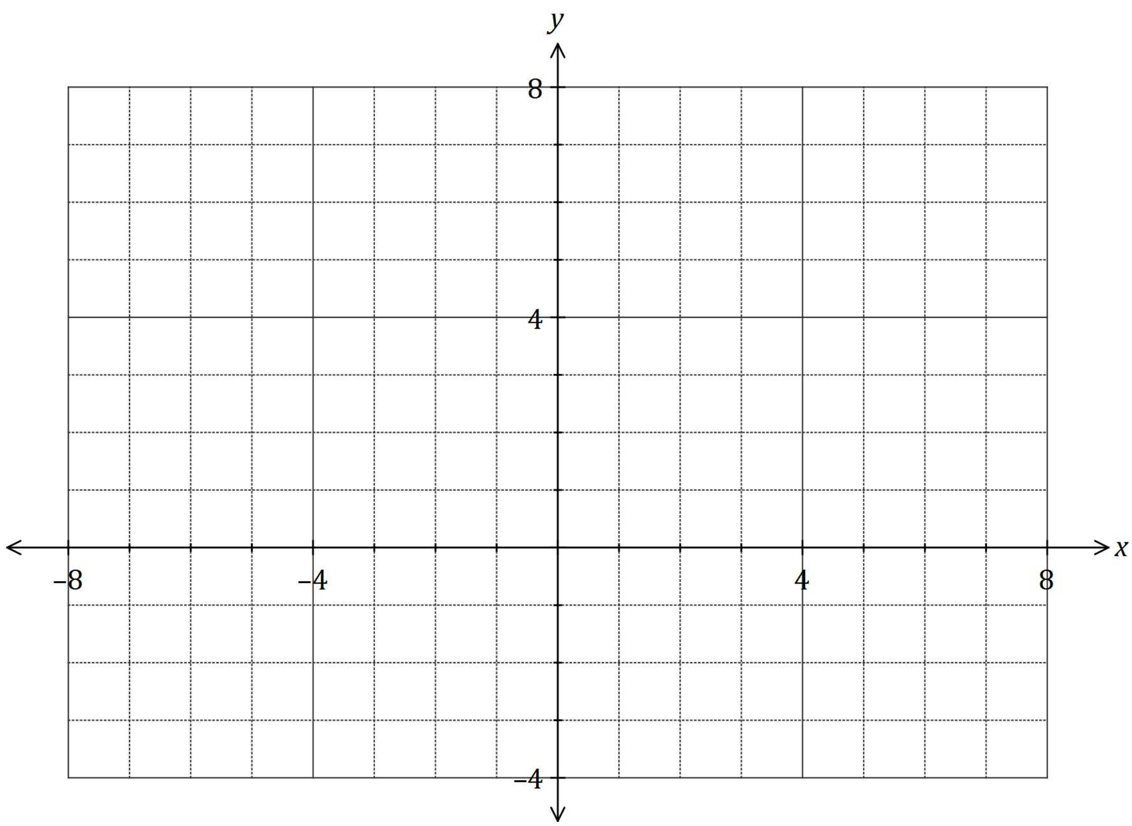
Question 12 **(9 marks)**

A function is defined by $f(x) = \frac{x^2+4x-12}{3x-7}$, $x \neq 0$.

(a) Determine the exact coordinates of all stationary points of the graph of $y = f(x)$. (2 marks)

(b) Determine the equation(s) of the asymptote(s) of the graph $y = f(x)$. (3 marks)

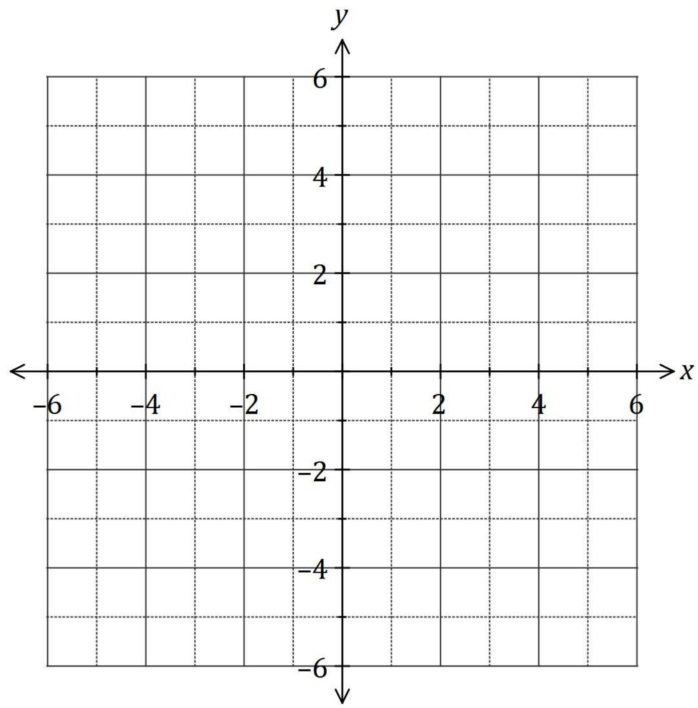
(c) Sketch the graph $y = f(x)$ on the axes below. (4 marks)



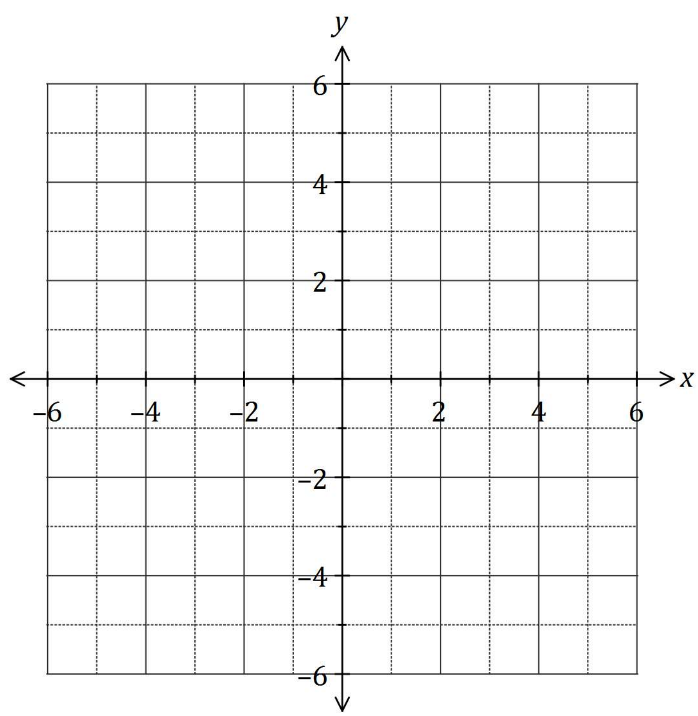
Question 13 **(11 marks)**

(a) On the Argand planes below, sketch the subsets of the complex plane determined by

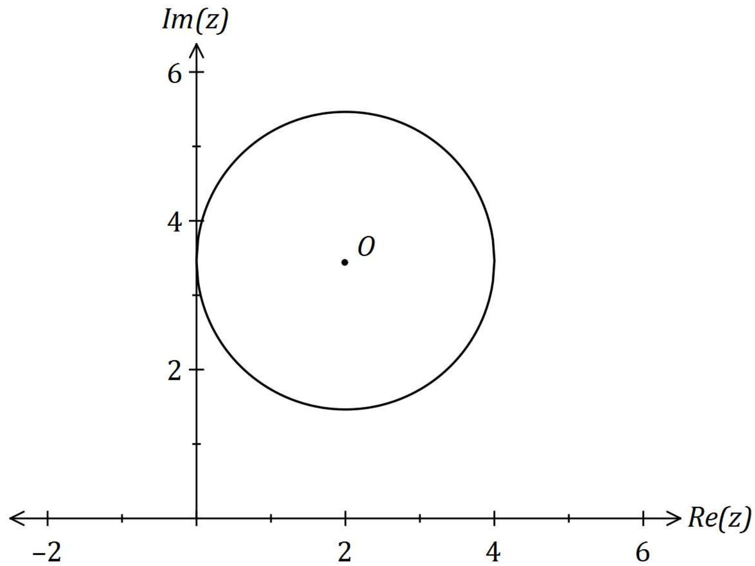
(i) $|z + 3i| = |z + 2 - i|$. (3 marks)



(ii) $|z + 3 + i| \leq 3$. (3 marks)



(b) A subset of the complex plane, a circle with centre O, is shown below.



(i) Mark the position on the subset where $|z|$ is maximised. Label this point (A).
(1 mark)

(ii) Mark the position on the subset where $|z-2|$ is minimised. Label this point (B).
(1 mark)

(iii) If the subset shown is $\left|z-2-2\sqrt{3}i\right|=2$, determine the maximum and minimum values of $\arg z$.
(3 marks)

Question 14 **(8 marks)**

The plane P has equation $\mathbf{r} \cdot \mathbf{n} = 11$, where $\mathbf{n} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and the point A has position vector $2\boldsymbol{i} + 5\mathbf{j} - 2\mathbf{k}$.

(a) Determine the Cartesian equation of plane Q that is parallel to P and passes through A. (2 marks)

(b) Determine the equation of the line L that passes through A and is perpendicular to P. (1 mark)

(c) Determine the position vector of B, the point of intersection of line L with plane P. (3 marks)

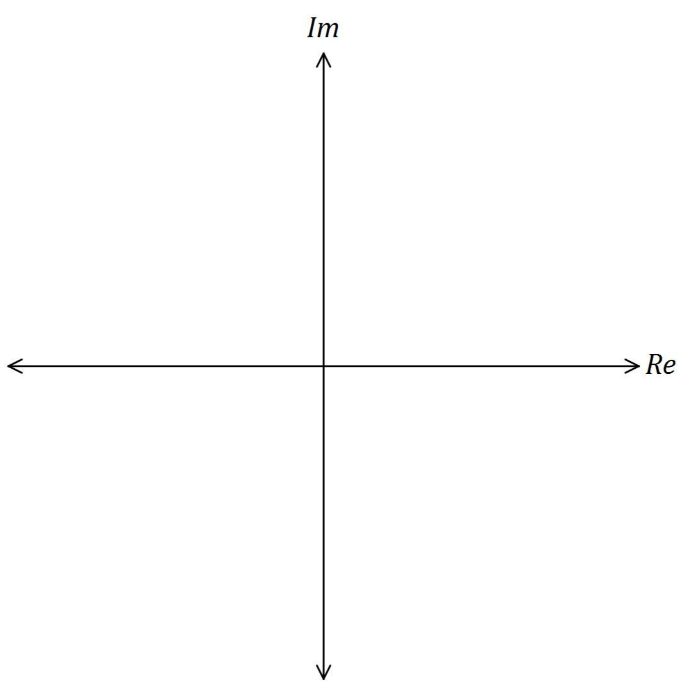
(d) Determine the exact distance between planes P and Q. (2 marks)

Question 15 **(8 marks)**

Consider the complex equation $z^5 = -16 + 16\sqrt{3}i$.

(a) Show use of De Moivre's theorem to solve the equation, giving all solutions in the form $r\,\text{cis}\,\theta\,,\ r > 0,\ -\pi \leq \theta \leq \pi$. (4 marks)

(b) Plot the solutions found in part (a) on the Argand diagram below, indicating all key features of the plot. (4 marks)



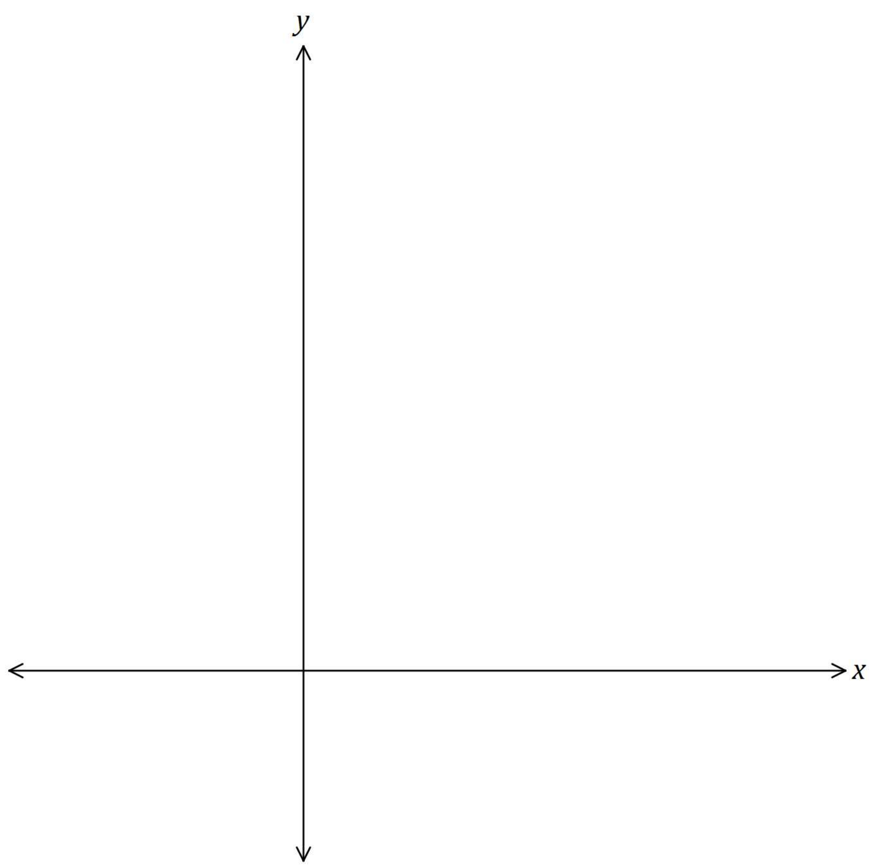
Question 16 **(13 marks)**

The position vector, in centimetres, of a particle at time t seconds is given below.

$$\mathbf{r}(t) = 2e^{t-1}\mathbf{i} + \frac{e^{2t}}{3}\mathbf{j}$$

(a) Show that the path of the particle can be expressed as a Cartesian equation in the form $y = ax^2$, and determine the value of a. (4 marks)

(b) Sketch the path of the particle for $0 \le t \le 2$. (3 marks)



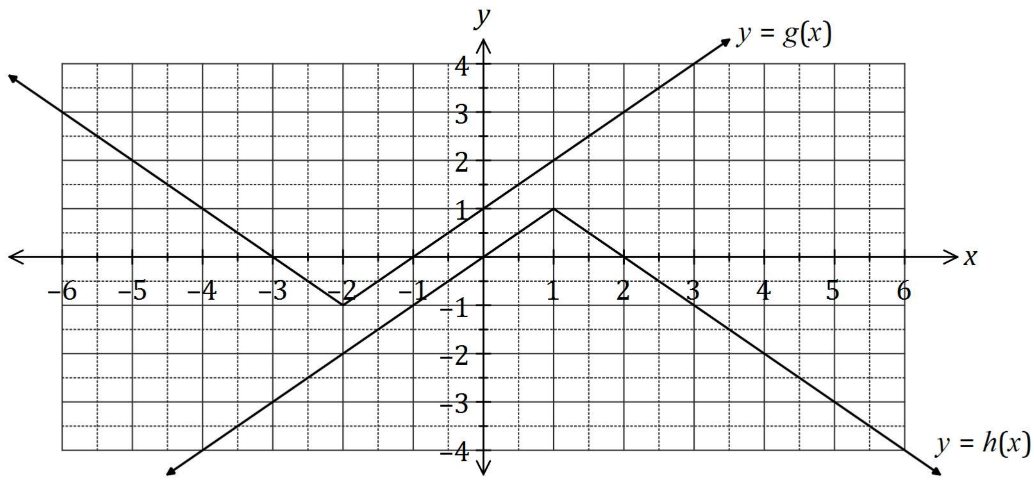
(c) Determine the speed of the particle when $t = 1$. (3 marks)

(d) Write an expression in terms of t for the speed of the particle. (1 marks)

(d) Hence write an expression in terms of t for the total distance travelled by the particle along its path between $t = 0$ and $t = 2$. Do **not** evaluate this expression. (2 marks)

Question 17 **(9 marks)**

(a) The graphs of the functions g and h are shown below.

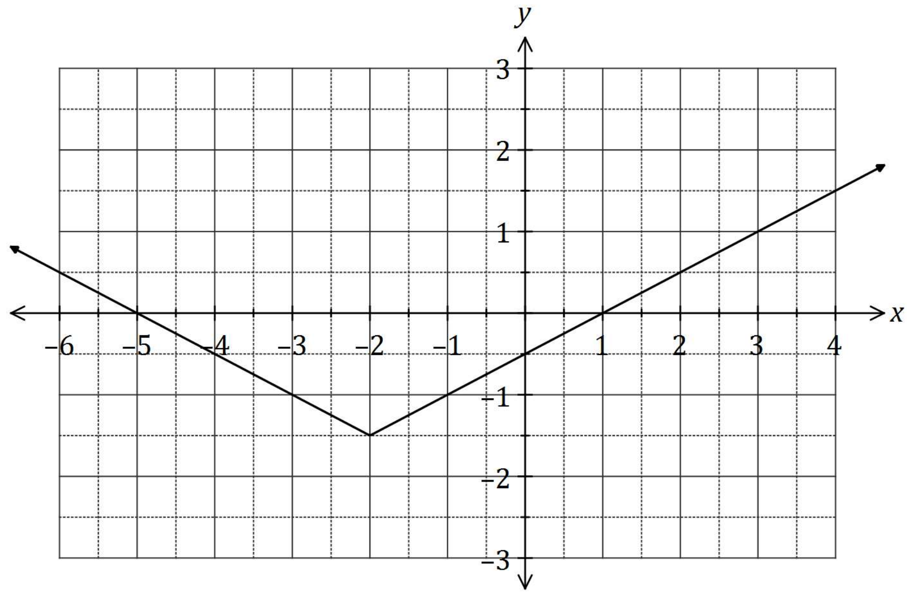


Determine the value(s) of k if

(i) $k = h \circ g(2)$. (1 mark)

(ii) $g\big(h(k)\big) = 1$. (2 marks)

(b) The graph of $f(x) = a|x - p| + q$ is shown below.

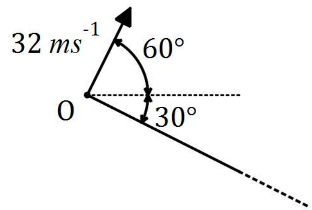


(i) Determine the value of the constants a, p and q. (3 marks)

(ii) $g(x) = mx + c$ is a line with positive constants m and c.
If the equation $|f(x)| = g(x)$ has an infinite number of solutions, determine the values of m and c. (3 marks)

Question 18 **(12 marks)**

A small body is projected upwards from the top of a hill with an initial velocity of 32 ms^{-1} at an angle of 60° to the horizontal. The hill slopes downwards at a constant angle of 30° to the horizontal. Let the origin O of a cartesian coordinate system be the top of the hill, with $\mathbf{i}$ a unit vector in the positive x direction and $\mathbf{j}$ a unit vector in the positive y direction. Displacement is measured in metres and time in seconds.



(a) Show that the initial velocity of the body is $16\mathbf{i} + 16\sqrt{3}\mathbf{j}$. (1 mark)

The acceleration of the body, t seconds after projection, is given by $\mathbf{a} = -0.2t\mathbf{i} + (0.2t - 10)\mathbf{j}$.

(b) Determine an expression for the position vector of the body after t seconds. (3 marks)

(c) Determine the time at which the body lands on the hillside. (3 marks)

(d) Calculate the distance of the body from O at the instant it lands. (2 marks)

(e) Determine the maximum vertical height attained by the particle above the hillside. (3 marks)

Question 19 **(6 marks)**

Determine, where possible, a unique solution for the following systems of equations. In each case, interpret the system of equations geometrically.

(a) $8x + y + z = 15$, $2x + y - z = 3$, and $x - y + 2z = 3$. (2 marks)

(b) $x + y - z = 0$, $x - y + 2z = 10$ and $3x - y + z = 16$. (2 marks)

(c) $x + y = z + 2$, $x - y + z = 1$ and $x + z = y + 3$. (2 marks)