SCOTCH
COLLEGE

12 Mathematics Methods 2020

Test 2 – Applications of Integration

Section 1: Calculator-free

Time allowed: 20 minutes **Maximum marks: 21**

Name: Solns.

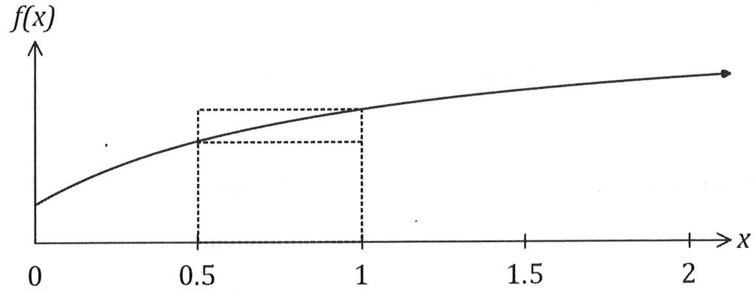
Teacher: Foster | Giese

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No calculators or personal notes are permitted.

Question 1 (6 marks)

The graph of $f(x) = \dfrac{6x+1}{x+1}$ is shown below.



Two rectangles are also shown on the graph, with dotted lines, and they both have corners just touching the curve. The smaller is called the inscribed rectangle and the larger is called the circumscribed rectangle.

a) Complete the missing values in the table below. [1]

x	0	0.5	1	1.5	2
$f(x)$	1	$\frac{8}{3}$	$\frac{7}{2}$	4	$\frac{13}{3}$

b) Complete the table of areas below and use the values to determine a lower and upper bound for $\int_0^2 f(x)\,dx$. [4]

x interval	0 to 0.5	0.5 to 1	1 to 1.5	1.5 to 2
Area of inscribed rectangle	$1/2$	$8/6$	$7/4$	2
Area of circumscribed rectangle	$\frac{4}{3}$	$7/4$	2	$13/6$

Lower: $67/12$ Upper: $87/12$

c) Explain how the bounds you found in (b) would change if a larger number of smaller intervals were used. [1]

lower increase
upper decrease

they would be closer together.

Question 2 (6 marks)

Determine the definite and indefinite integrals below. Fully simplify your answers where possible.

a) $\int \frac{2x+1}{(2x^2-5+2x)^3}\,dx$ [3]

b) $\int_1^4 (-x^2+3)\,dx$ [3]

$\int (2x+1)(2x^2-5+2x)^{-3}\,dx$

$\frac{(2x+1)(2x^2-5+2x)^{-2}}{(4x+2)(-2)} + C$

$= \frac{1}{-4(2x^2-5+2x)^2} + C$

$= \left[\frac{-x^3}{3} + 3x\right]_1^4$

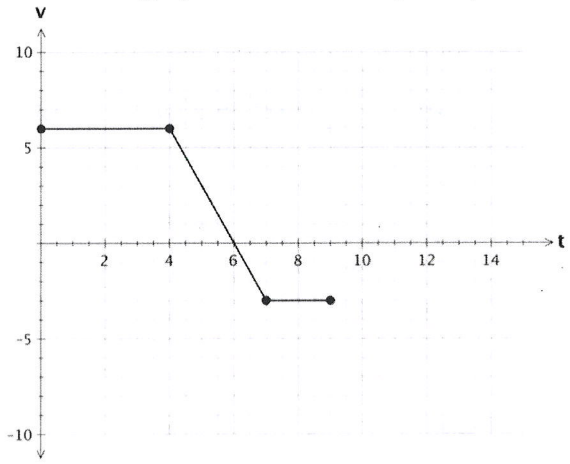
$= \left(\frac{-64}{3} + 12\right) - \left(-\frac{1}{3} + 3\right)$

$= \frac{-63}{3} + 9$

$= -12$

Question 3 (9 marks)

Below is a graph of the velocity of a particle moving, measure in metres per second.



a) What is the maximum speed of the particle? [1]

$6ms^{-1}$

b) Determine the acceleration at $t = 2$. [1]

0

c) How far has the particle travelled in the first 6 seconds? [2]

$4 \times 6 + \frac{1}{2} \times 2 \times 6 = 30m$

d) During what time interval(s) is the speed of the particle decreasing? [2]

$4 \to 6$ seconds.

① for $4 \to 7$

e) How long would the particle have to travel in total to return to its original starting position (assuming his velocity is the same as the last seen on the graph.) [3]

@ $t = 9$

$30 - \frac{1}{2} \times 1 \times 3 - 2 \times 3$

$= 30 - 1.5 - 6$

$= 22.5$

$\therefore \frac{22.5}{3} = \frac{45}{6}$

$= \frac{15}{2}$

$\therefore 9 + 7.5 = 16.5$ seconds

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12 Mathematics Methods 2020

Test 2 – Applications of Integration

Section 2: Calculator-Assumed

Time allowed: 25 minutes **Maximum marks: 24**

Name: Solns. **Teacher:** Foster | Giese

Instructions:
- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- Classpad Calculators and 1 page (both sides) of personal notes are permitted.

Question 4 (7 marks)

A fuel storage tank, initially containing 430 L, is being filled at a rate given by

$$\frac{dV}{dt} = \frac{t^2(120-3t)}{200}, \qquad 0 \le t \le 40$$

where V is the volume of fuel in the tank in litres and t is the time in minutes since filling began. The tank will be completely full after 40 minutes.

a) Calculate the volume of fuel in the tank after 20 minutes. [3]

$$\int_0^{20} \frac{t^2(120-3t)}{200} \cdot dt$$

$$= 1000$$

$$\therefore \quad V = 1430\,L$$

b) Determine the time taken for the tank to fill to one-quarter of its maximum capacity. [4]

$$430 + \int_0^{40} \frac{dV}{dt} = max$$

$$3630 = max$$

$$\therefore \tfrac{1}{4} \text{ capacity } 907.5.$$

$$907.5 - 430 = \int_0^{k} \frac{dV}{dt} \cdot dt$$

$$k = 14.91 \text{ or } 52.45$$

ignore

$\therefore$ 14.91 minutes.

Question 5 (7 marks)

Consider the following information about the function $f(x)$.

$\int_{-2}^{3} f(x)\,dx = 14$ $\int_{-2}^{5} f(x)\,dx = 6$ $\int_{5}^{7} f(x)\,dx = 9$ $\int_{7}^{10} f(x)\,dx = -4$

a) Use this information to determine the following;

i) $\int_{3}^{7} f(x)\,dx$ [2]

$\int_3^5 f(x)\,dx = 6 - 14$

$= -8$

$\therefore \int_3^7 f(x)\,dx = 9 + (-8)$

$= 1$

ii) The area between $f(x)$ and the x-axis between $x = 10$ and $x = 5$. [2]

13 units^2

iii) $\int_{5}^{7} (2x + f(x))\,dx$ [2]

$\int_5^7 2x + \int_5^7 f(x)$

$= \left. x^2 \right]_5^7 + 9$

$= 24 + 9 \Rightarrow 33.$

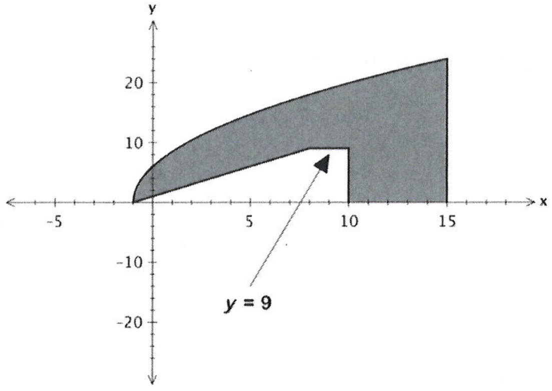
b) If $f(x)$ was a velocity function, describe in words what the following integral would represent. [1]

$$\int_0^9 |f(x)|\,dx$$

Total distance travelled in the first 9 "seconds".

Question 6 (5 marks)

Below is a graph with two vertical lines, one horizontal line and parts of the graphs of $y = 6\sqrt{x+1}$ and $y = x + 1$.



Showing use of calculus where applicable, determine the value of the shaded region.

$\int_{-1}^{15} 6\sqrt{x+1}\,dx$ - trapezium.

$= 256$

$A = \frac{11+2}{2} \times 9$

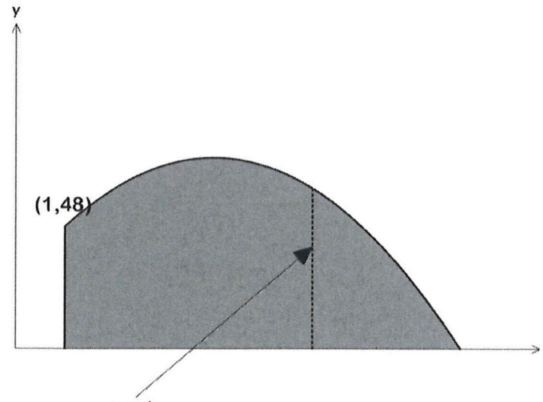
$= 58.5$

$\therefore 256 - 58.5$

$= 197.5$ units^2

Question 7 (5 marks)

Below is a shaded region partly created by the curve $y = -3(x+1)(x-9)$. Given that the dotted line $x = k$ cuts the area of the left-hand side to the right-hand side into the ratio 2:1, determine the value of k.



$$\int_1^k -3(x+1)(x-9)\,dx = 2\int_k^9 -3(x+1)(x-9)\,dx$$

$k = -5.266$, 5.377, 11.289

ignore ignore

$\therefore\ k = 5.377$

---END OF QUESTIONS---