



**Calculator Free
Differentiation and
Anti-Differentiation**

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 2, 2, 2, 3, 3 = 13 marks]

Determine the gradient function for each of the following functions:

(a) $f(x) = -x^4$

(b) $h(x) = 3x^2 + 6x - 5$

(c) $y = \sqrt{x} + 4\pi x$

(d) $g(x) = 2x(3x - 5)$

(e) $y = (x - 1)^2$

(f) $f(x) = \frac{3x^3 - 9x^4}{6x^2}$

Question Two: [2, 2, 3, 3 = 10 marks]

- (a) The gradient function of $f(x)$ is $f'(x) = x^3$. Determine an expression for $f(x)$.
- (b) The gradient function of $g(x)$ is $g'(x) = x(x-2)$. Determine an expression for $g(x)$.
- (c) The gradient function of $h(x)$ is $h'(x) = \frac{x^2 - 4x^3}{x}$. Determine an expression for $h(x)$.
- (d) The gradient function of $f(x)$ is $f'(x) = 2x - 3$. Determine $f(x)$ if it passes through the point $(-1, 6)$.

Question Three: [3, 5, 5 = 13 marks]

- (a) Determine the gradient of the function $f(x) = -x^3 - 5x$ at the point $(2, -18)$.
- (b) Determine the equation of the tangent to the curve $g(x) = 2x(x - 1)$ at $x = -2$.
- (c) Determine the coordinates of the point(s) on the curve $y = -2x^3$ where the gradient is -24 .

Question Four: [3 marks]

The function $y = ax + b$ has a gradient of -2 at the point (-1, -1). Determine the values of a and b .

Question Five: [6 marks]

The function $f(x) = ax^2 + bx + c$ passes through the point (0, -4) and has a gradient of -15 at the point (-2, 10).

Determine the values of a , b and c .



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Question One: [1, 2, 2, 2, 3, 3 = 13 marks]

Determine the gradient function for each of the following functions:

(a) $f(x) = -x^4$

$f'(x) = -4x^3$ ✓

(b) $h(x) = 3x^2 + 6x - 5$

$h'(x) = 6x + 6$ ✓✓

(c) $y = \sqrt{x} + 4\pi x$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + 4\pi$ ✓✓

(d) $g(x) = 2x(3x - 5)$

$g(x) = 6x^2 - 10x$ ✓

$g'(x) = 12x - 10$ ✓

(e) $y = (x - 1)^2$

$y = x^2 - 2x + 1$ ✓✓

$\frac{dy}{dx} = 2x - 2$ ✓

(f) $f(x) = \frac{3x^3 - 9x^4}{6x^2}$

$f(x) = \frac{x}{2} - \frac{3x^2}{2}$ ✓✓

$f'(x) = \frac{1}{2} - 3x$ ✓

Question Two: [2, 2, 3, 3 = 10 marks]

- (a) The gradient function of $f(x)$ is $f'(x) = x^3$. Determine an expression for $f(x)$.

$$f(x) = \frac{x^4}{4} + c \quad \checkmark \quad \checkmark$$

- (b) The gradient function of $g(x)$ is $g'(x) = x(x-2)$. Determine an expression for $g(x)$.

$$g'(x) = x^2 - 2x \quad \checkmark$$

$$g(x) = \frac{x^3}{3} - x^2 + c \quad \checkmark$$

- (c) The gradient function of $h(x)$ is $h'(x) = \frac{x^2 - 4x^3}{x}$. Determine an expression for $h(x)$.

$$h'(x) = x - 4x^2 \quad \checkmark \quad \checkmark$$

$$h(x) = \frac{x^2}{2} - \frac{4x^3}{3} + c \quad \checkmark$$

- (d) The gradient function of $f(x)$ is $f'(x) = 2x - 3$. Determine $f(x)$ if it passes through the point $(-1, 6)$.

$$f(x) = x^2 - 3x + c \quad \checkmark$$

$$6 = (-1)^2 - 3(-1) + c \quad \checkmark$$

$$c = 2$$

$$f(x) = x^2 - 3x + 2 \quad \checkmark$$

Question Three: [3, 5, 5 = 13 marks]

- (a) Determine the gradient of the function $f(x) = -x^3 - 5x$ at the point (2, -18).

$$f'(x) = -3x^2 - 5 \quad \checkmark \checkmark$$

$$f'(2) = -3(2)^2 - 5 = -17 \quad \checkmark$$

$$\checkmark$$

- (b) Determine the equation of the tangent to the curve $g(x) = 2x(x-1)$ at $x = -2$.

$$g(x) = 2x^2 - 2x \quad \checkmark$$

$$g'(x) = 4x - 2 \quad \checkmark$$

$$g'(-2) = -10 \quad \checkmark$$

$$g(-2) = 12 \quad \checkmark$$

$$y = -10x + c$$

$$12 = -10(-2) + c$$

$$c = -8$$

$$y = -10x - 8 \quad \checkmark$$

- (c) Determine the coordinates of the point(s) on the curve $y = -2x^3$ where the gradient is -24.

$$\frac{dy}{dx} = -6x^2 \quad \checkmark$$

$$-6x^2 = -24 \quad \checkmark$$

$$x^2 = 4$$

$$x = \pm 2 \quad \checkmark$$

$$(2, -16) \quad (-2, 16)$$

$$\checkmark \quad \checkmark$$

Question Four: [3 marks]

The function $y = ax + b$ has a gradient of -2 at the point $(-1, -1)$. Determine the values of a and b .

$$y = -2x + b$$

$$-1 = -2(-1) + b$$

$$-3 = b$$

Question Five: [6 marks]

The function $f(x) = ax^2 + bx + c$ passes through the point $(0, -4)$ and has a gradient of -15 at the point $(-2, 10)$.

Determine the values of a , b and c .

$$c = -4$$

$$10 = a(-2)^2 + b(-2) - 4$$

$$4a - 2b = 14$$

$$f'(x) = 2ax + b$$

$$2a(-2) + b = -15$$

$$-4a + b = -15$$

$$4a - 2b = 14$$

$$-4a + b = -15$$

$$-b = -1$$

$$b = 1$$

$$-4a + 1 = -15$$

$$-4a = -16$$

$$a = 4$$