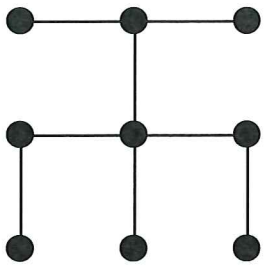
Eastern Goldfields College	Student Name MARKING KEY **Eastern Goldfields College** **Mathematics Applications U3&4 2017** **Test 3 – Calculator Free Section**
Working Time: 30 minutes	**Total Marks: 31 marks**

Question 1 [7 marks: 2, 1, 2, 2]

A **tree graph** is a planar graph with no cycles. It is an undirected graph in which any two vertices are connected by exactly one path. The following questions all refer to tree graphs.

a) For the following trees state the number of vertices and the number of edges.

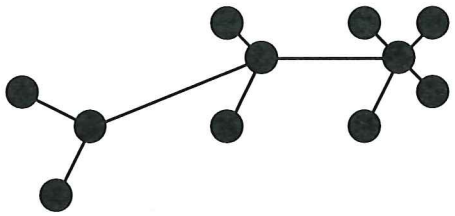
i)



Edges = 8 ✓R/W

Vertices = 9

ii)



Edges = 10 ✓R/W.

Vertices = 11

b) Draw a tree with 4 vertices and 3 edges.

✓ any reasonable answer.

c) Is it possible to draw a tree with 4 vertices and 5 edges? Explain your answer.

✓No.

reason ✓ - a cycle will be formed. - no cycles if
- no longer planar.
- more than one path connecting 2 nodes.

d) Use the tree in part a) i) above and Euler's formula to explain why tree graphs are all planar.

✓ $9 + 1 - 8 = 2$.

$10 - 8 = 2$

$2 = 2$ ✓ ⇒ trees are planar

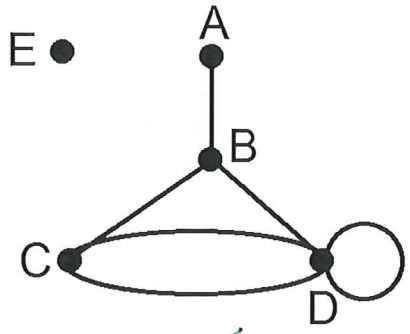
$v + f - e = 2$

$2 = 2$

$\therefore v = e + 1$

$+ f = 1$

Question 2 [8 marks: 5, 1, 2]



a) Consider the graph on the right.

i. Which vertex is adjacent to A? B ✓

ii. The loop in the graph connects which vertex to itself?

D ✓

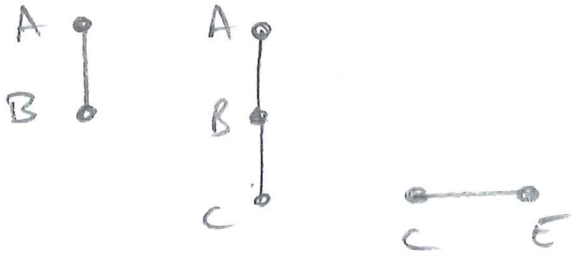
iii. Which pair of vertices is connected by multiple edges? C, D ✓R/W.

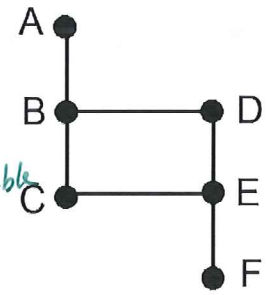
iv. Is this a connected graph? Explain.

✓ No

✓ reason { vertex E is isolated disconnected, ie no edge joining it to the graph.

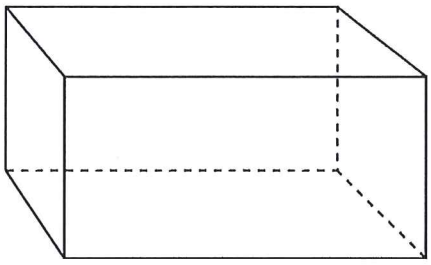
b) Draw a sub-graph of the graph on the right.

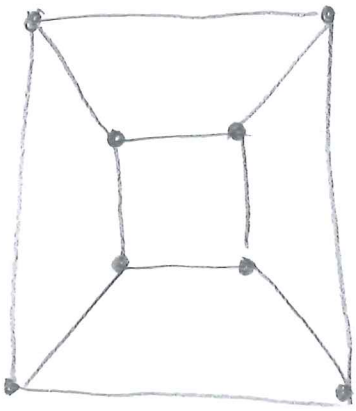




✓ Any reasonable answer

c) Represent the following three-dimensional prism as a planar graph.



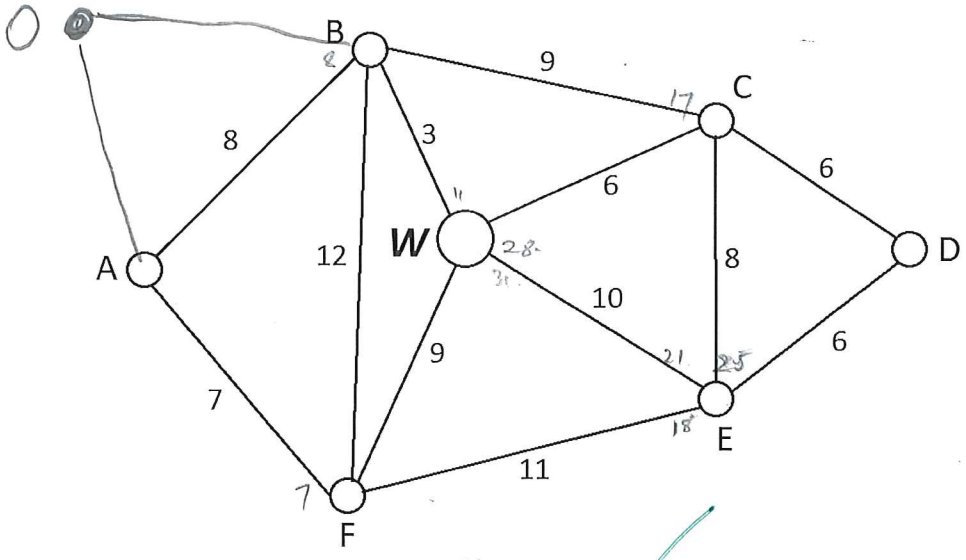


✓ 8 nodes drawn

✓ Degree of 3 ∀ node.

Question 3 [10 marks: 1, 2, 3, 4]

The graph on the right shows the location of a warehouse, *W*. This warehouse supplies timber to six factories *A, B, C, D, E* and *F*. The number along the edges indicates the length (in kilometres) of the connecting roads.



a) What is the degree of vertex *W*? 4 ✓

b) A delivery van is at vertex *A*. It must make a delivery to vertex *E* before returning to the warehouse. Determine the minimum distance travelled on this journey.

A F E W 7 + 11 + 10 = 28 km ✓ ✓

c) A salesman wants to leave the warehouse and visit every factory once before returning to the warehouse.

i. What is the mathematical term used to describe the route he is going to take?

Hamiltonian cycle OR cycle ✓ OR closed path

ii. Define a route that he could take.

W - B - A - F - E - D - C - W

OR

W - F - A - B - C - D - E - W

✓ start + end on W ✓ valid path.

d) The company wishes to lease an office which is adjacent to two of the factories.

i. Draw one possible location for the company on the graph, including one road to each of the adjacent factories. ✓ any valid

ii. What impact will these two extra roads have on the degree of the vertices?

Connecting vertices to Office will:
- change from 2 to 3 ✓
- even to odd. ✓

iii. The owner wishes to check for any competitors along the route. Will the owner have to complete an open or closed trail if she wants to travel along every edge only once?

Open ✓

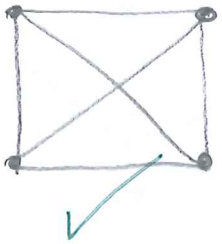
Question 4 [6 marks: 2, 2, 2]

In a netball competition there are four teams participating.

a) If every team needs to play each other exactly once, how many games are required for the season?

3, 2, 1 6 games. ✓✓

b) Draw the graph showing how each team can play each other, what type of graph would this be?



Complete ✓ ✓

c) If two more teams join the competition how many more games are required to be played in order for each team to still play each other exactly once?

5 4 3 2 1

Total 15 games

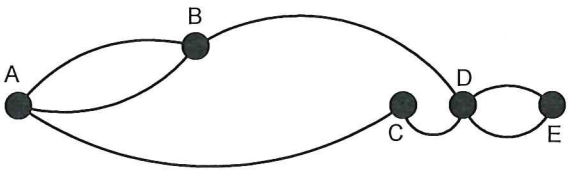
15 - 6 = 9 more games ✓✓

Student Name Marking Key

Eastern Goldfields College
Mathematics Applications U3&4 2017
Test 3 – Calculator Assumed Section

Working Time: 22 minutes | **Total Marks: 20 marks**

Question 1 [6 marks: 3, 2, 1]
Consider the following undirected graph.



a) Construct the adjacency matrix, M, for the above digraph.

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

✓ leading diag = 0

✓✓ rest

b) Calculate M^2 and explain the significance of the zero elements in this matrix.

$$M^2 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 5 & 0 & 0 & 3 & 0 \\ 0 & 5 & 3 & 0 & 2 \\ 0 & 3 & 2 & 0 & 2 \\ 3 & 0 & 0 & 6 & 0 \\ 0 & 2 & 2 & 0 & 4 \end{bmatrix} \end{array}$$

✓ F.T.

0 means no ✓ walks of length 2

eg. A–B no walks of length 2.

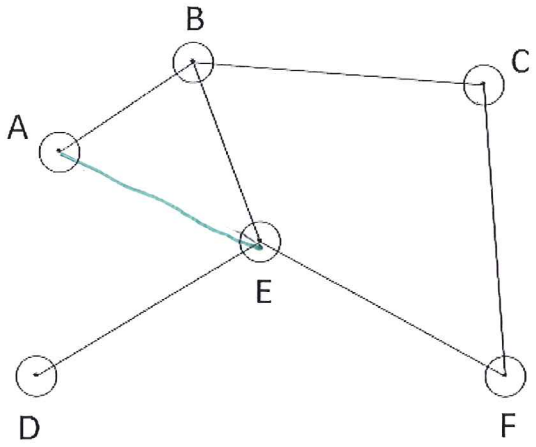
c) Identify a practical situation that could be represented by this network.

Any reasonable answer ✓

? X. Bus route or Train route would not have multiple edges?

Question 2 [3 marks: 1, 2]

A treasure hunt is being organised and a graph showing where prizes are hidden is shown right.



a) Show a semi-Hamiltonian path which you could walk in order to collect all the prizes.

A - B - C - F - E - D.

OR ←

✓

b) Is it possible to create an Eulerian circuit for the treasure hunt by adding exactly one edge? If so, what is the pathway, if not, explain why?

✓No

Still left with 2 odd. = semi Eulerian or open

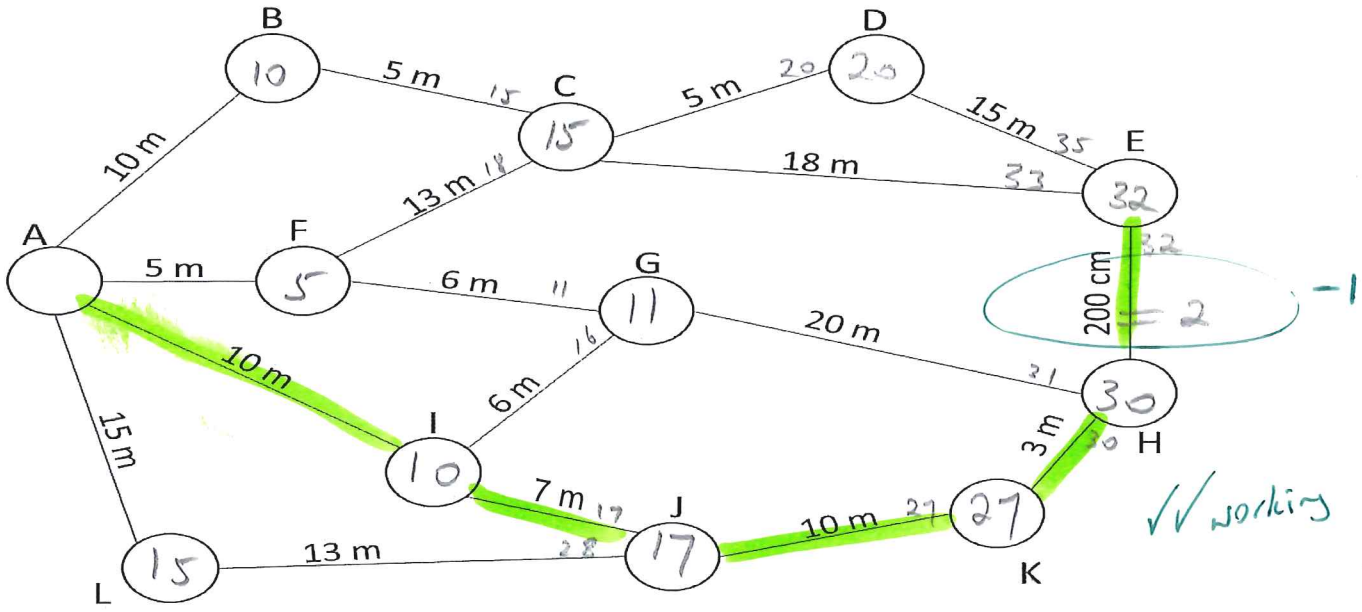
Not closed

circuit ⇒ is closed ⇒ all even nodes.

Question 3 [11 marks – 4, 1, 6]

The following network shows some of the corridors running through a large hospital. Most of the thoroughfare through this hospital is from A to E.

The hospital receives some funding to upgrade some of the finishings in the corridors. In order to maximise their budget they decided to upgrade the shortest path from A to E as these would be the most used corridors. The cost of upgrading is $95/metre.



a) What is the shortest distance from A to E and what is this pathway?

A - I - J - K - H - E = 32 m

b) What is the cost of upgrading the shortest path from A to E?

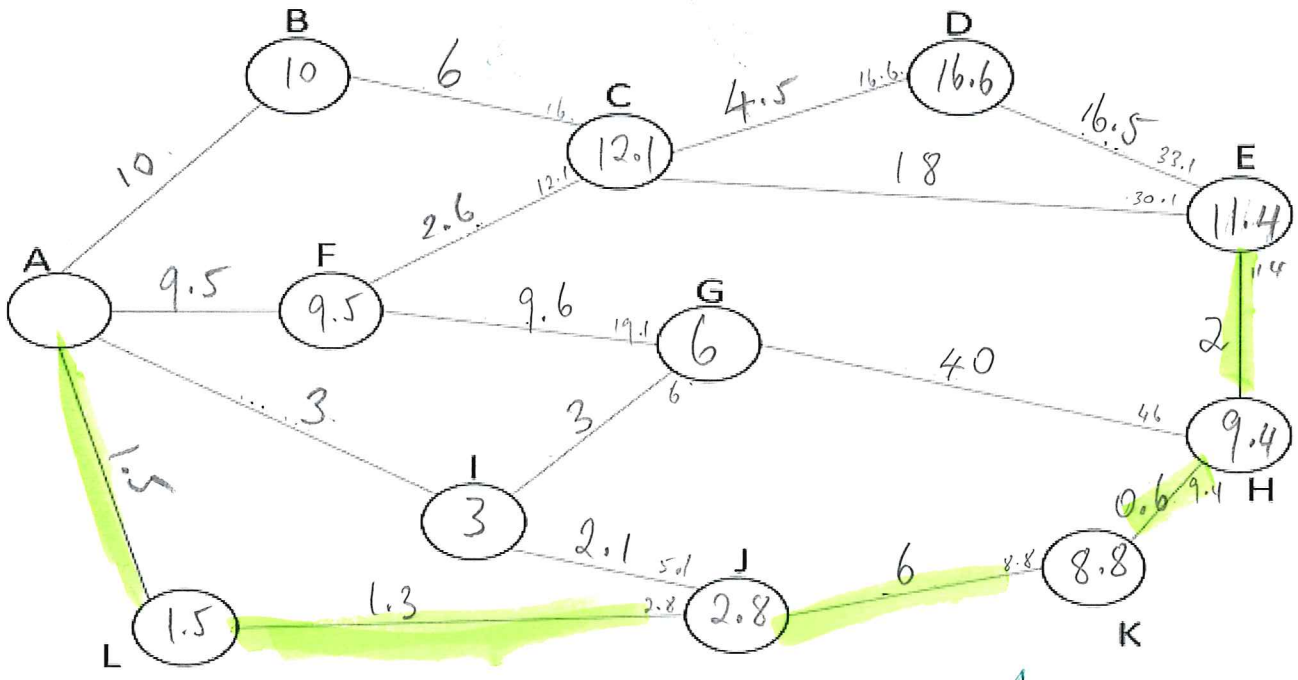
32 × 95 = $3040

if 30.2 = $2869

Upon further discussions it appears that not all the corridors require the same amount of work to upgrade and therefore the costs of upgrading each corridor are not equal. The following table shows the multiplication factor in the costs of upgrading each corridor relative to the lengths.

	A	B	C	D	E	F	G	H	I	J	K	L
A		1				1.9			0.3			0.1
B	1		1.2									
C		1.2		0.9	1	0.2						
D			0.9		1.1							
E			1	1.1				1				
F	1.9		0.2				1.6					
G						1.6		2	0.5			
H					1		2				0.2	
I	0.3						0.5			0.3		
J									0.3		0.6	0.1
K								0.2		0.6		
L	0.1									0.1		

c) Taking into account the multiplication factors, which path from A to E should the hospital upgrade in order to minimise costs? Clearly state this path and the total cost of upgrading it. Use the blank network below to assist with your answer.



A-L-J-K-H-E = 11.4 × 95 ✓f.t.

= $1083 ✓ft if reasonable

✓path

FT

✓ multiplication factors

✓✓ shortest path calc.

-1 ∓ error

END OF TEST