Name: SOLUTIONS

Mathematics: Methods Unit 1

Test 3, 2015

Trigonometric Functions and Equations, Probability

CALCULATOR FREE

25 marks

25 minutes

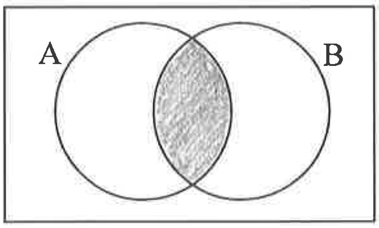
No Calculators Allowed

SCSA Formula Sheet

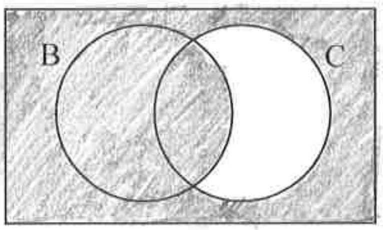
Question 1 **[1, 1, 1 = 3 marks]**

Shade in the region described by the following:

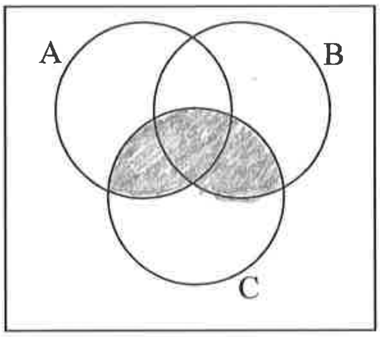
a) $A \cap B$



b) $B \cup C'$

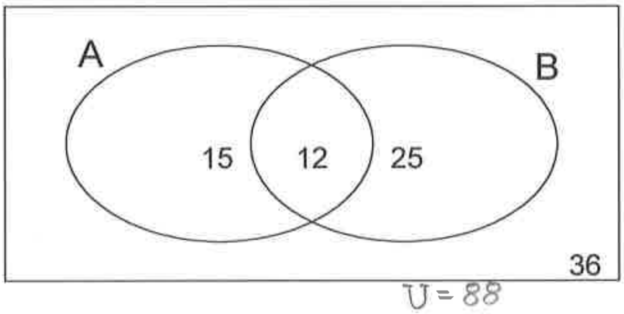


c) $(A \cup B) \cap C$



Question 2 **[3 marks]**

From the following Venn Diagram:



Find:

Leave your answer in **FRACTION** form

a) $P(A \cup B)$ $\frac{52}{88} = \frac{13}{22}$

b) $P(A/B)$ $\frac{12}{37}$

c) $P(A' \cap B)$ $\frac{25}{88}$

Question 3 **[8 marks]**

State the exact value of each of the following.

a) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

b) $\sin^2 3 + \cos^2 3 = 1$

c) $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$

d) $\cos 150° = -\frac{\sqrt{3}}{2}$

e) $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

f) $\cos(n\pi)$ where n is an odd integer $= -1$

g) $\tan 210° = \frac{1}{\sqrt{3}}$

h) $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

Question 4 **[2, 3, 3 =8 marks]**

Solve the following trigonometric equations. Show working.

a) $\sin x = -\frac{\sqrt{3}}{2}$ in the interval $0 \le x \le 2\pi$

$x = \frac{4\pi}{3}$ ✓ , $\frac{5\pi}{3}$ ✓

b) $2\sin^2 x = \frac{1}{2}$ in the interval $0° \le x \le 360°$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$ ✓

$x = 30°, 150°, 210°, 330°$ ✓ ✓

c) $\cos 2x = \frac{1}{2}$ in the interval $0° \le x \le 180°$

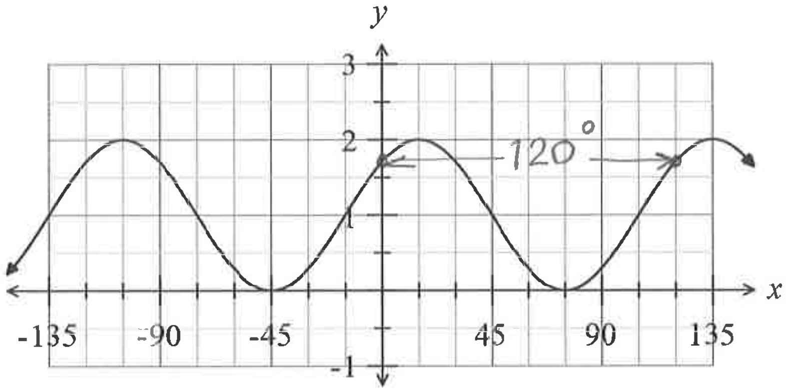
$0° \leq 2x \leq 360°$ ✓

$2x = 60°, 300°$ ✓

$x = 30°, 150°$ ✓

Question 5 **[3 marks]**

The graph below is of the general equation $y = \cos(a(x+b)) + c$.
Determine the values of a, b and c.



$c = 1$ ✓ translation up 1

$a = 3$ ✓ period is $\frac{1}{3}$ of $360°$

$b = -15$ ✓ phase shift right $15°$

$a = 3$, $b = -15$, $c = 1$

Name: SOLUTIONS

Mathematics: Methods Unit 1

Test 3, 2015

Trigonometric Functions and Equations, Probability

CALCULATOR ASSUMED

30 marks

30 minutes

Approved Calculators Allowed

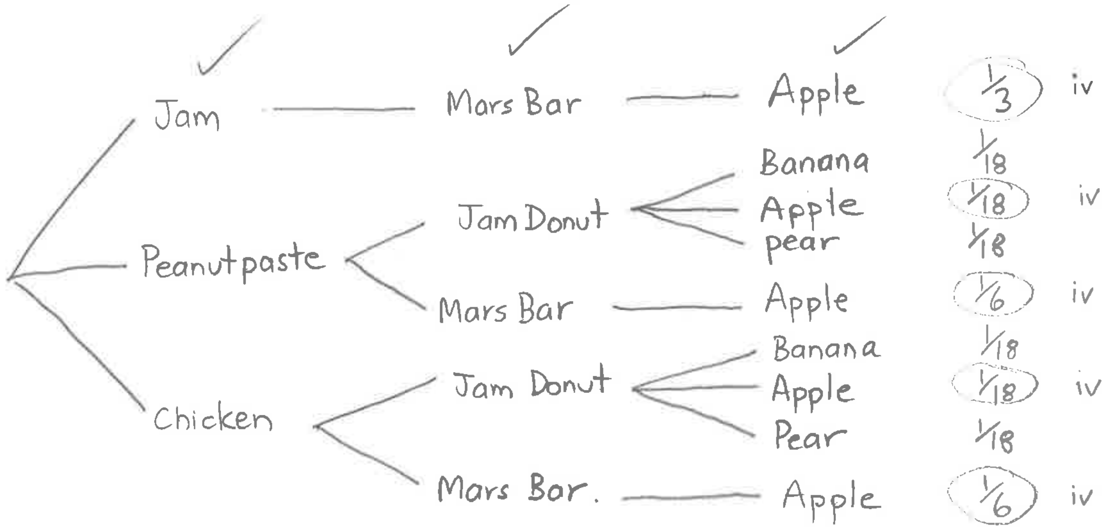
A4 Page of Notes

SCSA Formula Sheet

Question 6 **[3, 4 = 7 marks]**

Amos asked Lara what she would like for lunch and offered her several choices:
In her sandwich she could have jam, peanut paste or chicken.
As a sweet she could have a Mars Bar or a jam donut.
For fruit she could have a banana, an apple or a pear
Lara never has two jam items on the same day, and if she has a Mars Bar she always selects an apple for fruit.

a) Draw a tree diagram showing all possible combinations of lunches for Lara if it is known that she took a sandwich, a sweet and a piece of fruit.



b) Use the tree diagram to determine the following probabilities:

i. she chooses a jam sandwich

$$P(\text{Jam sandwich}) = \frac{1}{3} \times 1 \times 1 = \frac{1}{3}$$

ii. she has an apple and a Chicken sandwich

$$P(\text{Apple} \cap \text{Chicken sandwich}) = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{18}$$

iii. she chooses a jam donut or an apple

$$P(\text{Jam donut} \cup \text{apple}) = 1$$

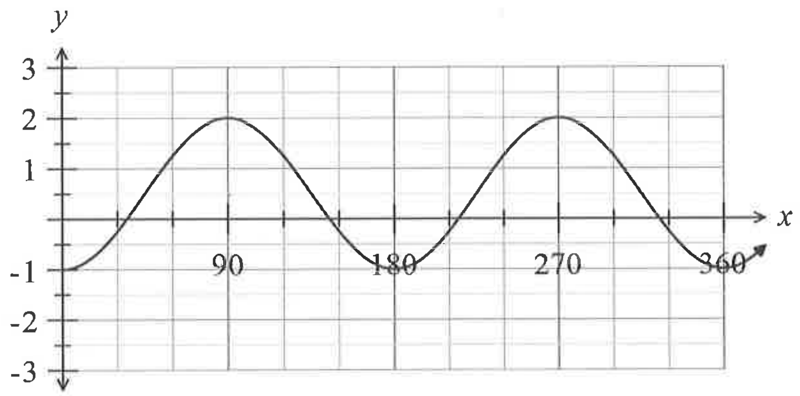
iv. she chose a chicken sandwich given that she chose an apple

$$P(\text{Chicken sandwich} \mid \text{Apple}) = \frac{P(\text{Chk} \cap \text{Apple})}{P(\text{Apple})} = \frac{2/9}{7/9} = \frac{2}{7}$$

Question 7 **[2, 2 =4 marks]**

State the period and amplitude for each of the following.

a)



period $= 180°$ ✓

amplitude $= 1\cdot5$ ✓

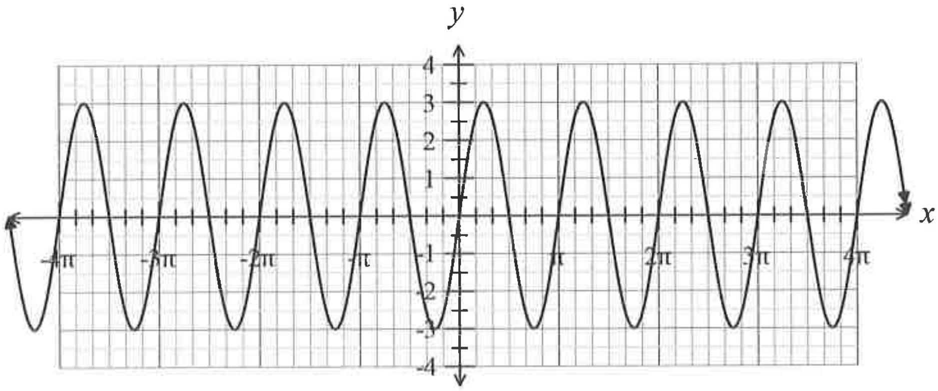
b) $y = -\tan\left(x + \frac{\pi}{4}\right)$

period $= \pi$ ✓

amplitude = indefinite ✓

Question 8 **[1, 1, 1 =3 marks]**

Given the graph below of a sinusoidal or sine function, determine its:



a) amplitude $= 3$ ✓

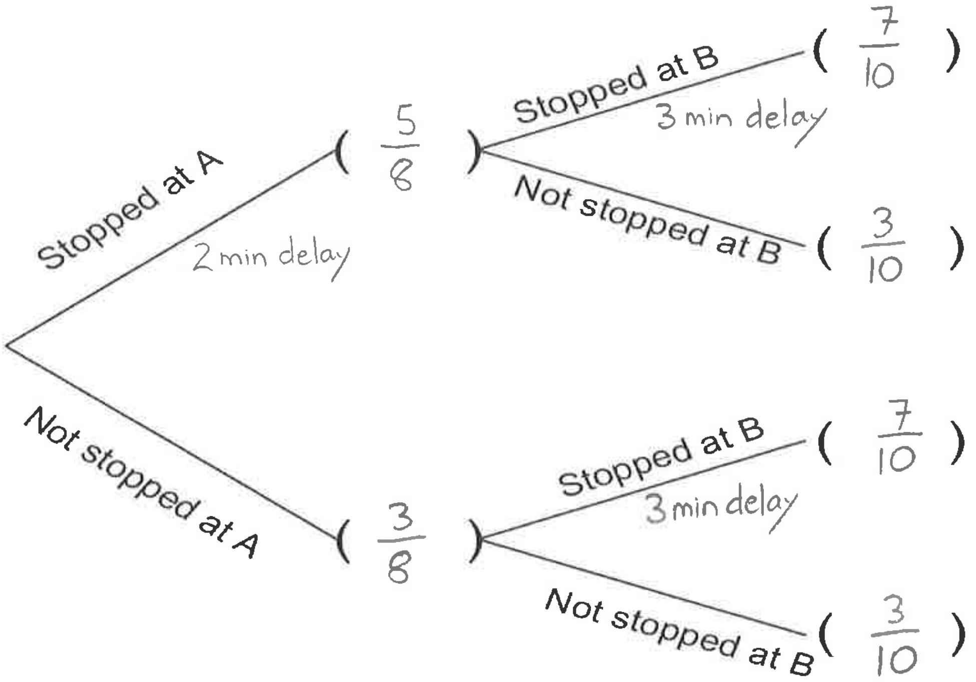
b) period $= \pi$ ✓

c) equation $y = 3\sin(2x)$ ✓

Question 9 **[2, 1, 2, 2 =7 marks]**

A student travelling to school has to pass through two sets of traffic lights A and B that operate independently of each other. The probabilities that she will be stopped at these lights are $\frac{5}{8}$ and $\frac{7}{10}$ respectively, with corresponding delays of 2 minutes and 3 minutes. Without these delays her journey takes 20 minutes.

a) Complete the tree diagram, entering the appropriate probabilities in the given brackets.



✓ correct P(stop) at each light

✓ correct P(not stopped) $\frac{3}{8}$ and $\frac{3}{10}$ respectively

b) Determine the probability that

(i) The journey takes no more than 20 minutes.

$$P(\text{no stopping}) = \frac{3}{8} \times \frac{3}{10} = \frac{9}{80} \checkmark$$

(ii) The student encounters just one delay

$$P(\text{one delay}) = P(\text{Stopped at A only}) + P(\text{stopped at B only})$$
$$= \frac{5}{8} \times \frac{3}{10} + \frac{3}{8} \times \frac{7}{10} \checkmark$$
$$= \frac{15}{80} + \frac{21}{80} = \frac{36}{80} \text{ or } \frac{9}{20} \checkmark$$

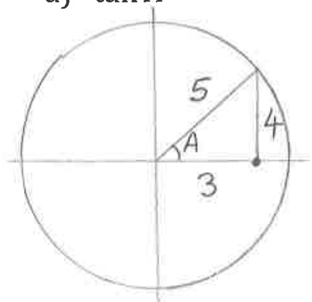
(iii) One morning the student has only 22 minutes to reach school on time. Determine the probability that she will be late.

$$P(\text{late}) = P(\text{Stopped at B only}) + P(\text{Stopped at A and B})$$
$$= \frac{3}{8} \times \frac{7}{10} + \frac{5}{8} \times \frac{7}{10} \checkmark$$
$$= \frac{21}{80} + \frac{35}{80} = \frac{56}{80} \text{ or } \frac{7}{10} \checkmark$$

Question 10 **[1, 4 =5 marks]**

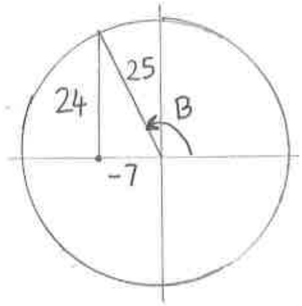
If $\sin A = \frac{4}{5}$ and $\cos B = -\frac{7}{25}$ where $0 \leq A \leq \frac{\pi}{2}$ and B is obtuse, determine exactly:

a) $\tan A$



$\tan A = \frac{4}{3}$ ✓

b) $\sin(A + B)$



$\cos A = \frac{3}{5}$ ✓ , $\sin B = \frac{24}{25}$ ✓

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$= \left(\frac{4}{5}\right)\left(-\frac{7}{25}\right) + \left(\frac{3}{5}\right)\left(\frac{24}{25}\right) \checkmark$$
$$= \frac{-28}{125} + \frac{72}{125} = \frac{44}{125} \checkmark$$

Question 11 **[4 marks]**

For two events A and B, $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(A \cup B) = 0.6$

Determine $\mathrm{P}(B)$ if events A and B are independent.

$P(A \cap B) = P(A) \times P(B)$ since A and B are independent

$= 0.3\, P(B)$ ✓

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ addition principle

$0.6 = 0.3 + P(B) - 0.3\, P(B)$ ✓

$0.6 - 0.3 = P(B)(1 - 0.3)$ ✓

$P(B) = \frac{0.3}{0.7}$

$P(B) = 0.4286$ ✓