Calculator Free
Applications of Arithmetic and Geometric Sequences

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 2, 2, 3 = 8 marks]

Rob is playing a computer game and he is increasing the number of points he scores on Level 50.

On his first attempt he scores 100 points. On his second attempt he scores 120 points and on his third attempt he scores 140 points. If this pattern continues:

(a)   What is his score on his fourth attempt?

(b)   What is his score on his tenth attempt?

(c)   He decides to move on to the next level when he scores 500 points in one attempt. How many games does he play before he moves on to Level 51?

(d)   Rob's sister tells him to move on to Level 51 when he has scored a total of 2000 points. After how many games does she suggest Rob should move on to the next level?

Question Two: **[2, 2, 1, 1, 3 = 9 marks]**

Tahlia's parents are helping her to save for a car. Tahlia suggested that she save up the first $1000 and then her parents can add to her savings in the following way.

In January they can add 10% to her savings so that in total she now has $1100.

In February they can add another 10% to her total savings so that she now has $1210.

Tahlia suggests that her parents continue to add 10% to her total savings each month.

   (a)   How much will her parents contribute in March?

   (b)   Write a recursive rule which defines Tahlia's total savings each month.

   (c)   Write a general rule which defines Tahlia's total savings each month.

Tahlia's parents aren't convinced by this plan, and instead agree to contribute $100 each month.

   (d)   How much will they have contributed by the end of the year?

Tahlia calculates that if they had agreed to her plan, she'd have a total of $3 138.43 by the end of the year.

   (e)   Calculate how much more Tahlia would have saved with her plan compared with her parents' plan and explain why Tahlia's plan results in more savings for Tahlia.

Question Three: [4, 3, 3 = 10 marks]

Josh wants to increase the amount of time he spends studying leading up to his Year 12 exams.

He decides he will spend a minutes studying the first week, $a + d$ minutes studying the second week, $a + 2d$ minutes studying the third week and so on.

In week 4 he studies for 120 minutes and in week 10 he studies for 5 hours.

(a)   Determine the value of a and d.

(b)   His parents tell him that 12 hours a week is enough study. In which week will he reach 12 hours?

(c)   Once he reaches 12 hours a week, he will maintain this for the rest of the year. If he studies for 40 weeks this year, write a calculation which can be used to calculate his total study for the year, in hours.

Question Four: [3, 2, 2, 3 = 10 marks]

Grace is collecting Colesworth animal cards for her grandson. On the first day she collects 1 card. On the second day she has a total of 3 cards and by the fourth day she has a total of 15 cards.

    (a)   How many cards did Grace collect on the second, third and fourth day?

    (b)   Write a general rule defining how many cards Grace collects each day.

    (c)   She sees her grandson on the seventh day and gives him all the cards she has collected. How many cards did he receive?

Nancy has also been collecting these animal cards for her grandson. She collects 10 cards on the first day and 15 on the second day, 20 on the third day and so on.

    (d)   How long will it take Nancy to collect the same number of cards as Grace did before she gave them to her grandson?

Question Five: **[1, 2, 3, 2 =8 marks]**

Michael plans to binge watch his favourite TV shows these holidays. He plans to watch 2 episodes on the first day and double the number of episodes he watches on each subsequent day.

(a)   How many episodes does he watch on the third day?

(b)   How many episodes does he watch on the eighth day?

(c)   If in fact this show ran for 9 seasons with 12 episodes in each season, during which day of his holidays will Michael finish the watching the series?

(d)   If each episode is an hour long, and Michael must sleep for 6 hours each day, how many days will it take him to watch the whole series?

SOLUTIONS
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Question One: [1, 2, 2, 3 = 8 marks]

Rob is playing a computer game and he is increasing the number of points he scores on Level 50.

On his first attempt he scores 100 points. On his second attempt he scores 120 points and on his third attempt he scores 140 points. If this pattern continues:

(a)   What is his score on his fourth attempt?

$$T_4 = 100 + 20(4-1) = 160 \text{ points} \quad \checkmark$$

(b)   What is his score on his tenth attempt?

$$T_{10} = 100 + 20(10-1) = 280 \text{ points} \quad \checkmark$$

(c)   He decides to move on to the next level when he scores 500 points in one attempt. How many games does he play before he moves on to Level 51?

$$500 = 100 + 20(n-1) \quad \checkmark$$
$$400 = 20(n-1)$$
$$20 = n - 1$$
$$n = 21$$
$$\therefore 21 \text{ games} \quad \checkmark$$

(d)   Rob's sister tells him to move on to Level 51 when he has scored a total of 2000 points. After how many games does she suggest Rob should move on to the next level?

$$S_n = 2000$$
$$S_{10} = 1900 \quad \checkmark\checkmark$$
$$S_{11} = 2200$$
$$\therefore 11 \text{ games} \quad \checkmark$$

Question Two: [2, 2, 1, 1, 3 = 9 marks]

Tahlia's parents are helping her to save for a car. Tahlia suggested that she save up the first $1000 and then her parents can add to her savings in the following way.

In January they can add 10% to her savings so that in total she now has $1100.

In February they can add another 10% to her total savings so that she now has $1210.

Tahlia suggests that her parents continue to add 10% to her total savings each month.

(a)   How much will her parents contribute in March?

$121 ✓ ✓

(b)   Write a recursive rule which defines Tahlia's total savings each month.

$T_n = 1.1 \times T_{n-1}; \ T_1 = 1100$

✓   ✓

(c)   Write a general rule which defines Tahlia's total savings each month.

$T_n = 1100 \times 1.1^{(n-1)}$

✓   ✓

Tahlia's parents aren't convinced by this plan, and instead agree to contribute $100 each month.

(d)   How much will they have contributed by the end of the year?

$12 \times 100 = \$1200$ ✓

Tahlia calculates that if they had agreed to her plan, she'd have a total of $3 138.43 by the end of the year.

(e)   Calculate how much more Tahlia would have saved with her plan compared with her parents' plan and explain why Tahlia's plan results in more savings for Tahlia.

$3138.43 - 2200 = \$938.43$ ✓

Tahlia's plan involves compounding the monthly contributions and this results in a geometric sequence or exponential pattern, rather than a linear arithmetic pattern, and thus her savings increase more rapidly. ✓✓

Question Three: [4, 3, 3 = 10 marks]

Josh wants to increase the amount of time he spends studying leading up to his Year 12 exams.

He decides he will spend a minutes studying the first week, $a + d$ minutes studying the second week, $a + 2d$ minutes studying the third week and so on.

In week 4 he studies for 120 minutes and in week 10 he studies for 5 hours.

(a)   Determine the value of a and d.

$$120 + 6d = 300 \checkmark$$
$$6d = 180$$
$$d = 30 \checkmark$$
$$120 = a + 30(4 - 1) \checkmark$$
$$30 = a$$
$$\checkmark$$

(b)   His parents tell him that 12 hours a week is enough study. In which week will he reach 12 hours?

$$30 + 30(n - 1) = 720 \checkmark$$
$$30(n - 1) = 690$$
$$n - 1 = 23 \checkmark$$
$$n = 24$$
$$\therefore week\ 24 \checkmark$$

(c)   Once he reaches 12 hours a week, he will maintain this for the rest of the year. If he studies for 40 weeks this year, write a calculation which can be used to calculate his total study for the year, in hours.

$$S = \frac{24}{2}\left[60 + 30(24 - 1)\right] + 16 \times 720$$
$$\checkmark \qquad\qquad \checkmark\ \checkmark$$

Question Four: **[3, 2, 2, 3 = 10 marks]**

Grace is collecting Colesworth animal cards for her grandson. On the first day she collects 1 card. On the second day she has a total of 3 cards and by the fourth day she has a total of 15 cards.

(a)   How many cards did Grace collect on the second, third and fourth day?

$$T_2 = 3 - 1 = 2\,cards \checkmark$$
$$T_3 = 4\,cards \checkmark$$
$$T_4 = 8\,cards \checkmark$$

(b)   Write a general rule defining how many cards Grace collects each day.

$$T_n = 1 \times 2^{(n-1)}$$

$\checkmark \checkmark$

(c)   She sees her grandson on the seventh day and gives him all the cards she has collected. How many cards did he receive?

$$S_7 = \frac{2^7 - 1}{2 - 1} = 127\ cards \checkmark\checkmark$$

Nancy has also been collecting these animal cards for her grandson. She collects 10 cards on the first day and 15 on the second day, 20 on the third day and so on.

(d)   How long will it take Nancy to collect the same number of cards as Grace did before she gave them to her grandson?

$$T_1 = 10 \quad S_1 = 10$$
$$T_2 = 15 \quad S_2 = 25$$
$$T_3 = 20 \quad S_3 = 45 \quad \checkmark \checkmark$$
$$T_4 = 25 \quad S_4 = 70$$
$$T_5 = 30 \quad S_5 = 100$$
$$T_6 = 35 \quad S_6 = 135$$
$$\therefore 6\ days$$

$\checkmark$

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Question Five: **[1, 2, 3, 2 = 8 marks]**

Michael plans to binge watch his favourite TV shows these holidays. He plans to watch 2 episodes on the first day and double the number of episodes he watches on each subsequent day.

(a)   How many episodes does he watch on the third day?

8 episodes ✓

(b)   How many episodes does he watch on the eighth day?

$T_8 = 2 \times 2^{(8-1)} = 256$ *episodes*

✓   ✓

(c)   If in fact this show ran for 9 seasons with 12 episodes in each season, during which day of his holidays will Michael finish the watching the series?

$9 \times 12 = 108$ ✓
$2 + 4 + 8 + 16 + 32 + 64 = 108$ ✓
$\therefore Day\ 6$ ✓

(d)   If each episode is an hour long, and Michael must sleep for 6 hours each day, how many days will it take him to watch the whole series?

$\dfrac{108}{18} = 6$ *days*

✓   ✓