Name: SOLUTIONS Date: ___________

Mathematics Methods Unit 1&2

Test 7, 2016

Chapters 8

Topic – Sequences

$\overline{57}$

= ______ %

Total Time: *60 minutes*

Weighting: *6% of the year.*

*This test comprises of **TWO sections**. The **first section** is **calculator free** where no calculators of any kind are to be used. The **second section** is **calculator assumed** where the CAS calculator may be used. All questions must be answered in both sections. **Answers should be in EXACT form or rounded appropriately**. All working should be shown in the space provided. Solutions without working may not be awarded full marks. Please take the marks for each question into account when answering the question.*

SECTION 1: CALCULATOR FREE

Time: *21 minutes* **Equipment Allowed:** *SCSA Formula sheets*

Marks for Section 1: *21 marks*

1. **[4 marks: 2, 2]**
Write down the first four terms of the sequence defined by:

a) $T_n = T_{n-1} - 5$ and $T_1 = -1$

$T_1 = -1$
$T_2 = -1 - 5 = -6$
$T_3 = -6 - 5 = -11$
$T_4 = -11 - 5 = -16$

½ each

$1, -6, -11, -16$

b) $T_n = 3T_{n-1} - 2$ and $T_1 = 2$

$T_1 = 2$
$T_2 = 3(2) - 2 = 4$
$T_3 = 3(4) - 2 = 10$
$T_4 = 3(10) - 2 = 28$

½ each

$2, 4, 10, 28$

2. **[5 marks: 3, 2]**
An arithmetic sequence is described by the rule: $T_n = 200 - 5n$, where $n = 1,2,3,4,5...$

a) Find the first three terms of the sequence

$n=1$, $T_1 = 200 - 5(1) = 195$ ✓

$n=2$, $T_2 = 200 - 5(2) = 190$ ✓

$n=3$, $T_3 = 200 - 5(3) = 185$ ✓

b) State the **recursive** rule for this sequence

$T_n = T_{n-1} - 5$ ✓

$T_1 = 195$ ✓

3. **[4 marks]**

If $t_8 = 24$ and $t_{12} = 40$, find the first term and the common difference, given the terms are from an arithmetic progression.

$t_n = a + (n-1)d$

$t_8 = 24$

$24 = a + (8-1)d$

$24 = a + 7d$

$t_{12} = 40$

$40 = a + (12-1)d$

$40 = a + 11d$

① $24 = a + 7d$ ✓

② $40 = a + 11d$ ✓ ② − ①

$16 = 4d$

$4 = d$ ✓

$\therefore 24 = a + 7(4)$

$24 = a + 28$

$-4 = a$ ✓

First term $= -4$

common difference $= 4$

4. [3 marks: 1, 1, 1]

a) The number of feral cats in a National park was initially 3500 but decreasing at an annual rate of 7.5%. Write a rule describing the number of cats after t years.

C = 3500×0.925^t ✓

b) The number of bacteria in a culture after t minutes is described by the rule $B = 50(1.12)^t$

i) Describe the percentage change in the number of bacteria each minute.

Increasing by 12%. ✓

½ ½

ii) Find how many bacteria were present initially.

50 ✓

5. [5 marks: 2, 2, 1]

Consider the sequence: 50, 42, 34, 26, …

-8 -8 -8

Determine:

a) The recursive rule for the sequence

$T_n = T_{n-1} - 8$ ✓

$T_1 = 50$ ✓

b) The general rule for any term in the sequence

$T_n = a + (n-1)d$

$T_n = 50 + (n-1)(-8)$ ✓

$= 50 - 8n + 8$

$= 58 - 8n$ ✓

c) Show using the general rule fom **b)** that -30 is a term in the sequence

$-30 = 58 - 8n$

$-88 = -8n$

$11 = n$ ✓

~ END OF SECTION ONE ~

Name: ______________________________ Date: ______________

SECTION 2: CALCULATOR ASSUMED

Time: *39 minutes* **Equipment Allowed:** *Curriculum Council Formula sheets, CAS calculator, 1 page of notes (A4 one side)*

Marks for Section 2: 36 *marks*

6. 6 **marks: 2, 2, 2]**

A geometric sequence is described by the rule $T_n = 5 \times 3^n$ where $n = 1,2,3,4,5\ldots$

a) Find the first four terms of the sequence

$n=1$, $T_1 = 5 \times 3^1 = 15$

$n=2$, $T_2 = 5 \times 3^2 = 45$ ½ each

$n=3$, $T_3 = 5 \times 3^3 = 135$

$n=4$, $T_4 = 5 \times 3^4 = 405$

b) How many terms less than 100 are there in this sequence?

solve$(100 < 5 \times 3^n, n)$ ✓ (or see above)

$n < 2$ I meant to change this to 10000 (sorry)

$\therefore$ 2 terms less than 10000 ✓

c) Find the sum of the first 11 terms, show your method

$$S_n = \frac{a(r^n - 1)}{r-1} \qquad a = 15, \quad r = \frac{45}{15} = 3$$

$$S_{11} = \frac{15(3^{11} - 1)}{3-1} \quad ✓$$

$$= 1328595 \quad ✓$$

7. **[4 marks: 2, 2]**

Given the sequence 6, 10, 14, 18, 22, …

$\underbrace{6, 10}_{4}, \underbrace{14}_{4}$

AP

a) Determine the sum of the first twenty terms

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}(2(6) + (20-1)(4))$ ✓

$= 880$ ✓

b) Does the number 624 belong in this sequence?

$T_n = a + (n-1)d$

$624 = 6 + (n-1)4$ ✓

$624 = 6 + 4n - 4$

$622 = 4n$

$155.5 = n$

No, 624 does not belong in the sequence. ✓

8. **[4 marks: 2, 2]**

a) Find the number of terms in the following sequence:

4, 11, 18, 25, … , 130

7 7 7

AP

$T_n = a + (n-1)d$

$130 = 4 + (n-1)(7)$ ✓

$130 = 4 + 7n - 7$

$130 = -3 + 7n$

$133 = 7n$

$19 = n$ ✓

b) Which term in the following sequence is the first that is greater than 1000 000:

9, 13.5, 20.25, …

$\times 3/2$ $\times 3/2$

GP

$T_n = a \times r^{n-1}$

$1000000 < 9 \times (3/2)^{n-1}$ ✓ (solve)

$n > 29.65$

$\therefore$ The 30th term is the 1st greater than 1000 000 ✓

9. [6 marks: 4, 2]

a) The first term of a geometric sequence is 2, and the fifth term is 162. Find the sum of the first 10 terms.

$t_n = a \times r^{n-1}$

$t_1 = 2 \quad t_5 = 162$

$2 = a \times r^0$ and $162 = a \times r^4$ (solve)

$a = 2$ ✓
$r = \pm 3$ ✓

$S_{10} = \frac{a(r^n - 1)}{r-1} = \frac{2(3^{10}-1)}{3-1} = 59048$ ✓

OR

$S_{10} = \frac{2((-3)^{10}-1)}{-3-1} = -29524$ ✓

b) If -1.1 , x , -4.4 are consecutive terms of a geometric sequence, find x.

GP $\therefore \frac{-4.4}{x} = \frac{x}{-1.1}$

working ✓

$x = \pm \frac{11}{5}$

$= \pm 2.2$ ✓

10. [3 marks]

A hiker sets out on a 100km hike. She walks 36 km on the first day and $\frac{2}{3}$ that distance on the second. Every day thereafter she walks $\frac{2}{3}$ of the distance she walked on the day before.

Will the hiker cover the distance of 100 km to complete the walk and if so, on what day will she complete the task?

GP

$a = 36$
$r = 2/3$ ✓
$S_n = 100$

$\therefore S_n = \frac{a(r^n - 1)}{r-1}$

$100 = \frac{36(2/3^n - 1)}{2/3 - 1}$ (solve) ✓

$n = 6.4$

36, 20, 13.33

$\therefore$ During the 7th day ✓

11. **[6 marks: 2, 1, 1, 2]**
Drilling tests show that in sinking a well, the distance drilled each hour decreases by 10%.
A depth of 20 metres is drilled in the first hour.

GP

a) Find how much is drilled in the second and third hours.

$t_1 = 20$

$t_2 = 20 \times 0.9^1 = 18\text{m}$ ✓

$t_3 = 20 \times 0.9^2 = 16.2\text{m}$ ✓

b) Explain why the distances drilled each hour will form a geometric progression.

$\frac{18}{20} = 0.9$

$\frac{16.2}{18} = 0.9$

Successive terms have a common ratio ✓

c) Find the distance drilled in the 10th hour, correct to the **nearest centimetre.**

$t_{10} = 20 \times 0.9^9 = 7.7484$

$\therefore 7.75\text{m}$ ✓

d) How long will it take to drill a depth of 100 metres? (**answer to the nearest minute**)

$S_n = 100$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$100 = \frac{20(0.9^n - 1)}{0.9 - 1}$ ✓ (solve)

$n = 6.57881$ hours.

6 h 35 min ✓

Yr 1 Rent \$300 /week
2 Rent \$320 /week

12. **[7 marks: 2, 2, 3]**
Kaya rents her house out at \$300 per week.
Each year she increases the weekly rent by \$20. AP

a) Find a recursive rule for the weekly rent after *n* years.

$$T_n = T_{n-1} + 20$$
$$T_1 = 300$$

b) How long will it take for the weekly rent to double?

$T_n = 600$ General Rule

$$T_n = 300 + 20n$$
$$600 = 300 + 20n$$
$$300 = 20n$$

$n = 15$ years

c) How many **years** would it take for the total rent to be collect to exceed \$200 000?

Yr 1 Rent $= 300 \times 52 = 15600$
2 Rent $= 320 \times 52 = 16640$
3 Rent $= 340 \times 52 = 17680$

$d = 1040$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$200000 = \frac{n}{2}(2(300) + (n-1)(1040))$$ (solve)

$n > 19.8$

By the end of the 20^{th} year.

~ **END OF TEST** ~