Name: MARKING GUIDE

Year 11 Mathematics Specialist
Test 3 – 2015

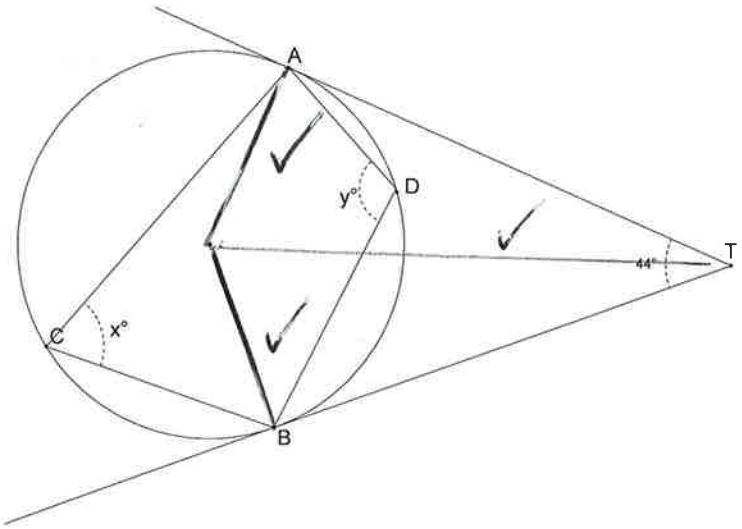
/51	%

Time allowed: 55 minutes

This test is Resource Rich

1. [7 marks]

Two tangents are drawn from point T to points A and B on a circle as shown in the diagram. Given the angle between the tangents is 44^0 calculate the size of $\angle ADB$ and $\angle ACB$ showing proof of each result.



$\triangle ATO \equiv \triangle BTO$ (RHS)

$\angle ATO = \angle BTO = 22^\circ$ (congruent $\triangle$s) ✓

$\angle AOT = \angle BOT = 68^\circ$ (right $\triangle$s)

$\angle AOB = 68^\circ + 68^\circ = 136^\circ$ (adjacent $\angle$s) ✓

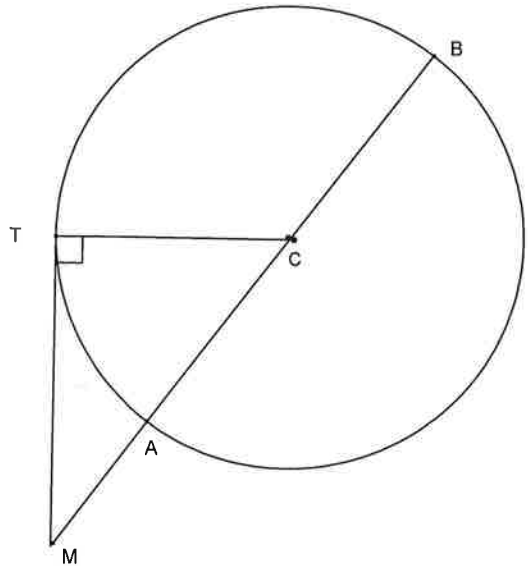
$\angle ACB\ (x^\circ) = 68^\circ$ ($\angle$ at circumference on arc AOB) ✓

$\angle ADB\ (y^\circ) = 112^\circ$ (opposite $\angle$s of cyclic quadrilateral) ✓

2. [8 marks]

Consider point M external to the circumference of the circle centre C. A secant is drawn from M passing through the centre C also intersecting the circumference of the circle at 2 points A and B. A line is also drawn from M to the circle at a point of tangency T.

Prove that $\overline{TM}^2 = \overline{AM} \times \overline{MB}$.



IN $\triangle MTC$: $\bar{MC}^2 = \bar{TM}^2 + \bar{CT}^2$ (1) (Pythagoras) ✓

$\bar{MC}^2 = (\bar{MA} + \bar{AC})^2$ ✓

$= \bar{AM}^2 + 2\bar{AC} \times \bar{AM} + \bar{AC}^2$ ✓

$= \bar{AM}^2 + \bar{AB} \times \bar{AM} + \bar{AC}^2$ ($\bar{AB} = 2\bar{AC}$ (radius))

$= \bar{AM}(\bar{MA} + \bar{AB}) + \bar{AC}^2$ ✓ ✓

$= \bar{AM}(\bar{MB}) + \bar{AC}^2$ (2)

Equating RHS of (1) = RHS of (2) ✓

$\bar{TM}^2 + \bar{CT}^2 = \bar{AM}(\bar{MB}) + \bar{AC}^2$ ✓

($\bar{AC} = \bar{CT}$ = radius)

$TM^2 = \bar{AM}(\bar{MB})$ ✓

3. [7 marks]

A group of 67 Year 9 students responded to a survey stating they owned one or more of the following items.

A: An internet connected mobile phone
B: A tablet or iPad
C: A laptop computer

The following information was recorded using set notation
n(A) = 37, n(B) = 28 and n(C) = 32 and the following notes were made.

There were as many students who owned all three as those who owned a phone and iPad but not a laptop.

There were as many students who owned all three as those who owned a laptop and iPad but not a phone.

There were twice as many students who owned a phone and laptop but not an iPad as those who owned all three.

Calculate how many students owned all three devices.
Calculate how many students owned only one of these devices.

$n(\overline{A \cup B \cup C}) = 0$

Let $n(A \cap B \cap C) = x$ ✓

then $n(A \cap B \cap \bar{C}) = n(B \cap C \cap \bar{A}) = x$ ✓

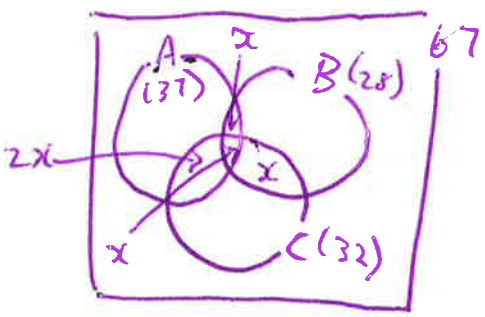
and $n(A \cap C \cap \bar{B}) = 2x$ ✓

$n(A) + n(B) + n(C) - n(A \cup B \cup C) = 97 - 67 = 5x$ ✓

$x = 6$

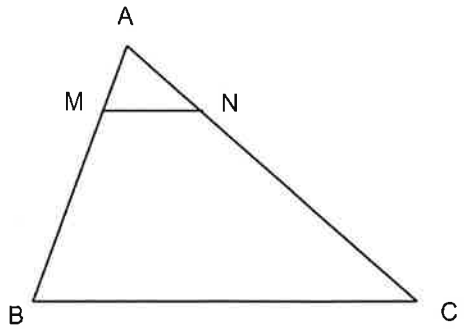
$n(A \cap B \cap C) = 6$ ✓

$n(\text{One only}) = 67 - 5x = 37$ ✓



4. [5 marks]

In ΔABC, the points M and N divide the sides AB and AC respectively in the ratio 1 : 3. Let AB = $\boldsymbol{u}$ and AC = $\boldsymbol{v}$. Find **BC** and **MN** in terms of $\boldsymbol{u}$ and $\underset{\sim}{v}$, and hence prove that **BC** = 4**MN**.



$\vec{BC} = \vec{BA} + \vec{AC}$

$= -\underset{\sim}{u} + \underset{\sim}{v}$ ✓

Since $AM : MB = 1 : 3$

$\Rightarrow AM = \frac{1}{4}AB = \frac{1}{4}\underset{\sim}{u}$ ✓

Also $AN : NC = 1 : 3$

$\Rightarrow AN = \frac{1}{4}AC = \frac{1}{4}\underset{\sim}{v}$ ✓

$MN = MA + AN$

$\Rightarrow MN = -\frac{1}{4}\underset{\sim}{u} + \frac{1}{4}\underset{\sim}{v}$

$= \frac{1}{4}(\underset{\sim}{v} - \underset{\sim}{u})$

$= \frac{1}{4}BC$ ✓

Hence $BC = 4MN$ ✓

5. [4 marks]

State whether each of the following are true (T) or false (F):

i. The contrapositive of a true statement is always true. T

ii. The converse of a true statement is always true. F

iii. The "equivalent" symbol used in proofs in Mathematics is $\Leftrightarrow$ T

iv. The converse to the statement, "A triangle is a polygon."
is "A polygon is a triangle." T

6. [12 marks: 7, 4, 1]

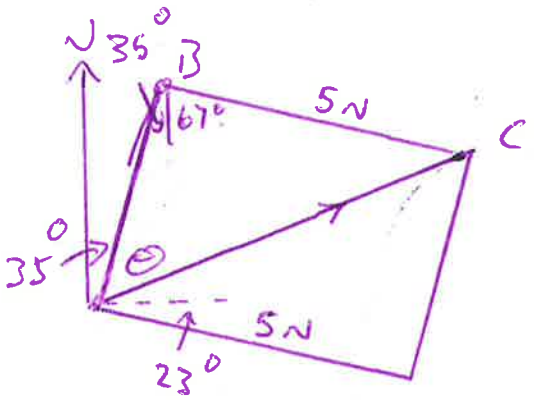
Two forces act on an object in a flat plane. $\mathbf{F}_1$ has a magnitude of 4 newtons and acts on a bearing of 035^0 and $\mathbf{F}_2$ with a magnitude of 5 newtons acts on a bearing 113^0.

(a) Use the triangle rule to calculate the magnitude and direction of the resultant force to an accuracy of two (2) decimal places.

(b) Calculate the **i** and **j** components of $\mathbf{F}_1$ and $\mathbf{F}_2$ given the unit vector **i** is on a bearing 0^0 T and the unit vector **j** is on a bearing 90^0 T

(c) Evaluate $\mathbf{F}_1 + \mathbf{F}_2$ in **i** and **j** component form.

a)



✓

In $\triangle ABC$

$AC^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \cos 102^\circ$ ✓ ✓

$AC \approx 7.02$ N ✓

$\frac{7.02}{\sin 102} = \frac{5}{\sin \theta}$

$\theta = 44.14^\circ$ ✓

$\therefore$ Bearing $35 + 44.14^\circ = 079.14^\circ$ ✓

b) $F_1 = 4\cos 55^\circ \underset{\sim}{i} + 4 \sin 55^\circ \underset{\sim}{j}$

$= 2.29\underset{\sim}{i} + 3.28\underset{\sim}{j}$

$F_2 = 5\cos(-23)^\circ \underset{\sim}{i} + 5\sin(-23)^\circ \underset{\sim}{j}$

$= 4.60\underset{\sim}{i} - 1.95\underset{\sim}{j}$ ✓ ✓

c) $F_1 + F_2 = 6.90\underset{\sim}{i} + 1.32\underset{\sim}{j}$ ✓

7. [3 marks]

Use proof by contradiction to show that a triangle with sides 8 cm, 15 cm and 17 cm is right angled.

Assume not right

$\therefore 8^2 + 15^2 \neq 17^2$ ✓

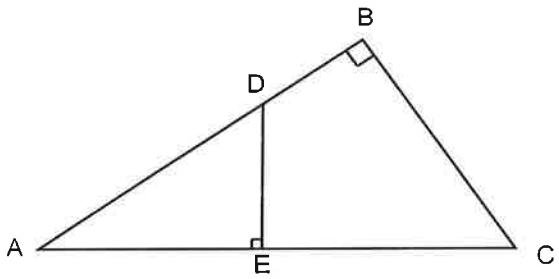
$64 + 225 \neq 289$

$289 \neq 289$ ✓

Hence 8, 15, 17 triangle is right ✓

8. [5 marks]

In Triangle ABC below, AD = 13 cm, DB = 5 cm, AE = 9 cm and DE = 5 cm.
Prove that BC = 10 cm.



$\angle ABC = \angle AED$ - GIVEN ✓

$\angle BAC = \angle EAD$ - Shared angle ✓

$\therefore \angle ACB = \angle ADE$

hence $\triangle ABC \sim \triangle AED$ ✓

$\therefore \frac{AB}{AE} = \frac{BC}{ED} = \frac{18}{9} = \frac{BC}{5}$ ✓

hence $BC = 10$ cm ✓

END OF TEST