ADJUSTMENTS

Student Name ______________________

Eastern Goldfields College
Mathematics Applications U3&4 2016
Test 2 – Calculator Free Section

Working Time: 25 minutes	Total Marks: 26 marks

Question 1 (6 marks: 2, 2, 2)

For the following sequences determine which are arithmetic, geometric or neither.
Provide a reason to support your answer.

a) 1, 2.5, 4, 5.5, …

AP constant difference of 1.5

b) 5, -5, 5, -5, 5, -5, …

GP constant ratio of -1

c) 2, 1, 2, 1, 2, 1 …

Neither – no common ratio or constant difference

Question 2 (11 marks: 3, 3, 3, 2)

a) A geometric sequence has $T_3 = 4$ and $T_6 = 32$. (3 marks: 2, 1)

i) Determine the recursive rule.

$T_{n+1} = T_n \times 2$ or $2T_n$ $T_1 = 1$

ii) Calculate the 5th term.

$T_5 = 16$

b) An arithmetic sequence has $T_3 = -5$ and $T_6 = 4$. (3 marks: 2, 1)

i) Determine the recursive rule.

$T_{n+1} = T_n + 3$ $T_1 = -11$

ii) Calculate the 5th term.

$T_5 = 1$

c) For the following sequence determine the recursive rule and term and T_7. (3 marks)

T_1	T_2	T_3	T_4	T_5
4	-8	16	-32	64

$T_{n+1} = -2T_n$ or $T_n \times -2$ $T_1 = 4$

$T_7 = 256$

d) The Lucas sequence is defined by $L_{n+1} = L_n + L_{n-1}$ with $L_5 = 11$ and $L_6 = 18$. Determine the first 4 terms in the sequence. (2 marks)

$\underbrace{1, 3, 4, 7}_{1^{st}\ 4}, 11, 18$

Question 3 (2 marks)

Renee bought a pair of dogs, and at the end of the year decides to breed them. If the dogs have three puppies every year, find how many dogs Renee will own in 5 years.

$2 + 3 \times 4$

$= 14$ dogs

Question 4 (5 marks: 2, 3)

The nth term of a sequence is given by the rule $T_n = 3^{n-1} + 5$

a) Find the first three terms in the sequence.

$T_1 = 6 \quad T_2 = 8 \quad T_3 = 14$ ✓✓ $\quad 6, 8, 14.$

$5, 8, 14.$

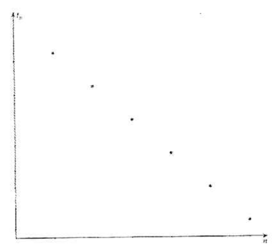
b) Find the first order recurrence relation that defines the sequence.

$T_{n+1} = 3T_n - 10$ ✓ ✓ $\quad T_1 = 6$ ✓ $\quad 2T_n - 2 \quad T_1 = 5$

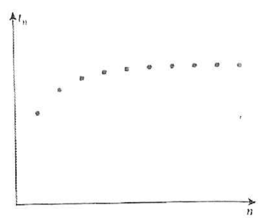
Question 5 (2 marks)

Match each of the following recursive rules with their respective graph.

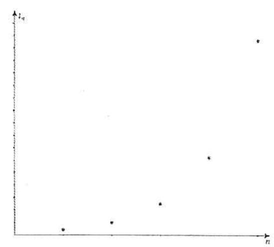
a) $T_{n+1} = T_n - 4;\ T_1 = 22$ b) $T_{n+1} = 2.5T_n;\ T_1 = 2$ c) $T_{n+1} = 0.5T_n + 3;\ T_1 = 4$



(a)



(c)



(b)

✓✓ -1 per error.

Student Name ____________

Eastern Goldfields College

Mathematics Applications U3&4 2016

Test 2 – Calculator Assumed Section

35

Working Time: 35 minutes	Total Marks: 34 marks

include Q5.

Question 1 [9 marks – 3, 3, 3]

The first and second terms of a sequence are 2 and 6 respectively.

(a) If these terms form part of a geometric sequence

(i) list the next two terms,

$2, 6, 18, 54 \quad T_3 = 18 \quad T_4 = 54$ ✓

(3)

(ii) state a recursive rule for the sequence.

$T_{n+1} = T_n \times 3$ ✓ $T_1 = 2$ ✓ or $T_{n+1} = 3T_n \quad T_1 = 2$

(b) If the two terms form part of an arithmetic sequence, find

(i) the fifth term of the sequence,

$T_5 = 18$ ✓

(ii) which term of the sequence is the first to exceed 100.

$T_n = 2 + (n-1)d$

$100 = 2 + (n-1)4$ solve

$100 = 4n - 2$

$n = 25.5$ ✓ $\therefore T_{26}$ ✓

(c) If the recursive rule for the sequence is given by $T_n = T_{n-1} - T_{n-2}$, find

(i) T_4, $= -2$ ✓ $\quad 2, 6, 4, -2, -6, -4, 2$ ✓

(ii) the smallest value of n, $n > 1$, for which $T_n = T_1$.

$n = 7$ ✓ ✓✓

Question 3 (6 marks – 3, 2, 1) 2 $2\frac{1}{2}$ 2

Elsa is negotiating with her mother as to how much pocket money she will get. Elsa suggests starting with $50 in the first month and increasing this by $5 every month.

a) With this scheme, how much pocket money will Elsa receive 12 months from the start?

50 55

1 2 3 4 5 . . . .

$50 + 11(5) = \$105$ ✓ ✓

Elsa's mother says that increasing the amount by 5% each month is better for Elsa in the long run.

b) Use the table below to show how much pocket money Elsa will receive with her scheme and her mother's scheme for the first 5 months of the year.

	Month 1	Month 2	Month 3	Month 4	Month 5
Elsa's scheme	50	55	60	65	70
Mother's scheme	50	52.50	55.13	57.88	60.78

✓

c) Is Elsa's mother correct? Justify your solution mathematically.

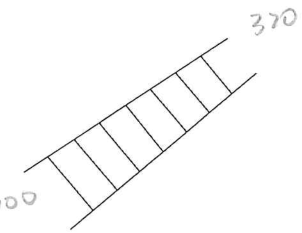
Yes ✓ – after 28 mths her method overtakes Elsa's scheme 185 v 186.67 ✓

d) If the amount of pocket money Elsa will be paid is capped $120/month, does this effect which scheme is better? Justify your solution.

Yes ✓ as Elsa will reach this after 15 mths & her Mum's scheme won't achieve this until 19 mths. ✓

Question 4 (5 marks – 2, 2, 1) 3 ?

A ladder has 21 rungs and from the bottom to the top each rung is shorter than the one before it by a constant amount. The bottom rung is 400 mm long and the top rung is 320 mm.



a) How much shorter is each rung than the rung below it?

$80 \div 20 = 4$ mm ✓ ✓

b) Give a recursive rule for calculating the length (Tn) of the nth rung from the rung below it.

$T_n = T_{n-1} - 4$ ✓ $T_1 = 400$ ✓

c) Give a non-recursive formula for calculating the length of any rung.

$T_n = a + (n-1)d$

$T_n = 400 + (n-1)(-4)$

$= 400 - 4n + 4$

$= 404 - 4n$ ✓

Question 5 4 (10 marks: 2, 4, 2, 2) 2 2 check sol^{ns}

Angelena has built a new school. When the school first opens she has 230 students across the school with the population of the school set to increase by 24% each year.

a) Assuming no students leave the school, state the recursive rule to describe the number of students at the end of each year.

$T_{n+1} = T_n \times 1.24$ ✓ $T_1 = 230$ ✓ $T_0 = 230$

For the first two years no students leave the school for other schools and no students are old enough to graduate yet. However, after the initial first two years, 50 students leave the school at the end of each year.

b) Calculate the number of students at the end of first 2 years.

$T_0 = 230$

$230 \times 1.24^2 = 353.6$ ✓

~ 354 ✓

Round up to the next whole no.

c) Calculate the number of students at the end of the 4[th] year assuming the population continues to grow at 24% each year.

$354 \times 1.24 - 50 = 388.96$ ✓

$388.96 \times 1.24 - 50 = 432.31$

∴ 432 students ✓

d) Explain why Angelena would not be worried about losing 50 students per year but would be very concerned if she was losing 100 students per year.

~~105~~

Losing 50 pop keeps rising as 24% of total will always be more than ✓ 50 (in fact will increase)

BUT losing 100/yr would mean ✓ student pop^n would decrease each yr.

9 years after Angelena opened her new school, Brad opened an even better school very nearby and Angelena's students begin flocking to Brad's new school. Her student population no longer increases but reduces by 24% each year.

e) Describe what happens to Angelana's school over the next few years.

After 9 yrs pop 865

$865 \times 0.76 = 657$ after 1 yr.

student nos. will continue to drop.

at decreasing rate.

657	500	380	289	219
1 yr	2 yrs	3 yrs	4 yrs	5 yrs

Question ~~6~~ 5 (4 marks – 3, 1)
The numbers 5, x and 49 are the first three terms of the sequence defined by the first order recurrence relation $T_{n+1} = rT_n + 1;\ T_1 = 5$

a) Find the values of r and x, given that $x > 0$.

$5 \times r + 1 = x$
$x \times r + 1 = 49$

$$\boxed{\begin{aligned} 5r+1 &= x \\ xr+1 &= 49 \end{aligned}}$$

*

?

difficult

include

$(5r+1)r+1 = 49$ solve.

$5r^2 + r = 48$

$r = 3$ $x = 16$

$5 \times 3 = 15$

b) Find the fourth term in the sequence.

$T_4 = 148$

END OF TEST