



APPLECROSS
SENIOR HIGH SCHOOL

Year 12 Mathematics TEST 3 APPLICATIONS UNIT 4

TERM 3, 2023

Test Date: Thursday 27 July

Name: Mark Ingkey

All working is to be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily so part marks may be awarded if the answer is incorrect. For any question worth more than 2 marks valid working or justification must be shown to be awarded full marks.

Resource Rich

Working Time: 50 minutes

Total Marks: 47

Question 1

(3 marks)

The seasonal indices for the sale of ice-cream for 2012-2014 first 3 quarters are:

Q1 : 115%

Q2 : 82%

Q3 : 124%

a) Determine the seasonal index Q4.

(1 mark)

79% ✓

b) What does the seasonal index for Q2 indicate about the sales of ice-cream?

(2 marks)

Sales 18% below average
✓ ✓

Question 2

(7 marks)

The table below shows the nightly revenue collected over a three-week period at a seafood restaurant in Bull Creek.

t	Week	Day	Revenue (\$)	7PMA
1	1	Monday	2530	-
2		Tuesday	3120	-
3		Wednesday	A	-
4		Thursday	4150	3410
5		Friday	5220	3430
6		Saturday	5100	3440
7		Sunday	2280	3480
8	2	Monday	2670	3500
9		Tuesday	3190	3580
10		Wednesday	1750	3640
11		Thursday	4290	3680
12		Friday	5780	B
13		Saturday	5520	3670
14		Sunday	2560	3690
15	3	Monday	2740	3700
16		Tuesday	3050	3740
17		Wednesday	1890	3780
18		Thursday	4360	3730
19		Friday	6060	-
20		Saturday	5800	-
21		Sunday	2210	-

- (a) Determine the values of A and B. Show clear working.

$$\frac{2530 + 3120 + A + 4150 + 5220 + 5100 + 2280}{7} = 3410 \quad (4 \text{ marks})$$

$$\therefore A = 1470 \quad \checkmark$$

$$B = \frac{3190 + 1750 + 4290 + 5780 + 5520 + 2560 + 2740}{7}$$

$$\therefore B = 3690 \quad \checkmark$$

- (b) Why would it make sense to consider a seven-point moving average to smooth this data?

Sensible

(2 marks)

Data follows a 7pt cycle.

✓✓

- (c) Does the smoothed time-series plot indicate increasing, decreasing or steady revenue for the restaurant?

(1 mark)

Increasing ✓

Question 3 [3 marks]

A philanthropist is considering funding 8 scholarships to a local college. Each student in receipt of a scholarship would receive a payment of \$6 000 on the yearly anniversary of the creation of the fund. Determine, to the nearest \$100, the initial sum of money that should be deposited in an account paying interest at a rate of 3.6% compounded monthly to create a perpetuity to fund all 8 scholarships.

(3 marks)

$$I = 3.6\%$$

$$CY = 12$$

$$PY = 1$$

$$PMT = 48000$$

$$Q = Pr$$

$$48000 = P \times \frac{0.036}{12}$$

$$P = \$16\,000\,000.$$

Question 4

(15 marks)

A local delicatessen owner recorded the number of ice-creams sold over a three-week period.

The information collected is displayed in the following table:

	Day (d)	Sales	Weekly Averages	Seasonal Proportions	Deseasonalise d Figures
Monday	1	210	158	132.9%	162
Tuesday	2	230		145.6%	151
Wednesday	3	100		63.3%	160
Thursday	4	90		57.0%	D
Friday	5	160		101.3%	156
Monday	6	190	B	128.4%	147
Tuesday	7	230		155.4%	151
Wednesday	8	90		60.8%	144
Thursday	9	80		54.1%	150
Friday	10	150		101.4%	146
Monday	11	180	142	126.8%	139
Tuesday	12	220		154.9%	145
Wednesday	13	A		C	144
Thursday	14	70		49.3%	131
Friday	15	150		105.6%	146

- (a) Determine the values of **A**, **B** and **C** (correct to 1 decimal place). Show clear working.

$$A = \frac{180 + 220 + A + 70 + 150}{5} = 142 \quad \checkmark \quad (6 \text{ marks})$$

$$\therefore A = 90 \quad \checkmark$$

$$B = \frac{190 + 220 + 90 + 80 + 150}{5} = B \quad \checkmark$$

$$\therefore B = 148 \quad \checkmark$$

$$C = \frac{A}{142} \quad \checkmark$$

$$= \frac{90}{142}$$

$$\therefore C = 63.4\% \quad \checkmark$$

- (b) Use the 'average percentage' method to calculate the seasonal index for **Thursdays** and explain what this index represents.

$$\frac{50.7 + 54.1 + 49.3}{3} = 53.46\% \text{ or } 53.5\% \text{ (2 marks)}$$

or 0.535

Icecreams sales on Thursdays are 46.5% lower than the average daily sales ✓

- (c) Determine the value of **D**. Show clear working.

$$D = \frac{90}{0.535} \quad \checkmark$$

(2 marks)

$$\therefore D = 168 \quad \checkmark$$

- (d) The equation of the least-squares regression line used to forecast the seasonally adjusted number of ice-creams is as follows:

$$\text{Seasonally adjusted number of ice-creams} = 162.42 - 1.64d$$

With reference to this line, describe the trend in the number of ice-cream sales over time.

Negative secular trend ✓ (2 marks)

Line has a negative gradient ✓

- (e) If the seasonal index for Friday is 1.028, predict the **actual** number of ice-cream sales for **Friday of Week 4**.

Show clear working.

(3 marks)

$$d = 20 \quad \checkmark$$

$$\begin{aligned} \text{SA value} &= 162.42 - 1.64(20) \\ &= 129.62 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{Predicted value} &= 129.62 \times 1.028 \\ &= 133 \text{ ICECREAMS} \end{aligned}$$

✓

Question 5**(7 marks)**

An annuity compounds interest annually and its value after n withdrawals can be modelled using the recurrence relation:

$$T_{n+1} = 1.065T_n - 42\,500, \quad T_0 = 485\,000.$$

(a) Use the relation to state

(i) the annual percentage interest rate.

(1 mark)

6.5% ✓

(ii) the annual withdrawal from the annuity.

(1 mark)

\$42 500 ✓

(b) Calculate the balance of the annuity, to the nearest dollar, after 3 withdrawals.

(1 mark)

$$T_3 = \$449\,889 \quad \checkmark$$

(c) The annuity is closed after 12 withdrawals. Calculate the total interest paid by the annuity up to this time.

(2 marks)

$$T_{12} = 294\,356.44$$

$$T_{12} - (T_0 - 12 \times 42\,500) \quad \checkmark$$

$$= \$319\,356.44 \quad \checkmark$$

(d) From the outset, the annual withdrawal could have been reduced so that the annuity becomes a perpetuity. Briefly explain what a perpetuity is and determine the withdrawal required to make this occur.

Perpetuity is an annuity in which the withdrawal equals the interest paid each time period so the withdrawals can be made forever. (2 marks) ✓

$$\text{Withdrawal} = \$31\,325 \quad \checkmark$$

Question 6

(8 marks)

A customer in a store is offered a reducible interest loan that attracts interest of 8.4% compounded monthly to purchase a \$2 545.39 computer and accessories. The monthly loan repayment is \$151.

The customer set up the spreadsheet below to analyse the loan, rounding the displayed figures to the nearest cent.

Month	Balance at start of month (\$)	Interest for month	Repayment	Balance at end of month (\$)
1	2 545.39	17.82	151.00	2 412.21
2	2 412.21	16.89	151.00	2 278.09
3	2 278.09	15.95	151.00	2 143.04

(a) Complete row 3 of the spreadsheet.

(3 marks)

See above

(b) Determine a simplified recurrence relation for B_n , the loan balance at the start of month n .

(2 marks)

$$T_{n+1} = T_n \left(1 + \frac{0.084}{12}\right) - 151, \quad T_1 = 2545.39$$

OR
$$T_{n+1} = T_n (1.007) - 151, \quad T_1 = 2545.39$$

(c) State the number of repayments required to pay off the loan.

(1 mark)

$$N \approx 18.0002$$

∴ 19 payments ✓

(d) Determine the total amount of interest paid over the life of the loan.

(2 marks)

$$PMT = 151$$

$$PMT = 19$$

$$I = 8.4$$

$$PV = -2545.39$$

$$PMT = 151$$

$$PMT = 12$$

$$CY = 12$$

$$\sum Int = \$172.61$$

Question 5

(6 marks)

An equestrian show is held in Bendigo each year over the Queen's Birthday long weekend, starting on the Friday and finishing on the public holiday Monday.

The attendance figures for four consecutive years are shown in the following table:

t	Year	Day	Attendance	4PCMA
1	2014	Friday	9100	
2		Saturday	14980	
3		Sunday	16240	14435
4		Monday	16840	14610
5	2015	Friday	10260	14705
6		Saturday	15220	14970
7		Sunday	16760	15065
8		Monday	18440	14930
9	2016	Friday	9420	14870
10		Saturday	14980	14755
11		Sunday	A	B
12		Monday	17760	14890
13	2016	Friday	10140	15045
14		Saturday	15300	15270
15		Sunday	17440	
16		Monday	18640	

- (a) Explain why the attendance on Fridays always seems to be much lower than on other days.

Most people work on Fridays ✓

(1 mark)

- (b) Express A in terms of B. Show clear working.

$$\frac{9420}{2} + \frac{14980 + A + 17760 + 10140}{2} = B$$

(3 marks)

$$A + 42520 = 4B$$

$$A = 4B - 42520$$

- (c) Should the long-term trend in attendance figures be cause for concern for the organisers of the show? Justify your answer.

(2 marks)

NO ✓

Positive secular trend ✓