MATHEMATICS METHODS **TEST 1** **RESOURCE FREE**

TIME ALLOWED : 20 MIN **TOTAL MARKS : 21**

NAME: ____________________

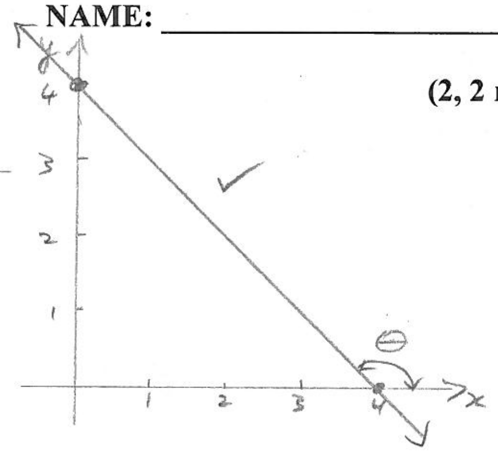
QUESTION 1 **(2, 2 marks)**

For the line $3x+3y=12$

(a) Sketch the line

x	0	4
y	4	0

✓



(b) Mark the angle of inclination the line makes with the x axis. Determine this angle. Justify your answer.

gradient of line $= -\frac{4}{4} = -1$

$\tan\theta = -1$

$\therefore \theta = 180 - 45$

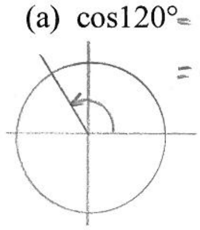
$\theta = 135^\circ$ ✓✓

QUESTION 2 **(1, 1 marks)**

Express the following in terms of angles between 0° and 90° and the state their exact value:

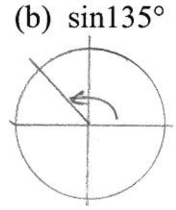
(a) $\cos 120^\circ = -\cos 60^\circ$

$= -\frac{1}{2}$ ✓



(b) $\sin 135^\circ = \sin 45$

$= \frac{1}{\sqrt{2}}$ ✓



QUESTION 3 **(4 marks)**

Find the exact value of $\dfrac{\cos 60^\circ \cos 45^\circ}{\cos 30^\circ}$ expressed with a rational denominator.

$$= \frac{\frac{1}{2} \times \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} = \frac{1}{2\sqrt{2}} \times \frac{2}{\sqrt{3}}$$ ✓✓✓

$$= \frac{1}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{\sqrt{6}}{6}$$ ✓

QUESTION 4 **(2, 3 marks)**

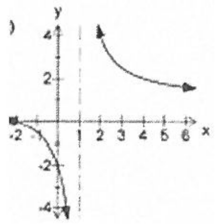
(a) Shown below are four rules and their corresponding graphs. For each, state whether or not it satisfies the conditions to be a function.

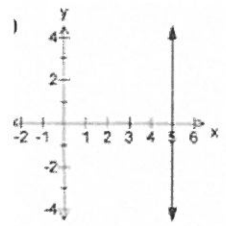
(i) $y = \dfrac{3}{x-1} + 1$ Yes/No

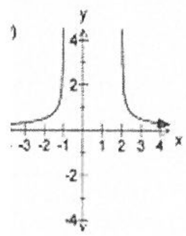
(ii) $x = 5$ Yes/No ✓✓

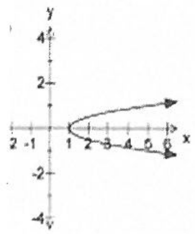
(iii) $\dfrac{1}{\sqrt{x^2 - x - 2}}$ Yes/No

(iv) $x - 4y^2 = 1$ Yes/No

(i) 

(ii) 

(iii) 

(iv) 

(b) For each function, state its domain and range.

RULE	DOMAIN	RANGE
$y = \dfrac{3}{x-1} + 1$	$x \in \mathbb{R}$ $x \neq 1$	$y \in \mathbb{R}$ $y \neq 1$ ✓
$x = 5$		
$\dfrac{1}{\sqrt{x^2 - x - 2}}$	$x \in \mathbb{R}$ $x < -1$, $x > 2$ ✓	$y \in \mathbb{R}$ $y > 0$ ✓
$x - 4y^2 = 1$		

QUESTION 5 **(3 marks)**

Simplify and express with a rational denominator: $\dfrac{\sqrt{7}-6}{\sqrt{7}+6}$

$$\frac{\sqrt{7}-6}{\sqrt{7}+6} \times \frac{\sqrt{7}-6}{\sqrt{7}-6} = \frac{(\sqrt{7}-6)(\sqrt{7}-6)}{(\sqrt{7}+6)(\sqrt{7}-6)}$$ ✓

$$= \frac{7-12\sqrt{7}+36}{7-36}$$ ✓

$$= \frac{43-12\sqrt{7}}{-29}$$ ✓

QUESTION 6 **(2 marks)**

Write 255^0 as an angle in radians as a simplified fraction in terms of π

π radians = 180°

$\frac{\pi}{180} = 1^\circ$

$\frac{\pi}{180} \times 255$ radians = 255°

$\frac{17\pi}{12}$ radians = 255° ✓✓

QUESTION 7 **(1 mark)**

A function is defined as $f(x) = x^2 + 2x - 8$. Write an expanded expression for $f(2x)$

$f(2x) = (2x)^2 + 2(2x) - 8$

$= 4x^2 + 4x - 8$ ✓

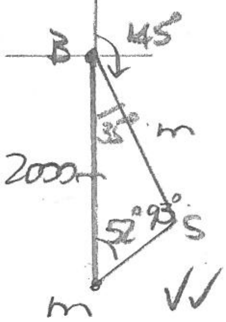
MATHEMATICS METHODS TEST 1 RESOURCE RICH

TIME ALLOWED : 40 MINUTES

TOTAL MARKS : 39 NAME: ______________________

QUESTION 8 (5 marks)

Three towns Brooks River (B), Fryman Mill (M) and Swann Place (S) are to cultivate the triangular piece of land that they immediately surround. M is 2000m due South of B. S is on a bearing of 145° from B and 052° from M. Calculate the area of this piece of land.



$\frac{m}{\sin 52} = \frac{2000}{\sin 93}$

$m = 1578.18\,m$ ✓

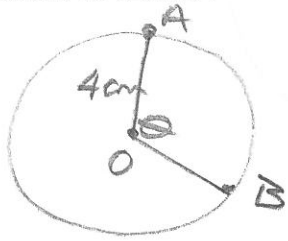
$Area(\Delta) = \frac{1}{2} \times 2000 \times 1578.18 \times \sin 35°$

$= 905209.36$

$= 905209\ m^2$ ✓✓

QUESTION 9 (3,1 marks)

The area of a sector AOB, in a circle centre O and radius 4 cm is $\frac{16\pi}{3}$ cm^2. Find the size of $\angle AOB$:



a. in radians in terms of π.

$Area(sector) = \frac{1}{2} r^2 \theta$ ✓✓

$\frac{16\pi}{3} = \frac{1}{2} \times 4^2 \times \theta \quad \angle AOB = \frac{2\pi}{3}$ radians. ✓

$\Downarrow$

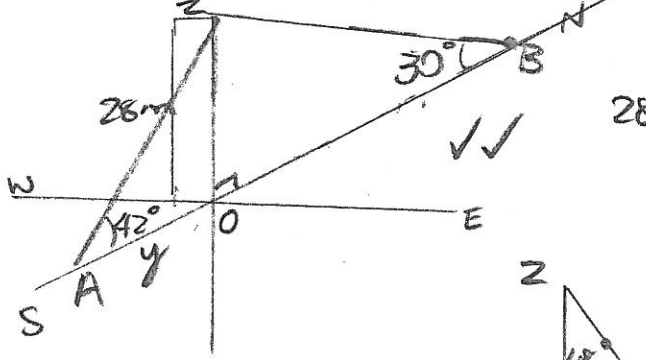
b. in degrees

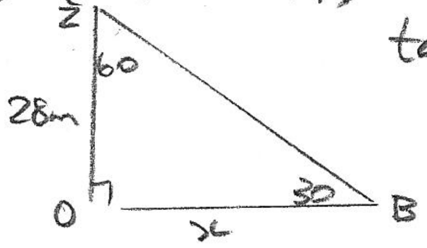
π radians $= 180°$

$\frac{2\pi}{3}$ rad $= 120°$ ✓

QUESTION 10 (6 marks)

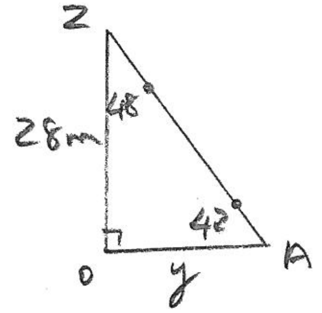
Two observers are on opposite sides of a tower, and collinear with its base. The first finds the angle of elevation of the top of the tower to be $42°$, the second finds it to be $30°$. If the tower is 28m high, how far apart are the observers? (ans to 1d.p)





$\tan 60 = \frac{z}{28}$

$z = 48.50m$



$\tan 48° = \frac{y}{28}$

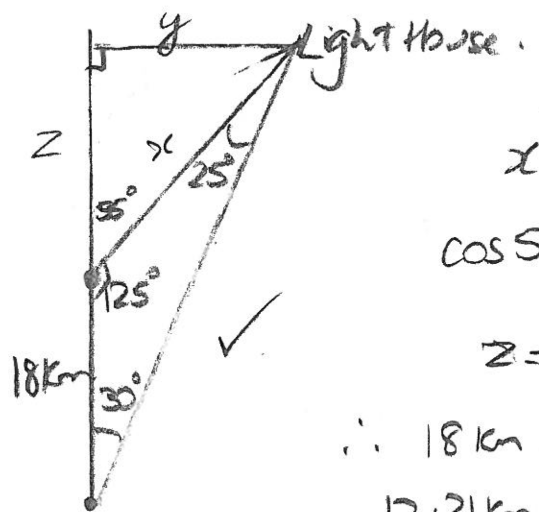
$y = 31.10m$

$\therefore AB = 48.5 + 31.1$

$= 79.6m$

QUESTION 11 (8 marks)

At 7pm a ship heading North and travelling at 18 km/hr observes a lighthouse on a bearing of $030°$. At 8pm the lighthouse is observed on a bearing of $055°$. At what time will the ship be due West of the lighthouse – and how close will it be at this time?



$\frac{x}{\sin 30} = \frac{18}{\sin 25}$

$x = 21.30$

$\cos 55° = \frac{z}{21.3}$

$z = 12.21$ Km

$\sin 55 = \frac{y}{21.30}$

$y = 17.45$ Km

$\therefore$ 18 Km : 1hr

12.21 Km : $\frac{12.21}{18} \times 60$

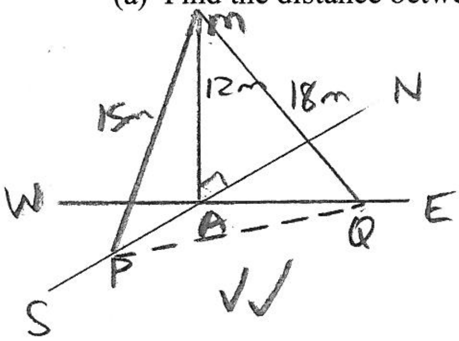
≈ 41 min

At 8.41pm ship will be due west of lighthouse and 17.45km from lighthouse.

QUESTION 12 (5, 2 marks)

A wireless mast is supported by two wires MP and MQ, each attached to it at the point M, 12m from the base A of the mast. Q, P and A are all on level ground, Q being due East and P due South of the mast. If MP and MQ are respectively 15m and 18m long:

(a) Find the distance between P and Q



In $\triangle MAP$, $AP = \sqrt{15^2 - 12^2}$

$= 9m$ ✓

In $\triangle MAQ$,

$AQ = \sqrt{18^2 - 12^2}$

$= 13.4\ m$ ✓

In $\triangle PAQ$

$PQ = \sqrt{9^2 + 13.4^2}$

$= 16.2\ m$ ✓

(b) Find the angle QMP between the wires

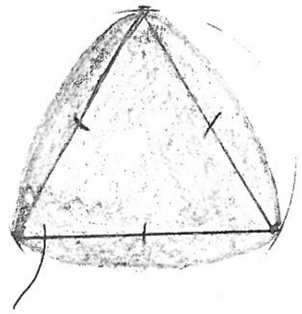
In $\triangle MQP$, $261 = 15^2 + 18^2 - 2 \times 15 \times 18 \times \cos M$

$\cos M = 0.53$

$\angle M = 57.8°$ ✓✓

QUESTION 13 (6 marks)

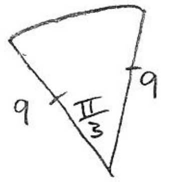
An equilateral triangle of side length 9 cm has circles with centres at each of the vertices drawn to pass through the other two vertices. Find the area common to the three circles.



$60° = \frac{\pi}{3}$

$\theta = \frac{\pi}{3}$

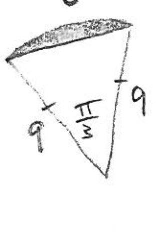
$\text{Area (sector)} = \frac{1}{2} r^2 \theta$



$= \frac{1}{2} \times 9^2 \times \frac{\pi}{3}$

$= 42.4\ cm^2$ ✓

$\text{Area (segment)} = \frac{1}{2} r^2 (\theta - \sin\theta)$



$= \frac{1}{2} \times 9^2 \left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$

$= 7.34\ cm^2$ ✓

$\therefore$ Total Area $= 42.4 + 2 \times 7.34$ ✓✓

$= 57.1\ cm^2$ ✓

QUESTION 14 (3 marks)

The owner of a shop that sells computers calculates that his total weekly profit is given by the rule:

$$\text{Total profit in dollars} = mx - c,$$

where $\$m$ is the profit per computer sold, x is the number of computers sold in the week and $\$c$ is the fixed weekly cost of running the shop.
If he sells ten computers in a week his total profit is \$360.
If he only sells five computers in the week he makes a loss of \$190.

(a) Calculate m and c.

$(10, 360) \quad (5, -190)$

$m = \dfrac{360-(-190)}{10-5}$

$m = \dfrac{550}{5}$

$m = 110$

Profit $= 110x - c$

sub $(10, 360)$

$360 = 110(10) - c$

$360 = 1100 - c$

$c = 1100 - 360$

$c = 740$

✓✓

(b) What is the least number of computers he can sell and still make a profit?

$P = 110x - 740$

$740 = 110x$

$\dfrac{740}{110} = x$

$6.73 = x$

$\therefore x = 7$ computers ✓