Mathematics Methods

Investigation 1

Heron's Rule

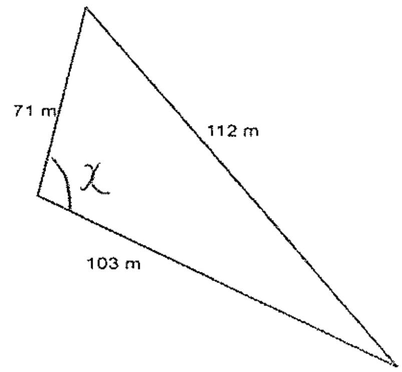
MINDARIE SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS NOW

In-Class Validation **Total marks: 36** **Time allowed: 45 mins**

NAME solutions.

Question 1: [3,3,3 marks]

a) Use Heron's formula to calculate the area of the scalene triangle shown below.
Show your working and give your answer to 2 decimal places.



$$S = \frac{71+112+103}{2}$$

$$= 143$$

$$A = \sqrt{143(143-71)(143-112)(143-103)}$$

$$= \sqrt{143 \times 72 \times 31 \times 40}$$

$$= 3573.10\,m^2$$

b) Use rules of trigonometry to find the area of the triangle in part (a) and to verify the solution found above. Show clearly the mathematics you used.

Use cosine rule to find angle:

$$\cos x = \frac{103^2 + 71^2 - 112^2}{2 \times 71 \times 103}$$

$$x = 77.74$$

$$\text{area} = \tfrac{1}{2} \times 71 \times 103 \times \sin 77.74$$

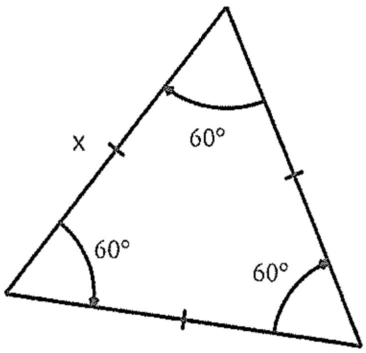
$$= 3573.1\,m^2$$

as required.

N.B may have calculated different angle.

c) This equilateral triangle has the *same area* as the triangle from part (a).
Determine the length of the side lengths, accurate to 2 decimal places.

Show working.



$$\tfrac{1}{2}x^2 \sin 60 = 3573.1$$

$$(x^2 = 8251.72)$$

$$x = 90.84\,m$$

from classpad.

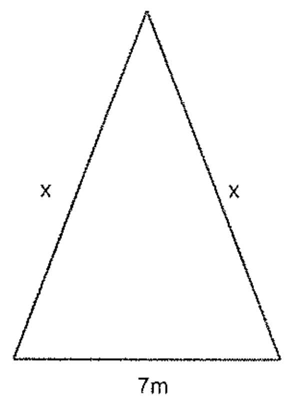
Question 2: [1,3,2 marks]

a) Show working to demonstrate that the semi-perimeter, s, of this triangle can be expressed as $s = x + 3.5$

$$s = \frac{x + x + 7}{2}$$

$$s = \frac{2x + 7}{2}$$

$$s = x + 3.5 \checkmark$$



b) Using Heron's rule, write a simplified algebraic expression for the area of this triangle, in terms of x.

$$A = \sqrt{(x+3.5)(x+3.5-x)(x+3.5-x)(x+3.5-7)} \checkmark$$
$$= \sqrt{(x+3.5)(3.5)(3.5)(x-3.5)}$$
$$= \sqrt{(x^2-12.25)(3.5)(3.5)}$$
$$= \sqrt{12.25x^2 - 150.0625} \checkmark\checkmark$$

c) Using the Solve function on your ClassPad or otherwise, determine the value of x when the area of the triangle is 25.18m².

$$\sqrt{12.25x^2 - 150.0625} = 25.18 \checkmark$$

$$\underline{x = 3.5m} \checkmark$$

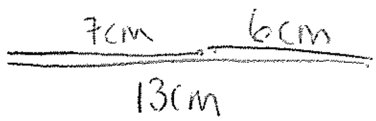
Question 3: [2,1 marks]

a) Use Heron's rule to find the area of a triangle with sides 6cm, 7cm and 13 cm.

$$s = \frac{6+7+13}{2} = 13 \checkmark \qquad A = \sqrt{13(13-6)(13-7)(13-13)} \qquad A = 0 \checkmark$$

b) Why does this solution occur? You may wish to draw a diagram to assist you.

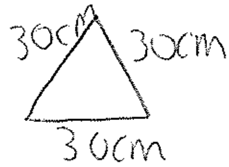
It has no area because the lengths do not form a triangle. ✓



Question 4: [3,2 marks]

a) Find the maximum area possible for a triangle with a perimeter of 90 cm. Show clearly how you did this and use a sketch to illustrate your solution.

Max. area → equilateral triangle



$S = \frac{30+30+30}{2} = 45$

$A = \sqrt{45(45-30)^3}$

$= 389.7 \text{ cm}^2$

b) The rule for the maximum area of a triangle with a semi-perimeter of s is equal to $\sqrt{\frac{s^4}{27}}$.

Show working to verify this rule is correct for the triangle in part (a).

$S = 45$ $\quad \sqrt{\frac{45^4}{27}} = 389.7 \text{ cm}^2$

Question 5: [3 marks]

Determine the dimensions of a triangle with an area of 1 hectare (10 000 m²) that has the smallest perimeter. Show how you did this.

Solve $\sqrt{\frac{S^4}{27}} = 10000$

$S = 227.95 \text{ m}$

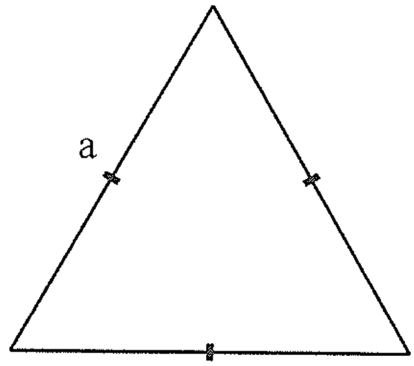
$\therefore a = 151.97 \text{ m}$

An equilateral triangle with lengths of 151.97m

Question 6: [1,2,3,2,2]

a) Given the following equilateral triangle, find an expression for s, in terms of a, the side length..

$S = \frac{3a}{2}$ ✓



b) Substitute this into Heron's rule to find an unsimplified expression for area in terms of a.

$A = \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)^3}$ ✓ ✓ $\underline{\underline{\text{or}}}$ $A = \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)}$ ✓ ✓

c) Show algebraic working to demonstrate that A is given by the expression $A = \sqrt{\frac{3a^4}{16}}$.

$A = \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - \frac{2a}{2}\right)^3}$

$= \sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)^3}$ ✓

$= \sqrt{\frac{3a}{2} \times \frac{a^3}{8}}$ ✓

$= \sqrt{\frac{3a^4}{16}}$ ✓

d) Investigate what happens to the area of an equilateral triangle when the side lengths are doubled. Show working and conclusions below.

Side length $a = 1$:

$A = \sqrt{\frac{3}{16}}$

$= \frac{\sqrt{3}}{4}$

✓ For examples

Side length $a = 2$:

$A = \sqrt{\frac{3 \times 2^4}{16}}$

$= \sqrt{\frac{48}{16}}$

$= \sqrt{3}$

side lengths doubled

$\Rightarrow$ area is 4 times as big.

✓ For conclusion.

e) Prove your findings above using algebra.
(Hint: Think about doubling the lengths in the triangle in part (a))

Replace a with 2a.

$A = \sqrt{\frac{3(2a)^4}{16}}$ ✓ $= \sqrt{3a^4}$

$= \sqrt{\frac{48a^4}{16}}$

which is 4 times ✓ bigger than $\sqrt{\frac{3a^4}{16}}$