Mathematics Applications Unit 3 2020	Year Weighting	Working Time	Total Time	YEAR 12
Test 1	8%	55 min		
	Resources Allowed	Marks	55	
Name: SOLUTIONS	No Calculator No notes	60		

Full working must be shown for all questions (or parts) worth more than 2 marks

Question One   (1, 2, 3, 2 = 8 marks)

The graph shown represents four buildings A, B, C and D with connecting walkways, at a local school.
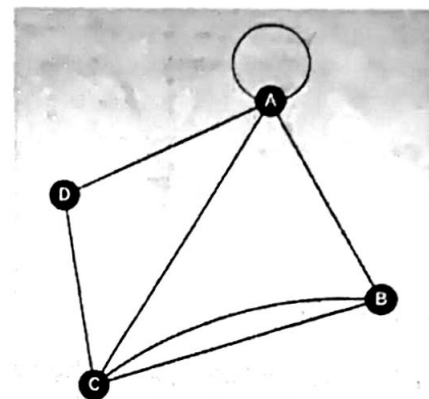
(a) Why is the graph planar?

Can be drawn with no two edges crossing. ✓

(b) Show that the graph satisfies Euler's formula.

$$V + F = E + 2$$
$$4 + 5 = 7 + 2 \checkmark$$
$$9 = 9 \checkmark$$

(c) Construct the adjacency matrix for the graph.

$$
\begin{array}{c}
 \\
A \\
B \\
C \\
D
\end{array}
\begin{array}{cccc}
A & B & C & D
\end{array}
\left[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
1 & 0 & 2 & 0 \\
1 & 2 & 0 & 1 \\
1 & 0 & 1 & 0
\end{array}
\right]
$$
✓✓✓

A student wishes to carry out closed walks of length two from Building A to Building B.

(d) List all his possible walks.

ACB   AAB
ACB   AAB
ABA

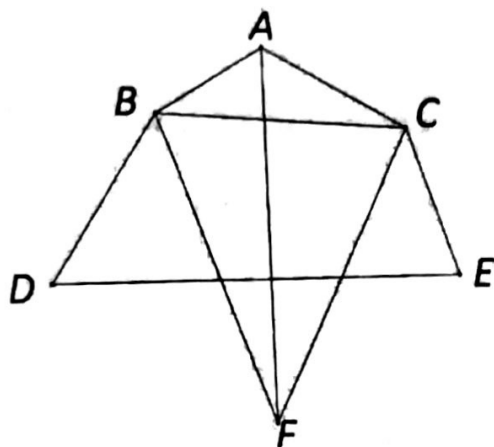✓✓

⑧

Question Two
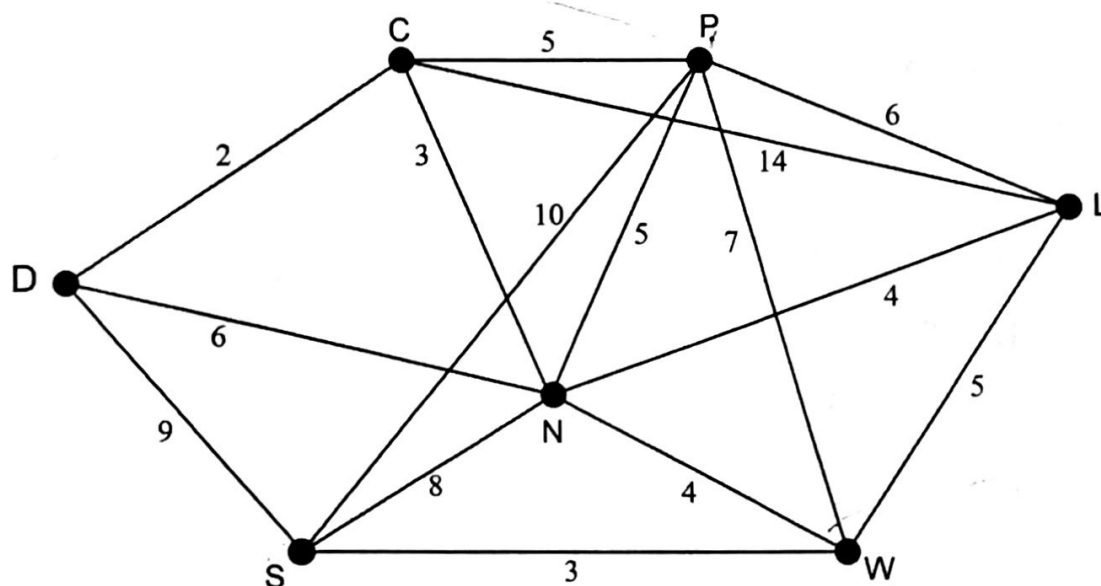
Redraw the following graph as a planar graph.

2

Question Three

(1, 1, 2, 2 = 6 marks)

Each morning Bert leaves the depot (D) to deliver milk to cafés in his area. He visits each café once and has produced a map showing the number of cafés, the roads linking them and the lengths (kilometres) of these roads. His map is represented by the graph below.
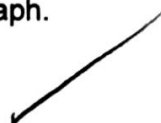

(a) (i) Identify the number of vertices on this graph.

7

(ii) Identify the number of edges on this graph.

15

) Is the graph planar? Justify your answer.

No, cant be drawn without edges crossing.

(b) This graph is described as a simple and connected graph. Explain why this description can be applied.

No loops and multiple edges

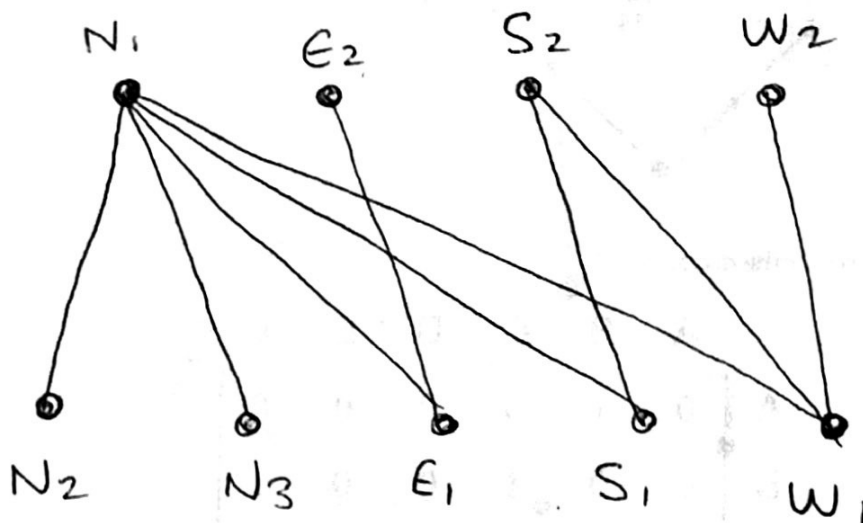Travel possible to all vertices

⑥

Question Four (2, 2, 1 = 5 marks)

In a commercial complex, buildings are named by their position (E.g. North 1 and East 2). North 1 connects with North 2, North 3, East 1, South 1 and West 1. East 1 connects with East 2. South 2 connects with South 1 and West 1. West 1 connects with West 2.

(a) Draw a graph of the complex.

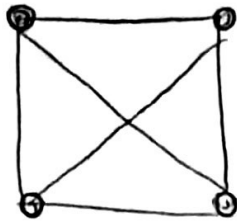(b) Which edges are bridges? Explain.

N1 – N2   E1 – E2   If removed will

N1 – N3   W1 – W2   become disconnected

N1 – E1

(c) Draw the graph K₄.

✓

⑤

## Question Five

A bus company conducts 'jump on, jump off' sightseeing tours, during which tourists can get on and off buses at any of the designated attractions as many times as they like during the same day. The weighted digraph below shows Attractions A to F, along with the time (in minutes) that a bus takes to travel between the attractions. The bus company operates two different circuits around the city, each shown differently below.
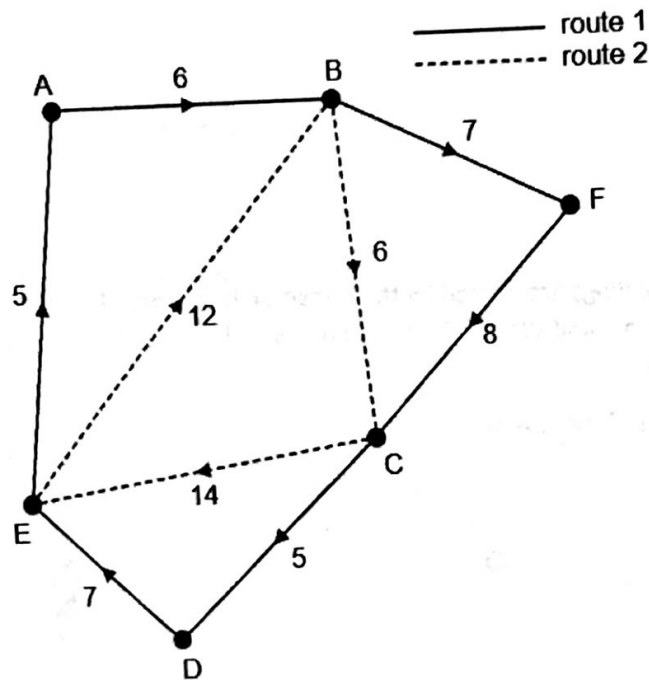
route 1
route 2

(a) Complete the adjacency matrix for the digraph.

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	1	0	0	1
C	0	0	0	1	1	0
D	0	0	0	0	1	0
E	1	1	0	0	0	0
F	0	0	1	0	0	0

(a)

✓✓

, Mai gets on a bus at E. She travels directly to B, changes bus, and continues travelling BFCDEA. Explain why her route is a trail.

It is a walk with no repeated edges

(c) Vinh has just visited Attraction C and wishes to visit Attraction B next. Determine the route he should take to arrive at B in the shortest travelling time. State the time taken.

CDEAB

23 minutes

The bus company plans to include another attraction, G. Instead of adding another circuit, they will provide shuttle buses between F and G (four minutes in each direction) and D and G (five minutes in each direction), as shown below.
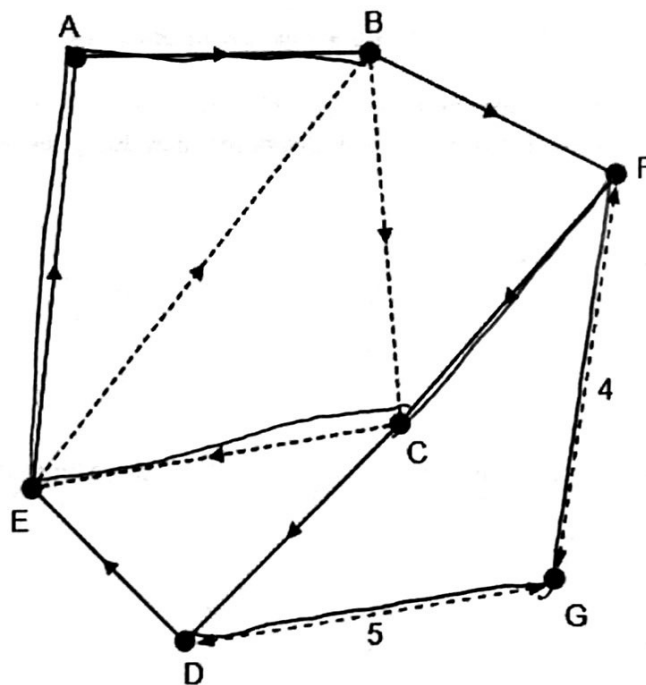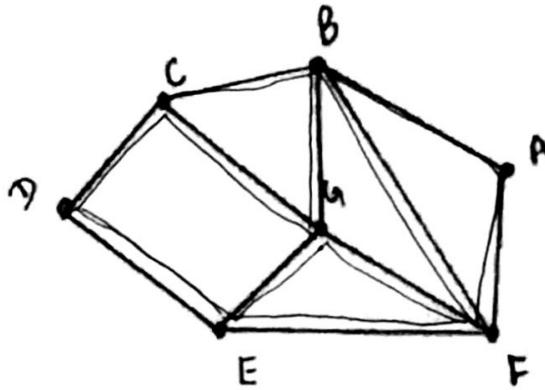

(d) Toshi is at Attraction D. He wants to complete a Hamiltonian path. State the route he should take.

DGFCEAB

7

Question Six

(a) Determine the degree of each vertex in the network above and use the table to record your results.

Vertex	A	B	C	D	E	F	G
Degree	2	4	3	2	3	4	4

(b) This network forms a semi-Eulerian trail

(i) What property of the vertices supports the claim that this network forms a semi-Eulerian trail?

Exactly 2 odd ✓

(ii) Describe the semi-Eulerian trail for the above network by listing the vertices of the trail in the order visited.

E F G E D C G B F A A B C ✓✓

⑤

Question Seven (2 marks)

The subjects studied by four students are shown in the table. Represent this using a bipartite graph.

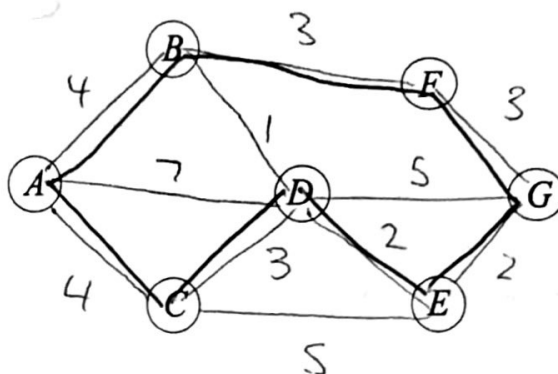| Student	Subjects
Amy	Maths, Chemistry, Media
Beth	Physics, Chemistry
Charles	Art, Media
Darren	Maths, Art

✓✓

The table below shows the distances, in kilometres, along direct footpaths between the checkpoints A, B, C, D, E, F and G at a national park.

|	B	C	D	E	F	G
A	4	4	7	-	-	-
B		-	1	-	3	-
C			3	5	-	-
D				2	-	5
E					-	2
F						3

(a) Complete the graph below to show this information.

A park worker is at Checkpoint D and needs to visit all the other checkpoints just once and then return to D.

(b) The worker leaves Checkpoint D, walks to Checkpoint A and then visits the rest of the checkpoints just once before returning to D. List the other checkpoints visited, in order, and state the total length of this route.

DACEGFBD ✓

35 Km ✓
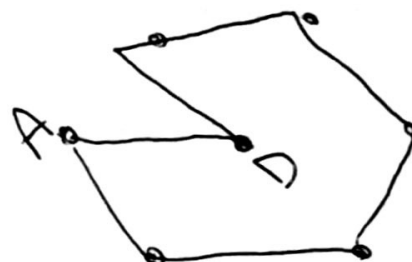
(c) Is the trail in (b) a Hamiltonian path or cycle? Explain your answer.

Cycle ✓

Same start/finish ✓

(d) A shorter trail, starting and finishing at D, is possible for the worker. List the order the checkpoints should be visited to achieve the smallest total length and state this length.

31 Km ✓

DEGFBACD ✓
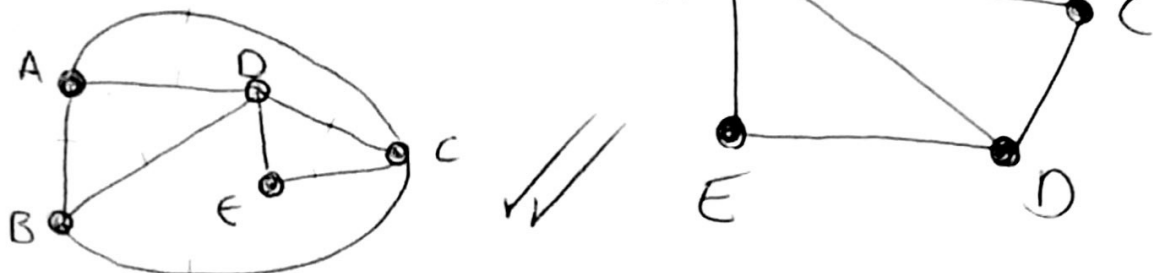
⑨

Question Nine

(a) A planar graph has five faces and five vertices, A, B, C, D and E.

(i) Determine the number of edges for this graph.

$$5 + 5 - e = 2$$
$$e = 8 \checkmark\checkmark$$

ii) Draw the planar graph in the space below.

(iii) Determine a Hamiltonian cycle for the graph, giving your answer as a sequence of vertices.

DECABO $\checkmark$

(iv) Is the graph Eulerian, semi-Eulerian or neither? Justify your answer.

Semi - Eulerian $\checkmark$ Two odd $\checkmark$

Depends on above Graph.

(b)   (i) A simple connected graph contains five vertices. Determine the minimum and the maximum number of edges it contains.

min $= 4 \checkmark$ max $= 10 \checkmark$

(ii) A simple connected graph contains n vertices. Determine the minimum number of edges it contains.

$n - 1 \checkmark$

(iii) What name is given to the simple connected graph with the maximum number of edges possible

Complete $\checkmark$

(11)