

Full Name: SOLUTIONS



## **MATHEMATICS**

### **Methods Units 1 & 2**

#### **Test 2 – Functions, Transformations and Relations**

#### **Chapters 8 and 9**

#### **Semester 1 2019**

#### **Section One - Calculator Free**

##### **Time allowed for this section**

Working time for this section: 25 minutes

Marks available: 25 marks

##### **Material required/recommended for this section**

###### **To be provided by the supervisor**

This Question/Answer booklet

Formula sheet

###### **To be provided by the candidate**

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

##### **Important note to candidates**

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (4 marks)

State two possible equations for a circle with radius 5 and passing through the point with coordinate (4, 4).

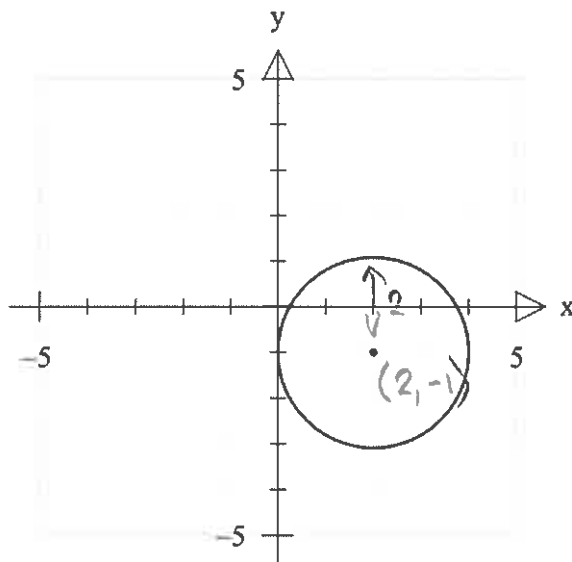
Any appropriate solutions

Mark for centre in equation

Mark for radius in equation

2. (3 marks)

Find the equation of the circle drawn below.



$$\begin{array}{ccccccc} (x-2)^2 & + & (y+1)^2 & = & 4 \\ \checkmark & & \checkmark & & \checkmark \end{array}$$

7

3. (8 marks)

Consider the function with equation  $y = 5 + \sqrt{9 - x^2}$ .

a) Explain why this curve exists only for  $-3 \leq x \leq 3$ . [2]

$9 - x^2$  must be non-negative ✓  
 $9 - (-3)^2 = 0$   
 $9 - (3)^2 = 0$  } all values between will give a positive ✓

b) Explain why the y-value must always be at least 5. [1]

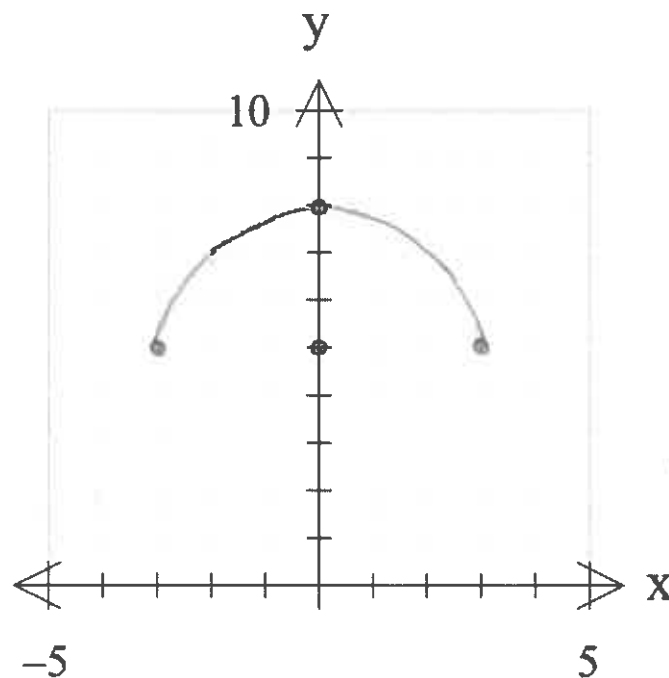
lowest value of  $\sqrt{9 - x^2}$  is zero

$0 + 5 = 5$  ∴ always greater or equal to 5 ✓

c) What is the largest possible value of y? [2]

$\sqrt{9 - 0} = 3$  ✓ (supportive working)  
 $5 + 3 = 8$  ✓ ∴ largest value of y is 8 ✓

d) On the axes provided, sketch this curve. [3]



✓ semicircle  
 ✓ centre (0, 5)  
 ✓ radius 3

4. (10 marks)

State the natural domain and range for each of the functions/relations below.

Function/Relation	Natural Domain	Natural Range
$y = (x + 1)^2 - 5$	$x \in \mathbb{R}$ ✓	$y \in \mathbb{R}, y \geq -5$ ✓
$y = \sqrt{x - 5}$	$x \in \mathbb{R}, x \geq 5$ ✓	$y \in \mathbb{R}, y \geq 0$ ✓
$y = \frac{1}{x - 1} + 3$	$x \in \mathbb{R}, x \neq 1$ ✓	$y \in \mathbb{R}, y \neq 3$ ✓
$(x + 1)^2 + (y + 1)^2 = 4$	$x \in \mathbb{R}, -3 \leq x \leq 1$ ✓	$y \in \mathbb{R}, -3 \leq y \leq 1$ ✓
$y^2 = 4(x - 1)$	$x \in \mathbb{R}, x \geq 1$ ✓	$y \in \mathbb{R}$ ✓

End of Section One

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## MATHEMATICS Methods Units 1 & 2

### Test 2 – Functions, Transformations and Relations

#### Chapters 8 and 9

Semester 1 2019

#### Section Two - Calculator Assumed

##### Time allowed for this section

Working time for this section: 30 minutes

Marks available: 35 marks

##### Material required/recommended for this section

###### To be provided by the supervisor

This Question/Answer booklet

Formula sheet

###### To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

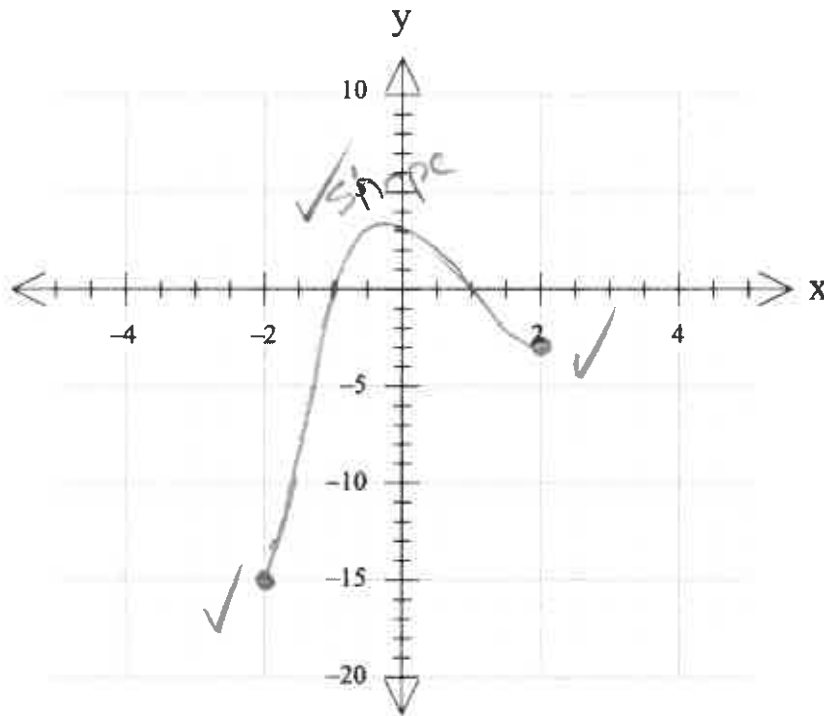
##### Important note to candidates

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1. (9 marks)

Consider the function  $f(x) = x^3 - 3x^2 - x + 3$  for  $-2 \leq x \leq 2$ .

a) On the axes provided below, sketch the graph of  $f(x)$  within the specified domain. [3]



b) State the range for  $f(x)$  for the domain specified. Give your answers correct to one decimal place. [2]

$$-15.0 \leq y \leq 3.1$$

c) State the coordinates of the horizontal intercept(s) of  $f(x)$  for the domain specified. [2]

$$(-1, 0) \text{ and } (1, 0)$$

(-1 if (3,0) is mentioned)

d) State the coordinates of the turning point(s) of  $f(x)$  for the domain specified. State the nature of this point. Give your answer correct to one decimal place. [2]

$$(-0.2, 3.1) \text{ Maximum}$$

(-1 if other tp mentioned)

2. (8 marks)

Describe a series of transformations required to convert  $f(x)$  into  $g(x)$ .

a)  $f(x) = x^2$  and  $g(x) = (x - 2)^2 + 4$

[2]

Translate right 2 ✓ then  
translate up 4 ✓

b)  $f(x) = x^3$  and  $g(x) = -(2x)^3$

[2]

Dilate parallel to x-axis factor of  $\frac{1}{2}$  ✓  
Reflect about x-axis. ✓

c)  $f(x) = \frac{1}{x}$  and  $g(x) = \frac{1}{1-x}$

[2]

Reflect about y-axis ✓  
Translate right 1 ✓

d)  $f(x) = (2x + 1)^2$  and  $g(x) = x^2$

[2]

Dilate parallel to x-axis factor of 2 ✓  
Translate right  $\frac{1}{2}$  ✓

3. (9 marks)

Consider the three graphs shown below.

a) State whether each is a relation or a function.

[3]

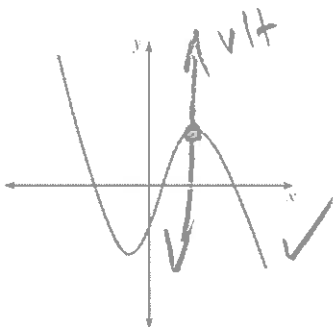
b) Show justification with an appropriate test.

[3]

c) Describe the graph using one of the terms one-to-one, one-to-many, many-to-one.

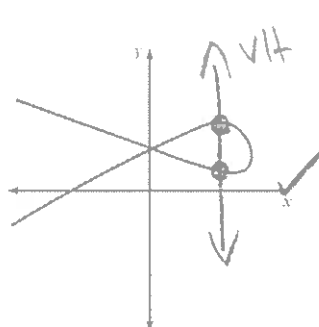
[3]

Graph A



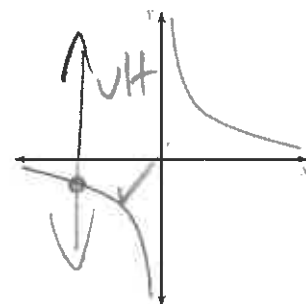
Function ✓  
many-to-one ✓

Graph B



Relation ✓  
one-to-many ✓

Graph C



Function ✓  
one-to-one ✓

4. (9 marks)

Given that  $f(x) = 3x - 1$ ,  $g(x) = x^2 - 2x + 1$  and  $h(x) = \begin{cases} x^2 & \text{for } x > 2 \\ 4x & \text{for } x \leq 2 \end{cases}$

a)  $f(2)$   $3 \times 2 - 1 = 5 \checkmark$  [1]

b)  $g(-2)$   $(-2)^2 - 2(-2) + 1 = 9 \checkmark$  [1]

c)  $h(2)$   $4(2) = 8 \checkmark$  [1]

d)  $g(f(x))$   $(3x-1)^2 - 2(3x-1) + 1 \checkmark$   
 $= 9x^2 - 6x + 1 - 6x + 2 + 1$   
 $= 9x^2 - 12x + 4 \checkmark$  [2]

e)  $h(f(x)) = \begin{cases} (3x-1)^2, & x > 2 \checkmark \\ 4(3x-1), & x \leq 2 \checkmark \end{cases}$  [2]

f)  $g(h(4))$   $h(4) = 16 \checkmark$   
 $g(16) = 16^2 - 2(16) + 1$   
 $= 225 \checkmark$  [2]

End of Test