12 ATAR Physics

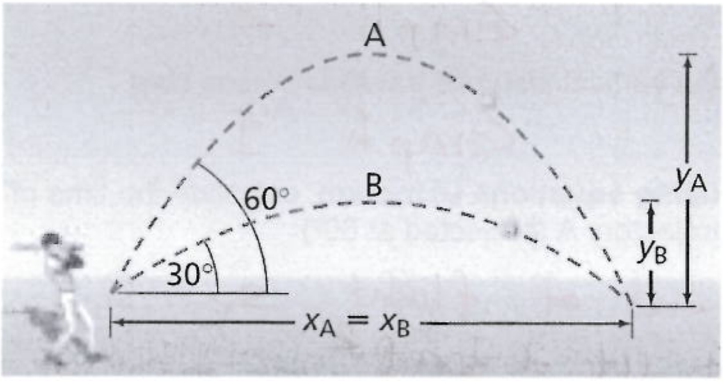
Projectile and Circular Motion Test 2016

Name: MARKING KEY Score: _______ / 56

Instructions:

Assume negligible air resistance unless question explicitly asks you to consider for real situations or qualitative arguments.

1. Two baseballs in the figure below were hit with the same initial speed of 25 ms^{-1}.



(a) For trajectory A, determine the magnitude of the

(i) horizontal component of the initial velocity [1]

$25 \cos 60° = 12.5 \text{ ms}^{-1}$

(ii) vertical component of the initial velocity. [1]

$25 \sin 60° = 21.7 \text{ ms}^{-1}$

(b) For trajectory B, determine the magnitude of the

(i) horizontal acceleration. [1]

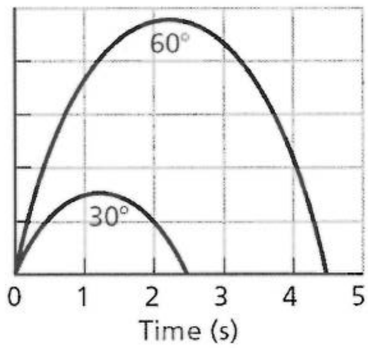
0 ms^{-2}

(ii) vertical acceleration [1]

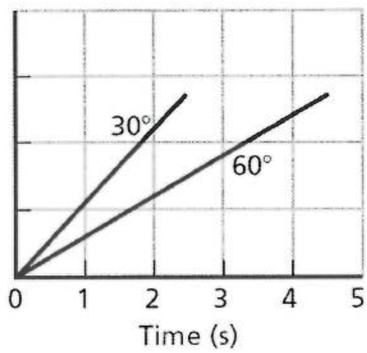
9.8 ms^{-2}

Question 1 (continued)

(c) For both trajectories, which graph shows



Graph 1



Graph 2

(i) the horizontal distance travelled versus time. [1]

Graph 2

(ii) the vertical distance travelled versus time. [1]

Graph 1

(iii) **using equations of motion**, calculate the time of flight of trajectory A (projected at 60°) [2]

For A, end of flight $s_Y = 0$

$s = ut + \frac{1}{2}at^2$ ✓

$0 = 21.7t - 4.9t^2$

$0 = t(21.7 - 4.9t)$

$t = 0$ or $t_A = 4.42\text{ s}$ ✓

↑ as +ve

time of flight $t_A = 4.42\text{ s}$

(iv) **using equations of motion**, calculate the maximum height reached in trajectory A (projected at 60°) [2]

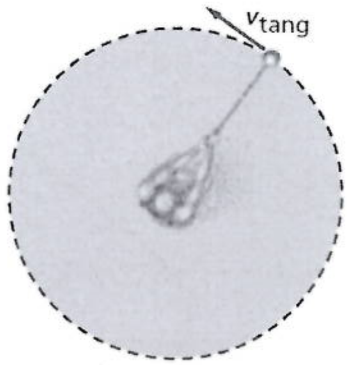
At max height

$v^2 = u^2 + 2as$ ✓

$0 = 21.7^2 + 2(-9.8)s$

$s = 24.025\text{ m}$ ✓

2. An athlete whirls a 7 kg hammer of radius 1.8 m from the axis of rotation in a horizontal circle as shown right.



(a) If the hammer makes one revolution in 1.0 s,

(i) calculate the centripetal acceleration of the hammer? [3]

$$a = \frac{v^2}{r}$$

$$= \left(\frac{2\pi r}{t}\right)^2 \div r$$

$$= \frac{4\pi^2 r^2}{t^2 r}$$

$$= \frac{4\pi^2 r}{t^2} = \frac{4\pi^2(1.8)}{1^2} = 71\ ms^{-2}$$

(ii) state the direction of the centripetal acceleration? [1]

towards the centre

(b) Determine the tension in the chain? [2]

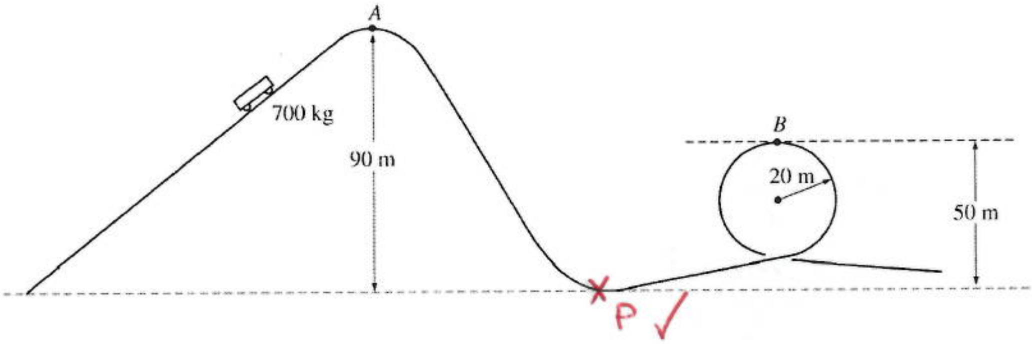
$$T = F_c$$

$$= m\frac{v^2}{r}$$

$$= 7(71)$$

$$= 497\ N$$

3.



A roller coaster at an amusement park lifts a car of mass M kg to point A at a height of 90 m above the lowest point on the track, as shown above.

The car starts from rest at point A, rolls with negligible friction down the incline and follows the track around a loop of radius 20m.

Point B, the highest point on the loop, is at a height of 50 m above the lowest point on the track.

(a)(i) Indicate on the figure the point P at which the maximum speed of the car is attained. [1]

shown

(ii) Calculate the value of this maximum speed. [3]

Using conservation of energy

$TE_A = TE_P$

$PE_A + KE_A = PE_B + KE_B$

$mgh_A + 0 = 0 + \frac{1}{2}mv_B^2$

$9.8(90) = \frac{1}{2}v_B^2$

$v_B^2 = 2(9.8)(90)$

$v_B^2 = 1764$

$v_B = 42\ ms^{-1}$

OR using equation of motion anything falling from A to P will achieve same speed as something is dropped

Question 3 (continued)

(b) Calculate the speed of the car at point B. [3]

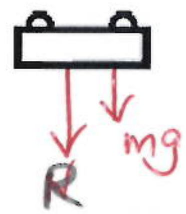
$TE_A = TE_B$

$\cancel{m}(9.8)(90) = \cancel{m}(9.8)50 + \frac{1}{2}\cancel{m}v_B^2$ ✓

$2(392) = v_B^2$ ✓

$v_B = 28\ ms^{-1}$ ✓

(c)(i) On the figure of the car below, draw and label vectors to represent the forces acting on the car when it is upside down at point B. [3]



✓ R↓

✓ mg↓

✓ no extraneous vectors

(ii) Calculate the magnitude of all the forces identified in (c)(i) given that M is 700 kg [3]

$W = mg = 700(9.8)$
$= 6860\,N$ ✓

$F_{net} = R + mg$ ✓

$F_c = R + 6860$

$\frac{mv^2}{r} = R + 6860$

$\frac{700(28)^2}{20} = R + 6860$

$R = 27440 - 6860$
$= 20580\ N$ ✓

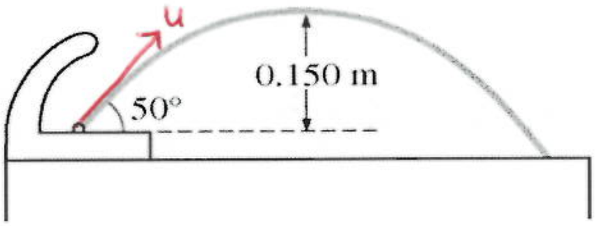
(d) Assume that friction is not negligible. How could the loop be modified to maintain the same speed at the top of the loop as found in (b)?

Justify your answer. [2]

Lower the height of B ✓

Work done by friction will reduce the total mechanical energy available at B so KE_B remains same, PE_B must be reduced ✓

4.



A drinking fountain projects water at an initial angle of 50° above the horizontal. The water reaches a maximum height of 0.150 m above the point of exit.

(a) Calculate the speed at which the water leaves the fountain. [4]

Vertical Motion (take ↑ as +ve)

$u_Y = u\sin 50^\circ$

$a_Y = -9.8$

At maximum height $v_Y = 0$ ✓

$v_Y^2 = u_Y^2 + 2a s_Y$ ✓

$0 = (u\sin 50)^2 - 2(9.8)0.15$

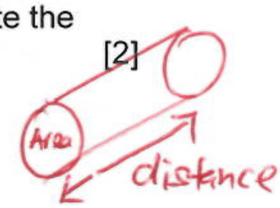
$2.94 = (u\sin 50)^2$ ✓

$u\sin 50 = \sqrt{2.94}$

$u = \dfrac{1.71}{\sin 50^\circ} = 2.24\ ms^{-1}$ ✓

It is moving $2.24\ ms^{-1}$ at 50° to horizontal

(b) The radius of the fountain's exit hole is 4.00×10^{-3} m. Calculate the rate of volume flow of the water. [2]



Volume = Area × distance travelled

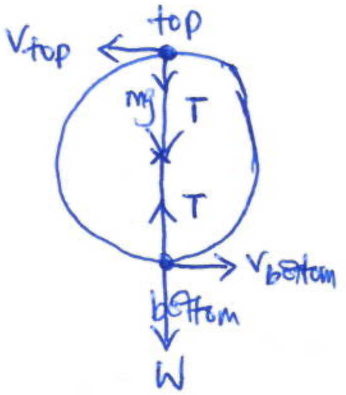
Rate of volume flow $= \text{Area} \times \dfrac{\text{distance}}{\text{time}}$ ✓

$= \pi r^2 v$

$= \pi (4\times10^{-3})^2 (2.24)$

$= 1.13 \times 10^{-4}\ m^3/s$ ✓

5. A ball on a light string moves in a vertical circle. Analyse and describe the motion of this system considering the effects of gravity and tension using free body diagrams. Is the system in uniform circular motion? Explain. [5]



At top $F_c = T + mg$ ✓

At bottom $F_c = T - mg$ ✓

with m and r constant

$V_{top} > V_{bottom}$ ✓✓

Non uniform circular motion ✓
(as speed is not constant)

6. A car is travelling with speed v around a curve of radius r (shown left).

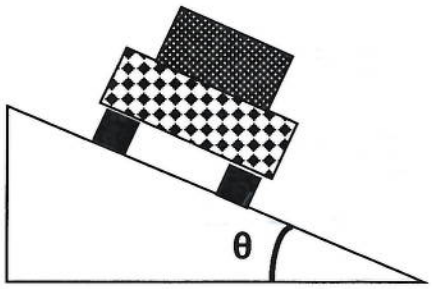
(a) Determine a formula for the angle θ at which a road should be banked so that no friction is required. Draw forces on the diagram below. [4]

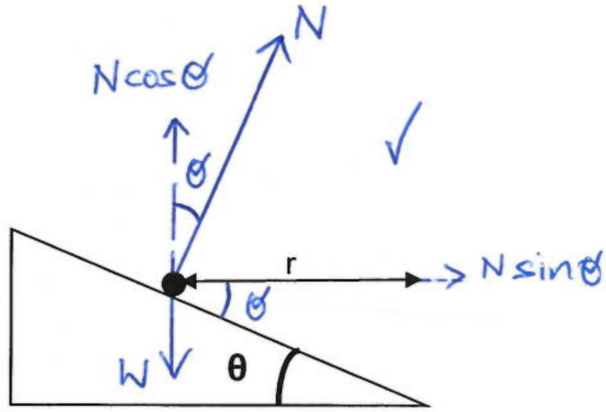
$N \sin\theta = F_c$ [1] ✓

$N \cos\theta = mg$ [2] ✓

$\tan\theta = \frac{mv^2}{r(mg)}$

$\tan\theta = \frac{v^2}{rg}$ ✓





✓

Free Body Diagram

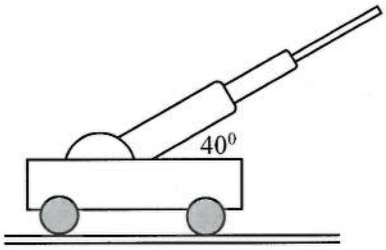
(b) Calculate the angle for a road which has a curve of radius 50 m with a design speed of 50 km h^{-1}. [2]

$50\ \text{km h}^{-1} = 13.89\ \text{ms}^{-1}$

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$= \tan^{-1}\left(\frac{13.89^2}{50 \times 9.8}\right) = 21.5°$ ✓✓

7.



Big Bertha was the name of a huge cannon used by the Germans in the First World War to fire across the Channel to England. This gun was mounted on a train and could be moved to different positions. The shell's muzzle velocity was 750 kmh^{-1}.

$V = 208.3\ ms^{-1}$

a) What would be the maximum height that the shell reached above the ground? [2]

$u_V = 208.3 \sin 40^\circ = 133.9\ ms^{-1}$ $\quad \uparrow +ve$

$v^2 = u^2 + 2as$ ✓

$0 = 133.9^2 - 19.6s$

$s = 915\ m$ ✓

b) What is the range of the gun if the angle of elevation is 40^0? [2]

$u_H = 208.3 \cos 40^\circ$

$= 160\ ms^{-1}$

Range $= 160(27.32)$ ✓

$= 4371.2 m = 4.37 \times 10^3\ m$

$s = ut + \frac{1}{2}at^2$

$0 = 133.9t - 4.9t^2$

$t = \frac{133.9}{4.9}$ ✓

$= 27.32\ s$

c) What would be the gun's maximum range when its angle of elevation can be increased or decreased? [2]

Max range when $\theta = 45^\circ$

verfically

$s = ut + \frac{1}{2}at^2$

$0 = 147.3t - 4.9t^2$

$t = 30.05\ s$ ✓

Horizontally

$s = ut = 147.3 \times 30.05$

$= 4.43 \times 10^3\ m$ ✓

Question 7 (continued)

d) What would be the velocity and angle of the shell 5 seconds after firing? Assume angle of elevation is 40° [2]

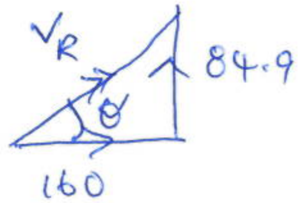
Vertically $\quad v_Y = 133.9 - 9.8 \times 5$

$= 84.9 \text{ ms}^{-1}$

horizontally $\quad v_X = 160 \text{ ms}^{-1}$

$$V_R^2 = \sqrt{160^2 + 84.9^2}$$

$$= 181 \text{ ms}^{-1}$$



$$\theta = \tan^{-1}\left(\frac{84.9}{160}\right)$$

$$\theta = 28^\circ$$

$V_R = 181 \text{ ms}^{-1}$, 28° above horizontal

END OF TEST

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