



Methods 11 Investigation 1 2018

Trigonometry

Total Marks: 56

Time Allowed: 60 minutes

Name: _____

Take home section and Classpad allowed

ALL working must be shown for full marks.

1. [3, 2, 1, 1, 2, 1, 1, 1, 1 = 13 marks]

Using your knowledge of unit circle, complete the following chart for $f(x) = \sin x$.

Use exact values. DO NOT ANSWER IN DECIMALS!!!!

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0

a) What do you notice about the values in the chart?

cyclic / repeat / form a pattern

Give FT marking

b) For which values of x does $\sin x = 1$? (There is more than one answer) $-\frac{3\pi}{2}, \frac{\pi}{2}$ $\frac{1}{2}$ each $-\frac{1}{2}$ if state any not correctc) For which values of x does $\sin x = -1$? $-\frac{\pi}{2}, \frac{3\pi}{2}$

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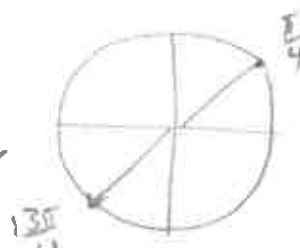
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d) For which values of x does $\sin x = 0$? $-2\pi, -\pi, 0, \pi, 2\pi$ $-\frac{1}{2}$ each not listed
 $-\frac{1}{2}$ each listed not correct

e) What is the domain and range for the table above?

 $\{x: -2\pi \leq x \leq 2\pi\}$ $\frac{1}{2}$ each $\{y: -1 \leq y \leq 1\}$ f) Describe the pattern of when $\sin x$ is a maximum? $\frac{\pi}{2} + k2\pi,$ multiples of $\frac{\pi}{2}$ plus multiples of 2π ✓

g) Describe the pattern of when the x-intercepts occur?

 π plus multiples of π ✓h) If the domain has no restrictions, find the value(s) of $\sin \frac{13\pi}{4}$. $4\sqrt{13}$ $3\frac{1}{4}\pi = \frac{\pi}{4} + 3\pi$ $\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\sin \frac{13\pi}{4} = -\frac{\sqrt{2}}{2}$ ✓

2. [2, 1, 1, 1, 1, 1, 1, 2 = 10 marks]

Using your knowledge of unit circle, complete the following chart for $f(x) = \cos x$. (Use exact values.)
DO NOT ANSWER IN DECIMALS!!!!

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
cos x	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1

✓✓

a) For which value(s) of x is $\cos x = 1$?

$-2\pi, 0, 2\pi$

$-\frac{1}{2}$ each wrong or missing

b) For which value(s) of x is $\cos x = 0$?

$-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

$-\frac{1}{2}$ each wrong or missing

c) What is the domain and range for the table above?

$\{x: -2\pi \leq x \leq 2\pi\}$

$\{y: -1 \leq \cos x \leq 1\}$

✓

d) Describe the pattern of when $\cos x$ is a maximum.

multiplier of 2π starting at 0 ✓

e) Describe the pattern of when the x-intercepts occur.

multiplier of π starting at $\frac{\pi}{2}$ ✓

it don't use words give '0'

f) If the domain is infinite, find the value of $\cos \frac{9\pi}{4}$

$\frac{\sqrt{2}}{2}$ ✓

g) Find all values of x, such that $\cos x = \frac{-\sqrt{2}}{2}$

$\frac{3\pi}{4} + n2\pi$ where n is an integer ✓

OR $-\frac{5\pi}{4}, -\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

Consider the functions $f(x) = a \sin(b(x + h)) + v$ and $g(x) = a \cos(b(x + h)) + v$.

a , b , h , and v have specific effects on the graphs of the function. In trigonometry we also have special names for the pronumerals based on their effect on the graph.

3. [2, 2 = 7 marks]

- a) The amplitude of the function is " a ". How is the amplitude related to the distance between the minimum and maximum values? (Hint: change the " a " value for each function to determine the effect of " a " on the distance between the minimum and maximum values.)

amplitude is half the distance between the maximum and minimum. ✓✓

OR - amplitude is the distance from the midpoint to the max or min. ✓✓

- b) What effect does " a " have if $a < 0$?

graph is reflected through the x axis. ✓✓

4. [2, 1, 1, 2 = 6 marks]

- a) The period of the function is " b ". Describe the effect of changing the " b " value.

period is distance required to complete one full cycle.
As b increases, the cycle distance decreases. ✓
As b decreases, the cycle distance increases. ✓

- b) When " b " is 2, how often does the function repeat itself in the usual period of 2π ?

Twice ✓

- c) When " b " is 4, how often does the function repeat itself in the usual period of 2π ?

4 times ✓

- d) Define the period of the sin and cosine functions in terms of " b " and 2π .

$\frac{2\pi}{b} = \text{period}$ ✓✓

5. [1, 1, 2, 1 = 5 marks]

A horizontal translation of a trigonometric function is called a **phase shift**. The "h" value shifts the graph horizontally, however, when calculating the phase shift you have to also consider the value of "b".
The phase shift = $\frac{-h}{b}$.

a) Graph $f(x) = \sin x$ and $g(x) = \sin(x + 90)$ using degrees. For what values of x is $f(x) = 0$?

$0, 180^\circ, 360^\circ, + \text{multiples of } 180^\circ \checkmark$

b) For what values is $g(x) = 0$?

$90^\circ, 270^\circ + \text{multiples of } 180^\circ \checkmark$

c) What do you notice about your answers?

$\sin(x+90^\circ)$ is translated 90° to the left of $\sin(x)$ \checkmark

d) Determine the value of "h" which will ensure
Give your answer in both degrees and radians.

$$\sin(x + h) = \cos x.$$

$h = 90^\circ$ or $\frac{\pi}{2}$ $\frac{1}{2}$ each \checkmark

6. [2, 2 = 4 marks]

Complete the following table.

Function	Period	Amplitude
$y = 2 \sin(2x^\circ)$	2 \checkmark	2 \checkmark
$y = -4 \cos(\frac{x}{2} + 30^\circ)$	$\frac{1}{2}$ \checkmark	4 \checkmark

7. [2, 1, 2 = 5 marks]

The following equation models the average monthly temperatures for Hobart. (The average monthly temperature is an average of the daily highs and daily lows.)

$$f(t) = 18.5 \sin\left(\frac{\pi}{6}t - 4\right) + 54.4 \quad \text{where } t = \text{represents January.}$$

a) State the values of a, ^h~~k~~, b, v in the equation.

$a = 18.5$
 $b = \frac{\pi}{6}$
 $h = -4$
 $v = 54.4$
 $\frac{1}{2}$ each

b) The maximum and minimum values of a periodic function oscillate about a horizontal line called the **midline**. What is the midline of the equation?

$$54.4$$



c) How is the value of the amplitude related to this midline?

maximum is midline plus amp

minimum is midline minus amp

$$54.4 + 18.5 = 72.9$$

$$54.4 - 18.5 = 35.9$$

8. [2, 4 = 6 marks]

Steven gets on a Ferris wheel at the Royal Show. His height, h metres, t seconds after the ride starts is given in the table below.

Instructions on calculator use to help with this question.

- Open the Statistics application
- Enter the data (t into list 1, h into list 2)
- Tap on graph icon to draw a scatter graph
- Select [Calc] Regression [Sinusoidal Reg]
- Save the function to $y1$ so you can work with the function in the Graph & Table application.

$t(s)$	0	1	2	3	4	5	6	7	8	9	10
$h(m)$	1	1.13	1.52	2.15	2.99	4	5.15	6.38	7.63	8.85	10

a) Write down the equation with rounding to whole numbers.

$$d = 6.0 \sin(12x + 90) + 7$$

$\frac{1}{2}$ each term correct

b) Use your model to determine the:

i) radius of the Ferris wheel

$$\text{radius} = \text{amplitude} = 6m$$

ii) minimum and maximum height of Tan above the ground

$$\text{min} = 1m$$

$$\text{max} = 13m$$

iii) time taken for one complete revolution.

$$\text{Period} = \frac{2\pi}{b}$$

$$= \frac{360}{12}$$

$$= 30 \text{ sec}$$

