MINDARIE SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS

MATHEMATICS: SPECIALIST 1 & 2

SEMESTER 1 2019
TEST 3

Name________________________

Calculator Free

Time allowed: 20 mins

Total marks: 19

1. [5 marks: 2, 3]

The position vectors of three points A ,B and C are $(1,3)$, $(-2,6)$ and $(-5,10)$ respectively.

a) Find ${}_B\boldsymbol{r}_C$

$${}_B\underset{\sim}{r}_C = \underset{\sim}{r}_B - \underset{\sim}{r}_C \quad ✓$$
$$= (-2,6) - (-5,10)$$
$$= (3,-4) \quad ✓$$

b) Given that ${}_D\boldsymbol{r}_A = (22,7)$, find ${}_B\boldsymbol{r}_D$

$$\underset{\sim}{r}_D - \underset{\sim}{r}_A = \underset{\sim}{r}_D - (1,3) = (22,7)$$
$$\underset{\sim}{r}_D = (23,10) \quad ✓$$

$${}_B\underset{\sim}{r}_D = \underset{\sim}{r}_B - \underset{\sim}{r}_D = (-2,6) - (23,10) \quad ✓$$
$$= (-25,-4) \quad ✓$$

2. [6 marks: 2, 2, 2]

Relative to a fixed point on the ground, Emirates flight EK16 has position vector $-3\boldsymbol{i} + 8\boldsymbol{j}$.

a) The aircraft has a velocity of $6\boldsymbol{i} + \boldsymbol{j}$ km/min. Give the position vector of the flight after 30 minutes.

$$-3\underline{i} + 8\underline{j} + 30(6\underline{i} + \underline{j})$$
$$= -3\underline{i} + 8\underline{j} + 180\underline{i} + 30\underline{j}$$
$$= 177\underline{i} + 38\underline{j}$$

b) From the same fixed point, Qantas flight QF4 has position vector $10\boldsymbol{i} - 7\boldsymbol{j}$.

i) Find the position vector of this aircraft relative to the original position of the Emirates flight.

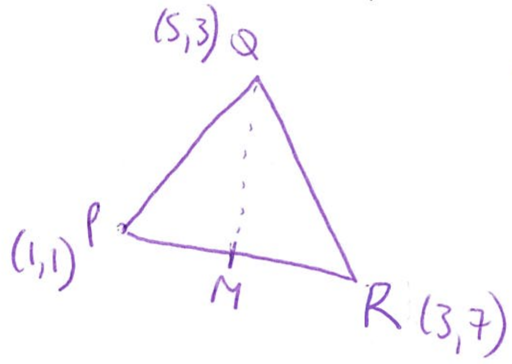
$${}_{Q}\underline{r}_{E} = \underline{r}_Q - \underline{r}_E$$
$$= 10\underline{i} - 7\underline{j} - (-3\underline{i} + 8\underline{j})$$
$$= 13\underline{i} - 15\underline{j}$$

ii) How far apart were the two aircraft? Give your answer as an exact value.

$$\sqrt{13^2 + 15^2} = \sqrt{169 + 225}$$
$$= \sqrt{394}$$

3. [6 marks: 3, 3]

(a) A triangle PQR has vertices $P(1, 1)$, $Q(5, 3)$ and $R(3, 7)$. Determine the vector, $\overrightarrow{QM}$ where M is the midpoint of side PR.

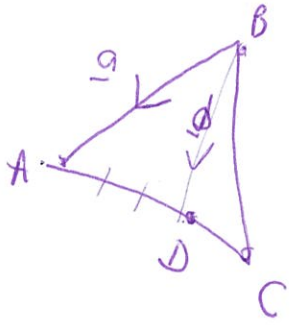


✓ $\overrightarrow{QM} = \overrightarrow{QP} + \overrightarrow{PM}$

✓ $= (-4, -2) + \frac{1}{2}(2, 6)$

✓ $= (-3, 1)$

(b) ABC is a triangle with point D on side AC such that $AD = \frac{3}{4}AC$. If $\overrightarrow{BA} = \boldsymbol{a}$ and $\overrightarrow{BD} = \boldsymbol{d}$, show that $\overrightarrow{BC} = \frac{1}{3}(4\boldsymbol{d} - \boldsymbol{a})$.



$\overrightarrow{AD} = \underline{d} - \underline{a}$ ✓

$\overrightarrow{AD} = \frac{3}{4}\overrightarrow{AC}$

$\underline{d} - \underline{a} = \frac{3}{4}\overrightarrow{AC}$ ✓

$\frac{4}{3}(\underline{d} - \underline{a}) = \overrightarrow{AC}$

So

$\overrightarrow{BC} = \underline{a} + \frac{4}{3}(\underline{d} - \underline{a})$

$= \frac{4}{3}\underline{d} - \frac{4}{3}\underline{a} + \underline{a}$

✓ $= \frac{4}{3}\underline{d} - \frac{1}{3}\underline{a}$

$= \frac{1}{3}(4\underline{d} - \underline{a})$

4. [2 marks]

Prove that:

$${}_m\boldsymbol{v}_n + {}_n\boldsymbol{v}_p = {}_m\boldsymbol{v}_p$$

$\underline{V}_m - \cancel{V_n} + \cancel{V_n} - \underline{V}_p$ ✓

$= \underline{V}_m - \underline{V}_p$ ✓

$= {}_m\underline{V}_p$

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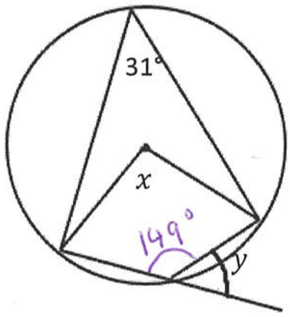
Calculator Assumed

Time allowed: 35 mins

Total marks: 28

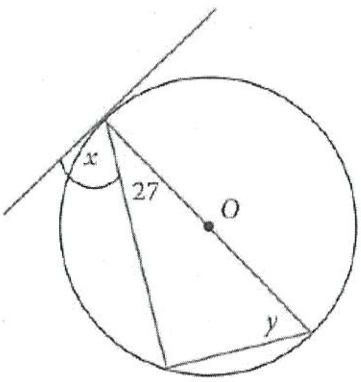
5. [6 marks: 3, 3]

In each of the following diagrams, find the values of the letters. You should give reasons for your answers.



$x = 62°$ – angle at the centre

$y = 31°$ – opp. angles in cyclic quad
– angle sum of a line.

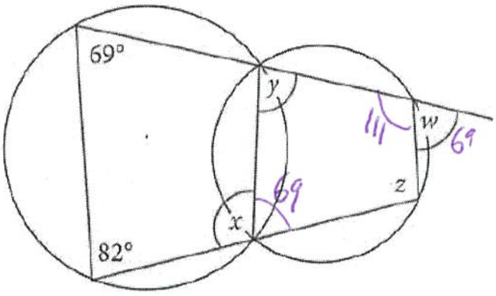


$x = 63°$ – tan/rad meet at 90°

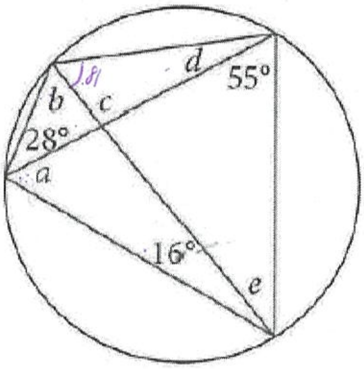
$y = 63°$ – alt/segment
or
– angle in semi-circle and angle sum of triangle.

6. [9 marks: 4, 5]

Find the angles indicated in the diagrams below.



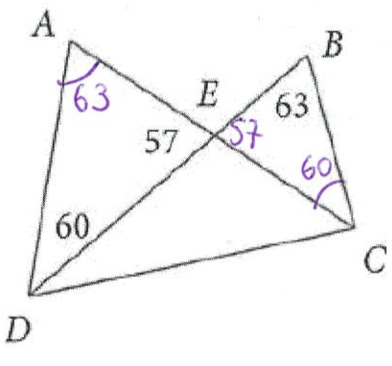
$x = 111°$ ✓
$y = 82°$ ✓
$z = 98°$ ✓
$w = 69°$ ✓



$a = 81°$ ✓
$b = 55°$ ✓
$c = 83°$ ✓
$d = 16°$ ✓
$e = 28°$ ✓

7. [3 marks]

Prove that A, B, C and D are concyclic, i.e. lie on the circumference of a circle.



$\angle DAC = \angle CBD = 63°$ ✓

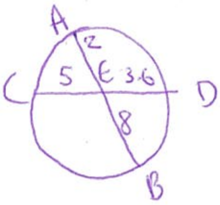
$\angle ADB = \angle BCA = 60°$ ✓

Using the theorem that states that angles in the same segment are equal then A, B, C and D must be concyclic. ✓

8. [4 marks]

Two straight lines, AB and CD, intersect at a point E. AE = 2cm, AB = 10cm, CE = 5cm and DE = 3.6cm. Using proof by contradiction, show that the two lines cannot be secants of a circle.

Assume that the lines are secants.



If they are secants then $AE \times EB = CE \times ED$

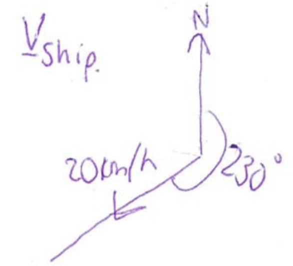
$2 \times 8 = 5 \times 3.6$

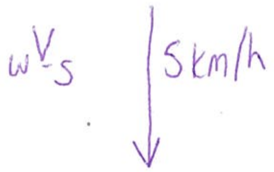
$16 \neq 18$

Hence the two lines cannot be secants and our assumption was false.

8. [6 marks]

To a person on a ship moving at 20km/h on a bearing 230° the wind appears to come from the North with speed 5km/h. Find the true velocity of the wind and the direction that it comes from.

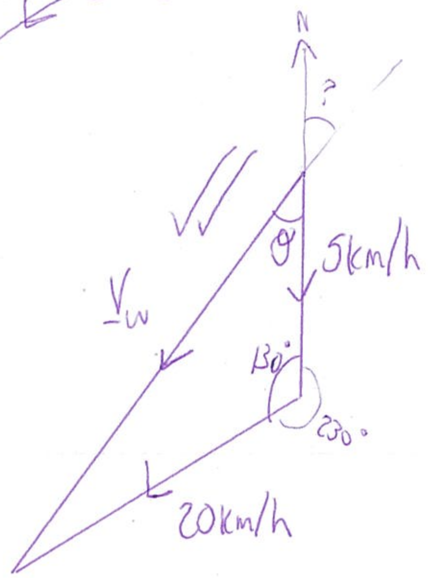




Need $\underline{V}_w$

$_w\underline{V}_s = \underline{V}_w - \underline{V}_{ship}$

$\underline{V}_w = {_w\underline{V}_s} + \underline{V}_{ship}$



$|\underline{V}_w|^2 = 5^2 + 20^2 - 2 \times 5 \times 20 \times \cos 130$

$|\underline{V}_w| = 23.5\text{km/h}$

$\dfrac{20}{\sin\theta} = \dfrac{23.5}{\sin 130}$ $\quad \theta = 40.7°$

True velocity of wind is 23.5km/h and from a bearing of 040.7°