EGC Eastern Goldfields College

MATHEMATICS:SPECIALIST
SEMESTER 1 2015

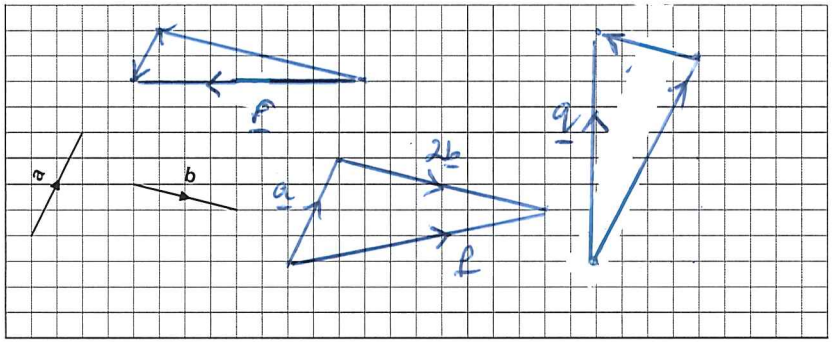
TEST 1

Calculator Assumed

Time Allowed: 60 minutes

Total Marks: 50

1. [4 marks -1, 1, 2]
Vectors **a** and **b** are as shown on the grid below.



On the grid above, sketch and label the vectors **p**, **q** and **r** where

$\mathbf{p} = \mathbf{a} + 2\mathbf{b}$
$\mathbf{q} = 2\mathbf{a} - \mathbf{b}$
$\mathbf{r} = -2\mathbf{b} - 0.5\mathbf{a}$

-1 any incorrect vectors
$-\frac{1}{2}$ each not having dirn.

-1 if no dirn marked

2. [7 marks -1, 1, 2, 2, 1]

(a) How many four letter 'words' can be formed using the letters from the word TRIANGLE if

(i) each letter may be used more than once?

$8 \cdot 8 \cdot 8 \cdot 8 = 4096$ ✓

(ii) each letter may only be used once?

$8\ 7\ 6\ 5 = 1680$ ✓

(b) (i) How many ways can the letters in the word MISSISSIPPI be arranged in a row?

$\frac{11!}{4!4!2!} = 34650$ ✓✓

(ii) How many of the ways from (b) (i) begin with the letter S?

$\frac{10!}{4!3!2!} = 12600$ ✓✓

(c) Towards the end of a game of Scrabble, only the letters A, T, Y, R, P, B and N remain in the bag. A player must select four letters from the bag. How many different groups of letters can the player select?

$\binom{7}{4} = 35$ ✓

3. [4 marks]

A three-letter code word is to be chosen from the letters of the word THERAPY or from the word TRIANGLE.

How many different code words are possible?

$7\,6\,5 + 8\,7\,6 - 4\,3\,2$

$= 210 + 336 - 24$

$= 522$

4. [6 marks - 2, 2, 2]

(a) A class of 22 Mathematics: Specialist students sit a test. The test has 20 questions that are either true or false. What can be concluded about the marks obtained from this test? Briefly explain your reasoning.

At least Two students get same score
(as 22 students + 21 scores)

(b) A school has 1000 students. What is the greatest number of students that we can be certain had their birthday in the same month?

$1000 \div 12 = 83\frac{1}{3}$

$\therefore 84$

(c) At the College sports carnival, every student belongs to one of the five houses: Arctic, Atlantic, Indian, Pacific or Southern. What is the least number of students that could be in a race to be certain of having at least three from the same house?

worst case 2 in each ie 10 students

$\therefore$ need 11 students

5. [3 marks]

In Year 11, 38 students study Mathematical Methods, 32 study Physics while 40 study Chemistry. 15 students study both Methods and Physics, 14 study Physics and Chemistry, while 17 study Methods and Chemistry. 12 students study all three subjects. An assembly is held for any student who studies Mathematical Methods, Physics, or Chemistry. How many students would be expected to attend the assembly?

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)$

$= 38 + 32 + 40 - 15 - 17 - 14 + 12$

$= 76$

6. [5 marks -2, 3]

(a) How many four digit even numbers can be made from the digits 1, 2, 3, 4, 5, 6, and 7 if no digit can be used more than once?

$6 \quad 5 \quad 4 \quad 3 = 360$

(b) How many of the even numbers from part (a) are greater than 5000?

$2 \quad 5 \quad 4 \quad 3 + 1 \quad 5 \quad 4 \quad 2$

$5,7 \qquad 246 \qquad 6 \qquad 34$

$= 120 + 40 = 160$

7. [10 marks - 1, 2, 2, 2, 1, 2]

The five members of One Direction (OMG! Swoon!) arrange themselves in a line for a photograph. The members are Niall, Zayn, Liam, Harry and Louis.

How many arrangements are there in which

(a) Zayn is in the middle?

$4 \quad 3 \quad 1 \quad 2 \quad 1 = 24$

$\quad\quad\quad\quad\quad 2$

(b) Liam and Louis are on each end?

$2 \quad 3 \quad 2 \quad 1 \quad 1 = 12$

$\quad 2 \qquad\qquad\qquad\quad 2$

(c) Harry is next to Zayn?

$4 \cdot 3 \quad 2 \quad 1 \times 2! = 48$

$\quad\quad\; 1+2$

(d) Niall and Louis are not next to each other?

$5! - 48 = 72$

Whilst in Perth for a concert, three of the members do a radio promotional interview.

(e) How many different groups could do the interview?

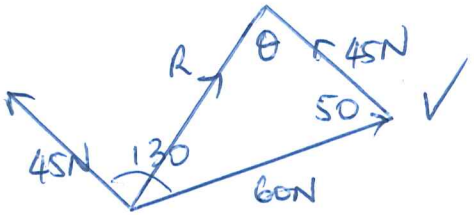
$\binom{5}{3} = 10$

(f) How many different groups are possible if their manager doesn't want both Niall and Harry to do the interview?

Both $\binom{2}{2} \times \binom{3}{1} = 3$ $\qquad 10 - 3 = 7$

8. [5 marks]

Two vectors have magnitudes of 45 N and 60 N and the angle between their directions is 130°. Sketch a diagram to show their sum and use trigonometry to calculate the magnitude of the resultant and the angle it makes with the smaller vector.



$R^2 = 45^2 + 60^2 - 2.45.60.\cos 50$ ✓

$R = 46.4\ N$ ✓

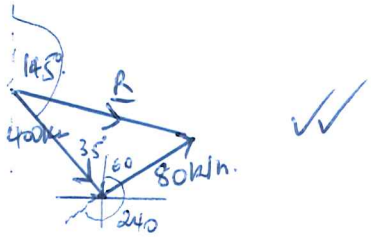
$\frac{\sin\theta}{60} = \frac{\sin 50}{R}$ ✓

$\theta = 82°$ ✓

9. [6 marks – 2, 4]

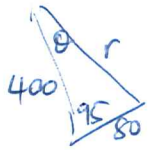
An aircraft is flying with a speed of 400 km/h along bearing 145°. The aircraft is buffeted by a strong wind of magnitude 80 km/h blowing from bearing 240°.

a) Draw a sketch to indicate the actual direction of the aircraft.



✓✓

b) Find the ground speed and direction of the aircraft



$r^2 = 400^2 + 80^2 - 2.400.80.\cos 95$

$r = 414.7\ km/h \approx 415\ km/h$ ✓✓

$\frac{\sin\theta}{80} = \frac{\sin 95}{r}$

$\theta = 11°$ ✓

$\text{Bearing} = 145 - 11 = 134°$ ✓