Topic: Calculations with Matrices

Time: 45 mins Marks: /45 marks

No calculator allowed

Question One: [1, 3: 4 marks]

If $A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 9 \\ 7 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 0 \\ 5 & 5 \end{bmatrix}$ evaluate:

a) $A + B$

b) $2A + 4B - C$

Question Two: [2, 2, 2, 2: 8 marks]

If $A = \begin{bmatrix} 4 & -2 \\ 6 & 1 \\ 7 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 1 & -1 & 4 \\ 9 & 10 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$ evaluate, where possible, each of the following. For those that cannot be calculated, clearly explain your reasoning.

a) AB

b) CA

c) B^2

d) DB

Question Three: [4, 2: 6 marks]

If $\boldsymbol{A} = \begin{bmatrix} 2 & 0 & 4 \\ 1 & 0 & 3 \\ -3 & 0 & -12 \end{bmatrix}$

a) Find **B** such that $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{I}$ $(The\ identity\ matrix)$

b) Find **C** such that $\boldsymbol{A} - \boldsymbol{C} = O$ $(The\ 3 \times 3\ zero\ matrix)$. Explain what you notice about matrix **C**.

Question Four: [7 marks]

Find the values of the variables given that $\begin{bmatrix} m^2 - 2 & 3m + n \\ 4 & p^3 \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 4 & 27 \end{bmatrix}$

Question Five: [1, 2, 4: 7 marks]

a) Find the value of P, a scalar, such that $P\begin{bmatrix} 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -12 \\ 9 \end{bmatrix}$

b) Find the value of matrix Q such that Q + $\begin{bmatrix} 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -12 \\ 9 \end{bmatrix}$

c) If $A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$, show that $A^2 - 3A = \begin{bmatrix} -3 & 2 \\ -2 & -1 \end{bmatrix}$

Question Six: [2,2: 4 marks]

In the AFL preseason, points are awarded slightly differently. 6 points are awarded for a goal, 1 point for a behind and 9 points are awarded for a super goal, which is a goal scored from a distance greater than fifty metres.

At the end of the preseason;

Ricky had scored 12 goals, 30 behinds and 2 super goals,

Kevin had scored 9 goals, 18 behinds and no super goals and

Wayne had scored 10 goals, 9 behinds and 3 super goals.

a) Construct an appropriate 3x3 matrix to multiply with $\begin{bmatrix} 6 \\ 1 \\ 9 \end{bmatrix}$ in order to calculate the total number of points scored by each player during the preseason.

b) Hence calculate the total number of points scored by each player.

Question Seven: [3, 4, 2: 9 marks]

Jules runs a hot dog food van. Customers can order a hot dog on its own for $3, a hot dog with a drink for $4, or a hot dog, drink and chips for $5. On a busy Saturday night he received 20 orders for hot dogs alone, 35 orders for hot dogs with a drink and 40 orders for a hot dog, drinks and chips.

a) Show, using matrix calculations, Jules' total revenue for that particular Saturday night.

b) If each hotdog costs Jules $1.00, each drinks costs him $0.50 and each serve of chips costs $0.50, show using matrix calculations, the total costs of the products sold on that particular Saturday night.

c) If Jules has no other expenses calculate his total profit for that Saturday night.

education equals

Calculations with Matrices SOLUTIONS

Time: 45 mins | Marks: /45 marks

No calculator allowed

Question One: [1, 3: 4 marks]

If $A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 9 \\ 7 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 0 \\ 5 & 5 \end{bmatrix}$ evaluate:

a) $A + B$

$\begin{bmatrix} 5 & 6 \\ 10 & 8 \end{bmatrix}$ ✓

b) $2A + 4B - C$

$\begin{bmatrix} 4 & -6 \\ 6 & 10 \end{bmatrix} + \begin{bmatrix} 12 & 36 \\ 28 & 12 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 17 & 30 \\ 29 & 17 \end{bmatrix}$ ✓ ✓ ✓

Question Two: [2, 2, 2, 2: 8 marks]

If $A = \begin{bmatrix} 4 & -2 \\ 6 & 1 \\ 7 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 1 & -1 & 4 \\ 9 & 10 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$ evaluate, where possible, each of the following. For those that cannot be calculated, clearly explain your reasoning.

a) AB

$$\begin{bmatrix} 2 & 10 \\ 23 & 27 \\ 36 & 37 \end{bmatrix}$$

dimensions ✓

calculations ✓

b) CA

$$\begin{bmatrix} 26 & 9 \\ 117 & 1 \end{bmatrix}$$

dimensions ✓

calculations ✓

c) B^2

$$\begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 29 & 24 \\ 30 & 29 \end{bmatrix}$$

dimensions ✓

calculations ✓

d) DB

Cannot be calculated. ✓

$$\begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}$$ ✓

$2 \times 1 \quad 2 \times 2$

Dimensions do not allow for multiplication. The number of columns in D does not equal the number of rows in B.

Question Three: [4, 2: 6 marks]

If $\boldsymbol{A} = \begin{bmatrix} 2 & 0 & 4 \\ 1 & 0 & 3 \\ -3 & 0 & -12 \end{bmatrix}$

a) Find **B** such that $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{I}$ $(The\ identity\ matrix)$

$$\begin{bmatrix} 2 & 0 & 4 \\ 1 & 0 & 3 \\ -3 & 0 & -12 \end{bmatrix} + \begin{bmatrix} -1 & 0 & -4 \\ -1 & 1 & -3 \\ 3 & 0 & 13 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) Find **C** such that $\boldsymbol{A} - \boldsymbol{C} = O$ $(The\ 3 \times 3\ zero\ matrix)$. Explain what you notice about matrix **C**.

$$\begin{bmatrix} 2 & 0 & 4 \\ 1 & 0 & 3 \\ -3 & 0 & -12 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 4 \\ 1 & 0 & 3 \\ -3 & 0 & -12 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

C = A

Question Four: [7 marks]

Find the values of the variables given that $\begin{bmatrix} m^2-2 & 3m+n \\ 4 & p^3 \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 4 & 27 \end{bmatrix}$

$m^2 - 2 = 2$ ✓

$m^2 = 4$ ✓

$m = \pm 2$ ✓

$3m + n = 12$

$6 + n = 12$

$n = 6$ ✓

Or

$-6 + n = 12$

$n = 18$ ✓

$p^3 = 27$

$p = 3$ ✓✓

Question Five: [1, 2, 4: 7 marks]

a) Find the value of P, a scalar, such that $P\begin{bmatrix} 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -12 \\ 9 \end{bmatrix}$

P = -3

b) Find the value of matrix Q such that Q + $\begin{bmatrix} 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -12 \\ 9 \end{bmatrix}$

$\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -12 \\ 9 \end{bmatrix}$

$\begin{bmatrix} -16 \\ 12 \end{bmatrix}$

c) If $A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$, show that $A^2 - 3A = \begin{bmatrix} -3 & 2 \\ -2 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} -3 & 2 \\ -2 & -1 \end{bmatrix}$

Question Six: [2,2 : 4 marks]

In the AFL preseason, points are awarded slightly differently. 6 points are awarded for a goal, 1 point for a behind and 9 points are awarded for a super goal, which is a goal scored from a distance greater than fifty metres.

At the end of the preseason;

Ricky had scored 12 goals, 30 behinds and 2 super goals,

Kevin had scored 9 goals, 18 behinds and no super goals and

Wayne had scored 10 goals, 9 behinds and 3 super goals.

a) Construct an appropriate 3x3 matrix to multiply with $\begin{bmatrix} 6 \\ 1 \\ 9 \end{bmatrix}$ in order to calculate the total number of points scored by each player during the preseason.

$$\begin{array}{l} \\ \text{Ricky} \\ \text{Kevin} \\ \text{Wayne} \end{array} \begin{array}{c} \begin{array}{ccc} \text{G} & \text{B} & \text{S} \end{array} \\ \begin{bmatrix} 12 & 30 & 2 \\ 9 & 18 & 0 \\ 10 & 9 & 3 \end{bmatrix} \end{array}$$

✓ values ✓ labels, columns and rows

b) Hence calculate the total number of points scored by each player.

$$\begin{bmatrix} 12 & 30 & 2 \\ 9 & 18 & 0 \\ 10 & 9 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \\ 9 \end{bmatrix} = \begin{bmatrix} 120 \\ 72 \\ 96 \end{bmatrix}$$ ✓

Ricky scored 120 points, Kevin scored 72 points and Wayne scored 96 points. ✓

Question Seven: [3, 4, 2: 9 marks]

Jules runs a hot dog food van. Customers can order a hot dog on its own for $3, a hot dog with a drink for $4, or a hot dog, drink and chips for $5. On a busy Saturday night he received 20 orders for hot dogs alone, 35 orders for hot dogs with a drink and 40 orders for a hot dog, drinks and chips.

a) Show, using matrix calculations, Jules' total revenue for that particular Saturday night.

$$\begin{bmatrix} 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 20 \\ 35 \\ 40 \end{bmatrix} = (60 + 200 + 140) = \$400$$

✓ ✓ ✓

b) If each hotdog costs Jules $1.00, each drinks costs him $0.50 and each serve of chips costs $0.50, show using matrix calculations, the total costs of the products sold on that particular Saturday night.

$$\begin{bmatrix} 1 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 95 \\ 75 \\ 40 \end{bmatrix} = (95 + 37.50 + 20) = \$152.50$$

✓ ✓ ✓ ✓

c) If Jules has no other expenses calculate his total profit for that Saturday night.

400 – 152.50 = $247.50 ✓✓