Manjimup SHS 2015 Year 11 Mathematics Methods Test 7 Introduction to Calculus	Name: ANSWERS Score: out of **29**

/55

Non-Calculator Section (No calculator nor notes, formula sheet is provided)

Time: 30 minutes   Marks: 29 marks

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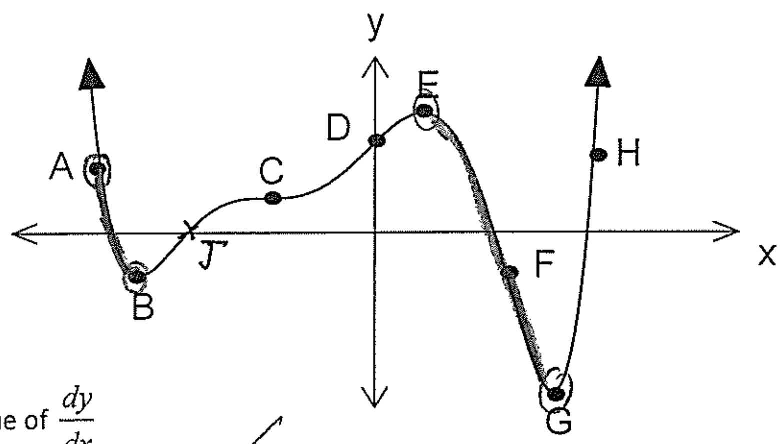
1. [4 marks]

Consider the graph of the function y = f(x). Use the features of this graph to answer the following questions.



a) List all stationary points.

B , C , E , G ✓

b) State the points of inflection.

C , D , F , J ✓

c) Highlight the sections with a negative value of $\frac{dy}{dx}$ ✓

d) Which point on this curve has the properties that f(x) > 0 and $f''(x) < 0$? E ✓

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2. [2,2,2 = 6 marks]

Find the derivative of the following functions. Express with positive indices.

a) y = $x^4 + \frac{x^2}{2} - x + 2$

$\boxed{y' = 4x^3 + x - 1}$ ✓✓

b) y = $\sqrt[3]{x}$

$y = x^{\frac{1}{3}}$

$\Rightarrow y' = \frac{1}{3}x^{-\frac{2}{3}}$

$\boxed{y' = \frac{1}{3x^{\frac{2}{3}}}}$ ✓✓

$= \frac{1}{3\sqrt[3]{x^2}}$

c) y = $\frac{1}{x^2} + \sqrt{x^3}$

$y = x^{-2} + x^{\frac{3}{2}}$

$\Rightarrow y' = -2x^{-3} + \frac{3}{2}x^{\frac{1}{2}}$

$\boxed{y' = \frac{-2}{x^3} + \frac{3\sqrt{x}}{2}}$ ✓✓

3. [4 marks]

Find the equation of the tangent line to the curve y = $x^2 + \frac{1}{x}$ at the point (1,2).

$$y = x^2 + x^{-1}$$

$$y' = 2x - \frac{1}{x^2} \quad \checkmark$$

$$y'(1) = 2 - 1$$
$$= 1 \quad \checkmark$$

$\Rightarrow$ gradient at $x=1$ is 1.

$$y = 1x + c$$

$$(1,2) \rightarrow \quad 2 = 1 + c$$
$$c = 1 \quad \checkmark$$

$\Rightarrow$ tangent line $\boxed{y = x + 1}$ ✓

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4. [4 marks]

Using first principles, $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, show that the derivative of y = 3x^2 is 6x.

$$f(x+h) = 3(x+h)^2$$
$$= 3x^2 + 6xh + 3h^2$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h} \quad \checkmark\checkmark$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h)}{\cancel{h}} \quad \checkmark$$

$$= 6x + 3(0)$$

$$= 6x \text{ as required.} \quad \checkmark$$

Manjimup SHS 2015 Year 11 Mathematics Methods Test 7 Introduction to Calculus	Name:			
	Score:		out of	26

Calculator Section (Calculators and 1 page (A4) of notes permitted, formula is sheet provided)

Time: 30 minutes **Marks: 26 marks**

7. [4,1 = 5 marks]

a) Determine the rules for the lines that are tangent to the curve $y = x^2 - 5x - 24$ where it crosses the x – axis.

$0 = x^2 - 5x - 24$

$0 = (x-8)(x+3)$ ✓

$x = 8$ or -3. ✓

tanline at $x = 8$. ✓

$\boxed{y = 11x - 88.}$

tanline at $x = -3$.

$\boxed{y = -11x - 33.}$ ✓

b) Find where these tangent lines meet.

$11x - 88 = -11x - 33$

$22x = 55$

$x = 2.5$

so at $y = -60.5$. ✓

Intersect at $\boxed{(2.5, -60.5)}$.

5. [1,2 = 3 marks]

A student types this expression on a Casio display.

$$\lim_{h\to 0}\left(\frac{\sqrt{4+h}-\sqrt{4}}{h}\right)$$

a) What function f(x) are they considering?

$y = f(x) = \sqrt{x}$. ✓

b) Find the value of the expression on the calculator display.

Find the derivative of $y = \sqrt{x}$ at $x = 4$.

$y' = \frac{1}{2\sqrt{x}}$ $\quad y'(4) = \frac{1}{2\sqrt{4}}$ ✓

$= \frac{1}{4}$. ✓

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6. [4,2,2 = 8 marks]

a) Use methods of calculus to find the coordinates of the stationary points of the function $f(x) = 3x^2 - x^3$

$f'(x) = 6x - 3x^2 = 0$ ✓

$\Rightarrow 3x(2-x) = 0$

$\Rightarrow x = 0$ and $x = 2$ ✓✓

$3(2)^2 - 2^3 = 4$

$(0, 0)$ ✓
$(2, 4)$ are the stationary points.

b) Find the coordinates of the local maximum point..

$f''(x) = 6 - 6x$ ✓

$f''(0) = 6 \Rightarrow$ concave up $\Rightarrow (0,0)$ is minimum point

$f''(2) = -6 \Rightarrow$ concave down $\Rightarrow$ $\boxed{(2,4) \text{ is the local maximum}}$ ✓

c) Find the coordinates of the point of inflection.

$f''(x) = 6 - 6x = 0$

$\Rightarrow x = 1$ ✓

$f(1) = 3(1)^2 - 1^3 = 2$

Po at $(1, 2)$ ✓

$\boxed{(1,2) \text{ is the the point of inflection.}}$

END OF SECTION 1

8. [1,2,1,2,2,3 = 11 marks]

A bullet is fired upwards. After t seconds the height of the bullet is found from the rule $H(t) = 150t - 4.9t^2 + 2$ where t is measured in seconds and H in metres.

a) Find the height of the bullet after 5 seconds.

$H(5) = \boxed{629.5\text{m.}}$ ✓

b) Determine the average speed of the bullet during the fifth second. Indicate your method.

$\dfrac{H(5) - H(4)}{5 - 4}$ ✓ $= \boxed{105.9 \text{ m/s}}$ ✓

The speed of the bullet is the instantaneous rate of change of the height of the bullet.

c) Find the speed of the bullet after 5 seconds.

Diff^{e} and sub in 5 i.e $\left.\dfrac{dH}{dt}\right|_{t=5}$ $\boxed{101 \text{ m/s}}$ ✓

d) Find a rule for the speed of the bullet at any time t.

$s(t) = H'(t) = \boxed{150 - 9.8t}$ ✓✓

e) Find the maximum height of the bullet, to the nearest metre. Indicate your method.

$f$max or $H'(t) = 0$. ✓ $\boxed{1150\text{m.}}$ ✓

f) Determine the bullet's speed as it hits the ground, on the way down, to 2 decimal places.

solve $(H(t) = 0)$. ✓

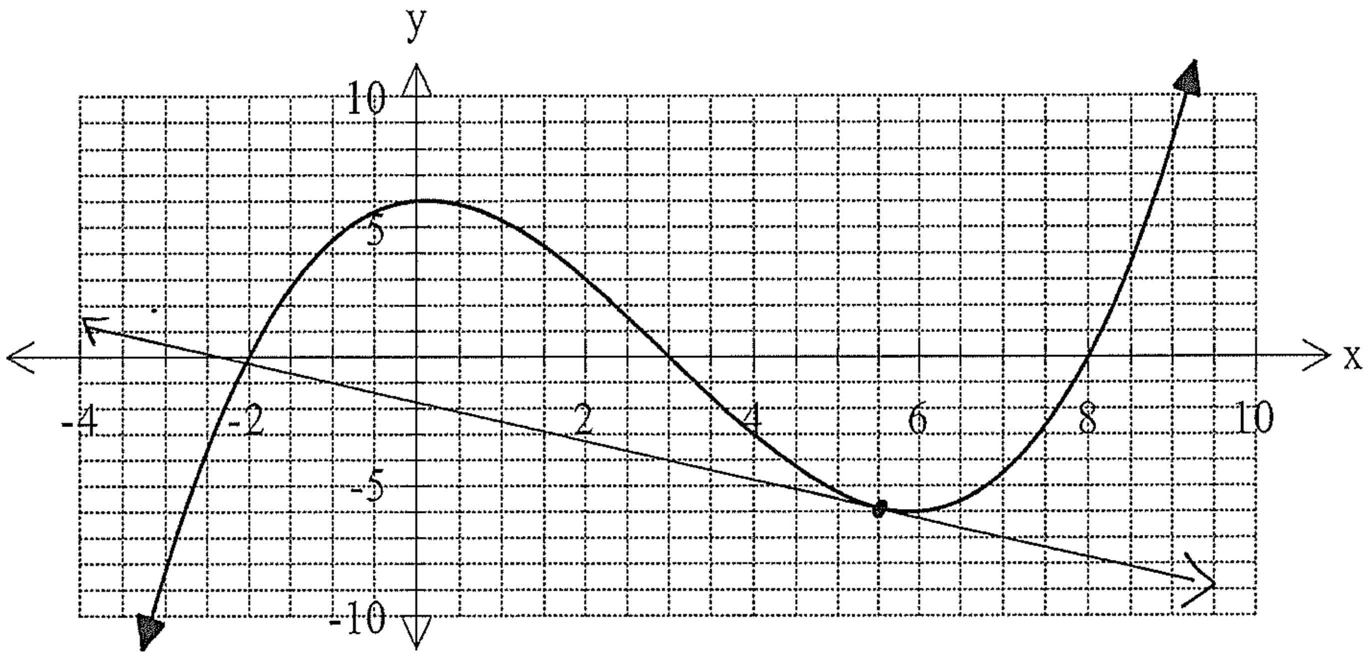
$\Rightarrow t = 30.626$ s. ✓

$s(30.626) = -150.13$ m/s.

speed $\boxed{150.13 \text{ m/s.}}$ ✓

9. [1,1,2,2 = 6 marks]

A sketch of f(x) = $\frac{(x+2)(x-3)(x-8)}{8}$ has been provided below.



a) What is the average (mean) of the two positive roots of f(x)?

$\frac{3+8}{2} = \boxed{5.5}$ ✓

b) On the graph above sketch a tangent line to the curve for the value of x that you calculated in part (a).

✓

c) Find the rule for this tangent line you drew above, that goes through the point where the x-value Is the average of the two positive roots.

tan line f(x) at x = 5.5. $\boxed{y = -\frac{25x}{32} - \frac{25}{16}.}$ ✓✓

A student has speculated that the tangent line to the curve at the average of two roots also intersects with the third root.

d) Use your answer from part c to show whether this statement is true or false for f(x).
Does this example support the student's speculation or not? Discuss.

When $y=0$, $x=-2$.

✓

check: $\frac{-25(-2)}{32} - \frac{25}{16} = 0$

Yes, it supports the student's conclusion ✓

10. [4 marks]

The function $y = x^3 + ax + b$ has a local minimum point at (2,3).
Use differentiation to find the values of a and b.

$$y' = 3x^2 + a.$$

$$y'(2) = 0 \Rightarrow 0 = 3(2)^2 + a$$ ✓

$$\Rightarrow \underline{a = -12}$$ ✓

$$(2,3) \rightarrow. \quad 3 = 2^3 - 12(2) + b.$$

$$\Rightarrow 3 = -16 + b.$$ ✓

$$\Rightarrow \underline{b = 19.}$$ ✓

END OF SECTION 2