Section One : Calculator Free **Marks 25**
Time Allowed 25 minutes

Question 1

[3 marks]

Express each of the following as the product of consecutive factors, fully simplifying your answer.

a. $4!$

$4 \times 3 \times 2 \times 1.$

b. $\frac{8!}{5!}$

$8 \times 7 \times 6.$

c. $\frac{n!}{(n-2)!}$

$n \times (n-1)$

Question 2

[3 marks]

Find the integer n if ${}^{n}P_{n-7} = 720$.

$\cancel{\frac{n!}{(n-7)!}} = \frac{n!}{n-(n-7)!} = \frac{n!}{7!} = 720$

$n! = 720 \times 7!$ $n! = 10!$ $n = 10$

Question 3

[3 marks]

Show that:

$${}^{n}C_{n-r} = {}^{n}C_{r}$$

${}^{n}C_{n-r} = \frac{n!}{(n-r)!\,(n-(n-r))!} = \frac{n!}{(n-r)!\,r!} = {}^{n}C_{r}.$

Question 4

[2 marks]

A class has 15 students. Explain why there must be at least 3 students who are born on the same day of the week.

Assume 1 born every day then
2 born every day gives 14 so 1 other
means three born on same day
pigeon hole principle

Question 5

[10 marks]

Let $\boldsymbol{u} = -\boldsymbol{i} + 2\boldsymbol{j}$ $\quad \boldsymbol{v} = -2\boldsymbol{i} - \boldsymbol{j}$ $\quad$ and $\boldsymbol{w} = 3\boldsymbol{i} + 9\boldsymbol{j}$

(a) Show that **u** and **v** are perpendicular.

$u.v = -1 \times -2 + 2 \times -1 = 2 - 2 = 0$

$\therefore$ Scalar product $= 0 \therefore$ perpendicular

(b) Find the acute angle between $\boldsymbol{u}$ and $\boldsymbol{w}$.

$\frac{a.b}{|a||b|} = \cos\theta \qquad \frac{u.w}{|u||w|} = \frac{-3+18}{\sqrt{5} \times \sqrt{90}} = \frac{15}{\sqrt{5} \times 3\sqrt{10}} = \frac{15}{15\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}} \quad \therefore \quad \theta = 45^\circ$

(c) Find the vector projection of $\boldsymbol{w}$ onto $\boldsymbol{v}$.

$\hat{v} = \frac{-2i}{\sqrt{5}} - \frac{j}{\sqrt{5}} \qquad (w.\hat{v})\hat{v}$

$= \left(\frac{-6}{\sqrt{5}} + \frac{-9}{\sqrt{5}}\right)\frac{-2i}{\sqrt{5}} - \frac{j}{\sqrt{5}}$

$= \frac{-15}{\sqrt{5}}\left(\frac{-2i}{\sqrt{5}} - \frac{j}{\sqrt{5}}\right) = 6\underline{i} + 3\underline{j}$

(d) $\boldsymbol{w}$ is the vector projection of $\lambda(2\boldsymbol{i} + \boldsymbol{j})$ onto $\boldsymbol{w}$. Find λ.

$\hat{w} = \frac{\underline{i}}{\sqrt{10}} + \frac{3}{\sqrt{10}}\underline{j} \qquad \frac{\lambda 2}{\sqrt{10}} + \frac{\lambda}{\sqrt{10}} = \frac{3\lambda}{\sqrt{10}}$

$\frac{3\lambda}{\sqrt{10}}\left(\frac{i}{\sqrt{10}} + \frac{3j}{\sqrt{10}}\right) = \frac{3\lambda}{10}i + \frac{9\lambda}{10}\underline{j} \Rightarrow \frac{3\lambda}{10} = 3 \quad \frac{9\lambda}{10} = 9$

$\underline{\lambda = 10}$

Question 6

[2 marks]

Circle the entries in Pascal's Triangle below that show:

a. $\binom{7}{4}$

b. $^{3}C_{2}$

							1							
						1		1						
					1		2		1					
				1		3		3		1				
			1		4		6		4		1			
		1		5		10		10		5		1		
	1		6		15		20		15		6		1	
1		7		21		35		35		21		7		1

Question 7

[2 marks]

In a group of students there are 15 who play basketball and 17 who play netball. If 8 students played both sports, how many play at least one spot?

$15+17-8 = 32-8 = \underline{\underline{24}}$

Section Two : Calculator Assumed **Mark 29**
Time Allowed 35 minutes **Name:______________________**

Question 8

[6 marks]

How many ways can the letters of the word MATHEMATICAL be arranged in a row?

$\frac{12!}{2!3!2!} = 19958400$

How many of these arrangements start with a C and end in an L?

$C \_ \frac{10!}{2!3!2!} \ldots L = 151200$

How many of these arrangements start or end with a C or L?

$2a(b) \quad 302400$

Question 9

[7 marks]

The year 11 student leaders consisting of 3 girls and 4 boys are to be arranged in a line for a photo.

(a) How many possible arrangements are there?

$7! = 5040$

(b) How many arrangements are there if all the girls are next to each other?

$3! \times 5! = 720$

(c) How many arrangements are there if all the girls are next to each other and all the boys are next to each other?

$3! \times 4! \times 2! = 288$

(d) How many arrangements if no two girls or no two boys are next to each other?

$3! \times 4! = 144$

Question 10

[6 marks]

How many positive integers less than 1000 are:

(a) divisible by 2?

499.

(b) divisible by 3?

333

(c) divisible by 2 and 3?

166.

(d) divisible by 2 or 3?

$499 + 333 - 166 = 666$.

(e) have 3 digits and divisible by 2 and 3?

<100 2. 49.

3 33

2 & 3 = 16.

2 or 3 = 66.

$\therefore$ ~~666 − 66 = 600~~

$166 - 16 = 150$

Question 11

[8 marks]

A committee of 6 students is to be formed from 10 Year 10 students and 11 Year 11 students. How many different committees can be formed if:

(a) there is no restriction on who is on the committee

$${}^{21}C_6 = 54264.$$

(b) the committee consists of only year 11 students

$${}^{11}C_6 = 462.$$

(c) the committee has 3 year 10 students and 3 year 11 students

$${}^{10}C_3 \times {}^{11}C_3 = 19800.$$

(d) the committee has at least 1 year 10 student?

$$54264 - 462 = 53802$$

(e) the committee has at least 4 year 11 students?

$${}^{11}C_4 \times {}^{10}C_2 + {}^{11}C_5 \times {}^{10}C_1 + {}^{11}C_6.$$

$$= 19932$$

Question 12

[8 marks]

Daniel needs to create a four digit password using the letters A, B, C, D, E and the digits 1, 2, 3, 4.

(a) How many passwords can be created if the characters can be repeated?

$9^4 = 6561$ ✓

(b) How many passwords can be created if the password has two letters followed by two digits the letters can be repeated but digits can only be used once?

$5 \times 5 \times 4 \times 3 = 300$ ✓

If no character is repeated.

(c) How many passwords can be created?

${}^9P_4 = 3024$ ✓

(d) How many passwords can be created if the password contains either only letters or only numbers?

${}^5P_4 + {}^4P_4 = 144$ ✓✓

(e) How many password have two letters followed by two digits?

$5 \times 4 \times 4 \times 3 = 240$ ✓✓

(f) How many passwords have both letters and digits?

$3024 - 144 = 2880$ ✓