Full Name: SOLUTIONS

MATHEMATICS
APPLICATIONS

Test 2 – Sequences

Chapter 1 and 2

Semester 1 2018

Section One - Calculator Free

Time allowed for this section
Working time for this section: 25 minutes
Marks available: 27 marks

Material required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

Important note to candidates
No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (2 marks)
   Calculate the sum of all the multiples of 5 **between** 1 and 50.

   5 10 15 20 25 30 35 40 45

   225

2. (5 marks)
   A sequence of numbers is described by the recursive equation $T_{n+1} = T_n - 8$, where $T_4 = 35$.

   a. Determine T_6 [1]

   $T_6 = 35 - 8 - 8$
   $= 19$

   b. Determine T_1 [1]

   $T_1 = 35 + 8 + 8 + 8$
   $= 59$

   c. State a rule for the n^th term of this sequence. [2]

   Brackets!

   $T_n = 59 + (n-1)(-8)$
   $= 67 - 8n$

   no simplification necessary this time

   d. Determine T_{1001} [1]

   $T_{1001} = -7941$

3. (5 marks)

a. A sequence is defined by $T_{n+1} = 2T_n$, where $T_1 = 9$.

i. State a rule for the n[th] term of this sequence in simplified form. [1]

$T_n = 9 \times 2^{n-1}$

ii. Determine T_5 [1]

$T_5 = 9 \times 2^4$

$= 144$

b. The first-order recurrence relation $t_{n+1} = bt_n + c$ was used with $t_1 = 3$ to calculate $t_2 = 4$ and $t_3 = 7$. Determine the values of *b, c* and t_4. [3]

$4 = 3b + c$

$7 = 4b + c$

$b = 3$

$c = -5$

$T_4 = 3 \times 7 - 5$

$= 16$

4. (8 marks)

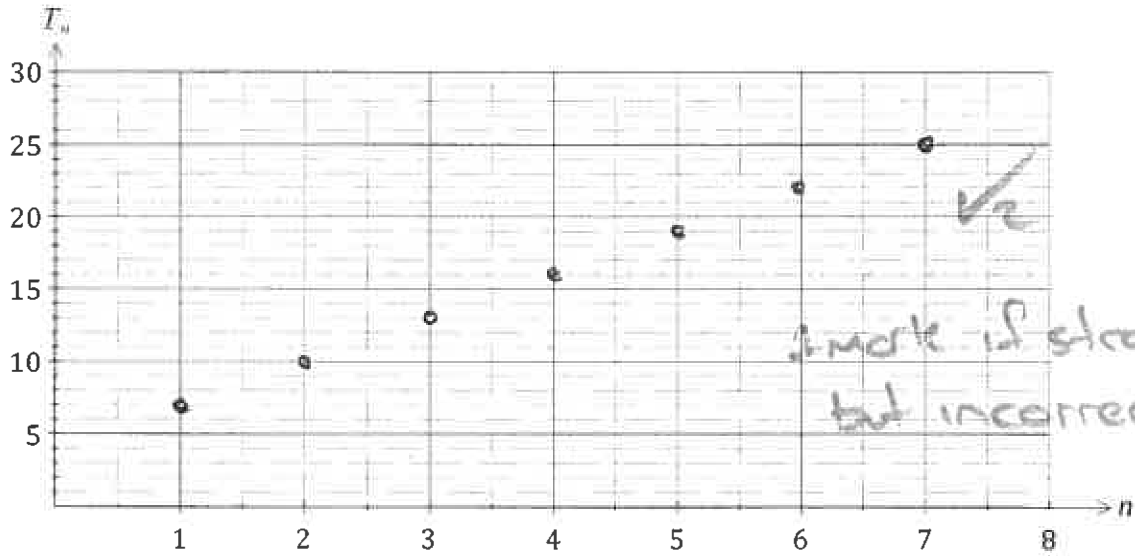
The number of laptop computers, T_n, that were brought to a school IT department for recharging during week n of the school year can be described recursively by the rule

$$T_{n+1} = T_n + 3, \qquad T_4 = 16$$

a. Use the rule to complete the table below. [2]

n	1	2	3	4	5	6	7
T_n	7	10	13	16	19	22	25

b. Display the terms of the sequence from the table on the graph below. [2]



1 mark if straight but incorrect

c. A rule to determine the number of laptops brought for recharging during week n can also be written in the form $T_n = an + b$. Determine the values of a and b. [2]

$T_n = 7 + (n-1) \times 3$

$= 3n + 4$

$a = 3$

$b = 4$

d. If the pattern continued, determine the number of the week during which the number of laptops brought in for recharging first exceeds 50. [2]

$50 = 3n + 4$

$46 = 3n$

$n = 15.\bar{3}$

$\therefore$ during week 16

5. (7 marks)

The sixth term of an arithmetic sequence is double its fourth term. The first term of the sequence is 20 and the common difference is d.

a. Show that [3]

$$T_4 = 2 \times d$$

$T_6 = 2T_4$ ✓ $\quad T_6 = T_4 + 2d$ ✓

$2T_4 = T_4 + 2d$

$T_4 = 2d$ ✓

b. Hence, find the recursive rule for the sequence. [4]

$T_n = 20 + (n-1)d$

For T_4

$2d = 20 + 3d$ ✓

$d = -20$ ✓

$\therefore T_{n+1} = T_n - 20, \quad T_1 = 20$

✓ ✓

Extra space for working if required

End of Section One

Full Name: SOLUTIONS

GREENWOOD COLLEGE

MATHEMATICS
APPLICATIONS

Test 2 – Sequences

Chapter 1 and 2

Semester 1 2018

Section Two - Calculator Assumed

Time allowed for this section

Working time for this section:	30 minutes
Marks available:	29 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items:	pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters
Special items:	drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

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4. (2 marks)

The nth term of an arithmetic sequence is given by the rule:

$$t_n = 10 - 4n$$

Determine the rule for the recurrence relation.

$T_1 = 6$

$T_2 = 2$ $\quad d = -4$

$T_3 = -2$ $\quad a = 6$

$T_{n+1} = T_n - 4 \ , \quad T_1 = 6$

5. (4 marks)

An arithmetic sequence has a second term of 5 and a ninth term of 26. Determine the rule for the nth term of the sequence and state T_{38}

$T_2 = 5$

$T_9 = 26$) 21 7 terms

$\frac{21}{7} = 3$ (d)

$T_1 = 2$

$T_n = 2 + (n-1) \times 3$

$= 3n - 1$

$T_{38} = 3(38) - 1$

$= 113$

6. (8 marks)

A plant grew from a seed to a height of 120 cm in its first year. The growth of the plant in subsequent years is expected to be 60% of its growth in the previous year.

a. Determine

i. The growth of the plant during the second year. [1]

72 cm

ii. The height of the plant after two years. [1]

192 cm

The growth of the plant during the n[th] year can be given by $T_{n+1} = 0.6T_n$, where $T_1 = 120$.

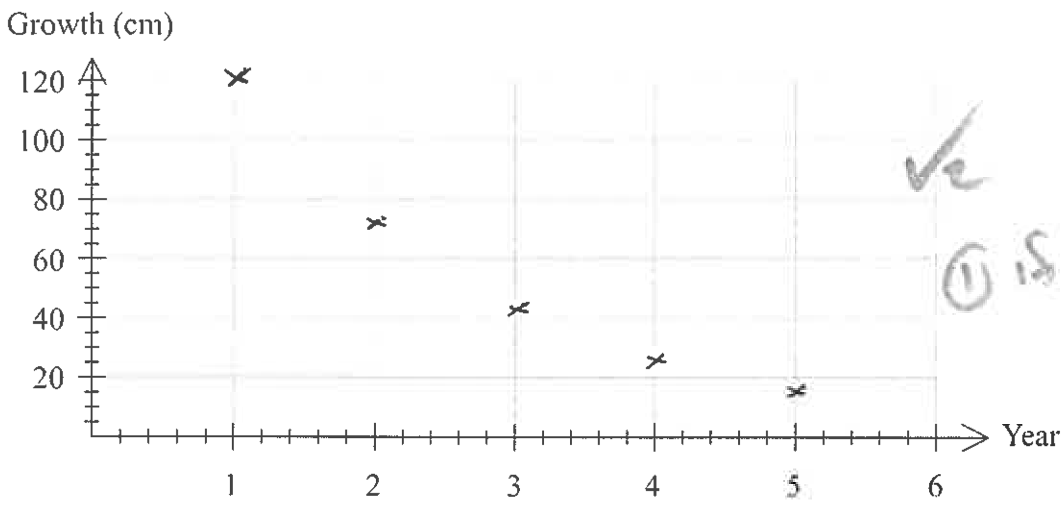
b. Complete the growth table below. [2]

Year	1	2	3	4	5
Growth (cm)	120	72	43.2	25.9	15.6

✓2

($\frac{1}{2}$ each)

c. Plot the annual growth of the plant on the axes below for the first five years. [2]



✓2

① if joined

d. In which year is the growth of the tree first less than 1 cm? [1]

11th year ✓ (after 10 years)

e. Describe height of the tree in the long-term. [1]

- approaching a maximum height of 3 m. ✓
- increasing in height but at a decreasing rate.
- something reasonable that indicates steady state.

7. (9 marks)

A fish farm is stocked with 5000 fish. The owners plan to sell 25% of the fish stock throughout the year and then to re-stock the farm with an extra 300 fish at the end of the year. The fish stock, F_n , at the start of year n can be modelled by $F_{n+1} = 0.75F_n + 300$, where $F_0 = 5000$.

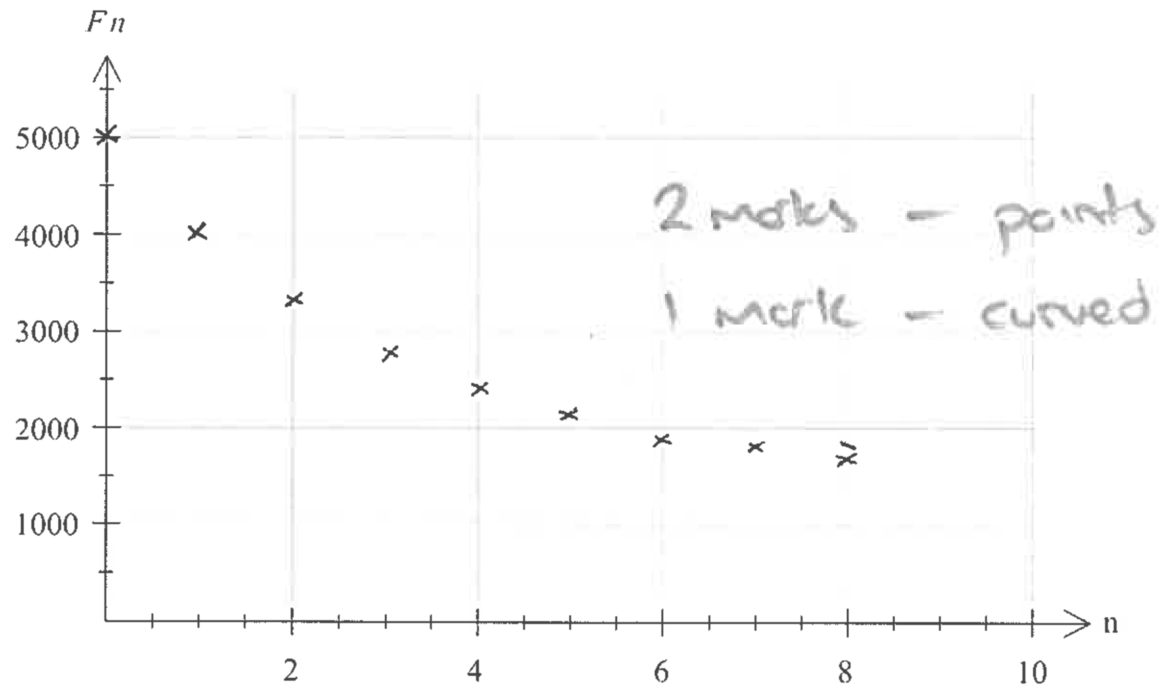
a. Explain the significance of the 0.75 in the model. [1]

75% left each year.
A reduction of 25% or similar.

b. Complete the table below for the first 8 years, **rounding values to the nearest ten.** [2]

n	0	1	2	3	4	5	6	7	8
F_n	5000	4050	3340	2800	2400	2100	1880	1710	1580
				2810	2410	2110	accept either		

c. Graph the fish stock at the start of the first 8 years on the axes below. [3]



d. Comment on how the size of the fish stock is changing over the first 8 years. [1]

Fish stock is decreasing but at a decreasing rate. or similar.

e. Calculate the expected fish stock after 20 years, and comment on the long-term size of the fish stock according to this model. [2]

From calculator, over time approaching 1200

(accept things like $F_{20} = 1212$)

8. (6 marks)
The sum of the first two terms of a geometric sequence is 90 and the sum of the first three terms of the same sequence is 105. Find the geometric sequence(s) which satisfy the stated conditions.

$$\left.\begin{array}{l} a + ar = 90 \\ a + ar + ar^2 = 105 \end{array}\right\} \Rightarrow ar^2 = 15$$

$$a = \frac{15}{r^2}$$

$$\frac{15}{r^2} + \frac{15}{r^2} \times r = 90$$

$$90r^2 - 15r - 15 = 0$$

$$r = -\frac{1}{3} \text{ or } \frac{1}{2}$$

when $r = -\frac{1}{3}$, $a = 135$

$135, -45, 15, \ldots$

when $r = \frac{1}{2}$, $a = 60$

$60, 30, 15, \ldots$

Extra space for working if required

End of Test