

Full Name: SOLUTIONS



Ellenbrook
CHRISTIAN COLLEGE

**MATHEMATICS
METHODS**

Test 3 – Counting and Probability

Chapters 2 and 5

Semester 1 2015

Calculator Assumed

Time allowed for this section

Working time for this section: 45 minutes

Marks available: 40 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (11 marks)

Consider the following sets:

$$U = \{\text{counting numbers from 1 to 20}\}$$

$$A = \{\text{prime numbers in the universal set } U\} \quad 2 \ 3 \ 5 \ 7 \ 11 \ 13 \ 17 \ 19$$

$$B = \{\text{factors of 20}\} \quad 1 \ 2 \ 4 \ 5 \ 10 \ 20$$

$$C = \{\text{multiples of 4 in the universal set}\} \quad 4 \ 8 \ 12 \ 16 \ 20$$

a. Determine:

i. $n(A)$

8

[1]

ii. $B \cap C$

{4, 20}

[2]

iii. $\overline{A \cup B}$

{6, 8, 9, 12, 14, 15, 16, 18}

[2]

iv. $n(\overline{C})$

15

[2]

b. State whether the following statements are True (T) or False (F)

i. $C \subset B$

F

[1]

ii. $A \cap B \cap C = \emptyset$

FT

[1]

iii. $9 \notin A$

T

[1]

iv. $\{4, 5\} \subset B$

T

[1]

2. (4 marks)

Two hundred (200) people were asked whether they had flown internationally (I) or domestically (D) in the past 12 months.

- The ratio of those who had flown domestically to those who had not was 3:1
- 10% had flown both internationally and domestically
- A fifth of those who had not flown domestically, had flown internationally

Use the above information to complete the two-way table below.

	Flown Internationally	Did not fly internationally	Total
Flown domestically	20	130	150
Did not fly domestically	10	40	50
Total	30	170	200

3. (4 marks)

Use your knowledge of Pascal's Triangle to answer the following question.

a. Expand and simplify $(x+m)^6$ 1 6 15 20 15 6 1

$$x^6 + 6x^5m + 15x^4m^2 + 20x^3m^3 + 15x^2m^4 + 6xm^5 + m^6$$

b. Expand and simplify $(n-3)^5$ 1 5 10 10 5 1

$$\begin{aligned} n^5 - 5n^4 \times 3 + 10n^3 \times 3^2 - 10n^2 \times 3^3 + 5n \times 3^4 - 3^5 \\ = n^5 - 15n^4 + 90n^3 - 270n^2 + 405n - 243 \end{aligned}$$

4. (7 marks)

Tom has 5 notes in his wallet: \$100, \$50, \$20, \$10 and \$5.

a. If he chooses two notes at random, how many combinations can he make?

[1]

$${}^5C_2 = \frac{5 \times 4}{2 \times 1} = 10$$

b. If he chooses three notes at random, how many combinations contain:

i. a \$5 note?

[2]

$$\frac{1 \quad 4 \quad 3}{2 \quad 1} = 6 \quad \text{or} \quad {}^1C_1 \cdot {}^4C_2 = 6$$

ii. no \$50 note?

[2]

10 combinations total $({}^5C_3)$

$$10 - 6 = 4 \text{ with no } \$50 \text{ note } ({}^4C_3)$$

iii. either a \$100 note or a \$50 note?

[2]

$${}^1C_1 \cdot {}^3C_2 + {}^1C_1 \cdot {}^3C_2 + {}^2C_2 \cdot {}^3C_1 = 9$$

5. (6 marks)

a. Hannah is getting married and wants to choose three of her 6 best friends to be her bridesmaids. How many choices does she have?

[2]

$${}^6C_3 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

b. Young Seth is going on a camp. He is allowed to pack four shirts, 3 pants and 2 pairs of shoes. If he has 10 shirts, 5 pants and 4 pairs of shoes to choose from, how many choices does he have?

[4]

$${}^{10}C_4 \times {}^5C_3 \times {}^4C_2 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \times \frac{5 \times 4 \times 3}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1}$$

$$= 2520$$

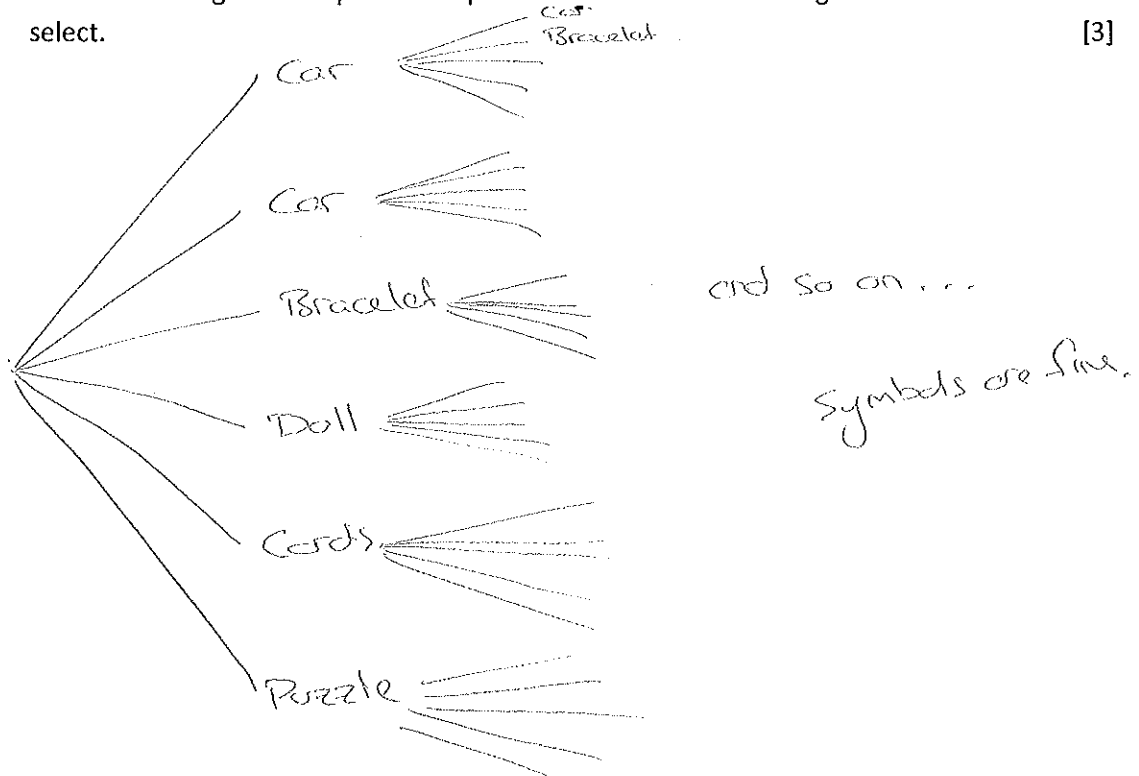
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6. (8 marks)

Tristan is the last one at a birthday party to choose two wrapped gifts from the 'lucky dip' bag. When it is his turn there are 2 toy cars, a bracelet, a doll, a pack of cards and a puzzle left in the bag.

He puts his hand in the bag without looking, chooses a wrapped gift, puts it aside and then returns his hand to the bag to choose his second wrapped gift.

a. Draw a tree diagram to represent all possible combinations of the gifts Tristan could select. [3]



b. Determine the probability that Tristan chooses

i. Two toy cars [1]

$$\frac{2}{30}$$

ii. A doll and a puzzle [1]

$$\frac{2}{30}$$

iii. A bracelet or a toy car [1]

$$\frac{24}{30}$$

iv. A puzzle if his first gift was a toy car [2]

$$\frac{2}{10}$$

End of Test

