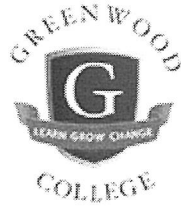Full Name: _____SOLUTIONS_____

MATHEMATICS
Methods Units 1 & 2

Test 1 – Relationships, Functions and Linear Functions

Chapters 3 and 4

Semester 1 2020

Section One - Calculator Free

Time allowed for this section
Working time for this section: 20 minutes
Marks available: 19 marks

Material required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

Important note to candidates
No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (7 marks: 1,1,1,1,1,2)

    Given that $f(x) = \sqrt{16 - x^2}$, determine:

    a) $f(0)$

    $f(0) = \sqrt{16} = \pm 4$ ✓

    b) $f(2)$

    $f(2) = \sqrt{12}$ ✓

    c) $f(-4)$

    $f(-4) = 0$ ✓

    d) the domain of $f(x)$

    $D\{x \in \mathbb{R} : -4 \le x \le 4\}$ ✓

    e) the range of $f(x)$

    $R\{f(x) \in \mathbb{R} : \cancel{f(x)} \ 0 \le f(x) \le 16\}$ ✓

    f) the value of x when $f(x) = \sqrt{12}$

    $\sqrt{12} = \sqrt{16 - x^2}$

    $12 = 16 - x^2$

    $x^2 = 4$

    $x = \pm 2$ ✓✓

2. (4 marks)

    The midpoint of $(a, 5)$ and $(7, b)$ is $(2, 11)$. Find a and b.

    $\dfrac{a + 7}{2} = 2 \implies a = -3$ ✓ ✓

    $\dfrac{b + 5}{2} = 11 \implies b = 17$ ✓ ✓

    $\dfrac{11}{}$

3. (8 marks: 3,3,2)

   Suggest **one** possible equation **each** for the lines $g(x)$ and $h(x)$ if:

   a) $g(x)$ and $h(x)$ are parallel to $x + 2y = 0$.   [3]

   $$y = -\frac{x}{2} \Rightarrow m = -\frac{1}{2}$$

   $$g(x) = -\frac{1}{2}x + 4$$
   $$h(x) = -\frac{1}{2}x - 7$$
   Same gradient as $x + 2y = 0$

   b) $g(x)$ and $h(x)$ meet at the point with coordinates $(0,4)$ and are perpendicular to each other.   [3]

   Easiest
   $$y = 4$$
   $$x = 0$$

   Another
   $$g(x) = 2x + 4$$
   $$h(x) = -\frac{1}{2}x + 4$$
   $M_1 M_2 = -1$
   Same y-int.

   c) $g(x)$ and $h(x)$ do not intersect.   [2]

   parallel

   $$g(x) = 4x - 1$$
   $$h(x) = 4x + 10$$
   Same gradient

End of Section One

Full Name: **SOLUTIONS**

MATHEMATICS
Methods Units 1 & 2

Test 1 – Relationships, Functions and Linear Functions

Chapters 3 and 4

Semester 1 2020

Section Two - Calculator Assumed

Time allowed for this section

Working time for this section: 35 minutes

Marks available: 36 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

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1. (3 marks)
   Find the equation of the line passing through $(-5, 4)$ and $(5, -16)$.

   $$\frac{y-4}{x+5} = \frac{-16-4}{5--5}$$

   $$y - 4 = -2(x+5) \Rightarrow y = -2x - 6$$

2. (5 marks)
   Determine the equation of the line that passes through the point $(-4, 2)$ and is perpendicular to the line with equation $3x - 4y - 5 = 0$. Give your answer in the form $ax + by + c = 0$.

   $$y = \frac{-3x+5}{-4} \Rightarrow m = \frac{3}{4} \quad m_\perp = -\frac{4}{3}$$

   $$y = -\frac{4}{3}x + b$$

   $$\text{sub } (-4, 2)$$

   $$2 = -\frac{4}{3}(-4) + b \qquad b = -\frac{10}{3}$$

   $$\therefore y = -\frac{4}{3}x - \frac{10}{3}$$

   $$4x + 3y + 10 = 0$$

3. (10 marks)
   State the natural domain and natural range for each of the functions/relations below.

Function/Relation	Natural Domain	Natural Range
$y = (x+1)^2 - 5$	$x \in \mathbb{R}$	$y \geq -5$
$y = \sqrt{x-5}$	$x \geq 5$	$y \geq 0$
$y = \dfrac{1}{x-1} + 3$	$x \in \mathbb{R}; x \neq 1$	$y \in \mathbb{R}; y \neq 3$
$(x+1)^2 + (y+1)^2 = 4$	$-3 \leq x \leq 1$	$-3 \leq y \leq 1$
$y^2 = 4(x-1)$	$x \geq 1$	$y \in \mathbb{R}$

4. (9 marks: 1,1,1,2,2,2)

Given that $f(x) = 3x - 1$, $g(x) = x^2 - 2x + 1$ and $h(x) = \begin{cases} x^2 & \text{for} \quad x > 2 \\ 4x & \text{for} \quad x \leq 2 \end{cases}$

a) $f(2)$

$3(2) - 1 = \underline{\underline{5}}$ ✓

b) $g(-2)$

$(-2)^2 - 2(-2) + 1 = \underline{\underline{9}}$ ✓

c) $h(2)$

$4(2) = \underline{\underline{8}}$ ✓

d) $g(f(x))$

$g(f(x)) = \underline{(3x-1)^2 - 2(3x-1) + 1}$ ✓✓

$= 9x^2 - 12x + 4$

e) $h(f(x))$

$h(f(x)) = \begin{cases} (3x-1)^2 & \checkmark \quad x > 2 \\ 4(3x-1), & x \leq 2 \end{cases}$ *

f) $g(h(4))$

$h(4) = 16$ ✓

$g(16) = 16^2 - 2(16) + 1$
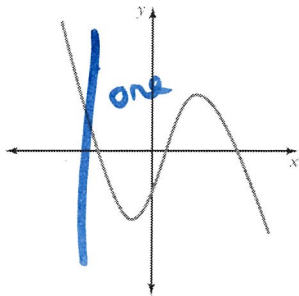
$= 256 - 32 + 1$

$= 225$ ✓

5. (9 marks: 3,3,3)

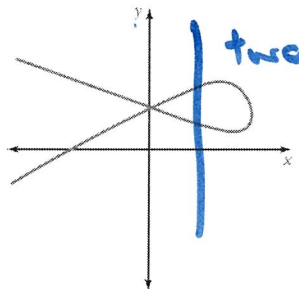Consider the three graphs shown below.

a) **State** whether each is a relation or a function.

b) **Show** justification with an appropriate test.

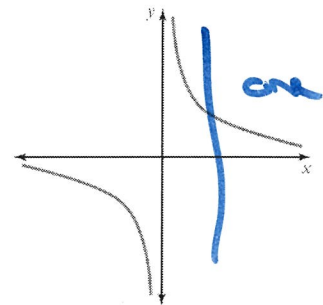c) **Describe** each graph using one of the terms one-to-one, one-to-many, many-to-one.

Graph A	Graph B	Graph C

Graph A: one

Function

many-to-one

Graph B: two

Relation

One-to-Many

Graph C: one

Function

One-to-one

End of Test