MINDARIE SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS NOW

MATHEMATICS: SPECIALIST 3 & 4

SEMESTER 2 2017

TEST 6

Resource Free

Reading Time: 2 minutes
Time Allowed: 24 minutes

Total Marks: 24

1. [1, 2, 3 marks]

A particle is travelling such that its acceleration is given by $\ddot{x} = -4x$. The particle has a maximum speed of 10 cm/sec. Initially, the particle has a negative velocity, a positive acceleration, and is 2.5 cm from its extreme point.

Determine the

(a) period of the motion.

$\ddot{x} = -k^2x \Rightarrow k = 2$

$T = \frac{2\pi}{2} = \pi$ ✓

(b) amplitude of the motion.

$v^2 = k^2(A^2 - x^2)$ ✓

$100 = 4(A^2 - 0)$ $\quad A = 5$ ✓

$25 = A^2$

(c) equation of motion, $x(t)$.

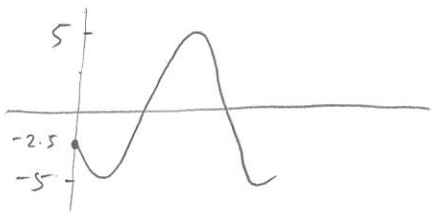
$x = -5\sin(2t + \alpha)$

$-2.5 = -5\sin\alpha$

$0.5 = \sin\alpha$

$\alpha = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\therefore x = -5\sin(2t + \frac{\pi}{6})$



-5 ✓

$2t$ ✓

$\frac{\pi}{6}/\frac{-\pi}{3}$ ✓

OR $x = -5\cos(2t + \alpha)$

$0.5 = \cos\alpha$

$\alpha = \frac{\pi}{3}$ or $\frac{-\pi}{3}$

$\therefore x = -5\cos(2t - \frac{\pi}{3})$

2. [2, 6 marks]

(a) A particle travels with a velocity, $v = \sin 2x$. Show that the acceleration of the particle is given by $a(x) = \sin 4x$.

$v \cdot \frac{dv}{dx} = \sin 2x \cdot 2\cos 2x$ ✓

$= 2\sin 2x \cos 2x$

$= \sin 4x$ ✓

(b) A second particle starts from the origin with a velocity of 6 metres/second. Its acceleration is given by $a(t) = e^{-0.2t}$.

Determine

(i) the limiting value of the velocity

$v(t) = \frac{-1}{0.2} e^{-0.2t} + C$ ✓

$= -5e^{-0.2t} + C$

$6 = -5 + C$

$C = 11$ ✓

$v(t) = 11 - 5e^{-0.2t}$

$t \to \infty \quad v \to 11$ m/s ✓

(ii) the exact value of the particle's position at $t = 5$ seconds,

$x(t) = 11t + 25e^{-0.2t} + C'$ ✓

$0 = 0 + 25 + C'$

$C' = -25$ ✓

$x(t) = 11t + 25e^{-0.2t} - 25$

$x(5) = 55 + 25e^{-1} - 25$

$= 30 + \frac{25}{e}$ ✓

3. [3 marks]

One hundred One-Eyed, One-Horned Flying Purple People-Eaters are captured, and the lengths of their horns are measured. These horns have a mean length of 27 cm with a standard deviation of 8 cm.

Given that $z_{87} \approx 1.5$, determine the 87% confidence interval for the mean length of the horns of all One-Eyed, One-Horned Flying Purple People-Eaters.

$27 \pm 1.5 \times \frac{8}{\sqrt{100}} = 27 \pm 1.5 \times 0.8$ ✓

$= 27 \pm 1.2$

$\therefore 25.8 \leq \bar{L} \leq 28.2$ ✓ ✓

4. [2, 5 marks]

The velocity of a particle is given by $v(x) = \sqrt{3x - 12}$.

Determine the

(a) acceleration when $x = 2$.

$\frac{1}{2}v^2 = \frac{3x-12}{2}$ ✓

$\frac{d(\frac{1}{2}v^2)}{dx} = \frac{3}{2}$ ✓

OR

or $v\frac{dv}{dx} = \sqrt{3x-12} \cdot \frac{3}{2}(3x-12)^{-\frac{1}{2}}$ ✓

$= \frac{3}{2}$ ✓

(b) displacement when $t = 2$ given that when $t = 0$, $x = 7$

$\frac{dx}{dt} = (3x-12)^{\frac{1}{2}}$

$\frac{dx}{(3x-12)^{\frac{1}{2}}} = dt$

$\int (3x-12)^{-\frac{1}{2}} dx = \int dt$ ✓

$\frac{(3x-12)^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} + c = t$ ✓

$\frac{\sqrt{9}}{\frac{3}{2}} + c = 0$

$c = \frac{-2 \cdot 3}{3}$ ✓

$= -2$

$t = \frac{2\sqrt{3x-12}}{3} - 2$

✓ • separates variables
✓ • integrates correctly
✓ • determines c
✓ • sets up equation
✓ • solves correctly for x

$2 = \frac{2\sqrt{3x-12}}{3} - 2$ ✓

$4 = \frac{2\sqrt{3x-12}}{3}$

$6 = \sqrt{3x-12}$

$36 = 3x - 12$

$48 = 3x$

$x = 16$ ✓

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Resource Assumed

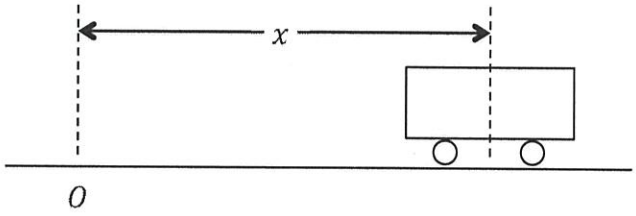
Reading Time: 2 minutes
Time Allowed: 29 minutes

Total Marks: 29

5. [2, 2, 4 marks]

A trolley is moving in simple harmonic motion about the origin, O. The displacement, x metres, of the centre of the trolley from O at t seconds is given by

$$x = 6\sin(2t + \frac{\pi}{4})$$



(a) Find the exact velocity of the trolley when $t = 0$.

$v = 12\cos(2t + \frac{\pi}{4})$ ✓

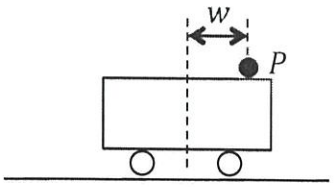
$v(0) = 6\sqrt{2}$ ✓ exact.

(b) Determine the exact first time after $t = 0$ when the centre of the trolley is at $x = 3$.

$3 = 6\sin(2t + \frac{\pi}{4})$ ✓

$t = \frac{7\pi}{24}$ ✓ exact.

A particle, P, on top of the trolley, is moving in simple harmonic motion about the centre of the trolley. Its displacement, w metres, from the centre of the trolley at time t seconds, is given by



$$w = \sin 2t$$

The displacement, y metres, of P from the origin is the sum of the displacements x and w.

(c) Show that P is moving with simple harmonic motion.

$y = x + w$

$= 6\sin(2t + \frac{\pi}{4}) + \sin 2t$ ✓

$\dot{y} = 12\cos(2t + \frac{\pi}{4}) + 2\cos 2t$ ✓

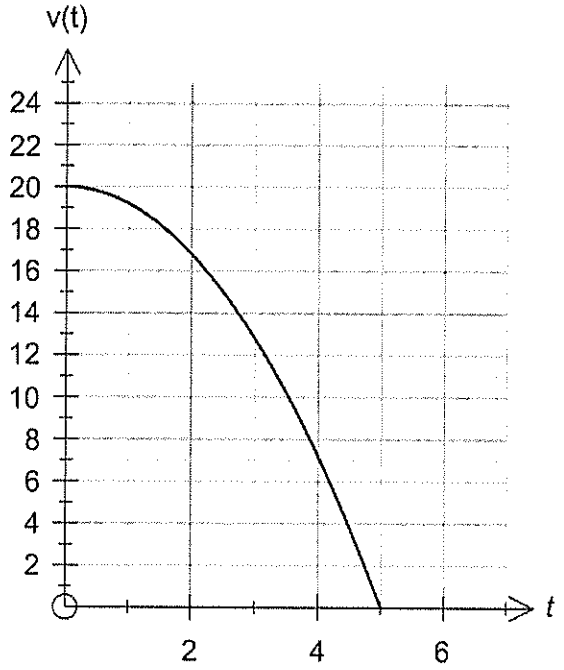
$\ddot{y} = -24\sin(2t + \frac{\pi}{4}) - 4\sin 2t$ ✓

$= -4(6\sin(2t + \frac{\pi}{4}) + \sin 2t)$

$= -4y$ ✓

6. [5, 2 marks]

A distracted driver doesn't notice a stop sign at an intersection, and has to apply the brakes rapidly in order to stop suddenly. The driver goes from 20 metres/second to stopping in 5 seconds, so that his velocity, $v(t)$, is shown in the graph below. His acceleration is given by the equation $a(t) = kt$.



(a) Using calculus, show that the equation of the curve is given by $v(t) = -0.8t^2 + 20$ m/sec.

$a(t) = kt$

$v(t) = \frac{1}{2}kt^2 + C$ ✓

$20 = 0 + C$

$v(t) = \frac{1}{2}kt^2 + 20$ ✓

$0 = \frac{1}{2}k(5)^2 + 20$ ✓

$-20 = 12.5k$

$k = -1.6$ ✓

$v(t) = \frac{1}{2}(-1.6)t^2 + 20$

$= -0.8t^2 + 20$ ✓

(b) Determine the distance the driver travels between applying the brakes and stopping.

$\text{Dist} = \int_0^5 -0.8t^2 + 20\,dt$ ✓

$= \frac{200}{3}$ m ✓

$= 66\frac{2}{3}$ m

7. [3, 3, 3, 3, 2 marks]

Andrew and Brad are both training to be in the cycling team at the Olympics. They are of equal ability, with their times to complete the races almost identical.

Andrew's coach has decided to include a lot of training drills that involve riding around an indoor cycle track of length 250 metres. Each time Andrew completes a lap his time is recorded. Over the duration of his training, the time taken for Andrew to complete one lap has been observed to be normally distributed with mean $\mu = 18.38$ seconds and standard deviation $\sigma = 0.36$ seconds.

During the final selection trials for the Olympics, Andrew has to complete a 2.5 kilometre (10 lap) time trial on the indoor cycle track.

Determine the probability that the:

(a) mean time per lap over the 2.5 kilometre time trial will be between 18.0 and 18.5 seconds. Give your answer to four decimal places.

$\bar{L} \sim N\left(18.38, \frac{0.36}{\sqrt{10}}\right)$ ✓ Indicates normal Dist

✓ states parameters

$P(18.0 < \bar{L} < 18.5) = 0.8537$ ✓ Determines prob.

(b) total time taken for the 2.5 kilometre time trial will be less than 182 seconds. Give your answer to four decimal places.

$\mu = 18.2$ ✓

$P(\bar{L} < 18.2) = 0.0569$ ✓

✓

Brad's coach believes the way to get stronger on the bike is to spend a lot of time doing long rides out on the road, and as such Brad does very little riding on the indoor track.

During the final selection trials for the Olympics, Brad completes the 2.5 kilometre (10 lap) time trial on the indoor cycle track in 186 seconds, and it was noted that the standard deviation for his lap times was 0.32 seconds.

(c) Determine the 98% confidence interval for the mean lap time for Brad, correct to 0.01 seconds.

$\mu = 18.6$

$Z_{98} = 2.3263$ ✓

$\bar{B} = 18.6 \pm 2.3263 \times \frac{0.32}{\sqrt{10}}$ ✓

$18.36 < \bar{B} < 18.84.$

✓

(d) How many complete laps would Brad have to complete to be 98% certain that the width of the confidence interval for his mean lap time is less than 0.3 seconds?

$$n = \left(\frac{z \cdot s}{d}\right)^2 \qquad d = 0.15$$

$$= \left(\frac{2.3263 \times 0.32}{0.15}\right)^2 \checkmark$$

$$= 24.6 \checkmark \qquad \therefore 25 \text{ laps} \checkmark$$

Andrew's coach claims that his coaching method is more effective than the method used by Brad.

(e) ~~Perform the calculations to~~ comment on this claim. stating any relevant statistics.

At a 98% confidence level, this claim is <u>not</u> supported as Andrew's mean lap time falls inside the confidence interval for Brad.

✓ states claim is not supported.

✓ justifies with reference to confidence interval