Name: T

SOLUTIONS

Baldivis
Secondary College

Year 12 Application

Test 2, 2017

Topic: Sequences

/60
= ____ %

Total Time: **60** **minutes**
Total Reading: 4 minutes
Total Working: 56 minutes
Weighting: *15% of the semester*

This test comprises of ***TWO sections****. The* ***first section*** *is* ***calculator free*** *where no calculators of any kind are to be used. The* ***second section*** *is* ***calculator assumed*** *where a CAS calculator may be used. All questions must be answered in both sections.* ***Answers should be rounded appropriately****. All working should be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.*

SECTION 1: CALCULATOR FREE

Time:	*24 minutes*	**Marks for Section 1:**	*25*
Reading:	*2 minutes*	**Equipment Allowed:**	*Nil*
Working:	*22 minutes*		

1. In the table below column 1, 2 and 3 represent 3 different series. State what *A*, *B* and *C* should be.

	Column 1	Column 2	Column 3
1^{st} term	−3	B	6
2^{nd} term	−7	8	8
3^{rd} term	−11	512	C
4^{th} term	−15	134217728	22
n^{th} term	$T_{n+1} = A - 4$	$T_{n+1} = (T_n)^3$ where $T_1 = 2$	$T_{n+1} = T_n + T_{n-1}$

$A = T_n$ $B = 2$ $C = 14$

(3 marks)

2. For each word description given below, select the appropriate recursive formula from the list provided.

List:

A: $T_{n+1} = nT_n + 2$

B: $T_{n+1} = 2T_{n-1} + T_n$

C: $T_{n+1} = 0.2T_n$

D: $T_{n+1} = T_{n-1} + T_n$

E: $T_{n+1} = (T_n + 2)^n$

a) Each term is obtained by adding the two previous terms together

D $(T_{n+1} = T_{n-1} + T_n)$

b) Each term is obtained by adding two to the previous term then multiplying this sum by itself n times.

(2 marks)

E $T_{n+1} = (T_n + 2)^n$

3. The third term of an arithmetic progression is 5 and the tenth term is -16

a) Write down an expression for the general term T_n

$T_n = 11 + (n-1)-3$ or $T_n = 11 - 3n + 3$

b) Find the thirtieth term. (2 marks)

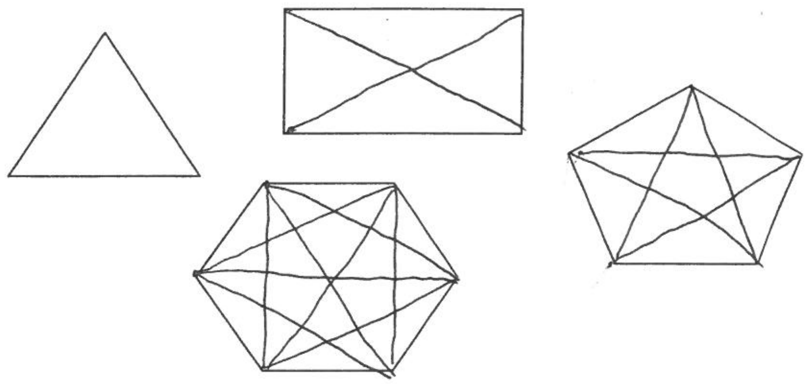
$5 = a + d(2)$ $21 = -7d$ $5 = a + (-3 \times 2)$ $T_{13} = 11 + (-3 \times 1$

$-16 = a + d(9)$ $d = -3$ $a = 11$ $\underline{T_{13} = -25}$

4. The diagonal of a geometric polygon is a line segment that connects two non-adjacent vertices. Bill and Hector need to find a rule for the number of diagonals in any polygon when given the number of sides. They drew several polygons to see how many diagonals each one had. Then they looked for a pattern. Using the polygons below, draw the diagonals and complete the table.

Diagonals of Polygons

# of Sides	# of Diagonals
3	0
4	2
5	5
6	9



a) Is the number of diagonals an arithmetic or geometric sequence?
Explain your answer. (2 marks)

Neither : no common ~~difference~~ ratio $\left(\frac{9}{5} \neq \frac{5}{2}\right)$

no common difference $(+2, +3, +4)$

b) Write a recursive rule. (2 marks)

$T_{n+1} = T_n + (T_n - T_{n-1} + 1)$

$\therefore T_{n+1} = 2T_n - T_{n-1} + 1$

5. Show that (x+1) + (2x+4) + (3x +7) + ……. Is an arithmetic sequence (2 marks)

$x+1$, $2x+4$, $3x+7$

$x+3$ $x+3$

common difference

6. Ryan is attempting to collect a set of 300 football cards. In the first month he collects 50 cards, and in each following month he collects 20 new cards.

a) Find the first term and common difference

50, 70, 90

First Term = 50 Common Difference = 20

b) How many cards will he collect in 6 months.

50, 70, 90, 110, 130, 150

150

c) How long will it take him to collect all 300 cards.

(2;2;2 marks)

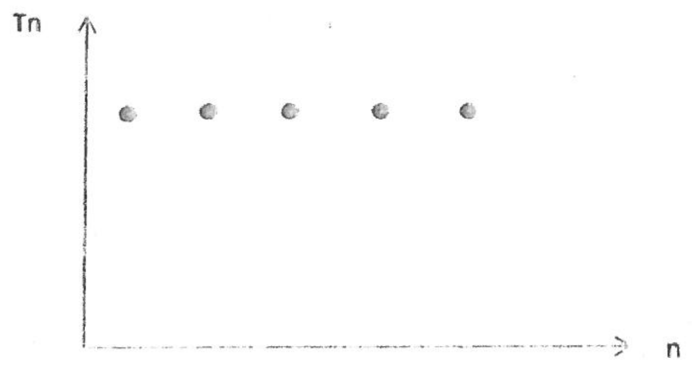
$T_n = a + (n-1)d$

$300 = 50 + 20n - 20$

$20n = 270$

$n = 13\frac{1}{2}$ months

7. This graph could be thought of as a special Geometric progression or a special Arithmetic progression



a) If this is a GP, state the common ratio.

1

b) If this is a AP state the common difference.

(2 marks)

0

8. A recursive sequence is defined by $T_n = T_{n-1} + T_{n-2} + 2$ with $T_1 = 2$ and $T_2 = -2$.

a) Determine the values of $T3$ and $T4$. (2 marks)

$T_3 = (-2) + 2 + 2$

$\underline{T_3 = 2}$

$T_4 = 2 + (-2) + 2$

$\underline{T_4 = 2}$

b) Paul states: *'The sequence will result in a positive two (2) when n is an odd number and will result in a negative two (–2) when n is an even number'.*

Comment on Paul's conjecture by using examples

(2 marks)

Paul is incorrect

Every term from the 3rd onwords will be positive

$T_5 = 2 + 2 + 2$
$= 6$

$T_6 = 6 + 2 + 2$
$= 10$

T

SECTION 2: CALCULATOR ASSUMED

Time:	~~37~~ 36 minutes	Marks for Section 2:	35
Reading:	2 minutes	Equipment Allowed:	½ page notes (A4 one side), CAS calculator
Working:	34 minutes		

1. Determine the recursive rule for

a) 90, 45, 22.5, 11.25……

$T_{n+1} = 0.5T_n$

b) $2 + a$, $3 + 3a$, $4 + 5a$, …

$T_{n+1} = T_n + 2a + 1$

c) The recursive rule for an ~~arithmetic~~ geometric sequence is given as:

$T_{n+1} = -3T_n + 2$ where $T_1 = 4$

Goran was asked to find the first 5 terms of this geometric sequence to which he got the answers 4, –10, 32, –96, 290.
One of these numbers is not correct. State which one it is, and give the correct value.

(2; 2; 2 marks)

-96 is incorrect
It should by -94

2. Consider the sequence 1, x, 25.

a) Use an appropriate method to find the value of x if the sequence is an arithmetic sequence.

$25-1=24$
$24 \div 2 = 12$
$1+12=13$

$x=13$

b) Use an appropriate method to find the value of x if the sequence is a geometric sequence where all the terms are positive.

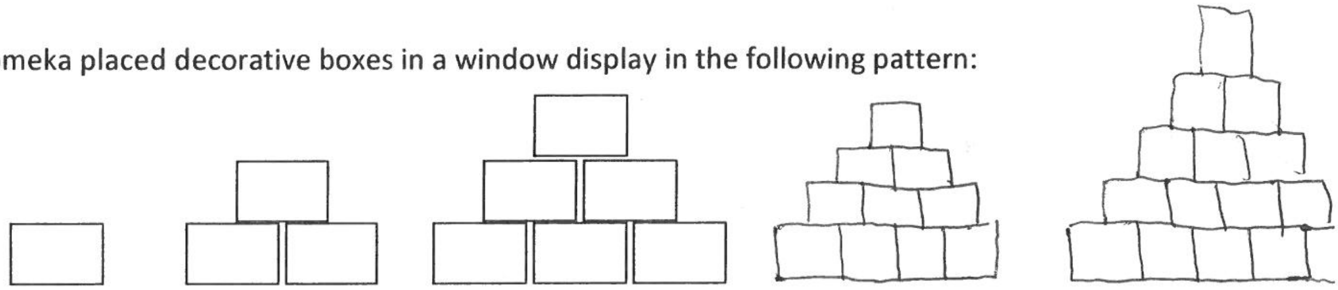
(2; 2 marks)

$\frac{25}{x} = \frac{x}{1}$

$\therefore x = \sqrt{25 \times 1}$

$x=5$

3. Tomeka placed decorative boxes in a window display in the following pattern:



a) Extend the pattern by sketching the next two displays in the space above. (1mark)

b) Complete the table below: (1mark)

Display	Number of boxes
1	1
2	3
3	6
4	10
5	15

c) Write the recursive rule for the pattern. (1mark)

$T_{n+1} = T_n + (T_n - T_{n-1} + 1) \therefore T_n = 2T_n - T_{n-1} + 1$

d) How many boxes would be needed to make a display that has 10 boxes in the **bottom** row? Explain and show your working. (1mark)

1, 3, 6, 10, 15, 21, 28, 36, 45, 55

$T_{10} = 55$

4. $20 000 is placed into a fixed term bank account for a period of 5 years. It is known that the value of this $20 000 will increase by 10% each year.

a) Fill out the table below to show how the value of the money is changing over the 5 years.

Time (years)	0	1	2	3	4	5
Value $	20 000	22 000	24 200	26 620	29 282	32 210.20

b) Determine the recursive formula for the value in dollars sequence.

$T_{n+1} = 1.1 T_n$

c) If the money continues to go at the current rate determine the value of the money after 10 years has passed by using the formula. (2; 2; 2 marks)

$T_n = ar^{n-1}$

$T_{10} = 22\,000 \times 1.1^9$

$T_{10} = \$51\,875$

5. Given the sequence $\frac{1}{4}, \frac{1}{2}, 1, \ldots.$

a) Define the sequence recursively. (1mark)

$T_{n+1} = 2T_n$

b) Explain why 12 is not a term of the sequence. (1mark)

If we keep multiplying by 2 from 1 onwords, it's impossible to reach 12. 1, 2, 4, 8, 16

6. A population of 16 rodents is expected to increase by 16% each year.

a) Define the population of rodents recursively. (1mark)

$T_{n+1} = 1.16\,T_n \quad T_1 = 16$

b) Write down a non-recursive formula for the population of rodents. (1mark)

$T_n = 16 \times 1.16^{(n-1)}$

c) Calculate the expected population of rodents in three years time. (1mark)

$T_3 = 16 \times 1.16^2 \quad T_3 = 21.5$

7. A layer of tinting for the walls of a conservatory lets in 95% of light

a) What percentage of light is let in by

(i) 2 layers

$0.95 \times 0.95 = 0.9025$

$= 90.25\%$

(ii) 10 layers of tinting (2 marks)

$0.95^{10} = 0.5987$

$= 59.87\%$

b) How many layers will let in 40% of light? (1 marks)

18

8. A rain tank contains 30 000L of water. A crack in the wall allows water to leak from the tank at a rate of 150L per minute. How much water is left in the tank after 2 hours.

(2 marks)

$T_n = a + d(n-1)$

$T_{120} = 30\,000 + (-150 \times 119)$

$T_{120} = 12\,150L$

9. Use your calculator or otherwise and state whether the following recurrence relations have long term increasing, decreasing or steady state solutions:

a) $t_{n+1} = 0.2t_n + 8$, with $t_1 = 4$

4, 8.8, 9.76 increasing

b) $t_{n+1} = 2t_n + 5$, with $t_1 = 2$ (2 marks)

2, 9, 23, increasing

10. A chicken farmer has 200 hens. Each month, 20% of the chicken are attacked and killed by foxes. The farmer has to buy an additional 32 hens to replace the lost stock.

a) Find a recurrence relation that models this situation. (2 marks)

$T_{n+1} = 0.8\,T_n + 32$

b) If this pattern continues how many hens will the farmer have in the long run? (1mark)

None