Eastern Goldfields College

MATHEMATICS:SPECIALIST
SEMESTER 1 2016

TEST 2

Calculator Free

Time Allowed: 20 minutes **Total Marks: 19**

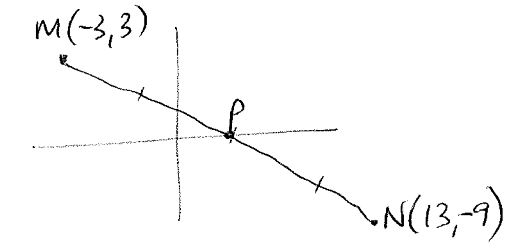
Question 1 **(5 marks)**

(b) Determine a unit vector perpendicular to the vector $8\mathbf{i} - 6\mathbf{j}$. (2 marks)

$6\underset{\sim}{i} + 8\underset{\sim}{j}$

$\frac{1}{10}(6\underset{\sim}{i} + 8\underset{\sim}{j}) = \frac{3}{5}\underset{\sim}{i} + \frac{4}{5}\underset{\sim}{j}$ OR $-\frac{3}{5}\underset{\sim}{i} - \frac{4}{5}\underset{\sim}{j}$

(c) The point P divides the line segment from M(-3, 3) to N(13, -9) in the ratio 1:3. Determine the position vector of point P. (3 marks)



$$\overrightarrow{OP} = \overrightarrow{OM} + \frac{1}{4}(\overrightarrow{MN})$$
$$= \begin{pmatrix} -3 \\ 3 \end{pmatrix} + \frac{1}{4}\left(\begin{pmatrix} 13 \\ -9 \end{pmatrix} - \begin{pmatrix} -3 \\ +3 \end{pmatrix}\right)$$
$$= \begin{pmatrix} -3 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$= \underset{\sim}{i}$$

Question 2 **(6 marks)**

The statement 'if two rectangles are congruent then they have the same area' is true.

(a) Write the inverse of the statement and explain whether or not the inverse is also true. (2 marks)

If two rectangles are not congruent then they do not have the same area. ✓

False eg. 2x6 and 3x4 ✓

(b) Write the contrapositive of the statement and explain whether or not the contrapositive is also true. (2 marks)

If two rectangles do not have the same area then they are not congruent. ✓

True - if original is true contrapositives are always true ✓

(c) Write the converse of the statement and explain whether or not the converse is also true. (2 marks)

If two rectangles have the same area then they are congruent. ✓

False - eg 2x6 and 3x4 ✓

only get second mark if they explain

Question 3 **(4 marks)**

If **a** = 2**i** +**j** and **b** = -3**i** +2**j**, find *m* and *n* such that *m***a** + *n***b** = -2**i** + 6**j**.

$$m\begin{pmatrix}2\\1\end{pmatrix} + n\begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}-2\\6\end{pmatrix}$$

$2m - 3n = -2$ ✓

$m + 2n = 6$ ✓

$2m - 3n = -2$

$-(2m + 4n = 12)$

$-7n = -14$

$n = 2$ ✓

$m = 2$ ✓

Question 4 **(4 marks)**

Use the method of contradiction to prove that a triangle with sides of 5 cm, 5 cm and 7 cm is not right angled.

Assume the triangle is right angled ✓
so Pythagoras' Theorem applies

$5^2 + 5^2 = 50$ $\quad 7^2 = 49$

$5^2 + 5^2 \neq 7^2$ ✓

Hence contradiction ✓

therefore triangle is not right angled. ✓

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Calculator Assumed

Time Allowed: 40 **Total Marks: 35**

Question 5 **(5 marks)**

Three vectors are given by $\mathbf{a} = 7\mathbf{i}$, $\mathbf{b} = 6\mathbf{i} + 9\mathbf{j}$ and $\mathbf{c} = x\mathbf{i} - 5\mathbf{j}$.

(a) Use your calculator to determine the angle between $\mathbf{a}$ and $\mathbf{b}$, to the nearest degree. (2 marks)

$$\text{angle}\left(\begin{pmatrix}7\\0\end{pmatrix}, \begin{pmatrix}6\\9\end{pmatrix}\right) = 56^\circ$$

✓✓

(b) Determine all possible values of x if $\mathbf{a} + \mathbf{c}$ and $\mathbf{b} + \mathbf{c}$ are perpendicular. (3 marks)

$$\left(\begin{pmatrix}7\\0\end{pmatrix} + \begin{pmatrix}x\\-5\end{pmatrix}\right) \cdot \begin{pmatrix}6+x\\9-5\end{pmatrix} = 0$$

$$\begin{pmatrix}7+x\\-5\end{pmatrix} \cdot \begin{pmatrix}6+x\\4\end{pmatrix} = 0 \quad ✓$$

$$(7+x)(6+x) + (-5)4 = 0$$

$$42 + 13x + x^2 - 20 = 0$$

$$x^2 + 13x + 22 = 0$$

$$(x+11)(x+2) = 0$$

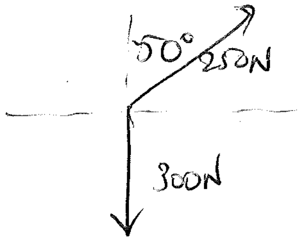
$$x = -11, -2$$

✓ ✓

Question 6 **(8 marks)**

Three forces are applied to a body. One has magnitude 300 N and acts due south. Another has magnitude 250 N and acts on a bearing of 050º.

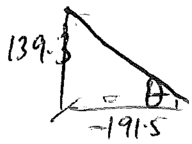
(a) If all three forces are in equilibrium, determine the magnitude and direction of the third force. (4 marks)



$250\cos 40\,\underset{\sim}{i} + 250\sin 40\,\underset{\sim}{j} - 300\,\underset{\sim}{j}$

$= 191.5\,\underset{\sim}{i} - 139.3\,\underset{\sim}{j}$ ✓✓

$\therefore$ Force : $-191.5\,\underset{\sim}{i} + 139.3\,\underset{\sim}{j}$



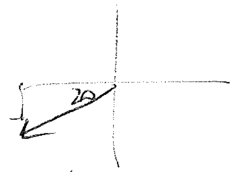
magnitude 237N ✓

Bearing 306° ✓

$\tan\theta = \dfrac{139.3}{191.5}$

$\theta = 36°$.

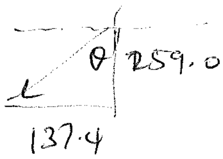
(b) If the third force has a magnitude of 350 N and acts on a bearing of 250º, determine the magnitude and direction of the resultant force. (4 marks)



$250\cos 40\,\underset{\sim}{i} + 250\sin 40\,\underset{\sim}{j} - 300\,\underset{\sim}{j}$

$- 350\cos 20\,\underset{\sim}{i} - 350\sin 20\,\underset{\sim}{j}$ ✓

$= -137.4\,\underset{\sim}{i} - 259.0\,\underset{\sim}{j}$ ✓



mag 293.2N ✓

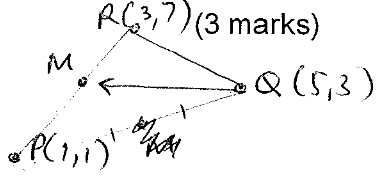
Bearing 207° ✓

$\tan\theta = \dfrac{137.4}{259}$

$\theta = 27.9°$.

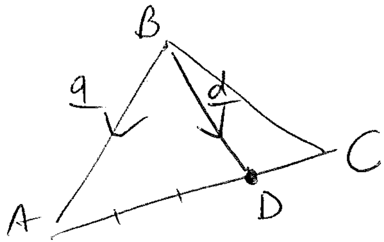
Question 7 **(6 marks)**

(a) A triangle PQR has vertices $P(1,\ 1)$, $Q(5,\ 3)$ and $R(3,\ 7)$. Determine the vector, $\overrightarrow{QM}$ where M is the midpoint of side PR. (3 marks)



$$\overrightarrow{QM} = \overrightarrow{QP} + \tfrac{1}{2}\overrightarrow{PR} \ \checkmark$$
$$= \begin{pmatrix}1\\1\end{pmatrix} - \begin{pmatrix}5\\3\end{pmatrix} + \tfrac{1}{2}\left(\begin{pmatrix}3\\7\end{pmatrix} - \begin{pmatrix}1\\1\end{pmatrix}\right)$$
$$= \begin{pmatrix}-4\\-2\end{pmatrix} + \tfrac{1}{2}\begin{pmatrix}2\\6\end{pmatrix} \ \checkmark$$
$$= \begin{pmatrix}-3\\1\end{pmatrix}$$
$$= -3\underline{i} + \underline{j} \ \checkmark$$

(b) ABC is a triangle with point D on side AC such that $AD = \frac{3}{4}AC$. If $\overrightarrow{BA} = \boldsymbol{a}$ and $\overrightarrow{BD} = \boldsymbol{d}$, show that $\overrightarrow{BC} = \frac{1}{3}(4\boldsymbol{d} - \boldsymbol{a})$ (3 marks)



$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$$
$$= \underline{a} + \tfrac{4}{3}\overrightarrow{AD}$$
$$= \underline{a} + \tfrac{4}{3}(-\underline{a} + \underline{d})$$
$$= \underline{a} - \tfrac{4}{3}\underline{a} + \tfrac{4}{3}\underline{d} \ \checkmark\checkmark\checkmark$$
$$= \tfrac{4}{3}\underline{d} - \tfrac{1}{3}\underline{a}$$
$$= \tfrac{1}{3}(4\underline{d} - \underline{a})$$

Question 8 **(9 marks)**

(a) A small body A has position (12, -3) m relative to another small body B. If a third small body C has position (-5, 6) relative to A, determine the position of B relative to C.

(2 marks)

$$_A r_B = \begin{pmatrix}12\\-3\end{pmatrix} \qquad _C r_A = \begin{pmatrix}-5\\6\end{pmatrix}$$

$$r_A - r_B = \begin{pmatrix}12\\-3\end{pmatrix} \qquad r_C - r_A = \begin{pmatrix}-5\\6\end{pmatrix}$$

$$r_A - \begin{pmatrix}12\\-3\end{pmatrix} = r_B \qquad r_C = \begin{pmatrix}-5\\6\end{pmatrix} + r_A$$

$$_B r_C = \cancel{r_A} - \begin{pmatrix}12\\-3\end{pmatrix} - \left(\begin{pmatrix}-5\\6\end{pmatrix} + \cancel{r_A}\right)$$

$$= \begin{pmatrix}-7\\-3\end{pmatrix}$$

✓✓

(b) To a cyclist moving with velocity (21, -5) km/h the wind appears to have velocity (-9, 3) km/h. Determine the true speed of the wind. (3 marks)

$$v_c = \begin{pmatrix}21\\-5\end{pmatrix} \qquad _w v_c = \begin{pmatrix}-9\\3\end{pmatrix}$$

$$_w v_c = v_w - v_c$$

$$\begin{pmatrix}-9\\3\end{pmatrix} = v_w - \begin{pmatrix}21\\-5\end{pmatrix}$$

$$\begin{pmatrix}-9\\3\end{pmatrix} + \begin{pmatrix}21\\-5\end{pmatrix} = v_w$$

$$\begin{pmatrix}12\\-2\end{pmatrix} = v_w$$

(c) A small ship is travelling with a constant speed of 14 knots on a bearing of 025º and another, larger ship is travelling with a constant speed of 17 knots on a bearing of 310º.

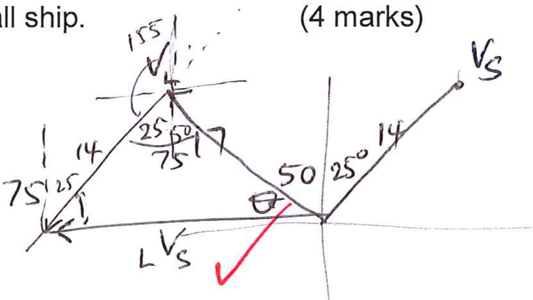
Determine the velocity of the large ship relative to the small ship. (4 marks)

$$_L V_S = V_L - V_S \checkmark$$

$$|_L V_S|^2 = 14^2 + 17^2 - 2.14.17.\cos 75$$

$$|_L V_S| = 19.0 \text{ knots.} \checkmark$$

Bearing 264.6° ✓



$$\frac{\sin\theta}{14} = \frac{\sin 75}{19}$$

$$\theta = 45.4^\circ.$$

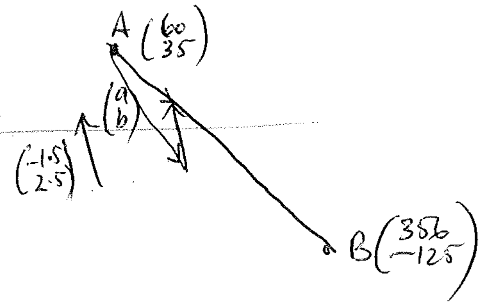
Question 9 **(7 marks)**

A small boat has to travel across a river from A to B, where $OA = 60\mathbf{i} + 35\mathbf{j}$ and $OB = 356\mathbf{i} - 125\mathbf{j}$. A uniform current of $-1.5\mathbf{i} + 2.5\mathbf{j}$ is flowing in the river and the boat can maintain a steady speed of 4 m/s.

(a) Determine, in the form $a\mathbf{i} + b\mathbf{j}$, the velocity vector the small boat should set to travel directly from A to B. (5 marks)

$$\left|\begin{pmatrix} a \\ b \end{pmatrix}\right| = \sqrt{a^2+b^2} = 4$$

$$a^2+b^2=16 \checkmark$$



$$t\begin{pmatrix} a-1.5 \\ b+2.5 \end{pmatrix} = \begin{pmatrix} 356-60 \\ -125-35 \end{pmatrix} \checkmark$$

$$t\begin{pmatrix} a-1.5 \\ b+2.5 \end{pmatrix} = \begin{pmatrix} 296 \\ -160 \end{pmatrix} \checkmark$$

$$\left.\begin{aligned} t(a-1.5) &= 296 \\ t(b+2.5) &= -160 \\ a^2+b^2 &= 16 \end{aligned}\right\} \text{Solve}$$

~~$a=-3.97, b=0.46, t=54$~~

$\checkmark$ $a = 2.56, b = -3.07, t = 279.14$

Velocity $2.56\underset{\sim}{i} - 3.07\underset{\sim}{j}$

$\checkmark$

(b) Calculate, to the nearest minute and second, how long the journey will take. (2 marks)

$t = 279.14$ sec

$= 4$ mins 39 sec $\checkmark$

$\checkmark$