



**Greenwood College**  
**Year 12 Applications**  
**Test 3 2019**  
**Resource-Free**

Name.....

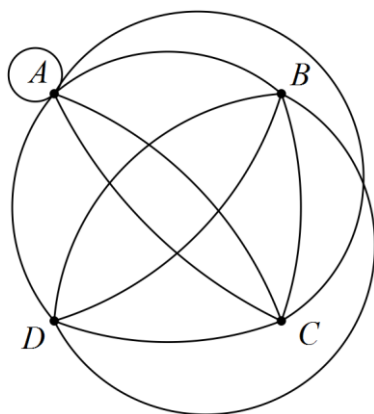
No calculators nor notes allowed.  
29 mark total.

Formula sheet allowed.  
30 minute time limit.

**Question 1**

**[7 marks: 3, 2, 2]**

- (a) Re-draw the following graph to clearly demonstrate that it is planar.



- (b) Draw a complete graph with five vertices.

**Question 1 cont.**

- (c) Do **all** complete graphs obey Euler's formula? Justify your answer.

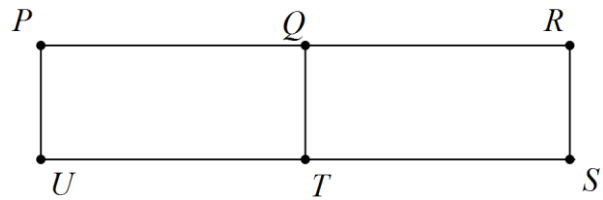
**Question 2****[6 marks: 2, 2, 2]**

- (a) Three hikers were sharing tips about walking on four famous routes. Ahn had walked the Inca Trail, Kilimanjaro and the Snowman Trek. Bi had walked the Inca Trail, Kilimanjaro and the Overland Track. Chris had walked the Overland Track and the Snowman Trek.

Display this information as a bipartite graph.

**Question 2 cont.**

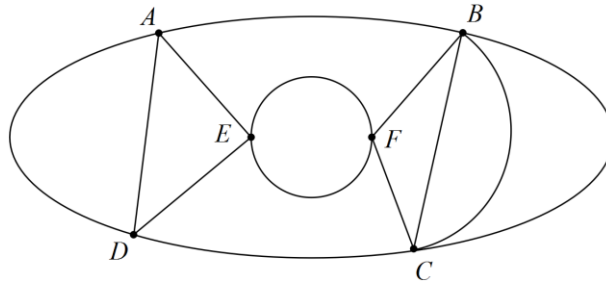
- (b) A bipartite graph is shown below, joining vertices in two disjoint sets. Clearly list the vertices belonging to each of the two sets.



- (c) A complete bipartite graph has six vertices. Determine the smallest possible number of edges.

**Question 3****[7 marks: 2, 2, 2, 1]**

The edges in the graph below represent the roads in a park that meet at the given vertices.



- (a) Show that Euler's formula applies to this graph.
- (b) Record the degree of each vertex shown in the graph in the table below.
- | Vertex | <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> |
|--------|----------|----------|----------|----------|----------|----------|
| Degree |          |          |          |          |          |          |
- (c) Without referring to the information in (b), clearly explain why the graph is semi-Eulerian.
- (d) A park ranger has to inspect every road in the park. List all possible starting points so that the ranger can complete this task without driving on the same road more than once.

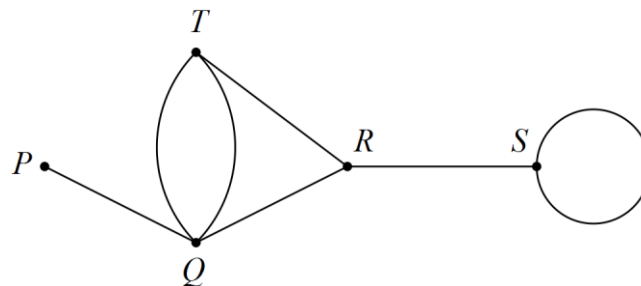
**Question 4****[9 marks: 2, 2, 2, 3]**

(a) A connected planar graph has one face and three edges.

(i) Determine the number of vertices the graph has.

(ii) Sketch a graph with these properties.

(b) Consider the graph below.



State the length of the longest possible walk in this graph and list, in order from first to last, the vertices visited, so that the walk...

(i) is an open trail.

(ii) is a closed path.



**Greenwood College**  
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**Resource-Allowed**

Name.....

Formula sheet, one A4 page single-sided of notes and calculators allowed.

27 mark total.

30 minute time limit.

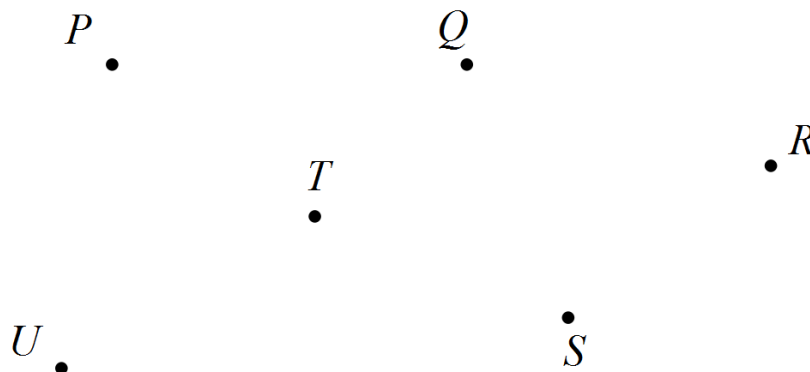
**Question 5**

**[9 marks: 3, 1, 1, 2, 2]**

A business has branches in six cities. The table below shows the time, in minutes, it takes for a package received at one branch to be transported to a branch in another city, where a direct route exists.

Q	40				
R	-	50			
S	-	25	30		
T	30	35	-	25	
U	25	-	-	65	35
	P	Q	R	S	T

- (a) Construct a weighted graph to show this information, using the cities placed below.

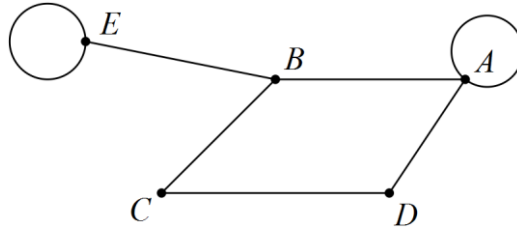


**Question 5 cont.**

- (b) Determine the shortest transport time for a package to travel from...
  - (i) P to S.
  
  
  
  
  
  
  
  
  
  
  - (ii) Q to U.
  
- (c) A document needs to be sent from branch U via branch R, where a customer will sign the document, to branch P. Determine the minimum transport time for the document to make this journey, listing all branches on the way.
  
  
  
  
  
  
  
  
  
  
- (d) Another business document requires signing by the manager of each branch. In planning a route for this document, would finding a Eulerian trail be more appropriate than finding a Hamiltonian trail? Explain your answer.

**Question 6****[9 marks: 3, 3, 3]**

An airline has flights between six cities as shown in the graph below. Two of the flights are sightseeing flights that return to the city from which they departed.

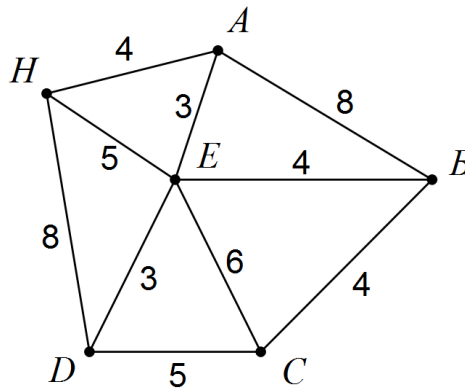


- (a) Determine  $M$ , the adjacency matrix for this graph.
- (b) Calculate  $M^2$  and explain the significance of the elements in this matrix that are zero.
- (c) Determine the number of zero elements in the matrix  $M + M^2$  and explain their significance in terms of specific flight(s).



**Question 7****[9 marks: 2, 2, 2, 3]**

The vertex  $H$  on the graph below represents a hotel and vertices  $A$  to  $E$  represent tourist attractions. The numbers on the edges of the graph below represent the walking times, in minutes, between the various attractions.



A group of tourists plan to leave the hotel at 10 am and visit all the attractions, spending 15 minutes at each one.

- (a) Given that the hotel bus will pick them up from the last attraction they visit,
- (i) determine the route they should take that involves the least possible walking time.
  - (ii) determine the time the bus should meet them at their last attraction.

**Question 7 cont.**

- (b) One member of the group knows a little about graph theory and suggests that the route that the group plan should be a Hamiltonian cycle.
  - (i) Explain what is meant by a Hamiltonian cycle.
  - (ii) Determine the Hamiltonian cycle the group of tourists should walk and state the time they will arrive back at their hotel.