education equals

Calculator Assumed
Applications of Differentiation – Rates of Change

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 3, 2 = 6 marks]

Give an expression for the instantaneous rate of change for each of the following functions, showing all working:

(a) $f(t) = 10t^3 - 6t^2$

(b) $P(x) = x(x-5)^2$

(c) $x(t) = \dfrac{12t^4 - 3t^3}{6t^2}$

Question Two: **[2, 2, 1, 2, 2, 2 = 11 marks]**

The profit function, in dollars, for producing and selling *x* hundred items is given by :
$P(x) = 6x^3 - 122x^2 + 490x - 150$

(a) Over what domain does this function adequately model the profit of producing and selling *x* items?

(b) What is the profit for producing zero items? Interpret this result.

(c) Determine an expression for the instantaneous rate of change for producing and selling *x* items.

(d) Calculate $P'(2)$ and interpret your result.

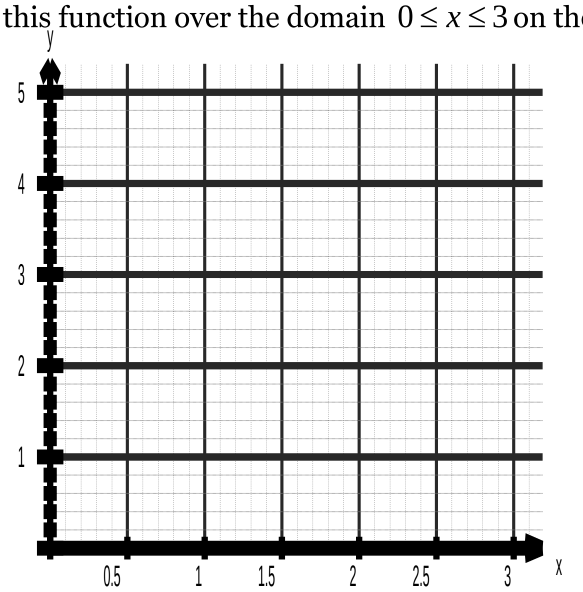
(e) Calculate $P'(3)$ and compare your result to your answer in part (d).

(f) Calculate the average profit for producing and selling the first 200 items.

Question Three: [3, 3, 1, 5 = 12 marks]

Consider the function $f(x) = (x-1)^2 + 1$

(a) Sketch this function over the domain $0 \le x \le 3$ on the axes below.



(b) Determine the gradient function of $f(x)$ and hence calculate $f'(2)$.

(c) Sketch the tangent to $f(x)$ at $x = 2$ on your graph above.

(d) Verify your answer to part (b) by using the limiting chord process and filling in the table below.

Point P	**Point Q**	**Gradient of segment PQ**
(2, 2)	(3, 5)	
(2, 2)	(2.5, _____)	
(2, 2)	(2.1, _____)	
(2, 2)	(_____,_______)	
(2, 2)	(_______,_______)	

Question Four: [2, 2, 2, 2, 2 = 10 marks]

The function $P(x)$ has the following table of values:

x	0	1	1.1	1.2	1.5	2	2.1	2.3	2.5
y	0	-3	-2.97	-2.88	-2.25	0	0.63	2.07	3.75

(a) Determine the approximate value of $\lim_{h \to 0} \frac{P(1.1+h)-P(1.1)}{h}$

(b) Determine when $P'(x) \approx 0$ and the value of $P(x)$ at this point.

(c) Calculate the average rate of change of $P(x)$ between $x=1.2$ and $x=2.5$.

(d) Determine the approximate instantaneous rate of change when $x=2$.

(e) Determine the approximate equation of the tangent to $P(x)$ when $x=2$.

Question Five: **[4, 2 = 6 marks]**

The instantaneous rates of change of $T(x)$ at $x=10$, $x=10.2$ and $x=11$ are given in the table below.

x	10	10.2	11
$T'(x)$	30	31.212	36.3

(a) Determine approximate values of $T(x)$ missing in the table below.

x	10	10.2	11
$T(x)$	100		

(b) Hence determine the approximate equation of the tangent to $T(x)$ when $x=11$.

SOLUTIONS
Calculator Assumed
Applications of Differentiation – Rates of Change

Time: 45 minutes
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Question One: **[1, 3, 2 = 6 marks]**

Give an expression for the instantaneous rate of change for each of the following functions, showing all working:

(a) $f(t) = 10t^3 - 6t^2$

$f'(t) = 30t^2 - 12t$ ✓

(b) $P(x) = x(x-5)^2$

$P(x) = x(x^2 - 10x + 25)$ ✓

$P(x) = x^3 - 10x^2 + 25x$ ✓

$P'(x) = 3x^2 - 20x + 25$ ✓

(c) $x(t) = \dfrac{12t^4 - 3t^3}{6t^2}$

$x(t) = 2t^2 - \dfrac{t}{2}$ ✓

$x'(t) = 4t - \dfrac{1}{2}$ ✓

Question Two: **[2, 2, 1, 2, 2, 2 = 11 marks]**

The profit function, in dollars, for producing and selling *x* hundred items is given by :
$P(x) = 6x^3 - 122x^2 + 490x - 150$

(a) Over what domain does this function adequately model the profit of producing and selling *x* items?

$0 \le x \le 5$ ✓ ✓

(b) What is the profit for producing zero items? Interpret this result.

$P(0) = -150$ ✓

Loss of $150 ✓

(c) Determine an expression for the instantaneous rate of change for producing and selling *x* items.

$P'(x) = 18x^2 - 244x + 490$ ✓

(d) Calculate $P'(2)$ and interpret your result.

$P'(2) = 74$ ✓

The profit for producing and selling the next item, i.e. the 201st item. ✓

(e) Calculate $P'(3)$ and compare your result to your answer in part (d).

$P'(3) = -80$ ✓

At the production of 200 items profit was still increasing for every extra item produced and sold, but by 300 items, a loss is being made of each extra item produced and sold. ✓

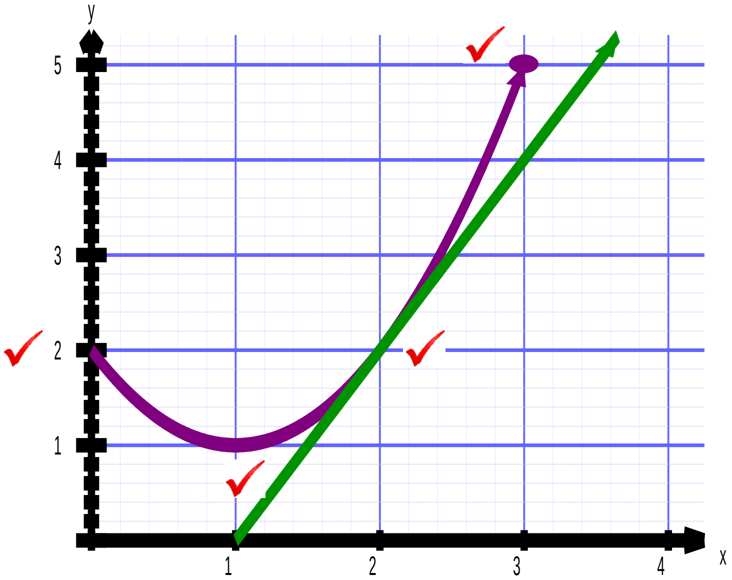
(f) Calculate the average profit for producing and selling the first 200 items.

$$\frac{P(2) - P(0)}{2 - 0} = \frac{390 - -150}{20} = \$270$$ ✓ ✓

Question Three: [3, 3, 1, 5 = 12 marks]

Consider the function $f(x) = (x-1)^2 + 1$

(a) Sketch this function over the domain $0 \le x \le 3$ on the axes below.



(b) Determine the gradient function of $f(x)$ and hence calculate $f'(2)$.

$f(x) = x^2 - 2x + 2$
$f'(x) = 2x - 2$
$f'(2) = 2$

(c) Sketch the tangent to $f(x)$ at $x = 2$ on your graph above.

(d) Verify your answer to part (b) by using the limiting chord process and filling in the table below.

Point P	Point Q	Gradient of segment PQ
(2, 2)	(3, 5)	3
(2, 2)	(2.5, 3.25)	2.5
(2, 2)	(2.1, 2.21)	2.1
(2, 2)	(2.05,2.1025)	2.05
(2, 2)	(2.001,2.002)	2

Question Four: [2, 2, 2, 2, 2 = 10 marks]

The function $P(x)$ has the following table of values:

x	0	1	1.1	1.2	1.5	2	2.1	2.3	2.5
y	0	-3	-2.97	-2.88	-2.25	0	0.63	2.07	3.75

(a) Determine the approximate value of $\lim_{h\to 0}\frac{P(1.1+h)-P(1.1)}{h}$

$$\frac{-2.88--2.97}{1.2-1.1}=0.9$$

(b) Determine when $P'(x)\approx 0$ and the value of $P(x)$ at this point.

$x=1$

$P(1)=-3$

(c) Calculate the average rate of change of $P(x)$ between $x=1.2$ and $x=2.5$.

$$\frac{3.75--2.88}{2.5-1.2}=5.1$$

(d) Determine the approximate instantaneous rate of change when $x=2$.

$$\frac{0.63}{0.1}=6.3$$

(e) Determine the approximate equation of the tangent to $P(x)$ when $x=2$.

$y=6.3x+c$

$0=6.3(2)+c$

$c=-12.6$

$y=6.3x-12.6$

Question Five: **[4, 2 = 6 marks]**

The instantaneous rates of change of $T(x)$ at $x=10$, $x=10.2$ and $x=11$ are given in the table below.

x	10	10.2	11
$T'(x)$	30	31.212	36.3

(a) Determine approximate values of $T(x)$ missing in the table below.

x	10	10.2	11
$T(x)$	100	106 ✓	131 ✓

$\frac{y-100}{0.2} \approx 30$ ✓

$y \approx 106$

$\frac{y-106}{0.8} \approx 31.212$ ✓

$y \approx 131$

(b) Hence determine the approximate equation of the tangent to $T(x)$ when $x=11$.

$y = 36.3x + c$

$131 = 36.3(11) + c$

$c = -268.3$

$y \approx 36.3x - 268.3$

✓ ✓