

Full Name: SOLUTIONS



MATHEMATICS

Applications Units 3 & 4

Test 3 – Graphs

Chapter 5

Semester 1 2017

Section Two – Calculator Assumed

Time allowed for this section

Working time for this section: 25 minutes

Marks available: 25 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

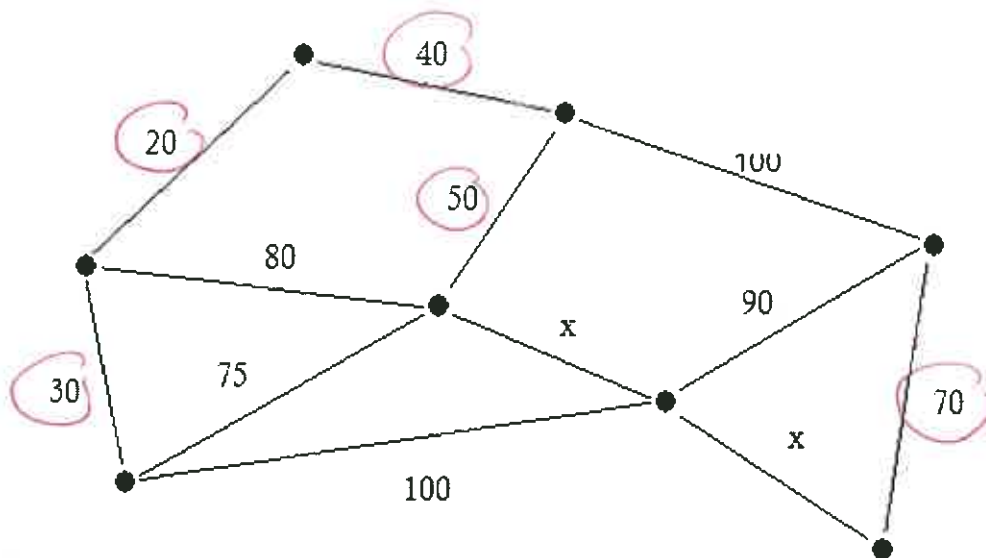
Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (3 marks)

The length of the minimum spanning tree is given in the following network. Use this information to find the value of x .



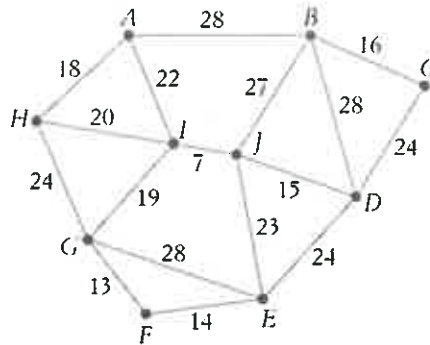
Length of minimum spanning tree = 260

$$260 - 210 = 50$$

$x = \underline{25}$

2. (6 marks)

A park has 10 large trees. The trees are denoted as vertices A to J on the following graph. Walking paths between the trees are indicated by the edges, with the numbers on the edges indicating the lengths of the paths in metres.



- a. Determine the sum of the degrees of the vertices in this network. [2]

34

- b. Alisha wishes to walk through the park on a route that will take her along each of the paths between the trees.

- i. State a vertex at which Alisha should begin her walk. [1]

A or H

- ii. Determine the total distance Alisha will walk. [1]

350 m

- c. Alisha's friend Alice has taken the following route: $H-A-I-J-B-C-D-E-F-G-H$.

- i. What is this route an example of? [1]

Hamiltonian Cycle

- ii. Determine the total distance Alice will walk. [1]

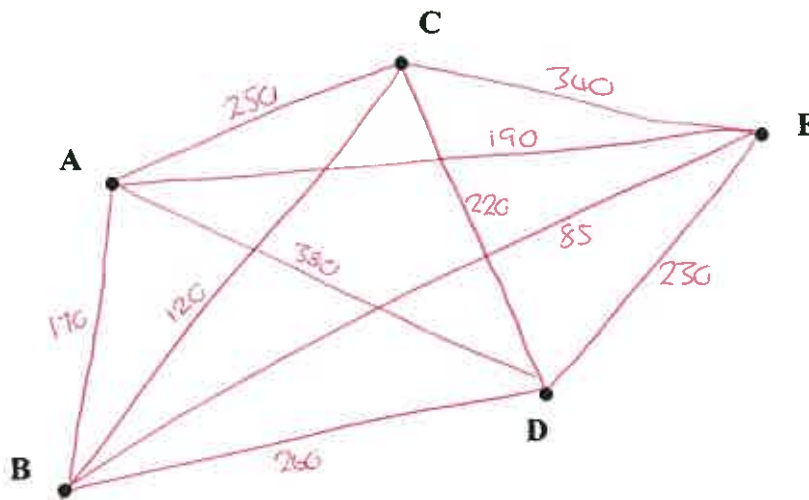
189 m

3. (8 marks)

- a. Use the nodes below to draw a network for the following table which shows the cost (in dollars) of the telecommunications infrastructure for the towns A, B, C, D, E

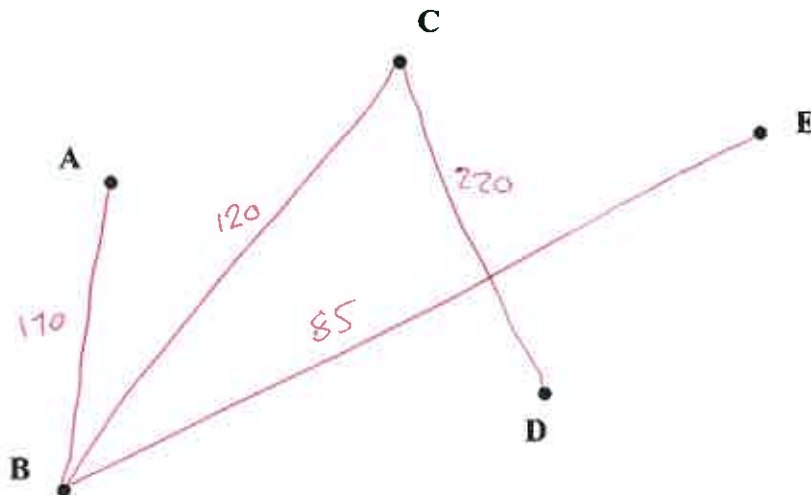
[3]

	A	B	C	D	E
A	-	170	250	380	190
B	170	-	120	260	85
C	250	120	-	220	340
D	380	260	220	-	230
E	190	85	340	230	-



- b. Use the nodes below to show how the towns can be connected to minimise the total cost of the project (Minimum Spanning Tree).

[3]



- c. What is the minimum cost?

\$595

[2]

4. (8 marks)

A city is to host an economic forum to be attended by the leaders of a number of nations. To ensure the safety of the leaders a sophisticated communication network, linking a number of control points, is to be set up. There are eight control points P_1, P_2, \dots, P_8 and the costs, in thousands of dollars, of establishing a direct link between points is given in the following table.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
P_1	-	25	18	16	5	12	19	8
P_2	25	-	22	20	24	25	22	23
P_3	18	22	-	10	15	16	20	21
P_4	16	20	10	-	19	21	28	27
P_5	5	24	15	19	-	15	20	10
P_6	12	25	16	21	15	-	28	30
P_7	19	22	20	28	20	28	-	18
P_8	8	23	21	27	10	30	18	-

The city needs to establish a minimal cost network.

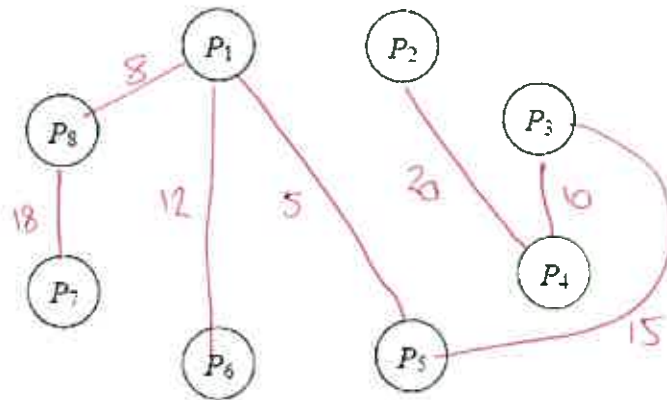
- a. Use Prim's algorithm to determine the minimal cost.

[4]

\$88 000

- b. Represent the solution found in Part (a) as a network.

[2]



- c. The planners realise that the cost of connecting the control points P_1 and P_2 can be reduced by \$8000 by using a remote device. By how much does the use of the remote device reduce the minimum cost of constructing the network? Justify your solution.

[2]

Affects P_2 to P_4 link (20)

$P_1 \rightarrow P_2$ was 25, now 17

End of Test

∴ reduces by \$3000

Full Name: SOLUTIONS



MATHEMATICS

Applications Units 3 & 4

Test 3 – Graphs

Chapter 5

Semester 1 2017

Section One – Calculator Free

Time allowed for this section

Working time for this section: 30 minutes
Marks available: 34 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

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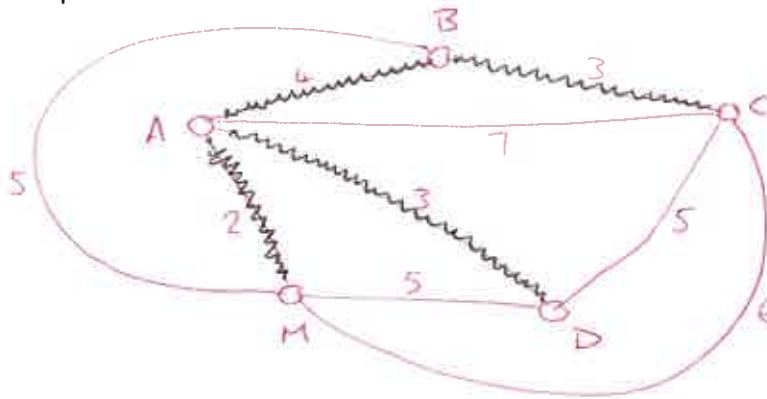
2. (5 marks)

Consider the table below showing the length of cable required to connect computer terminals A, B, C and D to each other and to the main terminal M.
(all measurements are in metres).

	A	B	C	D	M
A	-	4	7	3	2
B	4	-	3	-	5
C	7	3	-	5	6
D	3	-	5	-	5
M	2	5	6	5	-

a) Draw a possible network

[2]



b) Draw the minimum spanning tree. Mark it on your network in colour.

[2]

c) What is the minimum amount of cable needed?

[1]

12 metres

3. [1 mark]

A connected planar graph has 12 edges. This graph could have:
(select one of the answers)

a) 5 vertices and 6 faces

b) 5 vertices and 8 faces

c) 6 vertices and 8 faces

d) 6 vertices and 9 faces

e) 7 vertices and 9 faces

4. [1 mark]

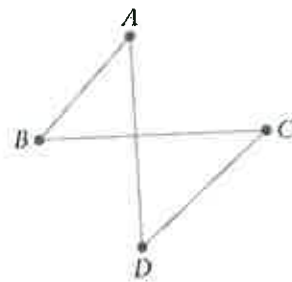
Find the sum of degrees in the following graph:



Sum is 10

5. [2 marks]

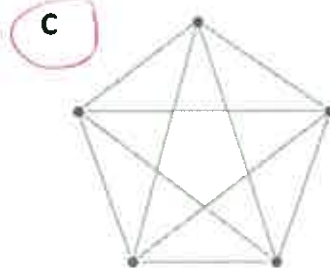
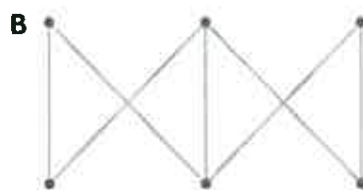
Write the adjacency matrices to represent the graph below?



$$\begin{array}{c}
 \begin{array}{cccc}
 & A & B & C & D \\
 \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}
 \end{array}
 \end{array}$$

6. [1 mark]

Which one of the following graphs is Eulerian? (select one)



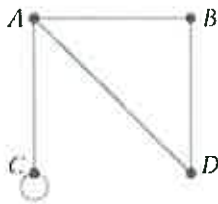
7. [1 mark]

A salesman is selling educational software to a number of different schools. He will visit every school once only and return to the school at which he started.

This situation can best be represented by: (select one)

- ☒ a) A Hamiltonian cycle
- ☐ b) A semi-Eulerian graph
- ☐ c) A Hamiltonian path
- ☐ d) A Eulerian graph
- ☐ e) A minimum spanning tree

8. [2 marks]



The following matrix represents the graph above. The matrix is missing two values k and m . State the values of k and m .

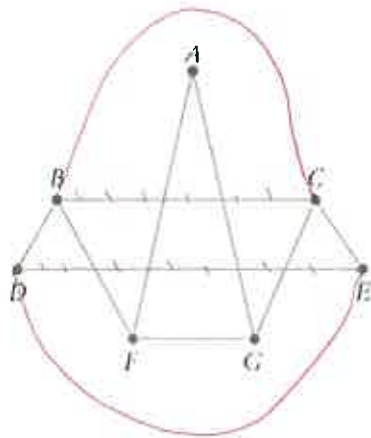
$$\begin{array}{c}
 \begin{array}{cccc}
 & A & B & C & D \\
 A & \begin{bmatrix} 0 & 1 & k & 1 \end{bmatrix} \\
 B & \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \\
 C & \begin{bmatrix} 1 & 0 & m & 0 \end{bmatrix} \\
 D & \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}
 \end{array}
 \end{array}$$

$k =$

$m =$

9. [2 marks]

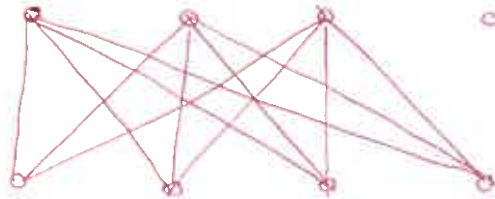
Redraw this graph as a planar graph:



10. [5 marks]

Two tennis teams are playing each other in a grand final. Each team has four players and each player on the team plays each player on the other team once. Unfortunately, one player from one team received an injury and could not play.

- a) Represent the games played using a bipartite graph with the eight players as the vertices of the graph and the edges the games played.

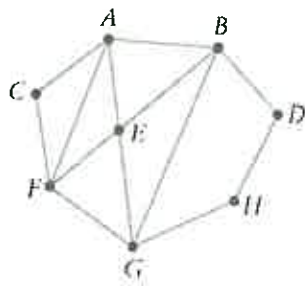


- b) How many games were played?

12 games

11. [1 mark]

Which edge/s need to be added or removed to make the following graph semi-Eulerian?



Remove AB or FG

12. [4 marks]

	A	B	C	D	E
A	1	0	1	0	0
B	0	0	0	0	0
C	1	0	0	2	1
D	0	0	2	0	1
E	0	0	1	1	0

For the above matrix:

a) What do all the zeros in row *B* indicate?

B is not connected

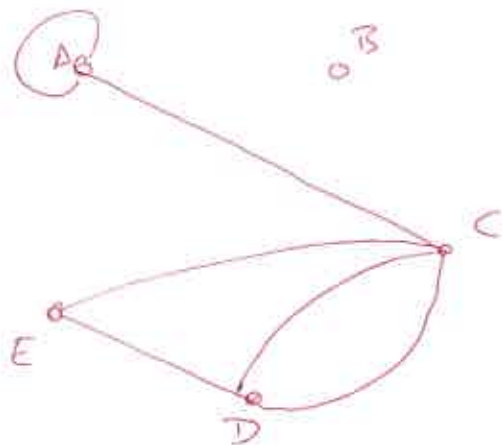
b) What does the '1' at the intersection of row *A* column *A* indicate?

Loop

c) What does the '2' in the matrix represent?

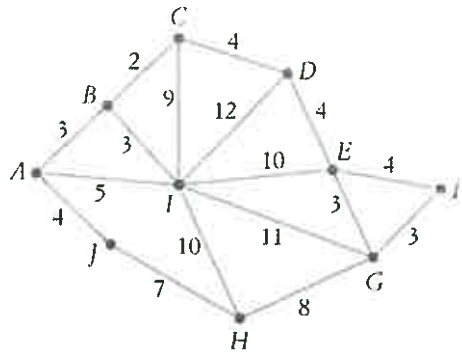
Multiple paths

d) Draw the graph that corresponds to this matrix.

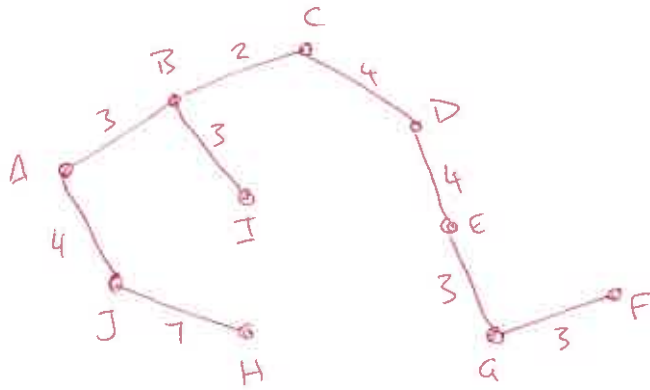


13. [5 marks]

For the following graph:



a) Use Prim's algorithm to find the minimum spanning tree.



b) What is the total weight of this minimum spanning tree?

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End of Section One