education equals

Calculator Assumed
Exponential Function Applications

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 2, 2, 2 = 7 marks]

The average rate of inflation from the beginning of 2010 to the beginning of 2015 in Australia has been 2.6% per year.

(a) The cost of buying 2L of milk cost $2.75 at the beginning of 2010. How much did it cost a year later?

(b) If this average rate of inflation remains constant for the foreseeable future, write a suitable function that models the yearly price of a 2L bottle of milk, since the beginning of 2010.

(c) If the model from part (b) remains valid, by the beginning of which year would you expect to pay double the 2010 price?

(d) Over time, the cost of a 256 MB USB drive has gone from $100 in 2003 to almost $0 in 2015. Give one reason why inflation doesn't tend to affect the prices of technology.

Question Two: **[1, 1, 3, 2, 3 = 10 marks]**

When an apple pie is taken out of the oven, its core temperature decreases by 20% every 10 minutes. The pie has been baking in the oven for 45 minutes at 180˚C.

(a) What is the core temperature of the pie immediately after it is taken out of the oven?

(b) What is the core temperature of the pie 10 minutes after it is taken out of the oven?

(c) Determine a suitable algebraic model which gives the temperature, T˚C, ***t*** **minutes** after the pie is taken from the oven.

(d) Use your model to determine the core temperature of the pie 1 hour after it is removed from the oven.

(e) The pie is ready to eat when the core temperature is 26˚C. If the pie was taken out of the oven at 11 am, when will it be ready to eat?

Question Three: [3 marks]

There's an ancient story about a king and a wise man who play a game of chess.

The king promises the wise man any prize he chooses if he wins the game.

The wise man does win the game and he asks the king to give him grains of rice in the following way: he asks the king to put one grain of rice on the first square, 2 on the second square, 4 on the third, 8 on the fourth and so on. There are 64 squares on the chess board.

The king thinks that the wise man can't be very wise at all, asking for such a cheap prize.

Explain mathematically why the king is mistaken.

Question Four: [3 marks]

Your parents offer to pay you pocket money for the entire year in one of the following two ways:

Option A: $50 the first week, $100 the next, $150 the third week and so on.

Option B: $0.01 the first week, $0.03 the second week, $0.09 the third week and so on.

Demonstrate, by sketching the function associated with each option, which is the wiser choice in the long term.

Question Five: **[3, 2, 2, 1, 3 = 11 marks]**

In 2014 there was an outbreak of the Ebola virus in West Africa and it was feared that without immediate medical intervention, the disease would spread throughout the world, killing millions of people.

On 26th June 2014, the number of reported cases in Sierra Leone was approximately 100 patients. By the 26th August 2014, the total number of reported cases in Sierra Leone was approximately 2500 patients.

(a) Assuming an exponential model of growth, determine the monthly rate of increase in the total number of patients reported with Ebola.

(b) Determine a suitable algebraic model which gives the total number of reported cases, P, of patients with Ebola, over time t, in months since 26th June 2014.

(c) Using your model or otherwise, determine the number of new cases of Ebola in Sierra Leone reported in September 2014.

(d) Based on this model, health organizations determined that 26th October 2014 was the latest date for significant intervention in Sierra Leone before the disease would become an epidemic. How many total cases had been reported by this date?

(e) With significant medical intervention beginning on 26th October 2014, the number of new cases being reported **each day** was predicted to reach 5 000 by 26th January 2015. What daily rate of decrease would this intervention have, if the number of new cases on 26th October 2014 was assumed to be 10 000 per day?

Question Six: [2, 2, 2, 2, 3 = 11 marks]

In January 1990 Telptus were providing mobile phone services to 200 people in Perth. The number of total customers was increasing by 75% each year from 1990 onwards.

(a) How many customers had a mobile phone service with Telptus by January 1995?

(b) If Telptus were the only company providing a mobile phone service, during which year could they expect to see a total of 500 000 customers?

(c) In 1990, Telptus noticed that their landline customers were decreasing. In January 1990 they had 800 000 Perth customers. In January 1991 they had 720 000 customers. What proportion of customers are they losing per year?

(d) How many landline customers will Telptus have by January 1995?

(e) During which year and month will the number of landline customers equal the number of mobile phone customers that have plans with Telptus?

SOLUTIONS
Calculator Assumed
Exponential Function Applications

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: **[1, 2, 2, 2 = 7 marks]**

The average rate of inflation from the beginning of 2010 to the beginning of 2015 in Australia has been 2.6% per year.

(a) The cost of buying 2L of milk cost \$2.75 at the beginning of 2010. How much did it cost a year later?

$= 2.75 \times 1.026 = \$2.82$ ✓

(b) If this average rate of inflation remains constant for the foreseeable future, write a suitable function that models the yearly price of a 2L bottle of milk, since the beginning of 2010.

$P = 2.75 \times 1.026^t$
✓ ✓

(c) If the model from part (b) remains valid, by the beginning of which year would you expect to pay double the 2010 price?

$t = 27,$ 2037
✓ ✓

(d) Over time, the cost of a 256 MB USB drive has gone from \$100 in 2003 to almost \$0 in 2015. Give one reason why inflation doesn't tend to affect the prices of technology.

Technology is becoming smaller and faster and hence cheaper.
✓ ✓

Question Two: **[1, 1, 3, 2, 3 = 10 marks]**

When an apple pie is taken out of the oven, its core temperature decreases by 20% every 10 minutes. The pie has been baking in the oven for 45 minutes at 180˚C.

(a) What is the core temperature of the pie immediately after it is taken out of the oven?

$180^{\circ}C$ ✓

(b) What is the core temperature of the pie 10 minutes after it is taken out of the oven?

$0.8\times180=144^{\circ}C$ ✓

(c) Determine a suitable algebraic model which gives the temperature, T˚C, ***t*** **minutes** after the pie is taken from the oven.

$T=180\times0.8^{\frac{t}{10}}$ ✓

✓ ✓

(d) Use your model to determine the core temperature of the pie 1 hour after it is removed from the oven.

$T=180\times0.8^{\frac{60}{10}}=47.19^{\circ}C$ ✓ ✓

(e) The pie is ready to eat when the core temperature is 26˚C. If the pie was taken out of the oven at 11 am, when will it be ready to eat?

$26=180\times0.8^{\frac{t}{10}}$ ✓

$t=86.7$ ✓

$12:27\ pm$ ✓

Question Three: [3 marks]

There's an ancient story about a king and a wise man who play a game of chess.

The king promises the wise man any prize he chooses if he wins the game.

The wise man does win the game and he asks the king to give him grains of rice in the following way: he asks the king to put one grain of rice on the first square, 2 on the second square, 4 on the third, 8 on the fourth and so on. There are 64 squares on the chess board.

The king thinks that the wise man can't be very wise at all, asking for such a cheap prize.

Explain mathematically why the king is mistaken.

The number of grains or rice are rising exponentially. By the time the king reaches the 64^{th} square he will need to give the wise man 2^{63} grains of rice, which is 9.2×10^{18} grains of rice, an enormous amount. Thus for the king to pay out his promise, it will be expensive to pay the wise man with all this rice.

Question Four: [3 marks]

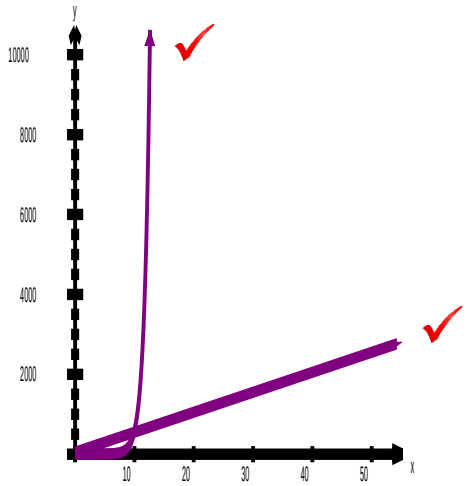
Your parents offer to pay you pocket money for the entire year in one of the following two ways:

Option A: \$50 the first week, \$100 the next, \$150 the third week and so on.

Option B: \$0.01 the first week, \$0.03 the second week, \$0.09 the third week and so on.

Demonstrate, by sketching the function associated with each option, which is the wiser choice in the long term.

The wiser choice is Option B as you earn far more through exponential growth in the long run.



Question Five: **[3, 2, 2, 1, 3 = 11 marks]**

In 2014 there was an outbreak of the Ebola virus in West Africa and it was feared that without immediate medical intervention, the disease would spread throughout the world, killing millions of people.

On 26th June 2014, the number of reported cases in Sierra Leone was approximately 100 patients. By the 26th August 2014, the total number of reported cases in Sierra Leone was approximately 2500 patients.

(a) Assuming an exponential model of growth, determine the monthly rate of increase in the total number of patients reported with Ebola.

$100 \times r^2 = 2500$ ✓

$r = 5$ ✓

Monthly increase of 500% ✓

(b) Determine a suitable algebraic model which gives the total number of reported cases, P, of patients with Ebola, over time t, in months since 26th June 2014.

$P = 100 \times 5^t$

✓ ✓

(c) Using your model or otherwise, determine the number of new cases of Ebola in Sierra Leone reported in September 2014.

$P = 100 \times 5^3 = 12500$ ✓

$12500 - 2500 = 10000$ *new cases* ✓

(d) Based on this model, health organizations determined that 26th October 2014 was the latest date for significant intervention in Sierra Leone before the disease would become an epidemic. How many total cases had been reported by this date?

$P = 100 \times 5^4 = 62500$ ✓

(e) With significant medical intervention beginning on 26th October 2014, the number of new cases being reported **each day** was predicted to reach 5 000 by 26th January 2015. What daily rate of decrease would this intervention have, if the number of new cases on 26th October 2014 was assumed to be 10 000 per day?

$10000 \times r^{92} = 5000$ ✓

$r = 0.9925$ ✓

$0.75\%\ decrease\ each\ day$ ✓

Question Six: **[2, 2, 2, 2, 3 = 11 marks]**

In January 1990 Telptus were providing mobile phone services to 200 people in Perth. The number of total customers was increasing by 75% each year from 1990 onwards.

(a) How many customers had a mobile phone service with Telptus by January 1995?

$= 200 \times 1.75^5 = 3282.62$ ✓

≈ 3300 *people*

(b) If Telptus were the only company providing a mobile phone service, during which year could they expect to see a total of 500 000 customers?

$500000 = 200 \times 1.75^x$ ✓

$x = 13.98$

$\therefore 2004$ ✓

(c) In 1990, Telptus noticed that their landline customers were decreasing. In January 1990 they had 800 000 Perth customers. In January 1991 they had 720 000 customers. What proportion of customers are they losing per year?

$\frac{720000}{800000} = 0.9$ ✓

$\therefore 10\%$ *per year* ✓

(d) How many landline customers will Telptus have by January 1995?

$800000 \times 0.9^5 = 472392$ ✓

$\therefore \approx 472000$ *customers* ✓

(e) During which year and month will the number of landline customers equal the number of mobile phone customers that have plans with Telptus?

$800000 \times 0.9^x = 200 \times 1.75^x$ ✓

$x = 12.47$ ✓

During 2002 ✓