

Semester One Examination, 2018

Question/Answer booklet

**MATHEMATICS
APPLICATIONS
UNIT 1**

**Section One:
Calculator-free**

SOLUTIONS

Student number: In figures

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| | | | | | | | |
|--|--|--|--|--|--|--|--|

In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|---------------------------|
| Section One: Calculator-free | 7 | 7 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
| Total | | | | | 100 |

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (52 Marks)

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(5 marks)

The rate of the Goods and Services Tax (GST) varies from country to country. In Australia it is 10% but in Thailand it is only 7% whilst in Sweden it is 25%.

- (a) Determine the amount of GST that must be added to goods valued at \$300 in Sweden.

(1 mark)

| Solution |
|--|
| $300 \times \frac{25}{100} = 3 \times 25 = \75 |
| Specific behaviours |
| ✓ states amount of GST |

- (b) The price of a TV set in Thailand is \$400, excluding GST. Calculate the GST inclusive price.

(2 marks)

| Solution |
|--|
| $400 \times \frac{7}{100} = 4 \times 7 = 28$ |
| Price = $400 + 28 = \$428$ |
| Specific behaviours |
| ✓ GST amount |
| ✓ GST inclusive price |

- (c) In another country, a \$20 CD costs \$23 when GST is included. Determine the rate of GST in this country.

(2 marks)

| Solution |
|-------------------------------------|
| $\frac{23 - 20}{20} = \frac{3}{20}$ |
| $\frac{3}{20} \times 100 = 15\%$ |
| Specific behaviours |
| ✓ expresses GST as fraction of cost |
| ✓ calculates rate as percentage |

Question 2**(8 marks)**

Four measurements are recorded on a production line at hourly intervals and then used to calculate the values of P_1 , P_2 and P_3 . These values are then reported to a supervisor. The formulas used are shown below.

$$P_1 = \frac{3}{2}a^2 + b^2, \quad P_2 = b\sqrt{c} + \frac{60}{d}, \quad P_3 = 100P_1 - 10P_2.$$

During one round of measurements, the values recorded were

$$a = 4, \quad b = 3, \quad c = 25, \quad d = 15.$$

Calculate the value of

(a) P_1 .

(3 marks)

| Solution |
|--|
| $P_1 = \frac{3}{2}4^2 + 3^2 = \frac{3}{2}(4)^2 + 9$ $P_1 = \frac{3}{2} \times 16 + 9 = 24 + 9$ $P_1 = 33$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ substitutes, simplifying b^2 ✓ simplifies product ✓ evaluates correctly |

(b) P_2 .

(3 marks)

| Solution |
|---|
| $P_2 = 3\sqrt{25} + \frac{60}{15} = 3\sqrt{25} + 4$ $P_2 = 3 \times 5 + 4 = 15 + 4$ $P_2 = 19$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ substitutes, simplifying quotient ✓ simplifies root ✓ evaluates correctly |

(c) P_3 .

(2 marks)

| Solution |
|--|
| $P_3 = 100(33) - 10(19) = 3300 - 190$ $P_3 = 3110$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ substitutes correctly, simplifying products ✓ evaluates correctly |

Question 3

(9 marks)

An agri-business employed casual labourers for fruit picking. The business pays casual labourers an hourly rate of \$20 per hour, with time and a half being paid for any weekend work.

One week, a casual labourer worked 6 hours on Tuesday, 5 hours on Wednesday, 4 hours on Saturday and 3 hours on Sunday.

(a) How much did the casual labourer earn

(i) on Tuesday?

| Solution |
|----------------------------------|
| Earnings = $6 \times 20 = \$120$ |
| Specific behaviours |
| ✓ correct earnings |

(1 mark)

(ii) over the weekend?

| Solution |
|--|
| Earnings = $(3 + 4) \times 20 \times 1.5$ = $7 \times 30 = \$210$ |
| Specific behaviours |
| ✓ uses time and a half |
| ✓ correct earnings |

(2 marks)

(iii) during the whole week?

| Solution |
|---|
| Weds earnings = $5 \times 20 = \$100$ Earnings = $120 + 100 + 210 = \$430$ |
| Specific behaviours |
| ✓ calculates Wednesday |
| ✓ correct earnings |

(2 marks)

(b) Calculate the **increase** in earnings the following week, if the casual labourer worked the same days but for 5 hours each day.

(2 marks)

| Solution |
|---|
| Paid hours = $5 + 5 + 7.5 + 7.5 = 25$ Earns $25 \times 20 = \$500$ Increase of \$70 |
| Specific behaviours |
| ✓ calculates wage |
| ✓ states increase |

(c) The business makes a superannuation contribution of 9.5% of weekly earnings for casual labourers. Calculate the weekly superannuation contribution for a casual labourer who earns \$880 per week.

(2 marks)

| Solution |
|---|
| 10% of 880 = 88 1% of 880 = 8.8 0.5% of 880 = 4.4 9.5% of 880 = $88 - 4.4 = \$83.60$ |
| Specific behaviours |
| ✓ indicates at least one correct, relevant % |
| ✓ correct amount |

Question 4

(8 marks)

The spreadsheet below shows some example costs (C in \$) to transport a package by air between two locations. The cost depends on the weight of the package (W kg) and the amount by which the longest dimension of the package exceeds one metre (E m).

| C (\$) | | E (m) | | | |
|----------|----|---------|-----|-----|-----|
| | | 0 | 1 | 2 | 3 |
| W (kg) | 10 | 30 | 42 | 58 | 78 |
| | 20 | 60 | 82 | 108 | 138 |
| | 30 | 90 | 122 | A | 198 |
| | 40 | 120 | 162 | 208 | 258 |

(a) Use the spreadsheet to determine C when

(i) $E = 1$ and $W = 40$.

(1 mark)

| Solution |
|--------------------------|
| (i) $C = 162$ |
| (ii) $C = 78$ |
| Specific behaviours |
| ✓ correct value for (i) |
| ✓ correct value for (ii) |

(ii) $W = 10$ and $E = 3$.

(1 mark)

The cost is calculated using the formula $C = 3W + 2E^2 + EW$.

(b) Show use of the above formula to determine the value of A in the spreadsheet. (3 marks)

| Solution |
|--|
| $W = 30, E = 2$ $A = C = 3 \times 30 + 2 \times 2^2 + 2 \times 30$ $A = 90 + 8 + 60 = 158$ |
| Specific behaviours |
| ✓ indicates both values ✓ substitutes correctly ✓ evaluates correctly |

(c) Calculate the cost of transporting a 15 kg package that has a longest dimension of 3 m.

(3 marks)

| Solution |
|---|
| $W = 15, E = 3 - 1 = 2$ $C = 3 \times 15 + 2 \times 2^2 + 2 \times 15$ $C = 45 + 8 + 30 = \$83$ |
| Specific behaviours |
| ✓ states both values ✓ substitutes correctly ✓ evaluates correctly |

Question 5

(10 marks)

- (a) Express $3\begin{bmatrix} 3 & 11 & -6 \end{bmatrix} - 2\begin{bmatrix} 1 & -5 & 4 \end{bmatrix}$ as a single matrix.

(3 marks)

| Solution |
|---|
| $\begin{bmatrix} 9 & 33 & -18 \end{bmatrix} - \begin{bmatrix} 2 & -10 & 8 \end{bmatrix} = \begin{bmatrix} 7 & 43 & -26 \end{bmatrix}$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ simplifies first matrix ✓ simplifies second matrix ✓ correct difference |

- (b) Matrices D and E , with dimensions 5×3 and 6×5 respectively, can be multiplied together. State the dimensions of the product, briefly justifying your answer. (2 marks)

| Solution |
|---|
| Product must be ED , with dimensions 6×3 . |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ correct dimensions ✓ indicates order of multiplication |

- (c) Calculate $\begin{bmatrix} 2 & -1 \end{bmatrix} \times \begin{bmatrix} 4 & 0 & 6 \\ 0 & -5 & 3 \end{bmatrix}$.

(2 marks)

| Solution |
|---|
| $\begin{bmatrix} 8 & 5 & 9 \end{bmatrix}$ |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ evaluates at least two values correctly ✓ evaluates all three values correctly |

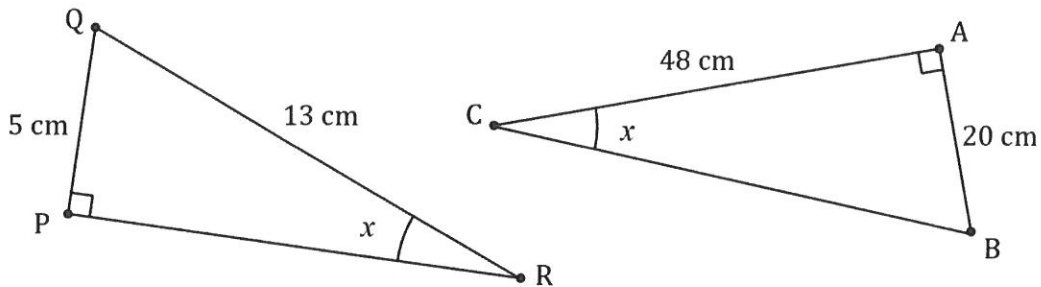
- (d) Determine the values of the constants a, b and c given that $10I + a\begin{bmatrix} -2 & 9 \\ 1 & b \end{bmatrix} = \begin{bmatrix} c & 27 \\ 3 & 31 \end{bmatrix}$, where I is the 2×2 identity matrix. (3 marks)

| Solution |
|---|
| $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} + a\begin{bmatrix} -2 & 9 \\ 1 & b \end{bmatrix} = \begin{bmatrix} c & 27 \\ 3 & 31 \end{bmatrix}$ <p>Using M_{21}: $0 + a = 3 \Rightarrow a = 3$ Using M_{22}: $10 + 3b = 31 \Rightarrow b = 7$ Using M_{11}: $10 + 3 \times -2 = c \Rightarrow c = 4$</p> |
| Specific behaviours |
| <ul style="list-style-type: none"> ✓ indicates use of $10I$ ✓ evaluates at least two constants ✓ evaluates all three constants |

Question 6

(7 marks)

(a) Two similar right-triangles (not drawn to scale) are shown below.

(i) Determine the lengths of the sides BC and PR . (3 marks)

| Solution |
|------------------------------------|
| $sf = 20 \div 5 = 4$ |
| $BC = 13 \times 4 = 52 \text{ cm}$ |
| $PR = 48 \div 4 = 12 \text{ cm}$ |
| Specific behaviours |
| ✓ indicates correct scale factor |
| ✓ BC correct |
| ✓ PR correct |

(ii) The area of $\triangle ABC$ is k times greater than the area of $\triangle PQR$. State the value of k . (1 mark)

| Solution |
|---------------------|
| $k = 4^2 = 16$ |
| Specific behaviours |
| ✓ correct value |

(b) A right-triangle has sides of lengths 20 cm, 29 cm and 21 cm.

(i) Between which two sides is the right-angle? (1 mark)

| Solution |
|---------------------------------|
| Between the 20 and 21 cm sides. |
| Specific behaviours |
| ✓ correct sides |

(ii) Calculate the area of the triangle. (2 marks)

| Solution |
|--|
| $A = \frac{1}{2} \times 20 \times 21 = 210 \text{ cm}^2$ |
| Specific behaviours |
| ✓ indicates use of correct formula |
| ✓ correct area |

Question 7

(5 marks)

Summary information about a small portfolio of shares is shown in the table below.

| Share code | Number of shares | Current market price | Last dividend per share | Price-Earnings ratio |
|------------|------------------|----------------------|-------------------------|----------------------|
| YBB | 200 | \$14.00 | \$0.45 | 16.5 |
| ZCC | 1000 | \$3.50 | \$0.12 | 11.0 |

- (a) Determine the total dividend of the portfolio, based on the last dividend paid. (2 marks)

| Solution |
|--|
| $2 \times 100 \times 0.45 = 2 \times 45 = 90$ $10 \times 100 \times 0.12 = 10 \times 12 = 120$ $\text{Total} = 90 + 120 = \210 |
| Specific behaviours |
| ✓ one dividend correct ✓ correct total |

- (b) The investor sells all the YBB shares at the current market price, paying a 2% commission for brokerage. Determine how much the investor receives. (3 marks)

| Solution |
|---|
| $\text{Value} = 200 \times 14 = 2800$ $\text{Brokerage} = 2800 \times \frac{2}{100} = 28 \times 2 = 56$ $\text{Net} = 2800 - 56 = \$2\,744$ |
| Specific behaviours |
| ✓ market value ✓ brokerage ✓ amount received |

Supplementary page

Question number: _____

Supplementary page

Question number: _____

