Eastern Goldfields College

# MATHEMATICS:SPECIALIST 1 & 2

## SEMESTER 1 2016

## TEST 3

## Resource Free

Time Allowed: 24 minutes Total Marks: 19

**1.** [1, 3 marks]

A line drawn from a point A forms a tangent to a circle at B. A second line from A cuts through the same circle at point C and D.

(a) State a relationship between the lengths of the line segments AB, AD and AC.

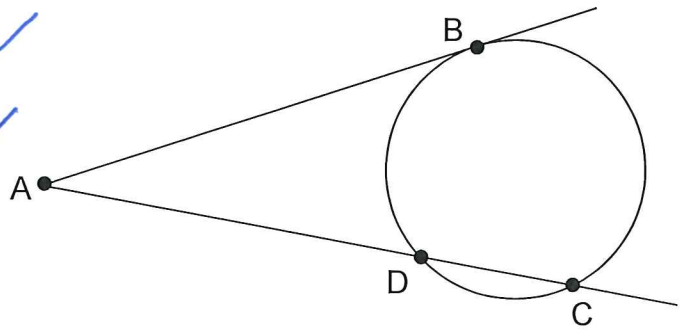
$AB^2 = AD \times AC$ ✓

(b) Hence prove that $\Delta ABD \sim \Delta ACB$.

$\frac{AB}{AC} = \frac{AD}{AB}$ same ratio ✓

$s\angle BAD = s\angle CAB$ same angle ✓

$\therefore \Delta ABD \sim \Delta ACB$ (SAS) ✓



**2.** [3, 1 marks]

Given vectors **m** = 5**i** – 2**j** and **n** = 4**i** + 3**j**, determine

(a) the scalar projection of **m** onto **n**.

$$\underset{\sim}{m}\cdot\hat{\underset{\sim}{n}} = (5\underset{\sim}{i}-2\underset{\sim}{j})\cdot\frac{(4\underset{\sim}{i}+3\underset{\sim}{j})}{5} \quad \checkmark\checkmark$$

$$= \frac{20-6}{5}$$

$$= \frac{14}{5} \quad \checkmark$$

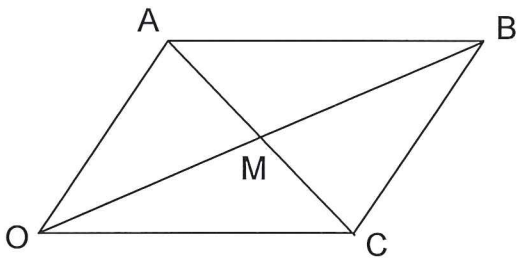
(b) the vector projection of **m** onto **n**.

$$(\underset{\sim}{m}\cdot\hat{\underset{\sim}{n}})\,\hat{\underset{\sim}{n}} = \frac{14}{5}\times\frac{(4\underset{\sim}{i}+3\underset{\sim}{j})}{5} \quad \checkmark$$

$$= \frac{56}{25}\underset{\sim}{i} + \frac{42}{25}\underset{\sim}{j}$$

**3.** [6 marks]

Prove that the diagonals of a parallelogram bisect each other.




OABC is a parallelogram with $\overrightarrow{OA}$ = **a** and $\overrightarrow{OC}$ = **c**. The diagonals OB and AC meet at M.

If $\overrightarrow{AM} = h\overrightarrow{AC}$ and $\overrightarrow{OM} = k\overrightarrow{OB}$, use the fact that $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$ to show that h = k = ½.

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$

$$k\overrightarrow{OB} = \underset{\sim}{a} + h\overrightarrow{AC} \quad \checkmark$$

$$k(\underset{\sim}{a}+\underset{\sim}{c}) = \underset{\sim}{a} + h(\underset{\sim}{c}-\underset{\sim}{a}) \quad \checkmark$$

$$k\underset{\sim}{a} + k\underset{\sim}{c} = \underset{\sim}{a} + h\underset{\sim}{c} - h\underset{\sim}{a}$$

$$k\underset{\sim}{c} - h\underset{\sim}{c} = \underset{\sim}{a} - h\underset{\sim}{a} - k\underset{\sim}{a}$$

$$\underset{\sim}{c}(k-h) = \underset{\sim}{a}(1-h-k) \quad \checkmark$$

$$k-h=0 \Rightarrow k=h \quad \checkmark$$

$$1-h-k=0$$

$$1-2h=0$$

$$h=\tfrac{1}{2}=k \quad \checkmark$$

**4.** [1, 1, 3 marks]

(a) Find a counter-example to show that the following conjecture is not true.

$\forall\, a, b \in \mathbb{Z}$ and $a > b$ then $a^2 > b^2$

$a = 1, b = -2 \quad 1 \not> 4$ ✓

(b) Find an example to show that the following conjecture is true.

$\exists\, a \in \mathbb{Q}$ such that $\frac{12}{a} \in \mathbb{Z}$

$a = 2$ ✓, $\frac{12}{2} = 6$

(c) Write the mathematical notation for the statement:

*For all rational numbers x, there exist integers y and w such that* $x = \frac{y}{w}$ *where w is non-zero.*

$\forall x \in \mathbb{Q}, \exists y, w \in \mathbb{Z} : x = \frac{y}{w}$ where $w \neq 0$.

½ ½ ½ ½ ½ ½

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# MATHEMATICS:SPECIALIST 1 & 2

## SEMESTER 1 2016

## TEST 3

## Calculator Assumed

Time Allowed: 27 minutes

Total Marks: 22

**5.** [2 marks]

The work done, in joules, by a force of $\mathbf{F}$ Newtons in changing the displacement of an object by $\mathbf{s}$ metres is given by the scalar product of $\mathbf{F}$ and $\mathbf{s}$.

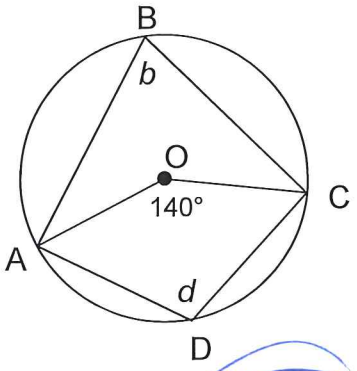
A force acting on a bearing of 160° does work of 1 200 joules. If the object moved a distance of 350 cm on a bearing of 135°, determine the magnitude of the force. (2 marks)

$1200 = |F| \times 3.5 \times \cos 25$ (½ ½)

$F = 378.3\,N$ ✓

**6.** [2, 4 marks]

(a) A circle centred at O has s∠AOC = 140°, as shown in the diagram. Determine the values of *b* and *d*. Justify your answers.




s∠ABC = 70° angle at centre twice angle at circumference. ✓

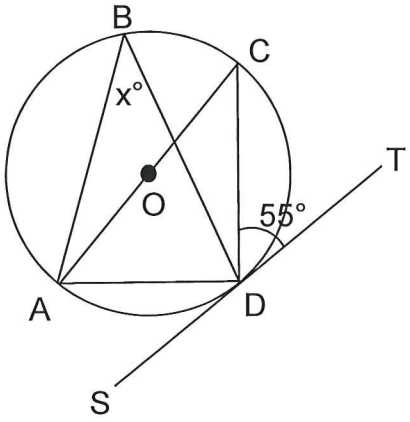
b = 70°

s∠ADC = 110° opp angles in cyclic quad are supplementary ✓

d = 110°

must have reason

(b) A circle centred at O has a tangent ST as shown in the diagram. Given that s∠CDT = 55°, determine the value of x. Justify your answer.




s∠CAD = 55° Angle in opp segment ✓

s∠ADC = 90° angles in semicircle are right angles. ✓

s∠ACD = 180 − 90 − 55 angle sum of a triangle ✓

= 35°

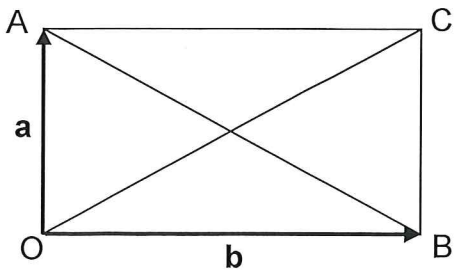
s∠ABD = 35 angles in same segment are equal ✓

x = 35

must have reason

**7.** [5 marks]

Prove that if the diagonals of a rectangle are perpendicular then the rectangle is a square.




$\overrightarrow{OC} = \underline{a} + \underline{b}$ ✓

$\overrightarrow{AB} = \underline{b} - \underline{a}$ ✓

$\overrightarrow{OC} \cdot \overrightarrow{AB} = (\underline{a} + \underline{b}) \cdot (\underline{b} - \underline{a})$ ✓

$= \underline{a} \cdot \underline{b} - a^2 + b^2 - \underline{a} \cdot \underline{b}$

$= b^2 - a^2$

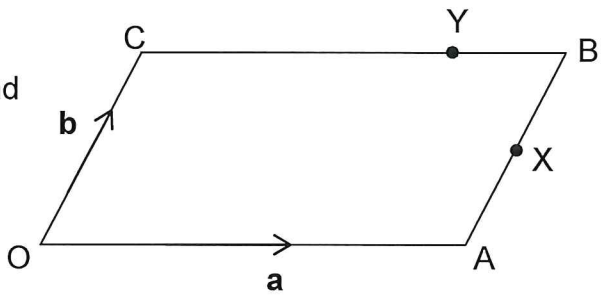
$= 0$

$\therefore b^2 = a^2$ ✓ $\Rightarrow |b| = |a|$

$\therefore$ OACB is a square ✓

**8.** [2, 3 marks]

OABC is a parallelogram, X is the midpoint of AB and Y is such that $\overrightarrow{CY} = \frac{2}{3}\overrightarrow{CB}$.




Let $\overrightarrow{OA}$ = **a** and $\overrightarrow{OC}$ = **b**.

(a) Express $\overrightarrow{OX}$ and $\overrightarrow{OY}$ in terms of **a** and/or **b**.

$\overrightarrow{OX} = \underline{a} + \frac{1}{2}\underline{b}$ ✓ $\qquad \overrightarrow{OY} = \underline{b} + \frac{2}{3}\underline{a}$ ✓

(b) Show that $\overrightarrow{OX} \bullet \overrightarrow{OY} = \frac{4}{3}\mathbf{a} \bullet \mathbf{b} + 8$, given $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 2$.

$\overrightarrow{OX} \cdot \overrightarrow{OY} = (\underline{a} + \frac{1}{2}\underline{b}) \cdot (\underline{b} + \frac{2}{3}\underline{a})$

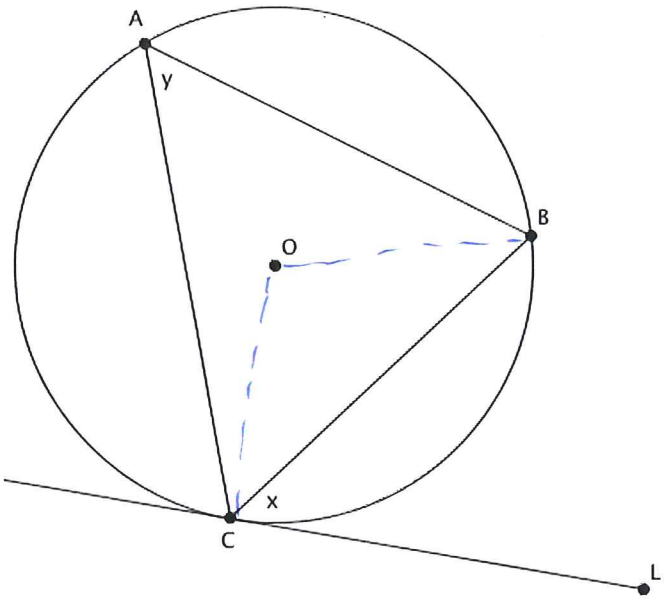
$= \underline{a} \cdot \underline{b} + \frac{2}{3}a^2 + \frac{1}{2}b^2 + \frac{1}{3}\underline{a} \cdot \underline{b}$ ✓

$= \frac{4}{3}\underline{a} \cdot \underline{b} + \frac{2}{3}3^2 + \frac{1}{2}2^2$ ✓

$= \frac{4}{3}\underline{a} \cdot \underline{b} + 6 + 2$

$= \frac{4}{3}\underline{a} \cdot \underline{b} + 8$ ✓

9. [4 marks]




In the diagram, CL is a tangent to a circle with centre O at C.

Angle BCL = $x$ and

Angle CAB = $y$.

Prove that $x = y$

Construction: OC and OB radii

$\angle COB = 2y$ (angle at centre twice angle on circumference)

$\angle OCB = \angle OBC$ (base angles isos. $\triangle$)

$2\angle OCB + \angle COB = 180$ (angle sum $\triangle$)

$2\angle OCB = 180 - 2y$

$\angle OCB = 90 - y$

$\angle OCL = 90°$ (angle radius + tangent 90°)

$\angle OCB = 90 - x$

$\therefore x = y$