Name: SOLUTIONS Date: _____

Methods Unit 1

Test 1, 2015

Topics – Equations, Polynomials and Conditional Probability

	51
	=____%

Total Time: *63 minutes*

Total Reading: *3 minutes*

Total Working: *60 minutes*

Weighting: *3.5% of the year.*

Equipment: *SCSA Formula Sheet; ½ page notes (A4 one side), CAS calculator; Scientific Calculator*

SECTION 1: CALCULATOR FREE

Time:	*24 minutes*	**Marks for Section 1:**	*23 marks*
Reading:	*1 minute*	**Equipment Allowed:**	*Nil*
Working:	*23minutes*		

1. [1 mark: 1]

A bag of 20 apples has 4 rotten ones in it. If the first two apples taken at random from the bag are good, what is the probability that the next one will be rotten?

$20-2=18$ apples left. 4 of 18 apples are rotten

$\therefore \frac{4}{18} = \frac{2}{9}$ ✓

2. [1 mark: 1]

The probability that a card drawn from a normal pack (52 cards) is a club or an Ace is:

club $= \frac{13}{52}$ (including 1 ace)

Aces $= \frac{3}{52}$ (excluding ace of clubs)

$\frac{13}{52} + \frac{3}{52} = \frac{16}{52}$ ✓

3. [2 marks: 2]

A 420-page novel is opened to a random page by 3 different students. What is the probability that they all open it to a page number between 1 and 105?

$\frac{105}{420} = \frac{1}{4}$ $\quad \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$ ✓✓

4. [6 marks: 3, 3]

Solve the following simultaneous equations

a) $5c + y = 10$ & $c + y = 10$

$y = 10 - 5c$ ✓

$c + (10 - 5c) = 10$

$c - 5c + 10 = 10$

$-4c = 10 - 10$

$-4c = 0$

$c = 0/-4$

$c = 0$ ✓

$0 + y = 10$

$y = 10$ ✓

b) $a - 6b = -6$ & $a - 2b = 2$

$a = 2 + 2b$ ✓

$2 + 2b - 6b = -6$

$2b - 6b = -6 - 2$

$-4b = -8$

$b = \frac{-8}{-4}$ $b = 2$ ✓

$a - 2(2) = 2$

$a - 4 = 2$

$a = 6$ ✓

5. [7 marks: 2, 2, 3]

Solve the following equations

a) $(3x - 2)(x + 9) = 0$

$3x - 2 = 0$

$3x = 2$

$x = 2/3$ ✓

$x + 9 = 0$

$x = -9$ ✓

b) $x^2 - 5x - 14$

$(x - 7)(x + 2)$

$x = 7$ ✓ $x = -2$ ✓

b) $3x^2 - 5x - 12 = 0$

$\frac{(3x + 4)(3x - 9)}{3} = 0$

$\frac{(3x + 4)\,3(x - 3)}{3} = 0$ ✓

$(3x + 4)(x - 3) = 0$

$3x + 4 = 0$ $x - 3 = 0$

$3x = -4$ $x = 3$ ✓

$x = \frac{-4}{3}$ ✓

6. [2 mark: 2]

Given that y is inversely proportional to the square root of x, and that $y = 24$ when $x = 36$, find the value of x when $y = 96$.

$y = \frac{k}{\sqrt{x}}$ ✓

$24 = \frac{k}{\sqrt{36}}$

$24 = \frac{k}{6}$

$24 \times 6 = k$

$144 = k$ ✓

$96 = \frac{144}{\sqrt{x}}$

$96\sqrt{x} = 144$

$\sqrt{x} = \frac{144}{96}$

$\sqrt{x} = \frac{3}{2}$

$x = \frac{9}{4}$ ✓

7. [4 marks: 1, 1, 1, 1]

The numbers of boys and girls doing some optional subjects on at the same time (on the same line of the timetable) in Year 11 at a school are shown here.

	Theatre	Workshop	Computer skills	History
Boys	13	28	32	19
Girls	34	7	35	13

a) Given that a student is taking Theatre, what is the probability that the student is female?

$34/47$ ✓

b) Given that a student is female, what is the probability that she is doing theatre?

$34/89$ ✓

c) What is the probability that a student selected at random is a girl taking theatre?

$34/181$ ✓

d) What is the probability that a boy on this line is in a workshop class?

$28/92 = 7/23$ ✓

~ END OF SECTION 1 ~

Name: _____ Date: _____

SECTION 2: CALCULATOR ASSUMED

Time: *40 minutes*

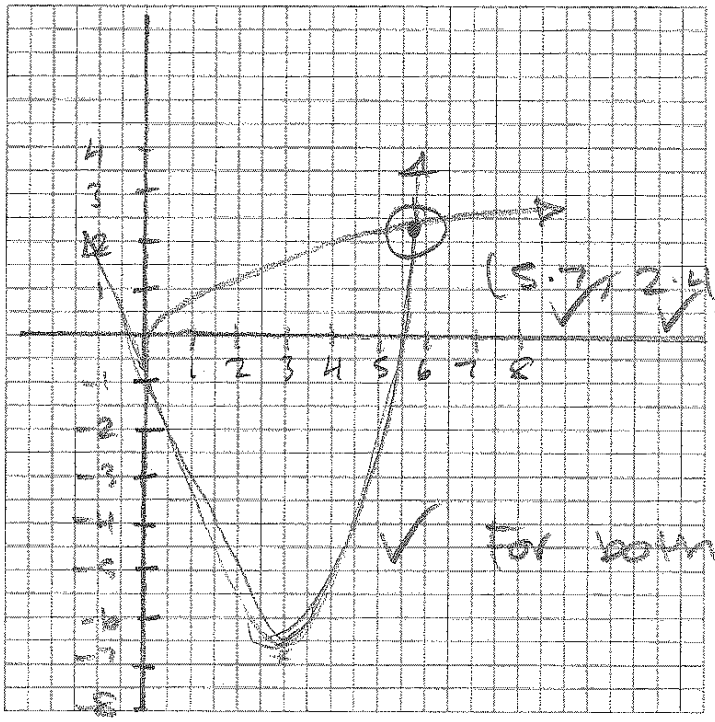
Equipment Allowed: *Curriculum Council Formula sheets, CAS calculator, 1 page of notes (A4 one side)*

Marks for Section 2: *29 marks*

8. [3 marks: 1, 1, 1]

Graph each equation then solve graphically.

a) $x^2 - 5x - 2 = \sqrt{x}$



9. [3 marks: 3]

Mike had a pen with chickens and rabbits in it. Between them, the animals had 42 heads and 142 feet.
Write equations and solve them to find out how many chickens and how many rabbits there were in the pen.

chickens = c rabbits = r

$c + r = 42$

$2c + 4r = 142$

$2(c + r = 42)$ ✓ for both formula

$2c + r = 84$

$2c + 4r = 142$

$-\ 2c + 2r = 84$

$2r = 58$

$r = \frac{58}{2}$

$r = 29$ ✓

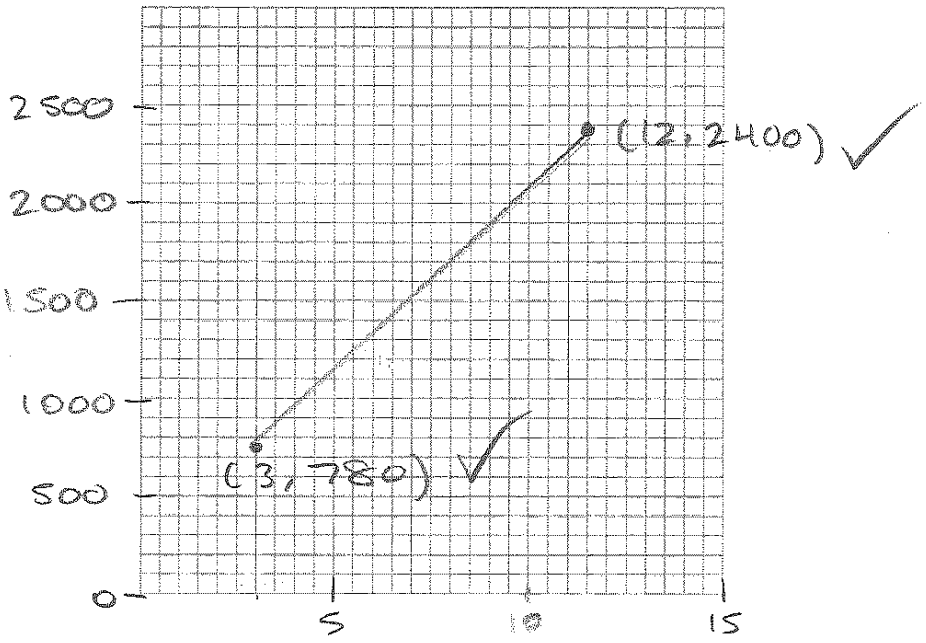
$c + 29 = 42$

$c = 13$ ✓

10. [5 mark: 5]

A Wilderness Trekking tour operator offers complete packages to the Tasmanian World Heritage areas from Launceston. The cycling tours may be from 3 to 12 days in length. They cost $780 for the 3-day tour and $180 for each additional day.

a) Draw a graph showing the cost of the cycling tours.



b) Find a relationship between the number of days and the cost of the tours.

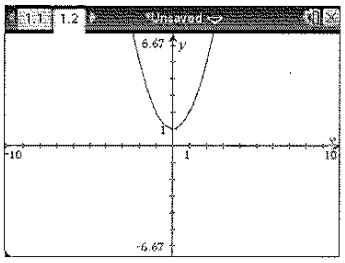
Cost of 3 days @ $180/day = $540

But 3 days cost $780 ie $540 + $240

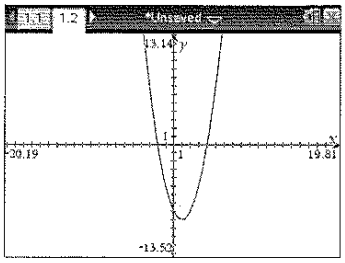
∴ costs of 3 days = $180 x 3 + $240

11. [3 marks: 1, 1, 1]

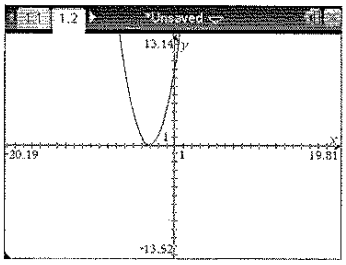
For the following graphs, match each with the correct discriminant



A



B



C

i) 0 C

ii) -3 A

iii) 6 B

12. [6 marks: 6]

Divide the polynomial $P(x) = 2x^3 + 4x^2 - x + 1$ by $x - 3$ and hence write $P(x)$ in the form $P(x) = (x - 3)Q(x) + R$.

$P(x) = 2x^3 + 4x^2 - x + 1$ ✓

$$(x-3)\overline{)\,2x^3 + 4x^2 - x + 1}\quad 2x^3 + 10x + 29$$ ✓

$2x^3 - 6x^2$ ✓

$10x^2 - x$

$10x^2 - 30x$ ✓

$29x + 1$

$29x - 87$

88

$\therefore 2x^3 + 4x^2 - x + 1 = (x-3)(2x^2 + 10x + 29) + 88$ ✓

13. [6 marks: 2, 2, 2]

A college library has a Boolean system of book classification which uses the symbols $\cap$ and $\cup$. The computer has 300 mathematical texts listed, and 405 books by Smith.

a) If M is the set of Mathematics texts and S is the set of Smith texts,

Complete the following statements:

(i) n(M) = 300 ✓

(ii) n(S) = 405 ✓

b) Matthew is looking for a mathematics text by Smith. He knows the library has at least one.

How many books are in the listing M $\cup$ S?

$n(M \cup S) = n(M) + n(S) - n(M \cap S)$

$300 + 405 - 1$ (at least) ✓

$= 704$ ✓

c) Which if the computers symbols $\cap$ or $\cup$, should he use to help him? Give reason.

looking $M \cap S$, means M and S ✓ ✓

~ END OF TEST ~