High School
Mathematics Test 2014

Year 10	*Congruence*	Non Calculator

Skills and Knowledge Assessed:

- Define congruence of plane shapes using transformations (ACMMG200)
- Develop the conditions for congruence of triangles (ACMMG201)
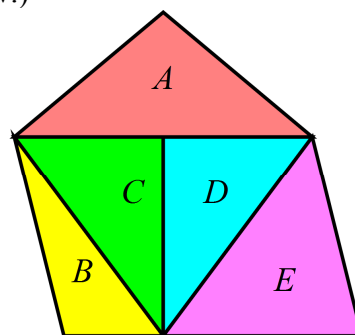- Formulate proofs involving congruent triangles and angle properties (ACMMG243)

Name_____

Section 1   Short Answer Section

Write all working and answers in the spaces provided on this test paper.

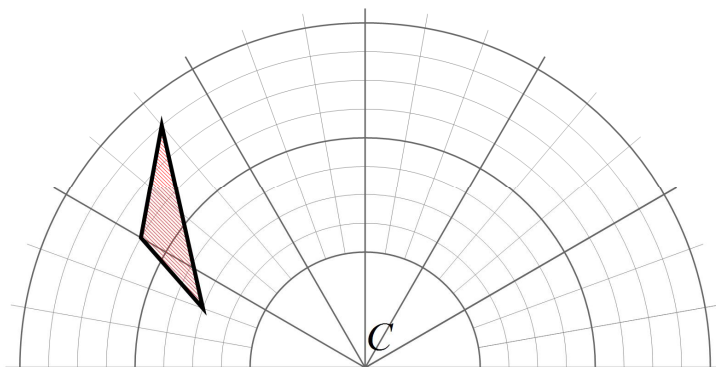| 1.	Which two triangles are congruent? (Write their letters in the space below.)

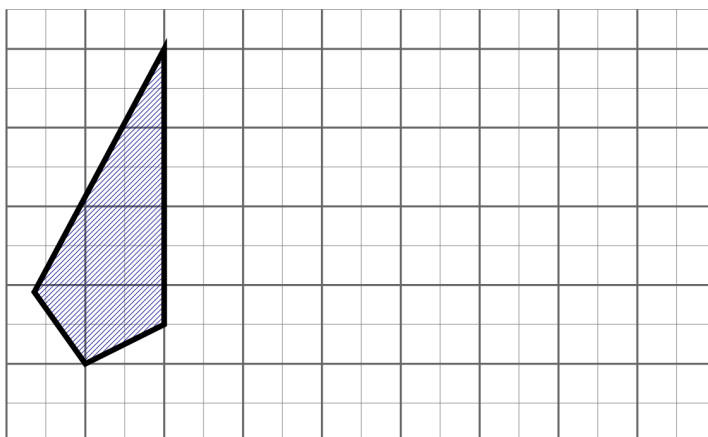TO SCALE

....

....

2.	Rotate the triangle through 90° about C in a clockwise direction.

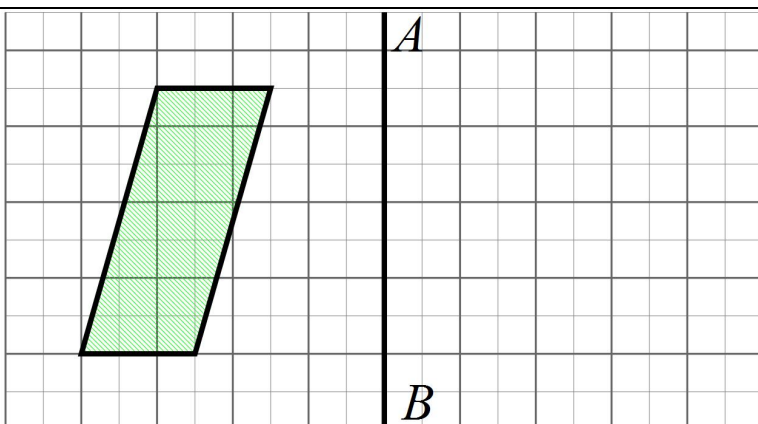Draw the triangle in its new position.

3.	Translate the kite 5 cm to the right.

Draw the kite in its new position.

4.	Reflect the parallelogram in the line *AB*. Draw the parallelogram in its new position.	

| 5.	Which two rectangles are congruent? (Write their letters in the space below.) .... ……………………………………………………………………	
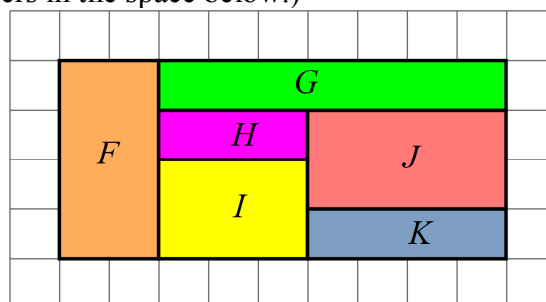
| 6.	AAS is an abbreviation for one of the tests for congruent triangles. It says: Two triangles are congruent if two angles and a side of one triangle are equal to two angles and a corresponding side of the other. What does the test which is abbreviated as SSS say? ....

7.	Which of the congruence test (AAS, RHS, SAS or SSS) could be used to show congruence of triangles PQR and RSP? 18 cm 18 cm 25 cm 25 cm ....
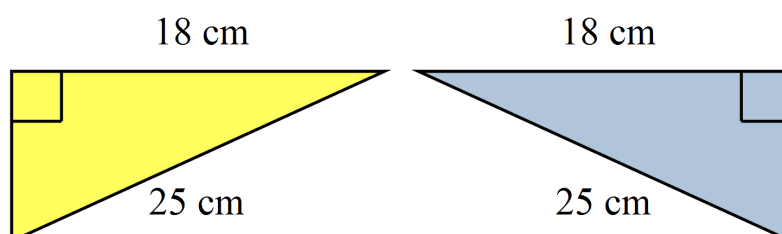
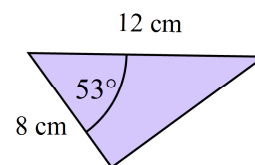**8.** Which two triangles below have enough information provided to show they are congruent to one another.
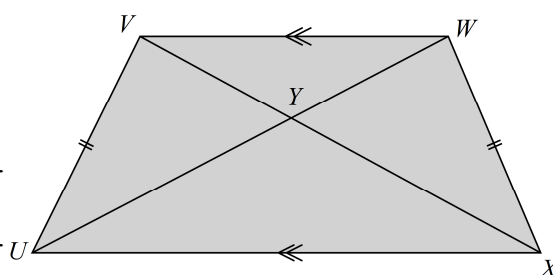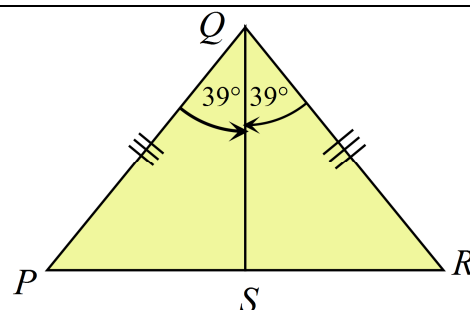Explain your answer.

12 cm 8 cm 12 cm

53° 53° 53°

8 cm 12 cm 8 cm
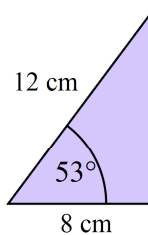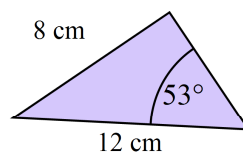
Triangle 1 Triangle 2 Triangle 3

....

....

9. *UVWX* is a trapezium which has a pair of equal sides.
Name a pair of congruent triangles in the diagram.

V *W*

Y

....

U *X*

....

10. Which of the conditions AAS, RHS, SAS or SSS could be used to show congruence of the triangles *PQS* and *RQS*?
Explain your answer.

Q

39° 39°

P *R*

S

....

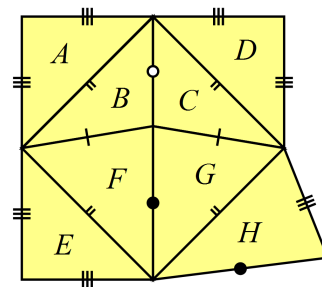...

11. In the diagram BE and AD are straight line segments and BC = CE.
What single piece of additional information would be sufficient to use the test SAS to prove that $\triangle ABC \equiv \triangle DEC$?

B

D

C

NOT TO SCALE

A
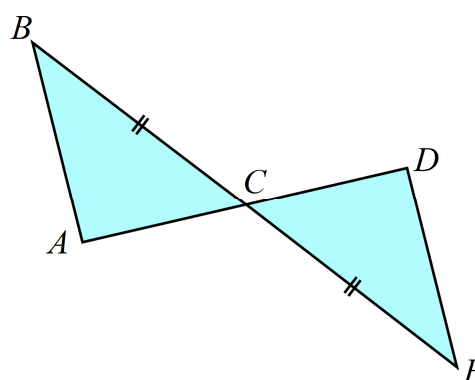
E

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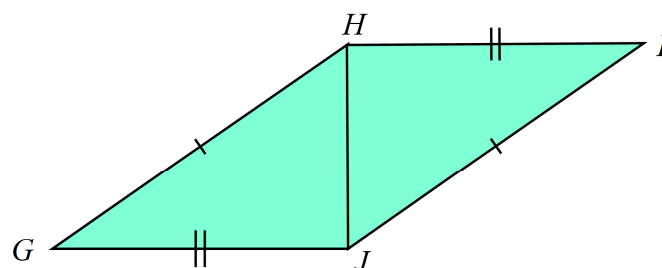|	
12.	Name two triangles which are congruent to triangle A. Explain why. ....

13.	Complete the proof below (including the congruence test used) to show that
	
	$\triangle GHJ \equiv \triangle IHJ$
	
	In $\triangle GHJ$ and $\triangle IHJ$
	  GJ = ... (given)
	  GH = ... (given)
	  HJ is ....
	
	  $\therefore \triangle GHJ \equiv \triangle IHJ$ ( )

NOT TO SCALE

14.	Complete the reasons in the congruence proof below:
	
	    Data : $\triangle STU$ is isosceles
	             with $ST = TU$
	    TV is perpendicular to SU
	    Aim : Prove $\triangle STV \equiv \triangle UTV$
	
	    Proof: In $\triangle STV$ and $\triangle UTV$
	             $\angle TVS = \angle TVU$ (.....................)
	             $\angle TSV = \angle TUV$ (.................................)
	                TV is ....
	    $\therefore$ $\triangle STV \equiv \triangle UTV$ (.................)

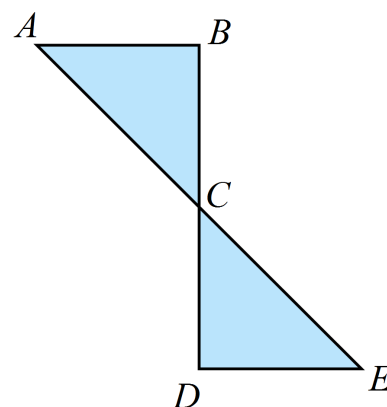|	
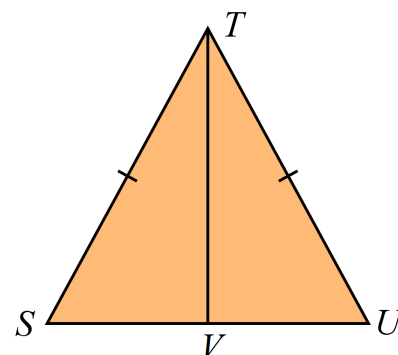
15.	Complete the congruence proof below.
	
	Data C bisects the interval AE
	        C bisects the interval BD
	Aim Prove $\triangle ABC \equiv \triangle EDC$
	Proof In $\triangle ABC$ and $\triangle EDC$
	        $CB = CD$ (C bisects BD)
	    $\angle$........ $=$ $\angle$........... (.............................)
	        $CA = CE$ (C bisects AE)
	  $\therefore$ $\triangle ABC \equiv \triangle EDC$ (.............)

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| Year	*Congruence*	Calculator Allowed
		Name_____

Section 2   Multiple Choice Section

Mark all your answers on the accompanying multiple choice answer sheet, not on this test paper. You may do any working out on this test paper. Calculators are allowed for this section.

1. The regular octagon has a number of diagonals drawn.
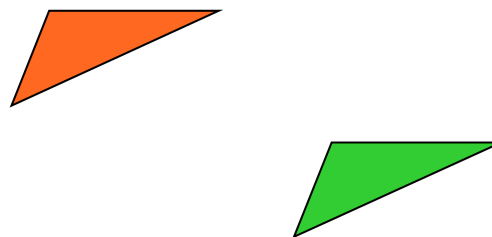
Which of these pairs of triangles are congruent?

   A.   *M* and *N*.   B.   *M* and *O*.

   C.   *N* and *R*.   D.   *N* and *S*.
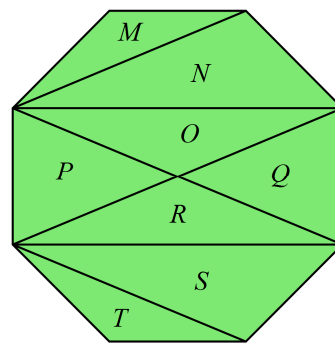
2. These two triangles are congruent.

Which transformation could move the first triangle to the second?
   A.   an enlargement.
   B.   a reflection.
   C.   a rotation.
   D.   a translation.

3. This figure seven is transformed to a congruent figure by a reflection.
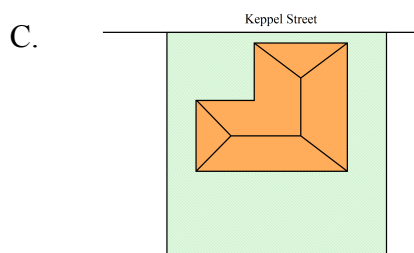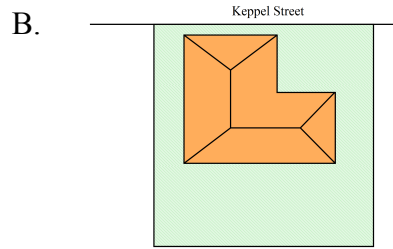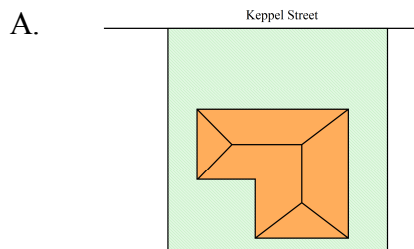
Which figure is the possible result?

   A.   B.   C.   D.

4.   Jack draws up a plan of a house on a block of land.

Keppel Street
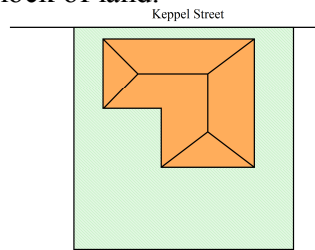
He decides to change the plan by rotating the house through 180º on the block of land.
Which could be his new plan?

A.   Keppel Street

B.   Keppel Street

C.   Keppel Street

D.   Keppel Street
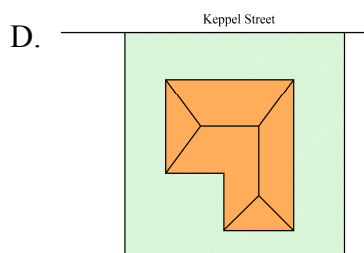
5.   Triangle A is a right angled triangle with the measurements shown at right.

Which of these triangles must be congruent to Triangle A?
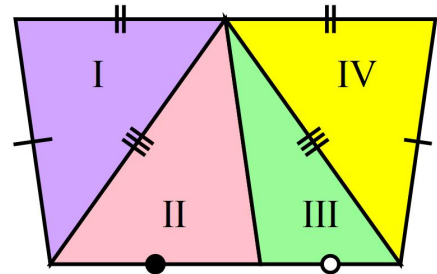
The diagrams are not to scale.

37°
5 cm
4 cm
53°
3 cm

A.   37°
4 cm
3 cm

B.   37°
53°

C.   4 cm
3 cm

D.   4 cm
53°
3 cm
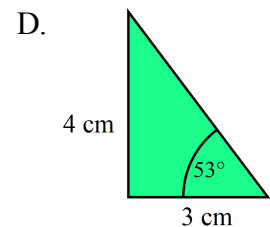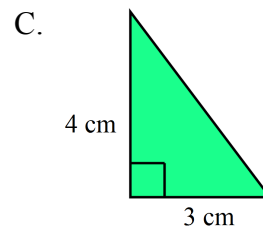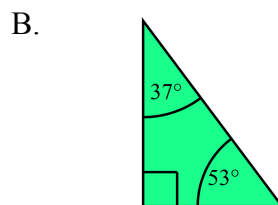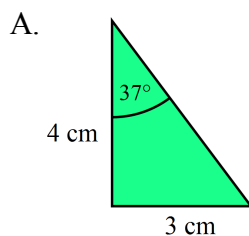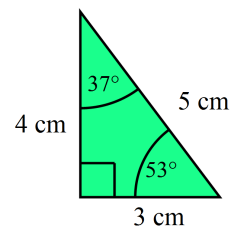
6. Which of these is not a test for a pair of congruent triangles?

    A.   Three pairs of corresponding angles are equal.

    B.   Three pairs of corresponding sides are equal.

    C.   Two pairs of corresponding sides and a pair of included angles are equal.

    D.   Two pairs of corresponding angles and a pair of corresponding sides are equal.
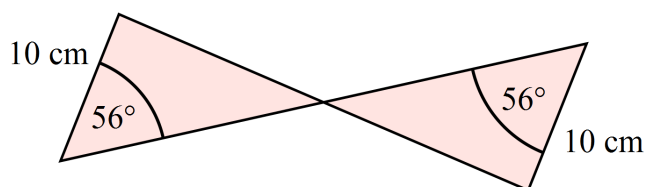
7. Which triangles are congruent?

    A.   All three triangles.

    B.   Triangles X and Y.

    C.   Triangles X and Z.

    D.   Triangles Y and Z.

3 cm   3 cm   3 cm

8 cm   8 cm   8 cm
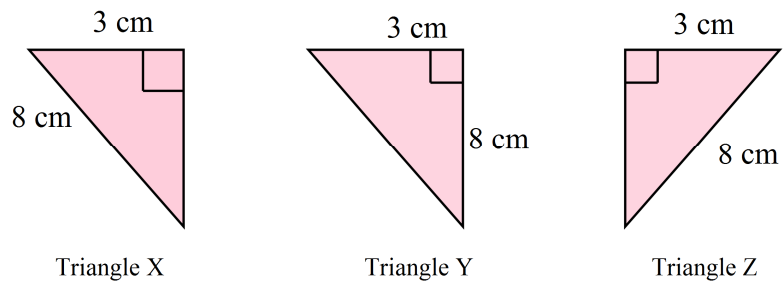
Triangle X   Triangle Y   Triangle Z

8. Which statement is true?

I   IV

II   III

    A.   Triangle *I* is congruent to triangle *II* only.

    B.   Triangle *I* is congruent to triangle *IV* only.

    C.   Triangle *I* is congruent to triangle *IV* and triangle *II* is congruent to triangle *III*.

    D.   Triangle *I* is congruent to triangle *II* and triangle *III* is congruent to triangle *IV*.

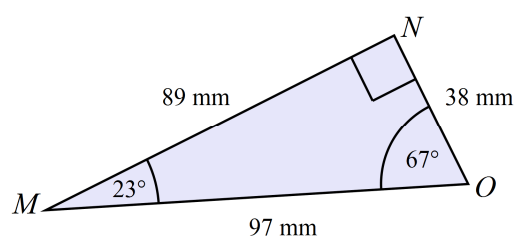9. Which of the congruence tests is sufficient to prove that $\Delta PQS \equiv \Delta RQS$?

10 cm

56°   56°

10 cm

    A. (AAS)   B. (RHS)   C. (SAS)   D. (SSS)
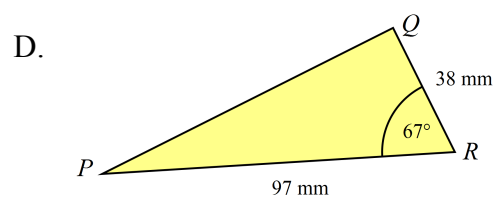
10. Which triangles are congruent ?
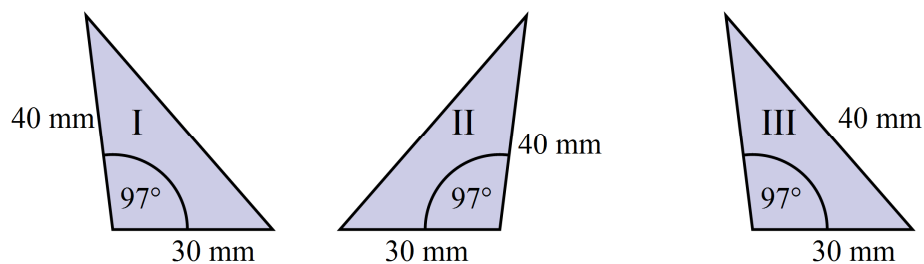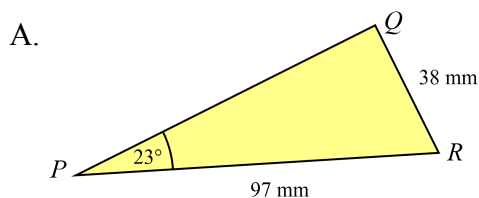

A. *I, II* and *III*.

B. *I* and *III* only.

C. *II* and *III* only.

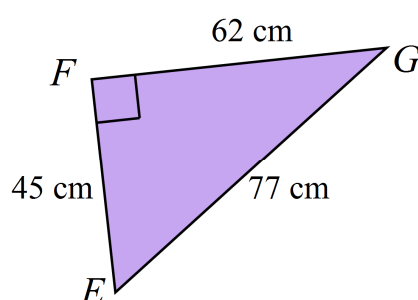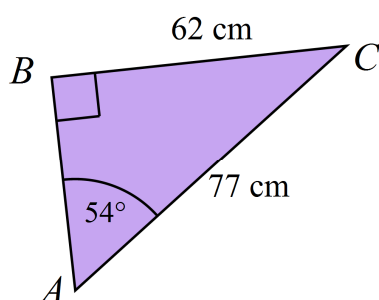D. *I* and *II* only.

11. In which triangle is there enough information to prove that $\Delta\,PQR\;\equiv\;\Delta\,MNO$?

DIAGRAMS
NOT TO
SCALE

A.

B.

C.

D.
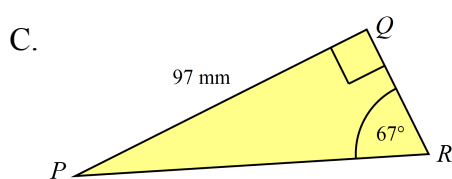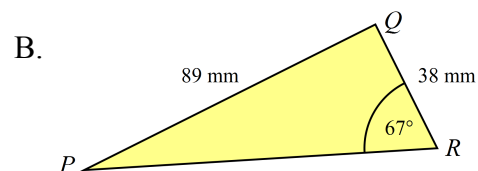
12. Which of the congruence tests is sufficient to prove that $\Delta\,ABC\;\equiv\;\Delta\,EFG.$?

A. AAS

B. RHS

C. SAS

D. SSS

13. The diagonals of the rhombus *UVWX* are drawn, intersecting at *Y*.
We can assume a rhombus to be a quadrilateral with all sides equal.
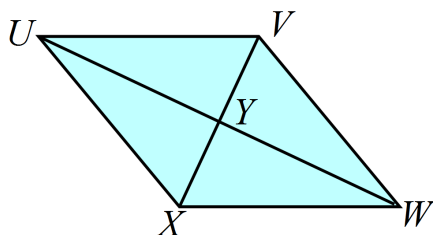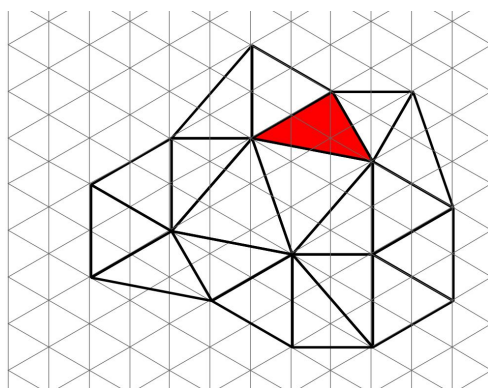Assume no other properties of the rhombus.
Which of the congruence tests is sufficient to prove that $\triangle UVX \equiv \triangle WVX$?

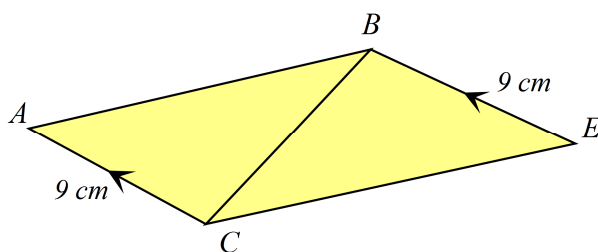A. (AAS)   B. (RHS)   C. (SAS)   D. (SSS)

14. Kate has been drawing a tiling pattern on isometric grid paper.
She has used several different sets of congruent triangles.

How many triangles has she drawn which are congruent to the shaded triangle? (including the shaded triangle).

A.   2   B.   7   C.   9   D.   19.

15. Kyle is writing a proof that $\triangle ABC \equiv \triangle ECB$.
He has left out a reason in the spot indicated by *******

In $\triangle ABC$ and $\triangle ECB$
     $AC = BE$ (given)
     $\angle ACB = \angle EBC$ (*******)
     BC is common.
     $\triangle ABC \equiv \triangle ECB$ (*SAS*)

Which reason should go in the spot?

     A.   Alternate angles on parallel lines.

     B.   Cointerior angles on parallel lines.

     C.   Equal supplementary angles.

     D.   Vertically opposite angles.

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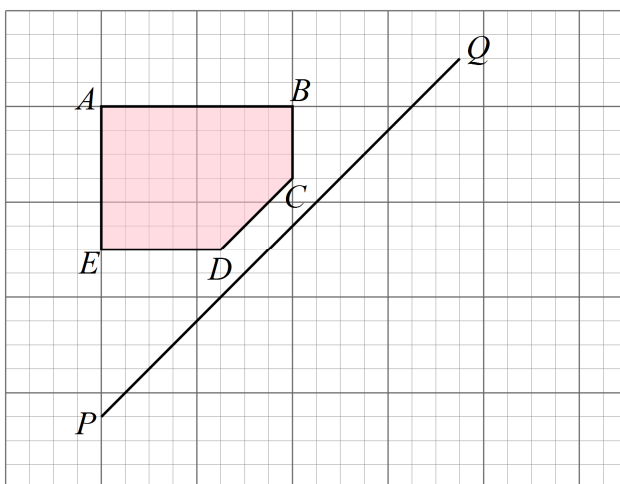| Year	*Congruence*	**Calculator Allowed**
		Name_____

Section 3   Longer Answer Section

Answers should be supported by relevant mathematical reasoning and/or calculations.
Write all working and answers in the spaces provided on this test paper.

Marks

1. (a)   Reflect the figure *ABCDE* in the line *PQ* and draw the image A'B'C'D'E'.   **2**

   (b)   Measure the distances DC and D'C'.
   What can you say about the two distances?   **1**

   ....

   ....

   (c)   What word describes the two figures *ABCDE* and A'B'C'D'E'   **1**

   ....
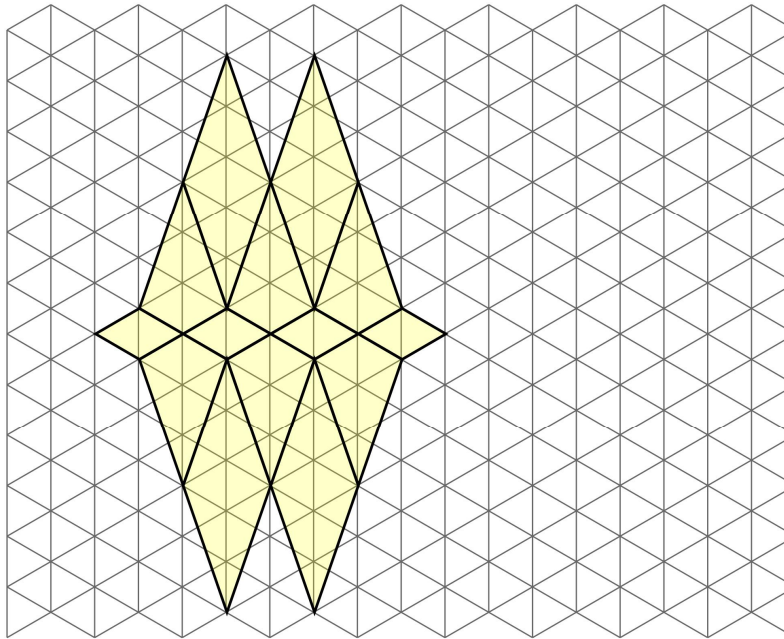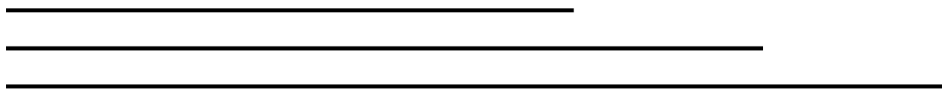
Marks

2. (a) A pattern of congruent triangles and kites has been started. **2**
    Continue the pattern by drawing at least five more shapes.

3. (a) A triangle *ABC* has sides whose lengths are identical to the three intervals below. **2**

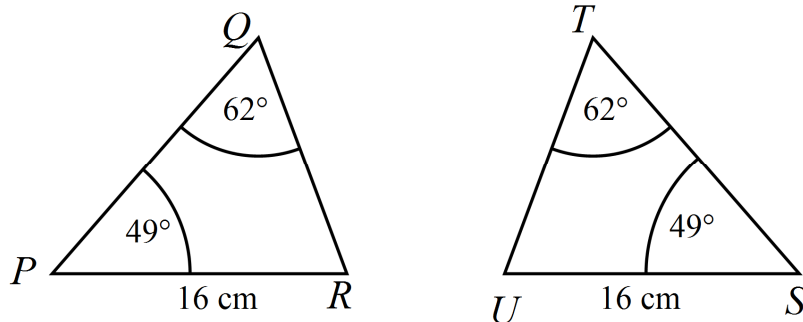   Use instruments to accurately draw a triangle congruent to *ABC*.

Marks

(b)   A triangle *PQR* has two sides which measure 6 cm and 8 cm, with an angle of 30°   **2**
between them.

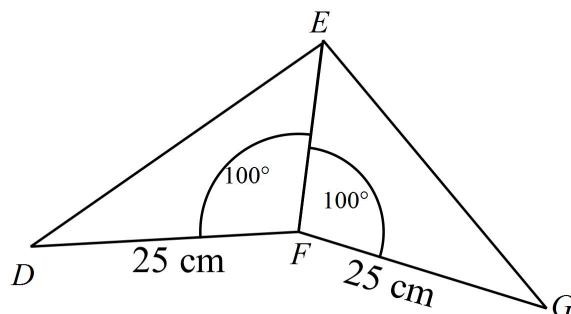Use instruments to accurately draw a triangle congruent to *PQR*.

4.   (a)   Prove that △ *PQR* ≡ △ *STU*.   **2**

....

....

....

....

....

....

Marks

(b)   Prove that $\Delta\,FED \equiv \Delta\,FEG$.   **2**

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(c)   In the quadrilateral *VWXY, VW* || *YX* and *VY* || *WX*.   **2**
        Prove that $\Delta\,VWY \equiv \Delta\,XYW$.

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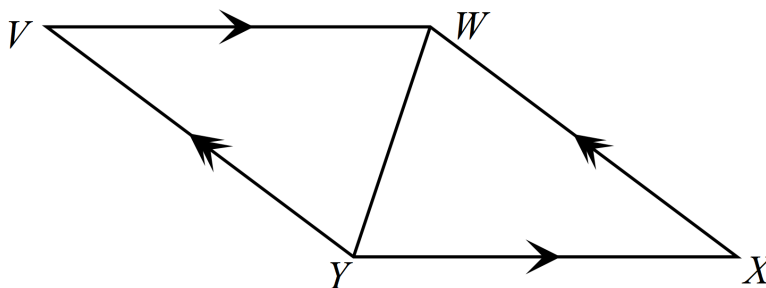
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Multiple Choice Answer Sheet

Name _____

Completely fill the response oval representing the most correct answer.

1. A ◯   B ◯   C ◯   D ◯
2. A ◯   B ◯   C ◯   D ◯
3. A ◯   B ◯   C ◯   D ◯
4. A ◯   B ◯   C ◯   D ◯
5. A ◯   B ◯   C ◯   D ◯
6. A ◯   B ◯   C ◯   D ◯
7. A ◯   B ◯   C ◯   D ◯
8. A ◯   B ◯   C ◯   D ◯
9. A ◯   B ◯   C ◯   D ◯
10. A ◯   B ◯   C ◯   D ◯
11. A ◯   B ◯   C ◯   D ◯
12. A ◯   B ◯   C ◯   D ◯
13. A ◯   B ◯   C ◯   D ◯
14. A ◯   B ◯   C ◯   D ◯
15. A ◯   B ◯   C ◯   D ◯

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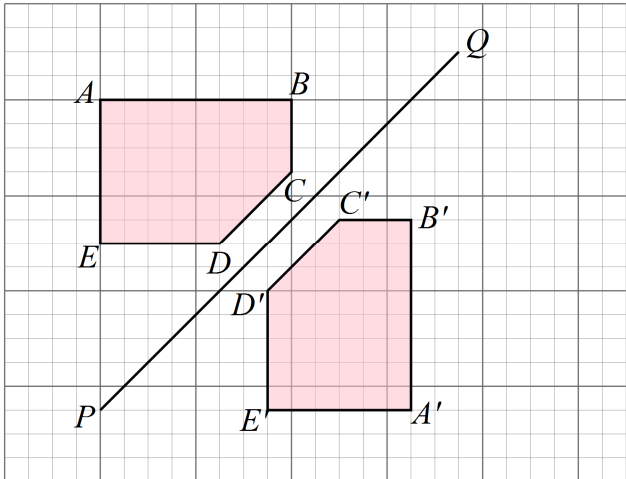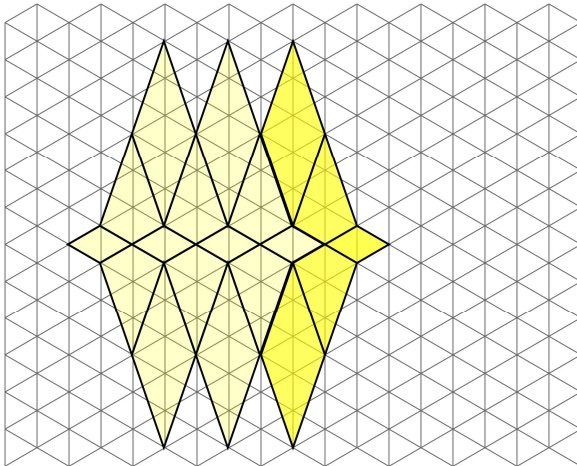
Congruence
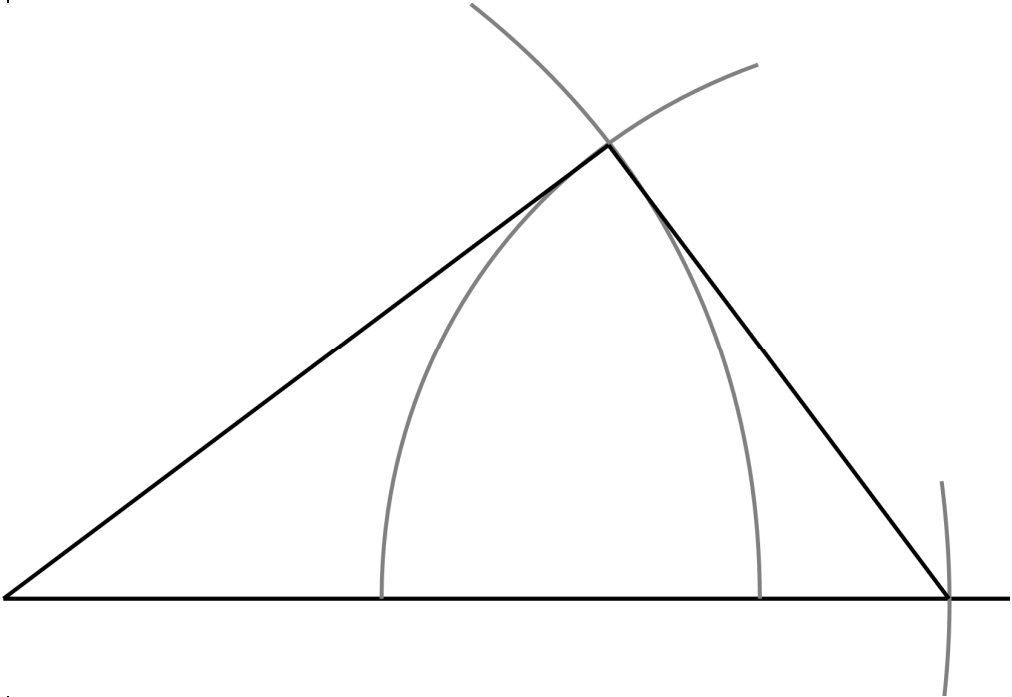
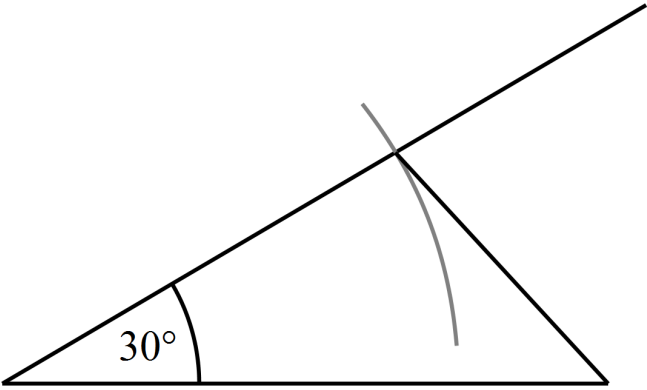## ANSWERS

<table>
<tr><td colspan="2">Section 1 (1 mark each)</td></tr>
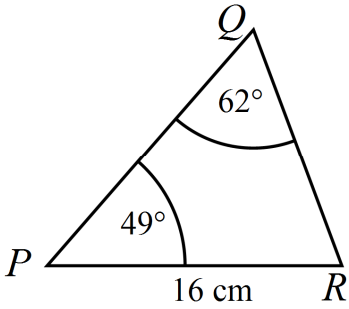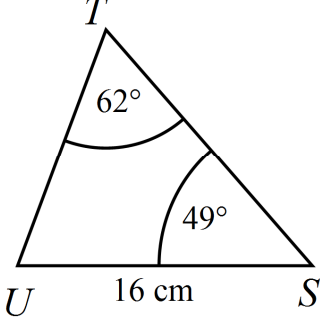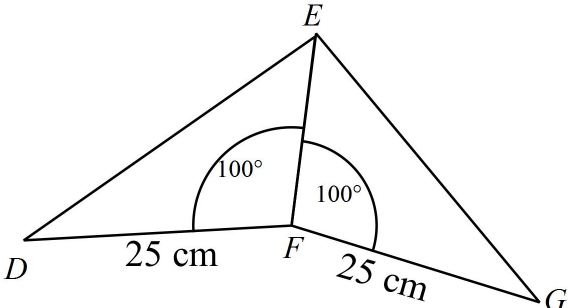<tr><td colspan="2">Working and Answers</td></tr>
<tr><td>1.</td><td>C and D are congruent.</td></tr>
<tr><td>2.</td><td></td></tr>
<tr><td>3.</td><td></td></tr>
<tr><td>4.</td><td></td></tr>
<tr><td>5.</td><td>Rectangles F and J. They both measure 4 by 2. One has been rotated 90º.</td></tr>
<tr><td>6.</td><td>Two triangles are congruent if the three sides of one triangle are equal to the three corresponding sides of the other.</td></tr>
</table>

| 7.	RHS (Right angle, Hypotenuse and Side.)
8.	Triangle 1 and Triangle 3 have enough information to use the SAS test to prove congruence.
9.	Three Possible Pairs of triangles (Only need one of these): $\Delta UVY \equiv \Delta XWY$, $\Delta UVX \equiv \Delta XWU$ and $\Delta UVW \equiv \Delta XWV$.
10.	SAS, using the common side and the data given.
11.	AC = CD (or C bisects AD) This gives sufficient data to prove congruence using SAS using the vertically opposite angles.
12.	Triangles D and E as they all have the same side lengths SSS
13.	$\Delta GHJ \equiv \Delta IHJ$ In ΔGHJ and ΔIHJ $GJ = $ HI (given) $GH = $ JI (given) HJ is common. $\therefore \Delta GHJ \equiv \Delta IHJ$ (SSS)
14.	Proof: In ΔSTV and Δ UTV $\angle TVS = \angle TVU$ (supplementary right angles) $\angle TSV = \angle TUV$ (given) TV is common. $\therefore \Delta STV \equiv \Delta UTV$ (AAS) **(Some students may try to use RHS as there is sufficient data, but would need to change given start)**

15.	Data : C bisects the interval BD
	          C bisects the interval AE
	 Aim : Prove $\Delta ABC \equiv \Delta EDC$
	 Proof : In ΔABC and ΔEDC
	      $CB = CD$ (C bisects BD)
	   $\angle ACB = \angle ECD$ (vertically opposite angles.)
	      $CA = CE$ (C bisects AE)
	 $\therefore$ $\Delta ABC \equiv \Delta EDC$ (SAS)

	Section 2 (1 mark each)	
	Working	Answers
1.	N and S are congruent (a reflection would transform N to S)	D
2.	A translation as there is no change of orientation or sense.	D
3.	Figure A is a reflection in a vertical line.	A
4.	Figure B is a rotation through 180º.	B
5.	Figure C has two sides and an included angles the same as the original. SAS	C
6.	Three pairs of corresponding angles only ensures we have similar figures, not congruent figures.	A
7.	Triangles X and Z have RHS corresponding.	C
8.	Only I and IV are congruent. SSS	B
9.	AAS using the vertically opposite angles and the two pieces of given data.	A
10.	Only I and II are congruent. SAS	D
11.	In D the arrangement gives a SAS arrangement which allows proof.	D
12.	The information which is common in both triangles is the sides of 77 cm (hypotenuse) and 62cm and the right angles. So use RHS.	B
13.	Since all sides are equal we have UV = WV and UX = WX and XV in common. Can use SSS.	D
14.		C
15.	Alternate angles on parallel lines.	A

Section 3	
Working and Answers	Marks
1. a)	2 marks for correct image, labelled. 1 mark if incomplete, incorrect or confused, but with some idea of correct method.
b) Both are approximately 13 mm in length, ie equal in length.	1 mark
c) CONGRUENT, but accept ½ mark for same shape or same size or words to that effect.	1 mark
2. At least five extra shapes added to the pattern, eg below. 	2 marks for correct pattern with at least 5 shapes. 1 mark if incomplete, incorrect or confused, but with some idea of correct method.

3. (a)

2 marks for correct triangle, with construction lines.

1 mark if incomplete, incorrect or confused, but with some idea of correct method.

(b)   A triangle *PQR* has two sides which measure 6 cm and 8 cm, with an angle of 30º between them.

Use instruments to accurately draw a triangle congruent to *PQR*.

30°

2 marks for correct triangle, with construction lines.

1 mark if incomplete, incorrect or confused, but with some idea of correct method.

4.	(a) Prove that $\triangle PQR \equiv \triangle STU$.	2 marks for complete and correct proof. 1 mark for partially complete proof or one with a minor error.

In $\triangle PQR$ and $\triangle STU$

$\angle QPR = \angle TSU$ (given)

$\angle PQR = \angle STU$ (given)

$PR = SU$ (given)

$\triangle PQR \equiv \triangle STU$ (AAS)

	(b) Prove that $\triangle FED \equiv \triangle FEG$.	2 marks for complete and correct proof. 1 mark for partially complete proof or one with a minor error.

In $\triangle FED$ and $\triangle FEG$

 $DF = GF$ (given)

$\angle DFE = \angle GFE$ (given)

FE is common

$\triangle FED \equiv \triangle FEG$ (SAS)

(c)   In the quadrilateral *VWXY*, *VW* || *YX* and *VY* || *WX*.
       Prove that $\triangle VWY \equiv \triangle XYW$.

2 marks for complete and correct proof.

1 mark for partially complete proof or one with a minor error.
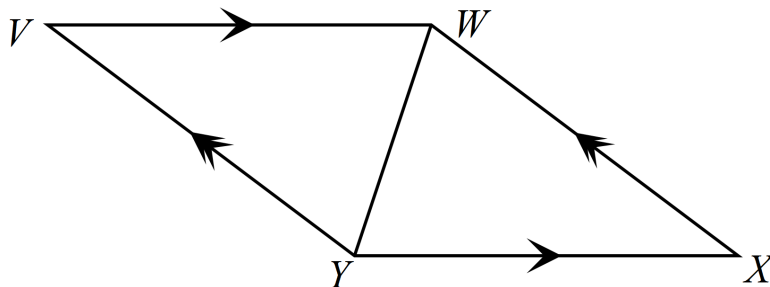
In $\triangle VWY$ and $\triangle XYW$

$\angle VWY = \angle XYW$ (alternate angles on || lines)

$\angle VYW = \angle XWY$ (alternate angles on || lines)

WY is common.

$\triangle VWY \equiv \triangle XYW$ (*AAS*)

High School
Mathematics Test 2014

Multiple Choice Answer Sheet

Name___**Marking Sheet**___

Completely fill the response oval representing the most correct answer.

1.   A ◯   B ◯   C ◯   D ⬤
2.   A ◯   B ◯   C ◯   D ⬤
3.   A ⬤   B ◯   C ◯   D ◯
4.   A ◯   B ⬤   C ◯   D ◯
5.   A ◯   B ◯   C ⬤   D ◯
6.   A ⬤   B ◯   C ◯   D ◯
7.   A ◯   B ◯   C ⬤   D ◯
8.   A ◯   B ⬤   C ◯   D ◯
9.   A ⬤   B ◯   C ◯   D ◯
10. A ◯   B ◯   C ◯   D ⬤
11. A ◯   B ◯   C ◯   D ⬤
12. A ◯   B ⬤   C ◯   D ◯
13. A ◯   B ◯   C ◯   D ⬤
14. A ◯   B ◯   C ⬤   D ◯
15. A ⬤   B ◯   C ◯   D ◯