Mathematics Department

11 Maths Methods Test 3 Odd

Trigonometry, Combinations, Transformation of Graphs and Circles.

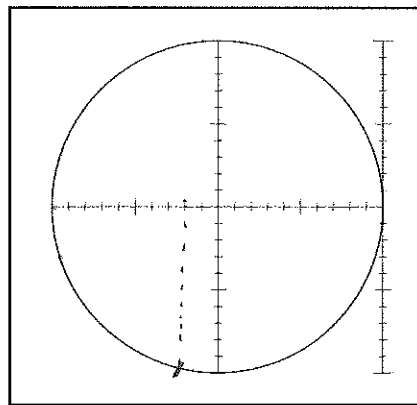Name _____

Section 1 – Resource Free

Marks : ~~26~~ 30   **Time: 30 minutes   (maximum)**

1.   [2 marks]

Use this unit circle below to estimate the value of sin 260°.

$Sin\ 260 = 0.2$

✓✓

2.   [3 marks]

a) Determine A   sin 35° = sin A° where 90° < A < 360°   A = $145°$ ✓

b) Determine B   Tan 75° = tan B ° where 90° < B < 360°   B = $255°$ ✓

c) Determine C   cos 35° = sin C ° where 90° < C < 360°   C = $325°$ ✓

3.   [3 marks]
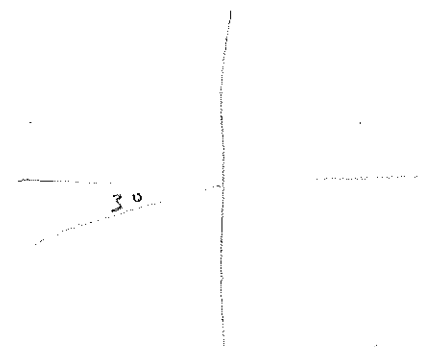
Use Exact values to evaluate:

(sin 60°)(cos 210°) + (tan 135°)²

$$= \frac{\sqrt{3}}{2} \times \frac{-\sqrt{3}}{2} + \left(\frac{-1}{\sqrt{2}}\right)^2$$

$$= \frac{-3}{4} + \frac{1}{2}$$

$$= -\frac{1}{4}$$

✓✓✓

$\boxed{\frac{8}{8}}$

4. [4 marks]

Find the 10th term of the expansion of $\left(x^2 + y^3\right)^{11}$

[you need to simplify your answer as much as possible]

$$\binom{11}{9}\left(x^2\right)^2\left(y^3\right)^9$$

$$= \frac{11 \times 10}{1 \times 2} \quad x^4 \quad y^{27}$$

$$= 55 \, x^4 \, y^{27}$$

✓
✓ Find $\binom{11}{9} = 55$
✓
✓

5. [4 marks]

Graph A shown in bold below has equation $y = \sqrt{x}$. Graphs B, C and D are all transformations of graph A. Write down the equations of B, C and D.
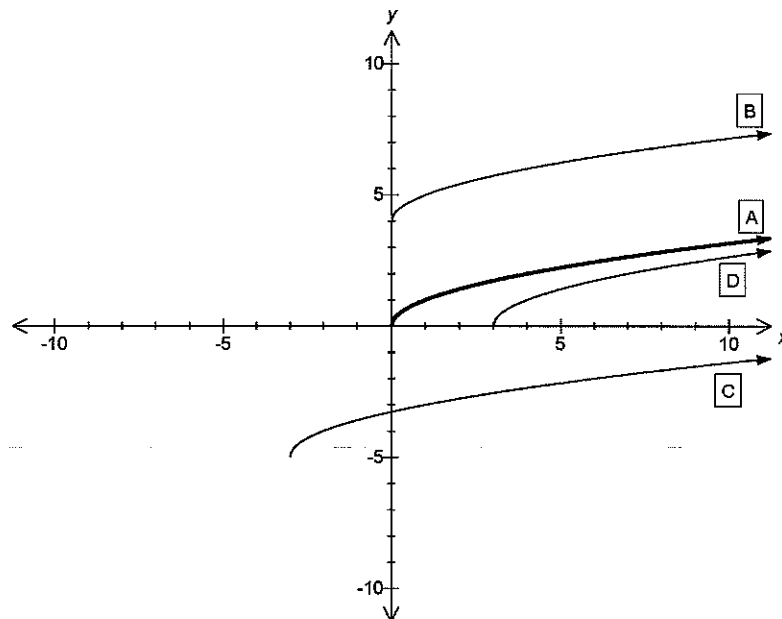
Graph B: $y = \sqrt{x} + 4$ ✓

Graph C: $y = \sqrt{x-3}$ ✓

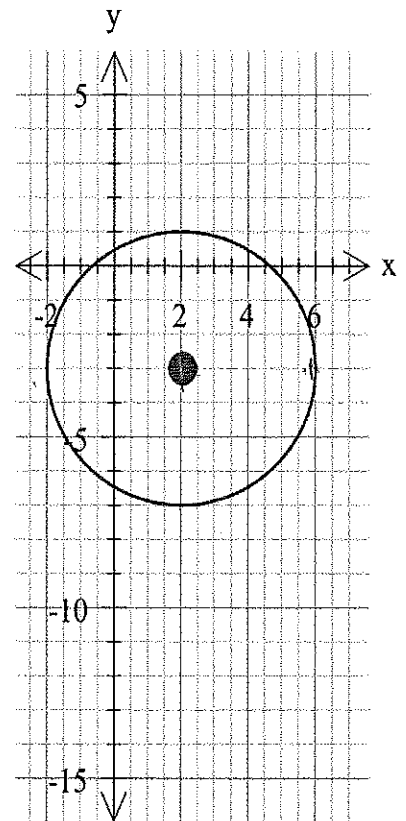Graph D: $y = \sqrt{x+3} - 5$ ✓

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6. [8 marks]

a)   i) Find the equation of the circle drawn:

$$(x-2)^2 + (y+3)^2 = 4^2$$ ✓✓

   ii) Give the domain and the range of this circle.

D $-2 \le x \le 6$ ✓

R $-7 \le y \le 1$ ✓

b)   Find the centre and radius of the circle defined by:

$$x^2 + y^2 - 8x + 6y - 24 = 0$$

$$(x-4)^2 - 16 \; (y+3)^2 - 9 - 24 = 0$$

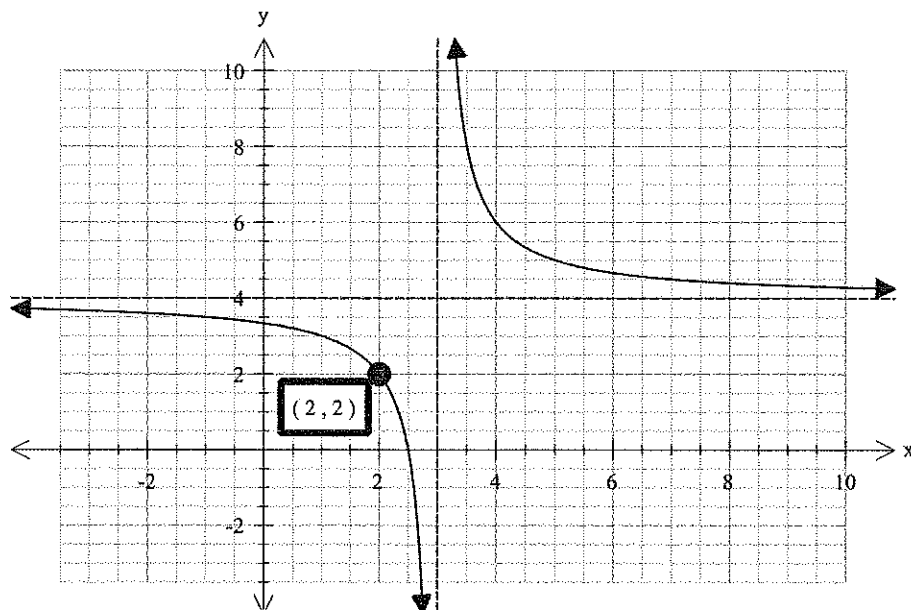$$(x-4)^2 + (y+3)^2 = 49$$ ✓✓

centre is $(4, -3)$ ✓

radius is 7 ✓

18

7. [3 marks]

Determine the equation of the graph below.

Equation: $y = \dfrac{a}{x-3} + 4$ ✓ $\Rightarrow$ $y = \dfrac{2}{x-3} + 4$ ✓✓

using $(2,2)$ $2 = \dfrac{a}{-1} + 4$

$a = 2$

8. [5 marks]
a) Describe the transformation that takes place when $y = x^3$ is changed to,
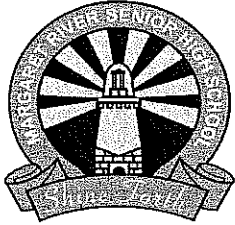
$$y = 2(3x-4)^3 + 5$$

translation 4 to right
dilation parallel to x-axis factor $\frac{1}{3}$
dilation parallel to y-axis factor 2 ✓✓✓
translation 5 up.

b) The original graph contains the point (5, 125), where will this point be transformed to in the graph of $y = 2(3x-4)^3 + 5$

$(5,125) \rightarrow (9,125) \rightarrow (3,125) \rightarrow (3,250)$
$\rightarrow (3,255)$ ✓✓

⑧

Name

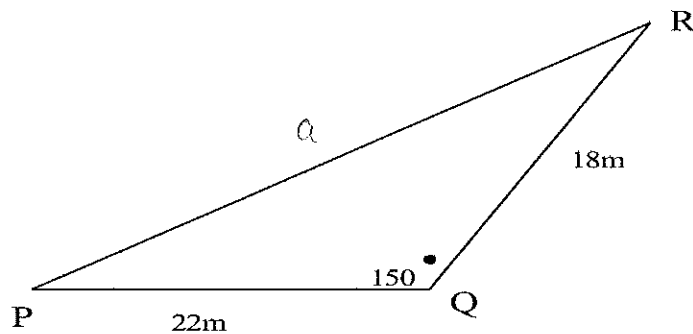**Section 2 – Resource Rich**

Marks : 26   **Time: 30 minutes   (minimum)**

10.   [2,2,2 = 6 marks]
A farmer has a paddock and he decides to fence off the corner PQR as shown in the diagram.

If PQ and QR are already fenced and are 22 metres and 18 metres respectively, find:

a)   the length of PR still to be fenced (accurate to nearest centimetre).

$$a^2 = 22^2 + 18^2 - 2 \cdot 22 \cdot 18 \cos 150$$

$$a = 38.65 \, m$$

b)   the area of the enclosed paddock PQR .

$$Area = \frac{1}{2} 22 \times 18 \times \sin 150$$

$$= 99 \, m^2$$

c)   the size of angle P, accurate to one decimal place.

$$\boxed{4}$$

11. [5 marks]

X, Y and Z are three points that all lie in a straight line with Y between X and Z. The distance from X to Y is 50 metres. A vertical tower, ZW, has its base at Z. The angle from the ground to the top of the tower (point W) at X is 30° and from Y is 50°. Find the height of the tower.
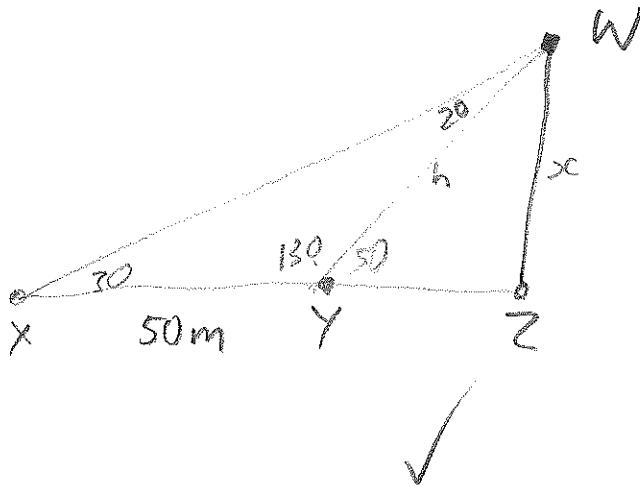
$$\frac{\sin 20}{50} = \frac{\sin 30}{h}$$

$$h = 73.095 \checkmark\checkmark$$

$$\sin 50 = \frac{x}{h}$$

$$x = 55.99 \, m. \checkmark\checkmark$$

$$\boxed{\tfrac{13}{}}$$

12. (8 marks; 1, 1, 2, 2, 2)

Fourteen girls and seven boys are applying for five student council positions. Determine how many ways the council positions can be filled if

a) there are no restrictions

$$\binom{21}{7} = 116\,280 \qquad \checkmark$$

b) three girls and two boys are selected

$$\binom{7}{2}\binom{14}{3} = 7644 \quad \checkmark$$

c) at least four girls are selected

$$\binom{14}{4}\binom{7}{1} + \binom{14}{5}\binom{7}{0} = 7007 + 2002 \qquad \checkmark\checkmark$$
$$= 9009$$

d) at least one girl and one boy are selected.

$$\binom{21}{7} - \binom{14}{5}\binom{7}{0} - \binom{14}{0}\binom{7}{5} = 116280 - 2002 - 21 \quad \checkmark\checkmark$$
$$= 114\,258$$

e) there needs to be more boys than girls on the council.

$$\binom{7}{3}\binom{14}{2} + \binom{7}{4}\binom{14}{1} + \binom{7}{5}\binom{14}{0} \qquad \checkmark\checkmark$$
$$3185 \qquad + \quad 490 \qquad + \quad 21$$
$$= 3696$$

only take off 1 mark if calculations wrong

13.   [6 marks]

The electrical resistance of a wire varies directly to its length and inversely to the square of its diameter. The resistance of 500cm of wire, with a diameter of 0.5 mm is 20 ohms.

a)   If the wire is increase to 800cm long, but the diameter stays at 0.5mm, what will be the new resistance?

$$R = kL$$
$$20 = 500k$$
$$k = 0.04$$

$$R = 800 \times 0.04$$
$$R = 32 \text{ ohms}$$

b)   If the diameter changes to 0.7mm, but the length stays at 500cm, what will be the resistance?

$$R = \frac{a}{d^2}$$
$$Rd^2 = a$$
$$20(0.5)^2 = a$$
$$a = 5$$

$$R = \frac{5}{0.7^2}$$
$$R = 10.2$$

c)   Find the resistance of 100cm of similar wire which has a diameter of 0.4mm.

$$R = \frac{cL}{d^2}$$
$$20 = \frac{c \cdot 500}{.5^2}$$
$$C = 0.01$$
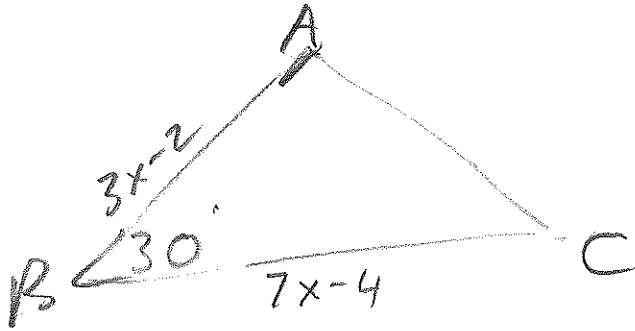
$$R = \frac{0.01 \times 100}{0.4^2}$$
$$R = 6.25 \Omega$$

$$R = \frac{0.01 L}{d^2}$$

$$\boxed{\tfrac{1}{6}}$$

14. [1, 2, 3 = 6 marks]

In triangle ABC the size of angle ABC is 30°, the side length of AB = 3x + 2 and BC = 7x - 4.

a) Write an expression for the area of the triangle in terms of x.

$$A = \frac{1}{2}(3x-2)(7x-4)\sin 30°$$ ✓

b) Write an expression for the length of AC in terms of x.

$$AC^2 = (3x-2)^2 + (7x-4)^2 - 2(3x-2)(7x-4)\cos 30°$$ ✓✓

c) Given that the area of the triangle is 131.75, find the value of x.

$$131.75 = \frac{1}{2}(3x-2)(7x-4)\sin 30$$

$$x = 5 \qquad \text{or} \qquad -5.09$$ ✓✓

but $x \neq -5$ since this makes
AC < 0

so $x = 5$

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