



12 Mathematics Methods 2022

Test 1 – Differentiation and Logarithms

Section 1: Calculator-free

Time allowed: 20 minutes

Maximum marks: 20

Name: Solutions

Teacher: Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No calculators or personal notes are permitted.

Question 1

[3 marks]

By firstly considering the result of 5^3 , use the *increments formula* to estimate the result of 4.99^3 .

$$y = x^3$$

$$\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx} \approx 3x^2 \quad \checkmark$$

$$\therefore \Delta y \approx 3(5)^2 \times (-0.01)$$

$$\approx -0.75 \quad \checkmark$$

$$\therefore 4.99^3$$

$$\approx 125 - 0.75$$

$$\approx 124.25 \quad \checkmark$$

Question 2

[2, 2 = 4 marks]

Solve the following equations exactly:

a) $9 \times 3^x - 20 = -3^x$

$$10 \cdot 3^x = 20 \quad \checkmark$$

$$3^x = 2 \quad \checkmark$$

$$x = \log_3 2 \quad \checkmark$$

b) $\log_3(2x - 3) = 4$

$$2x - 3 = 3^4 \quad \checkmark$$

$$2x = 84$$

$$x = 42 \quad \checkmark$$

Question 3 [2, 3 = 5 marks]

Differentiate the following with respect to x :

a) $\frac{-x^2}{2x+1}$

$$\frac{dy}{dx} = \frac{u'v - v'u}{v^2} \quad \checkmark \quad (\text{attempt to use quotient law})$$

$$= \frac{-2x(2x+1) - 2(-x^2)}{(2x+1)^2} \quad \checkmark \quad \left(= \frac{-2x^2 - 2x}{(2x+1)^2} \right)$$

b) $(3x^3 - 4)(2 - x)^7$

Leaving your answer in form $(2 - x)^n(ax^k + bx^{k-1} + \dots)$

$$y' = u'v + v'u \quad \checkmark \quad (\text{attempt to use product law})$$

$$= 9x^2(2-x)^7 + 7(2-x)^6(-1)(3x^3-4) \quad \checkmark$$

$$= 9x^2(2-x)^6 - 7(3x^3-4)(2-x)^6$$

$$= (2-x)^6(-30x^2 + 18x^2 + 28) \quad \checkmark$$

Question 4

[2, 2, 3 = 7 marks]

The tangent to $y = x^3 + kx + 2$ at $x = 1$ is parallel to the line $y = 4x + 9$.

a) Determine the value of k

$$y' = 3x^2 + k; \quad x=1, \quad y' = 3(1) + k = 4 \quad \checkmark$$

$$\therefore k = 1 \quad \checkmark$$

b)

i) Determine the equation of the tangent at $x = 1$.

$$x=1; \quad y = 1 + 1 + 2 = 4 \quad \checkmark$$

$$y - 4 = 4(x - 1); \quad y = 4x \quad \checkmark$$

ii) Determine the x - values of any points where this tangent meets the curve again.

$$x^3 + x + 2 = 4x \quad \checkmark \text{ (equating functions)}$$

$$x^3 - 3x + 2 = 0$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & \downarrow & & & \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$$\therefore (x-1)(x^2+x-2) = 0 \quad \checkmark$$

$$(x-1)(x+2)(x-1) = 0$$

$$\therefore x = -2 \quad \checkmark$$

END OF SECTION 1



12 Mathematics Methods 2022

Test 1 – Differentiation and Logarithms

Section 2: Calculator-assumed

Time allowed: 25 minutes

Maximum marks: 25

Name: Solutions

Teacher: Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- Calculators and 1xA4 double-sided page of personal notes are permitted.

Question 5**[1, 2, 2 = 5 marks]**Let $p = \log_5 2$ and $q = \log_5 3$.Write in terms of p and q :

a) $\log_5 6$

$$\begin{aligned} &= \log_5 2 + \log_5 3 \\ &= p + q \quad \checkmark \end{aligned}$$

b) $\log_5 \frac{16}{27}$

$$\begin{aligned} &= \log_5 2^4 - \log_5 3^3 \quad \checkmark \\ &= 4p - 3q \quad \checkmark \end{aligned}$$

c) $\log_5 2.4$

$$\begin{aligned} &= \log_5 \frac{12}{5} \quad \checkmark \\ &= \log_5 12 - \log_5 5 \quad \checkmark \\ &= 2p + q - 1 \quad \checkmark \end{aligned}$$

Question 6**[1, 2, 2 = 5 marks]**The weekly cost (in dollars) of producing x calculators is $C(x) = 40000 - 30x + 0.01x^2$

- a) Determine the cost of producing 200 calculators.

$$C(200) = \$34\,000 \quad \checkmark$$

- b) Using calculus methods, determine the value of
- x
- that minimises production costs.

$$C'(x) = -30 + 0.02x = 0 \quad \checkmark$$

$$x = 1500$$

$$C''(1500) > 0 \quad \checkmark \therefore 1500 \text{ is a min.}$$

- c) Each calculator is sold for \$40.

Determine the value of x that maximises profit and what this maximum profit is.

$$P(x) = 40x - (40000 - 30x + 0.01x^2) \quad \checkmark$$

$$P'(x) = 0 \quad ; \quad x = 3500$$

$$\therefore 3500 \text{ calculators, } \$82500 \text{ profit} \quad \checkmark$$

Question 7

[2, 2, 2, 2 = 8 marks]

A particle P moves in a straight line with its position from a fixed point, O, given by:

$$s(t) = 8t - \frac{40}{(t+1)^2} \text{ cm, where } t \text{ is the time in seconds and } t \geq 0.$$

a) Determine functions for the:

i) velocity of the particle.

$$v(t) = 8 + \frac{80}{(t+1)^3}$$



ii) acceleration of the particle.

$$a(t) = \frac{-240}{(t+1)^4}$$



b) Determine the average velocity during the first 5 seconds.

$$\frac{s(5) - s(0)}{5} = \frac{38.8 - (-40)}{5} = 15.7 \text{ cm/s}$$

c) Explain why the particle is never stationary.

$$\text{for } t \geq 0, v(t) \neq 0$$



\therefore never stationary

d) By considering both velocity and acceleration, describe what is happening to the particle at $t = 1$.

$$v(1) = 18 \text{ cm/s}$$



$$a(1) = -15 \text{ cm/s}^2$$

\therefore particle is "slowing down"



Question 8

[1, 3, 3 = 7 marks]

Properties of the function $f(x)$ are displayed in the table below.

x	-3	-2	-1	0	1	2
$f(x)$	-	0	0	-	0	-
$f'(x)$	+	+	-	+	0	-
$f''(x)$	-	-	0	+	-	-

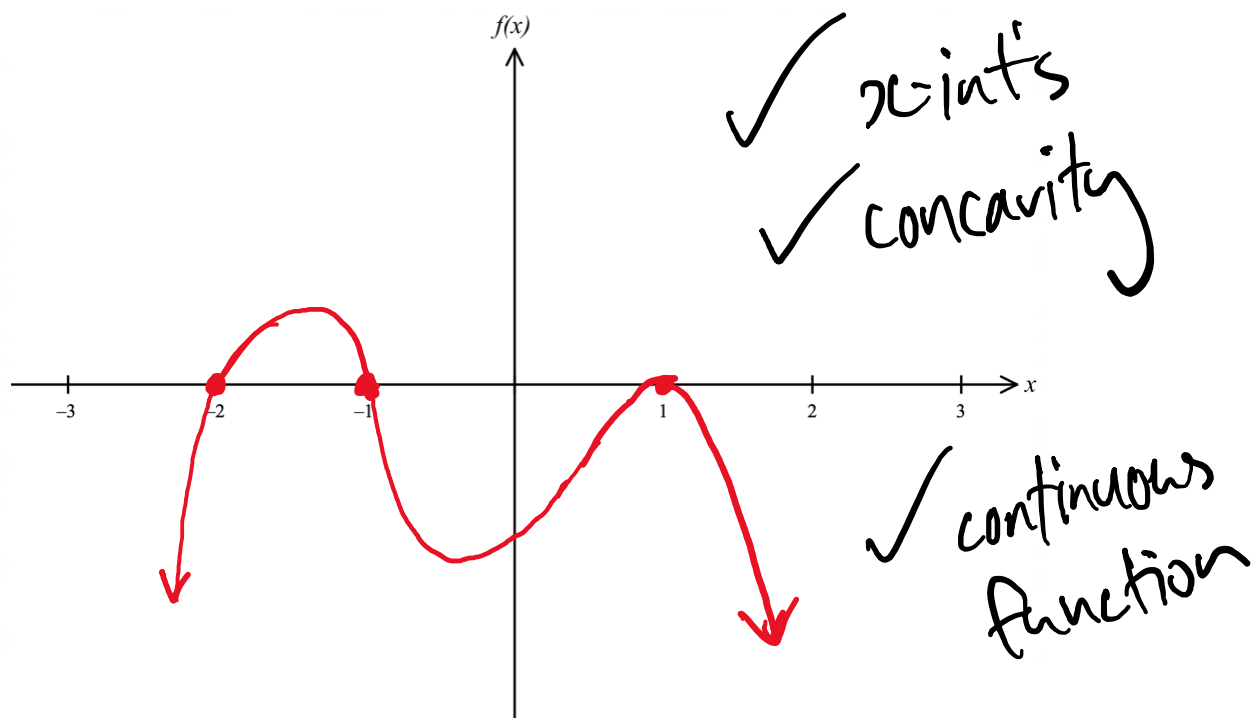
- a) Describe the concavity of the graph at $x = 0$. Justify your answer.

$f''(0) > 0$ ✓ ; concave up

- b) Fully describe all features of $f(x)$ at $x = 1$.

- x -int ✓
 - max t.p ($f'(1) = 0$ $f''(1) < 0$) ✓✓

- c) Sketch a possible graph of $f(x)$ below.



END OF TEST