Mathematics Specialist Unit 1
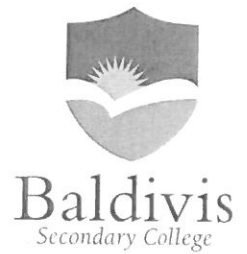
Test 1

Vectors

Name: _____ Total Marks:_____

|	
Task type:	**Response**
Time allowed for this task:	60 minutes, in-class, under test conditions
	Section One: Calculator-free 25 minutes (23 marks)
	Section Two: Calculator-assumed 35minutes (29 marks)
Materials required:	Calculator with CAS capability (to be provided by the student)
Standard items:	Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	Drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations
Marks available:	**marks**

Task weighting: 4%

Question 1 [1, 1, 1,2]
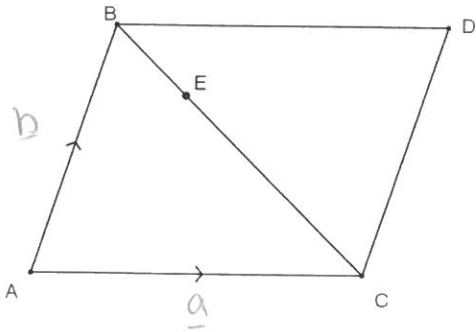
ABCD is a parallelogram with E a point on BC such that BE:EC = 1:3.
If $\overrightarrow{AC} = \boldsymbol{a}$ and $\overrightarrow{AB} = \boldsymbol{b}$. Express in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

a) $\overrightarrow{DC}$

$$-b \quad \checkmark$$

b) $\overrightarrow{AD}$

$$a+b \quad \checkmark$$

c) $\overrightarrow{BC} =$

$$a-b \quad \checkmark$$

d) $\overrightarrow{DE} =$

$$-a + \tfrac{1}{4}(a-b)$$
$$= -\tfrac{3}{4}a - \tfrac{1}{4}b \quad \checkmark$$

Question 2 [3]

The vector $\boldsymbol{a} = 12i + 5j$, Find the unit vector parallel to $\boldsymbol{a}$ but in the opposite direction.

$$|a| = 13 \quad \checkmark$$

$$\hat{a} = \tfrac{12}{13}i + \tfrac{5}{13}j \quad \checkmark$$

In opp direction $-\tfrac{12}{13}i + -\tfrac{5}{13}j$

$$\checkmark$$

2

Question 3 [1,1,1,2,2]
Given $\mathbf{a} = <2, 4>$, $\mathbf{b} = <5, -1>$ and $\mathbf{c} = <-7, 2>$. Find

   a) $\mathbf{a} - \mathbf{b}$

$$<-3, 5> \checkmark$$

   b) $-2\mathbf{b}$

$$<-10, 2> \checkmark$$

   c) $3\mathbf{c}$

$$<-21, 6> \checkmark$$

   d) $4\mathbf{a} - 3\mathbf{b}$

$$<8, 16> - <15, -3> \checkmark = <-7, 19> \checkmark$$

   e) $|\mathbf{a}| + |\mathbf{c}|$

$$|a| = \sqrt{20} = 2\sqrt{5} \qquad |a| + |c| = 2\sqrt{5} + \sqrt{53} \checkmark$$
$$|c| = \sqrt{53} \checkmark$$

Question 4 [2, 1, 1, 4]

   Given than $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{u} = 7\mathbf{i} - \mathbf{j}$, find:

   a) $|\mathbf{v} + \mathbf{u}|$.

$$v + u = 4i + 3j \checkmark$$
$$|v + u| = 5 \checkmark$$

   b) The unit vector parallel to $\mathbf{v} + \mathbf{u}$.

$$\frac{4}{5}i + \frac{3}{5}j \quad \checkmark$$

   c) A vector that is parallel to $\mathbf{v} + \mathbf{u}$ but with magnitude of 8.

$$\frac{32}{5}i + \frac{24}{5}j \quad \checkmark$$

   d) $\mathbf{v}$ in terms of $\mathbf{s}$ and $\mathbf{t}$ where $\mathbf{s} = -\mathbf{i} - 2\mathbf{j}$ and $\mathbf{t} = -\mathbf{i} + 8\mathbf{j}$.

$$4) \quad v = as + bt$$
$$-3 = -a - b \qquad \Rightarrow a = 3 - b$$
$$\checkmark \quad 4 = -2a + 8b$$
$$4 = -2(3 - b) + 8b$$
$$4 = -6 + 2b + 8b$$
$$10 = 10b \quad b = 1$$
$$\therefore a = 2 \qquad v = 2s + t \checkmark$$

Question 5 [5]

Vector $ai + bj$ is the same magnitude as $5i + 12j$ and is in the same direction as $-2i + 3j$.
Find the exact values of a and b.

$|5i + 12j| = 13$. ✓ $|-2i + 3j| = \sqrt{13}$ ✓

unit vector.

$\dfrac{-2i}{\sqrt{13}} + \dfrac{3j}{\sqrt{13}} = \dfrac{-2\sqrt{13}i}{13} + \dfrac{3\sqrt{13}j}{13}$ ✓

$ai + bj = -2\sqrt{13}i + 3\sqrt{13}j$
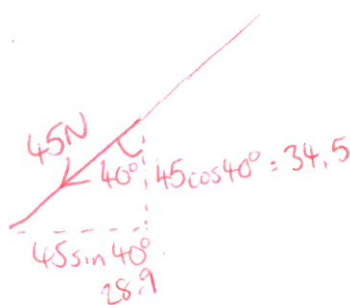
$a = -2\sqrt{13}$ ✓ $b = 3\sqrt{13}$ ✓

Question 6 [3, 2]
A force of 45N is acting on an object in the direction of 220°.

a)   Write this force in component form where i is a unit vector due east and j is a unit vector due north.

45N
40°, $45\cos 40° = 34.5$
$45\sin 40°$
28.9

$-28.9i - 34.5j$.
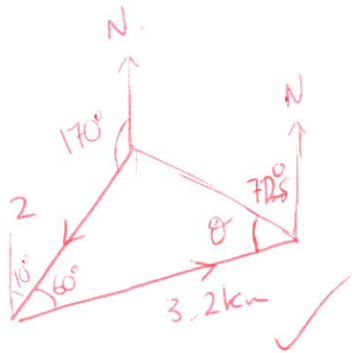
b)   A second force acts on the object to keep it in a state of equilibrium. What is the magnitude and direction of this force?

Mag = 45N.
direction = 040°.

4

Question 7 [6]

Adam leaves home and walks 2km on a bearing of 190°. He then travels 3.2km on a bearing of 070°. He then walks directly home from this point. How far will he have to walk home and on which bearing?

$$r^2 = 2^2 + 3.2^2 - 2 \times 2 \times 3.2 \cos 60°$$

$$r = \frac{14}{5} = 2.8km \checkmark$$
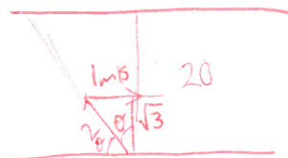
$$\frac{\sin\theta}{2} = \frac{\sin 60}{2.8} \checkmark$$

$$\theta = 38° \checkmark$$

Bearing $288°$ ✓

Question 8 [3, 3]

A boy intends to swim across a river of width 20 metres to the opposite bank. The river flows at a steady rate of 1 km/h. The boy can swim at a steady speed of 2 km/h.

a) In what direction should the boy be headed so that he ends up at the opposite bank directly opposite to where he started off?

$$\sin\theta = \frac{1}{2}$$

$$\theta = 30°$$

Direction $60°$ to the bank.

b) Find the time taken for the swim in part a).
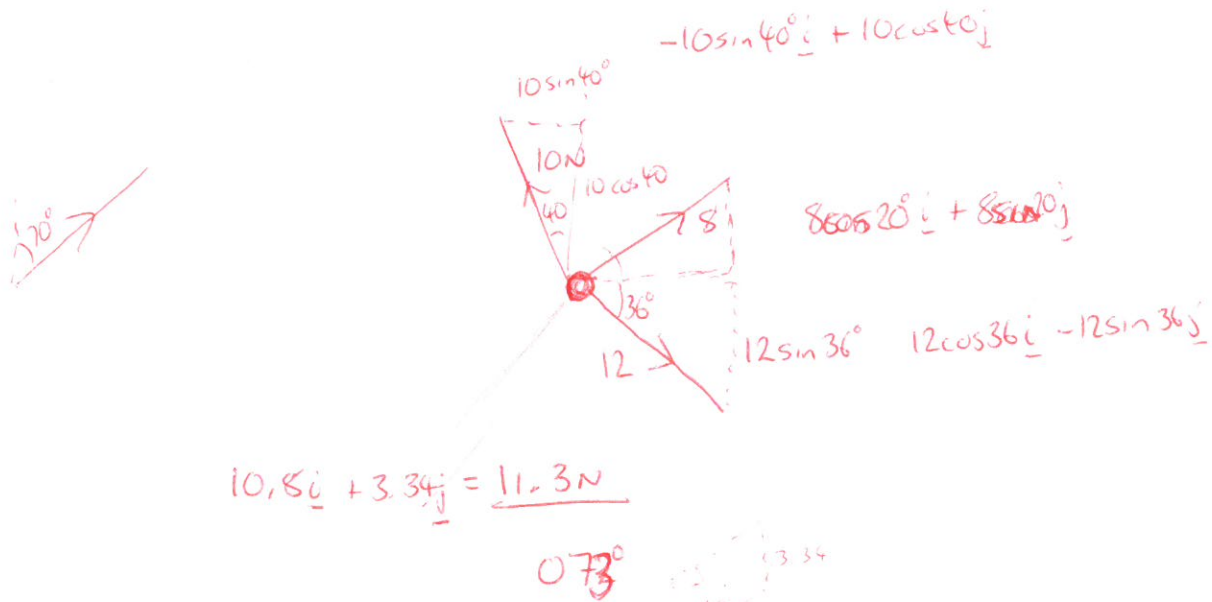
$$\sqrt{3} \, km/h = 1.73 km/h.$$

$$\frac{1.73 \times 1000}{3600} = 0.48 m/s$$

$$20 \div 0.48 = 41.57s \approx \underline{42s}$$

5

Question 9 [5,2]

Three forces act on an object in a flat plane. F_1 has a magnitude of 8 newtons and acts on a bearing of 070°, F_2 has a magnitude of 12 newtons and acts on a bearing 126° and F_3 has a magnitude of 10 newtons and acts on a bearing 320°.

a) Find the magnitude and the direction of the resultant force.

$-10\sin 40° \underline{i} + 10\cos 40 \underline{j}$

$10\sin 40°$

$10N$

$10\cos 40$

40

8

$8\cos 20° \underline{i} + 8\sin 20° \underline{j}$

$36°$

12

$12\sin 36°$ $12\cos 36 \underline{i} - 12\sin 36 \underline{j}$

$10.8\underline{i} + 3.34\underline{j} = \underline{11.3N}$

$073°$ 3.34

b) Find the magnitude and direction of the single force that will keep the object in equilibrium.

$11.3N$ $253°$

$8N$ $110°$ $126°$

$70°$ $124°$ $12N$

$34°$

$17.758N$

$11.3N$

$73°$ $10N$

$31°$

$36°$ $45°$

$17.8N$

6