



Calculator Free
Scientific Notation, Significant
Figures, Exponential Equations

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 4 = 8 marks]

(a) Write the following in expanded form:

(i) 1.07×10^8

(ii) 3.1×10^{-4}

(b) Write the following in standard form:

(i) 0.00002503

(ii) 10 200 000 000

(c) Round each of the following to 3 significant figures:

(i) 2457

(ii) 0.00005213

(iii) 12450

(iv) 0.004999

Question Two: [1, 2, 2, 2, 3, 4, 2, 4 = 20 marks]

Solve each of the following equations, showing full algebraic working:

(a) $3^x = 9$

(b) $4^y = 8$

(c) $3 \times 5^x = 375$

(d) $4^{2x} - 10 = 54$

(e) $10^m \times 100^{2m} = 0.1$

(f) $\frac{27^k}{9^{k+5}} = 81^{2k-3}$

(g) $2^{2x} - 16^x = 0$

(h) $5^{2x} - 26 \times 5^x = -25$

Question Three: [2, 2, 2 = 6 marks]

An ant travels a distance of $1.4 \times 10^{-3} \text{ km}$ from his nest, finds a crumb, and then returns to the nest.

- (a) How far did the ant travel in total? Give your answer in metres.
- (b) The trip to the crumb took 10 minutes. What was the ant's speed in km/h?
- (c) The total return trip took 30 minutes. What was the average speed of the ant, in km/h?

Question Four: [2, 3 = 5 marks]

A particular species of frog is declining in number due to a severe drought. At the beginning of 2015 a study finds that there are approximately 10 000 frogs in the area. Their numbers are halving each year.

When the population reaches 1250, conservationists will implement a plan to save this species.

- (a) Write an equation which can be used to model the above situation, where t is the number of years since the beginning of 2015.
- (b) Use your equation to determine at the beginning of which year conservationists will need to implement their plan.

This first group of humans will travel to Mars when the Earth and Mars are closest to each other in their orbit. At this point they will be approximately $5.6 \times 10^6 \text{ km}$ apart.

(a) How many kilometers will the astronauts travel per month?

(b) Light travels at approximately $3 \times 10^5 \text{ km/s}$. How many seconds would it take for people on Earth to see a very bright light shining from the surface of Mars?

(c) Give your answer in part (b) to 1 significant figure.



SOLUTIONS
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Question One: [2, 2, 4 = 8 marks]

(a) Write the following in expanded form:

(i) $1.07 \times 10^8 = 107\,000\,000$ ✓

(ii) $3.1 \times 10^{-4} = 0.00031$ ✓

(b) Write the following in standard form:

(i) $0.00002503 = 2.503 \times 10^{-5}$ ✓

(ii) $10\,200\,000\,000 = 1.02 \times 10^{10}$ ✓

(c) Round each of the following to 3 significant figures:

(i) $2457 = 2460$ ✓

(ii) $0.00005213 = 0.0000521$ ✓

(iii) $12\,450 = 12\,500$ ✓

(iv) $0.004999 = 0.00500$ ✓

Question Two: [1, 2, 2, 2, 3, 4, 2, 4 = 20 marks]

Solve each of the following equations, showing full algebraic working:

(a) $3^x = 9$

$$3^x = 3^2 \quad \checkmark$$

$$x = 2$$

(b) $4^y = 8$

$$2^{2y} = 2^3 \quad \checkmark$$

$$2y = 3$$

$$y = \frac{3}{2} \quad \checkmark$$

(c) $3 \times 5^x = 375$

$$5^x = 125 \quad \checkmark$$

$$5^x = 5^3$$

$$x = 3 \quad \checkmark$$

(d) $4^{2x} - 10 = 54$

$$4^{2x} = 64 \quad \checkmark$$

$$4^{2x} = 4^3$$

$$2x = 3$$

$$x = \frac{3}{2} \quad \checkmark$$

(e) $10^m \times 100^{2m} = 0.1$

$$10^m \times 10^{4m} = 10^{-1} \quad \checkmark$$

$$10^{5m} = 10^{-1} \quad \checkmark$$

$$5m = -1$$

$$m = \frac{-1}{5} \quad \checkmark$$

(f) $\frac{27^k}{9^{k+5}} = 81^{2k-3}$

$$\frac{3^{3k}}{3^{2k+10}} = 3^{8k-12} \quad \checkmark$$

$$3^{k-10} = 3^{8k-12} \quad \checkmark$$

$$k-10 = 8k-12 \quad \checkmark$$

$$-7k = -2$$

$$k = \frac{2}{7} \quad \checkmark$$

(g) $2^{2x} - 16^x = 0$

$$2^{2x} = 2^{4x} \quad \checkmark$$

$$2x = 4x$$

$$0 = 2x$$

$$x = 0 \quad \checkmark$$

(h) $5^{2x} - 26 \times 5^x = -25$

$$(5^x)^2 - 26(5^x) + 25 = 0$$

$$(5^x - 25)(5^x - 1) = 0 \quad \checkmark \quad \checkmark$$

$$5^x = 25 \quad 5^x = 1$$

$$5^x = 5^2 \quad 5^x = 5^0$$

$$x = 2 \quad x = 0$$

\checkmark

\checkmark

Question Three: [2, 2, 2 = 6 marks]

An ant travels a distance of $1.4 \times 10^{-3} \text{ km}$ from his nest, finds a crumb, and then returns to the nest.

- (a) How far did the ant travel in total? Give your answer in metres.

$$= 0.0014 + 0.0014 = 0.0028 \text{ km} = 2.8 \text{ m}$$

- (b) The trip to the crumb took 10 minutes. What was the ant's speed in km/h?

$$= \frac{0.0014}{\frac{1}{6}} = 0.0014 \times 6 = 0.0084 \text{ km/h}$$

- (c) The total return trip took 30 minutes. What was the average speed of the ant, in km/h?

$$= \frac{0.0028}{\frac{1}{2}} = 0.0056 \text{ km/h}$$

Question Four: [2, 3 = 5 marks]

A particular species of frog is declining in number due to a severe drought. At the beginning of 2015 a study finds that there are approximately 10 000 frogs in the area. Their numbers are halving each year.

When the population reaches 1250, conservationists will implement a plan to save this species.

- (a) Write an equation which can be used to model the above situation, where t is the number of years since the beginning of 2015.

$$1250 = 10000 \times 0.5^t$$

- (b) Use your equation to determine at the beginning of which year conservationists will need to implement their plan.

$$0.125 = 0.5^t$$

$$2^{-3} = 2^{-t}$$

$$-3 = -t$$

$$t = 3$$

$$\text{beginning of 2018}$$

Question Five: [2, 3, 1 = 6 marks]

In 2025, the Mars-One project aims to have the first group of humans land safely on Mars and inhabit the planet.

This first group of humans will travel to Mars when the Earth and Mars are closest to each other in their orbit. At this point they will be approximately $5.6 \times 10^6 \text{ km}$ apart.

It is estimated that it will take the astronauts approximately 7 months to arrive and land on Mars.

- (a) How many kilometers will the astronauts travel per month?

$$= \frac{5.6 \times 10^6}{7} = 0.8 \times 10^6 \text{ km} = 800\,000 \text{ km}$$

✓ ✓

- (b) Light travels at approximately $3 \times 10^5 \text{ km/s}$. How many seconds would it take for people on Earth to see a very bright light shining from the surface of Mars?

$$= \frac{5.6 \times 10^6}{3 \times 10^5} = 1.86666 \times 10 = 18 \frac{2}{3} \text{ s}$$

✓ ✓ ✓

- (c) Give your answer in part (b) to 1 significant figure.

20s

✓