

Full Name: SOLUTIONS



MATHEMATICS APPLICATIONS

Test 3 – Recurrence Relationships

Chapter 4

Semester 1 2017

Section Two - Calculator Assumed

Time allowed for this section

Working time for this section: 35 minutes

Marks available: 38 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

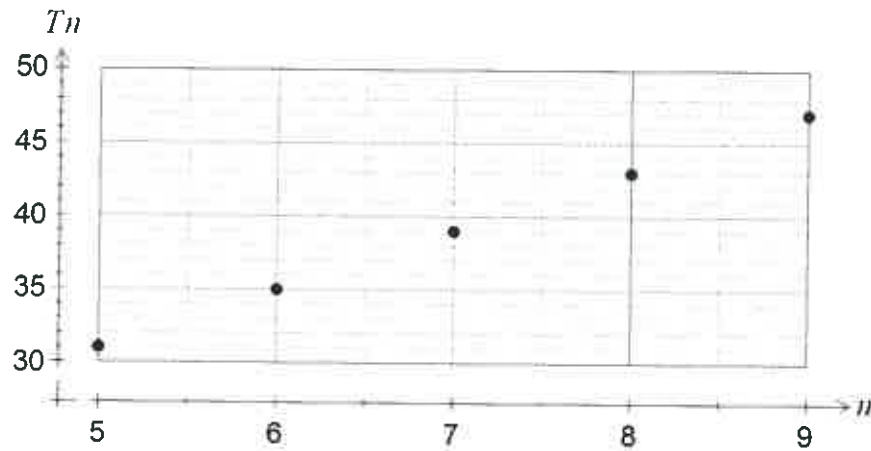
Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (6 marks)

The terms of a sequence are shown in the graph below.



- (a) Choose the best description of the sequence from geometric, arithmetic or neither, explaining your choice.

[2]

It appears linear \therefore arithmetic.

- (b) Determine

- (i) T_{10} .

[1]

$$\underline{\underline{51}} \quad (47 + 4)$$

- (ii) T_1 .

[1]

$$31 - 4 - 4 - 4 - 4 = \underline{\underline{15}}$$

- (c) Determine the recursive rule for this sequence.

[2]

$$T_n = 15 + (n-1) \times 4$$

$$= 4n + 11$$

* no simplification
is necessary
(preferred).

2. (9 marks)

a. A sequence is defined by $T_{n+1} = 2T_n$, where $T_1 = 9$.

i. Determine the next two terms of the sequence.

[2]

$$T_2 = 18 \quad \checkmark$$

$$T_3 = 36 \quad \checkmark$$

ii. State a rule for the n^{th} term of this sequence.

[1]

$$T_n = 9 \times 2^{n-1} \quad \checkmark$$

iii. Determine T_5

[2]

$$\begin{aligned} T_5 &= 9 \times 2^4 \\ &= 144 \end{aligned}$$

b. The first-order recurrence relation $t_{n+1} = bt_n + c$ was used with $t_1 = 3$ to calculate $t_2 = 4$ and $t_3 = 7$. Determine the values of b , c and t_4 .

[4]

$$4 = 3b + c$$

$$7 = 4b + c$$

$$3 = b \quad \checkmark$$

✓ process

$$\therefore c = -5 \quad \checkmark$$

$$\text{hence } t_{n+1} = 3t_n - 5$$

$$\begin{aligned} t_4 &= 3 \times 7 - 5 \quad \checkmark \\ &= 16 \end{aligned}$$

$$\begin{aligned} b &= 3 \\ c &= -5 \\ t_4 &= 16 \end{aligned}$$

3. ⁶ (5 marks)

(a) Some consecutive terms of an arithmetic sequence are shown in the table below.

n	4	5	6	7
T_n	21.5	24.2	26.9	29.6

(i) Determine the eighth term of this sequence.

(2 marks)

$$29.6 + \underbrace{2.7}_{\text{difference}} = 32.3 \quad \checkmark$$

(ii) Determine the first term of the sequence.

(1 marks)

$$21.5 - 3 \times \underbrace{2.7}_{\checkmark} = 21.5 - 8.1$$

$$= 13.4 \quad \checkmark$$

2

(iii) State a difference equation for this sequence.

(2 marks)

$$T_{n+1} = T_n + 2.7, \quad T_1 = 13.4$$

✓ ✓

End of Section One