



Name: SOLUTIONS

55 minutes working time

/47

Year 11 Mathematics Specialist Test 4

Calculators NOT allowed

1 A4 page of notes allowed

1. [7 marks: 2, 2, 3]

i. Expand and simplify $\sin\left(x + \frac{\pi}{4}\right)$.

$$\begin{aligned} & \sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} \quad \checkmark \\ &= \frac{\sqrt{2}}{2} (\sin x + \cos x) \quad \checkmark \end{aligned}$$

ii. Express $\sin\left(x + \frac{\pi}{4}\right) - \sqrt{2} \cos x$ exactly in the form $(a \sin x + b \cos x)$, where a and b are real numbers.

$$\begin{aligned} & \frac{\sqrt{2}}{2} (\sin x + \cos x) - \sqrt{2} \cos x \quad \checkmark \\ &= \frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x \quad \checkmark \end{aligned}$$

iii. Solve: $\sin\left(x + \frac{\pi}{4}\right) = \sqrt{2} \cos x$, for $0 \leq x \leq 2\pi$.

$$\sin\left(x + \frac{\pi}{4}\right) - \sqrt{2} \cos x = 0$$

$$\frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x = 0 \quad \checkmark$$

$$\frac{\sqrt{2}}{2} \sin x = \frac{\sqrt{2}}{2} \cos x$$

$$\tan x = 1 \quad \checkmark$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$\sqrt{2} \quad \sqrt{2}$

2. [4 marks]

Solve $2 - 2 \cos^2 x - 3 \sin x = 2$ for $-90^\circ \leq x \leq 90^\circ$

$$-2 \cos^2 x - 3 \sin x = 0$$

$$-2(1 - \sin^2 x) - 3 \sin x = 0$$

$$2 \sin^2 x - 3 \sin x - 2 = 0 \quad \checkmark$$

$$(2 \sin x + 1)(\sin x - 2) = 0$$

$$\sin x = 2 \Rightarrow \text{no sol}^n \quad \checkmark$$

$$\sin x = -\frac{1}{2}$$

$$x = \pm 45^\circ \quad \checkmark \checkmark$$

3. [6 marks]

Use appropriate trigonometric identities to solve $\sin 4x = \sin 2x$ between 0 and 360° .

$$\begin{aligned}\sin 4x - \sin 2x &= 0 \\ 2 \sin 2x \cos 2x - \sin 2x &= 0 \\ \sin 2x (2 \cos 2x - 1) &= 0 \quad \checkmark \\ \sin 2x &= 0 \quad \checkmark \\ 2x &= 0, 180, 360, 540, 720 \\ x &= 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ \\ 2 \cos 2x - 1 &= 0 \quad \checkmark \\ \cos 2x &= \frac{1}{2} \\ 2x &= 60, 300, 420, 660 \\ x &= 30^\circ, 150^\circ, 210^\circ, 330^\circ \\ \therefore x &= 0^\circ, 30^\circ, 90^\circ, 150^\circ, 180^\circ, 210^\circ, 270^\circ, 330^\circ, 360^\circ \\ &\quad \checkmark \checkmark \checkmark - \frac{1}{2} \text{ error or omission}\end{aligned}$$

4. [4 marks: 2, 2]

Evaluate:

i. $\cos 75^\circ \cos 15^\circ$

$$\frac{1}{2} (\cos 90 + \cos 60) = \frac{1}{2} (0 + \frac{1}{2}) = \frac{1}{4} \quad \checkmark$$

ii. $\sin \frac{11\pi}{12} \sin \frac{\pi}{12}$

$$\begin{aligned}\frac{1}{2} \left[\cos \left(\frac{10\pi}{12} \right) - \cos(\pi) \right] &\quad \checkmark \\ \frac{1}{2} \times \left(-\frac{\sqrt{3}}{2} - 1 \right) &= \frac{-\sqrt{3} - 2}{2} \quad \checkmark\end{aligned}$$

5. [3 marks]

Prove that $\cos \theta \sin 2\theta = 2 \sin \theta - 2 \sin^3 \theta$

$$\begin{aligned} \text{LHS} &= \cos \theta \sin 2\theta \\ &= \cos \theta (2 \sin \theta \cos \theta) \\ &= 2 \sin \theta \cos^2 \theta \\ &= 2 \sin \theta (1 - \sin^2 \theta) \\ &= 2 \sin \theta - 2 \sin^3 \theta \\ &= \text{RHS} \end{aligned}$$

Q.E.D.

6. [6 marks: 2, 2, 2]

Rewrite each of the following as an expression involving a single trigonometric function.

i. $6 \sin x \cos x$

$$3(2 \sin x \cos x) = 3 \sin 2x$$

ii. $2 - 4 \sin^2 4x$

$$2(1 - 2 \sin^2 4x) = 2 \cos 8x$$

$$\begin{cases} 1 - 2 \sin^2 y = \cos 2y \\ \text{Let } 4x = y \end{cases}$$

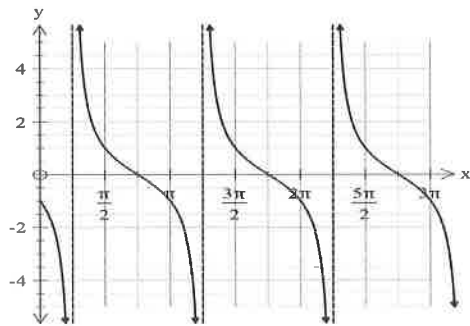
iii. $\cos 100^\circ \cos 40^\circ + \sin 100^\circ \sin 40^\circ$

$$\frac{1}{2} [\cos 140^\circ + \cos 60^\circ] + \frac{1}{2} [\cos 60^\circ - \cos 140^\circ] = \cos 60^\circ$$

7. [8 marks: 2, 2, 2, 2]

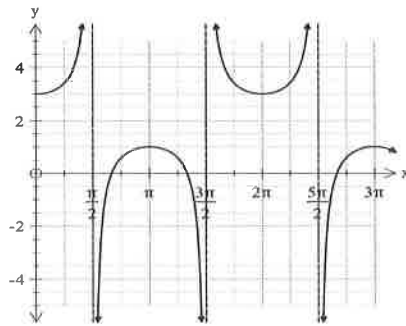
From the graphs below, determine the equation of the function.

i.



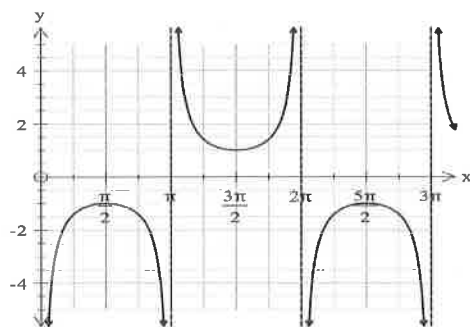
$$y = \cot(x - \frac{\pi}{4})$$

ii.



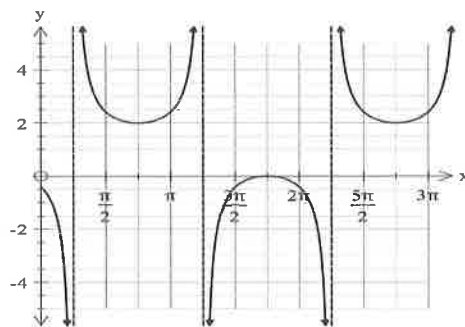
$$y = \sec x + 2$$

iii.



$$y = \sec(x + \frac{\pi}{2})$$

iv.



$$y = \operatorname{cosec}(x - \frac{\pi}{4}) + 1$$

8. [4 marks]

Prove that $\frac{\sin 6A + \sin 2A}{\sin 6A - \sin 2A} = \tan 4A \cot 2A$

$$\begin{aligned}
 LHS &= \frac{\sin 6A + \sin 2A}{\sin 6A - \sin 2A} \\
 &= \frac{2 \sin\left(\frac{6A+2A}{2}\right) \cos\left(\frac{6A-2A}{2}\right)}{2 \cos\left(\frac{6A+2A}{2}\right) \sin\left(\frac{6A-2A}{2}\right)} \\
 &= \frac{2 \sin 4A \cos 2A}{2 \cos 4A \sin 2A} \\
 &= \tan 4A \cot 2A \\
 &= RHS \\
 &\quad Q.E.D.
 \end{aligned}$$

9. [5 marks]

State the period and amplitude of the following functions.

Function	Period	Amplitude
$y = \sin x$	2π	1
$y = \sin (x+2)$	2π	1
$y = \frac{1}{2}\sin (x+2)$	2π	$\frac{1}{2}$
$y = \sec (3x+2)$	$\frac{2\pi}{3}$	undefined
$y = \cot \frac{1}{2}(x+2)$	2π	undefined

END OF TEST