Full Name: SOLUTIONS

MATHEMATICS
Methods Units 1 & 2

Test 6 – Differentiation

Chapter 8

Semester 2 2019

Section One - Calculator Free

Time allowed for this section

Working time for this section: 25 minutes
Marks available: 24 marks

Material required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the students
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

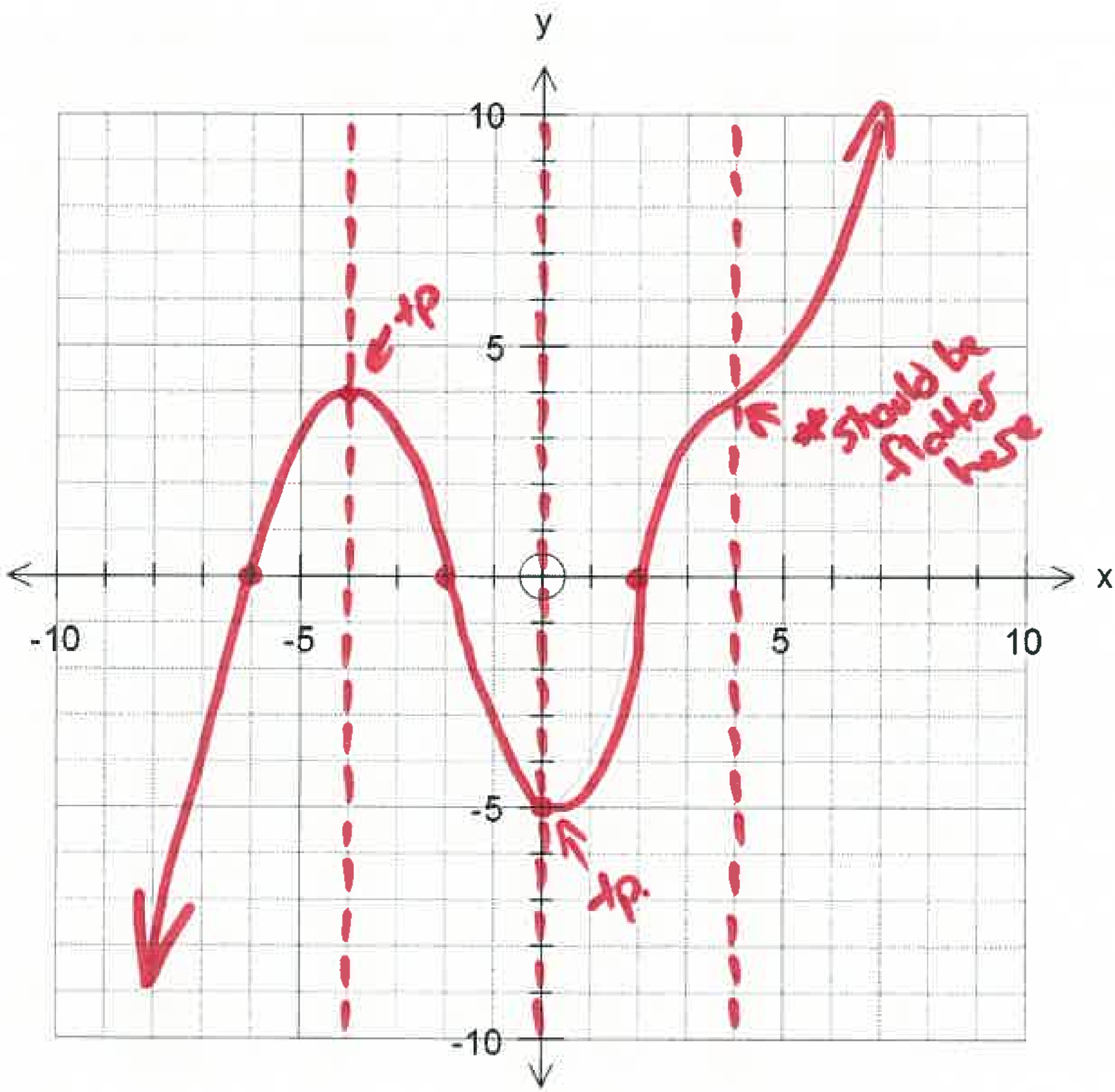
Special items: Nil

Important note to students
No other items may be used in this section of the assessment. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the assessment room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (5 marks)

Sketch the graph of a function that satisfies all of the conditions below. You do not need to determine the equation of such a function.

- The function cuts the x axis at (-6,0), (-2,0) and (2,0)
- The gradient of the function is zero at $x = -4$, $x = 0$ and $x = 4$
- The gradient is positive for $x < -4$, $0 < x < 4$, $x > 4$
- The gradient is negative for $-4 < x < 0$
- The graph cuts the y axis at -5



2. (8 marks)

Find the derivative of each of the following functions.

a. $y = 3x^4$ [1]

$\frac{dy}{dx} = 12x^3$ ✓

b. $y = 3x^2 - 2x + 5$ [2]

$\frac{dy}{dx} = 6x - 2$
✓ ✓

c. $y = (2x^2 - x + 3)(4x - 5)$ [3]

$y = 8x^3 - 4x^2 + 12x - 10x^2 + 5x - 15$
$= 8x^3 - 14x^2 + 17x - 15$ ✓

$\frac{dy}{dx} = 24x^2 - 28x + 17$
✓✓

d. $2y = 4x^2 + 6x - 8$ [2]

$y = 2x^2 + 6x - 4$ ✓

$\frac{dy}{dx} = 4x + 6$ ✓

3. (7 marks)

Given $y = 3x^2 - x$

a. Find the gradient of the tangent to $x = 3$. [3]

$\frac{dy}{dx} = 6x - 1$ ✓

sub $x = 3$ ✓

$\frac{dy}{dx} = 17$ ✓

b. Find the equation of the tangent at $x = 3$. [4]

$y = 17x + b$ ✓

when $x = 3$
$y = 24$ ✓

Sub $y = 24, x = 3$

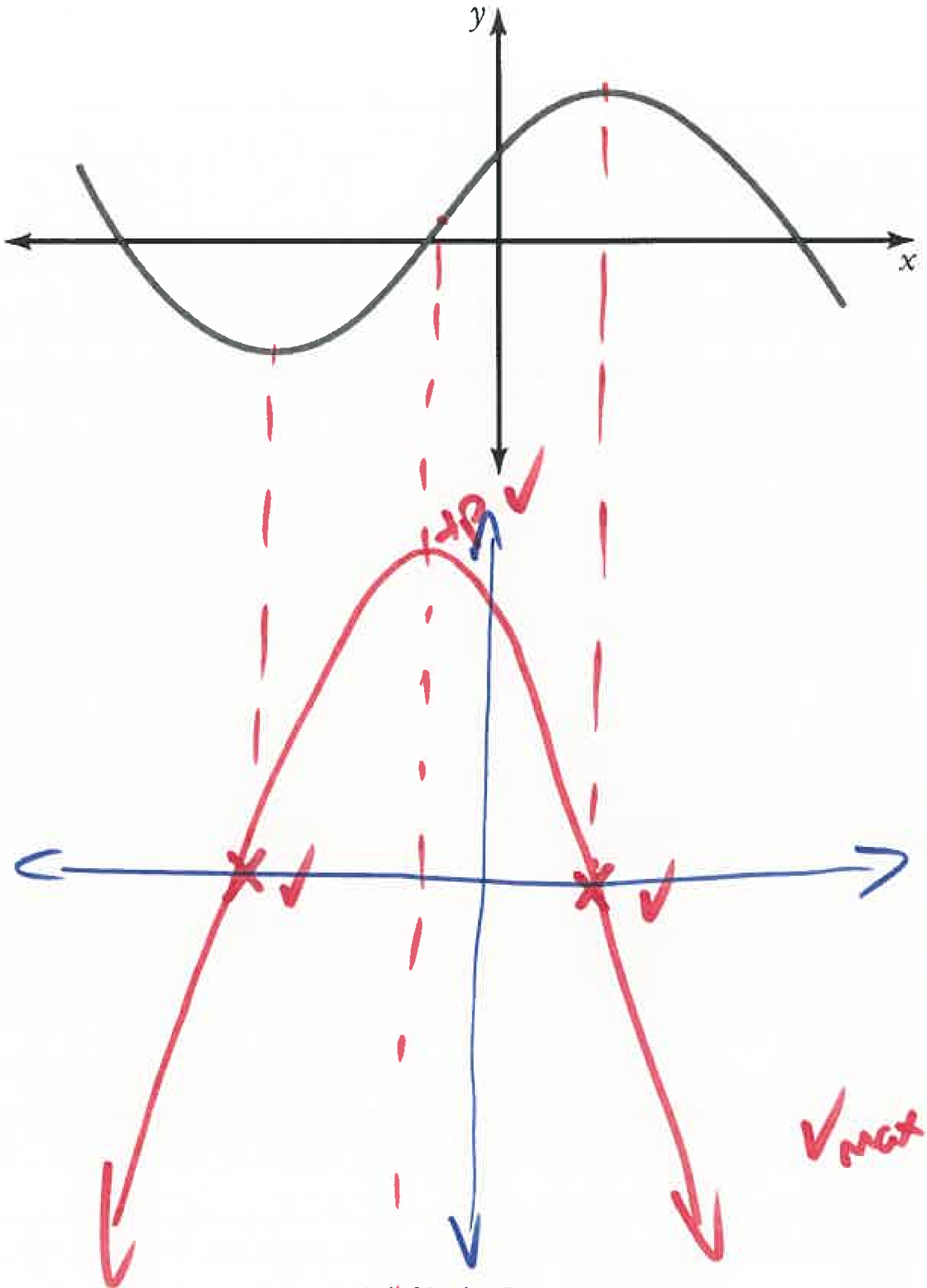
$24 = 17 \times 3 + b$

$b = 24 - 51$

$= -27$ ✓

$\therefore y = 17x - 27$ ✓

4. (4 marks)
The graph of a function is shown below.
Use it to sketch a graph of the corresponding gradient function.



End of Section One

Full Name: SOLUTIONS

MATHEMATICS
Methods Units 1 & 2

Test 6 – Differentiation

Chapter 8

Semester 2 2019

Section Two - Calculator Assumed

Time allowed for this section

Working time for this section: 30 minutes
Marks available: 31 marks

Material required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the students

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to students
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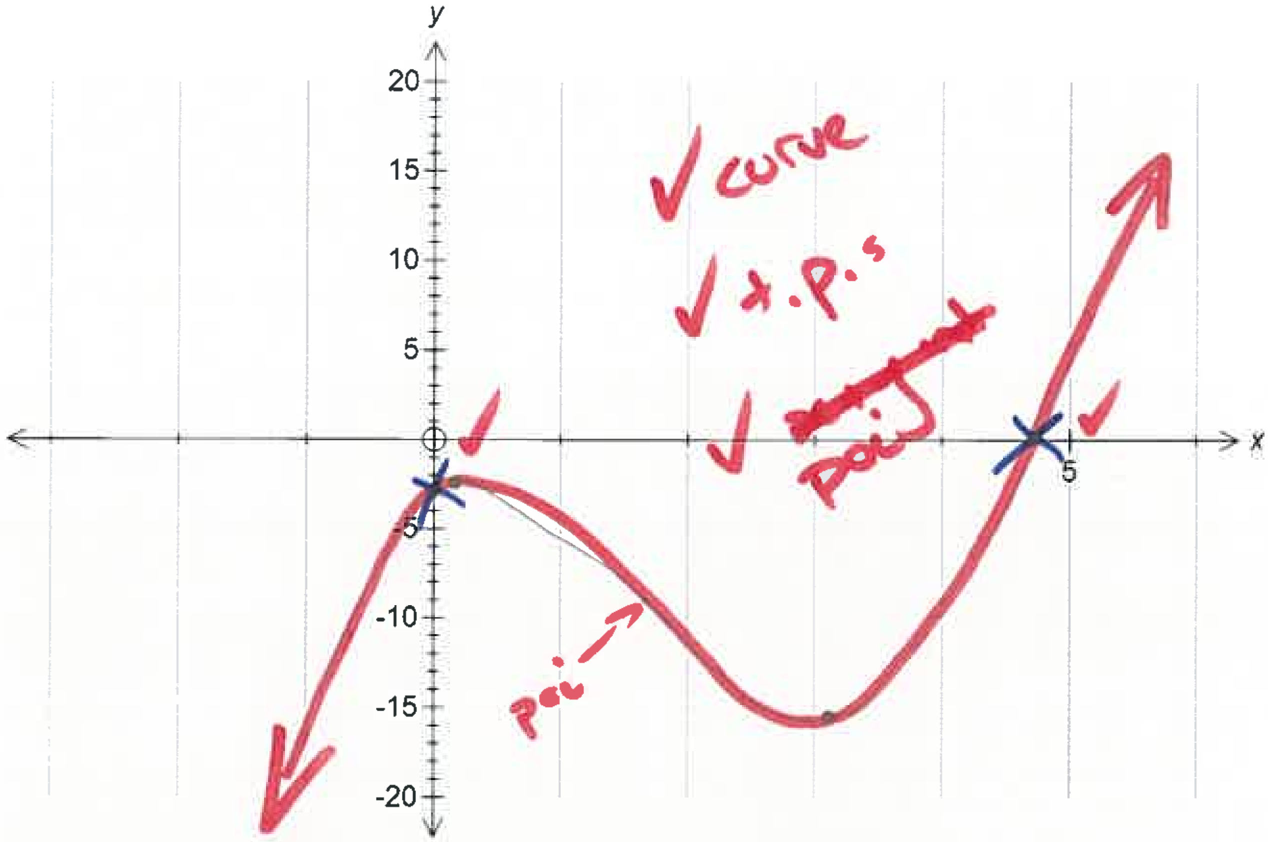
5. (9 marks)

a. On the axes below, sketch the graph of $y = x^3 - 5x^2 + 2x - 3$ [3]



b. Show all of the x-intercepts and y-intercepts on the graph above. [2]

c. Find $\frac{dy}{dx}$ [1]

$$\frac{dy}{dx} = 3x^2 - 10x + 2$$ ✓✓

d. For what values of x does the tangent to $y = x^3 - 5x^2 + 2x - 3$ have a negative slope? [1]

$$0.214 < x < 3.12$$ ✓✓

(-1) if equals signs

e. ~~Sketch the tangents to the curve, $y = x^3 - 5x^2 + 2x - 3$, at the points where $x = -1$ and $x = 4$.~~ [2]

Removed

6. (3 marks)

Given $g(x) = 3x^5 - 4x^3 + 3x - 7$ determine:

a. $g'(x)$ [1]

$g'(x) = 15x^4 - 12x^2 + 3$ ✓

b. $g'(1)$ [1]

$g'(1) = 15 - 12 + 3$
$= 6$ ✓

c. $g'(-2)$ [1]

$g'(-2) = 3(-2)^4 - 12(-2)^3 + 3$
$= 48 - 96 + 3$
$= -45$ ✓

7. (3 marks)

The volume of water flowing through a pipe is given by $V = 3t^2 + 4t$, where V is the volume in litres and t is the time in minutes. Find the rate of water flow after 7 minutes.

$V = 3t^2 + 4t$

$\frac{dV}{dt} = 6t + 4$ ✓

when $t = 7$ ✓

$\frac{dV}{dt} = 46$ ✓

$\therefore$ 46 litres per ~~minute~~ minute when $t = 7s$.

8. (8 marks)
Miss Bennett is organizing the Year 12 Awards Night. She believes that if the tickets are priced at \$80 each she will be able to sell 500 tickets. For each \$5 increase in the price of each ticket, she expects the sales to decrease by 10 tickets.

a) Find the number of tickets she expects to sell if the price of each ticket is increased by x lots of \$5. [1]

$500 - 10x$ ✓

b) Find the expected revenue when the price for each ticket is raised by x lots of \$5. [1]

$(80+5x)(500-10x)$ ✓

c) Use calculus to find the price per ticket that will maximize the revenue. State the maximum revenue. [6]

$R = (80+5x)(500-10x)$

$R' = -100x + 1700$ ✓

$0 = -100x + 1700$ ✓

$100x = 1700$

$x = 17$ ✓

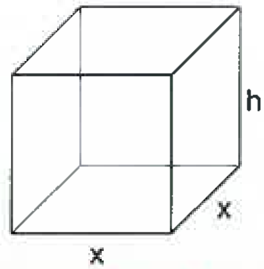
Price $80 + 5(17)$ ✓ $= \$165$ ✓

Revenue \$54450 ✓

9. (8 marks)
A rectangular box with a square base is to be made with a volume of 1500cm³.
Let x cm be the length of each side of the square base and h cm be the height of the box as shown in the diagram.

a) **Show** that the formula for the surface area is [4]

$$S.A. = 2x^2 + \frac{6000}{x}$$



$SA = 2x^2 + 4xh$ ✓

$V = 1500 = x^2 h$ ✓

$h = \frac{1500}{x^2} \Rightarrow 4xh = 4x \times \frac{1500}{x^2}$ ✓

$\therefore SA = 2x^2 + \frac{6000}{x}$ ✓

b) **Use calculus** to determine the value of x, correct to 1 decimal place, that will minimise the surface area. [4]

$SA = 2x^2 + \frac{6000}{x}$

$\frac{dSA}{dx} = 4x - \frac{6000}{x^2}$ ✓

$0 = 4x - \frac{6000}{x^2}$ ✓

$x = 11.48$

10	11.48	12
-ve	0	+ve

Min ✓

$\therefore x = 11.4$ cm ✓

End of Test