3 = 6 mark)

Unit 3

Semester 1 2019

Mathematics Methods Test 2

Integrals, fundamental theorem, applications of integration, Further differentiation and applications and integrals using exponential.

Baldivis
Secondary College

Solutions

Total time allowed: 55 minutes.
Section One: Calculator-free

Total marks: 57 marks

Time allowed for this section: 34 minutes
Total marks for this section: 34 marks

Materials allowed for this section:
SCSA Formula Sheet (provided)

Instructions to candidates
Show all of your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

Question 1

Determine each of the following, leaving your answers with positive indices. **(2, 3 = 5 Marks)**

$\int (5x+2)^4 \, dx$

$u = 5x+2$

$\frac{du}{dx} = 5$

$du = 5dx$

$dx = \frac{du}{5}$

$\int u^4 dx$

$\int u^4 \cdot \frac{du}{5}$ ✓

$= \frac{u^5}{5} \cdot \frac{1}{5}$

$= \frac{(5x+2)^5}{25} + c$ ✓

$\int \frac{3x+2}{x^4} \, dx$

$\int \frac{3x}{x^4} + \frac{2}{x^4} \, dx$

$\int \frac{3}{x^3} + \frac{2}{x^4} \, dx$

$= 3x^{-3} + 2x^{-4} dx$

$= \frac{3x^{-2}}{-2} + \frac{2x^{-3}}{-3}$

$= \frac{-3}{2x^2} - \frac{2}{3x^3} + C$

Question 2

Find the derivative of each of these with respect to x **(1, 1= 2 marks)**

$y = 2e^{3x+5}$

$\frac{d}{dx} 3x+5 = 3$

$\frac{dy}{dx} = 2e^{3x+5} \cdot 3$

$= 6e^{3x+5}$

$y = 4e^{2x} - \sqrt{e}$

$\frac{d}{dx} 4e^{2x} - \frac{d}{dx} e^{\frac{1}{2}}$

$= 8e^{2x} - 0$

$= 8e^{2x}.$

(3,

Question 3

Evaluate each of the following definite integrals, leaving your answers as exact values.

(a) $\int_2^4 3x^2 - x\,dx$

$\int 3x^2 dx - \int x\,dx$

$= x^3 - \frac{x^2}{3}$

$\int_0^4 \left(4^3 - \frac{4^3}{2}\right) - \int_0^2 \left(2^3 - \frac{2^2}{2}\right)$

$= 56 \quad - 6$

$= 50$

b) $\int_0^2 (2e^x + 3e^{3x})\,dx$

$2e^x + e^{3x}\Big]_0^2$ ✓

$(2e^2 + 3e^6) - 2e^0 + 3e^0$ ✓

$2e^2 + 3e^6 - 5$ ✓

(3 marks)

Question 4

Determine

a) $\cancel{\frac{d}{dx}\left(\int_2^x \left(1 - \frac{1}{t^3}\right) dt\right)}$

Calculate the gradient of the curve

$y = \frac{e^{-2x}}{5x}$ at $x = -1$

Quotient Rule

$\frac{dy}{dx} = \frac{5x(-2e^{-2x}) - e^{-2x}(5)}{(5x)^2}$ ✓ Correct diff ✓

$= \frac{5e^{-2x}(-2x - 1)}{5\,25x^2}$

$= \frac{-e^{-2x}(2x+1)}{5x^2}$

at $x = -1$ gradient

$= \frac{e^2}{5}$ ✓ Correct gradient

(3 marks)

Question 5

The change in cost in production of a type of table is given by $\frac{dC}{dt} = 0{\cdot}2t + 8$. Find the difference in the cost of production of 10 tables as opposed to 5.

$\frac{dc}{dt} = 0.2t + 8$

$\int_5^{10} 0.2t + 8\,dt$

$\frac{0.2t^2}{2} + 8t$

$0.1t^2 + 8t\Big]_5^{10}$

$0.1(10)^2 + 8(10) - (0.1(5)^2 + 8(5))$

$= 10 + 80 - (2.5 + 40)$

$= 47.5$

Diff in Cost is $47.50

Question 6 [3 marks]

Find the exact area between $y = e^{2x}$ and the x-axis from x = 2 to x = 3.

$$\int_2^3 e^{2x}\,dx$$

$$= \left.\frac{e^{2x}}{2}\right]_2^3$$

$$= \frac{1}{2}\left(e^6 - e^4\right)$$

Question 7 [3 marks]

A population of bacteria is decreasing at a continuous rate of 14% per month. If there were 500 million three months ago, find an expression for the number of bacteria five months from now.

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Question 8 ($\cancel{3}$ 2, 1,= $\cancel{4}$ 3 marks)

If $\frac{dA}{dt} = 4e^{3t}$ and when $t = 0$, $A = 1$. Determine:

a) A in terms of t.

$$A = \int 4e^{3t}\,dt$$

$$= \frac{4}{3}e^{3t} + C \quad ✓$$

$$1 = \frac{4}{3}e^0 + C$$

$$1 = \frac{4}{3} + C$$

$$C = -\frac{1}{3} \quad ✓$$

b) The exact value of A when $t = \frac{2}{3}$.

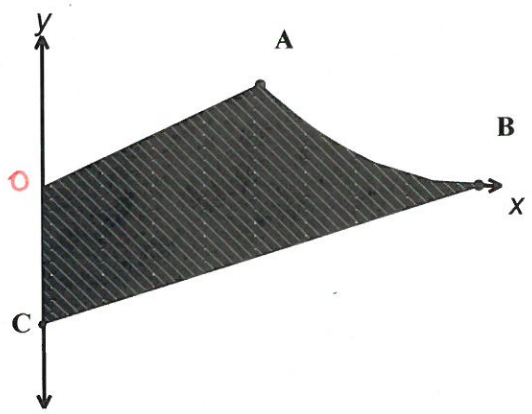
$$A\left(\frac{2}{3}\right) = \frac{4}{3}e^{3\left(\frac{2}{3}\right)} - \frac{1}{3}$$

$$= \frac{4}{3}e^2 - \frac{1}{3} \quad ✓$$

Question 9 **(3, 3 = 6 marks)**

Consider the following graphs of the functions $y = \frac{3x}{2}$, $y = \frac{3}{2}(x-2)^2$ and $y = x - 2$.

These graphs make up the shape of a special order of sail cloth as shown below.



Determine:

(a) the coordinates of A, B and C.

C = when $x = 0$ $(0, -2)$

B = when $y = 0$ $(2, 0)$

A = $1, \frac{3}{2}$

(b) definite integrals which, when added, will give the area of the sail cloth (shaded area).

(Do not evaluate)

$$\text{Area} = \int_0^1 \frac{3x}{2}\,dx + \int_1^3 \frac{3}{2}(x-2)^2\,dx + \int_2^0 x-2\,dx$$

Unit 3
Semester 1 2019
Mathematics Methods Test 2
Integrals, fundamental theorem, applications of integration, Further differentiation and applications and integrals using exponential.

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Secondary College

Name SOLUTIONS

Total time allowed: 55 minutes.
Section One: Calculator Assumed

Total marks: 60 marks

Time allowed for this section: 25 minutes
Total marks for this section: 25 marks

Materials allowed for this section:
SCSA Formula Sheet (provided)
1 Page front and back hand written notes
1 Scientific or Graphics calculator

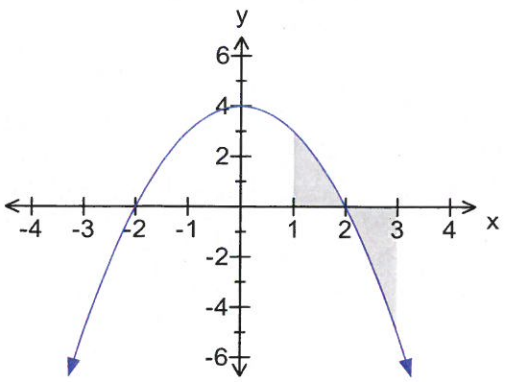
Instructions to candidates
Show all of your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

Question 6 **(1, 2, 1 = 4 marks)**

(a) Find $\int_1^3 4 - x^2 \; dx$

$= -\frac{2}{3}$

(b) Find the Area shaded below



Area $\int_1^3 |4 - x^2| \, dx$ ✓

$= 4$

(c) Explain why the answer for (a) is different than (b)

a) is not the area as it subtracts the Area below from Area above
where b) is the total area between curve and axis (shaded area) added

Question 7 **(3 marks)**

The area under the curve $f(x)=4e^{kx}$ over the domain $0 \le x \le 10$ is $\frac{40}{3}\left(-e^{-3}+1\right)$.

Determine the value of *k*.

$$\int_0^{10} 4e^{kx}\,dx = \frac{40}{3}\left(-e^{-3}+1\right)$$

$$\left[\frac{4e^{kx}}{k}\right]_0^{10} = \frac{40}{3}\left(-e^{-3}+1\right) \checkmark$$

$$\frac{4e^{10k}}{k} - \frac{4}{k} = \frac{40}{3}\left(-e^{-3}+1\right) \checkmark$$

$$k = -0.3 \checkmark$$

Question 8 **(5 marks)**

Determine the area between the curves $y=x^3$ and $y=3x^2+10x$. Show working for full marks.

$$\text{Solve}\left(x^3 = 3x^2+10x ; x\right)$$

$$x = -2, 0, 5 \checkmark$$

When $x=-1$, $x^3=-1$, $3x^2+10x=-7$

When $x=1$, $x^3=1$, $3x^2+10x=13$

$$\text{Area} = \int_{-2}^{0} x^3-(3x^2+10x)\,dx + \int_0^5 3x^2+10x-x^3\,dx \checkmark\checkmark$$

*

$$= 8 + 93.75 \checkmark$$

$$= 101.75 \text{ units} \checkmark$$

* Accept

$$\text{Area} = \int_{-2}^{5} \left|3x^2+10x-x^3\right|dx$$

or

$$\text{Area} = \int_{-2}^{5} \left|x^3-3x^2-10x\right|dx$$

but must have |.....| for any marks.

Question 9 **(3, 4 =7 marks)**

a) Use calculus to show that the function $y = a - 2ae^{2x}$,where *a* is a positive constant, only has one stationary point which is located at (0, -*a*) .

$\frac{dy}{dx} = -4axe^{x^2}$

$0 = -4axe^{x^2}$

$x = 0$

when $x = 0$

$y = a - 2ae^0$

$= -a$

stationary point at $(0, -a)$

✓ derivative using chain rule

✓ equates to zero

✓ Substitutes to determine y co-ordinate

b) Use the second derivate to determine the nature of the stationary point.

$\frac{d^2y}{dx^2} = -4ax(2xe^{x^2}) - 4ae^{x^2}$ ✓✓ (Second derivative using product & chain rule)

$\left.\frac{d^2y}{dx^2}\right|_{x=0} = -4a$ ✓ (finds value of 2nd derive when $x=0$)

since a is as positive constant the second derivative is negative

$\therefore$ Maximum ✓ (States nature of stationary point)

Question 10 **(2, 1, 2, 3 = 8 marks)**

The size of a population of birds is changing according to the rule $\frac{dP}{dt}$ = -0.08*P*, where *P* is the number of birds in the population and *t* is the time in years from the initial population measurement. There are initially 1000 birds in the population.

a) State an equation for *P* in terms of *t*.

$P = 1000e^{-0.08t}$

b) Determine

i) the number of birds in the population after 10 years.

$p = 449$

ii) the time taken (to the nearest month) for the population of birds to drop below 800.

Solve $800 = 1000e^{-0.08t}$

$t = 2.789$

After $t = 2$yrs 9months

c) What is the value of $\frac{dP}{dt}$ when t = 10.

Interpret this answer in terms of the bird population

$\frac{dP}{dt} = -0.08P$

$t = 10 \quad p = 449$

$\therefore \frac{dP}{dt} = -0.08 \times 449$

$= -35.92$

Approx 36.

End of Test