Year 12 Methods
Test 3
2019

Section 1

Calculus of Exponential, Logarithmic and Trigonometric Functions

15 Mins

Time: [redacted]

15 Marks

Total: 20[redacted]

Student Name: _____

Teacher: _____

Instructions: ***Show all working clearly.***
Sufficient detail must be shown for marks to be awarded for reasoning.
NO CALCULATOR AND NO PERSONAL NOTES ALLOWED

Question 1. **(7 marks)**

Differentiate the following functions. Simplify fully where possible.

a) $\left(\ln\left(\frac{x}{2}\right)\right)^3$ **[3 marks]**

b) $\sin^4(\pi - x^3)$ **[2 marks]**

c) xe^{2x^5} **[2 marks]**

5 Marks

Question 2. **(10 marks)**

Calculate the following.

a) $\int \frac{24-12x}{x^2-4x+1}\,dx$ **[2 marks]**

b) $\int \sin 3x$ [illegible] **[2 marks]**

c) [illegible] dx **[3 marks]**

d) $\int_{-1}^{0} \frac{e^{2x}+e^{4x}}{e^{3x}}\,dx$ **[3 marks]**

Question 3. **(3 marks)**

Find the equation of the tangent to the curve $y = \ln(3x^2)$ at the point $\left(\frac{e}{3}, \ln\left(\frac{e^2}{3}\right)\right)$.

*****END OF SECTION*****

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Section 2

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Total: [redacted]

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CALCULATOR AND 1xA4 PAGE PERSONAL NOTES ALLOWED

Question 4. **(6 marks)**

The water level of the ocean (in metres) in Broome *t* hours after midnight on a particular day is given by the equation $w(t) = 9.5\sin(\frac{\pi}{6}(t+4)) + 9.9$.

a) State the initial water level at midnight on this particular day **[1 mark]**

b) Find the water level at 10:45am **[1 mark]**

c) Find the minimum water level in Broome on this day **[1 mark]**

d) A tourist wants to visit a local wreck which is only visible when the water level is less than 2 metres. If the tourist is only free during the morning of this particular day, between what times (to the nearest minute) would he need to visit the wreck? **[3 marks]**

Question 5. **(9 marks)**

A doctor administers a 300mg dose of an antibiotic for a patient. The rate of change of the amount of antibiotics left in the patient's system after t hours is approximated by the following function.

$$\frac{dA}{dt} = -0.13A$$

The initial dose was administered at 8am.

a) Write an equation to represent the number of milligrams of antibiotics, A, left in the patient's system t hours later. Express your answer in the form $A = A_0 e^{-kt}$.
[2 marks]

b) Find the amount of antibiotic left in the patient's system at 1pm **[2 marks]**

c) Find the rate of change of the amount of antibiotics left in the patient's system at 10am **[2 marks]**

d) The doctor know that the antibiotics will only be effective if there is at least 60mg remaining in the patient's system. What is the latest time that he could administer another dose in order to make sure that the patient always has an effective amount of the drug in their system? **[3 marks]**

Question 6. 2 Marks

The velocity, v, in ms^{-1} of a particle moving along a straight line is given by the following equation.

$$v = 2\cos\left(\frac{\pi t}{4}\right) + 0.3t$$

The particle has an initial displacement of -2.5 metres.

a) Find the displacement of the particle after 4 seconds **[3 marks]**

b) Find the total distance travelled by the particle during the first 10 seconds **[2 marks]**

c) Find when the particle is not accelerating during the first 10 seconds. **[2 marks]**

Question 7. **[3 marks]**

The area between the functions $f(x) = \ln x$ and $g(x) = 3 - x^2$ and the line $x = k$ is 4 units^2. Find the value of k.

*****END OF SECTION*****