MATHEMATICS METHODS UNIT 1

TEST 3

RESOURCE FREE

NAME: _____

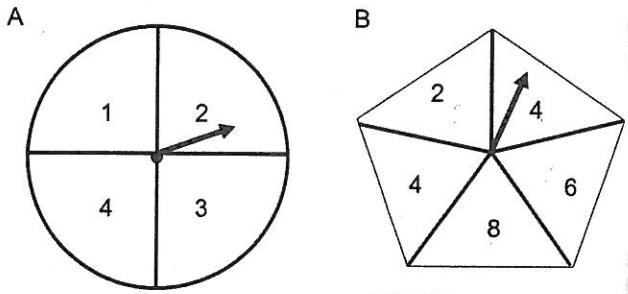
TIME ALLOWED: 20 MIN

Show all your working clearly. *Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.*

QUESTION 1

(6marks)

The spinners A and B are divided into 4 and 5 equal sectors respectively. They are spun at the same time and the numbers to which the arrows point are added.



(a) Use a suitable sample space to show all possible outcomes. (2 marks)

A \ B	2	4	4	6	8
1	3	5	5	7	9
2	4	6	6	8	10
3	5	7	7	9	11
4	6	8	8	10	12

✓✓

(b) What is the probability that the total is even? (1 mark)

$P(\text{even}) = \frac{10}{20} = \frac{1}{2}$ ✓

(c) What is the probability that the total is 12? (1 mark)

$P(\text{total } 12) = \frac{1}{20}$ ✓

(d) What is the probability that the total is even when the result on spinner A is an even number? (1 mark)

$\frac{10}{10} = 1$ ✓

(e) What is the probability that the total is at least 10 when it is known that the total is greater than or equal to 7? (1 mark)

$P(\text{total} \geq 10 \mid \text{total} \geq 7) = \frac{4}{12} = \frac{1}{3}$ ✓

QUESTION 2 **(8 marks)**

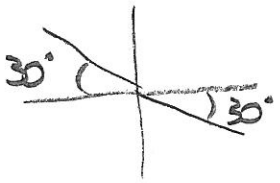
Solve for x within the given domain:

(a) $\cos x + \sqrt{3}\sin x = 0 \qquad 0 \le x \le 360^0$ (3 marks)

$\sqrt{3}\sin x = -\cos x$

$\frac{\sin x}{\cos x} = \frac{-1}{\sqrt{3}}$

$\tan x = \frac{-1}{\sqrt{3}}$



$x = 180 - 30°, 360 - 30°$

$x = 150°, 330°$

(b) $2\sin^2 x - 3\sin x - 2 = 0 \qquad 0 \le x \le 2\pi$ (5 marks)

let $y = \sin x$

$2y^2 - 3y - 2 = 0$

$(2y + 1)(y - 2) = 0$ ✓✓

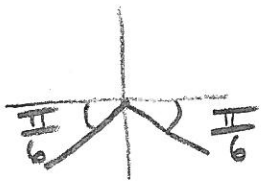
$2y + 1 = 0$ $\qquad$ $y - 2 = 0$

$y = -\frac{1}{2}$ ✓ $\qquad$ $y = 2$ ✓

but $y = \sin x$

$\sin x = -\frac{1}{2}$ $\qquad$ $\sin x = 2$

no soln



$x = \pi + \frac{\pi}{6}, \; 2\pi - \frac{\pi}{6}$

$x = \frac{7\pi}{6}, \; \frac{11\pi}{6}$ ✓

QUESTION 3 **(3 marks)**

Students in Year 11 were surveyed to determine the number who had played competition sport and undertaken paid work the previous weekend. The results are provided in the table.

		Played competition sport		
		Yes	No	Total
Undertaken paid work	Yes	54	11	65
	No	50	25	75
	Total	104	36	140

If a student is selected at random from this Year 11 group, what is the probability that:

(a) they played competition sport and undertook paid work? (1 mark)

$\frac{54}{140} = \frac{27}{70}$

(b) they either played competition sport or undertook paid work? (1 mark)

$\frac{104+11}{140} = \frac{115}{140}$

(c) they did not play competitive sport, given that they had undertaken paid work? (1 mark)

$\frac{11}{65}$

QUESTION 4 **(5 marks)**

The universal set is the set of integers from 1 to 20, A is the set of numbers divisible by 3, B is the set of square numbers and C is the set of prime numbers.

(a) List the elements of each set: $A = \{3,6,9,12,15,18\}$ $B = \{1,4,9,16\}$ $C = \{2,3,5,7,11,13,17,$

(b) Describe the set $A \cap B \cap C$ (1 mark)

Divisible by 3, and square and prime.
It is empty as all square n's are not prime.

(c) Determine $A \cup B$ (2 marks)

$\{1,3,4,9,12,15,16,18\}$

(c) Determine $n(\overline{A \cup B \cup C})$ (1 mark)

$\overline{A \cup B \cup C} = \{8,10,14,20\}$ $\therefore n(\overline{A \cup B \cup C}) = 4$

(d) A number is selected at random from the universal set, determine $P(A \mid B)$ (1 mark)

$\frac{P(A \cap B)}{P(B)} = \frac{1}{4}$

TOTAL MARKS: 22

MATHEMATICS METHODS UNIT 1 **TEST 3**

RESOURCE RICH **NAME:** _____

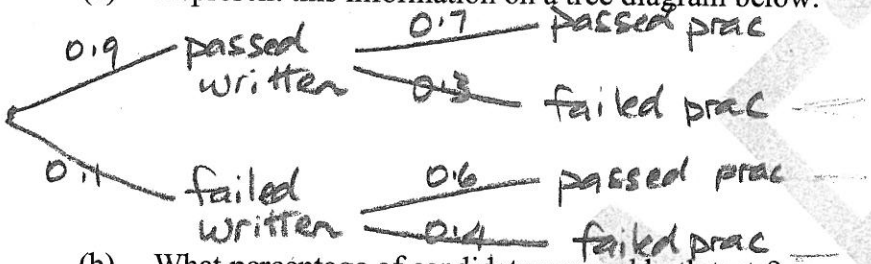
TIME ALLOWED: 40 MINUTES

Show all your working clearly. *Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.*

QUESTION 5 8 **(7 marks)**

To qualify as an umpire, candidates had to pass both a written test and a practical test. Data from previous tests indicated that 90% of candidates passed the written test and of these 70% passed the practical test. Of those who failed the written test, 40% also failed the practical test.

(a) Represent this information on a tree diagram below. (1 mark)



(b) What percentage of candidates passed both tests? (1 mark)

$P(\text{passed written} \cap \text{passed prac})$

$= 0.9 \times 0.7$

$= 0.63 = 63\%$

(c) What percentage of candidates passed at least one test? (2 marks)

$P(\text{passed one or more test}) = 0.9 \times 0.7 + 0.9 \times 0.3 + 0.1 \times 0.6$

$= 0.63 + 0.27 + 0.06$

$= 0.96$

$= 96\%$ OR $1 - 0.1 \times 0.4$

$= 0.96$

(d) Of those who did not qualify as an umpire what fraction of the candidates failed the written test? (2 marks)

$$\frac{0.1}{0.27 + 0.06 + 0.04} = \frac{0.1}{0.37} = \frac{10}{37}$$

$= 0.27$

6

(e) Are the events 'Pass the written test' and 'Pass the practical test' independent? Justify your answer. (2 marks)

If indept $P(W \cap P) = P(W) \cdot P(P)$ ✓

$0.63 = 0.9 \times (0.9 \times 0.7 + 0.1 \times 0.6)$

$= 0.9 \times (0.63 + 0.06)$

$\neq 0.621$ ✓not independent.

QUESTION 6 **(3 marks)**

A test containing five questions was given to a group of students. The table below shows the probability of the number of questions (out of 5) being answered correctly.

Number of questions answered correctly (n)	0	1	2	3	4	5
Probability that this number of questions were answered correctly	x	0.05	0.16	0.44	0.23	0.07

(a) Determine x (1 mark)

$1 - 0.95$

$= 0.05$ ✓

(b) Evaluate $P(3 \leq n \leq 4 \mid n > 2)$ (2 marks)

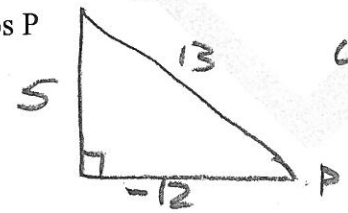
$\frac{0.44 + 0.23}{0.44 + 0.23 + 0.07} = \frac{0.67}{0.74} = \frac{67}{74}$ ✓✓

$= 0.905.$

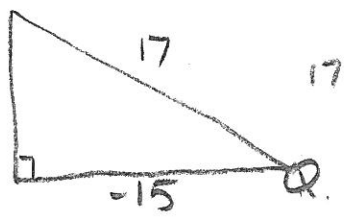
QUESTION 7 **(7 marks)**

Given that $\sin P = \frac{5}{13}$ and $\cos Q = -\frac{15}{17}$, where P and Q are each obtuse angles, find the exact value of:

(a) cos P (2 marks)



$\cos P = \frac{-12}{13}$ ✓✓



$17^2 - 15^2 = q^2$

$q = 8$

(b) sin Q (2 marks)

$\sin Q = \frac{8}{17}$ ✓✓

(c) $\sin(P-Q) = \sin P\cos Q - \cos P\sin Q$ ✓ (3 marks)

$= \frac{5}{13} \times -\frac{15}{17} - \left(-\frac{12}{13}\right)\left(\frac{8}{17}\right)$ ✓

$= \frac{21}{221}$ ✓

QUESTION 8 **(7 marks)**

(a) If the graphed function below has the equation of form $f(x) = a\cos(bx - c)$, determine the values of a, b, and c.

$a=2$ ✓ $b=2$ ✓ $c=-15$ ✓ 4

P108 OT Lee

(b) If the graphed function below has the equation of form $f(x) = \tan(bx + c)$ determine the values of b and c.

period $= 90°$ ✓ $b=1$ ✓ 3

phase $= 45°$ ✓ $c=-45°$ ✓

QUESTION 9 **(6 marks)**

For two events M and K, $P(M) = 0.4$ and $P(M \mid M \cup K) = 0.8$

Determine $P(K)$ if

(a) events M and K are mutually exclusive. (3 marks)

If mutually exclusive $P(M \cap K) = 0$ ✓

$P(M \mid M \cup K) = \frac{P(M)}{P(M \cup K)}$

$0.8 = \frac{0.4}{P(M \cup K)}$

$P(M \cup K) = 0.5$ ✓

$P(M \cup K) = P(M) + P(K)$

$0.5 = 0.4 + P(K)$

$\therefore P(K) = 0.1$ ✓

(b) events M and K are independent. (3 marks)

If independent $P(M \cap K) = P(M) \cdot P(K)$ ✓

$P(M \cup K) = P(M) + P(K) - P(M \cap K)$

$0.5 = 0.4 + P(K) - P(M)P(K)$ ✓

$0.1 = P(K) - 0.4P(K)$

$0.1 = 0.6P(K)$

$P(K) = \frac{1}{6}$ ✓

QUESTION 10 **(8 marks)**

Use an appropriate trigonometric identity to find the exact value of:

(a) $\sin 75°$ (4 marks)

$= \sin(45 + 30)$ ✓

$= \sin 45 \cos 30 + \cos 45 \sin 30$ ✓

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ ✓

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$ ✓

(b) $\tan(\frac{\pi}{3} + \frac{\pi}{4})$ (4 marks)

$= \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$ ✓

$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$ ✓

$= \frac{\sqrt{3} + 3 + 1 + \sqrt{3}}{1 - 3}$ ✓

$= \frac{4 + 2\sqrt{3}}{-2}$

$= -2 - \sqrt{3}$ ✓

TOTAL MARKS 38 39