MINDARIE SENIOR COLLEGE

WHERE YOUR FUTURE BEGINS NOW

MATHEMATICS: SPECIALIST 3AB

EXTENDED PIECE OF WORK 1

PART B

Different Number Bases

Time Allowed = 60 minutes

Marks Available = ~~40~~ 37 marks

Students may bring a graphic calculator and any notes into the Validation Test.

1. [2, 2, 2 marks]

Convert the following numbers into base 10:

(a) 24151_6

$2\times6^4+4\times6^3+1\times6^2+5\times6+1$ ✓

$=3523$ ✓

(b) 120012_3

$1\times3^5+2\times3^4+1\times3+2$ ✓

$=410$ ✓

(c) $38a5b_{12}$ (where a = 10 and b = 11)

$3\times12^4+8\times12^3+10\times12^2+5\times12+11$ ✓

$=77543$ ✓

2. [3, 3 marks]

Convert the indicated numbers to the given base:

(a) 2371_9 to base 5

625 125 25 5 1

$2\times9^3+3\times9^2+7\times9+9$ ✓

1765

2×625 ✓ 515

4×125

0×25 24030 ✓

3×5

0×1

(b) 302213_4 to base 11 (using a to represent 10)

1331 121 11 1

$3\times4^5+2\times4^3+2\times4^2+1\times4+3$ ✓

3239

2×11^3 ✓ 577

4×11^2 93

8×11 5 ✓

5×1 2485

3. [3, 3, 3, 3 marks]

Perform the following operations in the given base. Do NOT convert to base 10. Show ALL working

(a) $20112_3 + 222_3$

$$\begin{array}{r} {}^{1}{}^{1}{}^{1} \\ 20112 \\ 222 \\ \hline 21111 \end{array}$$ ✓✓ ✓ (3)

(b) $31021_4 - 3203_4$

$$\begin{array}{r} {}^{2}{}^{1}{}^{0}{}^{1}{}^{1} \\ 31021 \\ 3203 \\ \hline 21212 \end{array}$$ ✓✓ ✓ (4)

(c) $54_6 \times 13_6$

$$\begin{array}{r} {}^{2} \\ 54 \\ 13 \\ \hline 250 \\ 540 \\ \hline 1230 \end{array}$$ ✓ ✓ ✓ (6)

(d) $422_5 \div 13_5$

$$\begin{array}{r} 24 \\ 13\overline{)4222} \end{array}$$ ✓ ✓✓ (5)

8 16

25 51

4. [2, 3, 3 marks]

(a) Convert to base 10: $12{\cdot}32_5$

$5+2+\frac{3}{5}+\frac{2}{25} = 7\frac{17}{25}$ ✓ ✓

(b) Convert to base 4: $33{\cdot}24_5$

$15+3+\frac{2}{5}+\frac{4}{25} = 18\frac{14}{25}$

$= 18{\cdot}56$

.06

4 | 0.25 0.0625 0.015625

42.20

(c) Evaluate exactly, giving your answer in base 6: $24{\cdot}3_6 + 1{\cdot}25_6 + 345{\cdot}403_6$

$$\begin{array}{r} {}^{1}{}^{1} \\ 24.3 \\ 1.25 \\ \hline 345.403 \\ \hline 415.353 \end{array}$$ ✓ ✓✓ (6)

Modulo Arithmetic

Modulo arithmetic is closely related to number bases. A major difference is that we only worry about the units digit.

If we were to count (from zero) in base 3 it would look like:

0, 1, 2, 10, 11, 12, 20, 21, 22, 100, …

If we were to count (from zero) in modulo 3 it would look like:

0, 1, 2, 0, 1, 2, 0, 1, 2, …

An addition table in modulo 3 would look like:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

5. [2 marks]

Complete the following addition table in modulo 4.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

6. [3 marks]

Complete the following *multiplication* table in modulo 4.

x	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

7. [1, 2 marks]

The following calculations have been performed in modulo arithmetic. Determine what base has been used in each case.

(a) 5 + 4 = 1 base = 8

(b) 5 x 4 = 2 base = 6, 9, 18 [illegible]