

**SCOTCH
COLLEGE**



12 Mathematics Methods 2019

Test 1 – Differentiation and Logarithms

Section 1: Calculator-free

Time allowed: 25 minutes

Maximum marks: 24

Name: _____ **Teacher:** Foster | Giese | Reyhani

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No calculators or personal notes are permitted.

Question 1 [2, 2, 3 = 7 marks]

Differentiate the following with respect to x (do not simplify):

(a) $\frac{3}{x} - \frac{5}{x^3}$

(b) $(2 - x^5)^4$

(c) $x^3\sqrt{1 - x^2}$

Question 2 [4 marks]

Determine the equation of the tangent to $y = \frac{x^3}{x + 1}$ at $x = 1$.

Question 3 [4 marks]

Solve for x :

$$2 \ln(x) - 3 \ln\left(\frac{1}{x}\right) = 10$$

Question 4 [2, 3 = 5 marks]

Consider the function $f(x) = \log_2(x - 1) + 1$.

(a) State the domain and range of f .

(b) Find any asymptotes and co-ordinates of axes intercepts.

Question 5 [4 marks]

If $y = \sqrt[3]{x}$, use the incremental formula to determine the approximate value of $\sqrt[3]{1001}$.

END OF SECTION 1

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12 Mathematics Methods 2019

Test 1 – Differentiation and Logarithms

Section 2: Calculator-assumed

Time allowed: 25 minutes

Maximum marks: 21

Name: _____ **Teacher:** Foster | Giese | Reyhani

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- Calculators and 1A4 double-sided page of personal notes are permitted.

Question 6 [2, 1, 2 = 5 marks]

A particle moves in a straight line with its position from the origin given by

$$x(t) = 15t - \frac{60}{(t+1)^2} \text{ cm}$$

where t is the time in seconds, $t \geq 0$.

- (a) What is the initial velocity of the particle?

- (b) Determine the acceleration function for the particle's motion.

- (c) For what values of t is the particle's velocity increasing? Justify your answer.

Question 7 [2, 3 = 5 marks]

If $A = \log_5 2$ and $B = \log_5 3$, write the following in terms of A and B :

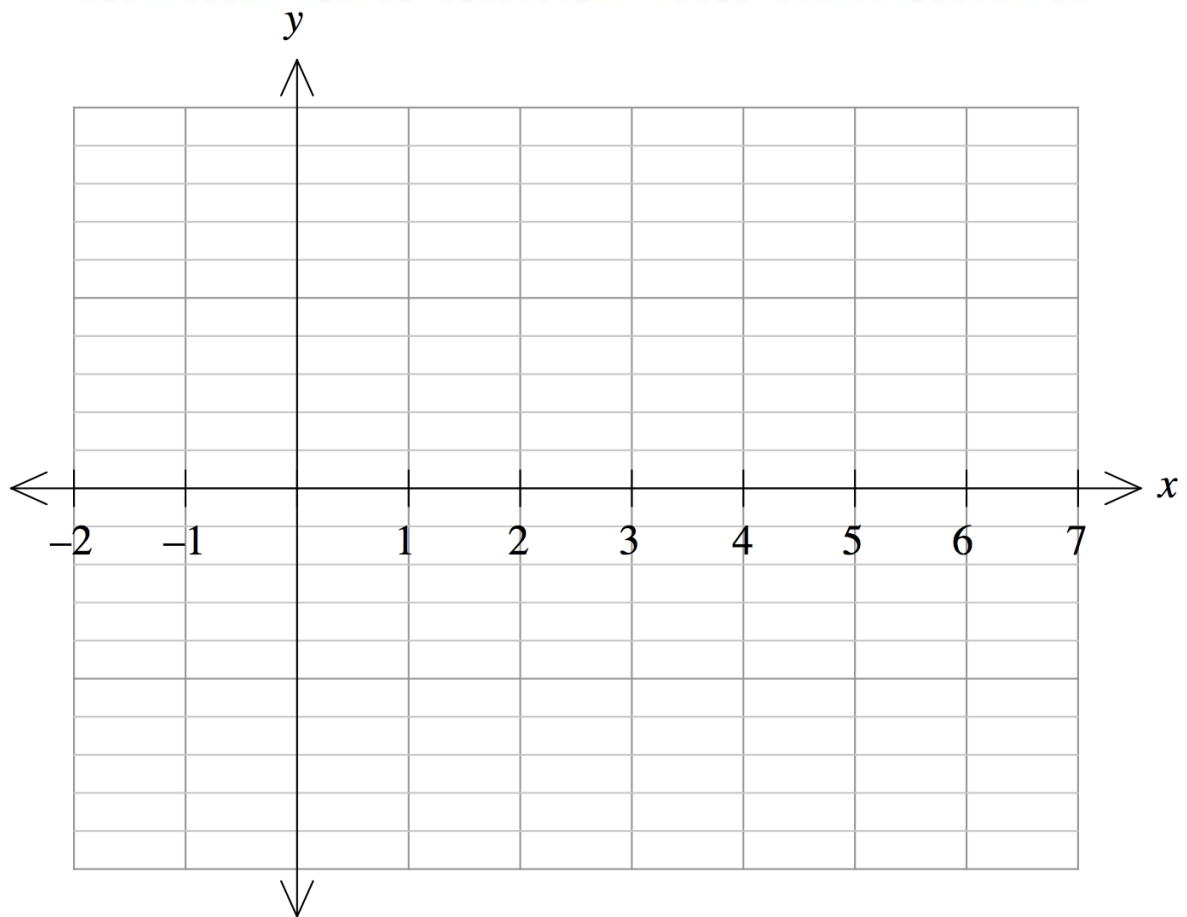
(a) $\log_5 1.5$

(b) $\log_5 60$

Question 8 [5 marks]

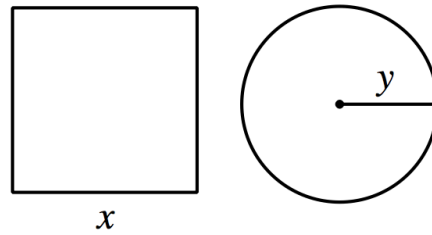
Sketch a function $y = f(x)$ with all of the following features, clearly indicating any stationary points and points of inflection:

- $f(0) = f(6) = 0$
- $f'(0) = f'(4) = 0$
- $f'(x) \geq 0$ strictly for $x < 4$
- $f''(x) > 0$ strictly for $0 < x < 2$



Question 9 [2, 4 = 6 marks]

The diagram below shows a square with side x cm and a circle with radius y cm.



The two shapes are made out of a piece of wire of length 8 cm.

- (a) Show that the total area of the two shapes is given by:

$$A = \left(2 - \frac{\pi y}{2}\right)^2 + \pi y^2$$

- (b) Showing the use of calculus techniques, determine the value of y which minimises the total area of the two shapes and confirm that it is a minimum area.

END OF TEST