MINDARIE
SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS NOW

MATHEMATICS: SPECIALIST 3 & 4

EXTENDED PIECE OF WORK 4

PART B

HYPERBOLIC FUNCTIONS

Time Allowed: 55 minutes

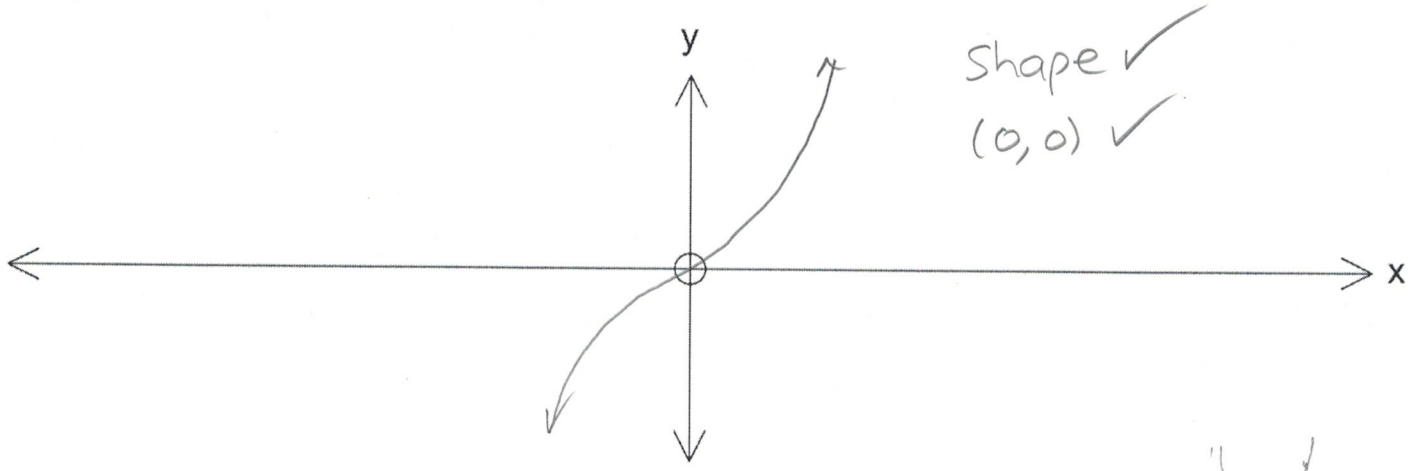
Total Marks: 40

No calculators allowed. Part A may be used for this Part.

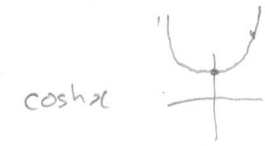
1. [6 marks]

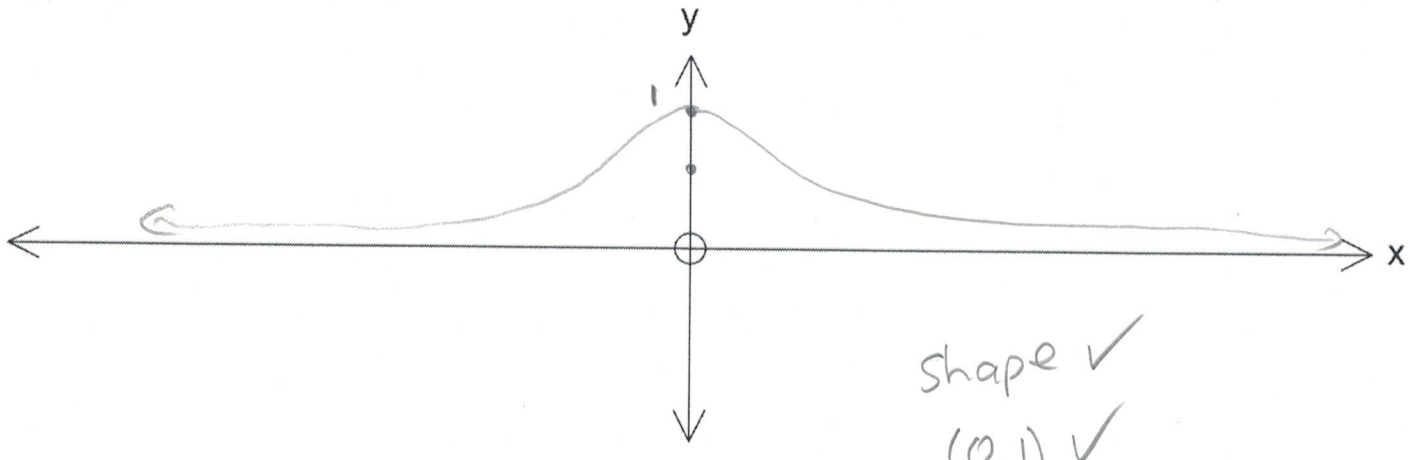
Sketch the following graphs. Show any important points on the axes.

(a) $f(x) = e^x - e^{-x}$

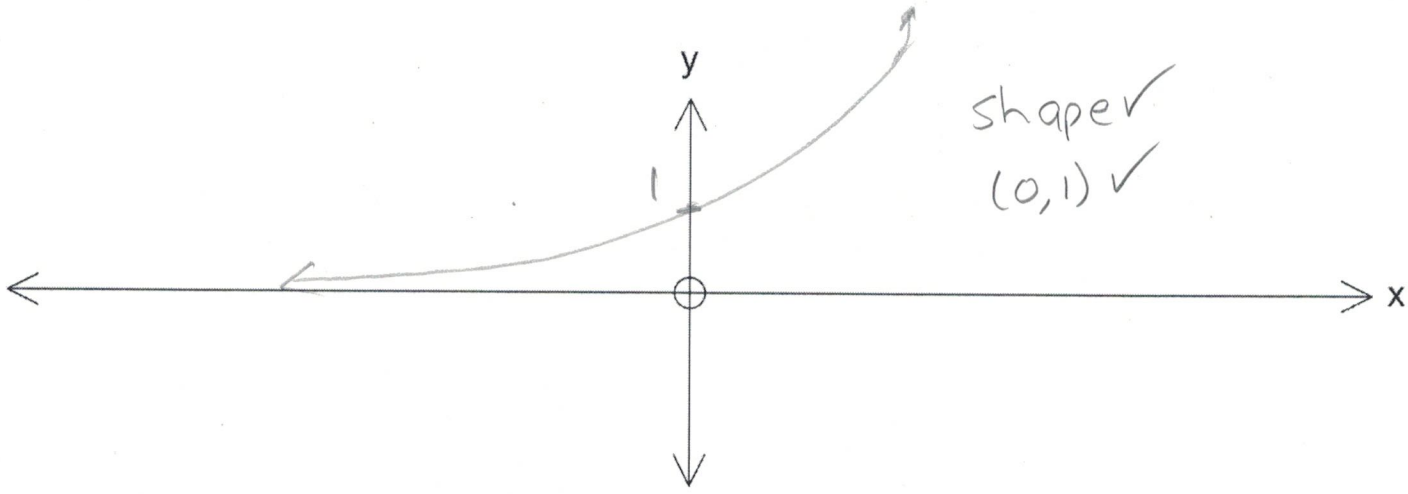


(b) $y = \operatorname{sech} x$, where $\operatorname{sech} x = \frac{1}{\cosh x}$





(c) $h(x) = \cosh x + \sinh x$



shape ✓
(0,1) ✓

2. [4 marks]

State whether each of the following are true or false.

(a) For x large and positive, $\cosh x$ and $\sinh x$ have approximately the same value. ✓

$\lim_{x\to\infty} \cosh x = \frac{1}{2}(e^{\infty} + 0)$ $\lim_{x\to\infty} \sinh x = \frac{1}{2}(e^{\infty} - 0)$ True.

(b) $\lim_{x\to\infty} \tanh x = 1$ $\frac{e^{\infty}}{e^{\infty}} = 1$ true ✓

(c) $\cosh(-x) = \cosh x$ true ✓

(d) For $x < 0$ and $|x|$ large, the graph of $y = \cosh x$ approximates the curve $y = \frac{1}{2}e^{-x}$.

True ✓

3. [7 marks]

Use the transformation suggested in Question 4 of your assignment to transform the following identities into identities involving hyperbolic functions.

(a) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cosh(A+B) = \cosh A \cosh B - i\sinh A \cdot i\sinh B$

$= \cosh A \cosh B + \sinh A \sinh B$ ✓

(b) $\cos 4A = 8\sin^4 A - 8\sin^2 A + 1$

$\cosh 4A = 8(i\sinh A)^4 - 8(i\sinh A)^2 + 1$

$= 8\sinh^4 A + 8\sinh^2 A + 1$ ✓ ✓

(c) $\cos^6 A + \sin^6 A = 1 - \frac{3}{4}\sin^2 2A$

$\cosh^6 A + (i\sinh A)^6 = 1 - \frac{3}{4}(i\sinh 2A)^2$

$\cosh^6 A - \sinh^6 A = 1 + \frac{3}{4}\sinh^2 2A.$ ✓ ✓

(d) $\sec^2 A - \tan^2 A = 1$

$\frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A} = 1$ ✓

$\frac{1}{\cosh^2 A} + \frac{\sinh^2 A}{\cosh^2 A} = 1$

$\operatorname{sech}^2 A + \tanh^2 A = 1$ ✓

4. [4 marks]

Given $\sinh x = -\frac{3}{4}$, show that $\cosh x = \frac{5}{4}$.

[Hint: use Question 2 (a) from Part A]

$\cosh^2 x - \left(-\frac{3}{4}\right)^2 = 1$ ✓

$\cosh^2 x - \frac{9}{16} = 1$ ✓

$\cosh^2 x = \frac{25}{16}$

$\cosh x = \pm\frac{5}{4}$ ✓

but $\cosh x > 0$ ✓

$\therefore \cosh x = 5/4$ ✓

IF miss out $\cosh x > 0$ but gives $\cosh x = 5/4$ max ✓✓

5. [6 marks]

Use the results from Question 2 (c) in Part A to prove

$$(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$$

LHS $= (\cosh x + \sinh x)^n$

$= (e^x)^n$ ✓

$= e^{nx}$ ✓

RHS $= \cosh nx + \sinh nx$ ✓

$= \frac{1}{2}(e^{nx} + e^{-nx}) + \frac{1}{2}(e^{nx} - e^{-nx})$

$= \frac{1}{2}(e^{nx} + e^{-nx} + e^{nx} - e^{-nx})$

$= \frac{1}{2}(2e^{nx})$

$= e^{nx}$ ✓

$\therefore$ LHS $=$ RHS ✓

6. [7, 6 marks]

(a) Find

(i) $\int \cosh x \, dx = \sinh x + C$ ✓

(ii) $\frac{d}{dx}(\cosh^2 x) = 2\cosh x \cdot \sinh x$ ✓

$= \sinh 2x$ ✓

(iii) $\frac{d}{dx}(\sinh 5x) = 5\cosh 5x$ ✓ ✓

(iv) $\frac{d}{dx}(\cosh x + \sinh x)^n$ $= \frac{d}{dx}(\cosh nx + \sinh nx)$ ✓

$= n\sinh nx + n\cosh nx$

$= n(\sinh nx + \cosh nx)$

$= n(\cosh x + \sinh x)^n$ ✓

(b) Show that $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

[Hint: Use Question 1 (b) from Part A and the quotient rule]

LHS $= \frac{d}{dx}(\tanh x)$

$= \frac{d}{dx}\left(\frac{\sinh x}{\cosh x}\right)$ ✓

$= \frac{\cosh x \cdot \cosh x - \sinh x \sinh x}{\cosh^2 x}$ ✓ ✓

$= \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$ ✓

$= \frac{1}{\cosh^2 x}$ ✓

$= \operatorname{sech}^2 x$ ✓