MATHEMATICS DEPARTMENT

Year 11 Methods - Test Number 3 2019

Relations, Function Transformations, Trig Functions & Counting

Resource Free

ALL SAINTS' COLLEGE

Name: SOLUTIONS

Marks: 45

Time Allowed: Reading: 2 minutes

Working: 43 minutes

Teacher: ____________________

Instructions: You ARE NOT permitted any notes or calculator.

The formula sheet will be provided.

This study source was downloaded by 100000846012161 from CourseHero.com on 09-12-2022 04:39:45 GMT -05:00

https://www.coursehero.com/file/163552763/2019-Year-11-Methods-Test-3-SOLUTIONSpdf/

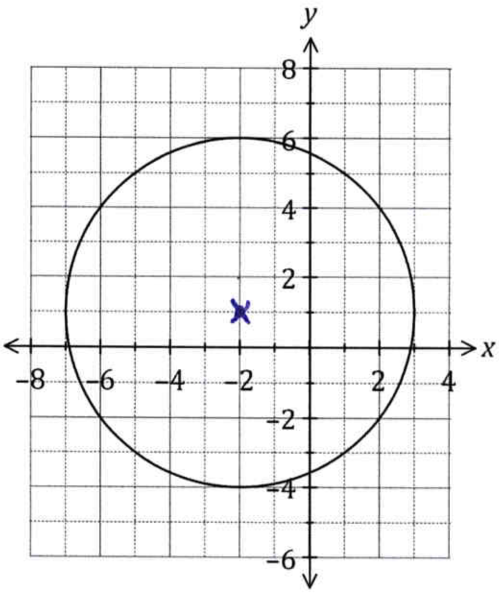
1. **[3, 4 = 7 marks]**

a) The graph of the relation $y^2 = x$ passes through the points $(36, p)$ and $(q, -9)$. Determine the values of p and q.

$y^2 = x$
$p^2 = 36$ ✓
$\therefore p = \pm 6$ ✓

$y^2 = x$
$(-9)^2 = q$
$\therefore q = 81$ ✓

b) The graph of a relationship is circular, as shown below.



Determine the equation of this circle in the form $x^2 + y^2 = a + bx + cy$, where a, b and c are constants.

Centre $(-2, 1)$ $\quad r = 5$

$(x+2)^2 + (y-1)^2 = 25$ ✓ ✓

$x^2 + y^2 + 4x + 4 - 2y + 1 = 25$ ✓ expand

$x^2 + y^2 = 20 - 4x + 2y$ ✓ correct form

This study source was downloaded by 100000846012161 from CourseHero.com on 09-12-2022 04:39:45 GMT -05:00

https://www.coursehero.com/file/163552763/2019-Year-11-Methods-Test-3-SOLUTIONSpdf/

2. **[2, 3 = 5 marks]**

a) Use an appropriate identity to show that $\cos(\pi - \theta) = -\cos\theta$

$\cos(\pi - \theta) = \cos\pi\cos\theta + \sin\pi\sin\theta$ ✓

$= (-1)\cos\theta + 0\sin\theta$ ✓ must show

$= \underline{-\cos\theta}$

b) Evaluate: $\sin(250°)\cos(25°) - \cos(250°)\sin(25°)$

$= \sin(250° - 25°)$ ✓

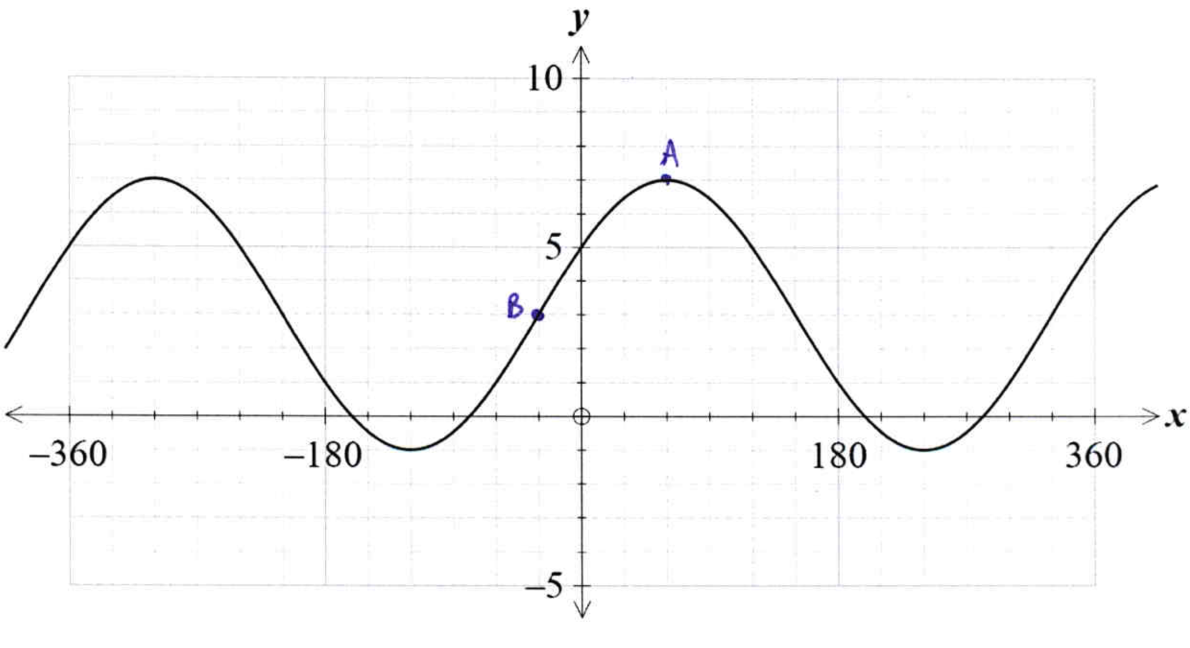
$= \sin 225°$

$= -\sin 45°$ ✓

$= -\dfrac{1}{\sqrt{2}}$ ✓ ✓

3. **[4 marks]**

The sketch of the curve $y = a\sin(x + b) + c$ is shown in the diagram below. Given that $a > 0$, find the values of a, b and c



amplitude $= \dfrac{7-(-1)}{2} = 4$ $\therefore a = 4$

At A, $y = 7$ $\therefore$ graph has moved up 3, ie $c = 3$

Graph has moved left 30° (point B) $\therefore b = 30°$

$a =$ 4 ✓ $b =$ 30° ✓✓ $c =$ 3 ✓

This study source was downloaded by 100000846012161 from CourseHero.com on 09-12-2022 04:39:45 GMT -05:00

https://www.coursehero.com/file/163552763/2019-Year-11-Methods-Test-3-SOLUTIONSpdf/

4. **[3, 3 = 6 marks]**

Use the Binomial theorem to answer the following:

a) Find the sixth term in the expansion of $(2x - \frac{1}{2})^9$. Give your answer in simplified form.

$$\binom{9}{5}(2x)^4\left(-\frac{1}{2}\right)^5 = 126 \times 2^4 x^4 \times -\frac{1}{2^5}$$

$$= -63x^4$$

b) Find the term independent of x (ie which contains no x) in the expansion of

$$[x + \left(\frac{1}{x^2}\right)]^{12}$$

5th term: $\binom{12}{4} x^8 \left(\frac{1}{x^2}\right)^4 = \frac{12!}{8!\,4!}$

$$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}$$

$$= \frac{990}{2}$$

$$= 495$$

This study source was downloaded by 100000846012161 from CourseHero.com on 09-12-2022 04:39:45 GMT -05:00

https://www.coursehero.com/file/163552763/2019-Year-11-Methods-Test-3-SOLUTIONSpdf/

5. **[1, 1, 3 = 5 marks]** (DO NOT SIMPLIFY YOUR ANSWERS)

An examination is divided into two sections, A and B, with 5 and 7 questions respectively. How many different combinations of questions are there if students are required to:

a) answer all the questions in each section?

1 ✓

b) answer all the questions in Section A and any two questions from Section B?

$$\binom{5}{5}\times\binom{7}{2} = \binom{7}{2}$$ ✓ either

c) answer a total of 5 questions with at least one question from each section?

1A,4B or 2A,3B or 3A,2B or 4A,1B ✓ possibilities

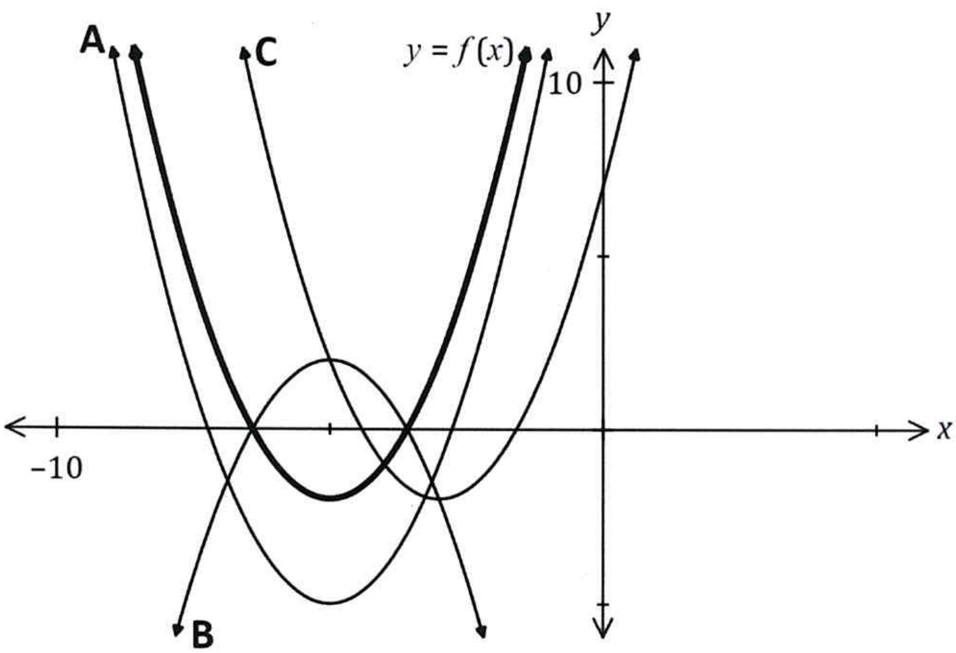
$$=\binom{5}{1}\binom{7}{4} + \binom{5}{2}\binom{7}{3} + \binom{5}{3}\binom{7}{2} + \binom{5}{4}\binom{7}{1}$$ ✓✓

This study source was downloaded by 100000846012161 from CourseHero.com on 09-12-2022 04:39:45 GMT -05:00

https://www.coursehero.com/file/163552763/2019-Year-11-Methods-Test-3-SOLUTIONSpdf/

6. [3 marks]

The graph of $y = f(x)$ is shown in bold below. The graphs of $y = -f(x)$, $y = f(x+p)$ and $y = f(x) + q$ are also shown, where p and q are constants.



Complete the table below, by identifying the defining equation of each graph:

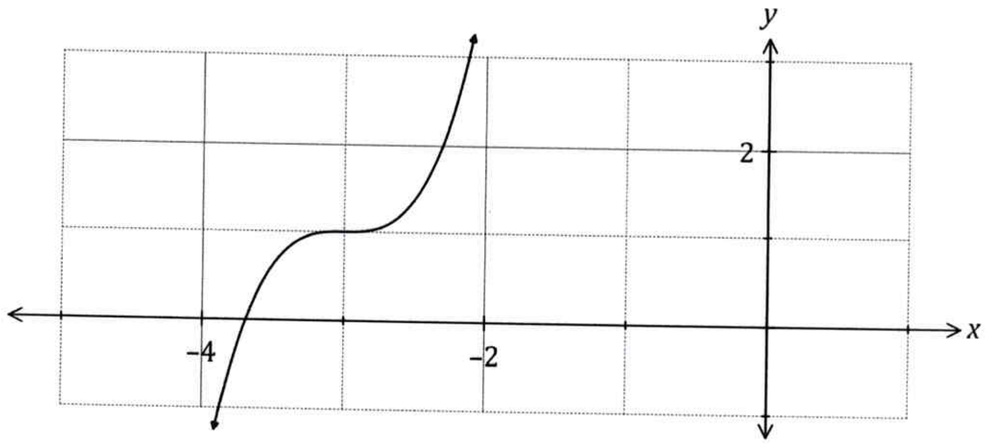
Graph	Equation
A	$y = f(x) + q$ ✓
B	$y = -f(x)$ ✓
C	$y = f(x+p)$ ✓

This study source was downloaded by 100000846012161 from CourseHero.com on 09-12-2022 04:39:45 GMT -05:00

https://www.coursehero.com/file/163552763/2019-Year-11-Methods-Test-3-SOLUTIONSpdf/

7. [1, 2, 2 = 5 marks]

(a) Part of the graph of $y = f(x)$ is shown below, where $f(x) = 3(x + b)^3 + c$, and b and c are constants.



(i) Determine the value of b.

$b = 3$ ✓

(ii) Determine $f(0)$.

$f(0) = 3(0+3)^3 + 1$ ✓ sub $x=0$

$= 82$ ✓

b) Another function, $g(x)$, is a transformation of $f(x)$, where $g(x) = f(2x - 3)$.

Describe how to obtain the graph of $y = g(x)$ from the graph of $y = f(x)$.

✓
Translate 3 units right then dilate ∥ to x axis, scale factor $\frac{1}{2}$ ✓

(−1 if wrong order)

This study source was downloaded by 100000846012161 from CourseHero.com on 09-12-2022 04:39:45 GMT -05:00

https://www.coursehero.com/file/163552763/2019-Year-11-Methods-Test-3-SOLUTIONSpdf/

8. **[3, 3, 4 = 10 marks]**

Solve the following trigonometric equations in the given domain:

a) $(\sin\theta - 2)(2\sin\theta - 1) = 0$, $0° \leq \theta \leq 360°$

$\sin\theta = 2$ or $\sin\theta = \frac{1}{2}$ ✓

no solution ✓

Ref $\angle = 30°$

$\therefore \underline{\theta = 30° \text{ or } 150°}$ ✓

b) $4\cos^2 x = 3$, $0 \leq x \leq 2\pi$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$ ✓

(if omit ±, max 2 for correct answers: 2 only)

Ref $\angle = \frac{\pi}{6}$ $\therefore x = \underline{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$ ✓✓

c) $2\sin 3x - 1 = 0$, $0 \leq x \leq \pi$

$\left.\begin{array}{l} \sin 3x = \frac{1}{2} \qquad 0 \leq 3x \leq 3\pi \\ \text{Ref } \angle = \frac{\pi}{6} \end{array}\right\}$ ✓

$\therefore 3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ ✓✓

$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$ ✓

****End of Test****

This study source was downloaded by 100000846012161 from CourseHero.com on 09-12-2022 04:39:45 GMT -05:00

https://www.coursehero.com/file/163552763/2019-Year-11-Methods-Test-3-SOLUTIONSpdf/