MINDARIE SENIOR COLLEGE

Methods Unit 3 Test 3, 2016
(Calculator Free) Name_____

Time: 18 minutes Marks: 18

1. [2, 1, 2, 2 marks]

Determine $\frac{dy}{dx}$ in terms of x, for the following (you do not need to simplify):

a) $y = \frac{2x+1}{\sin x}$ $\frac{dy}{dx} = \frac{2\sin x - (2x+1)\cos x}{\sin^2 x}$ ✓✓

b) $y = \cos^2(3x-1)$ $\frac{dy}{dx} = -6\cos(3x-1).\sin(3x-1)$ ✓

c) $y = x^2.\sin x.$ $\frac{dy}{dx} = 2x\sin x + x^2\cos x$ ✓✓

d) $y = \cos 4x.\cos 3x + \sin 4x.\sin 3x$

$y = \cos x$ ✓

$\frac{dy}{dx} = -\sin x$ ✓

2. [2, 2, 1 marks]

Determine the following indefinite integrals:

a) $\int 2\cos 3x \quad dx$ $= \frac{2\sin 3x}{3} + C$ ✓✓

b) $\int 20\cos^3 x \sin x \quad dx$ $= -5\cos^4 x + c$ ✓ −1 for no +c

c) $\int \frac{2}{\cos^2 x} \quad dx$ $= 2\tan x + c$ ✓

3. [2, 2, 2 marks]

A discrete probability distribution for the random variable X is given below.

x	1	2	3	4	5
P(X = x)	k	k + 0.3	0.1	2k	k + 0.1
	0.1	0.4	0.1	0.2	0.2

a) Determine k.

$k + k + 0.3 + 0.1 + 2k + k + 0.1 = 1$ ✓

$5k + 0.5 = 1$

$k = 0.1$ ✓

b) Determine the mean (or expected value) of X.

$0.1 + 0.8 + 0.3 + 0.8 + 1 = 3$ ✓

$E[X] = 3$ ✓

c) State P(X < 4/ X > 1)

$= \frac{0.5}{0.9}$ ✓ $= \frac{5}{9}$ ✓

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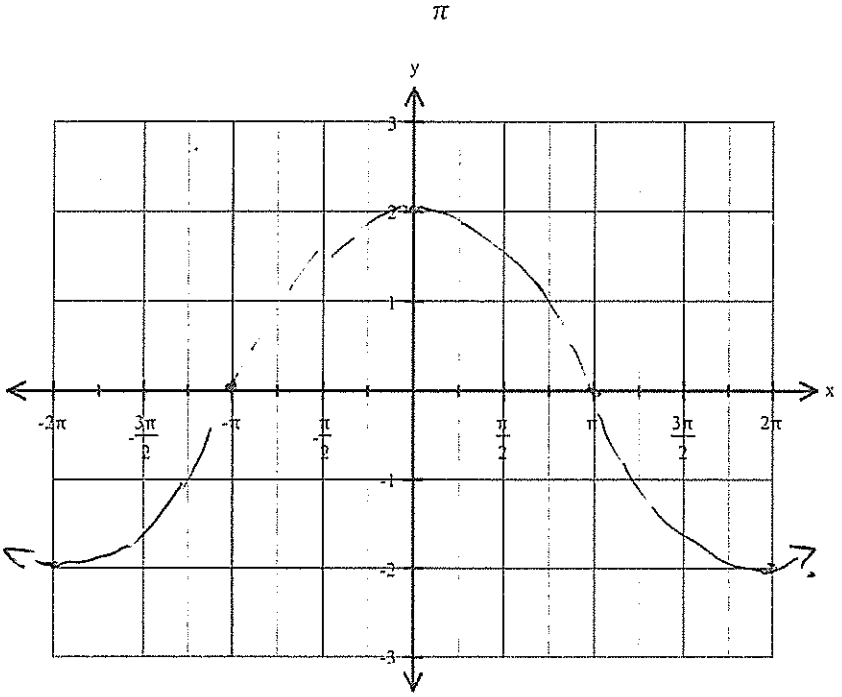
Time: 37 minutes Marks: 37

6. [2, 4 marks]

a) Sketch the curve y = 2 cos (0.5x) on the axes below.



√period
√ampl.

b) Determine the area between y = 2 cos (0.5x) and the x-axis for $-2\pi \leq x \leq 2\pi$

$$4 \times \int_0^{\pi} 2\cos 0.5x \, dx \quad \checkmark$$
$$= 4 \times \left[4 \sin \tfrac{x}{2}\right]_0^{\pi} \quad \checkmark$$
$$= 4 \times 4 \quad \checkmark$$
$$= 16 \text{ units}^2. \quad \checkmark$$

7. [5 marks]

Find the equation of the tangent to the curve y = 2 sin 2x at the point where $x = \frac{\pi}{2}$

$$\frac{dy}{dx} = 4\cos 2x \quad \checkmark$$
If $x = \frac{\pi}{2}$ $y = 0$ ✓
$$\frac{dy}{dx} = -4 \quad \checkmark$$
$$y = -4x + 2\pi \quad \checkmark\checkmark$$

8. [2, 1, 1, 2, 1 marks]

The probability function for a discrete random variable is

$$f(x) = \begin{cases} k(5-x) & for\ x = 1,2,3,4 \\ 0 & for\ all\ other\ values \end{cases}$$

Determine k, and thus find

x	1	2	3	4
P(X=x)	4k	3k	2k	k

$10k = 1$ ✓

$k = 0.1$ ✓

a) E[X]

$0.4 + 0.6 + 0.6 + 0.4 = 2$ ✓

b) E[3x - 1] $= 5$ ✓

c) Var[X] $0.4 \times (-1)^2 + 0.3 \times 0^2 + 0.2 \times 1^2 + 0.1 \times 2^2$

$= 1$ ✓

d) Var[1- 3X] 9 ✓

9. [1, 2, 1, 1, 2 marks]

Given that a discrete random variable is binomially distributed such that X~bin(8, 0.25), determine

a) the mean of the distribution $E[X] = 2$ ✓

b) the standard deviation of the distribution $\sigma_X = \sqrt{1.5} = 1.22$ ✓ ✓

c) P(X = 2) $= 0.3115$ ✓

d) P(X ≤ 3) $= 0.8862$ ✓

e) P (X = 3 / X < 6) $\frac{0.2076}{0.9958}$ ✓

$= 0.2085$ ✓

10. [1, 1, 1, 2, 3 marks]

A student who has not studied for his Biology test resorts to guessing every answer on the twenty multiple choice questions. Each question has 5 choices (where only one answer is correct).

a) Describe the probability distribution.

$X \sim Bin(20, 0.2)$ → binomial distribution with $n = 20$ and $p(\text{success}) = 0.2$ ✓

Determine the probability that the student has

b) at least 5 correct $P(X \geq 5) = 0.3704$ ✓

c) at most 4 correct $P(X \leq 4) = 0.6296$ ✓

d) 2 correct, given he has at least one correct. $\frac{0.1369}{0.9885} = 0.1385$ ✓ ✓

e) The student realizes that he can answer 5 questions correctly. What is the probability that he can achieve at least 50% for the test (i.e. at least 10 out of 20)?

$X \sim Bin(15, 0.2)$ ✓

$P(X \geq 5) = 0.1642$

✓ ✓

11. [4 marks]

In a Bernoulli trial, the standard deviation is 0.4
Determine E[X].

$\sqrt{p(1-p)} = 0.4$ ✓

$p(1-p) = 0.16$ ✓

$p = 0.8$ or 0.2 ✓

ie $E[X] = 0.8$ or 0.2 ✓