YEAR 12 PHYSICS
ASSIGNMENT 4 - GRAVITATION

Name: SOLUTIONS **Mark:** $\frac{\quad}{50}$

1. Use the information given in the Formulae and Data Booklet to calculate the orbital period, in seconds, of the Moon around the Earth. (5 marks)

$$F_g = F_c$$

$$\Rightarrow \frac{G M_E M_M}{r^2} = \frac{M_M v^2}{r} = \frac{4\pi^2 M_M r}{T^2} \quad (1)$$

$$\Rightarrow r^3 = \frac{G M_E T^2}{4\pi^2} \quad (1)$$

$$\Rightarrow T = \sqrt{\frac{4\pi^2 r^3}{G M_E}}$$

$$= \sqrt{\frac{4\pi^2 (3.84\times10^8)^3}{(6.67\times10^{-11})(5.97\times10^{24})}} \quad (1)$$

$$= \underline{2.37\times10^6\ s} \quad (1)$$

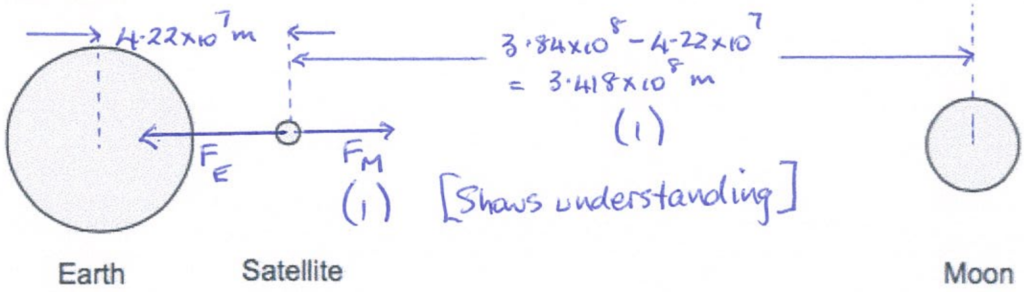
[Correct values - 1 mark]

2. When a satellite is launched it is placed in an initial circular orbit around the Earth. Later some small jets on board the satellite will fire compressed gas for a set period of time to move it to the precise final circular orbit required. These gas jets point backward relative to the satellite's motion only and **not** toward or away from the Earth.

How can backward facing gas jets be used to raise the satellite to a higher final circular orbit? (4 marks)

- Gas jet increases the speed and E_k of the satellite (1)
- This E_k is converted into E_p. (1)
- Satellite rises to a higher orbit. (1)
- It slows down since $r \propto \frac{1}{v^2}$ (from $v^2 = \frac{GM_E}{r}$). (1)

3. A satellite orbits 4.22 × 10^7 m above the Earth's centre. At a certain point in its orbit around the Earth, the satellite and the Moon line up as shown in the diagram below. Show that in this position the influence of the Moon on the satellite is negligible, compared with the influence of the Earth. (7 marks)



$$F_E = \frac{GM_E m_s}{r^2} = \frac{(6.67\times10^{-11})(5.97\times10^{24})m_s}{(4.22\times10^7)^2} \quad (1)$$

$$= 0.224\, m_s. \quad (1)$$

$$F_M = \frac{GM_M m_s}{r^2} = \frac{(6.67\times10^{-11})(7.35\times10^{22})m_s}{(3.418\times10^8)^2} \quad (1)$$

$$= 4.20\times10^{-5}\, m_s \quad (1)$$

∴ F_E is 5.33×10^3 times larger than F_m (1)

4. A GPS system uses the signals from four satellites to establish a position on the Earth's surface. The satellites have an orbital period of 12.0 hours but they are in different planes of orbit. Each satellite has an atomic clock that allows a signal to be emitted at prescribed intervals. The time difference between the four signals is used by the receiver to establish a position.

(a) By equating the relationship for centripetal force and gravitational force show that the orbital velocity of each satellite is close to 3.90 × 10^3 ms^{-1}. Show all your workings. (5 marks)

$$F_g = F_c$$

$$\Rightarrow \frac{GM_E M_s}{r^2} = \frac{M_s v^2}{r} = \frac{4\pi^2 M_s r}{T^2} \quad (1)$$

$$\Rightarrow r^3 = \frac{GM_E T^2}{4\pi^2}$$

$$\Rightarrow r = \sqrt[3]{\frac{(6.67\times10^{-11})(5.97\times10^{24})(12.0\times3.60\times10^3)^2}{4\pi^2}} \quad (1)$$

$$= 2.66\times10^7\text{ m} \quad (1)$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(2.66\times10^7)}{(12.0\times3.60\times10^3)} = 3.87\times10^3\text{ ms}^{-1} \quad (1)$$

(1)

(b) The manufacturers of the satellites deliberately build in a correction to the rate at which the clocks tick so that they run a little fast before they are put into orbit. Explain why they do this. (2 marks)

- Velocity is large. (1)
- Relativistic time dilation becomes significant to a very accurate atomic clock. (1)

5. An exoplanet is a planet that revolves around a star that is not our Sun. As one such exoplanet revolves around a distant star, it causes the star to oscillate, or wobble, in its path as the star and the exoplanet orbit their common centre of mass.

In the following calculations, assume that the centre of the exoplanet's orbit coincides with the star's centre of mass, and that the orbit is circular.

Some details of the star and the exoplanet are shown below:

Mass of star	$M_s = 2.15 \times 10^{30}$ *kg*
Mass of exoplanet	$M_p = 1.95 \times 10^{27}$ *kg*
Distance between centre of planet and centre of star	$d_{sp} = 7.50 \times 10^9$ m

(a) Show that the magnitude of the gravitational force acting on the exoplanet is 4.97×10^{27} N. (3 marks)

$$F = \frac{G M_s M_p}{r^2} \quad (1)$$

$$= \frac{(6.67\times10^{-11})(2.15\times10^{30})(1.95\times10^{27})}{(7.50\times10^{9})^2} \quad (1) \quad (1)$$

$$= \underline{4.97\times10^{27}\ \text{N}}$$

(b) Calculate the exoplanet's orbital velocity. Show all workings. (3 marks)

$$F_g = F_c = \frac{M_p v^2}{r}$$

$$\Rightarrow \quad v = \sqrt{\frac{F_g r}{M_p}} \qquad (1)$$

$$= \sqrt{\frac{(4.97\times10^{27})(7.50\times10^{9})}{(1.95\times10^{27})}} \qquad (1)$$

$$= \underline{1.38\times10^{5}\ ms^{-1}} \qquad (1)$$

(c) Calculate the exoplanet's orbital period, and express your answer in hours. Show all workings. (3 marks)

$$v = \frac{2\pi r}{T}$$

$$\Rightarrow \quad T = \frac{2\pi r}{v} \qquad (1)$$

$$= \frac{2\pi(7.50\times10^{9})}{(1.38\times10^{5})} \qquad (1)$$

$$= 3.415\times10^{5}\ s$$

$$= \underline{94.8\ hrs} \qquad (1)$$

(d) About 20% of exoplanets discovered so far have a period of 120 hours or less. Explain briefly how red shift and blue shift can be used to identify which stars have such exoplanets. (3 marks)

- Star wobbles as the planets revolve around it - causes it to move towards and away from an observer. (1)
- Towards - blue shifted. Away - red shifted. (1)
- Time between successive maximum red shifts (or blue shifts) is the orbital period. (1)

6. The Kepler NASA mission aims to search for planets orbiting stars in other solar systems. The star named Kepler 20 has been observed to have several planets orbiting it. Kepler 20 is 950 light-years from Earth.

Information about Kepler 20 and some of the planets orbiting it is summarised in the table below.

Astronomical object	Radius	Mass	Orbital period around Kepler 20
Star – Kepler 20	$0.944 \times \text{radius}_{SUN}$	$0.912 \times \text{mass}_{SUN}$	
Planet – Kepler 20b	$2.40 \times \text{radius}_{EARTH}$		290 days
Planet – Kepler 20e	$0.87 \times \text{radius}_{EARTH}$		6.1 days
Planet – Kepler 20f	$1.03 \times \text{radius}_{EARTH}$		19.6 days

(a) A light-year is an astronomical unit of distance. It is defined as the distance travelled by light in one year. Calculate the distance from Kepler 20 to Earth in kilometres.

(2 marks)

$s = vt$

$= (950)(3.00\times10^{8})(24.0)(60.0)(60.0)(365)$ (1)

$= 8.99 \times 10^{18}$ m

$= \underline{8.99 \times 10^{15} \text{ km}}$ (1)

(b) Astronomers express the mass of Kepler 20 as $(0.912 \pm 0.035) \times \text{mass}_{SUN}$. Calculate the maximum value astronomers expect for the mass of Kepler 20. (2 marks)

Max value : $(0.912 + 0.035)\, m_{sun}$ (1) $= (0.947)(1.99\times10^{30})$

$= 1.88\times10^{30}$ kg (1)

(c) Calculate the orbital radius of Kepler 20e around Kepler 20. You should use the mass for Kepler 20 quoted in the table and assume the orbit is circular. (4 marks)

$$F_g = F_c$$

$$\Rightarrow \frac{G M_S M_P}{r^2} = \frac{M_P v^2}{r} = \frac{4\pi^2 M_P r}{T^2}$$

$$\Rightarrow r^3 = \frac{G M_S T^2}{4\pi^2}$$

$$\Rightarrow r = \sqrt[3]{\frac{(6.67\times10^{-11})(0.912\times1.99\times10^{30})(6.1\times24\times60\times60)^2}{4\pi^2}}$$

$$= \underline{9.48\times10^{9}\ m}$$

(d) The mass of Kepler 20b is unknown but it has been speculated that it may have a density similar to that of Earth, 5520 kgm^{-3}. Calculate the surface gravity of Kepler 20b if its density is 5520 kgm^{-3}. (4 marks)

Reminder:

$$density = \frac{mass}{volume}$$

$$volume\ of\ a\ sphere = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(2.40\times6.38\times10^6)^3$$

$$= 1.50\times10^{22}\ m^3 \quad (1)$$

$$D = \frac{m}{V}$$

$$\Rightarrow m = DV$$

$$= (5520)(1.50\times10^{22})$$

$$= 8.28\times10^{25}\ kg \quad (1)$$

$$g = \frac{GM}{r^2}$$

$$= \frac{(6.67\times10^{-11})(8.28\times10^{25})}{(2.40\times6.38\times10^6)} \quad (1)$$

$$= \underline{23.6\ ms^{-2}} \quad (1)$$

The Kepler mission is particularly concerned with finding planets that lie within the habitable zones of stars. A planet in a star's habitable zone receives the right amount of energy from the star to maintain liquid water on its surface, provided it also has an appropriate atmosphere.

(e) By comparing the Kepler 20 system and our own solar system, suggest which planet in the Kepler 20 system is most likely to lie in the habitable zone. Explain your answer.

(3 marks)

- Kepler 20b (1)
- Given the star Kepler 20 is about the same size as our Sun: (1)

$\Rightarrow$ Choose a similar period of orbit for similar conditions. (1)

$\therefore$ Kepler 20b.