GREENWOOD COLLEGE

Balcatta Senior High School

West Coast Collaborative
Test 1 2016
Calculator Free Section

Name: ______________________ **Score:** _____ / 31

Section 1 is worth 50% of your final test mark.
<u>No</u> calculators or notes are to be used.
Access to approved Sample Mathematics Specialist formulae sheet is permitted. Time limit = 30 minutes.

Name: Marking Key

1. (4 marks)

Given that z is a complex number solve $\frac{z-6}{z+1} = 5z-3$, giving the solution(s) <u>exactly</u> in the form $a + bi$.

$\Rightarrow \quad z-6 = (z+1)(5z-3) \quad , \quad z \neq -1$ ✓

$\Rightarrow \quad z-6 = 5z^2+2z-3$

$\Rightarrow \quad 5z^2+z+3 = 0$ ✓

$a = 5$
$b = 1$
$c = 3$

$\Rightarrow \quad z = \dfrac{-1 \pm \sqrt{1^2-4\times5\times3}}{2\times5}$ ✓

$\Rightarrow \quad z = -\frac{1}{10} \pm \dfrac{\sqrt{59}}{10}\, i$ ✓

2. **[3 marks]**

Rewrite $\{ z : |z-(2+4i)| = |z-1| \}$ as a Cartesian equation.

$\Rightarrow \quad (x-2)^2 + (y-4)^2 = (x-1)^2 + y^2$ ✓

$\Rightarrow \quad \cancel{x^2} - 4x + 4 + \cancel{y^2} - 8y + 16 = \cancel{x^2} - 2x + 1 + \cancel{y^2}$ ✓

$\Rightarrow \quad -2x - 8y = -19$

so $\quad 2x + 8y = 19 \quad$ or $\quad y = -\frac{1}{4}x + \frac{19}{8}$ ✓

3. **[2,4,2 marks]**

The polynomials p(x) and Q(x) are defined as follows:

$P(x) = x^8 - 1, \qquad Q(x) = x^4 + 4x^3 + ax^2 + bx + 5$

a) Show that x-1 and x+1 are factors of P(x)

$f(1) = 1 - 1 = 0$ ✓

$f(-1) = 1 - 1 = 0$ ✓

b) Determine a and b if Q(x) leaves a remainder of 37 when it is divided by $x-2$, and a remainder of -19 when divided by $x+2$

$f(2) = 16 + 32 + 4a + 2b + 5 = 37$

$\Rightarrow \quad 53 + 4a + 2b = 37$

$\Rightarrow \quad 4a + 2b = -16 \quad \Rightarrow \quad 2a + b = -8$ ✓ — 1

$f(-2) = 16 - 32 + 4a - 2b + 5 = -19$

$\Rightarrow \quad 4a - 2b = -8 \quad \Rightarrow \quad 2a - b = -4$ ✓ — 2

$4a = -12$

$\therefore \ \underline{a = -3}$ ✓ $\ \underline{b = -2}$ ✓

c) With these values a and b, determine the remainder when the polynomial 4P(x) + 5Q(x) is divided by $x-1$

By remainder theorem f(1) is remainder

So $4x^8 - 4 + 5x^4 + 20x^3 - 15x^2 - 10x + 25 \,\big|\, x = 1$

$= 4 - 4 + 5 + 20 - 15 - 10 + 25$

$= 25$ ✓

4 ~~3~~. **[3,3 marks]**

a) Given $z = a - 3i$ and $z + c\bar{z} = 12 + 6i$, determine a and c.

$$a - 3i + c(a + 3i) = 12 + 6i$$

$\Rightarrow \quad a(c+1) = 12$ Real parts

$-3i + 3ci = 6i$ Imaginary parts ✓

$\Rightarrow \quad \underline{c = 3}$ ✓

$\Rightarrow \quad a(3+1) = 12$

$\Rightarrow \quad \underline{a = 3}$ ✓

b) Given $w = b\text{cis}\frac{5\pi}{6}$ and $|(\bar{w})^3| = 0.001$, determine b

and give the principal argument of w^3

$w = 0.1 \text{ cis}\left(\frac{5\pi}{6}\right)$ so $\underline{b = 0.1}$ ✓

$w^3 = (0.1)^3 \text{ cis}\left(\frac{5\pi}{6} \times 3\right)$

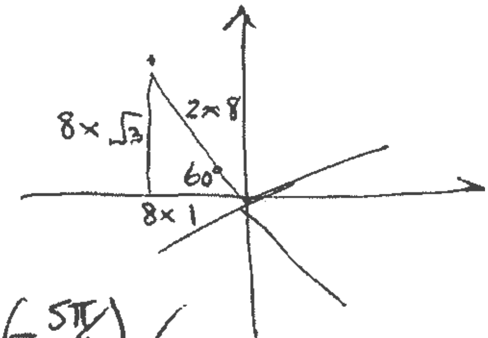
$= 0.001 \text{ cis } \frac{5\pi}{2}$ ✓

So principal argument is $\frac{\pi}{2}$ ✓

5 ~~6~~. **[5 marks]**

Use De Moivre's Theorem to find all the roots of $z^4 = -8 + 8\sqrt{3}i$. Give your answers exactly in polar form, with $r>0$ and $-\pi < \vartheta \le \pi$.

$-8 + 8\sqrt{3}i = 16 \text{ cis } \frac{2\pi}{3}$ ✓



So roots are $2 \text{ cis } \frac{\pi}{6}$ ✓

$2 \text{ cis } \frac{2\pi}{3}$ ✓

$2 \text{ cis } \frac{7\pi}{6} = 2 \text{ cis}\left(-\frac{5\pi}{6}\right)$ ✓

$2 \text{ cis}\left(-\frac{\pi}{3}\right)$ ✓

6 ~~7~~. **[5 marks]**

Find all the values, real and complex, of x for which G(x) = 0,

If G(x) = $5x^3-15x^2+5x-15$

$$5x^3-15x^2+5x-15$$

$$= 5(x^3-3x^2+x-3) \checkmark$$

$$f(3) = 27-27+3-3 = 0 \checkmark$$

So $(x-3)$ is a factor

$$\begin{array}{r|l} & x^2 \qquad\quad +1 \\ \hline x-3 & x^3-3x^2+x-3 \\ & \underline{x^3-3x^2} \quad \underline{x-3} \\ & \quad 0 \qquad\quad 0 \end{array}$$

The other factor is x^2+1 ✓

So roots are $x = 3, i, -i$ ✓✓ (-1 each omission)

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END OF SECTION 1
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7. [2, 2, 2, 2 marks]

Suppose that z is a complex number with modulus r and argument θ. Express in terms of r and θ the modulus and argument of each of the four complex numbers z_1, z_2, z_3, and z_4 where

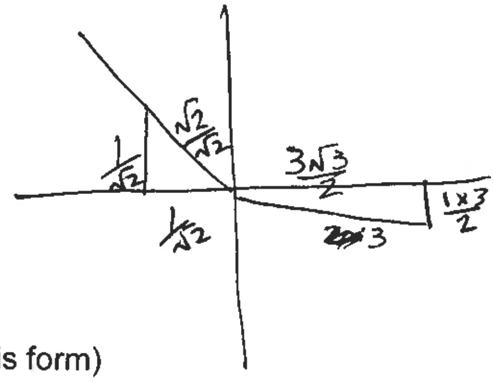
(i) $z_1 = z^2$ mod $= r^2$ arg $= 2\theta$

(ii) $z_2 = 2z$ mod $= 2r$ arg $= \theta$

(iii) $z_3 = z^{-1}$ mod $= 1/r$ arg $= -\theta$

(iv) $z_4 = -iz$ mod $= r$ arg $= \theta - \pi/2$

8. [2,2,2 marks]



The complex numbers of *z* and *w* are such that:

$z = 3\text{cis}(-\frac{\pi}{6})$ and $w = \text{cis}(\frac{3\pi}{4})$

Determine the following.

a) $z + w$ (in a + b*i* form)

$3cis(-\pi/6) + cis(3\pi/4)$

$= \frac{3\sqrt3}{2} - \frac{3i}{2} + -\frac{1}{\sqrt2} + \frac{1}{\sqrt2}i$

$= \left(\frac{3\sqrt3-\sqrt2}{2}\right)✓ + \left(\frac{\sqrt2-3}{2}\right)i$ ✓

$(1.89 - 0.798i)$

b) $w\,z$ (in cis form)

$= 3\ cis\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$

$=$ ✓ $3\ cis\left(\frac{7\pi}{12}\right)$ ✓

c) $\frac{1}{\bar{z}}$ (in cis form and a + b*i* form)

$\frac{1}{3cis(\pi/6)} = \frac{1\ cis\ 0}{3cis(\pi/6)} = \frac{1}{3}\ cis(-\pi/6)$ ✓

or $\frac{\sqrt3}{6} - \frac{i}{6}$ ✓

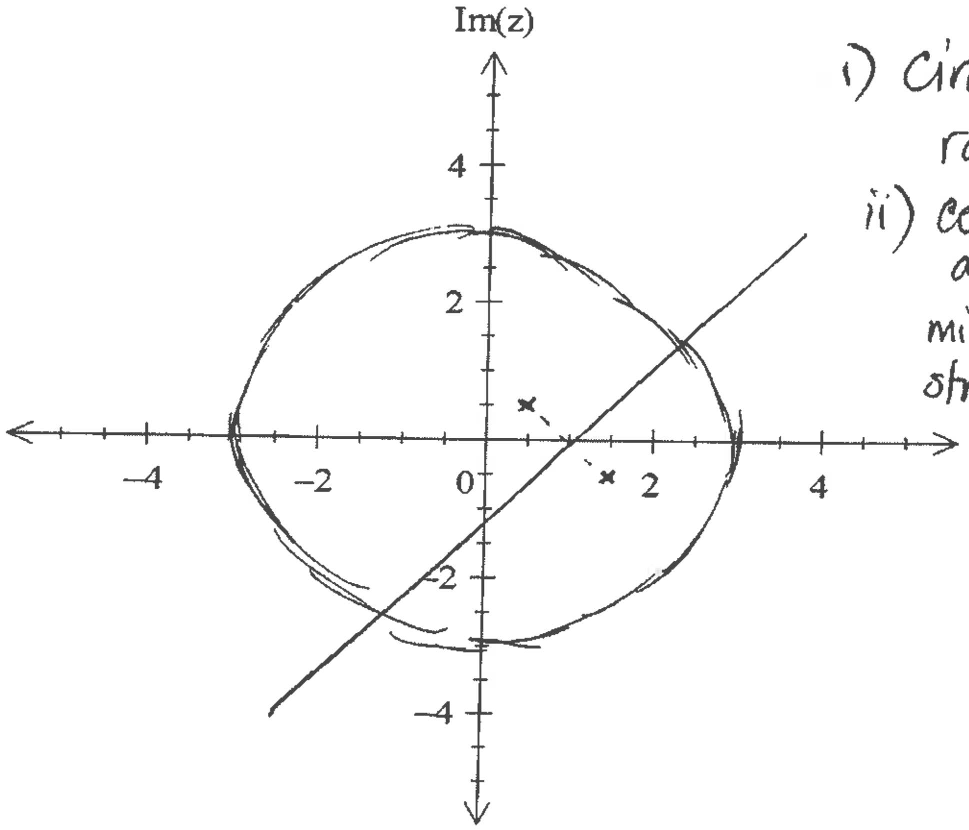
$(0.289 - 0.167i)$

9. ~~10.~~ **[5,5 marks]**

a) On the Argand Diagram below, sketch and label both of the sets of points specified:

i) $z\bar{z} = 9$

ii) $|z - 1 - i| = |z - 3 + i|$



i) circle shape ✓
radius = 3 ✓
ii) control points $(1, i)$ and $(3, -i)$ ✓
midpoint $(0, 2)$ ✓
straight line close to perpendicular ✓

b) Determine to four decimal places the largest possible argument, $-\pi < \theta \le \pi$, of the complex number that satisfies $\{z : z\bar{z} = 9\} \cap \{z : |z - 1 - i| = |z - 3 + i|\}$

$$(x-1)^2 + (y-1)^2 = (x-3)^2 + (y+1)^2 \checkmark \text{ — } (1)$$

Also $x^2 + y^2 = 9$ — (2) ✓

Solve using simultaneous equations

gives $x = \frac{\sqrt{14}}{2} + 1$, $y = \frac{\sqrt{14}}{2} - 1$ ✓

(2.870828693R) (0.8708286934R)

$$\tan^{-1} \frac{y}{x} = \frac{\frac{\sqrt{14}}{2} - 1}{\frac{\sqrt{14}}{2} + 1} \checkmark \therefore \theta = 0.2945154851 \text{ R}$$

(16.8744943°)

so 0.2945. ✓

10 ~~11.~~ **[4, 1 marks]**

a) Use de Moivre's theorem to show that $\cos 4x = \cos^4 x + \sin^4 x - 6\cos^2 x \sin^2 x$

$\cos 4x = (\cos x + i\sin x)^4$ ✓

$= \cos^4 x + 4\cos^3 x\, i\sin x + 6\cos^2 x\, i^2 \sin^2 x + 4\cos x\, i^3 \sin^3 x$ ✓
$+ \sin^4 x$

Using Real Parts ✓ $\cos 4x = \cos^4 x - 6\cos^2 x \sin^2 x + \sin^4 x$ ✓

b) Use this result to show that $\cos 4x = 1 - 8\cos^2 x \sin^2 x$

$\cos^4 x - 6\cos^2 x \sin^2 x + \sin^4 x$

$= (\cos^2 x + \sin^2 x)^2 - 2\cos^2 x \sin^2 x - 6\cos^2 x \sin^2 x$

$= 1^2 - 8\cos^2 x \sin^2 x$ ✓

$= 1 - 8\cos^2 x \sin^2 x$