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Topic: Perimeter, Surface Area and Volume (Circles, Cones, Cylinders, Spheres etc)

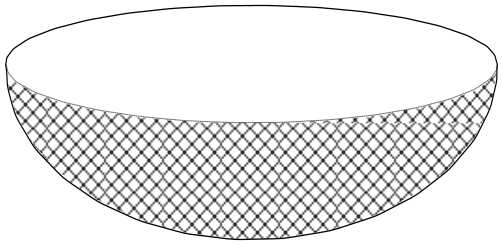
Time: 45 mins Marks: /45 marks

Calculator Assumed

Question One: [2, 2: 4 marks]

This is a picture of Tupperware's newest picnic salad bowl.

It is a hemisphere in shape and has a radius of 10 cm.



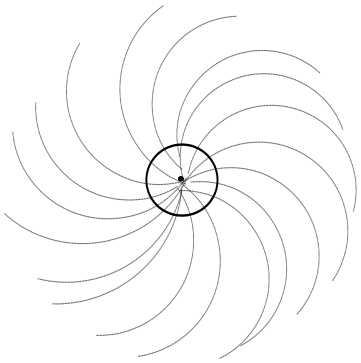
a) The lid overlaps the edge of the bowl by 0.2 cm and is 2 cm thick. Calculate the amount of material needed to make the lid.

b) There are various colours to choose from for the outside pattern and this is printed onto the outside of the plastic bowl. Calculate the surface area of this coloured print.

Question Two: [1, 2, 3, 2, 2: 10 marks]

Sprinklers are used for watering gardens. They spray water in a circular shape.

a) Calculate the area a sprinkler would cover with water if the water reaches 80 cm from the centre of the sprinkler.



Sprinklers don't always spray in a full revolution, they can spray a quarter or half revolution.

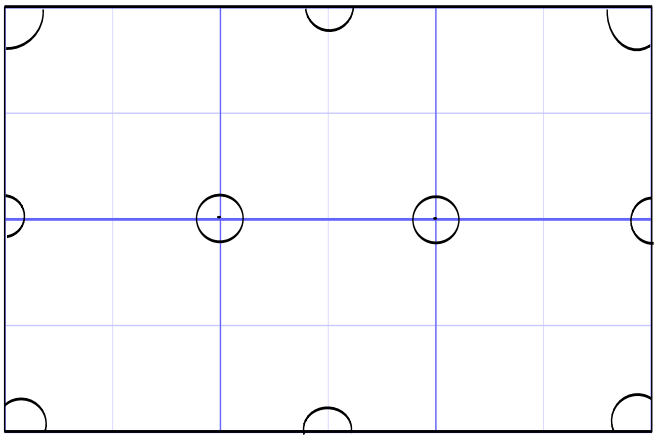
b) Calculate the area a quarter and a half revolution sprinkler would cover if they each had a radius reach of 50 cm.

Alisa and Lysandra are designing a backyard for their clients. Part of the design includes a rectangular patch of lawn. They are installing a reticulation sprinkler system to water the lawn.

The lawn is 6 m by 4 m. The diagram below shows the placement of the sprinklers, laid over a square grid. So as not to waste water, each corner sprinkler only sprays a quarter of a circle, the four perimeter sprinklers only spray half a circle and the two centre ones are the only sprinklers which spray the full circle.

There is the same vertical and horizontal distance between the centre of each neighbouring sprinkler.

The radius reach of each sprinkler is 1.2 m.



c) What is the total area which receives water from this sprinkler system?

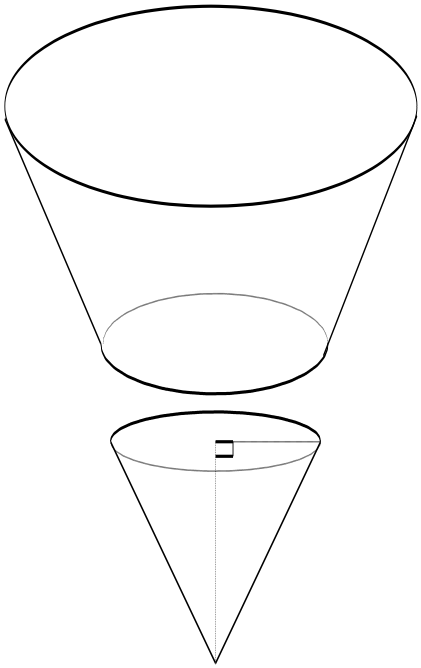
d) Is the entire grass area watered by the proposed sprinkler placement? Justify your answer mathematically.

e) As long as 95% of the grassed area receives water, the grass will flourish. Do Alisa and Lysandra need to revise their plan? Justify your answer mathematically.

Question Three: [2, 4, 3, 5, 3: 17 marks]

These delicious double choc waffle cones have chocolate in the bottom of the cone and an outer coating of chocolate. When they are being made the chocolate is poured into the cone up to exactly $\frac{1}{3}$ of the height of the cone.

a) If the radius of the cone is 30 mm and the height is 100mm, calculate the total volume of the cone, including the chocolate.



b) Calculate the surface area of the cone to be covered in chocolate if it is to be dipped in chocolate to cover the entire outside surface.

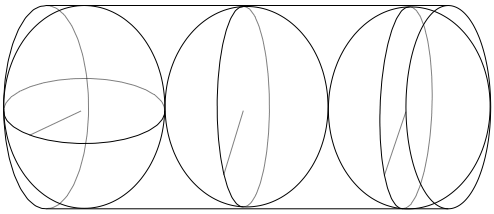
c) What is the diameter of the base of the cone formed by the chocolate once it has set in the bottom of the cone?

d) If the cone is coated 1 mm thick, how many mL of chocolate is needed for this cone?

e) If another waffle cone is made similar to this one but just a larger version, how many mL of chocolate is needed if the ratio of the smaller cone to the larger cone is 1 : 1.5?

Question Four: [3, 2, 2, 2, 5: 14 marks]

The official diameter of a tennis ball, as defined by the International Tennis Federation, is 6.54 cm. A standard tennis ball container is a cylinder in shape and snuggly fits three tennis balls so that the diameter of the cylindrical container is also 6.54 cm and the height is the exact height of the three tennis balls (spheres) inside.



a) Calculate the volume of the container.

b) Calculate the volume of one tennis ball.

c) Determine the volume of empty space inside the container.

A company comes up with a new design for the tennis ball container. It is a rectangular prism with the height and base such that the three tennis balls fit snuggly inside.

d) What are the dimensions of the new design tennis ball container?

e) Does this container use more or less material to produce than the original cylindrical container? Justify your answer mathematically.

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Perimeter, Surface Area and Volume (Circles, Cones, Cylinders, Spheres etc)
SOLUTIONS

Time: 45 mins Marks: /45 marks

Calculator Assumed

Question One: [2, 2: 4 marks]

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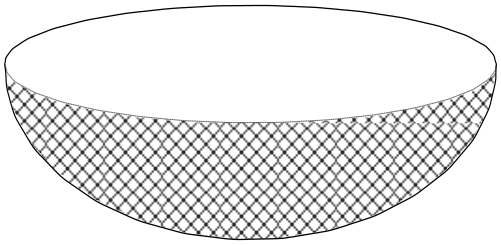
It is a hemisphere in shape and has a radius of 10 cm.



a) The lid overlaps the edge of the bowl by 0.2 cm and is 2 cm thick. Calculate the amount of material needed to make the lid.

$r = 10 + 0.2 = 10.2\ cm$ ✓

$V = \pi \times 10.2^2 \times 2 = 653.70\ cm^3\ (2dp)$ ✓

b) There are various colours to choose from for the outside pattern and this is printed onto the outside of the plastic bowl. Calculate the surface area of this coloured print.

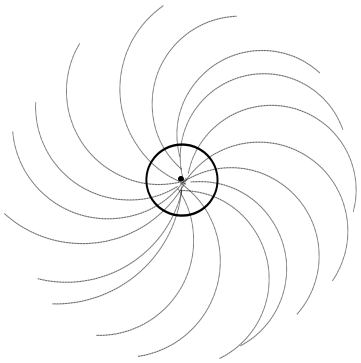
$r = 10\ cm$ ✓

$SA = 2 \times \pi \times 10^2 = 628.32\ cm^2\ (2\text{dp})$ ✓

Question Two: [1, 2, 3, 2, 2: 10 marks]

Sprinklers are used for watering gardens. They spray water in a circular shape.

a) Calculate the area a sprinkler would cover with water if the water reaches 80 cm from the centre of the sprinkler.



$A = \pi \times 80^2$

$= 20\ 106.19\ cm^2$ ✓

Sprinklers don't always spray in a full revolution, they can spray a quarter or half revolution.

b) Calculate the area a quarter and a half revolution sprinkler would cover if they each had a radius reach of 50 cm.

$\frac{1}{2}$ $A = \pi \times 50^2 \div 2$

$= 3926.99\ cm^2$ ✓

$\frac{1}{4}$ $A = \pi \times 50^2 \div 4$

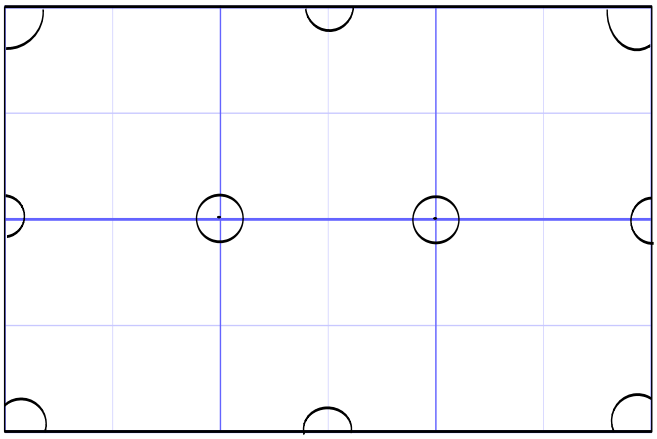
$= 1963.50\ cm^2$ ✓

Alisa and Lysandra are designing a backyard for their clients. Part of the design includes a rectangular patch of lawn. They are installing a reticulation sprinkler system to water the lawn.

The lawn is 6 m by 4 m. The diagram below shows the placement of the sprinklers, laid over a square grid. So as not to waste water, each corner sprinkler only sprays a quarter of a circle, the four perimeter sprinklers only spray half a circle and the two centre ones are the only sprinklers which spray the full circle.

There is the same vertical and horizontal distance between the centre of each neighbouring sprinkler.

The radius reach of each sprinkler is 1.2 m.



c) What is the total area which receives water from this sprinkler system?

$A = 5 \times \pi \times 1.2^2$

$= 22.62\ m^2$ ✓ ✓ ✓

d) Is the entire grass area watered by the proposed sprinkler placement? Justify your answer mathematically.

$A = 6 \times 4 = 24\ m^2$

No, $1.38\ m^2$ unwatered ✓ ✓

e) As long as 95% of the grassed area receives water, the grass will flourish. Do Alisa and Lysandra need to revise their plan? Justify your answer mathematically.

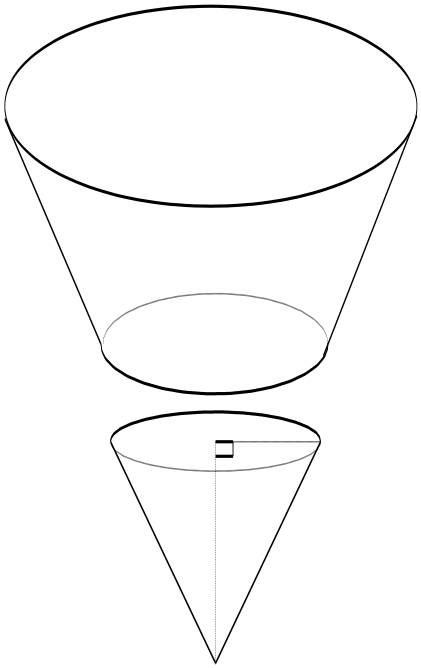
$Yes, \frac{22.62}{24} = 94.25\%$. This is not enough coverage.

✓ ✓

Question Three: [2, 4, 3, 5, 3: 17 marks]

These delicious double choc waffle cones have chocolate in the bottom of the cone and an outer coating of chocolate. When they are being made the chocolate is poured into the cone up to exactly $\frac{1}{3}$ of the height of the cone.

a) If the radius of the cone is 30mm and the height is 100 mm, calculate the total volume of the cone, including the chocolate.



✓

$$V = \frac{1}{3} \times \pi \times 30^2 \times 100$$

$$= 94\,247.78\ mm^2$$ ✓

b) Calculate the surface area of the cone to be covered in chocolate if it is to be dipped in chocolate to cover the entire outside surface.

$SA = \pi \times r \times s$

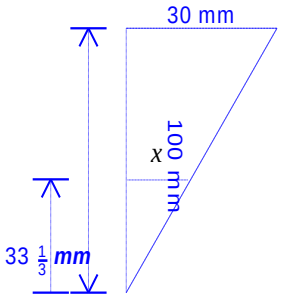
$s^2 = 30^2 + 100^2$ ✓

$s = 104.40\ mm$ ✓

$SA = \pi \times 30 \times 104.40$

$= 9839.47\ mm^2$ ✓ ✓

c) What is the diameter of the base of the cone formed by the chocolate once it has set in the bottom of the cone?



$$\frac{33\frac{1}{3}}{100} = \frac{x}{30}$$

$$x = 10$$

$$\text{diameter} = 20 \text{ mm}$$

d) If the cone is coated 1 mm thick, how many mL of chocolate is needed for this cone?

$$= 9839.47 \times 0.1 + \frac{1}{3} \times \pi \times 10^2 \times 33\frac{1}{3}$$

$$= 983.947 + 3490.6585$$

$$= 4474.61\ mm^3$$

$$4474.61 \div 10^3$$

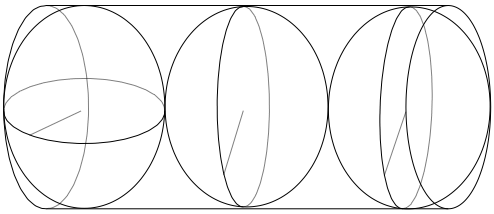
$$= 4.47\ cm^3 \ \therefore 4.47\ mL\ chocolate$$

e) If another waffle cone is made similar to this one but just a larger version, how many mL of chocolate is needed if the ratio of the smaller cone to the larger cone is 1 : 1.5?

$$4.47\ cm^3 \times 1.5^3 = 15.10\ mL$$

Question Four: [3, 2, 2, 2, 5: 14 marks]

The official diameter of a tennis ball, as defined by the International Tennis Federation, is 6.54 cm. A standard tennis ball container is a cylinder in shape and snuggly fits three tennis balls so that the diameter of the cylindrical container is also 6.54 cm and the height is the exact height of the three tennis balls (spheres) inside.



a) Calculate the volume of the container.

$$V = \pi \times 3.27^2 \times 19.62$$

$$= 659.09\ cm^3$$

b) Calculate the volume of one tennis ball.

$$V = \frac{4}{3} \times \pi \times 3.27^3$$

$$= 146.46\ cm^3$$

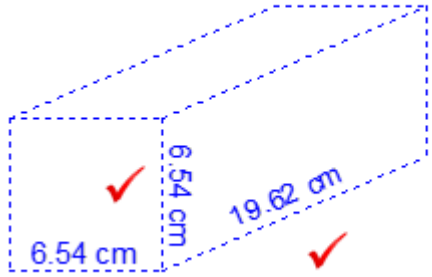
c) Determine the volume of empty space inside the container.

$$= 659.09 - 3 \times 146.46$$

$$= 219.71\ cm^3$$

A company comes up with a new design for the tennis ball container. It is a rectangular prism with the height and base such that the three tennis balls fit snuggly inside.

d) What are the dimensions of the new design tennis ball container?



e) Does this container use more or less material to produce than the original cylindrical container? Justify your answer mathematically.

$SA\ cylinder = 2 \times \pi \times 3.27^2 + 2 \times \pi \times 3.27 \times 19.62$ ✓

$= 470.298\ cm^2$ ✓

$SA\ prism = 6.54 \times 6.54 \times 2 + 6.54 \times 19.62 \times 4$ ✓

$= 598.8024\ cm^2$ ✓

∴ The original cylindrical design uses less material to produce ✓