

Full Name: SOLUTIONS



## MATHEMATICS Specialist Units 3 & 4

### Test 2 – Functions and Sketching Graphs

Semester 1 2019

#### Section One – Calculator Free

##### Time allowed for this section

Working time for this section: 24 minutes  
Marks available: 24 marks

##### Material required/recommended for this section

###### To be provided by the supervisor

This Question/Answer booklet  
Formula sheet

###### To be provided by the student

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

##### Important note to students

No other items may be used in this section of the assessment. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the assessment room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (8 marks: 3, 3, 1, 1)

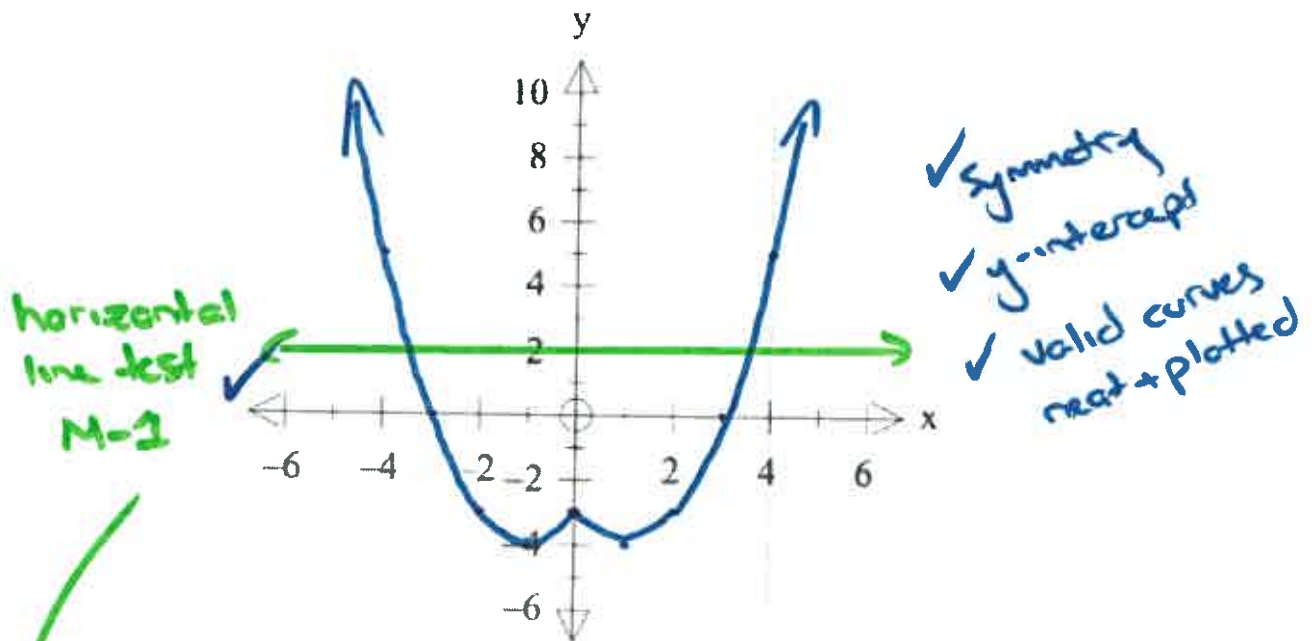
Let  $f(x) = x^2 - 2|x| - 3$

a. Rewrite  $f(x)$  in piecewise form.

critical pt  $x=0$

$$f(x) = \begin{cases} x^2 + 2x - 3, & x < 0 \\ x^2 - 2x - 3, & x \geq 0 \end{cases}$$

b. On the axes below, sketch the graph of  $f(x) = x^2 - 2|x| - 3$ .



c. Use the sketch to explain why  $f(x)$  does not have an inverse function.

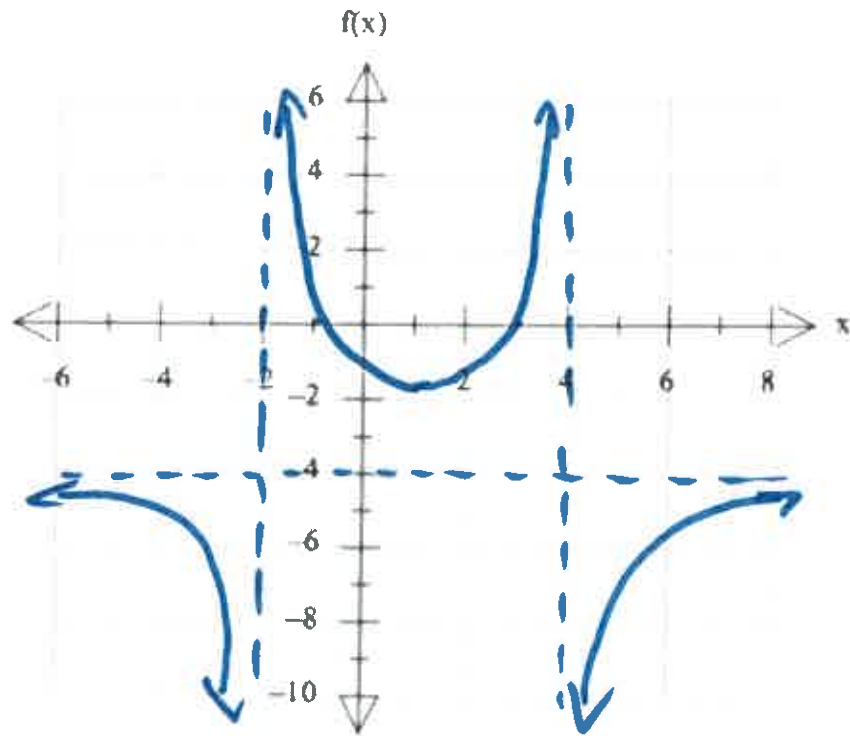
inverse is not a function. Fails HLT.

d. Suggest an appropriate restriction on the domain for the inverse of  $f(x)$  to be a function.

limit the function to  $x \geq 1$ ,  $x \leq -1$  or other alternative

✓

2. (6 marks)

Sketch the graph of  $f(x) = -\frac{4(x-3)(x+1)}{x^2-2x-8}$  on the axes below.

$$x^2 - 2x - 8 = (x - 4)(x + 2)$$

vertical asymptotes at  $x = -2, 4$  ✓

x-intercepts at  $x = -1, 3$  ✓

turning pt @  $x = 1$  ✓

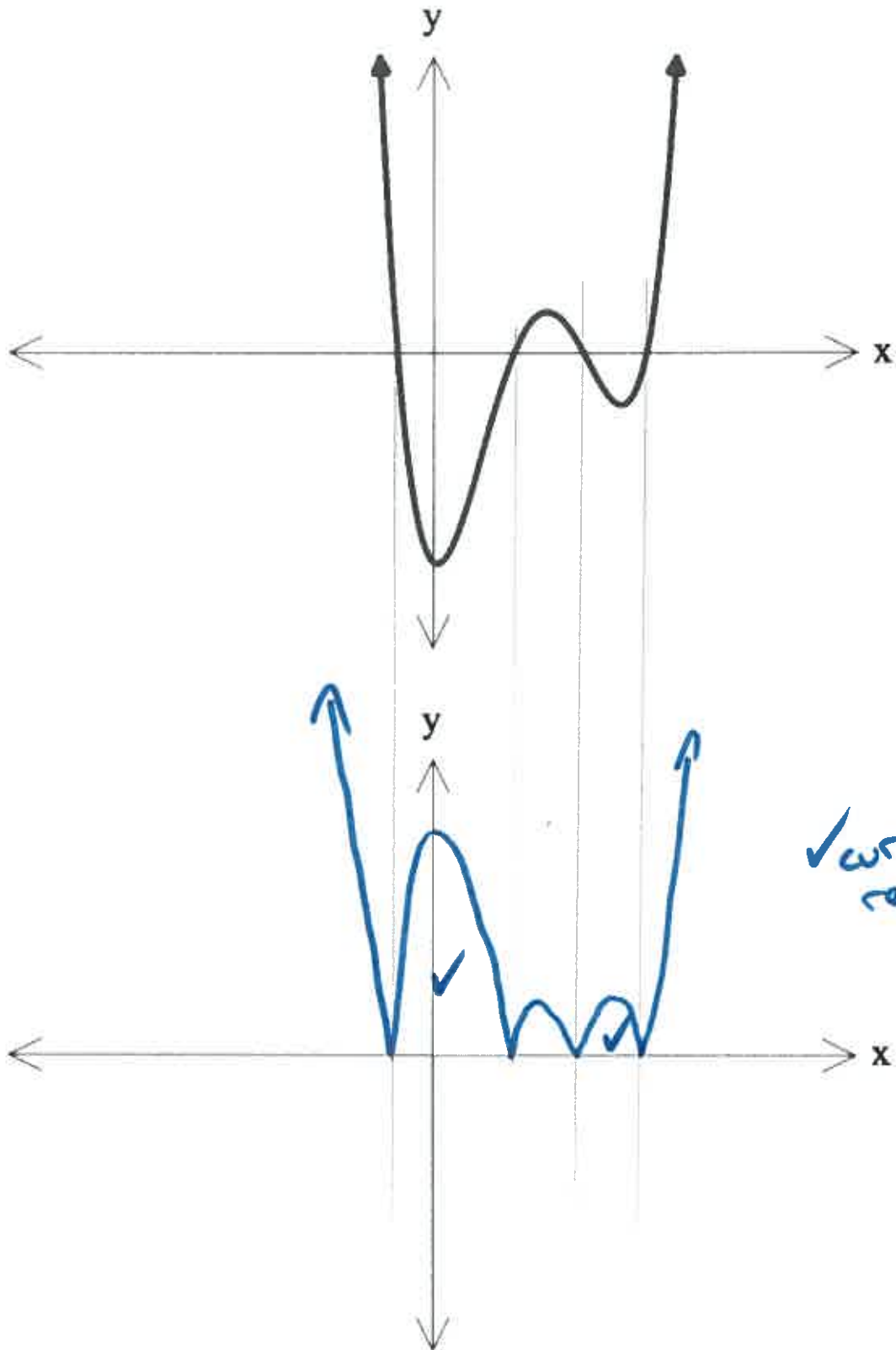
horizontal asymptote @  $y = -4$  ✓

behaviour as  $x \rightarrow \pm\infty, y \rightarrow \pm\infty$  ✓

curve shape and accuracy ✓

3. (3 marks)

Given the graph of  $g(x)$  below, sketch and label clearly  $y = |g(x)|$  on the second axes.



4. (7 marks: 3, 1, 3)

Suppose  $f(x) = 9 - \sqrt{x}$  and  $g(x) = x^2 + 4$ .

a. Find  $f \circ g(x)$  and state its domain and range.

$$f \circ g(x) = 9 - \sqrt{x^2 + 4} \quad \checkmark$$

$$D \{x: x \in \mathbb{R}\} \quad \checkmark$$

$$R \{y: y \leq 7\} \quad \checkmark$$

b. Find  $g \circ f(x)$

$$g \circ f(x) = (9 - \sqrt{x})^2 + 4 \quad \checkmark$$

c. Find  $f \circ f(x)$  and state its domain and range.

$$f \circ f(x) = 9 - \sqrt{9 - \sqrt{x}} \quad \checkmark$$

$$D: \{x: 0 \leq x \leq 81\} \quad \checkmark$$

$$R \{y: 6 \leq y \leq 9\} \quad \checkmark$$

End of Section One

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You may use this space to extend or re-attempt an answer to a question or questions and should you do so then number the question(s) attempted and cross out any previous unwanted working.

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## MATHEMATICS Specialist Units 3 & 4

### Test 2 – Functions and Sketching Graphs

Semester 1 2019

#### Section Two – Calculator Assumed

##### Time allowed for this section

Working time for this section: 26 minutes

Marks available: 26 marks

##### Material required/recommended for this section

###### To be provided by the supervisor

This Question/Answer booklet

Formula sheet

###### To be provided by the student

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

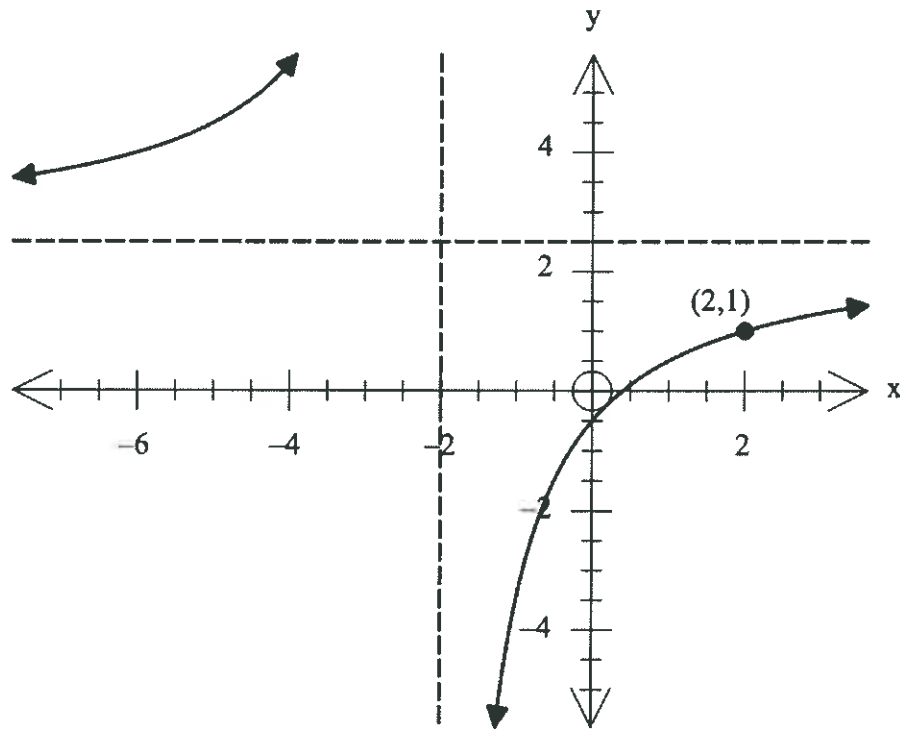
Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

##### Important note to candidates

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4. (5 marks)

The graph of  $y = \frac{ax+b}{cx+4}$ , where  $a$  and  $b$  are coefficients and  $c$  is a constant, is shown below.



The point  $(2, 1)$  lies on the graph, and the equations of its vertical and horizontal asymptotes are  $x = -2$  and  $y = 2.5$  respectively.

Find the values of  $a$ ,  $b$  and  $c$ .

$$cx+4 \Rightarrow c \times (-2) + 4 = 0 \\ c = 2 \quad \checkmark$$

$$\frac{a}{c} = 2.5 \Rightarrow a = 2.5 \times 2 = 5 \quad \checkmark$$

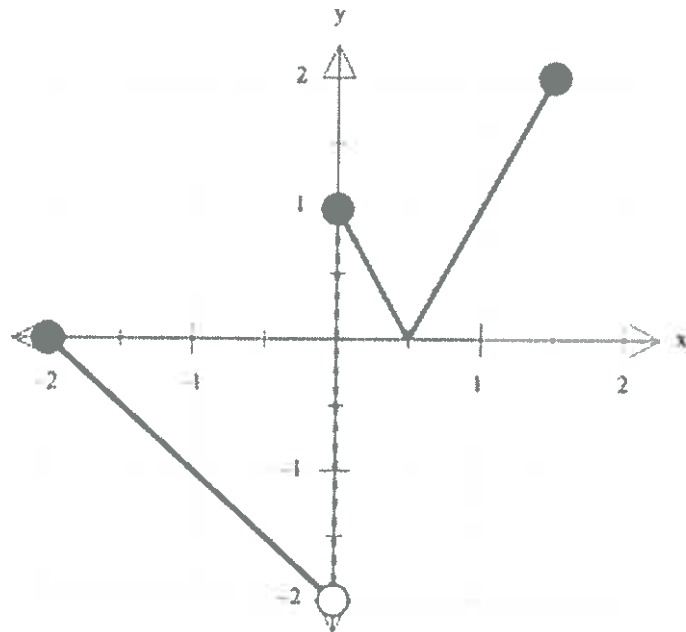
$$1 = \frac{2 \times 5 + b}{2 \times 2 + 4} \Rightarrow b = -2 \quad \checkmark$$

$$\therefore a = 5, b = -2 \text{ and } c = 2 \quad \checkmark$$

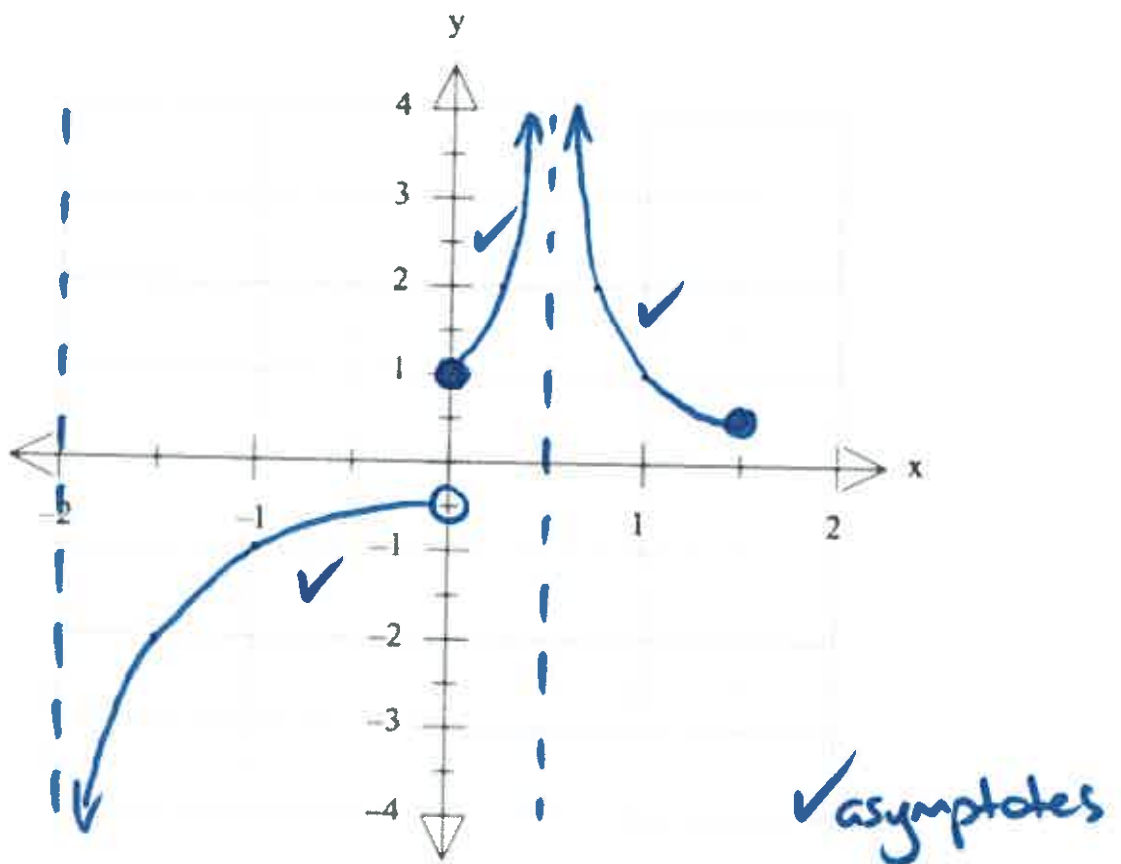


6. (9 marks: 4, 3, 2)

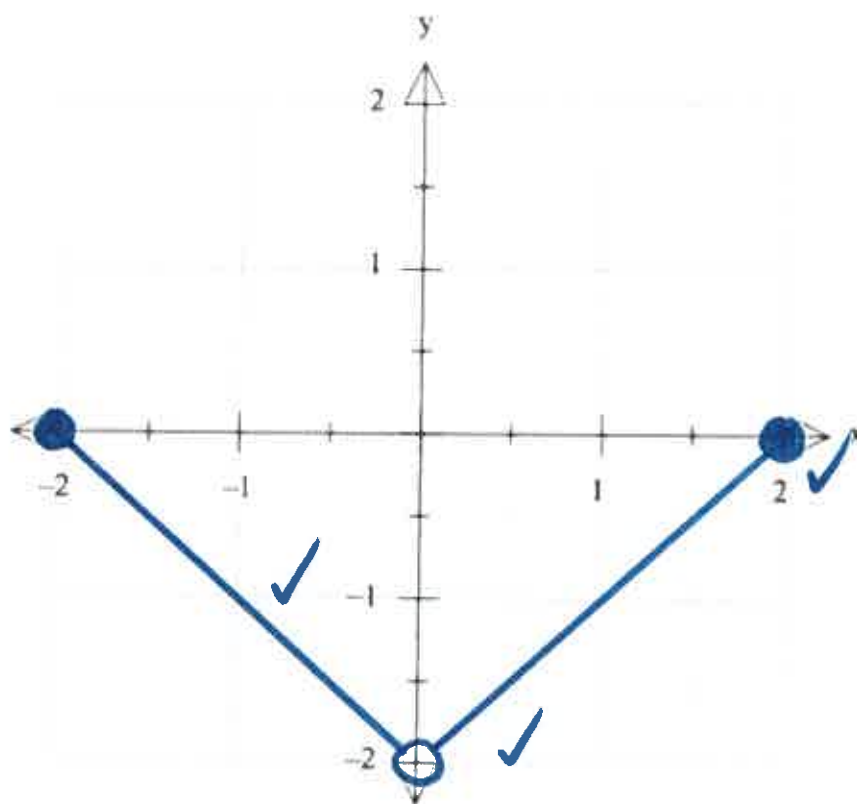
The graph of  $y = f(x)$  is shown below.



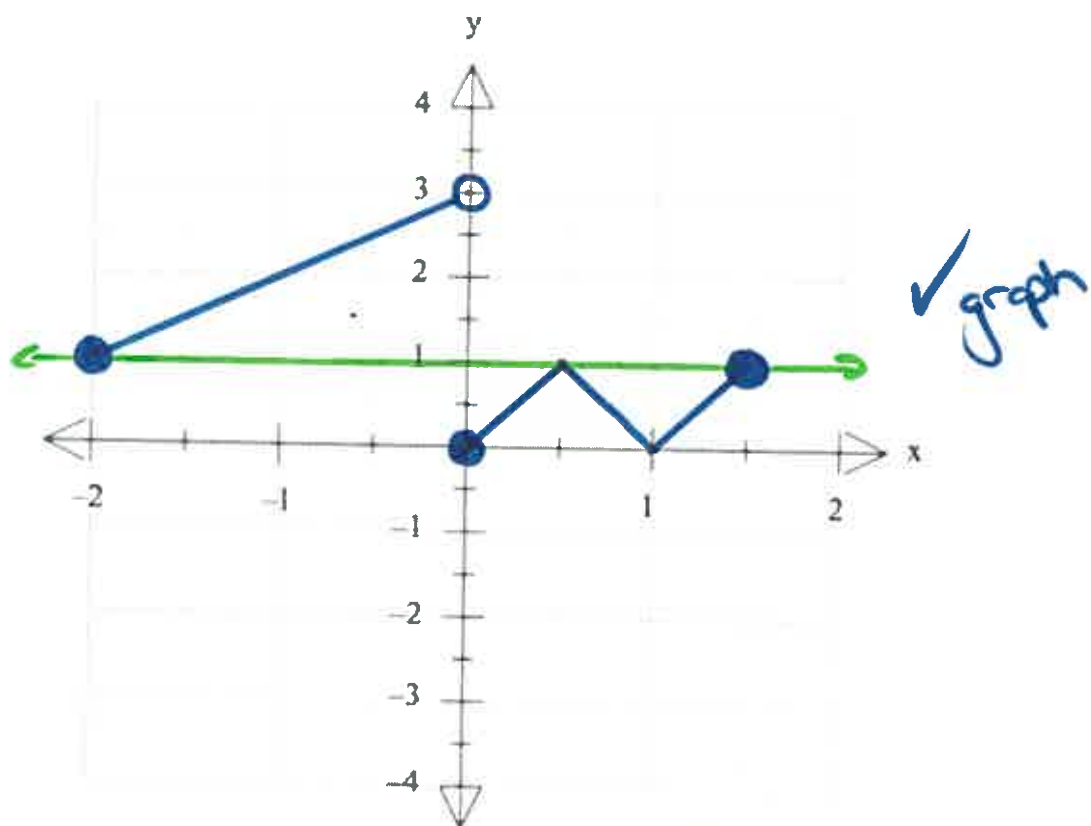
a. On the axes below, sketch the graph of  $y = \frac{1}{f(x)}$ .



- b. On the axes below, sketch the graph of  $y = f(-|x|)$ .



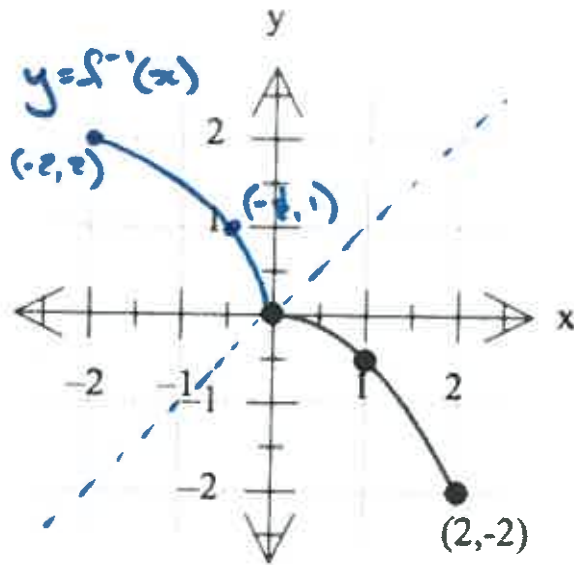
- c. Solve the equation  $|f(x) - 1| = 1$ .



$x = -2, \frac{1}{2}, \frac{3}{2}$  ✓ solutions

7. (7 marks: 2, 3, 2)

The graph of the function  $f(x) = -\frac{1}{2}x^2$ ,  $0 \leq x \leq 2$ , is shown below.



✓ correct points/  
reflection

✓ curve

- Sketch the graph of  $y = f^{-1}(x)$  on the axes with  $f(x)$  above.
- Determine the defining rule for  $y = f^{-1}(x)$  and state its domain.

$$\begin{aligned}
 y &= -\frac{1}{2}x^2 \\
 x &= -\frac{1}{2}y^2 \\
 -2x &= y^2 \quad \checkmark \text{ process} \\
 y &= \pm \sqrt{-2x} \\
 &\text{only + required}
 \end{aligned}$$

$$\therefore f^{-1}(x) = \sqrt{-2x} \quad \checkmark \quad D \{x: -2 \leq x \leq 0\} \quad \checkmark$$

c. Solve:

i.  $f(x) = -\frac{3}{2}$

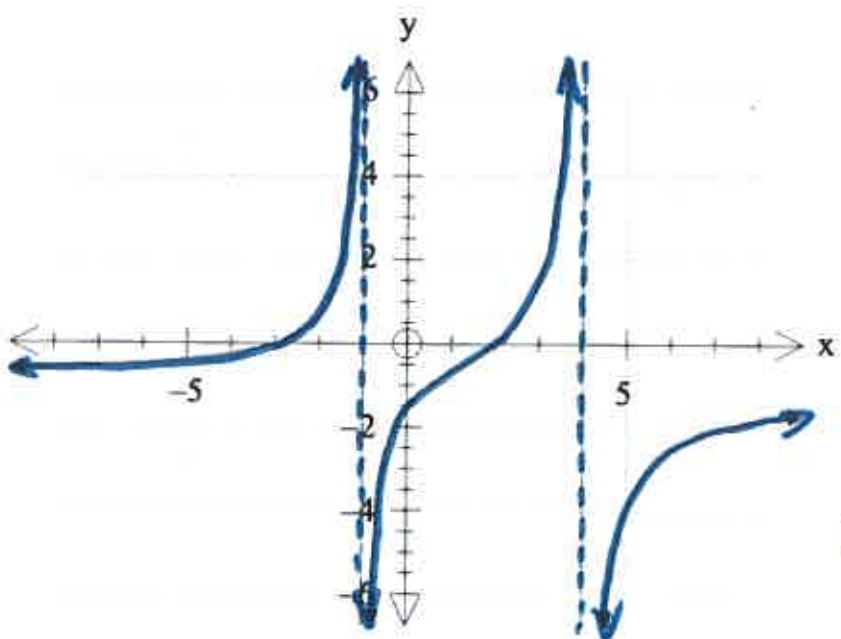
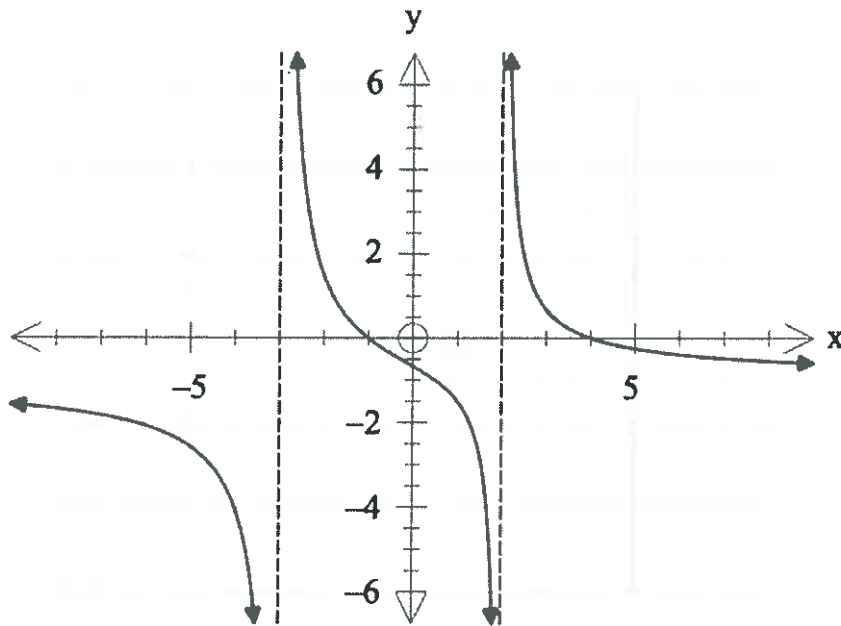
$$x = \sqrt{3} \quad \checkmark$$

ii.  $f^{-1}(x) = 1$

$$x = -\frac{1}{2} \quad \checkmark$$

8. (5 marks)

The graph of  $y = g(x)$  is shown below. It has asymptotes at  $x = -3, x = 2, y = -1$ . On the next set of axes draw the graph of  $y = \frac{1}{g(x)}$ , clearly showing any roots and asymptotes.



roots @  $x = -3, 2$  ✓

asymptotes at  $x = -1, 4$  and  $y = -1$  ✓

behaviour as  $x \rightarrow \pm\infty$

$y \rightarrow \pm\infty$  ✓

End of Section Two