Name: ______________________________

Calculator Free	/28	%
Calculator Assumed	/28	%
Total	/56	%

Mathematics Methods, Year 12, 2018

Test 4 – Logarithmic functions and calculus involving logarithmic functions.

30 ~~25~~ minutes working time.

Calculator Free Section (no notes, no calculators)

SCSA Formula sheet allowed

Question 1 **[6 marks: 1, 2, 3]**

Solve each of the following equations for x.

a) $log_2 x = 5$

$x = 2^5$

$= 32$ ✓

b) $ln(3x + 1) = 0$

$e^0 = 3x + 1$ ✓

$3x = 0$

$x = 0$ ✓

c) $\ln(x - 1) + \ln(x + 2) = \ln(6x - 8)$

$\ln(x-1)(x+2) = \ln 6x - 8$

$(x-1)(x+2) = 6x - 8$ ✓

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$ ✓

$x = 3$ or 2 ✓

Question 2 **[5 marks: 1, 2, 2]**

Express each of the following as the logarithm of a single term.

a) $log\ 9 - log\ 3$

$= \log \frac{9}{3}$

$= \log 3$ ✓

b) $\frac{1}{2}log36 - \frac{1}{3}log27 - \frac{2}{3}log8$

$= \log 6 - \log 3 - \log 4$

$= \log 2 - \log 4$ ✓

$= \log \frac{1}{2}$

$= -\log 2$ ✓

c) $2\log 5 + 2\log 2 + 1$

$= \log 25 + \log 4 + \log 10$

$= \log 100 + \log 10$ ✓

$= \log 1000$

$= 3$ ✓

Question 3 **[4 marks]**

For the following function, sketch the graph, labelling axis intercepts and asymptotes.

a) $y = log_4(x+4)+2$

Asymptote equation $x=-4$.

x intercept, $y=0$

$\therefore 0 = \log_4(x+4)+2$

$-2 = \log_4(x+4)$

$4^{-2} = x+4$

$x = -3^{15}/16$

$\therefore x$ Intercept $(-3\frac{15}{16}, 0)$ ✓

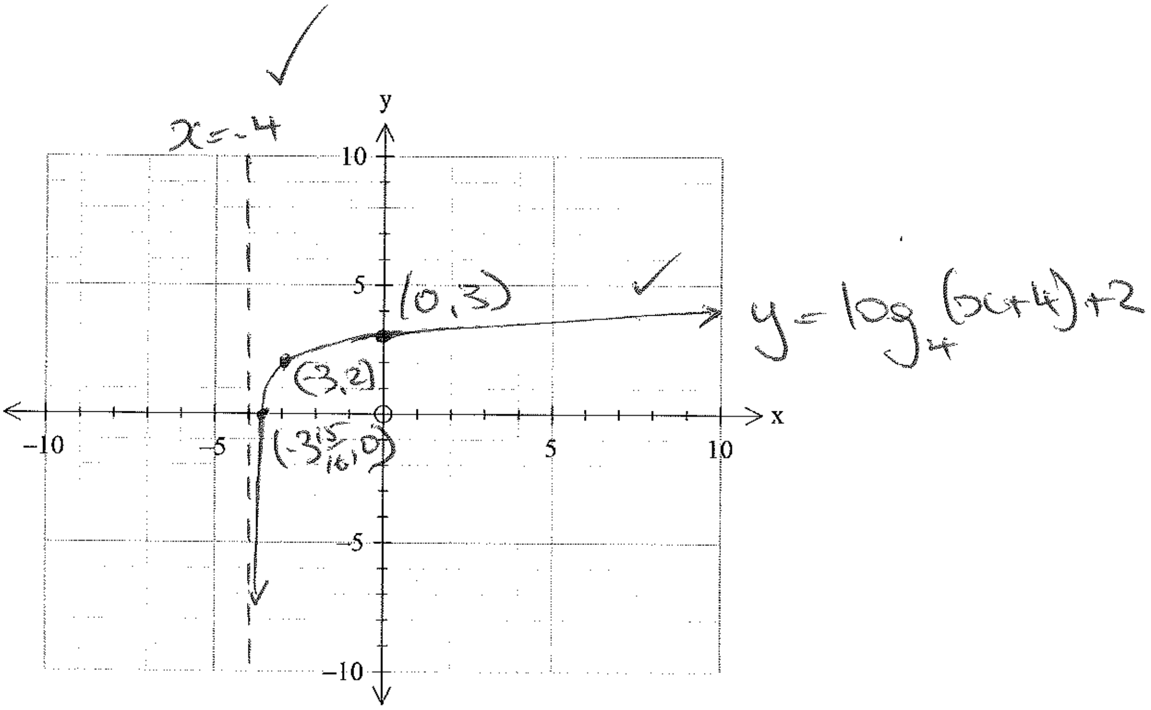
y intercept, $x=0$

$y = \log_4(0+4)+2$

$= \log_4 4 + 2$

$= 3$

$\therefore y$ intercept $(0,3)$ ✓



Question 4 **[7 marks: 1, 3, 3]**

Use common logarithms to solve for t.

a) $4^t = 17$

$\log 4^t = \log 17$

$t \log 4 = \log 17$

$t = \dfrac{\log 17}{\log 4}$ ✓

b) $3^t = 5^{1+2t}$

$\log 3^t = \log 5^{1+2t}$

$t \log 3 = (1+2t)\log 5$

$t \log 3 = \log 5 + 2t \log 5$ ✓

$t \log 3 - 2t \log 5 = \log 5$

$t(\log 3 - 2\log 5) = \log 5$ ✓

$t = \dfrac{\log 5}{\log 3 - 2\log 5}$ ✓

c) $3^{2x} + 3(3^x) - 4 = 0$ ot lee

let $y = 3^x$

$\therefore y^2 + 3y - 4 = 0$ ✓

$(y-1)(y+4) = 0$ ✓

$y = 1$ or -4. no solution for -4.

$\therefore 3^x = 1$

$\log 3^x = \log 1$

$x \log 3 = \log 1$

$x = \dfrac{\log 1}{\log 3}$ ✓

Question 5 **[4 marks: 1, 3]**

Differentiate the following:

a) $y = \ln x^4$

$y = 4 \ln x$

$= \frac{4}{x}$ ✓

b) $y = ln\frac{1+2x}{1+x^2}$

$= \ln(1+2x) - \ln(1+x^2)$ ✓

$= \frac{2}{1+2x} - \frac{2x}{1+x^2}$ ✓✓

Question 6 **[2 marks]**

~~Fine~~ Find the following integral.

$\int \frac{-\cos x}{\sin x}\,dx \quad = -\int \frac{\cos x}{\sin x}$ ✓

$= -\ln(\sin x) + c$ ✓

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Total	/28	%

Mathematics Methods, Year 12, 2016

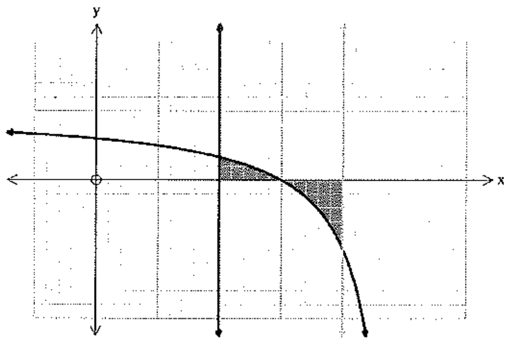
Test 4 – Logarithmic functions, Calculus involving logarithmic functions.

Calculator Assumed Section (notes allowed) 30 ~~25~~ minutes working time.

SCSA Formula sheet and calculators allowed

Question 7 **[3 marks]**

The shaded region shown in the accompanying diagram is trapped by the lines $x = 2\ and\ x = 4$ and curve with equation $y = \frac{2}{x-5} + 1$. Find the exact area of the shaded region.



x intercept (3,0).

$$A = \int_2^3 \frac{2}{x-5} + 1\, dx - \int_3^4 \frac{2}{x-5} + 1\, dx. \checkmark$$

$$= 2\ln\frac{4}{3} \quad \text{or} \quad -2\ln(3) + 4\ln(2).$$

$\checkmark\checkmark$

Question 8 **[4 marks]**

Given that $f(x) = 4x^2 ln\, x^2$. Use calculus techniques to find the exact coordinates of the stationary points on the curve $y = f(x)$

$y = 4x^2 \ln x^2$

$\frac{dy}{dx} = 8x \ln x^2 + 8x$ ✓

For stationary points.

$8x \ln x^2 + 8x = 0$

$x = 0, \pm e^{-1/2}$ ✓

$x \neq 0$ ✓

$\therefore$ Stationary points are

$(\pm e^{-1/2}, -4e^{-1})$ ✓

Question 9 **[5 marks]**

A particle moves in a straight line with acceleration (a) given by

$a = \frac{8}{(t+3)^2}\ m/s^2$

If the particle's initial displacement, from rest, is zero, find, correct to one decimal place, the displacement at $t = 10\ seconds$.

$$a = \frac{8}{(t+3)^2}$$

$$v = \int \frac{8}{(t+3)^2}\,dt \checkmark$$

$$= -\frac{8}{t+3} + c.$$

$$c = \frac{8}{3} \checkmark$$

$$s = \int \frac{-8}{t+3} + \frac{8}{3}\,dt.$$

$$= -8\ln(t+3) + \frac{8}{3}t + K \checkmark$$

$$s=0,\ t=0 \Rightarrow 0 = -8\ln 3 + K.$$

$$K = 8\ln 3. \checkmark$$

$$s = -8\ln(t+3) + \frac{8}{3}t + 8\ln 3.$$

$$s(10) = -8\ln 13 + \frac{80}{3} + 8\ln 3$$

$$= 14.9 \text{ metres.} \checkmark$$

Question 10 **[9 marks: 2, 2, 2, 3]**

Laura is starting a new fitness routine and she completes 2 sets of 5 repetitions of squats each day. Her aim is to get stronger and lift heavier each day.

Laura models her progress over t days by the function $f(t) = 5 + k\,ln(t + 10)$, where $f\ (t)$ is the weight in kilograms of her squat each day.

a) Calculate the value of k if initially Laura lifts 30 kg.

$30 = 5 + k\ln(10)$ ✓

$k = 10.86.$ ✓

b) After 2 weeks of training, by how many kilograms has her strength increased?

$f(14) = 5 + 10.86\ln 24 = 39.51$ ✓

$3951 - 30 = 9.51\,\text{kg}$ ✓

c) Calculate the rate of change of Laura's strength with respect to time.

$$f'(t) = \frac{10.86}{t+10}$$ ✓ ✓

d) Determine when Laura's increase in strength is half of what it was initially.

$$f'(0) = \frac{10.86}{10} = 1.086\,\text{kg/day}$$ ✓

$$0.543 = \frac{10.86}{t+10}$$ ✓

$t = 10$ ✓

Question 11 **[7 marks: 3, 2, 2]**

The instantaneous rate of change of the number of fish over t weeks, being farmed in a fish farm can be modeled by $P'(t) = \frac{-4970}{t+e}$ where $P(t)$ is the population after t weeks.

a) If after 5 weeks there are 12 000 fish left, determine an expression for $P(t)$.

$P(t) = -4970 \ln(t+e) + c$ ✓

$12000 = -4970 \ln(5+e) + c$

$c = 22156.65$ ✓

$P(t) = -4970 \ln(t+e) + 22156.65$ ✓

b) Calculate the initial number of fish when the study began.

$P(0) = 17186.65$ ✓✓

c) When the decline in fish each week falls below 500, the farmer is no longer as concerned for his fish stock. During which week does this occur?

$-500 > \frac{-4970}{t+e}$ ✓

$t = 7.22$

During the 8th week. ✓