HOLY CROSS COLLEGE

12 PHYSICS ATAR
TEST 5 - GRAVITATION

NAME: SOLUTIONS   MARK: —/47

1. An asteroid of mass 2.55 x 10^{22} kg and diameter of 772 km orbits the sun with a speed of 17.9 kms^{-1}.

(a) Calculate the gravitational field strength at any point on the surface of the asteroid?

[4 marks]

$$g = \frac{GM}{r^2} \quad (1)$$

$$= \frac{(6.67\times10^{-11})(2.55\times10^{22})}{(3.86\times10^{5})^2} \quad (1)$$

$$= \underline{11.4\ ms^{-2}} \quad (1) \qquad [\text{Units - 1 mark}]$$

(b) Calculate the orbital radius of the asteroid. [3 marks]

$$F_g = F_c$$

$$\Rightarrow \frac{GM_s \cancel{M_A}}{r^{\cancel{2}}} = \frac{\cancel{M_A} v^2}{\cancel{r}}$$

$$\Rightarrow r = \frac{GM_s}{v^2} \quad (1)$$

$$= \frac{(6.67\times10^{-11})(1.99\times10^{30})}{(17.9\times10^{3})^2} \quad (1)$$

$$= \underline{4.14\times10^{11}\ m} \quad (1)$$

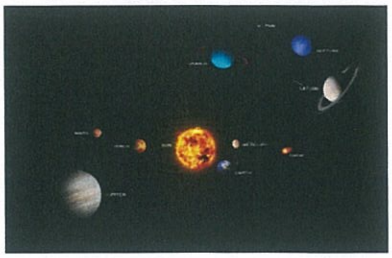
(c) With what force does the asteroid attract the sun? [4 marks]

$$F = \frac{GM_sM_A}{r^2} \quad (1)$$

$$= \frac{(6.67\times10^{-11})(1.99\times10^{30})(2.55\times10^{22})}{(4.14\times10^{11})^2} \quad (1)$$

$$= \underline{\underset{(1)}{1.98\times10^{19}\ N}\ \underset{(1)}{\text{towards the asteroid}}}.$$

2. The solar system consists of a number of planets in approximately circular orbits around the sun. The quotient, r^3/T^2, for each planet has the same value.



(a) Show, by using algebraic manipulation of the equations learned in class, that the relationship r^3/T^2, is a constant value. [4 marks]

$$F_g = F_c \quad (1)$$

$$\Rightarrow \frac{GM_s M_p}{r^2} = \frac{M_p v^2}{r} \quad (1)$$

$$\Rightarrow \frac{GM_s}{r^2} = \frac{4\pi^2 r}{T^2} \quad (1)$$

$$\Rightarrow \frac{r^3}{T^2} = \frac{GM_s}{4\pi^2} \quad (1)$$

$$= \text{constant}.$$

(b) What is the numerical value of this constant r^3/T^2? [3 marks]

$$\frac{r^3}{T^2} = \frac{GM_s}{4\pi^2}$$

$$= \frac{(6.67\times10^{-11})(1.99\times10^{30})}{4\pi^2} \quad (1)$$

$$= \underline{3.36\times10^{18}\ m^3 s^{-2}}$$

(1) (1)

(c) Mercury takes 88 days to orbit the sun, while Venus takes 225 days. Calculate the maximum distance that could ever exist between Mercury and Venus. (If you did not calculate a value in part (b), use a value of 4×10^{18}.)

[5 marks]

MERCURY $\quad r^3 = \text{constant} \times T^2$

$= (3.36 \times 10^{18})(88 \times 24.0 \times 3.60 \times 10^3)^2$ (1)

$= 1.94 \times 10^{32}$

$\Rightarrow r = 5.79 \times 10^{10}\ \text{m}$ (1)

VENUS $\quad r^3 = (3.36 \times 10^{18})(225 \times 24.0 \times 3.60 \times 10^3)^2$ (1)

$= 1.27 \times 10^{33}$

$\Rightarrow r = 1.08 \times 10^{11}\ \text{m}$ (1)

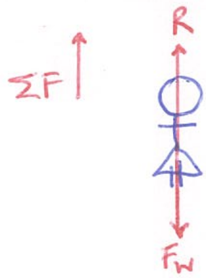
Max distance apart $= r_M + r_V$

$= 5.79 \times 10^{10} + 1.08 \times 10^{11}$

$= \underline{1.66 \times 10^{11}\ \text{m}}$ (1)

3. Just after lift-off a space shuttle rocket is accelerating vertically upwards. An astronaut inside states that she feels heavier.

(a) Explain, in terms of the forces acting on her, why she feels heavier. [3 marks]



$\Sigma F = R - F_W$

$\Rightarrow R = \Sigma F + F_W$ (1)

$= ma + mg$ (1)

$\therefore$ <u>She feels heavier than F_W by ΣF.</u> (1)

(b) As the shuttle continues to accelerate vertically upwards, at the same rate, she notices she feels her weight decreasing. Explain why. [3 marks]

- $g = \frac{GM_E}{r^2}$ (1)
- As r increases, g decreases. (1)
- $R = ma + mg$ ⇒ Her weight (mg) is decreasing. (1)

(c) The space shuttle launches from Cape Canaveral, Florida, USA. This is the location on mainland USA, closest to the equator. Explain how this might assist with the launch. [3 marks]

- Equator has maximum rotational speed. (1)
- The shuttle has some sideways speed due to this spin. (1)
- Less energy / fuel required to reach orbit. (1)

4. Callisto is the largest moon orbiting Jupiter. Callisto takes 16 days to complete each orbit, at a distance of 1.88 x 10^9 metres from the centre of Jupiter. Use this data to calculate the mass of Jupiter. [4 marks]

$$r^3 = \frac{GM_J T^2}{4\pi^2}$$

$$\Rightarrow M_J = \frac{4\pi^2 r^3}{GT^2} \quad (1)$$

$$= \frac{4\pi^2 (1.88\times10^9)^3}{(6.67\times10^{-11})(16\times24.0\times3.60\times10^3)^2} \quad (1) \quad (1)$$

$$= \underline{2.06\times10^{27}}\ kg \quad (1)$$

5. The planet Mercury has a radius of 1.30 x 10^6 m, a mass of 3.30 x 10^{23} kg and its day is 58.65 Earth days. A 25.0 kg satellite is positioned into a geostationary orbit. How high above the surface of Mercury is the satellite orbiting? [6 marks]

$$r^3 = \frac{GM_M T^2}{4\pi^2} \quad (1)$$

$$= \frac{(6.67\times10^{-11})(3.30\times10^{23})(58.65\times24.0\times3.60\times10^3)^2}{4\pi^2} \quad (1)$$

$$= 1.43\times10^{25} \quad (1)$$

$$\Rightarrow r = 2.43\times10^8\ m \quad (1)$$

$$r = r_M + h \quad (1)$$

$$\Rightarrow h = 2.43\times10^8 - 1.30\times10^6$$

$$= \underline{2.42\times10^8\ m} \quad (1)$$

6. The table shown below gives astronomical data for a planet currently orbiting the Sun.

Mass: 1.90×10^{27} kg (317.9 Earths)
Radius (equatorial): 71 492 km
Mean density: 1.33 gcm^{-3}
Distance from Sun: 778 330 000 km
Rotational period: 0.4135 days
Orbital Period: 4332.71 days
Apparent magnitude: -2.70
Surface temperature: -121°C (cloud)
Atmospheric composition: hydrogen (90%), helium (10%)

(a) What is the least massive planet in the solar system? [1 mark]

Mercury (1)

(b) Calculate the escape velocity of the planet Jupiter. [4 marks]

Escape velocity occurs when $E_p = E_K$

$$\Rightarrow \cancel{m}gh = \frac{1}{2}\cancel{m}v^2 \quad (1)$$

$$\Rightarrow v = \sqrt{2gh}$$

$$= \sqrt{\frac{2GM_J \cancel{r}}{r^{\cancel{2}}}} \quad (1)$$

$$= \sqrt{\frac{2(6.67 \times 10^{-11})(1.90 \times 10^{27})}{(71492 \times 10^{3})}} \quad (1)$$

$$= \underline{5.95 \times 10^{4} \ ms^{-1}} \quad (1)$$