

## **Topic: Statistical Calculations**

Time: 45 mins Marks: /45 marks

**Calculator Assumed** 

#### **Question One: [2, 2, 2, 3: 9 marks]**

Suppose the scores a, b, c, d, e and f are six positive real numbers with a mean of 5.3 and a standard deviation of 0.9.

- a) Determine the mean and standard deviation if:
  - i) each score is reduced by 2
  - ii) each score is multiplied by -4
  - iii) each of the scores are changed as follows 3a + 2, 3b + 2, 3c + 2, 3d + 2, 3e + 2, 3f + 2
- b) Determine the mean of a, b + 3, c 8, d + 1, e, f.

## Question Two: [2, 2: 4 marks]

The following set of 12 positive integers is arranged in ascending order and has a mean of 9.

a) Determine an expression for all possible values for m and n.

b) Determine the smallest possible value for the range.

## **Question Three: [4, 4: 8 marks]**

Abidina is calculating her end of year score. She has her results from her 5 tests, 5 investigations, and 2 exams. Her tests are each worth 5% of her final mark and the investigations are worth 7% each. Her first exam is worth 15% and her second exam is worth 25%.

Test scores						
1 2 3 4 5						
60%	65%	70%	45%	78%		

Investigation scores						
1 2 3 4 5						
80%	55%	65%	78%	70%		

Exam scores					
1 2					
67%	79%				

a) Calculate Abidina's end of year mark.

Harry is in Abidina's class. Harry's end of year mark was 65%.

He has kept all of his assessments from the year but cannot find his first exam.

b) If all of Harry's marks are listed below, calculate Harry's missing exam score.

Test scores						
1 2 3 4 5						
61%	65%	60%	35%	80%		

Investigation scores						
1 2 3 4 5						
70% 75% 55% 68% 76%						

Exam scores					
1	2				
	60%				

#### Question Four: [3, 4: 7 marks]

There are 200 people in an athletics squad. Out of the 200 people, some attend athletics on a Monday, some go on Tuesday and the rest go on Wednesdays.

All of the athletes run the 100m and their best time is recorded.

The 50 runners on Monday achieved an average time of 14 seconds. The 70 runners on Tuesday have an average time of 12.9 seconds and Wednesday's 80 runners have an average time of 13.5 seconds.

a) Calculate the average 100m running speed for the entire athletics squad.

Four new runners join the club and they are all training for the next Olympic Games. When the best average 100m time for these four athletes is included in the statistics for the entire club, the overall mean is reduced by 0.015 seconds.

b) Calculate the average 100m time for these four athletes.

#### Question Five: [3, 2: 5 marks]

Two classes sit the same test and the combined mean is 72%. One class has 50 students in it and the class mean is 69%.

a) How many students are in the combined group if the second class has a class mean of 78%?

Two students from the second class were absent on the day of the test and they sat the test on the next day.

b) If these two students scored 73% and 65% what is the new combined mean?

#### **Question Six: [4 marks]**

A farmer is trying to determine whether or not a new fertiliser is increasing the growth of his crops.

He has two separate fields where he is growing crops. He uses the new fertiliser on only one of the fields and not the other. He records the heights of the crops and analyses the statistics.

The heights of the 10 crops in metres are shown in the tables below.

#### Field A

1	2	3	4	5	6	7	8	9	10
5	8	12	14	16	6	5	8	18	6

#### Field B

1	2	3	4	5	6	7	8	9	10
5	6	5	7	8	7	8	5	4	6

The farmer concludes that the fertiliser does work but that it is not consistent.

Use statistical measures for reasoning to back-up the farmer's conclusion and state which field would be the one which received the new fertiliser.

## **Question Seven: [4 marks]**

The following is a box-plot representing scores 1, 1, a, 3, b, 5, c, d



Determine the values of a, b, c and d.

# **Question Eight: [4 marks]**

A set of 5 scores has a minimum of -2, a range of 10, median and mode of 5 and mean of 3.8. Determine the 5 scores.



# **Topic: Statistical Calculations SOLUTIONS**

Time: 45 mins Marks: /45 marks

**Calculator Assumed** 

#### **Question One: [2, 2, 2, 3: 9 marks]**

Suppose the scores a, b, c, d, e and f are six positive real numbers with a mean of 5.3 and a standard deviation of 0.9.

- a) Determine the mean and standard deviation if:
  - i) each score is reduced by 2

$$\bar{x} = 3.3 \checkmark \qquad \sigma = 0.9 \checkmark$$

ii) each score is multiplied by -4

$$\bar{x} = -21.2 \checkmark \qquad \sigma = 3.6 \checkmark$$

iii) each of the scores are changed as follows 3a + 2, 3b + 2, 3c + 2, 3d + 2, 3e + 2, 3f + 2

$$\bar{x} = 17.9 \checkmark \qquad \sigma = 2.7 \checkmark$$

b) Determine the mean of a, b + 3, c - 8, d + 1, e, f.

$$\frac{a+b+3+c-8+d+1+e+f}{6}$$

$$=\frac{(a+b+c+d+e+f)-4}{6} = \frac{31.8-4}{6} = 4.63$$

#### Question Two: [2, 2: 4 marks]

The following set of 12 positive integers is arranged in ascending order and has a mean of 9.

a) Determine an expression for all possible values for m and n.

$$m + n = 108 - 63 \checkmark$$

$$m + n = 45 \checkmark$$

b) Determine the smallest possible value for the range.

$$42 - 3 = 39$$

#### **Question Three: [4, 4: 8 marks]**

Abidina is calculating her end of year score. She has her results from her 5 tests, 5 investigations, and 2 exams. Her tests are each worth 5% of her final mark and the investigations are worth 7% each. Her first exam is worth 15% and her second exam is worth 25%.

Test scores						
1 2 3 4 5						
60%	65%	70%	45%	78%		

Investigation scores						
1 2 3 4 5						
80%	55%	65%	78%	70%		

Exam scores					
1 2					
67%	79%				

#### a) Calculate Abidina's end of year mark.

$$[0.05 \times 60 + 0.05 \times 65 + 0.05 \times 70 + 0.05 \times 45 + 0.05 \times 78]$$

$$+[0.07 \times 80 + 0.07 \times 55 + 0.07 \times 65 + 0.07 \times 78 + 0.07 \times 70]$$

$$+[0.15 \times 67 + \times 0.25 \times 79]$$

$$= 70.09\%$$

Harry is in Abidina's class. Harry's end of year mark was 65%.

He has kept all of his assessments from the year but cannot find his first exam.

b) If all of Harry's marks are listed below, calculate Harry's missing exam score.

Test scores						
1 2 3 4 5						
61%	65%	60%	35%	80%		

Investigation scores						
1	2	3	4	5		
70%	75%	55%	68%	76%		

Exam scores					
1	2				
	60%				

$$[0.05 \times 61 + 0.05 \times 65 + 0.05 \times 60 + 0.05 \times 35 + 0.05 \times 80]$$
  $\checkmark$   $+[0.07 \times 70 + 0.07 \times 75 + 0.07 \times 55 + 0.07 \times 68 + 0.07 \times 76]$   $\checkmark$   $+[0.15 \times x + \times 0.25 \times 60]$   $\checkmark$   $= 65\%$   $Semester 2 exam = 72.5\%$   $\checkmark$ 

#### Question Four: [3, 4: 7 marks]

There are 200 people in an athletics squad. Out of the 200 people, some attend athletics on a Monday, some go on Tuesday and the rest go on Wednesdays.

All of the athletes run the 100m and their best time is recorded.

The 50 runners on Monday achieved an average time of 14 seconds. The 70 runners on Tuesday have an average time of 12.9 seconds and Wednesday's 80 runners have an average time of 13.5 seconds.

a) Calculate the average 100m running speed for the entire athletics squad.

$$\frac{50 \times 14 + 70 \times 12.9 + 80 \times 13.5}{200}$$

$$= 13.415 seconds$$

Four new runners join the club and they are all training for the next Olympic Games. When the best average 100m time for these four athletes is included in the statistics for the entire club, the overall mean is reduced by 0.015 seconds.

b) Calculate the average 100m time for these four athletes.

$$13.40 = \frac{13.415 \times 200 + a + b + c + d}{204}$$

$$a + b + c + d = 50.6 \text{ seconds} \checkmark$$

$$average = \frac{50.6}{4} = 12.65 \text{ seconds} \checkmark$$

#### Question Five: [3, 2: 5 marks]

Two classes sit the same test and the combined mean is 72%. One class has 50 students in it and the class mean is 69%.

a) How many students are in the combined group if the second class has a class mean of 78%?

$$72 = \frac{50 \times 69 + x \times 78}{50 + x}$$

$$x = 25$$

$$total number of students = 75 students \checkmark$$

Two students from the second class were absent on the day of the test and they sat the test on the next day.

b) If these two students scored 73% and 65% what is the new combined mean?

$$\frac{[50 \times 69 + 25 \times 78] + [73 + 65]}{77}$$
= 71.92%  $\checkmark$ 

#### **Question Six: [4 marks]**

A farmer is trying to determine whether or not a new fertiliser is increasing the growth of his crops.

He has two separate fields where he is growing crops. He uses the new fertiliser on only one of the fields and not the other. He records the heights of the crops and analyses the statistics.

The heights of the 10 crops in metres are shown in the tables below.

#### Field A

1	2	3	4	5	6	7	8	9	10
5	8	12	14	16	6	5	8	18	6

#### Field B

1	2	3	4	5	6	7	8	9	10
5	6	5	7	8	7	8	5	4	6

The farmer concludes that the fertiliser does work but that it is not consistent.

Use statistical measures for reasoning to back up the farmer's conclusion and state which field would be the one which received the new fertiliser.

$$A: \bar{x} = 9.8 m$$

$$\sigma_x = 4.58 m$$

$$B: \bar{x} = 6.1 m$$

$$\sigma_x = 1.3 m$$

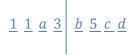
The fertiliser is likely to be used for field A since the mean height is much larger but the standard deviation indicates the inconsistency mentioned by the farmer.

#### **Question Seven: [4 marks]**

The following is a box-plot representing scores 1, 1, a, 3, b, 5, c, d



Determine the values of a, b, c and d.



d = 9

$$\frac{3+b}{2} = 4 \qquad \frac{1+a}{2} = 3$$

$$\frac{1+a}{2} = 3$$

$$\frac{5+c}{2}=6$$

$$b = 5$$
  $\checkmark$   $a = 3$   $\checkmark$ 

$$a = 3$$

$$c = 7$$

## **Question Eight: [4 marks]**

A set of 5 scores has a minimum of -2, a range of 10, median and mode of 5 and mean of 3.8. Determine the 5 scores.

$$-2 \quad 3 \quad 5 \quad 5 \quad 8$$