TEST 2 2022

CALC FREE /21 20 mins

QUESTION ONE (4 MARKS)

Determine the area of the region bounded by $y = x^2 - x - 3$ and $y = x - 3$.

QUESTION TWO (5 MARKS)

Determine the $x -$ coordinate(s) of the stationary point(s) for $y = \frac{e^x}{\sin x}$ over the domain $0 \leq x \leq \pi$.

QUESTION THREE (3, 1, 3 = 7 MARKS)

Consider $f'(x) = \cos x \sin^2 x$.

a) Determine $f''\left(\frac{\pi}{6}\right)$.

b) Explain the significance of your answer in part (a).

c) Determine $f(x)$ given that it passes through the point $\left(\frac{\pi}{6}, \frac{1}{2}\right)$.

QUESTION FOUR (1, 2, 2 = 5 MARKS)

The table below shows the values of $f(x)$ and $g(x)$, and their derivatives $f'(x)$ and $g'(x)$ respectively, when $x = 1, x = 2, x = 3$ and $x = 4$.

	$x = 1$	$x = 2$	$x = 3$	$x = 4$
$f(x)$	-1	0	3	8
$g(x)$	1	3	4.5	5.5
$f'(x)$	0	2	4	6
$g'(x)$	2	2	1	1

Determine:

a) $\int_1^4 g'(x)\,dx$

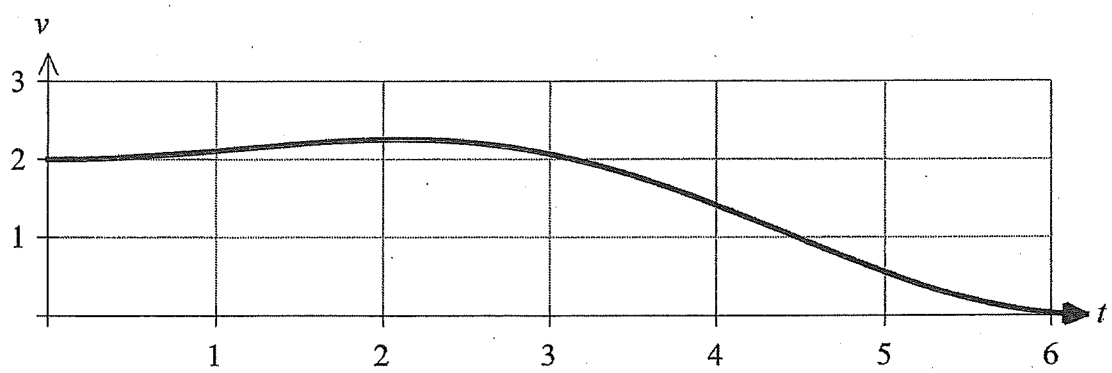
b) $\int_2^3 f'(x) - 1\,dx$

c) $\frac{d}{dx}\int_1^3 2f'(x)\,dx$

CALC ASS /29 30 mins

QUESTION FIVE (2, 3, 1, 2 = 8 MARKS)

The speed of an object in metres per second can be modelled by the equation $v = 1 + cos(0.5t) + sin^2(0.5t)$ where t represents the time in seconds and its graph is as shown below.



The area under the curve for any time interval represents the distance travelled by the object.

a) Complete the table below, rounding to two decimal places.

t	0	2	4	6
v	2			0.03

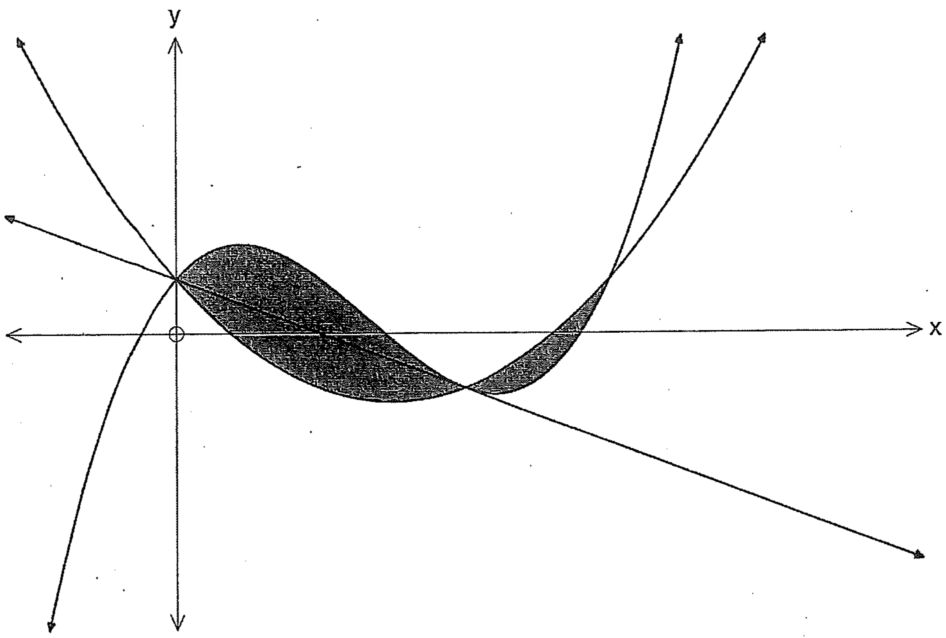
b) Estimate the distance travelled by the object during the first six seconds by calculating the circumscribed areas using three rectangular strips of width 2 seconds (i.e. by overestimation).

c) Suggest one way to improve the accuracy of the estimation.

d) Determine the actual distance travelled in the first six seconds.

QUESTION SIX (4 MARKS)

Two curves with equations $f(x) = x^3 - 4x^2 + 3x + 1$ and $g(x) = x^2 - 3x + 1$ intersect as shown below. The line passing through the points of intersection of the curves has the equation $h(x) = 1 - x$. Determine the fraction of the shaded area which lies above the line $h(x) = 1 - x$.



QUESTION SEVEN (2, 1, 1, 1, 2, 2 = 9 MARKS)

A group of medical researchers is investigating the spread of a virus in a province by keeping track of P, the population of people infected, t weeks after the virus is first discovered.

Initially, 60 people are infected with the virus. It is suspected that P and t are related by the differential equation:

$\frac{dP}{dt} = -6e^{-\frac{t}{5}} + K$, where K is a constant.

a) Determine an expression for the population of people infected, P, in terms of t and K.

b) In the long term, what happens to the population of infected people when:

i) $K = 1$?

ii) $K = 0$?

c) Consider when $K = 0$,

i) write an integral which can be used to determine the change in the population of people infected in the fifth week.

ii) hence, determine the change in the population of people infected in the fifth week.

d) Given that one week later the population of infected people reduces to half of its initial number, determine when the population of infected people will reduce to zero by first determining the value of K. $t=1$, go back to original equation using $t=1$

QUESTION EIGHT (2, 1, 5 = 8 MARKS)

The volume of a liquid (V in cubic metres) in a mixing tank varies depending on the height of the liquid (h in metres) in the mixing tank and is given by:

$$V = \int_0^h e^{-0.01x^2}\, dx$$

The height of the liquid in the mixing tank depends on time (t in minutes) as follows:

$$h = 2t^2 - t + 4$$

a) Determine $\frac{dV}{dh}$ when the height is 1.5 m.

b) Explain the significance of your answer in part a).

c) Determine $\frac{dV}{dt}$ when the height is 5 m.