Mathematics Applications
YEAR 12

Investigation 2 – Networks

Semester 1 2017

|	
Time allowed:	60 minutes
Marks Available:	56 marks
Materials required:	Writing implements, correction fluid/tape or eraser, ruler, Scientific or CAS calculator

Instructions:

1. Write your answers in the spaces provided in this Question/Answer Booklet.

2. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

A network or graph is a diagram consisting of interconnected points (vertices). It can represent many practical situations such as an electricity supply network or a series of roads connecting towns.
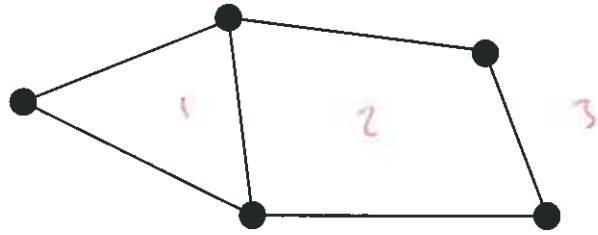
A network will have vertices (points), edges (joining lines) and faces (regions including the outside).

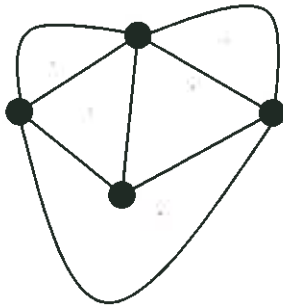The graph above can be described as having: 5 vertices
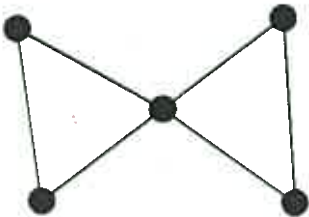
6 edges

3 faces

1. (12 marks)

   Identify the number of vertices, edges and faces in each of these graphs:
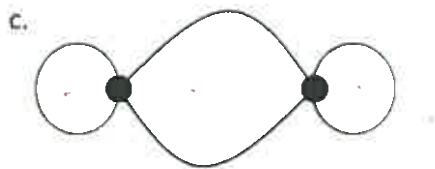
   a.

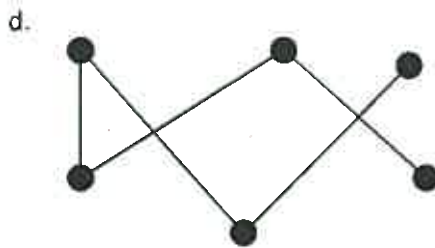   __4__ vertices, __8__ edges and __6__ faces

   b.

   __5__ vertices, __6__ edges and __3__ faces

c.

_2_ vertices, _4_ edges and _4_ faces

d.

_6_ vertices, _5_ edges and _3_ faces

2. (6 marks)
   Draw a network containing:

   a. Four vertices and six edges and state the number of faces that it has.
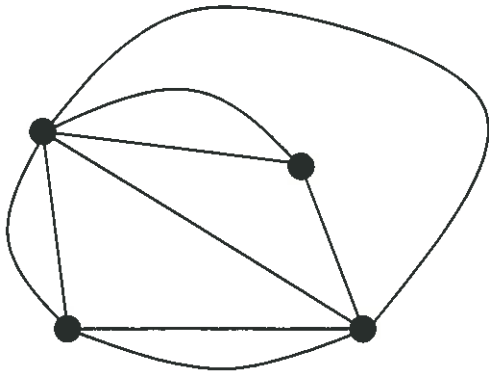
   ✓ 4 vertices

   ✓ six edges

   ✓ faces stated

   b. Seven vertices and ten edges and state the number of faces that it has.

   as above

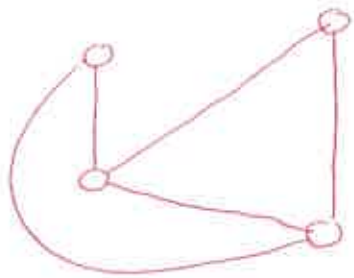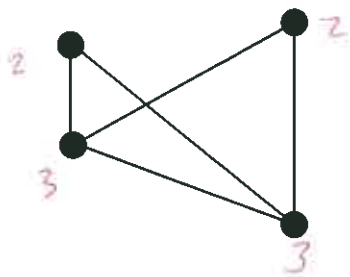A planar graph is one where no edges cross over.

For example:

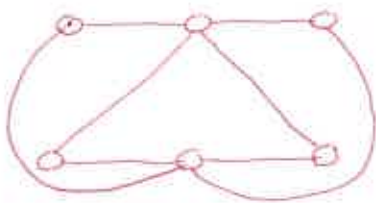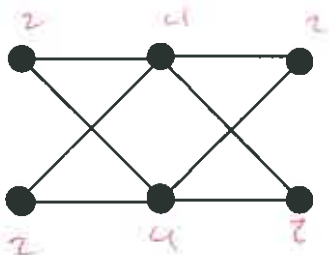Some graphs can be drawn as planar graphs.

3. (6 marks)
   In the space to the right, redraw the following as planar graphs.

   a.

   b.

   c.
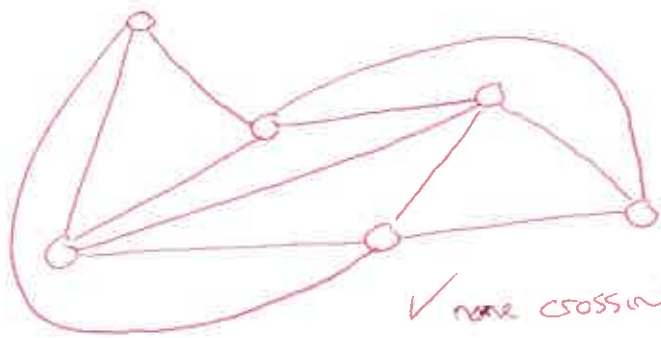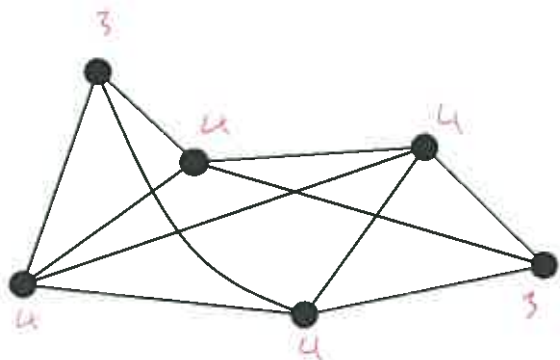

Any accurate
redraw is fine

Ruled lines
not necessary

✓ none crossing
✓ accurate

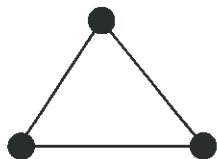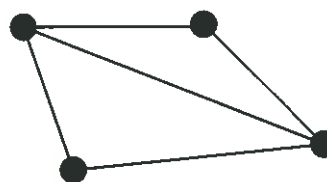✓ none crossing
✓ accurate.

✓ none crossing
✓ accurate

4. (6 marks)

Consider the planar graphs below and complete the given table.

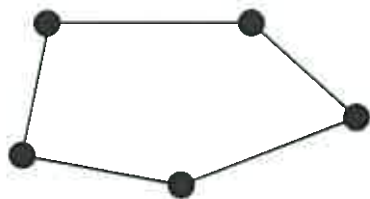a.

d.

b.

e.
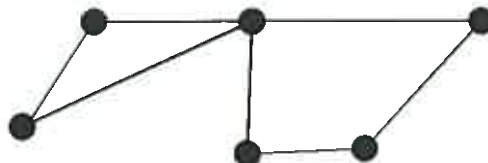
c.

f.

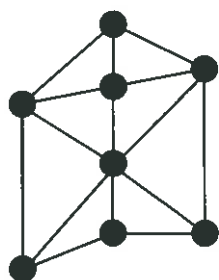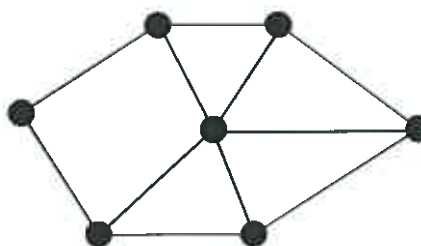|	Vertices	Edges	Faces	
a.	3	3	2	✓
b.	5	5	2	✓
c.	8	15	9	✓
d.	4	5	3	✓
e.	6	7	3	✓
f.	7	11	6	✓

There is a relationship between the number of edges, vertices and faces.

5. (2 marks)
   State this relationship using the pronumerals E, V and F.

   $$E - V = F + 2$$

   or variations ✓²

Networks can be used to solve situations.

Consider this problem:

6. (4 marks)
   The following graph represents a series of one-way streets.
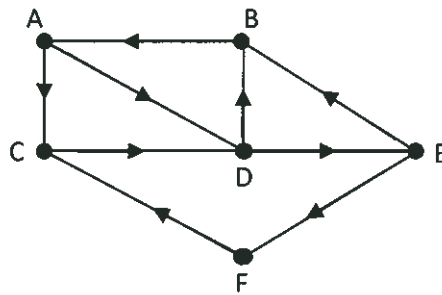
   Rory wants to travel by car from his apartment building at A to the cinemas at F.

   a. Describe a route Rory would have to take to travel to the cinemas by car.

   $$A \Rightarrow D \Rightarrow E \Rightarrow F$$ ✓²

   or alternative.

   b. What is the shortest route, determined by the least number of edges travelled on, for Rory to return home after his trip to the cinemas?

   $$F \Rightarrow C \Rightarrow D \Rightarrow B \Rightarrow A$$ ✓²

A network is an excellent tool for situations such as finding the shortest possible distance between towns, or even planning to lay pipe lines / reticulations in your garden. In this Problem Solving task, your job is to find the shortest possible distances from one place to another and shortest possible distances to connect pipelines:
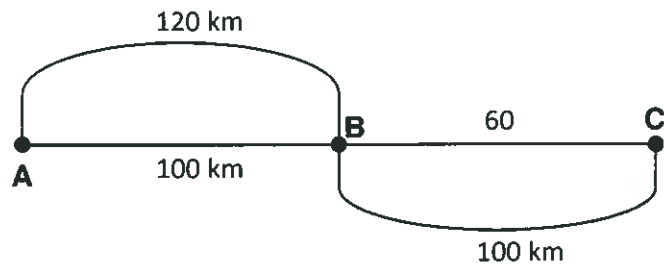
7. (2 marks)
   What is the shortest distance from A to C?

120 km

B   60   C

A   100 km

100 km

Shortest Distances from   A to B = ___100 km___

B to C = ___60___   [1]

Shortest Distance in total   A to C = _160 km_   [1]

8. (3 marks)
   What is the shortest distance from A to G?

   Show your working on the right. Try to use a systematic approach.
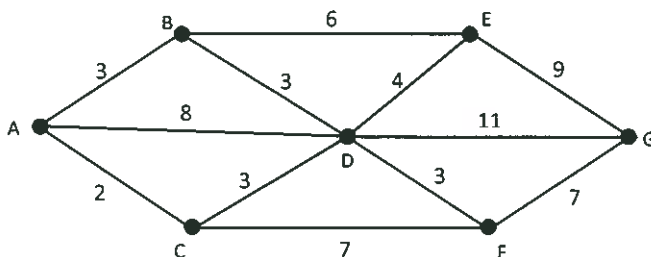
A → B → E → G   18

A → B → D → E → G   19

A → B → D → G   17

A → B → D → F → G   16

A → D → G   19

A → D → F → G   18

A → C → D → G   16

A → C → D → F → G   15   ⟸   shortest distance

A → C → F → G   16

✓ solution

✓₂ systematic approach.

not every option is needed but process is essential (evidence)

9. (4 marks)
    You are planning to travel from Wagin to Northam

    a. What is the shortest possible distance from Wagin to Northam?   [3]
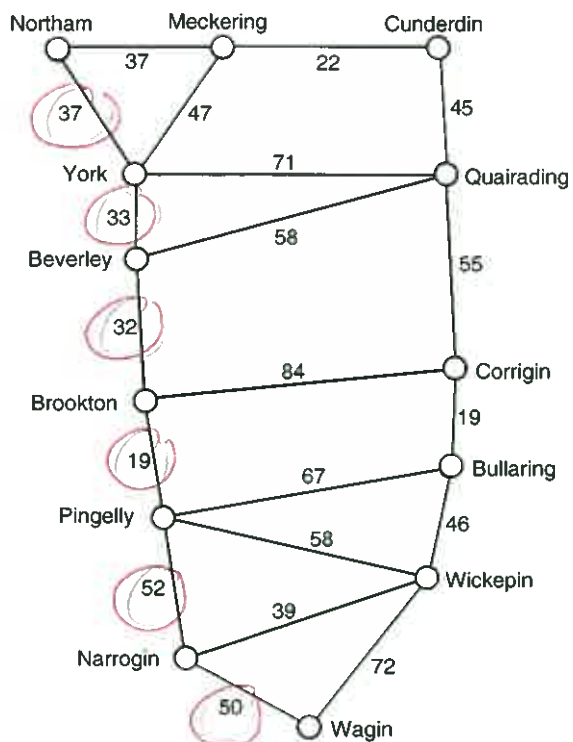
Looking for an answer ✓ and a process ✓✓

Shortest route marked

223 km

    b. Write down all the towns that you have to pass through to get from Wagin to Northam at the
        shortest possible distance.   [1]

pass through

Narrogin
Pingelly
Brookton
Beverley
York

✓

In this section, your task is to find out the shortest possible connections to all the points for the whole system of network shown.
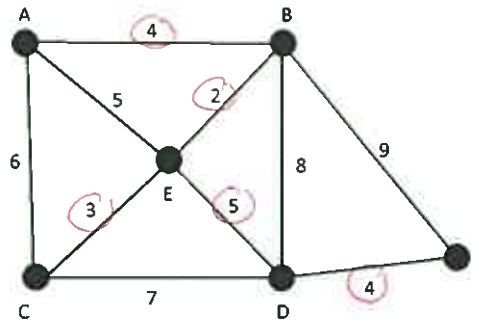
10. (7 marks)

in metres

After making a floor plan for the backyard with all the distances and all the important points for the sprinklers shown below, what is the shortest path to connect ALL the pipes to the different points in the backyard?

Start from A: Which is the closest point A?

Point: B   Distance = 4   [1]

From A and the point just found: Which is the closest point to either point?

Point: E   Distance = 2   [1]

From A and the other points found: Which is the closest point to any of the points?

Point: C   Distance = 3   [1]

From A and the other points found: Which is the closest point to any of the points?

Point: D   Distance = 5   [1]

From A and the other points found: Which is the closest point to any of the points?

Point: F   Distance = 4   [1]

All the points should now be connected. On the diagram, highlight the distances identified above to check that all the points are connected.   [1]

Total minimum length required for laying the pipes = 18 metres   [1]

11. (4 marks)

You are the town planner and you are given the task to find the shortest possible way to connect the gas pipes between the suburbs within your town council. Below is the Network collated by your surveyors.
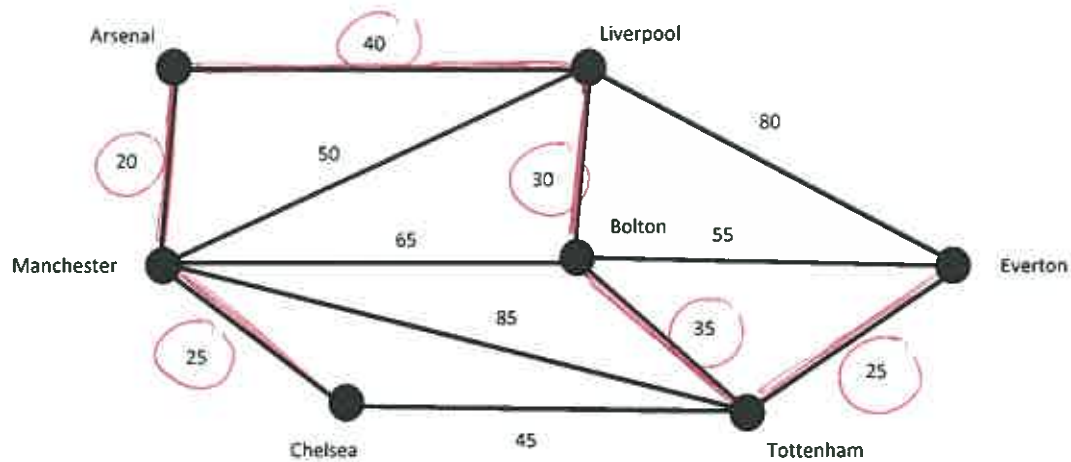
a. Find the shortest distance for the Network. Show your working. [3]

Looking for process.

MST — start at Arsenal

A → M	20
M → C	25
A → L	40
L → B	30
B → T	35
T → E	25

½₂   175 ✓

b. Highlight on the diagram the shortest distances connecting these towns. [1]

✓

End of Investigation