Year 11 Mathematics Specialist

Test – 5.5%

MELVILLE
SENIOR HIGH SCHOOL

Student Name : _____

Part One – Resource Free

Part One contains 6 questions worth 38 marks

Time Allowed : 40 minutes

INSTRUCTIONS TO STUDENTS:
You are required to attempt ALL questions,
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.

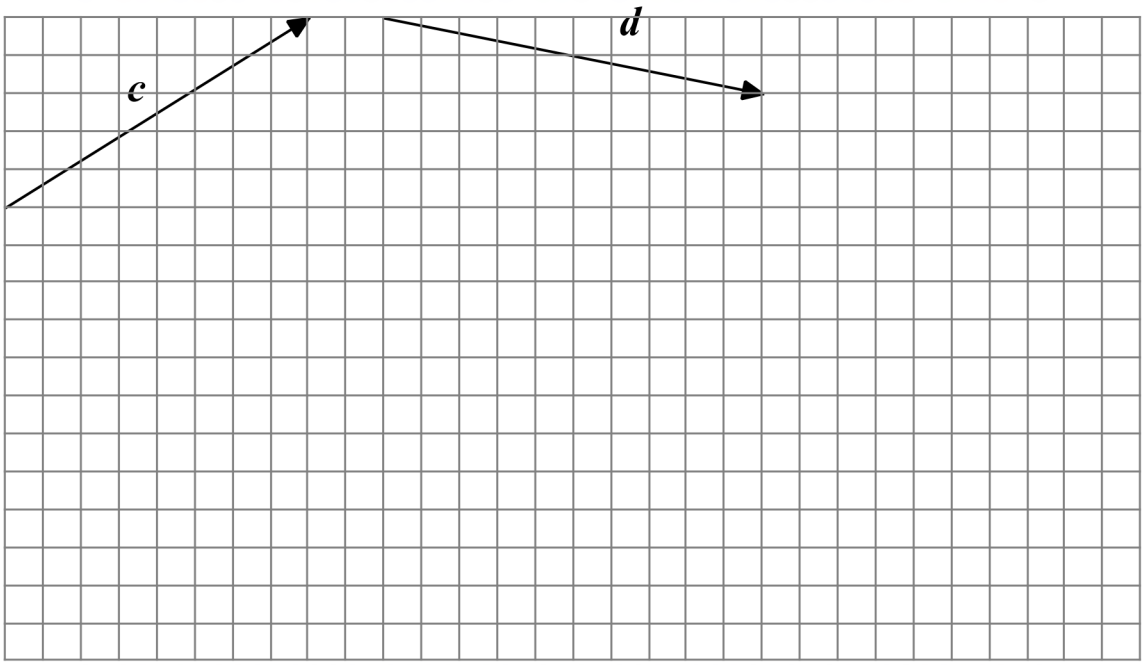
Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.

Question 1 **1, 1, 5 - 7 marks**

Using $\underset{\sim}{c}$ and $\underset{\sim}{d}$ as given on the diagram below, show the correct vector representing.

a) $\underset{\sim}{c}+\underset{\sim}{d}$

b) $\underset{\sim}{c}-\underset{\sim}{d}$



c) Given $\underset{\sim}{p}=-\underset{\sim}{i}+20\underset{\sim}{j}$, express $\underset{\sim}{p}$ in the form $\lambda\underset{\sim}{c}+\mu\underset{\sim}{d}$.

$-\underset{\sim}{i}+20\underset{\sim}{j}=\lambda(8\underset{\sim}{i}+5\underset{\sim}{j})+\mu(10\underset{\sim}{i}-2\underset{\sim}{j})$ ✓

$8\lambda+10\mu=-1$

$5\lambda-2\mu=20$ ✓

$8\lambda+10\mu=-1$

$25\lambda-10\mu=100$

$33\lambda=99$

$\lambda=3$ ✓

$5\times 3-2\mu=20$

$2\mu=-5$

$\mu=-2\frac{1}{2}$ ✓

$\underset{\sim}{p}=3\underset{\sim}{c}-2\frac{1}{2}\underset{\sim}{d}$ ✓

Question 2 — 4 marks

Points A, B and C have position vectors $\underset{\sim}{a} = x\underset{\sim}{i} - \underset{\sim}{j}$, $\underset{\sim}{b} = \underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{c} = 4\underset{\sim}{i} + y\underset{\sim}{j}$ respectively.

Given that AB:BC = 2:1, determine the value(s) of x and y.

$\overrightarrow{AB} =< 1-x, 4 >$ ✓

$\overrightarrow{BC} =< 3, y-3 >$

$2\overrightarrow{BC} = \overrightarrow{AB}$

$2 < 3, y-3 >=< 1-x, 4 >$ ✓

$6 = 1 - x$

$x = -5$ ✓

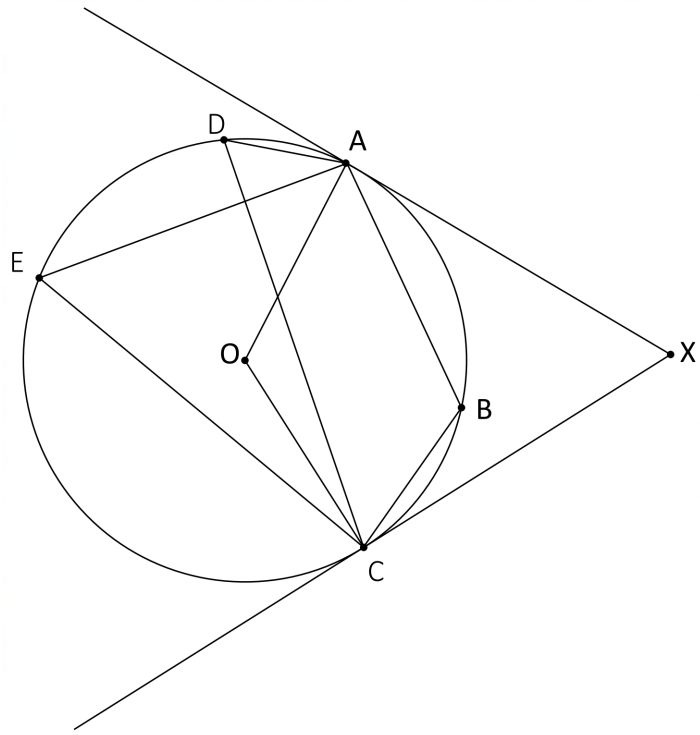
$2y - 6 = 4$

$y = 5$ ✓

Question 3 — 1, 1, 1, 1, 1, 2 – 7 marks

AX and CX are tangents to the circle ABCDE. O is the centre of the circle and the angle OCA = 30°.

Calculate each of the following angles giving reasoning for each.



a) OAC 30° – base angles of isosceles triangle

b) AOC 120° – angle sum of triangle

c) ADC 60° – angle in the arc is half the angle at the centre

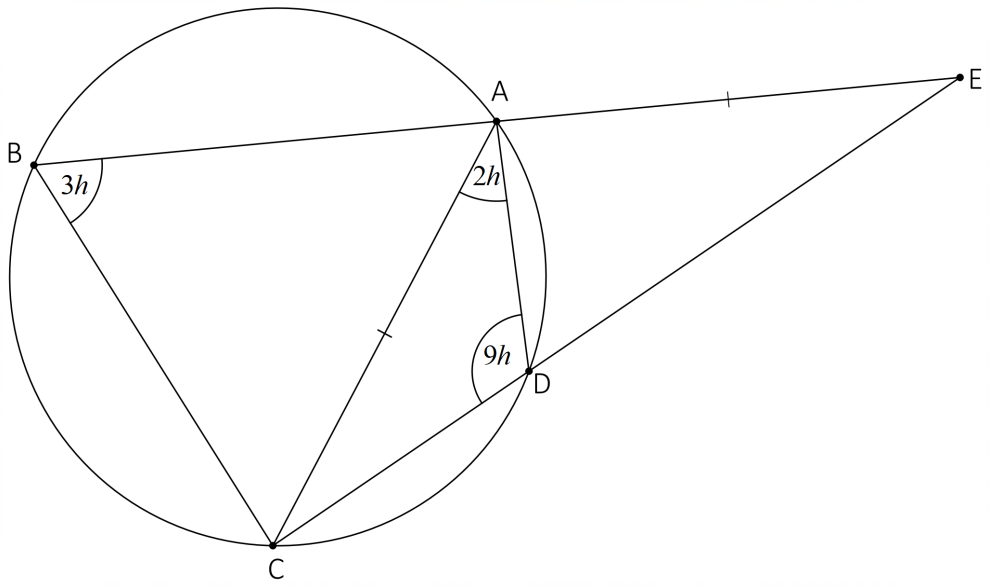
d) ACX 60° – OCX is 90° (tangent – radius intersection)

e) ABC 120° opposite angles in cyclic quad are supplementary

f) AXC 60° – Missing angle in quad AOXC, OAX=OX=90°, AOC=120°

Question 4 **5 marks**

The diagram shows the cyclic quadrilateral ABCD.
The extended lines of BA and CD meet at E, such that AE = AC.
$\angle ABC, \angle ADC$ and $\angle DAC$ are multiples of a constant h as shown.



a) Show that $h = 15°$.

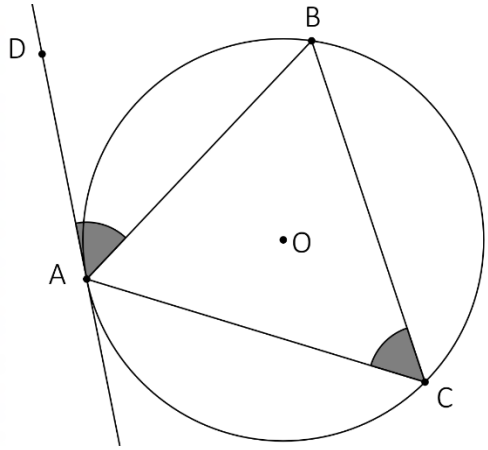
$3h + 9h = 180°$
$12h = 180°$
$h = 15°$

b) Determine the size of $\angle EAD$, clearly showing your method.

$\angle ACD = 180 - 11 \times 15$	
$= 15°$	Angle sum of triangle ✓
$\triangle ACE$ is isosceles	Given
$\angle ACE = \angle AEC = 15°$	Base angles of isosceles triangle ✓
$\angle CAE = 150°$	Angle sum of triangle✓
$\angle CAD + \angle DAE = \angle CAE$	Adjacent angles
$30° + \angle DAE = 150°$	
$\angle DAE = 120°$	✓

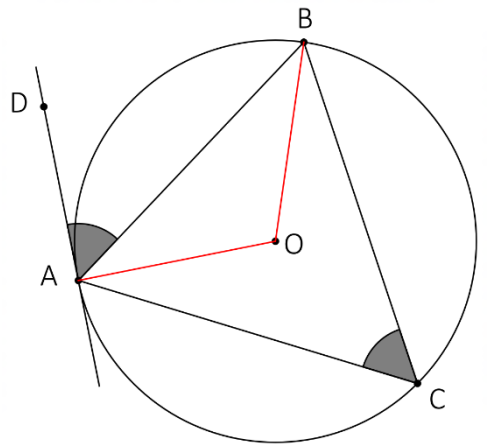
Question 5 **2, 6 – 8 marks**

a) Identify the relationship between the shaded angles and theorem involved in the given diagram.



The shaded angles are congruent ✓
Angle in the alternate segment ✓

b) Prove this relationship is always true.



RTP : $\angle DAB = \angle ACB$
Construction : Radii OA,OC ✓

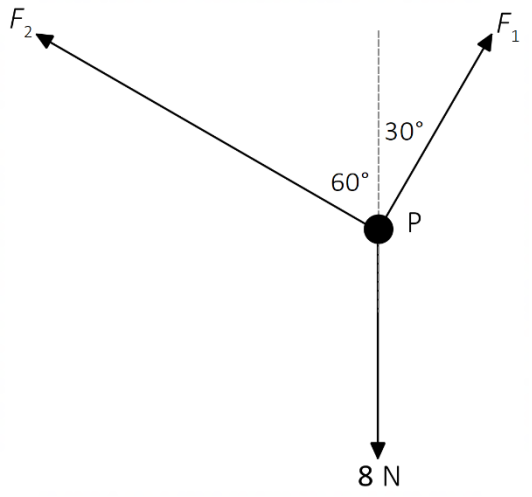
Let $\angle DAB = \theta$

$\angle DAO = 90°$	Pt of contact radius & tangent ✓
$\angle BAO = (90 - \theta)°$	Adjacent angles ✓
$\triangle AOB$ is isosceles	AO = OB
$\angle OBA = (90 - \theta)°$	Base angles of an isosceles triangle ✓
$\angle AOB = 2\theta$	Angle sum of a triangle ✓
$\angle ACB = \theta$	Angle at the circumference is half the central angle ✓

$\angle DAB = \angle ACB$ as required

Question 6 **7 marks**

The diagram shows a body P, of weight 8 N, supported in equilibrium by two wires.
Find the exact magnitudes of F_1 and F_2.



Horizontal components

$F_1\sin30° = F_2\sin60$ ✓

$0.5F_1 = \frac{\sqrt{3}}{2}F_2$

$F_1 = \sqrt{3}F_2$ ✓

Vertical components

$F_1\cos30° + F_2\cos60 = 8$ ✓

$\frac{\sqrt{3}}{2}F_1 + \frac{1}{2}F_2 = 8$ ✓

$\frac{\sqrt{3}}{2}\left(\sqrt{3}F_2\right) + \frac{1}{2}F_2 = 8$

$\frac{3}{2}F_2 + \frac{1}{2}F_2 = 8$ ✓

$2F_2 = 8$

$F_2 = 4$ ✓

$F_1 = 4\sqrt{3}$ ✓

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Student Name : _____

Part Two – Resource Allowed

Part Two contains 3 questions worth 17 marks

Time Allowed : 20 minutes

TO BE PROVIDED BY THE STUDENT

A maximum of one A4 page of notes, one sided.
Standard Items : Pens, pencils, eraser, sharpener, correction tape/fluid, highlighters, ruler.

Special Items: Drawing instruments, templates.
A maximum of three CAS calculators satisfying the conditions set by the SCSA.

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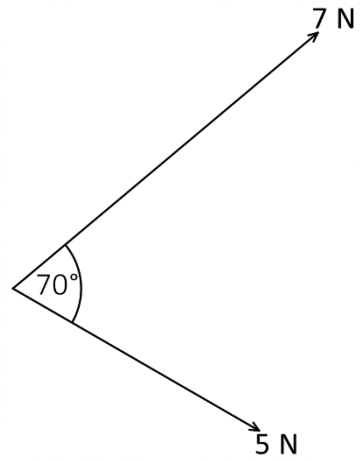
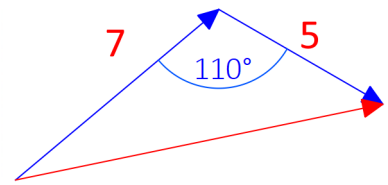
Question 1 **4 marks**

The forces, one of 7 N and one of 5 N, act on an object. The forces are separated by an angle of 70°, as shown on the diagram.



Calculate the magnitude (correct to 1 dp) and direction of the resultant force acting on the object. (Give the direction in terms of the angle it makes with larger force)



$|r|^2 = 7^2 + 5^2 - 2 \times 5 \times 7 \times \cos 110°$

$|r| = 9.9N$

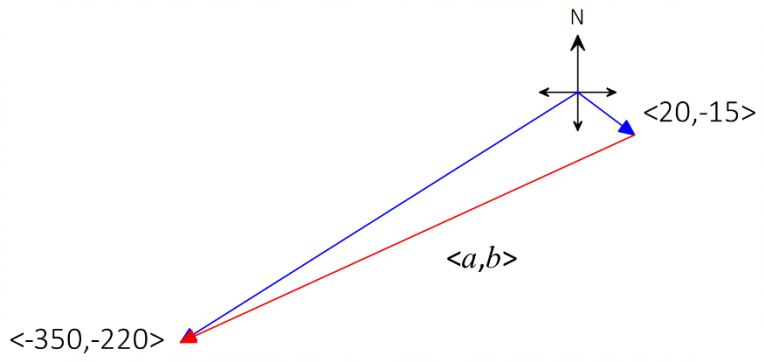
$\frac{\sin\theta}{5} = \frac{\sin 110}{9.9}$

$\theta = \frac{5 \times \sin 110}{9.9}$

$= 28°$

Question 2 **5 marks**

An aircraft capable of 100 km/h needs to travel to a destination located $(-350\underset{\sim}{i}-220\underset{\sim}{j})$ km from his starting location. The current wind speed is reported as $20\underset{\sim}{i}-15\underset{\sim}{j}$ km/h. Determine the velocity the pilot should set and his calculated travel time, to the nearest minute.



Let the velocity of the aircraft be <*a*,*b*>

$V_f = < a,b > + < 20,-15 >$

$= < a+20, b-15 >$

$< a+20, b-15 > t = < -350, -220 >$ ✓

$t = \dfrac{-350}{a+20} = \dfrac{-220}{b-15}$ ✓

$a^2 + b^2 = 100^2$ ✓

By classpad

$a = -94.75, b = -31.98$

The velocity to be set is $-97.75\underset{\sim}{i} - 31.98\underset{\sim}{j}$ ✓

$t = 4.68h$

$= 4\text{h } 41\text{min}$ ✓

Question 3 **2, 4, 2 – 8 marks**

Consider the vectors $\underset{\sim}{a} = <3,2>, \underset{\sim}{b} = <4,-3>$ and $\underset{\sim}{c} = <n,-0.5>$.

a) Determine $|2\underset{\sim}{b} - \underset{\sim}{a}|$ exactly.

$2<4,-3> - <3,2> = <5,-8>$ ✓

$|<5,-8>| = \sqrt{25+64}$

$= \sqrt{89}$ ✓

b) Find a vector parallel to $\underset{\sim}{a}$ and half the magnitude of $\underset{\sim}{b}$.

$|b| = 5$ & $|<3\lambda, 2\lambda>| = \frac{5}{2}$ ✓

$9\lambda^2 + 4\lambda^2 = \frac{25}{4}$

$13\lambda^2 = \frac{25}{4}$

$\lambda^2 = \frac{25}{52}$

$\lambda = \pm\frac{5}{2\sqrt{13}}$ ✓✓

vector required

$\pm\frac{5\sqrt{13}}{26} <3,2>$

$= \pm <\frac{15\sqrt{13}}{26}, \frac{10\sqrt{13}}{26}>$ ✓

c) Determine the value of n if $\underset{\sim}{c}$ is a unit vector.

$n^2 + \frac{1}{4} = 1$ ✓

$n^2 = \frac{3}{4}$

$n = \pm\frac{\sqrt{3}}{2}$ ✓