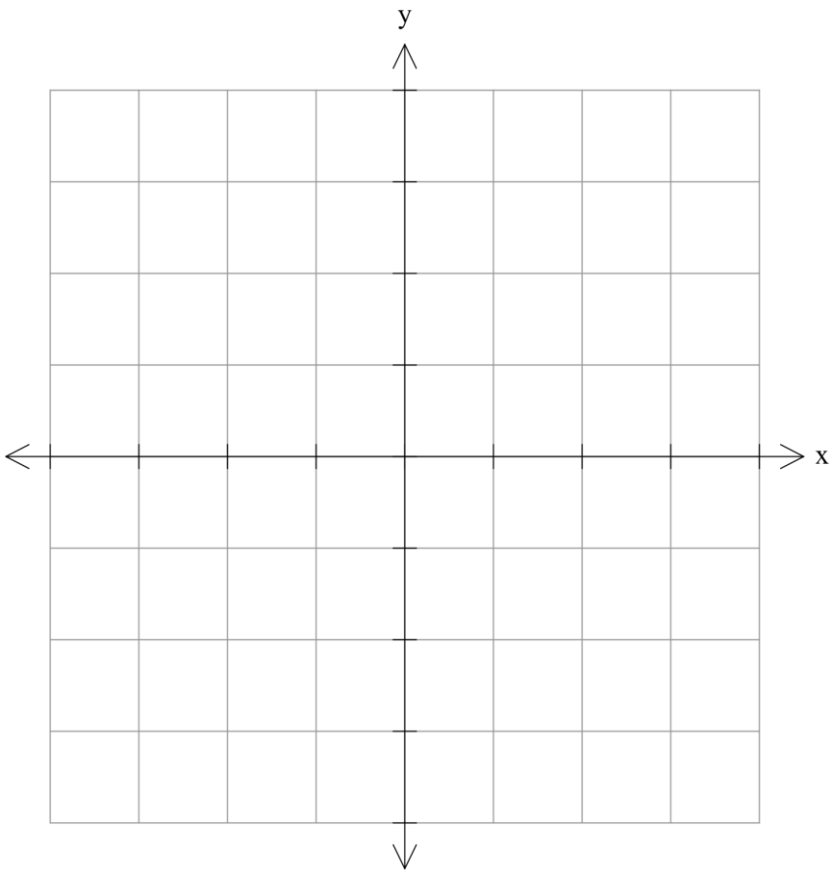
Investigating Graphs – Part A

In previous work you learnt that the graph of $y = a(x - b)^2 + c$ has the same shape as that of $y = ax^2$ but has been translated horizontally b units and translated vertically c units. The effect of the dilation factor, a, is to stretch or compress the curve depending on the value of a. Furthermore, if a was negative, the graph was inverted.

The aim of this investigation is to investigate translations and dilations in other functions.

1. [2 marks]
   On the axes below, accurately graph the following function: $y = f(x) = \frac{1}{x}$.



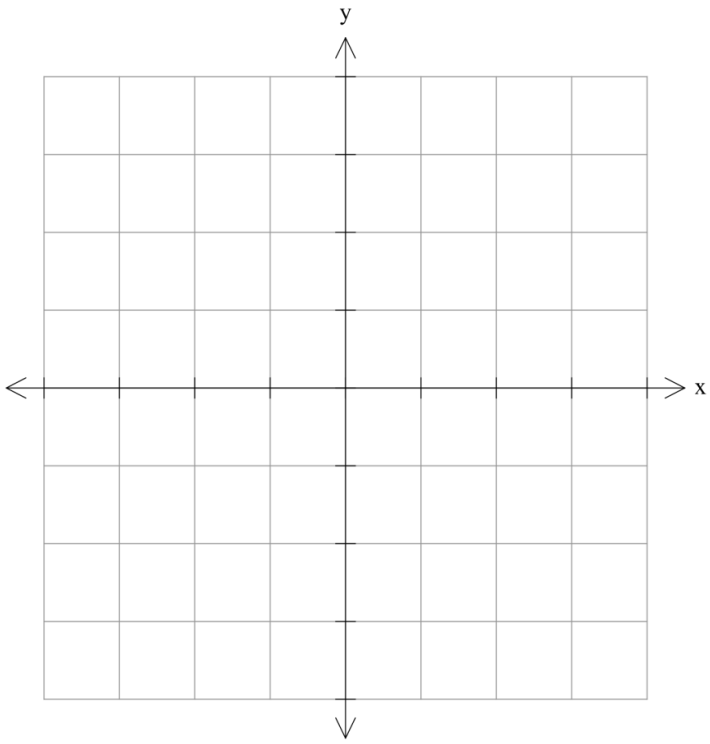
2. [5 marks]

On the following axes, labelling everything clearly, graph the following functions:

$$y = \frac{1}{x}$$

$$y = \frac{2}{x}$$

and $y = \frac{4}{x}$

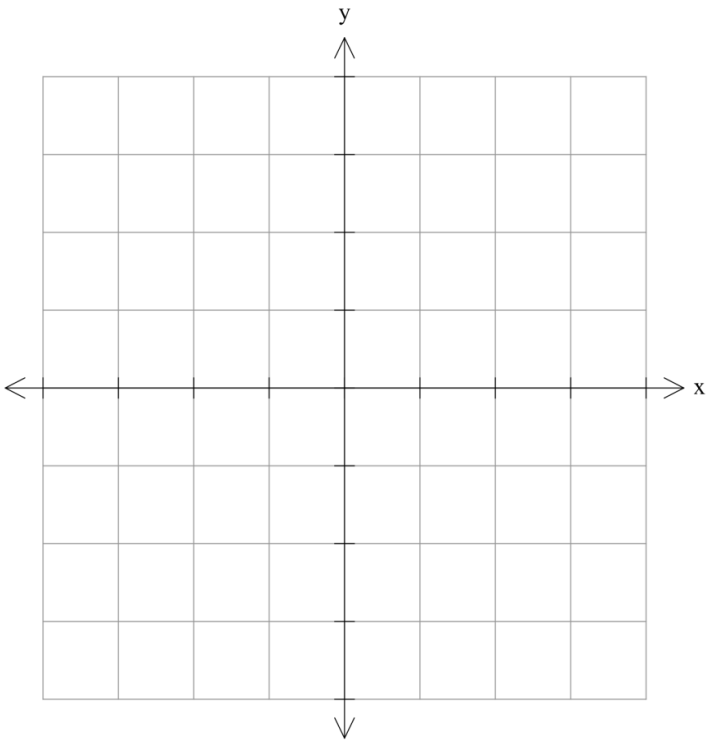


3. [4 marks]

On the following axes, labelling everything clearly, graph the following functions:

$$y = \frac{-1}{x}$$

and $y = \frac{-2}{x}$



4. [3 marks]

Comment on the effects that changes in a have on the graphs of $y = \frac{a}{x}$ $(y = af(x))$.

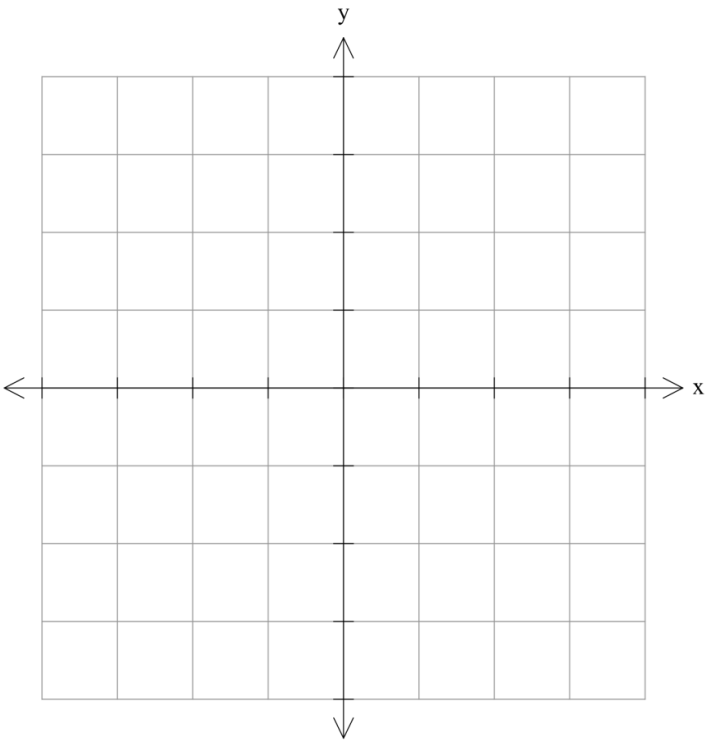
5. [5 marks]

On the following axes, labelling everything clearly, graph the following functions:

$y = \frac{1}{x}$

$y = \frac{1}{x} + 1$

and $y = \frac{1}{x} - 2$



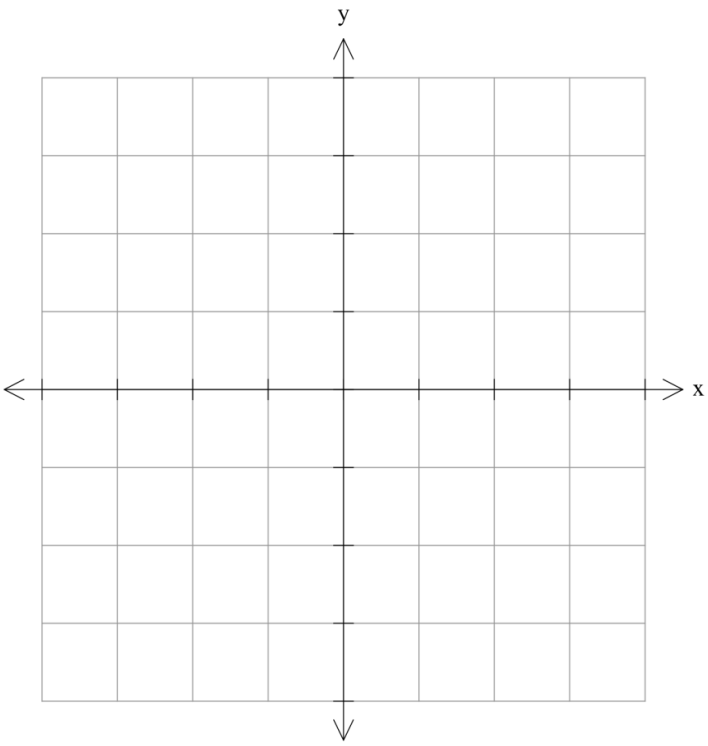
6. [5 marks]

On the following axes, labelling everything clearly, graph the following functions:

$y = \frac{1}{x}$

$y = \frac{1}{x+1}$

and $y = \frac{1}{x-2}$



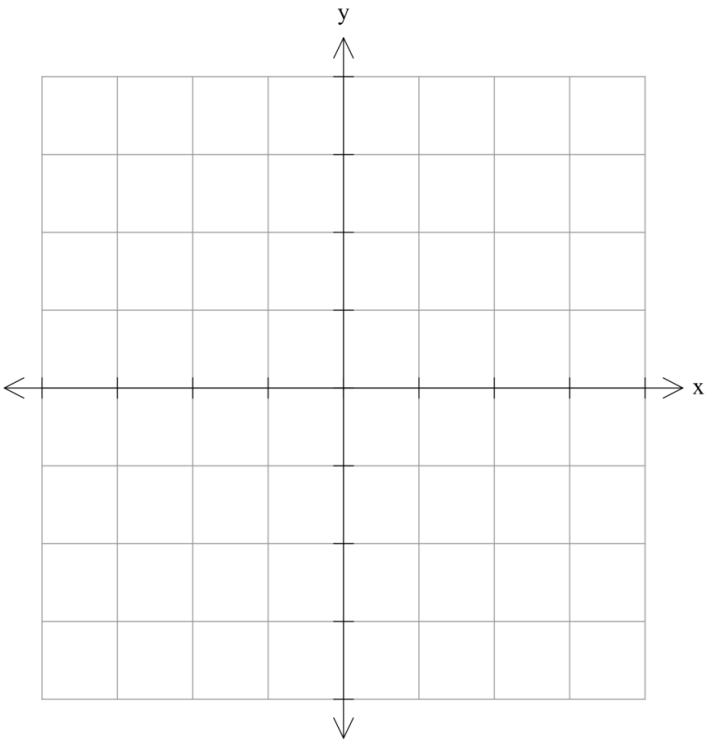
7. [4 marks]

Comment on the effects that changes in b and c have on the graphs of $y = \frac{1}{x-b} + c$ $(y = f(x-b) + c)$.

8. [3 marks]

On the following axes, labelling everything clearly, graph the following function:

$$y = \frac{1}{x-1} + 2$$

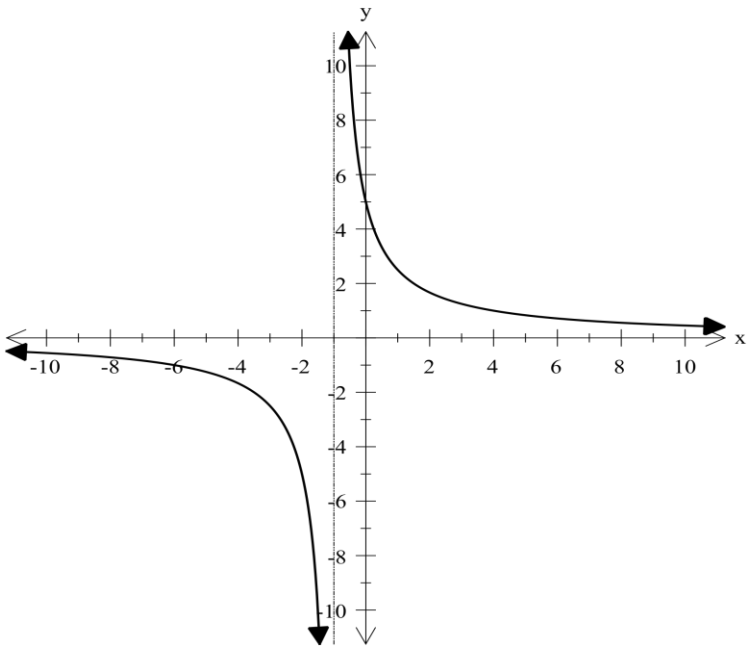


9. [4 marks]

(a) If the graph of $y = \frac{e}{x+1} + 4$ has a *y*-intercept of 7, determine the value of e.

(b) The graph shown has an equation given as $y = \frac{r}{x - s}$

Determine the values of *r* and *s*.

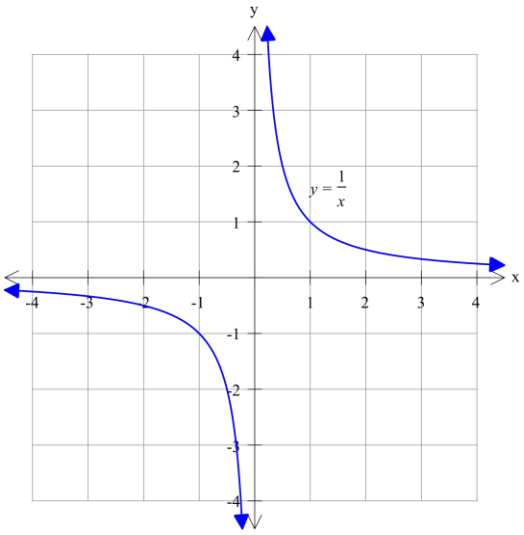


Solutions

Investigating Graphs – Part A

1. [2 marks]

On the axes below, accurately graph the following function: $y = f(x) = \frac{1}{x}$.



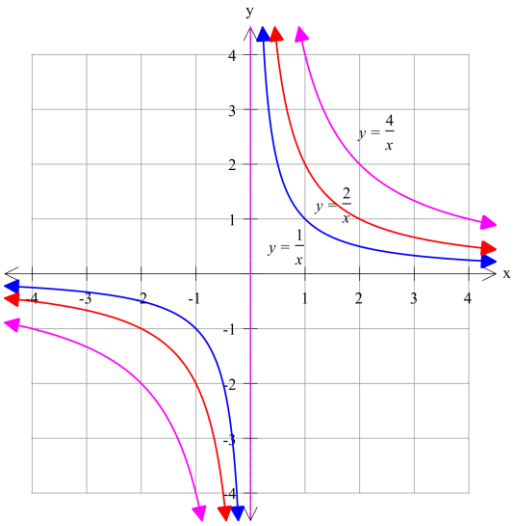
2. [5 marks]

On the following axes, labelling everything clearly, graph the following functions:

$y = \frac{1}{x}$

$y = \frac{2}{x}$

and $y = \frac{4}{x}$

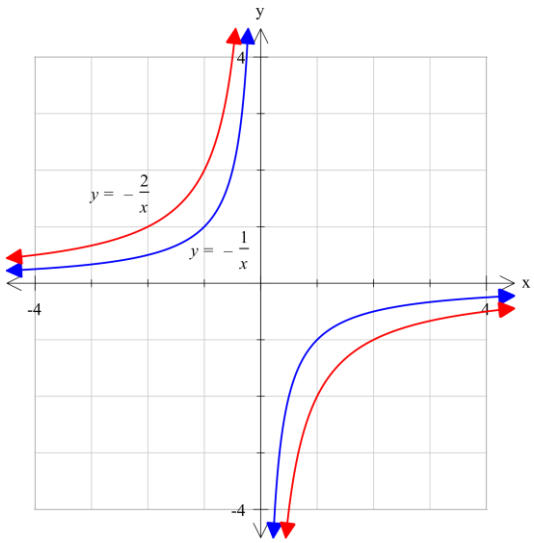


3. [4 marks]

On the following axes, labelling everything clearly, graph the following functions:

$$y = \frac{-1}{x}$$

and $y = \frac{-2}{x}$



4. [3 marks]

Comment on the effects that changes in a have on the graphs of $y = \frac{a}{x}$ $(y = af(x))$.

a is a dilation factor which stretches or compresses the graph of $y = \frac{1}{x}$ depending on the value of a. This affects the gradient of the curve and thus the distance the curve is from the axes. The larger the value of $|a|$, the further it is from each axis.

Furthermore, if a is negative, the graph is inverted. That is the curves are in the second and fourth quadrants, rather than in the first and third.

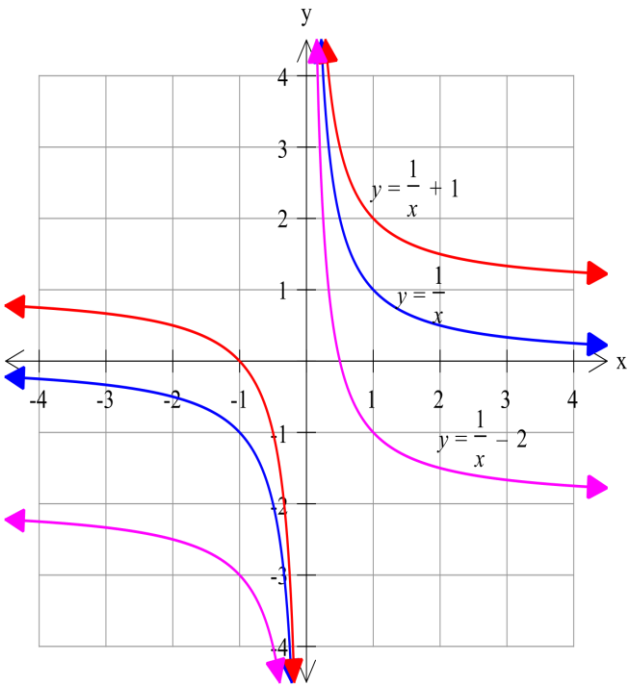
5. [5 marks]

On the following axes, labelling everything clearly, graph the following functions:

$$y = \frac{1}{x}$$

$$y = \frac{1}{x} + 1$$

and $y = \frac{1}{x} - 2$



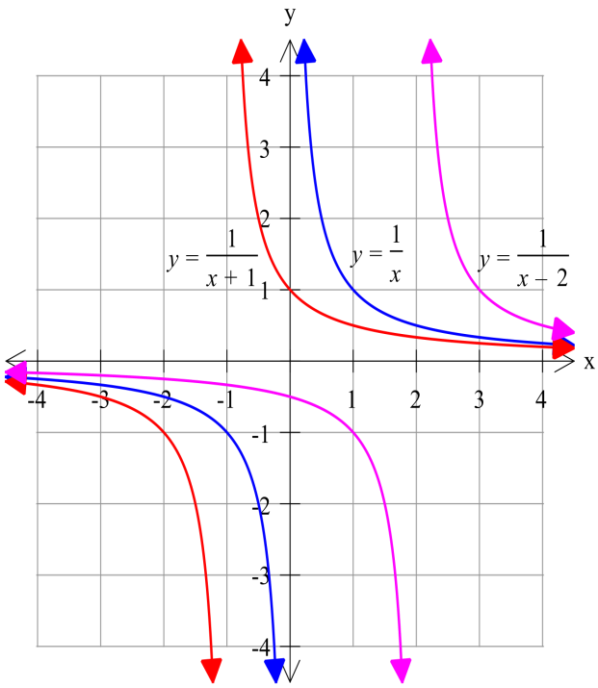
6. [5 marks]

On the following axes, labelling everything clearly, graph the following functions:

$y = \frac{1}{x}$

$y = \frac{1}{x+1}$

and $y = \frac{1}{x-2}$



7. [4 marks]

Comment on the effects that changes in b and c have on the graphs of $y = \frac{1}{x-b} + c$ $(y = f(x-b) + c)$.

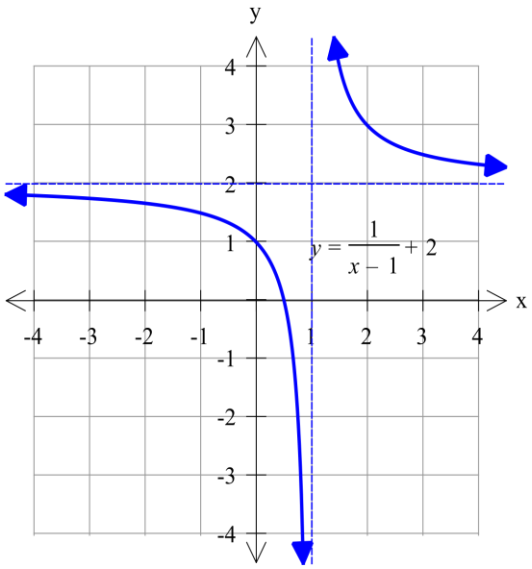
$y = \frac{1}{x-b} + c$ is congruent to $y = \frac{1}{x}$ but has been moved b units right and c units up.

That is b is the distance of the horizontal translation and c the vertical translation.

8. [3 marks]

On the following axes, labelling everything clearly, graph the following function:

$$y = \frac{1}{x-1} + 2$$



9. [4 marks]

(a) If the graph of $y = \frac{e}{x+1} + 4$ has a *y*-intercept of 7, determine the value of e.

(a)

$$y = \frac{e}{x+1} + 4$$

$$7 = \frac{e}{1} + 4$$

$$e = 3$$

(b) The graph shown has an equation given as $y = \frac{r}{x - s}$

Determine the values of *r* and *s*.

$$y = \frac{5}{x+1}$$

$r = 5$ and $s = -1$

Total: 35 marks