12 Mathematics Methods 2021

Test 1 – Differentiation and Logarithms

Section 1: Calculator-free

Time allowed: 25 minutes **Maximum marks: 25**

Name: Solutions **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No calculators or personal notes are permitted.

Question 1 **[2, 2 = 4 marks]**

Calculate the following;

a) $\log 1000\sqrt{10}$

$= \log 10^3 \times 10^{1/2}$ ✓

$= 3.5$ ✓

b) $\log_{81} 3 = x$

$3 = 81^x$

$3^1 = (3^4)^x$

$4x = 1$

$x = \frac{1}{4}$ ✓✓

Question 2 **[2, 3 = 5 marks]**

Differentiate the following (do not simplify your answers).

a) $y = \frac{3x^2-5x}{6x-5}$

$\frac{dy}{dx} = \frac{(6x-5)(6x-5) - 6(3x^2-5x)}{(6x-5)^2}$ ✓✓

(-1 per error)

b) $f(x) = (8-x)(7x^2+4x)^3$

$f'(x) = -1(7x^2+4x)^3 + 3(7x^2+4x)^2(14x+4)(8-x)$ ✓

✓✓ (-1 per error)

Question 3 **[5 marks]**

Determine the coordinates of any points on the function $y = -\frac{6}{(x-4)}$ whose tangents are parallel to the line $3x - 2y = 6$

$m = \frac{3}{2}$ ✓

$y' = -(6)(x-4)^{-2} = \frac{6}{(x-4)^2}$ ✓

(2, 3) (6, −3) ✓

$\frac{6}{(x-4)^2} = \frac{3}{2}$ ✓

$\frac{6}{(x-4)^2} = \frac{6}{4}$

$\therefore (x-4)^2 = 4$ ✓

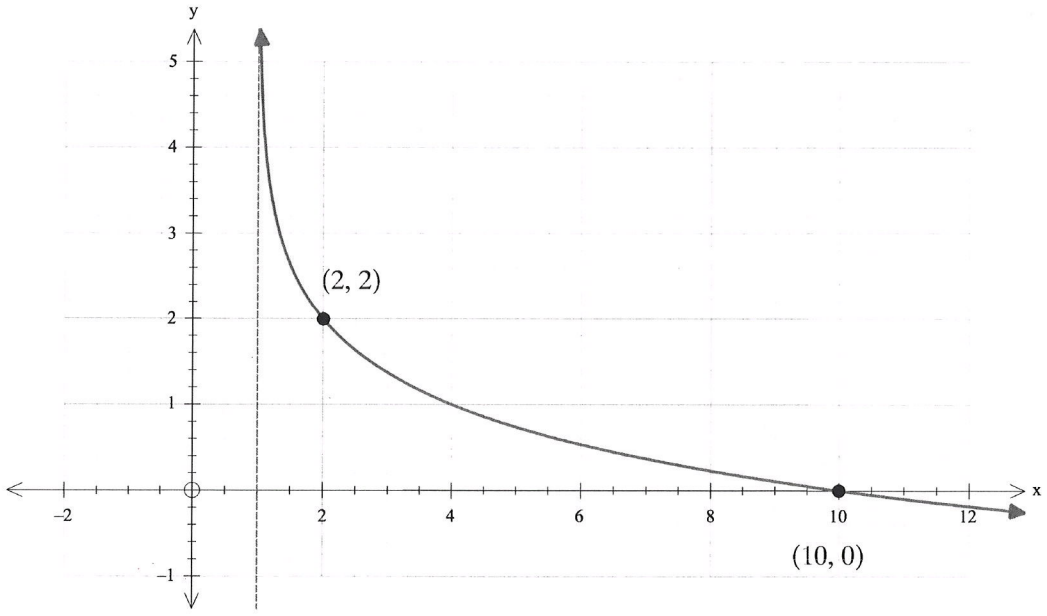
$x = \sqrt{4} + 4$

$x = \pm 2 + 4$

Question 4 **[4 marks]**

The graph of $y = -\log_b(x + c) + d$ is drawn below.

If there is a vertical asymptote at $x = 1$, determine the values of b, c and d.



$c = -1$ ✓ $d = 2$ ✓

$0 = -\log_b(10-1) + 2$

$\log_b 9 = 2$

$b = 3$ ✓✓

Question 5 **[3, 4 = 7 marks]**

Solve each of the following equations for x;

a) $16^{x+1} = (\sqrt{8})^{6x-2}$

$(2^4)^{x+1} = 2^{3/2(6x-2)}$ ✓

$2^{4x+4} = 2^{9x-3}$

$4x + 4 = 9x - 3$ ✓

$x = \frac{7}{5}$ ✓ (FT)

b) $12(2^x) = 7 + \frac{10}{2^x}$ (giving answer in form $a + \log_2 b$)

$12(2^x)^2 - 7(2^x) - 10 = 0$ ✓

$(4(2^x) - 5)(3(2^x) + 2) = 0$ ✓

$2^x = \frac{5}{4}$ or $2^x = -\frac{2}{3}$ ✓

no solution

$x = \log_2 \frac{5}{4}$

$= \log_2 5 - \log_2 4$

$= -2 + \log_2 5$ ✓

END OF SECTION 1

12 Mathematics Methods 2021

Test 1 – Differentiation and Logarithms

Section 2: Calculator-assumed

Time allowed: 20 minutes **Maximum marks: 20**

Name: Solutions **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- Calculators and 1xA4 double-sided page of personal notes are permitted.

Question 6 **[4 marks]**

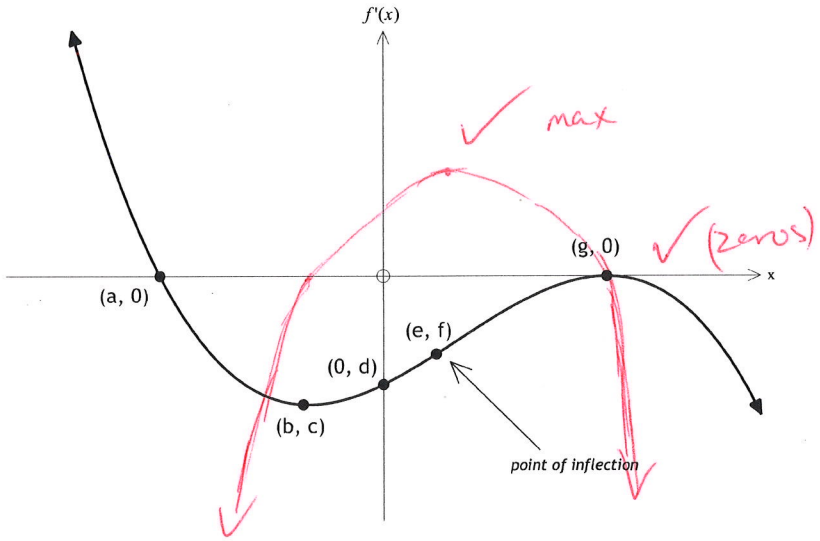
Use the *increments formula* to determine the percentage change in the radius of a sphere when its surface area decreases by 3%.

$\frac{\Delta A}{A} \approx \frac{dA}{dr} \times \frac{\Delta r}{A}$ ✓ $\quad \frac{-3}{100} \approx 2\frac{\Delta r}{r}$ ✓

$\frac{-3}{100} \approx 8\pi r \times \frac{\Delta r}{4\pi r^2}$ ✓ $\quad \therefore 1.5\%$ (decrease) ✓

Question 7 **[2, 2 = 4 marks]**

The graph of the **derivative**, $f'(x)$, is drawn below.



a) On the graph above, sketch a possible graph of the second derivative $f''(x)$.

b) Determine the x values of any stationary points, and their nature, on $f(x)$.

$x = a$ $\quad$ $x = g$

max t.p $\quad$ H.P.O.I

✓ $\quad$ ✓

Question 8 **[2, 3 = 5 marks]**

A closed cylindrical can, with base radius r and height h, has a volume of 250π cm^3.

a) Show that the total area, A cm^2, of metal required to make the can is given by $A = 2\pi r^2 + \frac{500\pi}{r}$

$V = \pi r^2 h$

$250\pi = \pi r^2 h$

$h = \frac{250}{r^2}$ ✓

$S.A. = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r \left(\frac{250}{r^2}\right)$

$= 2\pi r^2 + \frac{500\pi}{r}$ ✓

b) If the material for the curved side of the can costs \$0.001 per cm^2 and the material for each of the circular ends costs \$0.003 per cm^2, determine;

i. The area of material used to minimise cost.

$Cost = 0.003(2\pi r^2) + 0.001\left(\frac{500\pi}{r}\right)$

$C' = 0 \; ; \; r = 3.4668$ ✓

$\therefore \; S.A. = 528.61 \text{cm}^2$ ✓

ii. The minimum cost to produce a can.

$Cost \approx \$0.68 \text{ (per can)}$

✓ (FT)

Question 9 **[2, 2, 3 = 7 marks]**

Two particles, P and Q, both travel along the same straight line.

Their displacements, s metres, after t seconds ($t \geq 0$) from a fixed-point O on the line are given by;

$s_P = 3t^3 - 81t + 5$

$s_Q = -2(t-1)(t-4)$

a) Calculate the initial distance between the particles.

$t = 0$; $s_P = 5$, $s_Q = -8$ ✓

$\therefore$ 13m ✓ (FT)

b) At what time(s), does particle P change direction?

velocity; $s_P' = 9t^2 - 81 = 0$ ✓

$t = \pm 3$

$(t \geq 0)$ $\therefore$ $t = 3$ ✓

c) At $t = 4$, is particle Q speeding-up or slowing-down? Justify your answer.

velocity; $s_Q' = -4t + 10$

$t = 4$; $-4(4) + 10 = -6$ (< 0) ✓

acceleration; $s_Q'' = -4$ (< 0) ✓

$\therefore$ speeding up (velocity and acceleration have the same sign)

END OF TEST