Year 12 Mathematics
ATMAA UNIT 3 APPLICATIONS TEST 3
Graphs and Networks Calculator Assumed

Name: **SOLUTIONS**
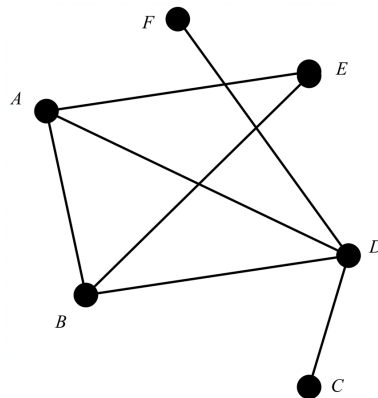
Mark   /40

Time Allowed: 30 minutes

Resources Allowed: Scientific Calculator, 1 side A4 notes. Formula sheet provided.

Solutions without working may not be allocated full marks.

Question One   [20 marks]

Use the network *Z* below to answer the following questions.

a)   Write the sum of degrees for the network *Z*.   (1 marks)

13

> 1 mark for correct answer

b)   Could network *Z*, be described as an Eulerian graph or a Hamilitonian graph? Justify your answer.
                                                                                             (4 marks)

The network is not Eulerian
It does not have all even nodes

The network is not Hamilitonian
All nodes can not be visited once only
without repeating node D.

> 1 mark for saying "Neither" or "Not Eulerian or Hamilitonian"
> 1 mark for justifying why it is not Eulerian.
> 1 mark for justifying why it is not Hamilitonian.
> 1 mark for saying that node D would be repeated.

c)   Highlight any bridges that exist in the network. Justify your anwer.   (3 marks)

Student highlights DF and CD
Student can write the edges instead of highlighting it.
Removing DF or CD will disconnect the graph,
or node C and F will have no edges.

> 1 mark for mentioning or highlighting DF
> 1 mark for mentioning or highlighting CD
> 1 mark for justifying why both of them are a bridge.

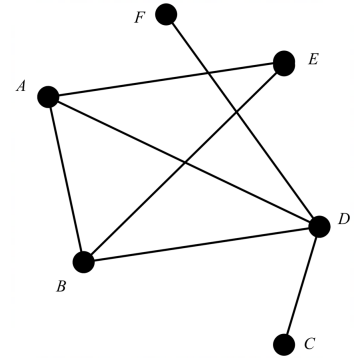d) Find the value x and y in the expression below. (2 marks)

$$Z + Z^2 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{bmatrix} A & B & C & D & E & F \\ 3 & & & & & \\ 3 & y & & & & \\ 1 & 1 & 1 & & & \\ 2 & 2 & 1 & 4 & & \\ 2 & 2 & x & 2 & 2 & \\ 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

1 mark for correct answer for x

1 mark for correct answer for y

$$x = 0 \ \& \ y = 3$$

e) What does the value of 4 in the matrix mean in the context of the the given network? (3 marks)

4 indicates the total number of 1 stage and 2 stage walks from node D back to node D.

1 mark for stating that it is a total number of walks
1 mark for correctly stating one stage and two stage walks
1 mark for correctly stating From node D back to node D.

f) Given matrix P represents a planar graph. Calculate the number of regions in the network.
(3 marks)

$$P = \begin{array}{c} \\ W \\ X \\ Y \\ Z \end{array} \begin{bmatrix} W & X & Y & Z \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
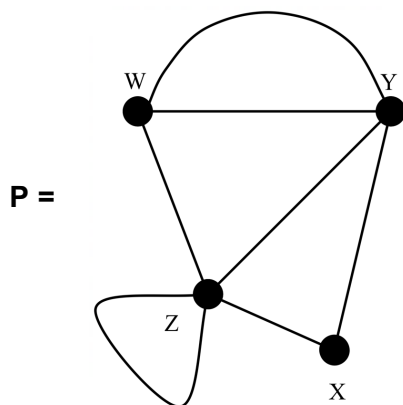
$E = 7, \qquad V = 4$

$F + V = E + 2$
$F + 4 = 7 + 2$
$F = 5$

1 mark for writing V = 4, E = 7
1 mark for using Euler's rule correctly and showing substitution..
1 mark for correct answer.

g) Draw the planar graph represented by matrix P. (4 marks)

$P =$

1 mark for all correct labels.
1 mark for planar (no edge is crossing any other edge).
Edges must touch the nodes.
Nodes are clearly defined

1 mark for correct Matrix labels including "P ="
1 mark for all correct edges.

Question Two   [11 marks]

a)   8 towns are connected by sealed roads. Determine the shortest journey from town G to K.

(4 marks)
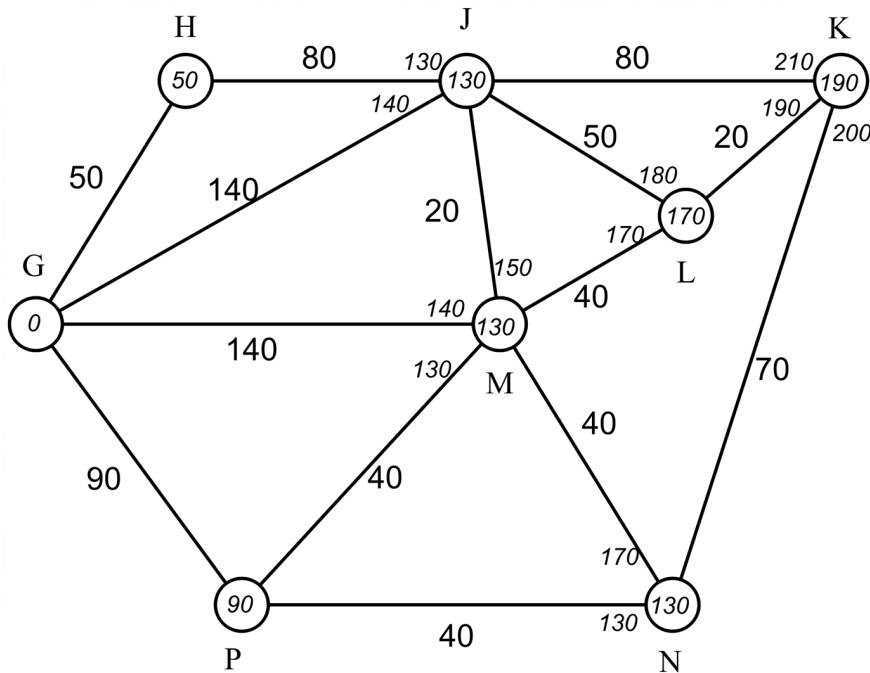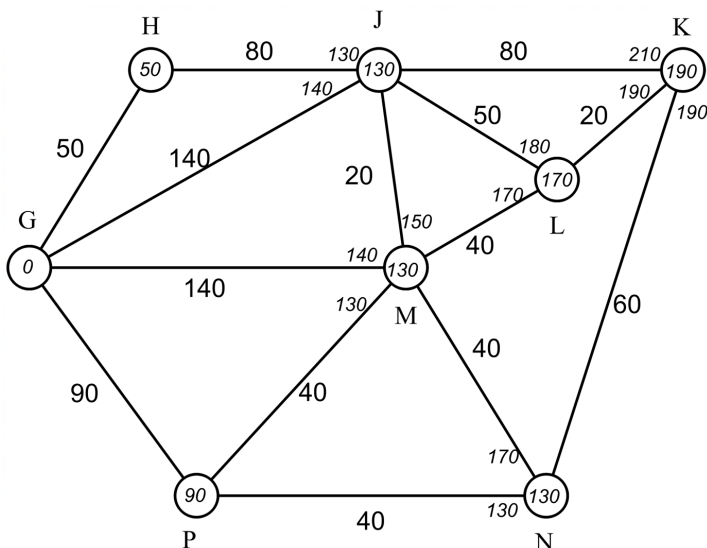

Shortest Path
G – P – M – L – K = 190

1 mark for showing some working (totals at different nodes)
1 mark for showing working at all nodes choosing the lowest number from the totals at each node
Accept any other method as long as the entire method is shown.

1 mark for correct walk
1 mark for correct length of the walk

b)   The road joining town N and town K improves and it is shortened by 10km. Comment in detail how this will impact the shortest journey from town G to K.   (3 marks)

1 mark for showing working out on Network or here.
1 mark for saying it is a shortest path as well.
1 mark for correct walk and weight.

Shows some calculation either on the graph or here.
The pathway GPNK is now 190.
It equals the shortest path.

This is a network showing businesses connected in town J.
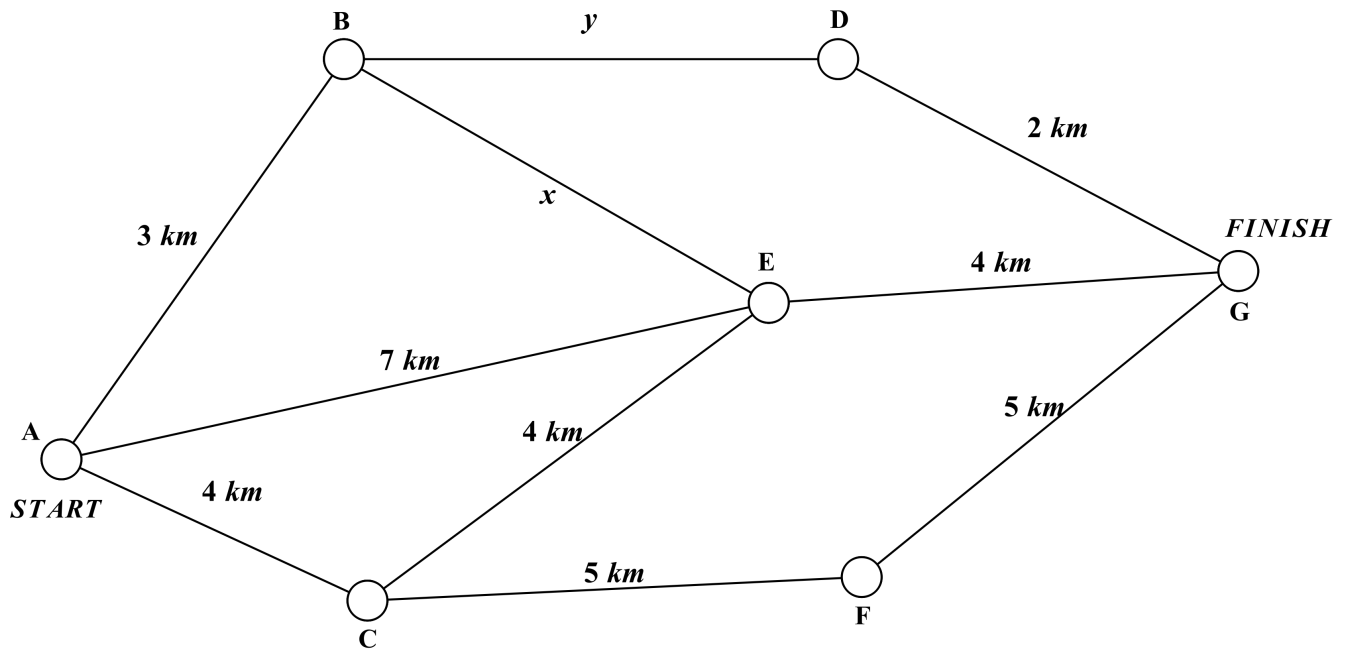
c) If the shortest path/s from start to finish is 9 km, state all possible values for x and y and the possible shortest path/s.   (4 marks)

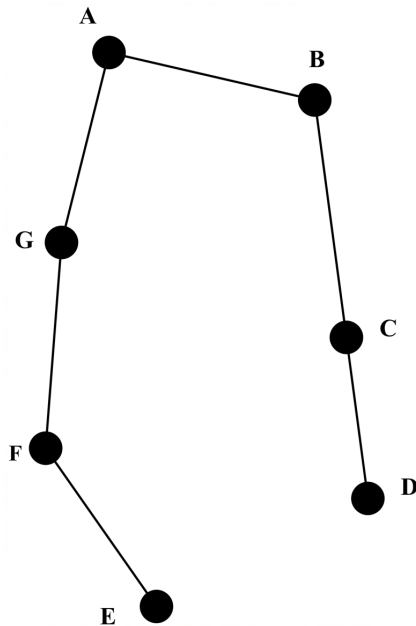|	
$3 + y + 2 = 9$ $y = 4$ $Path: A - B - D - G$ $y = 4 \ and \ x > 2$ OR $3 + x + 4 = 9$ $x = 2$ $Path: A - B - E - G$ $x = 2 \ and \ y > 4$	1 mark for showing y = 4 when x > 2 1 mark for stating path A – B – D – G 1 mark for showing x = 2 when y > 4 1 mark for stating path A – B – E – G

Question Three   [9 marks]

A network H is a tree with 7 vertices; A, B, C, D, E, F, G

a)   Draw network H as a Semi-Eulerian trail and a Semi-Hamilitonian walk.   (4 marks)

D-C-B-A-G-F-E = Semi-Hamilitonian walk

1 mark for a tree network with 7 vertices
1 mark for having a Semi-Eulerian trail.
(exactly two odd vertices)
1 mark for having all vertices labelled.
1 mark for writing a semi-Hamilitonian walk starting at vertex D

Edges must touch the vertices.

If network H can be changed into network P with 7 vertices; A, B, C, D, E, F, G.

b)   Network P is a planar non-simple graph with a Eulerian cycle, one loop, and it has 3 regions. Draw network P below. Clearly outline the Eulerian cycle as a walk starting at vertex A.

(5 marks)
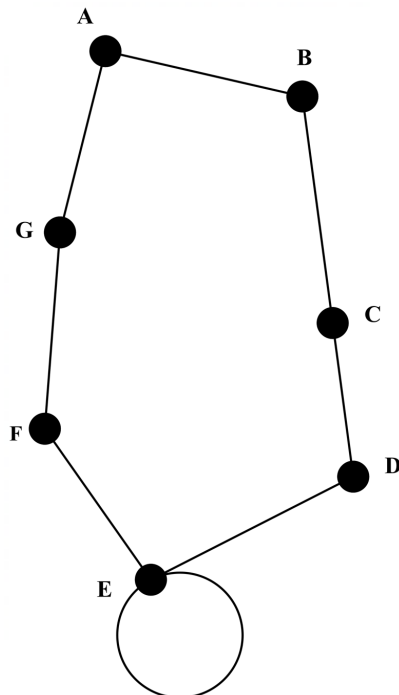
A-B-C-D-E-F-G-A = Eulerian trail

1 mark for a Planar network with 7 vertices
1 mark for having a Eulerian trail.
(all even vertices)
1 mark for having a loop.
1 mark for having exactly 3 regions.
1 mark for writing the Eulerian trail starting and finishing at A.

Edges must touch the vertices.

END OF SECTION TWO