

Full Name: SOLUTIONS



MATHEMATICS

Methods Units 1 & 2

Test 4 – Trigonometric Graphs and Equations

Chapter 14

Semester 1 2019

Section One - Calculator Free

Time allowed for this section

Working time for this section: 30 minutes

Marks available: 26 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the students

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

Important note to students

No other items may be used in this section of the assessment. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the assessment room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (4 marks)

Find the exact value of $\sin 75^\circ$.

$$75 = 45 + 30$$

$$\sin 75^\circ = \sin(45^\circ + 30^\circ) \checkmark$$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ \checkmark$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \checkmark$$

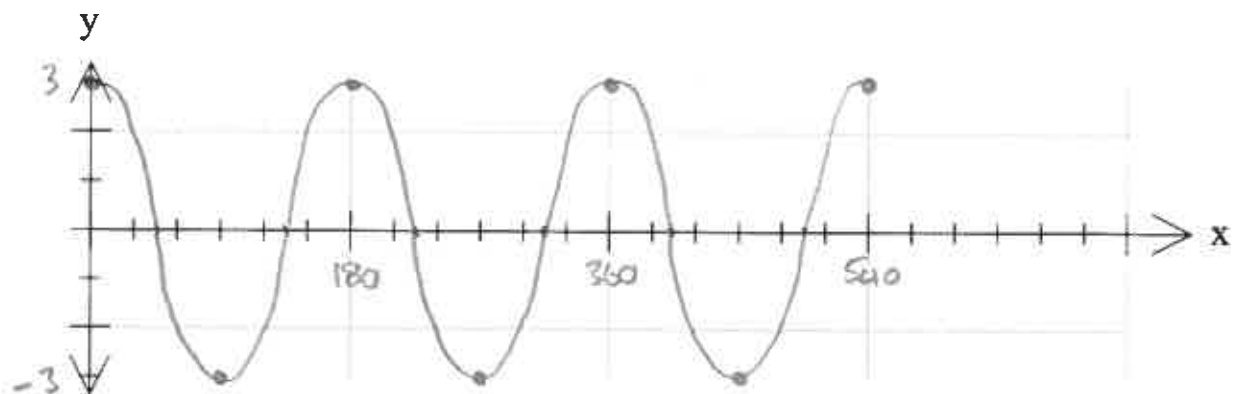
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}} \checkmark$$

alternatively

$$= \frac{\sqrt{2}(\sqrt{3} + 1)}{4} \quad \text{or} \quad = \frac{\sqrt{6} + \sqrt{2}}{4}$$

2. (4 marks)

Graph $y = 3\cos(2x)$ for $0^\circ \leq x \leq 540^\circ$ 

✓ amplitude

✓ period

✓ curve / shape

✓ within domain

3. (4 marks)

Given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{1}{4}$, where A and B are acute, find the exact value of $\cos(A - B)$

$$\begin{aligned}
 \cos(A - B) &= \cos A \cos B + \sin A \sin B \quad \checkmark \quad \begin{array}{c} 5 \\ \triangle A \\ 4 \end{array} \quad \begin{array}{c} 4 \\ \triangle B \\ \sqrt{15} \end{array} \\
 &= \frac{4}{5} \times \frac{1}{4} + \frac{3}{5} \times \frac{\sqrt{15}}{4} \quad \checkmark \\
 &= \frac{4 + 3\sqrt{15}}{20} \quad \checkmark
 \end{aligned}$$

4. (3 marks)

Show that $\cos 2x = 1 - 2\sin^2 x$

$$\begin{aligned}
 \cos 2x &= \cos(x + x) \quad \checkmark \\
 &= \cos x \cos x - \sin x \sin x \\
 &= \cos^2 x - \sin^2 x \quad \checkmark \\
 &= 1 - \sin^2 x - \sin^2 x \quad \checkmark \\
 &= 1 - 2\sin^2 x \quad \checkmark
 \end{aligned}$$

QED

5. (12 marks).

Solve the following equations for the given domain.

a) $\cos(x) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

✓ ✓



[3] 2



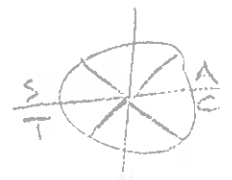
b) $2 \sin^2(2x) - 1 = 0$

for $0^\circ \leq x \leq 360^\circ$
 $0 \rightarrow 360^\circ$ ✓

$$\sin^2(2x) = \frac{1}{2}$$

$$\sin(2x) = \pm \frac{1}{\sqrt{2}}$$

✓



[4]

$$2x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

✓

$$x = 22.5^\circ, 67.5^\circ, 112.5^\circ, 157.5^\circ$$

✓

c) $2 \sin^2 x - 3 \sin x - 2 = 0$ for $[0^\circ, 720^\circ]$

[5]

$$2x^2 - 3x - 2 = 0 \quad \checkmark$$

$$\frac{(2x-4)(2x+1)}{2} = 0$$

$$(x-2)(2x+1) = 0 \quad \checkmark$$

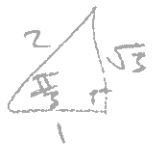
$\therefore \sin x = 2 = 0$
done

or $2 \sin x + 1 = 0$

$$\sin x = -\frac{1}{2} \quad \checkmark$$

$$x = 240^\circ, 300^\circ, 600^\circ, 660^\circ$$

\checkmark
 \checkmark



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MATHEMATICS Methods Units 1 & 2

Test 4 – Trigonometric Graphs and Equations

Chapter 14

Semester 1 2019

Section Two - Calculator Assumed

Time allowed for this section

Working time for this section: 20 minutes

Marks available: 20 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

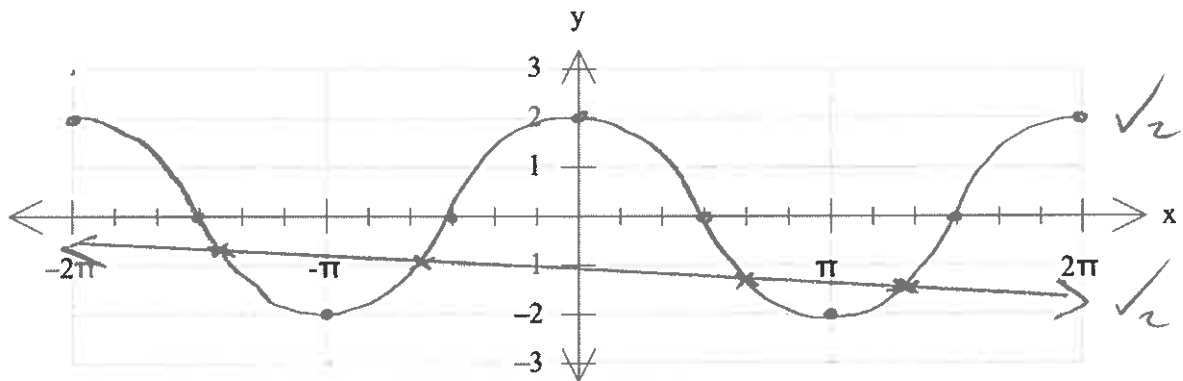
Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates

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1. (6 marks)

- a. Sketch the functions $y = 2\cos x$ and $x + 10y + 10 = 0$ on the same set of axes and clearly indicate any points of intersection. [4]



- b. Hence, graphically solve $2\cos x = -\frac{x+10}{10}$ for $-2\pi \leq x \leq 2\pi$. [2]

$$\begin{aligned} &(-4.43, -0.56) \left(\frac{1}{2}\right) \\ &(-1.96, -0.80) \left(\frac{1}{2}\right) \\ &(2.23, -1.22) \left(\frac{1}{2}\right) \\ &(3.94, -1.39) \left(\frac{1}{2}\right) \end{aligned}$$

2. (4 marks)

A trigonometric function has equation $P = a\cos(bt + \frac{\pi}{4})$. Find the values of a and b given that P has a maximum value of 4 and a period of 6π .

$$\text{max } 4 \checkmark \therefore a = 4 \checkmark$$

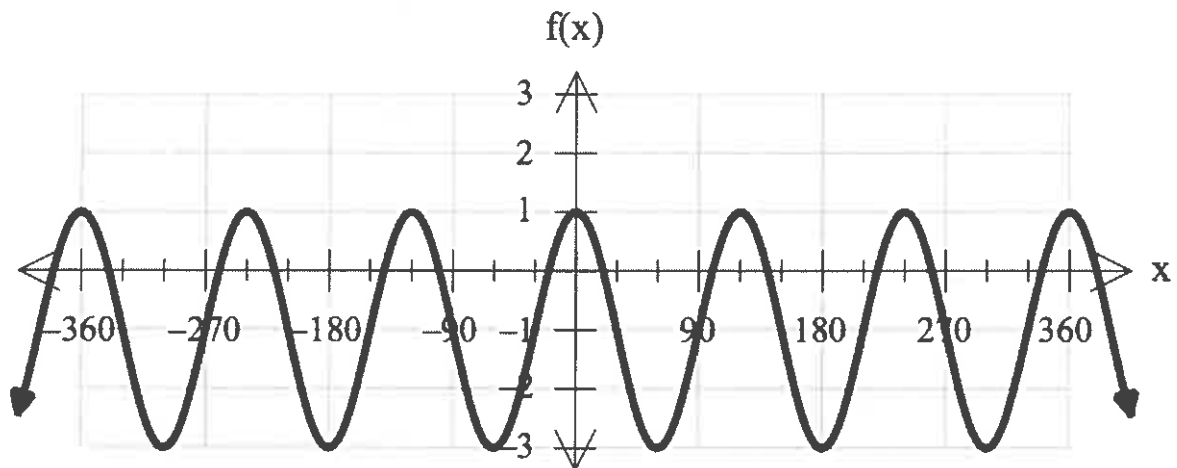
$$\text{period } 6\pi \Rightarrow T = \frac{2\pi}{6\pi} = \frac{1}{3} \therefore b = \frac{1}{3} \checkmark$$

3. (10 marks)

Find the equations of the following trigonometric functions.

a.

[4]

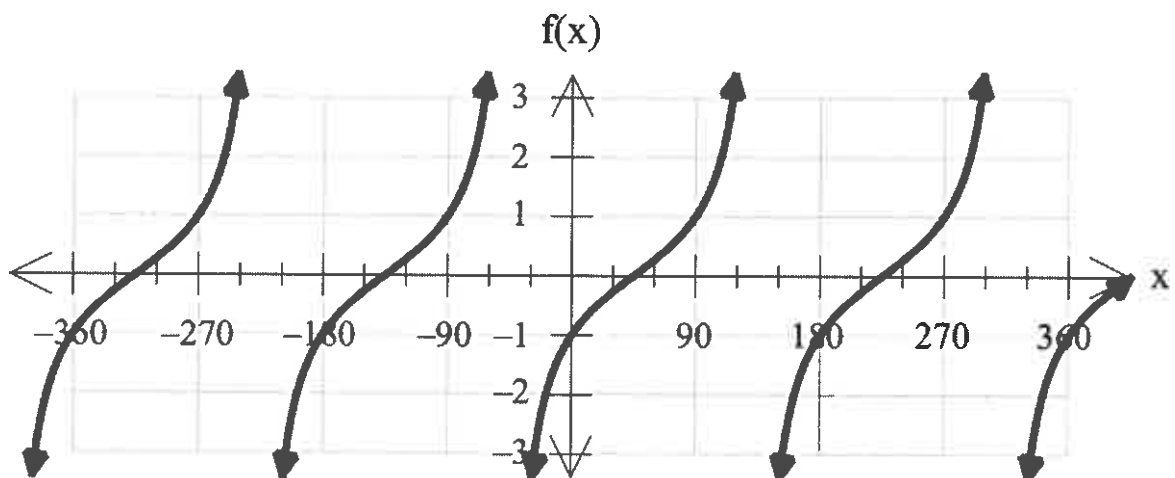


3 times.

$$f(x) = 2 \cos(3x) - 1$$

b.

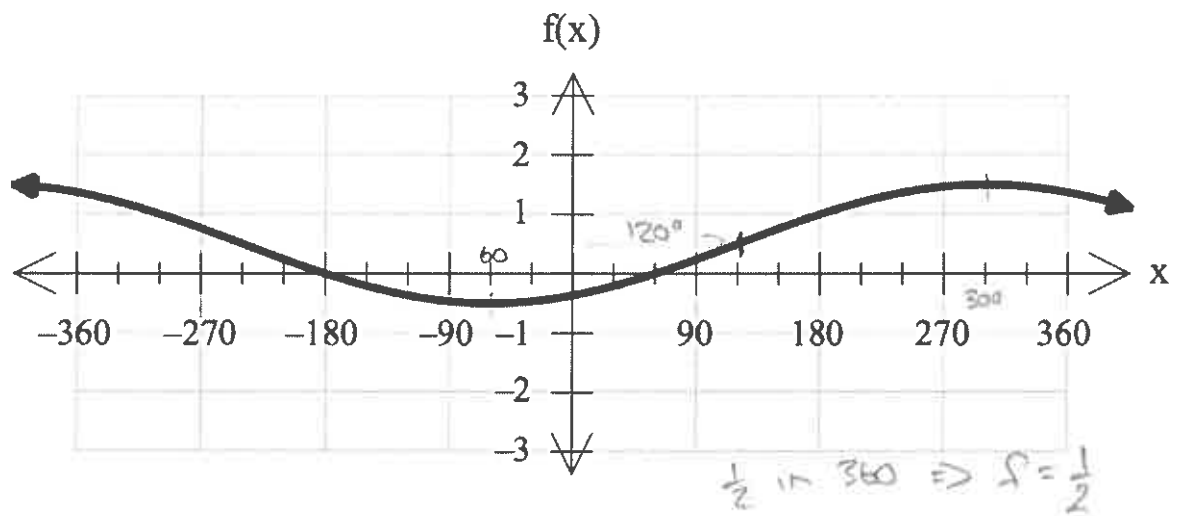
[2]



$$f(x) = \tan(x - 45^\circ)$$

c.

[4]



amplitude is 1

vertical shift $+\frac{1}{2}$ horizontal shift 120° right

$$y = \sin \frac{1}{2}(x - 120^\circ) + \frac{1}{2}$$

$$y = \sin \left(\frac{x}{2} - 60^\circ \right) + \frac{1}{2}$$

End of Test