WILLETTON SENIOR HIGH SCHOOL

FINAL DRAFT

Feedback by 18 Oct Monday Recess

Changes:

Calc Free – Marks allocation

Calc Assumed – Added one question (Q11)

MATHEMATICS METHODS – UNIT TWO

TEST FOUR 2021

SECTION ONE: Calculator Free

STUDENT NAME: ....

TOTAL MARKS: .... / 37

TIME ALLOWED: 30 mins

CIRCLE YOUR TEACHER'S NAME:

Mrs Kalotay	Ms Leow	Mr Riemer	Mrs Scoles
Ms Thompson	Ms Tsen	Mr Whiteley	

- Formulae sheet supplied.
- No calculators allowed.
- If a question is worth more than 2 marks, sufficient working must be shown to justify your answer, in order to receive full marks.

1. Consider the sequence: 15 , _____ , _____ , 405 , ...

   a. Given that these terms follow an arithmetic sequence, determine the value of T_2. [3]

   $T_4 = 405 = 15 + 3d$ ✓

   $390 = 3d$

   $d = 130$ ✓

   $T_2 = 15 + 130 = 145$ ✓

   b. Given that these terms follow a geometric sequence, determine the value of T_3. [3]

   $T_4 = 405 = 15r^3$ ✓

   $27 = r^3$

   $r = 3$ ✓

   $T_3 = \frac{405}{3} = 135$ ✓

2. Determine all the possible values of x given that 4, x, $(x^2 - 3)$ are consecutive terms of a geometric sequence. [4]

   $\frac{4}{x} = \frac{x}{x^2-3}$ ✓

   $4x^2 - 12 = x^2$

   $3x^2 = 12$

   $x^2 = 4$ ✓

   $x = 2$ or -2 ✓ ✓

3. Determine the derivative of the following functions.

   a. $f(x) = 3x^4 - 5$ [1]

   $f'(x) = 12x^3$

   b. $f(x) = 4\pi^2 - \sqrt{x}$ [2]

   $f(x) = 4\pi^2 - x^{\frac{1}{2}}$

   $\left.\begin{array}{l} f'(x) = -\frac{1}{2}x^{-\frac{1}{2}} \\ f'(x) = -\frac{1}{2\sqrt{x}} \end{array}\right\}$ ✓✓

   if 8π is in the answer (-1)

   c. $f(x) = \frac{5x^2-4}{x^3}$ [2]

   $f(x) = 5x^{-1} - 4x^{-3}$

   $\left.\begin{array}{l} f'(x) = -5x^{-2} + 12x^{-4} \\ f'(x) = \frac{-5}{x^2} + \frac{12}{x^4} \end{array}\right\}$ ✓✓

4. Determine the derivative of $y = x^3 - 12x + 3$ using first principles. [4]

$$\lim_{h \to 0} \frac{(x+h)^3 - 12(x+h) + 3 - (x^3 - 12x + 3)}{x+h \quad - \quad x}$$

$$\lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 12x - 12h + 3 - x^3 + 12x - 3}{h}$$

$$\lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 12h}{h}$$

$$\lim_{h \to 0} 3x^2 + 3xh + h^2 - 12$$

since $h \to 0$ $\quad \lim_{h \to 0} \Rightarrow 3x^2 - 12$

5. Find the equation of the tangent to the curve $y = x^2 + 4x - 1$ at the point where $x = 1$. [4]

$\frac{dy}{dx} = 2x + 4$ ✓ $\qquad y = 4$ ✓

$\frac{dy}{dx}\Big|_{x=1} = 2(1) + 4 = 6$ ✓

$y = 6x + c$

$4 = 6(1) + c$

$c = -2$

$y = 6x - 2$ ✓

6. Determine the anti-derivative, in simplified form for each of the following functions.

a. $3x^4 - 6$ [1]

$\frac{3}{5}x^5 - 6x + C$ ✓

b. $2x(3x-1)^2$ [2]

$2x(9x^2 - 6x + 1)$

$= 18x^3 - 12x^2 + 2x$ ✓

$\frac{18x^4}{4} - \frac{12x^3}{3} + \frac{2x^2}{2} + C$

$= \frac{9}{2}x^4 - 4x^3 + x^2 + C$ ✓

if forgotten $+C$ overall (-1)

7. A function is defined by $f(x) = x^4 - 8x^2 + 7$. After factorisation, $f(x) = (x^2 - 7)(x^2 - 1)$.

a. Given that $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ are the coordinates of the x-intercepts of $f(x)$, determine the coordinates of the remaining x-intercepts. [2]

$x^2 - 1 = 0$

$x = \pm 1$

$(1,0)$ ✓ and $(-1,0)$ ✓

b. Use calculus techniques to determine the nature and location of any stationary points. [6]

$f'(x) = 4x^3 - 16x$

$f'(x) = 0 \Rightarrow 4x^3 - 16x = 0$ ✓

$4x(x^2 - 4) = 0$

$x = 0,\ x = 2,\ x = -2$ ✓

$\boxed{\text{test}}$ ✓

$x = 0$

$y = 7$

x	<0	0	>0
$\frac{dy}{dx}$	+ve	0	−ve
	/	—	\

$(0,7)$ max ✓

$x = 2$

$y = -9$

x	<2	2	>2
$\frac{dy}{dx}$	−ve	0	+ve
	\	—	/

$(2,-9)$ min ✓

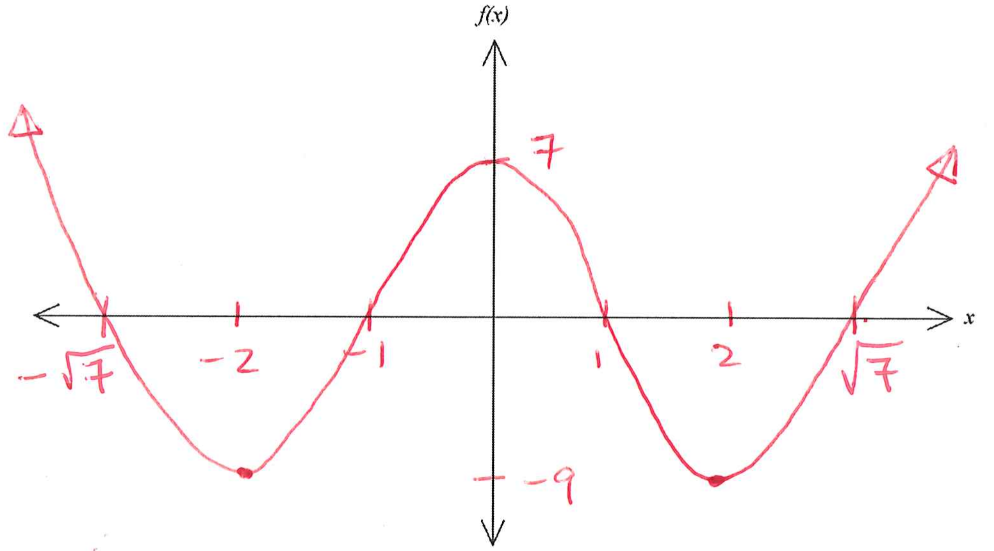
$x = -2$

$y = -9$

x	<-2	-2	>-2
$\frac{dy}{dx}$	−ve	0	+ve
	\	—	/

$(-2,-9)$ min ✓

c. In the space below, draw a sketch of the function, including the location of any intercepts, stationary points and indicate the behaviour as $x \to \pm\infty$. [3]



intercepts ✓

turning points ✓

shape ✓

WILLETTON SENIOR HIGH SCHOOL

MATHEMATICS METHODS – UNIT TWO

TEST FOUR 2021

SECTION TWO: Calculator Assumed

STUDENT NAME: ....

TOTAL MARKS: .... / 25

TIME ALLOWED: 20 mins

CIRCLE YOUR TEACHER'S NAME:

Mrs Kalotay	Ms Leow	Mr Riemer	Mrs Scoles
Ms Thompson	Ms Tsen	Mr Whiteley	

- Formulae sheet supplied.
- One page of A4 notes allowed.
- Classpads and scientific calculators are allowed.
- If a question is worth more than 2 marks, sufficient working must be shown to justify your answer, in order to receive full marks.

8. A fatal disease is caused by a particular bacteria increasing in such a way that the number of bacteria present after t hours is given by $N(t) = 5t^3 + 4t^2 + 300$. Determine:

a. The number of bacteria present initially. [1]

$N(0) = 300$ ✓

b. The average rate of increase of bacteria in the first 10 hours. [2]

$N(10) = 5700$

$\frac{5700 - 300}{10}$ ✓ $= 540$ bacteria/hour ✓

c. The expression for the instantaneous rate of change of bacteria. [1]

$N'(t) = 15t^2 + 8t$ ✓

d. The rate of change of the number of bacteria at:

i. t=150 minutes. [1]

150 mins = 2.5 hours

$N'(2.5) = 113.75$ bacteria/hour ✓

ii. t=1 day. [1]

1 day = 24 hours

$N'(24) = 8832$ bacteria/hour ✓

See next page for Q9

10. A sequence is defined by $T_{n+1} = T_n - 3.55,\ T_1 = 835$.

a. Determine T_{150}. [1]

$T_{150} = 306.05$ ✓

b. Determine the sum of the first 35 terms, S_{35}. [1]

$S_{35} = 27112.75$ ✓

c. Determine the value of n that will maximise S_n and state the corresponding value of S_n. Explain why this value of S_n is the maximum. [3]

$n = 236$ ✓

$S_n = 98618.5$ ✓

T_{237} onwards is negative ✓

9. A geometric sequence with $T_2 = 43.75$ has a sum to infinity of 400. Determine all possible values of T_1 for this sequence. [3]

$$\left.\begin{array}{l} T_2 = ar = 43.75 \\ S_\infty = \dfrac{a}{1-r} = 400 \end{array}\right\} ✓$$

$T_1 = 50$ ✓

$T_1 = 350$ ✓

Solve $a = 50$, $r = 0.875$

$a = 350$, $r = 0.125$

11. The gradient of the tangent of a graph is given by $m = x^2 + x + k$, where k is a constant. Given that $f(0) = -2$ and $f(-1) = 0$, determine the equation of the graph. [3]

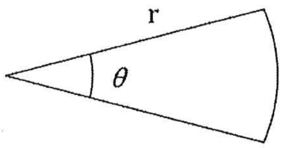
$f(x) = \frac{x^3}{3} + \frac{x^2}{2} + kx + c$ ✓

$$\left.\begin{array}{l} f(0) = -2 \Rightarrow c = -2 \\ f(-1) = 0 \Rightarrow -\frac{1}{3} + \frac{1}{2} - k + c = 0 \end{array}\right\} \text{Solve}$$

$k = -\frac{11}{6}$ ✓

$f(x) = \frac{x^3}{3} + \frac{x^2}{2} - \frac{11x}{6} - 2$ ✓

12. The perimeter of a sector of a circle, of radius r cm and angle θ radians, is 60 cm.



a. Show that $\theta = \frac{60}{r} - 2$. [2]

$P = 60 = 2r + r\theta$ ✓

$r\theta = 60 - 2r$

$\theta = \frac{60 - 2r}{r}$ ✓

$\theta = \frac{60}{r} - 2$

b. Hence, show that the area of the sector is given by $30r - r^2$. [2]

$A = \frac{1}{2} r^2 \theta$

$A = \frac{1}{2} r^2 \left(\frac{60}{r} - 2\right)$ ✓

$A = \frac{1}{2} r^2 \left(\frac{60}{r}\right) - \frac{1}{2} r^2 (2)$ ✓

$A = 30r - r^2$

c. Use calculus to determine the maximum area of the sector and state the corresponding values of r and θ. [4]

$\frac{dA}{dr} = 30 - 2r$

$\frac{dA}{dr} = 0 \Rightarrow r = 15$ ✓

max $A = 225\text{cm}^2$ ✓

$\theta = 2$ rad ✓

✓ test or explain that Area is a <u>negative</u> quadratic graph hence <u>max value</u>