Name: ANN

Esperance SHS

Year 12 MATHEMATICS METHODS
TEST 3 2016
Calculator Free

Total Marks: 39
Reading: 2 minutes Time Allowed: 40 minutes

Question 1 (5 marks)

(a) Simplify $\frac{3\log 100}{4\log 1000}$. (1 mark)

$= \frac{3\times 2}{4\times 3}$

$= \frac{1}{2}$

(b) Solve for x, where $\log_x 3 = \frac{1}{2}$. (2 marks)

$x^{\frac{1}{2}} = 3$

$x = 9$

(c) If $\log x = 0.313$, determine $\log\frac{1}{x^2}$. (2 marks)

$\log x^{-2}$

$= -2\log x \quad = -2\times 0.313$

$= -0.626$

Question 2 (5 marks)

Evaluate the following

(a) $\int \frac{3x}{5x^2-2}\,dx$ (2 marks)

$= \frac{3}{10}\int \frac{10x}{5x^2-2}\,dx$

$= \frac{3}{10}\ln(5x^2-2) + C$

(b) $\int_0^{\frac{\pi}{k}} \frac{\sin(x)}{1+\cos(x)}\,dx$ where k is a constant. (3 marks)

when $1+\cos x > 0$

$= \left[-\ln(1+\cos x)\right]_0^{\frac{\pi}{k}}$

$= -\ln(1+\cos\frac{\pi}{k}) + \ln 2$

Question 3 (8 marks)

Differentiate the following with respect to x, do not simplifying.

(a) $y = x^2 \ln(2x+3)$. (2 marks)

$$\frac{dy}{dx} = 2x \cdot \ln(2x+3) + x^2 \cdot \frac{2}{2x+3}$$

(b) $y = x \log_{10}(1+x)$ (3 marks)

$$= x \cdot \frac{\ln(1+x)}{\ln 10}$$

$$\frac{dx}{dx} = \frac{1}{\ln 10}\left(\ln(1+x) \cdot 1 + x \cdot \frac{1}{1+x}\right)$$

(c) $y = \dfrac{\ln(2x-1)}{x}$ (3 marks)

$$\frac{dy}{dx} = \frac{x \times \frac{2}{2x-1} - \ln(2x-1) \times 1}{x^2}$$

Question 4 (4 marks)

(a) Determine $\dfrac{d}{dx}[\ln(\cos^2 2x)]$ (2 marks)

$$= \frac{d}{dx}\left(2\ln(\cos 2x)\right)$$

$$= 2 \times \frac{-2\sin 2x}{\cos 2x}$$

Hence,

$$= -4\tan 2x$$

(b) determine $\int \tan 2x \; dx$ (2 marks)

$$\int -4\tan 2x = \ln(\cos^2 2x) + C$$

or use $c_1 + c_2$

$$\therefore \int \tan 2x = -\frac{1}{4}\left(\ln(\cos^2 2x) + c\right)$$

Question 5 **(8 marks)**

(a) Show that $\frac{5}{x+1}+\frac{x}{x^2-2}=\frac{6x^2+x-10}{(x+1)(x^2-2)}$ (2 marks)

$$LHS = \frac{5(x^2-2)+x(x+1)}{(x+1)(x^2-2)}$$

$$= \frac{6x^2+x-10}{(x+1)(x^2-2)}$$

$= RHS.$

Hence find,

(b) $\int \frac{6x^2+x-10}{(x+1)(x^2-2)}\,dx$ (3 marks)

$$= \int \frac{5}{x+1}\,dx + \int \frac{x}{x^2-2}\,dx$$

$$= 5\ln(x+1) + \frac{1}{2}\ln(x^2-2) + C$$

(c) Using your result from part (b), show that the exact value of

$\int_0^1 \frac{6x^2+x-10}{(x+1)(x^2-2)}\,dx = \frac{9}{2}\ln 2$ (3 marks)

needs 'absolute' values

$$= \left[5\ln(x+1) + \frac{1}{2}\ln|x^2-2|\right]_0^1$$

$$= 5\ln 2 - 5\ln 1 + \frac{1}{2}\ln|-1| - \frac{1}{2}\ln|-2|$$

$$= 5\ln 2 - \frac{1}{2}\ln 2$$

$$= \frac{9}{2}\ln 2.$$

Question 6 **(3 marks)**

Find an exact solution for x if $7^{2x} = 5^{x-3}$.

$7^{2x} = \frac{5^x}{5^3}$

$5^3 = \frac{5^x}{7^{2x}}$

$5^3 = \left(\frac{5}{49}\right)^x$

$\log_{\left(\frac{5}{49}\right)} 125 = x$

$\therefore \frac{\log 125}{\log\left(\frac{5}{49}\right)}$

$= \frac{\log 125}{\log 5 - \log 49}$

OR.

$2x \log 7 = (x-3) \log 5$

$2x \log 7 - x \log 5 = -3 \log 5$

$x(2\log 7 - \log 5) = -3 \log 5$

$x = \frac{-\log 125}{\log 49 - \log 5}$

Question 7 **(6 marks)**

Let $y = \ln\sqrt{\frac{1+x^2}{1-x^3}}$.

(a) Rewrite y as the difference of two logarithms without the radical sign. (3 marks)

$y = \frac{1}{2}\ln(1+x^2) - \frac{1}{2}\ln(1-x^3)$

(b) Hence, find $\frac{dy}{dx}$. You do not need to simplify your answers. (3 marks)

$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{2x}{1+x^2}\right) - \frac{1}{2}\left(\frac{-3x^2}{1-x^3}\right)$

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Question 8 (7 marks)

(a) Given that $a = log_3 2$ and $b = log_3 5$, find in terms of a and b:

(i) $log_3 0.4$ (2 marks)

$= \log_3 \frac{2}{5}$

$= a - b$

(ii) $log_3 30$ (2 marks)

$= \log_3 (2 \times 5 \times 3)$ $\log_3 (5 \times 4 \ldots)$

$= \log_3 2 + \log_3 5 + \log_3 3$

$= a + b + 1$

(b) For each of the following, express p in terms of q.

(i) $log_e p = 2 log_e q$ (1 mark)

$p = q^2$

(ii) $\dfrac{e^{2p}}{3} = q$ (2 marks)

$e^{2p} = 3q$

$2p = \ln 3q$

$p = \dfrac{\ln 3q}{2}$

Question 9 **(6 marks)**

The annual growth rate for an investment that is growing continuously is given by $r = \frac{1}{t}\ln\left(\frac{A}{P}\right)$ where P is the principal and A is the amount after t years. An investment of \$10 000 in Dell Computer stock in 2012 grew to \$31 800 in 2015.

a Assuming the investment grew continuously, what was the annual growth rate (to 4 decimal places)? (2 marks)

$$r = \frac{1}{3}\ln\left(\frac{31800}{10000}\right)$$
$$= 0.3856$$

b If Dell continues to grow at the same rate, what will the \$10 000 investment be worth in 2019? (2 marks)

$$0.3856 = \frac{1}{7}\ln\left(\frac{A}{10000}\right)$$

$A =$ ~~414 56 678.33~~

\$148706.50

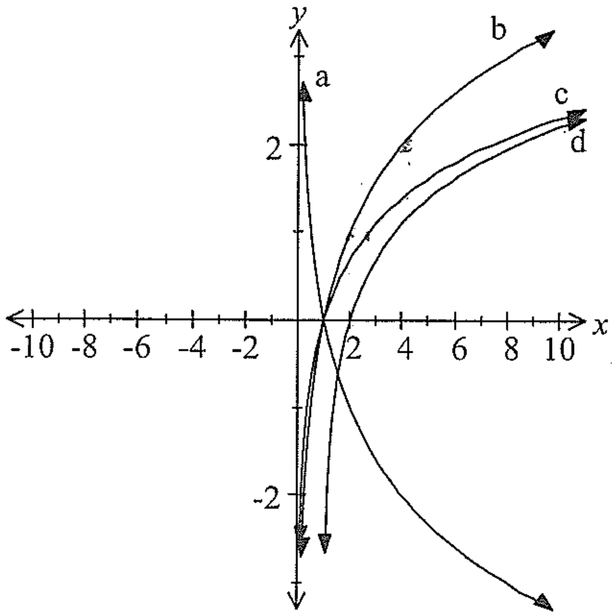
c Assuming the investment grew continuously at the same rate, how long will it take for the \$10 000 investment to grow to \$500 000? (2 marks)

$$0.3856 = \frac{1}{t}\ln\left(\frac{500\,000}{10\,000}\right)$$

$t = 10.1453$ years after 2012.

Question 10 **(4 marks)**

Match the equation with the graph. (not all equations are used)



$y = \ln x$	c
$y = \log_{0.5} x$	a
$y = \log_2 x$	b.
$y = \ln(x + 1)$	
$y = \ln(x - 1)$	d
$y = 2\ln x$	

Question 11 **(5 marks)**

The sound level L, in decibels (dB), for a single sound of pressure p, in millipascals (mPa), is calculated using the formula $L = 20\log\frac{p}{0.02}$, $p > 0$.

(a) Determine the sound level corresponding to a sound pressure of 0.02 mPa. (1 mark)

0

(b) Determine the sound pressure corresponding to a sound level of 80 dB. (2 marks)

$80 = 20 \log \frac{p}{0.02}$

$p = 200$ mPa

(c) Sketch the graph of the above function on the axes below with $\log\frac{p}{0.02}$ on the horizontal axis. Indicate the scale used on the vertical axis. (2 marks)