Unit 2
Semester 2 2018
Mathematics Methods Test 1

Baldivis Secondary College

32

Name: Solutions

Date: _____

Total Time: *60 minutes*

Weighting: *6% of the year.*

*This test comprises of **TWO sections**. The **first section** is **calculator free** where no calculators of any kind are to be used. The **second section** is **calculator assumed** where the CAS calculator may be used. All questions must be answered in both sections. **Answers should be in EXACT form or rounded appropriately**. All working should be shown in the space provided. Solutions without working may not be awarded full marks. Please take the marks for each question into account when answering the question.*

SECTION 1: CALCULATOR FREE

Time: *30 minutes* **Equipment Allowed:** *SCSA Formula sheets*

Marks for Section 1: *26 marks*

1. [4 marks: 1, 1, 1, 1]
Find the value of these expressions, writing your answer as a fraction where appropriate:

a) 2^{-3}

$\frac{1}{8}$

b) $5^0 + 2^{-2}$

$1 + \frac{1}{2^2} = 1\frac{1}{4}. \quad \frac{5}{4}$

c) $9^{\frac{3}{2}}$

$(\sqrt{9})^3 = 3^3 = 27.$

d) $\left(1\frac{2}{3}\right)^2$

$\left(\frac{5}{3}\right)^2 = \frac{25}{9}$

2. [4 marks: 2, 2]
Simplify the following indices, leaving your answer as positive indices:

a) $a^3b^3 \times a^{-2}b^{-7}$

$= ab^{-4}$

$= \frac{a}{b^4}$

b) $\frac{(6a^3b^2)^2}{(-3a^2b^3)^3}$

$= \frac{36a^6b^4}{-27a^6b^9}$

$= \frac{-4}{36^5}$

3. [4 marks: 2, 2]

Write the following using index notation, with positive indices:

c) $\sqrt[4]{y^{-3}} \times \sqrt{9x^3}$

More Space

$y^{-\frac{3}{4}} (9x^3)^{\frac{1}{2}}$

$\frac{(3^2 x^3)^{\frac{1}{2}}}{y^{\frac{3}{4}}} = \frac{3x^{\frac{3}{2}}}{y^{\frac{3}{4}}}$

d) $\sqrt[3]{x^5} \times \sqrt{x^3}$

$x^{\frac{5}{3}} \times x^{\frac{3}{2}}$

$x^{\frac{10}{6}+\frac{9}{6}}$

$x^{\frac{19}{6}}$.

4. [6 marks: 3, 3]

Simplify each of the following for x

a) $5^x \times 25^{(x+1)} = 0.04$

More Space

$5^x \times 5^{2(x+1)} = 0.04$.

$x^x \times x^{2x+2} = 0.04$

$0.04 = \frac{1}{25}$

$x^{3x+2} = \frac{1}{25} = \frac{1}{5^2}$

$x^{3x+2} = 5^{-2}$

$\therefore 3x+2 = -2$ $\therefore x = -\frac{4}{3}$

b) $9^{1-x} = 27^{x-2} \times 3^{x+2}$

$3^{2(1-x)} = 3^{3(x-2)} \times 3^{x+2}$

$3^{2-2x} = 3^{3x-6} \times 3^{x+2}$

$3^{2-2x} = 3^{4x-4}$.

$\therefore 2-2x = 4x-4$.

$6 = 6x$

$x = 1$

5. [2 marks: 1, 1]

When intercepted by the spacecraft Horizon recently, Pluto was about 4 864 100 000 km from Earth

a) Express this distance in scientific notation

4.8641×10^9

b) Write this distance in scientific notation correct to 2 significant figures

4.9×10^9.

M

6. **[2 marks: 1, 1]**

a) The number of feral cats in a National park was initially 3500 but decreasing at an annual rate of 7.5%. Write a rule describing the number of cats after t years.

N^0 of Cats $= 3500(0.925)^t$.

b) The number of bacteria in a culture after t minutes is described by the rule $B = 50(2.1)^{t+1}$. Describe the percentage change in the number of bacteria each minute.

110%. increasing every minute

7. **[1 mark]**

Estimate a solution to the equation $2^x = 17$, giving your estimate to one decimal place.

$2^1 = 2$ ∴ x is Between 4 & 5
$2^2 = 4$ but closer to 4.
$2^3 = 8$
$2^4 = 16$ ∴ $x \approx 4.1$
$2^5 = 32$.

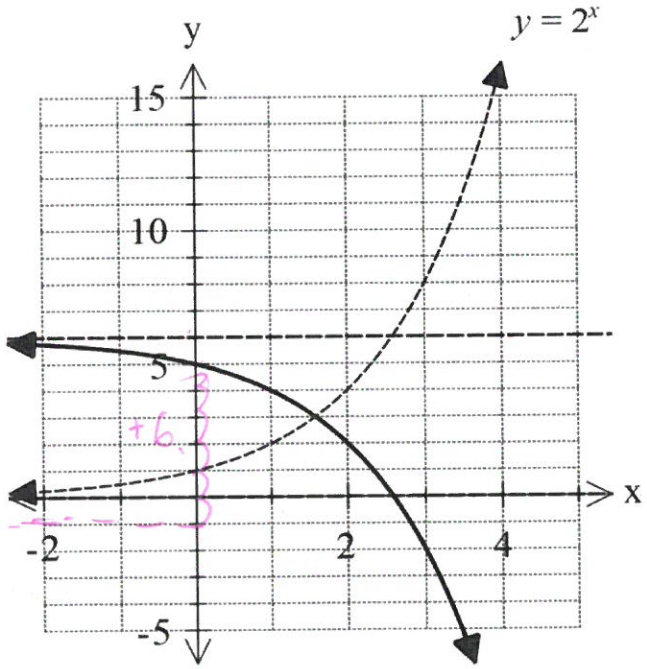
$x^{4.2} =$

8. **[3 marks: 1, 2]**

a) The curve $y = 2^x$ is transformed into $y = 2^{x+4}$.
Describe the transformation that has occurred.

H has been translated 4 units left

b) Find the rule below for Graph A if it represents a transformation of $y = 2^x$.



Graph A

-ve due to flip.

$y = -2^x + 6$

$y = ka^x + c$

$k = -1$

$a = 2$

$c = 6$.

~ END OF SECTION ONE ~

Name: _____

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SECTION 2: CALCULATOR ASSUMED

Time: *30 minutes*

Equipment Allowed: *Curriculum Council Formula sheets, CAS calculator, 1 page of notes (A4 one side)*

Marks for Section 2: *26 marks*

9. [4 marks: 1, 1, 2]

Scientists were trying to increase the population of a rare species of fish so they placed 50 fish in a small lake and monitored the population monthly. They discovered that the population of fish, ***P***, increased according to the rule:

$$P = 50 \times 1.15^{t}$$

where ***t***, was the time in months since the first fish was placed in the lake.

a) By what percentage did the population of the fish increase each month?

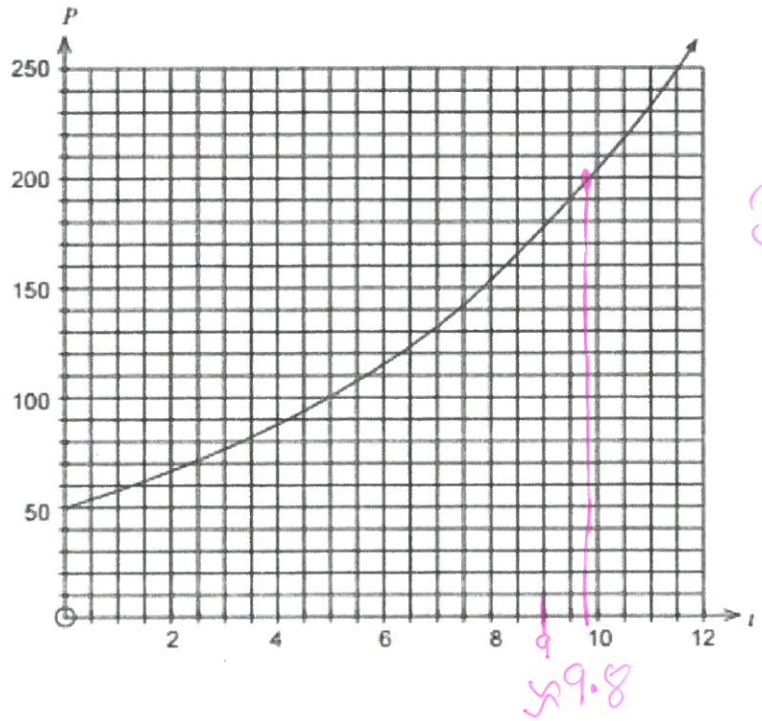
15 %

b) What was the population of the fish after 18 months?

$P = 50 \times 1.15^{18}$

$\approx$ 619 fish

c) A graph of the fish population is shown below. **Use the graph** to determine during which month the population of the fish first exceeded 200. Clearly **indicate all working on the graph** and state your solution.



During the 9th Month

10. [5 marks: 1, 2, 2]

The amount of grams of a dangerous radioactive substance remaining after a dangerous accidental spill in a laboratory, at time t minutes, is given by the rule:

$$A = 350(0.969)^t.$$

a) How much radioactive substance was spilled initially?

350g

b) How many grams remain after 1 hour?

$A = 350(0.969)^{60} = 52.9g$

c) For the laboratory to be safe there must be less than 1 gram of the substance left. Determine the time to the nearest minute, when it is safe to go back into the laboratory.

solve($350 \times 0.969^x = 1$

$x = 186.02$ → go up

∴ 187 minutes

11. [5 marks: 2, 3]

The changing population is shown below.

Year	2010	2011	2012	2013
Population (in millions)	20	12	7.2	4.32

a) Find the yearly percentage change in population.

$20r = 12$ $12r = 7.2$ or $1 - 0.6 = 0.4$

$r = 0.6$ $r = 0.6$

Decay of 40%

b) Create a rule for the population, and use it to find the population in 2017.

$P = 20(0.6)^{2017 - 2010}$

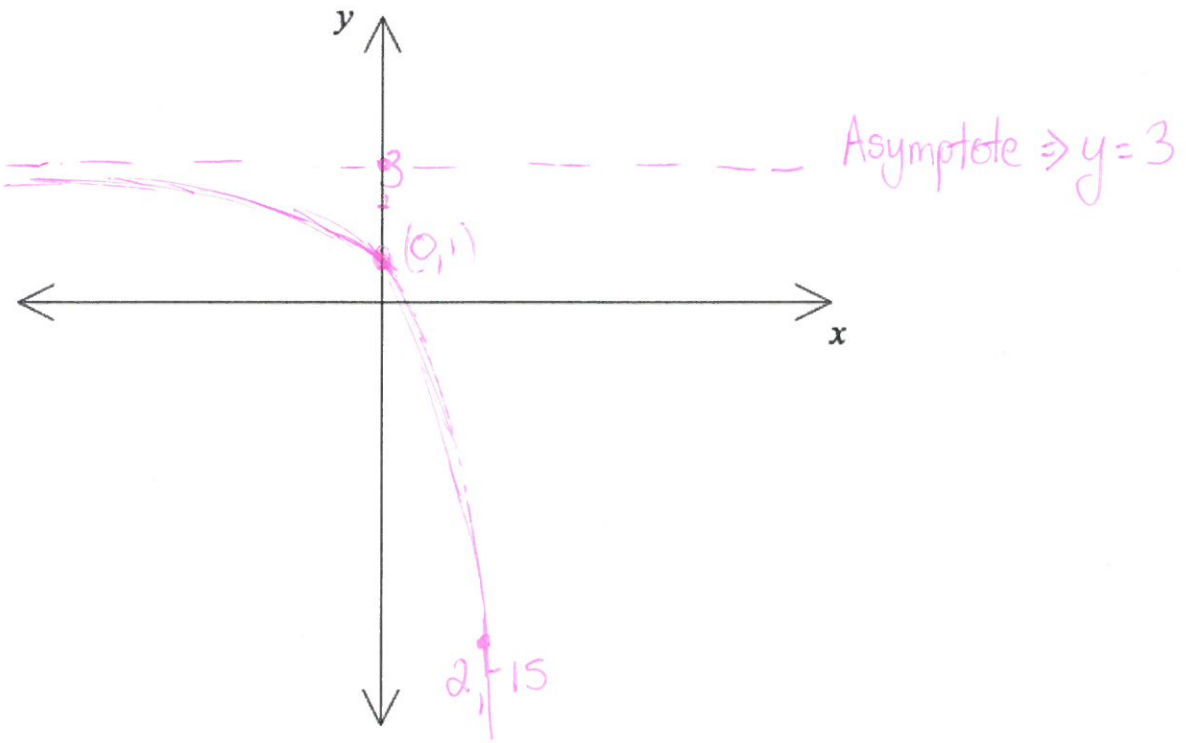
$= 0.55987$ million

≈ 559872 people

12. [4 marks: 2, 2]

The graph of an exponential function has a horizontal asymptote of y = 3, a y-intercept of (0,1) and goes through the point (2,-15).

a) Sketch the function below, indicating all key information..



b) Find a possible rule for the function in the form $y = Aa^x + c$.

Sub in (0,1)

$A = -2$

$y = Aa^x + 3$

no matter what a equals

a^0 will equal 1

Sub in (2, -15)

$a = \pm 3$

$a \neq 3$

$\therefore a = 3$

$\therefore$

$\boxed{y = -2a^x + 3}$

Asymptote at $y = 3$

$\therefore +3$

$y = -2a^x + 3$

$\therefore$

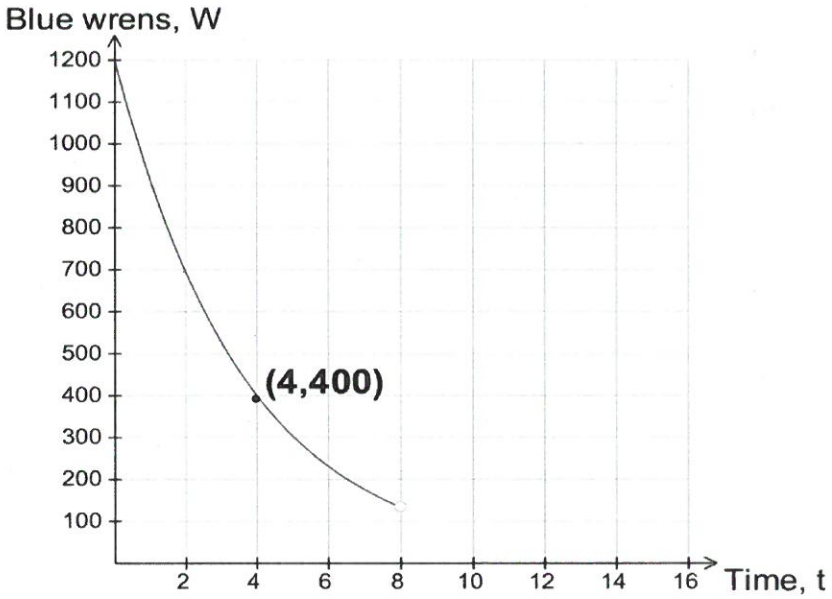
$1 = A \times 1 + 3$

$\therefore$ A must equal -2

$y = -2(3^x) + 3.$

13. [8 marks: 2, 2, 2, 2]

The population of blue wrens in a particular area of the South West has been studied over 8 years. The results are represented on the graph given below.



The population is modelled by the exponential function $W = a \times b^t$ and a point on the graph is indicated.

a) Find the value of b, to 2 decimal places.

$1200r^4 = 400$

$r = 0.76$

b) State the annual percentage change in the population of wrens.

$1 - 0.76 = 0.24$

24% decline

c) Find how many wrens there were after 13 years.

$W = 1200(0.76)^{13}$

$= 33.87$

$\approx$ 34 wrens left

d) When is it predicted there will be 10 wrens left in the area?

$10 = 1200(0.76)^t$

$8.33 \times 10^{-3} = 0.76^t$

$t = 17.44$

Approx halway between 17th & 18th years.

~ END OF TEST ~