MARKING KEY

Eastern Goldfields College

Year 12 MATHEMATICS METHODS

TEST 2 2016

Calculator Free

Total Marks: 36

Reading: 2 minutes Time Allowed: 36 minutes

1. (10 marks - 2, 3, 3, 2)

Find the following indefinite integrals:

(a) $\int (x^3+6x^2-7x+1)dx$

$= \frac{x^4}{4} + 2x^3 - \frac{7x^2}{2} + x + c$ ✓✓

(b) $\int \frac{20x^3-4}{x^2}dx$

$= \int 20x - 4x^{-2}dx = 10x^2 + 4x^{-1} + c$ ✓

$= 10x^2 + \frac{4}{x} + c$ ✓ ✓

(c) $\int (\sqrt{5x})dx$

$= \sqrt{5}\int x^{1/2}dx = \sqrt{5}\,\frac{2x^{3/2}}{3} + c = \frac{2\sqrt{5}}{3}x^{3/2} + c$ ✓ ✓

(d) $\int (4e^{2x} - \cos(2x))dx$

$= \frac{4e^{2x}}{2} - \frac{\sin 2x}{2}$

$= 2e^{2x} - \frac{\sin 2x}{2} + c$ ✓ ✓

2. (6 marks - 3, 3)

Evaluate

(a) $\int_1^3 x(9x-4)dx = \int_1^3 9x^2 - 4x\,dx$ ✓

$= \left[3x^3 - 2x^2\right]_1^3$ ✓

$= (54-18)-(3-2)$

$= 35.$ ✓

(b) $\int_{\pi/2}^{\pi}(\sin(x)-\cos(x))dx$

$= \left[-\cos x - \sin x\right]_{\pi/2}^{\pi}$ ✓

$= -\cos\pi - \cancel{\sin\pi} - \left(\left(-\cos\frac{\pi}{2}\right) - \sin\frac{\pi}{2}\right)$ ✓

$= 1 + 1$

$= 2$ ✓

3. (3 marks)

Evaluate, writing your answer with positive indices:

$\int_0^5 e^{-2x}dx$

$= \left[\frac{e^{-2x}}{-2}\right]_0^5 = \frac{e^{-10}}{-2} - \left(\frac{1}{-2}\right)$ ✓

$= \frac{1-e^{-10}}{2}$ ✓✓

4. (3 marks)

Find y in terms of x given that $\frac{dy}{dx} = 12x^2 - 15$, and that when $x = 2, y = 5$

$y = 4x^3 - 15x + c$ ✓

$(2,5)$ $5 = 32 - 30 + c$ ✓

$\underline{3 = c}$

$\therefore y = 4x^3 - 15x + 3.$ ✓

5. **[5 marks – 3, 2]**

(a) Find $\frac{dy}{dx}$ given $y = \int_x^1 (3t^3 - 2t)\,dt$ $= -\int_1^x (3t^3 - 2t)\,dt$ ✓

$\frac{dy}{dx} = -3x^3 + 2x$

✓✓

(b) Given $f(x) = 2x^2$, find $\int_2^4 f'(x)\,dx$ $= 2x^2\Big]_2^4$ ✓ $= 32 - 8 = 24.$ ✓

6. **[5 marks]**

A curve has a gradient function $\frac{dA}{dt} = 60 - 3bt^2$, where b is a constant.

Given that the curve has a maximum turning point when $t = 2$ and passes through the point (1,62),

Determine the equation of the curve.

✓ $t = 2,\ \frac{dA}{dt} = 0 = 60 - 12b.$

$\underline{5 = b}.$ ✓

$A = 60t - 5t^3 + c$ ✓

(1,62) $62 = 60 - 5 + c$ ✓

$c = 7$

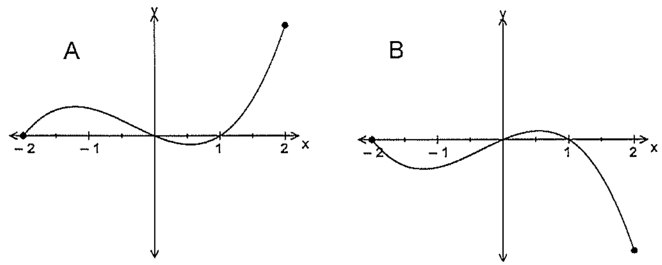
$\therefore A = 60t - 5t^3 + 7$ ✓

7. **(4 marks – 1, 3)**

Two functions; $f(x)$ and $g(x)$ exist such that:

$\int_1^0 f(x)\,dx = -1$ and $\int_{-2}^0 g(x)\,dx = 2$

(a) Determine which of the following functions are $f(x)$ and $g(x)$.



A = $g(x)$ and B = $f(x)$ ✓

(b) Place <, >, or = to make the statements below true.

(i) $\int_{-2}^{2} f(x)\,dx$ $<$ $\int_{-2}^{2} g(x)\,dx$ ✓

(ii) $\int_{0}^{2} f(x)\,dx$ $>$ $\int_{-2}^{2} f(x)\,dx$ ✓

(iii) $\int_{-2}^{2} g(x)\,dx$ $>$ 0 ✓

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Calculator Assumed

Total Marks: 25

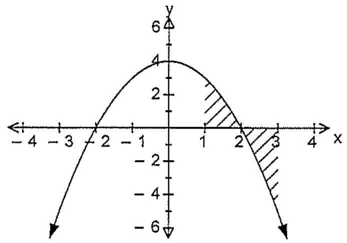
Reading: 2 minutes Time Allowed: 25 minutes.

Name:______________________________

8. [7 marks – 1, 2, 1, 3]

(a) Find $\int_1^3 4 - x^2 \; dx$ $= -\frac{2}{3}$

(b) Find the Area shaded below



$y = 4 - x^2$

$A = \int_1^3 |4-x^2| \, dx$

$= 4$

or

$\int_1^2 4-x^2 \, dx - \int_2^3 4 - x^2 \, dx$.

(c) Explain why the answer for (a) is different than (b)

(a) is the sum of signed areas (ie subtracts the area below from above)

(d) Determine the value of a to 3 decimal places so that $\int_{0.5}^{a} 4 - x^2 dx$ is 2.25.

$\left[4x - \frac{x^3}{3}\right]_{0.5}^{a} = 2.25$

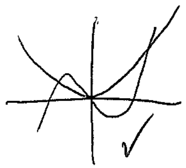
(can solve on calc)

$a = -3.903, 1.194, 2.709$

-1 per missing

9. (5 marks – 4, 1)

(a) Write an expression that would enable you to find the total area of the two closed regions formed by the intersection of the curves $y = x^3 - 2x$ and $y = x^2$.



Int: $-1, 0, 2$

$A = \int_{-1}^{0} (x^3 - 2x - x^2) \, dx + \int_0^2 (x^2 - x^3 + 2x) \, dx$

(b) What is the total area of the two closed regions formed by the intersection of the curves $y = x^3 - 2x$ and $y = x^2$?

$A = 3.083 \, u^2$

10. (5 marks – 2, 3)

Fuel was observed to leak from a damaged tank at a rate of $\frac{224}{5e^{0.1t}}$ litres per minute, where t is the number of minutes that have elapsed since the tank was ruptured.

(a) How much fuel leaked from the tank during the first two minutes?

$V = \int_0^2 \frac{224}{5e^{0.1t}} \, dt$

$= 87.2L$

(b) If the tank initially contained 350 litres of fuel, determine the time taken, to the nearest second, for the tank to empty.

$\int_0^t \frac{224}{5e^{0.1t}} \, dt = 350$.

$448 - \frac{448}{e^{0.1t}} = 350$

$t = 15.1982$

$= 15m \, 12s$

11. **(8 marks - 6, 2)**

A particle travels along a straight line such that its acceleration at time t seconds is equal to (2t - 8) m/s^2. When t = 3 the displacement is 9 metres and when t = 6 the displacement is -6 metres.

(a) Find the displacement and velocity when t = 4

$a = 2t - 8$

$v = t^2 - 8t + c$ ✓

$s = \frac{t^3}{3} - 4t^2 + ct + d$ ✓

$t = 3$ $\quad 9 = 9 - 36 + 3c + d$

$36 = 3c + d$ ✓

$t = 6$ $\quad -6 = 72 - 144 + 6c + d$

$66 = 6c + d$ ✓

$c = 10, d = 6.$ ✓

$t = 4$ $\quad S = 3\frac{1}{3}\,m$ ✓

(b) Find the distance traveled in the first 10 seconds.

$dist = \int_0^{10} |t^2 - 8t + 10|\, dt$ ✓✓

$= 72.53\,m$