MINDARIE
SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS NOW

MATHEMATICS: SPECIALIST 1 & 2

SEMESTER 1 2015

TEST 3

Resource Free

Time Allowed: 18 minutes

Total Marks: 16

1. [1, 1 marks]

Given that the position of particles A and B are given by the equations $\mathbf{r}_A(t) = 3\mathbf{i} - 5\mathbf{j} + t(\cancel{4\mathbf{j} - 2\mathbf{i}})$ $-2i + 4j$ and $\mathbf{r}_B(t) = 10\mathbf{i} + \mathbf{j} + t(2\mathbf{i} + 3\mathbf{j})$, determine

(a) ${}_A\mathbf{r}_B$ $(0) = r_A(0) - r_B(0)$

$= (3i - 5j) - (10i + j)$

$= -7i - 6j$ ✓

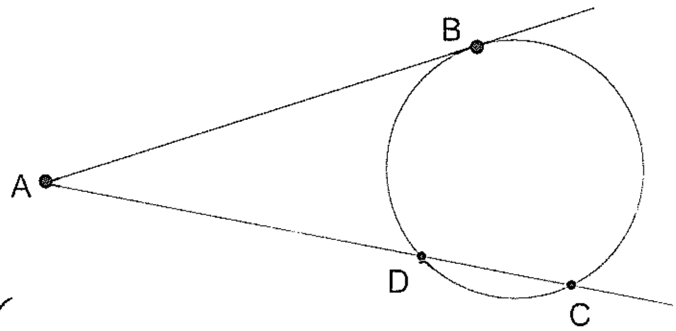
(b) ${}_B\mathbf{v}_A$ $= v_B - v_A$

$= (2i + 3j) - (-2i + 4j)$

$= 4i - j$ ✓

3. [1, 3 marks]

A line drawn from a point A forms a tangent to a circle at B. A second line from A cuts through the same circle at point C and D.



(a) State a relationship between the lengths of the line segments AB, AD and ~~D~~AC.

$AB^2 = AD \times AC$ ✓

(b) Hence prove that ΔABD ~ ΔACB.

$\frac{AB}{AC} = \frac{AD}{AB}$ Same ratio ✓

$\angle BAD = \angle CAB$ Same angle ✓

Must have reason

$\therefore \triangle ABD \sim \triangle ACB$ (SAS) ✓

4. [3, 1 marks]

Given vectors **m** = 5**i** – 2**j** and **n** = 4**i** + 3**j**, determine

(a) the scalar projection of **m** onto **n**.

$$\underset{\sim}{m} \cdot \hat{\underset{\sim}{n}} = \frac{(5\underset{\sim}{i} - 2\underset{\sim}{j}) \cdot (4\underset{\sim}{i} + 3\underset{\sim}{j})}{5} \checkmark \checkmark$$

$$= \frac{20-6}{5}$$

$$= \frac{14}{5} \checkmark$$

(b) the vector projection of **m** onto **n**.

$$(\underset{\sim}{m} \cdot \hat{\underset{\sim}{n}})\hat{\underset{\sim}{n}} = \frac{14}{5} \times \frac{4\underset{\sim}{i} + 3\underset{\sim}{j}}{5}$$

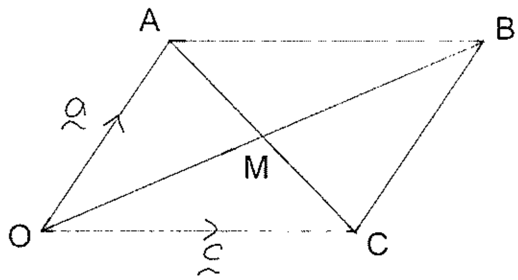
$$= \frac{14(4\underset{\sim}{i} + 3\underset{\sim}{j})}{25} \checkmark$$

$$= \frac{56\underset{\sim}{i} + 42\underset{\sim}{j}}{25}$$

5. [6 marks]

Prove that the diagonals of a parallelogram bisect each other.

OABC is a parallelogram with $\overrightarrow{OA}$ = **a** and $\overrightarrow{OC}$ = **c**. The diagonals OB and AC meet at M.



If $\overrightarrow{AM} = h\overrightarrow{AC}$ and $\overrightarrow{OM} = k\overrightarrow{OB}$, use the fact that $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$ to show that h = k = ½.

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$

$$k\overrightarrow{OB} = \underset{\sim}{a} + h\overrightarrow{AC} \checkmark$$

$$k(\underset{\sim}{a} + \underset{\sim}{c}) = \underset{\sim}{a} + h(\underset{\sim}{c} - \underset{\sim}{a}) \checkmark \checkmark$$

$$k\underset{\sim}{a} + k\underset{\sim}{c} = \underset{\sim}{a} + h\underset{\sim}{c} - h\underset{\sim}{a}$$

$$k\underset{\sim}{c} - h\underset{\sim}{c} = \underset{\sim}{a} - h\underset{\sim}{a} - k\underset{\sim}{a}$$

$$\underset{\sim}{c}(k-h) = \underset{\sim}{a}(1-h-k) \checkmark$$

$$k - h = 0 \Rightarrow k = h \checkmark$$

$$1 - h - k = 0$$

$$1 - 2h = 0$$

$$h = \tfrac{1}{2} = k \checkmark$$

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TEST 3

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Resource Allowed

Time Allowed: 40 minutes

Total Marks: 35

6. [3 marks]

To the rare Blue-breasted Speckled Mat bird, a similarly rare Red-breasted Speckled Mat bird appears to be flying at (-5**i** + 7**j**) m/s. However, to a nearby Yellow-breasted Speckled Mat bird, the Red-breasted Speckled Mat bird appears to be flying at (3**i** + 10**j**) m/s.

What would be the velocity of the Blue-breasted Speckled Mat bird according to it's Yellow-breasted cousin?

$_R\underset{\sim}{V}_B = -5\underset{\sim}{i} + 7\underset{\sim}{j}$

$_R\underset{\sim}{V}_Y = 3\underset{\sim}{i} + 10\underset{\sim}{j}$

$_B\underset{\sim}{V}_Y = {}_B\underset{\sim}{V}_R + {}_R\underset{\sim}{V}_Y$ ✓

$= 5\underset{\sim}{i} - 7\underset{\sim}{j} + 3\underset{\sim}{i} + 10\underset{\sim}{j}$

$= 8\underset{\sim}{i} + 3\underset{\sim}{j}$ ✓

7. [1, 1, 1 marks]

Calculate the following, given that **a** = 3**i** + 5**j**, **b** = 7**i** – 2**j**, and **c** = x**i** + 3**j**.

(a) **a•b** $= 21 - 10$
$= 11.$ ✓

(b) The angle between **a** and **b** $75°$ ✓

(c) The value of x such that **b** and **c** are perpendicular.

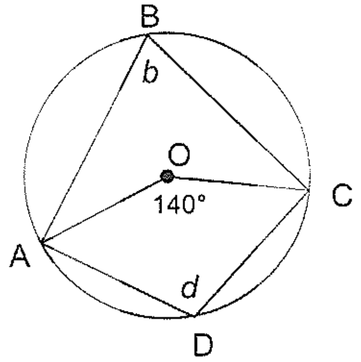
$7x - 6 = 0$

$7x = 6$

$x = \frac{6}{7}$ ✓

8. [2, 4 marks]

(a) A circle centred at O has s∠AOC = 140°, as shown in the diagram. Determine the values of *b* and *d*. Justify your answers.



s∠ABC = 70° angle at centre is twice angle at circumference ✓

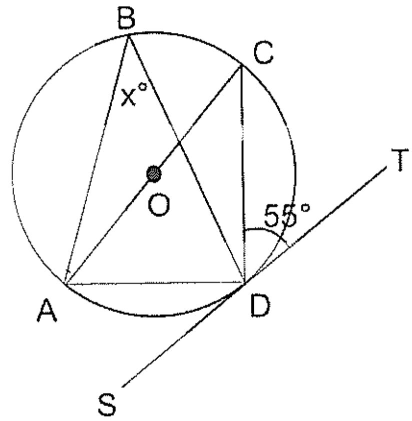
∴ b = 70°

s∠ADC = 110° Opposite angles in cyclic quadrilateral are supplementary ✓

d = 110°

must have reason

(b) A circle centred at O has a tangent ST as shown in the diagram. Given that s∠CDT = 55°, determine the value of x. Justify your answer.



s∠CAD = 55° Angle in opposite segment. ✓

s∠ADC = 90° Angles in semicircle are right angles ✓

s∠ACD = 180° − 90° − 55° Angle sum of a triangle ✓

= 35°

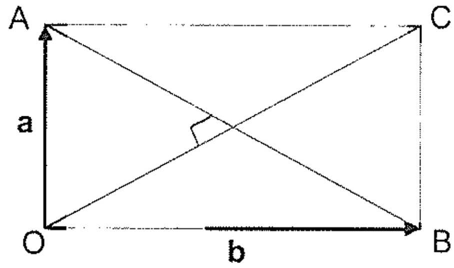
must have reason

s∠ABD = 35° Angles in same segment are equal. ✓

∴ x = 35°

9. [5 marks]

Prove that if the diagonals of a rectangle are perpendicular then the rectangle is a square.



$\overrightarrow{OC} = \underset{\sim}{a} + \underset{\sim}{b}$ ✓

$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ ✓

$\overrightarrow{OC} \circ \overrightarrow{AB} = (\underset{\sim}{a}+\underset{\sim}{b}) \circ (\underset{\sim}{b}-\underset{\sim}{a})$ ✓

$= \underset{\sim}{a} \circ \underset{\sim}{b} - a^2 + b^2 - \underset{\sim}{a} \cdot \underset{\sim}{b}$

$= b^2 - a^2$

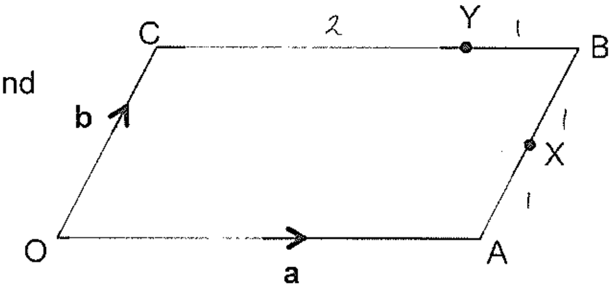
$= 0$

$\therefore b^2 = a^2$ ✓

$\therefore$ OACB is a square ✓

10. [2, 3 marks]

OABC is a parallelogram, X is the midpoint of AB and Y is such that $\overrightarrow{CY} = \frac{2}{3}\overrightarrow{CB}$.



Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$.

(a) Express $\overrightarrow{OX}$ and $\overrightarrow{OY}$ in terms of **a** and/or **b**.

$\overrightarrow{OX} = \underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$ ✓ $\overrightarrow{OY} = \underset{\sim}{b} + \frac{2}{3}\underset{\sim}{a}$ ✓

(b) Show that $\overrightarrow{OX} \bullet \overrightarrow{OY} = \frac{4}{3}\mathbf{a} \bullet \mathbf{b} + 8$, given $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 2$.

$\overrightarrow{OX} \circ \overrightarrow{OY} = (\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}) \circ (\underset{\sim}{b} + \frac{2}{3}\underset{\sim}{a})$

$= \underset{\sim}{a} \circ \underset{\sim}{b} + \frac{2}{3}a^2 + \frac{1}{2}b^2 + \frac{1}{3}\underset{\sim}{a} \circ \underset{\sim}{b}$ ✓

$= \frac{4}{3}\underset{\sim}{a} \circ \underset{\sim}{b} + \frac{2}{3}a^2 + \frac{1}{2}b^2$

$= \frac{4}{3}\underset{\sim}{a} \circ \underset{\sim}{b} + \frac{2}{3}(3)^2 + \frac{1}{2}(2)^2$ ✓

$= \frac{4}{3}\underset{\sim}{a} \circ \underset{\sim}{b} + 6 + 2$

$= \frac{4}{3}\underset{\sim}{a} \circ \underset{\sim}{b} + 8$ ✓

11. [~~4, 2~~ 2, 4 marks]

A football umpire (who is standing at the centre of Subiaco Oval at position (0, 0) notices a West Coast player at position (1, -9) metres, and running with velocity (2**i** + 4**j**) metres/second. At the same time he notices a Fremantle player at position (22, -16) metres, and running with velocity (-4**i** + 6**j**) metres/second.

(a) (i) Write an equation that gives the position of the West Coast player at any time, relative to the umpire, $\mathbf{r}_{WC}(t)$.

$$\underset{\sim}{r}_{wc}(t) = \underset{\sim}{i} - 9\underset{\sim}{j} + t(2\underset{\sim}{i} + 4\underset{\sim}{j})$$
$$= (1+2t)\underset{\sim}{i} + (4t-9)\underset{\sim}{j} \quad \checkmark$$

(ii) Write an equation that gives the position of the Fremantle player at any time, relative to the umpire, $\mathbf{r}_F(t)$.

$$\underset{\sim}{r}_F(t) = 22\underset{\sim}{i} - 16\underset{\sim}{j} + t(-4\underset{\sim}{i} + 6\underset{\sim}{j})$$
$$= (22-4t)\underset{\sim}{i} + (6t-16)\underset{\sim}{j} \quad \checkmark$$

(b) Determine how many seconds after the umpire first notices the two players that they collide, and how far (to 1 decimal place) from the umpire that the collision takes place.

$$1 + 2t = 22 - 4t \quad \checkmark$$
$$6t = 21$$
$$t = 3\tfrac{1}{2} \quad \checkmark$$

$$\underset{\sim}{r}(3\tfrac{1}{2}) = 8\underset{\sim}{i} + 5\underset{\sim}{j} \quad \checkmark$$

$$|\underset{\sim}{r}(3\tfrac{1}{2})| = \sqrt{8^2 + 5^2}$$
$$= 9.4\text{ m} \quad \checkmark$$

12. [1, 4, 2 marks]

Stupendous Man is at position 5**i** + 4**j** m from Evil Babysitter Girl, running with velocity 5**i** – 2**j** m/s when she notices him. She immediately takes off with velocity 6**i** – **j** m/s in order to intercept him.

(a) Determine Stupendous Man's velocity relative to Evil Babysitter Girl.

$$ {}_S\underset{\sim}{V}_E = \underset{\sim}{V}_S - \underset{\sim}{V}_E $$
$$ = -\underset{\sim}{i} - \underset{\sim}{j} \checkmark $$

(b) Using the dot product method, determine the time at which Stupendous Man and Evil Babysitter Girl are closest.

$$ {}_S\underset{\sim}{r}_E = 5\underset{\sim}{i} + 4\underset{\sim}{j} \qquad {}_S\underset{\sim}{V}_E = -\underset{\sim}{i} - \underset{\sim}{j} $$

$$ {}_S\underset{\sim}{r}_E(t) = 5\underset{\sim}{i} + 4\underset{\sim}{j} + t(-\underset{\sim}{i} - \underset{\sim}{j}) \checkmark $$
$$ = (5-t)\underset{\sim}{i} + (4-t)\underset{\sim}{j} $$

$$ {}_S\underset{\sim}{r}_E(t) \cdot {}_S\underset{\sim}{V}_E = -(5-t) - (4-t) \checkmark $$
$$ \checkmark 0 = -5 + t - 4 + t $$
$$ 9 = 2t $$
$$ t = 4.5 \text{ sec.} \checkmark $$

(c) If Evil Babysitter Girl's arms are 65 cm long, state whether or not she will be able to catch Stupendous Man. Justify your answer.

$$ {}_S\underset{\sim}{r}_E(4.5) = 0.5\underset{\sim}{i} - 0.5\underset{\sim}{j} \checkmark $$

$$ \text{dist} = \sqrt{0.5^2 + 0.5^2} $$
$$ = 0.707 \text{ m} $$

$\therefore$ She will miss ✓ him (by 5.7cm)