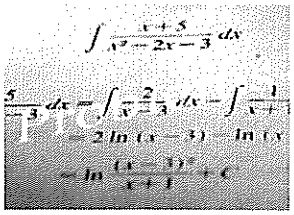
Rossmoyne SHS

METHODS 12 Mathematics
SPOT TEST 3, 2019

Week 8 Term 1
Time allowed: 20 minutes
No Calculator – Page 1 12 mks
Calculator – Page 2 8 mks



Name : _____

1. [12 mks -2,2,2,3,3]
Determine each of the following

(a) $\frac{d}{dx}(e^{3x+1}\cos 2x)$ (do not simplify)

(b) $\int \sin(\frac{\pi - 3x}{2})dx$

(c) $\int xe^{x^2}\left(1+e^{x^2}\right)^4 dx$

(d) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1 - sinx)dx$

(e) $f\left(\frac{\pi}{4}\right)$ *if* $f'(x) = \cos(2x)\,sin^2(2x)$ *and* $f(0) = 0$

2. **(a) [4 marks – 1,1,2]**

Certain medical tests require the patient to be injected with a solution containing 0.5 micrograms (μg) of the radioactive material Technetium-99. This material decays according to the rule:

$T = T_0 e^{-0.1155t}$ where t is the time (in hours) from injection.

(i) The rate of change of the amount of Technetium is proportional to the amount of Technetium remaining. That is ,

$$\frac{dT}{dt} = kT$$

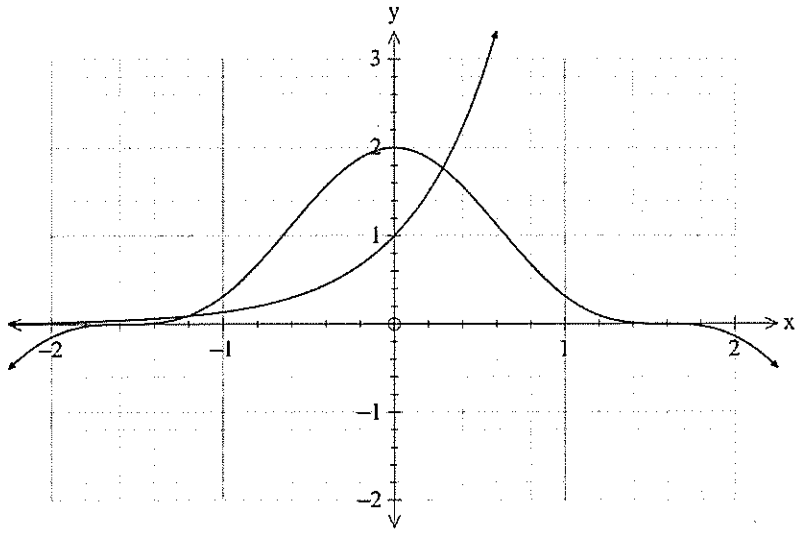
What is the value of k ?

(ii) What is the value of T_0 ?

(iii) What is the half-life of Technetium-99?

(b) [4 marks]

Determine to an accuracy of 3 sig figures, the area enclosed by $y = e^{2x}$ and $y = 2cos^3 x$



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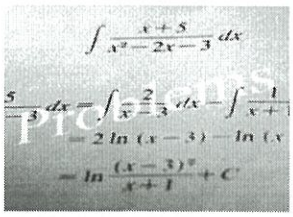


Name : Solutions

1. [12 mks -2,2,2,3,3]
Determine each of the following

(a) $\frac{d}{dx}(e^{3x+1}\cos 2x)$ (do not simplify)

$= e^{3x+1} \cdot (-\sin 2x)\,2 + \cos 2x \cdot e^{3x+1} \cdot 3$ ✓ ✓

(b) $\int \sin\left(\frac{\pi - 3x}{2}\right)dx$

Try $y = \cos\left(\frac{\pi-3x}{2}\right)$

$\Rightarrow \frac{dy}{dx} = -\sin\left(\frac{\pi-3x}{2}\right)\cdot\left(-\frac{3}{2}\right)$

$= \frac{2}{3}\int \frac{3}{2}\cdot \sin\left(\frac{\pi-3x}{2}\right)dx$ ✓

$= \frac{2}{3}\cos\left(\frac{\pi-3x}{2}\right) + C$ ✓

(c) $\int xe^{x^2}\left(1+e^{x^2}\right)^4 dx$

Try $y = (1+e^{x^2})^5$

$\Rightarrow \frac{dy}{dx} = 5(1+e^{x^2})^4 \cdot e^{x^2}\cdot 2x$

$= \frac{1}{10}\int 5(1+e^{x^2})^4 \cdot e^{x^2}\cdot 2x\,dx$ ✓

$= \frac{1}{10}(1+e^{x^2})^5 + C$ ✓

(d) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1-\sin x)dx$

$= \left[x + \cos x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ ✓

$= \left[\frac{\pi}{2}+0\right] - \left[\frac{\pi}{3}+\frac{1}{2}\right]$ ✓

$= \frac{\pi}{6} - \frac{1}{2}$ ✓

(e) $f\left(\frac{\pi}{4}\right)$ *if* $f'(x) = \cos(2x)\sin^2(2x)$ *and* $f(0) = 0$

Try $y = \sin^3 2x$

$\Rightarrow \frac{dy}{dx} = 3\sin^2 2x \cdot \cos 2x \cdot 2$

$f(x) = \int \cos 2x \cdot \sin^2 2x\,dx$

$= \frac{1}{6}\int 6\cos 2x \cdot \sin^2 2x\,dx$

$= \frac{1}{6}\sin^3 2x + C$ ✓

$f(0) = \frac{1}{6}\cdot 0 + C = 0 \Rightarrow C = 0$ ✓

$\therefore f\left(\frac{\pi}{4}\right) = \frac{1}{6}$ ✓

12

2. **(a) [4 marks – 1,1,2]**

Certain medical tests require the patient to be injected with a solution containing 0.5 micrograms (μg) of the radioactive material Technetium-99. This material decays according to the rule:

$T = T_0 e^{-0.1155t}$ where t is the time (in hours) from injection.

(i) The rate of change of the amount of Technetium is proportional to the amount of Technetium remaining. That is ,

$$\frac{dT}{dt} = kT$$

What is the value of k ? $k = -0.1155$ ✓

(ii) What is the value of T_0 ? $T_0 = 0.5$ ✓

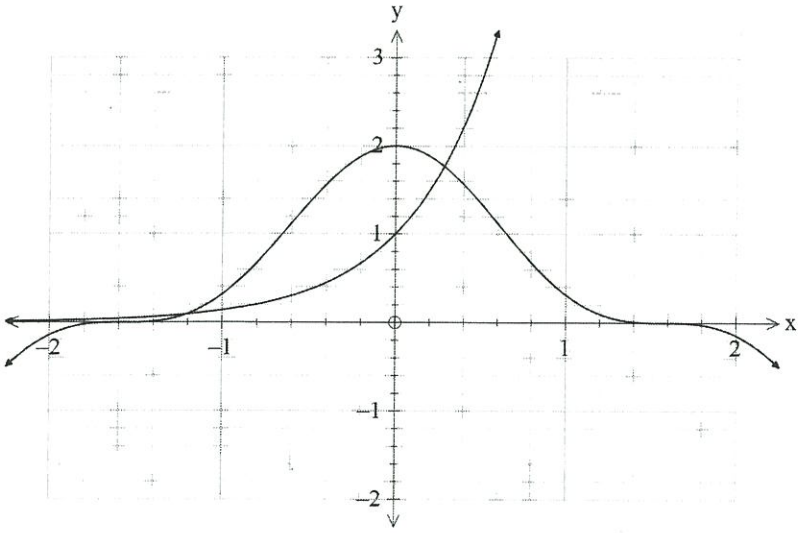
(iii) What is the half-life of Technetium-99? $0.5 = e^{-0.1155t}$ ✓ Solve CAS

$\Rightarrow t = 6.001$

$\therefore t = 6$ yrs ✓

(b) [4 marks]

Determine to an accuracy of 3 sig figures, the area enclosed by $y = e^{2x}$ and $y = 2cos^3 x$



Solve $e^{2x} = 2\cos^3 x$ ✓

$x = -1.2082, 0.2849$ ✓

$$\text{Area} = \int_{-1.2082}^{0.2849} 2\cos^3 x - e^{2x}\, dx = 1.03 \text{ units}^2$$ ✓ ✓

4