Year 11 Mathematics-Specialist
Test 6 - 2017

MELVILLE
SENIOR HIGH SCHOOL

This test is Resource Free
It contains 13 questions worth 65 marks in total

Time Allowed : 60 minutes

Student Name : _____

Question 1 **5 marks**

Use Proof by Exhaustion to prove p^2 is only a multiple of three if p is a multiple of 3.

Question 2 **1 mark**

If m is divisible by six and n is divisible by 15, then which of the following statements might be false?

a) $m \times n$ is divisible by 90

b) $m \times n$ is divisible by 30

c) $m \times n$ is divisible by 15

d) $m + n$ is divisible by 3

e) $m + n$ is divisible by 15

Question 3 **5 marks**

Prove by induction that $7^n - 4$ is divisible by 3 for all $n \in \mathbb{N}$.

Question 4 **1, 3 - 4 marks**

Consider the statement " If n is odd, then $n^2 - 1$ is divisible by eight".

a) Show using 3 examples it might be true.

b) Prove the statement to be true.

Question 5 **1, 4, 5 - 10 marks**

Let $n \in \mathbb{Z}$. Consider the statement "If n^3 is even, then n is even."

a) Write the contrapositive of this statement.

b) Prove the contrapositive.

c) Hence, prove by contradiction that $\sqrt[3]{6}$ is irrational.

Question 6 **2, 4 - 6 marks**

a) Prove that $0.15\overline{537}$ is rational.

b) Prove that $3.\dot{6}$ is rational by first expressing it as an infinite sum.

Question 7 **1,1, 2 - 4 marks**

Given $z = -11 + 7i$ state

a) Re(z) b) Im(z)

c) Evaluate $1 + i + i^6 - i^{11} + i^{16} + i^5$.

Question 8 **1, 1, 2, 1 - 5 marks**

Given that $z = 4 + 3i$, express in the form $a + bi$.

a) $\overline{z}$ b) $z + \overline{z}$

c) $z\overline{z}$ d) $\frac{1}{\overline{z}}$

Question 9 **2 marks**

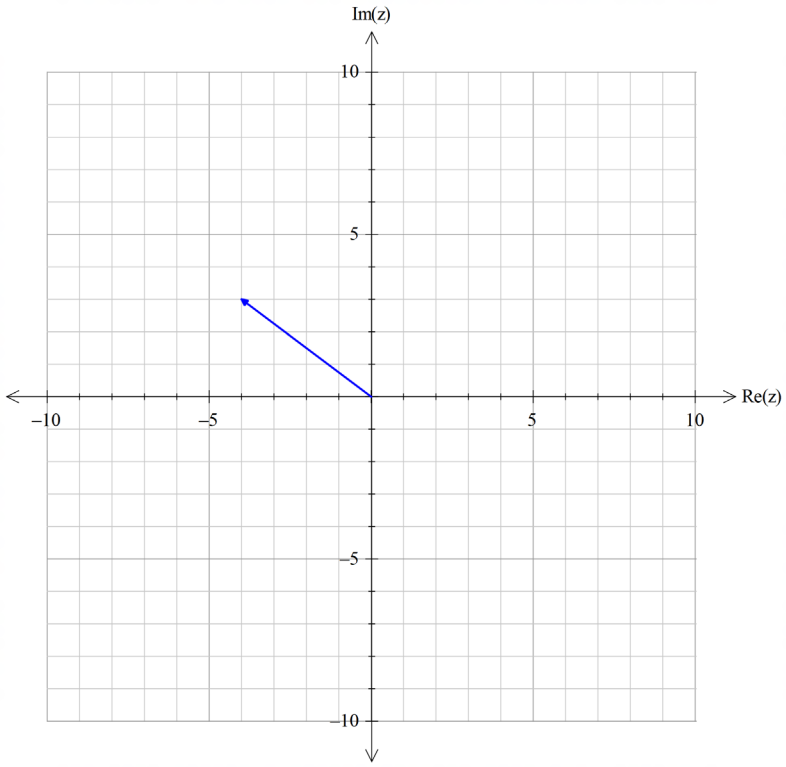
Find the complex number, w, formed when $z = 3 - 5i$ is rotated on the Argand diagram by

a) 180° clockwise

b) 270° anticlockwise

Question 10 **1, 1, 2, 2 - 6 marks**

Consider the Argand diagram shown.



a) State the complex number represented by z.

b) Show and label on the diagram the location of $\bar{z}$

c) Show how to determine the value of the following expressions on the diagram given.
State the value of each.

ii) $z + \bar{z}$

iv) $z - \bar{z}$

Question 11 **1,3, 3 - 7 marks**

a) Solve for x if $x^2 = -81$

b) Find the values of x and y in $w = x + yi$ if:

i) $w\overline{w} = 13$ and $w + \overline{w} = -6$.

ii) $5w - 7\overline{w} = 14 + 24i$

Question 12 **4 marks**

If one of the solutions to the equation $z^2 + bz + c = 0$ is $-6 + i$, find the values of b and c.

Question 13 **4, 2 - 6 marks**

a) Find the four roots of $f(x)=(4x^2+9)(x^2-6x+34)$, giving your answers in the form $x=a+bi,$ where a and b are real.

b) Show these four roots on a single Argand diagram.