Manjimup SHS 2015 Year 11 Mathematics Methods Test 8 Optimisation, Integration, Rectilinear Motion	Name: ANNE SIRS Score: ___ out of 28

Non-Calculator Section (No calculator nor notes, formula sheet is provided)

/58

Time: 28 minutes   Marks: 28 marks

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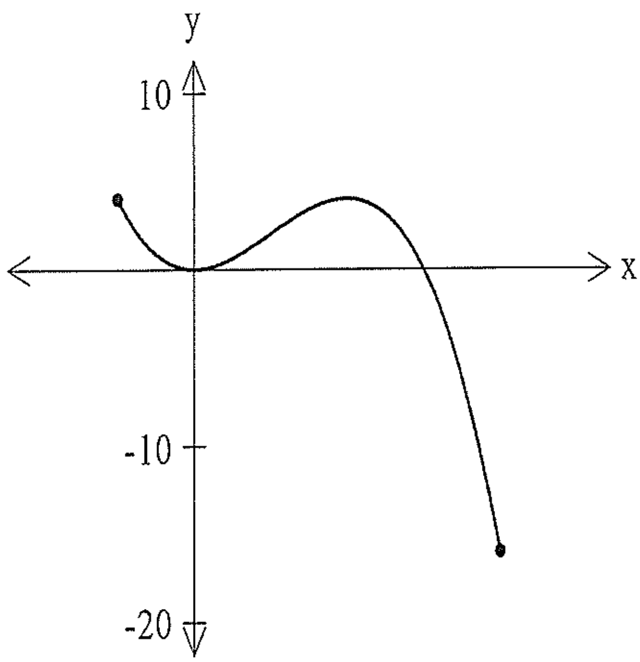
1. [1,2,1 = 4 marks]

Consider the function $f(x) = 3x^2 - x^3$ over the domain $-1 \le x \le 4$ sketched to the right



a) Find the derivative, $f'(x)$

$\boxed{f'(x) = 6x - 3x^2}$ ✓

b) Find the values of x where $f'(x) = 0$

$f'(x) = 0 = 6x - 3x^2$

$0 = 3x(2 - x)$

$\Rightarrow \boxed{x = 0 \text{ or } x = 2}$ ✓✓

c) Find the coordinates of the local maximum point.

$f(0) = 0$

$f(2) = 3(2)^2 - 2^3$

$= 4$

so maximum point $\boxed{(2, 4)}$ ✓

2. [2,2,3 = 7 marks]

Find expressions for the following anti-derivatives.

a) $\int(4x-3x^2+1)dx$

$= \boxed{2x^2-x^3+x+c}$ ✓✓

b) $\int(\sqrt{x})dx$

$= \int x^{\frac{1}{2}}dx$

$= \frac{x^{3/2}}{\frac{3}{2}} + c$

$= \boxed{\frac{2x^{3/2}}{3} + c}$ ✓✓

c) $\int(\frac{x^3}{2}+\frac{1}{x^3})dx$

$= \int(\frac{1}{2}x^3 + x^{-3})\,dx$

$= \frac{x^4}{8} + \frac{x^{-2}}{-2} + c$

$= \boxed{\frac{x^4}{8} - \frac{1}{2x^2} + c}$ ✓ ✓✓

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3. [3 marks]

Find the rule for a curve that goes through (1,-1) with a gradient function of $\frac{dy}{dx} = 6(1-x^2)$

$= 6-6x^{2}$

$y = \int(6-6x^2)\,dx$

$y = 6x - 2x^3 + c$ ✓

$(1,-1) \rightarrow -1 = 6-2+c$

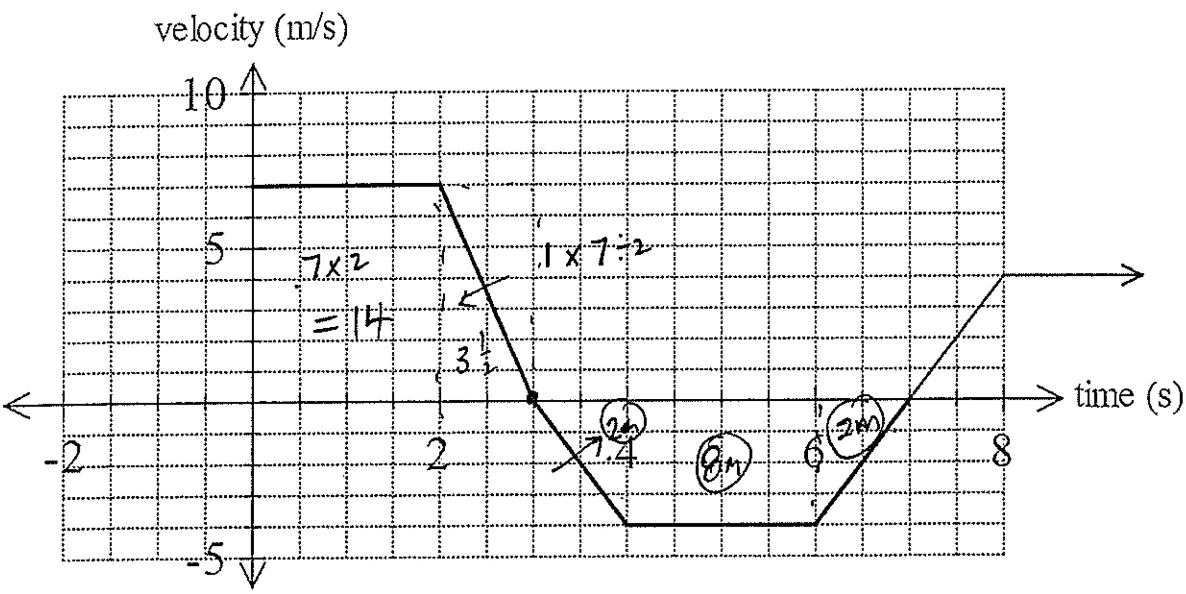
$-1 = 4 + c$ ✓

$-5 = c$

Rule: $\boxed{y = 6x - 2x^3 - 5.}$ ✓

4. [2,2,1,2 = 7 marks]

An owner is exercising her dog. The velocity-time graph of the dog is shown is shown below.



a) How far did the dog move in the first 3 seconds?

$14 + 3\frac{1}{2} = \boxed{17\frac{1}{2} \text{ metres.}}$ ✓✓

b) Describe the dog's motion between 3 seconds and 8 seconds.

3-7 sec. Dog is moving back towards its starting position. ✓
7 sec. Dog is not moving
7-8 seconds. Dog is running away from its starting positi ✓

c) Find the dog's acceleration when t = 7.

$a(t) = \frac{dv}{dt}$ = gradient at $t = 7$. $a(7) = \frac{4-(-4)}{2} = \boxed{4 m/s^2}$ ✓

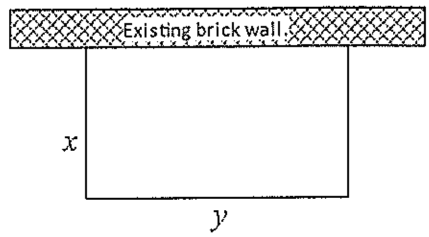
d) Find how far the dog was from its starting position at t = 7 seconds

3-4 seconds · moves 2m
4-6 seconds · moves 8m.
6-7 seconds - moves 2m.
12m ✓

$17\frac{1}{2} - 12 =$ $5\frac{1}{2}$ m from its starting position. ✓

5. [2,2,3 = 7 marks]

A person wants to fence off a rectangular enclosure for her chooks. She has 22 metres of fencing but only needs three sides of fencing as it will be located against a brick wall.



a) Find an expression for y in terms of x.

$$2x + y = 22.$$

$$\boxed{y = 22 - 2x.}$$

b) Find an expression for the area of the chooks enclosure, in terms of x only.

$$A = x \cdot y$$

$$= x \cdot (22 - 2x)$$

$$\boxed{A = 22x - 2x^2.}$$

c) Using calculus find the maximum area of the chook enclosure and the value of x that gives this.

$$\frac{dA}{dx} = 22 - 4x = 0$$

$$\Rightarrow 4x = 22$$

$$\Rightarrow x = 5.5m.$$

<u>Check:</u>

$$\frac{d^2A}{dx^2} = -4$$

$\Rightarrow$ Concave down

$\Rightarrow$ Maximum point.

Hence $A_{max} = x \cdot y$

$y = 22 - 2 \times 5.5$
$= 11m.$

$$= (5.5)(11)$$

$$= \boxed{60.5m^2} \text{ when } \boxed{x = 5.5m.}$$

END OF SECTION ONE

Manjimup SHS 2015 Year 11 Mathematics Methods Test 8 Optimisation, Integration, Rectilinear Motion	Name: ANNE SIRS Score: ___ out of 30

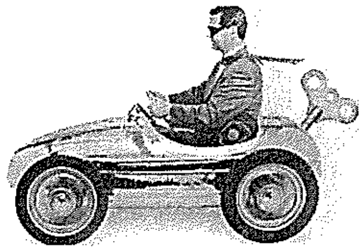
Calculator Section (Calculators and 1 page (A4) of notes permitted, formula is sheet provided)

Time: 30 minutes   Marks: 30 marks

6. [1,2,1,2,2 = 8 marks]

A wind-up toy car is designed so that when it is wound up; it moves forward for a certain distance, stops then reverses direction before coming to rest. It then does not move until wound up again.

It's motion for this time is found from the rule $x(t) = t^3 - 12t^2 + 36t$; $0 \le t \le T$ where T is the time it stops for the final time in seconds and x is the displacement of the car in cm.



a) What is the displacement at t = 5?

$x(5) = \boxed{5cm}$

b) Find the times when the car stops.

$v(t) = 3t^2 - 24t + 36 = 0$.

$\boxed{t = 2 \text{ seconds and } 6 \text{ seconds}}$

c) Determine the velocity of the car after 3 seconds.

$v(3) = \boxed{-9cm/s.}$

d) Will the car have zero acceleration? If so, at what time?

$a(t) = 6t - 24 = 0$

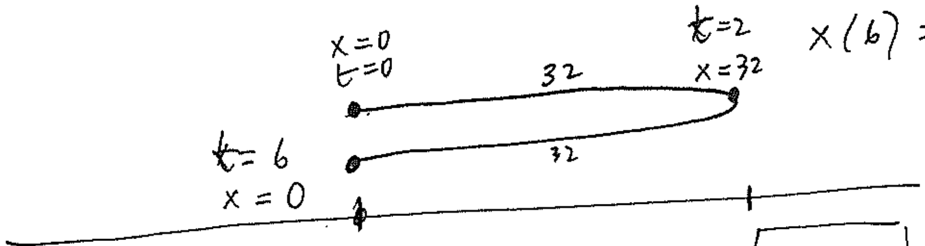
$\Rightarrow t = 4$

Yes it will, when time is 4 seconds.

e) Find the distance the car will travel.

$x(2) = 32.$

$x(6) = 0.$

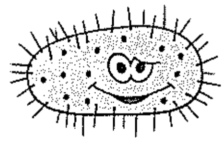


Distance $= 32 + 32 = \boxed{64cm}$

7. [1,2,4 = 7 marks]

The number of bacteria in a student's lunchbox, t minutes after being he puts his hand in is given by the rule:

$B(t) = 4200 + 900t + 30t^2 - t^3$ for $0 \le t \le 40$.



a) How many bacteria were in the lunchbox after 6 minutes?

$B(6) = \boxed{10464 \text{ bacteria}}$

b) Find the rate of change of bacteria after 10 minutes.

$\frac{dB}{dt} = 900 + 60t - 3t^2$

$\left.\frac{dB}{dt}\right|_{t=10} = \boxed{1200 \text{ bacter/min}}$

c) Use methods of calculus to determine the maximum number of bacteria and when this occurred.

$\frac{dB}{dt} = 0 = 900 + 60t - 3t^2$

reject

$\cancel{t = -10}$ or $t = 30$.

$\frac{d^2B}{dt^2} = 60 - 6t$.

$\left.\frac{d^2B}{dt^2}\right|_{t=30} = -120 \Rightarrow t = 30$ is a maximum.

$B_{max} = B(30) = \boxed{31200 \text{ when } t = 30 \text{ minutes}}$

8. [2,2,3 = 7 marks]

A train departs a station at 9 am, with its velocity given by the rule v(t) = $6t - t^2$ km/hour.

a) Determine the times when the train is stationary.

$v(t) = 0 = 6t - t^2$ ✓

$0 = t(6-t)$

$\Rightarrow \boxed{t = 0 \text{ hours or } 6 \text{ hours.}}$ ✓

$\boxed{9\text{am and } 3\text{pm.}}$

b) Find a rule for the displacement from the station, at any time t.

$x(t) = \int v(t)\, dt$

$= \int (6t - t^2)\, dt$

$= 3t^2 - \frac{t^3}{3} + c.$ ✓

· Train starts from station

$\Rightarrow t = 0,\ x = 0$

$\Rightarrow c = 0.$ ✓

Rule : $\boxed{x(t) = 3t^2 - \frac{t^3}{3}}$

c) To the nearest minute, find when the train was 15 kilometres from the station.

Solve : $15 = 3t^2 - \frac{t^3}{3}$

$\Rightarrow t = \cancel{-2.02 \text{ hours}}$ or $\underline{2.66527}$ ✓ | or 8.3554

$\downarrow$

501 min

= 8h 21m.

$\Rightarrow \boxed{5:21\text{pm.}}$

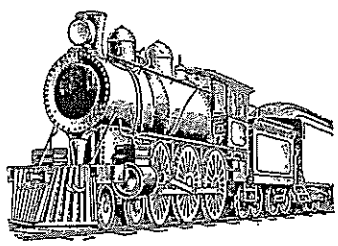
Accept this as well

2.66527 hours = 159.9 m

≈ 160 min ✓

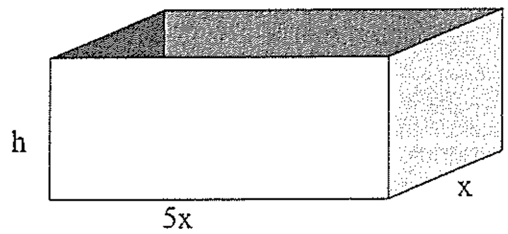
= 2 hours 40 min

$\Rightarrow$ Time is $\boxed{11:40\text{am.}}$ ✓



9. [2,2,4 = 8 marks]

A box is designed by a Year Eleven student to house their cool dude belly, nose and ear studs. It is to be open topped with the length five times the width. The volume of the box must be 400 cm^3.



a) Show working to show that $h = \dfrac{80}{x^2}$

$$\text{Volume: } \quad 400 = 5x \cdot x \cdot h$$
$$400 = 5x^2 h$$
$$\Rightarrow \quad h = \frac{400}{5x^2} = \frac{80}{x^2} \quad \text{as required.}$$

b) Show why the surface area of the box must be given by the rule $A = 5x^2 + \dfrac{960}{x}$

$$SA = \text{base} + 2\times \text{ends} + 2\times \text{front}$$
$$= 5x^2 + 2(hx) + 2\times(5xh)$$
$$= 5x^2 + 12xh.$$

$$SA = 5x^2 + 12x\frac{80}{x^2}$$
$$SA = 5x^2 + \frac{960}{x}$$
as required

c) Use your Casio and methods of calculus to find the dimensions of the box that will minimise the surface area of the box. State the minimum surface area needed.
Give all answers to one decimal place.

$$\frac{d(SA)}{dx} = 10x - \frac{960}{x^2} = 0$$

$$\Rightarrow x = 4.6 \text{ cm.}$$
$$\Rightarrow 5x = 22.9 \text{ cm.}$$
$$\Rightarrow h = 3.8 \text{ cm.}$$

$$SA_{max} = 5(4.6)^2 + \frac{960}{4.6}$$
$$\boxed{SA_{max} = 314.5 \text{ cm}^2}$$

END OF SECTION 2