Manjimup SHS 2015

Year 11 Mathematics Methods

Test 5

Indices, Exponential Functions

Name: ANSWERS

Score: **out of** 30

Non Calculator Section (No calculator or notes, formula sheet provided)

/58

Time: 30 minutes   Marks: 30 marks

1. [1,1 = 2 marks]

When intercepted by the spacecraft Horizon recently, Pluto was about 4 864 100 000 km from Earth.

a) Express this distance in scientific notation. 4.8641×10^9

b) Write this distance in scientific notation correct to 2 significant figures. 4.9×10^9

2. [3 marks]

Determine the rule connecting x and y.

x	0	1	2	3
y	20	12	7.2	4.32

$\times 0.6 \quad \times 0.6 \quad \times 0.6$

$y = 20(0.6)^x$

3. [4 marks]

Find the value of these expressions, writing your answer as a fraction where appropriate.

a) $2^{-3} = \frac{1}{8}$

b) $5^0 + 2^{-2} = 1 + \frac{1}{4} = 1\frac{1}{4}$

c) $9^{\frac{3}{2}} = 27$

d) $(1\frac{2}{3})^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

$2\frac{7}{9}$ $\cancel{2\frac{2}{3}}$

4. [1 mark]

Estimate a solution to the equation $2^x = 17$, giving your ~~answer~~ estimate to one decimal place.

$x = 4.1$ (Accept 4.0, 4.2)

5. [2,3,2 = 7 marks]

Simplify the following leaving your answers as positive indices.
Show all working and steps.

a) $a^3b^3 \times a^{-2}b^{-7}$

$= ab^{-4} = \frac{a}{b^4}$

b) $\sqrt{\left(\frac{x^2y^{-1}}{3y}\right)^2 \times \frac{9x^{-2}}{y^2}}$

$= \sqrt{\frac{x^4y^{-2}}{9y^2} \times \frac{9x^{-2}}{y^2}}$

$= \sqrt{x^2 \cdot y^{-6}}$

$= xy^{-3} = \frac{x}{y^3}$

c) $\frac{x^2 - 2x^5}{x}$

$= x - 2x^4$

6 [2,2,2 = 6 marks]

Solve for x in each case showing working clearly.

a) $4^{x-3} = \frac{1}{2}$

$2^{2x-6} = 2^{-1}$

$2x - 6 = -1$

$2x = 5$

$x = 2.5$

b) $\frac{3^{2x}}{9} = \sqrt{3}$

$\frac{3^{2x}}{3^2} = 3^{0.5}$

$3^{2x-2} = 3^{0.5}$

$2x - 2 = 0.5$

$2x = 2.5$

$x = 1.25$

c) $2(x-1)^{\frac{1}{3}} = 6$

$(x-1)^{\frac{1}{3}} = 3$

$x - 1 = 27$

$x = 28$

7. [1,2,1 = 4 marks]

a) The number of feral cats in a National park was initially 3500 but decreasing at an annual rate of 7.5%. Write a rule describing the number of cats after t years.

C = $3500(0.925)^t$

The number of bacteria in a culture after t minutes is described by the rule $B = 50(2.1)^{t+1}$.

b) Describe the percentage change in the number of bacteria each minute.

$2.1 = 210\%$

$= 100 + 110\%$ $\Rightarrow$ 110% increase.

c) Find how many bacteria were present initially.

$t = 0$

$\Rightarrow B = 50(2.1)^1 = 105.$

8. [1,2 = 3 marks]

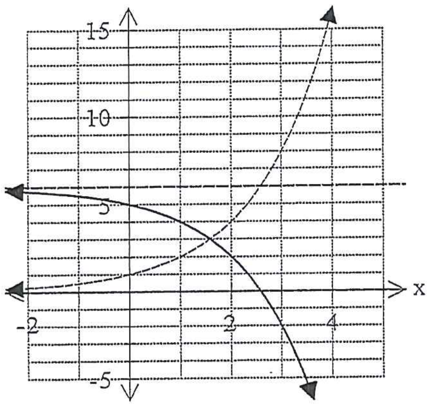
The curve $y = 2^x$ is transformed into $y = 2^{x+4}$.

a) State the transformation that has occurred.

Shift 4 units to the left.

b) Find the rule below for Graph A if it represents a transformation of $y = 2^x$. (dotted)

$y = 6 - 2^x$



Graph A

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Indices, Exponential Functions

Name:

Score: out of 28

Calculator Section (Calculators and 1 page (A4) of notes permitted, formula sheet provided)

Time: 30 minutes Marks: 28 marks

10. [2,2 = 4 marks]

The changing population of Saudi Arabia is shown below.

Year	2000	2010	2015
Population (in millions)	20.3	26.8	30.7

a) Find the yearly percentage growth in population. Show working.

$\left(\frac{30.7}{20.3}\right)^{\frac{1}{15}} = 1.02796 \approx 102.8\% \Rightarrow 2.8\%$ increase.

b) Find the population in 2040, to the nearest 0.1 of a million. Show all working to justify your answer.

$P = 20.3(1.02796)^{40} \approx 61.2$ million.

11. [1,2,3 = 6 marks]

The amount of grams of a dangerous radioactive substance remaining after a dangerous accidental spill in a laboratory, at time t minutes, is given by the rule

$A = 350(0.969)^t$.

a) How much radioactive substance was spilled initially?

350 grams.

b) How many grams remain after 1 hour? Show how you did this question.

$A(60) = 350(0.969)^{60}$

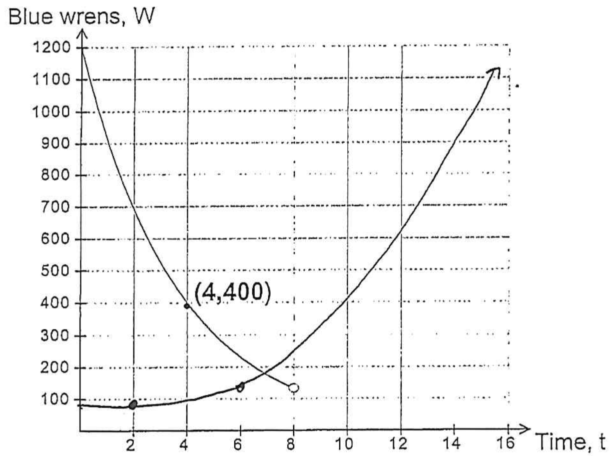
$= 52.9$ grams.

c) For the laboratory to be safe there must be less than 1 gram of the substance left. Determine the time to the nearest minute, when it is safe to go back into the laboratory. Show working clearly.

Solve $(1 = 350(0.969)^t \Rightarrow t = 186$ minutes

12 [2,1,1,2,2,2 = 10 marks]

The population of blue wrens in a particular area of the South West has been studied over 10 years. The results are represented on the graph given below.



The population is modelled by the exponential function $W = a \times b^t$ and a point on the graph is indicated.

a) Find the value of b, rounded to three decimal places.
Indicate clearly how you found this.

$b = \left(\frac{400}{1200}\right)^{\frac{1}{4}} = 0.7598 \approx \underline{0.760}$ ✓✓

b) State the annual percentage change in the population of wrens.

24% decrease ✓

c) Find how many wrens were there after 13 years.

$W = 1200(0.760)^{13} = 33.9 \Rightarrow$ 34 wrens left ✓

At the same time the number of numbats in the area has also been measured and found to be growing exponentially. After two years there were 96 numbats and after 6 years there were 138 numbats counted.

d) Mark these points on the axes above and sketch a curve showing how Numbats will increase. ✓✓

e) Find a general rule for the number of Numbats at any time t.

$N = 80(1.095)^t$.

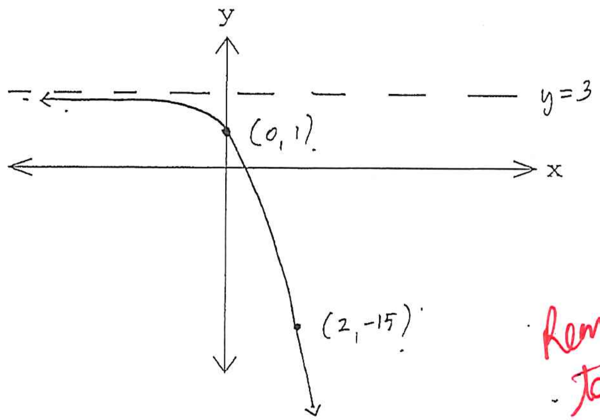
f) Determine to the nearest 0.1 of a year, when the population of Numbats will equal the population of Wrens.

solve $80(1.095)^t = 1200(0.76)^t \Rightarrow t = 7.4$ years

13. [2,2 = 4 marks]

The graph of an exponential function has a horizontal asymptote of y = 3, a y-intercept of (0,1) and goes through the point (2,-15).

a) Sketch the function below, indicating all key information..



Remember to dot asymptote

b) Find a possible rule for the function in the form $y = ka^x + b$.

$y = ka^x + b$

$b = 3$ (vertical shift)

$\Rightarrow y = ka^x + 3$.

$(0,1) \Rightarrow 1 = k + 3$

$\Rightarrow k = -2$

$\underline{y = -2a^x + 3}$.

$(2,-15) \Rightarrow$

$-15 = -2a^2 + 3$.

$\Rightarrow 2a^2 = 18$

$\Rightarrow a = 3$ (Reject -3)

Rule: ½ ½ ✓ ½

$y = -2(3)^x + 3$.

13 [2,1,1 = 4 marks]

The median house price in Sydney has increased from $648,000 in June 2010 to $1 million in June 2015. In the same time Melbourne house prices have jumped from $553,000 to $668,000.

(a) Determine the average percentage yearly growth of each city.

Sydney: $\left(\frac{1\,000\,000}{648\,000}\right)^{\frac{1}{5}}$

$= 1.0906$

$\Rightarrow 9.06\%$ growth

Melbourne: $\left(\frac{668\,000}{553\,000}\right)^{\frac{1}{5}}$

$= 1.0385$

$\Rightarrow 3.85\%$ growth.

(b) Some analysts have claimed that one of these cities is could experience a bubble, where prices rapidly increase and then the bubble will burst and the process then rapidly drop. Which city do you think they are referring to?

Sydney.

(c) Assuming Melbourne prices rise at the same average percentage rate, determine the year when they will first hit $1 million

$1\,000\,000 = 668,000(1.0385)^t$

$\Rightarrow t = 10.68$ years

i.e End of 2025

Start of 2026

2026.

9.

$\sin x \cos 10 - \cos x \sin 10 = 0.5$

$\sin(x-10) = 0.5$

$x - 10 = 30^\circ, 150^\circ$

$x = 40^\circ, 160^\circ$

$\cos x \cos 10 - \sin x \sin 10 = 0.5$

$\cos(x+10) = 0.5$

$x + 10 = 60^\circ, 300$

$x = 50, 290.$

14. (a) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= -\frac{4}{5} \times -\frac{2}{\sqrt{5}} + \frac{3}{5} \times \frac{1}{\sqrt{5}}$

$= \frac{11\sqrt{5}}{25}$

$\left(\frac{11}{5\sqrt{5}}\right)$

(b) $\sin 45 = \frac{1}{\sqrt{2}}$ $\cos 120 = -\frac{1}{2}$

$\sin(165) = \sin(45+120)$

$= \sin 45 \cos 120 + \cos 45 \sin 120$

$= \frac{1}{\sqrt{2}} \times -\frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\sqrt{2}(\sqrt{3}-1)}{4}$