SCOTCH COLLEGE

12 Mathematics Methods 2021

Test 2 – Integration and Area

Section 1: Calculator-free

Time allowed: 20 minutes **Maximum marks: 20**

Name: _____ **Teacher:** Foster | Kelly

Instructions:

- Show all working clearly.
- Sufficient detail must be shown for marks to be awarded for reasoning.
- A formula sheet will be provided.
- No calculators or personal notes are permitted.

Question 1 (6 marks)

a) If $f'(x) = 3x^2 - 2$ and $f(-2) = 4$, determine $f(x)$. [2]

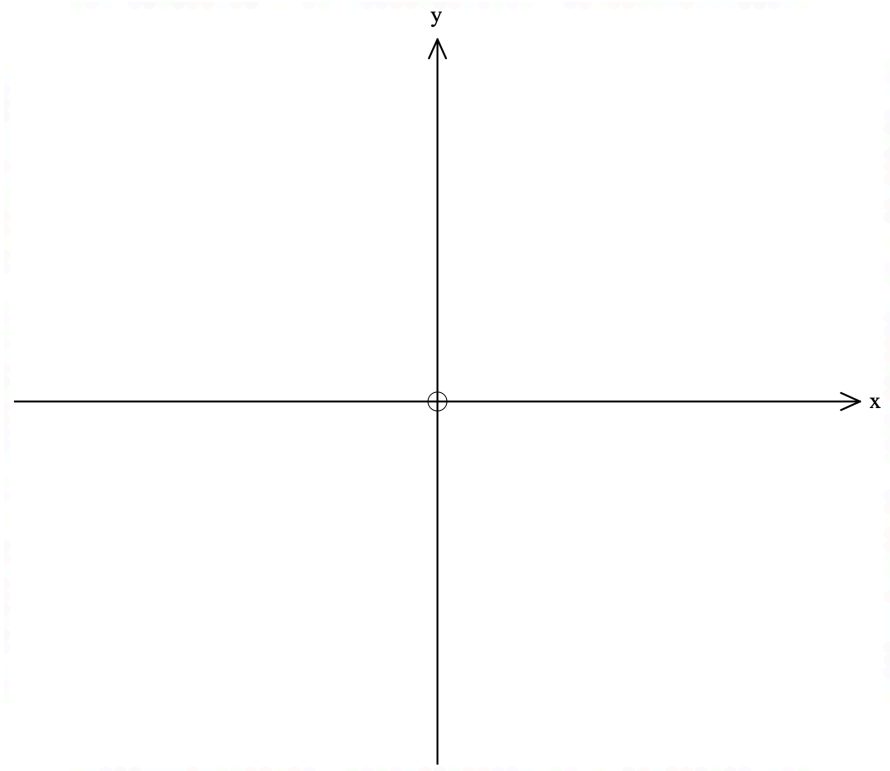
b) Calculate; [4]

$$\int_0^4 \frac{-x}{\sqrt{x^2+9}}\,dx$$

Question 2 (6 marks)

Consider the functions $f(x) = x + 2$ and $g(x) = x^2 + x - 2$

a) Sketch $f(x)$ and $g(x)$ on the axes below, showing key features. [2]

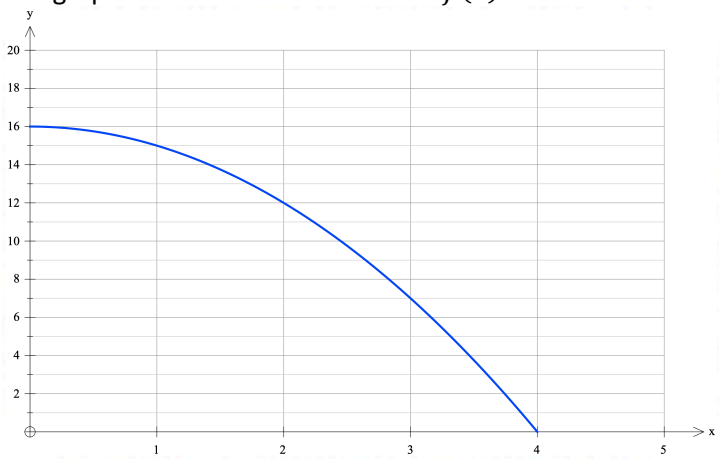


b) State the x values of the points of intersection of $f(x)$ and $g(x)$. [1]

c) Hence or otherwise, determine the area enclosed by $f(x)$ and $g(x)$. [3]

Question 3 (7 marks)

The graph below shows the function $f(x) = 16 - x^2$



a) An estimate for the area between the curve and the x −axis between $x = 0$ and $x = 4$ is required. [5]

i) Use 4 rectangles (each of width 1 $unit$) to find an overestimate for the area.

ii) Use 4 rectangles (each of width 1 $unit$) to find an underestimate for the area.

iii) Determine the mean of the overestimate and underestimate.

b) Use calculus techniques to find the exact area. [2]

c) If rectangles of $0.5\ units$ wide were used instead to estimate, (without calculations) suggest a possible a possible over-estimate and under-estimate for the area. [1]

END OF SECTION

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12 Mathematics Methods 2021

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Section 2: Calculator-assumed

Time allowed: 20 minutes **Maximum marks: 20**

Name: _____ **Teacher:** Foster | Kelly

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- Calculators and 1 A4 double-sided page of personal notes are permitted.

Question 4 (4 marks)

A train moves along a straight track with acceleration $\frac{t}{10} - 3\ ms^{-2}$. If the initial velocity of the train is $45ms^{-1}$, determine the total distance the train travels in the first $2\ mins$.

Question 5 (3 marks)

Find the total area enclosed by the graphs of $y = x^3 - 4x$ and $y = 3x + 6$.

Question 6 (4 marks)

The marginal cost of producing x items is given by $y = 4.15 - 0.03x + 0.0012x^2$ $(0 \leq x \leq 80)$.

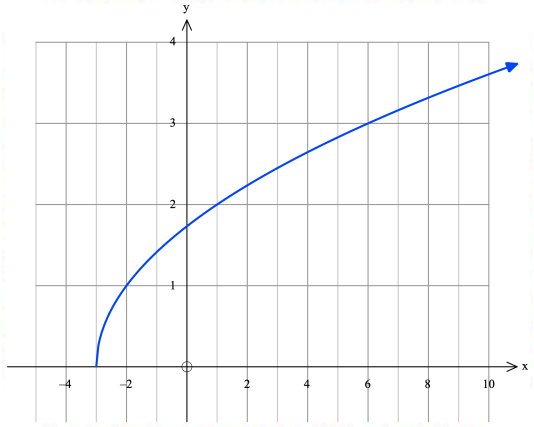
The initial costs are \$215 before production.

a) Determine the cost of producing 60 items. [2]

b) Determine the difference in cost of producing 65 items rather than 60 items. [2]

Question (9 marks)

The graph of $y = \sqrt{x+3}$ is drawn below.



a)

i) Calculate the area, A, enclosed between the curve, the $x-$axis and the line $x = 10$. [2]

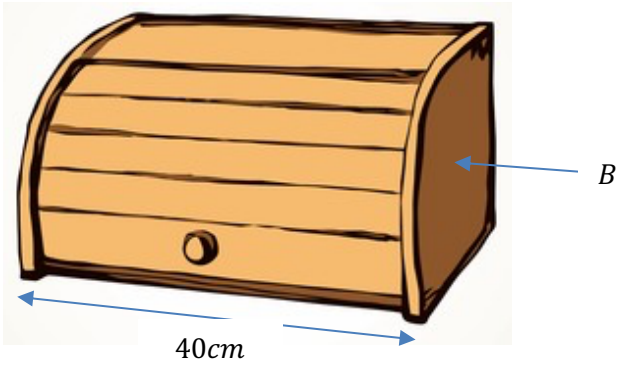
i) The line $x = k$ divides the region A into two regions B and C (where B lies left of $x = k$).

Determine the value of k if the ratio of $B : C$ is $2 : 3$ [3]

b)

B is used as a design for the cross-section of a $40cm$ long breadbin.

If each unit on the graph represents $7cm$, determine the volume of the breadbin.



[4]

END OF TEST