Mathematics Methods Unit 3

Test 1 - Differentiation

Baldivis
Secondary College

Name: ____________________ **Total Marks:**_________

Task type: **Response**

Time allowed for this task: 50 minutes, in-class, under test conditions
Calculator Free – 35 minutes
Calculator-assumed - 15 minutes

Materials required: Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred) , pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Marks available: **marks**

Task weighting: 6%

Section One - Calculator Free

Question 1 **[3 marks]**

Determine $\frac{dy}{dx}$ in terms of x where $y = u^3$ and $u = x^2 - 7$

$\frac{dy}{du} = 3u^2 \quad \frac{du}{dx} = 2x$

$\frac{dy}{dx} = 3(x^2-7)^2 \times 2x = 6x(x^2-7)^2$

Question 2 **[12 marks]**

Determine $\frac{dy}{dx}$ for each of the following.

a) $y = \frac{5x-4}{2x-1}$

Quotient rule

$\frac{dy}{dx} = \frac{5(2x-1) - 2(5x-4)}{(2x-1)^2} = \frac{10x - 5 - 10x + 8}{(2x-1)^2} = \frac{3}{(2x-1)^2}$

b) $y = 3x\sin(2x)$

$\frac{dy}{dx} = 3\sin(2x) + 6x\cos(2x)$

Product Rule

Answer

c) $y = \frac{1}{3x^2-1}$ $= (3x^2-1)^{-1}$ $= -6x(3x^2-1)^{-2} = \frac{-6x}{(3x^2-1)^2}$

d) $y = \cos^2(3x - \pi)$ $-2 \times 3\sin(3x-\pi)\cos(3x-\pi)$

Chain Rule

$= -6\sin(3x-\pi)\cos(3x-\pi$

Question 3 **[4 marks]**

a) Find the gradient of the curve $y = (2x^3 + 1)^5$ at the point where $x = 1$

$$\frac{dy}{dx} = 5 \times 6x^2 (2x^3+1)^4$$
$$= 30x^2(2x^3+1)^4$$

$$x=1 \quad \frac{dy}{dx} = 30(3)^4 = 30 \times 81 = 2430$$

Question 4 **[4 marks]**

Consider the function $f(x)=\sqrt{x+3}$

a) Calculate $f(6)$.

$$f(6) = \sqrt{9} = 3$$

b) Using your answer to a), the function $f(x)$ and the incremental formula, calculate the approximate value of $\sqrt{9.1}$.

$$y = f(x)$$

$$\delta y \approx \frac{dy}{dx} \times \delta x \qquad \delta x = 0.1$$

$$f'(x) = (x+3)^{1/2}$$
$$= \frac{1}{2}(x+3)^{-1/2} = \frac{1}{\sqrt{(x+3)}}$$

$$\delta y \approx \frac{1}{\sqrt{9}} \times 0.1 = \frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$$

$$\sqrt{9.1} \approx 3\frac{1}{30}$$

Question 5 6 ~~[7 marks]~~

Differentiate the following with respect to x

a) $2e^{3x+5}$

$6e^{3x+5}$ ✓✓

b) $e^x(2x-5)^2$

✓ ✓

$2\times 2e^x(2x-5) + (2x-5)^2 e^x$ ✓

$= e^x(2x-5)(4+(2x-5))$

$= e^x(2x-5)(2x-1)$ ✓

Question 6 6 ~~[5 marks]~~

Find the equation of the tangent to the curve $f(x)=\sqrt{e^{x^2} + 4x}$ at $x = 0$

chain ✓

$f(x) = u^{\frac{1}{2}}$ $\quad u = e^{x^2} + 4x$

$f'(x) = \dfrac{2xe^{x^2}+4}{2\sqrt{(e^{x^2}+4x)}}$ ✓ ✓ $\quad f'(x) = \frac{1}{2}u^{-\frac{1}{2}}$ $\quad \dfrac{du}{dx} = 2xe^{x^2}+4$

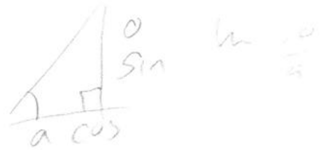
$x = 0$ $\quad y = 1$ ✓ $\quad \dfrac{dy}{dx} = \dfrac{4}{2} = 2$ ✓

$y = mx + c$

$1 = 2\times 0 + c$

$c = 1$ ✓

Eq $y = 2x + 1$



Section Two – Resources Allowed **Name:**__________________________

Question 7 **[3 marks]**

Use the quotient rule to show that $\frac{d}{dx}\tan x = \frac{1}{cos^2 x}$

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

Question 8 **[5 marks]**

Zebra mussels are an invasive species of shellfish recently discovered in some North American waterways. The mussel density, D, in shellfish per square metre, observed in a power station water supply pipe t days after a colony began, was modelled by the following equation, where k is a positive constant:

$$D = 200e^{kt}$$

(a) What was the mussel density in the colony when observations began? (1 mark)

$D = 200$. when $t = 0$ ✓

The mussel density was observed to double every eight days.

(b) Determine the value of k, rounded to four decimal places. (2 marks)

$t = 8$ $D = 400$. (Solve.

$400 = 200e^{k \times 8}$ ✓

$k = 0.0866$ ✓

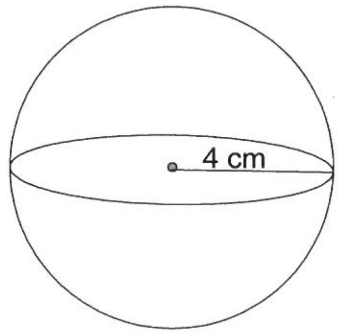
(c) The water supply pipe was seriously compromised when the mussel density reached 85 thousand shellfish per square metre. After how many days from the commencement of observations did this happen? (2 marks)

$85000 = 200e^{0.0866t}$

Solve $t = \cancel{68}69.88$ On 69th day

Question 9 **[5 marks]**

The radius of a sphere is measured as 4 cm with a possible error of 0.05 cm.



Using increments, determine the approximate error for the volume of the sphere.

$$V = \frac{4}{3}\pi r^3 \qquad \delta r = 0.05$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\delta V = \frac{dV}{dr} \times \delta r$$

$$\delta V = 4\pi r^2 \times 0.05$$

$$= 4 \times \pi \times 16 \times 0.05$$

$$\delta V = \underline{10.053 \text{ cm}^3}$$