Full Name: SOLUTIONS

Ellenbrook
CHRISTIAN COLLEGE

MATHEMATICS
METHODS

Test 4 – Sequences and Series
Chapter 8

Semester 2 2015

Section One – Calculator Free

Time allowed for this section
Working time for this section: 25 minutes
Marks available: 22 marks

Material required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates,

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (12 marks)

Define each of the following sequences recursively and state T_6 :

a. 10, 8, 6, 4... [3]

$T_{n+1} = T_n - 2$, $T_1 = 10$

$T_6 = 0$

b. 90, 100, 110, 120 ... [3]

$T_{n+1} = T_n + 10$, $T_1 = 90$

$T_6 = 140$

c. 10, 20, 40, 80... [3]

$T_{n+1} = 2T_n$, $T_1 = 10$

$T_6 = 320$

d. 1000, 500, 250, 125 ... [3]

$T_{n+1} = \frac{1}{2} T_n$, $T_1 = 1000$

$T_6 = 31.\cancel{8}25$

2. (10 marks)

Josh wants to increase the amount of time he spends studying leading up to his Year 12 exams.

He decides he will spend 'a' minutes studying the first week, 'a +d' minutes studying the second week, 'a +2d' minutes studying the third week and so on.

In week 4 he studies for 120 minutes and in week 10 he studies for 5 hours.

a. Determine the value of a and d. [4]

$120 + 6d = 300$ ✓
$6d = 180$
$d = 30$ ✓

$120 = a + 30(4-1)$ ✓
$a = 30$ ✓

b. His parents tell him that 12 hours a week is enough study. In which week will he reach 12 hours? [3]

$30 + 30(n-1) = 720$ ✓
$30(n-1) = 690$
$n-1 = 23$ ✓
$n = 24$ ✓ $\therefore$ week 24

e. Once he reaches 12 hours a week, he will maintain this for the rest of the year. If he studies for 40 weeks this year, write a calculation that can be used to calculate his total study for the year, in hours. [3]

$S = \frac{24}{2}\left(60 + 30(24-1)\right) + 16 \times 720$
✓ ✓ ✓

End of Section One

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MATHEMATICS METHODS

Test 4 – Sequences and Series

Chapter 8

Semester 2 2015

Section Two – Calculator Assumed

Time allowed for this section

Working time for this section: 40 minutes
Marks available: 37 marks

Material required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to candidates
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1. (9 marks)

The first term of an arithmetic sequence is 2 and the 5th term is 14.

a. Determine the common difference of this sequence. [2]

$d = 3$ $\left(\frac{14-2}{4} = 3\right)$

✓2

b. Hence or otherwise define this sequence recursively. [2]

$T_{n+1} = T_n + 3, \quad T_1 = 2$

✓ ✓

c. Calculate T_{10} [1]

$T_n = 2 + (n-1) \times 3$

$= 3n - 1$

$T_{10} = 29$ ✓

d. Calculate S_{10} [2]

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = 5(4 + (10-1) \times 3)$ ✓

$= 155$ ✓

e. Determine when the sequence first has a value greater than 100. [2]

$100 = 2 + (n-1) \times 3$

$100 = 3n - 1$

$101 = 3n$ ✓

$n = 33\frac{2}{3}$

when $n = 34$ ✓

2. (5 marks)

Eighty students participate in a fundraising event for charity. The student who raises the most money is rewarded with an $80 credit to spend at the canteen. The student who raises the second largest amount receives a $79 credit and this continues with the person who raises the least receiving just a $1 credit.

a. How much money is needed to give each of the 80 students their reward? [2]

$T_n = 80 - 1(n-1)$ ✓

$S_{80} = \$3240$ ✓

b. How much money is needed to give each of the bottom 20 students their reward? [2]

$S_{80} - S_{60} = 3240 - 3030$

$= \$216$

3. [5 marks]

The fourth term of a geometric sequence is 3125 and the sum of the 5th, 6th and 7th terms is 76171.875.

Determine a recursive rule for this geometric sequence.

$a \times r^{4-1} = 3125$ ✓

$a \times r^4 + a \times r^5 + a \times r^6 = 76171.875$ ✓✓

$a = 200 \ , \ r = 2.5$ ✓

$T_n = 2.5 \times T_{n-1} \ ; \ T_1 = 200$ ✓

4. (11 marks)
Winona is training for a half marathon. Her goal is to be able to run the full 21 km in 100 minutes.

On her first run she completes the half marathon in 150 minutes. On her second run it takes her 148 minutes to complete the 21 km, and on the third run it takes her 146 minutes. Her run times continue in this way.

a. How long will it take her to run the half marathon on her 10th training session? [2]

$T_{10} = 150 - 2(10-1) = 132$ min ✓ ✓

b. How long will it take her to run the half marathon on her nth training session? [2]

$T_n = 150 - 2(n-1)$ ✓ ✓

c. When will she be able to run the half marathon in less than 2 hours? [2]

$150 - 2(n-1) < 120$ ✓

$n = 17$ by the 17th run ✓

d. How long has she spent training during her first seven runs? [2]

$S_7 = 1008$ min ✓ ✓

e. If her training continues in this way, when will she achieve her goal? [1]

$150 - 2(n-1) = 100$

$n = 26$

after 26 runs ✓

f. ~~Winona wonders if her training can reap these sort of results indefinitely. Explain whether Winona can keep achieving these results.~~ [2]

5. (7 marks)
A ball is dropped from a height of 3 m. On each bounce back up the ball reaches a height 80% of its previous height.

a. Determine the height reached on the first bounce. [1]

$3 \times 0.8 = 2.4\text{m}$ ✓

b. Write a recursive rule defining the height of each bounce after the first bounce. [2]

$H_n = 0.8 \times H_{n-1}\ ;\ H_1 = 2.4$
✓ ✓

c. Calculate the total vertical distance travelled by the ball after 10 bounces. [2]

$S_{10} \times 2 + 3 = 10.71 \times 2 + 3$
✓ $= 24.42\text{m}$ ✓

d. If the ball bounces indefinitely, calculate the total vertical distance travelled. [2] 0

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End of Test