MOSQUITOES DI

Stage 1 Mathematics

Introduction:

Mosquito A and Mosquito B are moving in straight lines in 3D space with paths that can be given by lines in Vector, Parametric or Cartesian forms. Through vector geometry and algebra, these paths will be manipulated to determine the speed of the mosquitoes, and hence an equation giving the distance between the two mosquitoes at any time will be determined. From this, the shortest distance between the two will be identified. Finally, this information shall be used to determine the most effective method of exterminating the two mosquitoes with insecticide by developing conical models, with supporting discussions of the 'results' at the end.

PART 1:

Mosquito A has been given initial position (1, 2, 3) and has velocity vector [4,5,6]. Whereas Mosquito B has been given initial position (3,2,1) and has velocity vector [6,5,4]. Person A is located at the point (41,52,63) and Person B is located at the point (69,57,45)

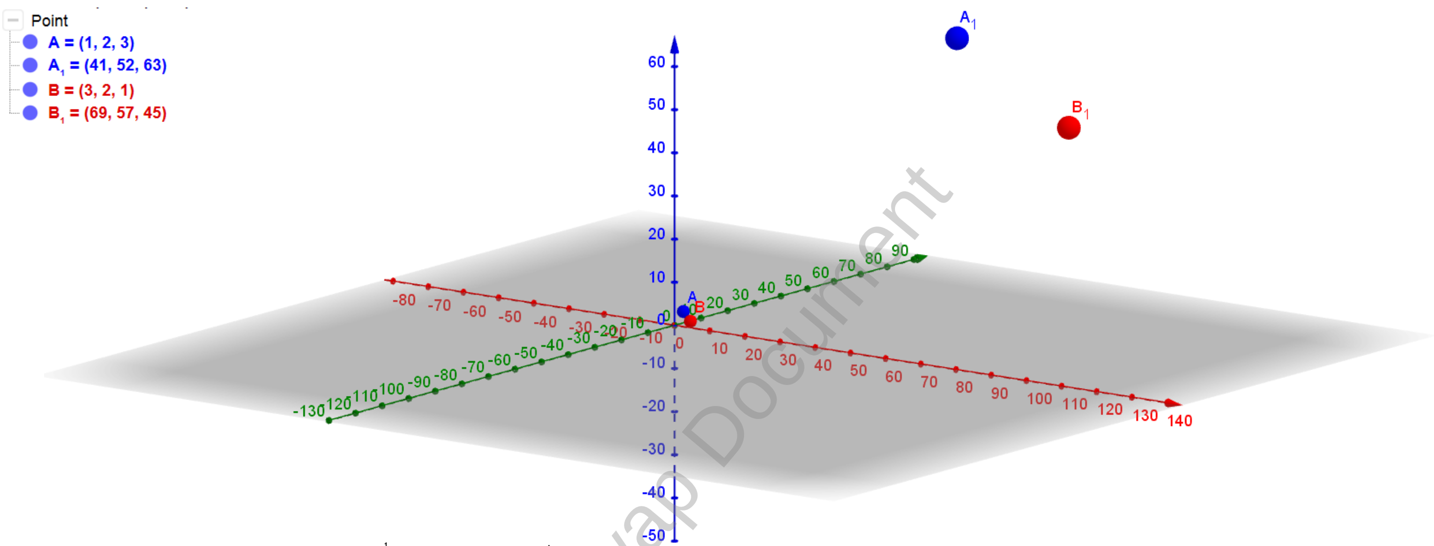


Figure 1: Positions of Mosquito A and B and Person A and B.

This would mean that the vector equation of the form $[x, y, z] = [x_1, y_1, z_1] + t[a, b, c]$ that links the location of the mosquitoes in regards to parameter 't' can be found for each mosquito:

Mosquito A:$[x, y, z] = [1,2,3] + t[4,5,6]$

Mosquito B:$[x, y, z] = [3,2,1] + t[6,5,4]$

These vector equations can now be converted into parametric equations of the form $x = x_1 + at, y = y_1 + at, z = z_1 + at$, simply by rearranging and equating matrices.

Mosquito A: $x = 1 + 4t, y = 2 + 5t, z = 3 + 6t$

Mosquito B: $x = 3 + 6t, y = 2 + 5t, z = 1 + 4t$

The units are in centimetres for these equations.

Now the speed of the mosquito would simply be the magnitude of the velocity vector. This is because the velocity vector is what determines the distance each mosquito will travel in interval 't'. As speed is a magnitude it is free of any direction and hence can be found through the modulus of the velocity vectors.

$$if\ \boldsymbol{a} = [a_1, a_2, a_3]\ then\ |\boldsymbol{a}| = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$$

$$\therefore Mosquito\ A = \sqrt{(4)^2 + (5)^2 + (6)^2} = \sqrt{77} \cong 8.77cm/t$$

$$\therefore Mosquito\ B = \sqrt{(6)^2 + (5)^2 + (4)^2} = \sqrt{77} \cong 8.77cm/t$$

If Person A is at (41, 52, 63), assuming it is in the path of the Mosquito A and Person B is at (69,57,45), assuming it is in the path of Mosquito B, using the vector equations for each mosquito, the value for t can be found at which the mosquito will meet the person simply by working backwards with the parametric equations for both mosquitoes.

$$\therefore Mosquito\ A: 41 = 1 + 4t, 52 = 2 + 5t, 63 = 3 + 6t\ \therefore t = \frac{40}{4} = 10$$

$$\therefore Mosquito\ B: 69 = 3 + 6t, 57 = 2 + 5t, 45 = 1 + 4t \therefore t = \frac{66}{6} = 11$$

This would mean that when t = 10 mosquito a would hit person a and when t = 11, mosquito b would hit person b.

PART 2:

A position point for each mosquito can be created of the form (a, b, c) using the parametric equations from part 1.

$\therefore Mosquito\ A = (1 + 4t, 2 + 5t, 3 + 6t) = Position\ P$
$\therefore Mosquito\ B = (3 + 6t, 2 + 5t, 1 + 4t) = Position\ Q$

These points can now be used to determine vector $\overrightarrow{PQ} = [\Delta x, \Delta y, \Delta z]$

$$\therefore \overrightarrow{PQ} = [(3 + 6t) - (1 + 4t), (2 + 5t) - (2 + 5t), (1 + 4t) - (3 + 6t)] = [(3 - 1 + 6t - 4t), 0, (1 - 3 + 4t - 6t)]$$

$$\therefore \overrightarrow{PQ} = [(2 + 2t), 0, (-2 - 2t)]$$

This vector can now be used to find the distance between the two mosquitoes for any given value of t, simply by calculating the modulus of the vector **PQ.**

$$|\overrightarrow{PQ}| = \sqrt{(2 + 2t)^2 + 0 + (-2 - 2t)^2} = \sqrt{4 + 8t + 4t^2 + 8t + 4t^2} = \sqrt{8t^2 + 16t + 8}$$

Hence this would mean that $distance\ between\ mosquitoes\ (d) = \sqrt{8t^2 + 16t + 8}\ or\ d^2 = 8t^2 + 16t + 8$

Theoretically, when the mosquitoes touch, $d^2 = 0$. Hence to find out when the mosquitoes touch, $0 = 8t^2 + 16t + 8$ can be solved.

$$\therefore 0 = 8(t^2 + 2t + 1) \therefore 0 = t^2 + 2t + 1 \therefore 0 = (t + 1)^2\ \therefore\ 0 = t + 1\ \therefore t = -1\ when\ d^2 = 0$$

However, t in this scenario represents a time interval, and a time interval cannot be negative, hence the mosquitoes may or may not touch so this may not be the shortest distance between the two.

This would mean that to find the smallest value of d, where it must be greater than or equal to zero $(t \geq 0)$, the equation $(d) = \sqrt{8t^2 + 16t + 8}$ would have to be graphed across the domain $\{t: t \geq 0\}$

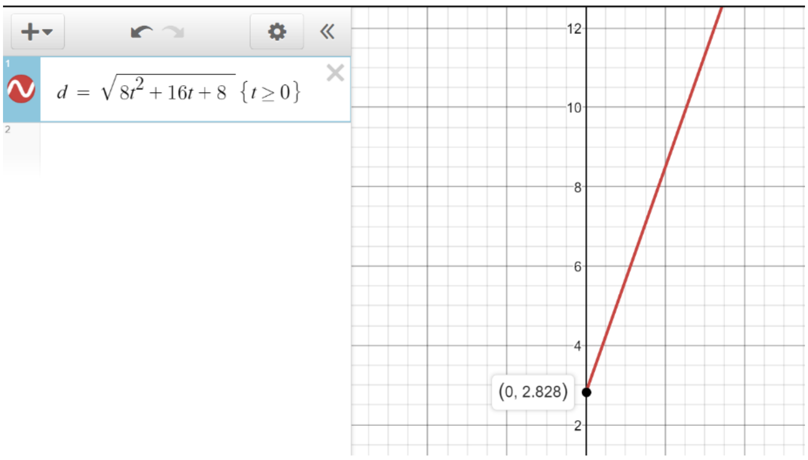


Figure 2: graph of $d = \sqrt{8t^2 + 16t + 8}$ across domain $\{t : t \geq 0\}$

In the graph above the minimum distance between the two mosquitoes is when t = 0 and the minimum distance is 2.828cm.

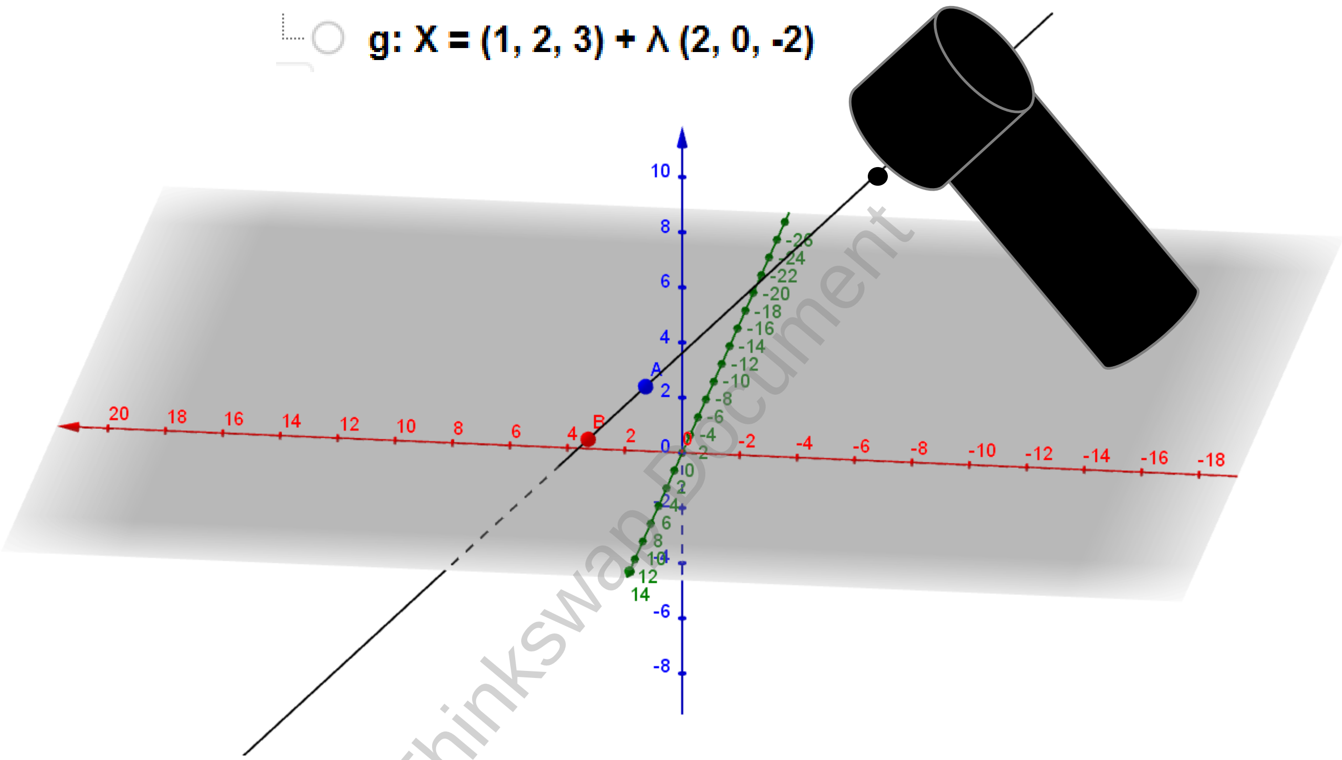
PART 3

If one were to supposedly kill the two mosquitoes using insect repellent spray, the best time for that would be when the mosquitoes would be closest to each other and hence when t = 0. This would mean that a line in space can be found which is collinear to the two mosquitoes when t = 0.

To find out this line where both mosquitoes lie on when t = 0, the vector PQ and the initial coordinate of any of the two mosquitoes can be used.

$Mosquito\ A = (1, 2, 3)\ when\ t = 0\ and\ [2, 0, -2]\ when = 0$
Hence the line on which both mosquitoes lie on when t = 0 is:

$$[x, y, z] = [1,2,3] + t[2,0,-2]$$



$$d = \sqrt{(-6-1)^2 + (2-2)^2 + (10-3)^2}$$

$$d = \sqrt{(-7)^2 + (0)^2 + (7)^2}$$

$$d = \sqrt{49 + 0 + 49}$$

$$d = \sqrt{98}$$

$$d = 9.899495$$

This would mean that the insect repellent would have to fired in this direction to perform a double kill from position (-6,2,10), being only a mere 10cm away from mosquito A. However, mosquito repellents or any spray do not particularly fire in a straight line, in fact they produce conical shapes.

PART 4

For Part 1:

Part 1 seems quite reasonable initially as the "top speed of a mosquito is given as 1.5miles per hour" which equates to approximately 2.4 km/h = 0.66m/s or 66 centimetres a second**.** Mosquito A and B both have speeds of around 9cms a second which seems highly realistic and plausible. However, one aspect of Part 1 that concerned the reasonableness of the results is associating a person being located at a point. The graphical scale for coordinates and equations is given in centimetres so how can a person that is billions of times larger than a mosquito be represented as a single point in space. The person(s), should in fact be represented as capacities in space that are similar to the volumes and shapes of an actual human. This would make more sense in terms of the human's location and the mosquito's location. As through this method, the correct distance between a human and mosquito can be established.

For Part 2:

Part 2 initially also seems fairly valid however do mosquitoes always travel in straight lines? This is one issue however not a huge issue in terms of reasonableness as the task sheet for this investigation does require us to assume they are going in a straight line. Part 2, graphically also establishes that the mosquitoes are travelling further and further away, and this does seem plausible as the both have velocity vectors heading in separate directions, making t proportional to the distance between the two. Also in this part, it is assumed that the mosquitoes have constant velocities, whereas in real life factors such as acceleration may come in to play, making it not 100% clear whether the mosquito(s) position at any given time interval is actually accurate.

For Part 3:

Part 3 is determining a way in which both mosquitoes can be killed at the same time, i.e. a double kill can be performed. Theoretically, the idea would be that the best time to kill both mosquitoes would be when they are closest to each other, as this increases the likelihood of success. But the mosquitos are closest when t = 0, meaning in real life the person would not have any time to react and to reach that position. Also, assumptions have been made on the strength and velocity of the spray and even the shape the spray will produce. Making assumptions often creates uncertainty in the reliability and reasonableness of results and a majority of this part is on the basis of assumptions, giving no real information on the insecticide spray.

Conclusion:

As it can be seen, many assumptions have been made throughout this investigation, making it one of the limitations of the results in this report. Due to this, none of the results can be labelled as accurate as it is very personalized to the values that I design. However, useful mathematical concepts can be seen throughout this investigation, that would be the same processes for every mosquito direction and point regardless of the real numerical values.