GREENWOOD COLLEGE

Methods 11 Investigation 3 2018
Calculus
Total Marks: 52 Time Allowed: 55 minutes

Name: Marking Key.

Classpad allowed.
All electronic devices must be switched off and in student bags.

ALL working must be shown for full marks.

1. [2 marks]

Describe a method that can be used to determine the gradient at a point P, for any given function. Eg. Describe how to determine the gradient to the tangent at the point x = 2 for the function $y = x^7$.

① For the function equation, make the ~~indice~~ index the coefficient, and reduce the ~~indice~~ index by one. Then put the x value into the new equation to find the gradient at that point on the function. ✓ ✓

OR ② test some values, as they get closer to the point P and determine the limit of these values.

2. [2 marks]

Using your description from part 2 to write a rule that links the function and the "gradient of the tangent" to the function at a point x for the function $y = x^n$

The "gradient of the tangent" can be determined by $y =$ nx^{n-1}. This is referred to as the "**gradient formula.**" (Hint: Use *y, x, and n in your formula)*

✓✓

$$m = \frac{(x+h)^n - x^n}{h} \quad \text{as } h \to 0$$

Values outside the box are finding the gradient at the points 2.1, 2.01 and 2.001. values not to 4 dp. lose one mark each. Values beyond 4 dp lose one mark only for this page.

3. [8 marks]

Repeat the table above for the functions below, determining the gradient of the tangent at the point, P, when x = 2. (Use x = 2.1, x = 2.01, x = 2.001 as your Q points) Place answers in the table below. (4 d.p.)

a) $y = 2x^3$

b) $y = 2x^4$

26.46 24.2406 24.0240

	Gradient when x = 2.1 (for Q)	Gradient when x = 2.01	Gradient when x = 2.001	Gradient Approaches
$y = 2x^3$	25.22	24.1202	24.0120	24
$y = 2x^4$	68.962	64.4816	64.0480	64

74.0880 64.9648 64.0960

✓ each value

4. [8 marks]

Repeat the table above for the functions below, determining the gradient of the tangent at the point, P, when x = 2. (Use x = 2.1, x = 2.01, x = 2.001 as your Q points) Place answers in the table below. (4 d.p.)

a) $y = 3x^3$

b) $y = 3x^4$

39.69 36.3609 36.0360

	Gradient when x = 2.1	Gradient when x = 2.01	Gradient when x = 2.001	Gradient Approaches
$y = 3x^3$	37.83	36.1803	36.0180	36
$y = 3x^4$	103.443	96.7224	96.0720	96

111.132 97.4472 96.1441

✓ each value

5. [8 marks]

Repeat the table above for the functions below, determining the gradient of the tangent at the point, P, when x = 2. (Use x = 2.1, x = 2.01, x = 2.001 as your Q points) Place answers in the table below. (4 d.p.)

a) $y = 10x^3$

b) $y = 10x^4$

✓ each value

132.3 121.203 120.1200

	Gradient when x = 2.1	Gradient when x = 2.01	Gradient when x = 2.001	Gradient Approaches
$y = 10x^3$	126.1	120.601	120.0600	120
$y = 10x^4$	344.81	320.4080	320.2401	320

370.44 324.8240 320.4802

24

6. **[6 marks]**

Fill in the table with the gradient of the tangent acquired in parts 3-5. (To nearest whole number)

	$y = 2x^3$	$y = 2x^4$	$y = 3x^3$	$y = 3x^4$	$y = 10x^3$	$y = 10x^4$
x = 2	24	64	36	96	120	320

✓ ✓ ✓ ✓ ✓ ✓

7. **[2 marks]**

Describe a method that can be used to determine the gradient at a point P, for any given function.

Eg. Describe how to determine the gradient to the tangent at the point x = 2 for the function $y = 3x^5$

multiple the coefficient by the index ✓ and reduce the ~~index~~ index by one. ✓

"Description" just involves example (one mark)

Example: $m = 5 \times 3x^{5-1}$

$= 15x^4$ when $x = 2$

$= 240$

8. **[2 marks]**

Using your description from part 8, modify your rule from part 4 to write a rule that links the function and the "gradient of the tangent" to the function at a point x when the coefficient to x is greater than 1 for the function $y = ax^n$

The "gradient of the tangent" can be determined by $y =$ ✓ nax^{n-1} ✓. This is referred to as the "**gradient formula.**" (Hint: Use *y, x, and n in your formula)*

9. **[2, 2, 2, 2 = 8 marks]**

Use your rule to determine the **gradient formula** for the following functions.

a) $y = 4x^5$ gradient formula $f'(x) = 20x^4$ ✓✓

b) $y = 3x^2$ gradient formula $f'(x) = 6x$ ✓✓

c) $y = x^5$ gradient formula $f'(x) = 5x^4$ ✓✓

d) $y = \frac{x^9}{3}$ gradient formula $f'(x) = 3x^8$ ✓✓

non-simplified answers count one mark only.

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10. **[2, 2, 2 = 6 marks]**

Use your rule to determine:

a) The gradient formula for the function $y = 2x^4$

$f'(x) = 8x^3$ ✓ ✓

b) the gradient of the tangent at the point (2, 32)

$f'(x) = 8(2)^3$ ✓

$= 8 \times 8$

$= 64$ ✓

c) the equation of the tangent line at the point (2, 32).

$y = mx + c$

$32 = 64(2) + c$

$c = -96$

$y = \cancel{6x}\ 64x - 96$ ✓ ✓