

High School Mathematics Test 2013

Year
10

Geometric Reasoning

Calculator Allowed

Name _____

Skills and Knowledge Assessed:

- Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)
- Formulate proofs involving congruent triangles and angle properties (ACMMG243)

Extended/Connected Answer Test.

Give reasons for all steps when finding unknown values and proving properties.

Write all working and answers in the spaces provided on this test paper.

Diagrams are not to scale unless otherwise stated.

Marks

1. Find the value of the pronumeral in each of the following diagrams.
Show step by step reasoning to explain your answer.

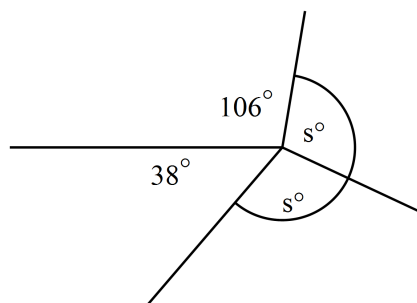
a)

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2

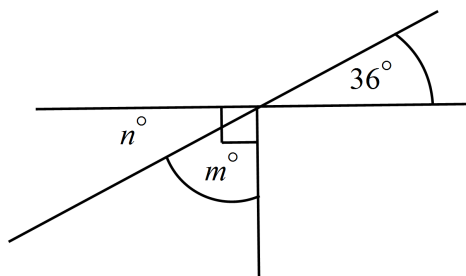
b)

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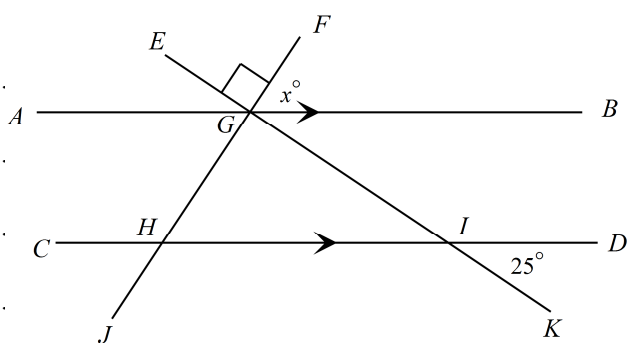


2

Marks

c)

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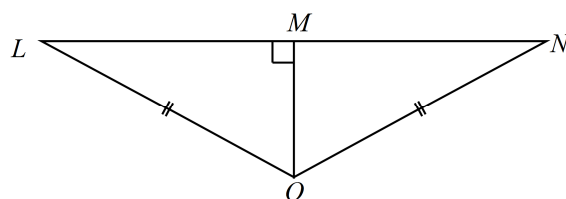


3

2. a) $OL = ON$

Prove that $\triangle LMO \equiv \triangle NMO$.

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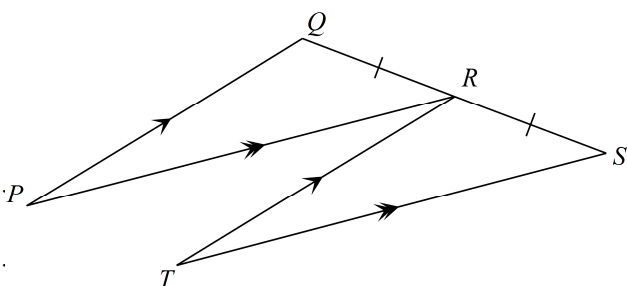
3

b) $PQ \parallel TR$ and $PR \parallel TS$.

$QR = RS$

Prove that $\triangle PQR \equiv \triangle TRS$

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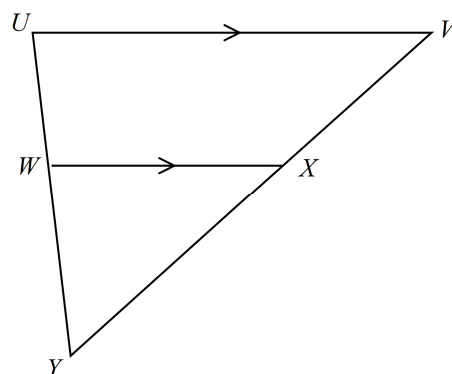


3

3. a) $UV \parallel WX$

Prove that $\triangle UVY \parallel \triangle WXY$.

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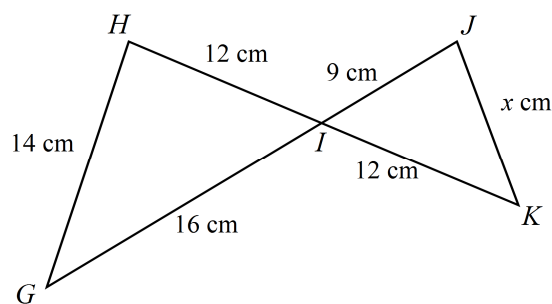


3

Marks

- b) i) Given the dimensions shown, prove that $\triangle GHI \parallel \triangle KJI$.

3



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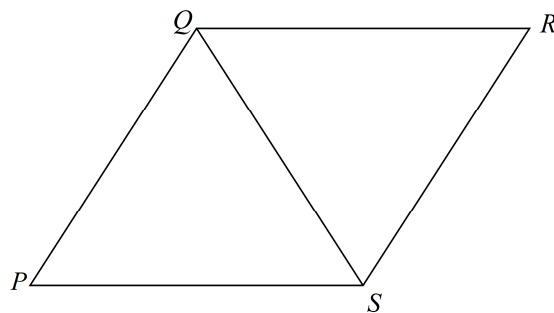
- ii) Find the value of x .

1

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4. a) In $PQRS$; $\angle QSP = \angle SQR$ and $SP = RQ$.
 Prove that $PQRS$ is a parallelogram.

3

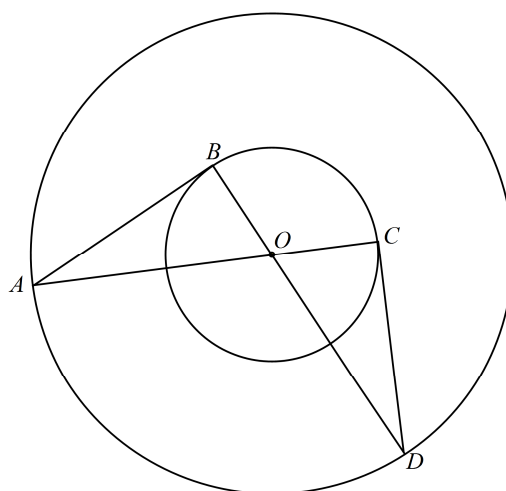


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Marks

- b) O is the centre of both circles.
 A and D are points on the larger circle.
 B and C are points on the smaller circle.
 Use congruent triangles to prove that $AB = CD$.

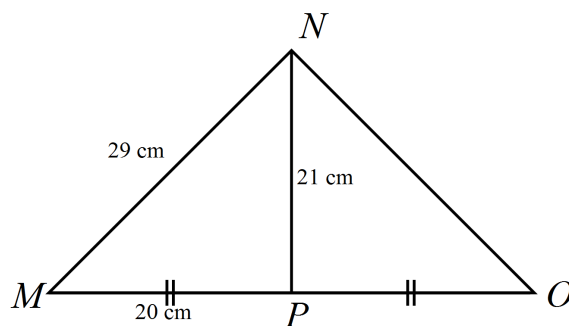
4



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- c) Prove that the triangle MNO is isosceles.

4



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High School Mathematics Test 2013 Geometric Reasoning

ANSWERS

1. a)	$2s^{\circ} + 38^{\circ} + 106^{\circ} = 360^{\circ}$ (angles at a point add to 360°) $2s^{\circ} = 360^{\circ} - 144^{\circ}$ $2s^{\circ} = 216^{\circ}$ $s = 216 \div 2 = 108$	1 mark for sum 1 for value of pronumeral with reason
b)	$n^{\circ} = 36^{\circ}$ (vertically opposite angles) $m^{\circ} + 36^{\circ} = 90^{\circ}$ (complementary angles) $m = 90 - 36$ $m = 54$	1 mark for each pronumeral with reason
c)	$\angle BGI = 25^{\circ}$ (corresponding to $\angle KID$ on \parallel lines) $\angle AGE = 25^{\circ}$ (vertically opposite to $\angle BGI$) $\angle FGB = x^{\circ} = 180^{\circ} - (25^{\circ} + 90^{\circ})$ (angles on a straight line) $\therefore x = 65$	1 mark for each step with reasons (or equivalent)
2 a)	<p>In $\triangle LMO$ and $\triangle NMO$</p> $\angle LMO = \angle NMO$ (equal supplementary angles) $OL = ON$ (given) MO is common $\therefore \triangle LMO \equiv \triangle NMO$ (RHS)	3 marks for correct & complete proof. 2 if a reason or statement is incorrect or missing 1 if one correct statement is made toward proof.
b)	<p>In $\triangle PQR$ and $\triangle TRS$</p> $\angle PQR = \angle TRS$ (corresponding angles on \parallel lines) $\angle PRQ = \angle TSR$ (corresponding angles on \parallel lines) $QR = RS$ (given) $\therefore \triangle PQR \equiv \triangle TRS$ (AAS)	3 marks for correct & complete proof. 2 if a reason or statement is incorrect or missing 1 if one correct statement is made toward proof.

3 a)	<p>In $\triangle UVY$ and $\triangle WXY$</p> <p>$\angle VUY = \angle XWY$ (Corresponding angles on \parallel lines)</p> <p>$\angle UYV = \angle WYX$ (Corresponding angles on \parallel lines)</p> <p>$\angle Y$ is common</p> <p>$\therefore \triangle UVY \parallel \triangle WXY$ (All corresponding angles equal)</p>	<p>3 marks for correct & complete proof.</p> <p>2 if a reason or statement is incorrect or missing</p> <p>1 if one correct statement is made toward proof.</p>
b) i)	<p>In $\triangle GHI$ and $\triangle KJI$</p> <p>$\frac{HI}{IJ} = \frac{12}{9} = \frac{4}{3}$ (Values Given)</p> <p>$\frac{GI}{IK} = \frac{16}{12} = \frac{4}{3}$ (Values Given)</p> <p>$\angle HIG = \angle JIK$ (Vertically Opposite angles)</p> <p>$\therefore \triangle GHI \parallel \triangle KJI$ (2 sides in same ratio and equal included angle)</p>	<p>3 marks for correct & complete proof.</p> <p>2 if a reason or statement is incorrect or missing</p> <p>1 if one correct statement is made toward proof.</p>
	<p>ii) $\frac{x}{14} = \frac{9}{12} = \frac{12}{16} = \frac{3}{4}$</p> <p>$\frac{x}{14} = \frac{3}{4}$</p> <p>$x = 14 \times \frac{3}{4}$</p> <p>$x = \frac{42}{4} = 10\frac{1}{2} \text{ cm}$</p>	<p>1 for correct answer</p>
4 a)	<p>$SP = RQ$ (given)</p> <p>$\angle QSP = \angle SQR$ (given)</p> <p>$\therefore SP \parallel RQ$ (equal alternate angles on \parallel lines)</p> <p>$\therefore PQRS$ is a parallelogram (Opposite sides are equal and parallel)</p>	<p>3 marks for correct & complete proof.</p> <p>2 if a reason or statement is incorrect or missing</p> <p>1 if one correct statement is made toward proof.</p>

b)	<p>In $\triangle ABO$ and $\triangle DCO$</p> <p>$AO = DO$ (equal radii of larger circle)</p> <p>$BO = CO$ (equal radii of smaller circle)</p> <p>$\angle AOB = \angle DOC$ (Vertically opposite angles)</p> <p>$\therefore \triangle ABO \equiv \triangle DCO$ (<i>SAS</i>)</p> <p>$\therefore AB = DC$ (corresponding sides of congruent triangles)</p>	<p>3 marks for correct & complete CONGRUENCE proof.</p> <p>2 if a reason or statement is incorrect or missing</p> <p>1 if one correct statement is made toward proof.</p> <p>1 mark for deduction of result from the congruence.</p>
c)	<p>Using Pythagoras Theorem</p> <p>$MN^2 = 29^2 = 841$</p> <p>$MP^2 + PN^2 = 20^2 + 21^2 = 841$</p> <p>$\therefore \triangle MPN$ is right angled.</p> <p>$\therefore \angle MPN = 90^\circ$</p> <p>$\therefore \angle NPO = 90^\circ$ (equal supplementary angles)</p> <p>In $\triangle MPN$ and $\triangle OPN$</p> <p>$MP = PO$ (given)</p> <p>$\angle MPN = \angle NPO = 90^\circ$ (from above)</p> <p>NP is common</p> <p>$\therefore \triangle MPN \equiv \triangle OPN$ (<i>SAS</i>)</p> <p>$\therefore MN = NO$ (corresponding sides in congruent triangles)</p> <p>$\therefore \triangle MNO$ is isosceles</p>	<p>1 mark for the Pythagoras proof</p> <p>2 marks for correct congruence proof, or 1 mark for partially correct congruence.</p> <p>1 mark for deduction of required result following from congruence.</p>