Baldivis
Secondary College

Year 11 Specialist Mathematics
Test 2 2020
Chapters 3, 4, 5

Name: ANSWERS **Time: 25 minutes Total ______ / 25**

INSTRUCTIONS:

Part A: Non-calculator

Notes not allowed

Full working must be shown for all questions (or parts) worth more than 2 marks.

Marks will be deducted for rounding and unit errors.

Question 1 **(7 marks)**

i horizontal unit vector
j vertical unit vector. ✓

$\begin{pmatrix} i \\ j \end{pmatrix}$ $pi + qj$ or (p, q) or $\begin{pmatrix} p \\ q \end{pmatrix}$

Given that $\underset{\sim}{a} = 2\underset{\sim}{i} + 5\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 5\underset{\sim}{j}$ find

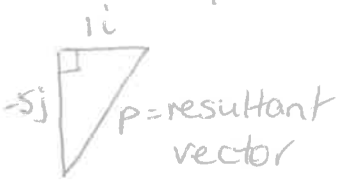
a) $2\underset{\sim}{a} - \underset{\sim}{b}$ [1]

$\begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} 2\times 2 \\ 2\times 5 \end{pmatrix} - \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

$\begin{pmatrix} 4 \\ 10 \end{pmatrix} - \begin{pmatrix} 1 \\ -5 \end{pmatrix} = \begin{pmatrix} 4-1 \\ 10--5 \end{pmatrix} = \begin{pmatrix} 3 \\ 15 \end{pmatrix} = 3i + 15j$ ✓

b) $\hat{\underset{\sim}{b}} = \dfrac{b}{|b|}$ [1]

$|p| = \sqrt{(-5)^2 + 1^2} = \sqrt{26}$



$\dfrac{1}{\sqrt{26}}(i - 5j) = \dfrac{1}{\sqrt{26}} i - \dfrac{5}{\sqrt{26}} j$ ✓

c) vector $\underset{\sim}{c}$ such that $|\underset{\sim}{c}| = 5$ and $\underset{\sim}{c} = \alpha \underset{\sim}{b}$ [2]

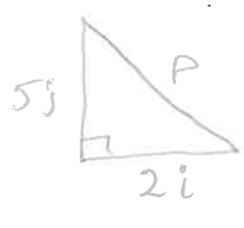
$|c|$ means the magnitude or modulus of c

$\underset{\sim}{c} = \dfrac{5}{\sqrt{26}}(i - 5j) = \dfrac{5}{\sqrt{26}} i - \dfrac{25}{\sqrt{26}} j$ ✓ ✓

d) a vector of magnitude 10 moving in the opposite direction to $\underset{\sim}{a}$ [3]

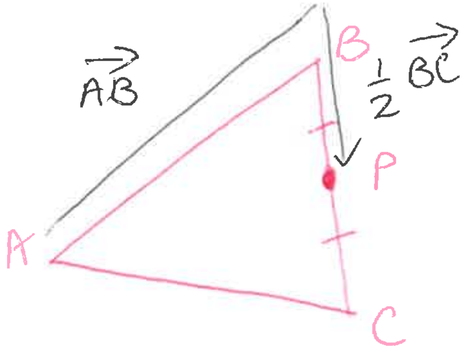
$|p| = \sqrt{5^2 + 2^2} = \sqrt{29}$ ✓

✓
$-\dfrac{10}{\sqrt{29}}(2i + 5j) = -\dfrac{20}{\sqrt{29}} i - \dfrac{50}{\sqrt{29}} j$ ✓



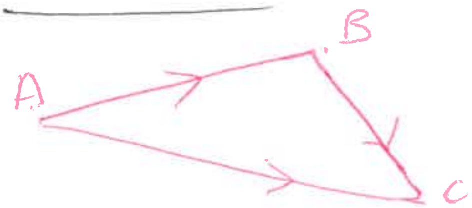
Question 2 **(5 marks)**

In a triangle ABC, P is the midpoint of BC. Prove that $\overrightarrow{AP} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$



$$\overrightarrow{AP} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$$
$$= \overrightarrow{AB} + \frac{1}{2}\left(\overrightarrow{AC} - \overrightarrow{AB}\right)$$
$$= \overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{AB}$$
$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC}$$
$$= \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$$

p61 Sadler

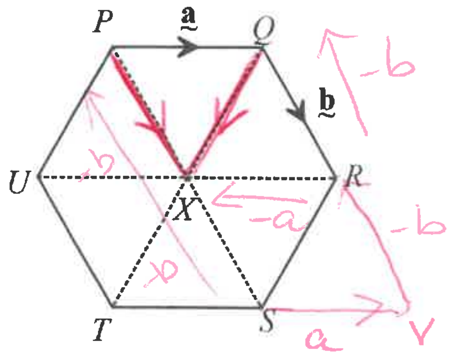


$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ So $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

must match in middle

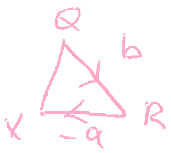
Question 3 **(1, 1, 2, 2 = 6 marks)**

PQRSTU is a regular hexagon. If $\overrightarrow{PQ} = \underset{\sim}{a}$ and $\overrightarrow{QR} = \underset{\sim}{b}$, find in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$

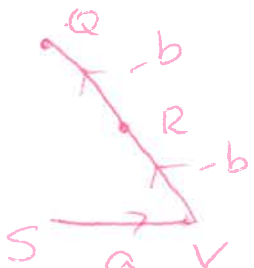


a) $\overrightarrow{PX}$ $\underset{\sim}{b}$

b) $\overrightarrow{SP}$ $-2\underset{\sim}{b}$



c) $\overrightarrow{QX}$ $\underset{\sim}{b} - \underset{\sim}{a}$



d) $\overrightarrow{SQ}$ $\underset{\sim}{a} - 2\underset{\sim}{b}$

Question 4 **(3 marks)**

In the diagram $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

Point P divides OA in the ratio OP : PA = 4 : 1.

Point Q divides AB in the ratio AQ : QB = 2 : 3.

Show that $\overrightarrow{PQ} = \frac{2}{5}\underset{\sim}{b} - \frac{1}{5}\underset{\sim}{a}$

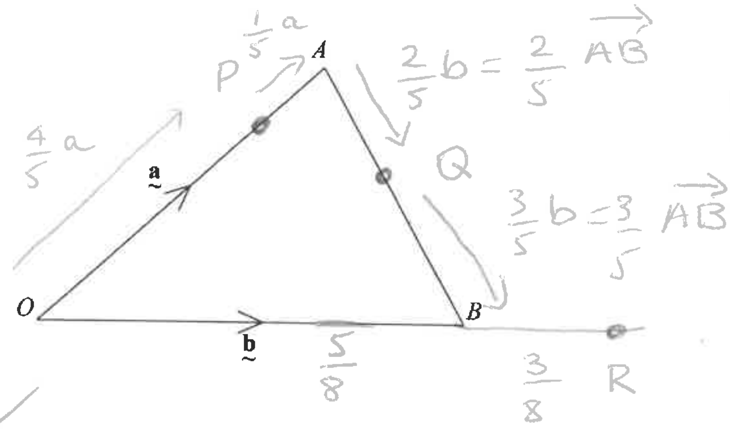
$$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ} \quad ✓$$

$$= \frac{1}{5}\underset{\sim}{a} + \frac{2}{5}\overrightarrow{AB} \quad ✓$$

$$= \frac{1}{5}\underset{\sim}{a} + \frac{2}{5}(\underset{\sim}{b} - \underset{\sim}{a}) \quad ✓$$

$$= \frac{1}{5}\underset{\sim}{a} + \frac{2}{5}\underset{\sim}{b} - \frac{2}{5}\underset{\sim}{a}$$

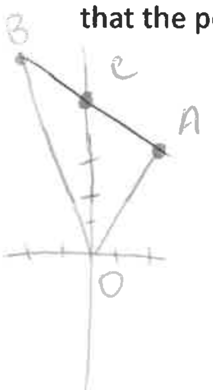
$$= \frac{2}{5}\underset{\sim}{b} - \frac{1}{5}\underset{\sim}{a}$$



Question 5 **(4 marks)**

A B $C(0i + 6j)$

Points A, B and C have position vectors $2\underset{\sim}{i} + 3\underset{\sim}{j}$, $-2\underset{\sim}{i} + 9\underset{\sim}{j}$ and $6\underset{\sim}{j}$ respectively. Use vectors to prove that the points A, B and C are collinear.



$$\overrightarrow{AB} = \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} -2 \\ 9 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2-2 \\ 9-3 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad ✓$$

$$\overrightarrow{BC} = \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} - \begin{pmatrix} -2 \\ 9 \end{pmatrix} = \begin{pmatrix} 0--2 \\ 6-9 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 2 \\ -3 \end{pmatrix}}_{\overrightarrow{BC}} \times -2 = \underbrace{\begin{pmatrix} -4 \\ 6 \end{pmatrix}}_{\overrightarrow{AB}}$$

Therefore collinear (in a line) as parallel and have B in common

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Name: ______________________________ **Time: 30 minutes** **Total /25**

INSTRUCTIONS:

Part B: Calculator allowed

1 page of A4 notes allowed

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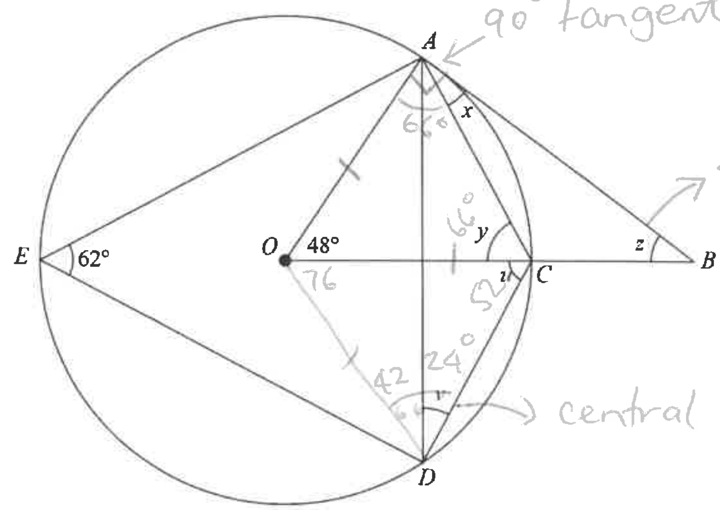
Question 1 **(5 marks)**

Find the value of all pronumerals, giving reasons (O is the centre of the circle, AB is a tangent)

90° tangent to radius



42° $(48+42+90^\circ) = 180^\circ$ Δ

central angle theorem

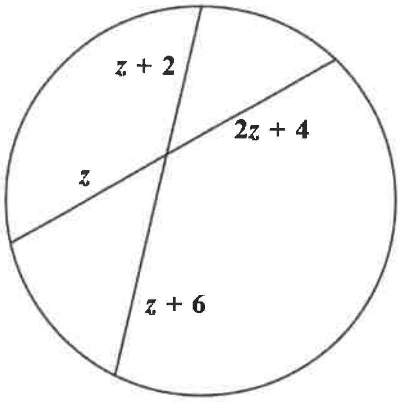
$62 + 62 = 124$
$124 - 48 = 76$
$76 + 66 = 142$
$180 - 142 = 38$

Angle/working	Reason
✓ $z = 42^\circ$	Third angle of right triangle
✓ $v = 24^\circ$	Angle at centre is double angle at circumference in same arc
✓ $y = 66^\circ$	isosceles Δ from two radii $180^\circ - 48^\circ = 132^\circ \div 2 = 66^\circ$
✓ $x = 24^\circ$	$90^\circ - 66^\circ = 24^\circ$
✓ $u = 52^\circ$	opposite angles of cyclic quadrilateral EDCA are supplementary $180 - 62^\circ = 118^\circ$

$118^\circ - y = 118^\circ - 66^\circ = 52^\circ$

Question 2 **(3 marks)**

Find the value of the pronumeral z



$z \times (2z+4) = (z+6) \times (z+2)$ ✓

$2z^2 + 4z = z^2 + 6z + 2z + 12$

$2z^2 - z^2 + 4z - 8z - 12 = 0$ ✓

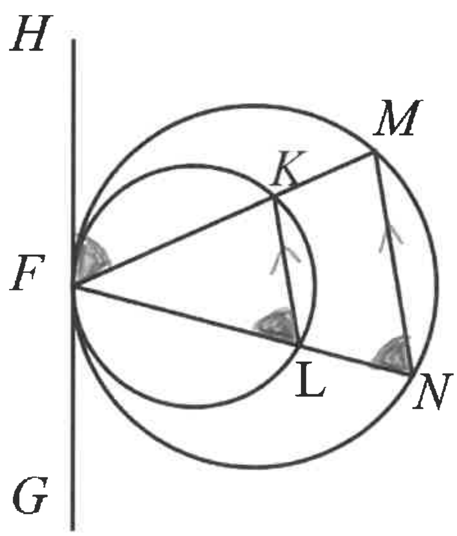
$z^2 - 4z - 12 = 0$

$(z-6)(z+2) = 0$

$z = 6, \quad z = -2$ (omit) ✓

Question 3 **(4 marks)**

GFH is a common tangent to both circles. Prove that *LK is parallel to NM*



$\angle HFM = \angle FLK$ (alternate segment theorem) ✓

$\angle HFM = \angle FNM$ (alternate segment theorem) ✓

$\therefore \angle FLK = \angle FNM$ ✓

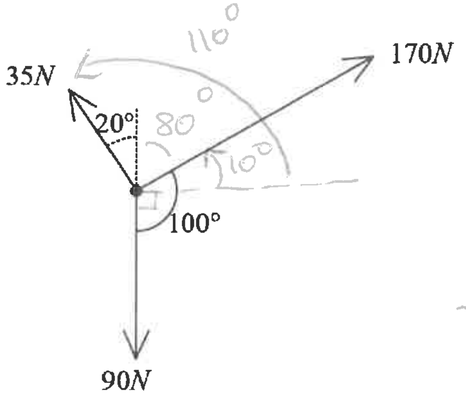
$\Rightarrow KL \parallel MN$ as corresponding angles are equal ✓

Question 4

use Classpad

(4 marks)

Find the resultant of the set of vectors below, giving your answer in the form $a\underset{\sim}{i} + b\underset{\sim}{j}$



$$\underset{\sim}{r} = 170\begin{pmatrix}\cos 10\\ \sin 10\end{pmatrix} + 35\begin{pmatrix}\cos 110\\ \sin 110\end{pmatrix} + \begin{pmatrix}0\\ -90\end{pmatrix}$$

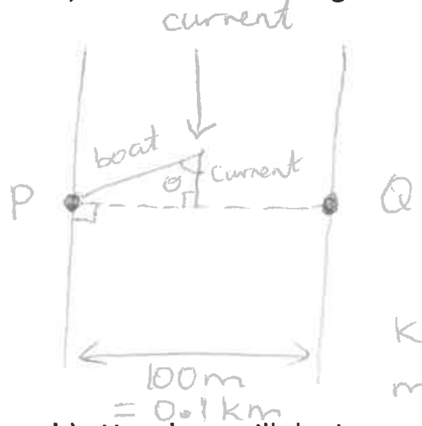
$$\underset{\sim}{r} = \begin{bmatrix}155.4\\ -27.6\end{bmatrix}$$

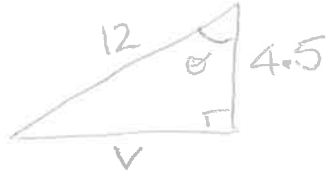
Question 5

(2, 3 = 5 marks)

A river of width 100 m where the current flows at 4.5 km/h is shown in the diagram. A boat is to be driven directly across the river from point P to Q. The boat has a speed of 12 k/h in still water.

a) Determine the angle to the bank in which the boat must be directed





km → m ÷ 1000

$\cos\theta = \frac{4.5}{12}$

$\theta = \cos^{-1}(4.5 \div 12)$

$\theta = 68^\circ$

b) How long will the journey take?

$\sin 68^\circ = \frac{v}{12}$

$v = \sin 68^\circ \times 12$

$v = 11.13$ km/h

$V = \frac{D}{T}$

$T = \frac{D}{V} = \frac{100\text{m}}{11.1} = \frac{0.1 \text{ km}}{11.1 \text{ km/h}}$

$T = 0.00898$ h) sec (×60×60)

$T = 32.4$ sec

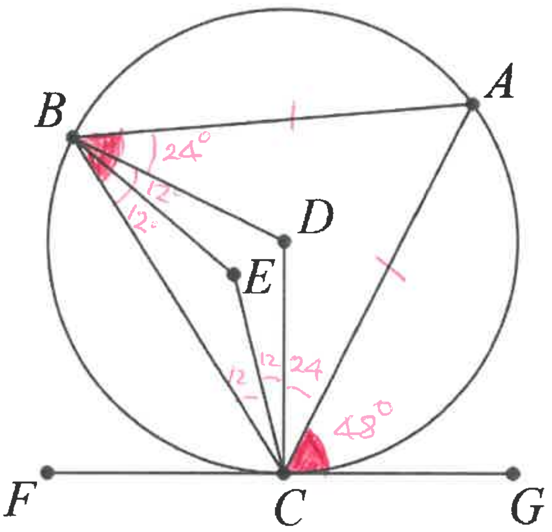
Question 6 **(5 marks)**

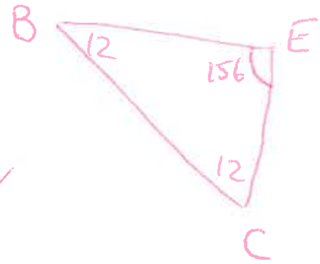
In the diagram below, FCG is a tangent to the circle ABC.

BD bisects $\angle ABC$ and CD bisects $\angle ACB$
BE bisects $\angle DBC$ and CE bisects $\angle DCB$

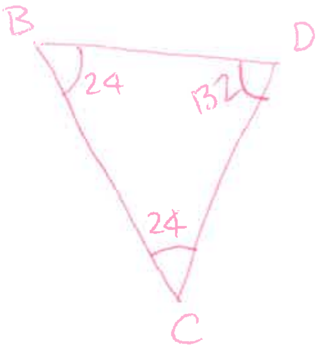
$\angle A \quad \angle D \quad \angle E$

If AB = AC and $\angle ACG = 48°$, determine the ratio, in simplest form, of $\angle BAC = \angle BDC = \angle BEC$

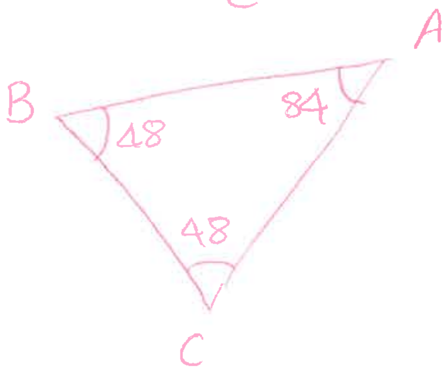




$$\begin{array}{r} 180 \\ -\ 24 \\ \hline 156° \end{array}$$



$$\begin{array}{r} 180 \\ -\ 48 \\ \hline 132° \end{array}$$



$$\begin{array}{r} 180 \\ -\ 96 \\ \hline 84° \end{array}$$

Ratio $\angle BAC : \angle BDC : \angle BEC$

$84° : 132° : 156°$

$\div 2$ $\quad 42 : 66 : 78$

$\div 2$ $\quad 21 : 33 : 39$

$\div 3$ $\quad 7 : 11 : 13$