High School
Mathematics Test 2015

Year 10	*Geometric Reasoning*	Non Calculator

Skills and Knowledge Assessed:

Name_____

- Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)
- Formulate proofs involving congruent triangles and angle properties (ACMMG243)

Extended Answer Test.

Answers should be supported by relevant mathematical reasoning and/or calculations
Marks will not be awarded for answers with no reasoning.
Complete any diagrams and write all working and answers in the spaces provided on this test paper.
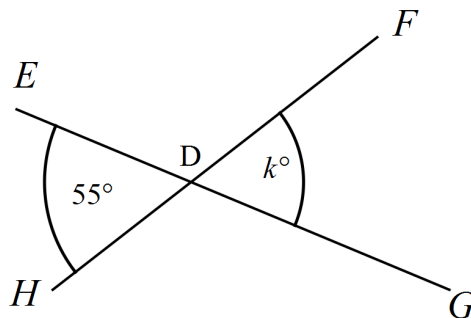
Marks

1. (a) Find the value of *k*.   **2**

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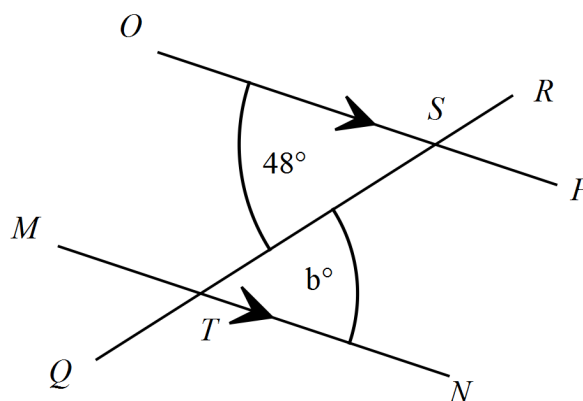
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(b) Find the value of *b*.   **2**

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(c) Find the value of *m*.   **2**

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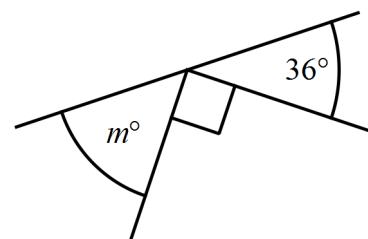
Marks

2.   (a)   Find the value of *g*.   **2**

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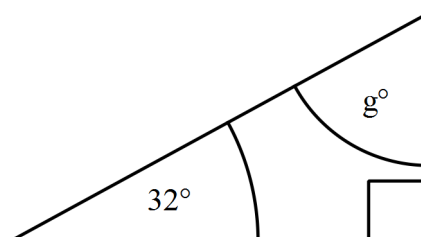
(b)   Find the value of *p*.   **2**

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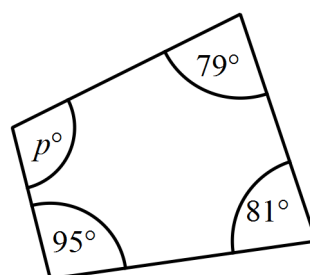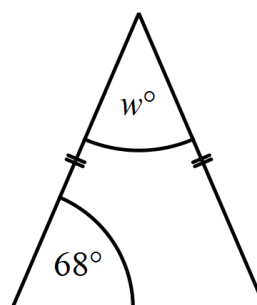
(c)   Find the value of *w*.   **3**

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3.   (a)   *KL, MN* and *QR* are straight lines
             which intersect at *P*.
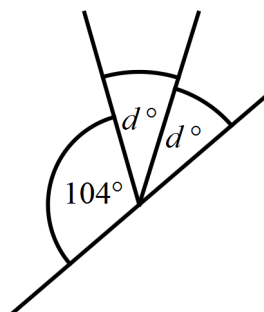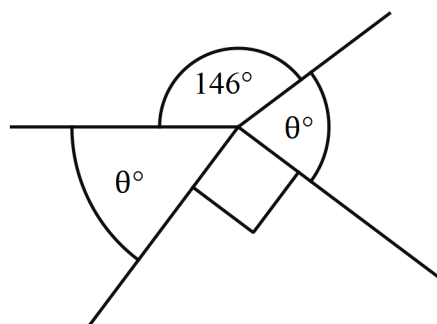   Find the value of *x*.   **3**
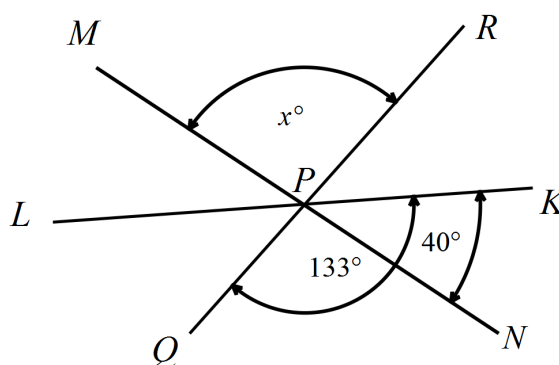
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Marks

(b)   Find the value of d.   3

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$104°$ $d°$ $d°$

(c)   Find the value of θ.   3

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$146°$ $\theta°$ $\theta°$

4.   (a)   AC ∥ DF.   3
      Find the value of g.

A   B   C

$130°$ $g°$

$55°$

D   E   F

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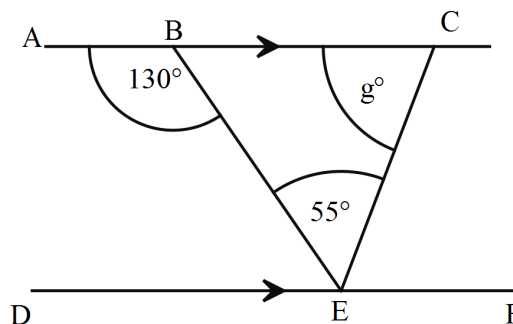
(b)   ABCD is a rectangle.   F is a point on BC.   ∠ FDC = 43°.   3
      Find the size of ∠ BFD.

A   B

F

$43°$

D   C

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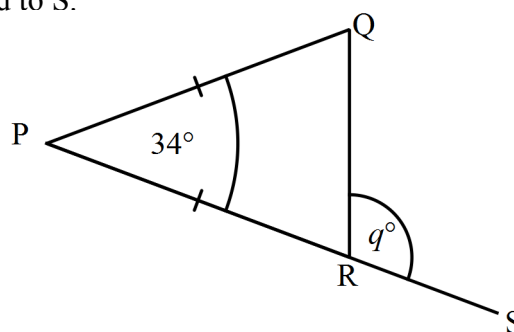
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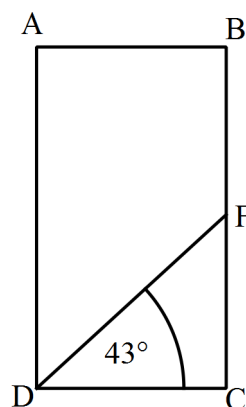
Marks

3

(c)   PQR is an isosceles triangle. PR is produced to S.

Find the value of q.

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5.   (a)   Prove that $\Delta\,ABC \equiv \Delta\,EFG$.

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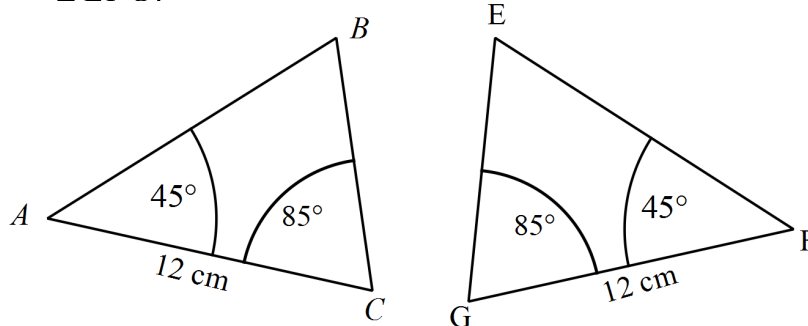
Marks

(b)   Δ ABC is isosceles, with AB = BC and ∠ ABC = 80°.   **3**
AC is produced to D so that ∠ BDC = 20°.
Find the size of ∠ DBC.

B

80°

NOT TO
SCALE

20°

D   C   A

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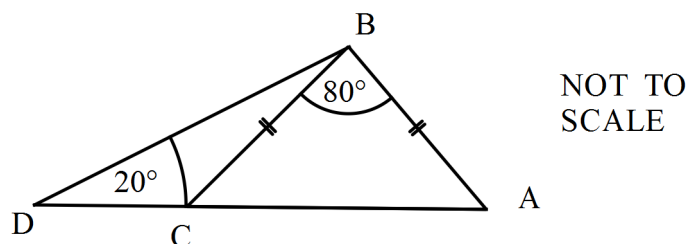
(c)   TS and RP are straight line segments which bisect one another at right angles at Q.   **3**
Prove that $\Delta\,PQT \equiv \Delta\,RQS$.

R

T

Q

S

P

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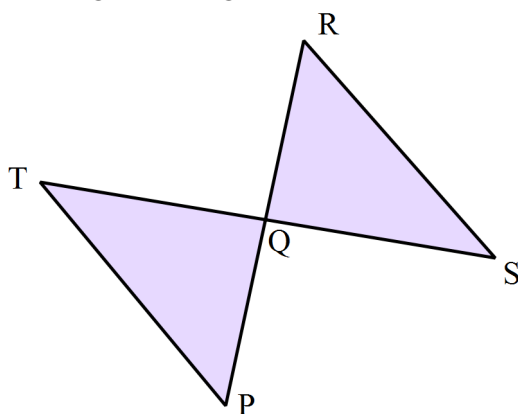
Marks

6. (a) Using only the property that *two pairs of adjacent sides of the kite ABCD are* **3**
    equal, prove the property that the diagonal AC bisects the angle BAD.

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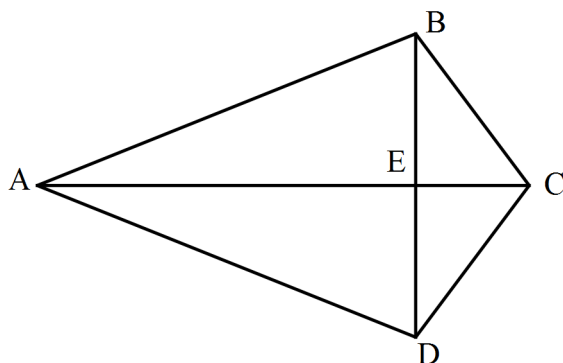
(b) IH ∥ KL, IH = 4.5 cm, KL = 12 cm and JK = 8 cm.

    (i) Prove that Δ *HIJ* ||| Δ *K*L*J*. **3**

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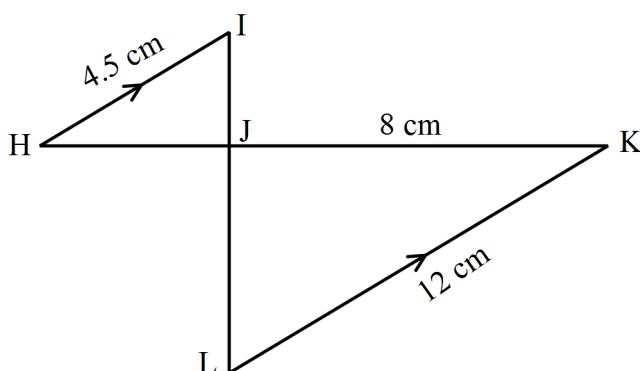
(ii) Find the length of HJ. **2**

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6

Marks

(c)   A, B C and D are three points on a circle centre O, such that BD is a diameter.

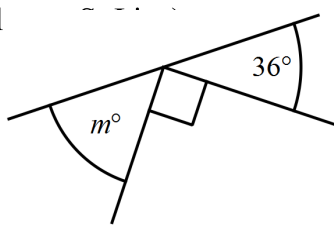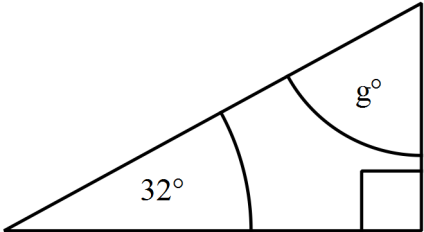   (i)   Prove that $\angle\,\text{AOD} = 2 \times \angle\,\text{ABD}$.   **2**
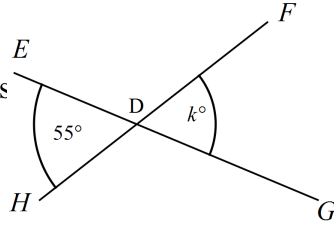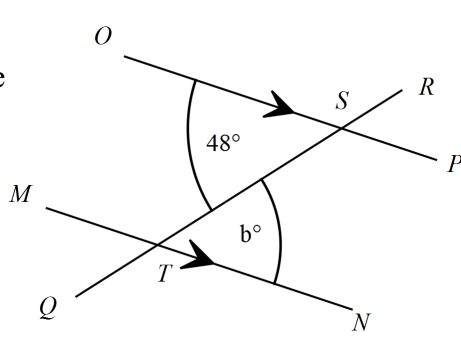
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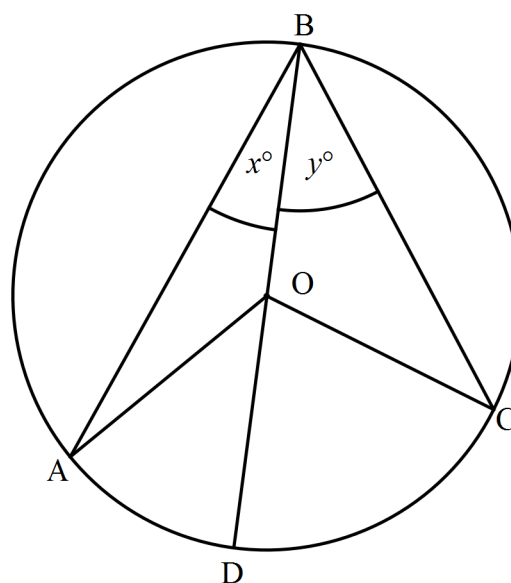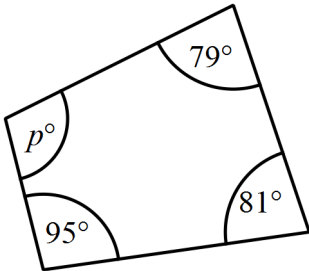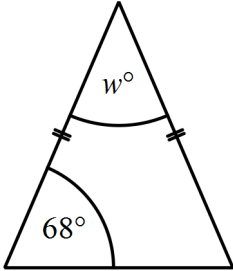
    (ii)   Prove that $\angle\,\text{AOC} = 2 \times \angle\,\text{ABC}$.   **2**
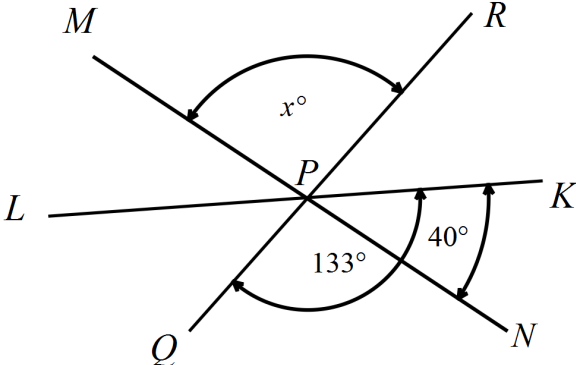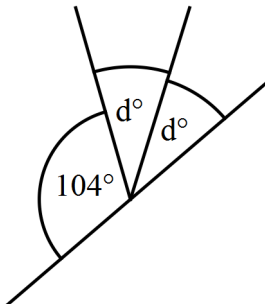
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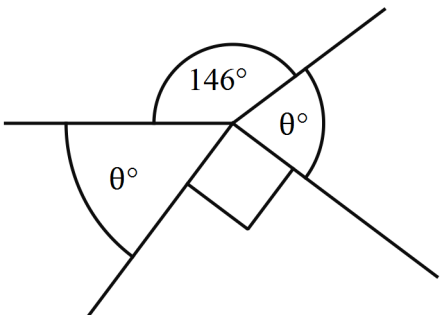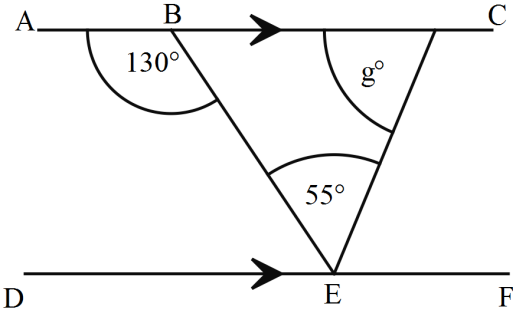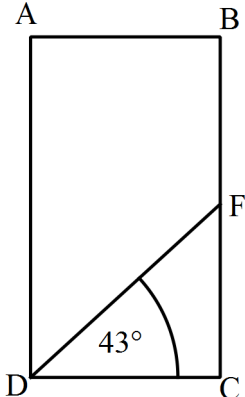
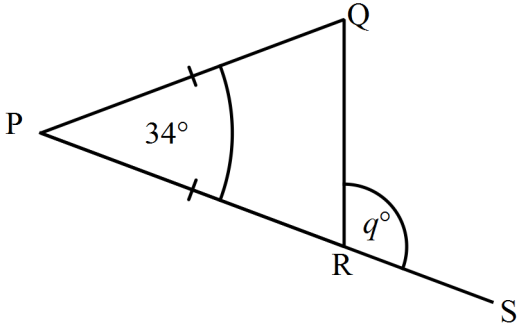...

High School
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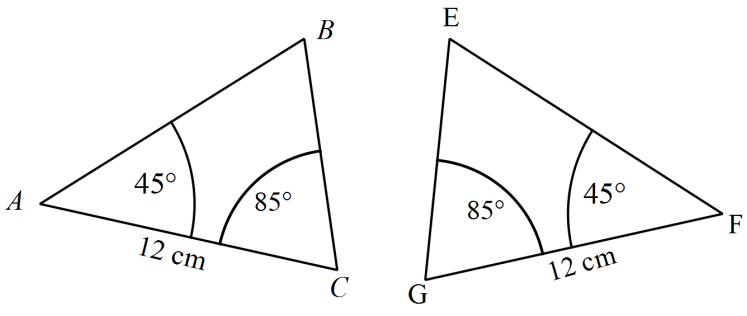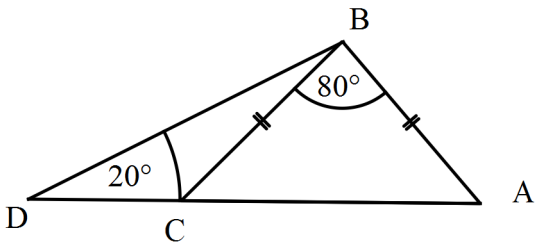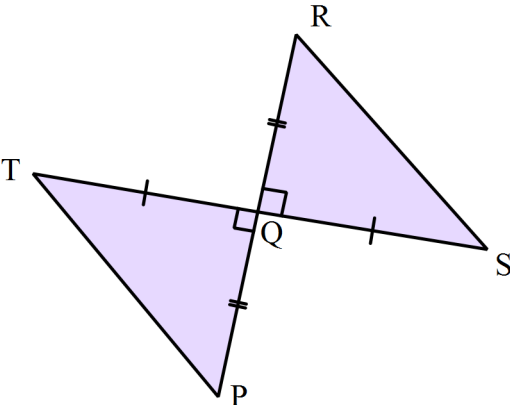
Year 10	*Geometric Reasoning*	Calculator Allowed

Extended Answer Test.

ANSWERS

		Marks
1.	(a) $k = 55$ (vertically opposite angles 	**2 marks for correct answer with reason.** **1 mark if answer or reason is wrong or not provided.**
	(b) $b = 48$ (alt angles on ∥ line 	**2 marks for correct answer with reason.** **1 mark if answer or reason is wrong or not provided.**
	(c) $m + 90 + 36 = 180$ (Angl $m = 180 - 126$ $m = 54$ 	**2 marks for correct answer with reason.** **1 mark if answer or reason is wrong or not provided.**

| 2.	(a)	2 marks for correct answer with reason.
	$g + 32 + 90 = 180$ (angle sum Δ) $g = 180 - 122$ $g = 58$	1 mark if answer or reason is wrong or not provided.
	(b) $p + 95 + 81 + 79 = 360$ (angle sum quadrilateral) $p = 360 - 255$ $p = 105$	2 marks for correct answer with reason. 1 mark if answer or reason is wrong or not provided
	(c) other base angle $= 68°$ (angles in isosceles Δ) $w + 68 \times 2 = 180$ (angle sum Δ) $w = 180 - 136$ $w = 44$	2 marks for correct answer with reasons. 1 mark if answer is calculated incorrectly or one line of reasoning is wrong or not provided

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3.	(a) $\angle\, QPN\ =\ 133\ -\ 40$ (adjacent angles) $\qquad\qquad =\ 93$ $\angle\, MPR\ =\ \angle\, QPN\ =\ x\ =\ 93$ (vert opp angles) 	**3 marks for correct answer with all reasons.** **2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided** **1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided**
	(b) $104 + 2 \times d\ =\ 180$ (angles on st line) $\qquad\quad 2d\ =\ 180 - 104$ $\qquad\quad 2d\ =\ 76$ $\qquad\quad d\ =\ \dfrac{76}{2}\ =\ 38$ 	**3 marks for correct answer with all reasons.** **2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided** **1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided**
	(c) $90\ +\ 146\ +\ 2 \times \theta\ =\ 360$ (angles at a point) $\qquad\qquad\qquad 2\theta\ =\ 360 - 236$ $\qquad\qquad\qquad 2\theta\ =\ 124$ $\qquad\qquad\qquad \theta\ =\ \dfrac{124}{2}\ =\ 62$	**3 marks for correct answer with all reasons.** **2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided** **1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided**

4.	(a)	3 marks for correct answer with all reasons.
	$\angle\ BEF\ =\ 130°$ (alt angles on ‖ lines) $\angle\ CEF\ =\ 130-55\ =\ 75$ (adjacent angles) $\angle\ CEF\ =\ \angle\ BCE\ =\ g\ =\ 75$ (alt angles on ‖ lines)	
		2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided
	A ——— B ——→ C 130° g° 55° D ——————→ E ——— F	1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided

	(b)	3 marks for correct answer with all reasons.
	$\angle\ FCD =\ 90°$ (angle in rectangle) $\angle\ BFD =\ 43\ +\ 90$ (exterior angle of Δ) $\angle\ BFD =\ 133°$	
		2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided
	A B F 43° D C	1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided

	(c)	3 marks for correct answer with all reasons.
	$\angle\ PRQ\times 2\ +\ 34\ =\ 180$ (angle sum isosceles Δ) $\angle\ PRQ =\ \dfrac{180-34}{2}\ =\ \dfrac{146}{2}\ =\ 73$ $\angle\ QRS\ =\ q\ =\ 180-73$ (angles on st line) $=\ 107°$	2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided
	Q P 34° q° R S	1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided

5.	(a)	3 marks for correct conclusion with right congruence test and all steps of reasoning given. 2 marks for correct conclusion with one step of reasoning incorrect or not provided or if conclusion uses wrong test 1 mark if at least one correct statement needed for the proof is provided

(a)

In $\triangle ABC$ and $\triangle FEG$
  $\angle A = \angle F = 45°$ (given)
  $\angle C = \angle G = 85°$ (given)
   $AC = FG = 12\text{cm}$ (given)
$\therefore \triangle ABC \equiv \triangle FEG \ (AAS)$

(b)

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$\angle BCA \times 2 + 80 = 180$ (angle sum isosceles $\triangle$)
$\angle BCA = \dfrac{180 - 80}{2} = 50$
$\angle DCB = 180 - 50 = 130$ (angles on st line)
$\angle DBC = 180 - (20 + 130)$ (angle sum $\triangle$)
$\angle DBC = 180 - 150 = 30$

3 marks for correct answer with all reasons.

2 marks if answer is calculated incorrectly or one line of reasoning is wrong or not provided

1 mark if answer is calculated incorrectly and one line of reasoning is wrong or more than one line of reasoning is wrong or not provided

(c)

In $\triangle PQT$ and $\triangle RQS$
   $TQ = QS$ (Q bisects TS)
  $\angle TQP = \angle SQR$ (lines cross at   90°)
   $PQ = RQ$ (Q bisects PR)
$\therefore \triangle PQT \equiv \triangle RQS \ (SAS)$

3 marks for correct conclusion with right congruence test and all steps of reasoning given.

2 marks for correct conclusion with one step of reasoning incorrect or not provided or if conclusion uses wrong test

1 mark if at least one correct statement needed for the proof is provided

6.	(a) Adjacent sides are equal.	**3 marks for correct conclusion with right congruence test and all steps of reasoning given, including the bisection of angle.**

(a)   Adjacent sides are equal.
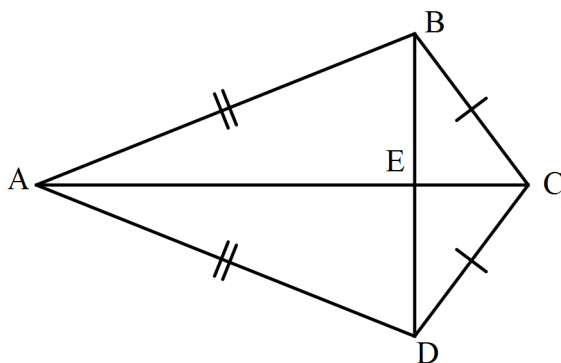
In $\Delta\, ABC$ and $\Delta\, ADC$

   $AB \;=\; AD$ (adjacent equal sides in kite   )

   $BC \;=\; DC$ (adjacent equal sides in kite   )

   AC is common

$\therefore \Delta\, ABC \;\equiv\; \Delta\, ADC \; (SSS)$

$\therefore \quad\quad \angle\, BAE \;=\; \angle\, DAE$ (corr angles in congruent Δ)

$\therefore AC$ bisects $\angle\, BAD$

3 marks for correct conclusion with right congruence test and all steps of reasoning given, including the bisection of angle.

2 marks for correct conclusion with one step of reasoning incorrect or not provided or if congruence uses wrong test or conclusion is incorrect or not provided

1 mark if at least two correct statements needed for the proof are provided

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(b)
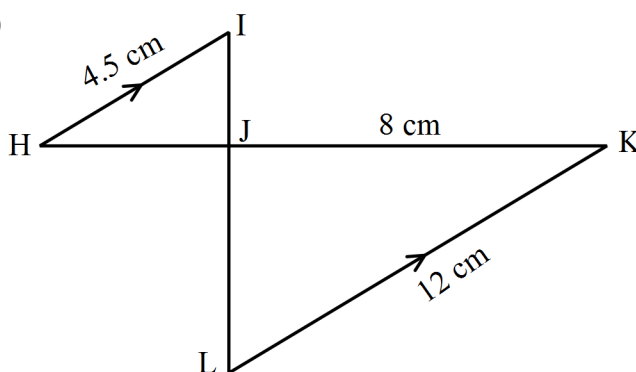
(i)

In $\Delta\, HIJ$ and $\Delta\, KLJ$

   $\angle\, HIJ \;=\; \angle\, JLK$ (alt $\angle$ on $\parallel$ lin

similarly $\angle\, IHJ \;=\; \angle\, LKJ$

   $\angle\, HJI \;=\; \angle\, KJ$L (vert opp $\angle$)

$\therefore \Delta\, HIJ \;\parallel\parallel\parallel\; \Delta\, KLJ$ (corr $\angle$ equal   )

$$\frac{HJ}{JK} \;=\; \frac{HI}{KL} \text{ (corr sides in same ratio)}$$

$$\frac{HJ}{8} \;=\; \frac{4.5}{12}$$

$$HJ \;=\; \frac{4.5}{12} \times 8 \;=\; 3$$

(ii)

(i)
3 marks for correct conclusion with right similarity test and all steps of reasoning given.

2 marks for correct conclusion with one step of reasoning incorrect or not provided or if congruence uses wrong test or conclusion is incorrect or not provided
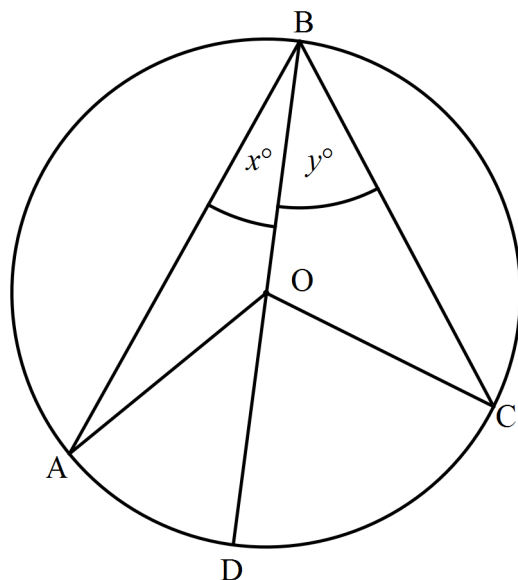
1 mark if at least two correct statements needed for the proof are provided

(ii)

2 marks if answer is calculated correctly with reasoning/working provided

1 mark if answer is calculated incorrectly with some correct working/reasoning

(c)

(i)

In $\Delta\, ABO$

$\angle\, BAO \;=\; x°$ (base angles of isosceles Δ)

$\angle\, AOD \;=\; x° + x° \;=\; 2x°$ (ext $\angle$ of $\Delta\, ABO$

$\therefore \angle\, AOD = \quad 2 \times \angle ABD = 2x°$

(ii)   Similarly $\angle\, COD = 2 \times \angle\, CBD \;=\; 2y°$

$\angle\, ABC \;=\; x \;+\; y \;=\; (x + y)°$ (adj angles)

$\angle\, AOC \;=\; 2x \;+\; 2y \;=\; 2(x + y) \;=\; 2 \times \angle\, ABC$

(i)   2 marks for correct conclusion with all reasons.

1 mark if some progress is made towards conclusion

(ii) 2 marks for correct conclusion with all reasons.

1 mark if some progress is made towards conclusion