GREENWOOD COLLEGE

Methods 11 Test 7 2018
Expontenital & Differentiation
Total Marks: 59 Time Allowed: 55 minutes

Name: Marking Key.

SECTION A - Resource Free
25 minutes - 30 marks

ALL working must be shown for full marks.

1. [4, 4 = 8 marks]

Solve the following **exactly**.

a) $8^{x+1} = \frac{4}{16^x}$

$\Rightarrow 2^{3x+3} = \frac{2^2}{2^{4x}}$ ✓

$\Rightarrow 2^{3x+3} = 2^{2-4x}$ ✓

$\Rightarrow 3x+3 = 2-4x$ ✓

$\Rightarrow 7x = -1$

$\Rightarrow x = -1/7$ ✓

b) $3^{x-2} = \frac{\sqrt[3]{3}}{9^x}$

$\Rightarrow 3^{x-2} = \frac{3^{1/3}}{3^{2x}}$ ✓

$\Rightarrow 3^{x-2} = 3^{1/3-2x}$ ✓

$\Rightarrow x-2 = 1/3 - 2x$ ✓

$\Rightarrow 3x = 7/3$

$\Rightarrow x = 7/9$. ✓

2. [2, 2, 2 = 6 marks]

State the derivative of each of the following:

a) $2x^3 - x + 1$

$\frac{dy}{dx} = 6x^2 - 1$

✓ ✓

b) $(x-1)(x+2)$

$= x^2 + x - 2$ ✓

$\Rightarrow \frac{dy}{dx} = 2x+1$ ✓

b) $\frac{5x^3-3x^5}{3x^2}$

$= 5/3\, x - x^3$ ✓

$\frac{dy}{dx} = 5/3 - 3x^2$ ✓

3. [2 marks]

Use an appropriate derivate to evaluate $\lim_{h\to 0}\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{h}\right)$

$= \frac{d}{dx}\sqrt{2}x^{\frac{1}{2}}$ ✓

$= \frac{\sqrt{2}}{2\sqrt{x}}$ ✓

0

4. **[3, 3 = 6 marks]**

State the equation of the tangent line for each of the following curves at the given point.

a) $y = 3x^4 + x,\ (1,4)$

$\frac{dy}{dx} = 12x^3 + 1 \,\big|\, x = 1$ ✓

$= 13$ ✓

At $(1,4)$ $y = 13x + c$

$\Rightarrow 4 = 13(1) + c$

$\Rightarrow c = -9$

So $y = 13x - 9$ ✓

b) $y = 2x(1 - x), (-2, -12)$

$y = 2x - 2x^2$

$y' = 2 - 4x \,\big|\, x = -2$ ✓

$= 10$ ✓

$y = 10x + c$

$\Rightarrow -12 = 10(-2) + c$

$\Rightarrow c = 8$

So $y = 10x + 8$ ✓

5. **[8 marks]**

Given that $y = ax^3 + bx^2 + 2$ has a tangent with equation $y = -4x + 5$ at the point where $x = 1$, find a, and b.

$\frac{dy}{dx} = 3ax^2 + 2bx$ ✓

At $x = 1$ $3a + 2b = -4$ —— ① ✓

At $y = -4x + 5$

given $x = 1$ $y = -4(1) + 5$ ✓

$= 1$

So $1 = a + b + 2$

$\Rightarrow a + b = -1$ —— ② ✓

① — $3a + 2b = -4$ ✓

② ×3 $3a + 3b = -3$

②×3 − ① $b = 1$ ✓

So $3a + 2 = -4$ ✓

$\Rightarrow a = -2$ ✓

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SECTION B - Calculators Allowed **30 minutes - 29 marks**

6. [1, 1, 2, 2 = 6 marks]

The function f is such that $f(x) = 1 - x + 2x^2 - x^3$

a) Determine f(2)

-1 ✓

b) Determine f' (2)

$f'(x) = -3x^2+4x-1$
$f'(2) = -5$ ✓

c) Determine x when f'(x) = 0

Solve $(-3x^2+4x-1)$ ✓
$\Rightarrow x=1$ or $x=1/3$ ✓

d) Determinethe coordinates of any point on the function f where the gradient is -1.

Solve $-3x^2+4x-1=-1$
$\Rightarrow x=0$ or $x=4/3$ ✓

So $(0,1)$ or $\left(4/3, \frac{23}{27}\right)$ ✓

7. [3 marks]

Use first principles to determine the derivative of $y = 5x^2$.

$$f'(x) = \lim_{h\to 0} \frac{5(x+h)^2-5x^2}{h}$$ ✓

$$= \lim_{h\to 0} \frac{5(x^2+2xh+h^2)-5x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{5x^2}+10xh+5h^2-\cancel{5x^2}}{h}$$

$$= \lim_{h\to 0} \frac{10x\cancel{h}+5h^{\cancel{2}}}{\cancel{h}}$$ ✓

$$= \lim_{h\to 0} 10x+5h$$

$$= 10x.$$ ✓

(-1 notation if $\lim_{h\to 0}$ not included)

8. **[1, 2, ~~2~~ 3, ~~2~~ 1, 1, 1, 1, 1, 2 = 13 marks]**

The population, P, of a country town in the South-West of WA may be modelled by the equation

$$P(t) = 400(1.15)^t,$$

where t is the number of years from 1st January 2003 to the 1st January 2015.

a) What is the population of this town on 1st January 2003?

400 ✓

b) Is the population increasing or decreasing? Justify your answer.

increasing ✓ because $r > 1$. ✓

c) Complete the following table:

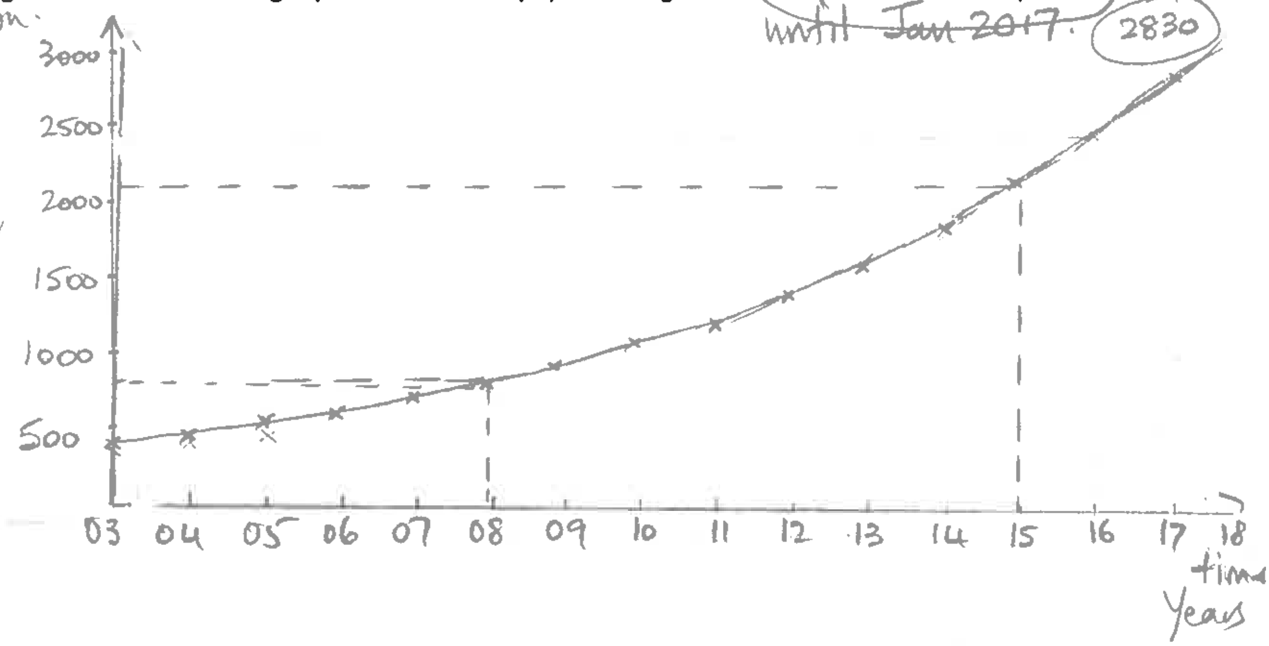
Year	Jan '03	Jan '04	Jan '05	Jan '06	Jan '07	Jan '08	Jan '09
Population	400	460	529	608	699/700	804/805	925

✓✓ (−1 each error)

$x = 14$

d) On the given axes sketch a graph to show the population growth for at least January 2017. ~~until Jan 2017.~~ 2830

Scales ✓
labels ✓
points ✓



e) Justify that the population of this town is increasing using your graph. – Popn → ∞ as $t \to \infty$ ✓

f) Show how to use your graph to estimate when the population of this town doubles.

Popn. doubles around Jan '07. ✓

g) Show how use your graph to estimate the population of this town in Jan 2015. (must show on graph)

~ 2400 see graph ✓

h) Use your calculator to refine the solution for ~~(f)~~ 9 above to the nearest ten.

~~800 = 400 × 1.15^t~~

400×1.15^{12}
$= 2140$ (nearest ten) ✓

i) Examine your graph, compare and comment on the rate at which the population is increasing on 1st January 2005 and 1st January 2015.

2015 has a higher gradient on the tangent. ✓
So higher rate of change at each point. ✓

9. [1, 1, 2, 2, 1 = 7 marks]

Soup is heated in the microwave, then removed and allowed to cool. Its temperature T, in °C, x minutes after being removed, is given by the formula: $T = 140\,(0.85)^x$

a) What was the temperature of the soup when it was taken from the microwave?

140°C ✓

b) At what % rate is the temperature decreasing?

15% per minute ✓

c) Predict the temperature of the soup after 5 minutes.

$T = 140\,(0.85)^5$ ✓

$= 62.11874375$°C ✓ (3 sf required)

d) Soup is drinkable at a temperature of 20° C. Determine, to the nearest 0.1 minute, when this will happen.

Solve $20 = 140\,(0.85)^t$ ✓

$\Rightarrow t = 11.97343691$

So 12.0 min. ✓

e) Another food item is place in the microwave, heated, and left to cool. It cools at a fixed % rate, until its temperature eventually remains relatively constant at k°C. Comment on the value of k.

K°C is the temperature of the room. ✓

(half mark if K is described as an assymptote).