MINDARIE
SENIOR COLLEGE

Semester Two Examination, 2018

Validation Test

Section One: Calculator-free

This section has **three (3)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 20 minutes. Marks: 20

Question 1 **(6 marks)**

(a) Determine n, if $\log_2 n - 1 = 2\log_2 3 + \log_2 6$. (3 marks)

$\log_2 n - \log_2 2 = \log_2 9 + \log_2 6$ ✓
$\log_2 n = \log_2 \cancel{54} + \log_2 2$
$= \log_2 108$ ✓
$n = 108$ ✓

(b) Determine the exact solution to $4(3)^{x-2} = 20$. (3 marks)

$4(3)^{x-2} = 20$
$3^{x-2} = 5$
$\log 3^{x-2} = \log 5$ ✓
$(x-2)\log 3 = \log 5$ ✓
$x - 2 = \log 5 / \log 3$
$x = \log 5 / \log 3 + 2$ ✓

Question 2 **(7 marks)**

The discrete random variable X is defined by

$$P(X = x) = \begin{cases} \dfrac{2k}{3x+1} & x = 0, 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Determine the value of the constant k. (3 marks)

$\frac{2k}{1} + \frac{k}{2} = 1$ ✓

$\frac{5}{2}k = 1$ ✓

$k = 0.4$ ✓

(b) Determine $E(X)$, thus determine

(i) $\mathrm{E}(5 - 2X)$. (2 marks)

x	0	1
$P(X=x)$	0.8	0.2

$E[X] = 0.2$ ✓

$E[5-2X] = 4.6$ ✓

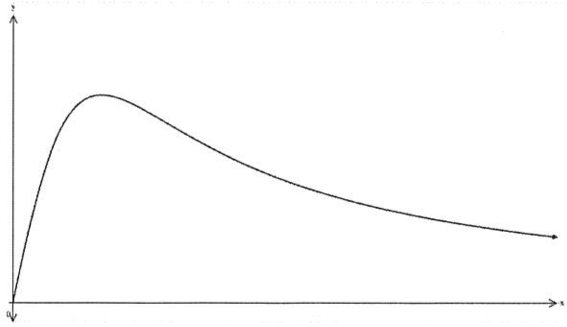
(ii) $\mathrm{Var}(5 - 2X)$. (2 marks)

$VAR[X] = 0.16$ ✓

$VAR[5-2X] = 0.64$ ✓

Question 3 **(7 marks)**

The graph of $y = f(x),\ x \geq 0$, is shown below, where $f(x) = \dfrac{4x}{x^2+3}$.



(a) Determine the gradient of the curve when $x = 1$. (3 marks)

$$f'(x) = \frac{4(x^2+3) - 4x(2x)}{(x^2+3)^2}$$

$$= \frac{4x^2+12-8x^2}{(x^2+3)^2}$$

$$= \frac{12-4x^2}{(x^2+3)^2}$$

$$f'(1) = 8/16$$

$$= 0.5.$$

(b) Determine the exact area bounded by the curve $y = f(x)$ and the lines y = 0, x = 1 and x = 2, simplifying your answer. (4 marks)

$$\int_1^2 \frac{4x}{x^2+3}\,dx$$

$$= \left[2\ln(x^2+3)\right]_1^2$$

$$= 2\ln 7 - 2\ln 4$$

$$= \ln 49 - \ln 16$$

$$= \ln 49/16.$$

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Section Two: Calculator-assumed

This section has **five (5)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 38 minutes. Marks: 38

Question 1 **(7 marks)**

The capacity, X mL, of aluminium cans made in a factory can be modelled by a normal distribution with mean μ and standard deviation 1.5 mL.

(a) If $\mu = 375$, determine

(i) $P(X \geq 376)$. (1 mark)

0.2525 ✓

(ii) $P(X > 376 \mid X < 378)$. (2 marks)

$\frac{0.2297}{0.9772} = 0.2350$ ✓ ✓

(iii) the value of x, if $P(X \geq x) = 0.2$ (1 mark)

$x = 376.3$ ✓

(b) Given that $P(X < k) = 0.059$,

(i) determine the value of μ in terms of k. (2 marks)

$\frac{k-\mu}{1.5} = -1.563$ ✓

$k - \mu = -2.345$

$\mu = k + 2.345$ ✓

(ii) determine μ if $k = 75$. (1 mark)

$\mu = 77.345$ ✓

Question 2 **(8 marks)**

From a random survey of 640 users of a free music streaming service, it was found that 472 would stop using it if they had to pay.

(a) Based on this survey, calculate the percentage of users who would stop using the service. (1 mark)

$\hat{p} = \frac{472}{640}$

$= 0.7375 = 73.75\%$ ✓

(b) Calculate the approximate margin of error for a 95% confidence interval estimate of the proportion of users who would stop using the service. (3 marks)

✓ $1.96\sqrt{\frac{(0.7375)(0.2625)}{640}}$ ✓

$=$ ~~[illegible]~~ ✓

0.0341

(c) Determine a 95% confidence interval for the proportion of users who would stop using the service. (2 marks)

✓ ✓

$0.7034 < p < 0.7716$

(d) If 50 identical surveys were carried out and a 95% confidence interval for the proportion was calculated from each survey, determine the probability that exactly 49 of the intervals will contain the true value of the proportion. (2 marks)

$X \sim bin(50, 0.95)$ ✓

$P(X = 49) = 0.2025$ ✓

Question 3 **(7 marks)**

The length, T minutes, of phone calls to a help line is a continuous random variable with probability density function given by

$$f(t) = 0.4e^{-0.4t}, \qquad 0 \le t < \infty.$$

(a) Determine the probability that a randomly chosen call lasts less than 5 minutes. (2 marks)

$$\int_0^5 0.4e^{-0.4t}\,dt \quad ✓$$
$$= 0.8647 \quad ✓$$

(b) An operator answers 20 calls, chosen at random. If call times are independent of each other, determine the probability that at least 3 of them will exceed 5 minutes. (2 marks)

$P(\text{exceeds } 5m) = 0.1353$

$X \sim Bin(20, 0.1353)$ ✓

$P(X \geq 3) = 0.200$ ✓

(c) An operator has been on a call for exactly 6 minutes. Determine the probability that the call will end within the next minute. (3 marks)

$P(6 < X < 7) = 0.0299$ ✓

$P(X > 6) = 0.0907$ ✓

$P(6 < X < 7 \mid X > 6) = 0.2206$ ✓

Question 4 **(9 marks)**

A fair die has one face numbered 1, two faces numbered 2 and three faces numbered 3.

(a) Determine the probability that the second odd number occurs on the ~~fourth~~ fifth throw of the dice. (3 marks)

$P(1) = 1/6$ $\Rightarrow$ $P(odd) = 2/3$

$P(2) = 2/6$ $\quad$ $P(even) = 1/3$

$P(3) = 3/6$

$P(3\text{ even}, 1\text{odd}) = 4\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^3 = 8/81$

$P(\text{odd on }5^{th}) = \frac{8}{81} \times \frac{2}{3}$

$= 16/243$

(b) The die is thrown twice and X is the sum of the two scores.

(i) Complete the table below to show the probability distribution of X. (2 marks)

x	2	3	4	5	6
$P(X = x)$	$\frac{1}{36}$	$\frac{1}{9}$	$5/18$	$\frac{1}{3}$	$1/4$

(ii) Determine $P(X = 4 \mid 3 \leq X \leq 6)$. (2 marks)

$\dfrac{5/18}{35/36} = 2/7$

(iii) Calculate $\mathrm{E}(X)$. (2 marks)

$1/18 + 1/3 + 10/9 + 5/3 + 3/2$

$= 4\frac{2}{3}$

Question 5 **(7 marks)**

A polynomial function $f(x)$ is such that $\int_1^8 4f(x)\,dx = 32$.

(a) Show that $\int_8^1 f(x)\,dx = -8$. (2 marks)

$$\int_1^8 4f(x)\,dx = 32$$

$$4\int_1^8 f(x)\,dx = 32 \quad \checkmark$$

$$\int_1^8 f(x)\,dx = 8$$

$$\int_8^1 f(x)\,dx = -8 \quad \checkmark$$

(b) Determine the value of $\int_1^2 (3 + f(x))\,dx + \int_2^8 (f(x) + x - 2)\,dx$. (5 marks)

$$\int_1^2 3 + \int_1^2 f(x)\,dx + \int_2^8 f(x) + \int_2^8 (x-2)\,dx \quad \checkmark\checkmark$$

$$= 3 + \int_1^8 f(x)\,dx + (18) \quad \checkmark$$

$$= 3 + 8 + 18. \quad \checkmark$$

$$= 29 \quad \checkmark$$