MATHEMATICS APPLICATIONS

YEAR 12 UNIT 3

TEST 2

2022

PART A

CALCULATOR FREE

TIME: 20 mins
MARKS: 25 marks

STUDENT'S NAME: Solutions.

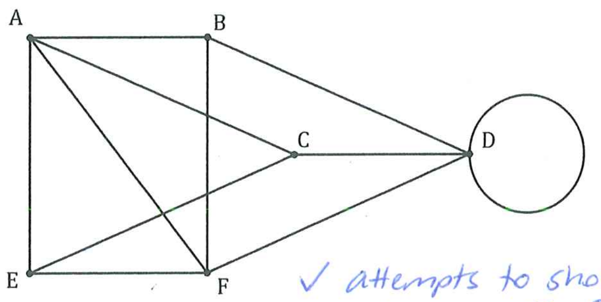
CIRCLE YOUR TEACHER'S NAME:

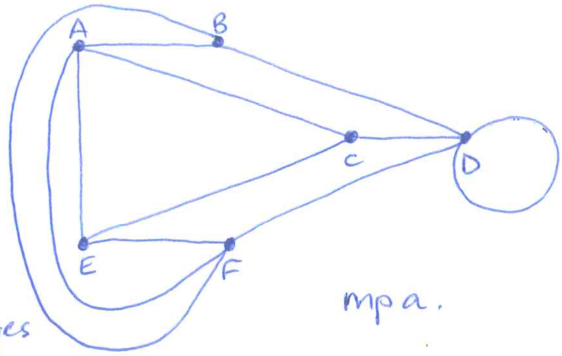
Mr Galbraith	Mr Ismail	Mrs Kalotay
Mr Lee	Mrs Smirke	Ms Thompson

MATERIALS SUPPLIED: Formula Sheet

Question 1 **(5 marks)**

(a) Re-draw the following graph to clearly demonstrate that it is planar. (2 marks)



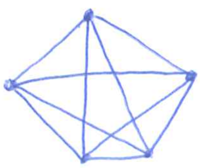


✓ attempts to show planar with 9+ edges

✓ fully correct.

mpa.

(b) Draw a complete graph with five vertices. (1 mark)



✓

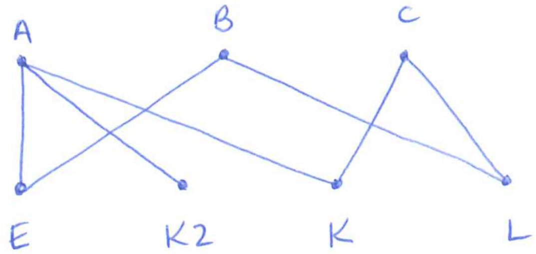
(c) Do **all** complete graphs obey Euler's formula? Explain your answer. (2 marks)

No ✓

Not all complete graphs are planar <u>and</u> Euler's rule only applies to planar graphs. ✓

Question 2 **(4 marks)**

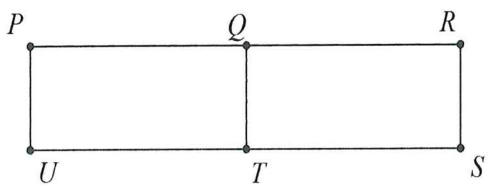
(a) Three hikers were sharing tips about climbing the world's highest mountains. Alison had climbed Everest (E), K2 and Kangchenjunga (K), Ben had climbed Everest and Lhotse (L) and Chris had climbed Kangchenjunga and Lhotse. Display this information as a bipartite graph. (2 marks)

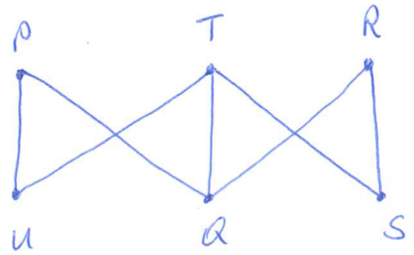


Displays 2 sets correctly ✓

Correctly joins vertices. ✓

(b) A bipartite graph is shown below, joining vertices in two disjointed sets. Redraw the graph, clearly showing the vertices that belong in each set. (2 marks)



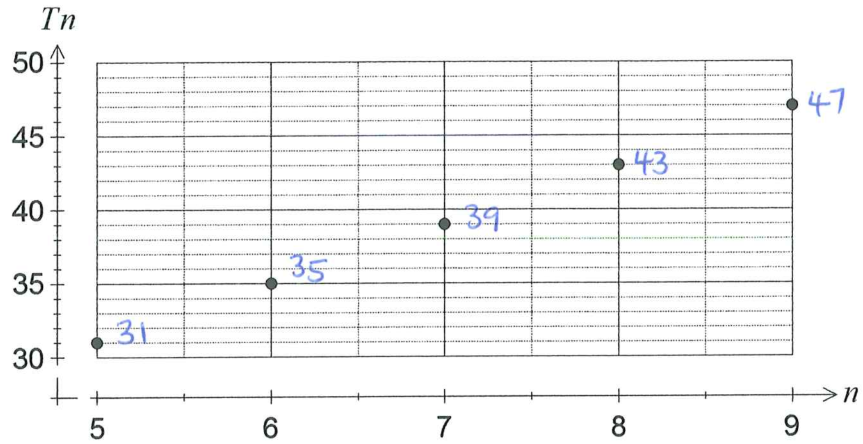


Correct sets ✓

correct edges ✓

Question 3 **(8 marks)**

The terms of a sequence are shown in the graph below.



(a) Choose the best description of the sequence from geometric, arithmetic or neither, explaining your choice. (2 marks)

Arithmetic ✓

The terms are increasing by a constant amount. ✓

(b) Determine

(i) T_{10} $= 47 + 4 = 51$ ✓ (1 mark)

(ii) T_1 (1 mark)

$T_1 + 4d = 31$

$T_1 = 31 - 4(4)$

$= 15$ ✓

(c) Deduce a rule for the n^{th} term of the sequence, simplifying your answer. (2 marks)

$T_n = 4(n-1) + 15$ ✓

$T_n = 4n + 11$ ✓ (or ✓✓)

(d) Determine the value of n such that $T_n = 415$. (2 marks)

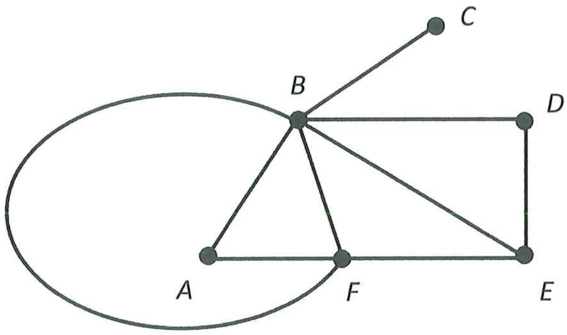
$415 = 4n + 11$ ✓

$404 = 4n$

$n = 101$ ✓

Question 4 **(8 marks)**

The edges in the graph below represent the roads in a park that meet at the given vertices.



(a) State a bridge that is found in this graph. (1 mark)

BC

(b) Show that Euler's formula applies to this graph. (2 marks)

$V + F = E + 2$

$6 + 5 = 9 + 2$ ✓

$11 = 11$ ∴ Euler's rule works ✓

(c) Record the degree of each vertex shown in the graph in the table below. (2 marks)

Vertex	*A*	*B*	*C*	*D*	*E*	*F*
Degree	2	6	1	2	3	4

−1 per error.

(d) Without referring to the information in (c), clearly explain why the graph is semi-Eulerian. (2 marks)

The graph contains a trail which passes over every edge exactly once, ✓

and starts and ends at a different vertex. ✓

(e) A park ranger has to inspect every road in the park. List all possible starting points so that the ranger can complete this task without driving on the same road more than once. (1 mark)

C and E ✓

MATHEMATICS APPLICATIONS

YEAR 12 UNIT 3

TEST 2

2022

PART B

CALCULATOR ASSUMED

TIME: 30 mins
MARKS: 30 marks

STUDENT'S NAME: Solutions.

CIRCLE YOUR TEACHER'S NAME:

Mr Galbraith	Mr Ismail	Mrs Kalotay
Mr Lee	Mrs Smirke	Ms Thompson

MATERIALS SUPPLIED: Formula Sheet

MATERIALS RECOMMENDED: Up to three approved calculators
One A4 single sided unfolded page of notes

Question 5 **(7 marks)**

The number of votes still to count at the end of an election, decreased by 72 votes every minute after 6 pm. At 6 pm, 2955 votes still needed counting.

(a) Show that by 6:02 pm, 2811 votes still needed counting. (1 mark)

6pm 2955

6:02pm $2955 - 2(72) = 2811$ ✓

(b) Determine the nth term rule for the number of votes still needing counting n minutes after 6 pm. (2 marks)

$T_n = 2955 - 72n$ ✓ ✓ or $T_n = 2883 - 72(n-1)$

(c) Determine how many votes still needed counting at 6:30 pm. (1 mark)

$T_{30} = 2955 - 7(30) = 795$ ✓

(d) At 6:30 pm, counting slowed so that only 36 votes were processed every minute. Determine the time, to the nearest minute, that counting finished. (3 marks)

$T_n = 795 - 36n$ ✓

$0 = 795 - 36n$

$n = 22.08$ ✓ $\therefore$ 6:52 pm ✓

Question 6 **(6 marks)**

Sequence T is defined given by $T_{n+1} = 1.25T_n$, $T_1 = 50$.

(a) Use the recursive rule to complete the table below, rounding values to one decimal place. (2 marks)

n	1	2	3	4	5	6
T_n	50	62.5	78.1	97.7	122.1	152.6

values ✓

rounding ✓

The first three terms of the geometric sequence U are 200, 160 and 128.

(b) Determine a rule for the n^{th} term of sequence U. (2 marks)

$\left.\begin{array}{l} 160 \div 200 = 0.8 \\ 128 \div 160 = 0.8 \end{array}\right\}$ ✓

$U_n = 0.8^{n-1} \times 200$

OR $U_n = 0.8^n \times 250$. ✓

(c) Determine the largest value of n so that $U_n > T_n$ and justify your answer. (2 marks)

n	$U_n > T_n$
4	$102.4 > 97.7$
5	$81.9 < 122.1$

✓ $n = 4$ ✓

Question 7 **(7 marks)**

A fish farmer initially stocked a tank with 50 small fish. Each month, the farmer caught some of the largest fish and sold them before adding more, smaller fish to the tank.

The number of fish in the tank at the start of the n^{th} month is given by the recurrence relation, F_n

where $$F_{n+1} = 0.7F_n + 120, \quad F_1 = 50.$$

(a) Use the recurrence relation to state

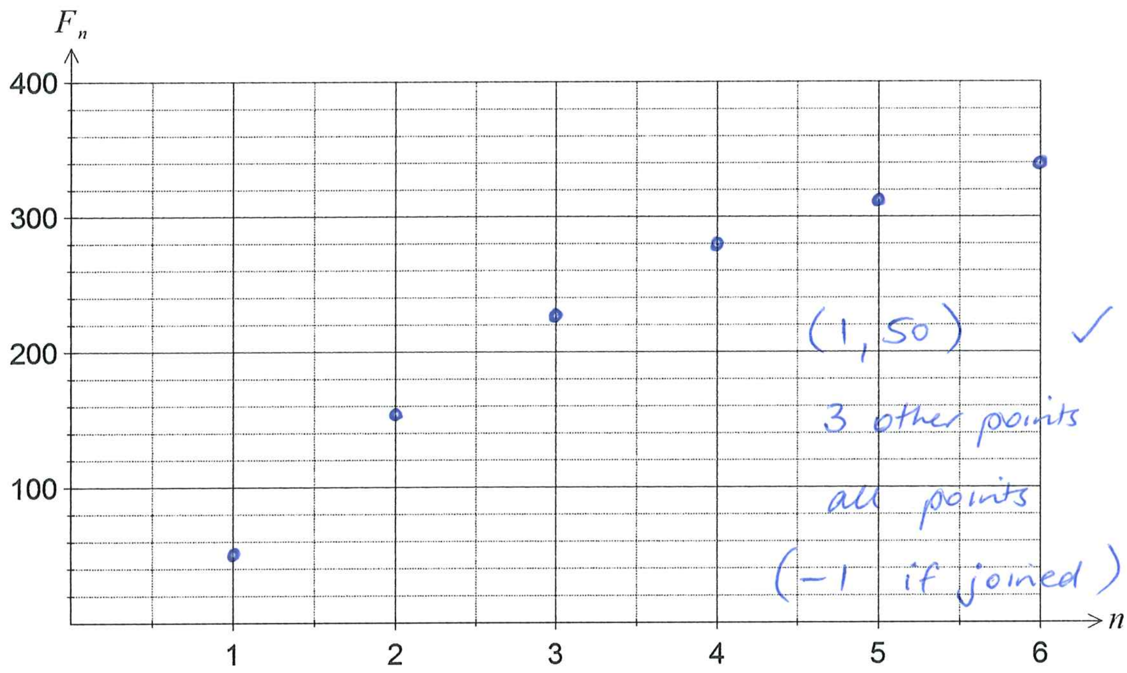
(i) the number of smaller fish added to the tank each month. (1 mark)

120 ✓

(ii) the percentage of the fish caught and sold each month. (1 mark)

30% ✓

(b) Graph F_n on the axes below for $1 \le n \le 6$. (3 marks)



Assuming this model continues, comment on how the number of fish in the tank changes

(c) over the next few months. (1 mark)

The fish are increasing at a slower rate ✓

(d) in the long term. (1 mark)

A steady state solution of 400 fish ✓

Question 8 **(5 marks)**

An art gallery records the value of all artworks at the start of each year for insurance purposes. The first valuation of a painting was \$4 800. The next two years the painting was valued at \$5 040 and \$5 292 respectively.

(a) Show that the painting values form a geometric sequence. (1 mark)

$$\left.\begin{aligned} 5040 \div 4800 &= 1.05 \\ 5292 \div 5040 &= 1.05 \end{aligned}\right\} \checkmark \quad \therefore \text{ same ratio.}$$

(b) Assuming that the value of the painting continues to increase in this way, and the insurance premium is 2.5% of the value of the painting,

(i) calculate the insurance premium for the painting at the start of the tenth year. (2 marks)

$T_{10} = 1.05^{9} \times 4800$ Premium $= 2.5\% \times 7446.38$

$= 7446.38$ ✓ $= \$186.16$ ✓

(ii) determine at the start of which year the insurance premium will first exceed \$300. (2 marks)

$300 = 1.05^{n-1} \times 4800 \times 0.025$ ✓

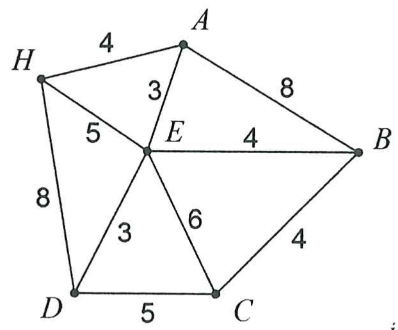
$n = 19.78$

$\therefore$ 20th year. ✓

Question 9 **(5 marks)**

The vertex *H* on the graph to the right, represents a hotel and vertices *A* to *E* represent tourist attractions. The numbers on the edges of the graph represent the walking times, in minutes, between the various attractions.

A group of tourists plan to leave the hotel at 9 am and visit all the attractions, spending 45 minutes at each one. One member of the group suggests that the route that the group plan should be a Hamiltonian cycle.



(a) Explain what is meant by a Hamiltonian cycle. (2 marks)

A path that passes through all vertices once ✓
and starts and ends at the same vertex. ✓

(b) Determine the Hamiltonian cycle the group of tourists should walk and state the time they will arrive back at their hotel.

(3 marks)

H A E B C D H   mpa ✓

$4 + 3 + 4 + 4 + 5 + 8 + 5 \times 45$ mins ✓   FT.

$= 28 + 225 = 253$ mins $\therefore$ 1:13 pm. ✓