

Investigation 2

Polynomial Division

In-Class Validation

Total marks: 45

Time allowed: 50 mins

NAME _____

Question 1: [2,2 marks]

Using the Remainder Theorem, find the remainder when:

a) $P(x) = x^3 + 5x^2 - 2x - 2$ is divided by $(x + 1)$

$$\begin{aligned} P(-1) &= (-1)^3 + 5(-1)^2 - 2(-1) - 2 \quad \checkmark \\ &= -1 + 5 + 2 - 2 \\ &= \underline{\underline{4}} \quad \checkmark \end{aligned}$$

b) $P(x) = x^3 - 3x^2 + x - 10$ is divided by $(x - 4)$

$$\begin{aligned} P(4) &= 4^3 - 3(4)^2 + 4 - 10 \quad \checkmark \\ &= 64 - 48 + 4 - 10 \\ &= \underline{\underline{10}} \quad \checkmark \end{aligned}$$

Question 2: [3 marks]

A polynomial is given by $f(x) = 2x^3 + ax^2 + 6x + 3$ where a is a constant.
When $f(x)$ is divided by $(x - 2)$ there is a remainder of 7.
Find the value of a .

$$\begin{aligned} f(2) : \quad 2(2)^3 + a(2)^2 + 6(2) + 3 &= 7 \quad \checkmark \\ 16 + 4a + 12 + 3 &= 7 \\ 4a + 31 &= 7 \quad \checkmark \\ 4a &= -24 \\ a &= -6 \quad \checkmark \end{aligned}$$

Question 3: [2,2 marks]

Using the Factor Theorem, show that:

a) $(x - 1)$ is a factor of $P(x) = x^3 + x^2 + 3x - 5$

$$\begin{aligned} P(1) &= 1^3 + 1^2 + 3(1) - 5 \\ &= 1 + 1 + 3 - 5 \\ &= 0 \end{aligned}$$

$\therefore (x-1)$ is a factor of $P(x)$

b) $(x + 2)$ is a factor of $P(x) = x^3 - 4x^2 - 7x + 10$

$$\begin{aligned} P(-2) &= (-2)^3 - 4(-2)^2 - 7(-2) + 10 \\ &= -8 - 16 + 14 + 10 \\ &= 0 \end{aligned}$$

$\therefore (x+2)$ is a factor of $P(x)$

Question 4: [4,1 marks]

a) Use the factor theorem to fully factorise $P(x) = x^3 + 6x^2 - 13x - 42$.

$$\begin{aligned} P(3) &= 0 \\ P(-2) &= 0 \\ P(-7) &= 0 \end{aligned}$$

✓✓ for working

$$P(x) = (x+7)(x+2)(x-3)$$

✓✓ for factorised form.

b) Hence solve the equation $x^3 + 6x^2 - 13x - 42 = 0$.

$$x = -7$$

$$x = -2$$

$$x = 3$$

✓ f.t. from above if necessary.

Question 5: [3 marks]

Given that $(x - 3)$ is a factor of $P(x) = ax^3 + 2ax^2 + 3ax - 54$, find the value of a .

$$\begin{aligned}
 P(3) : \quad & a(3)^3 + 2a(3)^2 + 3a(3) - 54 = 0 \quad \checkmark \\
 & 27a + 18a + 9a - 54 = 0 \\
 & 54a - 54 = 0 \quad \checkmark \\
 & 54a = 54 \\
 & a = 1 \quad \checkmark
 \end{aligned}$$

Question 6: [5,5 marks]

Complete the following divisions using one of the methods from parts D, E and F of the take home component.

$$a) \quad x^3 + 4x^2 - 7x - 10 \div (x + 5) = x^2 - x - 2 \quad \checkmark$$

	x^2	$-x$	-2
x	x^3	$-x^2$	$-2x$
$+5$	$5x^2$	$-5x$	-10

/// for method

$$= (x - 2)(x + 1) \quad \checkmark$$

$$b) \quad 2x^3 + 5x^2 - x - 6 \div (x - 1) = 2x^2 + 7x + 6 \quad \checkmark$$

	$2x^2$	$+7x$	$+6$
x	$2x^3$	$+7x^2$	$+6x$
-1	$-2x^2$	$-7x$	-6

///

$$= (2x + 3)(x + 2) \quad \checkmark$$

Question 7: [5,5 marks]

Solve the following equations by factorising. You should only use the Factor Theorem to find the first factor.

a) $x^3 - 4x^2 - 3x + 18 = 0$

$$(x+2)(x-3)^2 = 0$$

✓ for finding first factor
✓✓ for division

$x = -2$ or $x = 3$ ✓ for solutions.

b) $2x^3 + 5x^2 - x - 6 = 0$

$$(2x+3)(x+2)(x-1) = 0$$

✓ for first factor
✓✓ for division

$x = -\frac{3}{2}$ $x = -2$ $x = 1$ ✓ for solutions.

Question 8: [6 marks]

Find the coordinates of the points where $2x^4 - 9x^3 - x^2 + 18x + 8$ crosses the x axis.

$$\sqrt{(2x+1)(x+1)(x-2)(x-4)} = 0$$

✓✓✓✓ for method

$$\sqrt{x = -\frac{1}{2} \quad x = -1 \quad x = 2 \quad x = 4}$$

- allow using factor theorem to find all 4