GREENWOOD COLLEGE

Methods 11 Test 6 2018
Recursive Sequences and Series
Total Marks: 59 Time Allowed: 60 minutes

Name: Marking Key

SECTION A - Resource Free
25 minutes - ~~23~~ 22 marks

ALL working must be shown for full marks.

1. [3 marks]

The common ratio of a geometric series is 4 and the sum of the first 5 terms is 3069. Find the first term.

$$3069 = \frac{a(4^5-1)}{4-1} \checkmark$$

$$\Rightarrow 3069 = \frac{a(4^5-1)}{3}$$

$$\Rightarrow 9207 = a(1024-1)$$

$$\Rightarrow 9207 = 1023a \checkmark$$

$$\therefore a = 9 \checkmark$$

$256 = 2^4$

1024

$$1023 \overline{)9207}$$

9207

2. [3, 3 = 6 marks]

Find i) the common ratio ii) the limiting sum:

a) $2 + 1 + \frac{1}{2} + \frac{1}{4} + \ldots$

i) $r = \frac{1}{2} \checkmark$

ii) $S_\infty = \frac{2}{1-\frac{1}{2}} \checkmark$

$= 4 \checkmark$

b) $6 + 2 + \frac{2}{3} + \ldots$

i) $r = \frac{1}{3} \checkmark$

ii) $S_\infty = \frac{6}{1-\frac{1}{3}} \checkmark$

$= 6 \div \frac{2}{3}$

$= 6 \times \frac{3}{2}$

$= 9. \checkmark$

2 2 13 14

3. [2, 1, 3, ~~1~~, ~~3~~, 4 = ~~14~~ marks]

Consider sequence A defined as Sequence A: 2, ~~4.5~~ 4.5, 7, ...

a) Is Sequence A an arithmetic or geometric sequence? Justify your response.

not geometric because $\frac{4.5}{2} \neq \frac{7}{4.5}$ ✓

✓ ~~not~~ arithmetic because $7 - 4.5 = 4.5 - 2 = 2.5$ ✓

b) Define Sequence A using a non-recursive rule which will give the nth term of this sequence.

$T_n = 2 + (n-1)2.5$ or $T_n = 2.5n - 0.5$ ✓

c) Using the rule found in b), or otherwise, determine whether 50 is a term of Sequence A. **To earn marks you must show working.**

$2.5n - 0.5 = 50$ ✓

$\Rightarrow 2.5n = 50.5$ ✓

n is not an integer ✓

d) Determine Sequence A using a recursive rule.

$T_{n+1} = T_n + 2.5, \; T_1 = 2$ ✓ ✓

Sequence B is represented by $T_n = T_{n-1} - 3, \; T_4 = 59$ $T_{n+1} = T_n - 3, \; T_4 = 59$

e) Is Sequence B an arithmetic or geometric sequence? Justify your response.

Arithmetic ✓ because it has a common difference of -3 ✓

f) Using algebraic techniques show how to determine the value(s) of n for which Sequence A = Sequence B. **You must show working to earn marks.**

Sequence B: $T_4 = 59$

$T_3 = 62$

$T_2 = 65$

$T_1 = 68$

$T_0 = 71$ ✓

$71 + 3n = 2.5n - 0.5$ ✓

$\Rightarrow 5.5n = 71.5$ ✓

$\Rightarrow n = 13$ ✓

$55 \overline{)715}$ → 13, 55, 165

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SECTION B - Calculators Allowed **35 minutes - ~~37~~ marks**

33 marks

4. [6, 2, 2 = 10 marks]

The sum of the first two terms of a geometric sequence is 90 and the sum of the first three terms of the same sequence is 105.

a) Find the geometric sequence(s) which satisfy the above conditions.
YOU MUST SHOW WORKING.

$T_3 = S_3 - S_2$
$= 105 - 90$
$= 15$.

$ar^2 = 15$ —— (1) ✓

$a + ar = 90$
$\Rightarrow a(1+r) = 90$ —— (2) ✓

(2) ÷ (1) gives $\frac{r+1}{r^2} = 6$.

$\Rightarrow 6r^2 - r - 1 = 0$

$\Rightarrow r = \frac{1}{2}$ or $-\frac{1}{3}$. ✓ ✓

If $r = \frac{1}{2}$, $a = 60$
If $r = -\frac{1}{3}$, $a = 135$

So $T_n = 60 \times \left(\frac{1}{2}\right)^{n-1}$ ✓
or $135 \times \left(-\frac{1}{3}\right)^{n-1}$ ✓

b) Find S_{30} to S_{50}.

taken out of test

c) Find S_{30} to S_{50} inclusively.

taken out of test.

5. [3, 3 = 6 marks]

For what values of k are the following sequences a) arithmetic b) geometric

	i) 6, k, 54	ii) a, k, 2a	iii) $a^2b^3,\ k,\ a^6b^7$
AP	a)	a)	a)
GP	b)	b)	b)

6. [3, 2, 4, 1 = 10 marks]

The average annual earnings for workers in the hospitality industry in 1999 was $28000. If the average annual earnings of hospitality workers is expected to rise by 6.5% each year until the year 2009, find:

a) the average annual earnings for hospitality workers in 2005 to the nearest hundred dollars..

b) the total amount earned by a hospitality employee on the average wage between 1st January 2000 and 31st December 2005.

c) when the average annual wage will first exceed $70000.

d) Have any assumptions been made in determining the answers to this question?

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SECTION B - Calculators Allowed **40 minutes - 37 marks**

4. [4, 2, 2, 2 = 10 marks]

The sum, S_n of n terms of a series is given by $S_n = 50[1 - \left(\frac{3}{5}\right)^n]$ so $S_n = 20\left[1 - \left(\frac{3}{5}\right)^n\right]$ ✓ $\frac{2}{5}$.

a) Find the first three terms of this series.

$T_1 = 20$ ✓

$T_2 = 20 \times \left(\frac{3}{5}\right) = 12$ ✓

$T_3 = 12 \times \frac{3}{5} = 7.2$ ✓

b) Show that the series is geometric.

It is geometric because the common ratio is $\frac{3}{5}$ ✓

ie $\frac{12}{20} = \frac{3}{5}$ ✓

c) Find the recursive rule for this sequence.

$T_{n+1} = \frac{3}{5} T_n, \quad T_1 = 20.$

✓ ✓

5. [3, 3 = 6 marks]

For what values of k are the following sequences a) arithmetic b) geometric

i) 6, k, 54

a) $\frac{54-6}{2} = 24$

so $k = 30$ ✓

b) $\frac{k}{6} = \frac{54}{k}$

$\Rightarrow k^2 = 54 \times 6$ ✓

$k = 324$

$\therefore k = \sqrt{324}$

$= 18$ ✓

ii) a, k, 2a

a) $\frac{2a+a}{2} =$

$= \frac{3}{2}a$ ✓

b) $\frac{k}{a} = \frac{2a}{k}$ ✓

$\Rightarrow k^2 = 2a^2$

$\Rightarrow k = \sqrt{2}a$ ✓

iii) a^2b^3, k, a^6b^7

a) $\frac{a^6b^7 + a^2b^3}{2}$ ✓

b) $\frac{k}{a^2b^3} = \frac{a^6b^7}{k}$ ✓

$\Rightarrow k^2 = a^8b^{10}$

$\Rightarrow k = a^4b^5$ ✓

4 /4

6. [3, 2, 2, 2 = 9 marks]

The average annual earnings for workers in the hospitality industry in 1999 was $28000. If the average annual earnings of hospitality workers is expected to rise by 6.5% each eyar until the year 2009, find:

a) the average annual earnings for hospitality workers in 2005 to the nearest hundred dollars..

$a = 28000 \quad r = 1.065$

$T_7 = 28000 \times 1.065^6$ ✓

$= \$40855.98$ ✓ so $40900 ✓

b) the total amount earned by a hospitality employee on the average wage between 1st January 2000 and 31st December 2005.

$= S_7 - T_1$

$= \frac{28000(1.065^7 - 1)}{0.065} - 28000$ ✓

$= 238640.36 - 28000 = \$210640.36$ ✓

$T_2 + T_3 + T_4 + T_5 + T_6$

29820	T_2 2000
+ 31758.30	T_3 2001
33822.59	T_4 2002
36021.06	T_5 2003
38.362.143	T_6 2004
40855.99	T_7 2005
$210640.37	rounding discrepancy

c) when the average annual wage will first exceed $70000.

$70000 = 28000 \times 1.065^{n-1}$ ✓

$\Rightarrow 2.5 = 1.065^{n-1}$ ✓

$\Rightarrow n = 15.55$ so 16 yrs ie 2014 ✓

d) Have any assumptions been made in determining the answers to this question?

That pay increases will be the same each year

that

that inflation will mean wages rise higher

a 1999 — 28K

ar 2000

ar^2 2001

ar^3 2002

ar^4 2003

ar^5 2004

ar^6 2005

2 2 11

7. [4, 2, 2, 2, 1, 1 = 12 marks]

The sum of the first n terms of a sequence is given by $S_n = 3n^2 - 15n$

a) Find the first five terms of the sequence.

$S_1 = -12$ so $T_1 = -12$.

45
27
-18

$S_2 = -18$ so $T_2 = -6$

$S_3 = -18$ so $T_3 = 0$. ✓✓

$S_4 = -12$ so $T_4 = 6$

$S_5 = 0$ so $T_5 = 12$

b) Classify this sequence as either arithmetic or geometric. Justify your choice?

Arithmetic ✓ because $d = 6$. ✓

c) Find the general term rule for this sequence, in the form ~~ax^2~~ $an+b$

$T_n = -12 + 6(n-1)$ ✓ so $T_n = 6n - 18$ ✓

d) Find the recursive rule for this sequence.

$T_{n+1} = T_n + 6$ ✓, $T_1 = -12$ ✓

e) Find the smallest value of n such that $S_n > 1000$

✓

$S_n > 1000$ $\Rightarrow$ $n/2(-24 + 6(n-1))$ $\Rightarrow$ $n = 20.927$

So 21 ✓

f) Find the largest value of n such that $S_n < 700$

Solve $700 = n/2(-24 + 6(n-1))$

$\Rightarrow n = 17.978$

So 17 ✓