Year 11 Mathematics Specialist

Test 3 - Vectors

MELVILLE
SENIOR HIGH SCHOOL

Student Name : _____

Part One – Resource Free

Part One contains 6 questions worth 44 marks

Time Allowed : 45 minutes

INSTRUCTIONS TO STUDENTS:
You are required to attempt ALL questions,
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be answered for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.

Question 1 **2, 2 – 4marks**

a) Car A has position vector $(10\boldsymbol{i}-25\boldsymbol{j})$ km. Relative to an observer on A, a second car B has position vector $(-4\boldsymbol{i}+10\boldsymbol{j})$ km. Find the position vector of the car B.

$_B r_A = r_B - r_A$

$r_B = {}_B r_A + r_A$ ✓

$=<-4,10>+<10,-25>$

$=<6,-15>$ ✓

b) If $v_A = 4i - 8j$ and $v_B = 10\boldsymbol{i} + 2\boldsymbol{j}$, find ${}_B v_A$ and ${}_A v_B$.

${}_B v_A = v_B - v_A$

$=<10,2>-<4,-8>$

$=<6,10>$ ✓

${}_A v_B = <-6,-10>$ ✓

Question 2 **2, 2 – 4 marks**

a) Given $\boldsymbol{f} = 7\boldsymbol{i} + 4\boldsymbol{j}$ and $\boldsymbol{g} = -3\boldsymbol{i} + 5\boldsymbol{j}$, find

i) $\boldsymbol{f} \bullet \boldsymbol{g}$

$<7,4> \bullet <-3,5>$

$=-21+20$

$=-1$ ✓

ii) $10\boldsymbol{f} \bullet 7\boldsymbol{g}$

$70 \times \boldsymbol{f} \bullet \boldsymbol{g} = -70$ ✓

b) Find α if $\boldsymbol{h} = 5\boldsymbol{i} + \alpha\boldsymbol{j}$ and $\boldsymbol{f}$ and $\boldsymbol{h}$ are perpendicular.

$<7,4> \bullet <5,\alpha> = 0$

$35 + 4\alpha = 0$ ✓

$4\alpha = -35$

$\alpha = -\frac{35}{4}$ ✓

Question 3 **2, 2, 2 – 6 marks**

a) Given that vector $\boldsymbol{u}$ has magnitude 25 m/s in the direction 070°, $\boldsymbol{v}$ has magnitude 10 m/s in the direction 130° and $\boldsymbol{w}$ has magnitude 8 m/s in the direction 220°, find:

i) $\boldsymbol{v} \bullet \boldsymbol{u}$

$25 \times 10 \times \cos 60^\circ$

$= 250 \times \frac{1}{2}$

$= 125$ ✓✓

ii) $\boldsymbol{u} \bullet \boldsymbol{w}$

$25 \times 8 \times \cos(150^\circ)$

$= 200 \times (-\frac{\sqrt{3}}{2})$ ✓

$= -100\sqrt{3}$ ✓

iii) the magnitude and direction of $(\boldsymbol{u} \bullet \boldsymbol{v})\boldsymbol{w}$

$125w$

magnitude 1000 m/s ✓

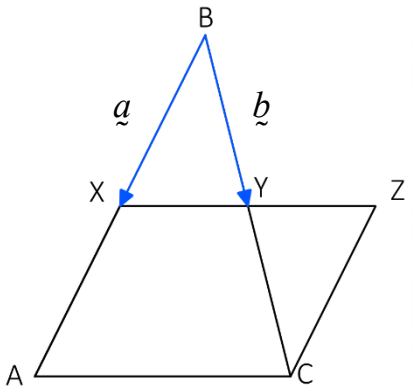
direction 220° ✓

Question 4 **1, 1, 1, 1, 2 – 6 marks**

$\triangle ABC$ is shown in the diagram.

X is the midpoint of AB and Y is the midpoint of BC.

XY is extended to Z so that $XY = YZ$



a) Find an expression for XY in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

$\boldsymbol{b-a}$ ✓

b) Find an expression for XZ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

$2(\boldsymbol{b-a})$ ✓

c) Find an expression for XC in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

$\overrightarrow{XC} = \overrightarrow{XY} + \overrightarrow{YC}$

$= \boldsymbol{b-a+b}$

$= 2\boldsymbol{b-a}$ ✓

d) Find an expression for ZC in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

$\overrightarrow{ZC} = \overrightarrow{ZY} + \overrightarrow{YC}$

$= \boldsymbol{a-b+b}$

$= \boldsymbol{a}$ ✓

e) Show that $XBZC$ is a parallelogram.

$\overrightarrow{ZC} = \overrightarrow{BA} = \boldsymbol{a}$ ✓(matches a pair of congruent parallel sides)

States a pair of congruent parallel sides is necessary for a parallelogram to be formed ✓

Question 5 **3, 1, 1, 1, 2, 3 – 11 marks**

a) Given $|\boldsymbol{m}|=4, |\boldsymbol{n}|=9$ and $\boldsymbol{m}\bullet\boldsymbol{n}=18\sqrt{3}$, find exactly:

i) the angle between $\boldsymbol{m}$ and $\boldsymbol{n}$

$$\cos\theta=\frac{\boldsymbol{m}\bullet\boldsymbol{n}}{|\boldsymbol{m}||\boldsymbol{n}|}$$
$$=\frac{18\sqrt{3}}{4\times 9}$$
$$=\frac{\sqrt{3}}{2}$$
$$\theta=30°$$

ii) the angle between $2\boldsymbol{m}$ and $5\boldsymbol{n}$

As only the magnitudes have changed the angle remains at 30°

iii) $\boldsymbol{m}\bullet\boldsymbol{m}$ 16

iv) $\boldsymbol{n}\bullet\boldsymbol{n}$ 81

v) $(\boldsymbol{m}-\boldsymbol{n})\bullet(\boldsymbol{m}-\boldsymbol{n})$

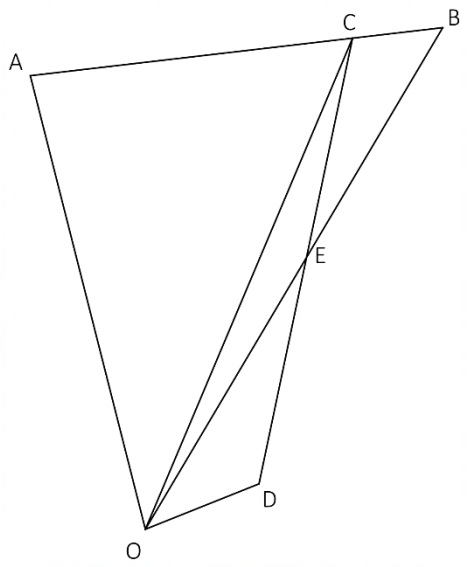
$$=\boldsymbol{m}\bullet\boldsymbol{m}-\boldsymbol{m}\bullet\boldsymbol{n}-\boldsymbol{n}\bullet\boldsymbol{m}+\boldsymbol{n}\bullet\boldsymbol{n}$$
$$=m^2-2\boldsymbol{m}\bullet\boldsymbol{n}+n^2$$
$$=16-2\times 18\sqrt{3}+81$$
$$=97-36\sqrt{3}$$

vi) $|\boldsymbol{m}+\boldsymbol{n}|$

$$=\boldsymbol{m}\bullet\boldsymbol{m}+\boldsymbol{m}\bullet\boldsymbol{n}+\boldsymbol{n}\bullet\boldsymbol{m}+\boldsymbol{n}\bullet\boldsymbol{n}$$
$$=m^2+2\boldsymbol{m}\bullet\boldsymbol{n}+n^2$$
$$=16+2\times 18\sqrt{3}+81$$
$$=97+36\sqrt{3}$$
$$|m+n|=\sqrt{97+36\sqrt{3}}$$

Question 6 **4, 3, 3, 3 – 13 marks**

In the diagram, $\overrightarrow{OA}=\boldsymbol{a}$, $\overrightarrow{OB}=\boldsymbol{b}$ and C divides AB in the ratio 5:1.



a) Write down, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$, expressions for

i) $\overrightarrow{AB}$ $\boldsymbol{b}-\boldsymbol{a}$

ii) $\overrightarrow{AC}$ $\frac{5}{6}(\boldsymbol{b}-\boldsymbol{a})=\frac{5}{6}\boldsymbol{b}-\frac{5}{6}\boldsymbol{a}$

iii) $\overrightarrow{OC}$

$$\overrightarrow{OC}=\overrightarrow{OA}+\overrightarrow{AC}$$
$$=\boldsymbol{a}+\frac{5}{6}\boldsymbol{b}-\frac{5}{6}\boldsymbol{a}$$
$$=\frac{1}{6}\boldsymbol{a}+\frac{5}{6}\boldsymbol{b}$$

b) Given that $\overrightarrow{OE}=\lambda\boldsymbol{b}$, where λ is a scalar, determine an expression for $\overrightarrow{CE}$ in terms of $\boldsymbol{a}$,$\boldsymbol{b}$ and λ.

$$OC+CE=OE$$
$$\frac{1}{6}\boldsymbol{a}+\frac{5}{6}\boldsymbol{b}+CE=\lambda\boldsymbol{b}$$
$$CE=\lambda\boldsymbol{b}-\frac{1}{6}\boldsymbol{a}-\frac{5}{6}\boldsymbol{b}$$
$$=-\frac{1}{6}\boldsymbol{a}+(\lambda-\frac{5}{6})\boldsymbol{b}$$

c) Given that $\overrightarrow{OD}=\mu(\boldsymbol{b}-\boldsymbol{a})$, where μ is a scalar, determine an expression for $\overrightarrow{ED}$ in terms of $\boldsymbol{a}$,$\boldsymbol{b}$, $\lambda$and μ.

$$\overrightarrow{OE}+\overrightarrow{ED}=\overrightarrow{OD}$$
$$\lambda\boldsymbol{b}+\overrightarrow{ED}=\mu(\boldsymbol{b}-\boldsymbol{a})$$
$$\overrightarrow{ED}=\mu(\boldsymbol{b}-\boldsymbol{a})-\lambda\boldsymbol{b}$$
$$=\mu\boldsymbol{b}-\mu\boldsymbol{a}-\lambda\boldsymbol{b}$$
$$=(\mu-\lambda)\boldsymbol{b}-\mu\boldsymbol{a}$$

d) Given that E is the midpoint of CD, deduce the values of λ and μ.

$$CE=ED$$
$$-\frac{1}{6}\boldsymbol{a}+(\lambda-\frac{5}{6})\boldsymbol{b}=(\mu-\lambda)\boldsymbol{b}-\mu\boldsymbol{a}\ \checkmark$$

Equating coefficients of $\boldsymbol{a}\Rightarrow\mu=\frac{1}{6}$ ✓

Equating coefficients of $\boldsymbol{b}$

$$\lambda-\frac{5}{6}=\mu-\lambda$$
$$\lambda=\frac{1}{2}\ \checkmark$$

Year 11 Mathematics Specialist

Test 3 - Vectors

MELVILLE
SENIOR HIGH SCHOOL

Student Name : _____

Part Two – Resource Allowed

Part Two contains 2 questions worth 16 marks

Time Allowed : 15 minutes

TO BE PROVIDED BY THE STUDENT

A maximum of one A4 page of notes, one sided.
Standard Items : Pens, pencils, eraser, sharpener, correction tape/fluid, highlighters, ruler.

Special Items: Drawing instruments, templates.
A maximum of three CAS calculators satisfying the conditions set by the SCSA.

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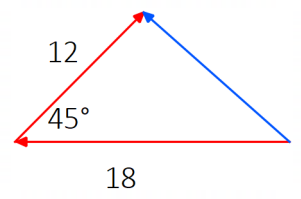
Question 1 **5,5 -11 marks**

a) A ship is travelling due West at 18 km/h. To an observer on the ship, a jetksi appears to be moving North-West at 12 km/h. Find the true velocity of the jetski.

$_h v_s = v_h - v_s \quad \Rightarrow v_h = {}_h v_s + v_s$ ✓

$v_s = 18\,\text{km/h}$



$|v_j|^2 = 12^2 + 18^2 - 2\times 12\times 18\times \cos 45$

$|v_j| = 15.526$ ✓✓

$\frac{\sin\theta}{12} = \frac{\sin 45}{15.526}$

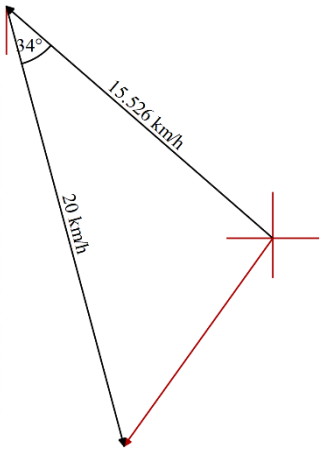
$\theta = 41^\circ$ ✓

jetski is travelling 15.5 km/h on a bearing of 311° ✓

b) The ship alters its course to travel at 20 km/h on a bearing of 345°. Find the velocity of the jetski relative to the ship's new course.

✓ ruled correct diagram

$_h v_s = v_h - v_s \quad \Rightarrow {}_h v_s = v_h + (-v_s) \quad {}_h v_s = v_h - v_s$ ✓



$|{}_j v_s|^2 = 15.526^2 + 20^2 - 2\times 15.526\times 20\times \cos 34$ ✓✓

$|{}_j v_s| = 11.233$ km/h

$\frac{\sin\theta}{15.526} = \frac{\sin 34}{11.233}$ ✓

$\theta = 50.6°$

Bearing : 180-(34+51)=95

311-95=216° ✓

Question 2 **2,3 – 5 marks**

a) Calculate the scalar projection of $\boldsymbol{m}$ = <5,10> on $\boldsymbol{n}$=<4,3>

$$\text{Proj}_{\boldsymbol{n}}\boldsymbol{m} = \frac{\boldsymbol{m} \bullet \boldsymbol{n}}{|\boldsymbol{n}|}$$
$$= \frac{50}{5}$$
$$= 10 \quad \checkmark\checkmark$$

b) Calculate the vector projection of $\boldsymbol{m}$ = <5,10> on $\boldsymbol{n}$=<4,3>

$$10 \times \frac{<4,3>}{|5|} = <8,6> \quad \checkmark$$

c) Calculate the scalar projection of vector $\boldsymbol{c}$, with magnitude 8 in direction 20° on to vector $\boldsymbol{d}$, with magnitude 12 in the direction 75°. (Directions measured from the positive x axis)

$$\frac{\boldsymbol{c} \bullet \boldsymbol{d}}{|\boldsymbol{d}|} = \frac{55.0633}{12} \quad \checkmark$$
$$= 4.589 \quad \checkmark$$