SHENTON COLLEGE

Examination Semester One 2018
Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two
(Calculator-assumed)

Your name ____

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section Two.
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
			Total	150	100

Section Two: Calculator-assumed **65% (97 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

Two complex numbers are $u = \sqrt{3} + i$ and $v = 3\,\text{cis}\left(-\frac{\pi}{4}\right)$.

(a) Determine the argument of uv. (2 marks)

(b) Simplify $|v \times \bar{v} \times u^{-1}|$. (2 marks)

(c) Determine z in polar form if $5zu = v^2$. (2 marks)

Question 10 **(5 marks)**

(a) The vector equation of a curve is given by $\mathbf{r}(\mu) = (\mu^2 + 3)\mathbf{i} + (\mu - 5)\mathbf{j}$. Determine the corresponding Cartesian equation for the curve. (2 marks)

(b) A sphere has Cartesian equation $x^2 + y^2 + z^2 + 8x - 12y + 2z = 0$. Determine the vector equation of the sphere. (3 marks)

Question 11 **(7 marks)**

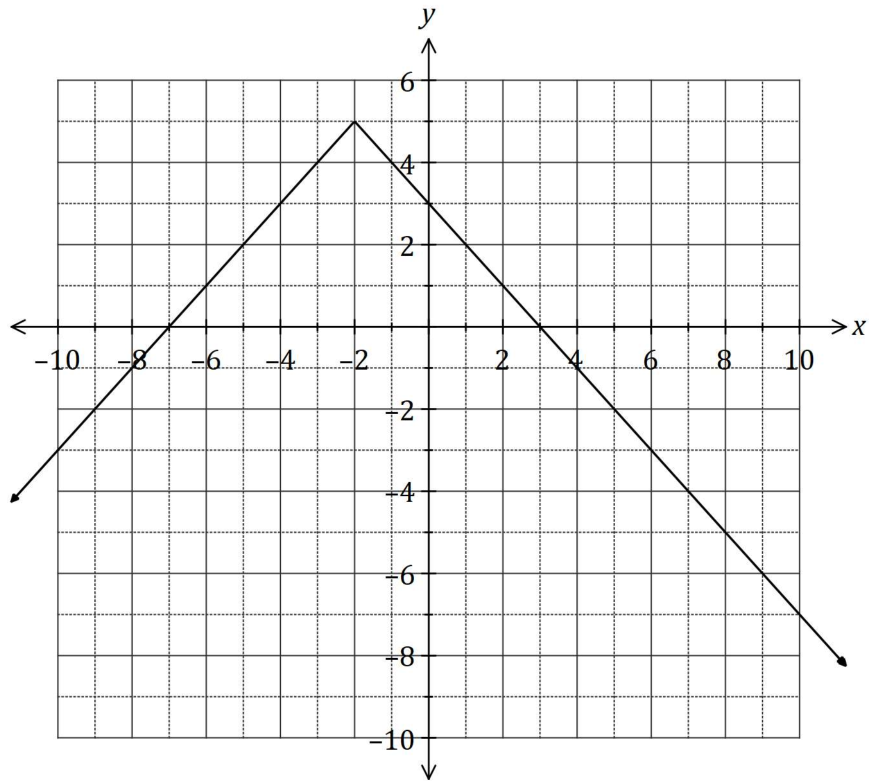
A particle, with initial velocity vector $(10, 3, -15)$ ms^{-1}, experiences a constant acceleration for 16 seconds. The velocity vector of the particle at the end of the 16 seconds is $(34, -37, -31)$ ms^{-1}.

(a) Determine the magnitude of the acceleration. (3 marks)

(b) Calculate the change in displacement of the particle over the 16 seconds. (4 marks)

Question 12 **(7 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = a|x + b| + c$, where a, b and c are constants.



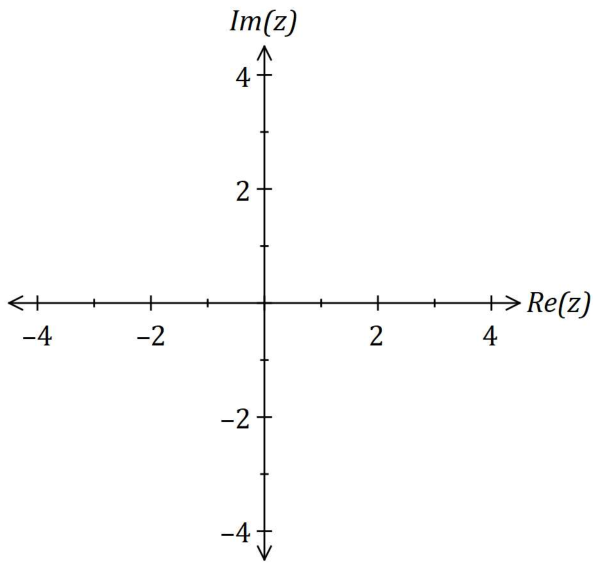
(a) Add the graph of $y = g(x)$ to the axes above, where $g(x) = 3|x - 1| - 10$. (2 marks)

(b) Determine the values of a, b and c. (3 marks)

(c) Using your graph, or otherwise, solve $f(x) + g(x) = 0$. (2 marks)

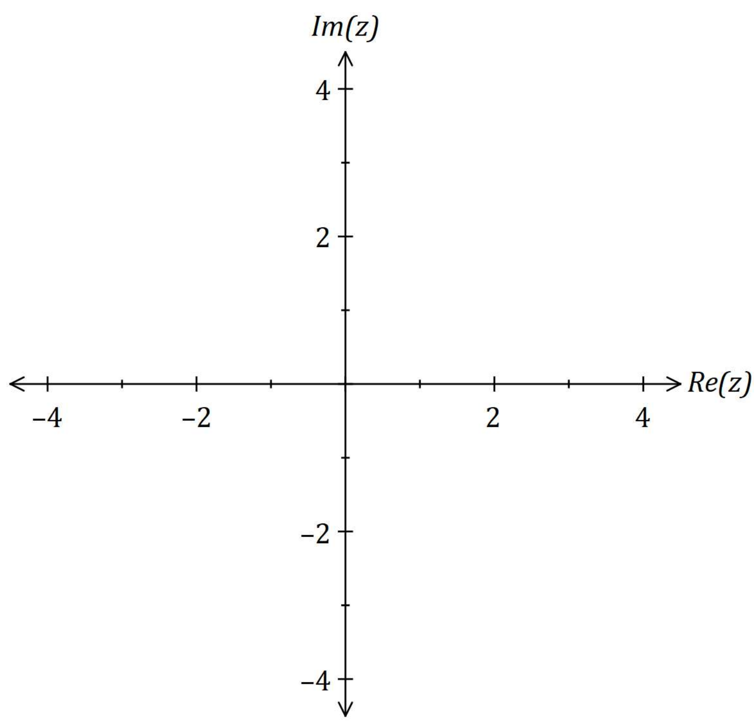
Question 13 **(9 marks)**

(a) On the Argand plane below, sketch the locus of $|z - 1 - i| = |z + 1 - 3i|$, where z is a complex number. (3 marks)



(b) Consider the three inequalities $|z + 2 + 2i| \leq 2$, $\arg(z) \geq -\frac{3\pi}{4}$ and $\mathrm{Re}(z) \leq -1$.

(i) On the Argand plane below, shade the region that represents the complex numbers satisfying these inequalities. (5 marks)



(ii) Determine the minimum possible value of $\mathrm{Re}(z)$ within the shaded region. (1 mark)

Question 14 **(8 marks)**

The position vectors of bodies L and M at times λ and μ are given by

$$\mathbf{r}_L = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$$

and

$$\mathbf{r}_M = 7\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - \mathbf{k})$$

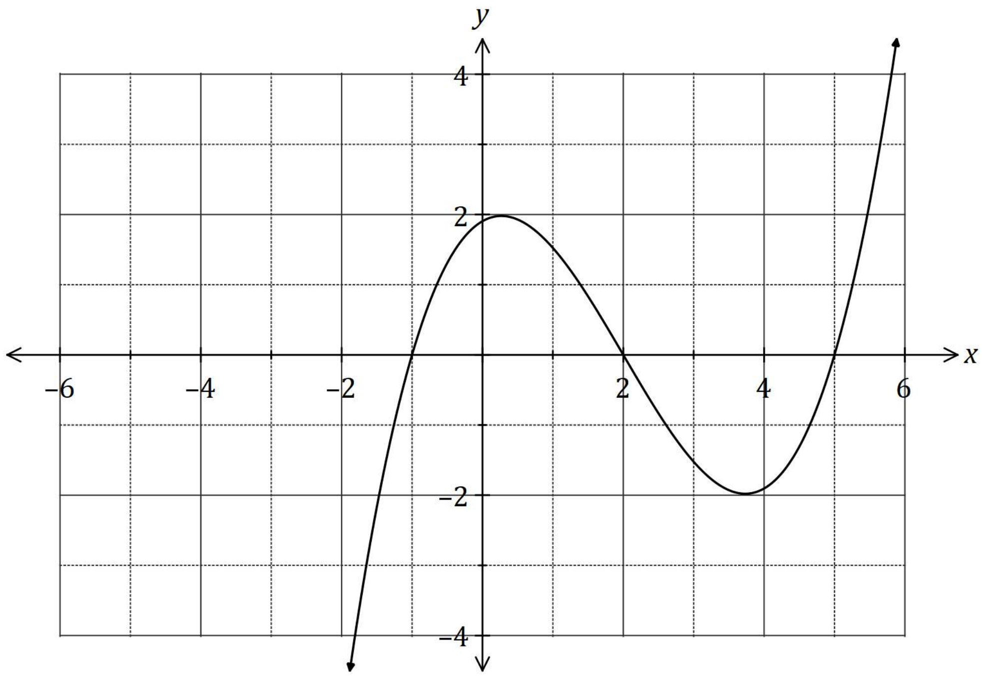
where a and b are constants, times are in seconds and distances are in metres.

(a) Given that the paths of L and M intersect, show that $a + 4b + 7 = 0$. (4 marks)

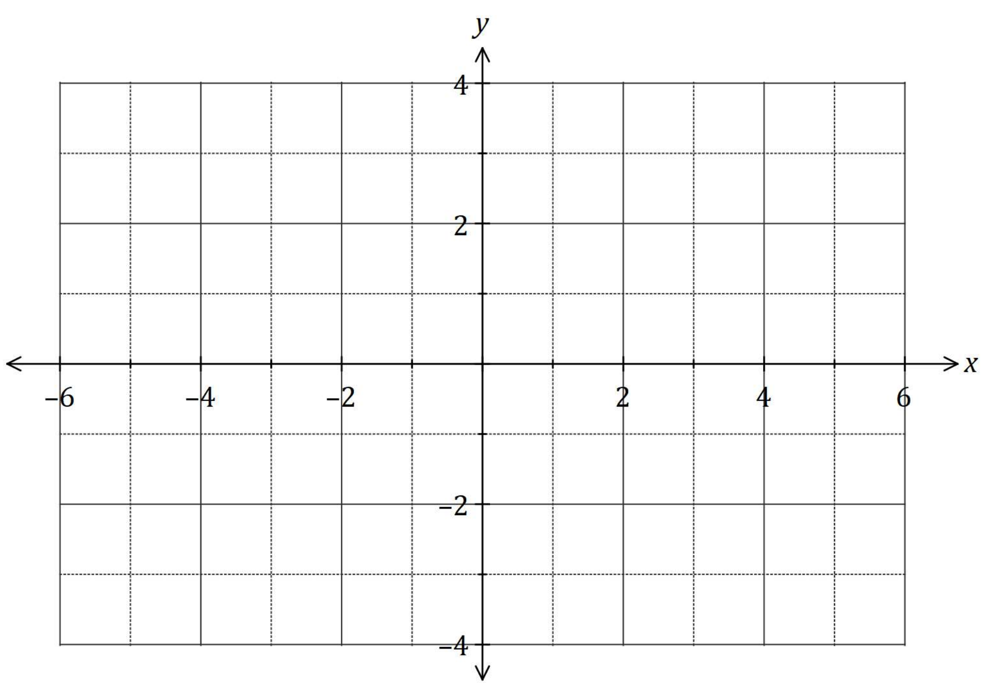
(b) Given that the paths of L and M are also perpendicular, determine the values of a and b, and the position vector of the point of intersection of the paths. (4 marks)

Question 15 **(8 marks)**

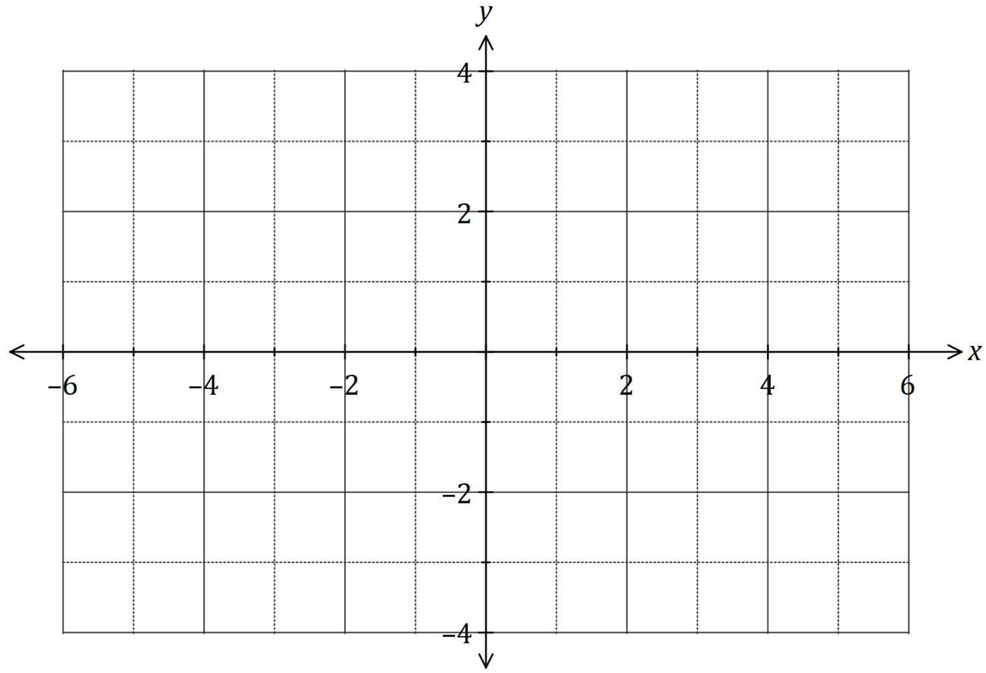
The graph of $y = f(x)$ is shown below.



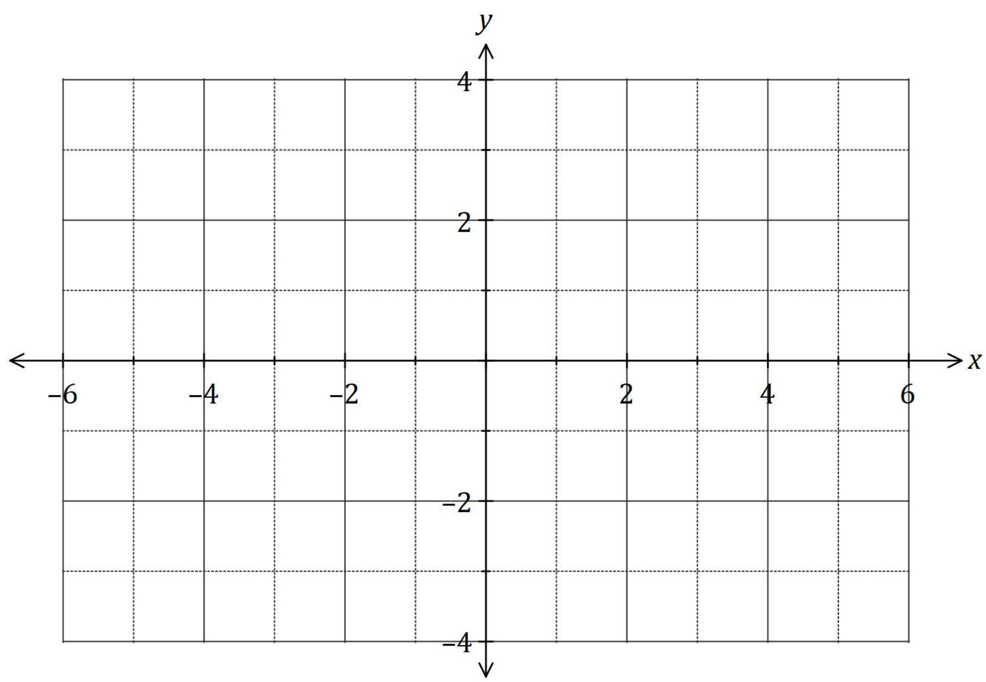
(a) Sketch the graph of $y = f(|x|)$ on the axes below. (2 marks)



(b) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the axes below. (4 marks)



(c) Sketch the graph of $y = |f(|x|)|$ on the axes below. (2 marks)



Question 16 **(8 marks)**

The velocity vector of a small body at time t seconds is $\mathbf{v}(t) = 6\cos(5t)\,\mathbf{i} - 2\sin(5t)\,\mathbf{j}$ ms^{-1}. Initially, the body has position vector $3\mathbf{i} - 4\mathbf{j}$.

(a) Determine the acceleration vector for the body when $t = \dfrac{2\pi}{15}$. (2 marks)

(b) Show that the maximum speed of the body is 6 ms^{-1}. (3 marks)

(c) Determine the distance the body travels between $t = 0$ and the first instant after this time that the body returns to its initial position, rounding your answer to the nearest cm. (3 marks)

Question 17 **(8 marks)**

(a) Let $r \operatorname{cis} \theta$ be a point in the complex plane. Determine, in terms of r and θ, the polar form of this point after it is rotated by $\frac{\pi}{4}$ about the origin and then reflected in the real axis.

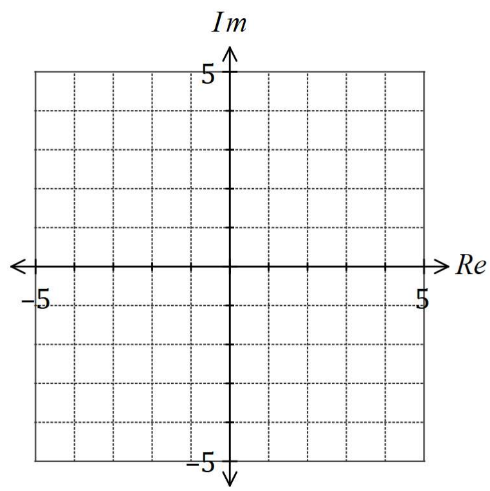
(2 marks)

(b) Let $f(w) = -i\bar{w} + 1 + i$.

(i) Complete the following table. (3 marks)

w	$1 + 2i$	$-3 + i$	$-1 - 4i$
$f(w)$			

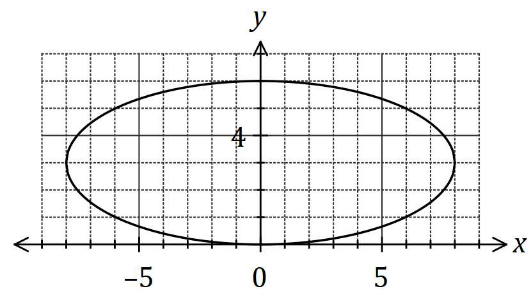
(ii) Sketch each point, w, and join it with a dotted line to its image, $f(w)$, on the diagram below. (1 mark)



(iii) Describe the geometric transformation that $f(w)$ represents. (2 marks)

Question 18 **(8 marks)**

The position vector of a boat motoring on a lake is given by $\mathbf{r}(t) = -8\sin(2t)\,\mathbf{i} + (6 - 6\cos^2(t))\mathbf{j}$, where t is the time, in hours, after it leaves $(0, 0)$ and distances are in kilometres. The path of the boat is shown below, where the shoreline is represented by the line $y = 0$.



(a) Express the path of the particle as a Cartesian equation. (3 marks)

(b) On the graph above, mark the position of the boat when it is first 4.5 km from the shoreline and indicate the direction it is travelling. (1 mark)

(c) Determine the speed of the boat when it is first 4.5 km from the shoreline. (4 marks)

Question 19 **(8 marks)**

Functions f and g are defined as $f(x) = x^2 + ax - 4a$ and $g(x) = \dfrac{x}{x+b}$, where a and b are constants.

(a) Let $a = 4$ and $b = -5$.

(i) State, with reasons, whether the composition $f(g(x))$ is a one-to-one function over its natural domain.

(2 marks)

(ii) Determine any domain restrictions required so that the composition $g(f(x))$ is defined. (3 marks)

(b) Determine the relationship between a and b so that the composition $g(f(x))$ is always defined for $x \in \mathbb{R}$. (3 marks)

Question 20 **(7 marks)**

Points A, B and C have position vectors $(a, 0, 0), (0, b, 0)$ and $(0, 0, c)$ respectively, where a, b and c are non-zero, real constants. Point M is the midpoint of B and C. Use a vector method to prove that $\overrightarrow{AM}$ is perpendicular to $\overrightarrow{BC}$ when $|\overrightarrow{OB}| = |\overrightarrow{OC}|$.

Question 21 **(8 marks)**

(a) Consider the complex equation $z^5 = -4 + 4i$.

Solve the equation, giving all solutions in the form $r \operatorname{cis} \theta$ where $r > 0$ and $-\pi \leq \theta \leq \pi$.
(4 marks)

(b) One solution to the complex equation $z^5 = 9\sqrt{3}i$ is $z = \sqrt{3} \operatorname{cis}\left(\frac{9\pi}{10}\right)$.

Let u be the solution to $z^5 = 9\sqrt{3}i$ so that $-\frac{\pi}{2} \leq \arg(u) \leq 0$. Determine $\arg(u - \sqrt{3})$ in exact form.
(4 marks)