Name: MARKING KEY. Date: ____________

METHODS MAT 11

Test 5 - 2015

____ 44
= ______ %

Topics: **Rates of Change ,Differentiation & Application of Differentiation**

Total Time: *50 minutes* **Weighting:** *6% of the year.*

Answers should be rounded to 2 decimal places unless specified. *All working should be shown in the space provided. Solutions without working may not be awarded full marks. Please take the marks for each question into account when answering the question.*

CALCULATOR FREE

Time: 25 minutes **Equipment Allowed:** Formula sheet

Marks for Section 1: 20 *marks*

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1. (3 marks: 1, 2)

Find: (a) $\lim_{x\to 5}\,(3x-2)$

$= (3(5)-2)$
$= 13$ ①

(b) $\lim_{x\to -2}\frac{x^2-4}{x+2}$ $= \frac{(x+2)(x-2)}{(x+2)}$ ①

$= (x-2)$
$= -2-2$
$= -4$ ①

2. (5 marks: 2, 3)

Differentiate with respect to x:

(a) $y = x^5 - x^3 + 7x + 2$

$\frac{dy}{dx} = 5x^4 - 3x^2 + 7$ ②

(b) $y = \frac{3x^4 + x^3}{x}$

$y = 3x^3 + x^2$ ①

$\frac{dy}{dx} = 9x^2 + 2x$ ②

3. (3 marks)
If $f(x) = 3x^2 - 4x$, find the value of x given that $f'(x) = 5$

$f'(x) = 6x - 4$ ①

$5 = 6x - 4$ (+4) ①

$9 = 6x$ (÷6)

$9/6 = x$

$3/2 = x$ ①

4. (4 marks)

Determine, using calculus, the coordinates on the curve $y = 2x^3 - 3x^2 + 4$ where the gradient is 0.

$dy/dx = 6x^2 - 6x$ ①

$0 = 6x^2 - 6x$

$0 = 6x(x - 1)$ ①

$6x = 0$ $\quad$ $x - 1 = 0$

$x = 0$ $\quad$ $x = 1$

At $x = 0$

$y = 2(0)^3 - 3(0)^2 + 4$

$= 4$

$(0, 4)$ ①

At $x = 1$

$y = 2(1)^3 - 3(1)^2 + 4$

$= 2 - 3 + 4$

$= 3$

$(1, 3)$ ①

5. (5 marks)
From first principles, find the gradient of the function $f(x) = 3x^2$ at the point $(-1, 3)$.

$$f(x+h) = 3(x+h)^2$$
$$= 3(x^2 + 2xh + h^2)$$
$$= 3x^2 + 6xh + 3h^2$$

$$f'(x) = \lim_{h \to 0} \frac{(\cancel{3x^2} + 6xh + 3h^2) - \cancel{3x^2}}{h}$$ 1

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h)}{\cancel{h}}$$ 2

$$= \lim_{h \to 0} 6x + 3h$$

$$= 6x + 3(0)$$

$$= 6x$$ 1

@ $f'(-1) = 6(-1)$
$$= -6$$ 1

~ END OF SECTION 1 ~

Name: ______________________ Date: __________

SECTION 2: CALCULATOR ASSUMED

Time: *35 minutes* **Equipment Allowed:** Formula sheet, *1 page of notes (A4), CAS and scientific calculators*

Marks for Section 2: 24 *marks*

6. (2 marks)

Find $\lim_{x\to 0}\frac{2x^2-3x^3}{x^2-x^3}$ $= 2$ In calc (2)

7. (3 marks)

Find the average rate of change for the expression $f(x) = x^2 - 6x + 10$ from $x = 4$ to $x = 4.001$

$f(4) = (4)^2 - 6(4) + 10 = 2$

$f(4.001) = (4.001)^2 - 6(4.001) + 10 = 2.002$

$$\text{Ave. R.O.C} = \frac{2.002 - 2}{4.001 - 4} = 2$$

(2) (1)

8. (3 marks)

Find the gradient of the curve $y = 2x^3 + 5$ at the point where $y = 3$

At $y = 3$

$3 = 2x^3 + 5$ (−5)

$-2 = 2x^3$ (÷2)

$-1 = x^3$

$-1 = x$

$(-1, 3)$ (1)

Gradient of curve @ $x = -1$

$dy/dx = 6x^2$ (1)

$\left.dy/dx\right|_{x=-1} = 6(-1)^2$

$= 6$ (1)

9. (4 marks)
Find the equation of the tangent to $y = x^3 - 5x^2 + 14$ at $x = 4$

At $x = 4$

$y = (4)^3 - 5(4)^2 + 14$

$= -2$ (1)

$(4, -2)$

Gradient $m \Rightarrow$

$dy/dx = 3x^2 - 10x$ (1)

$dy/dx \big|_{x=4} = 3(4)^2 - 10(4)$

$= 8$ (1)

Sub in $m = 8$ $(4, -2)$

$\therefore y = mx + c$

$-2 = 8(4) + c$

$-2 = 32 + c$

$-34 = c$

$\therefore$ Equation of the tangent

$y = 8x - 34$ (1)

10. (4 marks)
Find the coordinates of any point on the curve $y = x^3 + 6x^2 - 10x + 1$ where the gradient is 5

$dy/dx = 3x^2 + 12x - 10$ (1)

Solve $(5 = 3x^2 + 12x - 10)$

$\therefore x = -5$ OR

$x = 1$ (1)

Coordinates;

At $x = -5$

$y = (-5)^3 + 6(-5)^2 - 10(-5) + 1$ (1)

$= 76$

At $x = 1$

$y = (1)^3 + 6(1)^2 - 10(1) + 1$ (1)

$= -2$

Coordinates where the gradient $= 5$

$(-5, 76)$ and

$(1, -2)$

11. (8 marks: 1, 1, 2, 2, 2)

A colony of bacteria is increasing in such a way that the number of bacteria present after t hours is given by N where $N = 120 + 500t + 10t^3$. Find

(a) the number of bacteria present initially

When N = 0

$N = 120 + 500(0) + 10(0)^3$ ①

$= 120$

(b) the number of bacteria present when t= 5

$N(5) = 120 + 500(5) + 10(5)^3$ ①

$= 3870$

(c) the average rate of increase, in bacteria/hour, in the first 5 hours

Ave. Rate of Increase $= \frac{3870 - 120}{5 - 0} = 750$ bacteria/hour

① ①

(d) Determine an expression for the instantaneous rate of change of bacteria.

Instaneous R.o.c $\Rightarrow N'(t) = 500 + 30t^2$ ②

(e) the rate the colony is increasing, in bacteria/hour, when t= 10

$N'(10) = 500 + 30(10)^2$ ①

$= 3500$ bactera/hour ①

~ END OF SECTION 1 ~