

# TRIG CURVES

Name:

SOLUTIONS

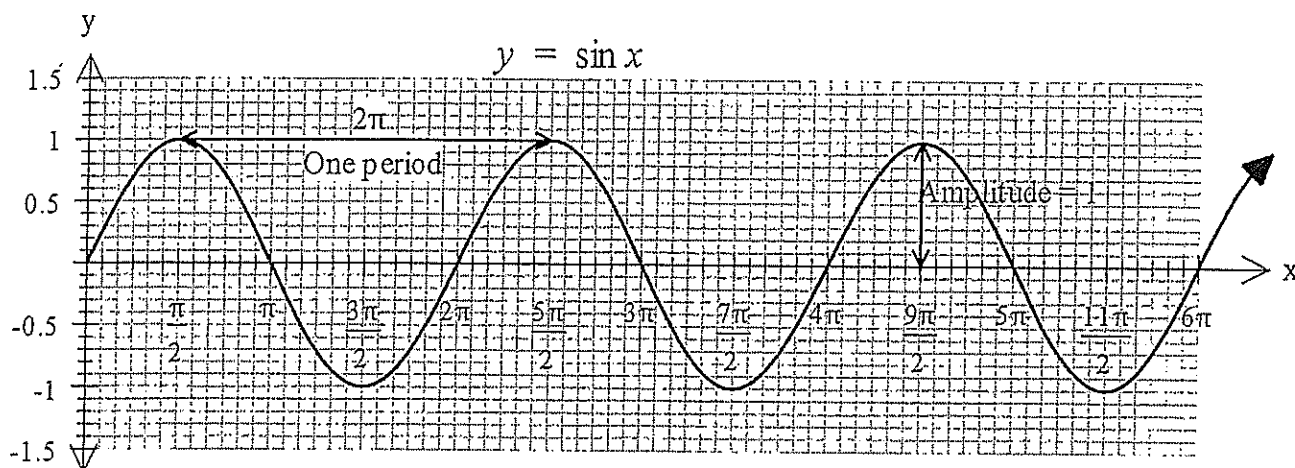
- The Out of Class Investigation is designed for you to learn the essentials needed for the In-Class validation.
- This is the "Take Home" part of the Investigation. It does not count towards your mark for this investigation.
- You will need your Casio Classpad.
- You are permitted to take this section into the "In Class" validation test to assist you.

Date of Validation: Tuesday 12<sup>th</sup> May.

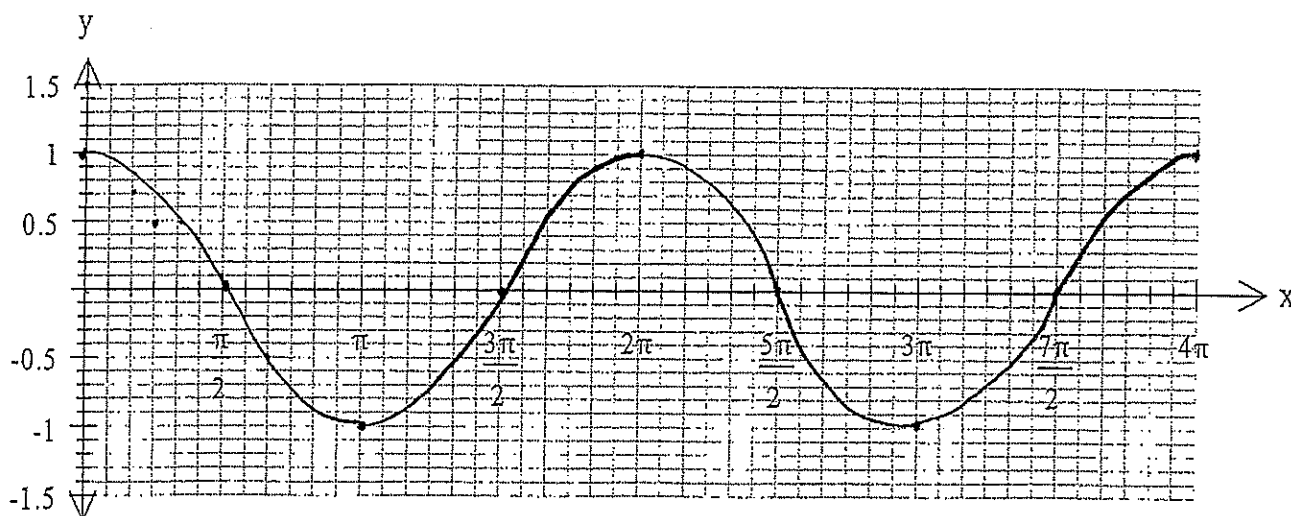
Shown below is a graph of  $y = \sin x$  for the domain  $0 \leq x \leq 6\pi$ . It shows the curve as it goes through 3 cycles.

The period is the length of one cycle of the curve. It is often best to measure from maximum to maximum. The amplitude of a curve is equal to *half* of the distance from the maximum value to the minimum value.

The sine curve below has a period of  $2\pi$  and an amplitude of 1.



- Using radians draw a graph of  $y = \cos x$  between 0 and  $4\pi$ . Use your calculator to help.



- State the period of  $y = \cos x$  and the amplitude of  $y = \cos x$ .

Period  $2\pi$  Amp = 1.

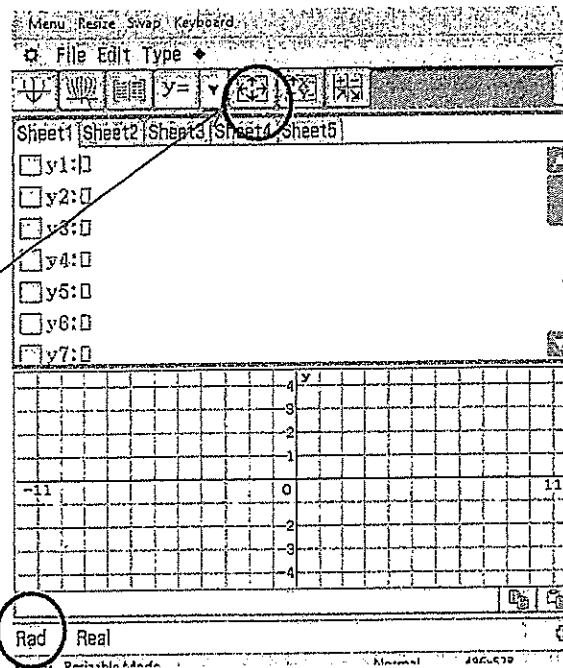
- Compare the differences and similarities of  $y = \cos x$  and  $y = \sin x$ .

• Same basic curve  
sine starts at 0, cosine at 1.

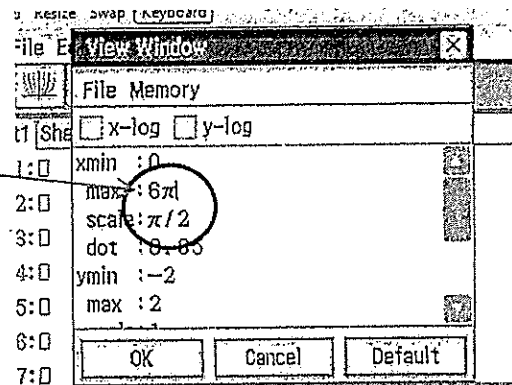
## HOW TO SET UP YOUR CALCULATOR TO DRAW TRIG GRAPHS

1. IN GRAPH & TAB set your calculator to radian mode.

2. Go to View Window.

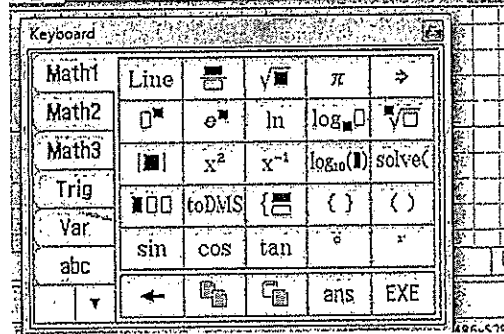
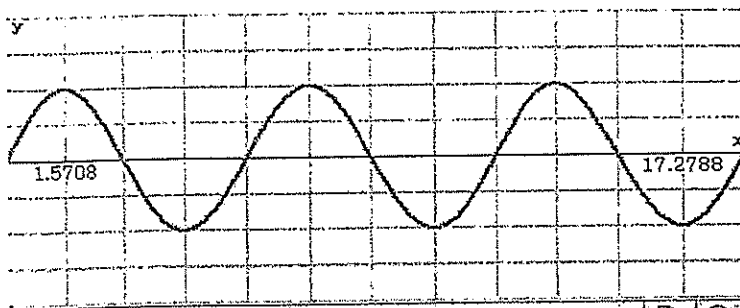


3. Set your x-axis from 0 to whatever value of  $\pi$  you need, using the  $\pi$  on the keyboard..

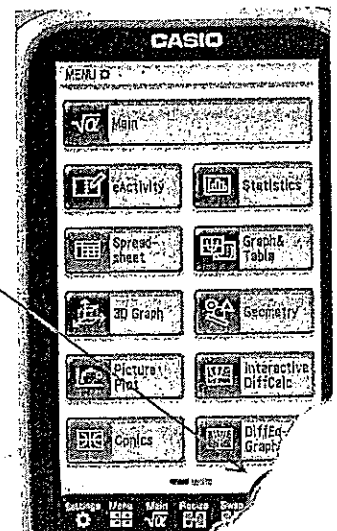
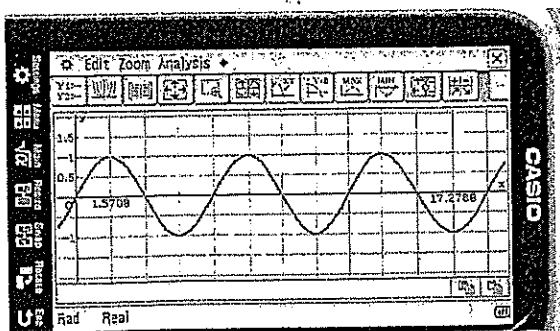


4. Set your scale as either  $\frac{\pi}{2}$  (about 1.57) or  $\frac{\pi}{6}$ .

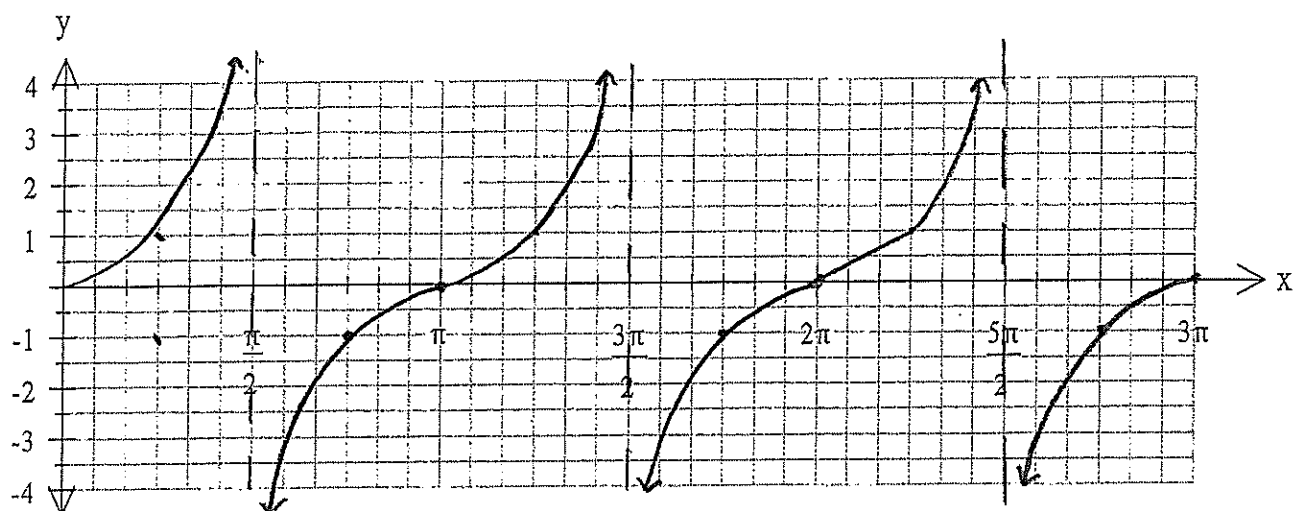
5. Your graph will appear as thus:



6. You may see the graph better if you rotate your screen to landscape view.



2. Plot the function  $y = \tan x$  for the domain  $0 \leq x \leq 3\pi$ , indicating the presence of vertical asymptotes.



- a) State the period of the  $y = \tan x$  curve.

Period =  $\pi$ .

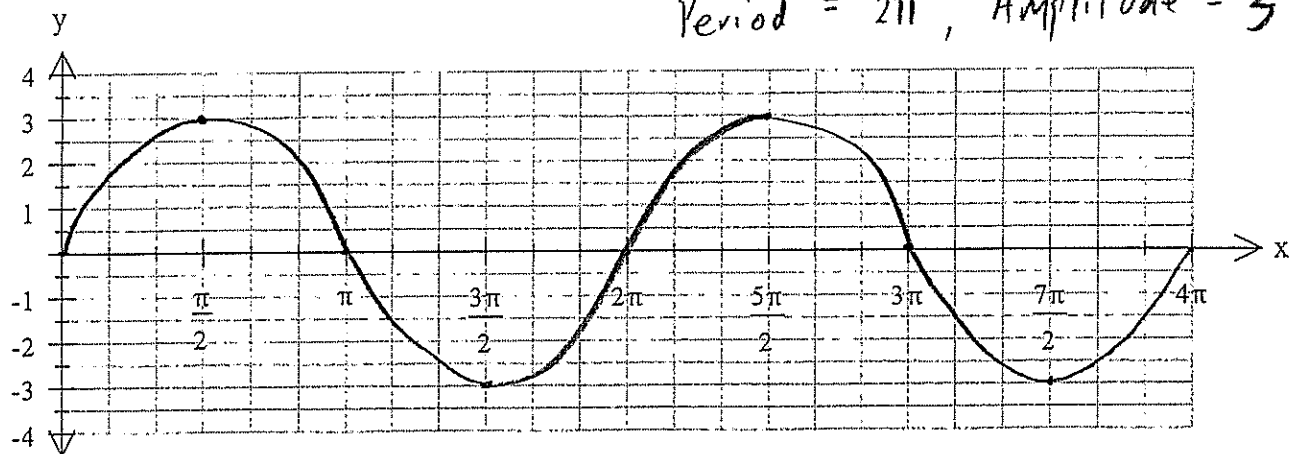
- b) Discuss the amplitude of the  $y = \tan x$  curve.

Amplitude is infinite.

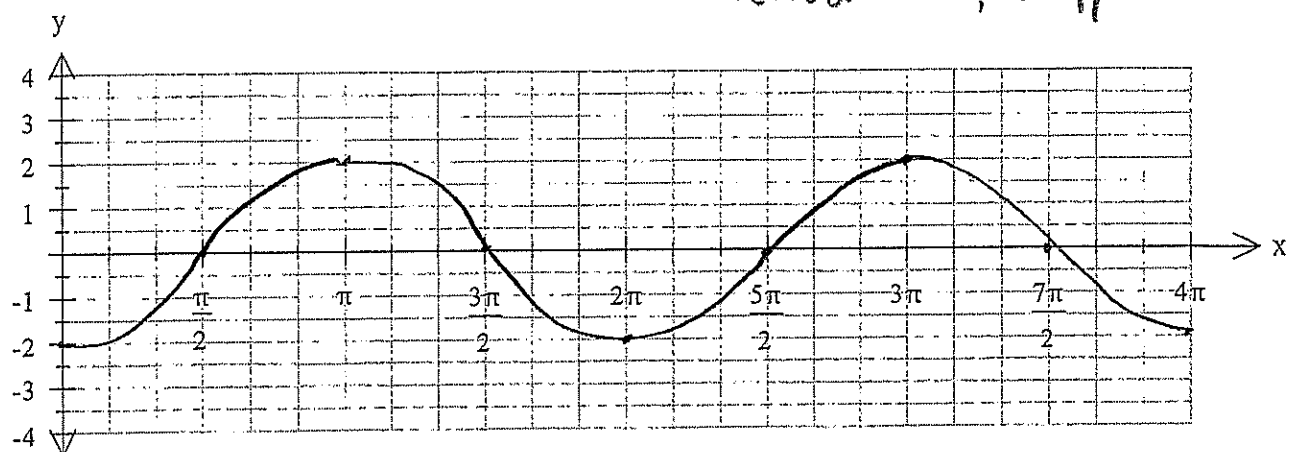
3. Plot the curve  $y = 3 \sin x$  and  $y = -2 \cos x$  on the axes below.

State the period and amplitude of each.

Period =  $2\pi$ , Amplitude = 3.

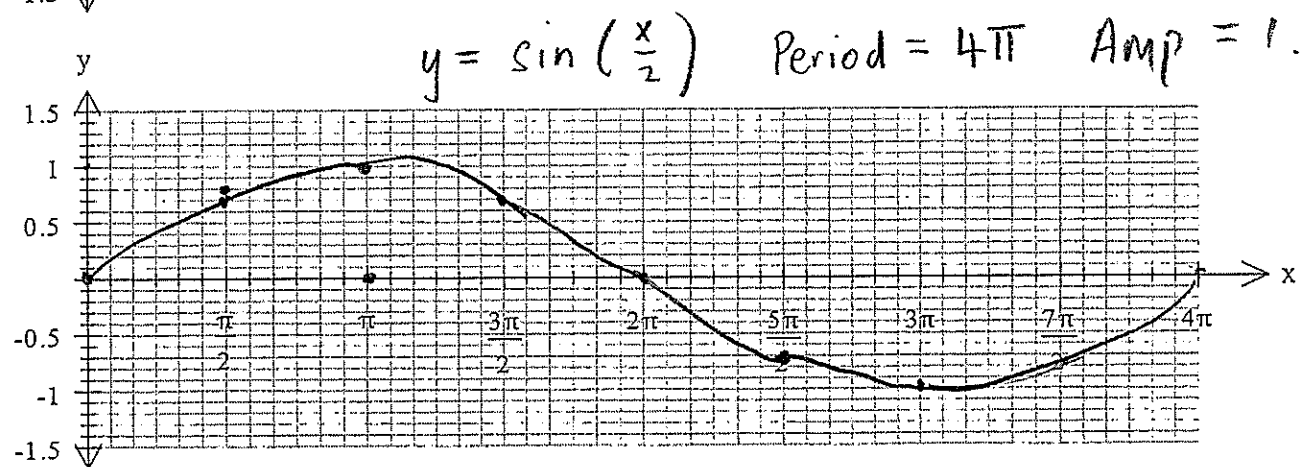
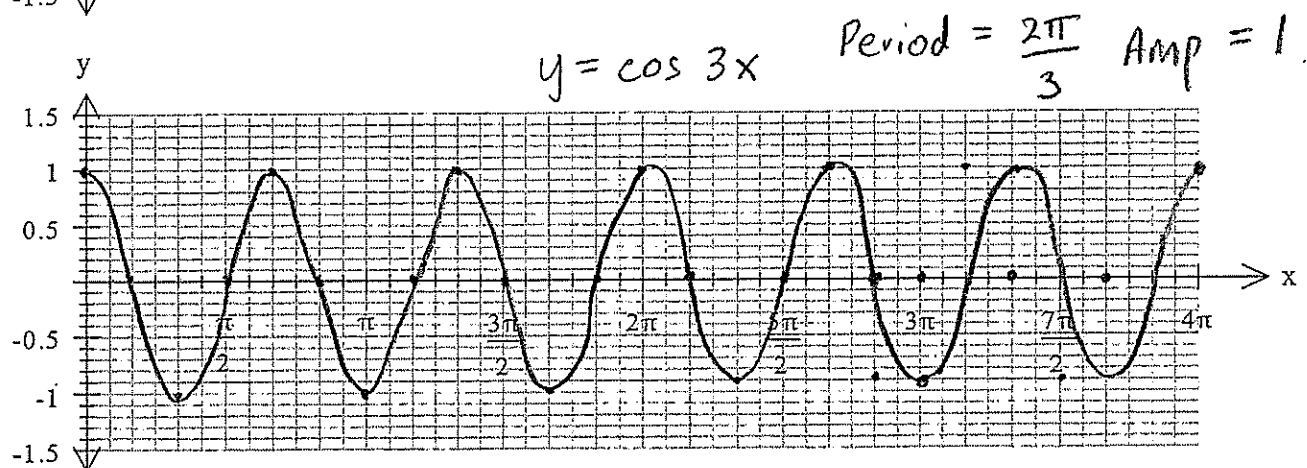
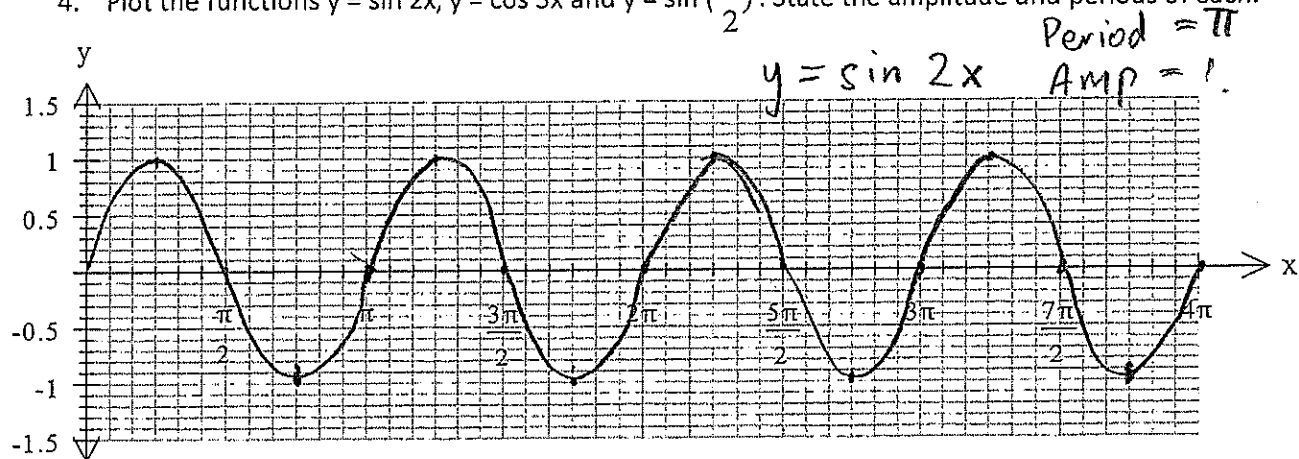


Period =  $2\pi$ , Amplitude = 2



Discuss the transformations that have occurred.

4. Plot the functions  $y = \sin 2x$ ,  $y = \cos 3x$  and  $y = \sin\left(\frac{x}{2}\right)$ . State the amplitude and periods of each.



5. State the effect of  $b$  in the rules  $y = \sin bx$  and  $y = \cos bx$ .

Change of Period.

6. State the period of the functions  $y = \sin 4x$ ,  $y = \cos 0.2x$  and  $y = \tan 2x$ .

$$y = \sin 4x \quad \left| \quad y = \cos 0.2x \quad \left| \quad y = \tan 2x \right. \right.$$

$$P = \frac{2\pi}{4} = \frac{\pi}{2} \quad \left| \quad P = \frac{2\pi}{0.2} = 10\pi \quad \left| \quad P = \frac{\pi}{2} \right. \right.$$

7. State the rule for the period of  $y = \sin bx$  and  $y = \cos bx$  in terms of  $b$ :

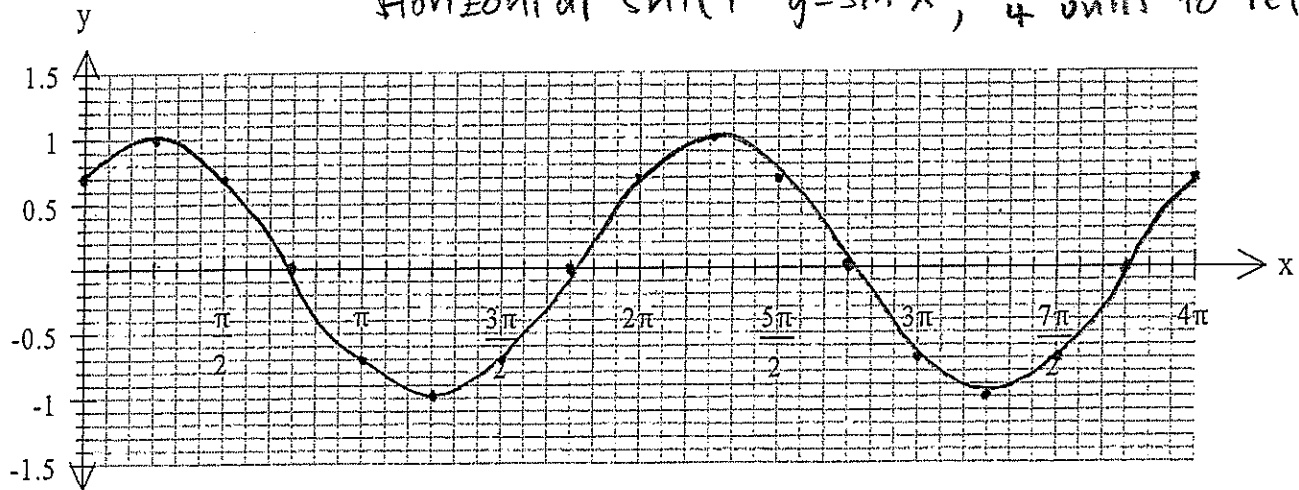
Period =  $\frac{2\pi}{b}$ .

8. State the rule for the period of the graph of  $y = \tan bx$  in terms of  $b$ .

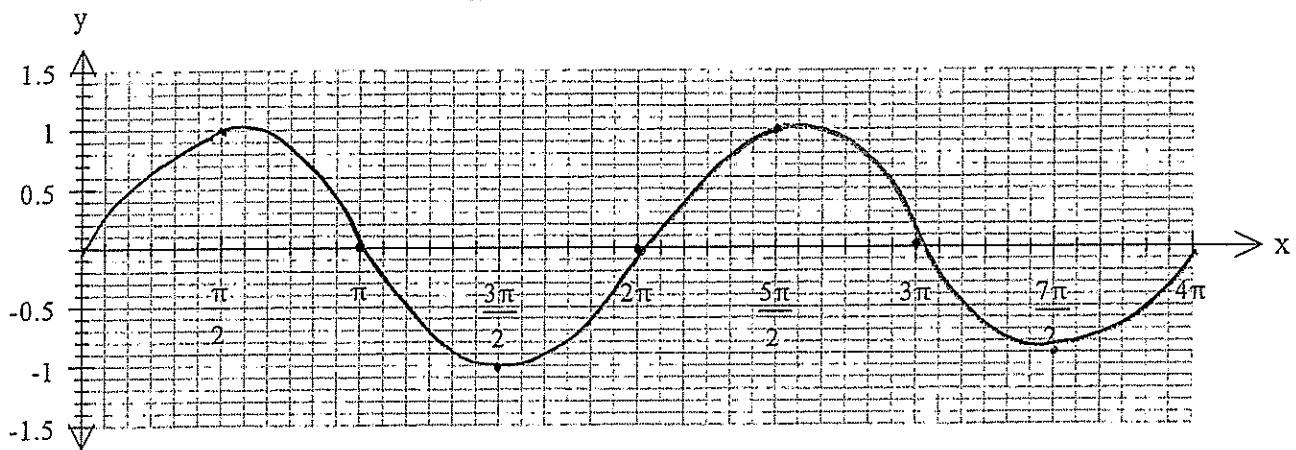
Period =  $\frac{\pi}{b}$ .

9. What is the effect on the  $\sin x$  graph if it is transformed into  $y = \sin(x + \frac{\pi}{4})$ . Draw it below.

Horizontal shift  $y = \sin x$ ,  $\frac{\pi}{4}$  units to left.



10. What is the effect on the  $\cos x$  graph if it transformed into the rule  $y = \cos(x - \frac{\pi}{2})$ ? Shift  $\frac{\pi}{2}$  unit to the right
11. Draw the graph of  $y = \cos(x - \frac{\pi}{2})$  below.



12. What is the  $y = \cos(x - \frac{\pi}{2})$  graph the same as?  $y = \sin x$

Explain why.

The  $\sin x$  graph is simply a  $\cos x$  graph translated  $\frac{\pi}{2}$  to the right.

13. What is the effect on a sine graph by transforming it into  $y = \sin x + 3$ . Graph it on your calculator and comment.  
(Hint; What will the new mean position be?)

$y = \sin x$  graph,  
translated 3 units up.

14. Describe what the graph of  $y = 2\cos x - 11$  will look like. (Hint; What will the new mean position be?)

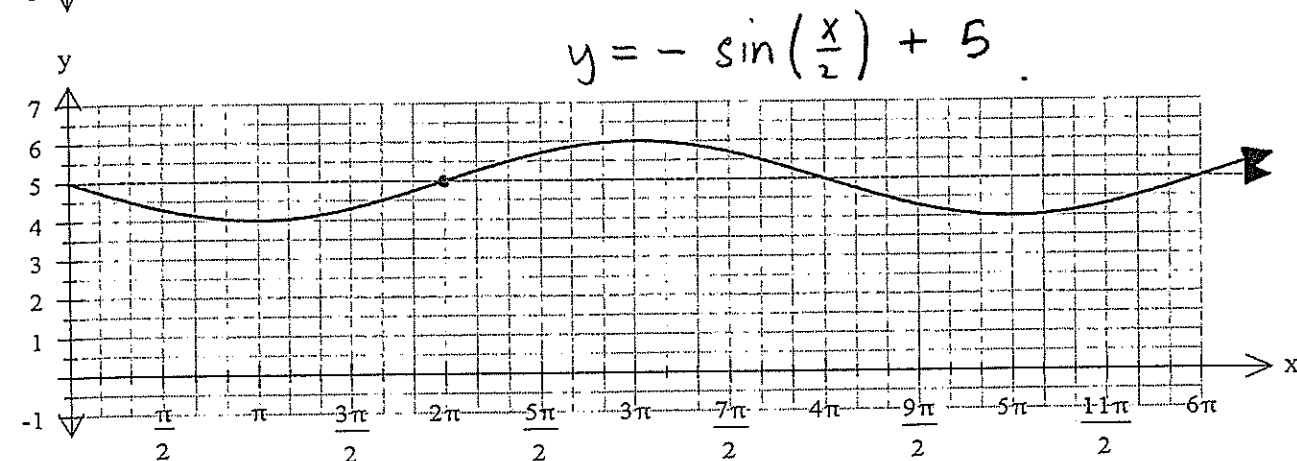
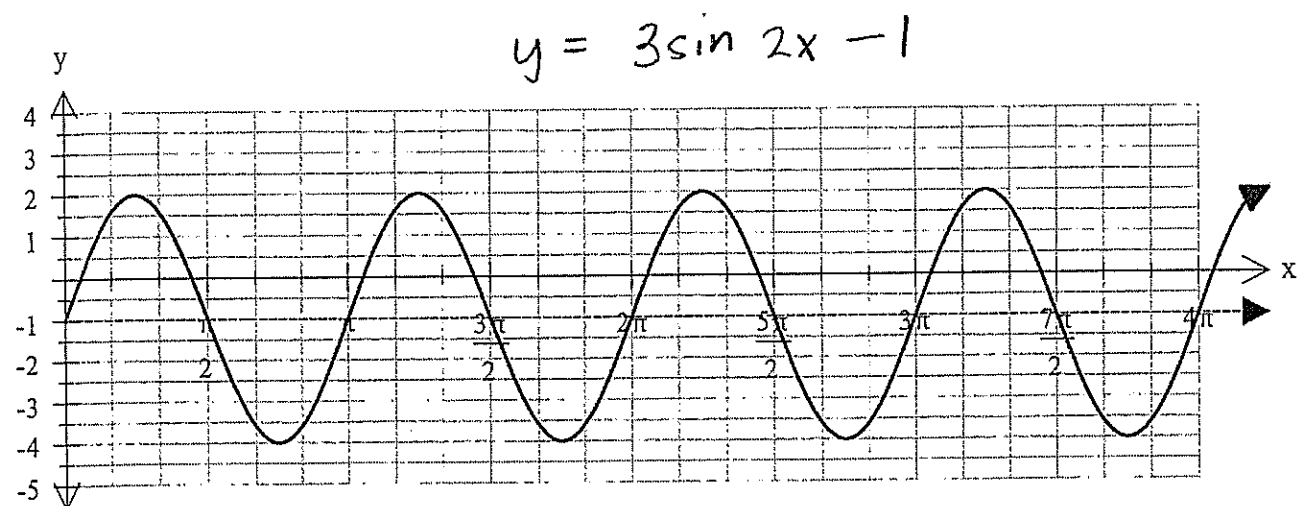
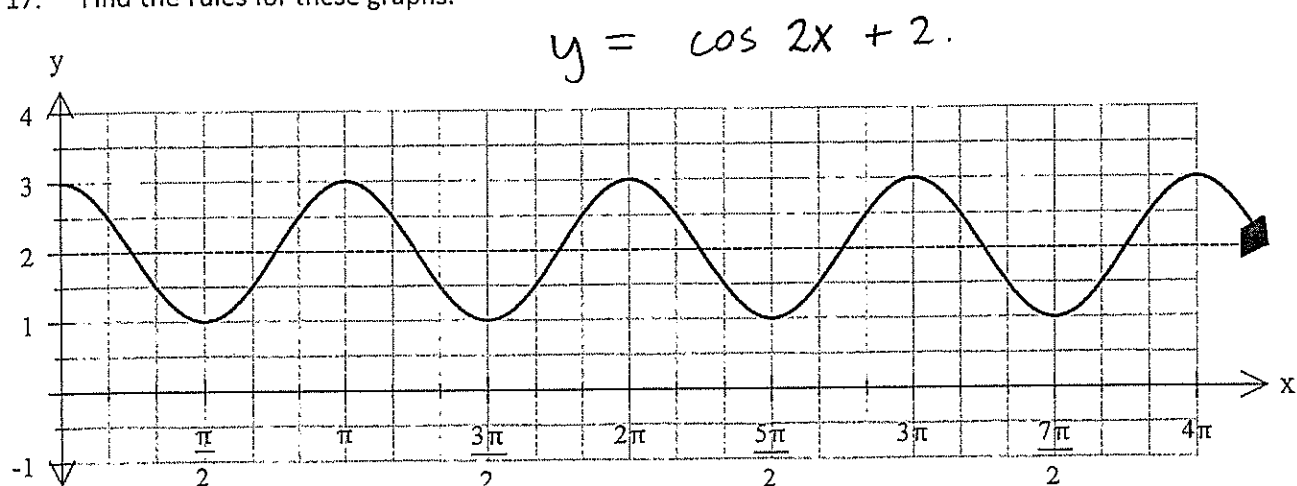
Vertical shift 11 units down, with an amplitude of 2.

15. Putting it all together: Explain the transformations that have occurred to  $y = \cos x$  to alter it to  $y = -4\cos(2x - \pi) + 3$
- affected about x-axis  
Amp = 4  
Period is now  $\pi$   
Translated  $\pi$  to the right  
up 3

16. Using the ideas of transformations complete the table for the following trigonometry rules.  
Use your calculator if you are not sure.

Rule	Period	Amplitude	Mean position	Maximum value	Minimum value	Horizontal shift
$y = 3 \sin 2x + 5$	$\pi$	3	$y = 5$	8	2	none
$y = -3 \cos 3x - 4$	$\frac{2\pi}{3}$	3	$y = -4$	1	-7	none
$y = 0.6 \sin (x - 2) + 1.2$	$2\pi$	0.6	$y = 1.2$	1.8	0.6	2 right
$y = -2 \sin (\pi x) - 1$	$\frac{2\pi}{\pi} = 2$	2	$y = -1$	1	-3	none
$y = \tan(0.5x + \frac{\pi}{2})$	$\frac{\pi}{0.5} = 2\pi$	Infinite	$y = 0$	$\infty$	$-\infty$	$\frac{\pi}{2}$ to left

17. Find the rules for these graphs.



The following questions will show you some examples of how trigonometrical rules are used.

18. Use your calculator and new found knowledge to answer this question.

Tides in a particular harbour are such that the depth of water (in metres) in the harbour is given by:

$$D = 3 \sin\left(\frac{\pi t}{12}\right) + 9$$

where  $t$  is the number of hours after midnight each day.

(a) At what time does low tide occur each day?

midnight + 18 = 6 pm.

(b) What is the depth of water in the harbour at high tide?

12 metres.

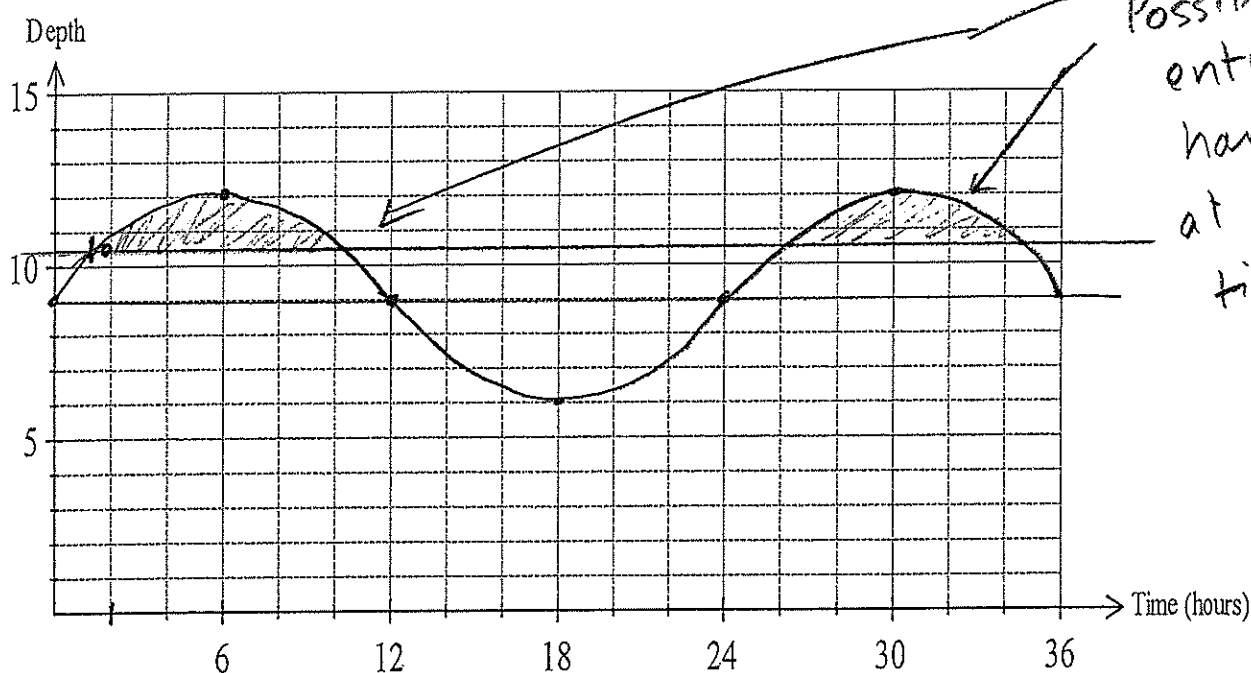
(c) What is the depth of the water in the harbour at 3.00 pm?

$t = 15 \Rightarrow D = 6.88\text{m}.$

(d) What is the maximum and minimum depth of the tide?

Max 12m Min 6m.

(e) Sketch the graph of  $D = 3 \sin\left(\frac{\pi t}{12}\right) + 9$  for  $0 \leq t \leq 36$



(f) A ship carrying a load has draught (i.e. the depth to which the ship sinks into the water when it is floating) of 10.5 metres.

(i) Show on your graph the times it is possible for the ship to enter the harbour?

(ii) Determine these times, for any day, correct to the nearest minute.

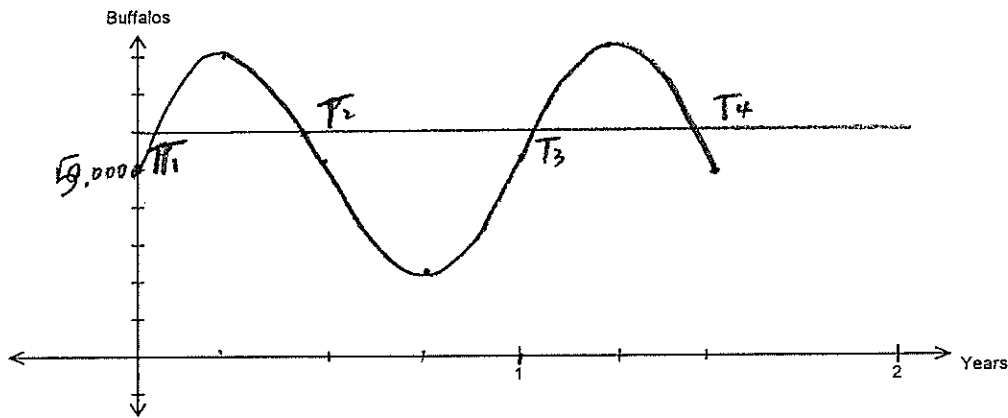
2am → 10pm

Between 2am and 10pm.

19. The population of a water buffalo herd in the Northern Territory is modelled by:

$$P(t) = 5000 + 3000 \sin(2\pi t) \quad \text{where } t \text{ is the number of years since the first estimate was made.}$$

- (a) Sketch the graph showing the population for  $0 \leq t \leq 2$



- (b) Find the maximum herd size and when it occurs

$$\text{Maximum} = 8,000 \text{ after } \frac{1}{2} \text{ year}$$

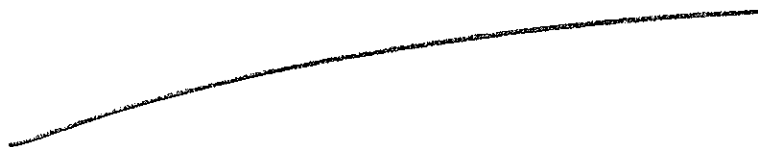
- (c) When is the herd size 6000?

$$T_1 = 0.054 \text{ years} = 20 \text{ days}$$

$$T_2 = 0.4459 \text{ years} = 163 \text{ days}$$

$$T_3 = 1.054 \text{ years} = 1 \text{ year and } 20 \text{ days}$$

$$T_4 = 1.4459 \text{ year} = 1 \text{ year and } 163 \text{ days}$$





20. The ultraviolet radiation rate,  $R(t)$ , from the sun  $t$  hours after sunrise on February 14 is modelled by:

$$R(t) = 9.2 + 5.8 \sin\left[\frac{\pi}{6}(t-2)\right], \quad 0 \leq t \leq 14$$

- (a) What is the UV rate at sunrise ( $t=0$ )?

$$t=0, R(0) = 4.1771$$

- (b) Find the greatest and least UV rates

$$\text{Max} = 15, \text{Min} = 3.4$$

- (c) What is the period of the function?

$$P = \frac{2\pi}{\frac{\pi}{6}} = 12 \text{ hours}$$

- (d) Find when the UV rate is 9.

$$9 = R(t) \Rightarrow 1 \text{ } 93 \text{ hours and } 8 \text{ } 07 \text{ hour.}$$

116 minutes      484 min

- (e) If sunrise is at 6:15 am, at what time does the maximum UV rate occur?

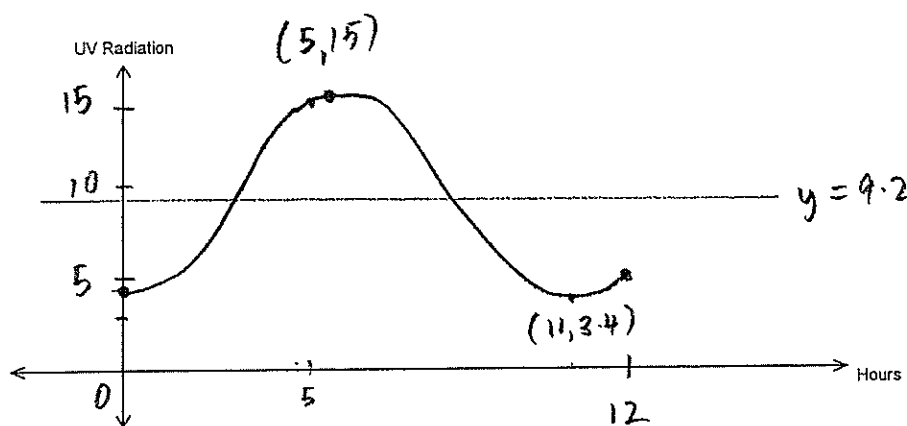
After 5 hours i.e At 11:15 am

- (f) The most dangerous times for sunburn occur when the UV rate is 12 or above. If sunrise is at 6:15 am, during what times of the day is the UV radiation most dangerous?

Between 2.962 hours and 7.0378h  
= 178min and 422min  
3h 58min — 7h 2min

9:13am —→ 1:17pm  
Above 12.

- (g) Sketch  $R(t)$  against time for  $0 \leq t \leq 14$



THE END