



MINDARIE
SENIOR COLLEGE
WHERE YOUR FUTURE BEGINS NOW

**MATHEMATICS:
SPECIALIST 1 & 2**

SEMESTER 2 2015

TEST 5

Resource Free

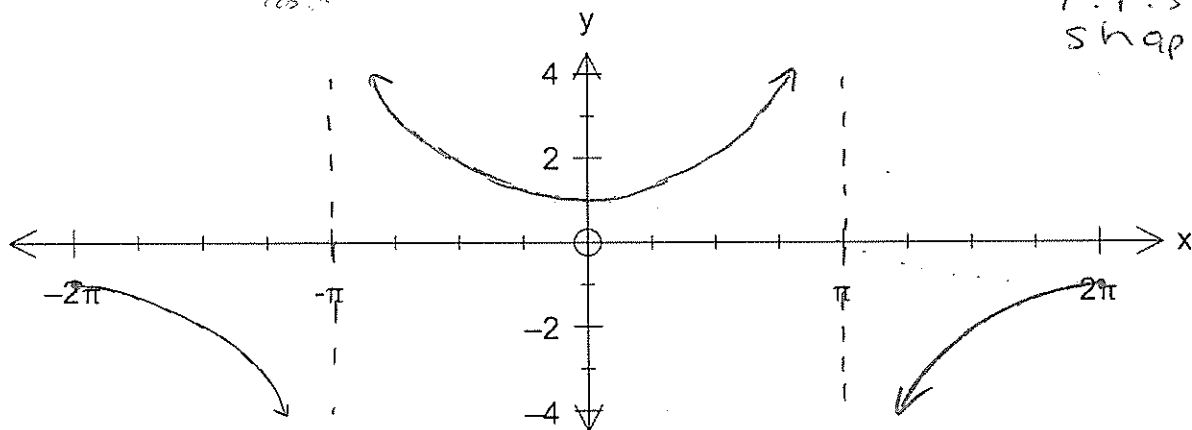
Time Allowed: 25 minutes

Total Marks: 18

1. [3, 3 marks]

(a) Sketch $y = \sec \frac{1}{2}x$ over the domain $-2\pi \leq x \leq 2\pi$

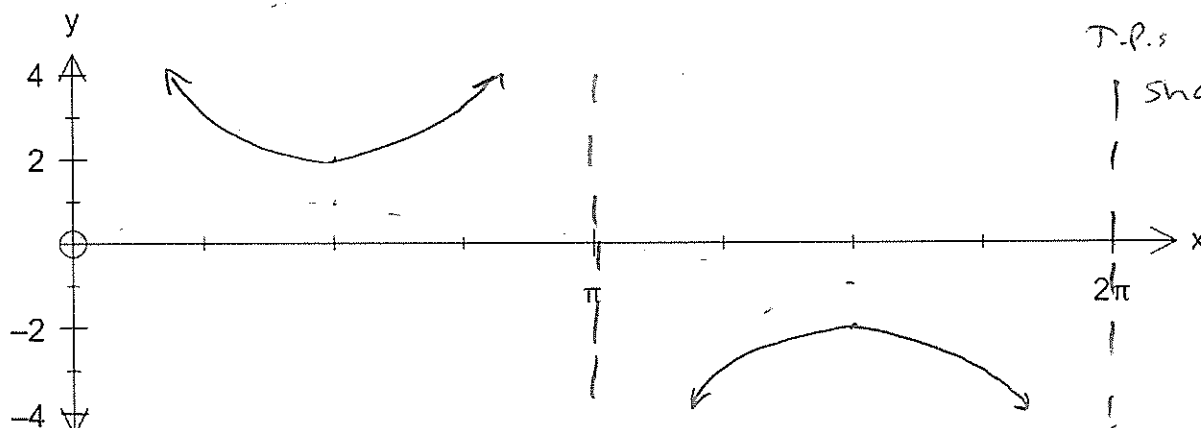
$= \frac{1}{\cos \frac{1}{2}x}$



Asymptotes ✓
T.P.s ✓
shape ✓

(b) Sketch $y = 2 \operatorname{cosec} x$ over the domain $0 \leq x \leq 2\pi$

$= \frac{2}{\sin x}$



Asymptotes ✓
T.P.s ✓
shape ✓

2. [4, 4 marks]

Solve the following equations over the given interval

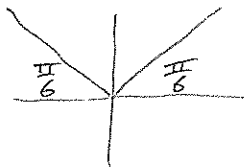
(a) $3\sec^2 x = 4$ for $0 \leq x \leq \pi$

$$\sec^2 x = \frac{4}{3} \quad \checkmark$$

$$\cos^2 x = \frac{3}{4} \quad \checkmark$$

$$\cos x = \pm \frac{\sqrt{3}}{2} \quad \checkmark$$

$$x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \quad \checkmark$$



(b) $6 \sin 5x \cos 4x = 3 \sin 9x - \frac{3\sqrt{2}}{2}$ for $0 \leq x \leq 2\pi$

$$3 \sin 9x \quad \checkmark \quad 3 \sin x = 3 \sin 9x - \frac{3\sqrt{2}}{2}$$

$$3 \sin x = -\frac{3\sqrt{2}}{2} \quad \checkmark$$

$$\sin x = -\frac{\sqrt{2}}{2} \quad \checkmark$$

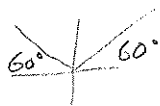


$$x = \frac{5\pi}{4} \text{ or } \frac{7\pi}{4} \quad \checkmark$$

3. [3, 2 marks]

Find all solutions to the following equations for x in degrees

(a) $\sin x = 0.5$



$$x = 60^\circ + k360^\circ \quad k \in \mathbb{Z}$$

$$\text{or } x = 120^\circ + k360^\circ \quad \checkmark$$

(b) $\cot x = \sqrt{3}$

$$\tan x = \frac{1}{\sqrt{3}} \quad \checkmark$$

$$x = 30^\circ + k180^\circ \quad k \in \mathbb{Z} \quad \checkmark$$



Resource Assumed

Time Allowed: 25 minutes

Total Marks: 35

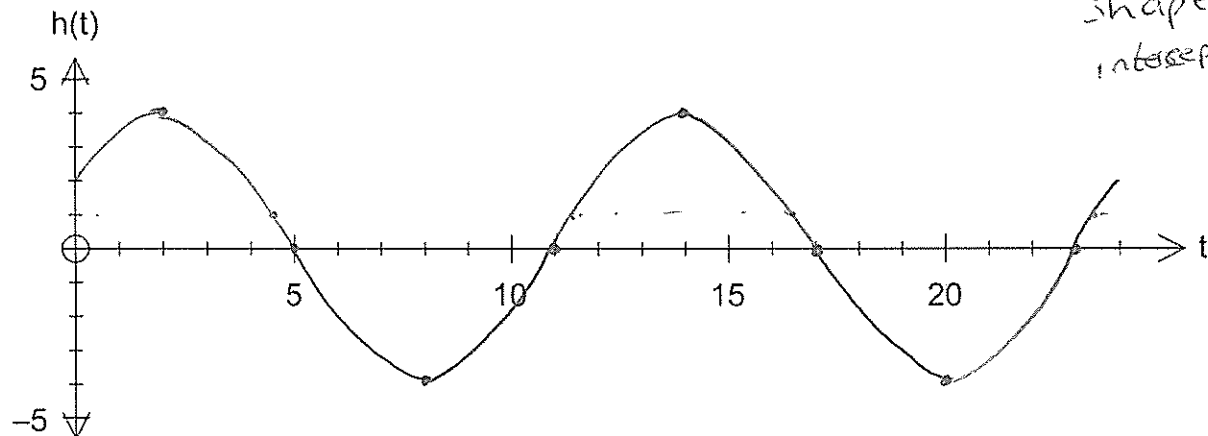
4. [2, 3, 1, 2 marks]

The height of the tide above mean sea level at a certain port has been modelled by the equation

$$h(t) = 4 \sin \frac{\pi(t+1)}{6}$$

where t is the number of hours after midnight on a particular day.

(a) Neatly sketch the graph for $h(t) = 4 \sin \frac{\pi(t+1)}{6}$ for $0 \leq t \leq 24$



(b) When was the high tide? What is its height above mean sea level at this time?

2am ✓ 2pm ✓ 4m ✓

(c) What was the height of the tide at 8 pm? -2m

2m below ✓ mean sea level

(d) A ship can only enter port when there is a depth of 3 metres of water above low tide. Between what times could a ship enter or leave port? Give answers to nearest 5 minutes.

5.5 → 10.5 17.5 → 22.5

5:30 am → 10:30 am & 5:30 pm → 10:30 pm

5. [2, 5 marks]

(a) Express the product $\cos 45^\circ \cos 15^\circ$ as an exact value.

$$\frac{1}{2} (\cos 30^\circ + \cos 60^\circ) = \frac{1}{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right) \\ = \frac{\sqrt{3}+1}{4}$$

(b) Prove that $\frac{\cos 6\theta \cos 3\theta + \sin 2\theta \sin \theta}{\cos 5\theta} = \cos 4\theta$

$$\begin{aligned} \text{LHS} &= \frac{\cos 6\theta \cos 3\theta + \sin 2\theta \sin \theta}{\cos 5\theta} \\ &= \frac{\frac{1}{2} (\cos 3\theta + \cos 9\theta) + \frac{1}{2} (\cos 5\theta - \cos 3\theta)}{\cos 5\theta} \\ &= \frac{\frac{1}{2} (\cos 9\theta + \cos \theta)}{\cos 5\theta} \\ &= \frac{\cos 5\theta \cos 4\theta}{\cos 5\theta} \\ &= \cos 4\theta \\ &= \text{RHS} \end{aligned}$$

6. [4, 4, 3 marks]

(a) Prove that 3 more than the square of an odd number is always divisible by 4.

$$\begin{aligned} \checkmark (2n+1)^2 + 3 &= 4n^2 + 4n + 1 + 3 \checkmark \\ &= 4n^2 + 4n + 4 \\ &= 4(n^2 + n + 1) \checkmark \\ &\text{which is divisible by 4} \end{aligned}$$

(b) Consider three consecutive numbers. Prove that the sum of the cube of the smallest number, the square of the middle number, and the largest number will always be a multiple of the middle number.

$$\begin{aligned} n, n+1, n+2 \checkmark \quad n^3 + (n+1)^2 + n+2 &= n^3 + n^2 + 2n + 1 + n + 2 \checkmark \\ &= n^3 + n^2 + 3n + 3 \\ &= (n^2 + 3)(n+1) \checkmark \\ \therefore (n^2 + 3) \times \text{middle number} \checkmark \end{aligned}$$

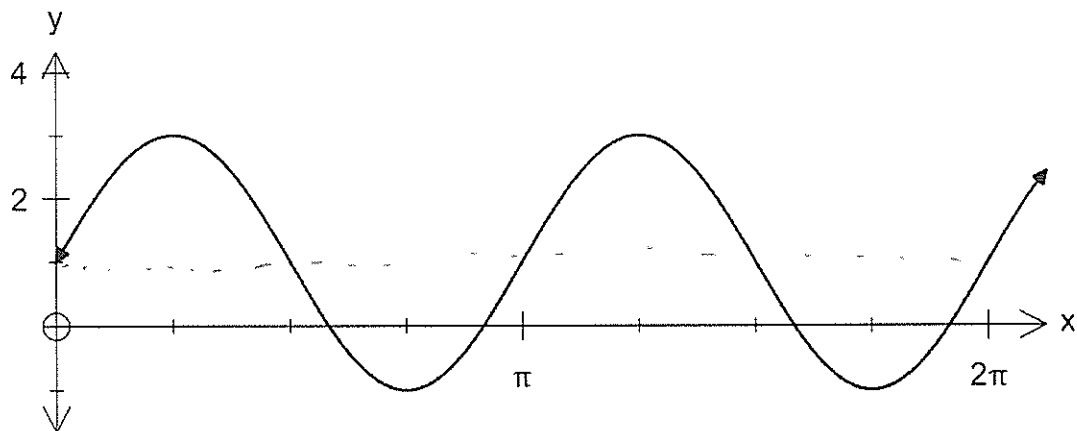
(c) Express 0.14527527527... as a fraction. Show full working.

$$\begin{aligned} x &= 0.14527527527... \\ 1000x &= 145.27527527 \checkmark \\ 1000x - x &= 145.13 \checkmark \\ 999x &= 145.13 \checkmark \end{aligned} \quad \begin{aligned} x &= \frac{145.13}{999} \\ &= \frac{14513}{99900} \checkmark \end{aligned}$$

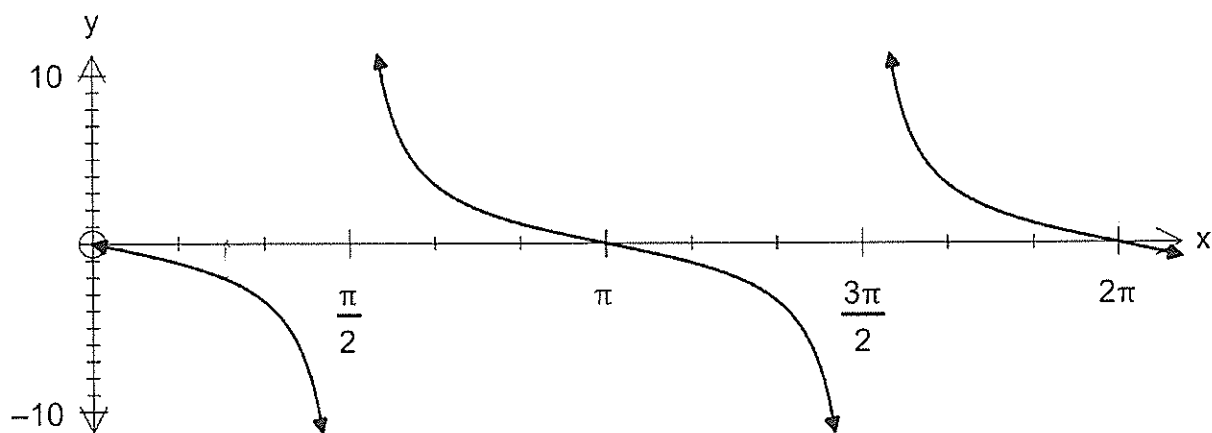
7. [3, 2, 4 marks]

Determine the equation of the following graphs

(a) $y = 2 \sin 2x$



(b) $y = -2 \tan x$



(c) $y = 3 \cos 2(x + 15)$

