education equals

Topic: Matrix Algebra and Operations

Time: 45 mins Marks: /45 marks

Calculator Assumed

Question One: [1, 1, 1, 1, 1, 1, 1, 1: 8 marks]

Use your calculator to perform the following calculations. If any of the following cannot be evaluated clearly, state this and provide an explanation as to why.

$A = \begin{bmatrix} 3 & 4 & 11 \\ 0 & 6 & -9 \\ 12 & -11 & 9 \end{bmatrix}$ $B = [7 \quad 11 \quad 24]$ $C = \begin{bmatrix} 22 & -1 & 14 \\ -9 & 6 & 3 \end{bmatrix}$ $D = \begin{bmatrix} 9 \\ 6 \\ -12 \end{bmatrix}$ $E = \begin{bmatrix} -1 \\ 3 \\ 22 \end{bmatrix}$

a) D + E

b) A + C

c) A × C

d) A × D × B

e) $13C$

f) $-3D - 2E$

g) $4(D + E)$

h) $(AD)^2$

Question Two: [2, 2, 2: 6 marks]

A 3x4 matrix **C** is determined by $\mathbf{C = 3A - 2B}$

Where $A = \begin{bmatrix} 5 & 2 & 9 & 5 \\ 18 & -4 & 6 & 47 \\ -21 & 4 & 2 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 11 & 15 \\ 6 & -12 & 8 & 7 \\ -1 & 1 & 1 & 2 \end{bmatrix}$

a) Determine matrix **C.**

b) Explain clearly why B^2 does not exist.

c) Calculate $C_{23} + C_{34}$

Question Three: [6 marks]

By equating corresponding elements, find the numbers m, n and a, if $-3\boldsymbol{J} = \boldsymbol{K}$ where:

$\boldsymbol{J} = \begin{bmatrix} 5 & -20 \\ -6 & m^2 \\ m & a \end{bmatrix}$ and $\boldsymbol{K} = \begin{bmatrix} -15 & m \times n \\ 18 & -300 \\ 30 & -3 \end{bmatrix}$ Show full working.

Question Four: [2, 2: 4 marks]

a) If $\begin{bmatrix} 3 & 1 \\ 5 & 6 \end{bmatrix}^x = \begin{bmatrix} 87 & 68 \\ 340 & 291 \end{bmatrix}$, determine the value of x

b) If $\boldsymbol{M}^4 = \begin{bmatrix} 16 & 0 \\ 87 & 625 \end{bmatrix}$ determine matrix $\mathbf{M}$

Question Five: [6 marks]

Matrices **A**, **B** and **C** are:

$$\boldsymbol{A} = \begin{bmatrix} -8 \\ 6 \\ 2 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \qquad \boldsymbol{C} = \begin{bmatrix} 10 \\ 7 \\ 2.5 \end{bmatrix}$$

Find scalars p, q and Matrix $\boldsymbol{R}$ such that:

$$p\boldsymbol{A} = \begin{bmatrix} \mathbf{12} \\ \mathbf{-9} \\ \mathbf{-3} \end{bmatrix} \qquad q(\boldsymbol{B} + \boldsymbol{C}) = \begin{bmatrix} 11.2 \\ 4 \\ 2.8 \end{bmatrix} \qquad 3\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{R} + \boldsymbol{C}$$

Show working to justify your solution.

Question Six: [2, 2: 4 marks]

Charlene was calculating with matrices. She got the following answers incorrect. Explain what she did wrong and write the correct solution.

a) $\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}^2 = \begin{bmatrix}4 & 9\\16 & 25\end{bmatrix}$

b) $\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix} \times \begin{bmatrix}5 & 4\\3 & 2\end{bmatrix} = \begin{bmatrix}10 & 12\\12 & 10\end{bmatrix}$

Question Seven: [2, 2, 2: 6 marks]

Are the following statements true or false? Explain your answer.

a) If $A = [1 \quad 1]$ and $B = [1 \quad 1]$ then $AB = [1]$

b) If $C = \begin{bmatrix}-2 & 5\\10 & -4\end{bmatrix}$ and D $= \begin{bmatrix}10 & -4\\-2 & 5\end{bmatrix}$ then $CD = DC$

c) If $E = \begin{bmatrix}4 & 1\\6 & 7\\-5 & -9\end{bmatrix}$ and $F = \begin{bmatrix}-4 & -1\\-6 & -7\\5 & 9\end{bmatrix}$ then $E - F = O$

Question Eight: [5 marks]

If $M = [a \quad b]$, $N = \begin{bmatrix} c \\ d \end{bmatrix}$ and $P = \begin{bmatrix} e \\ f \end{bmatrix}$ then prove the distributive law, $M(N + P) = MN + MP$

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Matrix Algebra and Operations
SOLUTIONS

Time: 45 mins Marks: /45 marks

Calculator Assumed

Question One: [1, 1, 1, 1, 1, 1, 1, 1: 8 marks]

Use your calculator to perform the following calculations. If any of the following cannot be evaluated clearly, state this and provide an explanation as to why.

$$A = \begin{bmatrix} 3 & 4 & 11 \\ 0 & 6 & -9 \\ 12 & -11 & 9 \end{bmatrix} \quad B = [7 \quad 11 \quad 24] \quad C = \begin{bmatrix} 22 & -1 & 14 \\ -9 & 6 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 9 \\ 6 \\ -12 \end{bmatrix} \quad E = \begin{bmatrix} -1 \\ 3 \\ 22 \end{bmatrix}$$

a) D + E

$\begin{bmatrix} 8 \\ 9 \\ 10 \end{bmatrix}$ ✓

b) A + C

Cannot add. Matrices A and C are different sizes and can only add matrices of the same size. ✓

c) A × C

Dimensions do not allow for multiplication. The number of columns in A does not equal the number of rows in C. ✓

d) A × D × B

$\begin{bmatrix} -567 & -891 & -1944 \\ 1008 & 1584 & 3456 \\ -462 & -726 & -1584 \end{bmatrix}$ ✓

e) $13C$

$\begin{bmatrix} 286 & -13 & 182 \\ -117 & 78 & 39 \end{bmatrix}$ ✓

f) $-3D - 2E$

$\begin{bmatrix} -25 \\ -24 \\ -8 \end{bmatrix}$ ✓

g) $4(D + E)$

$\begin{bmatrix} 32 \\ 36 \\ 40 \end{bmatrix}$ ✓

h) $(BE)^2$

[306916] ✓

Question Two: [2, 2, 2: 6 marks]

A 3x4 matrix **C** is determined by $\mathbf{C} = \mathbf{3A} - \mathbf{2B}$

Where $A = \begin{bmatrix} 5 & 2 & 9 & 5 \\ 18 & -4 & 6 & 47 \\ -21 & 4 & 2 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 11 & 15 \\ 6 & -12 & 8 & 7 \\ -1 & 1 & 1 & 2 \end{bmatrix}$

a) Determine matrix **C.**

$$C = \begin{bmatrix} 19 & 6 & 5 & -15 \\ 42 & 12 & 2 & 127 \\ -61 & 10 & 4 & 23 \end{bmatrix}$$

✓ ✓

b) Explain clearly why B^2 does not exist.

B^2 = B x B, but matrix B does not have the same number of rows and columns and therefore it cannot be multiplied by itself.

✓ ✓

c) Calculate $C_{23} + C_{34}$

$C_{23} = 2$ ✓

$C_{34} = 23$

$C_{23} + C_{34} = 25$ ✓

Question Three: [6 marks]

By equating corresponding elements, find the numbers m, n and a, if $-3\boldsymbol{J} = \boldsymbol{K}$ where:

$\boldsymbol{J} = \begin{bmatrix} 5 & -20 \\ -6 & m^2 \\ m & a \end{bmatrix}$ and $\boldsymbol{K} = \begin{bmatrix} -15 & m \times n \\ 18 & -300 \\ 30 & -3 \end{bmatrix}$ Show full working.

$-3m = 30$ ✓

$m = -10$ ✓

$60 = m \times n$ ✓

$60 = -10n$

$-6 = n$ ✓

$-3a = -3$ ✓

$a = 1$ ✓

Question Four: [2, 2: 4 marks]

a) If $\begin{bmatrix} 3 & 1 \\ 5 & 6 \end{bmatrix}^x = \begin{bmatrix} 87 & 68 \\ 340 & 291 \end{bmatrix}$, determine the value of x

x = 3 ✓✓

b) If $\boldsymbol{M}^4 = \begin{bmatrix} 16 & 0 \\ 87 & 625 \end{bmatrix}$ determine matrix **M**

$$M = \begin{bmatrix} 16 & 0 \\ 87 & 625 \end{bmatrix}^{\frac{1}{4}} \checkmark$$

$$= \begin{bmatrix} 2 & 0 \\ 0.42857 & 5 \end{bmatrix} \checkmark$$

Question Five: [6 marks]

Matrices **A**, **B** and **C** are:

$$\boldsymbol{A} = \begin{bmatrix} -8 \\ 6 \\ 2 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \qquad \boldsymbol{C} = \begin{bmatrix} 10 \\ 7 \\ 2.5 \end{bmatrix}$$

Find scalars p, q and Matrix $\boldsymbol{R}$ such that:

$$p\boldsymbol{A} = \begin{bmatrix} \mathbf{12} \\ \mathbf{-9} \\ \mathbf{-3} \end{bmatrix} \qquad q(\boldsymbol{B} + \boldsymbol{C}) = \begin{bmatrix} 11.2 \\ 4 \\ 2.8 \end{bmatrix} \qquad 3\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{R} + \boldsymbol{C}$$

Show working to justify your solution.

$p = -1.5$ ✓

✓

$$q \begin{bmatrix} 14 \\ 5 \\ 3.5 \end{bmatrix} = \begin{bmatrix} 11.2 \\ 4 \\ 2.8 \end{bmatrix}$$

$q = 0.8$ ✓

✓ ✓

$$\begin{bmatrix} -24 \\ 18 \\ 6 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} 10 \\ 7 \\ 2.5 \end{bmatrix}$$

$$\begin{bmatrix} -20 \\ 16 \\ 7 \end{bmatrix} = \begin{bmatrix} a + 10 \\ b + 7 \\ c + 2.5 \end{bmatrix}$$

$a = -30, b = 9, c = 4.5$

$$\boldsymbol{R} = \begin{bmatrix} -30 \\ 9 \\ 4.5 \end{bmatrix}$$ ✓

Question Six: [2, 2: 4 marks]

Charlene was calculating with matrices. She got the following answers incorrect. Explain what she did wrong and write the correct solution.

a) $\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}^2 = \begin{bmatrix}4 & 9\\16 & 25\end{bmatrix}$ ✗

She squared each element ✓

$\begin{bmatrix}16 & 21\\28 & 37\end{bmatrix}$ ✓

b) $\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix} \times \begin{bmatrix}5 & 4\\3 & 2\end{bmatrix} = \begin{bmatrix}10 & 12\\12 & 10\end{bmatrix}$ ✗

She only multiplied corresponding eleme ✓

$\begin{bmatrix}19 & 14\\35 & 26\end{bmatrix}$ ✓

Question Seven: [2, 2, 2: 6 marks]

Are the following statements true or false? Explain your answer.

a) If $A = [1 \quad 1]$ and $B = [1 \quad 1]$ then $AB = [1]$

✓ ✓

False AB is not possible.

b) If $C = \begin{bmatrix}-2 & 5\\10 & -4\end{bmatrix}$ and D $= \begin{bmatrix}10 & -4\\-2 & 5\end{bmatrix}$ then $CD = DC$

✓

False ✓

$CD = \begin{bmatrix}-30 & 33\\108 & -60\end{bmatrix}$ and $DC = \begin{bmatrix}-60 & 66\\54 & -30\end{bmatrix}$

c) If $E = \begin{bmatrix}4 & 1\\6 & 7\\-5 & -9\end{bmatrix}$ and $F = \begin{bmatrix}-4 & -1\\-6 & -7\\5 & 9\end{bmatrix}$ then $E - F = O$

False ✓

$E - F = \begin{bmatrix}0 & 0\\0 & 0\\-10 & -18\end{bmatrix}$ ✓

Question Eight: [5 marks]

If $M = [a \quad b]$, $N = \begin{bmatrix} c \\ d \end{bmatrix}$ and $P = \begin{bmatrix} e \\ f \end{bmatrix}$ then prove the distributive law, $M(N + P) = MN + MP$

$N + P = \begin{bmatrix} c + e \\ d + f \end{bmatrix}$ ✓

LHS: $[a \quad b]\begin{bmatrix} c + e \\ d + f \end{bmatrix}$

$= \boldsymbol{a(c + e) + b(d + f)}$ ✓

$MN = [a \quad b]\begin{bmatrix} c \\ d \end{bmatrix}$

$= ac + bd$ ✓

$MP = [a \quad b]\begin{bmatrix} e \\ f \end{bmatrix}$

$= ae + bf$ ✓

RHS: $MN + MP$

$= ac + bd + ae + bf$

$= \boldsymbol{a(c + e) + b(d + f)}$ ✓

$= LHS$