Name: ....

Resource Free	/25	%
Resource Rich	~~/30~~ 25	%
Total	~~/55~~ 50	%

Mathematics Methods, Year 12, 2016
Test 2 – Further differentiation and applications, Integrals

25 minutes working time.
Calculator Free Section (no notes, no calculators)
SCSA Formula sheet allowed

1. (3 marks)

Given $f(x)=e^x$ and $h(x)=-2x$

(a) find $y=f(h(x))$. (1)

$y = f(-2x)$

$y = e^{-2x}$ ✓

(b) hence find the expression for $\frac{d}{dx}(f(h(x)))$. (2)

$\frac{d}{dx}(f(h(x))) = -2e^{-2x}$ ✓✓

2. (7 marks)

(a) Find the following. Simplify your answers.

(i) $\int \sqrt{2x+1}\,dx$ (2)

$= \frac{2(2x+1)^{\frac{3}{2}}}{3 \times 2} + c$ ✓

$= \frac{\sqrt{(2x+1)^3}}{3} + c$ ✓

(ii) $\int (6\sqrt{x}-e^{6x})\,dx$ (3)

$= \int (6x^{\frac{1}{2}} - e^{6x})\,dx$

$= 6 \times \frac{2}{3} x^{\frac{3}{2}} - \frac{e^{6x}}{6} + c$ ✓

$= 4\sqrt{x^3} - \frac{e^{6x}}{6} + c$ ✓✓

(or $4x^{\frac{3}{2}} - \frac{1}{6}e^{6x} + c$)

(b) Given $\frac{dy}{dx} = 2x + 3x^2 - x^{\frac{1}{2}}$ find the function $y = f(x)$ given the point $\left(1, -\frac{2}{3}\right)$ belongs to the function. (2)

$$y = \frac{2x^2}{2} + \frac{3x^3}{3} - \frac{2x^{\frac{3}{2}}}{3} + c \quad \checkmark$$

sub $(1, -\frac{2}{3})$

$$-\frac{2}{3} = 1 + 1 - \frac{2}{3} + c$$

$$c = -2 \qquad \therefore \; y = x^2 + x^3 - \frac{2x^{\frac{3}{2}}}{3} - 2 \quad \checkmark$$

3. (4 marks)

Find $\frac{dy}{dx}$ for each of the following

(a) $y = (x + e^{3x})^2$ (2)

$$\frac{dy}{dx} = 2(x + e^{3x}) \times (1 + 3e^{3x})$$

$\checkmark$ $\checkmark$

(b) $\int_x^2 (3 - t^2)\, dt$ (2)

$$= -\frac{d}{dx}\int_2^x (3 - t^2)\, dt \quad \checkmark$$

$$= x^2 - 3 \quad \checkmark$$

4. (5 marks)

(a) If $F(x) = x^3 - x^2$

(i) find $F'(x)$. $\quad F'(x) = 3x^2 - 2x$ $\checkmark$ (1)

(ii) hence simplify $\int_0^p F'(x)dx$ $= \left[x^3 - x^2\right]_0^p$ $\checkmark$ (2)

$$= p^3 - p^2 \quad \checkmark$$

(b) Given $F(x) = \int_1^x t^3\, dt$ find an expression for $F'(x)$. (2)

$$F'(x) = x^3 \quad \checkmark\checkmark$$

5. (6 marks)

Calculate the area bounded by the functions $f(x) = (x-3)^2 - 2$ and $g(x) = 4 - 2x$

Solve $f(x) = g(x)$

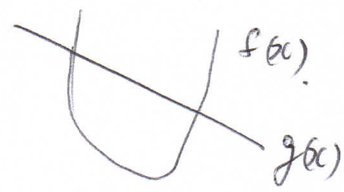
$$(x-3)^2 - 2 = 4 - 2x$$

$$x^2 - 6x + 9 - 2 = 4 - 2x$$

$$x^2 - 4x + 3 = 0$$

$\therefore \quad x = 3, \quad x = 1$

$$\begin{array}{cc} x & -3 \\ x & -1 \\ \hline -3x & -1x = -4x. \end{array}$$



$$\text{Area} = \int_1^3 g(x) - f(x) \, dx.$$

$$= \int_1^3 4 - 2x - (x-3)^2 + 2 \, dx.$$

$$= \int_1^3 4 - 2x - x^2 + 6x - 9 + 2 \, dx$$

$$= \int_1^3 -x^2 + 4x - 3 \, dx$$

$$= \left[-\frac{x^3}{3} + 2x^2 - 3x \right]_1^3$$

$$= \left(-\frac{3^3}{3} - 2(3)^2 - 3(3) \right) - \left(-\frac{1^3}{3} + 2(1)^2 - 3(1) \right)$$

$$= 0 - \left(-\frac{1}{3} + 2 - 3 \right).$$

$$= \underline{\underline{1\tfrac{1}{3}}} \text{ sq units}$$

END OF TEST

Name: ....

Mathematics Methods, Year 12, 2016

Test 2 – Further differentiation and applications, Integrals

Total	/~~30~~ 25	%

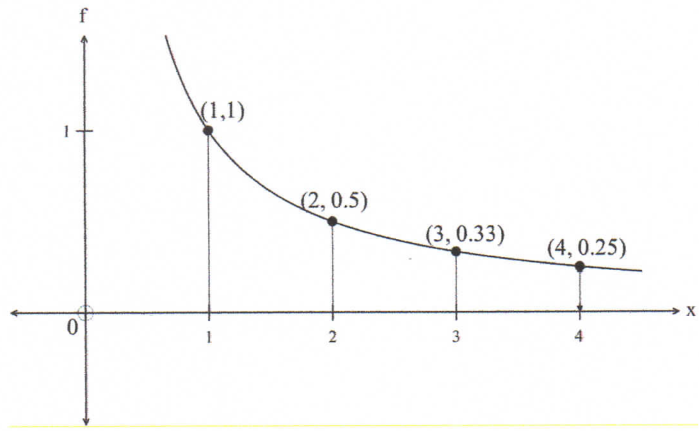
30 minutes working time.

Calculator Assumed Section (notes allowed)

SCSA Formula sheet and calculators allowed

1. (2 marks)

Consider the function $f(x) = \dfrac{1}{x}$ graphed below:



Estimate the area between the function $y = f(x)$, the x axis and $x = 1$ and $x = 4$ using rectangles from below and from above.

Estimate from below $= (1\times 0.5) + (1\times 0.33) + (1\times 0.25)$

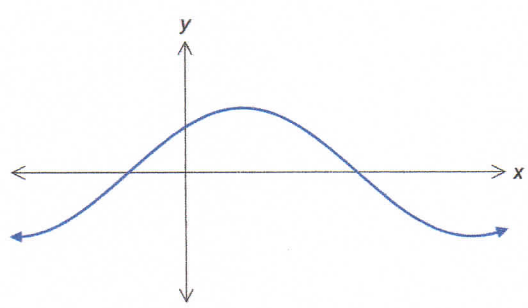
$= 1.08$ ✓

Estimate from Above $= (1\times 1) + (1\times 0.5) + (1\times 0.33)$

$= 1.83$ ✓

2. (5 marks)

The graph of $y = 3\sin\left(2x + \dfrac{\pi}{4}\right)$ is given below for $-1 \le x \le 2$.



(a) Determine the roots of the function $y = 3\sin\left(2x + \dfrac{\pi}{4}\right)$ for $-1 \le x \le 2$. (2)

$3\sin\left(2x + \frac{\pi}{4}\right) = 0$ $\therefore \sin\left(2x + \frac{\pi}{4}\right) = \sin(0)$ $\therefore 2x + \frac{\pi}{4} = 0$ $x = -\frac{\pi}{8}$ ✓

$\sin\left(2x + \frac{\pi}{4}\right) = \sin(\pi)$ OR $2x + \frac{\pi}{4} = \pi$ $x = \frac{3\pi}{8}$ ✓

(b) Calculate exact value for the area under the curve between the two roots. (3)

$A = \displaystyle\int_{-\frac{\pi}{8}}^{\frac{3\pi}{8}} 3\sin\left(2x + \frac{\pi}{4}\right) dx$ ✓✓✓

$A = \underline{\underline{3 \text{ sq}}}$ units.

3. (3 marks)

Given $f(x) = e^x \times sin(x)$

(a) find $f'(x)$. (1)

$f'(x) = \cos x\, e^x + \sin x\, e^x$

$f'(x) = e^x(\cos x + \sin x)$. ✓

(b) hence find $y = \int e^x(sin(x) + cos(x))dx$ given $y = 1$ when $x = 0$. (2)

$y = e^x \times \sin x + c$.

sub (0,1) $1 = e^0 \times \sin 0 + c$.

$1 = c$ ✓

$\therefore y = e^x \sin x + 1$ ✓

4. (6 marks)

Fuel was observed to leak from a damaged tank at a rate of $\frac{45}{e^{0.1t}}$. Litres per minute, where t is the number of minutes that have elapsed since the tank was ruptured.

(a) How much fuel leaked from the tank during the first two minutes? (to 2 decimal places) (2)

$= \int_0^2 \frac{45}{e^{0.1t}}\, dt$ ✓

$= 81.57$ L (2 d.p.) ✓

(b) If the tank initially contained 400 litres of fuel, determine the time taken, to the nearest second, for the tank to empty. (3)

$\int_0^t \frac{45}{e^{0.1t}}\, dt = 400$ ✓

$\int_0^t 45 e^{-0.1t}\, dt = 400$.

$\left[\frac{45 e^{-0.1t}}{-0.1}\right]_0^t = 400$. ✓

$\left[-450 e^{-0.1t} + 450\right] = 400$

$t = 21.97224$ ✓

$= 21$ m 58 s ✓

~~6~~.5. (9 marks)

The velocity of a body is $v = -3t + 6$ m/s and the initial displacement is $2\,m$.
Assume $t \geq 0$.

(a) Find the expression for the displacement. (2)

$$\int -3t + 6$$

$$x = \frac{-3t^2}{2} + 6t + c \checkmark$$

when $t = 0$ $x = 2$. $\therefore c = 2$

$$\therefore x = \frac{-3t^2}{2} + 6t + 2 \checkmark$$

(b) Find the expression for the acceleration. (1)

$$a = -3 \ m/s^2 \checkmark$$

(c) Find the <u>second</u> time when the displacement is equal to the initial displacement. (2)

Solve $\frac{-3t^2}{2} + 6t + 2 = 2 \checkmark$

$\therefore \frac{-3t^2}{2} + 6t = 0$

$\therefore$ Using calculator

$t = 0, \ t = 4$

$\therefore t = 4s \checkmark$

(d) Find the displacement when the body changes direction. (2)

Body changes direction when

$$-3t + 6 = 0$$
$$\therefore 3t = 6$$
$$t = 2 \checkmark$$

$$\therefore x = \frac{-3(2)^2}{2} + 6(2) + 2$$
$$= 8m \checkmark$$

(e) How far does the body travel in the first second after the body changes direction? (2)

when $t = 2$ $x = 8$

$t = 3$ $x = 6.5 \checkmark$

$$x = \frac{-3(3)^2}{2} + 6(3) + 2$$
$$= 6.5$$

$\therefore$ Distance travelled $= 1.5m \checkmark$

END OF TEST