



Name: _____

Full Test (Sections 1 and 2)

Total Time: 55 minutes

Total Marks: 43 marks

Student Result _____ / 49

MATHEMATICS METHODS Unit 4

TEST 4 -2022: Random Sampling and Sample Proportions

Calculator Free Section

Time: 18 minutes

Total Marks: _____ / 20 marks

Resources allowed: SCSA Formula Sheet

Instructions to candidates

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. **For any question or part question worth more than two marks, valid working or justification is required to receive full marks.** If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

Question 1

[2 marks]

Some children at a school in a large town are taking part in a project about magazines that are bought and read by the town's population. In order to obtain a sample of the town they decide to ask every child in the school to find out which magazines are regularly bought by his or her family.

Assuming a total response, would the sample be biased? Justify your answer.

✓ Yes, the sample is likely to be biased.

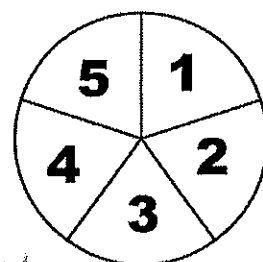
✓ Excludes people who buy magazines that don't have school aged children

Question 2

[4 marks]

Assume the rule that for normal distribution we would expect approximately 95% of the scores to be within 2 standard deviations of the means.

If we were to spin the spinner on the right, 96 times we would expect to get an odd number approximately 60% of the time.



Between what two values, situated symmetrically either side of the 60% long term average, we expect the proportion of odd numbers to lie for approximately 95% of the samples of size 96?

$$p = 60\% = 0.6$$

$$\sigma = \sqrt{\frac{0.6(1-0.6)}{96}} = \sqrt{\frac{0.24}{96}} = \sqrt{\frac{1}{400}} = \frac{1}{20}$$

95% of the samples of size 96 has to be between

$$0.6 \pm 2\sigma$$

$$0.6 \pm 0.1$$

$$\therefore 0.5 \leq \hat{p} \leq 0.7$$

Question 3**[2, 2 = 4 marks]**

The table below gives the number of students from a population of 540 belonging to various extracurricular clubs at a secondary school.

Clubs	Robotics	Sports	Art	Music	Debating	Drama
Boys	24	81	33	37	22	55
Girls	16	84	40	45	25	78

A survey is to be conducted looking at ways to best support students at the school belonging to these clubs.

Given that a stratified sample of size 60 is to be carried out,

- a) how many girls in the music club should be sampled?

$$\frac{45}{540} \times 60 = \frac{5}{60} \times 60 = 5 \text{ Girls}$$

- b) how many boys in the sports club should be sampled?

$$\begin{aligned} & \frac{81}{540} \times 60 \checkmark \\ &= \frac{9}{60} \times 60 \\ &= 9 \text{ Boys} \checkmark \end{aligned}$$

Question 4

[2, 2 = 4 marks]

A large sample is taken and a sample proportion of 0.74 is determined. A 95% confidence interval for the population proportion is calculated with an associated margin of error of 0.05.

- a) Determine the 95% confidence interval that was calculated.

$$0.74 \pm 0.05$$

$$= 0.69 \leq \hat{p} \leq 0.79$$

95% confidence interval for the population proportion is between 0.69 and 0.79.

- b) If the margin of error was deemed to be too large, what two considerations could be employed to reduce the margin of error associated with a confidence interval for the population proportion?

✓ increasing the sample size will reduce error
 ✓ Reducing the level of confidence for confidence interval

Question 5

[1, 2 = 5 marks]

Use the 68%, 95%, 99.7% rule to give approximate answers to the following. (2, 4 = 6 marks)

- a) If $\mu = 3$ and $\sigma = 2$, find

i) $P(X > 5) = 0.16$ ✓

ii) $P(X < 1 \mid X < 5) = \frac{0.16}{0.84}$ ✓

- b) If $X = 3$ has a z score of -2 and $X = 8$ has a z score of 2, find μ and σ

$$Z = \frac{X - \mu}{\sigma}$$

$$-2 = \frac{3 - \mu}{\sigma}$$

$$-2\sigma = 3 - \mu$$

$$-2\sigma + \mu = 3 \quad (1) \quad \checkmark$$

$$2 = \frac{8 - \mu}{\sigma}$$

$$2\sigma = 8 - \mu$$

$$2\sigma + \mu = 8 \quad (2) \quad \checkmark$$

$$2\sigma + \mu = 8$$

$$-2\sigma + \mu = 3$$

$$2\sigma = 11$$

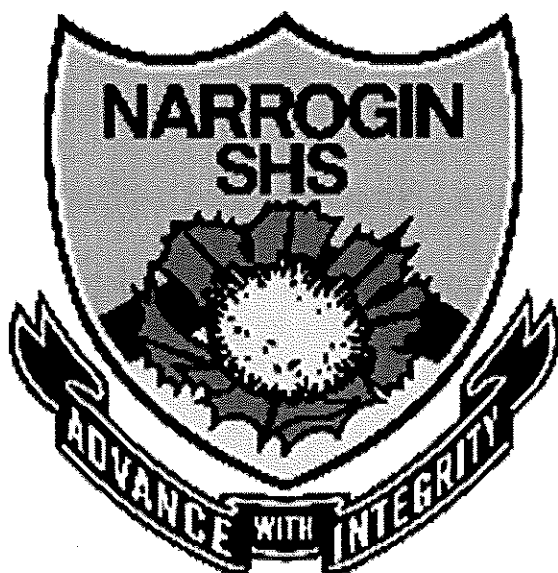
$$\checkmark \sigma = 5.5$$

$$2\sigma = 8 - 5.5$$

$$2\sigma = 2.5$$

$$\checkmark \sigma = 1.25$$

Name: _____



MATHEMATICS METHODS Unit 4

TEST 4 -2022: Random Sampling and Sample Proportions

Calculator Assumed Section

Time: 37 minutes

Total Marks: _____ / 29 marks

Resources allowed:

SCSA Formula Sheet

Up to three Calculators and

One A4 sheet, both sides of notes

Instructions to candidates

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. **For any question or part question worth more than two marks, valid working or justification is required to receive full marks.** If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

Question 5**[3 marks]**

Gypsy moth populations soar every few years in the North-eastern deciduous forests, causing great damage to the trees their larvae eat.

In order to determine the population of gypsy moths in a forest, 186 were trapped, marked, and released.

The next night, more moths were collected. Of the 234 that were collected, 17 were already marked. What is an estimate for the size of the population of gypsy moths in the forest?

$$\frac{186}{N} = \frac{17}{234}$$

$$N = \frac{17}{234} \times 186$$

$$\approx 2560.235$$

∴ There would be approximately
2560 gypsy moths. ✓

Question 6**[1, 3 = 4 marks]**

A mechanical component from a machine has its probability of a breakdown before t years given by:

$$P(\text{breakdown} < t) = \int_0^t 0.2e^{-0.2x} dx$$

- a) Determine the probability the mechanical component will breakdown before 5 years.

$$P(\text{breakdown} < 5) = \int_0^5 0.2e^{-0.2x} dx \\ = 0.632 \quad \checkmark$$

A sample of 50 of these mechanical components were put into operation until they each broke-down.

- b) Determine, correct to 4 decimal places, the probability that the sample proportion of components lasting longer than 5 years is at least a half.

$$p = 1 - 0.632 \Rightarrow 0.368 \quad \checkmark$$

$$\hat{p} \geq 0.5$$

$$\sigma = \sqrt{\frac{0.368 \times (1 - 0.368)}{50}} = 0.0682 \quad \checkmark$$

$$\text{normcdf}(0.5, \infty, 0.0682, 0.368)$$

$$= 0.026465$$

$$P(\hat{p} \geq 0.5) = 0.0265 \quad \checkmark$$

Question 7

[1, 1, 3, 2 = 7 marks]

The census for a particular country showed that 80% of the people who classified their occupation as chef or head cook were men.

According to a recent survey of 240 people who classified their occupation as chef / head cook, 200 were men and the rest women.

- a) Determine p , the population proportion of women who identify as chef / head cooks.

$$p = 0.2 \checkmark$$

- b) Determine the value of \hat{p} , the sample proportion of woman for the recent survey of 240 people mentioned. Correct to 4 decimal places.

$$\hat{p} = \frac{40}{240} = 0.1667 \checkmark$$

- c) Describe the sampling distribution of sample proportions of size 240 of people classifying their occupation as chef / head cook being women, stating its mean and standard deviation.

Sample is large enough, \therefore the \hat{p} is normally distributed.

$$\text{mean of } \hat{p} = p = 0.2 \checkmark$$

$$\sigma = \sqrt{\frac{0.2(1-0.2)}{240}} = 0.0258 \checkmark$$

- d) How many standard deviations from p is our value for \hat{p} ?

$$Z = \frac{0.1667 - 0.2}{0.0258} = -1.29 \checkmark$$

\therefore 1.29 std. deviations below the mean

Question 8**[3, 2 = 5 marks]**

Suppose a consumer advocacy group would like to conduct a survey to find the proportion of consumers who bought the newest generation iphone were happy with their purchase.

The advocacy group took a random sample of 880 consumers who recently purchased this iphone and found that 748 were happy with their purchase.

- a) Find a 95% confidence interval for the proportion of consumers who have purchased this iphone and were happy with their purchase.

$$\hat{p} = \frac{748}{880} = 0.85 \quad \checkmark$$

$$s = \sqrt{\frac{0.85(1-0.85)}{880}} = 0.012037 \quad \checkmark$$

$$0.85 \pm (1.96 \times 0.012037)$$

95% C.I. for the population proportion is
between 0.8826 and 0.8736

Apple, the iphone company, claims that "...no less than 90% of people who purchase their iphone product, love it."

- b) Based on the survey conducted, what is the likelihood of this claim being plausible? Calculate the probability a sample proportion having a value of at least 90% and comment on the result.

$$Z = \frac{0.90 - 0.85}{0.012037} \approx 4.15$$

$$P(Z > 4.15) \approx 0.000016 \quad \checkmark$$

OR

$$\text{normcdf}(0, 9, \infty, 0.012037, 0.85)$$

$$0.00001634$$

The claim by Apple is extremely unlikely based on the survey evidence. \checkmark

Question 9**[2, 2, 2, 4 = 10 marks]**

A random selection of 1650 Australian adults across the nation found that 363 have a particular disease.

Determine the following

- a) A 90% confidence interval, to three significant figures, for the proportion of Australian adults who have this particular disease.

$$\hat{p} = \frac{363}{1650} = 0.22$$

$$\sigma = \sqrt{\frac{0.22 \times (1 - 0.22)}{1650}} = 0.0102 \quad \checkmark$$

$$0.22 \pm (1.645 \times 0.0102)$$

$$0.203 \leq p \leq 0.237 \quad \checkmark$$

- b) The margin of error in this confidence interval, to three significant figures.

$$\begin{aligned} M.E &= 1.645 \times 0.0102 \\ &= 0.0168 \end{aligned}$$

- c) The margin of error is deemed to be too large for the original survey conducted. Determine the minimum sample size required for a 90% confidence interval with a margin of error to be no more than 0.01 for the proportion of adult Australians with this particular disease.

$$0.01 = 1.645 \sqrt{\frac{0.22(1-0.22)}{n}}$$

$$n = 4643.5389$$

Minimum Sample Size required is 4644.

Another three surveys of adults were taken, two more across Australia and one across an Asian country.

Survey 2	Survey 3	Survey 4
Having the disease 543 out of 2340 adults	Having the disease 332 out of 1845 adults	Having the disease 178 out of 896 adults

- d) Determine which of these surveys were more likely to have been taken from the Asian country? Justify your answer.

Survey 2: $\hat{p} = \frac{543}{2340} = 0.2321$

$$\sigma = \sqrt{\frac{0.2321 \times (1 - 0.2321)}{2340}} = 0.008727$$

90% C.I.

✓ $0.2321 \pm (1.645 \times 0.008727)$

90% C.I. interval is 0.221 to 0.243.

Survey 3: $\hat{p} = \frac{332}{1845} = 0.1799$

✓ $\sigma = \sqrt{\frac{0.1799 \times (1 - 0.1799)}{1845}} = 0.008943$

90% C.I.

$0.1799 \pm (1.645 \times 0.008943)$

90% C.I. is between 0.168 to 0.191.

Survey 4: $\frac{178}{896} = 0.1987$

✓ $\sigma = \sqrt{\frac{0.1987 \times (1 - 0.1987)}{896}} = 0.0133$

90% C.I.

$0.1987 \pm (1.645 \times 0.0133)$

90% C.I. is between 0.181 and 0.215

✓ Survey 2 is likely to be from the Asian country.