Yr 12 METHODS TEST 3 2018

CALCULUS INVOLVING EXPONENTIALS AND LOGARITHMS

Time: 25 minutes **Total: 24 marks**

Student Name: Solns.

Teacher: ____________

Instructions: *Show all working clearly.*
Sufficient detail must be shown for marks to be awarded for reasoning.
NO CALCULATOR AND NO PERSONAL NOTES ALLOWED

Question 1. **(7 marks)**

Determine an exact value for each of the following

a) The gradient of $y = \frac{x^2}{e^x+1}$ at the point $(2, \frac{4}{e^2+1})$ [4]

$$y' = \frac{(e^x+1)(2x) - x^2(e^x)}{(e^x+1)^2}$$

$$= \frac{(e^2+1)(4) - 4(e^2)}{(e^2+1)^2}$$

$$= \frac{4}{(e^2+1)^2}$$

b) $\int_0^4 \frac{4x+6}{x^2+3x+1} dx$ [3]

$$2\int_0^4 \frac{2x+3}{x^2+3x+1} dx$$

$$\left[2\ln|x^2+3x+1|\right]_0^4$$

$$= 2\ln|16+12+1| + 2\ln 1$$

$$= 2\ln 29.$$

Question 2. **(7 marks)**

a) i) Determine $\frac{dy}{dx}$ if $y = \ln \frac{x^2+3}{x-8}$ [2]

$y = \ln(x^2+3) - \ln(x-8)$

$y' = \frac{2x}{x^2+3} - \frac{1}{x-8}$

ii) given $\frac{dy}{dx}$ can be written in the form $\frac{ax^2+bx+c}{(x^2+3)(x-8)}$, determine the values of a, b and c. [3]

$\frac{2x(x-8) - (x^2+3)}{(x^2+3)(x-8)}$

$= \frac{2x^2 - 16x - x^2 - 3}{(x^2+3)(x-8)}$

$= \frac{x^2 - 16x - 3}{(x^2+3)(x-8)}$

$a = 1$

$b = -16$

$c = -3$

b) Differentiate $y = [\ln(3x-4)]^4$, do not simplify your answer [2]

$4[\ln(3x-4)]^3 \times \left[\frac{3}{3x-4}\right]$

Question 3. **(6 marks)**

A straight tunnel is going to be hollowed out of a large hill. The height, $h(x)$, of the tunnel at any distance, x metres, from the left hand edge can be modelled by the relationship;

$$h(x) = 6\sin(3x) \quad , \quad 0 \leq x \leq k$$

a) If the x axis represents ground level, determine the value of k in radians. [1]

$$k = \frac{2\pi}{3} \div 2$$

$$= \frac{\pi}{3}$$

b) Showing use of integration, determine the area of the tunnel's opening. [3]

$$\int_0^{\pi/3} 6\sin(3x)$$

$$= \left[-2\cos(3x)\right]_0^{\pi/3}$$

$$= \left[-2\cos\pi\right] - \left[-2\cos(0)\right] = 4m^2$$

$$= 2 \quad - [-2]$$

c) It is planned that 3 identical tunnels are to be cut from the same hill. If all 3 tunnels are 450m long, determine the amount of earth that will need to be removed to create the tunnels. [2]

$$4 \times 3 \times 450$$

$$= 5400m^3$$

Question 4. (4 marks)

Determine the exact value of $\int_0^{\frac{\pi}{3}} -\sin x \cos x \, dx$

$$\left[\frac{-\cancel{\sin x}\,(\cos x)^2}{-\cancel{\sin x} \times 2} \right]_0^{\pi/3}$$

$$\left[\frac{(\cos x)^2}{2} \right]_0^{\pi/3}$$

$$\frac{\left(\cos \frac{\pi}{3}\right)^2}{2} - \frac{(\cos 0)^2}{2}$$

$$= \frac{1}{8} - \frac{1}{2}$$

$$= -\frac{3}{8}.$$

--------END OF SECTION A--------

Yr 12 METHODS TEST 3 2018

CALCULUS INVOLVING EXPONENTIALS AND LOGARITHMS

Time: 30 minutes | Total: 26 marks

Student Name: Solutions

Teacher: ____________

Instructions: *Show all working clearly.*
Sufficient detail must be shown for marks to be awarded for reasoning.
CALCULATOR AND 1 PAGE OF PERSONAL NOTES ALLOWED

Question 5. (8 marks)

A group of biologists has decided that colonies of a native Australian animal are in danger if their populations are less than 1000. One such colony had a population of 2300 at the start of 2011. The population was growing continuously such that $\frac{dP}{dt} = 0.065P$, where P is the number of animals in the colony t years after the start of 2011.

a) Determine, to the nearest 10 animals, the population of the colony at the start of 2014. [2]

$P = 2300e^{0.065t}$

$P = 2300e^{0.065(3)}$

$= 2795.22$

$\therefore$ 2800 Animals.

b) Determine the rate of change of the colony's population when $t = 2.5$ [2]

$\frac{dP}{dt} = 149.5e^{0.065(2.5)}$

$= 175.88$ animals/year.

c) At the beginning of 2017 a disease caused the colony's population to decrease continuously at the rate of 8.25% of the population per year. If this rate continues, when will the colony become 'in danger'? Give your answer to the nearest month. [4]

$P = P_0 e^{-0.0825t}$

$P_0 = 2300e^{0.065(6)}$

$P = 3397e^{-0.0825t}$

$= 3397.06$

$1000 = 3397e^{-0.0825t}$

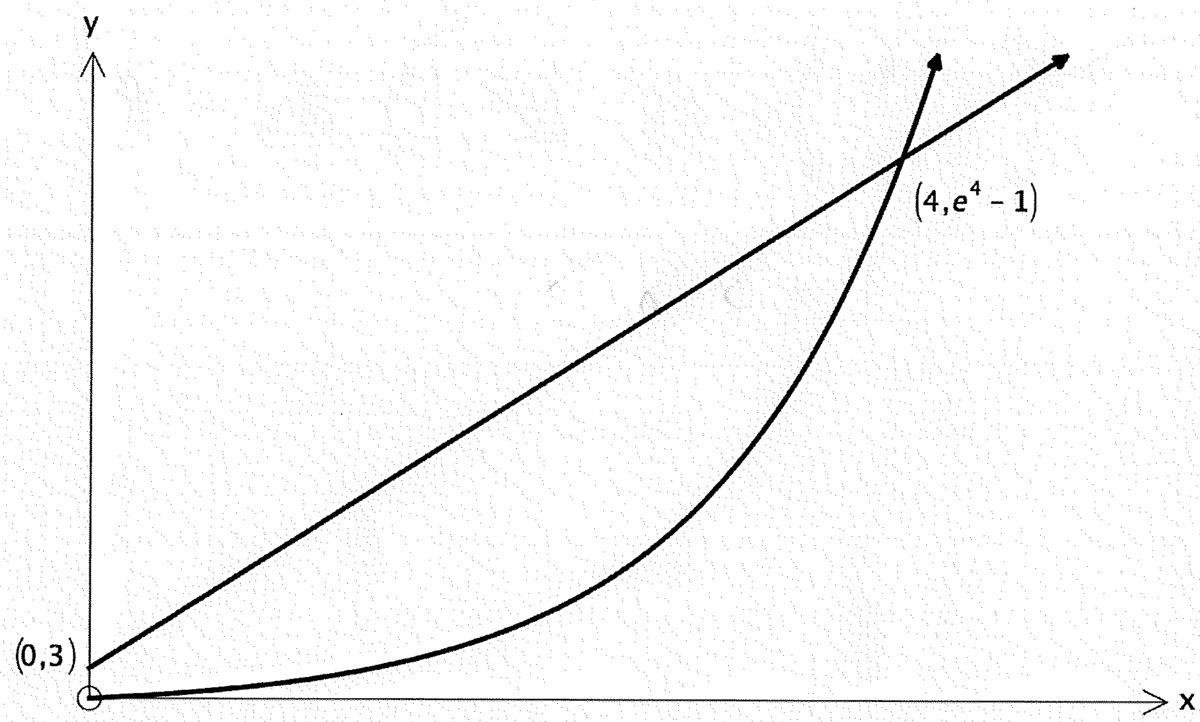
$t = 14.822$

$\therefore$ during Sept 2031

or at start of Oct 2031.

Question 6. (8 marks)

Consider the graph of two functions below. The curve can be described by $g(x) = e^x - 1$.



a) Show that the gradient of the line is $\frac{e^4}{4} - 1$. [2]

$$\frac{e^4 - 1 - 3}{4 - 0}$$

$$= \frac{e^4 - 4}{4}$$

$$= \frac{e^4}{4} - 1.$$

other techniques possible.

b) Write an integral that could be used to determine the area enclosed by the two graphs and the y axis. [3]
Note: there is no need to calculate this integral.

$$\int_0^4 \left(\frac{e^4}{4} - 1\right)x + 3 - (e^x - 1) \, . dx$$

c) A vertical line is added to the graph with equation $x = k$, given $0 \leq k \leq 4$. If the area bounded by this line and the two graphs is 8 square units, determine the value of k.

[3]

$$\int_k^4 \left(\frac{e^4-1}{4}\right)x + 3 - (e^x - 1)\,.\,dx = 8$$

$$k = 3\cdot 28$$

Question 7. (10 marks)

A particle travels in a straight line back and forth past a fixed origin such that it's displacement in metres at time t seconds is given by;

$$x(t) = 2 + 3\sin 2x$$

a) What is the maximum distance the particle gets from the origin? [1]

5 metres.

b) Determine how many times the particle crosses the origin in the first 2π seconds. [2]

4 times.

c) Showing use of calculus; [4]

i) determine the velocity of the particle at $t = 10$.

$v(t) = 6\cos 2x$

$v(10) = 6\cos 20$

$= 2.448$ m/sec.

ii) determine the distance travelled by the particle in the first 80 seconds.

$\int_0^{80} |6\cos 2x| \, dx = 305.34$m.

d) if $\int_0^{\pi} x(t)dt = k$, determine $\int_0^{10\pi} [x(t) + 3]dt$ in terms of k. [3]

$10k + 30\pi$.