Year 12 Physics	TEST # 5 – Special Relativity	2022

NAME: ....SOLUTIONS........................

Total Marks: 36 *Time Allowed: 45 minutes*

(Formula sheet and scientific calculator permitted)

Question 1 **(8 marks)**

(a) In a Christmas parade, elves on the back of a float moving at 9.00 ms^{-1} East are tossing a ball around. Pepper throws the ball at 7.00 ms^{-1}

(i) in the direction of the float's movement,
(ii) in the opposite direction to the float's movement.

Determine the velocity of the ball relative to a spectator watching the parade pass by in each case. [2]

(i) $16.00\ ms^{-1}$ East ✓
(ii) $2.00\ ms^{-1}$ East ✓

2

(b) In a futuristic parade, the float speed is 0.900 c and Pepper throws the ball at 0.700 c. Pepper throws the ball in the same two directions as in part (a).

Determine the velocity of the ball relative to a spectator watching the parade pass by in each case. [6]

(i)
$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$
$$= \frac{0.9c + 0.7c}{1 + 0.9 \times 0.7} \quad ✓✓$$
$$\approx \underline{0.982\,c \text{ East}} \quad ✓$$

3

(ii)
$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$
$$= \frac{0.9c + (-0.7c)}{1 + (0.9)(-0.7)} \quad ✓✓$$
$$\approx \underline{0.541\,c \text{ East}} \quad ✓$$

3

8

Question 2 **(5 marks)**

Answer T (true) or F (false) for each of the following statements.

(a) The amount of kinetic energy of an object measured in an inertial frame is the same as in the frame of a stationary observer. [1]

F

(b) The speed of light measured in an inertial frame is the same as in the frame of a stationary observer. [1]

T

(c) The speed of a moving ball measured in an inertial frame is the same as in the frame of a stationary observer. [1]

F

(d) The momentum of a moving ball measured in an inertial frame is the same as in the frame of a stationary observer. [1]

F

(e) The distance to a given destination measured in an inertial frame is the same as in the frame of a stationary observer. [1]

F

5

5

Question 3 **(6 marks)**

A proton is accelerated from rest to a speed of 0.970 c in a linear accelerator by a large voltage. What voltage is required?

$$E_K = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 \quad ✓$$

$$= mc^2\left(\frac{1}{\sqrt{1-0.97^2}} - 1\right) \quad ✓$$

$$\approx 3.11345\ mc^2 \quad ✓$$

$$= 3.11345 \times 1.67 \times 10^{-27} \times (3\times10^8)^2$$

$$\approx 4.6795 \times 10^{-10}\ \text{J} \quad ✓$$

Now $E_K = \Delta V q$

$$\therefore \Delta V = \frac{E_K}{q}$$

$$= \frac{4.6795\times10^{-10}}{1.6\times10^{-19}} \quad ✓$$

$$\approx \underline{2.92\times10^{9}\ \text{V}} \quad ✓$$

6

6/6

Question 4 **(12 marks)**

Two spaceships travelling at the same speed approach each other at a relative speed of 0.860 c, as measured by a stationary observer.

(a) Show that the common speed of the two spaceships, as measured by the stationary observer, is approximately 0.569 c. [4]

Let $u = xc, \quad v = -xc$

Then $u' = \dfrac{u - v}{1 - \frac{uv}{c^2}}$

i.e $0.86\,c = \dfrac{xc - (-xc)}{1 - x(-x)}$ ✓✓

$= \dfrac{2xc}{1+x^2}$

$\therefore \dfrac{2x}{1+x^2} = 0.86$

$2x = 0.86 + 0.86x^2$ ✓

$0.86x^2 - 2x + 0.86 = 0$

$\therefore x = \dfrac{2 \pm \sqrt{4 - 4 \times 0.86^2}}{1.72}$

$\therefore x \approx 0.569$ ✓ (reject $x = 1.76$)

i.e speed is $0.569\,c$.

4

4

The length of each spaceship is 28.0 m in its own frame.

(b) What is the length of one of the ships, as measured by

(i) the stationary observer? [2]

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$
$$= 28\sqrt{1-0.569^2} \checkmark$$
$$\simeq \underline{23.0\text{ m}} \checkmark$$

2

(ii) the other ship? [4]

Don't need this!

$$u' = \frac{u-v}{1-\frac{uv}{c^2}}$$
$$= \frac{0.569c - (-0.569c)}{1-(0.569)(-0.569)} \checkmark$$
$$= \frac{1.138c}{1+0.569^2} \simeq 0.860c \checkmark$$

$$\therefore l = l_0\sqrt{1-\frac{v^2}{c^2}}$$
$$= 28\sqrt{1-0.860^2} \checkmark$$
$$\simeq \underline{14.3\text{ m}} \checkmark$$

4

A period of 60.0 minutes elapses, as measured on one spaceship.

(c) What time has elapsed, as measured by the stationary observer? [2]

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$= \frac{60\text{ mins}}{\sqrt{1-0.569^2}} \checkmark$$
$$\simeq \underline{73.0\text{ mins}} \checkmark$$

2

/8

Question 5 **(5 marks)**

One of the experimental verifications of Einstein's Theory of Special relativity is the momentum change in accelerated high-speed electrons.

To what final speed must an electron initially travelling at 0.600 c be accelerated in order to achieve a 50.0 % increase in momentum?

New momentum = 1·5 × Old momentum ✓

$$\frac{\cancel{m}v}{\sqrt{1-\frac{v^2}{c^2}}} = 1{\cdot}5 \times \frac{\cancel{m} \times 0{\cdot}6c}{\sqrt{1-0{\cdot}6^2}}$$ ✓

(where v is new speed)

$$\frac{v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{0{\cdot}9c}{0{\cdot}8}$$ ✓

$$v = 1{\cdot}125\sqrt{1-\frac{v^2}{c^2}}\ c$$

$$v^2 = 1{\cdot}265\,625\left(1-\frac{v^2}{c^2}\right)c^2$$ ✓

$$v^2 = 1{\cdot}265\,625\,c^2 - 1{\cdot}265\,625\,v^2$$

$$2{\cdot}265\,625\,v^2 = 1{\cdot}265\,625\,c^2$$

$$\therefore v \approx \underline{0{\cdot}747c}$$ ✓

5

- End of Questions -

5