

Full Name: SOLUTIONS



MATHEMATICS
Applications Units 3 & 4

Test 3 – Graphs
Chapter 5

Semester 1 2018

Section One – Calculator Free

Time allowed for this section

Working time for this section: 25 minutes
Marks available: 21 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

Important note to candidates

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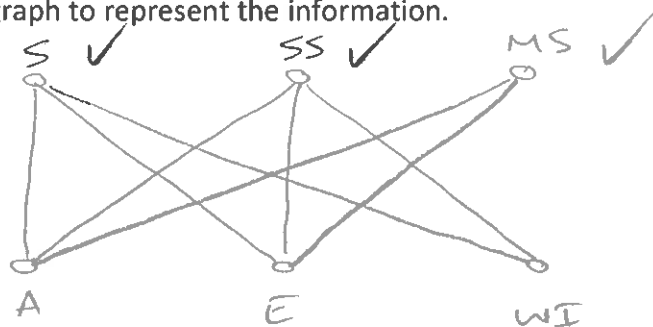
1. (5 marks)

The table shows whether some Big Bash clubs and some National Test teams share players.

	Scorchers	Sydney Sixers	Melbourne Stars
Australia	Yes	Yes	Yes
England	Yes	Yes	Yes
West Indies	Yes	Yes	No

a) Draw a bipartite graph to represent the information.

[3]



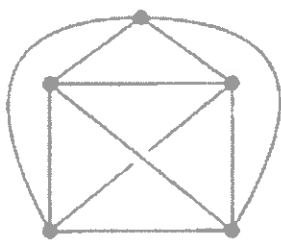
b) Is this a complete bipartite graph? Explain.

[2]

No, because MS is not connected to WI. To be complete each team must be connected to each country.

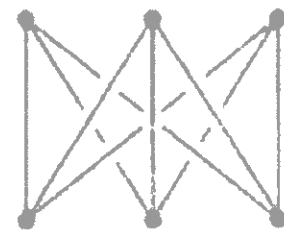
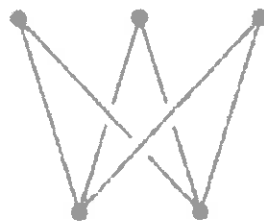
2. (4 marks)

Decide whether each graph is planar or non-planar. Where possible, draw a planar representation of the graph.



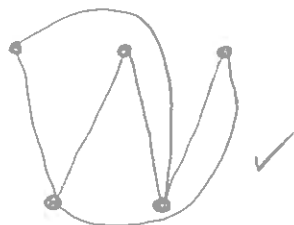
non-planar

✓



non-planar

✓



planar ✓

3. (9 marks)

Each of these networks contains a Semi-Eulerian path.

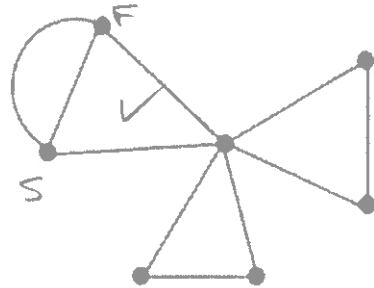
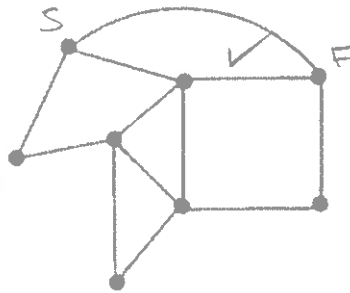
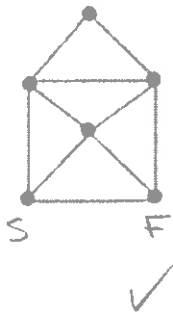
a) Define 'a Semi-Eulerian path'.

[1]

A path starting at one vertex, traveling along each edge and finishing at a different vertex ✓

b) Show a starting point, and a finishing point for each Semi-Eulerian path.

[3]



c) Complete this statement:

[1]

If ~~of~~ a network with two odd vertices is traversed, one odd vertex will be the starting point and the other odd vertex is the finishing point. ✓

d) Which of the networks given contain a Hamiltonian cycle?

[1]

The left network and the middle network ✓

e) Verify that Euler's rule applies to each of the given networks.

[3]

$$V + F - E = 2$$

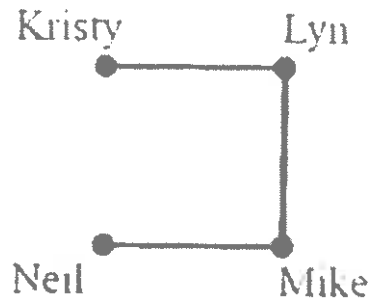
left $6 + 6 - 10 = 2$ True ✓

middle $8 + 6 - 12 = 2$ True ✓

right $7 + 5 - 10 = 2$ True ✓

4. (3 mark)

The members of one team are Kristy (K), Lyn (L), Mike (M) and Neil (N). In one of the challenges, these four team members are only allowed to communicate directly with each other as indicated by the edges of the following network.



The adjacency matrix below also shows the allowed lines of communication.

$$\begin{array}{c}
 \begin{array}{cccc}
 K & L & M & N \\
 \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & f & 0 & 1 \\ 0 & g & 1 & 0 \end{bmatrix} & \begin{array}{l} K \\ L \\ M \\ N \end{array}
 \end{array}
 \end{array}$$

a) Explain the meaning of the zero in the adjacency matrix.

[1]

It shows those who cannot communicate with each other ✓

b) Write down the values of f and g in the adjacency matrix.

[2]

$$\begin{array}{l}
 f = 1 \quad \checkmark \quad M_L = L_M \\
 g = 0 \quad \checkmark \quad N_L = L_N
 \end{array}
 \quad \text{symmetry}$$

End of Section One

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Full Name: SOLUTIONS



MATHEMATICS

Applications Units 3 & 4

Test 3 – Graphs

Chapter 5

Semester 1 2018

Section One – Calculator Free

Time allowed for this section

Working time for this section: 30 minutes
Marks available: 29 marks

Material required/recommended for this section

To be provided by the supervisor

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Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

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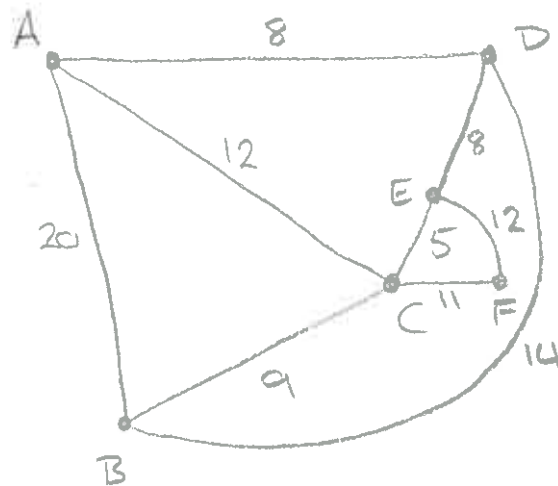
1. (10 marks)

The following table gives costs (in \$1000) of gas between country centres A to F.

	A	B	C	D	E	F
A	-	20	12	8	-	-
B		-	9	14	-	-
C			-	-	5	11
D				-	8	-
E					-	12
F						-

a) Draw the network.

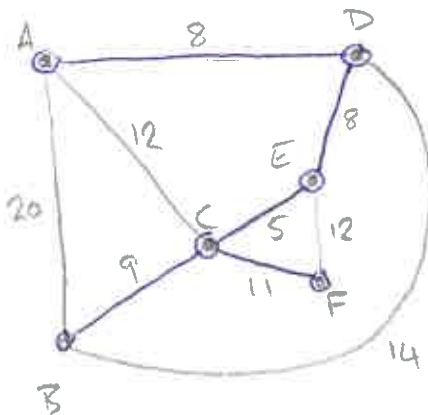
[3]



(-1 per error)
Verify each
given solution

b) Find the minimum cost of linking all towns to gas. Highlight the connections on the network.

[3]



$$8 + 8 + 5 + 11 + 9 = 41 \quad \checkmark \text{ total}$$

$$\therefore \$41\,000 \quad \checkmark \text{ context}$$

✓ spanning tree

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c) Verify the minimum cost by using Prim's Algorithm on the table.

[3]

	A	B	C	D	E	F
A	-	20	12	8	-	-
B	20	-	9	14	-	-
C	12	9	-	-	5	11
D	8	14	-	-	8	-
E	-	-	5	8	-	12
F	-	-	11	-	12	-

$$8 + 8 + 5 + 9 + 11 = 41$$

cost \$41 000

verified ✓

process ✓

The most expensive connection in the planned tree was not made due to a lack of funding.

d) Which town was left unconnected?

[1]

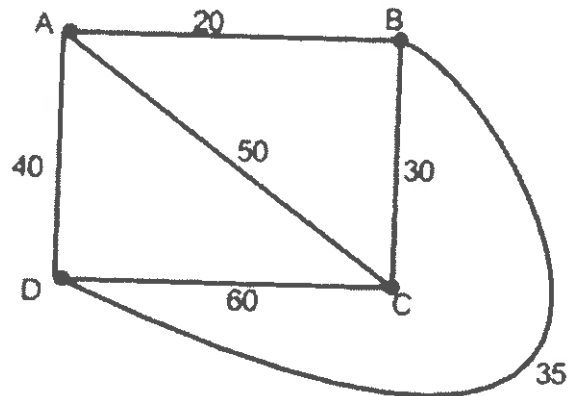
most expensive is 11

∴ town F left unconnected ✓

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2. (7 marks)

Before telephones, carrier pigeons were used to send messages. The following network shows four towns and the travel routes, in km.



A pigeon at town A flies directly to town B, then returns directly to town A. It then flies directly to C and directly back to A. It then flies directly to D and directly back to A.

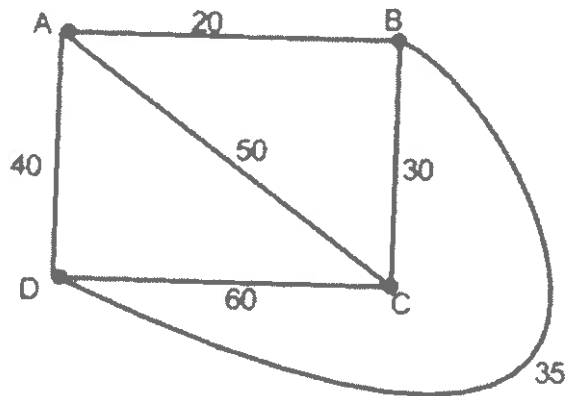
- a) How long does it take for the pigeon to complete this journey if it flies at an average speed of 40kph? [2]

$$\frac{40 + 100 + 80}{40} = 5.5 \text{ hours}$$

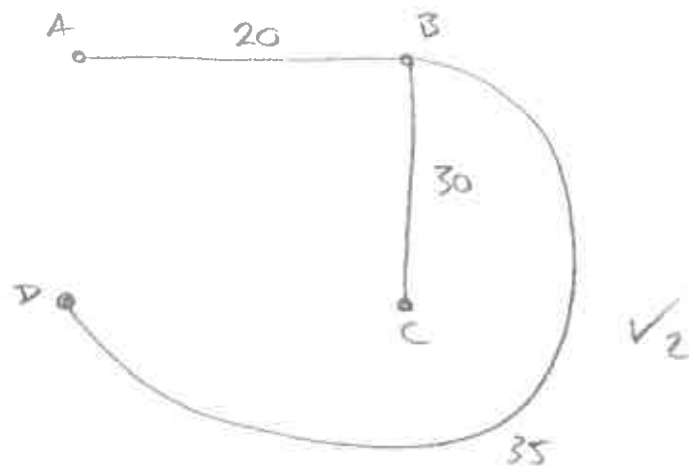
- b) Determine the minimum flying distance to fly from A to visit all the other towns, if the pigeon only returns to A after visiting the final town in its journey. [2]

$$20 + 30 + 60 + 40 = 150 \text{ km}$$

Once telephones were invented, lines were laid to connect towns together.



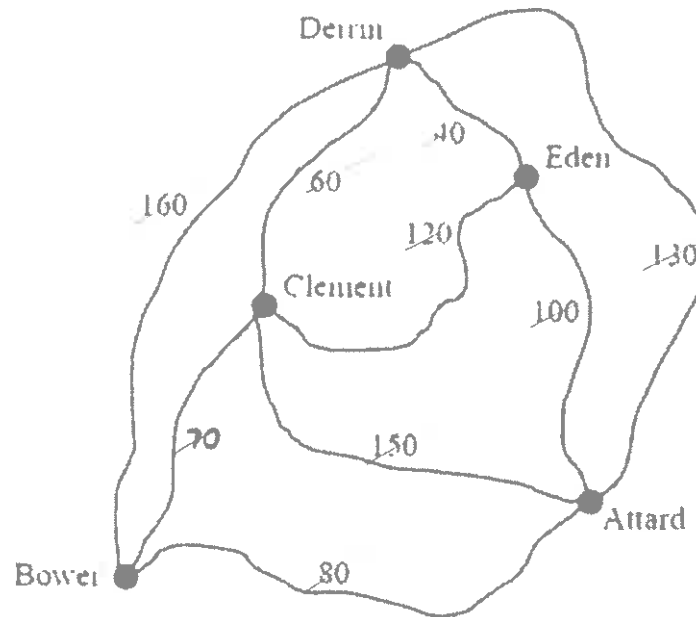
- c) State the minimum length of cable needed to connect the four towns and clearly show this connection of the diagram above. [3]



85 km of cable needed ✓

3. (5 marks)

The diagram below shows a network of train lines between five towns: Attard, Bower, Clement, Derrin and Eden. The numbers indicate the distances, in km, that are travelled by train between connected towns.



Charlie followed a Eulerian path through this network of train lines.

a) Write down the names of the towns at the start and at the end of Charlie's path. [2]

Bower and Eden
✓ ✓

b) What distance did he travel?

[1]

910 km ✓

Brianna will follow a Hamiltonian path from Bower to Attard.

c) What is the shortest distance that she can travel?

[1]

270 km ✓ (B → C → D → E → A)

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The train line between Derrin and Eden will be removed. If one other train line is removed from the network, Andrew would be able to follow a Eulerian circuit through the network of train lines.

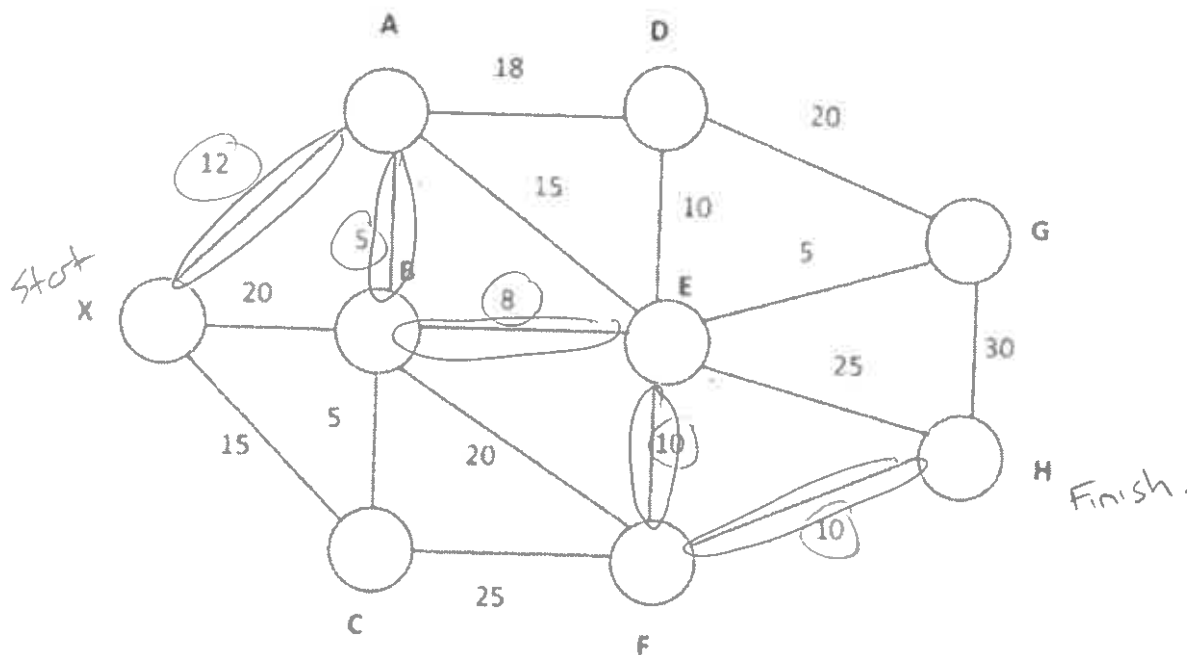
d) Which other train line should be removed?

[1]

Bower → Derrin ✓

4. (7 marks)

The network below represents the road transport network for a product distributor. The network consists of one distribution centre X and eight retail outlets A, B, C, ..., H. The number on each edge represents the distance, in km.



a) Determine and state the shortest route from the distribution centre X to H. You must demonstrate reasoning/process. [3]

X → A → B → E → F → H ✓

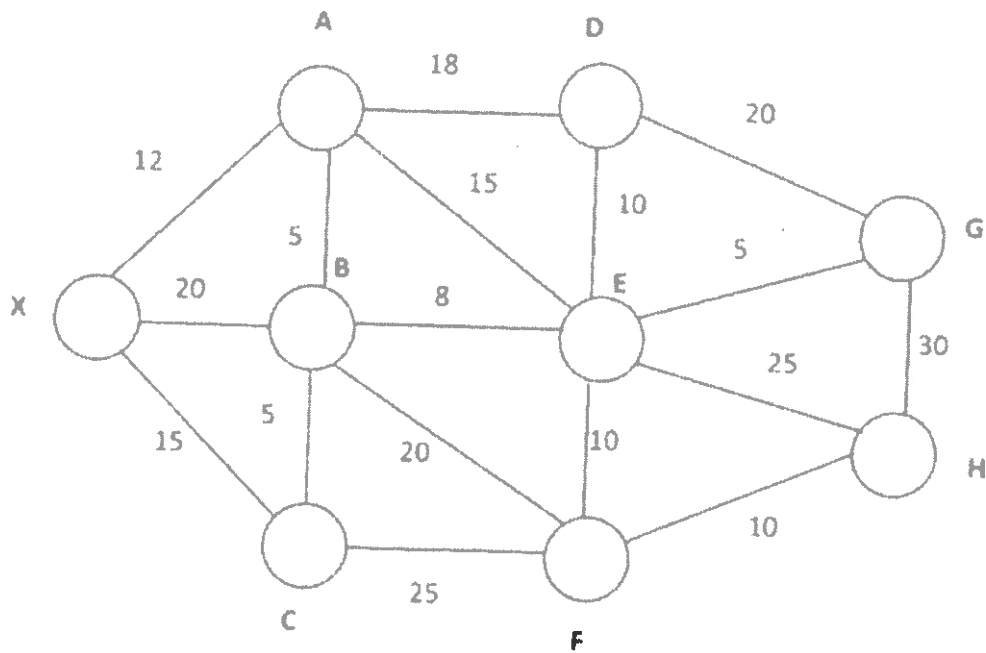
② reasoning/process

4

b) State the distance from the route found in a).

[1]

45 km ✓



c) The edge GH currently goes around a creek. A bridge is being proposed that can reduce the distance between G and H by x km. For what value(s) of x will the shortest route from X to H definitely use this bridge? Justify your answer.

[3]

Changes shortest route.

X - A - B - E - G - H ✓

this results in GH having to be less than 15 km. ✓

① reasoning

End of Test

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