MINDARIE
SENIOR COLLEGE
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MATHEMATICS: SPECIALIST 1 & 2

SEMESTER 2 2017

TEST 6

Reading Time: 4 minutes
Time Allowed: 51 minutes

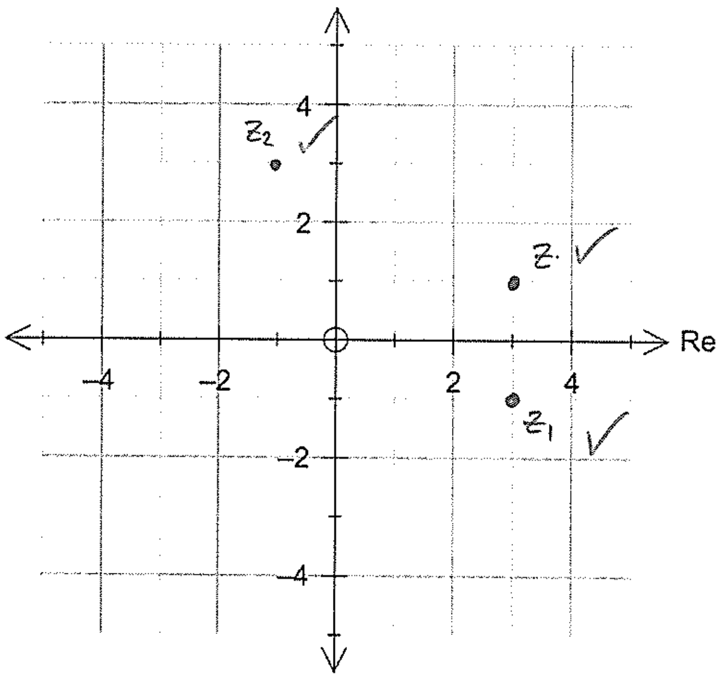
Total Marks: 51

1. [1, 1, 1 marks]

Given that $z = 3 + i$, plot (and label) the following points on the Argand diagram below.

(a) z (b) $z_1 = \bar{z}$ (c) $z_2 = iz$

$3i - 1$



2. [3 marks]

The transformation matrix $\begin{bmatrix} k & 2 \\ 5 & 3 \end{bmatrix}$ is such that it doubles the area of any shape that it transforms.

Determine the value(s) of k.

$3k - 10 = \pm 2$

$3k = 10 \pm 2$

$= 12$ or 8

$k = 4$ or $-8/3$

Δ ✓

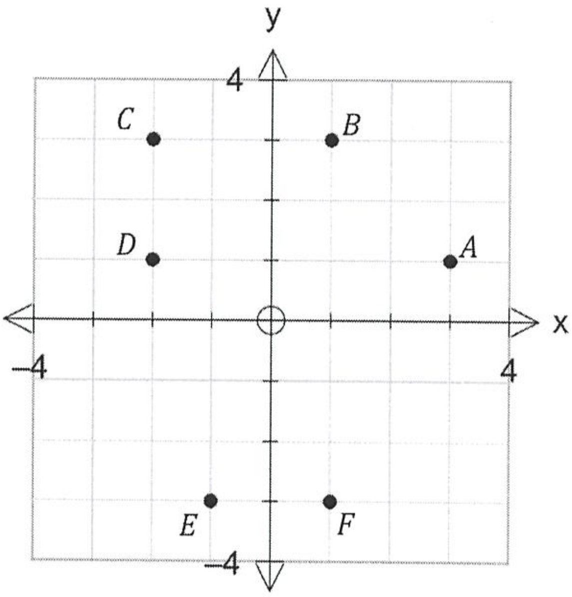
$k=4$ ✓

$k = -8/3$ ✓

3. [4 marks]

The Argand diagram on the right shows six complex numbers, A, B, C, D, E and F. Four complex numbers, Z_1, Z_2, Z_3 and Z_4 have the following information known about them.

- $Re(Z_1) = Im(Z_2)$
- $Z_3 = \overline{Z_4}$
- $Z_1 = iZ_3$



Complete the table below by allocating one of A, B, C, D, E and F to each of the complex numbers.

Complex Number	Z_1	Z_2	Z_3	Z_4
Point	A	C	F	B

✓ ✓ ✓ ✓

4. [4 marks]

The points $O(0,0), A(3,1), B(5,2)$ and $C(2,2)$ are transformed to the points $O'(0,0), A'(-5,8), B'(-8,13)$ and $C'(-2,4)$ by the matrix M. Determine M.

$$M\begin{bmatrix}3 & 5\\ 1 & 2\end{bmatrix} = \begin{bmatrix}-5 & -8\\ 8 & 13\end{bmatrix}$$

✓ sets up equation correctly

$$M = \begin{bmatrix}-5 & -8\\ 8 & 13\end{bmatrix}\begin{bmatrix}2 & -5\\ -1 & 3\end{bmatrix}$$

✓ post-multiplies

✓ inverse

$$= \begin{bmatrix}-2 & 1\\ 3 & -1\end{bmatrix}$$

✓ matrix

5. [5 marks]

By considering two rotations of angle A about the origin, prove the following identities:

$$\sin 2A = 2\sin A\cos A$$

and

$$\cos 2A = \cos^2 A - \sin^2 A$$

✓ ✓

$$\begin{bmatrix}\cos 2A & -\sin 2A\\ \sin 2A & \cos 2A\end{bmatrix} = \begin{bmatrix}\cos A & -\sin A\\ \sin A & \cos A\end{bmatrix}\begin{bmatrix}\cos A & -\sin A\\ \sin A & \cos A\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2 A - \sin^2 A & -\sin A\cos A - \sin A\cos A\\ \sin A\cos A + \sin A\cos A & -\sin^2 A + \cos^2 A\end{bmatrix}$$ ✓

$\therefore \cos 2A = \cos^2 A - \sin^2 A.$ ✓

$\sin 2A = \sin A\cos A + \sin A\cos A$

$= 2\sin A\cos A.$ ✓

6. [5 marks]

Determine the equation of the image line formed when all points on the line $y = 2x + 3$ are transformed by the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 2x+3 \end{bmatrix}$$

$$= \begin{bmatrix} x + 4x + 6 \\ 2x + 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5x + 6 \\ 2x + 3 \end{bmatrix}$$

$x' = 5x + 6$

$x = \dfrac{x' - 6}{5}$

$y' = 2x + 3$

$= 2\left(\dfrac{x'-6}{5}\right) + 3$

$= \dfrac{2}{5}x' - \dfrac{12}{5} + \dfrac{15}{5}$

$= \dfrac{2}{5}x' + \dfrac{3}{5}$

7. [3, 3 marks]

(a) Determine the value of the real constants m and n if $z = 3 - 2i$ is a solution of the equation $z^2 + mz + n = 0$.

$z^2 = (3 - 2i)(3 - 2i)$

$= 9 - 12i - 4$

$= 5 - 12i$

$5 - 12i + 3m - 2mi + n = 0$

Re: $5 + 3m + n = 0$

Im: $-12 - 2m = 0$

$\underline{m = -6}$

$5 - 18 + n = 0$ $\quad \underline{n = 13}$

(b) Determine the complex solutions to the equation: $x^2 + 5 = 3x$

$x^2 - 3x + 5 = 0$

$x = \dfrac{3 \pm \sqrt{9 - 20}}{2}$

$= \dfrac{3 \pm \sqrt{-11}}{2}$

$= \dfrac{3 \pm i\sqrt{11}}{2}$

8. [6 marks]

Let z_1 and z_2 be complex numbers such that $3z_1 + 2z_2 = 19$ and $z_1 + iz_2 = 3i$.

Determine z_1 and z_2 in the form $z = a + bi$, where $a, b \in \mathbb{Z}$.

$$3z_1 + 2z_2 = 19$$
$$-\quad 3z_1 + 3iz_2 = 9i$$
$$(2-3i)z_2 = 19 - 9i$$
$$z_2 = \frac{19-9i}{2-3i} \times \frac{2+3i}{2+3i}$$
$$= \frac{38 + 57i - 18i + 27}{4+9}$$
$$= \frac{65 + 39i}{13}$$
$$= 5 + 3i$$

$$3z_1 + 10 + 6i = 19$$
$$3z_1 = 9 - 6i$$
$$z_1 = 3 - 2i$$

9. [1, 2, 2, 3 marks]

Given that $z = 3 - 2i$ and $w = 1 + 3i$, determine

(a) $|w| = \sqrt{1+9}$
$= \sqrt{10}$

(b) $\bar{z} + 2w = 3 + 2i + 2 + 6i$
$= 5 + 8i$

(c) w^2

$= (1+3i)(1+3i)$

$= 1 + 6i - 9$

$= 6i - 8$

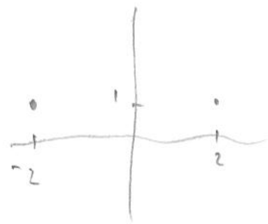
(d) $\frac{1}{z}$ in the form $a + bi$

$$\frac{1}{3-2i} \times \frac{3+2i}{3+2i} = \frac{3+2i}{9+4}$$
$$= \frac{3}{13} + \frac{2}{13}i$$

10. [2, 3, 2 marks]

A triangle has vertices $A(2, 1)$, $B(5, 1)$ and $C(4, 6)$.

(a) Triangle ABC is transformed to $A'(-2, 1)$, $B'(-5, 1)$ and $C'(-4, 6)$. Describe this transformation geometrically and state the 2×2 matrix that will transform ABC to $A'B'C'$.



Reflection in y-axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Triangle $A'B'C'$ is dilated by a scale factor of 3 parallel to the x-axis and then rotated 60° clockwise about the origin to form triangle $A"B"C"$.

(b) Determine the single matrix that will transform $A'B'C'$ to $A"B"C"$.

$$D = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \cos(-60°) & -\sin(-60) \\ \sin(-60) & \cos(-60) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3/2 & \sqrt{3}/2 \\ \frac{-3\sqrt{3}}{2} & 1/2 \end{bmatrix}$$

(c) Determine the single matrix that will transform $A"B"C"$ back to $A'B'C'$

$$\frac{1}{\frac{3}{4} + \frac{9}{4}} \begin{bmatrix} \frac{1}{2} & -\sqrt{3}/2 \\ \frac{3\sqrt{3}}{2} & \frac{3}{2} \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1/2 & -\frac{\sqrt{3}}{2} \\ \frac{3\sqrt{3}}{2} & \frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{6} & \frac{-\sqrt{3}}{6} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

-1/error