Mathematics Department

Course: ATMAA

Topic Title: Growth & Decay in Sequences

Test 1

Student Name: Answers.

Date: ___________

Special Instructions: **Calculator Allowed**

Formula Sheet Allowed

Time Allowed: 45 mins

Marks: / 40

Question 1. **(1, 2: 3 marks)**

A sequence has a recursive formula given by:

$$T_{n+1} = 0.6T_n, \quad T_1 = 150$$

a) Determine the first five terms of the sequence.

150, 90, 54, 32.4, 19.44

b) What percentage increase or decrease occurs with each successive term?

40% decrease.

Question 2. **(2, 2:4 marks)**

A new road 85km long is being laid. At the end of Stage 1, 35km of road has been laid.
It took 45 days to complete Stage 1.
For Stage 2, covering the remaining 50km, an extra 600m of new road is completed each day.

Let $t(n)$ be the length of road completed at the start of day n in Stage 2.

a) Write a recursive equation for the length of completed road at the start of day n.

$T_{n+1} = T_n + 0.6, \quad T_1 = 35$

or

$T_{n+1} = T_n + 600, \quad T_1 = 35000.$

b) Find how long it would take for the entire road to be laid.

CAS $T(26) = 50.$ ∴ 26 days to complete Stage 2.

∴ 45 + 26 days = 71 days to complete 85km

Question 3. **(2, 2:4 marks)**

In 2005, a recycling centre processed 2000 tonnes of plastic containers. The amount of plastic containers processed each year, increased by 8% p.a.
Let $a(n)$ be the amount of plastics containers recycled.

a) Write a recursive rule for the amount of plastic containers recycled.

$a_{n+1} = a_n \times 1.08 \quad , \quad a_1 = 2000$

b) In which year will the recycling centre for the first time recycle more than 5000 tonnes of plastic containers? Assume that the recycling centre is able to handle the growth in the amount of containers recycled.

$a(12) = 4663.3$

$a(13) = 5036.3$

$\therefore$ The 13th year (2017).

Question 4. **(4, 2: 6 marks)**

The fifth term of a geometric progression is 81 and the second term is 24.

$T_n = a \times r^{(n-1)}$

Find:

$T_5 = 81 = a \times r^4$

$T_2 = 24 = a \times r \quad \div$

$3.375 = r^3$

$1.5 = r$

$81 = a \times r^4$

$16 = a$

$T_n = 16 \times 1.5^{(n-1)}$

a) the fourth term

$T_4 = 16 \times 1.5^3$

$= 54$

b) the recursive formula for this sequence

$T_{n+1} = 1.5T_n \quad ; \quad T_1 = 16$

Question 6. (3, 1, 1, 2, 2: 9 marks)

A large swimming pool contains 80 000 litres of water. Each day 5% of the water is lost due to evaporation.
An additional 2 500 litres of water is added to the pool at the end of each day.
The recurrence relation is: $T_{n+1} = rT_n + d, \quad T_1 = a$
where T_n represents the amount of water in the pool at the start of the nth day.

a) Calculate the values of r, d and a.

$r = 0.95$ ✓
$d = 2500.$ ✓
$a = 80\,000$ ✓

b) How many litres of water will be in the swimming pool at the start of the 6th day?

~~T1=80000~~

$T_6 = 73213.43$ litres ✓

c) How long will it take for the water in the swimming pool to fall below 60 000 litres?

Start 23rd Day ✓

22 Days

d) Using the recurrence relation find the steady-state solution of the pool over time.

$T_{n+1} = 0.95T_n + 2500$
let $x = T_{n+1} = T_n$.
$x = 0.95x + 2500$
$0.05x = 2500$ ✓

$x = 50\,000.$

SS = 50 000 litres ✓

e) If the amount of water in the pool is not to fall below 65 000 litres on any given day, find the amount of water that needs to be added at the end of each day.

Let $x = 65000$
$65000 = 0.95(65000) + \cancel{250}d.$
$65000 = 61750 + d.$
$3250 = d$ ✓ ✓

∴ 3250 litres need to be added.

Question 5. **(1, 1, 1, 2: 5 marks)**

A plasma TV costs \$12 000 and John decides to borrow the money at a rate of 2.4% per month. He agrees to repay \$400 per month for the life of the loan. The amount owing after ($\mp$ 1) months is related to the amount owing after n months according to the rule:

$$T_{n+1} = 1.024T_n - 400 \qquad n \geq 0$$

where T_n is the amount owing after n month.

a) Determine T_0

$T_0 = \$12\,000$ ✓

b) Find the amount owing after the first month

$T_1 = \$11\,888$ ✓

c) Find the amount owing after six months

$T_6 = \$11\,286.37$ ✓

d) How many months will it take for John to pay out his loan?

54 months ✓✓

Question 7. **(3, 1, 1, 2, 2: 9 marks)**

Consider the sequences A, B and C defined by the recurrence relations.

A: $a_{n+1} = a_n + 3, \quad a_0 = 1$

B: $b_{n+1} = 2b_n, \quad b_0 = 1$

C: $c_{n+1} = 2c_n + 3, \quad c_0 = 1$

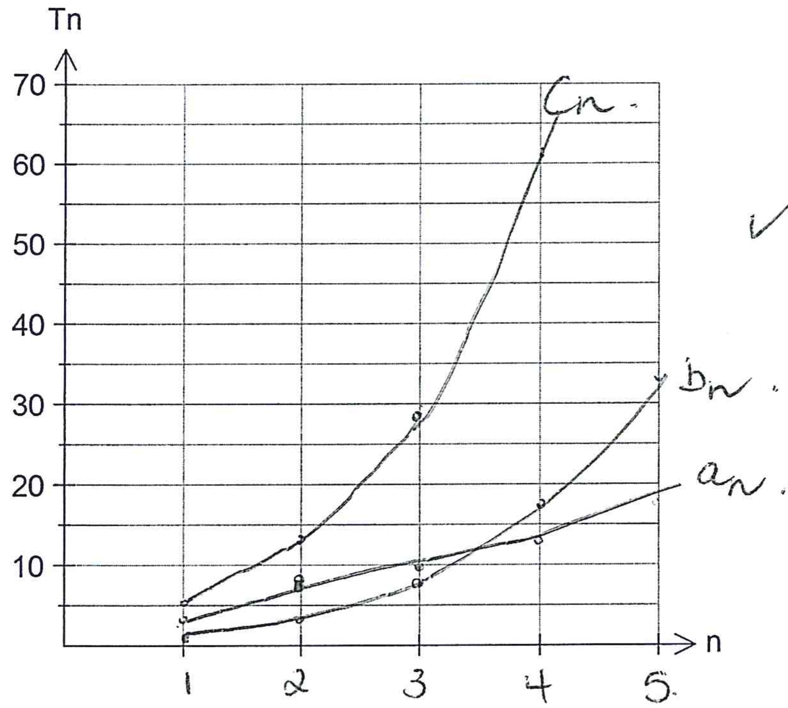
a) List the first six terms of each of the above sequences.

$a_n = 4, 7, 10, 13, 16, 19.$ ✓

$b_n = 2, 4, 8, 16, 32, 64.$ ✓

$c_n = 5, 13, 29, 61, 125, 253$ ✓

b) On the axes below, plot the terms of each of the given sequences with respect to n.



✓ ✓

c) Comment on the shape of each of these recurrence relations.

an is a straight line. ✓

bn + Cn are a curved lines ✓

d) Discuss the rate of growth between the terms in each of the given sequences.

an growth increases at a constant rate, ✓

bn Cn growth increases rapidly in comparison to bn. ✓