Question 6 (7, 3, 4 = 14 marks)

Consider the function $f(x) = -x^4 + 2x^3 + 11x^2 - 12x$

a) Use calculus to determine all stationary points of $f(x)$ and determine their nature.

$f'(x) = -4x^3 + 6x^2 + 22x - 12$ ✓

$0 = -4x^3 + 6x^2 + 22x - 12$ ✓

$x = -2,\ x = \frac{1}{2},\ x = 3$ ✓

$f''(x) = -12x^2 + 12x + 22$ ✓

✓ $f(-2) = 36$

$f''(-2) = -50 \therefore$ max TP

$f(3) = 36$

$f''(3) = -50 \therefore$ max TP ✓

$f(\frac{1}{2}) = -3.0625$ OR $-\frac{49}{16}$ ✓

$f''(\frac{1}{2}) = 25 \therefore$ min TP ✓

$\therefore$ max TP's $(-2, 36)$ and $(3, 36)$

min TP $(\frac{1}{2}, -\frac{49}{16})$

b) Determine the coordinates of any points of inflection.

$f''(x) = -12x^2 + 12x + 22$

$0 = 12x^2 + 12x + 22$ ✓

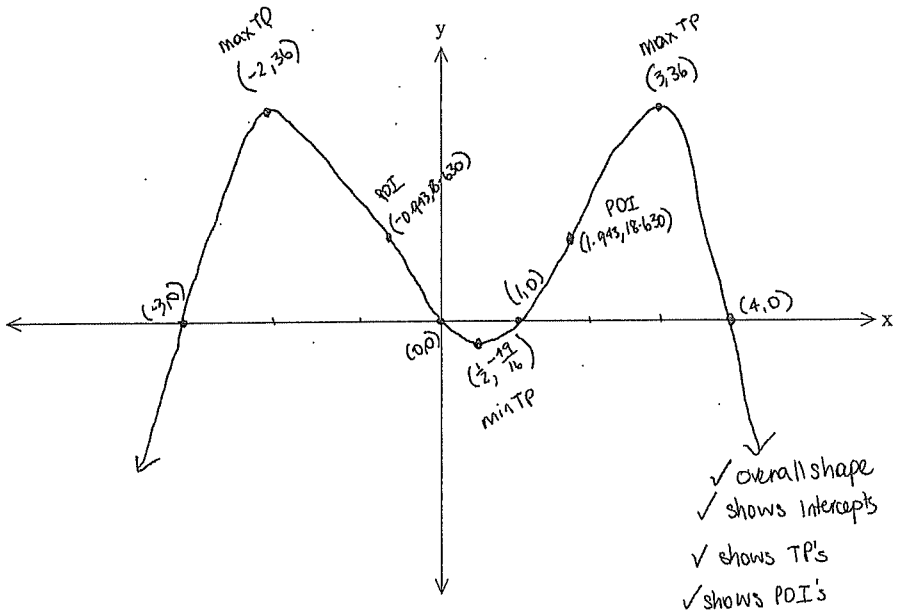
$x = -0.943$ OR $x = 1.943$ $(-0.943375673$ or $x = 1.943375673)$

$-\frac{3\sqrt{3}}{6} + \frac{1}{2}$ or $\frac{3\sqrt{3}}{6} + \frac{1}{2}$

$(-0.943, 18.639)$ and $(1.943, 18.639)$ ✓

✓ $, 18.638888889)$

c) Hence, sketch the graph of $f(x)$, clearly indicating the location of all intercepts, stationary points and points of inflection.



✓ overall shape

✓ shows intercepts

✓ shows TP's

✓ shows POI's

Question 7 (5 marks)

A spherical balloon has a volume $V = \frac{4\pi r^3}{3}$, where r is the radius of the balloon. Using the incremental change formula, find the approximate percentage increase of the balloon's volume when its diameter increases by 3%.

$\frac{dV}{dr} = 4\pi r^2$

$\frac{\delta V}{\delta r} \approx \frac{dV}{dr}$

$\delta V = 4\pi r^2 \times \delta r$ ✓

$\frac{\delta V}{V} = \frac{4\pi r^2 \times \delta r}{V}$

$= \frac{4\pi r^2 \times \delta r}{\frac{4}{3}\pi r^3}$ ✓

$= \frac{3\delta r}{r}$

$\frac{\delta r}{r} = 3\% = 0.03$ ✓

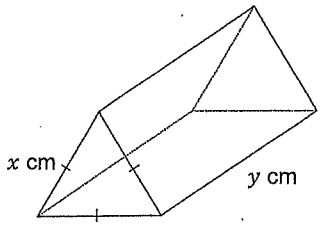
$\frac{\delta V}{V} = 3 \times 0.03$ ✓

$= 0.09$

$\therefore$ 9% increase in volume. ✓

Question 8 (7 marks)

A plastic block is made in the shape of a right triangular prism. The triangular end is an equilateral triangle with side length x cm and the length of the block is y cm. The volume of the block is 600 cm^3. Determine the dimensions of the block to minimise the total surface area of the block.



Method 1.

$V = \frac{1}{2}xhy$ $\quad h = \sqrt{x^2 - (\frac{x}{2})^2} = \sqrt{\frac{3x^2}{4}} = \frac{\sqrt{3}x}{2}$ ✓

$600 = \frac{1}{2}x(\frac{\sqrt{3}x}{2})y$ ✓

$600 = \frac{\sqrt{3}x^2}{4}y \Rightarrow y = \frac{2400}{\sqrt{3}x^2}$ ✓ or $\frac{800\sqrt{3}}{x^2}$

$SA = 3xy + \frac{2xh}{2}$

$= 3x(\frac{2400}{\sqrt{3}x^2}) + x(\frac{\sqrt{3}x}{2})$ ✓

$= \frac{2400\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}$

$SA' = \frac{-2400\sqrt{3}}{x^2} + \sqrt{3}x$ ✓

$0 = \frac{-2400\sqrt{3}}{x^2} + \sqrt{3}x$

$x = 13.39$cm ✓

$SA'' = \frac{4800\sqrt{3}}{x^3} + \sqrt{3}$

$SA''(13.39) = 5.195 > 0 \therefore$ min TP ✓

$\therefore x = 13.39$cm $\quad y = 7.73$cm ✓

Method 2

$V = \frac{1}{2}x^2 \sin 60\, y$

$600 = \frac{1}{2}x^2\frac{\sqrt{3}}{2}y$ ✓

$600 = \frac{\sqrt{3}x^2}{4}y \Rightarrow y = \frac{2400}{\sqrt{3}x^2}$ ✓

$SA = 2 \times \frac{1}{2}x^2 \sin 60 + 3xy$

$= \frac{\sqrt{3}x^2}{2} + 3x(\frac{2400}{\sqrt{3}x^2})$ ✓

$= \frac{\sqrt{3}x^2}{2} + \frac{7200}{\sqrt{3}x}$

$SA' = \sqrt{3}x - \frac{7200}{\sqrt{3}x^2}$ ✓

$0 = \sqrt{3}x - \frac{7200}{\sqrt{3}x^2}$

$x = \sqrt[3]{2400}$

$= 13.39$ cm ✓

$SA'' = \sqrt{3} + \frac{14400}{\sqrt{3}x^3}$

$SA''(\sqrt[3]{2400}) = \sqrt{3} + \frac{14400}{2400\sqrt{3}} > 0 \therefore$ min TP ✓

$\therefore x = 13.39$cm $\quad y = 7.73$cm ✓

END OF SECTION

WILLETTON SENIOR HIGH SCHOOL

YEAR 12 MATHEMATICS METHODS

TEST 1 2022

Section 2: Calculator Allowed

Student Name: Solutions

Circle your teacher's name

Miss Ahern **Ms Arora** **Mrs Gatland**

Mrs Sun **Mrs Tay**

Mark: _____ / 26

Time: 25 mins

For this test:

Scientific calculators and Classpads are allowed

One A4 single side of notes is allowed

Show any working in the spaces provided