Mathematics Methods Unit 4
Investigation 2: The usefulness of logarithms

Name: Solutions
Validation: Week 17 Thursday 7th of June

- *The Out of Class Investigation is designed for you to learn the essentials needed for the In-Class validation.*
- *This is the "Take Home" part of the Investigation. It does not count towards your mark for this investigation.*

The following chart shows the loudness levels, some of which can damage the human ear.

Chart of sound intensity levels (loudness)for environmental noise		
	Weakest sound heard	0 dB
	Rustling leaves	10 dB
	A whisper in library at 2 m	30 dB
	Converation at home	50 dB
	Conversation in restaurant	60 dB
	Passenger car at 80 kph at 6 m	70 dB
	Vacuum cleaner at 1m	70 dB
	Freeway at 20 m	73 dB
	Telephone dial tone	80 dB
At 90 - 95 dB sustained exposure may sustain hearing loss	Car wash at 6 m	90 dB
	Train whistle at 150 m	90 dB
	Hand drill	98 dB
	Lawn mower at 1M	105 dB
	Motorbike	100 dB
	Jet take off at 300 m	100 dB
	Sand blasting	115 dB
Threshold of discomfort	Thunderclap	120 dB
	Chain saw	120 dB
	Oxygen torch	120 dB
	Loud rock concert	115 dB
Pain threshold 130 dB	Pneumatic riveter	125 dB
	Aircraft carrier deck	140 dB
Eardrum rupture	Jet take off at 25 m	150 dB

The reference level of the intensity of sound, I_0 that all others are compared to is 10^{-12} watts/m ^{2}. It was chosen because it is the weakest intensity of sound that can be detected by the human ear.

Intensity, $I = \dfrac{Power}{Area}$ so I is measured in watts/m ^{2}.

$I_0 = 10^{-12}$ watts/m ^{2}.

The most intense sound that is not painful to humans is roughly 10 watts/m^2. Since the human pain threshold at 10 watts/m ^{2} is 10,000,000,000,000 times greater than the reference level, it makes sense to use a logarithmic scale to discuss the intensity of sound, I. The sound intensity level, L, is a logarithmic measure given as

$L = 10\log\left(\frac{I}{I_0}\right)$ and is measured in decibels (dB).

The lowest sound heard by man is	The loudest sound heard without pain (ie. the pain threshold) is 10 watts/m^2.
$L = 10\log\left(\frac{I_0}{I_0}\right)$ $= 10\log(1)$ $L = 0\ dB$	$L = 10\log\left(\frac{I}{I_0}\right)$ $= 10\log\left(\frac{10}{10^{-12}}\right)$ $= 10\log(10^{13})$ $= 130\log(10)$ $L = 130\ dB$

Question 1

Show that the pain threshold is 10^{13} times more intense than the lowest sound heard by man.

0 dB has $I = I_0 = 10^{-12}$ watts/m^2

130 dB has $I = 10$ watts/m^2

$$\frac{I}{I_0} = \frac{10}{10^{-12}} = 10^{13}$$

∴ pain threshold is 10,000,000,000,000 times more intense than the lowest sound heard by humans.

Question 2

a) What is the sound intensity level that corresponds to a sound that has an intensity of 10^{-2} watts/m^2?

$$\frac{I}{I_0} = \frac{10^{-2}}{10^{-12}} = 10^{10}$$

$$L = 10\log\left(\frac{I}{I_0}\right)$$

$$L = 10\log(10^{10})$$

$$L = 100\ dB$$

b) What sound could this be from the table?

The sound could be a motorbike or a jet taking off 300m away.

Question 3

a) How many times more intense is the sound of a loud rock concert than the sound of a conversation at home?

Hint : Find $\frac{I_{rock\,concert}}{I_{converstaion\ at\ home}}$

$I_o = 10^{-12}$ watt/m^2

Rock Concert $L = 115$ dB

$115 = 10 \operatorname{Log}\left(\frac{I}{I_o}\right)$

$11.5 = \log\left(\frac{I}{10^{-12}}\right)$

$10^{11.5} = \frac{I}{10^{-12}}$

$RC = 10^{-0.5}$

Conversation @ home $L = 50$ dB

$50 = 10 \log\left(\frac{I}{I_o}\right)$

$5 = \log\left(\frac{I}{10^{-12}}\right)$

$10^5 = \frac{I}{10^{-12}}$

$CH = 10^{-7}$

$= \frac{10^{-0.5}}{10^{-7}}$

$= 10^{6.5}$

$= 3,162,278$

∴ RC is about 3 million times more intense

b) How many times more intense is the sound of a conversation in a restaurant than the sound of a conversation at home?

$60 = 10 \log\left(\frac{I}{I_o}\right)$

$6 = \log\left(\frac{I}{10^{-12}}\right)$

$10^6 = \frac{I}{10^{-12}}$

$I = 10^{-6}$

$\frac{10^{-6}}{10^{-7}} = 10^1$

∴ Restaurant conversation is 10 times more intense than @ home.

Question 4

Determine the intensity in watts/m ^{2} of the following sounds

(a) (i) a chain saw

$120 = 10 \log\left(\frac{I}{I_o}\right)$

$12 = \log \frac{I}{10^{-12}}$

$10^{12} = \frac{I}{10^{-12}}$

$I = 1$ watt/m^2

(ii) a vacuum cleaner at 1m

$70 = 10\log\left(\frac{I}{I_o}\right)$

$7 = \log\left(\frac{I}{10^{-12}}\right)$

$10^7 = \frac{I}{10^{-12}}$

$I = 10^{-5}$ watt/m^2

(iii) rustling leaves

$10 = 10\log\left(\frac{I}{I_o}\right)$

$1 = \log\left(\frac{I}{10^{-12}}\right)$

$10^1 = \frac{I}{10^{-12}}$

$I = 10^{-11}$ watt/m^2

(iv) a telephone dial tone

$80 = 10\log\left(\frac{I}{I_o}\right)$

$8 = \log\left(\frac{I}{10^{-12}}\right)$

$10^8 = \frac{I}{10^{-12}}$

$I = 10^{-4}$ watt/m^2

(v) a hand drill

$$98 = 10\log\left(\frac{I}{I_o}\right)$$

$$9.8 = \log\left(\frac{I}{10^{-12}}\right)$$

$$10^{9.8} = \frac{I}{10^{-12}}$$

$$I = 10^{-2.2} \text{ watt}/m^2$$

(vi) an aircraft carrier deck

$$140 = 10\log\left(\frac{I}{I_o}\right)$$

$$14 = \log\left(\frac{I}{10^{-12}}\right)$$

$$10^{14} = \frac{I}{10^{-12}}$$

$$I = 10^{2} \text{ watt}/m^2$$

(vii) the ticking of a watch with a sound intensity level of 20 dB

$$20 = 10\log\left(\frac{I}{I_o}\right)$$

$$2 = \log\left(\frac{I}{10^{-12}}\right)$$

$$10^{2} = \frac{I}{10^{-12}}$$

$$I = 10^{-10} \text{ watt}/m^2$$

b) Explain the reason for using sound intensity levels in decibels rather than intensity in watts/m ^{2}.

Decibels are within a much more manageable range, human hearing between 0-130 dB. Humans can distinguish sounds in that range.

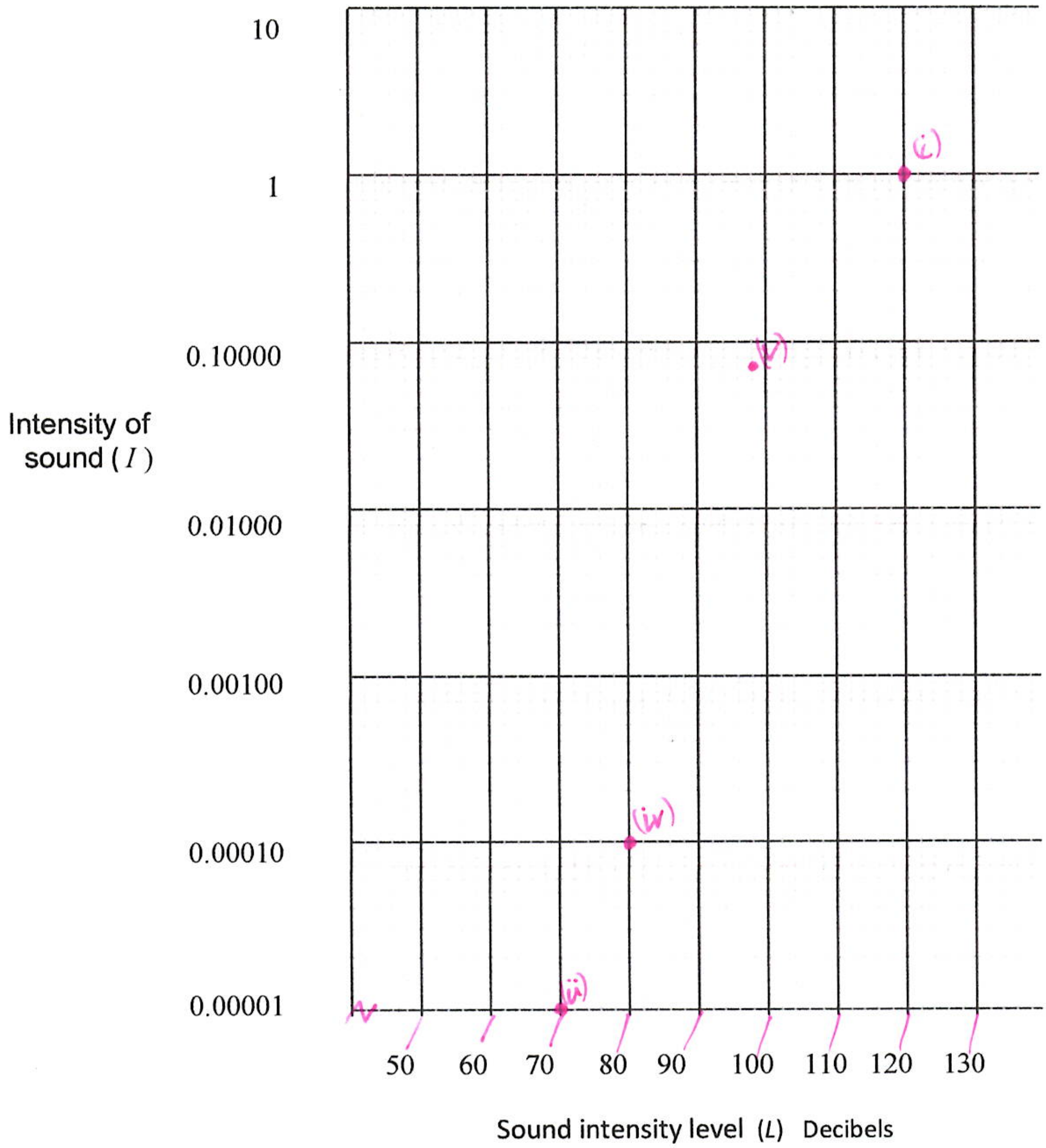
Question 5

Use the semi – logarithm graph paper (one axis has a logarithmic scale) to plot some of the intensities of the sounds in **Question 4** against their sound intensity levels and comment on the shape of the graph.

graph looks roughly linear

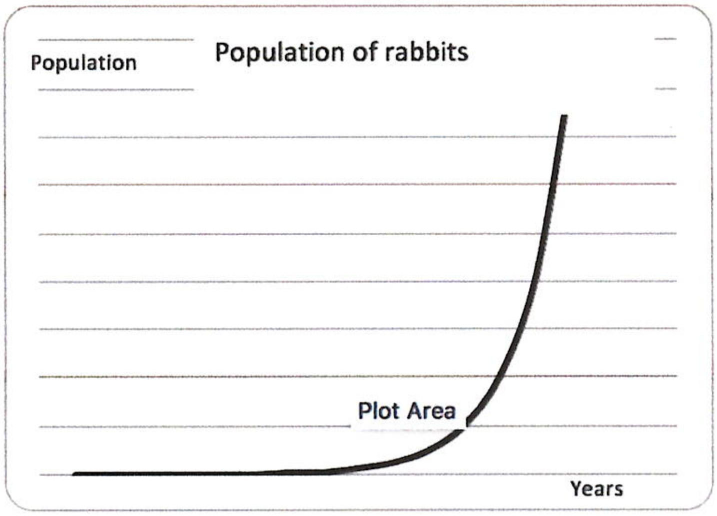
What is the advantage of using this graph paper for the data from Question 4?

Semi-logarithm paper squashes the y-axis into a manageable scale so it's possible to use all parts of the number range on the graph.

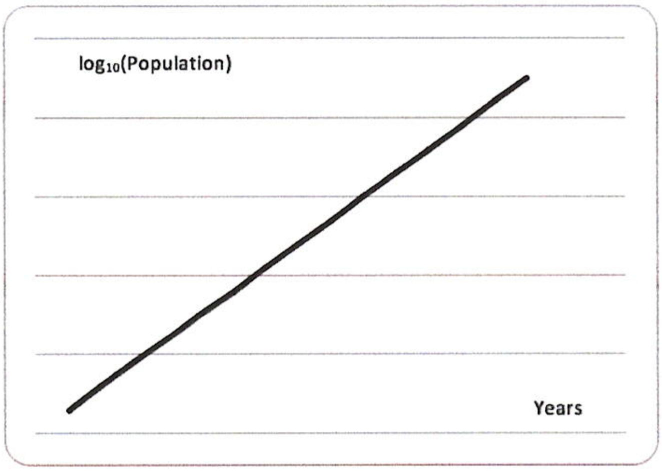


Semi-logarithm graphs are useful when graphing data that increases exponentially. For example, a population of rabbits that is doubling every year.

Year	Population
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128
9	256
10	512
11	1024
12	2048
13	4096
14	8192
15	16384



Year	$\log_{10}$(Population)
1	0.30103
2	0.60206
3	0.90309
4	1.20412
5	1.50515
6	1.80618
7	2.10721
8	2.40824
9	2.70927
10	3.0103
11	3.31133
12	3.61236
13	3.91339
14	4.21442
15	4.51545



If the function is exponential, then using semi-logarithm paper makes the graph linear.

Question 6

Show that if the function is exponential, say $y = A(b)^t$ then the function $\log_c(y)$ graphed against t is linear.

$$\log_c(y) = \log_c\left(A(b)^t\right)$$

$$= \log_c(A) + \log_c\left((b)^t\right)$$

$$\log_c(y) = \log_c(A) + t\log_c(b)$$

$$y = c + mx$$