

Full Name: SOLUTIONS



MATHEMATICS

Methods Units 1 & 2

Test 5 – Counting Techniques, Sets and Probability

Chapter 15, 16 and 17

Semester 1 2019

Section One - Calculator Free

Time allowed for this section

Working time for this section: 26 minutes

Marks available: 26 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the students

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: Nil

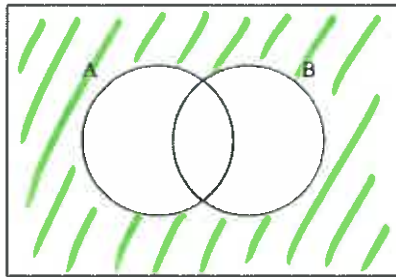
Important note to students

No other items may be used in this section of the assessment. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the assessment room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (4 marks: 2, 2)

Clearly shade the region indicated by,

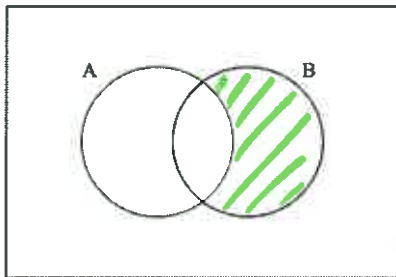
a) $\overline{A \cup B}$



✓✓

1 mark if not clear or
one extra region shaded.

b) $\overline{A} \cap B$

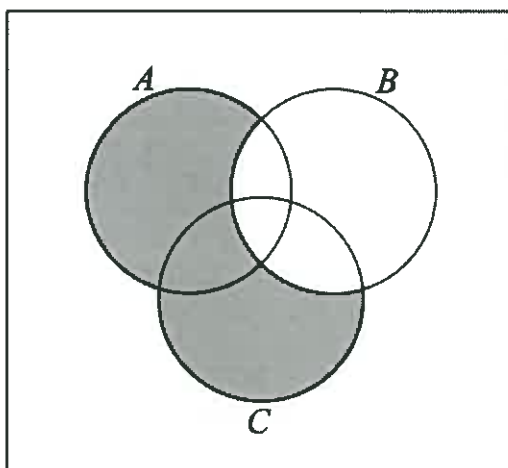


✓✓

as above

2. (2 marks)

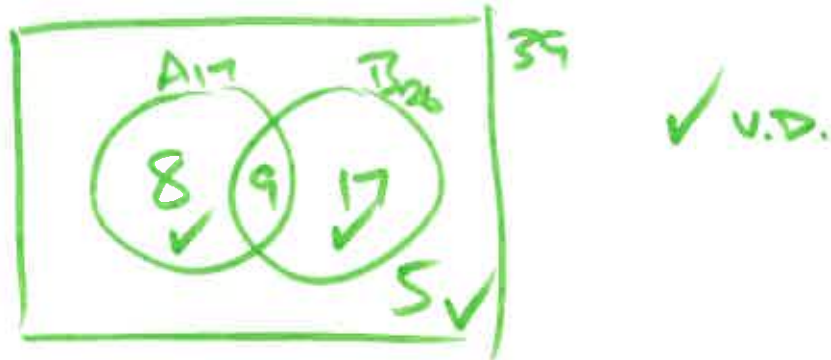
Use set notation to describe the shaded region represented in the Venn Diagram below.



$(A \cup C) \cap \overline{B}$
✓ ✓

3. (4 marks)

Sets A and B are such that $n(A) = 17$, $n(B) = 26$ and $n(A \cup B) = 34$. If there are 39 elements in the Universal Set, construct a Venn Diagram that represents this information.



4. (4 marks: 2, 2)

An experiment consists of drawing a number at random from $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Let $A\{1, 2, 3, 4\}$, $B\{1, 3, 5, 7\}$ and $C\{4, 6, 8\}$.

a) Are Set A and Set B mutually exclusive? Explain

No, not m.e. ✓

Both sets include 1 and 3 ✓

b) Are Set B and Set C mutually exclusive? Explain

Yes, m.e. ✓

No common elements ✓

5. (2 marks)

If $P(A) = 0.46$, $P(B) = 0.8$ and $P(A \cup B) = 0.528$, without calculating, show $P(A \cap B)$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) \quad \checkmark$$

$$= 0.46 + 0.8 - 0.528 \quad \checkmark$$

6. (5 marks: 2, 3)

Three students are to be chosen to represent the class in a debate. If the class consists of six boys and four girls, what is the probability that the team will contain,

a) exactly one girl?

$$\frac{\binom{4}{1} \times \binom{6}{2}}{\binom{10}{3}} = 0.5$$

b) at least two girls?

$$\begin{aligned} 2 \text{ girls } \binom{4}{2} \times \binom{6}{1} &= 36 \checkmark \\ 3 \text{ girls } \binom{4}{3} \times \binom{6}{0} &= 4 \checkmark \end{aligned} \quad \therefore \frac{40}{120} = \frac{1}{3} \checkmark$$

7. (5 marks: 1, 2, 2)

Samantha has a collection of 18 fluoro pens in her pink box and 24 fluoro pens in her blue box. Write mathematical expressions for the number of ways Amy can pick:

a) three pens from her pink box.

$$18C_3 \checkmark$$

b) three pens from the pink box and four pens from the blue box.

$$18C_3 \times 24C_4 \quad \begin{array}{l} \checkmark \text{ combinations} \\ \checkmark \text{ multiplied} \end{array}$$

c) a dozen pens from both boxes.

$$42C_{12} \checkmark$$

End of Section One

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MATHEMATICS Methods Units 1 & 2

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Section Two - Calculator Assumed

Time allowed for this section

Working time for this section: 28 minutes

Marks available: 28 marks

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the students

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important note to students

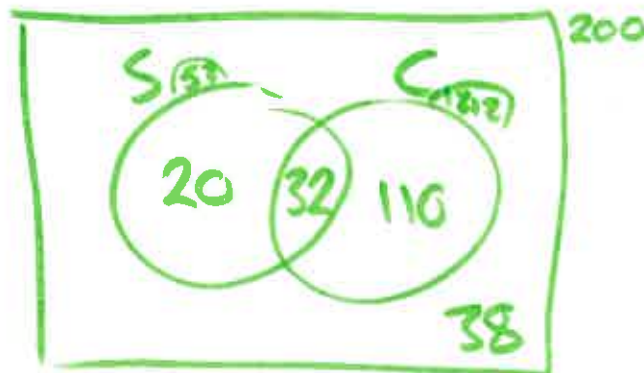
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1. (8 marks: 2, 2, 2, 2)

The Mathematics Department at a school conducted a random survey involving 200 senior students.

- 38 students did not have any calculator with them
- 142 students has a ClassPad with them
- 52 students had a scientific calculator with them

Use a Venn Diagram, or other method, to answer the following questions.



a) Find the probability that a student chosen at random had only a ClassPad.

$$\frac{110}{200} \checkmark$$

b) Find the probability that a student chosen at random had both a ClassPad calculator as well as a scientific calculator.

$$\frac{32}{200} \checkmark$$

c) Find the probability that a student chosen at random had either a ClassPad calculator as well as a scientific calculator.

$$\frac{162}{200} \checkmark$$

d) Find the probability that a student selected from those who had at least one type of calculator, had both types of calculator.

$$\frac{32}{110 + 32 + 20} = \frac{32}{162} \checkmark$$

2. (15 marks: 1, 3, 3, 4, 4)

Consider the digits 0 to 9 inclusive and all the letters of the alphabet. Ten characters consisting of digits and letters are chosen. Determine the number of ways of choosing:

a) all the even numbers and all the vowels.

$$1 \quad \checkmark$$

b) any six digits and any four letters.

$$10C_6 \times 26C_4 \quad \checkmark \checkmark$$

$$= 3139500 \quad \checkmark$$

c) exactly four vowels.

$$5C_4 \times 31C_6 \quad \checkmark \checkmark$$

$$= 3681405 \quad \checkmark$$

d) at least four odd digits.

$$5C_4 \times 31C_6 + 5C_5 \times 31C_5 \quad \checkmark \text{ addition}$$

$$= 3851316$$

e) Four vowels or four odd digits.

$$44 + 40 = 84 \quad \checkmark$$

$$= \underline{3681405 + 3681405} - 8125 \quad \checkmark$$

$$= 7354685 \quad \checkmark$$

3. (5 marks: 3, 2)

Given that $P(\overline{A \cup B}) = 0.2$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$,

a. calculate $P(B|A)$

$$P(B|A) = \frac{0.2}{0.6} \checkmark \checkmark$$

$$= \frac{1}{3} \checkmark$$

	A	\bar{A}	
B	0.2	0.2	0.4
\bar{B}	0.4	0.2	0.6
	0.6	0.4	1

b. determine, with reasons, if the events A and B are independent.

$$P(B) = 0.4$$

$$\text{but } P(B|A) = \frac{1}{3}$$

$$\text{hence } P(B) \neq P(B|A) \checkmark$$

∴ not independent ✓

End of Test