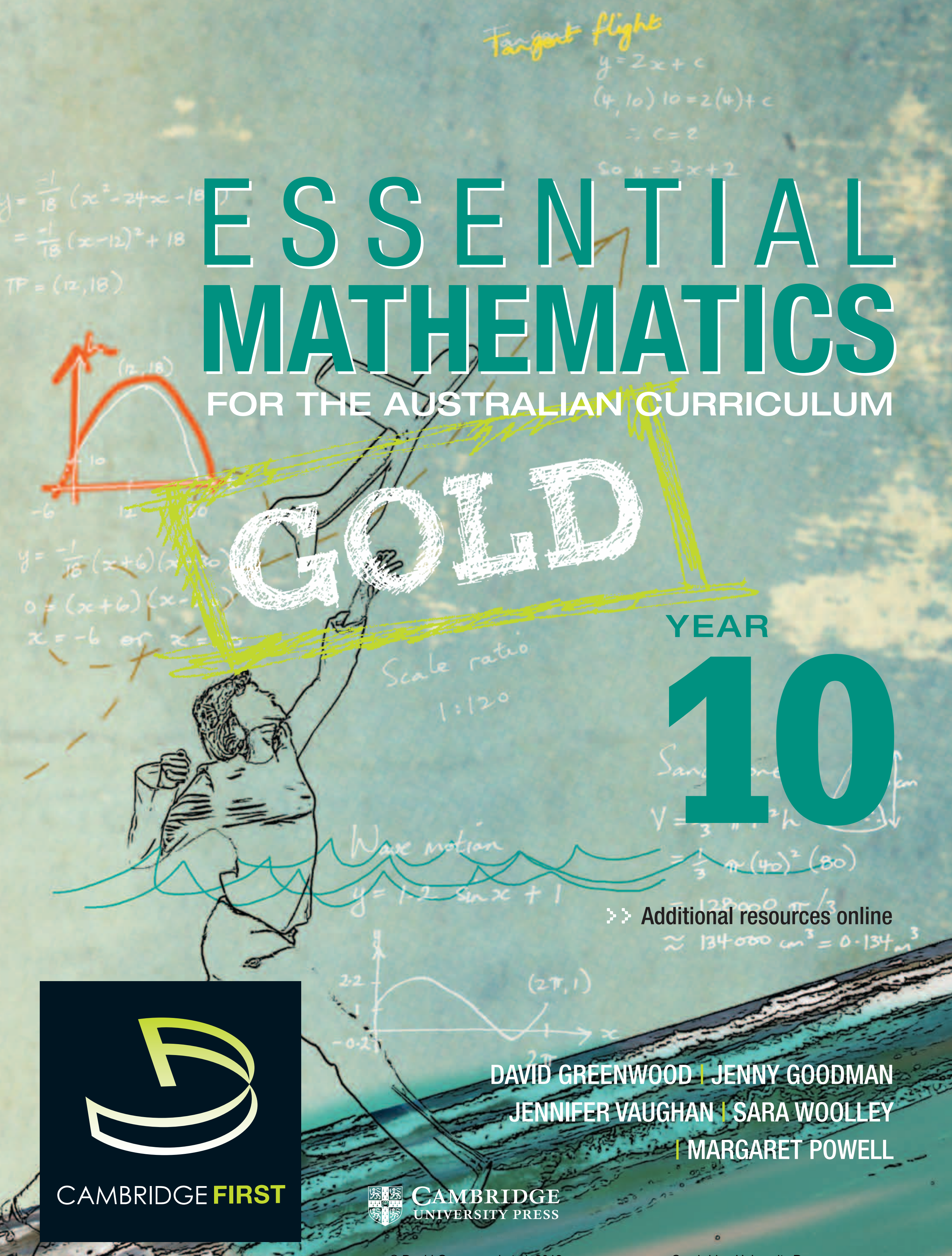


ESSENTIAL MATHEMATICS

FOR THE AUSTRALIAN CURRICULUM

GOLD

YEAR **10**

>> Additional resources online

DAVID GREENWOOD | JENNY GOODMAN
JENNIFER VAUGHAN | SARA WOOLLEY
| MARGARET POWELL

CAMBRIDGE FIRST

CAMBRIDGE UNIVERSITY PRESS

ISBN: 9781107687028 Cambridge University Press

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
477 Williamstown Road, Port Melbourne, VIC 3207, Australia

www.cambridge.edu.au
Information on this title: www.cambridge.org/9781107687028

First published 2012

Cover design by Denise Lane
Edited by Kelly Robinson
Typeset by Aptara Corp
Printed in China by C & C Offset Printing Co. Ltd.

A Cataloguing-in-Publication entry is available from the catalogue of the National Library of Australia at www.nla.gov.au

ISBN 978-1-107-68702-8 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

Contents

Strand and content description

ISBN: 9781107687028

Cambridge University Press

ISBN: 9781107687028 Cambridge University Press

ISBN: 9781107687028

ISBN: 9781107687028

ISBN: 9781107687028

Cambridge University Press

About the authors

David Greenwood is the head of Mathematics at Trinity Grammar School in Melbourne and has 19 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 20 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics in Victoria from Years 7 to 12 since 2006 and has a keen interest in the creation of resources that cater for a wide range of ability levels.

Jenny Goodman has worked for 20 years in comprehensive state and selective high schools in NSW and has a keen interest in teaching students of differing ability levels. She was awarded the Jones medal for education at Sydney University and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the *Spectrum* and *Spectrum Gold* series.

Jennifer Vaughan has taught secondary mathematics for more than 30 years in NSW, WA, QLD and New Zealand, and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible, hence facilitating student confidence, achievement and an enjoyment of maths.

Consultant

Margaret Powell has 23 years of experience in teaching special needs students in Sydney and London. She has been head teacher of the support unit at a NSW comprehensive high school for 12 years. She is one of the authors of *Spectrum Maths Gold Year 7* and *Year 8*. Margaret is passionate about ensuring that students with learning difficulties are provided with learning materials that are engaging and accessible, so as to give them the best chance to achieve in their adademic careers.

ISBN: 9781107687028 Cambridge University Press

Introduction and how to use this book

The *Essential Mathematics Gold for the Australian Curriculum* series has been developed especially to cater for students who require additional support in mathematics. Using the *Essential Mathematics for the Australian Curriculum* series as a starting point, the *Gold* series heavily revises that material and adds a wealth of new content to help students meet the Achievement Standards of the Australian Curriculum.

In addition to an author team of skilled teachers with experience teaching students at all levels, the *Gold* series has been carefully reviewed by Special Education advisors to make sure that the language and concepts are effective for struggling students.

The *Gold* series takes a practical interpretation of the Australian Curriculum that focuses on the Understanding and Fluency proficiency strands in order to help students learn core mathematical skills. These foundation abilities are developed through clear explanations of concepts that are repeated and closely linked to carefully graded exercises, which include linked examples and hints throughout. Questions for the Problem-solving and Reasoning proficiency strands, along with Gold Star enrichment problems, are provided to challenge students that want to improve their skills further.

Literacy and numeracy skills are the other major focus of the *Gold* series. Key words are defined multiple times within the books to help students become fluent with mathematical terms and concepts. Real-world contexts and applications of mathematics help students connect these concepts to everyday life. Each chapter has a Puzzles and games section for stimulus and motivation.

The *Essential Mathematics Gold for the Australian Curriculum* series complements the full *Essential Mathematics* series and Cambridge HOTmaths to provide a choice of mathematics resources that work well across all ranges of ability.

ISBN: 9781107687028

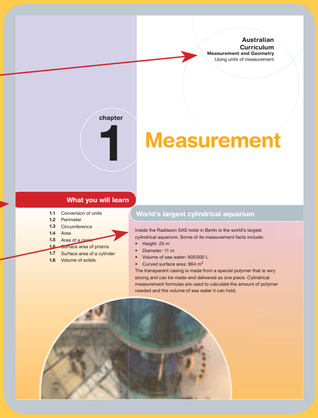
How to use this book

Features:

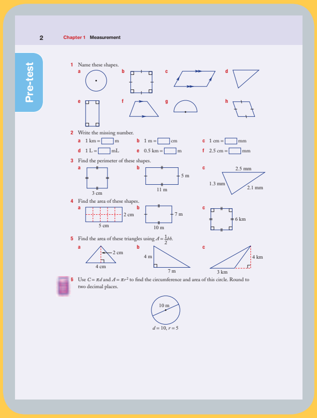
Australian Curriculum: strands and content descriptions for chapter (also available in a grid)

What you will learn: an overview of chapter contents

Chapter introduction: use to set a context for students



Pre-test: establishes prior knowledge (also available as a printable worksheet)

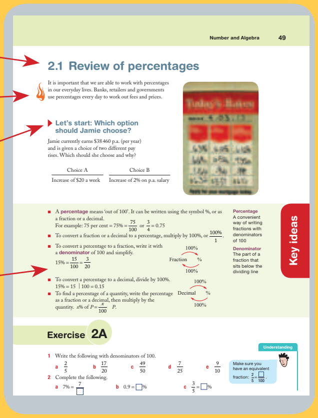


Topic introduction: use to relate the topic to mathematics in the wider world

HOTmaths icons: links to interactive online content via the topic number, 1.1 in this case (see next page for more)

Let's start: an activity (which can often be done in groups) to start the lesson

Key ideas: summarises the knowledge and skills for the lesson (digital version also available for use with IWB)

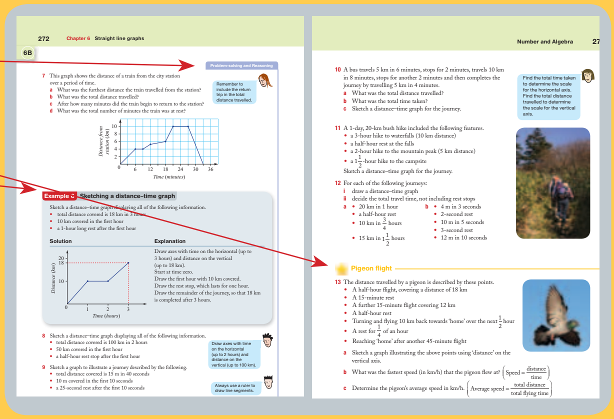


ISBN: 9781107687028

Cambridge University Press

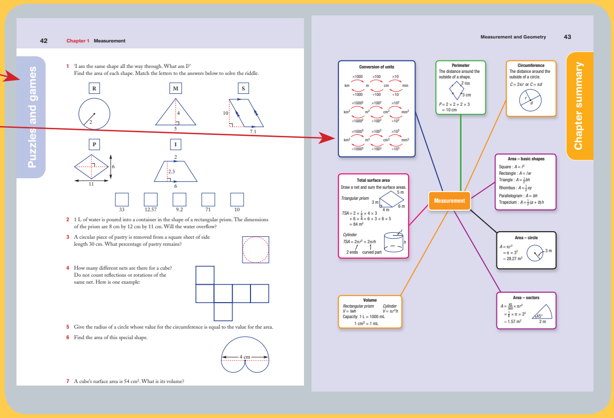
How to use this book

Exercise questions categorised by the four **proficiency strands** and **Gold star**



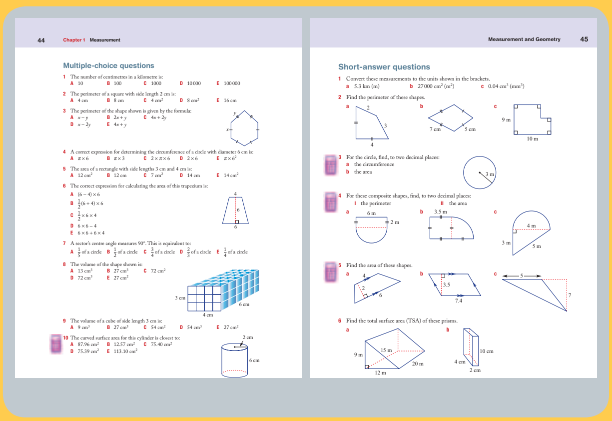
Questions are linked to **examples:** solutions with explanations and descriptive titles to aid searches

Puzzles and **games**



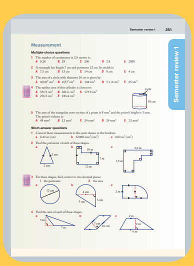
Chapter summary: mind map of key concepts and interconnections

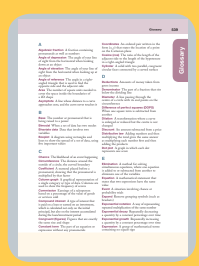
Chapter reviews with **multiple-choice**, **short-answer** and **extended-response questions**

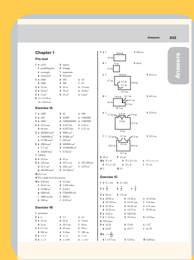


Each textbook also contains:

- Two **semester reviews**
- **Glossary**
- **Answers**







ISBN: 9781107687028 Cambridge University Press

This textbook is supported by online resources...

Cambridge GO

YOUR GATEWAY ONLINE

Digital resources and support material for schools.

About the free online resources...

Free additional student support resources are available online at *Cambridge GO* and include:

- the PDF Textbook – a downloadable version of the student text, with note-taking and bookmarking enabled
- extra material and activities
- links to other resources.

Available free for users of this textbook. Use the unique access code found in the front of this textbook to activate these resources.

www.cambridge.edu.au/GO

Access your online resources today at www.cambridge.edu.au/GO

1. **Log in** to your existing *Cambridge GO* user account

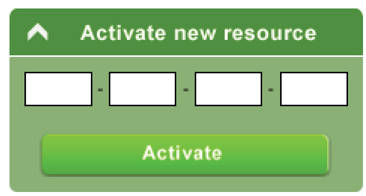
OR

Create a new user account by visiting:
www.cambridge.edu.au/GO/newuser

- All of your *Cambridge GO* resources can be accessed through this account.
- You can log in to your *Cambridge GO* account anywhere you can access the internet using the email address and password with which you're registered.

2. **Activate** *Cambridge GO* resources by entering the unique access code found in the front of this textbook.

- Once you have activated your unique code on *Cambridge GO*, it is not necessary to input your code again. Just log in to your account using the email address and password you registered with and you will find all of your resources.



3. **Go** to the My Resources page on *Cambridge GO* and access all of your resources anywhere, anytime.*

* Technical specifications: You must be connected to the internet to activate your account. Some material, including the PDF Textbook, can be downloaded. To use the PDF Textbook you must have the latest version of Adobe Reader installed.

For more information or help contact us on 03 8671 1400 or enquiries@cambridge.edu.au

ISBN: 9781107687028

Cambridge University Press

Acknowledgements

The author and publisher wish to thank the following sources for permission to reproduce material:

Images: ©Bill Bachman/Alamy, p.250; ©2012 iStockphoto LP/pxlar8, p.12/evirgen, p.13/Tommounsy, p.14/zooooma, p.18/laughingmango, p.23/ftwitty, p.25/FeodorKorolevsky, p.33/Sturti, p.47/Isitsharp, p.52/VM, p.54/ CraigRJD, p.72/H-Gall, p.74(b) /DNY59, p.80/LisaInGlasses, p.76/Maxian, p.82/Rorem, p.100(b) /Blend-Images, pg.100(t)/ georgeclerk, p.106/Opulent-Images, p.111/ermingut, p.243(l)/Vetta Collection, p.162/CEFutcher, p.211/CocoZhang, p.213/pamspix, p.216/mjbs, p.432/fstop123, p.442/Lisa-Blue, p.483/CelesteQuest, p.487/andipantz, p.488/stocknshares, p.501; ©Michael William/Shutterstock.com, p.394; Jan Kranendonk/Shutterstock.com, p.192; Kenneth William Caleno/Shutterstock.com, p.60; All other Images © 2012 Used under license from Shutterstock.com

Every effort has been made to trace and acknowledge copyright. The publisher apologises for any accidental infringement and welcomes information that would redress this situation.

ISBN: 9781107687028

Australian Curriculum
Measurement and Geometry
Using units of measurement

chapter 1

Measurement

What you will learn

1.1 Conversion of units
1.2 Perimeter
1.3 Circumference
1.4 Area
1.5 Area of a circle
1.6 Surface area of prisms
1.7 Surface area of a cylinder
1.8 Volume of solids

World's largest cylindrical aquarium

Inside the Radisson SAS hotel in Berlin is the world's largest cylindrical aquarium. Some of its measurement facts include:

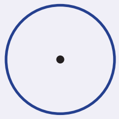
- Height: 25 m
- Diameter: 11 m
- Volume of sea water: 900 000 L
- Curved surface area: 864 m^2

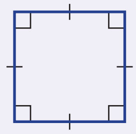
The transparent casing is made from a special polymer that is very strong and can be made and delivered as one piece. Cylindrical measurement formulas are used to calculate the amount of polymer needed and the volume of sea water it can hold.

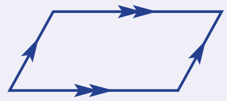
ISBN: 9781107687028 Cambridge University Press

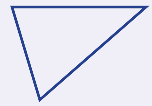
Pre-test

1 Name these shapes.

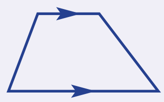
a 

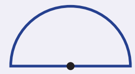
b 

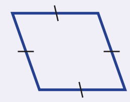
c 

d 

e 

f 

g 

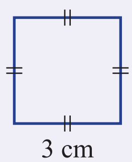
h 

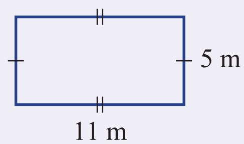
2 Write the missing number.

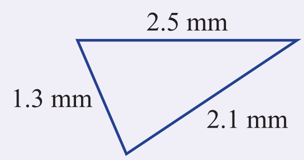
a 1 km = ☐ m **b** 1 m = ☐ cm **c** 1 cm = ☐ mm

d 1 L = ☐ mL **e** 0.5 km = ☐ m **f** 2.5 cm = ☐ mm

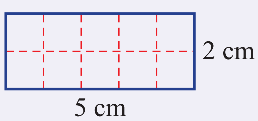
3 Find the perimeter of these shapes.

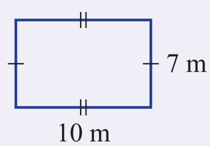
a 

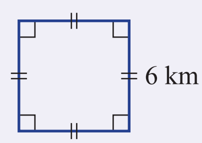
b 

c 

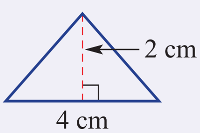
4 Find the area of these shapes.

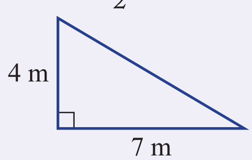
a 

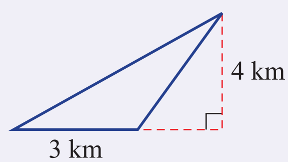
b 

c 

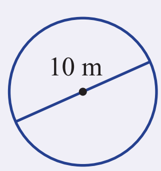
5 Find the area of these triangles using $A = \frac{1}{2}bh$.

a 

b 

c 

6 Use $C = \pi d$ and $A = \pi r^2$ to find the circumference and area of this circle. Round to two decimal places.



$d = 10, r = 5$

ISBN: 9781107687028

Cambridge University Press

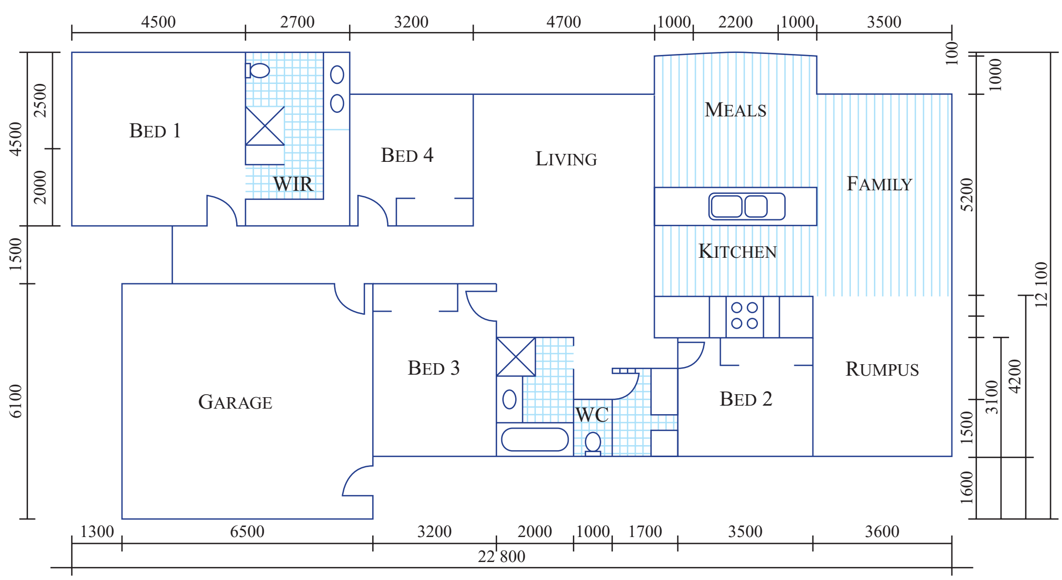
1.1 Conversion of units

To work with length, area or volume measurements, it is important to be able to convert between different units. Timber, for example, is widely used in buildings for frames, roof trusses and windows, to name a few things. It is important to order the correct amount of timber so that the cost of the house is minimised. Although plans give measurements in mm and cm, timber is ordered in metres (often referred to as lineal metres), so we have to convert all our measurements to metres.

Building a house also involves many area and volume calculations and conversions.

Let's start: House plans

All homes start from a plan, which is usually designed by an architect and shows most of the basic features and measurements that are needed to build the house. Measurements are given in millimetres.

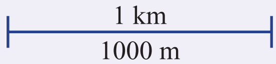


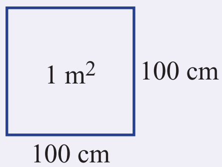
- How many bedrooms are there?
- What are the dimensions of the master bedroom (BED 1)?
- What are the dimensions of the master bedroom, in metres?
- Will the rumpus room fit a pool table that measures 2.5 m × 1.2 m, and still have room to play?
- How many cars do you think will fit in the garage?
- What do you think is going to cover the floor of the kitchen, meals and family rooms?

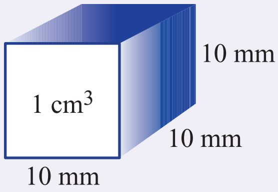
ISBN: 9781107687028

Key ideas

- To convert units, draw an appropriate diagram and use it to find the conversion factor. For example:







$1\text{ km} = 1000\text{ m}$

$$\begin{aligned} 1\text{ m}^2 &= 100 \times 100 \\ &= 10\,000\text{ cm}^2 \end{aligned}$$

$$\begin{aligned} 1\text{ cm}^3 &= 10 \times 10 \times 10 \\ &= 1000\text{ mm}^3 \end{aligned}$$

- Conversions:

Length
$$\begin{tikzcd} \text{km} \arrow[r, bend left, "1000"] & \text{m} \arrow[l, bend left, "|1000"] \arrow[r, bend left, "100"] & \text{cm} \arrow[l, bend left, "|100"] \arrow[r, bend left, "10"] & \text{mm} \arrow[l, bend left, "|10"] \end{tikzcd}$$

Area
$$\begin{tikzcd} \text{km}^2 \arrow[r, bend left, "1000^2"] & \text{m}^2 \arrow[l, bend left, "|1000^2"] \arrow[r, bend left, "100^2"] & \text{cm}^2 \arrow[l, bend left, "|100^2"] \arrow[r, bend left, "10^2"] & \text{mm}^2 \arrow[l, bend left, "|10^2"] \end{tikzcd}$$

Volume
$$\begin{tikzcd} \text{km}^3 \arrow[r, bend left, "1000^3"] & \text{m}^3 \arrow[l, bend left, "|1000^3"] \arrow[r, bend left, "100^3"] & \text{cm}^3 \arrow[l, bend left, "|100^3"] \arrow[r, bend left, "10^3"] & \text{mm}^3 \arrow[l, bend left, "|10^3"] \end{tikzcd}$$

- To multiply by 10, 100, 1000, etc. move the decimal point one place to the right for each zero; e.g. $3.425 \times 100 = 342.5$

- To divide by 10, 100, 1000 etc. move the decimal point one place to the left for each zero; e.g. $4.10 \div 1000 = 0.0041$

Exercise 1A

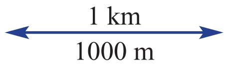
Understanding

1 Write the missing numbers in these sentences involving length.

a There are ☐ m in 1 km.

b There are ☐ mm in 1 cm.

c There are ☐ cm in 1 m.

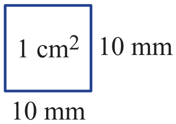


2 Write the missing numbers in these sentences involving area units.

a There are ☐ mm^2 in 1 cm^2.

b There are ☐ cm^2 in 1 m^2.

c There are ☐ m^2 in 1 km^2.



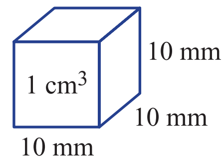
ISBN: 9781107687028
Cambridge University Press

3 Write the missing numbers in these sentences involving volume units.

a There are $\square$ mm^3 in 1 cm^3.

b There are $\square$ m^3 in 1 km^3.

c There are $\square$ cm^3 in 1 m^3.



Fluency

Example 1 Converting length measurements

Convert these length measurements to the units shown in the brackets.

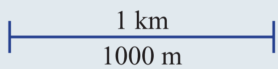
a 8.2 km (m) **b** 45 mm (cm)

Solution

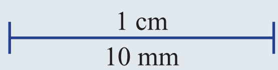
a $8.2 \text{ km} = 8.2 \times 1000$
$= 8200 \text{ m}$

b $45 \text{ mm} = 45 \div 10$
$= 4.5 \text{ cm}$

Explanation



1 km = 1000 m



1 cm = 10 mm

Divide if converting from a smaller unit to a larger unit.

4 Convert the following measurements of length into the units given in the brackets.

a 4.32 cm (mm) **b** 327 m (km)

c 834 cm (m) **d** 0.096 m (mm)

e 297.5 m (km) **f** 0.0127 m (cm)

If converting to a smaller unit, multiply. Otherwise, divide.

Example 2 Converting area measurements

Convert these area measurements to the units shown in the brackets.

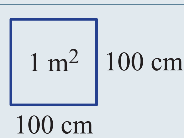
a 930 cm^2 (m^2) **b** 0.4 cm^2 (mm^2)

Solution

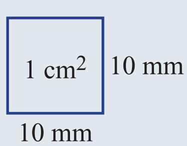
a $930 \text{ cm}^2 = 930 \div 10\,000$
$= 0.093 \text{ m}^2$

b $0.4 \text{ cm}^2 = 0.4 \times 100$
$= 40 \text{ mm}^2$

Explanation



$1 \text{ m}^2 = 100 \times 100$
$= 10\,000 \text{ cm}^2$



$1 \text{ cm}^2 = 10 \times 10$
$= 100 \text{ mm}^2$

ISBN: 9781107687028

5 Convert the following area measurements into the units given in the brackets.

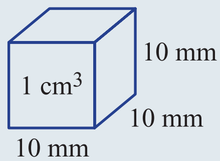
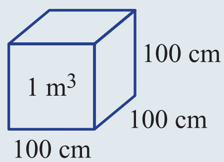
a 3000 cm^2 (mm^2) **b** 0.5 m^2 (cm^2)
c 5 km^2 (m^2) **d** 2 980 000 mm^2 (cm^2)
e 537 cm^2 (mm^2) **f** 0.023 m^2 (cm^2)

1 cm^2 = 100 mm^2
1 m^2 = 10 000 cm^2
1 km^2 = 1 000 000 m^2

Example 3 Converting volume measurements

Convert these volume measurements to the units shown in the brackets.

a 3.72 cm^3 (mm^3) **b** 4300 cm^3 (m^3)

Solution	Explanation
a $3.72\text{ cm}^3 = 3.72 \times 1000 = 3720\text{ mm}^3$	 $1\text{ cm}^3 = 10 \times 10 \times 10 = 1000\text{ mm}^3$
b $4300\text{ cm}^3 = 4300 \div 1\,000\,000 = 0.0043\text{ m}^3$	 $1\text{ m}^3 = 100 \times 100 \times 100 = 1\,000\,000\text{ cm}^3$

6 Convert these volume measurements into the units given in the brackets.

a 2 cm^3 (mm^3) **b** 0.2 m^3 (cm^3)
c 5700 mm^3 (cm^3) **d** 0.015 km^3 (m^3)
e 28 300 000 m^3 (km^3) **f** 762 000 cm^3 (m^3)

1 cm^3 = 1000 mm^3
1 m^3 = 1 000 000 cm^3
1 km^3 = 1 000 000 000 m^3

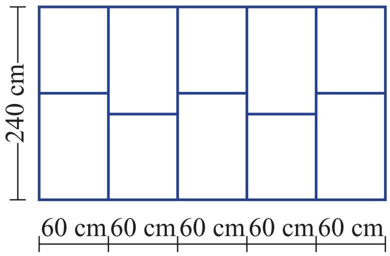
Problem-solving and Reasoning

7 An athlete has completed a 5.5 km run. How many metres did the athlete run?

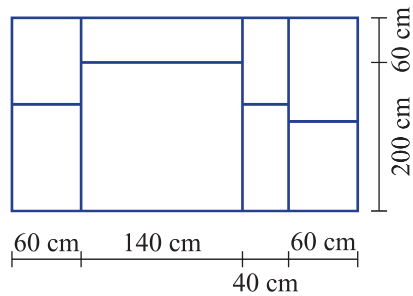
ISBN: 9781107687028

8 Determine the metres of timber needed to construct the following frames.

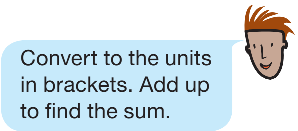
a



b



9 Find the total sum of the measurements given, expressing your answer in the units given in the brackets.



a 10 cm, 18 mm (mm) **b** 1.2 m, 19 cm, 83 mm (cm)
c 453 km, 258 m (km) **d** 400 mm^2, 11.5 cm^2 (cm^2)
e 0.3 m^2, 251 cm^2 (cm^2) **f** 0.000 03 km^2, 9 m^2, 37 000 000 cm^2 (m^2)
g 482 000 mm^3, 2.5 cm^3 (mm^3) **h** 0.000 51 km^3, 27 300 m^3 (m^3)

10 A snail is moving at a rate of 43 mm every minute. How many centimetres will the snail move in 5 minutes?

11 Why do you think that builders measure many of their lengths using only millimetres, even their long lengths?

Special units

12 Many units of measurement apart from those relating to mm, cm, m and km are used in our society. Some of these are described here.

Length	Inches	1 inch ≈ 2.54 cm = 25.4 mm
	Feet	1 foot = 12 inches ≈ 30.48 cm
	Miles	1 mile ≈ 1.609 km = 1609 m
Area	Squares	1 square = 100 square feet
	Hectares (ha)	1 hectare = 10 000 m^2
Volume	Millilitres (mL)	1 millilitre = 1 cm^3
	Litres (L)	1 litre = 1000 cm^3

Convert these special measurements into the units given in the brackets. Use the conversion information given earlier to help.

a 5.5 miles (km) **b** 54 inches (feet) **c** 10.5 inches (cm)
d 2000 m (miles) **e** 5.7 ha (m^2) **f** 247 cm^3 (L)
g 8.2 L (mL) **h** 5.5 m^3 (mL) **i** 10 squares (sq. feet)
j 2 m^3 (L) **k** 1 km^2 (ha) **l** 152 000 mL (m^3)

ISBN: 9781107687028 Cambridge University Press

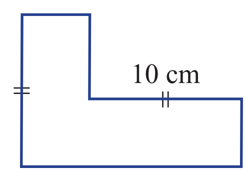
1.2 Perimeter

Perimeter is a measure of length around the outside of a shape. We calculate perimeter when ordering ceiling cornices for a room, or materials for fencing a paddock or building a television frame.

Let's start: L-shaped perimeters

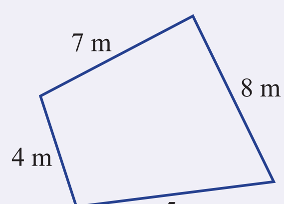
This L-shaped figure includes only right (90°) angles. Only two measurements are given.

- Can you figure out any other side lengths?
- Is it possible to find its perimeter? Why?

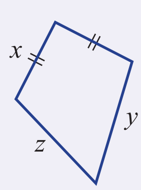


Key ideas

- **Perimeter** is the distance around the outside of a two-dimensional shape.
  - To find the perimeter we add all the lengths of the sides in the same units.
    $P = 4 + 5 + 7 + 8 = 24$ m



  - If two sides of a shape are the same length they are labelled with the same markings.
    $P = 2x + y + z$



Perimeter The total distance (length) around the outside of a figure

ISBN: 9781107687028 Cambridge University Press

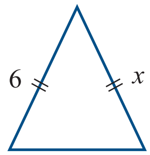
Exercise 1B

Understanding

1 Write the missing word: The distance around the outside of a shape is called the ____________.

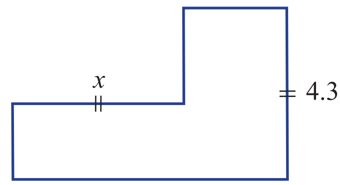
2 Write down the value of x for these shapes.

a



b

c

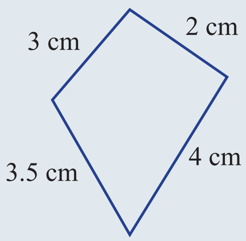


Fluency

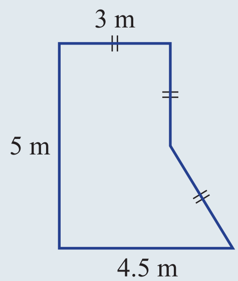
Example 4 Finding perimeters of basic shapes

Find the perimeter of these shapes.

a



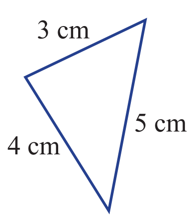
b



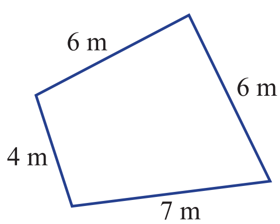
Solution	**Explanation**
a Perimeter $= 3 + 2 + 4 + 3.5$ $= 12.5$ cm	Add all the lengths of the sides together.
b Perimeter $= 5 + 4.5 + 3 \times 3$ $= 18.5$ m	Three lengths have the same markings and are therefore the same length.

3 Find the perimeter of these shapes.

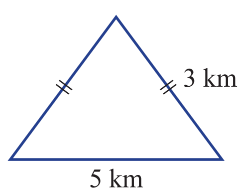
a

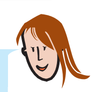


b



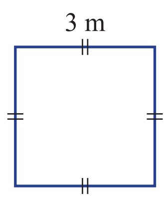
c



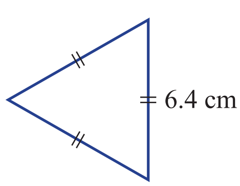


Sides with the same markings are the same length.

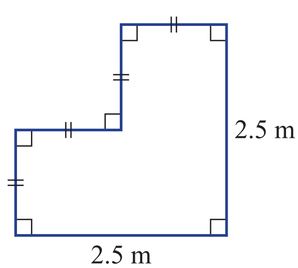
d



e



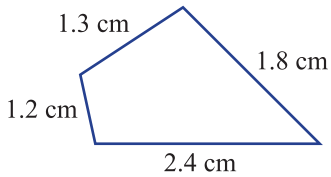
f

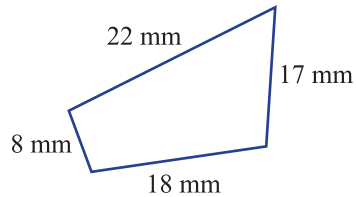


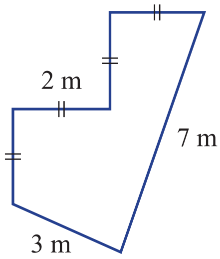
ISBN: 9781107687028

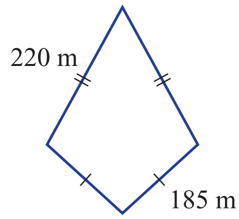
1B

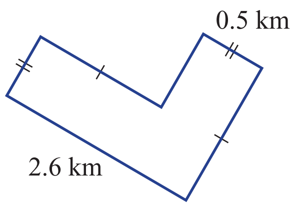
4 Find the perimeter of these shapes.

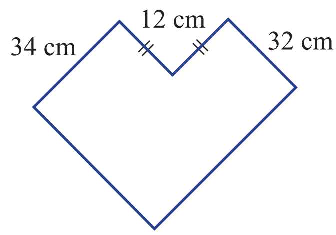
a


b


c


d


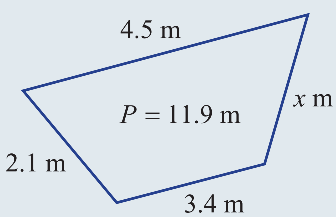
e


f


Problem-solving and Reasoning

Example 5 Finding a missing side length

Find the value of x for this shape with the given perimeter.



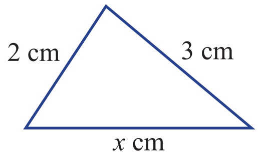
Solution	Explanation
$4.5 + 2.1 + 3.4 + x = 11.9$	All the sides add to 11.9 in length.
$10 + x = 11.9$	Simplify.
$x = 1.9$	Subtract 10 from both sides.

ISBN: 9781107687028

Cambridge University Press

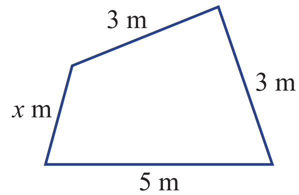
5 Find the value of x for these shapes with the given perimeters.

a



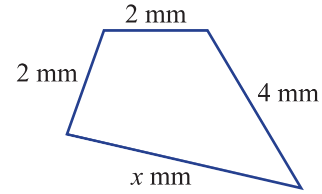
Perimeter = 9 cm

b



Perimeter = 13 m

c

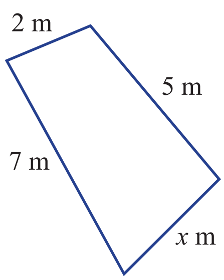


Perimeter = 14 mm

Add up all the sides then determine the value of x to suit the given perimeters.

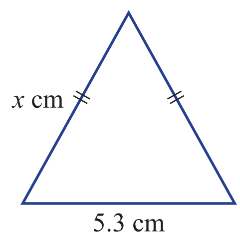
6 Find the value of x for these shapes with the given perimeters.

a



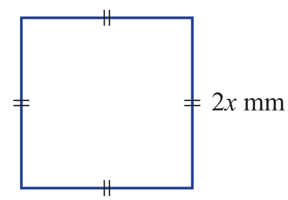
Perimeter = 17 m

b



Perimeter = 22.9 cm

c



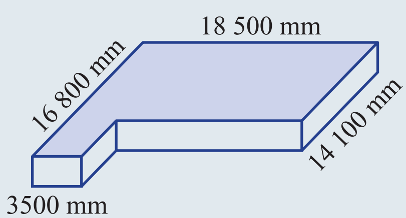
Perimeter = 0.8 mm

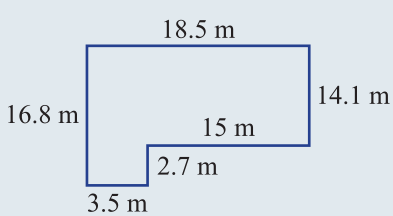
Example 6 Working with concrete slabs

For the concrete slab shown:

a draw a new diagram showing all the measurements in metres

b determine the lineal metres of timber needed to surround it



Solution	Explanation
a 	Convert your measurements and place them all on the diagram. $1 \text{ m} = 100 \times 10 = 1000 \text{ mm}$ Add or subtract to find the missing measurements.
b Perimeter $= 18.5 + 16.8 + 3.5 + 2.7 + 15 + 14.1$ $= 70.6$ m	Add all the measurements.
The lineal metres of timber needed is 70.6 m.	Write your answer in words.

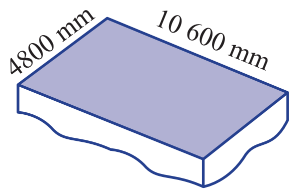
ISBN: 9781107687028

Cambridge University Press

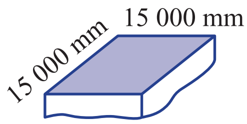
7 For the concrete slabs shown:

- **i** draw a new diagram with the measurements in metres
- **ii** determine the lineal metres of timber needed to surround it

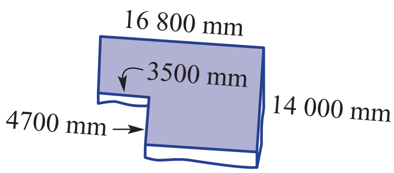
a



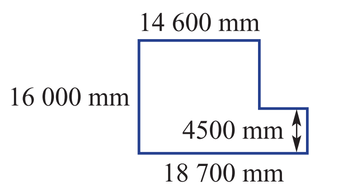
b



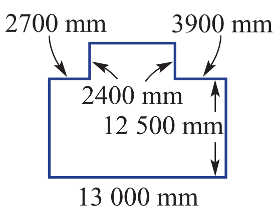
c



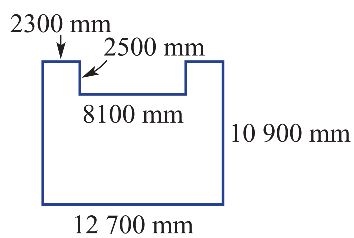
d



e

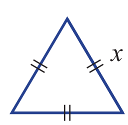


f



8 A rectangular paddock has perimeter 100 m. Find the width of the paddock if its length is 30 m.

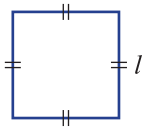
9 The equilateral triangle shown has perimeter 45 cm. Find its side length.

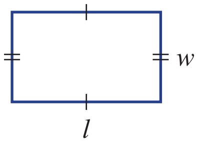


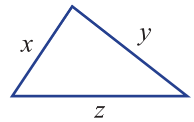
ISBN: 9781107687028

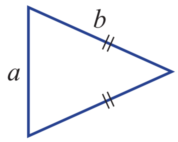
Cambridge University Press

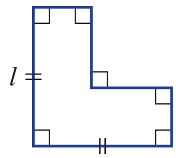
10 Find formulas for the perimeter of these shapes using the pronumerals given.

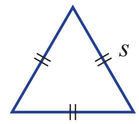
a


b


c


d


e


f


A formula for perimeter could be $P = l + 2w$ or $P = a + b + c$.

How many different tables?

11 A large dining table is advertised with a perimeter of 12 m. The length and width are a whole number of metres (e.g. 1 m, 2 m, …). How many different-sized tables are possible?

12 How many rectangles (using whole number lengths) have perimeters between 16 and 20 m inclusive?

ISBN: 9781107687028

1.3 Circumference

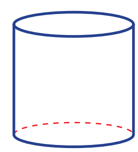
To find the distance around the outside of a circle – the circumference – we use the special number called pi (π). Pi provides a direct link between the diameter of a circle and the circumference of that circle.

The wheel is one of the most useful components in many forms of machinery, and its shape, of course, is a circle. One revolution of a vehicle's wheel moves the vehicle a distance equal to the wheel's circumference.

Let's start: When circumference = height

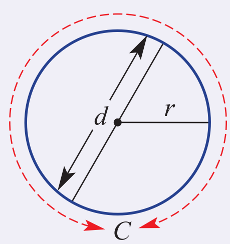
Here is an example of a cylinder.

- Try drawing your own cylinder so that its height is equal to the circumference of the circular top.
- How would you check that you have drawn a cylinder with the correct dimensions? Discuss.


height

Key ideas

- The **radius** is the distance from the centre of a circle to a point on the circle.
- The **diameter** is the distance across a circle through its centre.
  – Radius $= \frac{1}{2}$ diameter or diameter $= 2 \times$ radius
- **Circumference** is the distance around a circle.
  – Circumference $= 2\pi \times$ radius
  $= 2\pi r$
  or circumference $= \pi \times$ diameter
  $= \pi d$
  – π is a special number and can be found on your calculator. It can be approximated by $\pi \approx 3.142$



Radius The distance from the centre of a circle to its outside edge

Diameter A line passing through the centre of a circle with its end points on the circumference

Circumference The distance around the outside of a circle; the curved boundary

Exercise 1C

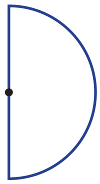
Understanding

1 Write the formula for the circumference of a circle using:
 a d for diameter
 b r for radius

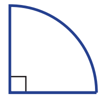
ISBN: 9781107687028

2 What fraction of a circle is shown here?

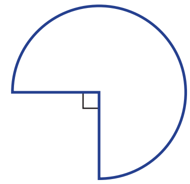
a



b



c



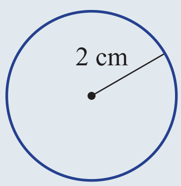
3 **a** What is the diameter of a circle if its radius is 4.3 m?
b What is the radius of a circle if its diameter is 3.6 cm?

Fluency

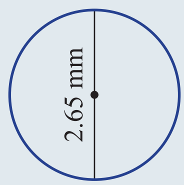
Example 7 Finding the circumference of a circle

Find the circumference of these circles to two decimal places.

a



b

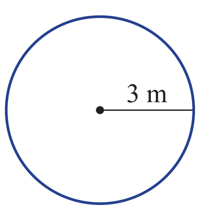


Solution	**Explanation**
a Circumference $= 2\pi r$	Write the formula involving radius.
$= 2\pi(2)$	Substitute $r = 2$
$= 12.57$ cm	Write your answer to two decimal places.
b $C = \pi d$	Write the formula involving diameter.
$= \pi(2.65)$	Substitute $d = 2.65$
$= 8.33$ mm	Write your answer to two decimal places.

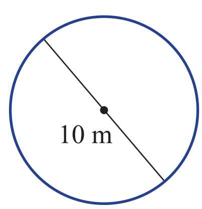
4 Find the circumference of these circles to two decimal places.

Use $C = 2\pi r$ or $C = \pi d$.

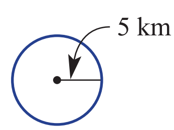
a



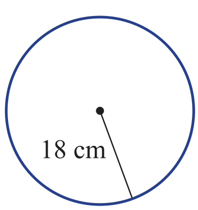
b



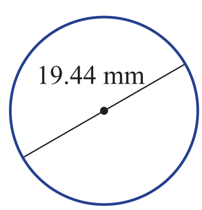
c



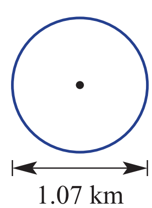
d



e



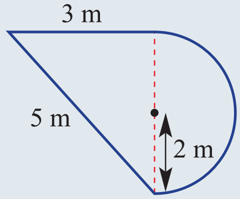
f



ISBN: 9781107687028 Cambridge University Press

Example 8 Finding perimeters of composite shapes

Find the perimeter of this composite shape to two decimal places.



Solution

$P = 3 + 5 + \frac{1}{2} \times 2\pi(2)$
$= 8 + 2\pi$
$= 14.28$ m

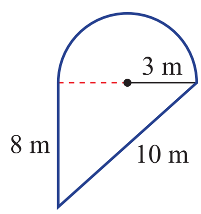
Explanation

Add all the sides, including half a circle.
Simplify.
Round as instructed.

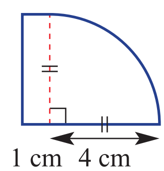
5 Find the perimeter of these composite shapes to two decimal places.

Don't forget to add the straight sides to the fraction $\left(\frac{1}{4}, \frac{1}{2} \text{ or } \frac{3}{4}\right)$ of the circumference.

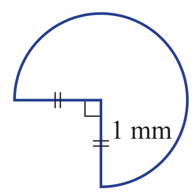
a



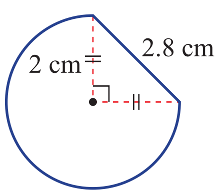
b



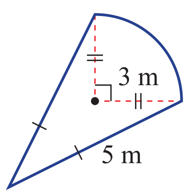
c



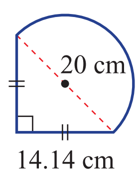
d



e



f



Problem-solving and Reasoning

6 David wishes to build a circular fish pond. The diameter of the pond is to be 3 m.

a How many linear metres of bricks are needed to surround it? Round your answer to two decimal places.

b What is the cost if the bricks are $45 per metre? (Use your answer from part **a**.)

7 The wheels of a bike have a diameter of 1 m.

For one revolution, use $C = \pi d$.

a How many metres will the bike travel (to two decimal places) after:

i one full turn of the wheels?

ii 15 full turns of the wheels?

b How many kilometres will the bike travel after 1000 full turns of the wheels? (Give your answer correct to two decimal places.)

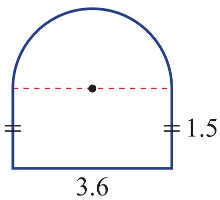
8 What is the minimum number of times a wheel of diameter 1 m needs to spin to cover a distance of 1 km? You will need to find the circumference of the wheel first. Answer as a whole number.

ISBN: 9781107687028 Cambridge University Press

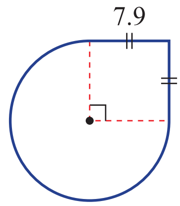
9 Find the perimeter of these composite shapes to two decimal places.

Make sure you know the radius or diameter of the circle you are dealing with.

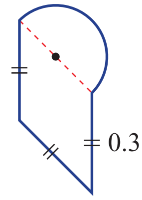
a



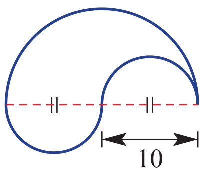
b



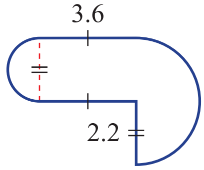
c



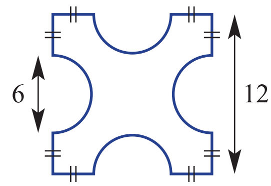
d



e



f



10 a Rearrange the formula for the circumference of a circle, $C = 2\pi r$, to write r in terms of C.

b Find, to two decimal places, the radius of a circle with the given circumference.

- **i** 35 cm
- **ii** 1.85 m
- **iii** 0.27 km

To make r the subject, divide both sides by 2π.

Target practice

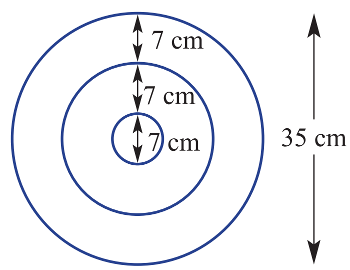
11 A target is made up of three rings, as shown.

a Find the radius of the smallest ring.

b Find, to two decimal places, the circumference of:

- **i** the smallest ring
- **ii** the middle ring
- **iii** the outside ring

c If the circumference of a different ring was 80 cm, what would its radius be, to two decimal places?

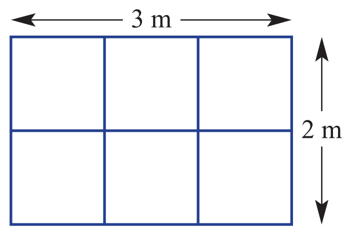


ISBN: 9781107687028

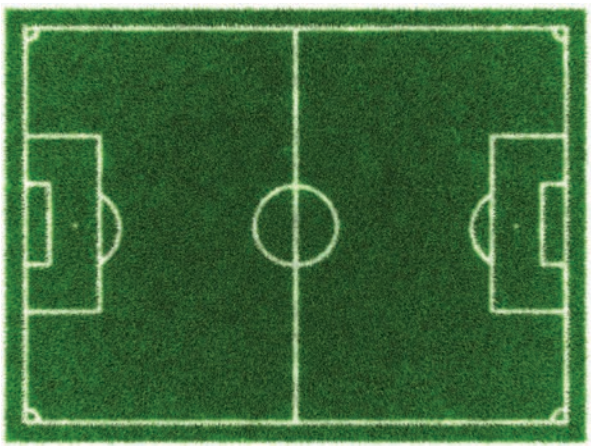
1.4 Area

In this simple diagram, a rectangle with side lengths 2 m and 3 m has an area of 6 square metres or 6 m^2. This is calculated by counting the number of squares (each a square metre) that make up the rectangle.

We use formulas to help us quickly count the number of square units contained within a shape. For this rectangle, for example, the formula $A = lw$ simply tells us to multiply the length by the width to find the area.



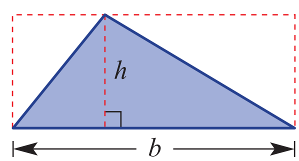
area = 6 m^2



Let's start: How does $A = \frac{1}{2}bh$ work for a triangle?

Look at this triangle, including its rectangular red dashed lines.

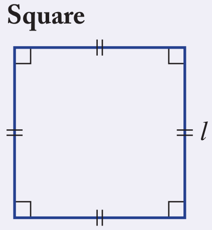
- How does the shape of the triangle relate to the shape of the outside rectangle?
- How can you use the formula for a rectangle to help find the area of the triangle (or parts of the triangle)?
- Why is the rule for the area of a triangle given by $A = \frac{1}{2}bh$?



Key ideas

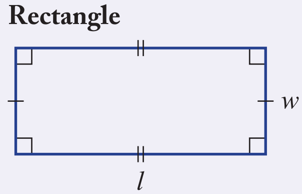
- The **area** of a two-dimensional shape is the number of square units contained within its boundaries.
- Some of the common area formulas are as follows.

Square



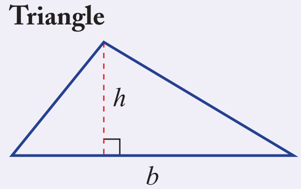
Area = l^2

Rectangle



Area = lw

Triangle

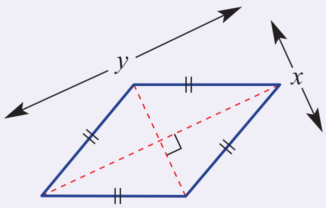


Area = $\frac{1}{2}bh$

Area The number of square units needed to cover the space inside the boundaries of a 2D shape

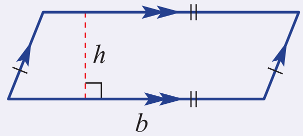
ISBN: 9781107687028

Rhombus



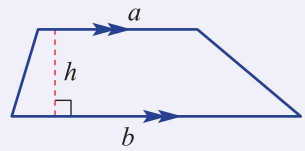
Area $= \frac{1}{2}xy$

Parallelogram



Area $= bh$

Trapezium



Area $= \frac{1}{2}(a+b)h$

- The 'height' in a triangle, parallelogram or trapezium should be perpendicular (at 90°) to the base.

Exercise 1D

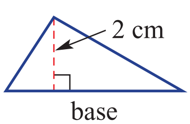
Understanding

1 Match each shape (**a**–**f**) with its area formula (**A**–**F**).

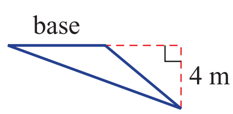
a	square	**A**	$A = \frac{1}{2}bh$
b	rectangle	**B**	$A = lw$
c	rhombus	**C**	$A = bh$
d	parallelogram	**D**	$A = \frac{1}{2}(a+b)h$
e	trapezium	**E**	$A = l^2$
f	triangle	**F**	$A = \frac{1}{2}xy$

2 These shapes show the base and a height length. What is the height of each shape?

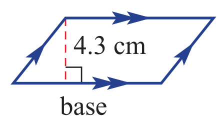
a



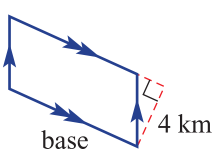
b



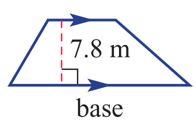
c



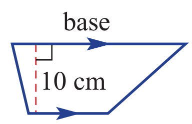
d



e



f

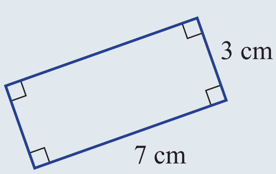


Fluency

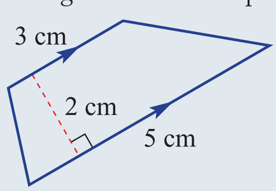
Example 9 Using area formulas

Find the area of these basic shapes, rounding to two decimal places where necessary.

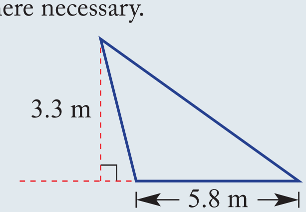
a



b



c



ISBN: 9781107687028

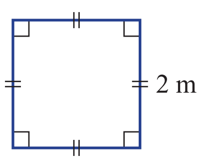
1D

Solution	Explanation
a Area $= lw$	Write the formula for a rectangle.
$= 7 \times 3$	Substitute the lengths $l = 7$ and $w = 3$.
$= 21\text{ cm}^2$	Simplify and add the units.
b Area $= \frac{1}{2}(a + b)h$	Write the formula for a trapezium.
$= \frac{1}{2}(3 + 5) \times 2$	Substitute the lengths $a = 3$, $b = 5$ and $h = 2$.
$= 8\text{ cm}^2$	Simplify and add the units.
c Area $= \frac{1}{2}bh$	Write the formula for a triangle.
$= \frac{1}{2}(5.8)(3.3)$	Substitute the lengths $b = 5.8$ and $h = 3.3$.
$= 9.57\text{ m}^2$	Simplify and add the units.

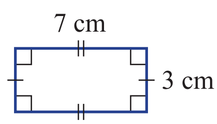
3 Find the area of these basic shapes, rounding to two decimal places where necessary.

First choose the correct formula and substitute for each pronumeral (letter).

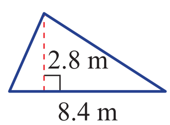
a



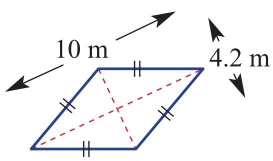
b



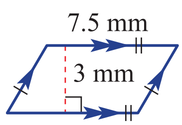
c



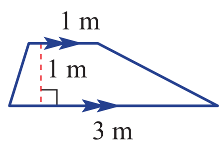
d



e

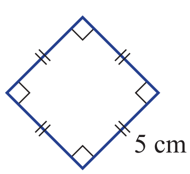


f

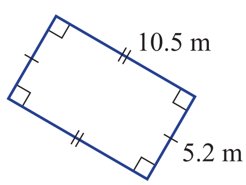


4 Find the area of these basic shapes, rounding to two decimal places where necessary.

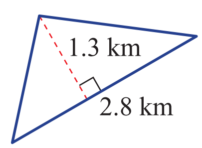
a



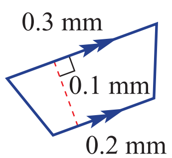
b



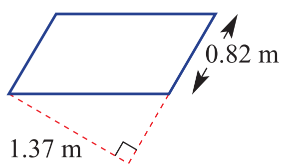
c



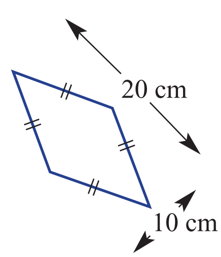
d



e



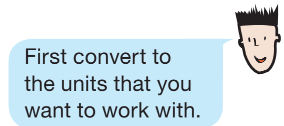
f



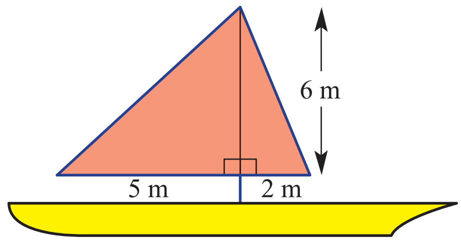
ISBN: 9781107687028

Problem-solving and Reasoning

5 A rectangular table top is 1.2 m long and 80 cm wide. Find the area of the table top using:
a square metres (m^2) **b** square centimetres (cm^2)

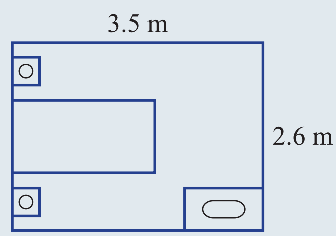


6 Two triangular sails have side lengths as shown. Find the total area of the two sails.



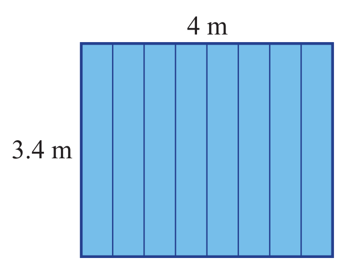
Example 10 Finding areas of floors

Christine decides to use carpet squares to cover the floor of her bedroom, shown at right. Determine:
a the area of floor to be covered
b the total cost if the carpet squares cost $32 a square metre

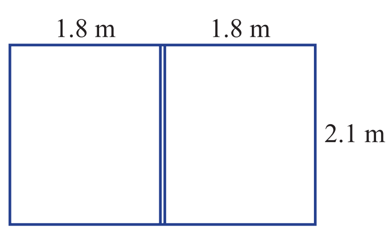


Solution	Explanation
a Area of floor $= l \times w$ $= 3.5 \times 2.6$ $= 9.1\ m^2$	The room is a rectangle, so use $A = l \times w$ to calculate the total floor space.
b Cost of carpet squares $= 9.1 \times 32$ $= \$291.20$	Every square metre of carpet squares costs $32.

7 Jack's shed is to have a flat roof, which he decides to cover with metal sheets.
a Determine the total area of the roof.
b If the metal roofing costs $11 a square metre, how much will it cost in total?



8 A sliding door has two glass panels. Each of these is 2.1 m high and 1.8 m wide.
a How many square metres of glass are needed?
b What is the total cost of the glass if the price is $65 per square metre?

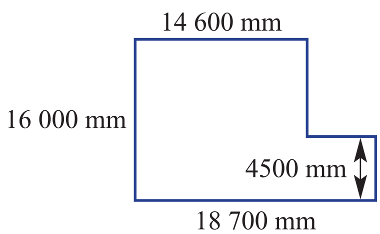


ISBN: 9781107687028 Cambridge University Press

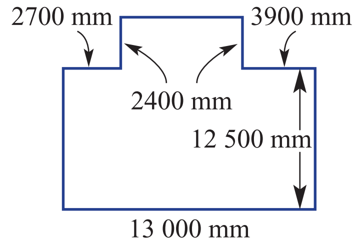
9 A rectangular window has a whole number measurement for its length and width and its area is 24 m^2. Write down the possible lengths and widths for the window.

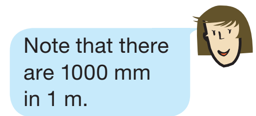
10 Determine the area of the houses shown, in square metres (correct to two decimal places).

a

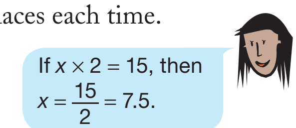


b

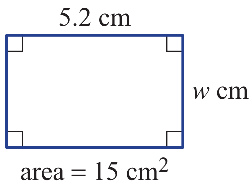




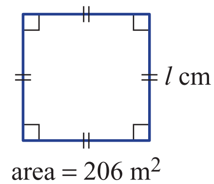
11 Find the value of the pronumeral in these shapes, rounding to two decimal places each time.



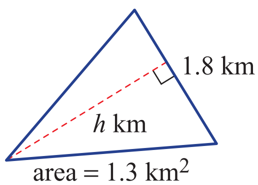
a



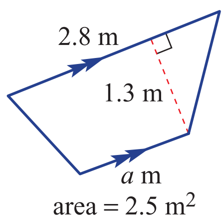
b



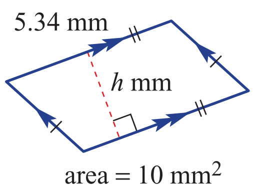
c



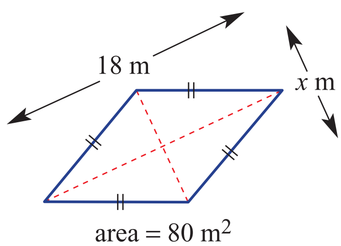
d



e



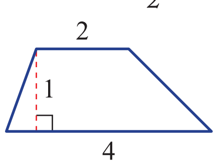
f



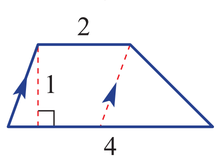
Four ways to find the area of a trapezium

12 Find the area of this trapezium using each of the suggested methods.

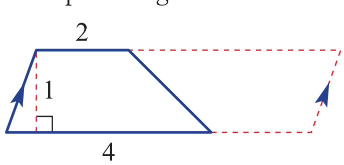
a Formula $A = \frac{1}{2}(a + b)h$



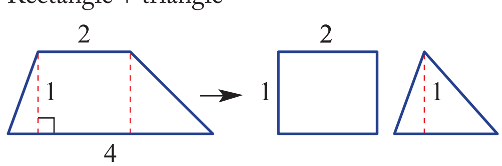
b Parallelogram and triangle



c Half-parallelogram



d Rectangle + triangle



ISBN: 9781107687028

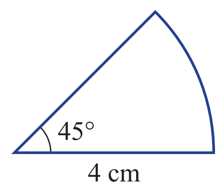
1.5 Area of a circle

Like its circumference, a circle's area is linked to the special number pi (π). The area is the product of pi and the square of the radius, so $A = \pi r^2$.

Knowing the formula for the area of a circle helps us build circular garden beds, plan water sprinkler systems and estimate the damage caused by an oil slick from a ship in calm seas.

Let's start: What fraction is that?

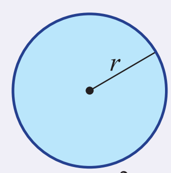
When finding areas of sectors, we first need to decide what fraction of a circle we are dealing with. This sector, for example, has a radius of 4 cm and a 45° angle.



- What fraction of a full circle is shown in this sector?
- How can you use this fraction to help find the area of this sector?
- How would you set out your working?

Key ideas

- The formula for finding the area (A) of a circle of radius r is given by the equation: $A = \pi r^2$
- If the diameter (d) of the circle is given, determine the radius before calculating the area of the circle: $r = d \div 2$

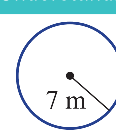


$A = \pi r^2$

Exercise 1E

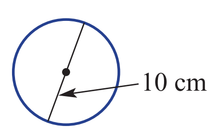
Understanding

1 Which is the correct working step for the area of this circle?



A $A = \pi(7)$ **B** $A = 2\pi(7)$ **C** $A = \pi(14)^2$ **D** $A = (\pi 7)^2$ **E** $A = \pi(7)^2$

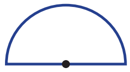
2 Which is the correct working step for the area of this circle?



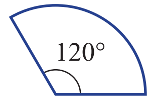
A $A = \pi(10)^2$ **B** $A = (\pi 10)^2$ **C** $A = \pi(5)^2$ **D** $A = 2\pi(5)$ **E** $A = 5\pi$

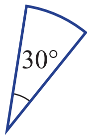
ISBN: 9781107687028

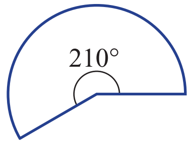
3 What fraction of a circle is shown by these sectors? Simplify your fraction.

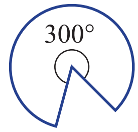
a


b


c


d


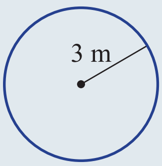
e


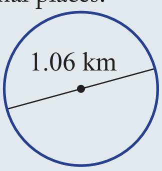
f


Fluency

Example 11 Finding areas of circles

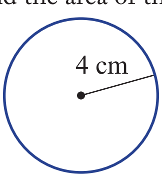
Find the area of these circles, correct to two decimal places.

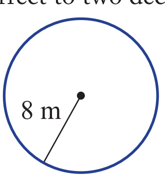
a


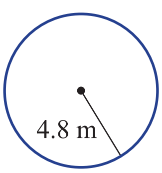
b


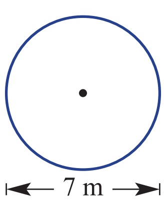
Solution	Explanation
a $A = \pi r^2$	Write the formula.
$= \pi(3)^2$	Substitute $r = 3$. Evaluate $3^2 = 9$ then multiply by π.
$= \pi \times 9$	
$= 28.27\ \text{m}^2$	
b Radius $r = 1.06 \div 2 = 0.53$ km	Find the radius, given the diameter of 1.06.
$A = \pi r^2$	Write the formula.
$= \pi(0.53)^2$	Substitute $r = 0.53$.
$= 0.88\ \text{km}^2$	Write your answer to two decimal places with units.

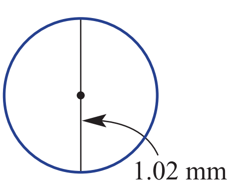
4 Find the area of these circles, correct to two decimal places.

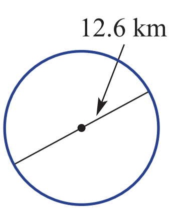
a


b


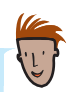
c


d


e


f


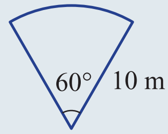
Remember:
$r = d \div 2$



ISBN: 9781107687028

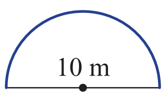
Example 12 Finding areas of sectors

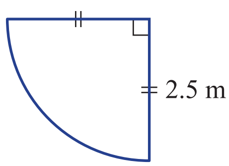
Find the area of this sector, correct to two decimal places.

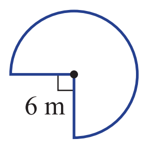


Solution	**Explanation**
Fraction of circle $= \dfrac{60}{360} = \dfrac{1}{6}$	The sector uses 60° out of the 360° in a whole circle.
$\text{Area} = \dfrac{1}{6} \times \pi r^2$	Write the formula, including the fraction part.
$= \dfrac{1}{6} \times \pi(10)^2$	Substitute $r = 10$.
$= 52.36\ \text{m}^2$	Write your answer to two decimal places.

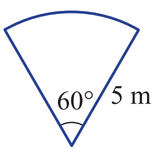
5 Find the area of these sectors, correct to two decimal places.

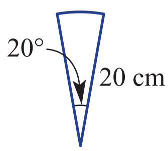
a


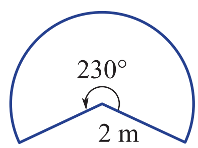
b


c


First determine the fraction of a full circle that you are dealing with.

d


e


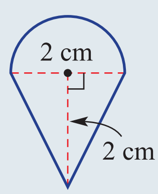
f


Problem-solving and Reasoning

6 A pizza with diameter 40 cm is divided into eight equal parts. Find the area of each portion, correct to one decimal place.

Example 13 Finding areas of composite shapes

Find the area of this composite shape, correct to two decimal places.



Solution	**Explanation**
$A = \dfrac{1}{2}\pi r^2 + \dfrac{1}{2}bh$	The shape is made up of a semicircle and a triangle. Write the formulas for both.
$= \dfrac{1}{2}\pi\,(1)^2 + \dfrac{1}{2}(2)\,(2)$	Substitute $r = 1$, $b = 2$ and $h = 2$
$= 1.5707\ldots + 2$	
$= 3.57\ \text{cm}^2$	Write your answer to two decimal places with units.

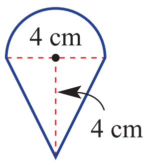
ISBN: 9781107687028

Cambridge University Press

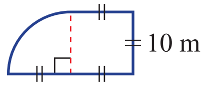
1E

7 Find the area of these composite shapes, correct to two decimal places.

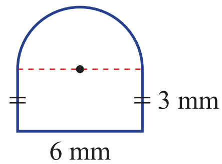
a



b

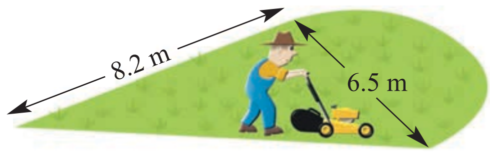


c



Find the area of each shape within the larger shape, then add them. For example, triangle + semicircle.

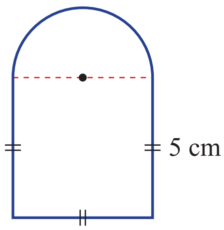
8 The lawn area in a backyard is made up of a semicircular region with diameter 6.5 m and a triangular region of length 8.2 m, as shown. Find the total area of lawn in the backyard, correct to two decimal places.



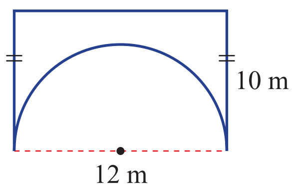
9 Find the area of these composite shapes, correct to one decimal place.

Use addition or subtraction depending on the shape given.

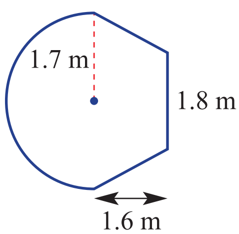
a



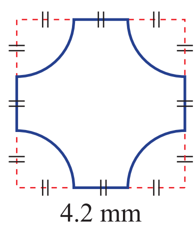
b



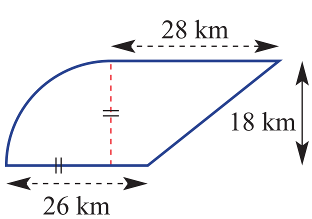
c



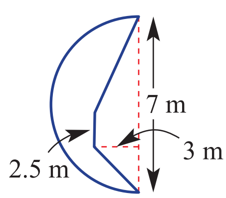
d



e



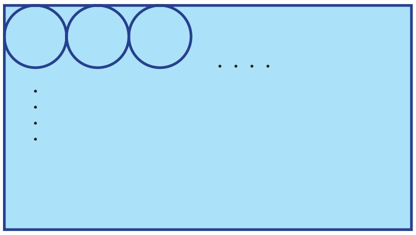
f



ISBN: 9781107687028 Cambridge University Press

Circular pastries

10 A rectangular piece of pastry is used to create small circular pastry discs for the base of Christmas tarts. The rectangular piece of pastry is 30 cm long and 24 cm wide, and each circular piece has a diameter of 6 cm.



- **a** How many circular pieces of pastry can be removed from the rectangle?
- **b** Find the total area removed from the original rectangle, correct to two decimal places.
- **c** Find the total area of pastry remaining, correct to two decimal places.
- **d** If the remaining pastry was collected and re-rolled to the same thickness, how many circular pieces could be cut? Assume that the pastry can be re-rolled many times.

ISBN: 9781107687028

Cambridge University Press

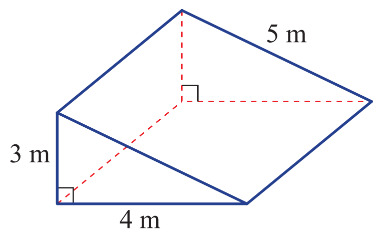
1.6 Surface area of prisms

The total surface area of a three-dimensional object can be found by finding the sum of the areas of each of the shapes that make up the surface of the object.

Let's start: Which net?

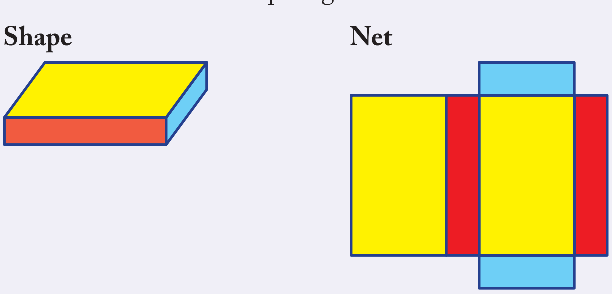
The solid below is a triangular prism with a right-angled triangle as its cross-section.

- How many different types of shapes make up its outside surface?
- What is a possible net for the solid? Is there more than one?
- How would you find the total surface area?



Key ideas

- To calculate the **total surface area (TSA)** of a solid:
  - draw a net (a two-dimensional drawing including all the surfaces)
  - determine the area of each shape inside the net
  - add the areas of each shape together.



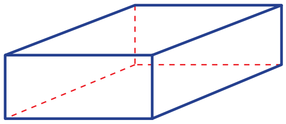
Total surface area (TSA) The total number of square units needed to cover the outside of a solid

ISBN: 9781107687028

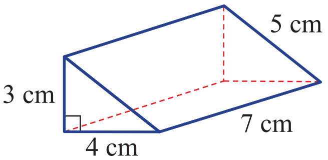
Exercise 1F

Understanding

1 For a rectangular prism, answer the following.
  a How many faces does the prism have?
  b How many *different* rectangles form the surface of the prism?



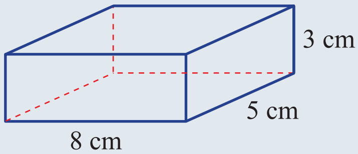
2 For this triangular prism, answer the following.
  a What is the area of the largest surface rectangle?
  b What is the area of the smallest surface rectangle?
  c What is the combined area of the two triangles?
  d What is the total surface area?



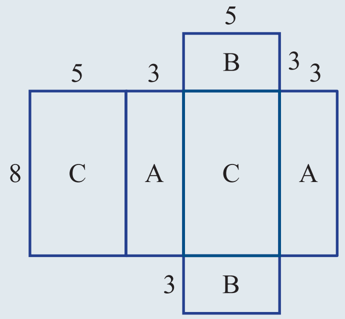
Fluency

Example 14 Finding the TSA of a rectangular prism

Find the total surface area (TSA) of this rectangular prism by first drawing its net.



Solution



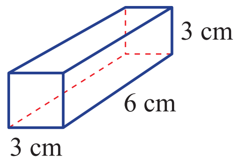
$$\begin{aligned} \text{TSA} &= 2 \times \text{area of A} + 2 \times \text{area of B} + 2 \times \text{area of C} \\ &= 2 \times (8 \times 3) + 2 \times (5 \times 3) + 2 \times (8 \times 5) \\ &= 158 \text{ cm}^2 \end{aligned}$$

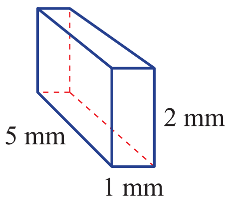
Explanation

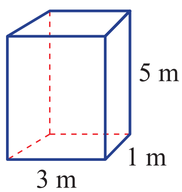
Draw the net of the solid, labelling the lengths and shapes of equal areas.

Describe each area.
Substitute the correct lengths.
Simplify and add units.

3 Find the total surface area (TSA) of these rectangular prisms by first drawing their nets.

a


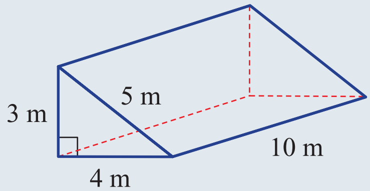
b


c


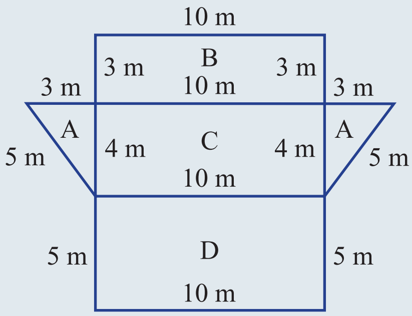
ISBN: 9781107687028

Example 15 Finding the TSA of a triangular prism

Find the surface area of the triangular prism shown.



Solution | **Explanation**



Draw a net of the object with all the measurements and label the sections to be calculated.

Total surface area
= 2 × area A + area B + area C + area D

There are two triangles with the same area and three different rectangles.

$= 2 \times \left(\frac{1}{2} \times 3 \times 4\right) + (3 \times 10) + (4 \times 10) + (5 \times 10)$

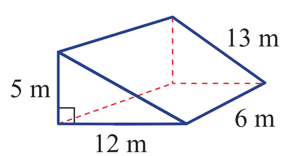
Substitute the correct lengths.

$= 12 + 30 + 40 + 50$
$= 132\text{ m}^2$

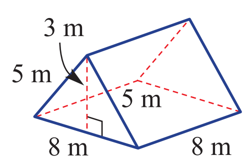
Calculate the area of each shape.
Add the areas together.

4 Find the total surface area of the following prisms.

a

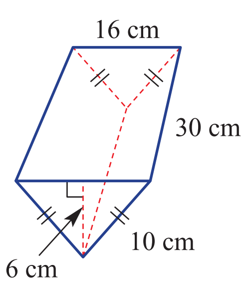


b



There are three rectangles and two identical triangles.

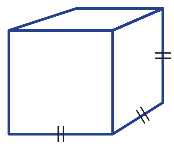
c



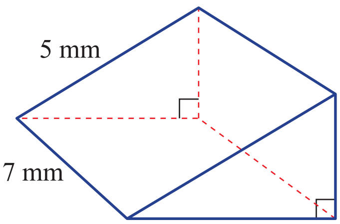
ISBN: 9781107687028 Cambridge University Press

5 Find the TSA of these objects by first drawing a net.

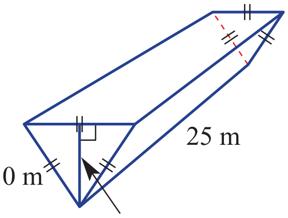
a



b



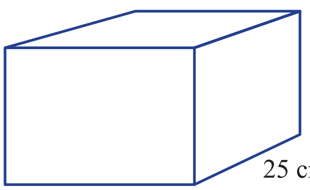
c



Problem-solving and Reasoning

6 A cube of side length 8 cm is to be painted all over with bright red paint. What is the total surface area that is to be painted?

7 What is the minimum amount of paper required to wrap a box with dimensions 25 cm wide, 32 cm long and 20 cm high?



8 An open-topped box is to be covered inside and out with a special material. If the box is 40 cm long, 20 cm wide and 8 cm high, find the minimum amount of material required to cover the box.

Count both inside and out but do not include the top.

9 David wants to paint his bedroom. The ceiling and walls are to be same colour. If the room measures 3.3 m × 4 m and the ceiling is 2.6 m high find the amount of paint needed:

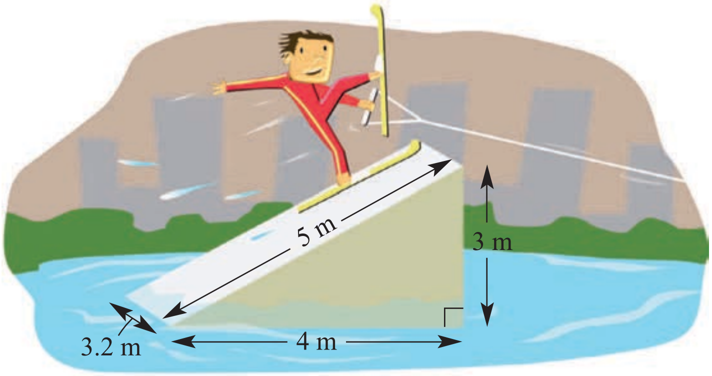
a if each litre covers 10 square metres

b if each litre covers 5 square metres

ISBN: 9781107687028

Cambridge University Press

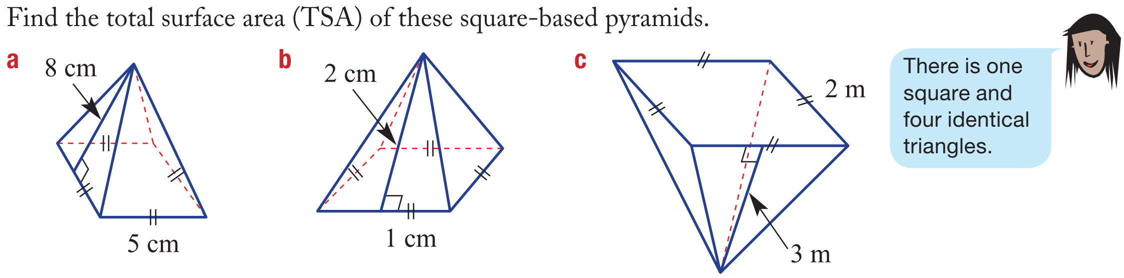
10 A ski ramp in the shape of a triangular prism needs to be painted before the Moomba Classic waterskiing competition in Melbourne. The base and sides of the ramp require a fully waterproof paint, which covers 2.5 square metres per litre. The top needs special smooth paint, which covers only 0.7 square metres per litre.



a Determine the amount of each type of paint required. Round to two decimal places where necessary.

b If the waterproof paint is \$7 per litre and the special smooth paint is \$20 per litre, calculate the total cost of painting the ramp, to the nearest cent. (Use the exact answers from part **a** to help.)

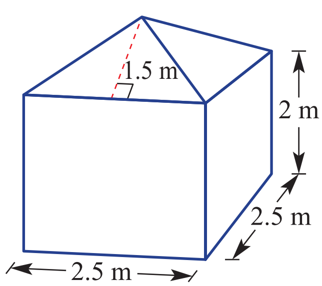
11 Find the total surface area (TSA) of these square-based pyramids.



Will I have enough paint?

12 I have 6 litres of paint and on the tin it says that the coverage is 5.5 m^2 per litre. I wish to paint the four outside walls of a shed and the roof, which has four triangular sections.

Will I have enough paint to complete the job?



ISBN: 9781107687028 Cambridge University Press

1.7 Surface area of a cylinder

Like a prism, a cylinder has a uniform cross-section with identical circles as its two ends. The curved surface of a cylinder can be rolled out to form a rectangle with a length equal to the circumference of the circle.

A can is a good example of a cylinder. We need to know the area of the ends and the curved surface area in order to cut sections from a sheet of aluminium to manufacture the can.

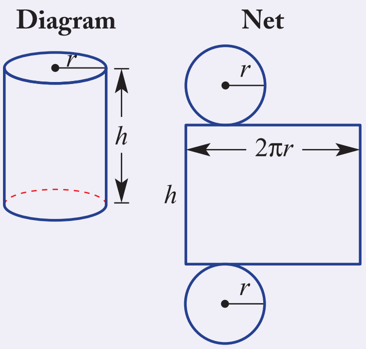
Let's start: Why $2\pi rh$?

We can see from the net of a cylinder (see below) that the total area of the two circular ends is $2 \times \pi r^2$ or $2\pi r^2$. For the curved part, though, consider the following.

- Why can it be drawn as a rectangle? Can you explain this using a piece of paper?
- Why are the dimensions of this rectangle h and $2\pi r$?
- Where does $\text{TSA} = 2\pi r^2 + 2\pi rh$ come from?

Key ideas

- A **cylinder** is a solid with a circular cross-section.
  - The net contains two equal circles and a rectangle. The rectangle has one side length equal to the circumference of the circle.
  - TSA = 2 circles + 1 rectangle
    $= 2\pi r^2 + 2\pi rh$
  - Another way of writing $2\pi r^2 + 2\pi rh$ is $2\pi r(r + h)$.



Cylinder A solid with two parallel, congruent circular faces connected by a curved surface

Exercise 1G

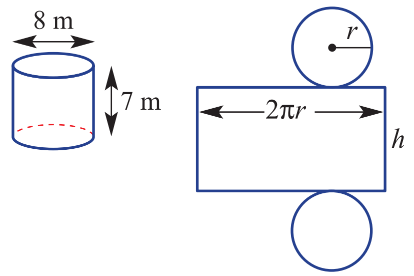
Understanding

1 Write the missing word/expression.
  a The cross-section of a cylinder is a __________.
  b The TSA of a cylinder is $\text{TSA} = 2\pi r^2 +$ __________.

ISBN: 9781107687028

1G

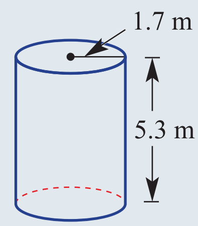
2 A cylinder and its net are shown here.
  a What is the value of:
    i r?
    ii h?
  b Find the value of $2\pi r$, correct to two decimal places.
  c Use $\text{TSA} = 2\pi r^2 + 2\pi rh$ to find the total surface area, correct to two decimal places.

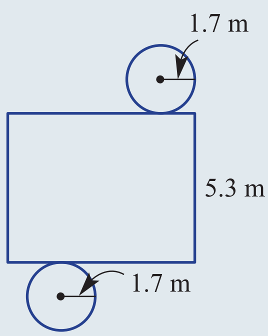


Fluency

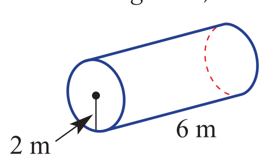
Example 16 Finding the surface area of a cylinder

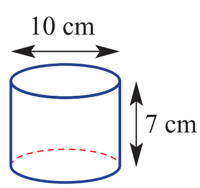
By first drawing a net, find the total surface area of this cylinder, to two decimal places.

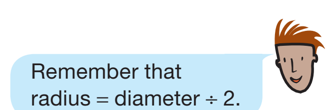


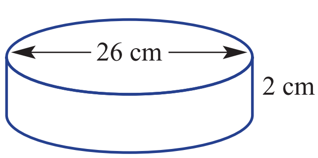
Solution	Explanation
	Draw the net and label the lengths.
TSA = 2 circles + 1 rectangle	Write what you need to calculate.
$= 2\pi r^2 + 2\pi rh$	Write the formula.
$= 2\pi(1.7)^2 + 2\pi(1.7)(5.3)$	Substitute the correct lengths: $r = 1.7$ and $h = 5.3$.
$= 74.77\ \text{m}^2$	Round to two decimal places.

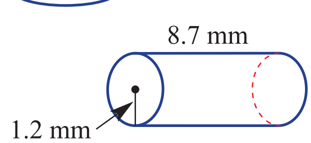
3 By first drawing a net, find the total surface area of these cylinders, to two decimal places.

a 

b 



c 

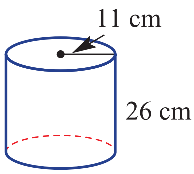
d 

ISBN: 9781107687028

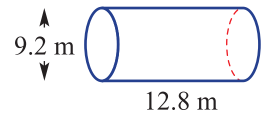
4 Use the formula $TSA = 2\pi r^2 + 2\pi rh$ to find the total surface area of these cylinders, to one decimal place.

Using $2\pi r^2 + 2\pi rh$ is the same as using $2\pi r(r + h)$.

a



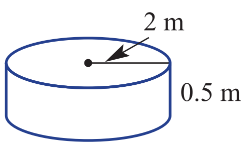
b



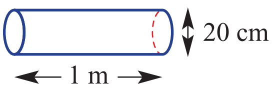
5 Find the area of the curved surface only of these cylinders, to one decimal place.

Find only the rectangular part of the net, so use $A = 2\pi rh$. Watch the units.

a

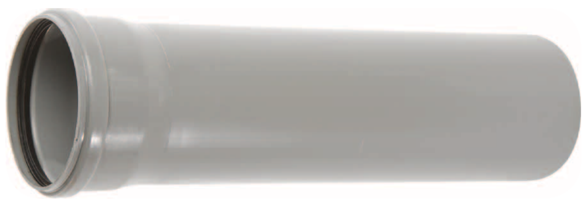


b



Problem-solving and Reasoning

6 Find the outside surface area of a pipe of radius 85 cm and length 4.5 m, to one decimal place. Answer in m^2.



7 The base and sides of a circular cake tin are to be lined on the inside with baking paper. The tin has a base diameter of 20 cm and is 5 cm high. What is the minimum amount of baking paper required, to one decimal place?

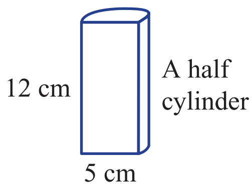
8 The inside and outside of an open-topped cylindrical concrete tank is to be coated with a special waterproofing paint. The tank has diameter 4 m and height 2 m. Find the total area to be coated with the paint. Round to one decimal place.

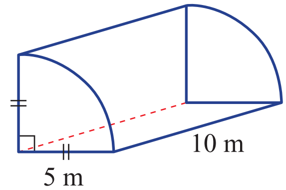
Include the base but not the top.

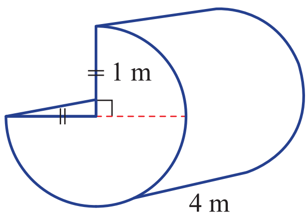
ISBN: 9781107687028

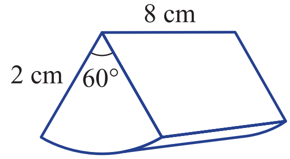
1G

9 Find the TSA of these cylindrical portions, to one decimal place.

a


b


c


d


Carefully consider the fraction of a circle made up by the ends, and the fraction of a full cylinder made up by the curved part.

The steamroller

10 A steamroller has a large, heavy cylindrical barrel that is 4 m wide and has a diameter of 2 m.

a Find the area of the curved surface of the barrel, to two decimal places.

b After 10 complete turns of the barrel, how much ground would be covered, to two decimal places?

c Find the circumference of one end of the barrel, to two decimal places.

d How many times would the barrel turn after 1 km of distance, to two decimal places?

e What area of ground would be covered if the steamroller travelled 1 km?

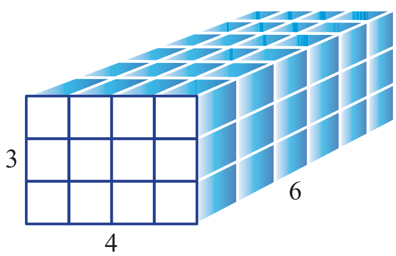
ISBN: 9781107687028

1.8 Volume of solids

The volume of a solid is the amount of space it occupies within its outside surface. It is measured in cubic units.

For solids with a uniform cross-section, the area of the cross-section multiplied by the perpendicular height gives the volume. Consider the rectangular prism below.

Number of cubic units (base) $= 4 \times 6 = 24$
Area (base) $= 4 \times 6 = 24$ units^2
Volume $=$ area (base) $\times 3 = 24 \times 3 = 72$ units^3

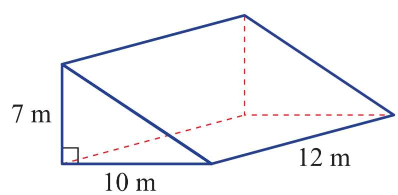



Knowing how to calculate volume is important in the shipping industry.

Let's start: Volume of a triangular prism

This prism has a triangular cross-section.

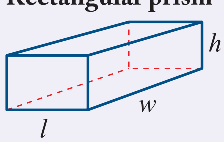
- What is the area of the cross-section?
- What is the 'height' of the prism?
- How can $V = A \times h$ be applied to this prism, where A is the area of the cross-section?



Key ideas

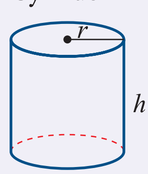
- The **volume** of a solid with a uniform cross-section is given by $V = A \times h$
  - A is the area of the cross-section.
  - h is the perpendicular (at 90°) height.

Rectangular prism



$V = lwh$

Cylinder



$V = \pi r^2 h$

- Units for capacity include:
  - $1\text{ L} = 1000\text{ mL}$
  - $1\text{ cm}^3 = 1\text{ mL}$

Volume
The amount of three-dimensional space within an object

ISBN: 9781107687028

Exercise 1H

Understanding

1 Match the solid (**a**, **b** or **c**) with the volume formula (**A**, **B** or **C**).

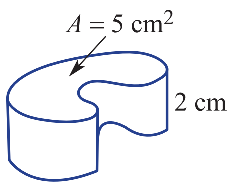
a	cylinder	**A**	$V = lwh$
b	rectangular prism	**B**	$V = \frac{1}{2}bh \times \text{length}$
c	triangular prism	**C**	$V = \pi r^2 h$

2 Write the missing number.

a There are _________ mL in 1 L.

b There are _________ cm^3 in 1 L.

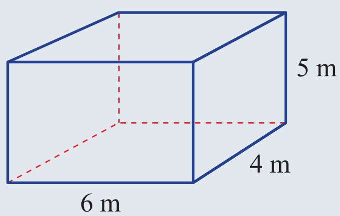
3 The area of the cross-section of this solid is given. Find the solid's volume using $V = A \times h$.



Fluency

Example 17 Finding the volume of a rectangular prism

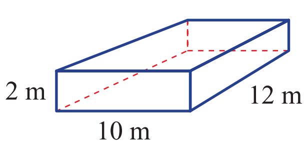
Find the volume of this rectangular prism.



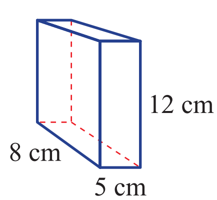
Solution	**Explanation**
$V = A \times h$	Write the general formula.
$= 6 \times 4 \times 5$	$A = 6 \times 4$ and $h = 5$
$= 120\ m^3$	Simplify and add units.

4 Find the volume of these rectangular prisms.

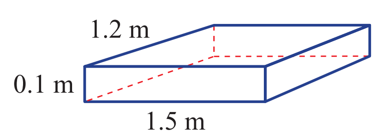
a

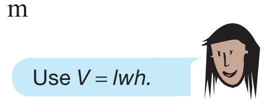


b



c

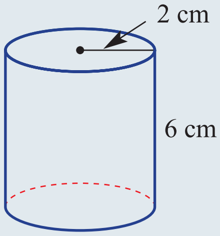




ISBN: 9781107687028

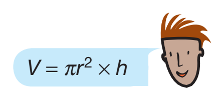
Example 18 Finding the volume of a cylinder

Find the volume of this cylinder, correct to two decimal places.

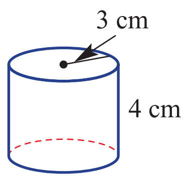


Solution	Explanation
$V = A \times h$	Write the general formula.
$= \pi r^2 \times h$	The cross-section is a circle.
$= \pi(2)^2 \times 6$	Substitute $r = 2$ and $h = 4$
$= 75.40\ \text{cm}^3$	Simplify and write your answer as an approximation, with units.

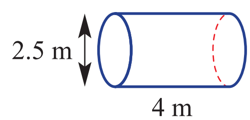
5 Find the volume of these cylinders, correct to two decimal places.



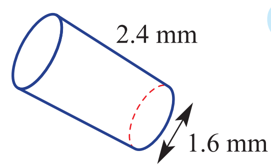
a



b



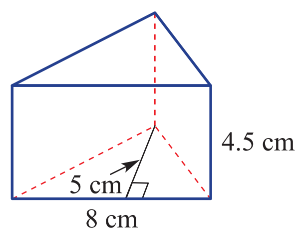
c



6 A triangle with base 8 cm and height 5 cm forms the base of a prism, as shown. If the prism stands 4.5 cm high:

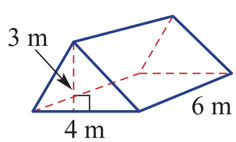
a find the area of the triangular base

b find the volume of the prism

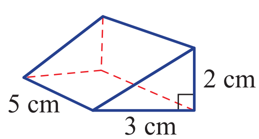


7 Find the volume of these triangular prisms.

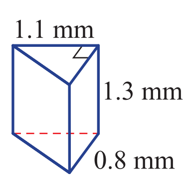
a

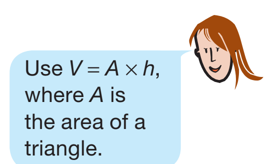


b



c





Problem-solving and Reasoning

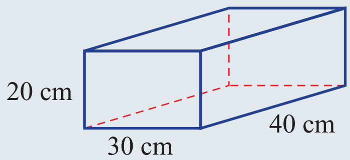
8 A cylindrical drum stands on one end with a diameter of 25 cm and water is filled to a height of 12 cm. Find the volume of water in the drum, in cm^3, correct to two decimal places.

ISBN: 9781107687028

Cambridge University Press

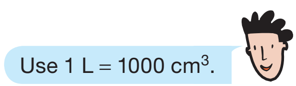
Example 19 Working with capacity

Find the number of litres of water that this container can hold.

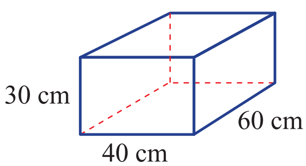


Solution	Explanation
$V = 30 \times 40 \times 20$	First work out the volume in cm^3.
$= 24\,000$ cm^3	Then divide by 1000 to convert to litres, since 1 cm^3 = 1 mL
$= 24$ L	and there are 1000 mL in 1 litre.

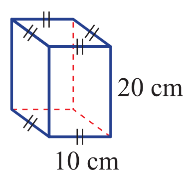
9 Find the number of litres of water that these containers can hold.



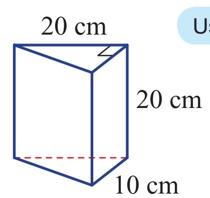
a



b

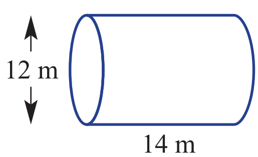


c

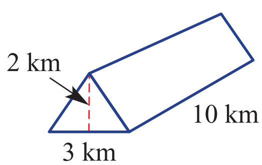


10 Find the volume of these prisms, rounding your answers to two decimal places where necessary.

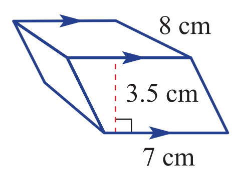
a

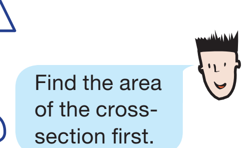


b

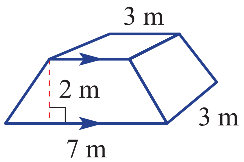


c

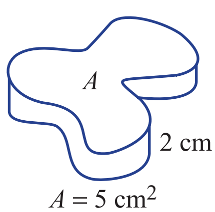




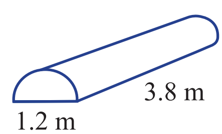
d



e



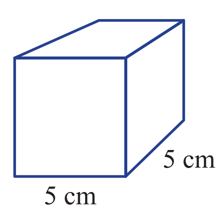
f



11 100 cm^3 of water is to be poured into this container.

a Find the area of the base of the container.

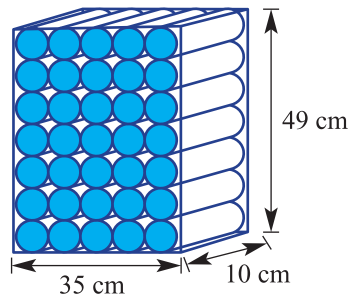
b Find the depth of water in the container.



ISBN: 9781107687028

Cambridge University Press

12 In a scientific experiment, solid cylinders of ice are removed from a solid block carved out of a glacier. The ice cylinders have diameter 7 cm and length 10 cm. The dimensions of the solid block are shown in the diagram.

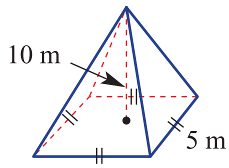


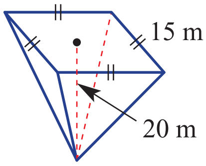
a Find the volume of ice in the original ice block.
b Find the volume of ice in one ice cylinder, to two decimal places.
c Find the number of ice cylinders that can be removed from the ice block using the configuration shown.
d Find the volume of ice remaining after the ice cylinders are removed from the block, to two decimal places.

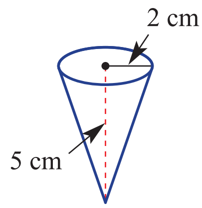
Volume of pyramids and cones

13 The volume of a pyramid or cone is exactly one-third the volume of the prism with the same base area and height; i.e. $V = \frac{1}{3} \times A \times h$.

Find the volume of these pyramids and cones. Round to one decimal place where necessary.

a


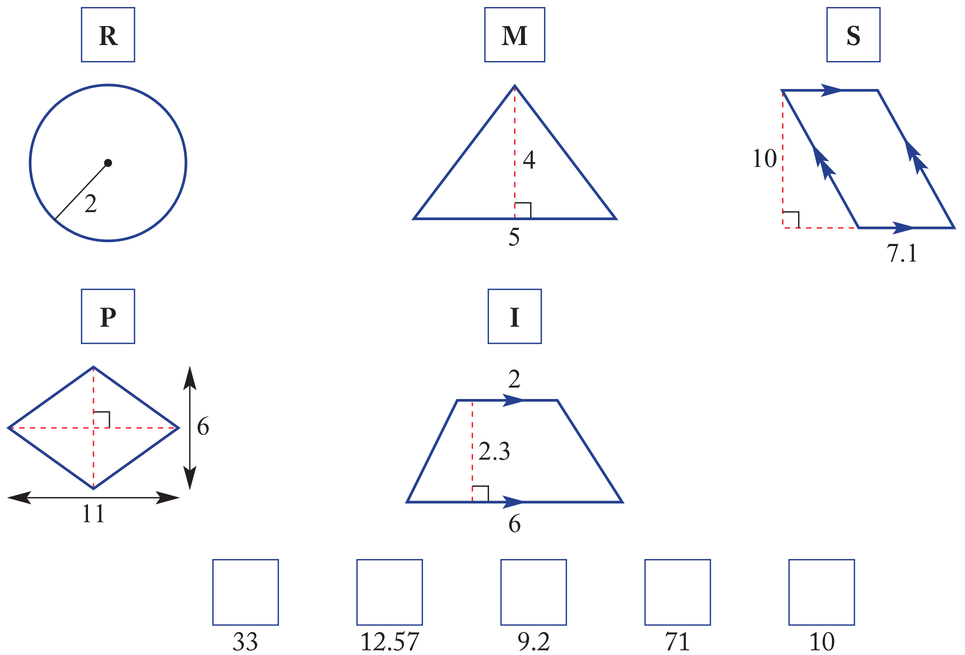
b


c


ISBN: 9781107687028 Cambridge University Press

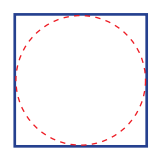
Puzzles and games

1 'I am the same shape all the way through. What am I?'
Find the area of each shape. Match the letters to the answers below to solve the riddle.

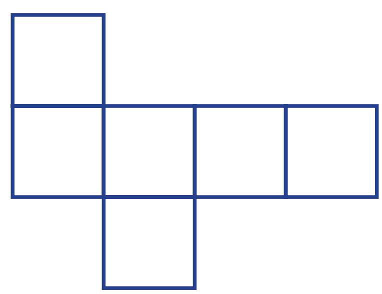


2 1 L of water is poured into a container in the shape of a rectangular prism. The dimensions of the prism are 8 cm by 12 cm by 11 cm. Will the water overflow?

3 A circular piece of pastry is removed from a square sheet of side length 30 cm. What percentage of pastry remains?

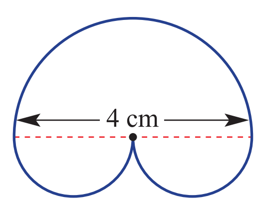


4 How many different nets are there for a cube? Do not count reflections or rotations of the same net. Here is one example:



5 Give the radius of a circle whose value for the circumference is equal to the value for the area.

6 Find the area of this special shape.



7 A cube's surface area is 54 cm^2. What is its volume?

ISBN: 9781107687028 Cambridge University Press

Chapter summary

Measurement

Conversion of units

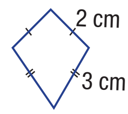
$$\text{km} \underset{\div 1000}{\overset{\times 1000}{\rightleftarrows}} \text{m} \underset{\div 100}{\overset{\times 100}{\rightleftarrows}} \text{cm} \underset{\div 10}{\overset{\times 10}{\rightleftarrows}} \text{mm}$$

$$\text{km}^2 \underset{\div 1000^2}{\overset{\times 1000^2}{\rightleftarrows}} \text{m}^2 \underset{\div 100^2}{\overset{\times 100^2}{\rightleftarrows}} \text{cm}^2 \underset{\div 10^2}{\overset{\times 10^2}{\rightleftarrows}} \text{mm}^2$$

$$\text{km}^3 \underset{\div 1000^3}{\overset{\times 1000^3}{\rightleftarrows}} \text{m}^3 \underset{\div 100^3}{\overset{\times 100^3}{\rightleftarrows}} \text{cm}^3 \underset{\div 10^3}{\overset{\times 10^3}{\rightleftarrows}} \text{mm}^3$$

Perimeter

The distance around the outside of a shape.

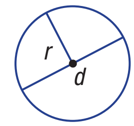


$P = 2 \times 2 + 2 \times 3$
$= 10$ cm

Circumference

The distance around the outside of a circle.

$C = 2\pi r$ or $C = \pi d$



Area – basic shapes

Square : $A = l^2$

Rectangle : $A = lw$

Triangle : $A = \frac{1}{2}bh$

Rhombus : $A = \frac{1}{2}xy$

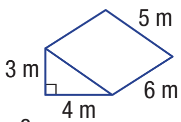
Parallelogram : $A = bh$

Trapezium : $A = \frac{1}{2}(a + b)h$

Total surface area

Draw a net and sum the surface areas.

Triangular prism

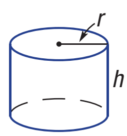


$TSA = 2 \times \frac{1}{2} \times 4 \times 3$
$+ 6 \times 4 + 6 \times 3 + 6 \times 5$
$= 84 \text{ m}^2$

Cylinder

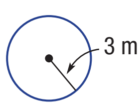
$TSA = 2\pi r^2 + 2\pi rh$

2 ends curved part



Area – circle

$A = \pi r^2$
$= \pi \times 3^2$
$\approx 28.27 \text{ m}^2$



Volume

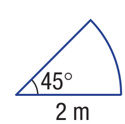
Rectangular prism
$V = lwh$

Cylinder
$V = \pi r^2 h$

Capacity: 1 L = 1000 mL
1 cm^3 = 1 mL

Area – sectors

$A = \frac{45}{360} \times \pi r^2$
$= \frac{1}{8} \times \pi \times 2^2$
$\approx 1.57 \text{ m}^2$



ISBN: 9781107687028

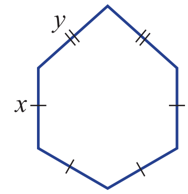
Cambridge University Press

Multiple-choice questions

1 The number of centimetres in a kilometre is:
A 10 **B** 100 **C** 1000 **D** 10 000 **E** 100 000

2 The perimeter of a square with side length 2 cm is:
A 4 cm **B** 8 cm **C** 4 cm^2 **D** 8 cm^2 **E** 16 cm

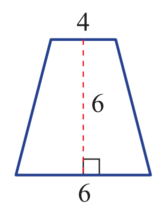
3 The perimeter of the shape shown is given by the formula:
A $x - y$ **B** $2x + y$ **C** $4x + 2y$
D $x - 2y$ **E** $4x + y$



4 A correct expression for determining the circumference of a circle with diameter 6 cm is:
A $\pi \times 6$ **B** $\pi \times 3$ **C** $2 \times \pi \times 6$ **D** 2×6 **E** $\pi \times 6^2$

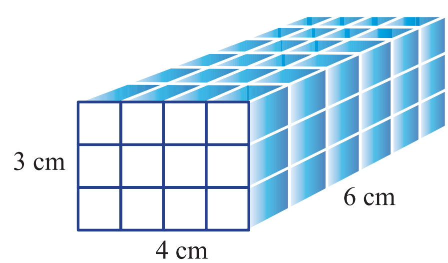
5 The area of a rectangle with side lengths 3 cm and 4 cm is:
A 12 cm^2 **B** 12 cm **C** 7 cm^2 **D** 14 cm **E** 14 cm^2

6 The correct expression for calculating the area of this trapezium is:
A $(6 - 4) \times 6$
B $\frac{1}{2}(6 + 4) \times 6$
C $\frac{1}{2} \times 6 \times 4$
D $6 \times 6 - 4$
E $6 \times 6 + 6 \times 4$



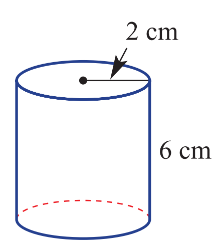
7 A sector's centre angle measures 90°. This is equivalent to:
A $\frac{1}{5}$ of a circle **B** $\frac{1}{2}$ of a circle **C** $\frac{3}{4}$ of a circle **D** $\frac{2}{3}$ of a circle **E** $\frac{1}{4}$ of a circle

8 The volume of the shape shown is:
A 13 cm^3 **B** 27 cm^3 **C** 72 cm^2
D 72 cm^3 **E** 27 cm^2



9 The volume of a cube of side length 3 cm is:
A 9 cm^3 **B** 27 cm^3 **C** 54 cm^2 **D** 54 cm^3 **E** 27 cm^2

10 The curved surface area for this cylinder is closest to:
A 87.96 cm^2 **B** 12.57 cm^2 **C** 75.40 cm^2
D 75.39 cm^2 **E** 113.10 cm^2



ISBN: 9781107687028 Cambridge University Press

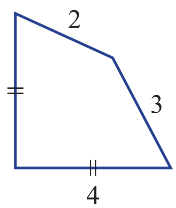
Short-answer questions

1 Convert these measurements to the units shown in the brackets.

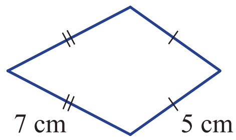
a 5.3 km (m) **b** 27 000 cm^2 (m^2) **c** 0.04 cm^3 (mm^3)

2 Find the perimeter of these shapes.

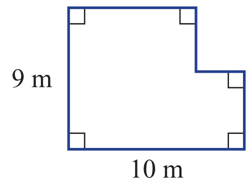
a



b



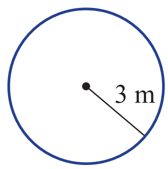
c



3 For the circle, find, to two decimal places:

a the circumference

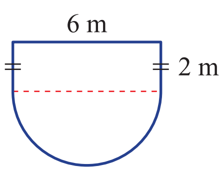
b the area



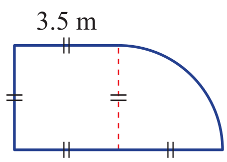
4 For these composite shapes, find, to two decimal places:

i the perimeter **ii** the area

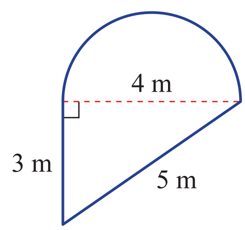
a



b

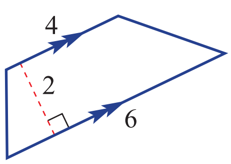


c

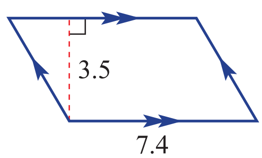


5 Find the area of these shapes.

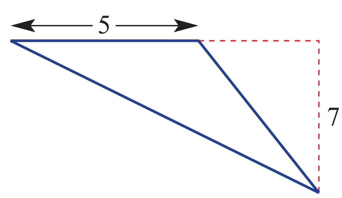
a



b

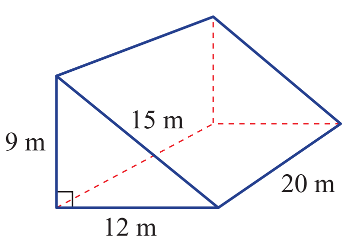


c

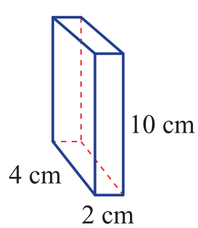


6 Find the total surface area (TSA) of these prisms.

a

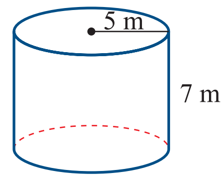


b



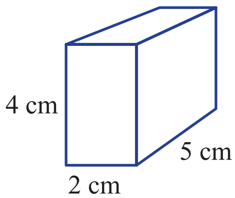
ISBN: 9781107687028

7 Determine the total surface area of this cylinder, to two decimal places.

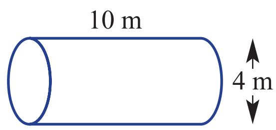


8 Find the volume of these solids, to two decimal places where necessary.

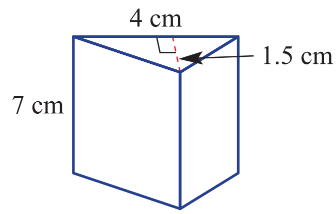
a



b



c



Extended-response question

1 A cylindrical tank has diameter 8 m and height 2 m.

a Find the surface area of the curved part of the tank, to two decimal places.

b Find the TSA, including the top and the base, to two decimal places.

c Find the total volume of the tank, to two decimal places.

d Find the total volume of the tank in litres, to two decimal places. Note: there are 1000 litres in 1 m^3.

ISBN: 9781107687028

Cambridge University Press

Australian Curriculum
Number and Algebra
Money and financial mathematics

chapter

2 Consumer arithmetic

What you will learn

Managing the household

As the people in charge of the family finances, parents or guardians need to budget in order to financially manage their household. They look at the money that is being earned through jobs, allowances and interest on bank accounts, and the money that needs to be outlaid to pay for goods and services. Money is spent on food, electricity, water, petrol and perhaps a mortgage or rent. The money that is left over may be spent on leisure activities or holidays, or put into savings.

ISBN: 9781107687028

Pre-test

1 Find the following totals.

a \$15.92 + \$27.50 + \$56.20 **b** \$134 + \$457 + \$1021 **c** \$457 × 6

d $\$56.34 \times 1\frac{1}{2}$ **e** \$87 560 ÷ 52 (to the nearest cent)

2 Express the following fractions with denominators of 100.

a $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{1}{5}$ **d** $\frac{17}{25}$ **e** $\frac{9}{20}$

3 Write each of the following fractions as decimals.

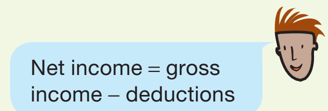
a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{5}$ **d** $\frac{7}{25}$ **e** $\frac{1}{3}$

4 Round the following decimals to two decimal places.

a 16.7893 **b** 7.347 **c** 45.3444 **d** 6.8389 **e** 102.8999

5 Copy and complete the following table.

Gross income (\$)	Deductions (\$)	Net income (\$)
4976	456.72	**a**
72 156	21 646.80	**b**
92 411	**c**	62 839
156 794	**d**	101 916
e	18 472.10	79 431.36



6 Calculate the following annual incomes for each of these people.

a Tom: \$1256 per week

b Sally: \$15 600 per month

c Anthony: \$1911 per fortnight

d Crystal: \$17.90 per hour, for 40 hours per week, for 50 weeks per year

7 Without a calculator, find:

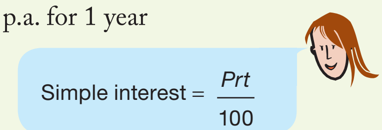
a 10% of \$400 **b** 5% of \$5000 **c** 2% of \$100

d 25% of \$844 **e** 20% of \$12.80 **f** 75% of \$1000

8 Find the simple interest on the following amounts.

a \$400 at 5% p.a. for 1 year **b** \$5000 at 6% p.a. for 1 year

c \$800 at 4% p.a. for 2 years



9 Complete the following table.

Cost price	Deduction	Sale price
\$34	\$16	**a**
\$460	\$137	**b**
\$500	**c**	\$236
d	\$45	\$67
e	\$12.65	\$45.27

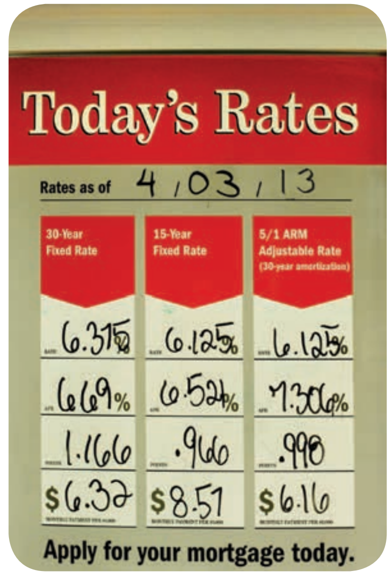
10 The following amounts include the 10% GST. By dividing each one by 1.1, find the original costs before the GST was added to each.

a \$55 **b** \$61.60 **c** \$605

ISBN: 9781107687028

2.1 Review of percentages

It is important that we are able to work with percentages in our everyday lives. Banks, retailers and governments use percentages every day to work out fees and prices.



Let's start: Which option should Jamie choose?

Jamie currently earns \$38 460 p.a. (per year) and is given a choice of two different pay rises. Which should she choose and why?

Choice A	Choice B
Increase of \$20 a week	Increase of 2% on p.a. salary

Key ideas

- A **percentage** means 'out of 100'. It can be written using the symbol %, or as a fraction or a decimal.
  For example: 75 per cent = 75% = $\frac{75}{100}$ or $\frac{3}{4} = 0.75$
- To convert a fraction or a decimal to a percentage, multiply by 100%, or $\frac{100\%}{1}$.
- To convert a percentage to a fraction, write it with a **denominator** of 100 and simplify.
  $15\% = \frac{15}{100} = \frac{3}{20}$

$$\text{Fraction} \underset{\div 100\%}{\overset{\times 100\%}{\rightleftarrows}} \%$$

- To convert a percentage to a decimal, divide by 100%.
  $15\% = 15 \div 100 = 0.15$
- To find a percentage of a quantity, write the percentage as a fraction or a decimal, then multiply by the quantity. $x\% \text{ of } P = \frac{x}{100} \times P$.

$$\text{Decimal} \underset{\div 100\%}{\overset{\times 100\%}{\rightleftarrows}} \%$$

Percentage A convenient way of writing fractions with denominators of 100

Denominator The part of a fraction that sits below the dividing line

Exercise 2A

Understanding

Make sure you have an equivalent fraction: $\frac{2}{5} = \frac{\square}{100}$.

1 Write the following with denominators of 100.

a $\frac{2}{5}$ b $\frac{17}{20}$ c $\frac{49}{50}$ d $\frac{7}{25}$ e $\frac{9}{10}$

2 Complete the following.

a $7\% = \frac{7}{\square}$ b $0.9 = \square\%$ c $\frac{3}{5} = \square\%$

ISBN: 9781107687028

Cambridge University Press

2A

3 Use mental strategies to find:

a 10% of \$7.50 **b** 20% of \$400 **c** 50% of \$98

d 75% of \$668 **e** 25% of \$412 **f** 2% of \$60

g 5% of \$750 **h** $33\frac{1}{3}$% of \$1200 **i** 30% of 15

$10\% = \frac{10}{100}$
'Of' means times.

Fluency

Example 1 Converting to a percentage

Write each of the following as a percentage.

a $\frac{19}{20}$ **b** $\frac{3}{8}$ **c** 0.07

Solution	Explanation
a $\frac{19}{20} = \frac{95}{100}$ $= 95\%$	Write using a denominator of 100. Alternatively, multiply the fraction by 100%. $\frac{19}{\cancel{20}_1} \times \frac{\cancel{100}^5}{1}\% = 19 \times 5\% = 95\%$
b $\frac{3}{8} = \frac{3}{\cancel{8}_2} \times \frac{\cancel{100}^{25}}{1}\%$ $= \frac{75}{2}\%$ $= 37.5\%$	Multiply the fraction by 100%. Cancel common factors then simplify.
c $0.07 = 0.07 \times 100\%$ $= 0.07$ $= 7\%$	Multiply the decimal by 100%. Move the decimal point two places to the right.

4 Convert each fraction to a percentage.

a $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{1}{4}$ **d** $\frac{1}{10}$

e $\frac{1}{100}$ **f** $\frac{7}{25}$ **g** $\frac{15}{50}$ **h** $\frac{3}{4}$

i $\frac{5}{8}$ **j** $\frac{19}{25}$ **k** $\frac{99}{100}$ **l** $\frac{47}{50}$

First write using a denominator of 100 or alternatively multiply by 100%.

5 Write these decimals as percentages.

a 0.17 **b** 0.73 **c** 0.48 **d** 0.09

e 0.06 **f** 0.13 **g** 1.13 **h** 1.01

i 0.8 **j** 0.9 **k** 0.99 **l** 0.175

To multiply by 100%, move the decimal point two places to the right.

ISBN: 9781107687028

Example 2 Writing percentages as simple fractions

Write each of the following percentages as a simple fraction.

a 37% **b** 58% **c** $6\frac{1}{2}\%$

Solution | **Explanation**

a $37\% = \frac{37}{100}$

Write the percentage with a denominator of 100.

b $58\% = \frac{58}{100}$
$= \frac{29}{50}$

Write the percentage with a denominator of 100.

Simplify $\frac{58}{100}$ by cancelling, using the HCF of 58 and 100, which is 2.

$$\frac{\cancel{58}^{29}}{\cancel{100}_{50}} = \frac{29}{50}$$

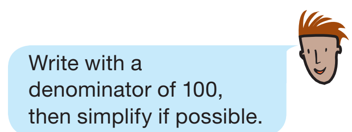
c $6\frac{1}{2}\% = \frac{6\frac{1}{2}}{100}$
$= \frac{13}{200}$

Write the percentage with a denominator of 100.

Double the numerator $\left(6\frac{1}{2}\right)$ and the denominator (100) so that the numerator is a whole number.

6 Write each percentage as a simple fraction.

a 71% **b** 80% **c** 25% **d** 55%
e 40% **f** 88% **g** 15% **h** $16\frac{1}{2}\%$
i $17\frac{1}{2}\%$ **j** $2\frac{1}{4}\%$ **k** $5\frac{1}{4}\%$ **l** $52\frac{1}{2}\%$



Example 3 Writing a percentage as a decimal

Convert these percentages to decimals.

a 93% **b** 7% **c** 30%

Solution | **Explanation**

a $93\% = 0.93$

Divide the percentage by 100. This is the same as moving the decimal point two places to the left.

$93 \div 100 = 0.93$

b $7\% = 0.07$

Divide the percentage by 100.

$7 \div 100 = 0.07$

c $30\% = 0.3$

Divide the percentage by 100.

$30 \div 100 = 0.30$

Write 0.30 as 0.3.

7 Convert into decimals:

a 61% **b** 83% **c** 75% **d** 45%
e 9% **f** 90% **g** 50% **h** 16.5%
i 7.3% **j** 200% **k** 430% **l** 0.5%

ISBN: 9781107687028

Example 4 Finding a percentage of a quantity

Find 42% of \$1800.

Solution	**Explanation**
42% of \$1800 $= 0.42 \times 1800$ = \$756	Remember that 'of' means multiply. Write 42% as a decimal or a fraction: $42\% = \frac{42}{100} = 0.42$ Then multiply by the amount. If using a calculator, type 0.42×1800 Without a calculator: $\frac{42}{\cancel{100}}\quad 18\cancel{00} = 42\quad 18$

8 Use a calculator to find:

a 10% of \$250 **b** 50% of \$300 **c** 75% of \$80

d 12% of \$750 **e** 9% of \$240 **f** 43% of 800 grams

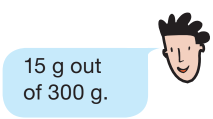
g 90% of \$56 **h** 110% of \$98 **i** $17\frac{1}{2}$% of 2000 m

Problem-solving and Reasoning

9 A 300 g pie contains 15 g of saturated fat.

a What fraction of the pie is saturated fat?

b What percentage of the pie is saturated fat?



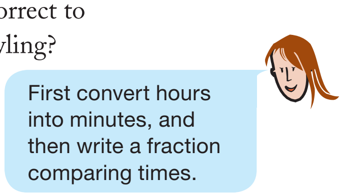
10 About 80% of the mass of a human body is water. If Hugo is 85 kg, how many kilograms of water are in his body?

11 Rema spends 12% of the 6.6 hour school day in maths. How many minutes are spent in the maths classroom?

12 In a cricket match, Brett spent 35 minutes bowling.

His team's total fielding time was $3\frac{1}{2}$ hours.

What percentage of the fielding time, correct to two decimal places, did Brett spend bowling?



ISBN: 9781107687028

Cambridge University Press

13 Malcom lost 8 kg, and now weighs 64 kg. What percentage of his original weight did he lose?

14 47.9% of a local council's budget is spent on garbage collection. If a rate payer pays $107.50 per quarter in total rate charges, how much do they contribute in a year to garbage collection?

Australia's statistics

15 Below is the preliminary data on Australia's population growth, as gathered by the Australian Bureau of Statistics for June 2011.

	Population at end June quarter 2011 ('000)	Change over previous year ('000)	Change over previous year (%, one decimal place)
New South Wales	7303.7	82.2	
Victoria	5624.1	84.2	
Queensland	4580.7	74.8	
South Australia	1657.0	12.8	
Western Australia	2346.4	55.8	
Tasmania	510.6	3.2	
Northern Territory	230.2	0.9	
Australian Capital Territory	365.4	6.8	
Australia	**22 618.1**	**320.8**	

a Calculate the percentage change for each state and territory shown using the previous year's population, and complete the table.

b What percentage of Australia's overall population, correct to one decimal place, is living in:

i NSW?

ii Vic?

iii WA?

You will need to calculate the previous year's population; e.g. for NSW, 7303.7 – 82.2.

c Use a spreadsheet to draw a pie chart (sector graph) showing the populations of the 8 states and territories in the table. What percentage of the total is represented by each state/territory?

d In your pie chart in part **c**, what is the angle size of the sector representing Victoria?

ISBN: 9781107687028

2.2 Applications of percentages

There are many applications of percentages. Prices are often increased by a percentage to create a profit, or decreased by a percentage when on sale.

When goods are purchased by a store, the cost to the owner is called the cost price. The price of the goods sold to the customer is called the selling price. This price will vary according to whether the store is having a sale or decides to make a certain percentage profit.

Let's start: Discounts

Discuss as a class:

- Which is better: 20% off or a \$20 discount?
- If a discount of 20% or \$20 resulted in the same price, what would the original price be?
- Why are percentages used to show discounts, rather than a flat amount?

Key ideas

- To increase by a given percentage, multiply by the sum of 100% and the given percentage.

  For example: To increase by 12%, multiply by 112% or 1.12.
- To decrease by a given percentage, multiply by 100% minus the given percentage.

  For example: To decrease by 20%, multiply by 80% or 0.8.
- Profits and discounts:
  - The normal price of the goods recommended by the manufacturer is called the retail price.
  - If there is a sale and the goods are less than the retail price, they are said to be **discounted**.
  - **Profit** = selling price − cost price
  - $\text{Percentage profit} = \dfrac{\text{profit}}{\text{cost price}} \quad 100\%$
  - $\text{Percentage discount} = \dfrac{\text{discount}}{\text{cost price}} \quad 100\%$

Discount An amount subtracted from a price

Profit The amount of money made by selling an item or service for more than its cost

ISBN: 9781107687028

Cambridge University Press

Exercise 2B

Understanding

1 By what percentage do you multiply to increase an amount by:
a 10%? b 20%? c 50%? d 2%? e 18%?

2 By what percentage do you multiply to decrease an amount by:
a 5%? b 30%? c 15%? d 50%? e 17%?

3 Decide how much profit or loss is made in each of the following situations.
a cost price = \$15 selling price = \$20
b cost price = \$17.50 selling price = \$20
c cost price = \$250 selling price = \$234
d cost price = \$147 selling price = \$158
e cost price = \$3.40 selling price = \$1.20

Fluency

Example 5 Increasing by a given percentage

Increase \$370 by 8%.

Solution	Explanation
$\$370 \times 1.08 = \$399.60$	100% + 8% = 108% Write 108% as a decimal (or fraction) and multiply by the amount. Remember that money has two decimal places.

4 a Increase \$90 by 5%. b Increase \$400 by 10%.
c Increase \$55 by 20%. d Increase \$490 by 8%.
e Increase \$50 by 12%. f Increase \$7000 by 3%.
g Increase \$49.50 by 14%. h Increase \$1.50 by 140%.

To increase by 5%, multiply by 100% + 5% = 1.05.

Example 6 Decreasing by a given percentage

Decrease \$8900 by 7%.

Solution	Explanation
$\$8900 \times 0.93 = \$8277.00$	100% – 7% = 93% Write 93% as a decimal (or fraction) and multiply by the amount. Remember to put the units in your answer.

5 a Decrease \$1500 by 5%. b Decrease \$400 by 10%.
c Decrease \$470 by 20%. d Decrease \$80 by 15%.
e Decrease \$550 by 25%. f Decrease \$49.50 by 5%.
g Decrease \$119.50 by 15%. h Decrease \$47.10 by 24%.

To decrease by 5%, multiply by 100% – 5% = 0.95.

ISBN: 9781107687028

Example 7 Calculating profits and percentage profit

The cost price for a new car is \$24 780 and it is sold for \$27 600.

a Calculate the profit.

b Calculate the percentage profit to two decimal places.

Solution	Explanation
a Profit = selling price − cost price	Write the rule.
= \$27 600 − \$24 780	Substitute the values and evaluate.
= \$2820	
b $\text{Percentage profit} = \dfrac{\text{profit}}{\text{cost price}}\ \ 100\%$	Write the rule.
$= \dfrac{2820}{24\,780}\ \ 100\%$	Substitute the values and evaluate.
$= 11.38\%$	Round your answer as instructed.

6 Copy and complete the table on profits and percentage profit.

	Cost price	Selling price	Profit	Percentage profit
a	\$10	\$16		
b	\$240	\$300		
c	\$15	\$18		
d	\$250	\$257.50		
e	\$3100	\$5425		
f	\$5.50	\$6.49		

$\text{Percentage profit} = \dfrac{\text{profit}}{\text{cost price}}\ \ 100\%$

Example 8 Finding the selling price

A retailer buys some calico material for \$43.60 a roll. He wishes to make a 35% profit.

a What will be the selling price per roll?

b If he sells 13 rolls, what profit will he make?

Solution	Explanation
a Selling price = 135% of \$43.60	For a 35% profit the unit price is 135%.
= 1.35 × \$43.60	Write 135% as a decimal (1.35) and evaluate.
= \$58.86 per roll	
b Profit per roll = \$58.86 − \$43.60 = \$15.26	Selling price − cost price
Total profit = \$15.26 × 13	There are 13 rolls at \$15.26 profit per roll.
= \$198.38	

ISBN: 9781107687028

Cambridge University Press

7 A retailer buys some snow globes for $41.80 each. He wishes to make a 25% profit.
  a What will be the selling price per snow globe?
  b If he sells a box of 25 snow globes, what profit will he make?

8 Ski jackets are delivered to a shop in packs of 50 for $3500. If the shop owner wishes to make a 35% profit:
  a what will be the total profit made on a pack?
  b what is the profit on each jacket?

9 A second-hand car dealer bought a trade-in car for $1200 and wishes to resell it for a 28% profit. What is the resale price?

Example 9 Finding the discounted price

A shirt worth $25 is discounted by 15%.
  a What is the selling price?
  b How much is the saving?

Solution	Explanation
a Selling price = 85% of $25 = 0.85 × 25 = $21.25	15% discount means there must be 85% left (100% – 15%). Convert 85% to 0.85 and multiply by the amount.
b Saving = 15% of $25 = 0.15 × 25 = $3.75	You save 15% of the original price. Convert 15% to 0.15 and multiply by the original price.
or saving = $25 – $21.25 = $3.75	Saving = original price – discounted price

10 Samantha buys a wetsuit from the sports store where she works. Its original price was $79.95. If employees receive a 15% discount:
  a what is the selling price?
  b how much will Samantha save?

11 A travel agent offers a 12.5% discount on airfares if you travel during May or June. If the normal fare to London (return trip) is $2446:
  a what is the selling price?
  b how much is the saving?

12 A store sells second-hand goods at 40% off the recommended retail price. For a lawn mower valued at $369:
  a what is the selling price?
  b how much would you save?

ISBN: 9781107687028

2B

Problem-solving and Reasoning

13 A pair of sports shoes is discounted by 47%. If the recommended price was \$179:

a what is the amount of the discount?

b what will be the discounted price?

14 Jeans are priced at a May sale for \$89. If this is a saving of 15% off the selling price, what do the jeans normally sell for?

85% of amount = \$89
Find 1% then × 100 to find 100%.

15 Discounted tyres are reduced in price by 35%. They now sell for \$69 each. Determine:

a the normal price of one tyre

b the saving if you buy one tyre

16 The local shop purchases a carton of containers for \$54. Each container is sold for \$4. If the carton had 30 containers, determine:

a the profit per container

b the percentage profit per container, to two decimal places

c the overall profit per carton

d the overall percentage profit, to two decimal places

ISBN: 9781107687028

17 A retailer buys a book for \$50 and wants to sell it for a 26% profit. The 10% GST must then be added onto the cost of the book.

a Calculate the profit on the book.

b How much GST is added to the cost of the book?

c What is the advertised price of the book, including the GST?

d Find the overall percentage increase of the final selling price compared to the \$50 cost price.

% increase = $\frac{\text{increase}}{\text{cost price}}$ 100%

Building a gazebo

18 Christopher designs a gazebo for a new house. He buys the timber from a retailer, who sources it at wholesale price and then marks it up before selling to Christopher at retail price. The table below shows the wholesale prices as well as the mark-up for each type of timber.

a Determine Christopher's overall cost for the material, including the markup.

b Determine the profit the retailer made.

c Determine the retailer's overall percentage profit, to two decimal places.

d If the retailer pays 27% of his profits in tax, how much tax does he pay on this sale?

Quantity	Description	Cost/unit	Mark-up
6	Treated pine posts	\$23	20%
11	300 × 50 oregon beams	\$75	10%
5	Sheet lattice work	\$86	15%
2	300 × 25 oregon fascias	\$46	12%
8	Laserlite sheets	\$32	10%

ISBN: 9781107687028

2.3 Income

You may have earned money for baby-sitting or delivering newspapers, or have a part-time job. As you move more into the workforce it is important that you understand how you are paid.

Let's start: Who earns what?

As a class, discuss the different types of jobs held by different members of each person's family, and discuss how they are paid.

- What are the different ways that people can be paid?
- What does it mean if you work fewer than full-time hours?
- What does it mean if you work longer than full-time hours?

What other types of income can people in the class think of?

Key ideas

Methods of payment

- Hourly **wages**: You are paid a certain amount per hour worked.
- **Commission**: You are paid a percentage of the total amount of sales.
- **Salary**: You are paid a set amount per year, regardless of how many hours you work.
- Fees: You are paid according to the charges you set; e.g. doctors, lawyers, contractors.
- Some terms you should be familiar with include:
  - **gross income**: the total amount of money you earn before taxes and other deductions
  - **deductions**: money taken from your income before you are paid; e.g. taxation, union fees, superannuation
  - **net income**: the amount of money you actually receive after the deductions are taken from your gross income

    net income = gross income − deductions

Payments by hourly rate

- If you are paid by the hour you will be paid an amount per hour for your normal working time. If you work overtime the rates may be different. Usually, normal working time is 38 hours per week.
  normal: 1.0 × normal rate
  time and a half: 1.5 × normal rate
  double time: 2.0 × normal rate
- If you work shift work the hourly rates may differ from shift to shift. For example:

6:00 a.m.–2:00 p.m.	$12.00/hour	(regular rate)
2:00 p.m.–10:00 p.m.	$14.30/hour	(afternoon shift rate)
10:00 p.m.–6:00 a.m.	$16.80/hour	(night shift rate)

Wages Earnings paid to an employee based on an hourly rate

Commission Earnings of a salesperson based on a percentage of the value of goods or services sold

Salary An employee's fixed agreed yearly income

Gross income Total income before any deductions (e.g. income tax) are made

Deductions Amounts of money taken from gross income

Net income Income remaining after deductions have been made from gross income

ISBN: 9781107687028

Taxation

Income earners pay tax according to the current tax instalment table. Your income determines the amount of tax you pay. These tables can vary from year to year, but for convenience we will adopt these figures in this section:

Income	Tax payable
Less than \$6000	Nil
\$6000–\$21 600	15 cents for each dollar over \$6000
\$21 601–\$70 000	\$2340 plus 30 cents for each dollar over \$21 600
\$70 001–\$125 000	\$16 860 plus 42 cents for each dollar over \$70 000
Greater than \$125 000	\$39 960 plus 47 cents for each dollar over \$125 000

Exercise 2C

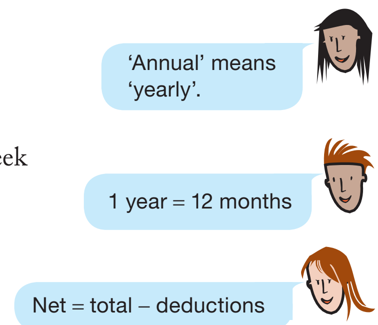
Understanding

1 If Tao earns \$570 for 38 hours' work, calculate his:

- **a** hourly rate of pay
- **b** time and a half rate
- **c** double time rate
- **d** annual income, given that he works 52 weeks a year, 38 hours a week

2 Which is better: \$5600 a month or \$67 000 a year?

3 Callum earns \$1090 a week and has annual deductions of \$19 838. What is Callum's net income for the year?



Fluency

Example 10 Finding gross and net income (including overtime)

Pauline is paid \$13.20 per hour at the local stockyard to muck out the stalls. Her normal hours of work are 38 hours per week. She receives time and a half for the next 4 hours worked and double time after that.

- **a** What will be her gross income if she works 50 hours?
- **b** If she pays \$220 per week in taxation and \$4.75 in union fees, what will be her weekly net income?

Solution	Explanation
a Gross income $= 38 \times \$13.20$ $+ 4 \times 1.5 \times \$13.20$ $+ 8 \times 2 \times \$13.20$ $= \$792$	Normal 38 hours Overtime rate for next 4 hours: time and a half $= 1.5 \times$ normal Overtime rate for next 8 hours: double time $= 2 \times$ normal
b Net income $= \$792 - (\$220 + \$4.75)$ $= \$567.25$	Net income = gross income – deductions

4 Copy and complete this table.

	Hourly rate	Normal hours worked	Time and a half hours	Double time hours	Gross income	Deductions	Net income
a	$15	38	0	0		$155	
b	$24	38	2	0		$220	
c	$13.15	38	4	1		$300	
d	$70	40	2	3		$510	
e	$17.55	35	4	6		$184	

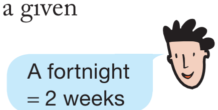
Example 11 Calculating shift work

Michael is a shift worker and is paid at $10.60 per hour for the morning shift, $12.34 per hour for the afternoon shift and $16.78 per hour for the night shift. Each shift is 8 hours. In a given fortnight he works four morning, two afternoon and three night shifts. Calculate his gross income.

Solution	Explanation
Gross income = $4 \times 10.60 \times 8$	4 morning shifts at $10.60 per hour for 8 hours
$+ 2 \times 12.34 \times 8$	2 afternoon shifts at $12.34 per hour
$+ 3 \times 16.78 \times 8$	3 night shifts at $16.78 per hour
= $939.36	Gross income as tax has not been paid

5 Greg works shifts at a processing plant. In a given rostered fortnight he works:

- 3 day shifts ($10.60 per hour)
- 4 afternoon shifts ($12.34 per hour)
- 4 night shifts ($16.78 per hour).



a If each shift is 8 hours long, determine his gross income for the fortnight.

b If the answer to part **a** was his average fortnightly income, what would his gross income be for a year (52 weeks)?

c If he is to be paid monthly, what would his gross income be for a month?

Example 12 Calculating income involving commission

Jeff sells memberships to a gym and receives $225 per week plus 5.5% commission on his sales. Calculate his gross income after a 5-day week.

Day	1	2	3	4	5
Sales ($)	680	450	925	1200	1375

Solution	Explanation
Total sales = $4630	Determine the total sales.
Commission = 5.5% of $4630	Determine the commission on the total sales at
$= 0.055 \times 4630$	5.5% by multiplying 0.055 by the total sales.
= $254.65	
Gross income = $225 + $254.65	Gross income is $225 plus commission.
= $479.65	

ISBN: 9781107687028 Cambridge University Press

6 A real estate agent receives 2.75% commission on the sale of a house valued at $125 000. Find the commission earned.

7 A car salesman earns $500 a month plus 3.5% commission on all sales. In the month of January his sales total $56 000. Calculate:

a his commission for January
b his gross income for January

8 Sally earns an annual salary of $27 000 plus 2% commission on all sales. Find:

a her weekly base salary before sales
b her commission for a week where her sales totalled $7500
c her gross weekly income for that week
d her annual gross income if over the year her sales totalled $571 250

Example 13 Calculating income tax

Fiona receives an annual salary of $47 842. Determine the amount of tax she pays over the year using the table below.

Income	Tax payable
Less than $6000	Nil
$6000–$21 600	15 cents for each dollar over $6000
$21 601–$70 000	$2340 plus 30 cents for each dollar over $21 600
$70 001–$125 000	$16 860 plus 42 cents for each dollar over $70 000
Greater than $125 000	$39 960 plus 47 cents for each dollar over $125 000

Solution	Explanation
Tax = $2340 + 0.3 × ($47 842 – $21 600)	Find the correct tax bracket: $47 842 fits into $21 601 – $70 000.
= $2340 + 0.3 × $26 242	Follow the instructions in the bracket.
= $10 212.60	Remember that 30 cents for each dollar means to multiply by 0.3.
Total tax is $10 212.60	Use your calculator to find the tax payable.

ISBN: 9781107687028

2C

Problem-solving and Reasoning

9 Determine the tax payable on the following yearly incomes, using the table below.

a \$5000
b \$10 000
c \$20 000
d \$35 000
e \$64 700
f \$37 450
g \$10 235
h \$47 632

Income	Tax payable
Less than \$6000	Nil
\$6000–\$21 600	15 cents for each dollar over \$6000
\$21 601–\$70 000	\$2340 plus 30 cents for each dollar over \$21 600
\$70 001–\$125 000	\$16 860 plus 42 cents for each dollar over \$70 000
Greater than \$125 000	\$39 960 plus 47 cents for each dollar over \$125 000

10 If Simone received \$2874 on the sale of a property worth \$95 800, calculate her rate of commission.

What percentage of \$95 800 is \$2874?

11 Jonah earns a commission on his sales of fashion items. For goods to the value of \$2000 he receives 6% and for sales over \$2000 he receives 9% on the amount in excess of \$2000. In a given week he sold \$4730 worth of goods. Find the commission earned.

12 An employee is paid a salary of \$52 400.

a Determine the tax payable on his annual salary using the table in question **9**.
b Determine his net pay (after tax) that he can expect if paid:
  i monthly **ii** weekly **iii** fortnightly
c If the employee wishes to pay 3% of his salary into superannuation, determine the amount he will contribute each year.

Superannuation is savings towards retirement.

13 Michael earns 1.75% commission on all sales at the electrical goods store where he works. If Michael earns \$35 in commission on the sale of one television, how much did the TV sell for?

1.75% is \$35. Find 1% then 100%.

ISBN: 9781107687028

Elmo's pay slip

14 Refer to the payslip below to answer the following questions.

Kuger Incorporated

Employee ID: 75403A	Page: 1
Name: Elmo Clowner	Pay Period: 21/05/2013

Pay Method: EFT — Tax Status: Gen Exempt
Bank account name: E. Clowner
Bank: Mathsville Credit Union
BSB: 102–196 Account No: 00754031

Payment Details this pay:

Amount	Days	Payment Description	Rate/Frequency
2777.15	14.00	Normal time	$72 454/annum

Before tax deductions:

This pay	Description
170	Salary sacrifice: car pre-tax deduction

Miscellaneous deductions:

This pay	Description
52.90	Health fund
23.10	Union fees
76.00	

Reconciliation details:

This pay	YTD	Description
2607.15	62 571.60	Taxable gross pay
616.00	14 784.00	less income tax
76.00	1 824.00	less miscellaneous deductions
1915.15	45 693.60	

a What company does Elmo work for?
b What is the name of Elmo's bank and what is his account number?
c How much gross pay does Elmo earn in 1 year?
d How often does Elmo get paid?
e How much, per year, does Elmo salary sacrifice?
f How much each week is Elmo's health fund contributions?
g Calculate 1 year's union fees.
h Using the information on this pay slip, calculate Elmo's annual tax and also his annual net income.
i If Elmo works Monday to Friday from 9 a.m. to 5 p.m. each day for an entire year, calculate his effective hourly rate of pay. Use Elmo's fortnightly payment as a starting point.

ISBN: 9781107687028

2.4 Budgeting

Once people have been paid their income for the week, fortnight or month, they must plan how to spend it. Most families work on a budget, allocating money for fixed expenses such as the mortgage or rent and the variable (changing) expenses of petrol, food and clothing.

Let's start: Expenses for the month

Write down everything that you think your family would spend money on for **a** the week and **b** the month, and estimate how much those things might cost for the entire year. Where do you think savings could be made?

Key ideas

- Managing money for an individual is similar to operating a small business. Expenses can be divided into two areas:
  - **fixed expenses**: payment of loans, mortgages, regular bills etc.
  - **variable expenses**: clothing, entertainment, food etc. (these are estimates)
- When your budget is completed you should always check that your figures are reasonable estimates.
- By looking at these figures you should be able to see how much money is remaining; this can be used as savings or to buy non-essential items.
- We often use percentages in budgets, so remember to change percentages to decimals by dividing by 100; e.g. 12% = 0.12.

Fixed expenses Expenses that are set and do not change during a particular time period

Variable expenses Expenses that may change during a particular period of time, or over time

ISBN: 9781107687028

Exercise 2D

Understanding

1 Binh has an income of \$956 a week. His expenses, both fixed and variable, total \$831.72 of his income. How much money can Binh save each week?

2 Roslyn has the following monthly expenses. Mortgage = \$1458, mobile phone = \$49, internet = \$60, council rates = \$350, water = \$55, electricity = \$190. What is the total of Roslyn's monthly expenses?

3 Last year, a households' four electricity bills were:
1st quarter = \$550, 2nd quarter = \$729, 3rd quarter = \$497, 4th quarter = \$661

Using these values as a guide, what should be the family budget, each week, for electricity for the following year? Round to the nearest dollar.

Fluency

Example 14 Budgeting using percentages

Christine has a net annual income of \$36 000 after deductions. She allocates her budget on a percentage basis.

	Mortgage	Car loan	Food	Clothing	Sundries	Savings
Expenses (%)	20	15	25	20	10	10

a Determine the amount of fixed expenses, including the mortgage and car loan.
b How much should Christine save?
c Is the amount allocated for food reasonable?

Solution	Explanation
a Fixed expenses = 35% of \$36 000 = $0.35 \times 36\,000$ = \$12 600	The mortgage and loan are 35% in total. Change 35% to a decimal and multiply by the net income.
b Savings = 10% of \$36 000 = $0.01 \times 36\,000$ = \$3600	Savings are 10% of the budget. Change 10% to a decimal and multiply by the net income.
c Food = 25% of \$36 000 = $0.25 \times 36\,000$ = \$9000 per year, or \$173 per week This seems reasonable.	Food is 25% of the budget. Change 25% to a decimal and calculate. Divide the yearly expenditure by 52 to make a decision on the reasonableness of your answer.

4 Paul has an annual income of \$25 000 after deductions. He allocates his budget on a percentage basis.

	Mortgage	Car loan	Personal loan	Clothing	Food	Other
Expenses (%)	20	10	25	5	10	30

a Determine the amount of fixed expenses, including the mortgage and loans.
b How much should Paul have left over after the expenses listed in his budget?
c Is the amount allocated for food reasonable?

ISBN: 9781107687028

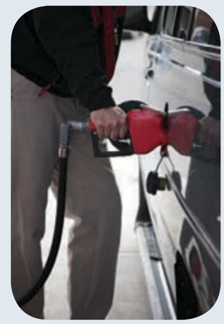
2D

5 Lachlan has an income of \$468.30 per month. If he budgets 13% for clothes, how much will he actually have to spend on clothes?

Example 15 Budgeting using fixed values

Running a certain type of car involves yearly, monthly and weekly expenditure. Consider the following vehicle's costs:

- lease \$210 per month
- registration \$475 per year
- insurance \$145 per quarter
- servicing \$1800 per year
- petrol \$37 per week

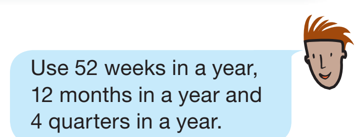


a Determine the overall cost to run this car for a year.
b What percentage of a \$70 000 budget would this be, correct to one decimal place?

Solution	Explanation
a Overall cost $= 210 \times 12$	Leasing cost: 12 months in a year
$+ \quad 475$	Registration cost
$+ \quad 145 \times 4$	Insurance cost: 4 quarters in a year
$+ \quad 1800$	Servicing cost
$+ \quad 37 \times 52$	Petrol cost: 52 weeks in a year
$= \quad \$7299$	The overall cost is found by adding the individual totals.
The overall cost to run the car is \$7299.	
b $\%\text{ of budget} = \dfrac{7299}{70\,000} \quad 100$ $= 10.4\%$	$\text{Percentage} = \dfrac{\text{car cost}}{\text{total budget}} \quad 100\%$

6 Lemona has the following expenses in her household budget.

- rent \$270 per week
- electricity \$550 per quarter
- phone and internet \$109 per month
- car \$90 per week
- food \$170 per week
- insurance \$2000 a year



a Determine the overall cost for running the household for a year.
b What percentage of Lemona's net annual salary of \$45 000 would this be, correct to one decimal place?

7 The costs of sending a student to Modkin Private College are as follows.

- fees per term (4 terms) \$1270
- subject levees per year \$489
- building fund per week \$35
- uniforms and books per year \$367

a Determine the overall cost per year.
b If the school bills twice a year, covering all the items above, what would be the amount of each payment?
c How much should be saved per week to make the biannual payments?

ISBN: 9781107687028

8 A small business owner has the following expenses to budget for.
- rent \$1400 a month
- phone line \$59 a month
- wages \$1200 a week
- electricity \$430 a quarter
- water \$120 a quarter
- insurance \$50 a month

a What is the annual budget for the small business?
b How much does the business owner need to make each week just to break even?
c If the business earns \$5000 a week, what percentage of this needs to be spent on wages?

Problem-solving and Reasoning

9 Francine's petrol budget is \$47 from her weekly income of \$350.
a What percentage of her budget is this? Answer to two decimal places.
b If petrol costs \$1.59 a litre, how many litres of petrol, correct to two decimal places, is Francine budgeting for in a week?

10 Grant works a 34-hour week at \$15.50 per hour. His net income is 65% of his gross income.
a Determine his net weekly income.
b If Grant spends 12% of his net income on entertainment, determine the amount he actually spends per year.
c He saves \$40 per week. What percentage of his net income is this (to two decimal places)?

11 Dario earns \$432 per fortnight at a takeaway pizza shop. He budgets 20% for food, 10% for recreation, 13% for transport, 20% for savings, 25% for taxation and 12% for clothing.
a Determine the actual amount budgeted for each category every fortnight.

Dario's wage increases by 30%.
b Determine how much he would now save each week.
c What percentage increase is this on the original amount saved?
d Determine the extra amount of money Dario saves per year after his wage increase.
e If transport is a fixed expense, its percentage of Dario's budget will change. Determine the new percentage.

ISBN: 9781107687028 Cambridge University Press

2D

Best buys

Families often need to consider value for money when working out their budgets.
Working out the best buy for groceries and phone deals is one way that parents can save money.
Best buys are calculated by finding the unit price per item; that is, the cost *per day*, *per kg*, *per gram* etc.

For example, consider this problem.
Soft drink is sold in three convenient packs at the local store:

- carton of 36 (375 mL) cans at $22.50
- a six-pack of (375 mL) cans at $5.00
- 2-litre bottles at $2.80.

Remember: 1 L = 1000 mL

We must determine the cheapest way to buy the soft drink.

Buying by the carton:
Cost = $22.50 ÷ (36 × 375)
= $0.0017 per mL

Determine the unit price of each type of purchase by dividing the total cost of items by the total volume.

Buying by the six-pack:
Cost = $5 ÷ (6 × 375)
= $0.0022 per mL

Buying by the bottle:
Cost = $2.80 ÷ 2000
= $0.0014 per mL

Compare the prices per mL and decide which is cheapest.

∴ the cheapest way to buy the soft drink is by the 2-litre bottle.

Apply similar techniques to answer the following questions.

12 Tea bags can be purchased from the supermarket in three forms:
- 25 tea bags at $2.36
- 50 bags at $4.80
- 200 bags at $15.00

What is the cheapest way to buy tea bags?

ISBN: 9781107687028 Cambridge University Press

13 A weekly train ticket costs $16. A day ticket costs $3.60. If you are going to work only 4 days next week, is it cheaper to buy one ticket per day or a weekly ticket?

14 A holiday resort offers its rooms at the following rates:
- $87 per night (Monday–Thursday)
- $187 for a weekend (Friday and Saturday)
- $500 per week

a Determine the nightly rate in each case.
b Which price is the best value?

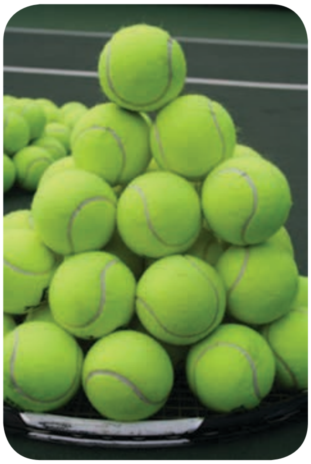
15 Tomato sauce is priced at:
- 200 mL bottle $2.35
- 500 mL bottle $5.24.

a Find the cost per mL of the tomato sauce in each case.
b Which is the cheapest way to buy tomato sauce?
c What would be the cost of 200 mL at the 500 mL rate?
d How much would be saved by buying the 200 mL bottle at this rate?
e Suggest why the 200 mL bottle is not sold at this price.

16 Safeserve has a sale on tennis balls for one month.
If you buy:
- 1 container, it costs $5
- 6 containers, it costs $28
- 12 containers, it costs $40
- 24 containers, it costs $60

If you need 90 containers for your club to have enough for a season:

a determine the minimum cost if you buy exactly 90 containers
b determine the overall minimum cost, and the number of extra containers you will have in this situation



ISBN: 9781107687028

Cambridge University Press

2.5 Simple interest and applications

Borrowed or invested money usually has an associated interest rate. The consumer needs to establish the type of interest they are paying and the effects it has on the amount borrowed or invested over time. Some loans or investments deliver the full amount of interest using only the initial loan or investment amount in the interest calculations. These types are said to use simple interest.

Let's start: How long to invest?

Tom and Brittney each have \$200 in their bank accounts. Tom earns \$10 a year in interest. Brittney earns 10% p.a. simple interest.

For how long must each of them invest their money for it to double in value?

Key ideas

- The terms needed to understand **simple interest** are:
  - **principal (P)**: the amount of money borrowed or invested
  - **rate of interest (r)**: the annual (yearly) percentage rate of interest (e.g. 3% p.a.)
  - time (t): the number of years for which the principal is borrowed or invested
  - interest (I): the amount of interest accrued over a given time.
- The formula for calculating simple interest is:

$$I = \text{principal} \times \text{rate} \times \text{time}$$

$$I = \frac{Prt}{100} \quad \text{(for yearly investment only)}$$

- Total repaid = amount + interest borrowed

Simple interest A type of interest that is paid on a loan or earned on an investment, which is always calculated on the principal amount loaned or invested

Principal (P) An amount of money invested in a financial institution or loaned to a person/business

Rate of interest (r) The annual percentage rate of interest paid or earned on a loan or investment

Exercise 2E

Understanding

1 Copy and complete:

a 12 months = ________ year

b $\frac{1}{2}$ year = _______ months

c ______ weeks = 1 year

d _____ quarters = 1 year

e 1 quarter = ______ months

f $2\frac{1}{2}$ years = _______ months

ISBN: 9781107687028

2 Interest on a loan is fixed at $60 a year. How much interest is due in:
a 2 years? **b** 7 years? **c** 6 months?

3 Simple interest on $7000 is 6% p.a. How much interest is earned in:
a 1 year? **b** 2 years? **c** 1 month?

Fluency

Example 16 Using the simple interest formula

Use the simple interest formula, $I = \frac{Prt}{100}$, to find:

a the interest (I) when $600 is invested at 8% p.a. for 18 months
b the annual interest rate (r) when $5000 earns $150 interest in 2 years

Solution	Explanation
a $P = 600$ $r = 8$ $t = 18\text{ months} = \frac{18}{12} = 1.5\text{ years}$	Write out the information that you know and the formula.
$I = \frac{Prt}{100}$ $= \frac{600 \quad 8 \quad 1.5}{100}$ $= 72$ The interest is $72 in 18 months.	Substitute into the formula using years for t.
b $P = 5000$ $I = 150$ $t = 2$ years $I = \frac{Prt}{100}$ $150 = \frac{5000 \quad r \quad 2}{100}$ $150 = 100 \times r$ $r = 1.5$	Write the formula and the information known. Substitute the values into the formula and solve the equation to find r. $\frac{50\cancel{0}\cancel{0} \quad r \quad 2}{1\cancel{0}\cancel{0}} = 100r$
The simple interest rate is 1.5% per year.	Write the rate as a percentage.

4 Copy and complete this table of values for I, P, r and t.

	P	Rate	Time	I
a	$700	5% p.a.	4 years	
b	$2000	7% p.a.	3 years	
c	$3500	3% p.a.	22 months	
d	$750	$2\frac{1}{2}$% p.a.	30 months	
e	$22 500		3 years	$2025
f	$1770		5 years	$354

ISBN: 9781107687028

Example 17 Calculating repayments with simple interest

\$3000 is borrowed at 12% p.a. simple interest for 2 years.

a What is the total amount owed over the 2 years?

b If repayments of the loan are made monthly, how much would each payment need to be?

Solution	Explanation
a $P = \$3000,\ r = 12,\ t = 2$	List the information you know.
$I = \dfrac{Prt}{100}$	Write the formula.
$= \dfrac{3000 \quad 12 \quad 2}{100}$	Substitute the values and evaluate.
$= \$720$	
Total amount $= 3000 + 720$	Total amount is the original amount *plus* the interest.
$= \$3720$	
b Amount of each payment $= 3720 \div 24$	2 years = 24 months
$= \$155$ per month	There are 24 payments to be made.
	Divide the total by 24.

5 \$5000 is borrowed at 11% p.a. simple interest for 3 years.

a What is the total amount owed over the 3 years?

b If repayments of the loan are made monthly, how much would each payment need to be?

Calculate the interest first.

6 Under hire purchase, John bought a new car for \$11 500. He paid no deposit and decided to pay the loan off in 7 years. If the simple interest was at 6.45%, determine:

a the total interest paid

b the total amount of the repayment

c the payments per month

7 \$10 000 is borrowed to buy a secondhand BMW. The interest is calculated at a simple interest rate of 19% p.a. over 4 years.

a What is the total interest on the loan?

b How much is to be repaid?

c What is the monthly repayment on this loan?

Problem-solving and Reasoning

8 How much interest will Georgio receive if he invests \$7000 in stocks at 3.6% p.a. simple interest for 4 years?

9 Rebecca invests \$4000 for 3 years at 5.7% p.a. simple interest paid yearly.

a How much interest will she receive in the first year?

b What is the total amount of interest Rebecca will receive over the 3 years?

c How much money will Rebecca have after the 3-year investment?

ISBN: 9781107687028 Cambridge University Press

10 An investment of \$15 000 receives an interest payment over 3 years of \$7200. What was the rate of simple interest per annum?

Substitute into the formula $I = \frac{Prt}{100}$ and solve the remaining equation.

11 Jonathon wishes to invest \$3000 at 8% per annum. How long will he need to invest for his total investment to double?

12 Grant wishes to invest some money for 5 years at 4.5% p.a. paid yearly. If he wishes to receive \$3000 in interest payments per year, how much should he invest? Round to the nearest dollar.

13 Gretta's interest payment on her loan totalled \$1875. If the interest rate was 5% p.a. and the loan had a life of 5 years, what amount did she borrow?

Which way is best?

14 A shed manufacturer offers finance with a rate of 3.5% p.a. paid at the end of 5 years with a deposit of 10%, or a rate of 6.4% p.a. repaid over 3 years with a deposit of 20%.

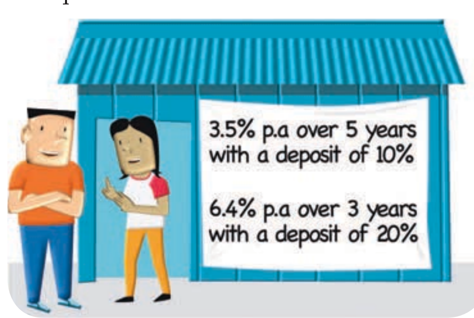
Christine and Donald decide to purchase a fully erected four-square shed for \$12 500.

a How much deposit will they need to pay in each case?

b What is the total interest they will incur in each case?

c If they decided to pay per month, what would be their monthly repayment?

d Discuss the benefits of the different types of purchasing methods.



2.6 Compound interest

For simple interest, the interest is always calculated on the principal amount. Sometimes, however, interest is calculated on the actual amount present in an account at each time period that interest is calculated. This means that the interest is added to the amount, then the next lot of interest is calculated again using this new amount. This process is called compound interest.

Compound interest can be calculated using updated applications of the simple interest formula or by using the compound interest formula.

Let's start: Investing using updated simple interest

Consider investing \$400 at 12% per annum. What is the balance at the end of 4 years if interest is added to the amount at the end of each year?
Copy and complete the table to find out.

Time	Amount (A)	Interest (I)	New amount
1st year	\$400	\$48	\$448
2nd year	\$448	\$53.76	\$501.76
3rd year	\$501.76		
4th year			

As you can see, the amount from which interest is calculated is continually changing.

Key ideas

- **Compound interest** can be found by using updated applications of the simple interest formula. For example, \$100 compounded at 10% p.a. for 2 years.

  Year 1: 100 + 10% of 100 = \$110

  Year 2: 110 + 10% of 110 = \$121, so compound interest = \$21
- The total amount in an account using compound interest for a given number of time periods is given by:

  $$A = P\left(1 + \frac{r}{100}\right)^n, \text{ where:}$$

  - principal (P) = the amount of money borrowed or invested
  - rate of interest (r) = the percentage applied to the principal per period of investment
  - periods (n) = the number of periods the principal is invested
  - amount (A) = the total amount of your investment.
- Interest = amount (A) − principal (P)

Compound interest A type of interest that is paid on a loan or earned on an investment, which is calculated not only on the initial principal, but also on the interest accumulated during the loan/investment period

ISBN: 9781107687028

Exercise 2F

Understanding

1 Consider \$500 invested at 10% p.a. compounded annually.
  a How much interest is earned in the first year?
  b What is the balance of the account once the first year's interest is added?
  c How much interest is earned in the second year?
  d What is the balance of the account at the end of the second year?
  e Use your calculator to work out $500(1.1)^2$.

For the second year, you need to use \$500 plus the interest from the first year.

2 Find the value of the following, correct to two decimal places.
  a $\$1000 \times 1.05 \times 1.05$
  b $\$1000 \times 1.05^2$
  c $\$1000 \times 1.05 \times 1.05 \times 1.05$
  d $\$1000 \times 1.05^3$

3 Fill in the missing numbers.
  a \$700 invested at 8% p.a. compounded annually for 2 years.
  $$A = \square\,(1.08)^{\square}$$
  b \$1000 invested at 15% p.a. compounded annually for 6 years.
  $$A = 1000\,(\square)^6$$
  c \$850 invested at 6% p.a. compounded annually for 4 years.
  $$A = 850\,(\square)^{\square}$$

For compound interest, $A = P\left(1 + \frac{r}{100}\right)^n$.

Example 18 Converting rates and time periods

Calculate the number of periods and the rates of interest offered per period for the following.
  a 6% p.a. over 4 years paid monthly
  b 18% p.a. over 3 years paid quarterly

Solution		Explanation
a $n = 4 \times 12$ $= 48$	$r = 6 \div 12$ $= 0.5$	4 years is the same as 48 months, as 12 months = 1 year. 6% p.a. = 6% in one year. Divide by 12 to find the monthly rate.
b $n = 3 \times 4$ $= 12$	$r = 18 \div 4$ $= 4.5$	There are 4 quarters in 1 year.

4 Calculate the number of periods (n) and the rates of interest (r) offered per period for the following (round the interest rate to three decimal places where necessary).
  a 6% p.a. over 3 years paid bi-annually
  b 12% p.a. over 5 years paid monthly
  c 4.5% p.a. over 2 years paid fortnightly
  d 10.5% p.a. over 3.5 years paid quarterly
  e 15% p.a. over 8 years paid quarterly
  f 9.6% p.a. over 10 years paid monthly

'Bi-annually' means 'twice a year'.
26 fortnights = 1 year.

ISBN: 9781107687028

2F

5 By considering an investment of \$4000 at 5% p.a. compounded annually, copy and complete the table below.

Year	Amount (\$)	Interest (\$)	New amount (\$)
1	4000	200	4200
2	4200		
3			
4			
5			

Fluency

Example 19 Using the compound interest formula

Determine the amount after 5 years if \$4000 is compounded annually at 8%.

Solution	Explanation
$P = 4000,\ n = 5,\ r = 8$	List the values for the terms you know.
$A = P\left(1 + \frac{r}{100}\right)^n$	Write the formula.
$= 4000\left(1 + \frac{8}{100}\right)^5$	Substitute the values.
$= 4000(1.08)^5$	Simplify and evaluate.
$= \$5877.31$	Write your answer to two decimal places.

6 Determine the amount after 5 years if:

a \$4000 is compounded annually at 5%
b \$8000 is compounded annually at 8.35%
c \$6500 is compounded annually at 16%
d \$6500 is compounded annually at 8%

$A = P\left(1 + \frac{r}{100}\right)^n$

7 Determine the amount if \$100 000 is compounded annually at 6% for:

a 1 year **b** 2 years **c** 3 years **d** 5 years **e** 10 years **f** 15 years

Example 20 Finding compounded amounts using months

Tony's investment of \$4000 is compounded at 8.4% p.a. over 5 years. Determine the amount he will have after 5 years if the interest is paid monthly.

Solution	Explanation
$P = 4000$ $n = 5 \times 12$ $= 60$	List the values of the terms you know. Convert the time in years to the number of periods (in this question, months). 60 months = 5 years.
$r = 8.4 \div 12$ $= 0.7$	Convert the rate per year to the rate per period (months) by dividing by 12.
$A = P\left(1 + \frac{r}{100}\right)^n$	Write the formula.
$= 4000(1 + 0.007)^{60}$ $= 4000(1.007)^{60}$	Substitute the values.
$= \$6078.95$	Simplify and evaluate.

ISBN: 9781107687028

8 Calculate the value of the following investments if interest is compounded monthly.

a $2000 at 6% p.a. for 2 years

b $34 000 at 24% p.a. for 4 years

c $350 at 18% p.a. for 8 years

d $670 at 6.6% p.a. for $2\frac{1}{2}$ years

e $250 at 7.2% p.a. for 12 years

Turn years into months and the annual rate into the monthly rate.

Problem-solving and Reasoning

9 An investment of $8000 is compounded at 12.6% over 3 years. Determine the amount the investor will have after 3 years if the interest is compounded monthly.

10 David invests $5000 compounded monthly at 18% p.a. Determine the value of the investment after:

a 1 month　　b 3 months　　c 5 months

18% p.a. = 1.5% each month

11 a Calculate the amount of compound interest paid on $8000 at the end of 3 years for each rate below.

i 12% compounded annually

ii 12% compounded bi-annually (twice a year)

iii 12% compounded monthly

iv 12% compounded weekly

v 12% compounded daily

b What is the interest difference between annual and daily compounding in this case?

12 The following are expressions relating to compound interest calculations. Determine the principal (P), number of periods (n), rate of interest per period (r), annual rate of interest (R) and the overall time (t).

a $300(1.07)^{12}$, bi-annually　　b $5000(1.025)^{24}$, monthly

c $1000(1.000\,36)^{65}$, fortnightly　　d $3500(1.000\,053)^{30}$, daily

e $10\,000(1.078)^{10}$, annually

13 Paula needs to decide whether to invest her $13 500 for 6 years at 4.2% p.a. compounded monthly or 5.3% compounded bi-annually. Decide which investment would be the best for Paula.

Double your money

14 You have $100 000 to invest and wish to double that amount. Use trial and error in the following.

a Determine, to the nearest whole number of years, the length of time it will take to do this using the compound interest formula at rates of:

i 12% p.a.　　ii 6% p.a.　　iii 8% p.a.

iv 16% p.a.　　v 10% p.a.　　vi 20% p.a.

b If the amount of investment is $200 000 and you wish to double it, determine the time it will take using the same interest rates as above.

c Are the lengths of time to double your investment the same in part **a** and part **b**?

ISBN: 9781107687028

2.7 Investments and loans

When you borrow money, interest is charged, and when you invest money, interest is earned. When you invest money, the institution in which you invest (e.g. bank or credit union) pays you interest. However, when you borrow money, the institution from which you borrow charges you interest, so that you must pay back the money you initially borrowed, plus the interest.

Credit cards charge high rates of interest if the full amount owing is not paid off every month.

Let's start: Credit card statements

Refer to Allan's credit card statement below.

a How many days were there between the closing balance and the due date?
b What is the minimum payment due?
c If Allan only pays the minimum, on what balance is the interest charged?
d How much interest is charged if Allan pays $475.23 on 25/5?

Statement Issue Date: 2/5/13		
Date of purchase	**Details**	**Amount**
3/4/13	Opening balance	314.79
5/4/13	Dean's Jeans	59.95
16/4/13	Tyre Warehouse	138.50
22/4//13	Payment made—thank you	–100.00
27/4/13	Cottonworth's Grocery Store	58.64
30/4/13	Interest charges	3.35
2/5/13	Closing balance	475.23
Percentage rate	**Due date**	**Min. payment**
18.95%	25/5/13	23.75

Key ideas

- Interest rates are associated with many loan and savings accounts.
- Bank accounts:
  – accrue interest each month on the minimum monthly balance
  – incur account-keeping fees each month
- **Loans** (money borrowed) have interest charged to them on the amount left owing (balance).
- **Repayments** are amounts paid to the bank, usually each month, to repay a loan plus interest within an agreed time period.

Loan Money borrowed and then repaid, usually with interest

Repayment An amount paid to a financial institution at regular intervals to repay a loan, with interest included

ISBN: 9781107687028 Cambridge University Press

Exercise 2G

Understanding

1 Donna can afford to repay \$220 a month. How much does she repay over:
a 1 year? b 18 months? c 5 years?

2 Sarah buys a new bed on a 'buy how, pay later' offer. No interest is charged if she pays for the bed in 2 years. Sarah's bed costs \$2490 and she pays it back in 20 months in 20 equal instalments. How much is each instalment?

3 A bank pays 0.3% interest on the minimum monthly balance in an account. Determine the interest due on accounts with the following minimum monthly balances.
a \$400 b \$570 c \$1000 d \$29.55

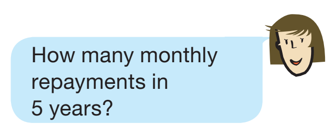
Fluency

Example 20 Repaying a loan

Wendy takes out a personal loan of \$7000 to fund her trip to South Africa. Repayments are made monthly for 3 years at \$275 a month. Find:
a the total cost of Wendy's trip
b the interest charged on the loan

Solution	Explanation
a Total cost = \$275 × 36 = \$9900	3 years = 3 × 12 = 36 months Cost = 36 lots of \$275
b Interest = \$9900 − \$7000 = \$2900	Interest = total paid − amount borrowed

4 Jason has a personal loan of \$10 000. He is repaying the loan over 5 years. The monthly repayment is \$310.
a Calculate the total amount Jason repays over the 5-year loan.
b How much interest is he charged?



5 Robert borrows \$5500 to buy a second-hand car. He repays the loan in 36 equal monthly instalments of \$155.
a Calculate the total cost of the loan.
b How much interest does Robert pay?

6 Alma borrows \$250 000 to buy a house. The repayments are \$1736 a month for 30 years.
a How many repayments does Alma make?
b What is the total Alma pays for the house?
c How much interest is paid over the 30 years?

ISBN: 9781107687028

Example 21 Paying off a purchase

Harry buys a new \$2100 computer on the following terms:

- 20% deposit
- monthly repayments of \$90 for 2 years

Find:

a the deposit paid
b the total paid for the computer
c the interest charged

Solution	Explanation
a Deposit $= 0.2 \times 2100$ $= \$420$	Find 20% of 2100.
b Repayments $= \$90 \times 24$ $= \$2160$	2 years = 24 months Repay 24 lots of \$90.
Total paid $= \$2160 + \$420$ $= \$2580$	Repay = deposit + repayments
c Interest $= \$2580 - \$2100$ $= \$480$	Interest = total paid − original price

7 George buys a car marked at \$12 750 on the following terms:
20% deposit and 36 monthly repayments of \$295.

a Calculate the deposit.
b How much is owed after the deposit is paid?
c Find the total of all the repayments.
d Find the cost of buying the car on those terms.
e Find the interest George pays on these terms.

Example 22 Calculating interest

An account has a minimum monthly balance of \$200 and interest is credited on this amount monthly at 1.5%.

a Determine the amount of interest to be credited at the end of the month.
b If the bank charges a fixed administration fee of \$5 per month and other fees totalling \$1.07, what will be the net amount credited or debited to the account at the end of the month?

Solution	Explanation
a Interest = 1.5% of \$200 $= 0.015 \times 200$ $= \$3$	Interest is 1.5% per month. Change 1.5% to a decimal and calculate.
b Net amount $= 3 - (5 + 1.07)$ $= -3.07$ \$3.07 will be debited from the account.	Subtract the deductions from the interest. A negative amount is called a debit.

ISBN: 9781107687028

8 A bank account has a minimum monthly balance of \$300 and interest is credited monthly at 1.5%.
  a Determine the amount of interest to be credited each month.
  b If the bank charges a fixed administration fee of \$3 per month and fees of \$0.24, what will be the net amount credited to the account at the end of the month?

9 An account has no administration fee. The monthly balances for May–October are in the table below. If the interest payable on the minimum monthly balance is 1%, how much interest will be added:
  a for each separate month?   b over the 6-month period?

May	June	July	August	September	October
\$240	\$300	\$12	\$500	\$208	\$73

Problem-solving and Reasoning

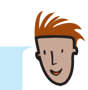
10 Supersound offers two deals on a sound system worth \$7500:
  - Deal A: no deposit, interest free and nothing to pay for 18 months
  - Deal B: 15% off for cash

  a Thomas chooses deal A. Find:
    i the deposit he must pay   ii the interest charged
    iii the total cost if Thomas pays the system off within the 18 months
  b Phil chooses deal B. What does Phil pay for the same sound system?
  c How much does Phil save by paying cash?

15% off is 85% of the original amount.

11 Camden finance company charges 35% flat interest on all loans.
  a Mei borrows \$15 000 from Camden finance over 6 years.
    i Calculate the interest on the loan.   ii What is the total repaid (loan + interest)?
    iii What is the value of each monthly repayment?
  b Lancelle borrows \$24 000 from the same company over 10 years.
    i Calculate the interest on her loan.   ii What is the total repaid?
    iii What is the value of each monthly instalment?

12 A list of transactions that Sally made over a 1-month period is shown. The bank calculates interest *daily* at 0.01% and adds the total to the account balance at the end of this period. It has an administrative fee of \$7 per month and other fees over this time total \$0.35.
  a Copy and complete the table.
  b Determine the amount of interest added over this month.
  c Determine the final balance after all calculations have been made.
  d Suggest what the regular deposits might be for.



In part b, interest is calculated on the end-of-the-day balance.

Date	Deposit	Withdrawal	Balance
1 May			\$3010
3 May	\$490		
5 May		\$2300	
17 May	\$490		
18 May		\$150	
20 May		\$50	
25 May		\$218	
31 May	\$490		

ISBN: 9781107687028

13 The following table shows the interest and monthly repayments on loans when the simple interest rate is 8.5% p.a.

a Use the table to find the monthly repayments for a loan of:

i \$1500 over 2 years **ii** \$2000 over 3 years **iii** \$1200 over 18 months

b Damien and his wife Lisa can afford monthly repayments of \$60. What is the most they can borrow and on what terms?

	18-month term		24-month term		36-month term	
Loan amount	Interest (\$)	Monthly payments (\$)	Interest (\$)	Monthly payments (\$)	Interest (\$)	Monthly payments (\$)
1000	127.50	62.64	170.00	48.75	255.00	34.86
1100	140.25	68.90	187.00	53.63	280.50	38.35
1200	153.00	75.17	204.00	58.50	306.00	41.83
1300	165.75	81.43	221.00	63.38	331.50	45.32
1400	178.50	87.69	238.00	68.25	357.00	48.81
1500	191.25	93.96	255.00	73.13	382.50	52.29
1600	204.00	100.22	272.00	78.00	408.00	55.78
1700	216.75	106.49	289.00	82.88	433.50	59.26
1800	229.50	112.75	306.00	87.75	459.00	62.75
1900	242.25	119.01	323.00	92.63	484.50	66.24
2000	255.00	125.28	340.00	97.50	510.00	69.72

14 Part of a credit card statement is shown here.

understanding your account

CLOSING BALANCE \$403.80	CLOSING BALANCE This is the amount you owe at the end of the statement period
MINIMUM PAYMENT DUE \$10.00	MINIMUM PAYMENT DUE This is the minimum payment which must be made towards this account
PAYABLE TO MINIMISE FURTHER INTEREST CHARGES \$403.80	PAYABLE TO MINIMISE FURTHER INTEREST CHARGES This amount you must pay to minimise interest charges for the next statememt period

a What is the closing balance?

b What is due on the card if only the minimum payment is made on the due date?

c This card charges 21.9% p.a. interest calculated daily on the unpaid balances. To find the daily interest, they multiply this balance by 0.0006. What does it cost in interest per day if only the minimum payment is made?

ISBN: 9781107687028 Cambridge University Press

15 Loans usually involve an establishment fee to set up the loan and an interest rate calculated monthly on your balance. You make a monthly or fortnightly payment, which reduces the balance. Bank fees also apply.
Consider the period for the loan statement shown below.

a What is the opening balance for this statement?
b What is the administrative fee charged by the bank for each transaction?
c What is the regular fee charged by the bank for servicing the loan?
d If the term of the loan is 25 years, what will be the total servicing fees charged by the bank?
e What is the regular fortnightly payment made?
f What will be the total fortnightly payments made over the term of the loan?
g If the loan was originally \$100 000, how much extra is paid in interest over the 25 years? (Note that interest does not include any fees.)

Complete Home Loan Transactions – Account number 33164 000				
Date	**Transaction description**	**Debits**	**Credits**	**Balance**
	Balance brought forward from previous page			98 822.90 Dr
15 Oct	Repayment/Payment		378.50	
	Administrative fee	0.23		98 444.63 Dr
24 Oct	Interest charged	531.88		98 976.51 Dr
24 Oct	Fee for servicing your loan	8.00		98 984.51 Dr
29 Oct	Repayment/Payment		378.50	
	Administrative fee	0.23		98 606.24 Dr
12 Nov	Repayment/Payment		378.50	
	Administrative fee	0.23		98 227.97 Dr
24 Nov	Interest charged	548.07		98 776.04 Dr
24 Nov	Fee for servicing your loan	8.00		98 784.04 Dr
26 Nov	Repayment/Payment		378.50	
	Administrative fee	0.23		98 405.77 Dr
→	Change in interest rate on 03/12/07 to 06.800% per annum			
10 Dec	Repayment/Payment		378.50	
	Administrative fee	0.23		98 027.50 Dr
24 Dec	Interest charged	543.08		98 570.58 Dr
24 Dec	Fee for servicing your loan	8.00		98 578.58 Dr
24 Dec	Repayment/Payment		378.50	
	Administrative fee	0.23		98 200.31 Dr
31 Dec	Closing balance			98 200.31 Dr

ISBN: 9781107687028 Cambridge University Press

Reducing the balance of loans (a spreadsheet approach)

When you take out loan from a lending institution you will be asked to make regular payments (usually monthly) for a certain period of time to repay the loan completely. The larger the repayment, the shorter the term of the loan.

Loans work mostly on a reducing balance and you can find out how much balance is owing at the end of each month from a statement, which is issued on a regular basis.

Let's look at an example of how the balance is reducing.

If you borrow \$15 000 at 17% p.a. and make repayments of \$260 per month, at the end of the first month your statement would be calculated as shown at the right.

$$\begin{aligned}\text{Interest due} &= \frac{150\,00 \times 0.17}{12}\\ &= \$212.50\\ \text{Repayment} &= \$260\\ \text{Amount owing} &= \$15\,000 + \$212.50 - \$260\\ &= \$14\,952.50\end{aligned}$$

This process would be repeated for the next month:

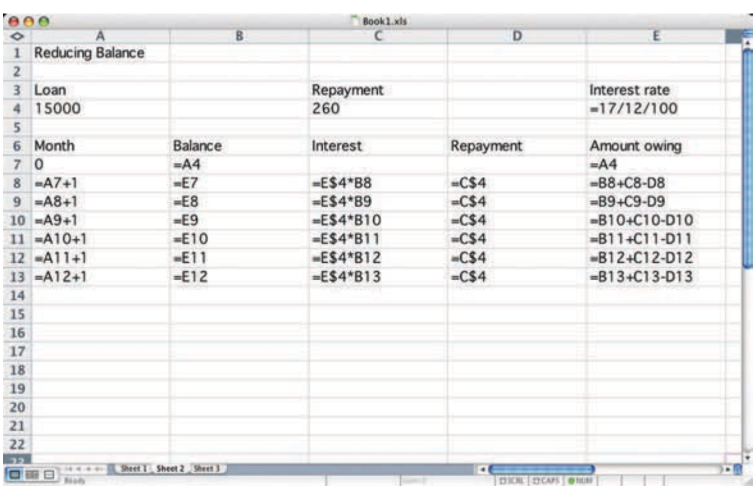
$$\begin{aligned}\text{Interest due} &= \frac{14\,952.50 \times 0.17}{12}\\ &= \$211.83\\ \text{Repayment} &= \$260\\ \text{Amount owing} &= \$14952.50 + \$211.83 - \$260\\ &= \$14\,904.33\end{aligned}$$

As you can see, the amount owing is decreasing and so is the interest owed each month. Meanwhile, more of your repayment is actually reducing the balance of the loan.

A statement might look like this:

Balance	Interest	Repayment	Amount owing
15 000	212.50	260	14 952.50
14 952.50	211.83	260	14 904.33
14 904.33	211.14	260	14 855.47
14 855.47	210.45	260	14 805.92
14 805.92	209.75	260	14 755.67

16 Check to see that all the calculations are correct on the statement above.
As this process is repetitive, the calculations are best done by means of a spreadsheet. To create a spreadsheet for the process, copy the following, extending your sheet to cover 5 years.



	A	B	C	D	E
1	Reducing Balance				
2					
3	Loan		Repayment		Interest rate
4	15000		260		=17/12/100
5					
6	Month	Balance	Interest	Repayment	Amount owing
7	0	=A4			=A4
8	=A7+1	=E7	=E$4*B8	=C$4	=B8+C8-D8
9	=A8+1	=E8	=E$4*B9	=C$4	=B9+C9-D9
10	=A9+1	=E9	=E$4*B10	=C$4	=B10+C10-D10
11	=A10+1	=E10	=E$4*B11	=C$4	=B11+C11-D11
12	=A11+1	=E11	=E$4*B12	=C$4	=B12+C12-D12
13	=A12+1	=E12	=E$4*B13	=C$4	=B13+C13-D13

ISBN: 9781107687028

2.8 Comparing interest using technology

In the following exercise we compare compound and simple interest and look at their applications to the banking world. You are expected to use technology to its best advantage in solving the problems in this section.

Let's start: Who earns the most?

- Ceanna invests \$500 at 8% p.a. compounded monthly over 3 years.
- Huxley invests \$500 at 10% p.a. compounded annually over 3 years.
- Loreli invests \$500 at 15% p.a. simple interest over 3 years.

– How much does each person have at the end of the 3 years?
– Who earned the most?

Key ideas

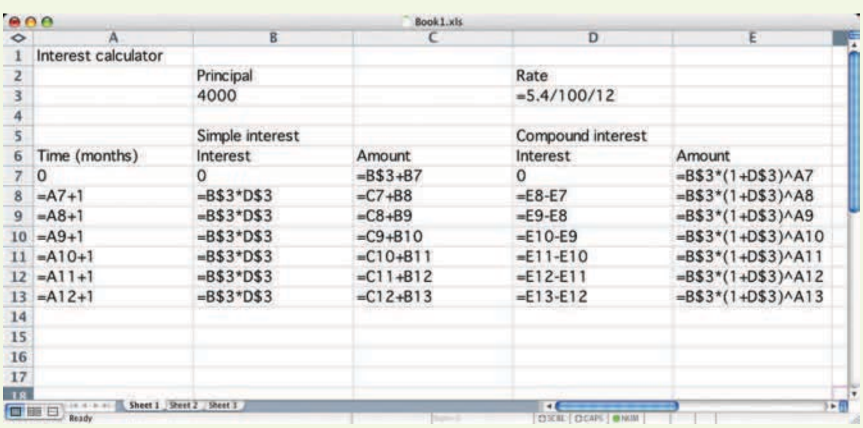
You can calculate the total amount of your investment for either form of interest using technology.

- **Graphics calculator**
  To create programs for the two types of interest, enter the following data.
  This will allow you to calculate both types of interest for a given time period. If you invest \$100 000 at 8% p.a. paid monthly for 2 years, you will be asked for P, $R = \frac{r}{100}$, t or n and the calculator will do the work for you.
  Note: Some modifications may be needed for the CAS or other calculators

```
PROGRAM:SIMPLE
:Prompt P,R,T
:PRT→I
:Disp "INTEREST"
,I
:I+P→A
:Disp "AMOUNT",A
```

```
PROGRAM:COMPOUND
:Prompt P,R,N
:P(1+R)^N→A
:Disp "AMOUNT",A
:A-P→I
:Disp "INTEREST"
,I
```

- **Spreadsheet**
  Copy and complete the spreadsheets as shown, to compile a simple interest and compound interest sheet.



	A	B	C	D	E
1	Interest calculator				
2		Principal		Rate	
3		4000		=5.4/100/12	
4					
5		Simple interest		Compound interest	
6	Time (months)	Interest	Amount	Interest	Amount
7	0	0	=B\$3+B7	0	=B\$3*(1+D\$3)^A7
8	=A7+1	=B\$3*D\$3	=C7+B8	=E8-E7	=B\$3*(1+D\$3)^A8
9	=A8+1	=B\$3*D\$3	=C8+B9	=E9-E8	=B\$3*(1+D\$3)^A9
10	=A9+1	=B\$3*D\$3	=C9+B10	=E10-E9	=B\$3*(1+D\$3)^A10
11	=A10+1	=B\$3*D\$3	=C10+B11	=E11-E10	=B\$3*(1+D\$3)^A11
12	=A11+1	=B\$3*D\$3	=C11+B12	=E12-E11	=B\$3*(1+D\$3)^A12
13	=A12+1	=B\$3*D\$3	=C12+B13	=E13-E12	=B\$3*(1+D\$3)^A13
14					
15					
16					
17					

Fill in the principal in B3 and the rate per period in D3. For example, for \$4000 invested at 5.4% monthly, B3 will be 4000 and D3 will be $\frac{0.054}{12}$.

ISBN: 9781107687028

Exercise 2H

Understanding

1 Which is better on an investment of \$100 for 2 years:
A simple interest calculated at 20% p.a.?
B compound interest calculated at 20% p.a. and paid annually?

2 Write down the values of P, r and n for an investment of \$750 at 7.5% p.a. compounded annually for 5 years.

3 Write down the values of I, P, r and t for an investment of \$300 at 3% p.a. simple interest over 300 months.

Recall: For simple interest $I = \frac{Prt}{100}$; for compound interest $A = P\left(1 + \frac{r}{100}\right)^n$

Fluency

Example 23 Using a spreadsheet

Find the total amount of the following investments using technology.
a \$5000 at 5% p.a. compounded annually for 3 years
b \$5000 at 5% p.a. simple interest for 3 years

Solution	Explanation
a \$5788.13	$A = P\left(1 + \frac{r}{100}\right)^n$
b \$5750	$I = \frac{Prt}{100}$

4 **a** Find the total amount of the following investments using technology.
i \$6000 at 6% p.a. compounded annually for 3 years
ii \$6000 at 3% p.a. compounded annually for 5 years
iii \$6000 at 3.4% p.a. compounded annually for 4 years
iv \$6000 at 10% p.a. compounded annually for 2 years
v \$6000 at 5.7% p.a. compounded annually for 5 years
b Which of the above yields the most interest?

5 **a** Find the total amount of the following investments, using technology where possible.
i \$6000 at 6% p.a. simple interest for 3 years
ii \$6000 at 3% p.a. simple interest for 6 years
iii \$6000 at 3.4% p.a. simple interest for 7 years
iv \$6000 at 10% p.a. simple interest for 2 years
v \$6000 at 5.7% p.a. simple interest for 5 years
b Which of the above yields the most interest?

Problem-solving and Reasoning

6 **a** Determine the total simple and compound interest accumulated on the following.
i \$4000 at 6% p.a. payable annually for:
I 1 year **II** 2 years **III** 5 years **IV** 10 years
ii \$4000 at 6% p.a. payable bi-annually for:
I 1 year **II** 2 years **III** 5 years **IV** 10 years
iii \$4000 at 6% p.a. payable monthly for:
I 1 year **II** 2 years **III** 5 years **IV** 10 years

6% p.a. paid biannually is 3% per 6 months.
6% p.a. paid monthly is $\frac{6}{12} = 0.5\%$ per month.

ISBN: 9781107687028 Cambridge University Press

b Would you prefer the same rate of compound interest or simple interest if you were investing money and paying off the loan in instalments?

c Would you prefer the same rate of compound interest or simple interest if you were borrowing money?

7 **a** Copy and complete the following table if simple interest is applied.

$I = \dfrac{Prt}{100}$

$A = P + I$

Principal	Rate	Overall time	Interest	Amount
$7000		5 years		$8750
$7000		5 years		$10 500
	10%	3 years	$990	
	10%	3 years	$2400	
$9000	8%	2 years		
$18 000	8%	2 years		

b Explain the effect on the interest when we double the:

i rate **ii** period **iii** overall time

8 Copy and complete the following table if compound interest is applied. You may need to use a calculator and trial and error to find some of the missing numbers.

Principal	Rate	Period	Overall time	Interest	Amount
$7000		Annually	5 years		$8750
$7000		Annually	5 years		$10 500
$9000	8%	Fortnightly	2 years		
$18 000	8%	Fortnightly	2 years		

Changing the parameters

9 If you invest $5000, determine the interest rate per annum (to two decimal places) if the total amount is approximately $7500 after 5 years and if:

a interest is compounded annually

b interest is compounded quarterly

c interest is compounded weekly

Comment on the effect of changing the period for each payment on the rate needed to achieve the same total amount in a given time.

10 **a** Determine, to one decimal place, the equivalent simple interest rate for the following investments over 3 years.

i $8000 at 4% compounded annually

ii $8000 at 8% compounded annually

b If you double or triple the compound interest rate, how is the simple interest rate affected?

ISBN: 9781107687028 Cambridge University Press

1 Find and define the 10 terms related to consumer arithmetic and percentages hidden in this wordfind.

C	O	M	M	I	S	S	I	O	N	Q	R	W
P	G	S	L	E	R	S	T	B	L	D	U	J
H	L	A	A	P	I	E	C	E	W	O	R	K
U	F	N	U	L	N	Q	B	D	Z	T	J	L
V	K	N	S	T	A	M	O	N	T	H	L	Y
B	H	U	A	I	G	R	O	S	S	U	B	S
N	E	A	C	Y	K	S	Y	E	T	Y	M	D
M	A	L	O	V	E	R	T	I	M	E	Q	T
S	F	O	R	T	N	I	G	H	T	L	Y	S

2 How do you stop a bull charging you? Answer the following problems and match the letters to the answers below to find out.

\$19.47 – \$8.53 E	5% of \$89 T	50% of \$89 I
$12\frac{1}{2}$% of \$100 A	If 5% = \$8.90 then 100% is? S	\$4.48 to the nearest 5 cents R
6% of \$89 W	Increase \$89 by 5% H	10% of \$76 O
\$15 monthly for 2 years D	$12\frac{1}{2}$% as a decimal K	\$50 – \$49.73 U
Decrease \$89 by 5% C	\$15.96 + \$12.42 Y	

\$28.38 \$7.60 27c

\$4.45 \$12.50 0.125 \$10.94

\$12.50 \$5.34 \$12.50 \$28.38

\$93.45 \$44.50 \$178

\$84.55 \$4.50 \$10.94 \$360 \$44.50 \$4.45

\$84.55 \$12.50 \$4.50 \$360

3 How many years does it take \$1000 to double if it is invested at 10% p.a. compounded annually?

4 The chance of Jayden winning a game of cards is said to be 5%. How many consecutive games should Jayden play to be 95% certain he has won at least one of the games played?

ISBN: 9781107687028

Chapter summary

Consumer arithmetic

Conversions

Fraction $\xrightarrow{\times 100\%}$ Percentage

Percentage $\xrightarrow{\div 100\%}$ Fraction

Decimal $\xrightarrow{\times 100\%}$ Percentage

Percentage $\xrightarrow{\div 100\%}$ Decimal

Percentage work

- 15% *of* 459

$= \frac{15}{100} \times 459$

Increase 20 by 8%

$= 20 \times 108\%$

$= 20 \times 1.08$

Decrease 20 by 8%

$= 20 \times 92\%$

$= 20 \times 0.92$

profit = selling price − cost price

$\%\text{ profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$

$\%\text{ discount} = \frac{\text{discount}}{\text{cost price}} \times 100\%$

Percentages

'Out of 100'

79 per cent = 79% = $\frac{79}{100}$

Income

hourly rate = \$/h

time and a half = $1\frac{1}{2} \times$ hrly rate

double time = 2 × hrly rate

commission is x% of total sales

gross income = total of all money earned

net income = gross income − deductions

taxation (income tax) = money given from wages to the government (Refer to the income tax table.)

Simple (flat rate) interest

$I = \frac{Prt}{100}$

I = simple interest

P = principal (\$ invested)

r = rate per year, as a percentage

t = number of years

Compound interest

$A = P(1 + \frac{r}{100})^n$

A = final balance

P = principal (\$ invested)

r = rate per time period, as a percentage

n = number of time periods money is invested for

Spreadsheets can be used to manage money or compare loans.

Budgets

Deciding how income is spent on *fixed* and *variable* expenses.

Loans

Balance owing = amount left to repay

Repayment = money given each month to repay the loan amount and the interest

ISBN: 9781107687028

Multiple-choice questions

1 28% of \$89 is closest to:
A \$28.00 **B** \$64.08 **C** \$113.92 **D** \$2492 **E** \$24.92

2 As a percentage, $\frac{21}{60}$ is:
A 21% **B** 3.5% **C** 60% **D** 35% **E** 12.6%

3 If a budget allows 30% for car costs, how much is allocated from a weekly wage of \$560?
A \$201 **B** \$145 **C** \$100 **D** \$168 **E** \$109

4 The gross income for 30 hours at \$5.26 per hour is:
A \$35.26 **B** \$389.24 **C** \$157.80 **D** \$249.20 **E** \$24.92

5 If Simone received \$2874 on the sale of a property worth \$195 800, her rate of commission, to one decimal place, was:
A 21% **B** 1.5% **C** 60% **D** 15% **E** 12.6%

6 In a given rostered fortnight, Bill works the following number of 8 hour shifts:
- three day shifts (\$10.60 per hour)
- three afternoon shifts (\$12.34 per hour)
- five night shifts (\$16.78 per hour).

His total income for the fortnight is:
A \$152.72 **B** \$1457.34 **C** \$1000 **D** \$168.84 **E** \$1221.76

7 An iPod is discounted by 26%. What is the price if it was originally \$56?
A \$14.56 **B** \$41.44 **C** \$26.56 **D** \$13.24 **E** \$35.22

8 A \$5000 loan is repaid by monthly instalments of \$200 for 5 years. The amount of interest charged is:
A \$300 **B** \$7000 **C** \$12 000 **D** \$2400 **E** \$6000

9 The simple interest on \$600 at 5% for 4 years is:
A \$570 **B** \$630 **C** \$120 **D** \$720 **E** \$30

10 The compound interest on \$4600 at 12% p.a. for 2 years is:
A \$1104 **B** \$5704 **C** \$4600 **D** \$5770.24 **E** \$1170.24

Short-answer questions

1 Find 15.5% of \$9000.

2 Increase \$968 by 12%.

3 Decrease \$4900 by 7%.

4 The cost price of an item is \$7.60. If the mark-up is 50%, determine:
a the retail price **b** the profit made

5 An airfare of \$7000 is discounted 40% if you fly off-peak. What would be the discounted price?

ISBN: 9781107687028

6 A couch is discounted to $375. If this is a 35% discount, find the recommended retail price.

7 Pina budgets 20% of her income for entertainment. If her yearly income is $37 000, how much could be spent on entertainment in:

a a year?
b a month?
c a week?

8 Maria works a 34-hour week at $13.63 per hour. Her net income is 62% of her wage.

a Work out her net income.
b If 15% is spent on clothing, determine the amount she can spend each week.
c If she saves $50, what percentage (to two decimal places) of her gross weekly income is this?

9 Frank has the following costs to run his car:

- hire purchase payment $350 per month
- registration $485 per year
- insurance $315 per quarter
- servicing $1700 per year
- petrol $90 per week

a Find the total cost of running his vehicle for 1 year.
b What percentage (to the nearest percentage) of the overall cost to run the car is the cost of the petrol?

10 Tom works 36 hours at $9.63 per hour. He pays $47.53 in tax and $8.50 in superannuation. Determine:

a his gross wage
b his net pay.

11 Lil receives an annual salary of $47 842. Using the tax table shown, calculate the amount of tax she pays over the year.

Income	Tax payable
Less than $6000	Nil
$6000–$21 600	15 cents for each dollar over $6000
$21 601–$70 000	$2340 plus 30 cents for each dollar over $21 600
$70 001–$125 000	$16 860 plus 42 cents for each dollar over $70 000
Greater than $125 000	$39 960 plus 47 cents for each dollar over $125 000

12 Paul receives 4.5% commission on sales of $790. Determine the amount of his commission.

ISBN: 9781107687028

13 A vehicle worth \$7000 is purchased on a finance package. The purchaser pays 15% deposit and \$250 per month over 4 years.

- a How much deposit is paid?
- b What are the total repayments?
- c How much interest is paid over the term of the loan?

14 Find the interest paid on a \$5000 loan under the following conditions.

- a 8% p.a. simple interest over 4 years
- b 7% p.a. simple interest over 3 years and 4 months
- c 4% p.a. compounded annually over 3 years
- d 9.75% p.a. compounded annually over 2 years

Extended-response questions

1 \$5000 is invested at 4% p.a. compounding annually for 3 years.

- a What is the value of the investment after the 3 years?
- b How much interest is earned in the 3 years?
- c Using $r = \dfrac{100I}{Pt}$, what simple rate of interest results in the same amount?
- d How much interest is earned on the investment if it is compounded monthly at 4% p.a. for the 3 years?

2 Your bank account has an opening July monthly balance of \$217.63. You have the following transactions over the month.

Date	Withdrawals	Date	Deposits
7 July	\$64.00	July 9th	\$140
11 July	\$117.34	July 20th	\$20
20 July	\$12.93	July 30th	\$140

- a Design a statement of your records if \$0.51 is taken out as a fee on 15 July.
- b Find the minimum balance.
- c If interest is credited monthly on the balance at 0.05%, determine the interest for July, rounded to the nearest cent.

ISBN: 9781107687028 Cambridge University Press

Australian Curriculum
Number and Algebra
Patterns and algebra

chapter
3 Algebra and indices

What you will learn

3.1 Algebraic expressions
3.2 Simplifying algebraic expressions
3.3 Expanding algebraic expressions
3.4 Factorising algebraic expressions
3.5 Multiplying and dividing algebraic fractions
3.6 Adding and subtracting algebraic fractions
3.7 Index notation and index laws 1 and 2
3.8 More index laws and the zero power
3.9 Scientific notation
3.10 Exponential growth and decay

Investment returns

Combined with indices, algebra plays an important role in simplifying calculations in the financial world. Consider this situation, where algebra and indices combine to determine the value of an investment.

Does an average investment return of 15% sound good to you? This return is compounded annually, so a \$10 000 investment would grow to more than \$40 000 after 10 years. This is calculated by multiplying the investment total by 1.15 (to return the original amount plus the 15%) for each year. Using indices, the total investment value after n years would be given by

Value $= 10\,000 \times 1.15^n$.

ISBN: 9781107687028

Pre-test

1 Write algebraic expressions for the following.

a 3 lots of x **b** one more than a
c 5 less than $2m$ **d** 4 times the sum of x and y

2 Find the value of the following if $x = 4$ and $y = 7$.

a $5x$ **b** $2y + 3$
c $xy - 5$ **d** $2(x + y)$

3 Decide whether the following pairs of terms are like terms.

a $6x$ and 8 **b** $3a$ and $7a$
c $4xy$ and $2yx$ **d** $3x^2$ and $10x$

4 Simplify:

a $3m + 5m$ **b** $8ab - 3ab$
c $4x + 3y + 2x + 5y$ **d** $2 \times 4 \times x$
e $5 \times a \times 3 \times b$ **f** $6y \div 2$

5 Expand:

a $2(x + 5)$ **b** $3(y - 2)$
c $4(2x - 3)$ **d** $x(3x + 1)$

6 Find the HCF (highest common factor) of these pairs of terms.

a 8, 12 **b** 18, 30
c $7a$, $14a$ **d** $2xy$ and $8xz$
e $5x$ and $8x^2$

7 Simplify:

a $\frac{3}{8} + \frac{2}{5}$ **b** $\frac{6}{7} - \frac{1}{3}$
c $\frac{5}{9} \quad \frac{6}{25}$ **d** $\frac{2}{3} \mid \frac{4}{9}$

8 Write each of the following in index form (e.g. $5 \times 5 \times 5 = 5^3$).

a $7 \times 7 \times 7 \times 7$ **b** $m \times m \times m$
c $x \times x \times y \times y \times y$ **d** $3a \times 3a \times 3a \times 3a \times 3a$

9 Evaluate:

a 7^2 **b** 3^3
c 1 **d** 4^3

10 Write the following as 3 raised to a single power.

a $3^4 \times 3^3$ **b** $3^7 \div 3^5$ **c** $(3^2)^5$
d 1 **e** $\frac{1}{3^2}$

11 Complete the following.

a $3.8 \times 10 =$ ______ **b** $2.31 \times 1000 =$ ________
c $17.2 \div 100 =$ ______ **d** $0.18 \div 100 =$ ________
e $3827 \div$ ______ $= 3.827$ **f** $6.49 \times$ ________ $= 64\,900$

ISBN: 9781107687028

3.1 Algebraic expressions

Algebra involves the use of pronumerals (also called variables), which are letters that represent numbers. Numbers and pronumerals connected by multiplication or division form *terms*, while *expressions* are one or more terms connected by addition or subtraction.

If a ticket to an art gallery costs \$12, then the cost for y visitors is the expression $12 \times y = 12y$. By substituting values for y we can find the costs for different numbers of visitors. For example, if there are five visitors, then $y = 5$ and $12y = 12 \times 5 = \$60$.

Let's start: Expressions at the gallery

Ben, Alea and Victoria are visiting the art gallery. The three of them combined have \$$c$ between them. Drinks cost \$$d$ and Ben has bought x postcards in the gift shop.
Write expressions for the following:

- The cost of two drinks
- The amount of money each person has if the money is shared equally
- The number of postcards Alea and Victoria bought if Alea bought three more than Ben and Victoria bought five less than twice the number Ben bought.

Key ideas

- Algebraic **expressions** are made up of one or more terms connected by addition or subtraction; e.g. $3a + 7b$, $\frac{x}{2} + 3y$, $3x - 4$.
  - A **term** is a group of numbers and pronumerals connected by multiplication and division; e.g. $2x$, $\frac{y}{4}$, $5x^2$.
  - A **constant term** is a number with no attached pronumerals; e.g. 7, -3.
  - The **coefficient** is the number multiplied by the pronumerals in the term; e.g. 3 is the coefficient of y in $2x + 3y$
    -4 is the coefficient of x in $5 - 4x$
    1 is the coefficient of x^2 in $2x + x^2$

  This expression has 3 terms: $3x$, $2y$ and 4

  $$3x - 2y + 4$$

  3 is the coefficient of x | -2 is the coefficient of y | 4 is the constant term

- Operations:
  - The operations for addition and subtraction are written with '+' and '–'.
  - Multiplication is written without the sign; e.g. $3 \times y = 3y$.
  - Division is written as a fraction; e.g. $y \div 4 = \frac{y}{4}$ or $\frac{1}{4}y$.

Expression A group of mathematical terms containing no equals sign

Term A number or pronumeral in an expression

Constant term The part of an equation or expression without any pronumerals

Coefficient A numeral placed before a pronumeral, showing that the pronumeral is multiplied by that factor

ISBN: 9781107687028

- The value of an expression can be found by **substituting** a value for each pronumeral. The order of operations (BODMAS) is followed. For example, if $x = 2$ and $y = 3$:

$$\begin{aligned} 4xy - y^2 &= 4 \times 2 \times 3 - 3^2 \\ &= 24 - 9 \\ &= 15 \end{aligned}$$

Substitute
To replace pronumerals with numerical values

Exercise 3A

Understanding

1 Fill in the missing word(s) in the sentences using these words:

expression, term, constant term, coefficient

a An algebraic ________ is made up of one or more terms connected by addition and subtraction.
b A term without a pronumeral part is a ________.
c A number multiplied by the pronumerals in a term is a ________.
d Numbers and pronumerals connected by multiplication and division form a ________.

2 Decide which mathematical operation (×, ÷, +, –) matches each of the following.

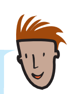
a sum **b** less than **c** product
d difference **e** more than **f** quotient

3 Substitute the value 3 for the pronumeral x in the following and evaluate.

a $x + 4$ **b** $5x$ **c** $8 - x$ **d** x^2 **e** $\dfrac{18}{x}$

4 Evaluate:

a $2 \times (-3)$ **b** -4×5 **c** $2 - 8$ **d** $4 - 11$
e $7 - (-2)$ **f** $8 - (-10)$ **g** $-9 + 3$ **h** $-9 + 16$
i $-3 - 4$ **j** $-6 - 7$ **k** $-8 \div 2$ **l** $20 \div (-4)$



Positive × negative = negative.
To subtract a negative, add its opposite:
2 – (-3) = 2 + 3.

Fluency

Example 1 Naming parts of an expression

Consider the expression $\dfrac{xy}{2} - 4x + 3y^2 - 2$. Determine:

a the number of terms **b** the constant term
c the coefficient of:
  i y^2 **ii** x

ISBN: 9781107687028

Cambridge University Press

Solution	Explanation
a 4	There are four terms with different combinations of pronumerals and numbers, separated by + or –.
b -2	The term with no pronumerals is -2. The negative is included.
c **i** 3	The number multiplied by y^2 in $3y^2$ is 3.
ii -4	The number multiplied by x in $-4x$ is -4. The negative sign belongs to the term that follows.

5 For these algebraic expressions, determine:
- **i** the number of terms
- **ii** the constant term
- **iii** the coefficient of y

a $4xy + 5y + 8$
b $2xy + \frac{1}{2}y^2 - 3y + 2$
c $2x^2 - 4 + y$

The coefficient is the number multiplied by the pronumerals in each term. The constant term has no pronumerals.

Example 2 Writing algebraic expressions

Write algebraic expressions for the following.
- **a** three more than x
- **b** 4 less than 5 times y
- **c** the sum of c and d is divided by 3
- **d** the product of a and the square of b

Solution	Explanation
a $x + 3$	More than means add (+).
b $5y - 4$	Times means multiply ($5 \times y = 5y$) and less than means subtract (–).
c $\frac{c+d}{3}$	Sum c and d first (+), then divide by 3 (÷). Division is written as a fraction.
d ab^2	'Product' means 'multiply'. The square of b is b^2 (i.e. $b \times b$). $a \times b^2 = ab^2$.

6 Write an expression for the following.
- **a** two more than x
- **b** four less than y
- **c** the sum of ab and y
- **d** three less than 2 lots of x
- **e** the product of x and 5
- **f** twice m
- **g** three times the value of r
- **h** half of x
- **i** three quarters of m
- **j** the quotient of x and y
- **k** the sum of a and b is divided by 4
- **l** the product of the square of x and y

Quotient is ÷
Product is ×
$\frac{1}{3}y = \frac{y}{3}$

ISBN: 9781107687028

3A

Example 3 Substituting values

Find the value of these expressions if $x = 2$, $y = 3$ and $z = -5$.

a $xy + 3y$ **b** $y^2 - \frac{8}{x}$ **c** $2x - yz$

Solution	**Explanation**
a $xy + 3y = 2 \times 3 + 3 \times 3$ $= 6 + 9$ $= 15$	Substitute for each pronumeral: $x = 2$ and $y = 3$. Recall: $xy = x \times y$ and $3y = 3 \times y$. Simplify, following order of operations, by multiplying first.
b $y^2 - \frac{8}{x} = 3^2 - \frac{8}{2}$ $= 9 - 4$ $= 5$	Substitute $y = 3$ and $x = 2$. $3^2 = 3 \times 3$ and $\frac{8}{2} = 8 \div 2$. Do subtraction last.
c $2x - yz = 2 \times 2 - 3 \times (-5)$ $= 4 - (-15)$ $= 4 + 15$ $= 19$	Substitute for each pronumeral. $3 \times (-5) = -15$. To subtract a negative number, add its opposite.

7 Find the value of these expressions if $a = 4$, $b = -2$ and $c = 3$.

a ac **b** $2a - 5$ **c** $3a - c$ **d** $a^2 - 2c$

e $ac + b$ **f** $3b + a$ **g** $ab + c^2$ **h** $\frac{a}{2} - b$

i $\frac{ac}{b}$ **j** $2a - b$ **k** $a + bc$ **l** $\frac{6bc}{a}$

$12 + (-2) = 12 - 2$
$2 - (-2) = 2 + 2$

Problem-solving and Reasoning

8 Write an expression for the following.

a The cost of 5 pencils at x cents each

b The cost of y apples at 35 cents each

c One person's share if \$500 is divided among n people

d The cost of a pizza (\$11) shared between m people

e Paul's age in x years' time if he is 11 years old now

9 A taxi in Sydney has a pick-up charge (flagfall) of \$3.40 and charges \$2 per km.

a Write an expression for the taxi fare for a trip of d kilometres.

b Use your expression in part **a** to find the cost of a trip that is:

i 10 km **ii** 22 km

The taxi fare has initial cost + cost per km × number of km

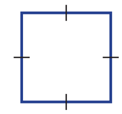
ISBN: 9781107687028

Cambridge University Press

10 **a** Ye thinks of a number, which we will call x.
Now write an expression for each of the following stages.
  - **i** He doubles the number.
  - **ii** He decreases the result by 3.
  - **iii** He multiplies the result by 3.

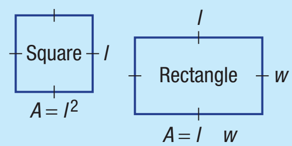
b If $x = 5$, use your answer to part **a iii** to find the final number.

11 A square of side length x is turned into a rectangle by increasing the length by 1 and decreasing the width by 1.

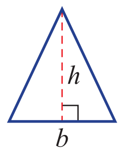


a Write an expression for the new length and width of the rectangle.
b Is there any change in the perimeter of the shape?
c **i** Write an expression for the area of the rectangle.
  ii Use trial and error to determine whether the area of the rectangle is more or less than the original square. By how much?

Perimeter is the sum of the side lengths.
Area:



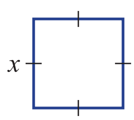
12 The area of a triangle is given by $\frac{1}{2}bh$.
a If $b = 6$ and $h = 7$, what is the area?
b If the area is 9, what are the possible whole number values for b if h is also a whole number?

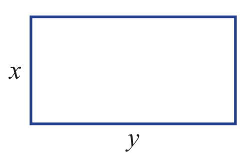


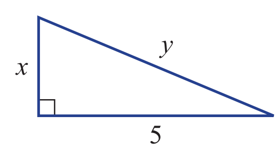
Area and perimeter

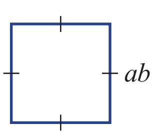
13 For the shapes shown, write an expression for:
  - **i** the perimeter
  - **ii** the area

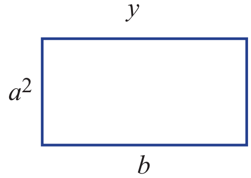
Perimeter = sum of the side lengths
Area of a rectangle = length × width
Area of triangle = $\frac{1}{2}$ × base × height

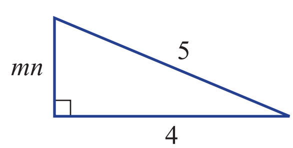
a


b


c


d


e


f


ISBN: 9781107687028 Cambridge University Press

3.2 Simplifying algebraic expressions

Many areas of finance and industry involve complex algebraic expressions. Often these base expressions can be made simpler by applying the rules of addition, subtraction, multiplication and division.

Just as we would write $3 + 3 + 3 + 3$ as 4×3, we write $x + x + x + x$ as $4 \times x$ or $4x$. Similarly, $3x + 2x = 5x$ and $3x - 2x = 1x$ ($1x$ is written as x). We also know that $2 \times 3 = 3 \times 2$ and $(2 \times 3) \times 4 = 2 \times 3 \times 4 = 3 \times 4 \times 2$ etc., so $2 \times x \times 4 = 2 \times 4 \times x = 8x$. By writing a division as a fraction we can also cancel common factors. For example, $9x \div 3 = \frac{9x}{3} = 3x$.

Let's start: Equivalent expressions

Split these expressions into two groups that are equivalent by simplifying them first.

$3x + 6x$	$17x - 5x$	$x + 7x + x$	$4x + 3 + 5x - 3$
$2 \times 6x$	$\frac{24xy}{2y}$	$3x \times 3$	$3x - 2y + 9x + 2y$
$8x + 6x - 2x$	$18x \div 2$	$\frac{9x^2}{x}$	$6x - (-6x)$

Key ideas

- **Like terms** have the exact same pronumeral factors, including powers; e.g. $3x$ and $7x$, and $4x^2y$ and $-3x^2y$.
  - Since $x \times y = y \times x$, $3xy$ and $2yx$ are like terms.
  - $7ab - 6ab = 1ab = ab$
- Addition and subtraction apply to like terms only.
  For example, $5x + 7x = 12x$
  $7ab - 6ab = 1ab = ab$
  $3x + 2y$ cannot be simplified
- Multiplication and division apply to all terms.
  - In multiplication, deal with numerals and pronumerals separately:
    $2 \times 8a = 2 \times 8 \times a = 16a$
    $6x \times 3y = 6 \times 3 \times x \times y = 18xy$
- When dividing, write as a fraction and cancel common factors:

$$\frac{\cancel{8}^{4}x}{\cancel{2}^{1}} = 4x \text{ and } 6x^2 \div (3x) = \frac{6x^2}{3x} = \frac{\cancel{6}^{2}\;\cancel{x}^{1}\;x}{\cancel{3}^{1}\;\cancel{x}^{1}} = 2x$$

Like terms Terms with the same pronumerals and same powers

ISBN: 9781107687028

Exercise 3B

Understanding

1 Are the following sets of terms like terms? Answer Yes (Y) or No (N).

a $3x, 2x, -5x$ b $2ax, 3xa, -ax$

c $2ax^2, 2ax, 62a^2x$ d $-3p^2q, 2pq^2, 4pq$

e $3ax^2y, 2ayx^2, -x^2ay$ f $\frac{3}{4}x^2, 2x^2, \frac{x^2}{3}$

2 Simplify the following.

a $8g + 2g$ b $3f + 2f$ c $12e - 4e$

d $3h - 3h$ e $5x + x$ f $14st + 3st$

g $7ts - 4ts$ h $4ab - ab$ i $9xy - 8xy$

Add or subtract the numerals in like terms.

3 Simplify the following.

a $3 \times 2x$ b $4 \times 3a$ c $2 \times 5m$ d $-3 \times 6y$

4 Simplify these fractions by cancelling.

a $\frac{4}{8}$ b $\frac{12}{3}$ c $\frac{24}{8}$

d $\frac{12}{18}$ e $\frac{14}{21}$ f $\frac{35}{15}$

g $\frac{27}{36}$ h $\frac{18}{45}$

Choose the highest common factor to cancel.

Fluency

Example 4 Identifying like terms

Write down the like terms in the following lists.

a $3x, 6a, 2ax, 3a, 5xa$ b $-2ax, 3x^2a, 3a, -5x^2a, 3x$

Solution	Explanation
a $6a$ and $3a$	Both terms contain a.
$5xa$ and $2ax$	Both terms contain ax. $x \times a = a \times x$
b $3x^2a$ and $-5x^2a$	Both terms contain x^2a.

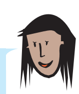
5 Write down the like terms in the following lists.

a $3ac, 2a, 5x, -2ac$ b $4pq, 3qp, 2p^2, -4p^2q$

c $7x^2y, -3xy^2, 2xy^2, 4yx^2$ d $2r^2, 3rx, -r^2, 4r^2x$

e $-2ab, 5bx, 4ba, 7xa$ f $3p^2q, -4pq^2, \frac{1}{2}pq, 4qp^2$

g $\frac{1}{3}lm, 2l^2m, \frac{lm}{4}, 2lm^2$ h $x^2y, yx^2, -xy, yx$



Like terms have the same pronumeral factors.
$x \times y = y \times x$, so $3xy$ and $5yx$ are like terms.

ISBN: 9781107687028

Example 5 Collecting like terms

Simplify the following.

a $4a + 5a + 3$ **b** $3x + 2y + 5x - 3y$
c $5xy + 2xy^2 - 2xy + xy^2$

Solution	Explanation
a $4a + 5a + 3 = 9a + 3$	Collect like terms ($4a$ and $5a$) and add coefficients.
b $3x + 2y + 5x - 3y = 3x + 5x + 2y - 3y$ $= 8x - y$	Collect like terms in x $(3 + 5 = 8)$ and y $(2 - 3 = -1)$. $-1y$ is written as $-y$.
c $5xy + 2xy^2 - 2xy + xy^2$ $= 5xy - 2xy + 2xy^2 + xy^2$ $= 3xy + 3xy^2$	Collect like terms. In xy, the negative belongs to $2xy$. In xy^2, recall that xy^2 is $1xy^2$.

6 Simplify the following by collecting like terms.

a $4t + 3t + 10$ **b** $5g - g + 1$ **c** $3x - 5 + 4x$
d $4m + 2 - 3m$ **e** $2x + 3y + x$ **f** $3x + 4y - x + 2y$
g $8a + 4b - 3a - 6b$ **h** $2m - 3n - 5m + n$ **i** $3de + 3de^2 + 2de + 4de^2$
j $6kl - 4k^2l - 6k^2l - 3kl$ **k** $3x^2y + 2xy^2 - xy^2 + 4x^2y$ **l** $4fg - 5g^2f + 4fg^2 - fg$

Example 6 Multiplying algebraic terms

Simplify the following.

a $2a \times 7d$ **b** $-3m \times 8mn$

Solution	Explanation
a $2a \times 7d = 2 \times 7 \times a \times d$ $= 14ad$	Multiply coefficients and collect the pronumerals: $2 \times a \times 7 \times d = 2 \times 7 \times a \times d$. Multiplication can be done in any order.
b $-3m \times 8mn = -3 \times 8 \times m \times m \times n$ $= -24m^2n$	Multiply coefficients $(-3 \times 8 = -24)$ and pronumerals. Recall: $m \times m$ can be written as m^2.

7 Simplify:

a $3r \times 2s$ **b** $2h \times 3u$ **c** $4w \times 4h$ **d** $2r^2 \times 3s$
e $-2e \times 4s$ **f** $5h \times (-2v)$ **g** $-3c \times (-4m^2)$ **h** $-7f \times (-5l)$
i $2x \times 4xy$ **j** $3ab \times 8a$ **k** $xy \times 3y$ **l** $-2a \times 8ab$
m $-3m^2n \times 4n$ **n** $-5xy^2 \times (-4x)$ **o** $5ab \times 4ab$ **p** $-8xy \times 6xy$

Multiply the numerals and collect the pronumerals.
$a \times b = ab$

ISBN: 9781107687028

Example 7 Dividing algebraic terms

Simplify the following.

a $\dfrac{18x}{6}$ **b** $12a^2b \div (8ab)$

Solution

a $\dfrac{\cancel{18}^{3}x}{\cancel{6}_{1}} = 3x$

b $12a^2b \div 8ab = \dfrac{12a^2b}{8ab}$

$= \dfrac{^{3}\cancel{12}\ a\ \cancel{a}_1\ \cancel{b}_1}{_{2}\cancel{8}\ \cancel{a}_1\ \cancel{b}_1}$

$= \dfrac{3a}{2}$

Explanation

a Cancel highest common factor of numerals; i.e. 6.

b Write division as a fraction.
Cancel the highest common factor of 12 and 8 and cancel an a and b.

8 Simplify by cancelling common factors.

Write each division as a fraction first where necessary.

a $\dfrac{7x}{14}$ **b** $\dfrac{6a}{2}$ **c** $3a \div 9$

d $2ab \div 8$ **e** $\dfrac{4ab}{2a}$ **f** $\dfrac{15xy}{5y}$

g $4xy \div (8x)$ **h** $28ab \div (35b)$ **i** $\dfrac{8x^2}{20x}$

j $\dfrac{12xy^2}{18y}$ **k** $30a^2b \div (10a)$ **l** $12mn^2 \div (36mn)$

Problem-solving and Reasoning

9 A rectangle's length is three times its width, x. Write a simplified expression for:

a the rectangle's perimeter

b the rectangle's area

Draw a rectangle and label the width x and the length $3 \times x = 3x$.

10 Fill in the missing term to make the following true.

a $8x + 4 - \square = 3x + 4$ **b** $3x + 2y - \square + 4y = 3x - 2y$

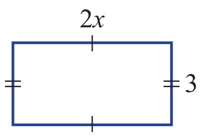
c $3b \times \square = 12ab$ **d** $4xy \times (\square) = -24x^2y$

e $12xy \div (\square) = 6y$ **f** $\square \div (15ab) = \dfrac{2a}{3}$

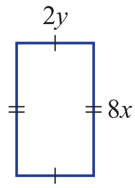
ISBN: 9781107687028

11 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes.

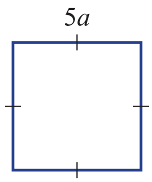
a



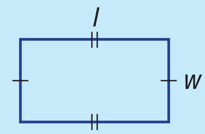
b



c

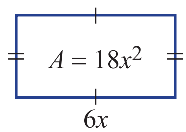


Perimeter is the sum of all the sides.
Area = $l \times w$



12 A rectangular garden bed has length given by $6x$ and area $18x^2$. What is the width of the garden bed?

The opposite of $\times$ is $\div$



Order of operations

13 Simplify the following expressions using order of operations.

a $4 \times 3x \div 2$

b $2 + 4a \times 2 + 5a \div a$

c $5a \times 2b \div a - 6b$

d $8x^2 \div (4x) + 3 \times 3x$

e $2x \times (4x + 5x) \div 6$

f $5xy - 4x^2y \div 2x + 3x \times 4y$

g $(5x - x) \times (16xy \div (8y))$

h $9x^2y \div 3y + 4x \times (-8x)$

ISBN: 9781107687028

Cambridge University Press

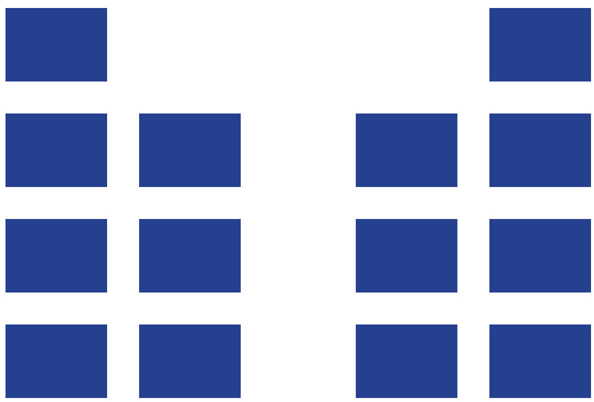
3.3 Expanding algebraic expressions

When an expression is multiplied by a term, each term in the expression must be multiplied by the term. Brackets are used to show this. For example, to double $4 + 3$ we write $2 \times (4 + 3)$, and each term within the brackets (both 4 and 3) must be doubled. The expanded version of this expression is $2 \times 4 + 2 \times 3$.

Similarly, to double the expression $x + 1$, we write $2(x + 1) = 2 \times x + 2 \times 1$. This expansion of brackets uses the distributive law.

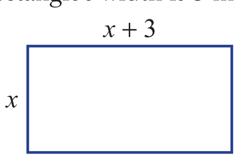
In this diagram, 7 blue blocks are doubled in groups of 4 and 3.

$2 \times (4 + 3) = 2 \times 4 + 2 \times 3$



Let's start: Rectangle brackets

A rectangle's width is 3 more than its height.



Write down as many expressions as you can, both with and without brackets, for:
- its perimeter
- its area.

Can you explain why all the expressions for the perimeter or area are equivalent?

Key ideas

- The **distributive law** is used to expand and remove brackets.
- The terms inside the brackets are multiplied by the term outside the brackets.

$$a(b + c) = ab + ac \qquad a(b - c) = ab - ac$$

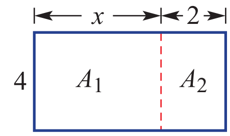
For example, $2(x + 4) = 2 \times x + 2 \times 4$
$= 2x + 8$

Distributive law Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the products

ISBN: 9781107687028

Exercise 3C

Understanding



1 Consider the diagram shown.
- **a** Write an expression for the area A_1.
- **b** Write an expression for the area A_2.
- **c** Add your results from parts **a** and **b** to give the area of the rectangle.
- **d** Write an expression for the total length of the rectangle.
- **e** Using part **d**, write an expression for the area of the rectangle.
- **f** Combine your results to complete this statement. $4(x + 2) = \square + \square$.

2 Multiply the following expressions involving negatives.

a $2 \times (-4)$	**b** $3 \times (-6)$	**c** $3 \times (-x)$
d $4 \times (-2x)$	**e** -4×5	**f** $-2 \times 8x$
g $-5 \times (-3)$	**h** $-6 \times (-4)$	**i** $-2x \times (-3)$

negative × positive = negative
negative × negative = positive

3 Complete the following.

a $3(x + 4) = 3 \times \square + 3 \times \square$
$= 3x + \square$

b $2(x - 5) = 2 \times \square + \square \times (-5)$
$= \square - 10$

c $2(4x + 3) = 2 \times \square + \square \times 3$
$= \square + 6$

d $x(x - 3) = x \times \square + \square \times \square$
$= \square - \square$

4 Simplify the following.

a $3 \times 2x$	**b** $4x \times 2y$	**c** $3x \times 5x$
d $5 + 2x + 4$	**e** $3x + 9 + 4x - 4$	**f** $5x + 10 - 2x - 14$

Fluency

Example 8 Expanding expressions with brackets

Expand the following.

a $2(x + 5)$ **b** $3(2x - 3)$ **c** $3y(2x + 4y)$

Solution	Explanation
a $2(x + 5) = 2 \times x + 2 \times 5$ $= 2x + 10$	Multiply each term inside the brackets by 2.
b $3(2x - 3) = 3 \times 2x + 3 \times (-3)$ $= 6x - 9$	Multiply $2x$ and -3 by 3. $3 \times 2x = 3 \times 2 \times x = 6x$.
c $3y(2x + 4y) = 3y \times 2x + 3y \times 4y$ $= 6xy + 12y^2$	Multiply $2x$ and $4y$ by $3y$. $3y \times 2x = 3 \times 2 \times x \times y$ and $3y \times 4y = 3 \times 4 \times y \times y$. Recall: $y \times y$ is written as y^2.

ISBN: 9781107687028

5 Expand the following.

a $2(x+4)$	**b** $3(x+7)$	**c** $4(y-3)$
d $5(y-2)$	**e** $2(3x+2)$	**f** $4(2x+5)$
g $3(3a-4)$	**h** $7(2y-5)$	**i** $5(2a+b)$
j $3(4a-3b)$	**k** $2x(x+5)$	**l** $3x(x-4)$
m $2a(3a+2b)$	**n** $2y(3x-4y)$	**o** $3b(2a-5b)$

Use the distributive law:

$a(b+c) = a \times b + a \times c$
$= ab + ac$

$a(b-c) = a \times b + a \times (-c)$
$= ab - ac$

Example 9 Expanding expressions with a negative out the front

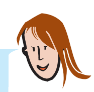
Expand the following.

a $-3(x-4)$ **b** $-2x(3x-2y)$

Solution	Explanation
a $-3(x-4) = -3 \times x + (-3) \times (-4)$ $= -3x + 12$	Multiply each term inside the brackets by -3. $-3 \times (-4) = +12$. If there is a negative sign outside the bracket, the sign of each term inside the brackets is changed when expanded.
b $-2x(3x-2y) = -2x \times 3x + (-2x) \times (-2y)$ $= -6x^2 + 4xy$	$-2x \times 3x = -2 \times 3 \times x \times x$ and $-2x \times (-2y)$ $= -2 \times (-2) \times x \times y$

6 Expand the following.

a $-2(x+3)$	**b** $-5(m+2)$	**c** $-3(w+4)$
d $-4(x-3)$	**e** $-2(m-7)$	**f** $-7(w-5)$
g $-(x+y)$	**h** $-(x-y)$	**i** $-2x(3x+4)$
j $-3x(2x+5)$	**k** $-4x(2x-2)$	**l** $-3y(2y-9)$
m $-2x(3x-5y)$	**n** $-3x(3x+2y)$	**o** $-6y(2x+3y)$



A negative out the front will change the sign of each term in the brackets when expanded.
$-2(x-3) = -2x + 6$

Example 10 Simplifying expressions by removing brackets

Expand and simplify the following.

a $8 + 3(2x-3)$ **b** $3(2x+2) - 4(x+4)$

Solution	Explanation
a $8 + 3(2x-3) = 8 + 6x - 9$ $= 6x - 1$	Expand the brackets: $3 \times 2x + 3 \times (-3) = 6x - 9$. Collect like terms: $8 - 9 = -1$.
b $3(2x+2) - 4(x+4) = 6x + 6 - (4x + 16)$ $= 6x + 6 - 4x - 16$ $= 2x - 10$	Expand the bracket first. Note that $-4(x+4) = -4 \times x + (-4) \times 4 = -4x - 16$. Collect like terms: $6x - 4x = 2x$ and $6 - 16 = -10$.

ISBN: 9781107687028

7 Expand and simplify the following.

a $2 + 5(x + 3)$ **b** $3 + 7(x + 2)$ **c** $5 + 2(x - 3)$
d $7 - 2(x + 3)$ **e** $21 - 5(x + 4)$ **f** $4 + 3(2x - 1)$
g $3 + 2(3x + 4)$ **h** $8 - 2(2x - 3)$ **i** $12 - 3(2x - 5)$
j $3(x + 2) + 4(x + 3)$ **k** $2(p + 2) + 5(p - 3)$ **l** $4(x - 3) + 2(3x + 4)$
m $3(2s + 3) - 2(s + 2)$ **n** $4(3f + 2) - 2(6f + 2)$ **o** $3(2x - 5) - 2(2x - 4)$

Expand first, then collect like terms.

Problem-solving and Reasoning

8 Fill in the missing term/number to make each statement true.

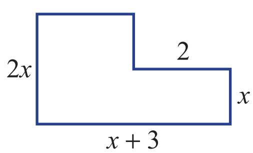
a $\square(x + 4) = 2x + 8$ **b** $\square(2x - 3) = 8x - 12$ **c** $\square(2x + 3) = 6x^2 + 9x$
d $4(\square + 5) = 12x + 20$ **e** $4y(\square - \square) = 4y^2 - 4y$ **f** $-2x(\square + \square) = -4x^2 - 6xy$

9 Four rectangular rooms in a house have floor side lengths listed below. Find an expression for the area of each floor in expanded form.

a 2 and $x - 5$ **b** x and $x + 3$
c $2x$ and $x + 4$ **d** $3x$ and $2x - 1$

Area of rectangle = length × width

10 The deck on a house is constructed in the shape shown. Find the area of the balcony in expanded form.



11 Virat earns $\$x$ but does not have to pay tax on the first \$6000.

a Write an expression for the amount of money Virat is taxed on.
b Virat is taxed 10% of his earnings in part **a**. Write an expanded expression for how much tax he pays.

To find 10% of an amount, multiply by $\frac{10}{100} = 0.1$.

Expanding binomial products

12 A rectangle has dimensions $(x + 2)$ by $(x + 3)$ as shown.

	x	3
x	x^2	$3x$
2	$2x$	6

The area can be found by summing the individual areas:

$$(x + 2)(x + 3) = x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

This can be done using the distributive law:

$$(x + 2)(x + 3) = x(x + 3) + 2(x + 3)$$
$$= x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

Expand and simplify these binomial products using this method.

a $(x + 4)(x + 3)$ **b** $(x + 3)(x + 1)$ **c** $(x + 2)(x + 5)$
d $(x + 2)(x - 4)$ **e** $(x + 5)(x - 2)$ **f** $(x + 4)(2x + 3)$
g $(2x + 3)(x - 2)$ **h** $(x - 3)(x + 4)$ **i** $(4x - 2)(x + 5)$

ISBN: 9781107687028

3.4 Factorising algebraic expressions

Factorising is an important step in solving many types of equations and in simplifying algebraic expressions.

Just as 15 can be expanded and written as 3×5, we can factorise to write an algebraic expression as the product of its factors. Factorising is therefore the opposite of expanding.

Let's start: Products of factors

- Expand the product $6(2x + 4)$.
- Write as many products as you can (using whole numbers) that give the same result as $6(2x + 4)$ when expanded.
- Which of your products has the highest number in front of the brackets? What is this number?
- How does this number relate to the two terms in the expanded form?
- Write a product of factors that expand to $18x + 24$ using the highest common factor.

Key ideas

- Factorisation is the opposite process of expansion.
- To **factorise** an expression, take out the highest common factor (HCF) of each of the terms. This may be a number, variable or product of these.
  - Divide each term by the HCF and leave the expression in the brackets.
  - A factorised expression can be checked by expanding to get the original expression.
  - If the HCF has been removed, the terms in the brackets should have no common factors; e.g. $2(x + 3)$ is fully factorised, but $2(4x + 6)$ is not because 2 can still be divided into both 4 and 6 within the brackets.

  For example: $3x + 12 = 3(x + 4)$ HCF: 3
  $2x^2 + 8x = 2x(x + 4)$ HCF: $2x$

Factorise To write an expression as a product, often involving grouping symbols

Exercise 3D

Understanding

1 Write down the highest common factor (HCF) of these pair of numbers.

a 10 and 16 **b** 4 and 12 **c** 9 and 27
d 18 and 30 **e** 14 and 35 **f** 36 and 48

ISBN: 9781107687028

Cambridge University Press

2 Write down the missing factor.

a $4 \times \square = 4x$ **b** $5 \times \square = 10x$ **c** $x \times \square = 3x^2$

d $3a \times \square = 6ab$ **e** $2x \times \square = 8x^2$ **f** $-4y \times \square = -12y^2$

3 Consider the expression $4x^2 + 8x$.

a Which of the following factorised forms uses the HCF?

A $2(2x^2 + 4x)$ **B** $4(x^2 + 8x)$ **C** $4x(x + 2)$ **D** $2x(2x + 4)$

b What can be said about the terms inside the brackets once the HCF is removed, that is not the case for the other forms?

Fluency

Example 11 Finding the HCF

Determine the HCF of the following.

a $8a$ and 20 **b** $3x$ and $6x$ **c** $10a^2$ and $15ab$

Solution	Explanation
a HCF of $8a$ and 20 is 4.	Compare numerals and pronumerals separately. The highest common factor (HCF) of 8 and 20 is 4. a is not a common factor.
b HCF of $3x$ and $6x$ is $3x$.	HCF of 3 and 6 is 3. x is also a common factor.
c HCF of $10a^2$ and $15ab$ is $5a$.	HCF of 10 and 15 is 5. HCF of a^2 and ab is a.

4 Determine the HCF of the following.

a $6x$ and 12 **b** 10 and $15y$ **c** $8a$ and $12b$

d $9x$ and $18y$ **e** $5a$ and $20a$ **f** $10m$ and $22m$

g $14x$ and $21x$ **h** $8a$ and $40ab$ **i** $3a^2$ and $9ab$

j $4x^2$ and $10x$ **k** $16y$ and $24xy$ **l** $15x^2y$ and $25xy$

Example 12 Factorising simple expressions

Factorise the following.

a $4x + 20$ **b** $6a - 15b$

Solution	Explanation
a $4x + 20 = 4(x + 5)$	HCF of $4x$ and 20 is 4. Place 4 in front of the brackets and divide each term by 4. Expand to check: $4(x + 5) = 4x + 20$.
b $6a - 15b = 3(2a - 5b)$	HCF of $6a$ and $15b$ is 3. Place 3 in front of the brackets and divide each term by 3.

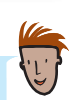
5 Factorise the following.

a $3x + 9$ **b** $4x - 8$ **c** $10y - 20$

d $6a + 30$ **e** $5x + 5y$ **f** $12a + 4b$

g $18m - 27n$ **h** $36x - 48y$ **i** $8x + 44y$

j $24a - 18b$ **k** $121m + 55n$ **l** $14k - 63l$



ISBN: 9781107687028

Example 13 Factorising expressions with pronumeral common factors

Factorise the following.

a $8y + 12xy$ **b** $4x^2 - 10x$

Solution	Explanation
a $8y + 12xy = 4y(2 + 3x)$	HCF of 8 and 12 is 4, HCF of y and xy is y. Place $4y$ in front of the brackets and divide each term by $4y$. Check that $4y(2 + 3x) = 8y + 12xy$.
b $4x^2 - 10x = 2x(2x - 5)$	HCF of $4x^2$ and $10x$ is $2x$. Place $2x$ in front of the brackets and divide each term by $2x$. Recall: $x^2 = x \times x$.

6 Factorise the following.

a $14x + 21xy$	**b** $6ab - 15b$	**c** $32y - 40xy$
d $5x^2 - 5x$	**e** $x^2 + 7x$	**f** $2a^2 + 8a$
g $12a^2 + 42ab$	**h** $9y^2 - 63y$	**i** $6x^2 + 14x$
j $9x^2 - 6x$	**k** $16y^2 + 40y$	**l** $10m - 40m^2$

Place the HCF in front of the brackets and divide each term by the HCF:
$14x + 21xy = 7x(\_\_ + \_\_)$

Example 14 Factorising expressions by removing a common negative

Factorise $-10x^2 - 18x$.

Solution	Explanation
$-10x^2 - 18x = -2x(5x + 9)$	The HCF of $-10x^2$ and $-18x$ is $-2x$, including the common negative. Place $-2x$ in front of the brackets and divide each term by $-2x$. Dividing by a negative changes the sign of each term.

7 Factorise the following, including the common negative.

a $-2x - 6$	**b** $-4a - 8$	**c** $-3x - 6y$
d $-7a - 14ab$	**e** $-x - 10xy$	**f** $-3b - 12ab$
g $-x^2 - 7x$	**h** $-4x^2 - 12x$	**i** $-2y^2 - 10y$
j $-8x^2 - 14x$	**k** $-12x^2 - 8x$	**l** $-15a^2 - 5a$

Dividing by a negative changes the sign of the term.

Problem-solving and Reasoning

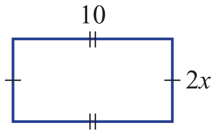
8 Factorise these mixed expressions.

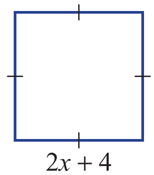
a $7a^2b + ab$	**b** $4a^2b + 20a^2$	**c** $xy - xy^2$
d $x^2y + 4x^2y^2$	**e** $6mn + 18mn^2$	**f** $5x^2y + 10xy^2$
g $-y^2 - 8yz$	**h** $-3a^2b - 6ab$	**i** $-ab^2 - a^2b$

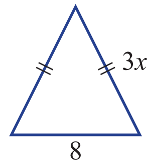
Be sure to find the highest common factor first.

ISBN: 9781107687028 Cambridge University Press

9 Give the perimeter of these shapes in factorised form.

a


b


c


Find the perimeter first, then factorise.

10 A square sand pit has perimeter $(4x + 12)$ metres. What is the side length of the square?

11 Common factors can be removed from expressions involving more than two terms in a similar way. Factorise these by taking out the HCF.

$4a + 6b + 10c$
$= 2(2a + 3b + 5c)$

a $2x + 4y + 6z$ **b** $3x^2 + 12x + 6$ **c** $4x^2 + 8xy + 12$
d $6x^2 + 3xy - 9x$ **e** $10xy - 5xz + 5x$ **f** $4y^2 - 18y + 14xy$

Taking out a binomial factor

A common factor may be a binomial term, such as $(x + 1)$.
For example, $3(x + 1) + x(x + 1)$ has HCF $= (x + 1)$, so $3(x + 1) + x(x + 1) = (x + 1)(3 + x)$, where $(3 + x)$ is what remains when $3(x + 1)$ and $x(x + 1)$ are divided by $(x + 1)$.

12 Use the above method to factorise the following.

a $4(x + 2) + x(x + 2)$ **b** $x(x + 3) + 2(x + 3)$ **c** $x(x + 4) - 7(x + 4)$
d $x(2x + 1) - 3(2x + 1)$ **e** $2x(y - 3) + 4(y - 3)$ **f** $2x(x - 1) - 3(x - 1)$

ISBN: 9781107687028 Cambridge University Press

3.5 Multiplying and dividing algebraic fractions

Since pronumerals represent numbers, the rules for algebraic fractions are the same as those for simple numerical fractions. This includes processes such as cancelling common factors to simplify the calculation and dividing by multiplying by the reciprocal of a fraction.

The process of cancelling requires cancelling of factors, for example:

$$\frac{8}{12} = \frac{2 \times \cancel{4}^1}{3 \times \cancel{4}_1} = \frac{2}{3}$$

For algebraic fractions, you need to factorise the expressions to identify and cancel common factors.

Let's start: Expressions as products of their factors

Factorise these expressions to write them as a product of their factors. Fill in the blanks and simplify.

$$\frac{2x+4}{2} = \frac{\boxed{}(\boxed{})}{2} = \boxed{} \qquad \frac{6x+9}{3} = \frac{\boxed{}(\boxed{})}{3} = \boxed{}$$

$$\frac{x^2+2x}{x} = \frac{\boxed{}(\boxed{})}{x} = \boxed{} \qquad \frac{4x+4}{4} = \frac{\boxed{}(\boxed{})}{4} = \boxed{}$$

Describe the errors made in these factorisations.

$$\frac{{}^1\cancel{3}x+2}{\cancel{3}^1} = x+2 \qquad \frac{{}^1\cancel{5}x+4}{{}^1\cancel{5}} = x+4$$

$$\frac{x^2+\cancel{3x}^1}{\cancel{3x}^1} = x^2+1 \qquad \frac{6\cancel{x}^1+6}{{}^1\cancel{x}+1} = \frac{12}{1} = 12$$

Key ideas

- Simplify **algebraic fractions** by cancelling common factors in factorised form.

  For example, $\frac{4x+6}{2} = \frac{\cancel{2}_1(2x+3)}{\cancel{2}_1}$
  $= 2x+3$

- To multiply algebraic fractions:
  - factorise expressions if possible
  - cancel common factors
  - multiply numerators and denominators together.

$$\frac{\cancel{(x+1)}^1}{\cancel{10}_2} \times \frac{{}^1\cancel{5}x}{4\cancel{(x+1)}_1} = \frac{x}{8}$$

- To divide algebraic fractions:
  - multiply by the reciprocal of the fraction following the division sign (the reciprocal of 6 is $\frac{1}{6}$, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$)
  - follow the rules for multiplication.

$$\frac{2}{(x-2)} \div \frac{8}{3(x-2)} = \frac{\cancel{2}_1}{\cancel{(x-2)}_1} \times \frac{3\cancel{(x-2)}_1}{\cancel{8}_4} = \frac{3}{4}$$

Algebraic fraction A fraction containing pronumerals as well as numbers

ISBN: 9781107687028

Exercise 3E

Understanding

1 Write these fractions in simplest form by cancelling common factors.

a $\frac{14}{21}$ **b** $\frac{9}{12}$ **c** $\frac{8x}{20}$ **d** $\frac{4x}{10}$

e $\frac{4(x+1)}{8}$ **f** $\frac{3(x-2)}{6}$ **g** $\frac{6(x+4)}{18}$ **h** $\frac{5(x+3)}{25}$

Be sure to cancel the highest common factor.

2 Simplify these fractions.

a $\frac{8}{9}\ \frac{3}{10}$ **b** $\frac{15}{21}\ \frac{14}{25}$ **c** $\frac{4}{27} \mid \frac{16}{9}$ **d** $\frac{18}{35} \mid \frac{9}{14}$

3 Write the reciprocal of these fractions.

a $\frac{3}{2}$ **b** $\frac{5x}{3}$ **c** 7 **d** $\frac{x+3}{4}$

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

4 Factorise these by taking out the highest common factor.

a $3x+6$ **b** $2x+4$ **c** $8x+12$ **d** $16-8x$

e x^2+3x **f** $4x^2+10x$ **g** $-2x-6$ **h** $-x^2-5x$

$3x + 6$ has a HCF of 3. Place 3 in front of the brackets and divide each term by 3.
$3x + 6 = 3(\_\_\_\_)$

Fluency

Example 15 Cancelling common factors

Simplify by cancelling common factors.

a $\frac{8xy}{12x}$ **b** $\frac{3(x+2)}{6(x+2)}$

Solution	Explanation
a $\frac{8xy}{12x} = \frac{{}^{2}\cancel{8} \times \cancel{x}^{1} \times y}{{}^{3}\cancel{12} \times \cancel{x}^{1}}$ $= \frac{2y}{3}$	Cancel the highest common factor of 8 and 12 (4) and cancel the x.
b $\frac{3(x+2)}{6(x+2)} = \frac{{}^{1}\cancel{3} \times \cancel{(x+2)}^{1}}{{}^{2}\cancel{6} \times \cancel{(x+2)}^{1}}$ $= \frac{1}{2}$	Cancel the highest common factors: 3 and $(x+2)$.

5 Simplify by cancelling common factors.

a $\frac{6xy}{12x}$ **b** $\frac{12ab}{30b}$ **c** $\frac{8x^2}{40x}$ **d** $\frac{25x^2}{5x}$

e $\frac{3(x+1)}{3}$ **f** $\frac{7(x-5)}{7}$ **g** $\frac{4(x+1)}{8}$ **h** $\frac{5(x-2)}{x-2}$

i $\frac{4(x-3)}{x-3}$ **j** $\frac{6(x+2)}{12(x+2)}$ **k** $\frac{9(x+3)}{3(x+3)}$ **l** $\frac{15(x-4)}{10(x-4)}$

Cancel the HCF of the numerals and pronumerals.

ISBN: 9781107687028

Example 16 Simplifying by factorising

Simplify these fractions by factorising first.

a $\dfrac{9x-12}{3}$ **b** $\dfrac{4x+8}{x+2}$

Solution

a $\dfrac{9x-12}{3} = \dfrac{{}^{1}\cancel{3}(3x-4)}{\cancel{3}_{1}}$

$= 3x-4$

Explanation

Factorise the expression in the numerator, which has HCF = 3. Then cancel the common factor of 3.

b $\dfrac{4x+8}{x+2} = \dfrac{4\cancel{(x+2)}_{1}}{\cancel{x+2}_{1}}$

$= 4$

4 is the HCF in the numerator.
After factorising, $(x+2)$ can be seen as a common factor and can be cancelled.

6 Simplify these fractions by factorising first.

Cancel after you have factorised the numerator.

a $\dfrac{4x+8}{4}$	**b** $\dfrac{6a-30}{6}$	**c** $\dfrac{8y-12}{4}$	**d** $\dfrac{14b-21}{7}$
e $\dfrac{3x+9}{x+3}$	**f** $\dfrac{4x-20}{x-5}$	**g** $\dfrac{6x+9}{2x+3}$	**h** $\dfrac{12x-4}{3x-1}$
i $\dfrac{x^2+2x}{x}$	**j** $\dfrac{x^2-5x}{x}$	**k** $\dfrac{2x^2+6x}{2x}$	**l** $\dfrac{x^2+4x}{x+4}$
m $\dfrac{x^2-7x}{x-7}$	**n** $\dfrac{2x^2-4x}{x-2}$	**o** $\dfrac{3x^2+6x}{x+2}$	**p** $\dfrac{2x^2+12x}{x+6}$

Example 17 Multiplying algebraic fractions

Simplify these products.

a $\dfrac{12}{5x}\ \dfrac{10x}{9}$ **b** $\dfrac{3(x-1)}{10} \times \dfrac{15}{x-1}$

Solution

a $\dfrac{\cancel{12}_{4}}{\cancel{5x}_{1}} \times \dfrac{\cancel{10x}^{2}}{\cancel{9}_{3}}$

$= \dfrac{8}{3}\left(= 2\dfrac{2}{3}\right)$

Explanation

Cancel common factors between numerators and denominators: $5x$ and 3.
Then multiply the numerators and the denominators.

b $\dfrac{3\cancel{(x-1)}^{1}}{\cancel{10}^{2}} \times \dfrac{\cancel{15}_{3}}{\cancel{x-1}_{1}}$

$= \dfrac{9}{2}\left(= 4\dfrac{1}{2}\right)$

Cancel the common factors, which are $(x-1)$ and 5.
Multiply numerators and denominators.

ISBN: 9781107687028 Cambridge University Press

3E

7 Simplify these products.

a $\frac{3}{x} \quad \frac{2x}{9}$ b $\frac{4x}{5} \quad \frac{15}{8x}$ c $\frac{9a}{14} \quad \frac{7}{6a}$

d $\frac{2x^2}{5} \quad \frac{25}{6x}$ e $\frac{4y^2}{7} \quad \frac{21}{8y}$ f $\frac{x+1}{6} \quad \frac{5}{x+1}$

g $\frac{x+3}{9} \quad \frac{4}{x+3}$ h $\frac{4(y-7)}{2} \times \frac{5}{y-7}$ i $\frac{10}{a+6} \quad \frac{3(a+6)}{4}$

j $\frac{4(x-2)}{7} \times \frac{14}{5(x-2)}$ k $\frac{3(x+2)}{2x} \quad \frac{8}{9(x+2)}$ l $\frac{4(2x+1)}{3x} \quad \frac{9x}{2x+1}$

Cancel any common factors between numerators and denominators before multiplying.

Example 18 Dividing algebraic fractions

Simplify the following.

a $\frac{3x^2}{8} \mid \frac{9x}{4}$ b $\frac{2(x-2)}{3} \div \frac{x-2}{6}$

Solution

a $\frac{3x^2}{8} \mid \frac{9x}{4} = \frac{\cancel{3}x^{\cancel{2}1}}{\cancel{8}_2} \quad \frac{\cancel{4}_1}{_3\cancel{9x}^1}$

$= \frac{x}{6}$

Explanation

Multiply by the reciprocal of the second fraction.

The reciprocal of $\frac{9x}{4}$ is $\frac{4}{9x}$.

Cancel common factors: $3x$ and 4.

Note: $\frac{3x^2}{9x} = \frac{\cancel{3}^1 \quad x \quad \cancel{x}^1}{_3\cancel{9} \quad \cancel{x}^1}$

Multiply the numerators and the denominators.

b $\frac{2(x-2)}{3} \div \frac{x-2}{6} = \frac{2\cancel{(x-2)}^1}{\cancel{3}^1} \times \frac{\cancel{6}^2}{\cancel{x-2}^1}$

$= 4$

The reciprocal of $\frac{x-2}{6}$ is $\frac{6}{x-2}$.

Cancel the common factors, $(x-2)$ and 3, and multiply. Recall: $\frac{4}{1} = 4$.

8 Simplify the following.

a $\frac{x}{5} \mid \frac{x}{15}$ b $\frac{3x}{10} \mid \frac{x}{20}$ c $\frac{4a^2}{9} \mid \frac{a}{18}$

d $\frac{3x^2}{10} \mid \frac{6x}{5}$ e $\frac{4a}{9} \mid \frac{5a^2}{6}$ f $\frac{2x}{7} \mid \frac{x^2}{14}$

g $\frac{x+4}{2} \mid \frac{x+4}{6}$ h $\frac{5(x-2)}{8} \div \frac{x-2}{4}$ i $\frac{3(x+4)}{10} \mid \frac{6(x+4)}{15}$

j $\frac{2}{5(2x-1)} \div \frac{10}{2x-1}$ k $\frac{2(x-3)}{x-4} \div \frac{x-3}{5(x-4)}$ l $\frac{3(x+1)}{14(x-1)} \div \frac{6(x+1)}{35(x-1)}$

To divide, multiply by the reciprocal of the fraction following the division sign.
$\frac{x}{5} \mid \frac{x}{15} = \frac{x}{5} \quad \frac{15}{x}$

ISBN: 9781107687028

Problem-solving and Reasoning

9 Find the error in the simplifying of these fractions and correct it.

a $\frac{3x+\cancel{6}_{2}}{\cancel{3}_{1}} = 3x+2$ **b** $\frac{x^2+2\cancel{x}_{1}}{\cancel{x}_{1}} = x^2+2$

c $\frac{4x}{5} | \frac{10x}{3} = \frac{4x}{\cancel{5}_{1}}\ \frac{{}^{2}\cancel{10}x}{3} = \frac{8x^2}{3}$ **d** $\frac{\cancel{x}_{1}+4}{\cancel{15}_{5}}\ \frac{\cancel{3}_{1}}{\cancel{x}_{1}} = \frac{4}{5}$

Remember that common factors can be easily identified when expressions are in factorised form.

10 Simplify these algebraic fractions by factorising expressions first.

a $\frac{7a+14a^2}{21a}$ **b** $\frac{4x+8}{5x+10}$ **c** $\frac{x^2+3x}{4x+12}$

d $\frac{2m+4}{15}\ \frac{3}{m+2}$ **e** $\frac{5-x}{12} \times \frac{14}{15-3x}$ **f** $\frac{x^2+2x}{4}\ \frac{8}{3x+6}$

g $\frac{2x-1}{10} \div \frac{4x-2}{25}$ **h** $\frac{2x+4}{6x} | \frac{3x+6}{x^2}$ **i** $\frac{2x^2-4x}{3x-6} \div \frac{6x}{x+5}$

11 By removing a negative factor, further simplifying is possible.

For example, $\frac{-2x-4}{x+2} = \frac{-2\cancel{(x+2)}_{1}}{\cancel{x+2}_{1}} = -2$

Use this idea to simplify the following.

a $\frac{-3x-9}{x+3}$ **b** $\frac{-4x-10}{2x+5}$ **c** $\frac{-x^2-4x}{x+4}$

d $\frac{-3x^2-6x}{-9x}$ **e** $\frac{-2x+12}{-2}$ **f** $\frac{-10x+15}{-5}$

Taking out a negative factor changes the sign of each term inside the brackets.

Cancelling of powers

12 Just as $\frac{x^{\cancel{2}1}}{\cancel{x}^{1}} = x$, $\frac{(x+1)^{\cancel{2}1}}{\cancel{x+1}_{1}} = x+1$. Use this idea to simplify these algebraic fractions. Some will need factorising first.

a $\frac{(x+1)^2}{8}\ \frac{4}{x+1}$ **b** $\frac{(x+1)^2}{7x}\ \frac{14x}{3(x+1)}$ **c** $\frac{9}{x-2} \div \frac{18}{(x-2)^2}$

d $\frac{(x+2)^2}{10}\ \frac{5}{4x+8}$ **e** $\frac{(x-3)^2}{9x} \times \frac{3x}{4x-12}$ **f** $\frac{15}{8x+4} | \frac{6}{(2x+1)^2}$

ISBN: 9781107687028

3.6 Adding and subtracting algebraic fractions

As with multiplying and dividing, the steps for adding and subtracting numeric fractions can be applied to algebraic fractions. A lowest common denominator is required before the fractions can be combined.

Let's start: Steps for adding fractions

- Write out the list of steps you would give to someone to show them how to add $\frac{3}{5}$ and $\frac{2}{7}$.
- Follow your steps to add the fractions $\frac{3x}{5}$ and $\frac{2x}{7}$.
- What is different when these steps are applied to $\frac{x+2}{5}$ and $\frac{x}{7}$?

Key ideas

- To add or subtract algebraic fractions:
  - determine the lowest common denominator (LCD).
    For example, the LCD of 3 and 5 is 15
    the LCD of 4 and 12 is 12
  - write each fraction as an equivalent fraction by multiplying the denominator(s) to equal the LCD. When denominators are multiplied, numerators should also be multiplied.
    For example, $\frac{x}{3}+\frac{2x}{5}$ (LCD of 3 and 5 = 15)
    $$=\frac{x_{(\ 5)}}{3_{(\ 5)}}+\frac{2x_{(\ 3)}}{5_{(\ 3)}}$$
    $$=\frac{5x}{15}+\frac{6x}{15}$$
    and $\frac{2x}{4}-\frac{x}{12}$ (LCD of 4 and 12 = 12)
    $$=\frac{2x_{(\times 3)}}{4_{(\times 3)}}-\frac{x}{12}$$
    $$=\frac{6x}{12}-\frac{x}{12}$$
  - add or subtract the numerators.
    For example, $\frac{5x}{15}+\frac{6x}{15}=\frac{11x}{15}$
    and $\frac{6x}{12}-\frac{x}{12}=\frac{5x}{12}$
- To express $\frac{x+1}{3}$ with a denominator of 12, both numerator and denominator must be multiplied by 4:
  $$\frac{(x+1)_{(\ 4)}}{3_{(\ 4)}}=\frac{4x+4}{12}$$

ISBN: 9781107687028

Cambridge University Press

Exercise 3F

Understanding

1 Write down the lowest common denominator for these pairs of fractions.

a $\frac{2}{3}, \frac{3}{4}$ **b** $\frac{1}{6}, \frac{4}{9}$ **c** $\frac{3}{4}, \frac{5}{8}$

d $\frac{x}{3}, \frac{x}{12}$ **e** $\frac{2x}{5}, \frac{x}{4}$ **f** $\frac{3x}{10}, \frac{2x}{15}$

The LCD is not always the two denominators multiplied together; e.g. 3 × 6 = 18 but the LCD of 3 and 6 is 6.

2 Simplify these fractions by adding or subtracting.

a $\frac{1}{6}+\frac{2}{3}$ **b** $\frac{2}{5}+\frac{3}{8}$ **c** $\frac{1}{4}+\frac{5}{6}$

d $\frac{2}{3}-\frac{3}{7}$ **e** $\frac{15}{16}-\frac{3}{4}$ **f** $\frac{5}{6}-\frac{4}{9}$

3 Complete these equivalent fractions by giving the missing term.

a $\frac{x}{4}=\frac{\square}{12}$ **b** $\frac{x}{8}=\frac{\square}{40}$ **c** $\frac{2x}{5}=\frac{\square}{15}$

d $\frac{3x}{2}=\frac{\square}{8}$ **e** $\frac{x+2}{3}=\frac{\square(x+2)}{9}$ **f** $\frac{x-1}{4}=\frac{\square(x-1)}{20}$

For equivalent fractions, whatever the denominator is multiplied by, the numerator must be multiplied by the same amount.

4 Expand and simplify.

a $4(x+2)+3x$ **b** $3(x-4)+2x$ **c** $4(x+1)-2x$

d $3(x+2)+4(x+3)$ **e** $5(x+2)+2(x-3)$ **f** $4(x-2)+3(x+6)$

$4(x+2) = 4 \times x + 4 \times 2$
$= 4x + 8$

Fluency

Example 19 Adding and subtracting algebraic fractions

Simplify the following.

a $\frac{x}{2}+\frac{x}{3}$ **b** $\frac{4x}{5}-\frac{x}{2}$ **c** $\frac{x}{2}-\frac{5}{6}$

Solution	Explanation
a $\frac{x_{(\ 3)}}{2_{(\ 3)}}+\frac{x_{(\ 2)}}{3_{(\ 2)}}=\frac{3x}{6}+\frac{2x}{6}$ $=\frac{5x}{6}$	The LCD of 2 and 3 is 6. Express each fraction with a denominator of 6 and add numerators.
b $\frac{4x_{(\times 2)}}{5_{(\times 2)}}-\frac{x_{(\times 5)}}{2_{(\times 5)}}=\frac{8x}{10}-\frac{5x}{10}$ $=\frac{3x}{10}$	The LCD of 5 and 2 is 10. Express each fraction with a denominator of 10 and subtract $5x$ from $8x$.
c $\frac{x_{(\times 3)}}{2_{(\times 3)}}-\frac{5}{6}=\frac{3x}{6}-\frac{5}{6}$ $=\frac{3x-5}{6}$	The LCD of 2 and 6 is 6. Multiply the numerator and denominator of $\frac{x}{2}$ by 3 to express with a denominator of 6. Write as a single fraction; $3x-5$ cannot be simplified.

ISBN: 9781107687028

5 Simplify the following.

a $\frac{x}{3}+\frac{x}{4}$ **b** $\frac{x}{5}+\frac{x}{2}$ **c** $\frac{x}{3}-\frac{x}{9}$ **d** $\frac{x}{5}-\frac{x}{7}$

e $\frac{2x}{3}+\frac{x}{5}$ **f** $\frac{3x}{4}+\frac{5x}{12}$ **g** $\frac{5x}{6}-\frac{4x}{9}$ **h** $\frac{7x}{10}-\frac{3x}{8}$

i $\frac{x}{7}-\frac{x}{2}$ **j** $\frac{x}{10}-\frac{2x}{5}$ **k** $\frac{5x}{6}-\frac{13x}{15}$ **l** $\frac{3x}{10}-\frac{3x}{2}$

Express each fraction with a common denominator using the LCD, then add or subtract numerators.

6 Simplify the following.

a $\frac{x}{2}+\frac{3}{4}$ **b** $\frac{x}{5}+\frac{2}{3}$ **c** $\frac{2x}{15}+\frac{7}{20}$

d $\frac{x}{4}-\frac{2}{5}$ **e** $\frac{2x}{3}-\frac{5}{9}$ **f** $\frac{5}{6}-\frac{x}{4}$

Example 20 Adding and subtracting with binomial numerators

Simplify the following algebraic expressions.

a $\frac{x+2}{4}-\frac{x}{6}$ **b** $\frac{x+3}{3}+\frac{x-4}{7}$

Solution	Explanation
a $\frac{x+2}{4}-\frac{x}{6}=\frac{3(x+2)}{12}-\frac{2x}{12}$ $=\frac{3x+6-2x}{12}$ $=\frac{x+6}{12}$	The LCD of 4 and 6 is 12. Express each fraction with a denominator of 12. When multiplying $(x + 2)$ by 3, brackets are required. Expand the brackets and collect the terms: $3x + 6 - 2x = 3x - 2x + 6$
b $\frac{x+3}{3}+\frac{x-4}{7}=\frac{7(x+3)}{21}+\frac{3(x-4)}{21}$ $=\frac{7x+21+3x-12}{21}$ $=\frac{10x+9}{21}$	The LCD of 3 and 7 is 21. Express each fraction with a denominator of 21. Expand each pair of brackets first and sum by collecting like terms.

7 Simplify these algebraic expressions.

a $\frac{x+2}{3}+\frac{x}{2}$ **b** $\frac{x+4}{5}+\frac{2x}{3}$ **c** $\frac{x-2}{4}+\frac{3x}{8}$

d $\frac{x+4}{3}-\frac{x}{6}$ **e** $\frac{x+2}{2}-\frac{2x}{5}$ **f** $\frac{6x+7}{12}-\frac{3x}{8}$

g $\frac{x+3}{5}+\frac{x+2}{4}$ **h** $\frac{2x+3}{7}+\frac{x+1}{2}$ **i** $\frac{x+8}{6}+\frac{x-3}{4}$

j $\frac{2x+5}{3}+\frac{x-2}{4}$ **k** $\frac{x-3}{5}+\frac{x+4}{10}$ **l** $\frac{2x+1}{8}+\frac{x-2}{3}$

LCD of 2 and 3 is 6,
$\frac{x+2}{3}+\frac{x}{2}=\frac{\square(x+2)}{6}+\frac{\square x}{6}$

ISBN: 9781107687028 Cambridge University Press

Problem-solving and Reasoning

8 Find the error in each of the following and then correct it.

a $\frac{2x}{3}+\frac{3x}{4}=\frac{5x}{12}$

b $\frac{3x}{5}-\frac{x}{2}=\frac{2x}{3}$

c $\frac{x+2}{5}+\frac{x+4}{3}=\frac{3x+2+5x+4}{15}$
$=\frac{8x+6}{15}$

d $\frac{x+4}{2}+\frac{x-3}{6}=\frac{3x+12+x+3}{6}$
$=\frac{4x+15}{6}$

9 Recall that the expansion $-5(x-2)=-5x+10$, so $6(x+1)-5(x-2)=6x+6-5x+10=x+16$. Use this method to simplify these subtractions.

a $\frac{x+1}{5}-\frac{x-2}{6}$

b $\frac{x+2}{3}-\frac{x-4}{5}$

c $\frac{x-3}{4}-\frac{x+2}{5}$

d $\frac{x+8}{2}-\frac{x+7}{4}$

10 The LCD of the fractions $\frac{4}{x}+\frac{2}{3}$ is $3\times x=3x$.

Use this to find the LCD and simplify these fractions.

a $\frac{4}{x}+\frac{2}{3}$

b $\frac{3}{4}+\frac{2}{x}$

c $\frac{2}{5}+\frac{3}{x}$

d $\frac{3}{7}-\frac{2}{x}$

e $\frac{1}{5}-\frac{4}{x}$

f $\frac{3}{x}-\frac{5}{8}$

$$\frac{4}{x}+\frac{2}{3}=\frac{\square}{3x}+\frac{\square}{3x}$$
$$=\frac{\boxed{}}{3x}$$

Pronumerals in the denominator

As seen in question **10**, pronumerals may form part of the LCD.

The fractions $\frac{5}{2x}$ and $\frac{3}{4}$ would have an LCD of $4x$, while the fractions $\frac{3}{x}$ and $\frac{5}{x^2}$ would have an LCD of x^2.

11 By first finding the LCD, simplify these algebraic fractions.

a $\frac{3}{4}+\frac{5}{2x}$

b $\frac{1}{6}+\frac{5}{2x}$

c $\frac{3}{10}-\frac{1}{4x}$

d $\frac{3}{x}+\frac{5}{x^2}$

e $\frac{4}{x}+\frac{1}{x^2}$

f $\frac{3}{x^2}-\frac{5}{x}$

g $\frac{3}{2x}+\frac{2}{x^2}$

h $\frac{4}{x}+\frac{7}{3x^2}$

i $\frac{3}{4x}-\frac{7}{2x^2}$

ISBN: 9781107687028

3.7 Index notation and index laws 1 and 2

If 9 is written as 3^2 then we are using index form. The 2 in 3^2 is called the index, exponent or power, and the 3 is called the base.

When the same factor is multiplied repeatedly, index form provides an efficient way to represent this. For example, $5 \times 5 \times 5 \times 5$ is written as 5^4 instead. For terms with the same base, calculations can be carried out in index form using the index laws.

Areas of banking such as calculating compound interest involve repeatedly multiplying by the same factor. These calculations are done using powers.

Let's start: Simplifying in index form

- Express the following calculations in expanded form and then simplify the result in index form.

  a $5^3 \times 5^4$ **b** $6^2 \times 6^3$

- Describe how you could write these in index form without first expanding them.
- Express the following in expanded form and then cancel common factors to write the result as a single term in index form.

  a $\dfrac{4^5}{4^2}$ **b** $\dfrac{7^6}{7^3}$

- Describe how you could write these in index form without first expanding them.

Key ideas

- For a number in **index form**, a^m we say 'this is a to the power of m', where a is the **base** and m is the index, exponent or power.
  - The exponent, index or power tells us how many times to multiply the base number by itself.

    For example:

index form		expanded form		basic numeral
5^3	=	$5 \times 5 \times 5$	=	125

  $a = a^1$

  The power button ($\boxed{\wedge}$) on a calculator allows you to evaluate 5^4 as $5 \boxed{\wedge} 4$.
- The first two index laws deal with multiplication and division of these types of numbers.
  - Index law 1: $a^m \times a^n = a^{m+n}$; for example, $5^3 \times 5^4 = 5^{3+4} = 5^7$
  - Index law 2: $a^m \div a^n = a^{m-n}$

    or $\dfrac{a^m}{a^n} = a^{m-n}$; for example, $7^5 \div 7^2 = 7^{5-2} = 7^3$

Index form A method of writing numbers that are multiplied by themselves

Base The number or pronumeral that is being raised to a power

ISBN: 9781107687028 Cambridge University Press

- Negative indices can be expressed as positive indices using the following rules.

$$a^{-m} = \frac{1}{a^m} \text{ and } a^m = \frac{1}{a^{-m}}$$

For $\frac{3^3}{3^5}$, we get $3^{3-5} = 3^{-2}$ and $\frac{3^3}{3^5} = \frac{\cancel{3}^1 \quad \cancel{3}^1 \quad \cancel{3}^1}{\cancel{3}^1 \quad \cancel{3}^1 \quad \cancel{3}^1 \quad 3 \quad 3} = \frac{1}{3^2}$

so $3^{-2} = \frac{1}{3^2}$

Exercise 3G

Understanding

1 Fill in the missing words.

a In 3^5, 3 is the __________ and 5 is the __________.

b 4^6 is read as 4 to the __________ of 6.

c 7^4 is the __________ form of $7 \times 7 \times 7 \times 7$.

d The power tells you how many times to __________ the base number by itself.

e $6 \times 6 \times 6$ is the __________ form of 6^3.

2 Write the following in expanded form.

a 8^3 **b** 7^5 **c** x^6 **d** $(ab)^4$

$$5^{4} = \underbrace{5 \times 5 \times 5 \times 5}_{4 \text{ times}}$$

index form expanded form

Example 21 Writing in index form

Write each of the following in index form.

a $5 \times 5 \times 5$ **b** $4 \times x \times x \times 4 \times x$ **c** $a \times b \times b \times a \times b \times b$

Solution	Explanation
a $5 \times 5 \times 5 = 5^3$	The factor 5 is repeated 3 times.
b $4 \times x \times x \times 4 \times x = 4 \times 4 \times x \times x \times x$	Group the factors of 4 and the factors of x together.
$= 4^2x^3$	The factor x is repeated 3 times; 4 is repeated twice.
c $a \times b \times b \times a \times b \times b$	Group the like pronumerals.
$= a \times a \times b \times b \times b \times b$ $= a^2b^4$	The factor a is repeated twice and the factor b is repeated 4 times.

3 Write the following in index form.

a $9 \times 9 \times 9 \times 9$ **b** $3 \times 3 \times 3 \times 3 \times 3 \times 3$

c $15 \times 15 \times 15$ **d** $5 \times x \times x \times x \times 5$

e $4 \times a \times 4 \times a \times 4 \times a \times a$ **f** $b \times 7 \times b \times b \times b$

g $x \times y \times x \times x \times y$ **h** $a \times b \times a \times b \times b \times b$

i $3 \times x \times 3 \times y \times x \times 3 \times y \times y$ **j** $4 \times x \times z \times 4 \times z \times x$

Group different bases together and write each base in index form.

4 Complete the following to write each as a single term in index form.

a $7^3 \times 7^4 = 7 \times 7 \times 7 \times \boxed{}$
$= 7^{\square}$

b $\frac{5^6}{5^2} = \frac{5 \quad 5 \quad \boxed{}}{5 \quad 5}$
$= 5^{\square}$

ISBN: 9781107687028 Cambridge University Press

Fluency

Example 22 Expressing negative indices in positive index form

Express the following with positive indices.

a x^{-2} **b** $4y^{-2}$ **c** $2a^{-3}b^2$

Solution	Explanation
a $x^{-2} = \frac{1}{x^2}$	Use $a^{-m} = \frac{1}{a^m}$.
b $4y^{-2} = 4 \quad \frac{1}{y^2}$	The negative index only applies to y. $y^{-2} = \frac{1}{y^2}$.
$= \frac{4}{y^2}$	$4 \quad \frac{1}{y^2} = \frac{4}{1} \quad \frac{1}{y^2} = \frac{4}{y^2}$.
c $2a^{-3}b^2 = 2 \quad \frac{1}{a^3} \quad b^2$	$a^{-3} = \frac{1}{a^3}, \frac{2}{1} \quad \frac{1}{a^3} \quad \frac{b^2}{1}$
$= \frac{2b^2}{a^3}$	Multiply the numerators and the denominators.

5 Express the following with positive indices.

a y^{-3}	**b** x^{-4}	**c** x^{-2}	**d** a^{-5}
e $3x^{-2}$	**f** $5b^{-3}$	**g** $4x^{-1}$	**h** $2m^{-9}$
i $2x^2y^{-3}$	**j** $3xy^{-4}$	**k** $3a^{-2}b^4$	**l** $5m^{-3}n^2$

Use $a^{-m} = \frac{1}{a^m}$.

Example 23 Using $\frac{1}{a^{-m}} = a^m$

Rewrite the following with positive indices only.

a $\frac{1}{x^{-3}}$ **b** $\frac{4}{x^{-5}}$ **c** $\frac{5}{a^2b^{-4}}$

Solution	Explanation
a $\frac{1}{x^{-3}} = x^3$	Use $\frac{1}{a^{-m}} = a^m$
b $\frac{4}{x^{-5}} = 4 \quad \frac{1}{x^{-5}}$	The 4 remains unchanged.
$= 4 \times x^5$	$\frac{1}{x^{-5}} = x^5$
$= 4x^5$	
c $\frac{5}{a^2b^{-4}} = \frac{5}{a^2} \quad \frac{1}{b^{-4}}$	The negative index applies to b only.
$= \frac{5}{a^2} \quad b^4$	$\frac{1}{b^{-4}} = b^4$
$= \frac{5b^4}{a^2}$	$\frac{5}{a^2} \quad b^4 = \frac{5}{a^2} \quad \frac{b^4}{1} = \frac{5b^4}{a^2}$

ISBN: 9781107687028

Cambridge University Press

6 Rewrite the following with positive indices only.

a $\dfrac{1}{b^{-4}}$ **b** $\dfrac{1}{x^{-7}}$ **c** $\dfrac{1}{y^{-1}}$

d $\dfrac{5}{m^{-3}}$ **e** $\dfrac{2}{y^{-2}}$ **f** $\dfrac{3}{x^{-4}}$

g $\dfrac{5a^2}{b^{-3}}$ **h** $\dfrac{4}{x^2y^{-5}}$ **i** $\dfrac{10}{a^{-2}b^4}$

Use $\dfrac{1}{a^{-m}} = a^m$.

Example 24 Using index law 1

Simplify the following using the first index law.

a $x^7 \times x^4$ **b** $a^2b^2 \times ab^3$ **c** $3x^2y^3 \times 4x^3y^4$

Solution	Explanation
a $x^7 \times x^4 = x^{7+4}$ $= x^{11}$	Use law 1, $a^m \times a^n = a^{m+n}$, to add the indices.
b $a^2b^2 \times ab^3 = a^{2+1}b^{2+3}$ $= a^3b^5$	Add the indices of base a and base b. Recall that $a = a^1$.
c $3x^2y^3 \times 4x^3y^4 = (3 \times 4)x^{2+3}y^{3+4}$ $= 12x^5y^7$	Multiply the coefficients and add indices of the common bases x and y.

7 Simplify the following using the first index law.

a $x^3 \times x^4$ **b** $p^5 \times p^2$ **c** $t^7 \times t^2$

d $d^4 \times d$ **e** $g \times g^3$ **f** $f^2 \times f$

g $2p^2 \times p^3$ **h** $3c^4 \times c^4$ **i** $2s^4 \times 3s^7$

j $a^2b^3 \times a^3b^5$ **k** $d^7f^3 \times d^2f^2$ **l** $v^3z^5 \times v^2z^3$

m $3a^2b \times 5ab^5$ **n** $2x^2y \times 3xy^2$ **o** $3e^7r^2 \times 6e^2r$

p $-4p^3c^2 \times 2pc$ **q** $-2r^2s^3 \times 5r^5s^5$ **r** $-3d^4f^2 \times (-2f^2d^2)$

Index law 1:
$a^m \times a^n = a^{m+n}$
Group common bases and add indices when multiplying.

Example 25 Using index law 2

Simplify the following using the second index law.

a $p^5 \div p^3$ **b** $12m^8 \div (6m^3)$ **c** $\dfrac{4x^2y^4}{8xy^2}$

Solution	Explanation
a $p^5 \div p^3 = p^{5-3}$ $= p^2$	Use law 2, $a^m \div a^n = a^{m-n}$, to subtract the indices.
b $12m^8 \,|\, (6m^3) = \dfrac{{}^{2}\cancel{12}m^8}{{}_{1}\cancel{6}m^3}$ $= 2m^{8-3}$ $= 2m^5$	Write in fraction form. Cancel the highest common factor of 12 and 6. Use law 2 to subtract indices.

ISBN: 9781107687028

c $\dfrac{4x^2y^4}{8xy^2} = \dfrac{{}^{1}\cancel{4} \times x^2 \times y^4}{{}^{2}\cancel{8} \times x \times y^2}$

$= \dfrac{x^{2-1}y^{4-2}}{2}$

$= \dfrac{xy^2}{2}$ $\left(\text{or } \dfrac{1}{2}xy^2\right)$

Cancel the common factors of the numerals and subtract the indices of base x and base y.

8 Simplify the following using the second index law.

a $a^4 \div a^2$ **b** $d^7 \div d^6$ **c** $r^3 \div r$ **d** $\dfrac{c^{10}}{c^6}$

e $\dfrac{l^4}{l^3}$ **f** $\dfrac{b^5}{b^2}$ **g** $\dfrac{4d^4}{d^2}$ **h** $\dfrac{f^3}{2f^2}$

i $\dfrac{9n^4}{3n}$ **j** $6p^4 \div (3p^2)$ **k** $24m^7 \div (16m^3)$ **l** $10d^3 \div (30d)$

m $\dfrac{8t^4r^3}{2tr^2}$ **n** $\dfrac{5b^6d^4}{3d^3b^2}$ **o** $\dfrac{2p^2q^3}{p^2q}$ **p** $\dfrac{4x^2y^3}{8xy}$

q $\dfrac{3r^5s^2}{9r^3s}$ **r** $6c^4d^6 \div (15c^3d)$ **s** $2a^4y^2 \div (4ay)$ **t** $13m^4n^6 \div (26m^4n^5)$

Index law 2:
$a^m \div a^n = a^{m-n}$
or
$\dfrac{a^m}{a^n} = a^{m-n}$
When dividing, subtract indices of common bases.

Problem-solving and Reasoning

Example 26 Combining laws 1 and 2 and negative indices

Simplify, using index laws 1 and 2. Express answers with positive indices.

a $\dfrac{2a^3b \quad 3a^2b^3}{12a^4b^2}$ **b** $\dfrac{x^4y^3 \quad x^{-2}y^5}{x^5y^4}$

Solution	Explanation
a $\dfrac{2a^3b \times 3a^2b^3}{12a^4b^2} = \dfrac{(2\times 3)a^{3+2}b^{1+3}}{12a^4b^2}$	Simplify numerator first by multiplying coefficients and using law 1 to add indices of a and b.
$= \dfrac{\cancel{6}^{1}a^5b^4}{{}^{2}\cancel{12}a^4b^2}$	Cancel common factor of numerals and use law 2 to subtract indices of common bases.
$= \dfrac{a^{5-4}b^{4-2}}{2}$	
$= \dfrac{ab^2}{2}$	
b $\dfrac{x^4y^3 \times x^{-2}y^5}{x^5y^4} = \dfrac{x^{4+(-2)}y^{3+5}}{x^5y^4}$	Use law 1 to add indices of x and y in numerator. $4 + (-2) = 4 - 2 = 2$.
$= \dfrac{x^2y^8}{x^5y^4}$	Use law 2 to subtract indices.
$= x^{2-5}y^{8-4}$	
$= x^{-3}y^4$	Express with positive indices, $x^{-3} = \dfrac{1}{x^3}$.
$= \dfrac{y^4}{x^3}$	$x^{-3}y^4 = \dfrac{1}{x^3} \quad \dfrac{y^4}{1}$.

ISBN: 9781107687028

9 Simplify the following using index laws 1 and 2.

a $\dfrac{x^2y^3 \quad x^2y^4}{x^3y^5}$ b $\dfrac{m^2w \quad m^3w^2}{m^4w^3}$ c $\dfrac{r^4s^7 \quad r^4s^7}{r^6s^{10}}$

d $\dfrac{16a^8b \quad 4ab^7}{32a^7b^6}$ e $\dfrac{9x^2y^3 \quad 6x^7y^7}{12xy^6}$ f $\dfrac{4e^2w^2 \quad 12e^2w^3}{12e^4w}$

Simplify the numerator first using index law 1, then apply index law 2.

10 Simplify the following, expressing answers using positive indices.

a $\dfrac{a^6b^2 \quad a^{-2}b^3}{a^7b}$ b $\dfrac{x^5y^3 \quad x^2y^{-1}}{x^3y^5}$

c $\dfrac{x^4y^7 \quad x^{-2}y^{-5}}{x^4y^6}$ d $\dfrac{a^5b^{-2} \quad a^{-3}b^4}{a^6b}$

Index laws 1 and 2 apply to negative indices also.
$x^5 \times x^{-2} = x^{5+(-2)} = x^3$
$\dfrac{x^4}{x^6} = x^{4-6} = x^{-2} = \dfrac{1}{x^2}$

11 When Billy uses a calculator to raise -2 to the power 4 he gets -16, when in fact the answer is actually 16. What has he done wrong?

Index laws and calculations

12 Consider the following use of negative numbers.

a Evaluate:
 i $(-3)^2$ ii -3^2

b What is the difference between your two answers in part **a**?

c Evaluate:
 i $(-2)^3$ ii -2^3

d What do you notice about your answers in part **c**? Explain.

$(-3)^2 = -3 \times (-3)$
$(-2)^3 = -2 \times (-2) \times (-2)$
Consider order of operations.

13 Use index law 2 to evaluate these expressions without the use of a calculator.

a $\dfrac{13^3}{13^2}$ b $\dfrac{18^7}{18^6}$ c $\dfrac{9^8}{9^6}$

d $\dfrac{3^{10}}{3^7}$ e $\dfrac{4^8}{4^5}$ f $\dfrac{2^{12}}{2^8}$

ISBN: 9781107687028 Cambridge University Press

3.8 More index laws and the zero power

Using index laws 1 and 2, we can work out four other index laws to simplify expressions, especially those using brackets.

For example, $(4^2)^3 = 4^2 \times 4^2 \times 4^2$

$= 4^{2+2+2} = 4^6$ (add indices using law 1)

Therefore, $(4^2)^3 = 4^{2 \times 3} = 4^6$

We also have a result for the zero power.

Consider $5^3 \div 5^3$, which clearly equals 1. Using index law 2, we can see that

$5^3 \div 5^3 = 5^{3-3} = 5^0$.

Therefore $5^0 = 1$, leading to the zero power rule: $a^0 = 1$.

Let's start: Indices with brackets

Brackets are used to show that the power outside the brackets applies to each factor inside the brackets.

Consider $(2x)^3 = 2x \times 2x \times 2x$.

- Write this in index form without using brackets.
- Can you suggest the index form of $(3y)^4$ without brackets?

Consider $\left(\frac{3}{5}\right)^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$.

- Write the numerator and denominator of this expanded form in index form.
- Can you suggest the index form of $\left(\frac{x}{4}\right)^5$ without brackets?

Write a rule for removing the brackets of the following.

- $(ab)^m$
- $\left(\frac{a}{b}\right)^m$

Key ideas

- Index law 3: $(a^m)^n = a^{m \times n}$
  Remove brackets and multiply indices:
  For example, $(x^3)^4 = x^{3 \times 4} = x^{12}$
- Index law 4: $(a \times b)^m = a^m \times b^m$
  Apply the index to each factor in the brackets:
  For example, $(3x)^4 = 3^4x^4$
- Index law 5: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
  Apply the index to the numerator and denominator:
  For example, $\left(\frac{y}{3}\right)^5 = \frac{y^5}{3^5}$
- The zero power: a^0
  Any number (except zero) to the power of zero is 1:
  For example, $5^0 = 1$, $y^0 = 1$, $4y^0 = 4 \times 1 = 4$

ISBN: 9781107687028

Exercise 3H

Understanding

1 Complete the following index laws.

a Any number (except 0) to the power of zero is ______.

b Index law 3 states $(a^m)^n =$ ______.

c Index law 4: $(a \times b)^m =$ ______ $\times$ ______.

d Index law 5: $\left(\dfrac{a}{b}\right)^m = \dfrac{\_\_\_\_}{\_\_\_\_}$.

2 Copy and complete the following.

a $(4^2)^3 = 4^2 \times \square \times \square$
$= 4^{\square}$

b $(2a)^3 = 2a \times \square \times \square$
$= 2 \times \square \times \square \times a \times \square \times \square$
$= 2^{\square}a^{\square}$

c $\left(\dfrac{4}{7}\right)^4 = \dfrac{4}{7} \times \square \times \square \times \square$
$= \dfrac{4 \times \square \times \square \times \square}{7 \times \square \times \square \times \square}$
$= \dfrac{4^{\square}}{7^{\square}}$

Fluency

Example 27 Using the zero power

Evaluate, using the zero power:

a $4^0 + 2^0$ **b** $3a^0$ **c** $(-3)^0 + 6x^0$

Solution	Explanation
a $4^0 + 2^0 = 1 + 1$ $= 2$	Zero power: $a^0 = 1$, any number to the power zero (except zero) is 1.
b $3a^0 = 3 \times a^0$ $= 3 \times 1$ $= 3$	The zero power only applies to a, so $a^0 = 1$.
c $(-3)^0 + 6x^0 = 1 + 6 \times 1$ $= 7$	Any number to the power of zero is 1. $(-3)^0 = 1$, $6x^0 = 6 \times x^0 = 6 \times 1$.

3 Evaluate, using the zero power:

a 4^0 **b** 5^0 **c** x^0 **d** a^0

e $3e^0$ **f** $4y^0$ **g** $3^0 + 6^0$ **h** $10 - 10x^0$

i $3d^0 - 2$ **j** $(-4)^0 + 2x^0$ **k** $\dfrac{2}{m^0}$ **l** $5a^0 + 4b^0$

Any number (except zero) to the power of 0 is 1.
$4a^0 = 4 \times a^0 = 4 \times 1$

ISBN: 9781107687028

Cambridge University Press

Example 28 Using index law 3: $(a^m)^n = a^{m \times n}$

Simplify the following using the third index law.

a $(x^5)^7$ **b** $3(f^4)^3$

Solution	Explanation
a $(x^5)^7 = x^{5 \times 7}$ $= x^{35}$	Apply index law 3: $(a^m)^n = a^{m \times n}$ to multiply indices.
b $3(f^4)^3 = 3f^{4 \times 3}$ $= 3f^{12}$	Apply index law 3 to the value inside the bracket only.

4 Simplify the following using the third index law.

a $(b^3)^4$ **b** $(f^5)^4$ **c** $(k^3)^7$
d $3(x^2)^3$ **e** $5(c^9)^2$ **f** $4(s^6)^3$

Index law 3:
$(a^m)^n = a^{m \times n}$

Example 29 Using index laws 4 and 5: $(a \times b)^m = a^m \times b^m$ and $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Simplify the following using the third, fourth and fifth index laws.

a $(2s)^4$ **b** $(x^2y^3)^5$ **c** $\left(\frac{x}{4}\right)^3$

Solution	Explanation
a $(2s)^4 = 2^4 \times s^4$ $= 16s^4$	Apply index law 4: $(a \times b)^m = a^m \times b^m$. Evaluate $2^4 = 2 \times 2 \times 2 \times 2$.
b $(x^2y^3)^5 = (x^2)^5 \times (y^3)^5$ $= x^{10}y^{15}$	Using index law 4, apply the index 5 to each factor in the brackets. Using index law 3, multiply indices: $(x^2)^5 = x^{2 \times 5}$, $(y^3)^5 = y^{3 \times 5}$.
c $\left(\frac{x}{4}\right)^3 = \frac{x^3}{4^3}$ $= \frac{x^3}{64}$	Apply index law 5: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$. Evaluate $4^3 = 4 \times 4 \times 4$.

5 Simplify, using the third, fourth and fifth index laws.

a $(3x)^2$ **b** $(4m)^3$ **c** $(5y)^3$
d $(2x^3)^4$ **e** $(x^2y)^5$ **f** $(3a^3)^3$
g $(x^4y^2)^6$ **h** $(a^2b)^3$ **i** $(m^3n^3)^4$
j $\left(\frac{x}{5}\right)^2$ **k** $\left(\frac{y}{3}\right)^4$ **l** $\left(\frac{m}{2}\right)^4$
m $\left(\frac{x^2}{y}\right)^3$ **n** $\left(\frac{x^3}{y^2}\right)^4$ **o** $\left(\frac{x}{y^5}\right)^3$

Index law 4 says
$(3 \times x)^2 = 3^2 \times x^2$

Index law 5 says
$\left(\frac{x}{5}\right)^2 = \frac{x^2}{5^2}$

ISBN: 9781107687028

Cambridge University Press

Example 30 Combining index laws and negative indices

Simplify, using index laws, and express with positive indices:

a $\dfrac{3x^2y \quad 2x^3y^2}{10xy^3}$ **b** $\left(\dfrac{2x^2}{y}\right)^4$ **c** $(2x^{-2})^3 + (3x)^0$

Solution	Explanation
a $\dfrac{3x^2y \quad 2x^3y^2}{10xy^3}$	Simplify the numerator by multiplying coefficients and adding indices using index law 1.
$= \dfrac{6x^5y^3}{10xy^3}$	Cancel the common factor of 6 and 10 and apply index law 2 to subtract indices of common bases.
$= \dfrac{3x^4y^0}{5}$	The zero power says $y^0 = 1$.
$= \dfrac{3x^4}{5}$	
b $\left(\dfrac{2x^2}{y}\right)^4 = \dfrac{(2x^2)^4}{y^4}$	Apply index law 5 to apply the index to the numerator and denominator.
$= \dfrac{2^4 \times (x^2)^4}{y^4}$	Apply laws 3 and 4 to multiply indices.
$= \dfrac{16x^8}{y^4}$	
c $(2x^{-2})^3 + (3x)^0 = 2^3 \times x^{-6} + 3^0 \times x^0$	Using index law 4, apply the power to each factor inside the brackets. $(x^{-2})^3 = x^{-2\times 3} = x^{-6}$.
$= 8x^{-6} + 1 \times 1$	Any number to the power of zero is 1.
$= \dfrac{8}{x^6} + 1$	Use $a^{-m} = \dfrac{1}{a^m}$ to express with a positive index.

6 Simplify, using index laws:

a $\dfrac{m^2w \quad m^3w^2}{m^4w^3}$

b $\dfrac{x^3y^2 \quad x^2y^7}{10x^5y^4}$

c $\dfrac{b^3c^5 \quad 4b^5c^3}{3b^4c^8}$

d $\dfrac{9c^4s^2 \quad 3c^3s^5}{2c^3s^7}$

e $\dfrac{(5r^6)^2}{3r^8}$

f $\dfrac{(2p^4)^3}{3p^7}$

g $\left(\dfrac{2s^2}{t^3}\right)^4$

h $\left(\dfrac{r^2}{5s^3}\right)^4$

First simplify the numerator, then combine the denominator.

ISBN: 9781107687028

Problem-solving and Reasoning

7 Simplify, using index laws, and express with positive indices:

a $(x^{-4})^2$ **b** $(x^3)^{-2}$ **c** $(x^{-2})^0$
d $(2y^{-2})^3$ **e** $(ay^{-3})^2$ **f** $(4x^{-3})^{-2}$
g $(m^{-4}) + 4(ab)^0$ **h** $(2a^{-2})^3 + (4a)^0$ **i** $(5a^{-2})^{-2} \times 5a^0$

Remove brackets using index laws then use $a^{-m} = \frac{1}{a^m}$ to express with a positive index.

8 Evaluate without the use of a calculator:

a 2^{-2} **b** $\frac{4}{3^{-2}}$ **c** $\frac{5}{2^{-3}}$
d $\frac{(5^2)^2}{5^4}$ **e** $\frac{36^2}{6^4}$ **f** $\frac{27^2}{3^4}$

$36^2 = (6^2)^2$

9 Simplify the following.

a $2p^2q^4 \times pq^3$ **b** $4(a^2b)^3 \times (3ab)^3$
c $(4r^2y)^2 \times r^2y^4 \times 3(ry^2)^3$ **d** $2(m^3n)^4 \div m^3$
e $\frac{(7s^2y)^2 \quad 3sy^2}{7(sy)^2}$ **f** $\frac{3(d^4c^3)^3 \quad 4dc}{(2c^2d)^3}$
g $\frac{4r^2t \quad 3(r^2t)^3}{6r^2t^4}$ **h** $\frac{(2xy)^2}{8xy} \quad \frac{2(x^2y)^3}{x^7y^3}$

All laws together

10 Simplify the following, expressing your answer with positive indices.

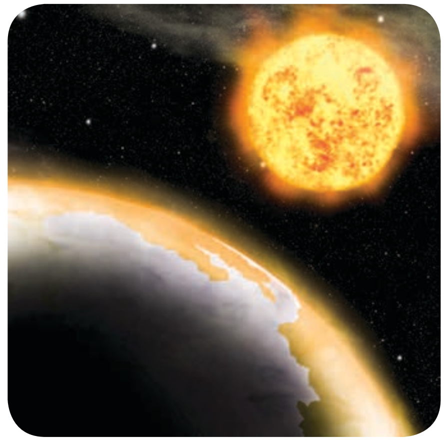
a $(a^3b^2)^3 \times (a^2b^4)^{-1}$ **b** $2x^2y^{-1} \times (3xy^4)^3$
c $2(p^2)^4 \times (3p^2q)^{-2}$ **d** $\frac{2a^3b^2}{a^{-3}} \quad \frac{2a^2b^5}{b^4}$
e $\frac{(3rs^2)^4}{r^{-3}s^4} \quad \frac{(2r^2s)^2}{s^7}$ **f** $\frac{4(x^{-2}y^4)^2}{x^2y^{-3}} \quad \frac{xy^4}{2s^{-2}y}$

$(3p^2q)^{-2} = \frac{1}{(3p^2q)^2}$

ISBN: 9781107687028

3.9 Scientific notation

Scientific notation is useful when working with very large or very small numbers. Combined with the use of significant figures, numbers can be written down with an appropriate degree of accuracy and without the need to write all the zeroes that define the position of the decimal point. The approximate distance between the Earth and the Sun is 150 million kilometres or 1.5×10^8 km, written in scientific notation. Negative indices can be used for very small numbers such as $0.000\,038\,2$ g $= 3.82 \times 10^{-5}$ g.



Let's start: Amazing facts large and small

Think of an object, place or living thing that is associated with a very large or small number.

- Give three examples of very large numbers.
- Give three examples of very small numbers.
- Can you remember how to write these numbers using scientific notation? List the rules you remember.

Key ideas

- A number written using **scientific notation** is of the form $a \times 10^m$, where $1 \le a < 10$ and m is an integer.
- To write numbers using scientific notation, place the decimal point after the first non-zero digit then multiply by the power of 10 corresponding to how many places the decimal point was moved.
  - Large numbers will use positive powers of 10.

    For example, $24\,800\,000 = 2.48 \times 10^7$

    $9\,020\,000\,000 = 9.02 \times 10^9$
  - Small numbers will use negative powers of 10.

    For example, $0.003\,07 = 3.07 \times 10^{-3}$

    $0.000\,001\,2 = 1.2 \times 10^{-6}$
- **Significant figures** are counted from left to right starting at the first non-zero digit. For example:
  - 47 086 000 has five significant figures
  - 2.037 has four significant figures
  - 0.0014 has two significant figures
  - 0.00140 has three significant figures

  Zeroes at the end of a number are counted for decimals (see 0.00140 above) but not whole numbers (see 47 086 000 above).
- When using scientific notation, the first significant figure sits to the left of the decimal point. For example:
  - $20\,190\,000 = 2.019 \times 10^7$ has four significant figures
- The EE or Exp keys on calculators can be used to enter numbers using scientific notation: 2.3E-4 means 2.3×10^{-4}.

Scientific notation A way to express very large and very small numbers using [a number between 1 and 10] $\times$ 10$^{\text{power}}$

Significant figure A digit that indicates how waccurate a number is

ISBN: 9781107687028

Exercise 3I

Understanding

1 Evaluate the following.

a 1.24×100 **b** 2.8×100 **c** 3.02×1000
d $4.5 \div 100$ **e** $3.75 \div 1000$ **f** $6 \div 100$

Move the decimal point as many places as there are zeroes.
$\times$ means move the decimal point right.
$\div$ means move the decimal point left.

2 Write these numbers as powers of 10.

a 1000 **b** 10 000 000 **c** 0.000 001 **d** $\dfrac{1}{1000}$

3 State whether these numbers would have positive or negative indices when written in scientific notation.

a 7800 **b** 0.0024 **c** 27 000 **d** 0.0009

4 Write the number 4.8721 using the following numbers of significant figures.

a 3 **b** 4 **c** 2

Fluency

Example 31 Converting from scientific notation to a basic numeral

Write these numbers as a basic numeral.

a 5.016×10^5 **b** 3.2×10^{-7}

Solution	Explanation
a $5.016 \times 10^5 = 501\,600$	Move the decimal point 5 places to the right, inserting zeroes after the last digit. 5.01600
b $3.2 \times 10^{-7} = 0.000\,000\,32$	Move the decimal point 7 places to the left, due to the -7, and insert zeroes where necessary.

5 Write these numbers as a basic numeral.

a 3.12×10^3 **b** 5.4293×10^4 **c** 7.105×10^5
d 8.213×10^6 **e** 5.95×10^4 **f** 8.002×10^5
g 1.012×10^4 **h** 9.99×10^6 **i** 2.105×10^8
j 4.5×10^{-3} **k** 2.72×10^{-2} **l** 3.085×10^{-4}
m 7.83×10^{-3} **n** 9.2×10^{-5} **o** 2.65×10^{-1}
p 1.002×10^{-4} **q** 6.235×10^{-6} **r** 9.8×10^{-1}

For a positive index, move the decimal point right (number gets bigger). For a negative index, move the decimal point left (number gets smaller).

Example 32 Writing numbers using scientific notation

Write these numbers in scientific notation.

a 5 700 000 **b** 0.000 000 6

Solution	Explanation
a $5\,700\,000 = 5.7 \times 10^6$	Place the decimal point after the first non-zero digit (5) then multiply by 10^6 as the decimal point has been moved 6 places to the left.
b $0.000\,000\,6 = 6 \times 10^{-7}$	6 is the first non-zero digit. Multiply by 10^{-7}, since the decimal point has been moved 7 places to the right.

ISBN: 9781107687028

6 Write these numbers in scientific notation.

a 43 000	**b** 712 000	**c** 901 200
d 10 010	**e** 23 900	**f** 703 000 000
g 0.000 78	**h** 0.001 01	**i** 0.000 03
j 0.030 04	**k** 0.112	**l** 0.001 92

For scientific notation, place decimal point after the first non-zero digit and multiply by the power of 10.

Example 33 Converting to scientific notation using significant figures

Write these numbers in scientific notation using 3 significant figures.

a 5 218 300 **b** 0.004 203 1

Solution	Explanation
a $5\,218\,300 = 5.22 \times 10^6$	Put the decimal point after 5 and multiply by 10^6: 5.218 300 The digit following the third digit (8) is at least 5, so round the 1 up to 2.
b $0.004\,203\,1 = 4.20 \times 10^{-3}$	Put the decimal point after 4 and multiply by 10^{-3}: 0.004 203 1 Round down in this case, since the digit following the third digit (3) is less than 5, but retain the zero to show the value of the third significant figure.

7 Write these numbers in scientific notation using 3 significant figures.

a 6241	**b** 572 644	**c** 30 248
d 423 578	**e** 10 089	**f** 34 971 863
g 72 477	**h** 356 088	**i** 110 438 523
j 0.002 423	**k** 0.018 754	**l** 0.000 125
m 0.007 866 3	**n** 0.000 708 2	**o** 0.113 96
p 0.000 006 403	**q** 0.000 078 92	**r** 0.000 129 983

For 3 significant figures, count from the first non-zero digit. Look at the digit after the third digit to determine whether you should round up or down.

Problem-solving and Reasoning

8 Write the following numerical facts using scientific notation.

a The area of Australia is about 7 700 000 km^2.

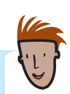
b The number of stones used to build the Pyramid of Khufu is about 2 500 000.

c The greatest distance of Pluto from the Sun is about 7 400 000 000 km.

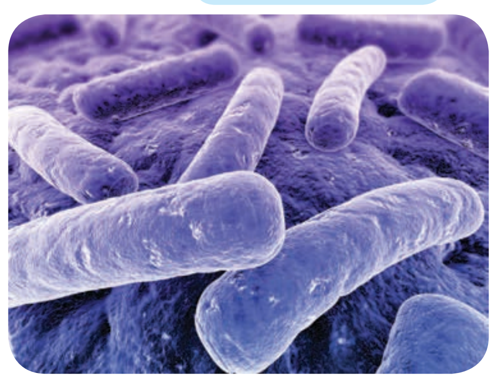
d A human hair is about 0.01 cm wide.

e The mass of a neutron is about 0.000000000000000000000000001675 kg.

f The mass of a bacteria cell is about 0.00000000000095 g.



Large numbers have a positive index. Small numbers have a negative index.



ISBN: 9781107687028 Cambridge University Press

9 Explain why 38×10^7 is not written using scientific notation and convert it to scientific notation.

10 Use a calculator to evaluate the following, giving the answers in scientific notation using 3 significant figures.

a $(2.31)^{-7}$ **b** $(5.04)^{-4}$

c $(2.83 \times 10^2)^{-3}$ **d** $5.1 \div (8 \times 10^2)$

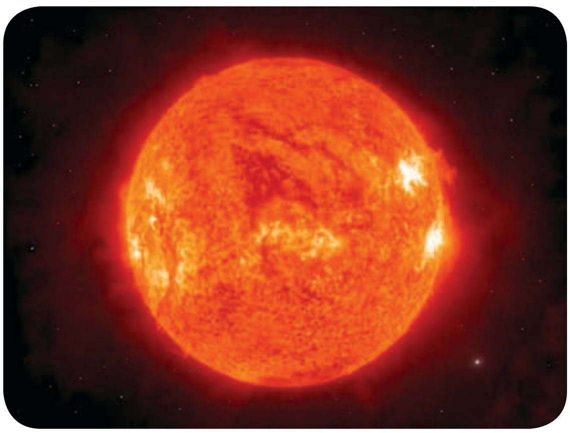
e $9.3 \times 10^{-2} \times 8.6 \times 10^8$ **f** $(3.27 \times 10^4) \div (9 \times 10^{-5})$

g $\sqrt{3.23 \quad 10^{-6}}$ **h** $\pi(3.3 \times 10^7)^2$

Locate the EE or Exp button on your calculator.

11 The speed of light is approximately 3×10^5 km/s and the average distance between Pluto and the Sun is about 5.9×10^9 km. How long does it take for light from the Sun to reach Pluto? Answer correct to the nearest minute. (Divide by 60 to convert seconds to minutes.)

$\text{Time} = \dfrac{\text{distance}}{\text{speed}}$



$E = mc^2$

12 $E = mc^2$ is a formula derived by Albert Einstein (1879–1955). The formula relates the energy (E joules) of an object to its mass (m kg), where c is the speed of light (approximately 3×10^8 m/s).

Use $E = mc^2$ to answer these questions using scientific notation.

a Find the energy, in joules, contained inside an object with the given masses.

i 10 kg **ii** 26 000 kg

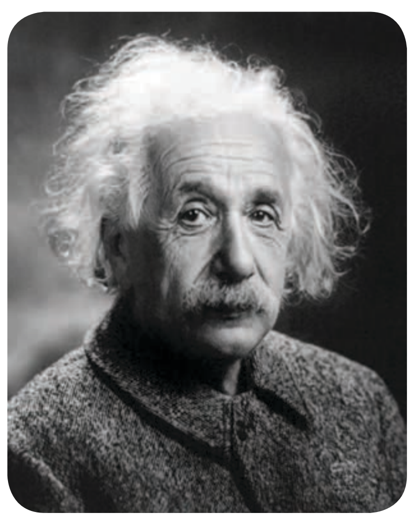
iii 0.03 kg **iv** 0.000 01 kg

b Find the mass in kilograms of an object that contains the given amounts of energy. Give your answer using 3 significant figures.

i 1×10^{25} J **ii** 3.8×10^{16} J

iii 8.72×10^4 J **iv** 1.7×10^{-2} J

c The mass of the Earth is about 6×10^{24} kg. How much energy does this convert to?



Albert Einstein

ISBN: 9781107687028 Cambridge University Press

3.10 Exponential growth and decay

Exponential change occurs when a quantity is continually affected by a constant multiplying factor. The change in quantity is not the same amount each time.

If you have a continual percentage increase, it is called exponential growth. If you have a continual percentage decrease, it is called exponential decay.

Some examples include:

- compound interest at a rate of 5% per year, where the interest is calculated as 5% of the investment value each year, including the previous year's interest
- a radioactive element has a 'half-life' of 5 years, which means the element decays at a rate of 50% every five years.

Let's start: A compound rule

Imagine that you have an investment valued at \$100 000 and you hope that it will return 10% p.a. (per annum).

The 10% return is to be added to the investment balance each year.

- Discuss how to calculate the investment balance in the first year.
- Discuss how to calculate the investment balance in the second year.
- Complete this table.

Year	0	1	2	3
Balance (\$)	100 000	100 000 × 1.1 = ______	100 000 × 1.1 × ___ = ______	______ = ______

- Recall how indices can be used to calculate the balance after the second year.
- Discuss how indices can be used to calculate the balance after the 10th year.
- What might be the rule connecting the investment balance (\$$A$) and the time, n years?

Key ideas

- **Exponential growth** and **decay** can be modelled by the rule $A = A_0\left(1 \pm \frac{r}{100}\right)^n$.
  - A is the amount
  - A_0 is the initial amount (the subscript zero represents time zero)
  - r is the percentage rate of increase or decrease
  - n is time; how many times the percentage increase/decrease is applied

  For example, for a population increasing at 2% per year, $P = P_0(1.02)^n$.
  For a population decreasing at 3% per year, $P = P_0(0.97)^n$.
- For a growth rate of r% p.a., use $1 + \frac{r}{100}$.

Exponential growth Repeatedly increasing a quantity by a constant percentage over time

Exponential decay Repeatedly decreasing a quantity by a constant percentage over time

ISBN: 9781107687028

Cambridge University Press

- For a decay rate of r% p.a., use $1-\dfrac{r}{100}$.
- Compound interest involves *adding* any interest earned to the balance at the end of each year or other period. The rule for the investment amount ($\$A$) is given by: $A = A_0\left(1+\dfrac{r}{100}\right)^n$.
  – A_0 is the initial amount.
  – r is the interest rate expressed as a percentage.
  – n is the time.

Exercise 3J

Understanding

1 An investment of \$1000 is to grow by 5% per year. Round your answers to the nearest cent.

5% of $x = 0.05 \times x$

a Find the interest earned in the first year; i.e. 5% of \$1000.
b Find the investment balance at the end of the first year, investment + interest.
c Find the interest earned in the second year, 5% of answer to part **b**.
d Find the interest earned in the third year.

2 The mass of a limestone 5 kg rock exposed to the weather is decreasing at a rate of 2% per annum.

For decrease, use $1-\dfrac{r}{100}$.

a Find the mass of the rock at the end of the first year.
b Copy and complete the rule for the mass of the rock (M kg) after n years.
$M = 5(1-\_\_\_)^n = 5 \times \_\_\_^n$
c Use your rule to calculate the mass of the rock after 5 years, correct to two decimal places.

3 Decide whether the following represent exponential *growth* or exponential *decay*.
a $A = 1000 \times 1.3^n$ **b** $A = 200 \times 1.78^n$ **c** $A = 350 \times 0.9^n$
d $P = 50\,000 \times 0.85^n$ **e** $P = P_0\left(1+\dfrac{3}{100}\right)^n$ **f** $T = T_0\left(1-\dfrac{7}{100}\right)^n$

Fluency

Example 33 Writing exponential rules

Form exponential rules for the following situations.
a John invests his \$100 000 in savings at a rate of 14% per annum.
b A city's initial population of 50 000 is decreasing by 12% per year.

Solution	Explanation
a Let A = the amount of money at any time n = the number of years the money is invested A_0 = initial amount invested $r = 14$ $A_0 = 100\,000$	Define your variables. The basic formula is $A = A_0\left(1\pm\dfrac{r}{100}\right)^n$.
$A = 100\,000\left(1+\dfrac{14}{100}\right)^n$ $\therefore A = 100\,000(1.14)^n$	Substitute $r = 14$ and $A_0 = 100\,000$ and use '+' since we have growth. $\dfrac{14}{100} = 0.14$.

ISBN: 9781107687028

b Let P = the population at any time
n = the number of years the population decreases
P_0 = starting population
$r = 12$

Define your variables.
The basic formula is $P = P_0\left(1 \pm \frac{r}{100}\right)^n$.

$$P = 50\,000\left(1 - \frac{12}{100}\right)^n$$

$$\therefore P = 50\,000(0.88)^n$$

Substitute $r = 12$ and $P_0 = 50\,000$ and use '–' since we have decay. $\frac{12}{100} = 0.12$ and $1 - 0.12 = 0.88$

4 Define variables and form exponential rules for the following situations.

a \$200 000 is invested at 17% per annum.

b A house initially valued at \$530 000 is losing value at 5% per annum.

c The value of a car, bought for \$14 200, is decreasing at 3% per annum.

d A population, initially 172 500, is increasing at 15% per year.

e A tank with 1200 litres of water is leaking at a rate of 10% of the water in the tank every hour.

f A cell of area 0.01 cm^2 doubles its size every minute.

g An oil spill, initially covering an area of 2 square metres, is increasing at 5% per minute.

h A substance of mass 30 g is decaying at a rate of 8% per hour.

The exponential rule is of the form

$$A = A_0\left(1 \pm \frac{r}{100}\right)^n$$

- A is the amount
- A_0 is the initial amount
- r is the percentage increase/decrease
- n is the time

Use '+' for growth and '–' for decay.

Example 34 Applying exponential rules

House prices are rising at 9% per year and Zoe's house is currently valued at \$145 000.

a Determine a rule for the value of Zoe's house (\$$V$) in n years' time.

b What will be the value of her house:

i next year? **ii** in 3 years' time?

c Use trial and error to find when Zoe's house will be valued at \$300 000, to one decimal place.

Solution

Explanation

a Let V = value of Zoe's house at any time
V_0 = starting value \$145 000
n = number of years from now
$r = 9$

Define your variables.
The basic formula is $V = V_0\left(1 \pm \frac{r}{100}\right)^n$.
Use '+' since we have growth.

$$V = V_0\left(1 + \frac{9}{100}\right)^n$$

$$\therefore V = 145\,000(1.09)^n$$

$\frac{9}{100} = 0.09$

ISBN: 9781107687028

Cambridge University Press

b **i** $n = 1$: $V = 145\,000\,(1.09)^1$
$= 158\,050$
Zoe's house would be valued at \$158 050 next year.

Substitute $n = 1$ for next year.
Answer in words.

ii $n = 3$: $V = 145\,000(1.09)^3$
$= 187\,779.21$
In 3 years time Zoe's house will be valued at about \$187 779.

For three years substitute $n = 3$.

c

n	8	9	8.5	8.3	8.4
V	288 922	314 925	301 643	296 489	299 055

Zoe's house will be valued at \$300 000 in about 8.4 years.

Try a value of n in the rule. If V is too low, increase your n value; if V is too high, decrease your n value. Continue this process until you get close to 300 000.

5 The value of a house purchased for \$500 000 is expected to grow by 10% per year. Let \$$A$ be the value of the house after n years.

a Copy and complete the rule connecting A and t.
$A = 500\,000 \times \_\_\_^n$

b Use your rule to find the expected value of the house after the following number of years. Round to the nearest cent.
i 3 years **ii** 10 years **iii** 20 years

c Use trial and error to estimate when the house will be worth \$1 million. Round to one decimal place.

An increase of 10% is $1 + \frac{10}{100}$.

6 A share portfolio initially worth \$300 000 is reduced by 15% p.a. over a number of years. Let \$$A$ be the share portfolio value after n years.

a Copy and complete the rule connecting A and n.
$A = \_\_\_\_\_\_ \times 0.85^n$

b Use your rule to find the value of the shares after the following number of years. Round to the nearest cent.
i 2 years **ii** 7 years **iii** 12 years

c Use trial and error to estimate when the share portfolio will be valued at \$180 000. Round to one decimal place.

A decrease of 15% is $1 - \frac{15}{100}$.

7 A water tank containing 15 000 L has a small hole that reduces the amount of water by 6% per hour.

a Determine a rule for the volume of water (V litres) left after n hours.

b Calculate (to the nearest litre) the amount of water left in the tank after:
i 3 hours **ii** 7 hours

c How much water is left after two days? Round to two decimal places.

d Using trial and error, determine when the tank holds less than 500 L of water, to one decimal place.

ISBN: 9781107687028

Problem-solving and Reasoning

8 A certain type of bacteria grows according to the equation $N = 3000(2.6)^n$, where N is the number of cells present after n hours.

a How many bacteria are there at the start?

b Determine the number of cells (round to the whole number) present after:

i 0 hours **ii** 2 hours **iii** 4.6 hours

c If 50 000 000 bacteria are needed to make a drop of serum, determine by trial and error how long you will have to wait to make a drop (to the nearest minute).

> 'At the start' is $n = 0$ and $a^0 = 1$.

9 A car tyre has 10 mm of tread when new. It is considered unroadworthy when there is only 3 mm left. The rubber wears at 12.5% every 10 000 km.

a Write an equation relating the depth of tread (D) for every 10 000 km travelled.

b If a tyre lasts 80 000 km ($n = 8$) before becoming unroadworthy, it is considered to be a 'good' tyre. Is this a good tyre?

c Using trial and error, determine when the tyre becomes unroadworthy ($D = 3$), to the nearest 10 000 km.

> Use $D = D_0\left(1 - \frac{12.5}{100}\right)^n$
> where n is $\frac{\text{number of km}}{10000}$
> and D_0 is initial tread.
> In part **b**, is D greater than 3 when $n = 8$?

10 A cup of coffee has an initial temperature of 90°C.

a If the temperature reduces by 8% every minute, determine a rule for the temperature of the coffee (T) after n minutes.

b What is the temperature of the coffee (to one decimal place) after:

i 2 minutes? **ii** 90 seconds?

c Using trial and error, when is the coffee suitable to drink if it is best consumed at a temperature of 68.8°C? Answer to the nearest second.

> The rule is of the form:
> $T = T_0\left(1 - \frac{r}{100}\right)^n$

Time periods

11 Interest on investments can be calculated using different time periods. Consider \$1000 invested at 10% p.a. over 5 years.

- If interest is compounded annually, then $r = 10$ and $t = 5$, so $A = 1000(1.1)^5$.
- If interest is compounded monthly, then $r = \frac{10}{12}$ and $t = 5 \times 12 = 60$, so $A = 1000\left(1 + \frac{10}{1200}\right)^{60}$.

a If interest is calculated annually, find the value of the investment, to the nearest cent, after:

i 5 years **ii** 8 years **iii** 15 years

b If interest is calculated monthly, find the value of the investment, to the nearest cent, after:

i 5 years **ii** 8 years **iii** 15 years

12 You are given \$2000 and you invest it in an account that offers 7% p.a. compound interest. What will the investment be worth, to the nearest cent, after 5 years if interest is compounded:

a annually? **b** monthly? **c** weekly (assume 52 weeks in the year)?

ISBN: 9781107687028

Puzzles and games

1 In this magic square, each row and column adds to a sum that is an algebraic expression. Complete the square to find the sum.

$\frac{4x^2}{2x}$	$-y$	$x + 3y$
$x - 2y$		$2y$

2 Write $3^{n-1} \times 3^{n-1} \times 3^{n-1}$ as a single power of 3.

3 You are offered a choice of two prizes:
- one million dollars right now, or
- you can receive 1 cent on the first day of a 30-day month, double your money every day for 30 days and receive the total amount on the 30th day.

Which prize offers the most money?

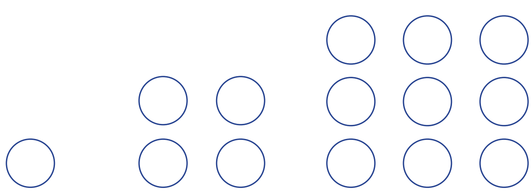
4 Simplify $\frac{25^6 \quad 5^4}{125^5}$ without the use of a calculator.

5 Write $(((2^1)^2)^3)^4$ as a single power of 2.

6 How many zeroes are there in 100^{100} in expanded form?

7 Simplify $\frac{x}{2} + \frac{3x}{5} - \frac{4x}{3} + \frac{x+1}{6}$.

8 Write a rule for the number of counters in the nth term of the pattern below. Use this to find the number of counters in the 15th term.



Term 1 Term 2 Term 3

ISBN: 9781107687028

Algebra

Simplifying expressions

Add/subtract like terms only. Like terms have the same pronumeral factors.
e.g. $3x$ and $7x$, $2xy$ and $4yx$,
not $2x$ and x^2 or $3y$ and $4xy$
For example,
$3x + 2y - x + 7y = 3x - x + 2y + 7y$
$= 2x + 9y$

Multiply/Divide

$3a \times 2b = 3 \times 2 \times a \times b$
$= 6ab$
$\frac{{}^{1}\cancel{7}x\cancel{y}^{1}}{{}_{2}\cancel{14}\cancel{y}_{1}} = \frac{x}{2}$

Algebraic expressions

$3xy + 7x - 4y + 3$

4 terms
3 is the constant term
7 is the coefficient of x
-4 is the coefficient of y

Expanding

$a(b + c) = ab + ac$
$a(b - c) = ab - ac$
e.g. $2(4x + 3) = 8x + 6$
$-3x(2x - y) = -6x^2 + 3xy$

Factorising

This is the opposite of expanding

Factorised form		Expanded form
$2(x + 4)$	=	$2x + 8$

Look for highest common factor of terms
e.g. $3ab + 6a$ HCF is $3a$
$= 3a(b + 2)$
$-2x^2 - 6x$ HCF is $-2x$
$= -2x(x + 3)$

Multiplying/Dividing algebraic fractions

Multiply: Cancel common factors in factorised form and then multiply.
e.g. $\frac{3\cancel{(x+4)}^{1}}{\cancel{2}_{1}} \times \frac{{}^{4}\cancel{8}x}{\cancel{x+4}_{1}}$
$= 12x$
Divide: Multiply by the reciprocal of the fraction following ÷ sign.
Reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

Adding/subtracting algebraic fractions

Find the lowest common denominator and combine.
e.g. $\frac{x}{2} + \frac{2x}{3}$ LCD is 6
$= \frac{3x}{6} + \frac{4x}{6}$
$= \frac{7x}{6}$

Indices

Index form a^m

a is the base
m is the index/power/exponent

Index form		Expanded form
5^4	=	$5 \times 5 \times 5 \times 5$

e.g. $3 \times a \times b \times a \times a \times b = 3a^3b^2$

Exponential growth and decay

For a repeated percentage increase or decrease, the rule can be
$A = A_0\left(1 \pm \frac{r}{100}\right)^n$, where
A_0 is the initial amount
r is % rate
n is the amount of time.
Use '+' for growth
and '–' for decay.

Scientific notation

Used for very large and small numbers in the form $a \times 10^m$, where $1 \le a < 10$
$3\,870\,000 = 3.87 \times 10^6$
$0.000\,21 = 2.1 \times 10^{-4}$

Negative indices

$a^{-m} = \frac{1}{a^m}$
e.g. $x^{-3} = \frac{1}{x^3}$
$\frac{1}{a^{-m}} = a^m$
e.g. $\frac{4}{2^{-3}} = 4 \times 2^3 = 32$

Index laws

Law 1 : $a^m \times a^n = a^{m+n}$
Law 2 : $a^m \div a^n = a^{m-n}$
Law 3 : $(a^m)^n = a^{m \times n}$
Law 4 : $(ab)^m = a^m \times b^m$
Law 5 : $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
Zero power: $a^0 = 1$ if $a \ne 0$

ISBN: 9781107687028

Multiple-choice questions

1 The coefficient of x in $3xy - 4x + 7$ is:
A 4 **B** 7 **C** -4 **D** 3 **E** -1

2 The simplified form of $7ab + 2b - 5ab + b$ is:
A $2ab + 2b^2$ **B** $2ab + 3b$ **C** $5ab$ **D** $2ab + b$ **E** $12ab + 3b$

3 The expanded form of $2x(3x - 5)$ is:
A $6x^2 - 5$ **B** $6x - 10$ **C** $6x^2 - 10x$ **D** $5x^2 - 10x$ **E** $-4x$

4 The fully factorised form of $8xy - 24y$ is:
A $4y(2x - 6y)$ **B** $8(xy - 3y)$ **C** $8y(x - 24)$ **D** $8y(x - 3)$ **E** $8x(y - 24)$

5 The simplified form of $\frac{2(x+1)}{5x} \quad \frac{15}{x+1}$ is:
A $\frac{6}{x+1}$ **B** $\frac{6}{x}$ **C** $\frac{3(x+1)}{x}$ **D** $6x$ **E** $\frac{3x}{x+1}$

6 The sum of the algebraic fractions $\frac{3x}{8} + \frac{x}{12}$ is:
A $\frac{x}{5}$ **B** $\frac{x}{6}$ **C** $\frac{x}{24}$ **D** $\frac{11x}{24}$ **E** $\frac{9x}{24}$

7 $3x^3y \times 2x^5y^3$ is equal to:
A $5x^{15}y^3$ **B** $6x^{15}y^3$ **C** $6x^8y^4$ **D** $5x^8y^4$ **E** $6x^8y^3$

8 $12a^4 \div (4a^7)$ simplifies to:
A $3a^3$ **B** $8a^3$ **C** $3a^{11}$ **D** $\frac{8}{a^3}$ **E** $\frac{3}{a^3}$

9 $(2x^4)^3$ can be written as:
A $2x^{12}$ **B** $2x^7$ **C** $6x^{12}$ **D** $8x^{12}$ **E** $8x^7$

10 $5x^0 - (2x)^0$ is equal to:
A 4 **B** 0 **C** 3 **D** 2 **E** -1

11 417 000 converted to scientific notation is:
A 4.17×10^{-5} **B** 417×10^3 **C** 4.17×10^5 **D** 0.417×10^6 **E** 41.7×10^{-2}

12 A rule for the amount of money, A, in an account after n years, if \$1200 is invested at 4% per year, is:
A $A = 1200(4)^n$ **B** $A = 1200(1.4)^n$ **C** $A = 1200(0.96)^n$
D $A = 1200(1.04)^n$ **E** $A = 1200(0.04)^n$

ISBN: 9781107687028 Cambridge University Press

Short-answer questions

1 Consider the expression $3xy - 3b + 4x^2 + 5$.

- **a** How many terms are in the expression?
- **b** What is the constant term?
- **c** State the coefficient of:
  - **i** x^2 **ii** b

2 Write an algebraic expression for the following.

- **a** 3 more than y
- **b** 5 less than the product of x and y
- **c** the sum of a and b is divided by 4

3 Evaluate the following if $x = 3$, $y = 5$ and $z = -2$.

a $3x + y$ **b** xyz **c** $y^2 - 5z$

4 Simplify the following expressions.

a $4x - 5 + 3x$ **b** $4a - 5b + 9a + 3b$ **c** $3xy + xy^2 - 2xy - 4y^2x$

d $3m \times 4n$ **e** $-2xy \times 7x$ **f** $\frac{8ab}{12a}$

5 Expand the following and collect like terms where necessary.

a $5(2x + 4)$ **b** $-2(3x - 4y)$ **c** $3x(2x + 5y)$

d $3 + 4(a + 3)$ **e** $3(y + 3) + 2(y + 2)$ **f** $5(2t + 3) - 2(t + 2)$

6 Factorise the following expressions.

a $16x - 40$ **b** $10x^2y + 35xy^2$

c $4x^2 - 10x$ **d** $-2xy - 18x$ (include the common negative)

7 Simplify the following algebraic fractions involving addition and subtraction.

a $\frac{2x}{3} + \frac{4x}{15}$ **b** $\frac{3}{7} - \frac{a}{2}$ **c** $\frac{x+4}{4} + \frac{x-3}{5}$

8 Simplify these algebraic fractions by first cancelling common factors in factorised form.

a $\frac{5x}{12} \quad \frac{9}{10x}$ **b** $\frac{x+2}{4} \quad \frac{16x}{x+2}$ **c** $\frac{12x-4}{4}$ **d** $\frac{x-3}{4} \div \frac{3(x-3)}{8}$

9 Simplify the following using index laws 1 and 2.

a $3x^5 \times 4x^2$ **b** $4xy^6 \times 2x^3y^{-2}$ **c** $\frac{b^7}{b^3}$ **d** $\frac{4a^3b^5}{6ab^2}$

10 Express the following using positive indices.

a $4x^{-3}$ **b** $3r^4s^{-2}$ **c** $\frac{2x^{-3}y^4}{3}$ **d** $\frac{4}{m^{-5}}$

11 Simplify the following using the third, fourth and fifth index laws.

a $(b^2)^4$ **b** $(2m^2)^3$ **c** $\left(\frac{x}{7}\right)^2$ **d** $\left(\frac{4y^2}{z^4}\right)^3$

ISBN: 9781107687028

12 Simplify the following using the zero power.

a 7^0 **b** $4x^0$ **c** $5a^0 + (2y)^0$ **d** $(x^2 + 4y)^0$

13 Simplify the following using index laws. Express all answers with positive indices.

a $\dfrac{3x^2y^4 \quad 5xy^7}{12x^3y^5}$ **b** $\dfrac{(5x^2y)^2 \quad 4xy^2}{8(xy)^2}$

c $\dfrac{2x^3y^2 \quad 5x^2y^5}{x^7y^4}$ **d** $\dfrac{4x^2y^5}{8(x^2)^{-3}y^8}$

14 Write these numbers as a basic numeral.

a 4.25×10^3 **b** 3.7×10^7 **c** 2.1×10^{-2} **d** 7.25×10^{-5}

15 Convert these numbers to scientific notation using three significant figures.

a 123 574 **b** 39 452 178 **c** 0.000 009 0241 **d** 0.000 459 86

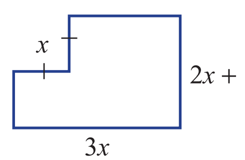
16 Form an exponential equation for the following.

a The population of a colony of kangaroos, which starts at 20 and is increasing at a rate of 10%

b The amount of petrol in a petrol tank fuelling a generator if it starts with 100 000 litres and uses 15% of its fuel every hour

Extended-response questions

1 A room in a house has the shape and dimensions, in metres, shown.



a Find the perimeter of the room in factorised form.

b If $x = 3$, what is the room's perimeter?

The floor of the room is to be recarpeted.

c Give the area of the floor in terms of x and in expanded form.

d If the carpet costs \$20 per square metre and $x = 3$, what is the cost of laying the carpet?

2 During the growing season, a certain type of water lily spreads by 9% per week. The water lily covers an area of 2 m^2 at the start of the growing season.

a Write a rule for the area, A m^2, covered by the water lily after t weeks.

b Calculate the area covered, correct to four decimal places, after:

i 2 weeks **ii** 5 weeks

c Use trial and error to determine, to one decimal place, when there will be a coverage of 50 m^2.

ISBN: 9781107687028 Cambridge University Press

Australian Curriculum
Statistics and Probability
Chance

chapter 4

Probability

What you will learn

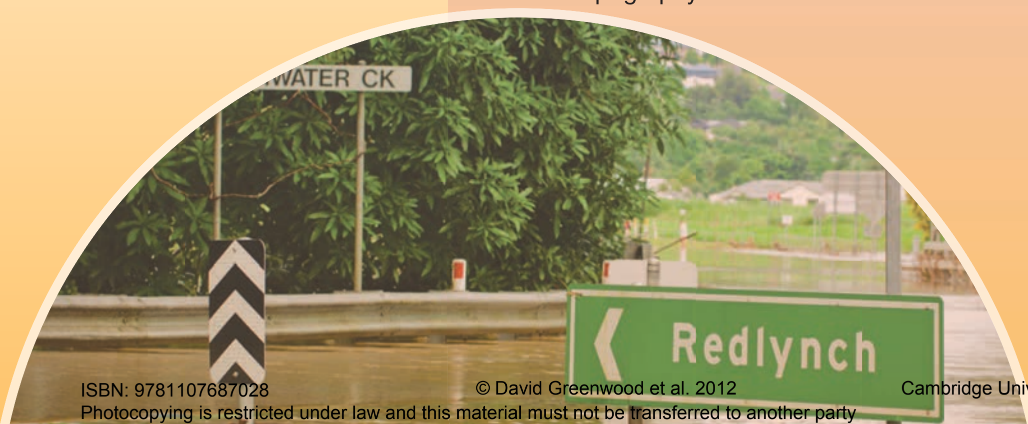
4.1 Review of probability
4.2 Venn diagrams
4.3 Two-way tables
4.4 Conditional probability
4.5 Multiple events using tables
4.6 Using tree diagrams
4.7 Independent events

Disaster probability

We have all heard about or experienced a natural disaster of some kind in Australia. Floods, bushfires or cyclones are very real possibilities in many parts of the country.

Because of this, most people take out insurance to cover them for any potential losses caused by such events. Insurance companies try to predict the likelihood of disasters and set their premiums to suit. This is called risk analysis, and involves calculating the experimental probability of certain disasters, depending on a number of factors. Some factors include:

- location
- historical records of previous disasters
- accessibility of local facilities, such as hospitals and power supply
- area topography.



ISBN: 9781107687028

Pre-test

1 A letter is selected from the word PROBABILITY.

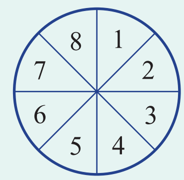
a How many letters are there in total?

b Find the chance (probability) of selecting:

i the letter R **ii** the letter B **iii** a vowel

iv not a vowel **v** a T or an I **vi** neither a B nor a P

2 A spinning wheel has 8 equal sectors numbered 1 to 8. On one spin of the wheel, find the following probabilities.



a Pr (5) **b** Pr (even)

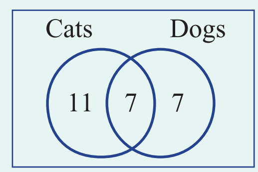
c Pr (not even) **d** Pr (multiple of 3)

e Pr (factor of 12) **f** Pr (odd or a factor of 12)

g Pr (both odd and a factor of 12)

3 Arrange from lowest to highest: $\frac{1}{2}$, 0.4, 1 in 5, 39%, $\frac{3}{4}$, 1, 0, $\frac{9}{10}$, 0.62, 71%

4 This Venn diagram shows the number of people in a group of 25 who own cats and/or dogs.



a State the number of people who own:

i a dog **ii** a cat or a dog (including both)

iii only a cat

b If a person is selected at random from this group, find the probability that they will own:

i a cat **ii** a cat and a dog **iii** only a dog

5 Drew shoots from the free-throw line on a basketball court. After 80 shots he counts 35 successful throws.

a Estimate the probability that his next throw will be successful.

b Estimate the probability that his next throw will not be successful.

6 Two 4-sided dice are tossed and the sum of the two numbers obtained is noted.

a Copy and complete this grid.

b What is the total number of outcomes?

c Find the probability that the total sum is:

i 2 **ii** 4

iii less than 5 **iv** less than or equal to 5

v at most 6 **vi** no more than 3

Toss 2 \ Toss 1	1	2	3	4
1				
2				
3				
4				

7 Two coins are tossed.

a Copy and complete this tree diagram.

b State the total number of outcomes.

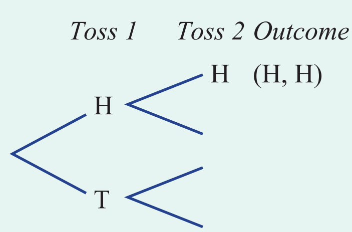
c Find the probability of obtaining:

i 2 heads **ii** no heads

iii 1 tail **iv** at least 1 tail

v 1 of each, a head and a tail

vi at most 2 heads



ISBN: 9781107687028

Cambridge University Press

4.1 Review of probability

Probability is an area of mathematics concerned with the likelihood of particular random events. In some situations, such as tossing a dice, we can determine theoretical probabilities because we know the total number of outcomes and the number of favourable outcomes. In other cases we can use statistics and experimental results to describe the chance that an event will occur. The chance that a particular soccer team will win its next match, for example, could be estimated using various results from previous games.

A soccer team could win, lose or draw the next match it plays, but these three outcomes do not necessarily have the same probability.

Let's start: Name the event

For each number below, describe an event that has that exact or approximate probability. If you think it is exact then give a reason.

$\frac{1}{2}$ 25% 0.2 0.00001 $\frac{99}{100}$

Key ideas

- Definitions
  - A **trial** is a single experiment, such as a single toss of a dice.
  - The **sample space** is the list of outcomes from an experiment.
  - An **outcome** is a possible result of an experiment.
  - An **event** is the list of favourable outcomes.
  - Equally likely outcomes are outcomes that have the same chance of occurring.
- In the study of probability, a numerical value based on a scale from 0 to 1 is used to describe levels of **chance**.

0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
impossible	unlikely				even chance			likely		certain

- The probability of an event in which outcomes are equally likely is calculated as follows:

$$\Pr(\text{Event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Trial One run of an experiment

Sample space All the possible outcomes of an event

Outcome One of the possibilities from a chance experiment

Event A situation involving chance or probability trials

Chance The likelihood of an event happening

ISBN: 9781107687028

- Experimental probability is calculated in the same way as theoretical probability but uses the results of an experiment:

$$\Pr(\text{Event}) = \frac{\text{number of favourable outcomes}}{\text{total number of trials}}$$

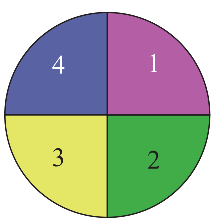
  – The long run proportion is the experimental probability for a sufficiently large number of trials.

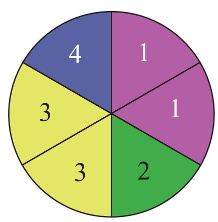
Exercise 4A

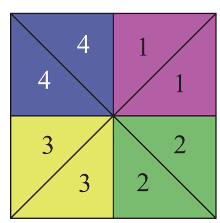
Understanding

1 Order these events (**A** to **D**) from least likely to most likely.
A The chance that it will rain every day for the next 10 days.
B The chance that a member of class is ill on the next school day.
C The chance that school is cancelled next year.
D The chance that the sun comes up tomorrow.

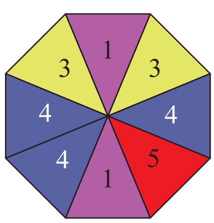
2 For the following spinners, find the probability that the outcome will be a 4.

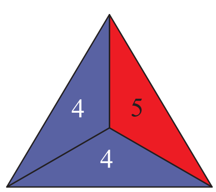
a


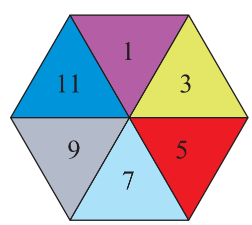
b


c


$\Pr(4) = \dfrac{\text{number of 4s}}{\text{total number of sections}}$

d


e


f


3 A coin is flipped once.
a How many different outcomes are possible from a single flip of the coin?
b What are the possible outcomes from a single flip of the coin (i.e. list the sample space)?
c Are the possible outcomes equally likely?
d What is the probability of obtaining a tail?
e What is the probability of not obtaining a tail?
f What is the probability of obtaining a tail or a head?

ISBN: 9781107687028 Cambridge University Press

Fluency

Example 1 Calculating simple theoretical probabilities

A letter is chosen from the word TELEVISION.

a How many letters are there in the word TELEVISION?

b Find the probability that the letter is:

i a V **ii** an E

iii not an E **iv** an E or a V

Solution	Explanation
a 10	The sample space includes 10 letters.
b i $\Pr(V) = \frac{1}{10}$ $(= 0.1)$	$\Pr(V) = \frac{\text{number of Vs}}{\text{total number of letters}}$
ii $\Pr(E) = \frac{2}{10}$	There are 2 Es in the word TELEVISION.
$= \frac{1}{5}$ $(= 0.2)$	Simplify the fraction.
iii $\Pr(\text{not an E}) = \frac{8}{10}$ $= \frac{4}{5}$ $(= 0.8)$	If there are 2 Es in the word TELEVISION with 10 letters, then there must be 8 letters that are not E.
iv $\Pr(\text{an E or a V}) = \frac{3}{10}$ $(= 0.3)$	The number of letters that are either E or V is 3.

4 A letter is chosen from the word TEACHER.

a How many letters are there in the word TEACHER?

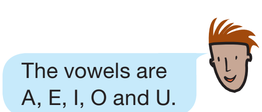
b Find the probability that the letter is:

i an R **ii** an E **iii** not an E **iv** an R or an E

5 A letter is chosen from the word EXPERIMENT. Find the probability that the letter is:

a an E **b** a vowel

c not a vowel **d** an X or a vowel



ISBN: 9781107687028

Cambridge University Press

Example 2 Calculating simple experimental probabilities

An experiment involves tossing three coins and counting the number of heads. Here are the results after running the experiment 100 times.

Number of heads	0	1	2	3
Frequency	11	40	36	13

a How many times did 2 heads occur?
b How many times did fewer than 2 heads occur?
c Find the experimental probability of obtaining:
  i 0 heads
  ii 2 heads
  iii fewer than 2 heads
  iv at least one head

Solution

a 36

b $11 + 40 = 51$

c **i** $\Pr(0 \text{ heads}) = \frac{11}{100} = 0.11$

ii $\Pr(2 \text{ heads}) = \frac{36}{100} = 0.36$

iii $\Pr(\text{less than } 2 \text{ heads}) = \frac{11+40}{100} = \frac{51}{100} = 0.51$

iv $\Pr(\text{at least one head}) = \frac{40+36+13}{100} = \frac{89}{100} = 0.89$

Explanation

a From the table you can see that 2 heads has a frequency of 36.

b Fewer than 2 means obtaining 0 heads or 1 head.

c i $\Pr(0 \text{ heads}) = \frac{\text{number of times 0 heads is observed}}{\text{total number of trials}}$

ii $\Pr(2 \text{ heads}) = \frac{\text{number of times 2 heads is observed}}{\text{total number of trials}}$

iii Fewer than 2 heads means to observe 0 or 1 head.

iv At least 1 head means that 1, 2 or 3 heads can be observed.

ISBN: 9781107687028

6 An experiment involves tossing three coins and counting the number of heads. Here are the results after running the experiment 100 times.

The total number of outcomes is 100.

Number of heads	0	1	2	3
Frequency	9	38	43	10

a How many times did 2 heads occur?
b How many times did fewer than 2 heads occur?
c Find the experimental probability of obtaining:
  i 0 heads
  ii 2 heads
  iii fewer than 2 heads
  iv at least one head

7 An experiment involves tossing two dice and counting the number of sixes. Here are the results after running the experiment 100 times.

Number of sixes	0	1	2
Frequency	62	35	3

Find the experimental probability of obtaining:
a 0 sixes **b** 2 sixes **c** fewer than 2 sixes **d** at least one six

Problem-solving and Reasoning

8 A 10-sided die numbered 1 to 10 is tossed once. Find these probabilities.

Prime numbers less than 10 are 2, 3, 5 and 7.

a Pr (8) **b** Pr (odd)
c Pr (even) **d** Pr (less than 6)
e Pr (prime) **f** Pr (3 or 8)
g Pr (8, 9 or 10)

1 2 3
4 5
6 7 8
9 10

9 Thomas is a prizewinner in a competition and will be randomly awarded a single prize chosen from a collection of 50 prizes. The type and number of prizes to be handed out are listed below.

Remember that the total number of prizes is 50.

Prize	Car	Holiday	iPad	Blu-ray player
Number	1	4	15	30

Find the probability that Thomas will be awarded the following.
a a car **b** an iPad
c a prize that is not a car

ISBN: 9781107687028

10 Many of the 50 cars inspected at an assembly plant contained faults. The results of the inspection were as follows.

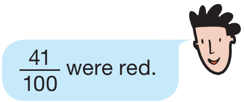
Number of faults	0	1	2	3	4
Number of cars	30	12	4	3	1

Find the experimental probability that a car selected from the assembly plant will have:

- **a** 1 fault
- **b** 4 faults
- **c** fewer than 2 faults
- **d** 1 or more faults
- **e** 3 or 4 faults
- **f** at least 2 faults

11 A bag contains red and yellow counters. A counter is drawn from the bag and then replaced. This happens 100 times and 41 of the counters drawn were red.

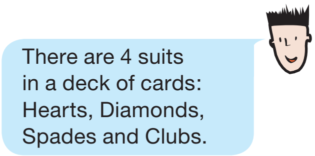
- **a** How many counters drawn were yellow?
- **b** If there were 10 counters in the bag how many do you expect were red? Give a reason.
- **c** If there were 20 counters in the bag how many do you expect were red? Give a reason.



Cards probability

12 A card is chosen from a standard pack of 52 playing cards that includes 4 aces, 4 kings, 4 queens and 4 jacks. Find the following probabilities.

- **a** Pr(heart)
- **b** Pr(king)
- **c** Pr(king of hearts)
- **d** Pr(heart or club)
- **e** Pr(king or jack)
- **f** Pr(heart or king)
- **g** Pr(not a king)
- **h** Pr(neither a heart nor a king)



ISBN: 9781107687028

Cambridge University Press

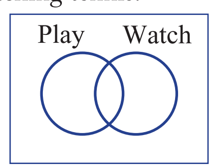
4.2 Venn diagrams

Sometimes we need to work with situations where there are overlapping events. A TV station, for example, might be collecting statistics regarding whether or not a person watches cricket and/or tennis or neither over a certain period of time. The estimated probability that a person will watch cricket *or* tennis will therefore depend on how many people responded *yes* to watching both cricket *and* tennis. Venn diagrams are a useful tool when dealing with such events.

Let's start: How many like both?

Of 20 students in a class, 12 people like to play tennis and 15 people like to watch tennis. Two people like neither playing nor watching tennis. Some like both playing and watching tennis.

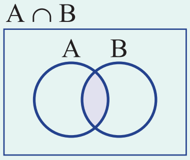
- Is it possible to represent this information in a Venn diagram?
- How many students like to play and watch tennis?
- How many students only like to watch tennis?
- From the group of 20 students, what would be the probability of selecting a person that likes watching tennis only?



Key ideas

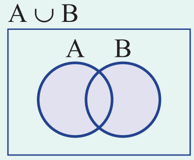
- A **Venn diagram** illustrates how all elements in the sample space are distributed among the events.
  - All elements that belong to both A *and* B make up the **intersection**: $A \cap B$.

$A \cap B$

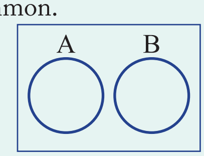


  - All elements that belong to either events A *or* B make up the **union**: $A \cup B$.

$A \cup B$



  - Two sets A and B are **mutually exclusive** if they have no elements in common.



  - For an event A, the complement of A is A′ (or 'not A'). $\Pr(A') = 1 - \Pr(A)$
  - A only is defined as all the elements in A but not in any other set.

Venn diagram A diagram using circles to show the relationships between two or more sets of data

Intersection The elements that are common to two or more sets of data

Union The combination of all elements from two or more sets of data

Mutually exclusive Two events that cannot both occur at the same time

ISBN: 9781107687028

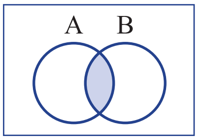
Exercise 4B

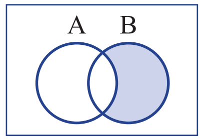
Understanding

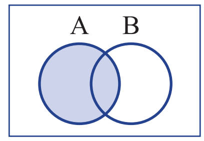
1 Match the symbols (**a**–**e**) to the pictures (**A**–**E**).

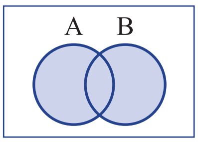
a $A \cup B$ **b** A **c** A'

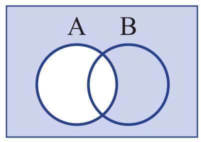
d $A \cap B$ **e** B only

A


B


C


D


E


2 Decide whether the events A and B are mutually exclusive.

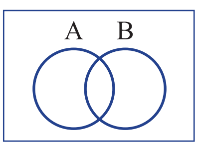
a $A = \{1, 3, 5, 7\}$
$B = \{5, 8, 11, 14\}$

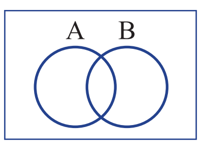
b $A = \{-3, -2, \ldots, 4\}$
$B = \{-11, -10, \ldots, -4\}$

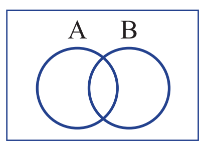
c A = {prime numbers}
B = {even numbers}

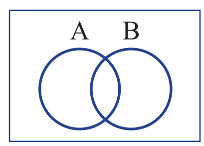
Mutually exclusive events have nothing in common.

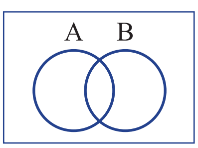
3 Copy these Venn diagrams and shade the region described by each of the following.

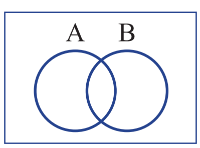
a A


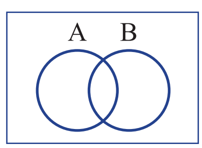
b B


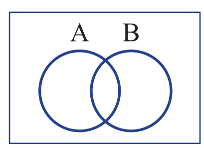
c $A \cap B$


d $A \cup B$


e A only


f B only


g A'


h neither A nor B


Fluency

Example 3 Listing sets

Consider the given events A and B that involve numbers taken from the first 10 positive integers.

$A = \{1, 2, 3, 4, 5, 6\}$ $B = \{1, 3, 7, 8\}$

a Represent the two events A and B in a Venn diagram.

b List the following sets:

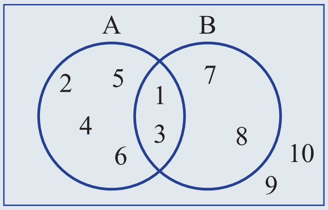
i $A \cap B$ **ii** $A \cup B$

c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.

i A **ii** $A \cap B$ **iii** $A \cup B$

d Are the events A and B mutually excusive? Why/why not?

ISBN: 9781107687028 Cambridge University Press

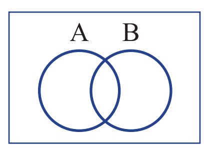
Solution	**Explanation**
a 	The elements 1 and 3 are common to both sets A and B. The elements 9 and 10 belong to neither set A nor set B.
b **i** $A \cap B = \{1, 3\}$	$A \cap B$ is the intersection of sets A and B.
ii $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$	$A \cup B$ contains elements in either A or B.
c **i** $\Pr(A) = \frac{6}{10} = \frac{3}{5}$	There are 6 numbers in A.
ii $\Pr(A \cap B) = \frac{2}{10} = \frac{1}{5}$	$A \cap B$ contains 2 numbers.
iii $\Pr(A \cup B) = \frac{8}{10} = \frac{4}{5}$	$A \cup B$ contains 8 numbers.
d The sets A and B are not mutually exclusive since there are numbers inside $A \cap B$.	The set $A \cap B$ contains at least one number.

4 Consider the given events A and B, which involve numbers taken from the first 10 positive integers.

$A = \{1, 2, 4, 5, 7, 8, 10\}$ $\qquad$ $B = \{2, 3, 5, 6, 8, 9\}$

$A \cap B$ means A *and* B
$A \cup B$ means A *or* B

a Represent events A and B in a Venn diagram.



b List the following sets.
 i $A \cap B$ **ii** $A \cup B$

c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.
 i A **ii** $A \cap B$ **iii** $A \cup B$

d Are the events A and B mutually exclusive? Why/why not?

5 The elements of the events A and B described below are numbers taken from the first 10 prime numbers.

$A = \{2, 5, 7, 11, 13\}$ $\qquad$ $B = \{2, 3, 13, 17, 19, 23, 29\}$

$A \cap B$ means A *and* B
$A \cup B$ means A *or* B

a Represent events A and B in a Venn diagram.

b List the elements belonging to the following:
 i A and B **ii** A or B

c If a number from the first 10 prime numbers is selected, find the probability that these events occur.
 i A **ii** B **iii** $A \cap B$ **iv** $A \cup B$

ISBN: 9781107687028

Cambridge University Press

Example 4 Using Venn diagrams

From a class of 30 students, 12 enjoy cricket (C), 14 enjoy netball (N) and 6 enjoy both cricket and netball.

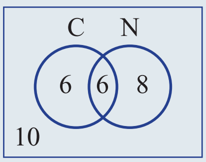
a Illustrate this information in a Venn diagram.

b State the number of students who enjoy:

  i netball only **ii** neither cricket nor netball

c Find the probability that a person chosen at random will enjoy:

  i netball **ii** netball only **iii** both cricket and netball

Solution	Explanation
a 	First place the 6 in the intersection (6 enjoy cricket and netball) then determine the other values according to the given information. The total must be 30, with 12 in the cricket circle and 14 in netball.
b **i** 8	Includes students in N but not in C.
ii 10	These are the students outside both C and N.
c **i** $\Pr(N) = \frac{14}{30} = \frac{7}{15}$	14 of the 30 students enjoy netball.
ii $\Pr(N \text{ only}) = \frac{8}{30} = \frac{4}{15}$	8 of the 30 students enjoy netball but not cricket.
iii $\Pr(C \cap N) = \frac{6}{30} = \frac{1}{5}$	6 students like both cricket and netball.

6 From a group of 50 adults, 35 enjoy reading fiction (F), 20 enjoy reading non-fiction (N) and 10 enjoy reading both fiction and non-fiction.

First enter the '10' in the intersection, then balance all the other regions.

a Illustrate the information in a Venn diagram.

b State the number of people who enjoy:

  i fiction only **ii** neither fiction nor non-fiction

c Find the probability that a person chosen at random will enjoy reading:

  i non-fiction **ii** non-fiction only **iii** both fiction and non-fiction

7 At a show, 45 children have the choice of riding on the Ferris wheel (F) and/or the Big Dipper (B). Thirty-five of the children wish to ride on the Ferris wheel, 15 children want to ride on the Big Dipper and 10 children want to ride on both.

a Illustrate the information in a Venn diagram.

b State the number of children who want to:

  i ride on the Ferris wheel only

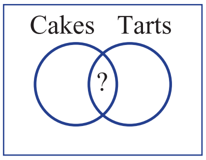
  ii ride on neither the Ferris wheel nor the Big Dipper

ISBN: 9781107687028 Cambridge University Press

c For a child chosen at random from the group, find the probability that they will want to ride on:
- **i** the Ferris wheel
- **ii** both the Ferris wheel and the Big Dipper
- **iii** the Ferris wheel or the Big Dipper
- **iv** not the Ferris wheel
- **v** neither the Ferris wheel nor the Big Dipper

Problem-solving and Reasoning

8 In a group of 12 chefs, all enjoy baking cakes and/or tarts. In fact, 7 enjoy baking cakes and 8 enjoy baking tarts. Find out how many chefs enjoy baking both cakes and tarts.



9 In a group of 32 car enthusiasts, all collect either vintage cars or modern sports cars. 18 collect vintage cars and 19 collect modern sports cars. How many collect both vintage cars and modern sports cars?

10 Mario and Elisa are choosing a colour to paint the interior walls of their house. They have six colours to choose from:

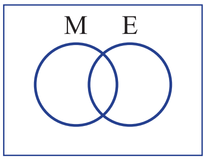
white (w), cream (c), navy (n), sky blue (s), maroon (m) and violet (v).

Mario would be happy with white or cream and Elisa would be happy with cream, navy or sky blue, but they can't decide, so a colour is chosen at random for them.

Let M be the event that Mario will be happy with the colour and E be the
event that Elisa will be happy with the colour.

a Represent the events M and E in a Venn diagram.

b Find the probability that the following events occur.
- **i** Mario will be happy with the colour choice, i.e. find Pr(M).
- **ii** Mario will not be happy with the colour choice.
- **iii** Both Mario and Elisa will be happy with the colour choice.
- **iv** Mario or Elisa will be happy with the colour choice.
- **v** Neither Mario nor Elisa will be happy with the colour choice.



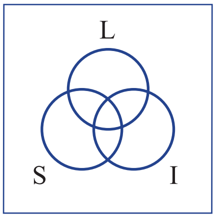
ISBN: 9781107687028

4B

Courier companies

11 Of 15 chosen courier companies, 9 offer a local service (L), 7 offer an interstate service (S) and 6 offer an international service (I). Two companies offer all three services, 3 offer both local and interstate services, 5 offer only local services and 1 offers only an international service.

a Draw a Venn diagram displaying the given information.



b Find the number of courier companies that offer neither local, interstate nor international services.

c If a courier is chosen at random from the 15 initially examined, find the following probabilities.

i Pr(L) **ii** Pr(L only)

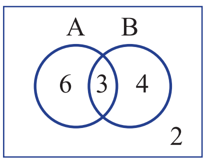
iii Pr(L or S) **iv** Pr(L and S only)

4.3 Two-way tables

Like a Venn diagram, two-way tables are useful tools for the organisation of overlapping events. The totals at the end of each column and row help to find the unknown numbers required to solve various problems.

Let's start: Comparing Venn diagrams with two-way tables

Here is a Venn diagram and an incomplete two-way table.



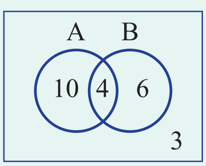
	A	A′	
B		4	
B′			8
	9		15

- First, can you complete the two-way table?
- Describe what each box in the two-way table means.
- Was it possible to find all the missing numbers in the two-way table without referring to the Venn diagram?

Key ideas

- Two-way tables use rows and columns to describe the number of elements in different regions of overlapping events.

Venn diagram



Two-way table

	A	A′		
B	4	6	10	Total for B
B′	10	3	13	Total for not B
	14	9	23	Total
	Total for A	Total for not A		

A ∩ B (4); B only (6); A only (10); Neither A nor B (3)

ISBN: 9781107687028 Cambridge University Press

Exercise 4C

Understanding

1 Match the shaded two-way tables (**A**–**D**) with each description (**a**–**d**).

a $A \cap B$ **b** B only **c** A **d** $A \cup B$

A

	A	A′	
B			
B′			

B

	A	A′	
B			
B′			

C

	A	A′	
B			
B′			

D

	A	A′	
B			
B′			

2 Look at this two-way table.

a State the number of elements in these events.

i A and B **ii** A only
iii B only **iv** neither A nor B
iv A **v** B
vi A′ **vii** B′

	A	A′	
B	4	3	7
B′	6	1	7
	10	4	14

b $A \cup B$ (A or B) includes $A \cap B$, A only and B only. Find the total number of elements in $A \cup B$.

Fluency

Example 5 Using two-way tables

The Venn diagram shows the distribution of elements in two sets A and B.

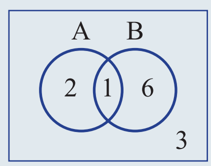
a Transfer the information in the Venn diagram into a two-way table.

b Find the number of elements for these regions.

i A and B **ii** B only **iii** A only
iv neither A nor B **v** A **vi** not B
vii A or B

c Find:

i $\Pr(A \cap B)$ **ii** $\Pr(A')$ **iii** A only



Solution

a

	A	A′	
B	1	6	7
B′	2	3	5
	3	9	12

Explanation

	A	A′	
B	$A \cap B$	B only	Total the row
B′	A only	Neither A nor B	Total the row
	Total the column	Total the column	Overall total

ISBN: 9781107687028

Cambridge University Press

b **i** 1 — In both A and B

ii 6 — In B but not A

iii 2 — In A but not B

iv 3 — In neither A nor B

v 3 — Total of A

vi 5 — Total not in B

vii $2 + 1 + 6 = 9$ — In A only or B only or both (3 regions)

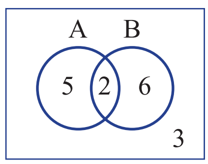
c **i** $\Pr(A \cap B) = \frac{1}{12}$

ii $\Pr(A') = \frac{9}{12} = \frac{3}{4}$

iii $\Pr(A \text{ only}) = \frac{2}{12} = \frac{1}{6}$

When calculating probabilities, you will need to divide the number of elements in each set by the number of elements in the sample space, which is 12.

3 The Venn diagram shows the distribution of elements in two sets A and B.



a Transfer the information in the Venn diagram into a two-way table.

b Find the number of elements in these regions.

i A and B **ii** B only **iii** A only **iv** neither A nor B

v A **vi** not B **vii** A or B

c Find:

i $\Pr(A \cap B)$ **ii** $\Pr(A')$ **iii** $\Pr(A \text{ only})$

4 From a total of 10 people, 5 like apples (A), 6 like bananas (B) and 4 like both apples and bananas.

a Draw a Venn diagram for the 10 people.

b Draw a two-way table.

c Find the number of people who like:

i only bananas **ii** apples

iii apples and bananas **iv** apples or bananas

d Find:

i $\Pr(B)$ **ii** $\Pr(A \cap B)$ **iii** $\Pr(A \text{ only})$

iv $\Pr(B')$ **v** $\Pr(A \cup B)$

Once you have your Venn diagram, you can transfer to the two-way table.

5 Of 12 people interviewed at a train station, 7 like staying in hotels, 8 like staying in apartments and 4 like staying in hotels and apartments.

a Draw a two-way table for the 12 people.

b Find the number of people who like:

i only hotels **ii** neither hotels nor apartments

c Find the probability that one of the people likes:

i hotels or apartments **ii** only apartments

ISBN: 9781107687028

Cambridge University Press

4C

Problem-solving and Reasoning

6 Complete the following two-way tables.

a

	A	A′	
B		3	6
B′			
		4	11

b

	A	A′	
B	2	7	
B′			3
	4		

All the rows and columns should add up correctly.

7 In a class of 24 students, 13 like Mathematics, 9 like English and 3 like both.
- a Find the probability that a randomly selected person likes both Mathematics and English.
- b Find the probability that a randomly selected person likes neither Mathematics nor English.

8 Two sets A and B are mutually exclusive.
- a Find $\Pr(A \cap B)$.
- b Now complete this two-way table.

	A	A′	
B		6	
B′			12
	10		18

9 Of 32 cars at a show, 18 cars have four-wheel drive, 21 are sports cars and 27 have four-wheel drive or are sports cars.
- a Find the probability that a randomly selected car is both four-wheel drive and a sports car.
- b Find the probability that a randomly selected car is neither four-wheel drive nor a sports car.

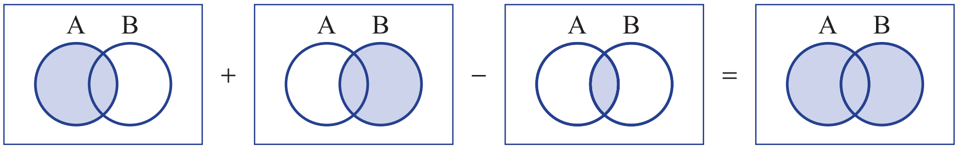
10 A card is selected from a pack of 52 playing cards. Find the probability that the card is:
- a a heart or a king
- b a club or a queen
- c a black card or an ace
- d a red card or a jack

Make sure you don't count some cards twice; e.g. the king of hearts in part **a**.

The addition rule

For some of the above problems you will have noticed the following, which is called the addition rule.

$$A \quad + \quad B \quad - \quad A \cap B \quad = \quad A \cup B$$



11 Use the addition rule to find $A \cup B$ in these problems.
- a Of 20 people at a sports day, 12 people like archery (A), 14 like basketball (B) and 8 like both archery and basketball ($A \cap B$). How many like archery or basketball?
- b Of 100 households, 84 have wide screen TVs, 32 have tube TVs and 41 have both. How many have wide screen or tube TVs?

ISBN: 9781107687028 Cambridge University Press

4.4 Conditional probability

The mathematics associated with the probability that an event occurs, given that another event has already occurred, is called conditional probability.

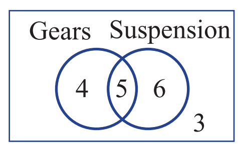
Consider, for example, a group of primary school students who own bicycles. Some of the bicycles have gears, some have suspension and some have both gears and suspension. Consider these two questions.

- What is the probability that a randomly selected bicycle has gears?
- What is the probability that a randomly selected bicycle has gears, given that it has suspension?

The second question is conditional, in that we already know that the bicycle has suspension.

Let's start: Gears and suspension

Suppose that, in a group of 18 bicycles, 9 have gears, 11 have suspension and 5 have both gears and suspension. Discuss the solution to the following question by considering the points below.

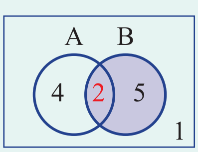


What is the probability that a randomly selected bicycle will have gears, given that it has suspension?

- First look at the information in a Venn diagram.
- How many of the bicycles that have suspension also have gears?
- Out of the 11 that have suspension, what is the probability that a bike will have gears?
- What would be the answer to the question in reverse; i.e. what is the probability that a bicycle will have suspension, given that it has gears?

Key ideas

- The probability of event A occurring given that event B has occurred is denoted by $\Pr(A|B)$, which reads 'the probability of A given B'.
- $\Pr(A \text{ given } B) = \dfrac{\text{number of elements in } A \cap B}{\text{number of elements in } B}$



$\Pr(A|B) = \frac{2}{7}$

	A	A′	
B	2	5	7
B′	4	1	5
	6	6	12

$\Pr(A|B) = \frac{2}{7}$

- $\Pr(B \text{ given } A) = \dfrac{\text{number of elements in } A \cap B}{\text{number of elements in } A}$

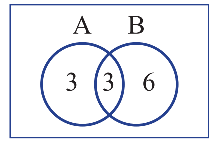
For the above diagrams, $\Pr(B|A) = \frac{2}{6} = \frac{1}{3}$.

ISBN: 9781107687028

Exercise 4D

Understanding

1 Consider this Venn diagram.



a What fraction of the elements in A are also in B? (This finds $\Pr(B \mid A)$.)

b What fraction of the elements in B are also in A? (This finds $\Pr(A \mid B)$.)

2 Use this two-way table to answer these questions.

	A	A′	
B	7	5	12
B′	3	1	4
	10	6	16

a What fraction of the elements in A are also in B? (This finds $\Pr(B \mid A)$.)

b What fraction of the elements in B are also in A? (This finds $\Pr(A \mid B)$.)

3 In a group of 20 people, 15 are wearing jackets and 10 are wearing hats; 5 are wearing both a jacket and a hat.

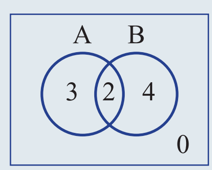
a What fraction of the people who are wearing jackets are wearing hats?

b What fraction of the people who are wearing hats are wearing jackets?

Fluency

Example 6 Finding conditional probabilities using a Venn diagram

Consider this Venn diagram displaying the number of elements belonging to the events A and B.



Find the following probabilities.

a $\Pr(A)$ b $\Pr(A \cap B)$

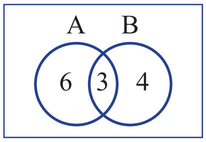
c $\Pr(A \mid B)$ d $\Pr(B \mid A)$

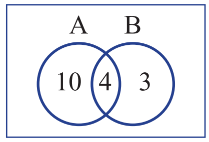
Solution	Explanation
a $\Pr(A) = \frac{5}{9}$	There are 5 elements in A and 9 in total.
b $\Pr(A \cap B) = \frac{2}{9}$	There are 2 elements common to A and B.
c $\Pr(A \mid B) = \frac{2}{6} = \frac{1}{3}$	2 of the 6 elements in B are in A.
d $\Pr(B \mid A) = \frac{2}{5}$	2 of the 5 elements in A are in B.

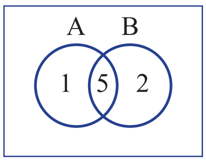
ISBN: 9781107687028

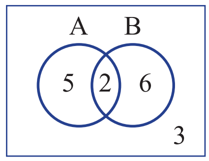
4 The following Venn diagrams display information about the number of elements associated with the events A and B.
For each Venn diagram, find:

i $\Pr(A)$ **ii** $\Pr(A \cap B)$
iii $\Pr(A|B)$ **iv** $\Pr(B|A)$

a


b


c


d


- $A \cap B$ means both A and B.
- $\Pr(A|B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$
- $\Pr(B|A) = \dfrac{\text{number in } A \cap B}{\text{number in } A}$

Example 7 Finding conditional probabilities using a two-way table

From a group of 15 hockey players at a game of hockey, 13 played on the field, 7 sat on the bench and 5 both played and sat on the bench.
A hockey player is chosen at random from the team.
Let A be the event 'the person played on the field' and B be the event 'the person sat on the bench'.

a Represent the information in a two-way table.
b Find the probability that the person only sat on the bench.
c Find the probability that the person sat on the bench, given that they played on the field.
d Find the probability that the person played on the field, given that they sat on the bench.

Solution

a

	A	A′	
B	5	2	7
B′	8	0	8
	13	2	15

$A \cap B$ has 5 elements, A has a total of 13 and B a total of 7. There are 15 players in total.

b $\Pr(\text{bench only}) = \dfrac{2}{15}$

Two people sat on the bench and did not play on the field.

c $\Pr(B|A) = \dfrac{5}{13}$

$\Pr(B|A) = \dfrac{\text{number in } A \cap B}{\text{number in } A}$

d $\Pr(A|B) = \dfrac{5}{7}$

$\Pr(A|B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$

Explanation

(Explanations are given beside each solution part above.)

ISBN: 9781107687028

4D

5 The following two-way tables show information about the number of elements in the events A and B. For each two-way table, find:

i $\Pr(A)$ **ii** $\Pr(A \cap B)$
iii $\Pr(A|B)$ **iv** $\Pr(B|A)$

First decide on the total that gives the denominator of your fraction.

a

	A	A′	
B	2	8	10
B′	5	3	8
	7	11	18

b

	A	A′	
B	1	4	5
B′	3	1	4
	4	5	9

c

	A	A′	
B	7	3	10
B′	1	6	7
	8	9	17

d

	A	A′	
B	4	2	6
B′	8	2	10
	12	4	16

Problem-solving and Reasoning

6 Of a group of 20 English cricket fans at a match, 13 purchased a pie, 15 drank beer and 9 purchased a pie *and* drank beer.
Let A be the event 'the fan purchases a pie'.
Let B be the event 'the fan drank beer'.

a Copy and complete this two-way table.

	A	A′	
B	9		
B′			
			20

b Find the probability that a fan only purchased a pie (and did not drink beer).
c Find the probability that a fan purchased a pie, given that they drank beer.
d Find the probability that a fan drank beer, given that they purchased a pie.

ISBN: 9781107687028 Cambridge University Press

7 Of 15 musicians surveyed to find out whether they play the violin or the piano, 5 play the violin, 8 play the piano and 2 play both instruments.

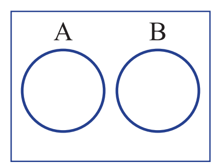
- **a** Represent the information in a Venn diagram.
- **b** How many of the people do not play either the violin or the piano?
- **c** Find the probability that one of the 15 people plays piano, given they play the violin.
- **d** Find the probability that one of the 15 people plays the violin, given they play piano.

8 A card is drawn from a pack of 52 playing cards. Find the probability that:

- **a** the card is a king given that it is a heart
- **b** the card is a jack given that it is a red card

13 of the cards are hearts. There are 4 kings, including one king of hearts.

9 Two events A and B are mutually exclusive. What can be said about the probability of A given B (i.e. $\Pr(A|B)$) or the probability of B given A (i.e. $\Pr(B|A)$)? Give a reason.



Cruise control and airbags

10 On a car production line, 30 cars are due to be completed by the end of the day. Fifteen of the cars have cruise control and 20 have airbags, while 6 have both cruise control and airbags.

- **a** Represent the information provided in a Venn diagram or two-way table.
- **b** Find the probability that a car chosen at random will contain the following:
  - **i** cruise control only
  - **ii** airbags only
- **c** Given that the car chosen has cruise control, find the probability that the car will have airbags.
- **d** Given that the car chosen has airbags, find the probability that the car will have cruise control.

ISBN: 9781107687028

4.5 Multiple events using tables

When an experiment involves two or more components, such as flipping a coin twice, we are dealing with multiple events. Multiple events are sometimes called multi-stage experiments or compound events. The outcomes for such an event depend on whether or not they are conducted with or without replacement. Other examples of multiple events include choosing two people to represent the student body on a school committee or selecting three chocolates from a box.

Let's start: Two prizes, three people

Two special prizes are to be awarded in some way to Bill, May and Li for their efforts in helping at the school fete. This table shows how the prizes might be awarded.

		2nd prize		
		Bill	May	Li
	Bill	(B, B)	(B, M)	(B, L)
1st prize	May	(M, B)		
	Li			

- Complete the table to show how the two prizes can be awarded.
- Does the table show that the same person can be awarded both prizes?
- What is the probability that Bill and Li are both awarded a prize?
- How would the table change if the same person could not be awarded both prizes?
- How do the words 'with replacement' and 'without replacement' relate to the above situation? Discuss.

Key ideas

- Tables are used to list the sample space for multiple events with two components.
- If replacement is allowed, then outcomes from each selection can be repeated.
- If selections are made without replacement, then outcomes from each selection cannot be repeated.
  For example: two selections are made from the digits {1, 2, 3}.

With replacement

		1st		
		1	2	3
	1	(1, 1)	(2, 1)	(3, 1)
2nd	2	(1, 2)	(2, 2)	(3, 2)
	3	(1, 3)	(2, 3)	(3, 3)

9 outcomes

Without replacement

		1st		
		1	2	3
	1	×	(2, 1)	(3, 1)
2nd	2	(1, 2)	×	(3, 2)
	3	(1, 3)	(2, 3)	×

6 outcomes

ISBN: 9781107687028 Cambridge University Press

Exercise 4E

Understanding

1 Two letters are chosen from the word DOG.

a Complete a table listing the sample space if selections are made:

i with replacement

		1st		
		D	O	G
	D	(D, D)	(O, D)	
2nd	O			
	G			

ii without replacement

		1st		
		D	O	G
	D	×	(O, D)	
2nd	O		×	
	G			×

b State the total numbers of outcomes if selection is made:

i with replacement **ii** without replacement

c If selection is made with replacement, find the probability that:

i the two letters are the same **ii** there is at least one D

iii there is not an O **iv** there is a D or a G

v there is a D and a G

Count up your favourable outcomes and divide by the total.

d If selection is made without replacement, find the probability that:

i the two letters are the same **ii** there is at least one D

iii there is not an O **iv** there is a D or a G

v there is a D and a G

2 Two digits are selected from the set {2, 3, 4} to form a two-digit number. Find the number of two-digit numbers that can be formed if the digits are selected:

a with replacement

	2	3	4
2	22	32	
3			
4			

b without replacement

	2	3	4
2	×	32	
3		×	
4			×

Fluency

Example 8 Constructing a table with replacement

A six-sided die is tossed twice.

a List all the outcomes using a table.

b State the total number of outcomes.

c Find the probability of obtaining the outcome (1, 5).

d Find:

i Pr (double) **ii** Pr (sum of at least 10) **iii** Pr (sum not equal to 7)

ISBN: 9781107687028

4E

Solution | **Explanation**

a Be sure to place the number from toss 1 in the first position for each outcome.

Toss 2

Toss 1	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

b 36 outcomes

There is a total of $6 \times 6 = 36$ outcomes.

c $\Pr(1, 5) = \frac{1}{36}$

Only one outcome is (1, 5).

d **i** $\Pr(\text{double}) = \frac{6}{36} = \frac{1}{6}$

Six outcomes have the same number repeated.

ii $\Pr(\text{sum of at least } 10) = \frac{6}{36} = \frac{1}{6}$

Six outcomes have a sum of either 10, 11 or 12.

iii $\Pr(\text{sum not equal to } 7) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$

This is the complement of having a sum of 7.
Six outcomes have a sum of 7.
$\Pr(\text{not } A) = 1 - \Pr(A)$

3 A 4-sided die is tossed twice.

a List all the outcomes using a table.
b State the total number of possible outcomes.
c Find the probability of obtaining the outcome (2, 4).
d Find the probability of:
  i a double
  ii a sum of at least 5
  iii a sum not equal to 4

Qs 3 and 4 are making selections 'with replacement' because outcomes can be repeated.

1st

2nd	1	2	3	4
1	(1, 1)	(2, 1)		
2				
3				
4				

ISBN: 9781107687028

4 Two coins are tossed, each landing with a head (H) or tail (T).
  a List all the outcomes using a table.
  b State the total number of possible outcomes.
  c Find the probability of obtaining the outcome (H, T).
  d Find the probability of obtaining:
    i exactly one tail
    ii at least one tail
  e If the two coins were tossed 1000 times, how many times would you expect to get two tails?

		1st	
		H	T
2nd	H	(H, H)	(T, H)
	T		

Example 9 Constructing a table without replacement

Two letters are chosen from the word KICK, without replacement.
  a Construct a table to list the sample space.
  b Find the probability of:
    i obtaining the outcome (K, C)
    ii selecting two Ks
    iii selecting a K and a C

Solution

a

		1st			
		K	I	C	K
	K	×	(I, K)	(C, K)	(K, K)
	I	(K, I)	×	(C, I)	(K, I)
2nd	C	(K, C)	(I, C)	×	(K, C)
	K	(K, K)	(I, K)	(C, K)	×

b i $\Pr(K, C) = \frac{2}{12}$

$= \frac{1}{6}$

ii $\Pr(K, K) = \frac{2}{12}$

$= \frac{1}{6}$

iii $\Pr(K \text{ and } C) = \frac{4}{12}$

$= \frac{1}{3}$

Explanation

Selection is without replacement, so the same letter (from the same position) cannot be chosen twice.

Two of the 12 outcomes are (K, C).

Two of the outcomes are K and K, which use different Ks from the word KICK.

Four outcomes contain a K and a C.

ISBN: 9781107687028

Cambridge University Press

5 Two letters are chosen from the word SET without replacement.

a Complete this table.

b Find the probability of:

i obtaining the outcome (E, T)

ii selecting one T

iii selecting at least one T

iv selecting an S and a T

v selecting an S or a T

		1st		
		S	E	T
	S	×	(E, S)	(T, S)
2nd	E		×	
	T			×

Problem-solving and Reasoning

6 A letter is chosen from the word LEVEL without replacement and then a second letter is chosen from the same word.

Remember that this is 'without replacement'.

a Draw a table displaying the sample space for the pair of letters chosen.

b State the total number of outcomes possible.

c State the number of outcomes that contain exactly one of the following letters.

i V **ii** L **iii** E

d Find the probability that the outcome will contain exactly one of the following letters.

i V **ii** L **iii** E

e Find the probability that the two letters chosen will be the same.

7 In a quiz, Min guessed that the probability of rolling a sum of 10 or more from two 6-sided dice is 10%. Complete the following to decide whether or not this guess is correct.

a Copy and complete the table representing all the outcomes for possible totals that can be obtained.

b State the total number of outcomes.

c Find the number of the outcomes that represent a sum of:

i 3 **ii** 7 **iii** less than 7

		Die 2					
		1	2	3	4	5	6
	1	2	3	…			
	2	3	…				
Die 1	3	4					
	4	⋮					
	5	⋮					
	6						

d Find the probability that the following sums are obtained.

i 7 **ii** less than 5 **iii** greater than 2 **iv** at least 11

e Find the probability that the sum is at least 10, and decide whether or not Min's guess is correct.

8 The 10 students who completed a special flying course are waiting to see if they will be awarded the one Distinction or the one Merit award available for their efforts.

For part **a**, you might want to start a table but not complete it.

a In how many ways can the two awards be given out if:

i the same person can receive both awards?

ii the same person cannot receive both awards?

b Assuming that a person cannot receive both awards, find the probability that a particular person receives:

i the Distinction award

ii the Merit award

iii neither award

c Assuming that a person can receive both awards, find the probability that they receive at least one award.

ISBN: 9781107687028 Cambridge University Press

9 Decide whether the following situations would naturally involve selections with replacement or without replacement.

- **a** Selecting two people to play in a team.
- **b** Tossing a coin twice.
- **c** Rolling two dice.
- **d** Choosing two chocolates to eat.

Random weights

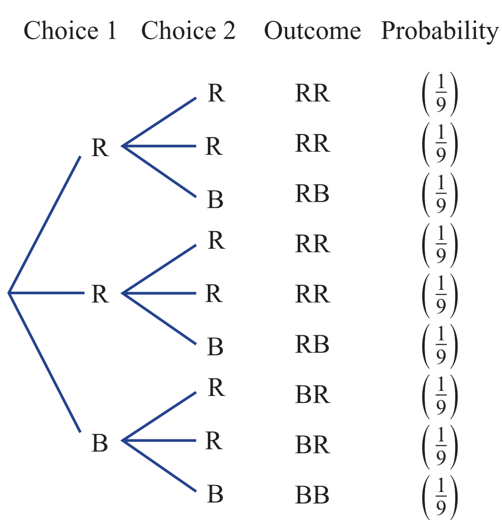
10 In a gym, Justine considers choosing two weights to fit onto a leg weights machine to make the load heavier. She can choose from 2.5 kg, 5 kg, 10 kg or 20 kg, and there are plenty of each weight available. Justine's friend randomly chooses both weights, with equal probability that she will choose each weight, and places them on the machine. Justine then attempts to operate the machine without knowing which weights were chosen.

- **a** Complete a table that displays all possible total weights that could be placed on the machine.
- **b** State the total number of outcomes.
- **c** How many of the outcomes deliver a total weight described by the following?
  i equal to 10 kg **ii** less than 20 kg **iii** at least 20 kg
- **d** Find the probability that Justine will be attempting to lift the following weight.
  i 20 kg **ii** 30 kg **iii** no more than 10 kg **iv** less than 10 kg
- **e** If Justine is unable to lift more than 22 kg, what is the probability that she will not be able to operate the leg weights machine?

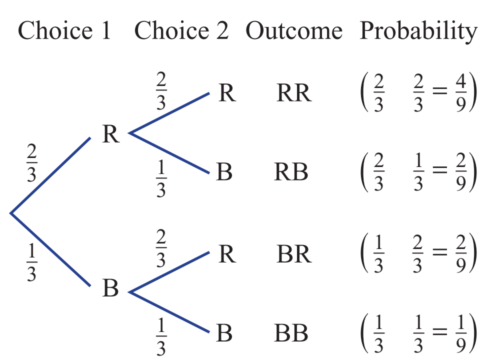
ISBN: 9781107687028

4.6 Using tree diagrams

Tree diagrams can also be used to help list outcomes for multiple events. Suppose that a bag contains two red counters and one blue counter and that two counters are selected at random with replacement. One way to display the outcomes is with a tree diagram in which all equally likely outcomes are listed in columns, as shown below left. A more efficient way, however, is to group similar outcomes and write their corresponding probabilities on the branches, as shown below right.



$\Pr(R, B) = \frac{2}{9}$



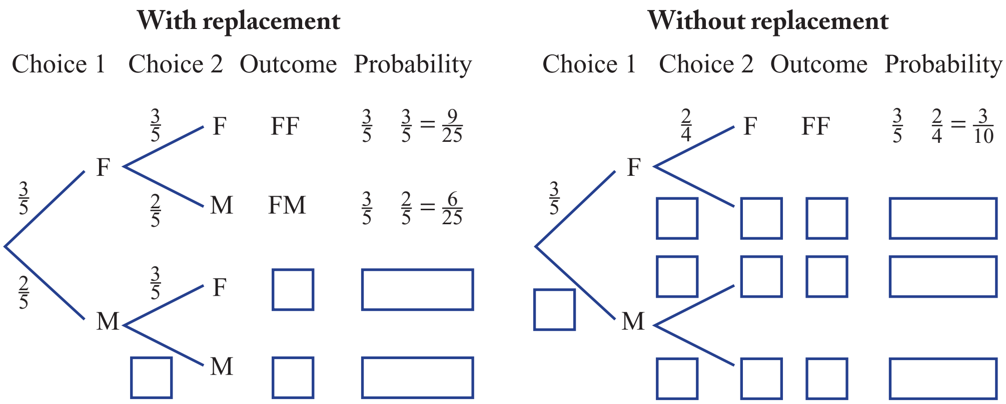
$\Pr(R, B) = \frac{2}{9}$

In the tree diagram on the right, the probability of each outcome is obtained by multiplying the branch probabilities. This also applies when selection is made without replacement.

Let's start: Trees with and without replacement

Suppose that two selections are made from a group of 2 male and 3 female workers to complete two extra tasks.

ISBN: 9781107687028



- Complete these two tree diagrams to show how these selections can be made, both with and without replacement.
- Explain where the branch probabilities come from on each branch of the tree diagrams.
- What is the total of all the probabilities on each tree diagram?

Key ideas

- Tree diagrams can be used to list the sample space for experiments involving two or more components.
  - Branch probabilities are used to describe the chance of each outcome at each step.
  - The probability of each outcome for the experiment is obtained by multiplying the branch probabilities.
  - Branch probabilities will depend on whether selection is made with or without replacement. For *with replacement*, probabilities do not change. For *without replacement*, probabilities do change.

Exercise 4F

Understanding

1 A coin is tossed three times and a head or tail is obtained each time.

a How many outcomes are there?

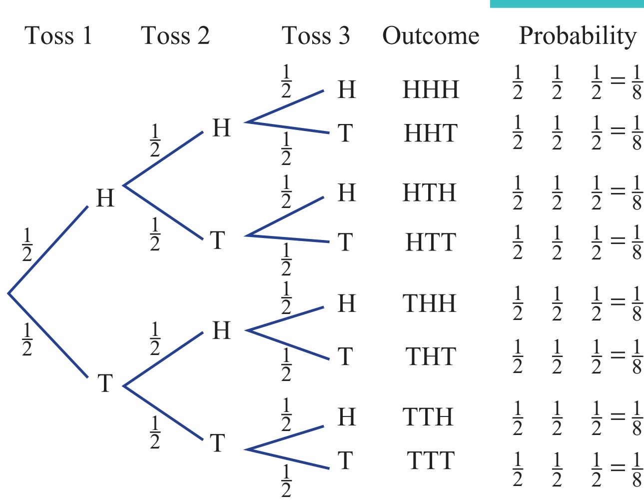
b What is the probability of the outcome HHH?

c How many outcomes obtain:

  i 2 tails?

  ii 2 or 3 heads?

d What is the probability of obtaining at least one tail?



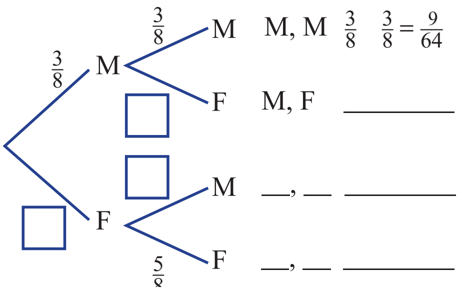
ISBN: 9781107687028

4F

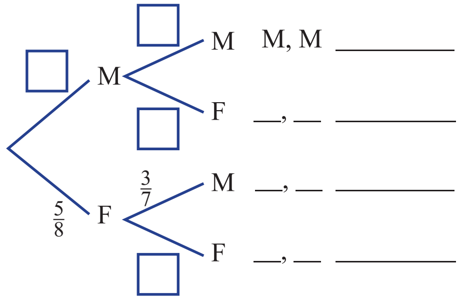
2 Two prizes are awarded to a group of 3 male (M) and 5 female (F) candidates. Copy and complete each tree diagram. Include the missing branch probabilities and outcome probabilities.

For 'without replacement' the second selection is out of 7, not 8.

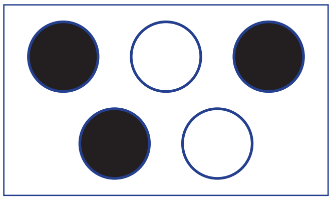
a with replacement



b without replacement



3 A box contains 2 white (W) and 3 black (B) counters.



a A single counter is drawn at random. Find the probability that it is:

i white ii black

b Two counters are now drawn at random. The first one is replaced before the second one is drawn. Find the probability that the second counter is:

i white ii black

c Two counters are drawn and the first counter is not replaced before the second one is drawn. If the first counter is white, find the probability that the second counter is:

i white ii black

After one white counter is taken out, how many of each remain?

Fluency

Example 10 Constructing a tree diagram for multiple events

Boxes A and B contain 4 counters each. Box A contains 2 red and 2 green counters and box B contains 1 red and 3 green counters. A box is chosen at random and then a single counter is selected.

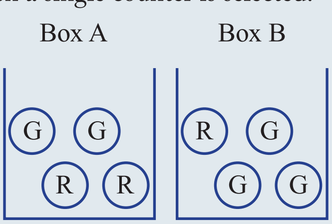
a What is the probability of selecting a red counter from box A?

b What is the probability of selecting a red counter from box B?

c Represent the options available as a tree diagram that shows all possible outcomes and related probabilities.

d What is the probability of selecting box B and a red counter?

e What is the probability of selecting a red counter?

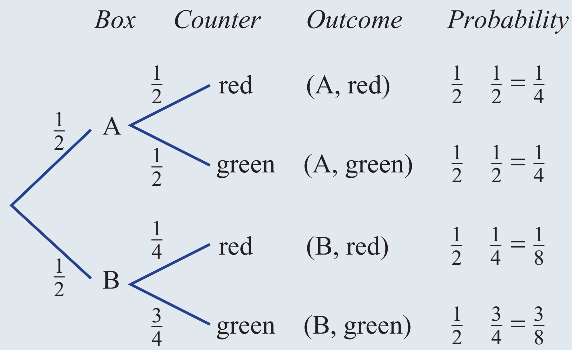


Solution	Explanation
a Pr (red from Box A) $= \frac{2}{4} = \frac{1}{2}$	Two of the 4 counters in box A are red.
b Pr (red from Box B) $= \frac{1}{4}$	One of the 4 counters in box B is red.

ISBN: 9781107687028

Cambridge University Press

c



First selection is a box followed by a counter.
Multiply each of the probabilities along the branch pathways to find the probability of each outcome.

d $\Pr(\text{B, red}) = \frac{1}{2}\ \ \frac{1}{4}$

$= \frac{1}{8}$

The probability of choosing box B is $\frac{1}{2}$ and a red counter from box B is $\frac{1}{4}$, so multiply the probabilities for these two outcomes together.

e $\Pr(1\text{ red}) = \frac{1}{2}\ \ \frac{1}{2} + \frac{1}{2}\ \ \frac{1}{4}$

$= \frac{1}{4} + \frac{1}{8}$

$= \frac{3}{8}$

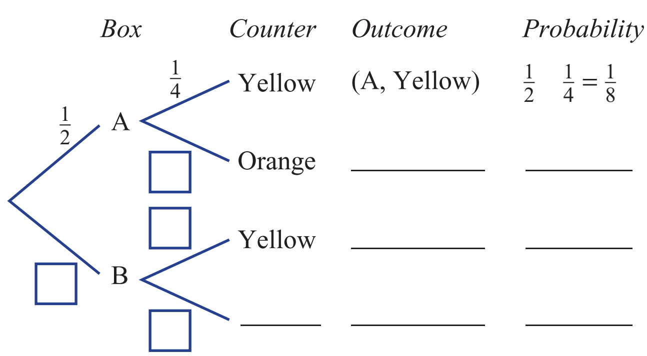
The outcomes (A, red) and (B, red) both contain 1 red counter, so add the probabilities for these two outcomes together.

4 Boxes A and B contain 4 counters each. Box A contains 1 yellow and 3 orange counters and box B contains 3 yellow and 1 orange counter. A box is chosen at random and then a single counter is selected.

a If box A is chosen, what is the probability of selecting a yellow counter?

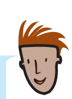
b If box B is chosen, what is the probability of selecting a yellow counter?

c Represent the options available by completing this tree diagram.



d What is the probability of selecting box B and a yellow counter?

e What is the probability of selecting 1 yellow counter?



For part **e**, add the probabilities for both outcomes that have a yellow counter.

ISBN: 9781107687028

Example 11 Using a tree diagram without replacement

A bag contains 5 blue (B) and 3 white (W) marbles and two marbles are selected without replacement.

a Draw a tree diagram showing all outcomes and probabilities.

b Find the probability of selecting:

  i a blue marble followed by a white marble (B, W)

  ii 2 blue marbles

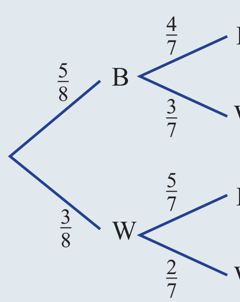
  iii exactly one blue marble

c If the experiment was repeated with replacement, find the answers to each question in part **b**.

Solution

a

Selection 1	*Selection 2*	*Outcome*	*Probability*
		(B, B)	$\frac{5}{8}\ \ \frac{4}{7} = \frac{5}{14}$
		(B, W)	$\frac{5}{8}\ \ \frac{3}{7} = \frac{15}{56}$
		(W, B)	$\frac{3}{8}\ \ \frac{5}{7} = \frac{15}{56}$
		(W, W)	$\frac{3}{8}\ \ \frac{2}{7} = \frac{3}{28}$



b **i** $\Pr(\text{B, W}) = \frac{5}{8}\ \ \frac{3}{7}$
$= \frac{15}{56}$

ii $\Pr(\text{B, B}) = \frac{5}{8}\ \ \frac{4}{7}$
$= \frac{5}{14}$

iii $\Pr(\text{1 blue}) = \frac{5}{8}\ \ \frac{3}{7} + \frac{3}{8}\ \ \frac{5}{7}$
$= \frac{15}{28}$

c **i** $\Pr(\text{B, W}) = \frac{5}{8}\ \ \frac{3}{8}$
$= \frac{15}{64}$

ii $\Pr(\text{B, B}) = \frac{5}{8}\ \ \frac{5}{8}$
$= \frac{25}{64}$

iii $\Pr(\text{1 blue}) = \frac{5}{8}\ \ \frac{3}{8} + \frac{3}{8}\ \ \frac{5}{8}$
$= \frac{15}{32}$

Explanation

After one blue marble is selected there are 7 marbles remaining: 4 blue and 3 white.

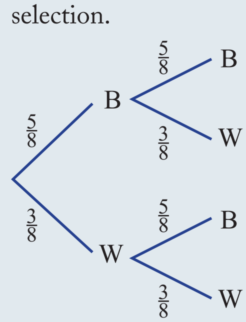
After one white marble is selected there are 7 marbles remaining: 5 blue and 2 white.

Multiply the probabilities on the (B, W) pathway.

Only 4 blue marbles remain after the first selection. Multiply the probabilities on the (B, B) pathway.

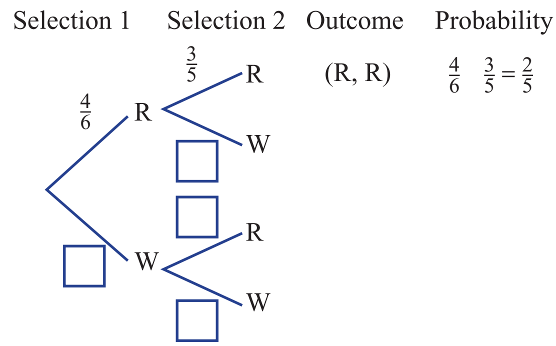
The outcomes (B, W) and (W, B) both have one blue marble. Multiply probabilities to find individual probabilities, then sum for the final result.

When selecting objects with replacement, remember that the number of marbles in the bag remains the same for each selection.



ISBN: 9781107687028

5 A bag contains 4 red (R) and 2 white (W) marbles, and two marbles are selected without replacement.



a Complete this tree diagram showing all outcomes and probabilities.

b Find the probability of selecting:

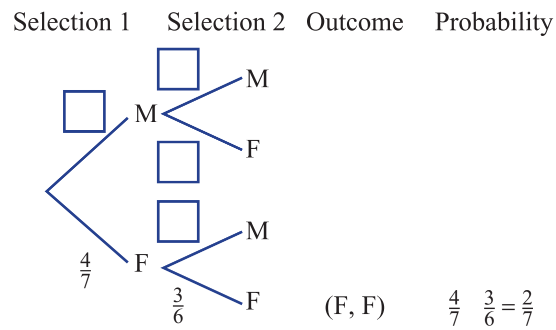
i a red marble and then a white marble (R, W)

ii 2 red marbles

iii exactly 1 red marble

c If the experiment is repeated with replacement, find the answers to each question in part **b**. You may need to redraw the tree diagram.

6 Two students are selected from a group of 3 males (M) and 4 females (F) without replacement.



a Complete this tree diagram to help find the probability of selecting:

i 2 males

ii 2 females

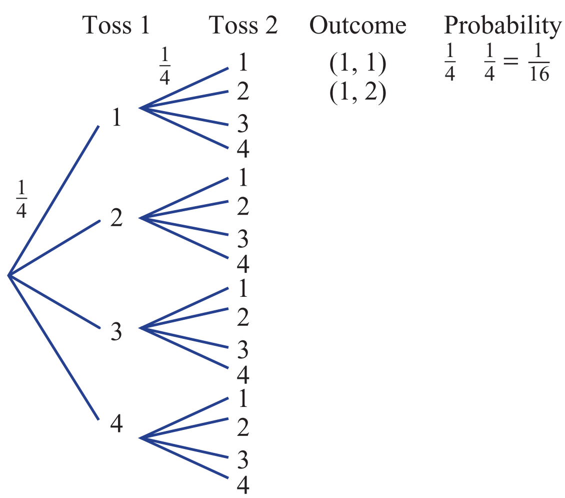
iii 1 male and 1 female

iv 2 people either both male or both female

b If the experiment is repeated with replacement, find the answers to each question in part **a**.

Problem-solving and Reasoning

7 A 4-sided die is tossed twice and the pair of numbers is recorded.



a Complete this tree diagram to list the outcomes.

b State the total number of outcomes.

c Find the probability of obtaining:

i a 4 then a 1, i.e. the outcome (4, 1)

ii a double

d Find the probability of obtaining a sum described by the following:

i equal to 2

ii equal to 5

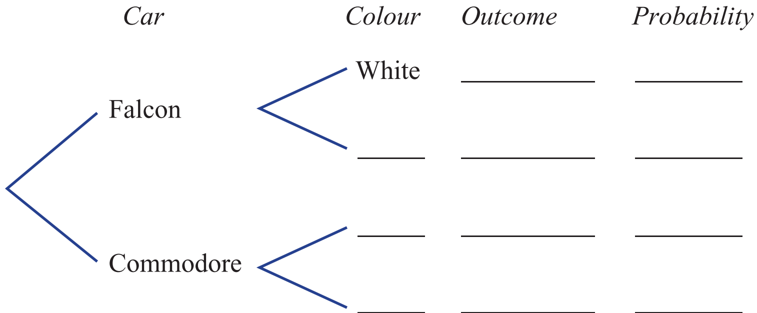
iii less than or equal to 5

ISBN: 9781107687028

4F

8 As part of a salary package a person can select either a Falcon or a Commodore. There are 3 white Falcons and 1 silver Falcon and 2 white Commodores and 1 red Commodore to choose from.

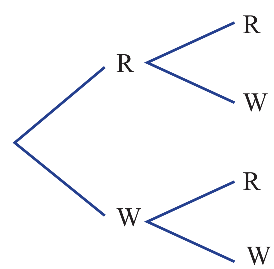
a Complete a tree diagram showing a random selection of a car type, then a colour.



b Find the probability that the person chooses:

i a white Falcon **ii** a red Commodore **iii** a white car
iv a car that is not white **v** a silver car or a white car **vi** a car that is not a Falcon nor red

9 Two bottles of wine are randomly selected for tasting from a box containing 2 red and 2 white wines. Use a tree diagram to help answer the following.



a If the first bottle is replaced before the second is selected, find:

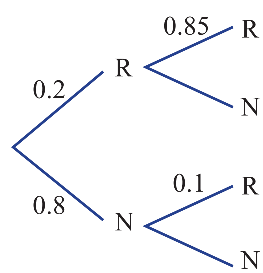
i Pr (2 red) **ii** Pr (1 red)
iii Pr (not two white) **iv** Pr (at least one white)

b If the first bottle is not replaced before the second is selected, find:

i Pr (2 red) **ii** Pr (1 red)
iii Pr (not two white) **iv** Pr (at least one white)

Rainy days

10 Imagine that the probability of rain next Monday is 0.2. The probability of rain on a day after a rainy day is 0.85, while the probability of rain on a day after a non-rainy day is 0.1.



a Next Monday and Tuesday, find the probability of having:

i two rainy days **ii** exactly one rainy day
iii at least one dry day

b Next Monday, Tuesday and Wednesday, find the probability of having:

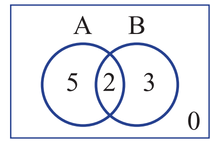
i three rainy days **ii** exactly one dry day
iii at most two rainy days

ISBN: 9781107687028

Cambridge University Press

4.7 Independent events

In previous sections we have looked at problems involving conditional probability. This Venn diagram, for example, gives the following results.

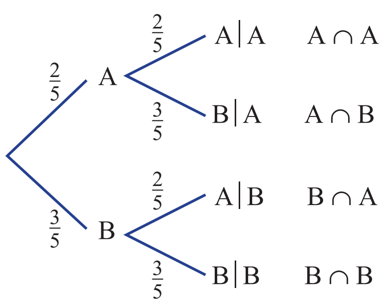


$$\Pr(A) = \frac{7}{10} \text{ and } \Pr(A \mid B) = \frac{2}{5}$$

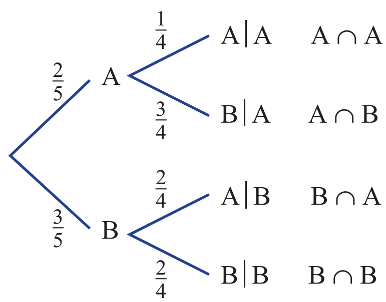
The condition B in $\Pr(A \mid B)$ has changed the probability of A. The events A and B are therefore not independent.

For multiple events we can consider events either with or without replacement. These tree diagrams, for example, show two selections of marbles from a bag of 2 aqua (A) and 3 blue (B) marbles.

With replacement



Without replacement

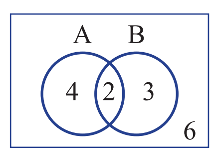


In the first tree diagram $\Pr(A \mid B) = \Pr(A)$, so the events are independent.
In the second tree diagram $\Pr(A \mid B) \neq \Pr(A)$, so the events are not independent.

Let's start: Is it the same to be mutually exclusive and independent?

Use the Venn diagram to consider the following questions.

- Are the events mutually exclusive? Why?
- Find $\Pr(A)$ and $\Pr(A \mid B)$. Does this mean that the events A and B are independent?



Key ideas

- Two events are **independent** if the outcome of one event does not change the probability of obtaining the other event.
  - $\Pr(A \mid B) = \Pr(A)$ or $\Pr(B \mid A) = \Pr(B)$
  - $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$
- For multiple events where selection is made with replacement, successive events are independent.
- For multiple events where selection is made without replacement, successive events are not independent.

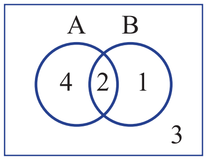
Independent events Two events that do not influence or affect each other

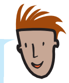
ISBN: 9781107687028

Exercise 4G

Understanding

1 This Venn diagram shows the number of elements in events A and B.





Recall:
$\Pr(B|A) = \dfrac{\text{number in } A \cap B}{\text{number in } A}$

a Find **i** $\Pr(B)$ **ii** Find $\Pr(B|A)$
b Is $\Pr(B|A) = \Pr(B)$?
c Are the events A and B independent?

2 Complete each sentence.

a For multiple events, successive events are independent if selections are made ____________ replacement.
b For multiple events, successive events are not independent if selections are made ____________ replacement.

Choose from ‘with’ or ‘without’.

3 A coin is tossed twice. Let A be the event ‘the first toss gives a tail’. Let B be the event ‘the second toss gives a tail’.

a Find **i** $\Pr(A)$ **ii** $\Pr(B)$
b Would you say that events A and B are independent?
c What is $\Pr(B|A)$?

Fluency

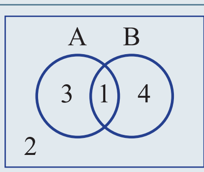
Example 12 Using Venn diagrams

A selection of 10 mobile phone plans includes 4 with free connection and 5 with a free second battery. 1 plan has both free connection and a free second battery.
Let A be the event ‘choosing a mobile phone plan with free connection’.
Let B be the event ‘choosing a mobile phone plan with a free second battery’.

a Summarise the information about the 10 mobile phone plans in a Venn diagram.
b **i** Find $\Pr(A)$. **ii** Find $\Pr(A|B)$.
c State whether or not the events A and B are independent.

Solution | **Explanation**

a



Start with the 1 element that belongs to both A and B and complete according to the given information.

b **i** $\Pr(A) = \dfrac{4}{10} = \dfrac{2}{5}$

4 of the 10 elements belong to A.

ii $\Pr(A|B) = \dfrac{1}{5}$

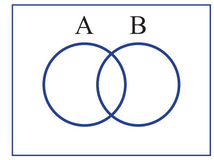
1 of the 5 elements in B belongs to A.

c The events A and B are not independent.

$\Pr(A|B) \neq \Pr(A)$

ISBN: 9781107687028 Cambridge University Press

4 A selection of 8 offers for computer printers includes 3 with a free printer cartridge and 4 with a free box of paper. 2 have both a free printer cartridge and a free box of paper.
Let A be the event 'choosing a printer with a free printer cartridge'.
Let B be the event 'choosing a printer with a free box of paper'.



- **a** Summarise the given information about the 8 computer printer offers in a Venn diagram.
- **b** **i** Find Pr(A). **ii** Find Pr(A|B).
- **c** State whether or not the events A and B are independent.

If Pr(A) = Pr(A|B), then the events A and B are independent.

5 A selection of 6 different baby strollers includes 3 with a free rain cover and 4 with a free sun shade. 2 offer both a free rain cover and free sun shade.
Let A be the event 'choosing a stroller with a free sun shade'.
Let B be the event 'choosing a stroller with a free rain cover'.

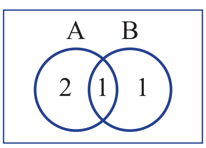
- **a** Summarise the given information about the six baby strollers in a Venn diagram.
- **b** **i** Find Pr(A). **ii** Find Pr(A|B).
- **c** State whether or not the events A and B are independent.

Problem-solving and Reasoning

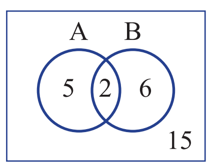
Remember:
$\Pr(A|B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$

6 From events A and B in the given Venn diagrams:
- **i** find Pr(A) and Pr(A|B)
- **ii** hence decide whether or not events A and B are independent

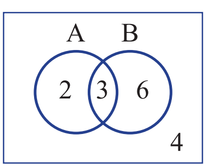
a



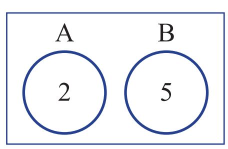
b



c



d



7 For the events A and B with details provided in the given two-way tables, find Pr(A) and Pr(A|B). Decide whether or not the events A and B are independent.

a

	A	A′	
B	1	1	2
B′	3	3	6
	4	4	8

b

	A	A′	
B	1	3	4
B′	2	4	6
	3	7	10

c

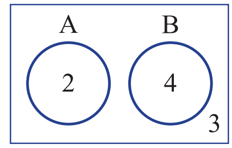
	A	A′	
B	3	17	20
B′	12	4	16
	15	21	36

d

	A	A′	
B	1		9
B′			
	5		45

ISBN: 9781107687028

8 Use this diagram to help decide if this statement is true or false: If two events A and B are mutually exclusive, then they are also independent.



9 A coin is tossed 5 times. Find the probability of obtaining:

a 5 heads
b at least one tail
c at least one head

Coin tosses are independent. From two coins, the probability of two heads is $\frac{1}{2} \times \frac{1}{2}$.

Tax and investment advice

10 Of 17 leading accountants, 15 offer advice on tax (T) while 10 offer advice on investment (I). Eight of the accountants offer advice on both tax and investment. One of the 17 accountants is chosen at random.

a Use a Venn diagram or two-way table to help find:
i $\Pr(T)$ **ii** $\Pr(T \text{ only})$ **iii** $\Pr(T|I)$
b Are the events T and I independent?

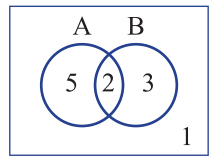
ISBN: 9781107687028 Cambridge University Press

Puzzles and games

1 'I have nothing in common'. Match the answers to the letters in parts **a** and **b** to to uncover the code.

___	___	___	___	___	___	___	___	
$\frac{5}{14}$	5	2	5	7	10	10	$\frac{7}{11}$	
___	___	___	___	___	___	___	___	___
$\frac{5}{11}$	$\frac{3}{14}$	$\frac{1}{2}$	10	5	$\frac{10}{11}$	$\frac{1}{7}$	3	$\frac{5}{11}$

a These questions relate to the Venn diagram at right.



- T How many elements in $A \cap B$?
- L How many elements in $A \cup B$?
- V How many elements in B only?
- Y Find $\Pr(A)$.
- S Find $\Pr(A \cup B)$.
- E Find $\Pr(A \text{ only})$.

b These questions relate to the two-way table at right.

	P	P′	
Q	U	4	9
Q′	2		
		A	14

- U What number should be in place of the letter U?
- A What number should be in place of the letter A?
- M Find $\Pr(P \cap Q)$.
- C Find $\Pr(P')$.
- X Find Pr(neither P nor Q).
- I Find $\Pr(P \text{ only})$.

2 What is the chance of rolling a sum of at least 10 from rolling two 6-sided dice?

3 *Game for two people*: You will need a bag or pocket and coloured counters.

- One person places 8 counters of 3 different colours in a bag or pocket. The second person must not look!
- The second person then selects a counter from the bag. The colour is noted, then the counter is returned to the bag. This is repeated 100 times.
- Complete this table.

Colour	Tally	Frequency	Guess
Total:	100	100	

- Using the experimental results, the second person now tries to guess how many counters of each colour are in the bag.

4 Two digits are chosen without replacement from the set {1, 2, 3, 4} to form a two-digit number. Find the probability that the two-digit number is:

a 32 **b** even **c** less than 40 **d** at least 22

5 A coin is tossed 4 times. What is the probability that at least one tail is obtained?

6 Two leadership positions are to be filled from a group of 2 girls and 3 boys. What is the probability that the positions will be filled by one girl and one boy?

7 The letters of the word DOOR are jumbled randomly. What is the probability that the final arrangement will spell DOOR?

ISBN: 9781107687028

Cambridge University Press

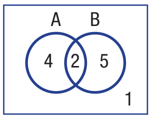
Chapter summary

Probability

Review

- Sample space is the list of all possible outcomes
- $\Pr(\text{Event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

Venn diagram

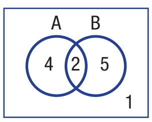


Two-way table

	A	A′	
B	2	5	7
B′	4	1	5
	6	6	12

Conditional probability

$\Pr(A \mid B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$



	A	A′	
B	2	5	7
B′	4	1	5
	6	6	12

$\Pr(A \mid B) = \frac{2}{7}$

$\Pr(B \mid A) = \frac{2}{6} = \frac{1}{3}$

Independent events

- $\Pr(A \mid B) = \Pr(A)$ or $\Pr(B \mid A) = \Pr(B)$
- $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

Tables

With replacement

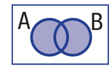
	A	B	C
A	(A, A)	(B, A)	(C, A)
B	(A, B)	(B, B)	(C, B)
C	(A, C)	(B, C)	(C, C)

Without replacement

	A	B	C
A	×	(B, A)	(C, A)
B	(A, B)	×	(C, B)
C	(A, C)	(B, C)	×

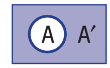
Notation

- Union $A \cup B$ (A or B)



- Intersection $A \cap B$ (A and B)

- Complement of A is A′ (not A)



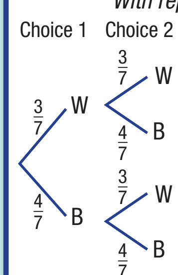
- A only

- Mutually exclusive events $\Pr(A \cap B) = 0$

Tree diagrams

3 white
4 black

With replacement

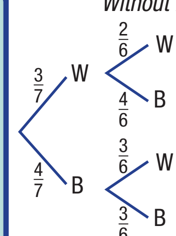


Outcome	Probability
(W, W)	$\frac{9}{49}$
(W, B)	$\frac{12}{49}$
(B, W)	$\frac{12}{49}$
(B, B)	$\frac{16}{49}$

$\Pr(W, B) = \frac{3}{7} \times \frac{4}{7}$

$= \frac{12}{49}$

Without replacement



Outcome	Probability
(W, W)	$\frac{1}{7}$
(W, B)	$\frac{2}{7}$
(B, W)	$\frac{2}{7}$
(B, B)	$\frac{2}{7}$

$\Pr(\text{one white}) = \frac{2}{7} + \frac{2}{7}$

$= \frac{4}{7}$

ISBN: 9781107687028

Multiple-choice questions

1 A letter is chosen from the word SUCCESS. The probability that the letter is not a C is:

A $\frac{2}{7}$ **B** $\frac{3}{5}$ **C** $\frac{5}{7}$ **D** $\frac{4}{7}$ **E** $\frac{3}{7}$

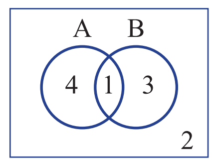
2 The number of manufacturing errors spotted in a car plant on 20 days is given by this table.

Number of errors	0	1	2	3	Total
Frequency	11	6	2	1	20

An estimate of the probability that on the next day no errors will be observed is:

A $\frac{3}{10}$ **B** $\frac{9}{20}$ **C** $\frac{11}{20}$ **D** $\frac{17}{20}$ **E** $\frac{3}{20}$

3 For this Venn diagram, $\Pr(A \cup B)$ is equal to:



A $\frac{4}{5}$ **B** $\frac{1}{2}$ **C** $\frac{5}{8}$

D $\frac{1}{4}$ **E** $\frac{1}{10}$

4 15 people like apples or bananas. Of those 15 people, 10 like apples and 3 like both apples and bananas. How many like only apples?

A 5 **B** 3 **C** 13 **D** 7 **E** 10

5 A letter is chosen from each of the words CAN and TOO.
The probability that the pair of letters will not have an O is:

	C	A	N
T	(C, T)	(A, T)	(N, T)
O	(C, O)	(A, O)	(N, O)
O	(C, O)	(A, O)	(N, O)

A $\frac{2}{3}$ **B** $\frac{1}{2}$ **C** $\frac{1}{3}$

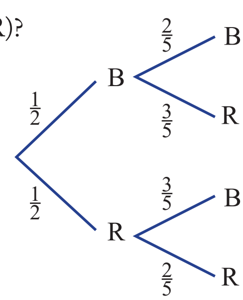
D $\frac{1}{9}$ **E** $\frac{5}{9}$

6 The sets A and B are known to be mutually exclusive. Which of the following is therefore true?

A $\Pr(A) = \Pr(B)$ **B** $\Pr(A \cap B) = 0$ **C** $\Pr(A) = 0$

D $\Pr(A \cap B) = 1$ **E** $\Pr(A \cup B) = 0$

7 For this tree diagram, what is the probability of the outcome (B, R)?



A $\frac{1}{5}$ **B** $\frac{3}{10}$ **C** $\frac{3}{7}$

D $\frac{1}{10}$ **E** $\frac{6}{11}$

8 For this two-way table, $\Pr(A \cap B)$ is:

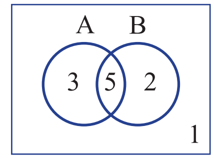
A $\frac{2}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{7}$

D $\frac{1}{3}$ **E** $\frac{2}{7}$

	A	A′	
B		1	3
B′			4
		4	

ISBN: 9781107687028

9 For this Venn diagram, $\Pr(A|B)$ is:



A $\frac{5}{7}$ **B** $\frac{5}{2}$ **C** $\frac{5}{8}$ **D** $\frac{5}{3}$ **E** $\frac{3}{11}$

10 Two events are independent if:

A $\Pr(A) = \Pr(B)$ **B** $\Pr(A') = 0$ **C** $\Pr(A \cup B) = 0$
D $\Pr(A|B) = \Pr(B)$ **E** $\Pr(A) = \Pr(A|B)$

Short-answer questions

1 A 6-sided die is tossed once. Find:

a Pr(4) **b** Pr(even) **c** Pr(at least 3)

2 A letter is chosen from the word INTEREST. Find the probability that the letter will be:

a I **b** E **c** a vowel
d not a vowel **e** E or T

3 An engineer inspects 20 houses in a street for cracks. The results are summarised in this table.

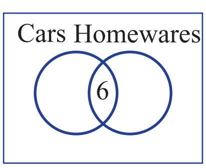
Number of cracks	0	1	2	3	4
Frequency	8	5	4	2	1

a From these results, estimate, the probability that the next house inspected in the street will have the following number of cracks:
i 0 **ii** 1 **iii** 2 **iv** 3 **v** 4

b Estimate the probability that the next house will have:
i at least one crack **ii** no more than 2 cracks

4 Of 36 people, 18 have an interest in cars, 11 have an interest in homewares and 6 have an interest in both cars and homewares.

a Complete this Venn diagram. **b** Complete this two-way table.



	C	C′	
H	6		
H′			

c State the number of people who do not have an interest in either cars or homewares.

d If a person is chosen at random from the group, find the probability that the person will:
i have an interest in cars and homewares
ii have an interest in homewares only
iii not have any interest in cars

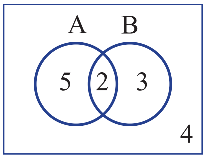
5 All 26 birds in a bird cage have clipped wings and/or a tag. In total, 18 birds have tags, 14 have clipped wings and 6 have both clipped wings and a tag.

a Find the number of birds that have only clipped wings.
b Find the probability that a bird chosen at random will have a tag only.

ISBN: 9781107687028

6 For these probability diagrams, find $\Pr(A|B)$.

a



b

	A	A′	
B	1	4	5
B′	2	2	4
	3	6	9

7 A letter is chosen at random from the word HAPPY and a second letter is chosen from the word HEY.

a List the sample space by completing this table.

b State the total number of outcomes.

c Find the probability that the two letters chosen will be:

i H then E **ii** the same **iii** not the same

	H	A	P	P	Y
H	(H, H)	(A, H)	(P, H)		
E					
Y					

8 A 4-sided die is tossed twice and the total is noted.

a Complete this tree diagram to list the sample space.

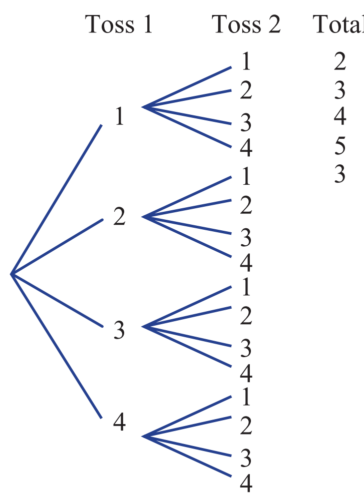
b Find these probabilities.

i $\Pr(2)$

ii $\Pr(5)$

iii $\Pr(1)$

iv $\Pr(\text{not } 1)$



9 Two people are selected from a group of 2 females and 3 males without replacement. Use a tree diagram to find the probability of selecting:

a a female on the first selection

b a male on the second selection given that a female was chosen on the first selection

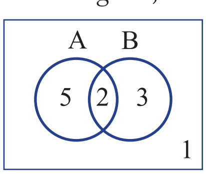
c two males

d one male

e at least one female

10 For each diagram, find $\Pr(A)$ and $\Pr(A|B)$, then decide if events A and B are independent.

a



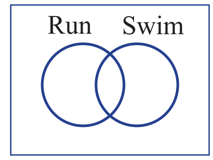
b

	A	A′	
B	3		6
B′			
	5		10

ISBN: 9781107687028

Extended-response questions

1 Of 15 people surveyed to find out if they run or swim for exercise, 6 said they run, 4 said they swim and 3 said they both run and swim.



a How many people neither run nor swim?

b One of the 15 people is selected at random. Find the probability that they:

i run or swim **ii** only swim

c Represent the information in a two-way table.

d Find the probability that:

i a person swims, given that they run

ii a person runs, given that they swim

2 A bakery sells three types of bread: raisin (R) at \$2 each, sourdough (S) at \$3 each, and white (W) at \$1.50 each. Judy is in a hurry. She randomly selects 2 loaves and quickly takes them to the counter. (Assume an unlimited loaf supply.)

a Complete this table showing the possible combination of loaves that Judy could have selected.

		1st		
		R	**S**	**W**
2nd	**R**	(R, R)	(S, R)	(W, R)
	S			
	W			

b Find the probability that Judy selects:

i 2 raisin loaves **ii** 2 loaves that are the same

iii at least one white loaf **iv** not a sourdough loaf

Judy has only \$4 in her purse.

c How many different combinations of bread will Judy be able to afford?

d Find the probability that Judy will not be able to afford her two chosen loaves.

ISBN: 9781107687028 Cambridge University Press

Australian Curriculum
Statistics and Probability
Data representation and interpretation

chapter

5 Statistics

What you will learn

5.1 Sorting data: frequency tables, column graphs and histograms
5.2 Graphical forms: dot plots and stem-and-leaf plots
5.3 Range and measures of centre
5.4 Quartiles and outliers
5.5 Boxplots
5.6 Time series data
5.7 Bivariate data and scatter plots
5.8 Line of best fit by eye

Census Australia

Data is collected in a variety of ways.

One large-scale collection of data is the Census that is run by the Australian Bureau of Statistics every five years. The most recent Census was held on 9 August 2011.

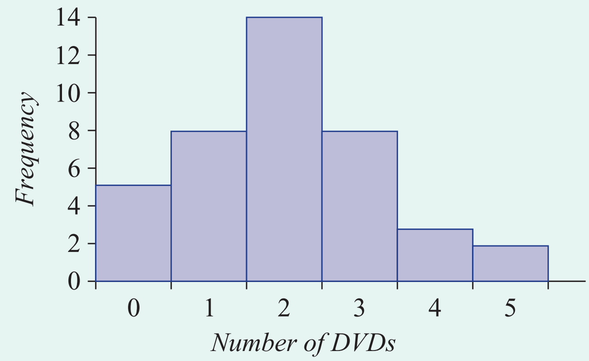
The Census counts all people who spend census night in Australia, but it is not only used to measure the number of people in the country. It also collects data on people's employment, citizenship, education level, use of public transport and a range of other topics.

This data is used by the Australian Government to allocate billions of dollars in funds. The population estimates are also used to determine the number of seats that each state and territory holds in the House of Representatives in Parliament.

ISBN: 9781107687028

Pre-test

1 The number of DVDs rented per person at a store is shown in this graph.



a How many customers rented three DVDs?
b How many customers were surveyed?
c How many DVDs were rented during the survey?
d How many customers rented fewer than two DVDs?

2 This table shows the frequency of scores in a test.

Score	Frequency
0–	2
20–	3
40–	6
60–	12
80–100	7

a How many scores were in the 40 to less than 60 range?
b How many scores were:
i at least 60? **ii** less than 80?
c How many scores were there in total?
d What percentage of scores were in the 20 to less than 40 range?

3 Calculate:
a $\frac{6+10}{2}$ **b** $\frac{8+9}{2}$ **c** $\frac{2+4+5+9}{4}$ **d** $\frac{3+5+8+10+14}{5}$

4 For each of these data sets find:
i the mean ('average')
ii the mode (most frequent)
iii the median (middle value of ordered data)
iv the range (difference between highest and lowest)
a 38, 41, 41, 47, 58
b 2, 2, 2, 4, 6, 6, 7, 9, 10, 12

5 This stem-and-leaf plot shows the weight in grams of some small calculators.
a How many calculators are represented in the plot?
b What is the mode (most frequent)?
c What is the minimum calculator weight and maximum weight?
d Find the range (maximum value − minimum value).

Stem	Leaf
9	8
10	2 6
11	1 1 4 9
12	3 6
13	8 9 9
14	0 2 5

13|6 means 136 grams

ISBN: 9781107687028

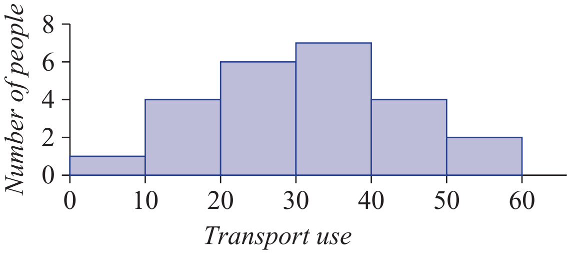
5.1 Sorting data: frequency tables, column graphs and histograms

Data is collected in a number of ways, including surveys, experiments, recording the performance of a sportsperson or just counting. As a simple list, data can be difficult to interpret. Sorting the data into a frequency table allows us to make sense of it and draw conclusions from it.

Statistical graphs are an essential part of the analysis and representation of data. By looking at statistical graphs, we can draw conclusions about the numbers or categories in the data set.

Let's start: Public transport analysis

A survey was carried out to find out how many times people in a particular group had used public transport in the last month. The results are shown in this histogram.



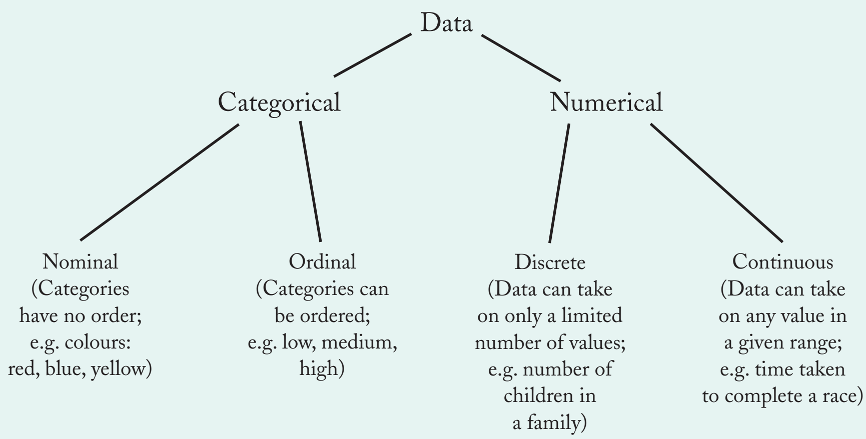
Discuss what the histogram tells you about this group of people and their use of public transport. You may wish to include these points:

- How many people were surveyed?
- Is the data symmetrical or skewed?
- Is it possible to work out all the data values from this graph?
- Do you think these people were selected from a group in your own community? Give reasons.

ISBN: 9781107687028 Cambridge University Press

Key ideas

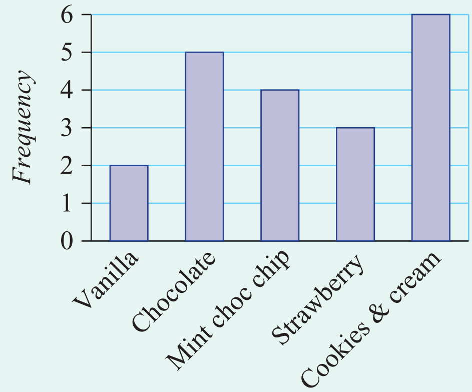
- **Statistical data** can be divided into sub-groups.



- A **frequency table** displays data by showing the number of values within a set of categories or class intervals. It may include a tally column to help count the data.

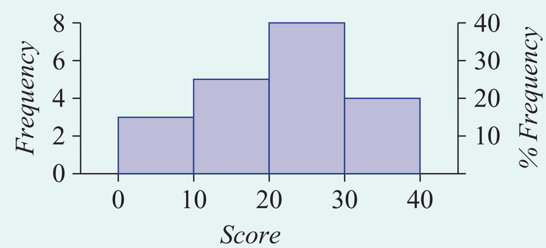
Favourite ice cream flavour	Tally	Frequency
Vanilla	||	2
Chocolate	~~||||~~	5
Mint choc chip	||||	4
Strawberry	|||	3
Cookies and cream	~~||||~~ |	6

- A **column graph** can be used for a single set of categorical or discrete data



- **Histograms** can be used for grouped discrete or continuous numerical data. The frequency of particular class intervals is recorded.
  - The interval 10– (below) includes all numbers from 10 (including 10) to less than 20.
  - The percentage frequency is calculated as $\%\text{Frequency} = \dfrac{\text{Frequency}}{\text{Total}} \times 100\%$.

Class interval	Frequency	Percentage frequency
0–	3	$\frac{3}{20} \times 100 = 15\%$
10–	5	$\frac{5}{20} \times 100 = 25\%$
20–	8	40%
30–40	4	20%
Total	20	100%



Statistical data Information gathered by observation, survey or measurement

Frequency table A table showing all possible scores in one column and the frequency of each score in another column

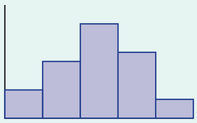
Column graph A graphical representation of a single set of categorical or discrete data, where columns are used to show the frequency of scores

Histogram A special type of column graph with no gaps between the columns; it can represent class intervals

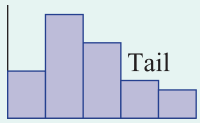
ISBN: 9781107687028

- Data can be symmetrical or skewed.

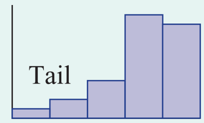
Symmetrical



Positively skewed



Negatively skewed



Exercise 5A

Understanding

1 Classify each set of data as categorical or numerical.

a 4.7, 3.8, 1.6, 9.2, 4.8

b red, blue, yellow, green, blue, red

c low, medium, high, low, low, medium

d 3 g, 7 g, 8 g, 7 g, 4 g, 1 g, 10 g

2 Complete these frequency tables.

In the tally, $\cancel{||||}$ is 5.

a

Car colour	Tally	Frequency
Red	|||	
White	$\cancel{||||}$	
Green	||	
Silver	||	
Total		

b

Class interval	Frequency	Percentage frequency
80–	8	$\frac{8}{50} \times 100 = 16\%$
85–	23	
90–	13	
95–100		
Total	**50**	

3 This frequency histogram shows how many competitors in a car rally race finished within a given time interval.

a How many competitors finished in a time between 150 and 160 minutes?

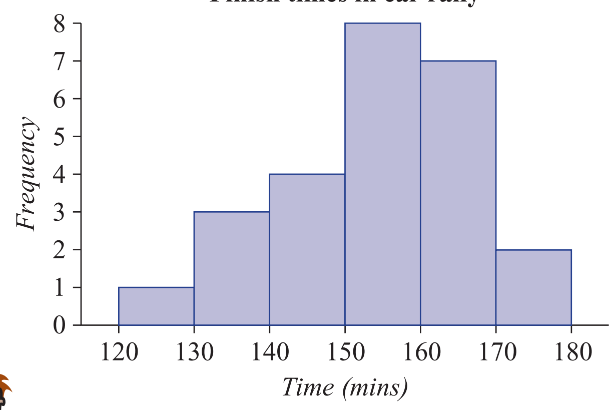
b How many cars were there in the race?

c Determine the following.

i How many competitors finished in fewer than 150 minutes?

ii What percentage of competitors finished in fewer than 150 minutes?

Finish times in car rally



$$\text{Percentage} = \frac{\text{number} < 150}{\text{total}} \times 100$$

ISBN: 9781107687028

Fluency

Example 1 Constructing a frequency table and column graph

Twenty people checking out of a hotel were surveyed on the level of service provided by the hotel staff. The results were:

Poor	First class	Poor	Average	Good
Good	Average	Good	First class	First class
Good	Good	First class	Good	Average
Average	Good	Poor	First class	Good

a Construct a frequency table to record the data, with headings Category, Tally and Frequency.
b Construct a column graph for the data.

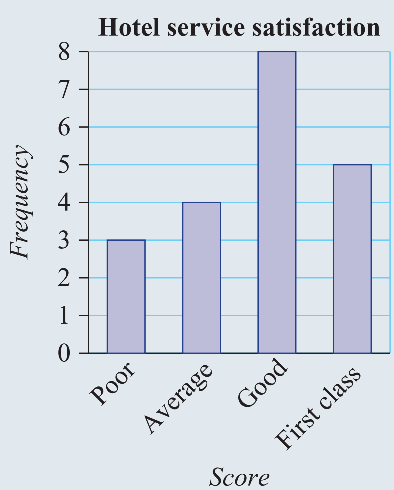
Solution / Explanation

a

Category	Tally	Frequency
Poor	|||	3
Average	||||	4
Good	𝍸 |||	8
First class	𝍸	5
Total	20	20

Construct a table with the headings Category, Tally, Frequency.
Fill in each category shown in the data. Work through the data in order, recording a tally mark (|) next to the category. It is a good idea to tick the data as you go, to keep track.
On the 5th occurrence of a category, place a diagonal line through the tally marks (𝍸). Then start again on the 6th. Do this every five values, as it makes the tally marks easy to count up.
Once all data is recorded, count the tally marks for the frequency.
Check that the frequency total adds up to the number of people surveyed (20).

b



Draw a set of axes with frequency going up to 8.
For each category, draw a column with height up to its frequency value.
Leave gaps between each column.
Give your graph an appropriate heading.

ISBN: 9781107687028

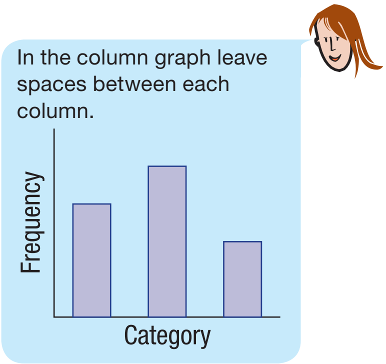
Cambridge University Press

4 For the data below obtained from surveys:

i copy and complete this frequency table

Category	Tally	Frequency
⋮	⋮	⋮

ii construct a column graph for the data and include a heading



a The results from 10 subjects on a student's school report are:

Good	Low	Good	Good	Excellent
Very Low	Low	Good	Good	Low

b The favourite sports of a class of students are:

Football	Tennis	Basketball	Tennis	Football
Netball	Football	Tennis	Football	Basketball
Basketball	Tennis	Netball	Football	Football
Football	Basketball	Football	Netball	Tennis

Example 2 Constructing and analysing a histogram

Twenty people were surveyed to find out how many times they use the internet in a week. The raw data is listed.

21, 19, 5, 10, 15, 18, 31, 40, 32, 25
11, 28, 31, 29, 16, 2, 13, 33, 14, 24

a Organise the data into a frequency table using class intervals of 10. Include a percentage frequency column.

b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.

c Which interval is the most frequent?

d What percentage of people used the internet 20 times or more?

ISBN: 9781107687028 Cambridge University Press

Solution

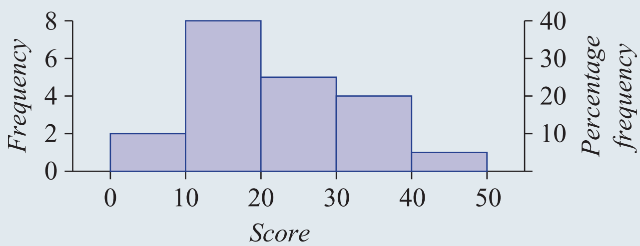
Explanation

a

Class interval	Tally	Frequency	Percentage frequency
0–	||	2	10%
10–	~~||||~~ |||	8	40%
20–	~~||||~~	5	25%
30–	||||	4	20%
40–49	|	1	5%
Total	20	20	100%

Work through the data and place a tally mark in the correct interval each time.
The interval 10– includes all numbers from 10 (including 10) to less than 20, so 10 is in this interval but 20 is not.
Count the tally marks to record the frequency.
Add the frequency column to ensure all 20 values have been recorded.
Calculate each percentage frequency by dividing the frequency by the total (20) and multiplying by 100%; i.e. $\frac{2}{20} \times 100 = 10$.

b Number of times the internet is accessed



Transfer the data from the frequency table to the histogram.
Axis scales are evenly spaced and the histogram bar is placed across the boundaries of the class interval.
There is no space between the bars.

c The 10– interval is the most frequent.

The frequency (8) is highest for this interval. It is the highest bar on the histogram.

d 50% of those surveyed used the internet 20 or more times.

Sum the percentages for the class intervals from 20– and above.
$25 + 20 + 5 = 50$.

5 The maths test results of a class of 25 students were recorded as:

74	65	54	77	85	68	93	59	75
71	82	57	98	73	66	88	76	
92	70	77	65	68	81	79	80	

a Organise the data into a frequency table using class intervals of 10. Include a percentage frequency column.
b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.
c Which interval is the most frequent?
d If an A is awarded for a score of 80 or more, what percentage of the class received an A?

Construct a frequency table like this:

Class interval	Tally	Frequency	Percentage frequency
50–	|||	3	$\frac{\text{freq.}}{\text{total}} \times 100\%$
60–			
70–			
80–			
90–99			
Total			

ISBN: 9781107687028 Cambridge University Press

6 The number of wins scored this season is given for 20 hockey teams. Here is the raw data.

4, 8, 5, 12, 15, 9, 9, 7, 3, 7
10, 11, 1, 9, 13, 0, 6, 4, 12, 5

a Organise the data into a frequency table using class intervals of 5, starting with 0–, then 5– etc. and include a percentage frequency column.
b Construct a histogram for the data showing both the frequency and percentage frequency on the one graph.
c Which interval is the most frequent?
d What percentage of teams scored 5 or more wins?

7 This frequency table displays the way in which 40 people travel to and from work.

Type of transport	Frequency	Percentage frequency
Car	16	
Train	6	
Tram	8	
Walking	5	
Bicycle	2	
Bus	3	
Total	**40**	

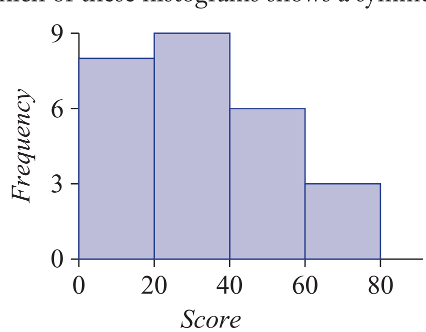
a Copy and complete the table.
b Use the table to find:
- **i** the frequency of people who travel by train
- **ii** the most popular form of transport
- **iii** the percentage of people who travel by car
- **iv** the percentage of people who walk or cycle to work
- **v** the percentage of people who travel by public transport, including trains, buses and trams.

Percentage frequency:
$$= \frac{\text{Frequency}}{\text{Total}} \times 100$$

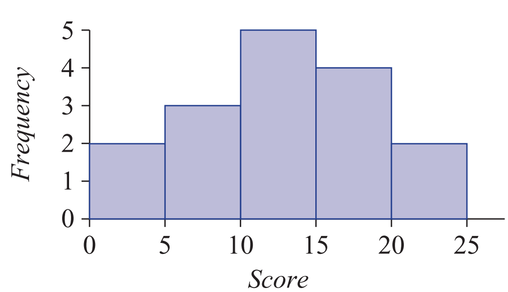
Problem-solving and Reasoning

8 Which of these histograms shows a symmetrical data set and which one shows a skewed data set?

a



b



ISBN: 9781107687028 Cambridge University Press

5A

9 This tally records the number of mice that were weighed and categorised into particular mass intervals for a scientific experiment.

- **a** Construct a table using these column headings: Mass, Frequency and Percentage frequency.
- **b** Find the total number of mice weighed in the experiment.
- **c** State the percentage of mice that were in the 20– gram interval.
- **d** Which was the most common weight interval?
- **e** What percentage of mice were in the most common mass interval?
- **f** What percentage of mice had a mass of 15 grams or more?

Mass (grams)	Tally
10–	|||
15–	𝍸 |
20–	𝍸 𝍸 𝍸 |
25–	𝍸 𝍸 𝍸 𝍸 |
30–34	||||

10 A school orchestra contains four musical sections: strings, woodwind, brass and percussion. The number of students playing in each section is summarised in this tally.

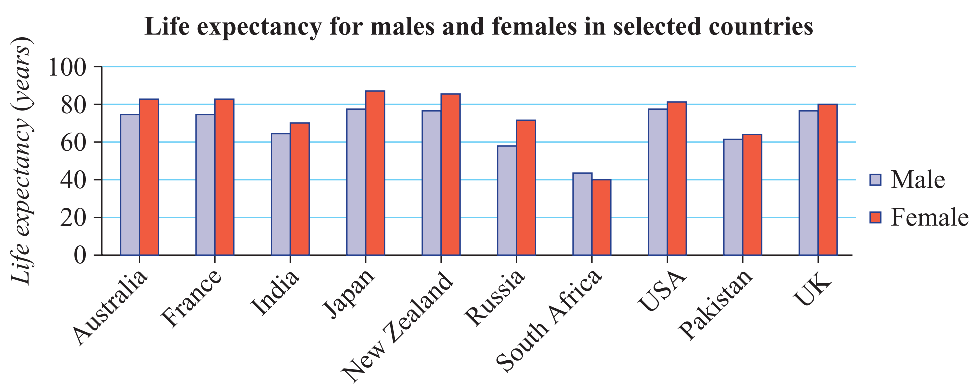
- **a** Construct and complete a percentage frequency table for this data.
- **b** What is the total number of students in the school orchestra?
- **c** What percentage of students play in the string section?
- **d** What percentage of students do not play in the string section?
- **e** If the number of students in the string section increased by three, what would be the percentage of students who play in the percussion section? Round to one decimal place.

Section	Tally
String	𝍸 𝍸 𝍸 𝍸 |
Woodwind	𝍸 |||
Brass	𝍸 ||
Percussion	||||

11 Describe the information that is lost when displaying data using a histogram.

Interpreting further graphical displays

12 The graph shown compares the life expectancy of males and females in 10 different countries. Use the graph to answer the following questions.



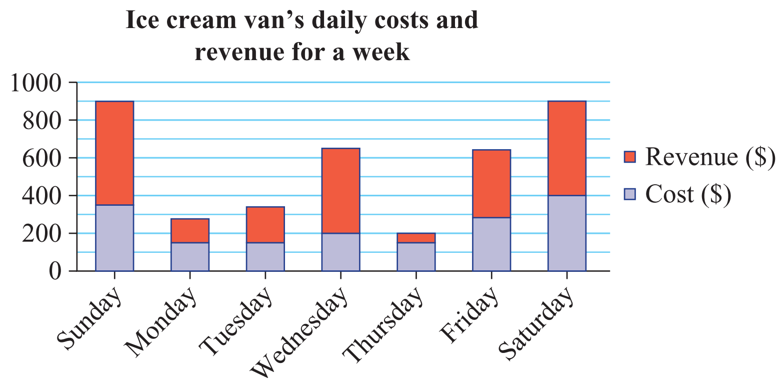
- **a** Which country has the biggest difference in life expectancy for males and females? Approximately how many years is this difference?
- **b** Which country appears to have the smallest difference in life expectancy between males and females?

ISBN: 9781107687028

c From the information in the graph, write a statement comparing the life expectancy of males and females.

d South Africa is clearly below the other countries. Provide some reasons why you think this may be the case.

13 This graph shows the amount spent (Cost) on the purchase and storage of ice cream each day by an ice cream vendor, and the amount of money made from the daily sales of ice cream (Revenue) over the course of a week.



a On which particular days was the cost highest for the purchase and storage of ice cream? Why do you think the vendor chose these days to spend the most?

b Wednesday had the greatest revenue for any weekday. What factors may have led to this?

c Daily profit is determined by the difference in revenue and cost. Identify:

  i on which day the largest profit was made and what this profit was (in dollars)

  ii on which day the vendor suffered the biggest financial loss

d Describe some problems associated with this type of graph.

Using technology 5.1: Using calculators to graph grouped data

This activity is available on the companion website as a printable PDF.

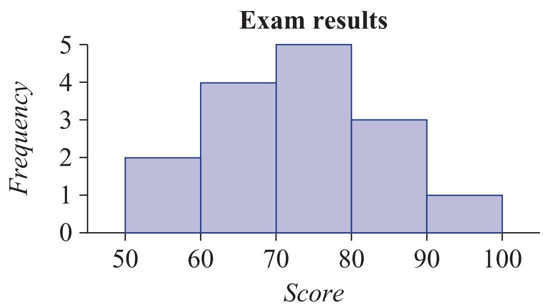
ISBN: 9781107687028 Cambridge University Press

5.2 Graphical forms: dot plots and stem-and-leaf plots

In addition to column graphs, dot plots and stem-and-leaf plots can be used to display categorical or discrete data. They can also display two related sets for comparison. Like a histogram, they help to show how the data is distributed. A stem-and-leaf plot has the advantage of still displaying all the individual data items.

Let's start: Alternate representations

The histogram and stem-and-leaf plot below represent the same set of data. They show the scores achieved by a class in an exam.



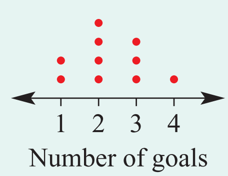
Stem	Leaf
5	1 3
6	5 7 8 8
7	1 2 4 4 6
8	3 4 7
9	6

6|8 means 68

- Describe the similarities in what the two graphs display.
- What does the stem-and-leaf provide that the histogram does not? What is the advantage of this?
- Which graph do you prefer?
- Discuss any other types of graphs that could be used to present this data.

Key ideas

- A **dot plot** records the frequency of each category or discrete value in a data set.
  - Each occurence of the value is marked with a dot.



- A **stem-and-leaf plot** displays each value in the data set using a stem number and a leaf number.
  - The data is displayed in two parts: a stem and a leaf.
  - The 'key' tells you how to interpret the stem and leaf parts.
  - The graph is similar to a histogram with class intervals, but the original data values are not lost.
  - The stem-and-leaf plot is ordered to allow for further statistical calculations.

Stem	Leaf
1	0 1 1 5
2	3 7
3	4 4 6
4	2 9

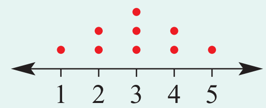
2|3 means 23
↑
key

Dot plot A graph in which each dot represents one score

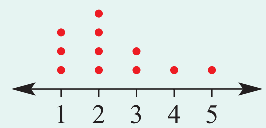
Stem-and-leaf plot A table that lists numbers in order, grouped in rows

ISBN: 9781107687028

- The shape of each of these graphs gives information about the distribution of the data.
  - A graph that is even either side of the centre is symmetrical.



  - A graph that is bunched to one side of the centre is skewed.

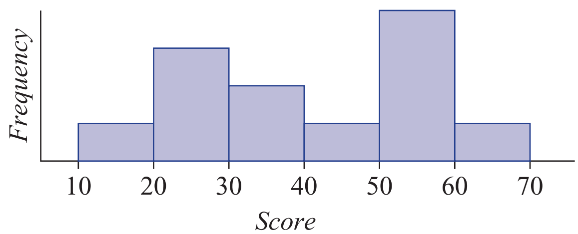


Exercise 5B

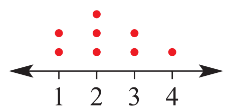
Understanding

1 Name each of these types of graphs.

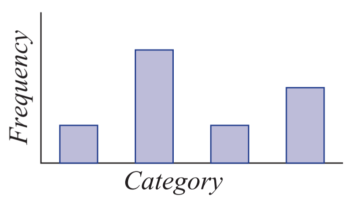
a



b



c



d

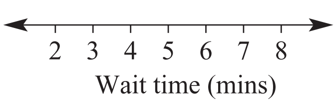
Stem	Leaf
0	1 1 3
1	2 4 7
2	0 2 2 5 8
3	1 3

2|5 means 25

2 A student records the following wait times in minutes for his school bus over 4 school weeks.

5 4 2 8 4 2 7 5 3 3
5 4 2 5 4 5 8 7 2 6

Copy and complete this dot plot of the data.



3 List the data shown in these stem-and-leaf plots.

a

Stem	Leaf
3	2 5
4	1 3 7
5	4 4 6
6	0 2
7	1 1

4|1 means 41

b

Stem	Leaf
0	2 3 7
1	4 4 8 9
2	3 6 6
3	0 5

2|3 means 2.3

Look at the key '4|1 means 41' to see how the stems and leaves go together.

ISBN: 9781107687028

4 Order this stem-and-leaf plot.

Stem	Leaf
12	7 2 3
10	1 4 8 1
13	9 0 2
11	3 0 3 6

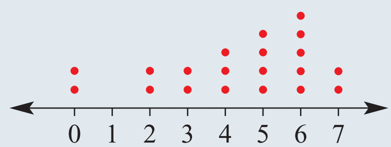
12|2 means 122

Stems and leaves need to be placed in numerical order.

Fluency

Example 3 Interpreting a dot plot

This dot plot shows the number of goals per game scored by a team during the soccer season.



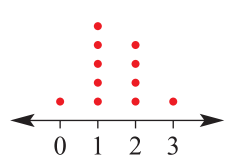
Number of goals per game

- **a** How many games were played?
- **b** What was the most common number of goals per game?
- **c** How many goals were scored for the season?
- **d** Describe the data in the dot plot.

Solution	Explanation
a There were 20 matches played.	Each dot represents a match. Count the number of dots.
b 6 goals in a game occured most often.	The most common number of goals has the most dots.
c $2 \times 0 + 2 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + 2 \times 7$ $= 0 + 4 + 6 + 12 + 20 + 30 + 14$ $= 86$ goals	Count the number of games (dots) for each number of goals and multiply by the number of goals. Add these together.
d Two games resulted in no goals but the data was generally skewed towards a higher number of goals.	Consider the shape of the graph; it is bunched towards the 6 end of the goal scale.

5 A number of families were surveyed to find the number of children in each. The results are shown in this dot plot.

- **a** How many families were surveyed?
- **b** What was the most common number of children in a family?
- **c** How many children were there in total?
- **d** Describe the data in the dot plot.

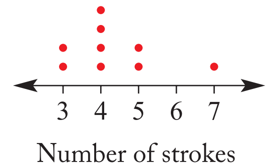


Number of children

4 families had 2 children (4 dots), so that represents 8 children from these families.

ISBN: 9781107687028 Cambridge University Press

6 This dot plot shows the number of strokes a golfer played, each hole, in his round of golf.
- **a** How many holes did he play?
- **b** How many strokes did he play in the round?
- **c** Describe his round of golf.



Example 4 Constructing a stem-and-leaf plot

For the following set of data:
- **a** organise the data into an ordered stem-and-leaf plot
- **b** describe the distribution of the data as symmetrical or skewed

22	62	53	44	35	47	51	64	72
32	43	57	64	70	33	51	68	59

Solution

a

Stem	Leaf
2	2
3	2 3 5
4	3 4 7
5	1 1 3 7 9
6	2 4 4 8
7	0 2

5|1 means 51

Explanation

For two-digit numbers, select the tens value as the stem and the units as the leaves.
The data ranges from 22 to 72, so the graph will need stems 2 to 7.
Work through the data and record the leaves in the order of the data.

Stem	Leaf
2	2
3	5 2 3
4	4 7 3
5	3 1 7 1 9
6	2 4 4 8
7	2 0

51 occurs twice, so the leaf 1 is recorded twice in the 5 stem row.
Once data is recorded, redraw and order the leaves from smallest to largest.
Include a key to explain how the stem and leaf go together; i.e. 5 | 1 means 51.

b The distribution of the data is symmetrical.

The shape of the graph is symmetrical (evenly spread) either side of the centre.

ISBN: 9781107687028

Cambridge University Press

7 For each of the following sets of data:
   i organise the data into an ordered stem-and-leaf plot
   ii describe the distribution of the data as symmetrical or skewed

a 46 22 37 15 26 38 52 24
  31 20 15 37 21 25 26

b 35 16 23 55 38 44 12 48 21 42
  53 36 35 25 40 51 27 31 40 36 32

c 153 121 124 117 125 118 135 137 162
  145 147 119 127 149 116 133 160 158

d 4.9 3.7 4.5 5.8 3.8 4.3 5.2 7.0 4.7
  4.4 5.5 6.5 6.1 3.3 5.4 2.0 6.3 4.8

Remember to include a key such as '4|6 means 46'.

Symmetrical

Stem	Leaf
1	1 2
2	1 2 3
3	1 2 3 4
4	1 2 7
5	3

Skewed

Stem	Leaf
1	2 5 7 8
2	3 4 6 6
3	1 2
4	5

Example 5 Constructing back-to-back stem-and-leaf plots

Two television salesman sell the following number of televisions each week over a 15-week period.

Employee 1
23 38 35 21 45 27 43 36
19 35 49 20 39 58 18

Employee 2
28 32 37 20 30 45 48 17
32 37 29 17 49 40 46

a Construct an ordered back-to-back stem-and-leaf plot.
b Describe the distribution of each employee's sales.

Solution

a

Employee 1 Leaf	Stem	Employee 2 Leaf
9 8	1	7 7
7 3 1 0	2	0 8 9
9 8 6 5 5	3	0 2 2 7 7
9 5 3	4	0 5 6 8 9
8	5	

3|7 means 37

Explanation

Construct an ordered stem-and-leaf plot with Employee 1's sales on the left hand side and Employee 2's sales on the right hand side. Include a key.

b Employee 1's sales are symmetrical, while Employee 2's sales are skewed.

Observe the shape of each employee's graph. If appropriate, use the words symmetrical (spread evenly around the centre) or skewed (bunched to one side of the centre).

ISBN: 9781107687028

8 For the following sets of data:

i draw a back-to-back stem-and-leaf plot

ii comment on the distribution of the two data sets

a Set 1: 61 38 40 53 48 57 64
39 42 59 46 42 53 43

Set 2: 41 55 64 47 35 63 61
52 60 52 56 47 67 32

b Set 1: 176 164 180 168 185 187 195 166 201
199 171 188 175 192 181 172 187 208

Set 2: 190 174 160 170 186 163 182 171
167 187 171 165 194 182 163 178

Problem-solving and Reasoning

9 Two football players, Nick and Jack, compare their personal tallies of the number of goals scored for their team over a 12-match season. Their tallies are as follows.

Game	1	2	3	4	5	6	7	8	9	10	11	12
Nick	0	2	2	0	3	1	2	1	2	3	0	1
Jack	0	0	4	1	0	5	0	3	1	0	4	0

a Draw a dot plot to display Nick's goal-scoring achievement.

b Draw a dot plot to display Jack's goal-scoring achievement.

c How would you describe Nick's scoring habits?

d How would you describe Jack's scoring habits?

10 This stem-and-leaf plot shows the times, in minutes, that Chris has achieved in the last 14 fun runs he competed in.

a What is the difference between his slowest and fastest times?

b Just by looking at the stem-and-leaf plot, what would you estimate to be Chris's average time?

c If Chris records another time of 24.9 minutes, how would this affect your answer to part **b**?

Stem	Leaf
20	5 7
21	1 2 6
22	2 4 6 8
23	4 5 6
24	3 6

22|4 means 22.4 mins

11 The data below shows the distances travelled (in km) by students at an inner-city and an outer-suburb school.

Inner city: 3 10 9 14 21 6
1 12 24 1 19 4

Outer suburb: 12 21 18 9 34 19
24 3 23 41 18 4

a Draw a back-to-back stem-and-leaf plot for the data.

b Comment on the distribution of distances for each school.

c Give a practical reason for the distribution of the data.

ISBN: 9781107687028 Cambridge University Press

5B

12 Determine the possible values of the pronumerals in the following ordered stem-and-leaf plots.

a

Stem	Leaf
1	2 4
2	3 6 9 *b*
a	1 4
4	7 *c* 8

2 | 3 means 2.3

b

Stem	Leaf
20	*a* 1 4
21	2 2 9
22	0 *b* 5 7
23	1 4

22 | 7 means 227

The stems and leaves are ordered from smallest to largest. A leaf can appear more than once.

Splitting stems

13 The back-to-back stem and leaf plot below shows the maximum daily temperature for two cities over a two-week period.

Maximum temperature

City A leaf	Stem	City B leaf
	0	
9 8 8	0*	
4 3 3 1 1 1	1	
8 8 6 6 5	1*	7 9
	2	0 2 2 3 4 4
	2*	5 6 7 7 8
	3	1

1 | 4 means 14
1* | 5 means 15

a Describe the difference between the stems 1 and 1*.

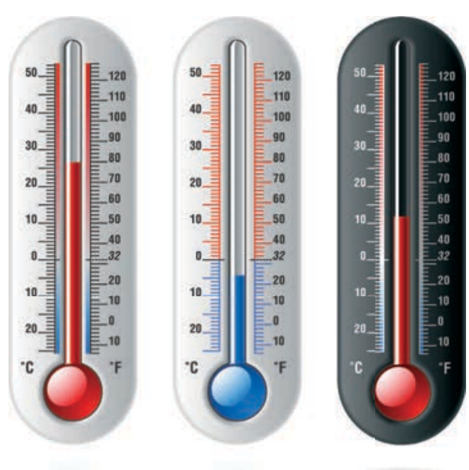
b To which stem would these numbers be allocated?

i 12°C **ii** 5°C

c Why might you use this process of splitting stems, like that used for 1 and 1*?

d Compare and comment on the differences in temperatures between the two cities.

e What might be a reason for these different temperatures?



ISBN: 9781107687028

5.3 Range and measures of centre

In the previous sections you have seen how to summarise data in the form of a frequency table and display data using graphs. Key summary statistics also allow us to describe the data using a single numerical value. The mean ('average'), for example, may be used to describe a student's performance over a series of tests. The median (middle value when data is ordered smallest to largest) is often used when describing the house prices in a suburb. These are termed *measures of centre*. Providing information about the spread of the data is the range, which measures the difference between the maximum and minimum values.

Let's Start: Mean, median or mode?

The following data represents the number of goals scored by Ellie in each game of a 9-game netball season.

24 18 25 16 3 23 27 19 25

It is known that the figures below represent, in some order, the mean, median and mode.

25 20 23

- Without doing any calculations, can you suggest which statistic is which? Explain.
- From the data, what gives an indication that the mean ('average') will be less than the median (middle value)?
- Describe how you would calculate the mean, median and mode from the data values.

Key ideas

- The **mean** (or average) is calculated by summing all the data values and dividing by the total number of values.

$$\text{Mean } (\bar{x}) = \frac{\text{sum of all data values}}{\text{number of data values}}$$

  - The mean is affected by extreme values in the data.
- The **mode** is the most commonly occurring value in the data set.
  - A data set can have two modes (called **bimodal**) or no unique mode at all.
- The **median** is the middle value of a data set when the data is arranged in order.
  - If the data set has an even number of values the median is the average of the two middle values.

    For example,

    2 3 6 8 12

    Median = 6

    4 7 8 10 13 17

    $$\text{Median} = \frac{8+10}{2} = 9$$

  - The mean is affected by extreme values in the data.

Mean An average value calculated by dividing the total of a set of numbers by the number of values

Mode The score that appears most often in a set of numbers

Bimodal When a set of data has two modes

Median The middle score when all the numbers in a set are arranged in order

ISBN: 9781107687028

- The **range** is a measure of how spread out the data is.
  - Range = maximum value − minimum value

Range The difference between the highest and lowest numbers in a set

Exercise 5C

Understanding

1 Use the words from the list below to fill in the missing word in these sentences.

mean, median, mode, bimodal, range

a The ________ is the most frequently occurring value in a data set.
b Dividing the sum of all the data values by the total number of values gives the _______.
c The middle value of a data set ordered from smallest to largest is the _________.
d A data set with two most common values is ________.
e A data set has a maximum value of 7 and a minimum value of 2. The _______ is 5.

2 Calculate the following.

a $\dfrac{1+4+5+8+2}{5}$ **b** $\dfrac{3+7+6+2}{4}$ **c** $\dfrac{3.1+2.3+6.4+1.7+2.5}{5}$

3 Circle the middle value(s) of these ordered data sets.

a 2 4 6 7 8 10 11
b 6 9 10 14 17 20

Recall that an even number of data values will have two middle values.

4 Michael drinks the following number of cups of coffee each day in a week:

4 5 3 6 4 3 3

a How many cups of coffee does he drink in the week (sum of the data values)?
b How many days are in the week (total number of data values)?
c What is the mean number of cups of coffee Michael drinks each day (part **a** ÷ part **b**)?

Fluency

Example 6 Finding the mean, mode and range

For the following data sets, find:

i the mean **ii** the mode **iii** the range

a 2, 4, 5, 8, 8 **b** 3, 15, 12, 9, 12, 15, 6, 8

Solution	Explanation
a **i** Mean $= \dfrac{2+4+5+8+8}{5}$	Mean $= \dfrac{\text{sum of all data values}}{\text{number of data values}}$
$= \dfrac{27}{5}$	Add all the data and divide by the number of values (5).
$= 5.4$	
ii The mode is 8.	The mode is the most common value in the data.
iii Range $= 8 - 2$	Range = maximum value − minimum value
$= 6$	

ISBN: 9781107687028

Cambridge University Press

b i Mean $= \dfrac{3 + 15 + 12 + 9 + 12 + 15 + 6 + 8}{8}$

$= \dfrac{80}{8}$

$= 10$

Mean $= \dfrac{\text{sum of all data values}}{\text{number of data values}}$

Add all the data and divide by the number of values (8).

ii There are two modes, 12 and 15.

The data is bimodal: 12 and 15 are the most common data values.

iii Range = 15 – 3

= 12

Range = maximum value – minimum value

5 For each of the following data sets, find:

i the mean **ii** the mode **iii** the range

a 2 4 5 8 8
b 5 8 10 15 20 12 10 50
c 55 70 75 50 90 85 50 65 90
d 27 30 28 29 24 12
e 2.0 1.9 2.7 2.9 2.6 1.9 2.7 1.9
f 1.7 1.2 1.4 1.6 2.4 1.3

Recall:
Mean $= \dfrac{\text{sum of data values}}{\text{number of data values}}$
Mode is the most common value
Range = maximum – minimum

Example 7 Finding the median

Find the median of each data set.

a 4, 7, 12, 2, 9, 15, 1 **b** 16, 20, 8, 5, 21, 14

Solution	Explanation
a 1 2 4 7 9 12 15 Median = 7	The data must first be ordered from smallest to largest. The median is the middle value. For an odd number of data values, there will be one middle value.
b 5 8 $\underbrace{14\quad 16}$ 20 21 Median $= \dfrac{14 + 16}{2}$ $= 15$	Order the data from smallest to largest. For an even number of data values, there will be two middle values. The median is the average of these two values (the value halfway between the two middle numbers).

6 Find the median of each data set.

a 1 4 7 8 12
b 1 2 2 4 4 7 9
c 11 13 6 10 14 13 11
d 62 77 56 78 64 73 79 75 77
e 2 4 4 5 6 8 8 10 12 22
f 1 2 2 3 7 12 12 18
g 30 36 31 38 27 40
h 2.4 2.0 3.2 2.8 3.5 3.1 3.7 3.9

First make sure that the data is in order. For two middle values, find their average.

ISBN: 9781107687028

Cambridge University Press

7 Nine people watch the following number of hours of television on a weekend.

4 4 6 6 6 8 9 9 11

a Find the mean number of hours of television watched.
b Find the median number of hours of television watched.
c Find the range of the television hours watched.
d What is the mode number of hours of television watched?

Problem-solving and Reasoning

8 Eight students compare the amount of pocket money they receive. The data is as follows.

\$12 \$15 \$12 \$24 \$20 \$8 \$50 \$25

a Find the range of pocket money received.
b Find the median amount of pocket money.
c Find the mean amount of pocket money.
d Why is the mean larger than the median?

Example 8 Calculating summary statistics from a stem-and-leaf plot

For the data in this stem and leaf plot, find:

a the range
b the mode
c the mean
d the median

Stem	Leaf
2	5 8
3	1 2 2 2 6
4	0 3 3
5	2 6

5|2 means 52

Solution	Explanation
a Minimum value = 25 Maximum value = 56 Range = 56 − 25 = 31	In an ordered stem-and-leaf plot the first data item is the minimum and the last is the maximum. Use the key '5 | 2 means 52' to see how to put the stem and leaf together. Range = maximum value − minimum value
b Mode = 32	The mode is the most common value. The leaf 2 appears three times with the stem 3.

ISBN: 9781107687028

Cambridge University Press

c Mean

$$= \frac{24 + 28 + 31 + 32 + 32 + 32 + 36 + 40 + 43 + 43 + 52 + 55}{12}$$

$$= \frac{450}{12}$$

$$= 37.5$$

Form each data value from the graph and add them all together. Then divide by the number of data values in the stem-and-leaf plot.

d $\text{Median} = \frac{32 + 36}{2}$

$= 34$

There is an even number of data values: 12. The median will be the average of the middle two values (the 6th and 7th data values).

9 For the data in these stem-and-leaf plots, find:

i the range
ii the mode
iii the mean (rounded to one decimal place)
iv the median

Use the key to see how the stem and leaf go together.

a

Stem	Leaf
2	1 3 7
3	2 8 9 9
4	4 6

3|2 means 32

b

Stem	Leaf
0	4 4
1	0 2 5 9
2	1 7 8
3	2

2|7 means 27

c

Stem	Leaf
10	1 2 4
11	2 6
12	5

11|6 means 116

d

Stem	Leaf
3	0 0 5
4	2 7
5	1 3 3
6	0 2

3|2 means 3.2

10 This back-to-back stem-and-leaf plot shows the results of two students, Hugh and Mark, on their end-of-year examination in each subject.

a For each student, find:

i the mean
ii the median
iii the range

b Compare the performance of the two students using your answers to part **a**.

Hugh leaf	Stem	Mark leaf
8 8 5	6	4
7 3	7	4 7
5 4 2 1 1	8	2 4 6 8
	9	2 4 5

7|4 means 74%

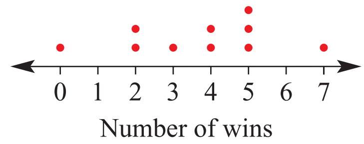
11 A real estate agent recorded the following amounts for the sale of five houses:

\$120 000 \$210 000 \$280 000 \$370 000 \$1 700 000

The mean is \$536 000 and the median is \$280 000.
Which is a better measure of the centre of the five house prices: the mean or the median? Give a reason.

ISBN: 9781107687028

12 This dot plot shows the number of wins recorded by a school sports team in the last 10 eight-game seasons.



a What was the median number of wins?
b What was the mean number of wins?
c The following season, the team records 3 wins. What effect will this have (increase/decrease/no change) on the:
  i median? **ii** mean?

13 Catherine achieves the following scores on her first four maths tests: 64 70 72 74
a What is her mean mark from the maths tests?
b In the fifth and final test, Catherine is hoping to raise her mean mark to 73. What mark does she need on the last test to achieve this?

A mean of 73 from 5 tests will need a five-test total of 73×5.

Moving run average

14 A moving average is determined by calculating the average of all data values up to a particular time or place in the data set.
Consider a batsman in cricket with the following runs scored from 10 completed innings.

Innings	1	2	3	4	5	6	7	8	9	10
Score	26	38	5	10	52	103	75	21	33	0
Moving average	26	32								

In the table, 26 is the average after 1 innings and 32 is the average after 2 innings.
a Complete the table by calculating the moving average for innings 3–10. Round to the nearest whole number where required.
b Plot the score and moving averages for the batsman on the same set of axes, with the innings number on the horizontal axis. Join the points to form two line graphs.
c Describe the behaviour of the:
  i score graph
  ii moving average graph
d Describe the main difference in the behaviour of the two graphs. Give reasons.

ISBN: 9781107687028 Cambridge University Press

5.4 Quartiles and outliers

In addition to the median of a single set of data, there are two related statistics called the upper and lower quartiles. If data are placed in order, then the lower quartile is central to the lower half of the data. The upper quartile is central to the upper half of the data. These quartiles are used to calculate the interquartile range, which helps to describe the spread of the data, and show whether or not any data points do not fit the rest of the data (outliers).

Let's start: House prices

A real estate agent tells you that the median house price for a suburb in 2012 was \$753 000 and the mean was \$948 000.

- Is it possible for the median and the mean to differ by so much?
- Under what circumstances could this occur? Discuss.

Key ideas

- The **five-figure summary** uses the following statistical measures.
  - Minimum value (Min) — the lowest value
  - Lower **quartile** (Q_1) — the number above 25% of the ordered data
  - Median (Q_2) — the middle value, above 50% of the ordered data
  - Upper quartile (Q_3) — the number above 75% of the ordered data
  - Maximum value (Max) — the highest value

Odd number of data values

$$1 \quad \underbrace{2 \quad 2}_{} \quad 3 \;) \; 5 \; (\; \underbrace{6 \quad 6}_{} \quad 7 \quad 9$$

$$Q_1 = \frac{2+2}{2} = 2 \qquad Q_2 \qquad Q_3 = \frac{6+7}{2} = 6.5$$

Even number of data values

$$2 \quad 3 \quad 3 \quad 4 \quad \underbrace{7 \,|\, 8}_{} \quad 8 \quad 9 \quad 9 \quad 9$$

$$Q_1 = 3 \quad Q_2 = 7.5 \quad Q_3 = 9$$

- Another measure of the spread of the data is the **interquartile range (IQR)**
  IQR = upper quartile − lower quartile
  $= Q_3 - Q_1$
- **Outliers** are data elements outside the vicinity of the rest of the data.
  A data point is an outlier if it is either:
  - less than $Q_1 - 1.5 \times \text{IQR}$ or
  - greater than $Q_3 + 1.5 \times \text{IQR}$

Five-figure summary A set of numbers that summarise a set of data: the minimum score, first quartile, median, third quartile and maximum score

Quartiles The three values that separate the scores when a set of ordered data is divided into four equal parts

Interquartile range A measure of spread giving the difference between the upper and lower quartiles

Outlier Any value that is much larger or much smaller than the rest of the data in a set

ISBN: 9781107687028

Exercise 5D

Understanding

1 **a** State the five values that need to be calculated for a five-figure summary.
b Explain the difference between the range and the interquartile range.
c What is an *outlier*?

2 Complete the following for calculating outliers.
a Numbers below _____ $- 1.5 \times \text{IQR}$
b Numbers above $Q_3 +$ ____ $\times \text{IQR}$.

3 This data shows, in order, the numbers of cars owned by 10 families surveyed.
0, 1, 1, 1, 1, 2, 2, 2, 3, 3
a Find the median (the middle value).
b By splitting the data in half, determine:
i the lower quartile Q_1 (middle of lower half)
ii the upper quartile Q_3 (middle of upper half)

4 For the data set with $Q_1 = 3$ and $Q_3 = 8$:
a find $\text{IQR} = Q_3 - Q_1$
b calculate $Q_1 - 1.5 \times \text{IQR}$ and $Q_3 + 1.5 \times \text{IQR}$
c identify the name that would be given to the value 18 in the data set

Fluency

Example 9 Finding quartiles and IQR for an even number of data values

Consider this data set.
2, 2, 4, 5, 6, 8, 10, 13, 16, 20
a Find the upper quartile (Q_3) and the lower quartile (Q_1).
b Determine the IQR.

Solution	Explanation
a 2 2 4 5 6 | 8 10 13 16 20 Q_1 (4), Q_3 (13) $Q_2 = \frac{6+8}{2}$ $= 7$ $Q_1 = 4$ and $Q_3 = 13$	The data is already ordered. Since there is an even number of values, split the data in half to locate the median. Q_1 is the middle value of the lower half: 2 2 4 5 6 Q_3 is the middle value of the upper half.
b $\text{IQR} = 13 - 4$ $= 9$	$\text{IQR} = Q_3 - Q_1$.

ISBN: 9781107687028

5 For these data sets, find:
  i the upper quartile (Q_3) and the lower quartile (Q_1)
  ii the IQR

a 3, 4, 6, 8, 8, 10
b 10, 10, 11, 14, 14, 15, 16, 18, 20, 21
c 41, 49, 53, 58, 59, 62, 62, 65
d 1.2, 1.7, 1.9, 2.2, 2.4, 2.5, 2.9, 3.2

For an even number of data values, split the ordered data in half:

2 4 7 | 8 10 12

Q_1 Q_3

$IQR = Q_3 - Q_1$

Example 10 Finding quartiles and IQR for an odd number of data values

Consider this data set.
2.2, 1.6, 3.0, 2.7, 1.8, 3.6, 3.9, 2.8, 3.8

a Find the upper quartile (Q_3) and the lower quartile (Q_1).
b Determine the IQR.

Solution	Explanation
a 1.6 1.8 | 2.2 2.7 2.8 3.0 3.6 | 3.8 3.9 $Q_2 = 2.8$ $Q_1 = \frac{1.8 + 2.2}{2} = \frac{4.0}{2} = 2.0$ $Q_3 = \frac{3.6 + 3.8}{2} = \frac{7.4}{2} = 3.7$	First order the data and locate the median (Q_2). Split the data in half; i.e. either side of the median. Q_1 is the middle value of the lower half; for two middle values, average the two numbers. Q_3 is the middle value of the upper half.
b $IQR = 3.7 - 2.0 = 1.7$	$IQR = Q_3 - Q_1$

6 For these data sets, find:
  i the upper quartile (Q_3) and the lower quartile (Q_1)
  ii the IQR

a 1, 2, 4, 8, 10, 11, 14
b 10, 7, 14, 2, 5, 8, 3, 9, 2, 12, 1
c 0.9, 1.3, 1.1, 1.2, 1.7, 1.5, 1.9, 1.1, 0.8
d 21, 7, 15, 9, 18, 16, 24, 33, 4, 12, 13, 18, 24

For an odd number of data values, split ordered data in half, leaving out the middle value.

0 2 4 7 9 14 16

Q_1 Q_3

Example 11 Finding the five-figure summary and outliers

The following data set represents the number of flying geese spotted on each day of a 13-day tour of England.

5, 1, 2, 6, 3, 3, 18, 4, 4, 1, 7, 2, 4

a For the data, find:
  i the minimum and maximum number of geese spotted
  ii the median
  iii the upper and lower quartiles
  iv the IQR
b Find any outliers
c Can you give a possible reason for why the outlier occurred?

ISBN: 9781107687028

Solution	Explanation
a **i** Min = 1, max = 18 **ii** 1, 1, 2, 2, 3, 3, 4, 4, 4, 5, 6, 7, 18 $\therefore$ median = 4	Look for the largest and smallest numbers and order the data:
iii Lower quartile $(Q_1) = \dfrac{2+2}{2}$ $= 2$ Upper quartile $(Q_3) = \dfrac{5+6}{2}$ $= 5.5$	1 1 2 | 2 3 3] 4 [4 4 5 | 6 7 18 Q_1 Q_2 Q_3 Since Q_2 falls on a data value, it is not included in the lower or higher halves when Q_1 and Q_3 are calculated.
iv IQR = 5.5 – 2 = 3.5	$\text{IQR} = Q_3 - Q_1$
b $Q_1 - 1.5 \times \text{IQR} = 2 - 1.5 \times 3.5$ $= 2 - 5.25$ $= -3.25$ $Q_3 + 1.5 \times \text{IQR} = 5.5 + 1.5 \times 3.5$ $= 5.5 + 5.25$ $= 10.75$	A data point is an outlier if it is less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$.
$\therefore$ the outlier is 18.	There are no numbers less than -3.25 but 18 is greater than 10.75.
c Perhaps a flock of geese was spotted that day.	

7 The following numbers of cars were counted on each day for 15 days, travelling on a quiet suburban street.

10, 9, 15, 14, 10, 17, 15, 0, 12, 14, 8, 15, 15, 11, 13

a For the given data, find:

i the minimum and maximum number of cars counted

ii the median

iii the lower and upper quartiles (Q_1 and Q_3)

iv the IQR

b Find any outliers.

c Give a possible reason for the outlier.

Outliers:
more than $Q_3 + 1.5 \times \text{IQR}$
or
less than $Q_1 - 1.5 \times \text{IQR}$

8 Summarise the data sets below by finding the:

i minimum and maximum values
ii median (Q_2)
iii lower and upper quartiles (Q_1 and Q_3)
iv IQR
v any outliers

a 4, 5, 10, 7, 5, 14, 8, 5, 9, 9

b 24, 21, 23, 18, 25, 29, 31, 16, 26, 25, 27

c 10, 13, 2, 11, 10, 8, 24, 12, 13, 15, 12

d 3, 6, 10, 11, 17, 4, 4, 1, 8, 4, 10, 8

ISBN: 9781107687028 Cambridge University Press

Problem-solving and Reasoning

9 Twelve different calculators had the following numbers of buttons.
36, 48, 52, 43, 46, 53, 25, 60, 128, 32, 52, 40

a For the given data, find:
- **i** the minimum and maximum number of buttons on the calculators
- **ii** the median
- **iii** the lower and upper quartiles (Q_1 and Q_3)
- **iv** the IQR
- **v** any outliers
- **vi** the mean

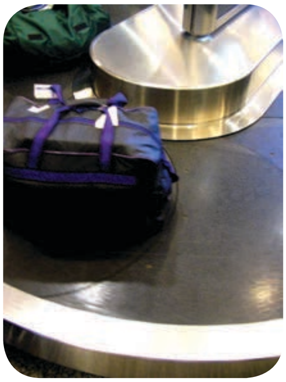
b Which is a better measure of the centre of the data: the mean or the median? Explain.

c Can you give a possible reason why the outlier has occurred?

10 At an airport Paul checks the weight of 20 luggage items. If the weight of a piece of luggage is an outlier, then the contents undergo a further check. The weights in kilograms are:

1 4 5 5 6 7 7 7 8 8
10 10 10 13 15 16 17 19 30 31

How many luggage items will undergo a further check?



11 The prices of nine fridges are displayed in a sale catalogue. They are:

\$350 \$1000 \$850 \$900 \$1100 \$1200 \$1100 \$1000 \$1700

How many of the fridge prices could be considered outliers?

12 For the data in this stem-and-leaf plot, find:

a the IQR

b any outliers

c any outliers if the number 32 was added to the list

Stem	Leaf
0	1
1	6 8
2	0 4 6 8
3	0

2|4 means 24

Some research

13 Use the internet to search for data about a topic that interests you. Try to choose a single set of data that includes between 15 and 50 values.

a Organise the data using:
- **i** a stem-and-leaf plot
- **ii** a frequency table and histogram

b Find the mean and the median.

c Find the range and the interquartile range.

d Write a brief report describing the centre and spread of the data, referring to parts **a** to **c** above.

e Present your findings to your class or a partner.

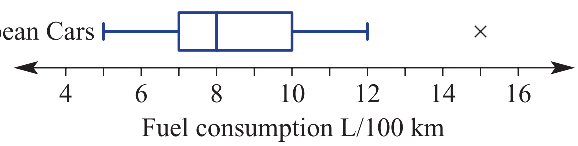
ISBN: 9781107687028

5.5 Boxplots

The five-figure summary (Min, Q_1, Q_2, Q_3, Max) can be represented in graphical form as a boxplot. Boxplots are graphs that summarise single data sets. They clearly display the minimum and maximum values, the median, the quartiles and any outliers. Q_1, Q_2 and Q_3 divide the data into quarters. Boxplots also give a clear indication of how data is spread, as the IQR (interquartile range) is shown by the width of the central box.

Let's start: Fuel consumption

This boxplot summarises the average fuel consumption (litres per 100 km) for a group of European-made cars.

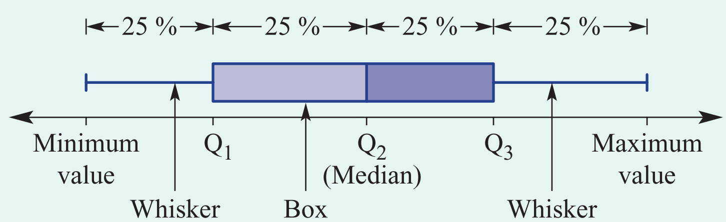


- What does each part of the boxplot represent in terms of the five-figure summary?
- What do you think the cross (×) represents?
- Describe how you can use the boxplot to find the IQR.
- What would you expect the fuel consumption to be above for the top 25% of cars?

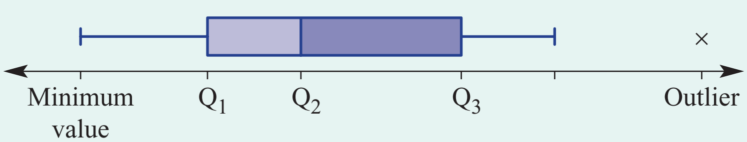
Key ideas

Boxplot A diagram using rectangles and ranges to show the spread of a set of data, using five important values

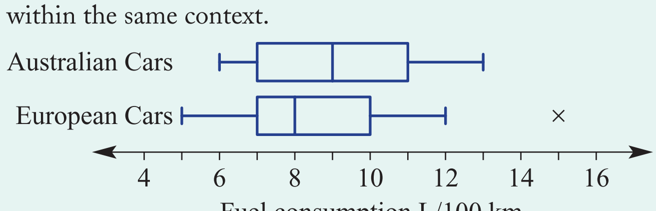
- A **boxplot** (also called a box-and-whisker plot) can be used to summarise a data set. It displays the five figure summary (Min, Q_1, Q_2, Q_3, Max), as shown.



- An outlier is marked with a cross (×).
  – An outlier is greater than $Q_3 + 1.5 \times \text{IQR}$ or less than $Q_1 - 1.5 \times \text{IQR}$.
  – $\text{IQR} = Q_3 - Q_1$.
  – The whiskers stretch to the lowest and highest data values that are not outliers.



- Parallel boxplots are boxplots drawn on the same scale. They are used to compare data sets within the same context.



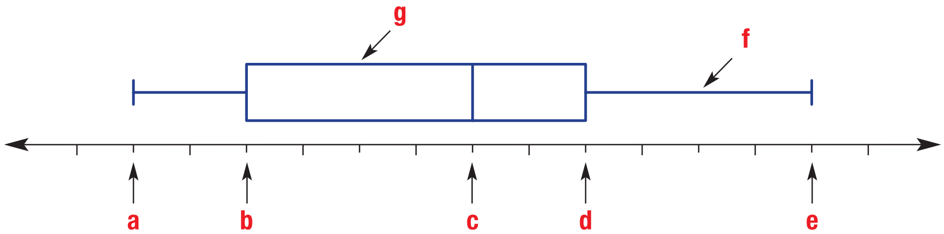
ISBN: 9781107687028

Cambridge University Press

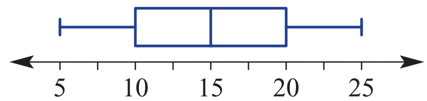
Exercise 5E

Understanding

1 Label the parts of the boxplot below.



2 For this simple boxplot, find:

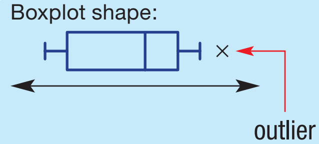


- **a** the median (Q_2)
- **b** the minimum
- **c** the maximum
- **d** the range
- **e** the lower quartile (Q_1)
- **f** the upper quartile (Q_3)
- **g** the interquartile range (IQR)

3 Construct a boxplot showing these features.

- **a** Min = 1, $Q_1 = 3$, $Q_2 = 4$, $Q_3 = 7$, max = 8
- **b** Outlier = 5, minimum above outlier = 10, $Q_1 = 12$, $Q_2 = 14$, $Q_3 = 15$, max = 17

Boxplot shape:



Include an even scale.

4 Select from the list below to fill in the blanks.

Minimum, Q_1, Q_2, Q_3, Maximum

- **a** The top 25% of data is above ________.
- **b** The middle 50% of data is between _____ and _____.
- **c** The lowest or first 25% of data is between the _____ and _____.
- **d** The highest or last 25% of data is between _____ and the _____.

ISBN: 9781107687028

5E

Fluency

Example 12 Constructing boxplots with no outliers

Consider the given data set.

12 26 14 11 15 10 18 17 21 27

a Find the five-figure summary (the minimum, lower quartile (Q_1), median (Q_2), upper quartile (Q_3) and the maximum).

b Draw a boxplot to summarise the data.

Solution

a 10 11 12 14 15 | 17 18 21 26 27

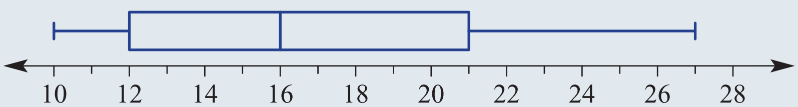
min: 10; Q_1: 12; $Q_2 = \dfrac{15 + 17}{2} = 16$; Q_3: 21; max: 27

Minimum = 10

$Q_1 = 12$, $Q_2 = 16$, $Q_3 = 21$

Maximum = 27

b



Explanation

a Order the data from smallest to largest.

Locate the median (Q_2) first. For 10 data values there are two middle values, 15 and 17. Average these to find Q_2.

Split the data in half at the median. Q_1 is the middle value of the lower half. Q_3 is the middle value of the upper half. The minimum is the smallest value and the maximum the largest value.

b Draw an even scale covering the minimum and maximum values.

Mark the minimum (10), Q_1 (12), Q_2 (16), Q_3 (21) and the maximum (27) to draw the boxplot.

5 For the given data sets:

i find the five-figure summary (minimum, Q_1, Q_2, Q_3, maximum)

ii draw a boxplot to summarise the data

a 11, 15, 18, 17, 1, 2, 8, 12, 19, 15

b 0, 1, 5, 4, 4, 4, 2, 3, 3, 1, 4, 3

c 124, 118, 119, 117, 120, 120, 121, 118, 122

d 62, 85, 20, 34, 40, 66, 47, 82, 25, 32, 28, 49, 41, 30, 22

Example 13 Constructing boxplots with outliers

Consider the given data set.

5, 9, 4, 3, 5, 6, 6, 5, 7, 12, 2, 3, 5

a Determine the quartiles Q_1, Q_2 and Q_3.

b Determine whether any outliers exist.

c Draw a boxplot to summarise the data, marking outliers if they exist.

ISBN: 9781107687028

Cambridge University Press

Solution

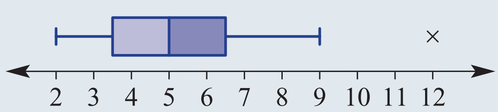
a $\boxed{2 \quad 3 \quad \underbrace{3 \quad 4}_{\uparrow\ Q_1} \quad 5 \quad 5} \quad \underset{\uparrow\ Q_2}{5} \quad \boxed{5 \quad 6 \quad \underbrace{6 \quad 7}_{\uparrow\ Q_3} \quad 9 \quad 12}$

$Q_1 = \dfrac{3+4}{2}$
$= 3.5$

$Q_3 = \dfrac{6+7}{2}$
$= 6.5$

b $\text{IQR} = 6.5 - 3.5 = 3$
$Q_1 - 1.5 \times \text{IQR} = 3.5 - 1.5 \times 3$
$= -1$
$Q_3 + 1.5 \times \text{IQR} = 6.5 + 1.5 \times 3$
$= 11$
$\therefore$ 12 is an outlier.

c



Explanation

a Order the data to help find the quartiles.
Locate the median Q_2 (the middle value), then split the data in half above and below this value.
Q_1 is the middle value of the lower half and Q_3 the middle value of the upper half. Average the two middle values to find the median.

b Determine $\text{IQR} = Q_3 - Q_1$.
Check for any outliers; i.e. numbers below $Q_1 - 1.5 \times \text{IQR}$ or above $Q_3 + 1.5 \times \text{IQR}$.
There is no data below -1 but $12 > 11$.

c Draw a line and mark in a uniform scale reaching from 2 to 12. Sketch the boxplot by marking the minimum 2 and the outlier 12, and Q_1, Q_2 and Q_3. The end of the five-point summary is the nearest value below 11; i.e. 9.

6 Consider the data sets below.
- **i** Determine the quartiles Q_1, Q_2 and Q_3.
- **ii** Determine whether any outliers exist.
- **iii** Draw a boxplot to summarise the data, marking outliers if they exist.

a 4, 6, 5, 2, 3, 4, 4, 13, 8, 7, 6
b 1.8, 1.7, 1.8, 1.9, 1.6, 1.8, 2.0, 1.1, 1.4, 1.9, 2.2
c 21, 23, 18, 11, 16, 19, 24, 21, 23, 22, 20, 31, 26, 22
d 37, 48, 52, 51, 51, 42, 48, 47, 39, 41, 65

Outliers:
more than $Q_3 + 1.5 \times \text{IQR}$
or
less than $Q_1 - 1.5 \times \text{IQR}$.
Mark with a X.
The next value above or below an outlier is used as the new end of the whisker.

Problem-solving and Reasoning

7 A butcher records the weight (in kilograms) of a dozen parcels of sausages sold on one morning.

1.6 1.9 2.0 2.0 2.1 2.2
2.2 2.4 2.5 2.7 3.8 3.9

a Write down the value of:
- **i** minimum
- **ii** Q_1
- **iii** Q_2
- **iv** Q_3
- **v** maximum
- **vi** IQR

b Find any outliers.
c Draw a boxplot for the weight of the parcels of sausages.

ISBN: 9781107687028

Cambridge University Press

8 Phillip the gardener records the number of days that it takes for 11 special bulbs to germinate. The results are:

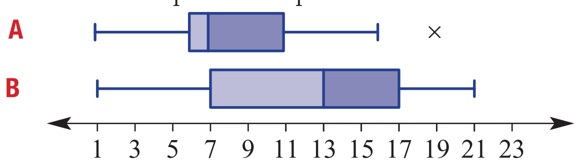
8 14 15 15 16 16 16 17 19 19 24

a Write down the value of:
i minimum **ii** Q_1 **iii** Q_2 **iv** Q_3 **v** maximum **vi** IQR

b Are there any outliers? If so, what are they?

c Draw a boxplot for the number of days it takes for the bulbs to germinate.

9 Consider these parallel boxplots A and B.



Parallel boxplots are two boxplots that can be compared using the same scale.

Compare the boxplots at each point of the five-figure summary.

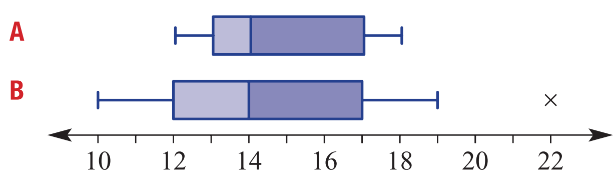
a What statistical measure do these boxplots have in common?

b Which data set (A or B) has a wider range of values?

c Find the IQR for: **i** data set A **ii** data set B

d How would you describe the main difference between the two sets of data from which the parallel boxplots have been drawn?

10 Two data sets can be compared using parallel boxplots on the same scale, as shown below.



a What statistical measures do these boxplots have in common?

b Which data set (A or B) has a wider range of values?

c Find the IQR for:
i data set A **ii** data set B

d How would you describe the main difference between the two sets of data from which the parallel boxplots have been drawn?

Creating parallel boxplots

11 Fifteen essays were marked for spelling errors by a particular examiner and the following numbers of spelling errors were counted.

3, 2, 4, 6, 8, 4, 6, 7, 6, 1, 7, 12, 7, 3, 8

The same 15 essays were marked for spelling errors by a second examiner and the following numbers of spelling errors were counted.

12, 7, 9, 11, 15, 5, 14, 16, 9, 11, 8, 13, 14, 15, 13

a Draw parallel boxplots for the data.

b Do you believe there was a major difference in the way the essays were marked by the two examiners? If yes, describe this difference.

Using technology 5.5: Using calculators to draw box plots

This activity is available on the companion website as a printable PDF.

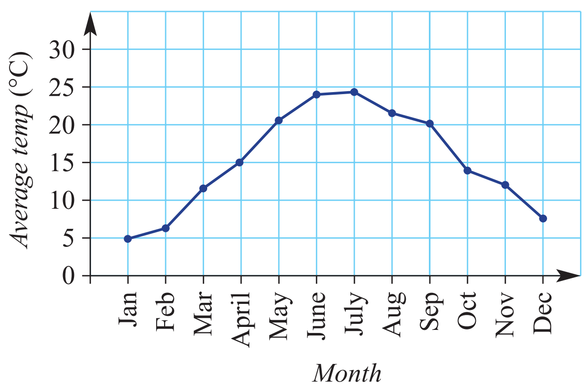
ISBN: 9781107687028 Cambridge University Press

5.6 Time series data

A time series is a sequence of data values that are recorded at regular time intervals. Examples include temperature recorded on the hour, speed recorded every second, population recorded every year and profit recorded every month. A line graph can be used to represent time series data. This can help to analyse the data, describe trends and make predictions about the future.

Let's start: Changing temperatures

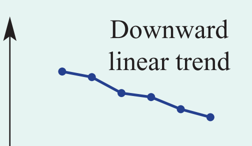
The average monthly maximum temperature for a city is illustrated by this graph.

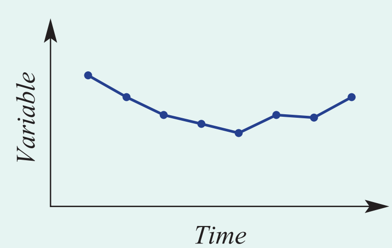


- Describe the trend in the data at different times of the year.
- Explain why the average maximum temperature for December is close to the average maximum temperature for January.
- Do you think this graph is for an Australian city? Explain.
- If another year of temperatures was included on this graph, what would you expect the shape of the graph to look like?
- Do you think this city is in the northern hemisphere or the southern hemisphere? Give a reason.

Key ideas

- **Time series data** are recorded at regular time intervals.
- The graph or plot of a time series uses:
  - time on the horizontal axis
  - line segments connecting points on the graph.
- If the time series plot results in points being on or near a straight line, then we say that the trend is linear.





Time series data A set of data collected in sequence over a period of time

ISBN: 9781107687028

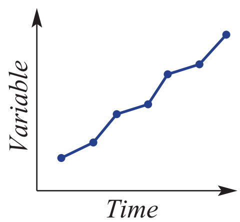
Cambridge University Press

Exercise 5F

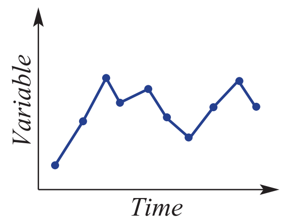
Understanding

1 Describe the following time series plots as having a linear (straight line) trend, non-linear trend (a curve) or no trend.

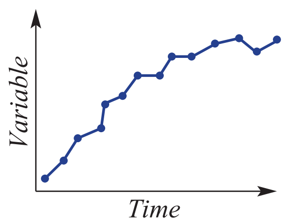
a



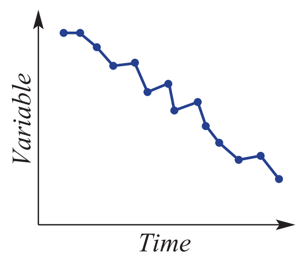
b



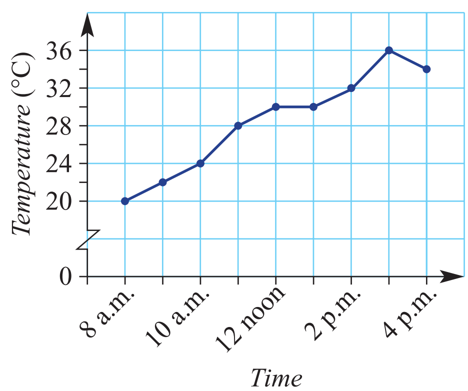
c



d



2 This time series graph shows the temperature over the course of 8 hours of a day.



- **a** State the temperature at:
  - **i** 8 a.m.
  - **ii** 12 noon
  - **iii** 1 p.m.
  - **iv** 4 p.m.
- **b** What was the maximum temperature?
- **c** During what times did the temperature:
  - **i** stay the same?
  - **ii** decrease?
- **d** Describe the general trend in the temperature for the 8 hours.

ISBN: 9781107687028

Fluency

Example 14 Plotting and interpreting a time series plot

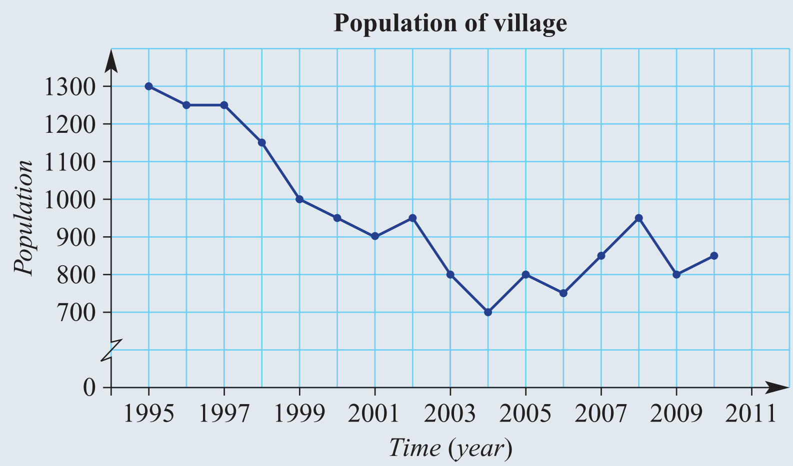
The approximate population of a small town was recorded from 1995 to 2010.

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Population	1300	1250	1250	1150	1000	950	900	950	800	700	800	750	850	950	800	850

a Plot the time series.
b Describe the trend in the data over the 16 years.

Solution

a



b The population declines steadily for the first 10 years. The population rises and falls in the last 6 years, resulting in a slight upwards trend.

Explanation

Use time on the horizontal axis. Break the y-axis so as to not include 0–700. Label an even scale on each axis. Join points with line segments.

Interpret the overall rise and fall of the lines on the graph.

3 The approximate population of a small town is recorded from 2000 to 2010.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Population	550	500	550	600	700	650	750	750	850	950	900

a Plot the time series graph. Break the y-axis so it does not include 0–500.
b Describe the general trend in the data over the 11 years.
c For the 11 years, what was the:
  i minimum population? **ii** maximum population?

The year will be on the horizontal axis. Place population on the vertical axis.

The vertical axis will need to range from 500 to 950. A scale going up in 100s would suit.

ISBN: 9781107687028

Cambridge University Press

4 A company's share price over 12 months is recorded in this table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Price ($)	1.30	1.32	1.35	1.34	1.40	1.43	1.40	1.38	1.30	1.25	1.22	1.23

The scale on the vertical axis will need to include from $1.20 to $1.43. Choose an appropriate scale.

a Plot the time series graph. Break the y-axis to exclude values from $0 to $1.20.
b Describe the way in which the share price has changed over the 12 months.
c What is the difference between the maximum and minimum share price in the 12 months?

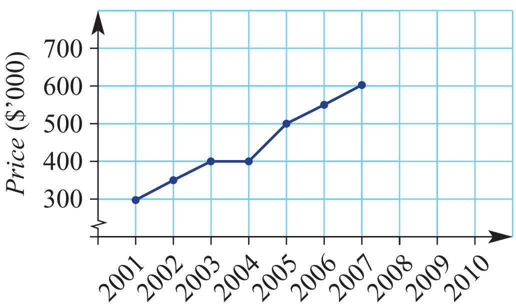
5 The pass rate (%) for a particular examination is given in a table over 10 years.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Pass rate (%)	74	71	73	79	85	84	87	81	84	83

a Plot the time series graph for the 10 years.
b Describe the way in which the pass rate for the examination has changed in the given time period.
c In what year was the pass rate a maximum?
d By how much had the pass rate improved from 2000 to 2004?

Problem-solving and Reasoning

6 This time series plot shows the upwards trend of house prices in an Adelaide suburb over 7 years from 2001 to 2007.



Recall that a linear trend has the points on or near a straight line.

a Would you say that the general trend in house prices is linear or non-linear?
b Assuming that the trend in house prices continued for this suburb, what would you expect the house price to have been in:
i 2008? **ii** 2010?

7 The following data shows the monthly sales of strawberries ($'000s) for a particular year.

$'000s means 22 represents $22 000

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales ($'000s)	22	14	9	11	12	9	7	9	8	10	18	25

a Plot the time series graph for the year.
b Describe any trends in the data over the year.
c Give a reason why you think the trends you observed may have occurred.

ISBN: 9781107687028

8 The two top-selling book stores for a company list their sales figures for the first six months of the year. Sales amounts are in thousands of dollars.

	July	August	September	October	November	December
City Central ($'000)	12	13	12	10	11	13
Southbank ($'000)	17	19	16	12	13	9

a What was the difference in the sales volume for:
  i August? **ii** December?

b In how many months did the City Central store sell more books than the Southbank store?

c Construct a time series plot for both stores on the same set of axes.

d Describe the trend of sales for the 6 months for:
  i City Central **ii** Southbank

e Based on the trend for the sales for the Southbank store, what would you expect the approximate sales volume to be in January?

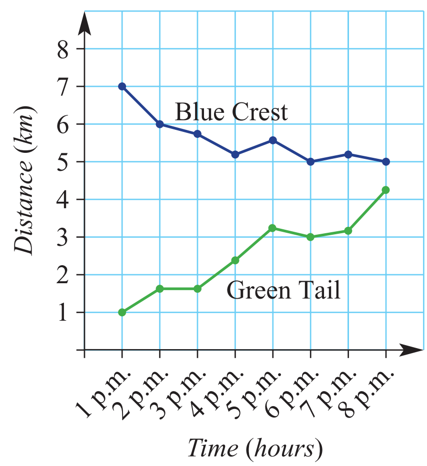
Use different colours for the two line graphs.

9 Two pigeons (Green Tail and Blue Crest) each have a beacon that communicates with a recording machine. The distance of each pigeon from the machine is recorded every hour for 8 hours.

a State the distance from the machine at 3 p.m. of:
  i Blue Crest **ii** Green Tail

b Describe the trend in the distance from the recording machine for:
  i Blue Crest **ii** Green Tail

c Assuming that the given trends continue, predict the time when the pigeons will be the same distance from the recording machine.

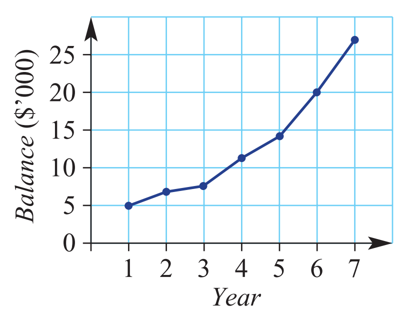


Non-linear trends

10 The balance of an investment account is shown in this time series plot.

a Describe the trend in the account balance over the 7 years.

b Give a practical reason for the shape of the curve that models the trend in the graph.



11 A drink at room temperature is placed in a fridge that is at 4°C.

a Sketch a time series plot that might show the temperature of the drink after it has been placed in the fridge.

b Would the temperature of the drink ever get to 3°C? Why?

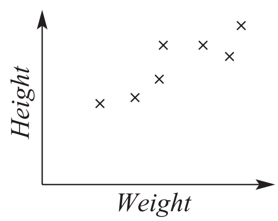
c Record the temperature at regular intervals of a drink at room temperature that is placed in a fridge. Plot your results and compare them to your answer in part **a**.

ISBN: 9781107687028

Cambridge University Press

5.7 Bivariate data and scatter plots

When we collect information about two variables in a given context we are collecting bivariate data. As there are two variables involved in bivariate data, we use a number plane to graph the data. These graphs are called scatter plots and are used to show a relationship that may exist between the variables. Scatter plots make it very easy to see the strength of the relationship between the two variables.



Let's start: A relationship or not?

Consider the two variables in each part below.

- Would you expect there to be some relationship between the two variables in each of these cases?
- If you feel that a relationship exists, would you expect the second listed variable to increase or to decrease as the first variable increases?

a Height of person and Weight of person
b Temperature and Life of milk
c Length of hair and IQ
d Depth of topsoil and Brand of motorcycle
e Years of education and Income
f Spring rainfall and Crop yield
g Size of ship and Cargo capacity
h Fuel economy and CD track number
i Amount of traffic and Travel time
j Cost of 2 litres of milk and Ability to swim
k Background noise and Amount of work completed

Key ideas

- **Bivariate data** is data that involves two variables.
  – The two variables are usually related; for example, height and weight.
- A **scatter plot** is a graph on a number plane in which the axis variables correspond to the two variables from the bivariate data. Points are marked with a cross.
- The words *relationship*, *correlation* and *association* are used to describe the way in which the variables are related.

Bivariate data Data that involves two variables

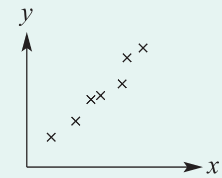
Scatter plot A diagram that uses coordinates to display values for two variables for a set of data

ISBN: 9781107687028

Cambridge University Press

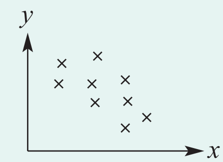
- Types of correlation:
  – The correlation is positive if the *y* variable generally increases as the *x* variable increases.
  – The correlation is negative if the *y* variable generally decreases as the *x* variable increases.
  Examples:

Strong positive correlation



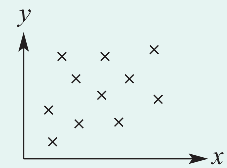
As *x* increases, *y* clearly increases.

Weak negative correlation



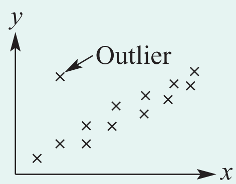
As *x* increases, *y* generally decreases.

No correlation



As *x* increases, there is no particular effect on *y*.

- An outlier can clearly be identified as a data point that is isolated from the rest of the data.



Exercise 5G

Understanding

1 Decide whether it is likely or unlikely that there will be a strong relationship between these pairs of variables.

a Height of door and width of door
b Weight of car and fuel consumption
c Temperature and length of phone calls
d Colour of flower and strength of perfume
e Amount of rain and size of vegetables in the vegetable garden

2 For each of the following sets of bivariate data with variables *x* and *y*:

i draw a scatter plot by hand
ii decide whether *y* generally increases or decreases as *x* increases

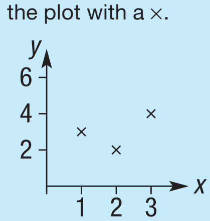
a

x	1	2	3	4	5	6	7	8	9	10
y	3	2	4	4	5	8	7	9	11	12

b

x	0.1	0.3	0.5	0.9	1.0	1.1	1.2	1.6	1.8	2.0	2.5
y	10	8	8	6	7	7	7	6	4	3	1

On a scatter plot, mark each point of the plot with a ×.



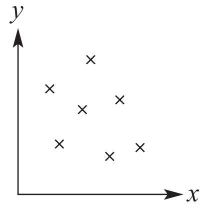
ISBN: 9781107687028

Cambridge University Press

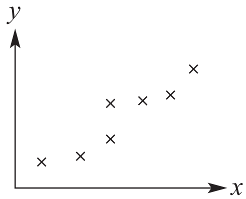
3 For these scatterplots, choose two words from those listed below to best describe the correlation between the two variables.

strong *weak* *positive* *negative*

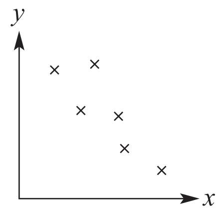
a



b



c



Fluency

Example 15 Constructing and interpreting scatter plots

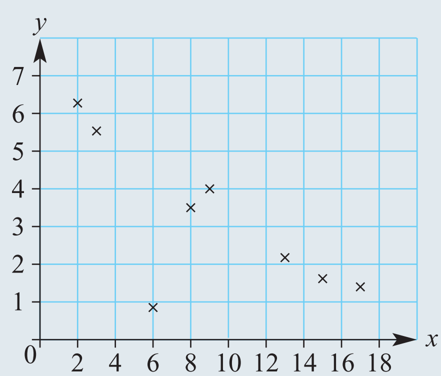
Consider this simple bivariate data set.

x	13	9	2	17	3	6	8	15
y	2.1	4.0	6.2	1.3	5.5	0.9	3.5	1.6

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

Solution | **Explanation**

a



Draw an appropriate scale on each axis by looking at the data:

- x is up to 17
- y is up to 6.2

The scale must be spread evenly on each axis.
Plot each point using a × symbol on graph paper.

b Negative correlation — Looking at the scatterplot, as x increases y decreases.

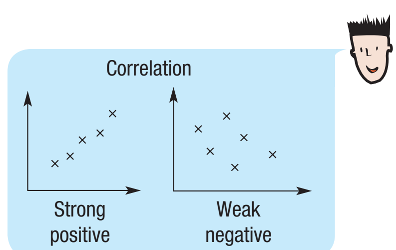
c Strong correlation — The downwards trend in the data is clearly defined.

d The outlier is (6, 0.9). — This point defies the trend.

4 Consider this simple bivariate data set.

x	1	2	3	4	5	6	7	8
y	1.0	1.1	1.3	1.3	1.4	1.6	1.8	1.0

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.



ISBN: 9781107687028

5 Consider this simple bivariate data set.

x	14	8	7	10	11	15	6	9	10
y	4	2.5	2.5	1.5	1.5	0.5	3	2	2

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

6 By completing scatter plots for each of the following data sets, describe the correlation between x and y as 'positive', 'negative' or 'none'.

a

x	1.1	1.8	1.2	1.3	1.7	1.9	1.6	1.6	1.4	1.0	1.5
y	22	12	19	15	10	9	14	13	16	23	16

b

x	4	3	1	7	8	10	6	9	5	5
y	115	105	105	135	145	145	125	140	120	130

c

x	28	32	16	19	21	24	27	25	30	18
y	13	25	22	21	16	9	19	25	15	12

Problem-solving and Reasoning

7 A tomato grower experiments with a new organic fertiliser and sets up five separate garden beds: A, B, C, D and E. The grower applies different amounts of fertiliser to each bed and records the diameter of each tomato picked.

The average diameter of a tomato from each garden bed and the corresponding amount of fertiliser are recorded below.

Bed	A	B	C	D	E
Fertiliser (grams per week)	20	25	30	35	40
Average diameter (cm)	6.8	7.4	7.6	6.2	8.5

a Draw a scatter plot for the data with 'Diameter' on the vertical axis and 'Fertiliser' on the horizontal axis. Label the points A, B, C, D and E.
b Which garden bed appears to go against the trend?
c According to the given results, would you be confident in saying that the amount of fertiliser fed to tomato plants does affect the size of the tomato produced?

ISBN: 9781107687028

8 For common motor vehicles, consider the two variables *Engine size* (cylinder volume) and *Fuel economy* (number of kilometres travelled for every litre of petrol).

a Do you expect there to be some relationship between these two variables?
b As the engine size increases, would you expect the fuel economy to increase or decrease?
c The following data was collected for 10 vehicles.

Car	A	B	C	D	E	F	G	H	I	J
Engine size	1.1	1.2	1.2	1.5	1.5	1.8	2.4	3.3	4.2	5.0
Fuel economy	21	18	19	18	17	16	15	20	14	11

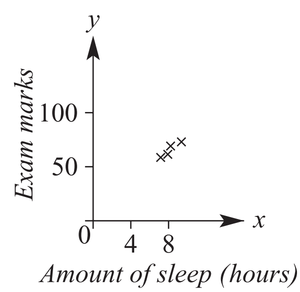
i Does the data generally support your answers to parts **a** and **b** above?
ii Which car gives a fuel economy reading that does not support the general trend?

9 On 14 consecutive days a local council measures the volume of sound heard from a freeway at various points in a local suburb. The volume (V) of sound is recorded against the distance (d m) between the freeway and the point in the suburb.

Distance (d)	200	350	500	150	1000	850	200	450	750	250	300	1500	700	1250
Volume (V)	4.3	3.7	2.9	4.5	2.1	2.3	4.4	3.3	2.8	4.1	3.6	1.7	3.0	2.2

a Draw a scatter plot of V against d, plotting V on the vertical axis and d on the horizontal axis.
b Describe the correlation between d and V as positive, negative or none.
c Generally as d increases, does V increase or decrease?

10 A person presents you with this scatter plot and suggests to you that there is a strong correlation between the amount of sleep and exam marks. What do you suggest is the problem with the person's graph and conclusions?

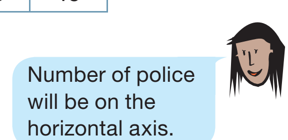


Crime rates and police

11 A government department is interested in convincing the electorate that a large number of police on patrol leads to lower crime rates. Two separate surveys are completed over a one-week period and the results are listed in this table.

	Area	A	B	C	D	E	F	G
Survey 1	Number of police	15	21	8	14	19	31	17
	Incidence of crime	28	16	36	24	24	19	21
Survey 2	Number of police	12	18	9	12	14	26	21
	Incidence of crime	26	25	20	24	22	23	19

a By using scatter plots, determine whether or not there is a relationship between the number of police on patrol and the incidence of crime, using the data in:
i survey 1 **ii** survey 2



b Which survey results do you think the government will use to make its point? Why?

Using technology 5.7: Using calculators to draw scatter plots

This activity is available on the companion website as a printable PDF.

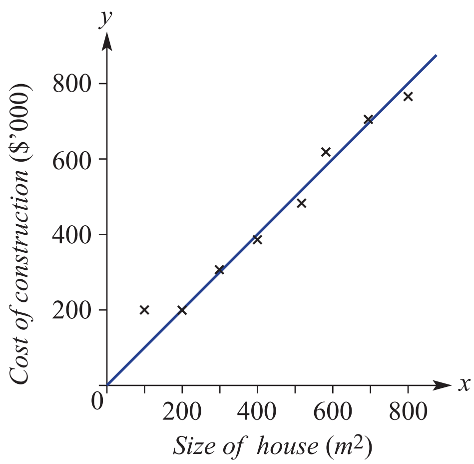
ISBN: 9781107687028

5.8 Line of best fit by eye

When bivariate data has a strong linear correlation, we can model the data with a straight line. This line is called a trend line or line of best fit. When we fit the line 'by eye', we try to balance the data points above the line with points below the line. This trend line can then be used to construct other data points inside and outside the existing data points.

Let's start: Size versus cost

This scatter plot shows the estimated cost of building a house of a given size by a building company. A trend line has been added to the scatter plot.

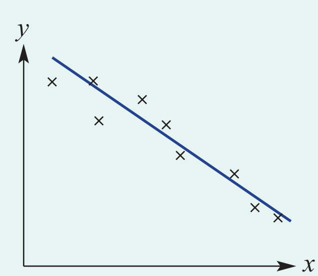


- Why is it appropriate to fit a trend line to this data?
- Do you think the trend line is a good fit to the points on the scatter plot? Why?
- How can you predict the cost of a house of 1000 m^2 with this building company?

Key ideas

- For bivariate data showing a clearly defined positive or negative correlation, a straight line can be fitted by eye.
- A **line of best fit** (or trend line) is positioned by eye by balancing the number of points above the line with the number of points below the line.
  - The distance of each point from the trend line also needs to be taken into account.
  - Outliers should be ignored.
- The line of best fit can be used for:
  - interpolation: finding unknown points within the given data range
  - extrapolation: finding points outside the given data range.

Line of best fit
A line that has the closest fit to a set of data points displayed in a scatter plot



ISBN: 9781107687028

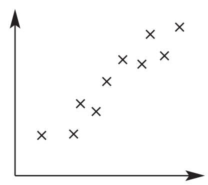
Cambridge University Press

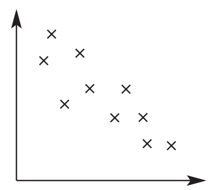
Exercise 5H

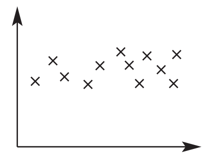
Understanding

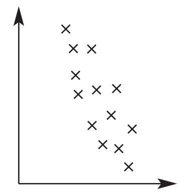
1 a When is it suitable to add a line of best fit to a scatterplot?
 b Describe the general guideline for placing a line of best fit.

2 Practise fitting a line of best fit on these scatter plots by trying to balance the number of points above the line with the number of points below the line. (Using a pencil might help.)

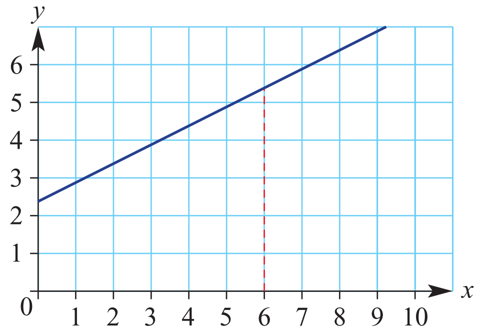
a 

b 

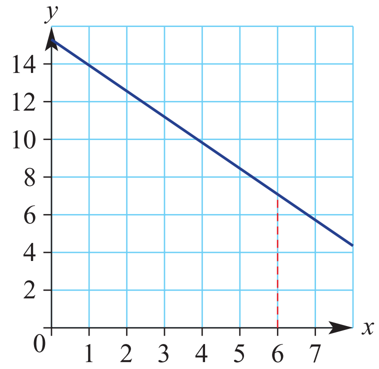
c 

d 

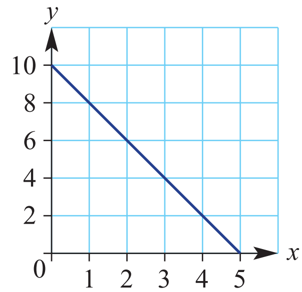
3 For each graph, use the line of best fit shown to estimate the y value when $x = 6$.

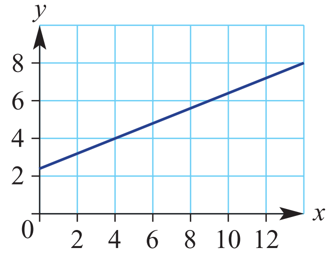
a 

Continue a horizontal line to the y axis from where the vertical dashed line touches the line of best fit.

b 

4 For each graph, use the line of best fit shown to find the x value when $y = 7$.

a 

b 

ISBN: 9781107687028 Cambridge University Press

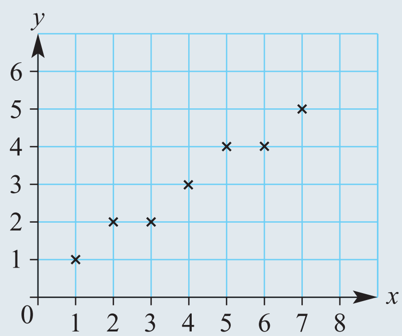
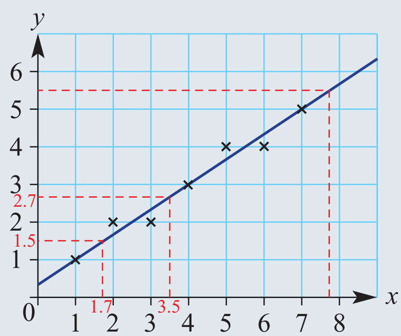
Fluency

Example 16 Fitting a line of best fit

Consider the variables x and y and the corresponding bivariate data.

x	1	2	3	4	5	6	7
y	1	2	2	3	4	4	5

a Draw a scatter plot for the data.
b Is there positive, negative or no correlation between x and y?
c Fit a line of best fit by eye to the data on the scatter plot.
d Use your line of best fit to estimate:
  i y when $x = 3.5$ **ii** y when $x = 0$
  iii x when $y = 1.5$ **iv** x when $y = 5.5$

Solution	Explanation
a 	Plot the points on graph paper.
b Positive correlation	As x increases, y increases.
c 	Since a relationship exists, draw a line on the plot, keeping as many points above as below the line (there are no outliers in this case).
d **i** $y \approx 2.7$ **ii** $y \approx 0.4$ **iii** $x \approx 1.7$ **iv** $x \approx 7.8$	Start at $x = 3.5$. Draw a vertical line to the line of best fit, then draw a horizontal line to the y-axis and read off your solution. Extend vertical and horizontal lines from the values given and read off your solution. As they are approximations, we use the ≈ sign and not the = sign.

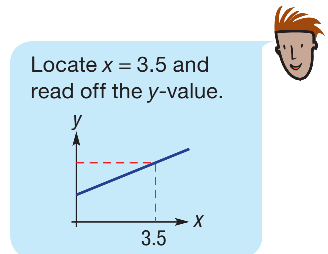
ISBN: 9781107687028

5H

5 Consider the variables x and y and the corresponding bivariate data.

x	1	2	3	4	5	6	7
y	2	2	3	4	4	5	5

a Draw a scatter plot for the data.
b Is there positive, negative or no correlation between x and y?
c Fit a line of best fit by eye to the data on the scatter plot.
d Use your line of best fit to estimate:
  i y when $x = 3.5$
  ii y when $x = 0$
  iii x when $y = 2$
  iv x when $y = 5.5$



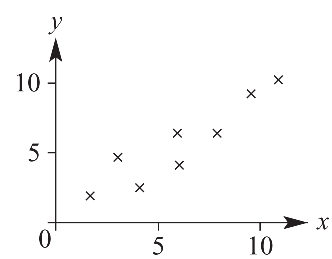
6 Consider the variables x and y and the corresponding data below:

x	1	2	4	5	7	8	10	12
y	20	16	17	16	14	13	9	10

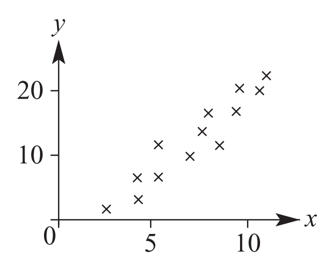
a Draw a scatter plot for the data.
b Is there a positive, negative or no correlation between x and y?
c Fit a line of best fit by eye to the data on the scatter plot.
d Use your line of best fit to estimate:
  i y when $x = 7.5$
  ii y when $x = 0$
  iii x when $y = 12$
  iv x when $y = 15$

7 For the following scatter plots, pencil in a line of best fit by eye, and then use your line to estimate the value of y when $x = 5$.

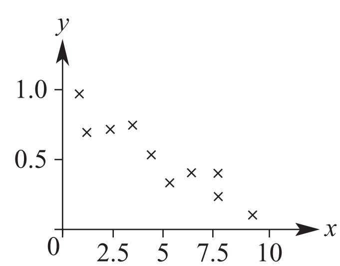
a



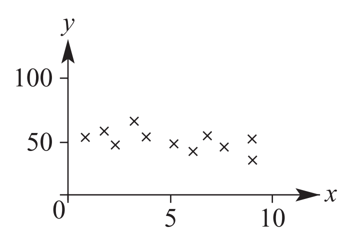
b



c

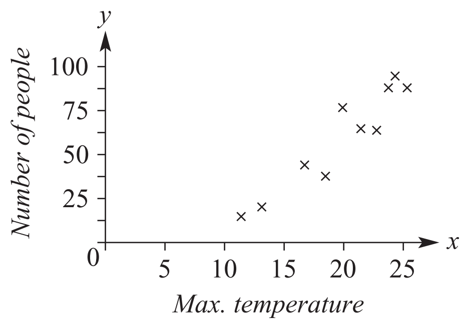


d



ISBN: 9781107687028

8 The following chart shows data for the *number of people* entering a suburban park and the corresponding *maximum temperature* for 10 spring days.



a Generally, as the maximum daily temperature increases, does the number of people who enter the park increase or decrease?

b Draw a line of best fit by eye on the given chart.

c Use your line of best fit to estimate:

i the number of people expected to enter the park if the maximum daily temperature is 20 degrees

ii the maximum daily temperature when the total number of people who visit the park on a particular day is 25.

Problem-solving and Reasoning

9 A small book shop records its profit and number of customers for the last 8 days.

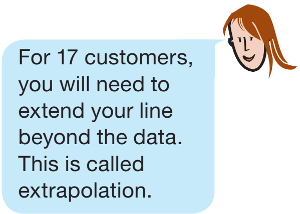
Number of customers	6	12	15	9	8	5	8
Profit ($)	200	450	550	300	350	250	300

a Draw a scatter plot for the data using profit on the vertical axis.

b Fit a line of best fit by eye.

c Use your line of best fit to predict the profit for 17 customers.

d Use your line of best fit to predict the number of customers for a $100 profit.



10 Over eight consecutive years, a city nursery has measured the growth of an outdoor bamboo species for that year. The annual rainfall in the area where the bamboo was growing was also recorded. The data is listed in the table.

Rainfall (mm)	450	620	560	830	680	650	720	540
Growth (cm)	25	45	25	85	50	55	50	20

a Draw a scatter plot for the data showing growth on the vertical axis.

b Fit a line of best fit by eye.

c Use your line of best fit to estimate the growth expected for the following rainfall readings.

i 500 mm ii 900 mm

d Use your line of best fit to estimate the rainfall for a given year if the growth of the bamboo was:

i 30 cm ii 60 cm

ISBN: 9781107687028 Cambridge University Press

5H

11 At a suburban sports club, the distance record for the hammer throw has increased over time. The first recorded value was 72.3 m in 1967. The most recent record was 118.2 m in 1996. Further details are in this table.

Year	1967	1968	1969	1976	1978	1983	1987	1996
New record (m)	72.3	73.4	82.7	94.2	99.1	101.2	111.6	118.2

a Draw a scatter plot for the data.
b Fit a line of best fit by eye.
c Use your line of best fit to estimate the distance record for the hammer throw for:
i 2000 **ii** 2015
d Would you say that it is realistic to use your line of best fit to estimate distance records beyond 2015? Why?

Heart rate and age

12 Two independent scientific experiments confirmed a correlation between *Maximum heart rate* and *Age*. The data for the two experiments is in this table.

Experiment 1													
Age	15	18	22	25	30	34	35	40	40	52	60	65	71
Max. heart rate	190	200	195	195	180	185	170	165	165	150	125	128	105
Experiment 2													
Age	20	20	21	26	27	32	35	41	43	49	50	58	82
Max. heart rate	205	195	180	185	175	160	160	145	150	150	135	140	90

a Sketch separate scatter plots for experiment 1 and experiment 2, with age on the horizontal axis.
b By fitting a line of best fit by eye to your scatter plots, estimate the maximum heart rate for a person aged 55, using the results from:
i experiment 1 **ii** experiment 2
c Estimate the age of a person who has a maximum heart rate of 190, using the results from:
i experiment 1 **ii** experiment 2
d For a person aged 25, which experiment estimates a lower maximum heart rate?
e Research the average maximum heart rate of people by age and compare with the results given above.

ISBN: 9781107687028 Cambridge University Press

1 The mean mass of 6 boys is 71 kg. The mean mass of 5 girls is 60 kg. Find the mean mass of all 11 people put together.

2 Sean has a current four-topic average of 78% for mathematics. What score does he need in the fifth topic to have an overall average of 80%?

3 I am a data set made up of five whole number values. My mode is 2 and both my mean and median are 5. What is my biggest possible range?

4 A single data set has 3 added to every value. Describe the change in:

a the mean b the median c the range

5 Find the interquartile range for a set of data if 75% of the data is above 2.6 and 25% of the data is above 3.7.

6 I am a data set with four whole number values.

- I have a range of 8.
- I have a mode of 3.
- I have a median of 6.

What are my four values?

7 A single-ordered data set includes the following data:

2, 4, 5, 6, 8, 10, x

What is the largest possible value of x if it is not an outlier?

8 Describe what happens to the mean, median and mode of a data set if each value in the set:

a is increased by 10

b is multiplied by 10

ISBN: 9781107687028 Cambridge University Press

Chapter summary

Statistics

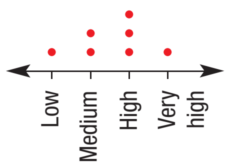
Data

Categorical
- Nominal (Red, blue ...)
- Ordinal (Low, medium ...)

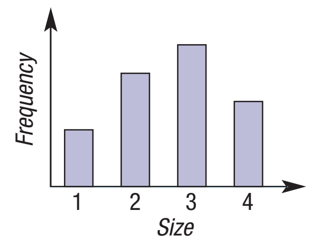
Numerical
- Discrete (1, 2, 3 ...)
- Continuous (0.31, 0.481 ...)

Graphs for single set of categorical or discrete data

Dot plot



Column graphs



Stem-and-leaf plot

Stem	Leaf
0	1 6
1	2 7 9
2	3 8
3	4

2|3 means 23

Measures of centre

- Mean $(\bar{x}) = \dfrac{\text{sum of all values}}{\text{number of scores}}$
- Median (Q_2) = middle value of ordered data

odd number: 1 4 6 9 12 — median = 6

even number: 1 2 4 6 7 11 — median $= \dfrac{4+6}{2} = 5$

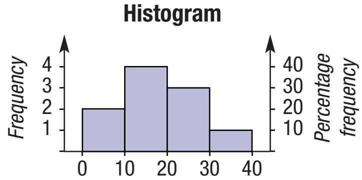
- Mode = most common value

Frequency tables

Class interval	Frequency	Percentage frequency
0–	2	20%
10–	4	40%
20–	3	30%
30–40	1	10%
Total	10	100%

$$\text{Percentage frequency} = \frac{\text{Frequency}}{\text{Total}} \times 100$$

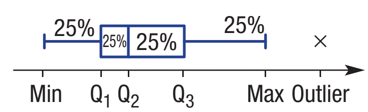
Histogram



Measures of spread

- Range = Max – Min
- Interquartile range (IQR) $= Q_3 - Q_1$

Boxplots



Quartiles

Q_1: above 25% of the data (middle value of lower half)

Q_3: above 75% of the data (middle value of upper half)

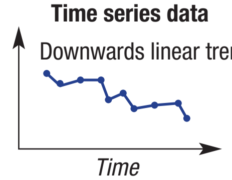
2 3 5 7 8 | 9 11 12 14 15

$Q_1 = 5$, $Q_2 = 8.5$, $Q_3 = 12$

1 2 3) 6 (7 10 13

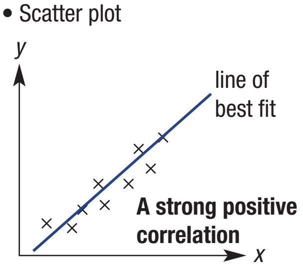
$Q_1 = 2$, $Q_2 = 6$, $Q_3 = 10$

Time series data

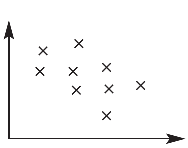


Bivariate data

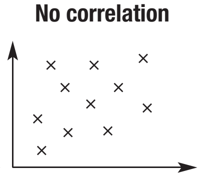
- Two related variables
- Scatter plot



Weak negative correlation



No correlation



Outliers

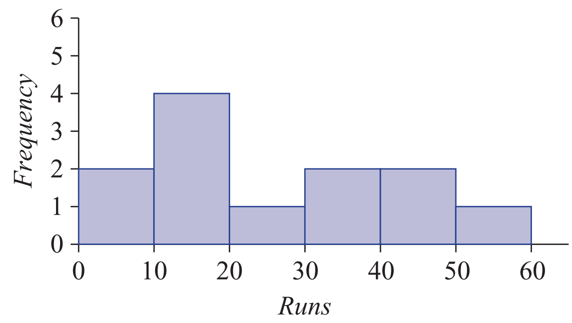
- Single data set
  - less than $Q_1 - 1.5 \times \text{IQR}$ or more than $Q_3 + 1.5 \times \text{IQR}$
- Bivariate
  - not in the vacinity of the rest of the data

ISBN: 9781107687028

Multiple-choice questions

1 This histogram shows the runs scored by an opening batsman in each match of the cricket season. The number of times he scored 30 or more runs was:

A 3 **B** 7
C 2 **D** 5
E 4



Questions **2–3** refer to the stem-and-leaf plot below.

Stem	Leaf
2	4 9
3	1 1 7 8
4	2 4 6
5	0 4

4|2 means 42

2 The minimum score in the data is:

A 4 **B** 0 **C** 24
D 38 **E** 54

3 The mode is:

A 3 **B** 31 **C** 4
D 38 **E** 30

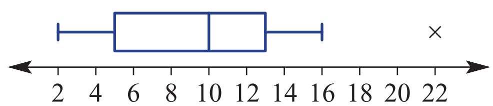
4 The range and mean of 2, 4, 3, 5, 10 and 6 are:

A range = 8, mean = 5 **B** range = 4, mean = 5 **C** range = 8, mean = 4
D range = 2–10, mean = 6 **E** range = 8, mean = 6

5 The median of 29, 12, 18, 26, 15 and 22 is:

A 18 **B** 22 **C** 20 **D** 17 **E** 26

Questions **6–8** refer to the boxplot below, at right.



6 The interquartile range (IQR) is:

A 8 **B** 5 **C** 3
D 20 **E** 14

7 The outlier is:

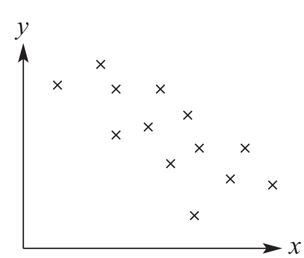
A 2 **B** 0 **C** 20
D 16 **E** 22

8 The median is:

A 2 **B** 3 **C** 10
D 13 **E** 16

9 The variables x and y in this scatter plot could be described as having:

A no correlation
B strong positive correlation
C strong negative correlation
D weak negative correlation
E weak positive correlation

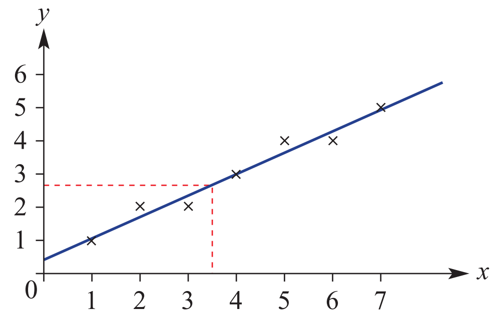


ISBN: 9781107687028

Cambridge University Press

10 According to this scatter plot, when x is 3.5, y is approximately:

A 4.4 **B** 2.7 **C** 2.5
D 3.5 **E** 5



Short-answer questions

1 A group of 16 people was surveyed to find the number of hours of television they watch in a week. The raw data is listed:
6, 5, 11, 13, 24, 8, 1, 12, 7, 6, 14, 10, 9, 16, 8, 3

a Organise the data into a table with class intervals of 5. Start at 0–, 5– etc. Include a tally, frequency and percentage frequency column.
b Construct a histogram for the data, showing both the frequency and percentage frequency on the graph.
c Would you describe the data as symmetrical or skewed?

2 A basketball team scores the following points per match for a season.

20, 19, 24, 37, 42, 34, 38, 49, 28, 15, 38, 32, 50, 29

a Construct an ordered stem-and-leaf plot for the data.
b Describe the distribution of scores.

3 For the following sets of data, determine:

i the mean **ii** the range **iii** the median

a 2, 7, 4, 8, 3, 6, 5
b 10, 55, 67, 24, 11, 16
c 1.7, 1.2, 1.4, 1.6, 2.4, 1.3

4 Thirteen adults compare their ages at a party. They are:

40, 41, 37, 32, 48, 43, 32, 76, 29, 33, 26, 38, 87

a Find the mean age of the adults, to one decimal place.
b Find the median age of the adults.
c Why do you think the mean age is larger than the median age?

5 Determine Q_1, Q_2 and Q_3 for these sets of data.

a 4, 5, 8, 10, 10, 11, 12, 14, 15, 17, 21
b 14, 6, 2, 23, 11, 6, 15, 14, 12, 18, 16, 10

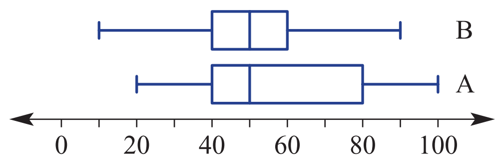
ISBN: 9781107687028

6 For each set of data below complete the following tasks.

- **i** Find the lower quartile (Q_1) and the upper quartile (Q_3).
- **ii** Find the interquartile range ($IQR = Q_3 - Q_1$).
- **iii** Locate any outliers.
- **iv** Draw a boxplot.

- **a** 2, 2, 3, 3, 3, 4, 5, 6, 12
- **b** 11, 12, 15, 15, 17, 18, 20, 21, 24, 27, 28
- **c** 2.4, 0.7, 2.1, 2.8, 2.3, 2.6, 2.6, 1.9, 3.1, 2.2

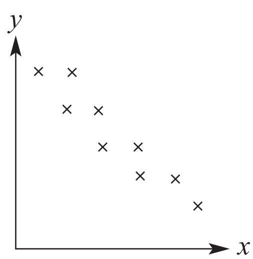
7 Compare these parallel boxplots, A and B, and answer the following as True or False.



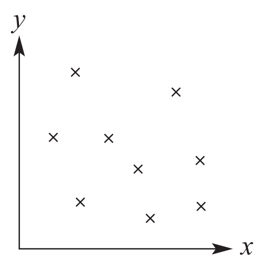
- **a** The range for A is greater than the range for B.
- **b** The median for A is equal to the median for B.
- **c** The interquartile range is smaller for B.
- **d** 75% of the data for A sits below 80.

8 For the scatter plots below, describe the correlation between x and y as positive, negative or none.

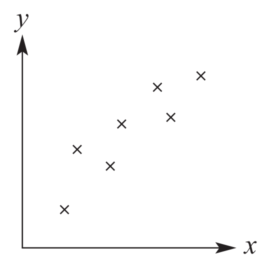
a



b



c



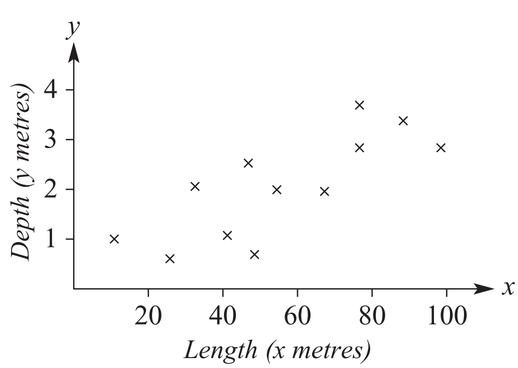
9 Consider the simple bivariate data set.

x	1	4	3	2	1	4	3	2	5	5
y	24	15	16	20	22	11	5	17	6	8

- **a** Draw a scatter plot for the data.
- **b** Describe the correlation between x and y as positive or negative.
- **c** Describe the correlation between x and y as strong or weak.
- **d** Identify any outliers.
- **e** Fit a line of best fit by eye.

10 The given scatter plot shows the maximum *length* (x metres) and *depth* (y metres) of 11 public pools around town.

- **a** Draw a line of best fit by eye.
- **b** Use your line to estimate the maximum depth of a pool that is 50 m in length.



ISBN: 9781107687028

Cambridge University Press

Extended-response questions

1 The number of flying foxes taking refuge in a fig tree was recorded over a period of 14 days. The data collected is given here.

Tree	73	50	36	82	15	24	73	57	65	86	51	32	21	39

a Find the IQR.
b Identify any outliers.
c Draw a boxplot for the data.

2 A newsagent records the *number of customers* and *profit* for 14 working days.

Number of customers	18	13	15	24	29	12	18	16	15	11	4	32	26	21
Profit ($)	150	70	100	210	240	90	130	110	120	80	30	240	200	190

a Draw a scatter plot for the data and draw a line of best fit by eye. Place number of customers on the horizontal axis.
b Use your line of best fit to predict the profit for:
  i 10 customers **ii** 20 customers **iii** 30 customers
c Use your line of best fit to predict the number of customers for a:
  i $50 profit **ii** $105 profit **iii** $220 profit

ISBN: 9781107687028

Measurement

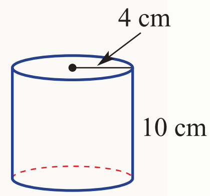
Multiple-choice questions

1 The number of centimetres in 2.8 metres is:
A 0.28 **B** 28 **C** 280 **D** 2.8 **E** 2800

2 A rectangle has length 7 cm and perimeter 22 cm. Its width is:
A 7.5 cm **B** 15 cm **C** 14 cm **D** 8 cm **E** 4 cm

3 The area of a circle with diameter 10 cm is given by:
A $\pi(10)^2$ cm^2 **B** $\pi(5)^2$ cm^2 **C** 10π cm^2 **D** $5 \times \pi$ cm^2 **E** 25 cm^2

4 The surface area of this cylinder is closest to:
A 351.9 cm^2 **B** 301.6 cm^2 **C** 175.9 cm^2
D 276.5 cm^2 **E** 183.4 cm^2



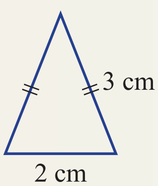
5 The area of the triangular cross-section of a prism is 8 mm^2 and the prism's height is 3 mm. The prism's volume is:
A 48 mm^3 **B** 12 mm^3 **C** 24 mm^2 **D** 24 mm^3 **E** 12 mm^2

Short-answer questions

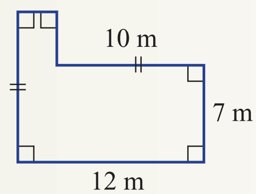
1 Convert these measurements to the units shown in the brackets.
a 0.43 m (cm) **b** 32 000 mm^2 (cm^2) **c** 0.03 m^3 (cm^3)

2 Find the perimeter of each of these shapes.

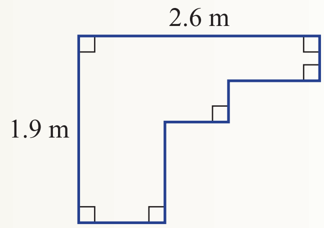
a



b

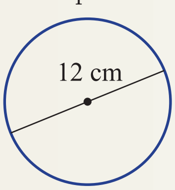


c

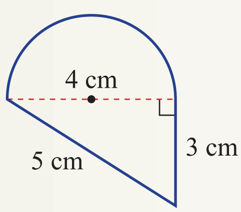


3 For these shapes, find, correct to two decimal places:
i the perimeter **ii** the area

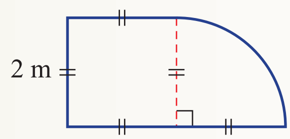
a



b

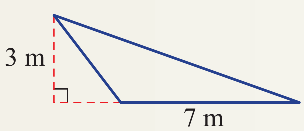


c

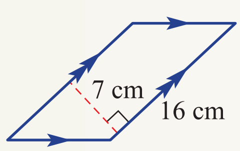


4 Find the area of each of these shapes.

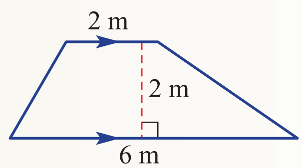
a



b



c

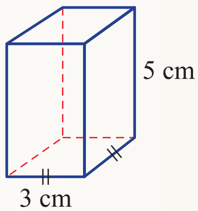


ISBN: 9781107687028 Cambridge University Press

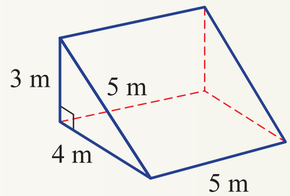
5 For these solids, find (correct to two decimal places where necessary):

i volume **ii** total surface area

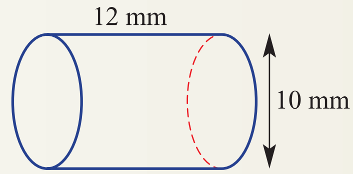
a



b

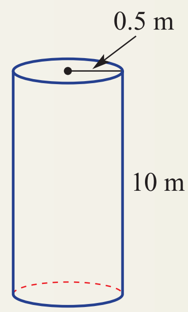


c



Extended-response question

1 A concrete cylindrical pole has radius 0.5 m and height 10 m. The outside curved surface only is to be painted. Answer the following to two decimal places.



a What volume of concrete is used to make the pole?

b What area is to be painted?

c A litre of paint covers 6 m^2. Paint costs $12 per litre and there are 18 poles to be painted. What is the cost of paint required? Round to the nearest $10.

Consumer arithmetic

Multiple-choice questions

1 Nigel earns $1256 a week. His annual income is:

A $24.15 **B** $32 656 **C** $65 312 **D** $15 072 **E** $12 560

2 Who earns the most?

A Sally: $56 982 p.a.
B Greg: $1986 per fortnight
C Chris: $1095 per week
D Paula: $32.57 per hour, 38-hour weeks for 44 weeks
E Bill: $20 000 p.a.

3 Adrian works 35 hours a week, earning $575.75. His wage for a 38-hour week is:

A $16.45 **B** $21 878.50 **C** $625.10 **D** $530.30 **E** $575.75

4 Jake earns a retainer of $420 per week plus a 2% commission on all sales. Find his fortnightly pay when his sales total $56 000 for the fortnight.

A $2240 **B** $840 **C** $1540 **D** $1960 **E** $56 420

5 Steve earns $4700 gross a month. He has annual deductions of $14 100 in tax and $1664 in health insurance. His net monthly income is:

A $3386.33 **B** $11 064 **C** $40 636 **D** $72 164 **E** $10 000

Short-answer questions

1 Sven earns $6.42 per hour working in a bar. What would he earn for working a 6-hour shift?

2 Theo's weekly wage is $875. Calculate:

a his annual wage

b his hourly rate if he works 35 hours a week

ISBN: 9781107687028 Cambridge University Press

3 Cara invests 10% of her net annual salary for one year into an investment account earning 4% p.a. simple interest for 5 years. Calculate the simple interest earned if her annual net salary is \$17 560.

4 Sean earns \$25.76 an hour as a mechanic. Calculate his:

a time and a half rate
b double time rate
c weekly wage for 38 hours at normal rate
d weekly wage for 38 hours at normal rate plus 3 hours at time and a half

5 Wendy earns \$15.40 an hour on weekdays and double time on the weekends. Calculate her weekly pay if she works 9 a.m.–3 p.m. Monday to Friday and 9 a.m. till 11:30 a.m. on Saturday.

6 Marina has a taxable income of \$42 600. Calculate her income tax if she falls into the following tax bracket.

\$4650 plus 30c for each \$1 over \$37 000

7 Darren earns \$372 per week plus 1% commission on all sales. Find his weekly income if his sales for the week total \$22 500.

8 Each fortnight, Raj earns \$1430 gross and pays \$34.94 in superannuation, \$23.40 in union fees and \$493.60 in tax.

a What is Raj's annual gross income?
b How much tax does Raj pay each year?
c What is Raj's net annual income?
d What is Raj's net weekly income?

9 Find the final value of an investment of \$7000 at 6% p.a. compounded annually for 4 years.

10 Jason earns \$664.20 in a week where he works 35 hours at his normal hourly rate and 4 hours at time and a half. Calculate Jason's normal hourly rate.

11 A \$120 Blu-ray player is discounted by 15%. What is the sale price?

Extended-response question

1 A tablet computer with a recommended retail price of \$749 is offered for sale in three different ways:

Method A	Method B	Method C
5% discount for cash	3% fee for a credit card payment	20% deposit and then \$18.95 per month for 3 years

a Jai buys a tablet for cash. How much does Jai pay?
b Talia buys a tablet using her mother's credit card. How much more does Talia pay for her tablet compared to Jai?
c Georgia needs to pay for her tablet using Method C.
  i Calculate the deposit Georgia needs to pay.
  ii What is the final cost of Georgia purchasing the tablet on terms?
  iii How much interest does Georgia pay on her purchase?
  iv What percentage of the recommended retail price is Georgia's interest? Round to two decimal places.

ISBN: 9781107687028

Algebra and indices

Multiple-choice questions

1 The expanded and simplified form of $4(2x - 3) - 4$ is:
A $8x - 7$ **B** $6x - 11$ **C** $8x - 16$ **D** $8x - 8$ **E** $6x - 7$

2 The fully factorised form of $4x^2 + 12x$ is:
A $4(x^2 + 3x)$ **B** $4x(x + 12)$ **C** $4(x^2 + 12x)$ **D** $4x(x + 3)$ **E** $2x(x + 6)$

3 $\dfrac{5(x-2)}{3} \times \dfrac{12}{x-2}$ simplifies to:
A 20 **B** -6 **C** $\dfrac{20}{x}$ **D** $16(x - 2)$ **E** $\dfrac{(x-2)}{6}$

4 Using index laws, $\dfrac{3x^2y \times 2x^3y^2}{xy^3}$ simplifies to:
A $\dfrac{5x^5}{y}$ **B** $6x^5$ **C** $\dfrac{6x^2}{y}$ **D** $6x^4$ **E** $\dfrac{6x^4}{y}$

5 $(a^3)^4b^{-2}$ expressed with positive indices is:
A $\dfrac{a^7}{b^2}$ **B** $\dfrac{a^{12}}{b^2}$ **C** $\dfrac{1}{a^7b^2}$ **D** $\dfrac{a^{12}}{b^{-2}}$ **E** a^7b^2

Short-answer questions

1 Simplify the following.
a $2xy + 7x + 5xy - 3x$ **b** $-3a \times 7b$ **c** $\dfrac{4a^2b}{8ab}$

2 **a** Expand and simplify the following.
i $-4(x - 3)$ **ii** $3x(5x + 2)$ **iii** $4(2x + 1) + 5(x - 2)$
b Factorise the following.
i $18 - 6b$ **ii** $3x^2 + 6x$ **iii** $-8xy - 12y$

3 Simplify these algebraic fractions.
a $\dfrac{6x + 18}{6}$ **b** $\dfrac{3(x-1)}{8x} \div \dfrac{x-1}{2x}$
c $\dfrac{x}{2} + \dfrac{2x}{5}$ **d** $\dfrac{x}{4} - \dfrac{3}{8}$

4 Use index laws to simplify the following. Express with positive indices.
a $2x^2 \times 5x^4$ **b** $\dfrac{12x^3y^2}{3xy^5}$ **c** $(2m^4)^3$ **d** $3x^0 + (4x)^0$
e $\left(\dfrac{3a}{b^4}\right)^2$ **f** $3a^{-5}b^2$ **g** $\dfrac{4}{t^{-5}}$ **h** $\dfrac{4x^5y^3 \times 5x^{-2}y}{10x^7y^2}$

5 **a** Write the following as basic numerals.
i 4.73×10^5 **ii** 5.21×10^{-3}
b Convert these to scientific notation using three significant figures.
i 0.000 027 561 **ii** 8 707 332

ISBN: 9781107687028

Extended-response question

1 Julie invests \$3000 at an interest rate of 6% per year.

a Write a rule for the amount of money, $\$A$, in her account after n years.

b How much will be in her account, correct to two decimal places, in:

i 2 years' time? ii 6 years' time?

c Use trial and error to determine how long it will take her to double her initial investment. Answer to one decimal place.

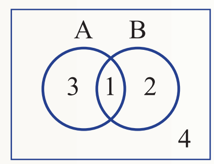
Probability

Multiple-choice questions

1 A letter is chosen from the word PROBABILITY. What is the probability that it will not be a vowel?

A $\frac{3}{11}$ B $\frac{4}{11}$ C $\frac{7}{11}$ D $\frac{1}{2}$ E $\frac{8}{11}$

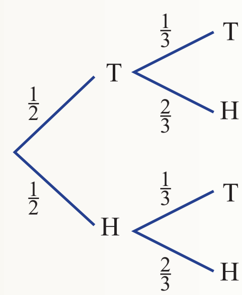
2 For this Venn diagram, $\Pr(A \cup B)$ is equal to:



A 1 B $\frac{1}{6}$ C $\frac{1}{10}$

D $\frac{3}{10}$ E $\frac{3}{5}$

3 For this tree diagram, what is the probability of the outcome (T, H)?



A $\frac{1}{3}$ B $\frac{1}{6}$ C $\frac{1}{2}$

D $\frac{1}{4}$ E $\frac{2}{3}$

4 The number of faults in a computer network over a period of 10 days is recorded in this table.

Number of faults	0	1	2	3
Frequency	1	5	3	1

An estimate for the probability that on the next day there would be at least two errors is:

A $\frac{3}{10}$ B $\frac{1}{5}$ C $\frac{4}{5}$ D $\frac{2}{5}$ E $\frac{1}{10}$

5 Two events are mutually exclusive if:

A $\Pr(A) = 0$ B $\Pr(A \cap B) = 0$ C $\Pr(A \cup B) = 0$

D $\Pr(A \mid B) = \Pr(A)$ E $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

Short-answer questions

1 A keen bird-watcher records the number of different species of birds in his backyard over a 20-day period.

Number of species	0	1	2	3	4	5	6
Frequency	0	2	3	8	4	2	1

From these results, estimate the probability that on the next day the bird-watcher will observe the following number of species.

a 3 b 2 or 3 c less than 5 d at least 2

ISBN: 9781107687028 Cambridge University Press

2 Of 25 students, 18 are wearing jackets, 14 are wearing hats and 10 are wearing both jackets and hats.

a Represent this information in a Venn diagram.
b Represent this information in a two-way table.
c How many students are wearing neither a hat nor a jacket?
d If a person is chosen randomly from the group, find the probability that the person will be wearing:
  i a hat and not a jacket
  ii a hat or a jacket
  iii a hat and a jacket
  iv a hat, given that they are wearing a jacket

3 Two 6-sided dice are tossed and the total is recorded.

a Complete the table to find the total number of outcomes.
b Find:
  i Pr(5)   **ii** Pr(7)
  iii Pr(at least 7)   **iv** Pr(at most 4)

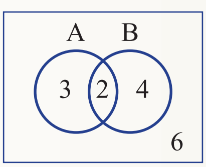
	1	2	3	4	5	6
1	2	3	4			
2	3	4				
3						
4						
5						
6						

4 Two people are chosen from a group of 2 males and 4 females without replacement. Use a tree diagram to help find the probability of selecting:

a two males
b one male and one female
c at least one female

5 For each diagram, find Pr(A) and Pr(A|B) then decide if events A and B are independent.

a



b

	A	A′	
B	3	2	5
B′	2	1	3
	5	3	8

Extended-response question

1 A hot dog stall produces two types of hot dogs: Traditional (T) at $4 each and Aussie (A) at $5 each. Gary randomly selects two hot dogs.

a Complete this table to show the possible selections.
b Find the probability of selecting:
  i two Aussie hot dogs
  ii at least one Aussie hot dog
c Gary only has $8. What is the probability that he will be able to afford two hot dogs?

	T	A
T	(T, T)	
A		

ISBN: 9781107687028

Statistics

Multiple-choice questions

1 The values of a and b in this frequency table are:

A $a = 3, b = 28$
B $a = 4, b = 28$
C $a = 4, b = 19$
D $a = 6, b = 20$
E $a = 3, b = 30$

Colour	Frequency	Percentage frequency (%)
Blue	4	16
Red	7	b
Green	a	12
White	6	24
Black	5	20
Total	25	

2 The mean, median and mode of the data set 3, 11, 11, 7, 1, 9 are:

A mean = 7, median = 9, mode = 11
B mean = 6 , median = 9, mode = 11
C mean = 7, median = 8, mode = 11
D mean = 7, median = 11, mode = 8
E mean = 8, median = 7, mode = 11

3 For the given stem-and-leaf plot the range and median respectively of the data are:

Stem	Leaf
0	2 2 6 7
1	0 1 2 3 5 8
2	3 3 5 7 9

1|5 means 15

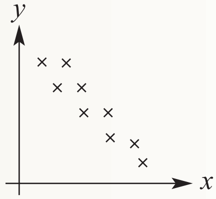
A 20, 12.5 **B** 7, 12 **C** 27, 12.5 **D** 29.3 **E** 27, 13

4 The interquartile range (IQR) for the data set 2, 3, 3, 7, 8, 8, 10, 13, 15 is:

A 5 **B** 8.5 **C** 7 **D** 13 **E** 8

5 The best description of the correlation between the variables for the scatter plot shown is:

A weak negative
B strong positive
C strong negative
D weak positive
E no correlation



Short-answer questions

1 Twenty people were surveyed to find out how many days in the past completed month they used public transport. The results were as follows:

7, 16, 22, 23, 28, 12, 18, 4, 0, 5, 8, 19, 20, 22, 14, 9, 21, 24, 11, 10

a Organise the data into a frequency table with class intervals of 5 and include a percentage frequency column.

b Construct a histogram for the data, showing both the frequency and the percentage frequency on the one graph.

ISBN: 9781107687028

Cambridge University Press

c i State the frequency of people who used public transport on 10 or more days.
  ii State the percentage of people who used public transport on fewer than 15 days.
  iii State the most common interval of days for which public transport was used. Can you think of a reason for this?

2 This data shows the number of DVDs owned by students in a school class.

12 24 36 17 8 24 9 4 15 32 41 26 15 18 7

a Display this data using a stem-and-leaf plot.
b Describe the distribution of the data as symmetrical or skewed.

3 For the data set 8, 10, 2, 17, 6, 30, 12, 7, 12, 15, 4:
a order the data
b determine:
  i the minimum and maximum values
  ii the median
  iii the lower quartile (Q_1) and the upper quartile (Q_3)
  iv IQR ($= Q_3 - Q_1$)
  v any outliers
c draw a box plot of the data

4 Farsan's bank balance over 12 months is recorded below.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Balance ($)	1500	2100	2300	2500	2200	1500	1200	1600	2000	2200	1700	2000

a Plot the time series for the 12 months.
b Describe the way in which the bank balance has changed over the 12 months.
c Between which consecutive months did the biggest change in the bank balance occur?
d What is the overall change in the bank balance over the year?

Extended-response question

1 The heights of plants in a group of the same species after a month of watering with a set number of millimetres of water per day are recorded below.

Water (mL)	8	5	10	14	12	15	18
Height (cm)	25	27	34	40	35	38	45

a Draw a scatter plot for the data using 'Water' for the x-axis.
b Describe the correlation between water and height as 'positive', 'negative' or 'none'.
c Fit a line of best fit by eye to the data on the scatterplot.
d Use your line of best fit to estimate:
  i the height of a plant watered with 16 mL of water per day
  ii the daily amount of water given to a plant of height 50 cm

ISBN: 9781107687028

Australian Curriculum
Number and Algebra
Linear and non-linear relationships

chapter

6 Straight line graphs

What you will learn

Roller coaster engineering

The features of straight lines such as length, angle of slope and gradient are all used in the construction industry. Construction workers, including carpenters, plumbers and crane operators, require an understanding of the properties of straight lines.

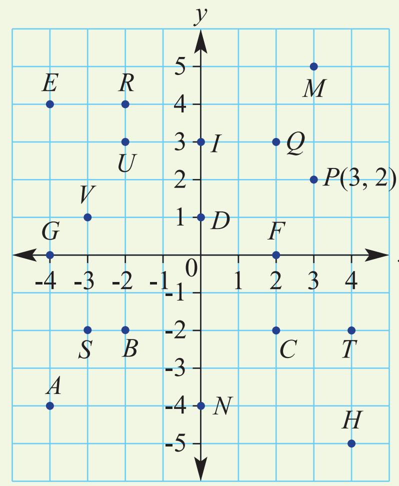
Engineers apply their knowledge of straight lines when they design buildings, bridges and roads. When designing a roller coaster ride, with its twists and turns, an engineer must calculate exactly where support poles need to be placed. The length of each pole and the angle it leans at are also vital measurements to be sure that the poles will safely support the massive weights of theme park rides.

ISBN: 9781107687028

Pre-test

1 The coordinates of P on this graph are (3, 2). Write down the coordinates of:

a M
b T
c A
d V
e C
f F



2 Name the point with coordinates:

a (-4, 0) **b** (0, 1)
c (-2, -2) **d** (-3, -2)
e (0, -4) **f** (2, 3)

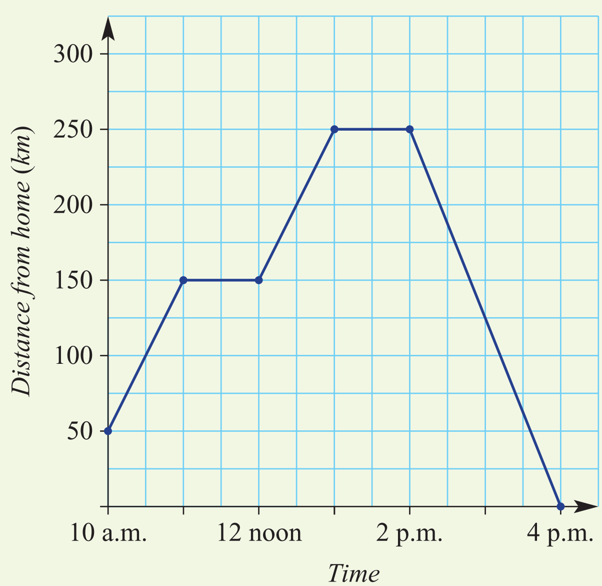
3 Draw up a four-quadrant number plane and plot the following points. What shape do they form?

a (0, 0), (0, 5), (5, 5), (5, 0)
b (-3, -1), (-3, 1), (4, 0)
c (-2, 3), (-4, 1), (-2, -3), (2, -3), (4, 1), (2, 3)

4 Find the mean ('average') of the following pairs.

a 10 and 12 **b** 15 and 23
c 6 and 14 **d** 3 and 4
e -6 and 6 **f** -3 and 1
g 0 and 7 **h** -8 and -10

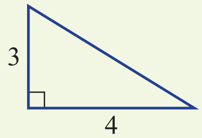
5 **a** For how many minutes did the Heart family stop on their trip if their journey is shown in this travel graph?

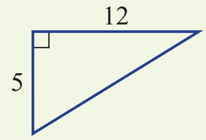


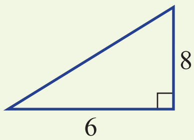
b How far had they travelled by 1 p.m., after starting at 10 a.m.?
c What was their speed in the first hour of travel?

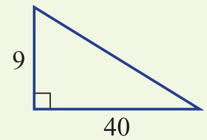
ISBN: 9781107687028

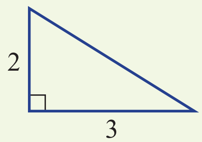
6 Find the length of the hypotenuse in each right-angled triangle. Use $a^2 + b^2 = c^2$. Round to two decimal places in parts **e** and **f**.

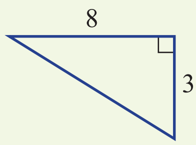
a


b


c


d


e


f


7 Copy and complete the table of values for each rule given.

a $y = x + 3$

x	0	1	2	3
y				

b $y = x - 2$

x	0	1	2	3
y				

c $y = 2x$

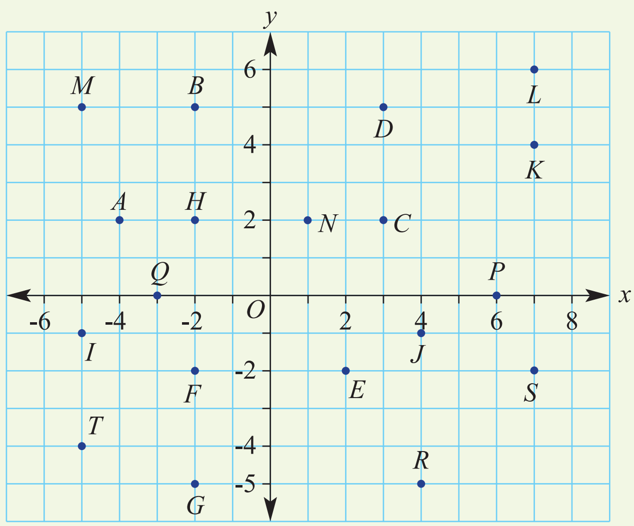
x	0	1	2
y			

d $y = 4 - x$

x	-2	-1	0
y			

8 Use the graph to find the following distances.

a OP **b** QP
c MB **d** FS
e BD **f** TM
g AC **h** LS
i AH **j** RJ
k LK **l** BG



ISBN: 9781107687028

Cambridge University Press

6.1 Interpretation of straight line graphs

When two variables are related, we can use mathematical rules to describe the relationship. The simplest kind of relationship forms a straight line graph and the rule is called a linear equation.

Information can be easily read from within a linear graph – this is called interpolation. A straight line can also be extended to determine information outside of the original data – this is called extrapolation.

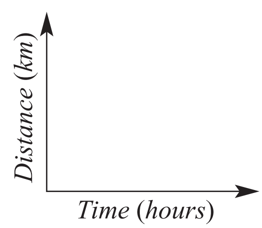
For example, if a swimming pool is filled at 1000 L per hour, the relationship between volume and time is linear because the volume is increasing at the constant rate of 1000 L/hr.

Volume = 1000 × number of hours

This rule is a linear equation and the graph of volume versus time will be a straight line.

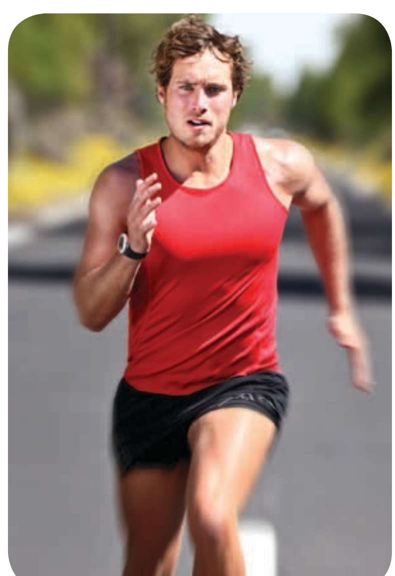
When a pool is filled with water at a constant rate, the graph of volume versus time will be a straight line.

Let's start: Graphing a straight line



Jozef is an athlete who trains by running 24 km in two hours at a constant rate. Draw a straight line graph to show this linear relation.

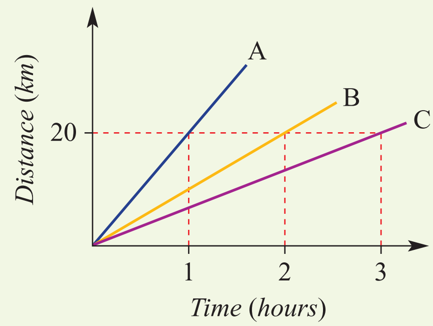
- Draw axes with time (up to 2 hours) on the horizontal axis and distance (up to 24 km) on the vertical axis.
- How far has Jozef run at zero hours? Mark this point on your graph.
- Mark the point on the graph that shows the end of Jozef's run.
- Join these two points with a straight line.
- Mark the point on the graph that shows Jozef's position after half an hour. How far had he run?
- Mark the point on the graph that shows Jozef's position after 18 km. For how long had Jozef been running?
- Name the variables shown on the graph.
- Discuss some advantages of showing information on a graph.



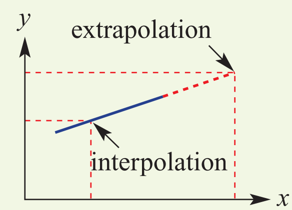
ISBN: 9781107687028

Key ideas

- A **variable** is an unknown that can take on many different values.
- When two variables have a **linear relationship** they can be represented as a straight line graph.
  Information about one of the variables based on information about the other variable is easily determined by reading from the graph:
  - A 20 km in 1 hour
  - B 20 km in 2 hours
  - C 20 km in 3 hours



- Information can be found from:
  - reading within a graph (**interpolation**) or
  - reading off an extended graph (**extrapolation**).



Variable An unknown, which can take on any value

Linear relationship The relationship between a variable and a constant term

Interpolation Reading information from within a graph

Extrapolation Determining information outside of the original data

Exercise 6A

Understanding

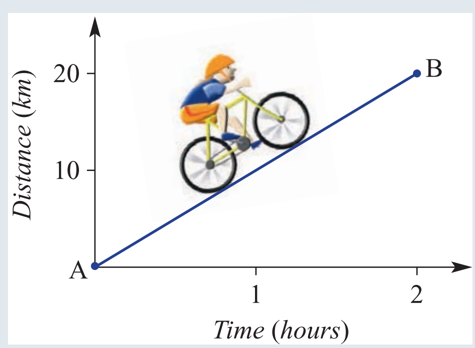
Example 1 Reading information from a graph

The graph shown here shows the journey of a cyclist from one place (A) to another (B).

a How far did the cyclist travel?

b How long did it take the cyclist to complete the journey?

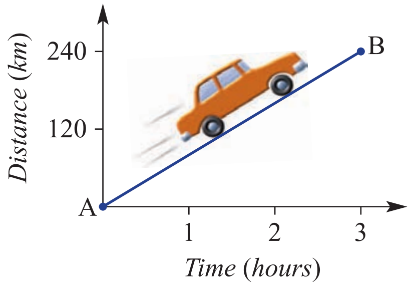
c If the cyclist rode from A to B and then halfway back to A, how far would the journey be?



Solution	Explanation
a 20 km	Draw an imaginary line from point B to the vertical axis; i.e. 20 km.
b 2 hours	Draw an imaginary line from point B to the horizontal axis; i.e. 2 hours.
c $20 + 10 = 30$ km	Ride 20 km out and 10 km back.

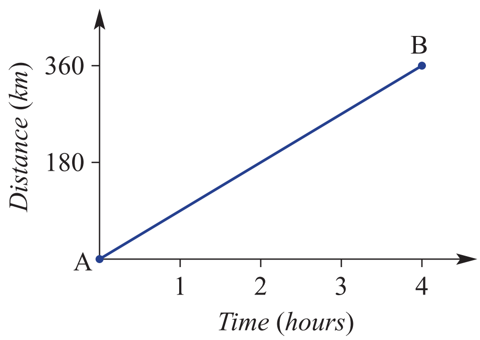
ISBN: 9781107687028

1 This graph shows a car journey from one place (A) to another (B).
- **a** How far did the car travel?
- **b** How long did it take to complete the journey?
- **c** If the car was driven from A to B, then halfway back to A, how far would the journey be?



Look on the distance scale that is level with point B on the line. This will be the total distance from A to B.

2 This graph shows a motorcycle journey from one place (A) to another (B).
- **a** How far did the motorcycle travel?
- **b** How long did it take to complete the journey?
- **c** If the motorcycle was driven from A to B, then halfway back to A, how far would the journey be?

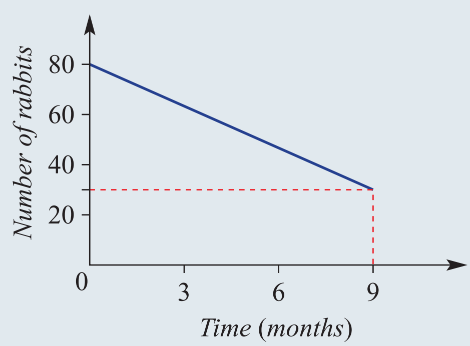


To find the total time taken to go from A to B, look on the time scale that is level with point B on the line.

Fluency

Example 2 Interpreting information from a graph

The number of rabbits in a colony has decreased according to this graph.
- **a** How many rabbits were there in the colony to begin with?
- **b** How many rabbits were there after 9 months?
- **c** How many rabbits disappeared from the colony during the 9-month period?



Solution	**Explanation**
a 80 rabbits	At $t = 0$ there were 80 rabbits.
b 30 rabbits	Read the number of rabbits from the graph at $t = 9$.
c $80 - 30 = 50$ rabbits	There were 80 rabbits at the start and 30 after 9 months.

ISBN: 9781107687028

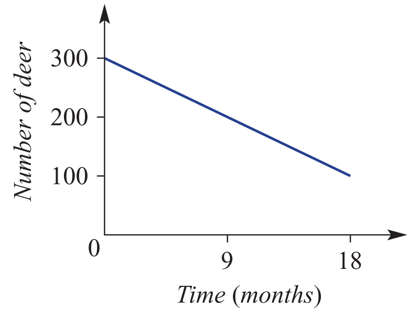
3 The number of deer in a particular forest has decreased over recent months according to the graph shown.

a How many deer were there to begin with?

b How many deer were there after 18 months?

c How many deer disappeared from the colony during the 18-month period?

To begin with, time = 0.

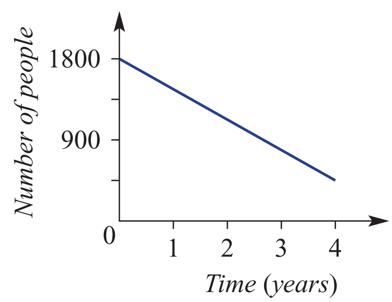


4 The number of people in a small village has decreased over recent years according to the graph shown.

a How many people were there to begin with?

b How many people were there after 4 years?

c How many people disappeared from the village during the 4-year period?



5 This graph shows the profit result for a company over a 5-month period.

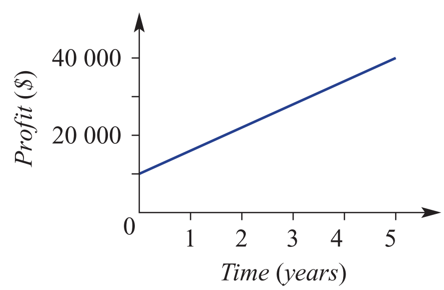
a What is the profit of the company at:

i the beginning of the 5-month period?

ii the end of the 5-month period?

b Has the profit increased or decreased over the 5-year period?

c How much has the profit increased over the 5 years?



6 A height versus weight graph for a golden retriever dog breed is shown.

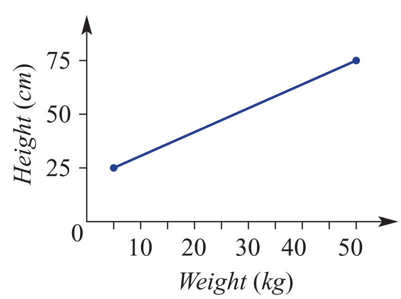
a From the smallest to the largest dog, use the graph to find the total increase in:

i height **ii** weight

b Fill in the missing numbers.

i The largest weight is ____ times the smallest weight.

ii The largest height is ____ times the smallest height.



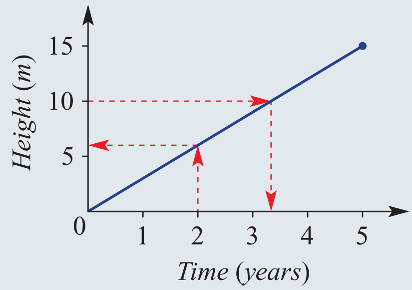
ISBN: 9781107687028 Cambridge University Press

Problem-solving and Reasoning

Example 3 Reading within a graph (interpolation)

This graph shows the growth of a tree over 5 years.

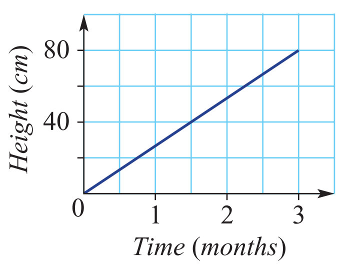
a How many metres has the tree grown over the 5 years?
b Use the graph to find how tall the tree is after 2 years.
c Use the graph to find how long it took for the tree to grow to 10 metres.

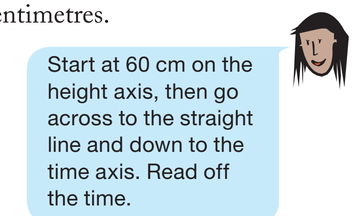


Solution	Explanation
a 15 metres	The end point of the graph is at 15 metres.
b 6 metres	Draw a dotted line at 2 years and read the height.
c 3.3 years	Draw a dotted line at 10 metres and read the time.

7 This graph shows the height of a tomato plant over 3 months.

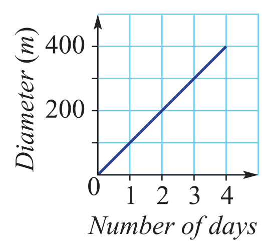
a How many centimetres has the tree grown over 3 months?
b Use the graph to find how tall the tomato plant is after $1\frac{1}{2}$ months.
c Use the graph to find how long it took for the plant to grow to 60 centimetres.





8 The diameter of an oil slick increased every day after an oil tanker hit some rocks. Use the graph to find:

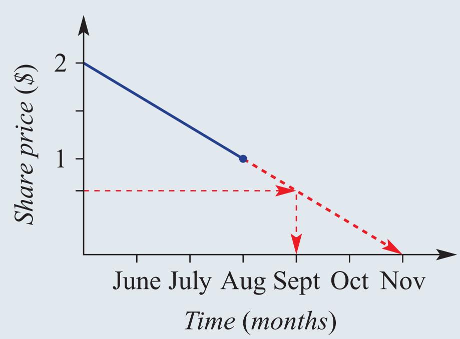
a how wide the oil slick is after 4 days?
b how wide the oil slick is after 2.5 days?
c how many days it took for the oil slick to reach a diameter of 350 m?



Example 4 Reading off an extended graph (extrapolation)

Due to poor performance, the value of a company's share price is falling.

a By the end of August, how much has the share price fallen?
b At the end of November what would you estimate the share price to be?
c Near the end of which month would you estimate the share price to be 70 cents?



ISBN: 9781107687028

Solution		Explanation
a	Price has dropped by $1.	By August the price has changed from $2 to $1.
b	$0	Use a ruler to extend your graph (shown by the dotted line) and read the share price for November.
c	September	Move across from 70 cents to the extended line and read the month.

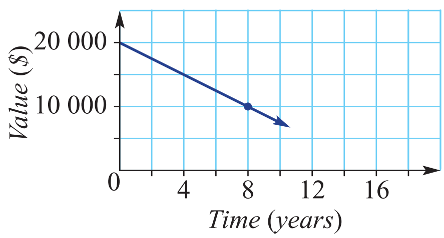
9 The value of a car decreases with time.

a By the end of 8 years, how much has the car's value fallen?

b At the end of 16 years, what would you estimate the car's value to be?

c Near the end of which year would you estimate the car's value to be $5000?

Use your ruler on the line to 'extend' it.

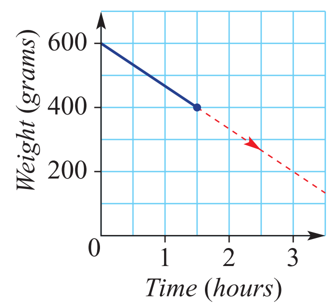


10 The weight of a wet sponge is reduced after it is left in the sun to dry.

a The weight of the sponge has been reduced by how many grams over the first 1.5 hours?

b What would you estimate the weight of the sponge to be after 3 hours?

c How many hours would it take for the sponge to weigh 300 g?



Submarine depth

11 A submarine goes to depths below sea level as shown in this graph.

a How long did it take for the submarine to drop from 40 to 120 m below sea level?

b At what time of day was the submarine at:

i -40 m? **ii** -80 m?

iii -60 m? **iv** -120 m?

c What is the submarine's depth at:

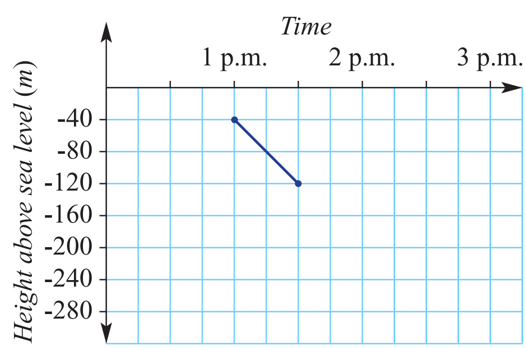
i 1:30 p.m.? **ii** 1:15 p.m.?

d Extend the graph to find the submarine's depth at:

i 12:45 p.m. **ii** 1:45 p.m. **iii** 2:30 p.m.

e Use your extended graph to estimate the time when the submarine was at:

i 0 m **ii** -200 m **iii** -320 m



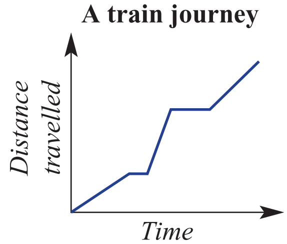
ISBN: 9781107687028

Cambridge University Press

6.2 Distance–time graphs

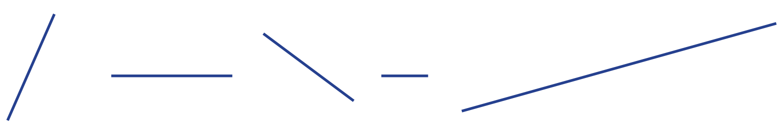
Some of the graphs considered in the previous section were distance–time graphs, which show the distance on the vertical axis and the time on the horizontal axis. Many important features of a journey can be displayed on such graphs. Each section of a journey that is at a constant rate of movement can be graphed with a straight line segment. Several different line segments can make up a total journey.

For example, a train journey could be graphed with a series of sloping line segments showing travel between stations and flat line segments showing when the train is stopped at a station.



Let's start: An imaginary journey

Here are five line segments.

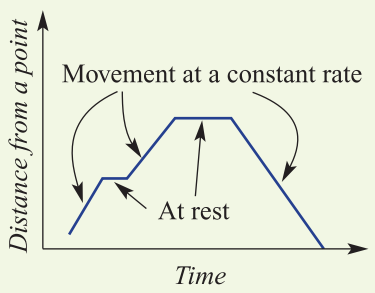


- Use five similar line segments arranged in any order you choose and draw a distance–time graph. Each segment must be joined to the one next to it.
- Write a summary of the journey shown by your distance–time graph.
- Swap graphs with a classmate and explain the journey that you think your classmate's graph is showing.

Key ideas

- Graphs of distance versus time usually consist of **line segments**.
- Each segment shows whether the object is moving or at rest.
- To draw a graph of a journey, use time on the horizontal axis and distance on the vertical axis.

Line segment A section of a straight line



ISBN: 9781107687028 Cambridge University Press

Exercise 6B

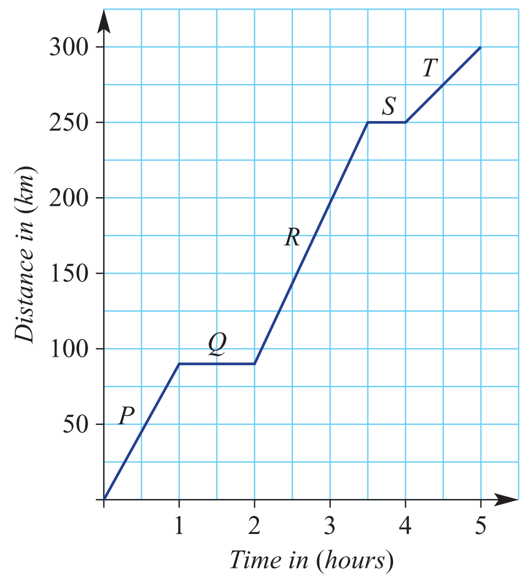
Understanding

1 The Martin family makes a 300-km car journey, which takes 5 hours. The distance–time graph of this journey is shown below. For each description below, choose the line segment of the graph that matches it. Some segments will have more than one descriptor.

A flat line segment shows that the car is stopped.

- **a** A half hour rest break is taken after travelling 250 km.
- **b** In the first hour the car travels 90 km.
- **c** The car is at rest for 1 hour, 90 km from the start.
- **d** The car takes 1.5 hours to travel from 90 km to 250 km.
- **e** The distance from 250 km to 350 km takes 1 hour.
- **f** The distance travelled stays constant at 250 km for half an hour.
- **g** A 1-hour rest break is taken after travelling 90 km.

Distance-time graph of car journey

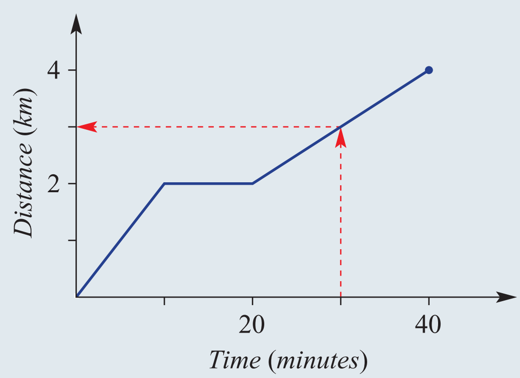


Fluency

Example 5 Interpreting a distance–time graph

This distance–time graph shows a car's journey from home, to school and then to the local shopping centre.

- **a** What was the total distance travelled?
- **b** How long was the car resting at the school?
- **c** What was the total distance travelled after 30 minutes?



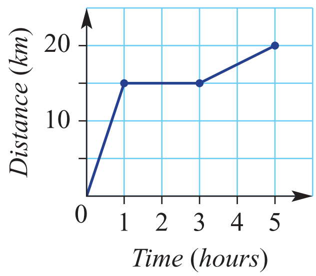
Solution	Explanation
a 4 km	Read the distance from the end point of the graph.
b 10 minutes	The rest starts at 10 minutes and finishes at 20 minutes.
c 3 km	Draw a line from 30 minutes and read off the distance.

ISBN: 9781107687028

2 A bicycle journey is shown on this distance–time graph.

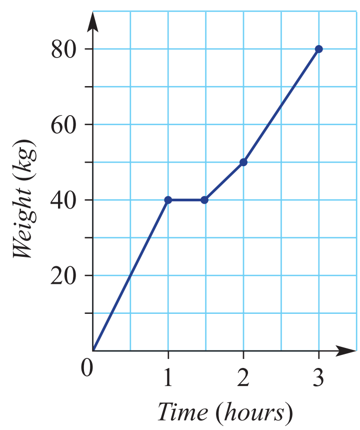
- **a** What was the total distance travelled?
- **b** How long was the cyclist at rest?
- **c** How far had the cyclist travelled after 4 hours?

From the end of the line, go across to the distance scale. This will show the total distance travelled.



3 The weight of a water container increases while water is poured into it from a tap.

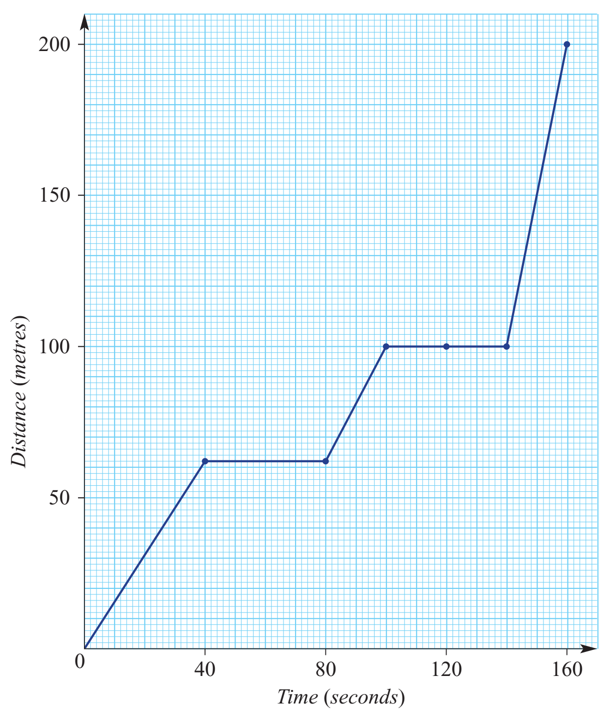
- **a** What is the total weight of the container after:
  - **i** 1 hour?
  - **ii** 2 hours?
  - **iii** 3 hours?
- **b** During the 3 hours, how long was the container not actually being filled with water?
- **c** During which hour was the container filling the fastest?



A flat line segment shows that the weight is not changing, so no water is being poured in at that time.

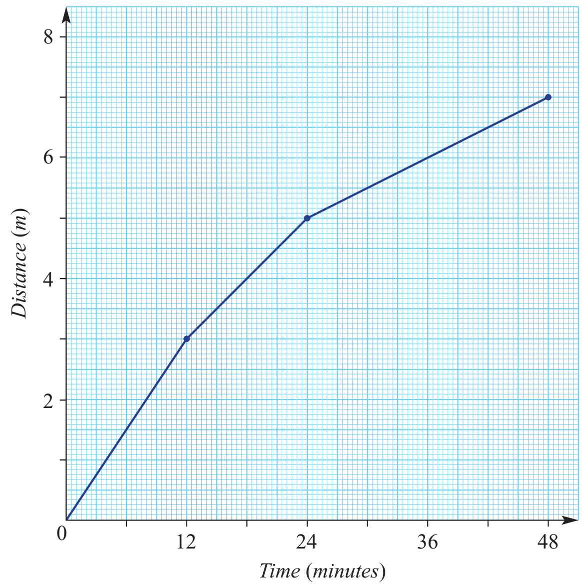
4 This graph shows a shopper's short walk in a shopping mall.

- **a** What is the total distance the shopper travelled?
- **b** How long was the shopper not walking?
- **c** What was the total distance the shopper travelled by the following times?
  - **i** 20 seconds
  - **ii** 80 seconds
  - **iii** 2.5 minutes



ISBN: 9781107687028 Cambridge University Press

5 A snail makes its way across a footpath, garden bed and lawn according to this graph.



a How far did the snail travel on:

i the footpath? **ii** the garden bed? **iii** the lawn?

b On which surface did the snail spend the most time?

c Use your graph to find how far the snail travelled after:

i 6 minutes **ii** 18 minutes **iii** 42 minutes

The line segment that has the largest horizontal change is the surface that the snail spent most time on.

6 The distance travelled during three phases of a rocket launch are shown on this graph.

a How long did it take for the rocket to get to:

i 50 m?

ii 100 m?

iii 200 m?

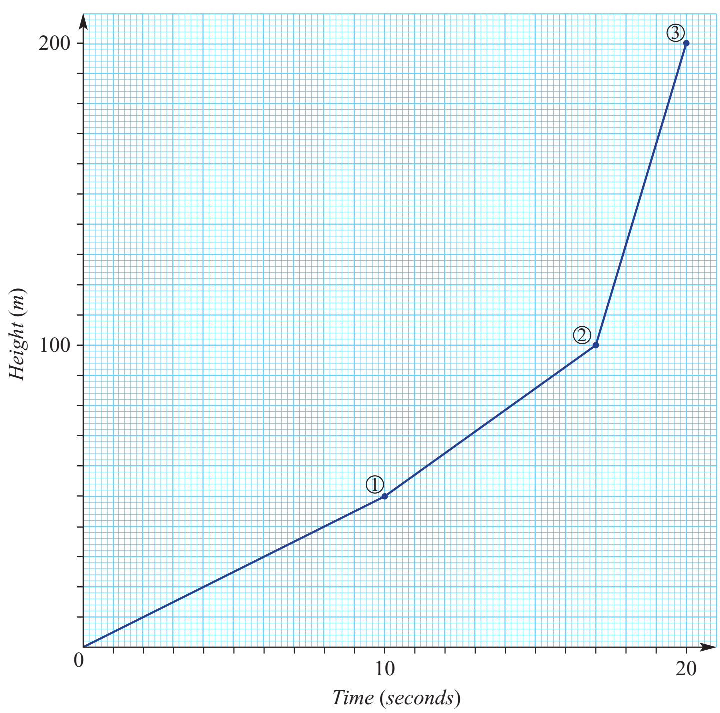
b During which of the three phases did the rocket gain the most height in the shortest time?

c Use your graph to find the height of the rocket after:

i 8 seconds

ii 15 seconds

iii 19 seconds



ISBN: 9781107687028 Cambridge University Press

6B

Problem-solving and Reasoning

7 This graph shows the distance of a train from the city station over a period of time.

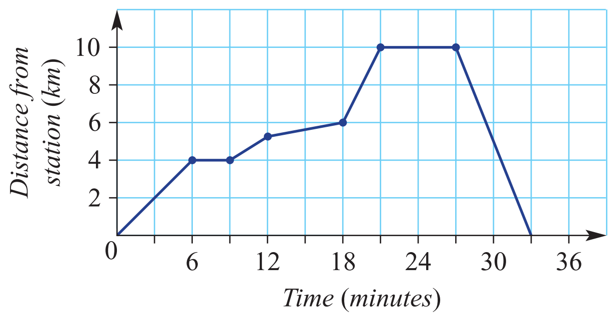
a What was the furthest distance the train travelled from the station?

b What was the total distance travelled?

c After how many minutes did the train begin to return to the station?

d What was the total number of minutes the train was at rest?

Remember to include the return trip in the total distance travelled.

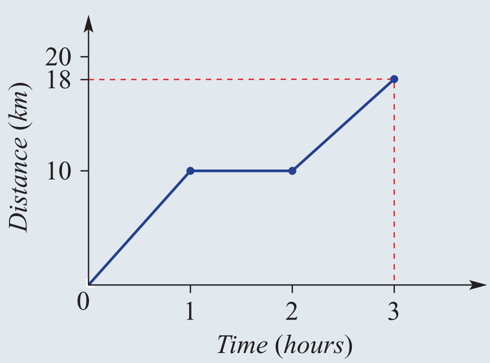


Example 6 Sketching a distance–time graph

Sketch a distance–time graph displaying all of the following information.

- total distance covered is 18 km in 3 hours
- 10 km covered in the first hour
- a 1-hour long rest after the first hour

Solution



Explanation

Draw axes with time on the horizontal (up to 3 hours) and distance on the vertical (up to 18 km).
Start at time zero.
Draw the first hour with 10 km covered.
Draw the rest stop, which lasts for one hour.
Draw the remainder of the journey, so that 18 km is completed after 3 hours.

8 Sketch a distance–time graph displaying all of the following information.

- total distance covered is 100 km in 2 hours
- 50 km covered in the first hour
- a half-hour rest stop after the first hour

Draw axes with time on the horizontal (up to 2 hours) and distance on the vertical (up to 100 km).

9 Sketch a graph to illustrate a journey described by the following.

- total distance covered is 15 m in 40 seconds
- 10 m covered in the first 10 seconds
- a 25-second rest after the first 10 seconds

Always use a ruler to draw line segments.

ISBN: 9781107687028 Cambridge University Press

10 A bus travels 5 km in 6 minutes, stops for 2 minutes, travels 10 km in 8 minutes, stops for another 2 minutes and then completes the journey by travelling 5 km in 4 minutes.

a What was the total distance travelled?

b What was the total time taken?

c Sketch a distance–time graph for the journey.

Find the total time taken to determine the scale for the horizontal axis. Find the total distance travelled to determine the scale for the vertical axis.

11 A 1-day, 20-km bush hike included the following features.

- a 3-hour hike to waterfalls (10 km distance)
- a half-hour rest at the falls
- a 2-hour hike to the mountain peak (5 km distance)
- a $1\frac{1}{2}$-hour hike to the campsite

Sketch a distance–time graph for the journey.

12 For each of the following journeys:

i draw a distance–time graph

ii decide the total travel time, not including rest stops

a
- 20 km in 1 hour
- a half-hour rest
- 10 km in $\frac{3}{4}$ hours
- 15 km in $1\frac{1}{2}$ hours

b
- 4 m in 3 seconds
- 2-second rest
- 10 m in 5 seconds
- 3-second rest
- 12 m in 10 seconds

Pigeon flight

13 The distance travelled by a pigeon is described by these points.

- A half-hour flight, covering a distance of 18 km
- A 15-minute rest
- A further 15-minute flight covering 12 km
- A half-hour rest
- Turning and flying 10 km back towards 'home' over the next $\frac{1}{2}$ hour
- A rest for $\frac{1}{4}$ of an hour
- Reaching 'home' after another 45-minute flight

a Sketch a graph illustrating the above points using 'distance' on the vertical axis.

b What was the fastest speed (in km/h) that the pigeon flew at? $\left(\text{Speed} = \frac{\text{distance}}{\text{time}}\right)$

c Determine the pigeon's average speed in km/h. $\left(\text{Average speed} = \frac{\text{total distance}}{\text{total flying time}}\right)$

ISBN: 9781107687028

Cambridge University Press

6.3 Plotting straight lines

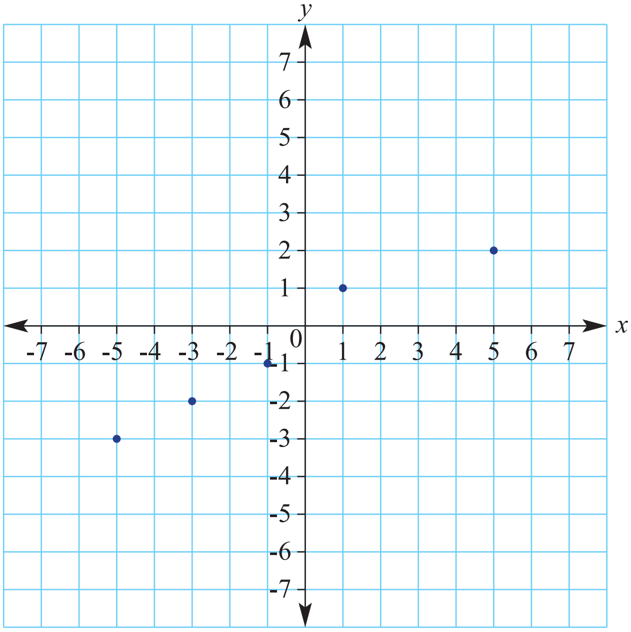
On a number plane, a pair of coordinates gives the exact position of a point. The number plane extends both a horizontal axis (x) and vertical axis (y) to include negative numbers. The point where these axes cross over is called the origin. It provides a reference point for all other points on the plane.

A rule that relates two variables can be used to generate a table that shows coordinate pairs (x, y). The coordinates can be plotted to form the graph. Rules that give straight line graphs are described as being linear.

Architects apply their knowledge of two-dimensional straight lines and geometric shapes to form interesting three-dimensional surfaces. Computers use line equations to produce visual models.

Let's start: What's the error?

- Which point is not in line with the rest of the points on this graph? What should its coordinates be so it is in line? List two other points that would be in line with the points on this graph.



- This table shows coordinates for the rule $y = 4x + 3$. Which y value has been incorrectly calculated in the table? What would the correct y value be?

x	0	1	2	3	4
y	3	7	11	12	19

- Which two points in this list would not be in the same line as the other points?

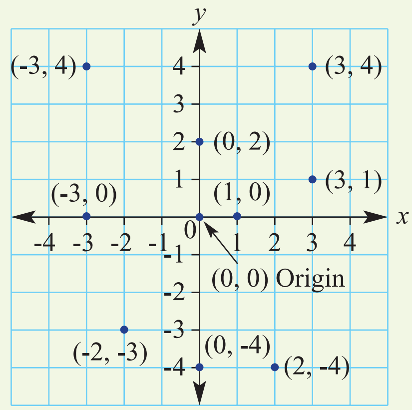
  (-2, 4), (-1, 2), (0, 0), (1, -2), (2, 4), (3, 6)

  What would the correct coordinates be for these two points?
- Points that follow a linear rule will always be in a straight line. Discuss some ways of checking whether the coordinates of a point have been incorrectly calculated.

ISBN: 9781107687028

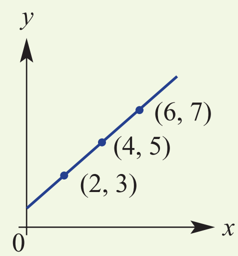
Key ideas

- A **number plane** includes a vertical y-axis and a horizontal x-axis intersecting at right angles.
- A point on a number plane has **coordinates** (x, y).
  - The x-coordinate is listed first, followed by the y-coordinate.
- The point (0, 0) is called the origin.



- $$(x, y) = \left(\begin{array}{c} \text{horizontal} \\ \text{units from} \\ \text{origin} \end{array}, \begin{array}{c} \text{vertical} \\ \text{units from} \\ \text{origin} \end{array}\right)$$
- A rule is an equation connecting two or more variables.
- A straight line graph will result from a rule that is linear.
- For two variables, a linear rule is often written with y as the subject. For example: $y = 2x - 3$ or $y = -x + 7$
- To graph a linear relationship using a rule.
  - Construct a table of values finding a y-coordinate for each given x-coordinate. Substitute each x-coordinate into the rule.
  - Plot the points given in the table on a set of axes.
  - Draw a line through the points to complete the graph.

x	2	4	6
y	3	5	7



- The **point of intersection** of two lines is the point where the lines cross over each other.

***x*-coordinate** The first coordinate of an ordered pair

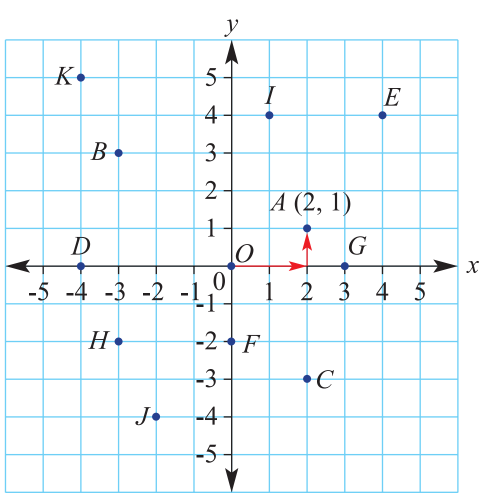
***y*-coordinate** The second coordinate of an ordered pair

Point of intersection The point at which two lines cross each other and therefore have the same coordinates

Exercise 6C

Understanding

1 **a** List the coordinates of each point plotted on this number plane.
b Which points are on the x-axis?
c Which points are on the y-axis?
d What are the coordinates of the point called the 'origin'?



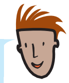
$$(x, y) = \left(\begin{array}{c} \text{right} \\ \text{or} \\ \text{left} \end{array}, \begin{array}{c} \text{up} \\ \text{or} \\ \text{down} \end{array}\right)$$

The 'origin' is the point where the x-axis and y-axis meet.

ISBN: 9781107687028

2 Ethan is finding the coordinates of some points that are on the line $y = -2x + 4$. Copy and complete these calculations, stating the coordinates for each point.

a $x = -3, y = -2 \times (-3) + 4 = 6 + 4 = \quad (-3, \quad)$
b $x = -2, y = -2 \times (-2) + 4 = \quad\quad = \quad (\quad , \quad)$
c $x = -1, y = -2 \times (-1) + 4 = \quad\quad = \quad (\quad , \quad)$
d $x = 0, y = -2 \times 0 + 4 = \quad\quad = \quad (\quad , \quad)$
e $x = 1, y = -2 \times 1 + 4 = \quad\quad = \quad (\quad , \quad)$
f $x = 2, y = -2 \times 2 + 4 = \quad\quad = \quad (\quad , \quad)$
g $x = 3, y = -2 \times 3 + 4 = \quad\quad = \quad (\quad , \quad)$



In the order of operations, first do any multiplication, then do addition or subtraction from left to right.

When multiplying, same signs make a positive and different signs make a negative.

3 Write the coordinates for each point listed in this table:

x	-2	-1	0	1	2
y	1	-1	-3	-5	-7

Coordinates are written as (x, y).

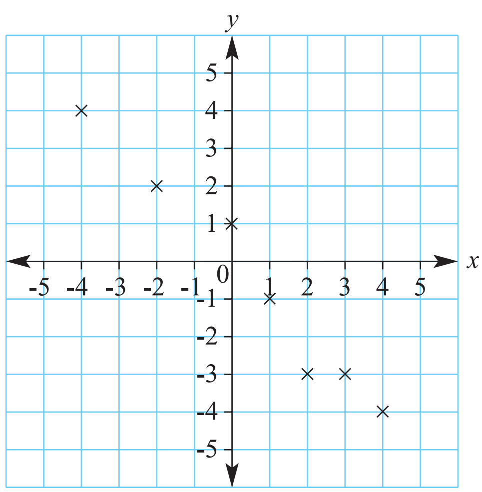
x	-2	(-2, 1)
y	1	

4 Jenna has plotted these points for the rule $y = -x$ and she knows they should all be in a straight line.

a State the coordinates of any points that are not in line with most of the other points.

b Using the rule $y = -x$, calculate the correct coordinates for these two points.

Place your ruler along the plotted points. Any point not in a straight line needs to be re-calculated.



Fluency

Example 7 Plotting a graph from a rule

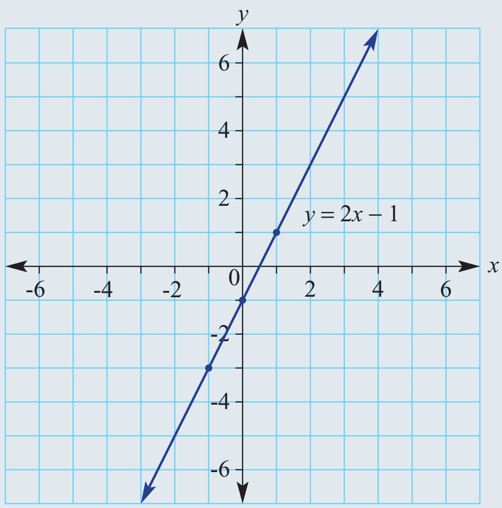
Plot the graph of $y = 2x - 1$ by first completing the table of values.

x	-1	0	1
y			

ISBN: 9781107687028 Cambridge University Press

Solution

x	-1	0	1
y	-3	-1	1



Explanation

Substitute each value into the equation:

$x = -1, y = 2 \times (-1) - 1 = -3$ $(-1, -3)$

$x = 0, y = 2 \times 0 - 1 = -1$ $(0, -1)$

$x = 1, y = 2 \times 1 - 1 = 1$ $(1, 1)$

Plot the points and draw the line with a ruler. When labelling axes, put the numbers on the grid lines, not in the spaces.

5 Complete the following tables, then plot the graph of each one on a separate number plane.

a $y = 2x$

x	-1	0	1
y			

b $y = x + 4$

x	0	1	2
y			

c $y = 2x - 3$

x	0	1	2
y			

d $y = -2x$

x	-1	0	1
y			

e $y = x - 4$

x	1	2	3
y			

f $y = 6 - x$

x	0	1	2
y			

When multiplying, same signs make a positive; e.g. $-2 \times (-1) = 2$

6 Complete the following tables, then plot the graph of each pair on the same axes.

a **i** $y = x + 2$

x	0	2	4
y			

ii $y = -x + 2$

x	0	2	4
y			

b **i** $y = x - 4$

x	0	4	6
y			

ii $y = 4 - x$

x	0	1	2
y			

c **i** $y = 2 + 3x$

x	-3	0	3
y			

ii $y = 3x - 4$

x	-3	0	3
y			

For each part, draw line **i** and line **ii** on the same axes.

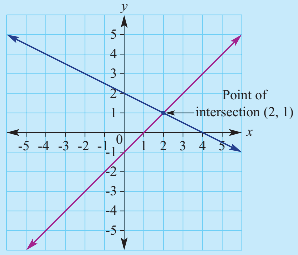
ISBN: 9781107687028

6C

7 By plotting the graphs of each of the following pairs of lines on the same axes, find the coordinates of the point of intersection. Use a table of values, with x from -2 to 2.

a $y = 2x$ and $y = x$
b $y = x + 3$ and $y = 2x + 2$
c $y = 2 - x$ and $y = 2x + 5$
d $y = 2 - x$ and $y = x + 2$
e $y = 2x - 3$ and $y = x - 4$

The point of intersection of two lines is where they cross each other. For example:



Problem-solving and Reasoning

Example 8 Interpreting a graph given a table of values

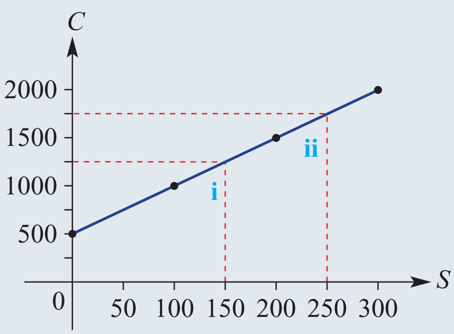
Jasmine is organising a school dance. The venue is chosen and the costs are shown in the table.

Number of students (S)	0	100	200	300
Total cost in dollars (C)	500	1000	1500	2000

a Plot a graph of the total cost against the number of students.
b Use the graph to determine:
   i the total cost for 150 students
   ii how many students could attend the dance if Jasmine has \$1750 to spend

Solution | **Explanation**

a



Construct a set of axes using S between 0 and 300 and C between 0 and 2000. Number of students is placed on the horizontal axis. Plot each point using the information in the table.

b i The total cost for 150 students is \$1250.

Draw a vertical dotted line at $S = 150$ to meet the graph, then draw another dotted line horizontally to the C-axis.

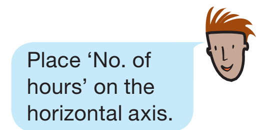
ii 250 students could attend the dance for \$1750.

Draw a horizontal dotted line at $C = 1750$ to meet the graph, then draw a dotted line vertically to the S-axis.

ISBN: 9781107687028

8 A furniture removalist charges by the hour. His rates are shown in the table below.

No. of hours (n)	0	1	2	3	4	5
Cost (C)	200	240	280	320	360	400



a Plot a graph of cost against hours.
b Use the graph to determine:
 i the total cost for 2.5 hours' work
 ii the number of hours the removalist will work for $380

9 Olive oil is sold in bulk for $8 per litre.

No. of litres (l)	1	2	3	4	5
Cost (C)	8	16	24	32	40

a Plot a graph of cost against number of litres.
b Use the graph to determine:
 i the total cost for 3.5 litres of oil
 ii the number of litres of oil you can buy for $20

Example 9 Constructing a table and graph for interpretation

An electrician charges $50 for a service call and $60 an hour for labour.

a Complete the table of values.

No. of hours (n)	1	2	3	4	5
Cost (C)					

b Plot a graph of cost against number of hours.
c Use the graph to determine:
 i the cost for 4.5 hours' work
 ii how long the electrician will work for $260

Solution

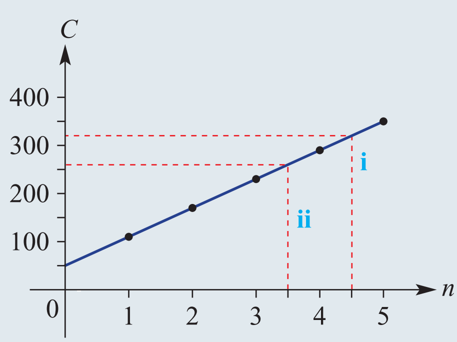
a

No. of hours (n)	1	2	3	4	5
Cost (C)	110	170	230	290	350

Explanation

Cost for 1 hour = $50 + $60 = $110
Cost for 2 hours = $50 + 2 × $60 = $170
Cost for 3 hours = $50 + 3 × $60 = $230 etc.

b



Plot the points from the table using C on the vertical axis and n on the horizontal axis. Join all the points to form the straight line.

c i The cost is $320.

Draw a vertical dotted line at $n = 4.5$ to meet the graph, then draw a line horizontally to the C-axis.

ii The electrician worked for 3.5 hours.

Draw a horizontal dotted line at $C = 260$ to meet the graph, then draw vertically to the n-axis.

ISBN: 9781107687028

10 A car rental firm charges \$200 plus \$1 for each kilometre travelled.

a Complete the table of values below.

No. of km (k)	100	200	300	400	500
Cost (C)					

b Plot a graph of cost against kilometres.

c Use the graph to determine:

i the cost if you travel 250 km

ii how many kilometres you can travel on a \$650 budget

11 Matthew delivers pizza for a fast food outlet. He is paid \$20 a shift plus \$3 per delivery.

a Complete the table of values below.

No. of deliveries (d)	0	5	10	15	20
Pay (P)					

b Plot a graph of Matthew's pay against number of deliveries.

c Use the graph to determine:

i the amount of pay for 12 deliveries

ii the number of deliveries made if Matthew is paid \$74

Which mechanic?

12 Two mechanics charge different rates for their labour. Paul charges \$75 for a service call plus \$50 per hour. Sherry charges \$90 for a service call plus \$40 per hour.

a Create a table for each mechanic for up to 5 hours of work.

b Plot a graph for the total charge against the number of hours worked for Paul and Sherry on the same axes.

c Use the graph to determine:

i the cost of hiring Paul for 3.5 hours

ii the cost of hiring Sherry for 1.5 hours

iii the number of hours of work if Paul charges \$100

iv the number of hours of work if Sherry charges \$260

v the number of hours of work if the cost from Paul and Sherry is the same

d Write a sentence describing who is cheaper for different hours of work.

ISBN: 9781107687028 Cambridge University Press

6.4 Midpoint and length of a line segment

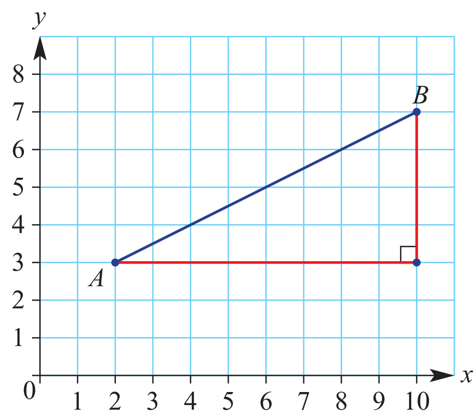
A line segment has a definite length and also has a point in the middle of the segment called the midpoint. Both the midpoint and length can be found by using the coordinates of the endpoints.

Builders use mathematical calculations to determine the length, midpoint and angle of inclination of wooden beams when constructing the timber frame of a house.

Let's start: Finding a method

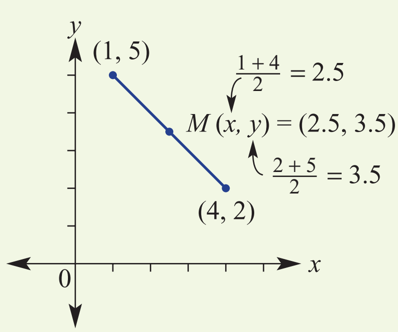
This is a graph of the line segment *AB*. A right-angled triangle has been drawn so that *AB* is the hypotenuse.

- How many units long are the horizontal and vertical sides of this right-angled triangle?
- Discuss and explain a method for finding the length of the line segment *AB*.
- What is the *x* value of the middle point of the horizontal side of the right-angled triangle?
- What is the *y* value of the middle point of the vertical side of the right-angled triangle?
- What are the coordinates of the point in the middle of the line segment *AB*?
- Discuss and explain a method for finding the midpoint of a line segment.



Key ideas

- The **midpoint** (M) of a line segment is the halfway point between the two endpoints.
  - Midpoint = (average of the two x values at the endpoints, average of the two y values at the endpoints)
  - $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$



  - When finding the average, add the values in the numerator before dividing by 2.

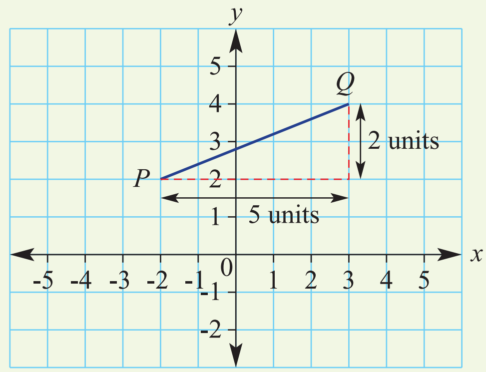
Midpoint The point on an interval that is equidistant from the end points of the interval

ISBN: 9781107687028

- The length of a line segment is found using **Pythagoras' theorem**.
  To find the length of the line segment PQ:
  – draw a right-angled triangle with the line segment PQ as the hypotenuse (longest side)
  – count the grid squares to find the length of each smaller side
  – apply Pythagoras' theorem

$$\begin{aligned} PQ^2 &= 5^2 + 2^2 \\ &= 25 + 4 \\ &= 29 \\ PQ &= \sqrt{29} \text{ units} \end{aligned}$$

  – $\sqrt{29}$ is the length of line segment PQ in square root form.

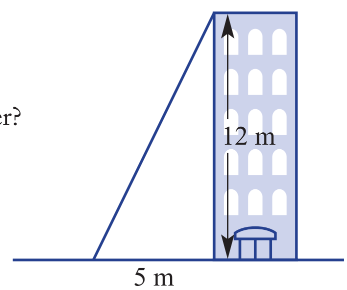


Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides

Exercise 6D

Understanding

1 Imagine that a fireman's ladder is extended to the top of a 12 m building, with the foot of the ladder on the ground 5 m out from the base of the building.



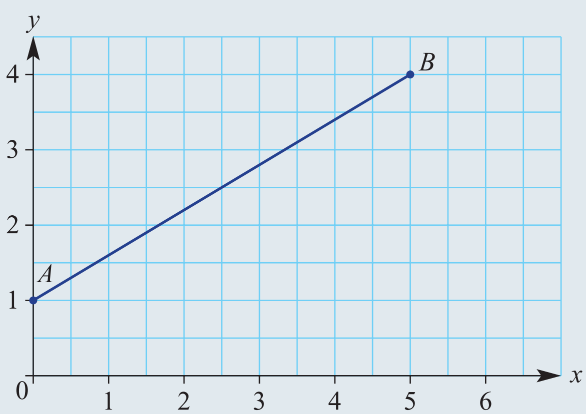
a What is the rule called that can be used to find the length of the ladder?
b Apply this method and calculate the length of this ladder.

Fireman Fred stands exactly halfway along the ladder.

c What height of the building is level with Fred's feet?
d How far out are his feet from the building?
e Write the missing words needed to complete this sentence:
The middle point of the hypotenuse is level with the __________ point of the horizontal side and the __________ point of the vertical side of a right-angled triangle.

Example 10 Finding the length of a line segment from a graph

Find the length of the line segment between $A(0, 1)$ and $B(5, 4)$.

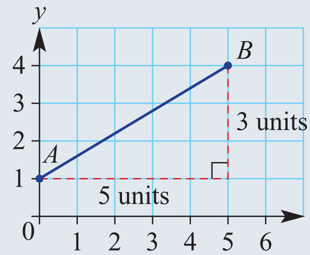


ISBN: 9781107687028 Cambridge University Press

Solution

$AB^2 = 5^2 + 3^2$
$AB^2 = 25 + 9$
$AB^2 = 34$
$AB = \sqrt{34}$

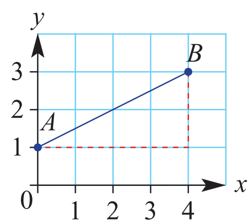
Explanation



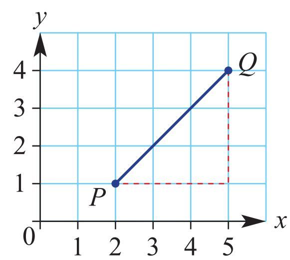
Create a right-angled triangle and use Pythagoras' theorem. For $AB^2 = 34$, take the square root of both sides to find AB. $\sqrt{34}$ is the exact answer.

2 Find the length of each of the following line segments. Leave each answer in square root form.

a



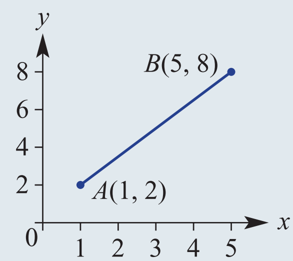
b



Count the 'spaces' to find the number of units for the horizontal and vertical sides.

Example 11 Finding the midpoint of a line segment from a graph

Find the midpoint of the interval between $A(1, 2)$ and $B(5, 8)$.



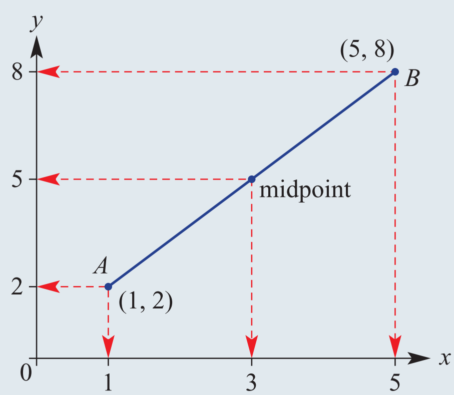
Solution

$$\text{Average of } x\text{-values} = \frac{1+5}{2} = \frac{6}{2} = 3$$

$$\text{Average of } y\text{-values} = \frac{2+8}{2} = \frac{10}{2} = 5$$

Midpoint (3, 5)

Explanation

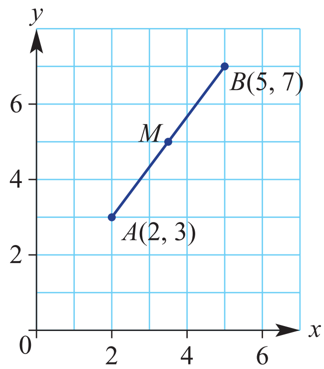


Midpoint (3, 5)

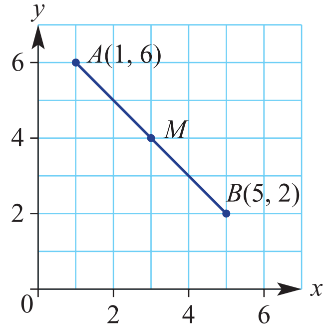
ISBN: 9781107687028 Cambridge University Press

3 Find the midpoint M of each of the following intervals:

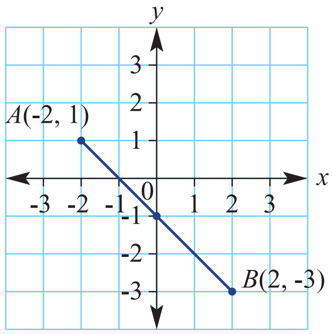
a



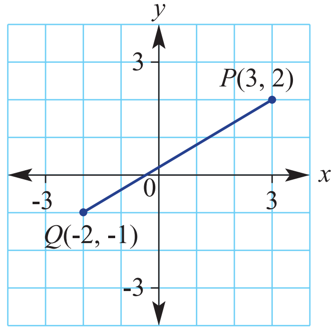
b



c



d

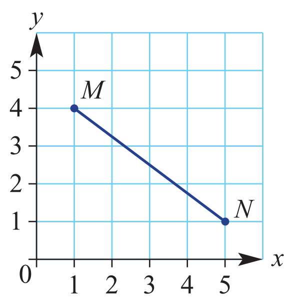


In finding the average, add the numerator values before dividing by 2.

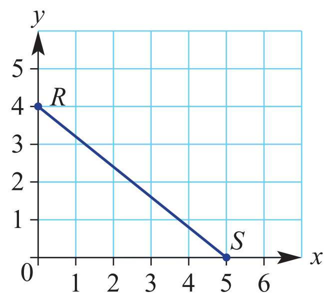
Fluency

4 Find the length of each of the following line segments.

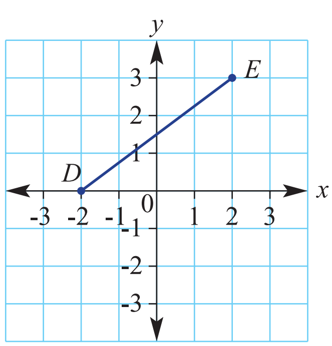
a



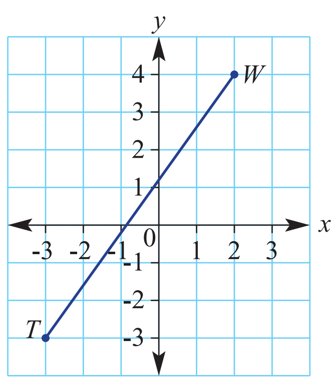
b



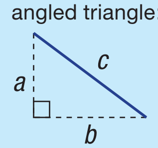
c



d



Use Pythagoras' theorem ($c^2 = a^2 + b^2$) by forming a right-angled triangle:

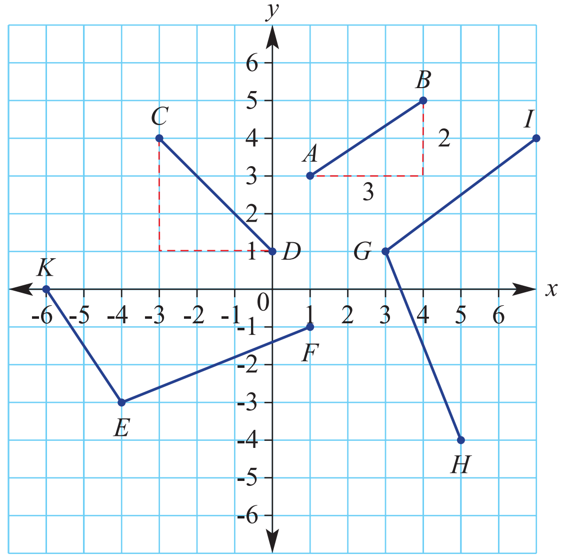


Write the answer in square root form if it is not a known square root.

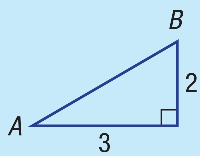
ISBN: 9781107687028 Cambridge University Press

5 Find the length of each line segment on the following number plane. Leave your answers in square root form.

a AB **b** CD
c EF **d** GH
e KE **f** GI

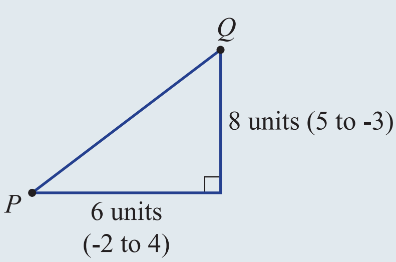
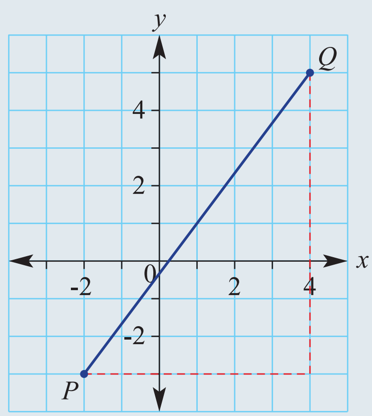


First sketch a right-angled triangle for each line segment, labelling the known sides. For example:



Example 12 Finding the length of a line segment given the coordinates of the end points

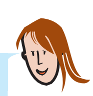
Find the distance between the points P and Q if P is at (-2, -3) and Q is at (4, 5).

Solution	**Explanation**
	Use Pythagoras' theorem to find PQ, the hypotenuse.
$PQ^2 = 6^2 + 8^2$ $PQ^2 = 36 + 64$ $PQ^2 = 100$ $PQ = \sqrt{100}$ $PQ = 10$ units	 If you know the value of the square root, write its value.

6 Plot each of the following pairs of points and find the distance between them, correct to one decimal place where necessary.

a (2, 3) and (5, 7) **b** (0, 1) and (6, 9)
c (0, 0) and (-5, 10) **d** (-4, -1) and (0, -5)
e (-3, 0) and (0, 4) **f** (0, -1) and (2, -4)

First rule up axes with x from -5 to 10 and y from -5 to 10.



ISBN: 9781107687028

Cambridge University Press

7 Find the exact length between these pairs of points.

a (1, 3) and (2, 2)
b (4, 1) and (7, 3)
c (-3, -1) and (0, 4)
d (-2, -3) and (3, 5)
e (-1, 0) and (-6, 1)
f (1, -3) and (4, -2)

Exact length means leave the $\sqrt{}$ sign in the answers.

Example 13 Finding the midpoint of a line segment given the coordinates of the end points

Find the midpoint of the line segment joining $P(-3, 1)$ and $Q(5, -4)$.

Solution	Explanation
$x = \frac{-3+5}{2}$ $= \frac{2}{2}$ $= 1$	Average the x coordinates. Calculate the numerator before dividing by 2. $-3 + 5 = 2$
$y = \frac{1+(-4)}{2}$ $= \frac{-3}{2}$ $= -1.5$	Average the y coordinates. Calculate the numerator before dividing by 2. $1 + (-4) = 1 - 4 = -3$
Midpoint (1, -1.5)	Write the coordinates of the midpoint.

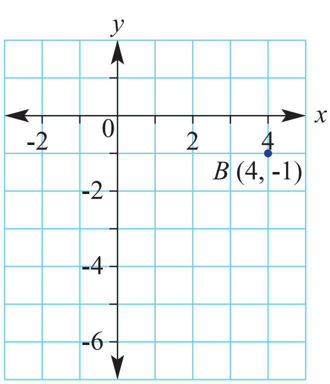
8 Find the midpoint of the line segment joining the following points.

a (1, 4) and (3, 6)
b (3, 7) and (5, 9)
c (0, 4) and (6, 6)
d (2, 4) and (3, 5)
e (7, 2) and (5, 3)
f (1, 6) and (4, 2)
g (0, 0) and (-2, -4)
h (-2, -3) and (-4, -5)
i (-3, -1) and (-5, -5)
j (-3, -4) and (5, 6)
k (0, -8) and (-6, 0)
l (3, -4) and (-3, 4)

Check that your answer appears to be halfway between the endpoints.

Problem-solving and Reasoning

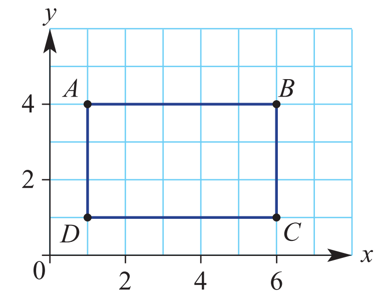
9 Copy the diagram on the right. Mark the point $B(4, -1)$, as shown, then mark on the point $M(1, -3)$. Find the coordinates of A if M is the midpoint of the interval AB.



ISBN: 9781107687028 Cambridge University Press

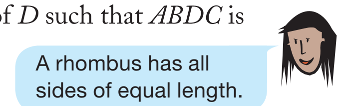
10 Copy the diagram of rectangle $ABCD$.

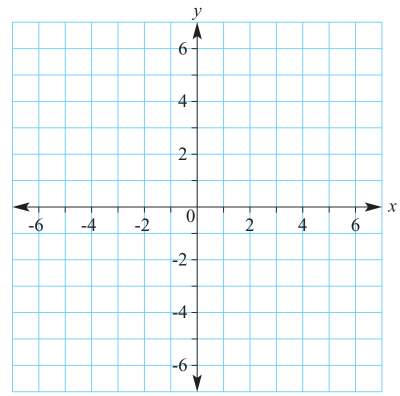
- **a** What are the coordinates of each vertex?
- **b** Find the midpoint of the diagonal AC.
- **c** Find the midpoint of the diagonal BD.
- **d** What does this tell us about the diagonals of a rectangle?



11 **a** Draw up a four-quadrant number plane like the one shown.

- **b** Plot the points $A(-4, 0)$, $B(0, 3)$ and $C(0, -3)$ and form the triangle ABC.
- **c** What is the length of:
  - **i** AB?
  - **ii** AC?
- **d** What type of triangle is ABC?
- **e** Calculate its perimeter and area.
- **f** Write down the coordinates of D such that $ABDC$ is a rhombus.

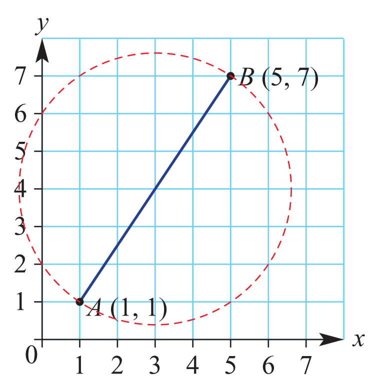




Features of a circle

12 The diameter of a circle is shown on this graph.

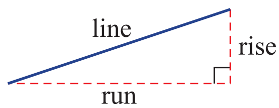
- **a** What are the coordinates of X, the centre of the circle? Mark this point on your graph.
- **b** What is the length of the radius XA?
- **c** Find the distance from X to the point (5, 1). How can we tell that (5, 1) lies on the circle?
- **d** Use $C = 2\pi r$ to find the circumference of the circle shown. Round to one decimal place.
- **e** Calculate the area of this circle using $A = \pi r^2$, correct to one decimal place.



ISBN: 9781107687028

6.5 Exploring gradient

The gradient of a line is a measure of its slope. It is a number that shows the steepness of a line. It is calculated by knowing how far a line rises or falls (called the *rise*) within a certain horizontal distance (called the *run*). The gradient is equal to the *rise* divided by the *run*. The letter m is used to represent gradient.

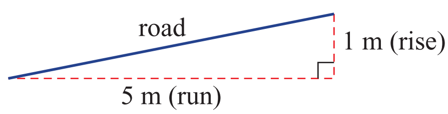


$$\textit{gradient of line} = m = \frac{rise}{run}$$

Engineers apply their knowledge of gradients when designing roads, bridges, railway lines and buildings. Some mountain railways have a gradient greater than 1, which is a slope far too steep for a normal train or even a powerful car.

For example, this train takes tourists to the Matterhorn, a mountain in Switzerland. To cope with the very steep slopes it has an extra wheel with teeth, which grips a central notched line.

Let's start: What's the gradient?



A road that rises by 1 m for each 5 m of horizontal distance has a gradient of 0.2 or 20%.
Trucks would find this gradient very steep.

The gradient is calculated by finding the rise divided by the run.

$$\textit{gradient} = \frac{rise}{run} = \frac{1}{5} = 0.2 = 20\%$$

1 Find the gradient for each of these roads. Give the answer as a decimal and a perentage.
  a Baldwin Street, Dunedin, New Zealand is known as the steepest street in the world. For each 2.86 m of horizontal (run) distance, the road rises by 1 m.
  b Gower Street, Toowong, is Brisbane's steepest street. For each 3.2 m of horizontal (run) distance, the road rises by 1 m.

2 The Scenic Railway, Katoomba, NSW has a maximum gradient of 122% as it passes through a gorge in the cliff. What is its vertical distance (rise) for each 1 metre of horizontal distance (run)?

ISBN: 9781107687028

Cambridge University Press

Use computer software (dynamic geometry) to produce a set of axes and grid.

- Construct a line segment with endpoints on the grid. Show the coordinates of the endpoints.
- Calculate the rise (vertical distance between the endpoints) and the run (horizontal distance between the endpoints).
- Calculate the gradient as the *rise* divided by the *run*.
- Now drag the endpoints and explore the effect on the gradient.
- Can you drag the endpoints but retain the same gradient value? Explain why this is possible.
- Can you drag the endpoints so that the gradient is zero or undefined? Describe how this can be achieved.

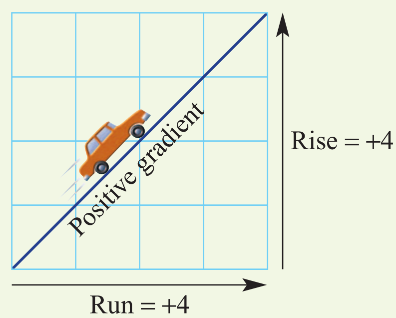
Key ideas

Gradient (m) The steepness of a slope

- **Gradient (m)** $= \dfrac{\text{rise}}{\text{run}}$

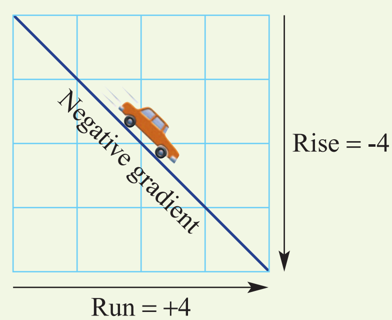
  Always move from left to right when considering the rise and the run.
  - The horizontal 'run' always goes to the right and is always positive. The vertical 'rise' can go up (positive) or down (negative).
  - If the line slopes up from left to right, the rise is positive and the gradient is positive.

    e.g. $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{+4}{+4} = 1$

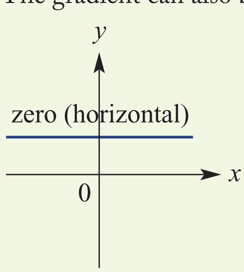


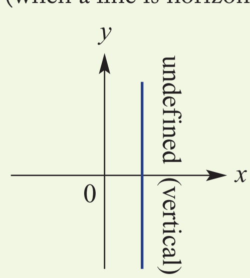
  - If the line slopes down from left to right, the rise is considered to be negative and the gradient is negative.

    e.g. $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{-4}{+4} = -1$



- The gradient can also be zero (when a line is horizontal) and undefined (when a line is vertical).





- Between two points (x_1, y_1) and (x_2, y_2), the gradient (m) is $m = \dfrac{y_2 - y_1}{x_2 - x_1} \dfrac{\text{(rise)}}{\text{(run)}}$.

ISBN: 9781107687028

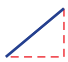
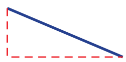
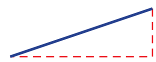
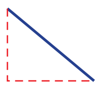
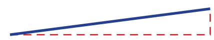
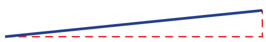
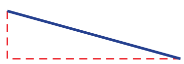
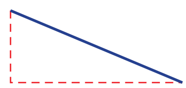
Exercise 6E

Understanding

1 Use the words 'positive', 'negative', 'zero' or 'undefined' to complete each sentence.
- **a** The gradient of a horizontal line is ____________.
- **b** The gradient of the line joining (0, 3) and (5, 0) is __________.
- **c** The gradient of the line joining (-6, 0) and (1, 1) is ___________.
- **d** The gradient of a vertical line is __________.

2 Decide whether each of the following lines would have a positive or negative gradient.

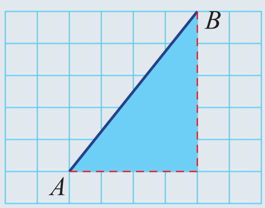
Lines going downhill from left to right have a negative gradient.

a 
b 
c 
d 
e 
f 
g 
h 
i 

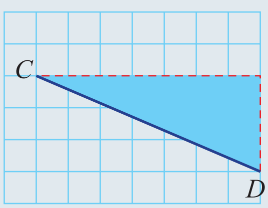
Example 14 Finding the gradient from a grid

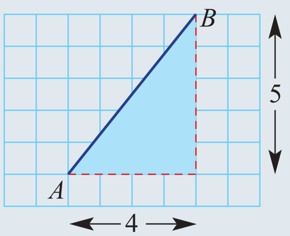
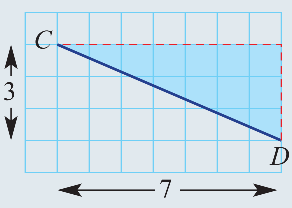
Find the gradient of the following line segments, where each grid box equals 1 unit.

a

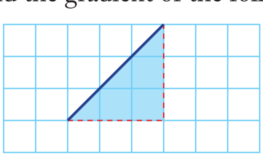
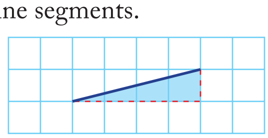
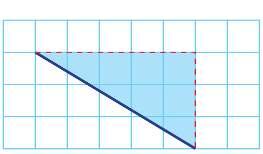


b



Solution	Explanation
a Gradient of AB $= \dfrac{\text{rise}}{\text{run}}$ $= \dfrac{5}{4}$	 The slope is upwards and therefore the gradient is positive. The rise is 5 and the run is 4.
b Gradient of CD $= \dfrac{\text{rise}}{\text{run}}$ $= \dfrac{-3}{7}$ $= -\dfrac{3}{7}$	 The slope is downwards and therefore the gradient is negative. The fall is 3 so we write rise = -3, and the run is 7.

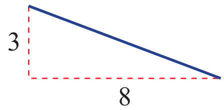
3 Find the gradient of the following line segments.

a 
b 
c 

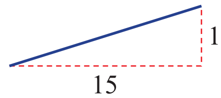
ISBN: 9781107687028

4 Find the gradient of the following.

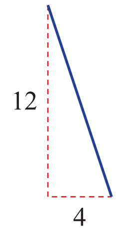
a

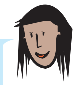


b



c





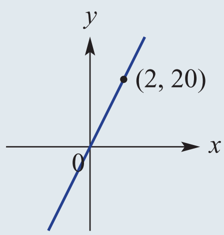
The gradient is written as a fraction or a whole number.

Fluency

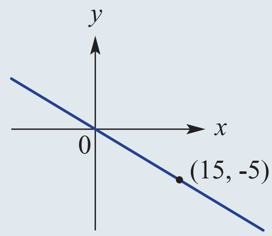
Example 15 Finding the gradient from graphs

Find the gradient of the following lines.

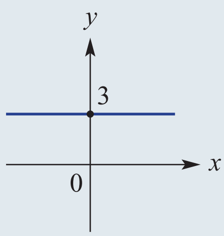
a



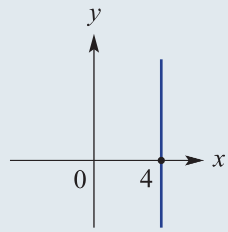
b



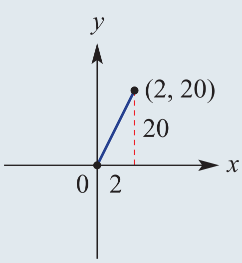
c



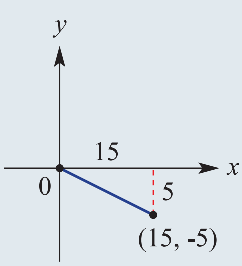
d



Solution	Explanation
a Gradient $= \dfrac{\text{rise}}{\text{run}}$	Write the rule each time.
$= \dfrac{20}{2}$	The rise is 20 and the run is 2 between the two points (0, 0) and (2, 20).
$= 10$	Simplify by cancelling.



Solution	Explanation
b Gradient $= \dfrac{\text{rise}}{\text{run}}$	Note this time that, when working from left to right, there will be a slope downwards.
$= \dfrac{-5}{15}$	The fall is 5 (rise = -5) and the run is 15.
$= \dfrac{-1}{3}$	Simplify.
$= -\dfrac{1}{3}$	
c Gradient $= 0$	Horizontal lines have a zero gradient.
d Gradient is undefined.	Vertical lines have an undefined gradient.



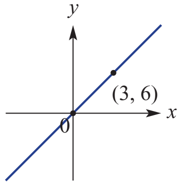
ISBN: 9781107687028

Cambridge University Press

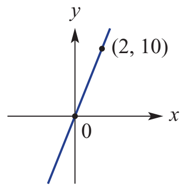
6E

5 Find the gradient of the following lines.

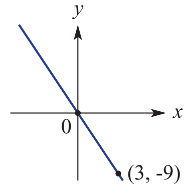
a



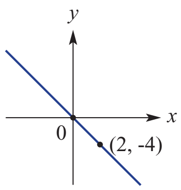
b



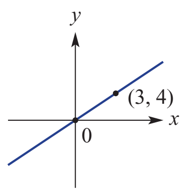
c



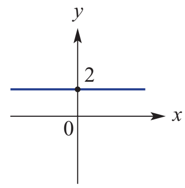
d

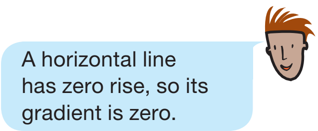


e

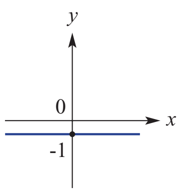


f

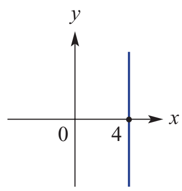




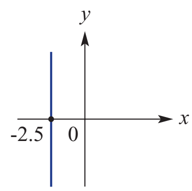
g

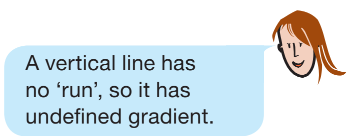


h

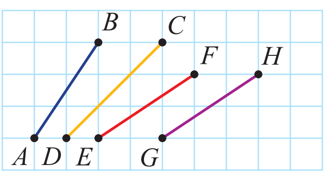


i



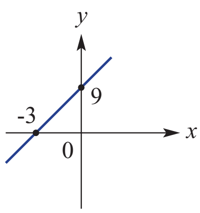


6 Use the grid to find the gradient of the following line segments. Then order the segments from least to greatest gradient.

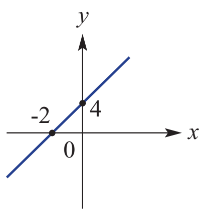


7 Determine the gradient of the following lines.

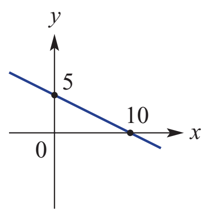
a

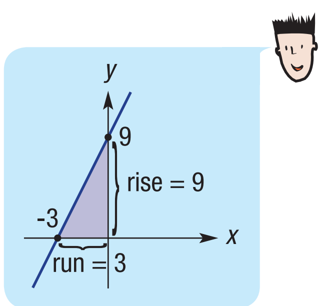


b

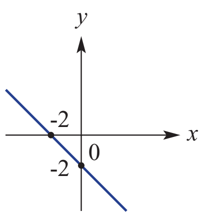


c

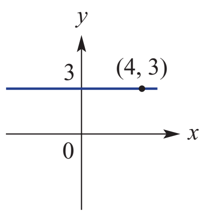




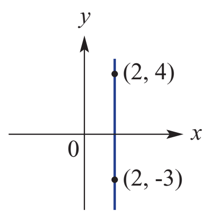
d



e



f



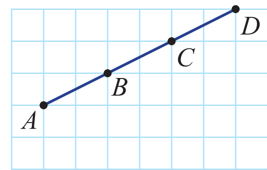
ISBN: 9781107687028

Cambridge University Press

Problem-solving and Reasoning

8 **a** Copy and complete the table below.

Line segment	Rise	Run	Gradient
AB			
AC			
AD			
BC			
BD			
CD			

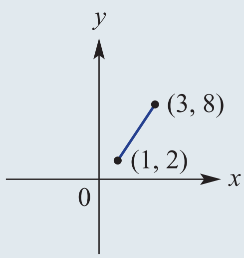


b What do you notice about the gradient between points on the same line?

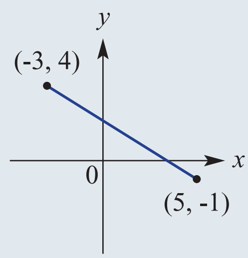
Example 16 Using a formula to calculate gradient

Use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line segments between the following pairs of points.

a



b



Solution

a $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{8-2}{3-1}$

$= \dfrac{6}{2}$

$= 3$

Explanation

Write the rule.

(1, 2) (3, 8)

x_1 y_1 x_2 y_2

It does not matter which point is labelled (x_1, y_1) and which is (x_2, y_2).

Solution

b $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-1-4}{5-(-3)}$

$= \dfrac{-5}{8}$

$= -\dfrac{5}{8}$

Explanation

Write the rule.

(5, -1) (-3, 4)

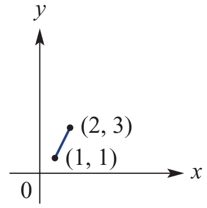
x_1 y_1 x_2 y_2

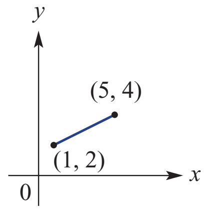
Remember that $5 - (-3) = 5 + 3 = 8$

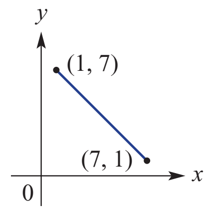
ISBN: 9781107687028

6E

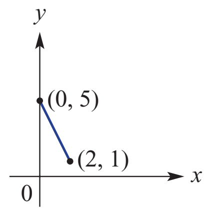
9 Use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient between these pairs of points.

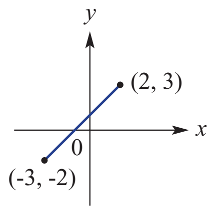
a


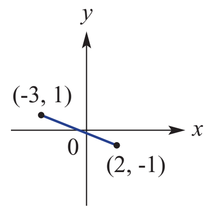
b


c


First copy the coordinates and label them.
E.g. (1, 1) (2, 3)
$x_1\ y_1 \quad x_2\ y_2$

d


e


f


You can choose either point to be (x_1, y_1).

10 Find the gradient between the following pairs of points:

a (1, 3) and (5, 7)
b (-1, -1) and (3, 3)
c (-3, 4) and (2, 1)
d (-6, -1) and (3, -1)
e (1, -4) and (2, 7)
f (-4, -2) and (-1, -1)

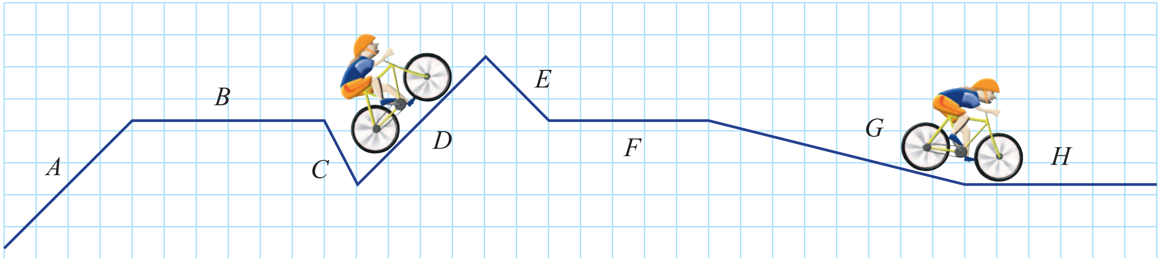
Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

11 The first section of the Cairns Skyrail travels from Caravonica terminal at 5 m above sea level to Red Peak terminal, which is 545 metres above sea level. This is across a horizontal distance of approximately 1.57 km. What is the overall gradient of this section of the Skyrail? Round the answer to three decimal places.

Both distances need to be in the same units.

From Bakersville to Rolland

12 A transversal map for a bike ride from Bakersville to Rolland is shown.



a Which sections A, B, C, D, E, F, G or H indicate travelling a positive gradient?
b Which sections indicate travelling a negative gradient?
c Which will be the hardest section to ride?
d Which sections show a zero gradient?
e Which section is the flattest of the downhill rides?

ISBN: 9781107687028 Cambridge University Press

6.6 Rates from graphs

The speed or rate at which something changes can be analysed by looking at the gradient (steepness) of a graph. Two common rates are kilometres per hour (km/h) and litres per second (L/s).

Graphs of a patient's records provide valuable information for a doctor. For example, from a graph of temperature versus time, the rate of temperature change in °C/minute can be calculated. This rate provides important information to help a doctor diagnose an illness.

Let's start: What's the rate?

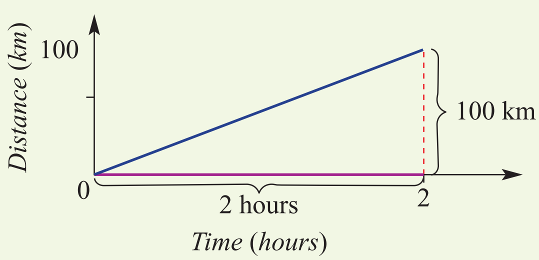
Calculate each of these rates.

a $60 000 for 200 tonnes of wheat.
b Lee travels 840 km in 12 hours.
c A foal grows 18 cm in height in 3 months.
d Petrol costs $96 for 60 litres.
e Before take-off, a hot-air balloon of volume 6000 m^3 is filled in 60 seconds.

Key ideas

- A **rate** compares two quantities. Many rates show how a quantity changes over *time*.

Rate A measure of one quantity against another



$$\text{rate} = \frac{\text{rise}}{\text{run}} = \frac{100}{2} = 50 \text{ km/h}$$

- Rate = change in quantity ÷ change in time
  (L, kg, …) (seconds, hours, …)
- A common rate is speed.
  – Speed = change in distance ÷ change in time
  (cm, km, …) (seconds, hours, …)
- The gradient of a line gives the rate.
- To determine a rate from a linear graph, calculate the gradient and include the units (y unit/x unit); e.g. km/h.

ISBN: 9781107687028

Exercise 6F

Understanding

1 Complete the sentences.

a A rate is found from a linear graph by calculating the ___________ of the line.

b A rate compares ___________ quantities.

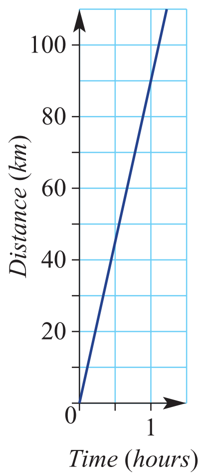
c A rate has two ___________.

d A speed of 60 kilometres per hour is written as 60 ___________.

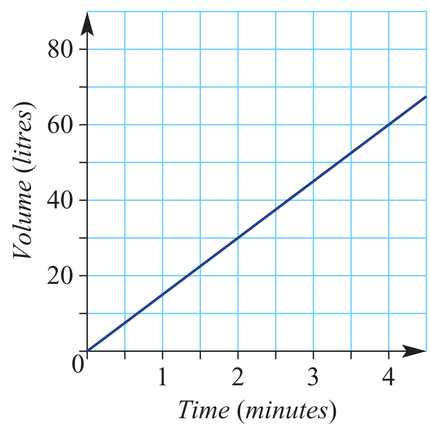
e If a rate of filling a bath is 50 litres per minute, this is written as 50 ___________.

2 Write down the rate by calculating the gradient of each line graph.

a



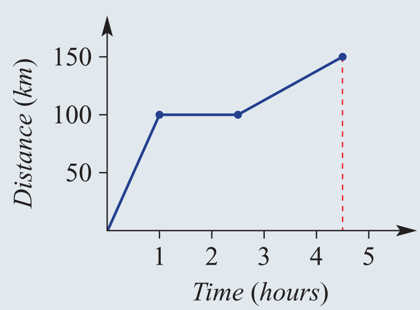
b



Fluency

Example 17 Calculating speed from a graph

A 4WD vehicle completes a journey, which is described by this graph.



a For the first hour, find:

i the total distance travelled

ii the speed

b How fast was the 4WD travelling during:

i the first hour?

ii the second section?

iii the third section?

Solution	Explanation
a i 100 km	Read the distance at 1 hour.
ii 100 km/1 h = 100 km/h	Speed = distance ÷ time
b i 100 km/1 h = 100 km/h	Speed = distance ÷ time
ii 0 km/1.5 h = 0 km/h	The vehicle is at rest.
iii (150 – 100) km/(4.5 – 2.5) h	Determine the distance travelled and the amount of time and apply the rate formula.
= 50 km/2 h	
= 25 km/h	Speed = distance ÷ time
	50 km in 2 hours is $\frac{50}{2} = 25$ km in 1 hour.

ISBN: 9781107687028

3 A car completes a journey, which is described by this graph.

a For the first hour find:

i the total distance travelled

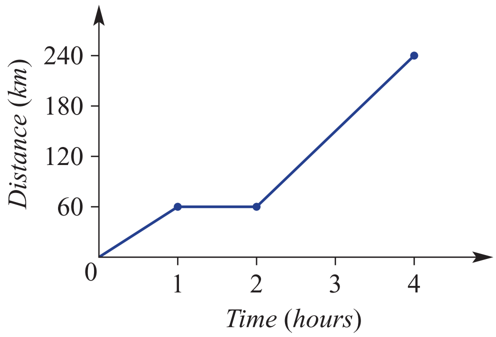
ii the speed

b How fast was the car travelling during:

i the first hour?

ii the second section?

iii the third section?



4 A bike rider training for a professional race includes a rest stop between two travelling sections.

a For the first hour, find:

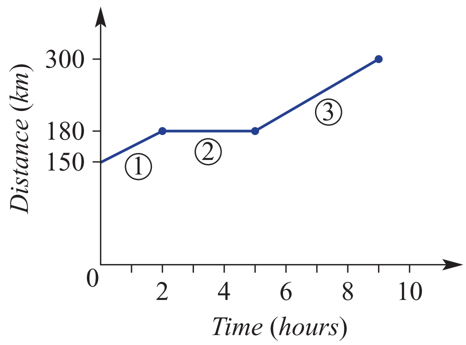
i the total distance travelled

ii the speed

b How fast was the bike travelling during:

i the second section?

ii the third section?



Example 18 Calculating the rate of change of volume in L/s

A container is being filled with water from a hose.

a How many litres were filled during:

i the first 10 seconds?

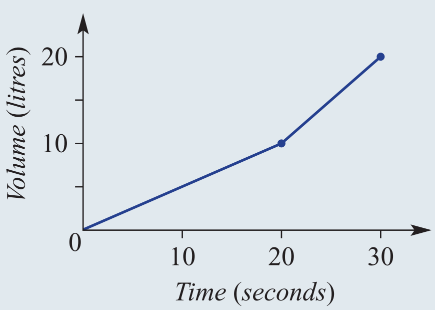
ii the final 10 seconds?

b How fast (what rate in L/s) was the container being filled:

i during the first 10 seconds?

ii during the final 10 seconds?

iii between the 10- to 20-second marks?

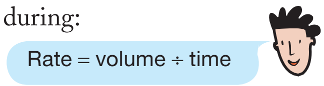


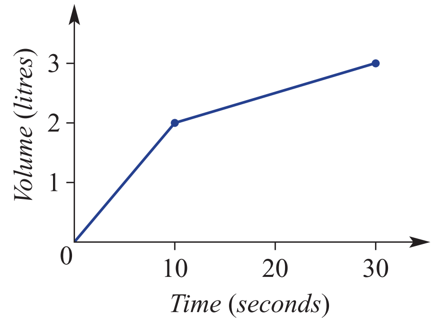
Solution			Explanation
a	**i**	5 litres	Read the number of litres after 10 seconds.
	ii	10 litres	Read the change in litres from 20 to 30 seconds.
b	**i**	5 L/10 s = 0.5 L/s	5 litres is added in the first 10 seconds.
	ii	10 L/10 s = 1 L/s	10 litres is added in the final 10 seconds.
	iii	5 L/10 s = 0.5 L/s	5 litres is added between 10 and 20 seconds.

ISBN: 9781107687028 Cambridge University Press

5 A large carton is being filled with milk.

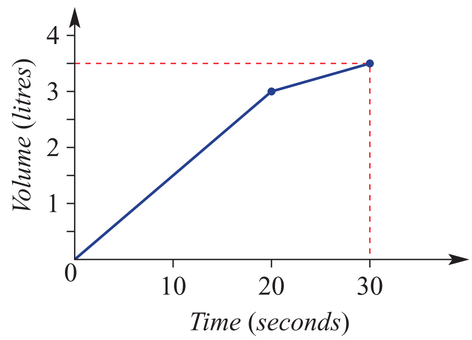
- **a** How many litres were filled during:
  - **i** the first 10 seconds?
  - **ii** the final 10 seconds?
- **b** How fast (what rate in L/s) was the container being filled:
  - **i** during the first 10 seconds?
  - **ii** during the final 10 seconds?
  - **iii** between the 10- and 20-second marks?





6 A large bottle with a long narrow neck is being filled with water.

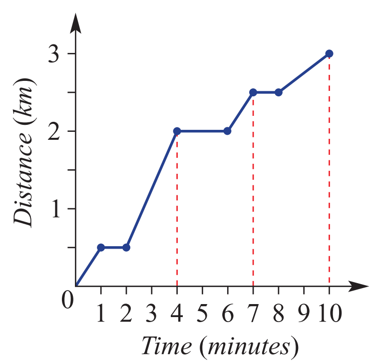
- **a** How many litres were filled during:
  - **i** the first 10 seconds?
  - **ii** the final 10 seconds?
- **b** How fast (what rate in L/s) was the bottle being filled:
  - **i** during the first 10 seconds?
  - **ii** during the final 10 seconds?
  - **iii** between the 10- and 20-second marks?



Problem-solving and Reasoning

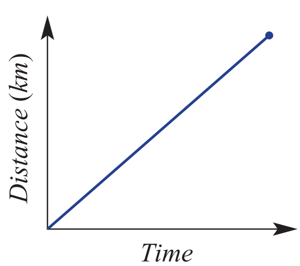
7 A postman stops to deliver mail to each of three houses along a country lane.

- **a** What was the total length of the country lane?
- **b** What was the total time the postman spent standing still?
- **c** Find the speed (use km/min) of the postman at the following times:
  - **i** before the first house
  - **ii** between the first and the second house
  - **iii** between the second and the third house
  - **iv** after his delivery to the third house

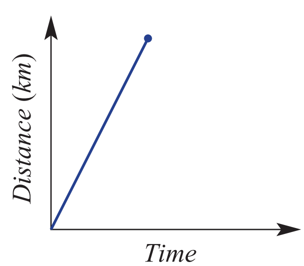


8 Three friends, Anna, Billy and Cianne, travel 5 km from school to the library. Their journeys are displayed in the three graphs below. All three graphs are drawn to the same scale.

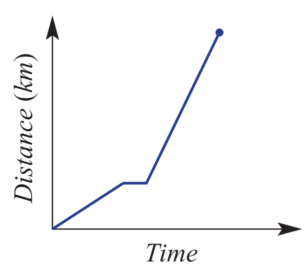
i



ii



iii



- **a** If Anna walked a short distance before getting picked up by her mum, which graph represents her trip?
- **b** If Cianne arrived at the library last, which graph best represents her journey?
- **c** Which graph represents the fastest journey? Explain your answer.

ISBN: 9781107687028

Cambridge University Press

9 **a** Draw your own graph to show the following journey: 10 km/h for 2 hours, then rest for 1 hour, and then 20 km/h for 2 hours.
b Then use your graph to find the total distance travelled.

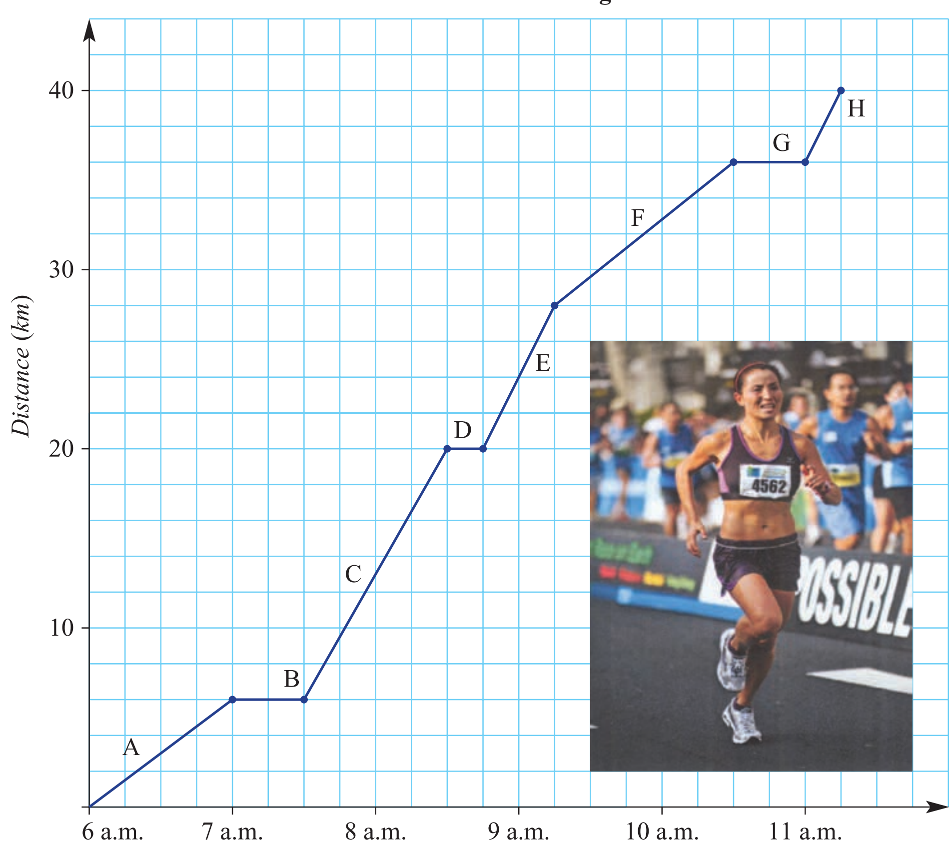
Mark each segment one at a time.
10 km/h for 2 hours covers a distance of $10 \times 2 = 20$ km.

10 A lift starts on the ground floor (height 0 m) and moves to floor 3 at a rate of 3 m/s for 5 seconds. After waiting at floor 3 for 9 seconds, the lift rises 45 m to floor 9 in 9 seconds. The lift rests for 11 seconds before returning to ground level at a rate of 6 m/s. Draw a graph to help find the total time taken to complete the above movements. Use time in seconds on the horizontal axis.

Sienna's training

11 Sienna is training for the Sydney Marathon. Her distance–time graph is shown below.
a How many stops did Sienna make?
b How far did she jog between:
  i 6 a.m. and 7 a.m.? **ii** 7.30 a.m. and 8.30 a.m.?
c Which sections of the graph have a zero gradient?
d Which sections of the graph have the steepest gradient?
e At what speed did Sienna run in section:
  i A? **ii** C? **iii** E? **iv** F? **v** H?
f In which sections is Sienna travelling at the same speed? How does the graph show this?
g How long did the training session last?
h What was the total distance travelled by Sienna during the training session?
i What was her average speed for the entire trip excluding rest periods?

Sienna's training



ISBN: 9781107687028

6.7 $y = mx + c$ and special lines

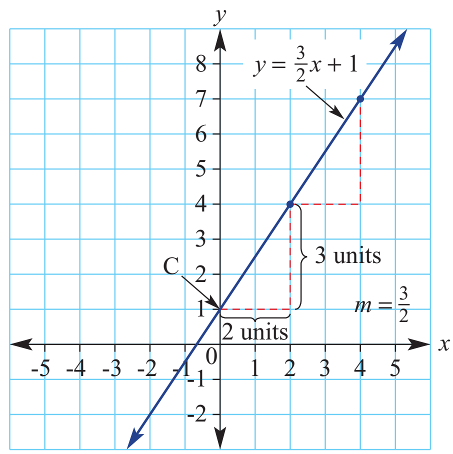
Most straight line graphs can be described by a linear equation $y = mx + c$.

The gradient, m, is the coefficient of x, and c is the y-intercept. This is why this rule is called the gradient–intercept form.

Here is a graph of $y = \frac{3}{2}x + 1$

The gradient $m = \frac{3}{2}$

The y-intercept $c = +1$



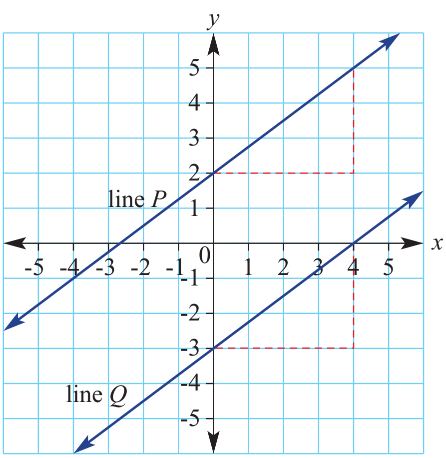
Mathematicians use rules and graphs to help determine how many items should be manufactured to make the maximum profit. For example, profit would be reduced by making too many of a certain style of mobile phone that was soon outdated. A knowledge of graphs is important in business.

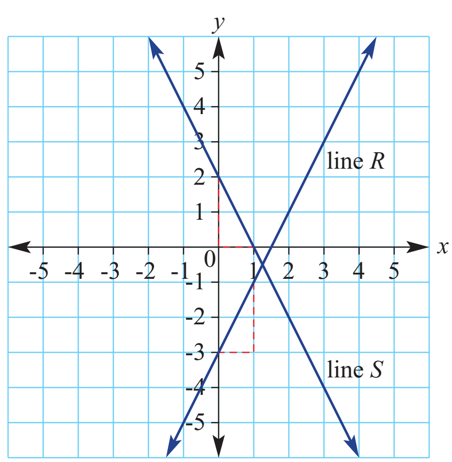
Let's start: Matching lines with equations

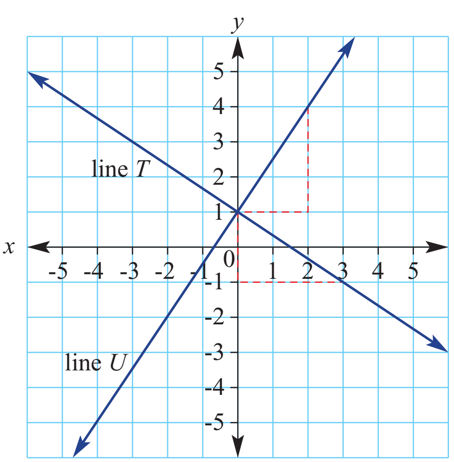
Below are some equations of lines and some graphs. Work with a classmate and help each other to match each equation with its correct line graph.

a $y = 2x - 3$ **b** $y = \frac{3}{4}x + 2$ **c** $y = -\frac{2}{3}x + 1$

d $y = -2x + 2$ **e** $y = \frac{3}{4}x - 3$ **f** $y = \frac{3}{2}x + 1$



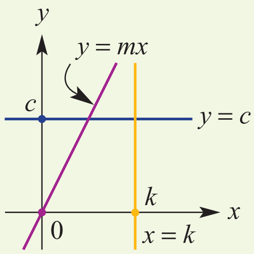




ISBN: 9781107687028 Cambridge University Press

Key ideas

- $y = mx + c$ is called **gradient–intercept form**, where m and c are constants.
  Examples are $y = 3x + 4$, $y = \frac{1}{2}x - 3$ and $y = 2$.
- The gradient or slope equals m (the **coefficient** of x).
- The **y-intercept** is the point where the line cuts the y-axis. In $y = mx + c$, it has coordinates $(0, c)$.
- Some special lines include:
  - horizontal lines: $y = c$ ($m = 0$)
  - vertical lines: $x = k$ (m is undefined)
  - lines passing through the origin $(0, 0)$: $y = mx$ ($c = 0$)



Gradient–intercept form The equation of a straight line, written with y as the subject of the equation

Coefficient A numeral placed before a pronumeral, showing that the pronumeral is multiplied by that factor

y-intercept The point at which a line or curve cuts the y-axis

Exercise 6G

Understanding

1 Complete the sentences.

a $y = mx + c$ is called the ________–________ form of a straight line.

b The symbol m stands for the ________.

c In the equation, the gradient, m, is the ________ of x.

d The symbol c stands for the ________.

e The coordinates of the y-intercept are ________.

Example 19 Reading the gradient and y-intercept from an equation

For the following equations, state:

i the gradient **ii** the y-intercept

a $y = 3x + 4$ **b** $y = -\frac{3}{4}x - 7$

Solution			Explanation
a	**i**	Gradient is 3.	The coefficient of x is 3, i.e. $m = 3$
	ii	y-intercept at $(0, 4)$.	The value of c is +4.
b	**i**	Gradient is $-\frac{3}{4}$.	The coefficient of x is $-\frac{3}{4}$, i.e. $m = -\frac{3}{4}$
	ii	y-intercept at $(0, -7)$.	The value of c is -7; don't forget to include the – sign.

ISBN: 9781107687028

2 For the following equations state the:

i gradient **ii** y-intercept

a $y = 2x + 4$ **b** $y = 6x - 7$ **c** $y = -\frac{2}{3}x + 7$

d $y = -7x - 3$ **e** $y = \frac{3}{5}x - 8$ **f** $y = 9x - 5$

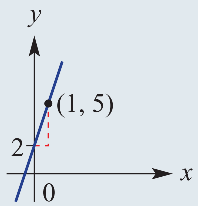
The gradient is the coefficient of x, which is the number multiplied by x. It does not include the x.

Write the coordinates of the y-intercept, including the sign of c (0, c).

Fluency

Example 20 Sketching a line using the y-intercept and gradient

Sketch the graph of $y = 3x + 2$ by considering the y-intercept and the gradient.

Solution	Explanation
	Consider $y = mx + c$; the value of c is 2 and therefore the y-intercept is (0, 2). The value of m is 3 and therefore the gradient is 3 or $\frac{3}{1}$. Start at the y-intercept (0, 2) and, with the gradient of $\frac{3}{1}$, move 1 right (run) and 3 up (rise) to the point (1, 5). Join the points in a line.

3 Sketch the graph of the following by considering the y-intercept and the gradient.

a $y = 2x + 3$ **b** $y = 3x - 12$

c $y = x + 4$ **d** $y = -2x + 5$

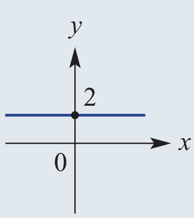
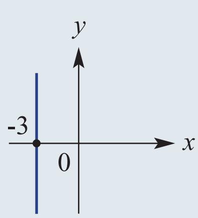
e $y = -5x - 7$ **f** $y = -x - 4$

Plot the y-intercept first.
For a line with $m = -2$:
$m = -2 = \frac{-2}{1} = \frac{\text{down } 2}{\text{right } 1}$.
From the y-intercept, go right 1 then down 2 to plot the next point.

Example 21 Sketching special lines

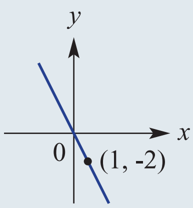
Sketch the graphs of these equations.

a $y = 2$ **b** $x = -3$ **c** $y = -2x$

Solution	Explanation
a 	Sketch a horizontal line passing through (0, 2).
b 	Sketch a vertical line passing through (-3, 0).

ISBN: 9781107687028

c When $x = 0$, $y = -2(0) = 0$
When $x = 1$, $y = -2(1) = -2$



The line passes through the origin (0, 0).
Use $x = 1$ to find another point.
Sketch the graph passing through (0, 0) and (1, -2)

4 Sketch the following lines.

a $y = 3x$ **b** $y = 6x$ **c** $y = -2x$
d $y = 4$ **e** $y = -2$ **f** $y = 5$
g $x = 5$ **h** $x = -2$ **i** $x = 9$

For the line equation $y = 2$, every point on the line has a y value of 2.
E.g. (-3, 2) (0, 2) (1, 2) (3, 2)
For the line equation $x = -3$, every point on the line has an x value of -3.
E.g. (-3, 1) (-3, 0) (-3, -4)

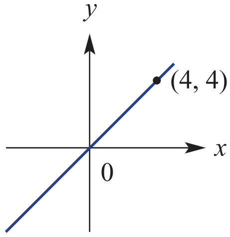
5 Write the equation of the following lines.

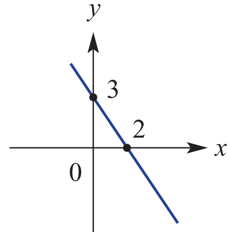
a gradient = 4, y-intercept at 2
b gradient = 3, y-intercept at -2
c gradient = 5, y-intercept at 0
d gradient = -3, y-intercept at 5
e gradient = -4, y-intercept at -3
f gradient = -2, y-intercept at 0

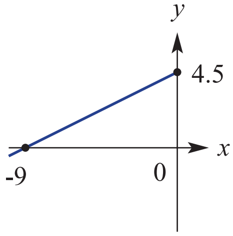
A line has equation
$y = mx + c$.
gradient y-intercept

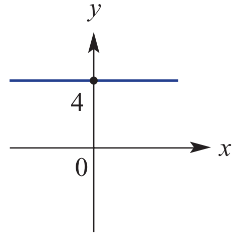
6 Determine the gradient and y-intercept for the following lines.

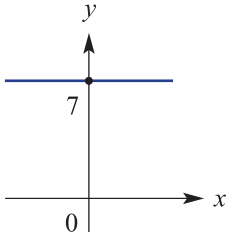
Use $m = \frac{\text{rise}}{\text{run}}$ for the gradient between two known points.

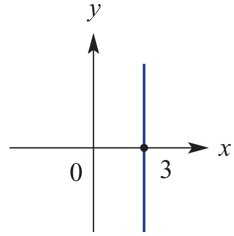
a


b


c


d


e


f


ISBN: 9781107687028

6G

Problem-solving and Reasoning

7 Match each of the following linear equations to one of the sketches shown.

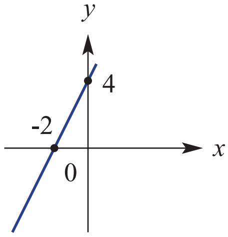
a $y = -\frac{2}{3}x + 2$ **b** $y = -x + 4$ **c** $y = x + 3$
d $y = 2x + 4$ **e** $y = 4$ **f** $y = 7x$
g $y = -3x + 6$ **h** $x = 2$ **i** $y = -3x$

A linear equation is an equation that gives a straight line graph.

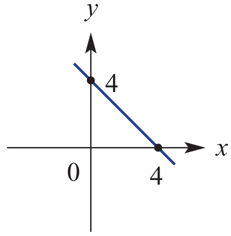
For a negative gradient, move the negative sign to the numerator.

$m = -\frac{2}{3} = \frac{-2}{3} = \frac{\text{down } 2}{\text{right } 3}$

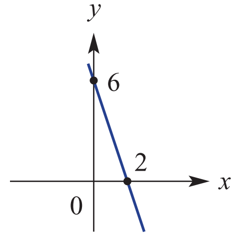
i



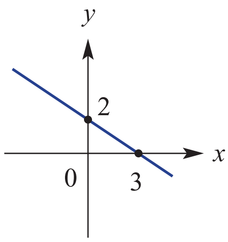
ii



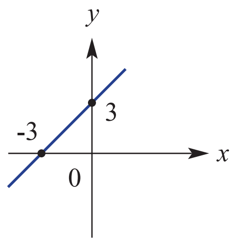
iii



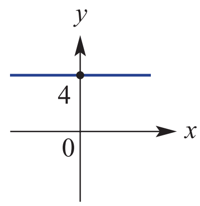
iv



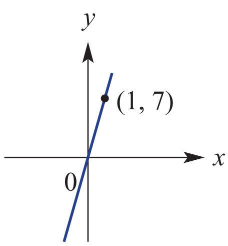
v



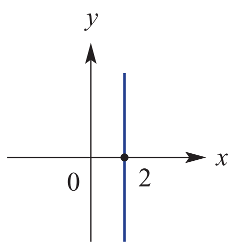
vi



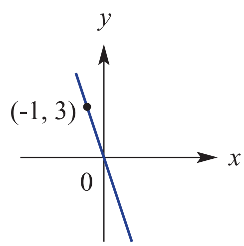
vii



viii



ix



8 **a** Write down three different equations that have a graph with a y-intercept of 5.
b Write down three different equations that have a graph with a y-intercept of -2.
c Write down three different equations that have a graph with a y-intercept of 0.

9 **a** Write down three different equations that have a graph with a gradient of 3.
b Write down three different equations that have a graph with a gradient of -1.
c Write down three different equations that have a graph with a gradient of 0.
d Write down three different equations that have a graph with an undefined gradient.

10 **a** Which of the following points lie on the line $y = 2$?
i (2, 3) **ii** (1, 2) **iii** (5, 2) **iv** (-2, -2)
b Which of the following points lie on the line $x = 5$?
i (5, 3) **ii** (3, 5) **iii** (1, 7) **iv** (5, -2)

ISBN: 9781107687028 Cambridge University Press

Example 22 Identifying points on a line

Does the point (3, -4) lie on the line $y = 2x - 7$?

Solution	Explanation
$y = 2x - 7$	Copy the equation and substitute $x = 3$.
$y = 2 \times 3 - 7$	The y value for $x = 3$ is $y = -1$.
$y = -1$	Compare the y values.
$\neq -4$	The point (3, -1) is *on* the line.
No, (3, -4) is not on the line.	So (3, -4) is *not* on the line.

11 **a** Does the point (3, 2) lie on the line $y = x + 2$?
b Does the point (-2, 0) lie on the line $y = x + 2$?
c Does the point (1, -5) lie on the line $y = 3x + 2$?
d Does the point (2, 2) lie on the line $y = x$?
e Does the line $y - 2x = 0$ pass through the origin?

Substitute the x value into the equation and compare the two y values. When the y values are the same, the point is on the line.

12 Draw each of the following on a number plane and write down the equation of the line.

a

x	0	1	2	3
y	4	5	6	7

b

x	0	1	2	3
y	-1	0	1	2

c

x	-2	0	4	6
y	-1	0	2	3

d

x	-2	0	2	4
y	-3	1	5	9

Use your graph to find the gradient between two points (m) and locate the y-intercept (c). Then use $y = mx + c$.

Graphs using technology

13 Use technology to sketch a graph of these equations.

a $y = x + 2$ **b** $y = -4x - 3$ **c** $y = \frac{1}{2}x - 1$ **d** $y = 1.5x + 3$
e $y = 2x - 5$ **f** $y = 0.5x + 5$ **g** $y = -0.2x - 3$ **h** $y = 0.1x - 1.4$

14 **a** On the same set of axes, plot graphs of $y = 2x$, $y = 2x + 1$, $y = 2x + 4$, $y = 2x - 2$ and $y = 2x - 3$ using a calculator.
Discuss what you see and describe the connection with the given equations.

b On the same set of axes, plot graphs of $y = x - 1$, $y = 2x - 1$, $y = 3x - 1$, $y = \frac{1}{2}x - 1$ and $y = \frac{3}{4}x - 1$ using a calculator.
Discuss what you see and describe the connection with the given equations.

c The equations of families of graphs can be entered into a calculator using one line only. For example, $y = 2x + 1$, $y = 2x + 2$ and $y = 2x + 3$ can be entered as $y = 2x + \{1, 2, 3\}$ using set brackets. Use this notation to draw the graphs of the rules in parts **a** and **b**.

ISBN: 9781107687028

Cambridge University Press

6.8 Sketching with *x*- and *y*-intercepts

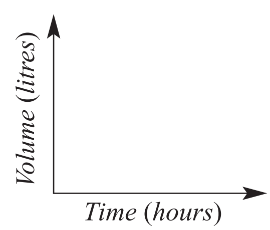
Only two points are required to define a straight line. Two convenient points are the x- and y-intercepts. These are the points where the graph crosses the x- and y-axis respectively.

The axis intercepts are quite significant in practical situations. For example, imagine that 200 m^3 of dirt needs to be removed from a construction site before foundations for a new building can be laid. The graph of volume remaining versus time taken to remove the dirt has a y-intercept of 200 showing the total volume to be removed, and an x-intercept showing the time taken for the job.

Let's start: Leaking water

A family that is camping has 20 litres of water in a plastic container. The water begins to slowly leak at a rate of 2 litres per hour.

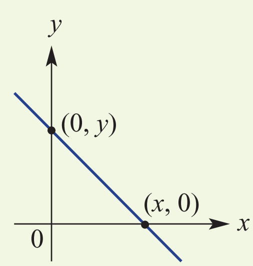
- Sketch two axes labelled 'Volume' and 'Time'.
- At $t = 0$, what is the volume of water in the tank? Mark and label this point on the Volume axis.
- How long will it take the water container to be empty? Mark this point on the Time axis.
- Join these two points with a straight line.
- Write the coordinates of the Volume axis intercept and the Time axis intercept. Follow the order (Time, Volume).
- Can you suggest a rule for finding the volume of the water in the tank after t hours?



Key ideas

To sketch a straight line by finding intercepts:

- The **x-intercept** is where $y = 0$.
  Find the x-intercept by substituting $y = 0$ into the equation.
- The **y-intercept** is where $x = 0$.
  Find the y-intercept by substituting $x = 0$ into the equation.



x-intercept The point at which a line or curve cuts the x-axis

y-intercept The point at which a line or curve cuts the y-axis

ISBN: 9781107687028 Cambridge University Press

Exercise 6H

Understanding

1 Copy and complete:

a The x-intercept is where ______ = 0. **b** The y-intercept is where ______ = 0.

2 a Perform these calculations.

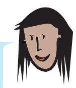
i $\frac{-21}{-7}$ **ii** $6 \times (-3)$ **iii** $-12 - 18$

iv $-12 + 18$ **v** $\frac{28}{-4}$

b Solve these equations for x.

i $0 = 3x - 12$ **ii** $0 = -4x + 2$

iii $0 = 5x - 2$ **iv** $0 = -2x - 13$



When multiplying and dividing, the same signs make a positive answer and different signs make a negative answer.

Example 23 Finding the x-intercept

Find the coordinates of the x-intercept for each line equation.

a $y = -2x - 7$ **b** $y = -\frac{3}{4}x + 6$

Solution	Explanation
a $y = -2x - 7$	
x-int ($y = 0$):	Substitute $y = 0$ for an x-intercept calculation.
$0 = -2x - 7$	The opposite of -7 is +7, so add 7 to both sides.
$7 = -2x$	Divide both sides by -2.
$x = \frac{7}{-2}$	When dividing two numbers with different signs, the answer is negative.
$x = -3\frac{1}{2}$	Write the answer as a mixed number.
x-intercept is $(-3\frac{1}{2}, 0)$	Write the coordinates.
b $y = -\frac{3}{4}x + 6$	Substitute $y = 0$.
x-int ($y = 0$):	The opposite of +6 is -6, so subtract 6 from both sides.
$0 = -\frac{3}{4}x + 6$	Change the fraction $-\frac{3}{4}$ to $\frac{-3}{4}$.
$-6 = \frac{-3}{4}x$	Multiply both sides by 4.
$-24 = -3x$	Divide both sides by -3.
$x = 8$	When dividing two numbers with the same sign, the answer is positive. $8 = x$ can be written with x as the subject: $x = 8$.
x-intercept is $(8, 0)$	Write the coordinates.

3 Find the coordinates of the x-intercept for each line equation.

a $y = 2x - 8$ **b** $y = -3x - 10$ **c** $y = \frac{3}{2}x + 9$

d $y = -\frac{1}{2}x + 9$ **e** $y = -\frac{3}{4}x + 9$ **f** $y = -\frac{5}{3}x - 10$

At the x-intercept, $y = 0$. Show all steps in each calculation.

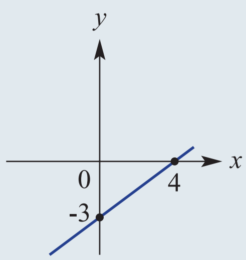
ISBN: 9781107687028

6H

Fluency

Example 24 Sketching lines in the form $ax + by = d$ using x- and y-intercepts

Sketch a graph of $3x - 4y = 12$ by finding the x- and y-intercepts.

Solution	Explanation
$3x - 4y = 12$	
x-intercept ($y = 0$): $3x - 4(0) = 12$	Find the x-intercept by substituting $y = 0$.
$3x = 12$	Simplify. Any number multiplied by 0 is 0.
$x = 4$	Divide both sides by 3 to solve for x.
y-intercept ($x = 0$): $3(0) - 4y = 12$	Find the y-intercept by substituting $x = 0$.
$-4y = 12$	Simplify and retain the negative sign.
$y = -3$	Divide both sides by -4 to solve for y.
	Sketch the graph by first marking the x-intercept (4, 0) and the y-intercept (0, -3) and join in a line.

4 Sketch graphs of the following equations by finding the x- and y-intercepts.

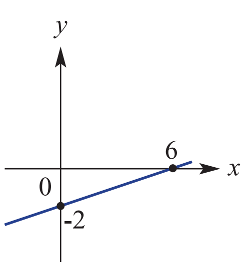
a $3x + 2y = 6$ **b** $2x + 6y = 12$ **c** $3x - 4y = 12$
d $5x - 2y = 20$ **e** $-2x + 7y = 14$ **f** $-x + 3y = 3$
g $-x - 2y = 8$ **h** $-5x - 9y = 90$ **i** $-6x - y = -24$

Two calculations are required:
Substitute $y = 0$ and solve for x-intercept.
Substitute $x = 0$ and solve for y-intercept.

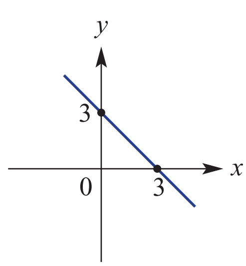
5 Match each of the following linear equations of the form $ax + by = d$ to one of the sketches shown.

a $x + y = 3$ **b** $2x - y = 4$ **c** $x - 3y = 6$

i

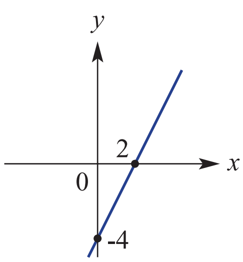


ii



First find the x- and y-intercepts for the line equation.

iii

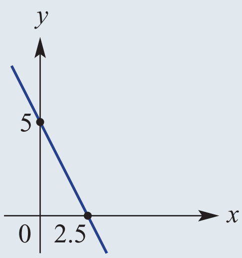


ISBN: 9781107687028 Cambridge University Press

Example 25 Sketching lines in the form $y = mx + c$ using x- and y-intercepts

Sketch the graph of $y = -2x + 5$ by finding the x- and y-intercepts.

Solution		Explanation
$y = -2x + 5$		
x-intercept ($y = 0$):	$0 = -2x + 5$	Substitute $y = 0$
	$-5 = -2x$	Subtract 5 from both sides.
	$x = 2.5$	Divide both sides by -2
y-intercept ($x = 0$):	$y = -2(0) + 5$	Substitute $x = 0$
	$= 5$	Simplify.



Sketch the graph by first marking the x-intercept (2.5, 0) and the y-intercept (0, 5).

6 Sketch graphs of the following equations by finding the x- and y-intercepts.

a $y = 2x + 1$ **b** $y = 3x - 2$ **c** $y = -4x - 3$

d $y = -x + 2$ **e** $y = -\frac{1}{2}x + 1$ **f** $y = \frac{3}{2}x - 3$

6e:

$0 = -\frac{1}{2}x + 1$

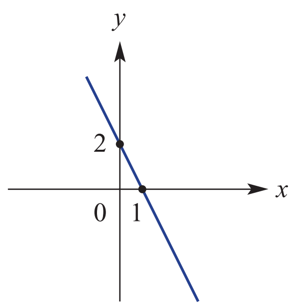
$\frac{1}{2}x = 1$

Now multiply both sides by 2.

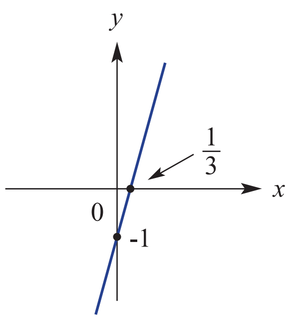
7 Match each of the following linear equations of the form $y = mx + c$ to one of the sketches shown.

a $y = x + 1$ **b** $y = 3x - 1$ **c** $y = -2x + 2$

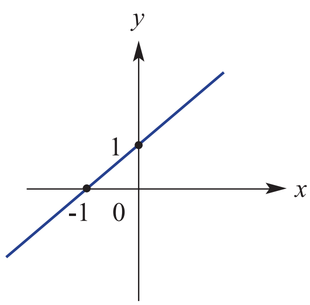
i



ii



iii



ISBN: 9781107687028

6H

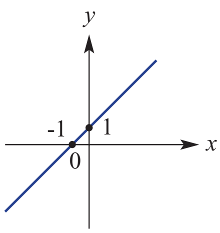
Problem-solving and Reasoning

8 Match each of the following linear equations to one of the sketches shown.

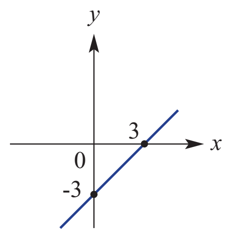
a $2x + y = 4$ **b** $x - y = 3$ **c** $y = x + 1$

d $y = -2x - 3$ **e** $3x - 5y = 15$ **f** $y = \frac{2}{5}x - 1$

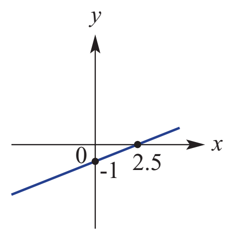
i



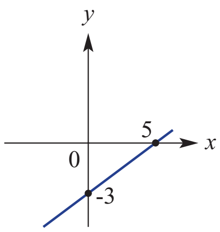
ii



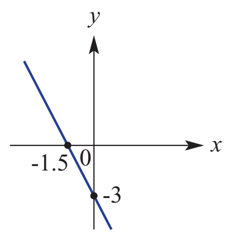
iii



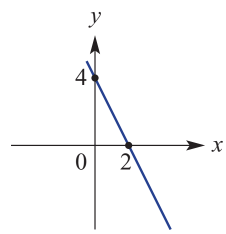
iv



v



vi



9 By first finding the x- and y-intercepts of the graphs of these equations, find the gradient in each case.

a $2x + y = 4$ **b** $x - 5y = 10$

c $4x - 2y = 5$ **d** $-1.5x + 3y = 4$

A quick sketch of each line and the axis intercepts will help to show the rise and run.

10 For the graphs of each of the following equations, find:

i the x- and y-intercepts

ii the area of the triangle enclosed by the x and y axes and the graph of each equation

Remember that the area of a triangle is $A = \frac{1}{2}\,bh$

a $2x - y = 4$ **b** $-3x + 3y = 6$

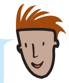
c $y = -2x - 3$ **d** $y = \frac{1}{2}x + 2$

11 The height, h, in metres, of a lift above ground after t seconds is given by $h = 90 - 12t$.

a How high is the lift initially (at $t = 0$)?

b How long does it take for the lift to reach the ground ($h = 0$)?

12 If $ax + by = d$, can you find a set of numbers a, b and d which give an x-intercept of (2, 0) and y-intercept of (0, 4).



Use trial and error to start.

Axis intercepts using technology

13 For the following rules, use technology to sketch a graph and find the x- and y-intercepts.

a $y = 2x - 4$ **b** $y = -2x - 10$ **c** $y = -x + 1$

d $y + 2x = 4$ **e** $2y - 3x = 12$ **f** $3y - 2x = 2$

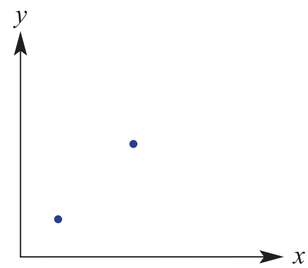
ISBN: 9781107687028

6.9 Linear modelling

Given at least two points, you can find the equation of a straight line.

If the relationship between two variables is linear, then:

- the graph of the relation is a straight line
- a rule can be written in the form $y = mx + c$.



Only one straight line can be drawn to pass through these two points.

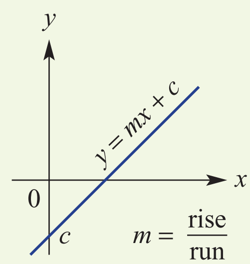
Let's start:

Isabella has a trainee scholarship to complete her apprenticeship as a mechanic. She is paid \$50 per week plus \$8/hr for work at the garage. Isabella's weekly wage can be modelled by the rule: Wage = $8t + 50$, where t is the number of hours worked in a week.

- Explain why the rule for Isabella's wage is Wage = $8t + 50$.
- Show how the rule can be used to find Isabella's wage after 10, 20 and 35 hours per week.
- Show how the rule can be used to find how long Isabella worked if she earned \$114, \$202 and \$370.

Key ideas

- The equation of a straight line can be determined using:
  - $y = mx + c$
  - gradient $= m = \dfrac{\text{rise}}{\text{run}}$
  - y-intercept $= (0, c)$
- If the y-intercept is not obvious, then it can be found by substituting a point.
- Vertical and horizontal lines:
  - Vertical lines have the equation $x = k$ where k is the x-intercept.
  - Horizontal lines have the equation $y = c$ where c is the y-intercept.
- Modelling may involve:
  - writing a rule linking two variables
  - sketching a graph
  - using the rule or the graph to help solve related problems.



Exercise 6I

Understanding

1 Each week Ava gets paid \$30 plus \$15 per hour. Decide which rule shows the relationship between Ava's total weekly pay, \$$P$, and the number of hours she works, n.

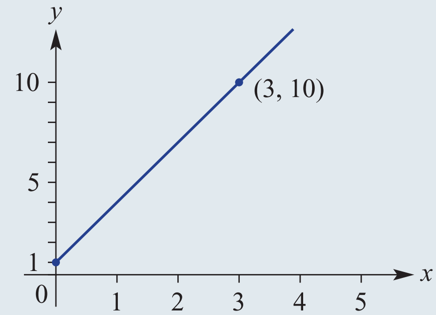
A $P = 30 + n$ **B** $P = 15n$ **C** $P = 30 + 15$ **D** $P = 30 + 15n$

ISBN: 9781107687028

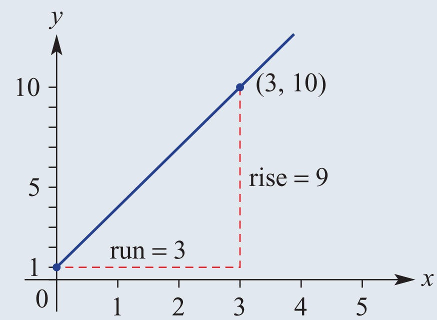
2 Riley is 100 km from home and is cycling home at 20 km/h. Decide which rule shows the relationship between Riley's distance from home, d km, and the number of hours he has been cycling, t.
A $P = 100t$ **B** $P = 100 - 20$ **C** $P = 100 - 20t$ **D** $P = 100 + 20t$

Example 26 Finding the equation of a line from a graph with a known y-intercept

For the straight line shown:
a determine its gradient
b find the y-intercept
c write the equation of the line

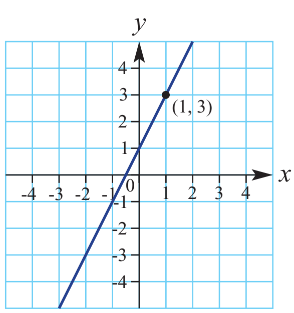


Solution	Explanation
a $m = \dfrac{\text{rise}}{\text{run}}$ $= \dfrac{9}{3}$ $= 3$	Draw a triangle on the graph and decide whether the gradient is positive or negative.
b The y-intercept is (0, 1), so $c = 1$.	Look at where the graph meets the y-axis.
c $y = 3x + 1$	Substitute m and c into $y = mx + c$

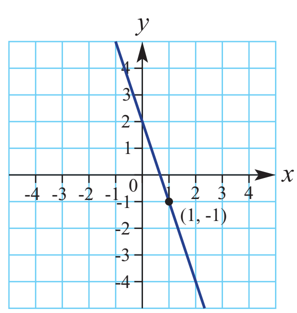


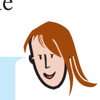
3 For the straight lines shown:
i determine the gradient **ii** find the y-intercept **iii** write the equation of the line

a



b

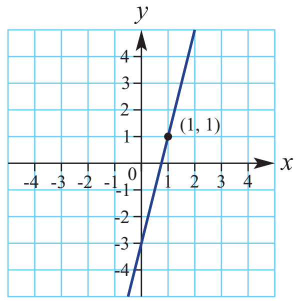




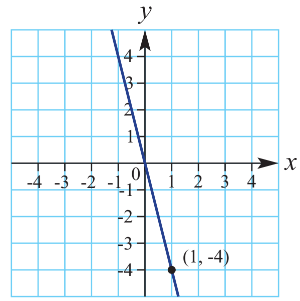
To find the rise and run. Form a right-angled triangle using the y-intercept and the second point.

ISBN: 9781107687028

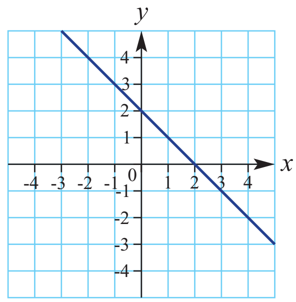
c



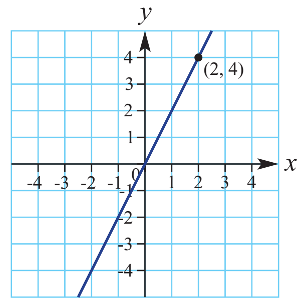
d



e



f

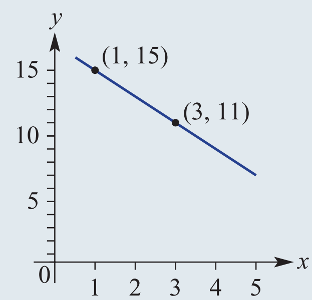


Fluency

Example 27 Finding the equation of a line given a graph with two known points

For the straight line shown:

a determine its gradient
b find the y-intercept
c write the equation of the line



Solution	Explanation
a $m = \dfrac{\text{rise}}{\text{run}}$ $= \dfrac{-4}{2}$ $= -2$	The gradient is negative. run = 3 – 1 = 2 rise = 11 – 15 = -4 A 'fall' of 4 means rise = -4. Simplify.
b $y = -2x + c$ $11 = -2(3) + c$ $11 = -6 + c$ $17 = c$ and the y-intercept is (0, 17).	Write $y = mx + c$ using $m = -2$. Substitute a chosen point into $y = -2x + c$ (use (3, 11) or (1, 15)). Here, $x = 3$ and $y = 11$. Simplify and solve for c.
c $y = -2x + 17$	Substitute $m = -2$ and $c = 17$ into $y = mx + c$.

ISBN: 9781107687028

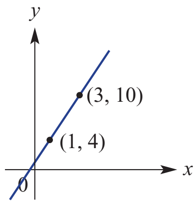
4 For the straight lines shown:

i determine the gradient

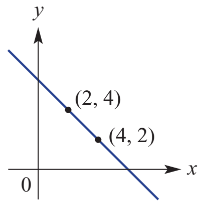
ii find the y-intercept

iii write the equation of the line.

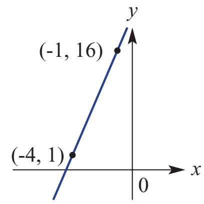
a



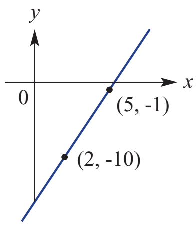
b



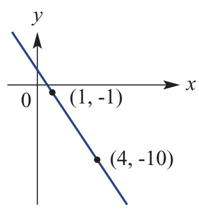
c



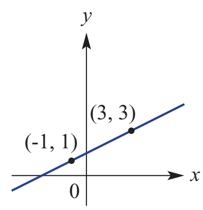
d



e



f



For m, find the rise and run between the two given points.

Choose either point to substitute when finding c. If $m = 4$ and $(3, 5) = (x, y)$:

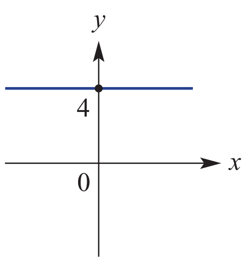
$y = mx + c$

$5 = 4 \times 3 + c$

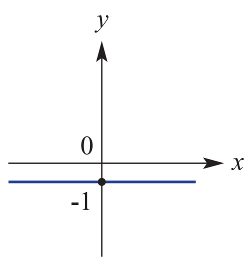
Solve for c.

5 Determine the equation of the following lines. Remember from Section 6.7 that vertical and horizontal lines have special equations.

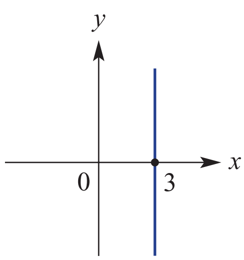
a



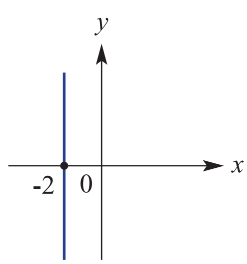
b



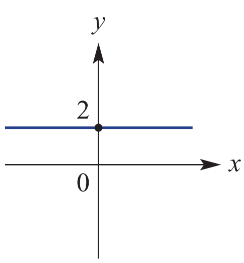
c



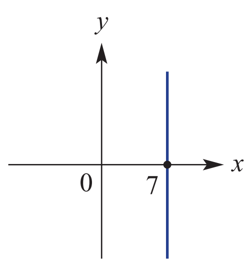
d



e



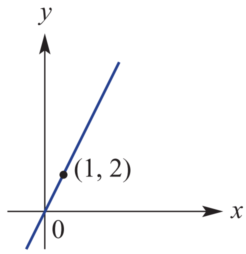
f

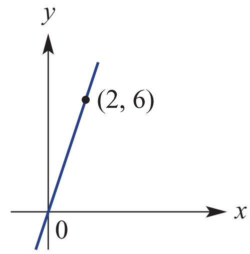


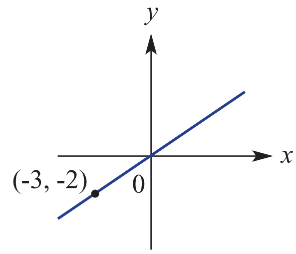
Vertical lines cut the x-axis and have an equation such as $x = 3$. Horizontal lines cut the y-axis and have an equation such as $y = -4$.

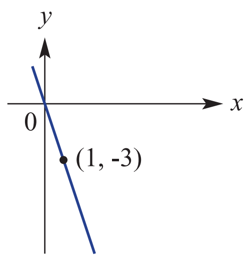
ISBN: 9781107687028

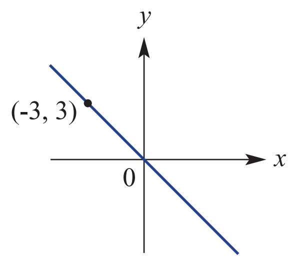
6 Remember that equations of graphs that pass through the origin are of the form $y = mx$ (since $c = 0$). Find the equation of these graphs.

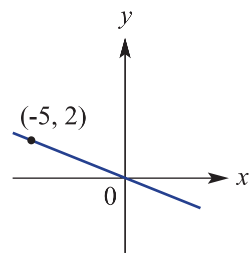
a


b


c


d


e


f


7 For the line joining the following pairs of points, find:

i the gradient **ii** the equation of the line

a (0, 0) and (1, 7) **b** (0, 0) and (2, -3)
c (-1, 1) and (1, 3) **d** (-2, 3) and (2, -3)
e (-4, 2) and (7, 2) **f** (3, -3) and (3, 1)

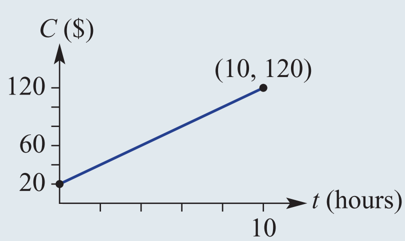
Substitute the gradient and a point in $y = mx + c$ to find c in part **ii**.

Problem-solving and Reasoning

Example 28 Modelling with linear graphs

A woman gets paid \$20 plus \$10 for each hour of work. If she earns \$$C$ for t hours work, complete the following.

a Write a rule for C in terms of t.
b Sketch a graph using t between 0 and 10.
c Use your rule to find:
  i the amount earned after working for 8 hours
  ii the number of hours worked if \$180 is earned

Solution	Explanation
a $C = 10t + 20$	\$10 is earned for each hour and \$20 is a fixed amount.
b 	20 is the y-intercept and the gradient is $10 = \frac{10}{1}$ For $t = 10$, $C = 10(10) + 20 = 120$

ISBN: 9781107687028

Cambridge University Press

c	**i** $C = 10(8) + 20$	Substitute $t = 8$ into $C = 10t + 20$
	$= 100$	Simplify.
	\$100 is earned.	Write your answer in words.
	ii $180 = 10t + 20$	Substitute $C = 180$ into $C = 10t + 20$.
	$160 = 10t$	Subtract 20 from both sides.
	$t = 16$	Divide both sides by 10.
	16 hours' work is completed.	Write your answer in words.

8 A boy gets paid \$10 plus \$2 per kg of tomatoes that he picks. If the boy earns \$$P$ for n kg of tomatoes picked complete the following:

- **a** Write a rule for P in terms of n.
- **b** Sketch a graph of P against n for n between 0 and 10.
- **c** Use your rule to find:
  - **i** the amount earned after picking 9 kg of tomatoes
  - **ii** the number of kilograms of tomatoes picked if he earns \$57

rate of pay

$y = mx + c$

P n fixed amount

9 An architect charges \$100 for the initial consultation plus \$60 per hour thereafter. If the architect earns \$$A$ for t hours of work, complete the following.

- **a** Write a rule for A in terms of t.
- **b** Sketch a graph of A against t for t between 0 and 15.
- **c** Use your rule to find:
  - **i** the amount earned after working for 12 hours
  - **ii** the number of hours worked if she earns \$700

Draw a line between the points at $t = 0$ and $t = 15$.

10 A man's weight when holding two empty buckets of water is 80 kg. 1 kg is added for each litre of water poured into the buckets. If the man's total weight is W kg with l litres of water, complete the following.

- **a** Write a rule for W in terms of l.
- **b** Sketch a graph of W against l for l between 0 and 20.
- **c** Use your rule to find:
  - **i** the man's weight after 7 litres of water are added
  - **ii** the number of litres of water added if the man's weight is 109 kg

ISBN: 9781107687028 Cambridge University Press

11 The amount of water (W litres) in a leaking tank after t hours is given by the rule $W = -2t + 1000$.

a State the gradient and y-intercept for the graph of the rule.

b Sketch a graph of W against t for t between 0 and 500.

c State the initial water volume at $t = 0$.

d Find the volume of water after:

i 320 hours **ii** 1 day **iii** 1 week

e Find the time taken, in hours, for the water volume to fall to:

i 300 litres **ii** 185 litres

Production lines

12 An assembly plant needs to order some new parts. Three companies can supply them but at different rates.

- Mandy's Millers charge: set-up fee \$0 + \$1.40 per part
- Terry's Turners charge: set-up fee \$3000 + \$0.70 per part
- Lenny's Lathes charge: set-up fee \$4000 + \$0.50 per part

a Complete a table of values similar to the following for each of the companies.

No. of parts (p)	0	1000	2000	3000	4000	5000	6000	7000	8000	9000
Total cost (C)										

b Plot a graph of the *total cost* against the *number of parts* for each company on the same set of axes. Make your axes quite large as there are three graphs to complete.

c Use the graphs to find the lowest price for:

i 1500 parts

ii 1000 parts

iii 6500 parts

iv 9500 parts

d Advise the assembly plant when it is best to use Mandy's, Terry's or Lenny's company.

ISBN: 9781107687028

1 What is not so devious? Solve the puzzle to find the answer.
Match the letter beside each question to the answers below.

Find where each line cuts the x axis:

O $y = 3x - 24$

! $y = -\frac{3}{2}x - 9$

N $4x - 2y = -20$

I This line joins (0, 4) to (5, -1)

S $x = -7$

Find the gradient of each line:

E $y = 3x - 4$

L $y = -\frac{5}{2}x + 7$

P This line joins the origin to (3, -6)

G This line joins (2, 5) to (-4, 11)

T $y = 5$

___ ___ ___ ___ ___ ___ ___ ___   ___ ___ ___ ___ ___ ___
-2 -2.5 8 -0 -0 4 -5 -1   -2.5 4 -5 3 -7 -6

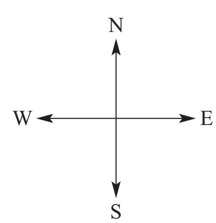
2 Solve the wordfind below.

T	F	Z	T	V	M	V	Z	J	H	E	R
O	W	I	M	J	G	R	J	O	A	L	A
T	M	G	T	E	R	K	R	U	T	B	T
E	C	N	A	T	S	I	D	N	E	A	E
S	T	M	P	R	Z	A	E	S	I	I	F
R	P	H	G	O	X	M	E	O	Z	R	E
D	Y	E	N	Z	G	J	U	R	X	A	F
H	G	T	E	E	J	Q	W	G	C	V	H
I	A	L	S	D	R	U	S	G	U	N	A
L	P	G	R	A	P	H	A	P	R	P	I

- DISTANCE
- GRAPH
- HORIZONTAL
- INCREASE
- RATE
- SEGMENT
- SPEED
- TIME
- VARIABLE

3 Cooper and Sophie are in a cycling orienteering competition.
- From the starting point, Cooper cycles 7 km east, then 3 km south to checkpoint 1. From there, Cooper cycles 5 km east and 8 km north to checkpoint 2.
- Sophie cycles 10 km north from the starting point to checkpoint 3.

Use calculations to show that the distance between where Sophie and Cooper are now is the same as the direct distance that Cooper is now from the starting point.



4 Lucas and Charlotte want to raise money for their school environment club so they have volunteered to run a strawberry ice-cream stall at their town's annual show.
It costs \$200 to hire the stall and they make \$1.25 profit on each ice-cream sold.

a How many ice-creams must be sold to make zero profit (i.e. not a loss)?

b If they make \$416.25 profit, how many ice-creams did they sell?

ISBN: 9781107687028

Cambridge University Press

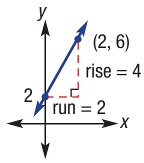
Straight line graphs

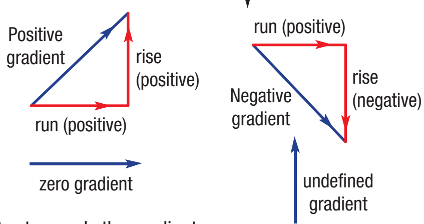
Gradient of a line

Gradient measures the slope of a line

Gradient $m = \frac{\text{rise}}{\text{run}}$ e.g.

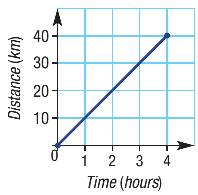
$m = \frac{4}{2} = 2$





A rate equals the gradient with units

E.g. speed $= \frac{40}{4} = 10$ km/h



Equation of a line

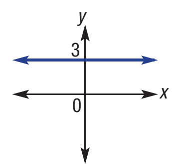
$y = mx + c$

gradient y-intercept

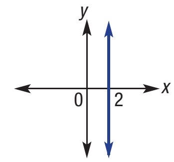
- The rule is a linear equation.
- The graph is made up of points in a straight line.

Special lines

Horizontal lines
E.g. $y = 3$



Vertical lines
E.g. $x = 2$



Sketching a line

Plotting straight line graphs

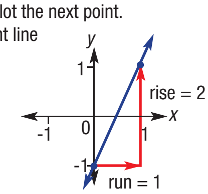
- Complete a table of values
- Plot points and join to form a straight line

Using the y-intercept and gradient

- Plot the y-intercept (c)
- Use the gradient to plot the next point.
- Join to form a straight line

E.g. $y = 2x - 1$

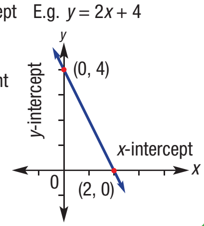
$c = -1$ $m = \frac{2}{1}$



Using the axis intercepts

- Plot each axis intercept
  x-intercept ($y = 0$)
  y-intercept ($x = 0$)
- Join to form a straight line

E.g. $y = 2x + 4$



Midpoint of a line segment

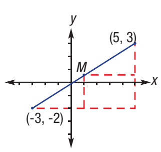
Find the average of the endpoint coordinates

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$x = \frac{-3 + 5}{2} = \frac{2}{2} = 1$$

$$y = \frac{-2 + 3}{2} = \frac{1}{2} = 0.5$$

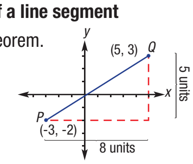
$\therefore M = (1, 0.5)$



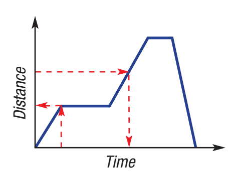
Length of a line segment

Use Pythagoras' theorem.

$PQ^2 = 8^2 + 5^2$
$PQ^2 = 64 + 25$
$PQ^2 = 89$
$PQ = \sqrt{89}$
$\sqrt{89}$ is an exact length.



Distance–time graph



- Flat segment means the object is at rest.

Reading a graph:

- Start on given distance; move across to line then down to time scale (or in reverse).

Linear modelling

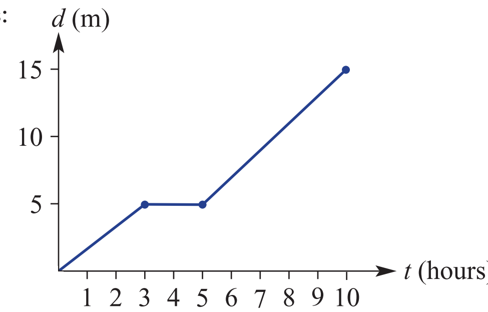
- Find a rule in the form $y = mx + c$ using the appropriate pronumerals.
- Sketch a graph.
- Apply the rule to solve problems.
- Answer the problem in words.

ISBN: 9781107687028

Cambridge University Press

Multiple-choice questions

Questions **1** to **4** refer to the following graph of the movement of a snail.



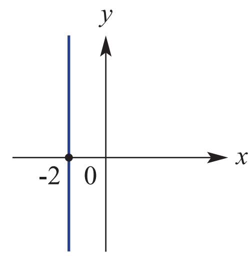
1 The total number of hours the Snail was at rest is:
A 2 **B** 4 **C** 5
D 6 **E** 10

2 The distance travelled in the first 3 hours was:
A 3 m **B** 3 hours **C** 7 m
D 4 m **E** 5 m

3 The speed of the Snail in the last 5 hours was:
A 5 hours **B** 10 m **C** 10 m/h
D 2 m/h **E** 5 m/h

4 The total distance travelled by the snail is:
A 15 m **B** 10 m **C** 5 m **D** 12 m **E** 8 m

5 The equation of the line shown at the right is:
A $x = 2$ **B** $x = -2$ **C** $y = 1$
D $y = -2$ **E** $y = -2x$



6 The graph of $C = 10t + 5$ would pass through which of the following points?
A (1, 10) **B** (1, 20) **C** (2, 20)
D (4, 50) **E** (5, 55)

7 The gradient of the line joining (0, 0) and (2, -6) is:
A 2 **B** 3 **C** -3 **D** 6 **E** -6

8 A vertical line has gradient:
A undefined **B** zero **C** positive **D** negative **E** 1

9 A line passes through (-2, 7) and (1, 2). The gradient of the line is:
A -3 **B** $-\frac{5}{3}$ **C** 3 **D** $\frac{5}{3}$ **E** $-\frac{3}{5}$

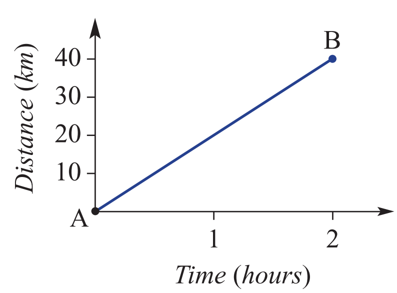
10 The x- and y-intercepts of the graph of the rule $3x - y = 4.5$ are respectively:
A (0, 3.5) and (4.5, 0) **B** (-1.5, 0) and (4.5, 0) **C** (1.5, 0) and (0, 4.5)
D (1.5, 0) and (0, -4.5) **E** (0, 3.5) and (-4.5, 0)

11 Which of the following equations has a gradient of 2 and a y-intercept of -1?
A $2y + x = 2$ **B** $y - 2x = 1$ **C** $y = -2x + 1$ **D** $y = 2x - 1$ **E** $2x + y = 1$

12 A line has x- and y-intercepts of respectively (1, 0) and (0, 2). Its equation is:
A $2x - y = 2$ **B** $y = -x + 2$ **C** $y = 2x + 2$ **D** $x + 2y = 1$ **E** $y = -2x + 2$

Short-answer questions

1 This graph shows the journey of a cyclist from place A to place B.
a How far did the cyclist travel?
b How long did it take the cyclist to complete the journey?
c If the cyclist rode from A to B and then halfway back to A, how far would the journey be?



ISBN: 9781107687028 Cambridge University Press

2 The value of a poor investment has decreased according to this graph.

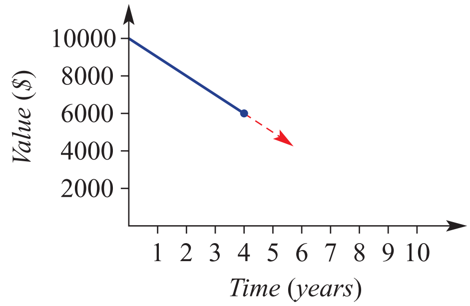
a Find the value of the investment after:

i 4 years **ii** 2 years **iii** 1 year

b Extend the graph and use it to estimate the value of the investment after:

i 8 years **ii** 6 years **iii** 5 years

c After how many years will the investment be valued at $0?



3 The distance travelled by a walker is described by this graph.

a What is the total distance walked?

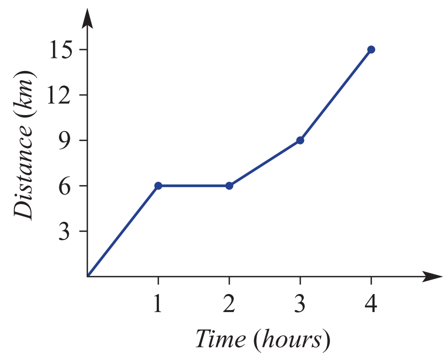
b How long was the person actually walking?

c How far had the person walked after:

i 1 hour? **ii** 2 hours?

iii 3 hours? **iv** 4 hours?

d How long did it take to walk a distance of 12 km?



4 Sketch a graph to show a journey described by:

- a total distance of 60 m in 15 seconds
- 30 m covered in the first 6 seconds
- a 5-second rest after the first 6 seconds.

5 Francene delivers burgers for a fast-food outlet. She is paid $10 a shift plus $5 per delivery.

a Complete the table of values.

No. of deliveries (d)	0	5	10	15	20
Payment (P)					

b Plot a graph of amount paid against number of deliveries.

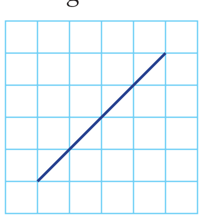
c Use the graph to determine:

i the amount of pay for 12 deliveries

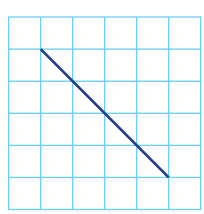
ii the number of deliveries made if Francene is paid $95.

6 Find the gradient of the following lines.

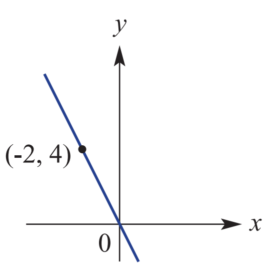
a



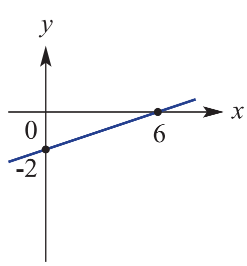
b



c



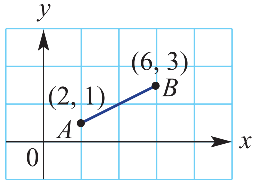
d



ISBN: 9781107687028

7 Find the midpoint of each line segment here.

a

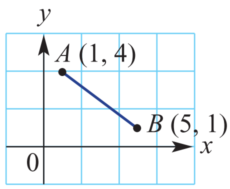


b P(5, 7) to Q(-1, -2)

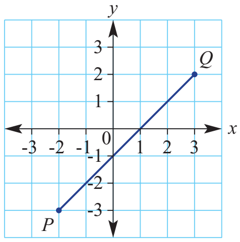
c G(-3, 8) to H(6, -10)

8 Find the length of each line segment.

a



b



9 State the gradient and y-intercept of the following lines.

a $y = 3x + 4$ **b** $y = -2x$

10 Sketch the following lines by considering the y-intercept and the gradient.

a $y = 2x + 3$ **b** $y = -4x$ **c** $y = 2$ **d** $x = -1$

11 Sketch the following lines by considering the x- and y-intercepts.

a $3x + 4y = 12$ **b** $2x - y = 6$ **c** $y = 3x - 9$

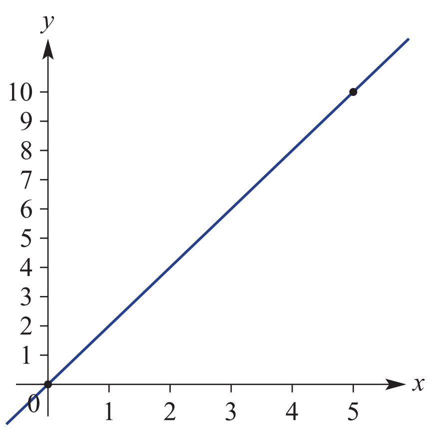
12 For each of the straight lines shown:

i determine its gradient

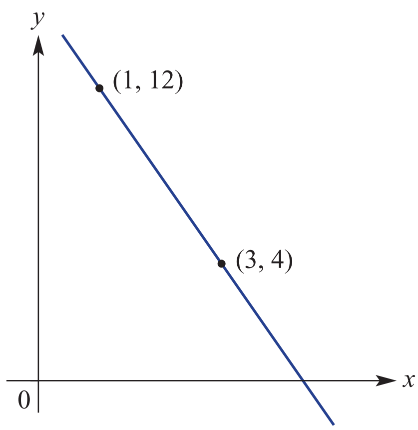
ii find the y-intercept

iii write the equation of the line

a



b



ISBN: 9781107687028

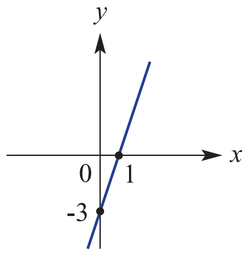
Cambridge University Press

13 Match each of the linear equations to lines shown.

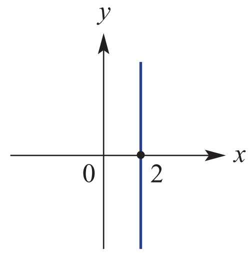
a $y = 3x - 3$ **b** $y = 5x$ **c** $5x + 4y = 20$

d $x = 2$ **e** $-2x + 5y = 10$ **f** $y = -4$

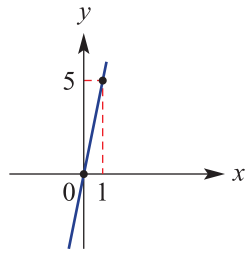
i



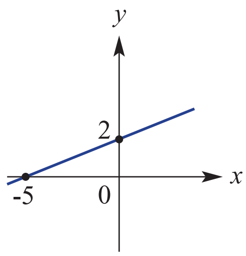
ii



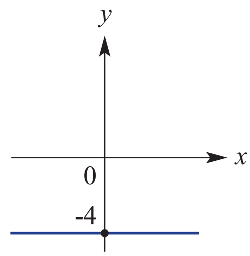
iii



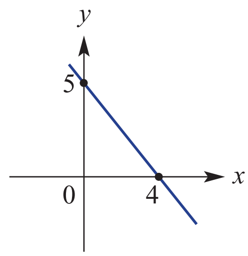
iv



v



vi



14 A fruit picker earns \$50 plus \$20 per bin of fruit picked. If the picker earns $\$E$ for n bins picked, complete the following:

a Write a rule for E in terms of n.

b Sketch a graph for n between 0 and 6.

c Use your rule to find:

i the amount earned after picking four bins of fruit

ii the number of bins of fruit picked if he earns \$160.

Extended-response questions

1 A courier van picks up goods from two different houses A and B as shown on the graph.

a Between house A and B find:

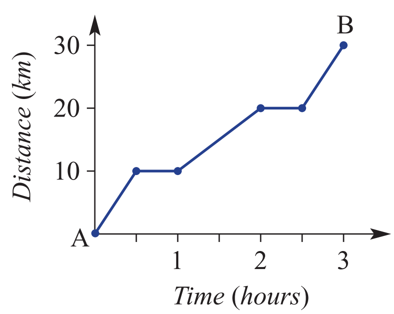
i the distance travelled

ii the average speed (not including stops)

b How fast was the courier van driving during:

i the first $\frac{1}{2}$ hour? **ii** the second $\frac{1}{2}$ hour?

iii the final $\frac{1}{2}$ hour?



ISBN: 9781107687028

Cambridge University Press

2 David and Kaylene travel from Melton to Moorbank army base to watch their son's march-out parade. The total distance for the trip is 720 km, and they travel an average of 90 km per hour.

a Complete the table of values below from 0 to 8 hours.

Time in hours (t)	0	2	4	6	8
Km from Moorbank	720				

b Plot a graph of the number of kilometres from Moorbank army base against time.

c They start their trip at 6 am. If they decide to stop for breakfast at Albury and Albury is 270 km from Melton, what time would they stop for breakfast?

d If the car they are driving needs refilling every 630 km, how long could they drive for before refilling the car?

e What would be the total driving time if they didn't stop at all?

f If the total number of breaks, including food and petrol stops, is 2 hours, when would they arrive at the army base?

3 A young maths whiz in the back of a car is counting down the distance to the nearest town, which initially is 520 km away. The car is travelling at an average speed of 80 km per hour.

a Find the distance to the town after:

i 1 hour **ii** 3 hours

b If D km is the distance to the town after t hours:

i write a rule for D in terms of t

ii sketch a graph for t between 0 and 6.5.

c Use your rule to find:

i the distance to the town after 4.5 hours

ii the time it takes for the distance to the town to be 340 km.

ISBN: 9781107687028

Australian Curriculum
Measurement and Geometry
Geometric reasoning

chapter

7 Geometry

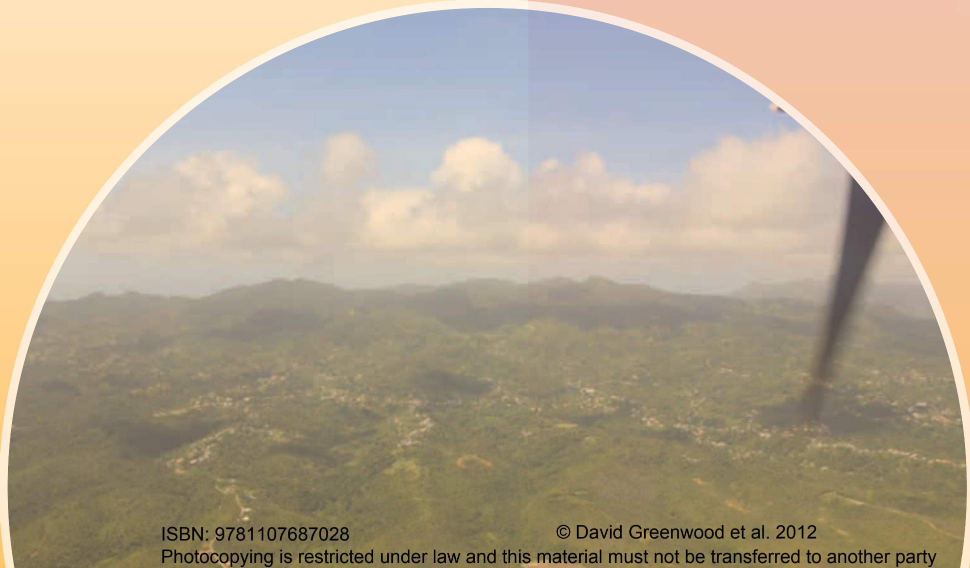
What you will learn

7.1 Parallel lines
7.2 Triangles
7.3 Quadrilaterals
7.4 Polygons
7.5 Congruent triangles
7.6 Similar triangles
7.7 Applying similar triangles
7.8 Applications of similarity in measurement

Aerial photography

Aerial photography involves flying a camera at a certain height above the ground. Using the focal length, frame width and desired ground distance, similar triangles are used to determine the height.

If, for example, a camera's focal length is 4 cm and frame width is 2 cm, then to photograph a ground distance of 100 m we use $\frac{h}{4} = \frac{10\,000}{2}$, so $h = 20\,000$, meaning that a height of 200 m is required.



ISBN: 9781107687028 Cambridge University Press

Pre-test

1 Write the missing word or number.

a ________ angles are between 0° and 90°.

b A right angle is _______.

c An obtuse angle is between 90° and _______.

d A 180° angle is called a _______ angle.

e A ________ angle is between 180° and 360°.

f A revolution is ______.

g Complementary angles sum to __________.

h __________ angles sum to 180°.

2 Name the type of triangle with the given properties.

a All sides of different length

b Two sides the same length

c One right angle

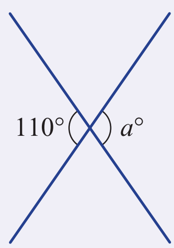
d One obtuse angle

e Three sides of equal length

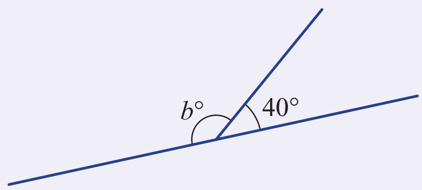
f All angles acute

3 Find the values of the pronumerals.

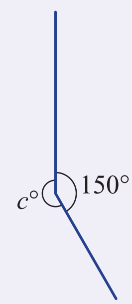
a



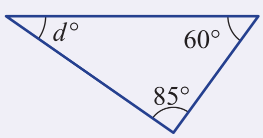
b



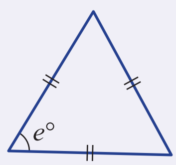
c



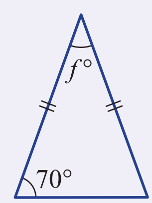
d



e

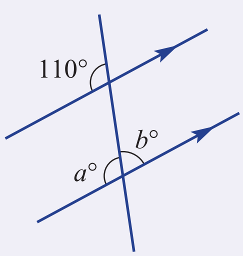


f

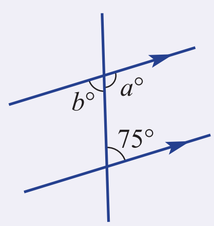


4 Find the value of the pronumerals in these sets of parallel lines.

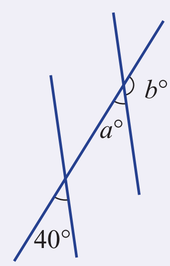
a



b



c

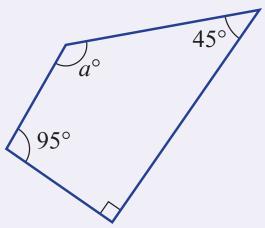


ISBN: 9781107687028 Cambridge University Press

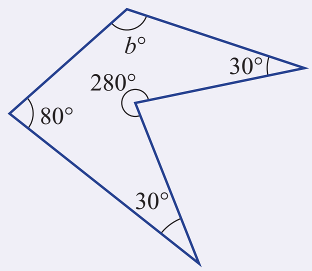
5 Name the quadrilaterals with these properties.
- **a** All sides equal and all angles 90°
- **b** Two pairs of parallel sides
- **c** Two pairs of parallel sides and all angles 90°
- **d** Two pairs of parallel sides and all sides equal
- **e** One pair of parallel sides
- **f** Two pairs of equal length sides and no sides parallel

6 Use the angle sum formula, $S = 180° \times (n - 2)$, to find the angle sum of these polygons and the value of the pronumeral.

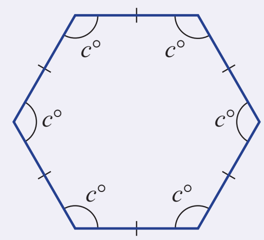
a



b



c



7 **a** What are the four tests for congruence of triangles?
- **b** What are the four tests for similarity of triangles?

ISBN: 9781107687028

7.1 Parallel lines

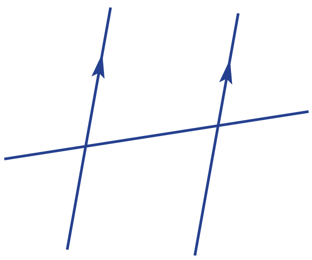
Parallel lines are everywhere: in buildings, in nature and on clothing patterns. Steel or concrete uprights at road intersections are an example.

Parallel lines are always the same distance apart and never meet. In diagrams, arrows are used to show that lines are parallel.

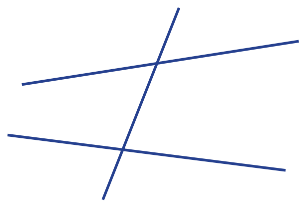
Let's start: 2, 4 or 8 different angles

Here are two pairs of lines crossed by a transversal. One pair is parallel and the other is not.

A



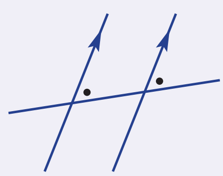
B



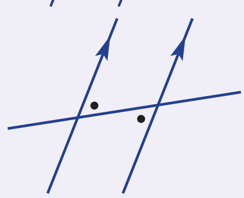
- How many angles of different size are in set A?
- How many angles of different size are in set B?
- If only one angle is known in set A, can you determine all other angles? Give reasons.

Key ideas

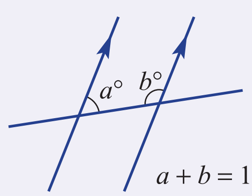
- A **transversal** is a line cutting two or more other lines.
- For parallel lines:
  - corresponding angles are equal



  - alternate angles are equal



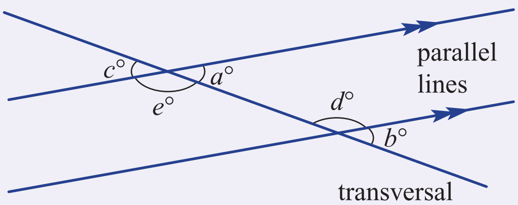
  - cointerior angles are supplementary



$a + b = 180$

Transversal A line that cuts two or more lines

ISBN: 9781107687028



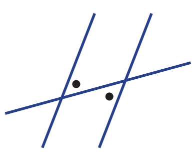
$a = b$	Corresponding angles
$a = c$	Vertically opposite angles
$d = e$	Alternate angles
$a + e = 180$	Supplementary angles
$a + d = 180$	Cointerior or allied angles

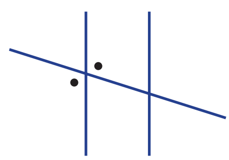
Exercise 7A

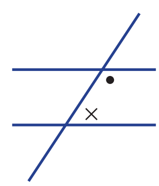
Understanding

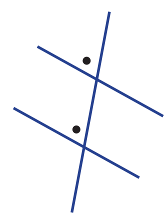
1 Write the missing word or number.
- **a** Supplementary angles add to ________.
- **b** Vertically opposite angles are _________.
- **c** If two lines are parallel and are crossed by a transversal, then:
  - **i** corresponding angles are _______.
  - **ii** alternate angles are _______.
  - **iii** cointerior angles are ________.

2 For the given diagrams, decide whether the given pair of marked angles are corresponding, alternate, cointerior or vertically opposite.

a


b


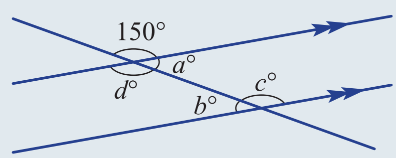
c


d


Fluency

Example 1 Finding angles in parallel lines

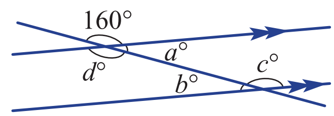
Find the values of the pronumerals in this diagram.
Write down the reason in each case.



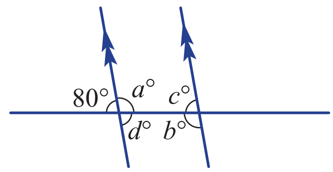
Solution	Explanation
$a = 180 - 150 = 30$ $a°$ and 150° are supplementary.	Two angles on a straight line sum to 180°.
$b = 30$ $b°$ is alternate to $a°$.	Alternate angles are equal in parallel lines.
$c = 150$ $c°$ is corresponding to 150° or cointerior to $a°$.	Corresponding angles are equal in parallel lines. Cointerior angles are supplementary in parallel lines.
$d = 150$ $d°$ is vertically opposite to 150°.	Vertically opposite angles are equal.

ISBN: 9781107687028 Cambridge University Press

3 Find the values of the pronumerals in this diagram. Write down the reason in each case.

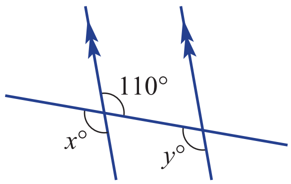


4 Find the values of the pronumerals in this diagram. Write down the reason in each case.

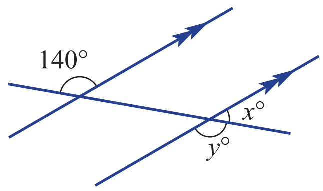


5 Find the value of x and y in these diagrams:

a

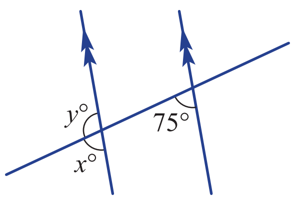


b

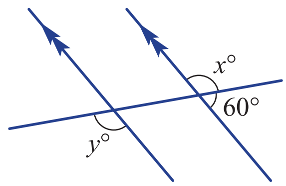


Corresponding angles are equal.
Alternate angles are equal.
Cointerior angles add to 180°.

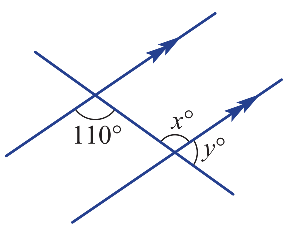
c



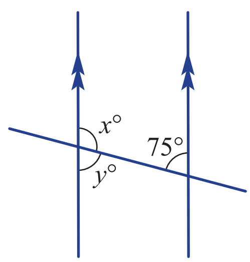
d



e



f

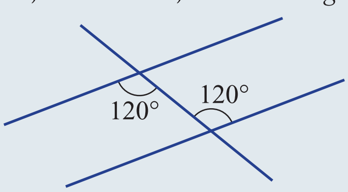


Problem-solving and Reasoning

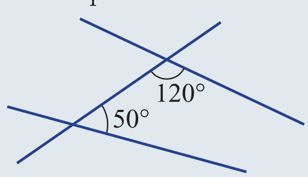
Example 2 Proving that two lines are parallel

Decide, with reasons, whether the given pairs of lines are parallel.

a



b

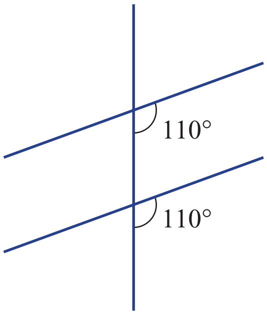


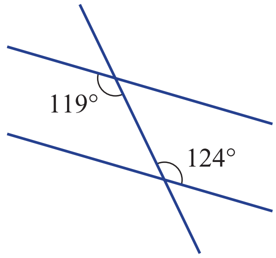
Solution	**Explanation**
a Yes – alternate angles are equal.	If alternate angles are equal, then lines are parallel.
b No – cointerior angles are not supplementary.	If lines are parallel, then cointerior angles should add to 180°, but 120° + 50° = 170°.

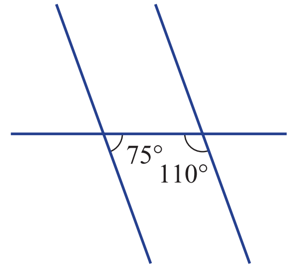
ISBN: 9781107687028

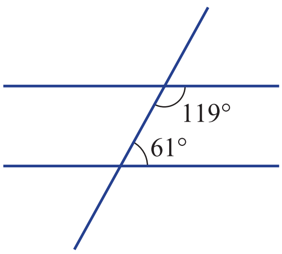
Cambridge University Press

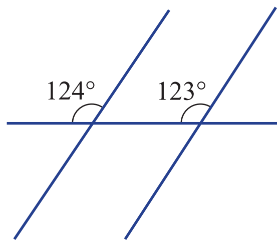
6 Decide, with reasons, whether the given pairs of lines are parallel.

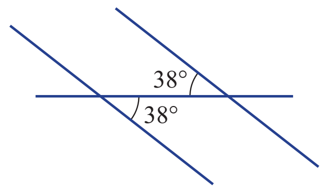
a


b


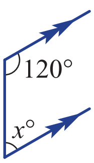
c


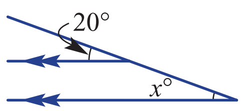
d


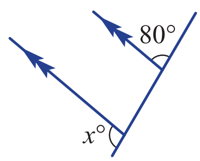
e


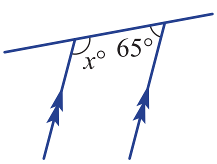
f


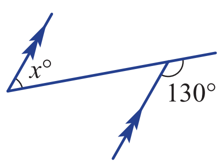
7 These diagrams have a pair of parallel lines. Find the unknown angle x.

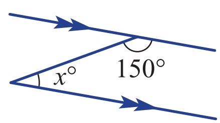
a


b


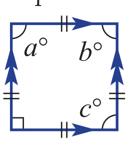
c


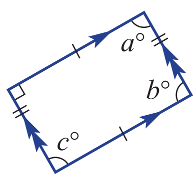
d


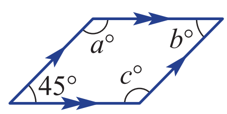
e


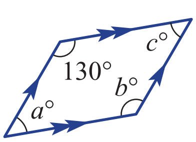
f


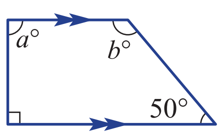
8 These common shapes consist of parallel lines. One internal angle is given. Find the values of the pronumerals.

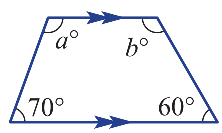
a


b


c


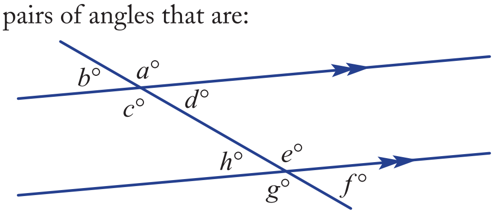
d


e


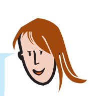
f


9 For this diagram, list all pairs of angles that are:

a corresponding
b alternate
c cointerior
d vertically opposite



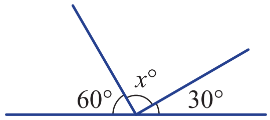
One example for part **a** is (a, e).



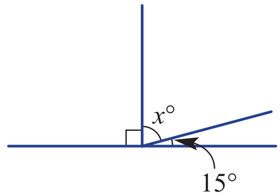
ISBN: 9781107687028

10 Find the unknown value x in each of these cases.

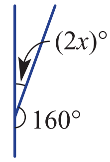
a



b

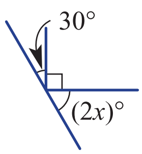


c

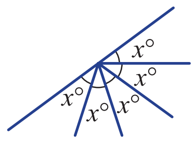


Angles on a straight line add to 180°.

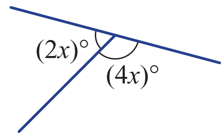
d



e



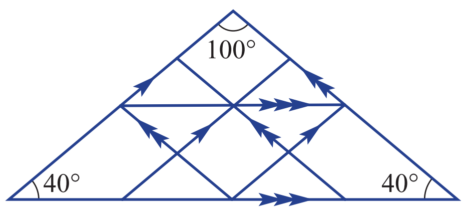
f



The roof truss

11 This diagram is of a roof truss with three groups of parallel supports.
How many of the angles are:

a 100° in size?
b 40° in size?
c 140° in size?



ISBN: 9781107687028 Cambridge University Press

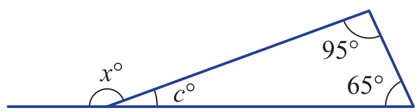
7.2 Triangles

The triangle is at the foundation of geometry, and its properties are used to work with more complex geometry.

One of the best known and most useful properties of triangles is the internal angle sum (180°). You can check this by measuring and adding up the three internal angles of any triangle.

Let's start: Exterior angle proof

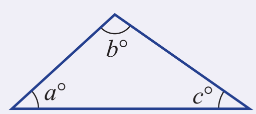
Consider this triangle with exterior angle $x°$.



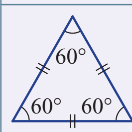
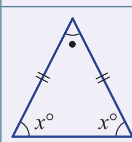
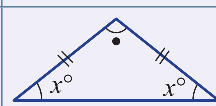
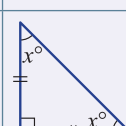
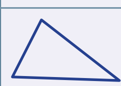
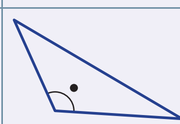
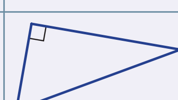
- Use the angle sum of a triangle to find the value of c.
- Now find the value of x.
- What do you notice about $x°$ and the two given angles? Is this true for other triangles? Give examples and reasons.

Key ideas

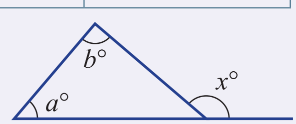
- The sum of all three internal angles of a triangle is 180°.
- Triangles can be classified by their side lengths or their internal angles.



$a + b + c = 180$

		Classified by internal angles		
		Acute-angled triangles (all angles acute, < 90°)	Obtuse-angled triangles (one angle obtuse, > 90°)	Right-angled triangles (one right angle, 90°)
Classified by side lengths	Equilateral triangles (three equal side lengths)		Not possible	Not possible
	Isosceles triangles (two equal side lengths)			
	Scalene triangles (no equal side lengths)			

- The **exterior angle theorem**:
  The exterior angle is equal to the sum of the two opposite interior angles.



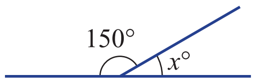
$x = a + b$

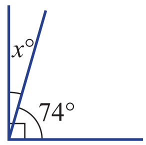
Exterior angle theorem The theorem that, in a triangle, the exterior angle is equal to the sum of the two opposite interior angles

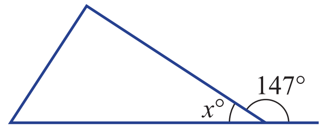
Exercise 7B

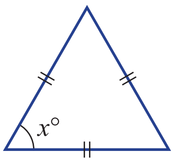
Understanding

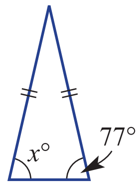
1 Give the value of x in these diagrams.

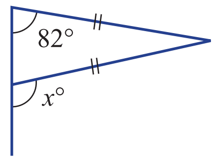
a


b


c


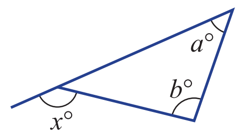
d


e


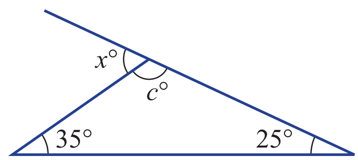
f


2 Choose the correct expression for this exterior angle.

A $a = x + b$ **B** $b = x + a$ **C** $x = a + b$
D $a + b = 180$ **E** $2a + b = 2x$



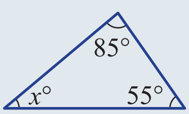
3 The two given interior angles for this triangle are 25° and 35°.
- **a** Use the angle sum (180°) to find the value of c.
- **b** Hence find the value of x.
- **c** What do you notice about the value of x and the two given interior angles?



Fluency

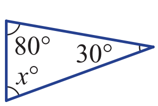
Example 3 Using the angle sum

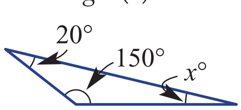
Find the value of the unknown angle (x) in this triangle.

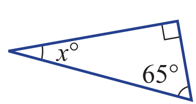


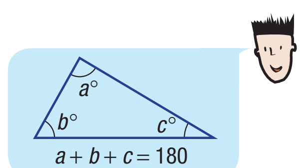
Solution	**Explanation**
$x + 85 + 55 = 180$	The sum of the three internal angles in a triangle is 180°.
$x + 140 = 180$	Simplify before solving for x.
$x = 40$	Solve for x by subtracting 140 from both sides of the 'equals' sign.
∴ the unknown angle is 40°	

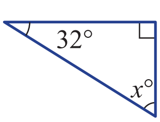
4 Find the value of the unknown angle (x) in these triangles:

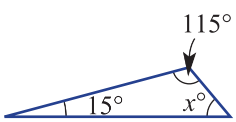
a


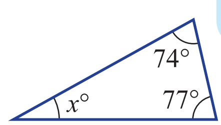
b


c




d


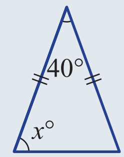
e


f


ISBN: 9781107687028 Cambridge University Press

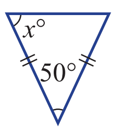
Example 4 Working with an isosceles triangle

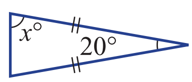
Find the value of x in this isosceles triangle.

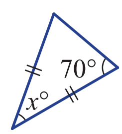


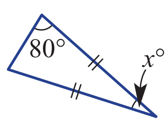
Solution	**Explanation**
$x + x + 40 = 180$	The triangle is isosceles and therefore the two base angles are equal.
$2x + 40 = 180$	Collect like terms.
$2x = 140$	Subtract 40 from both sides.
$x = 70$	Divide both sides by 2.
$\therefore$ the unknown angle is 70°	

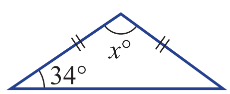
5 Find the value of the unknown angle (x) in these triangles:

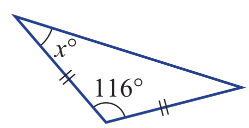
a


b


c


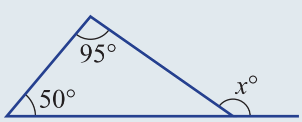
d


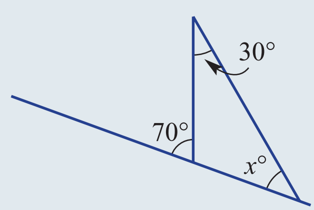
e


f


Example 5 Using the exterior angle theorem

Use the exterior angle theorem to find the value of x.

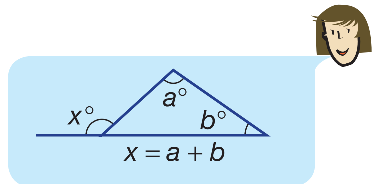
a


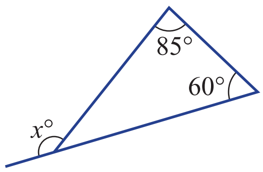
b


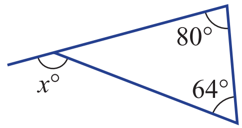
Solution	**Explanation**
a $x = 95 + 50$ $= 145$	The exterior angle $x°$ is the sum of the two opposite interior angles.
b $x + 30 = 70$ $x = 40$	The two opposite interior angles are $x°$ and 30°, and 70° is the exterior angle.

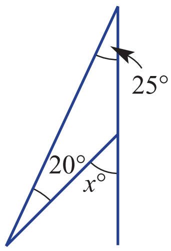
ISBN: 9781107687028

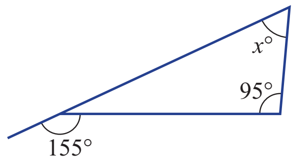
6 Use the exterior angle theorem to find the value of x.

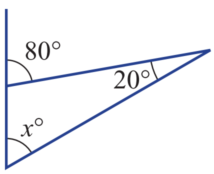


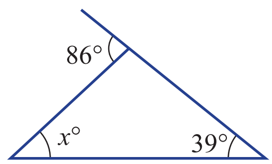
a


b


c


d


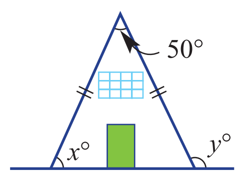
e


f


Problem-solving and Reasoning

7 Decide whether the following are possible. If so, make a drawing.
- a Acute scalene triangle
- b Acute isosceles triangle
- c Obtuse equilateral triangle
- d Acute equilateral triangle
- e Obtuse isosceles triangle
- f Obtuse scalene triangle
- g Right equilateral triangle
- h Right isosceles triangle
- i Right scalene triangle

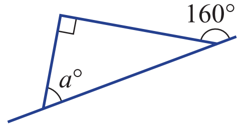
8 An architect draws the cross-section of a new ski lodge, which includes a very steep roof, as shown. The angle at the top is 50°. Find:
- a the acute angle the roof makes with the floor ($x°$)
- b the obtuse angle the roof makes with the floor ($y°$).



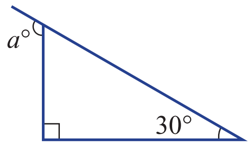
ISBN: 9781107687028 Cambridge University Press

9 Use your knowledge of parallel lines and triangles to find out the value of the pronumerals in these diagrams.

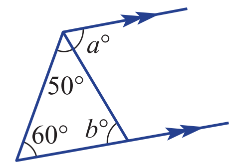
a



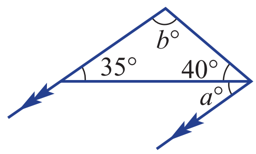
b



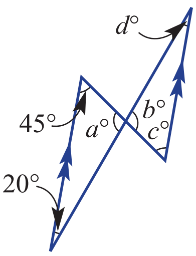
c



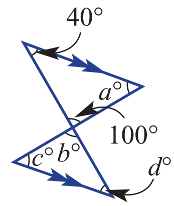
d



e



f

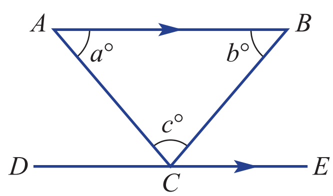


10 For this diagram, AB is parallel to DE.

a What is the size of $\angle ACD$? Use a pronumeral and give a reason.

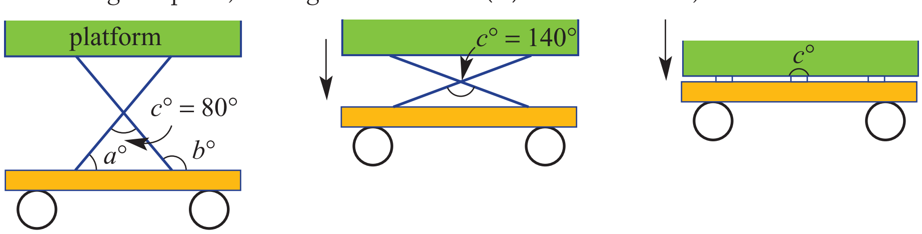
b What is the size of $\angle BCE$? Use a pronumeral and give a reason.

c Since $\angle DCE = 180°$, what does this tell us about a, b and c?



The hydraulic platform

11 A hydraulic platform includes a movable 'X' shape support system, as shown. When the platform is at its highest point, the angle at the centre ($c°$) of the 'X' is 80°, as shown.



a Find the following if the platform is at its highest position.

  i The acute angle the 'X' makes with the platform ($a°$)

  ii The obtuse angle the 'X' makes with the platform ($b°$)

b The platform now moves down so that the angle at the centre ($c°$) of the 'X' changes from 80° to 140°. With this platform position, find the values of:

  i the acute angle the 'X' makes with the platform ($a°$)

  ii the obtuse angle the 'X' makes with the platform ($b°$)

c The platform now moves down to the base so that the angle at the centre ($c°$) of the 'X' is now 180°. Find:

  i the acute angle the 'X' makes with the platform ($a°$)

  ii the obtuse angle the 'X' makes with the platform ($b°$)

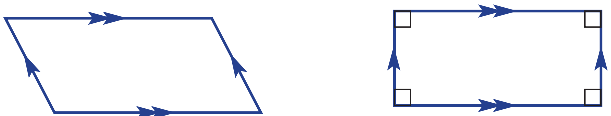
ISBN: 9781107687028

7.3 Quadrilaterals

Quadrilaterals are shapes that have four straight sides with a special angle sum of 360°. There are six special quadrilaterals, each with their own special set of properties. If you look around any old or modern building you will see examples of these shapes.

Let's start: Why is a rectangle a parallelogram?

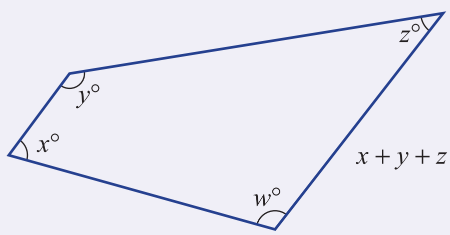
By definition, a parallelogram is a quadrilateral with two pairs of parallel sides.



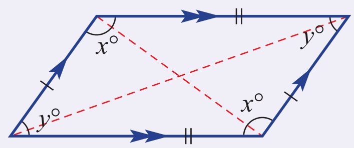
- Using this definition, do you think that a rectangle is also a parallelogram? Why?
- What properties does a rectangle have that a general parallelogram does not?
- What other special shapes are parallelograms? What are their properties?

Key ideas

- The sum of the interior angles of any quadrilateral is 360°.

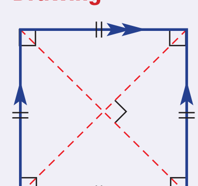


- **Parallelogram** Two pairs of parallel lines
  Two pairs of equal length sides
  Opposite angles equal



Parallelogram
A quadrilateral with both pairs of opposite sides parallel

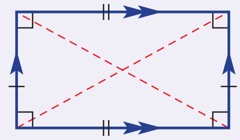
- Parallelograms are quadrilaterals with two pairs of parallel sides. These include the square, rectangle and rhombus.

Other quadrilaterals	Properties	Drawing
Square	Two pairs of parallel lines All sides of equal length All angles 90° Diagonals equal and intersect at right angles.	

ISBN: 9781107687028 Cambridge University Press

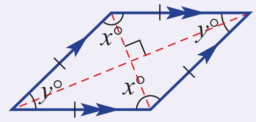
Rectangle

Two pairs of parallel lines
Two pairs of equal length sides
All angles 90°
Diagonals equal in length



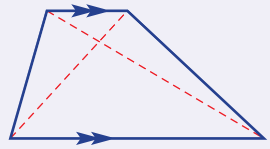
Rhombus

Two pairs of parallel lines
All sides of equal length
Opposite angles equal
Diagonals intersect at right angles



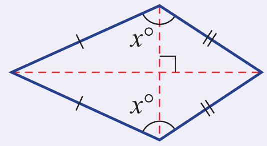
Trapezium

One pair of parallel sides



Kite

Two pairs of equal length adjacent sides
One pair of equal angles
Diagonals intersect at right angles



Exercise 7C

Understanding

1 Which special quadrilaterals are parallelograms?

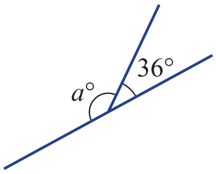
2 List all the quadrilaterals that have the following properties.

a two pairs of parallel sides
b two pairs of equal length sides
c equal opposite angles
d one pair of parallel sides
e one pair of equal angles
f all angles 90°
g equal length diagonals
h diagonals intersecting at right angles

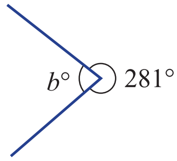
Refer to the Key ideas for help.

3 Find the value of the pronumerals.

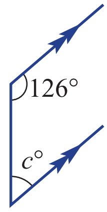
a



b



c

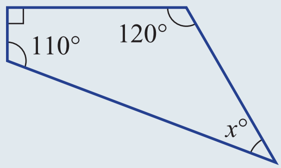


ISBN: 9781107687028

Fluency

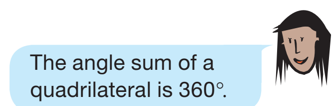
Example 6 Using the angle sum of a quadrilateral

Find the unknown angle in this quadrilateral.

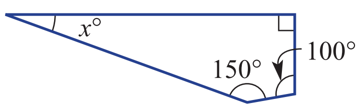


Solution	Explanation
$x + 110 + 120 + 90 = 360$	The sum of internal angles is 360°.
$x + 320 = 360$	Simplify.
$x = 40$	Subtract 320 from both sides.
∴ the unknown angle x is 40°.	

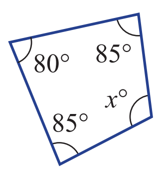
4 Find the unknown angles in these quadrilaterals.



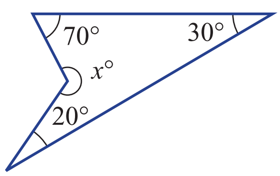
a



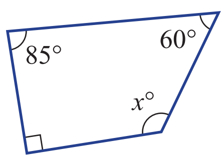
b



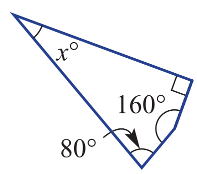
c



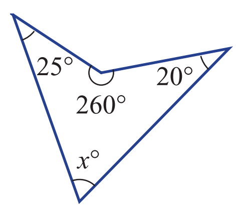
d



e



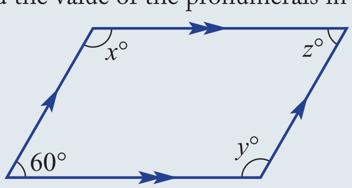
f



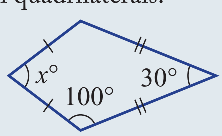
Example 7 Finding angles in special quadrilaterals

Find the value of the pronumerals in these special quadrilaterals.

a



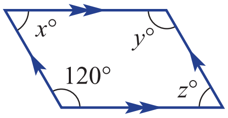
b

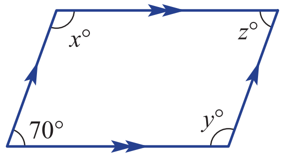


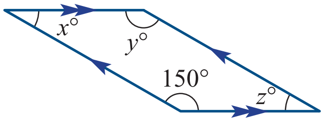
Solution	Explanation
a $x + 60 = 180$	$x°$ and 60° are cointerior angles and sum to 180°.
$x = 120$	Subtract 60 from both sides.
$\therefore y = 120$	$y°$ is opposite and equal to $x°$.
$\therefore z = 60°$	$z°$ is opposite and equal to 60°.
b $x + 100 + 100 + 30 = 360$	A kite has a pair of equal, opposite angles, so there are two 100° angles.
$x + 230 = 360$	The total sum is still 360°.
$x = 130$	

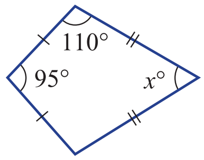
ISBN: 9781107687028

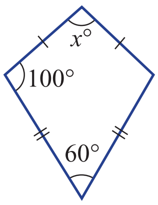
5 Find the value of the pronumerals in these special quadrilaterals.

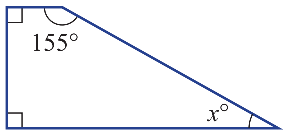
a


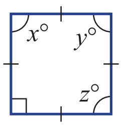
b


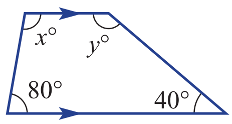
c


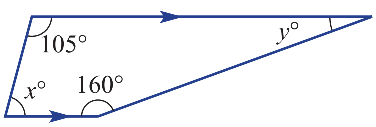
d


e


f


g


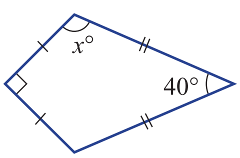
h


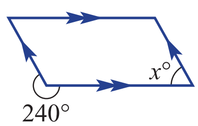
i


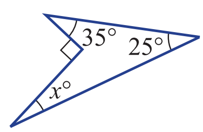
Refer to the properties of special quadrilaterals for help.

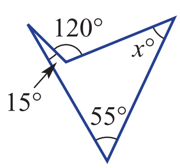
Problem-solving and Reasoning

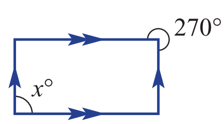
6 Find the value of the pronumerals in these shapes.

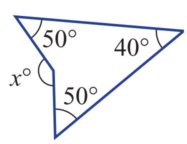
a


b


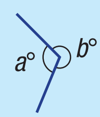
c


d


e


f


Angles in a revolution add to 360°.



$a + b = 360$

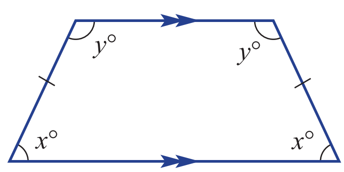
7 This shape is called an isosceles trapezium.

a Why do you think it is called an isosceles trapezium?

b i If $x = 60$, find y.

ii If $y = 140$, find x.

c List the properties of an isosceles trapezium.

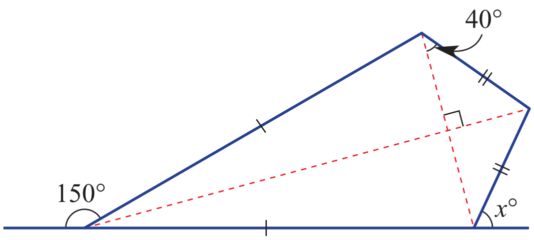


ISBN: 9781107687028 Cambridge University Press

8 A modern hotel is in the shape of a kite. Some angles are given in the diagram.

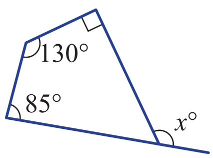
a Draw a copy of just the kite shape, including the diagonals.

b Find the angle that the right-hand wall makes with the ground ($x°$).

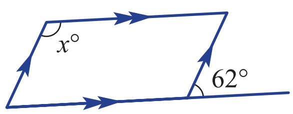


9 These quadrilaterals also include exterior angles. Find the value of x.

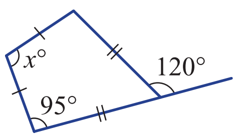
a



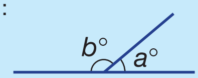
b



c



Recall:

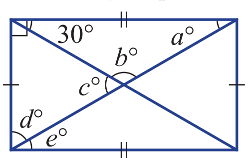


$a + b = 180$

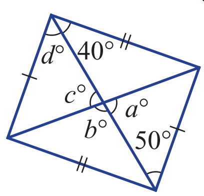
Quadrilaterals and triangles

10 The following shapes combine quadrilaterals with triangles. Find the value of the pronumerals.

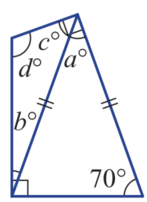
a



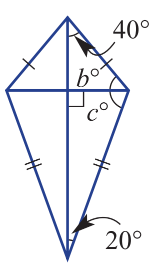
b



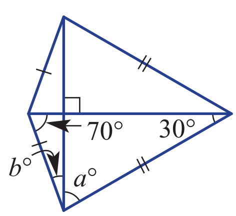
c



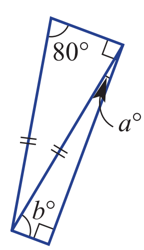
d



e



f

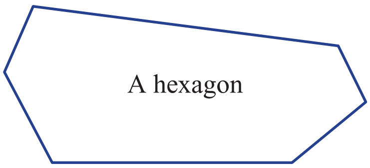


ISBN: 9781107687028

Cambridge University Press

7.4 Polygons

A closed shape with all straight sides is called a polygon. Like triangles and quadrilaterals (which are both polygons), they all have a special angle sum.



The Pentagon building in Washington, D.C.

Let's start: Remember the names

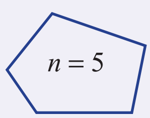
From previous years you should remember some of the names for polygons. See if you can remember them by completing this table.

Number of sides	Name
3	
4	
5	
6	
7	Heptagon

Number of sides	Name
8	
9	
10	
11	Undecagon
12	

Key ideas

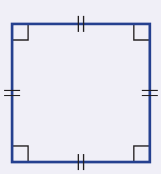
- The sum of internal angles (S) of a polygon is given by this rule:
  $S = 180° \times (n - 2)$
  where n is the number of sides



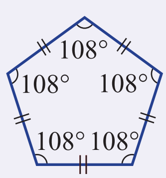
$$S = 180° \times (n - 2)$$
$$S = 180° \times (5 - 2)$$
$$= 180° \times 3$$
$$= 540°$$

- A **polygon** is a shape with straight sides.
  - They are named by their number of sides.
- A **regular polygon** has equal length sides and equal angles.

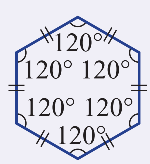
regular quadrilateral (square) (four sides)



regular pentagon (five sides)



regular hexagon (six sides)



Polygon A two-dimensional shape where three or more straight lines are joined together to form a closed figure

Regular polygon A polygon with all sides equal and all angles equal

ISBN: 9781107687028

Exercise 7D

Understanding

1 How many sides do these shapes have?

a quadrilateral **b** octagon **c** decagon **d** heptagon
e nonagon **f** hexagon **g** pentagon **h** dodecagon

2 Use the angle sum rule, $S = 180° \times (n - 2)$, to find the angle sum of these polygons.

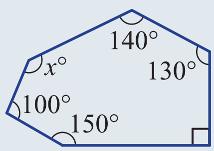
a pentagon ($n = 5$) **b** hexagon ($n = 6$) **c** heptagon ($n = 7$)
d octagon ($n = 8$) **e** nonagon ($n = 9$) **f** decagon ($n = 10$)

3 What is always true about a polygon that is regular?

Fluency

Example 8 Finding and using the angle sum of a polygon

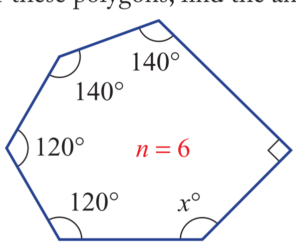
For this polygon, find the angle sum and then the value of x.



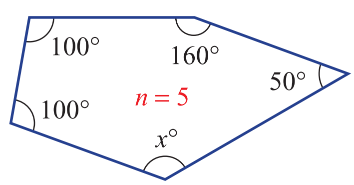
Solution	Explanation
$S = 180° \times (n - 2)$ $= 180° \times (6 - 2)$ $= 720°$	Use the angle sum rule first, with $n = 6$ since there are 6 sides. Find the angle sum.
$x + 100 + 150 + 90 + 130 + 140 = 720$ $x + 610 = 720$ $x = 110$	Use the total angle sum to find the value of x. Solve for the value of x.

4 For these polygons, find the angle sum then find the value of x.

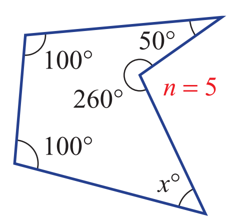
a

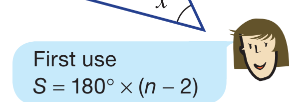


b

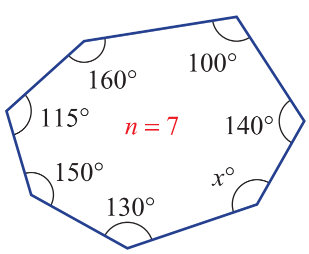


c

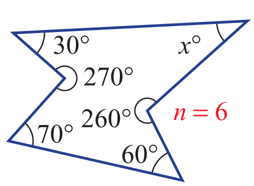




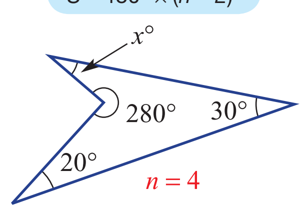
d



e



f

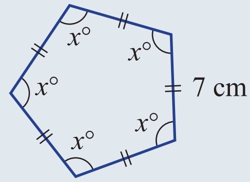


ISBN: 9781107687028

Example 9 Working with regular polygons

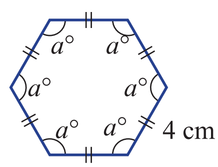
Shown here is a regular pentagon with straight edge side length 7 cm.

a Find the perimeter of the pentagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle $x°$.

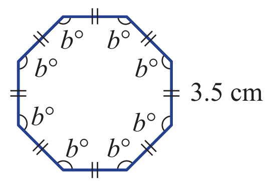


Solution	Explanation
a 35 cm	There are five sides at 7 cm each.
b $S = 180° \times (n - 2)$ $= 180° \times (5 - 2)$ $= 180° \times 3$ $= 540°$	Write the general rule for the sum of internal angles for a polygon. $n = 5$ since there are five sides. Simplify $5 - 2$. Evaluate 180×3.
c $540° \div 5 = 108°$ $\therefore x° = 108°$	There are five equally sized angles since it is a regular pentagon.

5 Shown here is a regular hexagon with straight edge side length 4 cm.
a Find the perimeter of the pentagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle $a°$.

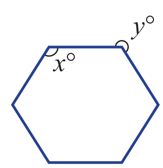


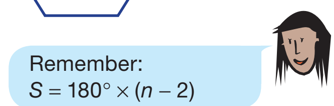
6 Shown here is regular octagon with straight edge side length 3.5 cm.
a Find the perimeter of the pentagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle $b°$.



Problem-solving and Reasoning

7 The cross-section of a pencil is a regular hexagon.
a Find the interior angle ($x°$).
b Find the outside angle ($y°$).





8 Find the total internal angle sum for a polygon with:
a 11 sides **b** 20 sides

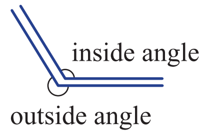
9 Find the size of a single internal angle for a regular polygon with:
a 10 sides **b** 25 sides

ISBN: 9781107687028

Cambridge University Press

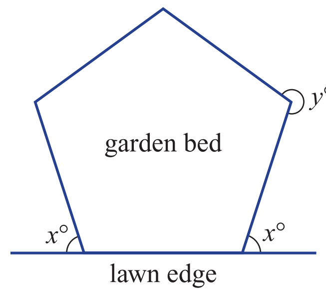
10 A castle turret is in the shape of a regular hexagon. At each of the six corners find:

- **a** the inside angle
- **b** the outside angle



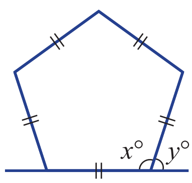
11 A garden bed is to be designed in the shape of a regular pentagon and sits adjacent to a lawn edge, as shown.

- **a** Find the angle the lawn edge makes with the garden bed (x).
- **b** Find the outside angle for each corner (y).

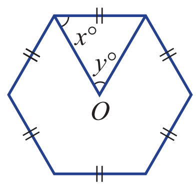


12 For these diagrams, find the values of the unknowns. The shapes are regular.

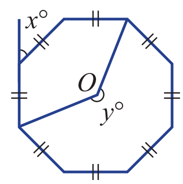
a



b



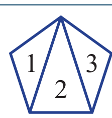
c



In parts **b** and **c**, the point O is the centre.

Develop the angle sum rule

13 a Copy and complete this table.

Polygon	Number of sides	Diagram	Number of triangles	Total angle sum (S)	Single internal angle (A)
Triangle					
Quadrilateral					
Pentagon	5		3	$3 \times 180 = 540$	$540 \div 5 = 108$
Hexagon					
…					
n-gon n					

b Complete these sentences by writing the rule.

- **i** For a polygon with n sides, the total angle sum, S, is given by $S =$ ______.
- **ii** For a polygon with n sides, a single internal angle, A, is given by $A =$ ______.

ISBN: 9781107687028 Cambridge University Press

7.5 Congruent triangles

In solving problems or in the building of structures, for example, it is important to know whether or not two expressions or objects are identical. The mathematical word used to describe identical objects is 'congruence.'

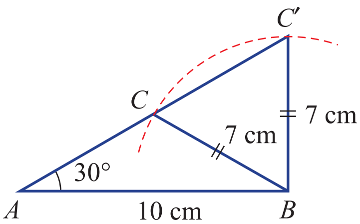
For congruent triangles there are four important tests that can be used to prove congruence.

Federation Square, Melbourne

Let's start: Why are AAA and ASS not tests for congruence?

AAA and ASS are not tests for the congruence of triangles.

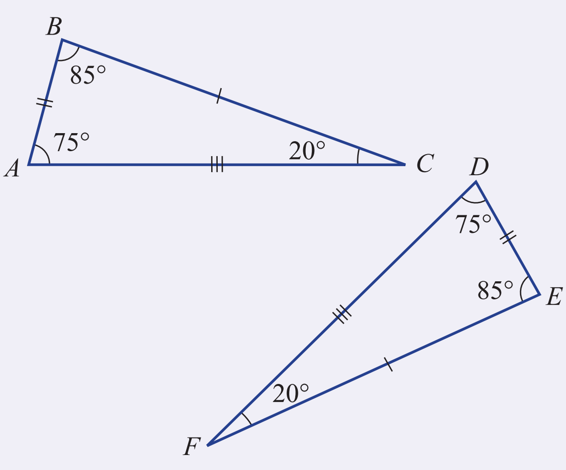
- For AAA, can you draw two different triangles using the same three angles? Why does this mean that AAA is not a test for congruence?
- Look at this diagram showing triangle *ABC* and triangle *ABC′*. Both triangles have a 30° angle and two sides of length 10 cm and 7 cm. Explain how this diagram shows that ASS is not a test for congruence of triangles.



Key ideas

- Two triangles are said to be **congruent** if they are exactly the same 'size' and 'shape'. Corresponding sides and angles will be of the same size, as shown in these triangles.
- If triangle *ABC* is congruent to triangle *DEF*, we write $\Delta ABC \equiv \Delta DEF$.
  - This is called a congruence statement.
  - Letters are usually written in matching order.
  - If two triangles are not congruent, we write: $\Delta ABC \not\equiv \Delta DEF$.

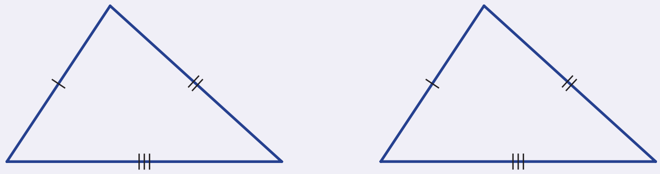
Congruent (figures) Figures that are exactly the same size and shape



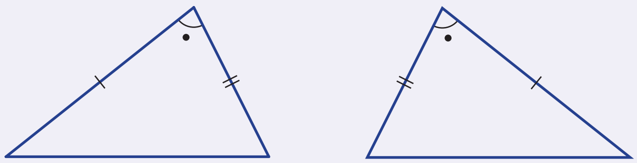
ISBN: 9781107687028

Cambridge University Press

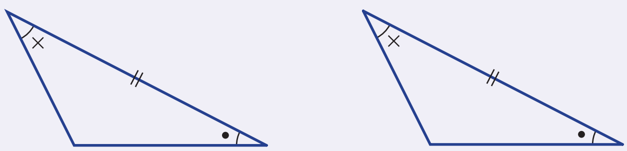
- Two triangles can be tested for congruence by considering the following necessary conditions.
  1. Corresponding sides are equal (SSS).



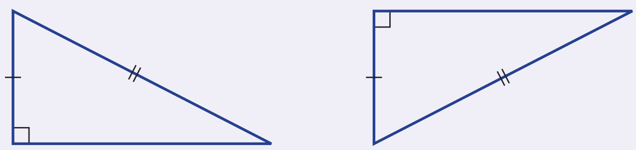
  2. Two corresponding sides and the angle between them are equal (SAS).



  3. Two angles and any corresponding side are equal (AAS).



  4. A right angle, the hypotenuse and one other pair of corresponding sides are equal (RHS).

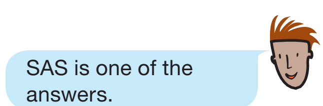


Exercise 7E

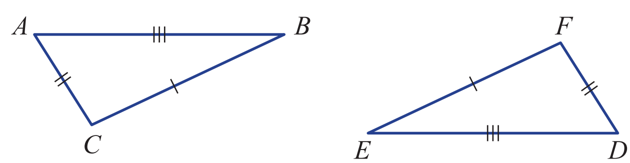
Understanding

1 True or false?
- **a** SSA is a test for the congruence of triangles.
- **b** AAA is a test for the congruence of triangles.
- **c** Two congruent triangles are the same shape and size.
- **d** If $\Delta ABC \equiv \Delta DEF$, then triangle ABC is congruent to triangle DEF.

2 Write the four tests for congruence using their abbreviated names.



3 Here is a pair of congruent triangles.
- **a** Which point on ΔDEF corresponds to point B on ΔABC?
- **b** Which side on ΔABC corresponds to side DF on ΔDEF?
- **c** Which angle on ΔDEF corresponds to $\angle BAC$ on ΔABC?



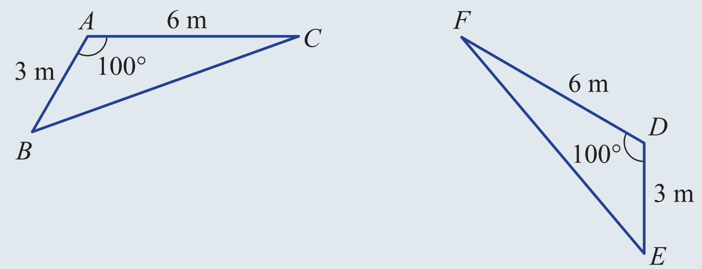
ISBN: 9781107687028 Cambridge University Press

Fluency

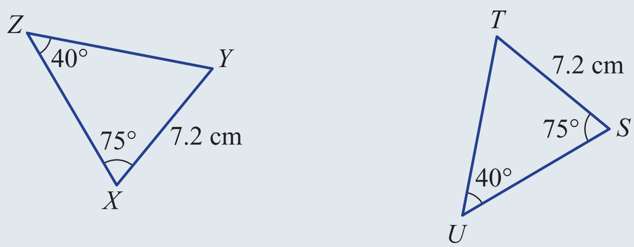
Example 10 Choosing a test for congruence

Write a congruence statement and the test to prove congruence for these pairs of triangles.

a



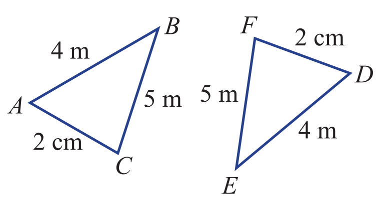
b



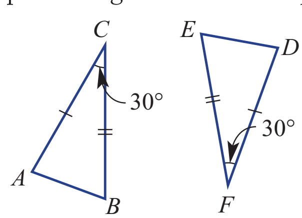
Solution	Explanation
a $\Delta ABC \equiv \Delta DEF$ (SAS)	Write letters in corresponding (matching) order. Two pairs of sides are equal as well as the angle between.
b $\Delta XYZ \equiv \Delta STU$ (AAS)	X matches S, Y matches T and Z matches U. Two angles and one pair of matching sides are equal.

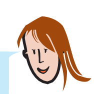
4 Write a congruence statement and the test to prove congruence for these pairs of triangles.

a



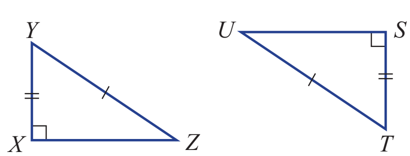
b



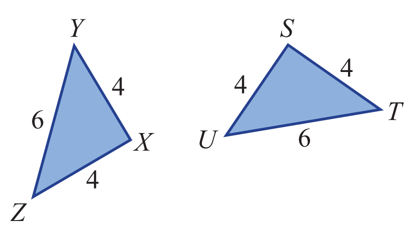


$\Delta ABC \equiv \Delta DEF$ is a congruence statement. Choose one of the tests SSS, SAS, AAS or RHS.

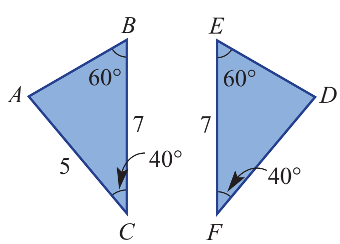
c



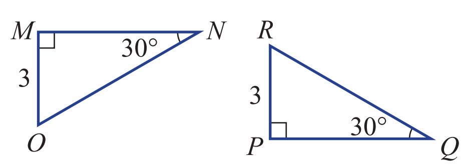
d



e



f

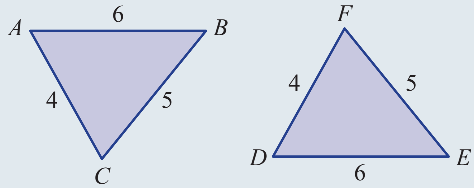


ISBN: 9781107687028

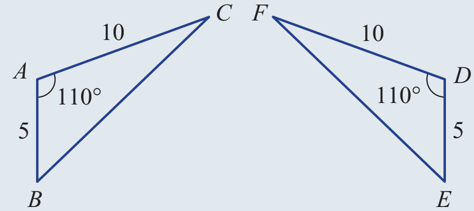
Example 11 Proving that a pair of triangles are congruent

Give reasons why the following pairs of triangles are congruent.

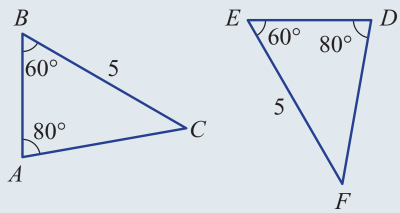
a



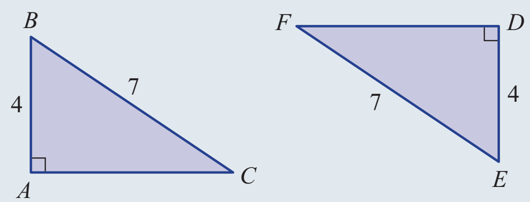
b



c



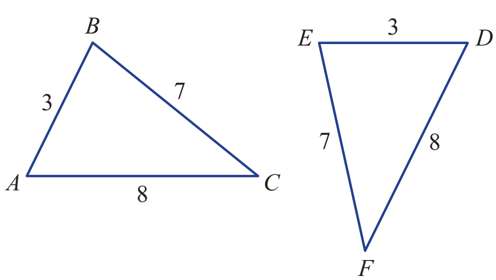
d



	Solution		Explanation
a	$AB = DE$	(S)	First choose all the corresponding side lengths.
	$AC = DF$	(S)	Corresponding side lengths will have the same length.
	$BC = EF$	(S)	
	$\therefore \Delta ABC \equiv \Delta DEF$	(SSS)	Write the congruence statement and the abbreviated reason.
b	$AB = DE$	(S)	Note that two corresponding side lengths are equal and the included angles are equal.
	$\angle BAC = \angle EDF$	(A)	
	$AC = DF$	(S)	
	$\therefore \Delta ABC \equiv \Delta DEF$	(SAS)	Write the congruence statement and the abbreviated reason.
c	$\angle ABC = \angle DEF$	(A)	Two of the angles are equal and one of the corresponding sides are equal.
	$\angle BAC = \angle EDF$	(A)	
	$BC = EF$	(S)	
	$\therefore \Delta ABC \equiv \Delta DEF$	(AAS)	Write the congruence statement and the abbreviated reason.
d	$\angle BAC = \angle EDF = 90^\circ$	(R)	Note that both triangles are right-angled, the hypotenuse of each triangle is of the same length and another corresponding side is of the same length.
	$BC = EF$	(H)	
	$AB = DE$	(S)	
	$\therefore \Delta ABC \equiv \Delta DEF$	(RHS)	Write the congruence statement and the abbreviated reason.

5 Give reasons why the following pairs of triangles are congruent.

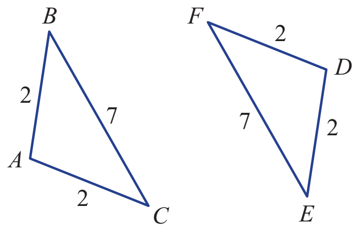
a



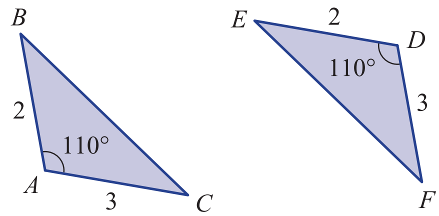
List reasons as in the examples to establish SSS, SAS, AAS or RHS.

ISBN: 9781107687028

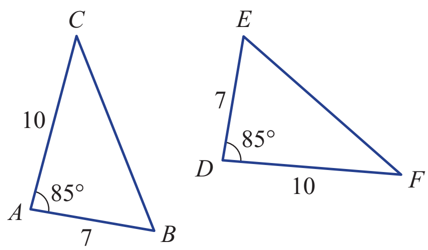
b



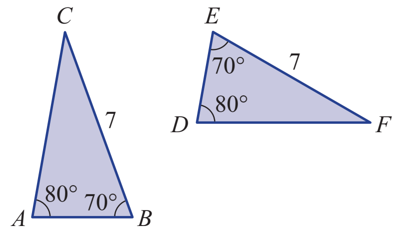
c



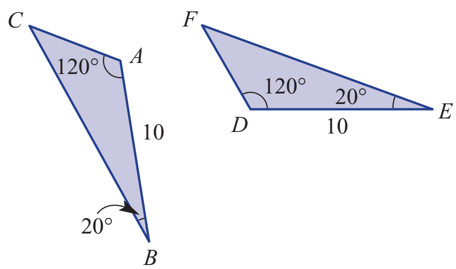
d



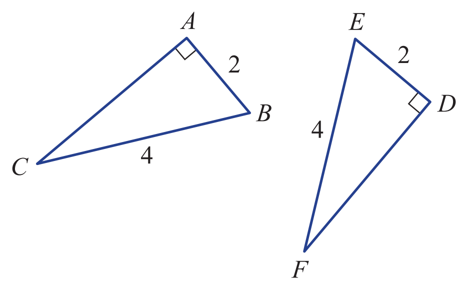
e



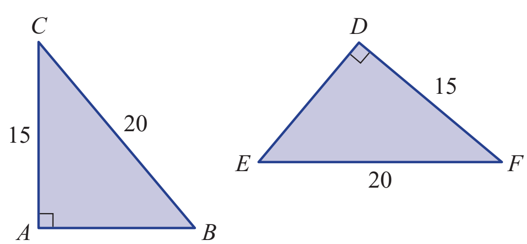
f



g



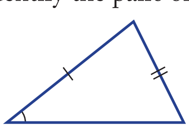
h



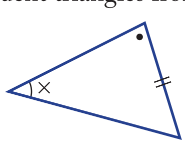
Problem-solving and Reasoning

6 Identify the pairs of congruent triangles from those below:

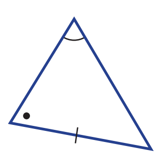
A



B

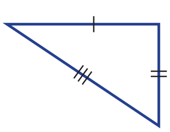


C

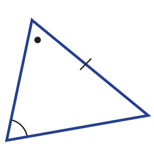


Sides with the same markings and angles with the same mark are equal.

D

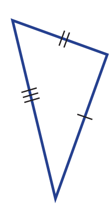


E

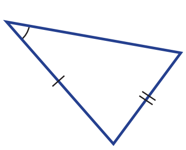


F

G



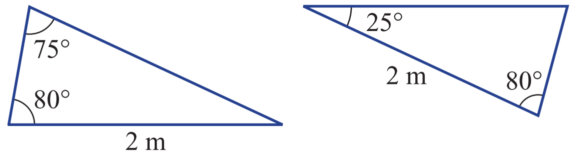
H



ISBN: 9781107687028

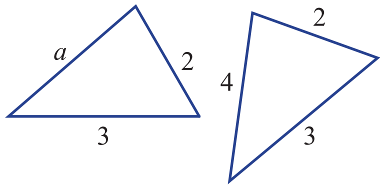
7 Two triangular windows have the given dimensions.

- **a** Find the missing angle in each triangle.
- **b** Are the two triangles congruent? Give a reason.

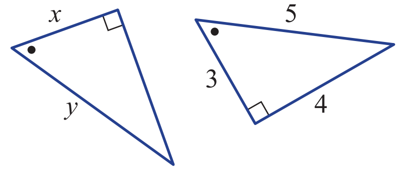


8 For the pairs of congruent triangles, find the values of the pronumerals.

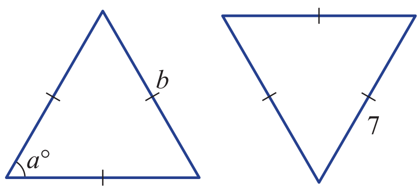
a



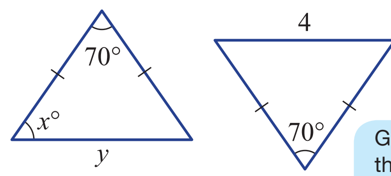
b



c

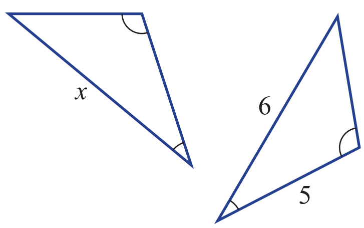


d

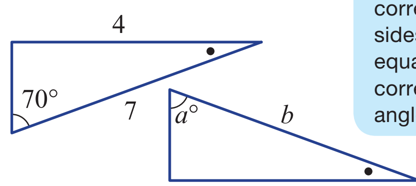


Given that these triangles are congruent, corresponding sides are equal, as are corresponding angles.

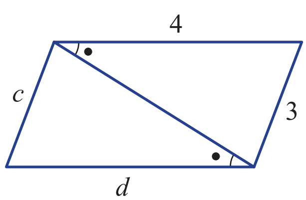
e



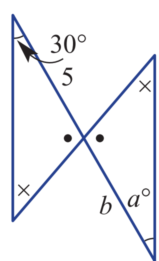
f



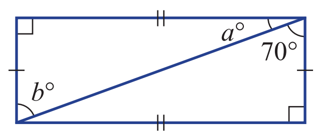
g



h



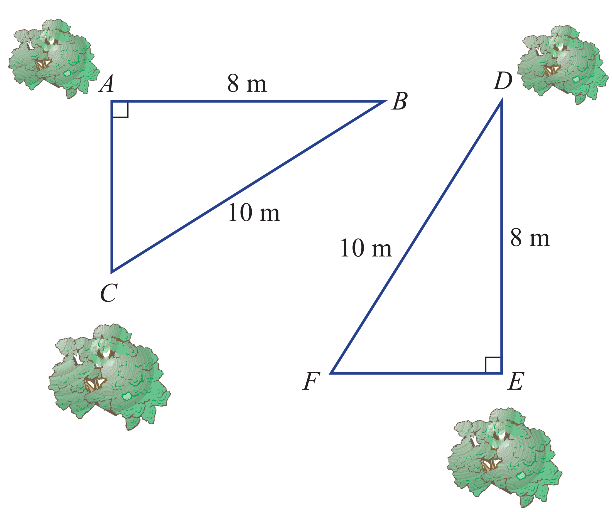
i



Lawn landscaping

9 A new garden design includes two triangular lawn areas, as shown.

- **a** Give reasons why the two triangular lawn areas are congruent.
- **b** If the length of AC is 6 m, find the length of EF.
- **c** If the angle $ABC = 37°$, find the angles:
  - **i** $\angle EDF$
  - **ii** $\angle DFE$



ISBN: 9781107687028 Cambridge University Press

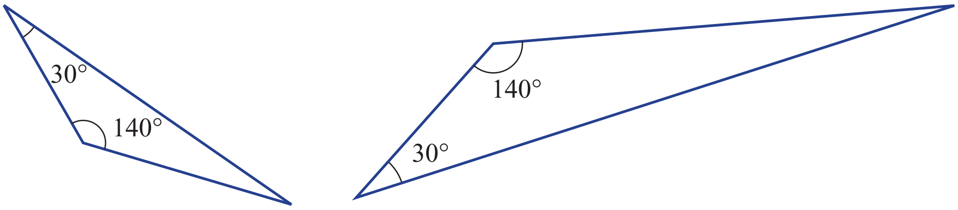
7.6 Similar triangles

When two objects are similar, they are the same shape but of different size. For example, a computer image reproduced on a large screen will show all aspects of the image in the same way except in size. The computer image and screen image are said to be similar figures.

An image on a laptop computer reproduced on a large TV screen is the same in all aspects except size.

Let's start: Is AA the same as AAA?

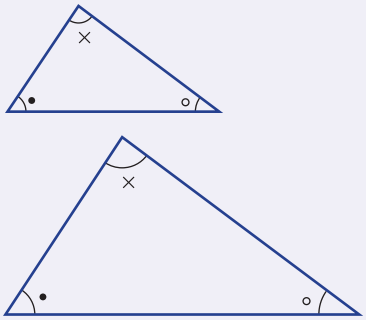
Look at these two triangles.



- What is the missing angle in each triangle?
- Do you think the triangles are similar? Why?
- Is the AA test equivalent to the AAA test?

Key ideas

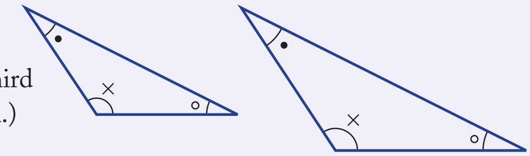
- Two triangles are said to be **similar** if they are the same shape but different in size. Corresponding angles will be equal and corresponding side lengths will be in the same ratio.
- If ΔABC is similar to ΔDEF, then we write $\Delta ABC \,|||\, \Delta DEF$ or $\Delta ABC \sim \Delta DEF$.



Similar (triangles) Triangles whose corresponding angles are equal and whose corresponding sides are in the same ratio

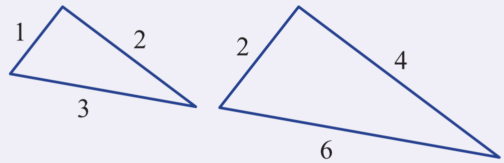
ISBN: 9781107687028

- Two triangles can be tested for similarity by considering the following necessary conditions:
  1. Corresponding angles are equal (AAA). (Remember that if two pairs of corresponding angles are equal then the third pair of corresponding angles are also equal.)



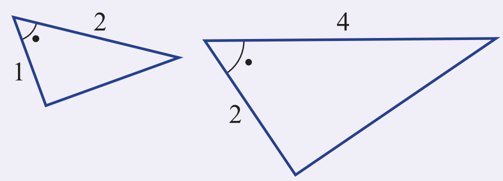
  2. Corresponding sides are in the same ratio (SSS).

$$\frac{6}{3} = \frac{4}{2} = \frac{2}{1} = 2$$



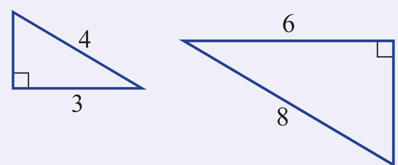
  3. Two pairs of corresponding sides are in the same ratio and the included corresponding angles between these sides are equal (SAS).

$$\frac{4}{2} = \frac{2}{1} = 2$$



  4. The hypotenuses and a pair of corresponding sides in a right-angled triangle are in the same ratio.

$$\frac{8}{4} = \frac{6}{3} = 2$$



- The **scale factor** is calculated using a pair of corresponding sides. In the above three examples the scale factor is 2.

Scale factor
The number you multiply each side length by to enlarge or reduce a shape

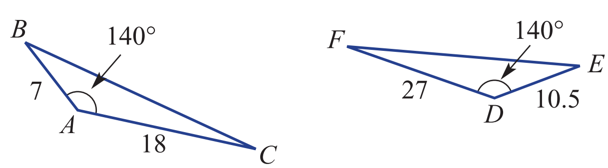
Exercise 7F

Understanding

1. Which of the following is not a test for the similarity of triangles?
SSS, SAS, RHS, SSA, AAA

2. Why is the AA test the same as the AAA test for similar triangles?

3. Consider this pair of triangles.



   a. Work out $\frac{DE}{AB}$.
   b. Work out $\frac{DF}{AC}$. What do you notice?
   c. What is the scale factor?
   d. Which of SSS, SAS, AAA or RHS would be used to explain their similarity?

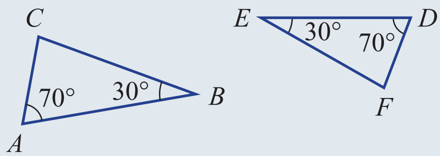
ISBN: 9781107687028 Cambridge University Press

Fluency

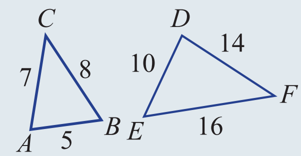
Example 12 Proving similar triangles

Decide whether the pairs of triangles are similar, giving reasons.

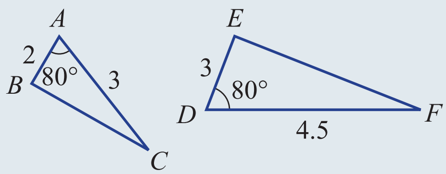
a



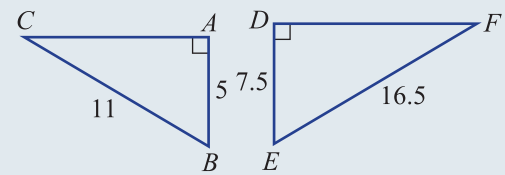
b



c



d



Solution / Explanation

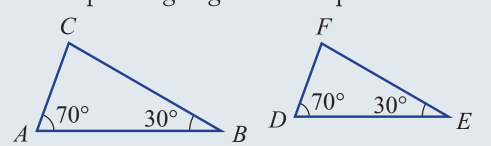
a $\angle BAC = \angle EDF$ (A)

$\angle ABC = \angle DEF$ (A)

$\angle ACB = \angle DFE$ (A)

$\therefore \Delta ABC \; ||| \; \Delta DEF$ (AAA)

Two corresponding angles are equal and therefore the third corresponding angle is also equal.



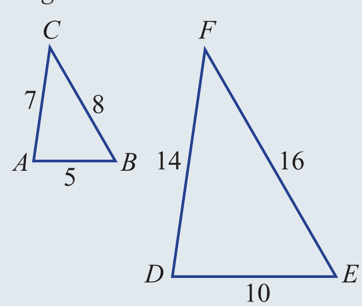
b $\frac{DE}{AB} = \frac{10}{5} = 2$ (S)

$\frac{EF}{BC} = \frac{16}{8} = 2$ (S)

$\frac{DF}{AC} = \frac{14}{7} = 2$ (S)

$\therefore \Delta ABC \; ||| \; \Delta DEF$ (SSS)

All three corresponding sides are in the same ratio or proportion.



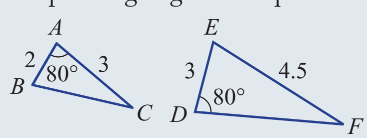
c $\frac{DE}{AB} = \frac{3}{2} = 1.5$ (S)

$\angle BAC = \angle EDF$ (A)

$\frac{DF}{AC} = \frac{4.5}{3} = 1.5$ (S)

$\therefore \Delta ABC \; ||| \; \Delta DEF$ (SAS)

Two corresponding sides are in the same ratio and the included corresponding angles are equal.



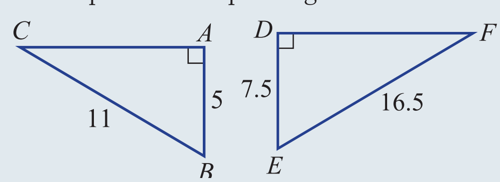
d $\angle BAC = \angle EDF = 90°$ (R)

$\frac{EF}{BC} = \frac{16.5}{11} = 1.5$ (H)

$\frac{DE}{AB} = \frac{7.5}{5} = 1.5$ (S)

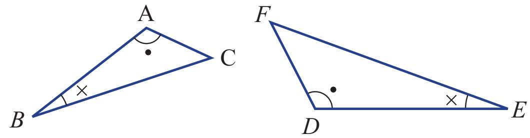
$\therefore \Delta ABC \; ||| \; \Delta DEF$ (RHS)

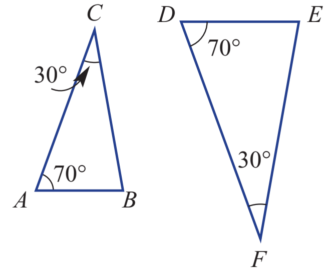
They are right-angled triangles with the hypotenuses and one other pair of corresponding sides in the same ratio.

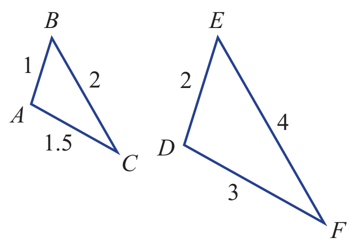


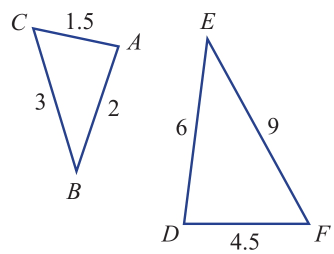
ISBN: 9781107687028

4 Decide whether the pairs of triangles are similar, giving reasons.

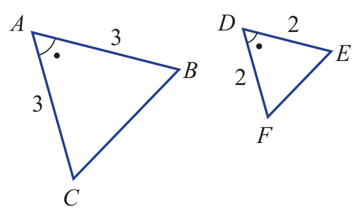
a


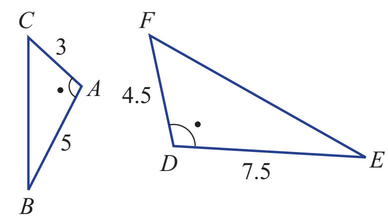
b


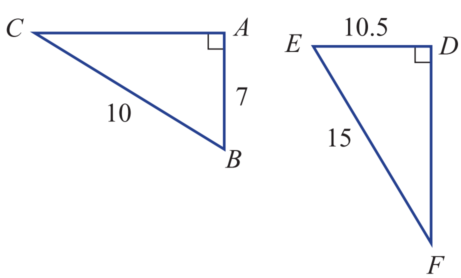
c


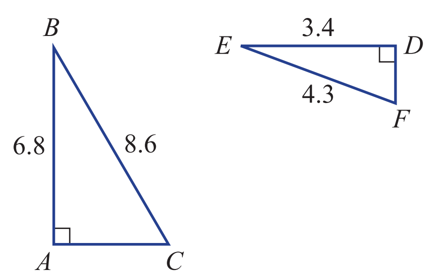
d


List all the equal angles and corresponding pairs of sides, as in Example 12.

e


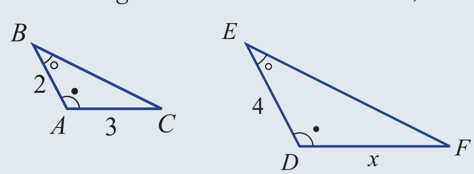
f


g


h


Example 13

If the given pair of triangles are known to be similar, find the value of x.

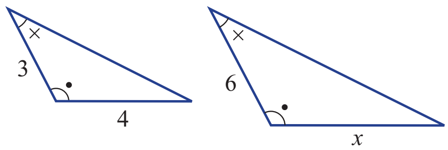


Solution	**Explanation**
Scale factor $= \dfrac{DE}{AB} = \dfrac{4}{2} = 2$ $x = 3 \quad 2$ $= 6$	First find the scale factor using a pair of corresponding sides. Divide the larger number by the smaller number. Multiply the corresponding length on the smaller triangle using the scale factor.

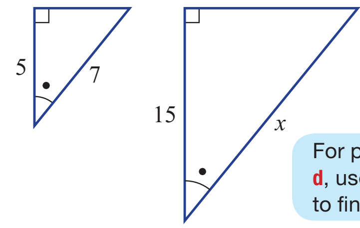
ISBN: 9781107687028

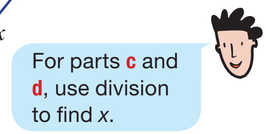
5 If the given pair of triangles are known to be similar, find the value of x.

a

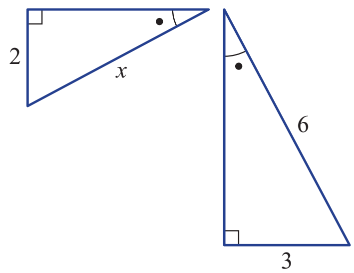


b

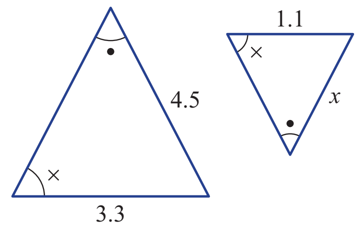




c



d



Problem-solving and Reasoning

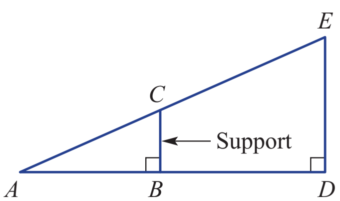
6 A ski ramp has a vertical support, as shown.

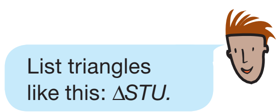
a List the two triangles that are similar?

b Why are the two triangles similar?

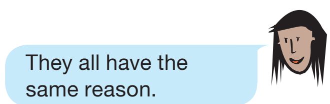
c If $AB = 4$ m and $AD = 10$ m, find the scale factor.

d If $BC = 1.5$ m, find the height of the ramp DE.

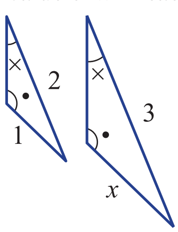




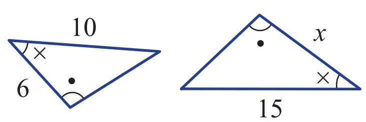
7 State why the pairs of triangles are similar (give the abbreviated reason) and determine the value of x in each case.



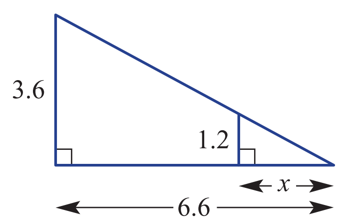
a



b



c



Triangles in parallel lines

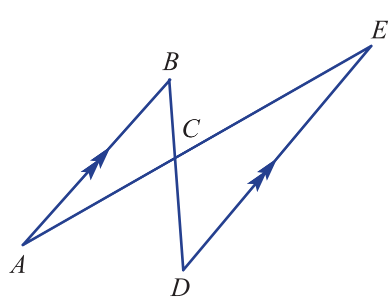
8 In the given diagram, AB is parallel to DE.

a List the three pairs of angles that are equal and give a reason.

b If $AB = 8$ cm and $DE = 12$ cm, find:

i DC if $BC = 4$ cm

ii AC if $CE = 9$ cm



ISBN: 9781107687028

Cambridge University Press

7.7 Applying similar triangles

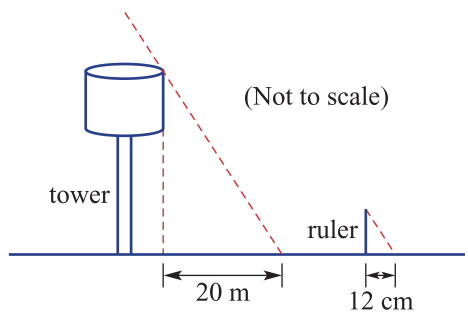
Once it is established that two triangles for a particular situation are similar, the ratio or scale factor between side lengths can be used to find unknown side lengths.

Similar triangles have many applications in the real world. One application is finding an inaccessible distance, like the height of a tall object or the distance across a deep ravine.

Let's start: The tower and the ruler

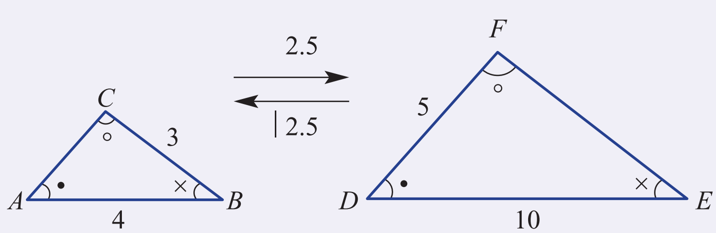
Franklin wants to know how tall a water tower is in his town. At a particular time of day he measures its shadow to be 20 m long. At the same time he stands a 30 cm ruler near the tower, which gives a 12 cm shadow.

- Explain why the two formed triangles are similar.
- What is the scale factor?
- What is the height of the tower?



Key ideas

- For two similar triangles, the ratio of the corresponding side lengths written as a single number is called the scale factor.



- Once the scale factor is known, it can be used to find unknown side lengths.

$$\frac{DE}{AB} = \frac{10}{4} = \frac{5}{2} = 2.5$$

$\therefore$ scale factor is 2.5

$\therefore EF = 3 \times 2.5 = 7.5$

$AC = 5 \div 2.5 = 2$

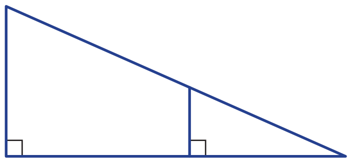
ISBN: 9781107687028 Cambridge University Press

Exercise 7G

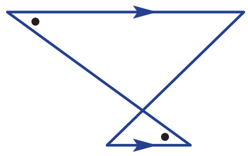
Understanding

1 Give reasons why the pairs of triangles in each diagram are similar.

a



b

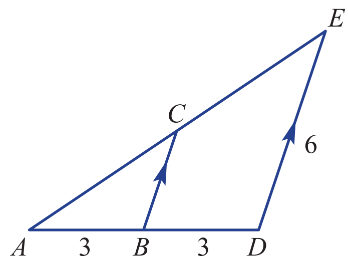


2 For the pair of triangles in the given diagram:

a which reason would be chosen to explain their similarity: SSS, SAS, AAA or RHS?

b what is the scale factor?

c what is the length BC?



Fluency

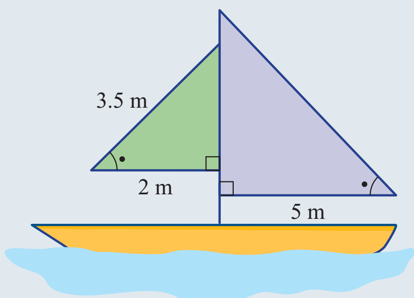
Example 14 Applying similar triangles

A homemade raft consists of two sails with measurements and angles as shown in this diagram.

a Give reasons why the two sails are similar in shape.

b Find the scale factor for the side lengths of the sails.

c Find the length of the longest side of the large sail.



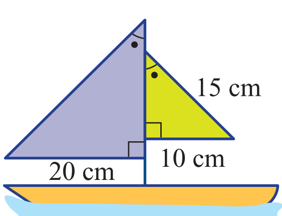
Solution	Explanation
a AAA	Two of the three angles are clearly equal, so the third must be equal.
b Scale factor $= \dfrac{5}{2} = 2.5$	Choose two corresponding sides with known lengths and divide the larger by the smaller.
c Longest side $= 3.5 \times 2.5$ $= 8.75$ m	Multiply the corresponding side on the smaller triangle by the scale factor.

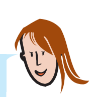
3 A toy yacht consists of two sails with measurements and angles as shown in this diagram.

a Give reasons why the two sails are similar in shape.

b Find the scale factor for the side lengths of the sails.

c Find the length of the longest side of the large sail.

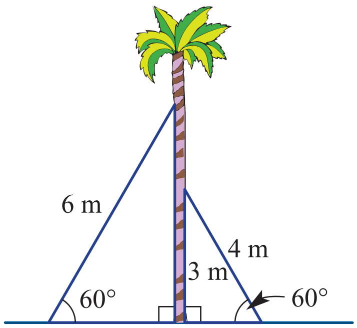


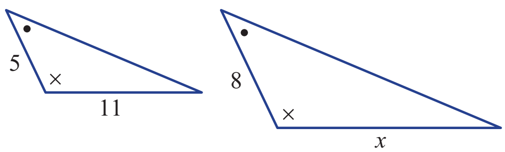


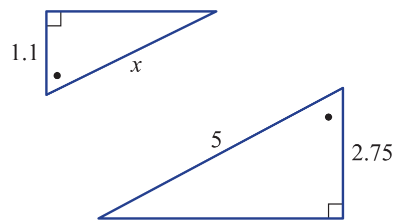
You can't choose SSS, SAS or RHS because only one pair of corresponding sides is given.

ISBN: 9781107687028

4 A tall palm tree is held in place with two cables of length 6 m and 4 m, as shown.
  a Give reasons why the two triangles created by the cables are similar in shape.
  b Find the scale factor for the side lengths of the cables.
  c Find the height of the point above the ground where the longer cable is attached to the palm tree.



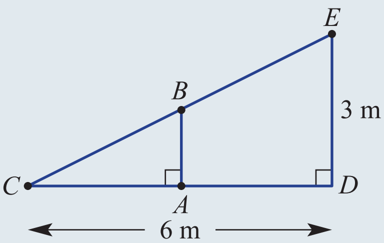
5 These pairs of triangles are known to be similar. By finding the scale factor, find the value of x.
  a


  b


Problem-solving and Reasoning

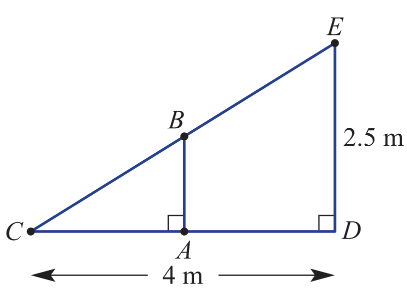
Example 15 Working with combined triangles

A ramp is supported by a vertical stud, AB, where A is at the centre of CD. It is known that $CD = 6$ m and that the ramp is 3 m high.
  a Using the letters given, name the two triangles that are similar and give your reason.
  b Find the length of the stud AB.



Solution	Explanation
a ΔABC and ΔDEC (AAA)	The angle at C is common to both triangles and they both have a right angle.
b $AC = 3$ m	Since A is in the centre of CD, then AC is half of CD.
Scale factor $= \dfrac{6}{3} = 2$	$CD = 6$ m and $AC = 3$ m
$\therefore AB = 3 \div 2$ $= 1.5$ m	Divide the larger side length, DE, by the scale factor.

6 A ramp is supported by a vertical stud AB, where A is at the centre of CD. It is known that $CD = 4$ m and that the ramp is 2.5 m high.
  a Using the letters given, name the two triangles that are similar and give your reason.
  b Find the length of the stud AB.



ISBN: 9781107687028

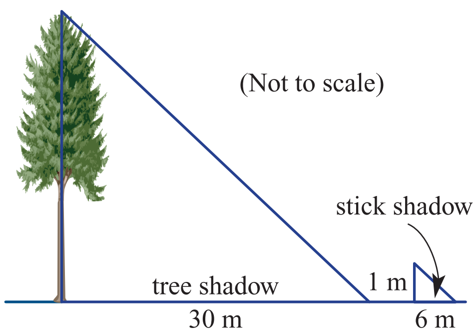
7 A 1 m vertical stick and a tree cast their shadows at a particular time in the day. The shadow lengths are shown in this diagram.

a Give reasons why the two triangles shown are similar in shape.

b Find the scale factor for the side lengths of the triangles.

c Find the height of the tree.

At the same time of day, the angle that the light makes with the ground will be same.



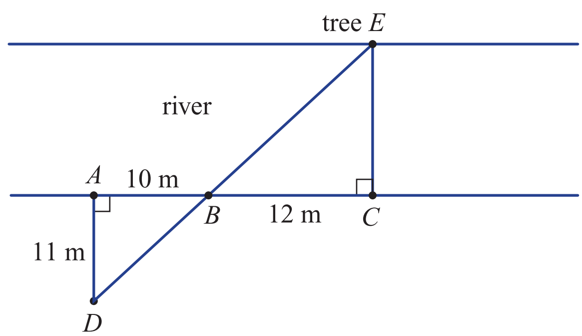
8 From a place on the river (C), a tree (E) is spotted on the opposite bank. The distances between selected trees A, B, C and D are measured as shown.

a List two similar triangles and give a reason why they are similar.

b Find the scale factor.

c Find the width of the river.

AB corresponds to *CB* and *AD* corresponds to *CE*.



9 At a particular time of day, Aaron casts a shadow 1.3 m long while Jack, who is 1.75 m tall, casts a shadow 1.2 m long. Find the height of Aaron to two decimal places.

Draw a diagram to find the scale factor.

10 Try this activity with a partner but ensure that at least one person knows their height.

a Go out into the sun and measure the length of each person's shadow.

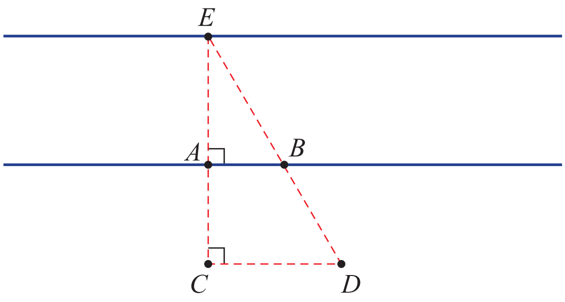
b Use these measurements plus the known height of one person to find the height of the other person.

ISBN: 9781107687028 Cambridge University Press

Gorge challenge

11 Mandy sets up a series of rocks alongside a straight section of a deep gorge. She places rocks A, B, C and D as shown. Rock E sits naturally on the other side of the gorge. She then measures the following distances.

- $AB = 10$ m
- $AC = 10$ m
- $CD = 15$ m



a Explain why $\Delta ABE \,|||\, \Delta CDE$.

b What is the scale factor?

c Use trial and error to find the distance across the gorge from rocks A to E.

d Can you instead find the length AE by setting up an equation?

ISBN: 9781107687028 Cambridge University Press

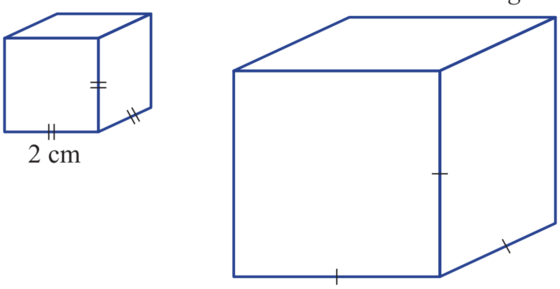
7.8 Applications of similarity in measurement

Shapes or objects that are similar have a distinct length, area and volume ratio relationship.

For example, if the lengths on a model of a building are one hundredth of the actual structure, then the length ratio is 1:100. From this, the surface area and volume ratios are $1^2 : 100^2$ (1 : 10 000) and $1^3 : 100^3$ (1 : 1 000 000) respectively. These ratios can be used to calculate the amount of material that is needed for the construction of the building.

Let's start: Cube analysis

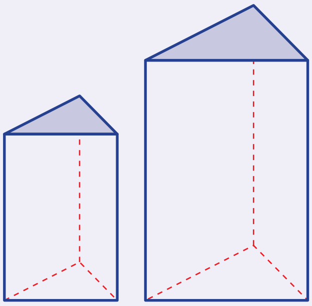
These two cubes have a 2 cm and 6 cm side length.



- What is the side length ratio comparing the two cubes?
- What are the surface areas of the two cubes?
- What is the surface area ratio? What do you notice?
- What are the volumes of the two cubes?
- What is the volume ratio? What do you notice?

Key ideas

- If two objects are similar and have a length ratio of $a : b$, then:
  - Length ratio $= a : b$ Scale factor $= \dfrac{b}{a}$
  - Area ratio $= a^2 : b^2$ Scale factor $= \dfrac{b^2}{a^2}$
  - Volume ratio $= a^3 : b^3$ Scale factor $= \dfrac{b^3}{a^3}$
  - one dimension: length ratio $= 2^1 : 3^1 = 2 : 3$
  - two dimensions: area ratio $= 2^2 : 3^2 = 4 : 9$
  - three dimensions: volume ratio $= 2^3 : 3^3 = 8 : 27$



ISBN: 9781107687028

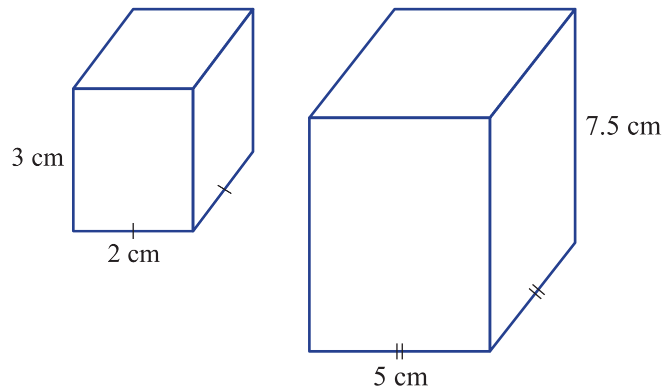
Exercise 7H

Understanding

Length ratio $a:b$
Area ratio $a^2:b^2$
Volume ratio $a^3:b^3$

1 The length ratio for two objects is 2:3.
- **a** What would be the area ratio?
- **b** What would be the volume ratio?

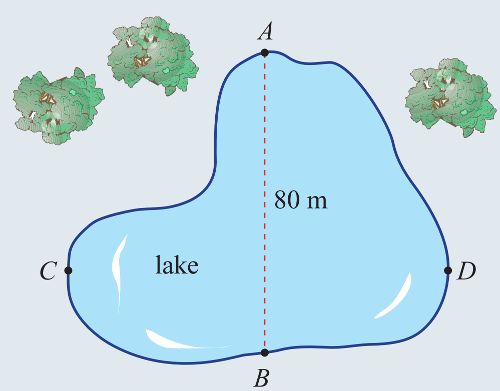
2 These two rectangular prisms are similar.
- **a** What is the side length ratio?
- **b** What is the surface area of the two prisms?
- **c** What is the surface area ratio? What do you notice?
- **d** What are the volumes of the two prisms?
- **e** What is the volume ratio? What do you notice?



Fluency

Example 16 Measuring to find actual lengths

The given diagram is a simple map of a park lake.
- **a** Use a ruler to measure the distance across the lake (AB). (Answer in cm.)
- **b** Find the scale factor between the map and ground distance.
- **c** Use a ruler to measure the map distance across the lake (CD). (Answer in cm.)
- **d** Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



Solution	Explanation
a 4 cm	Check with your ruler.
b $\frac{8000}{4} = 2000$	Using the same units, divide the real distance (80 m = 8000 cm) by the measured distance (4 cm).
c 5 cm	Check with your ruler.
d $5 \times 2000 = 10\,000$ cm = 100 m	Multiply the measured distance by the scale factor and convert to metres by dividing by 100.

ISBN: 9781107687028

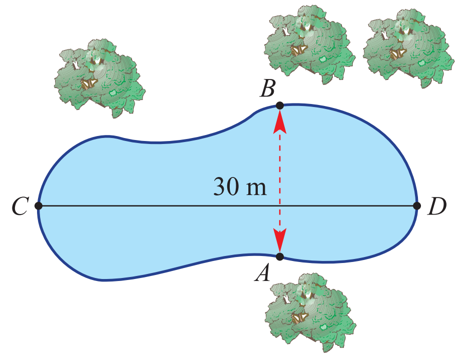
3 The given diagram is a simple map of a park lake.

a Use a ruler to measure the distance across the lake (AB). (Answer in cm.)

b Find the scale factor between the map and ground distance.

c Use a ruler to find the map distance across the lake (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



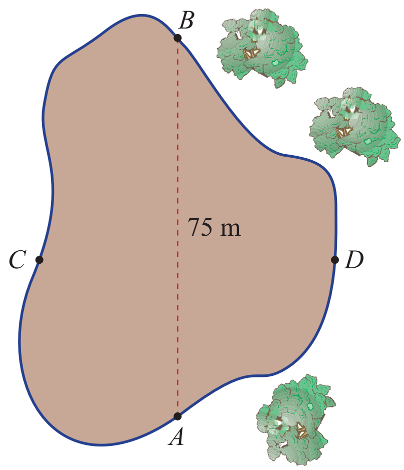
4 The given diagram is a simple map of a children's play area.

a Use a ruler to measure the distance across the children's play area (AB). (Answer in cm.)

b Find the scale factor between the map and ground distance.

c Use a ruler to find the map distance across the children's play area (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the children's play area (CD). (Answer in m.)

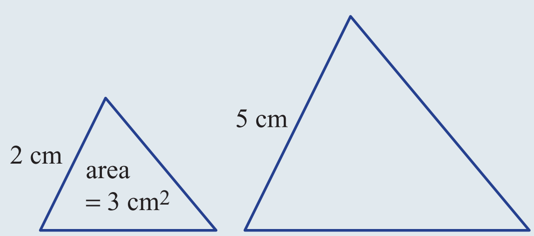


Use the measured distance AB and the actual distance AB to find the scale factor.

Example 17 Using similarity to find areas

The two given triangles are known to be similar.

Find the area of the larger triangle.



Solution	Explanation
Length ratio $= 2^1 : 5^1 = 2 : 5$	First, write the length ratio.
Area ratio $= 2^2 : 5^2 = 4 : 25$	Square each number in the length ratio to get the area ratio.
Area scale factor $= \dfrac{25}{4} = 6.25$	Divide the two numbers in the area ratio to get the scale factor.
$\therefore$ area of larger triangle $= 3 \times 6.25$ $= 18.75\text{ cm}^2$	Multiply the area of the smaller triangle by the scale factor.

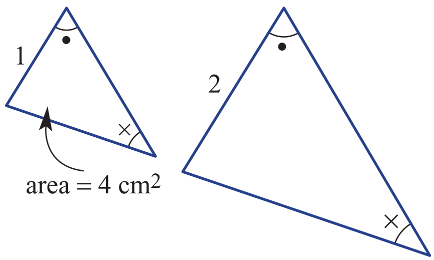
ISBN: 9781107687028

7H

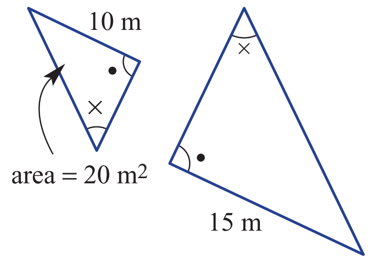
5 The two given triangles are known to be similar. Find the area of the larger triangle.

Length ratio = $a{:}b$
Area ratio = $a^2{:}b^2$
Area scale factor = $\frac{b^2}{a^2}$

a

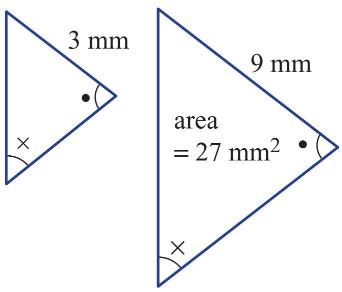


b

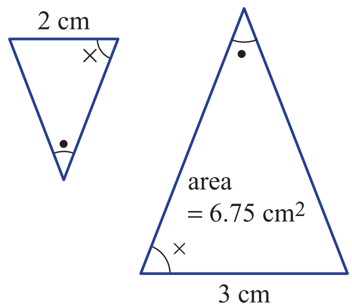


6 The two given triangles are known to be similar. Find the area of the smaller triangle.

a



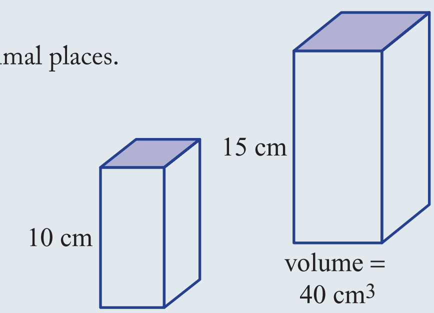
b



You will need to divide the larger area by the area scale factor.

Example 18 Using similarity to find volume

The two given prisms are known to be similar.
Find the volume of the smaller prism correct to two decimal places.



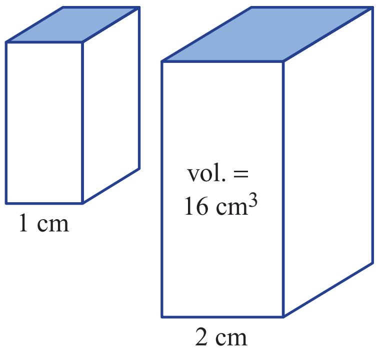
Solution	**Explanation**
Length ratio = $10^1{:}15^1 = 2{:}3$ Volume ratio = $2^3{:}3^3 = 8{:}27$	First, write the length ratio and simplify. Cube each number in the length ratio to get the volume ratio.
Volume scale factor = $\frac{27}{8} = 3.375$	Divide the two numbers in the volume ratio to get the scale factor.
$\therefore$ volume of smaller prism = $40 \div 3.375$ $= 11.85\text{ cm}^3$	Divide the volume of the larger triangle by the scale factor and round as required.

ISBN: 9781107687028

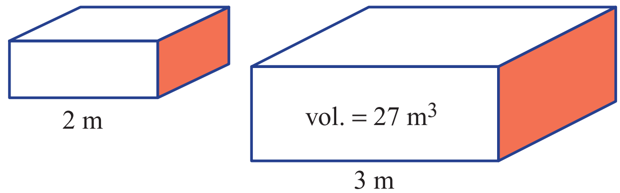
Cambridge University Press

7 The two given prisms are known to be similar. Find the volume of the smaller prism (to two decimal places).

a



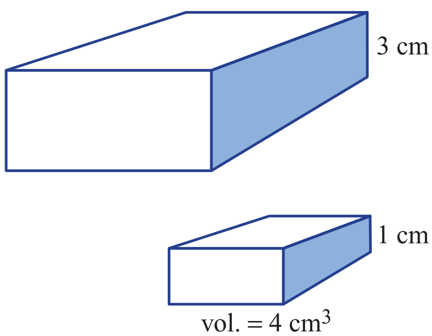
b



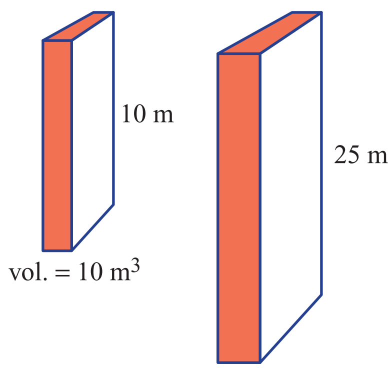
Volume scale factor = $\frac{b^3}{a^3}$ if the length ratio is $a:b$.

8 The two given prisms are known to be similar. Find the volume of the larger prism.

a



b



Problem-solving and Reasoning

9 The given map has a scale factor of 50 000 (ratio 1 : 50 000).

a How far on the ground, in km, is represented by these map distances?

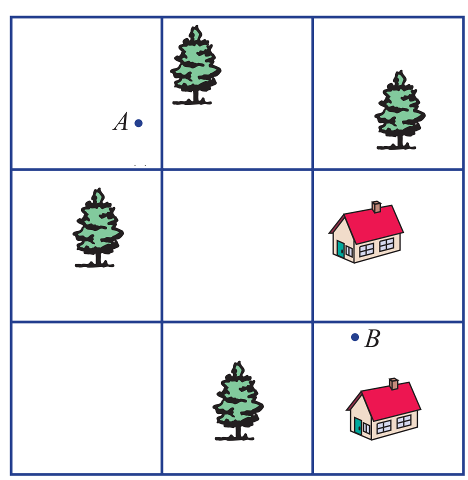
i 2 cm **ii** 6 cm

b How far on the map, in cm, is represented by these ground distances?

i 5 km **ii** 0.5 km

c What is the actual ground distance between the two points A and B? Use your ruler to measure the distance between A and B.

1 m = 100 cm
1 km = 1000 m



ISBN: 9781107687028

Cambridge University Press

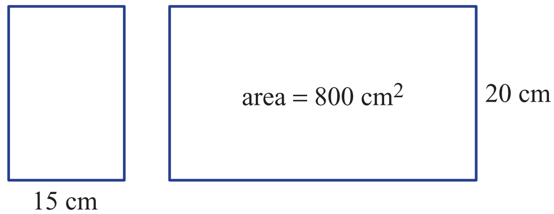
10 Two pieces of paper are similar in shape, as shown.

a What is:

i the length ratio?

ii the area ratio?

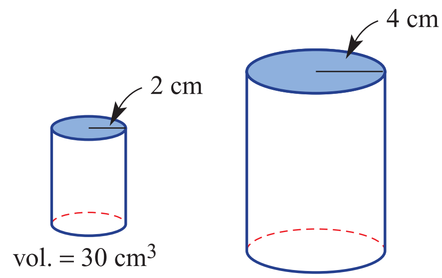
b Find the area of the smaller piece of paper.



11 Two cylinders are similar in shape, as shown.

a Find the volume ratio.

b Find the volume of the larger cylinder.



12 Two rectangular prisms are known to be similar.

a Find the following ratios.

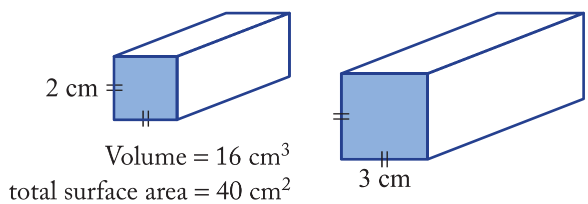
i Length

ii Area

iii Volume

b Find the total surface area of the larger prism.

c Find the volume of the larger prism.



Skyscraper model

13 A scale model of a skyscraper is 1 m tall and the volume is 2 m^3. The actual height of the skyscraper is 300 m tall.

a Find the volume ratio between the model and actual skyscraper.

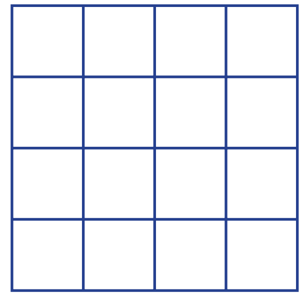
b Find the volume of the actual skyscraper.

c If the area of a window on the model is 1 cm^2, find the area of the actual window in m^2.

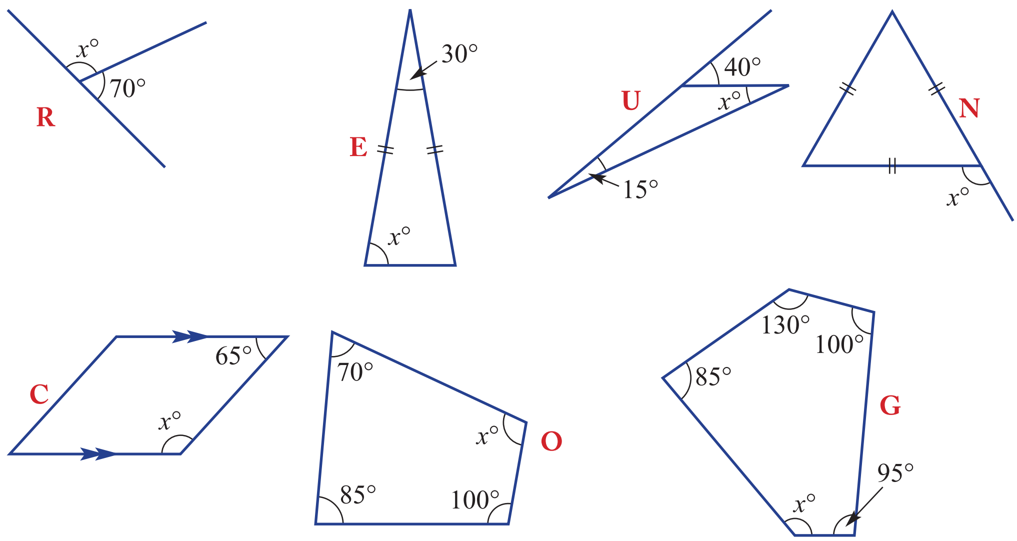
$1\text{ m}^2 = 100 \times 100$
$= 10000\text{ cm}^2$

ISBN: 9781107687028

1 How many squares can you see in this diagram?

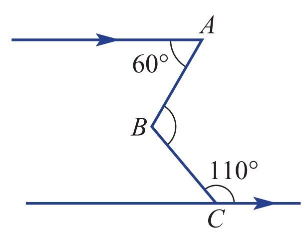


2 'I think of this when I look in the mirror'. Find the value of x in each diagram, then match the letters beside the diagrams to the answers below.



115	105	120	130	110	25	75	120	115	75

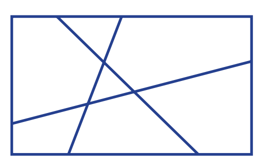
3 What is the size of the acute angle $\angle ABC$ in this diagram?



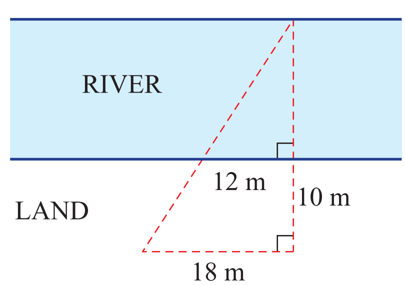
4 This rectangle is subdivided by three straight lines.

a How many regions are formed?

b What is the maximum number of regions formed if 4 lines are used instead of 3?



5 Find the distance across the river.

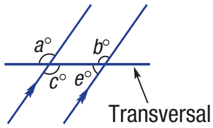


ISBN: 9781107687028 Cambridge University Press

Chapter summary

Geometry

Parallel lines



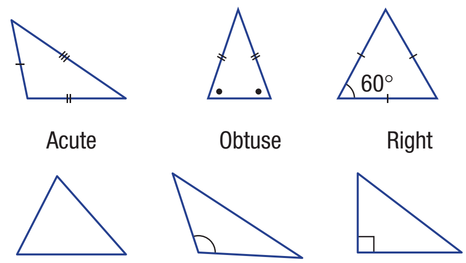
$a = b$ corresponding
$b = c$ alternate
$c + e = 180$ cointerior
$a = c$ vertically opposite

Triangles

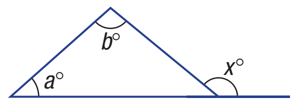
Angle sum = 180°

Scalene, Isosceles, Equilateral

Acute, Obtuse, Right



Exterior angles
$x = a + b$



Congruent triangles

Equal in size and shape

Tests: SSS, SAS, AAS, RHS

A congruence statement:
$\triangle ABC \equiv \triangle DEF$

Quadrilaterals

Angle sum = 360°

Special types include:
- parallelogram
- square
- rectangle
- rhombus
- kite
- trapezium

Similar triangles

Equal in shape and sides are in the same ratio

Tests: SSS, SAS, AAA, RHS

A similarity statement:
$\triangle ABC \mathrel{|||} \triangle DEF$

Applications of similarity

If $\triangle ABC \mathrel{|||} \triangle DEF$ then:
- Scale factor $= \frac{DE}{AB}$ or $\frac{DF}{AC}$ or $\frac{EF}{BC}$
- Length ratio a: b, Scale factor $= \frac{b}{a}$
- Area ratio a^2: b^2, Scale factor $= \frac{b^2}{a^2}$
- Volume ratio a^3: b^3, Scale factor $= \frac{b^3}{a^3}$

Polygons

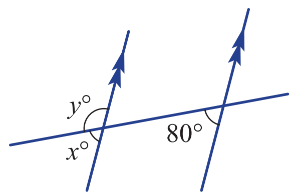
Angle sum $S = 180° \times (n - 2)$

Regular polygons have:
- equal side lengths
- equal angles.

ISBN: 9781107687028 Cambridge University Press

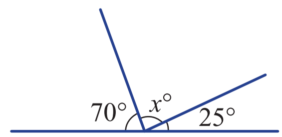
Multiple-choice questions

1 The values of x and y in this diagram are:



A 100, 100 **B** 80, 100 **C** 80, 80
D 60, 120 **E** 80, 60

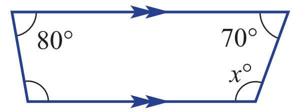
2 The unknown value x in this diagram is:



A 85 **B** 105 **C** 75
D 80 **E** 90

3 A triangle has one angle of 60° and another angle of 70°. The third angle is:
A 60° **B** 30° **C** 40°
D 50° **E** 70°

4 The value of x in this quadrilateral is:



A 130 **B** 90 **C** 100
D 120 **E** 110

5 The sum of the internal angles of a hexagon is:
A 180° **B** 900° **C** 360° **D** 540° **E** 720°

6 Which abbreviated reason is not relevant for proving congruent triangles?
A AAS **B** RHS **C** SSS **D** AAA **E** SAS

7 Two similar triangles have a length ratio of 2 : 3. If one side on the smaller triangle is 5 cm, the length of the corresponding side on the larger triangle is:
A 3 cm **B** 7.5 cm **C** 9 cm **D** 8 cm **E** 6 cm

8 A stick of length 2 metres and a tree of unknown height stand vertically in the sun. The shadow lengths cast by each are 1.5 m and 30 m respectively. The height of the tree is:
A 40 m **B** 30 m **C** 15 m **D** 20 m **E** 60 m

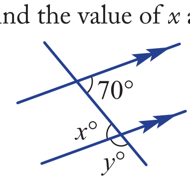
9 Two similar triangles have a length ratio of 1 : 3 and the area of the large triangle is 27 cm^2. The area of the smaller triangle is:
A 12 cm^2 **B** 1 cm^2 **C** 3 cm^2 **D** 9 cm^2 **E** 27 cm^2

10 Two similar prisms have a length ratio of 2 : 3. The volume ratio is:
A 4 : 9 **B** 8 : 27 **C** 2 : 27 **D** 2 : 9 **E** 4 : 27

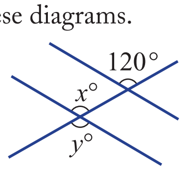
Short-answer questions

1 Find the value of x and y in these diagrams.

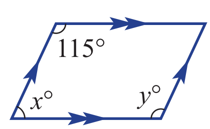
a



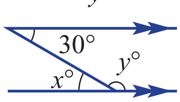
b



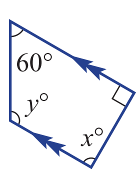
c



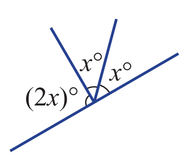
d



e

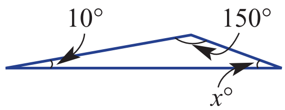


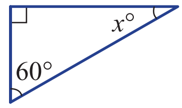
f

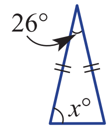


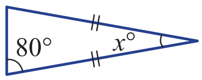
ISBN: 9781107687028

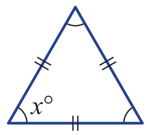
2 Find the value of x in these triangles.

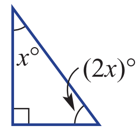
a


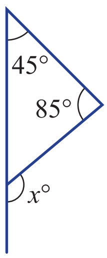
b


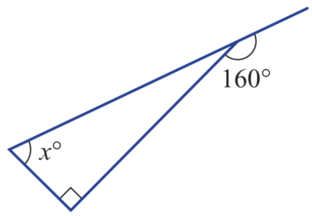
c


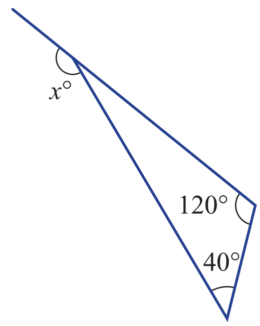
d


e


f


g


h


i


3 List all the quadrilaterals that have:

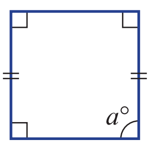
a two pairs of parallel lines

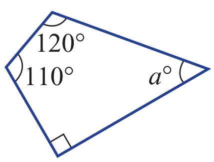
b opposite angles equal

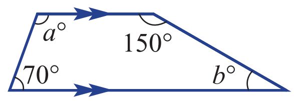
c one pair of equal angles

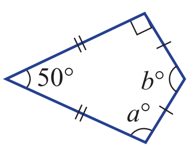
d diagonals intersecting at right angles

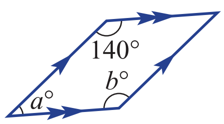
4 Find the values of the pronumerals in these quadrilaterals.

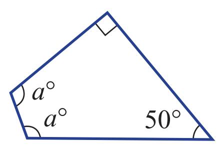
a


b


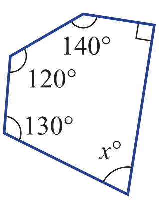
c


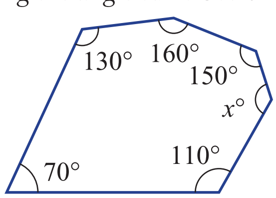
d


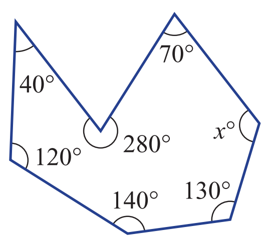
e


f


5 Find the value of x by first finding the angle sum. Use $S = 180° \times (n - 2)$.

a


b


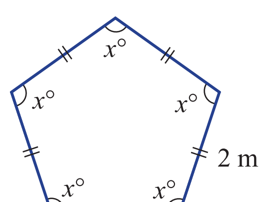
c


6 Shown here is an example of a regular pentagon ($n = 5$) with side length 2 m.

a Find the perimeter of the pentagon.

b Find the total internal angle sum (S).

c Find the size of each internal angle ($x°$).

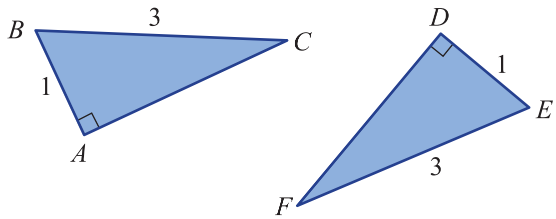


ISBN: 9781107687028

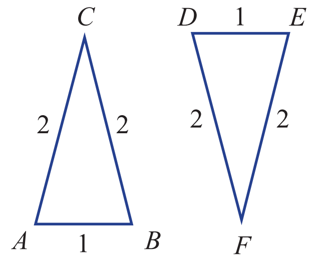
Cambridge University Press

7 Give reasons why the following pairs of triangles are congruent.

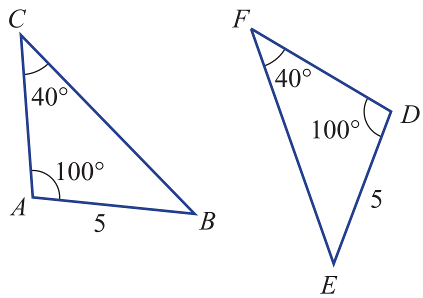
a



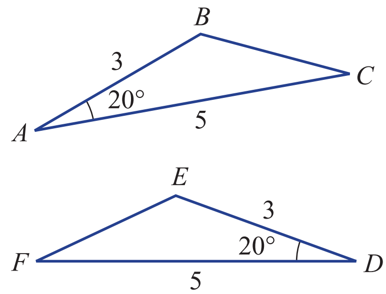
b



c

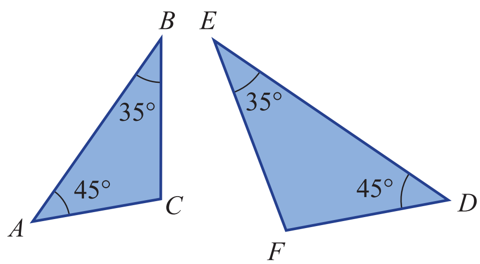


d

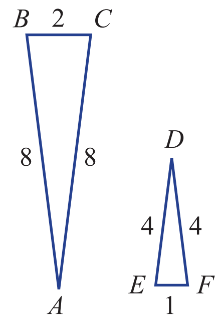


8 Decide whether the given pairs of triangles are similar and give your reasons.

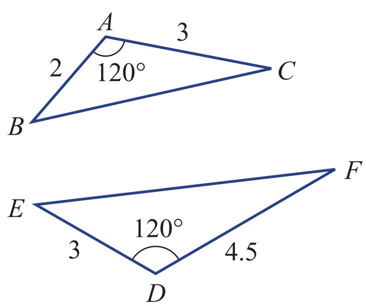
a



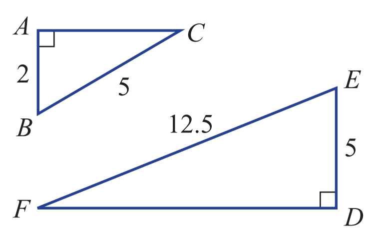
b



c



d



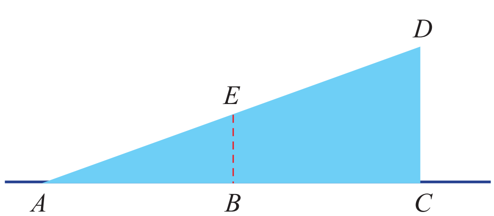
9 A skate board ramp is supported by two vertical struts BE (2 m) and CD (5 m).

a Name two triangles that are similar using the letters A, B, C, D and E.

b Give a reason why the triangles are similar.

c Find the scale factor from the smallest to the largest triangle.

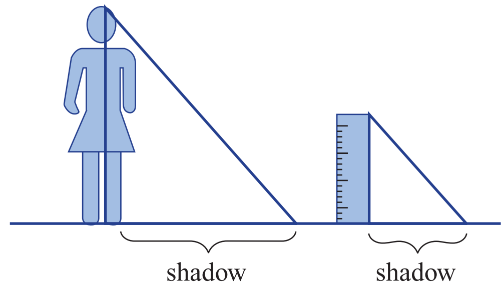
d If the length AB is 3 m find the horizontal length of the ramp AC.



ISBN: 9781107687028

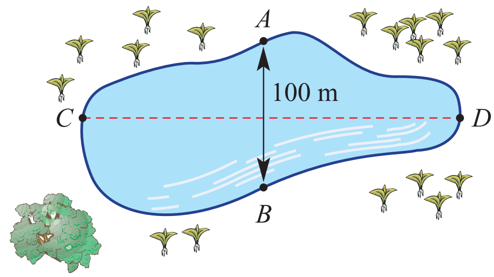
Cambridge University Press

10 The shadow of Mei standing in the sun is 1.5 m long, while the shadow of a 30 cm ruler is 24 cm.



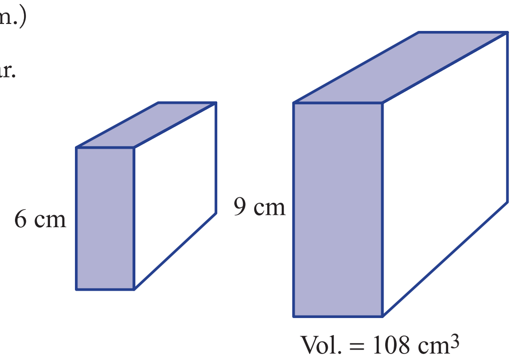
- **a** Give a reason why the two created triangles are similar.
- **b** Find the scale factor between the two triangles.
- **c** How tall is Mei?

11 The given diagram is a simple map of a swamp in bushland.



- **a** Use a ruler to measure the distance across the swamp (AB). (Answer in cm.)
- **b** Find the scale factor between the map and ground distance.
- **c** Use a ruler to find the map distance across the swamp (CD). (Answer in cm.)
- **d** Use your scale factor to find the real distance across the lake (CD). (Answer in m.)

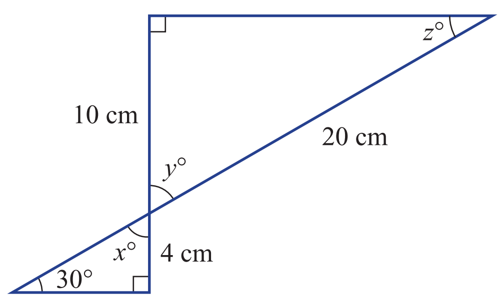
12 Two rectangular prisms are known to be similar.



- **a** Find:
  - **i** the length ratio
  - **ii** the area ratio
  - **iii** the volume ratio
- **b** Find the volume of the smaller prism.

Extended-response questions

1 A company logo contains two triangles, as shown.



- **a** Write down the value of x, y and z.
- **b** Explain why the two triangles are similar.
- **c** Write down the scale factor for length.
- **d** Find the length of the longest side of the smaller triangle.
- **e** Write down the area ratio of the two triangles.
- **f** Write down the area scale factor of the two triangles.

2 A toy model of a car is 8 cm long and the actual car is 5 m long.

- **a** Write down the length ratio of the toy car to the actual car.
- **b** If the toy car is 4.5 cm wide, what is the width of the actual car?
- **c** What is the surface area ratio?
- **d** If the actual car needed 5 litres of paint, what amount of paint would be needed for the toy car?

ISBN: 9781107687028

Australian Curriculum
Number and Algebra
Linear and non-linear relationships

chapter

8 Equations

What you will learn

Installing solar panels

Algebraic equations are used to analyse and solve a vast number of real life problems. Solving simultaneous equations involves solving two equations, each with two variables. This procedure is very useful as it finds the point at which two varying quantities become equal.

For example, a family thinking of installing solar panels for electricity could use simultaneous equations to calculate how many months it will take until the cost of solar power becomes equal to the cost of electrical power over that time. After this time the solar energy is a cheaper source of power.

ISBN: 9781107687028

Pre-test

1 If $a = 6$ and $b = -3$, evaluate the following.

a $a + b$ **b** $a - b$ **c** ab
d a^2 **e** b^2 **f** $3(a + 2b)$

2 If $m = 4$, $n = 7$ and $p = -2$, evaluate the following.

a $m + n + p$ **b** $4m + p$ **c** $p(4 - n)$
d $3m + 2n$ **e** $\dfrac{8m}{p}$ **f** $2m^2$

3 Simplify the following.

a $a + 2a$ **b** $4m - m$ **c** $6p + 2p$ **d** $7m - 7m$
e $2m - 7m$ **f** $8x + y - x$ **g** $8p + 4p - 3p$ **h** $7m - 4m + 3m$

4 Simplify the following.

a $5x \times 3$ **b** $4p \times 4$ **c** $8x \times 4y$
d $6a \times (-5)$ **e** $a \times b$ **f** $6x \div 6$
g $m \div m$ **h** $6a \div 3$ **i** $\dfrac{15a}{5a}$

5 Complete the following.

a $x + 5 - \square = x$ **b** $w - 3 + \square = w$ **c** $p - 5 + \square = p$
d $z + 1 - \square = z$ **e** $w \times 4 \div \square = w$ **f** $a \div 2 \times \square = a$
g $m - 3 + \square = m$ **h** $2m \div \square = m$ **i** $\dfrac{x}{4} \times \square = x$
j $\dfrac{m}{3} \times \square = m$ **k** $6a \div \square = a$ **l** $10x \div \square = x$

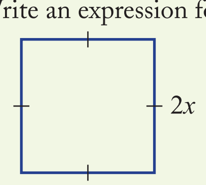
6 Simplify the following.

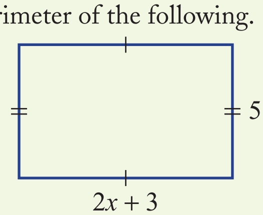
a $a + 6 - 6$ **b** $w + 9 - 9$ **c** $3w \div 3$
d $5z \div 5$ **e** $8n + 3 - 3$ **f** $6x \div 6$
g $2p - 3 + 3$ **h** $\dfrac{x}{2} + 1 - 1$ **i** $\dfrac{x}{7} \times 7$

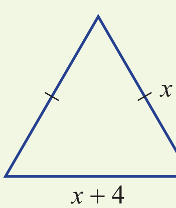
7 Write an expression for each of the following.

a the sum of x and 3 **b** six more than n
c double w **d** half of x
e six more than double x **f** seven less than x
g three more than x is then doubled **h** one more than triple x

8 Write an expression for the perimeter of the following.

a


b


c


9 Choose the equations from the following.

a $x + 3$ **b** $3x - 6 = 9$ **c** $x^2 - 8$
d $2x$ **e** $3a = 12$ **f** $x^2 = 100$
g $1 = x - 3$ **h** $m - m$ **i** $2p = 0$

ISBN: 9781107687028 Cambridge University Press

8.1 Solving linear equations

A cricket batsman will put on socks, then cricket shoes and finally pads in that order. When the game is over, these items are removed in reverse order: first the pads, then the shoes and finally the socks. Nobody takes their socks off before their shoes. A similar reversal occurs when solving equations.

We can undo the operations around x by doing the opposite operation in the reverse order to how they have been applied to x. To keep each equation balanced, we always apply the same operation to both sides of an equation.

For example:

Applying operations to $x = 7$

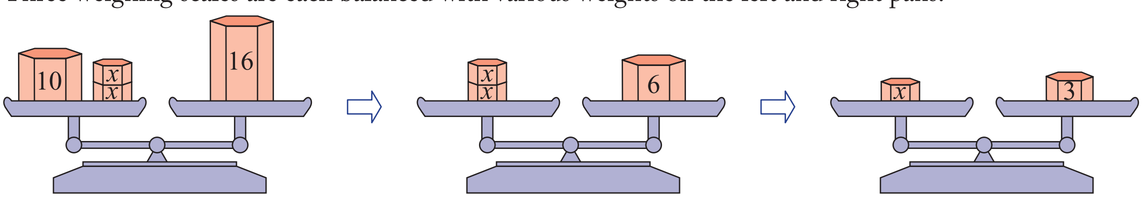
$$x = 7 \xrightarrow{\times 2} 2x = 14 \xrightarrow{+12} 2x + 12 = 26$$

Undoing the operations around x

$$2x + 12 = 26 \xrightarrow{-12} 2x = 14 \xrightarrow{\div 2} x = 7$$

Let's start: Keeping it balanced

Three weighing scales are each balanced with various weights on the left and right pans.



Balanced scales 1 Balanced scales 2 Balanced scales 3

- What weight has been removed from each side of scales 1 to get to scales 2?
- What has been done to both the left and right sides of scales 2 to get to scales 3?
- What equations are represented in each of the balanced scales shown above?
- What methods can you recall for solving equations?

Key ideas

- An **equation** has an equals sign. The equation will only be true for certain value(s) of the pronumeral(s) that make the left-hand side equal to the right-hand side.
  For example: $\frac{5x}{6} = -2$, $3p + 2t = 6$ are equations; $6x - 13$ is not an equation.
- A **linear equation** contains a variable (e.g. x) to the power of 1 and no other powers.
  For example: $3x - 5 = 7$, $4(m - 3) = m + 6$ are linear equations; $x^2 = 49$ is not linear.

Equation A mathematical statement that states that two expressions have the same value

Linear equation An equation whose pronumerals are always to the power of 1 and do not multiply or divide each other

ISBN: 9781107687028

Cambridge University Press

- To **solve** an equation, undo the operations built around x by doing the opposite operation in the reverse order.
  – Always perform the same operation to both sides of an equation so it remains balanced.
  For example:
  For $5x + 2 = 17$, we observe operations that have been applied to x:

$$x \xrightarrow{\times 5} 5x \xrightarrow{+2} 5x + 2$$

  So we solve the equation by 'undoing' them in reverse order on both sides of the equation:

$$5x + 2 \xrightarrow{-2} 5x \xrightarrow{\div 5} x \quad \text{and} \quad 17 \xrightarrow{-2} 15 \xrightarrow{\div 5} 3$$

  This gives the solution:

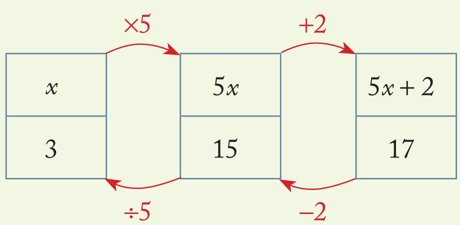
$$\begin{array}{ccc} & 5x + 2 = 17 & \\ -2 & & -2 \\ & 5x = 15 & \\ \div 5 & & \div 5 \\ & x = 3 & \end{array}$$

  – Alternatively, a solution need not show the operations applied to each side. These can be done mentally. For example:

$$\begin{aligned} 5x + 2 &= 17 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

Solve To find the value of an unknown quantity

- A flow chart can be used to solve equations. First, the equation is built up following the order of operations applied to x and then the solution for x is found by undoing these operations in the reverse order.
  For example, here is a flow chart solution to $5x + 2 = 17$.



Solution $x = 3$

  – Backtracking is the process of undoing the operations applied to x.
- To verify an answer means to check that the solution is correct by substituting the answer to see if it makes the equation true.
  e.g. Verify that $x = 3$ is a solution to $5x + 2 = 17$: $5 \times 3 + 2 = 17$

ISBN: 9781107687028

Cambridge University Press

Exercise 8A

Understanding

1 Decide whether $x = 2$ is a solution to these equations.

a $x + 3 = 5$ **b** $2x = 7$ **c** $x - 1 = 4$
d $2x - 1 = 10$ **e** $3x + 2 = 8$ **f** $2 - x = 0$

Substitute $x = 2$ to see whether LHS = RHS.

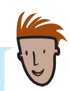
Example 1 Solving one-step equations involving addition

Solve $x + 7 = 12$.

Solution	Explanation
$x + 7 = 12$	Write the equation. The opposite of $+7$ is -7.
$x = 12 - 7$	Subtract 7 from both sides.
$x = 5$	Simplify.
Verify: $5 + 7 = 12$	Check that your answer is correct.

2 Solve the following.

a $t + 5 = 8$ **b** $m + 4 = 10$ **c** $8 + x = 14$
d $m + 7 = 0$ **e** $x + 3 = 11$ **f** $x + 6 = 2$
g $m + 8 = 40$ **h** $a + 1 = -5$ **i** $16 = m + 1$



$8 + x = 14$ is the same as $x + 8 = 14$.
$16 = m + 1$ is the same as $m + 1 = 16$.

Example 2 Solving one-step equations involving subtraction

Solve $x - 9 = 3$.

Solution	Explanation
$x - 9 = 3$	Write the equation. The opposite of -9 is $+9$.
$x = 3 + 9$	Add 9 to both sides.
$x = 12$	Simplify.
Verify: $12 - 9 = 3$	Check that your answer is correct.

3 Find the value of x.

a $x - 3 = 3$ **b** $x - 7 = 2$ **c** $x - 8 = 9$
d $x - 3 = 0$ **e** $x - 2 = -8$ **f** $x - 5 = 7$
g $x - 12 = 24$ **h** $x - 50 = 70$ **i** $x - 1 = 100$

ISBN: 9781107687028

Cambridge University Press

Example 3 Solving one-step equations involving multiplication

Solve $3x = 12$.

Solution	**Explanation**
$3x = 12$	Write the equation. The opposite of $\times 3$ is $\div 3$.
$x = \frac{12}{3}$	Divide both sides by 3.
$x = 4$	Simplify.
Verify: $3 \times 4 = 12$	Check that your answer is correct.

4 Solve the following.

a $8p = 24$ **b** $5c = 30$ **c** $27 = 3d$
d $2m = 16$ **e** $5z = 125$ **f** $9w = 81$
g $15p = 15$ **h** $6m = -42$ **i** $-10 = 20p$

27 = 3*d* is the same as 3*d* = 27.

Example 4 Solving one-step equations involving division

Solve $\frac{x}{4} = 20$.

Solution	**Explanation**
$\frac{x}{4} = 20$	Write the equation. The opposite of $\div 4$ is $\times 4$.
$x = 20 \times 4$	Multiply both sides by 4.
$x = 80$	Simplify.
Verify: $\frac{80}{4} = 20$	Check that your answer is correct.

5 Solve each of the following equations.

a $\frac{x}{5} = 10$ **b** $\frac{m}{3} = 7$ **c** $\frac{a}{6} = -2$

d $\frac{z}{7} = 0$ **e** $\frac{x}{8} = -1$ **f** $\frac{w}{9} = -3$

g $\frac{r}{7} = 8$ **h** $\frac{w}{3} = \frac{1}{2}$ **i** $\frac{m}{2} = \frac{1}{4}$

$3 \quad \frac{1}{2} = \frac{3}{1} \quad \frac{1}{2} = \frac{3}{2}$

Fluency

6 Solve the following equations.

a $x + 9 = 12$ **b** $x + 3 = 12$ **c** $x + 15 = 4$
d $x - 7 = 3$ **e** $x - 2 = 12$ **f** $x - 5 = 5$
g $3x = 9$ **h** $4x = 16$ **i** $2x = 100$
j $\frac{x}{5} = 4$ **k** $\frac{x}{3} = 7$ **l** $\frac{x}{7} = 1$

Carry out the 'opposite' operation to solve for *x*.

ISBN: 9781107687028

Cambridge University Press

Example 5 Solving two-step equations

Solve $4x + 5 = 17$.

Solution	Explanation
$4x + 5 = 17$	Write the equation.
$4x = 12$	Subtract 5 from both sides first.
$x = \frac{12}{4}$	Divide both sides by 4.
$x = 3$	Simplify.
Verify: $4(3) + 5 = 17$	Check your answer.

7 Solve the following equations.

a $2x + 5 = 7$ **b** $3x + 2 = 11$ **c** $4x - 3 = 9$
d $6x + 13 = 1$ **e** $8x + 16 = 8$ **f** $10x + 92 = 2$
g $3x - 4 = 8$ **h** $2x - 7 = 9$ **i** $5x - 4 = 36$
j $2x - 6 = -10$ **k** $7x - 3 = -24$ **l** $6x - 3 = 27$

When you have a number of steps to solve the equation, always go first to the number farthest from the pronumeral.

Example 6 Solving two-step equations involving simple fractions

Solve $\frac{x}{5} - 3 = 4$.

Solution	Explanation
$\frac{x}{5} - 3 = 4$	Write the equation. The number 'farthest' from x is the -3, so first add 3 to both sides.
$\frac{x}{5} = 7$	Multiply both sides by 5.
$x = 35$	Write the answer.
Verify: $\frac{35}{5} - 3 = 7 - 3 = 4$	Check that your answer is correct.

8 Solve the following equations.

a $\frac{x}{3} + 2 = 5$ **b** $\frac{x}{6} + 3 = 3$ **c** $\frac{x}{7} + 4 = 12$
d $\frac{x}{4} - 3 = 2$ **e** $\frac{x}{5} - 4 = 3$ **f** $\frac{x}{10} - 2 = 7$
g $\frac{x}{8} - 2 = -6$ **h** $\frac{x}{4} - 3 = -8$ **i** $\frac{x}{2} - 1 = -10$

When solving equations, the order of steps is important. For $\frac{x}{3} - 5$, undo the -5 first, then undo the $\div 3$.

ISBN: 9781107687028

8A

Example 7 Solving more two-step equations

Solve $\frac{x+4}{2}=6$.

Solution	**Explanation**
$\frac{x+4}{2}=6$	Write the equation.
$x+4=12$	$\frac{x+4}{2}=\frac{(x+4)}{2}$, so the number farthest from x is the 2, and we first multiply both sides by 2.
$x=8$	Subtract 4 from both sides.
Verify: $\frac{8+4}{2}=\frac{12}{2}=6$	Check that your answer is correct.

9 Solve the following.

a $\frac{m+1}{2}=3$ **b** $\frac{a-1}{3}=2$ **c** $\frac{x+5}{2}=3$

d $\frac{x+5}{3}=2$ **e** $\frac{n-4}{5}=1$ **f** $\frac{m-6}{2}=8$

g $\frac{w+4}{3}=-1$ **h** $\frac{m+3}{5}=2$ **i** $\frac{w-6}{3}=7$

j $\frac{a+7}{4}=2$ **k** $\frac{a-3}{8}=-5$ **l** $\frac{m+5}{8}=0$

When solving equations, the order of steps is important. For $\frac{x+7}{3}$, undo the $\div 3$ first, then undo the +7. Never cancel a number joined by + or – to an x. In $\frac{x+8}{4}$, you cannot cancel the 4 into the 8.

Problem-solving and Reasoning

Example 8 Writing equations from a word problem

For each of the following statements, write an equation and solve for the pronumeral.

a If 7 is subtracted from x, the result is 12.
b If x is divided by 5 then 6 is added, the result is 10.
c If 4 is subtracted from x and that answer is divided by 2, the result is 9.

Solution	**Explanation**
a $x-7=12$ $x=19$	Subtract 7 from x means to start with x then subtract 7. 'The result' means '='.
b $\frac{x}{5}+6=10$ $\frac{x}{5}=4$ $x=20$	Divide x by 5, then add 6 and make it equal to 10. Solve the equation by subtracting 6 from both sides first.
c $\frac{x-4}{2}=9$ $x-4=18$ $x=22$	Subtracting 4 from x gives $x-4$, and divide that answer by 2. Undo $\div 2$ by multiplying both sides by 2, then add 4 to both sides.

ISBN: 9781107687028 Cambridge University Press

10 For each of the following statements, write an equation and solve for the pronumeral.

- **a** If 4 is added to x, the result is 6.
- **b** If x is added to 12, the result is 8.
- **c** If 5 is subtracted from x, the result is 5.
- **d** If x is divided by 3 then 2 is added, the result is 8.
- **e** Twice the value of x is added to 3 and the result is 9.
- **f** $(x - 3)$ is divided by 5 and the result is 6.
- **g** 3 times x plus 4 is equal to 16.

> 5 subtracted from x is $x - 5$.

11 Write an equation and solve it for each of these questions.

- **a** The perimeter of a square is 52 cm. Determine the length of the side.
- **b** The perimeter of an isosceles triangle is 42 mm. If the equal sides are both 10 mm, determine the length of the other side.

> Draw a diagram and choose a pronumeral to represent the unknown side, then write an equation and solve it.

12 Convert the following into equations, then solve them for the unknown number.

- **a** n is multiplied by 2, then 5 is added. The result is 11.
- **b** Four times a certain number is added to 9 and the result is 29. What is the number?
- **c** Half of a number less two equals 12. What is the number?
- **d** A number plus 6 has been divided by 4. The result is 12. What is the number?
- **e** 12 is subtracted from a certain number and the result is divided by 5. If the answer is 14, what is the number?

> Choose a pronumeral to represent the unknown number, then write an equation using the pronumeral.
> $\frac{1}{2}$ of x can be written as $\frac{x}{2}$.

13 Write an equation and solve it for each of these questions.

- **a** The sum of two consecutive numbers is 23. What are the numbers?
- **b** If I add five to twice a number the result is 17. What is the number?
- **c** Three less than five times a number is 12. What is the number?
- **d** One person is 19 years older than another person. Their age sum is 69. What are their ages?
- **e** Andrew threw the shotput 3 m more than twice the distance Barry threw it. If Andrew threw the shotput 19 m, how far did Barry throw it?

> Consecutive numbers are one number apart; e.g. 3, 4, 5, 6 etc. The next consecutive number after x is $x + 1$.

Modelling with equations

14 A service technician charges $40 up front and $60 for each hour she works.

- **a** Find a linear equation for the total charge, $\$C$, of any job for h hours worked.
- **b** What will a 4-hour job cost?
- **c** If the technician works on a job for 3 days and averages 6 hours per day, what will be the overall cost?
- **d** If a customer is charged $400, how long did the job take?

15 A petrol tank holds 71 litres. It originally contained 5 litres. If a petrol pump fills it at 6 litres per minute, find:

- **a** a linear equation for the amount of fuel (V litres) in the tank at time t minutes
- **b** how long it will take to fill the tank to 23 litres
- **c** how long it will take to fill the tank

ISBN: 9781107687028

8.2 Solving more difficult linear equations

More complex linear equations may have variables on both sides of the equation and/or brackets. Examples are $6x = 2x - 8$ or $5(x + 3) = 12x + 4$.

Brackets can be removed by expanding. Equations with variables on both sides can be solved by collecting variables to one side using addition or subtraction of a term.

More complex linear equations of this type are used when constructing buildings and in science and engineering.

Let's start: Steps in the wrong order

The steps to solve $8(x+2) = 2(3x+12)$ are listed here in the incorrect order.

$$8(x+2) = 2(3x+12)$$
$$x = 4$$
$$2x + 16 = 24$$
$$8x + 16 = 6x + 24$$
$$2x = 8$$

- Arrange them in the correct order, working from the question to the solution.
- By considering all the steps in the correct order, write what has happened in each step.

Key ideas

- When solving complicated linear equations:
  1. First, **expand** any brackets.
     In this example, multiply the 3 into the first bracket and the -2 into the second bracket.
     $$3(2x - 1) - 2(x - 2) = 22$$
     $$6x - 6 - 2x + 4 = 22$$
  2. Collect any **like terms** on the LHS and any like terms on the RHS.
     Collecting like terms on the left side of this example:
     $5x - 3x = 2x$, and $-4 - 9 = -13$.
     $$5x - 4 - 3x - 9 = x - 5 + 2x + 10$$
     $$2x - 13 = 3x + 5$$

Expand Remove grouping symbols (such as brackets)

Like terms Terms with the same pronumerals and same powers

ISBN: 9781107687028

Cambridge University Press

3 If an equation has variables on both sides, collect to one side by adding or subtracting one of the terms.
For example, when solving the equation $12x + 7 = 5x + 19$, first subtract $5x$ from both sides: $LHS: 12x - 5x = 7x,\ RHS: 5x - 5x = 0$.

$$12x + 7 = 5x + 19$$
$$7x + 7 = 19$$

4 Start to perform the opposite operation to both sides of the equation.
5 Repeat step 4 until the equation is solved.
6 Verify that the answer is correct.

- To solve a word problem using algebra:
  – Read the problem and find out what the question is asking for.
  – Define a variable and write a statement such as: 'Let x be the number of …' The variable is often what you have been asked to find in the question.
  – Write an equation using your defined variable.
  – Solve the equation.
  – Answer the question in words.

Exercise 8B

Understanding

1 Expand brackets and collect like terms in each of these expressions.

a $3(x - 1)$
b $5(x + 3)$
c $-2(x + 2)$
d $-3(x - 4)$
e $-4(2x - 1)$
f $2(x + 5) + 3(x + 1)$
g $5(x + 4) + 2(x + 3)$
h $6(x + 2) + 3(x - 1)$
i $2(x - 8) - 3(x + 1)$
j $5(x - 4) - 11(x - 3)$

The number in front of the bracket needs to be multiplied to both terms inside the bracket.
$-5(2x - 3)$
$= -10x + 15$

2 For each of these equations, first collect the term from the right side to the left side and then solve for x.

a $5x = 2x + 12$
b $2x = x - 4$
c $8x = 3x + 25$
d $7x = -x + 8$
e $5x - 4 = 2x + 11$
f $8x + 2 = 3x + 47$
g $7x - 5 = -2x + 13$
h $2x + 3 = -3x + 38$

Add or subtract to remove the term containing x on the RHS.

Fluency

Example 9 Solving equations with brackets

Solve $4(x - 1) = 16$.

Solution	Explanation
$4(x - 1) = 16$	
$4x - 4 = 16$	Expand the brackets: $4 \times x$ and $4 \times (-1)$.
$4x = 20$	Add 4 to both sides.
$x = 5$	Divide both sides by 4.

ISBN: 9781107687028

3 Solve each of the following equations by first expanding the brackets.

a $3(x + 2) = 9$ **b** $4(x - 1) = 16$ **c** $3(x + 5) = 12$
d $4(a - 2) = 12$ **e** $5(a + 1) = 10$ **f** $2(x - 10) = 10$
g $6(m - 3) = 6$ **h** $3(d + 4) = 15$ **i** $7(a - 8) = 14$
j $10(a + 2) = 20$ **k** $5(3 + x) = 15$ **l** $2(a - 3) = 0$

Example 10 Solving equations with two sets of brackets

Solve $3(2x + 4) + 2(3x - 2) = 20$.

Solution	Explanation
$3(2x + 4) + 2(3x - 2) = 20$	Use the distributive law to expand each set of
$6x + 12 + 6x - 4 = 20$	brackets.
$12x + 8 = 20$	Collect like terms on the LHS.
$12x = 12$	Subtract 8 from both sides.
$x = 1$	Divide both sides by 12.

4 Solve the following equations.

a $3(2x + 3) + 2(x + 4) = 25$ **b** $2(2x + 3) + 4(3x + 1) = 42$
c $2(2x + 3) + 3(4x - 1) = 51$ **d** $3(2x - 2) + 5(x + 4) = 36$
e $4(2x - 3) + 2(x - 4) = 10$ **f** $2(3x - 1) + 3(2x - 3) = 13$
g $2(x - 4) + 3(x - 1) = -21$ **h** $4(2x - 1) + 2(2x - 3) = -22$

Expand each pair of brackets and collect like terms before solving.

5 Solve the following equations.

a $3(2x + 4) - 4(x + 2) = 6$ **b** $2(5x + 4) - 3(2x + 1) = 9$
c $2(3x - 2) - 3(x + 1) = -7$ **d** $2(x + 1) - 3(x - 2) = 8$
e $8(x - 1) - 2(3x - 2) = 2$ **f** $5(2x - 3) - 2(3x - 1) = -9$
g $5(2x + 1) - 3(x - 3) = 35$ **h** $4(2x - 3) - 2(3x - 1) = -14$

$-4(x + 2) = -4x - 8$
$-4(x - 2) = -4x + 8$

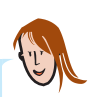
Example 11 Solving equations with variables on both sides

Solve $7x + 9 = 2x - 11$ for x.

Solution	Explanation
$7x + 9 = 2x - 11$	Subtract $2x$ from both sides.
$5x + 9 = -11$	Subtract 9 from both sides.
$5x = -20$	Divide both sides by 5.
$x = -4$	

6 Find the value of x in the following.

a $7x = 2x + 10$ **b** $10x = 9x + 12$ **c** $8x = 4x - 12$
d $6x = 2x + 80$ **e** $2x = 12 - x$ **f** $2x = 8 + x$
g $3x + 4 = x + 12$ **h** $4x + 9 = x - 3$ **i** $2x - 9 = x - 10$
j $6x - 10 = 12 + 4x$ **k** $9x = 10 - x$ **l** $1 - x = x + 3$



Remove the term containing x on the RHS. For parts **e**, **k** and **l**, you will need to add x to both sides.

ISBN: 9781107687028

Example 12 Solving equations with fractions

Solve $\dfrac{2x+3}{4}=2$ for x.

Solution	Explanation
$\dfrac{2x+3}{4}=2$	Multiply both sides by 4.
$2x+3=8$	Subtract 3 from both sides.
$2x=5$	Divide both sides by 2.
$x=2.5$	

7 Solve the following equations.

a $\dfrac{x+2}{3}=5$ **b** $\dfrac{x+4}{2}=5$ **c** $\dfrac{x-1}{3}=4$

d $\dfrac{x-5}{3}=2$ **e** $\dfrac{2x+1}{7}=3$ **f** $\dfrac{2x+2}{3}=4$

g $\dfrac{5x-3}{3}=9$ **h** $\dfrac{3x-6}{2}=9$ **i** $\dfrac{5x-2}{4}=-3$

First multiply by the denominator.

Example 13 Solving equations with more difficult fractions

Solve $\dfrac{3x}{2}-4=2$ for x.

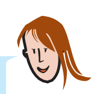
Solution	Explanation
$\dfrac{3x}{2}-4=2$	Add 4 to both sides.
$\dfrac{3x}{2}=6$	Multiply both sides by 2.
$3x=12$	Divide both sides by 3.
$x=4$	

8 Solve the following equations.

a $\dfrac{x}{3}+1=5$ **b** $\dfrac{x}{3}+1=7$ **c** $\dfrac{x}{4}-5=10$

d $\dfrac{3x}{4}-2=5$ **e** $\dfrac{2x}{5}-3=-1$ **f** $\dfrac{3x}{2}-5=-14$

First add or subtract from both sides.

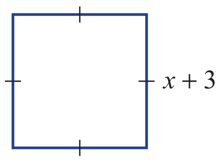


ISBN: 9781107687028

Problem-solving and Reasoning

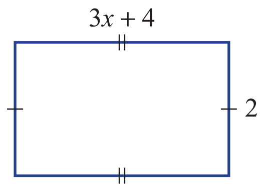
9 For each of these questions, write an equation and solve it for x.

a



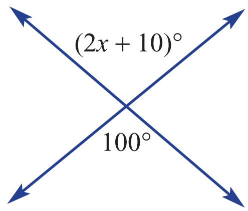
Perimeter = 52 cm

b



Perimeter = 22 cm

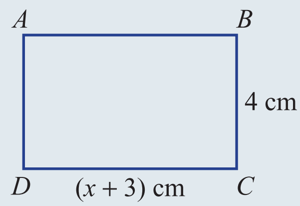
c



Vertically opposite angles are equal.

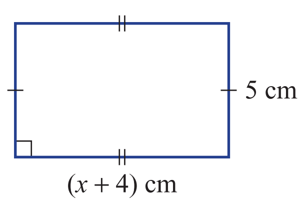
Example 14 Solving a word problem

Find the value of x if the area of rectangle $ABCD$ shown is 24 cm^2.

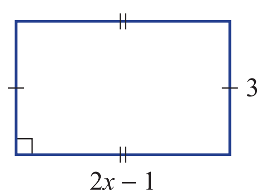


Solution	Explanation
$A = l \times w$	Write an equation for area.
$24 = (x + 3) \times 4$	Substitute: $l = (x + 3)$, $w = 4$, $A = 24$
$24 = 4x + 12$	Expand the brackets: $(x + 3) \times 4 = 4(x + 3)$
$12 = 4x$	Subtract 12 from both sides.
$3 = x$	Divide both sides by 4.
$x = 3$	Write the answer.

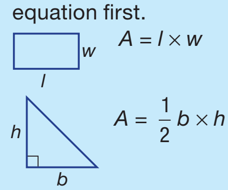
10 a Find the value of x if the area is 35 cm^2.



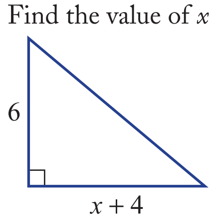
b Find the value of x if the area is 27 cm^2.



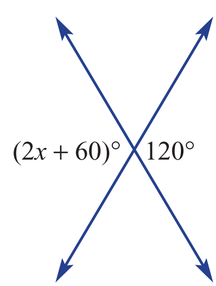
Form the area equation first.



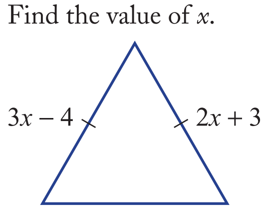
c Find the value of x if the area is 42 cm^2.



d Vertically opposite angles are equal. Find the value of x.



e Find the value of x.



ISBN: 9781107687028

Cambridge University Press

11 Using x for the unknown number, write down an equation then solve it to find the number.

- **a** The product of 5 and 1 more than a number is 40.
- **b** The product of 5 and 6 less than a number is -15.
- **c** When 6 less than 3 lots of a number is doubled, the result is 18.
- **d** When 8 more than 2 lots of a number is tripled, the result is 36.
- **e** 10 more than 4 lots of a number is equivalent to 6 lots of the number.
- **f** 5 more than 4 times a number is equivalent to 1 less than 5 times the number.
- **g** 6 more than a doubled number is equivalent to 5 less than 3 lots of the number.

- 'Product' means 'to multiply'
- The product of 5 and 1 more than a number means $5(x + 1)$
- '6 less than 3 lots of a number is doubled' will require brackets
- 'Tripled' means three times a number
- 'Equivalent' means 'equal to'

12 Sally and Steve are planning to hire a car for their wedding day. 'Vehicles For You' have the following deal: \$850 hiring fee plus a charge of \$156 per hour.

- **a** Write an equation for the cost (\$$C$) of hiring a car for h hours.
- **b** If Sally and Steve have budgeted for the car to cost a maximum of \$2000, find the maximum number of full hours they can hire the car.
- **c** If the car picks up the bride at 1.15 p.m., at what time must the event finish if the cost is to remain within budget?

More than one fraction

Consider:

$$\frac{4x-2}{3} = \frac{3x-1}{2}$$

$$\frac{{}^{2}\cancel{6}(4x-2)}{\cancel{3}_{1}} = \frac{{}^{3}\cancel{6}(3x-1)}{\cancel{2}_{1}}$$ (Multiply both sides by 6 to get rid of the fractions.)

(Simplify.)

$2(4x - 2) = 3(3x - 1)$ (Expand both sides.)

$8x - 4 = 9x - 3$ (Subtract $8x$ from both sides.)

$-4 = x - 3$ (Add 3 to both sides.)

$-1 = x$

$\therefore x = -1$

13 Solve the following equations.

a $\frac{x+2}{3} = \frac{x+1}{2}$	**b** $\frac{x+1}{2} = \frac{x}{3}$	**c** $\frac{3x+4}{4} = \frac{x+6}{3}$
d $\frac{5x+2}{3} = \frac{3x+4}{2}$	**e** $\frac{2x+1}{7} = \frac{3x-5}{4}$	**f** $\frac{5x-1}{3} = \frac{x-4}{4}$

Using technology 8.2: Solving linear equations

This activity is available on the companion website as a printable PDF.

ISBN: 9781107687028

8.3 Using formulas

A formula (or rule) is an equation that relates two or more variables. You can find the value of one of the variables if you are given the value of all other unknowns.

You will already be familiar with many formulas. For example:

$C = 2\pi r$ is the formula for finding the circumference, C, of a circle given its radius, r.

$F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.

A metal worker building pipes applies circle, area and volume formulas.

$s = \frac{d}{t}$ is the formula for finding the speed, s, given the distance, d, and time, t.

C, F and s are said to be the subjects of the formulas given above.

Let's start: Jumbled solution

Problem: The formula for the area of a trapezium is $A = \frac{h}{2}(a + b)$.

Xavier was asked to find a given that $A = 126$, $b = 10$ and $h = 14$, and to write the explanation beside each step of the solution.

Xavier's solution and explanation are below. His solution is correct, but he has jumbled up the steps in the explanation. Copy Xavier's solution and write the correct instruction(s) beside each step.

Solution	Explanation
$A = \frac{h}{2}(a + b)$	Write the answer.
$126 = \frac{14}{2}(a + 10)$	Subtract 70 from both sides. Divide both sides by 7.
$126 = 7(a + 10)$	Substitute the given values.
$126 = 7a + 70$	Copy the formula.
$56 = 7a$	Simplify $\frac{14}{2}$.
$a = 8$	Expand the brackets.

ISBN: 9781107687028 Cambridge University Press

Key ideas

- The **subject** of a **formula** is a variable that usually sits on its own on the left-hand side. For example, the C in $C = 2\pi r$ is the subject of the formula.
- A variable in a formula can be evaluated by substituting numbers for all other variables.
- A formula can be rearranged to make another variable the subject.
  $C = 2\pi r$ can be rearranged to give $r = \dfrac{C}{2\pi}$
- Note that $\sqrt{a^2} = a$ if $a \geq 0$ and $\sqrt{a^2 + b^2} \neq a + b$.

Subject The pronumeral or variable that is alone on one side of an equation

Formula A general rule for finding the value of one quantity given the values of others

Exercise 8C

Understanding

1 State the letter that is the subject in these formulas.

a $I = \dfrac{Prt}{100}$ **b** $F = ma$ **c** $V = \dfrac{4}{3}\pi r^3$

d $A = \pi r^2$ **e** $c = \sqrt{a^2 + b^2}$ **f** $P = 2x + 2y$

The subject of a formula is the letter on its own, on the left-hand side.

2 Substitute the given values into each of the following formulas to find the value of each subject. Round the answer to one decimal place where appropriate.

a $m = \dfrac{F}{a}$, when $F = 180$ and $a = 3$

b $A = lw$, when $l = 6$ and $w = 8$

c $A = \dfrac{1}{2}(a + b)h$, when $a = 6$, $b = 12$ and $h = 4$

d $v^2 = u^2 + 2as$, when $u = 6$ and $a = 12$ and $s = 7$

e $m = \sqrt{\dfrac{x}{y}}$, when $x = 56$ and $y = 4$

Copy each formula, substitute the given values then calculate the answer.

Fluency

Example 15 Substituting values and solving equations

If $v = u + at$, find t when $v = 16$, $u = 4$ and $a = 3$.

Solution	Explanation
$v = u + at$	Substitute each value into the formula. $v = 16$, $u = 4$, $a = 3$
$16 = 4 + 3t$	An equation now exists. Solve this equation for t.
$12 = 3t$	Subtract 4 from both sides.
$4 = t$	Divide both sides by 3.
$t = 4$	Answer with the pronumeral on the left-hand side.

ISBN: 9781107687028

3 If $v = u + at$, find t when:

- **a** $v = 16$, $u = 8$ and $a = 2$
- **b** $v = 20$, $u = 8$ and $a = 3$
- **c** $v = 100$, $u = 10$ and $a = 9$
- **d** $v = 84$, $u = 4$ and $a = 10$

4 If $P = 2(l + 2b)$, find b if:

- **a** $P = 60$ and $l = 10$
- **b** $P = 48$ and $l = 6$
- **c** $P = 96$ and $l = 14$
- **d** $P = 12.4$ and $l = 3.6$

First copy the formula. Then substitute the given values. Then solve the equation.

5 If $V = lbh$, find h when:

- **a** $V = 100$, $l = 5$ and $b = 4$
- **b** $V = 144$, $l = 3$ and $b = 4$
- **c** $V = 108$, $l = 3$ and $b = 12$
- **d** $V = 280$, $l = 8$ and $b = 5$

6 If $A = \frac{1}{2}bh$, find b when:

- **a** $A = 90$ and $h = 12$
- **b** $A = 72$ and $h = 9$
- **c** $A = 108$ and $h = 18$
- **d** $A = 96$ and $h = 6$

In $90 = \frac{1}{2} \times b \times 12$,

$$\frac{1}{2} \times b \times 12 = \frac{1}{2} \times 12 \times b = 6b$$

So, $90 = 6b$.
Solve for b.

7 If $A = \frac{h}{2}(a + b)$, find h when:

- **a** $A = 20$, $a = 4$ and $b = 1$
- **b** $A = 48$, $a = 5$ and $b = 7$
- **c** $A = 108$, $a = 9$ and $b = 9$
- **d** $A = 196$, $a = 9$ and $b = 5$

When solving the equation first undo the division by 2 by multiplying both sides by 2.

8 $E = mc^2$. Find m if:

- **a** $E = 100$ and $c = 5$
- **b** $E = 4000$ and $c = 10$
- **c** $E = 72$ and $c = 1$
- **d** $E = 144$ and $c = 6$

Square the c value before solving the equation.

9 If $V = \pi r^2 h$, find h (to one decimal place) when:

- **a** $V = 160$ and $r = 3$
- **b** $V = 400$ and $r = 5$
- **c** $V = 1460$ and $r = 9$
- **d** $V = 314$ and $r = 2.5$

For $160 = 9\pi h$, divide both sides by 9π to find h:
$h = \frac{160}{9\pi}$
Then evaluate on a calculator.

Problem-solving and Reasoning

10 The formula $F = \frac{9C}{5} + 32$ is used to convert temperature from degrees Celsius (°C) (which is used in Australia) to degrees Fahrenheit (°F) (which is used in the USA).

- **a** When it is 30°C in Sydney, what is the temperature in Fahrenheit?
- **b** How many degrees Celsius is 30° Fahrenheit? Answer to one decimal place.
- **c** Water boils at 100°C. What is this temperature in degrees Fahrenheit?
- **d** What is 0°F in degrees Celsius? Answer to one decimal place.

When finding C you will have an equation to solve.

ISBN: 9781107687028 Cambridge University Press

11 The cost (in dollars) of a taxi is $C = 3 + 1.45d$, where d is the distance travelled in kilometres.

a What is the cost of a 20 km trip?

b How many kilometres can be travelled for \$90?

12 $I = \dfrac{Prt}{100}$ calculates interest on an investment.

Find:

a P when $I = 60$, $r = 8$ and $t = 1$

b t when $I = 125$, $r = 5$ and $P = 800$

c r when $I = 337.50$, $P = 1500$ and $t = 3$

13 The number of tablets a nurse must give a patient is found by using the formula:

$$\text{tablets} = \frac{\text{strength required}}{\text{tablet strength}}$$

a 750 milligrams of a drug must be given to a patient. How many 500 milligram tablets should the nurse give the patient?

b If the nurse gives 2.5 of these tablets to another patient, how much of the drug did the patient take?

14 A drip is a way of pumping a liquid drug into a patient's blood. The flow rate of the pump in millilitres per hour is calculated using the formula: rate $= \dfrac{\text{volume (mL)}}{\text{time (h)}}$

a A patient needs 300 mL of the drug over 4 hours. Calculate the rate in mL/h which needs to be delivered by the pump.

b A patient received 100 mL of the drug at a rate of 300 mL/h. How long was the pump running?

Calculation challenges

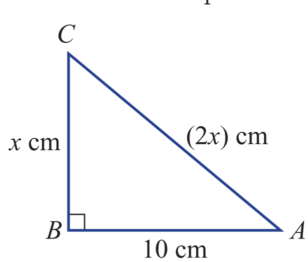
15 A tax agent charges \$680 for an 8-hour day. The agent uses the formula

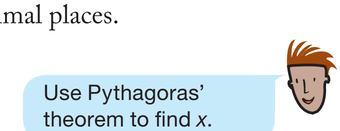
$F = \dfrac{680x}{8}$ to calculate a fee to a client in dollars.

a What does the x represent?

b If the fee charged to a client came to \$637.50, how many hours, to one decimal place, did the agent spend working on the client's behalf?

16 Find the area and perimeter of triangle ABC shown. Round to two decimal places.





17 Fatima is 10 years older than Yuri. In 3 years' time, she will be twice as old as Yuri. How old are they now?

ISBN: 9781107687028

Cambridge University Press

8.4 Inequalities

There are many situations where a solution to the problem is best described using one of the symbols <, ≤, > or ≥. For example, a medical company will publish the lowest and highest amounts for a safe dose of a particular medicine; e.g. 20 mg/day ≤ dose ≤ 55 mg/day.


The safe dosage range of a drug can be expressed as an inequality.

An inequality is a mathematical statement that uses a less than (<), a less than or equal to (≤), a greater than (>) or a greater than or equal to (≥) symbol. Inequalities may result in an infinite number of solutions. These can be illustrated using a number line.

You can solve inequalities in a similar way to solving equations.

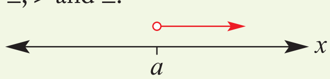
Let's start: What does it mean for *x*?

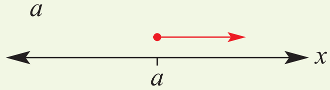
The following inequalities provide some information about the number x.

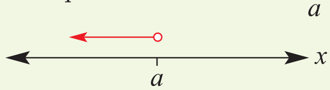
a $x < 6$ **b** $x \geq 4$ **c** $-5 \geq x$ **d** $-2 < x$

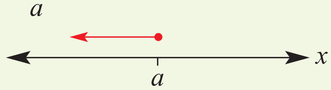
- Can you describe the possible values for x that satisfy each inequality?
- Test some values to check.
- How would you write the solution for x? Illustrate each on a number line.

Key ideas

- The four **inequality signs** are <, ≤, > and ≥.
  - $x > a$ means x is greater than a.
  

  - $x \geq a$ means x is greater than or equal to a.
  

  - $x < a$ means x is less than a.
  

  - $x \leq a$ means x is less than or equal to a.
  

- On the number line a closed circle (•) indicates that the number is included. An open circle (◦) indicates that the number is not included.
- Solving **linear inequalities** follows the same rules as solving linear equations, except:
  - we reverse an inequality sign if we multiply or divide by a negative number.
    For example, if $-5 < -3$ then $5 > 3$ and if $-2x < 4$ then $x > -2$.
  - we reverse the inequality sign if the sides are switched.
    For example, if $2 \geq x$, then $x \leq 2$.

Inequality sign A symbol that compares the size of two or more expressions or numbers by pointing to the smaller one

Linear inequality An inequality that involves a linear function

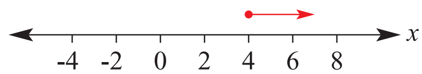
ISBN: 9781107687028

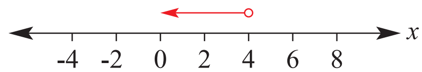
Cambridge University Press

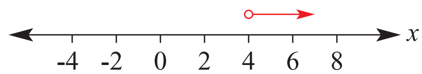
Exercise 8D

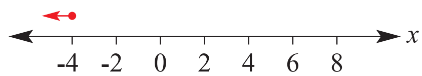
Understanding

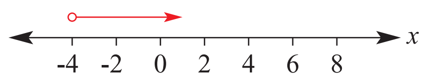
1 Match each inequality given with the correct number line.

a $x > 4$ **1** 

b $x < 4$ **2** 

c $x \geq 4$ **3** 

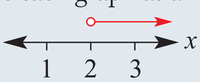
d $x > -4$ **4** 

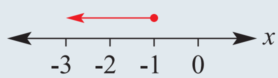
e $x \leq -4$ **5** 

Look back at the Key ideas. The direction of the arrowhead is the same as the direction of the inequality sign.

Example 16 Writing inequalities from number lines

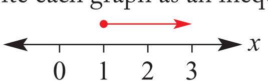
Write each graph as an inequality.

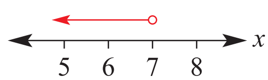
a 

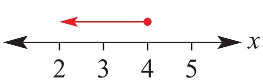
b 

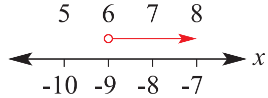
Solution	**Explanation**
a $x > 2$	An open circle means 2 is not included.
b $x \leq -1$	A closed circle means -1 is included.

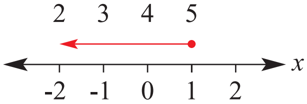
2 Write each graph as an inequality.

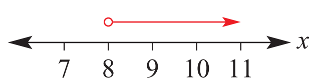
a 

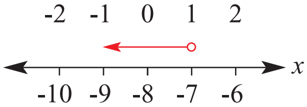
b 

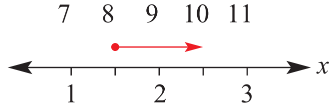
c 

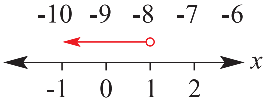
d 

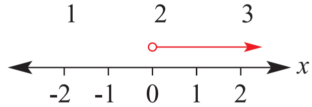
e 

f 

g 

h 

i 

j 

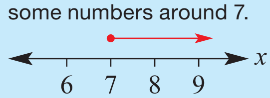
The inequality sign will have the same direction as the arrow.

Fluency

3 Show each of the following on separate number lines.

a $x \geq 7$ **b** $x > 1$ **c** $x < 1$
d $x \leq 1$ **e** $x \geq -1$ **f** $a \geq 0$
g $p \geq -2$ **h** $a > -15$ **i** $b < 5$

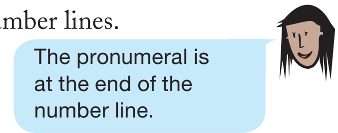
For $x \geq 7$, draw a number line showing some numbers around 7.

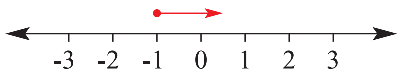


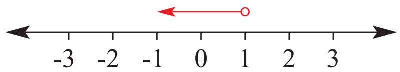
Use a closed circle (•) for ≥ and ≤. Use an open circle (◦) for > and <.

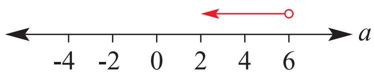
ISBN: 9781107687028

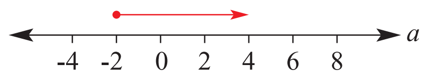
4 Write an inequality to describe what is shown on each of the following number lines.

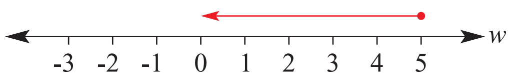


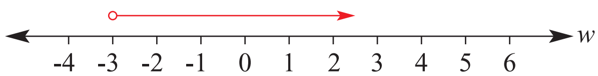
a 

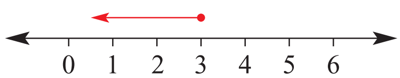
b 

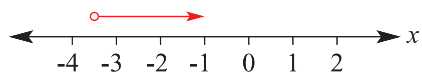
c 

d 

e 

f 

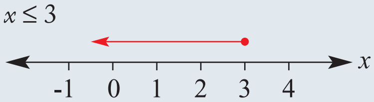
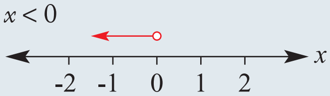
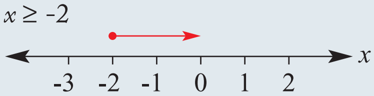
g 

h 

Example 17 Writing and graphing inequalities

Write each of the following as an inequality and then show each solution on a number line.

a x is less than or equal to 3 **b** x is greater than 1
c x is less than 0 **d** x is greater than or equal to -2

Solution	Explanation
a $x \leq 3$ 	Less than or equal to, ≤, closed circle
b $x > 1$ 	Greater than, >, open circle.
c $x < 0$ 	Less than, <, open circle
d $x \geq -2$ 	Greater than or equal to, ≥, closed circle

5 Write each of the following as an inequality and then show each solution on a number line.

a x is less than or equal to 6
b x is greater than 4
c x is less than 2
d x is greater than or equal to 5

ISBN: 9781107687028 Cambridge University Press

6 Write each of the following as an inequality using the pronumeral n.

a The number of people who visit the Sydney Opera House each year is more than 100 000.
b The number of lollies in a bag should be at least 50.
c A factory worker must pack more than three boxes a minute.
d More than 100 penguins take part in the nightly parade on Philip Island.
e The weight of a suitcase is 30 kg or less.

> 'At least 50' means '50 or more'

7 Write each of the following statements as an inequality and determine which of the numbers below make each inequality true.

$-6, -2, -\frac{1}{2}, 0, 2, 5, 7, 10, 15, 24$

a x is less than zero
b x is greater than 10
c x is greater than or equal to 10
d x is less than or equal to zero
e x is greater than or equal to -1
f x is less than 10

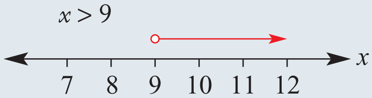
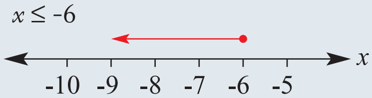
> Write the inequality, then list the given numbers that make it true.

Problem-solving and Reasoning

Example 18 Solving and graphing inequalities

Solve the following and show your solution on a number line.

a $2x - 1 > 17$
b $\frac{x}{3} \le -2$

Solution	Explanation
a $2x - 1 > 17$	Add 1 to both sides.
$2x > 18$	Divide both sides by 2.
$x > 9$	
	> uses an open circle.
b $\frac{x}{3} \le -2$	Multiply both sides by 3.
$x \le -6$	
	≤ uses a closed circle.

8 Solve each of the following inequalities and show your solution on a number line.

a $2x > 10$
b $x + 2 < 7$
c $3x > 15$
d $\frac{x}{2} \ge 8$
e $x - 3 > 4$
f $x - 3 < 4$
g $p + 8 \le 0$
h $3a > 0$
i $x - 7 < 0$
j $2x \le 14$
k $5m > -15$
l $d - 3 > 2.4$
m $\frac{x}{7} \le 0.1$
n $\frac{1}{2}x \le 6$
o $5 + x > 9$

> Keep the inequality sign the same when:
> - adding or subtracting a number from both sides
> - multiplying or dividing both sides by a positive number.

ISBN: 9781107687028

Cambridge University Press

8D

9 Solve the following.

a $2 + 4a \le 10$ **b** $5 + 2y > 11$ **c** $3p - 1 > 14$
d $3x - 2 \ge 10$ **e** $3x - 2 < 1$ **f** $5 + 2w \ge 8$
g $5x + 5 < 10$ **h** $5x - 5 \ge 0$ **i** $10p - 2 < 8$

10 Give the solution set for each of the following:

a $\frac{x+2}{4} \le 1$ **b** $\frac{a-3}{2} \le -1$ **c** $\frac{x}{4} - 1 \ge 6$
d $\frac{x}{3} + 7 > 2$ **e** $5 + \frac{x}{2} < 7$ **f** $\frac{x+2}{4} < 8$
g $\frac{2x-7}{3} > 4$ **h** $\frac{2x+1}{5} < 0$ **i** $\frac{3x}{2} + 1 \ge -3$
j $5x - 4 > 2 - x$ **k** $4(2x + 1) \ge 16$ **l** $3x + 7 < x - 2$

For $\frac{x+2}{4} \le 1$, first multiply both sides by 4.
For $\frac{x}{4} - 1 \ge 6$, first add 1 to both sides.

11 For each of the following, write an inequality and solve it to find the possible values of x.

a If a number, x, is multiplied by 3, the result is less than 9.
b If a number, x, is multiplied by 3 and the result divided by 4, it creates an answer less than 6.
c If a number, x, is doubled and then 15 is added, the result is greater than 20.
d Thuong is x years old and Gary is 4 years older. The sum of their ages is less than 24.
e Kaitlyn has x rides on the Ferris wheel at \$4 a ride and spends \$7 on food. The total amount she spends is less than or equal to \$27.

Example 19 Solving inequalities when the variable has a negative coefficient

Solve $4 - x \ge 6$.

Solution	Explanation
$4 - x \ge 6$	Subtract 4 from both sides.
$-x \ge 2$	Divide both sides by -1.
$x \le -2$	When we divide both sides by a *negative* number, the inequality sign is reversed.
Alternate solution:	
$4 - x \ge 6$	Add the x to both sides so that it is positive.
$4 \ge 6 + x$	Subtract 6 from both sides.
$-2 \ge x$	Switch the sides to have the x on the left-hand side.
$x \le -2$	Reverse the inequality sign. Note that the inequality sign still 'points' to the x.

ISBN: 9781107687028 Cambridge University Press

12 Choose an *appropriate strategy* to solve the following.

a $5 - x < 6$ **b** $7 - x \geq 10$
c $-p \leq 7$ **d** $9 - a < -10$
e $-w \geq 6$ **f** $-3 - 2p < 9$
g $5 - 2x < 7$ **h** $-2 - 7a \geq 4$

Remember to reverse the inequality sign if multiplying or dividing by a negative number.
E.g.

$$-x < 7 \xrightarrow{\times(-1)} x > -7$$
$$-2x \geq 20 \xrightarrow{\div(-2)} x \leq -10$$

Investigating inequalities

13 a Let us start with the numbers 4 and 6 and the true relationship 4 < 6. Copy and complete the following table.

4 and 6	4	6	4 < 6	True or false?
				True
Add 3	4 + 3	6 + 3	7 < 9	True
Subtract 3	4 – 3		1 < 3	True
Multiply by 2				
Divide by 2				
Multiply by -2				False (-8 > -12)
Divide by -2	4 ÷ (-2)	6 ÷ (-2)		

b Copy and complete the following.
When solving an inequality, you can add or _________ a number from both sides and the inequality remains true. You can multiply or _________ by a _________ number and the _________ also remains true. However, if you _________ or _________ by a negative _________ the inequality sign must be reversed for the inequality to remain _________.

ISBN: 9781107687028

8.5 Solving simultaneous equations graphically

When we approach an intersection while driving, we near the shared position of two or more roads.

Like two roads, two straight lines in the same plane will always intersect unless they are parallel.

If we try to find the point of intersection, we are said to be solving the equations simultaneously.

Let's start: Which job has better pay?

You start working as a delivery person for the Hasty Tasty Pizza Company. You're paid \$25 per shift and \$4 per pizza delivery.

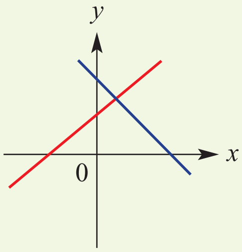
A second Pizza Company, More-2-Munch Pizzas, offers you a job at \$15 per shift and \$5 per pizza delivery.

- How much does each company pay for delivery of 7 pizzas in one shift? How much does each company pay for delivery of 12 pizzas in one shift?
- For each pizza company, draw up a table to show the money you could earn for delivery of up to 15 pizzas delivered in one shift.
- On the same sheet, draw a graph of the information in your tables for each pizza company. Draw the graph for each pizza company on the same set of axes.

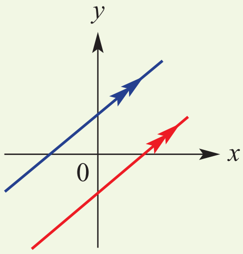
- What does the point of intersection show us?
- Write a sentence describing which job pays better for different numbers of pizzas delivered.
- Write down one advantage of using a graph to compare these two wages.

Key ideas

- At a point of intersection, two lines will have the same **coordinates**.
- If two lines are **parallel** they have the same gradient and there is no point of intersection.



1 point of intersection



0 points of intersection

Coordinates
An ordered pair written in the form (x, y) that states the location of a point on the Cartesian plane

Parallel lines
Lines in the same plane that are the same distance apart and never intersect

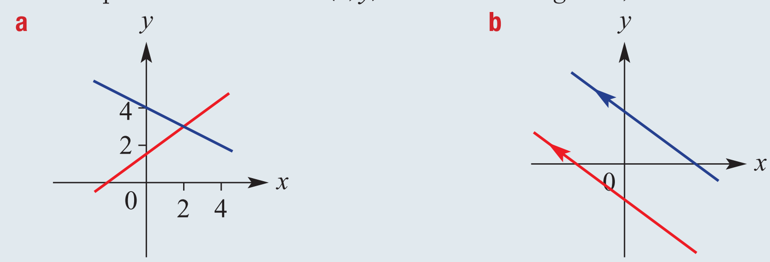
ISBN: 9781107687028 Cambridge University Press

Exercise 8E

Understanding

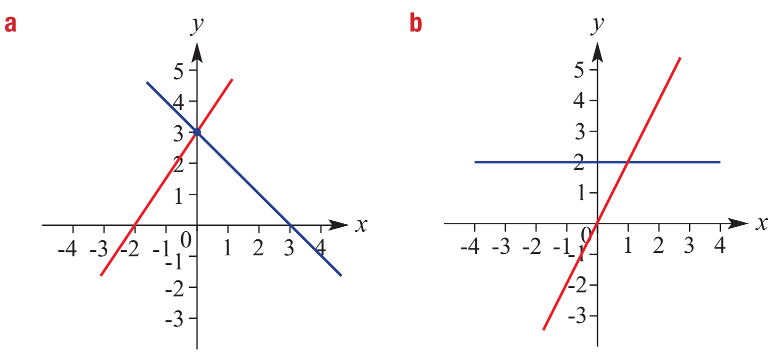
Example 20 Reading the coordinates of the point of intersection of two lines

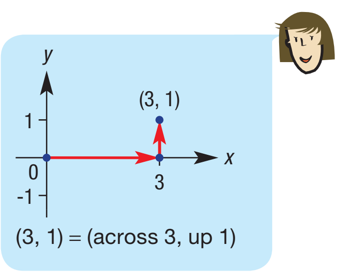
State the point of intersection (x, y) for the following lines, if there is one.

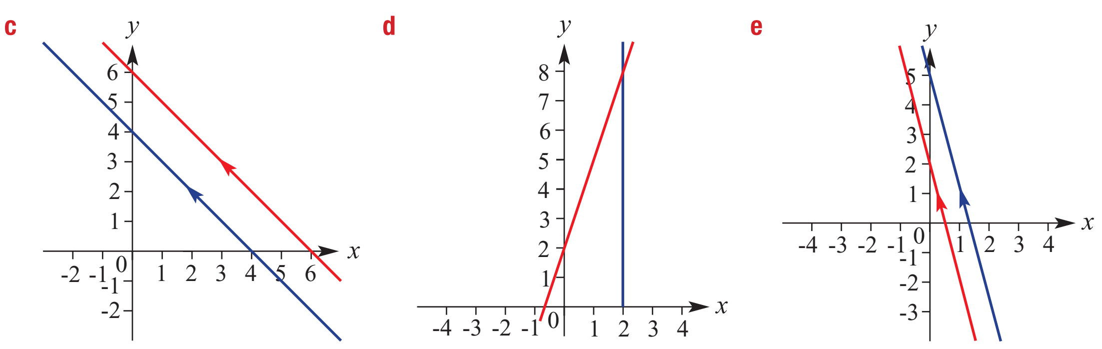


Solution	Explanation
a Point of intersection at (2, 3)	Read the x-coordinate, then the y-coordinate, directly from the graph.
b No point of intersection	Parallel lines have no point of intersection.

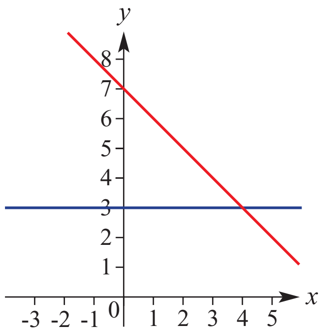
1 State the point of intersection (x, y) for the following lines, if there is one.



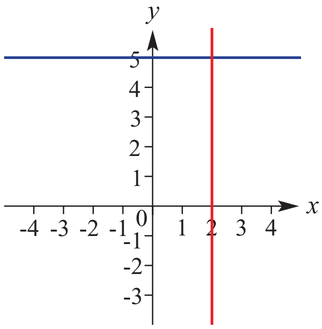




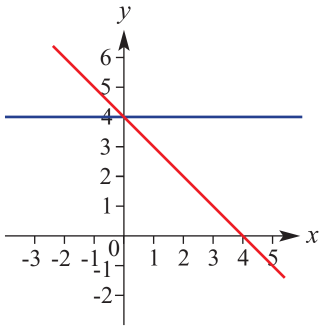
f



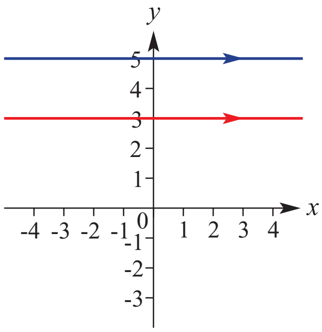
g



h



i



2 Complete the following.

a At the x-intercept, $y =$ ________.

b At the y-intercept, $x =$ ________.

Fluency

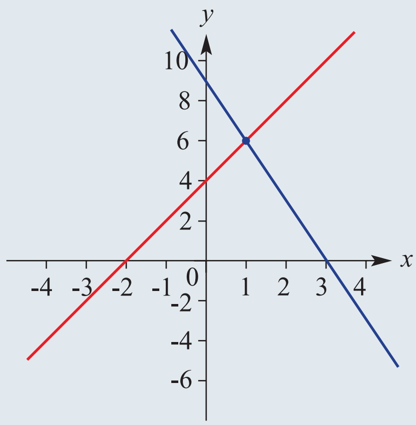
Example 21 Finding the point of intersection by graphing

Find the point of intersection (x, y) of $y = 2x + 4$ and $3x + y = 9$ by sketching accurate graphs on the same axes.

Solution	Explanation
$y = 2x + 4$	First, find the x- and y-intercepts of each graph.
y-intercept at $x = 0$: $y = 2(0) + 4 = 4$	Substitute $x = 0$
y intercept is $(0, 4)$.	State the y-intercept as an ordered pair.
x-intercept at $y = 0$: $0 = 2x + 4$	Substitute $y = 0$
$2x = -4$	Subtract 4 from both sides.
$x = -2$	Divide both sides by 2.
x intercept is $(-2, 0)$.	State the x-intercept as an ordered pair.
$3x + y = 9$.	
y-intercept at $x = 0$: $3(0) + y = 9$	Substitute $x = 0$
$y = 9$	Simplify.
y intercept is $(0, 9)$.	State the y-intercept as an ordered pair.
x-intercept at $y = 0$: $3x + (0) = 9$	Substitute $y = 0$
$3x = 9$	Simplify.
$x = 3$	Divide both sides by 3.
x-intercept is $(3, 0)$.	State the x-intercept as an ordered pair.

ISBN: 9781107687028

Cambridge University Press



Sketch the graphs using the x- and y-intercepts.

The point of intersection is (1, 6).

Read off the intersection point, listing x followed by y.

3 Find the point of intersection (x, y) of each pair of equations by plotting an accurate graph.

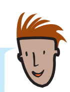
a $y = x + 1$ and $3x + 2y = 12$ **b** $y = 3x + 2$ and $2x + y = 12$

c $y = 2x + 9$ and $3x + 2y = 18$ **d** $y = x + 11$ and $4x + 3y = 12$

y-intercept: $x = 0$
x-intercept: $y = 0$

4 Find the point of intersection of each pair of equations by plotting an accurate graph.

a $y = 3$ and $x = 2$ **b** $y = -2$ and $x = 3$



$y = 3$ cuts the y-axis at 3 and is horizontal.
$x = 2$ cuts the x-axis at 2 and is vertical.

5 Find the point of intersection of each pair of equations by plotting an accurate graph.

a $y = 3x$ and $y = 2x + 3$ **b** $y = -3x$ and $y = 2x - 5$

6 Find the point of intersection of each pair of equations by plotting an accurate graph.

a $y = 2x - 6$ and $y = 3x - 7$ **b** $y = -2x + 3$ and $y = 3x - 2$

Problem-solving and Reasoning

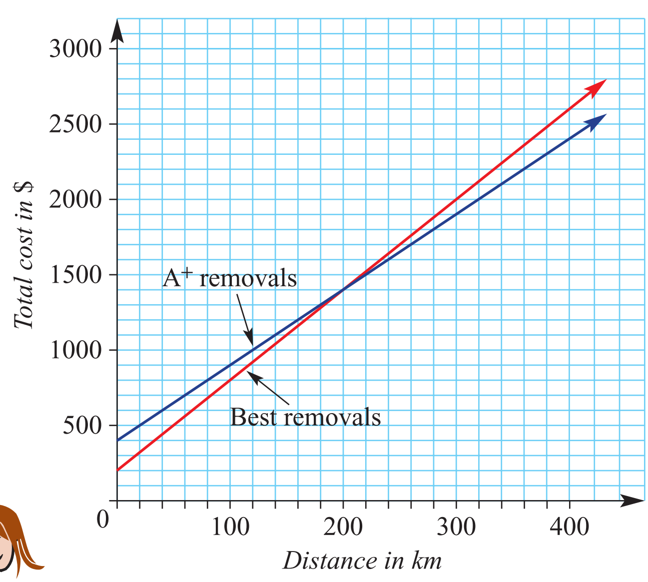
7 This graph represents the cost of hiring two different removalist companies to move a person's belongings for various distances.

a Determine the number of kilometres for which the total cost of the removalists is the same.

b What is the price when the total cost is equal?

c If a person wanted to move 100 km, which company would be cheaper and by how much?

d If a person wanted to move 400 km, which company would be cheaper and by how much?



The cost is the same at the point of intersection.

ISBN: 9781107687028 Cambridge University Press

8 The wage structures for baking companies A and B are given by the following.
Company A: \$20 per hour
Company B: \$45 plus \$15 per hour

a Complete two tables showing the wage for each company for up to 12 hours.
b Draw a graph of the wage for each company (on the vertical axis) versus time in hours (on the horizontal axis). Draw the graphs for both companies on the same set of axes.
c State the number of hours worked for which the earnings are the same for the two companies.
d State the amount earned when the earnings are the same for the two companies.

9 **a** Graph these three lines on the same coordinate axes by plotting the axis intercepts for each: $y = 3$, $y = x + 1$, $y = 1 - x$.
b Write the coordinates of the points of intersection.
c Find the length of each line segment formed between the intersection points.
d What type of triangle is formed by these line segments?

Use Pythagoras' theorem to find the length of a line segment.

10 The value of two cars is depreciating (decreasing) at a constant rate according to the information in this table.

Annual depreciation means how much the car's value goes down by each year.

Car	Initial value	Annual depreciation
Luxury sports coupe	\$70000	\$5000
Family sedan	\$50000	\$3000

a Complete two tables showing the value of each car every second year from zero to 12 years.
b Draw a graph of the value of each car (on the vertical axis) versus time in years (on the horizontal axis). Draw both graphs on the same set of axes.

ISBN: 9781107687028 Cambridge University Press

c From the graph, determine the time taken for the cars to have the same value.
d State the value of the cars when they have the same value.

Multiple intersections

Use a calculator to complete these questions.

11 On the same axis, plot the graphs of $y = 2x$, $y = 2x + 1$, $y = 2x + 2$ and $y = 2x + 3$.
- **a** Are there any points of intersection?
- **b** Suggest a reason for your answer to **a**.
- **c** Plot the graph of $y = 3x + 6$.
- **d** Determine the points of intersection of the graphs already drawn and $y = 3x + 6$.

12 On the same axis, plot $y = x - 1$, $y = 2x - 1$, $y = 3x - 1$ and $y = 4x - 1$.
- **a** Are there any points of intersection?
- **b** Suggest a reason for your answer to **a**.
- **c** Plot the graph of $y = 2x + 1$.
- **d** Determine the points of intersection of the graphs already drawn and $y = 2x + 1$.

Using technology 8.5: Finding intersections

This activity is available on the companion website as a printable PDF.

ISBN: 9781107687028

8.6 Solving simultaneous equations using substitution

Two simultaneous equations can be made when there are two unknown quantities (variables) and two lots of information relating these quantities. The solution gives the values that make both equations true.

In the last section, the solution was found from the point of intersection of two line graphs. In this section you will learn how to find the solution using the algebraic method of *substitution*.

An example of two variables is the cost of a wedding reception and the number of invited guests. Two simultaneous equations could be made from two different catering companies. The solution will be the number of guests that make the costs equal for the two companies. Using equations gives an accurate comparison of two deals.

Let's start: Equations and solutions

Match each set of simultaneous equations with the correct solution.

Simultaneous equations

J

$$a + b = 100$$
$$12a + 5b = 920$$

K

$$x = 4y$$
$$2x + 2y = 1000$$

L

$$3x + 2y = 29$$
$$2x - y = 3$$

M

$$y = 500 + 70x$$
$$y = 700 + 68x$$

N

$$a + b = 50$$
$$a - b = 6$$

O

$$a = b - 5$$
$$2a + b = 35$$

Solutions

a $x = 5$ and $y = 7$

b Elias' age (a) is 10 years and Maria's age (b) is 15 years.

c Nicole's age (a) is 28 years and Julian's age (b) is 22 years.

d The length (x) of a playing field is 400 m and the width (y) is 100 m.

e 60 adults (a) and 40 children (b) attended a rugby match. The tickets cost $12 per adult and $5 per child.

f A wedding reception with 100 guests (x) costs $7500 ($y$).

ISBN: 9781107687028

Key ideas

- The algebraic method of substitution is generally used when at least one of the linear equations has x or y as the subject:

  e.g. $y = 3x + 4$ or $y = -2x + 6$ or $x = 2$
  $3x + y = 2$ $\quad y = -x - 1$ $\quad 2x - y = 5$
- The method involves:
  – substituting one equation into the other
  – solving for the remaining variable
  – substituting to find the value of the second variable.
- When problem-solving with simultaneous linear equations:
  – define/describe two unknowns using pronumerals
  – write down two equations using your pronumerals
  – solve the equations using the method of substitution
  – answer the original question in words.

Exercise 8F

Understanding

1 Write the missing words to complete each statement.

a The x and y values that make two equations both true are called a ______________ solution.

b When two equations have been graphed, the x and y values that make both equations true are the coordinates of the point of ______________.

c If x (or y) is replaced with a number, then we have ______________ that number for x.

d If x (or y) is replaced with an algebraic expression, then we have ______________ that expression for x (or y).

e When we algebraically substitute one equation into another, this is called solving simultaneous equations by the method of ______________.

2 Solve these equations for the unknown variable.

a $5x = 2x - 9$ **b** $3x = 5x + 10$ **c** $3y - 12 = 2y + 1$

d $2(x + 3) + 5x = 20$ **e** $5(2y - 1) - 5y = 30$ **f** $7x + 3(x + 2) = 26$

g $4y - 2(3y - 1) = -12$ **h** $-3y - (3y - 6) = 12$ **i** $3x - 2(2x + 1) = 4$

Expand any brackets first, then collect like terms on one side and solve.

Fluency

Example 22 Using the substitution method to solve simultaneous equations

Determine the point of intersection of $y = 5x$ and $y = 2x + 6$.

Solution	Explanation
$y = 5x$ ....(1)	Label the two equations.
$y = 2x + 6$ ....(2)	
Substitute (1) into (2):	Explain how you are substituting the equations.
$5x = 2x + 6$	Replace y in the second equation with $5x$.
$3x = 6$	Subtract $2x$ from both sides.
$x = 2$	Divide both sides by 3.

ISBN: 9781107687028

8F

Solution	Explanation
Substitute $x = 2$ into (1):	Alternatively, substitute into equation (2).
$y = 5(2)$	Replace x with the number 2.
$y = 10$	Simplify.
The point of intersection is (2, 10).	Write the solution.
Check: $10 = 2(2) + 6$	Substitute your solution into the other equation to check.

3 Determine the point of intersection for the following pairs of lines.

a $y = 5x$
$y = 3x + 4$

b $y = 3x$
$y = 2x - 5$

c $y = 2x$
$y = 4x + 8$

d $y = 4x$
$y = -3x + 7$

e $y = x$
$y = -5x + 12$

f $y = 6x$
$y = -2x + 16$

$y = 5x$
$y = 3x + 4$
$5x = 3x + 4$

Example 23 Solving simultaneous equations with the substitution method

Solve the simultaneous equations $y = x + 3$ and $2x + 3y = 19$ using the substitution method; i.e. find the point of intersection.

Solution	Explanation
$y = x + 3$ ....(1)	Label the two equations.
$2x + 3y = 19$ ....(2)	
Substitute (1) into (2):	Explain how you are substituting the equations.
$2x + 3(x + 3) = 19$	Replace y in the second equation with $(x + 3)$.
$2x + 3x + 9 = 19$	Expand the brackets.
$5x + 9 = 19$	Simplify.
$5x = 10$	Subtract 9 from both sides.
$x = 2$	Divide both sides by 5.
Substitute $x = 2$ into (1):	Alternatively, substitute into equation (2).
$y = 2 + 3$	Replace x with the number 2.
$y = 5$	Simplify.
The point of intersection is (2, 5).	Write the solution.
Check: $2(2) + 3(5) = 19$	Substitute your solution into the other equation.

4 Solve the following pairs of simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = x + 3$ and $2x + 3y = 19$
b $y = x + 2$ and $3x + y = 6$
c $y = x - 1$ and $3x + 2y = 8$
d $y = x - 1$ and $3x + 5y = 27$
e $y = x + 2$ and $2x + 3y = -19$
f $y = x + 5$ and $5x - y = -1$
g $y = x - 3$ and $5x - 2y = 18$
h $y = x - 4$ and $3x - y = 2$

In part **a**, replace y in the second equation with $(x + 3)$.
It is important to use brackets.

$y = x + 3$

$2x + 3(y) = 19$
Remember that

$3(x + 3) = 3x + 9$.

ISBN: 9781107687028

5 Solve the following pairs of simultaneous equations; i.e. find the point of intersection.

a $y = 2$
$y = 2x + 4$

b $y = -1$
$y = 2x - 7$

c $y = 4$
$2x + 3y = 20$

Replace y in the second equation with 2.

$y = 2$

$y = 2x + 4$

6 Determine the point of intersection for the following.

a $x = 2$
$3x + 2y = 14$

b $x = -3$
$y = -2x - 4$

c $x = 7$
$4x - 3y = 31$

Replace x in the second equation with 2.

$x = 2$

$3x + 2y = 14$

Remember that $3x$ means $3 \times x$.

7 Solve the following pairs of simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = 2x + 3$
$11x - 5y = -14$

b $y = 3x - 2$
$7x - 2y = 8$

c $y = 3x - 5$
$3x + 5y = 11$

d $y = 4x + 1$
$2x - 3y = -23$

Be careful with signs when expanding brackets.

$-5 \times (+3) = -15$

$11x - 5(2x + 3)$
$= 11x - 10x - 15$

When multiplying numbers with different signs, the answer is negative.

Problem-solving and Reasoning

Example 24 Solving word problems with simultaneous equations (substitution)

Jade is 5 years older than Marian. If their combined age is 33, find their ages.

Solution	Explanation
Let j be Jade's age and m be Marian's age.	Define two pronumerals using words.
$j = m + 5$..................(1)	The first piece of information is that Jade is 5 years older than Marian.
$j + m = 33$................(2)	The second is that their combined age is 33.
$(m + 5) + m = 33$	Substitute $m + 5$ for j in the second equation.
$2m + 5 = 33$	Collect any like terms, so $m + m = 2m$
$2m = 28$	Subtract 5 from both sides.
$m = 14$	Divide both sides by 2.
$j = m + 5$............(1) $j = 14 + 5$ $j = 19$	Use the first equation, $j = m + 5$, to find j.
Jade is 19 years old and Marian is 14 years old.	Answer the original question in words.

ISBN: 9781107687028

8 Paul is 5 years older than Mary. If their combined age is 81, determine their ages.

First define a pronumeral for Paul's age and another pronumeral for Mary's age. Then write two equations before solving.

9 The length of a rectangle is three times the width. If the perimeter of the rectangle is 48 cm, determine its dimensions.

Draw a diagram to help form the perimeter equation.

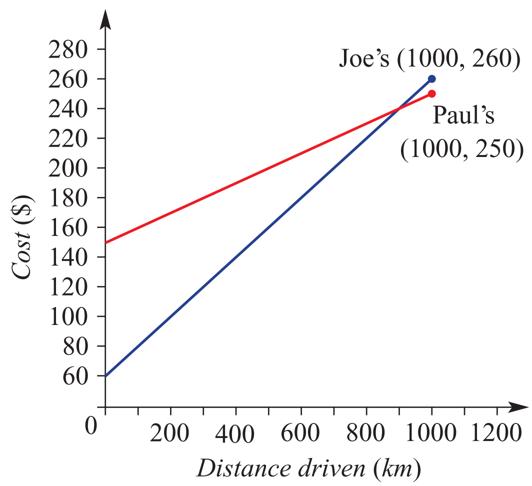
10 A vanilla thick shake is \$2 more than a fruity twirl. If three vanilla thick shakes and five fruity twirls cost \$30, determine their individual prices.

If a fruity twirl costs $\$x$, then 5 will cost $\$5x$.

11 Carlos is 3 more than twice Ella's age. If the sum of their ages is 54 years, determine their ages.

Rentals

12 The given graph represents the rental cost of a new car from two car rental firms called Paul's Motor Mart and Joe's Car Rental.



a Determine:
- **i** the initial rental cost from each company
- **ii** the cost per kilometre when renting from each company
- **iii** the linear equations for the total cost from each company
- **iv** the number of kilometres at which the total cost is the same from both rental firms using the method of substitution.

b Describe when you would use Joe's or Paul's rental firm.

ISBN: 9781107687028

8.7 Solving simultaneous equations using elimination

A second method for solving simultaneous equations, called elimination, can sometimes be more efficient, depending on how the equations are structured in the first place.

Using simultaneous equations, we can compare the overall cost of a diesel 4WD to that of a petrol 4WD.

When setting up equations for real situations, we should define the unknown quantities using pronumerals. When solving simultaneous linear equations there should be only two unknown quantities and two equations that can be formed from the given information.

For example, two related variables are the cost of owning a car and the number of kilometres driven. For two different cars, two equations could be made relating these variables. The simultaneous solution gives the number of kilometres that makes the total running costs of each car equal. Solving simultaneous equations provides information for an accurate comparison of costs between two vehicles.

Let's start: Eliminating a variable

One step in the elimination method involves adding or subtracting two equations in order to eliminate one of the variables. When adding, we write (1) + (2); when subtracting, we write (1) – (2).

- A student has either added or subtracted pairs of equations, but has many incorrect answers.
- Determine which answers are incorrect and write the correct answer for these. (Note: Do not solve the equations for x or y.)

A

$5x+3y=34$ (1)
$7x-3y=26$ (2)
(1)+(2)
$12x+0=60$

B

$3x+2y=18$ (1)
$2x-2y=2$ (2)
(1)+(2)
$5x-4y=20$

C

$3x-3y=9$ (1)
$2x-3y=4$ (2)
(1)–(2)
$5x+0=5$

D

$2x-2y=8$ (1)
$4x-2y=24$ (2)
(1)–(2)
$2x-4y=16$

E

$4x+3y=16$ (1)
$-4x+2y=3$ (2)
(1)+(2)
$0+y=19$

F

$3x+2y=25$ (1)
$2x+2y=18$ (2)
(1)–(2)
$x+0=43$

G

$5x+3y=31$ (1)
$5x-3y=19$ (2)
(1)+(2)
$0+0=12$

H

$x+3y=15$ (1)
$x+2y=12$ (2)
(1)–(2)
$2x+y=3$

ISBN: 9781107687028

Key ideas

- **Elimination** is generally used to solve simultaneous equations when both equations are in the form $ax + by = d$.
  e.g. $2x - y = 6$ or $-5x + y = -2$
  $3x + y = 10$ $6x + 3y = 5$
- Adding or subtracting multiples of these two equations allows one of the pronumerals to be eliminated.
- When problem-solving with simultaneous linear equations:
  - define/describe two unknowns using pronumerals
  - write down two equations using your pronumerals
  - solve the equations using the method of elimination
  - answer the original question in words.

Elimination A method for solving simultaneous equations, where one equation is added to or subtracted from another to eliminate one of the variables

Exercise 8G

Understanding

1 What operation (+ or –) will make these equations true?
a $2x$ ___ $2x = 0$ **b** $-3y$ ___ $3y = 0$ **c** $4x$ ___ $(-4x) = 0$

2 Multiply both sides of the equation $3x - 2y = -1$ by the following numbers. Write the new equations.
a 2 **b** 3 **c** 4

Example 25 Eliminating a variable by addition of equations then solving

Add equation (1) to equation (2), then solve for x and y.

$x + 2y = 10$ (1)
$x - 2y = 2$ (2)

Solution	Explanation
$x + 2y = 10$ (1) $x - 2y = 2$ (2)	Copy equations with the labels (1) and (2).
(1) + (2)	Write the instruction to add: (1) + (2)
$2x + 0 = 12$ $2x = 12$ $x = 6$	Add the x column, $x + x = 2x$ Add the y column, $2y + (-2y) = 0$ Add the RHS, $10 + 2 = 12$ Divide both sides by 2.
Substitute $x = 6$ into (1) $6 + 2y = 10$ $2y = 4$ $y = 2$	In equation (1) replace x with 6. Equation (2) could have been used also. Subtract 6 from both sides. Divide both sides by 2.
Solution is (6, 2)	Write the solution as an ordered pair.
Check: (2) $6 - 2 \times 2 = 2$, true	Check that the solution satisfies equation (2).

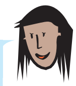
ISBN: 9781107687028 Cambridge University Press

3 Copy each pair of equations, add equation (1) to (2), then solve for x and y following the steps in Example 25.

a
$x + y = 7 \quad (1)$
$x - y = 5 \quad (2)$
$(1) + (2)$

b
$x + 2y = 11 \quad (1)$
$x - 2y = -5 \quad (2)$
$(1) + (2)$

c
$3x + 2y = 20 \quad (1)$
$-3x + y = 1 \quad (2)$
$(1) + (2)$



Adding equations:
(1) + (2)
$x + y = 7 \quad (1)$
$x - y = 5 \quad (2)$
$2x + 0 = 12$
Remember that
$+y + (-y) = +y - y = 0$

4 Copy each pair of equations, subtract equation (2) from equation (1), then solve for x and y showing all steps.

a
$2x + y = 16 \quad (1)$
$x + y = 9 \quad (2)$
$(1) - (2)$

b
$3x + 5y = 49 \quad (1)$
$3x + 2y = 25 \quad (2)$
$(1) - (2)$

c
$5x - 4y = 16 \quad (1)$
$2x - 4y = 4 \quad (2)$
$(1) - (2)$

Subtracting equations:
(1) – (2)
$5x - 2y = 16 \quad (1)$
$2x - 2y = 4 \quad (2)$
$3x + 0 = 12$
Remember that
$-2y - (-2y)$
$= -2y + 2y = 0$

Fluency

5 Determine the point of intersection of the following lines, using the elimination method.

a $x + y = 7$ and $5x - y = 5$
b $x + y = 5$ and $3x - y = 3$
c $x - y = 2$ and $2x + y = 10$
d $x - y = 0$ and $4x + y = 10$

Label the two equations, one under the other, and decide whether to eliminate x or y.
Remember that $+y + (-y) = 0$.
The point of intersection is the same as the simultaneous solution of the equations.

Example 26 Using the elimination method to find the coordinates of the point of intersection

Determine the point of intersection of the lines $x + y = 6$ and $3x + y = 10$ using the elimination method.

Solution	**Explanation**
$x + y = 6$ ....(1) $3x + y = 10$ ....(2)	Label the two equations and decide whether to eliminate x or y. Choose y, since there is $1y$ in each equation.
$(2) - (1)$: $2x = 4$	Subtract the two equations because $y - y = 0$ $3x - x = 2x$ and $10 - 6 = 4$
$x = 2$	Divide both sides by 2.
Substitute $x = 2$ into (1): $2 + y = 6$ $y = 4$	Alternatively, substitute into equation (2). Replace x with the number 2. Subtract 2 from both sides.
Point of intersection is (2, 4). Check: $3(2) + 4 = 10$	Write the solution as an ordered pair. Check that the solution satisfies the other equation.

ISBN: 9781107687028

6 Solve the following pairs of simultaneous equation, using the elimination method. You will need to subtract the equations to eliminate one of the variables.

a $3x + 4y = 7$
$2x + 4y = 6$

b $4x + 3y = 11$
$x + 3y = 5$

c $2x + 3y = 1$
$2x + 5y = -1$

> Always label the equations and write the instruction; e.g. (1) – (2) or (2) – (1).

Example 27 Solving simultaneous equations using the elimination method

Solve the simultaneous equations $3x + 2y = 6$ and $5x + 3y = 11$ using the elimination method.

Solution	Explanation
$3x + 2y = 6$ .... (1) $5x + 3y = 11$ .... (2)	Label the two equations and decide whether to eliminate x or y.
(1) × 3 $\quad 9x + 6y = 18$ .... (3) (2) × 2 $\quad 10x + 6y = 22$ .... (4)	Multiplying the first equation by 3 and the second by 2 results in a matching pair $6y$ in each equation.
(3) – (4) $\quad -x = -4$	Subtract the equations since $6y - 6y = 0$.
$x = 4$	Divide both sides by -1.
Substitute $x = 4$ into (1):	Alternatively substitute into equation (2).
$3(4) + 2y = 6$	Replace x with the number 4.
$2y = -6$	Subtract 12 from both sides, since $3 \times 4 = 12$.
$y = -3$	Divide both sides by 2.
Solution is (4, -3).	Write the solution as an ordered pair.
Check: $5(4) + 3(-3) = 11$	Check the solution with the other equation.

7 Solve the following pairs of simultaneous equation, using the elimination method.

a $3x + 2y = 6$ and $5x + 3y = 11$
b $3x + 2y = 5$ and $2x + 3y = 5$
c $2x + y = 4$ and $5x + 2y = 10$
d $2x + 5y = 7$ and $x + 3y = 4$

> When multiplying an equation by a number, multiply every term on the LHS and RHS by that number. Always write the instruction for multiplying; e.g. (2) × 4.

8 Solve the following pairs of simultaneous equation, using the elimination method.

a $3x + 5y = 8$
$x - 2y = -1$

b $2x + y = 10$
$3x - 2y = 8$

c $4x - 3y = 0$
$3x + 4y = 25$

> Choose to eliminate x or y. The coefficients need to be the same size (with + or –); e.g. $-4x$ and $4x$ or $-5y$ and $-5y$. Choose to add or subtract the equations to eliminate one variable.

ISBN: 9781107687028

9 Solve the following pairs of simultaneous equations.

a $5x + 3y = 18$ and $3y - x = 0$
b $3x - y = 13$ and $x + y = -9$
c $2x + 7y = -25$ and $5x + 7y = -31$
d $2x + 6y = 6$ and $3x - 2y = -2$
e $4x - 5y = -14$ and $7x + y = -5$
f $7x - 3y = 41$ and $3x - y = 17$

Problem-solving and Reasoning

Example 28 Solving word problems with simultaneous equations (elimination)

Kathy is older than Bill. The sum of their ages is 17 years and the difference is 5 years. Find Kathy and Bill's ages.

Solution	Explanation
Let k be Kathy's age and b be Bill's age.	Define two pronumerals.
$k + b = 17$ ....(1)	The first piece of information is 'the sum of their ages is 17'.
$k - b = 5$ ....(2)	The second is 'the difference is 5 and Kathy is older than Bill'.
(1) + (2) $\quad 2k = 22$ $k = 11$	Add the two equations to eliminate b or alternatively subtract to eliminate k.
Substitute $k = 11$ into (1) $\quad 11 + b = 17$	Alternatively substitute into (2).
$b = 6$	Subtract 11 from both sides.
Kathy is 11 years old and Bill is 6.	Answer the original question in words.

10 Bob is older than Francene. The sum of their ages is 56 years and the difference is 16 years. Use simultaneous equations to find Bob and Francene's ages.

Example 29 Problem solving with simultaneous equations

John purchases three daffodils and five petunias from the local nursery and the cost is \$25. Julia buys four daffodils and three petunias and the cost is \$26.

Determine the cost of each type of flower.

Solution	Explanation
Let d be the cost of a daffodil and p be the cost of a petunia.	Define your pronumerals.
$3d + 5p = 25$ ....(1)	Three daffodils and five petunias from the local nursery cost \$25.
$4d + 3p = 26$ ....(2)	Four daffodils and three petunias cost \$26.

ISBN: 9781107687028

8G

$(1)\times 4 \quad 12d+20p=100$ ....(3)	Multiply (1) by 4 and (2) by 3 to obtain a matching pair ($12d$ and $12d$).
$(2)\times 3 \quad 12d+9p=78$ ....(4)	
$(3)-(4) \quad 11p=22$	Subtract the equations to eliminate d.
$p=2$	Divide both sides by 2.
Substitute $p=2$ into (1):	Alternatively, substitute into (2).
$3d+5(2)=25$	Replace p with the number 2.
$3d+10=25$	Simplify.
$3d=15$	Subtract 10 from both sides.
$d=5$	Divide both sides by 3.
Check: $4(5)+3(2)=26$	Check your solutions by substituting into the second equation.
Daffodils cost \$5 and petunias cost \$2 each.	Answer the question in words.

11 Tickets to a basketball game cost \$3 for children and \$7 for adults. If 5000 people attended the game and the total takings at the door were \$25 000, determine the number of children and adults who attended the game.

What you are being asked to find is often what you define your variables as.

12 Chris the fruiterer sells two fruit packs:
Pack 1: 10 apples and 5 mangos (\$12.50)
Pack 2: 15 apples and 4 mangos (\$13.50)

a Define two pronumerals and set up a pair of linear equations to eventually find the cost of each fruit.
b Solve the two simultaneous equations to determine the individual prices of each piece of fruit.
c Determine the cost of one apple and five mangos.

13 A maths test contains multiple-choice questions worth 2 marks each and short-answer questions worth 3 marks each. The test is out of 50 marks and there are 22 questions.

a Define two pronumerals to represent the number of each question type.
b Set up two linear equations.
c Solve the two equations simultaneously to determine the number of multiple-choice questions.

Total marks is 50.
Number of questions is 22.

14 Let x and y be two numbers that satisfy the following statements. Set up two linear equations according to the information and solve them simultaneously to determine the numbers in each case.

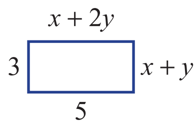
a Their sum is 16 but their difference is 2.
b Their sum is 30 but their difference is 10.
c Twice the larger number plus the smaller is 12 and their sum is 7.

ISBN: 9781107687028

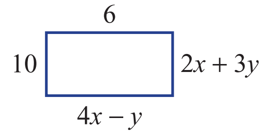
15 Find the value of x and y in the following rectangles. You will need to write two equations and solve using the elimination method.


Opposite sides of rectangles are equal.

a



b



16 Gary is currently 31 years older than his daughter. In 30 years' time he will be twice his daughter's age. Using g for Gary's current age and d for Gary's daughter's current age, complete the following.

a Write down expressions for:

i Gary's age in 30 years' time **ii** Gary's daughter's age in 30 years' time

b Write down two linear equations using the information at the start.

c Solve the equations to find the current ages of Gary and his daughter.

Using technology

17 Use technology to solve these simultaneous equations.

a $3x + 2y = 6$ and $5x + 3y = 11$ **b** $3x + 2y = 5$ and $2x + 3y = 5$

c $4x - 3y = 0$ and $3x + 4y = 25$ **d** $2x + 3y = 10$ and $3x - 4y = -2$

e $-2y - 4x = 0$ and $3y + 2x = -2$ **f** $-7x + 3y = 22$ and $3x - 6y = -11$

ISBN: 9781107687028 Cambridge University Press

1 The answers to these equations will form a magic square: each row, column and diagonal will add to the same number. Draw a 4 by 4 square for your answers and check that they do make a magic square.

$x-3=6$	$x+15=10$	$\frac{x}{2}=-2$	$5x=30$
$3x+7=1$	$\frac{x}{4}-8=-7$	$\frac{x+7}{2}=5$	$3(x+4)=x+14$
$\frac{x}{2}-5=-4$	$4x-9=-9$	$x+7=4x+10$	$2(3x-12)-5=1$
$\frac{9-3x}{3}=6$	$-2(3-x)=x+1$	$x-16=-x$	$5x+30-3x=-3x$

2 Write an equation and solve it to help you find the unknown number in these puzzles.

a Three quarters of a number plus 16 is equal to 64.

b A number is increased by 6 then that answer is doubled and the result is four more than triple the number.

c The average of a number and its triple is equal to 58.6.

d In four years' time, Ahmed's age will be double the age he was 7 years ago. How old is Ahmed now?

3 By applying at least two operations to x, write 3 different equations so that each equation has the solution $x = -2$. Verify that $x = -2$ makes each equation true.
For example: $3 \times (-2) + 10 = 4$, so one possible equation would be $3x + 10 = 4$.

ISBN: 9781107687028 Cambridge University Press

4 Which Australian city has its centre on the intersection of the Warrego Highway and the New England Highway?

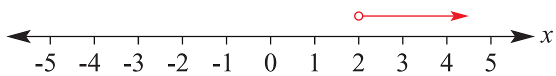
To decode this puzzle, solve the inequalities and simultaneous equations below, and match them to a number line or graph. Place the corresponding letters above the matching numbers to find the answer.

___	___	___	___	___	___	___	___	___
6	4	4	3	4	4	5	1	2

Solve these inequalities and match the solution to a number line (**1–3**).

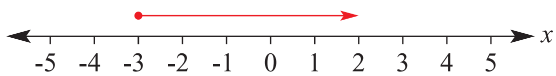
W $2 - 3x > 8$

1



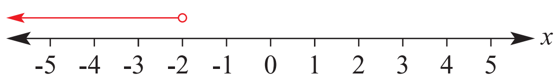
A $3x + 10 \geq 1$

2



B $x + 5 > 7$

3



Solve these simultaneous equations and match the solution to a graph (**4–6**).

M

$3x - y = 7$

$2x + y = 3$

O

$y = 2x + 1$

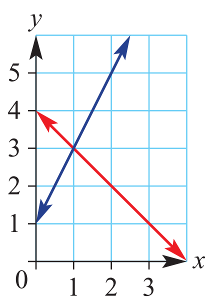
$x + y = 4$

T

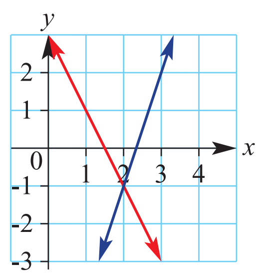
$2x - y = -1$

$x - 2y = 4$

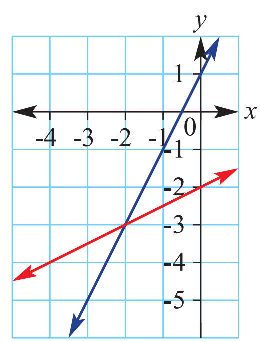
4



5



6



5 Write two sets of simultaneous equations so that each pair has the solution (3, -2).

6 Jules and Enzo have a long distance bike race. Jules rides at 18 km/h and has a 2-hour head start. Enzo travels at 26 km/h.

a How long does it take for Enzo to catch up to Jules? (Use distance = speed × time.)

b How far did they both ride before Enzo caught up to Jules?

7 Talia travelled a distance of 138 km by jogging for 2 hours and cycling for 5 hours. She could have travelled the same distance by jogging for 4 hours and cycling for 4 hours. Find the speed at which she was jogging and the speed at which she was cycling.

ISBN: 9781107687028 Cambridge University Press

Chapter summary

Solving linear equations that have brackets

- Expand all brackets
- Collect like terms on each side of the equation.
- Collect terms with a pronumeral to one side (usually the LHS)
- Solve for unknown.

e.g.

$$\begin{aligned} 12(x+1)-2(3x-3) &= 4(x+10) \\ 12x+12-6x+6 &= 4x+40 \\ 6x+18 &= 4x+40 \\ 2x+18 &= 40 \\ 2x &= 22 \\ x &= 11 \end{aligned}$$

Solving linear equations

Solving involves finding the value that makes an equation true

e.g. $2x+5=9$

$2x=4$ (subtract 5)

$x=2$ (divide by 2)

Equations with fractions

e.g.

$\frac{3x}{4}-2=7$ (*first* $+2$ *to both sides*)

$\frac{3x}{4}=9$ ($\times 4$ *both sides*)

$3x=36$ ($\div 3$ *both sides*)

$x=12$

e.g.

$\frac{2x-5}{3}=7$ (*first* $\times 3$ *to both sides*)

$2x-5=21$ ($+5$ *to both sides*)

$2x=26$ ($\div 2$ *to both sides*)

$x=13$

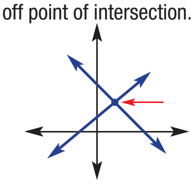
Solving word problems

1. Define variable(s)
2. Set up equation(s)
3. Solve equation(s)
4. Check each answer and write in words

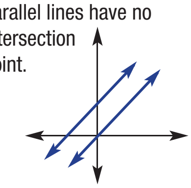
Equations

Graphical solutions of simultaneous equations

Graph each line and read off point of intersection.



Parallel lines have no intersection point.



Formulas

Some common formulas

e.g. $A=\pi r^2$, $C=2\pi r$

An unknown value can be found by substituting values for the other variables.

A formula can be rearranged to make a different variable the subject.

e.g. $E=mc^2$, *find* m *when* $E=320$ *and* $c=4$

$320=m\times 4^2$ (*substitute values*)

$320=16m$ ($4^2=16$)

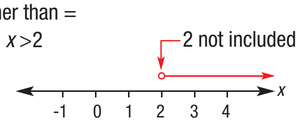
$20=m$ (*divide both sides by* 16)

$m=20$ (*write the answer with m on the left*)

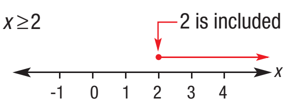
Inequalities

These can be represented using $>, <, \geq, \leq$ rather than $=$

e.g. $x>2$



e.g. $x\geq 2$



Solving inequalities uses the same steps as solving equations except when multiplying or dividing by a negative number. In this case, the inequality sign must be reversed.

e.g. $4-2x>10$ (-4)

$-2x>6$ ($\div$ -2)

$x>-3$ (reverse sign)

Simultaneous equations

Use substitution or elimination to find the solution that satisfies 2 equations.

Substitution

e.g. $2x+y=12$ (1)

$y=x+3$ (2)

In (1) replace y with (2)

$2x+(x+3)=12$

$3x+3=12$

$3x=9$

$x=3$

Sub. $x=3$ to find y

In (2) $y=3+3=6$

Solution (3, 6)

Elimination

Ensure both equations have a matching pair. Add 2 equations if matching pair has different sign; subtract if same sign.

e.g. $x+2y=2$ (1)

$2x+3y=5$ (2)

(1) $\times$ 2 $2x+4y=4$ (3)

(3) – (2) $y=-1$

In (1) $x+2(-1)=2$

$x-2=2$

$x=4$

Solution (4, -1)

ISBN: 9781107687028

Multiple-choice questions

1 The solution to $x + 7 = 9$ is:
A $x = 16$ **B** $x = -2$ **C** $x = 2$ **D** $x = 1$ **E** $x = -16$

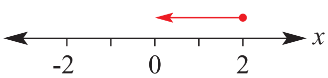
2 To solve the equation $3(2x + 4) - 4(x + 2) = 6$, you would first:
A divide both sides by 12
B expand the brackets
C subtract 6 from both sides
D multiply both sides by 6
E add $4(x + 2)$ to both sides

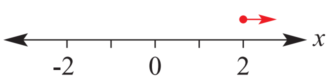
3 A number is increased by six and then doubled. The result is 36. This translates to:
A $6x + 2 = 36$ **B** $2x + 6 = 36$ **C** $2(x + 6) = 36$
D $2(x - 6) = 36$ **E** $x + 12 = 36$

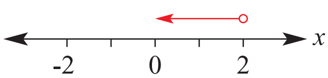
4 If $4a - 6 = 2a$, then a equals:
A -1 **B** 1 **C** 6 **D** 3 **E** -3

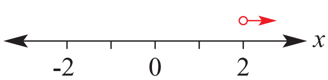
5 $x \le 4$ is a solution to:
A $x + 1 < 3$ **B** $3x - 1 \le 11$ **C** $\frac{x}{2} - 1 \ge 0$
D $x - 1 \ge 1$ **E** $-x \le -4$

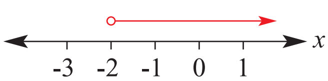
6 Which number line shows $x + 4 < 6$?

A


B


C


D


E


7 The solution to $\frac{5x}{9} - 4 = 1$ is:
A $x = 6$ **B** $x = -9$ **C** $x = -5$ **D** $x = 9$ **E** $x = 5$

8 If two lines are not parallel, the number of intersection points they will have is:
A 0 **B** 1 **C** 2 **D** 3 **E** 4

9 The intersection point for the graphs of $y = 2$ and $x = 3$ is:
A (-1, 2) **B** (2, 2) **C** (3, 2) **D** (3, 3) **E** (2, 3)

10 The solution to $3(x - 1) = 12$ is:
A $x = -1$ **B** $x = 2$ **C** $x = 0$ **D** $x = 5$ **E** $x = 4$

ISBN: 9781107687028 Cambridge University Press

11 $y = 3x$ and $x + y = 4$ has the solution:
A (1, 3) **B** (3, 1) **C** (2, 6) **D** (2, 2) **E** (-1, 5)

12 Substituting $y = x - 1$ into $x + 2y = 3$ gives:
A $x - 2x - 2 = 3$ **B** $x + 2y - 2 = 3$ **C** $x - x - 1 = 3$
D $x + 2x - 1 = 3$ **E** $x + 2(x - 1) = 3$

13 Adding $x + y = 3$ to $x - y = 4$ gives:
A $2x - 2y = 7$ **B** $2x = 7$ **C** $x = 7$ **D** $y = 7$ **E** $2y = 7$

14 Subtracting $2x + 3y = 10$ from $5x + 3y = 16$ gives:
A $3x = -6$ **B** $6y = 6$ **C** $6x = 26$ **D** $3x = 6$ **E** $8x = 26$

15 The solution to $2x - y = 3$ and $3x + y = 7$ is:
A (2, 1) **B** (1, -1) **C** (1, 4) **D** (3, 3) **E** (3, -2)

16 The sum of two numbers is 15 and their difference is 7. The two numbers are:
A (4, 11) **B** (5, 12) **C** (5, 10) **D** (3, 12) **E** (2, 13)

17 Two apples and three bananas cost \$3.40 while three apples and two bananas cost \$3.10. The cost of an apple is:
A \$0.70 **B** \$1.50 **C** 80 cents **D** \$1 **E** 50 cents

Short-answer questions

1 Solve the following.
a $4a = 32$ **b** $\frac{m}{5} = -6$ **c** $x + 9 = 1$
d $x + x = 16$ **e** $9m = 0$ **f** $w - 6 = 9$
g $8m = -1.6$ **h** $\frac{w}{4} = 1$ **i** $r - 3 = 3$

2 Find the solution to the following.
a $2m + 7 = 11$ **b** $3w - 6 = 18$ **c** $\frac{m}{2} + 1 = 6$
d $\frac{5w}{4} - 3 = 7$ **e** $\frac{m - 6}{2} = 4$ **f** $\frac{3m + 2}{6} = 1$
g $6a - 9 = 0$ **h** $4 - x = 3$ **i** $9 = x + 6$

3 Solve the following by first expanding the brackets.
a $3(m + 1) = 12$ **b** $4(a - 3) = 16$ **c** $5(2 + x) = 30$
d $4(2x + 1) = 16$ **e** $2(3m - 3) = 9$ **f** $2(1 + 4x) = 9$
g $2(2x + 3) + 3(5x - 1) = 41$ **h** $3(2x + 4) - 4(x - 7) = 56$

4 Find the value of p in the following.
a $7p = 5p + 8$ **b** $2p = 12 - p$ **c** $6p + 9 = 5p$
d $2p + 10 = p + 8$ **e** $3p + 1 = p - 9$ **f** $4p - 8 = p - 2$

ISBN: 9781107687028

5 **a** $A = \frac{1}{2}bh$. Find b if $A = 24$ and $h = 6$.

b $V = lwh$. Find w if $V = 84$, $l = 6$ and $h = 4$.

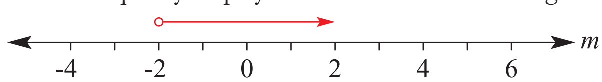
c $A = \frac{x + y}{2}$. Find x if $A = 3.2$ and $y = 4$.

d $E = mc^2$. Find m if $E = 40$ and $c = 2$.

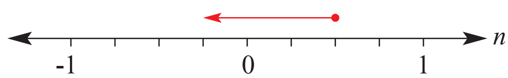
e $F = \frac{9}{5}C + 32$. Find C if $F = 95$.

6 Write the inequality displayed on each of the following number lines.

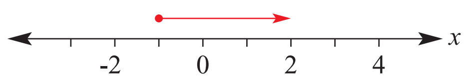
a



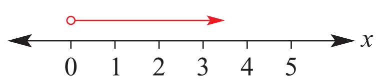
b



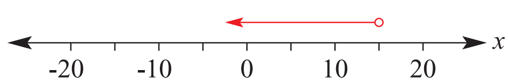
c



d



e



7 Solve the following.

a $x + 8 \geq -10$ **b** $2m < 7$ **c** $2x + 6 > 10$

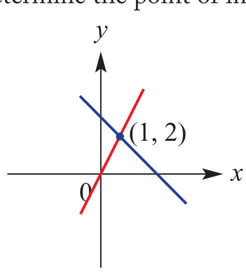
d $x - 3 < 0$ **e** $\frac{x}{4} + 1 \leq 3$ **f** $m - 6 \geq 4$

8 Solve the following.

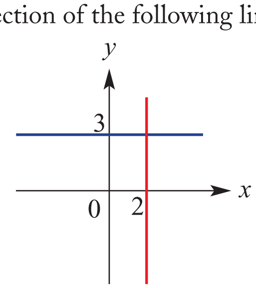
a $-6x \leq 12$ **b** $8 - x \leq 10$ **c** $-x > 0$

9 Determine the point of intersection of the following lines.

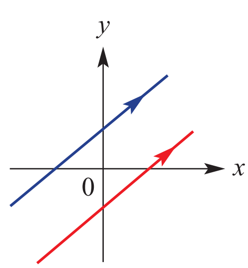
a



b



c



10 Find the point of intersection (x, y) of the following by plotting an accurate graph.

a $y = 2x + 4$
$3x + y = 9$

b $y = 2$
$x = 3$

c $y = 3x$
$y = -3x$

ISBN: 9781107687028

Cambridge University Press

11 Solve the simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = 5x - 13$
$2x + 3y = 12$

b $y = -1$
$y = 2x - 11$

12 Determine the point of intersection of the following lines, using the elimination method.

a $2x + 7y = -25$
$5x + 7y = -31$

b $3x + 2y = 8$
$x - 2y = 0$

13 Write an equation for the following and then solve it.

a Six times a number equals 420. What is the number?
b Eight more than a number equals 5. What is the number?
c A number divided by 9 gives 12. What is the number?
d Seven more than a number gives 3. What is the number?
e The sum of a number and 2.3 equals 7. What is the number?

14 Thomas earns \$96 a day as a portrait photographer, plus \$2 per photography package sold.

a How much does he earn if he sells 12 packages in a day?
b How much does he earn if he sells n packages in a day?
c If Thomas earns \$308 in one day, write an equation and find the number of photography packages sold.

15 A money box contains 20 cent and 50 cent coins. The amount in the money box is \$50 and there are 160 coins.

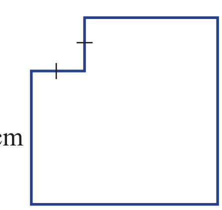
a Define two variables and set up a pair of linear equations.
b Solve the two simultaneous equations to determine the number of 20 cent and 50 cent coins.

16 There are twice as many adults as children at a local grand final football match. It costs \$10 for adults and \$2 for children to watch. If the football club collected \$1100 at the entrance gates, how many children went to see the match?

Extended-response questions

1 For the following shape:

a Determine the equation of its perimeter.
b **i** If the perimeter is 128 cm, determine the value of x.
ii Find the actual side lengths.
c Repeat part **b** for perimeters of:
i 152 cm **ii** 224 cm



2 Two computer consultants have an up-front fee plus an hourly rate. Rhys charges \$50 plus \$70 per hour while Agnes charges \$100 plus \$60 per hour.

a Using \$$C$ for the cost and t hours for the time, write a rule for the cost of hiring:
i Rhys **ii** Agnes
b By drawing a graph of C verses t for both Rhys and Agnes on the same set of axes, find the coordinates of the intersection point.
c Use the algebraic method of substitution to solve the simultaneous equations and confirm your answer to part **b**.

ISBN: 9781107687028

Australian Curriculum
Measurement and Geometry
Pythagoras and trigonometry

chapter
9

Pythagoras' theorem and trigonometry

What you will learn

Pythagoras and position location

Pythagoras was born in 582 BCE in Greece. His theorem relating the sides of right-angled triangles is still used in measurement and design today.

Trigonometry is the branch of mathematics that relates to right-angled triangles, linking the ratio of sides to angles. Trigonometry has many applications and is widely used. If you have a GPS (global positioning system) in your family's car, or if you have a map function on your mobile phone, these use trigonometry to help locate your position.

ISBN: 9781107687028

Cambridge University Press

Pre-test

1 Round the following decimals, correct to two decimal places.

a 15.843 12 **b** 164.8731 **c** 0.866 02 **d** 0.577 35
e 0.173 648 **f** 0.7071 **g** 12.990 38 **h** 14.301

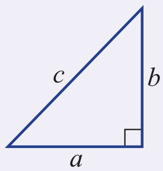
2 Find the value of each of the following.

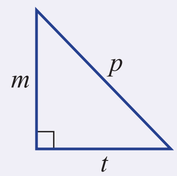
a 5^2 **b** 6.8^2 **c** 19^2 **d** $9^2 + 12^2$ **e** $3.1^2 + 5.8^2$ **f** $41^2 - 40^2$

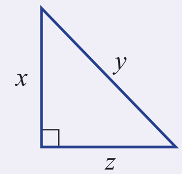
3 Find the following, correct to one decimal place.

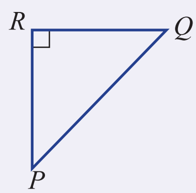
a $\sqrt{8}$ **b** $\sqrt{7}$ **c** $\sqrt{15}$ **d** $\sqrt{10}$
e $\sqrt{12.9}$ **f** $\sqrt{8.915}$ **g** $\sqrt{3.8}$ **h** $\sqrt{200}$

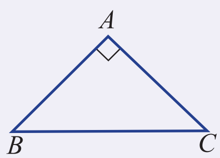
4 Write down the name of the hypotenuse (the side opposite the right angle) on the following triangles.

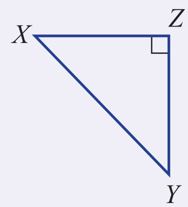
a


b


c


d


e


f


5 Solve for x.

a $3x = 9$ **b** $4x = 16$ **c** $5x = 60$ **d** $\frac{x}{5} = 7$ **e** $\frac{x}{12} = 9$ **f** $\frac{2x}{3} = 6$

6 Solve for m.

a $7m = 25.55$ **b** $9m = 10.8$ **c** $1.5m = 6.6$
d $\frac{m}{1.3} = 4$ **e** $\frac{m}{5.89} = 3.2$ **f** $\frac{m}{5.4} = 1.06$

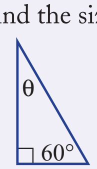
7 Solve each of the following equations, correct to one decimal place.

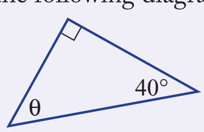
a $\frac{3}{x} = 5$ **b** $\frac{4}{x} = 17$ **c** $\frac{32}{x} = 15$
d $\frac{3.8}{x} = 9.2$ **e** $\frac{15}{x} = 6.2$ **f** $\frac{29.3}{x} = 3.2$

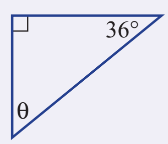
8 If x is a positive integer, solve:

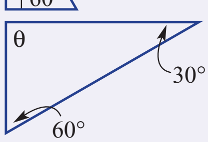
a $x^2 = 16$ **b** $x^2 = 400$ **c** $x^2 = 5^2 + 12^2$ **d** $x^2 + 3^2 = 5^2$

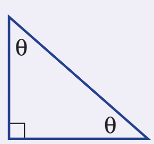
9 Find the size of the angle θ in the following diagrams.

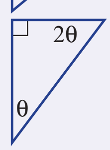
a


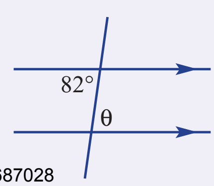
b


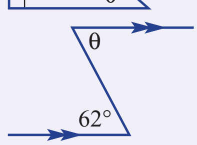
c


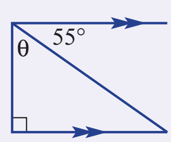
d


e


f


g


h


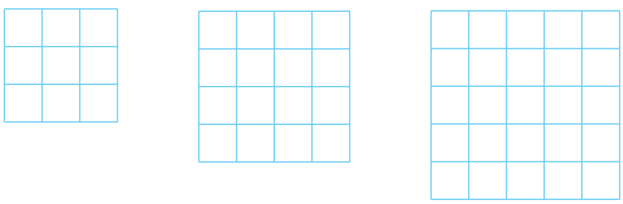
i


9.1 Reviewing Pythagoras' theorem

The Ancient Egyptians knew of the relationship between the numbers 3, 4 and 5 and how they could be used to form a right-angled triangle.

Greek philosopher and mathematician Pythagoras expanded on this idea and the theorem we use today is named after him.

Let's start: Three, four and five



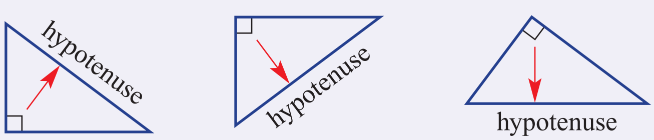
On square grid paper, construct three squares as above.

Cut them out and place the middle-sized square on top of the largest square. Then cut the smallest square into 9 smaller squares and also place them on the largest square to finish covering it.

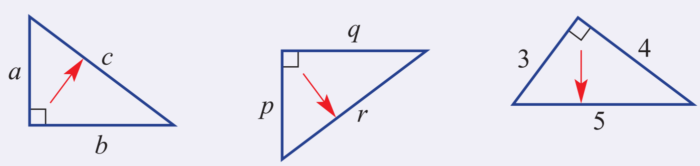
What does this show about the numbers 3, 4 and 5?

Key ideas

- A right-angled triangle has its longest side opposite the right angle. This side is called the **hypotenuse**.
  For example:



- Pythagoras' theorem states:
  The square of the hypotenuse is equal to the sum of the squares on the other two sides.
  For example:



$c^2 = a^2 + b^2$ $\qquad r^2 = p^2 + q^2$ $\qquad 5^2 = 3^2 + 4^2$

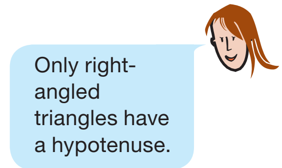
Hypotenuse
The longest side of a right-angled triangle (the side opposite the right angle)

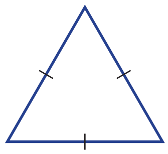
ISBN: 9781107687028

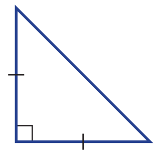
Exercise 9A

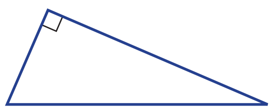
Understanding

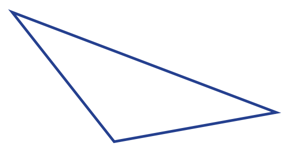
1 Which of the following triangles contain a side known as the hypotenuse?



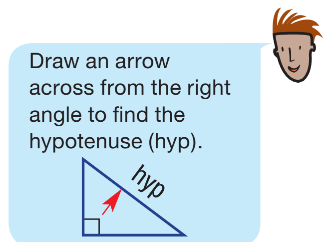
a


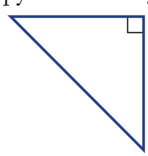
b


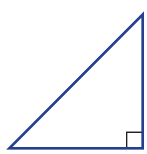
c


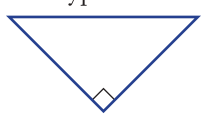
d


2 Copy these triangles into your workbook and label the hypotenuse.

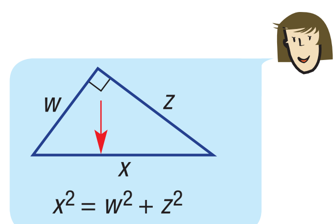


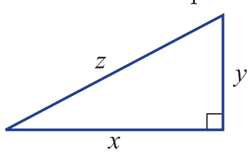
a


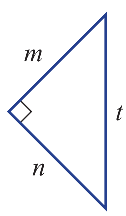
b


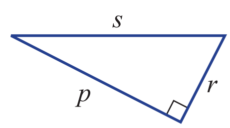
c


3 Write the relationship between the sides of these triangles.



a


b


c


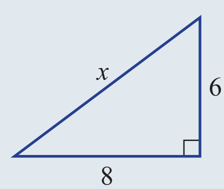
4 Find the value of $a^2 + b^2$ if:

a $a = 3$ and $b = 4$ **b** $a = 3$ and $b = 5$ **c** $a = 3$ and $b = 6$

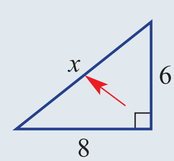
Fluency

Example 1 Finding the length of the hypotenuse

Find the length of the hypotenuse (x) of the triangle shown.



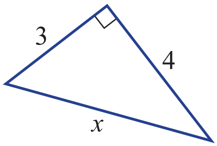
Solution	Explanation
$x^2 = 6^2 + 8^2$ $= 36 + 64$ $= 100$	Write the relationship for the given triangle using Pythagoras' theorem.
$x = \sqrt{100}$ $= 10$	Simplify. Take the square root to find x. Write your answer.

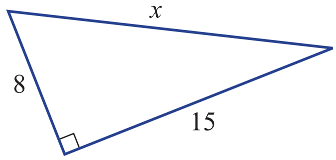


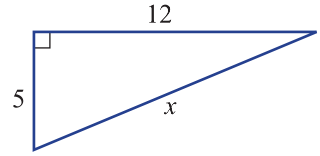
ISBN: 9781107687028

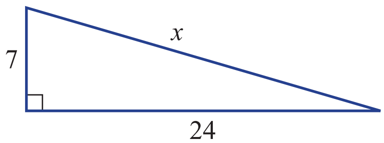
Cambridge University Press

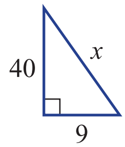
5 Find the length of the hypotenuse in these right-angled triangles.

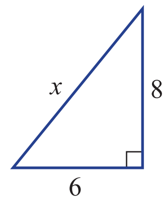
a


b


c


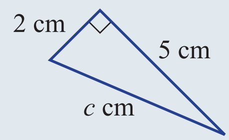
d


e


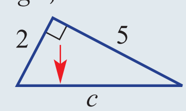
f


Example 2 Finding the length of the hypotenuse as a decimal

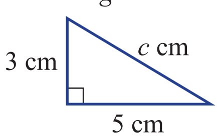
Find the length of the hypotenuse in this triangle, correct to one decimal place.

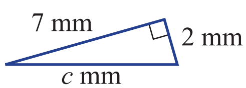


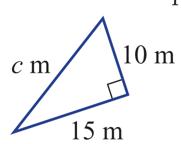
Solution	Explanation
$c^2 = 5^2 + 2^2$	Write the relationship for this triangle, where
$= 25 + 4$	c is the length of the hypotenuse.
$= 29$	
$c = \sqrt{29}$	Simplify.
$c = 5.38516\ldots$	Take the square root to find c.
$c = 5.4$	Round 5.38516 … to one decimal place.



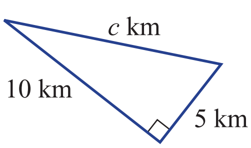
6 Find the length of the hypotenuse in these triangles, correct to one decimal place.

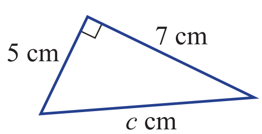
a


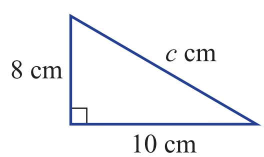
b


c


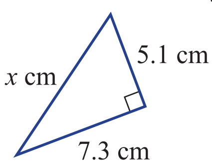
If $c^2 = 34$, then $c = \sqrt{34}$. Use a calculator to find the decimal.

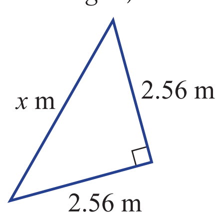
d


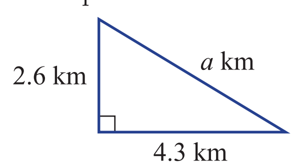
e


f


7 Find the value of the hypotenuse in these triangles, correct to two decimal places.

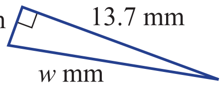
a


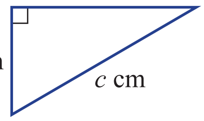
b


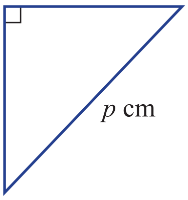
c


ISBN: 9781107687028

9A

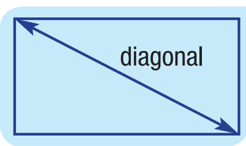
d 

e 

f 

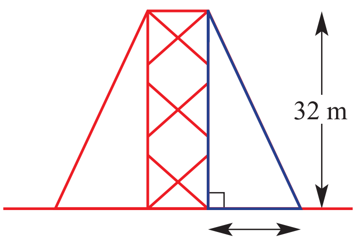
Problem-solving and Reasoning

8 A LCD plasma TV is 154 cm long and 96 cm high. Calculate the length of its diagonal, correct to one decimal place.

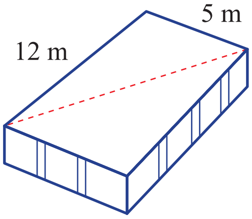


9 A 32 m tower is supported by cables from the top to a position on the ground 15 m from the base of the tower. Determine the length of each cable needed to support the tower, correct to one decimal place.

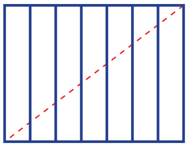
Set up and solve Pythagoras' theorem.



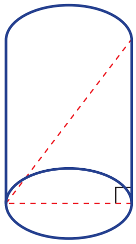
10 Boris the builder uses Pythagoras' theorem to check the corners of his concrete slab. What will be the length of the diagonal when the angle is 90°?



11 Find the length of the diagonal steel brace needed to support a gate of length 4.2 m and width 3.2 m, correct to two decimal places.

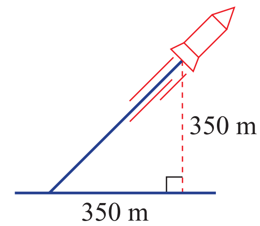


12 Find the length of the longest rod that will fit in a cylindrical container of height 2.1 m and diameter 1.6 m, correct to two decimal places.

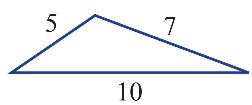


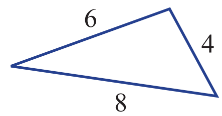
ISBN: 9781107687028 Cambridge University Press

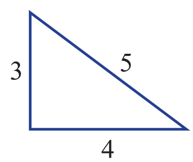
13 A rocket blasts off and after a few seconds it is 350 m above the ground. At this time it has covered a horizontal distance of 350 m. How far has the rocket travelled, correct to two decimal places?

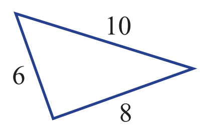


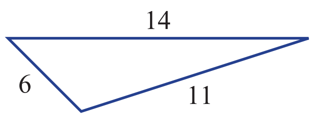
14 Determine whether these triangles contain a right angle.

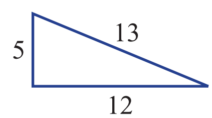
a


b


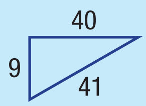
c


d


e


f


If Pythagoras' theorem works, then the triangle has a right angle.
For example:



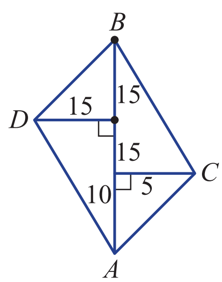
$41^2 = 1681$
$40^2 + 9^2 = 1681$
$\therefore 41^2 = 40^2 + 9^2$ and the triangle has a right angle, opposite the 41.

An offset survey

An offset survey measures distances perpendicular to the baseline offset. A notebook entry is made showing these distances and then perimeters and areas are calculated.

		B		
		40		
D	15	25		
		10	5	*C*
		0		
		A		

Notebook entry



Field diagram (Not to scale)

15 **a** Using the diagrams above, find these lengths, correct to one decimal place.
i *AC* **ii** *BC* **iii** *DB* **iv** *AD*
b Find the perimeter of the field *ACBD*, correct to the nearest metre.
c Find the area of the field.

16 At the right is a notebook entry. Draw the field diagram and find the perimeter of the field, to one decimal place.

		B		
		60		
		40	10	*E*
D	25	30		
C	15	10		
		0		
		A		

ISBN: 9781107687028

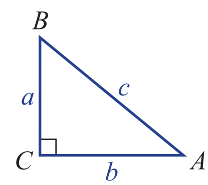
9.2 Finding the length of a shorter side

Using Pythagoras' theorem we can determine the length of the shorter sides of a right-angled triangle. The angled support beams on a rollercoaster ride, for example, create right-angled triangles with the ground. The vertical and horizontal distances are the shorter sides of the triangle.

Let's start: Choosing the correct numbers

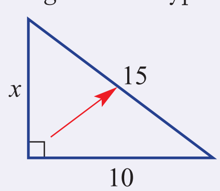
For the triangle ABC, Pythagoras' theorem is written $c^2 = a^2 + b^2$.
Choose the 3 numbers from each group that work for $c^2 = a^2 + b^2$.

Group 1 6, 7, 8, 9, 10
Group 2 15, 16, 20, 25
Group 3 9, 10, 12, 15
Group 4 9, 20, 21, 40, 41



Key ideas

- We can use Pythagoras' theorem to determine the length of one of the shorter sides if we know the length of the hypotenuse and the other side.
  For example:

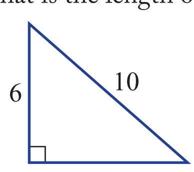


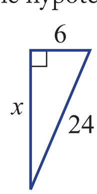
$15^2 = x^2 + 10^2$ becomes $x^2 = 15^2 - 10^2$
so $x^2 = 125$ and $x = \sqrt{125}$

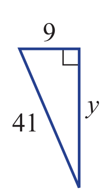
Exercise 9B

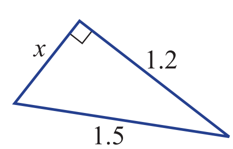
Understanding

1 What is the length of the hypotenuse in each of these triangles?

a


b


c


d


ISBN: 9781107687028

2 Copy and complete:

a If $10^2 = 6^2 + w^2$, then $w^2 = 10^2 - \square$.

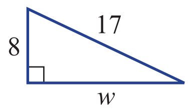
b If $13^2 = 5^2 + x^2$, then $x^2 = 13^2 - \square$.

c If $30^2 = p^2 + 18^2$, then $p^2 = \square - 18^2$.

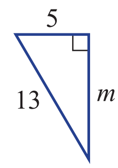
Follow a step as if you were solving an equation.

3 Write down Pythagoras' theorem for each of these triangles.

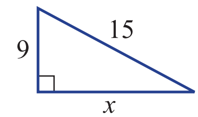
a



b



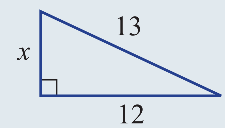
c



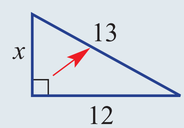
Fluency

Example 3 Calculating a shorter side

Determine the value of x in the triangle shown using Pythagoras' theorem.



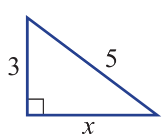
Solution	Explanation
$13^2 = x^2 + 12^2$	Write the relationship for this triangle using Pythagoras' theorem with 13 as the hypotenuse.
$x^2 = 13^2 - 12^2$	Rewrite the rule with the x^2 on the left-hand side.
$= 169 - 144$	Simplify.
$= 25$	
$x = \sqrt{25}$	Find the square root to find x.
$\therefore x = 5$	



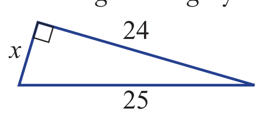
4 Determine the value of x in these triangles using Pythagoras' theorem.

In $c^2 = a^2 + b^2$, c is always the hypotenuse.

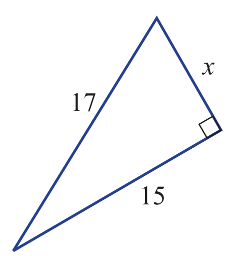
a



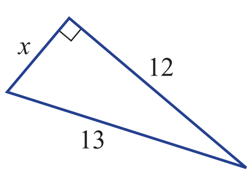
b



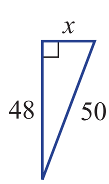
c



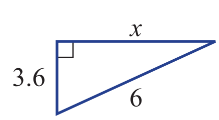
d



e



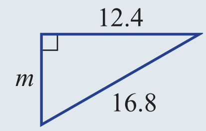
f



ISBN: 9781107687028

Example 4 Finding a shorter side length as a decimal value

Determine the value of m in the triangle correct to one decimal place.

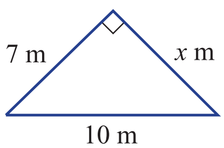


Solution	Explanation
$16.8^2 = m^2 + 12.4^2$	Write the relationship for this triangle.
$m^2 = 16.8^2 - 12.4^2$	Make m^2 the subject.
$= 128.48$	Simplify, using your calculator.
$m = \sqrt{128.48}$	Take the square root of both sides to find m.
$= 11.3349\ldots$	
$m = 11.3$	Round your answer to one decimal place.

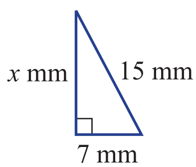
5 Determine the value of x in these triangles using Pythagoras' theorem. Answer correct to one decimal place.

To round to one decimal place, look at the 2nd decimal place. If it is 5 or more, round up. If it is 4 or less, round down. For example, 7.1414 … rounds to 7.1.

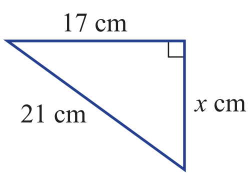
a



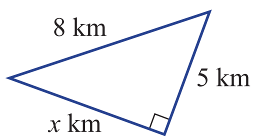
b



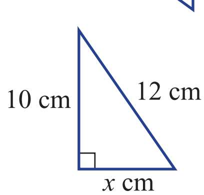
c



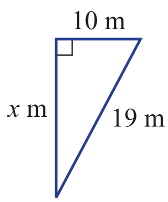
d



e

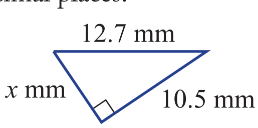


f

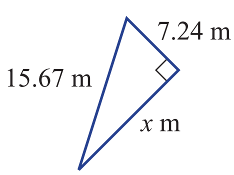


6 Determine the value of x in these triangles using Pythagoras' theorem. Answer correct to two decimal places.

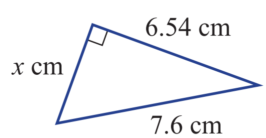
a



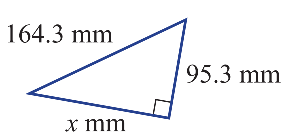
b



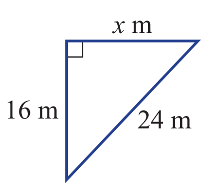
c



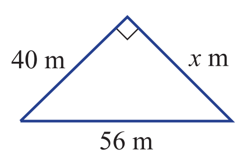
d



e



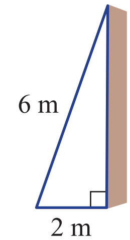
f



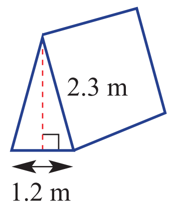
ISBN: 9781107687028

Problem-solving and Reasoning

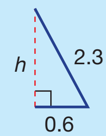
7 A 6 m ladder leans against a wall. If the base of the ladder is 2 m from the wall, determine how high the ladder is up the wall, correct to two decimal places.



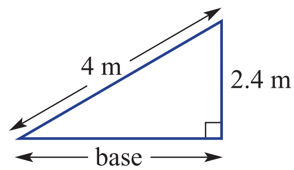
8 A tent has sloping sides of length 2.3 m and a base of 1.2 m. Determine the height of the tent pole, correct to one decimal place.



Identify the right-angled triangle.



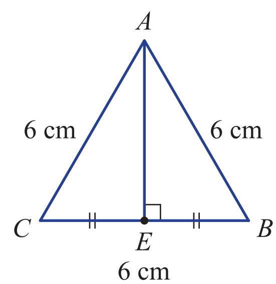
9 A city council wants to build a skateboard ramp 4 m long and 2.4 m high. How long should the base of the ramp be?



10 Triangle ABC is equilateral. AE is an axis of symmetry.

a Find the length of:
  i EB
  ii AE, to one decimal place

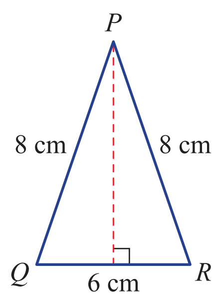
b Find the area of triangle ABC, to one decimal place.



An equilateral triangle has 3 equal sides.

Remember:
$A = \frac{1}{2}bh$ is the area of a triangle.

11 What is the height of this isosceles triangle, to one decimal place?

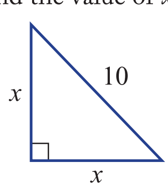


Pythagoras' theorem only applies to right-angled triangles.

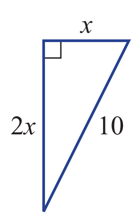
More than one pronumeral

12 Find the value of x in each of the following. Answer to one decimal place.

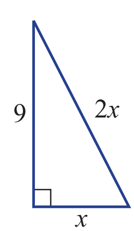
a



b



c



Remember to square the entire side.
The square of $2x$ is $(2x)^2$ or $4x^2$.

ISBN: 9781107687028 Cambridge University Press

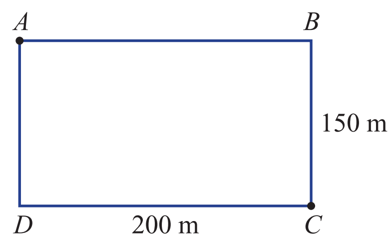
9.3 Applications of Pythagoras’ theorem

Pythagoras’ theorem has many applications, some of which you may have noticed already in this chapter. Some areas where Pythagoras’ theorem is useful include drafting, building and navigation.

Let’s start: Finding the shortest path

A rectangular field is 200 m by 150 m.

Marco wants to walk from the corner of the field marked A to the corner of the field marked C. How many metres are saved by walking along the diagonal AC rather than walking along AB then BC?



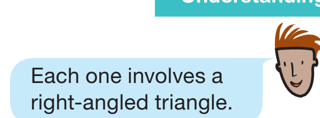
Key ideas

- When applying Pythagoras’ theorem:
  - identify and draw the right-angled triangle or triangles you need to solve the problem
  - label the triangle and place a pronumeral (letter) on the side length that is unknown
  - use Pythagoras’ theorem to find the value of the pronumeral
  - answer the question. (Written questions should have written answers.)

Exercise 9C

Understanding

1 Draw a diagram for each of the following questions. You don’t need to answer the question.



a A 2.4 metre ladder is placed 1 metre from the foot of a building. How far up the building will the ladder reach?

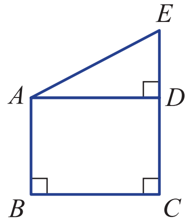
b The diagonal of a rectangle with length 18 cm is 24 cm. How wide is the rectangle?

c Tom walks 5 km North then 3 km West. How far is he from his starting point?

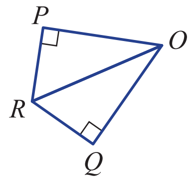
ISBN: 9781107687028 Cambridge University Press

2 Name the right-angled triangles in each of the following diagrams.

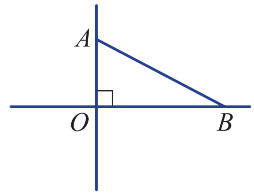
a



b

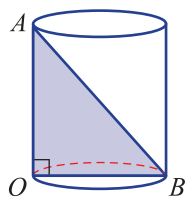


c

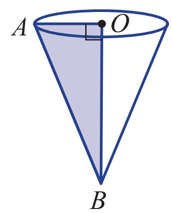


3 Name the hypotenuse in each of the shaded right-angled triangles found within these three-dimensional shapes.

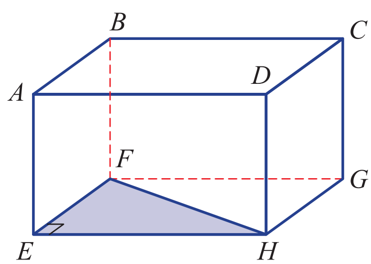
a



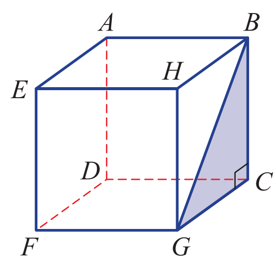
b



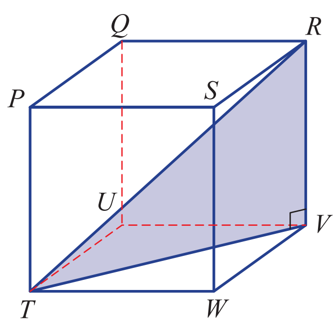
c



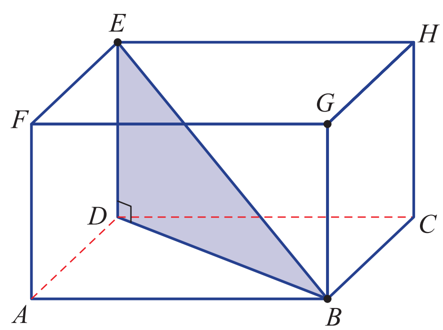
d



e



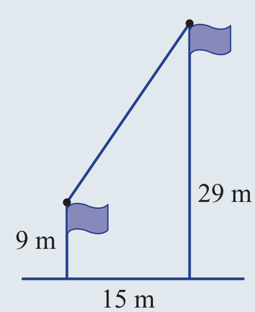
f



Fluency

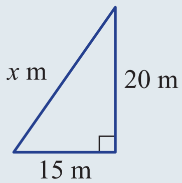
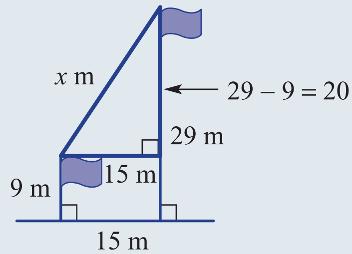
Example 5 Applying Pythagoras' theorem

Two flag poles are 15 metres apart and a rope links the tops of both poles. Find the length of the rope if one flag pole is 9 m and the other is 29 m.



ISBN: 9781107687028

Cambridge University Press

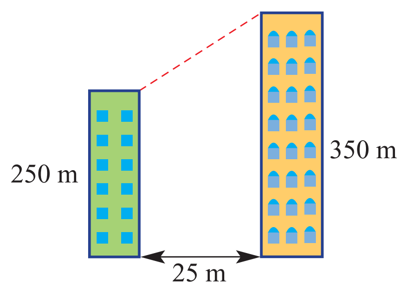
Solution	Explanation
Let x be the length of rope.	Locate and draw the right-angled triangle, showing all measurements. Introduce a pronumeral for the missing side.
	
$x^2 = 15^2 + 20^2$	Write the relationship, using Pythagoras' theorem.
$= 225 + 400$ $= 625$	Simplify.
$x = \sqrt{625}$ $= 25$	Take the square root to find x.
The rope is 25 m long.	Answer the question.

4 Two skyscrapers are 25 m apart and a cable runs from the top of one building to the top of the other. If one building is 350 m tall and the other 250 m:

a determine the difference in the heights of the buildings

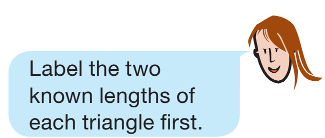
b draw an appropriate right-angled triangle you could use to find the length of the cable

c find out the length of the cable, correct to two decimal places

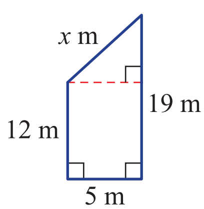


(Not to scale)

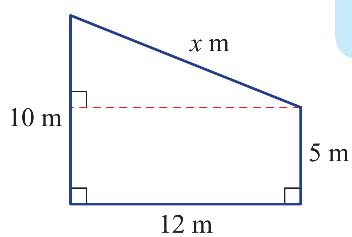
5 Find the value of x in each of the following, correct to one decimal place where necessary.



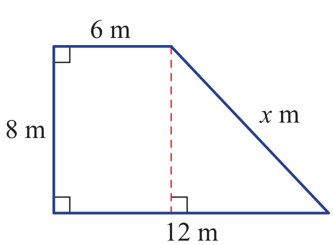
a



b



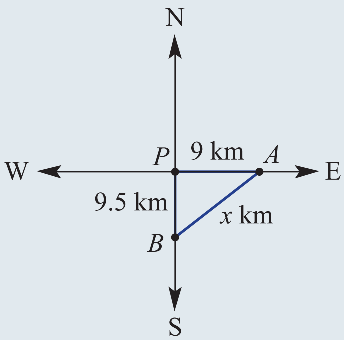
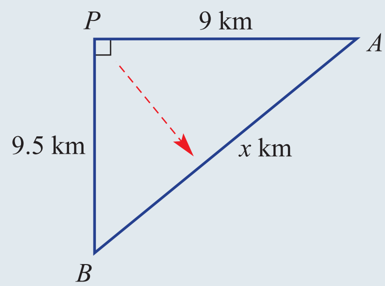
c



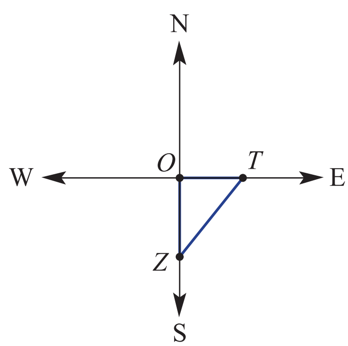
ISBN: 9781107687028

Example 6 Using direction with Pythagoras' theorem

Two hikers leave their camp (P) at the same time. One walks due East for 9 km; the other walks due South for 9.5 km. How far apart are the two hikers at this point (answer to one decimal place)?

Solution	Explanation
	Draw a diagram. Consider ΔPAB. 
$\therefore x^2 = 9^2 + 9.5^2$ $= 171.25$	Write Pythagoras' theorem and evaluate.
$x = \sqrt{171.25}$ $= 13.08\dot{6}$	Square root to find x.
$= 13.1$ (one decimal place)	Round to one decimal place.
$\therefore$ the hikers are 13.1 km apart.	Answer the question in words.

6 Tom (T) walks 4.5 km East while Zara (Z) walks 5.2 km South. How far from Tom is Zara? Answer to one decimal place.



7 Find the distance between Sui and Kevin if:

a Sui walks 6 km North from camp O and Kevin walks 8 km West from camp O.

b Sui walks 40 km East from point A and Kevin walks 9 km South from point A.

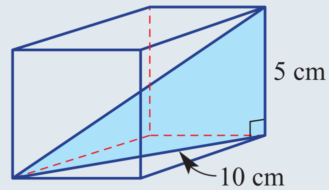
c Kevin walks 15 km North-west from O and Sui walks 8 km South-west also from O.

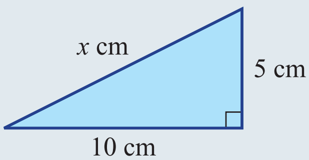
ISBN: 9781107687028

Cambridge University Press

Example 7 Using Pythagoras' theorem in 3D

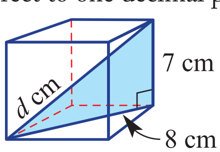
Find the distance from one corner of this rectangular prism to the opposite corner, correct to two decimal places.



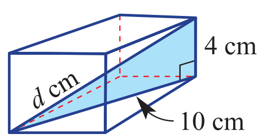
Solution	Explanation
	Draw the triangle you need and mark the lengths.
$x^2 = 5^2 + 10^2$	Write the relationship for this triangle.
$\quad = 25 + 100$	Simplify.
$x = \sqrt{125}$	Take the square root to find x.
$\quad = 11.18$ cm	Round your answer to two decimal places.
$\therefore$ the distance between the opposite corners is 11.18 cm.	Write the answer.

8 Find the distance of d from one corner to the opposite corner in the following rectangular prisms, correct to one decimal place.

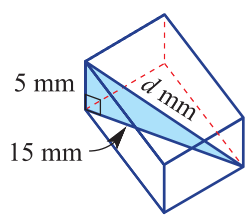
a



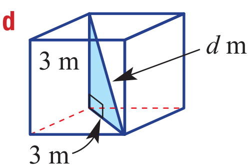
b



c

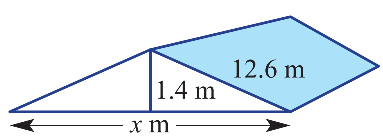


d



Problem-solving and Reasoning

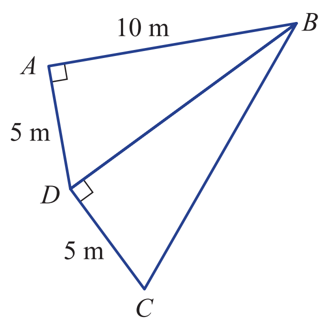
9 The height of a roof is 1.4 m. If the length of a gable (the diagonal) is to be 12.6 m, determine the length of the horizontal beam needed to support the roof, correct to two decimal places.



10 For the diagram find the lengths of:

a BD, correct to two decimal places

b BC, correct to one decimal place

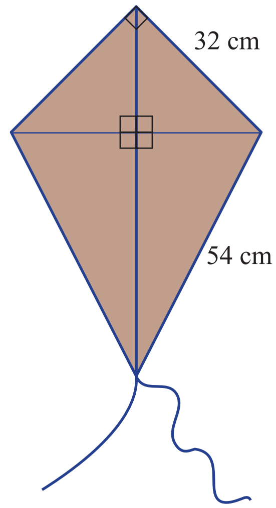


Find the base length of the right-angled triangle first.

ISBN: 9781107687028 Cambridge University Press

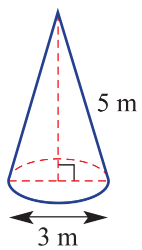
11 A kite is constructed with six pieces of wooden dowel and covered in fabric. The four pieces around the edge have two 32 cm rods and two 54 cm rods. If the top of the kite is right-angled, find the length of the horizontal and vertical rods, correct to two decimal places.

> Find the length of the horizontal rod first. What type of triangle is the top of the kite? Find the length of the vertical rod using two calculations.

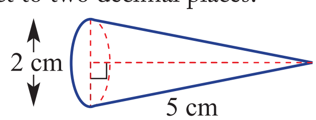


12 Find the height of the following cones, correct to two decimal places.

a

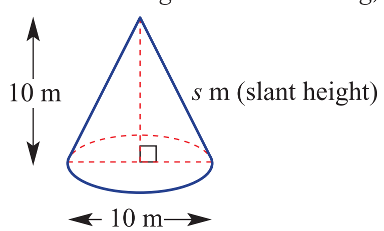


b

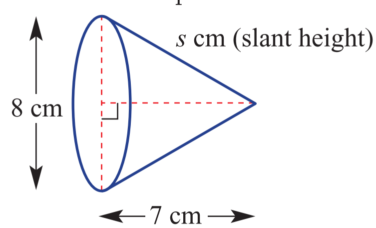


13 Find the slant height of the following, correct to one decimal place.

a

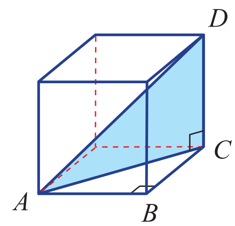


b



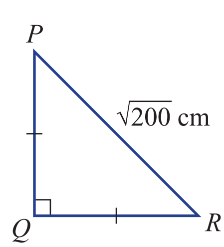
14 This cube has 1 cm sides. Find, correct to two decimal places, the lengths of:

a AC **b** AD



How much do you know?

15 Write down everything you know about ΔPQR.



ISBN: 9781107687028 Cambridge University Press

9.4 Trigonometric ratios

Trigonometry deals with the relationship between the sides and angles of triangles.

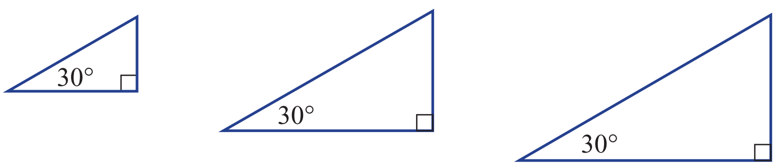
In right-angled triangles there are three trigonometric ratios: sine (sin) cosine (cos) and tangent (tan).

Using your calculator and knowing how to label the sides of right-angled triangles, you can use trigonometry to find missing sides and angles.

Surveyors use trigonometry to calculate accurate lengths.

Let’s start: Thirty degrees

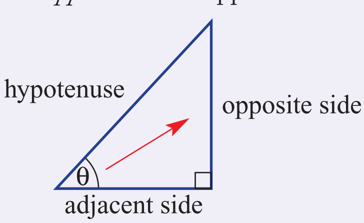
- Draw three different right-angled triangles that each have a 30° angle.

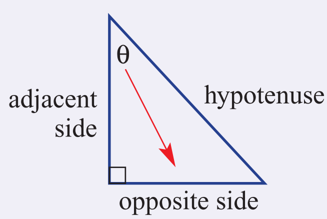


- Measure each side of each triangle, and add these measurements to your diagrams.
- The hypotenuse, as we know, is opposite the right angle. The side opposite the 30° is called the opposite side. For each of your three triangles, write down the ratio of the opposite side divided by the hypotenuse. What do you notice?
- Type ‘sin 30°’ into your calculator. What do you notice?

Key ideas

- Any right-angled triangle has three sides: the hypotenuse, adjacent and opposite.
  - The hypotenuse is always opposite the right angle.
  - The *adjacent* side is next to the **angle of reference**.
  - The *opposite* side is opposite the angle of reference.





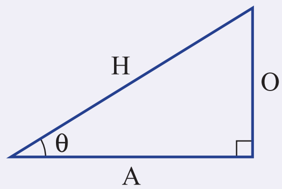
Angle of reference The angle in a right-angled triangle that is used to determine the opposite side and the adjacent side

ISBN: 9781107687028

- For a right-angled triangle with a given angle θ (theta), the three trigonometric ratios of **sine (sin)**, **cosine (cos)** and **tangent (tan)** are given by:
  - sine of angle θ: $\sin\theta = \dfrac{\text{length of opposite side}}{\text{length of the hypotenuse}}$
  - cosine of angle θ: $\cos\theta = \dfrac{\text{length of adjacent side}}{\text{length of the hypotenuse}}$
  - tangent of angle θ: $\tan\theta = \dfrac{\text{length of opposite side}}{\text{length of adjacent}}$
- When working with right-angled triangles:
  Label each side of the triangle O (opposite), A (adjacent) and H (hypotenuse).
- The three trigonometric ratios are:

  $\sin\theta = \dfrac{O}{H} \qquad \cos\theta = \dfrac{A}{H} \qquad \tan\theta = \dfrac{O}{A}$

  We can remember this as **SOH CAH TOA**.



Sine (sin) The ratio of the length of the opposite side to the length of the hypotenuse in a right-angled triangle

Cosine (cos) The ratio of the length of the adjacent side to the length of the hypotenuse in a right-angled triangle

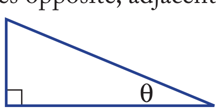
Tangent (tan) The ratio of the length of the opposite side to the length of the adjacent side in a right-angled triangle

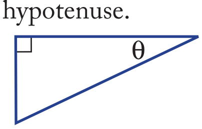
SOH CAHTOA A way of remembering the trigonometric ratios: Sine equals Opposite over Hypotenuse, Cosine equals Adjacent over Hypotenuse, Tangent equals Opposite over Adjacent

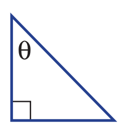
Exercise 9D

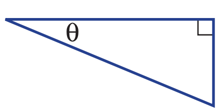
Understanding

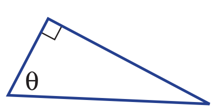
1 By referring to the angles marked, copy each triangle and label the sides opposite, adjacent and hypotenuse.

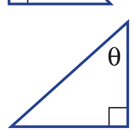
a 

b 

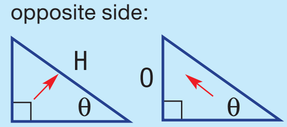
c 

d 

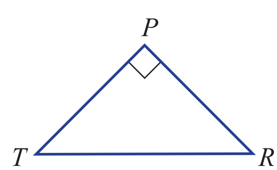
e 

f 

Arrows help you find the hypotenuse and the opposite side:



2 Referring to triangle PTR, name the:
   a side opposite the angle at T
   b side adjacent to the angle at T
   c side opposite the angle at R
   d side adjacent to the angle at R
   e hypotenuse
   f angle opposite the side PR



'Adjacent' means 'next to'.

ISBN: 9781107687028

Cambridge University Press

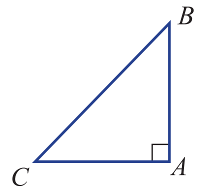
3 Referring to triangle ABC, name the:

a hypotenuse

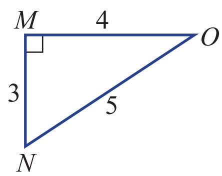
b side opposite the angle at B

c side opposite the angle at C

d side adjacent to the angle at B



4 In triangle MNO, write the ratio of:



a $\dfrac{\text{the side opposite angle } O}{\text{hypotenuse}}$

b $\dfrac{\text{the side opposite angle } N}{\text{hypotenuse}}$

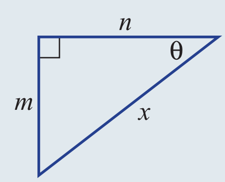
c $\dfrac{\text{the side adjacent angle } O}{\text{hypotenuse}}$

Fluency

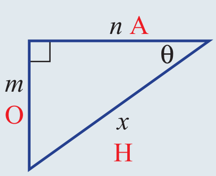
Example 8 Writing trigonometric ratios

Label the sides of the triangle O, A and H and write the ratios for:

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



Solution



a $\sin\theta = \dfrac{m}{x}$

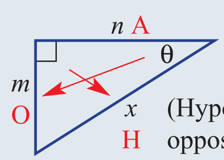
b $\cos\theta = \dfrac{n}{x}$

c $\tan\theta = \dfrac{m}{n}$

Explanation

Use arrows to label the sides correctly.

(opposite angle θ)



(Hypotenuse is always opposite the right angle.)

SOH CAH TOA

$$\sin\theta = \frac{O}{H} = \frac{m}{x}$$

$$\cos\theta = \frac{A}{H} = \frac{n}{x}$$

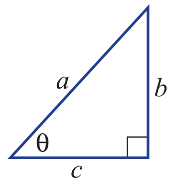
$$\tan\theta = \frac{O}{A} = \frac{m}{n}$$

ISBN: 9781107687028 Cambridge University Press

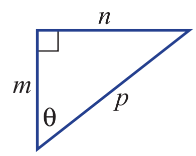
5 For each of the following triangles, write a ratio for:

i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$

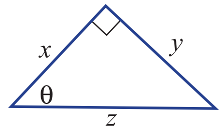
a

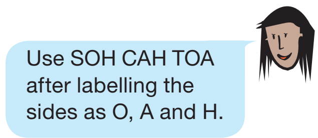


b

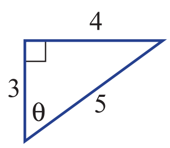


c

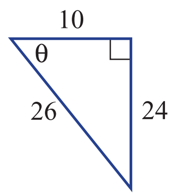




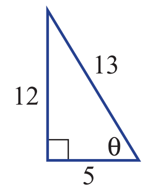
d



e

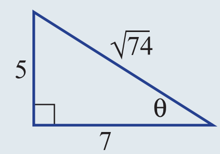


f

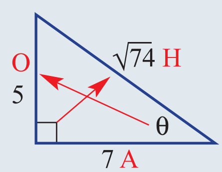


Example 9 Writing a trigonometric ratio

Write down the ratio of $\cos\theta$ for:



Solution	Explanation
$\cos\theta = \dfrac{A}{H}$	Label the sides of the triangle.
$\cos\theta = \dfrac{7}{\sqrt{74}}$	

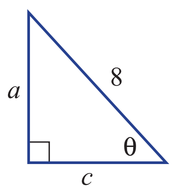


SOH CAH TOA tells us $\cos\theta$ is $\dfrac{\text{adjacent}}{\text{hypotenuse}}$.

Substitute the values for the adjacent (A) and hypotenuse (H).

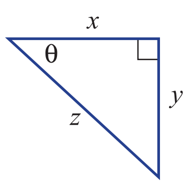
6 Write the trigonometric ratio asked for in each of the following.

a



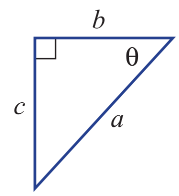
$\tan\theta =$

b



$\sin\theta =$

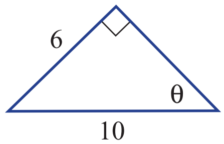
c



$\cos\theta =$

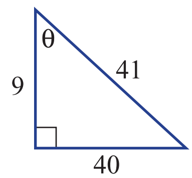
ISBN: 9781107687028

d



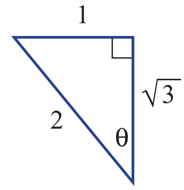
$\sin\theta =$

e



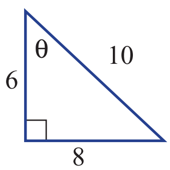
$\sin\theta =$

f



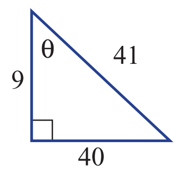
$\tan\theta =$

g



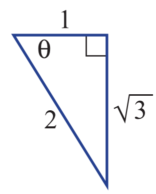
$\tan\theta =$

h



$\cos\theta =$

i

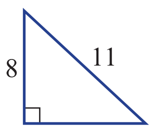


$\tan\theta =$

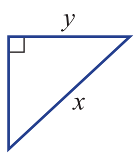
Problem-solving and Reasoning

7 Copy each of these triangles and mark the angle θ that will enable you to write a ratio for sin θ.

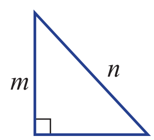
a



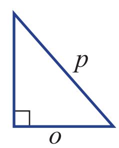
b



c



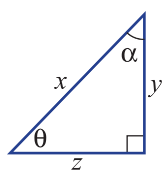
d



8 For the triangle shown on the right, write a ratio for:

a $\sin\theta$ **b** $\sin\alpha$ **c** $\cos\theta$

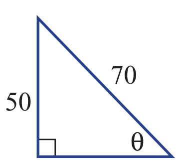
d $\cos\alpha$ **e** $\tan\theta$ **f** $\tan\alpha$



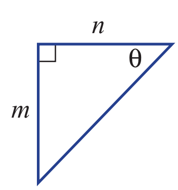
θ and α are letters of the Greek alphabet that are used to mark angles.

9 For each of the triangles below, decide which trigonometric ratio (sin, cos or tan) you would use.

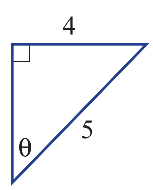
a



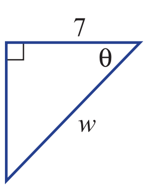
b



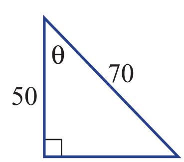
c



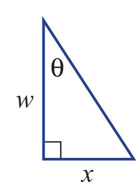
d



e



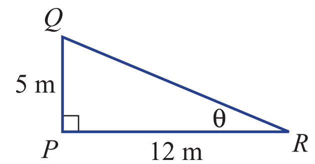
f



ISBN: 9781107687028 Cambridge University Press

10 Consider triangle PQR.

- **a** Use Pythagoras' theorem to find the length of QR.
- **b** Write down the ratio of $\sin\theta$.



11 For a given right-angled triangle, $\sin\theta = \dfrac{1}{2}$.

- **a** Draw up a right-angled triangle and show this information.
- **b** What is the length of the third side? Use Pythagoras' theorem.
- **c** Find the value of: **i** $\cos\theta$ **ii** $\tan\theta$

Relationship between sine and cosine

12 Use your calculator to complete the table, answering to three decimal places where necessary.

- **a** For what angle is $\sin\theta = \cos\theta$?
- **b** Copy and complete:
  - **i** $\sin 5° = \cos$ ____°
  - **ii** $\sin 10° = \cos$ ____°
  - **iii** $\sin 60° = \cos$ ____°
  - **iv** $\sin 90° = \cos$ ____°
- **c** Write down a relationship, in words, between sin and cos.
- **d** Why do you think it's called cosine?

For most calculators, you enter the values in the same order as they are written. That is, sin 30° → $\boxed{\text{sin}}$ 30 = 0.5.

Angle (θ)	$\sin\theta$	$\cos\theta$
0°		
5°		
10°		
15°		
20°		
25°		
30°		
35°		
40°		
45°		
50°		
55°		
60°		
65°		
70°		
75°		
80°		
85°		
90°		

ISBN: 9781107687028 Cambridge University Press

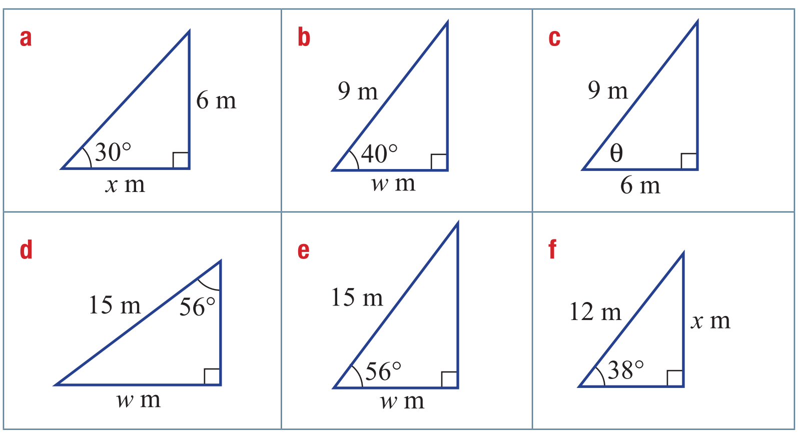
9.5 Finding side lengths

In any right-angled triangle, given one of the acute angles and a side length, you can find the length of the other two sides. This can help builders find special lengths in right-angled triangles if they know an angle and the length of another side.

Trigonometry can help builders work out lengths of timbers and angles of roofs.

Let's start: Is it sin, cos or tan?

Out of the six triangles below, only two provide enough information to use the sin ratio. Which two triangles are they?

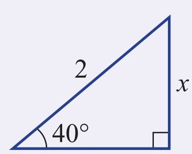


Key ideas

- To find a missing side given a right-angled triangle with one acute angle and one of the sides:
  - label the triangle using O (opposite), A (adjacent) and H (hypotenuse)
  - use SOH CAH TOA to decide on the correct trigonometric ratio
  - write down the relationship
  - solve the equation, using your calculator, to find the unknown.

Write $\rightarrow \dfrac{x}{2} = \sin 40°$

Solve $\rightarrow x = 2 \times \sin 40°$



ISBN: 9781107687028

Cambridge University Press

Exercise 9E

Understanding

1 Use a calculator to find the value of each of the following, correct to four decimal places.

a $\sin 10°$ **b** $\cos 10°$ **c** $\tan 10°$
d $\tan 30°$ **e** $\cos 40°$ **f** $\sin 70°$
g $\cos 80°$ **h** $\tan 40°$ **i** $\sin 80°$
j $\sin 60°$ **k** $\cos 50°$ **l** $\tan 60°$

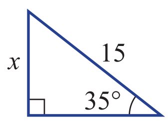
Locate the sin, cos and tan buttons on your calculator.

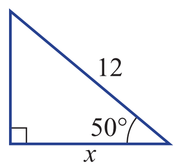
2 Evaluate each of the following, correct to two decimal places.

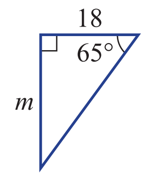
a $12 \tan 10°$ **b** $12 \sin 25°$ **c** $18 \tan 60°$
d $56 \sin 56°$ **e** $8 \tan 45°$ **f** $20 \sin 70°$
g $6 \cos 70°$ **h** $5 \cos 15°$ **i** $27.4 \sin 18°$

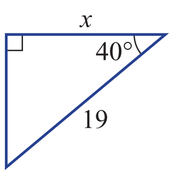
On your calculator, enter 12 tan 10° as 12 × tan 10.

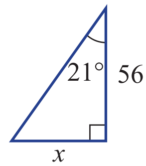
3 Decide which of the three trigonometric ratios is suitable for these triangles.

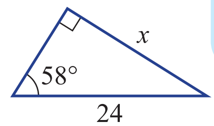
a


b


c


d


e


f


Remember to label the triangle and think SOH CAH TOA. Consider which two sides are involved.

Fluency

Example 10 Finding the value of *x* in a trigonometric equation

Find the value of x, correct to two decimal places, for $\cos 30° = \frac{x}{12}$.

Solution	Explanation
$\cos 30° = \frac{x}{12}$	Multiply both sides by 12 to get x on its own.
$x = 12 \times \cos 30°$	$12 \times \cos 30° = \frac{x}{\cancel{12}_1} \quad \cancel{12}_1$
$= 10.392\,30\ldots$	Use your calculator.
$= 10.39$ (two decimal places)	Round as indicated.

4 Find x in these equations, correct to two decimal places.

a $\sin 20° = \frac{x}{4}$ **b** $\cos 43° = \frac{x}{7}$ **c** $\tan 85° = \frac{x}{8}$
d $\tan 30° = \frac{x}{24}$ **e** $\sin 50° = \frac{x}{12}$ **f** $\cos 40° = \frac{x}{12}$

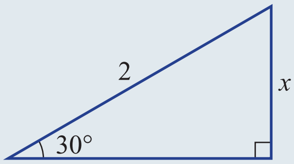
ISBN: 9781107687028

9E

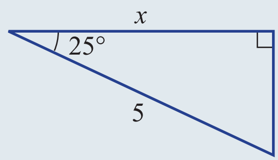
Example 11 Finding a missing side using SOH CAH TOA

Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

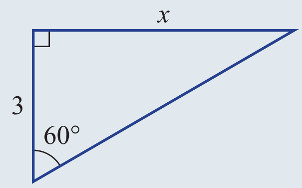
a



b



c



Solution

Explanation

a

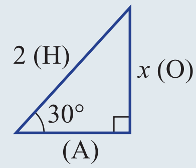
$$\sin\theta = \frac{O}{H}$$

$$\sin 30^\circ = \frac{x}{2}$$

$$2 \times \sin 30^\circ = x$$

$$\therefore\ x = 1$$

Label the triangle and decide on your trig ratio SOH CAH TOA.
Write the ratio.
Substitute values.
Solve the equation, using your calculator.



b

$$\cos\theta = \frac{A}{H}$$

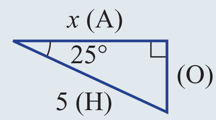
$$\cos 25^\circ = \frac{x}{5}$$

$$5 \times \cos 25^\circ = x$$

$$x = 4.5315\ldots$$

$$\therefore\ x = 4.53$$

Label the triangle.
SOH CAH TOA
Write the ratio.
Substitute values.
Solve the equation, using your calculator.
Round to two decimal places.



c

$$\tan\theta = \frac{O}{A}$$

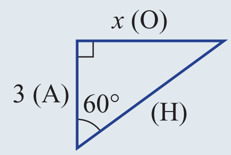
$$\tan 60^\circ = \frac{x}{3}$$

$$3 \times \tan 60^\circ = x$$

$$x = 5.1961\ldots$$

$$\therefore\ x = 5.20$$

Label the triangle.
SOH CAH TOA
Write the ratio.
Substitute values.
Solve the equation, using your calculator.
Round to two decimal places.



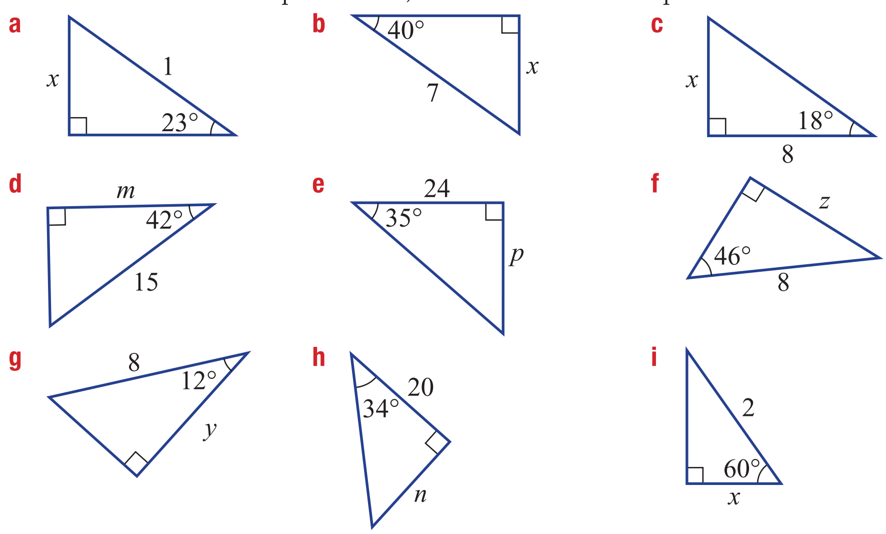
ISBN: 9781107687028

5 For the triangles given below:

i copy each one and label the three sides opposite (O), adjacent (A) and hypotenuse (H)

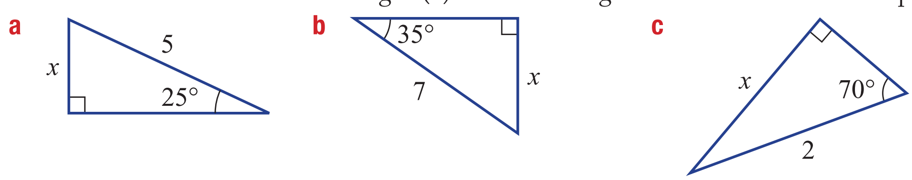
ii decide on a trigonometric ratio

iii find the value of each pronumeral, correct to two decimal places



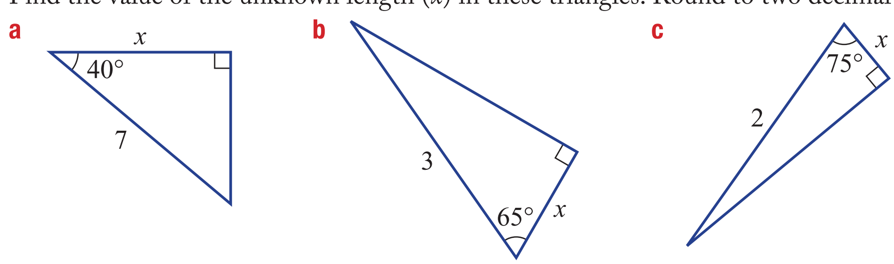
Use SOH CAH TOA to help you decide which ratio to use. If O and H are involved, use sin etc.

6 Find the value of the unknown length (x) in these triangles. Round to two decimal places.



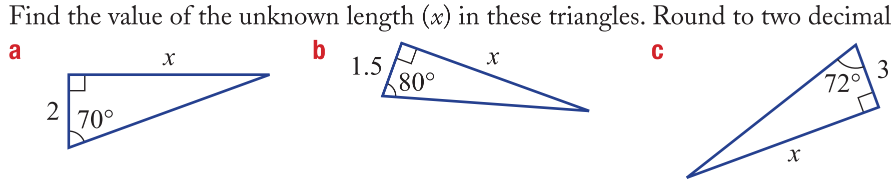
What ratio did you use for each of these?

7 Find the value of the unknown length (x) in these triangles. Round to two decimal places.



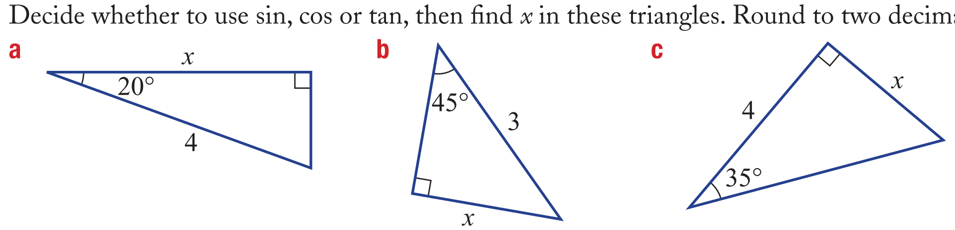
These three all use cos.

8 Find the value of the unknown length (x) in these triangles. Round to two decimal places.



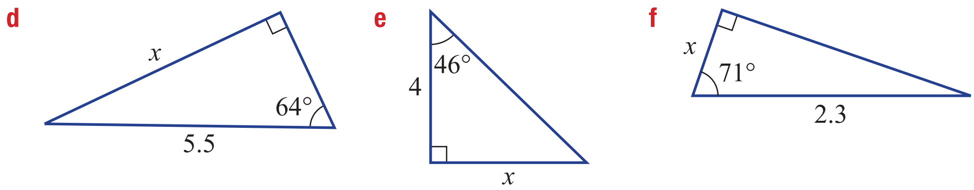
These all use tan.

9 Decide whether to use sin, cos or tan, then find x in these triangles. Round to two decimal places.



ISBN: 9781107687028

9E

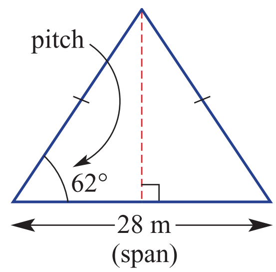


Problem-solving and Reasoning

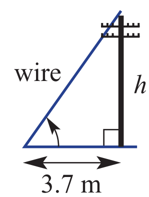
10 a Find the height of this isosceles triangle, which is similar to a roof truss, to two decimal places.

b If the span doubles to 56 m, what is the height of the roof, to two decimal places?

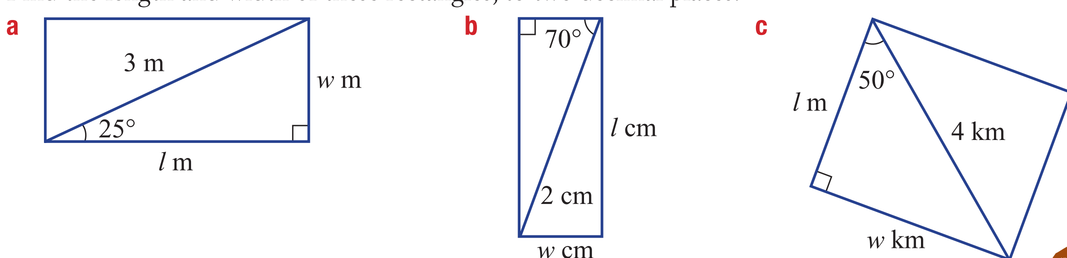
In an isosceles triangle, the perpendicular cuts the base in half.



11 The stay wire of a power pole joins the top to the ground. It makes an angle of 62° with the ground. It is fixed to the ground 3.7 m from the bottom of the pole. How high is the pole, correct to two decimal places?



12 Find the length and width of these rectangles, to two decimal places.



Use the hypotenuse in each calculation.

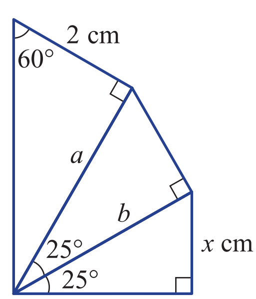
Accuracy and errors

13 Our aim is to find the value of x, correct to two decimal places, by first finding the value of a and b.

a Find the value of a then b then x, using one decimal place for a and b.

b Repeat this process, finding a and b correct to three decimal places each, before finding x.

c Does it make any difference to your final answer for x if you round off the values of a and b during calculations?



ISBN: 9781107687028

Cambridge University Press

9.6 Finding more side lengths

So far, we have been dealing with equations that have the pronumeral in the numerator. However, sometimes the unknown is in the denominator and these problems can be solved with an extra step in your mathematical working.

Sometimes builders need to find the value of the denominator when using trigonometry.

Let's start: Solving equations with x in the denominator

Consider the equations $\frac{x}{3} = 4$ and $\frac{3}{x} = 4$.

- Do the equations have the same solution?
- What steps are used to solve the equations?
- Now solve $\frac{4}{x} = \sin 30°$ and $\frac{2}{x} = \cos 40°$.

Key ideas

If the unknown value is in the **denominator**, you need to do two algebraic steps to find the unknown. For example,

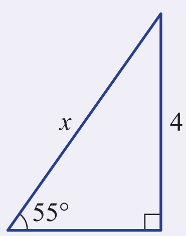
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 55° = \frac{4}{x}$$

$$x \times \sin 55° = 4$$

$$x = \frac{4}{\sin 55°}$$

$$= 4.88 \text{ to two decimal places}$$



Denominator The part of a fraction that sits below the dividing line

Exercise 9F

Understanding

1 Find the value, correct to two decimal places, of:

a $\frac{10}{\tan 30°}$

b $12 \div \sin 60°$

c $\frac{15}{\tan 8°}$

d $\frac{12.4}{\tan 32°}$

e $\frac{15.2}{\sin 38°}$

f $\frac{9}{\cos 47°}$

For part **a**, enter 10 ÷ tan 30 into your calculator.

ISBN: 9781107687028

2 Solve these equations for x.

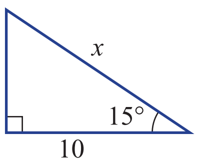
a $\frac{4}{x} = 2$ **b** $\frac{10}{x} = 2$ **c** $\frac{15}{x} = 30$

d $\frac{1.2}{x} = 1.2$ **e** $\frac{0.6}{x} = 6$ **f** $\frac{9}{x} = 90$

To solve these, you will need 2 steps.

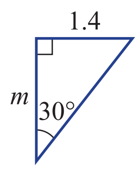
3 For each of these triangles, complete the required trigonometric ratio.

a



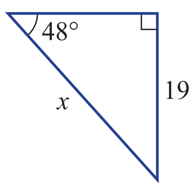
$\cos 15^\circ = \frac{\square}{\square}$

b



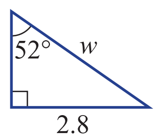
$\tan 30^\circ = \frac{\square}{\square}$

c



$\sin 48^\circ = \frac{\square}{\square}$

d



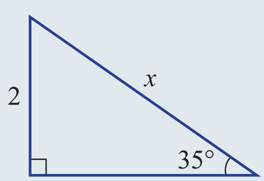
$\sin 52^\circ = \frac{\square}{\square}$

Fluency

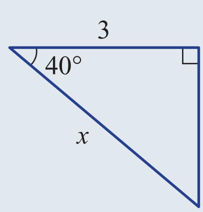
Example 12 Finding the denominator

Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.

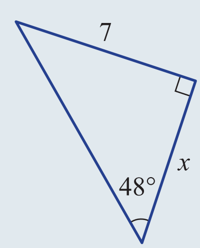
a



b



c



Solution	Explanation
a $\sin 35^\circ = \frac{2}{x}$	Use $\sin\theta = \frac{O}{H}$ as we can use the opposite (2) and hypotenuse (x).
$x \times \sin 35^\circ = 2$	Multiply both sides by x.
$x = \frac{2}{\sin 35^\circ}$	Divide both sides by $\sin 35^\circ$ to get x on its own. Recall that $\sin 35^\circ$ is just a number.
$\therefore x = 3.49$	Evaluate and round.

ISBN: 9781107687028 Cambridge University Press

b $\cos 40° = \dfrac{3}{x}$ — Use $\cos\theta = \dfrac{A}{H}$ as we can use the adjacent (3) and hypotenuse (x).

$x \times \cos 40° = 3$ — Multiply both sides by x.

$x = \dfrac{3}{\cos 40°}$ — Divide both sides by cos 40° to get x on its own.

$\therefore\ x = 3.92$ — Evaluate and round.

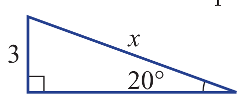
c $\tan 48° = \dfrac{7}{x}$ — Use $\tan\theta = \dfrac{O}{A}$ as we can use the adjacent (x) and opposite (7).

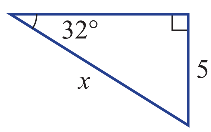
$x \times \tan 48° = 7$ — Multiply both sides by x.

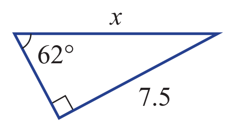
$x = \dfrac{7}{\tan 48°}$ — Divide both sides by tan 48° to get x on its own.

$\therefore\ x = 6.30$ — Evaluate and round.

4 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.

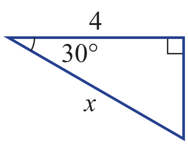
a


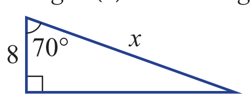
b


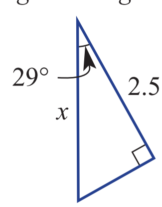
c


In $\sin 20 = \dfrac{3}{x}$, multiply both sides by x first.

5 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.

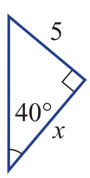
a


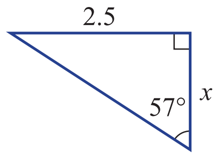
b


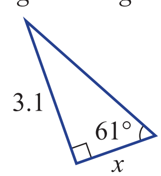
c


$\cos\theta = \dfrac{A}{H}$

6 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.

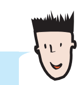
a


b


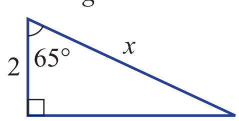
c


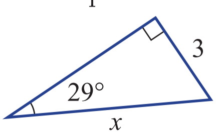
$\tan\theta = \dfrac{O}{A}$

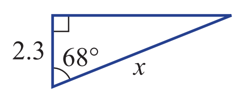
7 By first deciding whether to use sin θ, cos θ or tan θ, find the value of x in these triangles. Round to two decimal places.

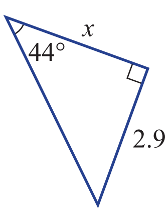


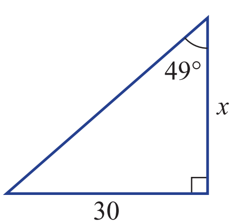
SOH CAH TOA

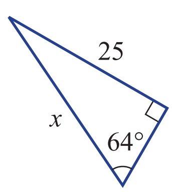
a


b


c


d


e


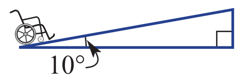
f


ISBN: 9781107687028

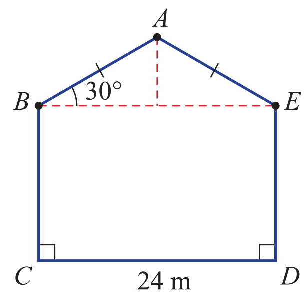
9F

Problem-solving and Reasoning

8 The recommended angle of a wheelchair ramp to the horizontal is approximately 10 degrees. How long is the ramp if the horizontal distance is 2.5 metres? Round to two decimal places.

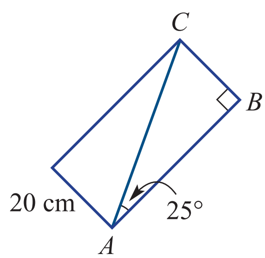


9 The roof of this barn has a pitch of 30°, as shown. Find the length of roof section AB, to one decimal place.

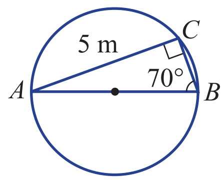


10 Find the length AB and BC in these shapes. Round to two decimal places.

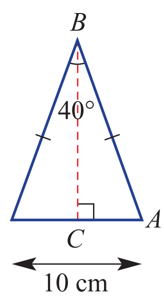
a



b



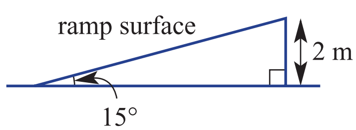
c

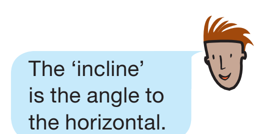


11 The ramp shown has an incline angle of 15° and a height of 2 m. Find, correct to three decimal places:

a the base length of the ramp

b the length of the ramp surface

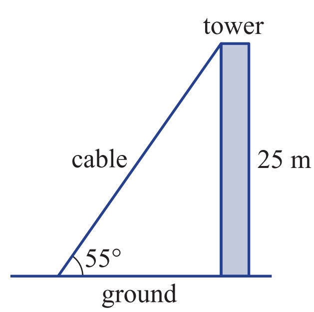




12 For this communications tower, find, correct to one decimal place:

a the length of the cable

b the distance from the base of the tower to the point where the cable is attached to the ground

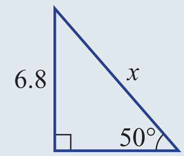


ISBN: 9781107687028 Cambridge University Press

Inverting the fraction

Shown below is another way of solving trigonometric equations with x in the denominator.

Find x to two decimal places.



$$\sin 50° = \frac{6.8}{x}$$

Invert both fractions so x is in the numerator.

$$\frac{1}{\sin 50°} = \frac{x}{6.8}$$

$$\frac{1}{\sin 50°}\ 6.8 = \frac{x}{6.8}\ 6.8$$

Multiply both sides by 6.8 to get x on its own.

$$x = \frac{1}{\sin 50°}\quad 6.8$$

$$x = \frac{6.8}{\sin 50°}$$

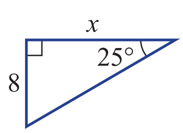
$x = 8.876\,76...$

$x = 8.88$ (to two decimal places)

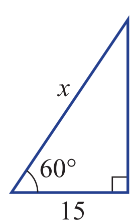
Use your calculator.
Round as required.

13 Use the method shown above to find the value of x, to two decimal places where necessary, in each of the following.

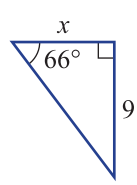
a



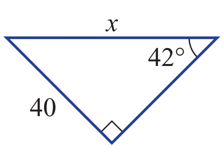
b



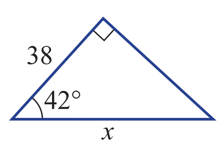
c



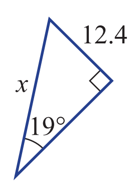
d



e



f



ISBN: 9781107687028

9.7 Finding angles

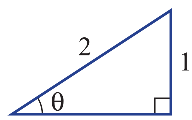
Given two side lengths of a right-angled triangle, you can find either of the acute angles. There are many different situations where you might be given two side lengths of a right-angled triangle and be asked to find the associated angles.

The wii can tell if you are standing straight or on a slight angle.

Let's start: Knowing the angle

Imagine a triangle that produces $\sin \theta = 0.5$.

- Use your calculator and trial and error to find a value of θ for which $\sin \theta = 0.5$.
- Repeat for $\tan \theta = 1$ and $\cos \theta = \dfrac{\sqrt{3}}{2}$.
- Do you know of a quicker method, rather than using trial and error?



Key ideas

- To find an angle, you use inverse trigonometric ratios on your calculator.

  If $\sin \theta = x$, then $\theta = \sin^{-1}(x)$; $\sin^{-1}$ is inverse sin

  $\cos \theta = y$, then $\theta = \cos^{-1}(y)$; $\cos^{-1}$ is inverse cos

  $\tan \theta = z$, then $\theta = \tan^{-1}(z)$; $\tan^{-1}$ is inverse tan

Exercise 9G

Understanding

1 Given $\sin 53° = 0.7986$, find $\sin^{-1}(0.7986)$.

2 Use your calculator to evaluate, correct to the nearest whole degree:

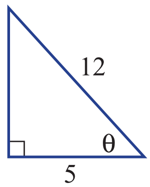
a $\sin^{-1}(0.71)$ **b** $\cos^{-1}(0.866)$ **c** $\tan^{-1}(1.6)$

Most calculators use $\boxed{\text{shift}}$ to access $\sin^{-1}$ or $\cos^{-1}$ or $\tan^{-1}$.

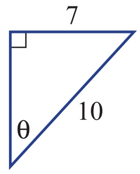
ISBN: 9781107687028

3 Write down the trigonometric ratio for these triangles. Is it sin, cos or tan?

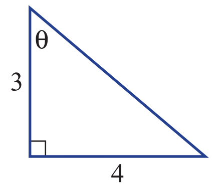
a



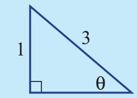
b



c



Look for: SOH CAH TOA



$\sin\theta = \frac{1}{3}$

Fluency

Example 13 Finding an angle

Find the angle θ, correct to the nearest degree, in each of the following.

a $\sin\theta = \frac{2}{3}$ **b** $\cos\theta = \frac{1}{2}$ **c** $\tan\theta = 1.7$

Solution	Explanation
a $\sin\theta = \frac{2}{3}$ $\theta = \sin^{-1}\left(\frac{2}{3}\right)$ $\theta = 42°$	Look for the $\sin^{-1}$ button on your calculator. Round as required.
b $\cos\theta = \frac{1}{2}$ $\theta = \cos^{-1}\left(\frac{1}{2}\right)$ $\theta = 60°$	Look for the $\cos^{-1}$ button on your calculator.
c $\tan\theta = 1.7$ $\theta = \tan^{-1}(1.7)$ $\theta = 60°$	Look for the $\tan^{-1}$ button on your calculator. Round to the nearest degree.

4 Find the angle θ, to the nearest degree, for the following.

a $\sin\theta = \frac{1}{2}$ **b** $\cos\theta = \frac{3}{5}$ **c** $\sin\theta = \frac{7}{8}$

d $\tan\theta = 1$ **e** $\tan\theta = \frac{7}{8}$ **f** $\sin\theta = \frac{8}{10}$

g $\cos\theta = \frac{2}{3}$ **h** $\sin\theta = \frac{1}{10}$ **i** $\cos\theta = \frac{4}{5}$

j $\tan\theta = 6$ **k** $\cos\theta = \frac{3}{10}$ **l** $\tan\theta = \sqrt{3}$

m $\sin\theta = \frac{4}{6}$ **n** $\cos\theta = \frac{4}{6}$ **o** $\tan\theta = \frac{4}{6}$

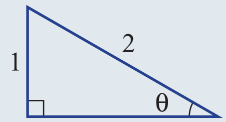
Remember: use $\boxed{\sin^{-1}}$, $\boxed{\cos^{-1}}$ or $\boxed{\tan^{-1}}$ on the calculator.

ISBN: 9781107687028

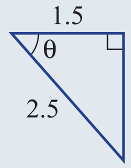
Example 14 Using SOH CAH TOA to find angles

Find θ in the following right-angled triangles, correct to two decimal places where necessary.

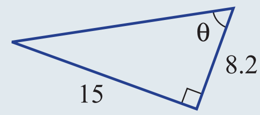
a



b



c

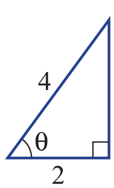


Solution	Explanation
a $\sin\theta = \dfrac{O}{H}$	Use sin θ since we know the **o**pposite and the **h**ypotenuse.
$\sin\theta = \dfrac{1}{2}$	Substitute **O** = 1 and **H** = 2
$\theta = \sin^{-1}\left(\dfrac{1}{2}\right)$ $= 30°$	Use your calculator to find $\sin^{-1}\left(\dfrac{1}{2}\right)$.
b $\cos\theta = \dfrac{A}{H}$	Use cos θ since we know the **a**djacent and the **h**ypotenuse.
$\cos\theta = \dfrac{1.5}{2.5}$	Substitute **A** = 1.5 and **H** = 2.5
$\theta = \cos^{-1}\left(\dfrac{1.5}{2.5}\right)$ $= 53.13°$	Use your calculator to find $\cos^{-1}\left(\dfrac{1.5}{2.5}\right)$. Round to two decimal places.
c $\tan\theta = \dfrac{O}{A}$	Use tan θ since we know the **o**pposite and the **a**djacent.
$\tan\theta = \dfrac{15}{8.2}$	Substitute **O** = 15 and **A** = 8.2
$\theta = \tan^{-1}\left(\dfrac{15}{8.2}\right)$ $= 61.34°$	Use your calculator to find $\tan^{-1}\left(\dfrac{15}{8.2}\right)$. Round to two decimal places.

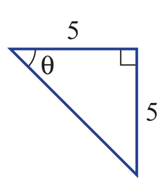
5 Use one of sin, cos or tan to find θ in these triangles, rounding to two decimal places where necessary.

SOH CAH TOA

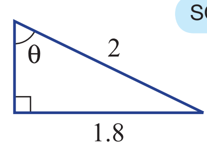
a



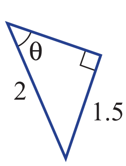
b



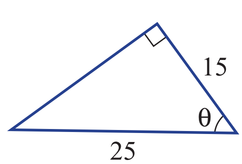
c



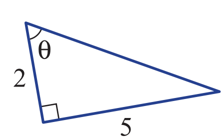
d



e

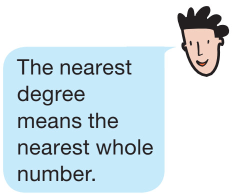


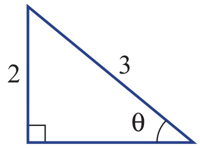
f

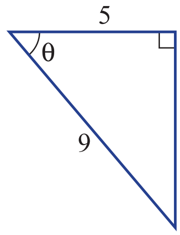


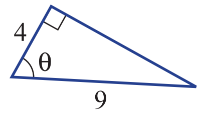
ISBN: 9781107687028

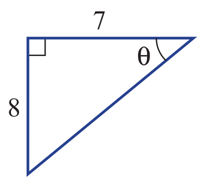
6 Find the angle θ, correct to the nearest degree, in these triangles. You will need to decide whether to use sin θ, cos θ or tan θ.

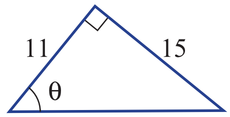


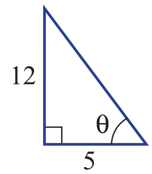
a


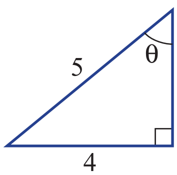
b


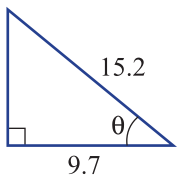
c


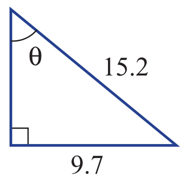
d


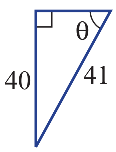
e


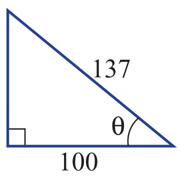
f


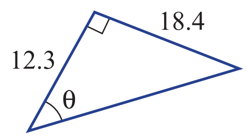
g


h


i


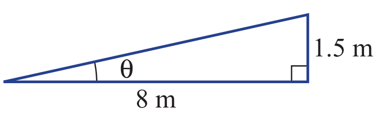
j


k


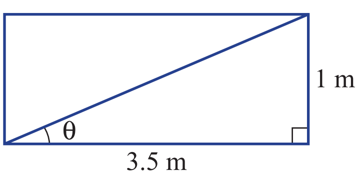
l


Problem-solving and Reasoning

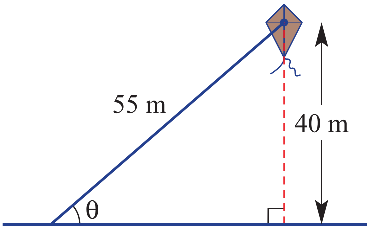
7 A ramp is 8 m long and 1.5 m high. Find the angle the ramp makes with the ground, correct to two decimal places.



8 A rectangular piece of wood 1 m wide and 3.5 m long is to be cut across the diagonal. Find the angle the cut makes with the long side (correct to two decimal places).



9 At what angle to the ground is a kite (shown) with height 40 m and string length 55 m? Round to two decimal places.

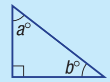


ISBN: 9781107687028

10 Find the two acute angles in a right-angled triangle with the given side lengths, correct to one decimal place.

- **a** hypotenuse 5 cm, other side 3 cm
- **b** hypotenuse 7 m, other side 4 m
- **c** hypotenuse 0.5 mm, other side 0.3 mm
- **d** the two shorter side lengths are 3 cm and 6 cm
- **e** the two shorter side lengths are 10 m and 4 m

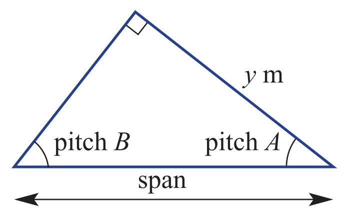
- Draw a picture.
- Use SOH CAH TOA.
- Find one acute angle using trigonometry.
- Remember: all triangles have an angle sum of 180°.



$a + b + 90 = 180$

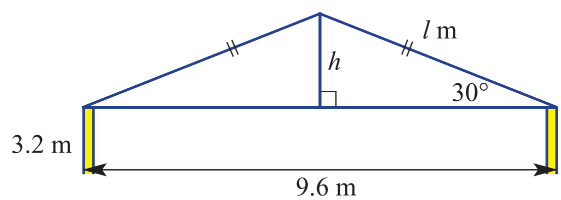
Building construction

11 A roof is pitched so that the angle at its peak is 90°. If each roof truss spans 10.5 m and distance y is 7.2 m, find the pitch angles A and B, to the nearest whole number.

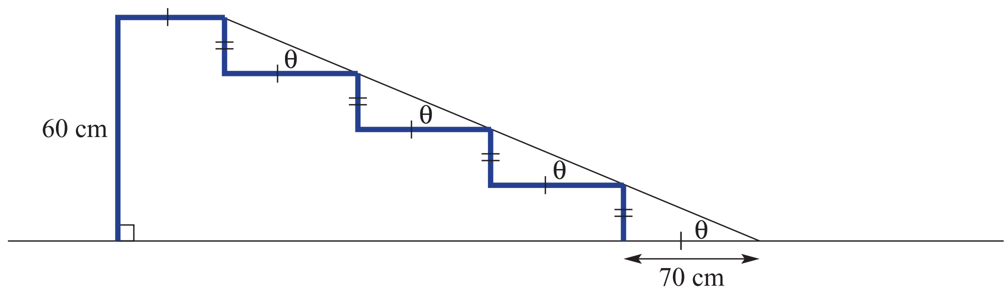


12 a Find the length of the slats (l metres) needed along each hypotenuse for this roof cross-section, correct to two decimal places.

b Find the height of the highest point of the roof above ground level, correct to two decimal places.



13 A ramp is to be constructed to allow disabled access over a set of existing stairs, as shown.



- **a** What angle does the ramp make with the ground, to the nearest degree?
- **b** Government regulations state that the ramp cannot be more than 13° to the horizontal. Does this ramp meet these requirements?
- **c** How long is the ramp? Round to one decimal place.

ISBN: 9781107687028

Cambridge University Press

9.8 Angles of elevation and depression

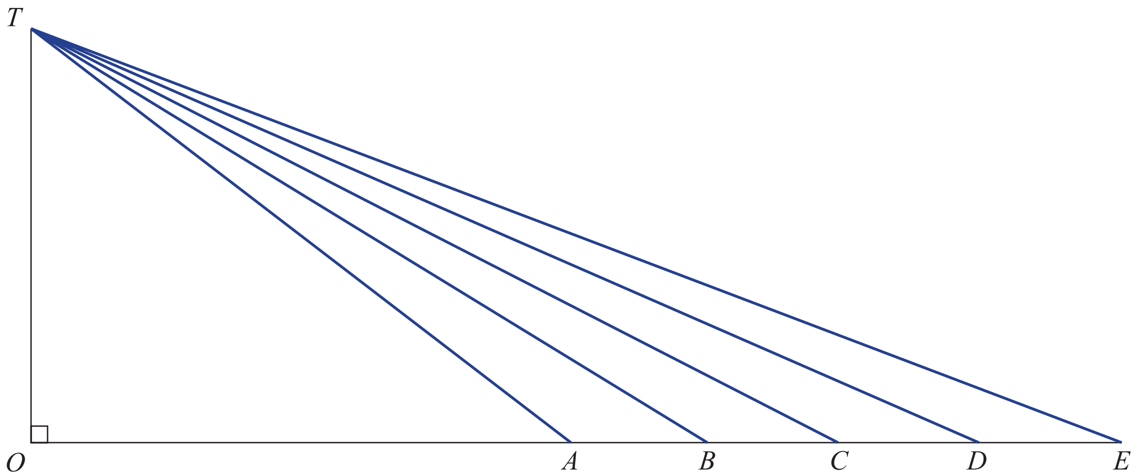
Many applications of trigonometry involve angles of elevation and angles of depression. These angles are measured up or down from a horizontal level.

Let's start: How close should you sit?

The diagram below shows an outdoor movie screen (OT). The point T is the top of the screen. The points A–E are the five rows of seats in the theatre, from which a person's line of sight is taken. The line OE is the horizontal line of sight.

Use your protractor to measure the angle of elevation from each point along the horizontal to the top of the movie screen.

Where should you sit if you wish to have an angle of elevation between 25° and 20° and not be in the first or last row of the theatre?

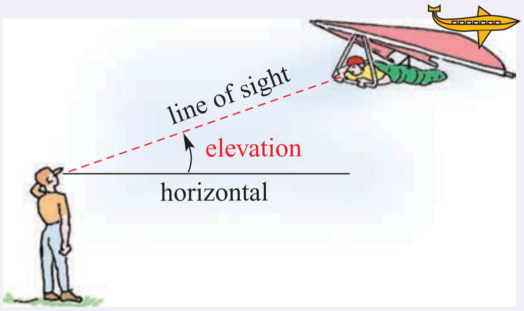


ISBN: 9781107687028

Cambridge University Press

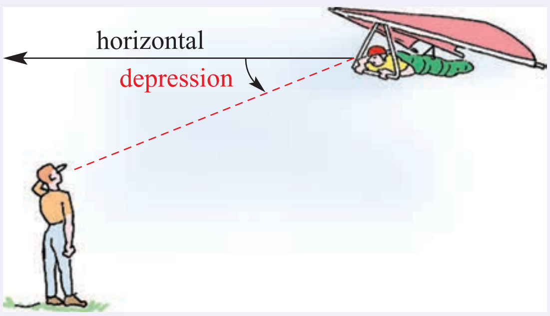
Key ideas

- Looking up to an object forms an **angle of elevation**.



Angle of elevation The angle of your line of sight from the horizontal when looking up at an object

- Looking down to an object forms an **angle of depression**.



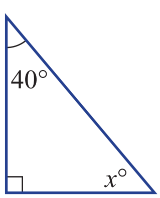
Angle of depression The angle of your line of sight from the horizontal when looking down at an object

Exercise 9H

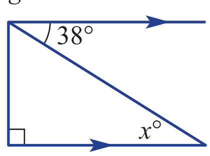
Understanding

1 Find the value of x in each triangle.

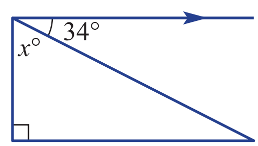
a



b



c

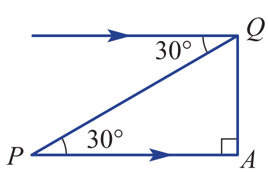


2 For this diagram:

a What is the angle of elevation of Q from P?

b What is the angle of depression of P from Q?

c What is the size of $\angle PQA$?

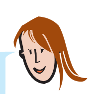


3 For each description, draw a triangle diagram that matches the information given.

a The angle of elevation to the top of a tower from a point 50 m from its base is 55°.

b The angle of depression from the top of a 200 m cliff to a boat out at sea is 22°.

c The angle of elevation of the top of a castle wall from a point on the ground 30 m from the castle wall is 33°.



Always measure angles of elevation from the horizontal.

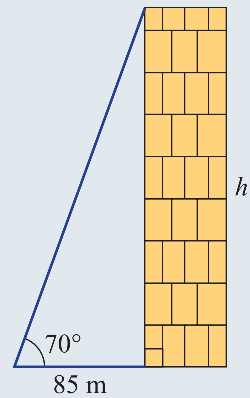
ISBN: 9781107687028

Cambridge University Press

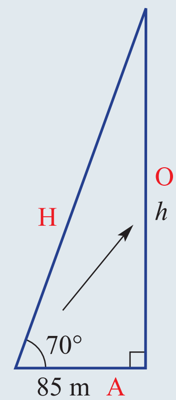
Fluency

Example 15 Using an angle of elevation

To find the height of a tall building, Johal stands 85 m away from its base and measures the angle of elevation at the top of the building as 70°. Find the height of the building correct to the nearest metre.

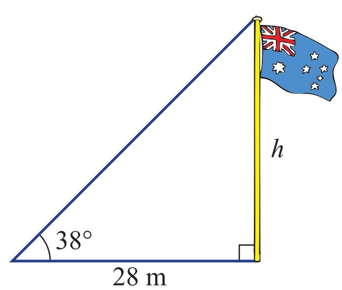


Solution	**Explanation**
$\tan\theta = \frac{O}{A}$	Label the triangle with O, A, and H.
$\tan 70^\circ = \frac{h}{85}$	Use tan, since the Opposite and Adjacent are given.
$h = 85 \times \tan 70^\circ$ $= 233.53558...$ $= 234$ m	Find h by solving the equation. $85 \quad \tan 70^\circ = \frac{h}{85} \quad 85$
∴ the building is 234 m tall.	Round to the nearest metre.

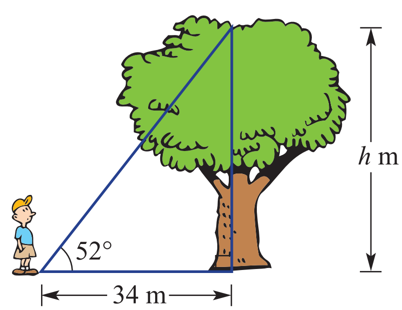


4 Answer the following questions about angles of elevation.

a The angle of elevation to the top of a flagpole from a point 28 m from its base is 38°. How tall is the flagpole, correct to two decimal places?

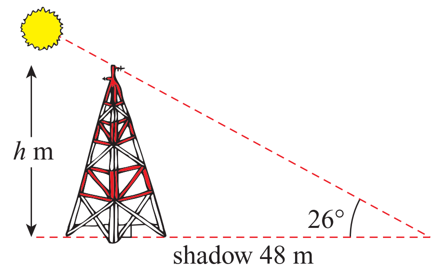


b Alvin is 34 m away from a tree and the angle of elevation to the top of the tree from the ground is 52°. What is the height of the tree, correct to one decimal place?

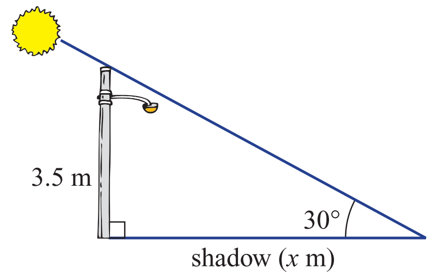


ISBN: 9781107687028

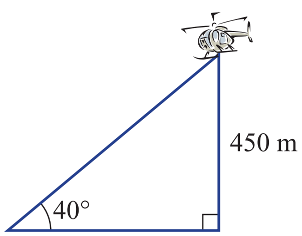
c The Sun's rays shining over a tower make an angle of elevation of 26° and casts a 48 m shadow on the ground. How tall, to two decimal places, is the tower?



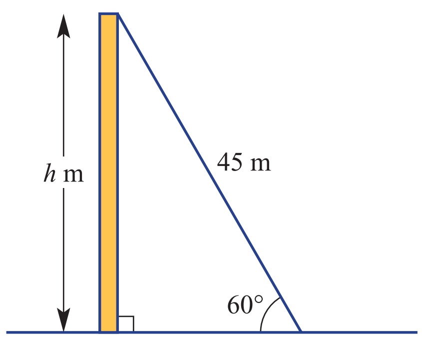
d The Sun makes an angle of elevation of 30° with a lamp post 3.5 m tall. How long is the shadow on the ground, correct to two decimal places?



e The altitude of a hovering helicopter is 450 m, while the angle of elevation from the helipad to the helicopter is 40°. Find the horizontal distance from the helicopter to the helipad, correct to two decimal places.

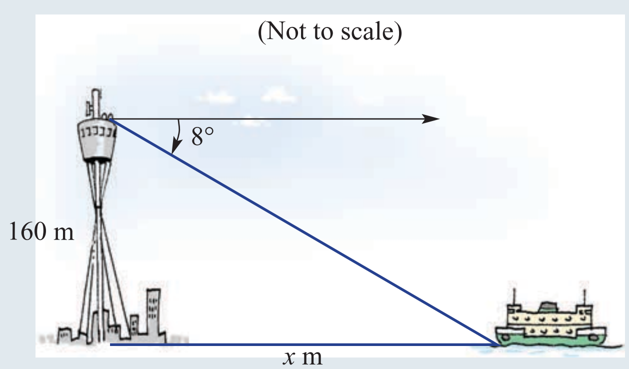


f A cable of length 45 m is anchored from the ground to the top of a communications mast. The angle of elevation of the cable to the top of the mast is 60°. Find the height of the communications mast, correct to two decimal places.



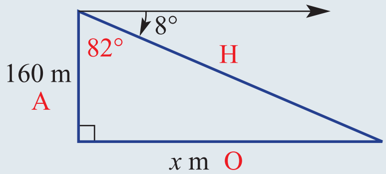
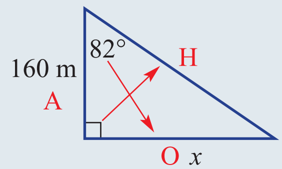
Example 16 Using an angle of depression

From the observation room of Centrepoint Tower in Sydney, height 160 m, the angle of depression of a boat moored at Circular Quay is observed to be 8°. How far from the base of the tower is the boat, correct to the nearest metre?



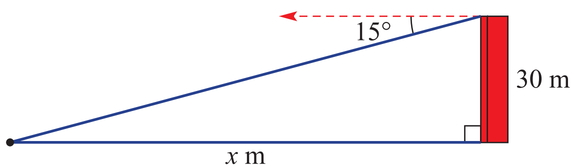
ISBN: 9781107687028

Cambridge University Press

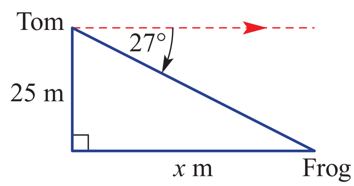
Solution	Explanation
	Draw the triangle and find the angle inside the triangle: $90° - 8° = 82°$ Use this angle to label the triangle. Use tan, since we have the Opposite and Adjacent. 
$\tan\theta = \dfrac{O}{A}$ $\tan 82° = \dfrac{x}{160}$ $x = 160 \times \tan 82°$ $= 1138.459\ldots$ ≈ 1138 $\therefore$ the boat is about 1138 m from the base of the tower.	Find x by solving the equation. Round to the nearest metre.

5 Answer these problems relating to angles of depression.

a The angle of depression from the top of a tower 30 m tall to a point x m from its base is 15°. Find the value of x, correct to one decimal place.

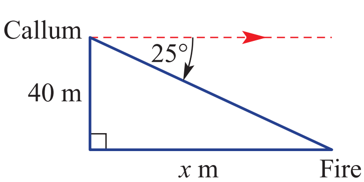


b From a bridge 25 m above a stream, Tom spots two frogs on a lilypad. He estimates the angle of depression to the frogs to be 27°. How far from the bridge are the frogs, to the nearest metre?



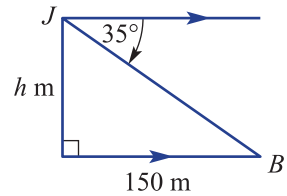
The angle of depression is the angle below the horizontal, looking down at an object.

c From a lookout tower, Callum spots a bushfire at an angle of depression of 25°. If the lookout tower is 40 m high, how far away (to the nearest metre) is the bushfire from the base of the tower?

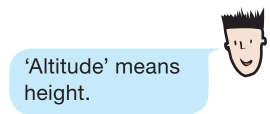


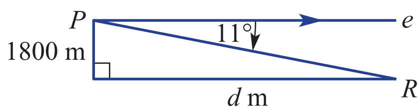
ISBN: 9781107687028

d From the top of a vertical cliff, Jung spots a boat 150 m out to sea. The angle of depression from Jung to the boat is 35°. How many metres (to the nearest whole number) above sea level is Jung?



e A plane is flying 1800 m above the ground. At the time the pilots spot the runway, the angle of depression to the edge of the runway is 11°. How far does the plane have to fly to be above the edge of the runway at its current altitude? Answer to the nearest whole number.

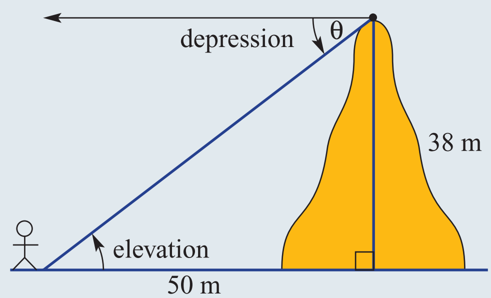




Example 17 Finding angles of elevation and depression

a Find the angle of depression from the top of the hill to a point on the ground 50 m from the middle of the hill. Answer to the nearest degree.

b What is the angle of elevation from the point on the ground to the top of the 38 m hill? Answer to the nearest degree.



Solution

a

$$\tan\theta = \frac{O}{A}$$

$$\tan\theta = \frac{38}{50}$$

$$\theta = \tan^{-1}\left(\frac{38}{50}\right)$$

$$\theta = 37.2348\ldots$$

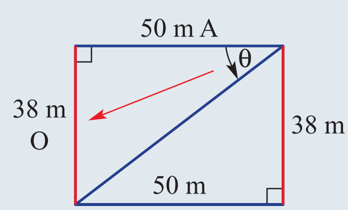
$$\theta \approx 37°$$

Angle of depression is 37°.

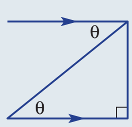
b Angle of elevation is 37°.

Explanation

Aim to find θ.
Redraw the diagram as a rectangle so that θ is inside the triangle.



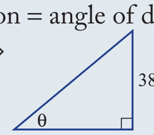
Label the triangle, opposite and adjacent.
Use tan.



Alternate angles are equal when lines are parallel.

angle of elevation = angle of depression

Alternatively ⇒



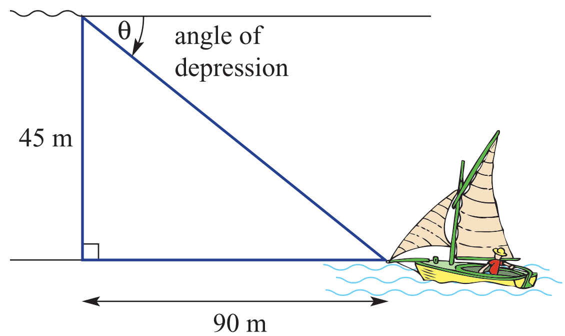
$\Rightarrow \tan\theta = \dfrac{38}{50}$

ISBN: 9781107687028

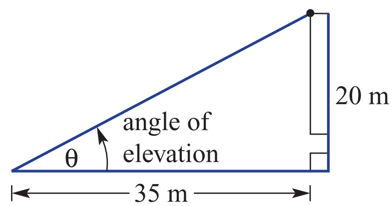
Cambridge University Press

6 Answer these questions about finding angles of elevation and depression. Round all answers to one decimal place.

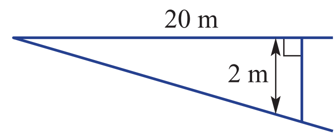
a From the top of a vertical cliff, Jacky spots a boat 90 m out to sea. If the top of the cliff is 45 m above sea level, find the angle of depression from the top of the cliff to the boat.



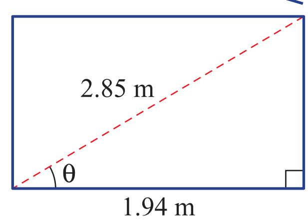
b Find the angle of elevation from a person sitting 35 m from a movie screen to the top of the screen 20 m above the ground.



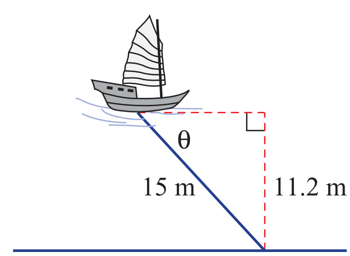
c A person sits 20 m away from a screen that is 2 m below the horizontal viewing level. Find the angle of depression of the person's viewing level to the screen.



d A diagonal cut 2.85 m long is to be made on a piece of plaster board attached to a wall, as shown. The base of the plaster board measures 1.94 m. Find the angle of elevation of the diagonal cut from the base.

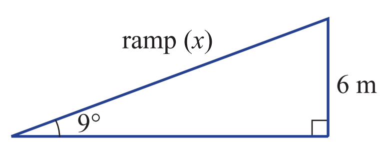


e A 15 m chain with an anchor attached as shown is holding a boat in a position against a current. If the water depth is 11.2 m, find the angle of depression from the boat to where the anchor is fixed to the sea bed.



Problem-solving and Reasoning

7 A ramp for wheelchairs is constructed to a footbridge 6 m high. The angle of elevation is to be 9°. What is the length of the ramp, correct to two decimal places?

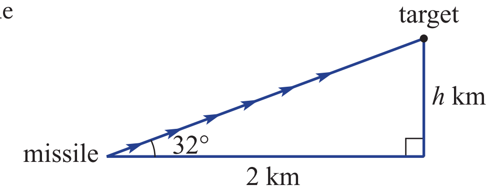


ISBN: 9781107687028

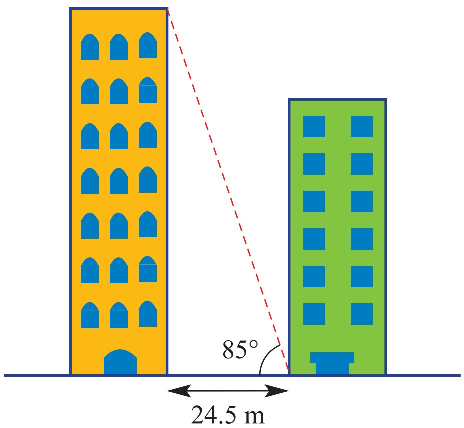
Cambridge University Press

9H

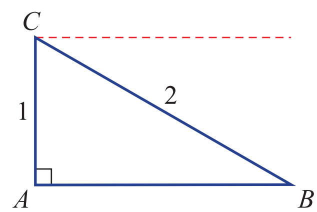
8 A missile is launched at an angle of elevation of 32°. If the target is 2 km away on the horizontal, how far above ground level is the target, correct to two decimal places?



9 The distance between two buildings shown is 24.5 m. Find the height of the taller building, correct to two decimal places, if the angle of elevation from the base of the shorter building to the top of the taller building is 85°.



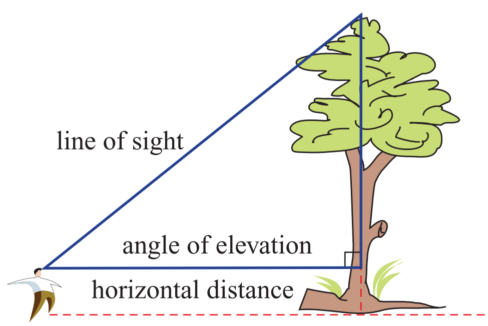
10 For this triangle:
- **a** find the angle of elevation from B to C
- **b** state the angle of depression from C to B
- **c** describe the relationship that exists between these two angles
- **d** find the length AB, correct to one decimal place



Practical trigonometry: measuring heights

11 It is not always possible or practical to measure the height of an object directly. Here you will find the height of a difficult object.

Select a building or other structure (e.g. a statue or flagpole) to calculate the height of. You must be able to measure right up to the base of the structure.
- **a** Choose a position from which you can see the top of your structure, and measure the angle of elevation, θ, from your eye level. (Use an inclinometer, if your teacher has one, or simply estimate the angle using a protractor.)
- **b** Measure the distance along the ground (d) from your location to the base of the structure.



- **c** Calculate the height of the structure. *Remember to make an adjustment for the height of your eye level from the ground.*
- **d** Move to another position and repeat the measurements. Calculate the height using your new measurements.
- **e** Was there much difference between the calculated heights? Suggest reasons for any differences.

ISBN: 9781107687028

9.9 Direction and bearings

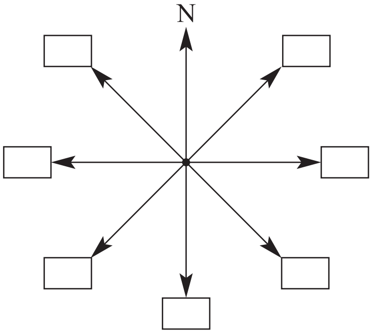
True bearings are used to communicate a direction, and are important in navigation. Ships, planes, bushwalkers and the military all use bearings when communicating direction.

Planes and ships need bearings to navigate the skys and seas.

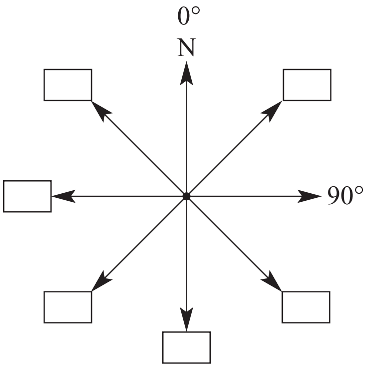
Let's start: Compass bearings

Work together as a class to label the 8-point compass rose using letters/words in **a** and angles in **b**.

a



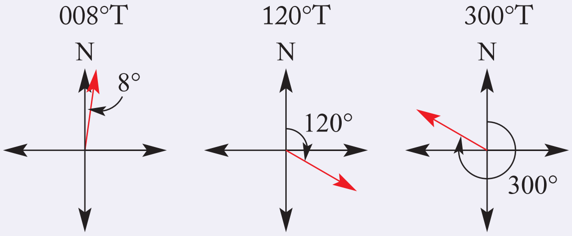
b

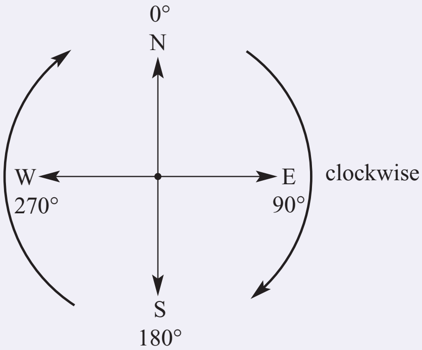


Key ideas

- A **true bearing** (°T) is an angle measured clockwise from North.
  - It is written using 3 digits.

  For example,

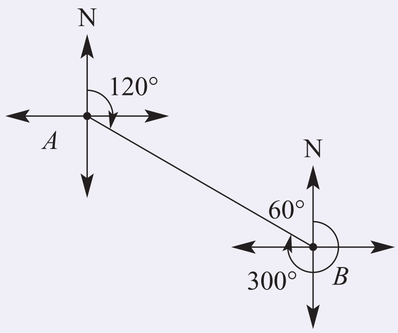




True bearing An angle that is measured clockwise from North

ISBN: 9781107687028 Cambridge University Press

– The word *from* indicates the direction from which a bearing is being taken. For example,



The bearing of B from A is 120°T.

The bearing of A from B is 300°T.

■ When solving problems relating to bearings, always draw a diagram using N, S, E and W each time a bearing is used.

Exercise 9I

Understanding

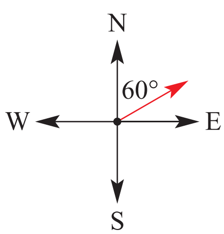
1 What is the opposite direction to:

a North (N)? **b** East (E)? **c** South (S)? **d** North-east (NE)?

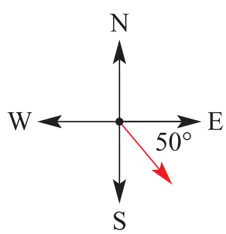
2 Match each diagram below with the correct true bearing from the list below.

i 300° **ii** 260° **iii** 225° **iv** 140° **v** 060° **vi** 350°

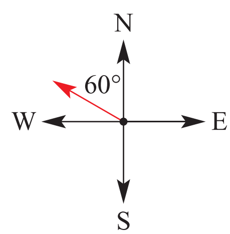
a



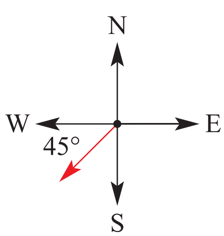
b



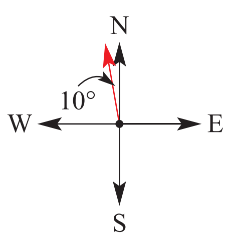
c



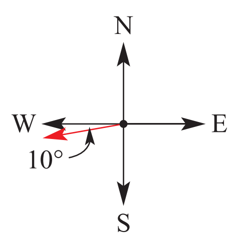
d



e



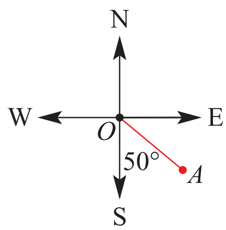
f



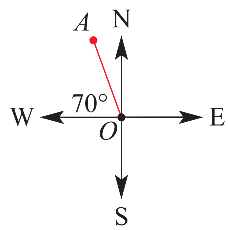
ISBN: 9781107687028

3 Write down the true bearings of A from O, as shown in these diagrams.

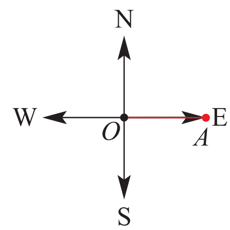
a



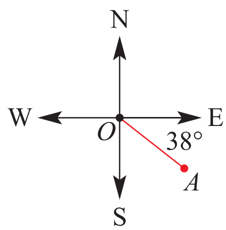
b



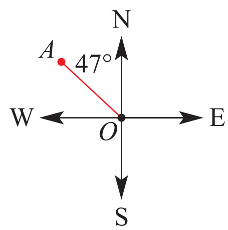
c



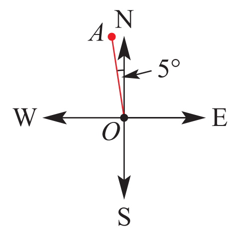
d



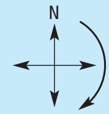
e



f



Remember to use 3 digits and to go clockwise from North.



4 Fill in the missing terms and values for the diagram shown.

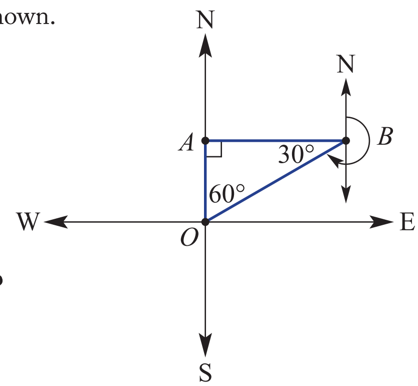
a A is due _________ of O.

b B is due _________ of A.

c A is due _________ of B.

d The bearing of B from O is _________.

e The bearing of O from B is _________.

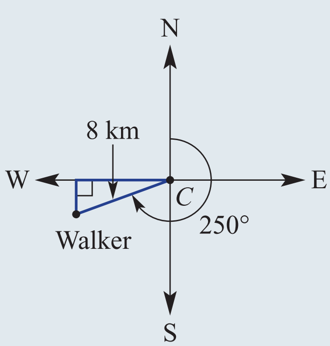
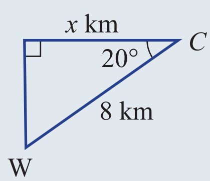


For part **e**, start from North and move clockwise to the line *BO*.

Fluency

Example 18 Drawing a diagram

A walker leaves camp (C) and walks on a bearing of 250°T for 8 km. How far West of camp (x km) is the walker? Show all this information on a right-angled triangle. You do not need to solve for x.

Solution	**Explanation**
 	Draw the compass points first. Start your diagram with the camp at the centre. Mark in 250° clockwise from North, 8 km. Draw a line from the walker to the West line at right angles. Redraw the triangle showing any angles and lengths known (270° – 250° = 20°). Place a pronumeral on the required side.

ISBN: 9781107687028

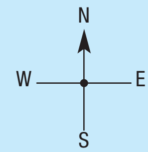
5 Draw a right-angled triangle for each of the situations outlined below.

a Zahra runs on a true bearing of 300° from her home for 6 km. How far North of home is she when she stops?

b Barry walks 12.5 km from camp C on a bearing of 135°T. How far South is he now from camp C?

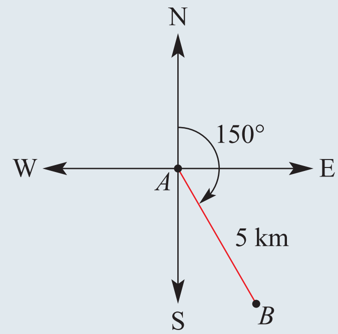
c Tom walks due South 10 km then turns and walks due East 12 km. What is his bearing from O, his starting point?

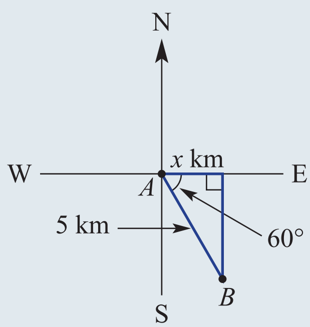
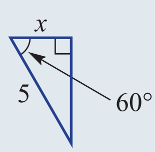
Start by marking the starting point at the centre of a compass.



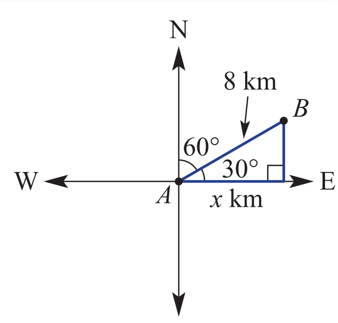
Example 19 Finding distances with bearings

A bushwalker walks 5 km on a true bearing of 150° from point A to point B. Find how far East point B is from point A.



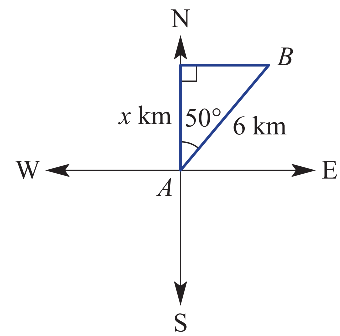
Solution	Explanation
 	Copy the diagram and draw a line from B up to the East line. Use the pronumeral x along the East line. Find the angle within the triangle: $150° - 90° = 60°$. Redraw the triangle.
$\cos\theta = \frac{A}{H}$	Since the Adjacent (A) and Hypotenuse (H) are given, use cos.
$\cos 60° = \frac{x}{5}$	
$x = 5 \times \cos 60°$	
$x = 2.5$	Solve the equation to find x.
∴ point B is 2.5 km East of point A.	Answer the question.

6 Tran walks 8 km from point A to point B on a true bearing of 060°. How far East, correct to one decimal place, is point B from point A?

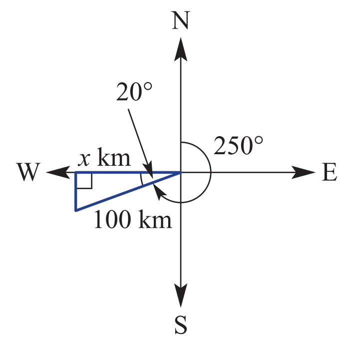


ISBN: 9781107687028 Cambridge University Press

7 A bushwalker walks 6 km on a true bearing of 050° from point A to point B. Find how far North point B is from point A, correct to two decimal places.



8 A speed boat travels 100 km on a true bearing of 250°. Find how far West of its starting point the speed boat is, correct to two decimal places.



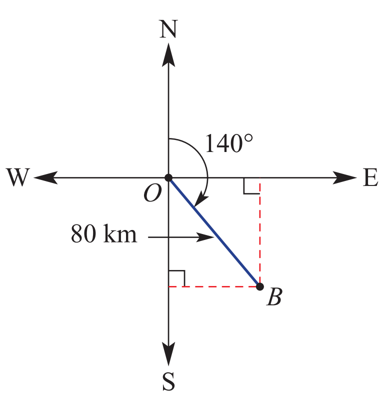
Problem-solving and Reasoning

9 A fishing boat starts from point O and sails 80 km on a true bearing of 140° to point B.

a How far East of point O is point B (answer to two decimal places)?

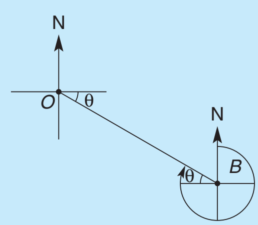
b How far South of point O is point B (answer to two decimal places)?

c What is the bearing of point O from point B?



Remember what 'from' means!

Remember: to find a bearing, face North and turn clockwise.



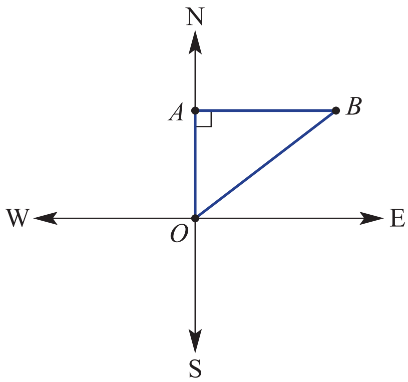
Bearings are given as a 3-digit angle.

ISBN: 9781107687028

Cambridge University Press

10 A plane flies from point O 50 km due North to point A, and then turns and flies 60 km east to point B.

a Copy the diagram below and mark in the lengths 50 km and 60 km.



Pythagoras' theorem: $c^2 = a^2 + b^2$.

b Use Pythagoras' theorem to find the distance of B from O, correct to two decimal places.

c Use trigonometry to find the size of angle AOB. Round to the nearest degree.

d What is the bearing of B from O?

11 William and Harry both leave camp O at the same time. William walks South from O for 10 km. Harry walks West from O for 8 km.

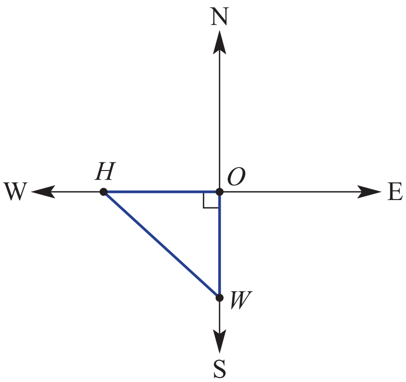
a Copy and complete the diagram for this question.

b How far is Harry from William (to one decimal place)?

c Find the size of angle OWH, correct to the nearest degree.

d What is the bearing of Harry from William?

You can use Pythagoras' theorem here.



Drawing your own diagrams

12 Tao walks on a true bearing of 210° for 6.5 km. How far West of his starting point is he?

13 A plane flies on a true bearing of 320° from airport A for 150 km. At this time how far North of the airport is the plane? Answer to the nearest kilometre.

14 Point A is 10 km due East of point O, and, point B is 15 km due South of point A.

a How far is it, correct to two decimal places, from point B to point O?

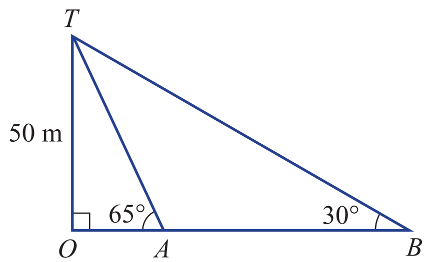
b What is the bearing, to the nearest degree, of point B from point O?

Remember: the word 'from' indicates where the bearing is being taken.

ISBN: 9781107687028

1 What is the opposite direction to:
a East? **b** NE? **c** SE? **d** 018°? **e** 300°?

2 Use two different right-angled triangles to find the distance from A to B in this diagram, correct to two decimal places.



3 Make up your own saying using SOH CAH TOA as the first letter of each word.
S ___ O ___ H ___ C ___ A ___ H ___ T ___ O ___ A ___

4 In the wordfind below there are 17 terms that were used in this chapter. See if you can locate all 17 terms and write a definition or draw a diagram for each of them.

S	I	D	R	T	Y	I	P	Y	T	S	O	H	T	H
I	D	E	P	R	E	S	S	I	O	N	D	Y	O	Y
N	T	E	I	N	T	E	W	P	Y	A	D	J	H	P
E	I	O	E	E	D	S	V	B	Y	T	P	U	W	O
Q	U	O	T	R	S	I	A	D	J	A	C	E	N	T
A	D	A	N	G	E	D	E	P	R	A	N	G	L	E
S	E	N	A	R	T	E	E	G	R	I	L	K	O	N
D	P	T	R	I	A	N	G	L	E	H	E	I	P	U
C	T	R	I	G	O	N	O	M	E	T	R	Y	P	S
B	A	C	G	B	L	I	N	O	Y	A	A	W	O	E
H	D	O	H	T	E	A	N	G	L	N	T	H	S	H
U	J	S	T	R	G	A	O	K	Y	G	I	I	I	Y
J	E	I	W	F	L	N	R	M	K	E	O	D	T	P
K	N	N	T	J	T	R	N	I	O	N	P	Z	E	O
E	L	E	V	A	T	I	O	N	N	T	P	A	M	B
E	T	B	A	S	P	Y	T	H	A	G	O	R	A	S

ISBN: 9781107687028 Cambridge University Press

Chapter summary

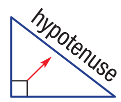
Pythagoras' theorem and trigonometry

Trigonometric ratios

S O H	$\sin\theta = \frac{O}{H}$
C A H	$\cos\theta = \frac{A}{H}$
T O A	$\tan\theta = \frac{O}{A}$

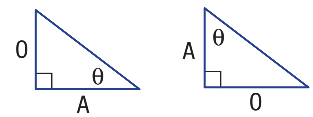
Hypotenuse

The *hypotenuse* is the longest side in a right-angled triangle. It is opposite the right angle.



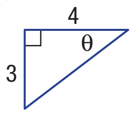
Opposite and Adjacent

In a right-angled triangle the sides marked 'opposite' and 'adjacent' are named according to the angle used.



Finding angles

⟨ Use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ ⟩



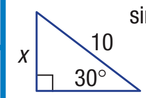
$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$= 37°$

(to the nearest degree)

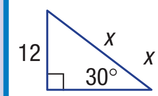
Finding lengths with trigonometry



$\sin 30° = \frac{x}{10}$

$x = 10\sin 30°$

$x = 5$



$\sin 30° = \frac{12}{x}$

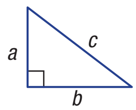
$x \times \sin 30° = 12$

$x = \frac{12}{\sin 30°}$

$x = 24$

Pythagoras' theorem

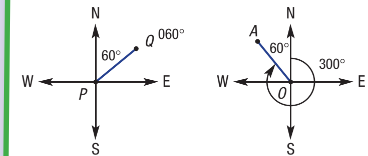
For the triangle shown, *Pythagoras' theorem* is



$c^2 = a^2 + b^2$

True bearings

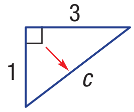
3-digit angle clockwise from North



Bearing of Q from P is 060°

Bearing of A from O is 300°

Finding the hypotenuse



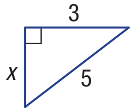
$c^2 = 1^2 + 3^2$

$c^2 = 10$

$c = \sqrt{10}$

$= 3.2$ (1 decimal place)

Finding a shorter side



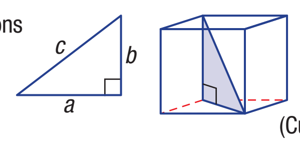
$5^2 = x^2 + 3^2$

$x^2 = 5^2 - 3^2$

$x^2 = 16$

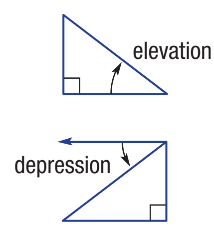
$x = 4$

Applications



(Cube)

Elevation and depression

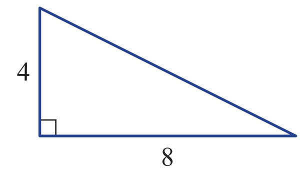


ISBN: 9781107687028

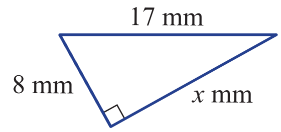
Cambridge University Press

Multiple-choice questions

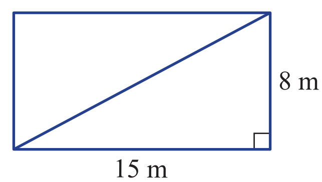
1 The length of the hypotenuse in the triangle shown is closest to:
A 10 **B** 9 **C** 4
D 100 **E** 64



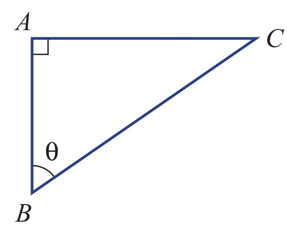
2 The length of the side marked x in the triangle shown is:
A 23 **B** 17 **C** 12
D 19 **E** 15



3 For the shape shown to be a rectangle, the length of the diagonal must be:
A 15 m **B** 8 m **C** 17 m
D 23 m **E** 32 m

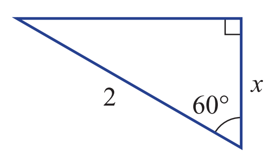


4 Which side (AB, AC or BC) is the adjacent to θ in this triangle?
A AC **B** AB **C** BC
D Hypotenuse **E** Opposite

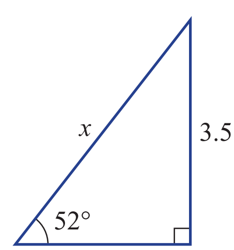


5 The value of $\cos 21°$ is closest to:
A -0.55 **B** 0.9 **C** 0.9336 **D** 0.93 **E** 0.934

6 The value of x in this triangle is:
A $2 \div \cos 60°$ **B** $2 \div \sin 60°$ **C** $2 \times \cos 60°$
D $2 \times \sin 60°$ **E** $2 \times \tan 60°$



7 The value of x in this triangle is closest to:
A 2.76 **B** 4.48 **C** 5.68
D 4.44 **E** 2.73

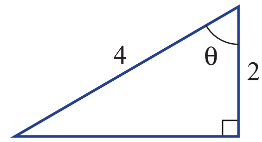


8 A metal brace sits at 55° to the horizontal and reaches 4.2 m up a wall. The distance between the base of the wall and the base of the brace is closest to:
A 6.00 m **B** 2.41 m **C** 7.32 m **D** 5.13 m **E** 2.94 m

ISBN: 9781107687028

9 The angle θ in this triangle is:

A $60°$ **B** $30°$ **C** $26.57°$
D $20°$ **E** none of the above

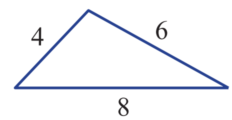


10 The angle of depression from a roof of a building to a trampoline is 75°. If the roof is 12 m above the level of the trampoline, then the distance of the trampoline from the building is closest to:

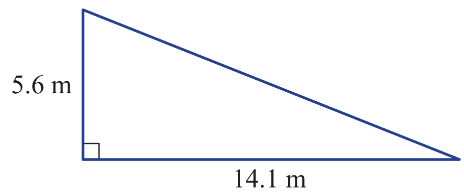
A 12.42 m **B** 11.59 m **C** 3.22 m **D** 44.78 m **E** 3.11 m

Short-answer questions

1 Determine whether the triangle shown contains a right angle.



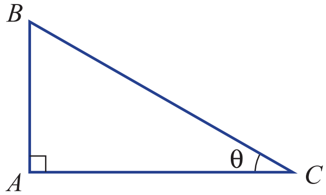
2 Find the length of the hypotenuse, correct to two decimal places, in the triangle shown.



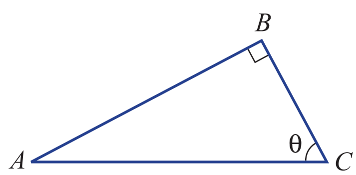
3 Which side (AB, AC or BC) of these triangles is:

i the hypotenuse? **ii** the opposite to θ? **iii** the adjacent to θ?

a



b

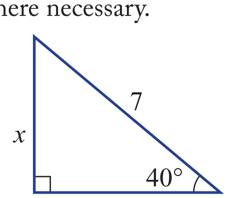


4 Use a calculator to find the value of the following, rounding to two decimal places.

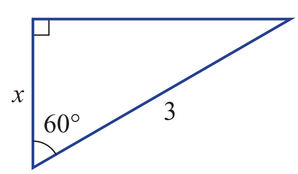
a $\sin 35°$ **b** $\cos 17°$ **c** $\tan 83°$

5 Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

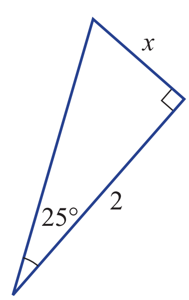
a



b



c

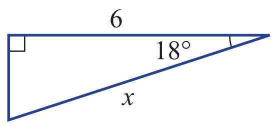


ISBN: 9781107687028

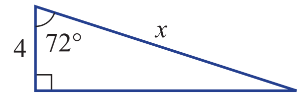
Cambridge University Press

6 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.

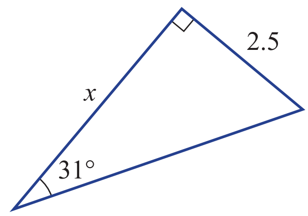
a



b



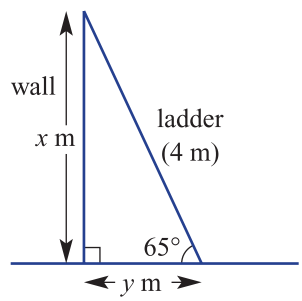
c



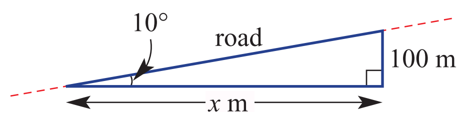
7 A 4 m ladder leans, as shown, against a wall at an angle of 65° to the horizontal.

a Find how high up the wall the ladder reaches (x m), correct to two decimal places.

b Find how far the bottom of the ladder is from the wall (y m), correct to two decimal places.

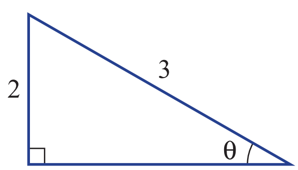


8 A section of road has slope 10° and gains 100 m in height. Find the horizontal length of the road (x m), correct to two decimal places.

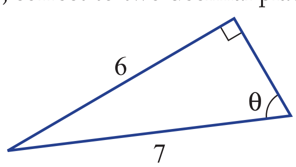


9 Find θ in the following right-angled triangles, correct to two decimal places.

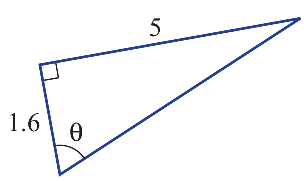
a



b



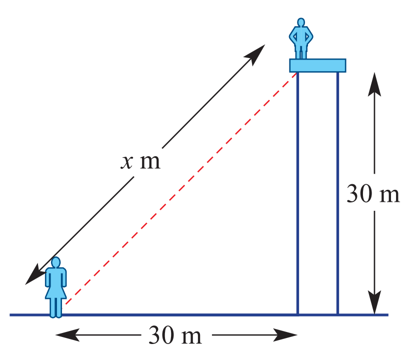
c



10 Barney and Mary view each other from two different places, as shown. Barney is on a viewing platform while Mary is 30 m from the base of the platform, on the ground. The platform is 30 m above the ground.

a Find the angle of elevation from Mary's feet to Barney's feet.

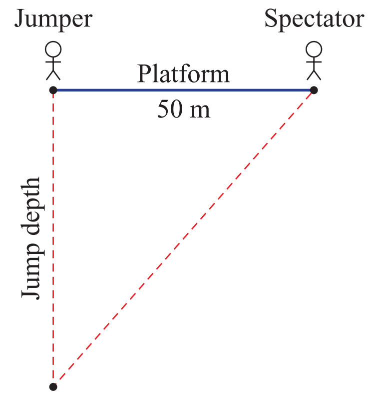
b Using your answer to part **a**, find the distance (x) between Mary and Barney, correct to one decimal place.



ISBN: 9781107687028 Cambridge University Press

Extended-response questions

1 A spectator is viewing bungee jumping from a point 50 m to the side but level with the jumping platform.



a The first bungee jumper has a maximum fall of 70 m. Find the angle of depression from the spectator to the bungee jumper at the maximum depth, correct to two decimal places.

b The second bungee jumper's maximum angle of depression from the spectator is 69°. Find the jumper's maximum depth, correct to two decimal places.

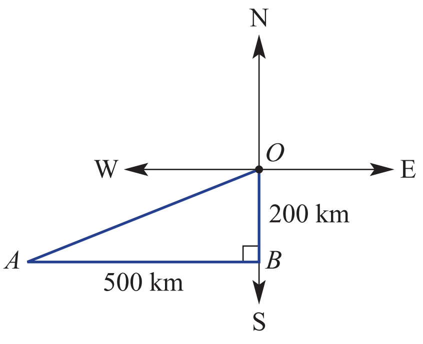
c The third jumper wants to do the 'Head Dunk' into the river below. This occurs when the spectator's angle of depression to the river is 75°. Find, correct to the nearest metre, the height of the platform above the river.

2 A military plane flies 200 km from point O to point B, then West 500 km to point A.

a How far is A from O (to the nearest kilometre)?

b What is angle BOA, correct to the nearest degree?

c What is the bearing of A from O?



ISBN: 9781107687028 Cambridge University Press

Australian Curriculum
Number and Algebra
Patterns and algebra
Linear and non-linear relationships

chapter

10 Quadratics and non-linear graphs

What you will learn

Solar reflectors

Collecting solar power involves reflecting the Sun's rays and capturing the heat generated. One of the most effective types of solar power plant uses parabolic reflectors. The shapes of these reflectors can be described by a quadratic relation. They direct all the rays reflected by the mirrors to the same place.

In a parabolic disc system, the Sun's rays are concentrated onto a glass tube receiver. This receiver is put at the focal point of the parabola. The fluid in the receiver is heated up, transferred to an engine and converted to electricity. Other types of solar power plants also use parabolic trough reflectors.

ISBN: 9781107687028

Pre-test

1 Consider the expression $5 + 2ab - b$.
  a How many terms are there?
  b What is the coefficient of the second term?
  c What is the value of the constant term?

2 Simplify each of the following by collecting like terms.
  a $7x + 2y - 3x$ b $3xy + 4x - xy - 5x$ c $4ab - 2ba$

3 Simplify:
  a $\dfrac{4a}{2}$ b $\dfrac{-24mn}{12n}$ c $6a \times 3a$
  d $-2x \times 3xy$ e $x \times (-3) \div (9x)$ f $4x^2 \div (2x)$

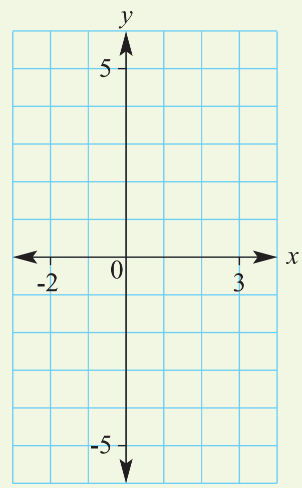
4 Expand and simplify by collecting like terms where possible.
  a $4(m + n)$ b $-3(2x - 4)$
  c $2x(3x + 1)$ d $4a(1 - 2a)$
  e $5 + 3(x - 4)$ f $5 - 2(x + 3) + 2$

5 Factorise each of the following by taking out a common factor.
  a $7x + 7$ b $-9x - 27x^2$ c $a^2 + ab$

6 Solve:
  a $2x + 1 = 0$ b $2(x - 3) = 0$ c $\dfrac{3x + 1}{4} = 4$

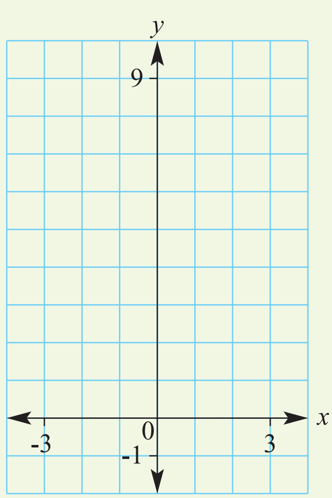
7 Complete the table and plot the graph of $y = 2x - 1$.

x	-2	-1	0	1	2	3
y						



8 Complete this table and plot the graph of $y = x^2$.

x	-3	-2	-1	0	1	2	3
y	9						



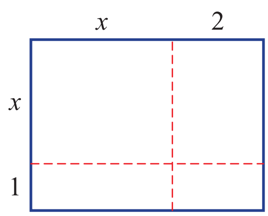
ISBN: 9781107687028

10.1 Expanding binomial products

Expressions that include numerals and variables (or pronumerals) are central to the topic of algebra. Sound skills in algebra are essential for solving most mathematical problems and this includes the ability to expand expressions involving brackets. This includes binomial products, perfect squares and the difference of perfect squares. Exploring how projectiles fly subject to the Earth's gravity, for example, can be modelled with expressions with and without brackets.

Let's start: Why does $(x + 1)(x + 2) = x^2 + 3x + 2$?

Look at this rectangle with side lengths $x + 1$ and $x + 2$.



- What are the areas of the four regions?
- Add up the areas to find an expression for the total area.
- Why does this explain that $(x + 1)(x + 2) = x^2 + 3x + 2$?

Key ideas

- **Like terms** have the same pronumeral part.
  - They can be collected (added and subtracted) to form a single term.
    For example: $7x - 11x = -4x$ and $4a^2b - 7ba^2 = -3a^2b$
- The **distributive law** is used to expand brackets.

$a(b + c) = ab + ac$

and $a(b - c) = ab - ac$

	a
b	ab
c	ac

$(a + b)(c + d) = ac + ad + bc + bd$

	a	b
c	ac	bc
d	ad	bd

$(a + b)(c + d)$ is called a binomial product because each expression in the brackets has two terms.

Like terms Terms with the same pronumerals and same powers

Distributive law Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the products

ISBN: 9781107687028

Cambridge University Press

- **Perfect squares**

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = (a-b)(a-b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$$

- **Difference of perfect squares (DOPS)**

$$(a+b)(a-b) = a^2 - \cancel{ab} + \cancel{ba} - b^2 = a^2 - b^2$$

Perfect square A quadratic trinomial that can be expressed as a single square

Difference of perfect squares When one square term is subtracted from another

Exercise 10A

Understanding

1 Decide whether the following are a perfect square (PS) or a difference of perfect squares (DOPS).
a $(x+1)^2$ **b** $x^2 - 16$ **c** $4x^2 - 25$ **d** $(2x-3)^2$

2 Simplify these expressions.
a $2 \times 3x$ **b** $-4 \times 5x$ **c** $x \times 2x$ **d** $-x \times 4x$
e $5x \div 10$ **f** $3x \div 9$ **g** $-4x^2 \div x$ **h** $-6x^2 \div (2x)$
i $3x - 21x$ **j** $12x - 5x$ **k** $-3x + 8x$ **l** $-5x - 8x$

Write ÷ as a fraction and cancel:
$5x \div 10 = \frac{5x}{10}$

3 Use each diagram to help expand the expressions.
a $x(x+2)$ **b** $(x+3)(x+1)$ **c** $(x+4)^2$

a

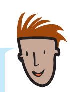
	x
x	x^2
2	$2x$

b

	x	3
x	x^2	$3x$
1	x	3

c

	x	4
x	x^2	$4x$
4	$4x$	16



Simply add up all the areas inside the rectangular diagram.

Fluency

Example 1 Expanding simple expressions

Expand and simplify where possible.
a $-3(x-5)$ **b** $-2x(1-x)$

Solution	Explanation
a $-3(x-5) = -3 \times x - (-3) \times 5$ $= -3x + 15$	Use the distributive law $a(b-c) = ab - ac$. A negative times a negative is a positive.
b $-2x(1-x) = -2x \times 1 - (-2x) \times x$ $= -2x + 2x^2$	Recall $x \times x = x^2$.

4 Expand and simplify where possible.
a $2(x+5)$ **b** $3(x-4)$ **c** $-5(x+3)$ **d** $-4(x-2)$
e $3(2x-1)$ **f** $4(3x+1)$ **g** $-2(5x-3)$ **h** $-5(4x+3)$
i $x(2x+5)$ **j** $x(3x-1)$ **k** $2x(1-x)$ **l** $3x(2-x)$
m $-2x(3x+2)$ **n** $-3x(6x-2)$ **o** $-5x(2-2x)$ **p** $-4x(1-4x)$

$a(b+c) = ab + ac$

ISBN: 9781107687028

Example 2 Expanding binomial products

Expand the following.

a $(x+5)(x+4)$ **b** $(2x-1)(3x+5)$

Solution	Explanation
a $(x+5)(x+4) = x^2 + 4x + 5x + 20$	For binomial products use $(a+b)(c+d) = ac + ad + bc + bd$.
$= x^2 + 9x + 20$	Simplify by collecting like terms. $4x + 5x = 9x$
b $(2x-1)(3x+5) = 6x^2 + 10x - 3x - 5$	Expand using the distributive law and simplify.
$= 6x^2 + 7x - 5$	Note that $2x \times 3x = 2 \times 3 \times x \times x = 6x^2$
	and $-1 \times 3x = -3x$, $-1 \times 5 = -5$.

5 Expand the following.

a $(x+2)(x+8)$ **b** $(x+3)(x+4)$ **c** $(x+7)(x+5)$
d $(x+8)(x-3)$ **e** $(x+6)(x-5)$ **f** $(x-2)(x+3)$
g $(x-7)(x+3)$ **h** $(x-4)(x-6)$ **i** $(x-8)(x-5)$
j $(2x+1)(3x+5)$ **k** $(4x+5)(3x+2)$ **l** $(5x+3)(2x+7)$
m $(3x+2)(3x-5)$ **n** $(5x+3)(4x-2)$ **o** $(2x+5)(3x-5)$

$(a+b)(c+d) =$
$ac + ad + bc + bd$

Example 3 Expanding perfect squares

Expand these perfect squares.

a $(x+2)^2$ **b** $(x-4)^2$

Solution	Explanation
a $(x+2)^2 = (x+2)(x+2)$	First write in expanded form, then use the distributive law.
$= x^2 + 2x + 2x + 4$	
$= x^2 + 4x + 4$	
OR $(x+2)^2 = x^2 + 2(x)(2) + 2^2$	$(a+b)^2 = a^2 + 2ab + b^2$ with $a = x$ and $b = 2$
$= x^2 + 4x + 4$	
b $(x-4)^2 = (x-4)(x-4)$	Rewrite and expand using the distributive law.
$= x^2 - 4x - 4x + 16$	$-4 \times (-4) = 16$
$= x^2 - 8x + 16$	
OR $(x-4)^2 = x^2 - 2(x)(4) + 4^2$	Alternatively for perfect squares $(a-b)^2 = a^2 - 2ab + b^2$.
$= x^2 - 8x + 16$	Here $a = x$ and $b = 4$.

6 Expand these perfect squares.

a $(x+5)^2$ **b** $(x+7)^2$ **c** $(x+6)^2$
d $(x-3)^2$ **e** $(x-8)^2$ **f** $(x-10)^2$
g $(2x+5)^2$ **h** $(5x+6)^2$ **i** $(7x-1)^2$

Recall:
$(x+5)^2 = (x+5)(x+5)$
$= \ldots$

ISBN: 9781107687028

Example 4 Expanding to form a difference of perfect squares

Expand to form a difference of perfect squares.

a $(x-3)(x+3)$ **b** $(2x+1)(2x-1)$

Solution	Explanation
a $(x-3)(x+3) = x^2 + 3x - 3x - 9$ $= x^2 - 9$ OR $(x-3)(x+3) = x^2 - 3^2$ $= x^2 - 9$	$x \times x = x^2$, $x \times 3 = 3x$, $-3 \times x = -3x$, $-3 \times 3 = -9$ Note that the two middle terms cancel. $(a-b)(a+b) = a^2 - b^2$
b $(2x+1)(2x-1) = 4x^2 - 2x + 2x - 1$ $= 4x^2 - 1$ OR $(2x+1)(2x-1) = (2x)^2 - (1)^2$ $= 4x^2 - 1$	Expand, recalling that $2x \times 2x = 4x^2$. Cancel the $-2x$ and $+2x$ terms. Alternatively for difference of perfect squares $(a-b)(a+b) = a^2 - b^2$. Here $a = 2x$ and $b = 1$ and $(2x)^2 = 2x \times 2x = 4x^2$.

7 Expand to form a difference of perfect squares.

a $(x+4)(x-4)$ **b** $(x+9)(x-9)$ **c** $(x+8)(x-8)$
d $(3x+4)(3x-4)$ **e** $(2x-3)(2x+3)$ **f** $(8x-7)(8x+7)$
g $(4x-5)(4x+5)$ **h** $(2x-9)(2x+9)$ **i** $(5x-7)(5x+7)$

The two middle terms will cancel to give $(a+b)(a-b) = a^2 - b^2$.

Problem-solving and Reasoning

8 Write the missing number.

a $(x+2)(x-3) = x^2 - x - \square$ **b** $(x-4)(x-3) = x^2 - \square x + 12$
c $(x-4)(x+4) = x^2 - \square$ **d** $(2x-1)(2x+1) = \square x^2 - 1$
e $(x+2)^2 = x^2 + \square x + 4$ **f** $(3x-1)^2 = 9x^2 - \square x + 1$

Expand if you need to.

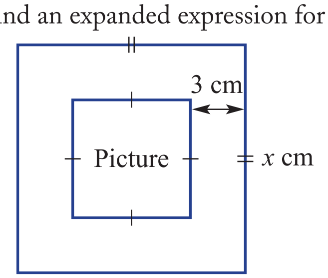
9 Write the missing number.

a $(x+\square)(x+2) = x^2 + 5x + 6$ **b** $(x+\square)(x+5) = x^2 + 8x + 15$
c $(x+7)(x-\square) = x^2 + 4x - 21$ **d** $(x+4)(x-\square) = x^2 - 4x - 32$
e $(x-6)(x-\square) = x^2 - 7x + 6$ **f** $(x-\square)(x-8) = x^2 - 10x + 16$

Notice how the two numerals in the brackets multiply to give the constant term.

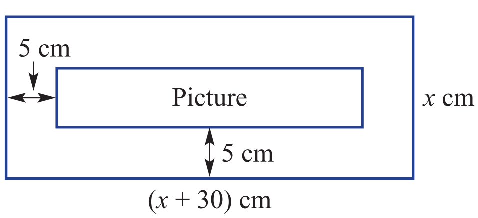
10 Find an expanded expression for the area of the pictures centred in these frames.

a



For part **a**, the side length of the picture will be $(x-6)$ cm.

b



ISBN: 9781107687028

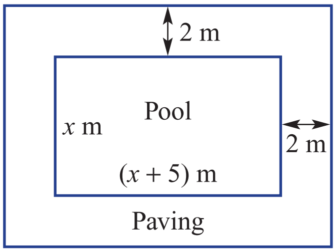
11 Each problem below has an incorrect answer. Find the error and give the correct answer.

a $-x(x - 7) = -x^2 - 7x$ **b** $3a - 7(4 - a) = -4a - 28$

c $(x - 9)(x + 9) = x^2 - 18x - 81$ **d** $(2x + 3)^2 = 4x^2 + 9$

Swimming pool algebra

12 A pool company builds rectangular pools that are 5 m longer than they are wide. The company then paves around the outside of the pool using a width of 2 m.



a Find an expanded expression for:

i the pool area

ii the total area (including the pool and paving)

iii the paved area

b Find the area of the following if $x = 4$.

i the pool **ii** the paved area

ISBN: 9781107687028

10.2 Factorising a difference of perfect squares

A common and key step in the simplification and solution of equations involves factorisation. Factorisation is the process of writing a number or expression as a product of its factors. For example: $6 = 2 \times 3$, $2x + 6 = 2(x + 3)$, $x^2 - x = x(x - 1)$ and $x^2 - 9 = (x + 3)(x - 3)$.

In this section we look at expressions in which each term has a common factor and expressions that are a difference of perfect squares.

Let's start: It's just a DOPS expansion in reverse

Complete each column to see the connection when expanding or factorising a DOPS.

Expand	**Factorise**
$(x + 2)(x - 2) = x^2 - 4$	$x^2 - 4 = (x$ ______$)(x$ ______$)$
$(x - 3)(x + 3) = x^2 - 9$	$x^2 - 9 = (x$ ______$)($______$)$
$(2x + 3)(2x - 3) = 4x^2 - 9$	$4x^2 - 9 = (2x$ ______$)($______$)$
$(7x - 6)(7x + 6) =$ _____	_____ $= ($______$)($______$)$

Key ideas

- **Factorise** expressions with common factors by 'taking out' the highest common factor.
  For example: $-5x - 20 = -5(x + 4)$ and $4x^2 - 8x = 4x(x - 2)$
- Factorise a difference of perfect squares (DOPS) using
  $$a^2 - b^2 = (a + b)(a - b)$$
  For example:
  $x^2 - 16 = x^2 - 4^2 = (x + 4)(x - 4)$ and $9x^2 - 25 = (3x)^2 - 5^2 = (3x + 5)(3x - 5)$

Factorise To write an expression as a product, often involving brackets

Exercise 10B

Understanding

1 Complete these statements.
  a $2(x + 3) = 2x + 6$ so $2x + 6 = 2($_____$)$
  b $-4(x - 1) = -4x + 4$ so $-4x + 4 = -4($____$)$
  c $(x + 2)(x - 2) = x^2 - 4$ so $x^2 - 4 = ($____$)($____$)$
  d $(3x + 2)(3x - 2) = 9x^2 - 4$ so $9x^2 - 4 = ($____$)($____$)$

2 Determine the highest common factor of these pairs of terms.
  a $7x$ and 14 b $12x$ and 30 c $-8y$ and 40
  d $-5y$ and -25 e $4a^2$ and $2a$ f $12a^2$ and $9a$
  g $-5a^2$ and $-50a$ h $-3x^2y$ and $-6xy$

Include a common negative.

ISBN: 9781107687028

Fluency

Example 5 Taking out common factors

Factorise by taking out the highest common factor.

a $-3x - 12$ **b** $20a^2 + 30a$

Solution	Explanation
a $-3x - 12 = -3(x + 4)$	-3 is common to both $-3x$ and -12.
b $20a^2 + 30a = 10a(2a + 3)$	The HCF of $20a^2$ and $30a$ is $10a$.

3 Factorise by taking out the highest common factor.

a $3x - 18$ **b** $4x + 20$ **c** $7a + 7b$ **d** $9a - 15$
e $-5x - 30$ **f** $-4y - 2$ **g** $-12a - 3$ **h** $-2ab - bc$
i $4x^2 + x$ **j** $5x^2 - 2x$ **k** $6b^2 - 18b$ **l** $14a^2 - 21a$
m $10a - 5a^2$ **n** $12x - 30x^2$ **o** $-2x - x^2$ **p** $-4y - 8y^2$

Find the highest common factor then take it out.

Example 6 Factorising a difference of perfect squares

Factorise the following differences of perfect squares.

a $x^2 - 16$ **b** $9a^2 - 4b^2$

Solution	Explanation
a $x^2 - 16 = (x)^2 - (4)^2$ $= (x - 4)(x + 4)$	Use $a^2 - b^2 = (a - b)(a + b)$ with $a = x$ and $b = 4$.
b $9a^2 - 4b^2 = (3a)^2 - (2b)^2$ $= (3a - 2b)(3a + 2b)$	$9a^2 = (3a)^2$ and $4b^2 = (2b)^2$.

4 Factorise the following differences of perfect squares.

a $x^2 - 9$ **b** $x^2 - 25$ **c** $y^2 - 49$ **d** $y^2 - 1$
e $a^2 - 16$ **f** $b^2 - 36$ **g** $y^2 - 144$ **h** $z^2 - 400$
i $4x^2 - 9$ **j** $36a^2 - 25$ **k** $1 - 81y^2$ **l** $100 - 9x^2$
m $25x^2 - 4y^2$ **n** $64x^2 - 25y^2$ **o** $9a^2 - 49b^2$ **p** $144a^2 - 49b^2$

$a^2 - b^2 = (a + b)(a - b)$

5 Factorise these differences of perfect squares.

a $4 - x^2$ **b** $9 - y^2$ **c** $36 - a^2$ **d** $100 - 9x^2$
e $b^2 - a^2$ **f** $400 - 25a^2$ **g** $4a^2 - 9b^2$ **h** $16y^2 - 121x^2$

Problem-solving and Reasoning

Example 7 Factorising by first taking out a common factor

Factorise $12y^2 - 1452$ by first taking out a common factor.

Solution	Explanation
$12y^2 - 1452 = 12(y^2 - 121)$ $= 12(y - 11)(y + 11)$	First take out the common factor of 12. $121 = (11)^2$, use $a^2 - b^2 = (a - b)(a + b)$.

ISBN: 9781107687028 Cambridge University Press

6 Factorise the following by first taking out a common factor.

a $2x^2 - 32$ **b** $5x^2 - 45$ **c** $6y^2 - 24$
d $3y^2 - 48$ **e** $3x^2 - 75y^2$ **f** $3a^2 - 300b^2$
g $12x^2 - 27y^2$ **h** $63a^2 - 112b^2$ **i** $108x^2 - 147y^2$

As a first step, take out a common factor then factorise the DOPS.

7 The height (in metres) of a falling object above ground level is given by $100 - t^2$, where t is in seconds.

a Find the height of the object:
  i initially (at $t = 0$)
  ii after 2 seconds
  iii after 8 seconds

b Factorise the expression $100 - t^2$.

c Use your factorised expression from part **b** to find the height of the object:
  i initially (at $t = 0$)
  ii after 2 seconds
  iii after 8 seconds

d How long does it take for the object to hit the ground?

8 We can work out problems such as $19^2 - 17^2$ without a calculator like this:

$$\begin{aligned} 19^2 - 17^2 &= (19 + 17)(19 - 17) \\ &= 36 \times 2 \\ &= 72 \end{aligned}$$

Factorise first, using $a^2 - b^2 = (a + b)(a - b)$, then evaluate.

Use this idea to evaluate the following by first factorising, without the use of a calculator.

a $16^2 - 14^2$ **b** $18^2 - 17^2$ **c** $13^2 - 10^2$ **d** $15^2 - 11^2$
e $17^2 - 15^2$ **f** $11^2 - 9^2$ **g** $27^2 - 24^2$ **h** $52^2 - 38^2$

Flexible framing

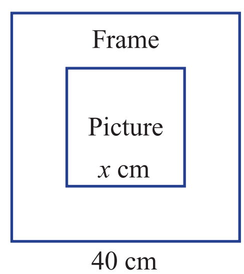
9 A special picture frame can hold a square picture of any size up to 40 cm.

a Using a picture side length of x cm, write expressions for the area of:
  i the picture **ii** the frame (not including the picture)

b Factorise your expression for the frame area.

c Find the frame area if: **i** $x = 20$ **ii** $x = 10$

d Using trial and error, what value of x is required if the frame area is to be 700 cm^2?



Using technology 10.2: Expanding and factorising

This activity is available on the companion website as a printable PDF.

ISBN: 9781107687028

10.3 Factorising trinomials of the form $x^2 + bx + c$

A quadratic trinomial of the form $x^2 + bx + c$ is called a monic quadratic because the coefficient of x^2 is 1. ('Monic' comes from the word 'mono', which means 'one'.)

Now consider:

$$\begin{aligned}(x + m)(x + n) &= x^2 + xn + mx + mn \\ &= x^2 + (m + n)x + mn\end{aligned}$$

We can see from this expansion that mn gives the constant term (c) and $m + n$ is the coefficient of x. This tells us that to factorise a monic quadratic, we should look for factors of the constant term (c) that add to give the coefficient of the middle term (b).

Let's start: So many choices of factors?

We know that to factorise $x^2 - 5x - 24$ we need to choose a pair of numbers that multiply to give -24. Look at the following equations and discuss which of them are true.

$x^2 - 5x - 24 = (x + 8)(x + 3)$

$x^2 - 5x - 24 = (x - 12)(x - 2)$

$x^2 - 5x - 24 = (x - 12)(x + 2)$

$x^2 - 5x - 24 = (x - 8)(x - 3)$

$x^2 - 5x - 24 = (x + 6)(x - 4)$

$x^2 - 5x - 24 = (x - 6)(x + 4)$

$x^2 - 5x - 24 = (x - 8)(x + 3)$

$x^2 - 5x - 24 = (x - 24)(x + 1)$

Key ideas

- **Monic quadratics** have a coefficient of x^2 equal to 1.
- Monic quadratics of the form $x^2 + bx + c$ can be factorised by finding the two numbers that multiply to give the constant term (c) and add to give the coefficient of x (b).

$x^2 + 5x + 6 = (x + 3)(x + 2)$

$2 + 3$ $\quad$ 2×3

$x^2 - 5x + 6 = (x - 3)(x - 2)$

$-2 + (-3)$ $\quad$ $-2 \times (-3)$

$x^2 - x - 6 = (x - 3)(x + 2)$

$-3 + 2$ $\quad$ -3×2

Monic quadratic A quadratic expression where the coefficient of the squared term is 1

ISBN: 9781107687028

Cambridge University Press

Exercise 10C

Understanding

1 Expand and simplify these expressions.

a $(x+2)(x+3)$ **b** $(x-4)(x+2)$ **c** $(x-7)(x-3)$
d $(x-1)^2$ **e** $(x+5)^2$ **f** $(x-6)^2$

2 Find two integers that multiply to give the first number and add to give the second number.

a 18, 11 **b** 20, 12 **c** -15, 2 **d** -12, 1
e -24, -5 **f** -30, -7 **g** 10, -7 **h** 36, -15

3 **a** **i** Which two numbers multiply to give 15 and add to give 8?
ii Complete $x^2 + 8x + 15 = (\_\_\_\_\_\_)(\_\_\_\_\_\_)$.
b **i** Which two numbers multiply to give -10 and add to give 3?
ii Complete $x^2 + 3x - 10 = (\_\_\_\_\_\_)(\_\_\_\_\_\_)$.
c **i** Which two numbers multiply to give 8 and add to give -6?
ii Complete $x^2 - 6x + 8 = (\_\_\_\_\_\_)(\_\_\_\_\_\_)$.

Fluency

Example 8 Factorising trinomials of the form $x^2 + bx + c$

Factorise:

a $x^2 + 7x + 12$
b $x^2 + x - 6$
c $x^2 - 5x + 6$

Solution	Explanation
a $x^2 + 7x + 12 = (x+4)(x+3)$	$3 \times 4 = 12$ and $3 + 4 = 7$ Check: $(x+4)(x+3) = x^2 + 3x + 4x + 12 = x^2 + 7x + 12$
b $x^2 + x - 6 = (x-2)(x+3)$	Since the numbers must multiply to -6, one must be positive and one negative. $-2 \times 3 = -6$ and $-2 + 3 = 1$ Check: $(x-2)(x+3) = x^2 + 3x - 2x - 6 = x^2 + x - 6$
c $x^2 - 5x + 6 = (x-3)(x-2)$	$-3 \times (-2) = 6$ and $-3 + (-2) = -5$ Check: $(x-3)(x-2) = x^2 - 2x - 3x + 6 = x^2 - 5x + 6$

4 Factorise these quadratic trinomials.

a $x^2 + 7x + 6$ **b** $x^2 + 5x + 6$ **c** $x^2 + 6x + 9$
d $x^2 + 7x + 10$ **e** $x^2 + 7x + 12$ **f** $x^2 + 11x + 18$
g $x^2 + 5x - 6$ **h** $x^2 + x - 6$ **i** $x^2 + 2x - 8$
j $x^2 + 3x - 4$ **k** $x^2 + 7x - 30$ **l** $x^2 + 9x - 22$
m $x^2 - 7x + 10$ **n** $x^2 - 6x + 8$ **o** $x^2 - 7x + 12$
p $x^2 - 2x + 1$ **q** $x^2 - 9x + 18$ **r** $x^2 - 11x + 18$
s $x^2 - 4x - 12$ **t** $x^2 - x - 20$ **u** $x^2 - 5x - 14$
v $x^2 - x - 12$ **w** $x^2 + 4x - 32$ **x** $x^2 - 3x - 10$

ISBN: 9781107687028

Example 9 Factorising perfect squares

Factorise $x^2 - 8x + 16$ to form a perfect square.

Solution	Explanation
$x^2 - 8x + 16 = (x - 4)(x - 4)$	$-4 \times (-4) = 16$ and $-4 + (-4) = -8$
$= (x - 4)^2$	$(x - 4)(x - 4) = (x - 4)^2$ is a perfect square.

5 Factorise these perfect squares.

a $x^2 - 4x + 4$ **b** $x^2 + 6x + 9$ **c** $x^2 + 12x + 36$
d $x^2 - 14x + 49$ **e** $x^2 - 18x + 81$ **f** $x^2 - 20x + 100$
g $x^2 + 8x + 16$ **h** $x^2 + 20x + 100$ **i** $x^2 - 30x + 225$

Factorise perfect squares just like any trinomial but finish by writing in the form $(x + a)^2$.

Problem-solving and Reasoning

Example 10 Factorising by first taking out a common factor

Factorise $2x^2 - 10x - 28$ by first taking out a common factor.

Solution	Explanation
$2x^2 - 10x - 28 = 2(x^2 - 5x - 14)$	First take out the common factor of 2.
$= 2(x - 7)(x + 2)$	$-7 \times 2 = -14$ and $-7 + 2 = -5$

6 Factorise by first taking out the common factor.

a $2x^2 + 14x + 20$ **b** $3x^2 + 21x + 36$
c $2x^2 + 22x + 36$ **d** $5x^2 - 5x - 10$
e $4x^2 - 16x - 20$ **f** $3x^2 - 9x - 30$
g $-2x^2 - 14x - 24$ **h** $-3x^2 + 9x - 6$
i $-2x^2 + 10x + 28$ **j** $-4x^2 + 4x + 8$
k $-5x^2 - 20x - 15$ **l** $-7x^2 + 49x - 42$

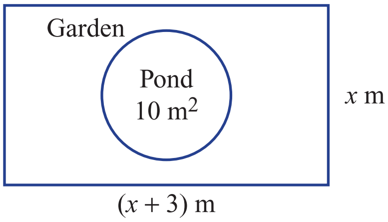
Factor out the coefficient of x^2.

7 Factorise these as perfect squares after first taking out the common factor.

a $2x^2 + 44x + 242$ **b** $3x^2 - 24x + 48$
c $5x^2 - 50x + 125$ **d** $-3x^2 + 36x - 108$
e $-2x^2 + 28x - 98$ **f** $-4x^2 - 72x - 324$

ISBN: 9781107687028 Cambridge University Press

8 A rectangular garden has length 3 m more than its width, x m. There is a pond of area 10 m^2 in the centre.



a Find an expression for:
  i the entire area (expand your answer) **ii** the garden area, excluding the pond

b Factorise your answer from part **a ii**.

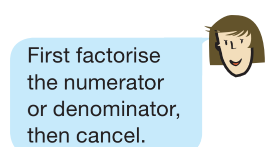
c What is the area of the garden, excluding the pond, if: **i** $x = 5$? **ii** $x = 7$?

Algebraic fractions

9 Some algebraic fractions can be simplified using factorisation.
Here is an example,

$$\frac{x^2 - x - 12}{x - 4} = \frac{\cancel{(x - 4)}(x + 3)}{\cancel{x - 4}} = x + 3.$$

Use this idea to simplify these fractions.



a $\dfrac{x^2 - 3x - 54}{x - 9}$ **b** $\dfrac{x^2 + x - 12}{x + 4}$ **c** $\dfrac{x^2 - 6x + 9}{x - 3}$

d $\dfrac{x + 2}{x^2 + 9x + 14}$ **e** $\dfrac{x - 3}{x^2 - 8x + 15}$ **f** $\dfrac{x + 1}{x^2 - 5x - 6}$

g $\dfrac{x^2 - 4x + 4}{x - 2}$ **h** $\dfrac{x^2 + 2x + 1}{x + 1}$ **i** $\dfrac{x^2 - 16x + 64}{x - 8}$

ISBN: 9781107687028 Cambridge University Press

10.4 Solving quadratic equations

In previous chapters you would have solved linear equations such as $3x = 9$ and $2x - 1 = 5$, and you may have used 'back tracking' or inverse operations to solve them.

For quadratic equations such as $x^2 - x = 0$ or $x^2 - x - 20 = 0$, we need a new method, because there are different powers of x involved and 'back tracking' isn't useful.

The result of multiplying a number by zero is zero. Therefore, if an expression equals zero then at least one of its factors must be zero. This is called the Null Factor Law and it provides us with an important method that can be utilised to solve a range of mathematical problems involving quadratic equations.

Parabolic arches (like this one supporting the weight of a bridge) can be modelled by quadratic equations.

Let's start: How does the Null Factor Law work?

Start this investigation by completing this table.

x	-5	-4	-3	-2	-1	0	1	2
$(x - 1)(x + 4)$	6							

- Which values of x made $(x - 1)(x + 4) = 0$? Why?
- Could you work out what values of x make $(x - 1)(x + 4) = 0$ without doing a table? Explain.
- What value of x makes $(x - 2)(x + 3) = 0$ or $(x + 5)(x - 7) = 0$?

Key ideas

- The **Null Factor Law** states that if the product of two numbers is zero then either or both of the two numbers is zero.
  – If $a \times b = 0$ then $a = 0$ and/or $b = 0$.
- To solve a quadratic equation, write it in standard form ($ax^2 + bx + c = 0$) and factorise. Then use the Null Factor Law.
  For example,
  $$\begin{aligned} x^2 - 2x - 8 &= 0 \\ (x - 4)(x + 2) &= 0 \\ x - 4 = 0 &\text{ or } x + 2 = 0 \\ x = 4 &\text{ or } x = -2 \end{aligned}$$
  – If the coefficients of all the terms have a common factor then first divide by that common factor.

Null Factor Law
If two numbers multiply to give zero, then one or both of those numbers must be zero

Exercise 10D

Understanding

1 Determine the value of x that makes each of the following true.

a $x - 1 = 0$ b $x + 3 = 0$ c $x + 7 = 0$ d $x - 5 = 0$
e $2x = 0$ f $-5x = 0$ g $2x + 4 = 0$ h $2x - 7 = 0$

2 a Complete this table for the given values of x.

x	-3	-2	-1	0	1	2
$(x + 2)(x - 1)$						

b What values of x made $(x + 2)(x - 1) = 0$?
c What values of x would make $(x + 3)(x - 2) = 0$?

3 Copy and complete:

a $x(x - 2) = 0$
$x = 0$ or ____ $= 0$
$x =$ ____

b $(x - 1)(x + 4) = 0$
$x - 1 = 0$ or ____ $= 0$
$x =$ ____ or $x =$ ____

c $(x + 6)(2x - 7) = 0$
____ $= 0$ or ____ $= 0$
$x =$ ____ or $2x =$ ____
$x =$ ____

Fluency

Example 11 Using the Null Factor Law

Use the Null Factor Law to solve these equations.

a $x(x - 1) = 0$ b $(x - 1)(2x + 5) = 0$

Solution	Explanation
a $x(x - 1) = 0$	
$x = 0$ or $x - 1 = 0$	Set each factor equal to zero.
$x = 1$	For $x - 1 = 0$, add one to both sides to finish.
b $(x - 1)(2x + 5) = 0$	Set each factor equal to zero then solve each linear equation.
$x - 1 = 0$ or $2x + 5 = 0$	
$x = 1$ or $2x = -5$	
$x = \frac{-5}{2}$	

4 Use the Null Factor Law to solve these equations.

a $x(x + 1) = 0$ b $x(x - 5) = 0$
c $2x(x - 4) = 0$ d $(x - 3)(x + 2) = 0$
e $(x + 5)(x - 4) = 0$ f $(x + 1)(x - 1) = 0$
g $(2x - 4)(x + 1) = 0$ h $(3x - 2)(x - 7) = 0$
i $3x(4x + 5) = 0$ j $(2x - 1)(3x + 7) = 0$
k $(4x - 5)(5x + 2) = 0$ l $(8x + 3)(4x + 3) = 0$

Null Factor Law: if $a \times b = 0$, then either $a = 0$ or $b = 0$.

ISBN: 9781107687028

Cambridge University Press

Example 12 Solving quadratic equations with a common factor

Solve $x^2 - 2x = 0$.

Solution	Explanation
$x^2 - 2x = 0$	Factorise by taking out the common factor x. Apply
$x(x - 2) = 0$	the Null Factor Law: if $a \times b = 0$ then $a = 0$ or $b = 0$.
$\therefore\ x = 0$ or $x - 2 = 0$	Solve for x.
$\therefore\ x = 0$ or $x = 2$	

5 Solve the following quadratic equations.

a $x^2 - 4x = 0$ **b** $x^2 - 3x = 0$
c $x^2 + 2x = 0$ **d** $3x^2 - 12x = 0$
e $2x^2 - 10x = 0$ **f** $4x^2 + 8x = 0$

First take out the common factor then use the Null Factor Law.

Example 13 Solving with DOPS

Solve $x^2 - 16 = 0$ by factorising the DOPS.

Solution	Explanation
$x^2 - 16 = 0$	Note that $x^2 - 16$ is a difference of perfect squares.
$(x + 4)(x - 4) = 0$	$x^2 - 16 = x^2 - 4^2 = (x + 4)(x - 4)$
$x + 4 = 0$ or $x - 4 = 0$	Solve each linear factor equal to zero to finish.
$x = -4$ or $x = 4$	

6 Solve the following by factorising the DOPS.

a $x^2 - 25 = 0$ **b** $x^2 - 36 = 0$ **c** $x^2 - 100 = 0$
d $4x^2 - 9 = 0$ **e** $9x^2 - 16 = 0$ **f** $49x^2 - 81 = 0$

Example 14 Solving quadratic equations

Solve the following quadratic equations.

a $x^2 - 5x + 6 = 0$ **b** $x^2 + 2x + 1 = 0$

Solution	Explanation
a $x^2 - 5x + 6 = 0$	Factorise by finding two numbers that multiply to 6
$(x - 3)(x - 2) = 0$	and add to -5: $-3 \times (-2) = 6$ and $-3 + (-2) = -5$.
$\therefore\ x - 3 = 0$ or $x - 2 = 0$	Apply the Null Factor Law and solve for x.
$\therefore\ x = 3$ or $x = 2$	
b $x^2 + 2x + 1 = 0$	$1 \times 1 = 1$ and $1 + 1 = 2$
$(x + 1)(x + 1) = 0$	$(x + 1)(x + 1) = (x + 1)^2$ is a perfect square.
$(x + 1)^2 = 0$	This gives one solution for x.
$\therefore\ x + 1 = 0$	
$\therefore\ x = -1$	

ISBN: 9781107687028 Cambridge University Press

7 Solve the following quadratic equations.

- **a** $x^2 + 3x + 2 = 0$
- **b** $x^2 + 5x + 6 = 0$
- **c** $x^2 - 6x + 8 = 0$
- **d** $x^2 - 7x + 10 = 0$
- **e** $x^2 + 4x - 12 = 0$
- **f** $x^2 + 2x - 15 = 0$
- **g** $x^2 - x - 20 = 0$
- **h** $x^2 - 5x - 24 = 0$
- **i** $x^2 - 12x + 32 = 0$
- **j** $x^2 + 4x + 4 = 0$
- **k** $x^2 + 10x + 25 = 0$
- **l** $x^2 - 8x + 16 = 0$
- **m** $x^2 - 14x + 49 = 0$
- **n** $x^2 - 24x + 144 = 0$
- **o** $x^2 + 18x + 81 = 0$

Parts **j** to **o** are perfect squares, so you will only find one solution.

Problem-solving and Reasoning

8 How many different solutions for x will these equations have?

- **a** $(x - 2)(x - 1) = 0$
- **b** $(x + 7)(x + 3) = 0$
- **c** $(x + 1)(x + 1) = 0$
- **d** $(x - 3)(x - 3) = 0$
- **e** $(x + \sqrt{2})(x - \sqrt{2}) = 0$
- **f** $(x + 8)(x - \sqrt{5}) = 0$
- **g** $(x + 2)^2 = 0$
- **h** $(x + 3)^2 = 0$
- **i** $3(2x + 1)^2 = 0$

9 The height of a paper plane above floor level in metres is given by $-\frac{1}{5}t(t - 10)$, where t is in seconds.

- **a** Find the height of the plane after:
  - **i** 2 seconds **ii** 6 seconds
- **b** Solve $-\frac{1}{5}t(t - 10) = 0$ for t.
- **c** How long does it take for the plane to hit the ground after it is launched?

10 Solve by first taking out a common factor.

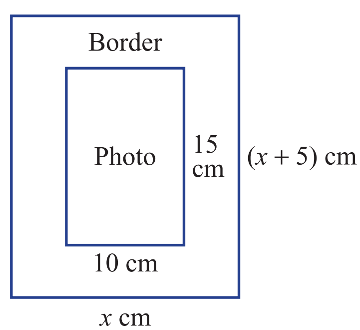
- **a** $2x^2 + 16x + 24 = 0$
- **b** $2x^2 - 20x - 22 = 0$
- **c** $3x^2 - 18x + 27 = 0$
- **d** $5x^2 - 20x + 20 = 0$

First take out the common factor, then factorise before using the Null Factor Law.

Photo albums

11 A printer produces photo albums. Each page has a length 5 cm more than the width, and includes a spot in the middle for a standard 10 cm by 15 cm photo.

- **a** Find the area of the photo.
- **b** Find an expression for:
  - **i** the total area of a page
  - **ii** the border area of a page (i.e. excluding the photo)
- **c** Factorise your expression for the border area.
- **d** For what value of x is the border area equal to zero?
- **e** For what value of x is the border area equal to 350 cm^2?



Using technology 10.4: Solving quadratic equations

This activity is available on the companion website as a printable PDF.

ISBN: 9781107687028

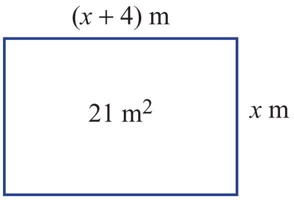
10.5 Applications of quadratics

Defining variables, setting up equations, solving equations and interpreting solutions are all important elements of applying quadratics in problem solving. For example, the area of a rectangular paddock that can be fenced off using a limited length of fencing can be found by setting up a quadratic equation, solving it and then interpreting the solutions.

Let's start: Rectangular quadratics

The length of a rectangular room is 4 m longer than its width, which is x metres. Its area is 21 m^2.

- Write an expression for the area using the variable x.
- Use the 21 m^2 fact to set up an equation equal to zero.
- Factorise and solve the equation to find x and then find the dimensions of the rectangle.



Key ideas

- When applying quadratic equations:
  - Define a variable. 'Let x be …'
  - Write an equation.
  - Solve the equation.
  - Choose the solution(s) that solves the equation and answers the question.

Exercise 10E

Understanding

1 Write expressions for each of the following.
  a The length of a rectangle if it is 4 more than its width, x.
  b The length of a rectangle if it is 10 more than its width, x.
  c The length of a rectangle if it is 7 less than its width, x.
  d The height of a triangle if it is 2 less than its base, x.
  e The height of a triangle if it is 6 more than its base, x.

ISBN: 9781107687028 Cambridge University Press

2 Rearrange these equations so that there is a zero on the right side. Do not try to solve the equation.

a $x^2 + 2x = 3$ **b** $x^2 - 3x = 5$ **c** $x^2 + 7x = 4$

Subtract from both sides to give a zero on the right side.

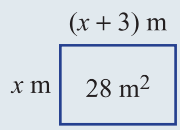
3 Factorise and solve these equations. Only give the positive answer for x.

a $x^2 - x - 6 = 0$ **b** $x^2 - 3x - 10 = 0$ **c** $x^2 + 2x - 24 = 0$

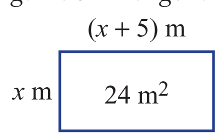
Fluency

Example 15 Finding dimensions

The area of a rectangle is fixed at 28 m^2 and its length is 3 metres more than its width. Find the dimensions of the rectangle.

Solution	Explanation
Let x m be the width of the rectangle. Length = $(x + 3)$ m	
$x(x + 3) = 28$ $x^2 + 3x - 28 = 0$ $(x + 7)(x - 4) = 0$ $x = -7$ or $x = 4$ Choose $x = 4$	Write an equation using the given information. Then write with a zero on the right and solve for x. Disregard $x = -7$ because x must be greater than zero.
Rectangle has width = 4 m and length = 7 m.	Answer the question in full.

4 A rectangle has an area of 24 m^2. Its length is 5 m longer than its width.



a Copy this sentence. 'Let x m be the width of the rectangle.'

b Write an expression for the rectangle's length.

c Write an equation using the rectangle's area.

d Write your equation from part **c** with a zero on the right hand side, and solve for x.

e Find the dimensions of the rectangle.

5 Repeat all the steps in Question **4** to find the dimensions of a rectangle with the following properties.

a Its area is 60 m^2 and its length is 4 m more than its width.

b Its area is 63 m^2 and its length is 2 m less than its width.

c Its area is 154 mm^2 and its length is 3 mm less than its width.

Carefully set out each step as in Example **15** and Question **4**.

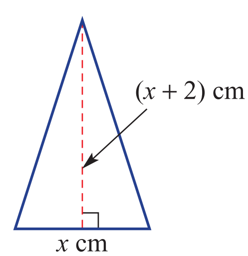
6 A triangle's area is 4 cm^2 and its height is 2 cm more than its base.

a Write an expression for the area of the triangle using $A = \frac{1}{2}bh$.

b Write an equation using the 4 cm^2 area fact.

c Multiply both sides by 2 and write your equation with a zero on the right side.

d Solve your equation to find the base and height dimensions of the triangle.



ISBN: 9781107687028

Cambridge University Press

7 Find the height and base lengths of a triangle that has an area of 7 m^2 and height 5 m less than its base.

Problem-solving and Reasoning

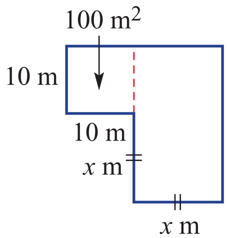
8 The product of two consecutive whole numbers (x and $x + 1$) is 132, so $x(x + 1) = 132$.

- **a** Expand the equation and make a zero on the right side.
- **b** Solve the equation to find two values of x.
- **c** List the two pairs of consecutive numbers that multiply to 132.

'Product' means 'times'. 'Consecutive' means 'next to'; e.g. 4 and 5 are consecutive whole numbers.

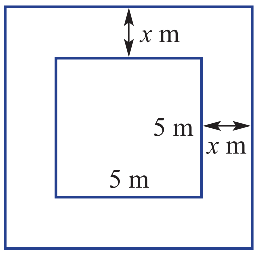
9 The product of two consecutive numbers is 72. Use a quadratic equation to find the two sets of numbers.

10 A 100 m^2 hay shed is to be expanded to give 475 m^2 of floor space in total, as shown in the diagram. Find the value of x.



11 A square hut of side length 5 m is to be surrounded with a verandah of width x metres. Find the width of the verandah if its area is to be 24 m^2.

What is the side length of the verandah?



Projectile maths

12 A ball is thrust vertically upwards from a machine on the ground. The height (h metres) after t seconds is given by $h = t(4 - t)$.

- **a** Find the height after 1.5 seconds.
- **b** Find when the ball is at a height of 3 m.
- **c** Why are there two solutions to part **b**?
- **d** Find when the ball is at ground level.
- **e** Find when the ball is at a height of 4 m.
- **f** Why is there only one solution for part **e**?
- **g** Is there a time when the ball is at a height of 5 m? Explain.

13 The height h (in metres) of a rocket is given by $h = -x^2 + 100x$ where x metres is the horizontal distance from where the rocket was launched.

- **a** Find the values of x if $h = 0$.
- **b** Interpret your answer from part **a**.
- **c** Find how far the rocket has travelled horizontally if the height is 196 m.

ISBN: 9781107687028

Cambridge University Press

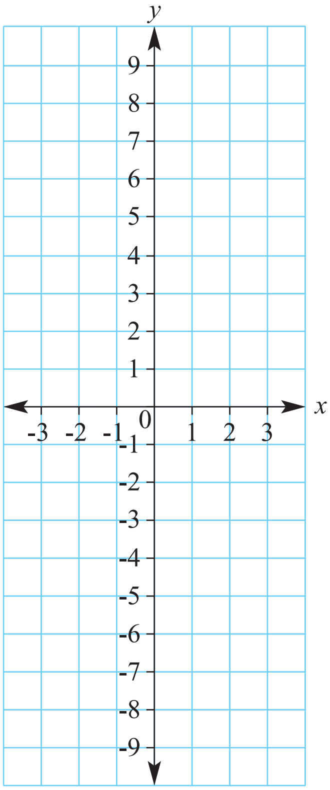
10.6 Exploring parabolas

One of the simplest and most important non-linear graphs is the parabola. When a ball is thrown or water streams up and out from a garden hose or fountain, the path followed has a parabolic shape. The parabola is the graph of a quadratic relation with the basic rule $y = x^2$. Quadratic rules such as $y = (x - 1)^2$ and $y = 2x^2 - x - 3$ also give graphs that are parabolas and are transformations of the graph of $y = x^2$.

Let's start: To what effect?

To see how different quadratic rules compare to the graph of $y = x^2$, complete this table and plot the graph of each equation on the same set of axes.

x	-3	-2	-1	0	1	2	3
$y_1 = x^2$	9	4					
$y_2 = -x^2$	-9						
$y_3 = (x - 2)^2$	25	16	9				
$y_4 = x^2 - 3$	6						



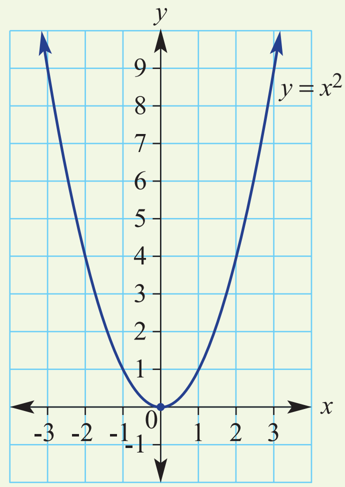
- For all the graphs, find such features as the:
  - turning point
  - axis of symmetry
  - y-intercept
  - x-intercepts.
- Discuss how each of the graphs of y_2, y_3 and y_4 compare to the graph of $y = x^2$. Compare the rule with the position of the graph.

ISBN: 9781107687028

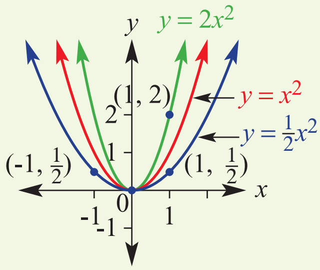
Cambridge University Press

Key ideas

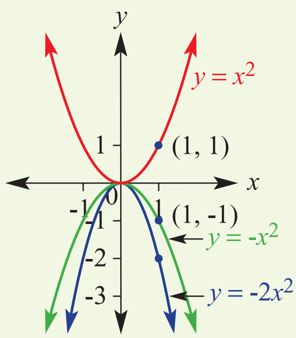
- A **parabola** is the graph of a quadratic relation. The basic parabola has the rule $y = x^2$.
  - The vertex (or turning point) is (0, 0).
  - It is a minimum turning point.
  - Axis of symmetry is $x = 0$.
  - y-intercept is (0, 0).
  - x-intercept is (0, 0).



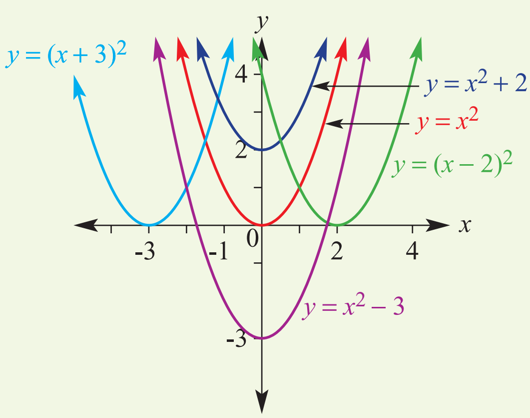
- Simple transformations of the graph of $y = x^2$ include:
  - **dilation**



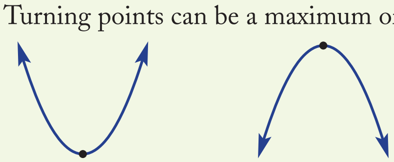
  - **reflection**



  - **translation**



- Turning points can be a maximum or a minimum.



Minimum turning point

Maximum turning point

Parabola
A smooth U-shaped curve with the basic rule $y = x^2$

Dilation
A transformation where a curve is enlarged or reduced but the centre is not changed

Reflection
A transformation where a curve is flipped across a line on the number plane

Translation
A transformation where a curve is moved a certain distance on the number plane

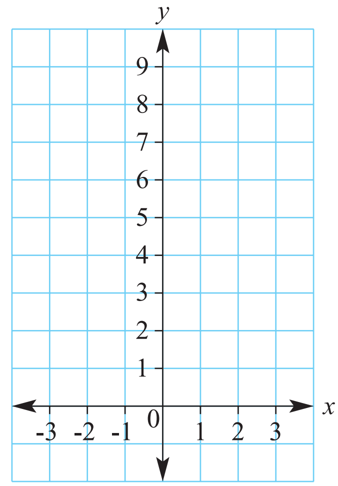
ISBN: 9781107687028

Exercise 10F

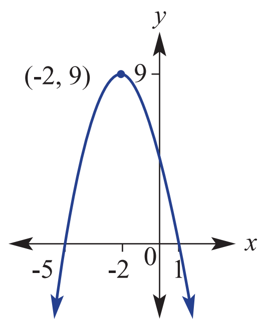
Understanding

1 Complete this table and grid to plot the graph of $y = x^2$.

x	-3	-2	-1	0	1	2	3
y	9						



2 Write the missing features for this graph.

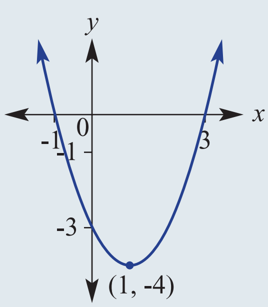


a The parabola has a __________ (maximum or minimum).
b The coordinates of the turning point are _____.
c The coordinates of the y-intercept are _____.
d The coordinates of the x-intercepts are _____ and _____.
e The axis of symmetry is ______.

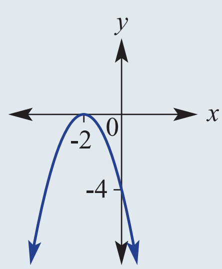
Example 16 Identifying key features of parabolas

Determine the following key features of each of the given graphs.
i Turning point and whether it is a maximum or minimum
ii Axis of symmetry **iii** x-intercepts **iv** y-intercept

a



b



ISBN: 9781107687028

Solution			Explanation
a	**i**	Turning point is a minimum at (1, -4).	Lowest point of graph is at (1, -4).
	ii	Axis of symmetry is $x = 1$.	Line of symmetry is through the x-coordinate of the turning point.
	iii	x-intercepts at (-1, 0) and (3, 0).	
	iv	y-intercept at (0, -3).	x-intercepts lie on the x-axis ($y = 0$) and the y-intercept on the y-axis ($x = 0$).
b	**i**	Turning point is a maximum at (-2, 0).	Graph has a highest point at (-2, 0).
	ii	Axis of symmetry is $x = -2$.	Line of symmetry is through the x-coordinate of the turning point.
	iii	x-intercept at (-2, 0).	
	iv	y-intercept at (0, -4).	Turning point is also the one x-intercept.

3 Determine these key features of the following graphs.

i turning point and whether it is a maximum or minimum

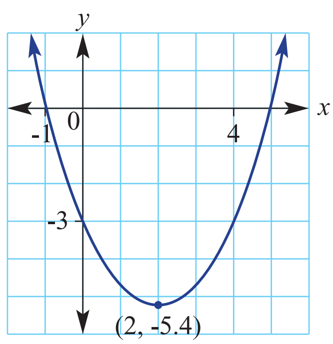
ii axis of symmetry

iii x-intercepts

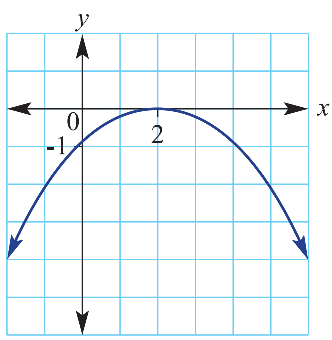
iv y-intercept

An axis of symmetry is described by a rule such as $x = 2$ or $x = -3$.

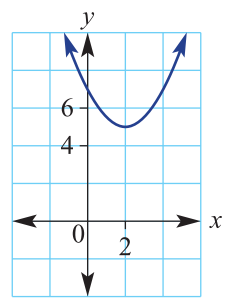
a



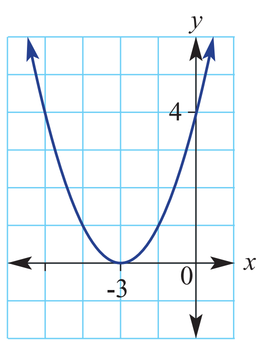
b



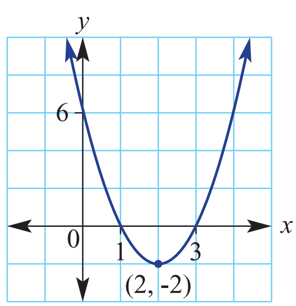
c



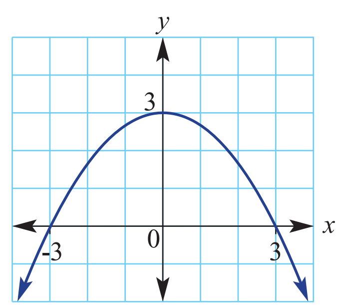
d



e



f



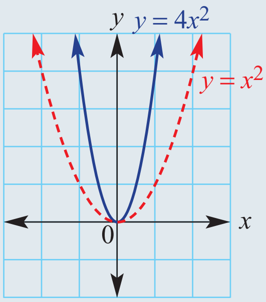
ISBN: 9781107687028 Cambridge University Press

Fluency

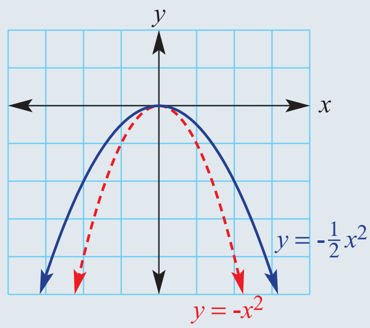
Example 17 Dilating and reflecting parabolas

Copy and complete the table for the following graphs.

a



b



	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 4x^2$					
b	$y = \frac{-1}{2}x^2$					

Solution

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 4x^2$	Minimum	No	(0, 0)	4	Narrower
b	$y = \frac{-1}{2}x^2$	Maximum	Yes	(0, 0)	$\frac{-1}{2}$	Wider

Explanation

Read features from graphs and consider the effect of each change in equation on the graph.

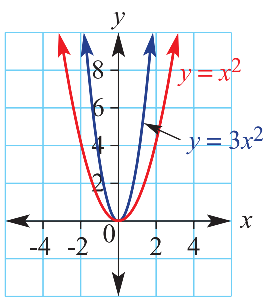
4 Copy and complete the table below for the graphs that follow.

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$					
b	$y = \frac{1}{2}x^2$					
c	$y = 2x^2$					
d	$y = -4x^2$					
e	$y = -\frac{1}{3}x^2$					
f	$y = -2x^2$					

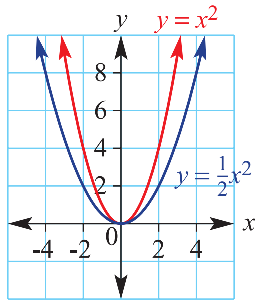
ISBN: 9781107687028

Cambridge University Press

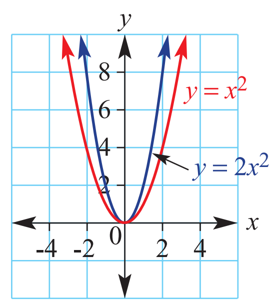
a



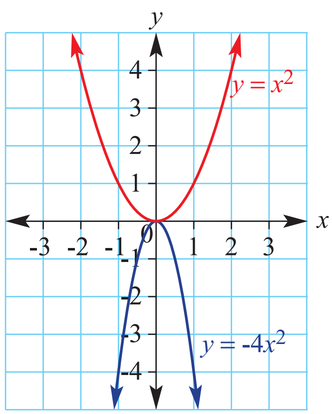
b



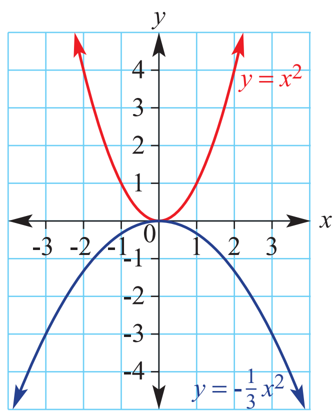
c



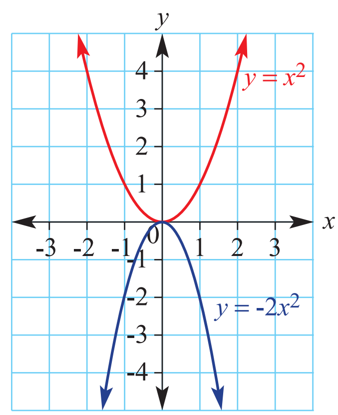
d



e



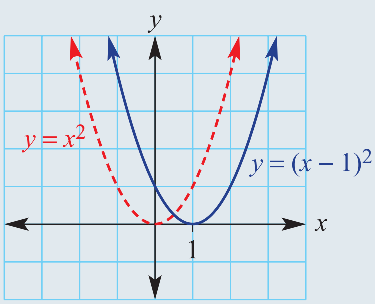
f



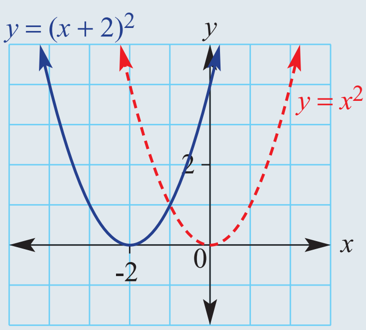
Example 18 Translating horizontally

Copy and complete the table for the following graphs.

a



b



	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = (x - 1)^2$					
b	$y = (x + 2)^2$					

Solution **Explanation**

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = (x - 1)^2$	Minimum	No	(1, 0)	0	Same
b	$y = (x + 2)^2$	Minimum	No	(-2, 0)	9	Same

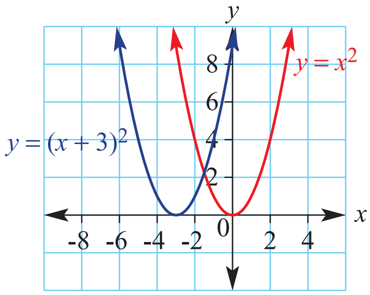
The effect is to shift right or left: Right for part **a** and left for part **b**.

ISBN: 9781107687028

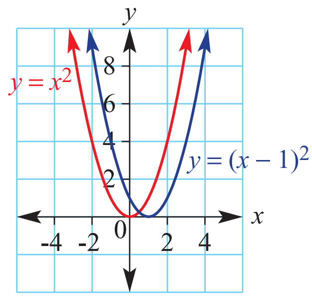
5 Copy and complete the table below for the graphs that follow.

	Formula	Turning point	Axis of symmetry	y-intercept ($x = 0$)	x-intercept
a	$y = (x + 3)^2$				
b	$y = (x - 1)^2$				
c	$y = (x - 2)^2$				
d	$y = (x + 4)^2$				

a

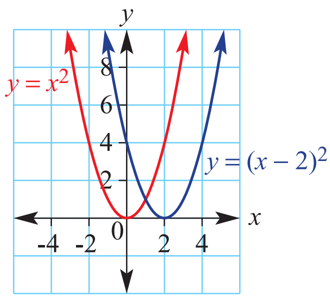


b

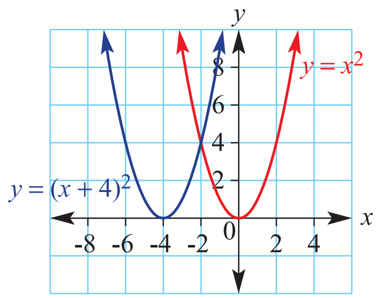


The axis of symmetry is a vertical line passing through the turning point of a parabola. The equation is given by the x coordinate of the turning point.

c



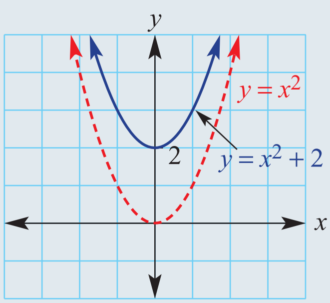
d



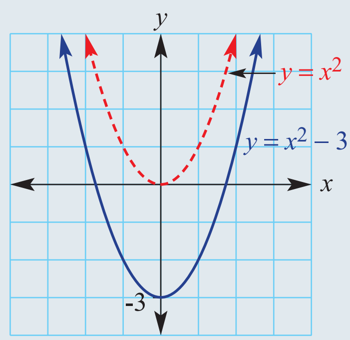
Example 19 Translating vertically

Copy and complete the table for the following graphs.

a



b



	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = x^2 + 2$					
b	$y = x^2 - 3$					

ISBN: 9781107687028

Solution **Explanation**

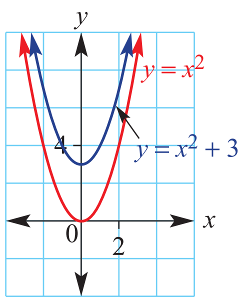
	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = x^2 + 2$	Minimum	No	(0, 2)	3	Same
b	$y = x^2 - 3$	Minimum	No	(0, -3)	-2	Same

The effect is to shift up or down: Up for $y = x^2 + 2$ and down for $y = x^2 - 3$.

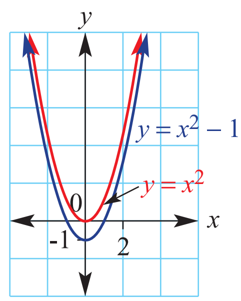
6 Copy and complete the table for the graphs that follow.

	Formula	Turning point	y-intercept ($x = 0$)	y value when $x = 1$
a	$y = x^2 + 3$			
b	$y = x^2 - 1$			
c	$y = x^2 + 2$			
d	$y = x^2 - 4$			

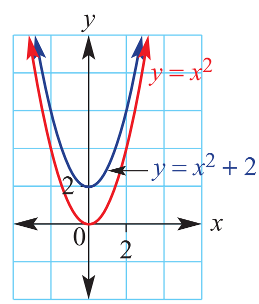
a



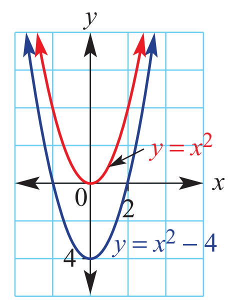
b



c



d



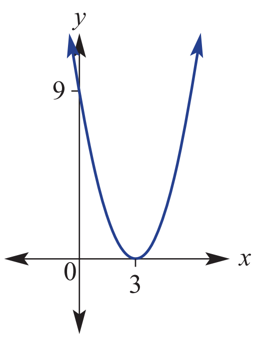
Problem-solving and Reasoning

7 Match each of the following equations to one of the graphs below.

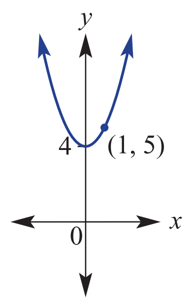
a $y = 2x^2$ **b** $y = x^2 - 6$ **c** $y = (x + 2)^2$

d $y = -5x^2$ **e** $y = (x - 3)^2$ **f** $y = x^2 + 4$

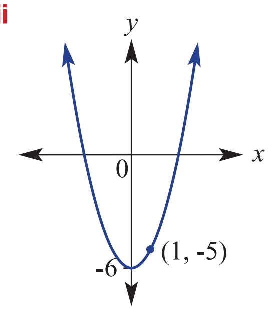
i



ii

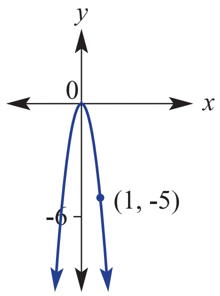


iii

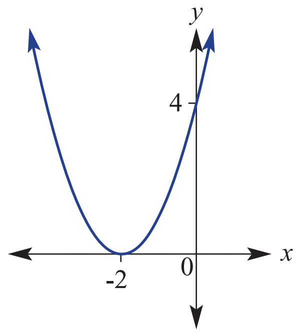


ISBN: 9781107687028 Cambridge University Press

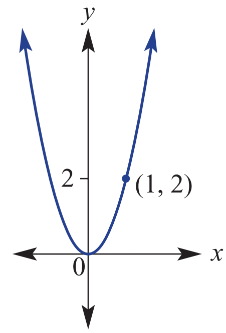
iv



v



vi



8 Write a rule for a parabola with each feature.

- **a** Same shape as $y = x^2$, minimum turning point (0, 2)
- **b** Same shape as $y = x^2$, maximum turning point (0, 0)
- **c** Same shape as $y = x^2$, minimum turning point (-1, 0)
- **d** Same shape as $y = x^2$, minimum turning point (5, 0)

What turns $y = x^2$ into a graph with a maximum turning point?

Parabolas with technology

9 a Using technology, plot the following pairs of graphs on the same set of axes for $-5 \le x \le 5$ and compare their tables of values.

- **i** $y = x^2$ and $y = 4x^2$
- **ii** $y = x^2$ and $y = \frac{1}{3}x^2$
- **iii** $y = x^2$ and $y = 6x^2$
- **iv** $y = x^2$ and $y = \frac{1}{4}x^2$
- **v** $y = x^2$ and $y = 7x^2$
- **vi** $y = x^2$ and $y = \frac{2}{5}x^2$

b Suggest how the constant a in $y = ax^2$ transforms the graph of $y = x^2$.

10 a Using technology, plot the following sets of graphs on the same set of axes for $-5 \le x \le 5$ and compare the turning point of each.

- **i** $y = x^2, y = (x + 1)^2, y = (x + 2)^2, y = (x + 3)^2$
- **ii** $y = x^2, y = (x - 1)^2, y = (x - 2)^2, y = (x - 3)^2$

b Explain how the constant h in $y = (x + h)^2$ transforms the graph of $y = x^2$.

11 a Using technology, plot the following sets of graphs on the same set of axes for $-5 \le x \le 5$ and compare the turning point of each.

- **i** $y = x^2, y = x^2 + 1, y = x^2 + 2, y = x^2 + 3$
- **ii** $y = x^2, y = x^2 - 1, y = x^2 - 3, y = x^2 - 5$

b Explain how the constant k in $y = x^2 + k$ transforms the graph of $y = x^2$.

Using technology 10.6: Sketching parabolas

This activity is available on the companion website as a printable PDF.

ISBN: 9781107687028

Cambridge University Press

10.7 Graphs of circles and exponentials

We know the circle as a common shape in geometry. We can also describe a circle using a rule and as a graph on the Cartesian plane.

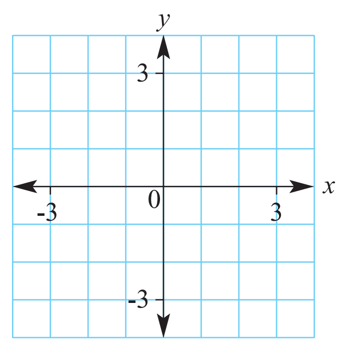
We can also use graphs to illustrate exponential relationships. The population of the world, for example, or the balance of an investment account, can be described using exponential rules that include indices. The rule $A = 100\,000(1.05)^t$ describes the account balance of \$100 000 invested at 5% p.a. compound interest for t years.

Let's start: Plotting non-linear curves

A graph has the rule $x^2 + y^2 = 9$.

- If $x = 0$ what are the two values of y?
- If $x = 1$ what are the two values of y?
- If $x = 4$ are there any values of y? Discuss.
- Complete this table of values.

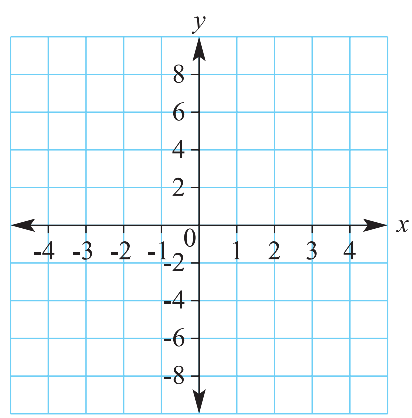
x	-3	-2	-1	0	1	2	3
y		$\pm\sqrt{5}$					



- Now plot all your points on a number plane and join them to form a smooth curve.
- What shape have you drawn and what are its features?
- How does the radius of your circle relate to the equation?

Complete this table and graph the rule $y = 2^x$ before discussing the points below.

x	-3	-2	-1	0	1	2	3
$y = 2^x$	$\frac{1}{8}$			1		4	

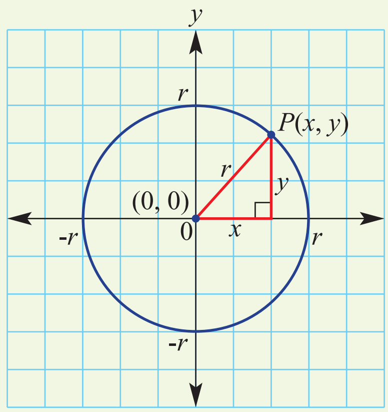


- Discuss the shape of the graph.
- Where does the graph cut the y-axis?
- Does the graph have an x-intercept? Why not?

ISBN: 9781107687028

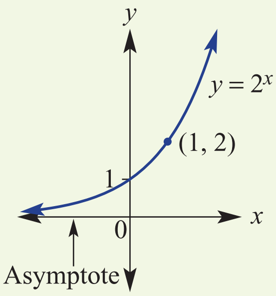
Key ideas

- The Cartesian equation of a circle with centre (0, 0) and radius r is given by $x^2 + y^2 = r^2$.



Using Pythagoras' theorem
$a^2 + b^2 = c^2$ gives $x^2 + y^2 = r^2$

- An **asymptote** is a line that a curve approaches but never touches. The curve gets closer and closer to the line so that the distance between the curve and the line approaches zero, but the curve never meets the line so the distance is never zero.



- A simple **exponential** rule is of the form $y = a^x$, where $a > 0$ and $a \neq 1$.
  - y-intercept is (0, 1).
  - $y = 0$ is the equation of the asymptote.

Asymptote A line whose distance to a curve approaches zero, and the curve never touches it

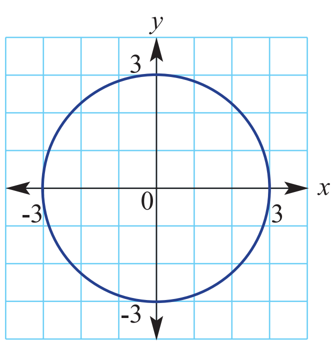
Exponential (notation) A way of representing repeated multiplication of the same number

Exercise 10G

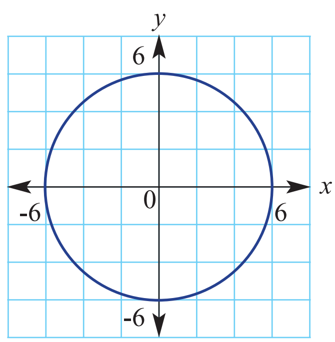
Understanding

1 Write the coordinates of the centre and give the radius of these circles.

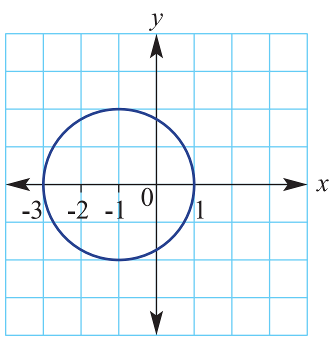
a



b

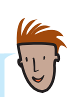


c



2 Solve these equations for the unknown variable, correct to one decimal place where necessary.

a $x^2 + 2^2 = 9$ **b** $x^2 + 3^2 = 25$ **c** $5^2 + y^2 = 36$



There are two answers for each. ± 4 means the answers are +4 and -4.

3 A circle has equation $x^2 + y^2 = r^2$. Complete these sentences.

a The centre of the circle is _______.
b The radius of the circle is _______.

4 Evaluate:

a 2^0 **b** 2^1 **c** 2^4 **d** 3^0 **e** 3^1
f 3^3 **g** 4^0 **h** 4^2 **i** 5^0 **j** 5^2

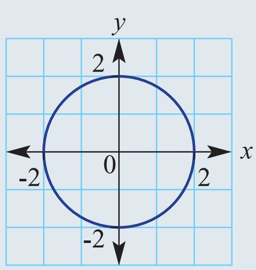
ISBN: 9781107687028 Cambridge University Press

Fluency

Example 20 Sketching a circle

Complete the following for the equation $x^2 + y^2 = 4$.

a State the coordinates of the centre.
b State the radius.
c Find the values of y when $x = 1$, correct to one decimal places.
d Find the values of x when $y = 0$.
e Sketch a graph showing intercepts.

Solution	Explanation
a $(0, 0)$	$(0, 0)$ is the centre for all circles $x^2 + y^2 = r^2$.
b $r = 2$	$x^2 + y^2 = r^2$ so $r^2 = 4$.
c $x^2 + y^2 = 4$ $1^2 + y^2 = 4$ $y^2 = 3$ $y = \pm 1.7$	Substitute $x = 1$ and solve for y. $y^2 = 3$, so $y = \pm\sqrt{3}$ $\sqrt{3} \approx 1.7$
d $x^2 + 0^2 = 4$ $x^2 = 4$ $x = \pm 2$	Substitute $y = 0$. Solve for x. Both $(-2)^2$ and $2^2 = 4$.
e 	Draw a circle with centre $(0, 0)$ and radius 2. Label intercepts.

5 A circle has equation $x^2 + y^2 = 9$. Complete the following.
  a State the coordinates of the centre.
  b State the radius.
  c Find the values of y when $x = 2$, correct to one decimal place.
  d Find the values of x when $y = 0$.
  e Sketch a graph showing intercepts.

If $x^2 + y^2 = r^2$ then r is the radius.

If $y^2 = 5$, then $y = \pm\sqrt{5}$.

6 Complete the following for the equation $x^2 + y^2 = 25$.
  a State the coordinates of the centre.
  b State the radius.
  c Find the values of y when $x = 4$.
  d Find the values of x when $y = 0$.
  e Sketch a graph showing intercepts.

ISBN: 9781107687028

Example 21 Plotting an exponential graph

For the rule $y = 2^x$:

a complete this table

b plot points to form its graph

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$				

Solution

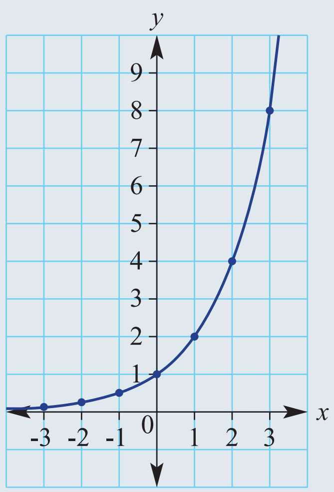
a

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Explanation

$2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8.$

b



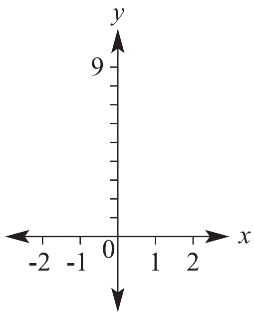
Plot each point and join to form a smooth curve.

7 Consider the exponential rule $y = 3^x$.

a Complete this table.

x	-2	-1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$			

b Plot the points in the table to form the graph of $y = 3^x$.



ISBN: 9781107687028

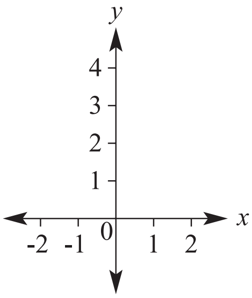
Cambridge University Press

8 Consider the exponential rule $y = 4^x$.

a Complete this table.

x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$			

b Plot the points in the table to form the graph of $y = 4^x$.



Problem-solving and Reasoning

9 **a** Sketch the graph of the following on the same set of axes, labelling the y-intercept and the point where $x = 1$.

i $y = 2^x$

ii $y = 4^x$

iii $y = 5^x$

b What do you notice about the y-intercept on each graph?

c What does increasing the base number do to each graph?

Use this table to help.

x	-1	0	1	2
$y = 2^x$	$\frac{1}{2}$	1		
$y = 4^x$	$\frac{1}{4}$			
$y = 5^x$	$\frac{1}{5}$			

10 Give the radius of the circles with these equations.

a $x^2 + y^2 = 36$ **b** $x^2 + y^2 = 81$ **c** $x^2 + y^2 = 144$

d $x^2 + y^2 = 5$ **e** $x^2 + y^2 = 14$ **f** $x^2 + y^2 = 20$

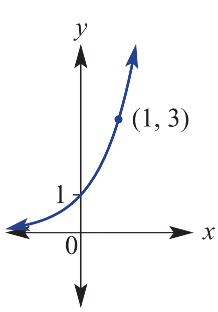
Remember: $x^2 + y^2 = r^2$

11 Write the equation of a circle with centre (0, 0) and radius 7.

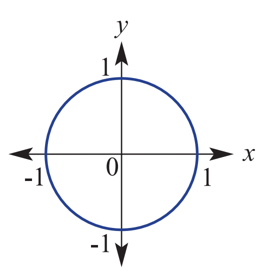
12 Match equations **a–c** with graphs **A–C**.

a $y = -x - 2$ **b** $y = 3^x$ **c** $x^2 + y^2 = 1$

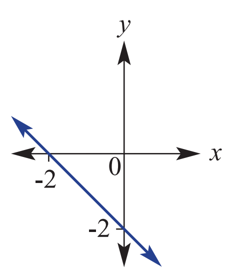
A



B



C



ISBN: 9781107687028

13 A study shows that the population of a town is modelled by the rule $P = 2^t$, where t is in years and P is in thousands of people.

- **a** State the number of people in the town at the start of the study ($t = 0$).
- **b** State the number of people in the town after:
  - **i** 1 year
  - **ii** 3 years
- **c** When is the town's population expected to reach:
  - **i** 4000 people?
  - **ii** 16 000 people?

If $P = 3$, there are 3000 people.

Graphs of hyperbolas

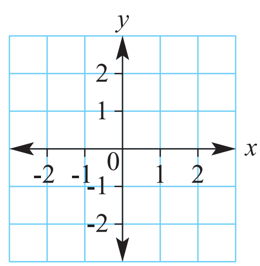
Another type of graph is called a hyperbola, and it comes from the rule $y = \frac{1}{x}$.

14 A hyperbola has the rule $y = \frac{1}{x}$.

a Complete this table.

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y						

b Plot the points to form the graph of $y = \frac{1}{x}$.

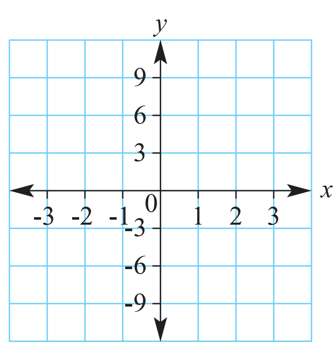


15 A hyperbola has the rule $y = \frac{3}{x}$.

a Complete this table

x	-3	-1	$-\frac{1}{3}$	$\frac{1}{3}$	1	3
y						

b Plot the points to form the graph of $y = \frac{3}{x}$.



Using technology 10.7: Graphing circles and other graphs

This activity is available on the companion website as a printable PDF.

ISBN: 9781107687028

1 I am a beautiful curve! Solve the equations then match the letters to the answers to find out what I am.

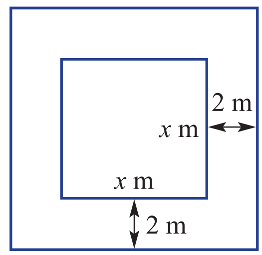
O	B	A	L	P	R
2^3	radius of $x^2 + y^2 = 16$	solution to $(x - 3)^2 = 0$	$x^2 - x$	$-2(x - 1)$	solution to $x^2 - 2x - 8 = 0$

___	___	___	___	___	___	___	___
$-2x + 2$	3	4, -2	3	4	8	$x(x - 1)$	3

2 A square pool of side length x metres is surrounded by a 2 m wide tiled edge.

a Find an expression for the total area.

b For what value of x is the total area equal to 100 m^2?



3 A book's length is 6 cm longer than its width and its total cover area is 280 cm^2. What are its dimensions?

4 The product of two consecutive even numbers is 168. Find the two numbers.

5 A father's age is the square of his son's age (x). In 20 years' time the father will be 3 times as old as his son. What are the ages of the father and son?

6 A rectangular painting is to have a total area (including the frame) of 1200 cm^2. The painting is 30 cm long and 20 cm wide, find the width of the frame.

7 Simplify these expressions.

a $4x - 3(2 - x)$ **b** $(x - 1)^2 - (x + 1)^2$ **c** $\dfrac{x^2 - x - 6}{x + 2}$

8 A cyclist in a charity ride rides 300 km at a constant average speed. If the average speed had been 5 km/h faster the ride would have taken 2 hours less. What was the average speed of the cyclist?

ISBN: 9781107687028 Cambridge University Press

Chapter summary

Quadratic equations

Factorising $x^2 + bx + c$

2 numbers × to b
+ to c

eg. $x^2 - 7x - 18 = (x - 9)(x + 2)$
$-9 \times 2 = -18$
$-9 + 2 = -7$

Expanding brackets

$a(b + c) = ab + ac$
$(a + b)(c + d) = ac + ad + bc + bd$
$(a + b)(a - b) = a^2 - b^2$ (DOPS)
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$

Solving quadratic equations

Null Factor Law:
If $ab = 0$ then $a = 0$ or $b = 0$.
Write each quadratic in standard form $ax^2 + bx + c = 0$, factorise then apply the Null Factor Law to solve.

e.g. **1** $x^2 - 4x = 0$
$x(x - 4) = 0$
$x = 0$ or $x - 4 = 0$
$x = 0$ or $x = 4$

2 $x^2 - 3x + 10 = 0$
$(x - 5)(x + 2) = 0$
$x - 5 = 0$ or $x + 2 = 0$
$x = 5$ or $x = -2$

Factorising and DOPS

Always take out common factors first.
Difference of perfect squares
$a^2 - b^2 = (a - b)(a + b)$

e.g. **1** $x^2 - 4 = x^2 - 2^2$
$= (x + 2)(x - 2)$

2 $4x^2 - 9 = (2x)^2 - (3)^2$
$= (2x - 3)(2x + 3)$

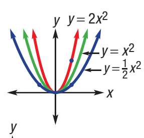
Applications

1. Define the variable.
2. Set up the equation.
3. Solve by factorising and using the Null Factor Law.
4. Determine the suitable answer(s).

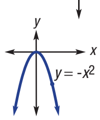
Parabolas

Transformations of a parabola

$y = ax^2$ dilates graph compared to $y = x^2$
$0 < a < 1$ graph is wider
$a > 1$ graph is narrower

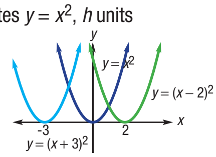


$a < 0$ graph is inverted
(reflected in x-axis)



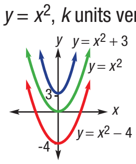
$y = (x - h)^2$ translates $y = x^2$, h units horizontally

$h > 0$ move right
$h < 0$ move left

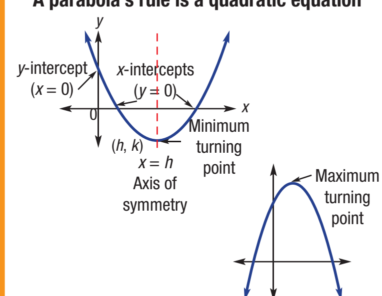


$y = x^2 + k$ translates $y = x^2$, k units vertically

$k > 0$ move up
$k < 0$ move down



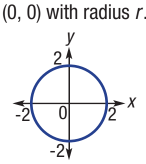
A parabola's rule is a quadratic equation



Other graphs

Circles

$x^2 + y^2 = r^2$ is a circle centred at (0, 0) with radius r.

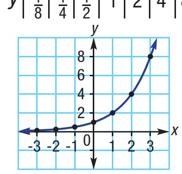


$x^2 + y^2 = 4$
$r^2 = 4$
$\therefore r = 2$

Exponential graphs

$y = 2^x$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



ISBN: 9781107687028

Cambridge University Press

Multiple-choice questions

1 $-2x(1 - x)$ expands to:
A $-2 + 2x^2$ **B** $-2x - 2x^2$ **C** $2x + 2x^2$
D $-3x^2$ **E** $-2x + 2x^2$

2 $(x + 5)^2$ is the same as:
A $x^2 + 25$ **B** $x^2 + 5x$ **C** $x^2 + 5x + 25$
D $x^2 + 10x + 25$ **E** $x^2 + 50$

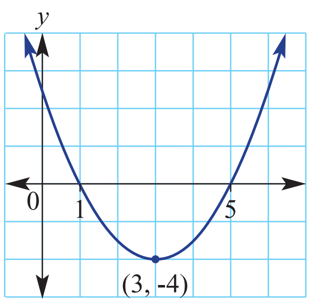
3 $(2x - 1)(x + 4)$ is equal to:
A $2x^2 + 11x - 2$ **B** $2x^2 + 7x - 4$ **C** $4x^2 + 14x - 8$
D $4x^2 + 9x - 2$ **E** $2x^2 + 5x + 4$

4 $4x^2 - 25$ in factorised form is:
A $4(x - 5)(x + 5)$ **B** $(2x - 5)^2$ **C** $(2x - 5)(2x + 5)$
D $(4x + 5)(x - 5)$ **E** $2(2x + 1)(x - 25)$

5 The solutions to $2x^2 - 8x = 0$ are:
A $x = 0, x = -4$ **B** $x = 2$ **C** $x = 0, x = 4$
D $x = 4$ **E** $x = 0, x = 2$

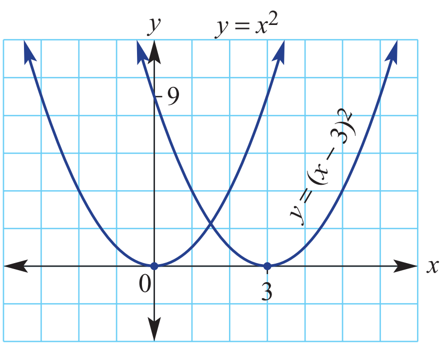
6 The solutions to $(2x - 1)(x + 1) = 0$ are:
A $x = 0, x = 1$ **B** $x = -1, x = \frac{1}{2}$ **C** $x = -1, x = 2$
D $x = -1, x = -2$ **E** $x = 1, x = \frac{1}{2}$

7 The equation of the axis of symmetry of the graph shown is:



A $y = -4$ **B** $x = 3$ **C** $x = -4$
D $y = 3$ **E** $y = 3x$

8 Compared to the graph of $y = x^2$, the graph of $y = (x - 3)^2$ is:
A 3 units down **B** 3 units left
C in the same place **D** 3 units right
E 3 units up



ISBN: 9781107687028

Cambridge University Press

9 The equation of a circle centred at the origin with radius 4 units is:

A $y = 4x^2$ **B** $x^2 + y^2 = 4$ **C** $x^2 + y^2 = 8$

D $y = 4^x$ **E** $x^2 + y^2 = 16$

10 The graph of $y = 3^x$ has y-intercept with coordinates:

A $(0, 3)$ **B** $(3, 0)$ **C** $(0, 1)$

D $(1, 3)$ **E** $\left(0, \frac{1}{3}\right)$

Short-answer questions

1 Expand the following and simplify where possible.

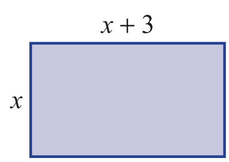
a $-2(x + 1)$ **b** $x(x + 3)$ **c** $(x + 2)(x - 1)$

d $(x + 5)(3x - 4)$ **e** $(x + 4)(x - 4)$ **f** $(5x - 2)(5x + 2)$

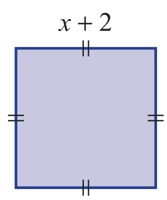
g $(x + 2)^2$ **h** $(x - 6)^2$ **i** $(3x - 2)(4x - 5)$

2 Write, in expanded form, an expression for the shaded areas.

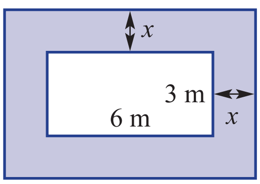
a



b



c



3 Factorise by removing a common factor.

a $3x - 9$ **b** $-4x - 16$ **c** $x^2 + 2x$

d $ab - b$ **e** $7x - 14x^2$ **f** $-a^2b - 6ab$

4 Factorise the following by using difference of perfect squares. Remember to look for a common factor first.

a $x^2 - 49$ **b** $9x^2 - 16$ **c** $4x^2 - 1$

d $3x^2 - 75$ **e** $2x^2 - 18$ **f** $4x^2 - 81$

5 Factorise these quadratic trinomials. Some are perfect squares.

a $x^2 + 5x + 6$ **b** $x^2 - x - 6$ **c** $x^2 - 8x + 12$

d $x^2 + 10x - 24$ **e** $x^2 + 5x - 50$ **f** $x^2 - 12x + 32$

g $x^2 - 6x + 9$ **h** $x^2 + 20x + 100$ **i** $x^2 + 40x + 400$

6 Solve using the Null Factor Law.

a $(x + 1)(x - 2) = 0$ **b** $(x - 3)(x + 7) = 0$ **c** $(2x - 1)(x + 4) = 0$

d $x(x - 3) = 0$ **e** $-4x(x + 6) = 0$ **f** $7x(2x - 5) = 0$

7 Solve these quadratic equations by factorising and applying the Null Factor Law.

a $x^2 + 4x = 0$ **b** $3x^2 - 9x = 0$ **c** $x^2 - 25 = 0$

d $9x^2 - 16 = 0$ **e** $x^2 + 8x + 15 = 0$ **f** $x^2 - 10x + 21 = 0$

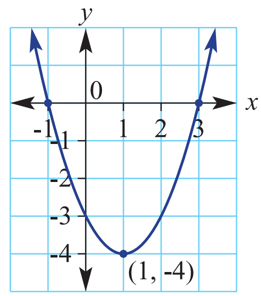
g $x^2 - 8x + 16 = 0$ **h** $x^2 + 10x + 25 = 0$ **i** $x^2 + 5x - 36 = 0$

8 A large rectangular sand pit is 2 m longer than it is wide. If it occupies an area of 48 m^2, determine the dimensions of the sandpit by solving a suitable equation.

ISBN: 9781107687028 Cambridge University Press

9 State the following features of the quadratic graph shown.

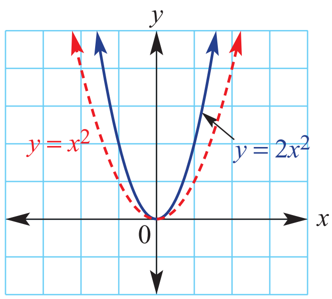
a Turning point and whether it is a maximum or a minimum
b Axis of symmetry
c x-intercepts
d y-intercept



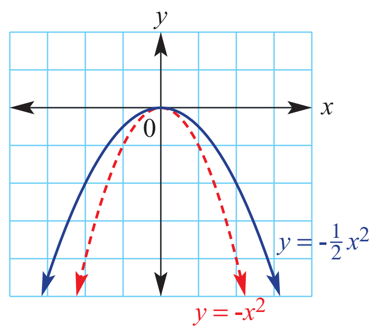
10 Copy and complete the table for the following graphs.

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 2x^2$					
b	$y = -\frac{1}{2}x^2$					
c	$y = (x - 2)^2$					
d	$y = x^2 - 2$					

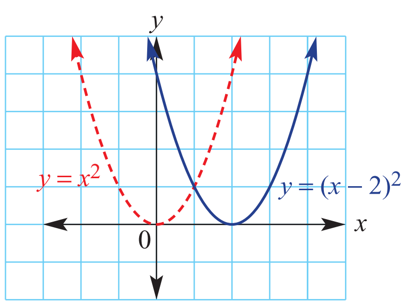
a



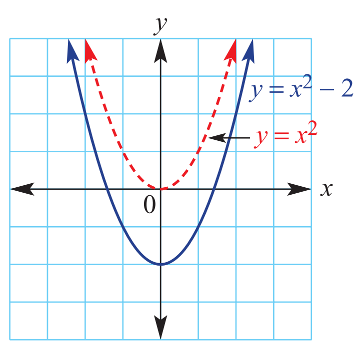
b



c



d



11 Sketch these circles. Label the centre and axes intercepts.

a $x^2 + y^2 = 25$
b $x^2 + y^2 = 4$

12 Sketch the following graphs, labelling the y-intercept and the point where $x = 1$.

a $y = 2^x$
b $y = 4^x$

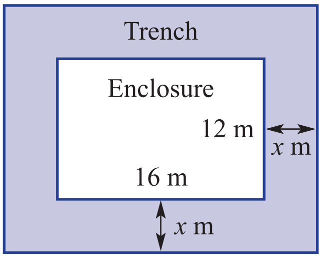
ISBN: 9781107687028 Cambridge University Press

Extended-response questions

1 A square spa is to be built in the middle of a 10 m by 10 m paved area. The builder does not yet know the size of the spa, so on the plan the spa size is variable. Its side length is x metres.

- **a** Write expressions for the area of:
  - **i** the spa
  - **ii** paving
- **b** Factorise your expression from part **a ii**.
- **c** What will be the area of the paving if:
  - **i** $x = 2$?
  - **ii** $x = 4$?
- **d** What value of x makes the paving area equal to 75 m^2?

2 A zoo enclosure for a rare tiger is rectangular in shape and has a trench of width x m all the way around it to ensure the tiger doesn't get far if it tries to escape. The dimensions are as shown.



- **a** Write an expression in terms of x for:
  - **i** the length of the enclosure
  - **ii** the width of the enclosure
- **b** Use your answers from part **a** to find the area of the enclosure (including the trench) in expanded form.
- **c** Hence, find an expression for the area of the trench alone.
- **d** Zoo restrictions state that the trench must have an area of at least 128 m^2. Find the minimum width of the trench.

ISBN: 9781107687028

Straight line graphs

Multiple-choice questions

1 If a straight line has a gradient of -3 and a y-intercept of 5, its equation is:
A $y = 5$ **B** $y = 3x + 5$ **C** $y = 5x - 3$ **D** $y = -3x + 5$ **E** $m = -3$

2 The gradient of the line joining (0, 6) and (2, -4) is:
A -2 **B** 6 **C** 5 **D** $\frac{1}{5}$ **E** -5

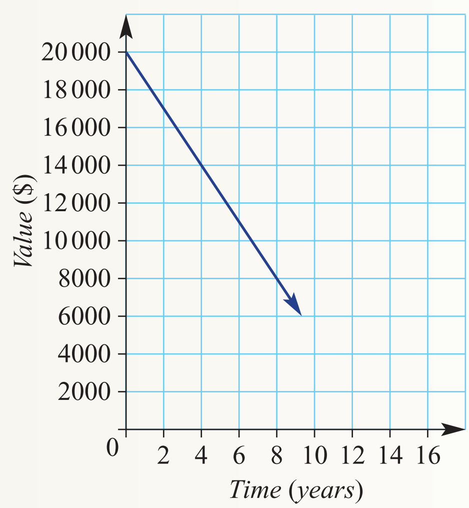
3 The midpoint of the line segment between (-3, 8) and (7, 2) has coordinates:
A (4, 10) **B** (2, 5) **C** (2.5, 4.5) **D** (0.5, 9) **E** (-5, 3)

4 The equation and gradient of the vertical line through the point (1, 3) are:
A $x = 1$; gradient undefined **B** $x = 1$; gradient zero **C** $y = 3$; gradient positive
D $y = 3$; gradient negative **E** $y = 3$; gradient undefined

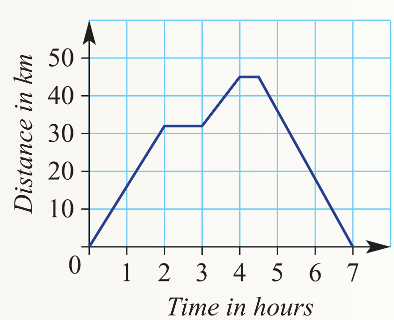
5 A landscape company charges \$80 delivery plus \$73 per cubic metre of soil. If \$$C$ is the cost of n cubic metres of soil, then:
A $C = 80n + 73$ **B** $C = 73n - 80$ **C** $n = 73C + 80$
D $C = 73n + 80$ **E** $C = 80n - 73$

Short-answer questions

1 This graph shows how the value of a car decreases with time.
a By the end of four years, how much has its value fallen?
b After how many years is the car worth \$8000?
c Estimate the car's worth after 13 years.



2 This distance–time graph shows the journey of a cyclist from home to a location and back again.
a How many km had the cyclist travelled after:
i 1 hour? **ii** 1.5 hours? **iii** 3 hours?
b Calculate the cyclist's speed over the first 2 hours.
c What was the total time in rest breaks?
d What was the cyclist's greatest distance from home?
e How long did the return trip take?
f Calculate the cyclist's speed for the return journey.
g What was the total distance cycled?



ISBN: 9781107687028 Cambridge University Press

3 **a** Copy and complete this table for the rule $y = 2x - 1$.

x	-2	-1	0	1	2	3
y						

b List the coordinates for each point in the table.

c Draw x- and y-axes each labelled between -5 and 5. Plot the points and join them with a ruler.

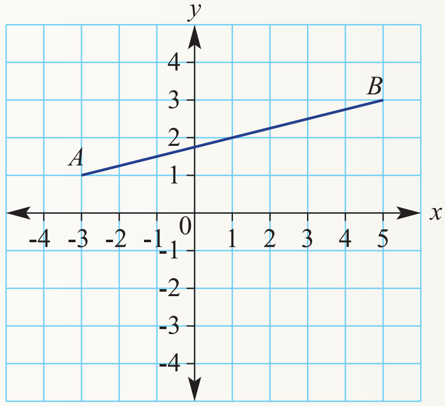
4 For each of the graphs below, find the:

i midpoint of the line segment

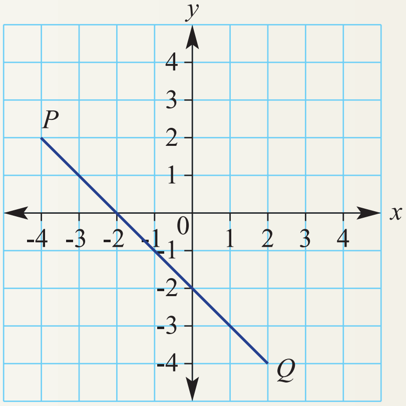
ii the exact length of the line segment (i.e. give answer in square root form)

iii the gradient of the line segment

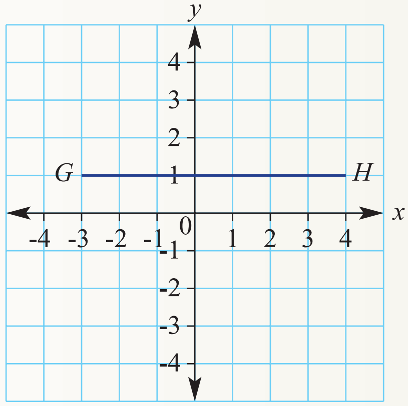
a



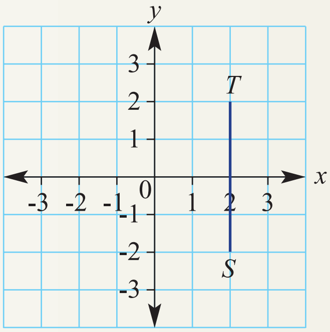
b



c



d



5 Plot and join each pair of points and find the gradient, m, of the line segment joining these points.

a $A(3, 2)$ $B(5, 6)$ **b** $K(1, -3)$ $L(-2, 6)$

6 Plot and join each pair of points and find the length of each line segment. Write the answer in square root form if it is not a whole number.

a $P(3, 4)$ $Q(-1, 9)$ **b** $R(-4, 2)$ $M(1, 10)$

ISBN: 9781107687028

Cambridge University Press

7 Without plotting these points, find the coordinates of the midpoint of the line segment joining each pair of points.

a $T(3, 4)$ and $K(5, 12)$ b $F(4, 5)$ and $G(-2, -9)$

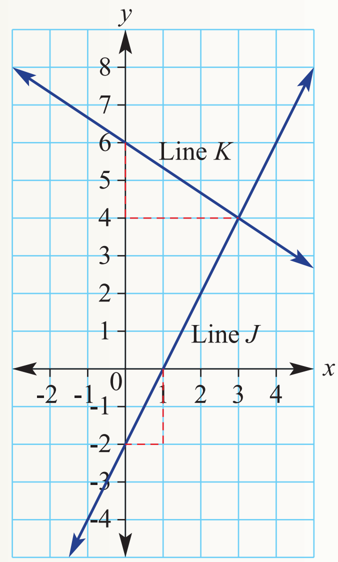
8 a For each of the lines J and K graphed here:

i find the y-intercept

ii determine its gradient

iii write the equation of the line

b State the coordinates of the point of intersection of lines J and K.



9 Sketch each of these lines by considering the y-intercept and gradient.

a $y = -3x + 4$ b $y = \frac{2}{3}x + 1$ c $y = -2$ d $x = -3$

10 Find the x- and y-intercepts for each of these lines and sketch. Label all axis intercepts.

a $y = 2x - 6$ b $2x - 3y = 12$

Extended-response question

1 You have a \$100 gift voucher for downloading movies from the internet. Each movie costs \$2.40. After downloading n movies you have a balance of \$$B$ on your voucher.

a Write a rule for the balance, B, on your voucher in terms of n.

b Use your rule to find:

i the balance on the voucher after 10 movies are downloaded

ii the number of movies bought that will leave a balance of \$28

c Copy and complete this table.

Number of movies, *n*	0	5	10	15	20
Balance, *B*					

d Sketch a graph of B versus n using the values in the table above.

e What is the maximum number of movies you could buy and how much would be left on your voucher?

ISBN: 9781107687028

Geometry

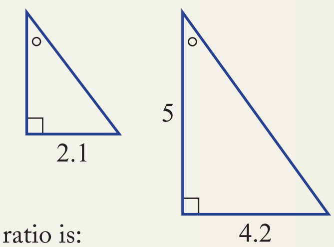
Multiple-choice questions

1 If two lines are parallel, then cointerior angles will:
A be equal **B** sum to 90° **C** sum to 180° **D** sum to 360° **E** sum to 270°

2 The sum of the internal angles of a hexagon is:
A 360° **B** 540° **C** 1080° **D** 900° **E** 720°

3 Which of the following is not a test for congruent triangles?
A SSS **B** SAS **C** AAA **D** AAS **E** RHS

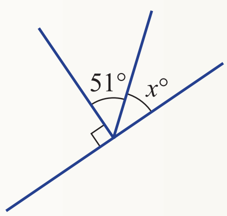
4 The scale factor for these similar triangles is:
A 2 **B** 4 **C** 5
D 0.1 **E** 0.4

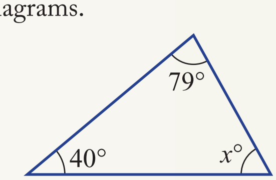


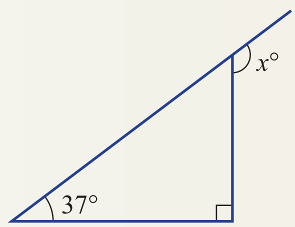
5 The length ratio for two similar solid objects is 2 : 3. The volume ratio is:
A 16 : 81 **B** 2 : 3 **C** 4 : 9 **D** 8 : 27 **E** 1 : 5

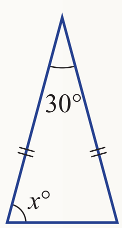
Short-answer questions

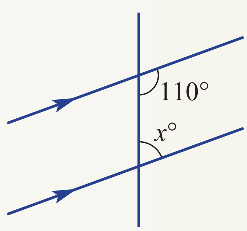
1 Find the value of x in these diagrams.

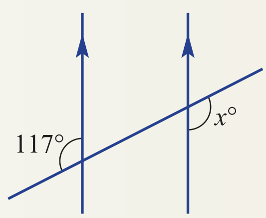
a


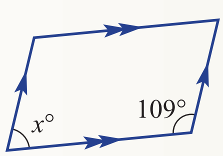
b


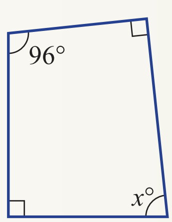
c


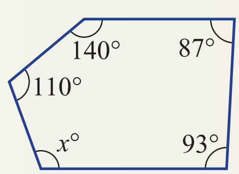
d


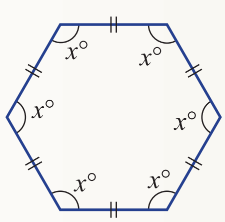
e


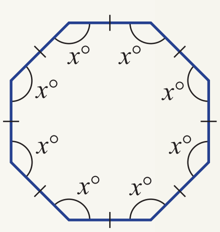
f


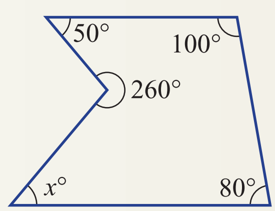
g


h


i


j


k


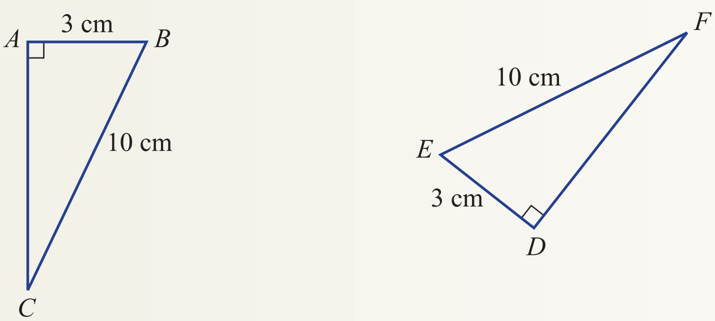
l


ISBN: 9781107687028

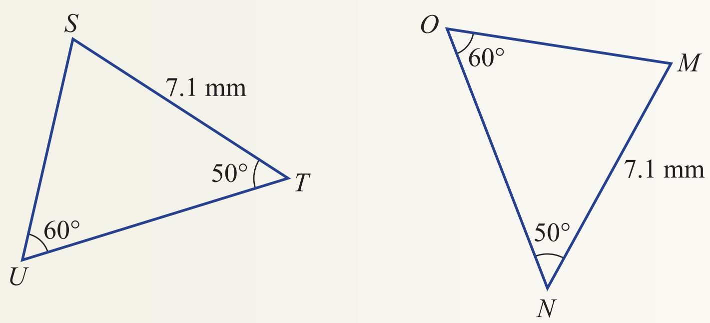
2 Write a congruence statement and the test to prove congruence in these pairs of triangles.

a

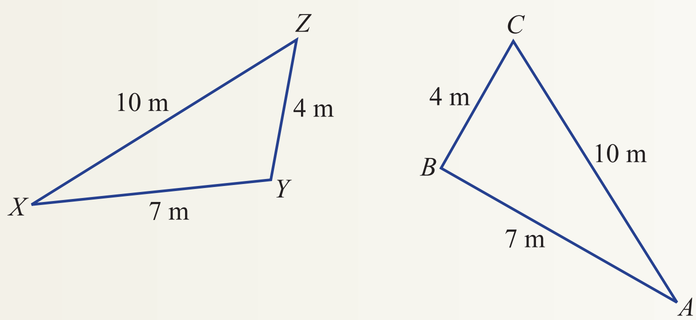
b



c

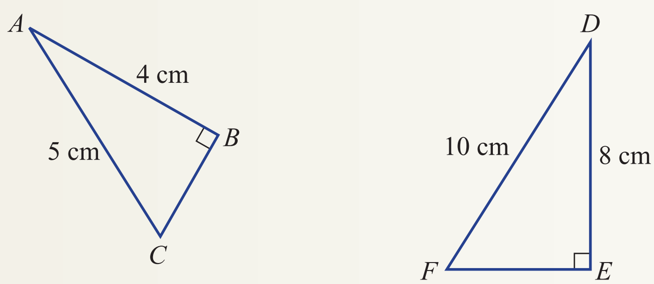


d



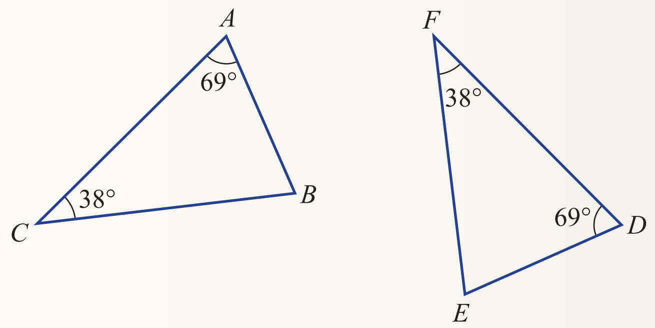
3 Decide whether the pairs of triangles are similar, giving reasons.

a

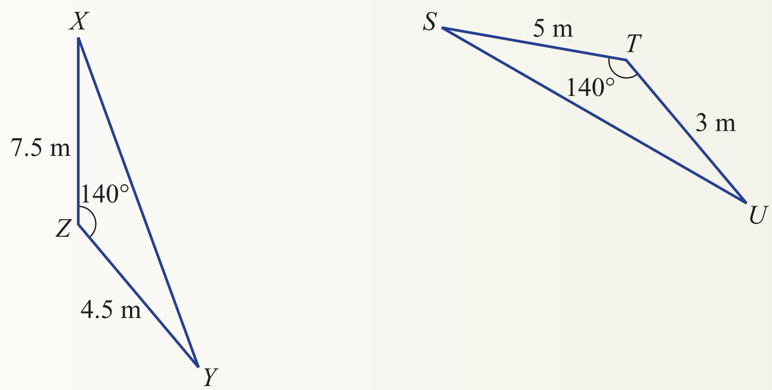


ISBN: 9781107687028

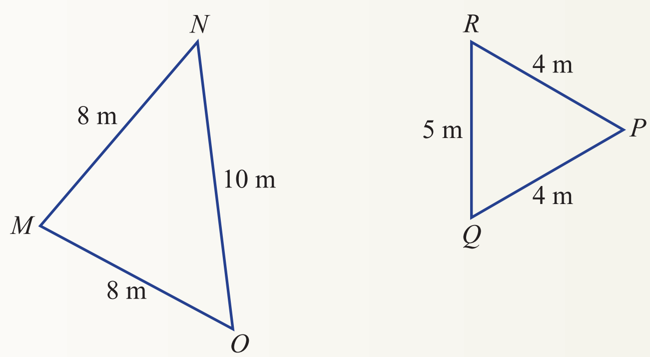
b



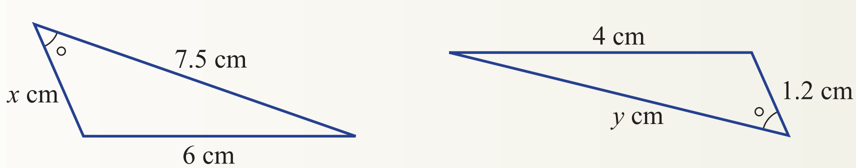
c



d



4 The given pair of triangles are known to be similar. Find the value of x and y.

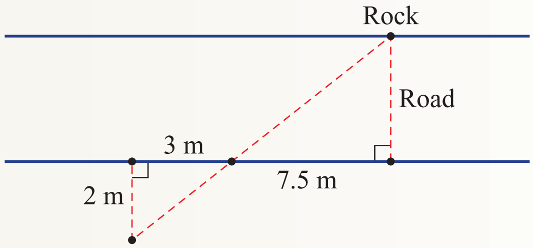


ISBN: 9781107687028

Cambridge University Press

Extended-response question

1 A chicken wants to know the distance across the road without having to cross it. The chicken places 4 pebbles in various positions on its own side of the road, as shown. There is a rock on the other side of the road aligned with one of the pebbles.



a What reason would be given to explain why the two triangles are similar?
b Find the scale factor.
c What is the distance across the road?

Equations

Multiple-choice questions

1 Which of the following is *not* an equation?
A $x - 3 = 5$ B $2x + 4 = 5x - 11$ C $y + 7x - 4$
D $y = 3x - 5$ E $y = 8$

2 A number is decreased by 8 and then doubled. The result is equal to 24. This can be written as:
A $2x - 8 = 24$ B $x - 8 \times 2 = 24$ C $x - 8 = 2 \times 24$
D $2(x - 8) = 24$ E $\frac{x-8}{2} = 24$

3 The solution to $\frac{x-9}{3} = 6$ is:
A $x = 27$ B $x = 45$ C $x = 9$ D $x = 11$ E $x = 3$

4 The solution to $3(x - 1) = 5x + 7$ is:
A $x = -4$ B $x = -5$ C $x = 5$ D $x = 3$ E $x = 1$

5 The solution to the inequality $1 - 2x > 9$ is:
A $x > 5$ B $x < 4$ C $x < -5$ D $x > -4$ E $x < -4$

Short-answer questions

1 Solve the following one-step equations.
a $x + 12 = 3$ b $x - 5 = 21$ c $\frac{m}{4} = 8$
d $5a = 20$ e $-2k = 12$ f $-x = 8$

2 Solve the following two-step equations.
a $2p + 3 = 7$ b $3a - 10 = 2$ c $\frac{x}{2} + 3 = 9$ d $\frac{x-8}{4} = 3$

3 Solve the following equations.
a $2(x - 4) = 8$ b $3(k - 2) + 4k = 15$ c $3m - 13 = m + 5$
d $\frac{3x+1}{2} = 8$ e $\frac{3a-2}{7} = -2$ f $4x + 7 + 3x - 12 = 5x + 3$

4 For each of the following statements, write an equation then solve it for the pronumeral.
a If 5 is subtracted from x, the result is 8.
b If 8 is added to the product of 4 and x, the result is 20.
c When 6 less than 3 lots of x is doubled, the result is 18.

ISBN: 9781107687028 Cambridge University Press

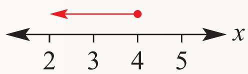
5 Find the value of the unknown in each of the following formulas.

a $A = \frac{1}{2}bh$, find b when $A = 120$ and $h = 24$

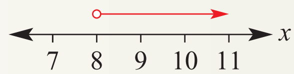
b $I = \frac{Prt}{100}$, find P when $I = 80$, $r = 5$ and $t = 4$

6 Write the inequality displayed on each of the following number lines.

a



b



7 Solve each of the following inequalities and graph the solution on a number line.

a $\frac{x}{3} \le 2$ **b** $3x - 2 > 4$ **c** $-3x \ge 12$

8 Find the point of intersection (x, y) of each pair of equations below by plotting an accurate graph. First draw x- and y-axes each labelled from -5 to 5.

a $x = 3, y = 2$ **b** $y = 2x - 4$ and $3x + 2y = 6$

9 Solve the following pairs of simultaneous equations by using the substitution method; i.e. find the point of intersection.

a $y = 2x$
$x + y = 3$

b $x + y = 12$
$y = x + 6$

c $y = 3 - x$
$3x + 2y = 5$

10 Solve the following pairs of simultaneous equations by using the elimination method; i.e. find the point of intersection.

a $x + 2y = 3$
$-x + 3y = 2$

b $3x + y = 10$
$x + y = 6$

c $2x - 3y = 3$
$3x - 2y = 7$

11 Oliver is older than Ruby. The sum of their ages is 45 years and the difference of their ages is 7 years.

a Define pronumerals to represent each person's age.

b Set up a pair of simultaneous equations based on the given information.

c Solve the simultaneous equations to find Oliver and Ruby's ages.

Extended-response question

1 Ishan and Mia normally had an electricity bill of \$200 per month.
Now that they have installed solar panels (which cost \$6000 including installation), the solar energy has provided for all their power usage with some left over. Their excess power is sold to the town's electricity supplier. On average, they receive a cheque for \$50 per month for the sale of this solar-generated power.

a Copy and complete this table, which compares the total cost of normal town electricity to solar power.

Number of months, n	0	6	12	18	24	30	36	42	48
Total cost of electricity (at \$200 per month), E	0								
Total cost of solar power (reducing by \$50 per month), S	\$6000								

ISBN: 9781107687028

Cambridge University Press

b Draw a graph of the above information.

c On the graph, show where the total cost of solar power is the same as the total cost of electricity for that period of time. State the value of n and the cost.

d Write an equation for E (total cost of electricity) in terms of n and another equation for S (total cost of solar power) in terms of n.

e Solve the equations in part **d** for n, when $E = S$ (i.e. the total costs of each power supply are equal).

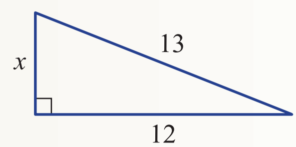
f After 4 years, how much money has been saved by using solar power?

g Suppose that, in a wet climate, the cheque for excess power sold to the town supply was reduced to \$25 per month. Using equations, find how many months it takes for the total costs to be equal. (Round to the nearest month.)

Pythagoras' theorem and trigonometry

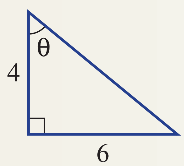
Multiple-choice questions

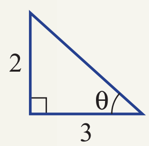
1 The value of x in the triangle shown is:

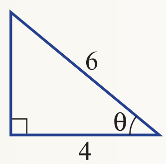


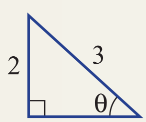
A 1 **B** 11 **C** 4
D 10 **E** 5

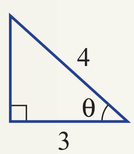
2 In which of the following triangles does $\cos\theta = \frac{2}{3}$?

a


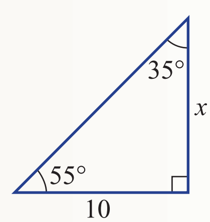
B


C


D


E


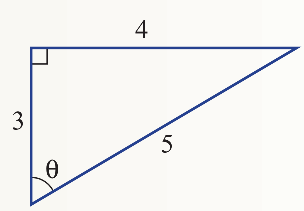
3 Choose the correct trigonometric statement for the diagram shown.



A $\tan 55° = \frac{x}{10}$ **B** $\tan 35° = \frac{x}{10}$ **C** $\sin 55° = \frac{x}{10}$

D $\sin 35° = \frac{x}{10}$ **E** $\cos 35° = \frac{x}{10}$

4 For the triangle shown, $\sin\theta$ is equal to:

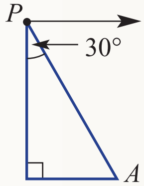


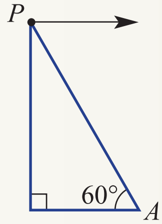
A $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{5}{3}$

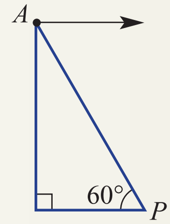
D $\frac{3}{4}$ **E** $\frac{4}{3}$

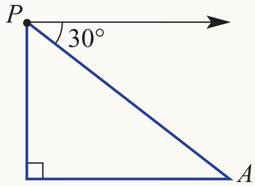
ISBN: 9781107687028

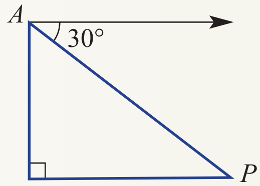
5 In which diagram is the angle of depression of A from P equal to 30°?

A


B


C


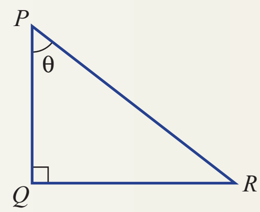
D


E


Short-answer questions

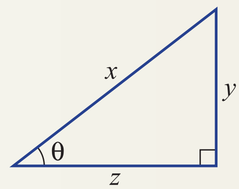
1 Refer to the angle marked θ and name:

a the hypotenuse
b the side opposite θ
c the side adjacent to θ



2 Use the triangle shown to help you write a fraction for:

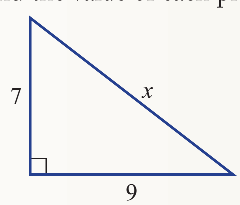
a $\sin\theta$ b $\cos\theta$
c $\tan\theta$

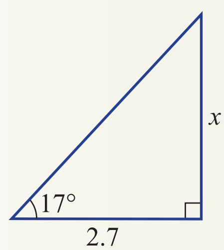


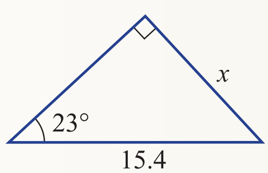
3 Use a calculator to find the value of each of the following, correct to two decimal places.

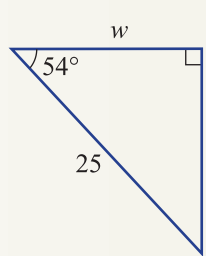
a $\sin 40°$ b $\cos 51°$ c $\tan 18°$
d $12\tan 32°$ e $40\tan 38°$ f $5.6\sin 55°$
g $\dfrac{15}{\sin 24°}$ h $\dfrac{28}{\cos 30°}$ i $\dfrac{12.5}{\tan 52°}$

4 Find the value of each pronumeral, correct to two decimal places.

a


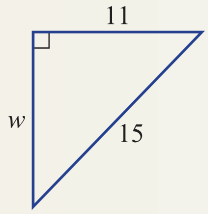
b


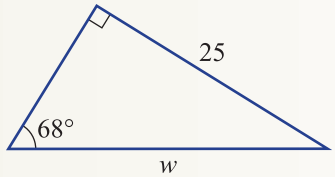
c


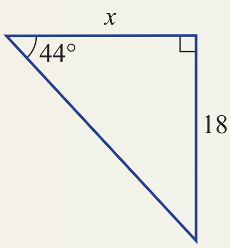
d


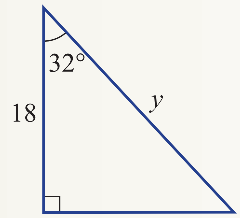
ISBN: 9781107687028 Cambridge University Press

5 Find the value of each pronumeral, correct to one decimal place.

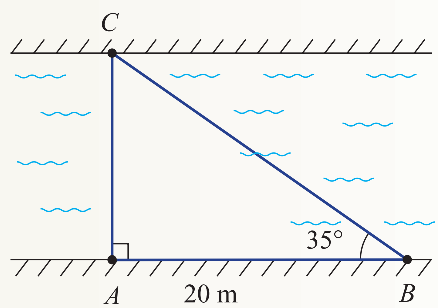
a


b


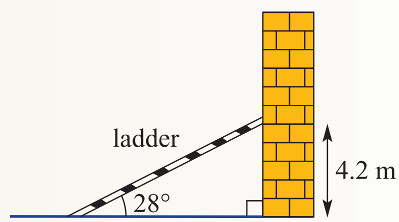
c


d


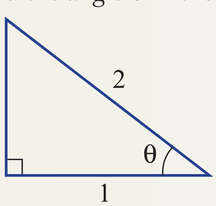
6 Kara wants to measure the width of a river. She places two markers, A and B, 20 m apart along one side. C is a point directly opposite marker A. Kara measures angle ABC as 35°. How wide is the river, to the nearest metre?

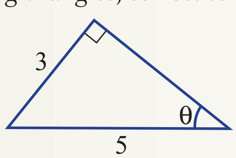


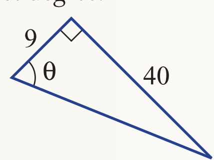
7 A ladder is inclined at an angle of 28° to the ground. If the ladder reaches 4.2 m up the wall, what is the length of the ladder, correct to two decimal places?



8 Find the angle θ in the following triangles, correct to the nearest degree.

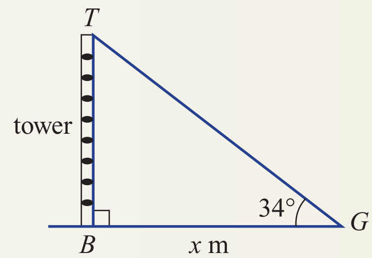
a


b


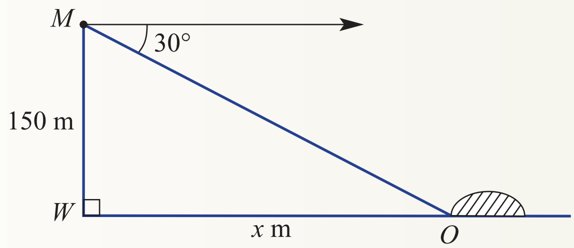
c


ISBN: 9781107687028 Cambridge University Press

9 Geoff measures the angle of elevation to the top of a 120 m tower to be 34°. How many metres is Geoff from the base of the tower? Round to one decimal place.



10 Malcolm is sitting on top of a bridge 150 m above the water level of the river. He notices an object floating on the river some distance away. If the angle of depression to the object is thought to be 30°, how many metres (x) from the bridge is the object? Round to one decimal place.



Extended-response question

1 A plane flies from the airport on a bearing of 136° for 450 km.

- **a** Draw a diagram showing the plane's flight.
- **b** How far East of the airport is the plane? Round to one decimal place.
- **c** How far South of the airport is the plane? Round to one decimal place.
- **d** What is the true bearing of the airport from the plane, correct to the nearest degree?

Quadratics and non-linear graphs

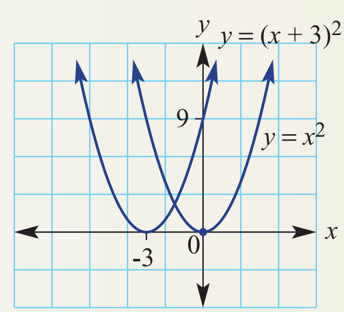
Multiple-choice questions

1 $-x(x - 1) + 2x^2$ simplifies to:

A $x^2 - 1$ **B** $x^2 + x$ **C** $3x^2 + 1$ **D** $3x^2 - x$ **E** $3x^2 + 3$

2 Compared to the graph of $y = x^2$, the graph of $y = (x + 3)^2$ is:

A 3 units down **B** 3 units up
C 3 units left **D** in the same place
E 3 units right



3 The solutions to $(2x - 1)(x + 3) = 0$ are:

A $x = -\frac{1}{2}, x = -3$ **B** $x = 2, x = 3$ **C** $x = 2, x = -3$ **D** $x = \frac{1}{2}, x = 3$ **E** $x = \frac{1}{2}, x = -3$

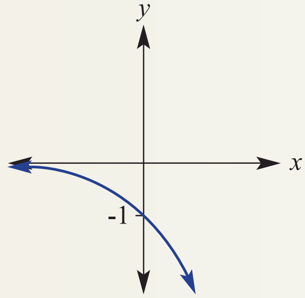
4 The radius of the circle with equation $x^2 + y^2 = 25$ is:

A 5 **B** 25 **C** -5 **D** 0 **E** 1

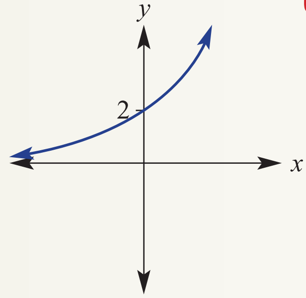
ISBN: 9781107687028 Cambridge University Press

5 The graph of $y = 2^x$ could be:

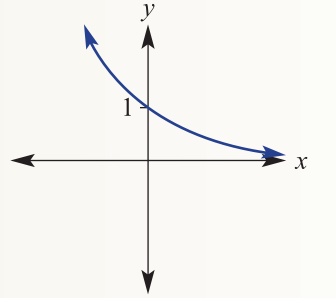
A



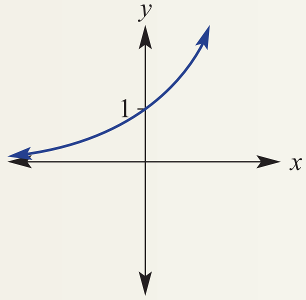
B



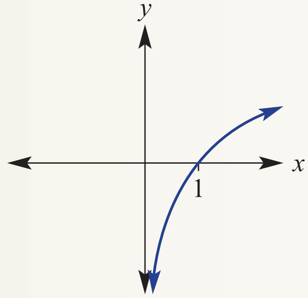
C



D



E



Short-answer questions

1 Expand the following expressions.

a $-2(x - 1)$ **b** $(x + 2)(x - 3)$ **c** $(2x - 7)(x + 3)$
d $(x + 2)(x - 2)$ **e** $(x - 3)^2$ **f** $(2x + 1)^2$

2 Factorise these expressions.

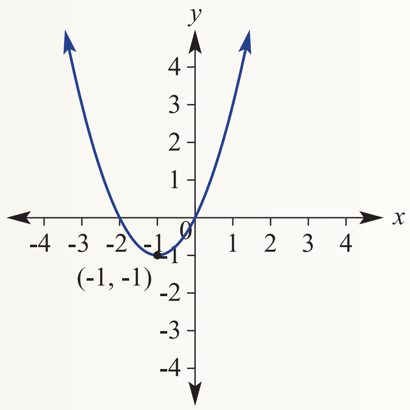
a $3x - 12$ **b** $-2x - x^2$ **c** $x^2 - 25$
d $9x^2 - 100$ **e** $x^2 + 7x + 12$ **f** $x^2 - x - 6$
g $x^2 + 2x - 8$ **h** $x^2 - 8x + 16$ **i** $x^2 + 6x + 9$

3 Solve these equations.

a $x(x - 3) = 0$ **b** $x^2 + 2x = 0$ **c** $x^2 - 4 = 0$
d $4x^2 - 9 = 0$ **e** $(x - 3)(2x - 1) = 0$ **f** $x^2 - x - 20 = 0$
g $x^2 + 10x + 21 = 0$ **h** $x^2 + 8x + 16 = 0$ **i** $x^2 - 14x + 49 = 0$

4 State the following features of the quadratic graph shown.

a Turning point and whether it is a minimum or a maximum
b Equation of the axis of symmetry
c x-intercepts
d y-intercept

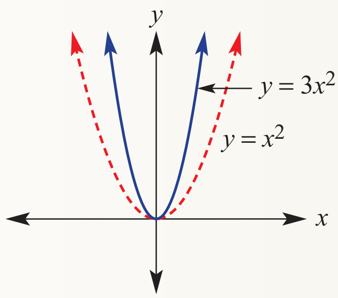


ISBN: 9781107687028 Cambridge University Press

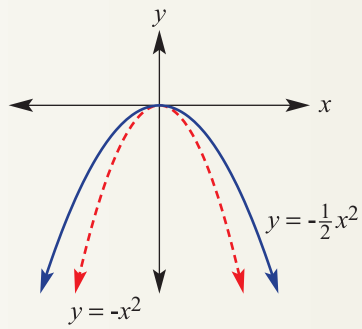
5 Copy and complete the table for these parabolas.

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$					
b	$y = -\frac{1}{2}x^2$					
c	$y = (x + 2)^2$					
d	$y = x^2 - 3$					

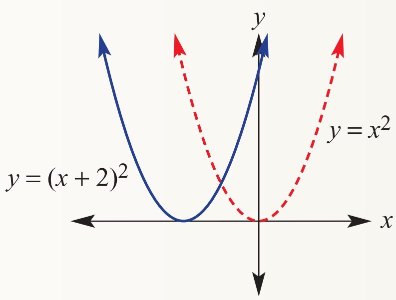
a



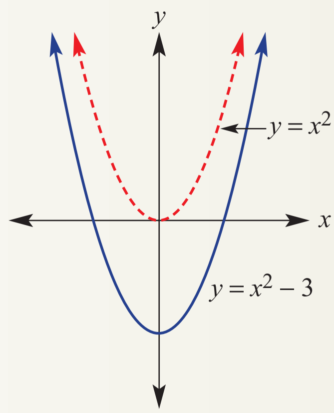
b



c



d



Extended-response question

1 In a garden, a square sitting area paved with stone is to be placed in the centre of a square area of lawn that is 20 m by 20 m.

a Write expressions for the area of:

i the sitting area

ii the lawn

b Factorise your expression from part **a ii**.

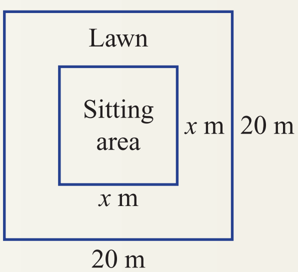
c What will be the lawn area if:

i $x = 4$? **ii** $x = 9$?

d What value of x makes the lawn area equal to:

i 175 m^2?

ii 75% of the total area?



ISBN: 9781107687028

A

Algebraic fraction A fraction containing pronumerals as well as numbers

Angle of depression The angle of your line of sight from the horizontal when looking down at an object

Angle of elevation The angle of your line of sight from the horizontal when looking up at an object

Angle of reference The angle in a right-angled triangle that is used to find the opposite side and the adjacent side

Area The number of square units needed to cover the space inside the boundaries of a 2D shape

Asymptote A line whose distance to a curve approaches zero, and the curve never touches it

B

Base The number or pronumeral that is being raised to a power

Bimodal When a set of data has two modes

Bivariate data Data that involves two variables

Boxplot A diagram using rectangles and lines to show the spread of a set of data, using five important values

C

Chance The likelihood of an event happening

Circumference The distance around the outside of a circle; the curved boundary

Coefficient A numeral placed before a pronumeral, showing that the pronumeral is multiplied by that factor

Column graph A graphical representation of a single category or type of data. Columns are used to show the frequency of scores

Commission Earnings of a salesperson based on a percentage of the value of goods or services sold

Compound interest A type of interest that is paid on a loan or earned on an investment, which is calculated not only on the initial principal, but also on the interest accumulated during the loan/investment period

Congruent (figures) Figures that are exactly the same size and shape

Constant term The part of an equation or expression without any pronumerals

Coordinates An ordered pair written in the form (x, y) that states the location of a point on the Cartesian plane

Cosine (cos) The ratio of the length of the adjacent side to the length of the hypotenuse in a right-angled triangle

Cylinder A solid with two parallel, congruent circular faces connected by a curved surface

D

Deductions Amounts of money taken from gross income

Denominator The part of a fraction that sits below the dividing line

Diameter A line passing through the centre of a circle with its end points on the circumference

Difference of perfect squares (DOPS) When one square term is subtracted from another

Dilation A transformation where a curve is enlarged or reduced but the centre is not changed

Discount An amount subtracted from a price

Distributive law Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the products

Dot plot A graph in which each dot represents one score

E

Elimination A method for solving simultaneous equations, where one equation is added to or subtracted from another to eliminate one of the variables

Equation A mathematical statement that states that two expressions have the same value

Event A situation involving chance or probability trials

Expand Remove grouping symbols (such as brackets)

Exponential notation A way of representing repeated multiplication of the same number

Exponential decay Repeatedly decreasing a quantity by a constant percentage over time

Exponential growth Repeatedly increasing a quantity by a constant percentage over time

Expression A group of mathematical terms containing no equals sign

ISBN: 9781107687028

Cambridge University Press

Exterior angle theorem The theorem that, in a triangle, the exterior angle is equal to the sum of the two opposite interior angles

Extrapolation Determining information outside of the original data

F

Factorise To write an expression as a product, often involving brackets

Five-figure summary A set of numbers that summarise a set of data: the minimum score, first quartile, median, third quartile and maximum score

Fixed expenses Expenses that are set and do not change during a particular time period

Formula A general rule for finding the value of one quantity given the values of others

Frequency table A table showing all possible scores in one column and the frequency of each score in another column

G

Gradient (*m*) The steepness of a slope

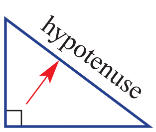
Gradient–intercept form The equation of a straight line, written with y as the subject of the equation

Gross income Total income before any deductions (e.g. income tax) are made

H

Histogram A special type of column graph with no gaps between the columns; it can represent class intervals

Hypotenuse The longest side of a right-angled triangle (the side opposite the right angle)



I

Independent events Two events that do not influence or affect each other

Index form A method of writing numbers that are multiplied by themselves

Inequality sign A symbol that compares the size of two or more expressions or numbers by pointing to the smaller one

Interpolation Reading information from within a graph

Interquartile range (IQR) A measure of spread giving the difference between the upper and lower quartiles

Intersection ($A \cap B$) The elements that are common to two or more sets of data

L

Like terms Terms with the same pronumerals and same powers

Line of best fit A line that has the closest fit to a set of data points displayed in a scatter plot

Line segment A section of a straight line

Linear equation An equation whose pronumerals are always to the power of 1 and do not multiply or divide each other

Linear inequality An inequality that involves a linear function

Linear relationship The relationship between a variable and a constant term

Loan Money borrowed and then repaid, usually with interest

M

Mean An average value calculated by dividing the total of a set of numbers by the number of values

Median The middle score when all the numbers in a set are arranged in order

Midpoint The point on an interval that is equidistant from the end points of the interval

Mode The score that appears most often in a set of numbers

Monic quadratic A quadratic expression where the coefficient of the squared term is 1

Mutually exclusive Two events that cannot both occur at the same time

N

Net income Income remaining after deductions have been made from gross income

Null Factor Law If two numbers multiply to give zero, then one or both of those numbers must be zero

O

Outcome One of the possibilities from a chance experiment

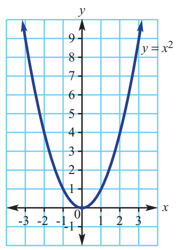
ISBN: 9781107687028

Cambridge University Press

Outlier Any value that is much larger or much smaller than the rest of the data in a set

P

Parabola A smooth U-shaped curve with the basic rule $y = x^2$



Parallel lines Lines in the same plane that are the same distance apart and never intersect

Parallelogram A quadrilateral with both pairs of opposite sides parallel

Percentage A convenient way of writing fractions with denominators of 100

Perfect square A quadratic trinomial that can be expressed as a single square

Perimeter The total distance (length) around the outside of a figure

Point of intersection The point at which two lines cross each other and therefore have the same coordinates

Polygon A two-dimensional shape where three or more straight lines are joined together to form a closed figure

Principal (*P*) An amount of money invested in a financial institution or loaned to a person/business

Profit The amount of money made by selling an item or service for more than its cost

Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides

Q

Quartiles The three values that separate the scores when a set of ordered data is divided into four equal parts

R

Radius The distance from the centre of a circle to its outside edge

Range The difference between the highest and lowest numbers in a set

Rate A measure of one quantity against another

Rate of interest (*r*) The annual percentage rate of interest paid or earned on a loan or investment

Reflection A transformation where a curve is flipped across a line on the number plane

Regular polygon A polygon with all sides equal and all angles equal

Repayment An amount paid to a financial institution at regular intervals to repay a loan, with interest included

S

Salary An employee's fixed agreed yearly income

Sample space All the possible outcomes of an event

Scale factor The number you multiply each side length by to enlarge or reduce a shape

Scatter plot A diagram that uses coordinates to display values for two variables for a set of data.

Scientific notation A way to express very large and very small numbers using [a number between 1 and 10] 10^{power}

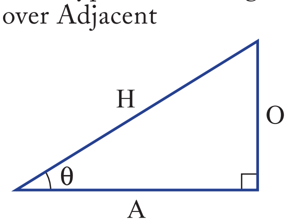
Significant figure A digit that indicates how accurate a number is

Similar (triangles) Triangles whose corresponding angles are equal and whose corresponding sides are in the same ratio

Simple interest A type of interest that is paid on a loan or earned on an investment, which is always calculated on the principal amount loaned or invested

Sine (sin) The ratio of the length of the opposite side to the length of the hypotenuse in a right-angled triangle

SOH CAH TOA A way of remembering the trigonometric ratios: Sine equals Opposite over Hypotenuse, Cosine equals Adjacent over Hypotenuse, Tangent equals Opposite over Adjacent



Solve To find the value of an unknown quantity

ISBN: 9781107687028

Statistical data Information gathered by observation, survey or measurement

Stem-and-leaf plot A table that lists numbers in order, grouped n rows

Subject The pronumeral or variable that is alone on one side of an equation

Substitute To replace pronumerals with numerical values

T

Tangent (tan) The ratio of the length of the opposite side to the length of the adjacent side in a right-angled triangle

Term A number or pronumeral in an expression

Time series data A set of data collected in sequence over a period of time

Total surface area (TSA) The total number of square units needed to cover the outside of a solid

Translation A transformation where a curve is moved a certain distance on the number plane

Transversal A line that cuts two or more lines

Trial One run of an experiment

True bearing (°T) An angle that is measured clockwise from North

U

Union ($A \cup B$) The combination of all elements from two or more sets of data

V

Variable An unknown, which can take on any value

Variable expenses Expenses that may change during a particular period of time, or over time

Venn diagram A diagram using circles to show the relationships between two or more sets of data

Volume The amount of three-dimensional space within an object

W

Wages Earnings paid to an employee based on an hourly rate

X

***x*-coordinate** The first coordinate of an ordered pair

***x*-intercept** The point at which a line or curve cuts the x-axis

Y

***y*-coordinate** The second coordinate of an ordered pair

***y*-intercept** The point at which a line or curve cuts the y-axis

ISBN: 9781107687028

Chapter 1

Pre-test

1 a circle b square
c parallelogram d triangle
e rectangle f trapezium
g semicircle h rhombus

2 a 1000 b 100 c 10
d 1000 e 500 f 25

3 a 12 cm b 32 m c 5.9 mm

4 a 10 cm^2 b 70 m^2 c 36 km^2

5 a 4 cm^2 b 14 m^2 c 6 km^2

6 $C = 31.42$ m
$A = 78.54$ m^2

Exercise 1A

1 a 1000 b 10 c 100

2 a 100 b 10 000 c 1 000 000

3 a 1000 b 1 000 000 000 c 1 000 000

4 a 43.2 mm b 0.327 km c 8.34 m
d 96 mm e 0.2975 km f 1.27 cm

5 a 300 000 mm^2 b 5000 cm^2
c 5 000 000 m^2 d 29 800 cm^2
e 53 700 mm^2 f 230 cm^2

6 a 2000 mm^3 b 200 000 cm^3
c 5.7 cm^3 d 15 000 000 m^3
e 0.0283 km^3 f 0.762 m^3

7 5500 m

8 a 23.4 m b 22 m

9 a 118 mm b 147.3 cm c 453.258 km
d 15.5 cm^2 e 3251 cm^2 f 3739 m^2
g 484 500 mm^3 h 537 300 m^3

10 21.5 cm

11 For a high level of accuracy

12 a 8.85 km b 4.5 feet
c 26.67 cm d 1.243 miles
e 57 000 m^2 f 0.247 L
g 8200 mL h 5 500 000 mL
i 1000 sq feet j 2000 L
k 100 ha l 0.152 m^3

Exercise 1B

1 perimeter

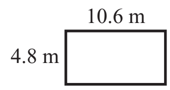
2 a 6 b 7.1 c 4.3

3 a 12 cm b 23 m c 11 km
d 12 m e 19.2 cm f 10 m

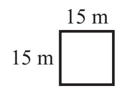
4 a 6.7 cm b 65 mm c 18 m
d 810 m e 9.4 km f 180 cm

5 a $x = 4$ b $x = 2$ c $x = 6$

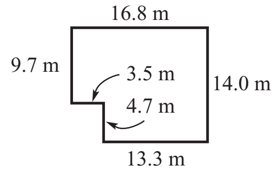
6 a $x = 3$ b $x = 8.8$ c $x = 0.1$

7 a i


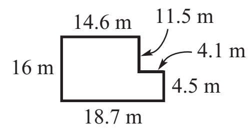
ii 30.8 m

b i


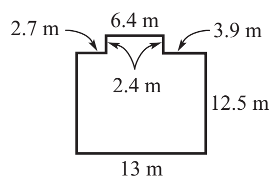
ii 60 m

c i


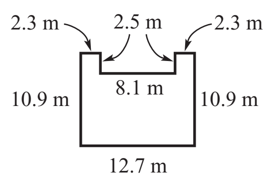
ii 61.6 m

d i


ii 69.4 m

e i


ii 55.8 m

f i


ii 52.2 m

8 20 m 9 15 cm

10 a $P = 4l$ b $P = 2l + 2w$ c $P = x + y + z$
d $P = a + 2b$ e $P = 4l$ f $P = 3s$

11 3 12 13

Exercise 1C

1 a $C = \pi d$ b $C = 2\pi r$

2 a $\frac{1}{2}$ b $\frac{1}{4}$ c $\frac{3}{4}$

3 a 8.6 m b 1.8 cm

4 a 18.85 m b 31.42 m c 31.42 km
d 113.10 cm e 61.07 mm f 3.36 km

5 a 27.42 m b 16.28 cm c 6.71 mm
d 12.22 cm e 14.71 m f 59.70 cm

6 a 9.42 m b $423.90

7 a i 3.14 m ii 47.12 m b 3.14 km

8 319 times

9 a 12.25 b 53.03 c 1.37
d 62.83 e 19.77 f 61.70

10 a $r = \frac{C}{2\pi}$
b i 5.57 cm ii 0.29 m iii 0.04 km

ISBN: 9781107687028 Cambridge University Press

Exercise 1C cont.

11 a 3.5 cm
 b i 21.99 cm ii 65.97 cm iii 109.96 cm
 c 12.73 cm

Exercise 1D

1 a E b B c F d C e D f A
2 a 2 cm b 4 m c 4.3 cm
 d 4 km e 7.8 m f 10 cm
3 a 4 m^2 b 21 cm^2 c 11.76 m^2
 d 21 m^2 e 22.5 mm^2 f 2 m^2
4 a 25 cm^2 b 54.6 m^2 c 1.82 km^2
 d 0.03 mm^2 e 1.12 m^2 f 100 cm^2
5 a 0.96 m^2 b 9600 cm^2
6 21 m^2
7 a 13.6 m^2 b $149.60
8 a 7.56 m^2 b $491.40
9 1 and 24, 2 and 12, 3 and 8, 4 and 6
10 a 252.05 m^2 b 177.86 m^2
11 a $w = 2.88$ b $l = 14.35$ c $h = 1.44$
 d $a = 1.05$ e $h = 1.87$ f $x = 8.89$
12 All answers = 3

Exercise 1E

1 E
2 C
3 a $\frac{1}{2}$ b $\frac{1}{4}$ c $\frac{1}{3}$
 d $\frac{1}{12}$ e $\frac{7}{12}$ f $\frac{5}{6}$
4 a 28.27 cm^2 b 201.06 m^2 c 72.38 m^2
 d 38.48 m^2 e 0.82 mm^2 f 124.69 km^2
5 a 39.27 m^2 b 4.91 m^2 c 84.82 m^2
 d 13.09 m^2 e 69.81 cm^2 f 8.03 m^2
6 157.1 cm^2
7 a 14.28 cm^2 b 178.54 m^2 c 32.14 mm^2
8 43.24 m^2
9 a 34.8 cm^2 b 63.5 m^2 c 8.7 m^2
 d 103.3 mm^2 e 578.5 km^2 f 5.0 m^2
10 a 20 b 565.49 cm^2
 c 154.51 cm^2 d 5

Exercise 1F

1 a 6 b 3
2 a 35 cm^2 b 21 cm^2
 c 12 cm^2 d 96 cm^2
3 a 90 cm^2 b 34 mm^2 c 46 m^2
4 a 240 m^2 b 168 m^2 c 1176 cm^2
5 a 8.64 cm^2 b 96 mm^2 c 836.6 m^2
6 384 cm^2
7 3880 cm^2
8 3520 cm^2
9 a 5.116 L b 10.232 L
10 a Waterproof 13.76 L
 Smooth paint 22.86 L
 b $553.46
11 a 105 cm^2 b 5 cm^2 c 16 m^2
12 Yes, only 5 L required

Exercise 1G

1 a circle b $2\pi rh$
2 a i 4 m ii 7 m b 25.13 m c 276.46 m^2
3 a 100.53 m^2 b 376.99 cm^2
 c 1225.22 cm^2 d 74.64 mm^2
4 a 2557.3 cm^2 b 502.9 m^2
5 a 6.3 m^2 b 6283.2 cm^2
6 24.0 m^2
7 628.3 cm^2
8 75.4 m^2
9 a 173.9 cm^2 b 217.8 m^2
 c 31.6 m^2 d 52.9 cm^2
10 a 25.13 m^2 b 251.33 m^2 c 6.28 m
 d 159.15 times e 4000 m^2

Exercise 1H

1 a C b A c B
2 a 1000 b 1000
3 10 cm^3
4 a 240 m^3 b 480 cm^3 c 0.18 m^3
5 a 113.10 cm^3 b 19.63 m^3 c 4.83 mm^3
6 a 20 cm^2 b 90 cm^3
7 a 36 m^3 b 15 cm^3 c 0.572 mm^3
8 5890.49 cm^3
9 a 72 L b 2 L c 2 L
10 a 1583.36 m^3 b 30 km^3 c 196 cm^3
 d 30 m^3 e 10 cm^3 f 2.15 m^3
11 a 25 cm^2 b 4 cm
12 a 17 150 cm^3 b 384.85 cm^3
 c 35 d 3680.42 cm^3
13 a 83.3 m^3 b 1500 m^3 c 20.9 cm^3

Puzzles and games

1 PRISM 2 No 3 21.46%
4 11 5 $r = 2$ 6 9.42 cm^2
7 27 cm^3

Multiple-choice questions

1 E 2 B 3 C 4 A 5 A
6 B 7 E 8 D 9 B 10 C

ISBN: 9781107687028

Short-answers questions

1 a 5300 m b 2.7 m^2 c 40 mm^3
2 a 13 b 24 cm c 38 m
3 a 18.85 m b 28.27 m^2
4 a i 19.42 m ii 26.14 m^2
b i 19.50 m ii 21.87 m^2
c i 14.28 m ii 12.28 m^2
5 a 10 b 25.9 c 17.5
6 a 828 m^2 b 136 cm^2
7 376.99 m^2
8 a 40 cm^3 b 125.66 m^3 c 21 cm^3

Extended-response question

1 a 50.27 m^2 b 150.80 m^2
c 100.53 m^3 d 100 530.96 L

Chapter 2

Pre-test

1 a \$99.62 b \$1612 c \$2742
d \$84.51 e \$1683.85
2 a $\frac{50}{100}$ b $\frac{75}{100}$ c $\frac{20}{100}$
d $\frac{68}{100}$ e $\frac{45}{100}$
3 a 0.5 b 0.25 c 0.2 d 0.28 e $0.\dot{3}$
4 a 16.79 b 7.35 c 45.34 d 6.84 e 102.90
5 a \$4519.28 b \$50 509.20 c \$29 572
d \$54 878 e \$97 903.46
6 a \$65 312 b \$187 200
c \$49 686 d \$35 800
7 a \$40 b \$250 c \$2
d \$211 e \$2.56 f \$750
8 a \$20 b \$300 c \$64
9 a \$18 b \$323 c \$264 d \$112 e \$57.92
10 a \$50 b \$56 c \$550

Exercise 2A

1 a $\frac{40}{100}$ b $\frac{85}{100}$ c $\frac{98}{100}$
d $\frac{28}{100}$ e $\frac{90}{100}$
2 a 100 b 90 c 60
3 a \$0.75 b \$80 c \$49 d \$501 e \$103
f \$1.20 g \$37.50 h \$400 i \$4.50
4 a 50% b 20% c 25% d 10%
e 1% f 28% g 30% h 75%
i $62\frac{1}{2}\%$ j 76% k 99% l 94%
5 a 17% b 73% c 48% d 9%
e 6% f 13% g 113% h 101%
i 80% j 90% k 99% l 17.5%
6 a $\frac{71}{100}$ b $\frac{4}{5}$ c $\frac{1}{4}$ d $\frac{11}{20}$
e $\frac{2}{5}$ f $\frac{22}{25}$ g $\frac{3}{20}$ h $\frac{33}{200}$
i $\frac{7}{40}$ j $\frac{9}{400}$ k $\frac{21}{400}$ l $\frac{21}{40}$
7 a 0.61 b 0.83 c 0.75 d 0.45
e 0.09 f 0.9 g 0.5 h 0.165
i 0.073 j 2 k 4.3 l 0.005
8 a \$25 b \$150 c \$60 d \$90 e \$21.60
f 344 grams g \$50.40 h \$107.80 i 350 m
9 a $\frac{15}{300} = \frac{1}{20}$ b 5%
10 68 kg
11 47.52 minutes
12 16.67%
13 $11\frac{1}{9}\%$
14 \$205.97
15 a 1.1 b i 32.3% c NSW – 32% d 90°
1.5 ii 24.9% Vic – 25%
1.7 iii 10.4% Qld – 20%
0.8 SA – 7%
2.4 WA – 10%
0.6 Tas – 2%
0.4 NT – 1%
1.9 ACT – 2%
1.4

Exercise 2B

1 a 110% b 120% c 150% d 102% e 118%
2 a 95% b 70% c 85% d 50% e 83%
3 a *P*: \$5 b *P*: \$2.50 c Loss: \$16
d *P*: \$11 e Loss: \$2.20
4 a \$94.50 b \$440 c \$66 d \$529.20
e \$56 f \$7210 g \$56.43 h \$3.60
5 a \$1425 b \$360 c \$376 d \$68
e \$412.50 f \$47.03 g \$101.58 h \$35.80
6

a	\$6	60%
b	\$60	25%
c	\$3	20%
d	\$7.50	3%
e	\$2325	75%
f	\$0.99	18%

ISBN: 9781107687028

Exercise 2B cont.

7 a $52.25 b $261.25
8 a $1225 b $24.50
9 $1536
10 a $67.96 b $11.99
11 a $2140.25 b $305.75
12 a $221.40 b $147.60
13 a $84.13 b $94.87
14 $104.71
15 a $106.15 b $37.15
16 a $2.20 b 122.22% c $66 d 122.22%
17 a $13 b $6.30 c $69.30 d 38.6%
18 a $1952.24 b $211.24 c 12.13% d $57.03

Exercise 2C

1 a $15 b $22.50 c $30 d $29 640
2 $5600 a month by $200
3 $36 842
4

	Gross income	Net income
a	$570	$415
b	$984	$764
c	$604.90	$304.90
d	$3430	$2920
e	$930.15	$746.15

5 a $1186.24 b $30 842.24 c $2570.19
6 $3437.50
7 a $1960 b $2460
8 a $519.23 b $150 c $669.23 d $38 425
9 a nil b $600 c $2100 d $6360
e $15 270 f $7095 g $635.25 h $10 149.60
10 3%
11 $365.70
12 a $11 580
b i $3401.67 ii $785 iii $1570
c $1572
13 $2000
14 a Kuger Incorporated
b Mathsville Credit Union, 00754031
c $72 454 d fortnightly
e $4420 f $26.45
g $600.60 h $16 016 tax, net = $49 793.90
i $34.71/h

Exercise 2D

1 $124.28
2 $2162
3 $47
4 a $13 750 b $11 250 c No, only $48 per week
5 $60.88
6 a $33 068 b 73.5%
7 a $7756 b $3878 c $149.15
8 a $82 708 b $1590.54 c 24%
9 a 13.43% b 29.56 L
10 a $342.55 b $2137.51 c 11.68%
11 a

food	$86.40
recreation	$43.20
transport	$56.16
savings	$86.40
taxation	$108.00
clothing	$51.84

b $56.16 c 30% d $673.92 e 10%
12 200 tea bags
13 daily
14 a Mon–Thurs – $87
Fri–Sat – $93.50
Weekly – $71.43
b Weekly
15 a 200 mL bottle $0.011 75, 500 mL bottle $0.01048
b 500 mL bottle c $2.10
d $0.25 e cost of packaging
16 a $248 b $240, 6 containers

Exercise 2E

1 a 1 b 6 c 52 d 4 e 3 f 30
2 a $120 b $420 c $30
3 a $420 b $840 c $35
4 a $140 b $420 c $192.50
d $46.88 e 3% p.a. f 4% p.a.
5 a $6650 b $184.72 per month
6 a $5192.25 b $16 692.25 c $198.72
7 a $7600 b $17 600 c $366.67
8 $1008
9 a $228 b $684 c $4684
10 16% 11 12.5 years 12 $66 667 13 $7500
14 a $1250, $2500 b $1968.75, $1920.00
c $220.31, $331.11

Exercise 2F

1 a $50 b $550 c $55 d $605 e $605
2 a $1102.50 b $1102.50 c $1157.63 d $1157.63
3 a $700(1.08)^2$ b $1000(1.15)^6$ c $850(1.06)^4$
4 a 6, 3% b 60, 1% c 52, 0.173%
d 14, 2.625% e 32, 3.75% f 120, 0.8%
5

2	4200	210	4410
3	4410	220.50	4630.50
4	4630.50	231.53	4862.03
5	4862.03	243.10	5105.13

6 a $5105.13 b $11 946.33
c $13 652.22 d $9550.63

ISBN: 9781107687028 Cambridge University Press

7 a \$106 000 **b** \$112 360 **c** \$119 101.60
d \$133 822.56 **e** \$179 084.77 **f** \$239 655.82
8 a \$2254.32 **b** \$87 960.39 **c** \$1461.53
d \$789.84 **e** \$591.63
9 \$11 651.92
10 a \$5075 **b** \$5228.39 **c** \$5386.42
11 a i \$3239.42 **ii** \$3348.15 **iii** \$3446.15
iv \$3461.88 **v** \$3465.96 **b** \$226.54
12 a $P = 300$, $n = 12$, $r = 7\%$, $R = 14\%$, $t = 6$ years **b** $P = 5000$, $n = 24$, $r = 2.5\%$, $R = 30\%$, $t = 2$ years
c $P = 1000$, $n = 65$, $r = 0.036\%$, $R = 0.936\%$, $t = 2.5$ years
d $P = 3500$, $n = 30$, $r = 0.0053\%$, $R = 1.9345\%$, $t = 30$ days
e $P = 10\,000$, $n = 10$, $r = 7.8\%$, $R = 7.8\%$, $t = 10$ years
13 5.3% compounded bi-annually
14 a i approx. 6 years **ii** approx. 12 years
iii approx. 9 years **iv** approx. 5 years
v approx. 7 years **vi** approx. 4 years
b Same answer as part **a**. **c** yes

Exercise 2G

1 a \$2640 **b** \$3960 **c** \$13 200
2 \$124.50
3 a \$1.20 **b** \$1.71 **c** \$3 **d** \$0.09
4 a \$18 600 **b** \$8600
5 a \$5580 **b** \$80
6 a 360 **b** \$624 960 **c** \$374 960
7 a \$2550 **b** \$10 200 **c** \$10 620
d \$13 170 **e** \$420
8 a \$4.50 **b** \$1.26
9 a

May	June	July	August	September	October
\$2.40	\$3.00	\$0.12	\$5.00	\$2.08	\$0.73

b \$13.33
10 a i \$0 **ii** \$0 **iii** \$7500
b \$6375 **c** \$1125
11 a i \$5250 **ii** \$20 250 **iii** \$281.25
b i \$8400 **ii** \$32 400 **iii** \$270
12 a

Date	Deposit	Withdrawal	Balance
1 May			\$3010
3 May	\$490		\$3500
5 May		\$2300	\$1200
17 May	\$490		\$1690
18 May		\$150	\$1540
20 May		\$50	\$1490
25 May		\$218	\$1272
31 May	\$490		\$1762

b \$4.90 **c** \$1759.55 **d** wages
13 a i \$73.13 **ii** \$69.72 **iii** \$75.17
b \$1700 over 3 years
14 a \$403.80 **b** \$393.80 **c** 24 cents a day
15 a \$98 822.90 **b** \$0.23 **c** \$8.00
d \$2400 **e** \$378.50 **f** \$246 025
g \$143 475.50

Exercise 2H

1 B
2 $P = 750$, $r = 7.5$, $n = 5$
3 $I = 225$ $P = 300$ $r = 3$ $t = 25$
4 a i \$7146.10 **ii** \$6955.64 **iii** \$6858.57
iv \$7260 **v** \$7916.37
b \$6000 at 5.7% p.a. for 5 years
5 a i \$7080 **ii** \$7080 **iii** \$7428
iv \$7200 **v** \$7710
b 6000 at 5.7% p.a., for 5 years
6 a i I \$240, \$240 II \$480, \$494.40
III \$1200, \$1352.90 IV \$2400, \$3163.39
ii I \$240, \$243.60 II \$480, \$502.04
III \$1200, \$1375.67 IV \$2400, \$3224.44
iii I \$240, \$246.71 II \$480, \$508.64
III \$1200, \$1395.40 IV \$2400, \$3277.59
b Compound interest **c** Compound interest
7 a

Principal	Rate	Overall time	Interest	Amount
\$7000	5%	5 years	\$1750	\$8750
\$7000	10%	5 years	\$3500	\$10 500
\$3300	10%	3 years	\$990	\$4290
\$8000	10%	3 years	\$2400	\$10 400
\$9000	8%	2 years	\$1440	\$10 440
\$18 000	8%	2 years	\$2880	\$20 880

b i interest is doubled
ii no change
iii interest is doubled
8

Principal	Rate	Period	Overall time	Interest	Amount
\$7000	4.56%	annually	5 years	\$1750	\$8750
\$7000	8.45%	annually	5 years	\$3500	\$10 500
\$9000	8%	fortnightly	2 years	\$1559.00	\$10 559.00
\$18 000	8%	fortnightly	2 years	\$3118.01	\$21 118.01

9 a 8.45% **b** 8.19% **c** 8.12%
The more often interest is calculated, the lower the required rate.
10 a i 4.2% **ii** 8.7%
b It increases by more than this factor

ISBN: 9781107687028

Puzzles and games

1 commission, fortnightly, overtime, piecework, annual, gross, net, monthly, casual, salary
2 You take away his credit card
3 7 years 4 months
4 59 games

Multiple-choice questions

1 E 2 D 3 D 4 C 5 B
6 E 7 B 8 B 9 C 10 E

Short-answer questions

1 \$1395
2 \$1084.16
3 \$4557
4 a \$11.40 b \$3.80
5 \$4200
6 \$576.92
7 a \$7400 b \$616.67 c \$142.31
8 a \$287.32 b \$43.10 c 10.79%
9 a \$12 325 b approx. 38%
10 a \$346.68 b \$290.65
11 \$10 212.60
12 \$35.55
13 a \$1050 b \$12 000 c \$6050
14 a \$1600 b \$1166.67 c \$624.32 d \$1022.53

Extended-response questions

1 a \$5624.32 b \$624.32 c 4.16% d \$636.36
2 a

Date	Deposit	Withdrawal	Balance
1st			217.63
7th		64.00	153.63
9th	140.00		293.63
11th		117.34	176.29
15th		0.51	175.78
20th	20.00	12.93	182.85
30th	140.00		322.85

b \$153.63 c \$0.08

Chapter 3

Pre-test

1 a $3x$ b $a+1$ c $2m-5$ d $4(x+y)$
2 a 20 b 17 c 23 d 22
3 a no b yes c yes d no
4 a $8m$ b $5ab$ c $6x+8y$ d $8x$
e $15ab$ f $3y$
5 a $2x+10$ b $3y-6$ c $8x-12$ d $3x^2+x$
6 a 4 b 6 c $7a$
d $2x$ e x
7 a $\frac{31}{40}$ b $\frac{11}{21}$ c $\frac{2}{15}$ d $\frac{3}{2}$
8 a 7^4 b m^3 c x^2y^3 d 3^5a^5
9 a 49 b 27 c 16 d 64
10 a 3^7 b 3^2 c 3^{10} d 3^0 e 3^{-2}
11 a 38 b 2310 c 0.172
d 0.0018 e 1000 f 10 000

Exercise 3A

1 a expression b constant term
c coefficient d term
2 a + b − c × d − e + f ÷
3 a 7 b 15 c 5 d 9 e 6
4 a -6 b -20 c -6 d -7
e 9 f 18 g -6 h 7
i -7 j -13 k -4 l -5
5 a i 3 ii 8 iii 5
b i 4 ii 2 iii -3
c i 3 ii -4 iii 1
6 a $x+2$ b $y-4$ c $ab+y$ d $2x-3$
e $5x$ f $2m$ g $3r$ h $\frac{1}{2}x$
i $\frac{3}{4}m$ j $\frac{x}{y}$ k $\frac{a+b}{4}$ l x^2y
7 a 12 b 3 c 9 d 10
e 10 f -2 g 1 h 4
i -6 j 10 k -2 l -9
8 a $5x$ c b $35y$ c c $\frac{\$500}{n}$ d $\frac{\$11}{m}$ e $11+x$
9 a $\$(3.40+2d)$ b i \$23.40 ii \$47.40
10 a i $2x$ ii $2x-3$ iii $3(2x-3)$ b 21
11 a $x+1$ and $x-1$ b No
c i $(x+1)(x-1)$ ii Less by one square metre.
12 a 21 sq. units b 1, 2, 3, 6, 9, 18
13 a i $4x$ ii x^2
b i $2x+2y$ ii xy
c i $x+y+5$ ii $\frac{5x}{2}$
d i $4ab$ ii a^2b^2
e i $2a^2+2b$ ii a^2b
f i $mn+9$ ii $2mn$

Exercise 3B

1 a Y b Y c N
d N e Y f Y

ISBN: 9781107687028

2 **a** $10g$ **b** $5f$ **c** $8e$
d 0 **e** $6x$ **f** $17st$
g $3ts$ **h** $3ab$ **i** xy

3 **a** $6x$ **b** $12a$ **c** $10m$ **d** $-18y$

4 **a** $\frac{1}{2}$ **b** 4 **c** 3 **d** $\frac{2}{3}$
e $\frac{2}{3}$ **f** $\frac{7}{3}$ **g** $\frac{3}{4}$ **h** $\frac{2}{5}$

5 **a** $3ac$ and $-2ac$ **b** $4pq$ and $3qp$
c $7xy^2$ and $4yx^2$, $-3xy^2$ and $2xy^2$
d $2r^2$ and $-r^2$ **e** $-2ab$ and $4ba$
f $3p^2q$ and $4qp^2$ **g** $\frac{1}{3}lm$ and $\frac{lm}{4}$
h x^2y and yx^2, $-xy$ and yx

6 **a** $7t + 10$ **b** $4g + 1$ **c** $7x - 5$
d $m + 2$ **e** $3x + 3y$ **f** $2x + 6y$
g $5a - 2b$ **h** $-3m - 2n$ **i** $5de + 7de^2$
j $3kl - 10k^2l$ **k** $7x^2y + xy^2$ **l** $3fg - fg^2$

7 **a** $6rs$ **b** $6hu$ **c** $16wh$ **d** $6r^2s$
e $-8es$ **f** $-10hv$ **g** $12cm^2$ **h** $35fl$
i $8x^2y$ **j** $24a^2b$ **k** $3xy^2$ **l** $-16a^2b$
m $-12m^2n^2$ **n** $20x^2y^2$ **o** $20a^2b^2$ **p** $-48x^2y^2$

8 **a** $\frac{x}{2}$ **b** $3a$ **c** $\frac{a}{3}$ **d** $\frac{ab}{4}$
e $2b$ **f** $3x$ **g** $\frac{y}{2}$ **h** $\frac{4a}{5}$
i $\frac{2x}{5}$ **j** $\frac{2xy}{3}$ **k** $3ab$ **l** $\frac{n}{3}$

9 **a** $8x$ **b** $3x^2$

10 **a** $5x$ **b** $8y$ **c** $4a$
d $-6x$ **e** $2x$ **f** $10a^2b$

11 **a** $P = 4x + 6, A = 6x$ **b** $P = 4y + 16x, A = 16xy$
c $P = 20a, A = 25a^2$

12 $3x$

13 **a** $6x$ **b** $8a + 7$ **c** $4b$ **d** $11x$
e $3x^2$ **f** $15xy$ **g** $8x^2$ **h** $-29x^2$

Exercise 3C

1 **a** $4x$ **b** 8 **c** $4x + 8$
d $x + 2$ **e** $4 \times (x + 2)$ **f** $4x + 8$

2 **a** -8 **b** -18 **c** $-3x$ **d** $-8x$ **e** -20
f $-16x$ **g** 15 **h** 24 **i** $6x$

3 **a** $3(x + 4) = 3 \times x + 3 \times 4$
$= 3x + 12$
b $2(x - 5) = 2 \times x + 2 \times (-5)$
$= 2x - 10$
c $2(4x + 3) = 2 \times 4x + 2 \times 3$
$= 8x + 6$
d $x(x - 3) = x \times x + x \times (-3)$
$= x^2 - 3x$

4 **a** $6x$ **b** $8xy$ **c** $15x^2$
d $2x + 9$ **e** $7x + 5$ **f** $3x - 4$

5 **a** $2x + 8$ **b** $3x + 21$ **c** $4y - 12$ **d** $5y - 10$
e $6x + 4$ **f** $8x + 20$ **g** $9a - 12$ **h** $14y - 35$
i $10a + 5b$ **j** $12a - 9b$ **k** $2x^2 + 10x$ **l** $3x^2 - 12x$
m $6a^2 + 4ab$ **n** $6xy - 8y^2$ **o** $6ab - 15b^2$

6 **a** $-2x - 6$ **b** $-5m - 10$ **c** $-3w - 12$
d $-4x + 12$ **e** $-2m + 14$ **f** $-7w + 35$
g $-x - y$ **h** $-x + y$ **i** $-6x^2 - 8x$
j $-6x^2 - 15x$ **k** $-8x^2 + 8x$ **l** $-6y^2 + 27y$
m $-6x^2 + 10xy$ **n** $-9x^2 - 6xy$ **o** $-12xy - 18y^2$

7 **a** $5x + 17$ **b** $7x + 17$ **c** $2x - 1$ **d** $1 - 2x$
e $1 - 5x$ **f** $1 + 6x$ **g** $6x + 11$ **h** $14 - 4x$
i $27 - 6x$ **j** $7x + 18$ **k** $7p - 11$ **l** $10x - 4$
m $4s + 5$ **n** 4 **o** $2x - 7$

8 **a** 2 **b** 4 **c** $3x$
d $3x$ **e** $y, 1$ **f** $2x, 3y$

9 **a** $2x - 10$ **b** $x^2 + 3x$ **c** $2x^2 + 8x$ **d** $6x^2 - 3x$

10 $2x^2 + 4x$

11 **a** $x - 6000$ **b** $0.1x - 600$

12 **a** $x^2 + 7x + 12$ **b** $x^2 + 4x + 3$ **c** $x^2 + 7x + 10$
d $x^2 - 2x - 8$ **e** $x^2 + 3x - 10$ **f** $2x^2 + 11x + 12$
g $2x^2 - x - 6$ **h** $x^2 + x - 12$ **i** $4x^2 + 18x - 10$

Exercise 3D

1 **a** 2 **b** 4 **c** 9 **d** 6 **e** 7 **f** 12

2 **a** x **b** $2x$ **c** $3x$ **d** $2b$ **e** $4x$ **f** $3y$

3 **a** C **b** They have no common factor.

4 **a** 6 **b** 5 **c** 4 **d** 9
e $5a$ **f** $2m$ **g** $7x$ **h** $8a$
i $3a$ **j** $2x$ **k** $8y$ **l** $5xy$

5 **a** $3(x + 3)$ **b** $4(x - 2)$ **c** $10(y - 2)$
d $6(a + 5)$ **e** $5(x + y)$ **f** $4(3a + b)$
g $9(2m - 3n)$ **h** $12(3x - 4y)$ **i** $4(2x + 11y)$
j $6(4a - 3b)$ **k** $11(11m + 5n)$ **l** $7(2k - 9l)$

6 **a** $7x(2 + 3y)$ **b** $3b(2a - 5)$ **c** $8y(4 - 5x)$
d $5x(x - 1)$ **e** $x(x + 7)$ **f** $2a(a + 4)$
g $6a(2a + 7b)$ **h** $9y(y - 7)$ **i** $2x(3x + 7)$
j $3x(3x - 2)$ **k** $8y(2y + 5)$ **l** $10m(1 - 4m)$

7 **a** $-2(x + 3)$ **b** $-4(a + 2)$ **c** $-3(x + 2y)$
d $-7a(1 + 2b)$ **e** $-x(1 + 10y)$ **f** $-3b(1 + 4a)$
g $-x(x + 7)$ **h** $-4x(x + 3)$ **i** $-2y(y + 5)$
j $-2x(4x + 7)$ **k** $-4x(3x + 2)$ **l** $-5a(3a + 1)$

8 **a** $ab(7a + 1)$ **b** $4a^2(b + 5)$ **c** $xy(1 - y)$
d $x^2y(1 + 4y)$ **e** $6mn(1 + 3n)$ **f** $5xy(x + 2y)$
g $-y(y + 8z)$ **h** $-3ab(a + 2)$ **i** $-ab(b + a)$

9 **a** $4(x + 5)$ **b** $8(x + 2)$ **c** $2(3x + 4)$

10 $(x + 3)$ metres

11 **a** $2(x + 2y + 3z)$ **b** $3(x^2 + 4x + 2)$ **c** $4(x^2 + 2xy + 3)$
d $3x(2x + y - 3)$ **e** $5x(2y - z + 1)$ **f** $2y(2y - 9 + 7x)$

12 **a** $(x + 2)(4 + x)$ **b** $(x + 3)(x + 2)$ **c** $(x + 4)(x - 7)$
d $(2x + 1)(x - 3)$ **e** $(y - 3)(2x + 4)$ **f** $(x - 1)(2x - 3)$

ISBN: 9781107687028

Cambridge University Press

Exercise 3E

1 a $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** $\frac{2x}{5}$ **d** $\frac{2x}{5}$
e $\frac{x+1}{2}$ **f** $\frac{x-2}{2}$ **g** $\frac{x+4}{3}$ **h** $\frac{x+3}{5}$

2 a $\frac{4}{15}$ **b** $\frac{2}{5}$ **c** $\frac{1}{12}$ **d** $\frac{4}{5}$

3 a $\frac{2}{3}$ **b** $\frac{3}{5x}$ **c** $\frac{1}{7}$ **d** $\frac{4}{x+3}$

4 a $3(x+2)$ **b** $2(x+2)$ **c** $4(2x+3)$ **d** $8(2-x)$
e $x(x+3)$ **f** $2x(2x+5)$ **g** $-2(x+3)$ **h** $-x(x+5)$

5 a $\frac{y}{2}$ **b** $\frac{2a}{5}$ **c** $\frac{x}{5}$ **d** $5x$
e $x+1$ **f** $x-5$ **g** $\frac{x+1}{2}$ **h** 5
i 4 **j** $\frac{1}{2}$ **k** 3 **l** $\frac{3}{2}$

6 a $x+2$ **b** $a-5$ **c** $2y-3$ **d** $2b-3$
e 3 **f** 4 **g** 3 **h** 4
i $x+2$ **j** $x-5$ **k** $x+3$ **l** x
m x **n** $2x$ **o** $3x$ **p** $2x$

7 a $\frac{2}{3}$ **b** $\frac{3}{2}$ **c** $\frac{3}{4}$ **d** $\frac{5x}{3}$
e $\frac{3y}{2}$ **f** $\frac{5}{6}$ **g** $\frac{4}{9}$ **h** 10
i $\frac{15}{2}$ **j** $\frac{8}{5}$ **k** $\frac{4}{3x}$ **l** 12

8 a 3 **b** 6 **c** $8a$ **d** $\frac{x}{4}$
e $\frac{8}{15a}$ **f** $\frac{4}{x}$ **g** 3 **h** $\frac{5}{2}$
i $\frac{3}{4}$ **j** $\frac{1}{25}$ **k** 10 **l** $\frac{5}{4}$

9 a Must factorise first, $x+2$
b Factorise first, $x+2$
c Need to multiply by the reciprocal of the fraction after division sign, $\frac{6}{25}$
d x is not a common factor, cannot cancel, $\frac{x+4}{5x}$

10 a $\frac{1+2a}{3}$ **b** $\frac{4}{5}$ **c** $\frac{x}{4}$
d $\frac{2}{5}$ **e** $\frac{7}{18}$ **f** $\frac{2x}{3}$
g $\frac{5}{4}$ **h** $\frac{x}{9}$ **i** $\frac{x+5}{9}$

11 a -3 **b** -2 **c** $-x$
d $\frac{x+2}{3}$ **e** $x-6$ **f** $2x-3$

12 a $\frac{x+1}{2}$ **b** $\frac{2(x+1)}{3}$ **c** $\frac{x-2}{2}$
d $\frac{x+2}{8}$ **e** $\frac{x-3}{12}$ **f** $\frac{5(2x+1)}{8}$

Exercise 3F

1 a 12 **b** 18 **c** 8
d 12 **e** 20 **f** 30

2 a $\frac{5}{6}$ **b** $\frac{31}{40}$ **c** $\frac{13}{12}=1\frac{1}{12}$
d $\frac{5}{21}$ **e** $\frac{3}{16}$ **f** $\frac{7}{18}$

3 a $3x$ **b** $5x$ **c** $6x$ **d** $12x$ **e** 3 **f** 5

4 a $7x+8$ **b** $5x-12$ **c** $2x+4$
d $7x+18$ **e** $7x+4$ **f** $7x+10$

5 a $\frac{7x}{12}$ **b** $\frac{7x}{10}$ **c** $\frac{2x}{9}$ **d** $\frac{2x}{35}$
e $\frac{13x}{15}$ **f** $\frac{7x}{6}$ **g** $\frac{7x}{18}$ **h** $\frac{13x}{40}$
i $\frac{-5x}{14}$ **j** $\frac{-3x}{10}$ **k** $\frac{-x}{30}$ **l** $\frac{-6x}{5}$

6 a $\frac{2x+3}{4}$ **b** $\frac{3x+10}{15}$ **c** $\frac{8x+21}{60}$
d $\frac{5x-8}{20}$ **e** $\frac{6x-5}{9}$ **f** $\frac{10-3x}{12}$

7 a $\frac{5x+4}{6}$ **b** $\frac{13x+12}{15}$ **c** $\frac{5x-4}{8}$
d $\frac{x+8}{6}$ **e** $\frac{x+10}{10}$ **f** $\frac{3x+14}{24}$
g $\frac{9x+22}{20}$ **h** $\frac{11x+13}{14}$ **i** $\frac{5x+7}{12}$
j $\frac{11x+14}{12}$ **k** $\frac{3x-2}{10}$ **l** $\frac{14x-13}{24}$

8 a Need to multiply numerators also when getting common denominator, $\frac{17x}{12}$.
b Need common denominator before subtracting, $\frac{x}{10}$.
c $3(x+2)=3x+6$ and $5(x+4)=5x+20$, $\frac{8x+26}{15}$
d – sign changed to +, $\frac{4x+9}{6}$

9 a $\frac{x+16}{30}$ **b** $\frac{2x+22}{15}$ **c** $\frac{x-23}{20}$ **d** $\frac{x+9}{4}$

10 a $\frac{12+2x}{3x}$ **b** $\frac{3x+8}{4x}$ **c** $\frac{2x+15}{5x}$
d $\frac{3x-14}{7x}$ **e** $\frac{x-20}{5x}$ **f** $\frac{24-5x}{8x}$

11 a $\frac{3x+10}{4x}$ **b** $\frac{x+15}{6x}$ **c** $\frac{6x-5}{20x}$
d $\frac{3x+5}{x^2}$ **e** $\frac{4x+1}{x^2}$ **f** $\frac{3-5x}{x^2}$
g $\frac{3x+4}{2x^2}$ **h** $\frac{12x+7}{3x^2}$ **i** $\frac{3x-14}{4x^2}$

ISBN: 9781107687028 Cambridge University Press

Exercise 3G

1 a Base, index or exponent or power
b power **c** index **d** multiply **e** expanded

2 a $8 \times 8 \times 8$ **b** $7 \times 7 \times 7 \times 7 \times 7$
c $x \times x \times x \times x \times x \times x$ **d** $ab \times ab \times ab \times ab$

3 a 9^4 **b** 3^6 **c** 15^3 **d** 5^2x^3
e 4^3a^4 **f** $7b^4$ **g** x^3y^2 **h** a^2b^4
i $3^3x^2y^3$ **j** $4^2x^2z^2$

4 a $7 \times 7 \times 7 \times 7, 7^7$ **b** $5 \times 5 \times 5 \times 5, 5^4$

5 a $\frac{1}{y^3}$ **b** $\frac{1}{x^4}$ **c** $\frac{1}{x^2}$ **d** $\frac{1}{a^5}$
e $\frac{3}{x^2}$ **f** $\frac{5}{b^3}$ **g** $\frac{4}{x}$ **h** $\frac{2}{m^9}$
i $\frac{2x^2}{y^3}$ **j** $\frac{3x}{y^4}$ **k** $\frac{3b^4}{a^2}$ **l** $\frac{5n^2}{m^3}$

6 a b^4 **b** x^7 **c** y **d** $5m^3$ **e** $2y^2$
f $3x^4$ **g** $5a^2b^3$ **h** $\frac{4y^5}{x^2}$ **i** $\frac{10a^2}{b^4}$

7 a x^7 **b** p^7 **c** t^9 **d** d^5
e g^4 **f** f^3 **g** $2p^5$ **h** $3c^8$
i $6s^{11}$ **j** a^5b^8 **k** d^9f^5 **l** v^5z^8
m $15a^3b^6$ **n** $6x^3y^3$ **o** $18e^9r^3$ **p** $-8p^4c^3$
q $-10r^7s^8$ **r** $6d^6f^4$

8 a a^2 **b** d **c** r^2 **d** c^4
e l **f** b^3 **g** $4d^2$ **h** $\frac{f}{2}$
i $3n^3$ **j** $2p^2$ **k** $\frac{3m^4}{2}$ **l** $\frac{d^2}{3}$
m $4t^3r$ **n** $\frac{5b^4d}{3}$ **o** $2q^2$ **p** $\frac{xy^2}{2}$
q $\frac{r^2s}{3}$ **r** $\frac{2cd^5}{5}$ **s** $\frac{a^3y}{2}$ **t** $\frac{n}{2}$

9 a xy^2 **b** m **c** r^2s^4
d $2a^2b^2$ **e** $\frac{9x^8y^4}{2}$ **f** $4w^4$

10 a $\frac{b^4}{a^3}$ **b** $\frac{x^4}{y^3}$ **c** $\frac{1}{x^2y^4}$ **d** $\frac{b}{a^4}$

11 He hasn't put brackets around -2 i.e. $(-2)^4$.

12 a i 9 **ii** -9
b i is $-3 \times (-3)$ and ii is $-(3 \times 3)$
c i -8 **ii** -8
d They are both -8 since $-2 \times (-2) \times (-2) = -8$

13 a 13 **b** 18 **c** 81 **d** 27 **e** 64 **f** 16

Exercise 3H

1 a 1 **b** a^{mn} **c** $a^m \times b^m$ **d** $\frac{a^m}{b^m}$

2 a $(4^2)^3 = 4^2 \times 4^2 \times 4^2$
$= 4^6$
b $(2a)^3 = 2a \times 2a \times 2a$
$= 2 \times 2 \times 2 \times a \times a \times a$
$= 2^3a^3$
c $\left(\frac{4}{7}\right)^4 = \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7}$
$= \frac{4 \times 4 \times 4 \times 4}{7 \times 7 \times 7 \times 7}$
$= \frac{4^4}{7^4}$

3 a 1 **b** 1 **c** 1 **d** 1
e 3 **f** 4 **g** 2 **h** 0
i 1 **j** 3 **k** 2 **l** 9

4 a b^{12} **b** f^{20} **c** k^{21} **d** $3x^6$ **e** $5c^{18}$ **f** $4s^{18}$

5 a $9x^2$ **b** $64m^3$ **c** $125y^3$ **d** $16x^{12}$
e $x^{10}y^5$ **f** $27a^9$ **g** $x^{24}y^{12}$ **h** a^6b^3
i $m^{12}n^{12}$ **j** $\frac{x^2}{25}$ **k** $\frac{y^4}{81}$ **l** $\frac{m^4}{16}$
m $\frac{x^6}{y^3}$ **n** $\frac{x^{12}}{y^8}$ **o** $\frac{x^3}{y^{15}}$

6 a m **b** $\frac{y^5}{10}$ **c** $\frac{4b^4}{3}$ **d** $\frac{27c^4}{2}$
e $\frac{25r^4}{3}$ **f** $\frac{8p^5}{3}$ **g** $\frac{16s^8}{t^{12}}$ **h** $\frac{r^8}{625s^{12}}$

7 a $\frac{1}{x^8}$ **b** $\frac{1}{x^6}$ **c** 1
d $\frac{8}{y^6}$ **e** $\frac{a^2}{y^6}$ **f** $\frac{x^6}{16}$
g $\frac{1}{m^4}+4$ **h** $\frac{8}{a^6}+1$ **i** $\frac{a^4}{5}$

8 a $\frac{1}{4}$ **b** 36 **c** 40
d 1 **e** 1 **f** 9

9 a $2p^3q^7$ **b** $108a^9b^6$ **c** $48r^9y^{12}$ **d** $2m^9n^4$
e $21s^3y^2$ **f** $\frac{3d^{10}c^4}{2}$ **g** $2r^6$ **h** y

10 a a^7b^2 **b** $54x^5y^{11}$ **c** $\frac{2p^4}{9q^2}$
d $4a^8b^3$ **e** $\frac{324r^{11}}{s}$ **f** $\frac{2y^{14}s^2}{x^5}$

Exercise 3I

1 a 124 **b** 280 **c** 3020
d 0.045 **e** 0.00375 **f** 0.06

2 a 10^3 **b** 10^7 **c** 10^{-6} **d** 10^{-3}

3 a positive **b** negative **c** positive **d** negative

4 a 4.87 **b** 4.872 **c** 4.9

5 a 3120 **b** 54 293 **c** 710 500
d 8 213 000 **e** 59 500 **f** 800 200
g 10 120 **h** 9 990 000 **i** 210 500 000
j 0.0045 **k** 0.0272 **l** 0.000 308 5
m 0.007 83 **n** 0.000 092 **o** 0.265
p 0.000 100 2 **q** 0.000 006 235 **r** 0.98

6 a 4.3×10^4 **b** 7.12×10^5 **c** 9.012×10^5
d 1.001×10^4 **e** 2.39×10^4 **f** 7.03×10^8

ISBN: 9781107687028

Exercise 3I cont.

g 7.8×10^{-4} h 1.01×10^{-3} i 3×10^{-5}
j 3.004×10^{-2} k 1.12×10^{-1} l 1.92×10^{-3}

7 a 6.24×10^{3} b 5.73×10^{5} c 3.02×10^{4}
d 4.24×10^{5} e 1.01×10^{4} f 3.50×10^{7}
g 7.25×10^{4} h 3.56×10^{5} i 1.10×10^{8}
j 2.42×10^{-3} k 1.88×10^{-2} l 1.25×10^{-4}
m 7.87×10^{-3} n 7.08×10^{-4} o 1.14×10^{-1}
p 6.40×10^{-6} q 7.89×10^{-5} r 1.30×10^{-4}

8 a 7.7×10^{6} km^2 b 2.5×10^{6} c 7.4×10^{9} km
d 1×10^{-2} cm e 1.675×10^{-27} kg f 9.5×10^{-13} g

9 The numeral part has to be between 1 and 10 i.e. 3.8×10^{8}

10 a 2.85×10^{-3} b 1.55×10^{-3} c 4.41×10^{-8}
d 6.38×10^{-3} e 8.00×10^{7} f 3.63×10^{8}
g 1.80×10^{-3} h 3.42×10^{15}

11 328 minutes

12 a i 9×10^{17} J ii 2.34×10^{21} J
iii 2.7×10^{15} J iv 9×10^{11} J
b i 1.11×10^{8} kg ii 4.22×10^{-1} kg
iii 9.69×10^{-13} kg iv 1.89×10^{-19} kg
c 5.4×10^{41} J

Exercise 3J

1 a \$50 b \$1050 c \$52.50 d \$55.13

2 a 4.9 kg b $\frac{2}{100}$, 0.98 c 4.52 kg

3 a Growth b Growth c Decay
d Decay e Growth f Decay

4 a A = amount of money at any time, n = number of years of investment
$A = \$200\,000 \times 1.17^n$
b A = house value at any time, n = number of years since initial valuation
$A = \$530\,000 \times 0.95^n$
c A = car value at any time, n = number of years since purchase
$A = \$14\,200 \times 0.97^n$
d A = population at any time, n = number of years since initial census
$A = 172\,500 \times 1.15^n$
e A = litres in tank at any time, n = number of hours elapsed
$A = 1200 \times 0.9^n$
f A = cell size at any time, n = number of minutes elapsed
$A = 0.01\text{ cm}^2 \times 2^n$
g A = Size of oil spill at any time, n = number of minutes elapsed
$A = 2\text{ m}^2 \times 1.05^n$
h A = mass of substance at any time, n = number of hours elapsed
$A = 30\text{ g} \times 0.92^n$

5 a 1.1
b i \$665 500 ii \$1 296 871.23 iii \$3 363 749.98
c After 7.3 years

6 a 300 000
b i \$216 750 ii \$96 173.13 iii \$42 672.53
c 3.1 years

7 a $V = 15\,000 \times 0.94^n$
b i 12 459 L ii 9727 L
c 769.53 L d 55.0 hours

8 a 3000
b i 3000 ii 20 280 iii 243 220
c 10 hours 10 minutes

9 a $D = 10 \times 0.875^t$, where t = number of 10 000 kms travelled
b Yes c 90 000

10 a $T = 90°\text{C} \times 0.92^t$
b i 76.2°C ii 79.4°C
c 3 minutes 13 seconds

11 a i \$1610.51 ii \$2143.59 iii \$4177.25
b i \$1645.31 ii \$2218.18 iii \$4453.92

12 a \$2805.10 b \$2835.25 c \$2837.47

Puzzles and games

1 magic square sum = $3x + 2y$

$\frac{4x^2}{2x}$	$-y$	$x + 3y$
$4y$	$x + y$	$2x - 3y$
$x - 2y$	$2x + 2y$	$2y$

2 3^{3n-3}

3 1c and then double each day

4 5

5 2^{24}

6 200

7 $\frac{5 - 2x}{30}$

8 n^2, 225

Multiple-choice questions

1 C 2 B 3 C 4 D
5 B 6 D 7 C 8 E
9 D 10 A 11 C 12 D

Short-answer questions

1 a 4 b 5 c i 4 ii -3

2 a $y + 3$ b $xy - 5$ c $\frac{a+b}{4}$

3 a 14 b -30 c 35

4 a $7x - 5$ b $13a - 2b$ c $xy - 3xy^2$
d $12mn$ e $-14x^2y$ f $\frac{2b}{3}$

ISBN: 9781107687028 Cambridge University Press

5 a $10x + 20$ **b** $-6x + 8y$ **c** $6x^2 + 15xy$
d $4a + 15$ **e** $5y + 13$ **f** $8t + 11$

6 a $8(2x - 5)$ **b** $5xy(2x + 7y)$
c $2x(2x - 5)$ **d** $-2x(y + 9)$

7 a $\frac{14x}{15}$ **b** $\frac{6-7a}{14}$ **c** $\frac{9x+8}{20}$

8 a $\frac{3}{8}$ **b** $4x$ **c** $3x - 1$ **d** $\frac{2}{3}$

9 a $12x^7$ **b** $8x^4y^4$ **c** b^4 **d** $\frac{2a^2b^3}{3}$

10 a $\frac{4}{x^3}$ **b** $\frac{3r^4}{s^2}$ **c** $\frac{2y^4}{3x^3}$ **d** $4m^5$

11 a b^8 **b** $8m^6$ **c** $\frac{x^2}{49}$ **d** $\frac{64y^6}{z^{12}}$

12 a 1 **b** 4 **c** 6 **d** 1

13 a $\frac{5y^6}{4}$ **b** $\frac{25x^3y^2}{2}$ **c** $\frac{10y^3}{x^2}$ **d** $\frac{x^8}{2y^3}$

14 a 4250 **b** 37 000 000
c 0.021 **d** 0.0000725

15 a 1.24×10^5 **b** 3.95×10^7
c 9.02×10^{-6} **d** 4.60×10^{-4}

16 a $P = 20(1.1)^n$ **b** $A = 100\,000(0.85)^n$

Extended-response questions

1 a $2(5x + 1)$ m **b** 32 m **c** $5x^2 + 3x$ **d** \$1080

2 a $A = 2(1.09)^t$
b i 2.3762 m^2 **ii** 3.0772 m^2
c 37.4 weeks

Chapter 4

Pre-test

1 a 11
b i $\frac{1}{11}$ **ii** $\frac{2}{11}$ **iii** $\frac{4}{11}$
iv $\frac{7}{11}$ **v** $\frac{3}{11}$ **vi** $\frac{8}{11}$

2 a $\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{4}$
e $\frac{5}{8}$ **f** $\frac{7}{8}$ **g** $\frac{1}{4}$

3 0, 1 in 5, 39%, 0.4, $\frac{1}{2}$, 0.62, 71%, $\frac{3}{4}$, $\frac{9}{10}$, 1

4 a i 14 **ii** 25 **iii** 11
b i $\frac{18}{25}$ **ii** $\frac{7}{25}$ **iii** $\frac{7}{25}$

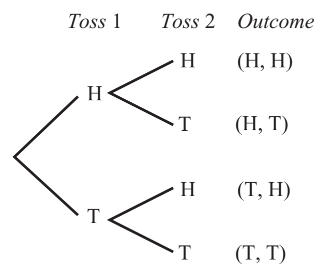
5 a $\frac{7}{16}$ **b** $\frac{9}{16}$

6 a

		Toss 1			
		1	2	3	4
Toss 2	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

b 16
c i $\frac{1}{16}$ **ii** $\frac{3}{16}$ **iii** $\frac{3}{8}$
iv $\frac{5}{8}$ **v** $\frac{13}{16}$ **vi** $\frac{3}{16}$

7 a



b 4
c i $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$
iv $\frac{3}{4}$ **v** $\frac{1}{2}$ **vi** 1

Exercise 4A

1 C, A, B, D

2 a $\frac{1}{4}$ **b** $\frac{1}{6}$ **c** $\frac{1}{4}$
d $\frac{3}{8}$ **e** $\frac{2}{3}$ **f** 0

3 a 2 **b** {H, T} **c** Yes
d $\frac{1}{2}$ **e** $\frac{1}{2}$ **f** 1

4 a 7
b i $\frac{1}{7}$ **ii** $\frac{2}{7}$ **iii** $\frac{5}{7}$ **iv** $\frac{3}{7}$

5 a $\frac{3}{10}$ **b** $\frac{2}{5}$ **c** $\frac{3}{5}$ **d** $\frac{1}{2}$

6 a 43 **b** 47
c i 0.09 **ii** 0.43 **iii** 0.47 **iv** 0.91

7 a 0.62 **b** 0.03 **c** 0.97 **d** 0.38

8 a $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$
e $\frac{2}{5}$ **f** $\frac{1}{5}$ **g** $\frac{3}{10}$

ISBN: 9781107687028 Cambridge University Press

Exercise 4A cont.

9 a $\frac{1}{50}$ **b** $\frac{3}{10}$ **c** $\frac{49}{50}$

10 a $\frac{6}{25}$ **b** $\frac{1}{50}$ **c** $\frac{21}{25}$

d $\frac{2}{5}$ **e** $\frac{2}{25}$ **f** $\frac{4}{25}$

11 a 59

b 4, as $\frac{41}{100}$ of 10 is closest to 4.

c 8, as $\frac{41}{100}$ of 20 is closest to 8.

12 a $\frac{1}{4}$ **b** $\frac{1}{13}$ **c** $\frac{1}{52}$ **d** $\frac{1}{2}$

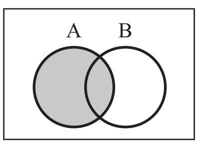
e $\frac{2}{13}$ **f** $\frac{4}{13}$ **g** $\frac{12}{13}$ **h** $\frac{9}{13}$

Exercise 4B

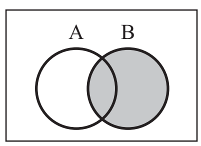
1 a D **b** C **c** E **d** A **e** B

2 a No **b** Yes **c** No

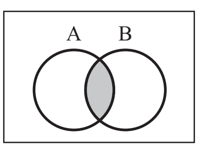
3 a



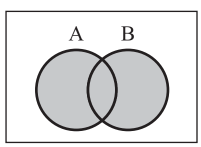
b



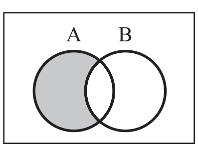
c



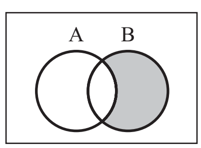
d



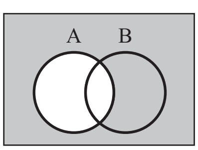
e



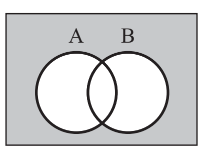
f



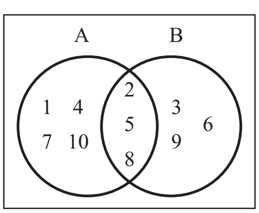
g



h



4 a



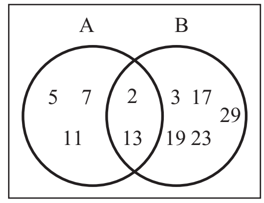
b i $A \cap B = \{2, 5, 8\}$

ii $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c i $\frac{7}{10}$ **ii** $\frac{3}{10}$ **iii** 1

d No, there is at least one number in $A \cap B$

5 a

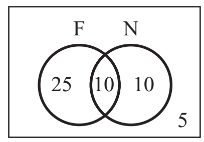


b i $A \cap B = \{2, 13\}$

ii $A \cup B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

c i $\frac{1}{2}$ **ii** $\frac{7}{10}$ **iii** $\frac{1}{5}$ **iv** 1

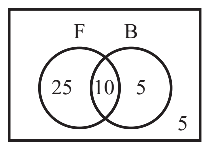
6 a



b i 25 **ii** 5

c i $\frac{2}{5}$ **ii** $\frac{1}{5}$ **iii** $\frac{1}{5}$

7 a



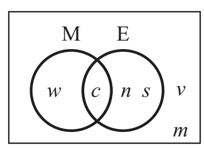
b i 25 **ii** 5

c i $\frac{7}{9}$ **ii** $\frac{2}{9}$ **iii** $\frac{8}{9}$ **iv** $\frac{2}{9}$ **v** $\frac{1}{9}$

8 3

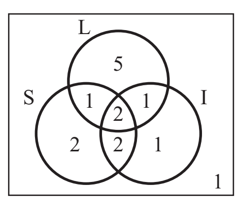
9 5

10 a



b i $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{6}$ **iv** $\frac{2}{3}$ **v** $\frac{1}{3}$

11 a



b 1

c i $\frac{3}{5}$ **ii** $\frac{1}{3}$ **iii** $\frac{13}{15}$ **iv** $\frac{1}{15}$

Exercise 4C

1 a B **b** A **c** D **d** C

2 a i 4 **ii** 6 **iii** 3 **iv** 1

v 10 **vi** 7 **vii** 4 **viii** 7

b 13

ISBN: 9781107687028

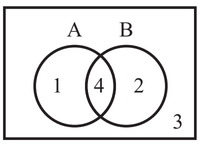
3 a

	A	A′	
B	2	6	8
B′	5	3	8
	7	9	16

b **i** 2 **ii** 6 **iii** 5 **iv** 3
v 7 **vi** 8 **vii** 13

c **i** $\frac{1}{8}$ **ii** $\frac{9}{16}$ **iii** $\frac{5}{16}$

4 a



b

	A	A′	
B	4	2	6
B′	1	3	4
	5	5	10

c **i** 2 **ii** 5 **iii** 4 **iv** 7

d **i** $\frac{3}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{1}{10}$ **iv** $\frac{2}{5}$ **v** $\frac{7}{10}$

5 a

	A	A′	
H	4	3	7
H′	4	1	5
	8	4	12

b **i** 3 **ii** 1

c **i** $\frac{11}{12}$ **ii** $\frac{1}{3}$

6 a

	A	A′	
B	3	3	6
B′	4	1	5
	7	4	11

b

	A	A′	
B	2	7	9
B′	2	1	3
	4	8	12

7 a $\frac{1}{8}$ **b** $\frac{5}{24}$

8 a 0

b

	A	A′	
B	0	6	6
B′	10	2	12
	10	8	18

9 a $\frac{3}{8}$ **b** $\frac{5}{32}$

10 a $\frac{4}{13}$ **b** $\frac{4}{13}$ **c** $\frac{7}{13}$ **d** $\frac{7}{13}$

11 a 18 **b** 75

Exercise 4D

1 a $\frac{1}{2}$ **b** $\frac{1}{3}$

2 a $\frac{7}{10}$ **b** $\frac{7}{12}$

3 a $\frac{1}{3}$ **b** $\frac{1}{2}$

4 a **i** $\frac{9}{13}$ **ii** $\frac{3}{13}$ **iii** $\frac{3}{7}$ **iv** $\frac{1}{3}$

b **i** $\frac{14}{17}$ **ii** $\frac{4}{17}$ **iii** $\frac{4}{7}$ **iv** $\frac{2}{7}$

c **i** $\frac{3}{4}$ **ii** $\frac{5}{8}$ **iii** $\frac{5}{7}$ **iv** $\frac{5}{6}$

d **i** $\frac{7}{16}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{4}$ **iv** $\frac{2}{7}$

5 a **i** $\frac{7}{18}$ **ii** $\frac{1}{9}$ **iii** $\frac{1}{5}$ **iv** $\frac{2}{7}$

b **i** $\frac{4}{9}$ **ii** $\frac{1}{9}$ **iii** $\frac{1}{5}$ **iv** $\frac{1}{4}$

c **i** $\frac{8}{17}$ **ii** $\frac{7}{17}$ **iii** $\frac{7}{10}$ **iv** $\frac{7}{8}$

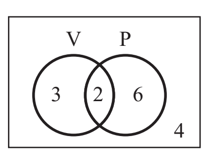
d **i** $\frac{3}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{2}{3}$ **iv** $\frac{1}{3}$

6 a

	A	A′	
B	9	6	15
B′	4	1	5
	13	7	20

b $\frac{1}{5}$ **c** $\frac{3}{5}$ **d** $\frac{9}{13}$

7 a



b 4 **c** $\frac{2}{5}$ **d** $\frac{1}{4}$

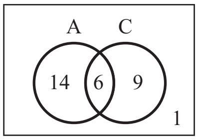
8 a $\frac{1}{13}$ **b** $\frac{1}{13}$

ISBN: 9781107687028

Exercise 4D cont.

9 $\Pr(A|B) = \Pr(B|A) = 0$ as $\Pr(A \cap B) = 0$

10 a



	A	A′	
C	6	9	15
C′	14	1	15
	20	10	30

b i $\frac{3}{10}$ **ii** $\frac{7}{15}$

c $\frac{2}{5}$

d $\frac{3}{10}$

Exercise 4E

1 a i

		1st		
		D	O	G
2nd	D	(D, D)	(O, D)	(G, D)
	O	(D, O)	(O, O)	(G, O)
	G	(D, G)	(O, G)	(G, G)

ii

		1st		
		D	O	G
2nd	D	X	(O, D)	(G, D)
	O	(D, O)	X	(G, O)
	G	(D, G)	(O, G)	X

b i 9 **ii** 6

c i $\frac{1}{3}$ **ii** $\frac{5}{9}$ **iii** $\frac{4}{9}$ **iv** $\frac{8}{9}$ **v** $\frac{2}{9}$

d i 0 **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** 1 **v** $\frac{1}{3}$

2 a 9 **b** 6

3 a

		1st toss			
		1	2	3	4
2nd toss	1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)

b 16 **c** $\frac{1}{16}$

d i $\frac{1}{4}$ **ii** $\frac{5}{8}$ **iii** $\frac{13}{16}$

4 a

		1st toss	
		H	T
2nd toss	H	(H, H)	(T, H)
	T	(H, T)	(T, T)

b 4 **c** $\frac{1}{4}$

d i $\frac{1}{2}$ **ii** $\frac{3}{4}$

e 250

5 a

		1st		
		S	E	T
2nd	S	X	(E, S)	(T, S)
	E	(S, E)	X	(T, E)
	T	(S, T)	(E, T)	X

b i $\frac{1}{6}$ **ii** $\frac{2}{3}$ **iii** $\frac{2}{3}$ **iv** $\frac{1}{3}$ **v** 1

6 a

		1st				
		L	E	V	E	L
2nd	L	X	(E, L)	(V, L)	(E, L)	(L, L)
	E	(L, E)	X	(V, E)	(E, E)	(L, E)
	V	(L, V)	(E, V)	X	(E, V)	(L, V)
	E	(L, E)	(E, E)	(V, E)	X	(L, E)
	L	(L, L)	(E, L)	(V, L)	(E, L)	X

b 20

c i 8 **ii** 12 **iii** 12

d i $\frac{2}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{3}{5}$

e $\frac{1}{5}$

7 a

		Die 2					
		1	2	3	4	5	6
Die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

ISBN: 9781107687028

b 36

c **i** 2 **ii** 6 **iii** 15

d **i** $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{35}{36}$ **iv** $\frac{1}{12}$

e $\frac{1}{6}$. Her guess is wrong.

8 **a** **i** 100 **ii** 90

b **i** $\frac{1}{10}$ **ii** $\frac{1}{10}$ **iii** $\frac{4}{5}$

c $\frac{19}{100}$

9 **a** Without **b** With **c** With **d** Without

10 **a**

		1st			
		2.5	5	10	20
2nd	2.5	5	7.5	12.5	22.5
	5	7.5	10	15	25
	10	12.5	15	20	30
	20	22.5	25	30	40

b 16

c **i** 1 **ii** 8 **iii** 8

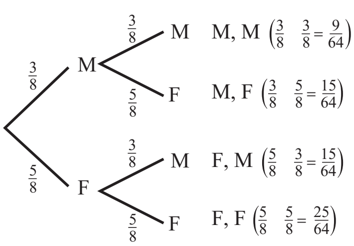
d **i** $\frac{1}{16}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{4}$ **iv** $\frac{3}{16}$

e $\frac{7}{16}$

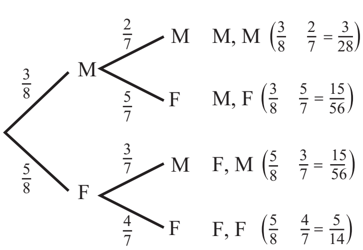
Exercise 4F

1 **a** 8 **b** $\frac{1}{8}$ **c** **i** 3 **ii** 4 **d** $\frac{7}{8}$

2 **a**



b



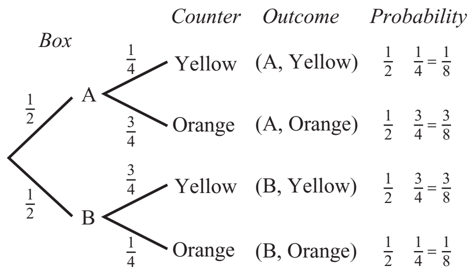
3 **a** **i** $\frac{2}{5}$ **ii** $\frac{3}{5}$

b **i** $\frac{2}{5}$ **ii** $\frac{3}{5}$

c **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$

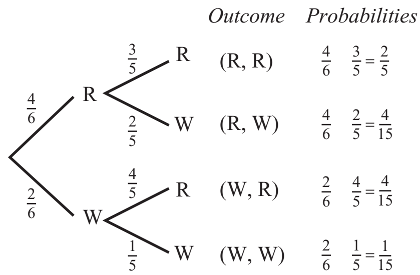
4 **a** $\frac{1}{4}$ **b** $\frac{3}{4}$

c



d $\frac{3}{8}$ **e** $\frac{1}{2}$

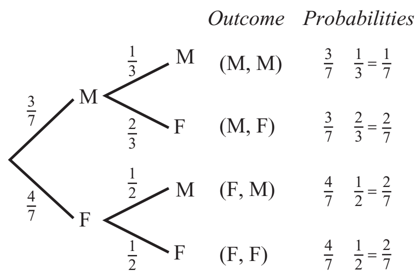
5 **a**



b **i** $\frac{4}{15}$ **ii** $\frac{2}{5}$ **iii** $\frac{8}{15}$

c **i** $\frac{2}{9}$ **ii** $\frac{4}{9}$ **iii** $\frac{4}{9}$

6 **a**



i $\frac{1}{7}$ **ii** $\frac{2}{7}$ **iii** $\frac{4}{7}$ **iv** $\frac{3}{7}$

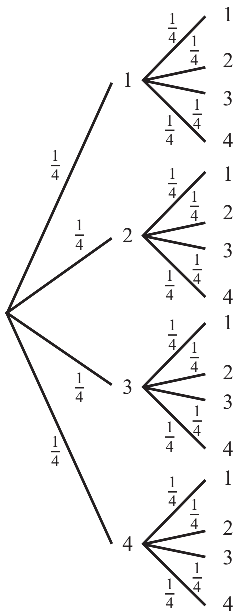
b **i** $\frac{9}{49}$ **ii** $\frac{16}{49}$ **iii** $\frac{24}{49}$ **iv** $\frac{25}{49}$

ISBN: 9781107687028

Cambridge University Press

Exercise 4F cont.

7 a



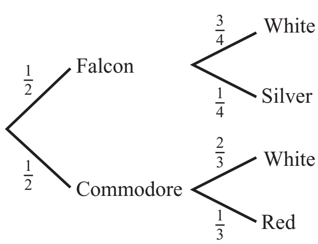
1st toss	2nd toss	Outcome	Probability
1	1	(1, 1)	$\frac{1}{16}$
	2	(1, 2)	$\frac{1}{16}$
	3	(1, 3)	$\frac{1}{16}$
	4	(1, 4)	$\frac{1}{16}$
2	1	(2, 1)	$\frac{1}{16}$
	2	(2, 2)	$\frac{1}{16}$
	3	(2, 3)	$\frac{1}{16}$
	4	(2, 4)	$\frac{1}{16}$
3	1	(3, 1)	$\frac{1}{16}$
	2	(3, 2)	$\frac{1}{16}$
	3	(3, 3)	$\frac{1}{16}$
	4	(3, 4)	$\frac{1}{16}$
4	1	(4, 1)	$\frac{1}{16}$
	2	(4, 2)	$\frac{1}{16}$
	3	(4, 3)	$\frac{1}{16}$
	4	(4, 4)	$\frac{1}{16}$

b 16

c **i** $\frac{1}{16}$ **ii** $\frac{1}{4}$

d **i** $\frac{1}{16}$ **ii** $\frac{1}{4}$ **iii** $\frac{5}{8}$

8 a

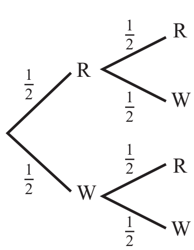


	Outcome	Probability
White	(Falcon, White)	$\frac{1}{2}\ \frac{3}{4}=\frac{3}{8}$
Silver	(Falcon, Silver)	$\frac{1}{2}\ \frac{1}{4}=\frac{1}{8}$
White	(Commodore, White)	$\frac{1}{2}\ \frac{2}{3}=\frac{1}{3}$
Red	(Commodore, Red)	$\frac{1}{2}\ \frac{1}{3}=\frac{1}{6}$

b **i** $\frac{3}{8}$ **ii** $\frac{1}{6}$ **iii** $\frac{17}{24}$

iv $\frac{7}{24}$ **v** $\frac{5}{6}$ **vi** $\frac{1}{3}$

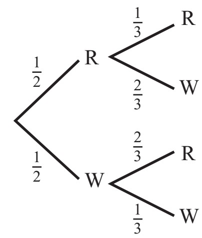
9 a



	Outcome	Probability
R	(R, R)	$\frac{1}{2}\ \frac{1}{2}=\frac{1}{4}$
W	(R, W)	$\frac{1}{2}\ \frac{1}{2}=\frac{1}{4}$
R	(W, R)	$\frac{1}{2}\ \frac{1}{2}=\frac{1}{4}$
W	(W, W)	$\frac{1}{2}\ \frac{1}{2}=\frac{1}{4}$

i $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$ **iv** $\frac{3}{4}$

b



	Outcome	Probability
R	(R, R)	$\frac{1}{2}\ \frac{1}{3}=\frac{1}{6}$
W	(R, W)	$\frac{1}{2}\ \frac{2}{3}=\frac{1}{3}$
R	(W, R)	$\frac{1}{2}\ \frac{2}{3}=\frac{1}{3}$
W	(W, W)	$\frac{1}{2}\ \frac{1}{3}=\frac{1}{6}$

i $\frac{1}{6}$ **ii** $\frac{2}{3}$ **iii** $\frac{5}{6}$ **iv** $\frac{5}{6}$

10 a **i** 0.17 **ii** 0.11 **iii** 0.83

b **i** 0.1445 **ii** 0.0965 **iii** 0.8555

Exercise 4G

1 a **i** $\frac{3}{10}$ **ii** $\frac{1}{3}$

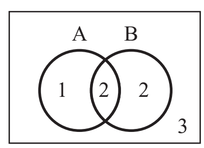
b No **c** No

2 a With **b** Without

3 a **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$

b Yes **c** $\frac{1}{2}$

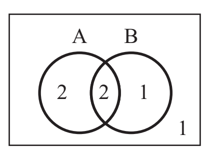
4 a



b **i** $\frac{3}{8}$ **ii** $\frac{1}{2}$

c Not independent

5 a



b **i** $\frac{2}{3}$ **ii** $\frac{2}{3}$

c Independent

6 a **i** $\frac{3}{4}, \frac{1}{2}$ **ii** Not independent

b **i** $\frac{1}{4}, \frac{1}{4}$ **ii** Independent

c **i** $\frac{1}{3}, \frac{1}{3}$ **ii** Independent

d **i** $\frac{2}{7}, 0$ **ii** Not independent

ISBN: 9781107687028 Cambridge University Press

7 a $\Pr(A) = \frac{1}{2}, \Pr(A \mid B) = \frac{1}{2}$, independent

b $\Pr(A) = \frac{3}{10}, \Pr(A \mid B) = \frac{1}{4}$, not independent

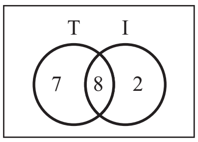
c $\Pr(A) = \frac{5}{12}, \Pr(A \mid B) = \frac{3}{20}$, not independent

d $\Pr(A) = \frac{1}{9}, \Pr(A \mid B) = \frac{1}{9}$, independent

8 False. $\Pr(A \mid B) = 0$, but $\Pr(A) = \frac{2}{9}$

9 a $\frac{1}{32}$ **b** $\frac{31}{32}$ **c** $\frac{31}{32}$

10 a



	T	T′	
I	8	2	10
I′	7	0	7
	15	2	17

i $\frac{15}{17}$ **ii** $\frac{7}{17}$ **iii** $\frac{4}{5}$

b No

Puzzles and games

1 MUTUALLY EXCLUSIVE

2 $\frac{1}{6}$

3 Results may vary.

4 a $\frac{1}{12}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$ **d** $\frac{2}{3}$

5 $\frac{15}{16}$

6 $\frac{3}{5}$

7 $\frac{1}{12}$

Multiple-choice questions

1 C **2** C **3** A **4** D **5** C
6 B **7** B **8** E **9** A **10** E

Short-answer questions

1 a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$

2 a $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$

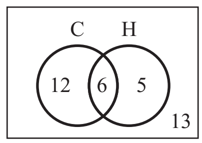
d $\frac{5}{8}$ **e** $\frac{1}{2}$

3 a i $\frac{2}{5}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{5}$

iv $\frac{1}{10}$ **v** $\frac{1}{20}$

b i $\frac{3}{5}$ **ii** $\frac{17}{20}$

4 a



b

	C	C′	
H	6	5	11
H′	12	13	25
	18	18	36

c 13

d i $\frac{1}{6}$ **ii** $\frac{5}{36}$ **iii** $\frac{1}{2}$

5 a 8 **b** $\frac{6}{13}$

6 a $\frac{2}{5}$ **b** $\frac{1}{5}$

7 a

		1st				
		H	A	P	P	Y
2nd	H	(H, H)	(A, H)	(P, H)	(P, H)	(Y, H)
	E	(H, E)	(A, E)	(P, E)	(P, E)	(Y, E)
	Y	(H, Y)	(A, Y)	(P, Y)	(P, Y)	(Y, Y)

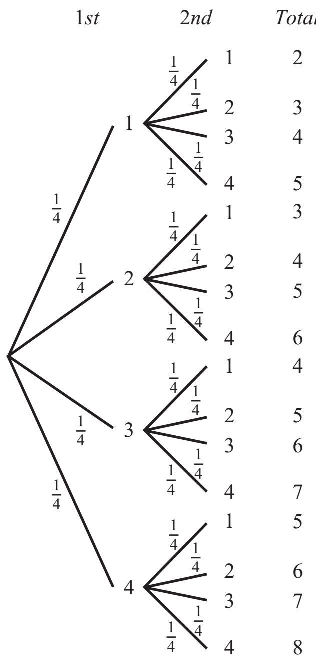
b 15

c i $\frac{1}{15}$ **ii** $\frac{2}{15}$ **iii** $\frac{13}{15}$

ISBN: 9781107687028

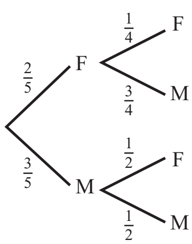
Short-answer questions cont.

8 a



b i $\frac{1}{16}$ **ii** $\frac{1}{4}$ **iii** 0 **iv** 1

9



a $\frac{2}{5}$ **b** $\frac{3}{4}$ **c** $\frac{3}{10}$

d $\frac{3}{5}$ **e** $\frac{7}{10}$

10 a $\frac{7}{11}, \frac{2}{5}$, No **b** $\frac{1}{2}, \frac{1}{2}$, Yes

Extended-response questions

1 a 8

b i $\frac{7}{15}$ **ii** $\frac{1}{15}$

c

	R	R′	
S	3	1	4
S′	3	8	11
	6	9	15

d i $\frac{1}{2}$ **ii** $\frac{3}{4}$

2 a

		1st		
		R	S	W
2nd	R	(R, R)	(S, R)	(W, R)
	S	(R, S)	(S, S)	(W, S)
	W	(R, W)	(S, W)	(W, W)

b i $\frac{1}{9}$ **ii** $\frac{1}{3}$ **iii** $\frac{5}{9}$ **iv** $\frac{4}{9}$

c 4

d $\frac{5}{9}$

Chapter 5

Pre-test

1 a 8 **b** 40 **c** 82 **d** 13

2 a 6

b i 19 **ii** 23

c 30 **d** 10%

3 a 8 **b** 8.5 **c** 5 **d** 8

4 a i Mean = 45 **ii** Mode = 41 **iii** Median = 41

iv Range = 20

b i Mean = 6 **ii** Mode = 2 **iii** Median = 6

iv Range = 10

5 a 15 **b** 111, 139 are most frequent.

c Min = 98 g, Max = 145 g **d** 47

Exercise 5A

1 a Numerical **b** Categorical (nominal)

c Categorical (ordinal) **d** Numerical

2 a

Car colour	Tally	Frequency
Red	|||	3
White	~~||||~~	5
Green	||	2
Silver	||	2
Total	**12**	**12**

b

Class interval	Frequency	Percentage frequency
80–	8	16%
85–	23	46%
90–	13	26%
95–100	6	12%
Total	**50**	**100%**

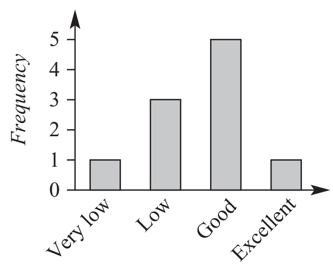
ISBN: 9781107687028

3 a 8 **b** 25 **c i** 8 **ii** 32%

4 a i

Application	Tally	Frequency
Very Low	I	1
Low	III	3
Good	~~IIII~~	5
Excellent	I	1
Total	**10**	**10**

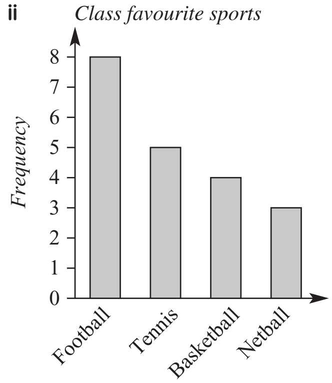
ii *Student level of application*



b i

Favourite sport	Tally	Frequency
Football	~~IIII~~ III	8
Tennis	~~IIII~~	5
Basketball	IIII	4
Netball	III	3
Total	**20**	**20**

ii *Class favourite sports*

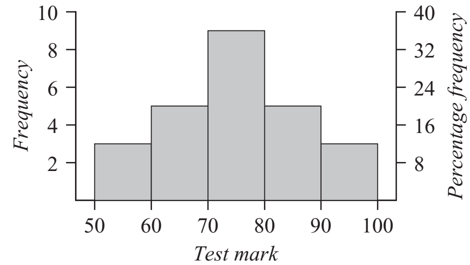


5 a

Class interval	Tally	Frequency	Percentage frequency
50–	III	3	12%
60–	~~IIII~~	5	20%
70–	~~IIII~~ IIII	9	36%
80–	~~IIII~~	5	20%
90–99	III	3	12%
Total	**25**	**25**	**100%**

b

Class test results



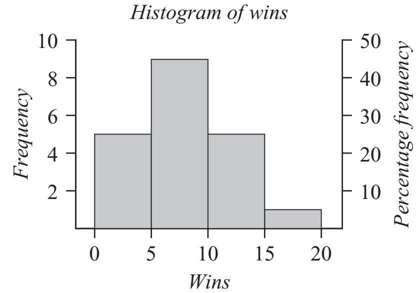
c The 70– interval

d 32%

6 a

Class interval	Tally	Frequency	Percentage frequency
0–	~~IIII~~	5	25%
5–	~~IIII~~ IIII	9	45%
10–	~~IIII~~	5	25%
15 –19	I	1	5%
Total		**20**	**100%**

b *Histogram of wins*



c The 5– interval

d 75%

7 a

Type of transport	Frequency	Percentage frequency
Car	16	40%
Train	6	15%
Tram	8	20%
Walking	5	12.5%
Bicycle	2	5%
Bus	3	7.5%
Total	**40**	**100%**

b i 6 **ii** car **iii** 40% **iv** 17.5% **v** 42.5%

8 a Skewed

b Symmetrical

ISBN: 9781107687028

Cambridge University Press

Exercise 5A cont.

9 a

Mass	Frequency	Percentage frequency
10–	3	6%
15–	6	12%
20–	16	32%
25–	21	42%
30–34	4	8%
Total	**50**	**100%**

b 50 **c** 32%
d At least 25 g but less than 30 g.
e 42% **f** 94%

10 a

Section	Frequency	Percentage frequency
Strings	21	52.5%
Woodwind	8	20%
Brass	7	17.5%
Percussion	4	10%
Total	**40**	**100%**

b 40 **c** 52.5% **d** 47.5% **e** 9.3%

11 Frequencies of individual scores are lost if the histogram displays only categories of scores.

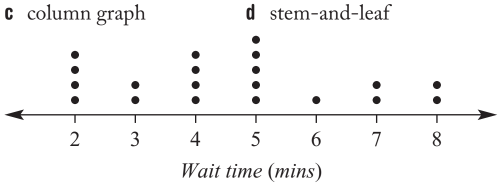
12 a Russia, ~ 14 years **b** Pakistan
c In nearly all countries, the female life expectancy is more than that for males.
d Living conditions in some areas; a high prevalence of HIV/AIDS.

13 a Saturday and Sunday; vendor would expect greater sales at the weekend
b May have been a particularly warm day or a public holiday
c i Wednesday; $250 **ii** Thursday
d The graph does not help us to visualise the profit and loss.

Exercise 5B

1 a histogram **b** dot plot
c column graph **d** stem-and-leaf

2



3 a 32, 35, 41, 43, 47, 54, 54, 56, 60, 62, 71, 71
b 0.2, 0.3, 0.7, 1.4, 1.4, 1.8, 1.9, 2.3, 2.6, 2.6, 3.0, 3.5

4

Stem	Leaf
10	1 1 4 8
11	0 3 3 6
12	2 3 7
13	0 2 9

5 a 11 **b** 1 **c** 16
d 1 family had 3 children but the data was generally symmetrical.

6 a 9 **b** 39
c He had one bad hole with 7 strokes, but generally the data was consistent, between 3 and 5 strokes.

7 a i

Stem	Leaf
1	5 5
2	0 1 2 4 5 6 6
3	1 7 7 8
4	6
5	2

1|5 means 15

ii Skewed

b i

Stem	Leaf
1	2 6
2	1 3 5 7
3	1 2 5 5 6 6 8
4	0 0 2 4 8
5	1 3 5

3|2 means 32

ii Symmetrical

c i

Stem	Leaf
11	6 7 8 9
12	1 4 5 7
13	3 5 7
14	5 7 9
15	3 8
16	0 2

13|5 means 135

ii Skewed

d i

Stem	Leaf
2	0
3	3 7 8
4	3 4 5 7 8 9
5	2 4 5 8
6	1 3 5
7	0

3|7 means 3.7

ii Symmetrical

ISBN: 9781107687028 Cambridge University Press

8 a i

Set 1 leaf	Stem	Set 2 leaf
9 8	3	2 5
8 6 3 2 2 0	4	1 7 7
9 7 3 3	5	2 2 5 6
4 1	6	0 1 3 4 7

5|2 means 52

ii Set 1 is symmetrical while set 2 is skewed with more data at the higher end.

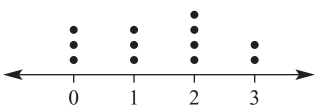
b i

Set 1 leaf	Stem	Set 2 leaf
8 6 4	16	0 3 3 5 7
6 5 2 1	17	0 1 1 4 8
8 7 7 5 1 0	18	2 2 6 7
9 5 2	19	0 4
8 1	20	

19|5 means 195

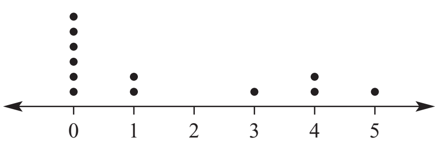
ii Set 1 is symmetrical while set 2 is skewed with most of the data at the lower numbers.

9 a



Nick's goal scoring

b



Jack's goal scoring

c Well spread performance

d Irregular performance, skewed

10 a 4.1 minutes **b** ~ 22.5 mins

c This would increase the average time.

11 a

Inner city leaf	Stem	Outer suburb leaf
9 6 4 3 1 1	0	3 4 9
9 4 2 0	1	2 8 8 9
4 1	2	1 3 4
	3	4
	4	1

2|1 means 21 km

b For the inner city, the data is closer together and bunched around the lower distances. The outer suburb data is more spread out.

c In the outer suburbs, students will be travelling further distances to their school, whereas at inner city schools they are more likely to live close to the school.

12 a $a = 3$, $b = 9$, $c = 7$ or 8

b $a = 0$ or 1, $b = 0, 1, 2, 3, 4$ or 5

13 a The stem 1 is allocated the leaves 0–4 (included) and 1* is allocated 5–9 (included).

b i 1 **ii** 0*

c For city B, for example, most temperatures are in the 20s; splitting into 20–24 and 25–29 allows better analysis of the data and still means that a stem-and-leaf is an appropriate choice of graph.

d City A experienced cooler weather, with temperatures between 8°C and 18°C. City B had warmer weather and a wider range of temperatures, between 17°C and 31°C.

e The cities may have been experiencing different seasons; maybe winter and summer.

Exercise 5C

1 a mode **b** mean **c** median
d bimodal **e** range

2 a 4 **b** 4.5 **c** 3.2

3 a 7 **b** 10 and 14

4 a 28 **b** 7 **c** 4

5 a i 5.4 **ii** 8 **iii** 6
b i 16.25 **ii** 10 **iii** 45
c i 70 **ii** 50, 90 **iii** 40
d i 25 **ii** no mode **iii** 18
e i 2.325 **ii** 1.9 **iii** 1
f i 1.6 **ii** no mode **iii** 1.2

6 a 7 **b** 4 **c** 11 **d** 75
e 7 **f** 5 **g** 33.5 **h** 3.15

7 a 7 **b** 6 **c** 7 **d** 6

8 a \$42 **b** \$17.50 **c** \$20.75
d Due to the \$50 value, which is much larger than the other amounts.

9 a i 25 **ii** 39 **iii** 34.3 **iv** 38
b i 28 **ii** 4 **iii** 17.2 **iv** 17
c i 24 **ii** no mode **iii** 110 **iv** 108
d i 3.2 **ii** 3.0, 5.3 **iii** 4.6 **iv** 4.9

10 a Mark: **i** 83.6 **ii** 85 **iii** 31
Hugh: **i** 76.4 **ii** 79 **iii** 20

b Mark's scores varied more greatly, with a higher range, whereas Hugh's results were more consistent. Mark had the higher mean and median, though, as he had several high scores.

11 The median, since the mean is affected by the one large value (\$1 700 000).

12 a 4 **b** 3.7
c i the median is unchanged in this case
ii the mean is decreased

13 a 70 **b** 85

ISBN: 9781107687028

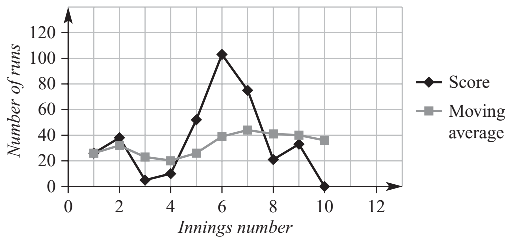
Cambridge University Press

Exercise 5C cont.

14 a

Innings	1	2	3	4	5	6	7	8	9	10
Score	26	38	5	10	52	103	75	21	33	0
Moving average	26	32	23	20	26	39	44	41	40	36

b



c **i** The score fluctuates wildly.
ii The graph is fairly constant with small increases and decreases.
d The moving average graph follows the trend of the score graph but the fluctuations are much less significant.

Exercise 5D

1 **a** Min, lower quartile (Q_1), median (Q_2), upper quartile (Q_3), max
b Range is max – min; IQR is $Q_3 - Q_1$. Range is the spread of all the data, IQR is the spread of the middle 50% of data.
c An outlier is a data point (element) outside the vicinity of the rest of the data.

2 **a** $Q_1 - 1.5 \times \text{IQR}$ **b** $Q_3 + 1.5 \times \text{IQR}$

3 **a** 1.5 **b** **i** 1 **ii** 2

4 **a** 5 **b** -4.5 and 15.5 **c** an outlier

5 **a** **i** $Q_1 = 4, Q_3 = 8$ **ii** 4
b **i** $Q_1 = 11, Q_3 = 18$ **ii** 7
c **i** $Q_1 = 51, Q_3 = 62$ **ii** 11
d **i** $Q_1 = 1.8, Q_3 = 2.7$ **ii** 0.9

6 **a** **i** $Q_1 = 2, Q_3 = 11$ **ii** 9
b **i** $Q_1 = 2, Q_3 = 10$ **ii** 8
c **i** $Q_1 = 1.0, Q_3 = 1.6$ **ii** 0.6
d **i** $Q_1 = 10.5, Q_3 = 22.5$ **ii** 12

7 **a** **i** Min = 0, max = 17 **ii** Median = 13
iii $Q_1 = 10, Q_3 = 15$ **iv** IQR = 5
b 0 is an outlier
c Road may have been closed that day

8 **a** **i** Min = 4, max = 14 **ii** 7.5
iii $Q_1 = 5, Q_3 = 9$ **iv** IQR = 4
v No outliers
b **i** Min = 16, max = 31 **ii** 25
iii $Q_1 = 21, Q_3 = 27$ **iv** IQR = 6
v No outliers
c **i** Min = 2, max = 24 **ii** 12
iii $Q_1 = 10, Q_3 = 13$ **iv** IQR = 3
v 24 and 2
d **i** Min = 1, max = 17 **ii** 7
iii $Q_1 = 4, Q_3 = 10$ **iv** IQR = 6
v No outliers

9 **a** **i** Min = 25, max = 128 **ii** 47
iii $Q_1 = 38, Q_3 = 52.5$ **iv** IQR = 14.5
v Yes, 128 **vi** 51.25
b Median as it is not affected dramatically by the outlier.
c A more advanced calculator was used.

10 2 bags; 30 and 31 will be checked

11 2 fridges; 350 and 1700 are outliers

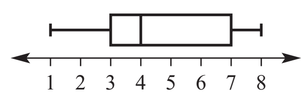
12 **a** IQR = 10 **b** Yes, 1 is an outlier **c** No

Exercise 5E

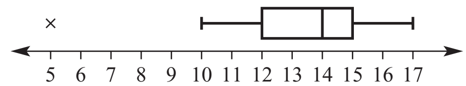
1 **a** minimum **b** Q_1 **c** Q_2(median) **d** Q_3
e maximum **f** whisker **g** box

2 **a** 15 **b** 5 **c** 25 **d** 20
e 10 **f** 20 **g** 10

3 **a**

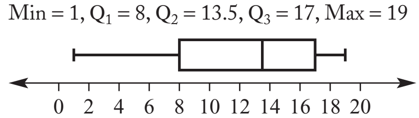


b

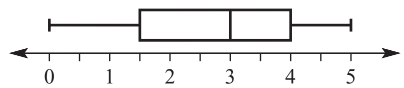


4 **a** Q_3 **b** Q_1, Q_3 **c** Minimum, Q_1 **d** Q_3, Maximum

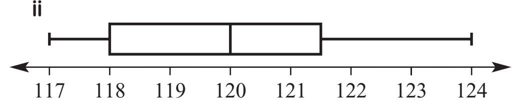
5 **a** **i** Min = 1, $Q_1 = 8$, $Q_2 = 13.5$, $Q_3 = 17$, Max = 19
ii



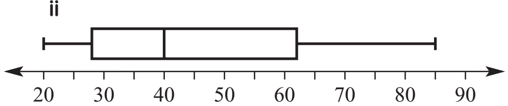
b **i** Min = 0, $Q_1 = 1.5$, $Q_2 = 3$, $Q_3 = 4$, Max = 5
ii



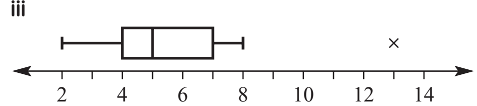
c **i** Min = 117, $Q_1 = 118$, $Q_2 = 120$, $Q_3 = 121.5$, Max = 124
ii



d **i** Min = 20, $Q_1 = 28$, Q2 = 40, $Q_3 = 62$, Max = 85
ii



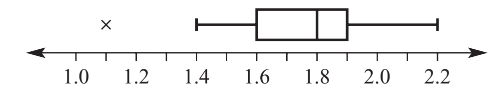
6 **a** **i** $Q_1 = 4$, $Q_2 = 5$, $Q_3 = 7$ **ii** outlier is 13
iii



ISBN: 9781107687028 Cambridge University Press

b **i** $Q_1 = 1.6, Q_2 = 1.8, Q_3 = 1.9$ **ii** outlier is 1.1

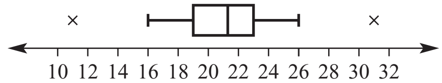
iii



c **i** $Q_1 = 19, Q_2 = 21.5, Q_3 = 23$

ii Outliers are 11 and 31

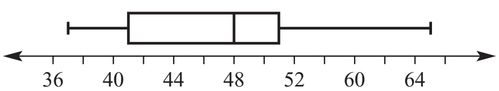
iii



d **i** $Q_1 = 41, Q_2 = 48, Q_3 = 51$

ii No outliers

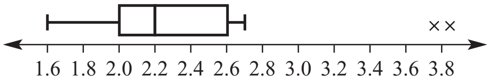
iii



7 **a** Minimum = 1.6, $Q_1 = 2.0$, $Q_2 = 2.2$, $Q_3 = 2.6$, maximum = 3.9, IQR = 0.6

b 3.8 and 3.9 are outliers.

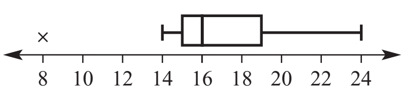
c



8 **a** **i** 8 **ii** 15 **iii** 16 **iv** 19

v 24 **vi** 4 **b** yes, 8

c



9 **a** Same minimum of 1.

b B **c** **i** 5 **ii** 10

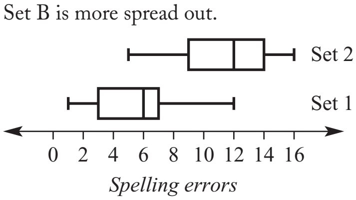
d Data points for B are more evenly spread than those for A.

10 **a** They have the same median and upper quartile.

b B **c** **i** 4 **ii** 5

d Set B is more spread out.

11 **a**



b Yes, examiner 2 found more errors.

Exercise 5F

1 **a** Linear **b** No trend **c** Non-linear **d** Linear

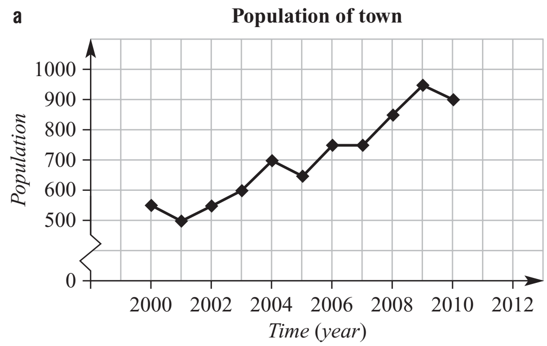
2 **a** **i** 20°C **ii** 30°C **iii** 30°C **iv** 34°C

b 36°C

c **i** 12 noon to 1:00 p.m. **ii** 3 to 4:00 p.m.

d Temperature is increasing from 8 a.m. to 3 p.m. in a generally linear way. At 3 p.m. the temperature starts to drop.

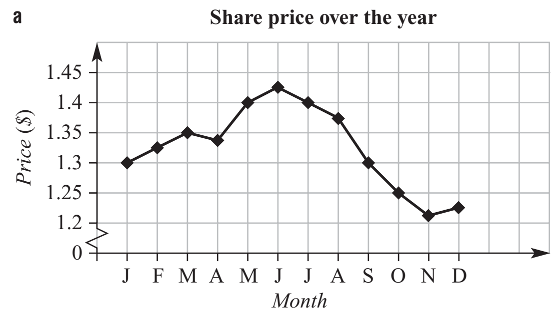
3 **a**



b Generally linear in a positive direction.

c **i** 500 **ii** 950

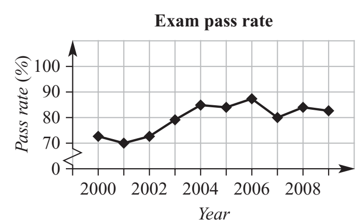
4 **a**



b The share price generally increased until it peaked in June and then continually decreased to a yearly low in November before trending upwards again in the final month.

c $0.21

5 **a**

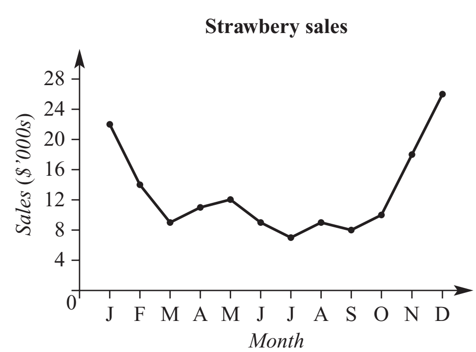


b The pass rate for the examination has increased marginally over the 10 years, with a peak in 2006.

c 2006 **d** 11%

6 **a** Linear **b** **i** $650 000 **ii** $750 000

7 **a**



ISBN: 9781107687028 Cambridge University Press

Exercise 5F cont.

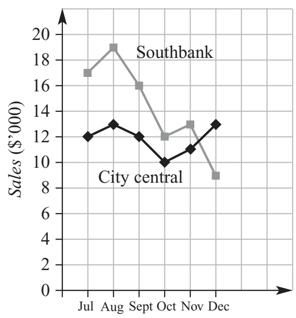
b The sales start high and decrease during the middle of the year, before increasing again towards the end of the year.

c Strawberries are in season in the warmer months, but not in the cooler winter months.

8 a i \$6000 **ii** \$4000

b 1

c



d i The sales trend for City Central for the 6 months is fairly constant.

ii Sales for Southbank peaked in August before taking a downturn.

e About \$5000

9 a i 5.8 km **ii** 1.7 km

b i Blue Crest slowly gets closer to the machine.

ii Green Tail starts near the machine and gets further from it.

c 8:30 p.m.

10 a Increases continually, rising more rapidly as the years progress.

b Compound interest – exponential growth

11 a Graphs may vary, but it should decrease from room temperature to the temperature of the fridge.

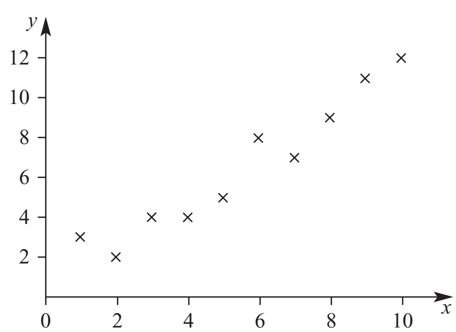
b No. Drink cannot cool to a temperature *lower* than that of the internal environment of the fridge.

Exercise 5G

1 a Likely **b** Likely **c** Unlikely

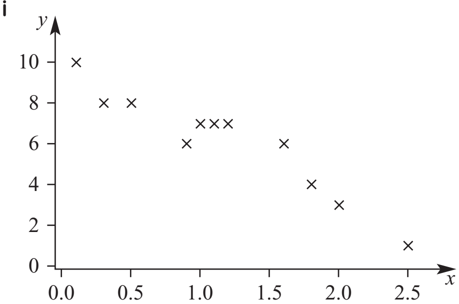
d Unlikely **e** Likely

2 a i



ii y generally increases as x increases.

b i



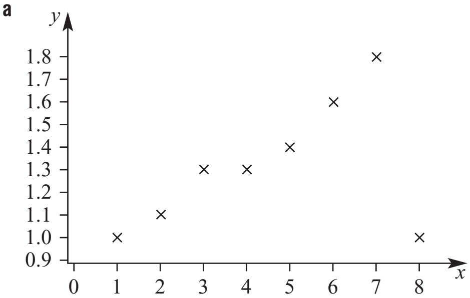
ii y generally decreases as x increases.

3 a weak negative

b strong positive

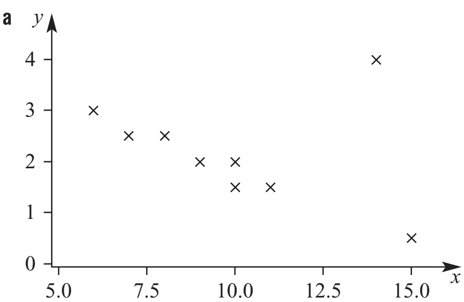
c strong negative

4 a



b Positive **c** Strong **d** (8, 1.0)

5 a

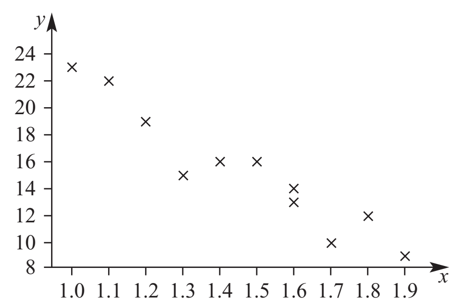


b Negative

c Strong

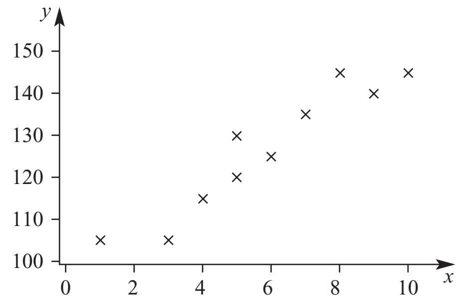
d (14, 4)

6 a Negative

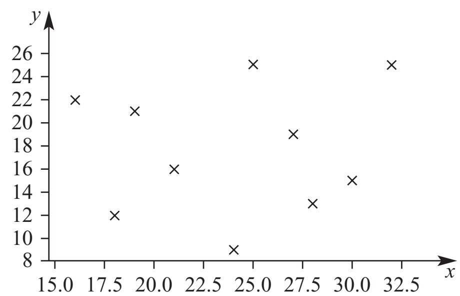


ISBN: 9781107687028

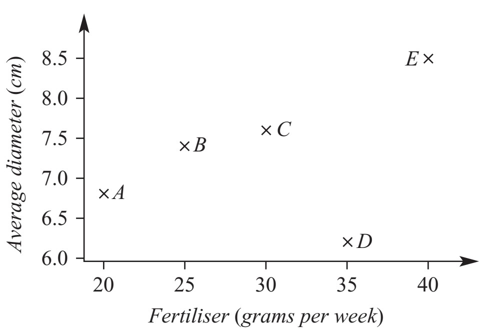
b Positive



c None



7 a



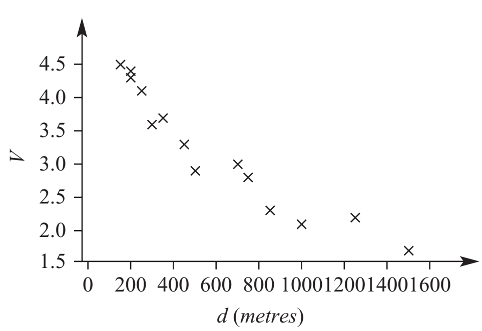
b D

c Yes, although small sample size does lead to doubt.

8 a Yes **b** Decrease

c i Yes **ii** Car H

9 a

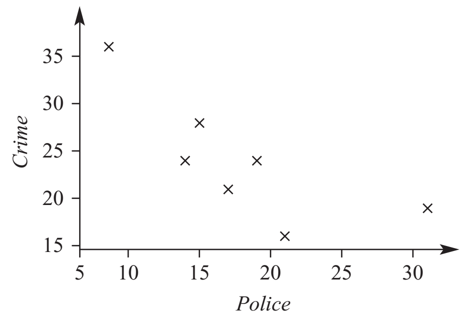


b Negative

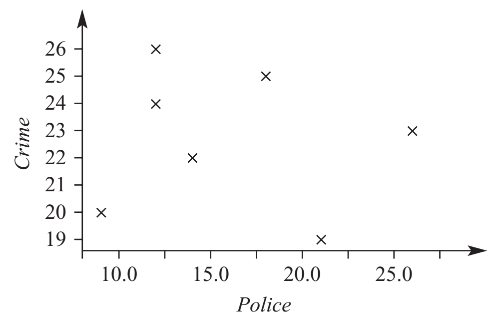
c As d increases, V decreases.

10 Each axis needs a better scale. All data is between 6 and 8 hours sleep and shows only a minimum change in exam marks. Also, there are only 4 observations presented on the plot, which is not enough to form strong conclusions.

11 a i Weak negative correlation



ii No correlation



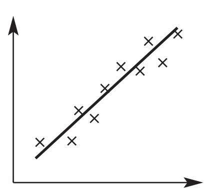
b Survey 1, as this shows an increase in the number of police has seen a decrease in the incidence of crime.

Exercise 5H

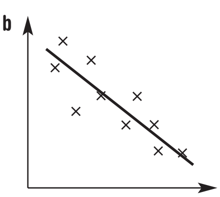
1 a When the data appears to fit on or near a straight line, it shows a definite linear trend.

b Balance points evenly either side of the line; ignore outliers when taking distance from the line into account.

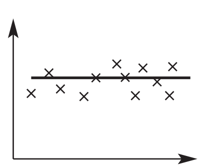
2 a



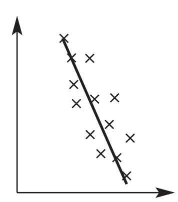
b



c



d



3 a ≈ 5.3 **b** ≈ 7

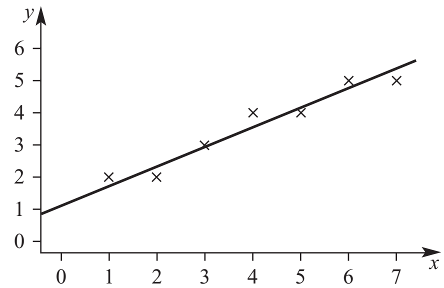
ISBN: 9781107687028

Cambridge University Press

Exercise 5H cont.

4 **a** $x \approx 1.5$ **b** $x \approx 11.7$

5 **a**

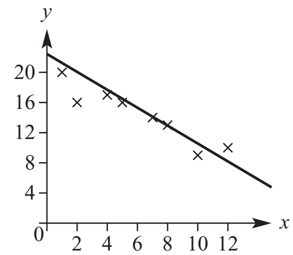


b Positive correlation **c** As above

d All answers are approximate.

i 3.2 **ii** 0.9 **iii** 1.8 **iv** 7.4

6 **a**



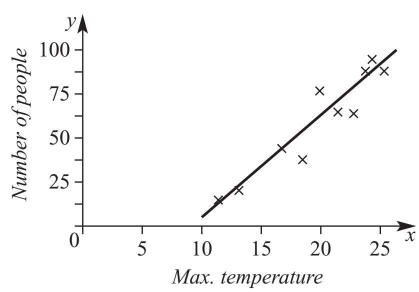
b negative **c** as above

d **i** 13.5 **ii** 23 **iii** 9 **iv** 7

7 **a** ≈ 4.5 **b** ≈ 6 **c** ≈ 0.5 **d** ≈ 50

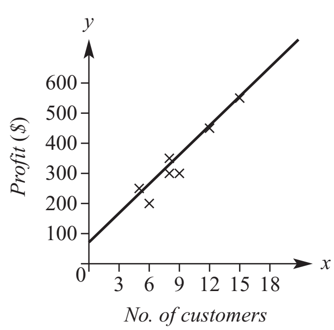
8 **a** increases

b



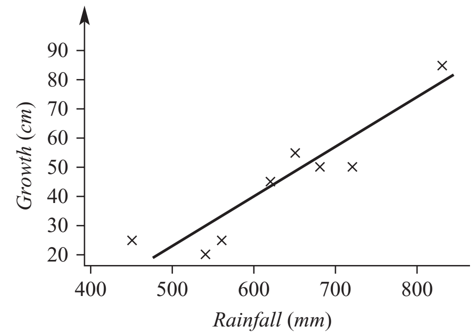
c **i** 65 **ii** 15 degrees

9 **a, b**



c \$600 **d** 2

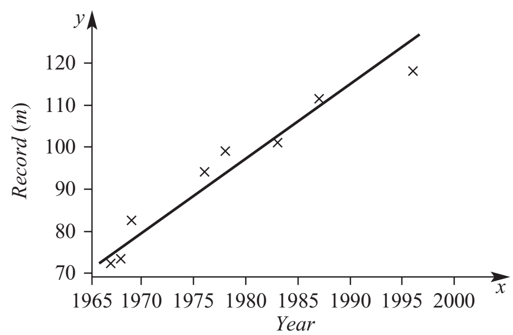
10 **a, b**



c **i** ≈25 cm **ii** ≈ 85 cm

d **i** ≈520 mm **ii** ≈720 mm

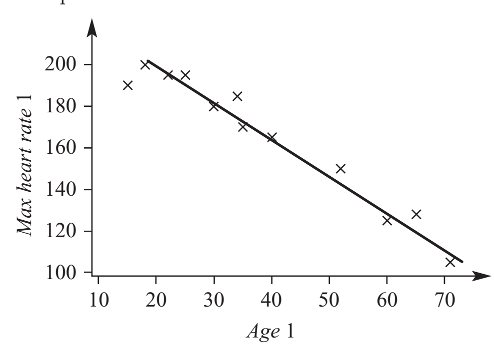
11 **a, b**



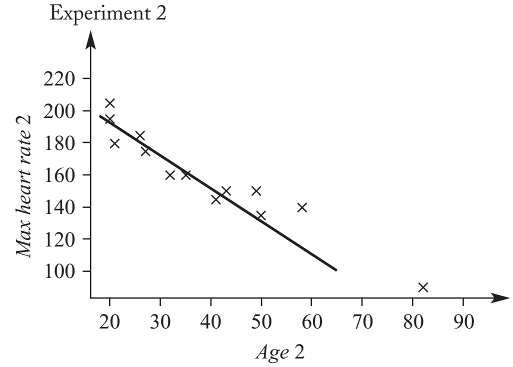
c **i** 130 m **ii** 170 m

d No, records are not likely to continue to increase at this rate.

12 **a** Experiment 1



Experiment 2



ISBN: 9781107687028

b **i** ≈140 **ii** ≈125
c **i** ≈25 **ii** ≈22
d Experiment 2 **e** Research

Puzzles and games

1 66 kg **2** 88% **3** 8
4 **a** Larger by 3 **b** Larger by 3 **c** No change
5 1.1
6 3, 3, 9, 11
7 19
8 **a** Mean ↑ by 10
Median ↑ by 10
Mode ↑ by 10
b Mean is × by 10
Median is × by 10
Mode is × by 10

Multiple-choice questions

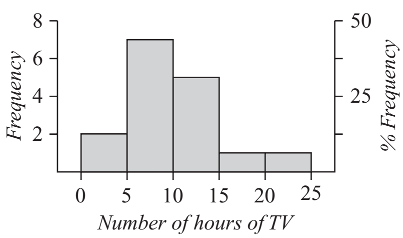
1 D	**2** C	**3** B	**4** A
5 C	**6** A	**7** E	**8** C
9 D	**10** B		

Short-answer questions

1 **a**

Class interval	Tally	Frequency	Percentage frequency
0–	||	2	12.5%
5–	~~||||~~ ||	7	43.75%
10–	~~||||~~	5	31.25%
15–	|	1	6.25%
20–24	|	1	6.25%
Total		**16**	**100%**

b



c It is skewed.

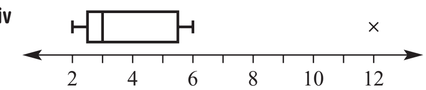
2 **a**

Stem	Leaf
1	5 9
2	0 4 8 9
3	2 4 7 8 8
4	2 9
5	0

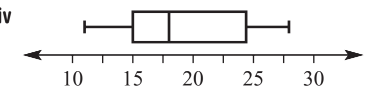
3|2 means 32

b The data is symmetrical about scores in the 30s.

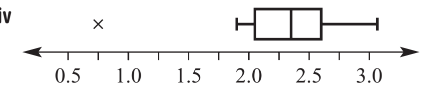
3 **a** **i** 5 **ii** 6 **iii** 5
b **i** 30.5 **ii** 57 **iii** 20
c **i** 1.6 **ii** 1.2 **iii** 1.5
4 **a** 43.2 years
b 38 years
c The mean is affected by the high ages 76 and 87
5 **a** $Q_1 = 8, Q_2 = 11, Q_3 = 15$
b $Q_1 = 8, Q_2 = 13, Q_3 = 15.5$
6 **a** **i** $Q_1 = 2.5, Q_3 = 5.5$ **ii** 3 **iii** 12
iv



b **i** $Q_1 = 15, Q_3 = 24$
ii 9 **iii** None
iv

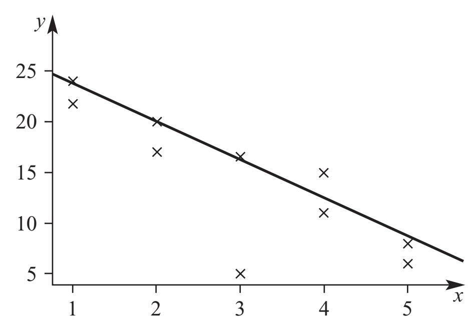


c **i** $Q_1 = 2.1, Q_3 = 2.6$
ii 0.5 **iii** 0.7
iv



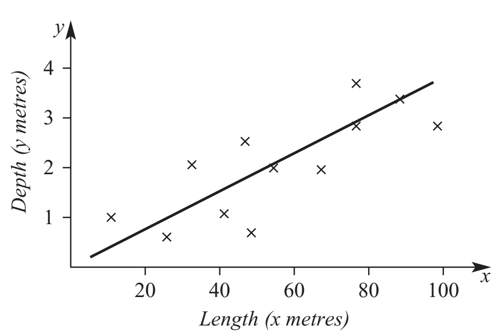
7 **a** False **b** True **c** True **d** True

8 **a** Negative **b** None **c** Positive
9 **a, e**



b Negative **c** Strong **d** (3, 5)

10 **a**



b 1.8 m

ISBN: 9781107687028

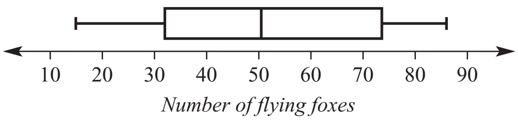
Cambridge University Press

Extended-response questions

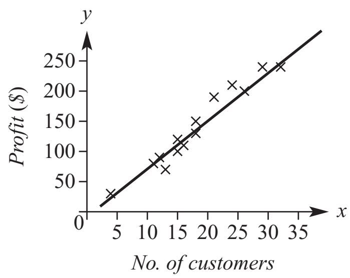
1 a 41

b No outliers

c



2 a



b i \$80 **ii** \$150 **iii** \$240

c i 7 **ii** 16 **iii** 27

Semester review 1

Measurement

Multiple-choice questions

1 C **2** E **3** B **4** A **5** D

Short-answer questions

1 a 43 cm **b** 320 cm^2 **c** 30 000 cm^3

2 a 8 cm **b** 44 m **c** 9 m

3 a i 37.70 cm **ii** 113.10 cm^2

b i 14.28 cm **ii** 12.28 cm^2

c i 11.14 m **ii** 7.14 m^2

4 a 10.5 m^2 **b** 112 cm^2 **c** 8 m^2

5 a i 45 cm^3 **ii** 78 m^2

b i 30 m^3 **ii** 72 m^2

c i 942.48 mm^3 **ii** 534.07 mm^2

Extended-response question

1 a 7.85 m^3 **b** 31.42 m^2 **c** \$1130

Consumer arithmetic

Multiple-choice questions

1 C **2** A **3** C **4** D **5** A

Short-answer questions

1 \$38.52

2 a \$45 500 **b** \$25

3 \$351.20

4 a \$38.64 **b** \$51.52 **c** \$978.88 **d** \$1094.80

5 \$539 **6** \$6330 **7** \$597

8 a \$37 180 **b** \$12 833.60 **c** \$22 829.56 **d** \$439.03

9 \$8837.34 **10** \$16.20 **11** \$102

Extended-response question

1 a \$711.55 **b** \$59.92

c i \$149.80 **ii** \$832 **iii** \$83 **iv** 11.08%

Algebra and indices

Multiple-choice questions

1 C **2** D **3** A **4** D **5** B

Short-answer questions

1 a $7xy + 4x$ **b** $-21ab$ **c** $\frac{a}{2}$

2 a i $-4x + 12$ **ii** $15x^2 + 6x$ **iii** $13x - 6$

b i $6(3 - b)$ **ii** $3x(x + 2)$ **iii** $-4y(2x + 3)$

3 a $x + 3$ **b** $\frac{3}{4}$ **c** $\frac{9x}{10}$ **d** $\frac{2x-3}{8}$

4 a $10x^6$ **b** $\frac{4x^2}{y^3}$ **c** $8m^{12}$ **d** 4

e $\frac{9a^2}{b^8}$ **f** $\frac{3b^2}{a^5}$ **g** $4t^5$ **h** $\frac{2y^2}{x^4}$

5 a i 473 000 **ii** 0.00521

b i 2.76×10^{-5} **ii** 8.71×10^6

Extended-response question

1 a $A = 3000(1.06)^n$

b i \$3370.80 **ii** \$4255.56

c 11.9 years

Probability

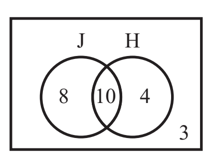
Multiple-choice questions

1 C **2** E **3** A **4** D **5** B

Short-answer questions

1 a $\frac{2}{5}$ **b** $\frac{11}{20}$ **c** $\frac{17}{20}$ **d** $\frac{9}{10}$

2 a



b

	H	H′	
J	10	8	18
J′	4	3	7
	14	11	25

ISBN: 9781107687028 Cambridge University Press

c 3

d **i** $\frac{4}{25}$ **ii** $\frac{22}{25}$ **iii** $\frac{2}{5}$ **iv** $\frac{5}{9}$

3 **a** 36

b **i** $\frac{1}{9}$ **ii** $\frac{1}{6}$ **iii** $\frac{7}{12}$ **iv** $\frac{1}{6}$

4 **a** $\frac{1}{15}$ **b** $\frac{8}{15}$ **c** $\frac{14}{15}$

5 **a** Yes, $\Pr(A) = \Pr(A|B) = \frac{1}{3}$

b No, $\Pr(A) = \frac{5}{8} \neq \Pr(A|B) = \frac{3}{5}$

Extended-response question

1 **a**

	T	A
T	(T, T)	(A, T)
A	(T, A)	(A, A)

b **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$ **c** $\frac{1}{4}$

Statistics

Multiple-choice questions

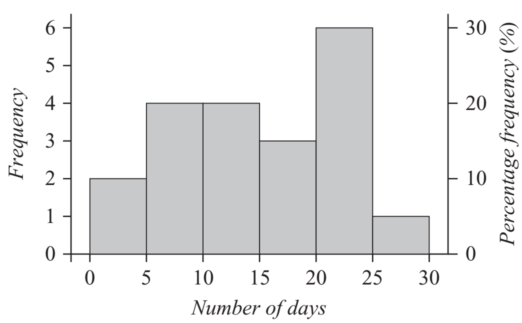
1 A **2** C **3** E **4** B **5** C

Short-answer questions

1 **a**

Class interval	Frequency	Percentage frequency
0–	2	10%
5–	4	20%
10–	4	20%
15–	3	15%
20–	6	30%
25–	1	5%
Total	**20**	**100%**

b



c **i** 14 **ii** 50%

iii 20–24 days, those that maybe catch public transport to work or school each week day

2 **a**

Stem	Leaf
0	4 7 8 9
1	2 5 5 7 8
2	4 4 6
3	2 6
4	1

3|6 means 36

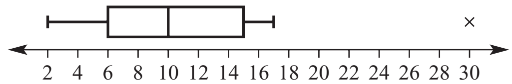
b Skewed

3 **a** 2 4 6 7 8 10 12 12 15 17 30

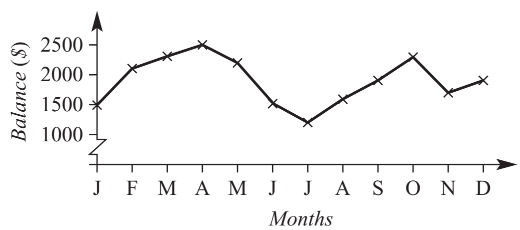
b **i** Min = 2, max = 30 **ii** 10

iii $Q_1 = 6, Q_3 = 15$ **iv** 9 **v** Yes, 30

c



4 **a**



b Balance fluctuated throughout the year but ended up with more money after 12 months.

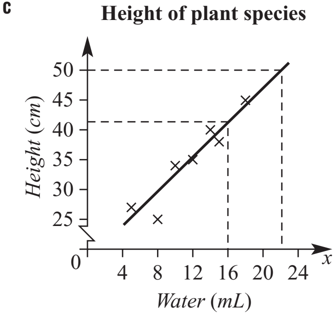
c May and June

d Increase of $500

Extended-response question

1 **a, c**

Height of plant species



b Positive

d **i** 41 cm **ii** 22 mL

Chapter 6

Pre-test

1 **a** (3, 5) **b** (4, -2) **c** (-4, -4)

d (-3, 1) **e** (2, -2) **f** (2, 0)

2 **a** G **b** D **c** B

d S **e** N **f** Q

ISBN: 9781107687028

Pre-test cont.

3 a Square **b** Isosceles triangle **c** Hexagon

4 a 11 **b** 19 **c** 10 **d** 3.5
e 0 **f** -1 **g** 3.5 **h** -9

5 a 120 minutes **b** 200 km **c** 100 km/h

6 a 5 **b** 13 **c** 10
d 41 **e** 3.61 **f** 8.54

7 a 3, 4, 5, 6 **b** -2, -1, 0, 1
c 0, 2, 4 **d** 6, 5, 4

8 a 6 **b** 9 **c** 3 **d** 9 **e** 5
f 9 **g** 7 **h** 8 **i** 2 **j** 4
k 2 **l** 10

Exercise 6A

1 a 240 km **b** 3 hours **c** 360 km

2 a 360 km **b** 4 hours **c** 540 km

3 a 300 deer **b** 100 deer **c** 200 deer

4 a 1800 people **b** 450 people **c** 1350 people

5 a i \$10 000 **ii** \$40 000
b increased **c** \$30 000

6 a i 50 cm **ii** 45 kg
b i 10 **ii** 3

7 a 80 cm **b** 40 cm **c** approx. $2\frac{1}{4}$ months

8 a 400 m **b** approx. 250 m
c approx. $3\frac{1}{2}$ days

9 a \$10 000 **b** \$0 **c** 12 years

10 a 200 g **b** 200 g
c $2\frac{1}{4}$ hours (2 hours 15 min)

11 a $\frac{1}{2}$ hour (30 min)
b i 1 p.m. **ii** 1:15 p.m.
iii approx. 1:08 p.m. **iv** 1:30 p.m.
c i -120 m **ii** approx. -80 m
d i 0 m **ii** -160 m **iii** -280 m
e i 12:45 p.m. **ii** 2 p.m. **iii** 2:45 p.m.

Exercise 6B

1 a *S* **b** *P* **c** *Q* **d** *R*
e *T* **f** *S* **g** *Q*

2 a 20 km **b** 2 hours **c** Approx. 17 km

3 a i 40 kg **ii** 50 kg **iii** 80 kg
b $\frac{1}{2}$ hour **c** 1st hour

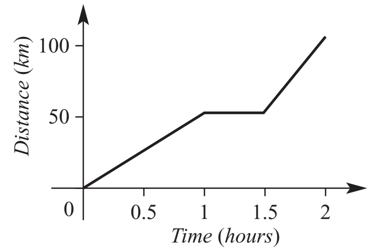
4 a 200 m **b** 80 s
c i Approx. 38 m **ii** 75 m **iii** Approx. 150 m

5 a i 3 m **ii** 2 m **iii** 2 m
b the lawn
c i 1.5 m **ii** 4 m **iii** 6.5 m

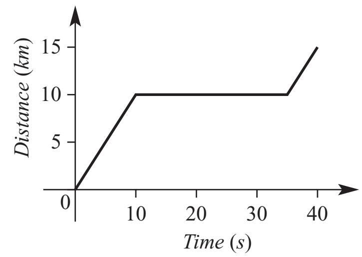
6 a i 10 s **ii** 17.5 s **iii** 20 s
b Phase 3
c i Approx. 40 m **ii** Approx. 85 m **iii** Approx. 160 m

7 a 10 km **b** 20 km **c** 27 min **d** 9 min

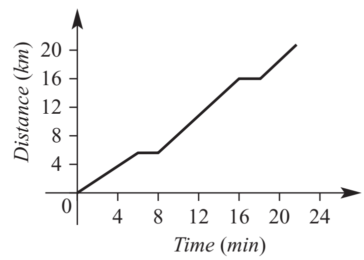
8



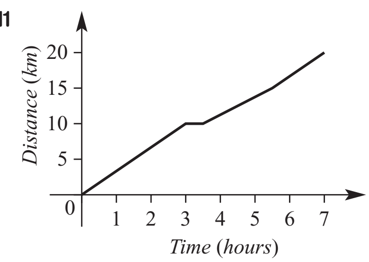
9



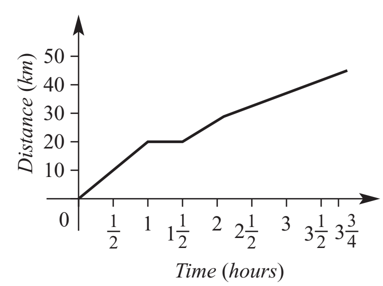
10 a 20 km **b** 22 min
c



11



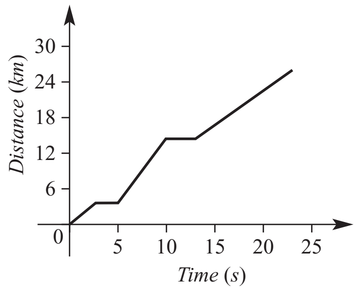
12 a i



ii $3\frac{1}{4}$ hours

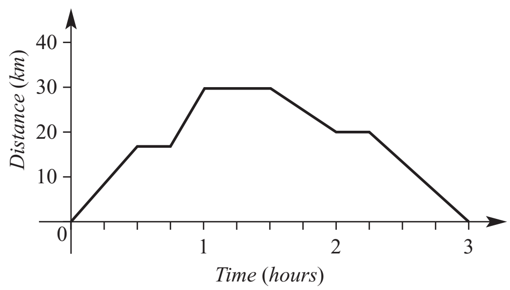
ISBN: 9781107687028

b i



ii 18 s

13 a



b 48 km/h **c** 30 km/h

Exercise 6C

1 a $A(2, 1)$ $B(-3, 3)$ $C(2, -3)$ $D(-4, 0)$ $E(4, 4)$ $F(0, -2)$ $G(3, 0)$ $H(-3, -2)$ $I(1, 4)$ $J(-2, -4)$ $K(-4, 5)$

b D, O, G

c O, F

d (0, 0)

2 a $6 + 4 = 10$, (-3, 10)

b $4 + 4 = 8$, (-2, 8)

c $2 + 4 = 6$, (-1, 6)

d $0 + 4 = 4$, (0, 4)

e $-2 + 4 = 2$, (1, 2)

f $-4 + 4 = 0$, (2, 0)

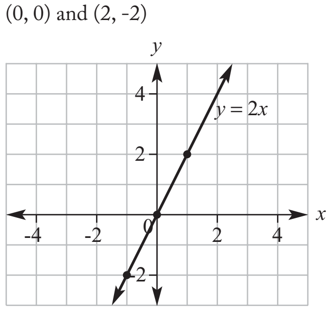
g $-6 + 4 = -2$, (3, -2)

3 (-2, 1) (-1, -1) (0, -3) (1, -5) (2, -7)

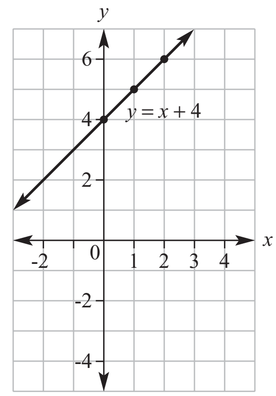
4 a (0, 1) and (2, -3) are not in line with the other points.

b (0, 0) and (2, -2)

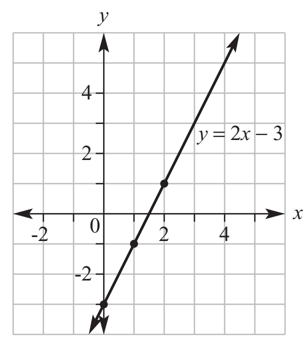
5 a



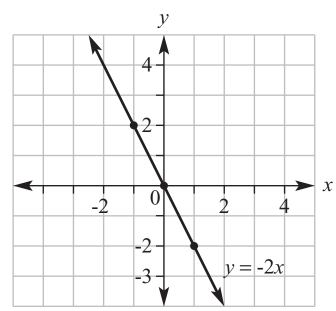
b



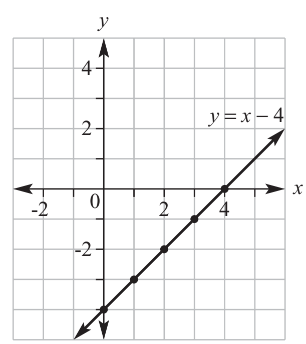
c



d



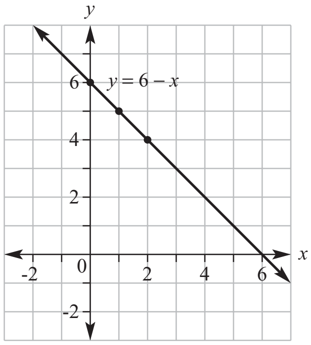
e



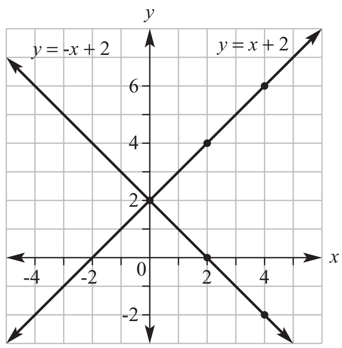
ISBN: 9781107687028 Cambridge University Press

Exercise 6C cont.

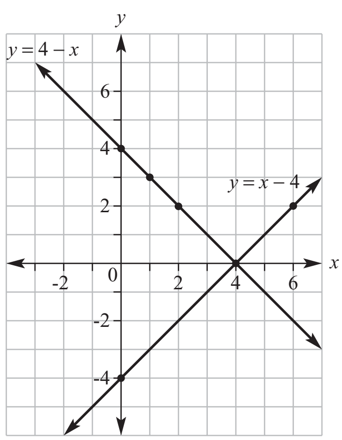
f



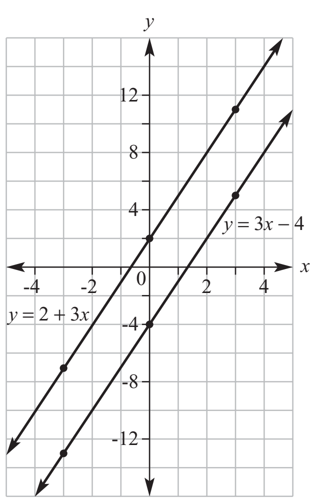
6 a



b

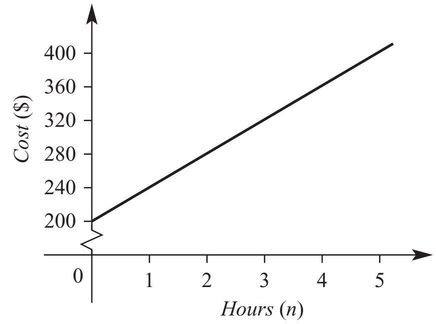


c



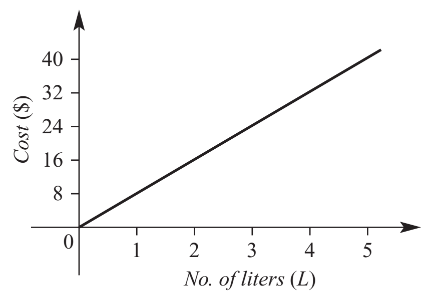
7 a (0, 0) **b** (1, 4) **c** (-1, 3)
d (0, 2) **e** (-1, -5)

8 a



b i $300 **ii** 4.5 hours

9 a

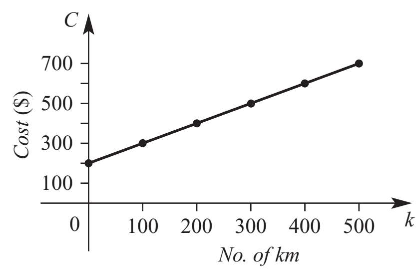


b i $28 **ii** 2.5 litres

10 a

No. of km (*k*)	100	200	300	400	500
Cost (*C*)	300	400	500	600	700

b

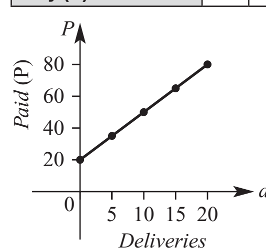


c i $450 **ii** 450 km

11 a

No. of deliveries (*d*)	0	5	10	15	20
Pay (*P*)	20	35	50	65	80

b



c i $56 **ii** 18

ISBN: 9781107687028

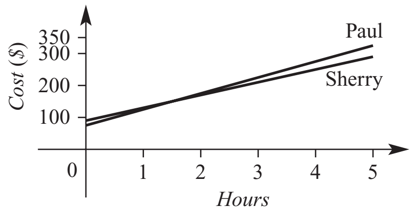
12 a Paul

No. of hours work	0	1	2	3	4	5
Cost (C)	75	125	175	225	275	325

Sherry

No. of hours work	0	1	2	3	4	5
Cost (C)	90	130	170	210	250	290

b



c **i** \$250 **ii** \$150 **iii** 0.5 hours
iv 4.25 hours **v** 1.5 hours

d Paul is cheaper only for 1.5 hours or less.

Exercise 6D

1 a Pythagoras' theorem: $c^2 = a^2 + b^2$
b 13 m **c** 6 m
d 2.5 m **e** middle, middle

2 a $\sqrt{20}$ **b** $\sqrt{18}$

3 a (3.5, 5) **b** (3, 4)
c (0, -1) **d** $\left(\frac{1}{2}, \frac{1}{2}\right)$

4 a 5 **b** $\sqrt{41}$ **c** 5 **d** $\sqrt{74}$

5 a $\sqrt{13}$ **b** $\sqrt{18}$ **c** $\sqrt{29}$
d $\sqrt{29}$ **e** $\sqrt{13}$ **f** $\sqrt{25} = 5$

6 a 5 **b** 10 **c** 11.2
d 5.7 **e** 5 **f** 3.6

7 a $\sqrt{2}$ **b** $\sqrt{13}$ **c** $\sqrt{34}$
d $\sqrt{89}$ **e** $\sqrt{26}$ **f** $\sqrt{10}$

8 a (2, 5) **b** (4, 8) **c** (3, 5)
d (2.5, 4.5) **e** (6, 2.5) **f** (2.5, 4)
g (-1, -2) **h** (-3, -4) **i** (-4, -3)
j (1, 1) **k** (-3, -4) **l** (0, 0)

9 (-2, -5)

10 a $D(1, 1)$, $A(1, 4)$, $B(6, 4)$, $C(6, 1)$
b (3.5, 2.5) **c** (3.5, 2.5)
d The diagonals of a rectangle bisect (cut in half) each other.

11 c **i** 5 **ii** 5 **d** isosceles
e $P = 16$ units, $A = 12$ units^2
f (4, 0)

12 a (3, 4) **b** $\sqrt{13}$
c $\sqrt{13}$; length of radius
d 22.7 units **e** 40.8 units^2

Exercise 6E

1 a zero **b** negative **c** positive **d** undefined

2 a + **b** – **c** + **d** –
e – **f** + **g** + **h** –
i –

3 a 1 **b** $\frac{1}{4}$ **c** $-\frac{3}{5}$

4 a $-\frac{3}{8}$ **b** $\frac{1}{15}$ **c** -3

5 a 2 **b** 5 **c** -3
d -2 **e** $\frac{4}{3}$ **f** 0
g 0 **h** undefined **i** undefined

6 $EF\ \frac{2}{3}$, $GH\ \frac{2}{3}$, DC 1, $AB\ \frac{3}{2}$

7 a 3 **b** 2 **c** $-\frac{1}{2}$
d -1 **e** 0 **f** undefined

8 a

Line segment	Rise	Run	Gradient
AB	1	2	$\frac{1}{2}$
AC	2	4	$\frac{1}{2}$
AD	3	6	$\frac{1}{2}$
BC	1	2	$\frac{1}{2}$
BD	2	4	$\frac{1}{2}$
CD	1	2	$\frac{1}{2}$

b They have the same gradient.

9 a 2 **b** $\frac{1}{2}$ **c** -1
d -2 **e** 1 **f** $-\frac{2}{5}$

10 a 1 **b** 1 **c** $-\frac{3}{5}$ **d** 0
e 11 **f** $\frac{1}{3}$

11 gradient = 0.344

12 a A, D **b** C, E, G **c** D
d B, F, H **e** G

ISBN: 9781107687028

Cambridge University Press

Exercise 6F

1 **a** gradient **b** two **c** units
d km/h **e** L/min

2 **a** 90 km/h **b** 15 L/min

3 **a i** 60 km **ii** 60 km/h
b i 60 km/h **ii** 0 km/h
iii 90 km/h

4 **a i** 15 km **ii** 15 km/h
b i 0 km/h **ii** 30 km/h

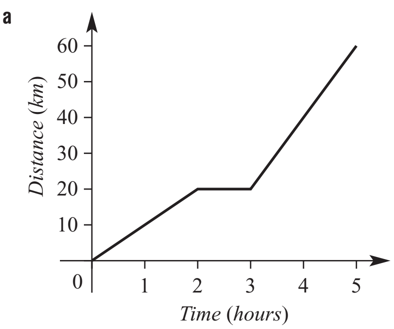
5 **a i** 2 L **ii** 0.5 L
b i 0.2 L/s **ii** 0.05 L/s **iii** 0.05 L/s

6 **a i** 1.5 L **ii** 0.5 L
b i 0.15 L/s **ii** 0.05 L/s **iii** 0.15 L/s

7 **a** 3 km **b** 4 min
c i 0.5 km/min **ii** 0.75 km/min
iii 0.5 km/min **iv** 0.25 km/min

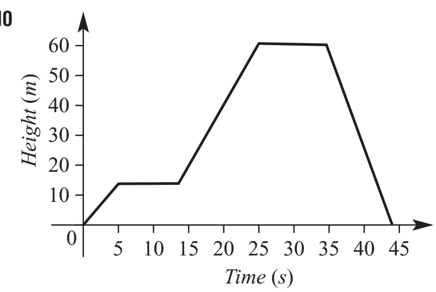
8 **a** iii **b** i **c** ii steepest

9 **a**



b 60 km

10



Time is 44 seconds.

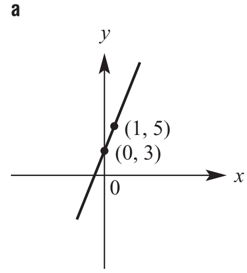
11 **a** 3
b i 6 km **ii** 14 km
c B, D, G
d E, H
e i 6 km/h **ii** 14 km/h **iii** 16 km/h
iv 6.4 km/h **v** 16 km/h
f E and H, same gradient
g $5\frac{1}{4}$ hours
h 40 km
i 10 km/h

Exercise 6G

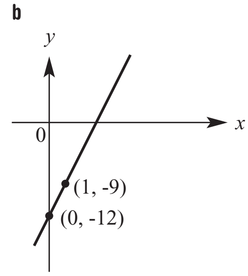
1 **a** gradient–intercept **b** gradient
c coefficient **d** y-intercept
e $(0, c)$

2 **a i** 2 **ii** (0, 4)
b i 6 **ii** (0, -7)
c i $-\frac{2}{3}$ **ii** (0, 7)
d i -7 **ii** (0, -3)
e i $\frac{3}{5}$ **ii** (0, -8)
f i 9 **ii** (0, -5)

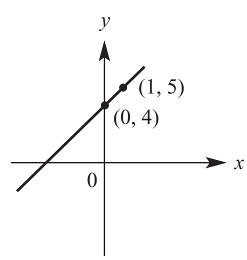
3 **a**



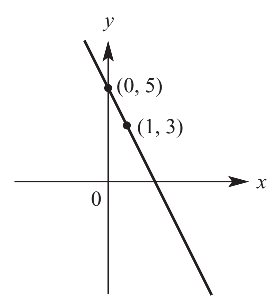
b



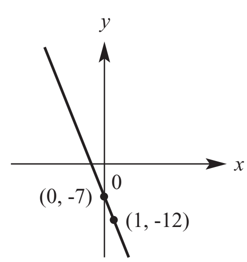
c



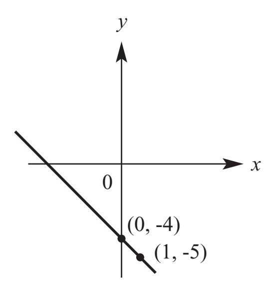
d



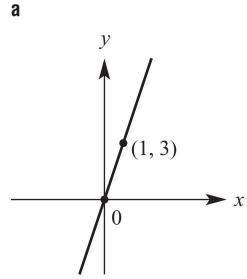
e



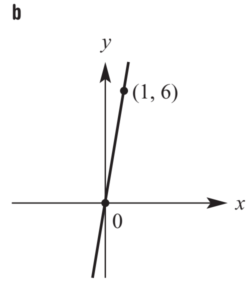
f



4 **a**

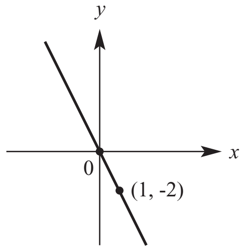


b

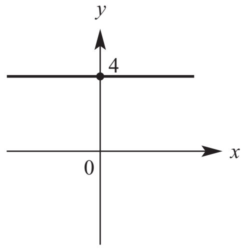


ISBN: 9781107687028

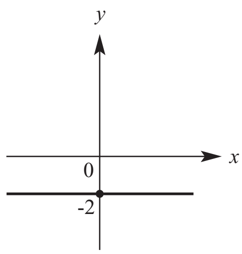
c



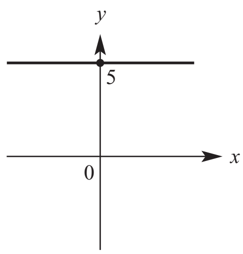
d



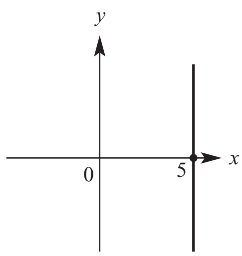
e



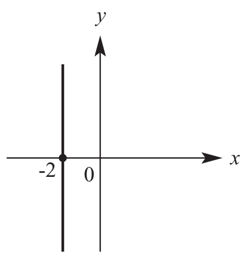
f



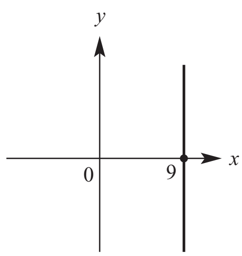
g



h



i



5 a $y = 4x + 2$ **b** $y = 3x - 2$ **c** $y = 5x$
d $y = -3x + 5$ **e** $y = -4x - 3$ **f** $y = -2x$

6 a 1, (0, 0) **b** $-\frac{3}{2}$, (0, 3) **c** $\frac{1}{2}$, (0, 4.5)
d 0, (0, 4) **e** 0, (0, 7) **f** undefined, none

7 a iv **b** ii **c** v **d** i **e** vi
f vii **g** iii **h** viii **i** ix

8 a $c = 5$ in each equation. E.g. $y = 2x + 5$, $y = -3x + 5$ etc.
b $c = -2$ in each equation. E.g. $y = 7x - 2$, $y = x - 2$ etc.
c $c = 0$ in each equation. E.g. $y = 2x$, $y = -5x$ etc.

9 a $m = 3$ in each equation. E.g. $y = 3x - 1$, $y = 3x$, $y = 3x + 4$ etc.
b $m = -1$ in each equation. E.g. $y = -x$, $y = -x + 7$, $y = -x - 3$ etc.
c $m = 0$ in each equation. E.g. $y = 4$, $y = -2$ etc.
d m is undefined in each equation. E.g. $x = -7$ etc.

10 a ii and iii **b** i and iv

11 a No **b** Yes **c** No **d** Yes **e** Yes

12 a $y = x + 4$ **b** $y = x - 1$
c $y = \frac{x}{2}$ **d** $y = 2x + 1$

14 a They all have the same gradient.
b They all have a y-intercept at (0, -1).

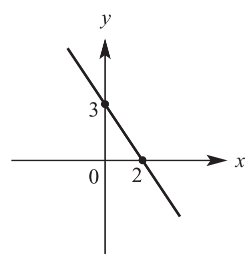
Exercise 6H

1 a The x-intercept is where $y = 0$.
b The y-intercept is where $x = 0$.

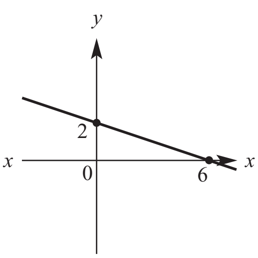
2 a i 3 **ii** -18 **iii** -30 **iv** 6 **v** -7
b i $x = 4$ **ii** $x = \frac{1}{2}$ **iii** $x = \frac{2}{5}$ **iv** $x = -6\frac{1}{2}$

3 a (4, 0) **b** $(-3\frac{1}{3}, 0)$ **c** (-6, 0)
d (18, 0) **e** (12, 0) **f** (-6, 0)

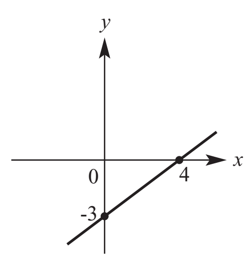
4 a



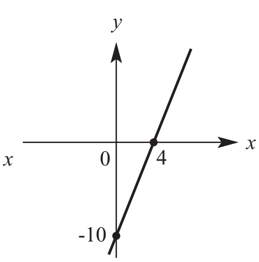
b



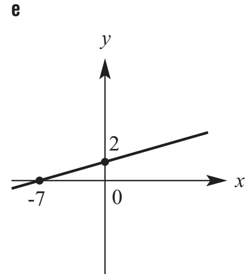
c



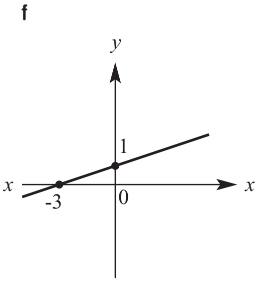
d



e



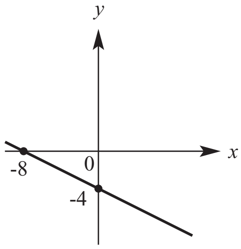
f



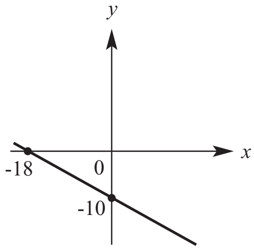
ISBN: 9781107687028

Exercise 6H cont.

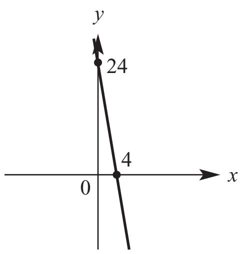
g



h

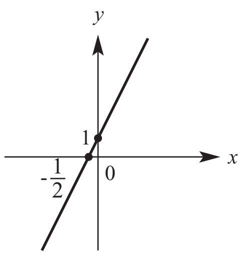


i

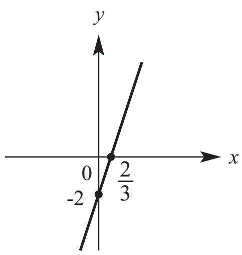


5 a ii **b** iii **c** i

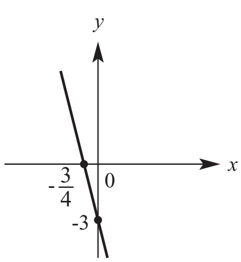
6 a



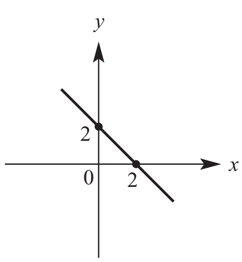
b



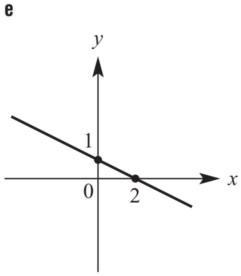
c



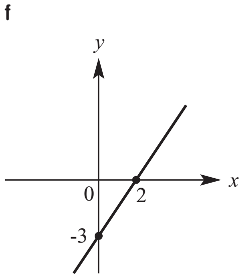
d



e



f



7 a iii **b** ii **c** i

8 a vi **b** ii **c** i
d v **e** iv **f** iii

9 a -2 **b** $\frac{1}{5}$ **c** 2 **d** $\frac{1}{2}$

10 a i (2, 0), (0, -4) **ii** $A = 4$ units^2
b i (-2, 0), (0, 2) **ii** $A = 2$ units^2
c i (-1.5, 0), (0, -3) **ii** $A = 2.25$ units^2
d i (-4, 0), (0, 2) **ii** $A = 4$ units^2

11 a 90 m
b $7\frac{1}{2}$ seconds

12 Many answers; e.g. $2x + y = 4$, $a = 2$, $b = 1$, $d = 4$

13 a (2, 0), (0, -4) **b** (-5, 0), (0, -10)
c (1, 0), (0, 1) **d** (2, 0), (0, 4)
e (-4, 0), (0, 6) **f** $(-1, 0), (0, \frac{2}{3})$

Exercise 6I

1 D

2 C

3 a i $m = 2$ **ii** (0, 1) **iii** $y = 2x + 1$
b i $m = -3$ **ii** (0, 2) **iii** $y = -3x + 2$
c i $m = 4$ **ii** (0, -3) **iii** $y = 4x - 3$
d i $m = -4$ **ii** (0, 0) **iii** $y = -4x$
e i $m = -1$ **ii** (0, 2) **iii** $y = -x + 2$
f i $m = 2$ **ii** (0, 0) **iii** $y = 2x$

4 a i 3 **ii** (0, 1) **iii** $y = 3x + 1$
b i -1 **ii** (0, 6) **iii** $y = -x + 6$
c i 5 **ii** (0, 21) **iii** $y = 5x + 21$
d i 3 **ii** (0, -16) **iii** $y = 3x - 16$
e i -3 **ii** (0, 2) **iii** $y = -3x + 2$
f i $\frac{1}{2}$ **ii** $\left(0, \frac{3}{2}\right)$ **iii** $y = \frac{1}{2}x + \frac{3}{2}$

5 a $y = 4$ **b** $y = -1$ **c** $x = 3$
d $x = -2$ **e** $y = 2$ **f** $x = 7$

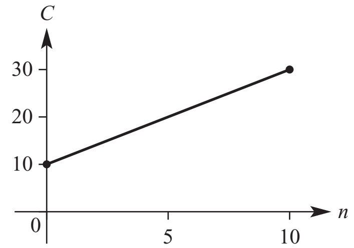
6 a $y = 2x$ **b** $y = 3x$ **c** $y = \frac{2}{3}x$
d $y = -3x$ **e** $y = -x$ **f** $y = -\frac{2}{5}x$

7 a i 7 **ii** $y = 7x$
b i $-\frac{3}{2}$ **ii** $y = -\frac{3}{2}x$
c i 1 **ii** $y = x + 2$
d i $-\frac{3}{2}$ **ii** $y = -\frac{3}{2}x$
e i 0 **ii** $y = 2$
f i undefined **ii** $x = 3$

ISBN: 9781107687028 Cambridge University Press

8 **a** $P = 2n + 10$

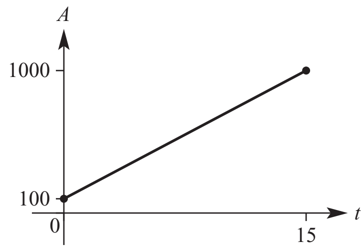
b



c **i** \$28 **ii** 23.5 kg

9 **a** $A = 60t + 100$

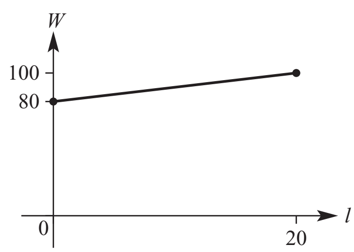
b



c **i** \$820 **ii** 10 hours

10 **a** $W = l + 80$

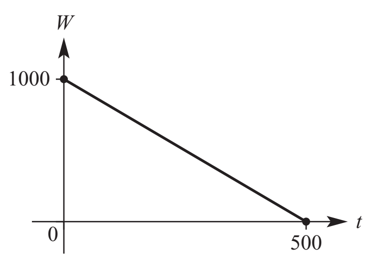
b



c **i** 87 kg **ii** 29 litres

11 **a** -2, (0, 1000)

b



c 1000 litres

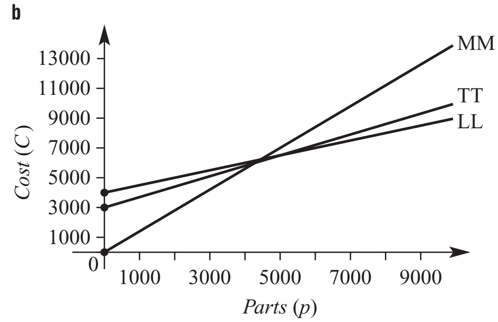
d **i** 360 litres **ii** 952 litres **iii** 664 litres

e **i** 350 hours **ii** 407.5 hours

12 **a**

p	0	1000	2000	3000	4000	5000	6000	7000	8000	9000
Cost, MM	0	1400	2800	4200	5600	7000	8400	9800	11 200	12 600
Cost, TT	3000	3700	4400	5100	5800	6500	7200	7900	8 600	9 300
Cost, LL	4000	4500	5000	5500	6000	6500	7000	7500	8 000	8 500

b



c **i** \$2100 **ii** \$1400 **iii** \$7250 **iv** \$8750

d Mandy's is best for parts less than or equal to 4285. Terry's is best for between 4286 and 5000 parts and equal to Lenny's at 5000. Lenny's is best for parts >5000.

Puzzles and games

1 PLOTTING LINES!

3 Both 13 km apart

4 **a** 160 ice creams for zero profit

b 493 ice creams sold

Multiple-choice questions

1 A	**2** E	**3** D	**4** A	**5** B
6 E	**7** C	**8** A	**9** B	**10** D
11 D	**12** E			

Short-answer questions

1 **a** 40 km **b** 2 hours **c** 60 km

2 **a** **i** \$6000 **ii** \$8000 **iii** \$9000

b **i** \$2000 **ii** \$4000 **iii** \$5000

c 10 years

3 **a** 15 km

b 3 hours

c **i** 6 km **ii** 6 km

iii 9 km **iv** 15 km

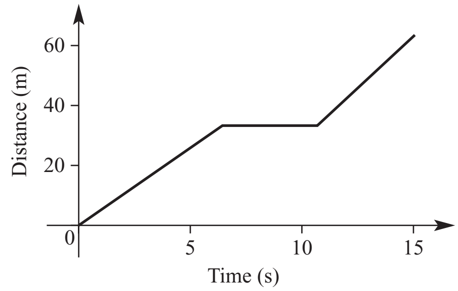
d $3\frac{1}{2}$ hours

ISBN: 9781107687028

Cambridge University Press

Short-answer questions cont.

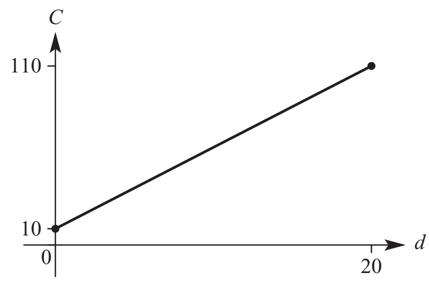
4



5 **a**

d	0	5	10	15	20
P	10	35	60	85	110

b



c **i** \$70 **ii** 17

6 **a** 1 **b** -1 **c** -2 **d** $\frac{1}{3}$

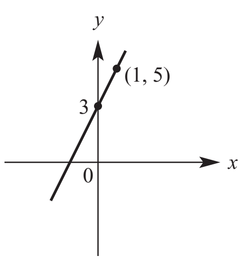
7 **a** (4, 2) **b** (2, 2.5) **c** (1.5, -1)

8 **a** $AB = 5$ **b** $PQ = \sqrt{50}$

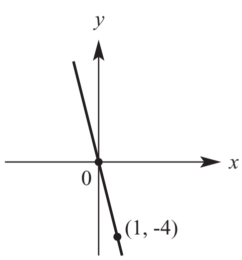
9 **a** gradient = 3, y-intercept = (0, 4)

b gradient = -2, y-intercept = (0, 0)

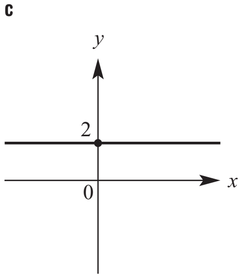
10 **a**



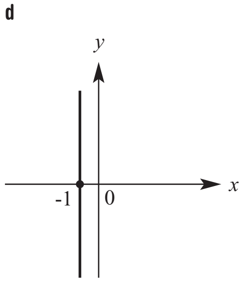
b



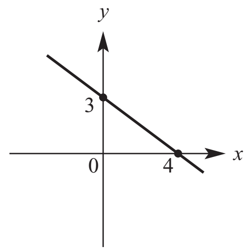
c



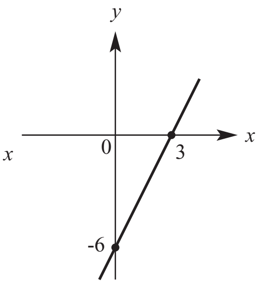
d



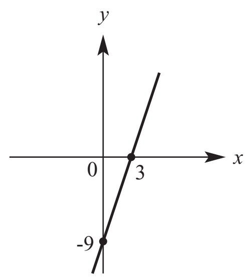
11 **a**



b



c



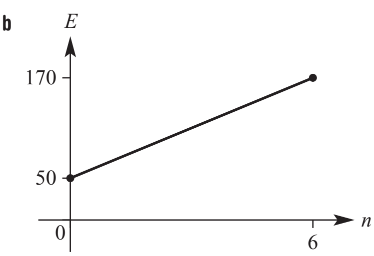
12 **a** **i** 2 **ii** (0, 0) **iii** $y = 2x$

b **i** -4 **ii** (0, 16) **iii** $y = -4x + 16$

13 **a** i **b** iii **c** vi **d** ii **e** iv **f** v

14 **a** $E = 20n + 50$

b



c **i** \$130 **ii** 5.5 bins

Extended-response questions

1 **a** **i** 30 km **ii** 15 km/hour

b **i** 20 km/hour **ii** 0 km/hour

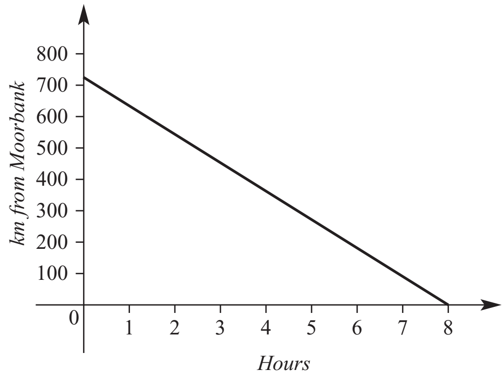
iii 20 km/hour

2 **a**

Time in hours (t)	0	2	4	6	8
Km from Moorbank	720	540	360	180	0

ISBN: 9781107687028

b

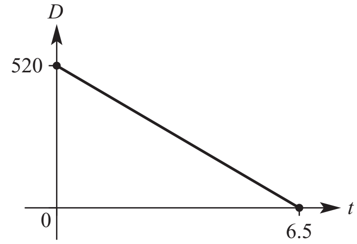


c 9:00 a.m. **d** 7 hours **e** 8 hours **f** 4 p.m.

3 a i 440 km **ii** 280 km

b i $D = -80t + 520$ or $D = 520 - 80t$

ii



c i 160 km **ii** 2.25 hours

Chapter 7

Pre-test

1 a Acute **b** 90° **c** 180° **d** straight
e reflex **f** 360° **g** 90° **h** supplementary

2 a Scalene **b** Isosceles **c** Right-angled
d Obtuse-angled **e** Equilateral **f** Acute-angled

3 a $a = 110$ **b** $b = 140$ **c** $c = 210$
d $d = 35$ **e** $e = 60$ **f** $f = 40$

4 a $a = 110, b = 70$ **b** $a = 105, b = 75$
c $a = 40, b = 140$

5 a Square
b Parallelogram incl. square, rectangle and rhombus
c Square, rectangle **d** Square, rhombus
e Trapezium **f** Kite

6 a $S = 360°, a = 130$ **b** $S = 540°, b = 120$
c $S = 720°, c = 120$

7 a SSS, SAS, AAS, RHS **b** SSS, SAS, AAA, RHS

Exercise 7A

1 a 180° **b** equal
c i equal **ii** equal **iii** supplementary

2 a alternate **b** vertically opposite
c cointerior **d** corresponding

3 $a = 20$ supplementary, $b = 20$ alternate, $c = 160$ corresponding, $d = 160$ vertically opposite

4 $a = 100$ supplementary, $b = 100$ alternate, $c = 80$ corresponding, $d = 80$ vertically opposite

5 a $x = 110, y = 110$ **b** $x = 40, y = 140$
c $x = 75, y = 105$ **d** $x = 120, y = 120$
e $x = 110, y = 70$ **f** $x = 105, y = 75$

6 a Yes, corresponding angles are equal.
b No, alternate angles are not equal.
c No, cointerior angles are not supplementary.
d Yes, cointerior angles are supplementary.
e No, corresponding angles are not equal.
f Yes, alternate angles are equal.

7 a 60 **b** 20 **c** 100
d 115 **e** 50 **f** 30

8 a $a = 90, b = 90, c = 90$
b $a = 90, b = 90, c = 90$
c $a = 135, b = 45, c = 135$
d $a = 50, b = 130, c = 50$
e $a = 90, b = 130$
f $a = 110, b = 120$

9 a $(a, e), (d, f), (b, h), (c, g)$ **b** $(d, h), (c, e)$
c $(c, h), (d, e)$ **d** $(a, c), (b, d), (e, g), (f, h)$

10 a 90 **b** 75 **c** 10 **d** 30 **e** 36 **f** 30

11 a 12 **b** 14 **c** 10

Exercise 7B

1 a 30 **b** 16 **c** 33 **d** 60 **e** 77 **f** 98

2 C

3 a $c = 120$ **b** $x = 60$ **c** $x = 25 + 35$

4 a 70 **b** 10 **c** 25 **d** 58 **e** 50 **f** 29

5 a 65 **b** 80 **c** 40 **d** 20 **e** 112 **f** 32

6 a 145 **b** 144 **c** 45 **d** 60 **e** 60 **f** 47

7 a Yes **b** Yes **c** No **d** Yes **e** Yes
f Yes **g** No **h** Yes **i** Yes

8 a 65° **b** 115°

9 a $a = 70$ **b** $a = 120$
c $a = 70, b = 70$ **d** $a = 35, b = 105$
e $a = 115, b = 115, c = 45, d = 20$
f $a = 40, b = 100, c = 40, d = 40$

10 a $a°$, alternate **b** $b°$, alternate **c** Sum to 180°

11 a i 50° **ii** 130°
b i 20° **ii** 160°
c i 0° **ii** 180°

Exercise 7C

1 a Parallelograms incl. squares, rectangles and rhombuses

2 a square, rectangle, rhombus, parallelogram
b rectangle, square, parallelogram, rhombus, kite

ISBN: 9781107687028 Cambridge University Press

Exercise 7C cont.

c rhombus, square, parallelogram, rectangle
d trapezium **e** kite
f square, rectangle **g** square, rectangle
h square, rhombus, kite

3 a $a = 144$ **b** $b = 79$ **c** $c = 54$

4 a $x = 20$ **b** $x = 110$
c $x = 240$ **d** $x = 125$
e $x = 30$ **f** $x = 55$

5 a $x = 60, y = 120, z = 60$
b $x = 110, y = 110, z = 70$
c $x = 30, y = 150, z = 30$ **d** $x = 45$
e $x = 100$ **f** $x = 25$
g $x = y = z = 90$ **h** $x = 100, y = 140$
i $x = 75, y = 20$

6 a 115 **b** 60 **c** 30
d 50 **e** 90 **f** 140

7 a It has two equal side lengths.
b i 120 **ii** 40
c It has two equal side lengths and two pairs of equal angles and one pair of parallel sides.

8 b 65°

9 a 125 **b** 118 **c** 110

10 a $a = 30, b = 120, c = 60, d = 60, e = 30$
b $a = 80, b = 100, c = 80, d = 50$
c $a = 40, b = 20, c = 50, d = 110$
d $b = 50, c = 70$
e $a = 60, b = 20$
f $a = 10, b = 80$

Exercise 7D

1 a 4 **b** 8 **c** 10 **d** 7
e 9 **f** 6 **g** 5 **h** 10

2 a 540° **b** 720° **c** 900°
d 1080° **e** 1260° **f** 1440°

3 All sides and all angles equal

4 a 720°, 110 **b** 540°, 130
c 540°, 30 **d** 900°, 105
e 720°, 30 **f** 360°, 30

5 a 24 cm **b** 720° **c** 120°

6 a 28 cm **b** 1080° **c** 135°

7 a 120° **b** 240°

8 a 1620° **b** 3240°

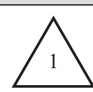
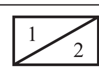
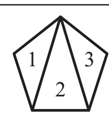
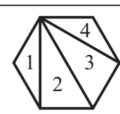
9 a 144° **b** 165.6°

10 a 120° **b** 240°

11 a 72° **b** 252°

12 a $x = 108, y = 72$ **b** $x = 60, y = 60$
c $x = 45, y = 225$

13 a *See table at bottom of page*
b i $S = 180° \times (n - 2)$ **ii** $A = \dfrac{180° \times (n - 2)}{n}$

Polygon	No. of sides	Diagram	No. of triangles	Total angle sum (S)	Single internal angle (A)
Triangle	3		1	180°	60°
Quadrilateral	4		2	360°	90°
Pentagon	5		3	540°	108°
Hexagon	6		4	720°	120°
…					
n-gon	n		$n - 2$	$180° \times (n - 2)$	$\dfrac{180°(n-2)}{n}$

ISBN: 9781107687028

Cambridge University Press

Exercise 7E

1 **a** False **b** False **c** True **d** True

2 SSS, SAS, AAS, RHS

3 **a** E **b** AC **c** $\angle EDF$

4 **a** $\Delta ABC \equiv \Delta DEF$ (SSS)
b $\Delta ABC \equiv \Delta DEF$ (SAS)
c $\Delta XYZ \equiv \Delta STU$ (RHS)
d $\Delta XYZ \equiv \Delta STU$ (SSS)
e $\Delta ABC \equiv \Delta DEF$ (AAS)
f $\Delta MNO \equiv \Delta PQR$ (AAS)

5 **a** $AB = DE$ (S)
$BC = EF$ (S)
$AC = DF$ (S)
$\therefore \Delta ABC \equiv \Delta DEF$ (SSS)
b $AC = DF$ (S)
$AB = DE$ (S)
$BC = EF$ (S)
$\therefore \Delta ABC \equiv \Delta DEF$ (SSS)
c $AB = DE$ (S)
$\angle BAC = \angle EDF$ (A)
$AC = DF$ (S)
$\therefore \Delta ABC \equiv \Delta DEF$ (SAS)
d $AB = DE$ (S)
$\angle BAC = \angle EDF$ (A)
$AC = DF$ (S)
$\therefore \Delta ABC \equiv \Delta DEF$ (SAS)
e $\angle CAB = \angle FDE$ (A)
$\angle CBA = \angle FED$ (A)
$BC = EF$ (S)
$\therefore \Delta ABC \equiv \Delta DEF$ (AAS)
f $\angle ABC = \angle DEF$ (A)
$\angle BAC = \angle EDF$ (A)
$AB = DE$ (S)
$\therefore \Delta ABC \equiv \Delta DEF$ (AAS)
g $\angle BAC = \angle EDF = 90°$ (R)
$BC = EF$ (H)
$AB = DE$ (S)
$\therefore \Delta ABC \equiv \Delta DEF$ (RHS)
h $\angle BAC = \angle EDF = 90°$ (R)
$BC = EF$ (H)
$AC = DF$ (S)
$\therefore \Delta ABC \equiv \Delta DEF$ (RHS)

6 (D, G), (C, E)

7 **a** 25°, 75° **b** Yes, AAS

8 **a** $a = 4$ **b** $x = 3, y = 5$
c $a = 60, b = 7$ **d** $x = 55, y = 4$
e $x = 6$ **f** $a = 70, b = 7$
g $c = 3, d = 4$ **h** $a = 30, b = 5$
i $a = 20, b = 70$

9 **a** $AB = ED$ (S)
$BC = DF$ (H)
$\angle BAC = \angle DEF = 90°$ (R)
$\therefore \Delta ABC \equiv \Delta EDF$ (RHS)
b 6 m
c i 37° **ii** 53°

Exercise 7F

1 SSA

2 If two pairs of angles are corresponding and equal, the third pair must be equal due to the angle sum of a triangle (180°).

3 **a** 1.5 **b** 1.5, the same
c 1.5 **d** SAS

4 **a** $\angle BAC = \angle EDF$ (A)
$\angle ABC = \angle DEF$ (A)
$\angle ACB = \angle DFE$ (A)
$\therefore \Delta ABC \;|||\; \Delta DEF$ (AAA)
b $\angle BAC = \angle EDF$ (A)
$\angle ACB = \angle DFE$ (A)
$\angle ABC = \angle DEF$ (A)
$\therefore \Delta ABC \;|||\; \Delta DEF$ (AAA)
c $\dfrac{DE}{AB} = 2$ (S)
$\dfrac{DF}{AC} = 2$ (S)
$\dfrac{EF}{BC} = 2$ (S)
$\therefore \Delta ABC \;|||\; \Delta DEF$ (SSS)
d $\dfrac{DE}{AB} = 3$ (S)
$\dfrac{DF}{AC} = 3$ (S)
$\dfrac{EF}{BC} = 3$ (S)
$\therefore \Delta ABC \;|||\; \Delta DEF$ (SSS)
e $\dfrac{AB}{DE} = 1.5$ (S)
$\angle BAC = \angle EDF$ (A)
$\dfrac{AC}{DF} = 1.5$ (S)
$\therefore \Delta ABC \;|||\; \Delta DEF$ (SAS)
f $\dfrac{DE}{AB} = 1.5$ (S)
$\angle BAC = \angle EDF$ (A)
$\dfrac{DF}{AC} = 1.5$ (S)
$\therefore \Delta ABC \;|||\; \Delta DEF$ (SAS)
g $\angle CAB = \angle FDE = 90°$ (R)
$\dfrac{EF}{BC} = 1.5$ (H)
$\dfrac{DE}{AB} = 1.5$ (S)
$\therefore \Delta ABC \;|||\; \Delta DEF$ (RHS)
h $\angle CAB = \angle FDE = 90°$ (R)
$\dfrac{BC}{EF} = 2$ (H)
$\dfrac{AB}{DE} = 2$ (S)
$\therefore \Delta ABC \;|||\; \Delta DEF$ (RHS)

5 **a** $x = 8$ **b** $x = 21$
c $x = 4$ **d** $x = 1.5$

6 **a** $\Delta ABC, \Delta ADE$ **b** AAA
c 2.5 **d** 3.75 m

7 **a** AAA, $x = 1.5$ **b** AAA, $x = 9$
c AAA, $x = 2.2$

8 **a** $\angle BAC = \angle DEC$ (alternate), $\angle ABC = \angle EDC$ (alternate), $\angle ACB = \angle ECD$ (vertically opposite)
b i $DC = 6$ cm **ii** $AC = 6$ cm

Exercise 7G

1 **a** AAA **b** AAA

2 **a** AAA **b** 2 **c** 3

3 **a** AAA **b** 2 **c** 30 cm

4 **a** AAA **b** 1.5 **c** 4.5 m

5 **a** $\dfrac{88}{5} = 17.6$ **b** 2

6 **a** $\Delta ABC, \Delta DEC$; AAA **b** 1.25 m

7 **a** AAA **b** 5 **c** 5 m

8 **a** $\Delta ABD, \Delta CBE$; AAA **b** $\dfrac{6}{5} = 1.2$ **c** 13.2 m

9 1.90 m

10 Answers will vary.

11 **a** AAA **b** 1.5 **c** 20 m
d Let $AE = x$
$1.5x = x + 10$
$\therefore x = 20$

ISBN: 9781107687028 Cambridge University Press

Exercise 7H

1 a 4:9 b 8:27
2 a 2:5 b 32 cm^2, 200 cm^2
c 4:25 (= 2^2:5^2)
d 12 cm^3, 187.5 cm^3
e 8:125 (= 2^3:5^3)
3 a 2 cm b 1500
c 5 cm d 75 m
4 a 5 cm b 1500
c 4 cm d 60 m
5 a 16 cm^2 b 45 m^2
6 a 3 mm^2 b 3 cm^2
7 a 2 cm^3 b 8 m^3
8 a 108 cm^3 b 156.25 m^3
9 a i 1 km ii 3 km
b i 10 cm ii 1 cm c 2 km
10 a i 3:4 ii 9:16 b 450 cm^2
11 a 1:8 b 240 cm^3
12 a i 2:3 ii 4:9 iii 8:27
b 90 cm^2 c 54 cm^3
13 a 1:27 000 000 b 54 000 000 m^3 c 9 m^2

Puzzles and games

1 30
2 CONGRUENCE
3 130°
4 a 7 b 11
5 20 m

Multiple-choice questions

1 B 2 A 3 D 4 E 5 E
6 D 7 B 8 A 9 C 10 B

Short-answer questions

1 a $x = 70, y = 110$ b $x = 120, y = 120$
c $x = 65, y = 115$ d $x = 30, y = 150$
e $x = 90, y = 120$ f $x = 45$
2 a 20 b 30 c 77 d 20 e 60
f 30 g 130 h 70 i 160
3 a square, rectangle, rhombus, parallelogram
b parallelogram, square, rhombus, rectangle
c kite d square, rhombus, kite
4 a $a = 90$ b $a = 40$ c $a = 110, b = 30$
d $a = 90, b = 130$ e $a = 40, b = 140$
f $a = 110$
5 a 540°, 60 b 720°, 100 c 900°, 120
6 a 10 m b 540° c 108°
7 a $\angle BAC = \angle EDF = 90°$ (R)
$BC = EF$ (H)
$AB = DE$ (S)
$\therefore \Delta ABC \equiv \Delta DEF$ (RHS)
b $AB = DE$ (S)
$AC = DF$ (S)
$BC = EF$ (S)
$\therefore \Delta ABC \equiv \Delta DEF$ (SSS)
c $\angle ACB = \angle DFE$ (A)
$\angle CAB = \angle FDE$ (A)
$AB = DE$ (S)
$\therefore \Delta ABC \equiv DEF$ (AAS)
d $AB = DE$ (S)
$\angle BAC = \angle EDF$ (A)
$AC = DF$ (S)
$\therefore \Delta ABC \equiv DEF$ (SAS)
8 a $\angle BAC = \angle EDF =$ (A)
$\angle ABC = \angle DEF$ (A)
$\angle ACB = \angle DFE$ (A)
$\therefore \Delta ABC \,|||\, \Delta DEF$ (AAA)
b $\frac{AB}{DE} = 2$ (S)
$\frac{AC}{DF} = 2$ (S)
$\frac{BC}{EF} = 2$ (S)
$\therefore \Delta ABC \,|||\, \Delta DEF$ (SSS)
c $\frac{DE}{AB} = 1.5$ (S)
$\angle EDF = \angle BAC$ (A)
$\frac{DF}{AC} = 1.5$ (S)
$\therefore \Delta ABC \,|||\, \Delta DEF$ (SAS)
d $\angle BAC = \angle EDF = 90°$ (R)
$\frac{EF}{BC} = 2.5$ (H)
$\frac{DE}{AB} = 2.5$ (S)
$\therefore \Delta ABC \,|||\, DEF$ (RHS)
9 a $\Delta ABE, \Delta ACD$ b AAA
c 2.5 d 7.5 m
10 a AAA b 6.25 c 187.5 cm
11 a 2 cm b 5000 c 5 cm d 250 m
12 a i 2:3 ii 4:9 iii 8:27
b 32 cm^3

Extended-response questions

1 a 60, 60, 30 b AAA
c 2.5 d 8 cm
e 4:25 f 6.25
2 a 2:125 b 281.25 cm
c 4:15 625 d 0.00128 L

Chapter 8

Pre-test

1 a 3 b 9 c -18
d 36 e 9 f 0
2 a 9 b 14 c 6
d 26 e -16 f 32
3 a $3a$ b $3m$ c $8p$
d 0 e $-5m$ f $7x + y$
g $9p$ h $6m$

ISBN: 9781107687028

4 **a** $15x$ **b** $16p$ **c** $32xy$
d $-30a$ **e** ab **f** x
g 1 **h** $2a$ **i** 3

5 **a** 5 **b** 3 **c** 5
d 1 **e** 4 **f** 2
g 3 **h** 2 **i** 4
j 3 **k** 6 **l** 10

6 **a** a **b** w **c** w
d z **e** $8n$ **f** x
g $2p$ **h** $\frac{x}{2}$ **i** x

7 **a** $x+3$ **b** $n+6$
c $2w$ **d** $\frac{x}{2}$
e $2x+6$ **f** $x-7$
g $2(x+3)$ **h** $3x+1$

8 **a** $8x$ **b** $4x+16$ **c** $3x+4$

9 b, e, f, g, i

Exercise 8A

1 **a** Yes **b** No **c** No
d No **e** Yes **f** Yes

2 **a** $t=3$ **b** $m=6$ **c** $x=6$
d $m=-7$ **e** $x=8$ **f** $x=-4$
g $m=32$ **h** $a=-6$ **i** $m=15$

3 **a** $x=6$ **b** $x=9$ **c** $x=17$
d $x=3$ **e** $x=-6$ **f** $x=12$
g $x=36$ **h** $x=120$ **i** $x=101$

4 **a** $p=3$ **b** $c=6$ **c** $d=9$
d $m=8$ **e** $z=25$ **f** $w=9$
g $p=1$ **h** $m=-7$ **i** $p=-\frac{1}{2}$

5 **a** $x=50$ **b** $m=21$ **c** $a=-12$
d $z=0$ **e** $x=-8$ **f** $w=-27$
g $r=56$ **h** $w=\frac{3}{2}$ **i** $m=\frac{1}{2}$

6 **a** $x=3$ **b** $x=9$ **c** $x=-11$
d $x=10$ **e** $x=14$ **f** $x=10$
g $x=3$ **h** $x=4$ **i** $x=50$
j $x=20$ **k** $x=21$ **l** $x=7$

7 **a** $x=1$ **b** $x=3$ **c** $x=3$
d $x=-2$ **e** $x=-1$ **f** $x=-9$
g $x=4$ **h** $x=8$ **i** $x=8$
j $x=-2$ **k** $x=-3$ **l** $x=5$

8 **a** $x=9$ **b** $x=0$ **c** $x=56$
d $x=20$ **e** $x=35$ **f** $x=90$
g $x=-32$ **h** $x=-20$ **i** $x=-18$

9 **a** $m=5$ **b** $a=7$ **c** $x=1$
d $x=1$ **e** $n=9$ **f** $m=22$
g $w=-7$ **h** $m=7$ **i** $w=27$
j $a=1$ **k** $a=-37$ **l** $m=-5$

10 **a** $x+4=6, x=2$
b $x+12=8, x=-4$
c $x-5=5, x=10$
d $\frac{x}{3}+2=8, x=18$
e $2x+3=9, x=3$
f $\frac{x-3}{5}=6, x=33$
g $3x+4=16, x=4$

11 **a** 13 cm **b** 22 mm

12 **a** 3 **b** 5 **c** 28 **d** 42 **e** 82

13 **a** 11, 12 **b** 6 **c** 3 **d** 25, 44 **e** 8 m

14 **a** $C=60h+40$ **b** \$280 **c** \$1120 **d** 6 hours

15 **a** $v=6t+5$ **b** 3 minutes **c** 11 minutes

Exercise 8B

1 **a** $3x-3$ **b** $5x+15$ **c** $-2x-4$ **d** $-3x+12$
e $-8x+4$ **f** $5x+13$ **g** $7x+26$ **h** $9x+9$
i $-x-19$ **j** $-6x+13$

2 **a** $x=4$ **b** $x=-4$ **c** $x=5$ **d** $x=1$
e $x=5$ **f** $x=9$ **g** $x=2$ **h** $x=7$

3 **a** $x=1$ **b** $x=5$ **c** $x=-1$
d $a=5$ **e** $a=1$ **f** $x=15$
g $m=4$ **h** $d=1$ **i** $a=10$
j $a=0$ **k** $x=0$ **l** $a=3$

4 **a** $x=1$ **b** $x=2$ **c** $x=3$
d $x=2$ **e** $x=3$ **f** $x=2$
g $x=-2$ **h** $x=-1$

5 **a** $x=1$ **b** $x=1$ **c** $x=0$
d $x=0$ **e** $x=3$ **f** $x=1$
g $x=3$ **h** $x=-2$

6 **a** $x=2$ **b** $x=12$ **c** $x=-3$
d $x=20$ **e** $x=4$ **f** $x=8$
g $x=4$ **h** $x=-4$ **i** $x=-1$
j $x=11$ **k** $x=1$ **l** $x=-1$

7 **a** $x=13$ **b** $x=6$ **c** $x=13$
d $x=11$ **e** $x=10$ **f** $x=5$
g $x=6$ **h** $x=8$ **i** $x=-2$

8 **a** $x=12$ **b** $x=18$ **c** $x=60$
d $x=9\frac{1}{3}$ **e** $x=5$ **f** $x=-6$

9 **a** $x=10$ **b** $x=\frac{5}{3}$ **c** $x=45$

10 **a** $x=3$ **b** $x=5$ **c** $x=10$
d $x=30$ **e** $x=7$

11 **a** $x=7$ **b** $x=3$ **c** $x=5$ **d** $x=2$
e $x=5$ **f** $x=6$ **g** $x=11$

12 **a** $C=850+156h$ **b** 7 hours **c** 8:15 p.m.

13 **a** $x=1$ **b** $x=-3$ **c** $x=2\frac{2}{5}$
d $x=8$ **e** $x=3$ **f** $x=-\frac{8}{17}$

ISBN: 9781107687028

Cambridge University Press

Exercise 8C

1 a I **b** F **c** V **d** A **e** c **f** P

2 a $m = 60$ **b** $A = 48$ **c** $A = 36$ **d** $v = 14.3$ **e** $m = 3.7$

3 a $t = 4$ **b** $t = 4$ **c** $t = 10$ **d** $t = 8$

4 a $b = 10$ **b** $b = 9$ **c** $b = 17$ **d** $b = 1.3$

5 a $h = 5$ **b** $h = 12$ **c** $h = 3$ **d** $h = 7$

6 a $b = 15$ **b** $b = 16$ **c** $b = 12$ **d** $b = 32$

7 a $h = 8$ **b** $h = 8$ **c** $h = 12$ **d** $h = 28$

8 a $m = 4$ **b** $m = 40$ **c** $m = 72$ **d** $m = 4$

9 a $h = 5.7$ **b** $h = 5.1$ **c** $h = 5.7$ **d** $h = 16.0$

10 a 86°F **b** -1.1°C **c** 212°F **d** -17.8°C

11 a \$32 **b** 60 km

12 a $P = 750$ **b** $t = 3.125$ **c** $r = 7.5$

13 a 1.5 tablets **b** 1250 mg

14 a 75 mL/h **b** $\frac{1}{3}$ h = 20 min

15 a number of hours **b** 7.5 hours

16 $P = 27.32$ cm, $A = 28.87$ cm^2

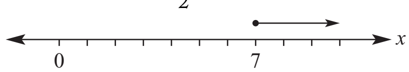
17 Fatima is now 17 and Yuri is 7.

Exercise 8D

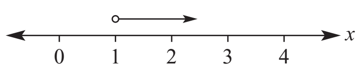
1 a 3 **b** 2 **c** 1 **d** 5 **e** 4

2 a $x \geq 1$ **b** $x < 7$ **c** $x \leq 4$ **d** $x > -9$ **e** $x \leq 1$ **f** $x > 8$ **g** $x < -7$ **h** $x \geq 1\frac{1}{2}$ **i** $x < 1$ **j** $x > 0$

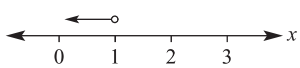
3 a



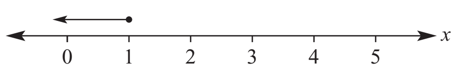
b



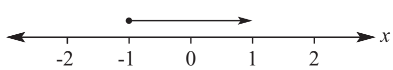
c



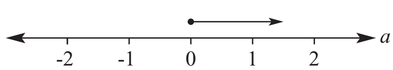
d



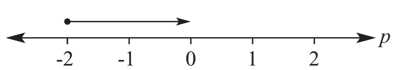
e



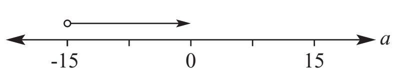
f



g



h

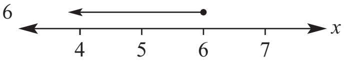


i

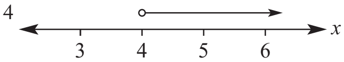
4 a $x \geq -1$ **b** $x < 1$ **c** $a < 6$ **d** $a \geq -2$

e $w \leq 5$ **f** $w > -3$ **g** $x \leq 3$ **h** $x > -3.5$

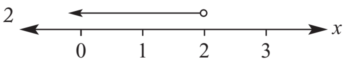
5 a $x \leq 6$



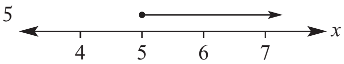
b $x > 4$



c $x < 2$



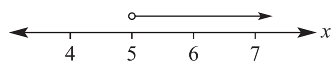
d $x \geq 5$



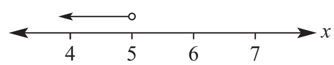
6 a $n > 100\,000$ **b** $n \geq 50$ **c** $n > 3$ **d** $n > 100$ **e** $n \leq 30$

7 a $x < 0$; -6, -2, $\frac{-1}{2}$ **b** $x > 10$; 15, 24 **c** $x \geq 10$; 10, 15, 24 **d** $x \leq 0$; -6, -2, $\frac{-1}{2}$, 0 **e** $x \geq -1$, 0, 2, 5, 7, 10, 15, 24 **f** $x < 10$; -6, -2, $\frac{-1}{2}$, 0, 2, 5, 7

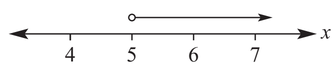
8 a $x > 5$



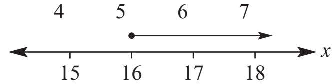
b $x < 5$



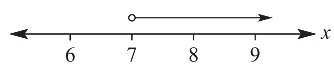
c $x > 5$



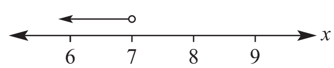
d $x \geq 16$



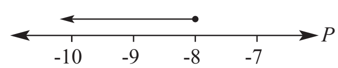
e $x > 7$



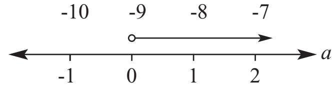
f $x < 7$



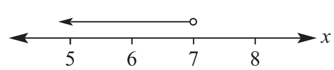
g $p \leq -8$



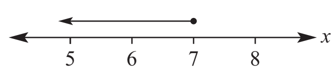
h $a > 0$



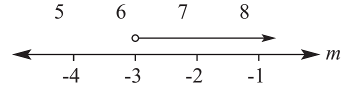
i $x < 7$



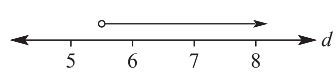
j $x \leq 7$



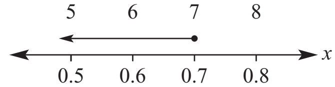
k $m > -3$



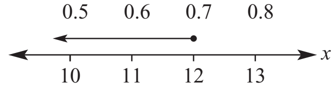
l $d > 5.4$



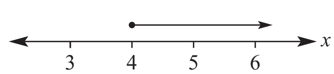
m $x \leq 0.7$



n $x \leq 12$



o $x > 4$



ISBN: 9781107687028

9 a $a \le 2$ **b** $y > 3$ **c** $p > 5$
d $x \ge 4$ **e** $x < 1$ **f** $w \ge \frac{3}{2}$
g $x < 1$ **h** $x \ge 1$ **i** $p < 1$

10 a $x \le 2$ **b** $a \le 1$ **c** $x \ge 28$
d $x > -15$ **e** $x < 4$ **f** $x < 30$
g $x > \frac{19}{2}$ **h** $x < \frac{-1}{2}$ **i** $x \ge \frac{-8}{3}$
j $x > 1$ **k** $x \ge \frac{3}{2}$ **l** $x < -\frac{9}{2}$

11 a $3x < 9,\ x < 3$
b $\frac{3x}{4} < 6,\ x < 8$
c $2x + 15 > 20,\ x > \frac{5}{2}$
d $x + x + 4 < 24,\ x < 10$
e $4x + 7 \le 27,\ x \le 5$

12 a $x > -1$ **b** $x \le -3$
c $p \ge -7$ **d** $a > 19$
e $w \le -6$ **f** $p > -6$
g $x > -1$ **h** $a \le --\frac{6}{7}$

13 a

4	6	4 < 6	T or F
4 + 3	6 + 3	7 < 9	T
4 − 3	6 − 3	1 < 3	T
4 × 2	6 × 2	8 < 12	T
4 ÷ 2	6 ÷ 2	2 < 3	T
4 × (-2)	6 × (-2)	-8 < -12	F
4 ÷ (-2)	6 ÷ (-2)	-2 < -3	F

b subtract, divide, positive, inequality, multiply, divide, number, true

Exercise 8E

1 a (0, 3) **b** (1, 2)
c no point of intersection **d** (2, 8)
e no point of intersection **f** (4, 3)
g (2, 5) **h** (0, 4)
i no point of intersection

2 a $y = 0$ **b** $x = 0$

3 a (2, 3) **b** (2, 8)
c (0, 9) **d** (-3, 8)

4 a (2, 3) **b** (3, -2)

5 a (3, 9) **b** (1, -3)

6 a (1, -4) **b** (1, 1)

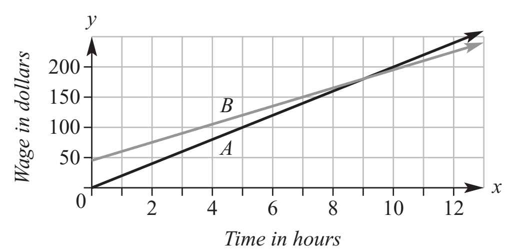
7 a 200 km
b $1400
c Best removals, $100 cheaper
d A$^+$ removals, $200 cheaper

8 a

Time in hours	0	1	2	3	4	5	6
Wage of company A	$0	$20	$40	$60	$80	$100	$120
Time in hours	7	8	9	10	11	12	
Wage of company A	$140	$160	$180	$200	$220	$240	

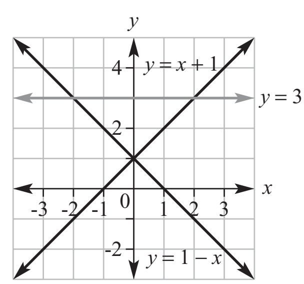
Time in hours	0	1	2	3	4	5	6
Wage of company B	$45	$60	$75	$90	$105	$120	$135
Time in hours	7	8	9	10	11	12	
Wage of company B	$150	$165	$180	$195	$210	$225	

b



c 9 hours
d $180

9 a



b (-2, 3) (0, 1) (2, 3)
c 4 units, $\sqrt{8}$ units, $\sqrt{8}$ units
d Isosceles right-angled triangle

10 a

Time in years	0	2	4	6
Value of luxury sports coupe	$70 000	$60 000	$50 000	$40 000

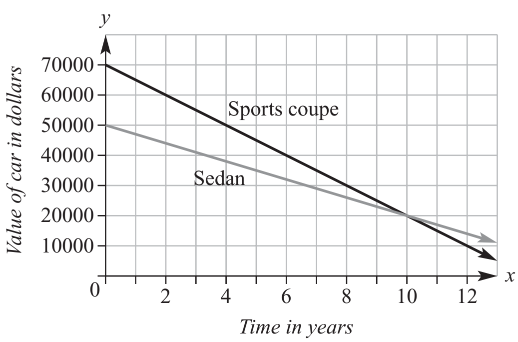
ISBN: 9781107687028

Exercise 8E cont.

Time in years	8	10	12
Value of luxury sports coupe	\$30 000	\$20 000	\$10 000

Time in years	0	2	4	6
Value of sedan	\$50 000	\$44 000	\$38 000	\$32 000
Time in years	8	10	12	
Value of sedan	\$26 000	\$20 000	\$14 000	

b



c 10 years **d** \$20 000

11 a no **b** parallel lines

d (-6, -12), (-5, -9), (-4, -6), (-3, -3)

12 a yes (0, -1) **b** same y-intercept

d (-2, -3), parallel, (2, 5), (1, 3)

Exercise 8F

1 a simultaneous **b** intersection
c substituted **d** substituted
e substitution

2 a $x = -3$ **b** $x = -5$ **c** $y = 13$
d $x = 2$ **e** $y = 7$ **f** $x = 2$
g $y = 7$ **h** $y = -1$ **i** $x = -6$

3 a (2, 10) **b** (-5, -15) **c** (-4, -8)
d (1, 4) **e** (2, 2) **f** (2, 12)

4 a (2, 5) **b** (1, 3) **c** (2, 1)
d (4, 3) **e** (-5, -3) **f** (1, 6)
g (4, 1) **h** (-1, -5)

5 a (-1, 2) **b** (3, -1) **c** (4, 4)

6 a (2, 4) **b** (-3, 2) **c** (7, -1)

7 a (1, 5) **b** (4, 10) **c** (2, 1) **d** (2, 9)

8 Paul: 43 years old, Mary: 38 years old

9 length = 6 cm, width = 18 cm

10 a vanilla thick shake: \$5, Fruity Twirl: \$3

11 Carlos: 37 years old, Ella: 17 years old

12 a i Joe's: \$60, Paul's \$150
ii Joe's: 20 c/km, Paul's 10 c/km
iii Joe's $C = 0.2k + 60$, Paul's $C = 0.1k + 150$
iv 900 km

b Joe's if you are travelling less than 900 km and Paul's for more than 900 km.

Exercise 8G

1 a – **b** + **c** +

2 a $6x - 4y = -2$ **b** $9x - 6y = -3$ **c** $12x - 8y = -4$

3 a (6, 1) **b** (3, 4) **c** (2, 7)

4 a (7, 2) **b** (3, 8) **c** (4, 1)

5 a (2, 5) **b** (2, 3) **c** (4, 2) **d** (2, 2)

6 a (1, 1) **b** (2, 1) **c** (2, -1)

7 a (4, -3) **b** (1, 1) **c** (2, 0) **d** (1, 1)

8 a (1, 1) **b** (4, 2) **c** (3, 4)

9 a (3, 1) **b** (1, -10) **c** (-2, -3) **d** (0, 1)
e (-1, 2) **f** (5, -2)

10 Bob: 36 years old, Francene: 20 years old

11 children: 2500, adults: 2500

12 a Let a be the number of apples and m be the number of mangoes. $10a + 5m = 1250$,
$15a + 4m = 1350$

b apples: 50*c*, mangos: \$1.50

c \$8.00

13 a Let m be the number of multiple-choice and s be the number of short-answer questions.

b $2m + 3s = 50$, $m + s = 22$

c 16 multiple-choice questions

14 a $x + y = 16$, $x - y = 2$, $x = 9$, $y = 7$

b $x + y = 30$, $x - y = 10$, $x = 20$, $y = 10$

c $2x + y = 12$, $x + y = 7$, $x = 5$, $y = 2$

15 a $x = 1, y = 2$

b $x = 2, y = 2$

16 a i $g + 30$ **ii** $d + 30$

b $g = d + 31$, $g + 30 = 2(d + 30)$

c Gary is 32 and his daughter is 1.

17 a (4, -3) **b** (1, 1) **c** (3, 4)
d (2, 2) **e** $\left(\frac{1}{2}, -1\right)$ **f** $\left(-3, \frac{1}{3}\right)$

Puzzles and games

1 Each row, column and diagonal adds to 6.

9	-5	-4	6
-2	4	3	1
2	0	-1	5
-3	7	8	-6

ISBN: 9781107687028

2 **a** 64 **b** 8 **c** 29.3 **d** 18 years old

3 Many possible equations.

$E.g.\ 3x+2=-4;\ \frac{5x}{2}=-5;\ 2(x+5)=6$

4 Toowoomba

5 Many possible simultaneous equations.

$E.g.\ x+y=1,\ 2x-y=8$

6 **a** 4.5 hours **b** 117 km

7 Talia was jogging at 11.5 km/h and cycling at 23 km/h.

Multiple-choice questions

1 C	**2** B	**3** C	**4** D	**5** B
6 C	**7** D	**8** B	**9** C	**10** D
11 A	**12** E	**13** B	**14** D	**15** A
16 A	**17** E			

Short-answer questions

1 **a** $a=8$ **b** $m=-30$ **c** $x=-8$
d $x=8$ **e** $m=0$ **f** $w=15$
g $m=-0.2$ **h** $w=4$ **i** $r=6$

2 **a** $m=2$ **b** $w=8$ **c** $m=10$
d $w=8$ **e** $m=14$ **f** $m=\frac{4}{3}=1\frac{1}{3}$
g $a=\frac{3}{2}=1\frac{1}{2}$ **h** $x=1$ **i** $x=3$

3 **a** $m=3$ **b** $a=7$ **c** $x=4$
d $x=\frac{3}{2}=1\frac{1}{2}$ **e** $m=2\frac{1}{2}$ **f** $x=\frac{7}{8}$
g $x=2$ **h** $x=8$

4 **a** $p=4$ **b** $p=4$ **c** $p=-9$
d $p=-2$ **e** $p=-5$ **f** $p=2$

5 **a** $b=8$ **b** $w=3.5$ **c** $x=2.4$
d $m=10$ **e** $C=35$

6 **a** $m>-2$ **b** $n\le 0.5$ **c** $x\ge -1$
d $x>0$ **e** $x<15$

7 **a** $x\ge -18$ **b** $m<3.5$ **c** $x>2$
d $x<3$ **e** $x\le 8$ **f** $m\ge 10$

8 **a** $x\ge -2$ **b** $x\ge -2$ **c** $x<0$

9 **a** (1, 2) **b** (2, 3)
c no point of intersection

10 **a** (1, 6) **b** (3, 2) **c** (0, 0)

11 **a** (3, 2) **b** (5, -1)

12 **a** (-2, -3) **b** (2, 1)

13 **a** $6x=420$, the number is 70
b $x+8=5$, the number is -3
c $\frac{a}{9}=12$, the number is 108
d $x+7=3$, the number is -4
e $x+2.3=7$, the number is 4.7

14 **a** \$120 **b** \$$(96+2n)$
c $96+2n=308$, 106 packages sold

15 **a** x: number of 20 cent coins, y: number of 50 cent coins;
$x+y=160,\ 20x+50y=5000$
b 100 20-cent coins and 60 50-cent coins

16 50 children

Extended-response questions

1 **a** $P=6x-4$
b **i** $x=22$
ii 20 cm, 27 cm, 37 cm, 7 cm, 30 cm
c **i** $x=26$; 24 cm, 31 cm, 45 cm, 7 cm, 38 cm
ii $x=38$; 36 cm, 43 cm, 69 cm, 7 cm, 62 cm

2 **a** **i** $C=70t+50$ **ii** $C=100+60t$
b (5, 400)

Chapter 9

Pre-test

1 **a** 15.84 **b** 164.87 **c** 0.87
d 0.58 **e** 0.17 **f** 0.71
g 12.99 **h** 14.30

2 **a** 25 **b** 46.24 **c** 361
d 225 **e** 43.25 **f** 81

3 **a** 2.8 **b** 2.6 **c** 3.9
d 3.2 **e** 3.6 **f** 3.0
g 1.9 **h** 14.1

4 **a** c **b** p **c** y
d PQ **e** BC **f** XY

5 **a** $x=3$ **b** $x=4$ **c** $x=12$
d $x=35$ **e** $x=108$ **f** $x=9$

6 **a** $m=3.65$ **b** $m=1.2$ **c** $m=4.4$
d $m=5.2$ **e** $m=18.848$ **f** $m=5.724$

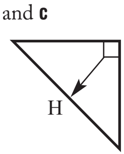
7 **a** $x=0.6$ **b** $x=0.2$ **c** $x=2.1$
d $x=0.4$ **e** $x=2.4$ **f** $x=9.2$

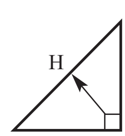
8 **a** $x=4$ **b** $x=20$ **c** $x=13$ **d** $x=4$

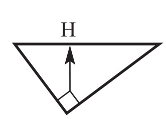
9 **a** 30° **b** 50° **c** 54° **d** 90° **e** 45°
f 30° **g** 82° **h** 62° **i** 35°

Exercise 9A

1 **b** and **c**

2 **a**


b


c


3 **a** $z^2=x^2+y^2$ **b** $t^2=m^2+n^2$ **c** $s^2=p^2+r^2$

4 **a** 25 **b** 34 **c** 45

5 **a** 5 **b** 17 **c** 13 **d** 25 **e** 41 **f** 10

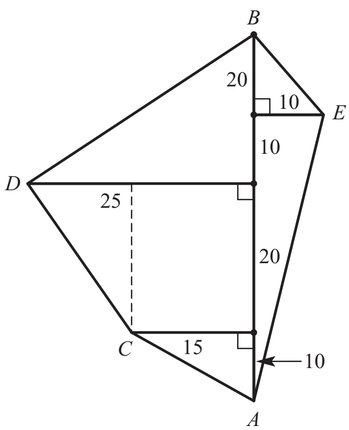
6 **a** 5.8 cm **b** 7.3 mm **c** 18.0 m
d 11.2 km **e** 8.6 cm **f** 12.8 cm

7 **a** 8.91 cm **b** 3.62 m **c** 5.02 km
d 13.95 mm **e** 17.49 cm **f** 24.21 cm

ISBN: 9781107687028

Exercise 9A cont.

8 181.5 cm
9 35.3 m
10 13 m
11 5.28 m
12 2.64 m
13 494.97 m
14 a No **b** No **c** Yes **d** Yes **e** No **f** Yes
15 a i $AC = 11.2$ **ii** $BC = 30.4$
iii $DB = 21.2$ **iv** $AD = 29.2$
b 92 m
c 400 m^2
16 $P = 143.0$ m

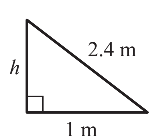


Exercise 9B

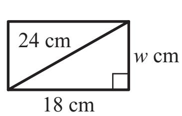
1 a 10 **b** 24 **c** 41 **d** 1.5
2 a $w^2 = 10^2 - 6^2$ **b** $x^2 = 13^2 - 5^2$
c $p^2 = 30^2 - 18^2$
3 a $17^2 = 8^2 + w^2$ **b** $13^2 = m^2 + 5^2$ **c** $15^2 = x^2 + 9^2$
4 a 4 **b** 7 **c** 8
d 5 **e** 14 **f** 4.8
5 a 7.1 **b** 13.3 **c** 12.3
d 6.2 **e** 6.6 **f** 16.2
6 a 7.14 **b** 13.90 **c** 3.87
d 133.84 **e** 17.89 **f** 39.19
7 5.66 m
8 2.2 m
9 3.2 m
10 a i 3 cm **ii** 5.2 cm
b 15.6 cm^2
11 7.4 cm
12 a 7.1 **b** 4.5 **c** 5.2

Exercise 9C

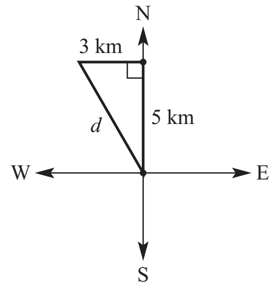
1 a



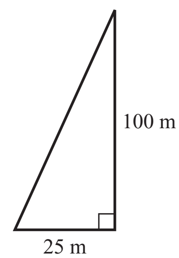
b



c



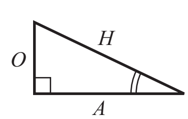
2 a ΔADE **b** ΔPRO and ΔRQO **c** ΔAOB
3 a AB **b** AB **c** FH **d** BG **e** RT **f** EB
4 a 100 m **b**



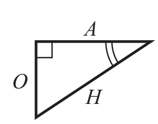
c 103.08 m
5 a 8.6 m **b** 13 m **c** 10 m
6 6.9 km
7 a 10 km **b** 41 km **c** 17 km
8 a 10.6 **b** 10.8 **c** 15.8 **d** 4.2
9 25.04 m
10 a 11.18 m **b** 12.2 m
11 Horizontal = 45.25 cm
Vertical = 71.66 cm
12 a 4.77 m **b** 4.90 cm
13 a 11.2 m **b** 8.1 cm
14 a 1.41 cm **b** 1.73 cm
15
- ΔPQR is a right-angled isosceles triangle
- With $\angle P = \angle R = 45°$
- the hypotenuse = $\sqrt{200}$ cm
- $PQ = QR = 10$ cm
- perimeter = 34.1 cm
- area = 50 cm^2

Exercise 9D

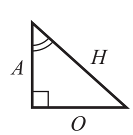
1 a



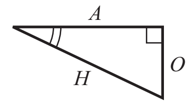
b



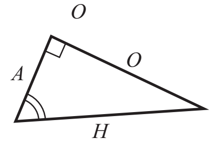
c



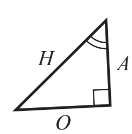
d



e



f



ISBN: 9781107687028 Cambridge University Press

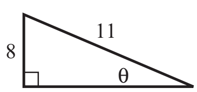
2 a PR **b** TP
c TP **d** PR
e TR **f** $\angle T$

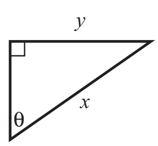
3 a BC **b** CA
c BA **d** BA

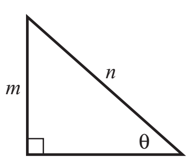
4 a $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** $\frac{4}{5}$

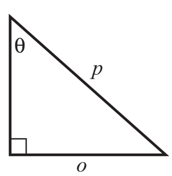
5 a i $\frac{b}{a}$ **ii** $\frac{c}{a}$ **iii** $\frac{b}{c}$
b i $\frac{n}{p}$ **ii** $\frac{m}{p}$ **iii** $\frac{n}{m}$
c i $\frac{y}{z}$ **ii** $\frac{x}{z}$ **iii** $\frac{y}{x}$
d i $\frac{4}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{4}{3}$
e i $\frac{24}{26}=\frac{12}{13}$ **ii** $\frac{10}{26}=\frac{5}{13}$ **iii** $\frac{24}{10}=\frac{12}{5}$
f i $\frac{12}{13}$ **ii** $\frac{5}{13}$ **iii** $\frac{12}{5}$

6 a $\frac{a}{c}$ **b** $\frac{y}{z}$ **c** $\frac{b}{a}$
d $\frac{6}{10}=\frac{3}{5}$ **e** $\frac{40}{41}$ **f** $\frac{1}{\sqrt{3}}$
g $\frac{8}{6}=\frac{4}{3}$ **h** $\frac{9}{41}$ **i** $\sqrt{3}$

7 a


b


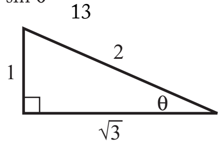
c


d


8 a $\frac{y}{x}$ **b** $\frac{z}{x}$ **c** $\frac{z}{x}$
d $\frac{y}{x}$ **e** $\frac{y}{z}$ **f** $\frac{z}{y}$

9 a $\sin\theta$ **b** $\tan\theta$ **c** $\sin\theta$
d $\cos\theta$ **e** $\cos\theta$ **f** $\tan\theta$

10 a $QR = 13$ m
b $\sin\theta = \frac{5}{13}$

11 a


b $\sqrt{3}$
c i $\cos\theta = \frac{\sqrt{3}}{2}$ **ii** $\tan\theta = \frac{1}{\sqrt{3}}$

12

Angle (θ)	sin θ	cos θ
0°	0	1
5°	0.087	0.996
10°	0.174	0.985
15°	0.259	0.966
20°	0.342	0.940
25°	0.423	0.906
30°	0.5	0.866
35°	0.574	0.819
40°	0.643	0.766
45°	0.707	0.707
50°	0.766	0.643
55°	0.819	0.574
60°	0.866	0.5
65°	0.906	0.423
70°	0.940	0.342
75°	0.966	0.259
80°	0.985	0.174
85°	0.996	0.087
90°	1	0

a 45°
b i 85 **ii** 80 **iii** 30 **iv** 0
c If angles θ and α sum to 90°, $\sin\theta = \cos\alpha$.
d It's the same as the complement of sine.

Exercise 9E

1 a 0.1736 **b** 0.9848 **c** 0.1763
d 0.5774 **e** 0.7660 **f** 0.9397
g 0.1736 **h** 0.8391 **i** 0.9848
j 0.8660 **k** 0.6428 **l** 1.7321

2 a 2.12 **b** 5.07 **c** 31.18
d 46.43 **e** 8 **f** 18.79
g 2.05 **h** 4.83 **i** 8.47

3 a sin **b** cos **c** tan
d cos **e** tan **f** sin

4 a $x = 1.37$ **b** $x = 5.12$ **c** $x = 91.44$
d $x = 13.86$ **e** $x = 9.19$ **f** $x = 9.19$

5 a 0.39 **b** 4.50 **c** 2.60
d 11.15 **e** 16.80 **f** 5.75
g 7.83 **h** 13.49 **i** 1

6 a 2.11 **b** 4.02 **c** 1.88

7 a 5.36 **b** 1.27 **c** 0.52

8 a 5.49 **b** 8.51 **c** 9.23

9 a 3.76 **b** 2.12 **c** 2.80
d 4.94 **e** 4.14 **f** 0.75

10 a 26.33 m **b** 52.66 m

11 6.96 m

12 a $w = 1.27, l = 2.72$ **b** $w = 0.68, l = 1.88$
c $w = 3.06, l = 2.57$

ISBN: 9781107687028 Cambridge University Press

Exercise 9E cont.

13 a $a = 3.5$, $b = 3.2$, $x = 1.4$
b $a = 3.464$, $b = 3.139$, $x = 1.327$
c It is better not to round off during the process as sometimes it can change the final answer.

Exercise 9F

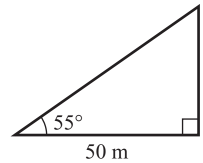
1 a 17.32 **b** 13.86 **c** 106.73
d 19.84 **e** 24.69 **f** 13.20
2 a $x = 2$ **b** $x = 5$ **c** $x = \frac{1}{2}$
d $x = 1$ **e** $x = 0.1$ **f** $x = 0.1$
3 a $\frac{10}{x}$ **b** $\frac{1.4}{m}$ **c** $\frac{19}{x}$ **d** $\frac{2.8}{w}$
4 a 8.77 **b** 9.44 **c** 8.49
5 a 4.62 **b** 23.39 **c** 2.86
6 a 5.96 **b** 1.62 **c** 1.72
7 a 4.73 **b** 6.19 **c** 6.14
d 3.00 **e** 26.08 **f** 27.82
8 2.54 m
9 13.9 m
10 a $AB = 42.89$ cm, $BC = 20$ cm
b $AB = 5.32$ m, $BC = 1.82$ m
c $AB = 14.62$ cm, $BC = 13.74$ cm
11 a 7.464 m **b** 7.727 m
12 a 30.5 m **b** 17.5 m
13 a 17.16 **b** 30 **c** 4.01
d 59.78 **e** 51.13 **f** 38.09

Exercise 9G

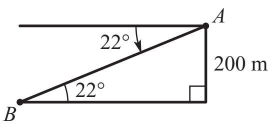
1 53°
2 a 45° **b** 30° **c** 58°
3 a $\cos\theta = \frac{5}{12}$ **b** $\sin\theta = \frac{7}{10}$ **c** $\tan\theta = \frac{4}{3}$
4 a 30° **b** 53° **c** 61°
d 45° **e** 41° **f** 53°
g 48° **h** 6° **i** 37°
j 81° **k** 73° **l** 60°
m 42° **n** 48° **o** 34°
5 a 60° **b** 45° **c** 64.16°
d 48.59° **e** 53.13° **f** 68.20°
6 a 42° **b** 56° **c** 64°
d 49° **e** 54° **f** 67°
g 53° **h** 50° **i** 40°
j 77° **k** 43° **l** 56°
7 10.62°
8 15.95°
9 46.66°
10 a 36.9°, 53.1° **b** 34.8°, 55.2° **c** 36.9°, 53.1°
d 26.6°, 63.4° **e** 68.2°, 21.8°
11 Pitch $A = 47°$, $B = 43°$
12 a 5.54 m **b** 5.97 m
13 a 12° **b** Yes **c** 286.4 cm

Exercise 9H

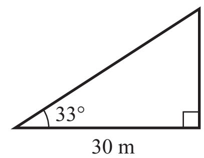
1 a 50 **b** 38 **c** 56
2 a 30° **b** 30° **c** 60°
3 a



b



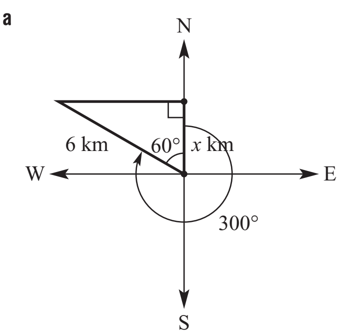
c



4 a 21.88 m **b** 43.5 m **c** 23.41 m
d 6.06 m **e** 536.29 m **f** 38.97 m
5 a 112.0 **b** 49 m **c** 86 m
d 105 m **e** 9260 m
6 a 26.6° **b** 29.7° **c** 5.7°
d 47.1° **e** 48.3°
7 38.35 m
8 1.25 km
9 280.04 m
10 a 30° **b** 30°
c equal due to parallel lines **d** 1.7
11 Answers will vary.

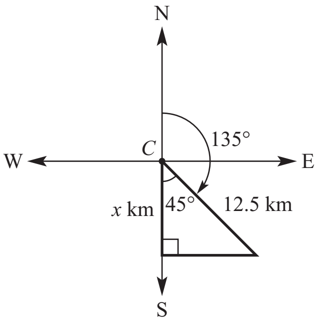
Exercise 9I

1 a S **b** W **c** N **d** SW
2 a v **b** iv **c** i **d** iii
e vi **f** ii
3 a 130° **b** 340° **c** 090°
d 128° **e** 313° **f** 355°
4 a North **b** East **c** West
d 060° **e** 240°
5 a

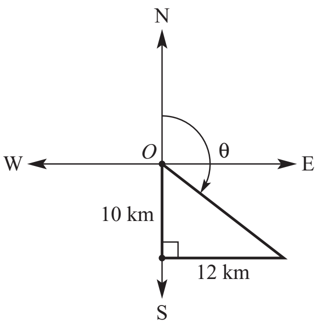


ISBN: 9781107687028 Cambridge University Press

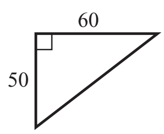
b



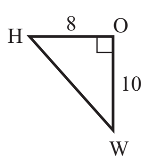
c



6 6.9 km
7 3.86 km
8 93.97 km
9 **a** 51.42 km **b** 61.28 km **c** 320°
10 **a**



b 78.10 km
c 50°
d 050°
11 **a**



b 12.8 km
c 39°
d 321°
12 3.25 km
13 115 km
14 **a** 18.03 km
b 146°

Puzzles and games

1 **a** West **b** SW **c** NW
d 198° **e** 120°
2 63.29 m
3 Answers will vary.
4

S														H
I	D	E	P	R	E	S	S	I	O	N				Y
N		E												P
E			G			S								O
				R		I	A	D	J	A	C	E	N	T
					E	D				A	N	G	L	E
						E							O	N
		T	R	I	A	N	G	L	E				P	U
	T	R	I	G	O	N	O	M	E	T	R	Y	P	S
		C	G	B						A	A		O	E
		O	H		E					N	T		S	
		S	T			A				G	I		I	
		I					R			E	O		T	
		N						I		N			E	
E	L	E	V	A	T	I	O	N	N	T				
					P	Y	T	H	A	G	O	R	A	S

Multiple-choice questions

1 B **2** E **3** C **4** B **5** C
6 C **7** D **8** E **9** A **10** C

Short-answer questions

1 No, $8^2 \neq 4^2 + 6^2$
2 15.17 m
3 **a** **i** BC **ii** AB **iii** AC
b **i** AC **ii** AB **iii** BC
4 **a** 0.57 **b** 0.96 **c** 8.14
5 **a** 4.50 **b** 1.5 **c** 0.93
6 **a** 6.31 **b** 12.94 **c** 4.16
7 **a** 3.63 m **b** 1.69 m
8 567.13 m
9 **a** 41.81° **b** 59.00° **c** 72.26°
10 **a** 45° **b** 42.4 m

Extended-response questions

1 **a** 54.46° **b** 130.25 m **c** 187 m
2 **a** 539 km **b** 68° **c** 248°

Chapter 10

Pre-test

1 **a** 3 **b** 2 **c** 5
2 **a** $4x + 2y$ **b** $2xy - x$ **c** $2ab$
3 **a** $2a$ **b** $-2m$ **c** $18a^2$ **d** $-6x^2y$
e $-\frac{1}{3}$ **f** $2x$
4 **a** $4m + 4n$ **b** $-6x + 12$ **c** $6x^2 + 2x$
d $4a - 8a^2$ **e** $3x - 7$ **f** $-2x + 1$

ISBN: 9781107687028

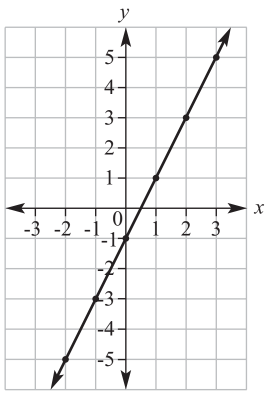
Pre-test cont.

5 **a** $7(x+1)$ **b** $-9x(1+3x)$ **c** $a(a+b)$

6 **a** $-\frac{1}{2}$ **b** 3 **c** 5

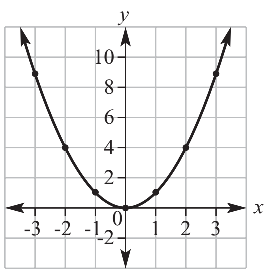
7

x	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5



8

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



Exercise 10A

1 **a** PS **b** DOPS **c** DOPS **d** PS

2 **a** $6x$ **b** $-20x$ **c** $2x^2$ **d** $-4x^2$
e $\frac{x}{2}$ **f** $\frac{x}{3}$ **g** $-4x$ **h** $-3x$
i $-18x$ **j** $7x$ **k** $5x$ **l** $-13x$

3 **a** x^2+2x **b** x^2+4x+3 **c** $x^2+8x+16$

4 **a** $2x+10$ **b** $3x-12$ **c** $-5x-15$ **d** $-4x+8$
e $6x-3$ **f** $12x+4$ **g** $-10x+6$ **h** $-20x-15$
i $2x^2+5x$ **j** $3x^2-x$ **k** $2x-2x^2$ **l** $6x-3x^2$
m $-6x^2-4x$ **n** $-18x^2+6x$
o $-10x+10x^2$ **p** $-4x+16x^2$

5 **a** $x^2+10x+16$ **b** $x^2+7x+12$
c $x^2+12x+35$ **d** $x^2+5x-24$
e x^2+x-30 **f** x^2+x-6
g $x^2-4x-21$ **h** $x^2-10x+24$
i $x^2-13x+40$ **j** $6x^2+13x+5$
k $12x^2+23x+10$ **l** $10x^2+41x+21$
m $9x^2-9x-10$ **n** $20x^2+2x-6$
o $6x^2+5x-25$

6 **a** $x^2+10x+25$ **b** $x^2+14x+49$
c $x^2+12x+36$ **d** x^2-6x+9
e $x^2-16x+64$ **f** $x^2-20x+100$
g $4x^2+20x+25$ **h** $25x^2+60x+36$
i $49x^2-14x+1$

7 **a** x^2-16 **b** x^2-81 **c** x^2-64
d $9x^2-16$ **e** $4x^2-9$ **f** $64x^2-49$
g $16x^2-25$ **h** $4x^2-81$ **i** $25x^2-49$

8 **a** 6 **b** 7 **c** 16
d 4 **e** 4 **f** 6

9 **a** 3 **b** 3 **c** 3
d 8 **e** 1 **f** 2

10 **a** $(x^2-12x+36)$ cm^2 **b** $(x^2+10x-200)$ cm^2

11 **a** $-x^2+7x$ **b** $10a-28$
c x^2-81 **d** $4x^2+12x+9$

12 **a** **i** x^2+5x **ii** $x^2+13x+36$ **iii** $8x+36$
b **i** 36 m^2 **ii** 68 m^2

Exercise 10B

1 **a** $2(x+3)$ **b** $-4(x-1)$
c $(x+2)(x-2)$ **d** $(3x+2)(3x-2)$

2 **a** 7 **b** 6 **c** 8 **d** -5
e $2a$ **f** $3a$ **g** $-5a$ **h** $-3xy$

3 **a** $3(x-6)$ **b** $4(x+5)$ **c** $7(a+b)$ **d** $3(3a-5)$
e $-5(x+6)$ **f** $-2(2y+1)$ **g** $-3(4a+1)$ **h** $-b(2a+c)$
i $x(4x+1)$ **j** $x(5x-2)$ **k** $6b(b-3)$ **l** $7a(2a-3)$
m $5a(2-a)$ **n** $6x(2-5x)$ **o** $-x(2+x)$ **p** $-4y(1+2y)$

4 **a** $(x+3)(x-3)$ **b** $(x+5)(x-5)$
c $(y+7)(y-7)$ **d** $(y+1)(y-1)$
e $(a+4)(a-4)$ **f** $(b+6)(b-6)$
g $(y+12)(y-12)$ **h** $(z+20)(z-20)$
i $(2x-3)(2x+3)$ **j** $(6a-5)(6a+5)$
k $(1+9y)(1-9y)$ **l** $(10-3x)(10+3x)$
m $(5x-2y)(5x+2y)$ **n** $(8x-5y)(8x+5y)$
o $(3a+7b)(3a-7b)$ **p** $(12a-7b)(12a+7b)$

5 **a** $(2+x)(2-x)$ **b** $(3+y)(3-y)$
c $(6+a)(6-a)$ **d** $(10+3x)(10-3x)$
e $(b+a)(b-a)$ **f** $(20+5a)(20-5a)$
g $(2a+3b)(2a-3b)$ **h** $(4y+11x)(4y-11x)$

6 **a** $2(x+4)(x-4)$ **b** $5(x+3)(x-3)$
c $6(y+2)(y-2)$ **d** $3(y+4)(y-4)$
e $3(x+5y)(x-5y)$ **f** $3(a+10b)(a-10b)$
g $3(2x+3y)(2x-3y)$ **h** $7(3a+4b)(3a-4b)$
i $3(6x-7y)(6x+7y)$

7 **a** **i** 100 m **ii** 96 m **iii** 36 m
b $(10+t)(10-t)$
c **i** 100 m **ii** 96 m **iii** 36 m
d 10 seconds

ISBN: 9781107687028

8 **a** 60 **b** 35 **c** 69 **d** 104
e 64 **f** 40 **g** 153 **h** 1260

9 **a** **i** x^2 cm^2 **ii** $(1600 - x^2)$ cm^2
b $(40 + x)(40 - x)$ cm^2
c **i** 1200 cm^2 **ii** 1500 cm^2
d $x = 30$

Exercise 10C

1 **a** $x^2 + 5x + 6$ **b** $x^2 - 2x - 8$
c $x^2 - 10x + 21$ **d** $x^2 - 2x + 1$
e $x^2 + 10x + 25$ **f** $x^2 - 12x + 36$

2 **a** 9, 2 **b** 10, 2
c 5, -3 **d** 4, -3
e -8, 3 **f** -10, 3
g -2, -5 **h** -12, -3

3 **a** **i** 5, 3 **ii** $(x + 5)(x + 3)$
b **i** 5, -2 **ii** $(x + 5)(x - 2)$
c **i** -4, -2 **ii** $(x - 4)(x - 2)$

4 **a** $(x + 6)(x + 1)$ **b** $(x + 3)(x + 2)$ **c** $(x + 3)^2$
d $(x + 5)(x + 2)$ **e** $(x + 4)(x + 3)$ **f** $(x + 9)(x + 2)$
g $(x - 1)(x + 6)$ **h** $(x + 3)(x - 2)$ **i** $(x + 4)(x - 2)$
j $(x - 1)(x + 4)$ **k** $(x + 10)(x - 3)$ **l** $(x + 11)(x - 2)$
m $(x - 2)(x - 5)$ **n** $(x - 4)(x - 2)$ **o** $(x - 4)(x - 3)$
p $(x - 1)^2$ **q** $(x - 6)(x - 3)$ **r** $(x - 2)(x - 9)$
s $(x - 6)(x + 2)$ **t** $(x - 5)(x + 4)$ **u** $(x - 7)(x + 2)$
v $(x - 4)(x + 3)$ **w** $(x + 8)(x - 4)$ **x** $(x - 5)(x + 2)$

5 **a** $(x - 2)^2$ **b** $(x + 3)^2$ **c** $(x + 6)^2$
d $(x - 7)^2$ **e** $(x - 9)^2$ **f** $(x - 10)^2$
g $(x + 4)^2$ **h** $(x + 10)^2$ **i** $(x - 15)^2$

6 **a** $2(x + 5)(x + 2)$ **b** $3(x + 4)(x + 3)$ **c** $2(x + 9)(x + 2)$
d $5(x - 2)(x + 1)$ **e** $4(x - 5)(x + 1)$ **f** $3(x - 5)(x + 2)$
g $-2(x + 4)(x + 3)$ **h** $-3(x - 2)(x - 1)$ **i** $-2(x - 7)(x + 2)$
j $-4(x - 2)(x + 1)$ **k** $-5(x + 3)(x + 1)$ **l** $-7(x - 6)(x - 1)$

7 **a** $2(x + 11)^2$ **b** $3(x - 4)^2$
c $5(x - 5)^2$ **d** $-3(x - 6)^2$
e $-2(x - 7)^2$ **f** $-4(x + 9)^2$

8 **a** **i** $(x^2 + 3x)$ m^2 **ii** $(x^2 + 3x - 10)$ m^2
b $(x + 5)(x - 2)$ m^2
c **i** 30 m^2 **ii** 60 m^2

9 **a** $x + 6$ **b** $x - 3$ **c** $x - 3$
d $\frac{1}{x+7}$ **e** $\frac{1}{x-5}$ **f** $\frac{1}{x-6}$
g $x - 2$ **h** $x + 1$ **i** $x - 8$

Exercise 10D

1 **a** 1 **b** -3 **c** -7 **d** 5
e 0 **f** 0 **g** -2 **h** $\frac{7}{2}$

2 **a**

x	-3	-2	-1	0	1	2
$(x + 2)(x - 1)$	4	0	-2	-2	0	4

b 1, -2 **c** -3, 2

3 **a** $x - 2$, 2 **b** $x + 4$, 1, -4
c $x + 6$, $2x - 7$, -6, 7, $\frac{7}{2}$

4 **a** $x = 0, -1$ **b** $x = 0, 5$ **c** $x = 0, 4$ **d** $x = 3, -2$
e $x = -5, 4$ **f** $x = -1, 1$ **g** $x = 2, -1$ **h** $x = \frac{2}{3}, 7$
i $x = 0, \frac{-5}{4}$ **j** $x = \frac{1}{2}, \frac{-7}{3}$ **k** $x = \frac{5}{4}, \frac{-2}{5}$ **l** $x = \frac{-3}{8}, \frac{-3}{4}$

5 **a** $x = 0, 4$ **b** $x = 0, 3$ **c** $x = 0, -2$
d $x = 0, 4$ **e** $x = 0, 5$ **f** $x = 0, -2$

6 **a** $x = -5, 5$ **b** $x = 6, -6$ **c** $x = 10, -10$
d $x = \frac{3}{2}, \frac{-3}{2}$ **e** $x = \frac{4}{3}, \frac{-4}{3}$ **f** $x = \frac{9}{7}, \frac{-9}{7}$

7 **a** $x = -2, -1$ **b** $x = -3, -2$ **c** $x = 2, 4$
d $x = 5, 2$ **e** $x = -6, 2$ **f** $x = -5, 3$
g $x = 5, -4$ **h** $x = 8, -3$ **i** $x = 4, 8$
j $x = -2$ **k** $x = -5$ **l** $x = 4$
m $x = 7$ **n** $x = 12$ **o** $x = -9$

8 **a** 2 **b** 2 **c** 1 **d** 1 **e** 2
f 2 **g** 1 **h** 1 **i** 1

9 **a** **i** 3.2 m **ii** 4.8 m
b 0, 10
c 10 seconds

10 **a** $x = -2, -6$ **b** $x = -1, 11$
c $x = 3$ **d** $x = 2$

11 **a** 150 cm^2
b **i** $x(x + 5)$ cm^2 = $(x^2 + 5x)$ cm^2 **ii** $(x^2 + 5x - 150)$ cm^2
c $(x + 15)(x - 10)$ cm^2
d 10 **e** 20

Exercise 10E

1 **a** $x + 4$ **b** $x + 10$ **c** $x - 7$ **d** $x - 2$ **e** $x + 6$

2 **a** $x^2 + 2x - 3 = 0$
b $x^2 - 3x - 5 = 0$
c $x^2 + 7x - 4 = 0$

3 **a** $x = 3$ **b** $x = 5$ **c** $x = 4$

4 **b** $(x + 5)$ m
c $x(x + 5) = 24$
d $x^2 + 5x - 24 = 0$, $x = -8, 3$
e Width = 3 m, length = 8 m

5 **a** Width = 6 m, length = 10 m
b Width = 9 m, length = 7 m
c Width = 14 mm, length = 11 mm

6 **a** $A = \frac{1}{2}x(x + 2)$ **b** $\frac{1}{2}x(x + 2) = 4$
c $x^2 + 2x - 8 = 0$ **d** $x = 2$, $h = 4$

ISBN: 9781107687028 Cambridge University Press

Exercise 10E cont.

7 Height = 2 m, base = 7 m

8 a $x^2 + x - 132 = 0$

b -12, 11

c -12, -11 and 11, 12

9 8 and 9 or -9 and -8

10 15

11 1 m

12 a 3.75 m

b $t = 1$ second, 3 seconds

c The ball will reach this height both on the way up and on the way down.

d $t = 0$ seconds, 4 seconds

e $t = 2$ seconds

f The ball reaches a maximum height of 4 m.

g No, 4 metres is the maximum height. If $h = 5$, there is no solution.

13 a $x = 0, 100$

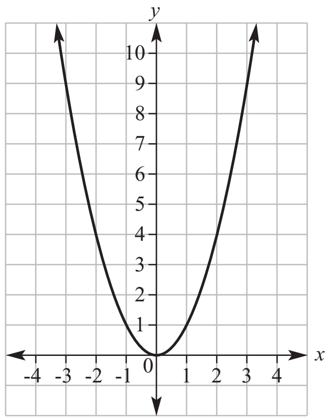
b The rocket starts at the launching site, i.e. at ground level, and hits the ground again 100 metres from the launching site.

c 2 m or 98 m

Exercise 10F

1

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



2 a maximum b (-2, 9) c (0, 5)

d (-5, 0), (1, 0) e $x = -2$

3 a i (2, -5.4), minimum ii $x = 2$

iii (-1, 0), (5, 0) iv (0, -3)

b i (2, 0), maximum ii $x = 2$

iii (2, 0) iv (0, -1)

c i (2, 5), minimum ii $x = 2$

iii no x-intercepts iv (0, 7)

d i (-3, 0), minimum ii $x = -3$

iii (-3, 0) iv (0, 4)

e i (2, -2), minimum ii $x = 2$

iii (1, 0), (3, 0) iv (0, 6)

f i (0, 3), maximum ii $x = 0$

iii (-3, 0), (3, 0) iv (0, 3)

4

	Formula	Max or min	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$	Min	No	(0, 0)	$y = 3$	Narrower
b	$y = \frac{1}{2}x^2$	Min	No	(0, 0)	$y = \frac{1}{2}$	Wider
c	$y = 2x^2$	Min	No	(0, 0)	$y = 2$	Narrower
d	$y = -4x^2$	Max	Yes	(0, 0)	$y = -4$	Narrower
e	$y = -\frac{1}{3}x^2$	Max	Yes	(0, 0)	$y = -\frac{1}{3}$	Wider
f	$y = -2x^2$	Max	Yes	(0, 0)	$y = -2$	Narrower

5

	Formula	Turning point	Axis of symmetry	y-intercept ($x = 0$)	x-intercept
a	$y = (x + 3)^2$	(-3, 0)	$x = -3$	(0, 9)	(-3, 0)
b	$y = (x - 1)^2$	(1, 0)	$x = 1$	(0, 1)	(1, 0)
c	$y = (x - 2)^2$	(2, 0)	$x = 2$	(0, 4)	(2, 0)
d	$y = (x + 4)^2$	(-4, 0)	$x = -4$	(0, 16)	(-4, 0)

ISBN: 9781107687028

6

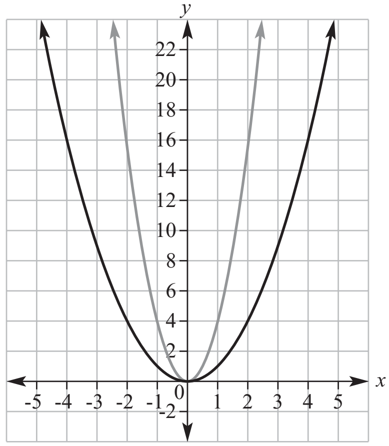
	Formula	Turning point	y-intercept ($x = 0$)	y value when $x = 1$
a	$y = x^2 + 3$	(0, 3)	(0, 3)	$y = 4$
b	$y = x^2 - 1$	(0, -1)	(0, -1)	$y = 0$
c	$y = x^2 + 2$	(0, 2)	(0, 2)	$y = 3$
d	$y = x^2 - 4$	(0, -4)	(0, -4)	$y = -3$

7 **a** vi **b** iii **c** v **d** iv **e** i **f** ii

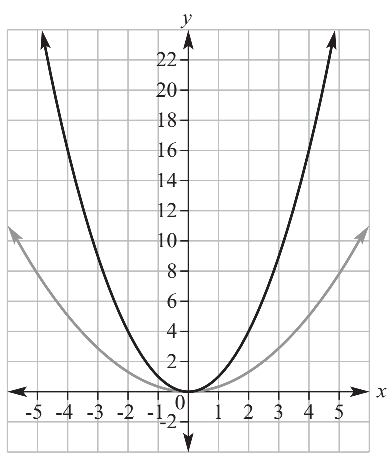
8 **a** $y = x^2 + 2$ **b** $y = -x^2$

c $y = (x + 1)^2$ **d** $y = (x - 5)^2$

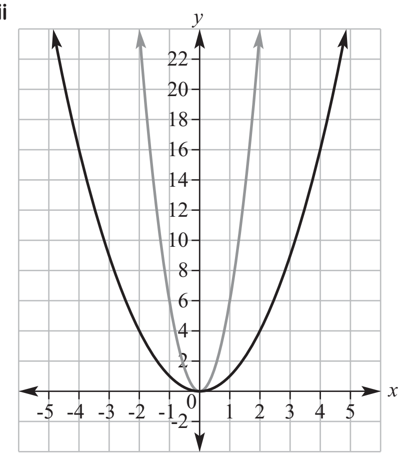
9 **a** **i**



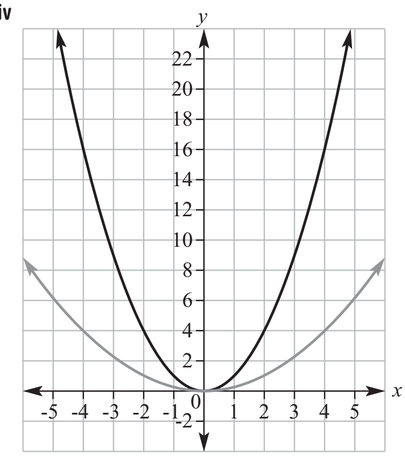
ii



iii



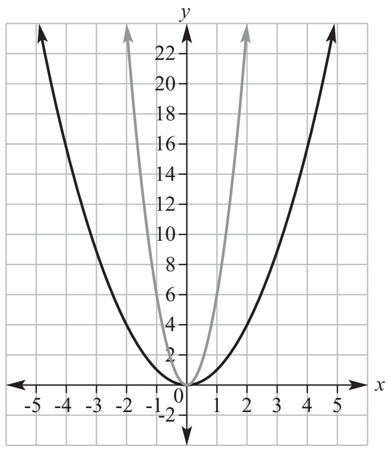
iv



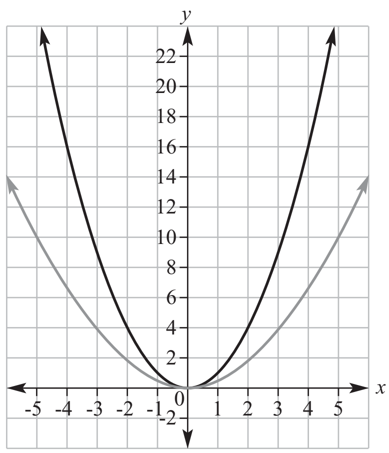
ISBN: 9781107687028

Exercise 10F cont.

v

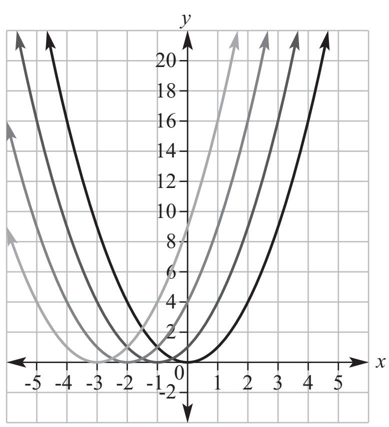


vi

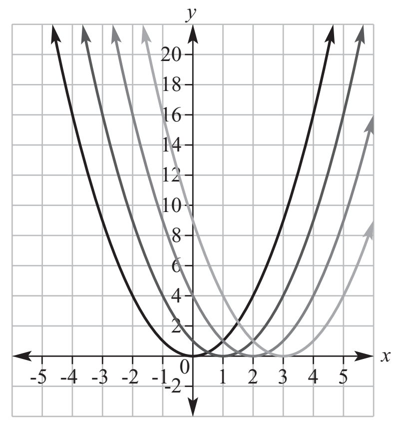


b The constant a determines the narrowness of the graph.

10 a i

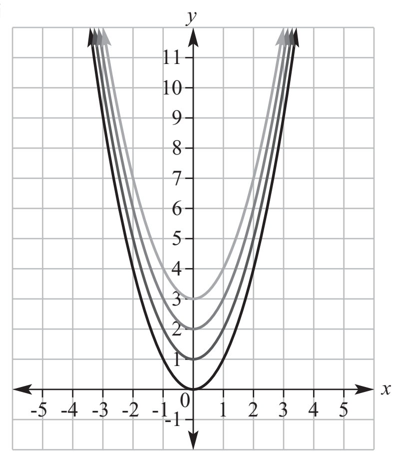


ii

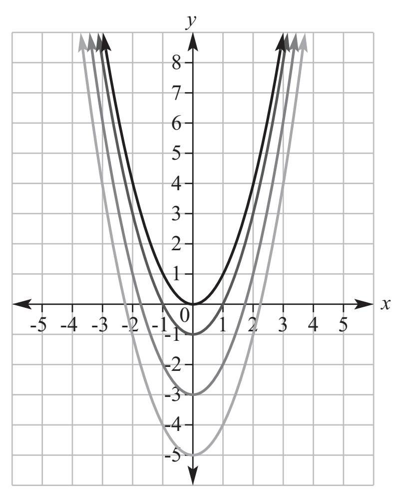


b The constant h determines whether the graph moves left or right from $y = x^2$.

11 a i



ii



b The constant k determines whether the graph moves up or down from $y = x^2$.

ISBN: 9781107687028

Cambridge University Press

Exercise 10G

1 a (0, 0), $r = 3$ **b** (0, 0), $r = 6$ **c** (-1, 0), $r = 2$

2 a ± 2.2 **b** ± 4 **c** ± 3.3

3 a (0, 0) **b** r

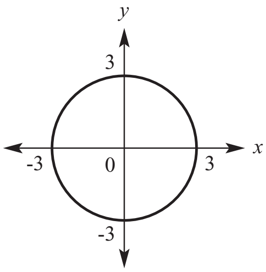
4 a 1 **b** 2 **c** 16 **d** 1 **e** 3

f 27 **g** 1 **h** 16 **i** 1 **j** 25

5 a (0, 0) **b** $r = 3$

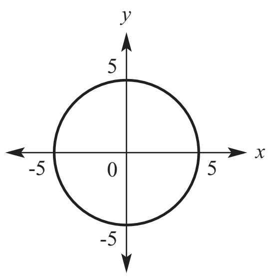
c $y = \pm 2.2$ **d** $x = \pm 3$

e



6 a (0, 0) **b** $r = 5$ **c** $y = \pm 3$

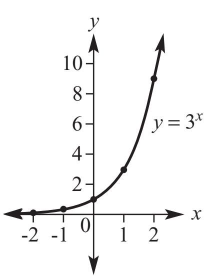
d $x = \pm 5$ **e**



7 a

x	-2	-1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

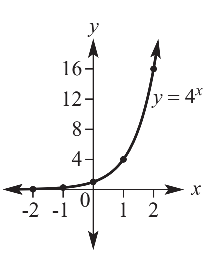
b



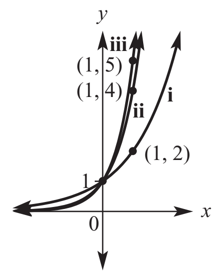
8 a

x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

b



9 a



b The same point (0, 1)

c Makes it rise more quickly

10 a $r = 6$ **b** $r = 9$ **c** $r = 12$

d $r = \sqrt{5}$ **e** $r = \sqrt{14}$ **f** $r = \sqrt{20}$

11 $x^2 + y^2 = 49$

12 a C **b** A **c** B

13 a 1000

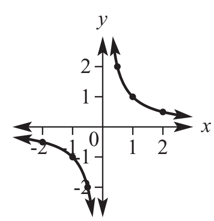
b i 2000 **ii** 8000

c i 2 years **ii** 4 years

14 a

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$

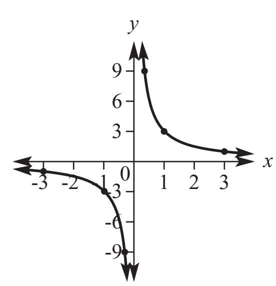
b



15 a

x	-3	-1	$-\frac{1}{3}$	$\frac{1}{3}$	1	3
y	-1	-3	-9	9	3	1

b



ISBN: 9781107687028

Puzzles and games

1 PARABOLA
2 a $(x + 4)^2\ \text{m}^2 = (x^2 + 8x + 16)\ \text{m}^2$ **b** 6
3 14 cm by 20 cm
4 12, 14 or -14, -12
5 64 and 8
6 5 cm
7 a $7x - 6$ **b** $-4x$ **c** $x - 3$
8 25 km/h

Multiple-choice questions

1 E **2** D **3** B **4** C **5** C
6 B **7** B **8** D **9** E **10** C

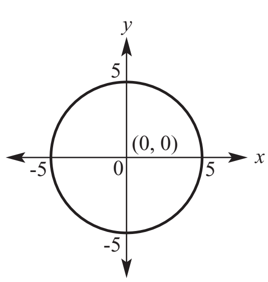
Short-answer questions

1 a $-2x - 2$ **b** $x^2 + 3x$ **c** $x^2 + x - 2$
d $3x^2 + 11x - 20$ **e** $x^2 - 16$ **f** $25x^2 - 4$
g $x^2 + 4x + 4$ **h** $x^2 - 12x + 36$
i $12x^2 - 23x + 10$
2 a $x^2 + 3x$ **b** $x^2 + 4x + 4$ **c** $4x^2 + 18x$
3 a $3(x - 3)$ **b** $-4(x + 4)$ **c** $x(x + 2)$
d $b(a - 1)$ **e** $7x(1 - 2x)$ **f** $-ab(a + 6)$
4 a $(x + 7)(x - 7)$ **b** $(3x + 4)(3x - 4)$
c $(2x + 1)(2x - 1)$ **d** $3(x + 5)(x - 5)$
e $2(x + 3)(x - 3)$ **f** $(2x + 9)(2x - 9)$
5 a $(x + 2)(x + 3)$ **b** $(x - 3)(x + 2)$ **c** $(x - 6)(x - 2)$
d $(x + 12)(x - 2)$ **e** $(x + 10)(x - 5)$ **f** $(x - 8)(x - 4)$
g $(x - 3)^2$ **h** $(x + 10)^2$ **i** $(x + 20)^2$
6 a $x = -1, 2$ **b** $x = 3, -7$ **c** $x = \frac{1}{2}, -4$
d $x = 0, 3$ **e** $x = 0, -6$ **f** $x = 0, \frac{5}{2}$
7 a $x = 0, -4$ **b** $x = 0, 3$ **c** $x = 5, -5$
d $x = \pm\frac{4}{3}$ **e** $x = -3, -5$ **f** $x = 3, 7$
g $x = 4$ **h** $x = -5$ **i** $x = -9, 4$
8 Length = 8 m, width = 6 m
9 a minimum at (1, -4) **b** $x = 1$
c (-1, 0) and (3, 0) **d** (0, -3)

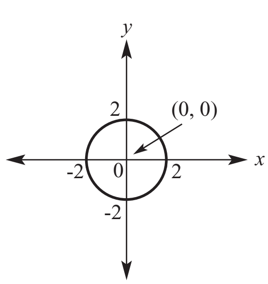
10

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y - x^2$
a	$y = 2x^2$	Min	No	(0, 0)	2	Narrower
b	$y = -\frac{1}{2}x^2$	Max	Yes	(0, 0)	$-\frac{1}{2}$	Wider
c	$y = (x - 2)^2$	Min	No	(2, 0)	1	Same
d	$y = x^2 - 2$	Min	No	(0, -2)	-1	Same

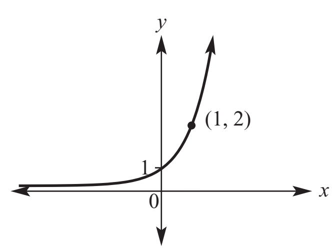
11 a



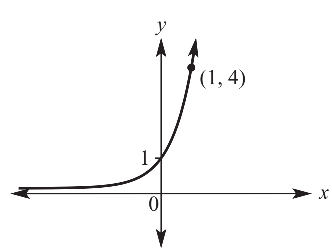
b



12 a



b



ISBN: 9781107687028

Extended-response questions

1 **a** **i** x^2 m^2 **ii** $(100 - x^2)$ m^2
b $(10 + x)(10 - x)$ m^2
c **i** 96 m^2 **ii** 84 m^2
d $x = 5$

2 **a** **i** $(16 + 2x)$ m **ii** $(12 + 2x)$ m
b Area = $(4x^2 + 56x + 192)$ m^2
c Trench = $(4x^2 + 56x)$ m^2
d Minimum width is 2 m.

Semester review 2

Straight line graphs

Multiple-choice questions

1 D **2** E **3** B **4** A **5** D

Short-answer questions

1 **a** \$6000 **b** 8 years **c** \$500

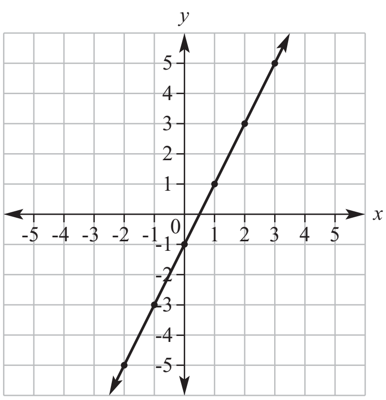
2 **a** **i** 16 km **ii** 24 km **iii** 32 km
b 16 km/h **c** 1.5 hours
d 45 km **e** 2.5 hours
f 18 km/h **g** 90 km

3 **a**

x	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5

b (-2, -5), (-1, -3), (0, -1), (1, 1), (2, 3), (3, 5)

c



4 **a** AB midpoint (1, 2), length = $\sqrt{68}$, gradient $m = \frac{1}{4}$
b PQ midpoint (-1, -1), length = $\sqrt{72}$, gradient $m = -1$
c GH midpoint (0.5, 1), length = 7, gradient $m = 0$
d ST midpoint (2, 0), length = 4, gradient undefined

5 **a** $m = 2$ **b** $m = -3$

6 **a** PQ length = $\sqrt{41}$ **b** RM length = $\sqrt{89}$

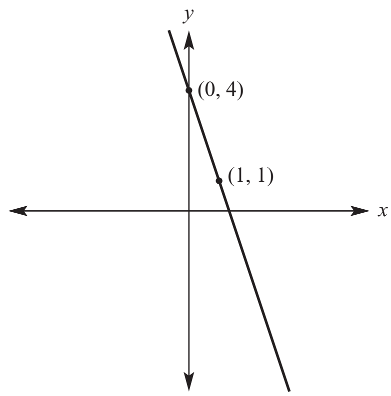
7 **a** TK midpoint (4, 8) **b** FG midpoint (1, -2)

8 **a** Line J: y-intercept = -2, $m = 2$, $y = 2x - 2$

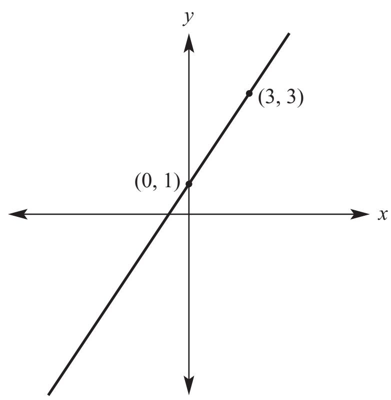
Line K: y-intercept = 6, $m = -\frac{2}{3}$, $y = -\frac{2}{3}x + 6$

b (3, 4)

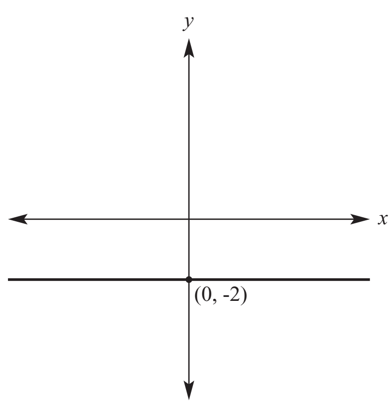
9 **a**



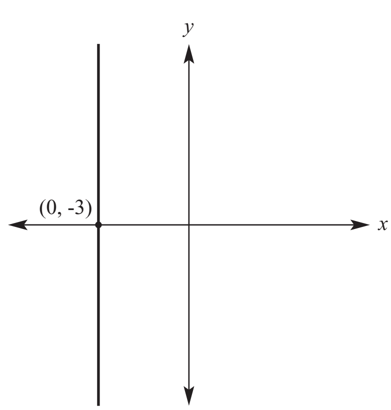
b



c



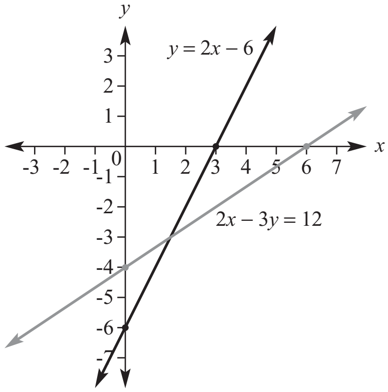
d



Short-answer questions cont.

10 a x-intercept (3, 0), y-intercept (0, -6)

b x-intercept (6, 0), y-intercept (0, -4)



Extended-response question

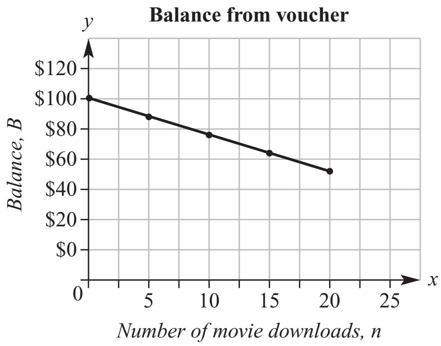
1 a $B = 100 - 2.4n$

b i \$76 **ii** 30 movies

c

Number of movies, n	0	5	10	15	20
Balance, B	\$100	\$88	\$76	\$64	\$52

d



e 41 movies, \$1.60 remaining on voucher

Geometry

Multiple-choice questions

1 C **2** E **3** C **4** A **5** D

Short-answer questions

1 a 39 **b** 61 **c** 127 **d** 75 **e** 70 **f** 117

g 71 **h** 84 **i** 110 **j** 120 **k** 135 **l** 50

2 a $\Delta ABC \equiv \Delta DEF$ (SAS)

b $\Delta ABC \equiv \Delta DEF$ (RHS)

c $\Delta STU \equiv \Delta MNO$ (AAS)

d $\Delta XYZ \equiv \Delta ABC$ (SSS)

3 a Yes, (RHS) **b** Yes, (AAA)

c Yes, (SAS) **d** Yes, (SSS)

4 $x = 1.8, y = 5$

Extended-response question

1 a AAA **b** 2.5 **c** 5 m

Equations

Multiple-choice questions

1 C **2** D **3** A **4** B **5** E

Short-answer questions

1 a $x = -9$ **b** $x = 26$ **c** $m = 32$

d $a = 4$ **e** $k = -6$ **f** $x = -8$

2 a $p = 2$ **b** $a = 4$

c $x = 12$ **d** $x = 20$

3 a $x = 8$ **b** $k = 3$ **c** $m = 9$

d $x = 5$ **e** $a = -4$ **f** $x = 4$

4 a $x - 5 = 8$; $x = 13$

b $4x + 8 = 20$; $x = 3$

c $2(3x - 6) = 18$; $x = 5$

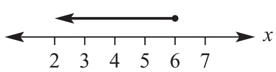
5 a $b = 10$

b $P = 400$

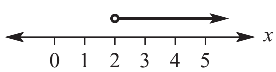
6 a $x \le 4$

b $x > 8$

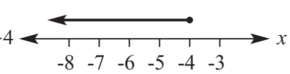
7 a $x \le 6$



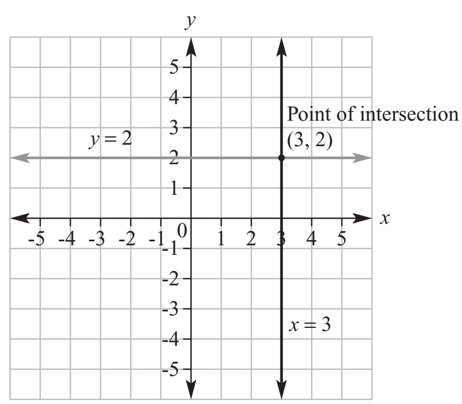
b $x > 2$



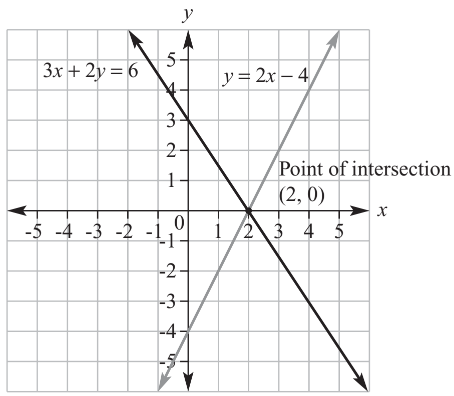
c $x \le -4$



8 a (3, 2)



b (2, 0)



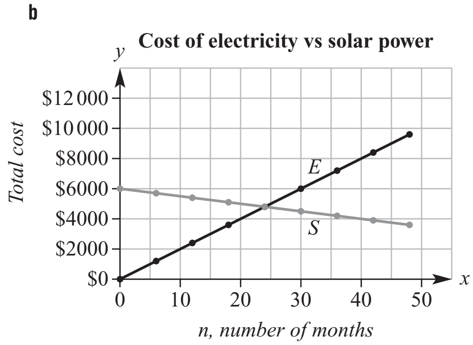
9 a (1, 2) **b** (3, 9) **c** (-1, 4)
10 a (1, 1) **b** (2, 4) **c** (3, 1)
11 a Let a = Oliver's age, b = Ruby's age (any pronumeral selection is correct)
b $a - b = 7$, $a + b = 45$
c Oliver is 26 years old, Ruby is 19 years old

Extended-response question

1 a

n	*E*	*S*
0	\$0	\$6000
6	\$1200	\$5700
12	\$2400	\$5400
18	\$3600	\$5100
24	\$4800	\$4800
30	\$6000	\$4500
36	\$7200	\$4200
42	\$8400	\$3900
48	\$9600	\$3600

b



c $n = 24$ months, $E = S = \$4800$
d $E = 200n$
$S = 6000 - 50n$
e 24 months **f** \$6000
g 27 months

Pythagoras' theorem and trigonometry

Multiple-choice questions

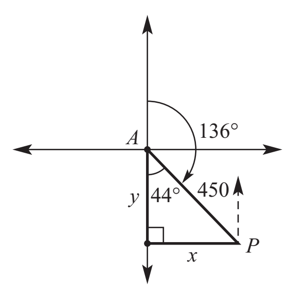
1 E **2** C **3** A **4** B **5** D

Short-answer questions

1 a PR **b** QR **c** PQ
2 a $\frac{y}{x}$ **b** $\frac{z}{x}$ **c** $\frac{y}{z}$
3 a 0.64 **b** 0.63
c 0.32 **d** 7.50
e 31.25 **f** 4.59
g 36.88 **h** 32.33
i 9.77
4 a 11.40 **b** 0.83
c 6.02 **d** 14.69
5 a 10.2 **b** 27.0
c 18.6 **d** 21.2
6 14 m **7** 8.95 m
8 a 60° **b** 37° **c** 77°
9 177.9 m
10 259.8 m

Extended-response question

1 a



b 312.6 km **c** 323.7 km **d** 316°

Quadratics and non-linear graphs

Multiple-choice questions

1 B **2** C **3** E **4** A **5** D

Short-answer questions

1 a $-2x + 2$ **b** $x^2 - x - 6$ **c** $2x^2 - x - 21$
d $x^2 - 4$ **e** $x^2 - 6x + 9$ **f** $4x^2 + 4x + 1$

ISBN: 9781107687028

Short-answer questions cont.

2 **a** $3(x-4)$ **b** $-x(2+x)$
c $(x+5)(x-5)$ **d** $(3x+10)(3x-10)$
e $(x+3)(x+4)$ **f** $(x-3)(x+2)$
g $(x+4)(x-2)$ **h** $(x-4)^2$ **i** $(x+3)^2$

3 **a** $x=0, 3$ **b** $x=0, -2$ **c** $x=-2, 2$
d $x=\frac{-3}{2}, \frac{3}{2}$ **e** $x=3, \frac{1}{2}$ **f** $x=5, -4$
g $x=-3, -7$ **h** $x=-4$ **i** $x=7$

4 **a** $(-1, -1)$, Minimum **b** $x=-1$
c $(-2, 0), (0, 0)$ **d** $(0, 0)$

5

	Formula	Maximum or minimum	Reflected in x-axis (yes/no)	Turning point	y value when $x=1$	Wider or narrower than $y=x^2$
a	$y=3x^2$	Minimum	No	(0, 0)	3	Narrower
b	$y=-\frac{1}{2}x^2$	Maximum	Yes	(0, 0)	$-\frac{1}{2}$	Wider
c	$y=(x+2)^2$	Minimum	No	(-2, 0)	9	Same
d	$y=x^2-3$	Minimum	No	(0, -3)	-2	Same

Extended-response question

1 **a** **i** x^2 m^2
ii $(400-x^2)$ m^2
b $(20+x)(20-x)$ m^2
c **i** 384 m^2
ii 319 m^2
d **i** 15
ii 10

ISBN: 9781107687028 Cambridge University Press