Chapter 13 – Graphs, networks and trees: travelling and connecting problems

Solutions to Exercise 13A

1 a i There are three edges connected to town A, so $\deg(A) = 3$.

ii There are two edges connected to town B, so $\deg(B) = 2$.

iii There is one edge connected to town H, so $\deg(H) = 1$.

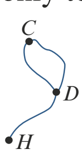
b sum of degrees

$= \deg(A) + \deg(B)$
$\quad + \deg(C) + \deg(D) + \deg(H)$
$= 3 + 2 + 4 + 4 + 1$
$= 14$

Alternatively: The total number of edges for this graph is 7.
The sum of the degrees of a graph
$= 2 \times$ the total number of edges
$= 2 \times 7 = 14$

c If the edge connecting towns D and H was removed, town H would not be connected to the other towns, directly or indirectly. Every other edge, if removed, would not result in one or more vertices being isolated from the other vertices. Therefore, for this graph one bridge exists between towns D and H.

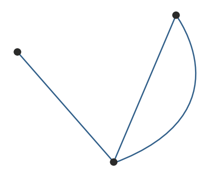
d A possible subgraph that contains only towns H, D and C is



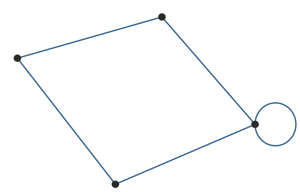
Note: Some subgraphs are possible where vertices are isolated. These subgraphs have not been shown.

2 *Note: only one alternative has been shown for the answers to the following questions. Others are possible.*

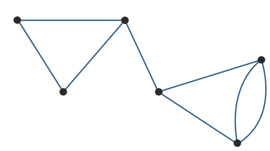
a



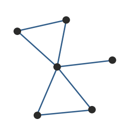
b



c



d



3 a The graphs in **ii**, **iii** and **iv** have all connections between vertices the same, but the graph in **i** does not. For example, they have two edges between A and C but the graph in i does not.

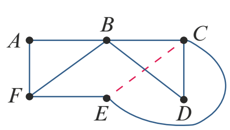
Graph **i** is not isomorphic to the others.

b The graphs in **i**, **iii** and **iv** have all connections between vertices the same, but the graph in **ii** does not. For example, they do not have an edge between A and C but the graph in **ii** does.
Graph **ii** is not isomorphic to the others.

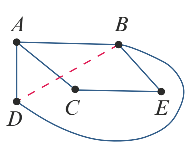
c The graphs in **i**, **iii** and **iv** have all connections between vertices the same, but the graph in **ii** does not. For example, they do not have an edge between E and C but the graph in **ii** does.
Graph **ii** is not isomorphic to the others.

4 *Note: In the graphs for this question, dotted edges show the edges that are repositioned in order to demonstrate the planar nature of the graphs. There are other solutions possible.*

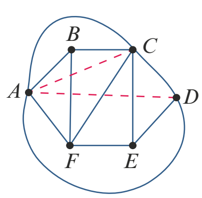
a



b



c



d This graph is non-planar and cannot be redrawn.

5 **a** **i** There are six vertices, so $v = 8$.
There are eight faces, so $f = 6$.
There are twelve edges, $e = 12$.

ii $v + f = e + 2$
$8 + 6 = 12 + 2$
$14 = 14$
Euler's formula is verified.

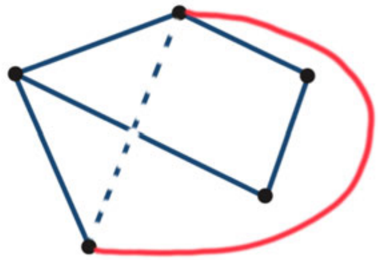
b **i** There are eight vertices, so $v = 6$.
There are six faces, so $f = 8$.
There are twelve edges, $e = 12$.

ii $v + f = e + 2$
$6 + 8 = 12 + 2$
$14 = 14$
Euler's formula is verified.

c **i** There are seven vertices, so $v = 7$.
There are seven faces, so $f = 7$.
There are twelve edges, $e = 12$.

ii $v + f = e + 2$
$7 + 7 = 12 + 2$
$14 = 14$
Euler's formula is verified.

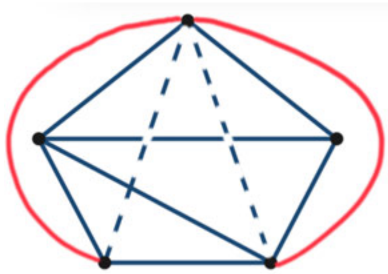
d **i** There are seven vertices, so $v = 5$.
There are seven faces, so $f = 3$.
Note: the graph must be redrawn without any edges crossing to identify the faces.



There are twelve edges, $e = 6$.

ii $v + f = e + 2$
$5 + 3 = 6 + 2$
$8 = 8$
Euler's formula is verified.

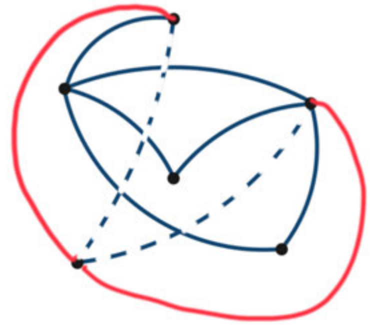
e **i** There are seven vertices, so $v = 5$.
There are seven faces, so $f = 6$.
Note: the graph must be redrawn without any edges crossing to identify the faces.



There are twelve edges, $e = 9$.

ii $v + f = e + 2$
$5 + 6 = 9 + 2$
$11 = 11$
Euler's formula is verified.

f **i** There are seven vertices, so $v = 6$.
There are seven faces, so $f = 4$.
Note: the graph must be redrawn without any edges crossing to identify the faces.



There are twelve edges, $e = 8$.

ii $v + f = e + 2$
$6 + 4 = 8 + 2$
$10 = 10$
Euler's formula is verified.

6 **a** $v + f = e + 2$
$8 + f = 10 + 2$
$8 + f = 12$
$f = 12 - 8$
$f = 4$

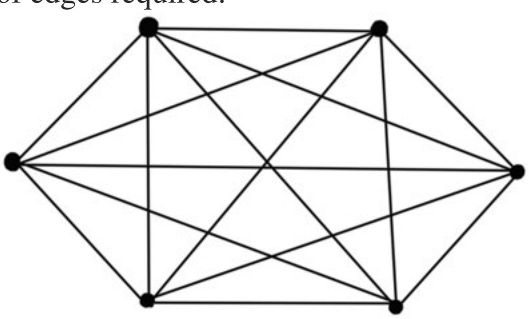
b $v + f = e + 2$
$v + 4 = 14 + 2$
$v + 4 = 16$
$v = 16 - 4$
$v = 12$

c $v + f = e + 2$
$10 + 11 = e + 2$
$21 = e + 2$
$21 - 2 = e$
$e = 19$

7 Connected planar graph, therefore Euler's formula will be verified:
$v + f = e + 2$
$v = 8$ and $e = 13$
$8 + f = 13 + 2$
$f = 15 - 8$
$f = 7$

8 The total number of edges for this graph is 7.
The sum of the degrees of a graph
$= 2 \times$ the total number of edges
$= 2 \times 7 = 14$

9 A complete graph is where all vertices are connected directly to all other vertices in the graph.
Method 1
Draw this graph and count the number of edges required:



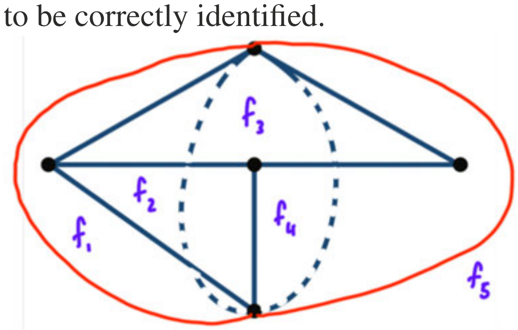
Method 2
A complete graph with n vertices will have $\frac{n(n-1)}{2}$ edges.
This graph has six vertices, so $n = 6$ and the graph will have $\frac{6(6-1)}{2} = \frac{6(5)}{2} = \frac{30}{2} = 15$ edges.

Exam 1 style questions

10 Identify the degree of each vertex (count how many edges are connected to each vertex). There is one vertex with degree 2, two vertices with degree 3 and three vertices with degree 4.
C

11 Planar graph, therefore Euler's formula applies: $v + f = e + 2$
For this graph $f = 4$.
$v + 4 = e + 2$
Test each option to find the correct number of vertices and edges that hold true for the formula above.
Option E: $v = 5$ and $e = 7$
$5 + 4 = 7 + 2$
$9 = 9$
Euler's formula is verified.
E

12 The graph must be redrawn without any edges crossing for the number of faces to be correctly identified.



There are five faces.
C

13 Consider each option:

- The graph is planar. *TRUE*; the graph is connected and it can be drawn so no edges are crossing.

- The graph contains a bridge. *FALSE*; if any one edge is removed, the graph remains connected.

- It is a simple graph. *FALSE*; the graph has multiple edges (two or more edges that connect the same vertices).

- The sum of degrees of the vertices is 16. *TRUE*; the total number of edges is 8 and the sum of the degrees of a graph $= 2 \times$ the total number of edges $= 2 \times 8 = 16$

- It is a complete graph. *FALSE*; every vertex is not connected directly to every other vertex in the graph.

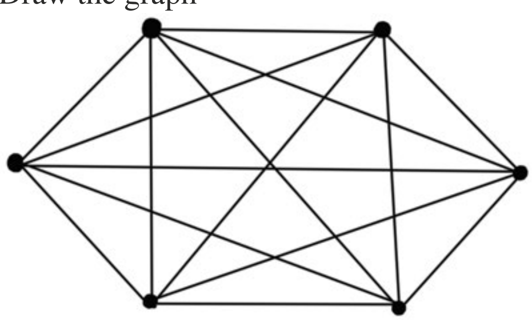
  There are two true statements.

  B

14 A complete graph must have every vertex connected directly to every other vertex in the graph with an edge.

Method 1

Draw the graph



Method 2

A complete graph with n vertices will have $\frac{n(n-1)}{2}$ edges.

This graph has six vertices, so $n = 6$ and the graph will have $\frac{6(6-1)}{2} = \frac{6(5)}{2} = \frac{30}{2} = 15$ edges. There are already 3 edges, therefore 12 more edges must be added to make this a complete graph.

E

15 This graph has 6 vertices. The minimum number of edges required for this graph to be connected is 5. The graph has 15 edges (this is found by either counting the number of edges in the graph or considering a complete graph with n vertices will have $\frac{n(n-1)}{2}$ edges, where this graph has six vertices, so $n = 6$ and the graph will have $\frac{6(6-1)}{2} = \frac{6(5)}{2} = \frac{30}{2} = 15$ edges.).

There are 15 edges, so 10 must be removed in order for the graph with 6 vertices to be connected with the minimum number of edges (5).

C

Solutions to Exercise 13B

1 *For each of the following, a square matrix is constructed where the number of rows and columns correspond to each of the vertices in the graph. Each row and column are labelled with the letter it represents.*

a
$$\begin{array}{cc} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

b
$$\begin{array}{cc} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

c
$$\begin{array}{cc} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

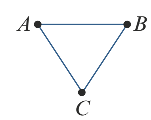
d
$$\begin{array}{cc} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

e
$$\begin{array}{cc} & \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

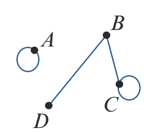
f
$$\begin{array}{cc} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

2 *For each of the following, a graph is drawn using a vertex for each row/column of the matrix.*

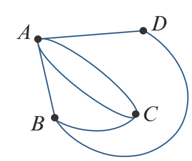
a



b



c



3 The zero in row C, column A show that vertex C is not connected to vertex A. There is a zero in row C, column B and row C, column C as well. This means that C is not connected to any other vertex, so it is isolated.

4 If every vertex has a loop, there will be a '1' in every position along the main diagonal (top left, to bottom right), that is in position (A,A), (B,B), . . .

5 The graph

- has no loops, so the diagonal will be all zeros
- has no duplicate edges, so there will only be '0' or '1'
- is complete, so every vertex is connected to every other vertex. Every position in the matrix will be a '1', except for the diagonal.

The adjacency matrix for the graph is

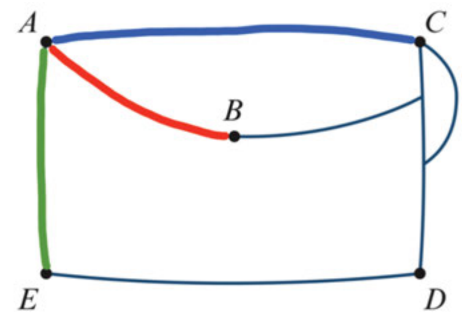
$$\begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Exam 1 style questions

6 Construct a matrix to represent the graph. It will have 5 rows and 5 columns for the 5 vertices:

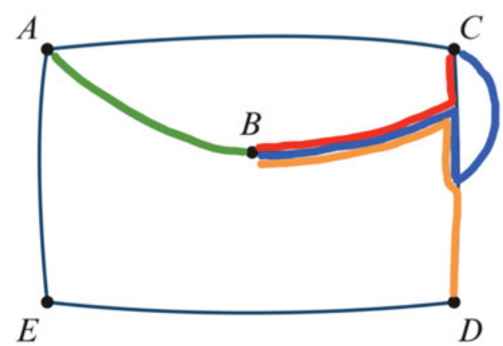
Vertex *A* has:

- one connection to *B*
- one connection to *C*
- no connection to *D*
- one connection to *E*



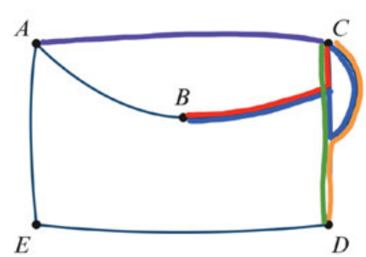
Vertex *B* has:

- one connection to *A*
- two different connections to *C*
- one connection to *D*
- no connection to *E*



Vertex *C* has:

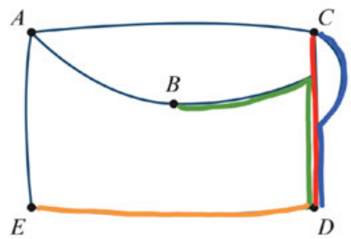
- one connection to *A*
- two different connections to *B*
- two different connections to *D*
- no connection to *E*



Vertex *D*

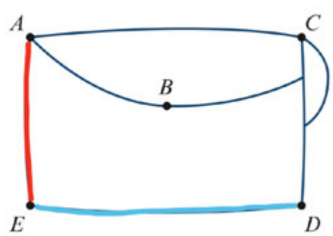
has:

- no connection to A
- one connection to B
- two different connections to C
- one connection to E



Vertex E has:

- one connection to A
- no connection to B
- no connection to C
- one connection to D



The adjacency matrix that represents this graph is:

$$\begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 & 0 \\ 1 & 2 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

E

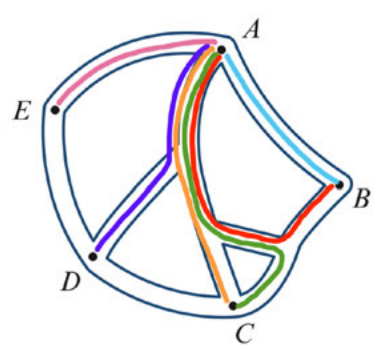
7 Look for key features in the matrix to eliminate incorrect options.

- Row A column B has a 3, thereofre there should be 3 edges between vertices A and B; eliminate options **B** and **C**
- Row B column B has a zero, therefore there should be no loop at vertex B; eliminate option **D**
- Row D column D has a zero, therefore there should be no loop at vertex D; eliminate option **E**

Final remaining graph is correct.
A

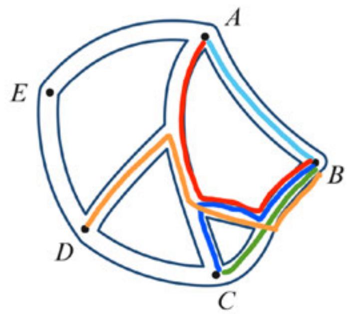
8 Construct a matrix to represent the graph. It will have 5 rows and 5 columns for the 5 vertices:
Vertex A has:

- two different connections to B
- two different connections to C
- one connection to D
- one connection to E



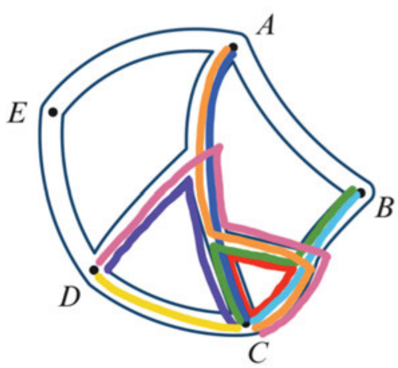
Vertex B has:

- two different connections to A
- two different connections to C
- one connection to D
- no connection to E



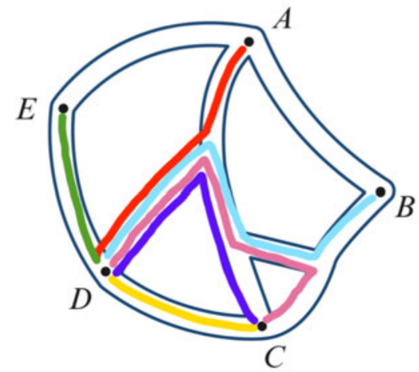
Vertex C has:

- two different connections to A
- two different connections to B
- a *loop*
- three different connections to D
- no connection to E



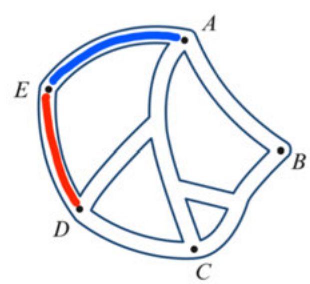
Vertex D has:

- one connection to A
- one connection to B
- three different connections to C
- one connection to E



Vertex E has:

- one connection to A
- no connection to B
- no connections to C
- one connection to D



From the information listed above, the matrix that represents the map is:

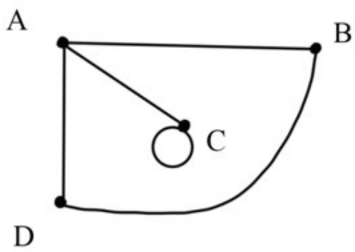
$$\begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 2 & 2 & 1 & 1 \\ 2 & 0 & 2 & 1 & 0 \\ 2 & 2 & 1 & 3 & 0 \\ 1 & 1 & 3 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

The number '1' appears 9 times.

C

9 *Use the matrix constructed for the solution to Question 8 above*. The numbers ’2’ and ’3’ appear 8 times.
C

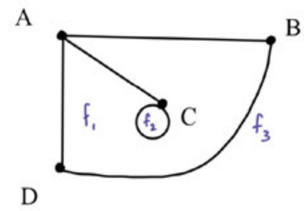
10 Draw the graph represented for the given adjacency matrix.



- the graph above is connected, as there are no isolated vertices; **A** is *TRUE*
- there is a loop at vertex *C*; **B** is *TRUE*
- there are no multiple edges (duplicate edges) between any of the vertices; **C** is *FALSE*
- the graph is connected and can be drawn with no edges crossing; **D** is *TRUE*
- the edge connecting vertices *A* and *C* is a bridge, because if it were removed, the vertex *C* would be isolated from the other vertices; **E** is *TRUE*

Only one option was not true.
C

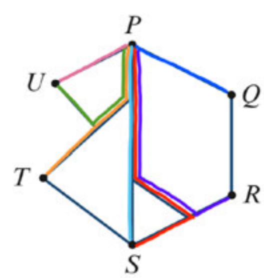
11 *Use the graph drawn for the solution to Question 10 above*:



B

12 Construct a matrix to represent the graph. It will have 6 rows and 6 columns for the 6 vertices:
Vertex *P* has:

- one connection to *Q*
- one connection to *R*
- two different connections to *S*
- one connection to *T*
- two different connections to *U*

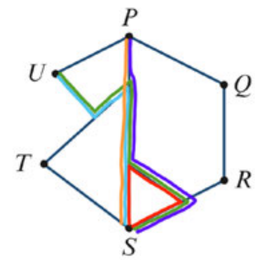


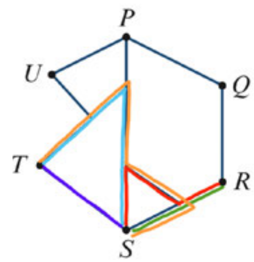
From this information, eliminate the incorrect options **A** and **B**

Rows *Q* and *R* are identical for the remaining options **C,D** and **E**.

Vertex S has:

- two different connections to P
- no connection to Q
- two different connections to R
- a loop
- three different connections to T
- two different connections to U





From this information, eliminate the remaining incorrect options **C,D**

Only one matrix correctly represents this map.

E

Solutions to Exercise 13C

Note: There are multiple possible answers to the questions in this exercise.

1 **a** This walk starts and ends at different vertices so it is not a cycle, nor circuit. The walk does not repeat edges, nor vertices, so it is a **path**.

b This walk starts and ends at different vertices so is not a cycle nor circuit. The walk has a repeated vertex, but not a repeated edge so the walk is a **trail**.

c This walk starts and ends at different vertices so it is not a cycle, nor circuit. The walk does not repeat edges, nor vertices, so it is a **path**.

d This walk starts and ends at the same vertex, so it could be a circuit or a cycle. however, there is a repeated vertex and a repeated edge, so it will be neither. It cannot be a trail or a path because of the repeated edge and vertex, so this walk is only a **walk**.

e This walk starts and ends at different vertices, so it is not a cycle, nor circuit. The walk has a repeated vertex but not a repeated edge, so this walk is a **trail**.

f This walk starts and ends at different vertices, so it is not a cycle, nor circuit. The walk has not repeated edge and no repeated vertex, so the walk is a **path**.

2 **a** This walk starts and ends at different vertices, so it is not a cycle, nor circuit. The walk has a repeated edge and vertex, so it is a **walk** only.

b This walk starts and ends at the same vertex, so it could be a cycle or a circuit. There is no repeated edge, nor vertex, so the walk is a **cycle**.

c This walk starts and ends at different vertices, so it is not a cycle, nor circuit, There are no repeated edges, nor vertices, so the walk is a **path**.

d This walk starts and ends at different vertices, so it is not a cycle, nor circuit. There are repeated edges and vertices, so the walk is a **walk** only.

e This walk starts and ends at different vertices, so it is not a cycle, nor circuit. There are no repeated edges, nor vertices, so the walk is a **path**.

f This walk starts and ends at different vertices, so it is not a cycle, nor circuit. There are repeated edges and vertices, so the walk is a **walk** only.

g This walk starts and ends at the same vertex, so it could be a cycle or a circuit. There is no repeated edge, however there is a repeated vertex, so the walk is a **circuit**.

h This walk starts and ends at the same vertex, so it could be a cycle or a circuit. There is no repeated edge, nor vertex, so the walk is a **cycle**.

3 **a** **i** This graph has two odd-degree vertices (A and E) and so it will have an eulerian trail.

ii One possible eulerian trail for this graph is $A–B–E–D–B–C–D–A–E$. There are other trails possible.

b **i** This graph has all vertices of odd degree. Neither an eulerian trail, nor eulerian circuit, are possible.

c **i** This graph has two odd-degree vertices (A and F) and so it will have an eulerian trail.

ii One possible eulerian trail for this graph is $A–C–E–C–B–D–E–F$. There are other trails possible

d **i** This graph has all vertices of even degree. Both an eulerian trail and an eulerian circuit are possible.

ii One possible eulerian circuit for this graph is $A–B–C–D–E–C–A$. There are other trails and circuits possible

e **i** This graph has all vertices of even degree. Both an eulerian trail and an eulerian circuit are possible.

ii One possible eulerian circuit for this graph is $F–E–A–B–E–D–C–B–D–F$. There are other trails and circuits possible.

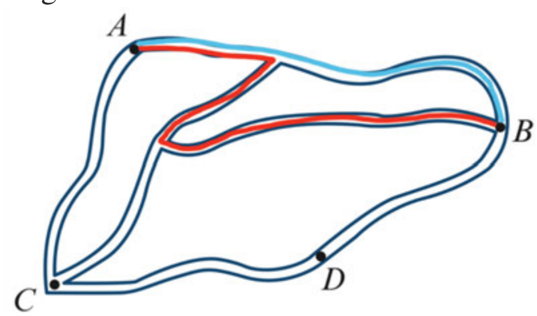
4 **a** A hamiltonian cycle for this graph is $A–B–C–F–I–H–E–G–D–A$.

b A hamiltonian cycle for this graph is $A–B–C–D–E–F–A$.

c A hamiltonian cycle for this graph is $A–B–D–C–E–A$.

5 $F–A–B–C–D–E–H–G$.

6 **a** A vehicle can travel between town A and town B in two ways, without visiting any other town. See the diagram below:



b By inspection, there are 7 different trails from town A to town D. Where there are different routes between two towns, the route is shown by a subscript.

$A–C_1–D$

$A–C_2–D$

$A–B_1–D$

$A–B_2–D$

$A–B_1–C–D$

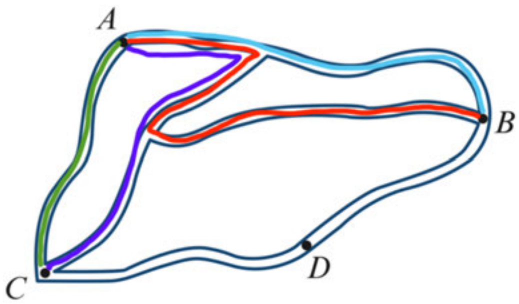
$A–C_1– B_1–D$

$A–C_1– B_2–D$

c Start with 4 vertices to represent the 4 towns.

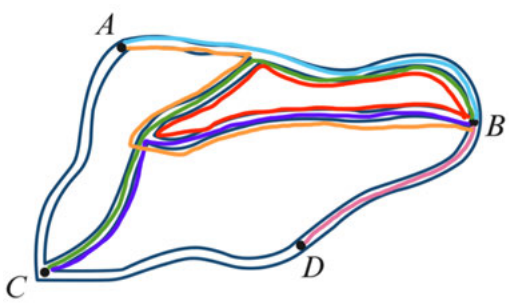
Vertex A has:

- two different connections to B
- two different connections to C
- no connections to D



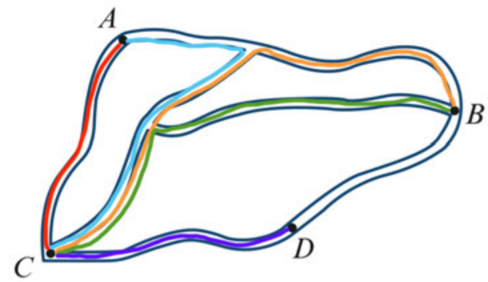
Vertex B has:

- two different connections to A
- two different connections to C
- one connection to D
- one loop



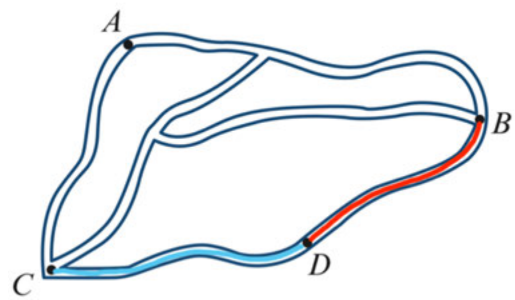
Vertex C has:

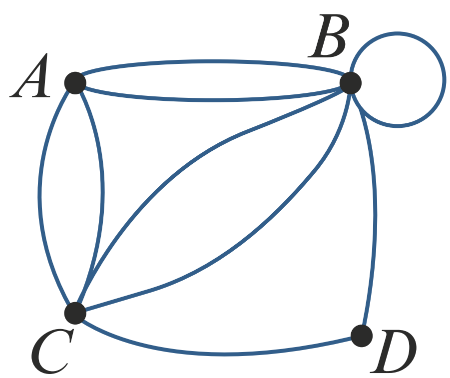
- two different connections to A
- two different connections to B
- one connection to D



Vertex D has:

- no connections to A
- one connection to B
- one connection to C





d An eulerian circuit is not possible through this network because there are some odd-degree vertices (B and C).

7 a $v = 9$, $e = 12$, $f = 5$; $v + f = e + 2$

$9 + 5 = 12 + 2$

$14 = 14$

Euler's formula is verified.

b **i** The walk described will start and end at the same vertex, so it could be either a circuit or a cycle. The walk will not repeat any vertex, therefore it is a cycle and because the organisers will visit *every* vertex the walk is classified as a **Hamiltonian cycle**.

ii Lake Bolac - Streatham - Nerrin Nerrin - Woorndoo - Mortlake - Hexham - Chatworth - Glen-thompson - Wickliffe - Lake Bolac. The reverse of this is the second possible option.

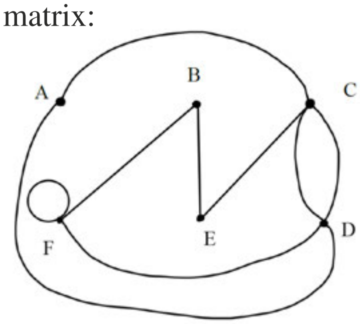
c **i** The walk described will start and end at the same vertex, so it could be either a circuit or a cycle. The walk will pass through every edge which describes an **Eulerian circuit**.

ii All vertices must have an *even* degree for an Eulerian circuit to be possible. The vertices repre-senting the towns of Wickliffe and Lake Bolac both have an *odd* degree, therefore an Eulerian circuit is not possible.

d **i** The proposed race planned in part **c** was an *Eulerian circuit*, however it was not possible because the vertices representing the towns of Wickliffe and Lake Bolac both have an *odd* degree; for an Eulerian circuit *all* vertices must have an *even* degree. For an Eulerian circuit to be possible, the road connecting **Wickliffe and Lake Bolac** could be travelled along twice. Travelling along a road twice can be interpreted as *adding* an extra edge to the graph, thus all vertices would have an even degree and the proposed Eulerian circuit from part **c** would be possible.

ii Lake Bolac - Streatham - Nerrin Nerrin - Woorndoo - Mortlake - Hexham - Chatsworth - Woorn-doo - Lake Bolac - Wickliffe - Chatsworth - Glenthompson - Wickliffe - Lake Bolac

8 Draw the graph represented by the matrix:



a Yes, as the graph above shows, all vertices are either directly or indirectly connected to all other vertices.

b Yes, the graph can be drawn with no edges crossing.

c No, there are no edges which, if removed, would result in a disconnected graph.

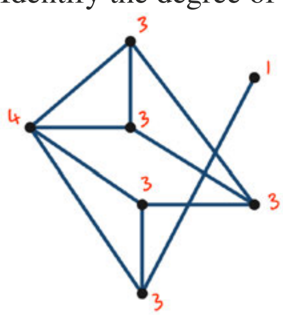
d Yes, as all vertices have an *even* degree.

e Yes, as all vertices have an *even* degree.

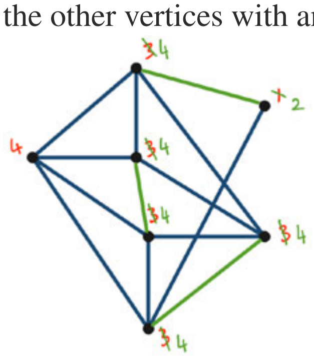
f Yes, a walk is possible where all vertices are visited without repeating any vertices.

g Yes, a walk is possible, where all vertices are visited without repeating any vertices, starting and ending at the same vertex.

9 For an Eulerian circuit to exist, all vertices must have an *even* degree. Identify the degree of each vertex:



Of the 7 vertices, 6 of them have an odd degree. When an edge is added between two vertices, the degree of each of those vertices increases by 1; when an edge is added to a vertex with an odd degree, it will change to an even degree as the number of edges connected to it has increased by 1. Consider connecting 3 vertices with an odd degree with one of the other vertices with an odd degree:



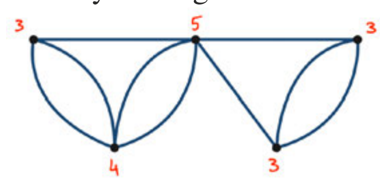
Adding 3 edges to this graph results in all vertices having an even degree; this could not be achieved with a smaller number of edges.

Exam 1 style questions

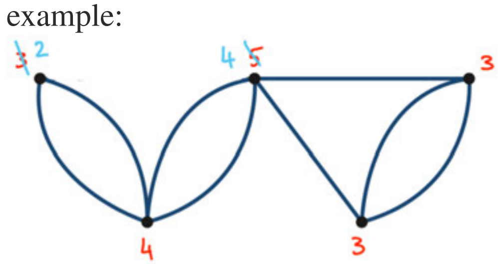
10 For an Eulerian circuit to exist, all vertices must have an *even* degree. Options A, B, D, E are graphs that have at least one vertex with an odd degree. *Note: A loop contributes 2 to the degree of a vertex.*
C

11 The walk $DEFACBD$ is not a Hamiltonian cycle for this graph as there is no edge connecting vertex A to vertex C.
D

12 Identify the degree of each vertex:



An Eulerian trail is possible if there are *zero* or *two* vertices with an odd degree. The original graph has 5 vertices and 4 of them have an odd degree. When removing an edge from the graph, the degree of two vertices will change. For example:



The vertex with an even degree should not have any of its edges removed, as it is only connected to vertices with an odd degree, therefore the removal of any of the edges connected to the vertex with an even degree in this graph will not result in a change in number of vertices with an odd degree.

There are five edges that connect two vertices with an odd degree; if any of these five edges were removed, the total number of vertices with an odd degree would decrease to 2 out of the 5 vertices, which would make an Eulerian trail possible for the new graph.

E

13 The walk $ABCDEF$ is the only Hamiltonian path for the graph. All other options either visit a vertex more than once (options **C,D,E**) or do not visit every vertex (option **A**).

B

14 For an Eulerian circuit to be possible, all vertices must have an *even* degree. Vertices C and E have an odd degree. If an edge was added between these vertices, the degree of these vertices would increase by 1 and then all vertices in the graph would have an *even* degree.

A

Solutions to Exercise 13D

1 **a** The edge showing a weight of 12 is between town D and town E.

b C to D via B, means C to B (8 minutes) followed by B to D (9 minutes) for a total of $9 + 8 = 17$ minutes.

c D to E direct is 12 minutes
D to E via B is $9 + 11 = 20$ minutes.
By driving direct, the motorist will save $20 - 12 = 8$ minutes.

d The options for travelling from A to E and visiting all towns exactly once are:
$A{-}C{-}D{-}B{-}E$ for a total time of 45 minutes.
$A{-}B{-}C{-}D{-}E$ for a total time of 36 minutes.
$A{-}C{-}B{-}D{-}E$ for a total time of 44 minutes.
The shortest time is 36 minutes.

2 By inspection, the shortest path from A to E will be $A{-}C{-}D{-}E$ for a length of 11.

3 **a** The path $A{-}B{-}E{-}H{-}I$ has length:
$5 + 9 + 12 + 8$
$= 34$ kilometres

b The circuit $F{-}E{-}D{-}H{-}E{-}A{-}C{-}F$ has length:
$6 + 6 + 10 + 12 + 8 + 4 + 10$
$= 56$ kilometres

c The shortest cycle starting and ending at E is via B and A, $E{-}B{-}A{-}E$ for a distance of:
$9 + 5 + 8$
$= 22$ kilometres

d Shortest path from A to I (by inspection) is either $A{-}C{-}F{-}G{-}I$ for a distance of $4 + 10 + 4 + 8$
$= 26$ kilometres
or $A{-}E{-}F{-}G{-}I$ for a distance of
$8 + 6 + 4 + 8$
$= 26$ kilometres

4 **a** The shortest path from S to F is $S{-}B{-}D{-}F$ with length 12.

b The shortest path from S to F is $S{-}A{-}C{-}D{-}F$ with length 10.

c The shortest path from S to F is $S{-}B{-}D{-}F$ with length 15.

d The shortest path from S to F is $S{-}A{-}E{-}G{-}F$ with length 19.

5 The length of the shortest path between town A and B is 19 kilometres.

Exam 1 style questions

6 The shortest path from C to E is
$C - D - A - E$
B

7 The shortest path from B to G is
$B - A - C - G$
C

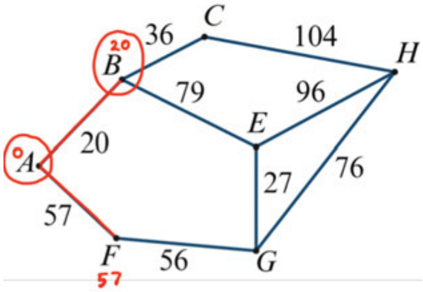
8 The shortest path from $Home$ to $School$

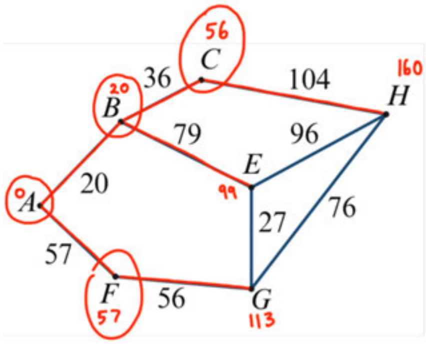
is $Home - E - C - B - D - School$ which equates to a length of $30 + 14 + 10 + 26 = 80$ minutes.
A

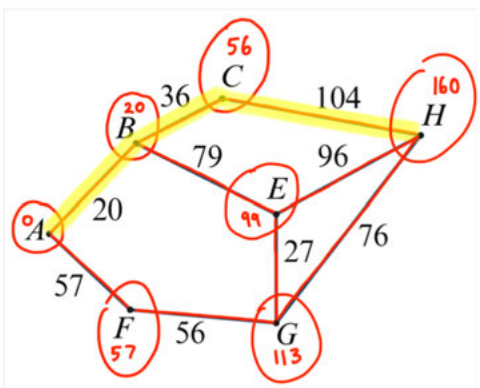
8 The shortest path from $Home$ to $School$ is either $Home - E - C - B - D - School$ or $Home - E - C - B - School$.
E

Solutions to Exercise 13E

1 a



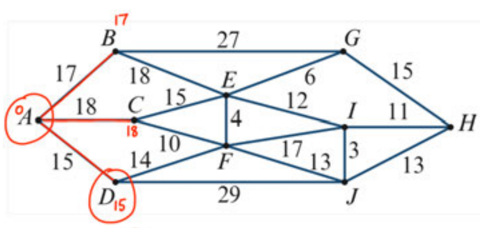


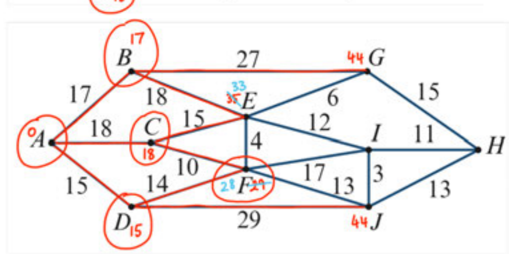


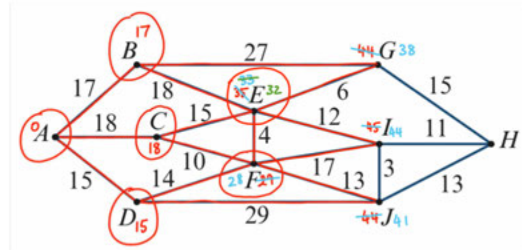
Shortest path= $A - B - C - H$

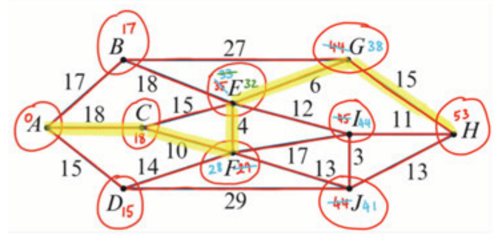
Length of shortest path = 160

b





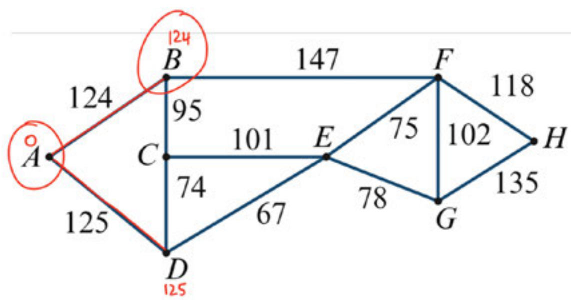


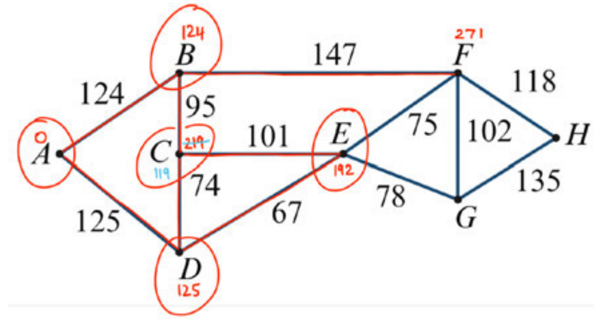


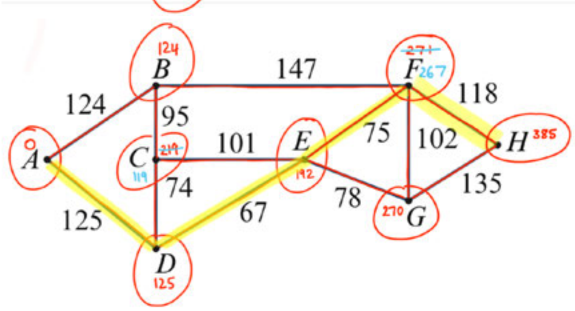
Shortest path= $A - C - F - E - G - H$

Length of shortest path = 53

c



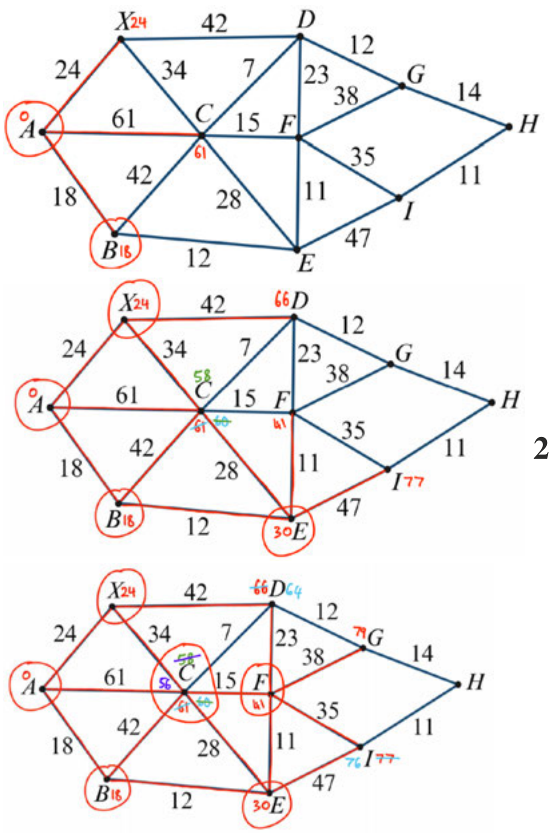


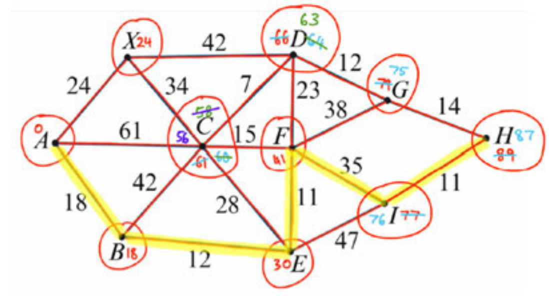


Shortest path= $A - D - E - F - H$

Length of shortest path = 385

d

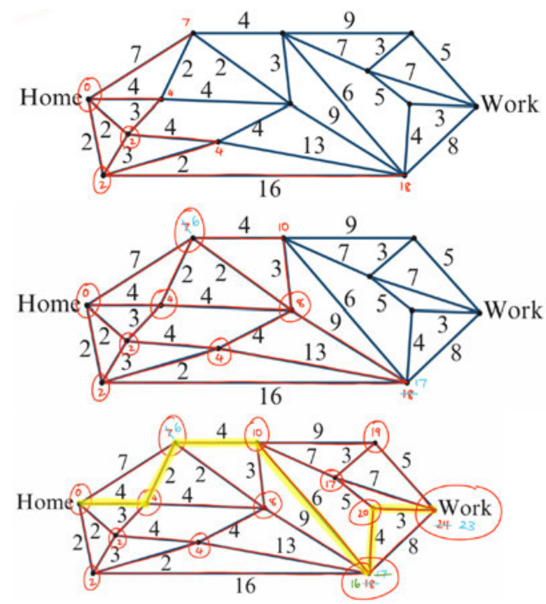




Shortest path= $A - B - E - F - I - H$

Length of shortest path = 87

2

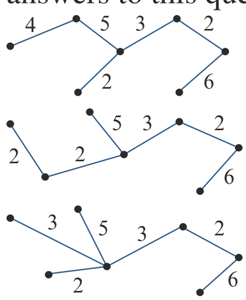


Shortest path = 23 minutes

Solutions to Exercise 13F

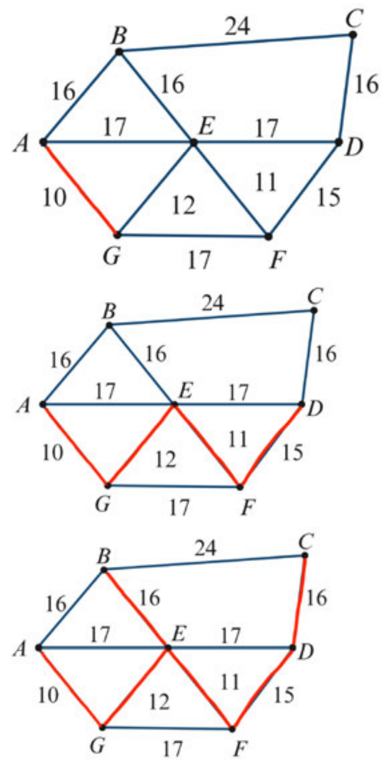
1 a There are 7 vertices, so the spanning tree will have 7 – 1 = 6 edges. The network has 12 edges, so 12 – 6 = 6 edges must be removed.

b Note: other trees are possible as answers to this question.



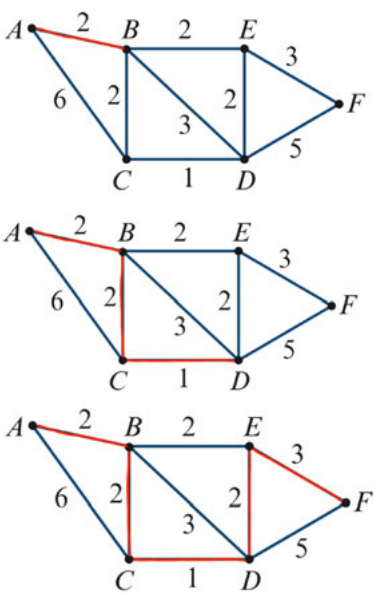
c The first graph has weight:
4 + 5 + 2 + 3 + 2 + 6 = 22
The second graph has weight:
2 + 2 + 5 + 3 + 2 + 6 = 20
The third graph has weight:
3 + 2 + 5 + 3 + 2 + 6 = 21

b



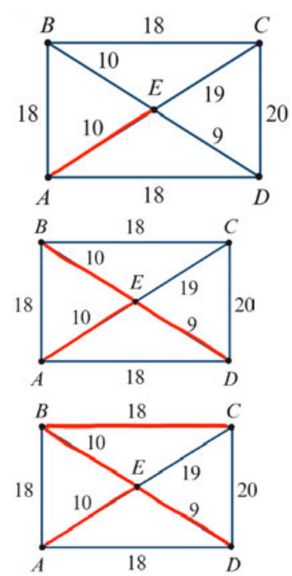
Total weight
= 16 + 10 + 12 + 11 + 15 + 16
= 80

2 a



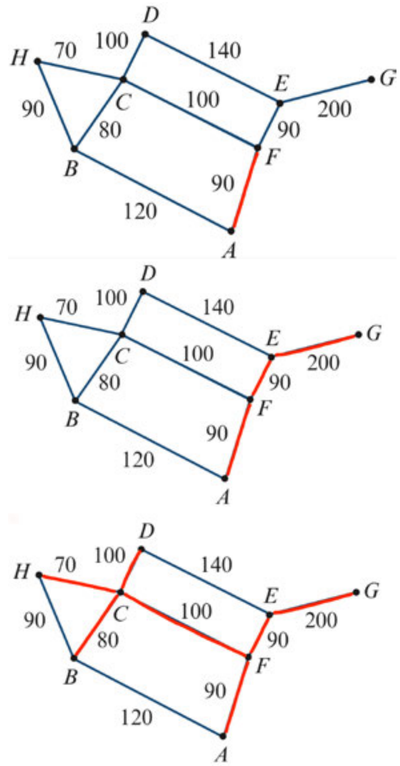
Total weight = 2 + 2 + 2 + 1 + 3
= 10

c



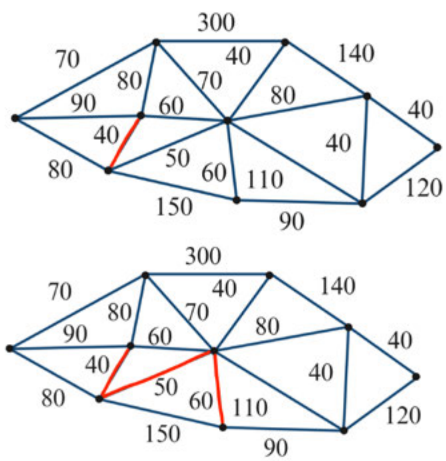
Total weight = 18 + 10 + 10 + 9
= 47

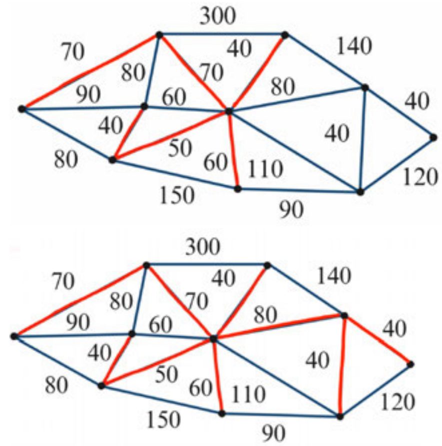
d



Total weight
= 70 + 80 + 100 + 100 + 90 + 90 + 200 = 730

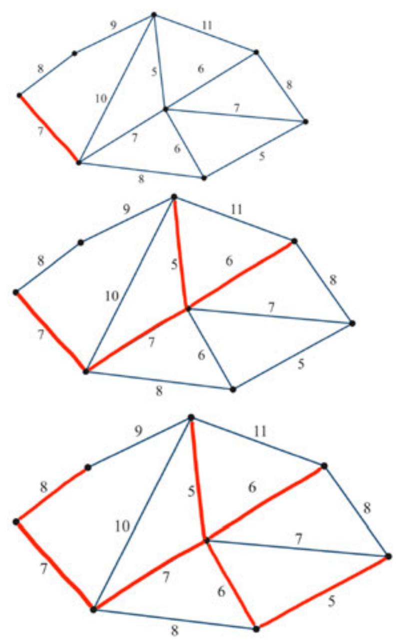
3 The shortest length of pipe required to connect all water storages will be the weight of the minimum spanning tree for the network.





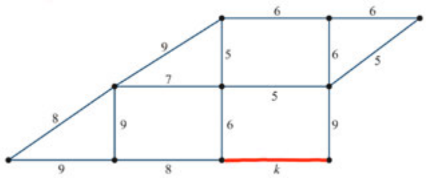
The weight of the minimum spanning tree, shown in red above, is
weight
= 70 + 70 + 40 + 50 + 40 + 60 + 80 + 40 + 40 = 490

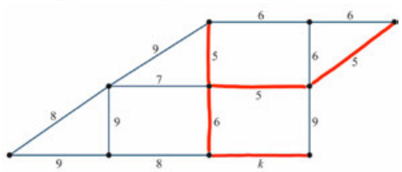
4 Using Prim's algorithm, determine a minimum spanning tree:

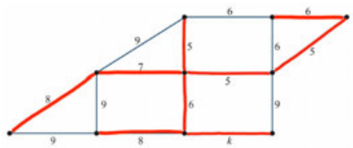


Length of minimum spanning tree= 8 + 7 + 7 + 5 + 6 + 6 + 5 = 44
A

5 Using Prim's algorithm, determine a minimum spanning tree. Start at the edge with weight *k*; although you do not know the value of *k* it is stated that it is included in the minimum spanning tree:







Total length of the minimum spanning tree = 58

$6 + 5 + 5 + 5 + 6 + 8 + 7 + 8 + k = 58$

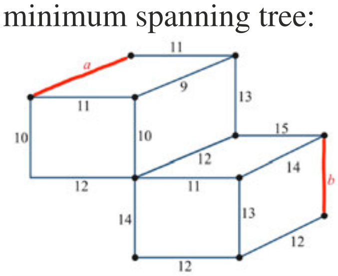
$51 + k = 58$

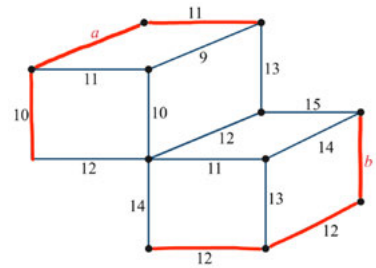
$k = 58 - 51$

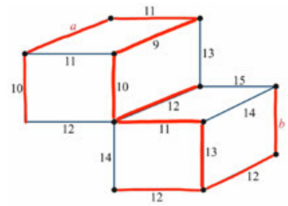
$k = 7$

B

6 Using Prim's algorithm, determine a minimum spanning tree. Start at the edges with weight *a* and *b*; although you do not know the values of *a* or *b*, it is stated that they are included in the minimum spanning tree:







Total length of the minimum spanning tree = 124

$a + b + 10 + 11 + 9 + 10 + 12 + 11 + 13 + 12 + 12 = 124$

$a + b + 100 = 124$

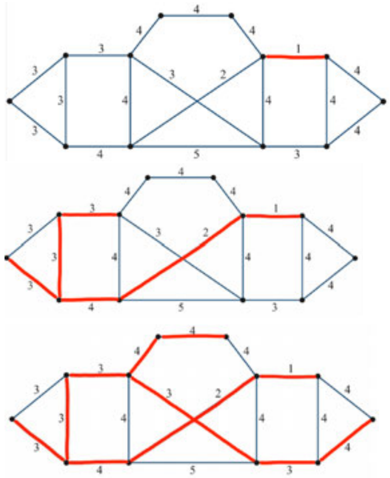
$a + b = 124 - 100$

$a + b = 24$

The sum of *a* and *b* must equal 24; eliminate the incorrect options **A,D,E**. Of the two remaining options, consider position of *a* and *b* and whether they would be included in the minimum spanning tree when applying Prim's algorithm. If $a = 12$, the edge with weight *a* would not be included in the minimum spanning tree, as the vertices that edge connects to could be included in the minimum spanning tree using edges with a smaller weight, thus option **B** is incorrect. From the given list of options, $a = 10$ and $b = 14$.

C

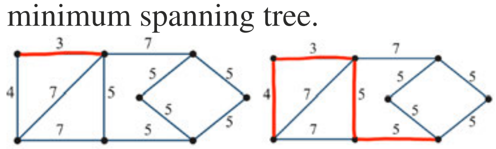
7 Using Prim's algorithm, determine a minimum spanning tree.



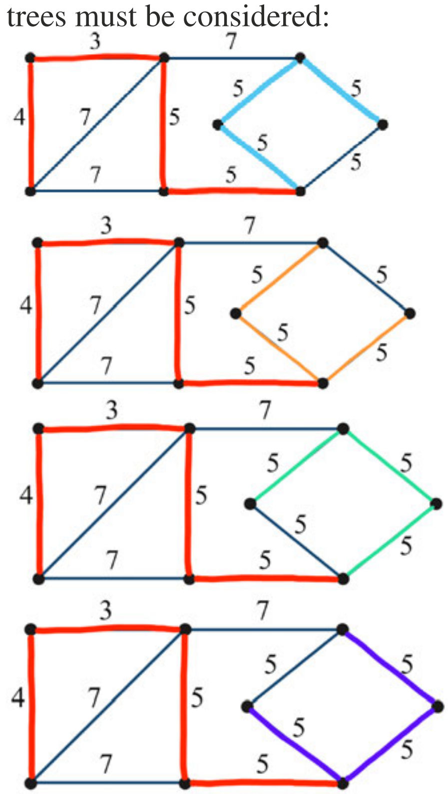
Count the number of edges with weight 4 that are not included in the minimum spanning tree found.

E

8 Using Prim's algorithm, determine a minimum spanning tree.



At this stage, there are multiple minimum spanning trees possible. For this question **all** possible minimum spanning trees must be considered:



There are four different minimum spanning trees possible.

D

Solutions to Review: Multiple-choice questions

1 Seven vertices can be connected with six edges, one less than the number of vertices. **C**

2 **A** This graph has a cycle so is not a tree.

B This graph has a cycle so is not a tree.

C This graph is a spanning tree.

D This graph has a cycle so is not a tree.

E This graph is a tree but does not include the vertex 2 so it is not a spanning tree. **C**

3 P has degree 2
Q has degree 5
R has degree 3
S has degree 4
T has degree 4
U has degree 2 **A**

4 $v = 15$ and $f = 12$
$v - e + f = 2$
$15 - e + 12 = 2$
$-e = 2 - 15 - 12$
$-e = -25$
$e = 25$ **D**

5 An eulerian circuit will exist if all of the vertices have an even degree.

A has two odd-degree vertices.

B has all even-degree vertices.

C has all even-degree vertices.

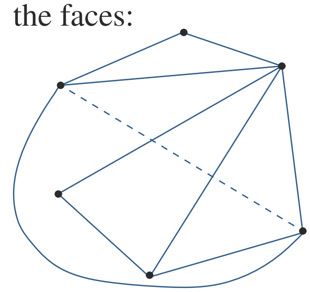
D has all even-degree vertices.

E has all even-degree vertices. **A**

6 $v = 8$ and $e = 13$
$v - e + f = 2$
$8 - 13 + f = 2$
$F = 2 - 8 + 13$
$f = 7$ **C**

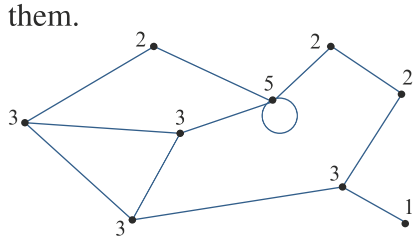
7 Hamiltonian cycle starts and ends at the same vertex, so it cannot be option E. Hamiltonian cycles pass through every vertex only once, so it cannot be option A (visits E multiple times) or option C (visits A multiple times), Hamiltonian cycles pass through every vertex in the graph, so it cannot be option D which does not visit vertex F. **B**

8 The graph is planar and must be redrawn without edges crossing before counting the faces:



There are five regions defined by the graph in planar form. **B**

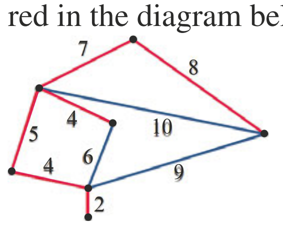
9 The graph is drawn below, with the degrees of each vertex written beside them.



The sum of the degrees is
$3 + 2 + 3 + 3 + 5 + 2 + 2 + 3 + 1$
$= 24$ **E**

10 An eulerian trail exists if there are exactly
two odd-degree vertices in a graph. The graph currently has four odd-degree vertices, that is A, E, C, D.
Joining two of these by an edge would make their degree even. **B**

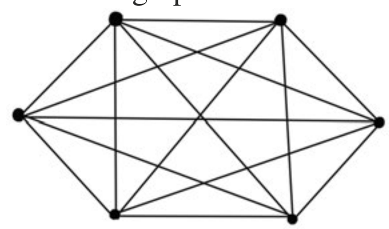
11 The minimum spanning tree is shown in red in the diagram below:



The length of the minimum spanning tree
$= 2 + 4 + 5 + 4 + 7 + 8 = 30$ **A**

12 Eulerian circuit will be possible if all of the vertices have an even degree, so it could be option **A** or **B**.
By inspection, a hamiltonian cycle is possible only in option **A**. **A**

13 A graph with six vertices, connected with the minimum number of edges, is a tree with 5 edges.
A complete graph has every vertex connected to every other vertex; the number of edges for a complete graph with six vertices can found by either:
Method 1
Draw the graph



Method 2
A complete graph with n vertices will have $\frac{n(n-1)}{2}$ edges. This graph has six vertices, so $n = 6$ and the graph will have $\frac{6(6-1)}{2} = \frac{6(5)}{2} = \frac{30}{2} = 15$ edges.
Starting with 5 edges, 10 extra edges must be added to the graph to make this a complete graph. **C**

14 In the given graph,

- A is directly connected to B in two ways (could be only option B)
- A is directly connected to C in one way
- A is directly connected to D in one way
- B is directly connected to C in one way
- B is directly connected to D in one way
- C is directly connected to D in two ways **B**

15 An eulerian circuit exists if all the vertices have an even degree.
Option **A** has two odd-degree vertices.
Option **B** has all even-degree vertices.
Option **C** has two odd-degree vertices.
Option **D** has two odd-degree vertices.
Option **E** has two odd-degree vertices. **B**

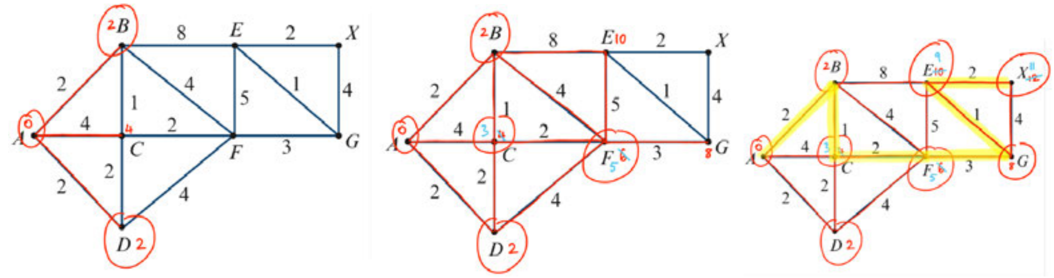
16 Consider some different paths from S to T:

- $S - A - F - T = 22 + 15 + 8 = 45$
- $S - A - D - G - T = 22 + 3 + 2 + 10 = 37$
- $S - B - D - G - T = 18 + 9 + 2 + 10 = 39$
- $S - B - E - G - T = 18 + 3 + x + 10 = 31 + x$
- $S - B - H - T = 18 + 3 + 7 + 11 = 39$
- $S - C - E - G - T = 17 + 6 + x + 10 = 33 + x$
- $S - C - E - H - T = 17 + 6 + 7 + 11 = 41$
- $S - C - H - T = 17 + 15 + 11 = 43$

The path with the shortest distance is $S - B - E - G - T$ with a distance of $31 + x$. Given the shortest distance is 36 metres, $x = 5$. **B**

Chapter Review: Extended-response questions

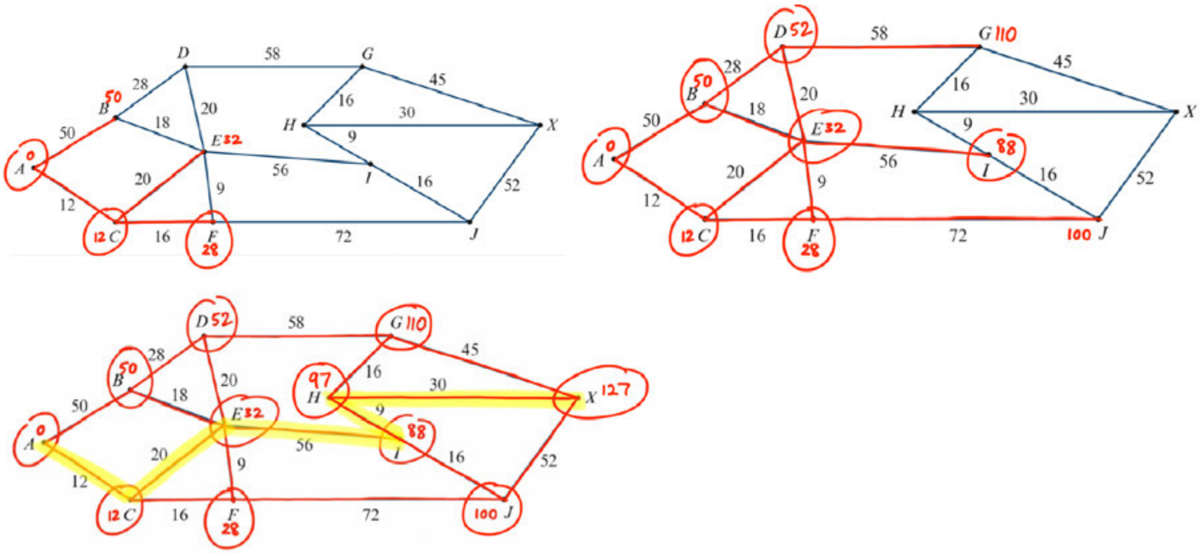
1 a



The shortest path from A to X is: $A - B - C - F - G - E - X$.

The length of the shortest path = 2 + 1 + 2 + 3 + 1 + 2 = 11.

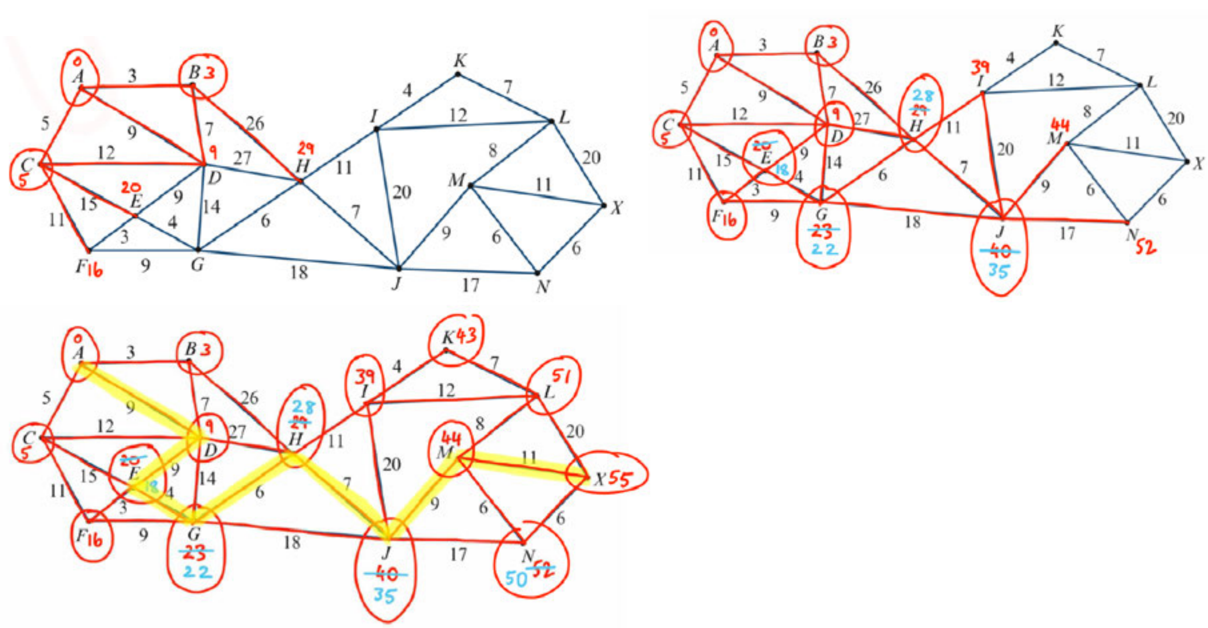
b



The shortest path from A to X is: $A - C - E - I - H - X$.

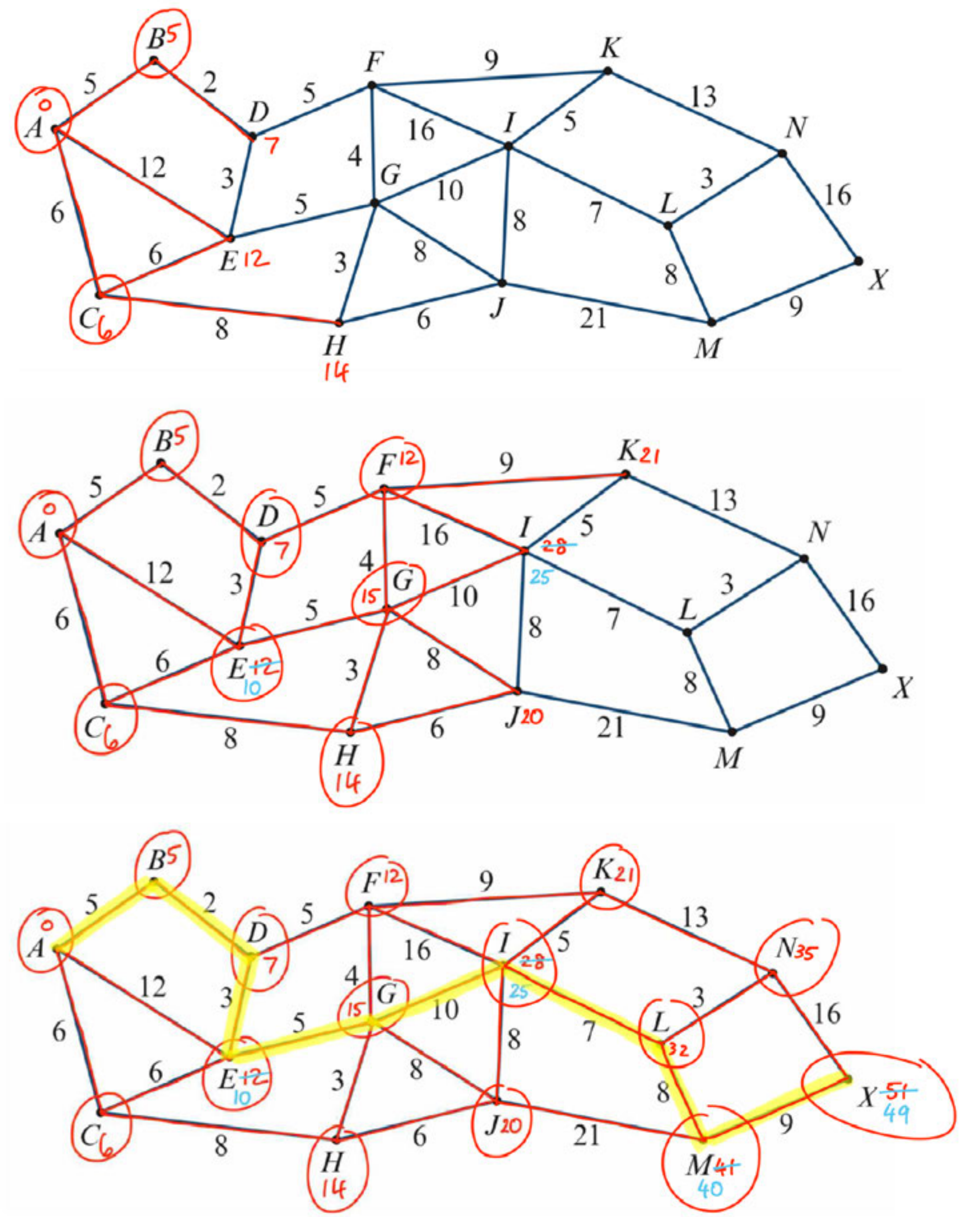
The length of the shortest path = 12 + 20 + 56 + 9 + 30 = 127.

c



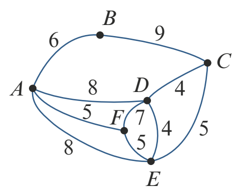
The shortest path from A to X is: $A - D - E - G - H - J - M - X$.
The length of the shortest path = 9 + 9 + 4 + 6 + 7 + 9 + 11 = 55.

d

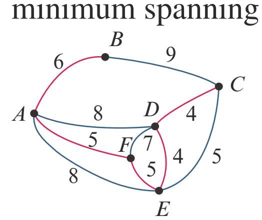


The shortest path from A to X is: $A - B - D - E - G - I - L - M - X$.
The length of the shortest path = 5 + 2 + 3 + 5 + 10 + 7 + 8 + 9 = 49.

2 a i The only edge missing from the graph is the direct connection between vertex E and vertex C. There is only one direct connection between these vertices, of length 5, so this must be added to the graph.



ii The cable should be laid along the minimum spanning tree for the graph. The minimum spanning tree is shown in red below:



The weight of the minimum spanning tree
= 4 + 5 + 4 + 5 + 6
= 24
The minimum length of cable required is 24 kilometres.

iii There is:

- one connection between *D* and *C*
- no loop at *D*
- one connection between *D* and *E*
- one connection between *D* and *F*
- one connection between *E* and *C*
- no loop at *E*
- one connection between *E* and *F*
- no connection between *F* and *C*
- no loop at *F*

The matrix is

$$\begin{array}{c|cccccc} & A & B & C & D & E & F \\ \hline A & 0 & 1 & 0 & 1 & 1 & 1 \\ B & 1 & 0 & 1 & 0 & 0 & 0 \\ C & 0 & 1 & 0 & 1 & 1 & 0 \\ D & 1 & 0 & 1 & 0 & 1 & 1 \\ E & 1 & 0 & 1 & 1 & 0 & 1 \\ F & 1 & 0 & 0 & 1 & 1 & 0 \end{array}$$

b **i** The route
$A - B - A - F - E - D - C - E - F - A$
has distance
$6 + 6 + 5 + (3+2) + 4 + 4 + 5 + (2+3) + 5$
$= 45$ kilometres

ii This route is not a hamiltonian cycle because some of the vertices are visited more than once, namely A, F and E.

iii There are many answers, but one possible route is:
$A - B - C - D - F - E - A$

iv The distance travelled will vary depending on the answer for part iii. The route in part **iii** above has distance
$6 + 9 + 4 + 7 + 5 + 8$
$= 39$ kilometres

c Starting at A and returning to A by
travelling each track once is an example of an eulerian circuit. This can only occur if all vertices are of even degree.
At the moment, vertex C and F both have odd degrees, so joining them by a new path will make them both have an even degree, making an eulerian circuit possible.

3 **a** The vertex representing Melville has 4 edges connected directly to it, so the degree of this vertex is 4.

b There is a total of 9 edges in this graph. The sum of the degrees of a graph
$= 2 \times$ the total number of edges
$= 2 \times 9 = 18$

c $v = 6, e = 9, f = 5$
$v + f = e + 2$
$6 + 5 = 9 + 2$
$11 = 11$
Euler's formula is verified for this graph.

d No, the salesperson has visited a vertex more than once (Melville), therefore this is not a Hamiltonian cycle.

e Hamiltonian path, because all vertices were visited without repeating any vertices and the starting vertex is different to the ending vertex.

f Bartow - Stratmoore - Melville - Kenton - Osburn - Croghon - Bartow.
The shortest distance is 71 kilometres.

g Melville - Croghon - Bartow - Stratmoore - Osburn - Kenton - Melville.
The shortest distance is 58 kilometres.

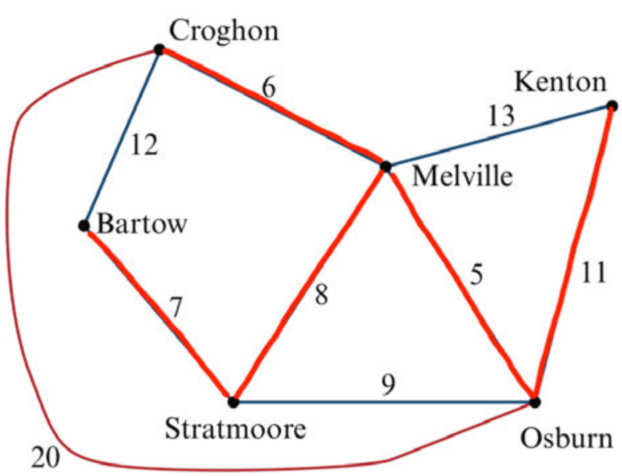
h An Eulerian circuit is possible is all vertices have an *even* degree. The vertices that represent Croghon and Stratmoore both have an *odd* degree.

i This walk described is an Eulerian trail. The inspector could start their route at either Croghon or Stratmoore, because these are the only two vertices with an *odd* degree.
One option is: Stratmoore - Osburn - Kenton - Melville - Osburn - Croghon - Bartow - Stratmoore - Melville - Croghon (the reverse is also acceptable)

j The total distance travelled by the inspector is
9 + 11 + 13 + 5 + 20 + 12 + 7 + 8 + 6 = 91 kilometres.
91 km at 60 km/hr
91 ÷ 60 = 1.51666... hours = 1 hour and (0.51666...× 60) minutes = 1 hour and 31 minutes.
1 hour and 31 minutes *BEFORE* 5:00pm.
Therefore 3:29 pm.

k Minimum spanning tree.

l

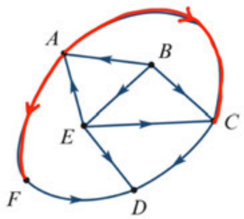


Total length of the minimum spanning tree = 11 + 5 + 8 + 7 + 6 = 37 kilometres.

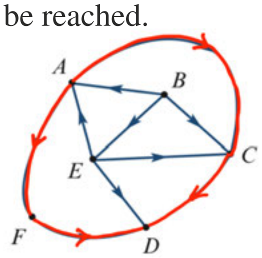
Chapter 14 – Flow, matching and scheduling problems

Solutions to Exercise 14A

1 a Starting at vertex A there are two edges, directed towards vertices C and F.

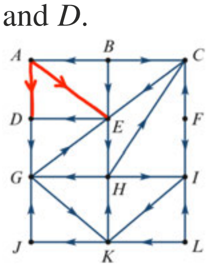


From vertices C and F, vertex D can be reached.

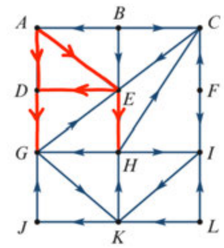


No other vertices can be reached from vertex A directly nor indirectly. There are 3 vertices that can be reached from vertex A (C, F, D).

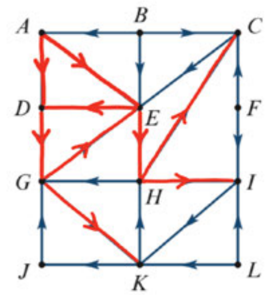
b Starting at vertex A, there are two edges, directed towards vertices E and D.



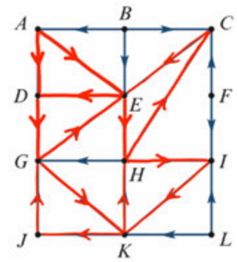
Vertex H is connected by a directed edge with E, likewise vertex G is connected by a directed edge with D.



Vertices K, I and C are connected by directed edges with vertices G and H.

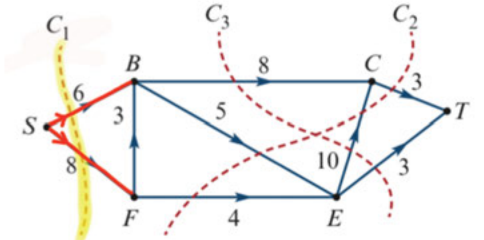


Finally, vertex J is connected by a directed edge with vertex K.

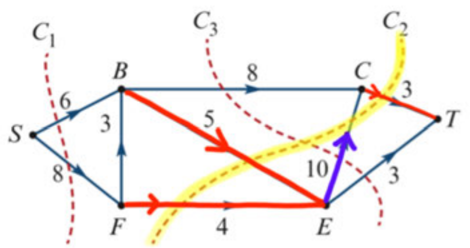


No other vertices can be reached from vertex A directly nor indirectly. There are 8 vertices that can be reached from vertex A (D, E, G, H, I, C, K, J).

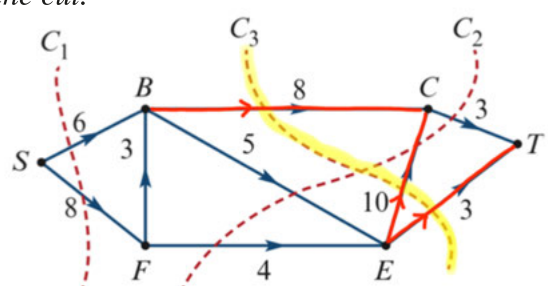
2



The capacity of *C*1
= 6 + 8
= 14

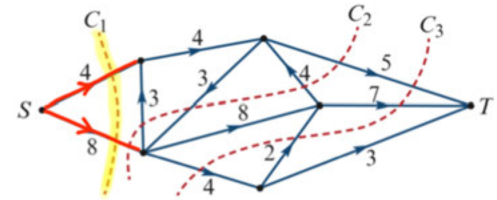


The capacity of *C*2
= 3 + 5 + 4
= 12
Note: the edge from E to C is not counted as the flow along this edge is from the sink side to the source side of the cut.

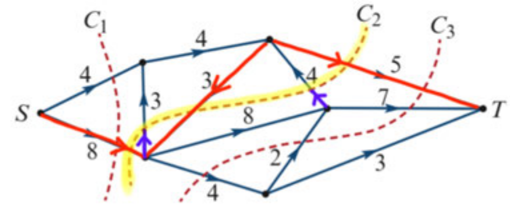


The capacity of *C*3
= 8 + 10 + 3
= 21

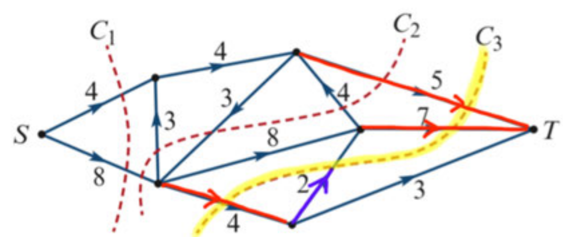
3



The capacity of *C*1
= 4 + 8
= 12

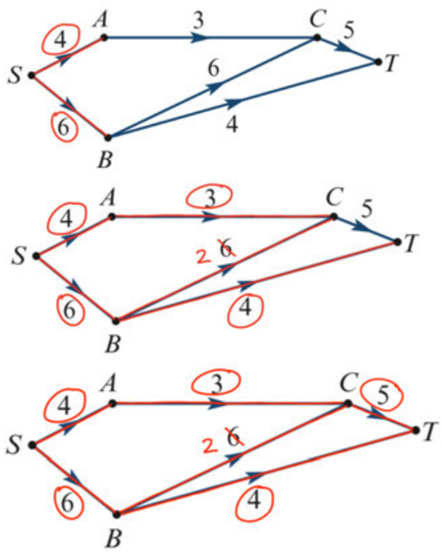


The capacity of *C*2
= 5 + 3 + 8
= 16
Note: there are two edges where the flow is from the sink side to the source side of the cut. These edges have capacity 4 and 3, and both are not counted in the calculation.



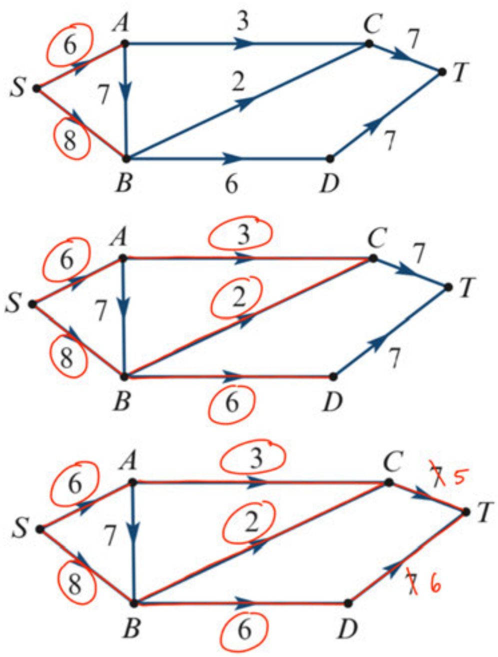
The capacity of *C*3
= 5 +7 + 4
= 16
Note: there is one edge where the flow is from the sink side to the source side of the cut. This edge has capacity 2 and this is not counted in the calculation.

4 a



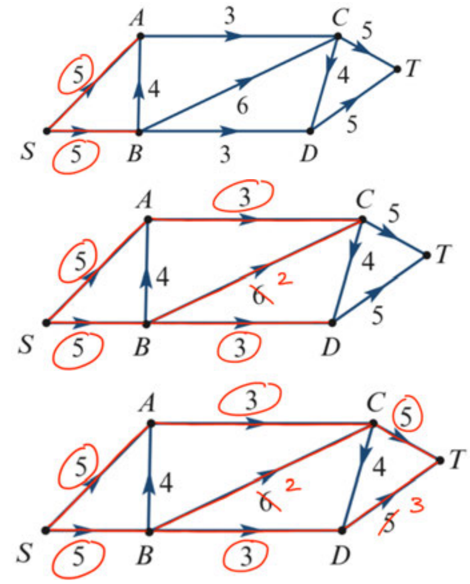
Maximum flow = 5 + 4 = 9

b



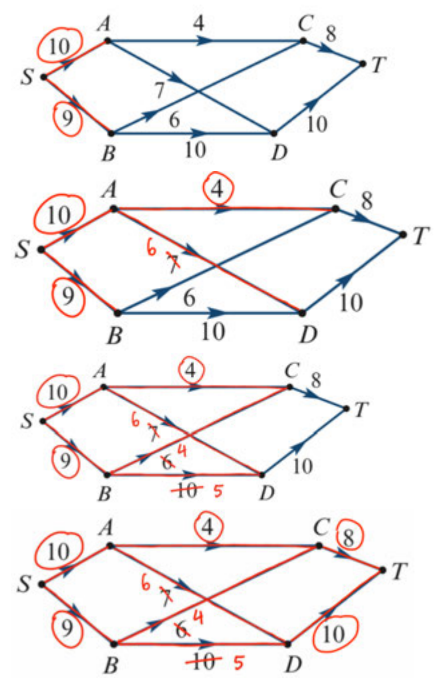
Maximum flow = 5 + 6 = 11

c



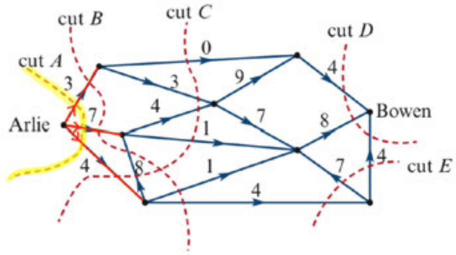
Maximum flow = 3 + 5 = 8

d

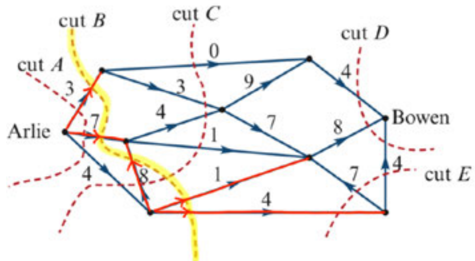


Maximum flow = 8 + 10 = 18

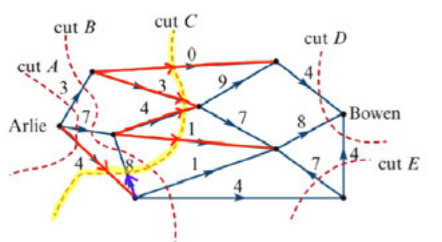
5 a



The capacity of cut *A*
= 3 + 7 + 4
= 14

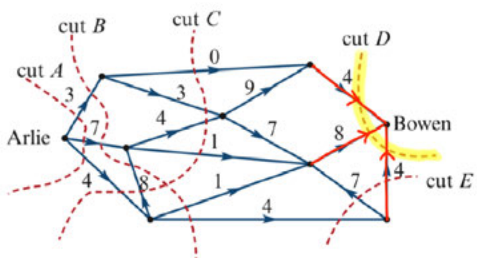


The capacity of cut *B*
= 3 + 7 + 8 + 1 + 4
= 23



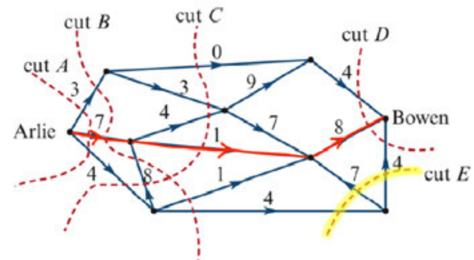
The capacity of cut *C*
= 0 + 3 + 4 + 1 + 4
= 12

Note: the edge with capacity 8 is not counted as the flow along this edge is from the sink side to the source side of the cut.

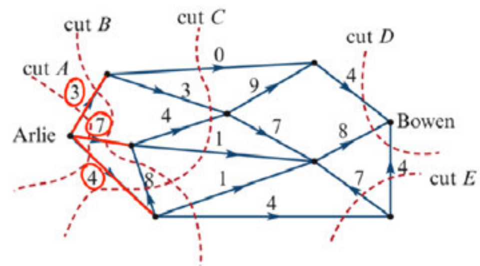


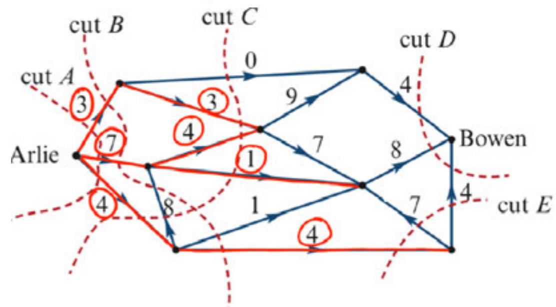
The capacity of cut *D*
= 4 + 8 + 4
16

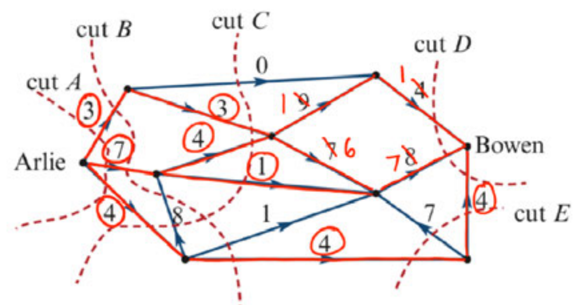
b Cut *E* is not valid because it does not isolate the flow from Arlie (source) to Bowen (sink).



c

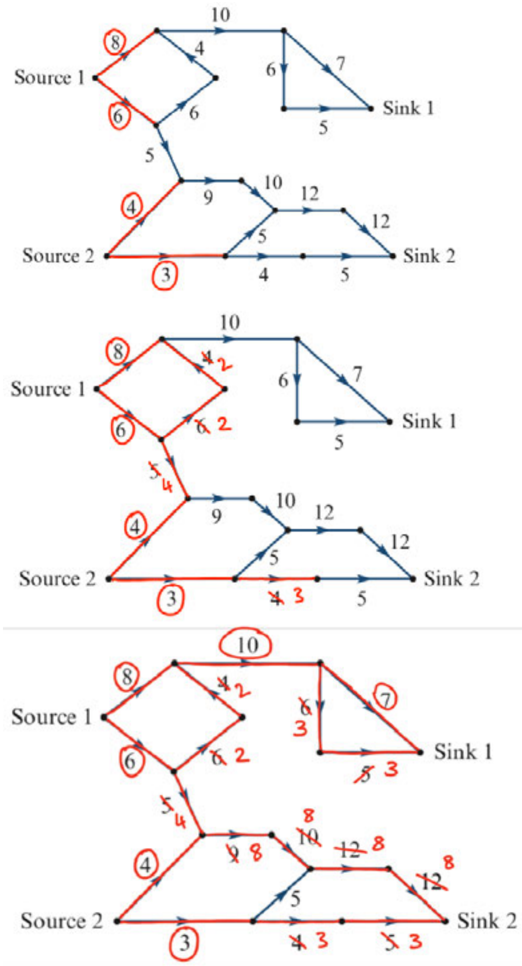






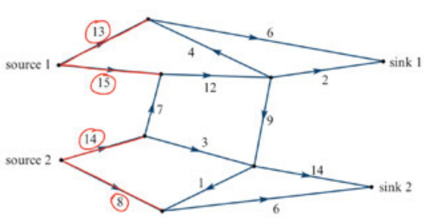
Maximum flow = 7 + 4 + 1 = 12.
The maximum number of seats from Arlie to Bowen is 12.

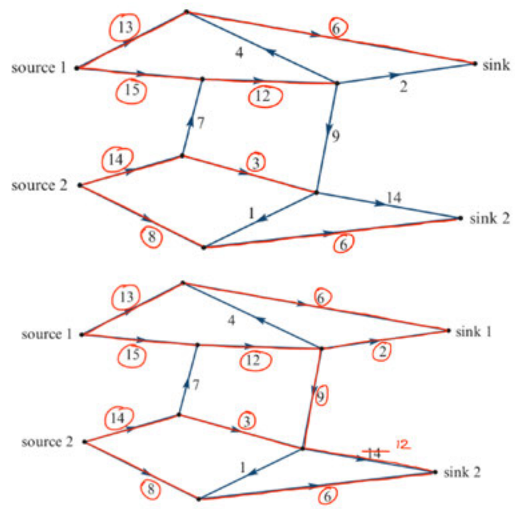
6 a



The maximum flow to sink 1
= 7 + 3
= 10
The maximum flow to sink 2
= 8 + 3
=11

b



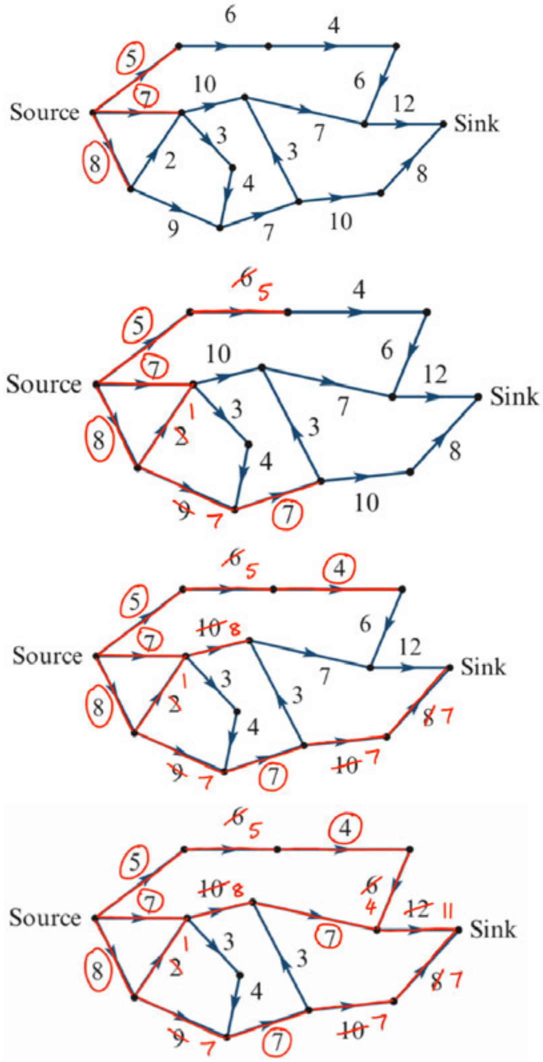


The maximum flow to sink 1
= 6 + 2
= 8
The maximum flow to sink 2
= 12 + 6
=18

7 a There are 9 different paths from the *source* to the *sink*:

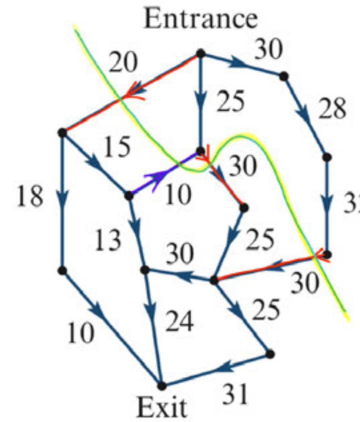
- Source - 5 - 6 - 4 - 6 - 12 - Sink
- Source - 7 - 10 - 7 - 12 - Sink
- Source - 7 - 3 - 4 - 7 - 3 - 7 - 12 - Sink
- Source - 7 - 3 - 4 - 7 - 10 - 8 - Sink
- Source - 8 - 2 - 10 - 7 - 12 - Sink
- Source - 8 - 2 - 3 - 4 - 7 - 3 - 7 - 12 - Sink
- Source - 8 - 2 - 3 - 4 - 7 - 10 - 8 - Sink
- Source - 8 - 9 - 7 - 3 - 7 - 12 - Sink
- Source - 8 - 9 - 7 - 10 - 8 - Sink

b



Maximum flow = 11 + 7 = 18.

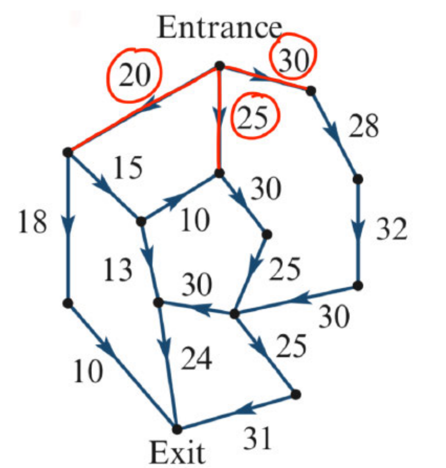
8 a

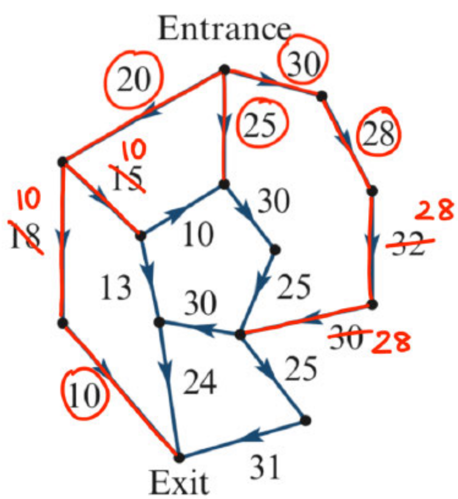


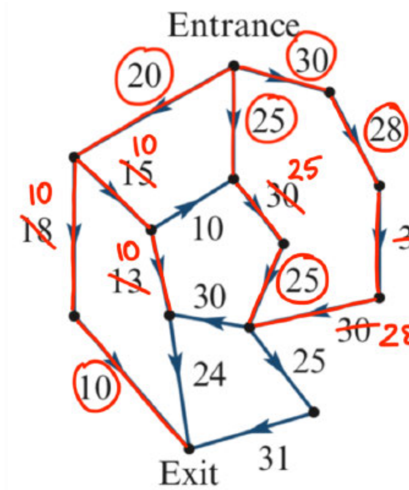
The capacity of the cut shown above
= 20 + 30 + 30 = 80

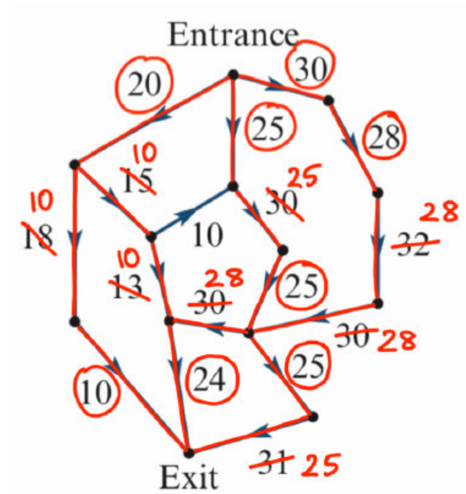
Note: the edge with capacity 10 is not counted as the flow along this edge is from the sink side to the source side of the cut.

b



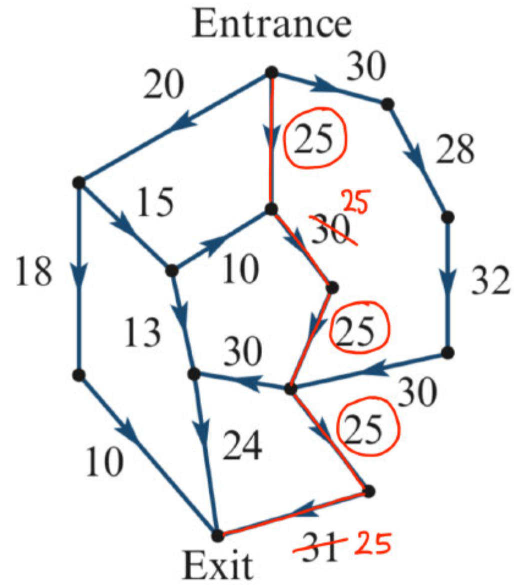


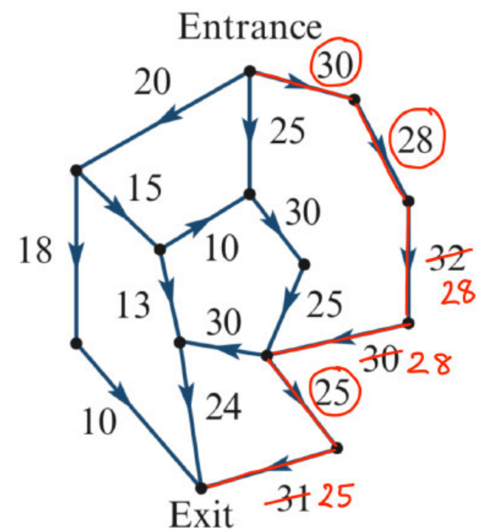




Maximum flow = 10 + 24 + 25 = 59.

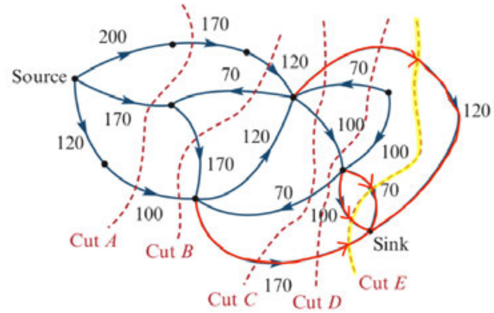
c Must consider the largest group of students that can follow one path from the *Entrance* down to the *Exit*. There are two potential paths that allow a maximum of 25 students to walk through the museum as one group every 30 minutes:





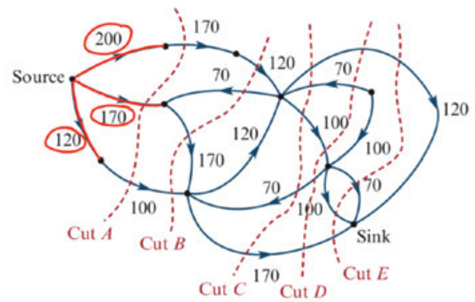
The largest group of students that can walk together through the museum is 25.

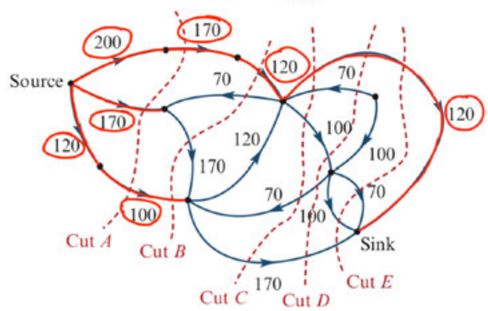
9

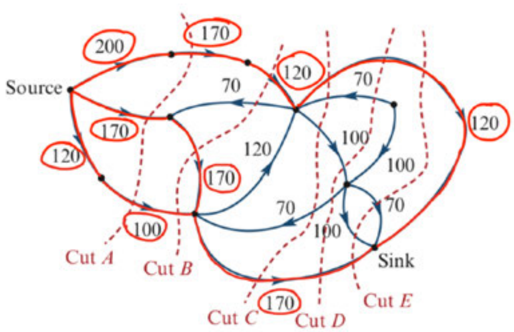


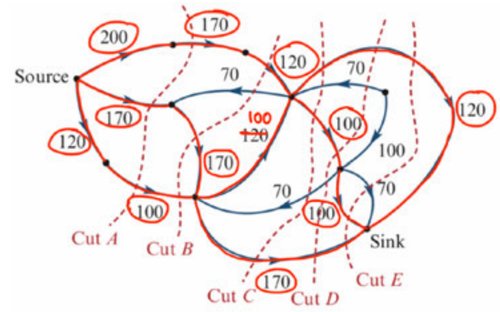
The capacity of *Cut E*
= 120 + 70 + 100 + 170 = 460
E

10









Maximum flow
= 120 + 100 + 170 = 390.

Capacity of *Cut A*
= 170 + 170 + 100 = 440
Capacity of *Cut B*
= 120 + 170 + 100 = 390
Capacity of *Cut C*
= 120 + 100 + 170 = 390
Capacity of *Cut D*
= 120 + 100 + 170 = 390
Capacity of *Cut E*
= 120 + 70 + 100 + 170 = 460 There-

fore 3 cuts (B, C, D) have a capacity equal to the maximum flow of this network.
D

11

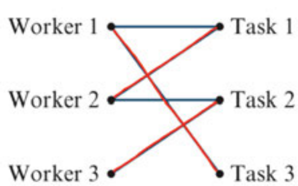
	if $x = 4$	if $x = 6$	if $x = 8$
Cut $A = 44$	44	44	**44**
Cut $B = 37 + x$	**41**	**43**	45
Cut $C = 41 + x$	45	47	49
Cut $D = 46$	46	46	46
Cut $E = 51$	51	51	51

If $x = 4$, the maximum flow is given by *Cut B* with the minimum capacity of 41.
If $x = 6$, the maximum flow is given by *Cut B* with the minimum capacity of 43.
If $x = 8$, the maximum flow is given by *Cut A* with the minimum capacity of 44.
B

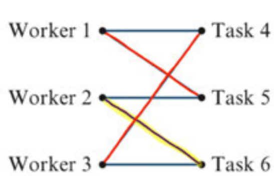
Solutions to Exercise 14B

1 **a** First allocate the Tasks that can only be completed by a certain Worker; consider vertices with only one edge connected to it. Then allocate Tasks to corresponding Workers with the remaining possible options.

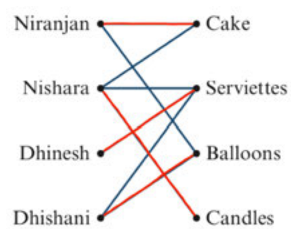
- Worker 1 must be allocated to Task 3, as there is only one edge connected to the vertex representing Task 3
- Worker 3 must be allocated Task 2, as there is only one edge connected to the vertex representing Worker 3
- As a result, Worker 2 must be allocated to Worker Task 1



b
- Worker 2 must be allocated Task 6
- Worker 3 cannot be allocated to Task 6, so they must be allocated to Task 4
- As a reuslt, Worker 1 must be allocated to Task 5



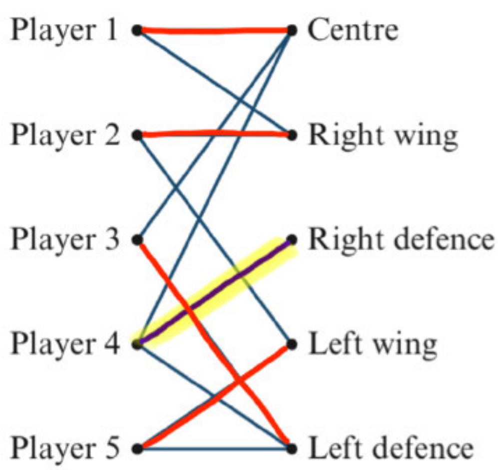
2
- Dhinesh must be allocated to the Serviettes, as there is one edge connected to the vertex representing Dhinesh
- Nishara must be allocated to the Candles, as there is one edge connected to the vertex representing Candles
- Dhishani cannot be allocated to the Serviettes, so she must be allocated Balloons
- As a result, Niranjan must be allocated to Cake



3 There is one position that must be allocated first; Player 4 must be allocated Right defence, as there is only one edge connected to the vertex representing Right defence. Player 4 could have been allocated to Centre; the choice of which player to allocate to Centre can determine the overall allocation of the team.

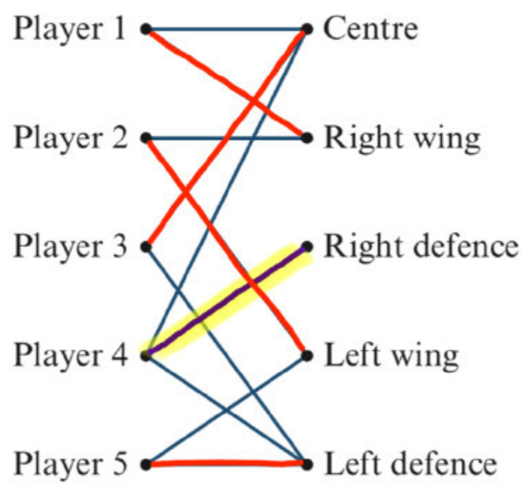
If Player 1 is allocated to Centre:

- Player 3 must be allocated Left defence, as Player 3 could not be allocated to Centre
- Player 5 must be allocated Left wing, as they cannot be allocated to their other possible option of Left defence
- Finally, Player 2 must be allocated Right wing, as there are no other positions available for allocation



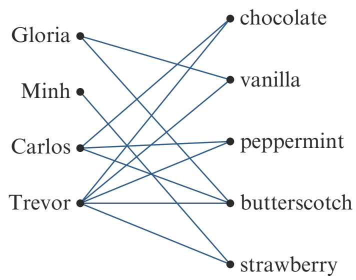
If Player 3 is allocated to Centre:

- Player 1 must be allocated Right wing, as Player 1 could not be allocated to Centre
- Player 2 must be allocated Left wing, as they cannot be allocated to their other possible option of Right wing
- Finally, Player 5 must be allocated Left defence, as there are no other positions available for allocation



4 **a** A bipartite graph can be used to display this information because there are two distinct groups of objects that are, in some way, connected to each other.
One group is the people and the other group is the flavours of ice-cream.
Each of the people are connected to one or more ice-cream flavours.

b List the people on the left and the flavours on the right with a vertex for each.
Gloria likes vanilla and butterscotch, so join Gloria's vertex to each of the vertices for these flavours.
Minh only likes strawberry, so join Minh's vertex to the vertex for strawberry.
Similarly, join Carlos' vertex to the vertex for chocolate, peppermint and butterscotch. Join Trevor's vertex to the vertex for every flavour.
The completed bipartite graph is below:



c Trevor is connected to all five ice-cream flavours. The degree of this vertex is 5.

5 a

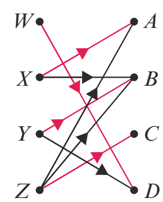
	A	**B**	**C**	**D**	
W	110	95	140	80	−80
X	105	82	145	80	−80
Y	125	78	140	75	−75
Z	115	90	135	85	−85

	A	**B**	**C**	**D**
W	30	15	60	0
X	25	2	65	0
Y	50	3	65	0
Z	30	5	50	0

– 25 – 2 –50

	A	**B**	**C**	**D**
W	5	13	10	0
X	0	0	15	0
Y	25	1	15	0
Z	5	3	0	0

	A	**B**	**C**	**D**
W	4	12	9	0
X	0	0	15	1
Y	24	0	14	0
Z	0	0	0	1



W must be allocated to *D*, so *Y* cannot.
Y must then be allocated to *B*, so *X* cannot.
X must be allocated to *A*
Z must be allocated to *C*
Allocation: *W* – *D*, *Y* – *B*, *X* – *A*, *Z* – *C*

b

	A	*B*	*C*	*D*	
W	2	4	3	5	−2
X	3	5	3	4	−3
Y	2	3	4	2	−2
Z	2	4	2	3	−2

	A	*B*	*C*	*D*
W	0	2	1	3
X	0	2	0	1
Y	0	1	2	0
Z	0	2	0	1

	A	*B*	*C*	*D*
W	0	1	1	3
X	0	1	0	1
Y	0	0	2	0
Z	0	1	0	1

	A	*B*	*C*	*D*
W	0	0	1	2
W	0	0	0	0
W	1	0	3	0
W	0	0	0	0

Multiple allocations are possible, but all will have the same minimum cost. One possible allocation is: *W* to *A* (2), *X* to *B* (5), *Y* to *D* (2) and *Z* to *C* (2)

Minimum cost = 2 + 5 + 2 + 2 = 11

6

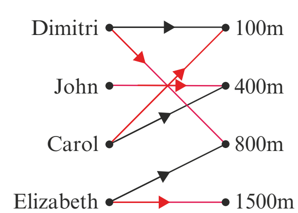
Student	*100 m*	*400 m*	*800 m*	*1500 m*	
Dimitri	11	62	144	379	−11
John	13	60	146	359	−13
Carol	12	61	149	369	−12
Elizabeth	13	63	142	349	−13

Student	*100 m*	*400 m*	*800 m*	*1500 m*
Dimitri	0	51	133	368
John	0	47	133	346
Carol	0	49	137	357
Elizabeth	0	50	129	336
		−47	−129	−336

Student	*100 m*	*400 m*	*800 m*	*1500 m*
Dimitri	0	4	4	32
John	0	0	4	10
Carol	0	2	8	21
Elizabeth	0	3	0	0

Student	*100 m*	*400 m*	*800 m*	*1500 m*
Dimitri	0	2	2	30
John	2	0	4	10
Carol	0	0	6	19
Elizabeth	2	3	0	0

Student	*100 m*	*400 m*	*800 m*	*1500 m*
Dimitri	0	2	0	28
John	2	0	2	8
Carol	0	0	4	17
Elizabeth	4	5	0	0



John must be allocated to 400 m, so Carol cannot.

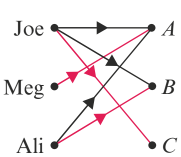
If Carol cannot be allocated to 400 m, she must be allocated to 100 m, so Dimitri cannot.
If Dimitri cannot be allocated to 100m, he must be allocated to 800 m, leaving 1500 m for Elizabeth
The “best” student allocation is:
Dimitri – 800 m, John – 400 m,
Carol – 100 m, Elizabeth – 1500 m

7

	Job			
Student	***A***	***B***	***C***	
Joe	20	20	36	−20
Meg	16	20	44	−16
Ali	26	26	44	−26

	Job		
Student	***A***	***B***	***C***
Joe	0	0	16
Meg	0	4	28
Ali	0	0	18
			−16

	Job		
Student	***A***	***B***	***C***
Joe	0	0	0
Meg	0	4	12
Ali	0	0	2



Meg must be allocated to job A, so Joe and Ali cannot.
If Ali cannot be allocated to job A, he must be allocated to job B, so Joe cannot.
If Joe cannot be allocated to job A, nor job B, he must be allocated to job C.

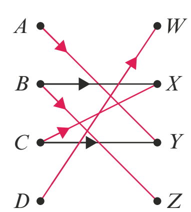
The allocation that minimises the time taken to complete the jobs is:
Meg – A, Ali – B, Joe – C

8

	Machine				
Operator	***W***	***X***	***Y***	***Z***	
A	38	35	26	54	−26
B	32	29	32	26	−26
C	44	26	23	35	−23
D	20	26	32	29	−20

	Machine			
Operator	***W***	***X***	***Y***	***Z***
A	12	9	0	28
B	6	3	6	0
C	21	3	0	12
D	0	6	12	9

	Machine			
Operator	***W***	***X***	***Y***	***Z***
A	9	6	0	28
B	3	0	6	0
C	18	0	0	12
D	0	6	15	12



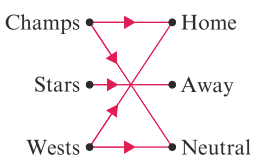
A must be allocated to Y, so C cannot.
If C cannot be allocated to Y, C must be allocated to X and so B cannot.
If B cannot be allocated to X, B must be allocated to Z.
D must be allocated to W.
The allocation of machinists to machines that minimises the total cost is:
$A – Y, B – Z, C – X, D – W$

9

Team	Home	Away	Neutral	
Champs	10	9	8	−8
Stars	7	4	5	−4
West	8	7	6	−6

Team	Home	Away	Neutral
Champs	2	1	0
Stars	3	0	2
West	2	1	0

Team	Home	Away	Neutral
Champs	0	1	0
Stars	1	0	2
West	0	1	2



Stars must play at the away ground. Both Champs and Wests can play at either Home or Neutral, so there are two possible allocations:

Champs – Home (10), Stars – Away (4) and Wests – Neutral (6) for a total of \$20 000.

or

Champs – Neutral (8), Stars – Away (4) and Wests – Home (8) for a total of \$20 000.

10

Service Vehicle	Motorist				
	Jess	Mark	Raj	Karla	
A	18	15	15	16	−15
B	7	17	11	13	−7
C	25	19	18	21	−18
D	9	22	19	23	−9

Service Vehicle	Motorist			
	Jess	Mark	Raj	Karla
A	3	0	0	1
B	0	10	4	6
C	7	1	0	3
D	0	13	10	14

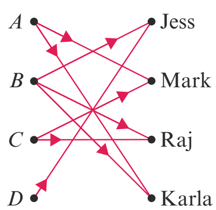
Service Vehicle	Motorist			
	Jess	Mark	Raj	Karla
A	3	0	0	1
B	0	10	4	6
C	7	1	0	3
D	0	13	10	14

Service Vehicle	Motorist			
	Jess	Mark	Raj	Karla
A	3	0	0	0
B	0	10	4	5
C	7	1	0	2
D	0	13	10	13

Service Vehicle	Motorist			
	Jess	Mark	Raj	Karla
A	7	0	0	0
B	0	6	0	1
C	11	1	0	2
D	0	9	6	9

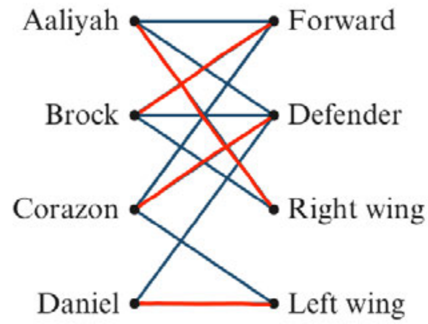
Service Vehicle	Motorist			
	Jess	Mark	Raj	Karla
A	7	0	0	0
B	0	6	0	1
C	11	1	0	2
D	0	9	6	9

Service Vehicle	Motorist			
	Jess	Mark	Raj	Karla
A	8	0	1	0
B	0	5	0	0
C	11	0	0	1
D	0	8	6	8

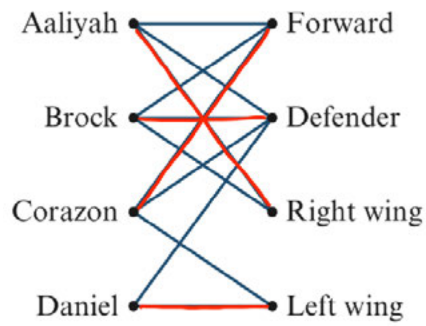


A must go to Mark or Karla. *B* can go to Jess, Raj or Karla. *C* must go to Mark or Raj. *D* must go to Jess.
The allocations of vehicle to motorist that minimise the total distance travelled are:
A – Karla (16), *B* – Raj (11), *C*– Mark (19) and *D* – Jess (9) OR
A – Mark (15), *B* – Karla (13), *C*– Raj (18) and *D* – Jess (9), both for a total of 55 km.

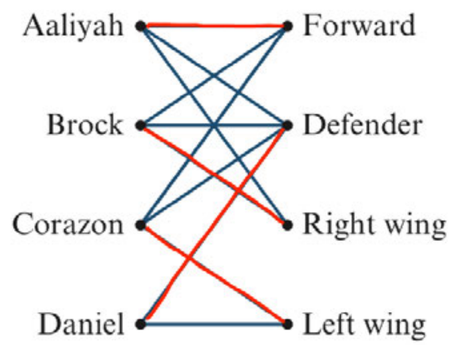
11 Option A is a viable allocation:



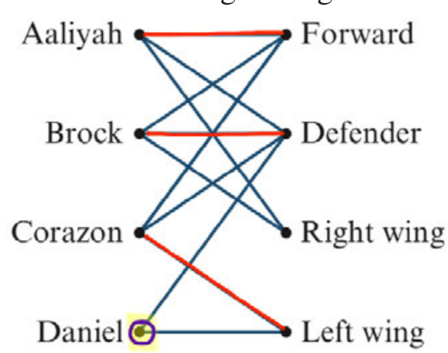
Option B is a viable allocation:



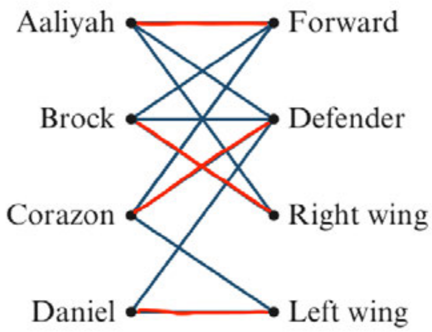
Option C is a viable allocation:



Option D is **not** a viable allocation, as Daniel cannot be allocated to Right wing:



Option E is a viable allocation:



D

12 Perform the Hungarian Algorithm to obtain the optimal allocation:

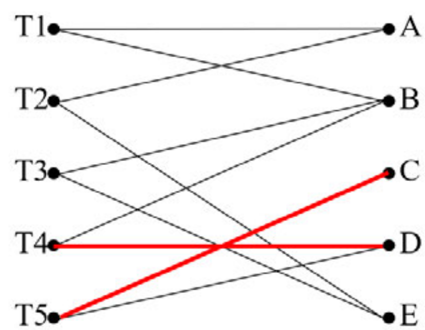
	A	B	C	D	E	
T1	1	2	2	5	4	−1
T2	4	9	7	11	6	−4
T3	5	3	3	9	4	−3
T4	8	5	6	6	7	−5
T5	5	8	4	6	9	−4

	A	B	C	D	E
T1	0	1	1	4	3
T2	0	5	3	7	2
T3	2	0	0	6	1
T4	3	0	1	1	2
T5	1	4	0	2	5
				−1	−1

	A	B	C	D	E
T1	0	1	1	3	2
T2	0	5	3	6	1
T3	2	0	0	5	0
T4	3	0	1	0	1
T5	1	4	0	1	4

	A	B	C	D	E
T1	0	0	1	2	1
T2	0	4	3	5	0
T3	3	0	1	5	0
T4	4	0	2	0	1
T5	1	3	0	0	3

	A	B	C	D	E
T1	0	0	1	2	1
T2	0	4	3	5	0
T3	3	0	1	5	0
T4	4	0	2	0	1
T5	1	3	0	0	3



Multiple allocations are possible (which will all result in the same overall minimum completion time for the tasks), however in every scenario Task 5 must be completed by Carmen and Task 4 must be completed by Dexter.
One allocation is: Task 1 = Anita = 1 hour, Task 2 = Electra = 6 hours, Task 3 = Brad = 3 hours, Task 4 = Dexter = 6 hours, Task 5 = Carmen = 4 hours. If this allocation is followed, then Anita will be the first person to finish. A second allocation is: Task 1 = Brad = 2 hours, Task 2 = Anita = 4 hours, Task 3 = Electra = 4 hours, Task 4 = Dexter = 6 hours, Task 5 = Carmen = 4 hours. If this allocation is followed, then Brad will be the first person to finish. In any other allocation, Anita or Brad will always finish their task before the ther people at the bank.

A

13 In the previous question, when the Hungarian algorithm was performed, the minimum total time for all tasks was found to be 20 hours.
Perform the Hungarian Algorithm with the new time for Task 5 to be completed by Electra, to obtain the optimal allocation:

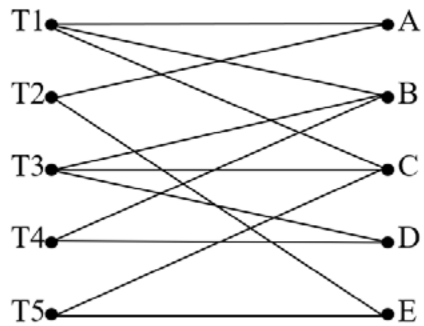
	A	B	C	D	E	
T1	1	2	2	5	4	−1
T2	4	9	7	11	6	−4
T3	5	3	3	9	4	−3
T4	8	5	6	6	7	−5
T5	5	8	4	6	4	−4

	A	B	C	D	E
T1	0	1	1	4	3
T2	0	5	3	7	2
T3	2	0	0	6	1
T4	3	0	1	1	2
T5	1	4	0	2	0
				−1	

	A	B	C	D	E
T1	0	1	1	3	2
T2	0	5	3	6	1
T3	2	0	0	5	0
T4	3	0	1	0	1
T5	1	4	0	1	0

	A	B	C	D	E
T1	0	0	0	2	1
T2	0	4	2	5	0
T3	3	0	0	5	0
T4	4	0	1	0	1
T5	2	4	0	1	0

	A	B	C	D	E
T1	0	0	0	2	1
T2	0	4	2	5	0
T3	3	0	0	5	0
T4	4	0	1	0	1
T5	2	4	0	1	0



Multiple allocations are possible, however every allocation will result in the same minimum total time for all tasks to be completed. One possible allocation is: Task 1 = Carmen = 2 hours, Task 2 = Anita = 4 hours, Task 3 = Brad = 3 hours, Task 4 = Dexter = 6 hours, Task 5 = Electra = 4 hours. This allocation takes a total of 19 hours, which is 1 hour less than the original allocation where Electra took 9 hours to complete Task 5.

E

14 If $p = 10$:

	Job 1	Job 2	Job 3	Job 4
Xena	5	3	7	10
Wilson	1	2	5	6
Yasmine	1	7	1	5
Zachary	4	7	6	10

The allocation by the manager will not produce the minimum total completion time. When $p = 10$, following the manager's allocation, Wilson = Job 1 = 1 hour and Zachary = Job 4 = 10 hours, giving a total of 11 hours. If this allocation was swapped whereby Wilson = Job 4 = 6 hours and Zachary = Job 1 = 4 hours, this would give a total of 10 hours, therefore when $p = 10$ the manager's inital allocation of jobs would not result in the minimum total completion time.

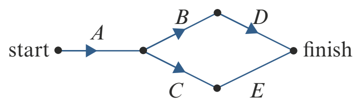
If $p = 9$:

	Job 1	Job 2	Job 3	Job 4
Xena	5	3	7	9
Wilson	1	2	5	6
Yasmine	1	7	1	5
Zachary	4	7	6	9

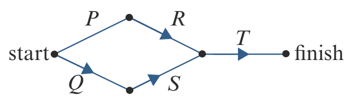
The allocation by the manager will produce the minimum total completion time. When $p = 9$, following the manager's allocation, Zachary's completion time of 9 hours for Job 4 cannot be swapped with any of the previously allocated jobs to result in a shorter completion time. For example, when $p = 9$, Zachary and Wilson take a total of 10 hours to complete their allocated jobs and if they swapped jobs, they would still take a total of 10 hours to complete each job (Wilson = Job 4 = 6 hours, Zachary = Job 1 = 4 hours). This will be true for all values of p less than or equal to 9, therefore the manager's initial allocation will only achieve the minimum completion time if the value of p is not greater than 9 hours.
D

Solutions to Exercise 14C

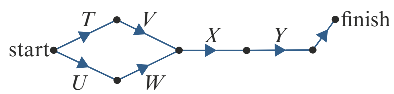
1 a



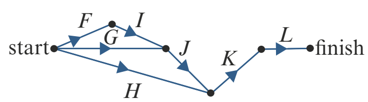
b



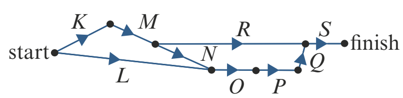
c



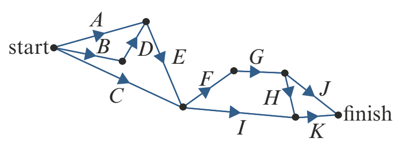
d



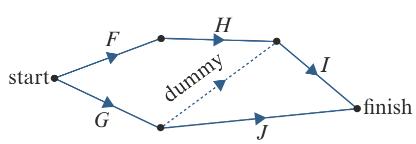
e



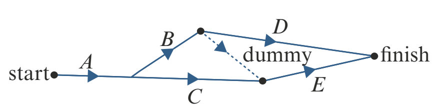
f



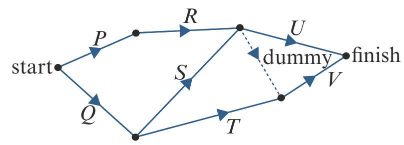
2 a



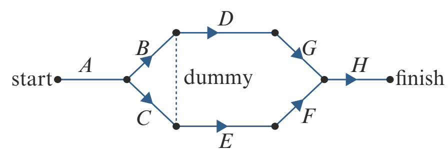
b



c



d



3 a

Activity	*Immediate predecessors*
A	-
B	-
C	*A*
D	*A*
E	*B,C*
f	*D*
G	*E*

b

Activity	*Immediate predecessors*
P	-
Q	*P*
R	*p*
S	*Q*
T	*Q*
U	*S,V*
V	*R*
W	*R*
X	*T,U*

c

Activity	*Immediate predecessors*
J	-
K	-
L	J
M	N
N	K
O	K
P	N
Q	L,M
R	P
S	Q,R
T	Q

d

Activity	*Immediate predecessors*
A	-
B	-
C	A
D	A
E	D,B
f	C,E
G	D,B
H	B

e

Activity	*Immediate predecessors*
P	-
Q	P
R	p
S	Q
T	Q
U	R
V	S
W	S,T
X	U
Y	W
Z	V,X,Y

f

Activity	*Immediate predecessors*
A	-
B	A
C	A
D	A
E	B
f	C,D
G	D
H	E,F,G
I	G
J	I
K	H

4 a "Remove broken component" is activity C.
Look at activity C in the activity network.
Activity C follows immediately from activity A.
Activity A is an immediate predecessor of activity C.
"Remove panel" is an immediate predecessor of "Remove broken component".

b "Install new component" is activity F.
Look at activity F in the activity network.
Activity F follows immediately from activity B and the dummy that follows activity D.
Activities B and D are immediate predecessors of activity F.
"Order component" and "Pound out dent" are immediate predecessors of "Install new component".

5 **a**

Activity	*Immediate predecessors*
A	–
B	–
C	–
D	A
E	B, F
F	C
G	B, F
H	D, E
I	H
J	I, K
K	G
L	G
M	H
N	J, L
O	N

b $A - D - H - M$
$A - D - H - I - J - N - O$

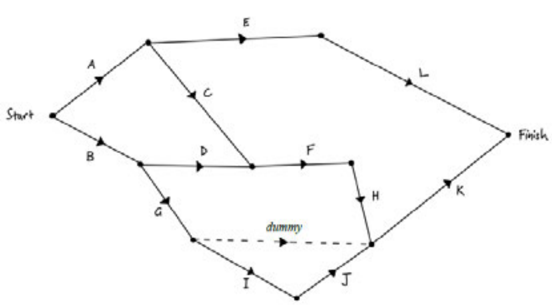
c $B - E - H - M$
$B - E - H - I - J - N - O$
$B - G - K - J - N - O$
$B - G - L - N - O$

d $C - F - E - H - M$
$C - F - E - H - I - J - N - O$
$C - F - G - K - J - N - O$
$C - F - G - L - N - O$

6 **a** D, F, H

b A, B, C, D, E, F, G, H

7 **a** Draw an acitvity network for this project:



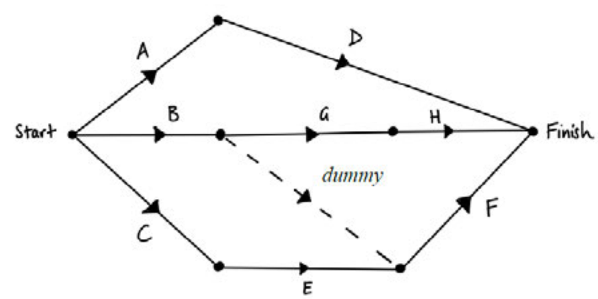
The dummy activity must be drawn from the end of activity G because activities I and K share activity G as an immediate predecessor.

b The dummy activity must be drawn to the start of activity K. As seen in the diagram above, activity K shares some but not all immediate predecessors with activity I.

c It is necessary to include a dummy activity because there are activities that share some, but not all, of their immediate predecessors. Activity I has activity G as an immediate predecessor, whereas activity K has activities H, J and G as immediate predecessors.

8 There are four paths from *start* to finish that start with activity B are:
$B - D - E - G - M$
$B - D - E - H - L$
$B - D - F - I - J - L$
$B - D - F - I - K$
D

9 Draw an acitvity network for this project:



The dummy activity will drawn from the end of activity B to the start of activity F
A

10 Using the activity network drawn for Question 9 above, there are 4 paths from *start* to *finish*:

$A - D$

$B - G - H$

$B - F$

$C - E - F$.

A

Solutions to Exercise 14D

1 **a** Use forward scanning,
$p = 8 + 4$
$p = 12$

b Use forward scanning,
$w = 4 + 6$
$w = 10$

c Using forward scanning,
$m + 4 = 12$
$m = 12 - 4$
$m = 8$
Using backward scanning,
$n = 12 - 4$
$n = 8$

d Using forward scanning,
$c = 6 + 5$
$c = 11$
Using backward scanning,
$a = 15 - 5$
$a = 10$
Using backward scanning,
$b - 3 = 15$
$b = 15 + 3$
$b = 18$

e Using forward scanning,
$f = 3 + 6$
$f = 9$
Using forward scanning, g is the largest of:
$g = 5 + 7$ or $g = f + 0$
$g = 12$ or $g = 9$
So, $g = 12$

f Using forward scanning,
$4 + q = 12$
$q = 12 - 4$
$q = 8$
Using backward scanning,
$9 - p = 4$
$p = 9 - 4$
$p = 5$
Using backward scanning,
$12 - r = 9$
$12 - 9 = r$
$r = 3$
Using forward scanning,
$n + r = 12$
$n + 3 = 12$
$n = 12 - 3$
$n = 9$

2 **a** Using forward scanning,
$6 +$ duration of $A = 9$
duration of $A = 9 - 6$
duration of $A = 3$

b The critical path follows activities that have no float time. The two numbers in the boxes at the start of these activities will be the same.
The critical path is: $A - C$

c Float time = LST – EST
= 11 – 6
= 5

d LST for D is the second number in the boxes at the start of activity D.
LST for activity $D = 13$

e Using backward scanning,
$15 -$ duration of $D = 13$
Duration of $D = 15 - 13$
Duration of $D = 2$

3 a Using forward scanning,
$0 + \text{duration of } B = 12$
Duration of $B = 12$

b LST for E is the second number in the boxes at the start of activity E.
LST for $E = 10$

c EST for E is the first number in the boxes at the start of activity E.
EST for $E = 9$

d Float time for E = LST – EST
$= 10 - 9$
$= 1$

e Using forward scanning,
$0 + \text{duration } A = 3$
duration $A = 3$

f Using forward scanning,
$0 + \text{duration } D = 9$
duration $D = 9$

4 a The critical path follows activities to boxes where the EST and LST are the same.
The critical path is: $D – E – F$

b Non-critical activities are A, B, C
Float A = LST (B) – duration of A
$= 4 – 3$
$= 1$
Float B = LST (B) – EST (B)
$= 4 – 3$
$= 1$
Float C = LST (F) – duration of C
$= 22 – 7$
$= 15$

5 a The critical path follows activities to boxes that have EST and LST the same.
The critical path is: $B – E – F – H – J$

b Non-critical activities are A, C, D, G, I
Float A = LST(D) – duration of A
$= 11 – 10$
$= 1$
Float C = LST(J) – duration of C
$= 17 – 3$
$= 14$
Float D = LST (D) – EST (D)
$= 11 – 10$
$= 1$
Float G = LST(I) – EST(G) – duration G
$= 15 – 13 – 1$
$= 1$
Float I = LST(J) – EST(I) – duration I
$= 17 – 14 – 2$
$= 1$

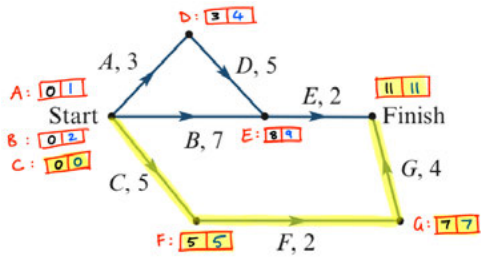
6 a Use information in the activity network.

Activity	*Duration(weeks)*	*Immediate predecessors*
A	3	-
B	6	-
C	6	A,B
D	5	B
E	7	C,D
f	1	D
G	3	E
H	3	F
I	2	B

b The critical path follows activities to boxes where the EST and LST are the same.

The critical path is: $B - C - E - G$

7



The answers for each of the following questions can be determined from inspection of the diagram above, where critical path analysis using forward and backward scanning have been used.

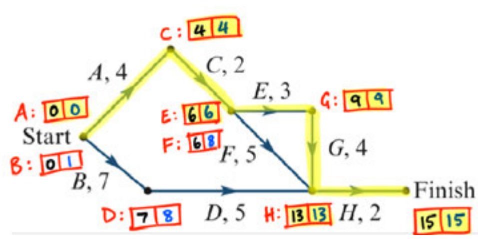
a EST of E = 8 days

b Minimum completion time = 11 days

c The critical path is the path where all activities have a float time of zero; EST = LST. The critical path is C-F-G

d Float time = LST − EST. Only Activity B has a float time of 2 days (2 − 0 = 2)

8



The answers for each of the following questions can be determined from inspection of the diagram above, where critical path analysis using forward and backward scanning have been used.

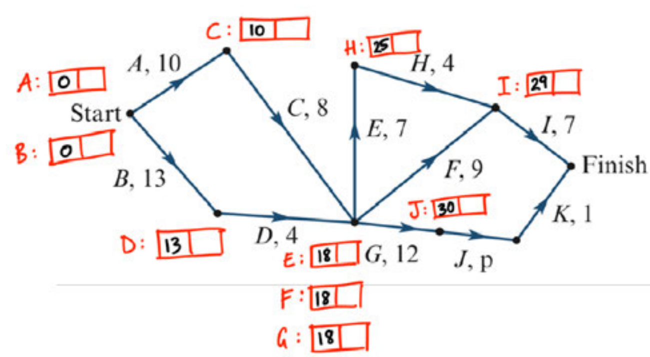
a The immediate predecessors of Activity H refers to the three edges directed towards the vertex where the edge representing Activity H begins; the three acticities are D, F, G.

b EST of H = 13

c Minimum completion time = 15

d The duration of time an activity can be delayed by, without affecting the minimum completion time of the project, is also referred to as the *Float time*. Float time for an activity = LST − EST; Activity F has the largest float time (8 − 6 = 2) which means is can be delayed longer than any other activity without affecting the minimum completion time of the project.

9



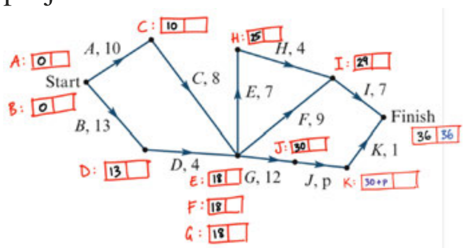
The answers for each of the following questions can be determined from inspection of the diagram above, where critical path analysis using forward scanning have been used. In this project, the duration of activity J is initially unknown, therefore the minimum completion time and backward scanning will only be considered for parts *b* and *c*.

a **i** EST for H = 25

ii EST for I = 29

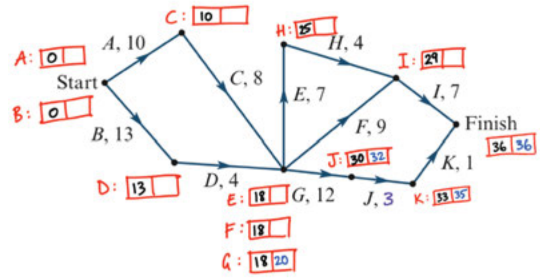
iii EST for J = 30

b Complete forward scanning for the diagram above, including p as an unknown value, knowing the minimum completion time for the project is 36:



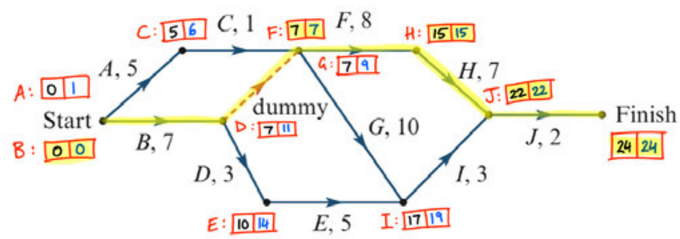
At the *Finish* vertex, there are two directed edges connected; activities I and K. Activity I has an EST of 29 and a duration of 7; it can be determined that the LST for activity I will be 29, resulting in a float time of zero and thus will be included in the critical path for this project. Activity K has a, EST of $30 + p$ and a duration of 1. The LST of activity K is 35, therefore to create more than one critical path, the value of p (the duration of activity J) must be 5. If it was greater than 5, the overall minimum completion time for the project would be greater than 36 and the path using activity I as the final activity would no longer be a critical path. $p = 5$.

c Complete forward and backward scanning for the diagram above, including $p = 3$ for the duration of activity J:



Float time of activity G
= LST − EST
= 20 − 18
= 2

10



The answers for each of the following questions can be determined from inspection of the diagram above, where critical path analysis using forward and backward scanning have been used.

a The immediate predecessors of activity G are the two edges directed towards the vertex where the edge representing activity G begins; be careful of the dummy activity, as this is not an activity. It is required if two activities share some, but not all, of their immediate predecessors. The dummy activity allows activity B to be an immediate predecessor for activity G. The immediate predecessors of activity G are C and B. *Note: we* *do not indicate the dummy activity is an immediate predecessor.*

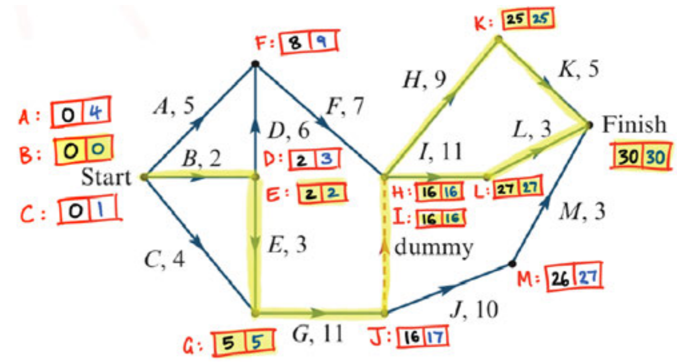
b *Note: this question is not exclusively asking about immediate predecessors.* To determine all activites that must be completed before activity I can begin, start by considering the immediate predecessors, then backtrack and identify the immediate predecessors for those activities. Activity I has G and E as immediate predecessors. Activity G has C and B as immediate predecessors. Activity C has A as an immediate predecessor. Activity E has D as an immediate predecessor. Therefore the activities that must be completed before activity I can begin are: activities A, B, C, D, E, G.

c The critical path is $B - F - H - J$

d Float time of activity E
$= \text{LST} - \text{EST}$
$= 14 - 10$
$= 4$

e Activities that have a float time of 2 or more can have their duration increase by 2 and not affect the minimum completion time of the project. There are four activities with a float time of 2 or more: D, E, G, I

11



The answers for each of the following questions can be determined from inspection of the diagram above, where critical path analysis using forward and backward scanning have been used.

a

Activity	Immediate predecessors
A	–
B	–
C	–
D	B
E	B
F	A, D
G	C, E
H	F, G
I	F, G
J	G
K	H
L	I
M	J

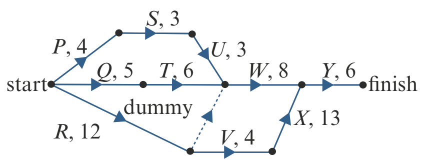
b 3 activities have an EST of 16 (activities H, I, J)

c LST of activity $F = 9$

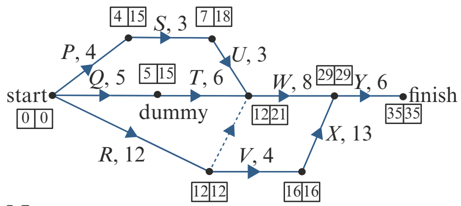
d The critical path includes all activities from $Start$ to $Finish$ that have a float time of zero. The two paths are: $B - E - G - H - K$ and $B - G - I - L$. Note, the dummy is never listed as part of the critical path.

e Activities that have a float time of 1 or more can be delayed by 1 and not affect the minimum completion time of the project. There are six activities with a float time of 1 or more: A, C, D, F, J, M

12 a



b



Note:

LST (P) = LST (S) – duration P

= 15 – 4

= 11

LST (Q) = LST (T) – duration Q

= 15 – 5

= 10

A table of EST and LST for each activity is shown below:

Activity	***EST***	***LST***
P	0	11
Q	0	10
R	0	0
S	4	15
T	5	15
U	7	18
V	12	12
W	12	21
X	16	16
Y	29	29

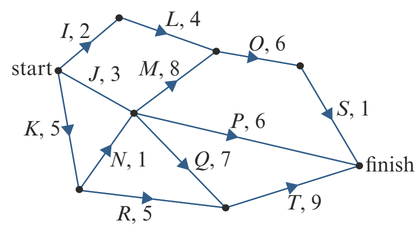
c The critical path follows activities to boxes where the EST and LST are the same.

The critical path is: $R – V – X – Y$

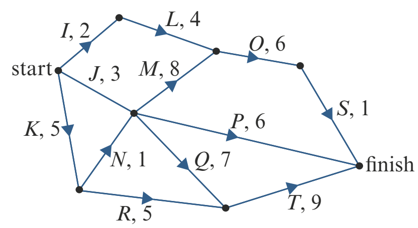
d The minimum time to complete the project is the value in the boxes at the finish.

Minimum project completion time is 35 weeks.

13 a



b



Note:

LST (I) = LST (L) – duration I

= 11 – 2

= 9

LST (J) = LST (M, P or Q) – duration J

= 6 – 3

= 3

LST (M) = LST (O) – duration M

= 15 – 8

= 7

LST (P) = LST (*finish*) – duration P

= 22 – 6

= 16

LST (Q) = LST (T) – duration Q

= 13 – 7

= 6

LST (R) = LST (T) – duration R

= 13 – 5

= 8

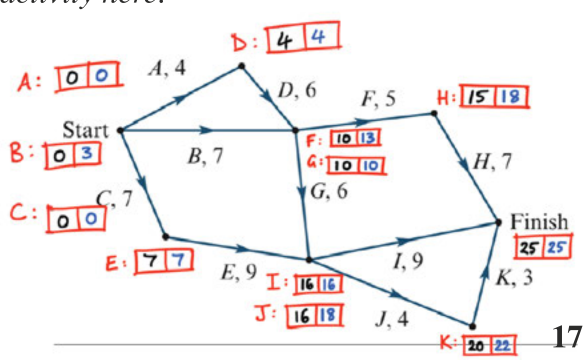
A table of EST and LST for each activity is shown below:

Activity	*EST*	*LST*
I	0	9
J	0	3
K	0	0
L	2	11
M	6	7
N	5	5
O	14	15
P	6	16
Q	6	6
R	5	8
S	20	21
T	13	13

c The critical path follows activities to boxes where the EST and LST are the same.
The critical path is: $K - N - Q - T$

d The minimum time to complete the project is the value in the boxes at the finish.
Minimum project completion time is 22 weeks.

14 *A crticial path anaylysis, using forward and backward scanning, was used on the activity network given for Questions 14, 15 and 16. Answers will be in reference to the EST and LST found for each activity here:*



EST for activity J = 16 days

E

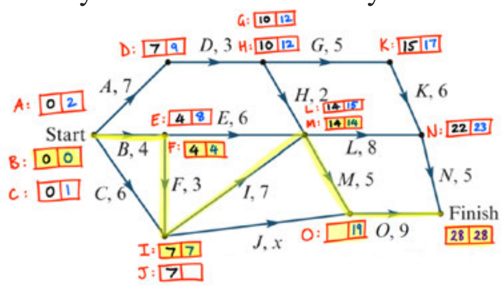
15
- Activities A, B, C each have zero immediate predecessors
- Activities D and E have one immediate predecessor each
- Activities F and G share two immediate predecessors (B, D)
- Activity H has one immediate predecessor
- Activities I and J share two immediate predecessors (G, E)
- Activity K has one immediate predecessor

Therefore 4 activities (F, G, I, J) are the only activities with two immediate predecessors each.
D

16 An activity can be delayed and not affect the minimum completion time of the project if it has a float time greater than zero (in other words, not along a critical path). There are two critical paths: $A - D - G - I$ and $C - E - I$.
Five activities have a float time greater than zero and are not included in either of the critical paths: B, F, H, J, K.
Five activites can be delayed without affecting the minimum completion time of the project.
C

17 Perform a critical analysis, using forward scanning and backward scanning.

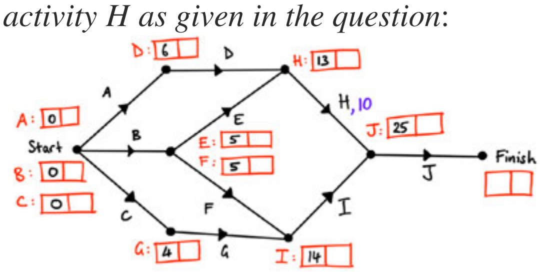
Although the duration of J is unknown, the minimum completion time for the project is given (28 weeks), therefore all but two values of the crticial analysis can intially be determined: EST of activity O and LST of activity J:



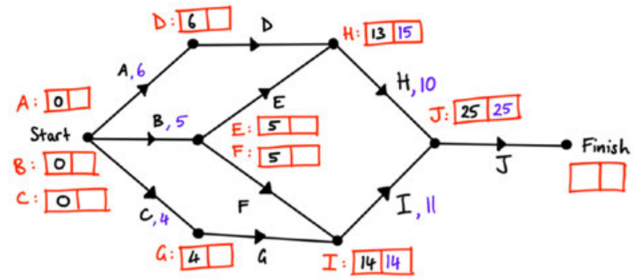
Activity J has an EST of 7 weeks and activity O has an LST of 19 week. If the duration of activity J was 12 weeks or less, the minimum completion time for the project would remain 28 weeks as the EST of activity O would remain 19 because of the duration of activity M and all activities leading to activity M are fixed. If the duration of activity J is 12 or less, the original critical path will still be the critical path for the project, as it will have the longest duration of all possible paths from $Start$ to $Finish$.

A

18 Using the precedence table, draw an activity network for the project and include the EST for each activity *Note: do not forget to include the duration of activity H as given in the question*:



Knowing activity H has a duration of 10 days, the LST of activity H can be determined (15 days) and given that activity H has a float time of 2 days, activity I must be along the critical path and have a float time of 0 days. Activities D, E, F and G are each connected to an acitivty with zero immediate predecessors (A, B, C), therefore the EST of D, E, F, G are the duration of activities A, B, C.



If Activity H has an EST of 13 days, activity D, with its EST of 6 days, can have a maximum duration of 7 days and a minimum of 1 day (a duration of zero days should never be considered). If the duration of activity D was 7 days, then its LST would be 8 days ($15 - 7 = 8$) and it could be delayed by 2 days (Delay or Float time = LST − EST = $8 - 6 = 2$). If the duration of activity D was 1 day, then its LST would be 14 days ($15 - 1 = 14$) and it could be delayed by 8 days (Delay or Float time = LST − EST = $14 - 6 = 8$). Therefore the maximum length of the delay for activity D is 8 days.

D

Solutions to Exercise 14E

1 a

Path	Duration (hours)
$A-D$	17
$B-E-F$	20
$B-E-G-I$	21 ©
$C-H-I$	16

b The critical path and minimum completion time refer to the path with the longest duration of time. From the table above, the critical path is $B-E-G-I$ with a duration of 21 hours.

c

Path	Duration (hours)	Max reduction: E by 3
$A-D$	17	17
$B-E-F$	20	17
$B-E-G-I$	21 ©	18 ©
$C-H-I$	16	16

The new minimum completion time for this project is 18 hours.

2 a

Path	Duration (days)
$A-B-D-G$	19
$A-B-E$	20
$A-B-F-H$	21 ©
$A-C-E$	19
$A-C-F-H$	20

The critical path is the path with the longest duration of time. From the table above, the critical path is $A-B-F-H$

b The minimum completion time for the project is the duration of the crtical path. 21 days.

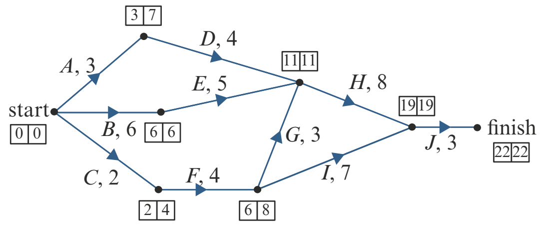
c

Path	Duration (days)	Max reduction: B by 3
$A-B-D-G$	19	16
$A-B-E$	20	17
$A-B-F-H$	21 ©	18
$A-C-E$	19	19
$A-C-F-H$	20	20 ©

The new minimum completion time for the project is 20 days.

d The new minimum completion time for the project is 20 days. Four of the five possible paths from $Start$ to $Finish$ originally had a duration of 20 days or less. There is no value in reducing the completion time for those four paths as this will incur an unnecessary cost. Paths that originally have a duration of 20 days or more must have activity B crashed by the minimum amount to achieve the mminimum completion time. The path $A-B-F-H$ originally had a duration of 21 days, therefore activity B would only need to be crashed by 1 day to achieve the greatest reduction in time taken to complete the project. As previously stated, reducing the completion time to less than 20 would incur an unnnecessary cost, as the minimum completion time of anoth path where crashing cannot take place prevents the overall completion time for the project to be achieved in a shorter amount of time. As the crashing will cost \$100 per day, the minimum cost to achieve the greatest reduction in time is \$100.

3 a



The critical path follows activities to boxes where the EST and LST are the same.
The critical path is: $B - E - H - J$

b Total minimum completion time for A and D is 3 + 4 = 7 hours.
Total minimum completion time for activities C, F and G is 2 + 4 + 3 = 9 hours
Reduction in completion time for activity E must not cause the total completion time for activities B and E to drop below 9 hours.
Completion time for activity E cannot be lower than 3.
The maximum reduction in completion time for activity E is 2 hours.

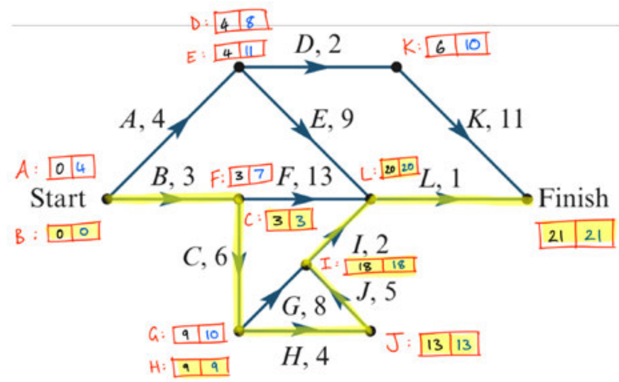
c Total minimum completion time for activities C, F and I is 2 + 4 + 7 = 13 hours.
Activity H can be reduced in duration to ensure duration of activity H + 11 is not lower than 13
Minimum duration of activity H is 2.
Maximum reduction in completion time for activity H is 6 hours.

d

Path	Duration (hours)	Max reduction: every activity by 2
$A - D - H - J$	18	10
$B - E - H - J$	22 ©	14 ©
$C - F - G - H - J$	20	10
$C - F - I - J$	16	8

New minimum completion time for the project is 14 hours.

4 a Perform a critical analysis of the activity network using forward and backward scanning:



b

Path	Duration (hours)
$A - D - K$	17
$A - E - L$	14
$B - F - L$	17
$B - C - G - I - L$	20
$B - C - H - J - I - L$	21 ©

Initially the minimum completion time for the project is 21 hours. Consider every activity and crash by its maximum number of hours (reduce to 1 hour) and examine the impact on the overall minimum completion time of the project:

- Activities A, D, E, F, G, K should not be considered as they do not lie along the critical path. If any of these activites were crashed, the minimum completion time would remain as 21 hours

- Activity *L* cannot be considered as it has a duration of 1 hour and cannot be reduced to a time less than 1 hour.

- Activity B can only be crashed by a maximum of 2 hours. This will result in a minimum completion time of 19 hours

- Activity C can only be crashed by a maximum of 5 hours. This will result in a minimum completion time of 17 hours, as an initial non-critical path will now become the critical path (both $A - D - K$ and $B - F - L$)

- Activity H can only be crashed by a maximum of 3 hours. This will result in a minimum completion time of 20 hours as there will be a new critical path $B - C - G - I - L$

- Activity I can only be crashed by a maximum of 1 hour. This will result in a minimum completion time of 20 hours

- Activity J can only be crashed by a maximum of 4 hours. This will result in a minimum completion time of 20 hours as there will be a new critical path $B - C - G - I - L$

The minimum time the project can be completed in, if only one activity can be crashed, is 17 hours. This is achieved by crashing activity C by a maximum of 5 hours.

c Activity G can be crashed by a maximum of 7 hours and activity J can be crashed by a maximum of 4 hours:

Path	**Duration (hours)**	**Max reduction: G by 7 and J by 4**
$A - D - K$	17	17 ©
$A - E - L$	14	14
$B - F - L$	17	17 ©
$B - C - G - I - L$	20	13
$B - C - H - J - I - L$	21 ©	17 ©

The new minimum completion time, after crashing, is 17 hours. There is at least one path that cannot be reduced to a shorter amount of time.

- The original critical path $B - C - H - J - I - L$ can be reduced from 21 hours to 17 hours if activity J is crashed by 4 hours.

- The path $B - C - G - I - L$ can be reduced from 20 hours to 17 hours if activity G is crashed by 3 hours. *Note: although this path can be reduced to 13 hours, this would incur unnecessary cost as the new overall minimum completion time for the project is 17 hours. At least one path cannot be reduced further than 17 hours, therefore we should not consider any reductions that results in a completion time less than 17 hours.*

- The three remaining paths do not have activities G or J along them, so they cannot be reduced by crashing. They also originally had completion times of 17 hours or less, therefore there is no needs to investigate further.

The minimum cost of reducing the completion time of this project as much as possible is

$= G \times 3 + J \times 4$
$= 200 \times 3 + 150 \times 4$
$= \$1200$

5 a

Path	**Duration (days)**	**Max reduction: D, E, H by 2**
$A-D-F-G$	23	21
$A-D-H$	22	20
$B-F-G$	22	22 ©
$B-H$	21	19
$C-E-F-G$	24 ©	22 ©
$C-E-H$	23	19

The new minimum completion time for this project is 22 days.

b
- The original critical path $C-E-F-G$ can be reduced from 24 to 22 days if activity E is crashed by 2 days
- The crashing of activity E will also reduce the time of the path $C-E-H$
- the path $A-D-F-G$ must be crashed by 1 day to reduce its completion time from 23 to 22 days. This can be achieved if activity D is crashed by 1 day.
- All other paths originally had a completion time of 22 days or less, so no further investigation is required

The minimum cost of reducing the completion time of this project as much as possible is
$= E \times 2 + D \times 1$
$= 350 \times 2 + 170 \times 1$
$= \$870$

6 a The immediate predecessors of activity G are C, D, H. The dummy activity is used because activities F and G share some, but not all immediate predecessors; activity F only has C as an immediate predecessor.

b

Path	**Duration (days)**
$A-C-F-J$	21
$A-C-G-I$ - J	23
$A-C-G-K$	22
$A-D-G-I-J$	22
$A-D-G-K$	21
$B-E-H-G-I-J$	24 ©
$B-E-H-G-K$	23

There is one critical path. If any of the activities along the critical path were crashed, more than one critical path would be created. The activities along the crtical path are: B, E, H, G, I, J

c i

Path	**Duration (days)**	**Max reduction: B, E, G, H, I by 1**
$A-C-F-J$	21	21 ©
$A-C-G-I$ - J	23	21 ©
$A-C-G-K$	22	21 ©
$A-D-G-I-J$	22	20
$A-D-G-K$	21	20
$B-E-H-G-I-J$	24 ©	20
$B-E-H-G-K$	23	20

The minimum number of hours in which the project could be completed, with crashing, is 21 hours.

ii All paths should have a duration of 21 hours or less. When investigating which activities to crash, start with the paths that have a duration closest to the minimum completion time, then work your way up to the paths that require the greatest reduction in completion time. Don't forget, when you choose to crash an activity, it will reduce the time of all paths that activity is used in.

- Do not consider the two paths that originally had a duration of 21 hours: $A-C-F-J$ and $A-D-G-K$.

- The path $A-C-G-K$ originally had a duration of 22 hours. To reduce this path to 21 hours, one activity must be crashed. The only activity that can be crashed along this path is activity G. Crash G by 1 hour; this will also reduce the duration of all paths with activity G by 1 hour.

- The path $A-C-G-I-J$ originally had a duration of 23 hours. As previously stated, activity G will be crashed by 1 hour and there is only one other option for crashing along this path; activity I must be crashed by 1 hour to reduce the duration of this path to 21 hours. All paths with activity I will also be reduced by 1 hour.

- The path $B-E-H-G-K$ originally had a duration of 23 hours. Activities E, G, H can be crashed by 1 hour each. Previously, it has been decided that acitivity G will be reduced by 1 hour. To reduce the duration of this path, either activity E or I can be crashed by 1 hour. The cost of crashing either of these activities is equal, so consider the final path to determine which activity to crash

- The path $B-E-H-G-I-J$ originally has a duration of 24 hours. Activities E, G, H, I can be crashed to reduce the duration to 21 hours. Previously, activities G and I were chosen to be crashed by 1 hour each. Similar to the previous path investigated, crashing either activity E or I by 1 hour would reduce the duration of this path to 21 hours. As the cost of crashing either of these activities is equal, the minimum cost of crashing can be determined by choosing either one of these activities; crash activitiy E by 1 hour.

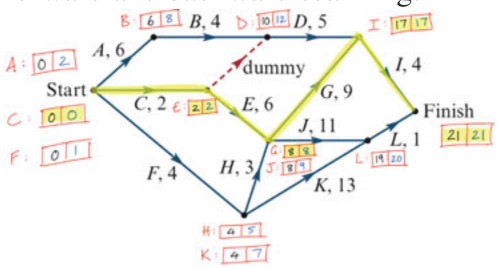
The minimum cost of completing this project in the minimum number of hours is
$= G \times 1 + I \times 1 + E \times 1$
$= 150 \times 1 + 150 \times 1 + 150 \times 1$
$= \$450$

7 a

Path	Duration (weeks)
$A-B-D-I$	19
$C-D-I$	11
$C-E-G-I$	21 ©
$C-E-J-L$	20
$F-H-G-I$	20
$F-H-J-L$	19
$F-K-L$	18

The project can be completed in 21 weeks.

b Perform a critical analysis using forward and backward scanning:



The EST for D is 10 weeks. This was a sum of the duration of activities A and B, as that was the path from the *Start* to activity D with the longest duration.

c The LST for H is 5 weeks. When backward scanning, if there are two LST options at a vertex, you must use the smallest value, before subtracting the duration of activity H.

d Activity K has a float time of 3 weeks and is the only activity with a float time greater than two weeks. Activity K has an EST of 4 weeks and a LST of 7 weeks.

e

Path	Duration (weeks)	Max reduction: D, E, G, H, J by 2
$A-B-D-I$	19	17
$C-D-I$	11	9
$C-E-G-I$	21 ©	17
$C-E-J-L$	20	16
$F-H-G-I$	20	16
$F-H-J-L$	19	15
$F-K-L$	18	18 ©

The minimum number of weeks in which the project could be completed, with crashing, is 18 weeks.

All paths should have a duration of 18 weeks or less. When investigating which activities to crash, start with the paths that have a duration closest to the minimum completion time, then work your way up to the paths that require the greatest reduction in completion time. Don't forget, when you choose to crash an activity, it will reduce the time of all paths that activity is used in.

- Do not consider the two paths that originally had a duration of 18 weeks or less: $C-D-I$ and $F-K-L$.
- The path $A-B-D-I$ originally had a duration of 19 weeks. Acitvity D is the only option for crashing to reduce the completion time of this path; crash activity D by 1 week.
- The path $F-H-J-L$ originaly had a duration of 19 weeks. Activities H or J can be crashed by 1 week to reduce the completion time of this path. Keep this in mind when investigating the other paths.
- The path $C-E-J-L$ originally had a duration of 20 weeks. Activities E and J can be crashed by 2 or 1 week each to reduce the completion time of this path.
- The path $F-H-G-I$ originally had a duration of 20 weeks. Activities G and H can be crashed by 2 or 1 week each to reduce the

completion time of this path.

- The path $C - E - G - I$ originally had a duration of 21 weeks. Activities E and G can be crashed by 2 or 1 week each to reduce the completion time of this path.

The information above, with all possible crashing scenarios, is summarised in the following table:

Path	Original duration	Crashing scenarios to achieve 18 weeks		
$A-B-D-I$	19	D by 1 = 2000		
$F-H-J-L$	19	H by 1 = 1500	J by 1 = 3000	
$C-E-J-L$	20	E by 2 = 2000	E by 1 J by 1 = 4000	J by 2 = 6000
$F-H-G-I$	20	G by 2 = 1000	G by 1 H by 1 = 2000	H by 2 = 3000
$C-E-G-I$	21	E by 2 G by 1 = 2500	E by 1 G by 2 = 2000	

Consider the cheapest options for each path and how there is overlap between crashing scenarios (e.g. crashing activities H, E and G will impact multiple paths). The cheapest scenario would be to crash D by 1, H by 1, E by 2 and G by 1. The minimum cost for the greatest reduction in time is:

$= D \times 1 + H \times 1 + E \times 2 + G \times 1$

$= 2000 \times 1 + 1500 \times 1 + 1000 \times 2 + 500 \times 1$

$= \$6000$

8 **a**

Path	Duration (days)
$A-C-H-I-N$	27
$A-C-H-J-M-O$	27
$A-C-G-M-O$	21
$A-C-F-K-M-O$	28
$A-C-F-L-O$	26
$B-D-H-I-N$	28
$B-D-H-J-M-O$	28
$B-D-G-M-O$	22
$B-D-F-K-M-O$	29 ©
$B-D-F-L-O$	27
$B-E-K-M-O$	28
$B-E-L-O$	26

The project can be completed in 29 weeks.

b The critical path is $B - D - F - K - M - O$. There are six activities along the critical path.

c From the table above, there are four paths with a duration of 28 days. They are: $A - C - F - K - M - O$, $B - D - H - I - N$, $B - D - H - J - M - O$, $B - E - K - M - O$.

d

Path	Duration (days)	Max reduction: H, J, K, L, M by 2
$A-C-H-I-N$	27	25
$A-C-H-J-M-O$	27	21
$A-C-G-M-O$	21	19
$A-C-F-K-M-O$	28	24
$A-C-F-L-O$	26	24
$B-D-H-I-N$	28	26 ©
$B-D-H-J-M-O$	28	22
$B-D-G-M-O$	22	20
$B-D-F-K-M-O$	29 ©	25
$B-D-F-L-O$	27	25
$B-E-K-M-O$	28	24
$B-E-L-O$	26	24

The new minimum completion time for the project is 26 days.
Four of the paths originally had a completion time of 26 days or less, so they do not need to be considered when crashing: $A - C - G - M - O$, $A - C - F - L - O$, $B - D - G - M - O$ and $B - E - L - O$.

Path	Duration (days)	Crashing scenarios to achieve 26 days					
$A-C-H-I-N$	27	H by 1					
$A-C-H-J-M-O$	27	H by 1	J by 1	M by 1			
$A-C-F-K-M-O$	28	K by 2	K by 1 M by 1	M by 2			
$B-D-H-I-N$	28	H by 2					
$B-D-H-J-M-O$	28	H by 2	H by 1 J by 1	H by 1 M by 1	J by 2	J by 1 M by 1	M by 2
$B-D-F-K-M-O$	29 ©	K by 2 M by 1	K by 1 M by 2				
$B-D-F-L-O$	27	L by 1					
$B-E-K-M-O$	28	K by 2	K by 1 M by 1	M by 2			

From the table above, activity H must be crashed by 2 and activity L must be crashed by 1, otherwise there will be paths that cannot be reduced to 26 days. Activity J will not be crashed as it only appears in one possible path to be crashed, whereas activities K and M are in common with multiple paths. Considering there is no difference in cost of crashing activities K and M there are mutiple options for crashing: K by 2 and M by 1 or K by 1 and M by 2 (as dictated by the original critical path $B-D-F-K-M-O$).

There are two possible crashing scenarios for this project, that would achieve the minimum completion time of 26 days:
H by 2, J by 0, K by 2, L by 1, M by 1
or
H by 2, J by 0, K by 1, L by 1, M by 2.

9

Path	Duration (hours)
$A-E-I$	17
$A-C-G$	19 ©
$B-D-F-G$	18
$B-D-H-J$	17

The critical path is the path with the longest duration of time; $A-C-G$.

B

10 Trial and error each option.

Path	Duration (hours)	Max reduction: C, G by 1
$A-E-I$	17	17 ©
$A-C-G$	19 ©	17 ©
$B-D-F-G$	18	17 ©
$B-D-H-J$	17	17 ©

B

11 Trial and error each option.

Path	Duration (weeks)	Max reduction: I by 3 and J by 1
$A-C-H-L$	21	21
$A-C-G-I-M$	29	26 ©
$B-D-F-I-M$	28	25
$B-E-J-I-M$	30 ©	26 ©
$B-E-K-M$	26	26 ©

D

Chapter Review: Solutions to Multiple-choice questions

1 Identify the shortest path using inspection, or Dijkstra's algorithm

	B	*C*	*D*	*E*	*F*	*G*	*H*	*Z*
A	10	12	×	×	×	×	×	×
B	10	12	17	19	15	×	×	×
C	10	12	17	19	15	22	×	×
F	10	12	17	19	15	18	22	26
D	10	12	17	19	15	18	22	26
G	10	12	17	19	15	18	22	26
E	10	12	17	19	15	18	22	26
H	10	12	17	19	15	18	22	26

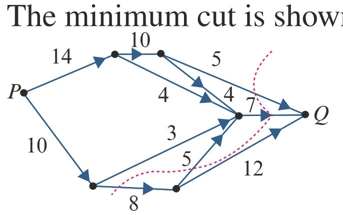
D

2 Capacity of cut = 3 + 4 + 3

= 10

Note: one of the edges with capacity of 4 is not counted as it flows from the sink side of the cut to the source side of the cut. **D**

3 The minimum cut is shown below:

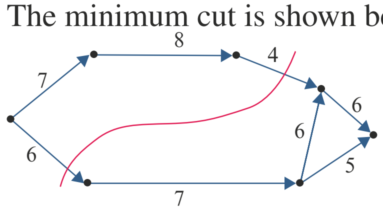


maximum flow = capacity of cut

= 5 + 7 + 8

= 20 **A**

4 The minimum cut is shown below:



maximum flow = capacity of cut

= 6 + 4

= 10 **A**

5 **A:** Travis – basketball and volleyball
Miriam – basketball, athletics, tennis
Swimming is missing so **A** is false.

B: Miriam played 3 sports
Fulvia played 2 sports
total: 5 sports

Andrew played 2 sports
Travis played 2 sports
total: 4 sports

Miriam and Fulvia played more sports than Andrew and Travis so **B** is false

C: Kieran played 3 sports
Miriam played 3 sports so **C** is true.

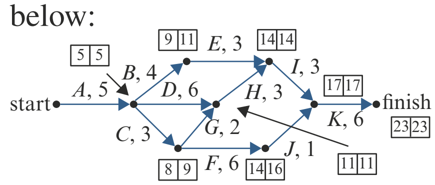
D: Kieran and Travis played basketball, volleyball, swimming, athletics and tennis (5 different sports).

Miriam and Fulvia played swimming, athletics, basketball and tennis (4 different sports).

Travis and Kieran played more sports than Miriam and Fulvia so **D** is false.

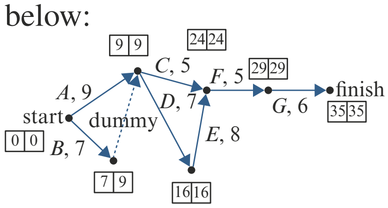
E: Andrew played the same number of sports as Travis and Fulvia so he did not play fewer sports than all others. **E** is false. **C**

6 The critical path analysis is shown below:



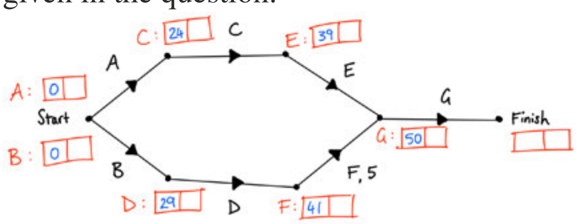
The critical path is: $A - D - H - I - K$ **B**

7 The critical path analysis is shown below:

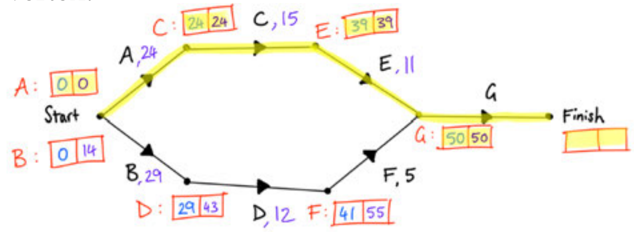


The earliest start time for activity F is 24. **E**

8 The information in the table has been used to construct the following incomplete activity network, in addition to the duration of activity F given in the question.

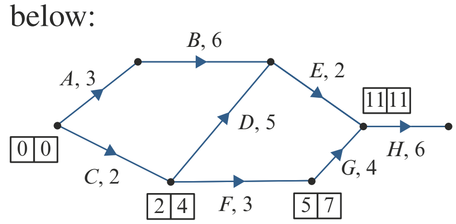


Given activity F has a duration of 5 hours and and an EST of 41, it cannot be on the critical path, as this sum does not equate to the EST of activity G. Likewise, the LST of activity G, minus the duration of activity F, does not equal the EST of activity F, thus there is a float time and if an activity is along the critical path, it must have a float time of zero. If activity F is not along the critical path, then activity E must be along the critical path with a duration of 11 hours to ensure its LST = 39 hours (LST of G − duration of E = 50 − 11 = 39 = EST of E). Although the completion time for this project is unknown, backward scanning can be used to find the LST of each activity as activity G is the only activity which leads to the *Finish* vertex.



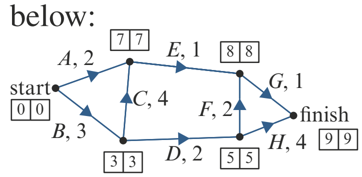
The EST of activity F can increase to 45 hours without affecting the completion time of the project, because activity F has a duration of 5 hours and activity G has an EST of 50 hours. Activity D has a duration of 12 hours. If increased by 4 hours to 16 hours, this would increase the EST of F from 41 to 45 hours (EST of D + new duration of D = 29 + 16 = 45). **D**

9 The critical path analysis is shown below:



The completion time of the project is 17 weeks. **D**

10 The critical path analysis is shown below:



The EST for activity G is 8. **E**

Chapter Review: Extended-response questions

1 **a** Perform the Hungarian algorithm:

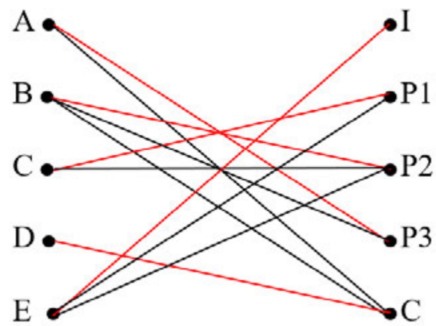
	I	P1	P2	P3	C	
A	16	14	19	9	9	−9
B	17	18	10	9	9	−9
C	9	8	6	15	8	−6
D	11	12	11	16	6	−6
E	10	10	8	15	8	−8

	I	P1	P2	P3	C
A	7	5	10	0	0
B	8	9	1	0	0
C	3	2	0	9	2
D	5	6	5	10	0
E	2	2	0	7	0
	−2	−2			

	I	P1	P2	P3	C
A	5	3	10	0	0
B	6	7	1	0	0
C	1	0	0	9	2
D	3	4	5	10	0
E	0	0	0	7	0

	I	P1	P2	P3	C
A	4	2	9	0	0
B	5	6	0	0	0
C	1	0	0	10	3
D	2	3	4	10	0
E	0	0	0	8	1

	I	P1	P2	P3	C
A	4	2	9	0	0
B	5	6	0	0	0
C	1	0	0	10	3
D	2	3	4	10	0
E	0	0	0	8	1

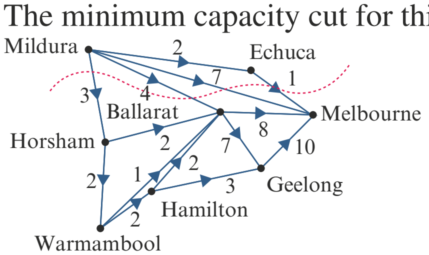


- Alvin should write **Paragraph 3**
- Billy should write **Paragraph 2**
- Chloe should write **Paragraph 1**
- Danielle should write the **Conclusion**
- Elena should write the **Introduction**

b A,P3 + B,P2 + C,P1 + D,Con + E,Into = 9 + 10 + 8 + 6 + 10 = 43 minutes

2 a All edges that this cut crosses flow from the source side of the cut to the sink side of the cut, so all of them are used in the capacity calculation.
Cut capacity = 1 + 7 + 8 + 10
= 26

b The minimum capacity cut for this network is shown below:



The maximum flow of passengers from Mildura to Melbourne is the capacity of the minimum capacity cut.
Maximum flow = 1 + 7 + 4 + 3
= 15

3 Use the Hungarian algorithm to determine the allocation for minimum time.

Swimmer	*Backstroke*	*Backstroke*	*Butterfly*	*Freestyle*	
Rob	76	78	70	62	−62
Joel	74	80	66	62	−62
Henk	72	76	68	58	−58
Sav	78	80	66	60	−60

Swimmer	*Backstroke*	*Backstroke*	*Butterfly*	*Freestyle*
Rob	14	16	8	0
Joel	12	18	4	0
Henk	14	18	10	0
Sav	18	20	6	0
	−12	−16	−4	

Swimmer	*Backstroke*	*Backstroke*	*Butterfly*	*Freestyle*
Rob	2	0	4	0
Joel	0	2	0	0
Henk	2	2	6	0
Sav	6	4	2	0

Swimmer	*Backstroke*	*Backstroke*	*Butterfly*	*Freestyle*
Rob	2	0	4	2
Joel	0	2	0	2
Henk	0	0	4	0
Sav	4	2	0	0

Rob must be allocated to Breaststroke, so Henk cannot.
There are two different allocations possible now:

1. If Joel is allocated to Backstroke, Henk cannot, so Henk must be allocated to Freestyle and Sav to Butterfly.

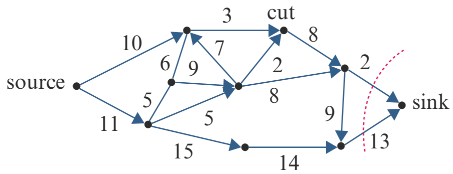
2. If Joel is allocated to Butterfly, Sav cannot, so Sav must be allocated to Freestyle and Henk to Backstroke.

Allocation 1:
Rob – Breaststroke, Joel – Backstroke, Henk – Freestyle, Sav – Butterfly
Allocation 2:
Rob – Breaststroke, Joel – Butterfly, Henk – Backstroke, Sav – Freestyle
Total time for both allocations is 276.

4 a The edge with capacity 9 flows from the sink side of the cut to the source side of the cut, so will not be counted in the capacity calculation.

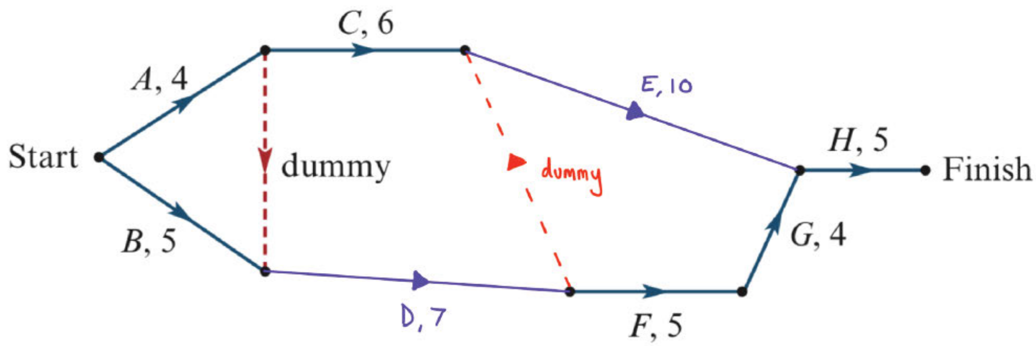
Cut capacity = 3 + 2 + 8 + 13
= 26

b The maximum flow through this network is equal to the minimum cut capacity.
The cut with minimum capacity is shown below:

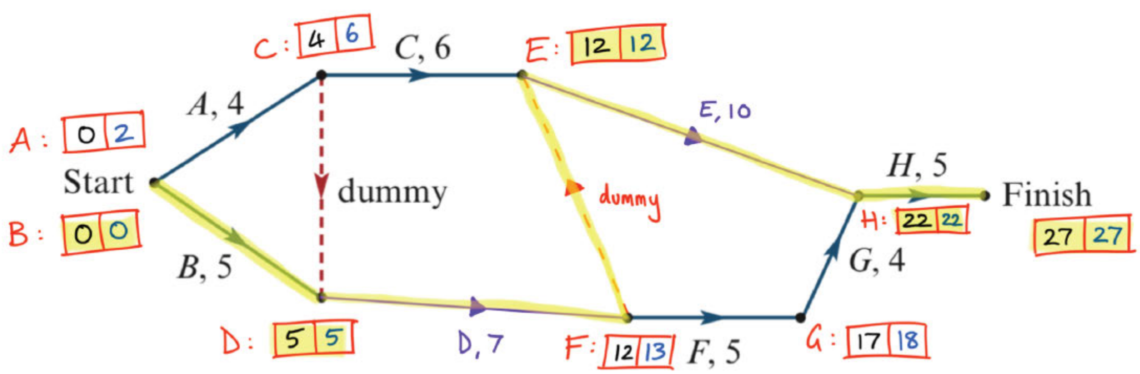


Cut capacity = 2 + 13
= 15

5 a Consider the immediate predecessors from the given table. Activity D must begin at the end of activities B and A (using the dummy activity) and go towards activity F. A dummy activity is needed because activities E and F share some, but not all, immediate predecessors. Activity E must start at the end of activity C and finish at the beginning of activity H.



b Perform a critical analysis of the activity network, using forward and backward scanning:



The EST of E = 12 hours

c Float time of G = LST − EST = 18 − 17 = 1 hour

d Using the diagram above, four activities have a non-zero float time (activities A, C, F, G).

e $B - D - E - H$

f 27 hours

g **i**

Path	Duration (hours)	Max reduction: E by 2
$A-C-E-H$	25	23
$A-D-E-H$	26	24
$A-D-F-G-H$	25	25
$B-D-E-H$	27 ©	25
$B-D-F-G-H$	26	26 ©

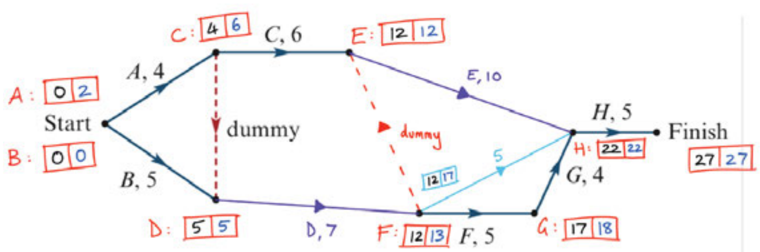
After crashing activity E, the critical path is $B - D - F - G - H$

ii

Path	Duration (hours)	Max reduction: E by 2 and A,B,D by 1
$A-C-E-H$	25	22
$A-D-E-H$	26	22
$A-D-F-G-H$	25	23
$B-D-E-H$	27 ©	23
$B-D-F-G-H$	26	24 ©

The minimum number of hours in which the project can now be completed in is 24 hours.

h



The new activity has an EST of 12 hours, therefore the new activity must start at the beginning of either activity E or activity F. The new activity has a duration of 5 hours and if it has an LST of 17 hours, it must finish at an acitivty with an LST of 22 hours (22 − 5 = 17); this can only be activity H. By convention, only straight edges are used in activity networks, as VCAA exams avoid networks where multiple activities have the same immediate predecessors and are directed towards the same activity. Therefore the new activity cannot be drawn as an identical line to activity E. The new activity must be drawn as a straight directed edge from the end of activity **D** to the start of activity **H**.