NATIONAL PHYSICS

PHYSICS CALCULATIONS

Don Humphrey

Science Press

This book is dedicated to the memory of my wife.
Without her patience and support I could not have completed the task.

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Science Press
Private Bag 7023 Marrickville NSW 1475 Australia
Tel: +61 2 9516 1122 Fax: +61 2 9550 1915
sales@sciencepress.com.au
www.sciencepress.com.au

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Introduction

A significant amount of the National Physics syllabus involves the solving of numerical problems. The object of this book is to provide sets of simple graded exercises which require the use of formulas mentioned in the syllabus. The sets of exercises are designed to be attempted immediately following the introduction of the theory or for homework. Although the formulas are given in the examination papers, only by practice will you develop the facility to recognise a problem and solve it.

Detailed answers are included. In some cases the solutions can be obtained via different pathways but should agree in the end with the answers provided.

Words to Watch

account, account for State reasons for, report on, give an account of, narrate a series of events or transactions.

analyse Interpret data to reach conclusions.

annotate Add brief notes to a diagram or graph.

apply Put to use in a particular situation.

assess Make a judgement about the value of something.

calculate Find a numerical answer.

clarify Make clear or plain.

classify Arrange into classes, groups or categories.

comment Give a judgement based on a given statement or result of a calculation.

compare Estimate, measure or note how things are similar or different.

construct Represent or develop in graphical form.

contrast Show how things are different or opposite.

create Originate or bring into existence.

deduce Reach a conclusion from given information.

define Give the precise meaning of a word, phrase or physical quantity.

demonstrate Show by example.

derive Manipulate a mathematical relationship(s) to give a new equation or relationship.

describe Give a detailed account.

design Produce a plan, simulation or model.

determine Find the only possible answer.

discuss Talk or write about a topic, taking into account different issues or ideas.

distinguish Give differences between two or more different items.

draw Represent by means of pencil lines.

estimate Find an approximate value for an unknown quantity.

evaluate Assess the implications and limitations.

examine Inquire into.

explain Make something clear or easy to understand.

extract Choose relevant and/or appropriate details.

extrapolate Infer from what is known.

hypothesise Suggest an explanation for a group of facts or phenomena.

identify Recognise and name.

interpret Draw meaning from.

investigate Plan, inquire into and draw conclusions about.

justify Support an argument or conclusion.

label Add labels to a diagram.

list Give a sequence of names or other brief answers.

measure Find a value for a quantity.

outline Give a brief account or summary.

plan Use strategies to develop a series of steps or processes.

predict Give an expected result.

propose Put forward a plan or suggestion for consideration or action.

recall Present remembered ideas, facts or experiences.

relate Tell or report about happenings, events or circumstances.

represent Use words, images or symbols to convey meaning.

select Choose in preference to another or others.

sequence Arrange in order.

show Give the steps in a calculation or derivation.

sketch Make a quick, rough drawing of something.

solve Work out the answer to a problem.

state Give a specific name, value or other brief answer.

suggest Put forward an idea for consideration.

summarise Give a brief statement of the main points.

synthesise Combine various elements to make a whole.

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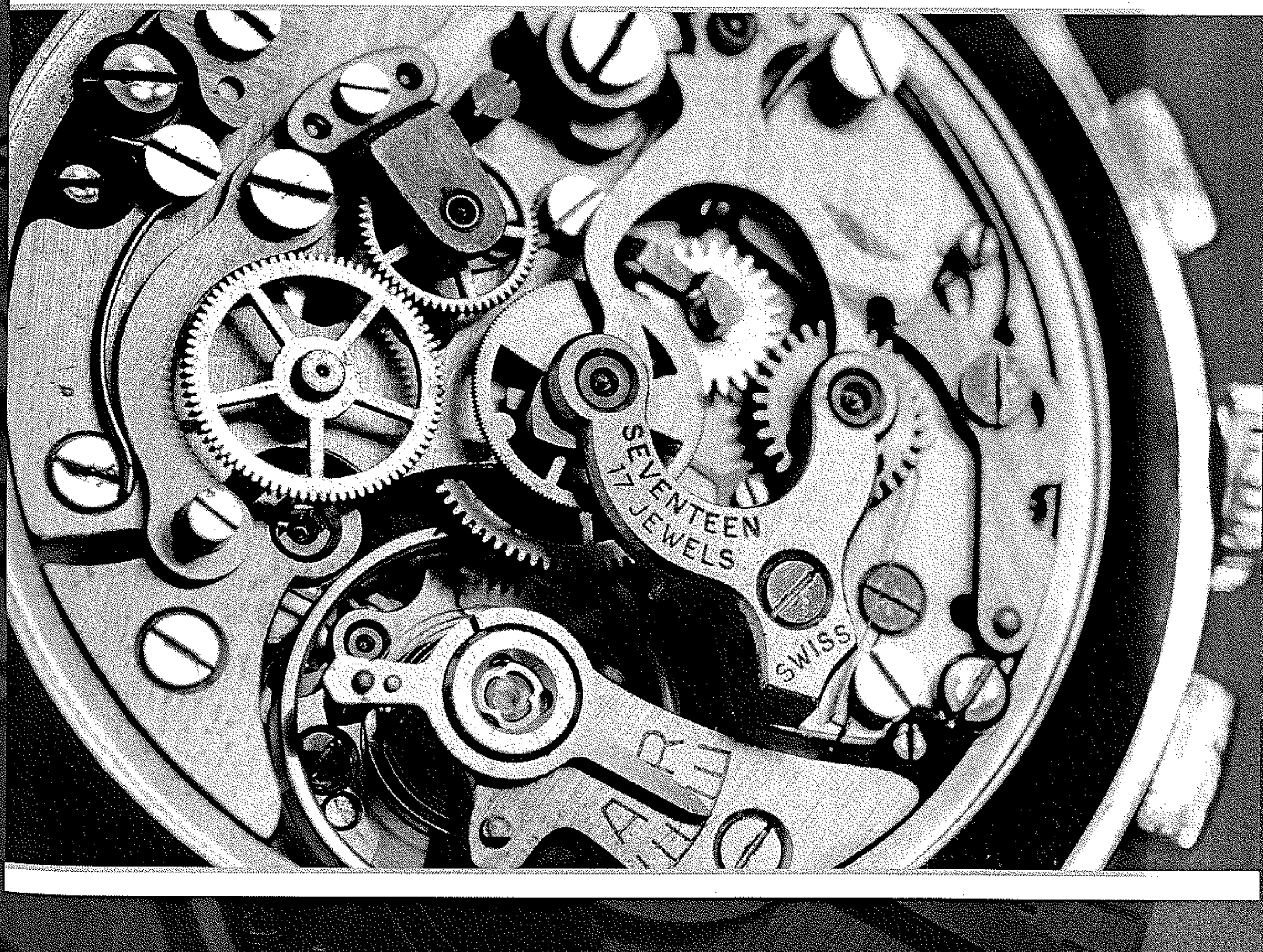
Physics Basics



1. Physics Basics

Units of measurement

In the SI system of units, the fundamental quantities are as follows.

Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Light Intensity	candela	cd
Amount of substance	mole	mol

These are compounded and subdivided as follows.

tera	10^{12}	T
giga	10^{9}	G
mega	10^{6}	M
kilo	10^{3}	k
*hecta	10^{2}	H
*deca	10^{1}	Da
*deci	10^{-1}	d
*centi	10^{-2}	c
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p
femto	10^{-15}	f
atto	10^{-18}	a

* Commonly used but not part of SI.

Questions

In each of the following, express your answer as a power of 10. How many:

1. Millimetres (mm) in 1 metre (m)?
2. Centigrams (cg) in 1 kilogram (kg)?
3. Microamperes (μA) in 1 milliampere (mA)?
4. Seconds (s) in 1 millisecond (ms)?
5. Megamoles (Mmol) in 1 kilomole (kmol)?
6. Kilocandelas (kcd) in 1 microcandela (μcd)?
7. Nanokelvins (nK) in 1 millikelvin (mK)?
8. Gigametres (Gm) in 1 kilometre (km)?
9. Picograms (pg) in a decigram (dg)?
10. Attoseconds (as) in 1 terasecond (Ts)?

Scientific notation and significant figures

Applying scientific notation to a number means expressing it as a number between 1 and 10 times 10 to the appropriate power, e.g. 40 000 = 4×10^4 and 0.000 005 = 5×10^{-6}.

Significant figures are those which are accurately known. A general rule is that the answer to a problem should not contain more significant figures than any of the data input.

Examples of correcting to two significant figures are $8.493 \times 10^6 \sim 8.5 \times 10^6$ and $7.0048 \times 10^{-2} \sim 7.0 \times 10^{-2}$.

Order of magnitude

The order of magnitude of a number is the power of 10 nearest to the number. It is often convenient to change a number to scientific notation before working out the order of magnitude.

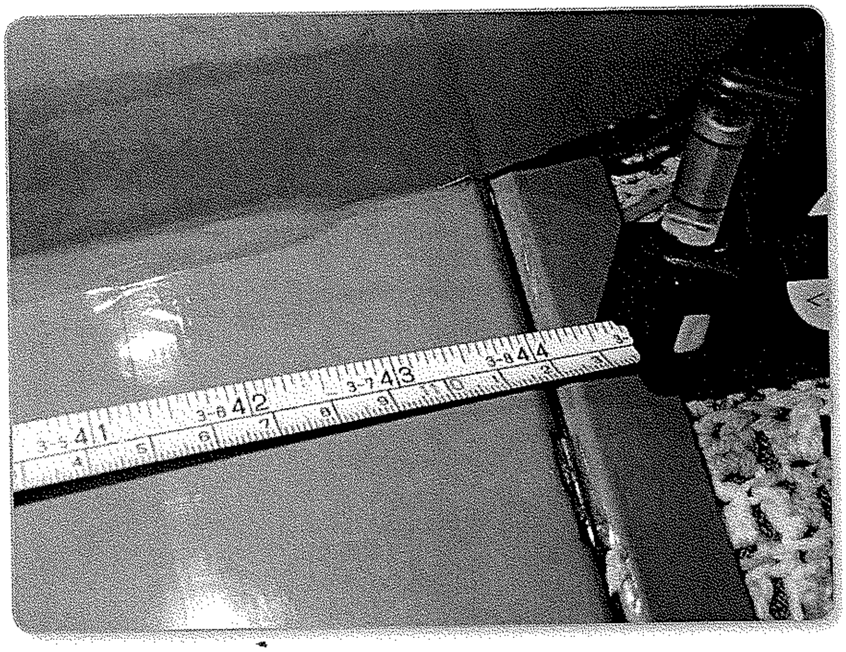
For example:

$7634.25 = 7.63425 \times 10^3$ which is approximately 10×10^3 or 10^4

$0.00236 = 2.36 \times 10^{-3}$ which is approximately 1×10^{-3} or 10^{-3}

Questions

For Questions 11 to 20, write each (or perform the calculation and write the answer) expressing the answer in scientific notation correct to *two* significant figures.



11. 7505.23

12. 0.003425

13. 70.2×34.2

14. $1578.3 + 0.400$

15. $\dfrac{0.600 \times 7020.0960}{0.0960}$

16. $\dfrac{1.53}{42.89 \times 8.4}$

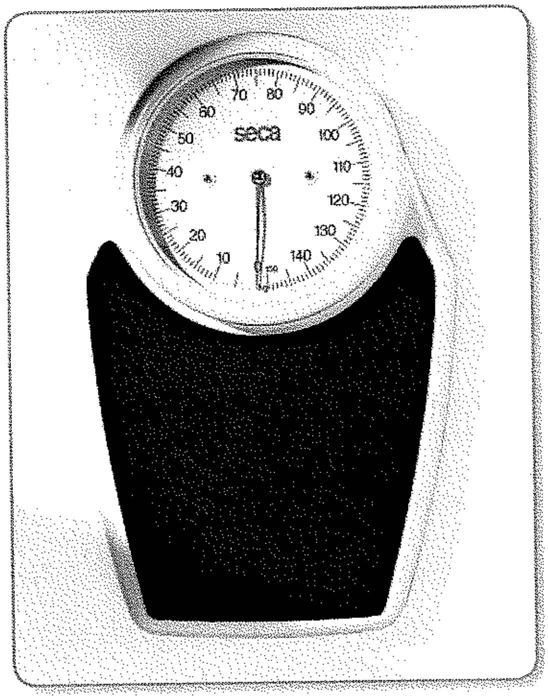
17. $\dfrac{0.8235 \times 445.7}{76.9 \times 0.00930}$

18. 20.4×10^{-7}

19. 134×10^{8}

20. $\dfrac{\left(2.4 \times 10^{-3}\right) \times \left(5.6 \times 10^{4}\right)}{\left(3.0 \times 10^{7}\right)^{4}}$

For Questions 21 to 30, identify the order of magnitude.



21. 8.2×10^{-4}

22. 3.5×10^{7}

23. 73.6

24. 0.00901

25. 863.7×10^{-5}

26. 0.01673×10^{6}

27. $3.5 \times 10^{6} \times 9.2 \times 10^{-4}$

28. $\dfrac{7.6 \times 10^{-5}}{1.9 \times 10^{4}}$

29. The number of seconds in a day.

30. The number of hours in a year.

31. How many millimetres (mm) in 4.5 metres (m)?

32. How many nanoseconds (ns) in 276 seconds (s)?

33. How many kilograms (kg) in 0.359 milligrams (mg)?

34. Express the following in scientific notation correct to two significant figures.

(a) 0.003778 (b) 70.2×34.2 (c) $\dfrac{2.64}{5.96 \times 24.2}$

35. Identify the order of magnitude of each of the following.

(a) 7.4×10^{-5} (b) $2.4 \times 10^{3} \times 1.8 \times 10^{-5}$ (c) $\dfrac{8.64 \times 10^{-7}}{2.95 \times 10^{3}}$

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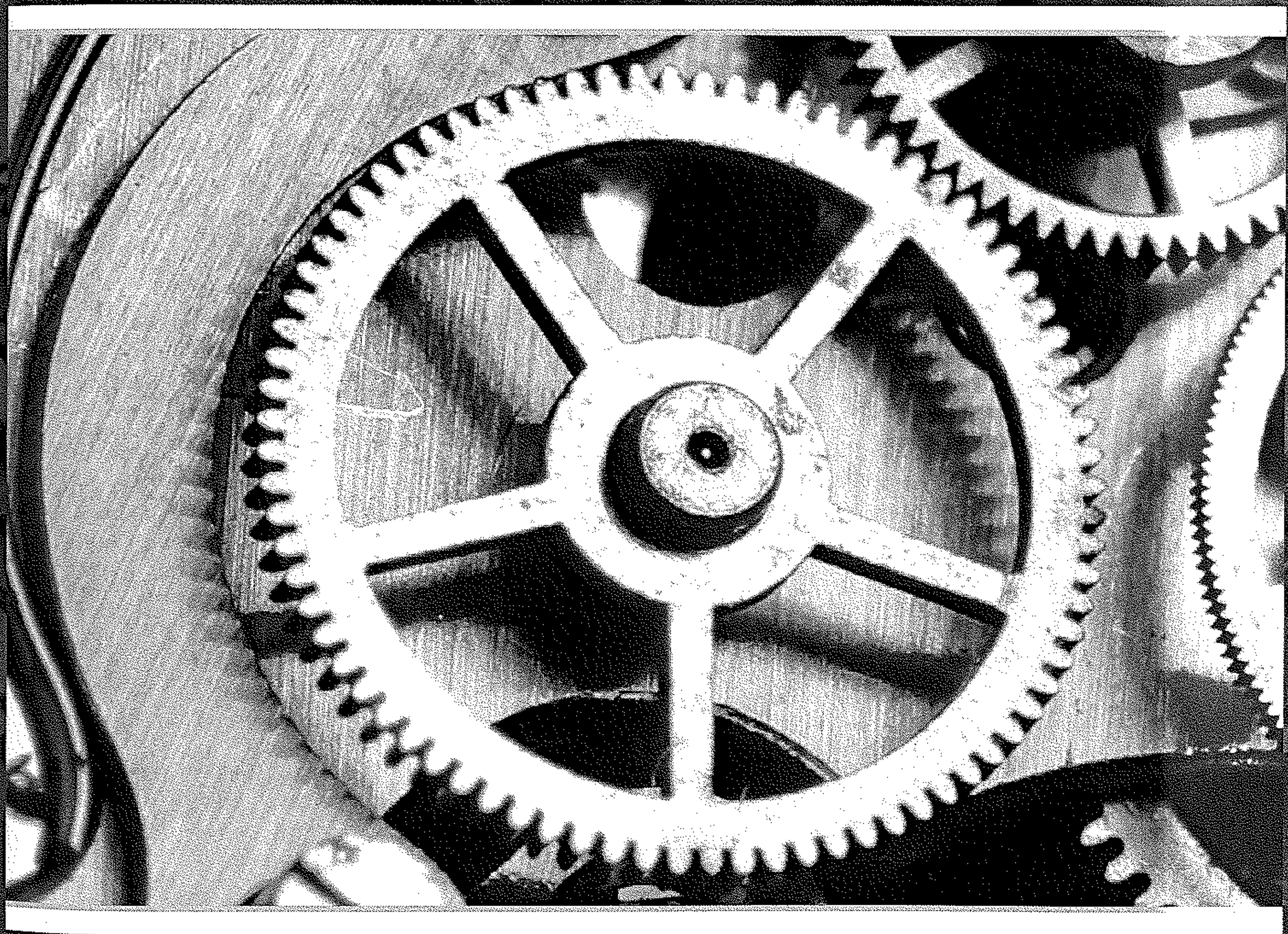
Thermal, Nuclear and Electrical Physics



2. Thermal Physics

$Q = mc\Delta T$	Q = heat transferred to or from an object (joules) J
$Q = mL$	m = mass of object (kg)
	c = specific heat capacity of the object (J kg^{-1} K^{-1} or J kg^{-1} °C^{-1})
	ΔT = temperature change
	L = latent heat capacity of the material (kJ kg^{-1})
	η = efficiency = (energy output / energy input) × 100%

Questions

Note that in these exercises changes of temperatures are quoted in kelvins (K) rather than in Celsius degrees whereas temperatures are quoted in degrees Celsius (°C).

1. Calculate the amount of heat needed to raise the temperature of 0.500 kg of water by 35 K given that the specific heat capacity of water is 4.18 kJ kg^{-1} K^{-1}.

2. 0.650 J of energy was required to raise the temperature of a sample of aluminium from 20°C to 45°C. Calculate the mass of the aluminium sample. (c = 0.900 kJ kg^{-1} K^{-1}.)

3. Calculate the specific heat capacity of copper given that 2.34 kJ raises the temperature of 0.200 kg of copper from 15°C to 45°C.

4. Porcelain has a specific heat capacity of 0.840 kJ kg^{-1} K^{-1}. A porcelain cup of mass 0.250 kg holds 0.250 kg of water. Calculate the amount of heat energy required to raise the temperature of a cupful of water from 20°C to 100°C. (c of porcelain = 0.897 kJ kg^{-1} K^{-1} and c of water = 4.18 kJ kg^{-1} K^{-1}.)

5. A sample of 0.480 kg of ethanol lost 6.00 kJ of heat. If the final temperature after cooling was 7.5°C calculate the initial temperature. (c of ethanol = 1.42 kJ kg^{-1} K^{-1}.)

6. For each kilojoule of heat supplied by a particular source of heat, 0.25 kJ is 'wasted' (not used to heat the sample). Calculate the amount of heat which would have to be supplied to 0.750 kg of lead to raise its temperature from 25°C to 200°C. (c of lead = 0.129 kJ kg^{-1} K^{-1}.)

7. Calculate how much energy would be needed to melt 1.2 kg of lead from the point where the lead has reached melting point to the point where it has completely melted. (Latent heat of fusion of lead is 22.4 kJ kg^{-1}.)

8. Calculate the mass of ethanol that would be vaporised by 1.500 kJ of energy. (Latent heat of vaporisation of alcohol is 885 kJ kg^{-1}.)

9. The efficiency of a heater is 20%. Calculate the energy (including the 'wasted' energy) that would be needed to melt 1.800 kg of solid nitrogen at the melting point of nitrogen using this heater. (Latent heat of vaporisation of nitrogen is 25.7 kJ kg^{-1}.)

10. The latent heat of fusion of water is 334 kJ kg^{-1} and its latent heat of vaporisation is 2260 kJ kg^{-1}. Calculate how much heat energy would be required to melt 0.200 kg of ice at 0°C, bring it up to boiling water at 100°C and turn it into vapour at 100°C. (Latent heat of fusion of water is 334 kJ kg^{-1}. Latent heat of vaporisation of water is 2260 kJ kg^{-1} and the specific heat capacity of water is 4.18 kJ kg^{-1} °C^{-1}.)

3. Radioactive Decay

Particle	Symbol	Atomic number	Mass number
Beta	${}^{0}_{-1}e$	−1	0
Positron	${}^{0}_{+1}e$	+1	0
Neutron	${}^{0}_{1}n$	0	1

$N = N_o(\frac{1}{2})^n$ N_o = original number or mass of radioactive nuclei

N = number or mass after decay

n = number of half-lives

Questions

You will need to refer to a copy of the periodic table for Questions 1 to 5.

1. An atom of uranium-239 disintegrated spontaneously emitting a beta particle. Into which element did it decay?

2. An atom of nitrogen-13 disintegrated leading to an atom of carbon-13. Name the other particle produced.

3. A certain element disintegrated producing atoms of nitrogen-15 together with an equal number of positrons. What was the original element?

4. Radioactive phosphorus-32 emits beta particles. Into which element does it transform?

5. After a period of time atoms of silicon-30 are found in a sample of radioactive phosphorus-30. Name the particle emitted to perform this transformation.

6. Polonium has a half-life of 140 days. If one started with 1.000 g of polonium, calculate the mass of polonium remaining after 420 days.

7. Bismuth-210 has a half-life of 5 days. An experiment was started with 32 micrograms of the element. Calculate how long it would take for this sample to be reduced to 1 microgram of the original sample.

8. If the activity of a radioactive sample dropped to 1/16 of its original value in 12.20 minutes, calculate the half-life of the sample.

9. Seaborgium-266 has a half-life of 30 seconds. Calculate the amount of a sample of 1.000 micrograms remaining after 1 hour.

4. Nuclear Reactions

Particle	Symbol	Atomic number	Mass number
Beta particle (electron)	$^{0}_{-1}e$	–1	0
Positron	$^{0}_{+1}e$	+1	0
Neutron	$^{0}_{1}n$	0	1
Alpha particle	$^{4}_{2}He$	2	4
Neutron	$^{0}_{1}n$	0	1
Deuteron	$^{2}_{1}H$	1	2

Questions

You will need to consult a periodic table to answer many of the questions which follow.
Write full nuclear reaction equations for all questions.

1. Beryllium-9 was bombarded with alpha particles. One neutron per atom of beryllium was ejected. What element remained after the reaction?

2. An atom of lithium-7 was struck by a proton which it absorbed. Two identical particles were produced as a result of this reaction. What were they?

3. Atoms of sulfur-32, on being struck by neutrons emitted protons. Atoms of which element remained as a result of this reaction?

4. Into which element would atoms of beryllium-7 change upon 'electron capture'?

5. Atoms of nitrogen-15 were bombarded with certain particles. Atoms of carbon-12 and alpha particles were emitted. Name the particles which caused the change.

6. A deuteron struck an atom of phosphorus-31 causing the emission of a proton. Into which element did the phosphorus turn?

7. When atoms of uranium-235 capture one neutron, they sometimes split to form atoms of lanthanum-148 and 3 neutrons. Which other element is formed in this case?

8. Atoms of copper-64 emitted positrons. What element was left?

9. When carbon-12 was bombarded by protons, atoms of another element were produced. These then disintegrated to emit positrons. Write out the full equations for these reactions and identify the other elements.

10. Write a reaction which could be used to transform mercury-200 to gold.

5. Mass and Energy

$\Delta E = \Delta mc^2$

ΔE = energy change (joules) J

Δm = mass change (kilograms) kg

c = velocity of light (3×10^8 m s^{-1})

Questions

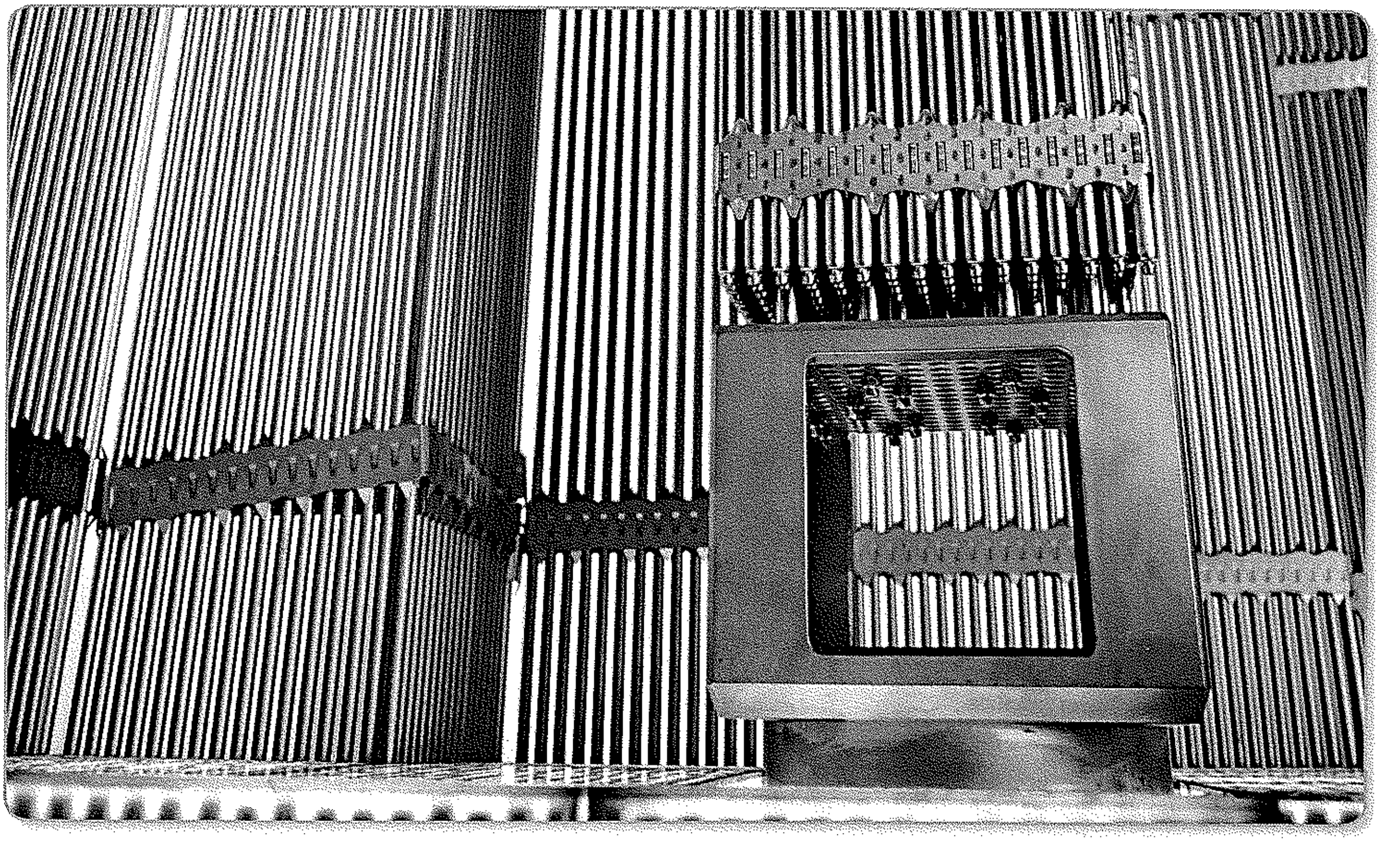
1. Calculate the amount of energy released in a nuclear reaction where 1.00×10^{-9} kg of matter was transformed into energy.

2. Calculate the reduction of mass of a fuel rod in a nuclear reactor after it had produced 1 kilojoule of energy.

3. A radioactive sample is producing energy at the rate of 1 kilowatt (1000 J s^{-1}). Calculate the time it would take to reduce the mass of the sample by 1 kg (answer in seconds).

4. 0.001 kg of mass is converted into energy. Calculate the time this loss of mass could run a 100 W light globe.

5. Calculate the loss of mass of a fuel rod when it produces enough energy to bring 1000 kg of water from 20°C to 100°C and then boil it. (For water, c = 4.18 kJ kg^{-1} K^{-1} and L = 2260 kJ kg^{-1}.)



6. Charge and Current

$$I = \frac{\Delta q}{t}$$

I = current (amperes) A

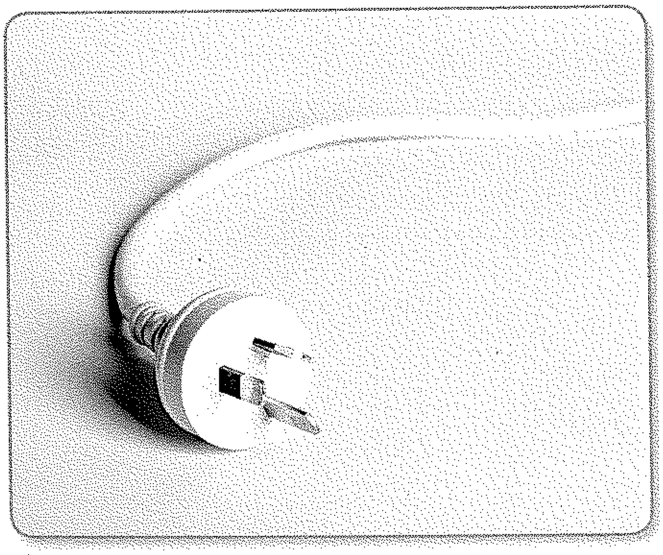
Δq = amount of charge (coulombs) C

t = time (seconds) s

Charge on the electron = 1.6×10^{-19} C

Questions

1. A charge of 7.5 C passes a point in a circuit in 0.25 s. Calculate the current.
2. A current of 9.6 A flowed in a circuit for 45 s. Calculate the charge passed through the circuit.
3. Calculate the time it would take for a current of 1.4 A to deliver a charge of 560 C.
4. Electricity is paid for by the unit. At normal household voltage, 1 unit represents 25 A flowing for 10 minutes. Calculate the number of coulombs this represents.
5. An ammeter registers a current of 8.0 A. Calculate the number of electrons which would pass through the meter in 1 second.
6. A current consists of 1.5×10^{20} electrons per second. Calculate the value of the current in amps.
7. To fully charge a car's battery, a charger operated on 2.5 A for 6.0 hours. Calculate the number of coulombs which were delivered to the battery.
8. Calculate the number of electrons which would travel in a circuit during a period of 5 minutes if a current of 8 milliamps was flowing.



7. Work, Charge and Electric Potential

$W = QV$	W = work (joules) J
	Q = charge (coulombs) C
	V = potential difference (volts) V

Questions

1. Calculate the work that would be done in bringing a charge of 1.6×10^{-8} C from infinity up to a body with a potential of 320 kV.

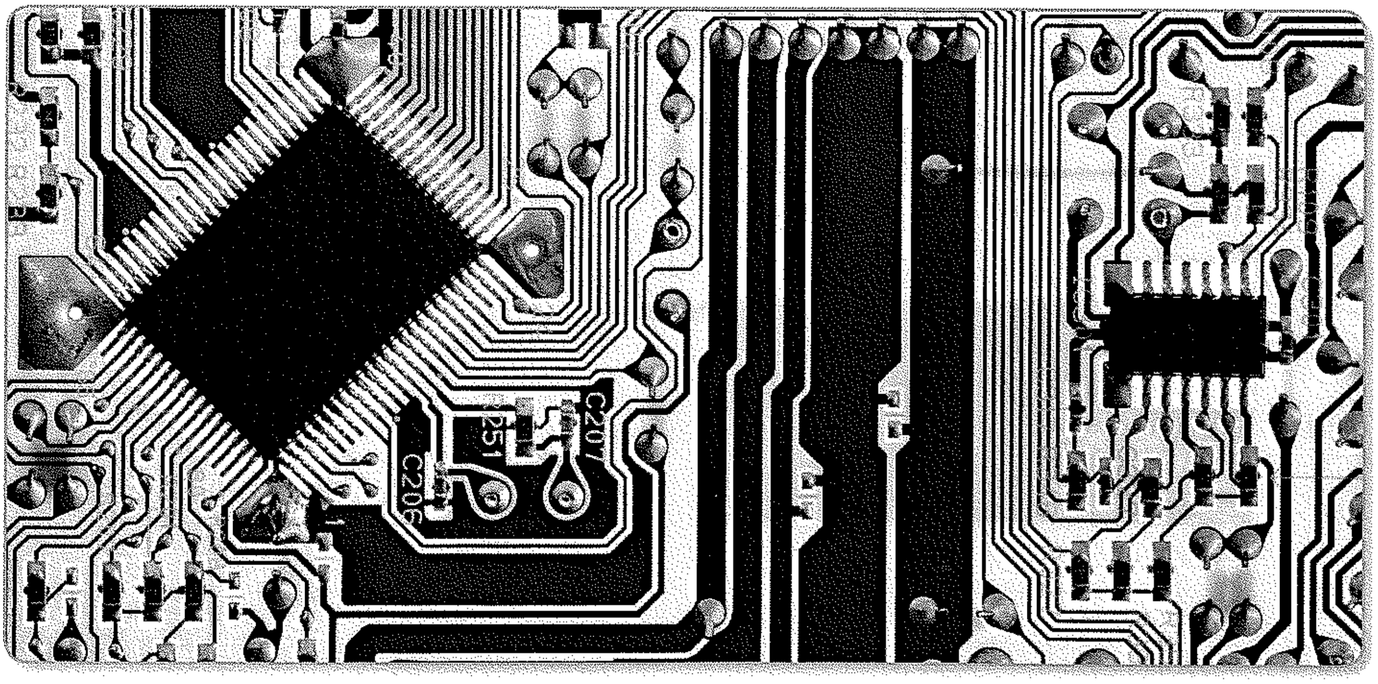
2. If it took 144 J of work to bring a charge of +18 C from infinity to a positively charged body, calculate the potential of the body.

3. Work of 13.2 J is done in moving a charge of 1.1 C around a circuit. Calculate the potential difference across the circuit.

4. A positive charge of 5.0 C moves from point A to point B acquiring 30.0 J of energy in so doing. If point A is at a potential of 12.0 V calculate the potential of point B.

5. The parallel plates of a condenser are at a potential difference of 250 V. Calculate the amount of work would be done in taking a charge of $+3.0 \times 10^{-7}$ C from one plate to the other against the potential difference.

6. The parallel plates of a condenser are at a potential difference of 1000 V. Calculate the work done due to taking one electron from one plate to the other.



8. Ohm's Law

$$R = \frac{V}{I}$$

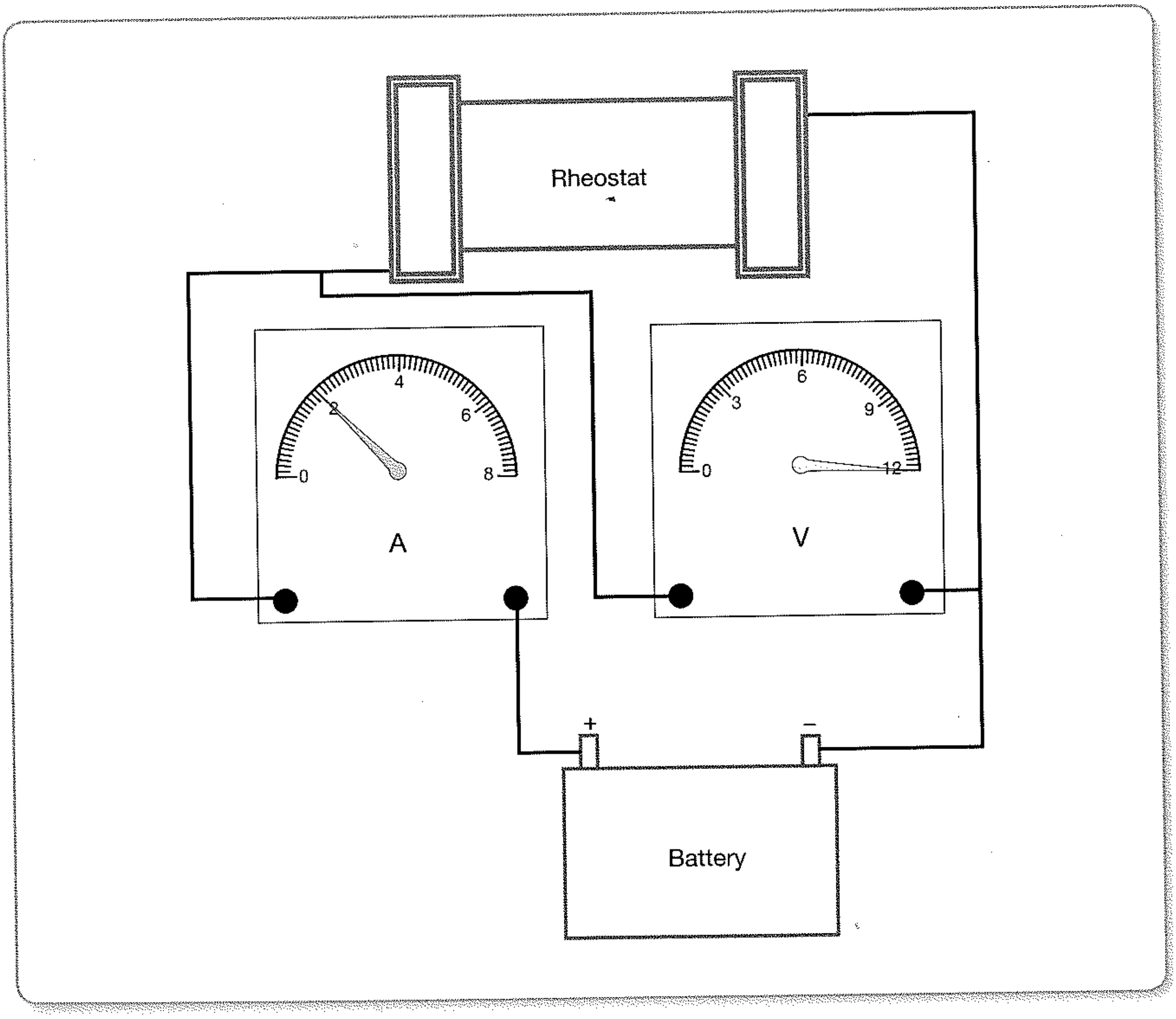
R = resistance (ohms) Ω

V = potential difference (volts) V

I = current (amperes) A

Questions

1. A rheostat is an adjustable resistor. Calculate the resistance of the rheostat shown in the diagram.



2. The element of an electric jug has a resistance of 32.0 ohms. If 240 V is applied, calculate the current it will carry.

3. A resistor is marked 1200 Ω and can carry a maximum of 0.0030 A. Calculate the potential difference that would cause it to carry its maximum current.

4. A 2.5 V light globe carries a current of 0.4 A. Calculate its resistance.

5. Calculate the current that would pass through a 5.5 megohm resistance connected to an 11 kilovolt source.

6. A 10.0 kΩ resistor carries a current of 0.6 mA. Calculate the potential difference operating across its ends.

7. A 12 V battery is connected to three resistors in series. The values of the resistors are 2 Ω , 3 Ω , and 4 Ω. Calculate the current that flows through the resistors. (*Hint:* Add the values of the three resistors to give the total resistance.)

8. Twenty Christmas tree light globes, each having a resistance of 0.4 Ω are connected in series to a transformer. The current passing through the light globes is 3.0 A. Calculate the voltage the transformer is producing to operate these globes.

9. A farm generator has an output of 240 V. It is intended to operate machinery some distance away. If the machinery has to operate on 25 A and the cables carrying the current have a total resistance of 5.0 Ω calculate the potential difference across the ends of the cables.

10. An engineer has the option of purchasing one of two generators of similar power output. Generator A puts out 30 A at 80 V while generator B puts out 10 A at 240 V. Whichever one is chosen has to supply electricity to machinery 2 km away. If the cable used to transport the electricity has a resistance of 500 $\mu\Omega$ per metre, calculate the potential difference across the ends of the cables in each case. The engineer will choose the generator which suffers the lower loss of potential difference. Using the data given, deduce which generator the engineer will choose.

9. Resistors in Series

$R_T = R_1 + R_2 + R_3 + \ldots$

R_T = total resistance

R_1, R_2 etc = resistances making up the circuit

Questions

1. Calculate the combined resistance of three resistors, 2, 4, and 5 ohms connected in series.

2. Resistances of 2.5 Ω, 4.9 Ω, and 7.6 Ω are connected in series. Calculate their combined resistance.

3. Calculate the size of the resistor that must be chosen to place in series with a resistance of 4700 Ω to make an effective resistance of 10.0 kΩ.

4. A current of 4.5 A is passed through two resistors, 0.7 Ω and 1.1 Ω in series. Calculate the potential difference across them.

5. A potential difference of 24 V is applied across four resistors, each 0.9 Ω in series. Calculate the current that will pass through them.

6. A potential difference of 10.0 V is applied across two resistors in series, 2.00 Ω and 3.0 Ω. Calculate:

(a) The current through both resistors.

(b) The current through each.

(c) The potential difference across each resistor.

7. Three resistors, 3.0 Ω, 4.0 Ω, and 5.0 Ω are connected in series. A current of 2.0 A flows through the 3 Ω resistor.
Calculate:

(a) The current through the 4 Ω resistor.

(b) The current through the 5 Ω resistor.

(c) The potential difference across the 3 Ω resistor.

(d) The potential difference across the 4 Ω resistor.

(e) The potential difference across the 5 Ω resistor.

(f) The potential difference across the combined three resistors.

8. Two resistors, one of which is 4.7 Ω, the other unknown, are connected in series. When a current of 3.2 A flows through the known resistor, the potential difference across the unknown resistor is found to 16.0 V. Calculate the value of the unknown resistor.

9. An electrician wanted to run a 110 V, 12 Ω appliance from a 240 V power supply. Determine the size of the resistance that the electrician would need to connect in series with the appliance to ensure that it ran properly.

10. An electric motor of resistance 8.00 Ω is designed to run properly on 12.0 V. It is connected to a generator by cables with a combined resistance of 1.00 Ω. Calculate the potential difference the generator must produce so that the motor will have a potential difference of 12.0 V across it.

10. Resistors in Parallel

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

R_T = total resistance

R_1, R_2 etc = resistances making up the circuit

Questions

1. Calculate the combined resistance of two resistors, 2.0 ohms and 5.0 ohms connected in parallel.

2. Resistances of 2.5 Ω, 0.75 Ω, and 1.50 Ω are connected in parallel. Calculate their effective resistance.

3. Calculate the size of the resistor that must be chosen to place in parallel with a resistor of 1.0 kΩ to make an effective resistance of 200.0 Ω.

4. A current of 5.0 A is passed through a circuit consisting of two resistors, 4.0 Ω and 6.0 Ω in parallel. Calculate the potential difference across the circuit.

5. A battery supplying a potential difference of 90.0 V is applied across four parallel resistors, each of 12 Ω. Calculate the current that is drawn from the battery.

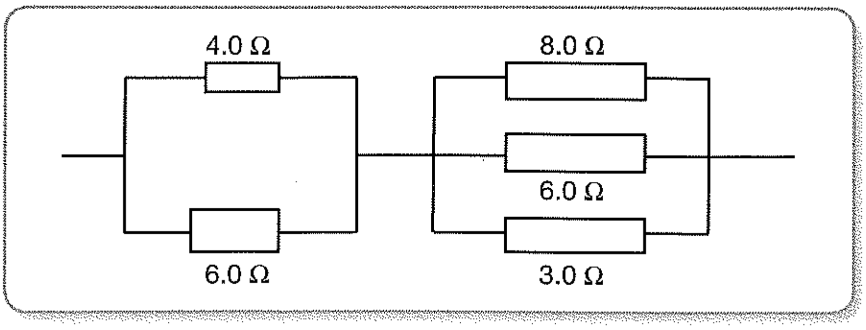
6. A generator supplies a potential difference of 120 V and this is applied across two resistors in parallel: 20 Ω and 30 Ω.
   Calculate the:
   (a) Potential difference across each resistor.
   (b) Current through each resistor.
   (c) Current drawn from the generator.

7. Three resistors, 3.0 Ω, 4.0 Ω, and 5.0 Ω are connected in parallel. A current of 2.0 A flows through the 3.0 Ω resistor.
   Calculate the:
   (a) Potential difference across the 3 Ω resistor.
   (b) Potential difference across the 4 Ω resistor.
   (c) Current through the 4 Ω resistor.
   (d) Potential difference across the 5 Ω resistor.
   (e) Current through the 5 Ω resistor.
   (f) Current through the combined resistors.

8. Two resistors, one of which has a resistance of 25 Ω are connected in parallel. When a current of 40.0 A flows through the 25 Ω resistor, the total current in the circuit is 90.0 A. Calculate the value of the other resistor.

9. Calculate the total resistance of the combination of resistors shown in the diagram.

10. Given that the current in the 4.0 Ω resistor in Question 9 above is 20.0 A, calculate the current in the 8.0 Ω resistor.



11. Electrical Power and Energy

$P = VI$

Energy $= VIt$

$R = \dfrac{V}{I}$

P = power (watts) W

V = potential difference (volts) V

I = current (amperes) A

t = time (seconds) s

R = resistance (ohms) Ω

Energy is measures in joules (J)

Questions

1. A 12 V lamp uses a current of 3.0 A. Calculate the power of the lamp.

2. A jug element has a power of 1800 Ω and runs on 240 V. Calculate the current it consumes.

3. The wiring of a machine carries a current of 6.4 A. If the power output is 48 W, calculate the voltage the machine runs on.

4. A motor car headlamp is marked 12 V, 48 W. Calculate its resistance.

5. Calculate the power needed to pass a current of 4.0 A through a resistor marked 5.0 W.

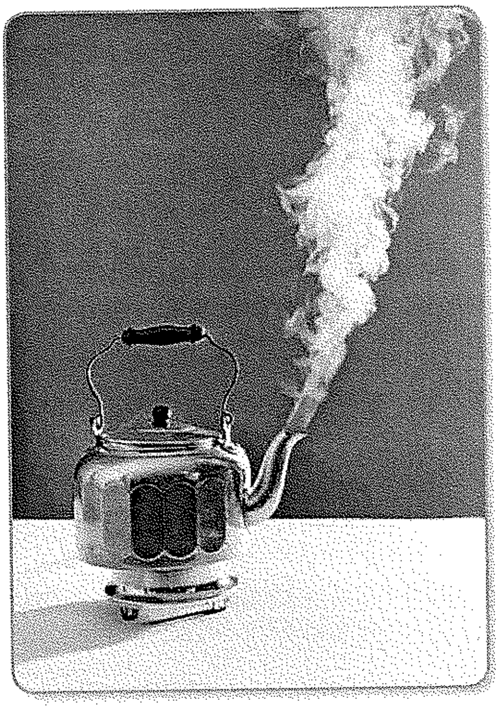
6. A certain radio component has a resistance of 4.8 Ω. Calculate the power dissipated when a potential difference of 1.6 mV is applied across it.

7. To boil a jug of water needs 3.2×10^5 J of heat energy. If the rating of the jug element is 1.6×10^3 W and there is no loss of energy in the system, calculate the time it would take to boil the water.

8. In 5.0 minutes, an appliance uses 2.7×10^3 J of energy. If the current passing through it was 2.0 A calculate its resistance.

9. Calculate the current needed to produce an output of 48.0 W from a resistance of 3.0 Ω.

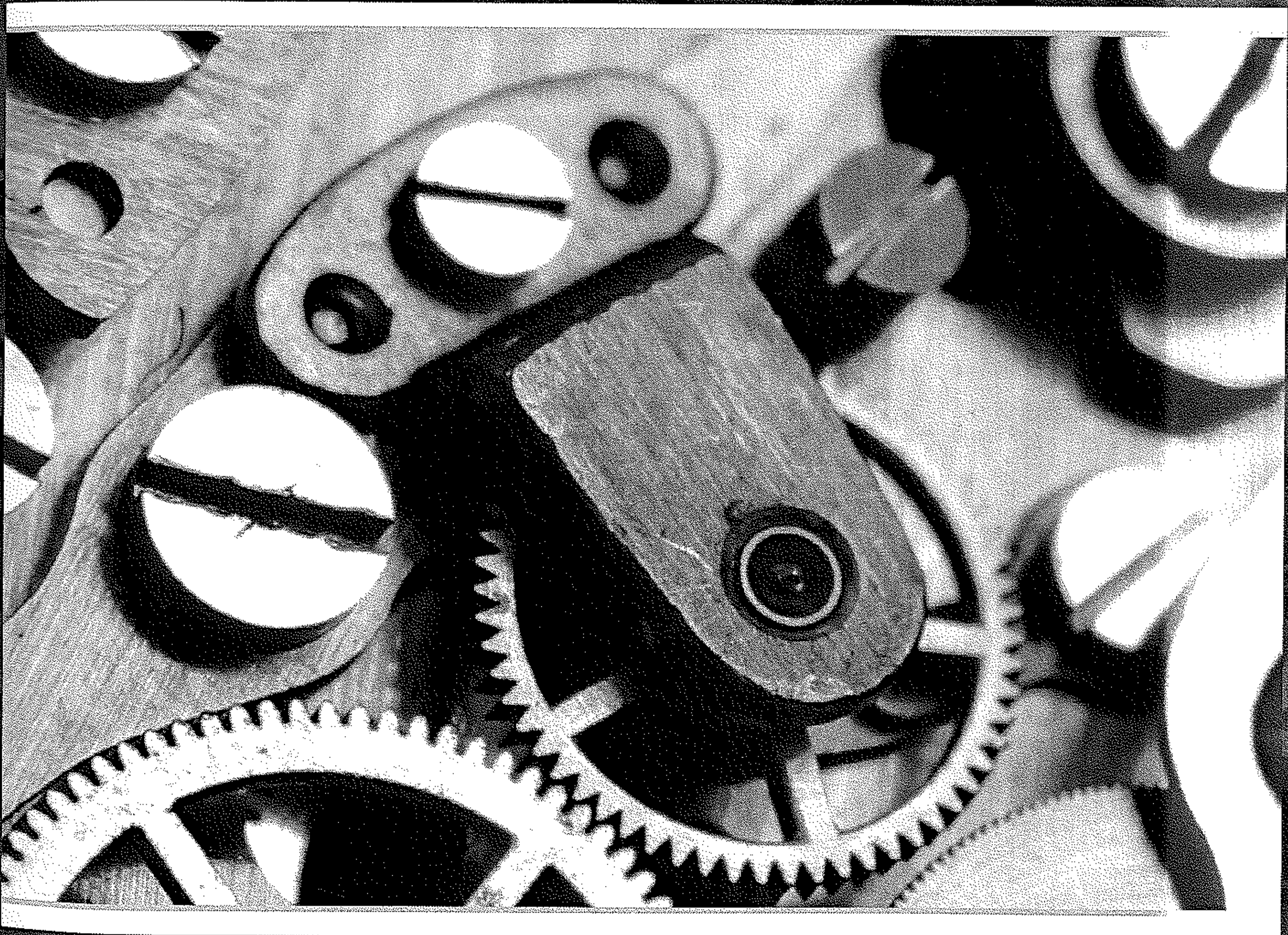
10. A certain light globe radiates a power of 120 W. Its resistance is 480 Ω. Calculate the voltage on which it runs.



NATIONAL PHYSICS

PHYSICS CALCULATIONS

Linear Motion and Waves



12. Uniform Velocity

$$v_{av} = \frac{\Delta s}{\Delta t}$$

v_{av} = velocity (metres per second) m s^{-1}

Δs = displacement (metres) m

Δt = time (seconds) s

Questions

1. A car travelled from Brisbane to Cairns, a distance of approximately 1700 km in 20 hours. Calculate its average speed during this journey.

2. At an average speed of 900 km per hour, calculate the length of time it would take for an aeroplane to travel from Frankfurt, Germany to Vancouver, Canada, a distance of approximately 8100 km.

3. A ship has a velocity of 12 m s^{-1} north. Calculate the distance it will travel in 3 minutes.

4. The 'four minute mile' was a goal of long distance athletes for many years. Given that 1 mile is approximately 1.6 km, calculate the average speed of an athlete who completed the mile distance in 4 minutes. Answer in metres per second.

5. At 8.45 am, a train was 2 km north of Central Station. At 9.25 am it was 48 km north.

Calculate the:

(a) Change of displacement during this time interval (in kilometres).

(b) Change in time (in hours).

(c) Average velocity (in kilometres per hour).

6. At 11.20 am, an aeroplane passed over a town 40 km east of Canberra. At 1.35 pm, the aeroplane was 600 km west of Canberra.

Calculate the:

(a) Change of displacement during this time interval (in kilometres).

(b) Change in time (in hours).

(c) Average velocity (in kilometres per hour).

7. A passenger in a car started his stopwatch as the car passed a sign which read '45 km to Canberra'. The driver kept the car at a constant 90 km h^{-1} until she passed the '55 km to Canberra' sign.

Calculate the:

(a) Change of displacement during this time interval (in kilometres).

(b) Velocity in kilometres per second.

(c) Time on the stopwatch in minutes and seconds as the car passed the second sign.

8. A hiker walked at 2.10 m s^{-1} towards the east for 20.0 minutes, then ran at 4.50 m s^{-1} towards the east for 10.0 minutes. She then returned to her start at a steady pace, taking 40.0 minutes.

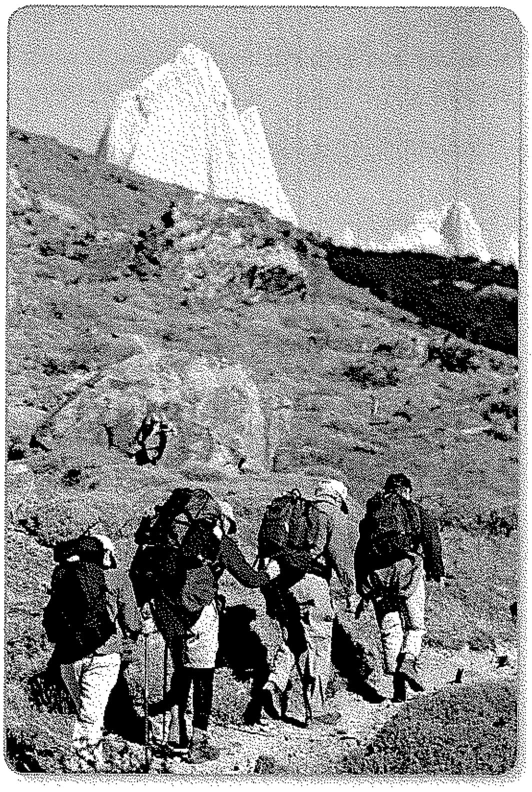
Calculate:

(a) Her displacement after the first 20 minutes.

(b) Her displacement during the 10 minute run.

(c) Her total displacement on the outward journey.

(d) Her velocity during the return journey.



9. A toy car moved 40.0 m north in 3.00 s, then 50.0 m south in 4.00 s.

Calculate:

(a) The car's velocity during the first 3.00 s.

(b) The car's velocity during the next 4.00 s.

(c) The total distance travelled (regardless of direction) in 7.00 s.

(d) The average speed (regardless of direction) during this time.

(e) The total displacement after 7.00 s.

(f) The average velocity of the toy car during this time.

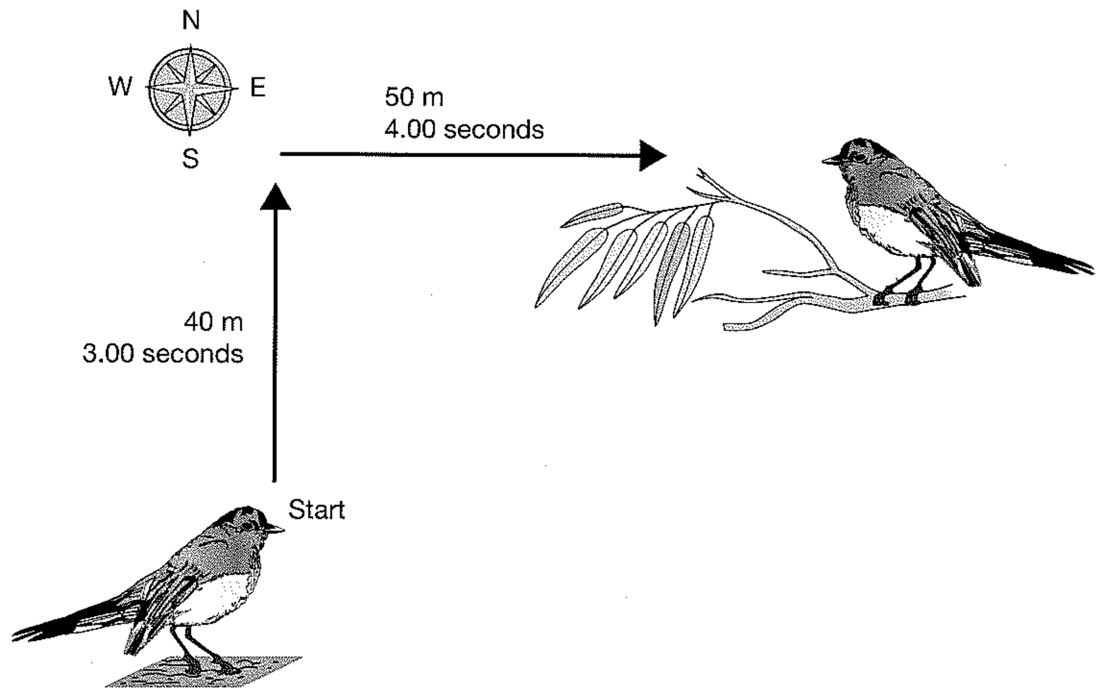
10. A bird moved as shown in the diagram.

Calculate the:

(a) Final displacement of the bird from the start.

(b) Time taken overall.

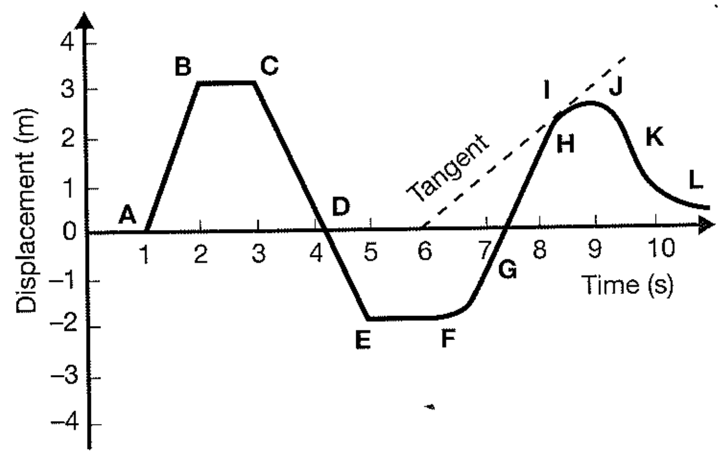
(c) Average velocity overall.



13. Displacement-Time Graphs

On a displacement-time graph, displacement is plotted against the vertical axis and time against the horizontal axis. Plots below the horizontal axis represent negative displacements.

The slope of the line obtained represents the velocity. Instantaneous velocity may be found by calculating the slope of the tangent at that instant.



OA	Stationary at starting position
AB	Constant velocity $= \frac{3-0}{2-1} = \frac{3}{1} = 3.0\ \text{m s}^{-1}$
BC	Stationary
CD	Constant velocity $= \frac{0-3}{4.2-3} = \frac{-3}{1.2} = -2.5\ \text{m s}^{-1}$ (backwards to start)
DE	Constant velocity $= -2.5\ \text{m s}^{-1}$ (backwards, behind start)
EF	Stationary
FG	Changing velocity, increasing, positive acceleration
GH	Constant velocity $= \frac{2-(-1)}{8-7} = \frac{3}{1} = 3.0\ \text{m s}^{-1}$
HJ	Changing velocity, decreasing, negative acceleration
I	Instantaneous velocity $= \frac{3-0}{9-6} = \frac{3}{3} = 1.0\ \text{m s}^{-1}$
JK	Changing velocity, increasingly negative, negative acceleration
KL	Changing velociity, decreasingly negative, positive acceleration

Questions

1. On 1 cm graph paper, using a scale of 1 cm to represent 1 unit on both the horizontal and vertical scales, plot the following displacement-time graph (join the points with straight lines).

Time (s)	0	1	2	4	6	8	10
Displacement (m)	0	1	3	4	4	2	0

2. Study this displacement-time graph and answer the following questions.

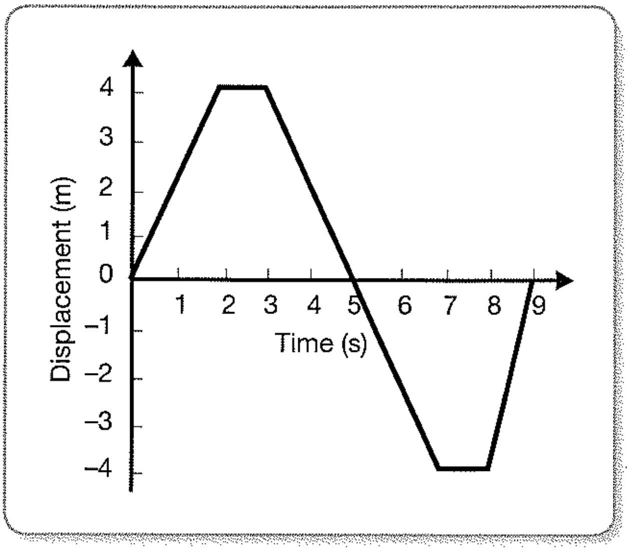
(a) Identify the displacement of the object at time $t = 5.0$ s.

(b) Identify the periods of time the object was stationary.

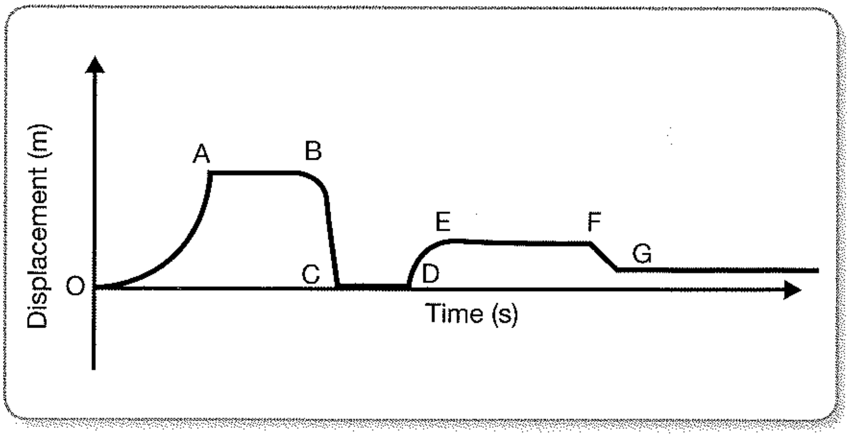
(c) Calculate its velocity during the period 0 to 2 s.

(d) Identify the period of time the velocity was negative.

(e) Identify the period of time the velocity was the greatest.



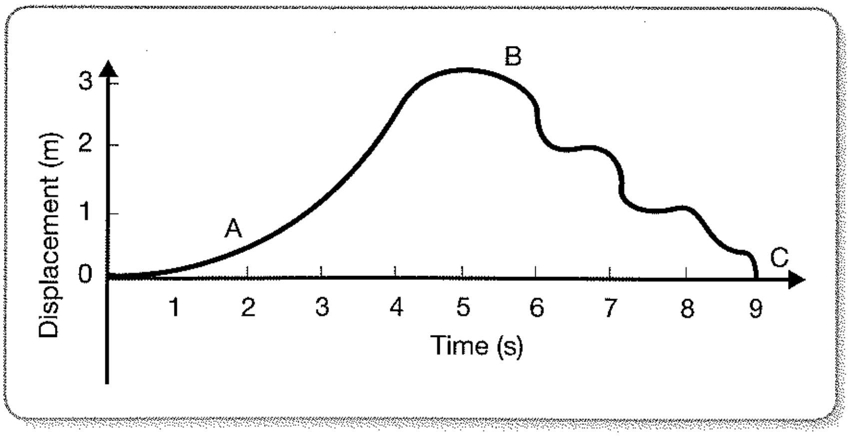
3. Study the following displacement-time graph and describe the changes of velocity which occur in each of the intervals OA, BC, DE, FG. In each case determine whether the acceleration is positive or negative.



4. Look at this displacement-time graph and answer the following questions.

(a) Estimate the instantaneous velocity at A.

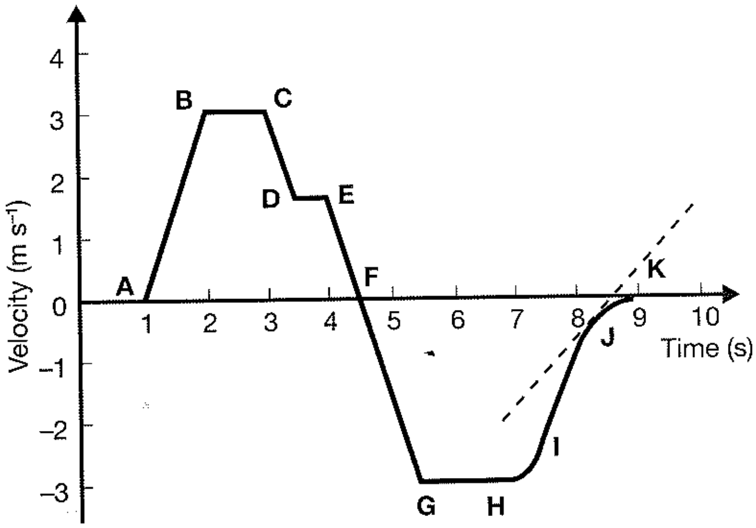
(b) Calculate the average velocity between B and C.



14. Velocity-Time Graphs

On a velocity-time graph, velocity is plotted against the vertical axis, and time against the horizontal axis. Plots below the horizontal axis represent negative velocities. The slope of the line represents the acceleration. Instantaneous acceleration may be found by calculating the slope of the tangent at that instant.

The area 'under' the curve or line represents the displacement. If the curve or line goes under the horizontal axis, the area between the curve or line and the axis is negative displacement and has to be subtracted from any positive displacement when calculating total displacement.



OA	Stationary at starting point
AB	Constant acceleration $= \frac{3-0}{2-1} = \frac{3}{1} = 3.0$ m s^{-2}
BC	Constant velocity = 3.0 m s^{-1}
CD	Constant acceleration $= \frac{1.5-3}{3.5-3} = \frac{1.5}{0.5} = -3.0$ m s^{-2}
DE	Constant velocity = 1.5 m s^{-1}
EF	Constant acceleration = –3.0 m s^{-2}
FG	Constant acceleration = –3.0 m s^{-2}
GH	Constant velocity = –3.0 m s^{-1}
HI	Non-uniform acceleration
J	Instantaneous acceleration $= \frac{1.5-(-2)}{10-7} = \frac{3.5}{3} = 1.2$ m s^{-2}
JK	Non-uniform acceleration

Science Press

Displacement from: A to B = $\frac{1}{2} \times 1 \times 3 = 1.5$ m

B to C = $1 \times 3 = 3.0$ m

C to D = $\frac{1}{2} \times 0.5 \times 1.5 + 1.5 \times 0.5 = 1.1$ m

D to F = $0.5 \times 1.5 + \frac{1}{2} \times 0.5 \times 1.5 = 1.1$ m

F to G = $\frac{1}{2} \times 1.0 \times (-3) = -1.5$ m

G to H = $2 \times (-3) = -6.0$ m

A to H = $1.5 + 3.0 + 1.1 + 1.1 - 1.5 - 6.0 = -0.8$ m

Questions

1. Using 1 cm graph paper, using a scale of 1 cm to represent 1 unit on both the horizontal and vertical scales, plot the following velocity-time graph. (Join the points with straight lines.)

Time (s)	0	2	5	7	10	11
Velocity (m s^{-1})	0	1	3	3	1	0

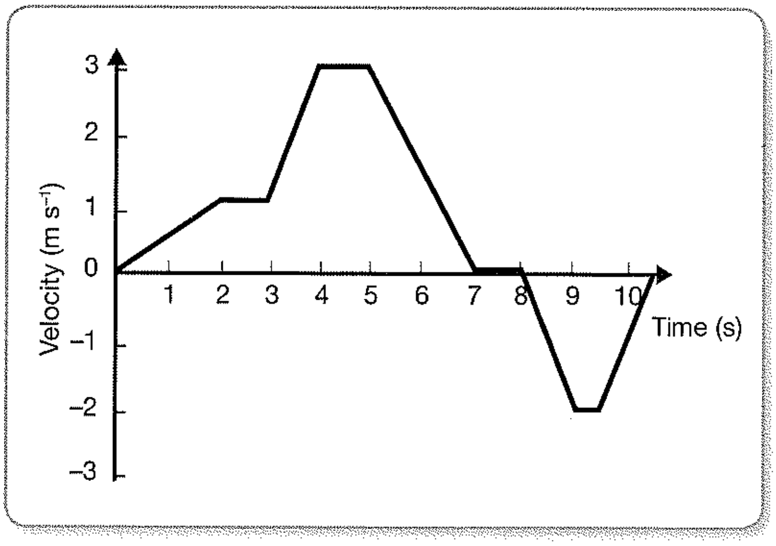
2. This velocity-time graph refers to the questions below.

(a) Deduce the velocity at 4 s.

(b) During which periods was the object stationary?

(c) Deduce its acceleration during the period 3 s to 4 s.

(d) During which period of time was the velocity negative?



3. Study the following velocity-time graph.

Identify the displacement from:

(a) 0 s to 2 s

(b) 2 s to 3 s

(c) 3 s to 4 s

(d) 4 s to 5 s

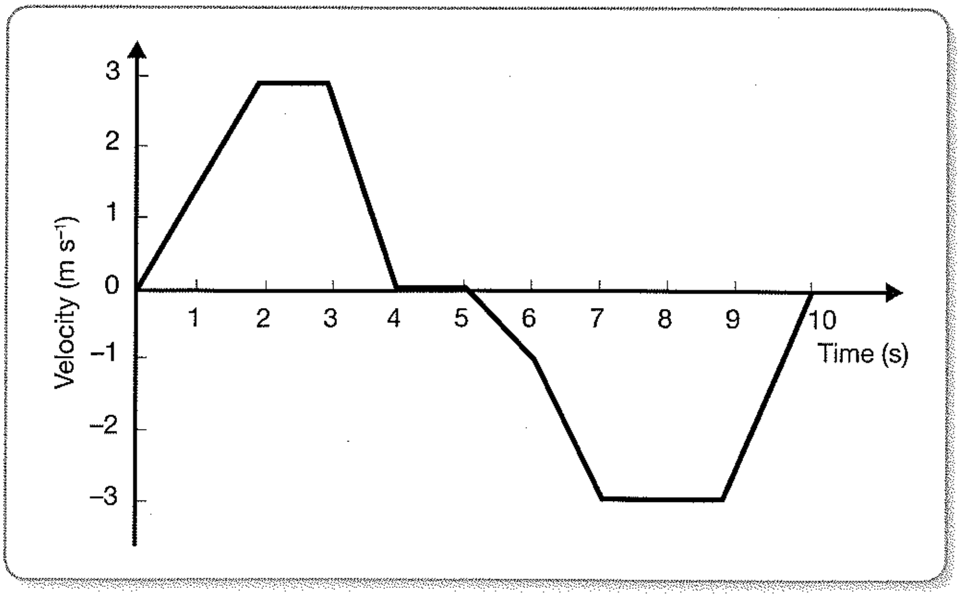
(e) 5 s to 6 s

(f) 6 s to 7 s

(g) 7 s to 9 s

(h) 9 s to 10 s

(i) 0 s to 10 s



15. Uniform Acceleration

$$v_{av} = \frac{\Delta s}{\Delta t}$$

$$a_{av} = \frac{\Delta v}{\Delta t}$$

$$v = u + at$$

v_{av} = average velocity (metres/second) m s^{-1}

Δs = displacement (metres) m

Δt = t = time (seconds) s

a_{av} = average acceleration (metres/sec^2) m s^{-2}

v = final velocity

u = initial velocity

Δv = $v - u$ = change of velocity

Questions

1. A car moving at 10.0 m s^{-1} accelerated at 15 m s^{-2} for 4 seconds. Calculate its new velocity.

2. A cyclist started downhill at 5.0 m s^{-1}. 10.0 s later, his velocity was 25 m s^{-1}. Calculate his average acceleration during this time.

3. A skyrocket accelerated at 19.6 m s^{-2} for 7.50 s. Calculate its change of velocity.

4. Calculate the time it would take a boat accelerating at 9.6 m s^{-2} to go from 4.7 m s^{-1} to 47.9 m s^{-1}.

5. After applying the brakes, a car accelerated (decelerated) at –15.5 m s^{-2}. Calculate its change of velocity in 0.800 s.

6. A boat accelerated at 2.6 m s^{-2} for 9.0 s reaching a velocity of 35.0 m s^{-1}. Calculate its initial velocity before this acceleration.

7. An athlete accelerated at 14.7 m s^{-2} from a standing start for 0.650 s. Calculate the velocity she attained.

8. A car accelerated from a standing start to 120 km h^{-1} in 10.0 s. Calculate its acceleration in m s^{-2}.

9. A ball moving at 30 m s^{-1} bounced against a wall, the collision taking one tenth of a second. The speed of the ball after it bounced was 20 m s^{-1}. Calculate its acceleration away from the wall.

10. At 9.26 am a train was moving north at 15.0 m s^{-1}. At 9.29 am, it was moving south at 45.0 m s^{-1}. Calculate its average acceleration during this time interval.

16. Equations of Motion

$v = u + at$	v = final velocity
	u = initial velocity
$v^2 = u^2 + 2as$	a = acceleration
	t = time
$s = ut + \frac{1}{2}at^2$	s = displacement

Questions

1. A certain car has a maximum acceleration of 3.0 m s^{-2}. At this maximum acceleration, calculate the time it would take for the car to accelerate from 15.0 m s^{-1} to 31.5 m s^{-1}.

2. The same car (see Question 1) took 1.5 s to slow down from 31.5 m s^{-1} to stop. Calculate its acceleration during its slowing down.

3. At an acceleration of 3.0 m s^{-2} calculate how far the car would travel if it began from a standing start and reached a velocity of 28.0 m s^{-1}.

4. A rocket travelling at 42.6 m s^{-1} accelerated at 15.2 m s^{-2} for a distance of 1800 m. Calculate its velocity then.

5. A boat, initially travelling at 21.4 m s^{-1} accelerated at 1.6 m s^{-2} for 6.5 s. Calculate how far it travelled in this time.

6. An aeroplane flying at 200.0 m s^{-1} accelerated for 6.2 s during which time it travelled 2000.0 m. Calculate its acceleration.

7. At an acceleration of 12.0 m s^{-2} calculate the time it would take for an object to travel 180.0 m if its velocity was originally 6.0 m s^{-1} .

8. A cyclist whose initial velocity was 5.0 m s^{-1} accelerated at 0.8 m s^{-2} for 4.3 s. Calculate the distance travelled by the cyclist during this time.

9. An object originally moving at 15.0 m s^{-1} accelerated to 105 m s^{-1} in 8.0 s. Calculate the distance it travelled during this time.

10. A car 4.0 m long caught up with a semitrailer truck 20.0 m long, travelling at a steady 72 km h^{-1}. The front of the car was kept at 10.0 m behind the truck. The driver of the car decided to overtake the truck. If the car pulled out from its front being 10.0 m behind the truck accelerating at 2.0 m s^{-2} until the back of the car was 10.0 m in front of the truck, calculate how far the car would travel and what time it would take.

17. Addition of Vectors

Directions are given as degrees from a cardinal compass point, e.g. north 25° east or as degrees clockwise from north, e.g. 25°.

Questions

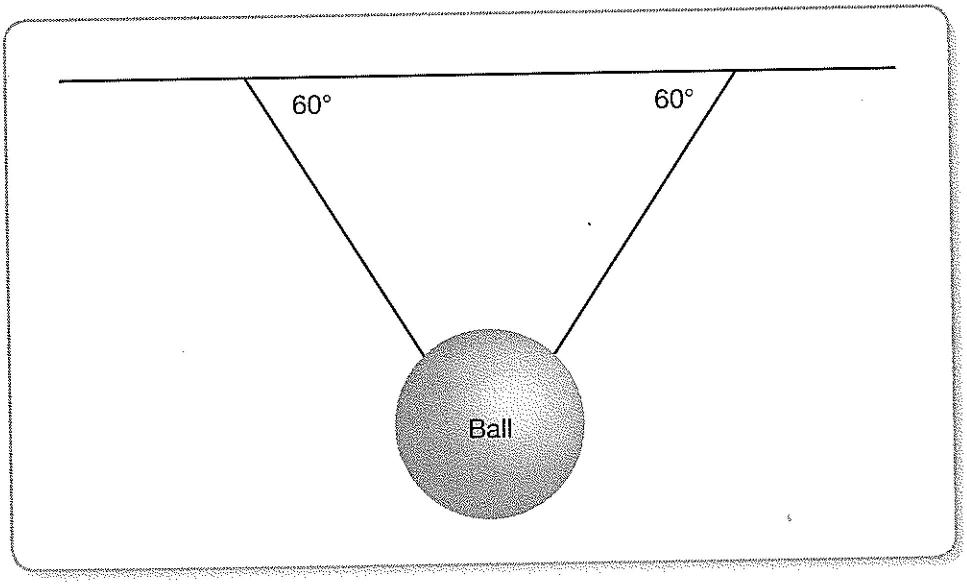
1. Calculate the resultants of the following combinations of vectors. Your answer must include both magnitude and direction.
   (a) 30 km north + 50 km south.
   (b) 15 newtons east + 5 newtons west.
   (c) 20 m s^{-1} north + 20 m s^{-1} east.
   (d) Acceleration of 30 m s^{-2} west and 40 m s^{-2} south.

2. A ship sailed 15.0 km north, then 20.0 km east, followed by 10.0 km south. Calculate its final displacement from its starting point.

3. A motor boat heads due east at 10.0 m s^{-1} on a current moving towards south 30° west at 5.0 m s^{-1}. Find the resultant velocity of the boat.

4. Two cars collide on an icy road. Car A had a momentum of 3.0×10^4 kg m s^{-1} in a direction south 20° east and car B had a momentum of 4.0×10^4 kg m s^{-1} in a direction east 20° north. After the collision the cars locked together and slid off with a momentum equal to the vector sum of the original momenta. Calculate this vector sum.

5. A steel ball weighing 100 newtons is supported by two wires as shown in the diagram, each wire having the same tension. Calculate the tension in each wire.



18. Subtraction of Vectors

Vector subtraction is performed by reversing the direction of the vector to be subtracted and adding it.

Directions may be references to the cardinal points of the compass, e.g. north 20° east or as number of degrees from north, e.g. 110° which is the same direction as north 20° east.

Some of these questions may involve the use of the cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{or} \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Questions

1. Calculate the resultants of the following. Your answers must contain both magnitude and direction.

(a) 26 m east – 13 m west.

(b) 15 m s^{-1} south – 24 m s^{-1} north.

(c) 40.0 kg m s^{-1} south – 30.0 kg m s^{-1} east.

(d) 20.0 newtons at 200° – 15.0 newtons at 110°.

(e) 48 m s^{-2} west 30° south – 36 m s^{-2} north 40° west.

2. Two forces act on an object. One force is 35 newtons towards the west. Calculate the other force given that the resultant of the two forces is 35 newtons towards 240°.

3. An aeroplane is to fly 200 km to the east in 1 hour. There is, however a wind blowing at 50 km h^{-1} from the south. Calculate the direction and speed the aeroplane has to fly to reach its destination on time.

4. A car travelling north at 16 m s^{-1} turned a corner to the east maintaining the same speed. Calculate the change in velocity.

5. In order to keep going in a direction of north 10° east at 15 km h^{-1} a boat had to sail on a heading of north 20° east at I8 km h^{-1} to counteract the current. Calculate the strength of the current in magnitude and direction.

19. Relative Velocity

Calculating relative velocity involves subtraction of vectors.

Questions

1. Two trains pass each other, the first travelling at 110 km h^{-1} west, the second at 90 km h^{-1} east. Calculate the velocity of the second train relative to the first.

2. A truck travelling east at 60 km h^{-1} is overtaken by a car travelling at 80 km h^{-1} in the same direction. Calculate the velocity of the truck relative to the car.

3. A boat travelling at 40 km h^{-1} due north passes a stationary buoy. On the boat a boy is walking at 2 km h^{-1} towards the stern of the boat. Calculate the boy's velocity relative to the buoy.

4. Two cars A and B approach an intersection. A is travelling west at 60.0 km h^{-1} and B is travelling south at 40.0 km h^{-1}. Calculate B's velocity relative to A.

5. A river is 350.0 m wide and the stream in it is flowing at 7.0 m s^{-1}. Calculate which direction relative to the stream would a boat need to travel if it was to reach the bank directly opposite its starting point in a time of 35.0 s?

6. Two aeroplanes flying on a collision course are 10.0 km apart. Aeroplane A is flying at 500 km h^{-1} on a heading of north 30° east. Aeroplane B is flying at 600 km h^{-1} on a heading of west 30° north. If neither deviated from its course calculate the time it would take before they collided.

20. Force and Motion

$\sum F = ma$

$v = u + at$

$v_y^2 = u_y^2 + 2a_y\Delta y$

$\Delta y = u_y t + \frac{1}{2}a_y t^2$

$\sum F$ = vector sum of all forces (newtons) N

m = mass (kilograms) kg

a = acceleration (metres/sec^2) m s^{-2}

v = final velocity

u = initial velocity

t = time (seconds) s

Questions

1. Calculate the force needed to make an object of mass 3.0 kg accelerate at 1.6 m s^{-2}.

2. A force of 9.1 N was applied to a box of mass 0.70 kg resting on a smooth horizontal surface. Calculate the acceleration that would be expected.

3. An acceleration of 17. 0 m s^{-2} was produced when a force of 0.85 N was applied to a body. Calculate the mass of the body.

4. A sports car of mass 550 kg accelerated from rest to 45 m s^{-1} in 5.0 s. Calculate the net force that must have been applied to produce this acceleration.

5. A rocket travelling at 5.50×10^1 m s^{-1} fired its motor which gave a thrust of 1.880×10^3 N. At the end of 6.2 s its velocity was 6.378×10^2 m s^{-1}. Calculate the average mass of the rocket during this process.

6. The force applied by a fan to an air track glider of mass 850 g was 0.34 N. Calculate the time taken for the glider to travel 0.80 m if it started from rest.

7. A small van of mass 840 kg was brought from 15.0 m s^{-1} to a stop in a distance of 11.25 m. Calculate the net force applied for this to happen.

8. A mass of 1.96 kg is pulled along a rough horizontal tabletop by a force of 10.00 N producing an acceleration of 3.50 m s^{-2}. Calculate the force of friction between the mass and the table.

9. A crate of mass 240.0 kg was lifted by a crane with an initial acceleration of 1.20 m s^{-2}. Calculate the force the rope would have to supply during this part of the lift.

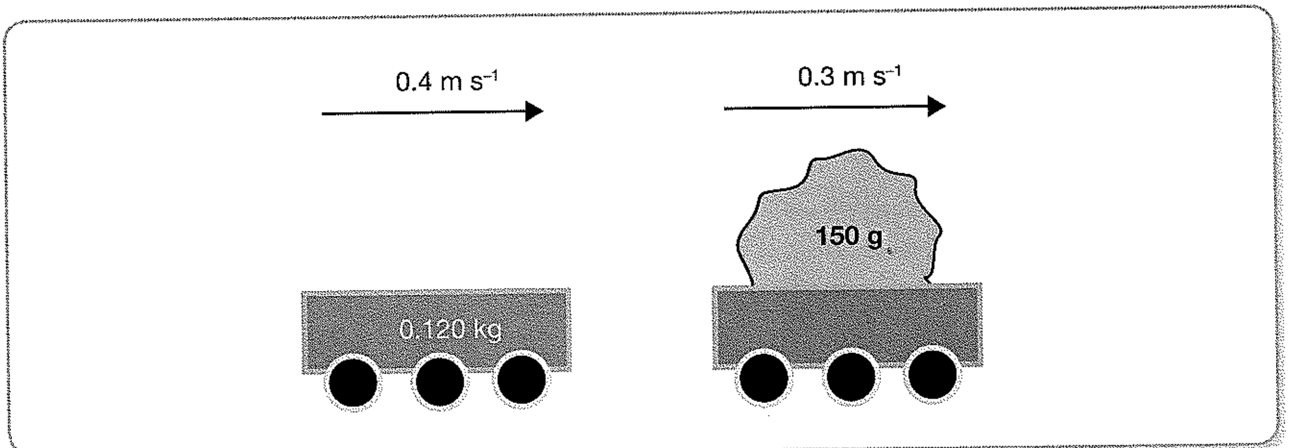
10. A box of mass 14.4 kg was sliding smoothly across the floor at a velocity of 5.00 m s^{-1} when it hit a rough patch 3.00 m wide. After crossing this patch, the box had slowed to 1.00 m s^{-1}. Calculate the size of the force of friction applied over the rough patch.

21. Momentum

$p = mv$	p = momentum (kilogram metre per second) kg m s^{-1}
	m = mass (kilograms) kg
	v = velocity (metres per second) m s^{-1}

Questions

1. Calculate the momentum of an object of mass 21 kg at 17 m s^{-1}.
2. A car of mass 740 kg has a momentum of 18 500 kg m s^{-1} north. Calculate the velocity of the car.
3. A ball moving at 16 m s^{-1} had a momentum of 12 kg m s^{-1} west. Calculate the mass of the ball.
4. A truck of mass 1500 kg changed its velocity from 17 m s^{-1} south to 29 m s^{-1} south. Calculate the change in momentum.
5. A ball of mass 0.500 kg struck a wall at 12.0 m s^{-1}, rebounding at 11.0 m s^{-1}. Calculate the change in momentum.
6. A loaded railway coal truck, total mass 4.0×10^4 kg moving at 3.5 m s^{-1} towards the north-east dumps 1.0×10^4 kg of coal as it moves over a hopper, continuing on at the same speed. Calculate the change in momentum that occurs.
7. A rocket of mass 550 kg moving at 150 m s^{-1} upwards fired its motor, using 240 kg of fuel to achieve a velocity of 425 m s^{-1} upwards. Calculate the change in momentum of the rocket.
8. A person of mass 100 kg was standing beside a path when a trolley of mass 50 kg, moving at 3.0 m s^{-1} came past. As the trolley drew level with the person, they jumped onto the trolley causing it to slow down. If the combined momentum of the person and the trolley remained the same as that of the trolley before this incident, calculate the final velocity of the trolley with the person on it.
9. A dynamics trolley ran into the back of another identical trolley loaded with 150 g moving in the same direction as shown in the diagram. After the collision, the two trolleys joined onto each other. Calculate the speed that the two trolleys move after the collision.

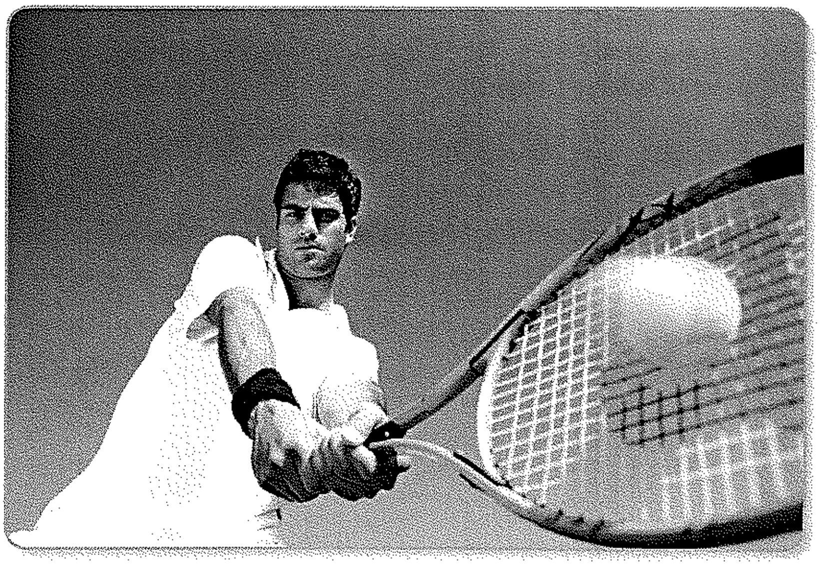


22. Impulse

$\Delta p = F\Delta t$ $\quad$ Δp = impulse = change of momentum (kilogram metre per second) kg m s^{-1} or (newton second) N s
F = force (newtons) N
t = time (seconds) s

Questions

1. A tennis racquet applied a force of 110 N for a period of 0.30 s to a ball initially at rest. Calculate the applied impulse.
2. Calculate the size of the force needed to apply an impulse of 720 N s over a period of 0.060 s.
3. Calculate the time during which a force of 1.6×10^2 N must act to supply an impulse of 8.0×10^{-1} N s.
4. Calculate the change in momentum one could expect from a force of 1.4×10^6 N acting for a period of 5.0×10^{-3} s.
5. A ball of mass 0.40 kg was acted on by an impulse of 4.8 N s. Calculate the change of velocity that occurred.
6. A car of mass 840 kg travelling at 15.0 m s^{-1} accelerated to 25.0 m s^{-1}. Calculate the impulse that was applied to cause this change of speed.
7. A ball of mass 0.600 kg struck a net at 12.0 m s^{-1} and rebounded at 10.0 m s^{-1} a quarter of a second later. Calculate the average force exerted by the ball on the net.
8. Calculate the time a force of 2.7 N must act to bring an object of mass 3.4 kg to rest from a velocity of 3.0 m s^{-1}.
9. A force of 8.6 N acted for 2.5 s on an object of mass 1.8 kg initially moving at 50.0 m s^{-1}. Calculate the object's new speed.
10. A tennis ball of mass 50 g served at 35 m s^{-1} was returned at the same speed. If the ball was in contact with the receiver's racquet during the return of service for one fiftieth of a second, calculate the force that was exerted on the ball.



23. Conservation of Momentum

$\Sigma mv_{\text{before}} = \Sigma mv_{\text{after}}$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

In the case of inelastic collision, only momentums conserved. In the case of elastic collision, both momentum and kinetic energy are conserved.

$\Sigma\frac{1}{2}mv^2_{\text{before}} = \Sigma\frac{1}{2}mv^2_{\text{after}}$

$\frac{1}{2}m_1u_1^{\ 2} + \frac{1}{2}m_2u_2^{\ 2} = \frac{1}{2}m_1v_1^{\ 2} + \frac{1}{2}m_2v_2^{\ 2}$

m_1 = mass of object 1

m_2 = mass of object 2

u_1 = velocity of object 1 before

u_2 = velocity of object 2 before

v_1 = velocity of object 1 after

v_2 = velocity of object 2 after

Questions

1. Two identical air track gliders moving from left to right on a linear air track collided and then separated. Before the collision, one had a velocity of 4.0 m s^{-1} to the right while the other had a velocity of 2.0 m s^{-1} to the right. After the collision, the first glider had a velocity of 2.0 m s^{-1} to the right. Calculate the velocity of the second glider after the collision and its direction.

2. A rocket of mass 75.0 kg moving towards the east at 3.0×10^3 m s^{-1} separated into two parts. One part, mass 50.0 kg moved off towards the east at 2.0×10 m s^{-1}. Calculate the velocity of the other part after the separation.

3. Two skaters, one of mass 80.0 kg moving at 6.0 m s^{-1} to the west, the other, 60.0 kg moving at 4.0 m s^{-1} to the east collided and clung to each other. Calculate their velocity after the collision.

4. A piece of modelling clay, 2.0 kg travelling north at 5.0 m s^{-1} collided and stuck to another piece of clay of mass 3.0 kg travelling south at 4.0 m s^{-1}. Calculate their velocity after the collision.

5. A stationary trolley of mass 2.5 kg was struck by a 1.5 kg trolley travelling south at 12.0 m s^{-1}. The two trolleys locked together and moved off. Calculate their velocity.

6. A railway truck of mass 2.0×10^3 kg moving freely at 4.50 m s^{-1} passed under a coal hopper where a load of coal was dumped into it. This caused a reduction of speed to 0.75 m s^{-1}. Calculate the mass of the coal.

7. A ball of mass 0.5 kg moving at 40.0 m s^{-1} struck a stationary hanging steel plate of mass 7.0 kg and rebounded at 30.0 m s^{-1}. Calculate the velocity of the steel plate as it recoils.

8. A bullet of mass 5.0 g was fired from a rifle of mass 6.0 kg with a speed of 480 m s^{-1}. Calculate the speed of recoil of the rifle.

24. Work

$W = Fs$	W = work (joules) J
	F = force (newtons) N
	s = displacement (metres) m

Questions

1. A force of 6.4 N moves an object 5.5 m. Calculate the work done.

2. 15.0×10^3 J of work is done in moving an object through a distance of 2.5 m. Calculate the average force required.

3. 14.4 J of work is done on an object by a force of 4.8 N. Calculate the displacement of the object.

4. A force of 4.2 N does work of 210.0 J on a body. Calculate the displacement of the body.

5. Calculate the work done in lifting a concrete block weighing 96.0 N to a height of 8.6 m.

6. A box was pushed across a floor a distance of 6.5 m against a frictional resistance of 7.3 N. Calculate the work done.

7. A machine does 450 J of work in 5.0 s. Calculate the time it would take for the machine to do 1.8 kJ of work.

8. A motor boat moving through the water at a constant 15 m s^{-1} is overcoming a force of resistance of 1200 N. Calculate the work being done by the boat each second.

9. A car of mass 800 kg moving at 20 m s^{-1} accelerated uniformly to 30 m s^{-1} in a time of 5.0 s. Calculate:

(a) The acceleration.

(b) The net force needed to accelerate the car.

(c) The distance travelled by the car during this acceleration.

(d) The work done on the car.

10. A motor car accelerated from a standing start to 30 m s^{-1} in 10.0 s. If the acceleration was constant and the car's engine supplied a constant force of 5.0 kN, calculate the work done by the engine in this time. (*Hint:* To work out the distance, multiply the average speed by the time.)

25. Kinetic Energy

$E_k = \frac{1}{2}mv^2$

E_k = kinetic energy (joules) J
m = mass (kilograms) kg
v = velocity (metres per second) m s^{-1}

Questions

1. Calculate the kinetic energy of a body of mass 7.0 kg moving at 1.2 m s^{-1}.

2. 28.8 J of work was done in accelerating a mass of 1.8 kg from rest. Calculate its final velocity.

3. The kinetic energy of a ball moving at 40 m s^{-1} was 60.0 J. Calculate the mass of the ball.

4. An athlete of mass 95 kg ran a 100 m sprint race in 10.0 s. Calculate the kinetic energy of the athlete.

5. A car of mass 550 kg accelerated from 12.0 m s^{-1} to 25 m s^{-1}. Calculate its increase in kinetic energy.

6. A spacecraft of mass 3.5×10^2 kg was moving at 1.5×10^2 m s^{-1} when its motor supplied 4.20×10^3 kJ of energy. Calculate the new velocity acquired by the spacecraft.

7. A force of 96 N acted on an object initially at rest of mass 12 kg for 5.0 s.
Calculate:
(a) The acceleration.
(b) The distance moved.
(c) The velocity at the end of 5.0 s.
(d) The work done by the force.
(e) The kinetic energy at the end of 5.0 s.

8. The engine of a car of mass 1400 kg supplied energy to the car at the rate of 140 kJ per second for 3.00 s causing the car to accelerate from rest.
Calculate:
(a) The work done in that time.
(b) The *KE* acquired after 3.00 s.
(c) The velocity at the end of 3.00 s.

9. A dynamics trolley of mass 120 g was set moving at 2.5 m s^{-1} towards a patch of sand which slowed the trolley. The trolley emerged from the sand at 1.5 m s^{-1}.
Calculate:
(a) The *KE* before and after the trolley hit the sand.
(b) The loss of *KE*.
(c) The work done on the trolley by the sand.

10. A skier of mass 80.0 kg, moving at 12.0 m s^{-1}, ran into a snow bank which stopped her completely. Calculate the work done in bringing her to a stop.

26. Characteristics of Waves

$v = f\lambda$

$T = \frac{1}{f}$

v = velocity (m s^{-1})

f = frequency (Hz)

λ = wavelength (m)

T = period (s)

Velocity of sound in air = 330 m s^{-1}

Velocity of light = 3×10^8 m s^{-1}

Questions

Note: In the following questions, *period* is the time interval between successive waves. It is the inverse of frequency.

1. A surfer noticed that the waves swept past approximately 10 seconds apart. Calculate the frequency of the waves.

2. The musical note 'middle C' has a frequency of 256 Hz. Calculate the time interval (period) between the arrival of successive waves at a receptor.

3. A tuning fork delivered soundwaves with a frequency of 512 Hz. This sound has a wavelength of 10 m in a steel rod. Calculate the velocity of the sound in the steel rod.

4. Whales are known to emit low frequency sounds in their communication. If the velocity of sound in sea water is 1533 m s^{-1}, calculate the wavelength of sounds of frequency 40 Hz in water.

5. Waves with a period of 0.01 s had a wavelength of 1.2 m. Calculate their velocity.

6. Middle C on the piano has a frequency of 256 Hz. Calculate the wavelength of the soundwaves of this frequency in air.

7. Radio waves from a popular radio station have a frequency of 104.1 MHz. If these waves travel at the speed of light, calculate their wavelength.

8. The wavelength of the D-line of the sodium spectrum is 5.893×10^{-7} m. Calculate the period of these waves.

9. Research is currently concerned with the medical effects of mobile telephone radio emissions on the human body. Radic waves travel at the speed of light. Calculate the period of radio waves of wavelength 0.36 m being received by the human body.

10. Cathode ray tubes can be used to display a person's heartbeat. The display consists of a series of complex waves on the screen of a cathode ray oscilloscope. Another monitor records the pulse rate. The diagram represents one full tracing of a person's heartbeats with a rate of 72 beats per minute. Calculate the speed of the trace as it moved across the screen, in this case 18 cm wide.

27. Refraction of Waves

$$\frac{v_1}{v_2} = \frac{\sin i}{\sin r}$$

v_1 = velocity of waves in medium 1

v_2 = velocity of waves in medium 2

i = angle of incidence from 1 to 2

r = angle of refraction

$$v = f\lambda$$

v_1 = velocity of waves in medium 1 (m s^{-1})

v_2 = velocity of waves in medium 2 (m s^{-1})

v = velocity of waves (m s^{-1})

f = frequency (Hz)

λ = wavelength (m)

Note: The ratio $\frac{v_1}{v_2}$ and $\frac{\sin i}{\sin r}$ is known as the refractive index ($_1n_2$) so that the refractive index from medium 1 to medium 2 can be represented as: $_1n_2 = \frac{v_1}{v_2} = \frac{\sin i}{\sin r}$

Questions

1. A train of straight waves of frequency 4.0 MHz passing through soft tissue in the human body at 1540 m s^{-1} crosses into bone where its speed is 4216 m s^{-1}.
Calculate:

(a) The wavelength of the waves in the soft tissue.

(b) The frequency of the waves in the bone.

(c) The wavelength of the waves in the bone.

(d) The refractive index of the bone relative to the soft tissue.

2. Straight soundwaves pass from one medium to another as shown in the diagram.
The angles (in degrees) made by the waves as they pass the dividing line are as shown.
Calculate:

(a) The angle of incidence.

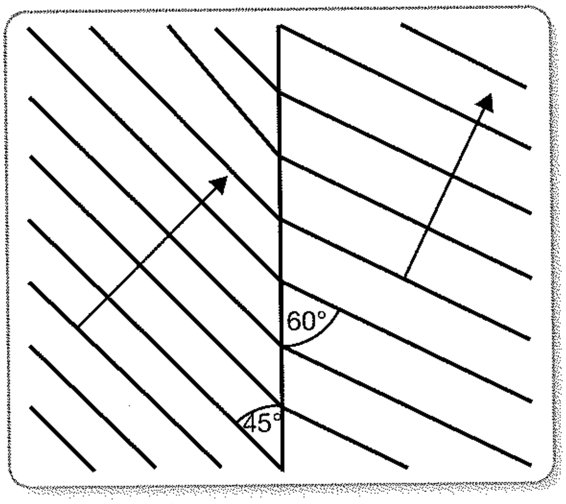
(b) The angle of refraction.

(c) The refractive index.

(d) If the wavelength of the incoming waves is 1.5 m and their frequency is 220 Hz, find the velocity of the incoming wave.

(e) The velocity of the outgoing waves.

(f) The wavelength of the outgoing waves.



3. It is found that soundwaves passing from fresh water to salt water are totally internally reflected if their angle of incidence is 77° or more.
Calculate:

(a) The refractive index from salt water to fresh water.

(b) The speed of waves in the salt water if their speed in fresh water is 1493 m s^{-1} at 25°C.

28. Refraction of Light

Refractive indices of selected substances to red and violet light:

Substance	Red light wavelength 6.708×10^{-7} m	Violet light wavelength 4.047×10^{-7} m
Water	1.3308	1.3428
Crown glass	1.5140	1.5318
Flint glass	1.6434	1.6852

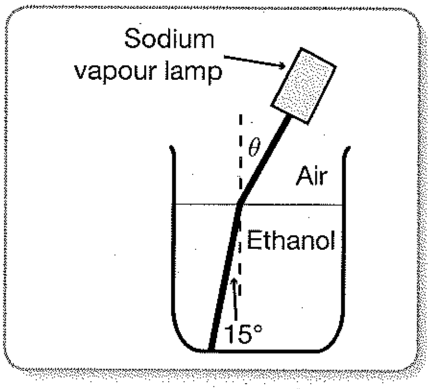
Refractive indices of selected substances to yellow light (D-line of sodium (wavelength 5.893×10^{-7}m):
Ice = 1.31, ethanol = 1.362, benzene = 1.504, glycerine = 1.47, olive oil = 1.46, water = 1.33.

Questions

Use the data given above to work out the following problems.

1. A ray of yellow light passed from air into glycerine at an angle of incidence of 25°. Calculate the angle of refraction.

2. The diagram shows a sodium vapour lamp shining a ray of light into a beaker of ethanol. Calculate the value of θ in the diagram.



3. When yellow light was incident at 20.0° onto a diamond, the angle of refraction was 8.14°. Calculate the refractive index of diamond.

4. Calculate the velocity of yellow light in ice. How does this compare to the velocity of light in water (less or more)?

5. The velocity of yellow light in ruby is 1.7×10^{8} m s^{-1}. Calculate the refractive index of ruby.

6. The speed of yellow light in olive oil is 2.05×10^{8} m s^{-1}. Calculate the refractive index of yellow light passing from olive oil to water.

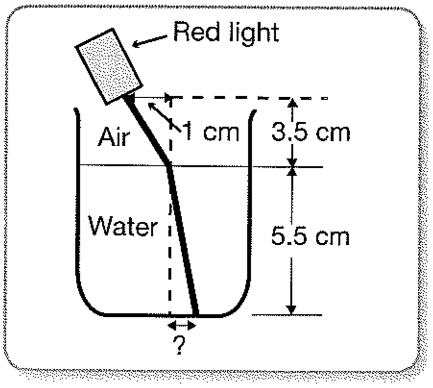
7. Calculate the ratio of the velocity of red light in flint glass to the velocity of violet light in flint glass.

8. If red light passes from crown glass to flint glass at an angle of incidence of 25.0°, calculate the angle of refraction.

9. The critical angle for yellow light leaving mica is 38.7°. Calculate the refractive index of mica.

10. A beaker contains benzene to a depth of 5.00 cm. Calculate its apparent depth when viewed from above.

11. The diagram shows a ray of red light passing from air into water. The source of the light was displaced 1 cm from the centre line as shown in the diagram. Calculate the displacement of the ray from the centre line when it hits the bottom of the beaker as shown.



29. Stationary Waves

The nth harmonic has a frequency given by:

For strings and open pipes: $f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = nf_1$

For closed pipes: $f_n = \frac{v}{\lambda_n} = \frac{nv}{4L} = nf_1$

f = frequency

v = velocity of waves

n = number of harmonics

L = length

Closed pipes have only odd harmonics.

Questions

In these questions (except Question 8) the velocity of sound in air may be taken as 340 m s^{-1}.

1. A stationary wave is produced by two identical transverse progressive waves, each of wavelength 0.50 m and amplitude 0.010 m travelling in opposite directions in the same medium at 50.0 m s^{-1}. Calculate:

(a) The wavelength of the stationary wave produced.

(b) Its amplitude.

(c) Its frequency.

2. A string, 1.00 m long is stretched out to a tension such that the speed of a transverse wave in it is 5.0 m s^{-1}. It is shaken such that the string vibrates in four segments (four antinodes). Calculate the frequency at which the rope is shaken.

3. A string 1.6 m long vibrates in its fundamental mode at a frequency of 480 Hz. Calculate:

(a) The wavelength of the wave in the string.

(b) The wavelength of the sound produced in the air.

(c) The frequency of the first overtone (second harmonic) of the note produced by the string.

4. An open organ pipe is 0.85 m long. Calculate the frequencies of its fundamental and first overtone (second harmonic).

5. The organ pipe in Question 4 above is closed at one end. Calculate the frequencies of its fundamental and first overtone (third harmonic).

6. A tube is closed at one end by a piston which is slowly withdrawn as a tuning fork of frequency 512 Hz is sounded over it. Calculate the shortest length of pipe which will resonate with the fork.

7. Calculate the shortest length of open organ pipe which will resonate with the third harmonic of a tuning fork whose fundamental is 256 Hz.

8. An electrically driven tuning fork was made to sound continuously at 384 Hz while being held over a glass tube full of water. As the water ran out of a tap at the bottom of the tube, resonance occurred, firstly when the water level had dropped to 22.0 cm and then when it had dropped to 67.6 cm. Calculate the speed of sound in air from this experiment.

30. Strings and Pipes

$L = n\frac{\lambda}{2}$ for strings attached at both ends and pipes open at both ends

$L = (2n - 1)\frac{\lambda}{4}$ for pipes closed at one end

n = whole numbers related to the harmonic

L = length of string or pipe (metres) m

λ = wavelength of soundwaves (metres) m

The speed of sound in air may be taken as 340 m s^{-1}.

Questions

1. A string of length 0.75 m on a musical instrument is vibrating at its fundamental frequency. Calculate the wavelength of the waves in the string.

2. A string, 1.0 m long attached at both ends is shaken so that it is vibrating in its fourth harmonic. Calculate the wavelength of the waves in the string.

3. An organ pipe open at both ends is 0.85 m long. Calculate the wavelengths of its fundamental and second harmonic.

4. If the organ pipe in Question 3 is closed at one end, calculate the wavelengths of its fundamental and second harmonic.

5. A pipe in an organ is closed at one end. It is 2.0 m long. Calculate the frequency of the sound it would produce as its fundamental.

6. An organ pipe, closed at one end is producing a sound as its second harmonic with a wavelength of 1.5 m. Calculate the length of the pipe and the frequency of the sound in air.

7. A tuning fork, vibrating at a frequency of 256 Hz (middle C) is held over the top of a long tube of water which is fitted with a tap so that the water can be slowly drained from the tube. Calculate the distance from the top of the tube that the column of air within the tube will resonate with the tuning fork.

31. Diffraction and Interference

Double slit

For maximum: $n\lambda = \frac{dx}{L} = d \sin\theta$

n = order of bright band

λ = wavelength

d = slit separation

x = distance of bands fròm centre

L = distance of slits from screen

θ = angular distance of band from centre

Single slit

For complete annulment: $\sin\theta = \frac{m\lambda}{d}$

θ = angular distance of minimum from centre

m = order of minimum

λ = wavelength

d = width of slit

Diffraction grating

For maximum: $\sin\theta = \frac{m\lambda}{d}$

θ = angular distance of bright line from centre

m = order of the line

d = distance apart of lines on grating

Questions

1. Light of wavelength 4.30×10^{-7} m falls on a double slit whose separation is 4.00×10^{-4} m. Calculate the angular distance between the central bright band and the second bright band on one side of the interference pattern.

2. A double slit, separation 6.00×10^{-5} m is illuminated by monochromatic light of wavelength 5.40×10^{-7} m. Calculate the linear distance from the central bright band to the fifth bright band on one side of a screen 1.20 m from the double slit.

3. Light of wavelength 6.328×10^{-7} m falls on a single slit 1.0×10^{-6} m wide. Calculate the angular distance of the first minimum to the centre of the interference pattern.

4. Monochromatic light is incident on a single slit 1.50×10^{-6} m wide. The width of the central maximum pattern (from first minimum on one side to first minimum on the other) is 60.0°. Calculate the wavelength of the light causing this pattern.

5. A diffraction grating has 6000 lines per centimetre ruled on it. Monochromatic light of wavelength 4.050×10^{-7} m falls normally on it. Calculate the angular distance of the first bright line from the central line.

6. A diffraction grating ruled with 2000 lines to the centimetre is used to find the wavelength of monochromatic light. When the beam of this light is shone normally onto the grating, the angular distance of the fifth bright line from the central line is 28.7°. Calculate the wavelength of the light.

32. The Inverse Square Law

$$I \propto \frac{1}{d^2}$$ d = distance (metres) m

Questions

Note: In Questions 1 and 2 the *width* or *diameter* of the image or the beam is directly proportional to the distance from the source. All other questions are based on the inverse square law.

1. A flashlight, having a circular lens, delivers a beam of light which has a diameter of 1 m at a distance of 10 m from the lens. Calculate the diameter of the beam at a distance of 30 m from the flashlight.

2. A slide projector produces a rectangular image which has the dimensions 0.300 m wide and 0.200 m high when the projector is 0.500 m from the screen. Calculate the distance from the screen the projector would have to be placed in order to fill a screen 1.500 m wide.

3. A particular surface is illuminated by a light shining on it. Deduce what will happen to the intensity of the light falling on the surface if the light is moved to twice its original distance from the surface.

4. A reading lamp was placed 0.5 m from a page of a book but it was too bright. Calculate the distance from the page the lamp should be placed in order to reduce the intensity of the light on the page to two-thirds of its original value.

5. On a certain suburban street, the lamp posts are situated 100 m apart. A photographer, standing next to one lamp post took a photo of the next lamp using a 0.5 seconds exposure. Calculate the time of the exposure needed so that the light from the next lamp after that would appear on the film to be equally as bright. (Assume that film exposure is inversely proportional to the brightness.)

6. The intensity of the light falling on a book was reduced to one sixteenth of its original value by moving the light source away from its original position. If the light was originally 2 m from the book, calculate the new distance away from the book.

7. The intensity of the light falling on the Earth from the Sun is approximately 1400 W m^{-2}. The distance of the Earth from the Sun is approximately 1.5×10^{11} m. The distance of the planet Mars from the Sun is approximately 2.28×10^{11} m. Calculate the intensity of the light falling on Mars.

8. Using the data of Question 7 above, calculate the distance from the Sun a planet would have to be to receive a light intensity of 700 W m^{-2} .

9. Light from a spot lamp falls onto a sheet of cardboard, 4.00 m from the lamp at the rate of 1600 W m^{-2}. Calculate the intensity of light falling on the sheet if the spot lamp was moved to be 6.25 m away.

10. In order to be readable, the intensity of the light from a slide projector showing the words of a song needs to be 900 W m^{-2}. When the projector was placed 5.00 m from the screen, the intensity was 1600 W m^{-2}. Calculate the maximum distance the projector can be from the screen so that the words are still readable.

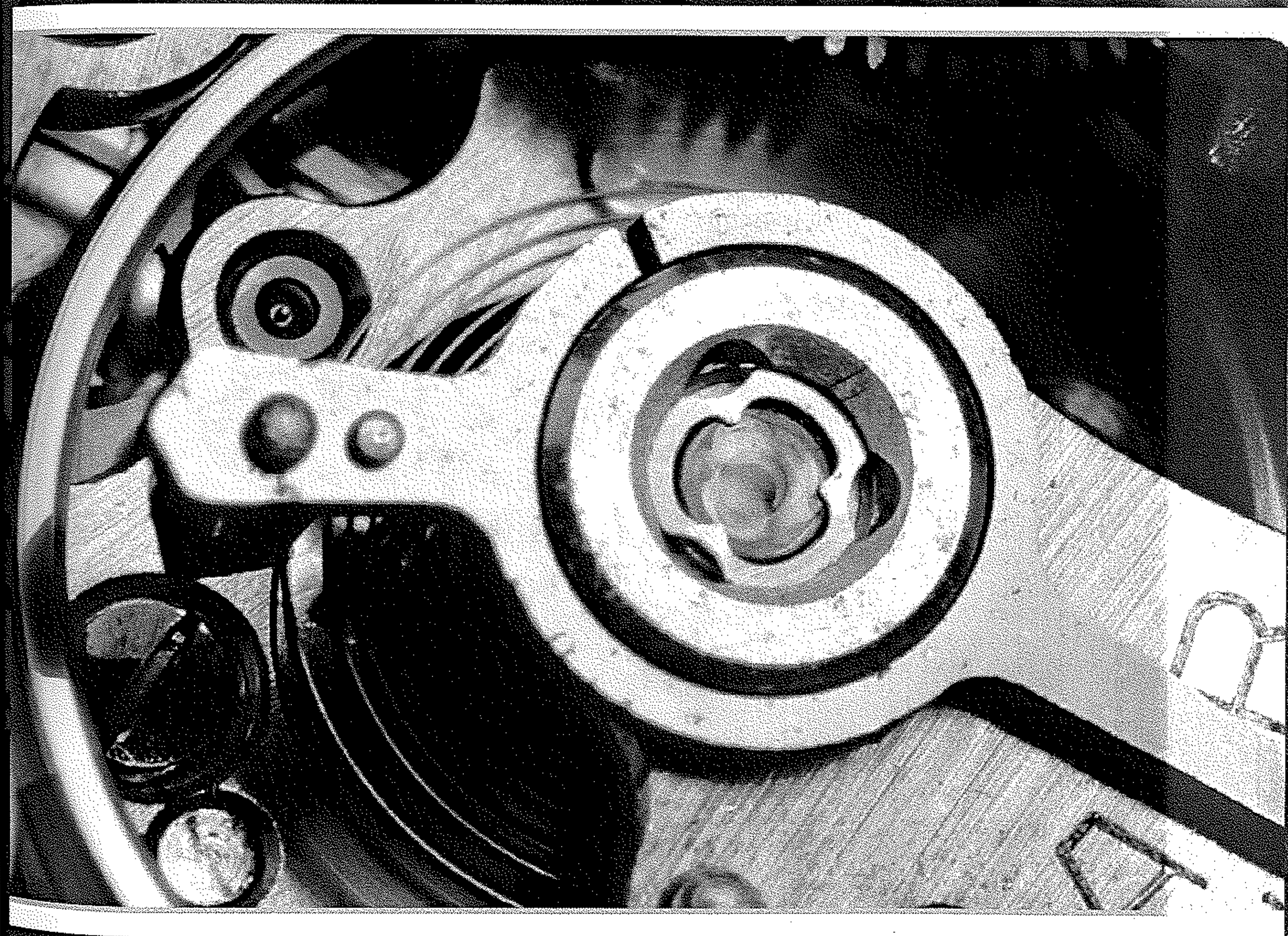
Notes

Science Press

NATIONAL PHYSICS

PHYSICS CALCULATIONS

Gravity and Electromagnetism



33. Mass and Weight

The weight of an object is the force of gravity on the object.

$$F = mg$$

Objects in frames of reference freely falling in the Earth's gravitational field appear to be weightless because they experience inertial forces equal in magnitude to their own weight.

Questions

In the questions which follow take $g = 9.8\ \text{m s}^{-2}$ on the surface of the Earth.

1. Calculate the weight force acting on an object of mass 5.0 kg.

2. A child weighs 200 N. Calculate the child's mass.

3. A diver of mass 70 kg is standing on a diving platform. Calculate the upward force the platform is exerting on the diver.

4. An elevator begins to rise with an acceleration of $5.0\ \text{m s}^{-2}$.
Inside the elevator are a number of people whose combined mass is 950 kg.

(a) Calculate the force needed to accelerate the people.

(b) Calculate the weight force due to the people.

(c) Calculate the combined upward force applied by the floor of the elevator to accomplish this acceleration.

5. In the last stage of its descent to the Earth, a space capsule is decelerating at the rate of 50 m s^{-2}. Calculate the force applied to the body of an astronaut of mass 90 kg by the seat of the capsule during this deceleration, taking into consideration their weight as well as their deceleration.

6. On the surface of the Moon, the acceleration due to gravity is about one sixth that of the Earth. Calculate the weight of a person of mass 110 kg standing on the surface of the Moon.

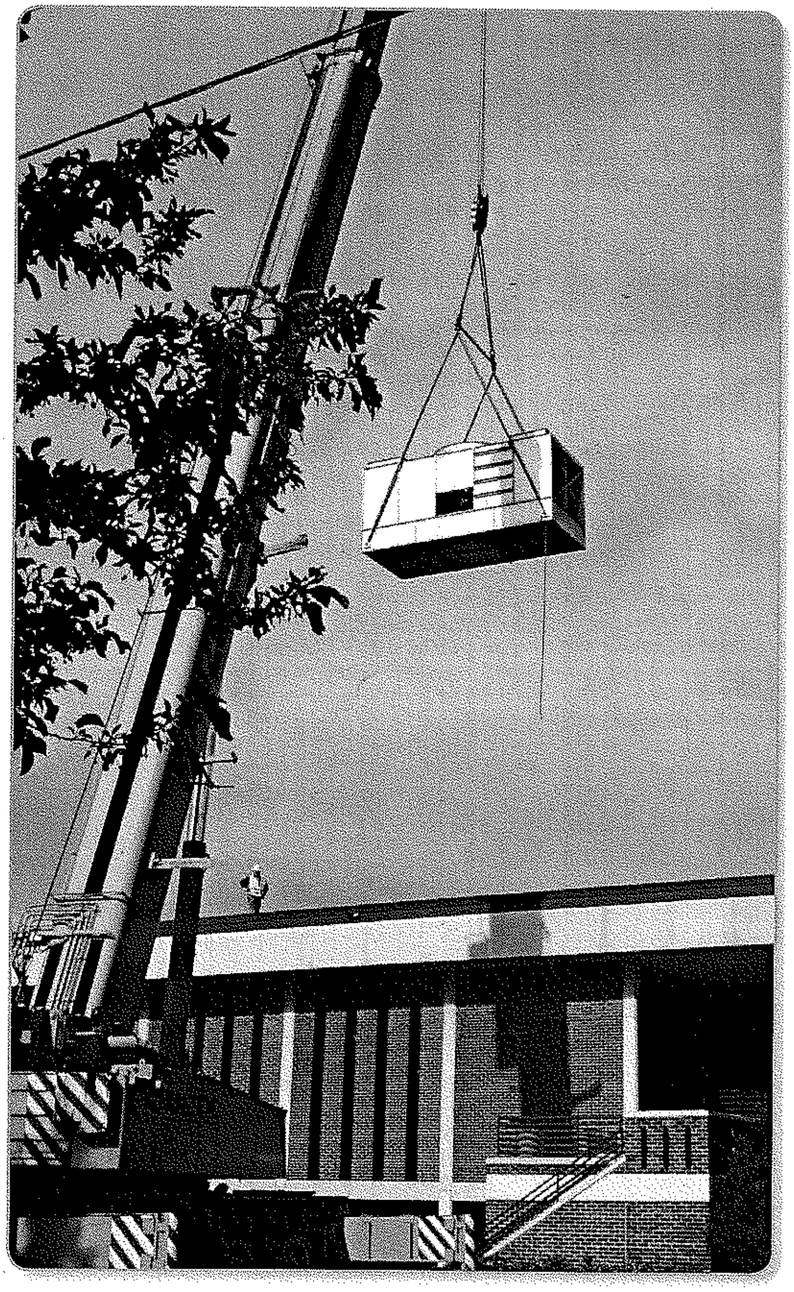
7. While pulling out from a dive in an aeroplane, the centripetal force acting on the pilot of the aeroplane was described as being of the order of 5 'g's'. Including the weight of the pilot, in this case of mass 95 kg, calculate the force that would be applied by the seat of the aircraft to the body of the pilot.

8. We have all experienced that 'dropping' sensation as an elevator begins its descent.

   A person standing on a set of scales in an elevator stated that her 'weight' as recorded by the scales was 56 kg. (*Note:* This means that the weight force was actually 56 × 9.8 N.) As the elevator began to descend she noted that her 'weight' fell to 28 kg before returning to normal.

   Calculate the acceleration of the elevator during this period.

9. A crane fitted with a weight measuring scale recorded a load of building material as having a 'weight' of 800 kg. As the crane began to lift the load, the operator noted that the scale registered a 'weight' of 850 kg for a short period of time before settling down to its original reading.
   (a) Calculate the original weight force in newtons.
   (b) Calculate the weight force during the initial lift.
   (c) Explain this force increase.
   (d) Calculate the value of the acceleration experienced during the initial lifting.



10. In a particular 'bungee' jump, a 75 kg person 'free-falls' for 5 seconds (ignore air resistance) before the bungee brings the person to a stop in a further 3 seconds. Calculate the average deceleration force acting during this time and compare it to the person's normal weight.

34. Motion in the Earth's Gravitational Field

The weight of an object is the force of gravity on the object.

$$F = mg$$

Objects in frames of reference freely falling in the Earth's gravitational field appear to be weightless because they experience inertial forces equal in magnitude to their own weight.

Questions

In the questions which follow take $g = 9.8$ m s^{-2}.

1. A stone fell from the top of a high cliff taking 5.0 s to reach the ground. Calculate its speed when it hit.
2. Calculate the time it would take for an object to fall from rest from a height of 490 m.
3. A stone was thrown straight up at a speed of 24.5 m s^{-1}. Calculate the time it took before it began to fall back to the ground.
4. Someone wanted to throw a brick onto the roof of a building 4.9 m above them. Calculate the least speed at which the brick could be thrown to just reach the roof.
5. A rocket fired vertically rose to a height of 240 m. Calculate the velocity at which it was fired.
6. Another rocket was fired upwards at a speed of 98 m s^{-1}. Calculate the time it took to fall back to the Earth.
7. A parachutist descending at a constant 4.9 m s^{-1} dropped his keys when he was 98.0 m above the ground. Calculate the time it took for the keys to fall to the ground.

8. A balloon rising at a constant speed of 10 m s^{-1} released a marker which took 7.0 s to reach the ground. Calculate the height off the ground the marker was when it was released.
9. A cricket ball thrown vertically upwards took 6.0 s to come back to the ground. Calculate the original upward speed.
10. A cliff stands 49.0 m above a beach. If a projectile were to be fired almost straight up from the top of the cliff at 19.6 m s^{-1}, calculate the time it would take for the projectile to fall back onto the beach.

35. Projectile Motion

$v_x^2 = u_x^2$	v_x = final horizontal velocity
$v = u + at$	u_x = initial horizontal velocity
$v_y^2 = u_y^2 + 2a_y\Delta y$	v = final velocity
$\Delta x = u_x t$	u = initial velocity
$\Delta y = u_x t + \frac{1}{2}a_y t^2$	a = acceleration
$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v-u}{t}$	t = time
	v_y = final vertical velocity
	u_y = initial vertical velocity
	a_y = vertical acceleration
	Δy = change of vertical displacement
	Δx = change of horizontal displacement
In addition to the above the following will also be useful:	a_{av} = average acceleration
	Δv = change of velocity
$u_x = u \cos \theta$	Δt = change of time
$u_y = u \sin \theta$	θ = angle of projection

Questions

1. An object is thrown up at a velocity of 20 m s^{-1}. Ignoring air resistance, calculate:

(a) How high the object will go to where its velocity is reduced to zero.

(b) How long it will take for the object to return to its original position.

(c) Its velocity as it returns to its original position.

2. A rocket was launched at 15.0 m s^{-1} at an angle of 45° to the horizontal. Calculate the horizontal and vertical components of the rocket's initial velocity.

3. A rocket was launched at an angle of 60.0° from the horizontal at 12.5 m s^{-1}.

(a) Calculate its initial velocity in the vertical direction.

(b) Calculate the time it stays in the air before returning to the level from which it came.

4. A projectile was launched from ground level at an angle of 40.0° from the horizontal at 16.0 m s^{-1}.

(a) Calculate its initial velocity in the vertical direction.

(b) Calculate the time it stays in the air before returning to ground level.

(c) Calculate its initial velocity in the horizontal direction.

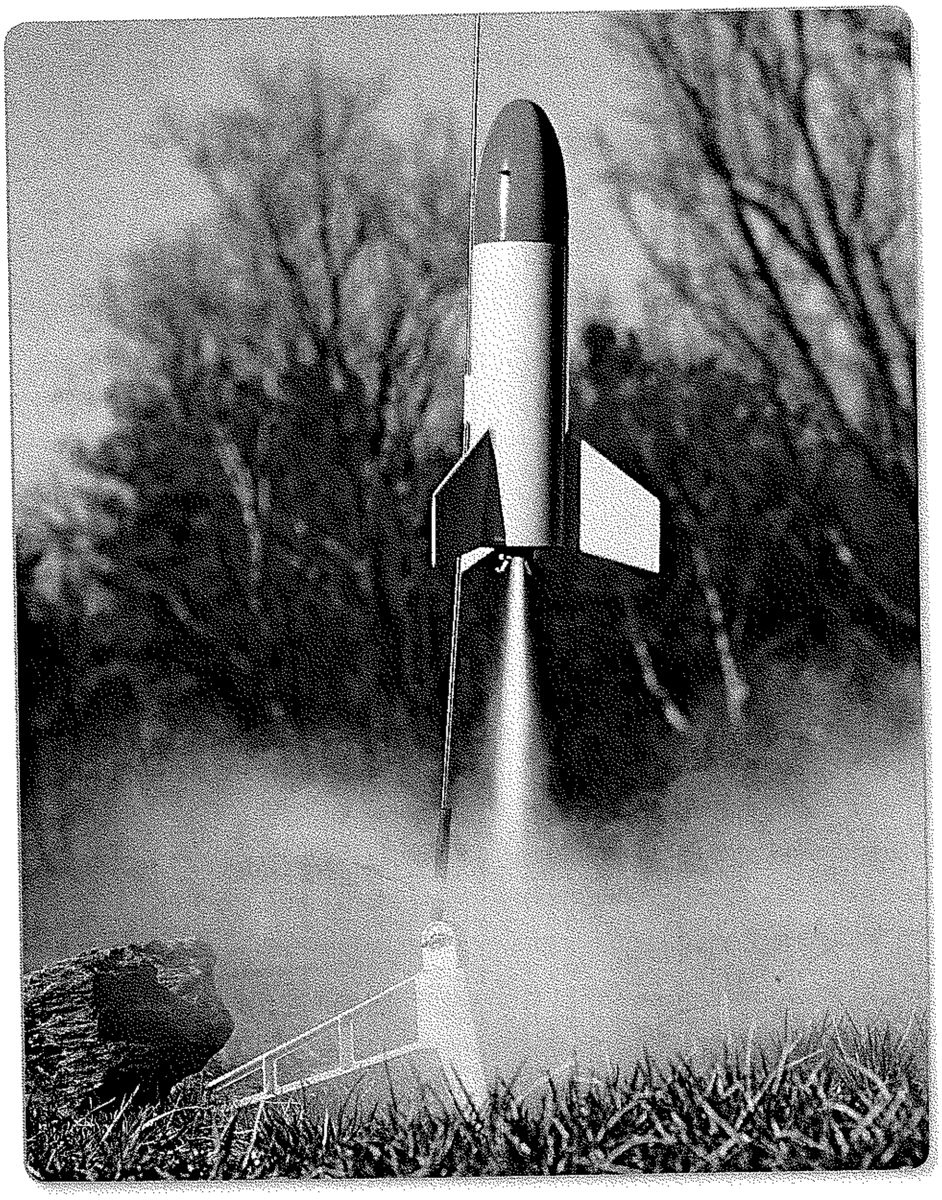
(d) Calculate the distance the projectile will travel in the horizontal direction before it hits the ground.

5. A projectile started on its way at 350 m s^{-1} at an angle of 10.0° from the horizontal. Calculate the greatest height to which the projectile would rise.

6. A rocket, initially on the ground, aimed at an angle of 15.0° from the horizontal was in the air 3.0 s before falling back to the ground. Calculate its original speed.

7. A launcher sent a projectile horizontally from the top of a cliff overlooking the ocean at 20 m s^{-1}.
It took 2.5 s before it was seen to hit the water.
Calculate:
(a) The height of the cliff (and the launcher) above the water.
(b) The horizontal distance travelled by the projectile.
(c) The velocity of the projectile as it hit the water (this should be given in magnitude and direction).

8. (a) A launcher sent a projectile from the ground at 18 m s^{-1} at an angle of 30°. Calculate how far it travels before it hits the ground again.
(b) The same projectile was launched again at the same speed and angle of elevation towards a wall 15 m away. Calculate how far above the ground the projectile hits the wall.



36. Circular Motion

$$F = \frac{mv^2}{r}$$

F = centripetal force (newtons) N

m = mass (kilograms) kg

v = velocity (metres per second) m s^{-1}

r = radius (metres) m

Questions

1. Calculate the centripetal force acting on an object of mass 10 kg travelling in a circle at 16 m s^{-1} in a circle of radius 8 m.

2. A force of 270 N was needed to keep an object of mass 3.0 kg travelling in a circle 20 m in diameter. Calculate the speed of the object.

3. In order to maintain motion in a circle at 6.0 m s^{-1} a body of mass 2.0 kg needed a centripetal force of 12.0 N. Calculate the radius of the circle in which it travelled.

4. A stone was attached to a string 1.0 m long and was whirled in a circle at a speed of 10.0 m s^{-1}. The centripetal force needed to hold the stone in the circle was 25.0 N. Calculate the mass of the stone.

5. A carnival roundabout of radius 10.0 m rotates at a radial speed of 5 m s^{-1}. Calculate the centripetal force on a child of mass 30 kg riding on the roundabout.

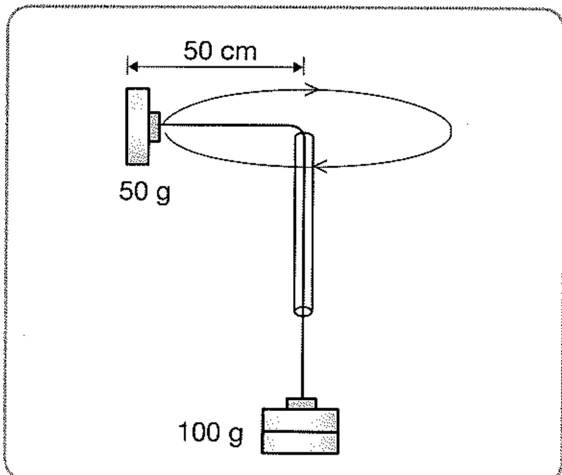
6. A motorcycle rounded a corner of radius 15 m at a speed of 15 m s^{-1}. Calculate the centripetal force on a rider of mass 80 kg.

7. A satellite of mass 2000 kg is to travel in an orbit of radius 7.2×10^6 m where its centripetal acceleration is 9.8 m s^{-2}. Calculate the speed at which it should travel.

8. A car of mass 600 kg travels on a circular track of radius 50.0 m. Calculate the centripetal force that would be required to enable it to travel at 30 m s^{-1}.

9. A piece of fishing line 2.0 m long, of breaking strain 40.0 N is fixed to a steel ball of mass 0.50 kg. Calculate the greatest angular tangential speed with which the ball could be whirled in a circle before the line broke.

10. In the diagram, a mass carrier is attached by a length of fishing line to another mass carrier, the line passing through a tube which is moved in such a way as to keep the first mass carrier moving in a circle. Given the data on the diagram, calculate the tangential speed of the moving mass carrier.



37. Universal Gravitation

$$F = \frac{Gm_1m_2}{d^2}$$

F = gravitational force of attraction (newtons) N

G = universal gravitational constant

$= 6.67 \times 10^{-11}$ N m^2 kg^{-2}

m_1 = mass of one object (kg)

m_2 = mass of other object (kg)

Questions

1. Calculate the force of attraction between a mass of 4.00×10^3 kg and another of mass 9.00×10^3 kg whose distance apart, centre to centre, is 6.00 m.

2. Calculate the force of gravity acting between two masses, each 1.2×10^2 kg, if the distance between their centres of mass is 1.6×10^{-2} m.

3. The force of attraction between two objects, one of mass 3.6×10^3 kg and the other of mass 2.8×10^3 kg is 1.0×10^{-4} N. Calculate the distance apart of the objects.

4. A body of mass 7.5×10^2 kg attracts another body 5.0×10^{-1} m from it with a force of 1.5×10^{-8} N. Calculate the mass of the second body.

5. Two objects, one twice the mass of the other, attract each other with a force of 4.8×10^{-6} N when they are placed 6.0×10^{-2} m apart. Calculate the mass of the lighter object.

6. The mass of the Earth is approximately 5.977×10^{24} kg, and its radius is 6.378×10^6 m. Calculate the force of attraction on an object of mass 1.000 kg if it was 6.378×10^6 m from the centre of the Earth, i.e. on the surface of the Earth.

7. Using the data of Question 6 above, calculate the force of attraction on an object of mass 1.000 kg if it was 1000 km above the surface of the Earth.

8. Referring to Question 7 above, calculate what the object would weigh.

9. Calculate how far above the Earth's surface will the weight of an object be half of what it is on the surface.

10. Calculate the force of gravity holding the Earth in orbit around the Sun.
(Mass of Sun = 1.991×10^{30} kg, mass of Earth = 5.977×10^{24} kg, orbital radius 1.496×10^{11} m.)

38. Inclined Planes

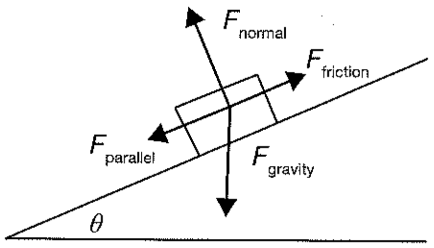


Diagram A

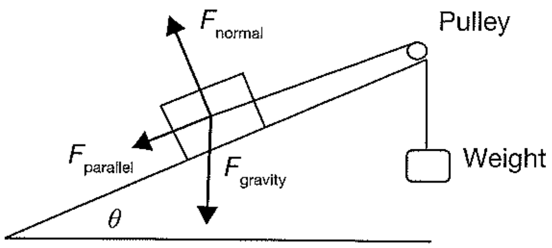


Diagram B

Questions

The following questions refer to diagram A or B.

1. (Diagram A) An object of mass 10.0 kg is situated on a frictionless inclined plane with a slope of 30°. Calculate the acceleration of the object down the plane.

2. (Diagram A) An object of mass 5.0 kg is situated on an inclined plane which is on a slope of 45° where the friction acting on the object is 2.0 N. If the object is free to slide and the plane is very long, calculate the distance that the object will have moved in 2 seconds.

3. (Diagram A) Calculate the magnitude of the friction force which would be necessary to stop an object of mass 2.0 kg from sliding down the slope of 30°.

4. (Diagram A) An object of mass 1.0 kg is situated on an inclined plane whose angle can be varied. Friction between this object and the plane is 1.0 N. Calculate the angle the plane would need to be tilted to just overcome the friction and begin to move.

5. (Diagram B) An object of mass 5.0 kg on a frictionless inclined plane whose slope is 60° is connected via a string and a pulley to a weight of mass 2.0 kg which is hanging down. Calculate the acceleration of the object and indicate the direction it will travel (up or down the plane).

6. (Diagram B) An object of mass 5.0 kg is situated on an inclined plane which is on a slope of 45° where the friction acting on the object is 2.0 N. It is connected via a string and a pulley to a weight of mass 3.0 kg which is hanging down. If it is released and allowed to slide down the plane, calculate the velocity it will acquire after 1 second.

39. Coulomb's Law

$$F = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$F = \frac{k q_1 q_2}{r^2}$$

F = force (newtons) N

$k = 9.00 \times 10^9$ N m^2 C^{-2}

$= \dfrac{1}{4\pi\,\varepsilon_0}$ where $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

q_1 and q_2 = electric charges (coulombs) C

r = distance separating the charges (metres) m

Questions

1. Calculate the force between charges of 3.0 C and 4.0 C separated by a distance of 2.0 m.

2. Charges of 6.0×10^{-8} C and 2.7×10^{-8} C are situated 3.0×10^{-2} m apart. Calculate the force that would act between them.

3. Two charges repel each other with a force of 1.8×10^{-4} N. They have the values of 4.0×10^{-9} C and 8.0×10^{-9} C. Calculate the distance separating them.

4. A charge of 1.2×10^{-8} C is placed 5.0 cm from another charge and a force of 9.6×10^{-1} N acts between them. Calculate the value of the other charge.

5. Two identical charges positioned 2.0 cm apart repel each other with a force of 8.1 N. Calculate the size of each charge.

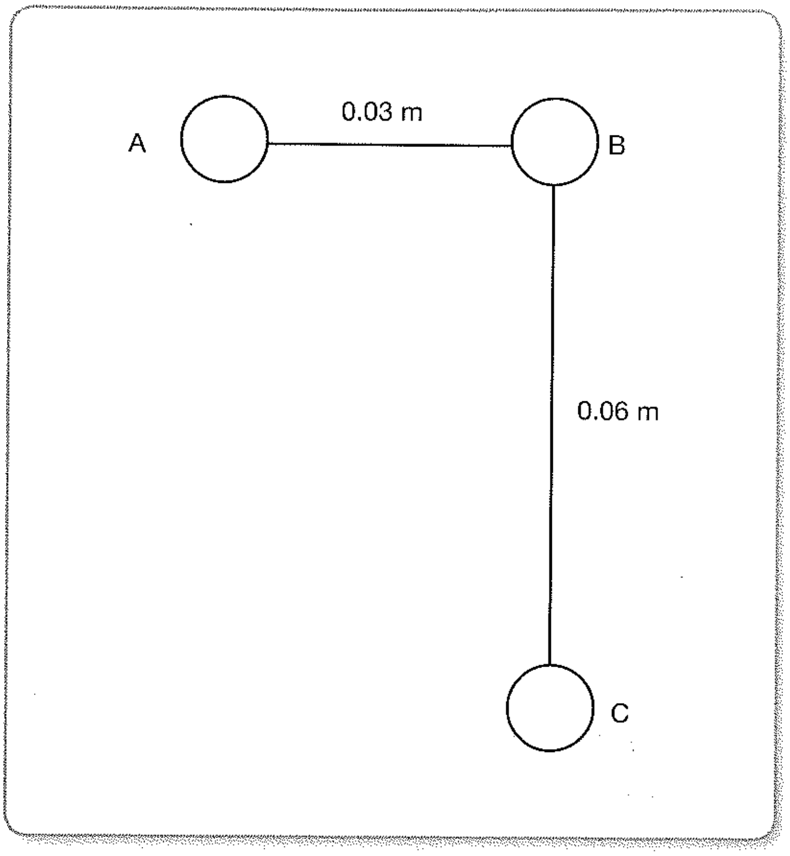
6. A small charged sphere A carries four times the charge of another sphere B. When these are placed 4.5 cm apart, they experience a force of 0.016 N attraction between them. Calculate the charge on B.

7. Two spheres, each of mass 5.0 g were charged with charges of 3.5×10^{-8} C and one was placed above the other so that the upper one was supported by the repulsion between it and the lower one. Calculate how far apart the two will be when the upper one is floating.

8. A sphere of mass 10.0 g was charged negatively. It was held, floating 2.5 cm above an object carrying a charge of -1.4×10^{-7} C. Calculate the charge that would be carried by the sphere.

9. A small sphere which carries a charge of $+2.4 \times 10^{-8}$ C is placed halfway between a charge of -3.6×10^{-8} C and another of $+4.8 \times 10^{-8}$ C which are 5.0 cm apart. What is the net force acting on the sphere.

10. Three charges A (-8.0×10^{-8} C), B ($+5.0 \times 10^{-8}$ C) and C (-2.4×10^{-7} C) are placed as shown in the diagram. Calculate the magnitude of the resultant force on B due to A and C.



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39. Coulomb's Law

$$F = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q_1 q_2}{r^2}$$

$$F = \frac{k q_1 q_2}{r^2}$$

F = force (newtons) N

$k = 9.00 \times 10^9$ N m^2 C^{-2}

$= \dfrac{1}{4\pi\,\varepsilon_0}$ where $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

q_1 and q_2 = electric charges (coulombs) C

r = distance separating the charges (metres) m

Questions

1. Calculate the force between charges of 3.0 C and 4.0 C separated by a distance of 2.0 m.

2. Charges of 6.0×10^{-8} C and 2.7×10^{-8} C are situated 3.0×10^{-2} m apart. Calculate the force that would act between them.

3. Two charges repel each other with a force of 1.8×10^{-4} N. They have the values of 4.0×10^{-9} C and 8.0×10^{-9} C. Calculate the distance separating them.

4. A charge of 1.2×10^{-8} C is placed 5.0 cm from another charge and a force of 9.6×10^{-1} N acts between them. Calculate the value of the other charge.

5. Two identical charges positioned 2.0 cm apart repel each other with a force of 8.1 N. Calculate the size of each charge.

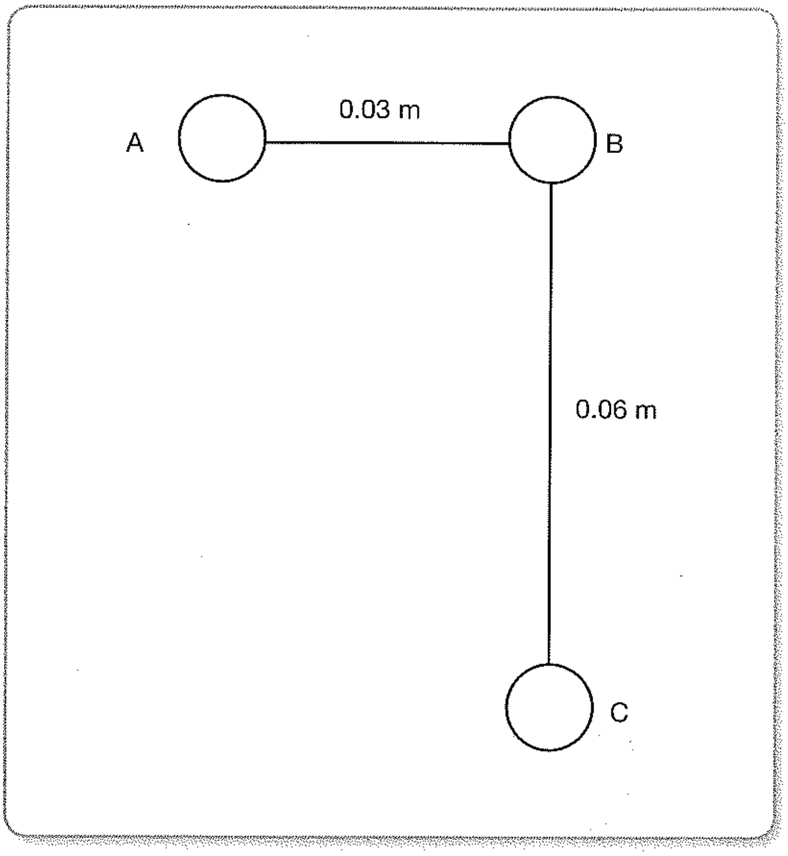
6. A small charged sphere A carries four times the charge of another sphere B. When these are placed 4.5 cm apart, they experience a force of 0.016 N attraction between them. Calculate the charge on B.

7. Two spheres, each of mass 5.0 g were charged with charges of 3.5×10^{-8} C and one was placed above the other so that the upper one was supported by the repulsion between it and the lower one. Calculate how far apart the two will be when the upper one is floating.

8. A sphere of mass 10.0 g was charged negatively. It was held, floating 2.5 cm above an object carrying a charge of -1.4×10^{-7} C. Calculate the charge that would be carried by the sphere.

9. A small sphere which carries a charge of $+2.4 \times 10^{-8}$ C is placed halfway between a charge of -3.6×10^{-8} C and another of $+4.8 \times 10^{-8}$ C which are 5.0 cm apart. What is the net force acting on the sphere.

10. Three charges A (-8.0×10^{-8} C), B ($+5.0 \times 10^{-8}$ C) and C (-2.4×10^{-7} C) are placed as shown in the diagram. Calculate the magnitude of the resultant force on B due to A and C.



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40. Electric Field

$$E = \frac{kq}{r^2}$$

$$F = Eq$$

E = electric field strength (newtons coulomb^{-1}) N C^{-1}

k = Coulomb constant = 9.00×10^9 N m^2 C^{-2}

q = charges (coulombs) C

r = separation of charges (metres) m

F = force (newtons) N

Questions

1. Calculate the magnitude of the electric field 0.05 m from a charge of 2.5×10^{-8} C.

2. The electric field 0.0060 cm from a charged object was 1.0×10^5 N C^{-1}. Calculate the size of the charge on the object.

3. Calculate the distance from an object carrying a charge of 4.50×10^{-8} C where the electric field strength would be 1.25×10^6 N C^{-1}.

4. In the diagram below, A has the charge 2.0×10^{-7} C and B has the charge -3.0×10^{-8} C. Calculate the resultant field at point X.

A^+ ------ 0.0030 m ---- X ----- 0.0020 m ------ B^-

5. In the diagram below, calculate the resultant field at X due to A ($+4.8 \times 10^{-8}$ C) and B (-3.0×10^{-9} C).

A^+ ------0.0030 m ---- B^- ----- 0.0010 m ---- X

6. Calculate the force that an object carrying a charge of 8.0×10^{-8} C would experience in an electric field with strength 4.0×10^7 N C^{-1}.

7. A charge of 3.6×10^{-8} C experienced a force of 1.08 N in an electric field. Calculate the magnitude of the field at that point.

8. An object of mass 0.0060 kg carries a charge of 3.5×10^{-9} C in a field of 4.0×10^9 N C^{-1}. Calculate the acceleration the object would experience if free to move.

9. Calculate how much work is done by an electric field of 1.25×10^9 N C^{-1} which moves a charge of 6.0×10^{-8} C a distance of 0.015 m.

10. Calculate the acceleration an electron of mass 9.1×10^{-31} kg and charge 1.6×10^{-19} C would experience in an electric field of magnitude 1.0×10^6 N C^{-1}.

41. Electric Potential and Potential Difference

$W = qV$	W = work (joules) J
$V = \dfrac{kq}{r}$	q = charge (coulombs) C
	v = potential difference (volts) V
$E = \dfrac{V}{d}$	k = Coulomb constant = 9.00×10^9 N m^2 C^{-2}
	r = distance (metres) m
$m_e = 9.1 \times 10^{-31}$ kg	E = electric field strength (newtons coulomb^{-1}) N C^{-1}
$e = 1.6 \times 10^{-19}$ C	d = distance between plates (metres) m

Questions

1. If it took 144.0 J of work to bring a charge of +18.0 C from infinity to a positively charged body, calculate the potential of the body.

2. Calculate the work that would be done in bringing a charge of 1.60×10^{-8} C from infinity up to a point with a potential of 320.0 kV.

3. Calculate the electric potential at a distance of 0.02 m from a charge of 4.0×10^{-8} C.

4. Calculate the distance from a charge of 6.4×10^{-7} C where the electric potential is 2.4×10^3 V.

5. Work of 1.32×10^{-6} J is done as a charge of 1.1×10^{-7} C moves around a circuit. Calculate the potential difference across the circuit.

6. A positive charge of 5.0 C moves from point A to point B acquiring 30.0 J of energy in doing so. If point A is at a potential of –24 V calculate the potential of B.

7. The parallel plates of a condenser are at a potential difference of 250 V. Calculate the work that would be done in taking a charge of $+3.0 \times 10^{-7}$ C from one plate to the other against the potential difference.

8. The parallel plates of a condenser are 0.0020 m apart and are at a potential difference of 100.0 V. Calculate the electric field between the plates.

9. The parallel plates of a condenser are at a potential difference of 1000 volts.
   Calculate:
   (a) The work done in taking an electron from one plate to the other.
   (b) The speed acquired by an electron in moving from the negative plate to the positive plate.

10. The potential difference between two parallel plates 0.0050 m apart is 150 volts. If an oil drop carrying three additional electrons enters the space between the plates, calculate the force applied by the field of the condenser on the oil drop.

42. Magnetic Induction

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \frac{kI}{r}$$

B = magnetic induction or field intensity or flux density (teslas) T

I = current (amperes) A

$k = \frac{\mu_0}{2\pi}$ = magnetic constant = 2.00×10^{-7} T A^{-1} m

r = distance (metres) m

Questions

1. Calculate the magnetic induction at a distance of 3.0 m from a long straight wire carrying a current of 48 A.

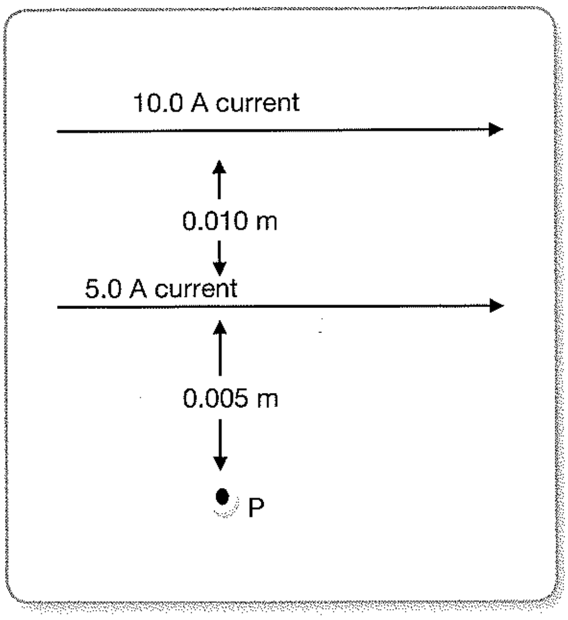
2. The magnetic induction at a distance of 3.0×10^{-2} m from a long straight conductor is 6.0×10^{-5} T. Calculate the current flowing in the conductor.

3. Calculate the distance from a conductor carrying a current of 36 A where one would experience a magnetic field of intensity 1.8×10^{-4} T.

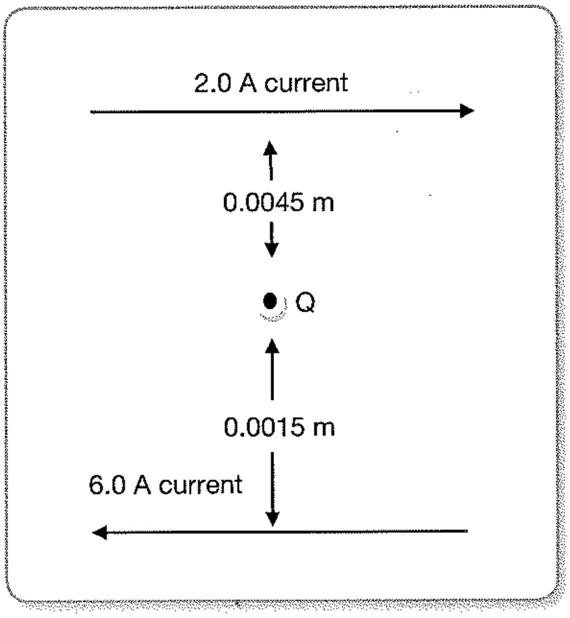
4. Two parallel straight wires 0.0060 m apart carry currents of 15 A and 18 A in opposite directions. Calculate the magnitude of the field intensity at a point midway between the wires.

5. Two long straight conductors carry currents as shown in the diagrams below. Calculate the magnitudes of the field intensities at points P and Q.

(a)



(b)



43. Ampere's Law

$$\frac{F}{L} = \frac{I_1 I_2}{d}$$

F = force (newtons) N

L = length of conductors which run parallel (metres) m

$k = \frac{\mu_o}{2p} = 2 \times 10^{-7}$ N A^{-2}

I_1 = current in one conductor (amperes) A

I_2 = current in other conductor (amperes) A

d = distance separating conductors (metres) m

Questions

1. Two parallel conductors 0.16 m apart are carrying currents of 24 A and 32 A. Calculate the force per unit length on each.

2. A wire carries a current of 12.0 A to a motor. The return wire, 2.00 mm away, carries the current back. If the wires are 2.0 m long calculate the total force of repulsion between the two wires.

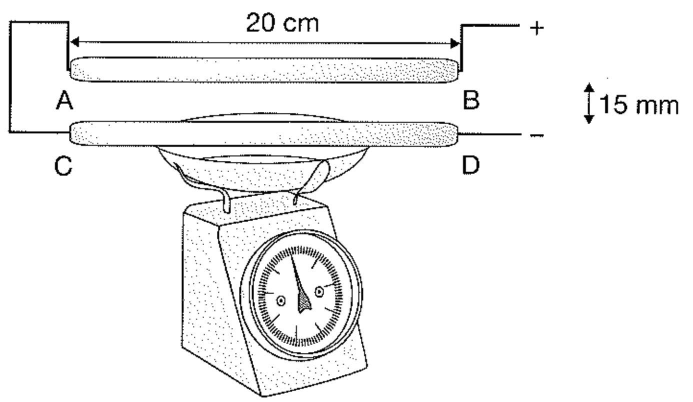
3. Two parallel conductors 2.5 m long and 4.0 mm apart repel each other with a force of 1.2×10^{-4} N. One conductor carries a current of 2.4 A. Calculate the current the other carries.

4. Calculate how far apart two parallel conductors 750 mm long, each carrying a current of 15 A, have to be for the force between them to be 4.5×10^{-3} N.

5. Two long conductors ran side by side 15 mm apart for part of their length. If the currents in the wires were 4.5 A and 3.6 A and the force between them was 1.8×10^{-3} N calculate the length running side by side.

6. The force per unit length between two conductors carrying identical currents 8.0 mm apart is 6.4×10^{-3} N m^{-1}. Calculate the size of the current.

7. Two parallel copper conductors joined together are situated as shown. The top conductor (AB) is fixed while the lower conductor (CD) lies across the pan of a balance. With no current flowing through the conductors, the balance shows a reading of 25.000 g for the conductor CD. When the current was switched on, the balance registered a mass of 25.010 g. Calculate the size of the current.



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44. Magnetic Forces on Currents

$F = BIL \sin\theta$	F = force (newtons) N	θ = angle between current and field
$F = qvB \sin\theta$	B = magnetic flux density (teslas) T	q = sixe of charge (coulombs) C
	I = current (amperes) A	v = velocity of charge (metres per second) m s^{-1}
	L = length of conductor (metres) m	e = charge on the electron = 1.6×10^{-19} C

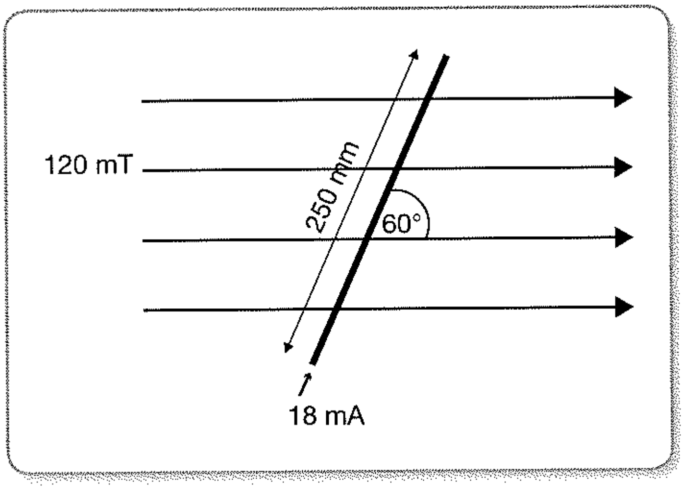
Questions

1. Calculate the force experienced by a conductor of length 5.0 m carrying a current of 8.0 A at right angles to a field of intensity 2.0 T.

2. A conductor 64 mm long, carrying a current of 2.5 A, experienced a force of 1.6 N when placed at right angles to a magnetic field. Calculate the flux density of the field.

3. A piece of wire carrying a current of 1.2 A at right angles to a magnetic field of 0.15 T experienced a force of 1.8 N. Calculate the length of the wire.

4. Calculate the force experienced by the conductor as shown in the diagram.



5. A conductor, 9.6×10^{-2} m long, carried a current of 8.0 A in a magnetic field of intensity 6.4×10^{-2} T. Calculate the angle to the field it would have to be inclined to experience a force of 3.2×10^{-2} N.

6. The mass of a piece of wire 4.9 m long is 8.4 g. It is placed at right angles to a magnetic field of flux density 3.5×10^{-2} T and a current passed through the wire. If the force on the wire due to the current's interaction with the magnetic field is just enough to balance its weight, calculate the value of the current which was needed.

7. If a charge of +16 C passed at 45 m s^{-1} at right angles through a vertical field of intensity 12 T, calculate the force that would act on it.

8. An oil drop carrying a charge of 3.6×10^{-8} C moved at 6.4×10^{-1} m s^{-1} through a field of 4.8×10^{-2} T inclined at 50° to the direction of motion of the drop. Calculate the force that would act on it.

9. An electron moving at 2.5×10^{6} m s^{-1} passed at right angles to a magnetic field of 4.0×10^{-1} T. Calculate the force that would act on it.

10. Calculate the speed at which an electron would have to move in a path inclined at 35° to a field of 7.2 T in order to experience a force of 1.3×10^{-10} N.

45. Torque on Coils

$\tau = Fd$

τ = torque (newton metres) N m

F = force (newtons) N

d = perpendicular distance from point of application of force to fulcrum (m)

$\tau = nBIA \cos\theta$

τ = torque (newton metres) N m

n = number of coils

B = magnetic field intensity (teslas) T

I = current (amperes) A

A = area of coil (square metres) m^2

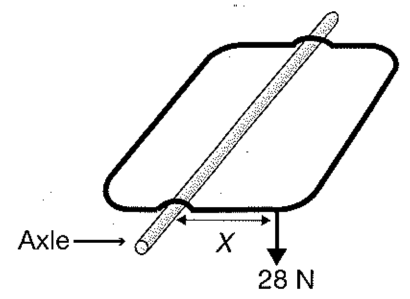
θ = inclination of coil to field (degrees)

Questions

1. A force of 7.5 N was applied to the edge of a disc 0.40 m in radius. Calculate the torque.

2. Calculate the force that one would need to apply to the rim of a wheel 5.4 m in radius to produce a torque of 36 N m.

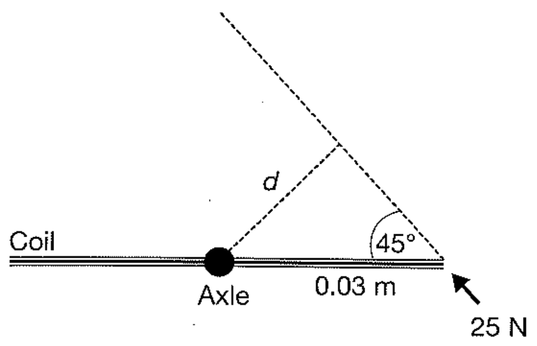
3. The diagram shows a rectangular coil pivoting on an axle. Calculate how far from the axle a force of 28 N would need to be applied to produce a torque of 42 N m.



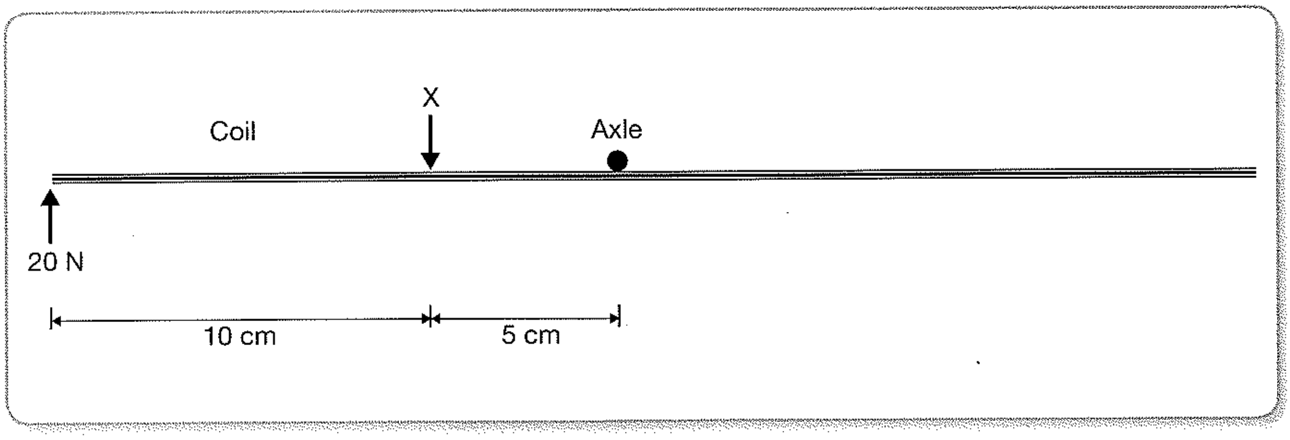
4. A force of 25 N is applied obliquely to a coil as shown in the diagram.

(a) Calculate the perpendicular distance between the line of operation of the force (d in the diagram) and the axle.

(b) Calculate the torque produced by the force.



5. Calculate the force that would need to be applied at point X in the diagram below to balance the force of 20 N.



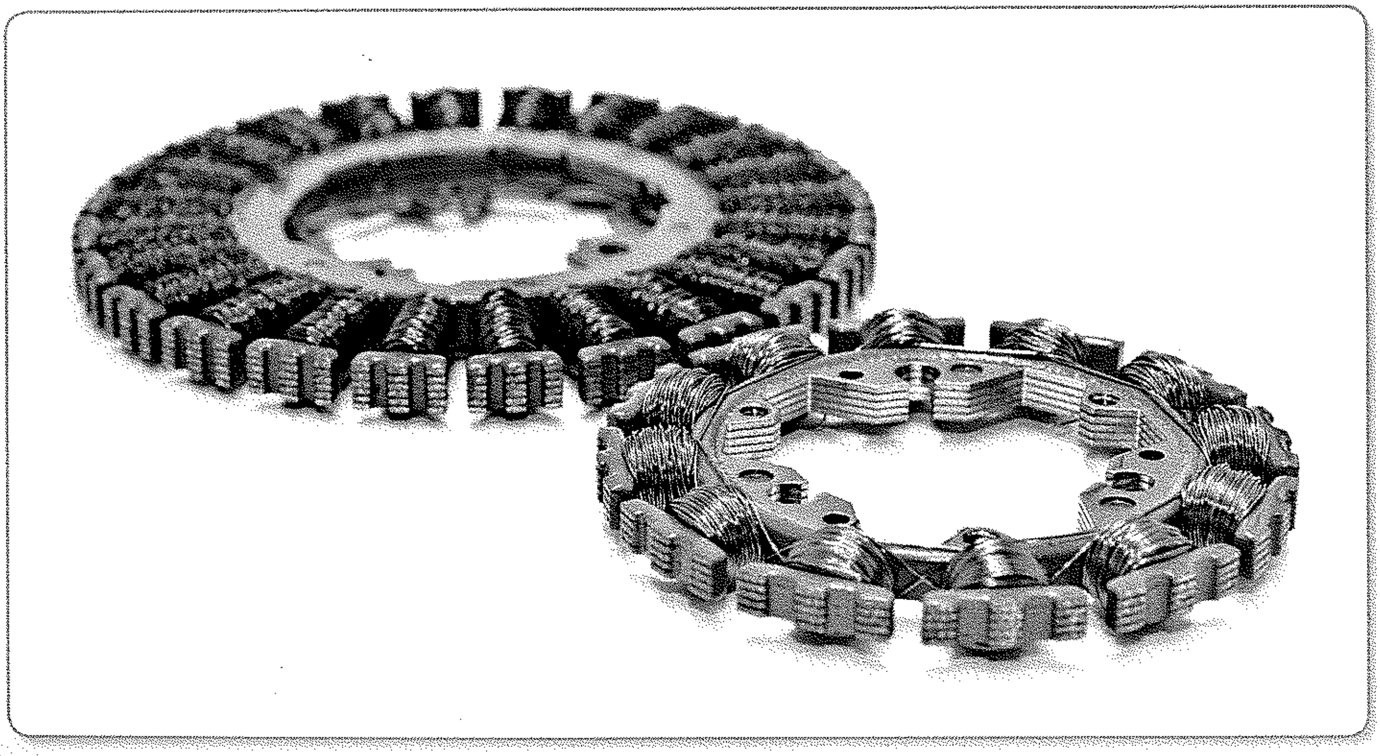
6. Calculate the torque on a coil of rectangular section 50 mm long, 20 mm wide, containing 10 turns and carrying a current of 25 mA if it is placed parallel to a field of 4 mT.

7. A torque of 1.5×10^{-1} N m was needed to hold a coil of 50 turns, cross-sectional area 1.2×10^{-3} m^2, carrying a current of 2.0 A at an angle of 60° to a magnetic field. Calculate the intensity of the field.

8. A circular coil of 250 turns has a radius of 75 mm. A couple of 0.50 N m is needed to maintain the plane of the coil parallel to a field of intensity 1.6 T. Calculate the current in the coil.

9. A circular coil of radius 0.10 m consists of 24 turns of wire. It is suspended in a magnetic field of flux density 0.36 T and a current of 5.0 A is passed through it. A couple of 1.2 N m is applied to the coil. Calculate at what angle the coil will be held.

10. The coil of an electric meter is rectangular in shape, 12 mm wide and 16 mm long and contains 60 turns. The intensity of the radial field supplied by the field magnets is 0.080 T. When a current is passed through the coil a torque of 0.0064 N m is set up. Calculate the value of the current passing through the coil.



46. Flux, Flux Density and Induced Emf

$\phi = BA$

$V_i = \frac{-n\Delta\phi}{\Delta t}$

$= BLv$

ϕ = flux (webers) Wb

B = flux density (teslas) T

A = area (m^2)

V_i = induced emf (volts) V

n = number of coils

$\Delta\phi$ = change in flux

Δt = change in time (s)

L = length of conductor (m)

v = velocity (m s^{-1})

Questions

1. Calculate the magnetic flux passing through a loop of wire of area 1.2 m^2 in a magnetic field of intensity 3.0 T.
2. 8.4 Wb of magnetic flux pass through a coil producing a flux density of 1.2 T. Calculate the area of the coil.
3. A magnetic flux of 6.4 Wb is built up from zero in a single coil of wire in 1.6 s. Calculate the induced emf.
4. A single coil of wire of area 2.4 m^2 was situated in a magnetic field of intensity 3.0 T when the magnetic field was reduced to zero in 2.5 s. Calculate the induced emf.
5. A coil of 240 turns of wire had a flux of 1.6 Wb passing at right angles through it. Calculate the emf that would be induced if this flux was to be completely reversed in 4.8 s.
6. If a coil of 150 turns of wire, having an area of 0.080 m^2 was to be rotated l80° from being at right angles to a field of intensity 0.20 T in 3,0 s, calculate the emf that would be induced.
7. A straight conductor 2.0 m long is moved at 3.0 m s^{-1}at right angles through a magnetic field of 5.0 T. Calculate the emf that would be induced.
8. A brass rod 0.075 m long passed at right angles through a magnetic field at a speed of 20.0 m s^{-1} producing an emf of 1.5 V. Calculate the intensity of the magnetic field.

47. Transformers

$$\frac{V_S}{V_P} = \frac{n_S}{n_P} = \frac{I_P}{I_S}$$

V_S = potential difference of secondary (output) coil (volts) V

V_P = potential difference of primary (input) coil (volts) V

n_S = number of turns in secondary coil

n_P = number of turns in primary coil

I_S = current in secondary coil (amperes) A

I_P = current in primary coil (amperes) A

In step-up transformers: $V_S > V_P$

In step-down transformers: $V_S < V_P$

Questions

1. A transformer has 1000 turns in its secondary wiring and 250 in its primary wiring. If a potential difference of 80 V is applied to the primary, calculate its output voltage.

2. The output of a transformer with 600 turns in its secondary wiring is 120 V. If the primary coil has 450 turns, calculate the input voltage.

3. A 10 to 1 step-up transformer has 500 turns in its primary coil. Calculate the number of turns in its secondary coil.

4. A step-down transformer has 4000 turns in its primary coil and 2500 turns in its secondary. If the current in the primary is 5 A, calculate the expected current in its secondary coil.

5. In a transformer designed to convert 32 V to 240 V, the input current is 36 A. Calculate the output current.

6. The output current from the secondary coil of a transformer having 600 turns in its wiring is 20 A. If the input current was 4 A, calculate the number of turns in the primary coil.

7. A certain transformer has an input of 0.25 A at 240 v. If its output voltage is 12 V, calculate its output current.

8. A 1 to 3 step-down transformer produces a current of 15 A. Calculate its input current.

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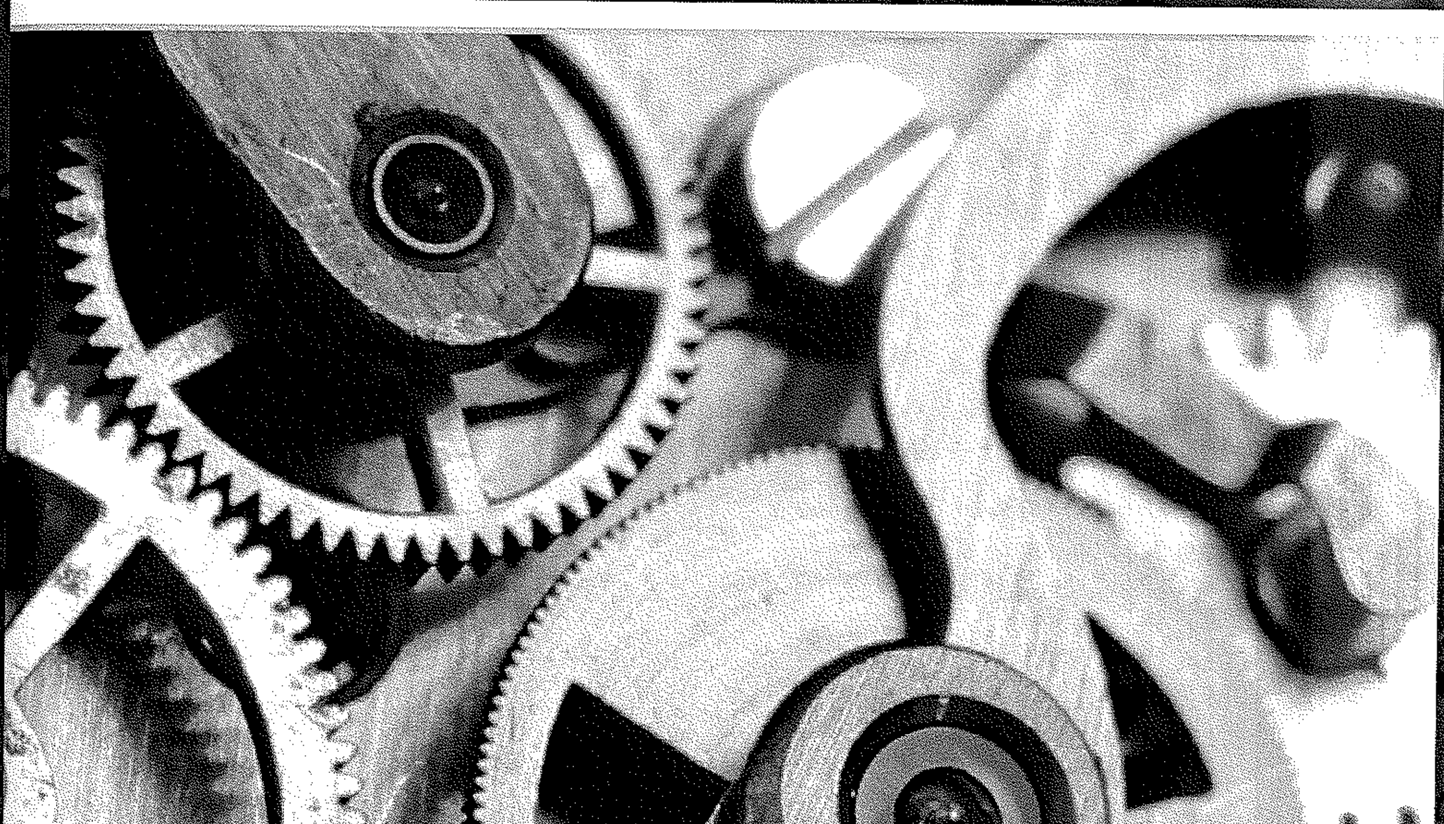
Revolutions in Modern Physics



48. Relativity

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} \qquad t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad p_v = \frac{mv}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$E = mc^2$

L = moving length (m)

L_0 = rest length (m)

v = velocity of moving object (m s^{-1})

c = velocity of light = 3×10^8 m s^{-1}

t = moving time (s)

t_0 = rest time (s)

m = moving mass (kg)

m_0 = rest mass (kg)

E = energy (J)

p_v = relativistic momentum

Questions

1. A spaceship which had a length of 12 m according to the pilot flew past an observer on the Earth at a speed of 0.8 *c*. The observer took a very fast photo and measured the length of the ship. Calculate their measurement.

2. While travelling at this speed (0.8 *c*) the pilot started his stopwatch and stopped it after counting off 2.0 seconds. If the observers on Earth started and stopped their watches as they saw the watch on the spaceship start and stop, calculate the time they would have counted off.

3. The rest mass of an electron is 9.1×10^{-31} kg. Calculate the mass it would be if it was to be measured by a stationary observer if it was moving at a speed of 0.9 *c*.

4. A certain subatomic particle had a rest mass of 1.67×10^{-27} kg. Having been speeded up by a linear accelerator, its mass was recorded as 2.5×10^{-27} kg. Calculate the velocity of the particle then.

5. Calculate the speed that a particle must travel so that its relativistic mass is 10 times its rest mass.

6. A spaceship made a journey to a star 10.0 light years away taking 12 years to do so as measured by observers on Earth. Calculate the time by which the pilot aged during the journey as measured by the pilot.

7. In a nuclear reactor, 16.0 g of fuel was converted to energy. Calculate how much energy was produced.

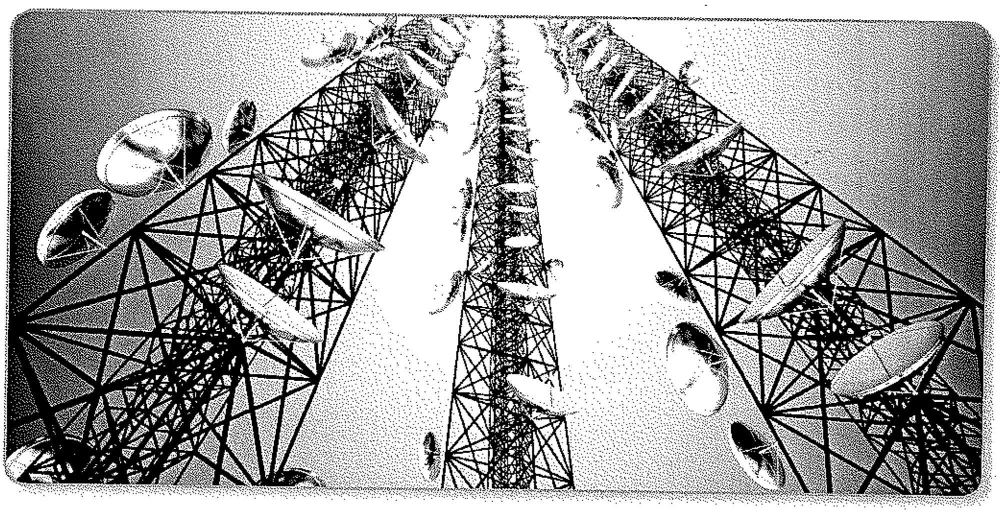
8. Calculate how much fuel would need to be converted to energy in order to produce 1.0 MJ of energy.

49. Photons and Quanta

$E = hf$	E = energy of photon or quantum (joules) J
	h = Planck's constant = 6.6×10^{-34} J s
$c = f\lambda$	f = frequency (hertz) Hz
	c = speed of light = 3.00×10^{8} m s^{-1}
	λ = wavelength (metres) m
	m_e = mass of electron = 9.12×10^{-31} kg

Questions

1. Calculate the energy of a photon of ultraviolet light of frequency 1.4×10^{15} Hz.
2. Calculate the energy of a photon of yellow light of wavelength 6.0×10^{-7} m.
3. Calculate how many quanta of red light of wavelength 7.5×10^{-7} m would be needed to make up one joule of energy.
4. The quanta of a certain radio station have an energy of 7.7×10^{-28} J. Calculate the frequency of the radio station.
5. A light globe emits energy at the rate of 60 watts. If the average frequency of the light emitted is 5.0×10^{14} Hz, calculate how many photons are being emitted per second.
6. Ultraviolet rays of frequency 8.4×10^{15} Hz strike a zinc plate ejecting electrons. If all the energy of each photon goes into accelerating one electron, calculate the velocity of the electron.
7. If energy of 2.5×10^{-18} J is needed to remove one electron from a certain atom, calculate the longest wavelength of light required to just remove electrons from a sheet made from these atoms.
8. The quanta of certain microwaves have an energy of 9.25×10^{-22} J. Calculate their wavelength.



50. Wien's Displacement Law

$$\lambda_{max} = \frac{b}{T}$$

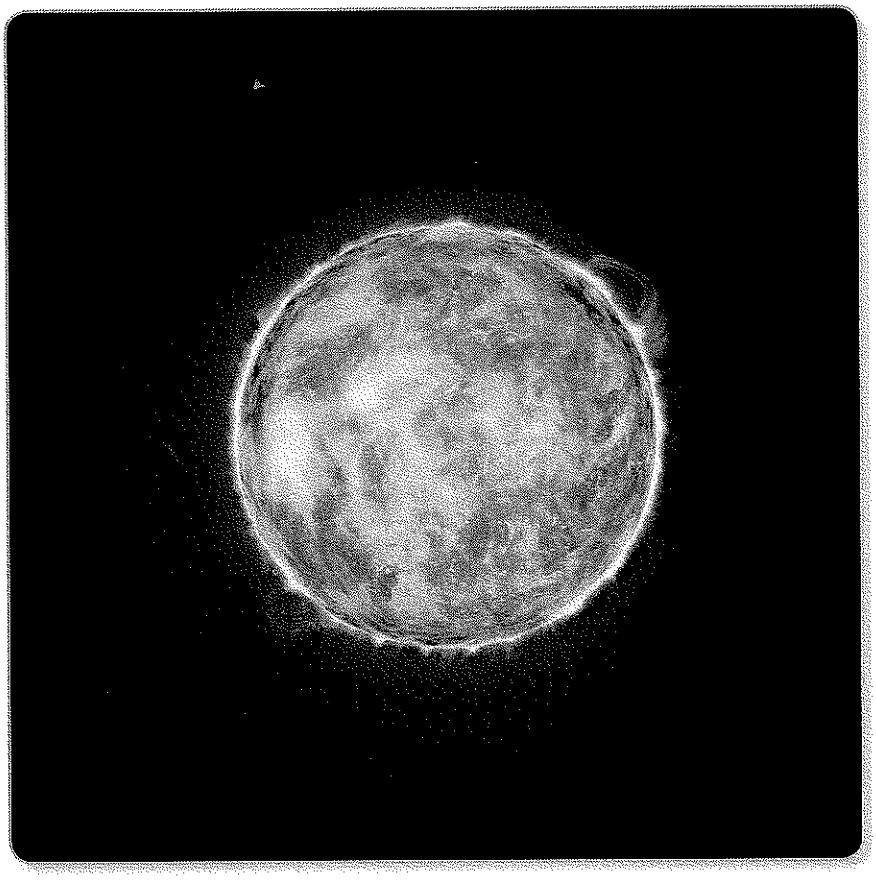
λ_{max} = peak wavelngth of emitted radiation in metres (m)

b = Wien's displacement constant = 2.9×10^{-3} m K

T = temperatures in kelvins (K)

Questions

1. Antares has a temperature of 3100 K. Calculate the peak wavelength of the light emitted by this star.
2. Human skin has a temperature of approximately 32°C. Calculate the peak wavelength of the radiation emitted by human skin.
3. The peak wavelength of the light emitted by the Sun is 5.02×10^{-7} m. Calculate the temperature of the outer atmosphere of the Sun.



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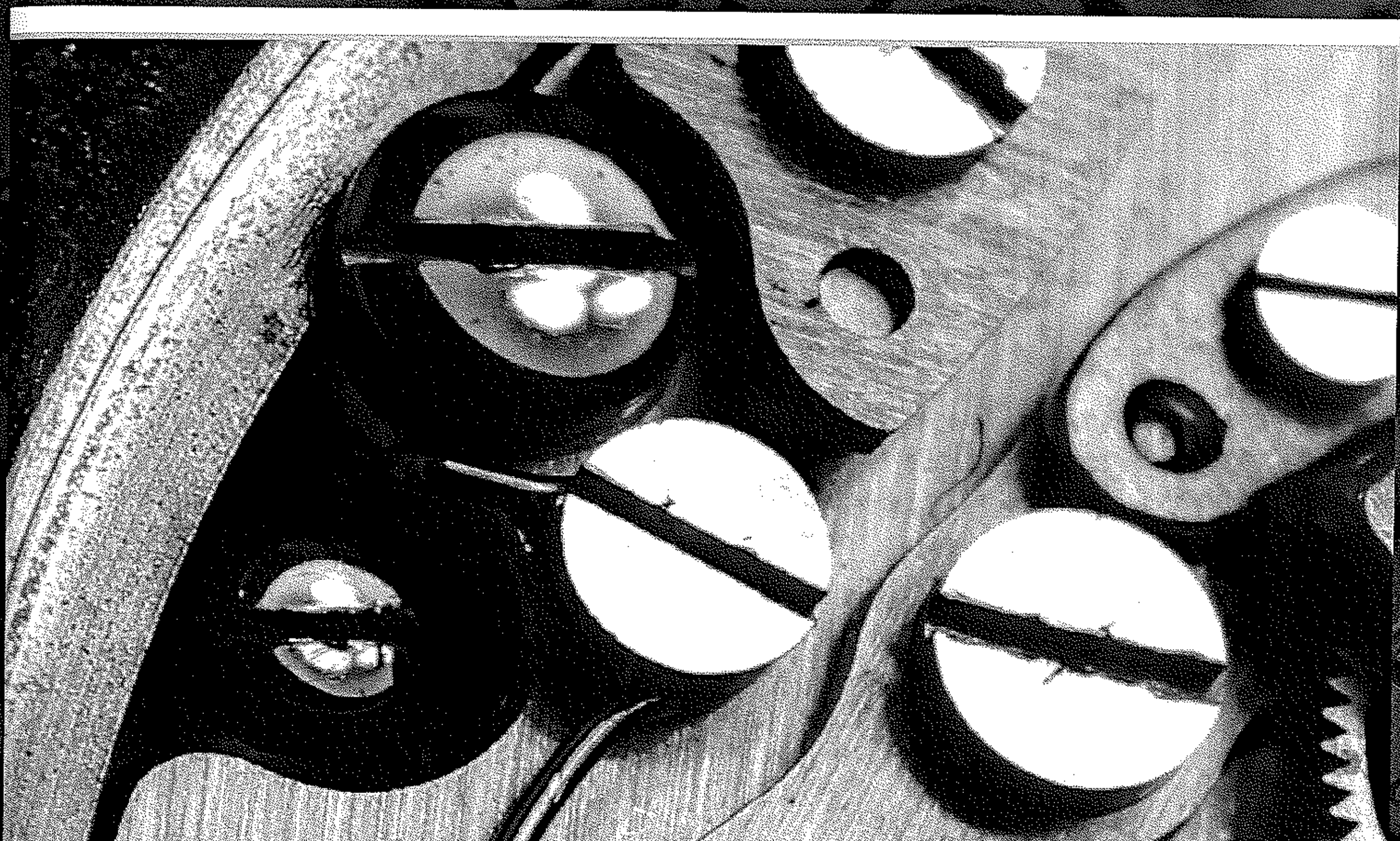
Extension Questions



51. Extension Questions

1. Waves with a period of 0.05 s had a wavelength of 24.3 m. Calculate their velocity.

2. How will the intensity of the light falling on a surface be affected if the source of the light is moved to three times its original distance?

3. A train of straight waves of frequency 5.0 Hz travels across the surface of deep water at 15.0 cm s^{-1} and crosses into a region of shallow water where the speed is 10.0 cm s^{-1}.
Calculate:

(a) The wavelength of the waves in the deep water.

(b) The frequency of the waves in the shallow water.

(c) The wavelength of the waves in the shallow water.

(d) The refractive index of the shallow water relative to the deep water.

4. C above middle C on the piano has a frequency of 512 Hz. Calculate the wavelength of the soundwaves of this frequency in air (speed of sound in air = 330 m s^{-1}).

5. The intensity of the light on a screen was reduced to one ninth of its original value by moving the light source away from its original position. If the light was originally 1.5 m from the book, calculate its new distance away from the screen.

6. Radio waves from a popular radio station have a frequency of 702 kHz. If these waves travel at the speed of light, calculate their wavelength.

7. When light was incident at 10.0° onto a mineral, the angle of refraction was 9.0°. Calculate the refractive index of the mineral.

8. Light from a spot lamp falls onto a sheet of cardboard, 5.00 m from the lamp at the rate of 1200 W m^{-2}. Calculate the intensity of light falling on the sheet if the spot lamp was moved to be 7.5 m away.

9. The angle of refraction of a ray of yellow light having passed from air into benzene was 12° (refractive index of benzene = 1.504). Calculate the angle of incidence.

10. Given that the refractive index of yellow light (wavelength 5.893×10^{-7} m) passing from air to benzene is 1.504 and that the speed of light in air is approximately 3.0×10^{8} m s^{-1}, calculate the speed of this light in benzene.

11. A charge of 3.5 C passes a point in a circuit in 7.0 s. Calculate the current.

12. A charge of 1.44×10^{-7} C experienced a force of 1.2×10^{-2} N in an electric field. Calculate the magnitude of the field at that point.

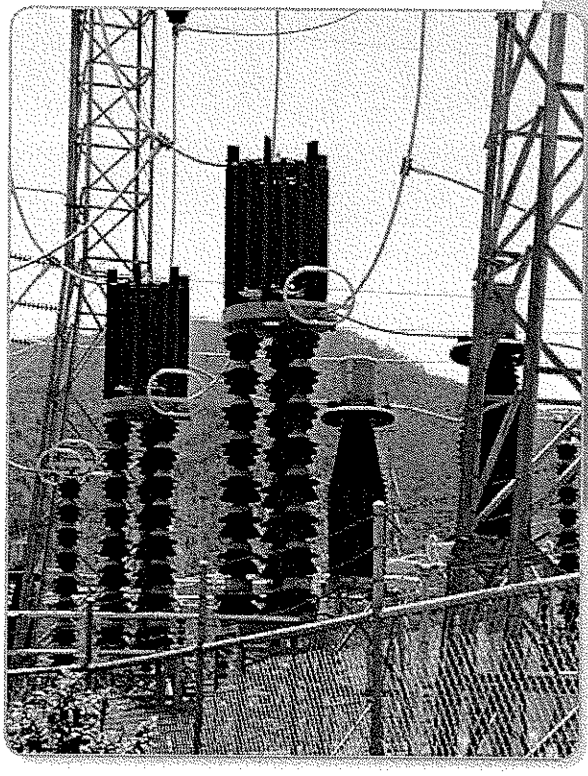
13. A 6 V lamp uses a current of 1.2 A. Calculate the power of the lamp.

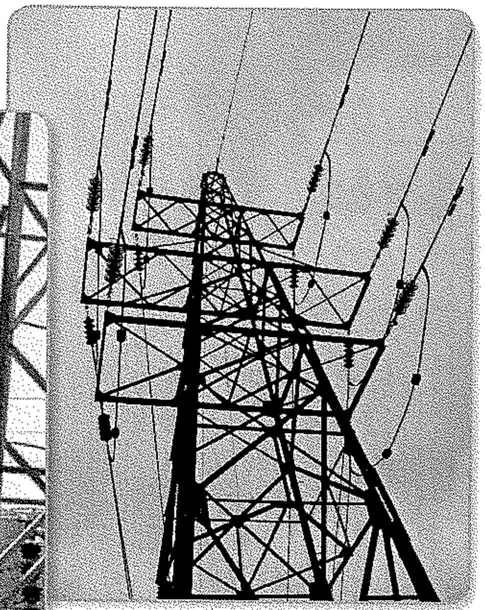
14. A resistor is designed to carry 10.0 A at 6.0 V. Calculate its resistance.

15. Calculate the energy that would be transformed by an appliance operating on 32 V, carrying a current of 2.5 A for a period of 10 minutes.

16. A current of 18.0 A flowed in a circuit for 27 s. Calculate the charge that passed through the circuit.

17. An electric frypan is designed to produce 57 600 J of heat energy in a minute. If the frypan operates on 240 V, calculate the current it is designed to carry.





18. A jug element has a power of 1800 W and runs on 240 V. Calculate the current it consumes.

19. The element of an electric toaster has a resistance of 64.0 ohms. If 240 V is applied, calculate the current it is designed to carry.

20. Calculate the force a charge of 4.0×10^{-9} C experiences in a field of strength 6.0×10^{9} N C^{-1}.

21. A car has a velocity of 35 m s^{-1} east. Calculate how far it will travel in 5 minutes.

22. A boat moving at 15.0 m s^{-1} accelerated at 10 m s^{-2} for 5 seconds. Calculate its new velocity.

23. Calculate the force needed to make an object of mass 4.8 kg accelerate at 2.6 m s^{-2}.

24. A child weighs 150 N. Calculate the child's mass.

25. A car of mass 850 kg has a momentum of 17 000 kg m s^{-1} north. Calculate the velocity of the car.

26. Calculate the size of the force needed to apply an impulse of 850 N s over a period of 0.050 s.

27. A force of 6.4 N moves an object 5.5 m. Calculate the work done.

28. Calculate the kinetic energy of a body of mass 12.5 kg moving at 1.5 m s^{-1}.

29. The kinetic energy of a ball moving at 30 m s^{-1} was 120.0 J. Calculate the mass of the ball.

30. An 800 kg car moving at 60 km h^{-1} is brought to rest in 10 m. Calculate the work done by the brakes in stopping the car.

31. A rocket can accelerate at 5.0 m s^{-2}. At this acceleration, calculate the time taken for the rocket to accelerate from 14.5 m s^{-1} to 45.6 m s^{-1}.

32. At an acceleration of 5.0 m s^{-2} calculate the rocket's speed after it had travelled 200 m from a standing start.

33. A spacecraft, initially travelling at 21.4 m s^{-1} accelerated at 1.6 m s^{-2} for 10 seconds. Calculate how far it travelled in that time.

34. A rocket fired vertically rose to a height of 300 m. Calculate at what velocity it was fired.

35. A rocket started on its way at 500 m s^{-1} at an angle of 30.0° from the horizontal. Calculate the greatest height to which the rocket would rise.

36. A force of 150 N was needed to keep an object of mass 5.0 kg travelling in a circle 20 m in diameter. Calculate the speed of the object.

37. Calculate the force of attraction between a mass of 3.00×10^3 kg and another of mass 7.00×10^3 kg whose distance apart, centre to centre is 5.00 m.

38. A rocket ship which had a length of 50 m according to the pilot flew past an observer on the Earth at a speed of 0.9 *c*. The observer took a very fast photo and measured the length of the ship. Calculate their measurement.

39. Calculate the mass of fuel that would be needed to be converted to energy in order to produce 25 MJ of energy.

40. Calculate the gravitational potential energy possessed by a body of mass 15 kg on the Earth's surface (mass of the Earth = 5.98×10^{24} kg, radius = 6.38×10^6 m).

41. Two conductors, running parallel to each other, 5 mm apart carry currents of 5 A and 10 A. Calculate the force per metre acting on each conductor.

42. The force acting on each of two parallel conductors 20 mm apart, both carrying a current of 5.0 A was 1.5×10^{-3} N. Calculate the length of the conductors which ran side by side.

43. Calculate the force experienced by a conductor 50 mm long carrying a current of 25 mA at an angle of 60° to a field of intensity 100 mT.

44. A particle carrying a charge of 2.5×10^{-15} C passed through a field of 5.0×10^{-2} T inclined at 45° to the direction of the particle's travel. If the speed of the particle was 1.5×10^8 m s^{-1}, calculate the force that would act on it.

45. Calculate the force that would be needed to apply a torque of 40 N m to a coil at a point 50 cm from its axle if the force acted at right angles to the coil.

46. Calculate the torque that would be experienced by a circular coil, 25 mm in diameter containing 500 turns of wiring and carrying a current of 100 mA if it was inclined at 45° to a field of 15 mT.

47. A transformer has 3000 turns of wire in its secondary coil and 1000 turns in its primary. If a potential difference of 12 V is applied to the primary coil, calculate the output potential difference.

48. A 5 : 1 step-up transformer has 750 turns in its secondary coil. Calculate the number of turns in its primary coil.

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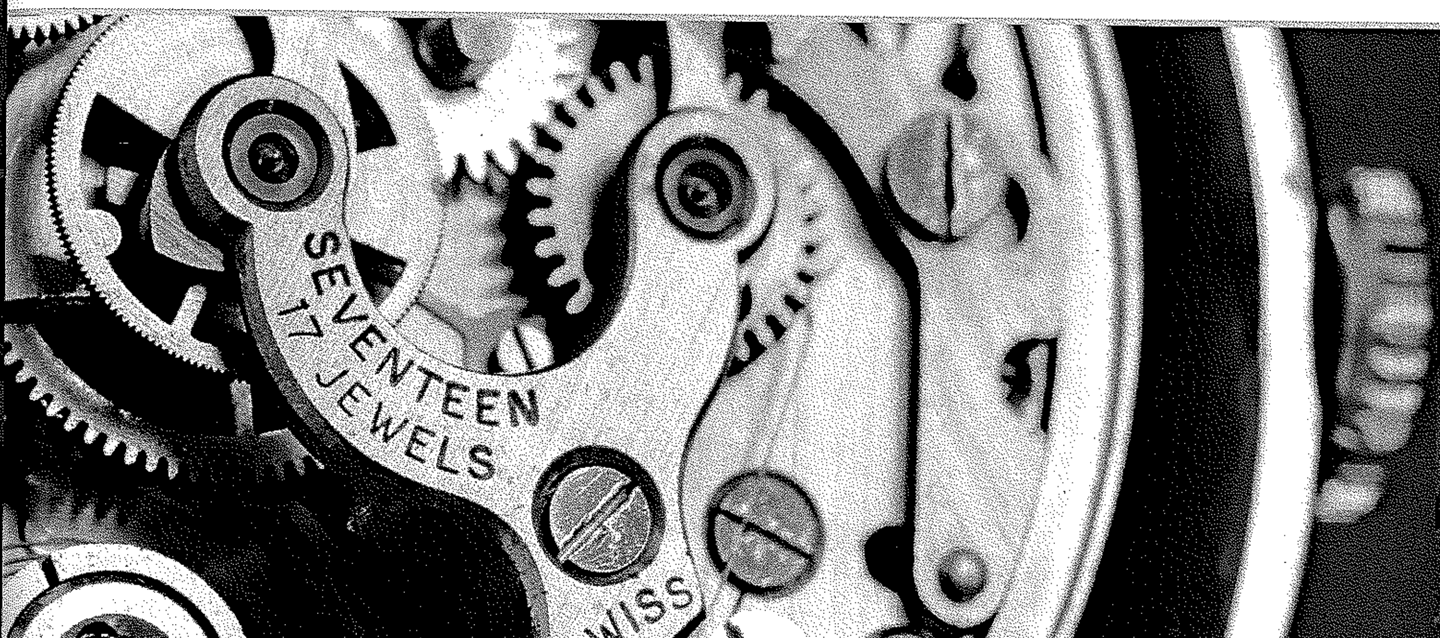
Answers



1 Physics Basics

1. $1 \div 10^{-3} = 10^3$
2. $1 \times 10^3 \div 10^{-2} = 10^5$
3. $1 \times 10^{-3} \div 10^{-6} = 10^3$
4. $1 \times 10^{-3} \div 1 = 10^{-3}$
5. $1 \times 10^3 \div 10^6 = 10^{-3}$
6. $1 \times 10^{-6} \div 10^3 = 10^{-9}$
7. $1 \times 10^{-3} \div 10^{-9} = 10^6$
8. $1 \times 100^3 \div 10^9 = 10^{-6}$
9. $1 \times 10^{-2} \div 10^{-12} = 10^{10}$
10. $1 \times 10^{12} \div 10^{-18} = 10^{30}$
11. $7505.23 \approx 7500.00 = 7.5 \times 10^3$
12. $0.003425 \approx 0.0034 = 3.4 \times 10^{-3}$
13. $70.2 \times 34.2 = 2400.84 \approx 2400 = 2.4 \times 10^3$
14. $1578.3 + 0.400 = 1578.700 \approx 1600 = 1.6 \times 10^3$
15. $\dfrac{0.600 \times 7020.0960}{0.0960} = 43875\ 6 \approx 44000 = 4.4 \times 10^4$
16. $\dfrac{1.53}{42.89 \times 8.4} = 4.246744163 \times 10^{-3} = 4.2 \times 10^{-3}$
17. $\dfrac{0.8235 \times 445\ 7}{76.9 \times 0.00930} = 513.21217 \approx 510 = 5.1 \times 10^2$
18. $20.4 \times 10^{-7} = 2.04 \times 10^1 \times 10^{-7} \approx 2.0 \times 10^{-6}$
19. $134 \times 10^8 = 1.34 \times 10^2 \times 10^8 \approx 1.3 \times 10^{10}$
20. $\dfrac{(2.4 \times 10^{-3}) \times (5.6 \times 10)^4}{(3.0 \times 10^7)^4} = 1.659259259 \times 10^{-28} \approx 1.7 \times 10^{-28}$
21. $8.2 \times 10^{-4} \approx 10 \times 10^{-4} = 10^{-3}$
22. $3.5 \times 10^7 \approx 1 \times 10^7 = 10^7$
23. $73.6 = 7.36 \times 10^1 \approx 10 \times 10^1 = 10^2$
24. $0.00901 \approx 0.010 = 10^{-2}$
25. $863.7 \times 10^{-5} = 8.673 \times 10^2 \times 10^{-5} \approx 10 \times 10^2 \times 10^{-5} = 10^{-2}$
26. $0.01673 \times 10^6 = 1.673 \times 10^{-2} \times 10^6 \approx 1 \times 10^4 = 10^4$
27. $3.5 \times 10^6 \times 9.2 \times 10^{-4} = 3220 = 3.220 \times 10^3 \approx 1 \times 10^3 = 10^3$
28. $\dfrac{7.6 \times 10^{-5}}{1.9 \times 10^4} = 4.0 \times 10^{-9} \approx 1 \times 10^{-9} = 10^{-9}$
29. $24 \times 60 \times 60 = 86400 = 8.64 \times 10^4 \approx 10 \times 10^4 = 10^5$
30. $365.25 \times 24 = 5766 = 8.766 \times 10^3 = 10 \times 10^3 = 10^4$
31. $4.5 \div 10^{-3} = 4.5 \times 10^3$
32. $276 \div 10^{-9} = 2.76 \times 10^{11}$
33. $0.359 \times 10^{-3} \div 10^3 = 3.59 \times 10^{-7}$
34. (a) $0.0038 = 3.8 \times 10^{-3}$

    (b) $2400.84 \approx 2400 = 2.4 \times 10^3$

    (c) $0.018303843 \approx 0.018 = 1.8 \times 10^{-2}$
35. (a) $7.4 \times 10^{-5} \approx 10 \times 10^{-5} = 10^{-4}$

    (b) $2.4 \times 10^3 \times 1.8 \times 10^{-5} = 0.0432$
    $= 4.32 \times 10^{-2} \approx 1.0 \times 10^{-2} = 10^{-2}$

    (c) $\dfrac{8.64 \times 10^{-7}}{2.95 \times 10^3} = 2.9 \times 10^{-10} \approx 1.0 \times 10^{-10} = 10^{-10}$

2 Thermal Physics

1. $Q = mc\Delta T$
   $= 0.500 \times 4.18 \times 35$
   $= 73.15$ kJ
2. $Q = mc\Delta T$
   $m = \dfrac{Q}{c\Delta T}$
   $= \dfrac{0.650}{0.900 \times (45 - 20)}$
   $= 0.029$ kg
3. $Q = mc\Delta T$
   $c = \dfrac{Q}{m\Delta T}$
   $= \dfrac{2.34}{0.200 \times (45 - 15)}$
   $= 0.39$ kJ kg^{-1} K^{-1}
4. $Q = mc\Delta T$
   For the porcelain:
   $Q_1 = 0.250 \times 0.840 \times (100 - 20)$
   $= 16.80$ kJ
   And for the water:
   $Q_2 = 0.250 \times 4.18 \times (100 - 20)$
   $= 83.60$ kJ
   Total heat required:
   $= 16.80 + 83.60$
   $= 100.4$ kJ

5. $Q = mc\Delta T$

$$\Delta T = \frac{Q}{mc}$$
$$= \frac{6.00}{0.480 \times 1.42}$$
$$= 8.8^\circ C$$

Initial temperature = 7.50 + 8.8
= 16.3°C

6. $Q = mc\Delta T$

$$= 0.750 \times 0.129 \times (200 - 25)$$
$$= 16.93$$

$$\text{Total heat needed} = \frac{100}{75} \times 16.93$$
$$= 22.57 \text{ kJ}$$

7. $Q = mL$

$$= 1.2 \times 22.4$$
$$= 26.9 \text{ kJ}$$

8. $Q = mL$

$$m = \frac{Q}{L}$$
$$= \frac{1.500}{885}$$
$$= 1.7 \times 10^{-3} \text{kg}$$

9. $Q = mL$

$$= 1.800 \times 25.7$$
$$= 46.26$$

$$\text{Total heat required} = \frac{100}{20} \times 46.26$$
$$= 231.3 \text{ kJ}$$

10. $Q_1 = mL_1$

$$= 0.200 \times 334$$
$$= 66.8$$

$Q_2 = mc\Delta T$
$$= 0.200 \times 4.18 \times 100$$
$$= 83.6$$

$Q_3 = mL_2$
$$= 0.200 \times 2260$$
$$= 452.0$$

$$\text{Total heat needed} = Q_1 + Q_2 + Q_3$$
$$= 66.8 + 83.6 + 452$$
$$= 602.4 \text{ kJ}$$

3 Radioactive Decay

1. Uranium atoms have 92 protons. When they emit beta particles, the number of protons increases by 1.{92–(–1) = 93} The remaining element has 93 protons and so is neptunium-239.
2. Nitrogen atoms have 7 protons. Carbon atoms have 6 protons. The particles emitted must have been positrons.{7–(+1) = 6}
3. Disintegration involves the production of nitrogen atoms which have 7 protons in their nuclei together with positrons. The original element must have been oxygen with 8 protons in their nuclei. {8 = 7+(+1)}
4. Radioactive phosphorus atoms have 15 protons in their nuclei. When they emit beta particles, the number of protons in each atom increases by 1. {15–(–1) = 16} The remaining element is therefore sulfur.
5. Radioactive phosphorus atoms have 15 protons in their nuclei whereas silicon atoms have 14 protons. The particles emitted must have been positrons {15–(+1) = 14}.
6. 420 days are 3 half-lives.

$$N = N_o\left(\tfrac{1}{2}\right)^n$$
$$= 1.000\left(\tfrac{1}{2}\right)^3$$
$$= 0.125 \text{ g}$$

7. $N = N_o\left(\tfrac{1}{2}\right)^n$

$$\frac{N}{N_o} = \left(\tfrac{1}{2}\right)^n$$
$$\frac{1}{32} = \left(\tfrac{1}{2}\right)^n$$
$$n = 5$$

Time taken is 5 half-lives
= 5 × 5 = 25 days

8. $N = N_o\left(\tfrac{1}{2}\right)^n$

$$\frac{N}{N_o} = \left(\tfrac{1}{2}\right)^n$$
$$\frac{1}{16} = \left(\tfrac{1}{2}\right)^4$$

Thus 12.20 minutes = 4 half-lives.
So the half-life is 3.05 minutes.

9. Note that 1 hour = 60 × 60 = 3600 s
No. of half-lives = 3600/30 = 120

$$N = N_o\left(\tfrac{1}{2}\right)^n$$
$$= 1\left(\tfrac{1}{2}\right)^{120}$$
$$= 7.523 \times 10^{-37} \mu g$$

4 Nuclear Reactions

1.	${}^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{13}_{6}C$	Carbon-13
2.	${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow 2\,{}^{4}_{2}He$	Alpha
3.	${}^{32}_{16}S + {}^{1}_{0}n \rightarrow {}^{32}_{15}P + {}^{1}_{1}H$	Phosphorus-32
4.	${}^{9}_{4}Be + {}^{0}_{-1}e \rightarrow {}^{9}_{3}Li$	Lithium-9
5.	${}^{15}_{7}N + {}^{1}_{1}H \rightarrow {}^{12}_{6}C + {}^{4}_{2}He$	Proton

6. $^{31}_{15}P + ^{2}_{1}H \rightarrow ^{32}_{15}P + ^{1}_{1}H$ Deuteron

7. $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{148}_{57}La + 3^{1}_{0}n + ^{85}_{35}Br$ Bromine-85

8. $^{64}_{29}Cu \rightarrow ^{0}_{+1}e + ^{64}_{28}Ni$ Nickel-64

9. $^{12}_{6}C + ^{1}_{1}H \rightarrow ^{13}_{7}N$ Nitrogen-13 and carbon-13

$^{13}_{7}N \rightarrow ^{0}_{+1}e + ^{13}_{6}C$

10. $^{200}_{80}Hg + ^{0}_{-1}e \rightarrow ^{200}_{79}Au$

5 Mass and Energy

1. $\Delta E = \Delta mc^2$

$= 1\times10^{-9}\times(3\times10^8)^2$

$= 9\times10^7$ kJ

2. $\Delta E = \Delta mc^2$

$\Delta m = \dfrac{\Delta E}{c^2}$

$= \dfrac{1}{(3\times10^8)^2}$

$= 1.1\times10^{-17}$ kg

3. $\Delta E = \Delta mc^2$

$\Delta m = \dfrac{\Delta E}{c^2}$

$= \dfrac{1}{(3\times10^8)^2}$

$= 1.1\times10^{-17}$ kg

So loss of mass in one second is 1.1×10^{-17} kg and number of seconds to reduce mass by 1 kg

$= \dfrac{1}{1.1\times10^{-17}}$

$= 9.1\times10^{16}$ seconds

4. $\Delta E = \Delta mc^2$

$= 0.001 \times (3.0 \times 10^8)^2$

$= 9.0 \times 10^{13}$ J

The number of joules being consumed by the light globe per second is 100 000.

So the number of seconds is:

$= \dfrac{9.00 \times 10^{13}}{100\ 000}$

$= 9.0 \times 10^8$ seconds

5. Amount of energy to bring water to the boil:

$Q_1 = mc\Delta T$

$= 1000 \times 4.18 \times (100 - 20)$

$= 334\ 400$ J

Amount of energy to boil the water

$Q_2 = 1000 \times 2260$

$= 2\ 260\ 000$ J

Total energy $= Q_1 + Q_2$

$= 2\ 594\ 400$ J

$E = mc^2$

$m = \dfrac{E}{c^2}$

$= \dfrac{2\ 594\ 400}{(3\times10^8)^2}$

$= 2.88\times10^{-11}$ kg

6 Charge and Current

1 $I = \dfrac{\Delta q}{t} = \dfrac{7.5}{0.25} = 30.0$ A

2. $\Delta q = It = 9.6 \times 45 = 432$ C

3. $t = \dfrac{\Delta q}{I} = \dfrac{560}{1.4} = 400$ s

4. $\Delta q = It = 25 \times 10 \times 60 = 15\ 000$ C

5. $\Delta q = It = 8.0 \times 1 = 8$ C

No. of electrons $= 8 \div (1.6 \times 10^{-19}) = 5 \times 10^{19}$

6. Charge $= 1.5\times10^{20}\times1.6\times10^{-19} = 24$ C

$I = \dfrac{\Delta q}{t} = \dfrac{24}{1} = 24$ A

7. $\Delta q = It = 2.5 \times 6.0 \times 60 \times 60 = 54\ 000$ C

8. $\Delta q = It = 8 \times 10^{-3} \times 5 \times 60 = 2.4$ C

No. of electrons $= 2.4 \div 1.6 \times 10^{-19} = 1.5 \times 10^{19}$

7 Work, Charge and Electric Potential

1. $W = qV$

$= 1.6\times10^{-8}\times320\times1000$

$= 5.12\times10^{-3}$ J

2. $W = qV$

$V = \dfrac{W}{q}$

$= \dfrac{144}{18}$

$= 8$ V

3. $W = qV$

$V = \dfrac{W}{q}$

$= \dfrac{13.2}{1.1}$

$= 12.0$ V

4. $W = qV$

$V = \dfrac{W}{q}$

$= \dfrac{30.0}{5.0}$

$= 6.0\ \text{V}$

Potential difference = 6.0 V
Thus potential = 12.0 + 6.0 = 18.0 V

5. $W = qV$

$= 3.0 \times 10^{-7} \times 250$

$= 7.5 \times 10^{-5}\ \text{J}$

6. $W = qV$

$= 1.6 \times 10^{-19} \times 1000$

$= 1.6 \times 10^{-16}\ \text{J}$

8 Ohm's Law

1. $R = \dfrac{V}{I} = \dfrac{12.0}{2.0} = 6\ \Omega$

2. $I = \dfrac{V}{R} = \dfrac{240}{32.0} = 7.5\ \text{A}$

3. $V = IR = 0.0030 \times 1200 = 3.6\ \text{V}$

4. $R = \dfrac{V}{I} = \dfrac{2.5}{0.4} = 6.25\ \Omega$

5. $I = \dfrac{V}{R} = \dfrac{11 \times 10^3}{5.5 \times 10^6} = 2.0 \times 10^{-3}\ \text{A}$

6. $V = IR = 0.6 \times 10^{-3} \times 10.0 \times 10^3 = 6.0\ \text{V}$

7. $R = 2 + 3 + 4 = 9$

$I = \dfrac{V}{R} = \dfrac{12}{9} = 1.33\ \text{A}$

8. $R = 20 \times 0.4 = 8.0$

$V = IR = 3.0 \times 8.0 = 24.0\ \text{V}$

9. Potential difference $V = IR = 25 \times 5.0 = 125$ V
Potential difference accross ends = 240 − 125 = 115 V

10. Total resistance of cables
$= 2 \times 2 \times 10^3 \times 500 \times 10^{-6} = 2\ \Omega$
A: Potential drop $V = IR = 30 \times 2 = 60$ V
B: Potential drop $V = IR = 10 \times 2 = 20$ V
Thus B is chosen.

9 Resistors in Series

1. $R = R_1 + R_2 + R_3$

$= 2 + 4 + 5$

$= 11\ \Omega$

2. $R = R_1 + R_2 + R_3$

$= 2.5 + 4.9 + 7.6$

$= 15.0\ \Omega$

3. $R = R_1 + R_2$

$R_2 = R - R_1$

$= 10.0 \times 1000 - 4700$

$= 5300\ \Omega$

4. $R = R_1 + R_2$

$= 0.7 + 1.1$

$= 1.8\ \Omega$

$R = \dfrac{V}{I}$

$V = RI$

$= 1.8 \times 4.5$

$= 8.1\ \text{V}$

5. $R = 4R$

$= 4 \times 0.9$

$= 3.6\ \Omega$

$R = \dfrac{V}{I}$

$I = \dfrac{V}{R}$

$= \dfrac{24}{3.6}$

$= 6.7\ \Omega$

6. (a) $R = R_1 + R_2$

$= 2.0 + 3.0$

$= 5.0\ \Omega$

$R = \dfrac{V}{I}$

$I = \dfrac{V}{R}$

$= \dfrac{10.0}{5.0}$

$= 2.0\ \text{A}$

(b) 2.0 A

(c) $V_1 = R_1 I$
$= 2.0 \times 2.0$
$= 4.0$ V
$V_2 = R_2 I$
$= 3.0 \times 2.0$
$= 6.0$ V
The 2.0 resistor has a potential difference of 4.0 V across it whereas the potential difference across the 3.0 resistor is 6.0 V.

7. (a) 2.0 A
(b) 2.0 A
(c) $V_1 = R_1 I$
$= 3.0 \times 2.0$
$= 6.0$ V
(d) $V_2 = R_2 I$
$= 4.0 \times 2.0$
$= 8.0$ V
(e) $V_3 = R_3 I$
$= 5.0 \times 2.0$
$= 10.0$ V
(f) $V = V_1 + V_2 + V_3$
$= 24$ V

8. Because the resistors are in series, the same current flows through each.
$R = \frac{V}{I}$
$= \frac{16.0}{3.2}$
$= 5.0\ \Omega$

9. The current which will flow through the appliance is given by:
$P = VI$
$I = \frac{P}{V} = \frac{12}{110} = 0.10909$ A
The resistance of the appliance is given by:
$R = \frac{V}{I} = \frac{110}{0.10909} = 1008.34\ \Omega$
This current (0.10909 A) will have to pass through the resistance and the appliance. The total resistance is given by:
$R = \frac{V}{I} = \frac{240}{0.10909} = 2200.02\ \Omega$
Since the appliance has a resistance of 1008.34 Ω, the resistance needed to be connected in series is
$2200.02 - 1008.34 = 1191.7\ \Omega$
SAFETY NOTICE: DON'T TRY THIS, IT WOULD BE DANGEROUS.

10. To run properly, the motor must have a current of 1.5 A passing through it (see below).
$I = \frac{V}{R}$
$= \frac{12.0}{8.00}$
$= 1.5$ A
When the motor is running properly, the combined resistance of the circuit is (8.00 + 1.0 =) 9.0 Ω.
Thus the potential difference the generator must produce is given by:
$V = RI$
$= 9.0 \times 1.5$
$= 13.5$ V

10 Resistors in Parallel

1. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
$= \frac{1}{2.0} + \frac{1}{5.0}$
$= 0.7$
$R = \frac{1}{0.7}$
$= 1.4\ \Omega$

2. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
$= \frac{1}{2.5} + \frac{1}{0.75} + \frac{1}{1.50}$
$= 2.4$
$R = \frac{1}{2.4}$
$= 0.42\ \Omega$

3. $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$
$\frac{1}{R_2} = \frac{1}{R_P} - \frac{1}{R_1}$
$= \frac{1}{200.0} - \frac{1}{1000}$
$= 0.004$
$R_2 = \frac{1}{0.004} = 250\ \Omega$

4. $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$

$= \frac{1}{4.0} + \frac{1}{6.0}$

$= 0.4167$

$R_P = \frac{1}{0.4167} = 2.4\ \Omega$

$V = IR$

$= 5.0 \times 2.4$

$= 12.0\ V$

5. $\frac{1}{R_P} = 4 \times \frac{1}{R_1}$

$= 4 \times \frac{1}{12}$

$= 0.33$

$R_P = \frac{1}{0.33} = 3\ \Omega$

$I = \frac{V}{R}$

$= \frac{90.0}{3}$

$= 30.0\ A$

6. (a) 120 V across each.

(b) $I_1 = \frac{V}{R_1} = \frac{120}{20}$

$= 6$ A through the 20 Ω resistor

$I_2 = \frac{V}{R_2} = \frac{120}{30}$

$= 4$ A through the 30 Ω resistor

(c) Current = 6 + 4 = 10 A

7. (a) $V = R_1 I_1 = 3.0 \times 2.0 \times 6.0\ V$

(b) 6.0 V

(c) $I_2 = \frac{V}{R_2} = \frac{6.0}{4.0} = 1.5\ A$

(d) 6.0 V

(e) $I_3 = \frac{V}{R_3} = \frac{6.0}{5.0} = 1.2\ A$

(f) Current = 2.0 + 1.5 + 1.2 = 4.7 A

8. $V = I_1 R_1 = 40.0 \times 25 = 1000\ V$

Thus the potential difference across both of the resistors = 1000 V and the current through the other resistor = 50.0 A.

$R = \frac{V}{I_2} = \frac{1000}{50.0} = 20\ \Omega$

9. Resistance of left hand circuit is given by:

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4.0} + \frac{1}{6.0} = 0.4167$

$R = \frac{1}{0.4167} = 2.4\ \Omega$

Resistance of right hand circuit is given by:

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{8.0} + \frac{1}{6.0} + \frac{1}{3.0} = 0.625$

$R = \frac{1}{0.625} = 1.6\ \Omega$

Total resistance of both sides of the circuit is given by $R = 2.4 + 1.6 = 4.0\ \Omega$

10. The potential difference across the 4.0 Ω resistor is given by:

$V = RI = 4.0 \times 20.0 = 80.0\ V$

This is also the potential difference across the 6.0 Ω resistor in the left hand circuit. The current flowing through this resistor is given by:

$I = \frac{V}{R} = \frac{80.0}{6.0} = 13.3\ A$

The total current flowing through the left hand circuit is given by:

$I = 20.0 + 13.3 = 33.3\ A$

This current also flows through the right hand circuit. The resistance of the right hand circuit is 1.6 Ω (see Question 9 above).

The potential difference across the right hand circuit is given by:

$V = IR = 33.3 \times 1.6 = 53.3\ V$

The current passing through the 8.0 Ω resistor is given by $I = \frac{V}{R} = \frac{53.3}{8.0} = 6.7\ A$

11 Electrical Power and Energy

1. $P = VI = 12 \times 3.0 = 36\ W$

2. $I = \frac{P}{V} = \frac{1800}{240} = 7.5\ A$

3. $V = \frac{P}{I} = \frac{48}{6.4} = 7.5\ V$

4. $P = VI$

$I = \frac{P}{V} = \frac{48}{12} = 4\ A$

$R = \frac{V}{I} = \frac{12}{4} = 3.0\ \Omega$

5. $R = \frac{V}{I}$

$V = IR = 4.0 \times 5.0 = 20.0\ V$

$P = VI = 20.0 \times 4.0 = 80.0\ W$

6. $R = \frac{V}{I}$

$I = \frac{V}{R} = \frac{1.6 \times 10^{-3}}{4.8}$

$P = VI = 1.6 \times 10^{-3} \times \frac{1.6 \times 10^{-3}}{4.8} = 5.3 \times 10^{-7}$ W

7. Energy $= VIt$

$VI = \frac{\text{Energy}}{t}$

$P = VI = \frac{\text{Energy}}{t}$

$t = \frac{\text{Energy}}{P} = \frac{3.2 \times 10^5}{1.6 \times 10^3} = 200.0$ s

8. Energy $= VIt$

$V = \frac{\text{Energy}}{It} = \frac{2.7 \times 10^3}{2.0 \times 5.0 \times 60}$

$R = \frac{V}{I} = \frac{2.7 \times 10^3}{2.0 \times 5.0 \times 60 \times 2.0} = 2.25\ \Omega$

9. $R = \frac{V}{I}$

$V = IR = I \times 3.0 = 3I$

$P = VI = 3I \times I = 3I^2$

$I = \sqrt{\frac{P}{3}} = \sqrt{\frac{4.8}{3}} = 1.26$ A

10. $R = \frac{V}{I}$

$I = \frac{V}{R} = \frac{V}{480}$

$P = VI$

$120 = V \times \frac{V}{480}$

$V = \sqrt{120 \times 480} = 240$ V

12 Uniform Velocity

1. $v_{av} = \frac{\Delta s}{\Delta t} = \frac{1700}{20} = 85$ km h^{-1} ($=23.61$ m s^{-1})

2. $v_{av} = \frac{\Delta s}{\Delta t}$

$\Delta t = \frac{\Delta s}{v_{av}} = \frac{8100}{900} = 9$ hours

3. $\Delta s = v_{av}\Delta t = 12 \times 3 \times 60 = 2.160 \times 10^3$ m

4. $v_{av} = \frac{\Delta s}{\Delta t} = \frac{1.6 \times 10^3}{4 \times 60} = 6.67$ m s^{-1}

5. (a) 48 north − 2 north = 46 km north

(b) 9.25 am − 8.45 am = 15 + 25 = 40 min
= 0.67 hours

(c) $v_{av} = \frac{\Delta s}{\Delta t} = \frac{46}{0.67} = 69$ km h^{-1} north

6. (a) 600 west − 40 east = 640 km west

(b) 1.35 pm − 11.20 am = 40 + 95 = 135 min
= 2.25 hours

(c) $v_{av} = \frac{\Delta s}{\Delta t} = \frac{640}{2.25} = 284.4$ km h^{-1} west

7. (a) $55 - 45 = 10$ km

(b) $v = 90$ km h^{-1} $= \frac{90}{60 \times 60} = 0.025$ km s^{-1}

(c) $\Delta t = \frac{\Delta s}{v_{av}} = \frac{10}{0.025} = 400$ s = 6 min 40 sec

8. (a) $\Delta y_1 = 2.10 \times 20 \times 60 = 2520$ m east

(b) $\Delta y_2 = 4.50 \times 10.0 \times 60 = 2700$ m east

(c) $\Delta y = \Delta y_1 + \Delta y_2 = 2520 + 2700 = 5220$ m east

(d) $v_{av} = \frac{\Delta s}{\Delta t} = \frac{5220}{40.0 \times 60} = 2.175$ m s^{-1} west

9. (a) $v_{av} = \frac{\Delta s}{\Delta t} = \frac{40.0}{3.00} = 13.33$ m s^{-1} north

(b) $v_{av} = \frac{\Delta s}{\Delta t} = \frac{50.0}{4.0} = 12.5$ m s^{-1} south

(c) $40.0 + 50.0 = 90.0$ m

(d) $v_{av} = \frac{\Delta s}{\Delta t} = \frac{90.0}{7.0} = 12.86$ m s^{-1}

(e) 40.0 north + 50.00 south = 10.0 m south

(f) $v_{av} = \frac{\Delta s}{\Delta t} = \frac{10}{7.00} = 1.43$ m s^{-1} south

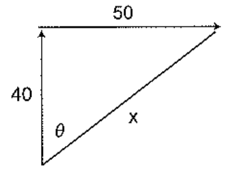
10. (a) $x = \sqrt{40.00^2 + 50.0^2} = 64.0$ m

$\theta = \tan^{-1}\frac{50}{40} = 51.3°$

Final displacement
= 64.0 m at N 51.3° E or E 38.7° N

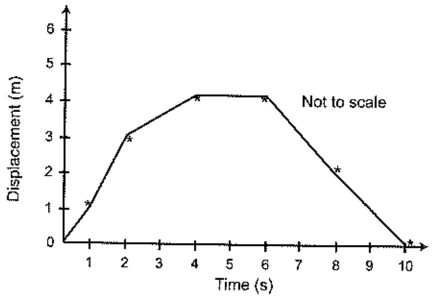
(b) Time = 3.00 + 4.00 = 7.00 s

(c) $v_{av} = \frac{\Delta s}{\Delta t} = \frac{64.0}{7.00}$
= 9.14 m s^{-1} at E 38.7° N



13 Displacement-Time Graphs

1.



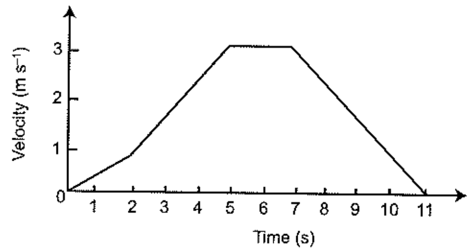
2. (a) 0
   (b) 2 to 3 s and 7 to 8 s
   (c) $v = \frac{4-0}{2-0} = 2\text{ m s}^{-1}$
   (d) 3 to 7 s
   (e) 8 to 9 s

3. OA positive acceleration
   BC negative acceleration
   DE positive acceleration
   FG negative acceleration

4. (a) 0.5 m s^{-1}
   (b) 0.8 m s^{-1}

14 Velocity-Time Graphs

1.



2. (a) 3 m s^{-1}
   (b) 7 to 8 s
   (c) $a = \frac{v-u}{t} = \frac{3-1}{4-3} = 2\text{ m s}^{-2}$
   (d) 8 to 10 s

3. (a) Displacement from 0 to 2 s = area of triangle
   $s = \frac{1}{2} \times \text{base} \times \text{height} = 0.5 \times 2 \times 3 = 3.0\text{ m}$
   (b) Displacement from 2 to 3 s = area of triangle
   $s = \text{base} \times \text{height} = 1 \times 3 = 3.0\text{ m}$
   (c) Displacement from 3 to 4 s = area of triangle
   $s = \frac{1}{2} \times \text{base} \times \text{height} = 0.5 \times 1 \times 3 = 1.5\text{ m}$
   (d) Displacement from 4 to 5 s = 0 m
   (e) Displacement from 5 to 6 s = area of triangle
   $s = \frac{1}{2} \times \text{base} \times \text{height} = 0.5 \times 1 \times 1 = 0.5\text{ m}$
   (f) Displacement from 6 to 7 s = area of trapezium
   $s = 1 \times \frac{1}{2} \times (1+3) = 2\text{ m}$
   (g) Displacement from 7 to 9 s = area of triangle
   $s = 2 \times 3 = 6\text{ m}$
   (h) Displacement from 9 to 10 s = area of triangle
   $s = \frac{1}{2} \times 1 \times 3 = 1.5\text{ m}$
   (i) Total displacement from 0 to 10 s
   $= 3+3+1.5+0-0.5-2-6-1.5 = -2.5\text{ m}$

15 Uniform Acceleration

1. $v = u + at = 10 + 15 \times 4 = 70\text{ m s}^{-1}$

2. $v = u + at$
   $a = \frac{v-u}{t} = \frac{25-5.0}{10.0} = 2\text{ m s}^{-2}$

3. $v - u = at = 19.6 \times 7.50 = 147\text{ m s}^{-1}$

4. $t = \frac{v-u}{a} = \frac{47.9-4.7}{9.6} = 4.5\text{ s}$

5. $v - u = at = -15.5 \times 0.800 = -12.4\text{ m s}^{-1}$

6. $u = v - at = 35.0 - 2.6 \times 9.0 = 11.6\text{ m s}^{-1}$

7. $v = u + at = 0 + 14.7 \times 0.650 = 9.56\text{ m s}^{-1}$

8. $120\text{ km h}^{-1} = \frac{120 \times 10^3}{60 \times 60} = 33\ 33\text{ m s}^{-1}$
   $a = \frac{v-u}{t} = \frac{33.33-0}{10.0} = 3.33\text{ m s}^{-2}$

9. $a = \frac{v-u}{t} = \frac{20-(-30)}{\frac{1}{10}}$
   $= 500\text{ m s}^{-2}$ away from the wall

10. $a = \frac{v-u}{t} = \frac{45.0\text{ south} - 15.0\text{ north}}{3 \times 60} = \frac{60\text{ south}}{180}$
   $= 0.33\text{ m s}^{-2}$ towards south

16 Equations of Motion

1. $v = u + at$
   $t = \frac{v-u}{a}$
   $= \frac{31.5-15.0}{3.0}$
   $= 5.5\text{ s}$

2. $v = u + at$
   $a = \frac{v-u}{t}$
   $= \frac{0-31.5}{1.5}$
   $= -21.0\text{ m s}^{-2}$

3. $v^2 = u^2 + 2as$

$s = \dfrac{v^2 - u^2}{2a}$

$= \dfrac{28.0^2 - 0}{2 \times 3.0}$

$= 130.7$ m

4. $v^2 = u^2 + 2as$

$= (42.6)^2 + 2 \times 15.2 \times 1800$

$= 56534.76$

$v = \sqrt{56534.76}$

$= 237.8$ m s^{-1}

5. $s = ut + \dfrac{1}{2}at^2$

$= 21.4 \times 6.5 + \dfrac{1}{2} \times 1.6 \times 6.5^2$

$= 173$ m

6. $s = ut + \dfrac{1}{2}at^2$

$a = \dfrac{2(s - ut)}{t^2}$

$= \dfrac{2(2000 - 200.0 \times 6.2)}{6.2^2}$

$= 39.5$ m s^{-2}

7. $s = ut + \dfrac{1}{2}at^2$

$at^2 + 2ut - 2s = 0$

$12t^2 + 2 \times 6 \times t - 2 \times 180 = 0$

$t^2 + t - 30 = 0$

$(t + 6)(t - 5) = 0$

$t = -6$ or 5

Since time cannot be negative, time is 5 s.

8. $s = ut + \dfrac{1}{2}at^2$

$= 5.0 \times 4.3 + 0.5 \times 0.8 \times 4.3^2$

$= 28.9$ m

9. $a = \dfrac{v - u}{t}$

$= \dfrac{105.0 - 15.0}{8.0}$

$= 11.25$

$s = \dfrac{v^2 - u^2}{2a}$

$= \dfrac{105.0^2 - 15.0^2}{2 \times 11.25}$

$= 480$ m

10. During its acceleration, the extra distance the car has to travel is 10 m (to travel up to the rear of the truck) + 40 m (to reach the front of the truck) + 10 + 4 (to get the rear of the car 10 m in front of the truck). This distance adds up to 64 m.

First calculate the time taken to overtake the truck from a standing start:

$s = ut + \dfrac{1}{2}at^2$

$at^2 + 2ut - 2s = 0$

$2.0 \times t^2 + 2 \times 0 \times t - 2 \times 64 = 0$

$2.0t^2 = 128$

$t = 8$ s

$72 \text{ km h}^{-1} = \dfrac{72 \times 1000}{24 \times 60 \times 60} = 0.83 \text{ m s}^{-1}$

During these 8 seconds, the total distance travelled by the car is given by:

$s = ut + \dfrac{1}{2}at^2$

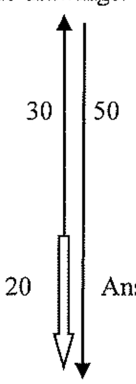
$= 0.83 \times 8 + 0.5 \times 2.0 \times 8^2$

$= 70.7$ m and 8 seconds

17 Addition of Vectors

The diagrams which follow are sketches only and do not represent scale drawings.

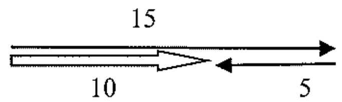
1. (a)



Resultant: 20

Answer: 20 km south.

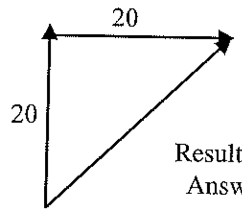
(b)



Resultant: 10

Answer: 10 newtons east.

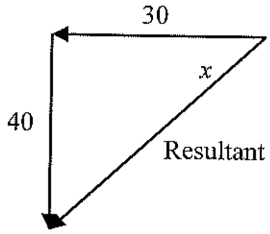
(c)



Resultant:
(Right-angled triangle)

Resultant: $\sqrt{20^2 + 20^2} = 28.3$
Answer: 28 m s^{-1} north-east.

(d)

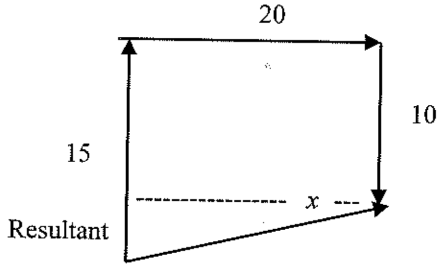


Resultant:
(Right-angled triangle)
$\sqrt{30^2 + 40^2} = 50$

Direction:
Tan $x = 40/30 = 1.33$
$x = 53°$

Answer: 50 m s^{-2} at W 53° S or S 37° W or 217°.

2.

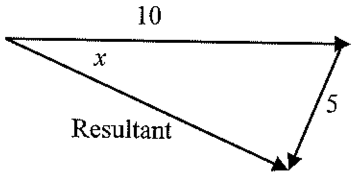


Resultant:
$\sqrt{20^2 + 5^2} = 20.6$

Direction:
Tan $x = 5/20 = 0.25$

Answer: 21 km at E 14° N or 76°.

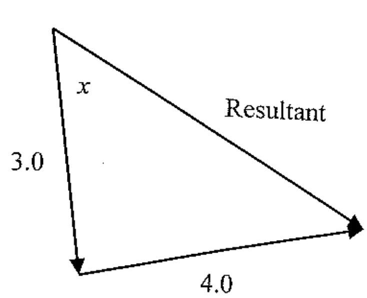
3.



Resultant:
$a^2 = b^2 + c^2 - 2bc \cos A$
$R^2 = 10^2 + 5^2 - 2 \times 10 \times 5 \times \cos 60°$
$R = 8.7$ m s^{-1}
Direction:
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{R}{\sin 60°} = \frac{5}{\sin x}$$
$$\operatorname{Sin} x = \frac{5 \sin 60°}{8.7} = 0.50$$
$$x = 30$$

Answer: 8.7 m s^{-1} at E 30° S or 120°.

4.

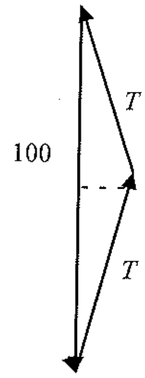


Resultant:
(Right-angled triangle)
5.0×10^4

Direction:
Tan $x = 4.0/3.0 = 1.33$
$x = 53°$

Answer: 5.0×10^4 kg m s^{-1} at S 73° E or E 17° S or 107°.

5. The three forces: weight of ball and tension in each wire add together to zero. Each wire supports half the weight of the ball. The tension is given by:

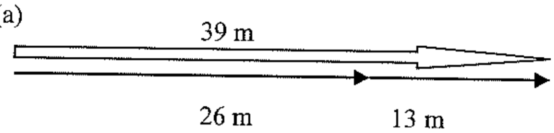


$50/T = \tan 30° = 0.577$
$T = 87$ newtons
Answer: 87 newtons.

18 Subtraction of Vectors

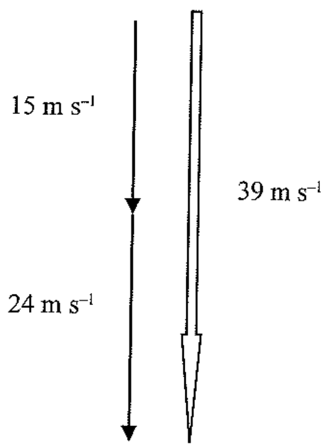
The diagrams which follow are sketches only and do not represent scale drawings.

1. (a)



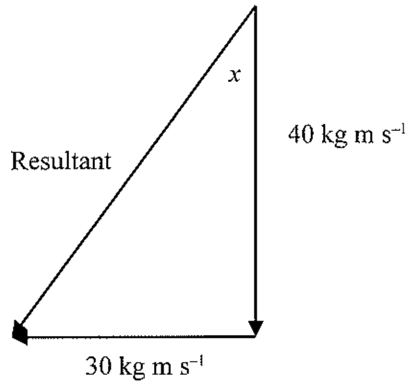
Resultant 39 m east.

(b)



Resultant 39 m s^{-1} south.

(c) 45.0 kg m s^{-1} south – 30.0 kg m s^{-1} east
$= 45.0 \text{ kg m s}^{-1}$ south + 30.0 kg m s^{-1} west



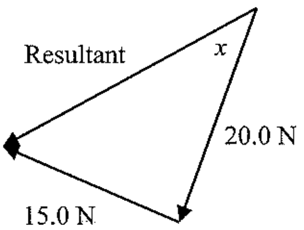
Resultant:
(345 right-angled triangle)
50 kg m s^{-1}
Direction:

$$\text{Tan } x = \frac{30}{40} = 0.75$$

$$x = 36.9°$$

Answer: 50 kg m s^{-1} at S 37° W.

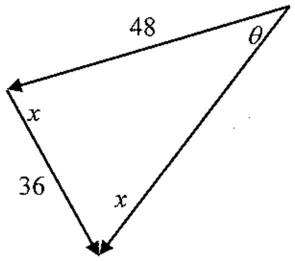
(d) 20.0 newtons at 200° – 15.0 newtons at 110°
= 20.0 newtons at 200° + 15.0 newtons at 290°.



Resultant:
20.0 N
(345 right-angled triangle)
25 N
Direction:
200.0 + 36.9 = 236.9°
Answer: 25.0 newtons at 236.9°.

Tan x = 15.0/20.0
= 0.75
$x = 36.9°$

(e) 48 m s^{-2} W 30° S – 36 m s^{-2} N 40° W
= 48 m s^{-2} W 30° S + 36 m s^{-2} S 40° E



Resultant:
$x = 80°$

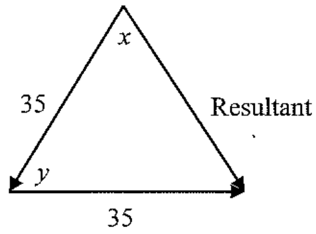
$$\text{Resultant}^2 = 48^2 + 36^2 - 2 \times 48 \times 36 \cos 80°$$
$$= 2999.87$$
$$\text{Resultant} = 55 \text{ m s}^{-2}$$

Direction:

$$\text{Cos } \theta = \frac{48^2 + 55^2 - 36^2}{2 \times 48 \times 55}$$
$$= 0.7638$$
$$\theta = 40.2°$$

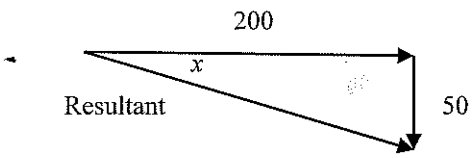
Answer is 55 m s^{-2} at W 70 2°S or S 19.8°W or 199.2°.

2. 35 newtons at 240° – 35 newtons west
= 35 newtons at 240° + 35 newtons east



$x = y = 60°$
Resultant:
(equilateral triangle)
35 newtons
Answer is 35 newtons at 150°.

3. 200 km h^{-1} east – 50 km h^{-1} from south
= 200 km h^{-1} east + 50 km h^{-1} from north

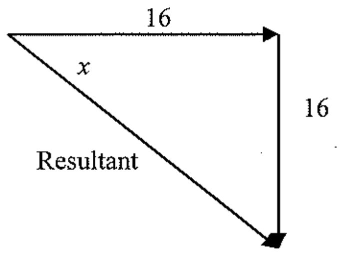


Resultant:
(Right-angled triangle)
$\text{Resultant}^2 = 200^2 + 50^2$
Resultant = 206

Direction:
Tan x = 50/200 = 0.25
$x = 14°$

Answer is 206 km h^{-1} at E 14° S.

4. 16 m s^{-1} towards east – 16 m s^{-1} towards north
= 16 m s^{-1} towards east + 16 m s^{-1} towards south

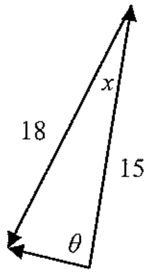


Resultant:
(Right -angled triangle)
$\text{Resultant}^2 = 16^2 + 16^2$
Resultant = $\sqrt{512}$
Resultant = 22.6

Direction:
$x = 45°$

Answer is 22.6 m s^{-1} at E 45° S.

5. 15 km h^{-1} at N 10° E – 18 km h^{-1} at N 18° E
= 15 km h^{-1} at N 10° E + 18 km h^{-1} at S 18° W



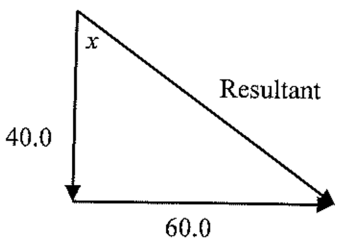
$x = 8°$
Resultant:
$\text{Resultant}^2 = 18^2 + 15^2 - 2 \times 18 \times 15 \times \cos 8°$
$= 103.4$
Resultant = 10.2
Direction:

$$\cos\theta = \frac{15^2 + 10.2^2 - 18^2}{2 \times 15 \times 10.2}$$
$$= 0.014$$
$$\theta = 89°$$

Answer is 10.2 km h^{-1} at N 79° W.

19 Relative Velocity

1. 90 km h^{-1} east – 110 km h^{-1} west
= 90 km h^{-1} east + 110 km h^{-1} east
= 200 km h^{-1} east.
2. 60 km h^{-1} east – 80 km h^{-1} east
= 60 km h^{-1} east + 80 km h^{-1} west
= 20 km h^{-1} west.
3. The boat's velocity relative to the stationary buoy is 40 km h^{-1} north. However, the boy's velocity relative to the boat is 2 km h^{-1} south. Thus the boy's velocity relative to the buoy is (40 – 2) km h^{-1} north, i.e. 38 km h^{-1} north.
4. 40.0 km h^{-1} south – 60.0 km h^{-1} west
= 40.0 km h^{-1} south + 60.0 km h^{-1} east.

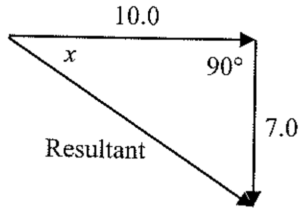


Resultant:
(Right-angled triangle)
$R^2 = \sqrt{60^2 + 40^2} = 72.1$

Direction:
Tan $x = 60/40 = 1.5$
$x = 56.3°$

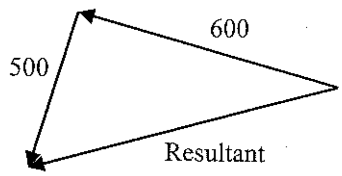
Answer: 72.1 km h^{-1} at S 56.3° E or E 33.7° S or 123.7°.

5. The boat's speed relative to the Earth is 10.0 m s^{-1}. For the sake of simplicity, assume that the river is flowing towards the north and the boat has to cross towards the east.
10.0 m s^{-1} east – 7.0 m s^{-1} north
= 10.0 m s^{-1} east + 7.0 m s^{-1}south



Direction:
Tan $x = 7.0/10.0 = 0.7$
$x = 35°$

Answer: 35° upstream.
6. In solving this problem, calculate the speed of one aeroplane relative to the other and from the fact that they 'start' 10 km apart, calculate the time.
600 km h^{-1} at W 30° N – 500 km h^{-1} at N 30° E
= 600 km h^{-1} at W 30° N + 500 km h^{-1} at S 30° W



Resultant:
(Right-angled triangle)
$\sqrt{500^2 + 600^2} = 781.0$
Time = distance/speed = 10/781 = 0.013 hr
= 0.77 mins = 46.1 seconds.

20 Force and Motion

1. $\Sigma F = ma = 3.0 \times 1.6 = 4.8$ N
2. $a = \dfrac{\Sigma F}{m} = \dfrac{9.1}{0.7} = 13 \text{ m s}^{-2}$
3. $m = \dfrac{\Sigma F}{a} = \dfrac{0.85}{17.0} = 0.05$ kg
4. $a = \dfrac{v-u}{t} \quad \dfrac{45-0}{5.0} = 9.0$
$\Sigma F = ma = 550 \times 9.0 = 4950$ N
5. $a = \dfrac{v-u}{t} = \dfrac{6.378 \times 10^2 - 5.50 \times 10^1}{6.2} = 94.0$
$m = \dfrac{\Sigma F}{a} = \dfrac{1.880 \times 10^3}{94.0} = 20$ kg
6. $a = \dfrac{\Sigma F}{m} = \dfrac{0.34}{850 \times 10^{-3}} = 0.4$
$\Delta y = u_y t + \frac{1}{2} a_y t^2$
$0.80 = 0 \times t + \frac{1}{2} \times 0.4 \times t^2$
$t = \sqrt{\dfrac{0.80}{0.5 \times 0.4}} = 2$ s

7. $v_y^2 = u_y^2 + 2a_y \Delta y$

$a_y = \frac{v_y^2 - u_y^2}{2\Delta y} = \frac{0^2 - 15.0^2}{2 \times 11.25} = 10$

$\sum F = ma_y = 840 \times 10 = 8400$ N

8. Force required to produce acceleration:
$\sum F = ma = 1.96 \times 3.50 = 6.86$
$\therefore$ Force of friction = 10.00 – 6.86 = 3.14 N

9. Force required to produce acceleration:
$\sum F = ma = 240.0 \times 1.20 = 288$ N
Force on crate due to its weight:
$\sum F = mg = 240.0 \times 9.8 = 2352$ N
Total force during this part of the lift:
$\sum F = 288 + 2352 = 2640$ N

10. Acceleration of box while moving across the rough patch:
$\sum F = ma_y = m \times \frac{v_y^2 - u_y^2}{2\Delta y} = 14.4 \times \frac{1.00^2 - 5.00^2}{2 \times 3.00} = -57.6$
Thus force of friction = 57.6 N

21 Momentum

1. $p = mv = 21 \times 17 = 357$ kg m s^{-1}

2. $v = \frac{p}{m} = \frac{18\,500}{740} = 25$ m s^{-1} north

3. $m = \frac{p}{v} = \frac{12}{16} = 0.75$ kg

4. $\Delta p = m \times \Delta v = 1500 \times (29 - 17) = 18\,000$
$= 1.8 \times 10^4$ kg m s^{-1} south

5. $\Delta p = m \times \Delta v = 0.500(11.0 - [-12.0])$
$= 0.500 \times 23.0 = 11.5$ kg m s^{-1} away from the wall

6. $\Delta p = \Delta m \times v = (-1.0 \times 10^4) \times 3.5 = -3.5 \times 10^4$ kg m s^{-1}

7. Original momentum = $550 \times 150 = 82\,500$
Final momentum = $(550 - 240) \times 425 = 131\,750$
Change of momentum = 49 250 kg m s^{-1}

8. Original momentum of person and trolley
$= 100 \times 0 + 50 \times 3 = 150$
Final momentum of person and trolley = 150
$v = \frac{p}{m} = \frac{150}{100 + 50} = 1$ m s^{-1}

9. Total original momentum = $m_1u_1 + m_2u_2$
$= 0.120 \times 0.4 + (0.120 + 0.150) \times 0.3 = 0.129$ kg m s^{-1}
Final momentum = original momentum = $(m_1 + m_2)v$
$v = \frac{0.129}{(0.120 + 0.120 + 0.150)} = 0.331$ m s^{-1}

22 Impulse

1. $\Delta p = F\Delta t = 110 \times 0.3 = 33$ kg m s^{-1} (N s)

2. $F = \frac{\text{impulse}}{\Delta t} = \frac{720}{0.060} = 12\,000$ N

3. $\Delta t = \frac{\text{impulse}}{F} = \frac{8.0 \times 10^{-1}}{1.6 \times 10^2} = 5.0 \times 10^{-3}$ s

4. $\Delta p = F\Delta t = 1.4 \times 10^6 \times 5.0 \times 10^{-3} = 7000$ kg m s^{-1} (N s)

5. $p = mv$
$\Delta p = m\Delta v$
$\Delta v = \frac{\Delta p}{m} = \frac{4.8}{0.40} = 12$ m s^{-1}

6. $\Delta v = 25.0 - 15.0$
$= 10.0$ m s^{-1}
$\Delta p = m\Delta v$
$= m(v - u)$
$= 840 \times 10 = 8400$ N s

7. Change of velocity = 10.0 – (–12.0) = 22.0 m s^{-1}
$\Delta p = m\Delta v = 0.600 \times 22.0 = 13.2$ kg m s^{-1}
$\Delta p = F\Delta t$
$F = \frac{\Delta p}{\Delta t} = \frac{13.2}{0.25} = 52.8$ N

8. $\Delta p = m\Delta v = 3.4 \times (0 - 0.3) = -1.02$ kg m s^{-1}
$\Delta t = \frac{\Delta p}{F} = \frac{1.02}{2.7} = 0.38$ s
(Negative sign has no meaning in this case.)

9. $\Delta p = F\Delta t = 8.6 \times 2.5 = 21.5$ kg m s^{-1}
$\Delta p = m\Delta v$
$21.5 = 1.8(v - 50.0)$
$v = \frac{21.5}{1.8} + 50.0 = 61.9$ m s^{-1}

10. $\Delta v = 35$ m s^{-1} away from receiver's racquet
– 35 m s^{-1} towards receiver's racquet
= 70 m s^{-1} away from receiver's racquet
$\Delta p = m\Delta v = 0.050 \times 70 = 3.5$ kg m s^{-1}
Impulse = $F\Delta t$
$F = \frac{\text{impulse}}{\Delta t} = \frac{\Delta p}{\Delta t} = \frac{3.5}{\frac{1}{50}} = 175$ N

23 Conservation of Momentum

1. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
Because the gliders are identical, the masses may be eliminated from the equation:
$v_2 = u_1 + u_2 - v_1$
$= 4.0 + 2.0 - 2.0$
$= 4.0\text{ m s}^{-1}$ to the right

2. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
m_1 and m_2 are combined before the separation and have the same velocity:
$MU = m_1v_1 + m_2v_2$
$75.0 \times 3.0 \times 10^3 = 50.0 \times 2.0 \times 10^3 + 25.0 \times v_2$
$v_2 = (75.0 \times 3.0 \times 10^3 - 50.0 \times 2.0 \times 10^3) \div 25.0$
$= 5.0 \times 10^3\text{ m s}^{-1}$

3. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
m_1 and m_2 are combined after the collision and have the same velocity:
$m_1u_1 + m_2u_2 = MV_2$
$80.0 \times 6.0 + 60.0 \times 4.0 = 140 \times V_2$
$V_2 = 720.0 \div 140.0$
$= 5.1\text{ m s}^{-1}$

4. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
In this solution, north is taken as the positive direction, south as the negative:
m_1 and m_2 are combined after the collision and have the same velocity:
$m_1u_1 + m_2u_2 = MV_2$
$2.0 \times 5.0 + (-3.0 \times 4.0) = 5.0 \times V_2$
$V_2 = -2.0 \div 5.0$
$= 0.4\text{ m s}^{-1}$ south

5. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
$2.5 \times 0 + 1.5 \times 12.0 = MV_2$
$V_2 = 18.0 \div 4.0$
$= 4.5\text{ m s}^{-1}$ south

6. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
$2.0 \times 10^3 \times 4.50 + m_2 \times 0 = (m_2 + 2.0 \times 10^3) \times 0.75$
$m_2 = \dfrac{2.0 \times 10^3 \times 4.50}{0.75} - 2.0 \times 10^3$
$= 10\ 000\text{ kg}$

7. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
$0.5 \times 40.0 + 7.0 \times 0 = -0.5 \times 30 + 7.0 \times v_2$
$v_2 = \dfrac{35.0}{7.0}$
$= 5.0\text{ m s}^{-1}$

8. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
$5 \times 10^{-3} \times 0 + 6.0 \times 0 = 5 \times 10^{-3} \times 480 + 6.0 \times v_2$
$v_2 = \dfrac{-2.4}{6.0}$
$= -0.4\text{ m s}^{-1}$, i.e. 0.4 m s^{-1} recoil

24 Work

1. $W = Fs = 6.4 \times 5.5 = 35.2\text{ J}$

2. $F = \dfrac{W}{s} = \dfrac{15.0 \times 10^3}{2.5} = 6.0 \times 10^3\text{ N}$

3. $s = \dfrac{W}{F} = \dfrac{14.4}{4.8} = 3.0\text{ m}$

4. $s = \dfrac{W}{F} = \dfrac{210.0}{4.2} = 50.0\text{ m}$

5. $W = Fs = 96.0 \times 8.6 = 8.256 \times 10^2\text{ J}$

6. $W = Fs = 7.3 \times 6.5 = 47.45\text{ J}$

7. Work per second $= 450\text{ J} \div 5.0\text{ s} = 90\text{ J s}^{-1}$
Time taken $= 1.8 \times 10^3 \div 90 = 20\text{ s}$

8. In 1 second, the boat moves 15 m.
$W = Fs = 1200 \times 15 = 18\,000\text{ J}$

9. (a) $v = u + at$
$a = \dfrac{v-u}{t} = \dfrac{30-20}{5.0} = 2.0\text{ m s}^{-2}$
(b) $\Sigma F = ma$
$= 800 \times 2.0 = 1600\text{ N}$
(c) $\Delta y = u_y t + \dfrac{1}{2}a_y t^2$
$= 20 \times 5.0 + \dfrac{1}{2} \times 2.0 \times 5.0^2$
$= 125\text{ m}$
(d) $W = Fs = 1600 \times 125 = 200000$
$= 2.0 \times 10^5\text{ J}$

10. Average speed $= \dfrac{0+30}{2} = 15\text{ m s}^{-1}$
Distance travelled $= 15 \times 10.0 = 150\text{ m}$
$W = Fs = 5.0 \times 10^3 \times 150 = 7.5 \times 10^5\text{ J}$

25 Kinetic Energy

1. $E_K = \frac{1}{2}mv^2 = 0.5 \times 7.0 \times (1.2)^2 = 5.04$ J

2. $E_K = \frac{1}{2}mv^2$

$v^2 = \frac{2E_K}{m} = \frac{2 \times 28.8}{1.8} = 32$

$v = \sqrt{32} = 5.66$ m s^{-1}

3. $E_K = \frac{1}{2}mv^2$

$m = \frac{2E_K}{v^2} = \frac{2 \times 60.0}{(40)^2} = 0.075$ kg

4. Velocity of the athlete = $100 \div 10 = 10$ m s^{-1}

$E_K = \frac{1}{2}mv^2 = 0.5 \times 95 \times (10)^2 = 4.750 \times 10^3$ J

5. E_K at 12.0 m s^{-1} = $0.5 \times 550 \times (12.0)^2 = 39600$

E_K at 25.0 m s^{-1} = $0.5 \times 550 \times (25.0)^2 = 171875$

Increase = 171875 − 39600 = 132 275 or 1.32×10^5 J

6. Initial energy of spacecraft $= 0.5 \times 3.5 \times 10^2 \times (1.5 \times 10^2)^2$

$= 3937500$ J

Final energy of spacecraft $= 3937500 + 4.20 \times 10^6$

$= 8137500$ J

New velocity of spacecraft:

$E_K = \frac{1}{2}mv^2$

$v^2 = \frac{2E_K}{m}$

$v = \sqrt{\frac{2 \times 8137500}{3.5 \times 10^2}} = 215.64$ m s^{-1}

7. (a) $a = \frac{\Sigma F}{m} = \frac{96}{12} = 8$ m s^{-2}

(b) $\Delta y = u_y t + \frac{1}{2} a_y t^2$

$= 0 \times 5.0 + 0.5 \times 8 \times (5.0)^2 = 100$ m

(c) $v = u + at = 0 + 8 \times 5.0 = 40$ m s^{-1}

(d) $W = Fs = 96 \times 100 = 9600$ or 9.600×10^3 J

(e) $E_K = \frac{1}{2}mv^2 = 0.5 \times 12 \times 40^2 = 9600$ or 9.6×10^3 J

8. (a) Work done $= 140 \times 10^3 \times 3.00$

$= 420000 = 4.2 \times 10^5$ J

(b) E_K = work done

$= 4.2 \times 10^5$ J

(c) $E_K = \frac{1}{2}mv^2$

$v^2 = \frac{2E_K}{m}$

$v = \sqrt{\frac{2E_K}{m}}$

$= \sqrt{\frac{2 \times 4.2 \times 10^5}{1400}}$

$= 24.49$ m s^{-1}

9. (a) Before: $E_k = \frac{1}{2}mv^2 = 0.5 \times 0.120 \times 2.5^2 = 0.375$ J

After: $E_k = 0.5 \times 0.120 \times 1.5^2 = 0.135$ J

(b) Loss: $E_k = 0.375 - 0.135 = 0.240$ J

(c) Work done = 0.240 J

10. Skier's kinetic energy before hitting the snow bank:

$E_K = \frac{1}{2}mv^2 = 0.5 \times 80.0 \times (12.0)^2 = 5760$ J

Thus, amount of work done = 5760 J or 5.76×10^3 J

26 Characteristics of Waves

1. Period = $\frac{1}{f}$ $\quad f = \frac{1}{10} = 0.1$ Hz

2. Period = $\frac{1}{f} = \frac{1}{256} = 3.9 \times 10^{-3}$ s

3. $v = f\lambda = 512 \times 10 = 5120 = 5.12 \times 10^3$ m s^{-1}

4. $\lambda = \frac{v}{f} = \frac{1533}{40} = 38.325$ m

5. Period = $\frac{1}{f}$

$v = f\lambda = \frac{1}{0.01} \times 1.2 = 120 = 1.2 \times 10^2$ m s^{-1}

6. $\lambda = \frac{v}{f} = \frac{330}{256} = 1.2890625 = 1.29$ m

7. $\lambda = \frac{v}{f} = \frac{3 \times 10^8}{104.1 \times 10^6} = 2.88184438 = 2.88$ m

8. Period $= \frac{1}{f} = \frac{1}{v/\lambda} = \frac{\lambda}{v} = \frac{5.893 \times 10^{-7}}{3 \times 10^8}$

$= 1.964333333 \times 10^{-15} = 1.96 \times 10^{-15}$ s

9. Period $= \frac{1}{f} = \frac{1}{v/\lambda} = \frac{\lambda}{v} = \frac{0.36}{3 \times 10^8} = 1.2 \times 10^{-9}$ s

10. $f = 72$ min^{-1} $= \frac{72}{60} = 1.2$ Hz

$\lambda_{average} = 18 \div 5 = 3.6$ cm

$v = f\lambda = 1.2 \times 3.6 = 4.32$ cm s^{-1}

27 Refraction of Waves

1. (a) $v = f\lambda$

$\lambda = \frac{v}{f} = \frac{1540}{4.0 \times 10^6} = 3.85 \times 10^{-4}$ m

(b) 4.0 MHz

(c) $\lambda = \frac{v}{f} = \frac{4216}{4.0 \times 10^6} = 1.054 \times 10^{-3}$ m

(d) ${}_2n_1 = \frac{v_2}{v_1} = \frac{4216}{1540} = 2.74$

2. (a) 45°

(b) 60°

(c) ${}_1n_2 = \frac{\sin i}{\sin r} = \frac{\sin 45^\circ}{\sin 60^\circ} = 0.816$

(d) $v = f\lambda = 220 \times 1.5 = 330$ m s^{-1}

(e) ${}_1n_2 = \frac{v_1}{v_2}$

$v_2 = \frac{v_1}{{}_1n_2} = \frac{330}{0.816} = 404$ m s^{-1}

(f) $\lambda = \frac{v}{f} = \frac{404}{220} = 1.837$ m

3. For total internal reflection, $r \geq 90^\circ$
At critical angle, $r = 90^\circ$

(a) ${}_1n_2 = \frac{\sin i}{\sin r} = \frac{\sin 77^\circ}{\sin 90^\circ} = 0.97$

(b) ${}_1n_2 = \frac{v_1}{v_2}$

$v_2 = \frac{v_1}{{}_1n_2} = \frac{1493}{0.97} = 1539$ m s^{-1}

28 Refraction of Light

1. ${}_1n_2 = \frac{\sin i}{\sin r}$

$\sin r = \frac{\sin i}{{}_1n_2} = \frac{\sin 25^\circ}{1.47} = 0.287$

$r = 16.7^\circ \approx 17^\circ$

2. ${}_1n_2 = \frac{\sin i}{\sin r}$

$\sin i = {}_1n_2 \times \sin r = 1.362 \sin 15^\circ = 0.3525$

$i = 20.6^\circ \approx 21^\circ$

3. ${}_1n_2 = \frac{\sin i}{\sin r} = \frac{\sin 20.0^\circ}{\sin 8.14^\circ} = 2.42$

4. $v_2 = \frac{v_1}{{}_1n_2} = \frac{3 \times 10^8}{1.31} = 2.29 \times 10^8$ m s^{-1}

It is less.

5. ${}_1n_2 = \frac{v_1}{v_2} = \frac{3.0 \times 10^8}{1.7 \times 10^8} = 1.76$

6. ${}_An_O = \frac{v_A}{v_O}$ (A = air, O = olive oil)

$v_O = \frac{v_A}{{}_An_O} = \frac{3 \times 10^8}{1.33} = 2.26 \times 10^8$ m s^{-1}

${}_On_W = \frac{v_O}{v_W}$ (O = olive oil, W = water)

${}_On_W = \frac{v_O}{v_W} = \frac{2.05 \times 10^8}{2.26 \times 10^8} = 0.9$

7. ${}_{Air}n_{Flint} = \frac{Air}{v_{Flint(red)}}$

$v_{Flint(red)} = \frac{v_{Air}}{{}_{Air}n_{Flint}} = \frac{3.0 \times 10^8}{1.6434} = 1.825 \times 10^8$

${}_{Air}n_{Flint} = \frac{v_{Air}}{v_{Flint(violet)}}$

$v_{Flint(violet)} = \frac{v_{Air}}{{}_{Air}n_{Flint}} = \frac{3.0 \times 10^8}{1.6852} = 1.780 \times 10^8$

$v_{Flint(red)} : v_{Flint(violet)} = \frac{1.825 \times 10^8}{1.780 \times 10^8} = 1.025$

8. ${}_{Crown}n_{Air} = \frac{v_{Crown}}{v_{Air}} \therefore v_{Crown} = {}_{Crown}n_{Air} \times v_{Air}$

Similarly:

${}_{Flint}n_{Air} = \frac{v_{Flint}}{v_{Air}} \therefore v_{Flint} = {}_{Flint}n_{Air} \times v_{Air}$

${}_{Crown}n_{Flint} = \frac{v_{Crown}}{v_{Flint}} = \frac{{}_{Crown}n_{Air} \times v_{Air}}{{}_{Flint}n_{Air} \times v_{Air}}$

$= \frac{{}_{Crown}n_{Air}}{{}_{Flint}n_{Air}}$

But ${}_{Crown}n_{Air} = \frac{1}{{}_{Air}n_{Crown}} = \frac{1}{1.5140}$

And ${}_{Flint}n_{Air} = \frac{1}{{}_{Air}n_{Flint}} = \frac{1}{1.6434}$

$\therefore {}_{Crown}n_{Flint} = \frac{1.6434}{1.5140} = 1.0855$

$\frac{\text{Sin } i}{\text{Sin } r} = 1.0855$

$\text{Sin } r = \frac{\sin i}{1.0855} = \frac{\sin 25.0^\circ}{1.0855} = 0\ 3893$

$r = 22.9^\circ$

9. At the critical angle (c), $r = 90^\circ$

${}_{Mica}n_{Air} = \frac{\sin i}{\sin r} = \frac{\sin 38.7^\circ}{\sin 90.0^\circ} = 0.6252$

${}_{Air}n_{Mica} = \frac{1}{{}_{Mica}n_{Air}} = \frac{1}{0.6252} = 1.60$

Alternatively:

${}_{Air}n_{Mica} = \frac{1}{\sin c} = \frac{1}{\sin 38.7^\circ} = 1.60$

10. $_1n_2 = \frac{v_1}{v_2}$

Since $_{\text{Air}}n_{\text{Benzene}} = 1.504$

$\frac{v_{\text{Air}}}{v_{\text{Benzene}}} = 1.504$

$= \frac{\text{distance in air}}{\text{time}} \div \frac{\text{distance in benzene}}{\text{time}}$

$= \frac{\text{distance in air}}{\text{distance in benzene}}$

Distance in benzene $= \frac{\text{distance in air}}{1.504}$

$= 3.32$ cm

11. Angle of incidence $= \tan^{-1}\frac{1}{3.5} = 15.945°$

$_1n_2 = \frac{\sin i}{\sin r}$

$\text{Sin } r = \frac{\sin i}{_1n_2} = \frac{\sin 15.945°}{1.3308} = 0.2064$

$r = \sin^{-1} 0.2064 = 11.913°$

Displacement at bottom $= 5.5 \times \tan 11.913° = 1.16$ cm.

29 Stationary Waves

1. (a) 0.50 m
   (b) 0.20 m
   (c) $v = f\lambda$
   $f = v/\lambda$
   $= 50.0/0.50$
   $= 100$ Hz

2. With four segments, the string will be vibrating in its fourth harmonic.

$f_n = \frac{nv}{2L}$

$= \frac{4 \times 5.0}{1.00}$

$= 20$ Hz

3. (a) $\lambda_n = 2L$
   $\lambda_1 = 2 \times 1.6$
   $= 3.2$ m
   (b) $v = f\lambda$
   $\lambda = \frac{v}{f}$
   $= \frac{340}{480}$
   $= 0.71$ m
   (c) $f_2 = 2 \times f_1$
   $= 2 \times 480$
   $= 960$ Hz

4. $f_1 = \frac{nv}{2L}$

$= \frac{1 \times 340}{2 \times 0.85}$

$= 200$ Hz

$f_2 = nf_1$

$= 2 \times 200$

$= 400$ Hz

5. $f_n = \frac{nv}{4L}$

$= \frac{1 \times 340}{4 \times 0.85}$

$= 100$ Hz

$f_3 = nf_1$

$= 3 \times 100$

$= 300$ Hz

6. $f_n = \frac{nv}{4L}$

$L = \frac{nv}{4f_1}$

$= \frac{1 \times 340}{4 \times 512}$

$= 0.17$ m

7. $f_3 = 3f_1 = 3 \times 256 = 768$ Hz

$f_3 = \frac{nv}{2L}$

$L = \frac{nv}{2f_3}$

$= \frac{1 \times 340}{2 \times 768}$

$= 0.221$ m

8. First resonance occurs as the air space is vibrating in its fundamental mode, one quarter of the wavelength down.
The second resonance occurs as the air space is vibrating in its third harmonic, three-quarters of the wavelength down.
Thus the wavelength is $2 \times (67.6 - 22.0) = 91.2$ cm or 0.912 m.

$v = f\lambda$

$= 384 \times 0.912$

$= 350 \text{ m s}^{-1}$

30 Strings and Pipes

1. $L = n\frac{\lambda}{2}$

$\lambda = \frac{2L}{n}$

$= \frac{2 \times 0.75}{1}$

$= 1.5 \text{ m}$

2. $L = n\frac{\lambda}{2}$

$\lambda = \frac{2L}{n}$

$= \frac{2 \times 1.0}{4}$

$= 0.5 \text{ m}$

3. $L = n\frac{\lambda}{2}$

$\lambda = \frac{2L}{n}$

For its fundamental:

$= \frac{2 \times 0.85}{1}$

$= 1.7 \text{ m}$

And for its second harmonic:

$= \frac{2 \times 0.85}{2}$

$= 0.85 \text{ m}$

4. $L = (2n-1)\frac{\lambda}{4}$

$\lambda = \frac{4 \times L}{(2n-1)}$

$= \frac{4 \times 0.85}{(2 \times 1 - 1)}$

$= 3.4$ m for the fundamental and

$\lambda = \frac{4 \times L}{(2n-1)}$

$= \frac{4 \times 0.85}{(2 \times 3 - 1)}$

$= 0.68$ m for the third harmonic

5. $L = (2n-1)\frac{\lambda}{4}$

$\lambda = \frac{4L}{(2n-1)}$

$= \frac{4 \times 2.0}{2 \times 1 - 1}$

$= 8.0 \text{ m}$

$v = f\lambda$

$f = \frac{v}{\lambda}$

$= \frac{340}{8.0}$

$= 42.5 \text{ Hz}$

6. $L = (2n-1)\frac{\lambda}{4}$

$= (2 \times 2 - 1) \times \frac{1.5}{4}$

$= 0.75$ m is the length of the pipe and

$v = f\lambda$

$f = \frac{v}{\lambda}$

$= \frac{340}{1.5}$

$= 226.7$ Hz is the frequency

7. $v = f\lambda$

$\lambda = \frac{v}{f}$

$= \frac{340}{256}$

$= 1.33 \text{ m}$

$L = (2n-1)\frac{\lambda}{4}$

$= (2 \times 1 - 1)\frac{1.33}{4}$

$= 0.33 \text{ m}$

31 Diffraction and Interference

1. $n\lambda = d \sin\theta$

$\sin\theta = \frac{n\lambda}{d}$

$= \frac{2 \times 4.30 \times 10^{-7}}{4.00 \times 10^{-4}}$

$= 2.15 \times 10^{-3}$

$\theta = 0.12^\circ$

2. $n\lambda = \dfrac{dx}{L}$

$x = \dfrac{n\lambda L}{d}$

$= \dfrac{5\times 5.40\times 10^{-7}\times 1.20}{6.00\times 10^{-5}}$

$= 5.40\times 10^{-2}$ m

3. $\text{Sin}\,\theta = \dfrac{m\lambda}{d}$

$= \dfrac{1\times 6.328\times 10^{-7}}{1.0\times 10^{-6}}$

$= 6.328\times 10^{-1}$

$\theta = 39.3°$

4. The angular distance from the centre of the first bright lines is $60° \div 2 = 30°$.

$\text{Sin}\ \theta = \dfrac{m\lambda}{d}$

$\lambda = \dfrac{d\sin\theta}{m}$

$= \dfrac{1.50\times 10^{-6}}{1}\times \sin 30°$

$= 7.5\times 10^{-7}$ m

5. 6000 lines/cm equates to the distance apart of the lines being 1.67×10^{-6} m.

$\text{Sin}\ \theta = \dfrac{m\lambda}{d}$

$= \dfrac{1\times 4.050\times 10^{-7}}{1.67\times 10^{-6}}$

$= 2.43\times 10^{-1}$

$\theta = 14.1°$

6. $\text{Sin}\,\theta = \dfrac{m\lambda}{d}$

$\lambda = \dfrac{d\sin\theta}{m}$

$= \dfrac{5.0\times 10^{-6}\sin 28.7°}{5}$

$= 4.8\times 10^{-7}$ m

32 The Inverse Square Law

1. Diameter α distance.
30 m is 3 times 10 m.
Thus the diameter of the beam at 30 m is 3 times its diameter at 10 m,
i.e. it is $3\times 1 = 3$ m.

2. Width α distance.
1.500 m is 5 times 0.300 m.
Thus the distance of the projector from the screen should be 5 times its original distance,
i.e. it is $5\times 0.500 = 2.5$ m.

3. $I \propto \dfrac{1}{d^2}$

Since distance is doubled, intensity is $I \propto \dfrac{1}{2^2}$,
i.e. the intensity is one quarter of the original.

4. $I \propto \dfrac{1}{d^2}$

$\dfrac{I_1}{I_2} = \dfrac{d_2^{\,2}}{d_1^{\,2}}$

$d_2^{\,2} = \dfrac{I_1}{I_2}\times d_1^{\,2}$

$= \dfrac{3}{2}\times 0.5^2$

$= 0.375$

$d = \sqrt{0.375}$

$= 0.61$ m

5. Since the next is 100 m away and the next after that is 200 m away, the distance is doubled.
Thus the intensity of light from the 3rd pole is $\dfrac{1}{2^2}$ $(=\dfrac{1}{4})$ the intensity of the light from the 2nd pole.
The exposure would need to be 4 times as much,
i.e. $4\times 0.5 = 2.0$ s.

6. Since the intensity of the light was reduced to $\dfrac{1}{16}$,
the book must be 4 times $(\dfrac{1}{16} = \dfrac{1}{4^2})$ as far away,
i.e. $4\times 2 = 8$ m.

7. $I \propto \dfrac{1}{d^2}$

$\dfrac{I_1}{I_2} = \dfrac{d_2^{\,2}}{d_1^{\,2}}$

$I_2 = \dfrac{I_1 d_1^{\,2}}{d_2^{\,2}}$

$= \dfrac{1400\times(1.50\times 10^{11})^2}{(2.28\times 10^{11})^2} = 605.96$

Thus Mars received 606 W m^{-2}.

8. $\dfrac{I_1}{I_2} = \dfrac{d_2^{\,2}}{d_1^{\,2}}$

$d_2 = \sqrt{\dfrac{I_1 d_1^{\,2}}{I_2}}$

$= \sqrt{\dfrac{1400\times(1.50\times 10^{11})^2}{700}}$

$= 2.12\times 10^{11}$ m

9. $I \propto \frac{1}{d^2}$

$$\frac{I_1}{I_2} = \frac{d_2^{\ 2}}{d_1^{\ 2}}$$

$$I_2 = \frac{I_1 \times d_1^{\ 2}}{d_2^{\ 2}}$$

$$= \frac{1600 \times 4^2}{6.25^2} = 655.36 \text{ W m}^{-2}$$

$$= 655 \text{ W m}^{-2}$$

10. $I \propto \frac{1}{d^2}$

$$\frac{I_1}{I_2} = \frac{d_2^{\ 2}}{d_1^{\ 2}}$$

$$d_2^{\ 2} = \frac{I_1}{I_2} \times d_1^{\ 2}$$

$$= \frac{1600}{900} \times 5.00^2$$

$$d = \sqrt{\frac{1600}{900} \times 5.00^2}$$

$$= 6.67 \text{ m}$$

33 Mass and Weight

1. $F = mg = 5.0 \times 9.8 = 49$ N

2. $m = \frac{F}{g} = \frac{200}{9.8} = 20.4$ kg

3. $F = mg = 70 \times 9.8 = 686$ N

4. (a) $\Sigma F = ma = 950 \times 5.0 = 4750$ N
   (b) $F = mg = 950 \times 9.8 = 9310$ N
   (c) Total $F = 4750 + 9310 = 14\ 060$ N

5. Total force $= ma + mg$

$$= 90 \times 50 + 90 \times 9.8 = 5382 \text{ N}$$

6. $F = mg = 110 \times \frac{1}{6} \times 9.8 = 179.67$ N

7. Total force $= ma + mg = m(a + g)$

$$= 95[5 \times 9.8 + 1 \times 9.8] = 5586 \text{ N}$$

8. Because her 'weight' fell to half its original value, the downward force on her body must have been halved, i.e. 'gravity' was effectively 4.9 m s^{-2}. Thus the acceleration must have been 4.9 m s^{-2} down.

9. (a) Original weight force:
   $F = mg = 800 \times 9.8 = 7840$ N
   (b) Weight force during initial lift:
   $F = mg = 850 \times 9.8 = 8330$ N
   (c) Force during initial lift was due to load's own weight plus force needed to accelerate the load to lifting speed.
   (d) Force needed to accelerate the load to lifting speed $= 8330 - 7840 = 490$ N

$$a = \frac{\Sigma F}{m} = \frac{490}{800} = 0.6125 \text{ m s}^{-2}$$

10. Speed after 5 seconds: $(v) = u + at$

$$= 0 + 9.8 \times 5$$

$$= 49 \text{ m s}^{-1}$$

Deceleration: $(a) = \frac{v - u}{t}$

$$= \frac{0 - 49}{3} = -16.3 \text{ m s}^{-2}$$

Force: $(\Sigma F) = ma$

$$= 75 \times (-16.3) = -1225 \text{ N}$$

or 1225 N retarding force

Normal weight: $(F) = mg$

$$= 75 \times 9.8 = 735 \text{ N}$$

Thus the retarding force is 1.67 times the person's normal weight.

34 Motion in the Earth's Gravitational Field

1. $v = u + at = 0 + (-9.8) \times 5.0 = -49$

$= 49$ m s^{-1} down

2. $\Delta y = u_y t + \frac{1}{2} a_y t^2$

$$-490 = 0 \times t + \frac{1}{2}(-9.8)t^2$$

$$t = \sqrt{\frac{-490}{\frac{1}{2}(-9.8)}} = 10 \text{ s}$$

3. When the stone begins to fall back, its velocity is 0.

$v = u + at$

$$t = \frac{v - u}{a} = \frac{0 - 24.5}{(-9.8)} = 2.5 \text{ s}$$

4. To just reach the roof, the speed of the brick would be 0.

$$v_y^{\ 2} = u_y^{\ 2} + 2a_y \Delta y$$

$$u_y^{\ 2} = v_y^{\ 2} - 2a_y \Delta y = 0 - 2 \times (-9.8) \times 4.9 = 96.04$$

$$u_y = \sqrt{96.04} = 9.8 \text{ m s}^{-1}$$

5. At the top of its trajectory, the rocket's speed would be 0.

$v_y^2 = u_y^2 + 2a_y\Delta y$

$u_y^2 = v_y^2 - 2a_y\Delta y = 0 - 2\times(-9.8)\times 240 = 4704$

$u_y = \sqrt{4704} = 68.6 \text{ m s}^{-1}$

6. When the rocket falls back to Earth, its speed would be -98 m s^{-1}.

$v = u + at$

$t = \frac{v-u}{a} = \frac{(-98) - 98}{(-9.8)} = 20 \text{ s}$

7. At the moment he dropped his keys, he and the keys were moving down at 4.9 m s^{-1}.

$\Delta y = u_y t + \frac{1}{2}a_y t^2$

$98.0 = 4.9\times t + \frac{1}{2}\times(-9.8)t^2$

Rearranging and dividing through by 4.9:

$20 = t + t^2$

$t^2 + t - 20 = 0$

$(t+5)(t-4) = 0$

$t = -5$ or 4

Obviously, time is 4 seconds.

8. At the moment the marker was dropped, the balloon and the marker were rising at 10 m s^{-1}.

$\Delta y = u_y t + \frac{1}{2}a_y t^2$

$= 10\times 7.0 + \frac{1}{2}\times(-9.8)(7.0)^2$

$= -170.1 \text{ m}$

That is, the balloon was 170.1 m above the ground.

9. The speed down = (–) the speed up

i.e. $v = -u$

$v = u + at$

$(-u) = u + at$

$u = \frac{-at}{2} = \frac{-(-9.8)\times 6.0}{2} = 29.4 \text{ m s}^{-1}$

10. $\Delta y = u_y t + \frac{1}{2}a_y t^2$

$-49.0 = 19.6\times t + \frac{1}{2}(-9.8)t^2$

Dividing through by 4.9 and rearranging:

$t^2 - 4t - 10 = 0$

$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4\times 1\times(-10)}}{2\times 1}$

$= \frac{4 - 7.48}{2} = 5.74$ or -1.74

Obviously, the time is 5.74 s.

35 Projectile Motion

1. (a) $v_y^2 = u_y^2 + 2a_y\Delta y$

$\Delta y = \frac{v_y^2 - u_y^2}{2a_y} = \frac{0 - 20^2}{2(-9.8)} = 20.41 \text{ m}$

(b) When it returns to its original position,

$v_y = -u_y$

$v = u + at$

$t = \frac{v-u}{a} = \frac{(-20) - (20)}{(-9.8)} = 4.08 \text{ s}$

(c) At this point, $v_y = -20 \text{ m s}^{-1}$ or 20 m s^{-1} down.

2. $u_x = u\cos\theta = 15.0\cos 45° = 10.61 \text{ m s}^{-1}$

$u_y = u\sin\theta = 15.0\sin 45° = 10.61 \text{ m s}^{-1}$

3. (a) $u_y = 12.5\sin 60° = 10.83 \text{ m s}^{-1}$

(b) When it returns to the level from which it came, $v_y = (-u_y)$

$v = u + at$

$t = \frac{v-u}{a} = \frac{(-10.83) - 10.83}{(-9.8)} = 2.21 \text{ s}$

4. (a) $u_y = u\sin\theta = 16\times\sin 40° = 10.28 \text{ m s}^{-1}$

(b) $v_y = (-u_y)$

$v = u + at$

$t = \frac{v-u}{a} = \frac{(-10.28) - 10.28}{(-9.8)} = 2.10 \text{ s}$

(c) $u_x = u\cos\theta = 16.0\times\cos 40° = 12.26 \text{ m s}^{-1}$

(d) $\Delta x = u_x t$

$= 12.26\times 2.10 = 25.75 \text{ m}$

5. At its greatest height, $v_y = 0$

$v_y^2 = u_y^2 2a_y\Delta y$

$\Delta y = \frac{v_y^2 - u_y^2}{2a} = \frac{0 - (350\sin 10.0°)^2}{2\times(-9.8)} = 188.46 \text{ m}$

6. When the ball hits the ground, $v_y = (-u_y) = (-u\sin 15°)$

$v = u + at$

$t = \frac{v-u}{a}$

$3.0 = \frac{(-u\sin 15°) - u\sin 15°}{(-9.8)}$

$u = \frac{3.0\times 9.8}{2\times\sin 15°} = 56.80 \text{ m s}^{-1}$

7. (a) $u_x = 20 \text{ m s}^{-1}$

$u_y = 0$

$\Delta y = u_y t + \frac{1}{2}a_y t^2 = 0 + (0.5)(-9.8)(2.5)^2$

$= -30.63 \text{ m}$

Thus the height of the cliff was 30.63 m.

(b) Horizontal distance travelled by the stone $= 20 \times 2.5 = 50$ m.

(c) The horizontal velocity of the projectile $= 20 \text{ m s}^{-1}$.

The vertical velocity of the projectile:

$v = u + at = 0 + (-9.8) \times 2.5 = -24.5$

or 24.5 m s^{-1} down.

The combined velocity of the projectile as it hits the water:

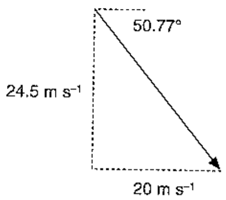
$v^2 = u_x{}^2 + v_y{}^2 = (-24.5)^2 + (20)^2 = 1000.25$

$v = \sqrt{1000.25} = 31.63 \text{ m s}^{-1}$

The angle at which it hit the water:

$\text{Tan}\ \theta = \dfrac{u_y}{u_x} = \dfrac{-24.5}{20} = -1.225$

$\theta = -50.77°$



Thus the velocity of the stone as it hits the water was 31.63 m s^{-1} at an angle of depression of $50.77°$.

8. (a) $u_x = u \cos\theta = 18 \cos 30° = 15.59 \text{ m s}^{-1}$

$u_y = u \sin\theta = 18 \sin 30° = 9.00 \text{ m s}^{-1}$

Time to return to the ground $(v_y = (-u_y))$:

$v = u + at$

$t = \dfrac{v - u}{a} = \dfrac{(-9) - 9}{(-9.8)} = 1.84 \text{ s}$

Horizontal distance travelled in that time:

$\Delta x = 15.59 \times 1.84 = 28.63 \text{ m}$

(b) $u_x = 15.59 \text{ m s}^{-1}$

$u_y = 9.00 \text{ m s}^{-1}$

Time to reach the wall:

$t = 15 \div 15.59 = 0.96 \text{ s}$

Height of the ball after this time:

$\Delta y = u_y t + \frac{1}{2} a_y t^2$

$= 9 \times (0.96) + (0.5) \times (-9.8) \times (0.96)^2 = 4.12 \text{ m}$

36 Circular Motion

1. $F = \dfrac{mv^2}{r} = \dfrac{10 \times (16)^2}{8} = 320 \text{ N}$

2. $F = \dfrac{mv^2}{r}$

$v = \sqrt{\dfrac{Fr}{m}} = \sqrt{\dfrac{270 \times 10}{3.0}} = 30 \text{ m s}^{-1}$

3. $F = \dfrac{mv^2}{r}$

$r = \dfrac{mv^2}{F} = \dfrac{2.0 \times (6.0)^2}{12.0} = 6 \text{ m}$

4. $F = \dfrac{mv^2}{r}$

$m = \dfrac{Fr}{v^2} = \dfrac{25.0 \times 1.0}{10^2} = 0.25 \text{ kg}$

5. $F = \dfrac{mv^2}{r} = \dfrac{30 \times (5)^2}{10.0} = 75 \text{ N}$

6. $F = \dfrac{mv^2}{r} = \dfrac{80 \times (15)^2}{15}$

$= 1.20 \times 10^3 \text{ N}$

7. $F = \dfrac{mv^2}{r}$

$F = ma = 2000 \times 9.8 = 19600$

$v = \sqrt{\dfrac{Fr}{m}} = \sqrt{\dfrac{19600 \times 7.2 \times 10^6}{2000}}$

$= 8400 \text{ m s}^{-1}$

8. $F = \dfrac{mv^2}{r} = \dfrac{600 \times (30)^2}{50.0} = 1.08 \times 10^4 \text{ N}$

9. $F = \dfrac{mv^2}{r}$

$v = \sqrt{\dfrac{Fr}{m}} = \sqrt{\dfrac{40.0 \times 2.0}{0.50}} = 12.65 \text{ m s}^{-1}$

10. Centripetal force = weight of hanging mass carrier

$= mg = 100 \times 10^{-3} \times 9.8 = 0.98 \text{ N} = \dfrac{mv^2}{r}$

$v = \sqrt{\dfrac{Fr}{m}} = \sqrt{\dfrac{0.98 \times 50 \times 10^{-2}}{50 \times 10^{-3}}} = 3.13 \text{ m s}^{-1}$

37 Universal Gravitation

1. $F = \dfrac{Gm_1m_2}{d^2}$

$= \dfrac{6.67 \times 10^{-11} \times 4.00 \times 10^3 \times 9.00 \times 10^3}{6.00^2} = 6.67 \times 10^{-5} \text{ N}$

2. $F = \dfrac{Gm_1m_2}{d^2}$

$= \dfrac{6.67 \times 10^{-11} \times 1.2 \times 10^2 \times 1.2 \times 10^2}{\left(1.6 \times 10^{-2}\right)^2} = 3.75 \times 10^{-3} \text{ N}$

3. $F = \dfrac{Gm_1m_2}{d^2}$

$d = \sqrt{\dfrac{Gm_1m_2}{F}}$

$= \sqrt{\dfrac{6.67 \times 10^{-11} \times 3.6 \times 10^3 \times 2.8 \times 10^3}{1.0 \times 10^{-4}}} = 2.59 \text{ m}$

4. $F = \dfrac{Gm_1m_2}{d^2}$

$$m_2 = \frac{Fd^2}{Gm_1} = \frac{1.5 \times 10^{-8} \times \left(5.0 \times 10^{-1}\right)^2}{6.67 \times 10^{-11} \times 7.5 \times 10^2} = 7.5 \times 10^{-2}\ \text{kg}$$

5. $F = \dfrac{Gm_1m_2}{d^2}$

In this case $m_1 = 2m_2$

$$F = \frac{G \times 2 \times m_2 \times m_2}{d^2}$$

$$m_2 = \sqrt{\frac{Fd^2}{2G}}$$

$$= \sqrt{\frac{4.8 \times 10^{-6} \times \left(6.0 \times 10^{-2}\right)^2}{2 \times 6.67 \times 10^{-11}}} = 11.4\ \text{kg}$$

6. $F = \dfrac{Gm_1m_2}{d^2}$

$$= \frac{6.67 \times 10^{-11} \times 5.977 \times 10^{24} \times 1.000}{\left(6.378 \times 10^6\right)^2} = 9.8003\ \text{N}$$

7. $F = \dfrac{Gm_1m_2}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 5.977 \times 10^{24} \times 1.000}{\left(6.378 \times 10^6 + 1000 \times 10^3\right)^2} = 7.324\ \text{N}$$

8. 7.324 N

9. From the formula:

$$F \propto \frac{1}{d^2} \text{ or } d \propto \sqrt{\frac{1}{F}}$$

So, if the force is to be halved, then the distance from the centre of the Earth would be increased by a factor of $\sqrt{\dfrac{1}{\frac{1}{2}}}$ or $\sqrt{2}$.

So its distance from the centre of the Earth $= 6.378 \times 10^6 \times \sqrt{2} = 9.020 \times 10^6$ m

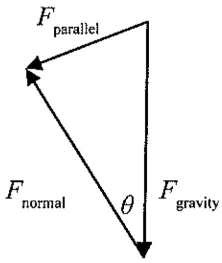
And its height above the Earth's surface $= 9.020 \times 10^6 - 6.378 \times 10^6 = 2.642 \times 10^6$ m

10. $F = \dfrac{Gm_1m_2}{d^2}$

$$= \frac{6.67 \times 10^{-11} \times 1.991 \times 10^{30} \times 5.977 \times 10^{24}}{\left(1.496 \times 10^{11}\right)^2}$$

$= 3.546 \times 10^{22}$ N

38 Inclined Planes



1. $F_{parallel}$(force parallel to plane)
$= mg \sin\theta$
$= 10 \times 9.8 \times \sin 30°$
$= 49$ N
$a = F/m$
$= 49/10$
$= 4.9$ m s^{-2}

2. $F_{parallel} = mg \sin\theta - F_{friction}$
$= 5 \times 9.8 \times \sin 45° - 2$
$= 32.6$ N
$a = F/m$
$= 32.6/5$
$= 6.53$ m s^{-2}
$s = ut + \frac{1}{2}at^2$
$= 0 \times 2 + \frac{1}{2} \times 6.53 \times 2^2$
$= 13$ m

3. $F_{parallel} = mg \sin\theta - F_{friction}$
$0 = 2 \times 9.8 \times \sin 30° - F_{friction}$
$F_{friction} = 9.8$ N

4. At the point where the object has just overcome friction the force down the plane = the friction force
$F_{parallel} = mg \sin\theta - F_{friction}$
$F_{friction} = 1 = mg \sin\theta$
$\sin\theta = 1/mg = 1/(1.0 \times 9.8)$
$= 0.102$
$\theta = 5.9°$

5. Resultant force parallel to plane
= weight $- mg \sin\theta$
$F_{parallel} = Mg - mg \sin\theta$
$= 2 \times 9.8 - 5 \times 9.8 \times \sin 60°$
$= -22.9$ N
$a = F/m$
$= -22.9/(5 + 2)$
$= -3.26$
The object will accelerate at 3.3 m s^{-2} down the plane.

6. Resultant force down plane
$= mg \sin\theta - F_{friction} - Mg$
$= 5 \times 9.8 \times \sin 45° - 2 - 3 \times 9.8$
$= 3.25$ N
$a = F/m$
$= 3.25/(5 + 3)$
$= 0.41$
$v = u + at$
$= 0 + 0.413 \times 1$
$= 0.4$ m s^{-1}

39 Coulomb's Law

1. $F = \frac{kq_1q_2}{r^2}$

$= \frac{9\times10^9 \times 3.0\times4.0}{2.0^2}$

$= 2.7\times10^{10}$ N

2. $F = \frac{kq_1q_2}{r^2}$

$= \frac{9\times10^9 \times 6.0\times10^{-8} \times 2.7\times10^{-8}}{3.0\times10^{-2}}$

$= 0.016$ N

3. $F = \frac{kq_1q_2}{r^2}$

$r = \sqrt{\frac{kq_1q_2}{F}}$

$= \sqrt{\frac{9\times10^9 \times 4.0\times10^{-9} \times 8.0\times10^{-9}}{1.8\times10^{-4}}}$

$= 0.040$ m

4. $F = \frac{kq_1q_2}{r^2}$

$q_2 = \frac{Fr^2}{kq_1}$

$= \frac{9.6\times10^{-1} \times (5.0\times10^{-2})^2}{9\times10^9 \times 1.2\times10^{-8}}$

$= 2.2\times10^{-5}$ C

5. $F = \frac{kq_1q_2}{r^2}$

$q_1 = \sqrt{\frac{Fr^2}{k}}$

$= \sqrt{\frac{8.1\times\left(2.0\times10^{-2}\right)^2}{9\times10^9}}$

$= 6.0\times10^{-7}$ C

6. $F = \frac{kq_1q_2}{r^2}$

$q_1 = \sqrt{\frac{Fr^2}{k\times4}}$

$= \frac{0.016\times\left(4.5\times10^{-2}\right)^2}{9\times10^9\times4}$

$= 3.0\times10^{-8}$ C

7. $F = mg = 0.005 \times 9.8 = 0.049$

$F = \frac{kq_1q_2}{r^2}$

$r = \sqrt{\frac{9\times10^9\times\left(3.5\times10^{-8}\right)^2}{0.049}}$

$= 0.015$ m

8. $F = mg = 10 \times 10^{-3} \times 9.8 = 0.098$

$F = \frac{kq_1q_2}{r^2}$

$q_1 = \frac{Fr^2}{kq_2}$

$= \frac{0.098\times\left(2.5\times10^{-2}\right)^2}{9\times10^9\times-1.4\times10^{-7}}$

$= -4.9 \times 10^{-8}$ C

9. $F = \frac{kq_1q_2}{r^2}$ (both forces act in the same direction)

$= \frac{9\times10^9\times\left(+2.4\times10^{-8}\right)\times\left(3.6\times10^{-8}\right)}{2.5\times10^{-2}}$

$+ \frac{9\times10^9\times\left(+2.4\times10^{-8}\right)\times\left(4.8\times10^{-8}\right)}{2.5\times10^{-2}}$

$= 7.26 \times 10^{-4}$ N

10. Let the force exerted by C on B be represented by F_{CB} and by A on B by F_{AB}:

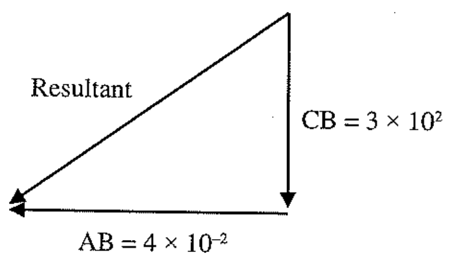
$F = \frac{kq_1q_2}{r^2}$

$F_{CB} = \frac{9\times10^9\times(-2.4\times10^{-7})\times5.0\times10^{-8}}{0.06^2}$

$= 3.0\times10^{-2}$ N down

$F_{AB} = \frac{9\times10^9\times(-8.0\times10^{-8})\times5.0\times10^{-8}}{0.03^2}$

$= 4.0\times10^{-2}$ N to the left



Resultant:
(Right-angled triangle)
$R^2 = 3^2 + 4^2 = 25$
$R = 5$
Answer: 5.0×10^{-2} N.

40 Electric Fields

1. $E = \frac{kq}{r^2}$

$= \frac{9\times10^9\times2.5\times10^{-8}}{0.05^2}$

$= 9.0\times10^4$ N C^{-1}

2. $E = \frac{kq}{r^2}$

$q = \frac{Er^2}{k}$

$= \frac{1.0\times10^5 \times 0.0060^2}{9.00\times10^9}$

$= 4.0\times10^{-10}$ C

3. $E = \frac{kq}{r^2}$

$r^2 = \frac{kq}{E}$

$= \frac{9\times10^9 \times 4.50\times10^{-8}}{1.25\times10^6}$

$r = 1.8\times10^{-2}$ m

4. Let field strength due to A be E_A and that due to B be E_B:

$E = \frac{kq}{r^2}$

$E_A = \frac{9\times10^9 \times 2.0\times10^{-7}}{0.0030^2}$

$= 2.0\times10^8$

$E_B = \frac{9\times10^9 \times 3.0\times10^{-8}}{0.0020^2}$

$= 6.75\times10^7$

Since A is repelling X to the right and B is attracting X to the right, the two fields should be added.

$E_A + E_B = 2.675\times10^8$ N C^{-1} to the right.

5. Let field strength due to A be E_A and that due to B be E_B:

$E = \frac{kq}{r^2}$

$E_A = \frac{9\times10^9 \times 4.8\times10^{-8}}{0.0040^2}$

$= 2.7 \times 10^8$

$E_B = \frac{9\times10^9 \times 3.0\times10^{-9}}{0.0010^2}$

$= 2.7\times10^7$

Since A is repelling X to the right and B is attracting X to the left, the two fields should be subtracted.

$E_A - E_B = 2.43 \times 10^8$ N C^{-1} to the left

6. $F = Eq$

$= 4.0\times10^7 \times 8.0\times10^{-8}$

$= 3.2$ N

7. $F = Eq$

$E = \frac{F}{q}$

$= \frac{1.08}{3.6\times10^{-8}}$

$= 3.0\times10^7$ N C^{-1}

8. $F = Eq$

$= 4.0\times10^8 \times 3.5\times10^{-9}$

$= 1.4$ N

$F = ma$

$a = \frac{F}{m}$

$= \frac{1.4}{0.0060}$

$= 233.3$ m s^{-2}

9. $F = Eq$

$= 1.25\times10^9 \times 6.0\times10^{-8}$

$= 75$ N

$W = Fs$

$= 75\times 0.015$

$= 1.1$ J

10. $F = Eq$

$= 1.0\times10^6 \times 1.6\times10^{-19}$

$= 1.6\times10^{-13}$ N

$a = \frac{F}{m}$

$= \frac{1.6\times10^{-13}}{9.1\times10^{-31}}$

$= 1.8\times10^{17}$ m s^{-2}

41 Electric Potential and Potential Difference

1. $W = qV$

$V = W/q$

$= 144.0/18.0$

$= 8.0$ volts

2. $W = qV$

$= 1.60 \times 10^{-8} \times 320.0 \times 1000$

$= 5.1 \times 10^{-3}$ J

3. $V = kq/r$

$= 9.00 \times 10^9 \times 4.0 \times 10^{-8}/0.02$

$= 1.8 \times 10^4$ V

4. $V = kq/r$

$r = kq/V$

$= 9.00 \times 10^9 \times 6.4 \times 10^{-7}/2.4 \times 10^3$

$= 2.4$ m

5. $W = qV$

$V = W/q$

$= 1.32 \times 10^{-6}/1.1 \times 10^{-7}$

$= 12$ volts

6. $V = W/q$
$= 30.0/5.0$
$= 6$ volts
Potential of point B = +18 volts

7. $W = qV$
$= 3.0 \times 10^{-7} \times 250$
$= 7.5 \times 10^{-5}$ J

8. $E = V/d$
$= 100/0.0020$
$= 5.0 \times 10^{4}$ V m^{-1}

9. (a) $W = qV$
$= 1.6 \times 10^{-19} \times 1000.0$
$= 1.6 \times 10^{-16}$ J

(b) $W = KE = \frac{1}{2}mv^2$
$$v = \sqrt{\frac{2 \times 1.60 \times 10^{-16}}{9.1 \times 10^{-31}}}$$
$= 1.8 \times 10^{7}$ m s^{-1}

10. $E = V/d$
$= 150/0.005$
$= 3.00 \times 10^{4}$ V m^{-1}
$F = Eq$
$= 3.00 \times 10^{4} \times 3 \times 1.6 \times 10^{-19}$
$= 1.44 \times 10^{-14}$ N

42 Magnetic Induction

1. $B = \frac{kI}{r}$
$$= \frac{2.00 \times 10^{-7} \times 48}{3.0}$$
$= 3.2 \times 10^{-6}$ teslas

2. $B = \frac{kI}{r}$
$$I = \frac{Br}{k} = \frac{6.0 \times 10^{-5} \times 3.0 \times 10^{-2}}{2.00 \times 10^{-7}}$$
$= 9.0$ A

3. $B = \frac{kI}{r}$
$$r = \frac{kI}{B} = \frac{2.00 \times 10^{-7} \times 36}{1.8 \times 10^{-4}}$$
$= 4.0 \times 10^{-2}$ m

4. The field of each wire will be in the same direction, and thus will be added. Let the field intensity of the 15 A current be B_{15} and that of the 18 A current be B_{18}.
$$B_{15} = \frac{2.00 \times 10^{-7} \times 15}{0.003} = 10 \times 10^{-4} \text{ T}$$
$$B_{18} = \frac{2.00 \times 10^{-7} \times 18}{0.003} = 12 \times 10^{-4} \text{ T}$$
$$B_{15} + B_{18} = 22 \times 10^{-4} = 2.2 \times 10^{-3} \text{ T}$$

5. (a) The field of each conductor will be in the same direction, and thus will be added. Let the field intensity of the 10 A current be B_{10} and that of the 5 A current be B_5.
$$B_{10} = \frac{2.00 \times 10^{-7} \times 10}{0.015} = 1.3 \times 10^{-4} \text{ T}$$
$$B_5 = \frac{2.00 \times 10^{-7} \times 5}{0.005} = 2.0 \times 10^{-4} \text{ T}$$
$$B_{10} + B_5 = 3.3 \times 10^{-4} \text{ T}$$

(b) The field of each conductor will be in the same direction, and thus will be added. Let the field intensity of the 2 A current be B_2 and that of the 6 A current be B_6.
$$B_2 = \frac{2.00 \times 10^{-7} \times 2.0}{0.0045} = 8.9 \times 10^{-5} \text{ T}$$
$$B_6 = \frac{2.00 \times 10^{-7} \times 6.0}{0.0015} = 8.0 \times 10^{-4} \text{ T}$$
$$B_2 + B_6 = 8.9 \times 10^{-5} + 8.0 \times 10^{-4}$$
$$= 0.89 \times 10^{-4} + 8.0 \times 10^{-4}$$
$$= 8.9 \times 10^{-4} \text{ T}$$

43 Ampere's Law

1. $\frac{F}{L} = k\frac{I_1 I_2}{d}$
$$= 2 \times 10^{-7} \times \frac{24 \times 32}{0.16}$$
$= 9.6 \times 10^{-4}$ N m^{-1}

2. $\frac{F}{L} = \frac{kI_1 I_2}{d}$
$$F = \frac{kI_1 I_2 L}{d}$$
$$= \frac{2 \times 10^{-7} \times 12.0 \times 12.0 \times 2.0}{2.0 \times 10^{-3}} = 2.88 \times 10^{-2} \text{ N}$$

3. $\frac{F}{L} = k\frac{I_1 I_2}{d}$
$$I_2 = \frac{Fd}{kI_1 L}$$
$$= \frac{1.2 \times 10^{-4} \times 4.0 \times 10^{-3}}{2 \times 10^{-7} \times 2.4 \times 2.5} = 0.4 \text{ A}$$

4. $\frac{F}{L} = k\frac{I_1 I_2}{d}$
$$d = k\frac{I_1 I_2 L}{F}$$
$$= 2 \times 10^{-7} \times \frac{15 \times 15 \times 750 \times 10^{-3}}{4.5 \times 10^{-3}}$$
$= 7.5 \times 10^{-3}$ m

5. $\frac{F}{L} = k\frac{I_1 I_2}{d}$

$L = \frac{Fd}{kI_1 I_2}$

$= \frac{1.8 \times 10^{-3} \times 15 \times 10^{-3}}{2 \times 10^{-7} \times 4.5 \times 3.6} = 8.33 \text{ m}$

6. In this case, $I_1 = I_2$.

$\frac{F}{L} = k\frac{I_1 I_2}{d}$

$I_1 = \sqrt{\frac{F}{L} \times \frac{d}{k}}$

$= \sqrt{6.4 \times 10^{-3} \times \frac{8.0 \times 10^{-3}}{2 \times 10^{-7}}} = 16 \text{ A}$

7. Force of repulsion = $mg = 0.01 \times 10^{-3} \times 9.8$

$= 9.8 \times 10^{-5}$ N

$\frac{F}{L} = \frac{kI_1 I_2}{d}$

Since the current is the same in both conductors:

$\frac{F}{L} = \frac{kI^2}{d}$

$I = \sqrt{\frac{Fd}{kL}} = \sqrt{\frac{9.8 \times 10^{-5} \times 15 \times 10^{-3}}{2 \times 10^{-7} \times 20 \times 10^{-2}}} = 6.06 \text{ A}$

44 Magnetic Forces on Currents

1. $F = BIL \sin \theta$

$= 2.0 \times 8.0 \times 5.0 \times \sin 90^\circ = 80 \text{ N}$

2. $F = BIL \sin \theta$

$B = \frac{F}{IL \sin \theta}$

$= \frac{1.6}{2.5 \times 64 \times 10^{-3} \times \sin 90^\circ} = 10 \text{ T}$

3. $F = BIL \sin \theta$

$L = \frac{F}{BI \sin \theta}$

$= \frac{1.8}{0.15 \times 1.2 \times \sin 90^\circ} = 10 \text{ m}$

4. $F = BIL \sin \theta$

$= 120 \times 10^{-3} \times 18 \times 10^{-3} \times 250 \times 10^{-3} \times \sin 60^\circ$

$= 4.68 \times 10^{-4}$ N

5. $F = BIL \sin \theta$

$\text{Sin } \theta = \frac{F}{BIL}$

$= \frac{3.2 \times 10^{-2}}{6.4 \times 10^{-2} \times 8.0 \times 9.6 \times 10^{-2}} = 0.651$

$\theta = 40.6^\circ$

6. Force needed $= 8.4 \times 10^{-3} \times 9.8 = 0.08232$ N

$F = BIL \sin \theta$

$I = \frac{F}{BL \sin \theta}$

$= \frac{0.08232}{3.5 \times 10^{-2} \times 4.9 \times \sin 90^\circ} = 0.48 \text{ A}$

7. $F = Bqv \sin \theta$

$= 16 \times 45 \times 12 \times \sin 90^\circ$

$= 8.64 \times 10^3$ N

8. $F = Bqv \sin \theta$

$= 3.6 \times 10^{-8} \times 6.4 \times 10^{-1} \times 4.8 \times 10^{-2} \times \sin 50^\circ$

$= 8.47 \times 10^{-13}$ N

9. $F = Bqv \sin \theta$

$= 1.6 \times 10^{-19} \times 2.5 \times 10^6 \times 4.0 \times 10^{-1} \times \sin 90^\circ$

$= 1.6 \times 10^{-13}$ N

10. $F = qvB \sin \theta$

$v = \frac{F}{Bq \sin \theta}$

$= \frac{1.3 \times 10^{-10}}{7.2 \times 1.6 \times 10^{-19} \times \sin 35^\circ} = 1.97 \times 10^8 \text{ m s}^{-1}$

45 Torque on Coils

1. $\tau = Fd$

$= 7.5 \times 0.40 = 3.0$ N m

2. $F = \frac{\tau}{d}$

$= \frac{36}{5.4} = 6.67 \text{ N}$

3. $d = \frac{\tau}{F}$

$= \frac{42}{28} = 1.5 \text{ m}$

4. (a) $d = 0.03 \sin 45^\circ = 0.0212$ m

(b) $\tau = Fd = 25 \times 0.0212 = 0.53$ N m

5. The two forces form a couple, the result of which is zero torque:

$\tau_1 + \tau_2 = 0$

$F_1 d_1 + F_2 d_2 = 0$

$F_2 = \frac{-F_1 d_1}{d_2}$

$= \frac{-20 \times 15}{5} = -60$ N or 60 N down

6. $\tau = nBIA\cos\theta$
$= 10\times4\times10^{-3}\times25\times10^{-3}$
$\times(50\times10^{-3}\times20\times10^{-3})\times\cos 0°$
$= 1\times10^{-6}$ N m

7. $B = \dfrac{\tau}{nIA\cos\theta}$

$= \dfrac{1.5\times10^{-1}}{50\times2.0\times1.2\times10^{-3}\times\cos 60°} = 2.5$ T

8. $I = \dfrac{\tau}{nBA\cos\theta}$

$= \dfrac{0.50}{250\times1.6\times\pi\times(75\times10^{-3})^2\times\cos 0°} = 0.071$ A

9. $\cos\theta = \dfrac{\tau}{nBIA}$

$= \dfrac{1.2}{24\times0.36\times5.0\times\pi(0.10)^2} = 0.8842$

$\theta = 27.8°$

10. $I = \dfrac{\tau}{nBA\cos\theta}$

$= \dfrac{0.0064}{60\times0.080\times(12\times10^{-3}\times16\times10^{-3})\cos 0°}$

$= 6.944$ A

46 Flux, Flux Density and Induced Emf

1. $\phi = BA$
$= 3.0\times1.2$
$= 3.6$ Wb

2. $\phi = BA$
$A = \dfrac{\phi}{B}$
$= \dfrac{8.4}{1.2}$
$= 7.0\ \text{m}^2$

3. $V_i = \dfrac{-n\Delta\phi}{\Delta t}$
$= \dfrac{-1\times6.4}{1.6}$
$= -4.0$ V

4. $\phi = BA$
$= 3.0\times2.4$
$= 7.2$ Wb
$V_i = \dfrac{-n\Delta\phi}{\Delta t}$
$= \dfrac{-1\times7.2}{2.5}$
$= 2.9$ V

5. $V_i = \dfrac{-n\Delta\phi}{\Delta t}$
$= \dfrac{-240\times3.2}{4.8}$
$= -160$ V

6. $\phi = BA$
$= 0.20\times0.80$
$= 0.16$ Wb
$V_i = \dfrac{-n\Delta\phi}{\Delta t}$
$= \dfrac{-150\times0.32}{3.0}$
$= 16$ V

7. $V_i = BLv$
$= 5.0\times2.0\times3.0$
$= 30$ V

8. $V_i = BLv$
$B = \dfrac{V_i}{Lv}$
$= \dfrac{1.5}{0.075\times20.0}$
$= 1.0$ T

47 Transformers

1. $\dfrac{V_S}{V_P} = \dfrac{n_S}{n_P}$
$V_S = \dfrac{n_S}{n_P}\times V_P$
$= \dfrac{1000}{250}\times80$
$= 320$ V

2. $\dfrac{V_S}{V_P} = \dfrac{n_S}{n_P}$
$V_P = \dfrac{n_P\times V_S}{n_S}$
$= \dfrac{450\times120}{600}$
$= 90$ V

3. $\dfrac{V_S}{V_P} = \dfrac{n_S}{n_P}$
$V_S = \dfrac{n_S\times V_P}{n_P}$
$= \dfrac{10\times500}{1}$
$= 5000$ V

4. $$\frac{n_S}{n_P} = \frac{I_P}{I_S}$$

$$I_S = \frac{n_P \times I_P}{n_S}$$

$$= \frac{4000 \times 5}{2500}$$

$$= 8 \text{ A}$$

5. $$\frac{V_S}{V_P} = \frac{I_P}{I_S}$$

$$I_S = \frac{V_P \times I_P}{V_S}$$

$$= \frac{32 \times 36}{240}$$

$$= 4.8 \text{ A}$$

6. $$\frac{n_S}{n_P} = \frac{I_P}{I_S}$$

$$n_P = \frac{n_S \times I_S}{I_P}$$

$$= \frac{600 \times 20}{4}$$

$$= 3000 \text{ turns}$$

7. $$\frac{V_S}{V_P} = \frac{I_P}{I_S}$$

$$I_S = \frac{V_P \times I_P}{V_S}$$

$$= \frac{240 \times 0.25}{12}$$

$$= 5 \text{ A}$$

8. $$\frac{n_S}{n_P} = \frac{I_P}{I_S}$$

$$I_P = \frac{n_S \times I_S}{n_P}$$

$$= \frac{1 \times 15}{3}$$

$$= 5 \text{ A}$$

48 Relativity

1. $$L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$= 12\sqrt{\left(1 - \frac{0.8^2}{1.0^2}\right)} = 7.2 \text{ m}$$

2. $$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$= \frac{2.0}{\sqrt{\left(1 - \frac{0.8^2}{1.0^2}\right)}} = 3.33 \text{ s}$$

3. $$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$= \frac{9.1 \times 10^{-31}}{\sqrt{\left(1 - \frac{0.9^2}{1.0^2}\right)}} = 2.09 \times 10^{-30} \text{ kg}$$

4. $$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$\sqrt{\left(1 - \frac{v^2}{c^2}\right)} = \frac{m_0}{m}$$

$$\left(1 - \frac{v^2}{c^2}\right) = \left(\frac{m_0}{m}\right)^2$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{m_0}{m}\right)^2$$

$$v^2 = c^2\left(1 - \frac{m_0^{\ 2}}{m^2}\right)$$

$$v = c\sqrt{1 - \frac{m_0^{\ 2}}{m^2}} = 3 \times 10^8\sqrt{1 - \frac{(1.67 \times 10^{-27})^2}{(2.5 \times 10^{-27})^2}}$$

$$= 2.23 \times 10^8 \text{ m s}^{-1}$$

5. $$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$\sqrt{\left(1 - \frac{v^2}{c^2}\right)} = \frac{m_0}{m}$$

$$\left(1 - \frac{v^2}{c^2}\right) = \left(\frac{m_0}{m}\right)^2$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{m_0}{m}\right)^2$$

$$v^2 = c^2\left(1 - \frac{m_0^{\ 2}}{m^2}\right)$$

$$v = c\sqrt{1 - \frac{m_0^{\ 2}}{m^2}} = 3 \times 10^8\sqrt{1 - \left(\frac{1}{10}\right)^2} = 2.98 \times 10^8 \text{ m s}^{-1}$$

6. Distance to star = 10 light years.

Speed of spaceship = 10 light years , 12 years = $\frac{10}{12}c$

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$t_0 = t\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$= 12\sqrt{\left(1 - \frac{10^2}{12^2}\right)} = 6.63 \text{ years}$$

7. $E = mc^2$
$= 16\times10^{-3}\times(3\times10^8)^2$
$= 1.44\times10^{15}$ J

8. $E = mc^2$
$m = \dfrac{E}{c^2}$
$= \dfrac{1.0\times10^6}{(3\times10^8)^2}$
$= 1.11\times10^{-11}$ kg

49 Photons and Quanta

1. $E = hf$
$= 6.6\times10^{-34}\times1.4\times10^{15} = 9.2\times10^{-19}$ J

2. $c = f\lambda$
$f = \dfrac{c}{\lambda}$
$E = hf = h\dfrac{c}{\lambda} = 6.6\times10^{-34}\times\dfrac{3.00\times10^8}{6.0\times10^{-7}} = 3.3\times10^{-19}$ J

3. $c = f\lambda$
$f = \dfrac{c}{\lambda}$
One quantum of red light has energy:
$E = hf = h\dfrac{c}{\lambda} = 6.6\times10^{-34}\times\dfrac{3.00\times10^8}{7.5\times10^{-7}} = 2.64\times10^{-19}$ J
Number of quanta to make one joule:
$= \dfrac{1}{2.64\times10^{-19}} = 3.78\times10^{18}$

4. $E = hf$
$f = \dfrac{E}{h} = \dfrac{7.7\times10^{-28}}{6.6\times10^{-34}} = 1.167\times10^6$ Hz

5. Energy emitted per second = 60 J
Average energy of each photon:
$E = hf = 6.6\times10^{-34}\times5.0\times10^{14} = 3.3\times10^{-19}$ J
Number of photons $= 60 \div 3.3\times10^{-19} = 1.8\times10^{20}$

6. $E = \frac{1}{2}mv^2$
$E = hf$
$\frac{1}{2}mv^2 = hf$
$v^2 = \dfrac{2hf}{m} = \dfrac{2\times6.6\times10^{-34}\times8.4\times10^{15}}{9.1\times10^{-31}} = 1.218\times10^{13}$
$v = 3.49\times10^6$ m s^{-1}

7. $E = hf = h\dfrac{c}{\lambda}$
$\lambda = \dfrac{hc}{E} = \dfrac{6.6\times10^{-34}\times3.0\times10^8}{2.5\times10^{-18}} = 7.92\times10^{-8}$ m

8. $E = hf = h\dfrac{c}{\lambda}$
$\lambda = \dfrac{hc}{E} = \dfrac{6.6\times10^{-34}\times3\times10^8}{9.25\times10^{-22}} = 2.14\times10^{-4}$ m

50 Wien's Displacement Law

1. $\lambda_{max} = b/T$
$= 2.9\times10^{-3}/3100$
$= 9.4\times10^{-7}$ m

2. $32°C = 273 + 32 = 305$ K
$\lambda_{max} = b/T$
$= 2.9\times10^{-3}/305$
$= 9.5\times10^{-6}$ m

3. $\lambda_{max} = b/T$
$T = b/\lambda_{max}$
$= 2.9\times10^{-3}/5.02\times10^{-7}$
$= 5.8\times10^3$ K

51 Extension Questions

1. $v = f\lambda = \dfrac{1}{0.05}\times24.3 = 486$ m s^{-1}

2. $I \propto \dfrac{1}{d^2}$
Since distance is $(\times3)$, intensity is given by:
$I \propto \dfrac{1}{3^2}$
i.e. the intensity is one ninth of the original.

3. (a) $\lambda = \dfrac{v}{f} = \dfrac{15.0\times10^{-2}}{5.0} = 3.0\times10^{-2}$ m
(b) 5.0 Hz (same)
(c) $\lambda = \dfrac{v}{f} = \dfrac{10.0\times10^{-2}}{5.0} = 2.0\times10^{-2}$ m
(d) $n = \dfrac{v_{deep}}{v_{shallow}} = \dfrac{10\times10^{-2}}{15.0\times10^{-2}} = 0.67$

4. $\dfrac{v}{f} = \dfrac{330}{512} = 0.64$ m

5. $I \propto \dfrac{1}{d^2}$
Since intensity is $(\times\frac{1}{9})$, distance is given by:
$\dfrac{1}{9} \propto \dfrac{1}{d^2} \quad \therefore d = 3$
i.e. the distance is three times that of the original.

6. $\lambda = \dfrac{v}{f} = \dfrac{3\times10^8}{702\times10^3} = 427.35 \text{ m} = 4.27\times10^2$ m

7. $n = \dfrac{\sin i}{\sin r} = \dfrac{\sin 10°}{\sin 9°} = 1.11$

8. $I \propto \dfrac{1}{d^2}$
$\dfrac{I_1}{I_2} = \dfrac{d_2^2}{d_1^2}$
$I_2 = \dfrac{I_1\times d_1^2}{d_2^2} = \dfrac{1200\times(5.00)^2}{(7.5)^2} = 533.3$ W m^{-2}

9. $n = \dfrac{\sin i}{\sin r}$
$\sin i = n\times\sin r = 1.504\times\sin 12° = 0.313$
$i = 18.2°$

10. ${}_1n_2 = \dfrac{v_1}{v_2}$
$v_2 = \dfrac{v_1}{{}_1n_2} = \dfrac{3.0\times10^8}{1.504} = 1.99\times10^8$ m s^{-1}

11. $i = \frac{Q}{t} = \frac{3.5}{7.0} = 0.5$ A

12. $E = \frac{F}{q} = \frac{1.2 \times 10^{-2}}{1.44 \times 10^{-7}} = 8.33 \times 10^4$ N C^{-1}

13. $P = VI = 6 \times 1.2 = 7.2$ W

14. $R = \frac{V}{I} = \frac{6.0}{10.0} = 0.6\ \Omega$

15. Energy $= VIt = 32 \times 2.5 \times 10 \times 60 = 48\,000$ J

16. $q = It = 18.0 \times 27 = 4.86 \times 10^2$ C

17. Energy $= VIt$

$I = \frac{\text{Energy}}{Vt} = \frac{57600}{240 \times 1 \times 60} = 4.0$ A

18. $P = VI$

$I = \frac{P}{V} = \frac{1800}{240} = 7.5$ A

19. $I = \frac{V}{R} = \frac{240}{64.0} = 3.75$ A

20. $F = Eq = 6.0 \times 10^9 \times 4.0 \times 10^{-9} = 24$ N

21. $v = \frac{\Delta s}{\Delta t}$

$\Delta s = v\Delta t = 35 \times 5 \times 60 = 1.0500 \times 10^4$ m

22. $v = u + at = 15 + 10 \times 5 = 65$ m s^{-1}

23. $\Sigma F = ma = 4.8 \times 2.6 = 12.48$ N

24. $m = \frac{F}{g} = \frac{150}{9.8} = 15.3$ kg

25. $v = \frac{p}{m} = \frac{17000}{850} = 20$ m s^{-1} north

26. $F = \frac{p}{t} = \frac{850}{0.050} = 17000\text{ N} = 1.7 \times 10^4$ N

27. $W = Fs = 6.4 \times 5.5 = 35.2$ J

28. $E_K = \frac{1}{2}mv^2 = 0.5 \times 12.5 \times (1.5)^2 = 14.06$ J

29. $E_K = \frac{1}{2}mv^2$

$m = \frac{2E_K}{v^2} = \frac{2 \times 120.0}{30^2} = 0.267$ kg

30. Average speed of car $= \frac{60 + 0}{2} = 30$ km h^{-1}

$= \frac{30 \times 1000}{60 \times 60} = 8.33$ m s^{-1}

Acceleration of car $(a_y) = \frac{v_y^2 - u_y^2}{2\Delta y} = \frac{0 - 8.33^2}{2 \times 10}$

$= -3.47$ m s^{-2}

Force on car $(\Sigma F) = ma = 800 \times (-3.47)$

$= -2777.8$ N

Work on car $(W) = Fs = -2777.8 \times 10$

$= 2.78 \times 10^4$ J (no negative sign since work is not a vector)

31. $v = u + at$

$t = \frac{v - u}{a} = \frac{45.6 - 14.5}{5.0} = 6.22$ s

32. $v_y^2 = u_y^2 + 2a_y\Delta y$

$= 0^2 + 2 \times 5.0 \times 200 = 2000$

$v = 44.72$ m s^{-1}

33. $\Delta y = u_y t + \frac{1}{2}a_y t^2$

$= 21.4 \times 10 + \frac{1}{2} \times 1.6 \times 10^2 = 294$ m

34. At the top of its trajectory, $v = 0$

$v_y^2 = u_y^2 + 2a_y\Delta y$

$u_y^2 = v_y^2 - 2a_y\Delta y$

$= 0^2 - 2 \times (-9.8) \times 300$

$= 5880$

$u_y = 76.7$ m s^{-1}

35. $u_y = u\sin\theta$

$= 500\sin 30° = 250$ m s^{-1}

$v_y^2 = u_y^2 + 2a_y\Delta y$

$\Delta y = \frac{v_y^2 - u_y^2}{2a_y} = \frac{0 - 250^2}{2(-9.8)} = 3188.8$ m

36. $F = \frac{mv^2}{r}$

$v^2 = \frac{Fr}{m} = \frac{150 \times 10}{5.0} = 300$

$v = 17.3$ m s^{-1}

37. $F = \frac{Gm_1m_2}{d^2}$

$= \frac{6.67 \times 10^{-11} \times 3.00 \times 10^3 \times 7.00 \times 10^3}{5.00^2}$

$= 5.60 \times 10^{-5}$ N

38. $l_v = l_o\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$

$= 50\sqrt{1 - 0.9^2} = 21.79$ m

39. $E = mc^2$

$m = \frac{E}{c^2} = \frac{25 \times 10^6}{(3.0 \times 10^8)^2} = 2.78 \times 10^{-10}$ kg

40. $E_P = -G\frac{m_1m_2}{r} = -\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 15}{6.38 \times 10^6}$

$= -9.38 \times 10^8$ J

41. $\frac{F}{L} = \frac{kI_1I_2}{d} = \frac{2 \times 10^{-7} \times 5 \times 10}{5 \times 10^{-3}} = 2.0 \times 10^{-3}$ N m^{-1}

42. $\frac{F}{L} = \frac{kI_1I_2}{d}$

$L = \frac{Fd}{kI_1I_2} = \frac{1.5 \times 10^{-3} \times 20 \times 10^{-3}}{2 \times 10^{-7} \times 5.0 \times 5.0} = 6.0$ m

43. $F = BIL\sin\theta$

$= 100 \times 10^{-3} \times 25 \times 10^{-3} \times 50 \times 10^{-3} \times \sin 60°$

$= 1.08 \times 10^{-4}$ N

44. $F = Bqv \sin\theta$

$= 2.5 \times 10^{-15} \times 1.5 \times 10^{8} \times 5.0 \times 10^{-2} \times \sin 45°$

$= 1.33 \times 10^{-8}$ N

45. $\tau = Fd$

$F = \frac{\tau}{d} = \frac{40}{50 \times 10^{-2}} = 80$ N

46. $\tau = nBIA \cos\theta$

$= 500 \times 15 \times 10^{-3} \times 100 \times 10^{-3}$

$\times \pi \times \left(12.5 \times 10^{-3}\right)^{2} \times \cos 45° = 2.60 \times 10^{-4}$ N m

47. $\frac{V_S}{V_P} = \frac{n_S}{n_P}$

$V_S = \frac{n_S V_P}{n_P} = \frac{3000 \times 12}{1000} = 36$ V

48. $\frac{V_S}{V_P} = \frac{n_S}{n_P}$

$n_P = \frac{n_S V_P}{V_S} = \frac{750 \times 1}{5} = 150$

Acceleration of free fall, g	9.81 m s^{-2}
Gravitational constant, G	6.67×10^{-11} N m^2 kg^{-2}
Avogadro constant, N_A	6.02×10^{23} mol^{-1}
Gas constant, R	8.31 J K^{-1} mol^{-1}
Boltzmann constant, k	1.38×10^{-23} J K^{-1}
Stefan-Boltzmann constant, σ	5.67×10^{-8} W m^{-2} K^{-4}
Coulomb constant, k	8.99×10^{9} N m^2 C^{-2}
Permittivity of free space, ε_0	8.85×10^{-12} C^2 N^{-1} m^{-2}
Permeability of free space, μ_0	$4\pi \times 10^{-7}$ T m A^{-1}
Speed of light in a vacuum, c	3.00×10^{8} m s^{-1}
Planck constant, h	6.63×10^{-34} J s
Elementary charge, e	1.60×10^{-19} C
Electron rest mass, m_e	9.110×10^{-31} kg = 0.000549 u = 0.511 MeV c^{-2}
Proton rest mass, m_p	1.673×10^{-27} kg = 1.007276 u = 938 MeV c^{-2}
Neutron rest mass, m_n	1.675×10^{-27} kg = 1.008665 u = 940 MeV c^{-2}
Atomic mass unit, u	1.661×10^{-27} kg = 931.5 MeV c^{-2}

1 light year (ly) = 9.46×10^{15} m

1 parsec (pc) = 3.26 ly

1 astronomical unit (AU) = 1.50×10^{11} m

1 radian (rad) = $\frac{180°}{\pi}$

1 kilowatt hour (kWh) = 3.60×10^{6} J

1 atm = 1.01×10^{5} N m^{-2} = 101 kPa = 760 mmHg

Heating Processes

- $Q = mc\Delta T$

  Q = heat transferred to or from the object, m = mass of object, c = specific heat capacity of the object, ΔT = temperature change
- $Q = mL$

  Q = heat transferred to or from the object, L = latent heat capacity of the material, m = mass of object
- $\eta = \dfrac{\text{energy output}}{\text{energy input}} \times \dfrac{100}{1}\%$

  η = efficiency

Ionising Radiation and Nuclear Reactions

- $N = N_0(\frac{1}{2})^n$ (for whole numbers of half-lives only)

  N = number of nuclides remaining in a sample, n = number of whole half-lives, N_0 = original number of nuclides in the sample
- $\Delta E = \Delta mc^2$

  ΔE = energy change, Δm = mass change, c = speed of light (3×10^8 m s^{-1})

Electrical Circuits

- $I = \dfrac{\Delta q}{t}$

  I = current, Δq = the amount of charge that passes a point in the circuit, t = time interval
- $V = \dfrac{W}{q}$

  V = potential difference, W = work, q = charge
- $R = \dfrac{V}{I}$

  R = resistance, V = potential difference, I = current
  For ohmic resistors, resistance, R, is a constant
- $P = \dfrac{W}{t} = VI$

  P = power, W = work = energy transformed, t = time interval, V = potential difference, I = current
  - Equivalent resistance for series components, I = constant
  - $V_t = V_1 + V_2 + \ldots V_n$
  - $R_t = R_1 + R_2 + \ldots R_n$

  I = current, V_t = total potential difference, V_n = the potential difference across each component, R_t = equivalent resistance, R_n = resistance of each component
  - Equivalent resistance for parallel components, V = constant
  - $I_t = I_1 + I_2 + \ldots I_n$
  - $\dfrac{1}{R_t} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots \dfrac{1}{R_n}$

  V = potential difference, I_t = total current, I_n = current in each of the components, $\dfrac{1}{R_t}$ = the reciprocal of the equivalent resistance, $\dfrac{1}{R_n}$ = the reciprocal of the resistance of each component

Linear Motion and Force

- $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$
  s = displacement, t = time interval, u = initial velocity, v = final velocity, a = acceleration
- $a = \frac{F}{m}$
  a = acceleration, F = force, m = mass
- $W = \Delta E$; where the applied force is in the same direction as the displacement, $W = Fs$,
  W = work, F = force, s = displacement, ΔE = change in energy
- $p = mv$, $\Delta p = F\Delta t$
  p = momentum, v = velocity, m = mass, F = force, Δp = change in momentum,
  Δt = time interval over which force F acts
- $E_k = \frac{1}{2}mv^2$
  E_k = kinetic energy, m = mass, v = speed
- $\Delta E_p = mg\Delta h$
  ΔE_p = change in potential energy, m = mass, g = acceleration due to gravity, Δh = change in vertical distance
- $\Sigma mv_{\text{before}} = \Sigma mv_{\text{after}}$
  $\Sigma mv_{\text{before}}$ = vector sum of the momenta of all particles before the collision, Σmv_{after} = vector sum of the momenta of all particles after the collision
- For elastic collisions: $\Sigma\frac{1}{2}mv^2_{\text{before}} = \Sigma\frac{1}{2}mv^2_{\text{after}}$
- $\Sigma\frac{1}{2}mv^2_{\text{before}}$ = sum of the kinetic energies before the collision, $\Sigma\frac{1}{2}mv^2_{\text{after}}$ = sum of the kinetic energies after the collision

Waves

- $v = f\lambda$
  v = speed, f = frequency, λ = wavelength
- angle of incidence = angle of reflection
- $L = n\frac{\lambda}{2}$ for strings attached at both ends and for pipes open at both ends
- $L = (2n - 1)\frac{\lambda}{4}$ for pipes closed at one end
  n = whole numbers 1, 2, 3 … relating to the harmonic, L = length of string or pipe,
  λ = wavelength of soundwave
- $I \propto \frac{1}{r^2}$
  I = intensity, r = distance from the source
- $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$
  i = incident angle (relative to the normal), r = angle of refraction (relative to the normal),
  v_1 = velocity in medium 1, v_2 = velocity in medium 2, λ_1 = wavelength in medium 1,
  λ_2 = wavelength in medium 2

Gravity and motion

- $w = mg$
  w = weight force, m = mass,
  g = acceleration due to gravity (gravitational field strength)
- $F = \frac{GMm}{r^2}$ and $g = \frac{F}{m} = \frac{GM}{r^2}$
  F = gravitational force,
  G = universal constant of gravitation (6.67×10^{-11} N m^2 kg^{-2}),
  M = mass of first body,
  m = mass of second body,
  r = separation between the centres of mass of the two bodies, g = acceleration due to gravity
- $v_y = gt + u_y$, $y = \frac{1}{2}gt^2 + u_y t$, $v_y^2 = 2gy + u_y^2$, $v_x = u_x$ and $x = u_x t$
  y = vertical displacement,
  x = horizontal displacement,
  u_y = initial vertical velocity,
  v_y = vertical velocity at time t,
  u_x = initial horizontal velocity,
  v_x = horizontal velocity at time t,
  g = speed of light acceleration due to gravity,
  t = time into flight
- $v = \frac{2\pi r}{T}$
  v = tangential velocity, T = period
- $a_c = \frac{V^2}{r}$
  a_c = centripetal acceleration,
  v = tangential velocity,
  r = radius of the circle
- $F_{net} = \frac{mv^2}{r}$
  F_{net} = net force, m = mass of body undergoing uniform circular motion,
  v = tangential velocity,
  r = radius of the circle
- $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$
  T = period of satellite,
  M = mass of the central body,
  r = orbital radius,
  G = universal constant of gravitation (6.67×10^{-11} N m^2 kg^{-2})

Electromagnetism

- $F = \frac{1}{4\pi\varepsilon_o}\frac{Qq}{r^2}$
  F = force, $\frac{1}{4\pi\varepsilon_o}$ = Coulomb constant (9×10^9 N m^2 C^{-2}),
  q = charge on the first object, Q = charge on the second object, r = separation between the charges
- $E = \frac{F}{q} = \frac{1}{4\pi\varepsilon_o}\frac{g}{r^2}$
  E = electric field strength, F = force,
  q = charge, r = distance from the charge,
  $\frac{1}{4\pi\varepsilon_o}$ = Coulomb constant (9×10^9 N m^2 C^{-2})
- $V = \frac{\Delta U}{q}$
  V = electrical potential difference,
  ΔU = change in potential energy, q = charge
- $B = \frac{\mu_0 I}{2\pi r}$
  B = magnetic flux density, I = current in wire,
  r = distance from the centre of the wire,
  $\frac{\mu_0}{2\pi}$ = magnetic constant (2×10^{-7} T A^{-1} m)
- For a straight, current carrying wire perpendicular to a magnetic field $F = BIL$
  B = magnetic flux density, F = force on the wire,
  L = length of wire in the magnetic field,
  I = current in the wire
- For a charge moving perpendicular to a magnetic field $F = qvB$
  F = force on a charge moving in an applied magnetic field,
  q = charge, v = velocity of the charge, B = magnetic flux density
- $\Phi = BA_\perp$
  Φ = magnetic flux, $A_\perp$ = area of current loop perpendicular to the applied magnetic field,
  B = magnetic flux density
- Emf $= -\frac{n\Delta(BA_\perp)}{\Delta t} = -n\frac{\Delta\Phi}{\Delta t}$
  Emf = induced potential difference, $\Delta\Phi$ = change in magnetic flux, n = number of windings in the loop,
  $A_\perp$ = area of current loop perpendicular to the applied magnetic field, Δt = time interval over which the magnetic flux change occurs, B = magnetic flux density
- $\frac{V_p}{V_s} = \frac{n_p}{n_s}$
  V_p = potential difference across the primary coil,
  V_s = potential difference across the secondary coil,
  n_p = number of turns on primary coil,
  n_s = number of turns on secondary coil
- $I_p V_p = I_s V_s$
  I_p = current in primary coil, V_p = potential difference across primary coil, I_s = current in secondary coil,
  V_s = potential difference across secondary coil

Special Relativity

- $t = \dfrac{t_0}{\sqrt{\left(1 - \dfrac{v^2}{c^2}\right)}}$

  t = time interval in the moving frame as measured by the observer in the proper frame, t_0 = proper time interval (time interval for a clock at rest in the observer's frame), v = relative speed of the two inertial frames, c = speed of light in a vacuum 3×10^8 m s^{-1}
- $L = L_0\sqrt{\left(1 - \dfrac{v^2}{c^2}\right)}$
- L = length interval in the frame moving at velocity (v) with respect to the observer, L_0 = proper length (length in a frame at rest with respect to the observer), c = speed of light 3×10^8 m s^{-1}
- $p_v = \dfrac{mv}{\sqrt{\left(1 - \dfrac{v^2}{c^2}\right)}}$
- p_v = relativistic momentum for an object moving with velocity, v, with respect to the observer, m = mass, c = speed of light 3×10^8 m s^{-1}
- $\Delta E = \Delta mc^2$
- ΔE = change in energy, Δm = change in mass, c = speed of light 3×10^8 m s^{-1}

Quantum Theory

- $E = hf$

  E = energy of photon,
  f = frequency,
  h = Planck's constant 6.626×10^{-34} J s
- $\lambda_{max} = \dfrac{b}{T}$

  λ_{max} = peak wavelength,
  t = absolute temperature,
  b = Wien's displacement constant 2.898×10^{-3} m K
- $E_k = hf - W$

  E_k = kinetic energy of photoelectron,
  hf = energy of incident photon,
  W = work function of the material
- $\lambda = \dfrac{h}{p}$

  λ = wavelength associated with particle,
  p = momentum of particle,
  h = Planck's constant 6.626×10^{-34} J s
- $n\lambda = 2\pi r$

  n = an integer 1, 2, 3, 4...,
  λ = wavelength of electron,
  r = orbital radius of electron
- $mvr = \dfrac{nh}{2\pi}$

  m = mass of electron,
  v = velocity of electron,
  r = orbital radius of electron,
  n = an integer 1, 2, 3, 4, etc,
  h = Planck's constant $(6.626 \times 10^{-34}\text{ J s})$
- $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$

  λ = wavelength of spectral line,
  n_i = principal quantum number of initial electron state,
  n_f = principal quantum number of final electron state,
  R = Rydberg's constant 1.097×10^7 m^{-1}

PERIODIC TABLE OF THE ELEMENTS

KEY

Atomic Number	79
Symbol	Au
Standard Atomic Weight	197.0
Name	Gold

1 H 1.008 Hydrogen																	2 He 4.003 Helium
3 Li 6.941 Lithium	4 Be 9.012 Beryllium											5 B 10.81 Boron	6 C 12.01 Carbon	7 N 14.01 Nitrogen	8 O 16.00 Oxygen	9 F 19.00 Fluorine	10 Ne 20.18 Neon
11 Na 22.99 Sodium	12 Mg 24.31 Magnesium											13 Al 26.98 Aluminium	14 Si 28.09 Silicon	15 P 30.97 Phosphorus	16 S 32.07 Sulfur	17 Cl 35.45 Chlorine	18 Ar 39.95 Argon
19 K 39.10 Potassium	20 Ca 40.08 Calcium	21 Sc 44.96 Scandium	22 Ti 47.87 Titanium	23 V 50.94 Vanadium	24 Cr 52.00 Chromium	25 Mn 54.94 Manganese	26 Fe 55.85 Iron	27 Co 58.93 Cobalt	28 Ni 58.69 Nickel	29 Cu 63.55 Copper	30 Zn 65.38 Zinc	31 Ga 69.72 Gallium	32 Ge 72.64 Germanium	33 As 74.92 Arsenic	34 Se 78.96 Selenium	35 Br 79.90 Bromine	36 Kr 83.80 Krypton
37 Rb 85.47 Rubidium	38 Sr 87.61 Strontium	39 Y 88.91 Yttrium	40 Zr 91.22 Zirconium	41 Nb 92.91 Niobium	42 Mo 95.96 Molybdenum	43 Tc Technetium	44 Ru 101.1 Ruthenium	45 Rh 102.9 Rhodium	46 Pd 106.4 Palladium	47 Ag 107.9 Silver	48 Cd 112.4 Cadmium	49 In 114.8 Indium	50 Sn 118.7 Tin	51 Sb 121.8 Antimony	52 Te 127.6 Tellurium	53 I 126.9 Iodine	54 Xe 131.3 Xenon
55 Cs 132.9 Caesium	56 Ba 137.3 Barium	57–71 Lanthanoids	72 Hf 178.5 Hafnium	73 Ta 180.9 Tantalum	74 W 183.9 Tungsten	75 Re 186.2 Rhenium	76 Os 190.2 Osmium	77 Ir 192.2 Iridium	78 Pt 195.1 Platinum	79 Au 197.0 Gold	80 Hg 200.6 Mercury	81 Tl 204.4 Thallium	82 Pb 207.2 Lead	83 Bi 209.0 Bismuth	84 Po Polonium	85 At Astatine	86 Rn Radon
87 Fr Francium	88 Ra Radium	89–103 Actinoids	104 Rf Rutherfordium	105 Db Dubnium	106 Sg Seaborgium	107 Bh Bohrium	108 Hs Hassium	109 Mt Meitnerium	110 Ds Darmstadtium	111 Rg Roentgenium	112 Cn Copernicium						

Lanthanoids

57 La 138.9 Lanthanum	58 Ce 140.1 Cerium	59 Pr 140.9 Praseodymium	60 Nd 144.2 Neodymium	61 Pm Promethium	62 Sm 150.4 Samarium	63 Eu 152.0 Europium	64 Gd 157.3 Gadolinium	65 Tb 158.9 Terbium	66 Dy 162.5 Dysprosium	67 Ho 164.9 Holmium	68 Er 167.3 Erbium	69 Tm 168.9 Thulium	70 Yb 173.1 Ytterbium	71 Lu 175.0 Lutetium

Actinoids

89 Ac Actinium	90 Th 232.0 Thorium	91 Pa 231.0 Protactinium	92 U 238.0 Uranium	93 Np Neptunium	94 Pu Plutonium	95 Am Americium	96 Cm Curium	97 Bk Berkelium	98 Cf Californium	99 Es Einsteinium	100 Fm Fermium	101 Md Mendelevium	102 No Nobelium	103 Lr Lawrencium

Elements with atomic numbers 113 and above have been reported but not fully authenticated.
Standard atomic weights are abridged to four significant figures.
Elements with no reported values in the table have no stable nuclides.