Question 12 **(16 marks)**

The Larje Machine Co manufactures metal rods for large industrial equipment. Their standard manufacturing process produces rods whose lengths are normally distributed with a mean of 400 cm, and a standard deviation of 5 cm. A rod is considered 'useable' if its length is between 395 cm and 405 cm.

Let X be a random variable denoting the length of a rod manufactured by the Larje Machine Co.

(a) Determine the probability that a rod manufactured by the Larje Machine Co is useable. Round your answer to three decimal places. (3 marks)

Recently the Larje Machine Co introduced a new manufacturing process that industry experts claim will improve the percentage of useable rods produced to 80%. The quality control department decides to investigate whether this standard is being achieved and plan to collect a random sample of rods manufactured using the new process.

(b) What condition must the sample satisfy in order to use a normal distribution to model the sample proportion of useable rods? (1 mark)

The quality control department collects a sample of 100 rods.

(c) What is the approximate distribution of the sample proportion of useable rods? (2 marks)

Upon measuring the sample of 100 rods, it is found that 75 are useable.

(d) Calculate a 95% confidence interval for the population proportion of useable rods. (3 marks)

(e) The quality control department would like to obtain a confidence interval with a smaller margin of error. State **two** methods that it could use to achieve this. (2 marks)

(f) The quality control department decides to select a new sample for which the maximum possible margin of error for a 95% confidence interval is 0.05. What sample size will achieve this requirement? (3 marks)

(g) The new sample yields the 95% confidence interval $(0.717, 0.803)$. On the basis of this sample, is the proportion of useable rods different from what was claimed by the industry experts? Justify your answer. (2 marks)