Question 3

(7 marks)

Consider the definite integral $\int_{0}^{1} \frac{x^2}{(1+x^2)^2} dx$.

(a) By using the substitution $x = \tan u$, show that $\int_0^1 \frac{x^2}{(1+x^2)^2} dx = \int_a^b \sin^2 u \, du$ and state the values of a, b. (4 marks)

(b) Hence evaluate
$$\int_{0}^{1} \frac{x^2}{(1+x^2)^2} dx$$
 exactly. (3 marks)