Question 6 **(11 marks)**

The table of values below may be used to assist you in answering part (b) of this question.

$\sin(0) = 0$	$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$	$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	$\sin\left(\frac{\pi}{2}\right) = 1$
$\cos(0) = 1$	$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$	$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	$\cos\left(\frac{\pi}{2}\right) = 0$

(a) (i) Determine $\frac{d}{dx}\left(x \sin\left(\frac{\pi x}{4}\right)\right)$. (2 marks)

(ii) Hence show that

$$\int \frac{\pi x}{4} \cos\left(\frac{\pi x}{4}\right) dx = x \sin\left(\frac{\pi x}{4}\right) + \frac{4}{\pi} \cos\left(\frac{\pi x}{4}\right) + c$$

where c is a constant. (3 marks)

(b) The time in minutes, T, between incoming phone calls at a call centre is a random variable with probability density function

$$p(t) = \begin{cases} \frac{\pi}{4}\cos\left(\frac{\pi t}{4}\right), & 0 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the probability that the time between two consecutive phone calls is less than 40 seconds. State your answer exactly. (3 marks)

(ii) Use the result from part (a)(ii) to determine the expected time between consecutive phone calls. (3 marks)