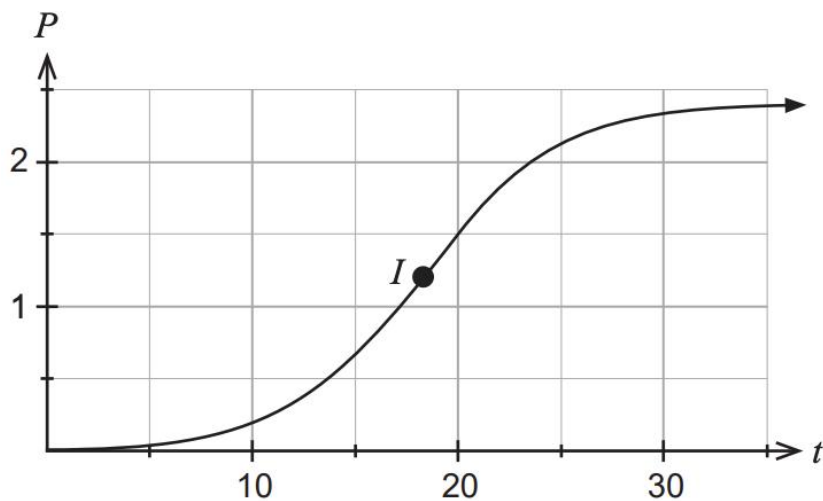**Question 19** **(11 marks)**

The population $P(t)$ of sardines in an ocean, measured in million tonnes after t years, was modelled by the logistic equation:

$$P(t) = \frac{2.4}{1 + 239\,e^{-0.3t}}$$

The graph of this model is shown below. This graph contains a point of inflection at point I.

(a)   Calculate the size of the sardine ocean population at $t = 0$.   (2 marks)

(b)   Rewrite the logistic equation in the form $\dfrac{dP}{dt} = rP(k-P)$, stating clearly the values for r and k.

(2 marks)

(c)   When the sardine population is 500 000 tonnes, use the technique of increments to calculate the approximate change in population in the next month.   (3 marks)

(d)   Determine the maximum rate of growth of the sardine population.   (2 marks)

Suppose that the initial population of sardines was 1.3 million tonnes.

(e) Assuming that the rate of growth is still given by $\dfrac{dP}{dt} = rP(k-P)$ sketch the graph of the population growth on the axes below. Explain your graph. (2 marks)