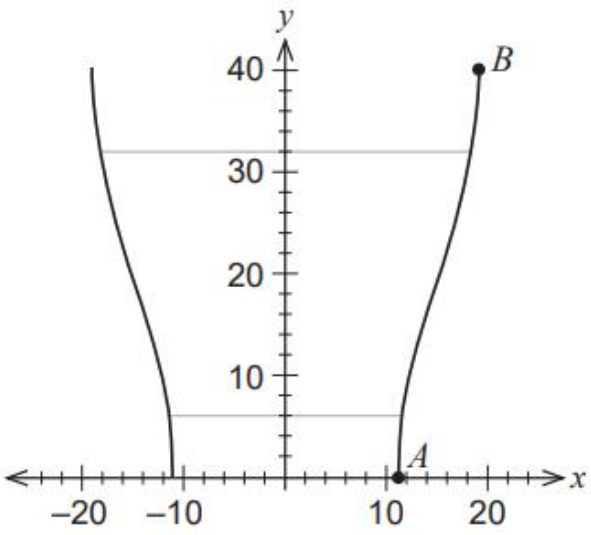
Question 17 **(10 marks)**

A vase has water with a depth of 6 cm and needs to be filled to a depth of 32 cm. The cross-section of the vase is modelled by the curve AB where

$x = 15 - 4\cos\left(\frac{\pi y}{40}\right),\ 0 \le y \le 40,$ and this curve is revolved about the y axis.
All dimensions are in centimetres.

Give all answers in this question to the nearest appropriate unit of measurement.



(a) Calculate the volume of water that needs to be added to increase the depth of water from 6 cm to 32 cm. (3 marks)

Josie, an interior designer, uses a hose to add water to the vase. This hose has a water-saving device that regulates the rate at which water flows into the vase. This rate is given by:

$$\frac{dV}{dt} = 300e^{-\frac{V}{12\,000}}$$

where $V(t)$ = the volume of water (cm^3) poured into the vase after t seconds of flow.

(b) If Josie has already poured 6000 cm^3, use the increments formula to calculate an approximation for the volume of water she will pour in the next 0.5 seconds. (2 marks)

To prevent an overflow of water, the device can be calibrated to switch off the flow after a set length of time using an in-built timer.

(c) Calculate the rate of flow into the vase at the instant when the depth becomes 32 cm. (2 marks)

(d) Using separation of variables, obtain the defining rule for $V(t)$. (3 marks)