

Question 13**(11 marks)**

A factory produces boxes of breakfast cereal with a labelled weight of 1.00 kg.

Let μ denote the population mean and σ denote the population standard deviation of the weights of the boxes. The factory sets the packaging process to a mean weight $\mu = 1.01$ kg with a standard deviation $\sigma = 0.05$ kg.

To maintain quality, a random sample of 400 boxes is taken each day and weighed. Let \bar{X} denote the sample mean weight.

(a) State the distribution for \bar{X} and its parameters. (3 marks)

(b) Determine the probability that the sample mean is more than 5 g above the labelled weight. (2 marks)

The sample mean on a particular day is $\bar{x} = 1.05$ kg, while the sample standard deviation is $s = 50$ g.

(c) Determine a 95% confidence interval, correct to 0.001 kg, for the population mean weight based on this sample. (2 marks)

Anja, a quality control officer, wants a 95% confidence interval based on a sample size of 100 with a width of no more than 0.1 kg.

- (d) What is the maximum standard deviation for this confidence interval? (2 marks)

Over the next 50 days, Ben, who is a data collection agent, takes random samples of size 100 each day and a 95% confidence interval is calculated for each sample. Ten of these 50 intervals (20% of the intervals) have a lower bound that is less than 1.00 kg. Ben claims that this indicates that the mean weight of the packaging is set too low.

- (e) Is Ben correct? Justify your response. (2 marks)