Question 3 **(10 marks)**

A company has four small workshops that each produce four different types of outdoor furniture. The annual cost of production of the furniture at each workshop is shown in the table below, with all values in thousands of dollars.

	Type 1 $'000	**Type 2 $'000**	**Type 3 $'000**	**Type 4 $'000**
Workshop A	25	43	50	39
Workshop B	33	31	56	39
Workshop C	28	47	59	38
Workshop D	36	32	56	41

The cost matrix is given by

$$\begin{bmatrix} 25 & 43 & 50 & 39 \\ 33 & 31 & 56 & 39 \\ 28 & 47 & 59 & 38 \\ 36 & 32 & 56 & 41 \end{bmatrix}$$

The company is interested in knowing what the minimum annual cost would be if each furniture type was allocated to its own individual workshop. The Hungarian Algorithm is to be used to determine the allocation and the minimum annual cost. The first step of the Hungarian Algorithm, where the smallest number in each row is subtracted from all other numbers in that row, is shown below.

(a) Continue the steps of the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type and state the **minimum** annual cost. (5 marks)

$$\begin{bmatrix} 0 & 18 & 25 & 14 \\ 2 & 0 & 25 & 8 \\ 0 & 19 & 31 & 10 \\ 4 & 0 & 24 & 9 \end{bmatrix}$$

Type	Type 1	Type 2	Type 3	Type 4
Workshop				

Total minimum annual cost _____

The revenue matrix, in thousands of dollars, for the sale of the furniture produced annually at each workshop is given by

$$\begin{bmatrix} 37 & 61 & 60 & 53 \\ 45 & 52 & 73 & 50 \\ 38 & 65 & 75 & 55 \\ 44 & 54 & 76 & 45 \end{bmatrix}$$

(b) Given that $Profit = Revenue - Cost$, complete the Profit matrix below. (1 mark)

$$\text{Profit matrix} = \begin{bmatrix} 12 & 18 & 10 & 14 \\ 12 & 21 & 17 & 11 \\ 10 & 18 & & \end{bmatrix}$$

(c) Use the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type that will produce the **maximum** annual profit. (4 marks)

Type	Type 1	Type 2	Type 3	Type 4
Workshop				