Question 9 **(14 marks)**

Andrew plans to run a game called Lucky Cup for a school fundraising event. All profits go toward the school's fundraising efforts. The game consists of three standard dice, each placed into a red cup. The red cup is shaken, the dice rolled, and the number of sixes recorded.

Let X be a random variable denoting the number of sixes rolled in a game of Lucky Cup.

(a) State the distribution of X. (2 marks)

An incomplete probability distribution for X is shown in the table below.

x	0	1	2	3
$P(X=x)$	$\frac{125}{216} = 0.5787$			

(b) Complete the table above, providing the missing probabilities. (2 marks)

Lucky Cup costs \$1 to play. If a player rolls one 6 they win \$1, if they roll two 6s they win \$2, and if they roll three 6s they win \$3.

(c) Determine the school's expected profit/loss for each game of Lucky Cup. (3 marks)

(d) Determine the probability that a player will make a profit in a game of Lucky Cup.

(2 marks)

Andrew wants to increase the attraction of the game by providing the opportunity for larger winnings. He modifies the rules of the game so that players only win money when two or more 6s are rolled, and the winnings for rolling three 6s is three times as much as the winnings for rolling two 6s. Each game still costs $1 to play. He estimates that he should be able to run 500 games and wants to make a profit of $200 for the school.

Andrew calls this game Lucky Cup II.

(e) Determine the winnings Andrew should set for rolling three 6s in Lucky Cup II. (3 marks)

Andrew decides to make the game even more dynamic and exciting. He adds a green cup with a die that he rolls at the beginning of each game. The value that Andrew rolls becomes the target value, and players must roll this target value to win. For example, if Andrew rolls a 2 from the green cup and a player rolls three 2s, the player wins the top prize.

Andrew calls this game Lucky Cup III.

(f) Explain how this change affects a player's chance of winning compared with Lucky Cup II.

(2 marks)