Question 13 **(14 marks)**

A carnival game involves five buckets, each containing 5 blue balls and 15 red balls. A player blindly selects a ball from each bucket and wins the game if they select at least 4 blue balls. Let X denote the number of blue balls selected.

(a) State the distribution of X, including its parameters. (2 marks)

(b) What is the probability of a player winning the game on any given attempt? (2 marks)

(c) Players are charged \$2 for each attempt at the game and offered a \$150 prize if they win the game. By providing appropriate numerical justification, explain why this is not a good idea for the carnival organisers. (2 marks)

An observer records the outcome of 100 consecutive games and determines the 90% and 95% confidence intervals for the proportion of wins, p. The confidence intervals are (0.04, 0.16) and (0.05, 0.15).

(d) Which of these intervals is the 95% confidence interval for p ? Justify your answer. (2 marks)

(e) How many wins were observed out of the 100 games? (2 marks)

(f) Determine what you would expect to happen to the width of the confidence intervals if 400 games had been observed. (2 marks)

(g) The true proportion of wins does not lie within either of the above confidence intervals. Does this suggest that a sampling error was made? Justify your answer. (2 marks)