Question 17 **(11 marks)**

After t seconds, the displacement x centimetres of a small mass attached to a spring, oscillates about a fixed point O according to the differential equation $\frac{d^2x}{dt^2} = -\pi^2 x$.

The initial velocity is 8π centimetres per second and the initial displacement is zero.

(a) Determine the function $x(t)$ that gives the displacement of the mass at time t. (3 marks)

(b) Calculate the distance the mass travels during the first 5 seconds. (3 marks)

The differential equation $\frac{d^2x}{dt^2} = -\pi^2 x$ assumes that the amplitude of oscillation A is a constant over time.

Now assume that friction reduces the amplitude of the oscillation according to the equation $\frac{dA}{dt} = -0.4A$. Also assume $A(0) = 8$ centimetres.

(c) Determine the function $A(t)$ that gives the amplitude of the mass. (2 marks)

As time passes, the amplitude continues to decrease to the point at which the small mass appears to stop oscillating. This occurs when the amplitude is less than 0.01 cm.

(d) Determine, correct to the nearest 0.1 seconds, how long it takes for the small mass to appear to stop oscillating. (3 marks)