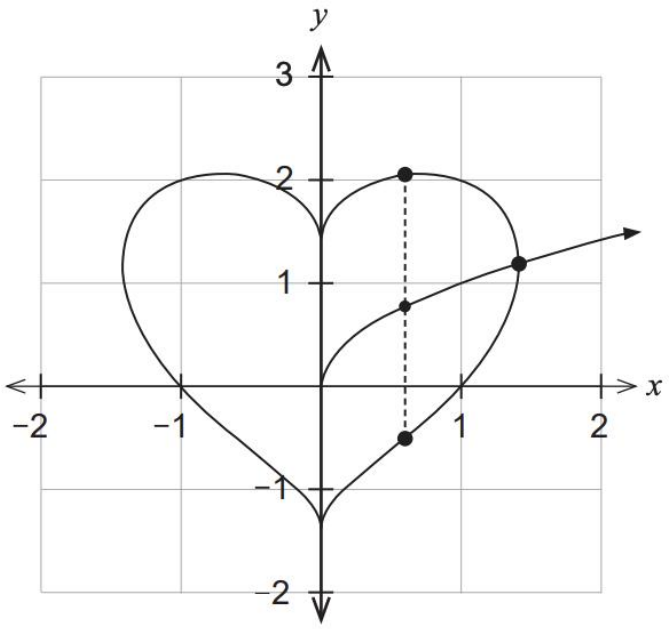
Question 8 **(9 marks)**

The heart-shaped figure shown is given by the equation $x^2 + \left(y - \sqrt{|x|}\right)^2 = 2$.

For $x \geq 0$, this equation becomes $x^2 + \left(y - \sqrt{x}\right)^2 = 2$. The curve $y = \sqrt{x}$ is also drawn.
This heart-shaped curve has the special property that for each x coordinate in its domain its two y coordinates are an equal vertical distance from the curve $y = \sqrt{x}$.



(a) Explain why the domain for the curve given by $x^2 + \left(y - \sqrt{x}\right)^2 = 2$ is $0 \leq x \leq \sqrt{2}$.
(2 marks)

(b) Show that the total area enclosed by the heart-shaped figure is given by:

$$Area = 4\int_0^{\sqrt{2}} \sqrt{2 - x^2}\, dx.$$
(2 marks)

(c) By using the substitution $x = \sqrt{2}\sin\theta$, evaluate the total area enclosed by the heart-shaped figure, and hence see why it can be said that 'π is at the heart of mathematics'. (5 marks)