

**Question 16****(12 marks)**

An analyst was hired by a large company at the beginning of 2021 to develop a model to predict profit. At that time, the company's profit was \$4 million. The model developed by the analyst was:

$$P(x) = \frac{20 \ln(x + a)}{x + 5},$$

where  $P(x)$  is the profit in millions of dollars after  $x$  weeks and  $a$  is a constant.

(a) Show that  $a = e$ . (2 marks)

(b) What does the model predict the profit will be after five weeks? (1 mark)

(c) Showing use of the quotient rule, determine an equation that, when solved, will give the time when the model predicts the profit will be maximised. (3 marks)

(d) What is this maximum profit and during which week will it occur? (2 marks)

- (e) According to the model, during which week will the company's profit fall below its value at the beginning of 2021? (1 mark)

The model proved accurate and after 10 weeks the company implemented some changes. From this time the analyst used a new model to predict the profit:

$$N(y) = 2e^{b(10+y)},$$

where  $N(y)$  is the profit in millions of dollars  $y$  weeks from this point in time and  $b$  is a constant.

- (f) The company is projecting its profit to exceed \$5 million. During which week does the new model suggest this will happen? (3 marks)