Question 19 **(17 marks)**

Kepler's Laws

Kepler's three laws of planetary motion can be stated as follows:

I All planets move about the Sun in elliptical orbits, having the Sun as one of the foci.

II A radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time.

III The squares of the periods (of revolution) of the planets are directly proportional to the cubes of their mean distances from the Sun.

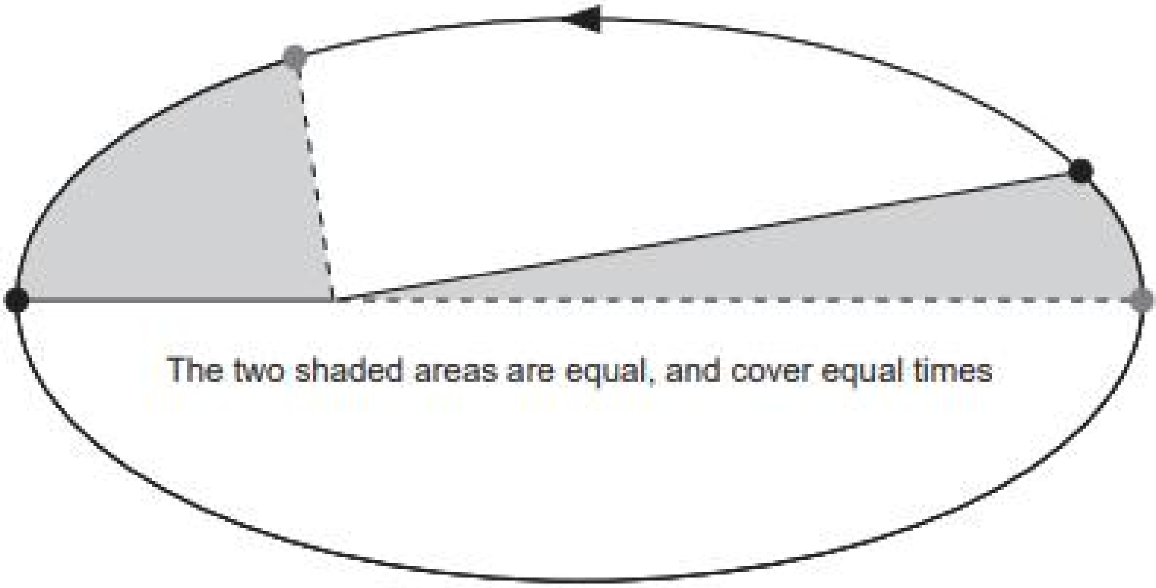

Figure 1: Kepler's 2nd Law

Kepler's 3rd Law, which appears on the Formulae and Data booklet, can be derived for a circular orbit from first principles: the centripetal force between the planet and the Sun is provided by Newton's Law of Gravitation, and $S = vT$, where S is the orbiting circumference and T is the period.

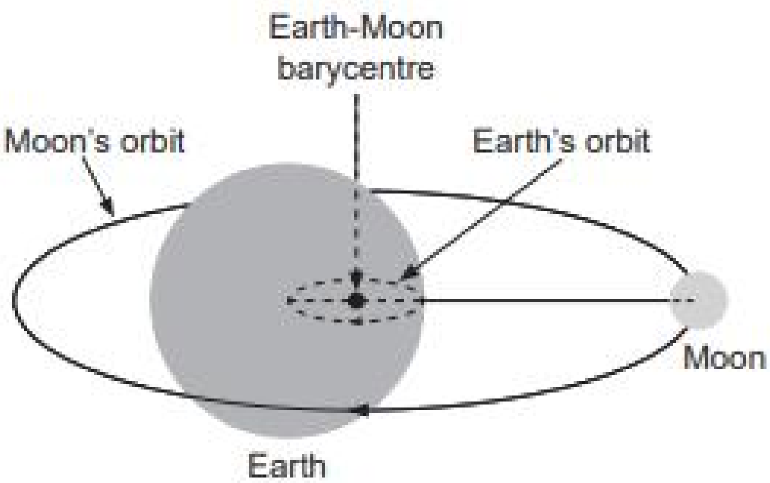

Figure 2: Location of Earth-Moon barycentre

The system of any large celestial body, and its satellite, orbits around a 'barycentre'. This represents the centre of mass of the system. The barycentre of the Earth–Moon system is shown in Figure 2. The system could be seen as a balance beam, with the barycentre located where the fulcrum would be placed to achieve equilibrium. The gravitational field strength due to the Sun is identical for both the Earth and Moon and therefore cancels out when calculating moments around the barycentre.

When the mass of the satellite represents a significant percentage of the system, the barycentre is outside either body. When its mass is significantly less, the barycentre is usually found within the more massive body which appears to 'wobble'. When the mass of the orbiting satellite is insignificant compared to the mass of the body it is orbiting, the barycentre can be assumed to be the centre of mass of the larger body.

(a) (i) Using Kepler's 2nd Law, describe the relationship between the distance a planet is from the Sun it orbits and its orbiting speed by filling in the blank below. (1 mark)

As the distance from the planet increases, the orbiting speed ________________.

(ii) Without completing a calculation, justify this relationship with reference to Figure 1. (3 marks)

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(b) Using the instructions given in the article, derive Kepler's 3rd Law from first principles, showing each step of the derivation. The final expression must match the equation in the Formulae and Data booklet. Assume the orbit is perfectly circular and the mass of the satellite is insignificant compared to the mass of the body it is orbiting. (5 marks)

(c) Using moments, estimate how far the barycentre of the Earth–Moon system is from the centre of the Earth. (4 marks)

Answer: ______________________ m

(d) With the use of a calculation and your answer to part (c), show that the Moon is travelling roughly 81 times faster than Earth as they orbit the barycentre. If you could not get an answer to part (c), use 4.81×10^6 m and show that the ratio of the Moon's orbiting velocity to that of the Earth is roughly 80. (4 marks)