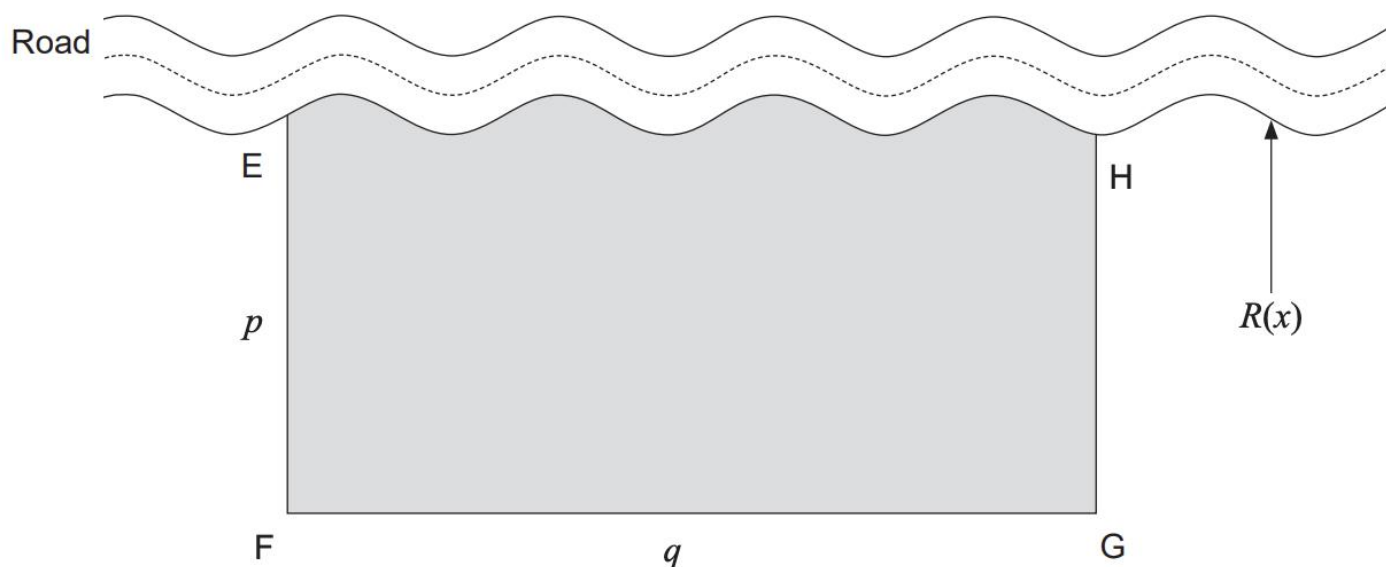


Question 17

(12 marks)

David and Katrina have a small farm and wish to fence off an area of their land so they can raise sheep. The area they have chosen has one border along a road as shown in the diagram below.



The enclosure is shown as the shaded area above and has right angles at points F and G. David and Katrina want the combined lengths of the fencing from E to F and F to G to equal 500 metres. Let the length of fence EF be equal to p metres and the length of fence FG be equal to q metres. If we locate the origin at point F and the x -axis along the line FG, the equation defining the fence along the road is given by:

$$R(x) = 10 \sin\left(\frac{x}{15}\right) + p$$

- (a) Show that the equation defining the area of the enclosure, $A(q)$, can be given in terms of q as follows:

$$A(q) = 500q - 150 \cos\left(\frac{q}{15}\right) - q^2 + 150$$

(4 marks)

- (b) Determine, to the nearest metre, the value of q that will allow the sheep to graze over the maximum area and state this maximum area. (4 marks)

The length of the fence from E to H is given by the equation:

$$L_{EH} = \int_0^q \sqrt{1 + (R'(x))^2} \, dx, \text{ where } R'(x) \text{ is the first derivative of } R(x).$$

- (c) (i) Determine $R'(x)$. (1 mark)

- (ii) Hence determine the total length of fencing required by David and Katrina to enclose their sheep with maximum area for grazing. (3 marks)