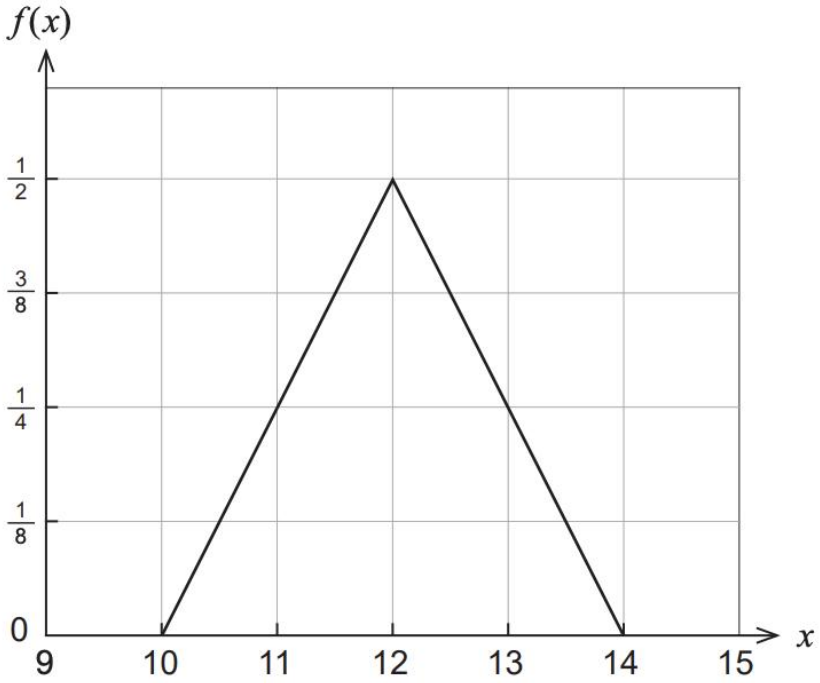
Question 3 **(11 marks)**

Arnold would like to purchase a toy for his child's birthday. The Isosceles Toy Company claims that the number of weeks until delivery, X, is a random variable whose probability density function is displayed in the graph below.



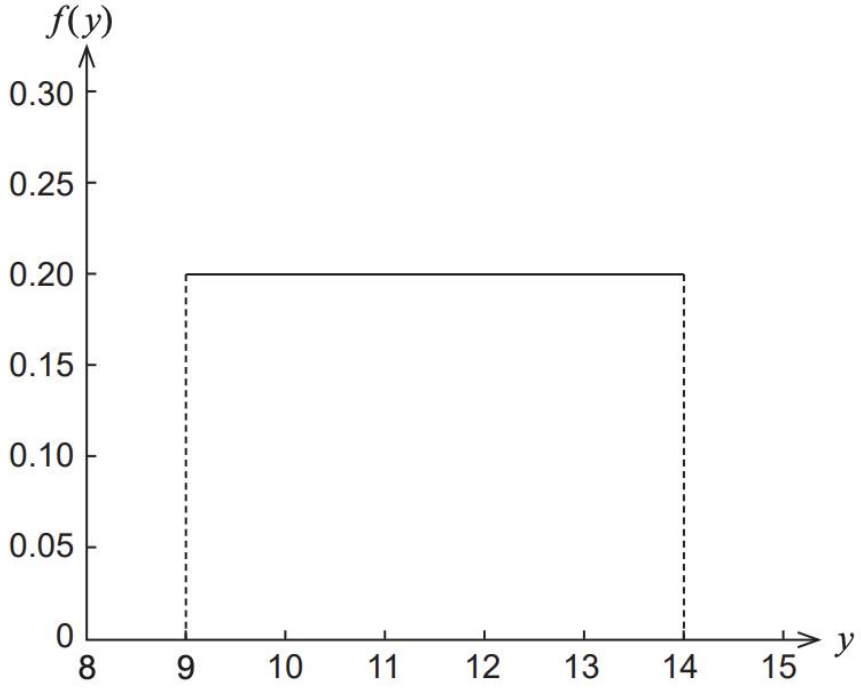
(a) What is the expected time for the toy to be delivered? (1 mark)

His child's birthday is 13 weeks away.

(b) What is the probability that the Isosceles Toy Company will deliver the toy in time for his child's birthday? (2 marks)

(c) Given that the toy arrives in time for his child's birthday, what is the probability that it arrives at least one week early? (2 marks)

Uniform Toys, a rival toy company, claims that the number of weeks until delivery of the same toy, Y, is a random variable whose distribution is displayed in the graph below.



(d) Which toy company should Arnold choose if he would like to maximise the chance that the toy will be delivered in time for his child's birthday? Why? (2 marks)

Suppose that five people order the toy from Uniform Toys and let Z be a random variable that denotes the number of those people who receive the toy within 13 weeks.

(e) State the distribution for Z. (2 marks)

(f) What is the probability that four out of the five people receive the toy within 13 weeks? (2 marks)