

Question 18**(7 marks)**

- (a) Show that for all positive integers n and complex numbers z where $0 \leq \theta \leq \frac{\pi}{2}$,

$$(z^n - \operatorname{cis}(\theta))(z^n + \operatorname{cis}(-\theta)) = z^{2n} - (2i \sin \theta) z^n - 1. \quad (3 \text{ marks})$$

- (b) Hence, using the result from part (a), obtain all the solutions to the equation $z^6 - (i)z^3 - 1 = 0$ in exact polar form. (4 marks)