Question 4   **(8 marks)**

A company produces rolls of shade cloth. Today, there are three different machines that can be used (1, 2 and 3) and four workers who can operate these machines (Peter, Quentin, Rebecca and Sasha). Each machine will have one worker assigned to it for the whole day.

The table below shows the number of metres of shade cloth that can be produced in a day by each machine operator.

		Machines		
		1	2	3
	Peter	300	250	270
Workers	Quentin	290	410	320
	Rebecca	190	240	120
	Sasha	310	410	280

(a)   Draw the weighted bipartite graph below, showing the possible allocations for each of the workers.   (2 marks)

Peter   Quentin   Rebecca   Sasha
•   •   •   •

•   •   •
Machine 1   Machine 2   Machine 3
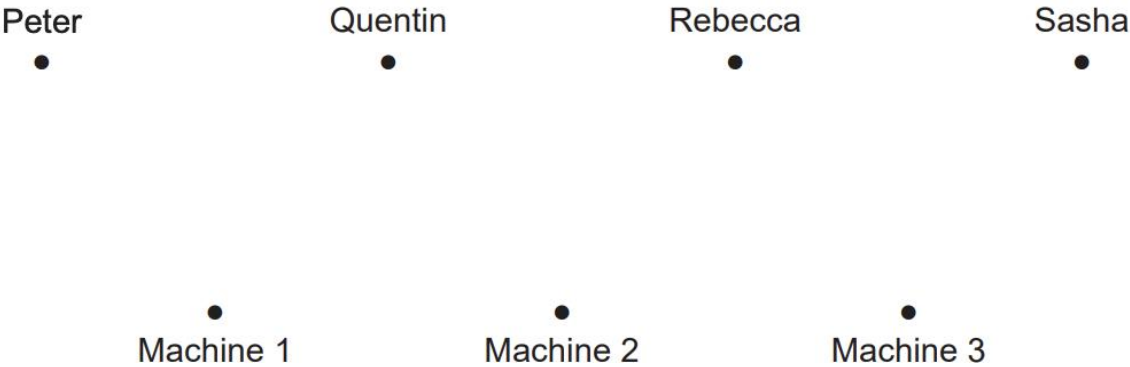
The company manager wants to allocate the workers to the machines so that the production for the day is at a maximum. She decides to use the Hungarian algorithm to determine the allocation. Her first step is to rewrite the information in matrix form, adding in a column containing all zeros.

$$\begin{bmatrix} 300 & 250 & 270 & 0 \\ 290 & 410 & 320 & 0 \\ 190 & 240 & 120 & 0 \\ 310 & 410 & 280 & 0 \end{bmatrix}$$

(b)   Why has she added the column of zeros?   (1 mark)

(c)   Continue the steps of the Hungarian algorithm, showing the optimum allocation of workers to machines in the table below. State the maximum total length of shade cloth that can be produced in the day.   (5 marks)

Worker	Peter	Quentin	Rebecca	Sasha
Machine				

Maximum total length of shade cloth _____