Question 18 **(13 marks)**

A rumour that the Federal Government plans to cut university funding begins to spread around a campus. There is a combined total of 1600 students and staff at this university.

One hundred people know of this rumour via a social media post at 8 am one morning.

Let $N(t)$ = the number of people at the university who have heard the rumour at t hours after 8 am. It is found that the rate at which the rumour spreads is given by the equation:

$$\frac{dN}{dt} = kN(1600 - N).$$

At 8 am the rumour was spreading at a rate of 60 people per hour.

(a) Show that $k = 0.0004$. (2 marks)

(b) At 11 am there were approximately 500 people who had heard the rumour. Using the increments formula, determine the approximate number of people who learn of this rumour between 11 am and 11.15 am. (3 marks)

(c) Given that $\frac{1}{N(1600-N)} = \frac{1}{1600}\left(\frac{1}{N} + \frac{1}{1600-N}\right)$, use the separation of variables technique to show that $N(t)$ is given by $\frac{N}{1600-N} = \frac{e^{0.64t}}{15}$. (4 marks)

(d) At what time, correct to the nearest minute, does the rumour spread at the fastest rate?

(4 marks)