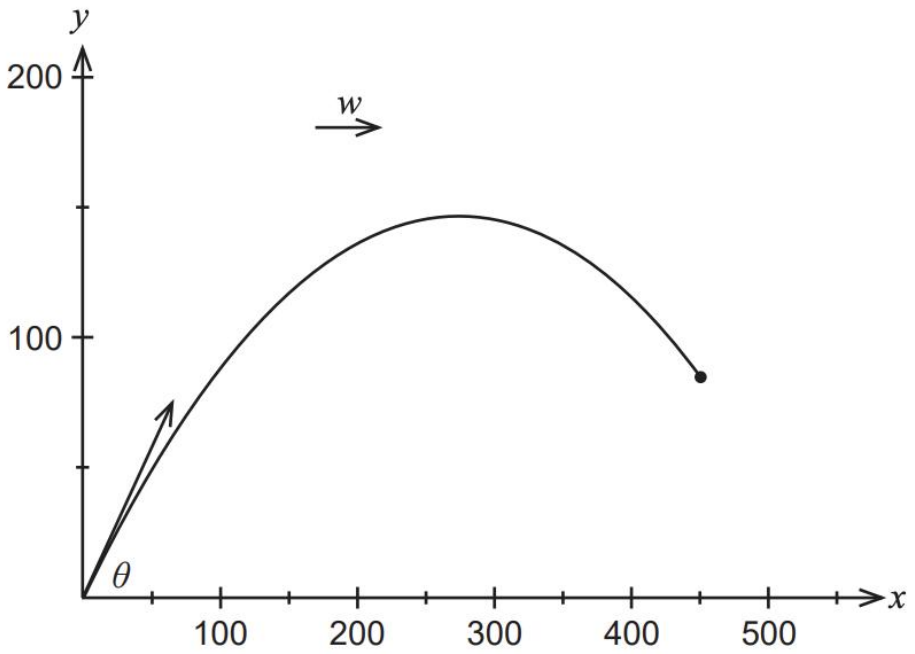
Question 19 **(9 marks)**

A small rocket is fired from the ground at an angle of $\theta°$ to the horizontal with a speed of 70 metres per second. The rocket has the assistance of a steady wind that is blowing horizontally at w metres per second.

A coordinate system is set up to track the path of the rocket as shown below.

Let t = the number of seconds elapsed after the rocket is fired
$\underset{\sim}{r}(t)$ = the position vector (metres)
$\underset{\sim}{v}(t)$ = the velocity vector (ms^{-1})
$\underset{\sim}{a}(t)$ = the acceleration vector (due to gravity) $= \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$ (ms^{-2})



(a) Given $\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$, show that $\underset{\sim}{r}(t) = \begin{pmatrix} (70\cos\theta + w)t \\ (70\sin\theta)t - 4.9t^2 \end{pmatrix}$. (3 marks)

(b) Obtain the Cartesian equation for the path of the rocket, in terms of θ and w. (2 marks)

The range of the rocket is defined as the horizontal distance travelled from its launch to the point at which it strikes the ground.

(c) Assuming that the wind speed $w = 2$ metres per second, determine the optimum angle θ so that the range of the rocket is maximised, correct to the nearest 0.1 degree. (4 marks)