**Question 15** (14 marks)

A wall in a new Western Australian hotel is to feature a rolling, wave-shaped window. Engineers have modelled the top edge of the wave shape by joining together two functions,

$$h_1(x) = 4 - 4(x-1)^2, \quad 0 \le x \le 1 \text{ and}$$

$$h_2(x) = a(\cos(x-1) + 1), \quad 1 < x \le d \quad a,d \text{ constants.}$$

The functions give the height, h, above ground level of the top edge of the window measured in metres. The origin is defined as the leftmost point of the window which is at ground level and x is the horizontal distance to the right of the origin measured in metres. The graph of the two functions is shown below.
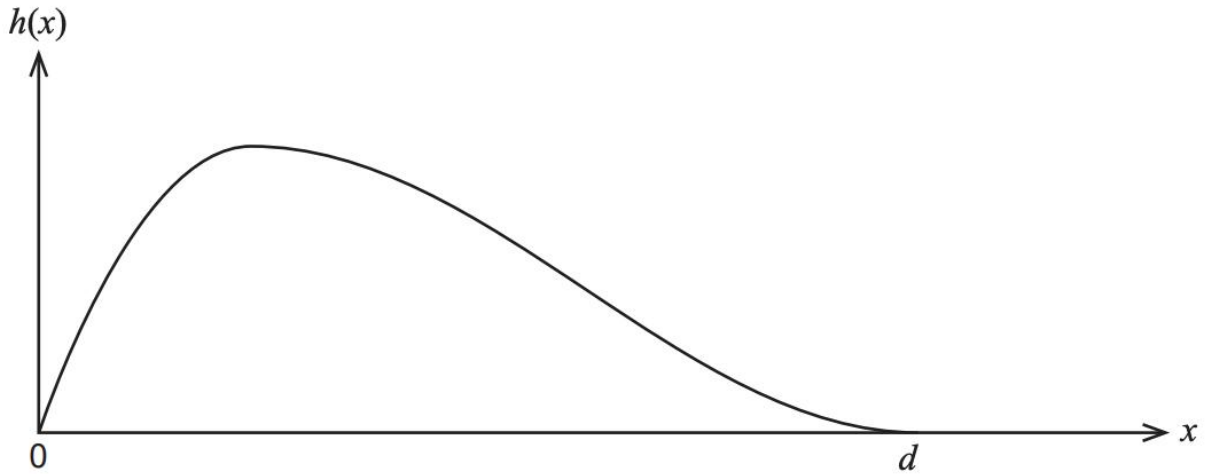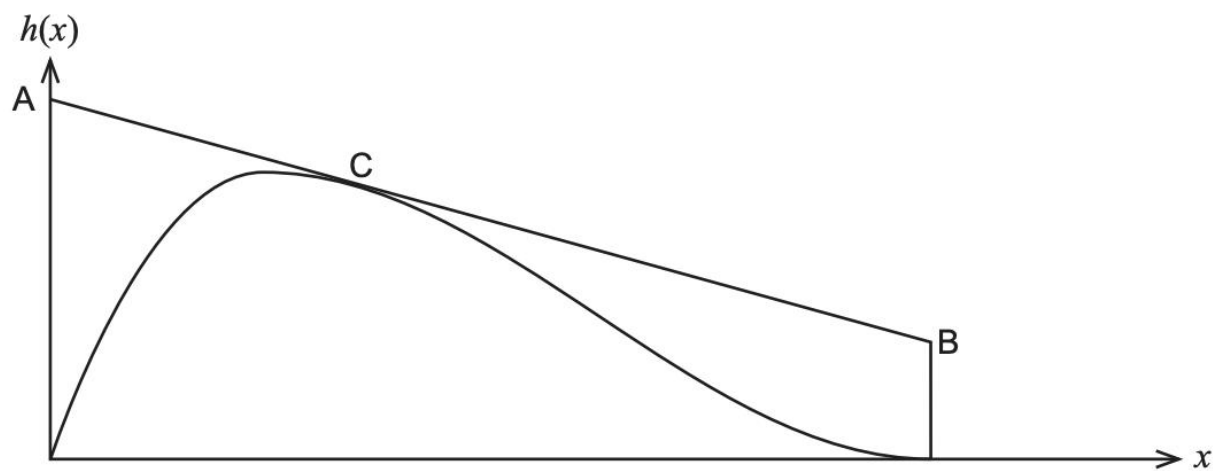

(a) Determine the value of the constant a in the function $h_2(x) = a(\cos(x-1) + 1)$. (3 marks)

(b)   Determine the length of the bottom edge of the window.   (2 marks)

(c)   Determine the volume of glass required for the window if it has a uniform thickness of 3 cm. (5 marks)

The top edge of the wall, shown as the line AB below, is to just touch the window at the point C shown below. Point A is 1.39 m above the point B.

(d)   How high is point C above the ground? (4 marks)