

Question 17**(8 marks)**

In Australia, the killing of humpback whales was banned in 1963.

At the end of 2018, 45 years later, the population P of migrating humpback whales off the coast of Western Australia was estimated at 30 000, i.e. $P(45) = 30\,000$.

- (a) Assuming that the population of humpback whales had been increasing at an instantaneous rate equal to 10% of the population, estimate the number of humpback whales at the end of 1963. (3 marks)

To model the growth in the population from the end of 2018, a marine biologist suggests that the rate of growth be modelled by the equation below.

$$\frac{dP}{dt} = 0.1P - \frac{P^2}{700\,000}$$

The biologist re-defines $P(0) = 30\,000$, i.e. t = number of years from the end of 2018.

- (b) If $P(t)$ is written in the form $P(t) = \frac{a}{1 + be^{-ct}}$, determine the values of the constants a , b and c . (2 marks)

(c) Hence determine the year during which the population of humpback whales off the coast of Western Australia will reach double that estimated at the end of 2018. (2 marks)

(d) State the major difference in the variation in the population $P(t)$ using the model in part (b) compared with that in part (a). (1 mark)