Question 16 **(13 marks)**

(a) Alex is about to retire and is planning to take an annuity from his pension fund. He sets up the pension fund on his 65th birthday with $500 000 and he estimates the fund can generate a growth rate of 6% per year. He plans to start withdrawing an annuity of $40 000 starting on his following birthday.

(i) Write a recurrence relation to calculate the total amount in the fund directly after each withdrawal. (3 marks)

(ii) For how many years will Alex be able to receive his annuity of $40 000? (2 marks)

(iii) Assuming that all other conditions are the same, explain what would happen if Alex decided to withdraw $30 000 per year instead of $40 000 per year. (2 marks)

(b) Abbey sets up her pension fund on July 1 2016 with a principal of $850 000. The fund guarantees an annual growth rate of 7.5% compounded monthly and she plans to take an annuity of $75 000 each year on July 1, starting in 2017.

(i) Calculate the balance in the fund after the annuity is withdrawn in July 2020.
(2 marks)

The investment fund revised its annual interest rate to 9% compounded monthly on July 1 2020 guaranteed for the period to July 2025 and Abbey continued withdrawing $75 000 as usual.

(ii) Calculate the balance in the fund after a withdrawal is made on July 1 2025.
(2 marks)

(iii) Calculate, to the nearest $100, the maximum amount Abbey could withdraw annually, starting in 2020, without decreasing her balance. (2 marks)