Question 18 (10 marks)

A young child rides on a merry-go-round at a carnival. The merry-go-round has a radius of 5 metres and completes one revolution every 12 seconds. The parent of the young child stands and watches at point P, exactly 3 metres away from point B.

The ride begins at point B, when the child is closest to the parent, and the merry-go-round rotates in an anti-clockwise direction at a constant speed. At any point in time, point C is the position of the child on the merry-go-round.
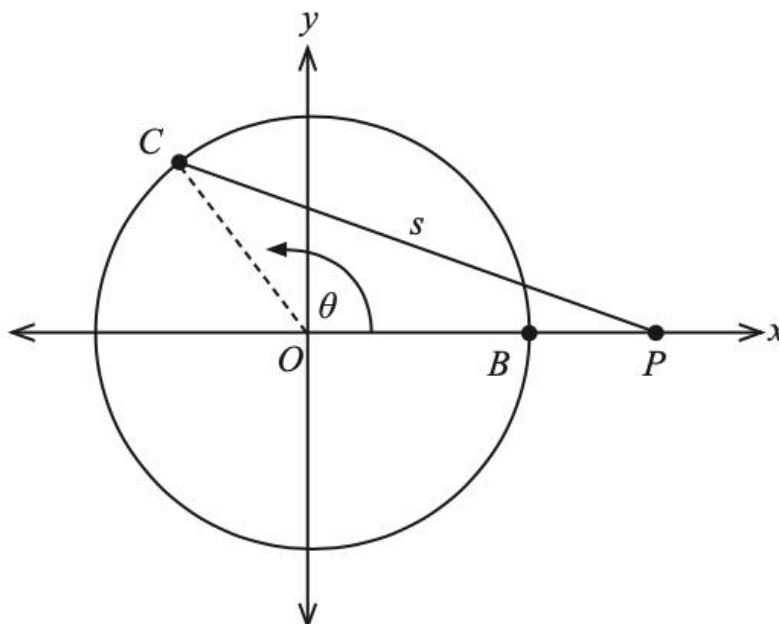
Let t = the number of seconds the ride has been in progress (from starting at point B)
       $s = PC$ = the distance that the child is from the parent (metres)
       θ = size of $\angle BOC$ (radians)

(a)   Show that $\dfrac{d\theta}{dt} = \dfrac{\pi}{6}$ radians per second.   (1 mark)

(b)   Show that $s^2 = 89 - 80\cos\theta$.   (1 mark)

(c)   By differentiating $s^2 = 89 - 80\cos\theta$ implicitly with respect to time t, determine, correct to the nearest 0.01 metre per second, the rate at which the child is moving away from the parent when the ride has been in progress for 4 seconds.   (4 marks)

The parent notices that the child appears to move away from point P at varying speeds.

(d)   Determine the value for $\cos\theta$ when the rate $\dfrac{ds}{dt}$ is a maximum.   (4 marks)