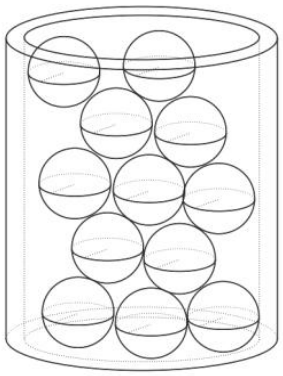
Question 16 **(10 marks)**

A cylindrical glass vase is filled with 20 spherical Christmas decorations as shown below (not all the decorations are visible). All the decorations have a diameter of one-third the internal diameter of the vase and they are completely contained within the vase. For design purposes the sum of the internal diameter of the base of the vase and the vase's internal height is to be 42 cm.



(a) Show that the volume of unused space in the vase, V, can be expressed as a function of the internal radius of the vase, r, and is given below as (3 marks)

$$V(r) = 2\pi\left(21r^2 - \frac{121}{81}r^3\right).$$

(b) Use calculus to determine the dimensions of the vase that will maximise the unused space in it. Give your answers rounded to the nearest millimetre. (4 marks)

(c) Can more than 20 of the spherical decorations fit inside the vase in part (b)? Use calculations to verify your answer. (3 marks)