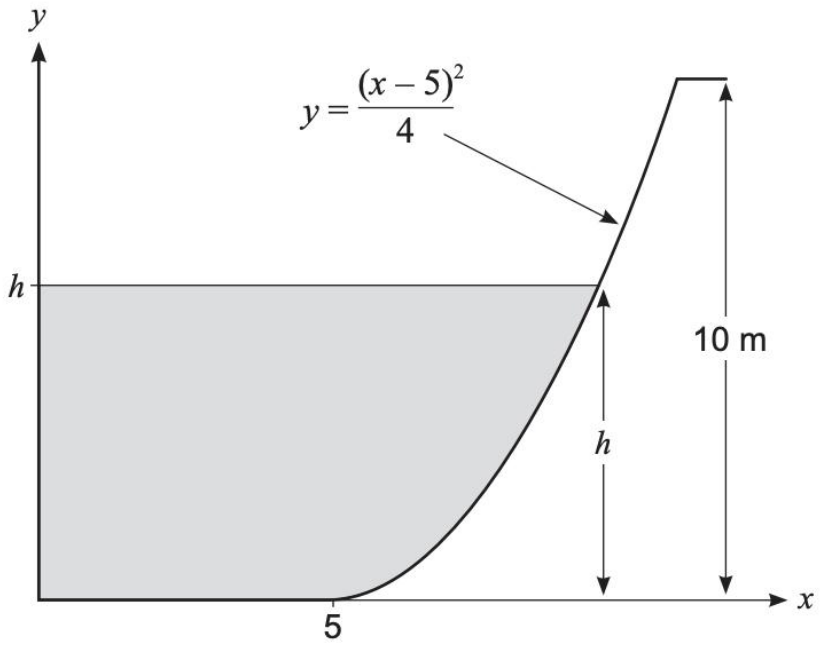
Question 14 **(11 marks)**

A small dam on an agricultural property has a length of 20 m, and a uniform cross-section shown below where x and y are in metres. The base of the dam is flat for $0 \leq x \leq 5$, and the right side is given by $y = \dfrac{(x-5)^2}{4}$ for $5 < x \leq 11.325$. The shaded region on the graph below represents the cross-section of a volume of water V (m^3) in the dam with water level h (m).



(a) Using calculus, show that the volume of water in the dam is given by

$$V(h) = 100h + \frac{80}{3}h^{\frac{3}{2}}.$$ (5 marks)

(b) Use the increments formula to estimate the change in water volume if the water level rises from 6 m to 6.1 m. (3 marks)

Suppose the water volume at the start of winter is 1000 m^3. On the basis of rainfall data from previous years, the volume of water V_R (m^3) that will flow into the dam over winter is normally distributed with a mean of 600 m^3 and a standard deviation of 200 m^3.

(c) Assuming that there are no other sources of water and no losses, determine the probability that the dam will reach full capacity (i.e. depth of 10 m) during winter. (3 marks)