Question 12 (11 marks)

The lifetime of an electronic device is distributed as an exponential random variable with mean $\mu = 20$ years and standard deviation $\sigma = 20$ years. A random sample of 50 of these devices is selected. Tam, a graduate electronics engineer, is interested in the sample mean lifetime $\overline{X}$ of these 50 devices.

(a) State the distribution of the sample mean lifetime $\overline{X}$. Justify your answer. (3 marks)

(b) Determine the probability that the sample mean lifetime is between 15 and 25 years. (2 marks)

Jai, the chief engineer, informs Tam that the lifetimes may not be exponentially distributed but could be a more complicated distribution, yet still having mean $\mu = 20$ years and standard deviation $\sigma = 20$ years.

(c)   If Jai is correct, will your answer to part (b) change? Explain.   (2 marks)

A different random sample of size n of these devices was selected. Repeated sampling with this sample size shows that there is a 3% chance of obtaining a sample mean greater than 25 years.

(d)   Determine the value of n.   (4 marks)