Question 2 **(7 marks)**

A supermarket provides a delivery service to its customers. This morning, there are four deliveries (1, 2, 3 and 4) to be made. Each of four drivers, John, Kerry, Liam and Max, is available to do one of the deliveries.

The table below shows the time, in minutes, that each driver would take to complete each of the four deliveries.

Table 1

Delivery Driver

Deliveries	John	Kerry	Liam	Max
1	35	31	41	36
2	25	26	33	36
3	32	28	25	24
4	27	30	31	28

The store manager will allocate the deliveries so that the total delivery time is at a minimum. He decides to use the Hungarian algorithm to determine the allocation of deliveries to the drivers.

His first step is to subtract the minimum entry in each row from each element, ensuring that each row contains at least one zero.

Table 2

Delivery Driver

Deliveries	John	Kerry	Liam	Max
1	4	0	10	5
2	0	1		11
3	8	4	1	0
4	0	3	4	1

(a) What is the number missing from the shaded cell? (1 mark)

The second step is to ensure that all columns contain at least one zero. The numbers that result from this step are shown in the table below.

Table 3

		Delivery Driver			
		John	Kerry	Liam	Max
Deliveries	1	4	0	9	5
	2	0	1	7	11
	3	8	4	0	0
	4	0	3	3	1

(b) The smallest number of horizontal and vertical lines that can be drawn to cover all the zeros is three.

(i) Draw in these lines on **Table 3** on the previous page. (1 mark)

(ii) State why an allocation of delivery drivers cannot be made yet. (1 mark)

(c) Continue the steps of the Hungarian algorithm to determine the optimum allocation of deliveries to the drivers. Complete the table at the bottom of the page and state the minimum total delivery time. (4 marks)

Delivery Driver	John	Kerry	Liam	Max
Delivery				

Minimum total delivery time _____