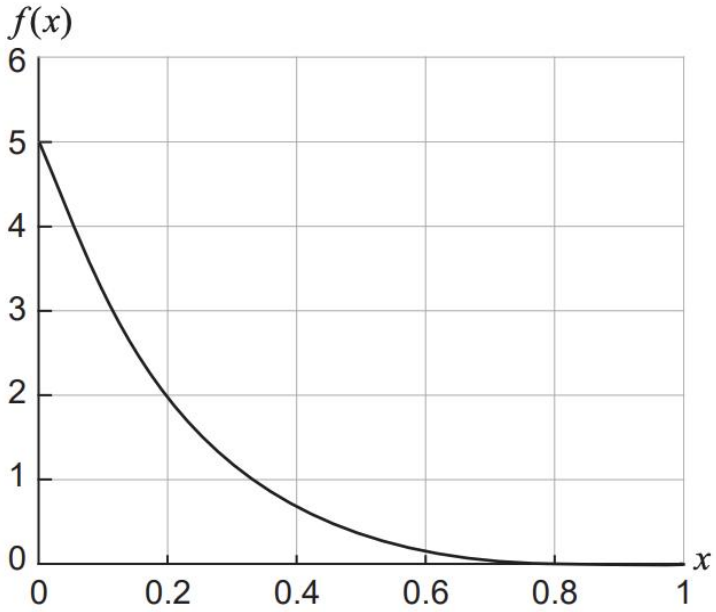
Question 8 **(10 marks)**

A small outback petrol station receives a weekly delivery of petrol. The volume of petrol sold in a week, X, (in units of 10 000 litres) is a random variable with probability density function

$$f(x) = 5(1-x)^4, \quad 0 \leq x \leq 1$$

as shown in the graph below.



(a) Determine, using appropriate units, the expected value and variance of the amount of fuel sold in a week. (4 marks)

(b) What storage tank capacity will ensure that there is only a 1% chance of running out of petrol in a given week? State your answer to the nearest litre. (3 marks)

(c) When the petrol is delivered, it is pumped into the storage tank. The rate of change of the petrol level in the tank, $h(t)$, (measured in metres) at time t (measured in minutes) is given by

$$h'(t) = \frac{5}{2t + 3}$$

Determine the height of the storage tank if it takes 20 minutes to fill. (3 marks)