Question 12 **(21 marks)**

It is estimated that 20% of small businesses fail in the first year. A business advisory group takes a random sample of 500 new businesses that started in January 2018. An analyst employed by the group suggests the use of the binomial distribution is appropriate in this case.

(a) What is the probability that at most 120 of the businesses fail in the first year? (2 marks)

(b) What is the approximate distribution of the sample proportion of small businesses that fail by the end of the year in this sample? Justify your answer. (3 marks)

(c) What is the probability that the sample proportion of businesses that fail by the end of the year is less than 0.18? (2 marks)

(d) By January 2019, 90 of the 500 new businesses had failed. Calculate a 95% confidence interval for the proportion of new businesses that fail in the first year. (2 marks)

The business advisory group believes that the proportion of new businesses that fail within a year can be reduced by providing financial advice. They took another random sample of 500 businesses that started in January 2019 and provided them with regular financial advice. In this random sample, at the end of the year 80 businesses had failed.

(e) Calculate the sample proportion and its margin of error at the 95% confidence level.
(2 marks)

(f) Calculate a 95% confidence interval for the proportion of businesses that failed. What do you conclude regarding the value of the financial advice provided to the new businesses?
(4 marks)

(g) If the sample size was reduced, what would be the effect on the confidence interval? Justify your answer.
(2 marks)

(h) State **two** assumptions that the analyst made in recommending the use of the binomial model in this case and discuss whether they are valid. (4 marks)