Question 6 (13 marks)

Consider $f(x) = 2\tan(x)$ where $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

Let $g(x) = f^{-1}(x)$ be the inverse of function f.

(a) Determine the defining rule for $y = g(x)$. (2 marks)

(b) By using implicit differentiation show that $g'(x)$ can be written in the form $\dfrac{a}{x^2 + b}$.

(4 marks)

(c)   Show that $\dfrac{3x^2 + 2x + 6}{(x^2 + 4)(x - 3)}$ can be expressed as $\dfrac{q}{x^2 + 4} + \dfrac{r}{x - 3}$ and hence determine the values for q and r. (3 marks)

(d)   Hence determine $\displaystyle\int \dfrac{3x^2 + 2x + 6}{(x^2 + 4)(x - 3)}\, dx$. (4 marks)