Question 9 **(8 marks)**

Deborah is purchasing mealworms for her pet lizard, Lizzy, to eat.

Deborah starts by buying 50 mealworms. She then buys an additional 15 at the start of each subsequent week. She feeds 12 mealworms to Lizzy each week, and each week a certain percentage of the mealworms dies.

Deborah has found that the approximate number of mealworms at the start of the n^{th} week can be modelled by M_n, where $M_{n+1} = 0.9\ (M_n - 12) + 15,\ M_1 = 50$.

(a) What percentage of the mealworms dies each week? (1 mark)

(b) Determine the approximate number of mealworms Deborah has at the start of the fifth week. (1 mark)

(c) Deborah claims that she will never run out of mealworms using this model. Justify her claim. (2 marks)

After 10 weeks, hot weather results in a larger percentage of the mealworms dying, so Deborah alters the model to:

$$N_{n+1} = 0.8\,(N_n - 12) + 15,\ N_1 = c$$

(d) (i) Determine the value of c. (1 mark)

(ii) Determine the approximate number of mealworms Deborah has at the start of the thirtieth week. (1 mark)

Deborah's vet recommends feeding Lizzy 10 mealworms a week. She would also like to maintain a constant number of 30 mealworms at the start of each week, so she changes the above model to:

$$P_{n+1} = 0.8\,(P_n - 10) + k$$

(e) Determine the value of k, the number of mealworms she must buy each week, to ensure this occurs. (2 marks)