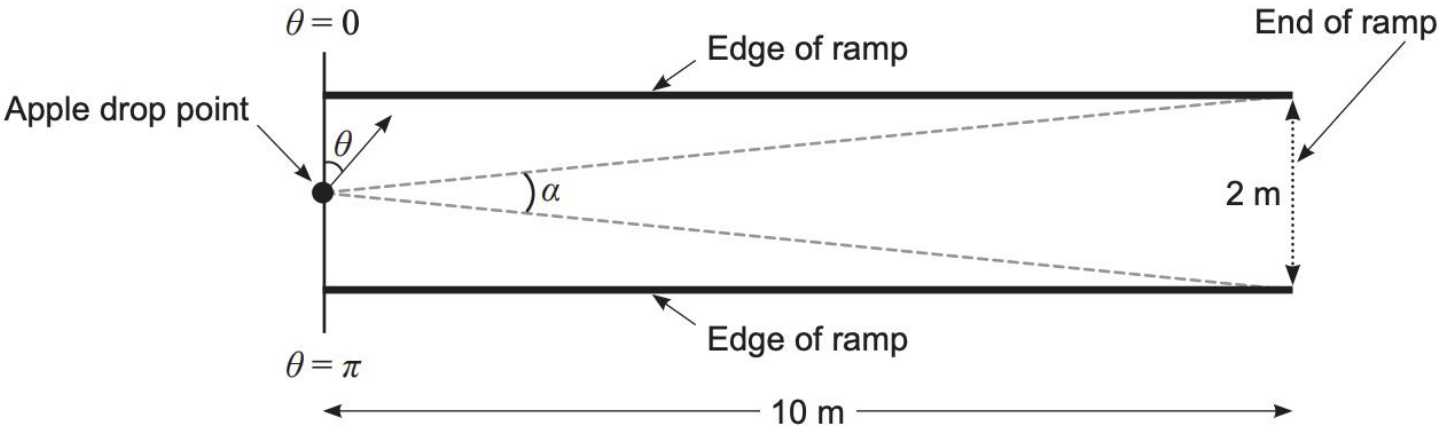
Question 13 **(9 marks)**

While leaving a shopping centre a mathematician accidentally drops a bag of apples at the top of a ramp of length 10 m and width 2 m. The diagram below shows the top view of the ramp. Four of the apples roll safely to the end of the ramp, while six roll off an edge and splatter on the ground below.

The mathematician decides to create a simple model by assuming that the:

- apples roll independently of one another along straight lines from the apple drop point
- direction each apple rolls, θ, is an angle measured about the apple drop point and is uniformly distributed over $0 \leq \theta \leq \pi$.



Apples that roll along a line within the sector marked by α will arrive safely at the end of the ramp, while others will roll off the edge.

(a) (i) Determine the value of α. (2 marks)

(ii) Hence show that the probability, p, of an apple rolling safely to the end of the ramp is $p = 0.063$ (rounded to three decimal places). (1 mark)

(b) Determine the probability that, of the 10 apples, four or more make it safely to the end of the ramp. (2 marks)

The mathematician decides to purchase another 20 bags of apples, i.e. 200 apples, return to the top of the ramp, and break each bag open one at a time. After the experiment a total of 63 apples have rolled safely to the end of the ramp.

(c) Using the sample of 200 apples, calculate a 99% confidence interval for the population proportion of apples that will roll safely to the end of the ramp. (2 marks)

(d) What does the confidence interval from part (c) suggest about the validity of the model assumptions used to calculate the probability in part (a)(ii)? (2 marks)