

**Question 15****(10 marks)**

The volume  $V(h)$  in cubic metres of a liquid in a large vessel depends on the height  $h$  (metres) of the liquid in the vessel and is given by

$$V(h) = \int_0^h e^{\left(-\frac{x^2}{100}\right)} dx, \quad 0 \leq h \leq 15.$$

(a) Determine  $\frac{dV}{dh}$  when the height is 0.5 m. (2 marks)

(b) What is the meaning of your answer to part (a)? (1 mark)

(c) The height of the liquid depends on time  $t$  (seconds) as follows:

$$h(t) = 3t^2 - t + 4, \quad t \geq 0.$$

(i) Determine  $\frac{dh}{dt}$  when the height is 6 m. (2 marks)

- (ii) Use the chain rule to determine  $\frac{dV}{dt}$  when the height is 6 m. (2 marks)

- (iii) Given the volume of the liquid at 2 seconds is  $8.439 \text{ m}^3$ , use the incremental formula to estimate the volume 0.1 second later. (3 marks)