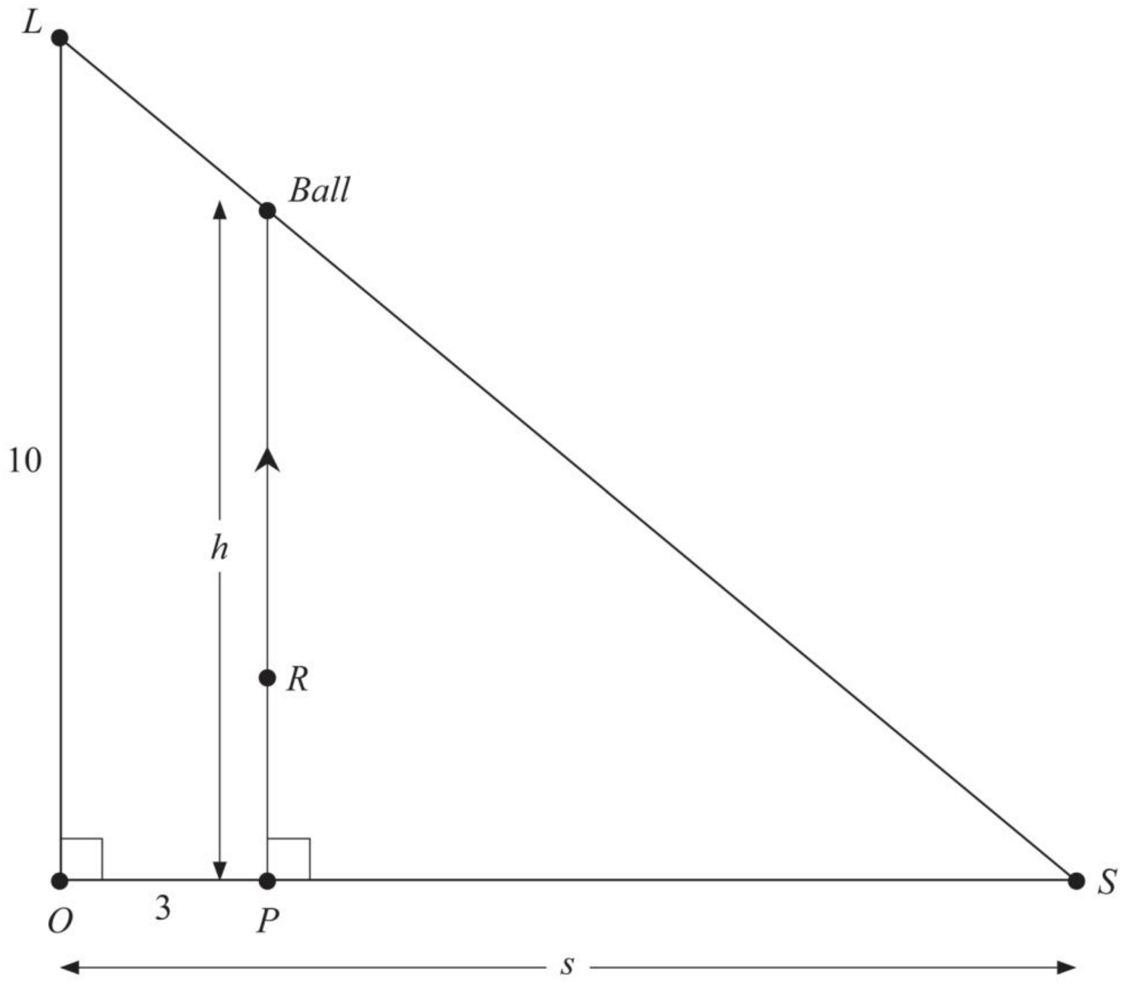
Question 19 **(16 marks)**

A ball is projected vertically into the air from point R, so that it will eventually hit the ground at point P, 3 metres from the base of a 10 metre high light at L.

At any time t seconds, when the ball is h metres above the ground, it casts a shadow on the ground at point S at a distance s metres from the base of the light.



At any time t it can be shown that $s(10-h)=30$.

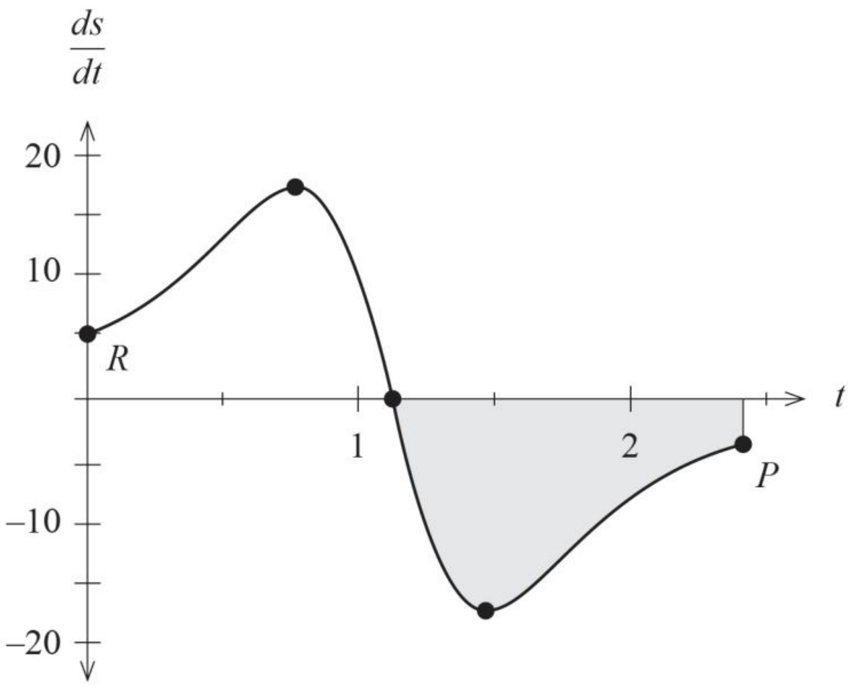
(a) Using implicit differentiation, show that $\dfrac{ds}{dt}=\dfrac{30}{(10-h)^2}\times\dfrac{dh}{dt}$. (3 marks)

At $t = 0.5$ seconds, it is found that the ball is 6.275 metres above the ground and moving upwards at 6.1 metres per second.

(b) By assuming $h''(t) = -9.8$ ms^{-2}, show that $h(t) = 2 + 11t - 4.9t^2$. (3 marks)

(c) Determine the initial speed of the ball's shadow, correct to the nearest 0.01 metres per second. (3 marks)

The graph of the function $\frac{ds}{dt}$ against time t is shown below. Point R of this graph corresponds to the ball being thrown into the air, while point P corresponds to the ball hitting the ground.



The definite integral $\int_a^b \left(\frac{ds}{dt}\right) dt$ was evaluated so that the area for the shaded region could be determined. This area is 13.4258 square units.

(d) Determine the values for a and b (correct to 0.01 seconds) and describe what this definite integral represents in terms of the motion of the shadow. (4 marks)

(e) Determine the fastest rate at which the shadow moves (correct to the nearest 0.01 metres per second) and the time when this occurs (correct to the nearest 0.01 seconds). (3 marks)