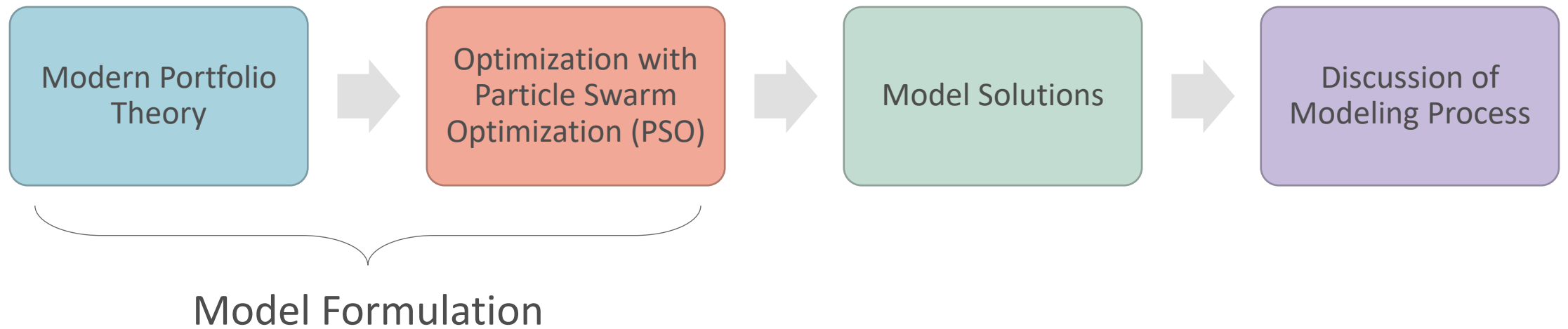


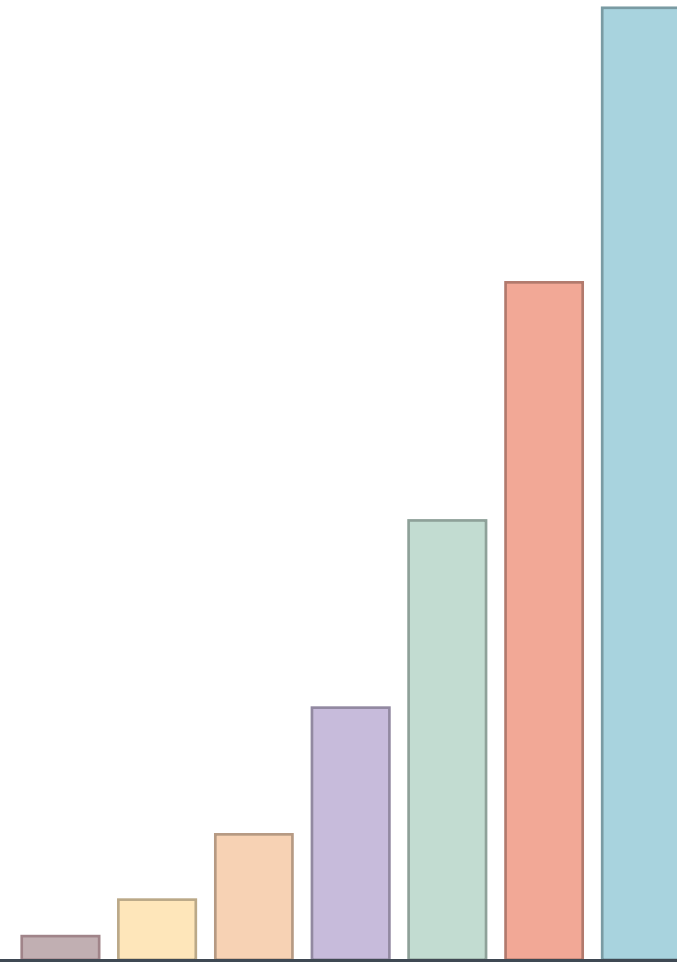


Modern Portfolio Theory with Particle Swarm Optimizer

Daniel Carpenter
Financial Engineering Analysis

Overview





Model Formulation: Modern Portfolio Theory

Modern Portfolio Theory (MPT) Overview

Get Stock Data (`yfinance`)

Calculations to set up MPT Modeling

Meet an Expected Return by Minimizing Risk of Portfolio

Calculate the Returns

- Calculate the percentage return of each stock's adjusted close price

$$Returns_{stock,t} = \ln \left(\frac{AdjClose_{stock,t}}{AdjClose_{stock,t-1}} \right),$$
$$\forall stock \in StockList, \forall t \in Periods$$

"StockList" contains the list stock tickers to consider investing in *

"Periods" are the monthly periods that associate with a stock's adjusted close price **

Calculate the Expected Returns of Each Stock

- Find the expected, or mean, return for each stock
- Pitfalls of MPT: predictive analysis

$$ExpectedReturns_{stock} = \frac{1}{numPeriods} \sum_{t \in Periods} Returns_{stock,t},$$

$\forall stock \in StockList$

“StockList” contains the list stock tickers to consider investing in *

“Periods” are the monthly periods that associate with a stock’s adjusted close price **

Calculate the Excess Returns

- Return in excess of the average return of the stock

$$\begin{aligned} \text{ExcessReturns}_{stock,t} &= \text{Returns}_{stock,t} - \text{ExpectedReturns}_{stock,t}, \\ &\forall stock \in \text{StockList}, \forall t \in \text{Periods} \end{aligned}$$

“StockList” contains the list stock tickers to consider investing in *

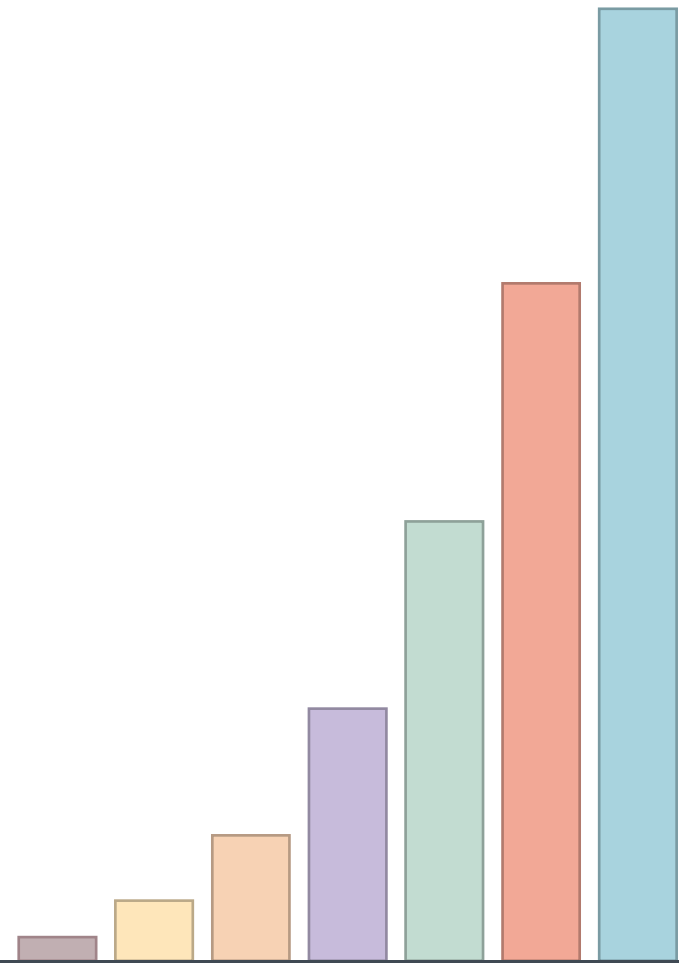
“Periods” are the monthly periods that associate with a stock’s adjusted close price **

Account for Covariances Among Stocks

- Goal: understand how stock returns covary
- Higher risk associated with higher covariance
- Note non-linearity of variance-covariance matrix

$$VarCov = \frac{ExcessReturns^T \cdot ExcessReturns}{N - 1}$$

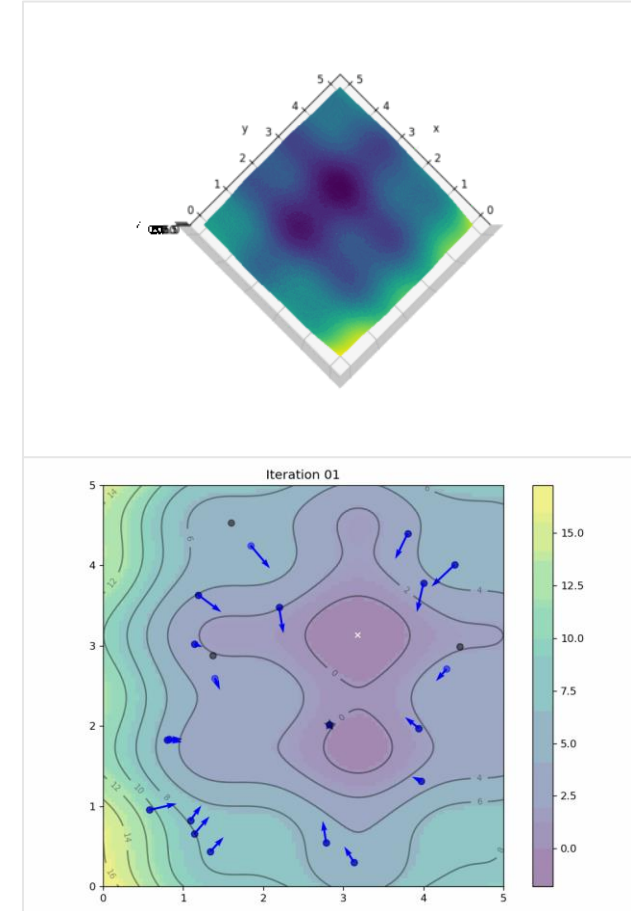
“N” represents the number of stocks in the “StockList” *



Model Formulation: Particle Swarm Optimizer

PSO Overcomes Non-Linear Objectives

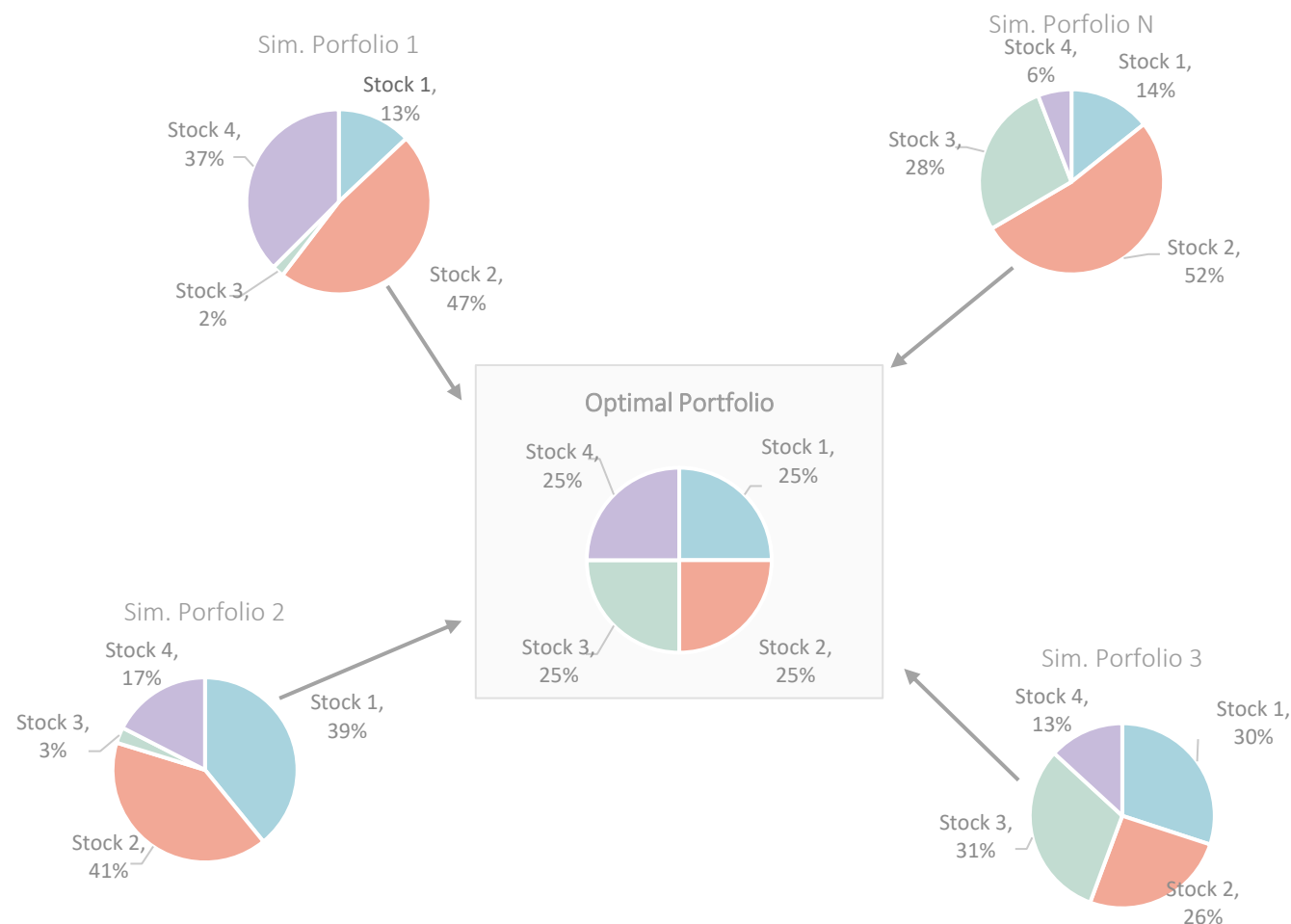
1. Metaheuristic algorithm that emulates organisms, or “particles” moving in swarms
2. Helps overcome local minima for non-linear modeling
3. Basic idea:
 - Swarm of particles starting random locations
 - Each particle has no understanding of where the overall swarm is headed
 - Each particle only knows where their immediate surrounding environment is headed
 - Collectively will arrive at global optima



Above model animation adapted from [Machine Learning Mastery](#)

PSO Used to Minimize Risk of Portfolio

1. Minimizing Risk involves non-linear objective
2. Randomly use PSO algorithm to adjust swarm of stock weights to find the minimum risk portfolio*



PSO Velocity and Position update equation used for modeling *

Minimize the Risk of Portfolio

- Decision Variable: Weights to invest into each stock
- Risk Defined as

$$risk = \sqrt{12} \times \sqrt{Weights.VarCov.Weights^T}$$

- Optimization Problem:

Minimize: $risk$

Subject to:

$$\sum_{stock \in StockList} Weights_{stock} = 1$$

$$\sum_{stock \in StockList} (Weights_{stock} \times ExpectedReturns_{stock}) \geq minDesiredReturn$$

Note that “12” represents the number of months in a single calendar year *

Maximize the Sharpe Ratio

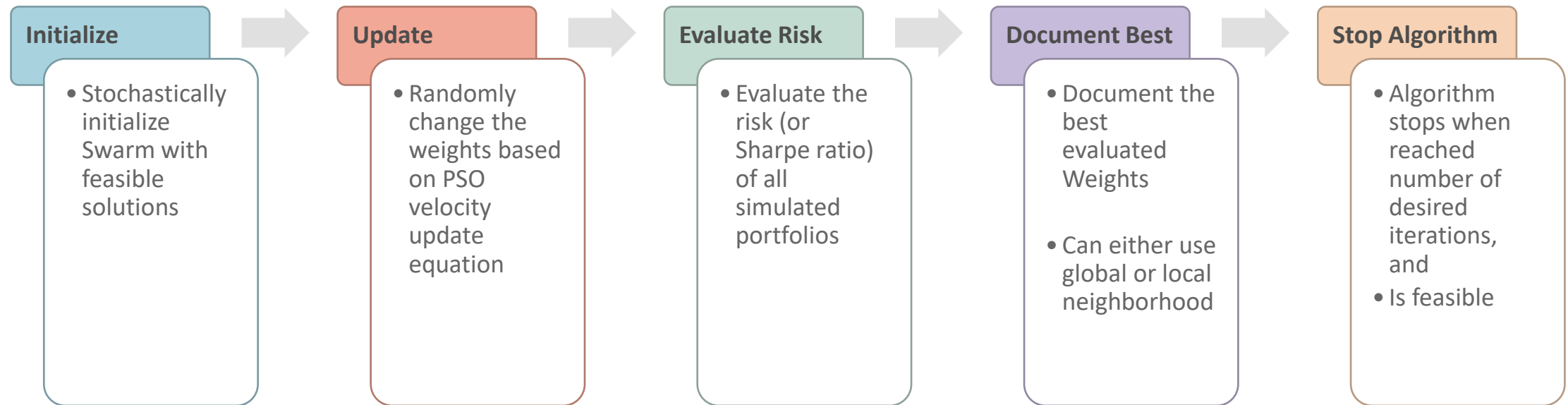
- Note algorithm can solve by maximizing the Sharpe Ratio
- Sharpe Ratio defined as:

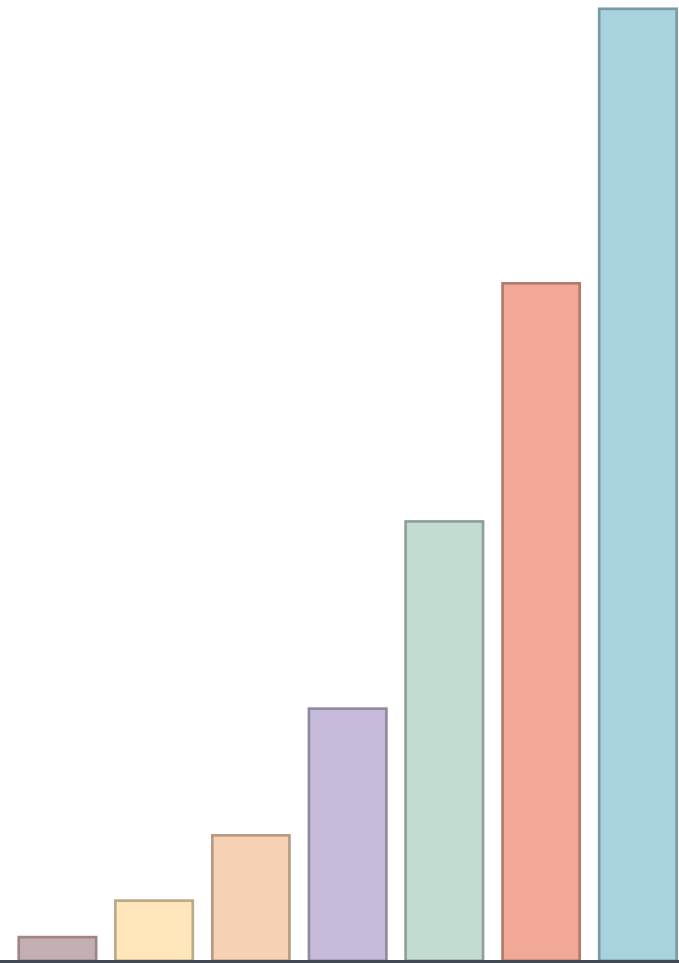
$$\textit{sharpeRatio} = \frac{\textit{expectedReturn} - \textit{riskFreeRate}}{\textit{risk}}$$

- Limitations of maximization of Sharpe Ratio with metaheuristic

“riskFreeRate” contains a proxy for the risk-free rate. *
It includes the previous period’s value of the U.S. Treasury Bill **

PSO Conceptual Process Overview





Model Solutions

Base Solution

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 3,000 Iterations
- 30 Simulated Portfolios
- Minimizing Portfolio Risk
- Expected Return $\geq 7.5\%$
- Local Best Ring Structure
- Sample Period:
 - May 2012 – May 2022

Key Summary Statistics -----

Global Best Annualized Risk: 10.0%
Annualized Expected Return: 15.0%
Sharpe Ratio: 1.44
Expected Return over 9.4 Years: 373.9%

Global best weights in each stock (Only includes stocks to invest in):

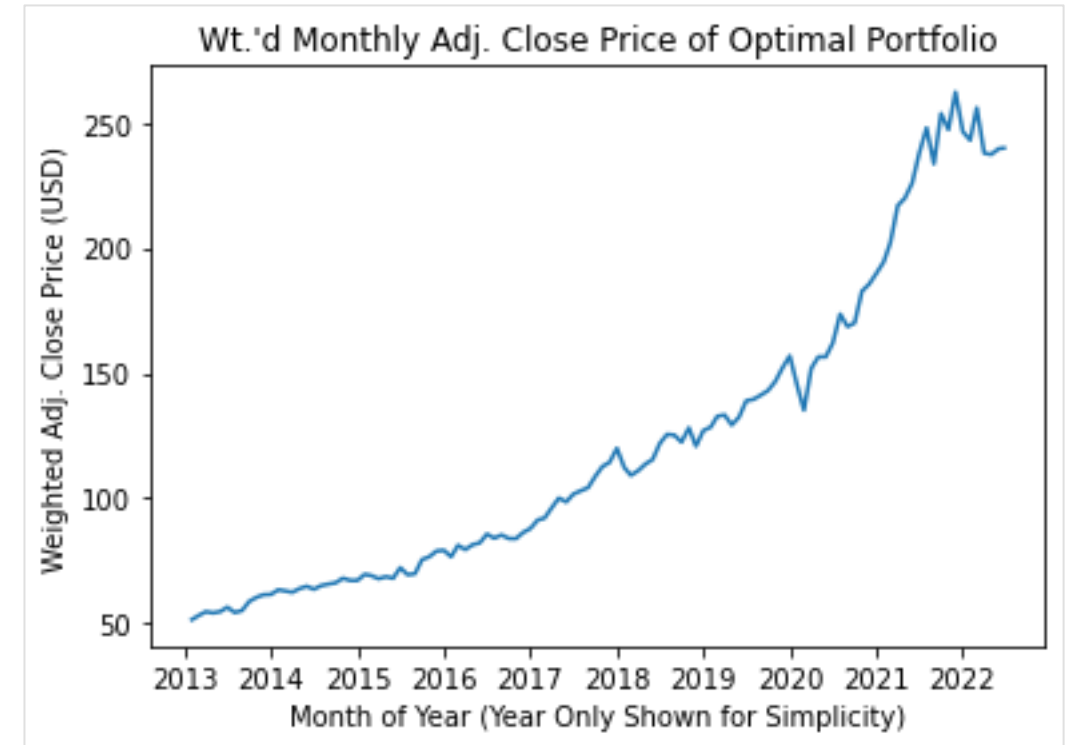
Ticker	Opt. Weight
V	0.3%
NVDA	0.7%
PG	0.3%
MA	3.4%
PFE	3.7%
KO	0.4%
ABBV	11.6%
PEP	12.2%
VZ	0.5%
COST	0.6%
AVGO	11.5%
MCD	0.1%
CSCO	9.7%
DIS	1.0%
TMUS	13.5%
UPS	5.0%
INTC	6.6%
WFC	9.0%
RTX	9.5%

S&P 500 inclusion list as of June 22, 2022 *

Base Solution Cont'd

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 3,000 Iterations
- 30 Simulated Portfolios
- Minimizing Portfolio Risk
- Expected Return $\geq 7.5\%$
- Local Best Ring Structure
- Sample Period:
 - May 2012 – May 2022



S&P 500 inclusion list as of June 22, 2022 *

Test 1:

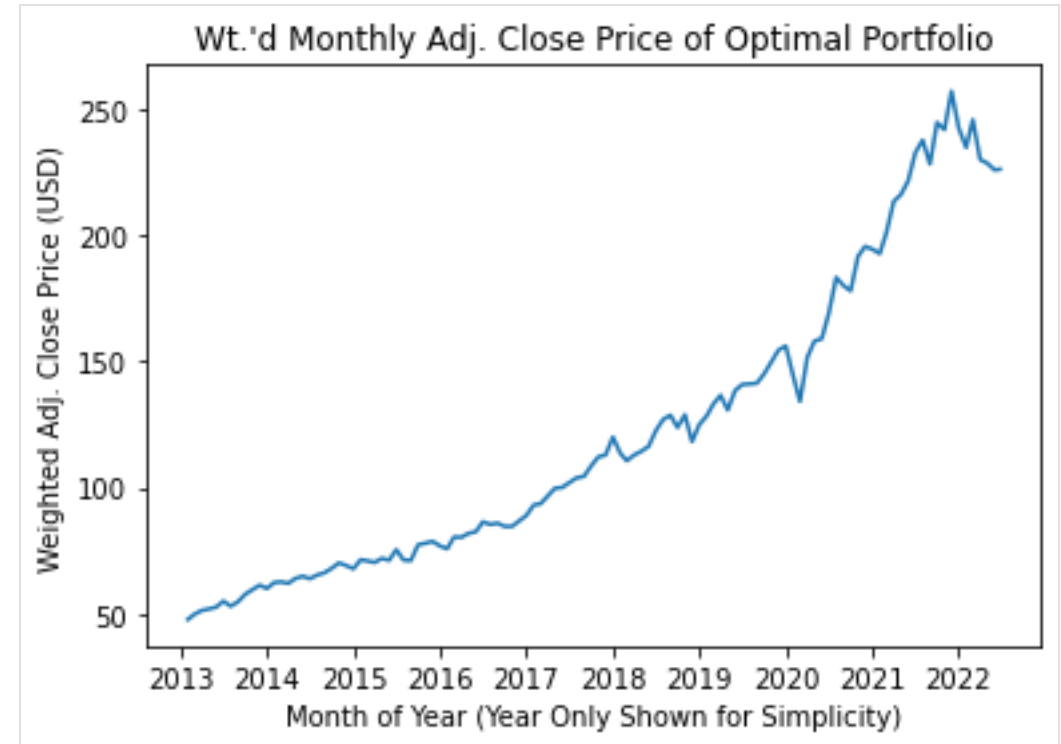
More Iterations and Less Simulated Portfolios: Worse

Jun-22

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 10,000 Iterations**
- 10 Simulated Portfolios***
- Minimizing Portfolio Risk
- Expected Return $\geq 7.5\%$
- Local Best Ring Structure
- Sample Period:
 - May 2012 – May 2022

Key Summary Statistics -----
Global Best Annualized Risk: 12.0%
Annualized Expected Return: 18.1%
Sharpe Ratio: 1.45
Expected Return over 9.4 Years: 477.7%



S&P 500 inclusion list as of June 22, 2022 *

Changed from base model using 3,000 iterations **

Changed from base model using 30 portfolio simulations **

Total of 35 stocks recommended to invest in for optimal portfolio ***

Test 2:

Maximizing the Sharpe Ratio : *Worse*

Parameters

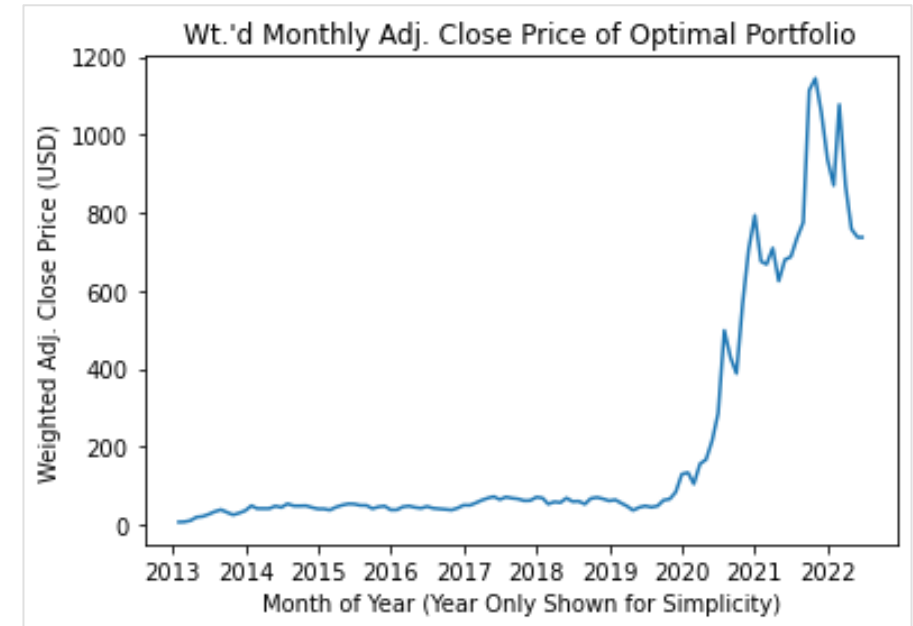
- Top 50 Market Cap. Firms in S&P 500*
- 3,000 Iterations
- 30 Simulated Portfolios
- Maximize Portfolio Sharpe Ratio**
- Expected Return $\geq 7.5\%$
- Local Best Ring Structure
- Sample Period:
 - May 2012 – May 2022

Key Summary Statistics -----

Global Best Annualized Risk: 68.4%
Annualized Expected Return: 68.4%
Sharpe Ratio: 0.99
Expected Return over 9.4 Years: 13544.6%

Global best weights in each stock (Only includes stocks to invest in):

Ticker	Opt. Weight
PM	100.0%



S&P 500 inclusion list as of June 22, 2022 *

Changed from base model which minimized the risk **

Note not recommended to use the Sharpe Ratio with this model **

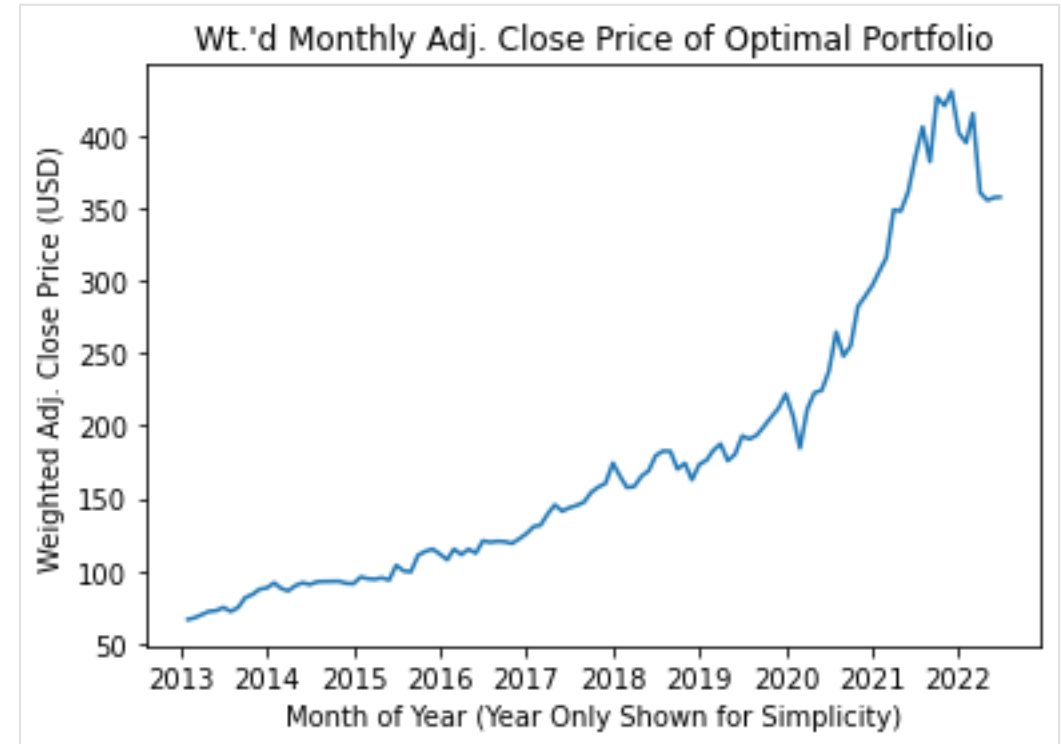
Test 3:

Global Best Method : *Worse*

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 3,000 Iterations
- 30 Simulated Portfolios
- Minimizing Portfolio Risk
- Expected Return $\geq 7.5\%$
- **Global Best Structure****
- Sample Period:
 - May 2012 – May 2022

Key Summary Statistics -----
Global Best Annualized Risk: 12.7%
Annualized Expected Return: 19.0%
Sharpe Ratio: 1.45
Expected Return over 9.4 Years: 513.2%



S&P 500 inclusion list as of June 22, 2022 *

Changed from base model using local best ring structure **

Not all stocks shown, assume weights total 100% as seen in detailed report **

Total of 47 stocks recommended to invest in for optimal portfolio ***

Test 4:

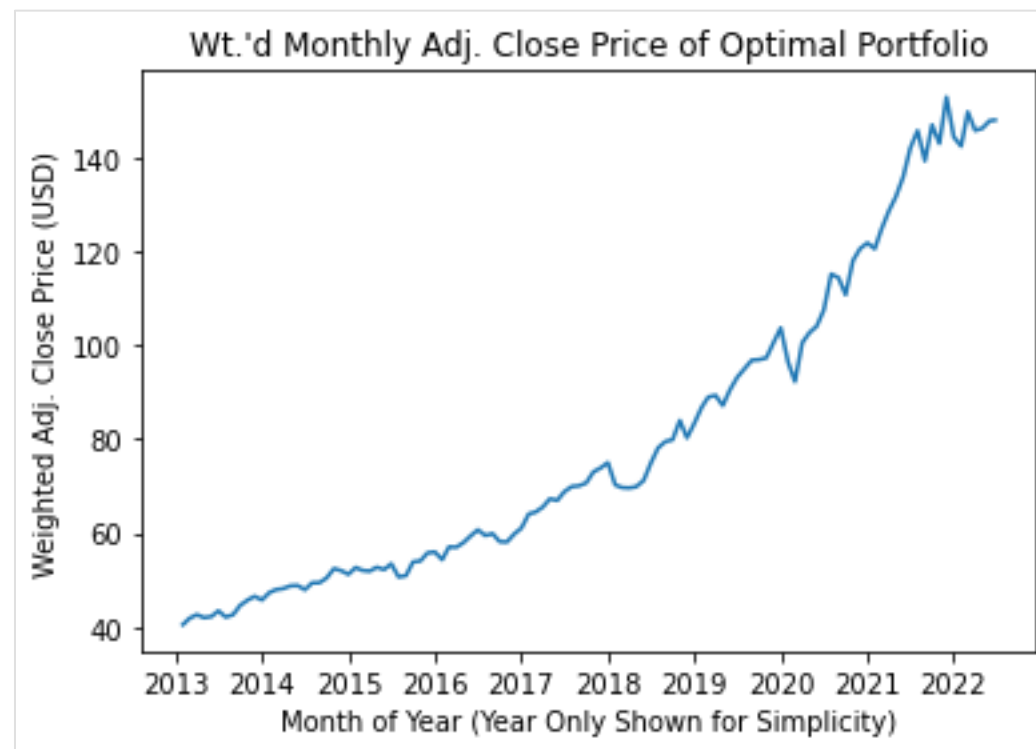
More Iterations and Simulated Portfolios: *Better*

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 10,000 Iterations**
- 100 Simulated Portfolios***
- Minimizing Portfolio Risk
- Expected Return $\geq 7.5\%$
- Local Best Ring Structure
- Sample Period:
 - May 2012 – May 2022

Key Summary Statistics -----

Global Best Annualized Risk: 9.5%
Annualized Expected Return: 14.4%
Sharpe Ratio: 1.45
Expected Return over 9.4 Years: 355.4%



S&P 500 inclusion list as of June 22, 2022 *

Changed from base model using 3,000 iterations **

Changed from base model using 30 portfolio simulations **

Total of 15 stocks recommended to invest in for optimal portfolio ***

Test 5:

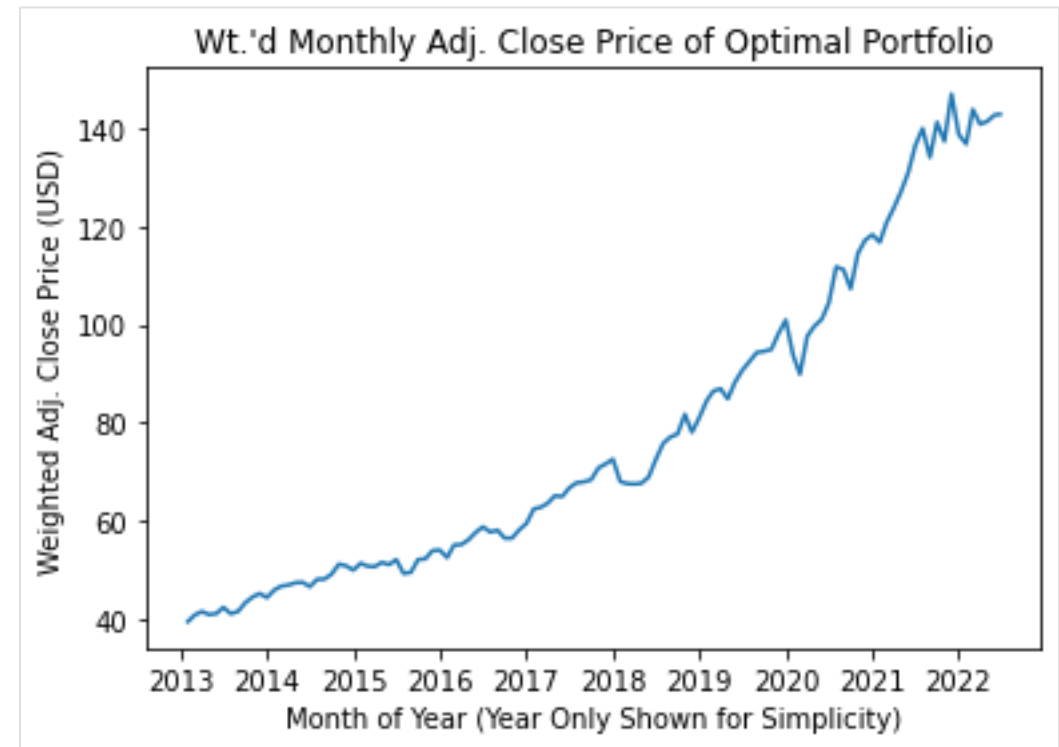
More Iterations and Simulated Portfolios:

Better

Parameters

- Top 50 Market Cap. Firms in S&P 500*
- 10,000 Iterations**
- 200 Simulated Portfolios***
- Minimizing Portfolio Risk
- Expected Return $\geq 7.5\%$
- Local Best Ring Structure
- Sample Period:
 - May 2012 – May 2022

Key Summary Statistics -----
Global Best Annualized Risk: 9.5%
Annualized Expected Return: 14.5%
Sharpe Ratio: 1.46
Expected Return over 9.4 Years: 357.8%



S&P 500 inclusion list as of June 22, 2022 *

Changed from base model using 3,000 iterations **

Changed from base model using 30 portfolio simulations **

Total of 15 stocks recommended to invest in for optimal portfolio ***

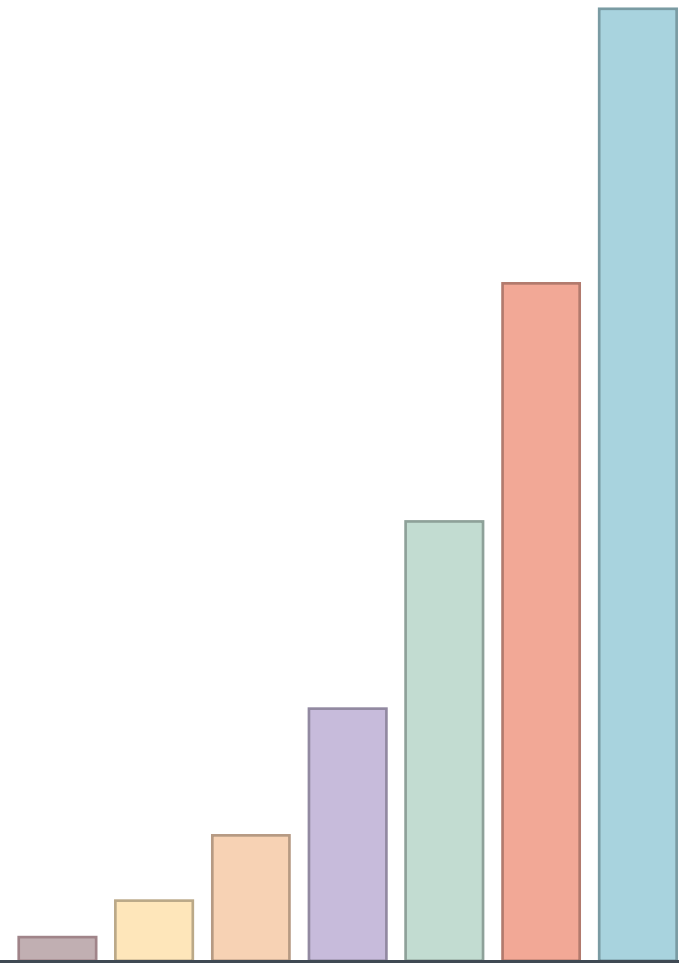
Summary of Model Solutions



Increasing iterations and simulations is better

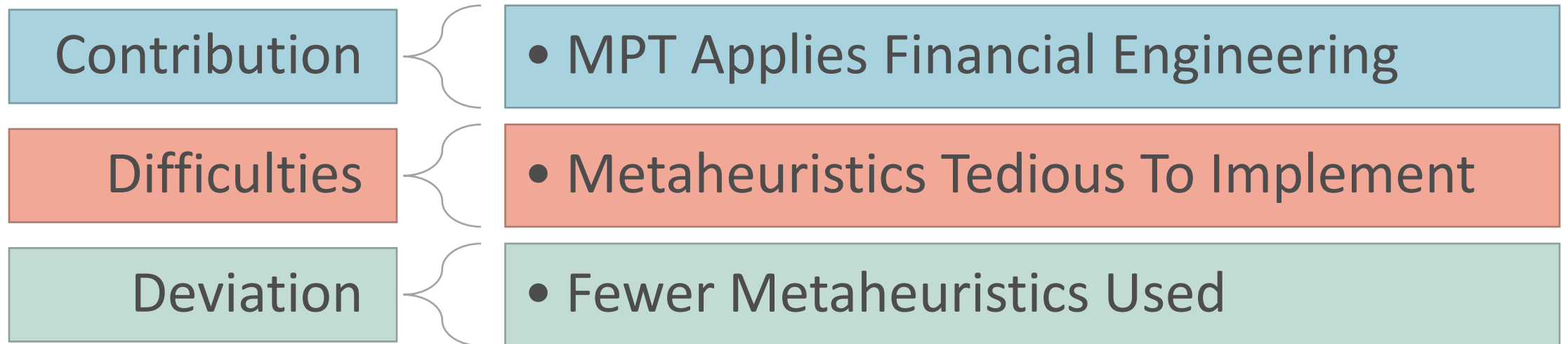
Local best is better than global best

Maximizing Sharpe ratio has future opportunity



Discussion of Modeling Process

Discussion of Modeling Process



Summary of Results

