常微分方程

分为四种常见的方法，主要通过判断其类型判断

可分离变量

$$y' = f(x)g(y)$$

$$\Rightarrow \frac{dy}{dx} = f(x)g(y)$$

$$\Rightarrow \int g(y)dy = \int f(x)dx$$

$$\text{左右同时积分}$$

$$\Leftrightarrow G(y) = F(x) + C$$

一阶线性方程

$$y' + p(x)y = \theta(x)$$

$$\text{对}p(x)\text{积分，左右同时乘上}e\text{的积分过后的次方}$$

$$\Rightarrow e^{\int p(x)dx}.y' + p(x).e^{\int p(x)dx}y = \theta(x)e^{\int p(x)dx}$$

$$\text{化简}$$

$$\Leftrightarrow [y.e^{\int p(x)dx}]' = \theta(x)e^{\int p(x)dx}$$

$$\Rightarrow y.e^{\int p(x)dx} = \int \theta(x)e^{\int p(x)dx}dx + C$$

$$\Rightarrow y = e^{-\int p(x)dx}(\int \theta(x)e^{\int p(x)dx}dx + C)$$

可化为可分离变量

$$y' = \Phi(\frac{y}{x})$$

$$\text{令}\frac{y}{x} = u \Leftrightarrow y = ux$$

$$\therefore y' = (ux)' = u + x.\frac{du}{dx}$$

$$\Rightarrow u + x.\frac{du}{dx} = \Phi(u)$$

$$\Leftrightarrow \int \frac{1}{\phi(u) - u}du = \int \frac{1}{x}dx$$

二阶线性齐次方程

$$y'' + py' + q = 0$$

$$\text{写出特征方程}$$

$$r^2 + pr + q = 0$$

情况一 两个不同实根 r1 r2

$$y_1 = e^{r1x}$$
$$y_2 = e^{r2x}$$
$$y = C_1 y_1 + C_2 y_2$$

情况二 重根 r1=r2

$$y = (C_1 x + C_2) e^{rx}$$

情况三 共轭复根

$$r_{1,2} = \alpha \pm \beta i$$
$$y = e^{\alpha x}[C_1 sin(\beta x) + C_2 cos(\beta x)]$$

非齐次

与齐次的区别在于多了求特解

求特解的情况

$$y'' + py' + q = \Phi(x)$$
$$\Phi(x) = e^{\lambda x}\theta(x)$$
$$y^* = U(x)e^{\lambda x}\text{带入原方程}$$
$$U^{''}(x) + (2\lambda + p)U^{'}(x) + (\lambda^2 + p\lambda + q)U(x) = \theta(x)$$
$$f(x) = Asin\omega x + Bcos\omega x$$
$$y^* = Asinx + bcosx$$
$$\text{如果不行就}$$
$$y^* = [Asinx + bcosx]x$$
$$\text{然后带入原式求}$$