Q1

$$P(open|u) = \frac{P(u|open)P(open)}{P(u|open)P(open)+P(u|close)P(close)} = \frac{1*0.5}{1*0.5+0.8*0.5} = 0.556$$

Q2

As we don't know the initial state of the door, we believe.

$bel(X_0 = open) = bel(X_0 = closed) = 0.5$.

From (4) (5) (6), we can know the process noise

$P(X_t = open \mid U_t = push, X_{t-1} = open) = 1$

$P(X_t = close \mid U_t = push, X_{t-1} = open) = 0$

$P(X_t = open \mid U_t = push, X_{t-1} = close) = 0.9$

$P(X_t = close \mid U_t = push, X_{t-1} = close) = 0.1$

$P(A|B) = \frac{P(AB)}{P(B)}$

$P(B|A) = \frac{P(AB)}{P(A)}$

$P(X_t = open \mid U_t = nothing, X_{t-1} = open) = 1$

$P(X_t = close \mid U_t = nothing, X_{t-1} = open) = 0$

$P(X_t = open \mid U_t = nothing, X_{t-1} = close) = 0$

$P(X_t = close \mid U_t = nothing, X_{t-1} = close) = 1$

z observation
u action
x real state.

$\frac{P(x_1) P(x_1|open)}{P(open)}$

$P(z_1 = open | x_1) =$

From the input, we know $u_1 = do\_nothing$, $z_1 = open$

Then $\overline{bel}(x_1) = \sum_{x_0} P(x_1|u_1, x_0)\, bel(x_0)$

$= P(X_1 \mid U_t = do\_nothing, X_0 = open)\, bel(X_0 = open)$
$+ P(X_1 \mid U_t = do\_nothing, X_0 = closed)\, bel(X_0 = closed)$

$\overline{bel}(X_1 = open) = 1 \times 0.5 + 0 \times 0.5 = 0.5$

$\eta = (0.4 + 0.15)^{-1}$

$\overline{bel}(X_1 = closed) = 0 \times 0.5 + 1 \times 0.5 = 0.5$

$bel(X_1) = \eta\, P(z_1 = open \mid x_1)\, \overline{bel}(x_1)$

$bel(X_1 = open) = 0.8 \times 0.5 = 0.4$

$bel(X_1 = closed) = 0.3 \times 0.5 = 0.15$

$bel(X_1 = open) = 0.727$

$bel(X_1 = closed) = 0.273$

$\therefore \overline{bel}(X_2 = open) = 0.727 \times 1 + 0.273 \times 0.9 = 0.973$

$\overline{bel}(X_2 = close) = 0.727 \times 0 + 0.273 \times 0.1 = 0.0273$

$bel(X_2 = open) = 0.990$

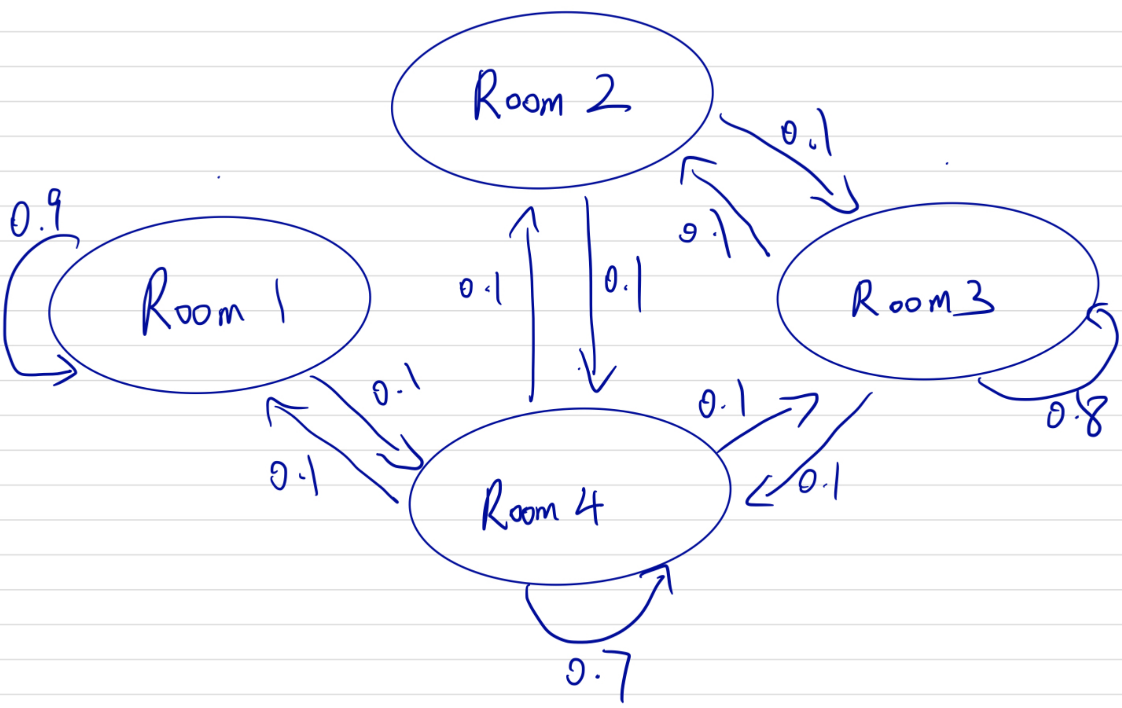
$bel(X_2 = closed) = 0.010$

$\therefore \eta\, bel(X_2 = open) = 0.973 \times 0.8 = 0.778$ $\eta\, bel(X_2 = close) = 0.0273 \times 0.3 = 0.00819$

Q3

(1)

↓ Room 2 to room 2,0.8



(2)

According to the room number in (1), the transition matrix is

$$\begin{bmatrix} 0.9\ 0\ 0\ 0.1 \\ 0\ 0.8\ 0.1\ 0.1 \\ 0\ 0.1\ 0.8\ 0.1 \\ 0.1\ 0.1\ 0.1\ 0.7 \end{bmatrix}$$

The stationary distribution of a Markov Chain with transition matrix P is some vector, ψ, such that ψP = ψ.

Then we know

$$.9\phi_1 + .1\phi_4 = \phi_1$$

$$.8\phi_2 + .1\phi_3 + .1\phi_4 = \phi_2$$

$$.1\phi_2 + .8\phi_3 + .1\phi_4 = \phi_3$$

$$.1\phi_1 + .1\phi_2 + .1\phi_3 + .7\phi_4 = \phi_4$$

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 = 1$$

Solve, get $\phi = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$, so the probability is all 0.25 for every room.

(3)

(3)

From left to right, top to buttom we mark the doors as door 1-4

let $A_i = the\ robot\ is\ at\ room\ i\ B = the\ robot\ is\ crossing\ a\ door$

Then $P(A_1|B) = \frac{P(B|A_1)P(B)+P(B|A_2)P(B)}{\sum_1^4 P(B|A_i)P(B)} = \frac{0.25*(0.1+0.3*\frac{1}{3})}{0.25(0.1+0.2+0.2+0.3)} = 0.25$

Q4

$$
\begin{aligned}
bel(x_t) &= P(x_t|u_1, z_1, \ldots, u_t, z_t) \\
&= \eta P(z_t|x_t, u_1, z_1, \ldots, u_t)P(x_{t-1}, u_1, z_1, \ldots, u_t) \\
&= \eta P(z_t|x_t)P(x_t|u_1, z_1, \ldots, u_t) \\
&= \eta P(z_t|x_t) \int P(x_t|u_1, z_1, \ldots, u_t, x_{t-1})P(x_{t-1}|u_1, z_1, \ldots, u_t)dx_{t-1} \\
&= \eta P(z_t|x_t) \int P(x_t|u_t, x_{t-1})P(x_{t-1}|u_1, z_1, \ldots, u_t)dx_{t-1} \\
&= \eta P(z_t|x_t) \int P(x_t|u_t, x_{t-1})P(x_{t-1}|u_1, z_1, \ldots, z_{t-1})dx_{t-1} \\
&= \eta P(z_t|x_t) \int P(x_t|u_t, x_{t-1})bel(x_{t-1})dx_{t-1}
\end{aligned}
$$