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ELL-225 PROJECT

1 Abstract Idea

The aim of this course project is to develop an autopilot for a rider-less bicycle to keep it in vertical upright position. We are considering the following quantities as states: i) roll angle,ii) roll rate, iii)steer angle iv) steer rate and as input we have steering torque and our output is roll angle, the dynamics of the bicycle can be expressed as a fourth order linear model. For this project, we have considered following three fixed velocities: v1 = 0 m/s, v2=1m/s and v3 =5m/s.

2 Introduction

Bicycles are statically unstable like the inverted pendulum, but it can be under some conditions, be stable in forward motion. A detailed model of a bicycle is complex because the system has many degrees of freedom and the geometry is very complicated. We assume that the bicycle consists of four rigid parts, specifically, two wheels, a frame, and a front fork with handlebars. We have not taken the influence of other moving parts, such as pedals, chain, and brakes on the dynamics. We often assume that the forward velocity is constant. To summarize, we simply assume that the bicycle moves on a horizontal plane and that the wheels always maintain contact with the ground.

3 Methodology

Using Carvallo Whipple bicycle model, $\mathrm{M}\ddot{\mathrm{q}} + \mathrm{CV}\dot{q} + (\mathrm{K}_o + K_2 v_2)q = T$

$\dot{x}$(t)=A(v(t))$\cdot x(t) + B \cdot u(t)$,   (1)

y(t)= C $\cdot x(t) + D \cdot u(t)$,   (2)

where   x(t)=$\begin{pmatrix} \phi \\ \delta \\ \dot{\phi} \\ \dot{\delta} \end{pmatrix}$ u(t)=T$_\delta(t)$ $y(t) = x(t)$.

$$A=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 13.67 & 0.225 - 1.319\, v^2(t) & -0.164\, v(t) & -0.552\, v(t) \\ 4.857 & 10.81 - 1.125\, v^2(t) & 3.621\, v(t) & -2.388\, v(t) \end{pmatrix} \qquad B=\begin{pmatrix} 0 \\ 0 \\ -0.339 \\ 7.457 \end{pmatrix}$$

$$C=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad D=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$ Hence, G(s)=C(sI-A)$^{-1}B + D$

In the given state space representation we examine the output for different velocities. Consider different cases v=0, v=0.5 and v=5 m/s. We shall be investigating each case individually by putting appropriate value of v in the state space representation and finding out G(s) and then finding the unit step response.

3.1 Case 1: v=0 m/s

3.1.1 State Space Model

$$\begin{pmatrix} \dot{\phi} \\ \dot{\delta} \\ \ddot{\phi} \\ \ddot{\delta} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 13.6700 & 0.2250 & 0 & 0 \\ 4.8570 & 10.8100 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \dot{\phi} \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -0.3390 \\ 7.4570 \end{pmatrix} \mathrm{T}$$

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \dot{\phi} \\ \dot{\delta} \end{pmatrix}$$

3.1.2 Transfer Function

The transfer function comes out to be $T(s) = \frac{-0.339s^2+5.342}{s^4+3.553e^{-15}s^3-24.48s^2-2.842e^{-14}s+146.7}$

3.1.3 Eigenvalues

Eigenvalues in this case are -3.7432, -3.2355, 3.7432, 3.2355

3.1.4 Poles

Poles of transfer function are -3.7432,-3.2355,3.7432,3.2355

3.1.5 Zeroes

Zeroes of transfer function are 3.9698, -3.9698

3.1.6 Laplace Transform of Unit Step Response

The Laplace Transform comes out to be $L(s) = \frac{-0.339s^2+5.342}{s^5+3.553e^{-15}s^4-24.48s^3-2.842e^{-14}s^2+146.7s}$

3.1.7 Unit Step Response

$0.005969e^{-3.743t} - 0.02418e^{3.236t} - 0.02418e^{-3.236t} + 0.005969e^{3.743t} + 0.03642$

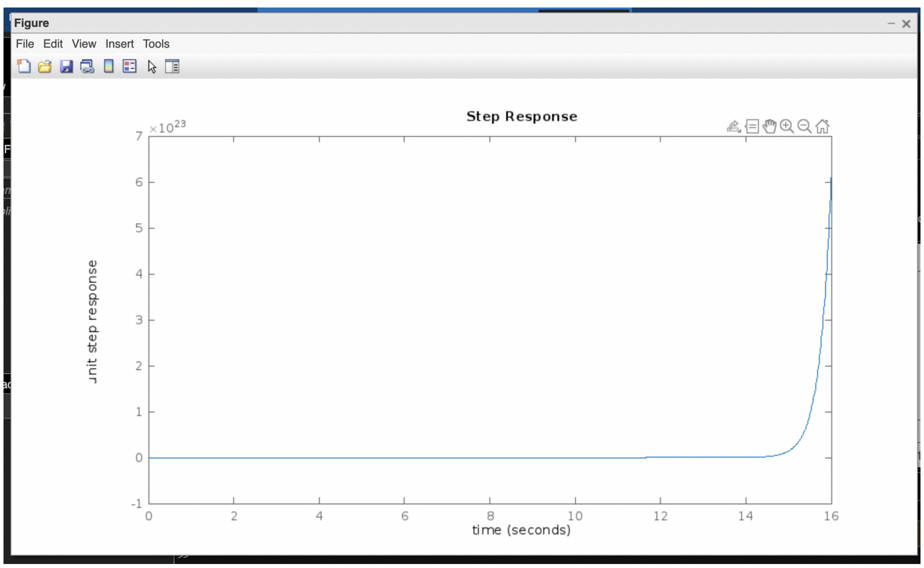

Figure 1: Caption

3.1.8 Zero Input Response

In Laplace domain the frequency Response is as follows $Y(s) = -\frac{(40(-200s^3-200s^2+2117s+2117)}{8000s^4-195840s^2+1173439}$

$y(t) = 0.0113e^{-3.24t} + 0.0215e^{3.24t} + 0.354e^{-3.74t} + 0.613e^{3.74t}$

Zero input response is shown in the following plot

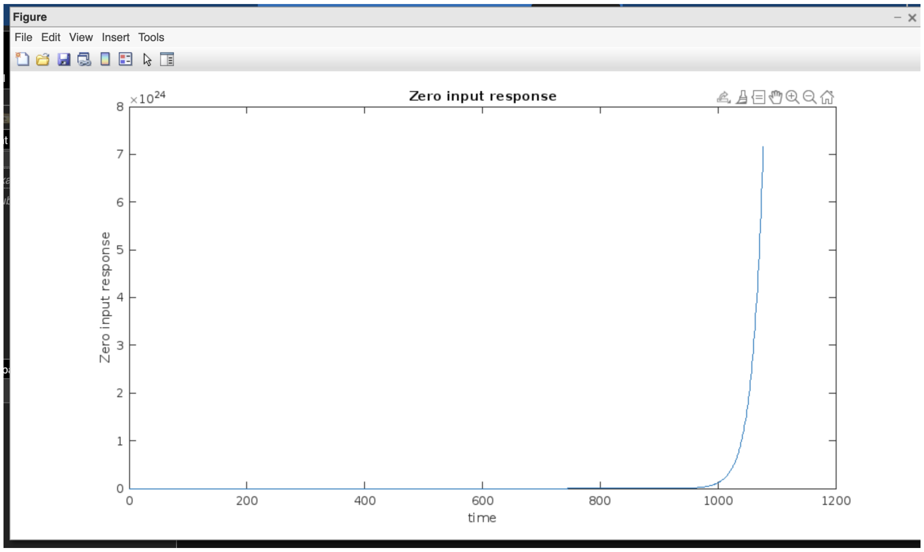

Figure 2: Caption

3.2 Case 2: v=1 m/s

3.2.1 State Space Model

$$\begin{pmatrix} \dot{\phi} \\ \dot{\delta} \\ \ddot{\phi} \\ \ddot{\delta} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 13.6700 & -1.0940 & -0.1640 & -0.5520 \\ 4.8570 & 9.6850 & 3.6210 & -2.3880 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \dot{\phi} \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -0.3390 \\ 7.4570 \end{pmatrix} \mathrm{T}$$

$$\mathrm{y} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \dot{\phi} \\ \dot{\delta} \end{pmatrix}$$

3.2.2 Transfer Function

$T(s) = \frac{-0.339s^2 - 4.926s - 4.875}{s^4 + 2.552s^3 - 20.96s^2 - 27.59s + 137.7}$

3.2.3 Eigenvalues

Eigenvalues in this case are -4.0875 + 0.3256i, -4.0875 - 0.3256i, 2.8115 + 0.5343i, 2.8115 - 0.5343i

3.2.4 Poles

Poles of transfer function are -4.0875 + 0.3256i, -4.0875 - 0.3256i, 2.8115 + 0.5343i, 2.8115 - 0.5343i

3.2.5 Zeroes

Poles of transfer function are -13.4622, -1.0682

3.2.6 Laplace Transform of Unit Step Response

The Laplace Transform comes out to be $T(s) = \frac{-0.339s^2-4.926s-4.875}{s^5+2.552s^4-20.96s^3-27.59s^2+137.7s}$

3.2.7 Unit Step Response

$-e^{-4.088t}cos(0.3256t)(0.007565+0.07499i) - e^{-4.088t}cos(0.3256t)(0.007565-0.07499i)$

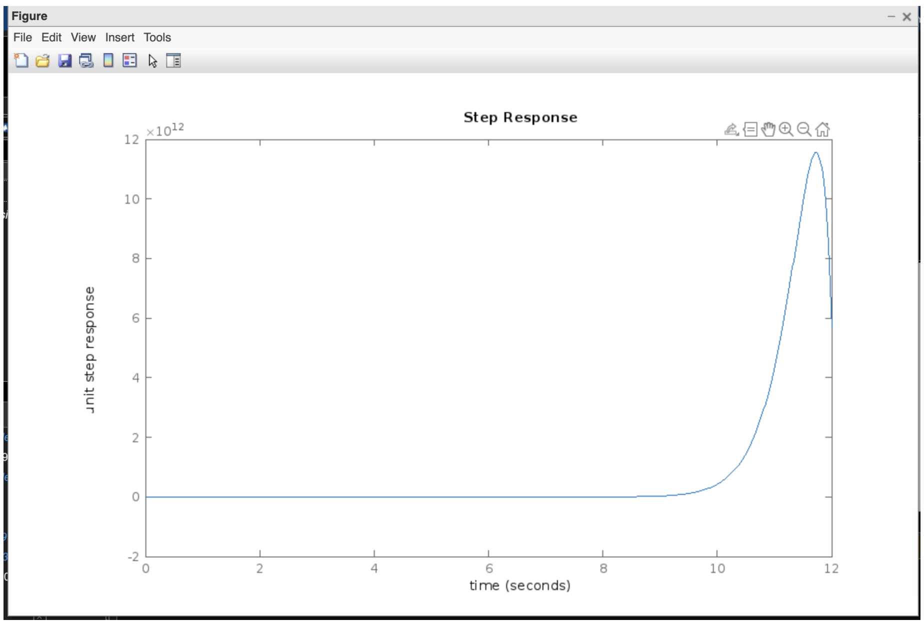

Figure 3: Caption

3.2.8 Zero Input Response

In Laplace domain the frequency response $Y(s) = \frac{-(-500000s^3-1776000s^2+3276288s+8182279)}{(500000s^4+1276000s^3-10482288*s^2-13794931s+68853754)}$

$y(t) = e^{-4.09t}cos(0.326t)(0.157+0.0622i) + e^{-4.09t}cos(0.326t)(0.157-0.0622i) + e^{4.09t}cos(0.326t)(0.157-0.0622i)+e^{2.81t}cos(0.534t)(0.343-0.292i)+e^{4.09t}cos(0.326t)(0.157-0.0622i)+e^{2.81t}cos(0.534t)(0.343-0.292i)+e^{-4.09t}sin(0.326t)(0.0622-0.157i)+e^{-4.09t}sin(0.326t)(0.0622+0.157i) + e^{2.81t}sin(0.534t)(0.292-0.343i) + e^{2.81t}sin(0.534t)(0.292+0.343i)$

Zero input response is shown in the following plot

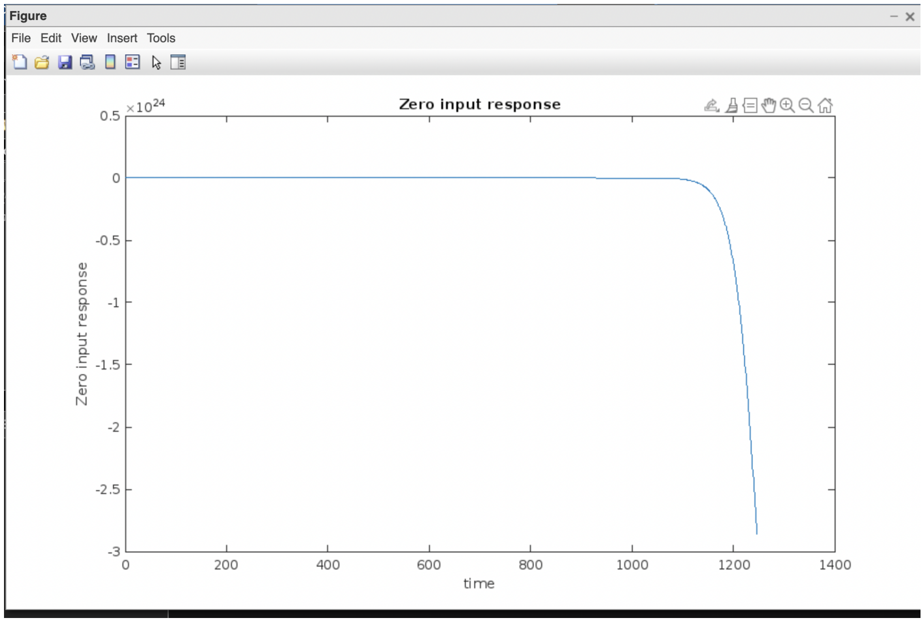

Figure 4: Caption

3.3 Case 3: v=5 m/s

3.3.1 State Space Model

$$\begin{pmatrix} \dot{\phi} \\ \dot{\delta} \\ \ddot{\phi} \\ \ddot{\delta} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 13.6700 & -32.7500 & -0.8200 & -2.7600 \\ 4.8570 & -17.3150 & 18.1050 & -11.9400 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \dot{\phi} \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -0.339 \\ 7.457 \end{pmatrix} \mathrm{T}$$

$$\mathrm{y} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \dot{\phi} \\ \dot{\delta} \end{pmatrix}$$

3.3.2 Transfer Function

$T(s) = \frac{-0.339s^2 - 24.63s - 250.1}{s^4 + 12.76s^3 + 63.41s^2 + 457.3s - 77.63}$

3.3.3 Eigenvalues

Eigenvalues in this case are -10.8598 , -1.0330 + 6.4842i, -1.0330 - 6.4842i, 0.1658

3.3.4 Poles

Eigenvalues in this case are -10.8598, -1.0330 + 6.4842i, -1.0330 - 6.4842i, 0.1658

3.3.5 Zeroes

Zeroes of transfer function are -60.4476, -12.2043

3.3.6 Laplace Transform of Unit Step Response

The Laplace Transform comes out to be $L(s) = \frac{-0.339s^2-24.63s-250.1}{s^5+12.76s^4+63.41s^3+457.3s^2-77.63s}$

3.3.7 Unit Step Response

$3.222-0.001362e^{-10.86t}-e^{-1.033t}cos(6.484t)(0.0113+0.03794i)-e^{-1.033t}cos(6.484t)(0.0113-0.03794i)+e^{-1.033t}sin(6.484$

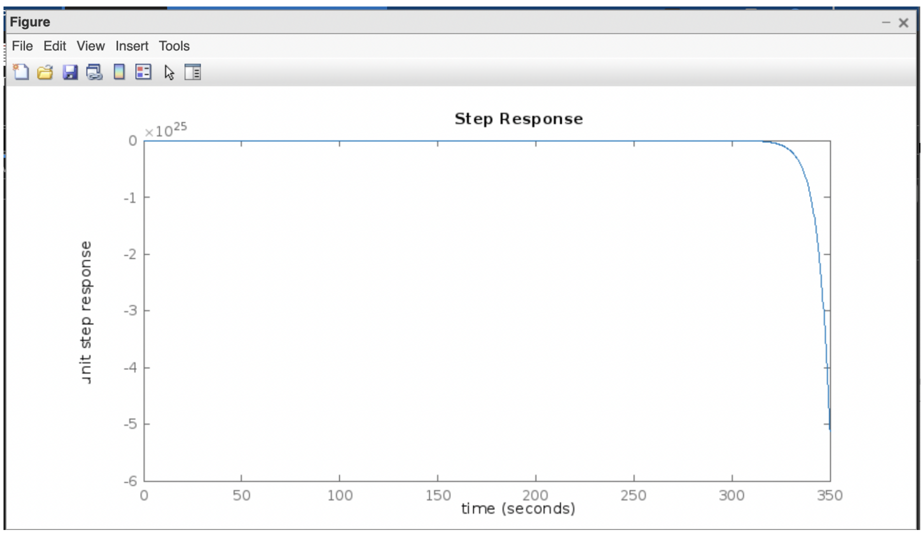

Figure 5: Caption

3.3.8 Zero Input Response

In Laplace domain the frequency response

$Y(s) = \frac{(5(20000s^3+275200s^2+1070112s+4969129)}{100000s^4+1276000s^3+6340560s^2+45732257s-7762930}$

$y(t) = 0.538e^{0.166t}-0.00617exp-10.9t+e^{-1.03t}cos(6.48t)(0.234-0.102i)+e^{-1.03t}cos(6.48t)(0.234+0.102i) + e^{-1.03t}sin(6.48t)(0.102+0.234i) + e^{-1.03t}sin(6.48t)(0.102-0.234i)$

Zero input response is shown in the following plot

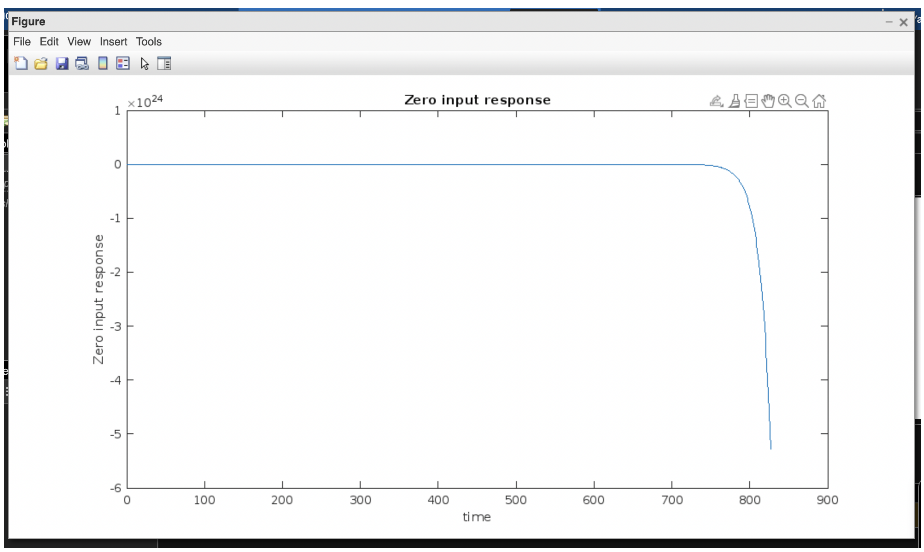

Figure 6: Caption