

$$\begin{aligned}
 & \dots + n + n + n/2 + n/4 + \dots + 1 + 1 + \\
 & = n + n/2 + n/4 + \dots + n/n \\
 & = n \left(1 + 1/2 + 1/4 + \dots + 1/n \right) \\
 & = n \left(\log n \right) = \underline{n \log n}
 \end{aligned}$$

~~Q13:-~~

Q13:-

void fun (int n)

{ int j=1, i=0;

while (i < n)

{ i=i+j;

j++; }

i=0 i=1 3 7

0 1 3 7 12 ...

1

1+2.

1+2+4

1+2+4+5

last term would be

$$R(R+1) = n$$

R

$$\begin{aligned}
 R^2 &= n \\
 R &= \sqrt{n}
 \end{aligned}$$

time complexity :- $O(\sqrt{n})$

~~Q13:-~~

~~1) $n \log n$~~

Q14:-

$$T(n) = T(n/4) + T(n/2) + cn^2$$

~~as $T(n/2)$~~

assuming $T(n/2) > T(n/4)$

$$T(n) = T(n/2) + cn^2$$

Applying master's theorem,

$$a=1 \quad b=2$$

$$f(n) = cn^2 \quad | \quad n^{\log_2 1}$$

$$b, cn^2 > n^{\log 2}$$

$$\text{So, } T(n) = O(f(n))$$

$$T(n) = n^2 \underline{\underline{\text{Ans}}}$$

$$= O(n^2)$$

Q15:-

$$n + n/2 + n/3 + n/4 + \dots$$

$$n/1 + n/2 + n/3 + n/4 + \dots$$

$$= n(\log n) = n \log n \underline{\underline{\text{Ans}}}$$

Q16:-

$$n = i^K$$

$$\log n = K \log i$$

$$K = \log_i n$$

$$O(\log n) \underline{\underline{\text{Ans}}}$$

Q18:-

a) $100 < \log \log n < \log n < \log(n!) < \text{root } n < n < n \log n < n^2 < 2^n < 2^{2^n} = 4^n < n!$

b) ~~$\log(n)$~~ $1 < \log(\log(n)) < \sqrt{\log(n)} < \log(n) < \log^2 n < 2 \log n < n < 2n < 4n < n \log n < \log(n!) < n^2 < 2(2^n) < n!$

c) $96 < \log_8 n < \log_8 n < \log(n!) < 5n < n \log_8 n < 8n^2 < 7n^3 < 8 \cdot 2^{n^6} < n!$