Mapping the relations between computational thinking and mathematics in terms of problem-solving.

Kaup, Camilla Finsterbach

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Camilla Finsterbach Kaup
Aalborg University and
University College of Northern Denmark

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Mapping the relations between computational thinking and mathematics in terms of problem-solving

Abstract
Over the last two decades, computational thinking has grown popular in educational settings. This article aims to unfold computational thinking in relation to mathematics. The study reviewed 19 articles published between 2014 and 2021 and provides new opportunities for understanding how to get involved and introduce computational thinking into mathematics teaching. It was found that computational thinking and mathematical thinking had a relationship and that problem-solving could be an approach to relating them. The findings reveal that the relationship is primarily theoretical and that teachers found it challenging to make a connection between computational thinking and mathematics. In addition to providing a structured example of research conducted in the field and identifying gaps, the study raises new research opportunities.

Keywords: computational thinking, mathematics, primary level, mediating artifacts

Kortlægning af relationen mellem computationel tankegang og matematik med udgangspunkt i problemløsning

Sammendrag
I løbet af de sidste to årtier er computationel tankegang blevet mere populært i undervisningsmiljøer. Denne artikel har til formål at udfolde computationel tankegang i relation til matematik. Undersøgelsen gennemgik 19 artikler publiceret mellem 2014 og 2021 og giver nye muligheder for at forstå, hvordan man kan involvere sig i og introducere computationel tankegang i matematikundervisningen. Det blev fundet, at computationel tankegang og matematisk tænkning havde en relation, og at problemløsning kunne være en tilgang til at relatere dem. Resultaterne afslører, at forholdet primært er teoretisk, og at lærerne finder det udfordrende at skabe en forbindelse mellem computationel tankegang og matematik. Udover at give et struktureret eksempel på forskning, der er udført på området, og identificere mangler, rejser undersøgelsen også nye forskningsmuligheder.

Nøgleord: computationel tankegang, matematik, indskolingen, medierende artefakter

Introduction

Computational thinking (CT) has become central to the discussion on utilizing technology's potential for education, and several Nordic countries have implemented CT in the curriculum (Bocconi et al., 2016). Indeed, many now acknowledge these skills as fundamental as numeracy and literacy (Wing, 2006; Barr & Stephenson, 2011; Grover & Pea, 2018). CT can be considered a set of thinking skills and an approach connected to solving complex problems using informatics structures (Caspersen et al., 2018; Wing, 2006). The origins of CT can be traced to Papert's 1980 book *Mindstorms*; however, the present discussion on CT originated with Wing (2006). In 2006, Wing described CT as a way of "solving problems, designing systems, and understanding human behavior by drawing on the concepts fundamental to computer science" (p. 33). This approach addresses how humans or computers are able to solve problems. A natural, historical link between CT and mathematics exists due to logical structures and mathematical modeling problems (Gadanidis et al., 2017). In 2020, Li et al. clarified that CT is "a model of thinking that is more about thinking than computing" (p. 4) and argued that it is generally an essential ability for all students in the 21st century. Students allowed to study mathematical phenomena using computational problem-solving strategies, such as programming, algorithmic thinking, and creating computational abstractions, can develop a deeper understanding of the phenomena (Weintrop et al., 2016; Pérez, 2018). From this perspective, it is important to learn not only arithmetic but also how to think like a mathematician (Schoenfeld, 2016; Grover & Pea, 2018). Highlighting core concepts and practices from CT in the context of mathematical learning can help students understand mathematical concepts as ideas that can also be used in domains other than mathematics (Pérez, 2018). CT can also help strengthen problem-solving and abstraction, as demonstrated by a meaningful correlation between CT and mathematical understanding, (Li et al., 2020). Several studies have highlighted the strong correlation between mathematics achievement and students' engagement in CT. Strong mathematical reasoning combines high levels of abstraction and algorithmic skills, leading to rapid progress in CT through computer programming courses (Bocconi et al., 2016; Buitrago Flórez et al., 2017). As emphasized above, there is a movement to teach CT integrated within disciplines such as mathematics (Israel & Lash, 2020). Through this integration, CT can be introduced within authentic, real-world examples that can inspire students' mathematical and computational thinking (Weintrop et al., 2016; Pérez, 2018). A growing interest is emerging in integrating CT and mathematics; however, it is still a challenge to find a way to integrate the two disciplines in a meaningful manner (Israel & Lash, 2020; Grover & Pea, 2018; Bocconi et al., 2016).

In this article, I present a literature review of current research on the use of CT in mathematics. The purpose of this article is not to define CT. Instead, this article explains how CT has been integrated into mathematics teaching and how it has influenced problem-solving approaches to learning mathematics. The mapping review can be helpful for teachers, educators, and researchers who wish to improve the quality of teaching or professional development programs for teachers; this will enable them to relate CT to mathematics better.

A mapping review with a two-sources scope

The literature review aims to map existing research on CT in mathematics education at the K–12 level. This type of review seeks to map and categorize the existing literature on a particular topic and contextualize the findings within broader literature (Booth, 2016). The purpose of the literature review is to map the relation between CT and mathematics. This will enable me to gain a deeper understanding of CT's relation to mathematics and how this is embedded in research. The review focused on the following research question: *What is the relationship between CT and mathematics, and how is it characterized in relation to problem-solving?*

I strived for a structured and transparent approach to the mapping process, following Hart's (1998) description of the purpose of mapping:

> The main use of mapping a topic is to acquire sufficient knowledge of the subject to develop the necessary understanding of the methodology and research techniques, to comprehend the history and diffusion of interest in the topic, and to undertake an analytical evaluation of the main arguments, concepts, and theories relevant to the topic in order to synthesize from the analysis an approach or thesis that is unique, that is, your work. (Hart, 1998, p. 142)

As Hart (1998) stated, to map is to obtain an overview of a diverse body of research on a specific subject to synthesize a unique approach to the investigation. I, therefore, assembled a literature search strategy that aimed to obtain an overview of the broader strands of research, not a comprehensive review of all existing literature in the field.

Planning the review

I focused on finding research articles from two primary sources in educational research limited to publications from 2011 to 2021, first in Scopus with "computational thinking" and math* in the title and then in Eric with the same search strategy but limited to only full access. The relevance of each research article was assessed by the inclusion and exclusion criteria presented in Table 1.

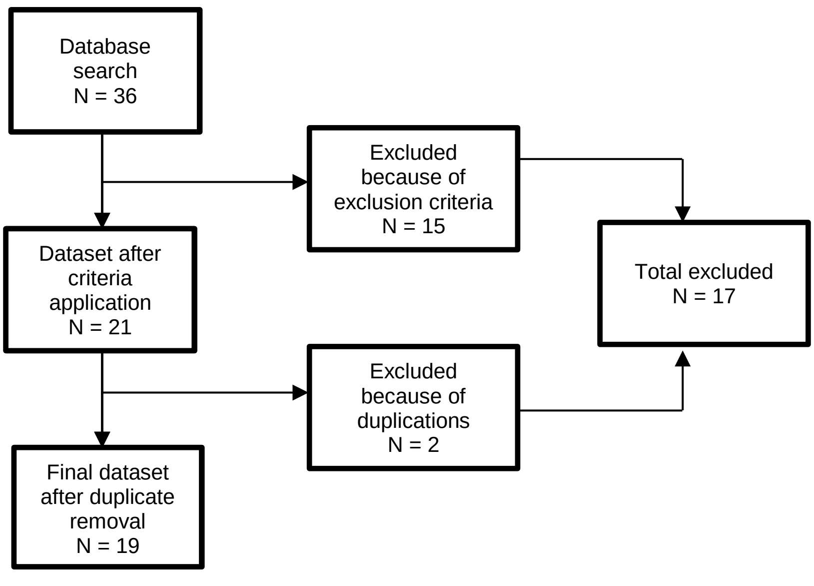
Table 1. Inclusion and exclusion criteria

Inclusion	Exclusion
• Empirical investigation of CT in mathematics education at the K–12 level • The study should consider and discuss how CT can be embedded in mathematics education at the K–12 level • Peer-reviewed article	• Empirical investigation of CT in science other than mathematics at the K–12 level • Empirical investigation of programming excluding CT

Conducting the mapping review

The study was conducted in October 2021. Altogether, 36 articles were identified and then classified as relevant for the review or not; 19 articles were found to match the inclusion criteria defined in Table 1. Figure 1 gives an overview of the process. Titles and abstracts of articles were screened for relevance, and full-text articles of potentially relevant publications were obtained. Full-text articles were screened according to the pre-determined inclusion criteria. For all included publications, the following data were abstracted: use and definition of CT and mathematics, references used, publication years, and problem-solving approaches. To recognize each article's research question, I read each article carefully to consider the connection between CT and mathematics and the link to problem-solving.

Figure 1. Literature review process



The review begins by comparing the CT definitions used in the selected studies. Second, the different pedagogical perspectives are considered. Finally, the review examines how various studies integrated CT, mathematics, and problem-solving.

CT and definitions

Mathematics and CT have been described in numerous ways that often vary depending on the perspective and understanding of the natures of mathematics and computer science. According to Kallia et al. (2021), different definitions emphasize the contextualization of mathematics or computer science, where the connection is between real-world situations and mathematical and computational concepts. The included articles are based primarily on known definitions of CT as a basis for their studies and findings, for example, Reichert et al. (2020) and the definition that Barr and Stephenson (2011) provided:

> an approach to solving problems in a way that can be implemented with a computer: students become not merely tool users but tool builders. They use a set of concepts, such as abstraction, recursion, and iteration, to process and analyze data and create real and virtual artifacts. (Barr & Stephenson, 2011, p. 51)

This definition emphasizes users' ability to build and create tools – not just problem-solving. Chongo et al. (2020), based on Wing (2006), Grover & Pea (2018), and Weintrop et al. (2016), offered the following definition: "a process of thinking and a tool for solving problems using computer concepts either with a computer (plugged-in) or without one (unplugged)" (Chongo et al. 2020, p. 160). Similar to Barr & Stephenson (2011), this definition seems to relate to a mental one that leans close to the systemic understanding but differs from the definitions above by highlighting that CT can occur without using a computer. The definitions mentioned include problem-solving and relate to how computers work; however, Barr and Stephenson's (2011) definition focuses more on the process, analysis, and creation of artifacts rather than solely using artifacts.

Table 2 shows the articles included in the review and the definitions they draw from in their research. When three or more of the articles used the same reference, I mapped it in Table 2. The authors who are marked in bold, use the term *mathematical thinking* when they relate CT to mathematics. The x's marked with bold use some of the same references when describing CT, mathematics, and problem-solving. The objective was to get an overview and map out the references and definitions used in the selected articles. Almost all articles use Wing's (2006) description of CT when they describe their understanding of CT. Only 2 out of 21 articles have not included Wing's (2006) perspective but instead draw on Papert's (1980) perspective and Grover and Pea's (2013). Promraksa et al. (2014) do not use any well-known definition but rather refer to other research studies. Papert (1980) is referred to in 13 of the 21 articles, demonstrating that CT supports the assertion made by Papert (1980) that computers can be used as educational tools.

Table 2. The selected articles and their use of references. Authors marked in bold use mathematical thinking to relate CT to mathematics, while x's in bold use similar references when describing CT, mathematics, and problem-solving.
References: (1) Barr & Stephenson (2011); (2) Wing (2006); (3) Wing (2017); (4) Grover & Pea (2013); (5) Grover & Pea (2018); (6) Weintrop et al. (2016); (7) Brennan & Resnick (2012); (8) Papert (1980); (9) Sneider et al. (2014); (10) Lye & Koh (2014).

No	Selected articles	1	2	3	4	5	6	7	8	9	10
1	Chongo, Osman, & Nayan (2020)		x			x	x				
2	**Columba (2020)**	x	x						x		
3	**Cui & Ng (2021)**		x				x	x	x	x	
4	Gadanidis, Cendros, Floyd, & Namukasa (2017)		x						x		
5	Gadanidis, Clements, & Yiu (2018)		x						x		
6	Israel & Lash (2020)				x		x	x	x	x	
7	**Kallia, Borkulo, Drijvers, Barendsen, & Tolboom (2021)**	x	x				x	x			
8	**Lockwood, DeJarnette, Asay, & Thomas (2016)**		x		x		x				
9	Pei, Weintrop, & Wilensky (2018)		x		x		x		x		
10	Promraksa, Sangaroon, & Inprasitha (2014)										
11	Rafiepour & Farsani (2021)		**x**			**x**			**x**		**x**
12	Reichert, Dante, & Milton (2020)	x	x						x		
13	**Rich, Spaepen, Strickland, & Moran (2020a)**	x	x		x		x	x			
14	Rich, Yadav, & Larimore (2020b)		x		x						
15	**Rodríguez-Martínez, González-Calero, & Sáez-López (2020)**		**x**			**x**	**x**		**x**		**x**
16	**Soboleva, Sabirova, Babieva, Sergeeva, & Torkunova (2021)**	x		x					x		
17	**Sung, Ahn, & Black (2017)**		**x**			**x**			**x**		**x**
18	**Sung & Black (2021)**		**x**				**x**		**x**	**x**	**x**
19	**Weintrop, Beheshti, Horn, Orton, Jona, Trouille, & Wilensky (2016)**	x	x		x			x	x		

Weintrop et al. (2016) formulated a taxonomy with four categories of practice for framing and defining CT for mathematics and science from a more theoretical perspective. The taxonomy can be used to analyze and incorporate CT in mathematics curricula, which helps enhance an experiential approach to learning mathematics. The four categories include data practices, modeling and simulation practices, computational problem-solving practices, and systems thinking practices. Many of the selected articles draw on Weintrop et al.'s (2016) perspectives. Lockwood et al. (2016) began by examining some existing broad definitions of CT but contributed to teaching mathematics with new views on CT; based on their study, they deduced a definition of algorithmic thinking, which relates to procedural thinking, where thinking follows a logical order and use of means to achieve a particular goal. The researchers found that algorithmic thinking helps strengthen problem-solving, mathematical communication skills, and prerequisites for assessing whether something is right or wrong. Lockwood et al. (2016) found that combining procedural knowledge with algorithmic thinking helped teachers improve their mathematics instruction and that students could benefit from it in their mathematical thinking.

Pedagogical perspectives

The teacher's role

The study from the review indicates that CT can strengthen students' mathematical abilities (Chongo et al., 2020; Reichert et al., 2020; Sung et al., 2017; Sung & Black, 2021). Among other things, CT has been found effective in facilitating problem-solving in mathematics (Lockwood et al., 2016; Promraksa et al., 2014; Weintrop et al., 2016; Cui & Ng, 2021; Sung & Black, 2021; Kallia et al., 2021). Another study reveals that in addition to having a significant impact on learning, teachers' work with CT generates reflections on the methods used for teaching and learning (Reichert et al., 2020). Gadanidis et al. (2018), Rafiepour and Farsani (2021), and Columba (2020) also find the teacher's role necessary, arguing that the teacher should coach with the appropriate amount of gentle encouragement and support. Rodríguez-Martínez et al. (2020) found that teachers must be explicit in their instructions to develop basic computational concepts. They discovered that Scratch, a programming application, improves students' mathematical understanding and CT by solving word problems. Using mathematical word problems enables CT and mathematical concepts to be addressed simultaneously (Rodríguez-Martínez et al., 2020). According to Rich et al. (2020b), for students to become proficient with CT, teachers must provide opportunities to learn how to work with CT; this also indicates the need to support teachers in translating CT into their mathematics curricula and then into practice.

Student-centered activities

Pei et al. (2018) argued that CT practices and mathematical habits of mind are distinct. Still, through the careful design of computational learning environments and curricula, the two could be implemented in mathematics classrooms. Pei et al. (2018) use a student-centered approach, where students use inductive reasoning rather than deductive reasoning by experimenting, collecting data, and building hypotheses. In Pei et al.'s (2018) study, students explored a microworld called Lattice Land. The microworld lets students explore and understand a concept, specifically the relationship between the shape of a triangle (and the length of its sides) and the resulting area. Students were to find at least one example of every triangle area in a 4x4 lattice (there are 16 possible areas). When students work with this activity, they connect the open-ended, exploratory microworld and the mathematics they typically study in class (Pei et al., 2018, p. 83). Using a computational tool to understand a concept is a CT practice in the Modeling and Simulation part of Weintrop et al.'s (2016) taxonomy.

Sung et al. (2017) indicated that focusing on full-embodied activities combined with computational perspective-taking in solving mathematics problems can promote mathematical understanding and develop students' programming skills. The programming environment can provide a practical setting for "cognitive processing by focusing on how to think rather than on what to think, which is fundamental to computational thinking" (Sung et al., 2017, p. 24). As per Sung and Black (2021), working with a computational perspective can support thinking by making thought processes explicit, whereas the embodied approach helps make thought processes visible. This perspective prepares students by supporting bodily engagement and computational perspectives before introducing them to digital tools, allowing them to take different stances in problem-solving activities, using mathematical and computational perspectives. Rich et al. (2020a) compared differences and synergies between mathematical thinking and CT by analyzing K–5 curricular documents. The result confirms synergies, and they suggest that the relationship between CT and mathematics can be built from a spiral curricula design. Integrating a spiral curriculum can prepare students to engage in activities based on CT and mathematics aligned with Weintrop et al. (2016). According to the spiral curriculum, mathematics activities that do not directly address CT can support later CT learning (Rich et al., 2020a). Introducing gamification to teach students how to use computers to solve a mathematical problem can allow them to begin with small tasks, increasing the complexity of the task (Soboleva et al., 2021). Israel and Lash (2020) highlighted this complexity, which must be considered in a learning environment. First, a digital artifact's level of integration and whether it fits the mathematical curriculum should be considered, followed by how the teaching structure can build on sufficient knowledge in mathematics and CT and create a more authentic integration.

Computational thinking and the relation to mathematical thinking

All the authors in Table 2 marked with bold use the term mathematical thinking when they relate CT to mathematics. According to mathematical thinking, the reviewed articles draw on Schoenfeld (2016) and Sneider et al. (2014). Sneider et al. (2014) have created a Venn diagram that shows reciprocal relationships between CT and mathematics. The concepts used in CT and mathematical thinking are similar, such as problem-solving, modeling, analyzing, and interpreting data. Sung and Black (2021) argue that using CT in mathematics can provide a more realistic perspective into the field of mathematics education and support students' learning of mathematics content. In line with that, Schoenfeld (2016) emphasized that mathematics can be considered a subject that seeks to understand patterns that infuse the world around us and our minds. Although the language of mathematics is based on rules that are important for students to develop, students must be motivated to move beyond rules to express activities mathematically. Learning to think mathematically means that students should develop a mathematical perspective, apply the processes of mathematization and abstraction, and develop competence with mathematical tools, using them to capture the goal of understanding. The overlap between the disciplines was described similarly by Pei et al. (2018). In their view, CT and mathematics habits of mind are distinct areas that mutually support each other and can also be instructionally related: Although they are closely linked, they do not often appear together in instruction.

Computational thinking in mathematics and the connection to problem-solving

Kallia et al. (2021) revealed that the thinking processes involved in problem-solving are the common ground between computational and mathematical thinking. The researchers conducted a Delphi study that agreed with corresponding literature that CT in mathematics should highlight the following aspects: abstraction, decomposition, pattern recognition, algorithmic thinking, modeling, logical thinking, and automatization, followed by analytical thinking, generalization, and evaluation of solutions and strategies. According to Cui and Ng (2021), problems must be reframed to apply existing computational tools. Problems should thus be decomposed into sub-problems and reframed new issues into a known problem-solving strategy. The computational solutions are more accessible and support procedural and conceptual challenges (Cui & Ng, 2021; Rich et al., 2020b; Weintrop et al., 2016).

The selected articles all address CT as a problem-solving method. Weintrop et al. (2016) use CT as a tool for problem-solving and exploring mathematics concepts. Using computers to solve mathematical problems requires a problem-solving strategy, so that students can think about a computational approach to solving problems (Chongo et al., 2020). Rich et al. (2020b) indicated that when students analyze representations and note differences, they can use CT skills to choose an appropriate representation for a specific problem.

The ability to make connections and make relations between CT and mathematics can assist teachers in integrating CT into mathematics. The Delphi study that Kallia et al. (2021) conducted, highlights the following CT process between computational and mathematical thinking. Barr and Stephenson (2011) emphasized some of the same processes, which provide examples of how teachers can embed CT in mathematics. Table 3 illustrates the result of the efforts between CT and mathematics.

Kallia et al. (2021) specified the importance of problem-solving as a central goal of mathematics education in which CT can be embedded. They highlighted the thinking processes provided in Table 3, among others. The solution to the mathematical problem should be explained in “such a way that it can be transferred/outsourced to another person or a machine (transposition)” (Kallia et al., 2021, p. 179).

Table 3. CT processes and the relation to mathematics

Computational thinking (Kallia et al., 2021)	**Mathematics (Barr & Stephenson, 2011)***
Abstraction	Using variables in Algebra; identifying essential facts in a word problem
Decomposing	Applying the order of operations in an expression
Pattern recognition/generalization	Using a histogram, pie chart, or bar chart
Logical and algorithmic thinking	Completing long division and factoring; performing carries in addition or subtraction
Modeling	Graphing a function in a Cartesian plane and modifying the variables' values
Automatization	Using tools such as Geometer, Star Log, and Scratch
Analytical thinking	Counting the number of occurrences of flips and dice throws and analyzing the results
Generalization and evaluation	Executing guess and check

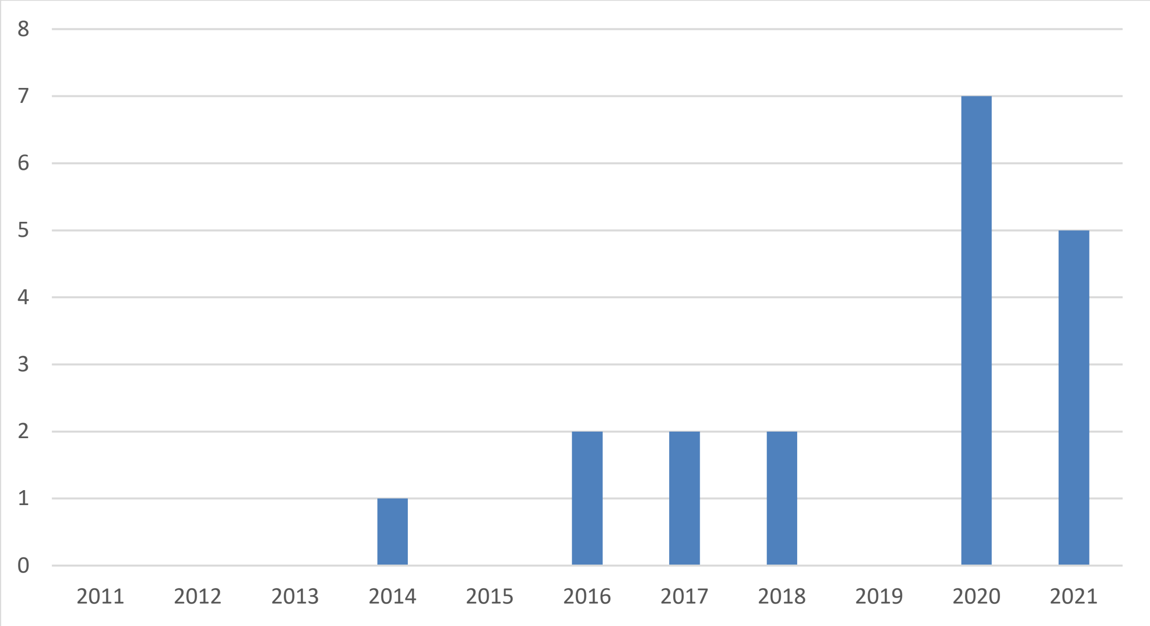
* See Barr and Stephenson (2011) for other disciplines and CT concepts

Four selected articles (Rafiepour & Farsani, 2021; Rodríguez-Martínez et al., 2020; Sung et al., 2017; Sung & Black, 2021) use Lye and Koh (2014) in their descriptions of CT. Those four articles are connected because they also use Wing (2006), Papert (1980), and Grover and Pea (2018) in their descriptions of CT and the connection to mathematics (see Table 2). The four articles, among others, use programming to develop CT, but not only; they also recognize that CT can be learned, for example, through unplugged and embodied activities (Chongo et al., 2020; Sung et al., 2017; Sung & Black, 2021). Lye and Koh (2014) have reviewed 27 intervention studies to present current trends in using CT in an educational setting. They refer to the Logo programming language proposed by Mayer (1992) as an appropriate way to understand how to work with programming and problem-solving among K–12 students. Programming goes beyond coding alone; it reinforces concepts of abstraction and decomposition and teaches students how to work through problems (Lye & Koh, 2014). It is consistent with Sung and Black's (2021) concept that CT consists of solving problems systematically, sequentially organizing the solution steps, and expressing solutions in ways an information-processing agent like a computer or other tool can understand.

Summary of the literature review

The literature review was performed as a mapping review; I thus identified a more manageable number of articles than I would have by searching in increasingly more extensive databases. I restricted the search to articles published after 2010 to ensure timeliness and a certain degree of applicability for CT in mathematics. However, applying such a narrow scope entails an obvious risk of bias and neglect (Booth, 2016). The intention was to form a basic understanding of the field to investigate the research question. The literature review reveals an increase in the number of articles from 2016 to 2021, which indicates the relevance of and interest in examining CT in mathematics. This is illustrated in Figure 2.

Figure 2. Number of identified articles by publishing year



The findings from the included studies provide a relationship between CT and mathematics. CT emphasizes the importance of students considering and understanding the underlying structures of mathematics (Sung & Black, 2021; Pei et al., 2018). When working with CT in mathematics, students can learn about problem-solving processes (Kallia et al., 2021; Cui & Ng, 2021; Chongo et al., 2020). Furthermore, using CT in mathematics helps students develop procedural and strategic knowledge, which allows them to plan and solve problems (Lockwood et al., 2016; Rich et al., 2020b).

Lockwood et al. (2016) argue that CT strengthens mathematical problem-solving and the student's understanding and learning of mathematics. It is crucial that CT is facilitated and that teachers conceptualize factual and conceptual knowledge to help students develop an understanding of a given mathematical problem (Lockwood et al., 2016). Teachers can support the work by facilitating a spiral curriculum that supports the student's CT and mathematical thinking (Rich et al., 2020b). It underlines the significance of the teacher's awareness of students' metacognitive knowledge and when students are prepared to proceed to the next step or level of the problem-solving process (Lockwood et al., 2016; Rich et al., 2020b). Working with CT in mathematics can help students understand the primary approach that can support problem-solving. Moreover, working with CT in mathematics allows students to develop procedural and strategic knowledge to solve mathematical problems. With the teacher's support, the students can develop their metacognitive knowledge and, thus, their self-regulation process (Lockwood et al., 2016; Weintrop et al., 2016; Reichert et al., 2020; Kallia et al., 2021).

Nevertheless, the review also highlights the need to support teachers in developing their teaching practices for mathematics when introducing CT. Teachers need to learn how to work with CT to support students' computational and mathematical understanding (Lockwood et al., 2016; Weintrop et al., 2016; Reichert et al., 2020).

Conclusive discussion

The analysis of the selected studies reveals a relationship between CT and mathematics. The concepts used in CT and mathematical thinking are related, including problem-solving, modeling, analyzing, and interpreting data (Sneider et al., 2014). Especially, problem-solving is the common ground between CT and mathematics (Kallia et al., 2021). The primary purpose of CT is to solve problems efficiently, organize solution steps sequentially, and express such steps efficiently so that processing agents like computers, other tools, or humans can understand them. CT in mathematics should cover the following aspects: abstraction, decomposing, pattern recognition, generalization, logical and algorithmic thinking, modeling, automatization, analytical thinking, generalization, and evaluation (Kallia et al., 2021). As students work with those aspects, they can learn more about approaches to problem-solving. However, the connections between CT and mathematics are primarily made theoretically. The studies marked in bold in Table 2 indicate that integrating CT into a mathematical curriculum can be challenging, especially because teachers have not learned about CT in their initial education (Pei et al., 2018; Bocconi et al., 2016).

Furthermore, the literature review confirms that the teacher has a specific role in facilitating and mediating the connection between CT and mathematics in teaching activities. Still, it is challenging to make the transfer from CT to mathematics. Therefore, there is a need to help teachers integrate CT and see the connection to mathematics. The literature review indicated that teachers found it challenging to integrate CT into their mathematics curriculum (Israel & Lash, 2020; Pei et al., 2018). Despite this, the studies showed that careful selection between a teacher- or student-centered perspective could help teachers integrate CT and mathematics. CT's effectiveness in mathematics education depends on the balance between teacher- and student-centered activities, and whether they are presented as traditional lessons or as more problem-based approaches.

The problem-solving process and knowledge are compared with CT in Table 4. Tables 3 and 4 can both help guide teachers in their understanding and development of CT integration in mathematics.

Table 4. Problem-solving and CT processes

Problem-solving process	Problem-solving knowledge	Computational thinking
Representing	Facts/concepts	Understanding the problem and choosing the proper facts and concepts to solve it
Planning/ monitoring	Strategies	Decomposition, pattern recognition, generalization, and abstraction
Executing	Procedures	Logical and algorithmic thinking; automatization; and evaluation
Self-regulating	Metacognitive knowledge	Analytical thinking

Given the problem-solving perspective of CT and mathematics, it is possible to identify processes that support problem-solving. Mayer and Wittrock (2006) state that different processes can be linked to specific kinds of knowledge. In Table 4, those processes and knowledge are linked with CT.

- Representation is about understanding the problem and using facts such as one precedes two, while conceptual knowledge involves understanding concepts such as classification, models, and principles. Factual and conceptual knowledge represent knowledge to understand a given problem (Mayer & Wittrock, 2006). Using the different CT and mathematical processes provided by Kallia et al. (2021) and Barr and Stephenson (2011) summarized in Table 3, it is possible to be aware of students' knowledge needed to solve a given problem.
- Monitoring and planning relate to formulating strategies for solving problems. Strategic knowledge includes general methods such as decomposing to separate a problem into parts; it concerns planning and monitoring the procedures. As Rich et al. (2020b) pointed out, students can use CT skills to choose an appropriate strategy when analyzing different representations.
- Executing is about solving the exact problem or part of a problem. It involves specific procedures for executing a task, such as how to use the subtraction algorithm; it helps one perform step-by-step solutions in the problem-solving process. Using procedural knowledge and algorithmic thinking can assist teachers in teaching CT and mathematics to students (Lockwood et al., 2016).
- The self-regulation process depends on metacognitive knowledge. Metacognitive knowledge refers to the awareness and control of one's thinking process, such as if the student is unprepared to proceed to the next step in the problem-solving process. Through Scratch, students can develop critical, metacognitive, and reflexive skills closely related to mathematics. (Rodríguez-Martínez et al., 2020).

Lye & Koh (2014) state that predetermined categories can help researchers comprehend a context fully; otherwise, they may fail to identify key categories. In addition to overcoming the weakness of conventional content analysis, predetermined categories, as in Tables 3 and 4, will benefit researchers and teachers in analyzing and understanding the integration of CT in mathematics (Lye & Koh, 2014).

The mapping review can be helpful for teachers, educators, and researchers who wish to improve the quality of teaching or professional development programs for teachers; this will enable them to relate CT to mathematics better. It can be used for comparing and evaluating the various problem-solving processes that occur when CT is integrated into mathematics education.

Future research should explore how teachers can use the table of *Problem-solving and CT processes* (Table 4) to describe and analyze their understandings regarding CT in mathematics and help them incorporate CT into mathematics education. Additionally, future research could explore how teachers develop competencies to work with CT in mathematics and to explore how teachers can use the theoretical perception and connection between CT and mathematics to assist them in creating this relationship. Finally, studies should also focus on investigating how students enhance their problem-solving approaches when working with CT and mathematics.

About the author

Camilla Finsterbach Kaup is a Ph.D. student at Aalborg University. Her research focuses on students' development of computational thinking regarding mathematics at the primary level.
Institutional affiliation: Aalborg University, Faculty of Social Studies and Humanities, Department of Culture and learning, Kroghstræde 3, 9220 Aalborg, and University College of Northern Denmark, Department of Social Education, Mylius Erichsens Vej 137, 9210 Aalborg, Denmark.
Email: cfk@learning.aau.dk

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