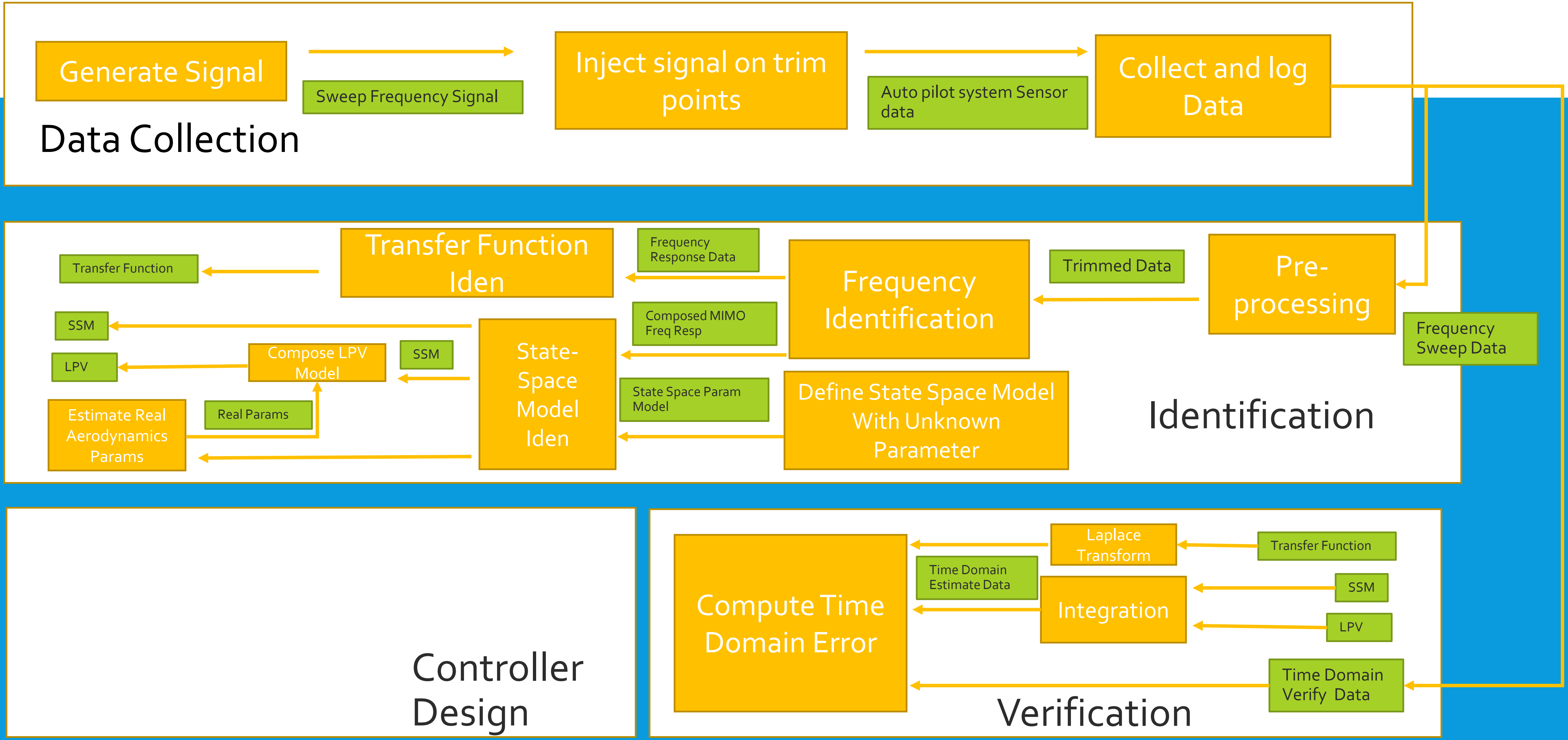
AIRCRAFT IDENTIFICATION

Using Frequency Method

TARGET FOR AIRCRAFT IDENTIFICATION

- Obtain System Frequency Response show it with Bode Plot
  - Design PID controller with Bode plot
  - Analyze system resonace/flexible mode
- Obtain Transfer function
  - Use for controller design
  - To understand system dynamics
- Obtain State Space Model
  - Understand system dynamics
  - Design controller with modern control theory e.g. using optimize method. Controller can work fun on operating point.
  - Analyze real aircraft parameter from several SSM at different operating point.
- Obtain Linear Parameter Varying Model
  - Design controller using LPV can make controller working fun when state is not on tested trimming point.
  - Simulation with LPV model.
- Simulation using model below
  - Test controller performance
  - Training reinforce controller

WORK FLOW



Generate Signal
Sweep Frequency Signal
Inject signal on trim points
Auto pilot system Sensor data
Collect and log Data
Data Collection
Transfer Function
Transfer Function Iden
Frequency Response Data
Frequency Identification
Trimmed Data
Pre-processing
Frequency Sweep Data
SSM
Composed MIMO Freq Resp
LPV
Compose LPV Model
SSM
State-Space Model Iden
Estimate Real Aerodynamics Params
Real Params
State Space Param Model
Define State Space Model With Unknown Parameter
Identification
Controller Design
Laplace Transform
Transfer Function
Time Domain Estimate Data
Compute Time Domain Error
Integration
SSM
LPV
Time Domain Verify Data
Verification

FREQUENCY-SWEEP SIGNAL GENERATION

Collecting data need to have information on wide frequency domain, we use below formula to generate data.

$$\delta_{sweep} = A\, sin[\theta(t)]$$

$$\theta(t) \equiv \int_{0}^{T_{rec}} \omega(t)\, dt$$

$$\omega = \omega_{min} + K(\omega_{max} - \omega_{min})$$

$$K = C_2[exp(C_1 t / T_{rec}) - 1]$$

By chosing Omega we can specify the range which we are interesting in

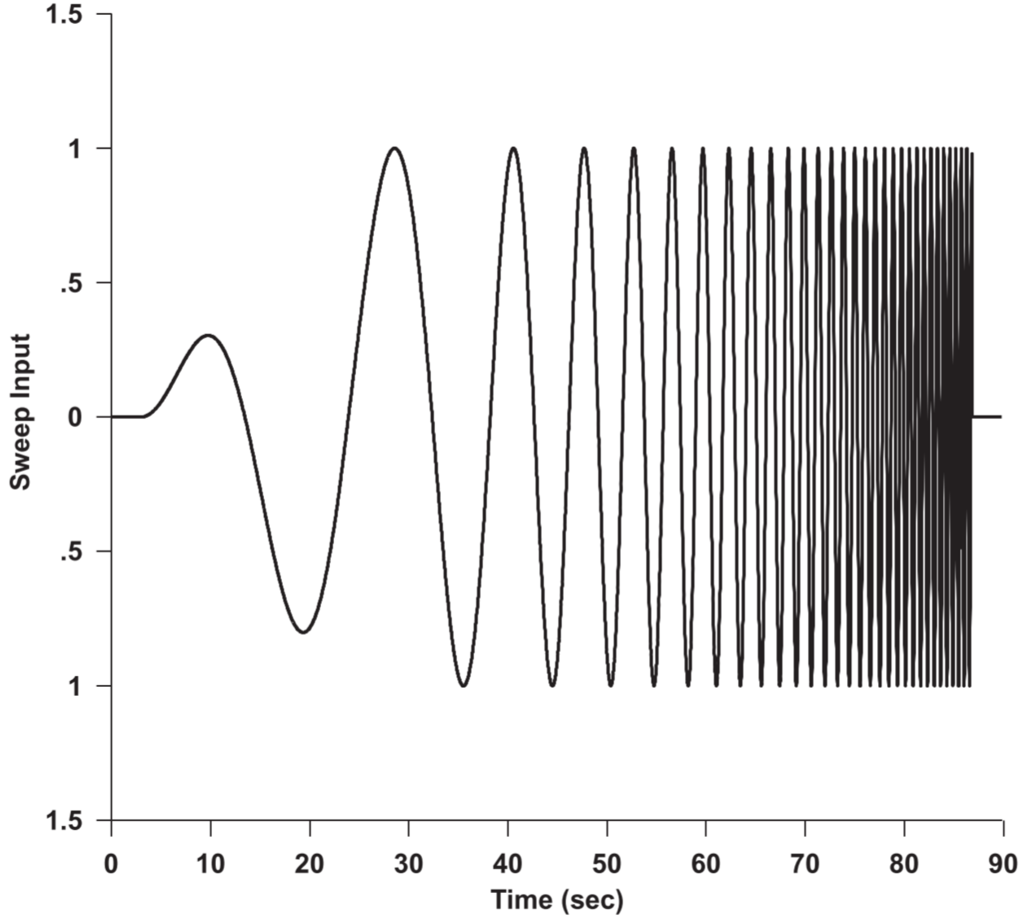
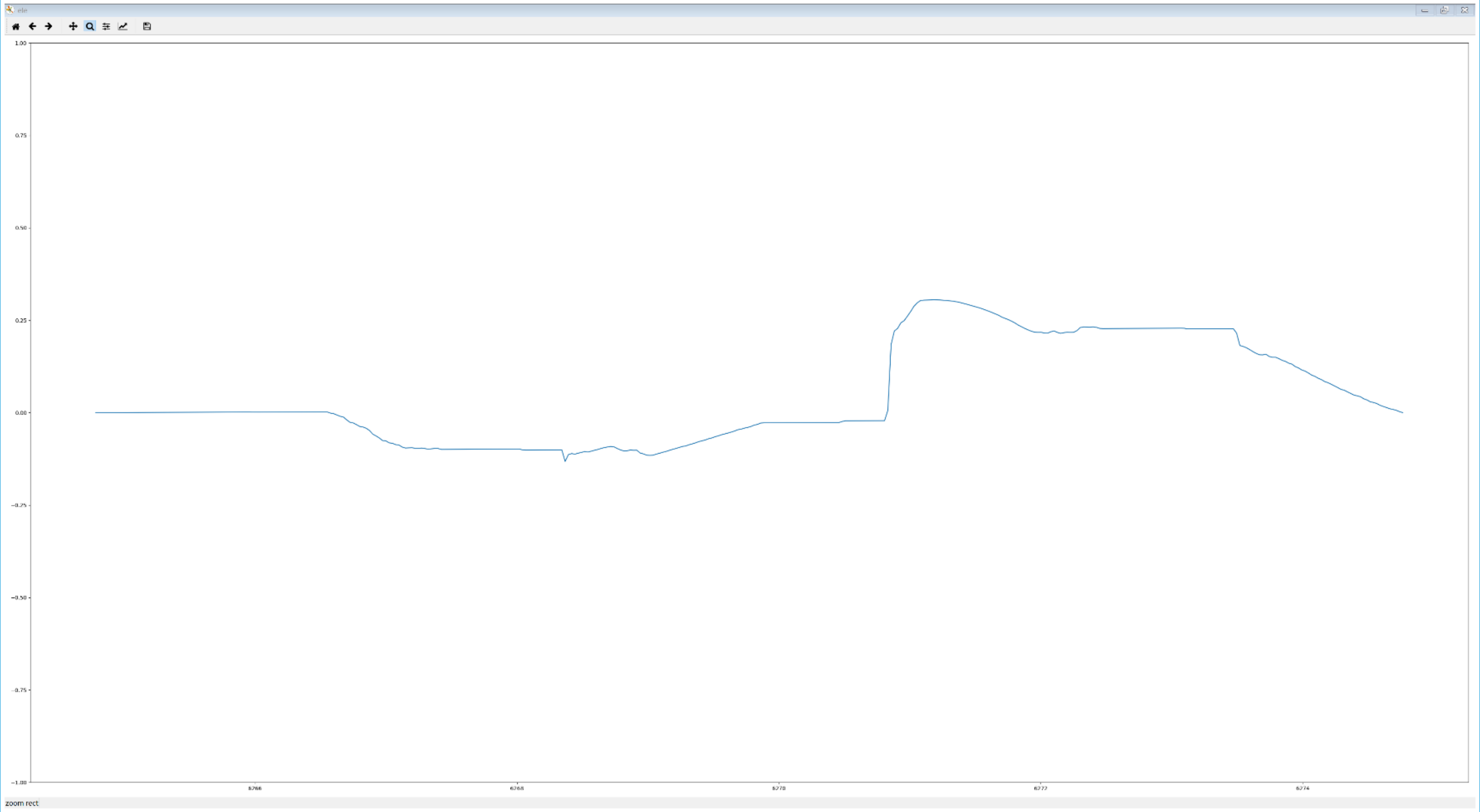


Fig. 5.11 Typical automated frequency-sweep input [Eq. (5.17)].

TIME-DOMAIN VERIFICATION DATA GENERATION

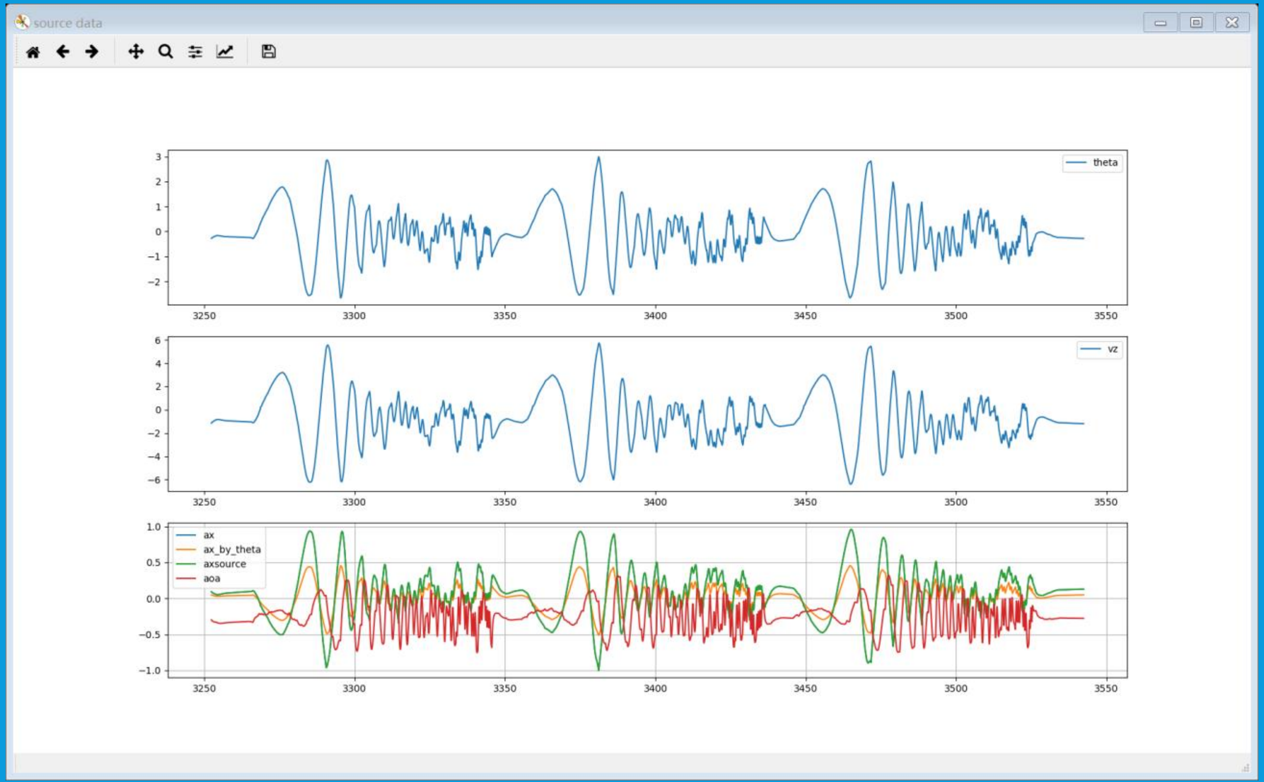
Colleting data for time-domain verification is much more flexible. We can easily use manual input or auto generation data.
The key point is that data must not make plant far away far away from linear point. And the data don’t need too long. 10 second data provide enough information we need.



SENSOR DATA PREPROCESS

Source Data grab from Flight Controller often have some problem:

1. Some system like PX4 using nonuniform sampling, we must do resampling to ensure the subsequent analyze could be applied.
2. For frequency analyze, the data must be trimmed first.



FREQUENCY IDENTIFICATION:BASIC IDEA

The basic idea of frequency-response estimate is

$$\text{Frequency-response estimate} = \hat{H}(f) = \frac{\hat{G}_{xy}(f)}{\hat{G}_{xx}(f)}$$

Gxx(f) is autospectrum estimate
Gxy(f) is cross-spectrum estimate

Where Gxx and Gxy can be defined by

$$\tilde{G}_{xy}(f) = \frac{2}{T}[X^*(f)Y(f)]$$

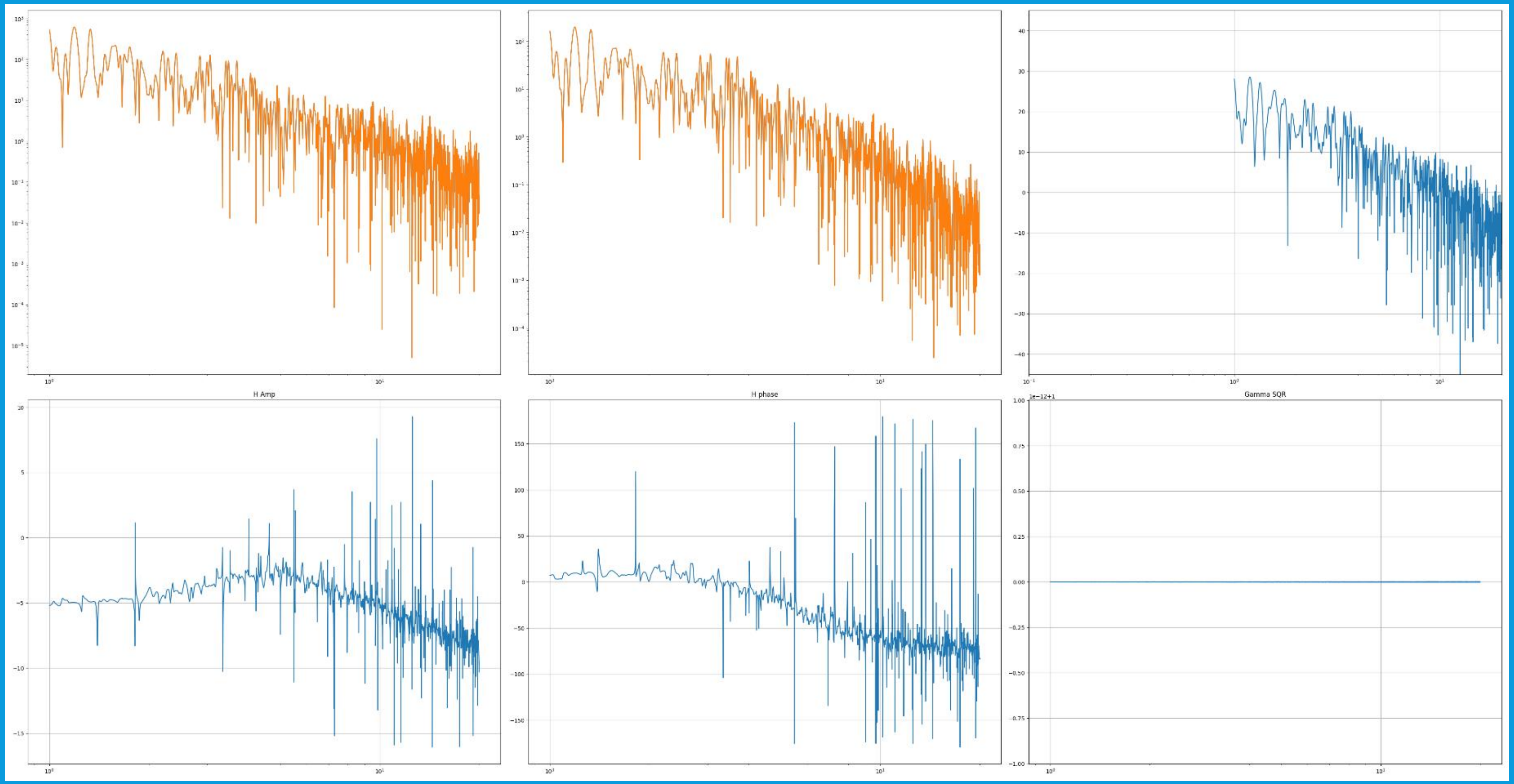
X(f) and Y(f) is the fourier coefficient of frequency f

$$\tilde{G}_{xx}(f) = \frac{2}{T}|X(f)|^2$$

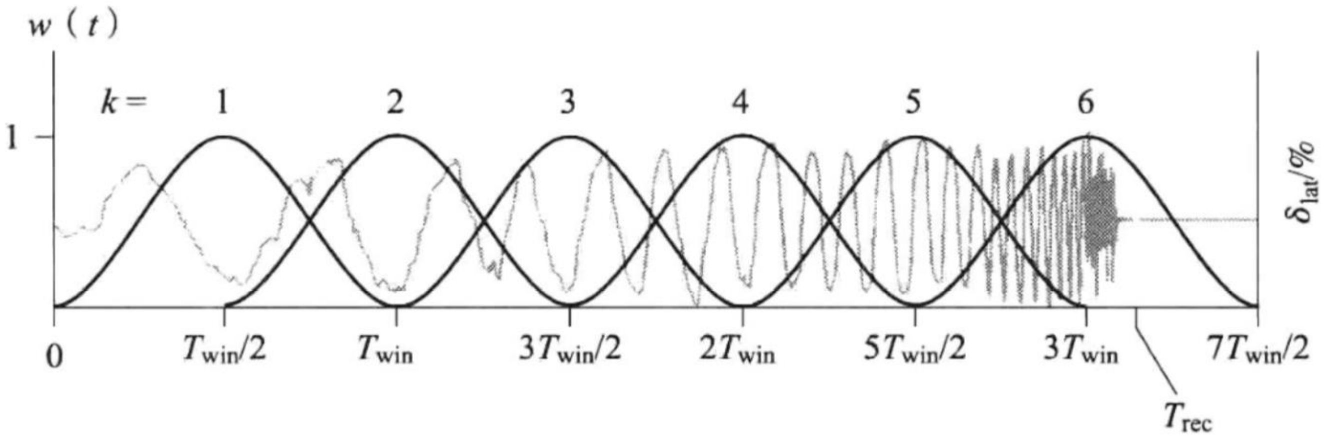
Because of the noise and Discrete Sample, the computing of frequency-response is very complex and use a lot of empirical conclusion. A lot of work should be done with spectrum analyze method, like periodograms method to calculate Gxx Gxy.

WELCH'S METHOD

- Direct using Fourier Coefficient will produce very noised result, which is hard to use in the subsequence analyze.
- Right image is a example which direct use Fourier Coefficients to compute Gxx(Left Up) Gyy(Up middle), Gxy(Up Right). And use these result to obtain Frequency Response(Down Left 2 images)



WELCH'S METHOD



$$\hat{G}_{xx}(f) = \left(\frac{1}{Un_r}\right)\sum_{k=1}^{n_r}\hat{G}_{xx,k}(f)$$

$$\hat{G}_{yy}(f) = \left(\frac{1}{Un_r}\right)\sum_{k=1}^{n_r}\hat{G}_{yy,k}(f)$$

$$\hat{G}_{xy}(f) = \left(\frac{1}{Un_r}\right)\sum_{k=1}^{n_r}\hat{G}_{xy,k}(f)$$

We cut data to lots of periods with overlapped, and multiply data with Hanning Window to get Windowed Data period. The new Gxx, Gyy, Gxy is compute by average the Gxx, Gxy, Gyy of Windowed Data periods.

In physics, engineering, and applied mathematics, Welch's method, named after P.D. Welch, is used for estimating the power of a signal at different frequencies: that is, it is an approach to spectral density estimation. The method is based on the concept of using periodogram spectrum estimates, which are the result of converting a signal from the time domain to the frequency domain. Welch's method is an improvement on the standard periodogram spectrum estimating method and on Bartlett's method, in that it reduces noise in the estimated power spectra in exchange for reducing the frequency resolution. Due to the noise caused by imperfect and finite data, the noise reduction from Welch's method is often desired.(Copy From Wikipedia)

COHERENCE

- When using Welch's Method to compute frequency response, it led to a new concept called coherence which defined by

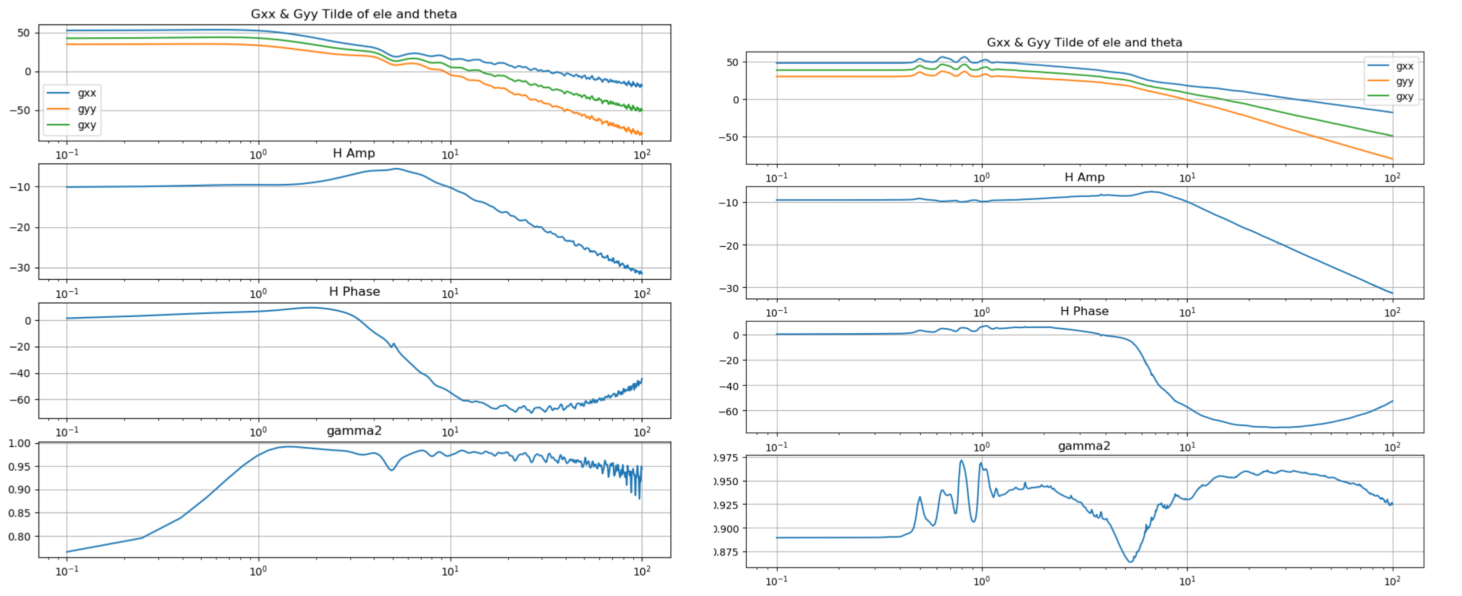
$$\hat{\gamma}_{xy}^2(f) = \frac{|\hat{G}_{xy}(f)|^2}{|\hat{G}_{xx}(f)|\,|\hat{G}_{yy}(f)|}$$

When coherence satisfy

$$\gamma_{xy}^2 \geqslant 0.6$$

We say that the frequency response at this frequency is good enough.

COHERENCE AND WINDOW NUMBER



When using Welch's Method, we must specific a window number, but different window number will cause coherence in different frequency be different.
Smaller window number performance better in low frequency, bigger window number performance better in high frequency.

COMPOSITED WINDOW

Also we can try a lot of window number to found out the best window number, there is also a way called composited window method to solve this problem.

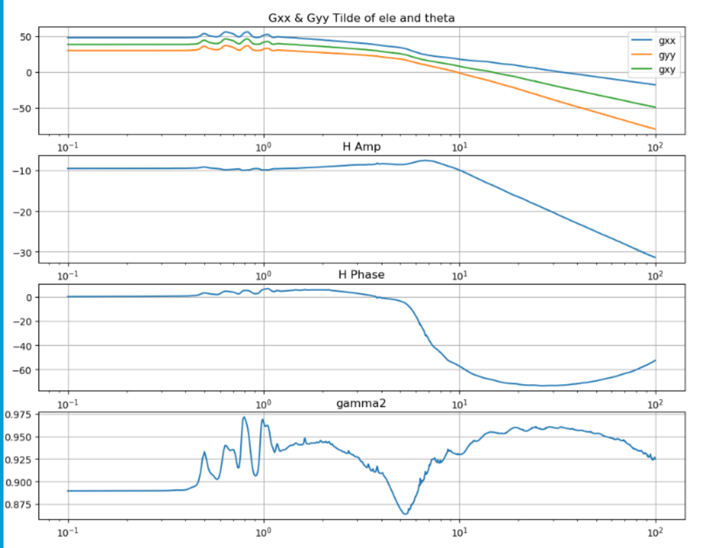
The Basic Idea of Composited Window is first we using a series of different windows number to calculate the Gxxi Gxyi Gyyi at each frequency point. Next we use optimization method to found out the best Gxxc Gxyc Gyyc which make the error function minimized. And we will use this Gxxc Gxyc Gyyc as our Gxx

The result shows this method actually make sense.

$$J(f) = \sum_{i=1}^{n_w} W_i \left\{ \left(\frac{\hat{G}_{xx_c} - \hat{G}_{xx_i}}{\widehat{G}_{xx}} \right)^2 + \left(\frac{\hat{G}_{yy_c} - \hat{G}_{yy_i}}{\widehat{G}_{yy}} \right)^2 + \left[\frac{\mathrm{Re}(\hat{G}_{xy_c}) - \mathrm{Re}(\hat{G}_{xy_i})}{\mathrm{Re}(\widehat{G}_{xy})} \right]^2 + \left[\frac{\mathrm{Im}(\hat{G}_{xy_c}) - \mathrm{Im}(\hat{G}_{xy_i})}{\mathrm{Im}(\widehat{G}_{xy})} \right]^2 + 5.0 \left(\frac{\hat{\gamma}^2_{xy_c} - \hat{\gamma}^2_{xy_i}}{\widehat{\gamma}_{xy}} \right)^2 \right\} \qquad (10-4)$$

$$\widehat{\gamma}^2_{xy}(f) = \frac{|\widehat{G}_{xy}(f)|^2}{|\widehat{G}_{xx}(f)|\,|\widehat{G}_{yy}(f)|}$$

$$\widehat{G}_{xx} = \frac{\sum_{i=1}^{n_w} W_i^2 \hat{G}_{xx_i}}{\sum_{1}^{n_w} W_i^2}$$



DEAL WITH MIMO SYSTEM

- In some case, it’s not convenient to collect data in just one channel.
- In longitude dynamics sweep test, we need to keep altitude and velocity not varying a lot.
- So when deal with the, we use elevator as main input and throttle as aux input. Using right forumal we can eliminate the influence of aux input

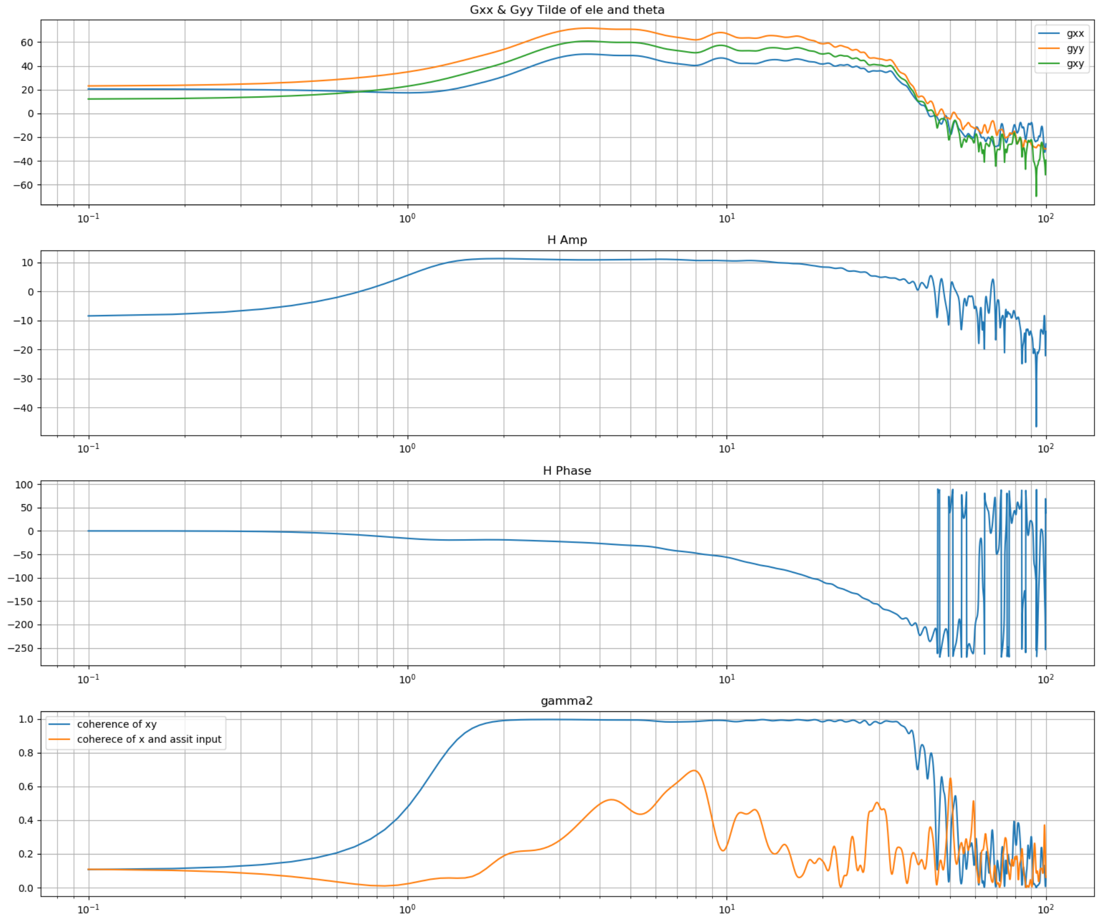
$$G_{\delta_r p} = \left(\frac{p}{\delta_r}\right) G_{\delta_r \delta_r} + \left(\frac{p}{\delta_a}\right) G_{\delta_r \delta_a}$$

$$G_{\delta_a p} = \left(\frac{p}{\delta_r}\right) G_{\delta_a \delta_r} + \left(\frac{p}{\delta_a}\right) G_{\delta_a \delta_a}$$

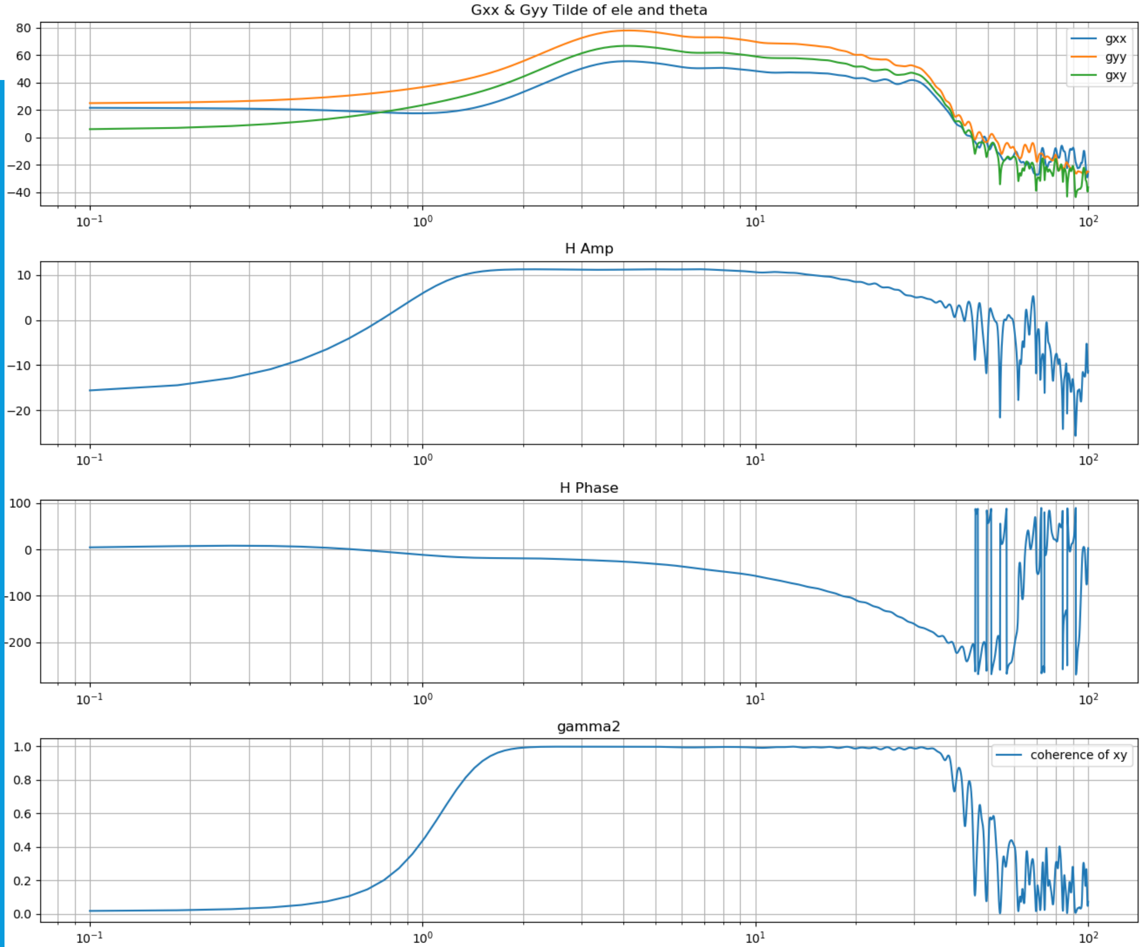
$$\frac{p}{\delta_r} = \frac{G_{\delta_r p}\left[1 - (G_{\delta_r \delta_a} G_{\delta_a p}) / (G_{\delta_a \delta_a} G_{\delta_r p})\right]}{G_{\delta_r \delta_r}(1 - \gamma^2_{\delta_r \delta_a})}$$

$$\frac{p}{\delta_a} = \frac{G_{\delta_a p}\left[1 - (G_{\delta_a \delta_r} G_{\delta_r p}) / (G_{\delta_r \delta_r} G_{\delta_a p})\right]}{G_{\delta_a \delta_a}(1 - \gamma^2_{\delta_r \delta_a})}$$

MIMO SYSTEM



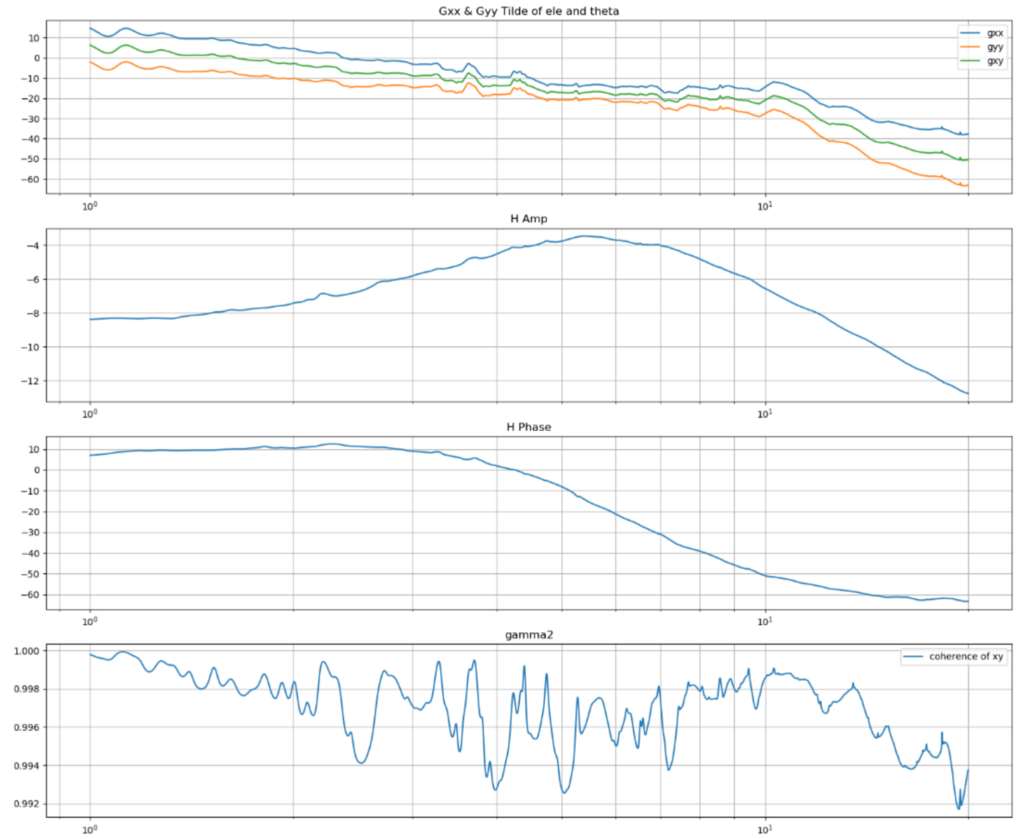
Use MIMO



Direct use SISO Analyze

FREQUENCY IDENTIFICATION

Left image shows the frequency identification result of a Cessna 172sp in X-Plane simulator. Top image shows the raw signal power density, second and third image show the frequency identification result.



TRANSFER FUNCTION FITTING

Fitting transfer function is basically a optimize problem, we have those transfer function define

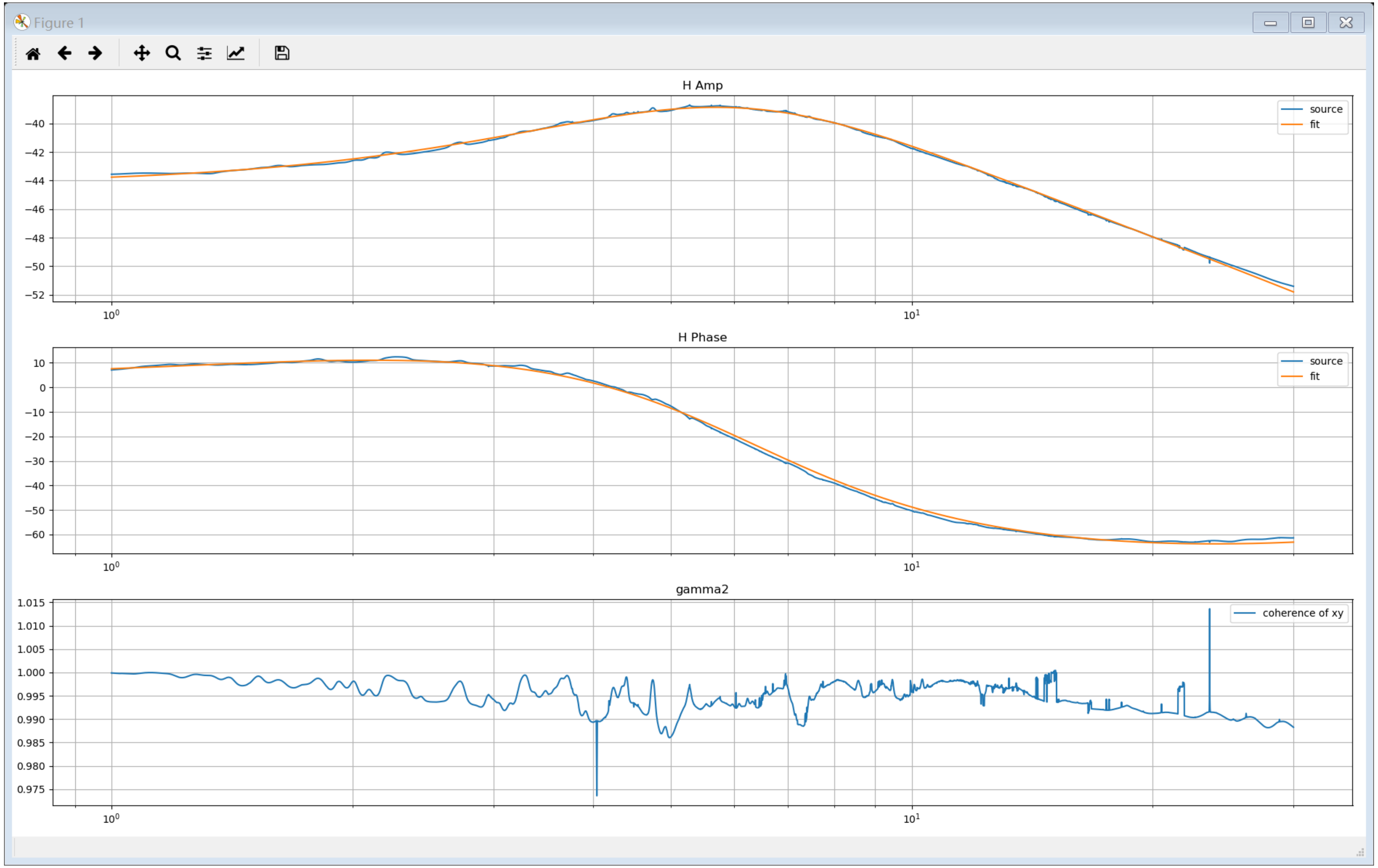
$$T(s) = \frac{(b_0 s^m + b_1 s^{m-1} + \cdots + b_m) e^{-\tau_{eq} s}}{(s^n + a_1 s^{n-1} + \cdots + a_n)}$$

And using the cost function, where Tc is the frequency estimate

$$J = \frac{20}{n_\omega} \sum_{\omega_1}^{\omega_{n_\omega}} W_\gamma [W_g (|\hat{T}_c| - |T|)^2 + W_p (\angle \hat{T}_c - \angle T)^2] \qquad (11.4)$$

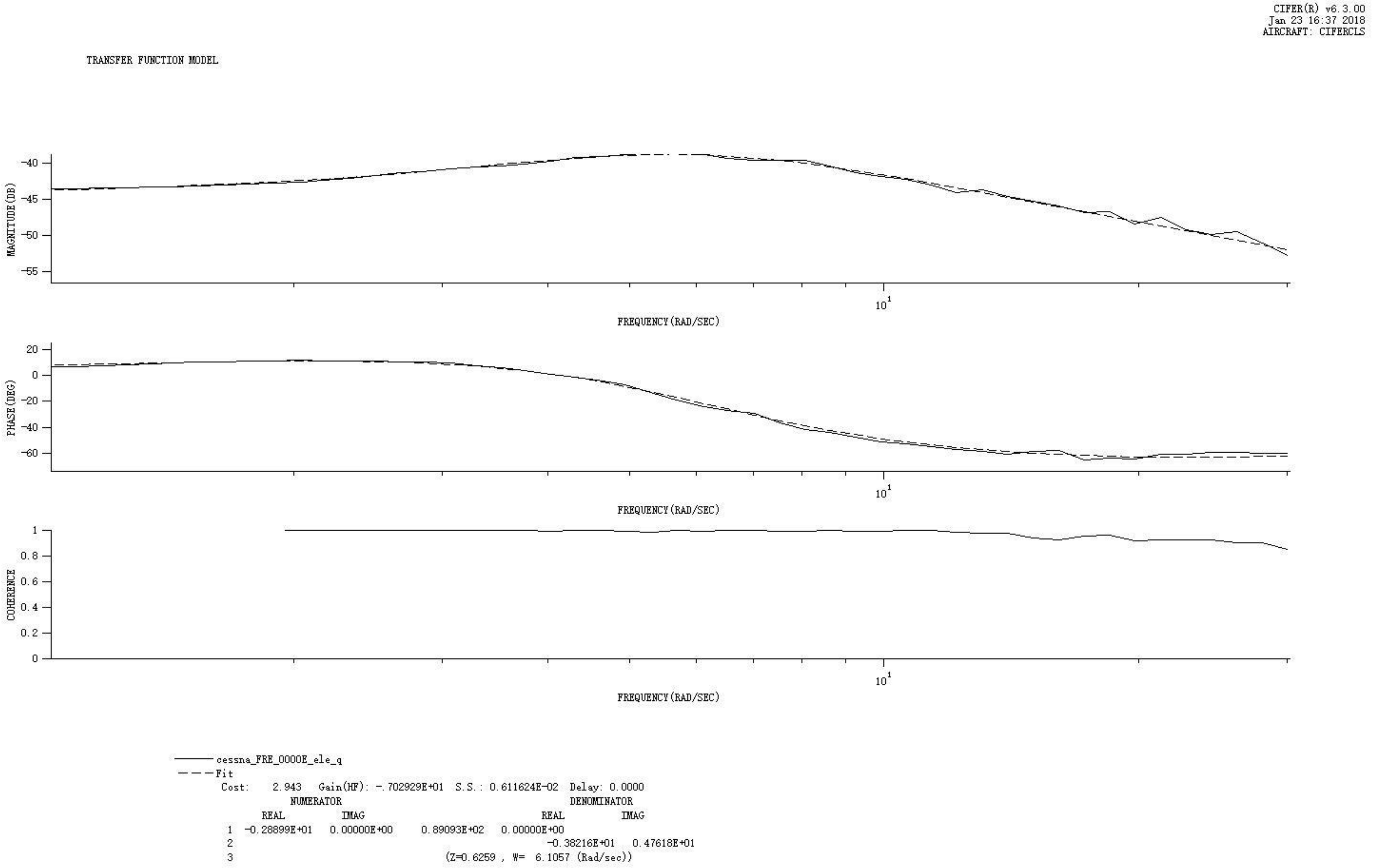
$$W_\gamma(\omega) = \left[1.58\left(1 - e^{-\gamma_{xy}^2}\right)\right]^2$$

TRANSFER FUNCTION FITTING



The result of fitting elevator to q transfer function.
The J is 0.63

CIFER FITTING RESULT



	CIFER	pyAircraften
Num S1	-7.0	-7.7
NUM s0	-20.3	-22.6
DEN s3	1	1
DEN s2	-81.6	-88.3
DENs1	-645	-710
DENS0	-3323.2	-3696
cost	2.943	0.62

STATE SPACE IDENFICATION

We define a state space model as

$$M\dot{x} = Fx + Gu\ (t - \tau)$$

$$y = H_0 x + H_1 \dot{x}$$

Convert is to frequency is

$$T(s) = [H_0 + sH_1]\,[\,(sI - M^{-1}F)^{-1}M^{-1}G] \qquad (12-12)$$

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \\ \vdots \\ Y_{n_0}(s) \end{bmatrix} = \begin{bmatrix} T_{11}(s) & T_{12}(s) & \cdots & T_{1n_c}(s) \\ T_{21}(s) & T_{22}(s) & \cdots & T_{2n_c}(s) \\ \vdots & \vdots & & \vdots \\ T_{n_0}1(s) & T_{n_0}2(s) & \cdots & T_{n_0 n_c}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \\ \vdots \\ U_{n_c}(s) \end{bmatrix} \qquad (12-11)$$

Minimize cost function J

$$J = \sum_{l=1}^{n_{TF}} \left\{ \frac{20}{n_\omega} \sum_{\omega_1}^{\omega_{n_\omega}} W_\gamma \{ W_g (|\hat{T}_c| - |T|)^2 + W_p (\angle \hat{T}_c - \angle T)^2 \} \right\}_l$$

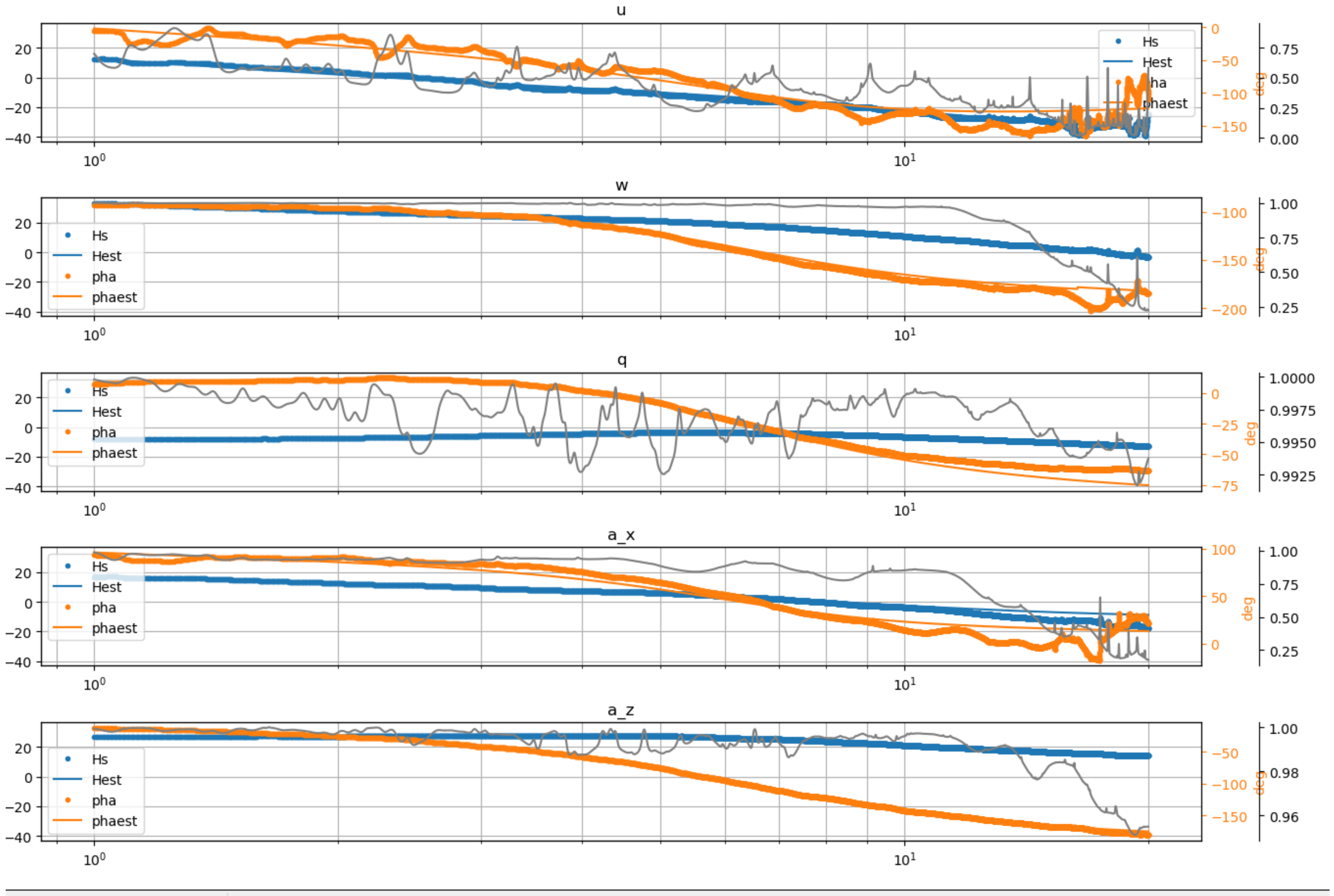
The basic idea of estimate ssm is almost the same with estimate transfer function.
A useful trick is that we can give 20-30 random initial value for the optimization.
Multi-thread technology makes it not very slow.

EXAMPLE: LATITUDINAL DYNAMICS OF CESSNA

$$\begin{bmatrix} 1 & -X_{\dot{w}} & 0 & 0 \\ 0 & 1-Z_{\dot{w}} & 0 & 0 \\ 0 & -M_{\dot{w}} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & X_q - W_0 & -g \cdot \cos(\theta_0) \\ Z_u & Z_w & Z_q + U_0 & -g \cdot \sin(\theta_0) \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_E} & X_{\delta_{RPM}} \\ Z_{\delta_E} & Z_{\delta_{RPM}} \\ M_{\delta_E} & M_{\delta_{RPM}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_E \\ \delta_{RPM} \end{bmatrix}$$

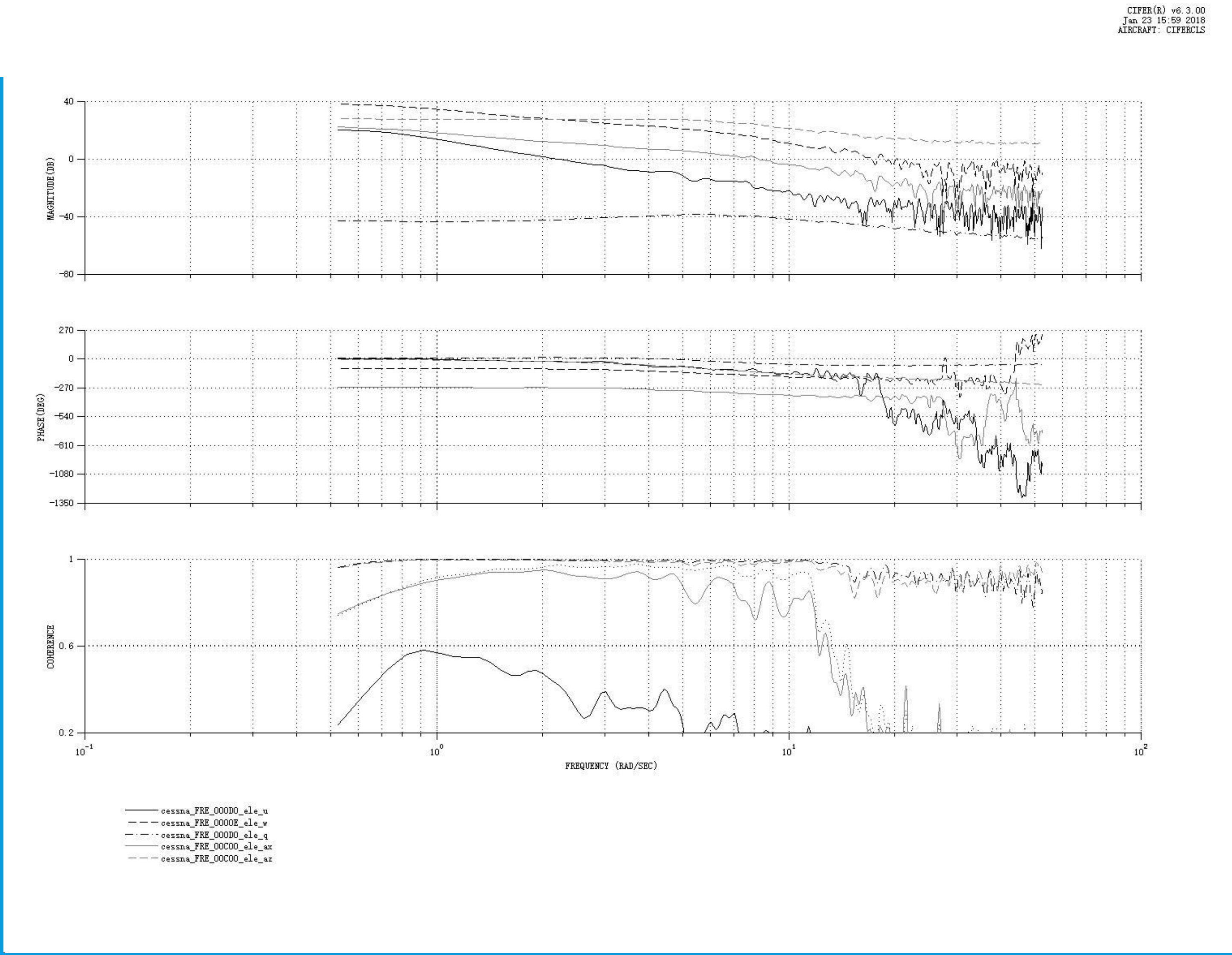
$$\begin{bmatrix} V_T \\ \alpha \\ q \\ A_x \\ A_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1/V_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s & 0 & W_0 & g \cdot \cos(\theta_0) \\ 0 & s & -U_0 & g \cdot \sin(\theta_0) \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}$$

STATE-SPACE MODEL IDENTIFICATION



Here is the ssm identification of a Cessna corresponding to each channel

CIFER RESULT



TIME DOMAIN DATA VERIFICATION

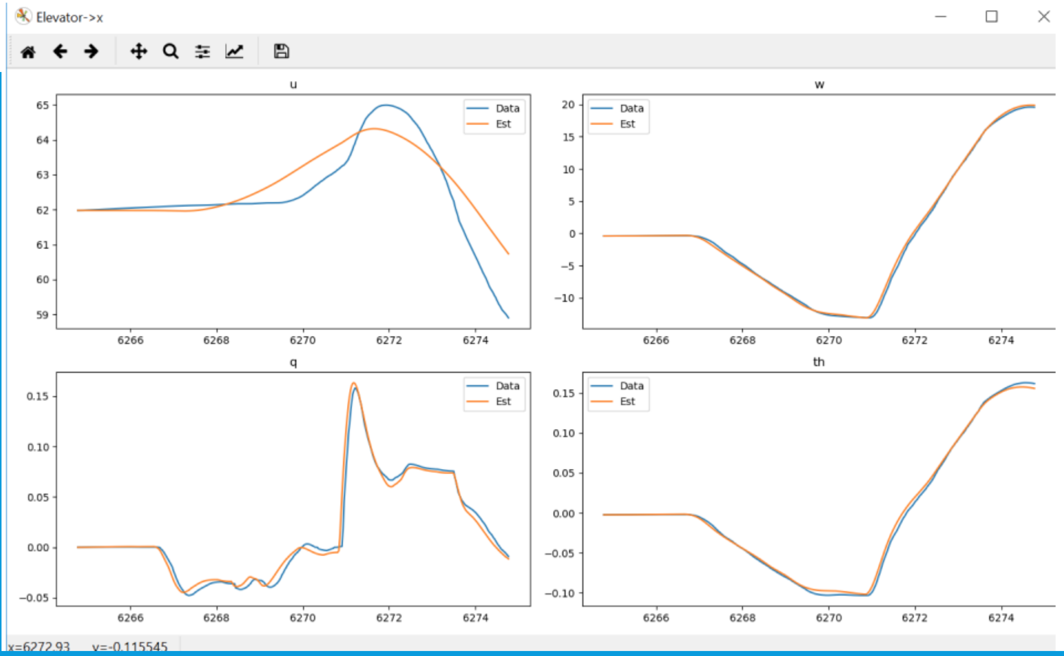
$$M\dot{x} = Fx + Gu\ \ (t-\tau)$$

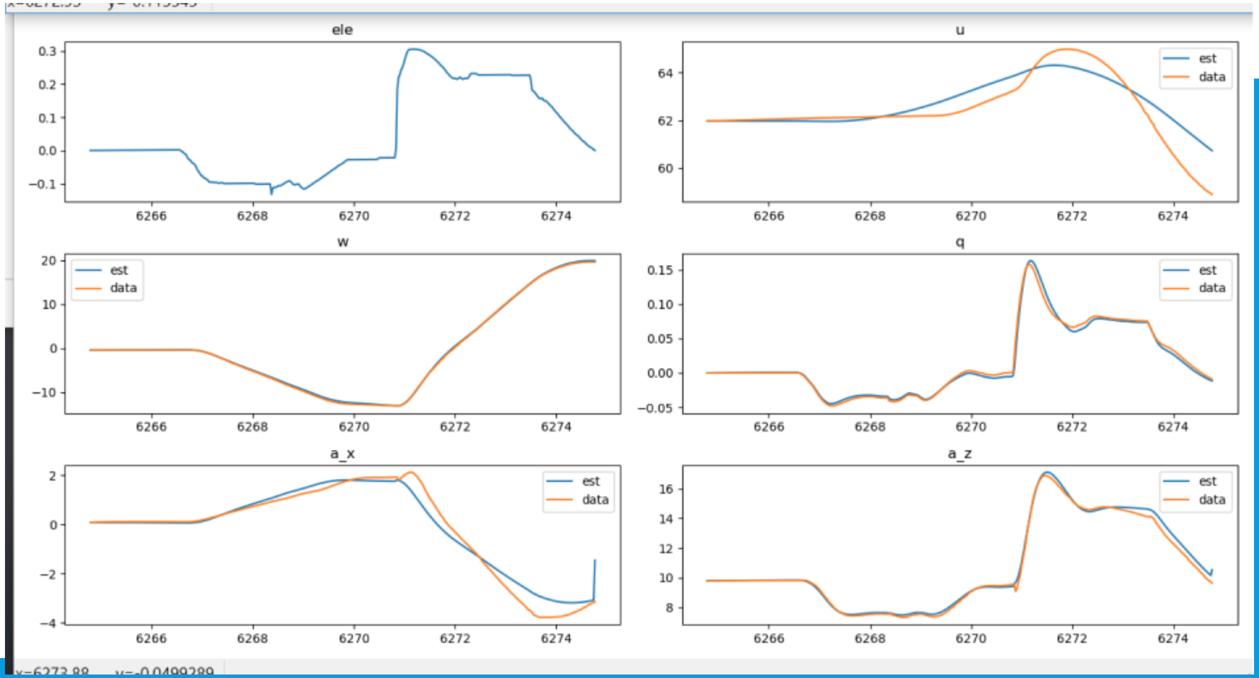
$$y = H_0 x + H_1 \dot{x}$$

By solving the State Space Mode with the parameter we calculate before ,we can verification is our identification, and on which state we have good enough identification result, which we can only get useless result.

We solve the linear equation, we need to use the delta value from time-domain data to linear point as initial value.

TIME DOMAIN DATA VERIFICATION





In this example, the airspeed(u) and body x acc identification is not very good, the result shows that the relationship between elevator control and airspeed is not very tight for this Cessna on cruising speed. It can be explained on high speed cruising fly, the induced drag is not so strong that the change of angle-of-attack cause a little drag variation