ProAdvice 1: TAIL ARM OPTIMIZATION

Introduction

Deciding the length of an airplane's tail arm is based on a number of considerations. These include:

- Stability Derivatives such as C_{ma}, C_{mde}, C_{mq}, C_{yb}, C_{nb}, C_{ndr}, C_{nr}, and others.
- Impact on the non-linear behavior of the above SDs.
- Structures (structural weight, aeroelasticity, etc).
- Aesthetics.
- Operation (e.g. tail length may interfere with T-O rotation, require excessive control surface deflection, etc).

Provided no other requirements are violated, the designer should always try to minimize drag. The largest part of drag during cruise is caused by skin friction. The method shown here minimizes the wetted area of the tail boom, HT, and VT in a manner that will result in a stable aircraft (provided correct tail volume is chosen). The method assumes a fixed tail volume and returns the tail arm length that results in the minimum wetted area of the design. The method shown assumes a simplified geometry, but can easily be adapted for more complicated ones than shown here.

STEP 1: Choose Desired values for V_{HT} and V_{VT}

Table 6.4: Tail Volume Coefficient

	V_{HT}	V_{VT}
Sailplane	0.50	0.02
Homebuilt	0.50	0.04
General aviation - single engine	0.70	0.04
General aviation - twin engine	0.80	0.07
Agricultural aircraft	0.50	0.04
Twin turboprop	0.90	0.08
Flying boat	0.70	0.06
Jet trainer	0.70	0.06
Jet fighter	0.40	0.07
Military cargo/bomber	1.00	0.08
Jet transport	1.00	0.09

SOURCE: Raymer, "Aircraft Design: A Conceptual Approach".

The definition of horizontal and vertical tail volumes:

$$V_{HT} = \frac{L_{HT} \times S_{HT}}{\overline{C}_{REF} \times S_{REF}} \quad and \quad V_{HT} = \frac{L_{VT} \times S_{VT}}{b_{REF} \times S_{REF}}$$

Where: C_{REF} = Mean Aerodynamic Chord (or Mean Geometric Chord), ft or m
          b_{REF} = Reference span (e.g. wing span), ft or m

S_{REF} = Reference area (e.g. wing area), ft² or m²
L_{HT} = Horizontal tail arm, ft or m
L_{VT} = Vertical tail arm, ft or m
S_{HT} = Horizontal tail area, ft² or m²
S_{VT} = Vertical tail area, ft² or m²
V_{HT} = Horizontal tail volume
V_{VT} = Vertical tail volume

STEP 2: Compute Required Tail Arm that Minimizes the Wetted Area

Tail arm:

$$L_T = \sqrt{\frac{2 \cdot S_{REF}\left(V_{HT} \cdot C_{REF} + V_{VT} \cdot b_{REF}\right)}{\pi(R_1 + R_2)}}$$

STEP 3: Compute Required HT and VT Areas

HT area:

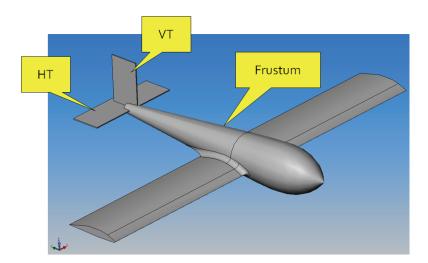$$S_{HT} = \frac{V_{HT} \cdot S_{REF} \cdot C_{REF}}{L_T}$$

VT area:

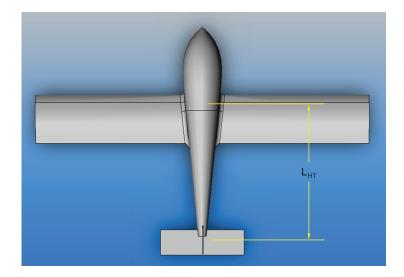$$S_{VT} = \frac{V_{VT} \cdot S_{REF} \cdot b_{REF}}{L_T}$$

That's it. You have defined your required geometries. Use the values of L_T, S_{HT}, and S_{VT} when you size your tail using SURFACES.
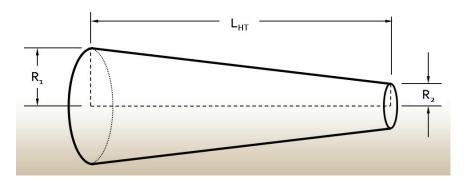
THEORY

Geometric Properties

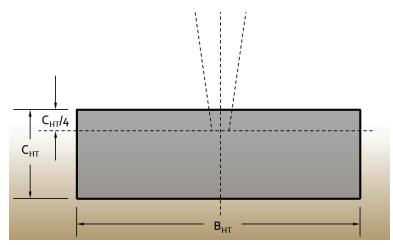Consider the empennage shown below, which consists of a frustum and flat horizontal and vertical tail:

Consider the dimensions of the frustum:

L_{HT} is assumed to extend from $C_{REF}/4$ to $C_{HT}/4$

Consider the dimensions of the HT. If the HT is tapered calculate the Mean Geometric Chord and use that in place of C_{HT}:

Tail Geometry - Standard Definitions

Horizontal tail area:

$$S_{HT} = B_{HT} \cdot C_{HT}$$

Horizontal tail volume:

$$V_{HT} = \frac{S_{HT} \cdot L_{HT}}{S_{REF} \cdot C_{REF}}$$

Vertical tail area:

$$S_{VT} = B_{VT} \cdot C_{VT}$$

Vertical tail volume:

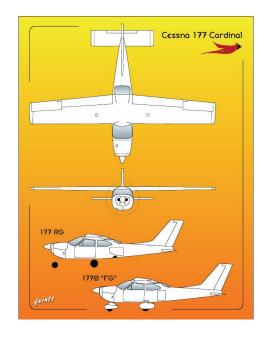$$V_{VT} = \frac{S_{VT} \cdot L_{VT}}{S_{REF} \cdot b_{REF}}$$

Frustum Geometry

Surface area:

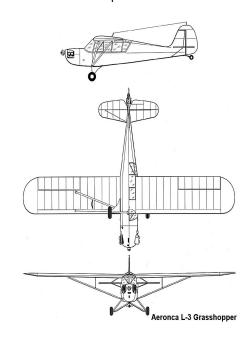$$S_F = 2\pi \frac{(R_1 + R_2)}{2} \cdot L_{HT} = \pi(R_1 + R_2) \cdot L_{HT}$$

Wetted Area

Total wetted area:

$$S_{WET} \approx \pi(R_1 + R_2) \cdot L_{HT} + 2S_{HT}$$

A Combined HT and VT Optimization of the Tail Arm

Many airplane configurations feature a HT and VT that are in a close enough proximity with one another to consider a simultaneous optimization for both. This section formulates such an optimization.

Cessna 177 Cardinal

177 RG

177B "FG"

Aeronca L-3 Grasshopper

Formulate expressions for the S_{HT} and S_{VT} using a combined tail arm L_T. In other words, once we have determined the required tail arm LT, we use that value with the following equations to calculate the planform areas of the HT and VT, such that the chosen VHT and VVT result:

HT area:
$$S_{HT} = \frac{V_{HT} \cdot S_{REF} \cdot C_{REF}}{L_T}$$

VT area:
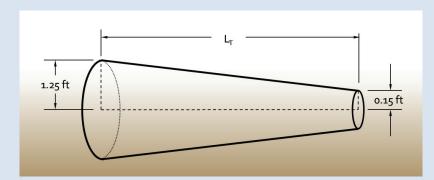$$S_{VT} = \frac{V_{VT} \cdot S_{REF} \cdot b_{REF}}{L_T}$$

Wetted area:
$$S_{WET} \approx \pi(R_1 + R_2) \cdot L_T + 2\frac{V_{HT} \cdot S_{REF} \cdot C_{REF}}{L_T} + 2\frac{V_{VT} \cdot S_{REF} \cdot b_{REF}}{L_T}$$

Next differentiate with respect to LT and set equal to zero to find the minimum. The steps are omitted from here, but the resulting expression is:

$$L_T = \sqrt{\frac{2 \cdot S_{REF}(V_{HT} \cdot C_{REF} + V_{VT} \cdot b_{REF})}{\pi(R_1 + R_2)}}$$

EXAMPLE

A small airplane has a reference area S_{REF} = 130 ft², an Aspect Ratio (AR) of 16, and Taper Ratio (TR) of 0.5. If the tail cone has the dimensions shown, determine L_T for V_{HT} = 0.75 and V_{VT} = 0.04 such that the total wetted area is minimized. Size the HT for an AR of 4 and the VT for an AR of 2, assuming a rectangular planform. Here use the wing's Mean Geometric Chord (MGC) for C_{ref}.

SOLUTION:

Wing span:
$$AR = \frac{b^2}{S} = \frac{b}{C_{avg}} \iff b = \sqrt{AR \times S} = \sqrt{16 \times 130} = 45.6\,\text{ft}$$

Average chord:
$$AR = \frac{b}{C_{avg}} \iff C_{avg} = \frac{b}{AR} = \frac{45.6}{16} = 2.85\,\text{ft}$$

Root chord:
$$C_R = \frac{2C_{avg}}{(1+\lambda)} = \frac{2(2.85)}{(1+0.5)} = 3.80\,\text{ft}$$

| Tip chord: | $C_T = \lambda C_R = 0.5 \times 3.80 = 1.90\,\text{ft}$ |

| MGC: | $MGC = \left(\dfrac{2}{3}\right)C_r\left(\dfrac{1+\lambda+\lambda^2}{1+\lambda}\right) = \left(\dfrac{2}{3}\right)(3.80)\left(\dfrac{1+0.5+0.25}{1+0.5}\right) = 2.956\,\text{ft}$ |

Determine the optimum tail arm:

$$L_T = \sqrt{\frac{2\cdot S_{REF}\left(V_{HT}\cdot C_{REF} + V_{VT}\cdot b_{REF}\right)}{\pi\left(R_1 + R_2\right)}}$$

$$= \sqrt{\frac{2\cdot 130\left(0.75\times 2.956 + 0.02\times 45.6\right)}{\pi\left(1.25 + 0.15\right)}} = 13.60\,ft$$

Determine tail area:

$$S_{HT} = \frac{V_{HT}\cdot S_{REF}\cdot C_{REF}}{L_T} = \frac{0.75\cdot 130\cdot 2.956}{13.60} = 21.19\ \text{ft}^2$$

| HT span: | $b_{HT} = \sqrt{AR_{HT}\times S_{HT}} = \sqrt{4\times 21.19} = 9.74\ \text{ft}$ |

| Average chord: | $C_{avg} = \dfrac{b_{HT}}{AR_{HT}} = \dfrac{9.74}{4} = 2.44\ \text{ft}$ |

Determine tail area:

$$S_{VT} = \frac{V_{VT}\cdot S_{REF}\cdot b_{REF}}{L_T} = \frac{0.020\cdot 130\cdot 45.6}{13.60} = 8.72\ \text{ft}^2$$

| VT span: | $b_{VT} = \sqrt{AR_{VT}\times S_{VT}} = \sqrt{2\times 8.72} = 4.18\ \text{ft}$ |

| Average chord: | $C_{avg} = \dfrac{b_{VT}}{AR_{VT}} = \dfrac{4.18}{2} = 2.09\ \text{ft}$ |

WHAT IS ProAdvice?

Pro-Advices are short and simplified excerpts from Professor Gudmundsson's design handbook *Aircraft Preliminary Design Handbook* and are intended to provide the aircraft designer with clear and concise analysis methods for the aircraft designer. This handbook is currently in development. Snorri Gudmundsson is an Assistant Professor of Aerospace Engineering at Embry-Riddle Aeronautical University in Daytona Beach, Florida, where he teaches Aircraft Preliminary Design to senior engineering students.