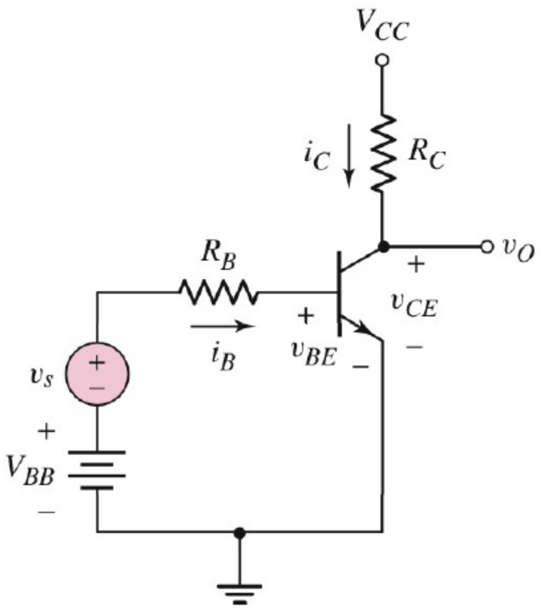
Basic Electronic Circuits (IEC-103)

Lecture-18

Small Signal Analysis

Small Signal Voltage Gain



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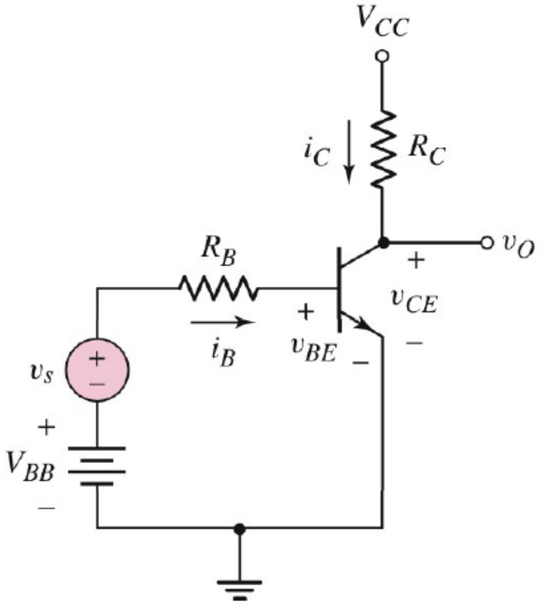

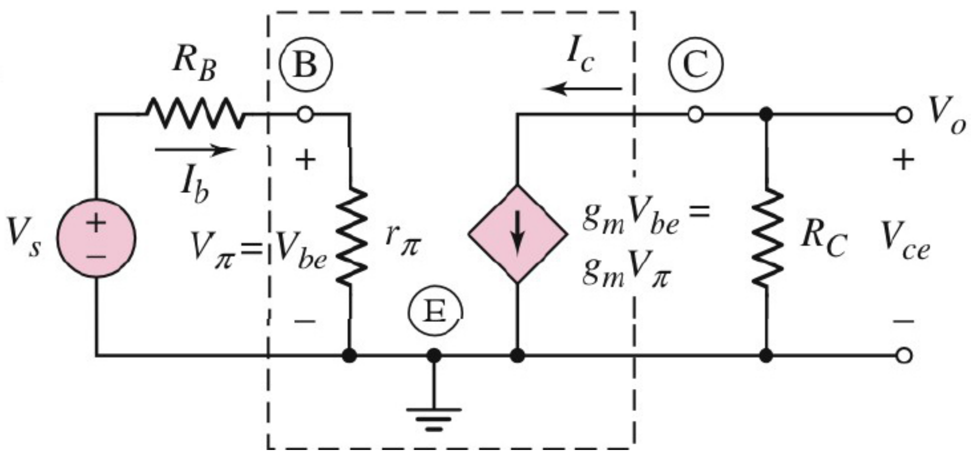

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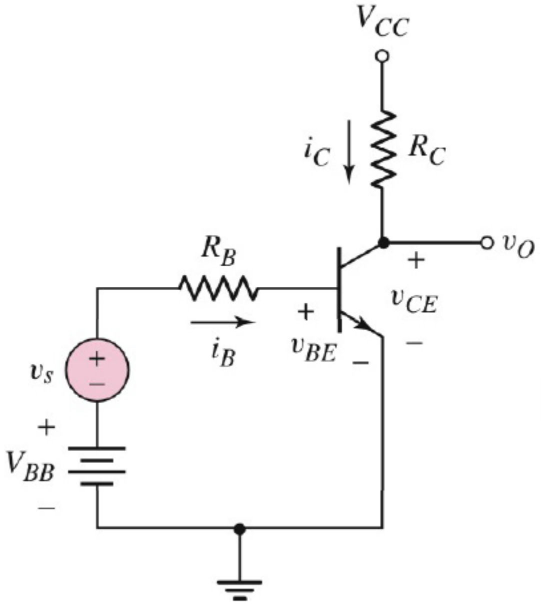
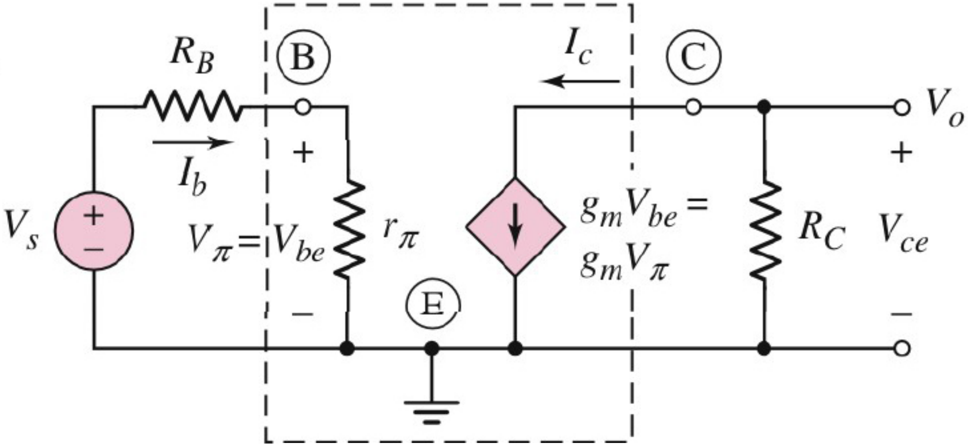
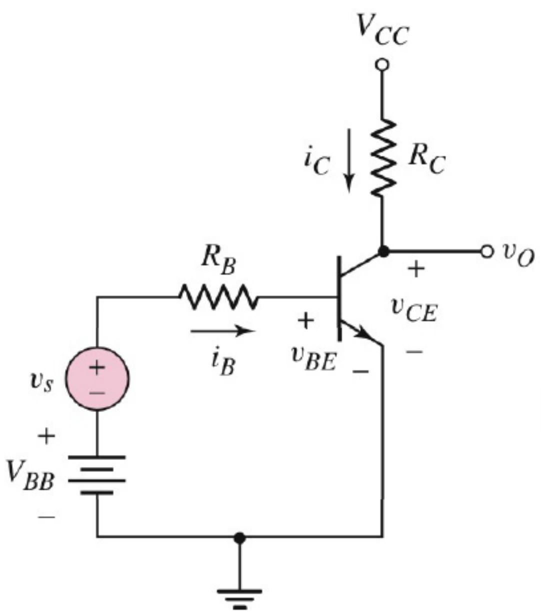


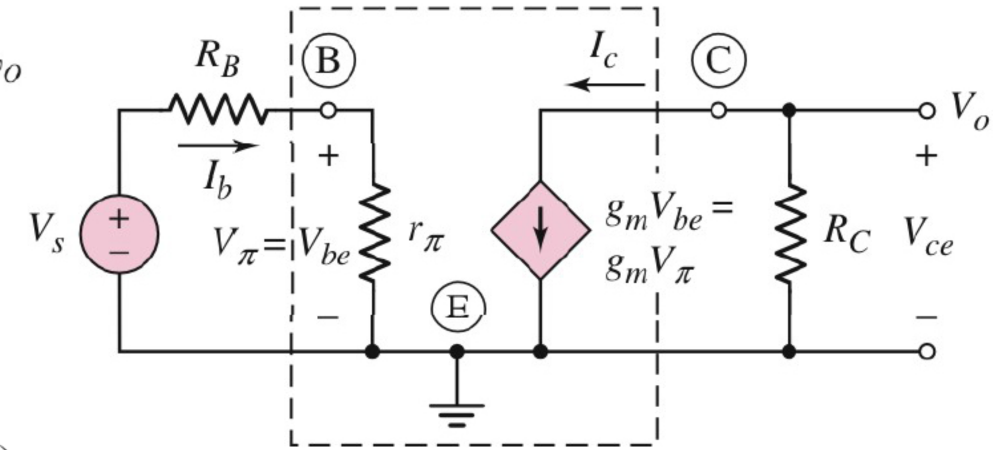


$$V_\pi = V_{be} = V_s \left(\frac{r_\pi}{r_\pi + R_B} \right)$$

$$V_o = V_{ce} = -(g_m V_\pi) R_C = -(g_m V_{be}) R_C$$

Small Signal Voltage Gain





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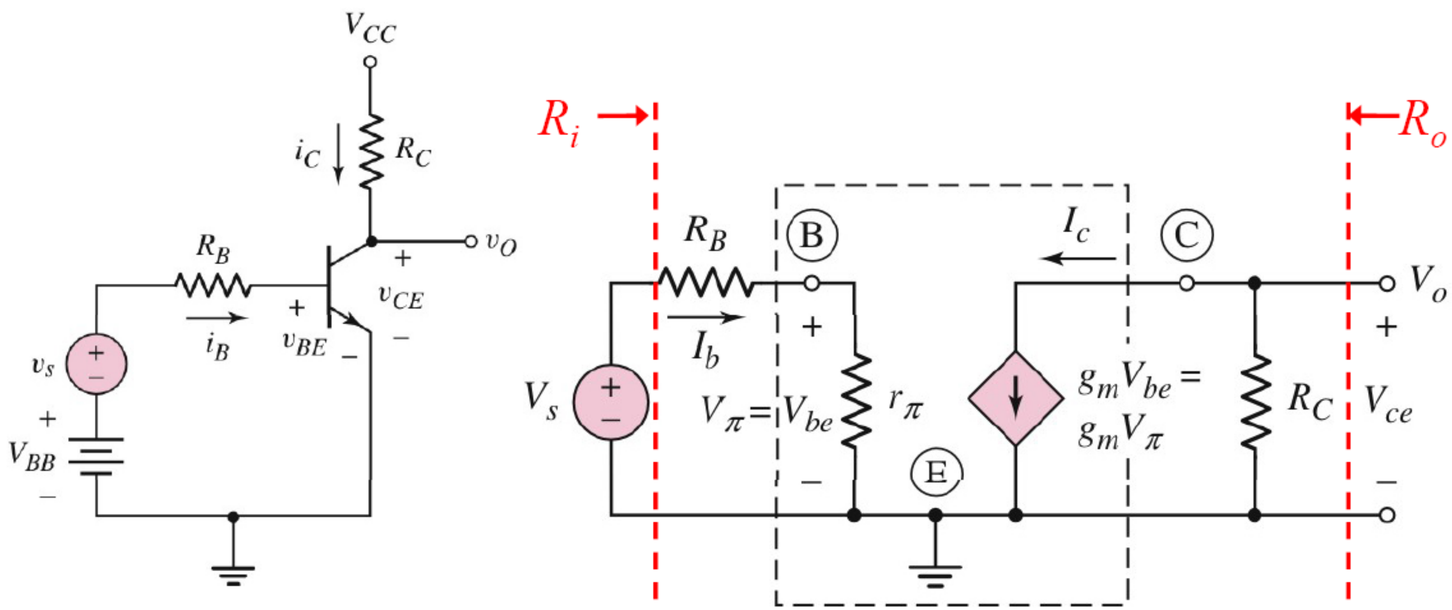
$$V_o = V_{ce} = -(g_m V_\pi) R_C = -(g_m V_{be}) R_C$$

Small-signal voltage gain:

$$A_v = \frac{V_o}{V_s} = -(g_m R_C) \left(\frac{r_\pi}{r_\pi + R_B} \right)$$

Input and Output Resistances



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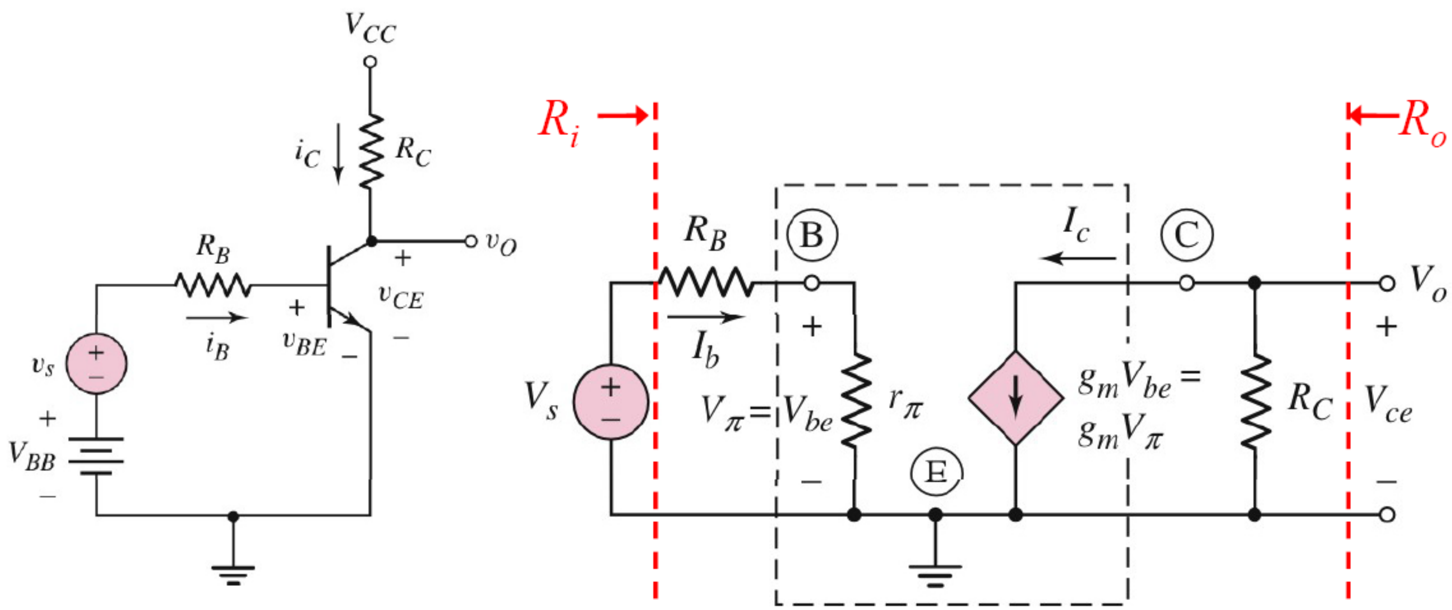


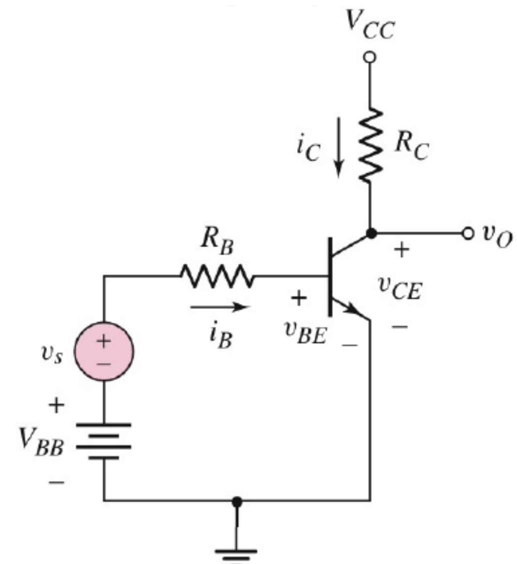
Input resistance: $R_i = R_B + r_\pi$

Output resistance: $R_o = R_C$

[Setting $V_s = 0$ (short), then $V_\pi = 0$ & $g_m V_\pi = 0$ (open)]

Example

- Calculate the small-signal voltage gain, input resistance & output resistance of the BJT amplifier circuit at 300 K. Assume that the BJT & circuit parameters are: $\beta = 100$, $V_{CC} = 12$ V, $V_{BE} = 0.7$ V, $R_C = 6$ kΩ, $R_B = 50$ kΩ & $V_{BB} = 1.2$ V.



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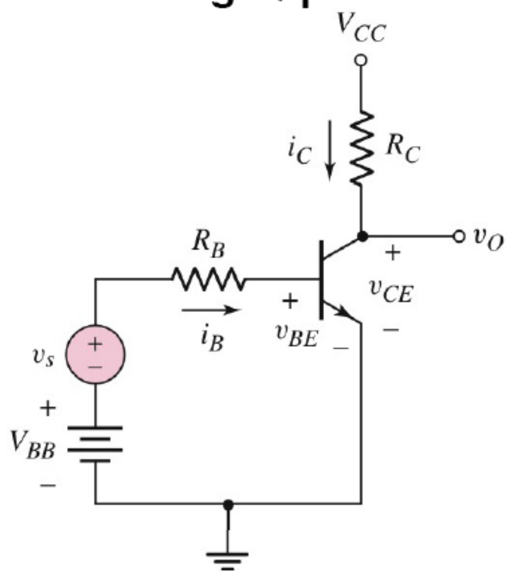
Step 1: Perform dc analysis to determine the dc biasing Q-point:



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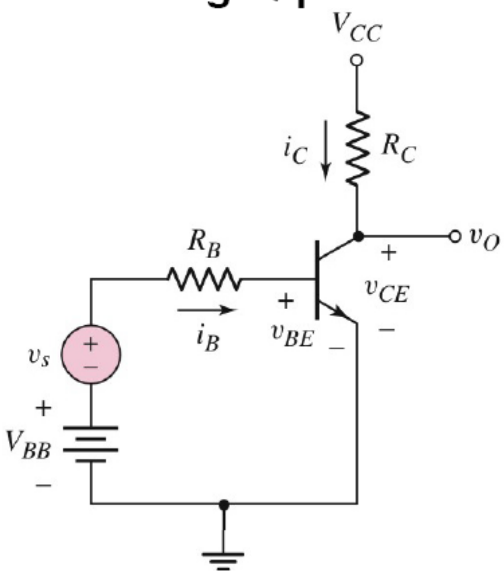
Step 1: Perform dc analysis to determine the dc biasing Q-point:

$$I_{BQ} = \frac{V_{BB} - V_{BE}(\text{on})}{R_B} = \frac{1.2 - 0.7}{50\text{k}} = 10\,\mu\text{A}$$

$$I_{CQ} = \beta\, I_{BQ} = (100)(10\mu) = 1\,\text{mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C = 12 - (1\text{m})(6\text{k}) = 6\,\text{V}$$



Since $I_{BQ} > 0$ A, $V_{BB} > V_{BE}(\text{on})$ & $V_{CEQ} > V_{BE}(\text{on})$, the BJT amplifier is biased in forward-active mode about the Q-point ($V_{CEQ} = 6$ V, $I_{CQ} = 1$ mA, $I_{BQ} = 10\,\mu$A).

Example (continued)

Step 2: Perform ac analysis using small-signal hybrid-π equiv. cct:

Example (continued)

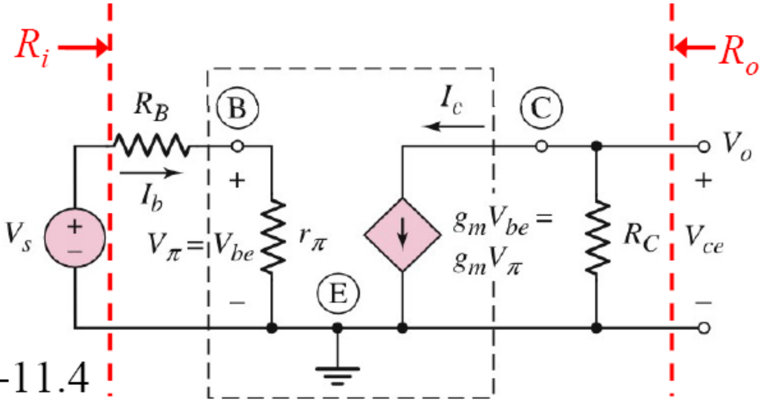
Step 2: Perform ac analysis using small-signal hybrid-π equiv. cct:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{\beta(kT/e)}{I_{CQ}} = \frac{(100)\left[(86\times10^{-6}e)(300)/e\right]}{1\text{m}} = 2.6\,\text{k}\Omega$$

$$g_m = \frac{\beta}{r_\pi} = \frac{100}{2.6\text{k}} = 38.5\,\text{mA/V}$$

$$A_v = \frac{V_o}{V_s} = -(g_m R_C)\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

$$= -(38.5\text{m})(6\text{k})\left(\frac{2.6\text{k}}{2.6\text{k}+50\text{k}}\right) = -11.4$$



The BJT amplifier is capable of amplifying the input signal amplitude by 11.4 times. The –ve sign indicates a phase reversal of 180º.

$$R_i = R_B + r_\pi = 50\text{k} + 2.6\text{k} = 52.6\,\text{k}\Omega \qquad R_o = R_C = 6\,\text{k}\Omega$$

Example (continued)

Further Discussion:

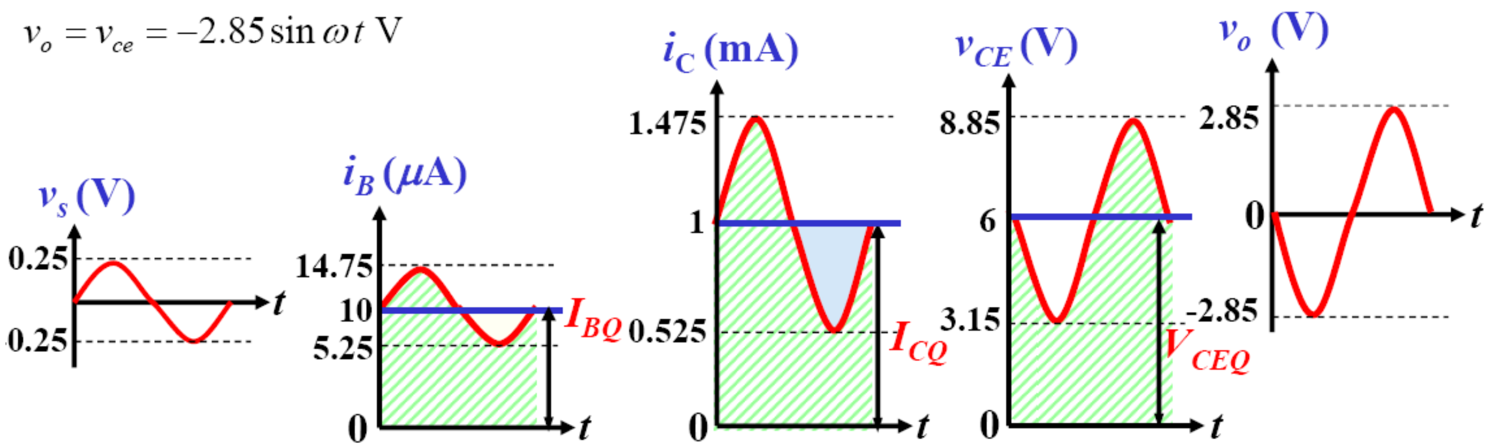
Let $v_s = 0.25 \sin \omega t$ V

$$i_b = \frac{v_s}{R_B + r_\pi} = \frac{0.25 \sin \omega t}{50\text{k} + 2.6\text{k}} = 4.75 \sin \omega t \ \mu\text{A} \quad \Rightarrow \quad i_B = I_{BQ} + i_b = 10 + 4.75 \sin \omega t \ \mu\text{A}$$

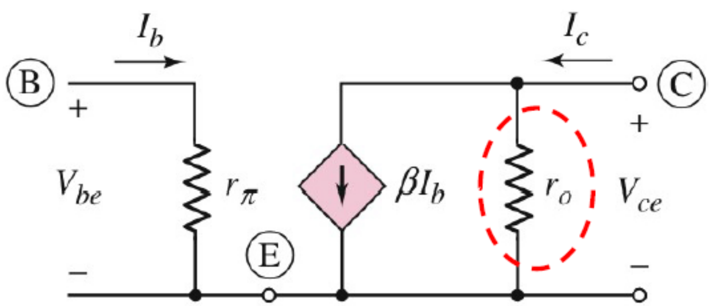
$$i_c = \beta i_b = (100)(4.75\mu \sin \omega t) = 0.475 \sin \omega t \text{ mA} \quad \Rightarrow \quad i_C = I_{CQ} + i_c = 1 + 0.475 \sin \omega t \text{ mA}$$

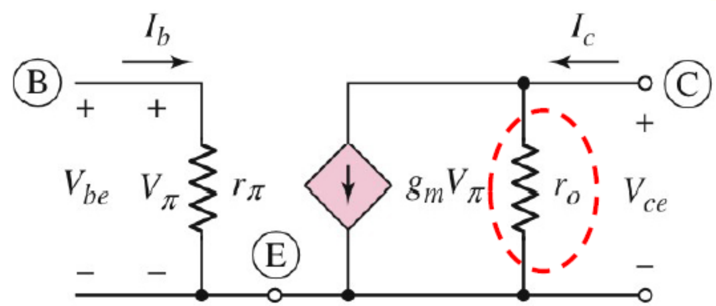
$$v_{ce} = -i_c R_C = -(0.475\text{m})(6\text{k}) \sin \omega t = -2.85 \sin \omega t \text{ V} \quad \Rightarrow \quad v_{CE} = V_{CEQ} + v_{ce} = 6 - 2.85 \sin \omega t \text{ V}$$

$$v_o = v_{ce} = -2.85 \sin \omega t \text{ V}$$



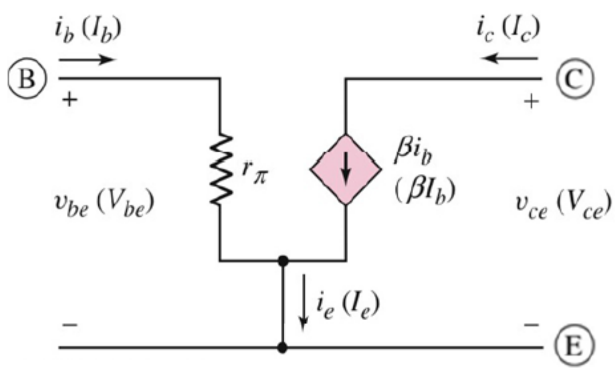
Small Signal Hybrid-π Equivalent with Early Effect



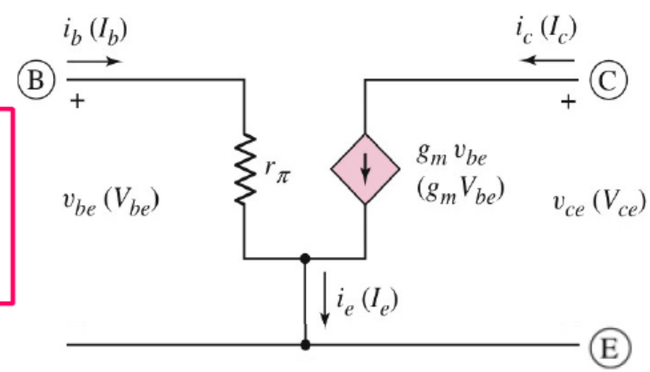


$$r_o = \left.\frac{\partial v_{CE}}{\partial i_C}\right|_{Q-point} \cong \frac{V_A}{I_{CQ}}$$

- r_o = small-signal transistor output resistance
- V_A = early voltage ($50 < V_A < 300$)



No Early Effect



Example

Determine the small-signal voltage gain, input resistance, and output resistance of the BJT amplifier circuit in previous example with the early effect. Assume that the early voltage is 50 V.

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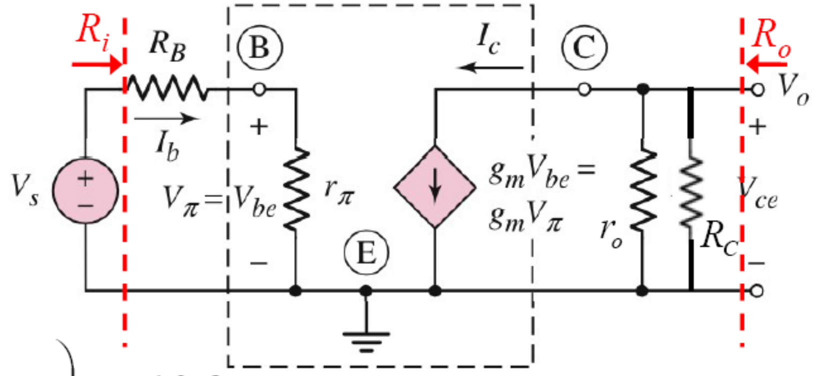


$$r_o = \frac{V_A}{I_{CQ}} = \frac{50}{1\text{m}} = 50\,\text{k}\Omega$$

$$A_v = \frac{V_o}{V_s} = -g_m\left(r_o \,//\, R_C\right)\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

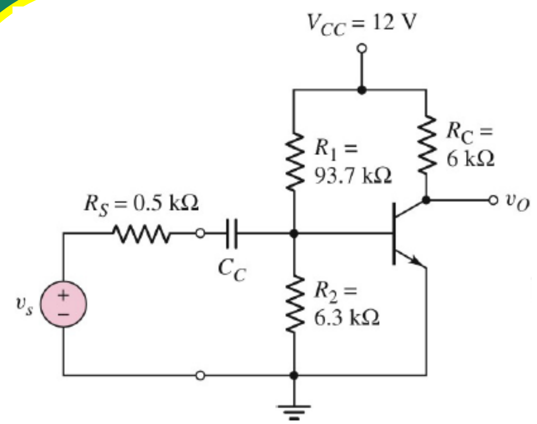
$$= -(38.5\text{m})(50\text{k}//6\text{k})\left(\frac{2.6\text{k}}{2.6\text{k}+50\text{k}}\right) = -10.2$$

$$R_i = R_B + r_\pi = 50\,\text{k} + 2.6\,\text{k} = 52.6\,\text{k}\Omega$$

$$R_o = r_o \,//\, R_C = 50\,\text{k} \,//\, 6\,\text{k} = 5.4\,\text{k}\Omega$$

r_o reduces both $|A_v|$ from 11.4 to 10.2 & R_o from 6 to 5.4 kΩ.

CE amplifier (with voltage-divider biasing & coupling capacitor)



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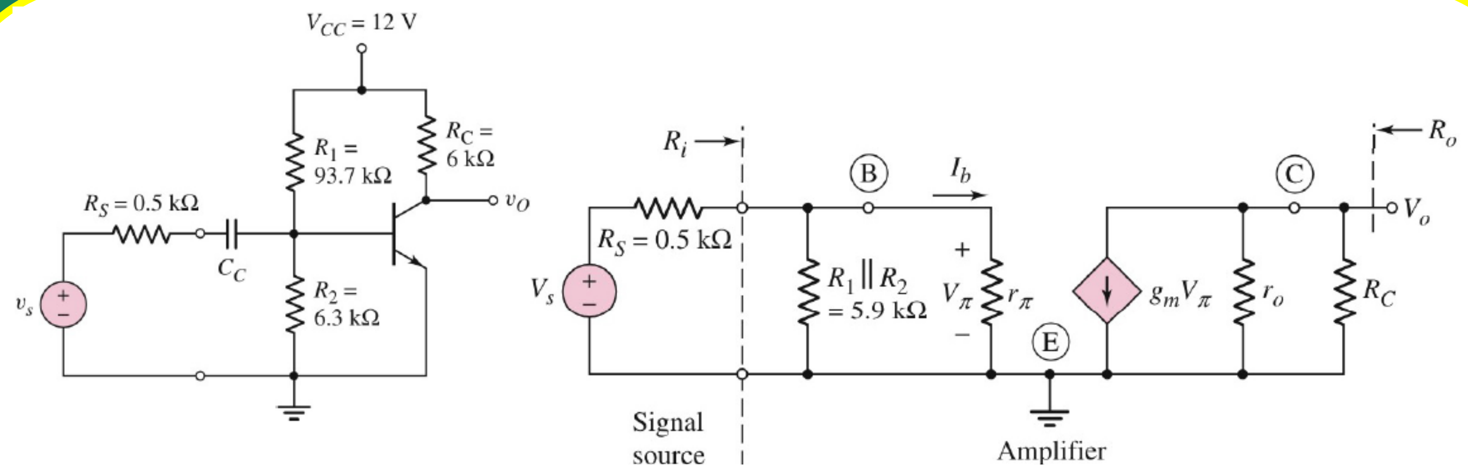

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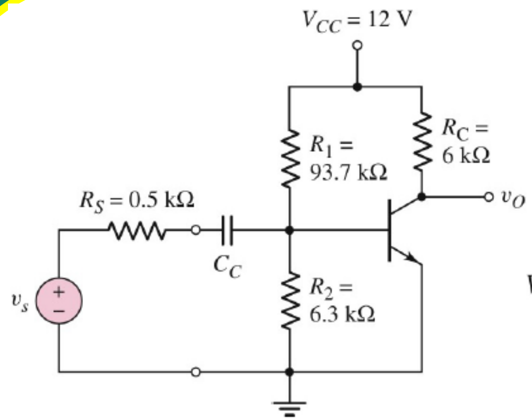

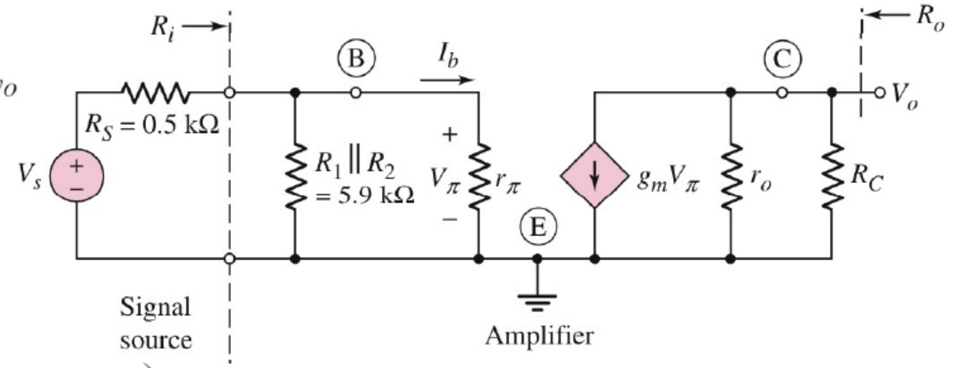

$$V_\pi = V_{be} = V_s \left(\frac{R_1 // R_2 // r_\pi}{R_1 // R_2 // r_\pi + R_S} \right)$$

$$V_o = -g_m V_\pi (r_o // R_C) = -g_m V_s \left(\frac{R_1 // R_2 // r_\pi}{R_1 // R_2 // r_\pi + R_S} \right) (r_o // R_C)$$

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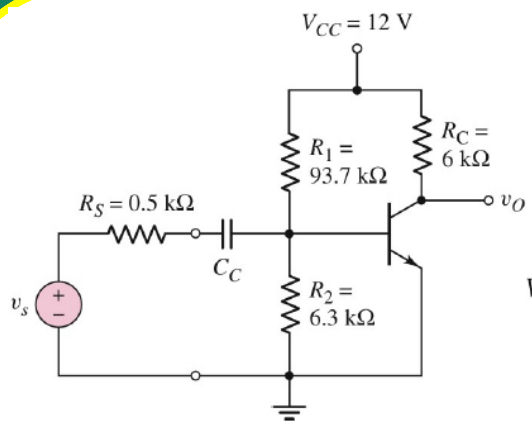

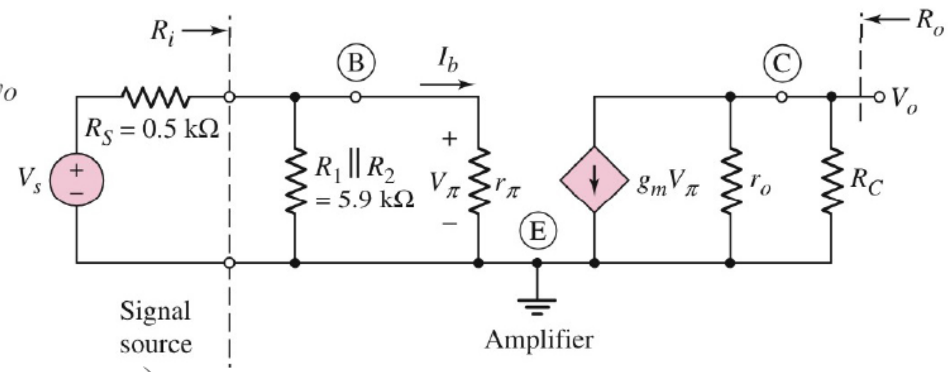

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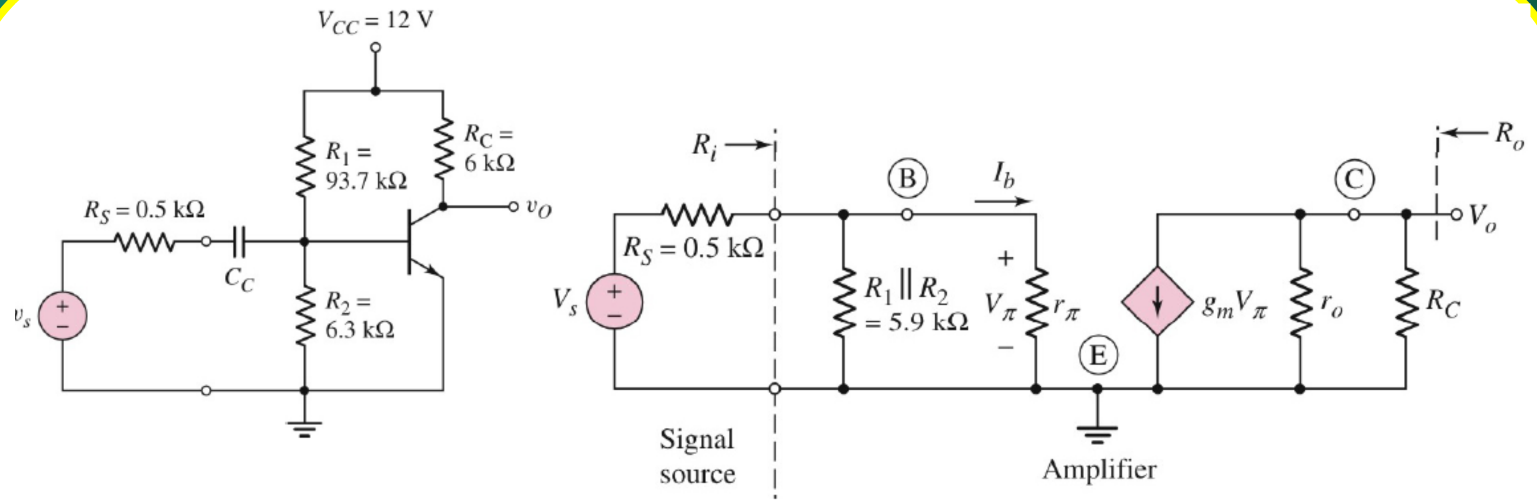
Small-signal voltage gain: $A_V = \dfrac{V_o}{V_s} = -g_m \left(\dfrac{R_1 // R_2 // r_\pi}{R_1 // R_2 // r_\pi + R_S} \right) (r_o // R_C)$

CE amplifier (with voltage-divider biasing & coupling capacitor)



Input resistance: $R_i = R_1 // R_2 // r_\pi$

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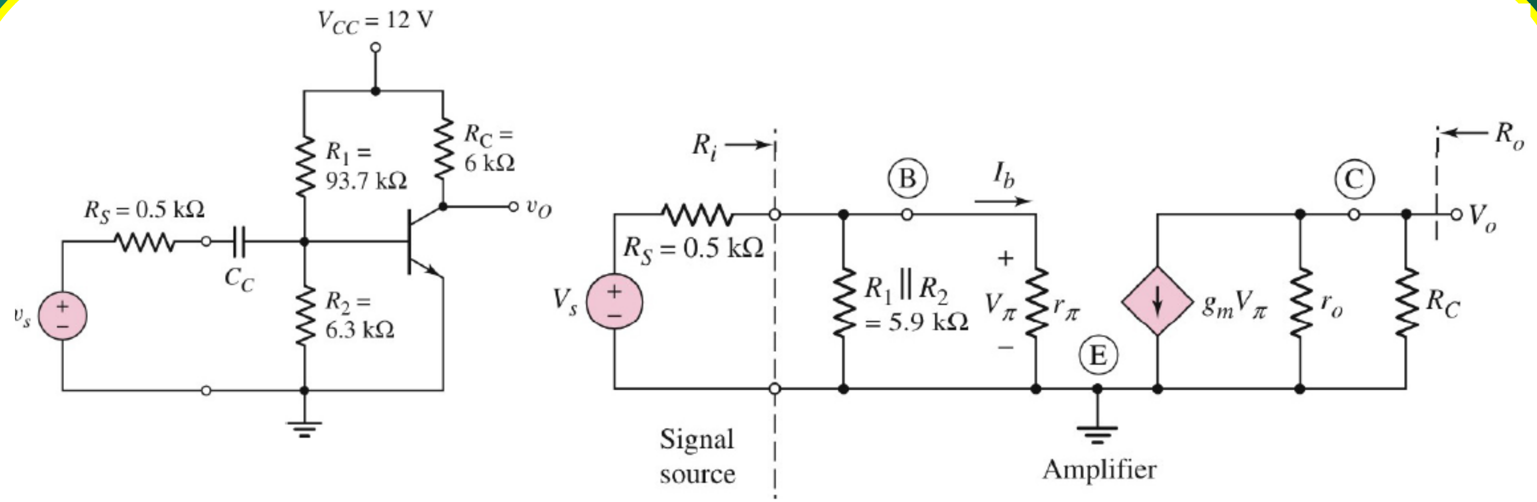

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Output resistance: $R_o = r_o \,//\, R_C$

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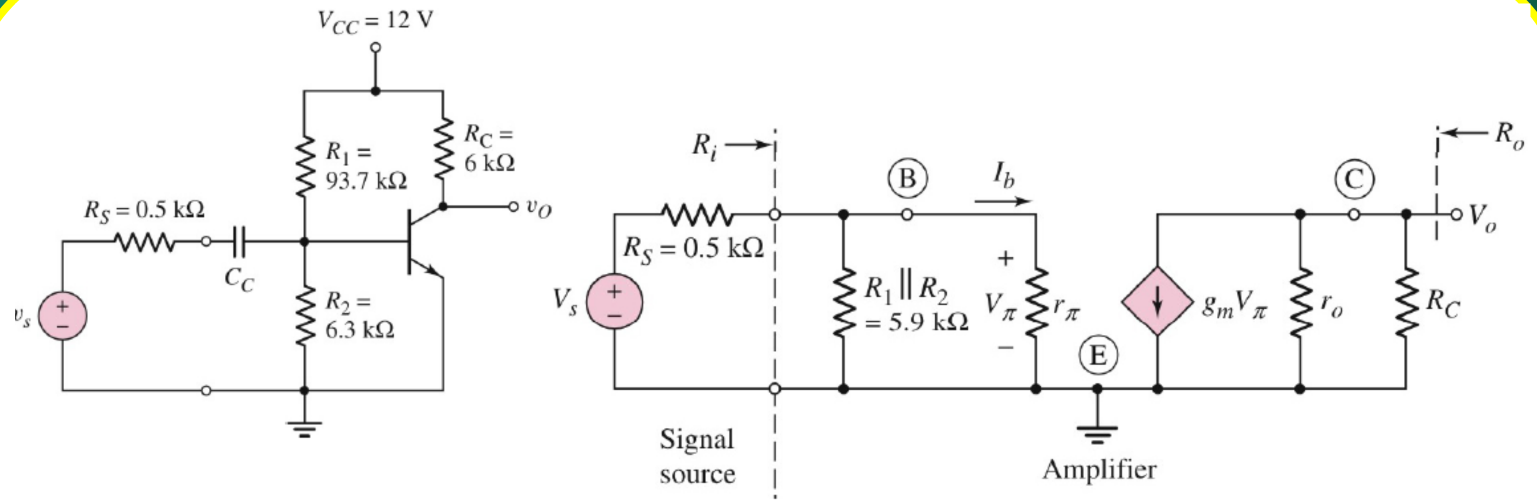

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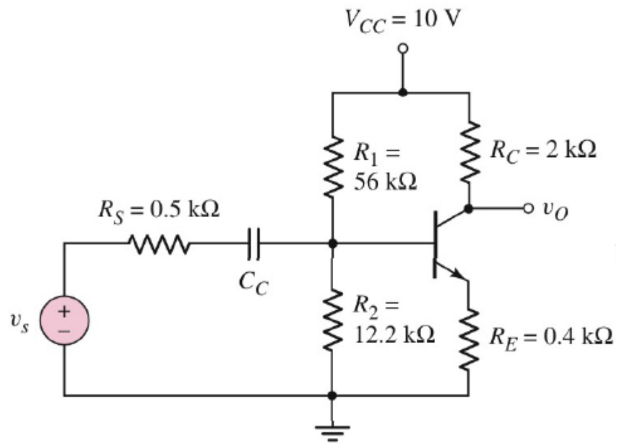
CE amplifier (with voltage-divider biasing, coupling capacitor & R_E)



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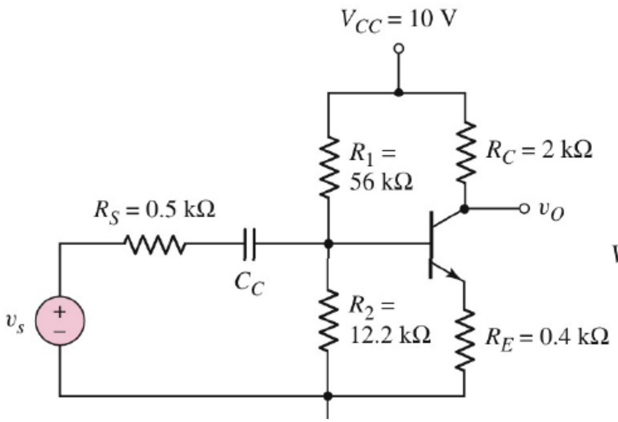

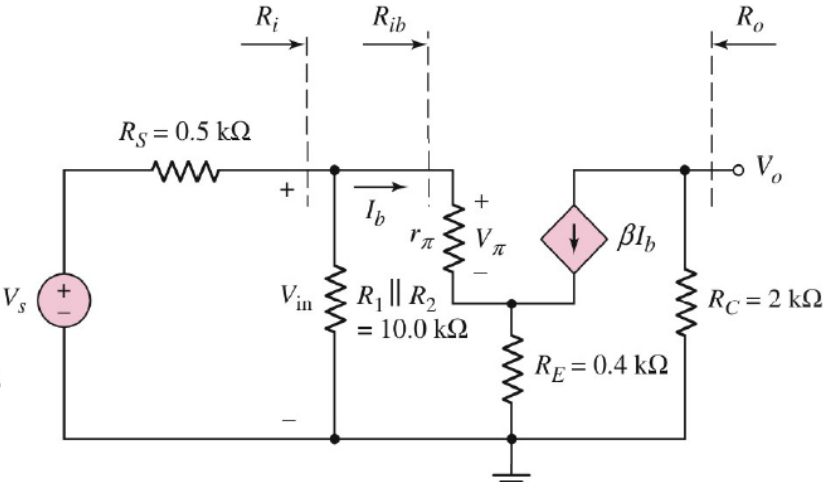

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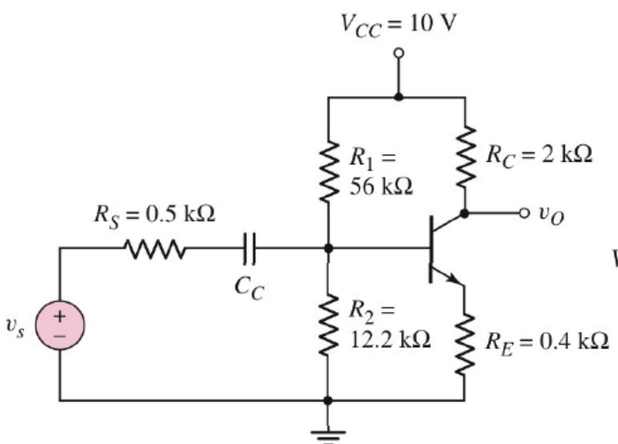
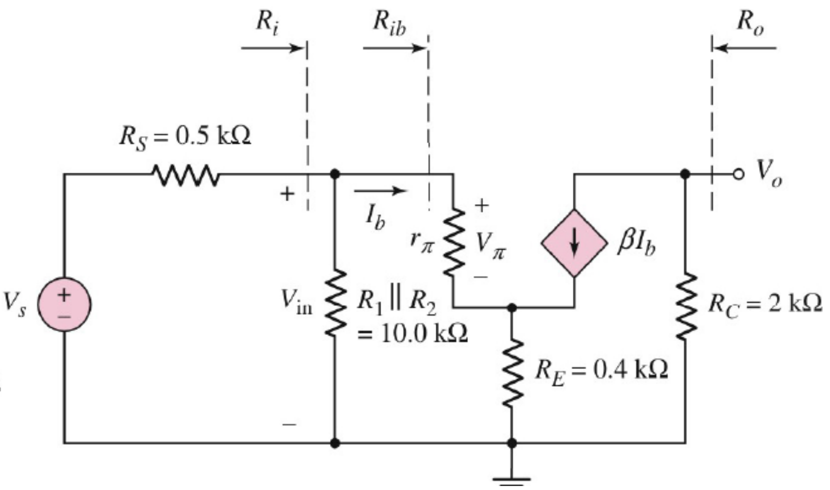




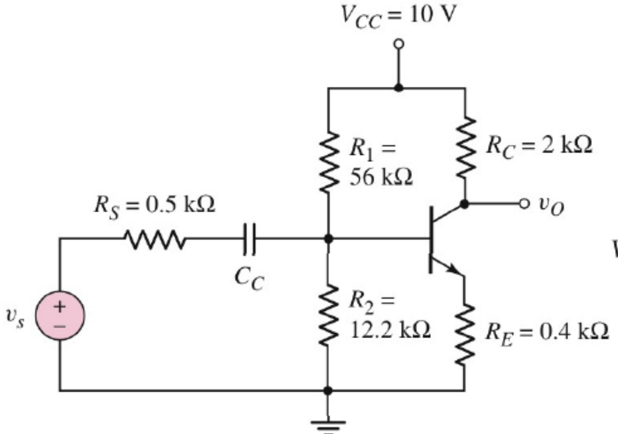
$$V_o = -(\beta I_b) R_C$$

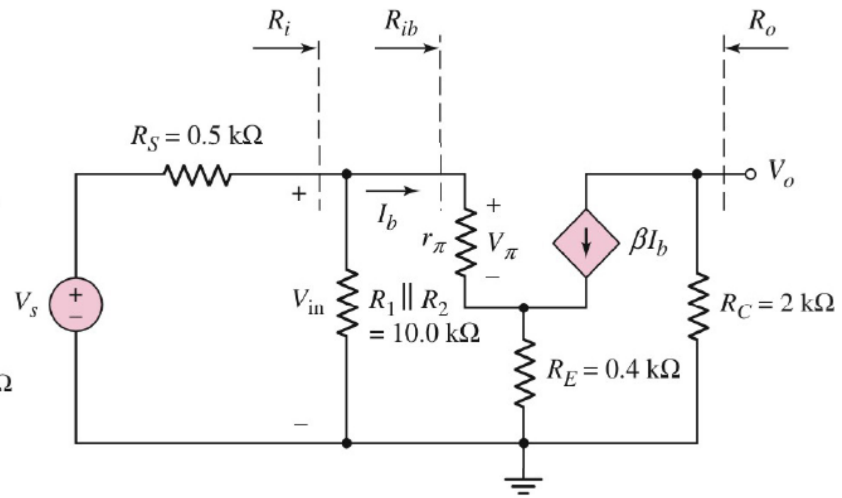
$$V_{in} = I_b r_\pi + (I_b + \beta I_b) R_E$$

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1+\beta) R_E$$

$$R_i = R_1 \,//\, R_2 \,//\, R_{ib}$$

CE amplifier (with voltage-divider biasing, coupling capacitor & R_E)





$$V_o = -(\beta I_b) R_C$$

$$V_{in} = I_b r_\pi + (I_b + \beta I_b) R_E$$

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1+\beta) R_E$$

$$R_i = R_1 // R_2 // R_{ib}$$

$$V_{in} = V_s \left(\frac{R_i}{R_S + R_i} \right)$$

$$A_v = \frac{V_o}{V_s} = \frac{-\beta R_C}{r_\pi + (1+\beta) R_E} \left(\frac{R_i}{R_i + R_S} \right)$$

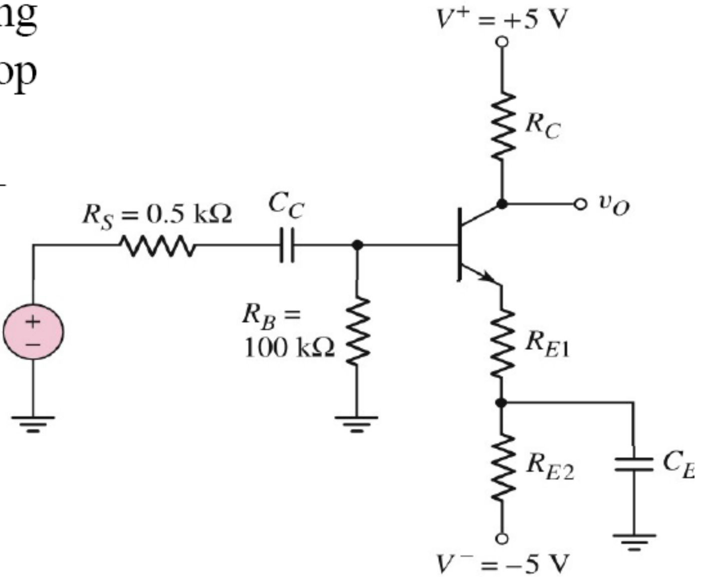
DC and AC Load Lines

- DC load line is used to find Q-point
- AC load line is used to determine graphically the operation of a BJT amplifier
- DC and AC load lines are essentially different since capacitors appear as an open circuit for a DC operation but a short circuit for an AC operation

DC Load Line

- The DC load line is found by writing a dc KVL equation for the C-E loop as follows:

$$V^+ = I_C R_C + V_{CE} + I_E (R_{E1} + R_{E2}) + V^-$$



- Noting that $I_E = [\,(1+\beta)/\beta\,]\, I_C$,

$$V_{CE} = (V^+ - V^-) - I_C\left[R_C + \left(\frac{1+\beta}{\beta}\right)(R_{E1} + R_{E2})\right]$$

- If $\beta >> 1$, the $(1+\beta)/\beta \approx 1$. The slope of the dc load line is $\dfrac{-1}{R_C + R_{E1} + R_{E2}}$

AC Load Line

- The ac load line is found by writing an ac KVL equation for the C-E loop as follows:

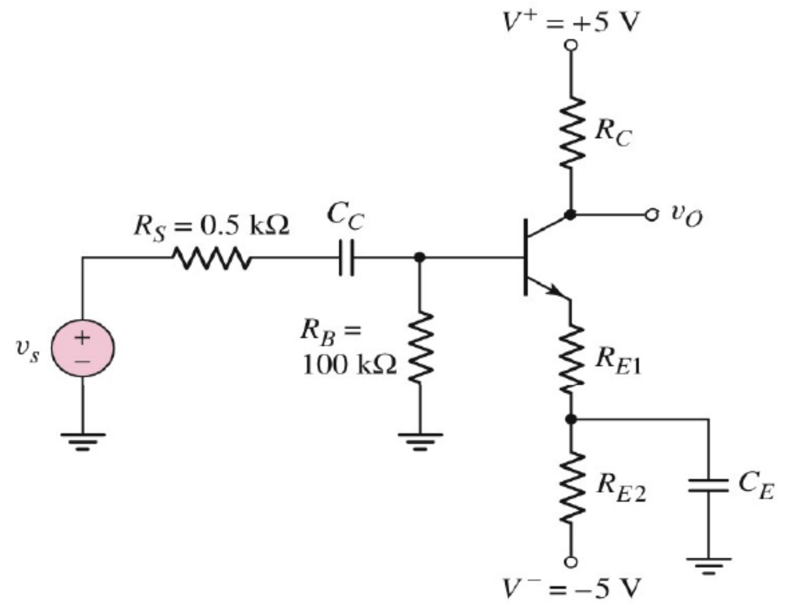
$$i_c R_C + v_{ce} + i_e R_{E1} = 0$$

- Assuming $(i_c \cong i_e)$

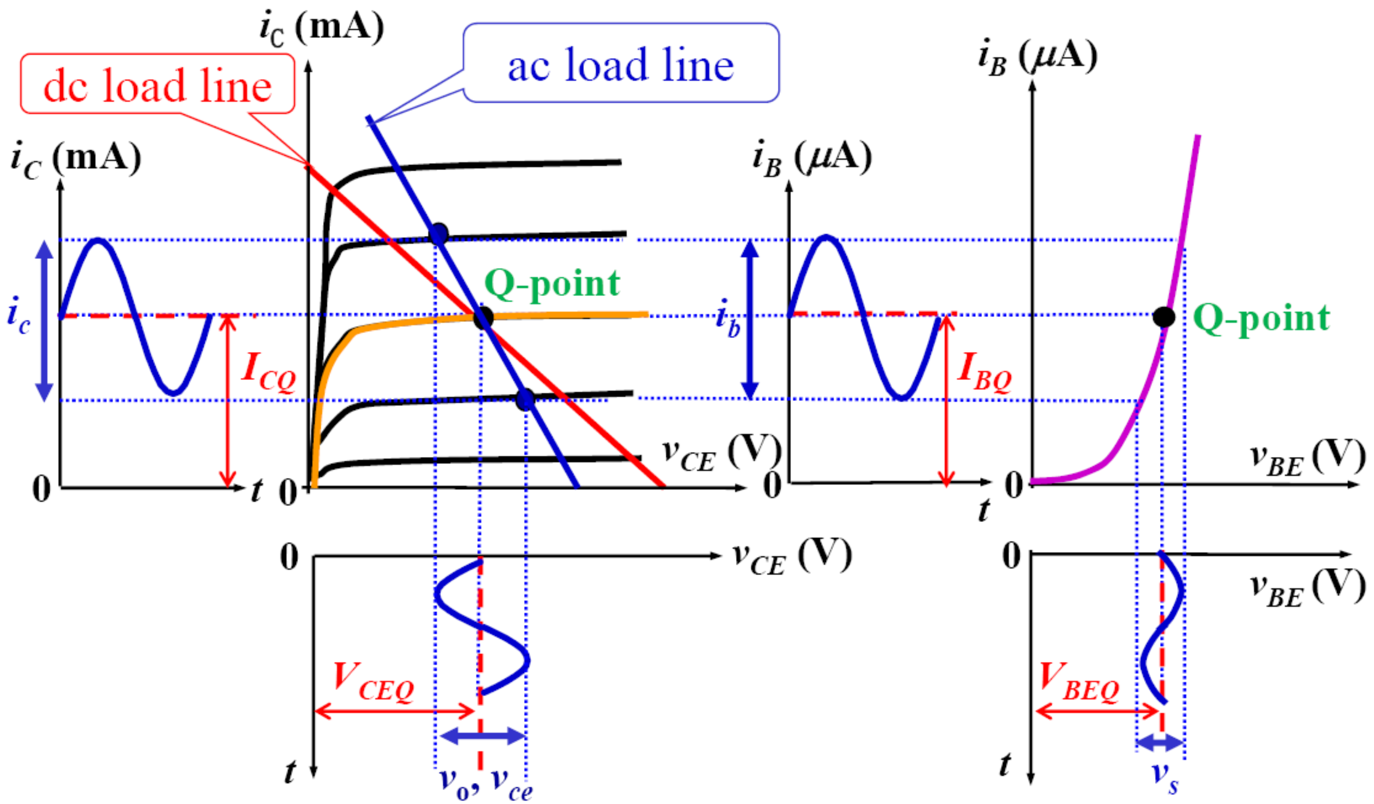
$$v_{ce} = -i_c (R_C + R_{E1})$$

- The slope of the ac load line is

$$\frac{-1}{R_C + R_{E1}}$$

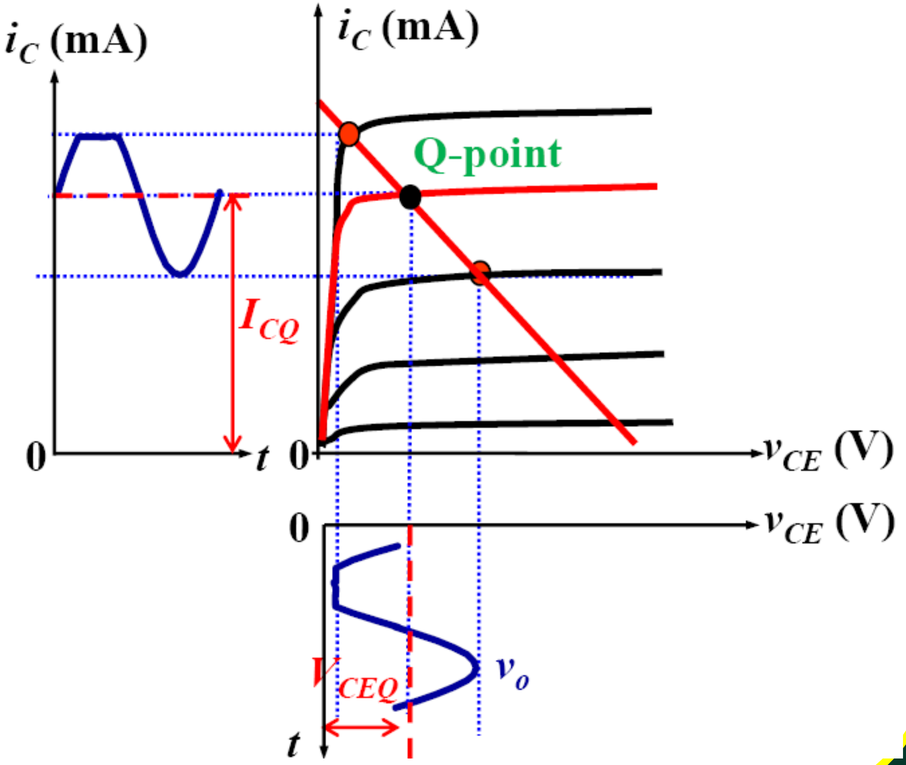


Maximum Output Symmetrical Swing

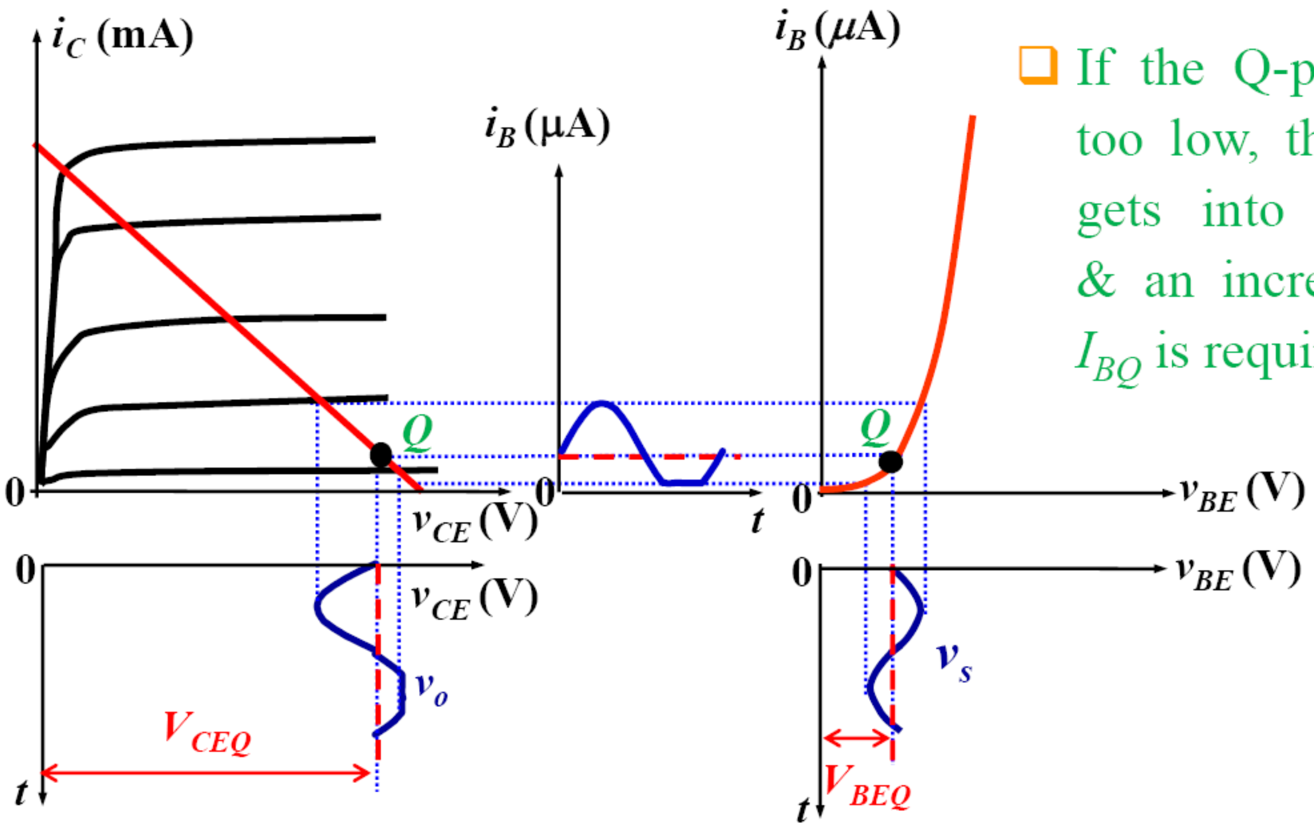


Saturation

- If the Q-point is not set properly, the BJT may get into saturation or cutoff, resulting in nonlinear distortion.
- If the Q-point is set too high, the BJT gets into saturation, & a reduction in I_{BQ} is required.
- Even with a proper Q-point setting, if the signal amplitude is too large, distortion will also result, & a reduction of signal amplitude is required.



Cutoff



- If the Q-point is too low, the BJT gets into cutoff, & an increase in I_{BQ} is required.