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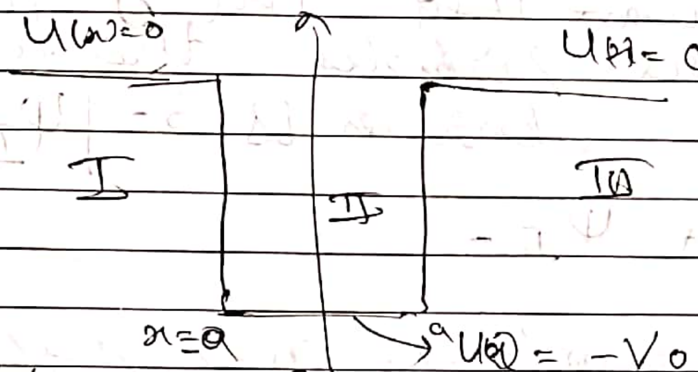
2016170

Science - 2 Assignment - 3

Particle inside a finite well



$$U(x) = \begin{cases} -V_0 & 0 \le x \le a \\ 0 & \text{o.w} \end{cases}$$

Case 1 => $E < 0$ (Total energy is less than 0.)

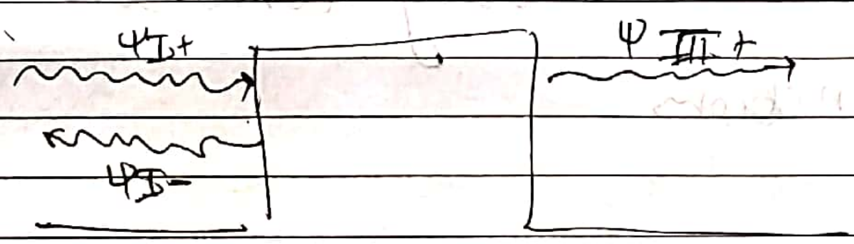
Hence the probability of the particle being outside the well is non-zero.

$$\psi_I = Ae^{ikx} + Be^{-ikx}$$

$$\psi_{III} = Fe^{ikx} + Ge^{-ikx} \qquad k = \sqrt{\frac{2mE}{\hbar}}$$

$$\psi_{I+} = Ae^{ikx} \text{ (incoming wave)}$$

$$\psi_{I-} = Ae^{-ikx} \text{ (outgoing wave)}$$



Let ψ_{I+} be the grp velocity of the incoming wave then the flux of the particles that arrive at the barrier is $S = |\psi_{I+}|^2 V_{I+}$

$\psi_I = \psi_{I+} \; \psi_{I-}$

As there won't be any reflection at III $\Rightarrow \psi_{III-} \; \psi_{III}(\epsilon)$ hence $G = 0$

The transmission probability T for a particle to pass through the barrier is

$$T = \frac{|\psi_{III+}|^2 \, V_{III+}}{|\psi_{I+}|^2 \, V_{I+}}$$

$$= \frac{FF^*}{AA^*} \; \frac{V_{III+}}{V_{I+}}$$

$$\psi_{II} = C e^{-Kx} + D e^{Kx}$$

$$K = \sqrt{\frac{2m(V_0 - E)}{\hbar}}$$

from boundary conditions of the functions

① $\psi_I(x=-a) = \psi_{II}(x=-a)$

② $\left.\dfrac{\partial \psi_I}{dx}\right|_{x=-a} = \left.\dfrac{d\psi_{II}}{dx}\right|_{x=a}$

③ $\psi_{II}(x=a) = \psi_{III}(x=a)$

④ $\left.\dfrac{\partial \psi_{II}}{dx}\right|_{x=a} = \left.\dfrac{d\psi_{II}}{dx}\right|_{x=a}$

By applying the above 4 condition

⑤ $Ae^{-ika} + Be^{ik_1a} = Ce^{ika} + De^{-ika}$

$ikAe^{ika} - ik_1e^{ika} = -iCke^{ka} + iDke^{ka}$

$Ce^{-ka} + De^{ka} = Fe^{ik_1a} + Ge^{-ika}$

$-Cke^{-ka} + Dke^{ka} = iFk_1e^{ik_1a}$

$-ikG\,e^{-ika}$

Assuming $|V_0| >> E$

$\dfrac{k_2}{k_1} - \dfrac{k_1}{k_2} \approx \dfrac{k_2}{k_1}$

Assuming ψ_{II} is widely weakened ~~by~~ at the barrier

$k_2L >> 1 \Rightarrow e^{+k_2L} >> e^{-k_2L}$

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$$\Rightarrow \frac{A}{F} = \left(\frac{1}{2} + \frac{iR_2}{4R_1}\right) e^{i(k_1+k_2)L}$$

$$\left(\frac{A}{F}\right)^* = \left(\frac{1}{2} - \frac{ik_2}{4k_1}\right) e^{-i(k_1+k_2)2}$$

$$\Rightarrow \frac{AA^*}{FF^*} = \left(\frac{1}{4} + \frac{1k_2}{16k_1}\right.$$

Here $V_{III} = V_I^+$

hence trasibia probality $= \left(\frac{AA^*}{FF^*}\right)^{-1}$

$$\Rightarrow \frac{16 \, e^{-2R_2 L}}{4 + (R_2/k_1)^2}$$

$$\frac{R_2}{k_1} = \frac{V_0}{E} - 1 \gg 1$$

$$\Rightarrow T \approx e^{-2k_2 L}$$

$$R_2 = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

where $\Rightarrow T \propto e^{-\sqrt{V_0}}$ for some $E = -E_0$

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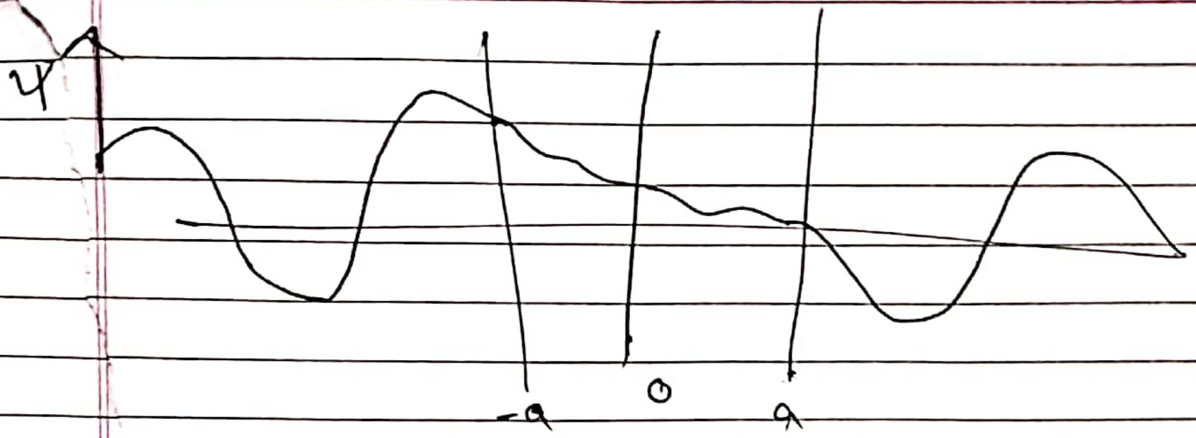
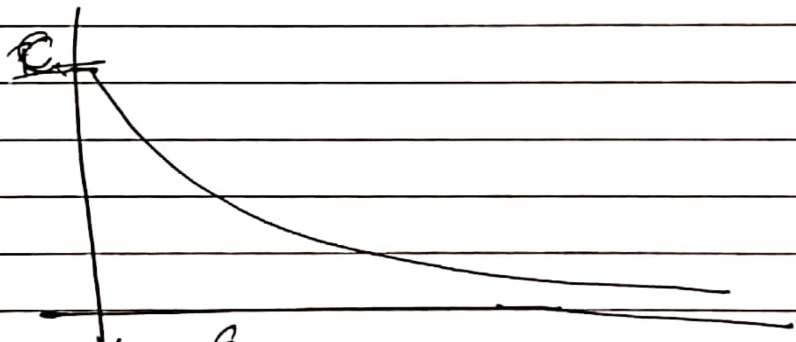


If we take V_0 to be the heigh to the varilr.

$T \propto e^{-\sqrt{V_0 - E_0}}$ E_0 is a constant



Hence it is evident that smallel the barrier potential, higher the chance that a particle will jump the barrier

And as $V_0 \to \infty$ $T \to 0$
which is similiar to the paricle in a 1D where we assume $\psi \to 0$ as $U(x) \to \infty$ outside the box.