INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY HYDERABAD

Monsoon Semester, 2017
Algorithms (ISC211)

Assign. II
August 15, 2017

Due: **22.08.17**

Instructor: Dr. Pawan Kumar

Maximum Marks: 100

Instructions: Write answers on your own. Don't write unnecessary text. Solutions to this assignment will be shown during the tutorial on 29/08/17.

1. Consider the following algorithm

```
foo (a, k)
n = len ( a )
if ( n == 0)
  return False
if ( n <= 2)
   for i in range (0, n )
      if ( a [ i ] == k )
         return True
   return False
t1 = n / 3
t2 = 2 * n / 3
if ( a [0] >= k and k < a [ t1 ])
   return foo ( a [0: t1 ], k )
elseif ( a [ t1 ] <= k and k < a [ t2 ])
   return foo ( a [ t1 : t2 ], k )
elseif ( a [ t2 ] <= k and k < a [n -1])
   return foo ( a [ t2 : n ], k )
else
   return False
```

Now answer the following. [20=10+10]

   1. Derive the recurrence relation for the running time $T(n)$.
   2. Solve the recurrence relation found above.

2. A list of size M is all but s sorted if by removing these s elements the list becomes sorted. For insertion sort applied to such array, prove that the running time is bounded by $O(M \times s)$.

3. Consider the following code snippet

```
sum = 0;
for(i = 1; i < f(n); i++)
   sum += i;
```

Here $f(n)$ is a function call. Determine a big-oh running time estimate for the following cases [20=5+5+5+5]

   1. The running time of $f(n)$ is $O(n)$, and the value of $f(n)$ is $n!$

2. The running time of $f(n)$ is $O(n)$, and the value of $f(n)$ is n.
3. The running time of $f(n)$ is $O(n^2)$, and the value of $f(n)$ is n.
4. The running time of $f(n)$ is $O(1)$, and the value of $f(n)$ is 0.

4. Solve the following recurrences. [30=10+10+10]

   1. $T(n) = \sqrt{n}\,T(\sqrt{n}) + 100n$
   2. $T(n) = T(n/5) + T(4n/5) + \Theta(n)$
   3. $T(n) = T(n-2) + \log n$

5. Consider an array $A[1 \ldots n]$ which consists of increasing sequence followed by a decreasing sequence, i.e., there is an integer $m \in \{1 \ldots n\}$ such that

   - $A[i] < A[i+1]$ for all $1 \leq i < m$
   - $A[i] > A[i+1]$ for all $m \leq i < n$

   Now answer the following: [30=10+10+10]

   1. Show a divide and conquer algorithm to compute maximum element of array A.
   2. Find the recurrence relation for the running time $T(n)$, and find the solution of this recurrence.
   3. Find a loop invariant, and use that to prove the correctness of your algorithm.