INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

MATHS II

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Problem Set - I

Q 1. Prove the Schwarz inequality:

$$|A.B| \leq \|A\|\|B\|$$

where A and B are two vectors in $\mathbb{R}^n$.

Q 2. Prove the triangle inequality:

$$\|A+B\| \leq \|A\| + \|B\|$$

where A and B are vectors.

Q 3. Prove that:

$$\varepsilon_{ijk}\varepsilon_{lmn} = \det \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

Further prove that, when $n = k$

$$\varepsilon_{ijk}\varepsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

where ε and δ are Levi-Civita and Kroenecker-delta respectively with appropriate dimensions.

Q 4. Find the projection of A along B for the cases:

(a) $A = (-1,-2,3), B = (-1,3,-4)$

(b) $B = (\pi,3,-1), B = (2\pi,-3,7)$

Q 5. Let $A_1,\ldots,A_r$ be non zero vectors which are mutually perpendicular. Let $c_1,\ldots,c_r$ be numbers such that

$$c_1A_1 + \ldots c_rA_r = O$$

Show that all $c_i = 0$.

Q 6. Let A, B, C be 3 non zero vectors. If $A.B = A.C$, show by a counter example that we do not necessarily have $B = C$

Q 7. Let A be the matrix

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Find A^2, A^3. Generalize to 4x4 matrices.

Q 8. (a) Find a 2x2 matrix A such that $A^2 = -I = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(b) Determine all 2x2 matrices such that $A^2 = O$.

Q 9. Let $R(\theta)$ be tha matrix given by

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

a) Show for any 2 numbers θ_1 and θ_2 we have

$$R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$$

b) Show that $R(\theta)$ has inverse and compute it's inverse.

c) Show that the square of $R(\theta)$ is given by

$$\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

Q 10. Prove that 2-D rotation matrices preserves dot products.

Q 11. Are the following collections of vectors in $\mathbb{R}^3$ linearly independent? Why or why not?

a) $\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \}$

b) $\{ \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} \}$

c) $\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 17 \\ 0 \\ 0 \end{pmatrix} \}$