20161170
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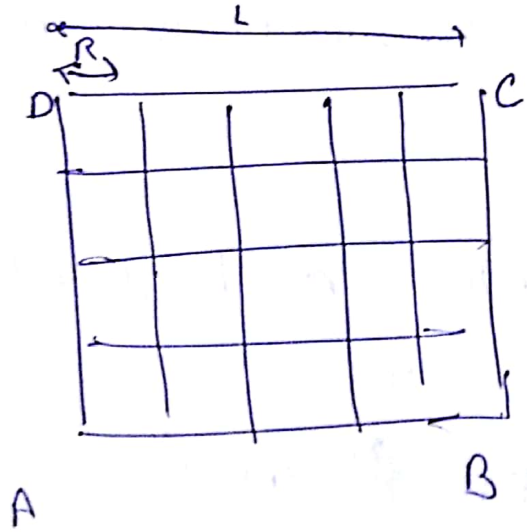
Science - I

Project-I

PART - I (Grid Problem)

Given: A lattice of length L a1
R is a grid size
N is the total no. of cells.

$N = L^2/R^2$



* Cat and Rat are on opposite sides.

Let N be the total number of steps.
Let $l = L/R$

Assumptions :
a) To simply the model, whenever both cat/rat are at the boundary/corner, they have a provision to wait/not move corresponding to the wall
b) So i) probability of wait a boundary = $1/4$
ii) probability of wait at corner = $1/2$
(c) $N > \frac{L}{R}\sqrt{2}$. The number of steps should be greater than minimum required to meet.

Let us assume the cat and rat to be 2 molecules, which are doing a random walk.

Hence the probaliby of being at n_1, n_2 at the 2D-Space. out of N steps can be.

if it takes x steps in x-dir : then it takey $N-x$ steps in y direction.

$P(n_1)$ or Prohaliby of being at n_1 after x steps.

$$\cancel{= \frac{x!}{n_1!(x-n_1}}$$

x_1 = steps taken in right, x_2 to steps taken in left.

$x_1 - x_2 = n_1$; $x_1 + x_2 = N$

$$P(n_1) = \frac{x!}{x_1!\, x_2!} (p_{right})^{x_1} (p_{left})^{x_2}.$$

similarly for y-axis.

y_R = steps up and y_2 = steps down.

$y_1 + y_2 = N-x$; $y_1 - y_2 = n_2$

$$P(n_2) = \frac{(N-x)!}{y_1!\, y_2!} (P_{up})^{y_1} (P_{down})^{y_2}$$

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As the movement is mutually exclusive

$$P(n_1, n_2) = P(n_1) \cap P(n_2) = P(n_1) \times P(n_2)$$

$$P(n_1, n_2) = \frac{x!}{(x_1!)(x_2)} \times \left(\frac{1}{2}\right)^n \times \frac{(N-x_1)}{(y_1!)!(y_2)!} \times \left(\frac{1}{2}\right)^n$$

Actual probability would be

$$P = \sum_{x=n_1}^{n_1-n_2} P(n_1, n_2) \Rightarrow \$$$

when cat and Rat meet

$$P(\text{meet}) = P_{rat}(n_1, n_2) \times P_{cat}(n_1, n_2)$$

$$P_{cat}(n_1, n_2) = P_{rat}(L-n_1, L-n_2)$$

$x_{rat} - x_{cat} = L-n_1$

$x_1\ cat + x_2\ cat = \cancel{X}$

$L = L/R$ ☆☆

$$P(\text{meet}) = P_{rat}(n_1, n_2) \times P_{rat}(L-n_1, L-n_2)$$

$$P(\text{meet, at all points}) = \sum_{n_1=0}^{L} \sum_{n_2=0}^{L} P_{rat}(n_1, n_2) \times P_{rat}(L-n_1, L-n_2)$$

for constant x

$$\Rightarrow \sum_{x_1=2}^{N} \sum \sum \left(\frac{1}{2}\right)^{4N} \frac{x!_{rat}}{(x_1!)_{rat}(x_2)_{rat}!} \times \frac{(N-x)!^{rat}}{(y_1^{rat}!)(y_2^{rat})!} \times \frac{x_{cat}!}{x_1^{cat}!\, x_2^{cat}!} \times \frac{(N-x)^{cat}!}{(y_1^{cat}!\, y_2^{cat}!)}$$

$$\Rightarrow \cancel{\sum \sum \left(\frac{1}{4}\right)^{4N}}$$

Hence $$P(\text{meet}) = \sum_{x}^{N} \sum_{n_1=0}^{L} \sum_{n_2=0}^{L} \left(\frac{1}{2}\right)^{4N} \frac{x_{rat}!\,(N-x_{rat}) \times x_{cat}! \times (N-x_{cat})!}{(x_1^{rat})!\,(x_2^{rat})!\,(y_1^{rat})!\,(y_2^{rat})!\,x_1^{cat}!\,(x_2^{cat})! \times y_1^{cat}!\,(y_2^{cat})!}$$

Summation for x_{rat}, x_{cat}

where $\Rightarrow$ ~~$n_1+n_2=N$~~

$x_{1\,rat} + x_{2\,rat} = \cancel{X}_{rat}$ $\quad x_1\,cat + x_2\,cat = \cancel{L}\ x_{cat}$

$x_{1\,rat} - x_{2\,rat} = n_1 \quad x_{1\,cat} - x_{2\,cat} = L - n_1$

$y_{1\,rat} + y_2^{rat} = N - x_{rat} \quad y_{1\,cat} + y_{2\,cat} = N - x_{cat}$

$y_{1\,rat} - y_{2\,rat} = n_2 \quad y_{1\,cat} - y_{2\,cat} = L - n_2$

By substituting everything in terms of n_1, n_2 and N followed by calculating the summation.

We would get the value of $P(meet)$.

$$P(meet) = \cancel{\sum\sum\sum \left(\frac{1}{4}\right)^{4N} (n_1+n_2)!\ (N}$$

$$\Rightarrow \cancel{\sum_{x_{cat}}\sum_{x_{rat}}\sum_{n_1}\sum_{n_2}} \frac{x_{rat}!\ \cancel{(N-x_{rat})!}}{\left(\frac{n_1+x_{rat}}{2}\right)!\ \cancel{\left(\frac{x_c-n_{rat}}{2}\right)!}} \times \frac{1}{\ }$$

$$\Rightarrow \sum\sum\sum\sum \left(\frac{1}{2}\right)^{4N} \frac{(x_{rat}!)\ (N-x_{rat})!}{\left(\frac{n_1+x}{2}\right)!\ \left(\frac{x-n_1}{2}\right)!\ \left(\frac{n_2+N-x}{2}\right)!\ \left(\frac{N-x-n_2}{2}\right)!}$$

$$\times \frac{(x_{cat}!)\ (N-x_{cat})!}{\left(\frac{L-n_1+x_{cat}}{2}\right)!\ \left(\frac{x_{cat}+n_1-L}{2}\right)!\ \left(\frac{L-n_2+N-x_{cat}}{2}\right)!\ \left(\frac{N-x_{cat}-L+n_2}{2}\right)!}$$

We can calculate this using a PC and Hence

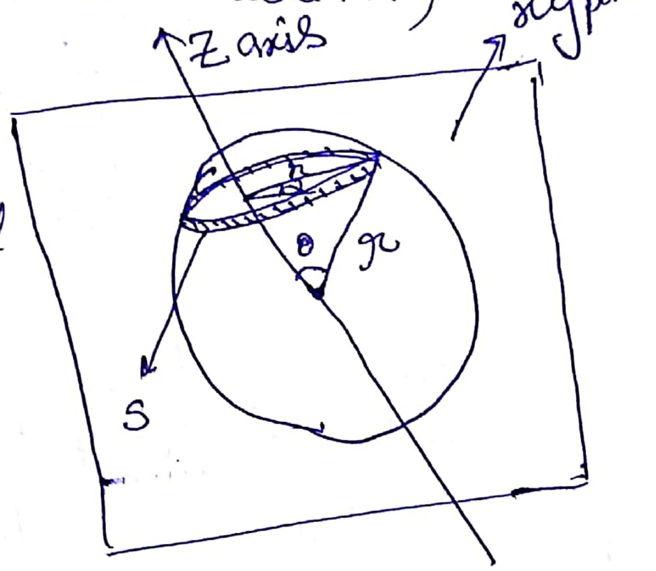
$P(\text{Survival of Rat}) = 1 - P(meet)$

$\Rightarrow$

Hence Probability of meet depends of N and L

if $N < L\sqrt{2}$ the particles will never meet.

PART-2 (Sphere Problem)



→ Z-axis is the Line cutting the sphere and emerging out of the sphere at another point.

→ Also, there is a plane xy, that has ~~Z~~ axis on it, which cuts the sphere into 2 hemisphere

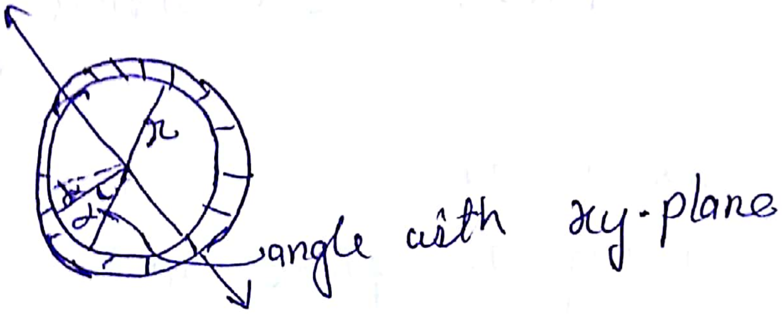
→ Blind cat and a blind rat are present on the sphere. We show their position using

i) θ : angle with the Z-axis

ii) α : angle with the line of intersection of plane with the plane.

Because : a set of points can have angle θ with Z-axis, as in the above example, all the points are on the circumference S.

The typical circles cut by the xy plane of the sphere will look like



Probability of being at a point which make angle α with the xy plane.

$$P(\alpha) = \frac{r\,d\alpha}{2\pi r} = \frac{d\alpha}{2\pi}$$

Probability of being at a point which makes an angle θ with the z axis

$\Rightarrow \dfrac{\text{Area occupied by the circular strip}}{\text{total surface area of the sphere.}}$

$$\Rightarrow \frac{(r\,d\theta)(2\pi r\sin\theta)}{4\pi r^2} = \frac{\sin\theta . d\theta}{2}$$

Hence the probability of being at a point is.

$$P(\theta,\alpha) = \frac{\sin\theta\, d\theta\, d\alpha}{4\pi}$$

Anology:

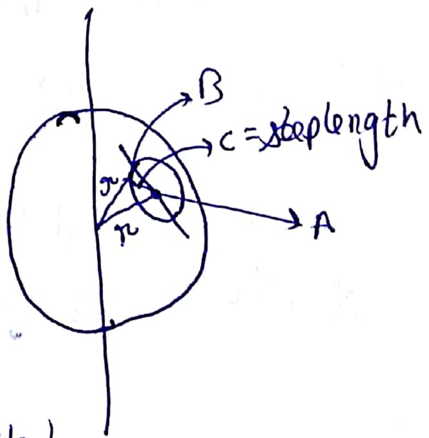
Displacement.

A: original position

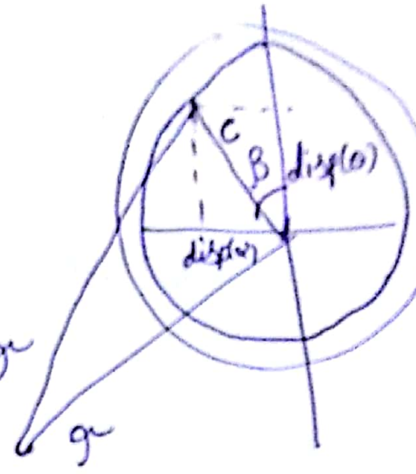
B: new position

C = step length

angle of displacement = $\dfrac{\text{arc length (step length)}}{\text{radius.}}$



$$\text{disp}'\theta' = \frac{\text{dis}\,\theta}{r}$$



$$\theta_{new} = \theta - \frac{c\cos B}{r}$$

$$\text{dis}'\alpha' = \frac{\text{disp}}{r\sin\theta}$$

$$\alpha_{new} = \alpha + \frac{c\sin B}{r\sin\theta}$$

Assumption:

i) Basically assuming that B will be small enough so that the helix cat and rat travel individually happen to cover the total surface of the sphere.

Probability Density Function, $\omega(\theta,\alpha)\, d\theta d\alpha = ?$

$$\text{Probability of a point} = \frac{\sin\theta\, d\theta d\alpha}{4\pi}$$

$$d\alpha \Rightarrow \frac{c\sin B}{r\sin\theta} \qquad d\theta = \frac{c\cos B}{r}$$

$$\omega(\theta,\alpha)\, d\theta d\alpha = \frac{\sin\theta}{4\pi} \times \frac{c\sin B\, c\cos B\, d\theta d\alpha}{r\sin\theta\, r}$$

$$= \frac{c^2}{4\pi r^2} \sin B\cos B\, d\theta d\alpha$$

$$\text{Probability of meet} = \int_\theta \int_\alpha \omega(\theta,\alpha)^2\, d\theta d\alpha$$

$$\Rightarrow \int_{r=0}^{2\pi} \int_{\theta=0}^{2\pi} \frac{c^4}{16\pi^2 r^4} \sin^2 B \cos^2 B \, d\theta \, dr$$

$$\Rightarrow \frac{c^4}{16\pi^2 r^2} \sin^2 B \cos^2 B \iint_{r=0,\ \theta=0}^{2\pi,\ 2\pi} d\theta \, dr \cdot$$

$$\Rightarrow \frac{c^4 \sin^2 B \cos^2 B}{8 r^4}$$

Hence Probability of meet $\Rightarrow$ $P(\text{meet}) = \frac{c^4 \cos^2 B \sin^2 B}{r^4}$

where B is the angle of displacement.

Probability of Survival of the rab $\Rightarrow$

$1 - P(\text{meet}) \Rightarrow 1 - \frac{c^4 \cos^2 B \sin^2 B}{r^4}$

Conclusion: Answer to Questions.

a) Survival/meeting is going to vary with time according to the above equation.

Since the probability of survival decreases with increase in time i.e.

probability of survival $\propto \frac{1}{(\text{time})^{c}}$; c is a constant.

Because at large time, more number of steps have been taken.

b) i) it depends of L of the square. Hence as L => length of the grid increases; chance of survival increases.

ii) ~~It~~ depends on the radius of the sphere.

meeting $\propto \frac{1}{r^4}$. Hence r increases, chance of survival increases.

c) i) ~~If~~ the number of steps is too large; probability for each cat and rat to be at a point = $1/L^2$

Hence probability of meet = $1/L^4$

Probability of survival = $1 - 1/L^4$

ii) Similarly for sphere portion. If a step is ~~in~~ very small probability of being at a point => $1/4\pi r^2$

Survival Probability = $1 - \left(\frac{1}{4\pi r^2}\right)^2 = 1 - \frac{1}{16\pi^2 r^4}$