Linear Algebra

Assignment - 1

1. Show that if every element of the group G is its own inverse, then G is abelian.

2. Prove that if G is an abelian group, then for all $a, b \in G$ and all integers n, $(a \,.\, b)^n = a^n . \, b^n$

3. If the order of a finite group is divisible by a prime p, then show that the group must have an element of order p.

4. Prove that every group with 5 or less elements is abelian.

5. If G has no proper subgroups, then prove that G is cyclic of order p, where p is a prime number.

6. If G is a group in which $(ab)^i = a^i b^i$ for 3 consecutive integers i for all $a,b \in G$, show that G is abelian.

7. If N is a normal subgroup of G and H is any subgroup of G, prove that NH is a subgroup of G.

8. If M, N are such that $x^{-1}Mx = M$ and $x^{-1}Nx = N$ for all x $\in$ G, and if $M \cap N = \{e\}$, prove that $mn = nm$ for any $m \in M$, $n \in N$.