

$$\alpha_k = \frac{\langle v_1, v_k \rangle}{\langle v_k, v_k \rangle}$$

EX: $v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$v_1 = \begin{bmatrix} 1/3\sqrt{2} \\ 1/3\sqrt{2} \\ -4/3\sqrt{2} \end{bmatrix} \quad v_2 = \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$$

$\langle v_1, v_2, v_3 \rangle$
 \downarrow
 orthogonal

$$\alpha_1 = \frac{\langle \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1/3\sqrt{2} \\ 1/3\sqrt{2} \\ -4/3\sqrt{2} \end{bmatrix} \rangle}{1} = -2/3\sqrt{2}, \quad \alpha_2 = 5/3, \quad \alpha_3 = 0$$

$$v = -2/3\sqrt{2} v_1 + 5/3 v_2 + 0 \cdot v_3$$

DISCRETE FOURIER TRANSFORM →

$$v_k[n] = e^{j \frac{2\pi k n}{N}}, \quad n \in [0, \dots, N-1]$$

$$v_0 = [1 \quad 1 \quad 1 \quad \dots \quad 1]$$

$$v_1 = [1 \quad e^{j \frac{2\pi}{N}} \quad e^{j \frac{4\pi}{N}} \quad \dots \quad e^{j \frac{2\pi(N-1)}{N}}]$$

$$\begin{aligned} \langle v_k, v_l \rangle &= \sum_{n=0}^{N-1} v_k(n) \cdot \bar{v}_l(n) \\ &= \sum_{n=0}^{N-1} e^{j \frac{2\pi k n}{N}} \cdot e^{-j \frac{2\pi l n}{N}} \\ &= \sum_{n=0}^{N-1} e^{j \frac{2\pi n}{N} (k-l)} \end{aligned}$$

$$\Rightarrow \langle v_k, v_l \rangle = \sum_{n=0}^{N-1} e^{j \frac{2\pi n}{N} (k-l)}$$

① $K=l$;

$$\langle v_K, v_l \rangle = \sum_{n=0}^{N-1} 1 = N$$

② $K \neq l$;

$$= e^{j2\pi 0 \frac{(K-l)}{N}} + e^{j2\pi 1 \frac{(K-l)}{N}} + \dots + e^{j2\pi (N-1) \frac{(K-l)}{N}}$$

$$= \frac{1 - \left(e^{j2\pi \frac{(K-l)}{N}} \right)^N}{1 - e^{j2\pi \frac{(K-l)}{N}}} = 0$$

③

$$x = \sum_{K=0}^{N-1} X_K v_K$$

$$X_K = \frac{\langle x, v_K \rangle}{\langle v_K, v_K \rangle}$$

$$x = X_0 v_0 + X_1 v_1 + X_2 v_2 + \dots + X_{N-1} v_{N-1}$$

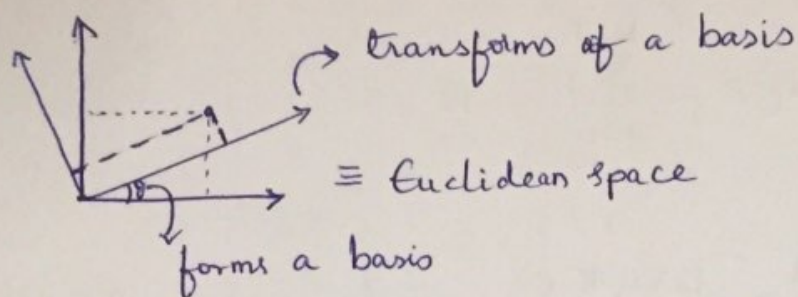
$$x[n] = \sum_{K=0}^{N-1} X_K \cdot e^{j2\pi \frac{Kn}{N}}$$

$$X_K = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi \frac{Kn}{N}}$$

$$\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix} = \frac{1}{N} \begin{bmatrix} z^0 & z^0 & z^0 & z^0 & \dots & z^0 \\ z^0 & z^1 & z^2 & & & z^{N-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ z^0 & z^{N-1} & z^{2(N-1)} & \dots & z^{(N-1)(N-1)} & z^0 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}$$

$$e^{-j2\pi \frac{Kn}{N}} = z^{Kn} \Rightarrow z = e^{-j2\pi \frac{1}{N}}$$

$$\Rightarrow \boxed{z = e^{-j2\pi \frac{1}{N}}}$$



$$X_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-j \frac{2\pi kn}{N}} \quad \left(\text{Discrete Fourier Transform} \right)$$

$$x(n) = \sum_{k=0}^{N-1} X_k \cdot e^{j \frac{2\pi kn}{N}}$$

Ex: Let the signal be \equiv $(N=4)$

$$x(t) = 5 + 2 \cos\left(2\pi t - \frac{\pi}{2}\right) + 3 \cos \frac{4\pi t}{2\pi(2)t}$$

$$f_{\text{max}} = 2 \text{ Hz}$$

$$\text{Sampling frequency} = 4 \text{ Hz} \quad , \quad t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$$

$$x[0] = 5 + 2 \cos\left(-\frac{\pi}{2}\right) + 3 \cos 0 = 8$$

$$x[1] = 5 + 2 \cos\left(\frac{\pi}{2} - \frac{\pi}{2}\right) + 3 \cos \pi = 4$$

$$x[2] = 5 + 2 \cos \pi + 3 \cos 2\pi = 8$$

$$x[3] = 5 + 2 \cos \pi + 3 \cos 3\pi = 0$$

$$x[n] = \{8, 4, 8, 0\}$$

$$\frac{1}{4} \begin{bmatrix} 20 \\ -4j \\ 12 \\ 4j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 8 \\ 4 \\ 8 \\ 0 \end{bmatrix} \quad \begin{matrix} k=0 \\ k=1 \\ k=2 \end{matrix}$$

↓
roots of unity $= e^{j \frac{2\pi}{4}} = 1$

Ans $\equiv [5 -j \quad 3 \quad j]$ $\boxed{X[K] = X^*[N-K]}$

sinusoid of 1Hz
amplitude $\equiv 2$

sinusoid of 2Hz where
amplitude is 3.

since it
is sampled
at 4Hz,
Max
freq $\} = 2\text{Hz}$

since it is complex; there may be a phase shift
in the 1Hz signal.

$$[-\pi/2]$$

First Value \equiv Mean of the signal.
[No frequency term]
DC Value.

Magnitude of 1Hz frequency \equiv distributed between the two values
[-j, j]

> length of the signal, {8, 4, 8, 0} $\equiv 4$.

> 2Hz value \Rightarrow It is real; No phase shift
Amplitude = 3.
Frequency = 2Hz. } By convention
It is \cos^4

frequency vs time (just like Heisenberg's Uncertainty Principle).

If we take more samples \equiv frequency shifts accordingly

CONTINUOUS FOURIER TRANSFORM [FT]

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} d\omega$$

① $\delta(t)$.

$$X(f) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j2\pi ft} dt ; \text{ at } t=0; 1$$

$$X(f) = 1$$

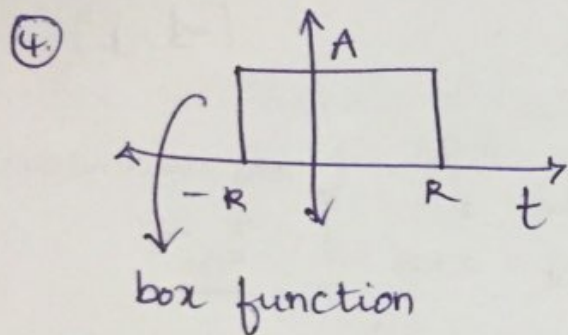
② $\delta(t-t_0)$

$$X(f) = \int_{-\infty}^{\infty} \delta(t-t_0) \cdot e^{-j2\pi ft} dt, \text{ at } t=t_0, \delta(t-t_0) \rightarrow 1$$

$$= e^{-j2\pi ft_0} = e^{-j\omega t_0}$$

③ $\delta(t-t_0) + \delta(t+t_0)$ (a symmetric impulse responses)

$$\hookrightarrow e^{-j\omega t_0} + e^{j\omega t_0} = 2\cos\omega t_0$$



"can be reduced from $(-\infty, \infty)$ to $(-R, R)$ ".

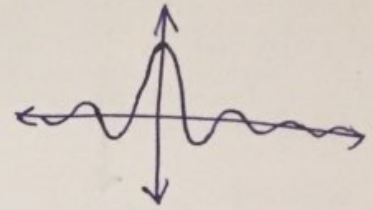
$$\begin{aligned} &\equiv \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt \\ &\equiv \int_{-R}^R A \cdot e^{-j\omega t} dt \\ &\equiv A \int_{-R}^R e^{-j\omega t} dt \\ &\equiv \frac{A}{-j\omega} \left[e^{-j\omega t} \right]_{-R}^R \\ &\equiv \frac{A}{-j\omega} \left[e^{-j\omega R} - e^{j\omega R} \right] \\ &\equiv \frac{A}{j\omega} \left[e^{j\omega R} - e^{-j\omega R} \right] \\ &\equiv \frac{A}{j\omega} \left[2j\sin\omega R \right] \equiv \frac{2A}{\omega} \sin\omega R \\ &\equiv 2AR \cdot \left(\frac{\sin\omega R}{\omega R} \right) \end{aligned}$$

$$\equiv 2AR \left(\underbrace{\frac{\sin \omega R}{\omega R}} \right)$$

$$\equiv 2AR \operatorname{sinc}(\omega R)$$

light - periodic waves.

$$\operatorname{sinc}(a) = \frac{\sin a}{a}$$



dft \rightarrow takes a lot of time to do the large matrix multiplication
 \downarrow
 FFT
 very fast

FAST FOURIER TRANSFORM \rightarrow

$$X_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-j \frac{2\pi K n}{N}}$$

$$(1) \quad K = \frac{N}{2} K_1 + K_0 \quad ; \quad K_1 = 0 \text{ (or) } 1$$

$$0 \leq K_0 \leq \frac{N}{2} - 1$$

$K_1 = 0$ \rightarrow first half of the signal
 $K_1 = 1$ \rightarrow next half of the signal
 \uparrow
 K

$$X_K = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-2\pi j \left(\frac{N}{2} K_1 + K_0 \right) \frac{n}{N}}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-\frac{2\pi j N K_1 n}{2N}} \cdot e^{-2\pi j \frac{K_0 n}{N}}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-\pi j K_1 n} \cdot e^{-2\pi j \frac{K_0 n}{N}}$$

$$\therefore e^{-\pi j} = -1$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot (-1)^{K_1 n} \cdot e^{-2\pi j \frac{K_0 n}{N}}$$

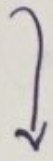


divide into even & odd indices

$$X_K = \frac{1}{N} \sum_{n=0}^{\frac{N}{2}-1} x[2n] \cdot e^{-\frac{2\pi j K_0 (2n)}{N}} \cdot (-1)^{K_1 (2n)} + \sum_{n=0}^{\frac{N}{2}-1} x[2n+1] \cdot e^{-\frac{2\pi j K_0 (2n+1)}{N}} \cdot (-1)^{K_1 (2n+1)}$$

$$X_K = \frac{1}{N} \sum_{n=0}^{\frac{N}{2}-1} x[2n] \cdot e^{-\frac{2\pi j K_0 n}{N/2}} + \sum_{n=0}^{\frac{N}{2}-1} x[2n+1] \cdot e^{-\frac{2\pi j K_0 (2n+1)}{N}} \cdot (-1)^{K_1}$$

$$X_K = \frac{1}{N} \sum_{n=0}^{\frac{N}{2}-1} x(2n) \cdot e^{-\frac{2\pi j K_0 n}{N/2}} + (-1)^{K_1} e^{-\frac{2\pi j K_0}{N}} \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) \cdot e^{-\frac{2\pi j K_0 n}{N/2}}$$

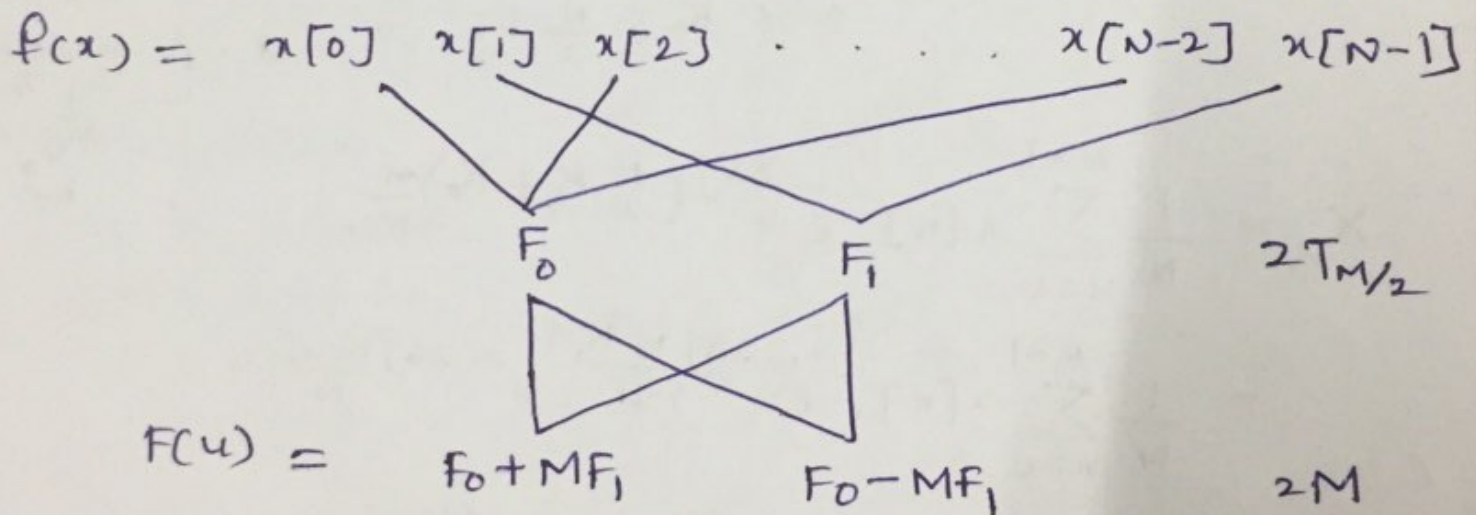


computing just the
fourier
transforms of half signals.

$$X_K = F_0 + (-1)^{K_1} Q F_1$$

$$X_{K_1=0} = F_0 + Q F_1$$

$$X_{K_1=1} = F_0 - Q F_1$$



Complexity $\equiv T(N) = 2 T(N/2) + 2N$

$O[N \log N]$

FFT Computation times

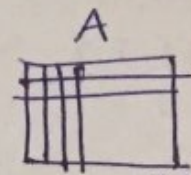
2D FOURIER TRANSFORM →

physical intuition?

$$X = W_N x^T \quad (1D \text{ case})$$

$$\hat{A} = W_{M \times M} (A_{M \times N} W_{N \times N})$$

if it rotates, it changes
but on translating, it doesn't change.



row wise ← Fourier transform

& then the output is applied Fourier transform column wise.

CONVOLUTION THEOREM →

$$y[n] = \sum_{k=0}^{N-1} x[k] \cdot h[n-k]$$

$$x[n] = [1 \ 2 \ 0 \ 1] \quad h[n] = [2 \ 2 \ 1 \ 1]$$

$$\begin{array}{cccccccccc} \downarrow & & & & & & & & & \\ 1 & 2 & 0 & 1 & = & 0 & 2 & 6 & 5 & 4 & 1 & 1 & 0 & * \\ & & & \uparrow & & & & & & & \uparrow & & & \\ & & & & & & & & & & \text{Convolution} & & & \end{array}$$

Circular Convolution: $y[n] = \sum_{k=0}^{N-1} x[k] \cdot h[(n-k) \bmod N]$

? what is happening

$$y[0] = x[0] \cdot h[0] + x[1] \cdot h[3] + x[2] \cdot h[2] + x[3] \cdot h[1] = 6$$

$$y[1] = x[0] \cdot h[1] + x[1] \cdot h[0] + x[2] \cdot h[3] + x[3] \cdot h[2] = 7$$

$$\begin{array}{cccc} 1 & 2 & 0 & 1 \\ \hline 2 & 1 & 1 & 2 \\ \hline 2 & 2 & 1 & 1 \\ \hline 1 & 2 & 2 & 1 \\ \hline 1 & 1 & 2 & 2 \end{array}$$

$$\begin{array}{c} \star \\ \uparrow \\ \text{Circular Convolution} \end{array} = 6 \ 7 \ 6 \ 5$$

2 2 1 1
↑

... 1 2 0 1 1 2 0 1 1 2 0 1 ...
1 1 2 2

↪ to result in the size of "output".

$$\begin{aligned} & 1 \ 2 \ 0 \ 1 \ 0 \ 0 \ 0 \\ 2 \ (\Rightarrow & 2 \ 0 \ 0 \ 0 \ 1 \ 2 \\ 6 \ (\Rightarrow & 2 \ 2 \ 0 \ 0 \ 0 \ 1 \ 1 \\ & 1 \ 0 \ 2 \ 0 \ 0 \ 0 \ 1 \end{aligned}$$

$$\begin{aligned} & \mathcal{L}[n] \\ & \begin{bmatrix} 2 & 2 & 1 & 1 & 0 & 0 & 0 \\ & 1 & 2 & 2 & 1 & 5 & 0 \end{bmatrix} \\ & \begin{matrix} 0 & 6 & 5 & 4 & 3 & 2 & 1 \end{matrix} \\ & \swarrow \text{circular convolution} \searrow \end{aligned}$$

$\rightarrow y[n] = x[n] \otimes h[n]$ (Discrete Domain)
[\therefore Circular Convolution]
 $y[n] = F^{-1}(F(x[n]) \cdot F(h[n])) \Rightarrow$ DFT is used.

$\vec{F_T}$ is used

$$n - k = m$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} x(k) \cdot h(m) dk \right) e^{-j\omega(k+m)} dm$$

$$u = k + m \\ du = dm$$

$$= \int_{-\infty}^{\infty} h(m) \cdot e^{-j\omega m} dm \cdot \int_{-\infty}^{\infty} x(k) \cdot e^{-j\omega k} dk$$

$$= FT(x) \cdot FT(h)$$

Discrete Domain:

$$y(n) = \sum_{r=0}^{N-1} x(r) \cdot h(n-r) \quad (\text{convolution formula})$$

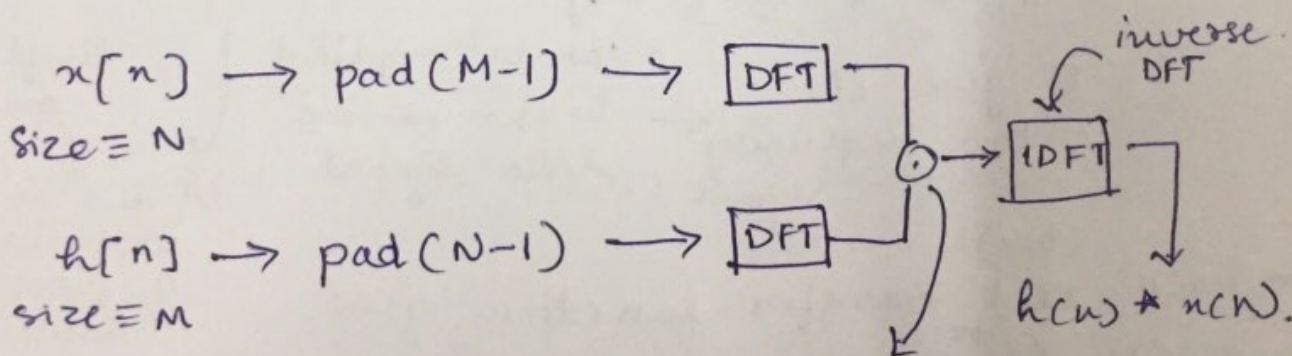
$$Y_K = \frac{1}{N} \sum_{n=0}^{N-1} y(n) \cdot e^{-j \frac{2\pi K n}{N}}$$

$$n-r=m$$

$$= \sum_{n=0}^{N-1} \left(\sum_{r=0}^{N-1} x(r) \cdot h(n-r) \right) \cdot e^{-j \frac{2\pi K n}{N}}$$

$$= \sum_{n=0}^{N-1} \left(\sum_{r=0}^{N-1} x(r) \cdot h(m) \right) e^{-j \frac{2\pi K (m+r)}{N}}$$

$$= \sum_{n=0}^{N-1} h(m) \cdot e^{-j \frac{2\pi K m}{N}} \sum_{r=0}^{N-1} x(r) \cdot e^{-j \frac{2\pi K r}{N}}$$

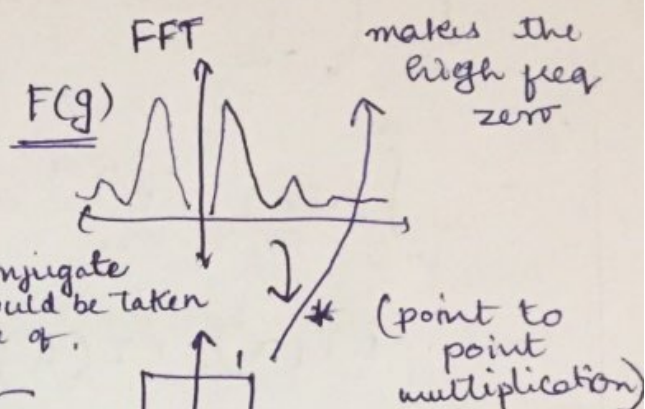
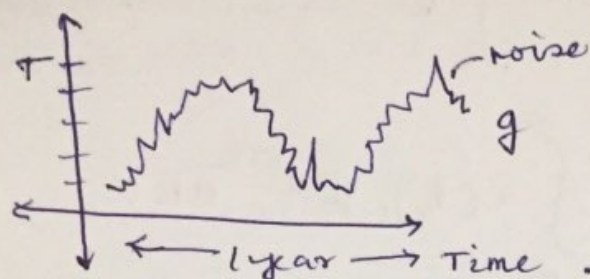


(Circulant matrix)

operation must have same size output signals.

Transfer Function?

Real Life Examples: - Turbidity of lake (data)



Fourier transform of that is similar to convolution with sinc

low pass filtering transfer function.

box function

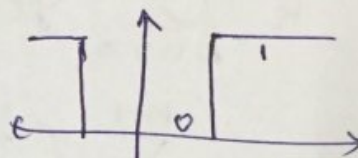
Conjugate should be taken care of.

(point to point multiplication)

- Band pass filtering

fixed band width size.

high pass filtering



(removing the lower frequencies).

inverse Fourier
you'll get the ~~noise~~ filtered signal.

Moving Average Filter

$$n(n) = \frac{1}{N} \sum_{k=0}^{N-1} n[n-k]$$

Ex: $\left[\frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{5} \right]$

to give the low frequency signal

can be applied to the above data signal

Shift occurs due to the contribution of previous values.

Since rect transfer function gives ripples (like convoluting with sinc function) \Rightarrow we can do multiply with an "exponentially decaying function" \Rightarrow Gaussian function

pepper noise = max filter
 salt noise = min filter
 salt and pepper noise = median filter

Leaky Integrator -

moving average = $y_M[n] = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$ ↗ M previous values

$$\rightarrow y_M[n-1] = \frac{1}{M} \sum_{k=0}^{M-1} x(n-1-k)$$

$$\star y_M[n-1] = \frac{1}{M} \sum_{k=1}^M x(n-k) \quad \leftarrow \begin{matrix} K+1=k \end{matrix}$$

$$\rightarrow y_{M-1}[n] = \frac{1}{M-1} \sum_{k=0}^{M-2} x(n-k)$$

$$\begin{aligned} \rightarrow y_{M-1}[n-1] &= \frac{1}{M-1} \sum_{k=0}^{M-2} x(n-1-k) \\ &= \frac{1}{M-1} \sum_{k=1}^{M-1} x(n-k) \end{aligned}$$

$$\star y_{M-1}[n-1] = \frac{1}{M-1} \sum_{k=1}^{M-1} x[n-k]$$

$$\sum_{k=0}^{M-1} x(n-k) = x(n) + \sum_{k=1}^{M-1} x(n-k)$$

from above;

$$M y_M[n] = x[n] + (M-1) y_{M-1}[n-1]$$

$$y_M[n] = \frac{x[n]}{M} + \frac{M-1}{M} y_{M-1}[n-1]$$

$$\text{let } \frac{M-1}{M} = \lambda$$

$$1 - \frac{1}{M} = \lambda \Rightarrow \frac{1}{M} = 1 - \lambda$$

$$y_M[n] = (1 - \lambda) x[n] + \lambda y_{M-1}[n-1]$$

Suppose M is very large, $\lambda \approx 1$ & that results in
(window is large)

$y_M[n-1] = y_{M-1}[n-1]$
mean of M values is equal to mean
of $M-1$ values. (just the position)

Leaky Integrator \rightarrow

$$y[n] = (1 - \lambda) x[n] + \lambda y_M[n-1]$$

* current value * weight + previous (mean) value = mean.

Though you're integrating the values, it gives
slight weightage to the current value (there's a leak).

used for Analog signals.

Ex: $y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$

Impulse Response $\rightarrow h[n] = \frac{1}{M} \sum_{k=0}^{M-1} \delta(n-k)$

for $n=0$; $k=0 \Rightarrow \frac{1}{M}$
 $n=1$; $k=1 \Rightarrow \frac{1}{M}$

$$h[n] = \begin{cases} \frac{1}{M} & ; 0 \leq n < M \\ 0 & ; \text{otherwise} \end{cases}$$

FIR filter (finite values)

{ finite Impulse Response }

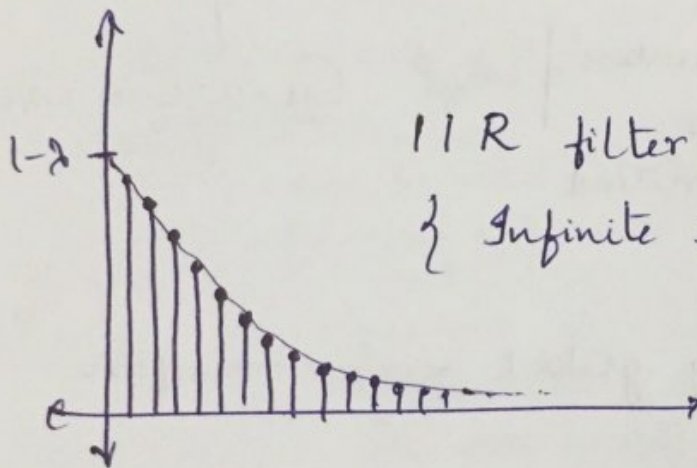
leaky integrator - impulse response?

$$h[n] = (1-\lambda) \delta(n) + \lambda h[n-1]$$

for ex: $n=0$; $h[0] = (1-\lambda) \delta(0) + \lambda h[-1] = 1-\lambda$

$$h[1] = (1-\lambda) \delta(1) + \lambda h[0] = \lambda(1-\lambda)$$

$$h[n] = \begin{cases} 1-\lambda & ; n=0 \\ \lambda(1-\lambda) & ; n=1 \\ \vdots \\ \lambda^n(1-\lambda) & ; n=K \end{cases}$$



IIR filter

{ Infinite Impulse Response }

Leaky Integrator \equiv IIR filter. CDE.

$$\sum_{k=0}^{p-1} a_k x(n-k) = \sum_{r=0}^{q-1} b_r y(n-r)$$

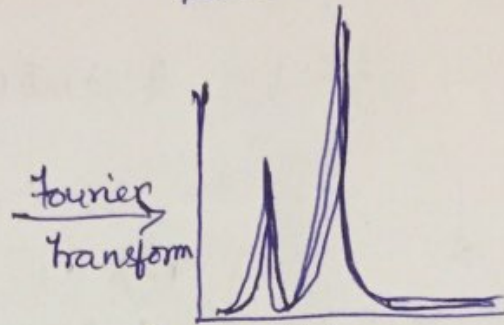
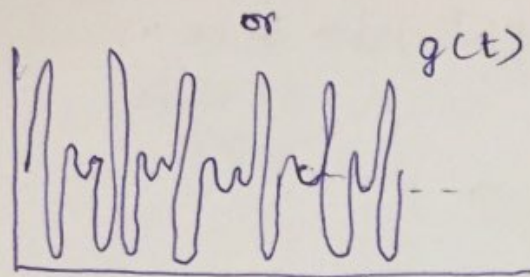
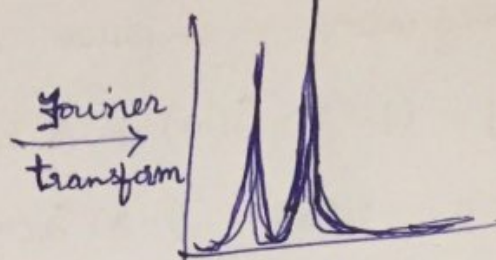
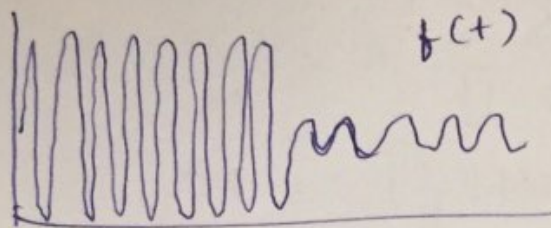
leaky integrator can be expressed like this.

$$y[n] - \lambda y[n-1] = (1-\lambda) x[n]$$

{ λ shouldn't be too high, as the current value contribution decreases. }

Short Time Fourier Transform:

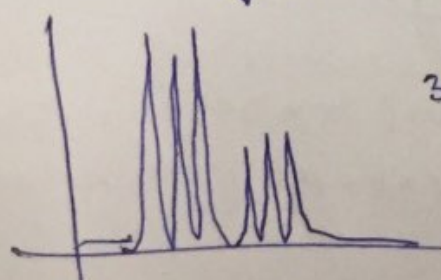
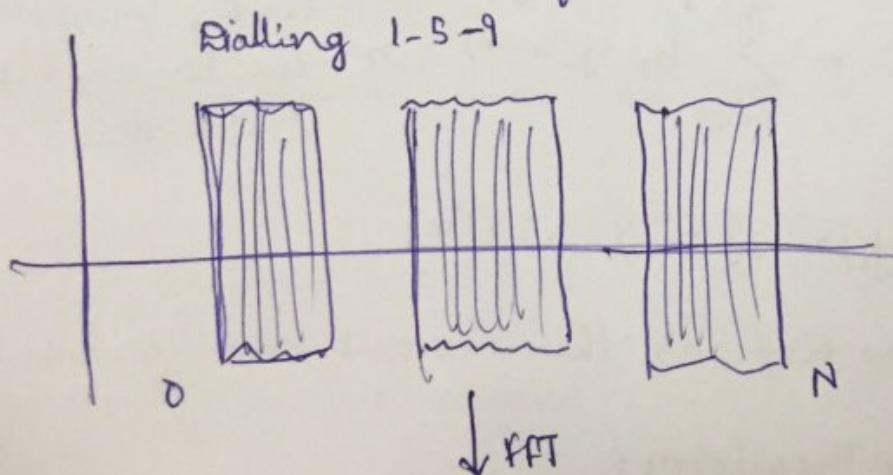
For Ex: $g(t) = \begin{cases} 2 \times \sin(2\pi \times 39t) & , 0 < t < 1/2 \\ \sin(2\pi \times 15t) & , 1/2 < t < 1 \end{cases}$



$f(t) \& g(t) =$ though are different in time domain
 \downarrow
 give same Fourier

Ideophones, on dialling numbers, why? GLOBAL IN NATURE
 different frequencies are
 (very carefully chosen) transmitted
 coprime numbers

On taking global fourier transform
 - Limitations -
 - order of dialling } can't be
 - Number of dials } figured out.



3 pairs for 3 ranges

To resolve this issue ; \rightarrow sliding window fourier transform.

but we lose on "resolution in fourier transform"

intervals - less, frequency is roughly calculated.
(samples are less)

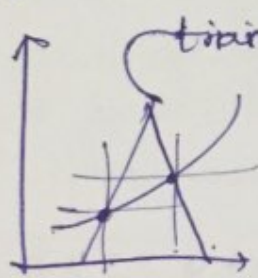
uncertainty
 $\Delta t \cdot \Delta f = 2\pi$

resolution in time & frequencies

Windowing \rightarrow break the signal into blocks "windows" in time domain.

Apply DFT to each window independently

Adjacent windows may overlap.

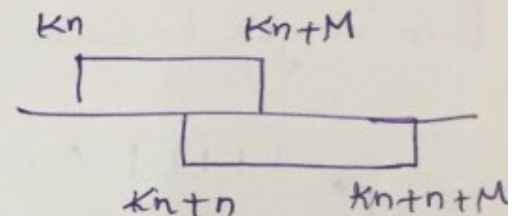


triangular window can be used

[instead of rect]

* gaussian is not advisable

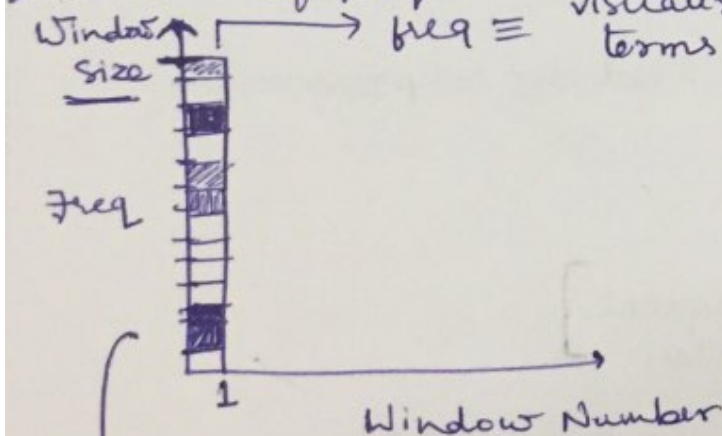
for ~~analog~~ analog



$$n = M$$

(No overlap).

resolution of freq. \rightarrow freq \equiv visualised in terms of colour | as per each window.



\rightarrow graph \equiv Spectrogram.

can be used in Music also

lighter \equiv \uparrow DFT Magnitude
darker \equiv \downarrow DFT Magnitude

Compression →

$$\begin{aligned} 1 \text{ movie (2hrs)} &= 2 \times 60 \times 60 \times 30 \times 1920 \times 1080 \times 24 = \\ &1.074 \times 10^{13} = \underline{10 \text{ TB}} \end{aligned}$$

→ temporary redundancy

→ spatial redundancy

→ perceptual redundancy

What parameters can be reduced to reduce the size?

How come you are able to store it on the computer?

frames that are not at all perceived.

Run length Encoding → RLE

1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 0 0

Encoding

(1,5) (0,8) (1,5) (0,2)

W W W A A A S S S B S S

13 x 8

[W A S B S] 5 x 8

[4 3 3 1 2] 5 x 8

W W 4 A A 3 S S 3 B S S 2

Signal with high entropy - hard to compress.

Huffman Coding -

[Idea: smaller encoding - more frequent characters]

Ex:

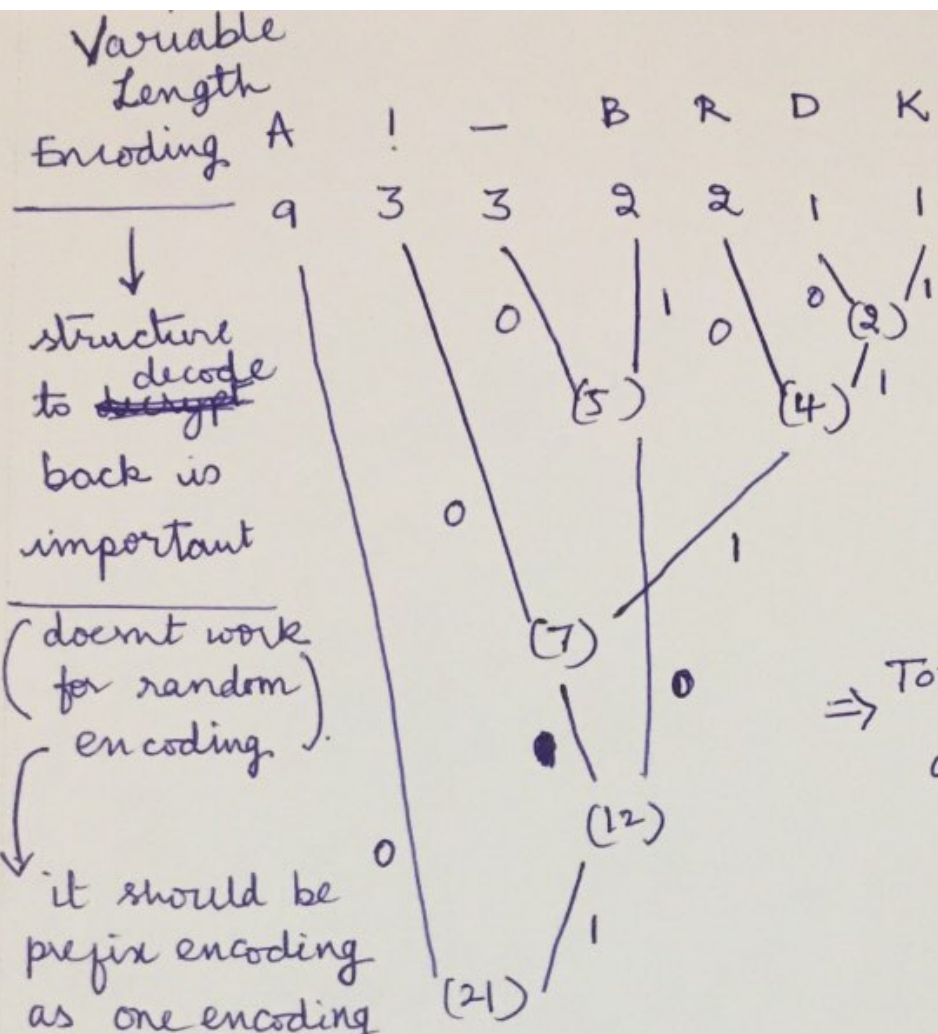
A A B R A A - K A - D A A B R A A - ! ! !

20 x 8 bit encoding

frequency: A ! - B R D K F

9 3 3 2 2 1 1

min: add them up



structure to ~~decode~~ back is important

(doesn't work for random encoding)

it should be prefix encoding as one encoding should be within the other encoding.

* prefix encoding
 * dictionary coding
 A - 0
 ! - 110
 - - 100
 B - 101
 R - 1110
 K - 11110
 D - 11111

↓
to decode it back.

⇒ Total Encoding =

$$9 \times 1 + 3 \times 3 + 3 \times 3 + 2 \times 3 + 2 \times 4 + 1 \times 5 + 1 \times 5 = 51 \text{ bits}$$

⇒ To encode, we need the dictionary

Initial = 21×8 , since there are only 7 characters we can use 3 bits only so, possible bits = 21×3 .

Compression: $\frac{21 \times 3}{51}$

view
 "frequentist coding"
 ↓
 only looking at the frequency of the characters.