Assignment-2

Q1) Let V(R) be the vector space of all functions from set R of all the real numbers into itself and let $U = \{ f \epsilon v \mid f(-x) = -f(x)\}$, $W = \{ f \epsilon v \mid f(-x) = f(x)\}$. Then prove that $V = U \oplus W$.

Q2) Show that a linear transformation f from V_n (F) to W(F) is determined by the effect of f on any basis of V_n (F).

Q3) Let V be the set of all ordered pairs (x, y) of real numbers and Let (F, +, .) be a field of real numbers. Define: $(x, y) + (x1, y1) = (3y + 3y1, -x -x1)$, $c(x, y) = (3cy, -cx)$. Verify that with these operations V is not a vector space over the field of real numbers.

Q4) Let X(F) and W(F) be subspaces of vector space V(F). Prove that $(X \cup W)(F)$ forms a subspace of V(F) iff $W \subseteq X$ or $X \subseteq W$.

Q5) Let U(F) and W(F) be subspaces of $V_n(F)$ such that dim U > n/2 and dim W > n/2. Show that $U \cap W \neq \{ 0 \}$.

Q6) A linear mapping $f \epsilon L(V,V)$ is said to be nilpotent if $f^n = 0$ for some $n \epsilon N$. If f is nipotent and if $f^{n-1}(x_0) \neq 0$, prove that $\{ x_0, f(x_0), f^2(x_0), \ldots, f^{n-1}(x_0) \}$ is linearly independent set of vectors.

Q7) Let V(F), U(F) and W(F) be vector spaces over F. Let f , f1 be linear mappings from V into U and let g, g1 be linear mappings from U to W. Then prove that

1) $g \circ (f + f1) = g \circ f + g \circ f1$
2) $(g + g1) \circ f = g \circ f + g1 \circ f$

Q8) Prove that if the mapping $f \epsilon L(V,V)$ is such that $\ker(f) = \mathrm{Ker}(f^2)$, then $V = \ker(f) \oplus f(V)$.

Q9) For any linear transformation $f : V \rightarrow W$, show that $r(f) \leq \min \{ \dim V, \dim W\}$.

Q10) Suppose the mapping $f \epsilon L(V,W)$ with dim V > dim W. Show that there exists a non zero vector $x_0 \epsilon V$ for which $f(x_0) = 0$.

Q11) If S and T are linear operators on R^2, defined by $S(x,y) = (y,x)$ and $T(x,y) = (0,x)$. Find ST, TS, S^2 and T^2.

Q12) If we regard the complex number as a vector space over the real field, is the conjugate mapping defined as $f(a + i b) = a - i b$, a linear transformation?