Formal Methods – Assignment 2

13/04/2018

1. Give CFG's for the following languages
   i The language $\{0^n 1^{3n} \mid n \geq 0\}$
   ii The language of properly nested loops of parenthesis, braces $(\{\})$ and square brackets $(\square)$
2. Give context-free grammars that generate the following languages:
   i { w | the length of w is odd}
   ii { $a^i b^j c^k$ | i, j, k ≥ 0, and i = j or i = k }
3. Build Context Free Grammars for the following languages:
   i {w | length of w is a multiple of 3} . Consider alphabet as $\{0, 1\}$
   ii {w | w has the words 'formal' and 'methods' as sub-strings in it} . Consider alphabet as the set of lowercase English alphabets, i.e, {a,b,c, ..., z}.
4. Find a CFGs that generates the languages
   i L(G) = $\{a^n b^m \mid 0 \leq n \leq m \leq 2n\}$.
   ii L(G) = $\{a^n b^m c^k : k = n + m\}$
5. Give CFG for following language
   i L= { w | w belongs to $\{0, 1\}^*$ and w is odd length palindrome }
   ii L = { $a^i b^j c^k$ | k $\leq$ i or k $\leq$ j}
6. Convert the following CFG into an equivalent CFG in Chomsky normal form:
   $S \to S_1 \mid S_2$
   $S_1 \to 0S_1 1 \mid \varepsilon$
   $S_2 \to 1S_2 0 \mid \varepsilon$
7. Convert the following CFG into its equivalent in CNF:
   $S \to ASB$
   $A \to aAS \mid a \mid \varepsilon$
   $B \to SbS \mid A \mid bb$
8. Find a CNF for the following CFG:
   $S \to aXbY$
   $X \to aX \mid \varepsilon$
   $Y \to bY \mid \varepsilon$
9. Design a PDA to recognize the language: L = { w | w $\in \{0, 1\}^*$ is a palindrome of odd length }.

10. Create a pushdown automaton that accepts the language $\{0^{2n}1^n \mid n > 0\}$. Show that your PDA accepts 000011 and that it rejects 0001.

11. Design a PDA to accept the language L = $\{w^r w \mid w \in \{a, b, c\}^*\}$

12. Construct pushdown automata for the following languages. Acceptance either by empty stack or by final state.

    i $\{a^i b^j c^k \mid i, j, k \in N, i + k = j\}$

    ii $\{ a^n b^m \mid n \leq m \leq 2n \}$

13. Construct PDA for following languages L = $\{w \in \{a, b\}^*$: #a(w) = #b(w)$\}$, where #a(w) = the number of occurrences of a in w and #b(w) = the number of occurrences of b in w.

14. Use the Pumping Lemma and prove that the following languages are not context free

    i L = $\{ a^n b^n a^n b^n \mid n \geq 0\}$

    ii L = $\{ a^n bba^{2n} bba^{4n} \mid n \geq 0\}$

    iii L = $\{ww^R w \mid$ w is a string of 0s and 1s$\}$

15. Using pumping lemma, prove that the following language is not context-free: L = $\{a^i b^{2i} a \mid i \geq 0^i\}$

16. Prove using pumping lemma that the following language is not context-free: L = $\{a^n$: n is prime $\}$

17. Using Pumping Lemma, show that the following language is not context free: L = $\{1^{n^2}, n \geq 0\}$

18. Give one example to show that the set of context-free languages over a given alphabet is not closed under intersection operation. Note that you should create 2 context free languages A and B such that their intersection is not context free.

19. Prove or disprove : The context-free languages are closed under union and concatenation.

20. Prove: The context-free languages are not closed under complementation.

21. Prove or disprove : The context-free languages are closed under Kleene star.

22. Prove/disprove that L = $\{ a^n b^n$: n $\neq 100$, n $\geq 0 \}$ is context-free.