

Science-1

Project-2

Double Pendulum

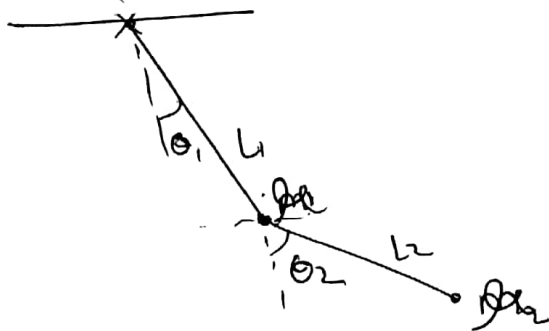
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Youtube Link:

[https://www.youtube.com/watch?](https://www.youtube.com/watch?v=bGFOVoyv7LI&feature=youtu.be)

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Double PendulumFind the motion of the trajectorySolutional Matrix

$$x_1 = l_1 \sin \theta_1 \quad y_1 = -l_1 \cos \theta_1$$

$$x_2 = l_1 \sin \theta_1 + l_2 \sin \theta_2 \quad y_2 = -l_1 \cos \theta_1 - l_2 \cos \theta_2$$

Let x_1, y_1 = co-ordinates of m_1 and x_2, y_2 be co-ordinates of m_2 , m_1, m_2 be their mass.

$$V = m_1 g y_1 + m_2 g y_2$$

$$T = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$$

Lagrangian of the System is

$$L = T - V$$

$$\frac{d}{dt} \left(\frac{dL}{d\dot{\theta}_i} \right) = \frac{dL}{d\theta_i}$$

$$T = \frac{1}{2} m_1 \dot{\theta}_1^2 l_1^2 + \frac{1}{2} m_2 (\dot{\theta}_1^2 l_1^2 + \dot{\theta}_2^2 l_2^2 + 2\dot{\theta}_1 \dot{\theta}_2 l_1 l_2 \cos(\theta_1 - \theta_2))$$

$$V = -(m_1 + m_2) g l_1 \cos \theta_1 - m_2 l_2 g \cos \theta_2$$

$$\frac{dL}{d\theta_1} = -Lg(m_1+m_2)\sin\theta_1 - m_2l_1\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = (m_1+m_2)l_1^2\dot{\theta}_1 + m_2l_1l_2\dot{\theta}_2\cos(\theta_1-\theta_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = 2(m_1+m_2)l_1\ddot{\theta}_1 + m_2l_1l_2\sin(\theta_1-\theta_2)(\ddot{\theta}_1-\ddot{\theta}_2)$$

$$+ m_2l_1l_2\cos(\theta_1-\theta_2)\ddot{\theta}_2$$

$$2(m_1+m_2)l_1\ddot{\theta}_1 - m_2l_1l_2\sin(\theta_1-\theta_2)\ddot{\theta}_2(\ddot{\theta}_1-\ddot{\theta}_2)$$

$$+ m_2l_1l_2\cos(\theta_1-\theta_2)\ddot{\theta}_2 = -Lg(m_1+m_2)\sin\theta_1$$

$$- m_2l_1l_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)$$

$$\ddot{\theta}_1 = \frac{-m_2l_2\ddot{\theta}_2\cos(\theta_1-\theta_2) - m_2l_2\dot{\theta}_2^2\sin(\theta_1-\theta_2) - (m_1+m_2)g\sin\theta_1}{m_1+m_2l_1}$$

Similarly

$$\frac{\partial L}{\partial \theta_2} = m_2l_1l_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2) - l_2m_2g\sin\theta_2$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2l_2^2\dot{\theta}_2 + m_2l_1l_2\dot{\theta}_1\cos(\theta_1-\theta_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = m_2l_2^2\ddot{\theta}_2 + m_2l_1l_2\dot{\theta}_1\cos(\theta_1-\theta_2)$$

$$- m_2l_1l_2\dot{\theta}_1\sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2)$$

$$\ddot{\theta}_2 = \frac{-l_1\dot{\theta}_1^2\cos(\theta_1-\theta_2) + l_1\dot{\theta}_1^2\sin(\theta_1-\theta_2) - g\sin\theta_2}{l_2}$$

We can easily solve these types of Equation using Runge Kutta approximation Method.

To simplify

$$\ddot{\theta}_1 + \alpha_1(\theta_1, \theta_2) \ddot{\theta}_2 = f_1(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$$

$$\ddot{\theta}_2 + \alpha_2(\theta_1, \theta_2) \ddot{\theta}_1 = f_2(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$$

$$\alpha_1(\theta_1, \theta_2) = \frac{L_2}{L_1} \left(\frac{m_2}{m_1 + m_2} \right) \cos(\theta_1 - \theta_2)$$

$$\alpha_2(\theta_1, \theta_2) = \frac{L_1}{L_2} \cos(\theta_1 - \theta_2)$$

$$f_1(\quad) = -\frac{L_2}{L_1} \left(\frac{m_2}{m_1 + m_2} \right) \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - \frac{g}{L_1} \sin \theta_1$$

$$f_2(\quad) = \frac{L_1}{L_2} \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - \frac{g}{L_2} \sin \theta_2$$

Hence we get $\Rightarrow \begin{pmatrix} 1 & \alpha_1 \\ \alpha_2 & 1 \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} 1 & -\alpha_1 \\ -\alpha_2 & 1 \end{pmatrix} = \frac{\begin{pmatrix} 1 & -\alpha_1 \\ -\alpha_2 & 1 \end{pmatrix}}{1 - \alpha_1 \alpha_2}$$

Now $\det(A) \neq 0$ as $\alpha_1 \alpha_2$ is always less than 1

$$\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \frac{1}{1 - \alpha_1 \alpha_2} \begin{pmatrix} f_1 - \alpha_1 f_2 \\ -\alpha_2 f_1 + f_2 \end{pmatrix}$$