FORMAL METHODS – ASSIGNMENT 1

28/03/2018

1. Prove that e is not rational.
2. Let $S(n) = 1 + 2 + 3 + \ldots = n(n+1)/2$. Let $C(n) = 1^3 + 2^3 + 3^3 + \ldots = (n^4 + 2n^3 + n^2)/4$. Prove by induction that $S(n)^2 = C(n)$ for every n.
3. Prove that if $2^n - 1$ is prime, then n is prime.
4. Prove that :
   i 5n + 4 is odd whenever n is odd. (Use Proof by Contradiction)
   ii 3 divides $n^3 + 2n$ for all $n \in N$. (Use Proof by Induction)
5. Show that $2^{3n+1} + 5$ is always a multiple of 7. (using induction)
6. Show that in any group of n people, there are two who have an identical number of friends within the group.
7. Prove the following by contrapositive:
   i An integer n is even if and only if n+1 is odd.
   ii If n and m have the same parity then n+m is even.
   iii If x^2 is even then x is even.
8. State the alphabet Σ for the following languages:
   i $L = \Sigma^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, ...\}$
   ii $L = \Sigma^+ = \{a, aa, aaa, ...\}$
   iii $L = \Sigma^+ = \{\varepsilon\}$
9. For each of the following languages, give three strings that are part of the language and three strings that are not part of the language. Assume the alphabet is $\{a, b\}$ in all the languages:
   i $\Sigma^* a \Sigma^* b \Sigma^* a \Sigma^*$
   ii $(\varepsilon \cup a)b$
10. Prove that $(L^*)^* = L^*$ for any language L.
11. Of the following, select the language which has no pair of consecutive 1's $(\Sigma = \{0, 1\})$:
    i $(0 \cup 10)^*(1 \cup \varepsilon)$
    ii $(0 \cup 10)^*(1 \cup \varepsilon)^*$
    iii $(0 \cup 101)^*(0 \cup \varepsilon)^*$

iv $(1 \cup 010)^*(1 \cup \varepsilon)^*$

12. Define Kleene closure of L for following cases:

    i $L = \{0, 1\}$

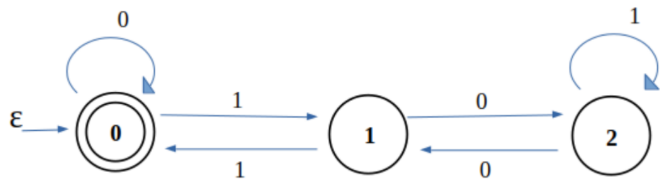
    ii $L = \{\varepsilon\}$

13. Given a language L over Σ, we define the reverse of L as $L^R = \{w^R \mid w \in L\}$. For each of the following, either prove equality or provide a counter example. Which of the false equalities can be made true by replacing = with a containment sign

    i $(S \cup T)^R = S^R \cup T^R$

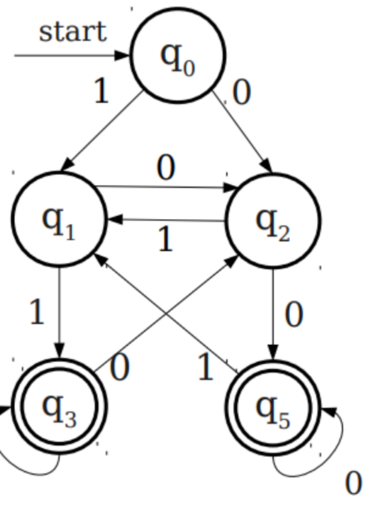
    ii $(ST)^R = T^R.S^R$.

14. Observe the following DFA and give the numbers (binary form) accepted by it



15. Draw DFA which accepts binary numbers divisible by 5.

16. Which language does the below DFA recognise? Make the transition table for the DFA. Assume alphabet is $\{0, 1\}$.



17. Design DFA for the following languages:

    i { w | w has either an even number of 1's or an odd number of 0's, but not both}. The alphabet is $\{0, 1\}$.

    ii {w | w has 'aab' as a substring}. The alphabet is $\{a, b, c\}$

18. Design a state diagram of a DFA for the given language. $(\Sigma = \{a, b\})$

i {w|w has even length and an odd number of a's}

ii {w|w contains neither the substrings ab nor ba}

19. Design a DFA to accept the language L = { w | w has exactly even number of 0's and 1's }

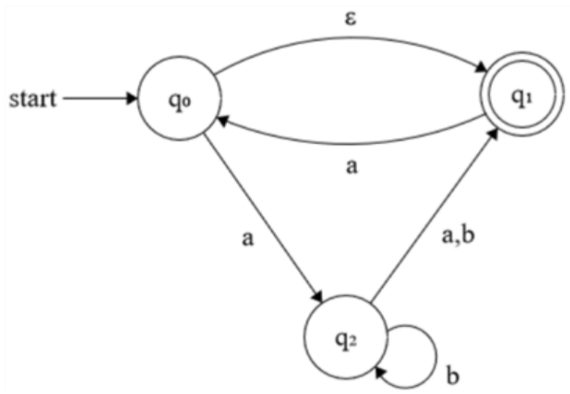
20. Design a DFA to accept the language string with regular expression $(111 \cup 11111)^{*}$.

21. Construct a DFA for the following regular expressions

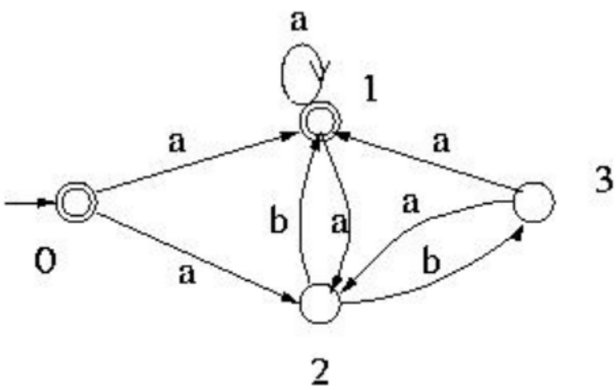
i $a^{*}(ab)^{*}b^{*}$, alphabet = {a, b}

ii $(0 \cup 1)^{*}10(0 \cup 1)^{*}$, alphabet = {0, 1}

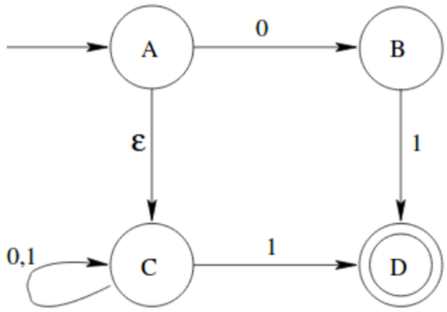
22. Convert NFA to DFA



23. Convert the below NFA to a DFA.



24. Convert below NFA to DFA. Please remove the irrelevant states(states which can't be reached from start state) in the final DFA.



25. For the following, construct DFA and then give the state diagram of the NFA recognizing the union of the two languages.

   i {w|w begins with a 1 and ends with a 0}

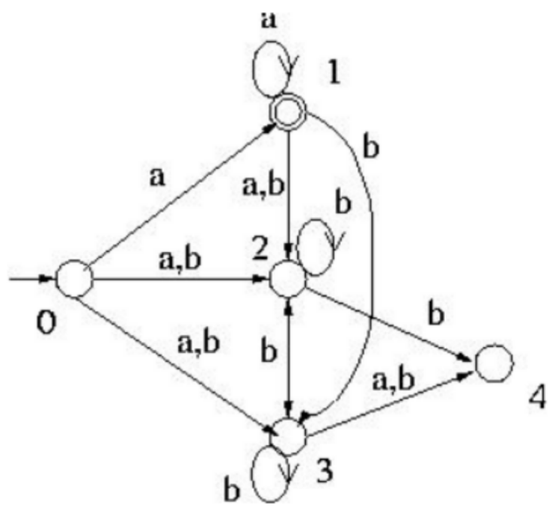
   ii {w|w contains at least three 1's}

26. Convert the following NFA to DFA and informally describe the language it accepts.

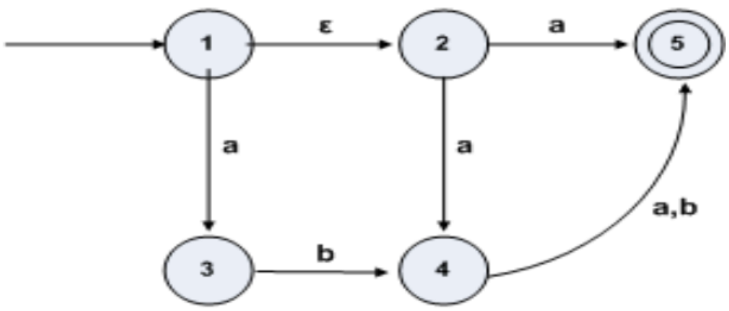
	0	1
$\rightarrow p$	$\{p, q\}$	$\{p\}$
q	$\{r, s\}$	$\{t\}$
r	$\{p, r\}$	$\{t\}$
$*s$	$\emptyset$	$\emptyset$
$*t$	$\emptyset$	$\emptyset$

Here * represents final state.

27. Convert the following NFA to DFA



28. Convert the following NFA to DFA



29. Prove that the language { string w such that w has an odd number of a's and an even number of b's} over the alphabet {a,b} is regular.

30. Prove that for two regular languages L_1 and L_2, $L_1 \cap L_2$ is also regular where $L_1 \cap L_2 = \{$Strings that are a part of both L_1 and L_2 $\}$.

31. Prove:

    i Set of regular languages is closed under complement operation. (Hint: Any regular language have an NFA and every NFA can be represented as a DFA. So prove that every DFA has a complement DFA)

    ii Prove that if L_1 is regular and L_2 is regular then so is $L_1 - L_2$ (the set of all strings in L_1 but not in L_2).

32. If L^* is regular, is L regular? Justify.

33. If L, M and N are any languages, prove that L(M $\cup$ N) = LM $\cup$ LN.

34. True or False: If L_1 and L_2 are two languages such that $L_1.L_2$ and $L_2.L_1$ are all regular,then L_1 must be regular.

35. If L is a regular language then so is L^R (Reverse of L) .

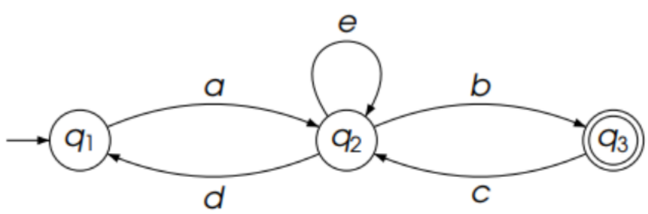
36. Give regular expressions: for all cases, the alphabet is {0,1}.

    i The language of strings of length at least two that begin with 0 and end in 1.

    ii The language of strings of length at least two that have a 1 as their second symbol.

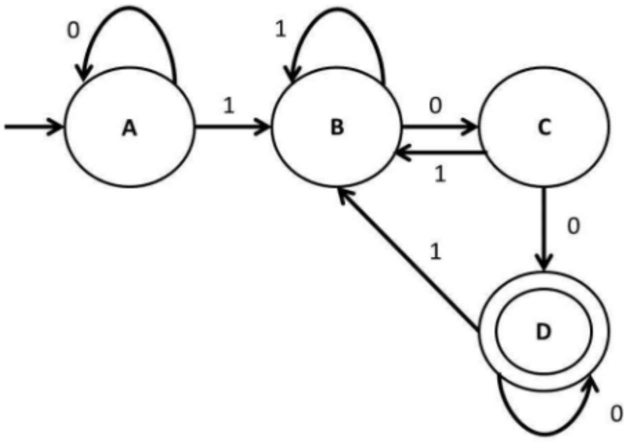
    iii The language of strings of length at least k that have a 1in position k. Do this in general, for every $k \geqslant 1$.

37. Convert below NFA to its corresponding regular expression. Assume alphabet is $\{a, b, c, d, e\}$.



38. In Unix shell and other related places, a regular expression can also optionally contain the $[n_1 - n_2]$ feature which corresponds to repeats of the pattern between n_1 and n_2 times, both inclusive. Can this type of regular expression be represented using an NFA? If yes, how?

39. The following DFA corresponds to the language of binary strings which represent all natural numbers that give remainder 4 when divided by 5.
    (L = $\{w$|w $\in$ {0, 1}* , w = <n>, n $\in$ N, n $\equiv$ 4 mod 5$\}$)
    Write the regular expression for this DFA.



40. Write regular expression for the following languages:

    i The set of strings over alphabet {a, b, c} containing atleast one a and atleast one b.

    ii The set of strings of 0's and 1's whose tenth symbol from the right end is 1.

    iii The set of strings of 0's and 1's with atmost one pair of consecutive 1's.

41. Let S and T be language over $\Sigma = \{a, b\}$ represented by the regular expressions $(a \cup b^*)^*$ and $(a \cup b)^*$, respectively. Which of the following is true?

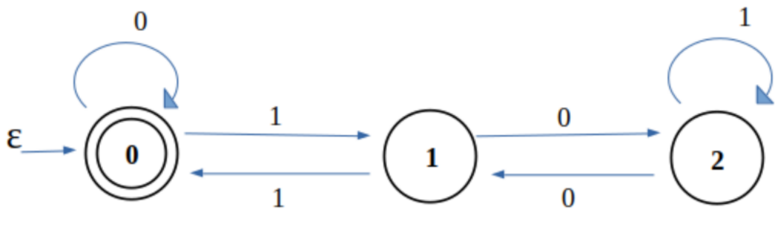
    i $S \subset T$

    ii $T \subset S$

    iii $S = T$

    iv $S \cap T = \phi$

42. Convert the following DFA to regular expression using GNFA



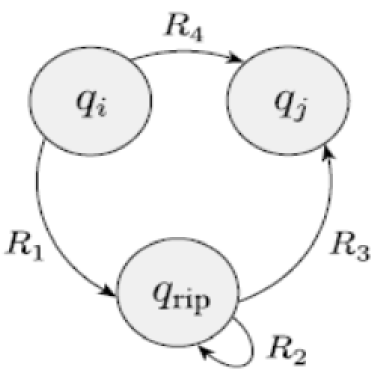
43. By using equivalence between R.E. and NFA, convert the following regular expressions into NFA's.

    i $0\Sigma^*0$

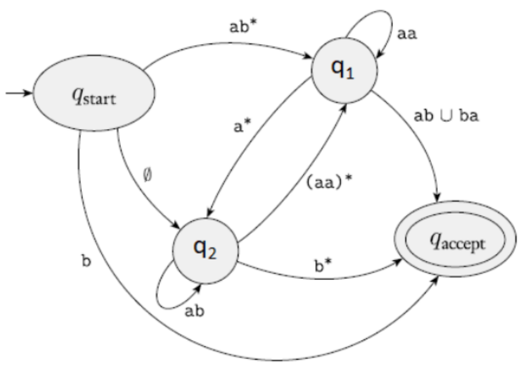
    ii $(11)^*$

44. Convert each of the following GNFAs with k states to their corresponding GNFAs with $k-1$ states.

    i .

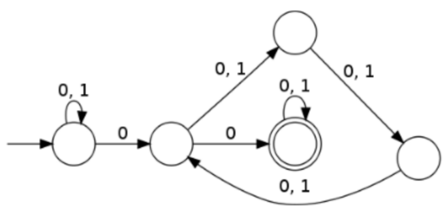


In the above GNFA, q_i is the start state, q_j is the end state. R_1, R_2, R_3, R_4 are arbitrary regular expressions



ii

45. Give regular expression for the following NFA:



The alphabet is $\{0, 1\}$

46. Give regular expression corresponding to the complement of : $(a \cup ab)^*$ (Hint : Find NFA from the regular expression, and then DFA)

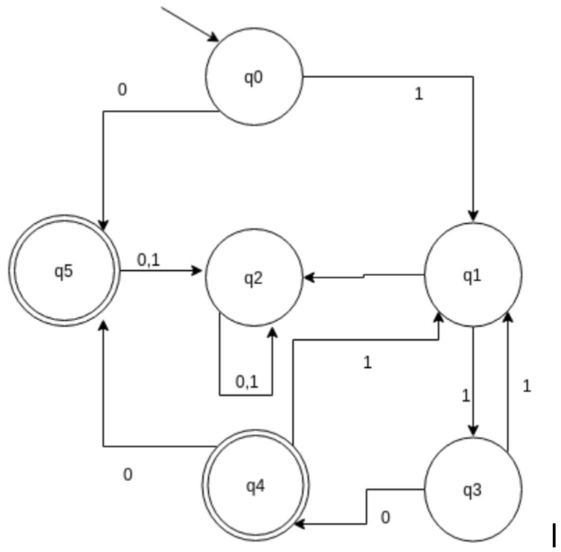
47. Convert the following regular expressions to NFA's with ε-transition.

   i 01^*

   ii $(0 \cup 1)01$

   iii $00(0 \cup 1)^*$

48. Which of the given regular expressions correspond to the automata shown? Explain it in a sentence.



i $(110 \cup 1)^{*}0$

ii $(11 \cup 110)^{*}1$

iii $(110 \cup 11)^{*}0$

iv $(1 \cup 110)^{*}1$

49. Convert the following regular expressions to NFA

 i $0^{*}(10^{*})^{*}$

 ii $a \cup (a \cup b)^{*}b(ab)^{*}$

50. Use the Pumping Lemma to show that the language $\{1^{i}\#1^{j}\#1^{i+j}\}$ is not regular. The alphabet is $\{1, \#\}$.

51. State whether the below statements are True or False with justification using pumping lemma:

 i $L = \{w \mid w = w^{R}, w \in \{0,1\}^{*}\}$ is not a regular language.

 ii $L = \{ww^{R} \mid w \in \{0,1\}^{*}\}$ is not a regular language.
 where w^{R} is the reverse of w.

52. Prove that the following language is not regular:

$$L = \{0^{m}1^{n}0^{m+n} : m, n > 0\}$$

53. Prove that the following language is not regular using the pumping lemma :
 $\{w \mid w \in \{0,1\}^{*}$ is not a palindrome$\}$.

54. Prove that the following languages are not regular.

 i $\{\ 0^{n} \mid$ n is a perfect square$\}$

 ii $\{\ 0^{n} \mid$ n is power of 2$\}$

55. i Prove that $L = \{ww \mid w \in \{0,1\}^*\}$ is not regular.

ii Prove that L ={w | No of 0's in w $\neq$ No of 1's in w, w in $\{0,1\}^*$} is not regular.

56. Prove that the following languages are not regular

i L = { 1^n | n is prime number }

ii L= $\{0^i 1^j \mid i > j\}$