

# *Concepts of Modern Physics*

**Sixth Edition**

Looking back, it may seem odd that two decades passed between the 1905 discovery of the particle properties of waves and the 1924 speculation that particles might show wave behavior. It is one thing, however, to suggest a revolutionary concept to explain otherwise mysterious data and quite another to suggest an equally revolutionary concept without a strong experimental mandate. The latter is just what Louis de Broglie did in 1924 when he proposed that moving objects have wave as well as particle characteristics. So different was the scientific climate at the time from that around the turn of the century that de Broglie's ideas soon received respectful attention, whereas the earlier quantum theory of light of Planck and Einstein had been largely ignored despite its striking empirical support. The existence of de Broglie waves was experimentally demonstrated by 1927, and the duality principle they represent provided the starting point for Schrödinger's successful development of quantum mechanics in the previous year.

### 3.1 DE BROGLIE WAVES

*A moving body behaves in certain ways as though it has a wave nature*

A photon of light of frequency  $\nu$  has the momentum

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

since  $\lambda\nu = c$ . The wavelength of a photon is therefore specified by its momentum according to the relation

Photon wavelength 
$$\lambda = \frac{h}{p} \quad (3.1)$$

De Broglie suggested that Eq. (3.1) is a completely general one that applies to material particles as well as to photons. The momentum of a particle of mass  $m$  and velocity  $v$  is  $p = \gamma mv$ , and its de Broglie wavelength is accordingly

De Broglie wavelength 
$$\lambda = \frac{h}{\gamma mv} \quad (3.2)$$



Louis de Broglie (1892–1987), although coming from a French family long identified with diplomacy and the military and initially a student of history, eventually followed his older brother Maurice in a career in physics. His doctoral thesis in 1924 contained the proposal that moving bodies have wave properties that complement their particle properties: these “seemingly incompatible conceptions can each represent an

aspect of the truth. . . . They may serve in turn to represent the facts without ever entering into direct conflict.” Part of de Broglie’s inspiration came from Bohr’s theory of the hydrogen atom, in which the electron is supposed to follow only certain orbits around the nucleus. “This fact suggested to me the idea that electrons . . . could not be considered simply as particles but that periodicity must be assigned to them also.” Two years later Erwin Schrödinger used the concept of de Broglie waves to develop a general theory that he and others applied to explain a wide variety of atomic phenomena. The existence of de Broglie waves was confirmed in diffraction experiments with electron beams in 1927, and in 1929 de Broglie received the Nobel Prize.

The greater the particle's momentum, the shorter its wavelength. In Eq. (3.2)  $\gamma$  is the relativistic factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

As in the case of em waves, the wave and particle aspects of moving bodies can never be observed at the same time. We therefore cannot ask which is the "correct" description. All that can be said is that in certain situations a moving body resembles a wave and in others it resembles a particle. Which set of properties is most conspicuous depends on how its de Broglie wavelength compares with its dimensions and the dimensions of whatever it interacts with.

### Example 3.1

Find the de Broglie wavelengths of (a) a 46-g golf ball with a velocity of 30 m/s, and (b) an electron with a velocity of  $10^7$  m/s.

#### Solution

(a) Since  $v \ll c$ , we can let  $\gamma = 1$ . Hence

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.046 \text{ kg})(30 \text{ m/s})} = 4.8 \times 10^{-34} \text{ m}$$

The wavelength of the golf ball is so small compared with its dimensions that we would not expect to find any wave aspects in its behavior.

(b) Again  $v \ll c$ , so with  $m = 9.1 \times 10^{-31}$  kg, we have

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.1 \times 10^{-31} \text{ kg})(10^7 \text{ m/s})} = 7.3 \times 10^{-11} \text{ m}$$

The dimensions of atoms are comparable with this figure—the radius of the hydrogen atom, for instance, is  $5.3 \times 10^{-11}$  m. It is therefore not surprising that the wave character of moving electrons is the key to understanding atomic structure and behavior.

### Example 3.2

Find the kinetic energy of a proton whose de Broglie wavelength is  $1.000 \text{ fm} = 1.000 \times 10^{-15} \text{ m}$ , which is roughly the proton diameter.

#### Solution

A relativistic calculation is needed unless  $pc$  for the proton is much smaller than the proton rest energy of  $E_0 = 0.938 \text{ GeV}$ . To find out, we use Eq. (3.2) to determine  $pc$ :

$$pc = (\gamma mv)c = \frac{hc}{\lambda} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{1.000 \times 10^{-15} \text{ m}} = 1.240 \times 10^9 \text{ eV} \\ = 1.2410 \text{ GeV}$$

Since  $pc > E_0$  a relativistic calculation is required. From Eq. (1.24) the total energy of the proton is

$$E = \sqrt{E_0^2 + p^2 c^2} = \sqrt{(0.938 \text{ GeV})^2 + (1.2410 \text{ GeV})^2} = 1.555 \text{ GeV}$$

The corresponding kinetic energy is

$$KE = E - E_0 = (1.555 - 0.938) \text{ GeV} = 0.617 \text{ GeV} = 617 \text{ MeV}$$

De Broglie had no direct experimental evidence to support his conjecture. However, he was able to show that it accounted in a natural way for the energy quantization—the restriction to certain specific energy values—that Bohr had had to postulate in his 1913 model of the hydrogen atom. (This model is discussed in Chap. 4.) Within a few years Eq. (3.2) was verified by experiments involving the diffraction of electrons by crystals. Before we consider one of these experiments, let us look into the question of what kind of wave phenomenon is involved in the matter waves of de Broglie.

### 3.2 WAVES OF WHAT?

#### *Waves of probability*

In water waves, the quantity that varies periodically is the height of the water surface. In sound waves, it is pressure. In light waves, electric and magnetic fields vary. What is it that varies in the case of matter waves?

The quantity whose variations make up matter waves is called the **wave function**, symbol  $\Psi$  (the Greek letter psi). The value of the wave function associated with a moving body at the particular point  $x, y, z$  in space at the time  $t$  is related to the likelihood of finding the body there at the time.



Max Born (1882–1970) grew up in Breslau, then a German city but today part of Poland, and received a doctorate in applied mathematics at Göttingen in 1907. Soon afterward he decided to concentrate on physics, and was back in Göttingen in 1909 as a lecturer. There he worked on various aspects of the theory of crystal lattices, his “central interest” to which he often returned in later years. In 1915, at

Planck’s recommendation, Born became professor of physics in Berlin where, among his other activities, he played piano to Einstein’s violin. After army service in World War I and a period at Frankfurt University, Born was again in Göttingen, now as professor of physics. There a remarkable center of theoretical physics developed under his leadership: Heisenberg and Pauli were among his assistants and Fermi, Dirac, Wigner, and Goepfert were among those who worked with him, just to name future Nobel Prize winners. In those days, Born wrote, “There was complete freedom of teaching and learning in German universities, with no class examinations, and no control of students. The University just offered lectures and the student had to decide for himself which he wished to attend.”

Born was a pioneer in going from “the bright realm of classical physics into the still dark and unexplored underworld of the new quantum mechanics;” he was the first to use the latter term. From Born came the basic concept that the wave function  $\Psi$  of a particle is related to the probability of finding it. He began with an idea of Einstein, who “sought to make the duality of particles (light quanta or photons) and waves comprehensible by interpreting the square of the optical wave amplitude as probability density for the occurrence of photons. This idea could at once be extended to the  $\Psi$ -function:  $|\Psi|^2$  must represent the probability density for electrons (or other particles). To assert this was easy; but how was it to be proved? For this purpose atomic scattering processes suggested themselves.” Born’s development of the quantum theory of atomic scattering (collisions of atoms with various particles) not only verified his “new way of thinking about the phenomena of nature” but also founded an important branch of theoretical physics.

Born left Germany in 1933 at the start of the Nazi period, like so many other scientists. He became a British subject and was associated with Cambridge and then Edinburgh universities until he retired in 1953. Finding the Scottish climate harsh and wishing to contribute to the democratization of postwar Germany, Born spent the rest of his life in Bad Pyrmont, a town near Göttingen. His textbooks on modern physics and on optics were standard works on these subjects for many years.

The wave function  $\Psi$  itself, however, has no direct physical significance. There is a simple reason why  $\Psi$  cannot be interpreted in terms of an experiment. The probability that something be in a certain place at a given time must lie between 0 (the object is definitely not there) and 1 (the object is definitely there). An intermediate probability, say 0.2, means that there is a 20% chance of finding the object. But the amplitude of a wave can be negative as well as positive, and a negative probability, say  $-0.2$ , is meaningless. Hence  $\Psi$  by itself cannot be an observable quantity.

This objection does not apply to  $|\Psi|^2$ , the square of the absolute value of the wave function, which is known as **probability density**:

The probability of experimentally finding the body described by the wave function  $\Psi$  at the point  $x, y, z$ , at the time  $t$  is proportional to the value of  $|\Psi|^2$  there at  $t$ .

A large value of  $|\Psi|^2$  means the strong possibility of the body's presence, while a small value of  $|\Psi|^2$  means the slight possibility of its presence. As long as  $|\Psi|^2$  is not actually 0 somewhere, however, there is a definite chance, however small, of detecting it there. This interpretation was first made by Max Born in 1926.

There is a big difference between the probability of an event and the event itself. Although we can speak of the wave function  $\Psi$  that describes a particle as being spread out in space, this does not mean that the particle itself is thus spread out. When an experiment is performed to detect electrons, for instance, a whole electron is either found at a certain time and place or it is not; there is no such thing as a 20 percent of an electron. However, it is entirely possible for there to be a 20 percent chance that the electron be found at that time and place, and it is this likelihood that is specified by  $|\Psi|^2$ .

W. L. Bragg, the pioneer in x-ray diffraction, gave this loose but vivid interpretation: "The dividing line between the wave and particle nature of matter and radiation is the moment 'now.' As this moment steadily advances through time it coagulates a wavy future into a particle past. . . . Everything in the future is a wave, everything in the past is a particle." If "the moment 'now'" is understood to be the time a measurement is performed, this is a reasonable way to think about the situation. (The philosopher Søren Kierkegaard may have been anticipating this aspect of modern physics when he wrote, "Life can only be understood backwards, but it must be lived forwards.")

Alternatively, if an experiment involves a great many identical objects all described by the same wave function  $\Psi$ , the *actual density* (number per unit volume) of objects at  $x, y, z$  at the time  $t$  is proportional to the corresponding value of  $|\Psi|^2$ . It is instructive to compare the connection between  $\Psi$  and the density of particles it describes with the connection discussed in Sec. 2.4 between the electric field  $E$  of an electromagnetic wave and the density  $N$  of photons associated with the wave.

While the wavelength of the de Broglie waves associated with a moving body is given by the simple formula  $\lambda = h/\gamma mv$ , to find their amplitude  $\Psi$  as a function of position and time is often difficult. How to calculate  $\Psi$  is discussed in Chap. 5 and the ideas developed there are applied to the structure of the atom in Chap. 6. Until then we can assume that we know as much about  $\Psi$  as each situation requires.

### 3.3 DESCRIBING A WAVE

#### *A general formula for waves*

How fast do de Broglie waves travel? Since we associate a de Broglie wave with a moving body, we expect that this wave has the same velocity as that of the body. Let us see if this is true.

If we call the de Broglie wave velocity  $v_p$ , we can apply the usual formula

$$v_p = \nu\lambda$$

to find  $v_p$ . The wavelength  $\lambda$  is simply the de Broglie wavelength  $\lambda = h/\gamma mv$ . To find the frequency, we equate the quantum expression  $E = h\nu$  with the relativistic formula for total energy  $E = \gamma mc^2$  to obtain

$$h\nu = \gamma mc^2$$

$$\nu = \frac{\gamma mc^2}{h}$$

The de Broglie wave velocity is therefore

De Broglie phase velocity  $v_p = \nu\lambda = \left(\frac{\gamma mc^2}{h}\right)\left(\frac{h}{\gamma mv}\right) = \frac{c^2}{v}$  (3.3)

Because the particle velocity  $v$  must be less than the velocity of light  $c$ , the de Broglie waves always travel faster than light! In order to understand this unexpected result, we must look into the distinction between **phase velocity** and **group velocity**. (Phase velocity is what we have been calling wave velocity.)

Let us begin by reviewing how waves are described mathematically. For simplicity we consider a string stretched along the  $x$  axis whose vibrations are in the  $y$  direction, as in Fig. 3.1, and are simple harmonic in character. If we choose  $t = 0$  when the displacement  $y$  of the string at  $x = 0$  is a maximum, its displacement at any future time  $t$  at the same place is given by the formula

$$y = A \cos 2\pi\nu t$$
 (3.4)

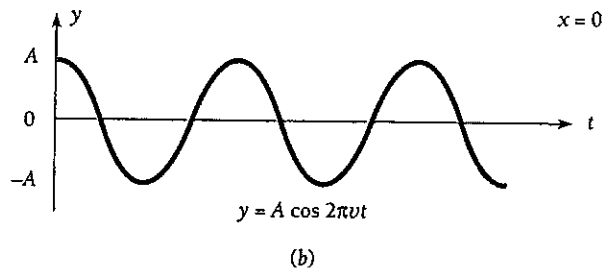
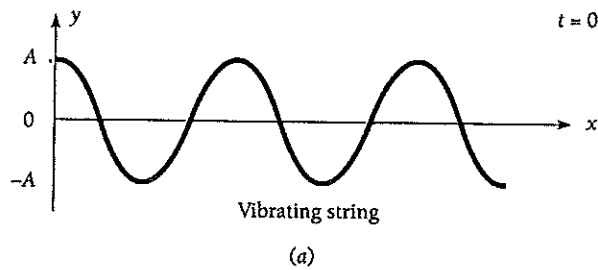


Figure 3.1 (a) The appearance of a wave in a stretched string at a certain time. (b) How the displacement of a point on the string varies with time.

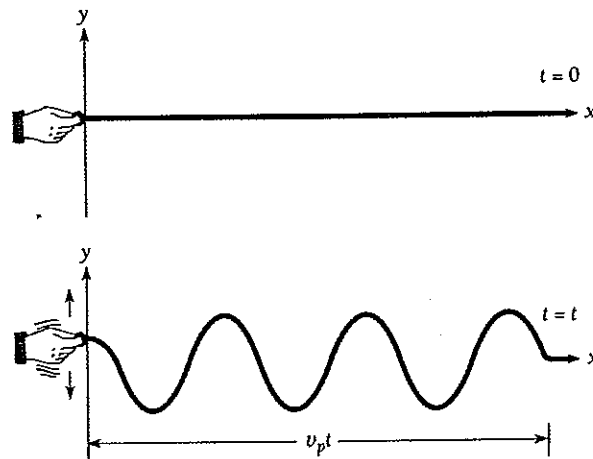


Figure 3.2 Wave propagation.

where  $A$  is the amplitude of the vibrations (that is, their maximum displacement on either side of the  $x$  axis) and  $\nu$  their frequency.

Equation (3.4) tells us what the displacement of a single point on the string is as a function of time  $t$ . A complete description of wave motion in a stretched string, however, should tell us what  $y$  is at *any* point on the string at *any* time. What we want is a formula giving  $y$  as a function of both  $x$  and  $t$ .

To obtain such a formula, let us imagine that we shake the string at  $x = 0$  when  $t = 0$ , so that a wave starts to travel down the string in the  $+x$  direction (Fig. 3.2). This wave has some speed  $v_p$  that depends on the properties of the string. The wave travels the distance  $x = v_p t$  in the time  $t$ , so the time interval between the formation of the wave at  $x = 0$  and its arrival at the point  $x$  is  $x/v_p$ . Hence the displacement  $y$  of the string at  $x$  at any time  $t$  is exactly the same as the value of  $y$  at  $x = 0$  at the *earlier time*  $t - x/v_p$ . By simply replacing  $t$  in Eq. (3.4) with  $t - x/v_p$ , then, we have the desired formula giving  $y$  in terms of both  $x$  and  $t$ :

$$\text{Wave formula} \quad y = A \cos 2\pi\nu \left( t - \frac{x}{v_p} \right) \quad (3.5)$$

As a check, we note that Eq. (3.5) reduces to Eq. (3.4) at  $x = 0$ .

Equation (3.5) may be rewritten

$$y = A \cos 2\pi \left( \nu t - \frac{\nu x}{v_p} \right)$$

Since the wave speed  $v_p$  is given by  $v_p = \nu\lambda$  we have

$$\text{Wave formula} \quad y = A \cos 2\pi \left( \nu t - \frac{x}{\lambda} \right) \quad (3.6)$$

Equation (3.6) is often more convenient to use than Eq. (3.5).

Perhaps the most widely used description of a wave, however, is still another form of Eq. (3.5). The quantities **angular frequency**  $\omega$  and **wave number**  $k$  are defined by the formulas

$$\text{Angular frequency} \quad \omega = 2\pi\nu \quad (3.7)$$

$$\text{Wave number} \quad k = \frac{2\pi}{\lambda} = \frac{\omega}{v_p} \quad (3.8)$$

The unit of  $\omega$  is the radian per second and that of  $k$  is the radian per meter. Angular frequency gets its name from uniform circular motion, where a particle that moves around a circle  $\nu$  times per second sweeps out  $2\pi\nu$  rad/s. The wave number is equal to the number of radians corresponding to a wave train 1 m long, since there are  $2\pi$  rad in one complete wave.

In terms of  $\omega$  and  $k$ , Eq. (3.5) becomes

$$\text{Wave formula} \quad y = A \cos(\omega t - kx) \quad (3.9)$$

In three dimensions  $k$  becomes a vector  $\mathbf{k}$  normal to the wave fronts and  $x$  is replaced by the radius vector  $\mathbf{r}$ . The scalar product  $\mathbf{k} \cdot \mathbf{r}$  is then used instead of  $kx$  in Eq. (3.9).

### 3.4 PHASE AND GROUP VELOCITIES

*A group of waves need not have the same velocity as the waves themselves*

The amplitude of the de Broglie waves that correspond to a moving body reflects the probability that it will be found at a particular place at a particular time. It is clear that de Broglie waves cannot be represented simply by a formula resembling Eq. (3.9), which describes an indefinite series of waves all with the same amplitude  $A$ . Instead, we expect the wave representation of a moving body to correspond to a **wave packet**, or **wave group**, like that shown in Fig. 3.3, whose waves have amplitudes upon which the likelihood of detecting the body depends.

A familiar example of how wave groups come into being is the case of **beats**. When two sound waves of the same amplitude but of slightly different frequencies are produced simultaneously, the sound we hear has a frequency equal to the average of the two original frequencies and its amplitude rises and falls periodically. The amplitude fluctuations occur as many times per second as the difference between the two original frequencies. If the original sounds have frequencies of, say, 440 and 442 Hz, we will hear a fluctuating sound of frequency 441 Hz with two loudness peaks, called beats, per second. The production of beats is illustrated in Fig. 3.4.

A way to mathematically describe a wave group, then, is in terms of a superposition of individual waves of different wavelengths whose interference with one another results in the variation in amplitude that defines the group shape. If the velocities of the waves are the same, the velocity with which the wave group travels is the common phase velocity. However, if the phase velocity varies with wavelength, the different individual waves do not proceed together. This situation is called **dispersion**. As a result the wave group has a velocity different from the phase velocities of the waves that make it up. This is the case with de Broglie waves.

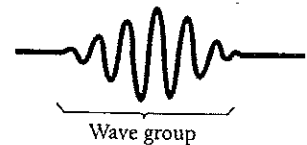


Figure 3.3 A wave group.



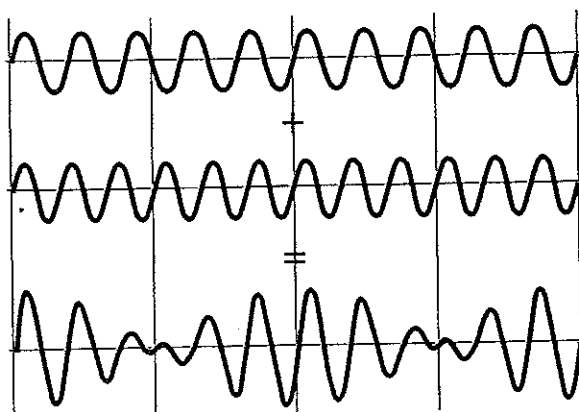


Figure 3.4 Beats are produced by the superposition of two waves with different frequencies.

It is not hard to find the velocity  $v_g$  with which a wave group travels. Let us suppose that the wave group arises from the combination of two waves that have the same amplitude  $A$  but differ by an amount  $\Delta\omega$  in angular frequency and an amount  $\Delta k$  in wave number. We may represent the original waves by the formulas

$$y_1 = A \cos(\omega t - kx)$$

$$y_2 = A \cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

The resultant displacement  $y$  at any time  $t$  and any position  $x$  is the sum of  $y_1$  and  $y_2$ . With the help of the identity

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

and the relation

$$\cos(-\theta) = \cos \theta$$

we find that

$$\begin{aligned} y &= y_1 + y_2 \\ &= 2A \cos \frac{1}{2}[(2\omega + \Delta\omega)t - (2k + \Delta k)x] \cos \frac{1}{2}(\Delta\omega t - \Delta k x) \end{aligned}$$

Since  $\Delta\omega$  and  $\Delta k$  are small compared with  $\omega$  and  $k$  respectively,

$$2\omega + \Delta\omega \approx 2\omega$$

$$2k + \Delta k \approx 2k$$

and so

$$\text{Beats} \quad y = 2A \cos(\omega t - kx) \cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) \quad (3.10)$$

Equation (3.10) represents a wave of angular frequency  $\omega$  and wave number  $k$  that has superimposed upon it a modulation of angular frequency  $\frac{1}{2}\Delta\omega$  and of wave number  $\frac{1}{2}\Delta k$ .

The effect of the modulation is thus to produce successive wave groups, as in Fig. 3.4. The phase velocity  $v_p$  is

$$\text{Phase velocity} \quad v_p = \frac{\omega}{k} \quad (3.11)$$

and the velocity  $v_g$  of the wave groups is

$$\text{Group velocity} \quad v_g = \frac{\Delta\omega}{\Delta k} \quad (3.12)$$

When  $\omega$  and  $k$  have continuous spreads instead of the two values in the preceding discussion, the group velocity is instead given by

$$\text{Group velocity} \quad v_g = \frac{d\omega}{dk} \quad (3.13)$$

Depending on how phase velocity varies with wave number in a particular situation, the group velocity may be less or greater than the phase velocities of its member waves. If the phase velocity is the same for all wavelengths, as is true for light waves in empty space, the group and phase velocities are the same.

The angular frequency and wave number of the de Broglie waves associated with a body of mass  $m$  moving with the velocity  $v$  are

$$\begin{aligned} \omega &= 2\pi\nu = \frac{2\pi\gamma mc^2}{h} \\ \text{Angular frequency of de Broglie waves} &= \frac{2\pi mc^2}{h\sqrt{1-v^2/c^2}} \end{aligned} \quad (3.14)$$

$$\begin{aligned} k &= \frac{2\pi}{\lambda} = \frac{2\pi\gamma mv}{h} \\ \text{Wave number of de Broglie waves} &= \frac{2\pi mv}{h\sqrt{1-v^2/c^2}} \end{aligned} \quad (3.15)$$

Both  $\omega$  and  $k$  are functions of the body's velocity  $v$ .

The group velocity  $v_g$  of the de Broglie waves associated with the body is

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$$

Now

$$\begin{aligned} \frac{d\omega}{dv} &= \frac{2\pi mv}{h(1-v^2/c^2)^{3/2}} \\ \frac{dk}{dv} &= \frac{2\pi m}{h(1-v^2/c^2)^{3/2}} \end{aligned}$$

which is indeed the same as R. Bohr's model of the hydrogen atom is therefore in accord with the spectral data.

### Example 4.4

Find the longest wavelength present in the Balmer series of hydrogen, corresponding to the  $H_\alpha$  line.

**Solution**

In the Balmer series the quantum number of the final state is  $n_f = 2$ . The longest wavelength in this series corresponds to the smallest energy difference between energy levels. Hence the initial state must be  $n_i = 3$  and

$$\frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = R \left( \frac{1}{2^2} - \frac{1}{3^2} \right) = 0.139R$$

$$\lambda = \frac{1}{0.139R} = \frac{1}{0.139(1.097 \times 10^7 \text{ m}^{-1})} = 6.56 \times 10^{-7} \text{ m} = 656 \text{ nm}$$

This wavelength is near the red end of the visible spectrum.

## 4.6 CORRESPONDENCE PRINCIPLE

*The greater the quantum number, the closer quantum physics approaches classical physics*

Quantum physics, so different from classical physics in the microworld beyond reach of our senses, must nevertheless give the same results as classical physics in the macroworld where experiments show that the latter is valid. We have already seen that this basic requirement is true for the wave theory of moving bodies. We shall now find that it is also true for Bohr's model of the hydrogen atom.

According to electromagnetic theory, an electron moving in a circular orbit radiates em waves whose frequencies are equal to its frequency of revolution and to harmonics (that is, integral multiples) of that frequency. In a hydrogen atom the electron's speed is

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 m r}}$$

according to Eq. (4.4), where  $r$  is the radius of its orbit. Hence the frequency of revolution  $f$  of the electron is

$$f = \frac{\text{electron speed}}{\text{orbit circumference}} = \frac{v}{2\pi r} = \frac{e}{2\pi\sqrt{4\pi\epsilon_0 m r^3}}$$

The radius  $r_n$  of a stable orbit is given in terms of its quantum number  $n$  by Eq. (4.13) as

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

and so the frequency of revolution is

Frequency of revolution  $f = \frac{me^4}{8\epsilon_0^2 h^3} \left( \frac{2}{n^3} \right) = \frac{-E_1}{h} \left( \frac{2}{n^3} \right)$  (4.19)

### Example 4.5

(a) Find the frequencies of revolution of electrons in  $n = 1$  and  $n = 2$  Bohr orbits. (b) What is the frequency of the photon emitted when an electron in an  $n = 2$  orbit drops to an  $n = 1$  orbit? (c) An electron typically spends about  $10^{-8}$  s in an excited state before it drops to a lower state by emitting a photon. How many revolutions does an electron in an  $n = 2$  Bohr orbit make in  $1.00 \times 10^{-8}$  s?

**Solution**

(a) From Eq. (4.19),

$$f_1 = \frac{-E_1}{h} \left( \frac{2}{1^3} \right) = \left( \frac{2.18 \times 10^{-18} \text{ J}}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} \right) (2) = 6.58 \times 10^{15} \text{ rev/s}$$

$$f_2 = \frac{-E_1}{h} \left( \frac{2}{2^3} \right) = \frac{f_1}{8} = 0.823 \times 10^{15} \text{ rev/s}$$

(b) From Eq. (4.17),

$$\nu = \frac{-E_1}{h} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = \left( \frac{2.18 \times 10^{-18} \text{ J}}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} \right) \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = 2.88 \times 10^{15} \text{ Hz}$$

This frequency is intermediate between  $f_1$  and  $f_2$ .

(c) The number of revolutions the electron makes is

$$N = f_2 \Delta t = (8.23 \times 10^{14} \text{ rev/s})(1.00 \times 10^{-8} \text{ s}) = 8.23 \times 10^6 \text{ rev}$$

The earth takes 8.23 million y to make this many revolutions around the sun.

Under what circumstances should the Bohr atom behave classically? If the electron orbit is so large that we might be able to measure it directly, quantum effects ought not to dominate. An orbit 0.01 mm across, for instance, meets this specification. As we found in Example 4.3, its quantum number is  $n = 435$ .

What does the Bohr theory predict such an atom will radiate? According to Eq. (4.17), a hydrogen atom dropping from the  $n_i$ th energy level to the  $n_f$ th energy level emits a photon whose frequency is

$$\nu = \frac{-E_1}{h} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Let us write  $n$  for the initial quantum number  $n_i$  and  $n - p$  (where  $p = 1, 2, 3, \dots$ ) for the final quantum number  $n_f$ . With this substitution,

$$\nu = \frac{-E_1}{h} \left[ \frac{1}{(n-p)^2} - \frac{1}{n^2} \right] = \frac{-E_1}{h} \left[ \frac{2np - p^2}{n^2(n-p)^2} \right]$$

When  $n_i$  and  $n_f$  are both very large,  $n$  is much greater than  $p$ , and

$$2np - p^2 \approx 2np$$

$$(n-p)^2 \approx n^2$$

so that

Frequency of  
photon

$$\nu = \frac{-E_1}{h} \left( \frac{2p}{n^3} \right) \quad (4.20)$$

When  $p = 1$ , the frequency  $\nu$  of the radiation is exactly the same as the frequency of rotation  $f$  of the orbital electron given in Eq. (4.19). Multiples of this frequency are radiated when  $p = 2, 3, 4, \dots$ . Hence both quantum and classical pictures of the hydrogen atom make the same predictions in the limit of very large quantum numbers. When  $n = 2$ , Eq. (4.19) predicts a radiation frequency that differs from that given by Eq. (4.20) by almost 300 percent. When  $n = 10,000$ , the discrepancy is only about 0.01 percent.

The requirement that quantum physics give the same results as classical physics in the limit of large quantum numbers was called by Bohr the **correspondence principle**. It has played an important role in the development of the quantum theory of matter.

Bohr himself used the correspondence principle in reverse, so to speak, to look for the condition for orbit stability. Starting from Eq. (4.19) he was able to show that stable orbits must have electron orbital angular momenta of

$$\text{Condition for} \quad mvr = \frac{nh}{2\pi} \quad n = 1, 2, 3, \dots \quad (4.21)$$

orbital stability

Since the de Broglie electron wavelength is  $\lambda = h/mv$ , Eq. (4.21) is the same as Eq. (4.12),  $n\lambda = 2\pi r$ , which states that an electron orbit must contain an integral number of wavelengths.

## 4.7 NUCLEAR MOTION

*The nuclear mass affects the wavelengths of spectral lines*

Thus far we have been assuming that the hydrogen nucleus (a proton) remains stationary while the orbital electron revolves around it. What must actually happen, of course, is that both nucleus and electron revolve around their common center of mass, which is very close to the nucleus because the nuclear mass is much greater than that of the electron (Fig. 4.17). A system of this kind is equivalent to a single particle of mass  $m'$  that revolves around the position of the heavier particle. (This equivalence is

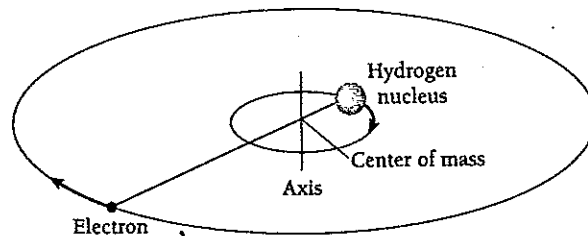


Figure 4.17 Both the electron and nucleus of a hydrogen atom revolve around a common center of mass (not to scale).