4 - Jan 2019.

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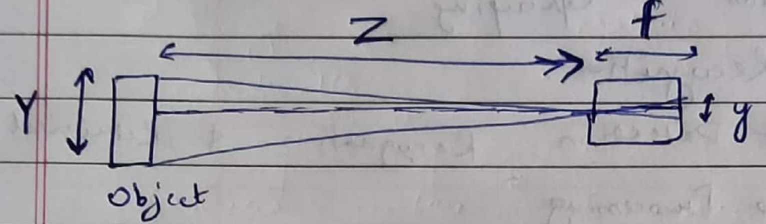
Imaging and Camera Model

Pinhole Camera.

$$\boxed{y = f\left(\frac{Y}{Z}\right)}$$

focal length



⇒ Brightness, sharpness, Exposure Time

VS → PINHOLE

Aperture ↓ DOF ↑
(Depth of field)

Focal Ratio = (f/d)

Geometric Distortions

→ Pincushion, Barrel
Chromatic Abberation

Resolution ⇒ $\boxed{\text{Columns} - \text{Rows.}}$

Q. $\frac{7}{50} = \frac{\cancel{1.75}\ \cancel{7}}{x}\ 4$

$x = \frac{\cancel{50}^{25} \times 8.2}{4 \times 7} \left(\frac{100}{7}\right) = 12.25 \text{ mm} = 1.25 \text{ cm}$

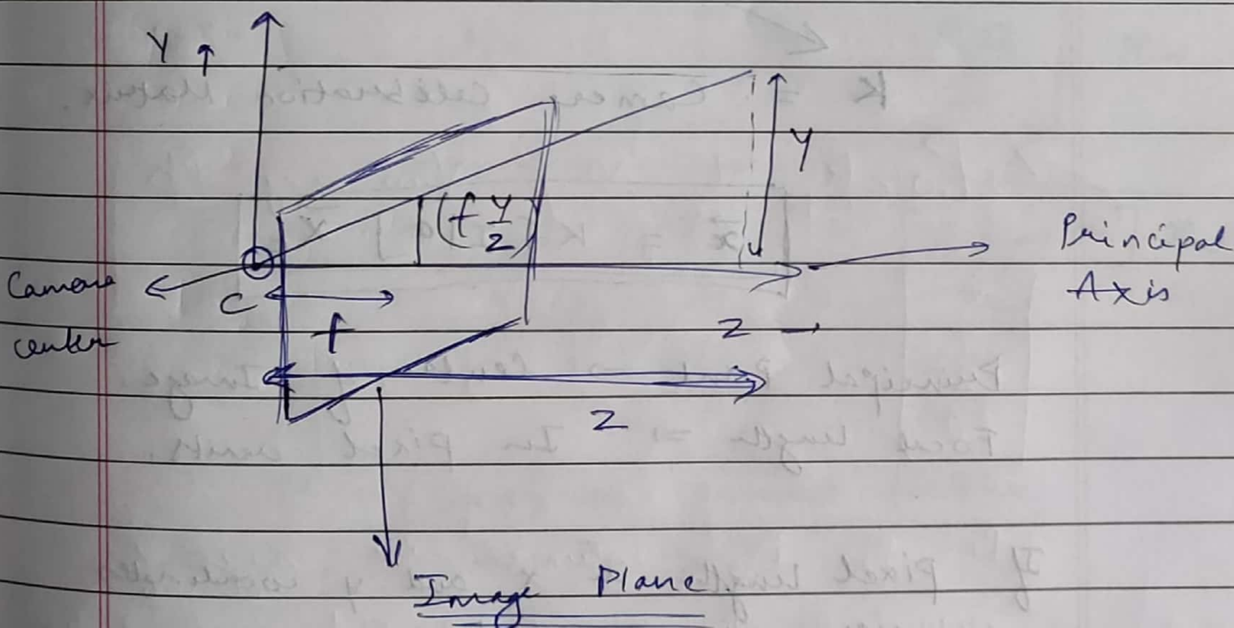
30 mm — 3000

1 → 10.

$\boxed{\cancel{128 \text{ pixels}}}$ $\boxed{\cancel{367.5 \text{ px}}}$

→ $\boxed{1250 \text{ pixels.}}$

8-1-19. Perspective Projection



$x = \frac{fX}{Z}$ $y = \frac{fY}{Z}$

Non-linear.

HOMOGENEOUS COORDINATES

Convert Non-linear Eq. to linear

$$\begin{bmatrix} x/w \\ y/w \end{bmatrix} \leftarrow \underbrace{\begin{bmatrix} x \\ y \\ w \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}}_{\vec{X}_c}$$

Image coordinates — Homogeneous Image coordinates — Camera Matrix — World coordinates

Camera center is assumed to be at origin and Z-Axis is principal Axis.

$$P = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} [I|0]$$

K = Camera Calibration Matrix.

$$\boxed{\vec{x} = K[I|0]\vec{X}_c}$$

Principal Point ⇒ Center of Image.
Focal length ⇒ In pixel units.

If pixel length in x and y coordinates is different then focal length f_x and f_y exists.

General Camera:

→ Image Centre at (x_0, y_0)

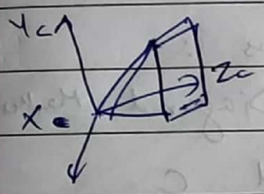
Non orthogonal axes with skew s.

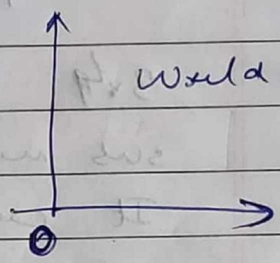
$$K = \begin{bmatrix} f_x & s & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

↙ Upper Diagonal Matrix

(5 DOF)

Camera system





$$X_c = \begin{bmatrix} R & -RC \\ 0 & 1 \end{bmatrix} x_w$$

$$\boxed{x = K[I|0]X_c = \underbrace{K[R|-RC]}_{P} X_w}$$

$P = (3 \times 4) \Rightarrow$ Camera Matrix.

→ $[P_1 \; P_2 \; P_3 \; P_4]$

→ $\begin{bmatrix} p^1 \\ p^2 \\ p^3 \end{bmatrix}$

→ $[KR \,|\, -KRC]$

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Interest Point Detection & Description

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General Camera Eqn.

$$x = K\underbrace{[R \mid -RC]}_{P} X_w$$

projects World C to Image - C

* Orthographic projection ⇒ Left submatrix is singular.

* 3×4 matrix P with Non-Singular left sub matrix represents a camera.

It can be decomposed as:

→ Non - Singular Upper Diagonal Matrix K

→ Orthonormal vector R and C.

Points at ∞ / Vanishing Pts.

$$\begin{bmatrix} X \\ Y \\ Z \\ 0 \end{bmatrix}$$

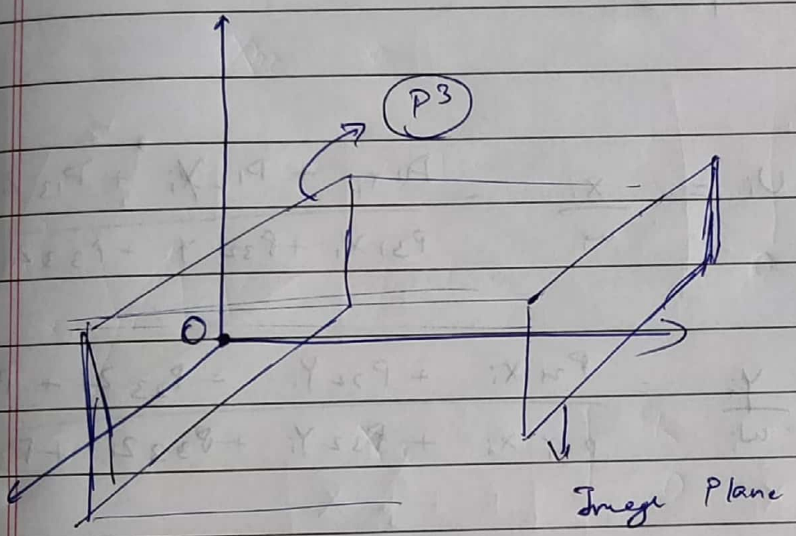
* P_1, P_2, P_3 columns are images of vanishing points of world X, Y, Z dir.

P_4 represents origin coordinates in Homogeneous frame.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

If $w = 0$ then image pt. lies at ∞

$\Rightarrow$ Row vector P^3 is the principal plane.



Q. Image centre is given by $x_0 = M m_3$

where m_3 is the third row vector of Matrix M.

Q. $\det(M)\, m_3$ gives the principal axis as a vector from camera centre through principal pt. to the front of camera.

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Camera Calibration

1. 3D Reference Object based calibration
2. Calibration from precisely Moving plane.
3. Calibration using a plane with unknown motion.
4. Collibration from set of collinear point that moves s.t. lines passing through fixed pt.

$$u_i = \frac{x_i}{w_i} = \frac{P_{11} x_i + P_{12} Y_i + P_{13} Z_i + P_{14}}{P_{31} X_i + P_{32} Y_i + P_{33} Z_i + P_{34}}$$

$$v_i = \frac{y_i}{w_i} = \frac{P_{21} X_i + P_{22} Y_i + P_{23} Z_i + P_{24}}{P_{31} X_i + P_{32} Y_i + P_{33} Z_i + P_{34}}$$

Decomposing P

$$P = K [R \mid t]$$

$$P = [M \quad p_4]$$

$$M = KR \qquad p_4 = Kt$$

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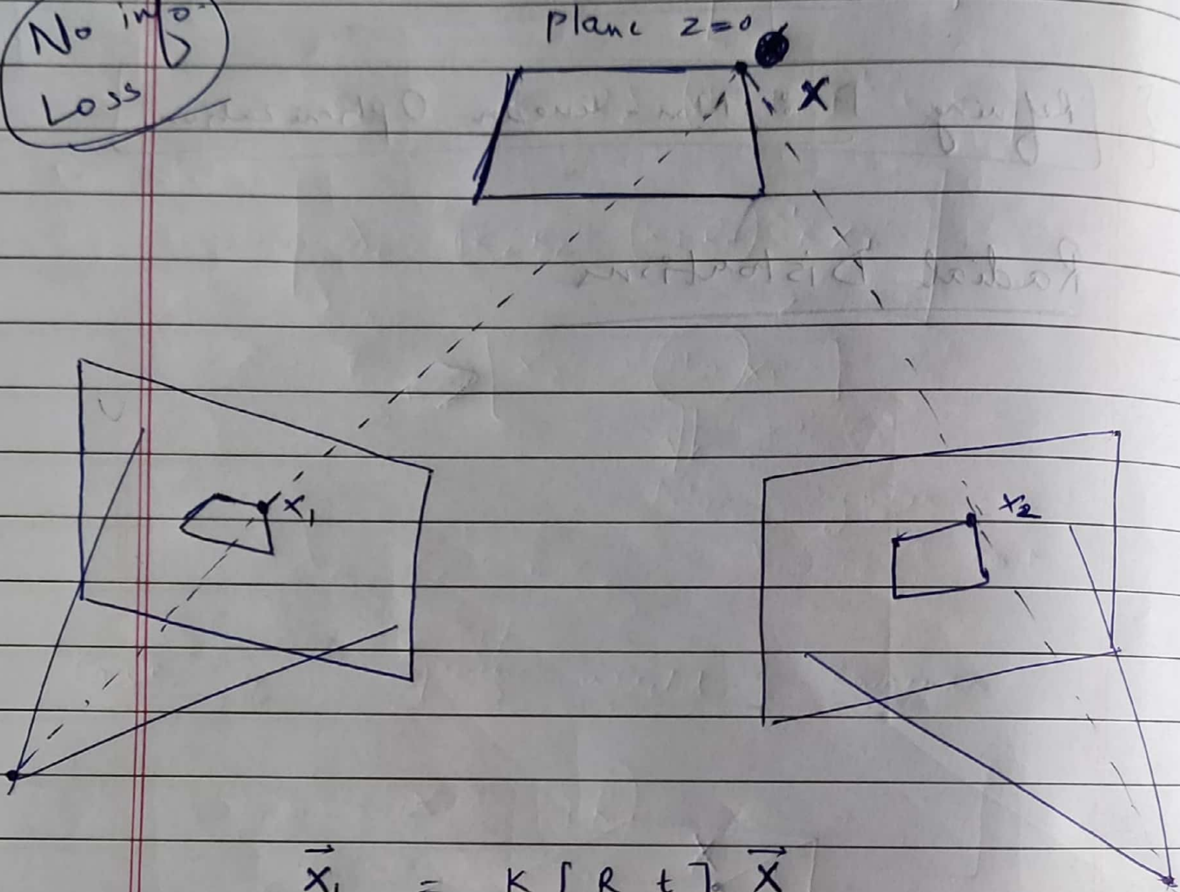
2 View Geometry

Case 1 Planar World

No info Loss



$$\vec{x}_1 = K[R \;\; t]\,\vec{X}$$

$$= K\begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{x}_1 = \underbrace{K\begin{bmatrix} r_1 & r_2 & t \end{bmatrix}}_{H}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\boxed{\vec{x}_1 = H_{3\times 3}\,\vec{X}}$$

$\Downarrow$

Invertible Matrix

$$\vec{x}_1 = H_1 \vec{X}$$
$$\vec{x}_2 = H_2 \vec{X}$$

Since H is an invertible matrix

$$\Rightarrow \boxed{x_1 = H_1 H_2^{-1} x_2}$$

$$\boxed{\vec{x}_1 = H_{12} \vec{x}_2}$$
$$\boxed{\vec{x}_2 = H_{21} \vec{x}_1}$$

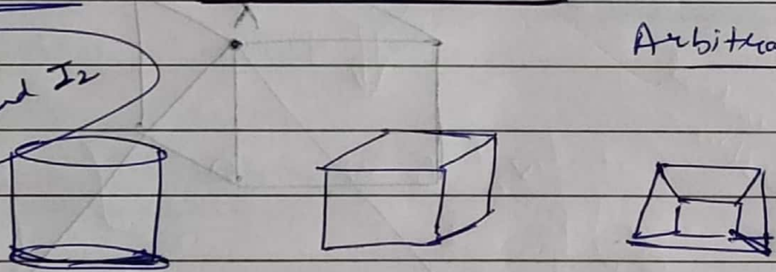
$\boxed{H \longrightarrow \text{HOMOGRAPHY MATRIX.}}$

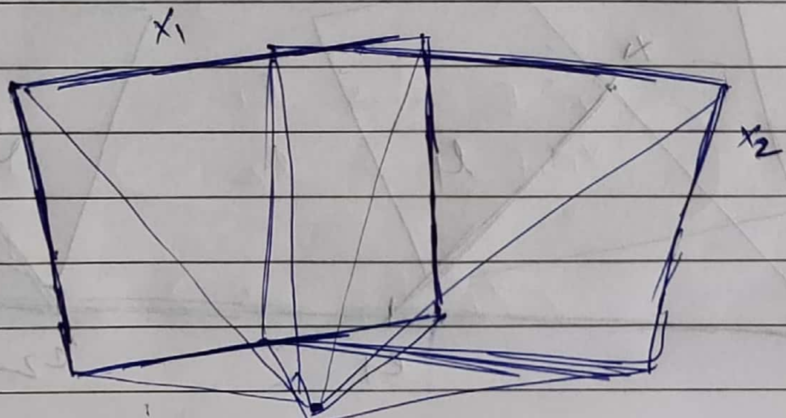
→ If H is singular then it means that the camera centre lies in the same plane as planar world.

Case-2. Same camera centre.

Arbitrary world

Same info loss in I_1 and I_2





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$$x_1 = K_1 R_1 \begin{bmatrix} I & -c \end{bmatrix} X$$

$$x_2 = K_2 R_2 \begin{bmatrix} I & -c \end{bmatrix} X$$

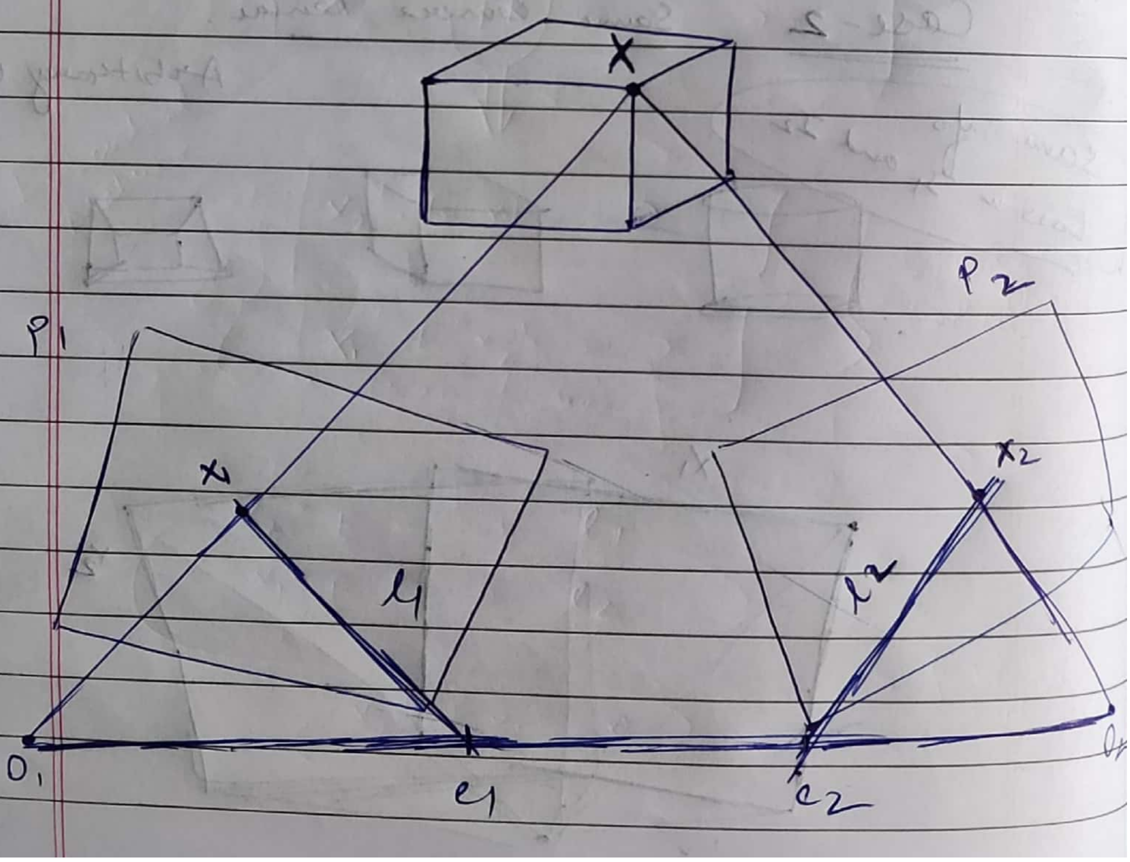
$$x_2 = K_2 R_2 (K_1 R_1)^{-1} K_1 R_1 \begin{bmatrix} I & -c \end{bmatrix} X$$

$$\boxed{x_2 = (K_2 R_2)(K_1 R_1)^{-1} x_1}$$

$$\boxed{x_2 = H_{21} x_1}$$

Non - Singular 3x3 Matrix

Case - 3 Generic World & Cameras



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$XO_1O_2 \longrightarrow$ Plane

e_1 lies on P_1

e_2 lies on P_2

$\Rightarrow$ Any pt on l_1 lies on line L_2.

e_1 is image of O_2 ~~on plane~~ from O_1

$\Rightarrow$ e_1 will be a constant.

Similarly e_2 is also constant.

If pt X is varied then the plane XO_1O_2 ~~...~~ varies, but e_1 and e_2 does not change.

All world pts that map to x_1 in I_1 (pre-image of x_1) map to line L_2 in I_2 is called epipolar line.

Epipoles.

Epipolar Plane

Epipolar constraints.

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Considering $K_1, K_2 = I$.

Assuming origin to be at O,

$$\lambda_1 x_1 = X$$
$$\lambda_2 x_2 = RX + T$$

$$\lambda_2 x_2 = R \lambda_1 x_1 + T$$

$$\boxed{\lambda_2 \hat{T} x_2 = \lambda_1 \hat{T} R x_1}$$

$\hat{T}$ → matrix formulated s.t $\hat{T} x T$.
→ cross product Matrix

$$\underbrace{\lambda_2 \vec{x}_2^T \hat{T} \vec{x}_2}_{0} = \lambda_1 \vec{x}_2^T \hat{T} R \vec{x}_1$$

$$\boxed{\vec{x}_2^T (\hat{T}R) \vec{x}_1 = 0}$$

E → Essential Matrix

$$\boxed{x_2^T E x_1 = 0}$$
$$\boxed{x_1^T E x_2 = 0}$$
} Strong Calibration

$x_2^T E \longrightarrow l_1$
$x_1^T E \longrightarrow l_2$

$x_2^T F x_1 = 0$ } Weak callibration

Scanned by CamScanner

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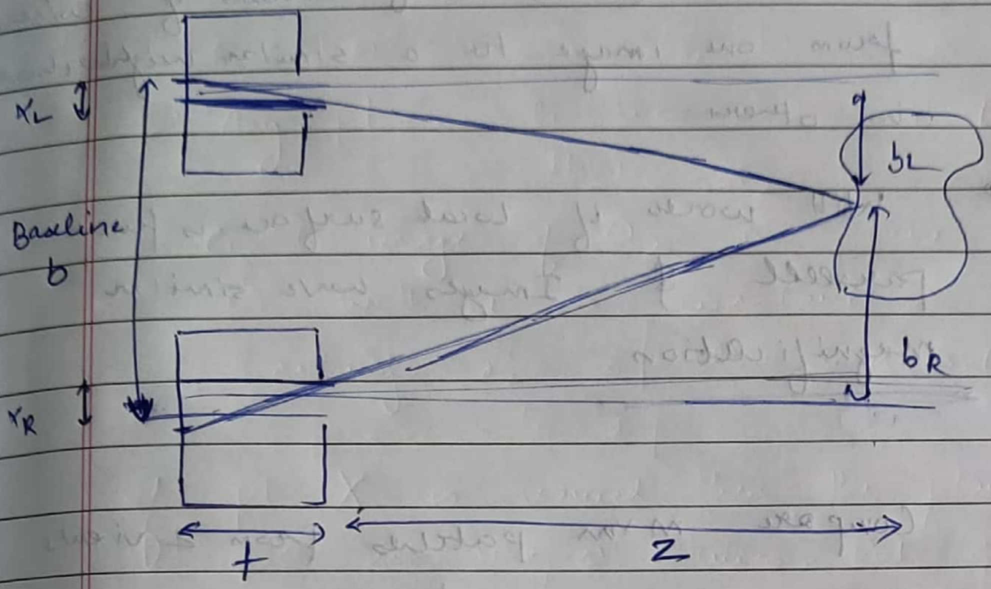
Stereo



$$\frac{b_R}{2} + \frac{b_L}{2} = \frac{x_R}{f} + \frac{x_L}{f}$$

$$\left(\frac{b}{Z}\right) = \frac{x_L + x_R}{f}$$

$$Z = \left(\frac{b \cdot f}{x_L + x_R}\right)$$

A Larger baseline can give more reliable depth, however matching becomes harder.

Scanned by CamScanner

Matching

Match a (small) neighborhood of colors from one image to a similar neighborhood in others.

Will work if local surface is fronto-parallel & Images have similar magnification.

Compare $m \times m$ patches from 2 views.
Form vector v & v'.

Scores ⇒ Sum of Absolute Diff $= \|v - v'\|_1$
Sum of squared dist $= \|v - v'\|_2$

Correlation ⇒ $\dfrac{v'^T v}{\sqrt{v^T v}\ \sqrt{v'^T v'}}$

Normalized Correlation ⇒ $\dfrac{\bar{v}'^T \bar{v}}{\sqrt{\bar{v}^T \bar{v}}\ \sqrt{\bar{v}'^T \bar{v}'}}$
Range $[-1, 1]$

$\boxed{\text{Census Transform}}$ → speed

Use Epipolar constraints

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Constraints on Matchings

→ Epipolar:
→ Color constancy:
→ Uniqueness:
→ Order of or Montonicity.

If pt A is to left of B in view 1 it will to the left of B in view 2 also. Eg: DP.

Continuity ⇒ Disparity values vary smoothly.

Sparse correspondence ⇒ Good feature pts.
Dense "

Rectification ⇒ A rotation of same camera and change in K matrix.

→ Can be represented using aligning one image to another using H.

→ Reproject images onto common parallel plane.

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SIFT INTEREST POINT DETECTOR

LOG vs DOG

$$DOG(x_1, y_1) = \frac{1}{2\pi\sigma_1} e^{-\left(\frac{x_1^2+y_1^2}{2\sigma_1^2}\right)} - \frac{1}{2\pi\sigma_2} e^{-\left(\frac{x_1^2+y_1^2}{2\sigma_2^2}\right)}$$

MOPS Feature Descriptor

Description is based on nearby image region

- Intensity Values
- Moments
- Derivatives
- MOPS | SIFT

MOPS ⟹ 64 length.