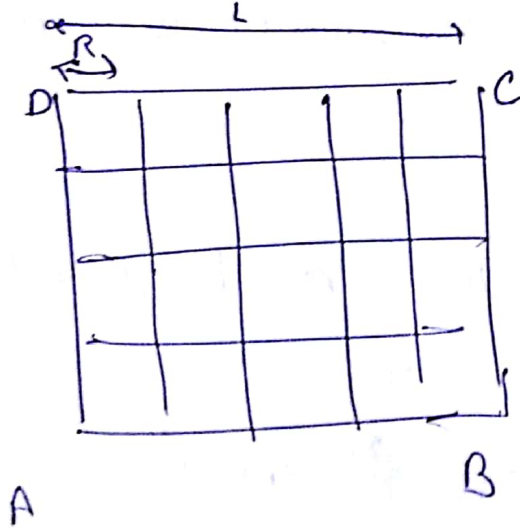


## Project-I

## PART-I (Grid Problem)

Given: A lattice of length  $L$  and  
 $R$  is a grid size  
 $N$  is the total no. of cells.

$$N = L^2 / R^2$$



\* Cat and Rat are on opposite sides.

Let  $N$  be the total number of steps.

$$\text{Let } L = L/R$$

## Assumptions :

- a) To simplify the model, whenever both cat/rat are at the boundary/corner, they have a provision to wait / not move corresponding to the wall.
- b) So i) probability of wait at a boundary =  $1/4$   
 ii) probability of wait at corner =  $1/2$
- (c)  $N > \frac{L\sqrt{2}}{R}$ . The number of steps should be greater than minimum required to meet.

Let us assume the cat and rat to be 2 molecules, which are doing a random walk.

Hence the probability of being at  $n_1, n_2$  at the 2D-space. out of  $N$  steps can be.

if it takes  $x$  steps in  $x$ -dir: then it takes  $N-x$  steps in  $y$  direction.

$$P(n_1) \text{ or Probability of being at } n_1 \text{ after } x \text{ steps.}$$

$$= \frac{x!}{n_1! (x-n_1)!}$$

$x_1$  = steps taken in right,  $x_2$  to steps taken in left

$$x_1 - x_2 = n_1 ; \quad x_1 + x_2 = N$$

$$P(n_1) = \frac{x!}{x_1! x_2!} (p_{\text{right}})^{x_1} (p_{\text{left}})^{x_2}$$

similarly for  $y$ -axis.

$y_1$  = steps up and  $y_2$  = steps down.

$$y_1 + y_2 = N - x ; \quad y_1 - y_2 = n_2$$

$$P(n_2) = \frac{(N-x)!}{y_1! y_2!} (p_{\text{up}})^{y_1} (p_{\text{down}})^{y_2}$$

As the movement is mutually exclusive.

$$P(n_1, n_2) = P(n_1) \cap P(n_2) = P(n_1) \times P(n_2)$$

$$P(n_1, n_2) = \frac{x!}{(x_1! x_2!)} \times \left(\frac{1}{2}\right)^n \times \frac{(N-x)!}{(y_1! y_2!)} \times \left(\frac{1}{2}\right)^n$$

Actual probability would be

$$P = \sum_{x=n_1}^{n_1+n_2} P(n_1, n_2) \Rightarrow$$

when cat and Rat meet

$$P(\text{meet}) = P_{\text{rat}}(n_1, n_2) \times P_{\text{cat}}(n_1, n_2)$$

$$P_{\text{cat}}(n_1, n_2) = P_{\text{rat}}(L-n_1, L-n_2)$$

$$L = L/R \star \star$$

$$x_{\text{cat}} + x_{\text{rat}} = L-n_1$$

$$P(\text{meet}) = P_{\text{rat}}(n_1, n_2) \times P_{\text{rat}}(L-n_1, L-n_2)$$

$$P(\text{meet, at all points}) = \sum_{n_1=0}^L \sum_{n_2=0}^L P_{\text{rat}}(n_1, n_2) \times P_{\text{rat}}(L-n_1, L-n_2)$$

$$\Rightarrow \sum_{n_1=0}^L \sum_{n_2=0}^L \left(\frac{1}{2}\right)^{4N} \frac{x!}{(x_1! x_2!)} \times \frac{(N-x)!}{(y_1! y_2!)} \times \frac{x_{\text{cat}}!}{(x_{1,\text{cat}}! x_{2,\text{cat}}!)} \times \frac{(N-x_{\text{cat}})!}{(y_{1,\text{cat}}! y_{2,\text{cat}}!)}$$

$$\text{Hence } P(\text{meet}) = \sum_{n_1=0}^L \sum_{n_2=0}^L \left(\frac{1}{2}\right)^{4N} \frac{x_{\text{rat}}!}{(x_{1,\text{rat}}! x_{2,\text{rat}}!)} \times \frac{(N-x_{\text{rat}})!}{(y_{1,\text{rat}}! y_{2,\text{rat}}!)} \times \frac{x_{\text{cat}}!}{(x_{1,\text{cat}}! x_{2,\text{cat}}!)} \times \frac{(N-x_{\text{cat}})!}{(y_{1,\text{cat}}! y_{2,\text{cat}}!)}$$

$$\text{where } \Rightarrow \cancel{n_1 + n_2 = N}$$

$$\begin{aligned} x_{\text{rat}} + x_{\text{cat}} &= \cancel{N} \quad x_{1,\text{cat}} + x_{2,\text{cat}} = \cancel{N} - x_{\text{rat}} \\ x_{\text{rat}} - x_{\text{cat}} &= n_1 \quad x_{\text{rat}} - x_{\text{cat}} = L - n_1 \\ y_{1,\text{rat}} + y_{2,\text{rat}} &= N - x_{\text{rat}} \quad y_{1,\text{cat}} + y_{2,\text{cat}} = N - x_{\text{rat}} \\ y_{1,\text{rat}} - y_{2,\text{rat}} &= n_2 \quad y_{1,\text{cat}} - y_{2,\text{cat}} = L - n_2 \end{aligned}$$



By substituting everything in terms of  $n_1, n_2$  and  $N$  followed by calculating the summation.

We would get the value of  $P(\text{meet})$ .

$$\begin{aligned}
 P(\text{meet}) &= \sum_{n_1=0}^L \sum_{n_2=0}^L \sum_{n_{\text{rat}}=0}^L \left(\frac{1}{2}\right)^{4N} \frac{(n_1+n_2)! (N-n_{\text{rat}})!}{(n_1+n_{\text{rat}})! (n_2+n_{\text{rat}})!} \times \frac{1}{(L-n_1+n_{\text{rat}})! \left(\frac{n_{\text{rat}}+n_1+L}{2}\right)! \left(\frac{L-n_2+N-n_{\text{rat}}}{2}\right)!} \\
 &\Rightarrow \sum_{n_{\text{rat}}=0}^L \sum_{n_1=0}^L \sum_{n_2=0}^L \left(\frac{1}{2}\right)^{4N} \frac{(n_{\text{rat}})! (N-n_{\text{rat}})!}{\left(\frac{n_1+n_{\text{rat}}}{2}\right)! \left(\frac{n_2+n_{\text{rat}}}{2}\right)! \left(\frac{L-n_1+n_{\text{rat}}}{2}\right)! \left(\frac{n_{\text{rat}}+n_1+L}{2}\right)! \left(\frac{L-n_2+N-n_{\text{rat}}}{2}\right)!} \\
 &\Rightarrow \sum_{n_{\text{rat}}=0}^L \sum_{n_1=0}^L \sum_{n_2=0}^L \left(\frac{1}{2}\right)^{4N} \frac{(n_{\text{rat}})! (N-n_{\text{rat}})!}{\left(\frac{n_1+n_{\text{rat}}}{2}\right)! \left(\frac{n_2+n_{\text{rat}}}{2}\right)! \left(\frac{L-n_1+n_{\text{rat}}}{2}\right)! \left(\frac{n_{\text{rat}}+n_1+L}{2}\right)! \left(\frac{L-n_2+N-n_{\text{rat}}}{2}\right)!}
 \end{aligned}$$

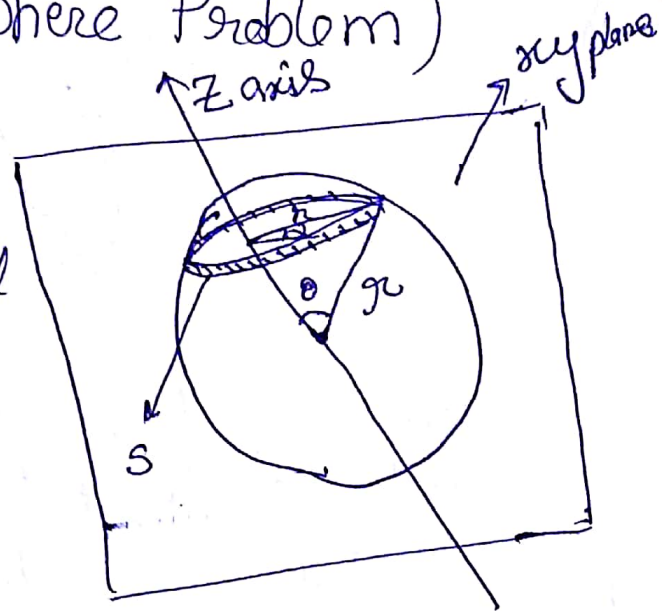
We can calculate this using a PC and Hence

$$P(\text{Survival of Rat}) = 1 - P(\text{meet})$$

Hence Probability of meet depends of  $N$  and  $L$   
if  $N < L\sqrt{2}$  the particles will never meet.

## PART-2 (Sphere Problem)

→ Z-axis is the line cutting the sphere and emerging out of the sphere at another point.



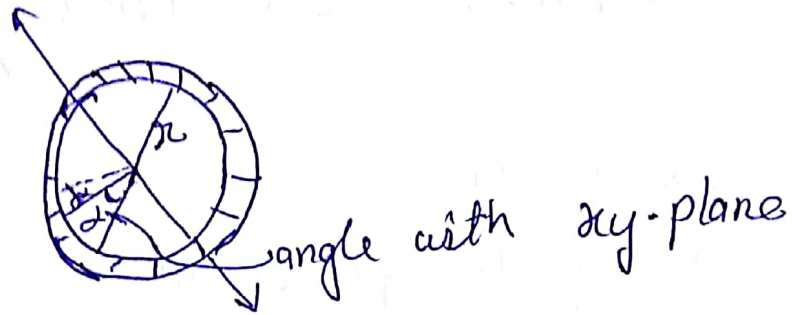
→ Also, there is a plane  $xy$ , that has ~~Z~~ axis on it, which cuts the sphere into 2 hemispheres.

→ Blind cat and a blind rat are present on the sphere. We show their position using

- i)  $\theta$  : angle with the Z-axis
- ii)  $\phi$  : angle with the line of intersection of plane with the plane.

Because: a set of points can have angle  $\theta$  with Z-axis, as in the above example, all the points are on the circumference  $S$ .

The typical circles cut by the  $xy$  plane of the sphere will look like



Probability of being at a point which make angle  $\alpha$  with the  $xy$  plane.

$$P(\alpha) = \frac{r d\alpha}{2\pi r} = \frac{d\alpha}{2\pi}$$

Probability of being at a point which makes an angle  $\theta$  with the  $Z$  axis

$\Rightarrow$  Area occupied by the circular strip  
total area of the sphere.

$$\Rightarrow \frac{(r d\theta) (2\pi r \sin\theta)}{4\pi r^2} = \frac{\sin\theta \cdot d\theta}{2}$$

Hence the probability of being at a point is.

$$P(\theta, \alpha) = \frac{\sin\theta \cdot d\theta \cdot d\alpha}{4\pi}$$

Analogy:

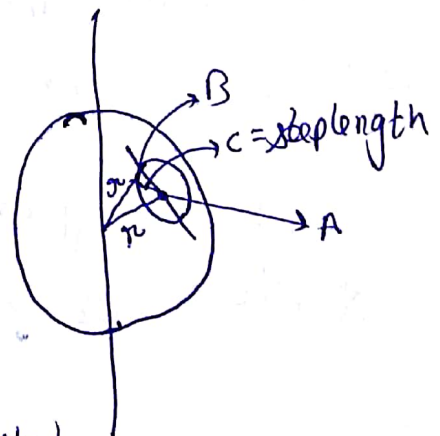
Displacement.

A: original position

B: new position

C = step length

angle of displacement =  $\frac{\text{arc length (step length)}}{\text{radius}}$



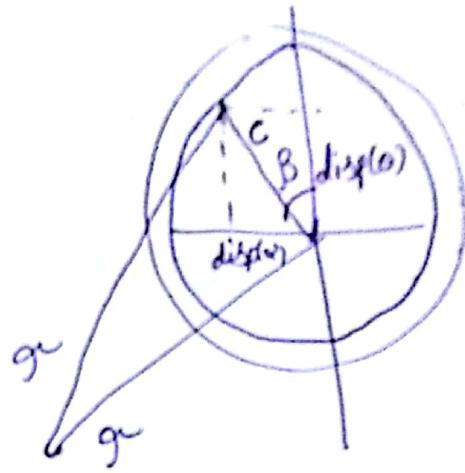


$$\text{disp}'\theta' = \frac{\text{disp}}{r}$$

$$\theta_{\text{new}} = \theta - \frac{c \cos B}{r}$$

$$\text{dis}'r' = \frac{\text{disp}}{r \sin \theta}$$

$$\varphi_{\text{new}} = \varphi + \frac{c \sin B}{r \sin \theta}$$



Assumption:

i) Basically assuming that  $B$  will be small enough so that the helix cat and rat travel individually happens to cover the total surface of the sphere.

Probability Density Function,  $w(\theta, \varphi) d\theta d\varphi = ?$

$$\text{Probability at a point} = \frac{\sin \theta d\theta d\varphi}{4\pi}$$

$$d\varphi \Rightarrow \frac{c \sin B}{r \sin \theta} \quad d\theta = \frac{c \cos B}{r}$$

$$\begin{aligned} w(\theta, \varphi) d\theta d\varphi &= \frac{\sin \theta}{4\pi} \times \frac{(c \sin B \cos B d\theta d\varphi)}{r \sin \theta r} \\ &= \frac{c^2}{4\pi r^2} \sin B \cos B d\theta d\varphi \end{aligned}$$

$$\text{Probability of meet} = \int_0^\pi \int_0^{2\pi} w(\theta, \varphi)^2 d\theta d\varphi$$

$$\Rightarrow \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{2\pi} \frac{c^4}{16\pi^2 r^4} \sin^2 B \cos^2 B \, d\theta \, d\varphi$$

$$\Rightarrow \frac{c^4}{16\pi^2 r^4} \sin^2 B \cos^2 B \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{2\pi} d\theta \, d\varphi$$

$$\Rightarrow \frac{c^4 \sin^2 B \cos^2 B}{8 r^4}$$

Hence Probability of meet  $\Rightarrow P(\text{meet}) = \frac{c^4 \cos^2 B \sin^2 B}{r^4}$

where  $B$  is the angle of displacement.

Probability of Survival of the rab  $\Rightarrow$

$$1 - P(\text{meet}) \Rightarrow 1 - \frac{c^4 \cos^2 B \sin^2 B}{r^4}$$

~~Con~~



## Conclusion: Answer to Questions.

a) Survival/meeting is going to vary with time according to the above equation.

Since the probability of survival decreases with increase in time i.e. probability of survival  $\propto \frac{1}{(\text{time})^2}$ ;  $C$  is a constant.

Because at large time, more number of steps have been taken.

b) i) it depends of  $L$  of the square. Hence as  $L \Rightarrow$  length of the grid increases; chance of survival increases.

ii) It depends on the radius of the sphere.

meeting  $\propto \frac{1}{r^4}$ . Hence  $r$  increases, chance of survival increases.

c) i) If the number of steps is too large; probability for each cat and rat to be at a point =  $\frac{1}{L^2}$

Hence probability of meet =  $\frac{1}{L^4}$

Probability of survival =  $1 - \frac{1}{L^4}$

ii) Similarly for sphere portion. If a step is ~~in~~ very small probability of being at a point  $\Rightarrow \frac{1}{4\pi r^2}$

'Survival Probability' =  $1 - \left(\frac{1}{4\pi r^2}\right)^2 = 1 - \frac{1}{16\pi^2 r^4}$

# SCIENCE-1 Project

## Survival Probability of Blind Rat

### COMPUTATIONAL:

For lattice:

```
import random
from matplotlib import pyplot as plt

# size of the grid
N = 8
# No of iterations
EXPERIMENTS = 1000

# No of steps
TIME = 1000

time_array = [i for i in range(0, TIME)]

def valid(row, col):
    if(row < 0 or row >= N or col < 0 or col >= N):
        return False
    return True

def solve(mouseStartRow, mouseStartCol, catStartRow, catStartY):

    frq = [0] * TIME
    probability = [0] * TIME
    diff = [[1, 0], [-1, 0], [0, 1], [0, -1]]

    for i in range(0, EXPERIMENTS):

        mouseRow = mouseStartRow
        mouseCol = mouseStartCol
        catRow = catStartRow
        catCol = catStartY

        for j in range(1, TIME):

            mouse_movement = []
            cat_movement = []

            mouse_movement = random.choice(diff)
            cat_movement = random.choice(diff)
```

```

        if(valid(mouseRow + mouse_movement[0], mouseCol +
mouse_movement[1])):
            mouseRow = mouseRow + mouse_movement[0]
            mouseCol = mouseCol + mouse_movement[1]

        if(valid(catRow + cat_movement[0], catCol + cat_movement[1])):
            catRow = catRow + cat_movement[0]
            catCol = catCol + cat_movement[1]

        if(mouseRow == catRow and mouseCol == catCol):
            frq[j] = frq[j] + 1
            break

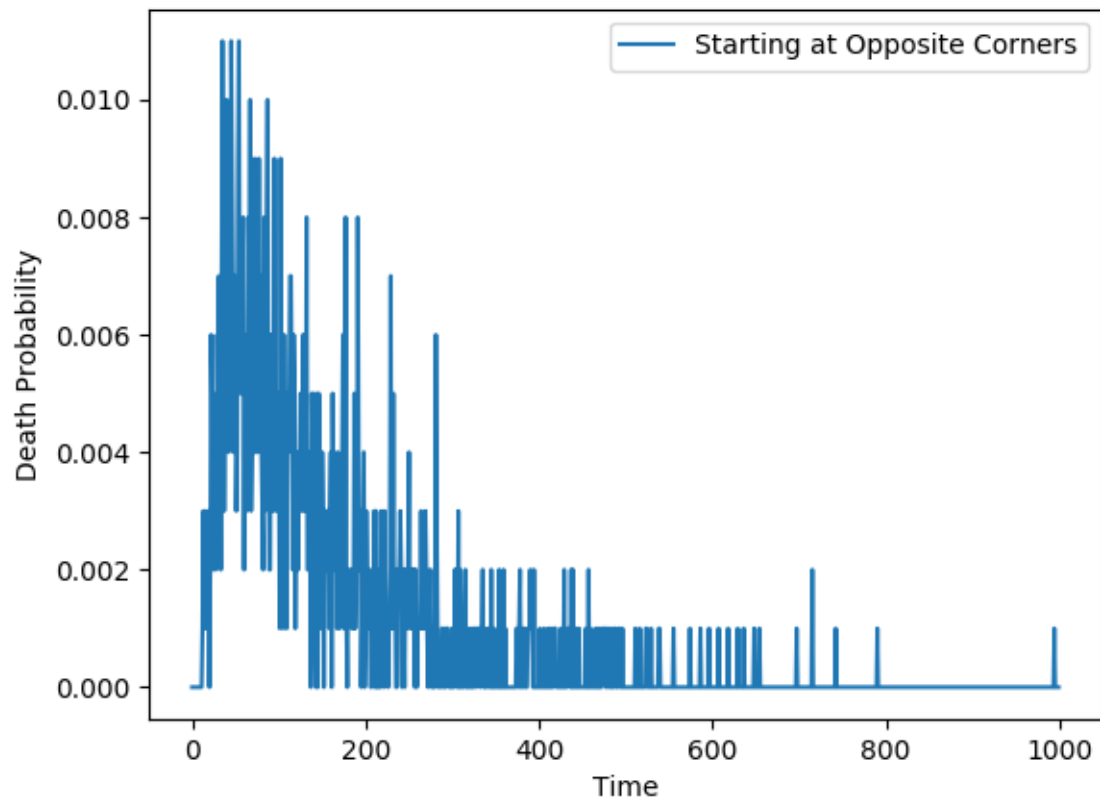
    return [(frq[i] / EXPERIMENTS) for i in range(0, TIME)]

MID = N // 2
Opposite = solve(0, 0, N - 1, N - 1)
Center = solve(MID, MID, MID, MID)

plt.plot(time_array, Opposite, label = "Starting at Opposite Corners")
plt.plot(time_array, Center, label = "Starting at Center")
plt.legend()
plt.ylabel("Death Probability")
plt.xlabel("Time")
plt.show()
plt.savefig("Center_Grid.png")
plt.close()

```





Probability of Meet when CAT and RAT start at opposite ends

Probability of Meet when CAT and RAT start at from center

