Digital Image Processing (CSE/ECE 478)

Lecture 5 : Introduction to Spatial Filters

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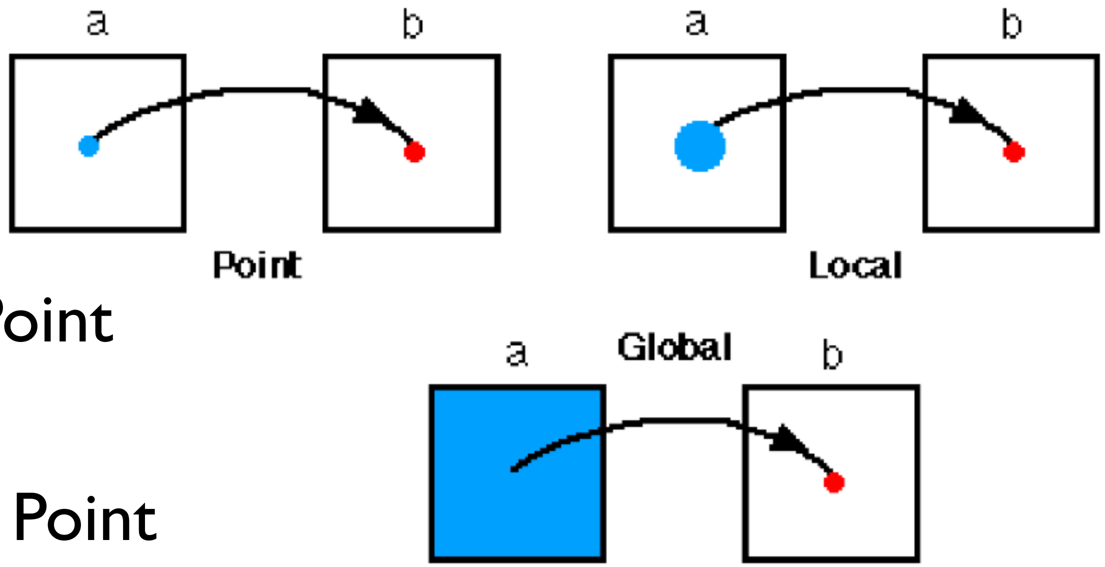
Recap…

Spatial Domain Processing

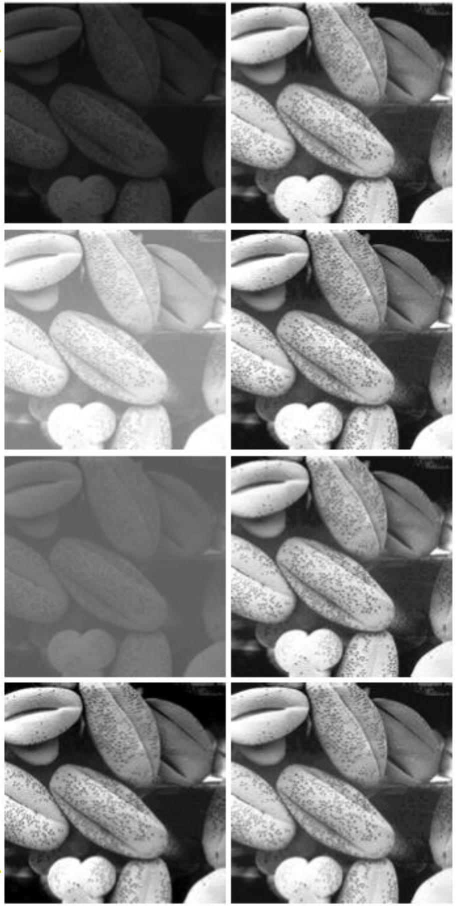
- Manipulating Pixels Directly in Spatial Domain
- Point to Point
- Neighborhood to Point
- Global Attribute to Point

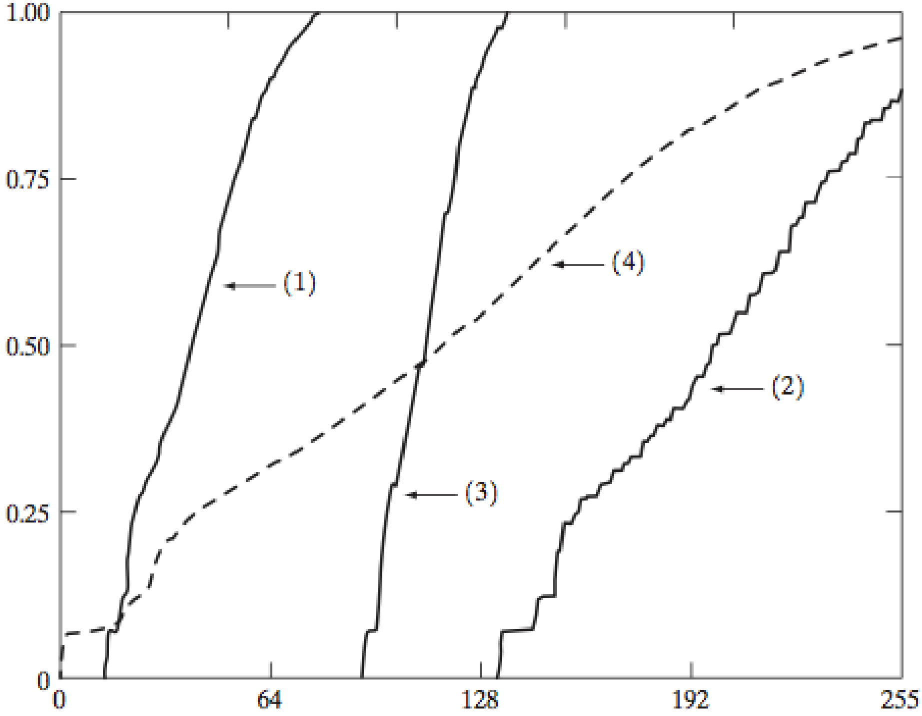


Histogram Processing

- Histogram Stretching/ Contrast Stretching
- Histogram Equalization
- Histogram Specification
- Local Histogram Equalization

Histogram Equalization



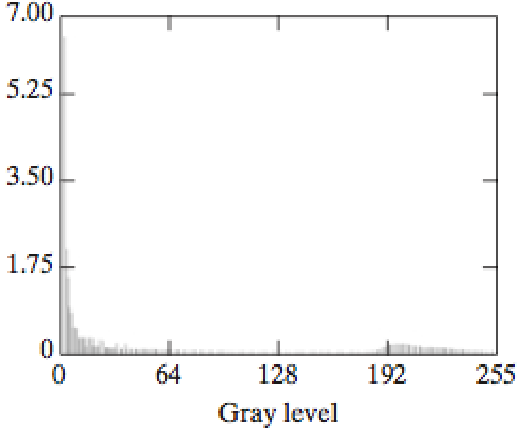


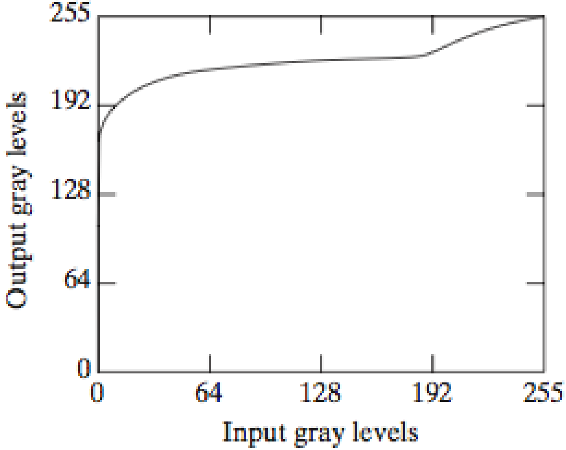
1 through 4 : top to bottom

Image Courtesy: Gonzalez and Woods

Histogram Equalization

Histogram Equalization





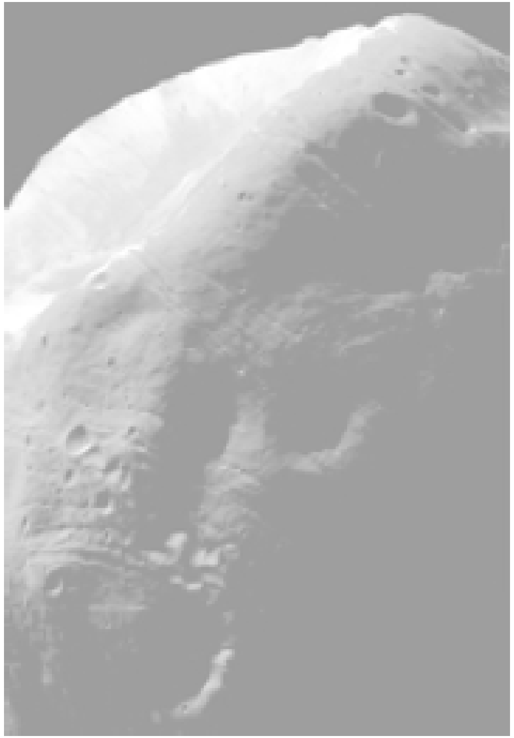


Image Courtesy: Gonzalez and Woods

Histogram Specification / Matching

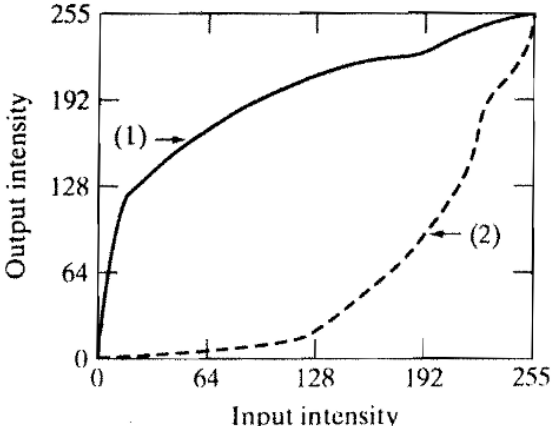
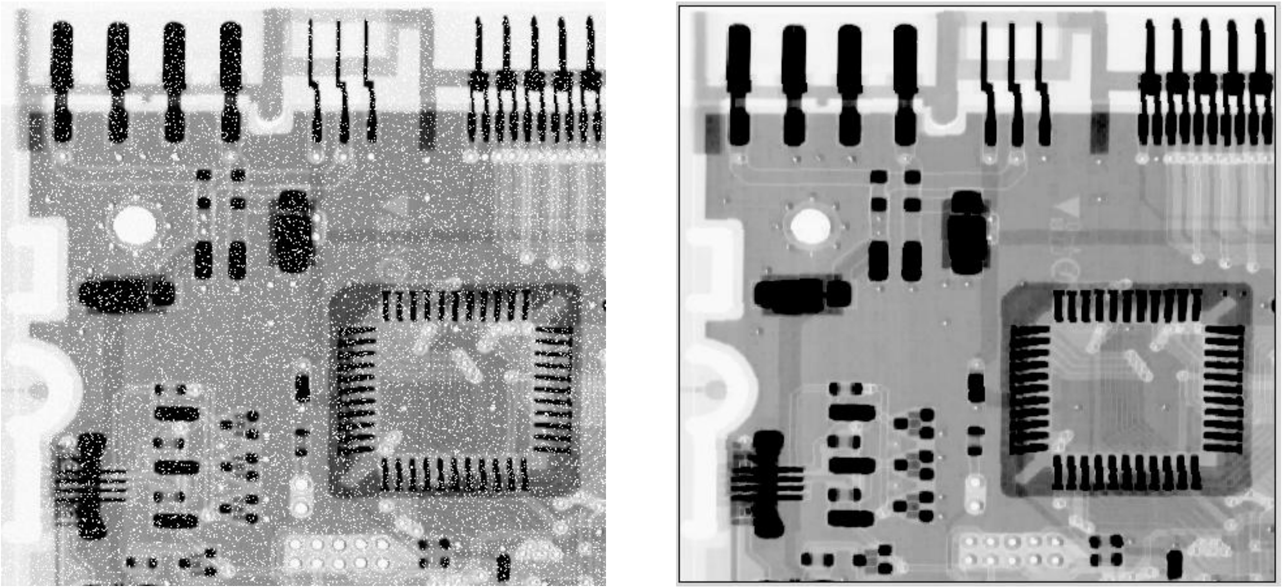


Image Courtesy: Gonzalez and Woods

What point operations can't do?

Image Sharpening

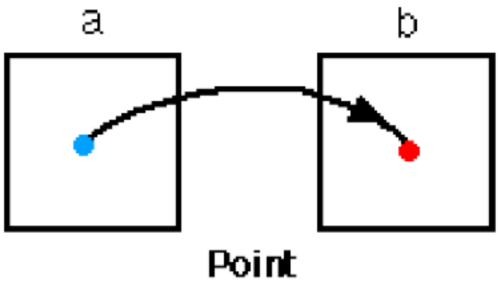
What point operations can't do?

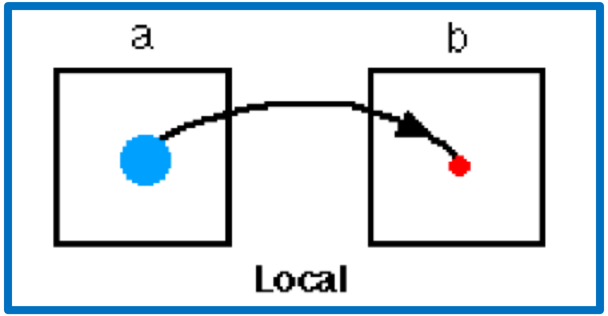


Noise removal

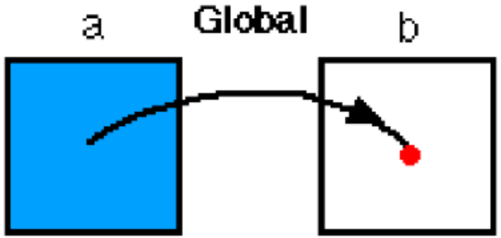
Spatial Domain Processing

- Manipulating Pixels Directly in Spatial Domain
- Point to Point

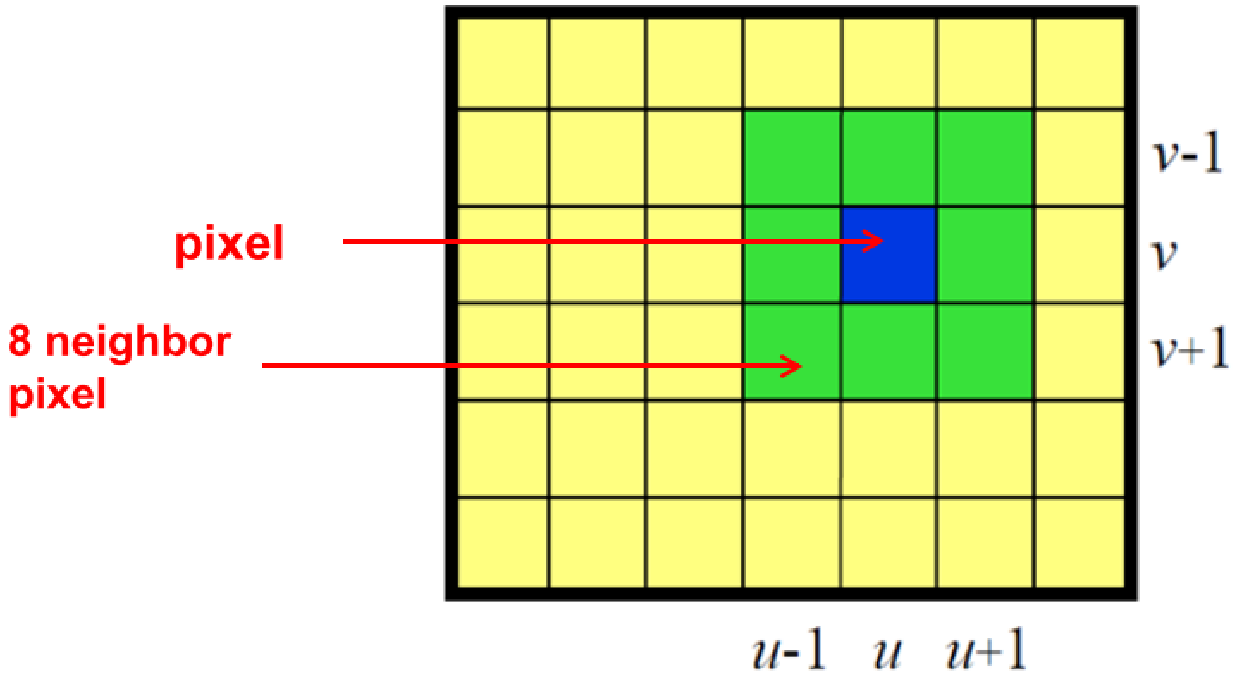




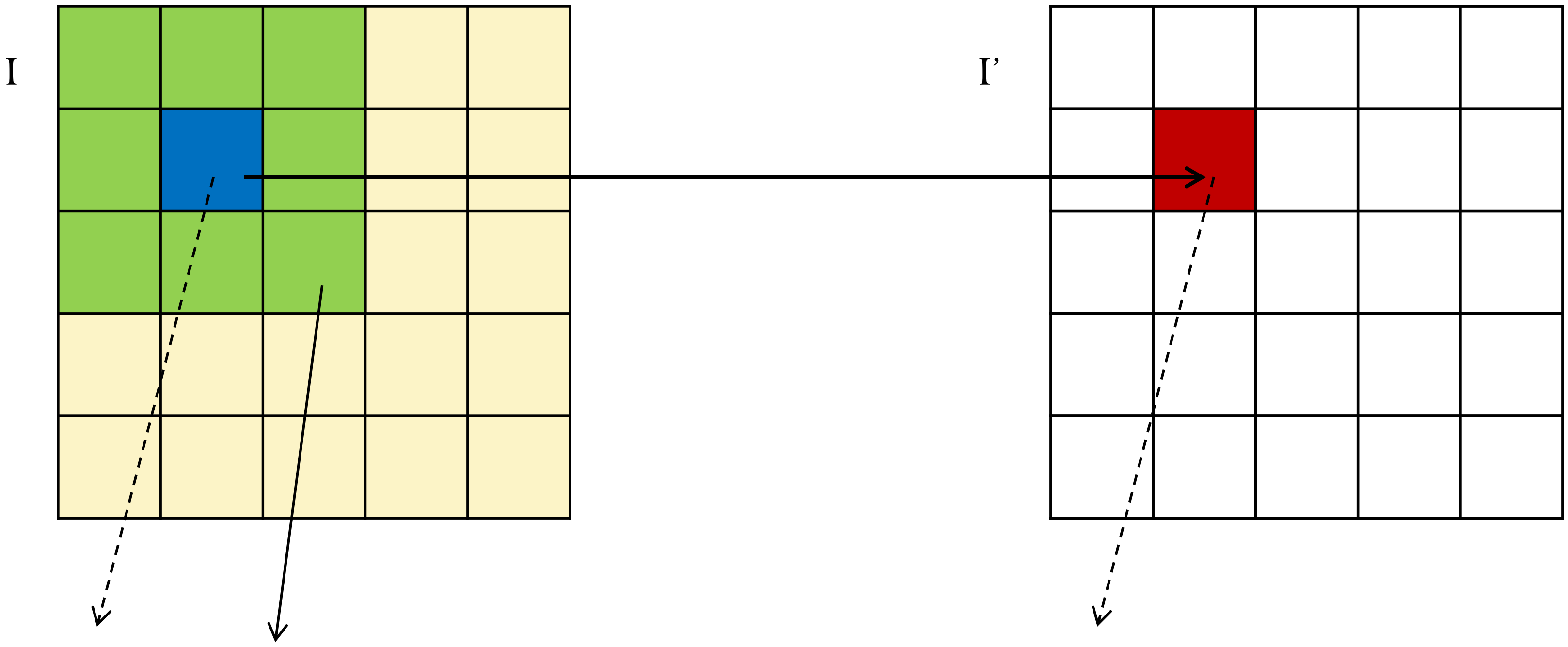
- **Neighborhood to Point**
- Global Attribute to Point



Neighborhood Operation



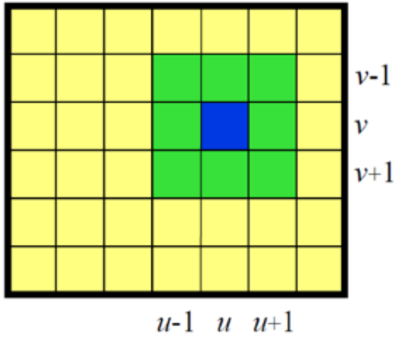
Smoothing Operation

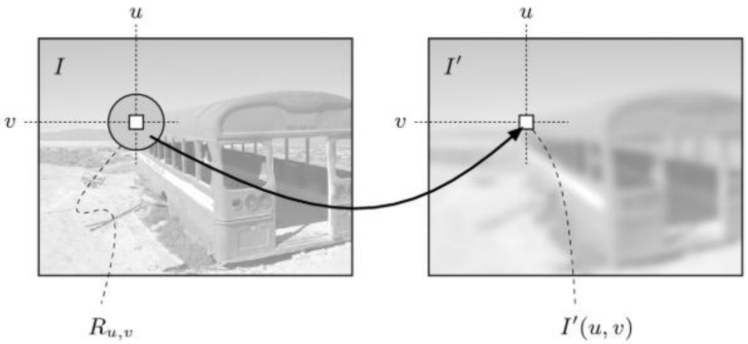


$$I(u, v) \cup N_8(p) = \{ I(u-1, v-1),\ I(u-1, v),\ I(u-1, v+1),$$
$$I(u, v-1),\ I(u, v),\ I(u-1, v+1),$$
$$I(u+1,v-1),\ I(u+1, v),\ I(u+1, v+1) \}$$

$I'(u, v) = ?$

Smoothing Operation





$$I'(u,v) \leftarrow \frac{p_0+p_1+p_2+p_3+p_4+p_5+p_6+p_7+p_8}{9}$$

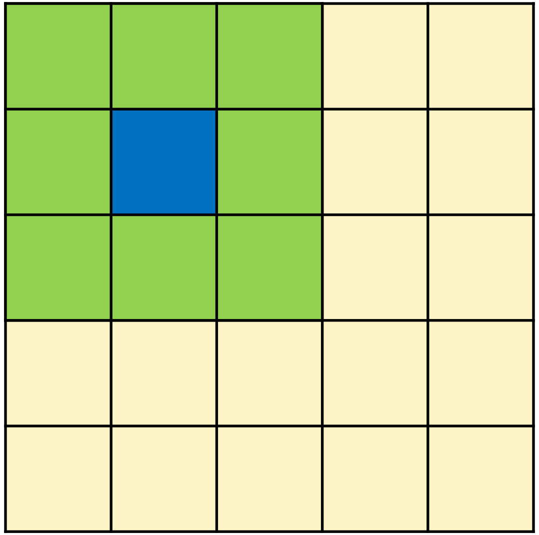
$$\begin{array}{lllll} I'(u,v) \leftarrow \frac{1}{9} \cdot [\, I(u-1,v-1) & +\, I(u,v-1) & +\, I(u+1,v-1) & + \\ \phantom{I'(u,v) \leftarrow \frac{1}{9} \cdot [\,} I(u-1,v) & +\, I(u,v) & +\, I(u+1,v) & + \\ \phantom{I'(u,v) \leftarrow \frac{1}{9} \cdot [\,} I(u-1,v+1) & +\, I(u,v+1) & +\, I(u+1,v+1)\,] & \end{array}$$

$$I'(u,v) \leftarrow \frac{1}{9} \cdot \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i,v+j)$$

Continued …

Smoothing as Averaging

I



H → Weight Mask

Kernel

Filter

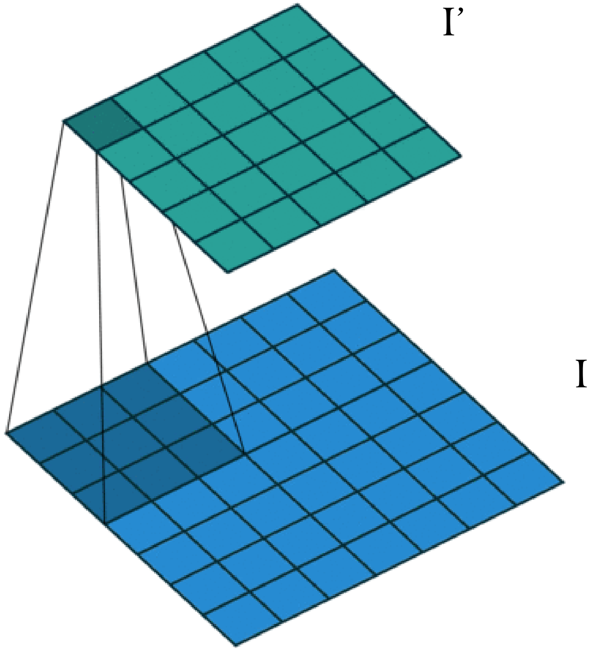
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$$I'(u,v) \leftarrow \frac{1}{9} \cdot \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i, v+j)$$

$$I'(u,v) \leftarrow \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i, v+j)$$

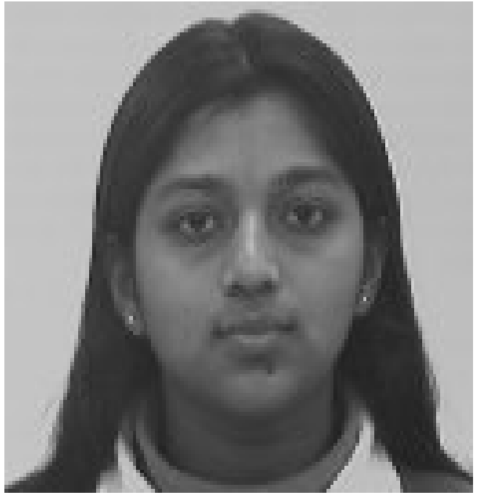
$\boxed{\cdot\ H(i,j)}$

Smoothing Operation



Output size is smaller

Effect of Repeated Smoothing



Before

After

After repeated averaging

Effect of Mask Size

Original Image

[3x3]

[5x5]

[7x7]

Weighted Averaging

$$I'(u,v) = \frac{\sum_{(j=-a)}^{a} \sum_{(i=-b)}^{b} I(u+i, v+j) \cdot H(i,j)}{\sum_{(j=-a)}^{a} \sum_{(i=-b)}^{b} H(i,j)}$$

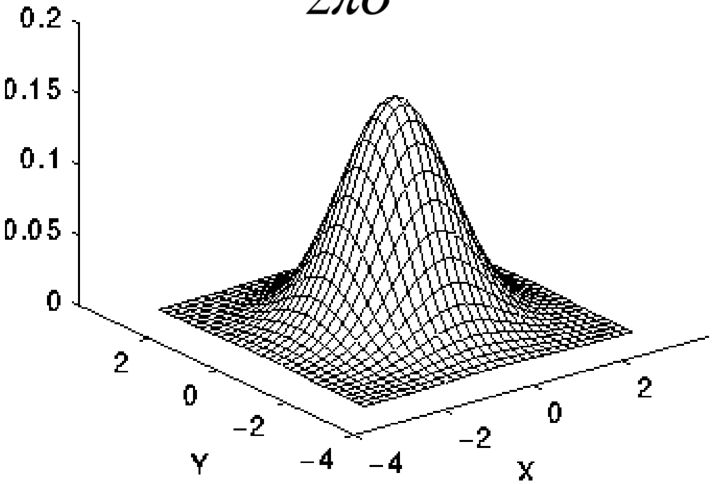
$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Standard average   Weighted average

Gaussian Smoothing

- Mask weights are samples of a Gaussian Function

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2 + y^2)/2\sigma^2\}$$



$\frac{1}{265}$

1	4	6	4	1
4	16	26	16	4
6	26	43	26	6
4	16	26	16	4
1	4	6	4	1

5×5 Gaussian filter, σ=1

- Relation between σ and smoothing?

Spatial Domain Filtering

Notion of Frequency and Filtering

Slowly varying intensities vs.
Rapidly varying intensities

- What is a filter?
- What is filtering operation?

Sharpening Filter

- Objective of sharpening is to highlight fine detail in an image or to enhance detail that has been blurred.
- Smoothing → Averaging → Summation → Integration
- Sharpening → Difference

1-D Derivatives

- First Derivative
  $$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$
  - Zero in flat segments
  - Nonzero at the onset of a step or ramp
  - Nonzero along ramps

- Second Derivative
  $$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x).$$
  - Zero in flat areas;
  - Nonzero at the onset and end of a gray-level step or ramp;
  - Zero along ramps of constant slope

Image Derivatives

f(x-1,y)	*f(x,y)*	*f(x+1,y)*

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

f(x,y-1)
f(x,y)
f(x,y+1)

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

Laplacian Filter

$$\nabla^2 f = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	-4	1
0	1	0

Laplacian Filter

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a	b
c	d

FIGURE 3.37
(a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Implementation

$$I'(u,v) = \begin{cases} I(u,v) - \nabla^2 I(u,v) \\ I(u,v) + \nabla^2 I(u,v) \end{cases}$$

If the center coefficient is negative

If the center coefficient is positive

Where $I(u,v)$ is the original image

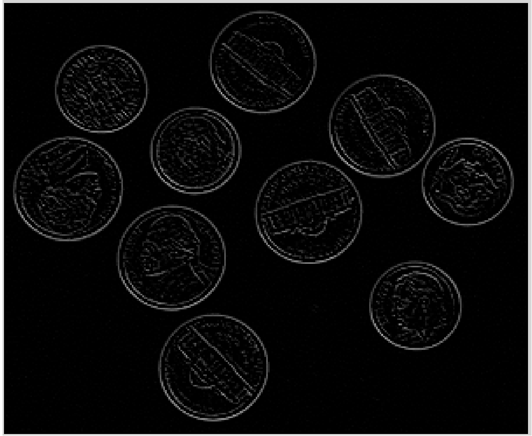
$\nabla^2 I(u,v)$ is Laplacian filtered image

$I'(u,v)$ is the sharpened image

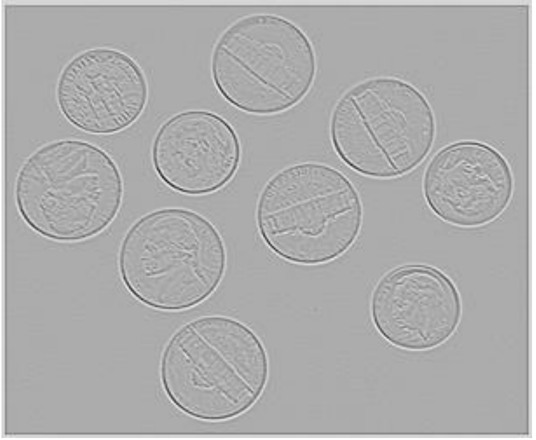
sharpened = range_normalize(input + filtered)

Sharpening with Laplacian Filters

$I(u,v)$



$\nabla^2 I(u,v)$

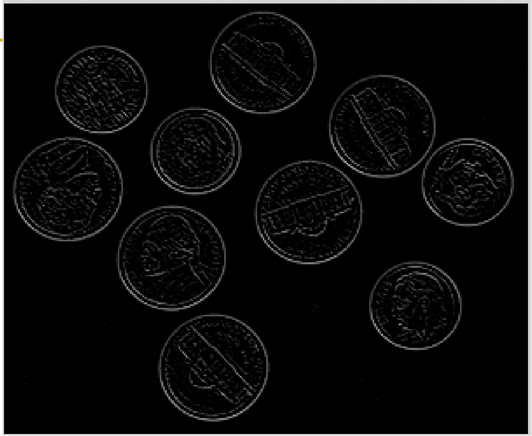


$\nabla^2 I(u,v) + 128$

(For Visualization)

Laplacian Filters

$\nabla^2 I(u,v)$

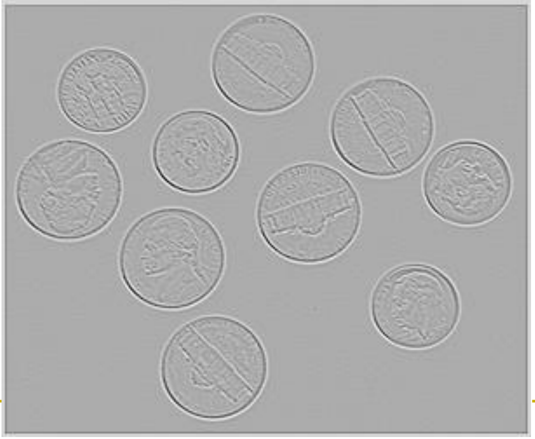


$I(u,v)$

$I'(u,v)$

$\nabla^2 I(u,v) + 128$

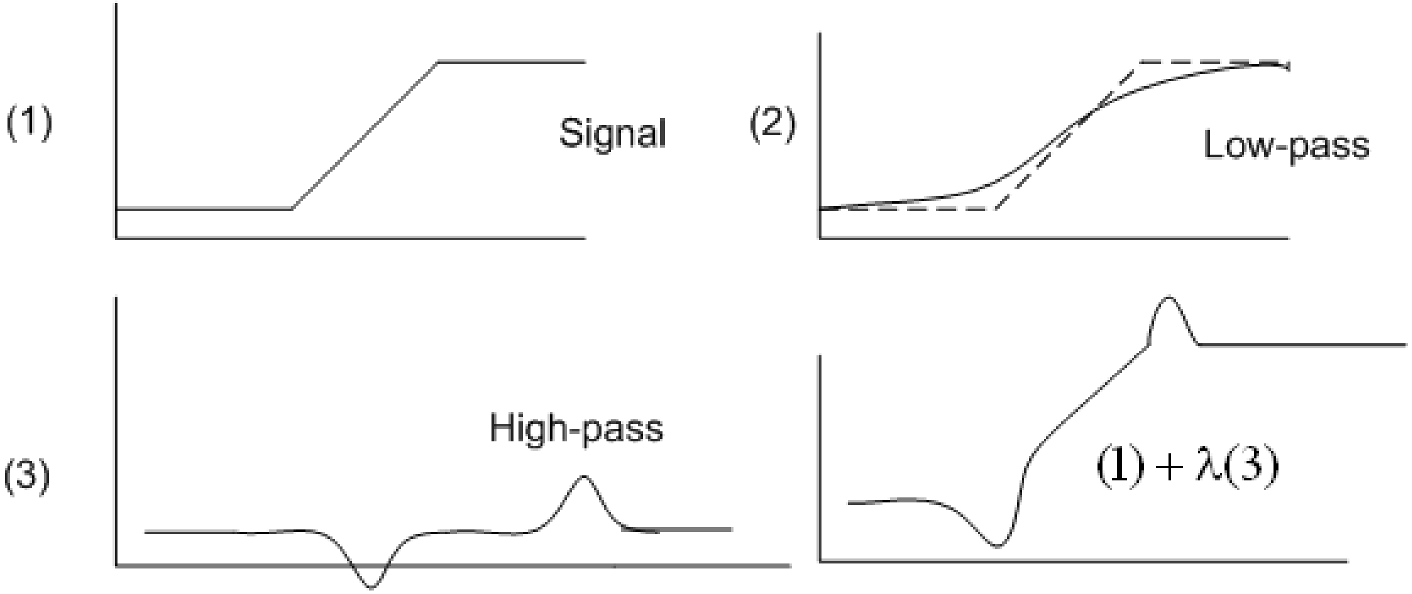
(For Visualization)



Sharpening with Laplacian Filters

Image Courtesy: NASA

Unsharp Masking (and Highboost Filtering)



$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y), \quad g(x,y) = f(x,y) + k * g_{mask}(x,y).$$

Unsharp Masking (and Highboost Filtering)

- **High boost filter**: amplify input image, then subtract a lowpass image

$$Highboost = A\ Original - Lowpass$$

$$= (A-1)\ Original + Original - Lowpass$$

$$= (A-1)\ Original + Highpass$$

(A-1) 

+ 

= 

Unsharp Masking / Highboost Filtering

A>=1
w = 9A-1

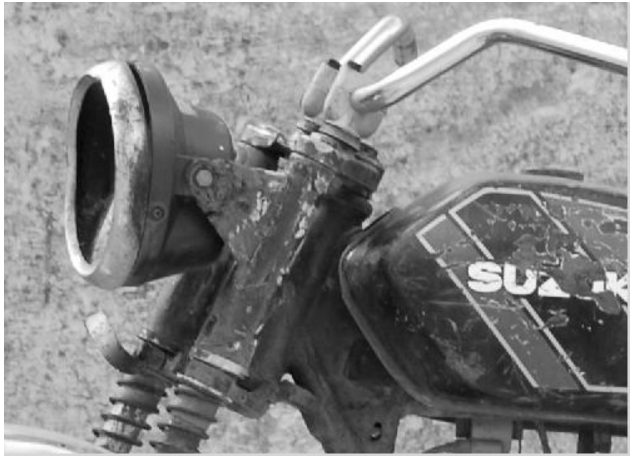
-1	-1	-1
-1	w	-1
-1	-1	-1

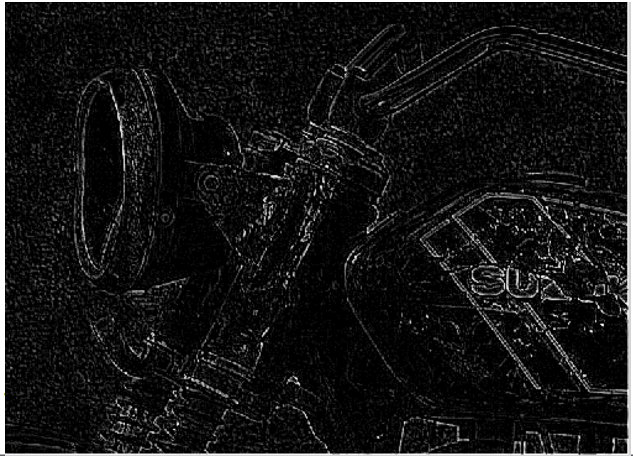
A=2
w = 17

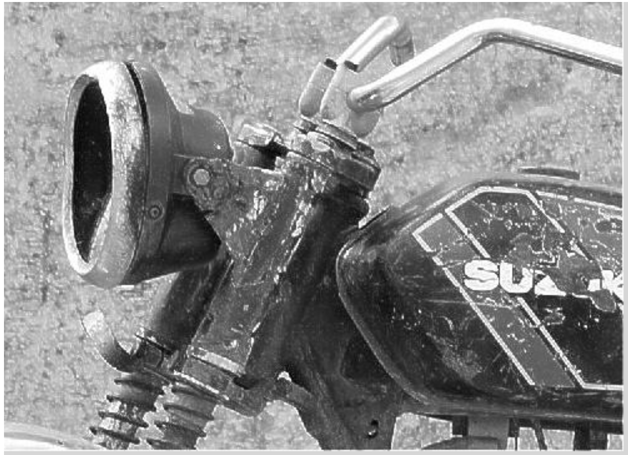
-1	-1	-1
-1	17	-1
-1	-1	-1

- If **A=1**, we get unsharp masking.
- If **A>1**, part of the original image is added back to the high pass filtered image.

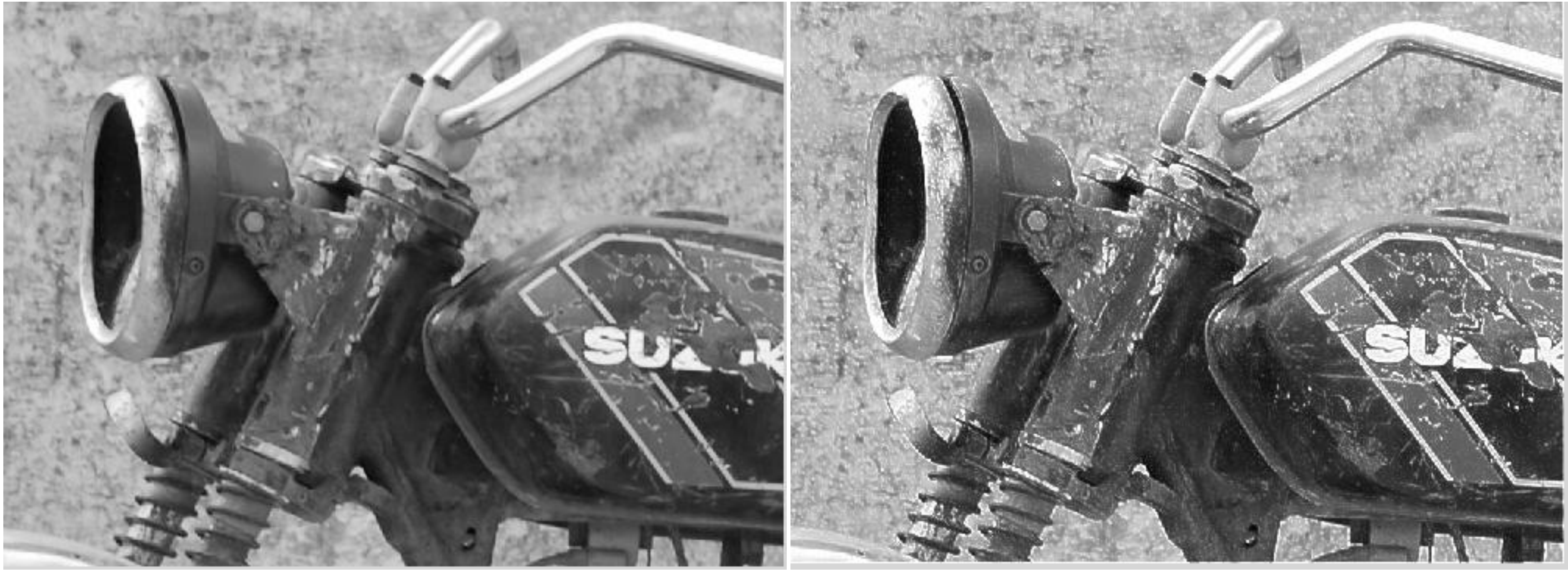
Unsharp Masking (and Highboost Filtering)





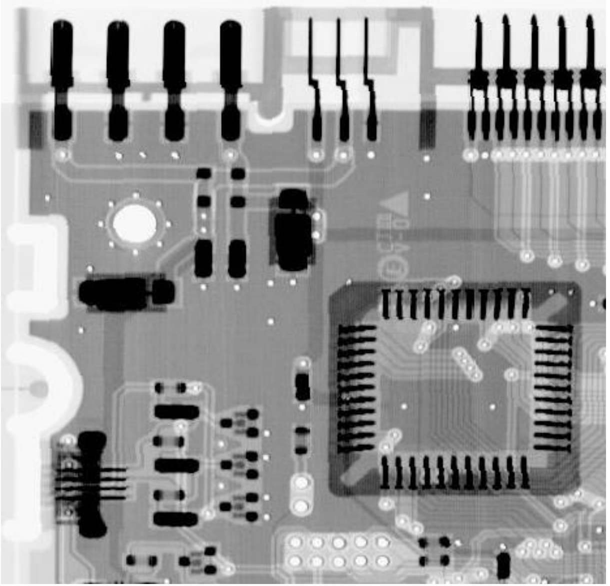


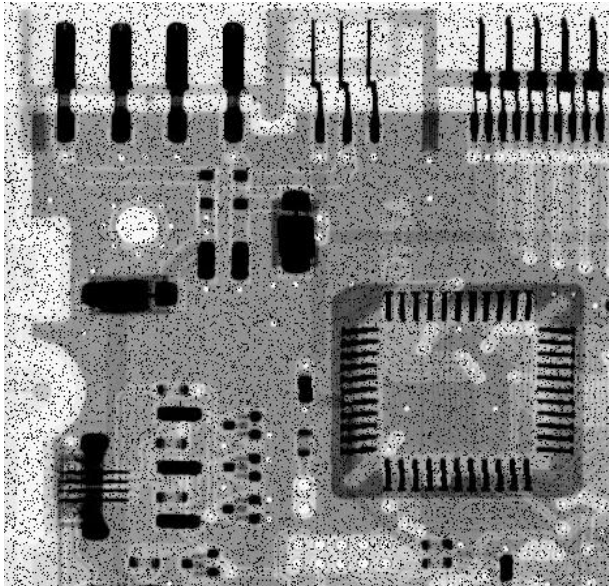
Unsharp Masking (and Highboost Filtering)

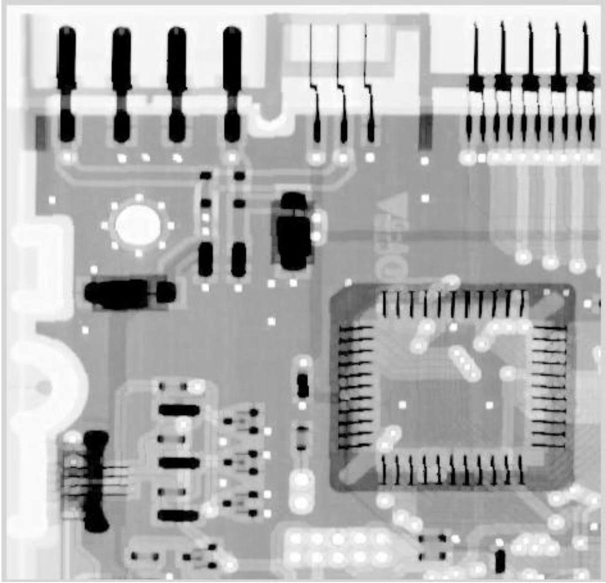


Unsharp Masking (and Highboost Filtering)

Other Spatial Filters (non linear)

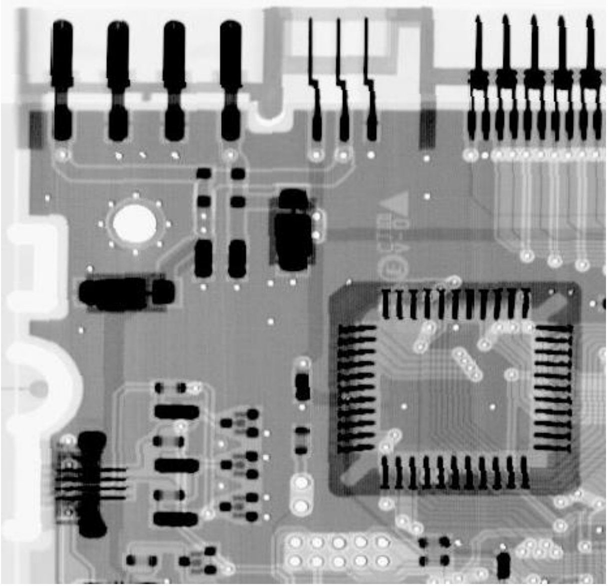
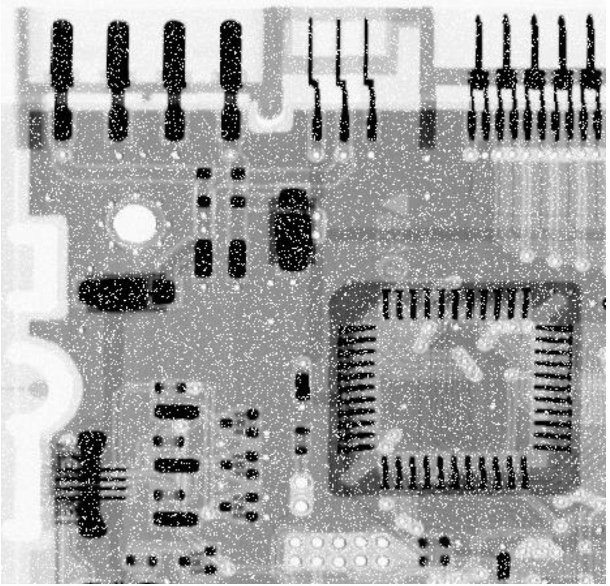
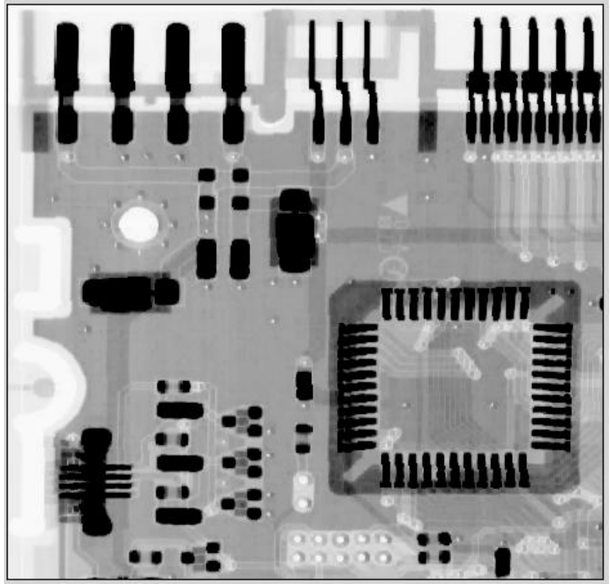






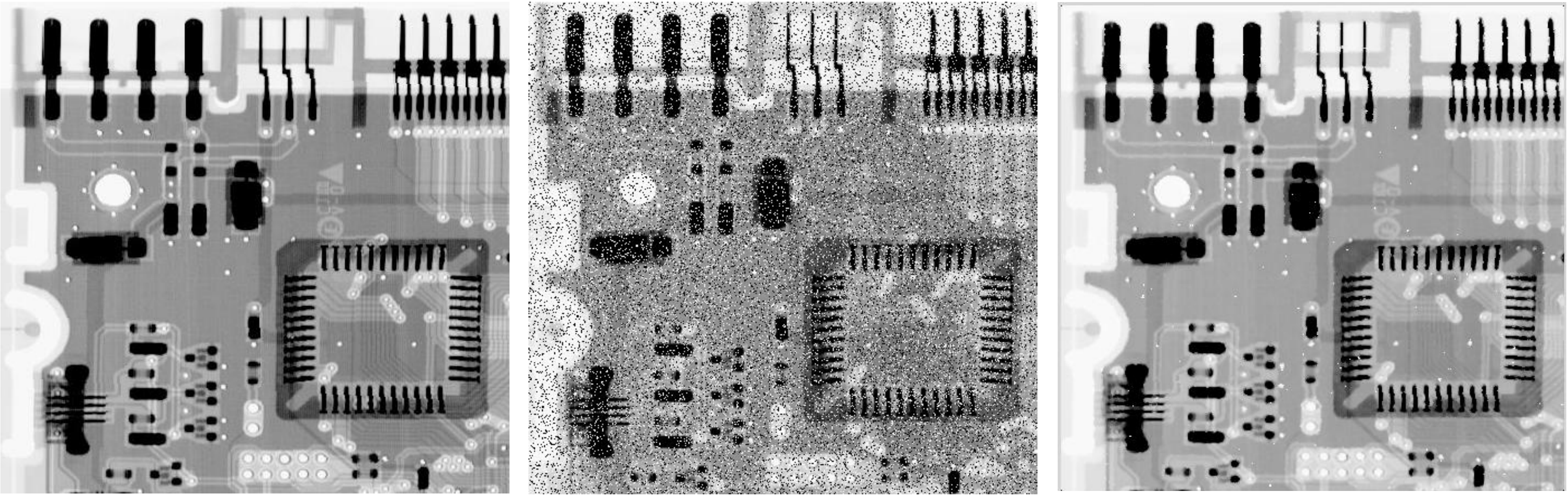
max filter

Other Spatial Filters (non linear)

min filter

Other Spatial Filters (median filter – non linear)



max, min, median → also known as order statistic filters

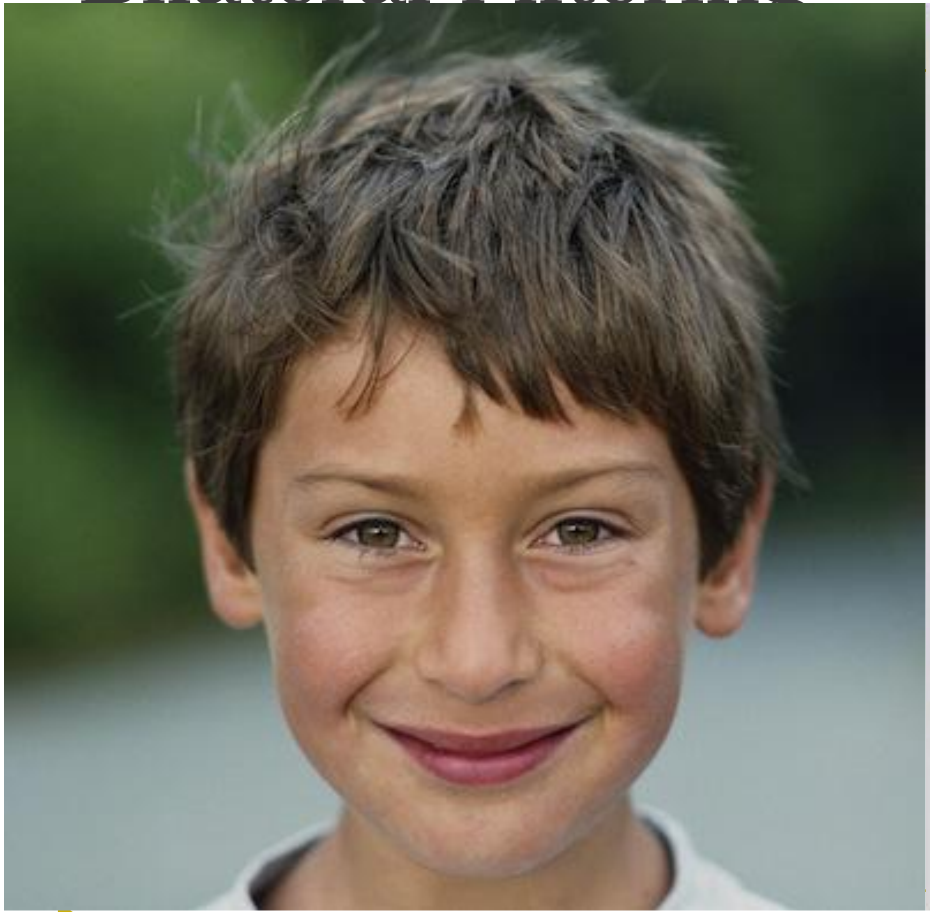
Other Spatial Filters

- Geometric mean
- Harmonic mean
- Contra harmonic mean
- Mid Point filter
- Alpha trimmed mean filter
- …..

Bilateral Filtering

Original image taken from cs.cityu.edu.hk

Bilateral Filtering



Original image from mfullywoodco.hol.es

Bilateral Filtering

References

- GW Chapter – 3.4.1, 3.5, 3.6