SCIENCE-1 Project

Survival Probality of Blind Rat

YOUTUBE LINK: https://youtu.be/5Iw_vimtYN0

COMPUTATIONAL:

For lattice grid:

```
import random
from matplotlib import pyplot as plt


# size of the grid
N = 8
# No of iterations
EXPERIMENTS = 1000

# No of steps
TIME =1000

time_array = [i for i in range(0, TIME)]

def valid(row, col):
	if(row < 0 or row >= N or col < 0 or col >= N):
		return False
	return True

def solve(mouseStartRow, mouseStartCol, catStartRow, catStartY):

	frq = [0] * TIME
	probability = [0] * TIME
	diff = [[1, 0], [-1, 0], [0, 1], [0, -1]]

	for i in range(0, EXPERIMENTS):

		mouseRow = mouseStartRow
		mouseCol = mouseStartCol
		catRow = catStartRow
		catCol = catStartY

		for j in range(1, TIME):

			mouse_movement = []
			cat_movement = []

			mouse_movement = random.choice(diff)
```

```
			cat_movement = random.choice(diff)

			if(valid(mouseRow + mouse_movement[0], mouseCol + mouse_movement[1])):
				mouseRow = mouseRow + mouse_movement[0]
				mouseCol = mouseCol + mouse_movement[1]

			if(valid(catRow + cat_movement[0], catCol + cat_movement[1])):
				catRow = catRow + cat_movement[0]
				catCol = catCol + cat_movement[1]

			if(mouseRow == catRow and mouseCol == catCol):
				frq[j] = frq[j] + 1
				break

	return [(frq[i] / EXPERIMENTS) for i in range(0, TIME)]

MID = N // 2
Opposite = solve(0, 0, N - 1, N - 1)
Center = solve(MID, MID, MID, MID)

plt.plot(time_array, Opposite, label = "Starting at Opposite Corners")
plt.plot(time_array, Center, label = "Starting at Center")
plt.legend()
plt.ylabel("Death Probability")
plt.xlabel("Time")
plt.show()
plt.savefig("Center_Grid.png")
plt.close()
```

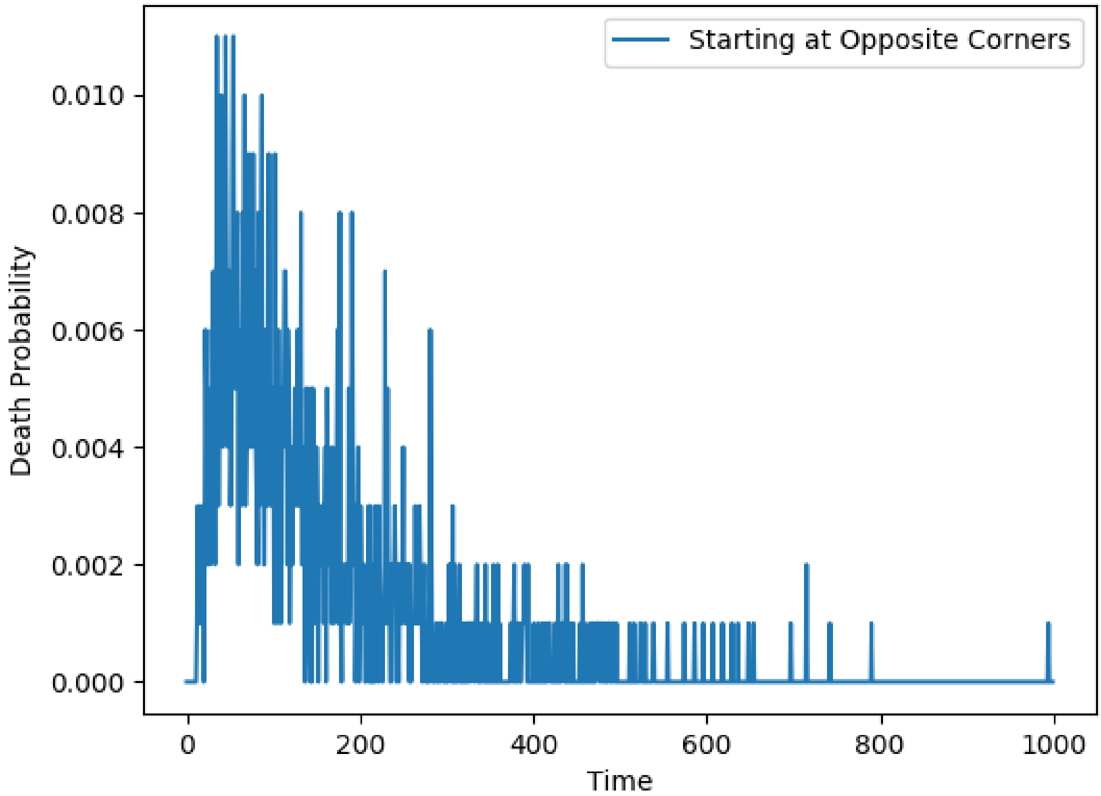
For Sphre:

```
#include<stdio.h>
#include<stdlib.h>
#include<time.h>
#include<math.h>
#define PI 3.1415926
#define dbl double
dbl fn[100005];
dbl save[100005];
int main(){
	dbl rad;
	int i,j;
	dbl count = 0.00,x,y,z;
	dbl rats[3];
	dbl cats[3],fl=0;
rep:
	fprintf(stderr,"R??");
	scanf("%lf",&rad);
	dbl range = rad/10;
```

```
	srand(time(0));
	for(i=0;i<10000;i++){
		rats[0] = -rad;
		rats[1] = rats[2] = cats[1] = 0;
		cats[0] = rad;
		cats[2] = 1;
		fprintf(stderr,"%d\n",i);
		for(j=1;j<=10000;j++){
			int flag=0;
			do{
				dbl theta = (dbl)rand()/(dbl)RAND_MAX;
				theta*=PI;
				dbl phi = (dbl)rand()/(dbl)RAND_MAX;
				phi*=2*PI;
				x = cos(theta);
				y = sin(theta) * cos(phi);
				z = sin(theta) * sin(phi);
				if(sqrt(pow(rats[0]+x,2) + pow(rats[1]+y,2) + pow(rats[2]+z,2)) <=
rad){
					flag=1;
					rats[0]+=x;
					rats[1]+=y;
					rats[2]+=z;
				}
			}
			while(flag==0);
			flag=0;
			do{
				dbl theta = (dbl)rand()/(dbl)RAND_MAX;
				theta*=PI;
				dbl phi = (dbl)rand()/(dbl)RAND_MAX;
				phi*=2*PI;
				x = cos(theta);
				y = sin(theta) * cos(phi);
				z = sin(theta) * sin(phi);
				if(sqrt(pow(cats[0]+x,2) + pow(cats[1]+y,2) + pow(cats[2]+z,2)) <=
rad){
					flag=1;
					cats[0]+=x;
					cats[1]+=y;
					cats[2]+=z;
				}
			}
			while(flag==0);
			if(sqrt(pow(cats[0]-rats[0],2) + pow(cats[1]-rats[1],2) +pow(cats[3]-
rats[3],2)) < range) break;
			else if(fl==0) fn[j]+=1.0;
			else save[j]+=1.0;
		}
	}
	if(fl==0){
		fl=1;
```

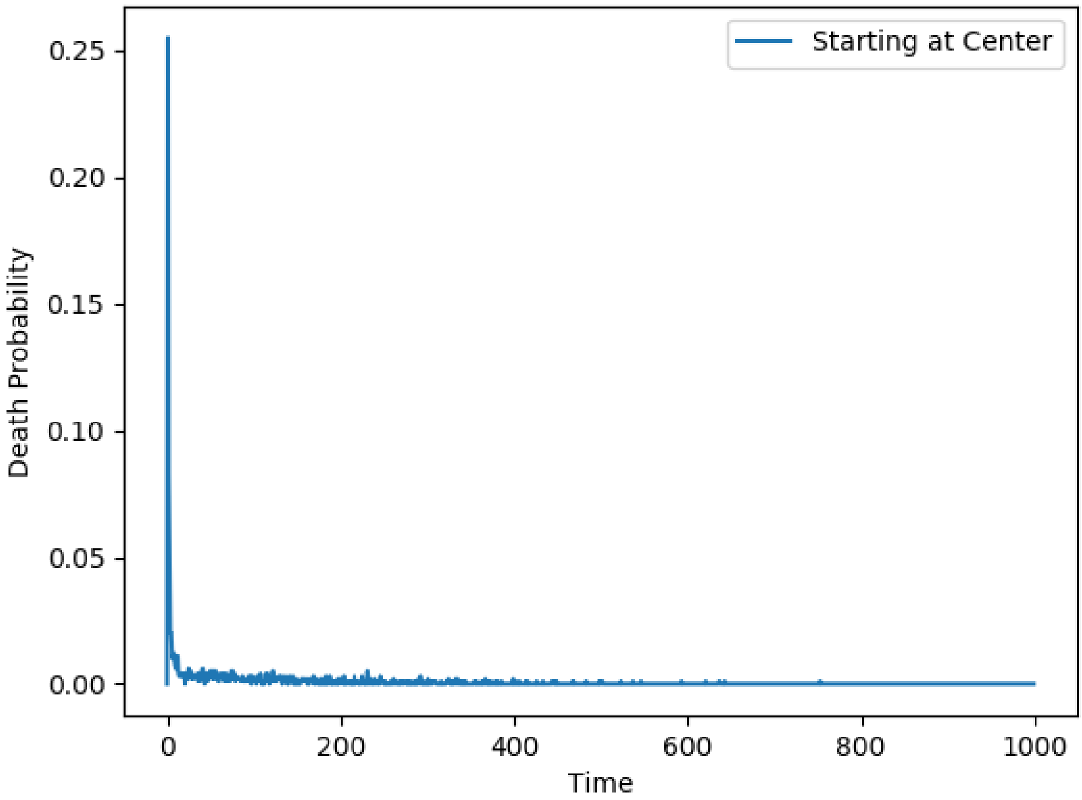
```
            goto rep;
    }
    for(i=1;i<=10000;i++){
            printf("%d %lf %lf\n",i,(dbl)(fn[i]/10000.00),(dbl)(save[i]/10000.00));
    }
    return 0;
}
```

Probablity of Meet when CAT and RAT start at opposite ends



Probablity of Meet when CAT and RAT start at opposite ends

Probablity of Meet when CAT and RAT start at from center



Science - I

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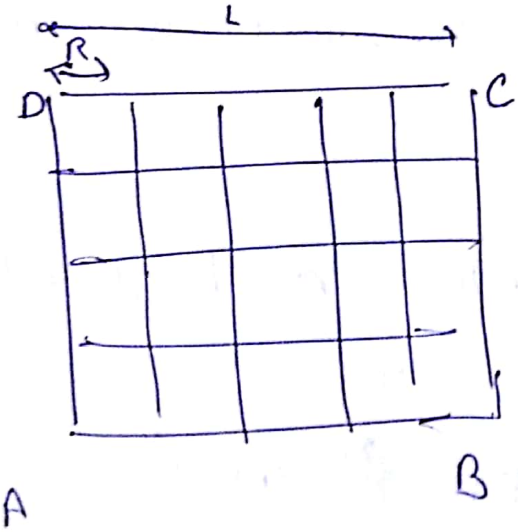
Project-I

PART-I (Grid Problem)

Given: A lattice of length L ai
R is a grid size
N is the total no. of cells.

$N = L^2/R^2$



* Cat and Rat are on opposite sides.

Let N be the total number of steps.

Let $L = L/R$

Assumptions :

a) To simply the model, whenever both cat/rat are at the boundary/corner, they have a provision to wait / not move corresponding to the wall

b) So i) probability of wait a boundary = $1/4$

ii) probablity of wait at corner = $1/2$

(C) $N > \frac{L}{R}\sqrt{2}$. The number of steps should be greater than minimum required to meet.

Let us assume the cat and rat to be 2 molecules, which are doing a random walk.

Hence the probability of being at n_1, n_2 at the 2D-Space. out of N steps can be.

if it takes x steps in x-dir : then it takes $N-x$ steps in y direction.

$P(n_1)$ or Probability of being at n_1 after x steps.

$$\cancel{= \frac{x!}{n_1!\,(x-n_1}}$$

x_1 = steps taken in right, x_2 to steps taken in left.

$x_1 - x_2 = n_1$; $x_1 + x_2 = N$

$$P(n_1) = \frac{x!}{x_1!\,x_2!}\,(p_{right})^{x_1}\,(p_{left})^{x_2}.$$

similarly for y-axis.

y_R = steps up and y_2 = steps down.

$y_1 + y_2 = N-x$; $y_1 - y_2 = n_2$

$$P(n_2) = \frac{(N-x)!}{y_1!\,y_2!}\,(P_{up})^{y_1}\,(P_{down})^{y_2}$$

As the movement is mutually exclusive.

$$P(n_1, n_2) = P(n_1) \cap P(n_2) = P(n_1) \times P(n_2)$$

$$P(n_1, n_2) = \frac{x!}{(x_1!)(x_2)} \times \left(\frac{1}{2}\right)^n \times \frac{(N-x_1)}{(y_1!)!(y_2)!} \times \left(\frac{1}{2}\right)^n$$

Actual probability would be

$$P = \sum_{x=n_1}^{n_1-n_2} P(n_1, n_2) \Rightarrow$$

when cat and Rat meet

$$P(\text{meet}) = P_{rat}(n_1, n_2) \times P_{cat}(n_1, n_2)$$

$$P_{cat}(n_1, n_2) = P_{rat}(L-n_1, L-n_2)$$

$x_{1\,cat} - x_{2\,cat} = L-n_1$

$x_{1\,cat} + x_{2\,cat} = \cancel{x}$

$$L = L/R \;\star\star$$

$$P(\text{meet}) = P_{rat}(n_1, n_2) \times P_{rat}(L-n_1, L-n_2)$$

$$P(\text{meet, at all points}) = \sum_{n_1=0}^{L} \sum_{n_2=0}^{L} P_{rat}(n_1, n_2) \times P_{rat}(L-n_1, L-n_2)$$

for constant x

$$\Rightarrow \sum_{x_{rat}}^{N} \sum \sum \left(\frac{1}{2}\right)^{4N} \frac{x_{rat}!}{(x_{1\,rat}!)(x_{2\,rat})!} \times \frac{(N-x_{rat})!}{(y_1^{rat}!)(y_2^{rat})!} \times \frac{x_{cat}!}{x_1^{cat}!\, x_2^{cat}!} \times \frac{(N-x^{cat})!}{(y_1^{cat}!\, y_2^{cat})!}$$

$$\cancel{\Rightarrow \sum \sum} \left(\frac{1}{4}\right)^{4N}$$

$$\text{Hence } P(\text{meet}) = \sum_{x}^{N} \sum_{n_1=0}^{L} \sum_{n_2=0}^{L} \left(\frac{1}{2}\right)^{4N} \frac{x_{rat}!\,(N-x_{rat}) \times x_{cat}! \times (N-x_{cat})!}{(x_{1\,rat})!\,(x_{2\,rat})!\,(y_{1\,rat})!\,(y_{2\,rat})!\,(x_{1\,cat})!\,(x_{2\,cat})! \times (y_{1\,cat})!\,(y_{2\,cat})!}$$

Summation for x_{rat}, x_{cat}

where $\Rightarrow$ $\cancel{n_1 + n_2 = N}$

$x_{1\,rat} + x_{2\,rat} = x_{rat}$ $\qquad x_{1\,cat} + x_{2\,cat} = x_{cat}$

$x_{1\,rat} - x_{2\,rat} = n_1$ $\qquad x_{1\,cat} - x_{2\,cat} = L - n_1$

$y_{1\,rat} + y_{2\,rat} = N - x_{rat}$ $\qquad y_{1\,cat} + y_{2\,cat} = N - x_{cat}$

$y_{1\,rat} - y_{2\,rat} = n_2$ $\qquad y_{1\,cat} - y_{2\,cat} = L - n_2$

By substituting everything in terms of n_1, n_2 and N followed by calculating the summation.

We would get the value of $P(meet)$.

$P(meet)$ ~~$= \sum\sum\sum \left(\frac{1}{4}\right)^{4N} (n_1+n_2)! \; (N$~~

$$\Rightarrow \cancel{\sum_{x_{rat}}\sum_{x_{rat}}\sum_{n_1}\sum_{n_2}} \frac{x_{rat}! \; \cancel{(N-x_{rat})!}}{\left(\frac{n_1+x_{rat}}{2}\right)! \left(\frac{x_{rat}-n_1}{2}\right)!} \times \frac{1}{}$$

$$\Rightarrow \sum\sum\sum\sum \left(\frac{1}{2}\right)^{4N} \frac{(x_{rat}!)\,(N-x_{rat})!}{\left(\frac{n_1+x}{2}\right)! \left(\frac{x-n_1}{2}\right)! \left(\frac{n_2+N-x}{2}\right)! \left(\frac{N-x-n_2}{2}\right)!}$$

$$\times \frac{(x_{cat}!)\,(N-x_{cat})!}{\left(\frac{L-n_1+x_{cat}}{2}\right)! \left(\frac{x_{cat}+n_1-L}{2}\right)! \left(\frac{L-n_2+N-x_{cat}}{2}\right)! \left(\frac{N-x_{cat}-L+n_2}{2}\right)!}$$

We can calculate this using a PC and Hence

$P(\text{Survival of Rat}) = 1 - P(meet)$

$\Rightarrow$

Hence Probability of meet depends of N and L

if $N < L\sqrt{2}$ the particles will never meet.

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Case: 2

Assuming they start from center.

$$P(\text{meet}) = P(n_1, n_2) \times P(n_1 \times n_2)$$

$$= P(n_1 \times n_2)^2$$

$$= \sum_{n_1} \sum_{n_2} \left(\frac{1}{2}\right)^{4N} \left(\frac{x! \, (N-x)!}{x_1! \, x_2! \, y_1! \, y_2!} \right)^2$$

x = Steps taken in x direction

x_1 = steps in right $\quad x_2$ = steps in left

y_1 = step up $\quad y_2$ = steps down

$x_1 + x_2 = x \quad y_1 + y_2 = N - x$

$x_1 - x_2 = n_1 \quad y_1 - y_2 = n_2$

$P(\text{survival}) = 1 - \text{Probability}(\text{meet})$

$$P(\text{survival}) = 1 - \left(\frac{1}{2}\right)^{4N} \frac{x! \, (N-x)!}{(x_1!)(x_2!) \, y_1! \, (y_2)!}$$

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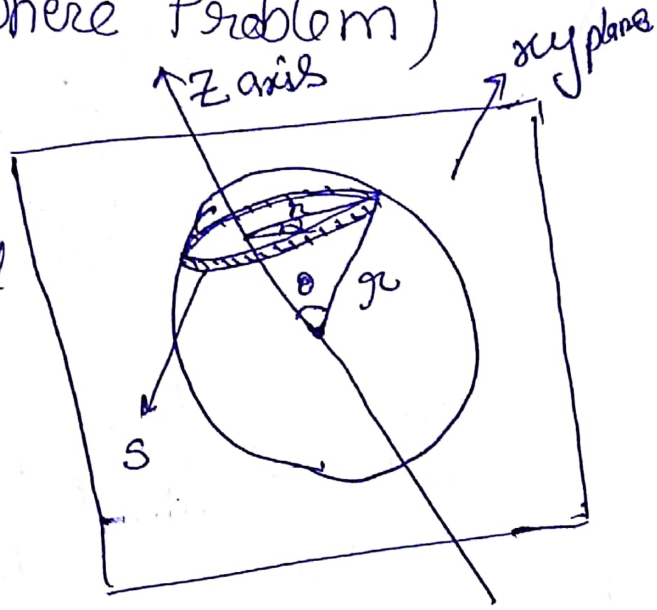
PART-2 (Sphere Problem)



→ Z-axis is the Line cutting the sphere and emerging out of the sphere at another point.

→ Also, there is a plane xy, that has ~~Z~~ axis on it, which cuts the sphere into 2 hemisphere

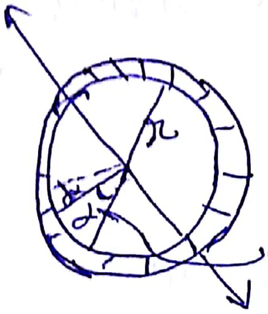
→ Blind cat and a blind rat are present on the sphere. We show their position using

i) θ : angle with the Z-axis

ii) α : angle with the line of intersection of plane with the plane.

Because : a set of points can have angle θ with Z-axis, as in the above example, all the points are on the circumference S.

The typical circles cut by the xy plane of the sphere will look like



Probability of being at a point which make angle α with the xy plane.

$$P(\alpha) = \frac{r\,d\alpha}{2\pi r} = \frac{d\alpha}{2\pi}$$

Probability of being at a point which makes an angle θ with the Z axis

$\Rightarrow \dfrac{\text{Area occupied by the circular strip}}{\text{total surface area of the sphere.}}$

$$\Rightarrow \frac{(r\,d\theta)(2\pi r \sin\theta)}{4\pi r^2} = \frac{\sin\theta \cdot d\theta}{2}$$

Hence the probability of being at a point is.

$$P(\theta,\alpha) = \frac{\sin\theta\, d\theta\, d\alpha}{4\pi}$$

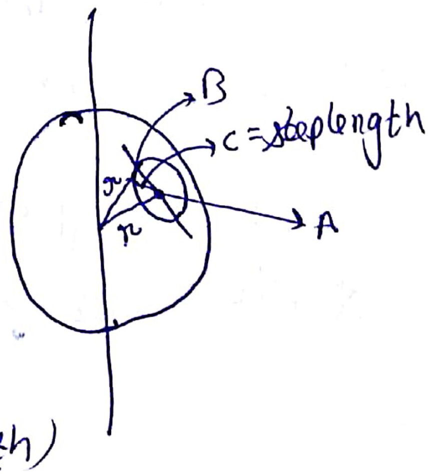
Anology:

Displacement.

A: original position

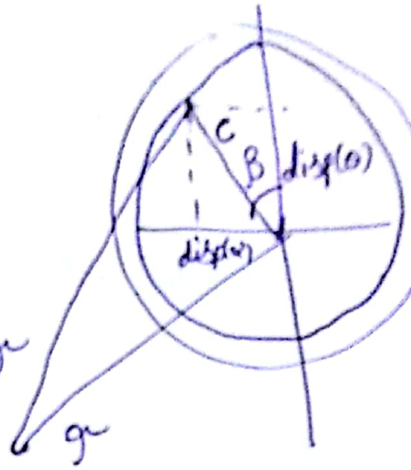
B: new position

C = step length



angle of displacement = $\dfrac{\text{arc length (step length)}}{\text{radius.}}$

$\text{disp}'\theta' = \frac{\text{dis}\,\theta}{r}$



$$\theta_{new} = \theta - \frac{c\cos B}{r}$$

$$\text{dis}'\alpha' = \frac{\text{disp}}{r\sin\theta}$$

$$\alpha_{new} = \alpha + \frac{c\sin B}{r\sin\theta}$$

Assumption:

i) Basically assuming that B will be small enough so that the helix cat and rat travel individually happen to cover the total surface of the sphere.

Probability Density Function, $w(\theta,\alpha)\,d\theta\,d\alpha = ?$

Probability of a point $= \frac{\sin\theta\,d\theta\,d\alpha}{4\pi}$

$d\alpha \Rightarrow \frac{c\sin B}{r\sin\theta}$ $\quad d\theta = \frac{c\cos B}{r}$

$$w(\theta,\alpha)\,d\theta\,d\alpha = \frac{\sin\theta}{4\pi} \times \frac{c\sin B\,c\cos B\,d\theta\,d\alpha}{r\sin\theta\, r}$$

$$= \frac{c^2}{4\pi r^2}\sin B\cos B\,d\theta\,d\alpha$$

$$\text{Probability of meet} = \int_\theta\int_\alpha w(\theta,\alpha)^2\,d\theta\,d\alpha$$

$$\Rightarrow \int_{r=0}^{2\pi} \int_{\theta=0}^{2\pi} \frac{c^4}{16\pi^2 r^4} \sin^2\beta \cos^2\beta \, d\theta \, d\alpha$$

$$\Rightarrow \frac{c^4}{16\pi^2 r^2} \sin^2\beta \cos^2\beta \int_{r=0}^{2\pi}\int_{\theta=0}^{2\pi} d\theta \, d\alpha \, .$$

$$\Rightarrow \frac{c^4 \sin^2\beta \cos^2\beta}{8 r^4}$$

Hence Probality of meet $\Rightarrow$ $P(\text{meet}) = \dfrac{c^4 \cos^2\beta \sin^2\beta}{r^4}$

where β is the angle of displacment.

Probality of Survival of the rab $\Rightarrow$

$$1 - P(\text{meet}) \Rightarrow 1 - \frac{c^4 \cos^2\beta \sin^2\beta}{r^4}$$

Con

Conclusion: Answer to Questions.

a) Survival/meeting is going to vary with time according to the above equation.

Since the probability of survival decreases with increase in time i.e.

probability of survival $\propto \frac{1}{(\text{time})^{c}}$; c is a constant.

Because at large time, more number of steps have been taken.

b) i) it depends of L of the square. Hence as L => Length of the grid increases; chance of survival increases.

ii) It depends on the radius of the sphere.

meeting $\propto \frac{1}{r^4}$. Hence r increases, chance of survival increases.

c) i) If the number of steps is too large; probability for each cat and rat to be at a point = $1/L^2$

Hence probability of meet = $1/L^4$

Probability of survival = $1 - 1/L^4$

ii) Similarly for sphere portion. If a step is very small probability of being at a point => $1/4\pi r^2$

Survival Probability = $1 - \left(\frac{1}{4\pi r^2}\right)^2 = 1 - \frac{1}{16\pi^2 r^4}$

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