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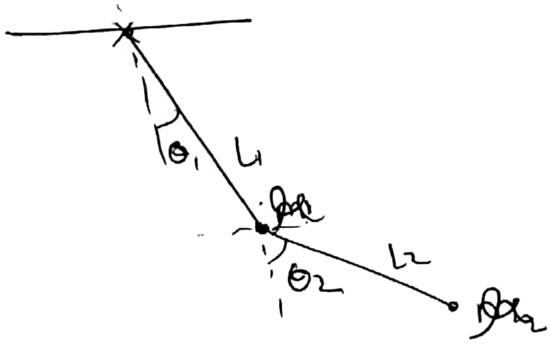
Project - 2

Double Pendulum

Find the motion of the trajectory



Solutional Matrix

$x_1 = L_1 \sin\theta_1 \qquad y_1 = -L_1\cos\theta_1$

$x_2 = L_1 \sin\theta_1 + l_2 \sin\theta_2 \qquad y_2 = -L_1\cos\theta_1 - l_2\cos\theta_2$

Let $x_1 y_1$ = co-ordinate of m_1 and x_2, y_2 be cordinate of P_2, m_1, m_2 be their mass.

$$U = m_1 g y_1 + m_2 g y_2$$

$$T = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$$

Lagrangium of the System is

$$L = T - U$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}_i}\right) = \frac{dL}{d\theta_i}$$

$$T = \frac{1}{2} m_1 \dot{\theta}_1^2 L^2 + \frac{1}{2} m_2 (\dot{\theta}_1^2 L_1^2 + \dot{\theta}_2^2 l_2^2 + 2\dot{\theta}_1\dot{\theta}_2 L_1 l_2 \cos(\theta_1 - \theta_2)$$

$$U = -(m_1 + m_2) g L_1 \cos\theta_1 - m_2 l_2 g \cos\theta_2$$

$$\frac{dL}{d\theta_1} = -Lg(m_1+m_2)\sin\theta_1 \; - m_2 l_2 L_1 \dot\theta_1 \dot\theta_2 \sin(\theta_1-\theta_2)$$

$$\frac{\partial L}{\partial \dot\theta_1} = (m_1+m_2) L_1^2 \dot\theta_1^2 + m_2 L_1 l_2 \dot\theta_2 \cos(\theta_1-\theta_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_1}\right) = 2(m_1+m_2) L_1^2 \dot\theta_1 \ddot\theta_1 - m_2 L_1 l_2 \sin(\theta_1-\theta_2)(\dot\theta_2)(\dot\theta_1-\dot\theta_2)$$
$$+ m_2 L_1 l_2 \cos(\theta_1-\theta_2)\ddot\theta_2$$

$$2(m_1+m_2) L_1^2 \dot\theta_1 \ddot\theta_1 - m_2 L_1 l_2 \sin(\theta_1-\theta_2)\dot\theta_2(\dot\theta_1-\dot\theta_2)$$
$$+ m_2 L_1 l_2 \cos(\theta_1-\theta_2)\ddot\theta_2 = -L_1 g(m_1+m_2)\sin\theta_1$$
$$- m_2 l_2 l_1 \dot\theta_1 \dot\theta_2 \sin(\theta_1-\theta_2)$$

$$\ddot\theta_1 = \frac{-m_2 L_2 \ddot\theta_2 \cos(\theta_1-\theta_2) - m_2 l_2 \dot\theta_2^2 \sin(\theta_1-\theta_2) - (m_1+m_2) g \sin\theta_1}{m_1 + m_2 l_1}$$

Similary

$$\frac{\partial L}{\partial \theta_2} = m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \sin(\theta_1-\theta_2) \; - l_2 m_2 g \sin\theta_2$$

$$\frac{\partial L}{\partial \dot\theta_2} = m_2 l_2^2 \dot\theta_2 \quad + m_2 l_1 l_2 \dot\theta_1 \cos(\theta_1-\theta_2)$$

$$\frac{\partial}{dt}\left(\frac{dL}{d\dot\theta}\right) = m_2 l_2^2 \ddot\theta_2 + m_2 l_1 l_2 \ddot\theta_1 \cos(\theta_1-\theta_2)$$
$$- m_2 l_1 l_2 \dot\theta_1 \sin(\theta_1-\theta_2)(\dot\theta_1-\dot\theta_2)$$

$$\ddot\theta_2 = \frac{-l_1 \ddot\theta_1 \cos(\theta_1-\theta_2) + l_1 \dot\theta_1^2 \sin(\theta_1-\theta_2) - g\sin\theta_2}{l_2}$$

We can easily solve these types of Equation using Runge Kutta approximation Method.

To simplify

$$\ddot{\theta}_1 + \alpha_1(\theta_1 \theta_2)\ddot{\theta}_2 = f_1(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$$

$$\ddot{\theta}_2 + \alpha_2(\theta_1, \theta_2)\ddot{\theta}_1 = f_2(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$$

$$\alpha_1(\theta_1, \theta_2) = \frac{L_2}{L_1}\left(\frac{m_2}{m_1+m_2}\right)\cos(\theta_1 - \theta_2)$$

$$\alpha_2(\theta_1, \theta_2) = \frac{L_1}{L_2}(\cos(\theta_1 - \theta_2))$$

$$f_1(\ ) = \frac{-L_2}{L_1}\left(\frac{m_2}{m_1+m_2}\right)\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - \frac{g}{L_1}\sin\theta_1$$

$$f_2(\ ) = \frac{L_1}{L_2}\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - \frac{g}{L_2}\sin\theta_2$$

Hence we get $\Rightarrow \begin{pmatrix} 1 & \alpha_1 \\ & 1 \end{pmatrix}\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$

$$A\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \alpha_2$$

$$A^{-1} = \frac{1}{\det(A)}\begin{pmatrix} 1 & -\alpha_1 \\ -\alpha_2 & 1 \end{pmatrix} = \frac{\begin{pmatrix} 1 & -\alpha_1 \\ -\alpha_2 & 1 \end{pmatrix}}{1-\alpha_1\alpha_2}$$

Now $\det(A) \neq 0$ as $\alpha_1\alpha_2$ is alway less than 1

$$\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \cancel{f_1 - \alpha}\ A^{-1}\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

$$= \frac{1}{1-\alpha_1\alpha_2}\begin{pmatrix} f_1 - \alpha_1 f_2 \\ -\alpha_2 f_1 + f_2 \end{pmatrix}$$