1)

i) $G = (\{S\}, \{0, 1\}, R, S)$

$R = S \to 0S111 \mid \varepsilon$

ii) $G = \langle \{S\}, \{[,], (,)\}, P, S \rangle$

$R = S \to [S] \mid (S) \mid SS \mid \varepsilon$

2) i) $\cancel{S \to 0A \mid 1A}$ $G = \langle \{S, A\}, \{0, 1\}, R, S \rangle$

$R = S \to 0A \mid 1A$

$A \to 0S \mid 1S \mid \varepsilon$

ii) $G = \langle \{S, X, Y, Z, W\}, \{a, b, c\}, R, S \rangle$

$R = S \to XZ \mid W$

$X \to aXb \mid \varepsilon$

$Z \to Zc \mid \varepsilon$

$Y \to Yb \mid \varepsilon$

$W \to aWc \mid Y$

3) i) $G = \langle \{S, X, Y\}, \{0, 1\}, R, S \rangle$

$R = S \to 0X \mid 1X \mid \varepsilon$

$X \to 0Y \mid 1Y$

$Y \to 0S \mid 1S$

ii) $G = \langle \{S, X\}, \{a, b, c, \ldots, z\}, R, S \rangle$

$R = S \to X\,formal\,X\,methods\,X \mid X\,methods\,X\,formal\,X$

$X \to aX \mid bX \mid cX \mid \ldots \mid zX \mid \varepsilon$

i) $L(G) = \{a^n b^m \mid 0 \leq n \leq m \leq 2n\}$

$G = \langle \{X\}, \{a,b\}, R, X \rangle$

R =

$X \rightarrow aXb \mid aXbb \mid \varepsilon$

ii) $L(G) = \{a^n b^m c^k \mid k = n+m\}$

$G = \langle \{S,X\}, \{a,b,c\}, R, S \rangle$

R = $S \rightarrow aSc \mid X$

$X \rightarrow bXc \mid \varepsilon$

5) i) $G = \langle \{S\}, \{0,1\}, R, S \rangle$

~~$S \rightarrow X \mid Y$~~

~~$X \rightarrow 0X0 \mid 1X1 \mid 0$~~

~~$Y \rightarrow 0Y0 \mid 1Y1 \mid 1$~~

~~OR~~

R = $S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1$

ii) $G = \langle \{S, S_1, S_2, A, B, C, A', B', C'\}, \{a,b,c\}, R, S \rangle$

$i \geq k$ or $j \geq k$: $S \rightarrow S_1 \mid S_2$

$j \geq k$:

$S_1 \rightarrow ABC$

$C \rightarrow bCc \mid \varepsilon$

$B \rightarrow Bb \mid \varepsilon$

$A \rightarrow Aa \mid \varepsilon$

$i \geq k$:

$S_2 \rightarrow$ ~~$A'Bc$~~ $A'C' \mid$ ~~ε~~

$B' \rightarrow B'b \mid \varepsilon$

$C' \rightarrow aC'c \mid B'$

$A' \rightarrow A'a \mid \varepsilon$

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6) $S \rightarrow S_1 | S_2$

$S_1 \rightarrow 0S_11 | \epsilon$

$S_2 \rightarrow 1S_20 | \epsilon$

Remove $S_1 \rightarrow \epsilon$ & $S_2 \rightarrow \epsilon$

$\Rightarrow$ $S \rightarrow S_1 | S_2 | \epsilon$

$S_1 \rightarrow 0S_11 | 01$

$S_2 \rightarrow 1S_20 | 10$

Here, $S \rightarrow \epsilon$ is allowed in CNF.

$\Rightarrow$ Remove unitary transition $S \rightarrow S_1$, $S \rightarrow S_2$

$S \rightarrow 0S_11 | 01 | 1S_20 | 10 | \epsilon$

$S_1 \rightarrow 0S_11 | 01$

$S_2 \rightarrow 1S_20 | 10$

$\Rightarrow$ Add $S_z \rightarrow 0$ and $S_o \rightarrow 1$

$S \rightarrow S_zS_1S_o | S_zS_o | S_oS_2S_z | S_oS_z | \epsilon$

$S_1 \rightarrow S_zS_1S_o | S_zS_o$

$S_2 \rightarrow S_oS_2S_z | S_oS_z$

$S_z \rightarrow 0$

$S_o \rightarrow 1$

$\Rightarrow$ Add $S' \rightarrow S_zS_1$ and $S'' \rightarrow S_2S_z$

$S \rightarrow S'S_o | S_zS_o | S_oS'' | S_oS_z | \epsilon$

$S_1 \rightarrow S'S_o | S_zS_o$

$S_2 \rightarrow S_oS'' | S_oS_z$

$S_z \rightarrow 0$, $S' \rightarrow S_zS_1$

$S_o \rightarrow 1$, $S'' \rightarrow S_2S_z$

This is in CNF form.

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7) $S \to ASB$

$A \to aAS \mid a \mid \varepsilon$

$B \to SbS \mid A \mid bb$

i) Add $S_0 \to S$ (S_0 be the starting variable)

$\Rightarrow S_0 \to S$

$S \to ASB$

$A \to aAS \mid a \mid \varepsilon$

$B \to SbS \mid A \mid bb$

ii) Remove $A \to \varepsilon$

$\Rightarrow S_0 \to S$

$S \to ASB \mid SB$

$A \to aAS \mid a \mid aS$

$B \to SbS \mid bb \mid \varepsilon \mid A$

iii) Remove $B \to \varepsilon$

$\Rightarrow S_0 \to S$

$S \to ASB \mid SB \mid AS$

$A \to aAS \mid a \mid aS$

$B \to SbS \mid bb \mid A$

iv) Remove $S_0 \to S$ and $B \to A$

$\Rightarrow S_0 \to ASB \mid SB \mid AS$

$S \to ASB \mid SB \mid AS$

$A \to aAS \mid a \mid aS$

$B \to SbS \mid bb \mid aAS \mid a \mid aS$

v) Add $A_a \to a$, $A_b \to b$

$\Rightarrow S_0 \to ASB \mid SB \mid AS$

$S \to ASB \mid SB \mid AS$

$A \to A_aAS \mid a \mid A_aS$

$B \to SA_bS \mid A_bA_b \mid A_aAS \mid A_aS \mid a$

$A_a \to a$, $A_b \to b$

vi) Add $A' \to AS$, $B' \to A_bS$

$\Rightarrow S_0 \to A'B \mid SB \mid AS$, $A' \to AS$

$S \to A'B \mid SB \mid AS$, $B' \to A_bS$

$A \to A_aA' \mid A_aS \mid a$

$B \to SB' \mid A_bA_b \mid A_aA' \mid A_aS \mid a$

$A_a \to a$, $A_b \to b$

This is in CNF form.

8) $S \to aXbY$
$X \to aX \mid \varepsilon$
$Y \to bY \mid \varepsilon$

$\to$ Remove $X \to \varepsilon$ & $Y \to \varepsilon$

$S \to aXbY \mid abY \mid aXb \mid ab$
$X \to aX \mid a$
$Y \to bY \mid b$

$\to$ Add $S_a \to a$
$S_b \to b$

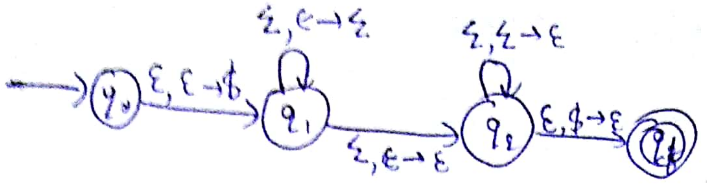
$S \to S_a X S_b Y \mid S_a S_b Y \mid S_a X S_b \mid S_a S_b$
$X \to S_a X \mid a$, $S_a \to a$
$Y \to S_b Y \mid b$, $S_b \to b$

$\to$ Add $S' \to X S_b$, $S'' \to S' Y$, $S''' \to S_b Y$

$S \to S_a S'' \mid S_a S''' \mid S_a S' \mid S_a S_b$
$X \to S_a X \mid a$
$Y \to S_b Y \mid b$
$S_a \to a$
$S_b \to b$

This is in CNF form

9)



$Q = \{q_0, q_1, q_2, q_f\}$

$F = \{q_f\}$

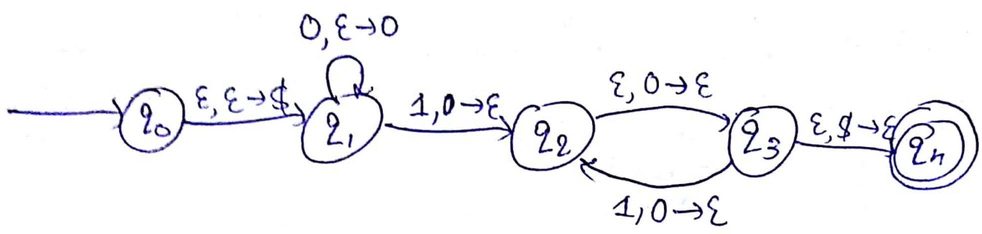
$q_0 = q_0$

$\Sigma = \{0, 1\}$

$T = \Sigma \cup \{\$\}$

δ as shown above

10)



$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$F = \{q_4\}$

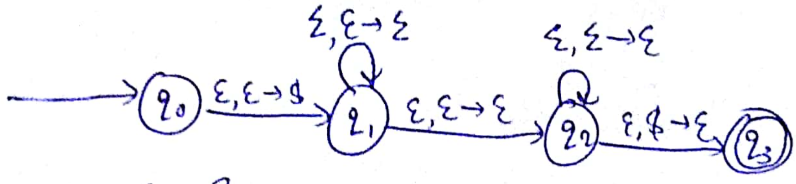
$q_0 = q_0$

$\Sigma = \{0, 1\}$

$T = \{0, \$\}$

δ as above

11)



$Q = \{q_0, q_1, q_2, q_3\}$

$F = \{q_3\}$

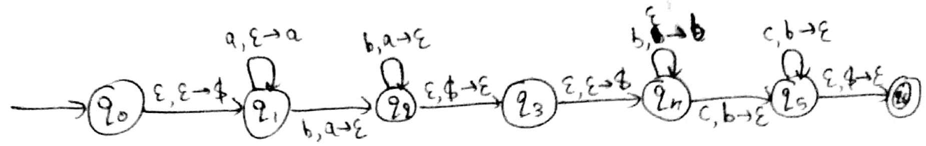
$q_0 = q_0$

$\Sigma = \{a, b, c\}$

$T = \Sigma \cup \{\$\}$ δ as shown

12)

i)



$Q = \{q_0, q_1, \dots, q_6\}$

$\Sigma = \{a, b, c\}$

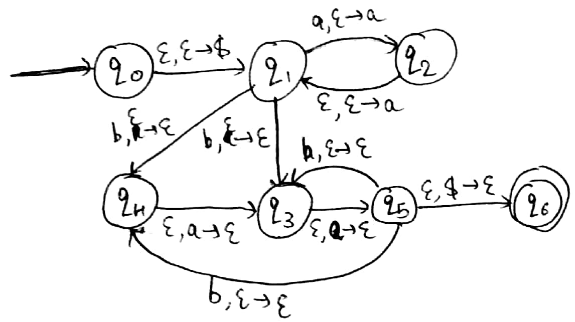
$\Gamma = \{a, b, \$\}$

$q_0 = q_0$

$F = \{q_6\}$

δ as shown in figure

ii)



$Q = \{q_0, q_1, \dots, q_6\}$

$\Sigma = \{a, b\}$

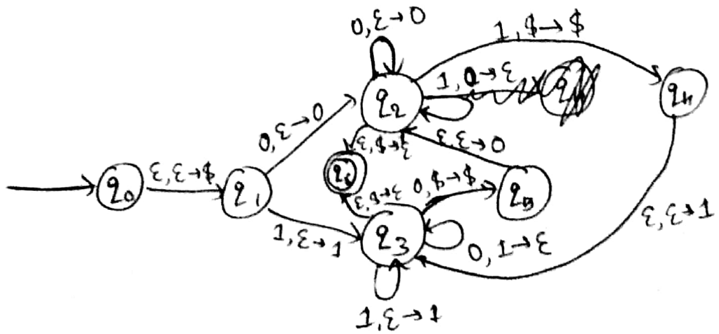
$\Gamma = \{a, \$\}$

$q_0 = q_0$

$F = \{q_6\}$

δ as shown in figure

13)



$Q = \{q_0, q_1, \ldots, q_6\}$

$q_0 = q_0$

$F = \{q_6\}$

$\Sigma = \{0, 1\}$

$\Gamma = \Sigma \cup \{\$\}$

δ as in figure

14)

i) $L = \{a^n b^n a^n b^n \mid n \geq 0\}$

Let s be $uvxyz = a^p b^p a^p b^p$, p is the pumping lt.

Now, $|vxy| \leq p$ & $|vy| > 0$

Let s be AB, such that $A = a^p b^p$ & $B = a^p b^p$

~~If $vxy \subseteq A$,~~

Case 1:

$vxy \subseteq A$

If vxy contains only a's or b's, on pumping up to uv^2xy^2z the #(a)'s in A ≠ #(b)'s in A

If vxy contains both a's & b's, s.t v contains a's & y contains b's, pumping to uv^2xy^2z, #(a)'s in A ≠ #(a)'s in B.

Similarly for case 2 where $vxy \subseteq B$,

we can conclude $uv^ixy^iz \notin L \quad \forall i \geq 0$

$\Rightarrow s \notin L \Rightarrow L$ is not context free.

Similarly for case 3 when vxy lies in centre, the #(a)'s in A & #(b)'s in B remain same while #(a) in A ≠ #(b) in A

Case 2 :

If vxy contains both ~~vxy~~ $0 \& 1$ of A such that v has only 0 & y has only 1^n, then, $\#(0) \& \#(1)$ in A > $\#(0) \& \#(1)$ in B & C on pumping to uv^2xy^2z

Same applies for B & C.

$\Rightarrow uv^ixy^iz \notin L \quad \forall i \geq 0$

$\Rightarrow S \notin L$

$\Rightarrow$ L isn't context free.

15) $L = \{a^i b^{2i} a \mid i \geq 0\}$

$S = uvxyz = \underbrace{a^p}_{A} \underbrace{b^{2p}}_{B} a$, where p is the pumping lt.

Now $|vxy| \leq p$ & $|vy| > 0$

Case 1:

vxy has only ~~initial~~ a's in the starting substring.

On pumping to uv^2xy^2z, $\#(a)$ in A $\neq \frac{1}{2} \#(b)$ in B

$\because$ the no. of a's increase while b's remain same.

Similarly, if vxy has only b's $2 \times \#(a)$ in A < $\#(b)$

Case 2 :

If vxy has some a's in A & some b's in B.

Here v has only a's & y has only b's, otherwise the pattern $a^n b^{2n} a$ gets disturbed.

On pumping to uv^ixy^iz, we get $a^{n-k} \underbrace{a^{ki}}_{\text{Due to } v} b^{2n-l} \underbrace{b^{li}}_{\text{Due to } y} a$

$2(n + k(i-1)) \neq 2n + l(i-1)$

$\because$ 2K

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ii) $L = \{a^n bba^{2n} bba^{4n} \mid n \geq 0\}$

$S = uvxyz = a^p bba^{2p} bba^{4p}$, p is the pumping length

~~b∈ b can't belong~~ $|vxy| \leq p$ & $|vy| > 0$

Case 1: $b \notin vxy$

$\Rightarrow$ ~~b∈~~ $vxy \subseteq a^p$ or $vxy \subseteq a^{2p}$ or $vxy \subseteq a^{4p}$

On pumping up, the geometric series of $p, 2p$ & $4p$ will be disturbed as either of the length increases.

Case 2: $b \in vxy$

But $b \notin v$ ~~or~~ and $b \notin y$

$\therefore$ #(b) is constant.

$\Rightarrow$ Either v and y can have a's from a^p & a^{2p} or a^{2p} and a^{4p}

$\Rightarrow$ Atleast one of the a series remain constant while the length of others increase on pumping up.

$\Rightarrow$ The series ratio of $1:2:4$ get disturbed.

$\Rightarrow$ $uv^ixy^iz \notin L$

$\Rightarrow$ $S \notin L$

$\Rightarrow$ L is not context-free.

iii) $L = \{ww^Rw \mid w \in \{0,1\}^*\}$

$S = uvxyz = \underbrace{0^p 1^p}_{A} \underbrace{1^p 0^p}_{B} \underbrace{0^p 1^p}_{C}$

$|vxy| \leq p$ & $|vy| > 0$.

Case 1

Let vxy contain only 0's in A.

On pumping up to uv^2xy^2z, #(0) in A $\neq$ #(0) in B and C

Similarly if vxy contains only one type of character the number of that character $\neq$ the no. of that character in other substrings

16)

$S = a^p = uvxyz$, p is the pumping lt.

$|vxy| \leq p$ & $|vy| > 0$

Let s be pumped to $uv^{p+1}xy^{p+1}z$

$\Rightarrow |uv^{p+1}xy^{p+1}z| = |uvxyz| + |v^{p}y^{p}|$

$= p + p|vy|$

$= p(1 + |vy|)$

$|vy| > 0$

$\Rightarrow |uv^{p+1}xy^{p+1}z| \neq$ a prime no.

$\Rightarrow uv^{p+1}xy^{p+1}z \notin L$

$\Rightarrow S \notin L$

$\Rightarrow L$ is not context free.

17)

$S = 1^{p^2}$, p is the pumping lt.

$|vxy| \leq p$ & $|vy| > 0$

$\Rightarrow |vy| < p$

$\Rightarrow uv^0xy^0z = uxz$

$\Rightarrow |uxz| \geq p^2 - p$ & $|uxz| < p^2$

$> p^2 - 2p + 1$

~~No perf~~ $> (p-1)^2$

$\Rightarrow (p-1)^2 < |uxz| < p^2$

$\Rightarrow uxz \notin L$

$\Rightarrow L$ is not context free

19) Consider L_1 & L_2 to be two context free languages

~~Let S_1 be the starting corresp~~ If a language is context-free, there is a CFG corresponding to it.

Let G_1 be the grammar corresponding to L_1 & G_2 corresponding to L_2.

Here S_1 & S_2 are the starting variables.

In the CFGs, just change the terminals in G_2 to avoid confusion with G_1.

For $L_1 \cup L_2$,

$$G = \langle V_1 \cup V_2 \cup \{S\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{S \to S_1 | S_2\}, S \rangle$$

$\Rightarrow \exists$ a ~~gram~~ CFG G corresponding to $L_1 \cup L_2$

$\Rightarrow$ $L_1 \cup L_2$ is context free.

For $L_1 \cdot L_2$,

$$G' = \langle V_1 \cup V_2 \cup \{S\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{S \to S_1 S_2\}, S \rangle$$

$\Rightarrow$ $L_1 \cdot L_2$ is also context free, as $\exists\ G'$ for $L_1 L_2$.

18) Let $L_1 = \{a^i b^j c^k \mid i \le j\}$ & $L_2 = \{a^i b^j c^k \mid i \le k\}$

For L_1, let G_1 be

$S \to ABC$
$A \to aAb | \varepsilon$
$B \to Bb | \varepsilon$
$C \to Cc | \varepsilon$

For L_2, $G_2 =$

$S \to \cancel{ABC}\, AC$
$A \to aAc | B$
$B \to Bb | \varepsilon$
$C \to Cc | \varepsilon$

$\Rightarrow$ L_1 & L_2 are context free.

$L_1 \cap L_2 = \{a^i b^j c^k \mid i \leq j \;\&\; i \leq k\}$

Let $s \in L_1 \cap L_2$ be $a^p b^p c^p$, p is the pumping lt.

Now, $s = uvxyz$

$|vxy| \leq p$ & $|vy| > 0$

i) Now v & y can't have mixture of $\{a, b, c\}$ as the pattern is disrupted.

ii) Now if v&y both have either a or b or c, then ~~there will be difference between what v&y are (and the other 2.~~ inequality will not follow if v&y are pumped up in case 1 & pumped down in case 2 and case 3.

$\#(a) > \#b > \#c$ $\qquad \#(b) < \#(a)$ $\qquad \#(a) > \#(c)$

iii) Also v can have only a & y can have only b or v has b's & y has c's.

~~On pumping up, the number of c & a respectively are different than other two.~~

In case 1, on pumping up $\#(a) > \#(c)$

In case 2, on pumping down $\#(a) > \#(b)$ & $\#(a) > \#(c)$

$\Rightarrow$ $uv^ixy^iz \notin L \quad \forall i \geq 0$

$\Rightarrow$ $L_1 \cap L_2$ isn't context free.

20) Let us assume if L_1 is CFL, $\overline{L_1}$ is also CFL

We know that $L_1 \cup L_2$ is CFL, if L_1 & L_2 are CFL

$\Rightarrow$ $\overline{\overline{L_1} \cup \overline{L_2}}$ should also be CFL (by assumption being true)

$\Rightarrow$ $L_1 \cap L_2$ should be CFL.

$\Rightarrow$ But we proved that $L_1 \cap L_2$ may not be a CFL. or ~~intersection~~ CFL are closed under intersection.

$\Rightarrow$ CFLs aren't closed under complement as our assump is false.

21) Let L_1 be a CFL with CFG G.

$$G = \{V_1, \Sigma_1, R_1, S_1\}$$

Let G^* be CFG for L_1^*

$$G^* = \{V_1 \cup \{S\}, \Sigma_1, R_1 \cup \{S \to S_1 S \mid \varepsilon\}, S\}$$

$\Rightarrow$ Each word generated is either ε or seq. of word in L_1^*.

$\Rightarrow$ Every word in L_1^* is decribed by G^*.

$\Rightarrow$ CFLs are closed under Kleene star.

22) Let $L_1 = \{a^n b^n : n \geq 0\}$

$L_2 = \{a^{100} b^{100}\}$

L_1 is CFL & L_2 is regular.

$\Rightarrow \overline{L_2}$ is also regular

Now $L = \{a^n b^n : n \neq 100, n \geq 0\} = L_1 \cap \overline{L_2}$

Now if L_1 is CFL & L_2' is regular, then $L_1 \cap L_2'$ is CFL.

Let Q for $L_1 \cap L_2'$ be $Q(L_1) \times Q(L_2')$

$q_0 = (q_0(L_1), q_0(L_2'))$

$F = F(L_1) \times F(L_2')$

δ is also crossproduct of δ_1 & δ_2. $\Rightarrow \exists$ a PDA for L. $\Rightarrow L$ is CFL