Mathematics - 3

Practice Assignment - 1

1. Locate the numbers $z_1 + z_2$ and $z_1 - z_2$ vectorically when

(i) $z_1 = (-\sqrt{3}, 1)$ and $z_2 = (\sqrt{3}, 0)$
(ii) $z_1 = x_1 + iy_1$ and $z_2 = x_1 - iy_1$

2. Use established properties of moduli to show that when $|z_3| \neq |z_4|$

$\frac{Re(z_1+z_2)}{|z_3+z_4|} \leq \frac{|z_1|+|z_2|}{||z_3|-|z_4||}$

3. Sketch the set of points determined by the given condition

(i) $|z + i| \leq 3$
(ii) $|z - 4i| \geq 4$

4. Using the fact that $|z_1 - z_2|$ is the distance between two points z_1 and z_2, give a geometric argument that

$|z - 1| = |z + i|$ represents the line through the origin with slope -1.

5. Use properties of conjugates and moduli to show that

(i) $|(2\overline{z} + 5)(\sqrt{2} - i)| = \sqrt{3}|2z + 5|$
(ii) $\overline{(2 + i)}^2 = 3 - 4i$

6. Sketch the set of points determined by condition

(i) $Re(\overline{z} - i) = 2$
(ii) $|2\overline{z} + i| = 4$

7. By factoring $z^4 - 4z^2$+3 into two quadratic factors and using inequality $|z_1 \pm z_2| \geq ||z_1| - |z_2||$, show that if z lies on the circle $|z|$=2, then

$|\frac{1}{(z^4-4z^2+3)}| \leq 1/3$

8. Prove that z is either real or pure imaginary if and only $\overline{z}^2 = z^2$

9. Use mathematical induction to show that when n=2,3,.....

(i) $\overline{z_1 + z_2 + ... + z_n} = \overline{z_1} + \overline{z_2} + ... + \overline{z_n}$
(ii) $\overline{z_1 z_2 ... z_n} = \overline{z_1}\,\overline{z_2} ....\overline{z_n}$

10. Let $a_0, a_1, a_2, ..., a_n$ (n≥1) denote real numbers and let z by any complex number. Show that

$\overline{a_0 + a_1 z + a_2 z^2 + ... + a_n z^n} = a_0 + a_1\overline{z} + a_2\overline{z}^2 + ... + a_n\overline{z}^n.$

11. Follow the steps below to give an algebraic derivation of the triangle inequality $|z_1 + z_2| \leq |z_1| + |z_2|$.

$$|z_1 + z_2|^2 = (z_1 + z_2)(\overline{z_1} + \overline{z_2}) = z_1\overline{z_1} + (z_1\overline{z_2} + \overline{z_1\overline{z_2}}) + z_2\overline{z_2}.$$

12. Show that (a) $|e^{i\theta}| = 1$, (b) $\overline{e^{i\theta}} = e^{-i\theta}$.

13. Using the fact that the modulus $|e^{i\theta} - 1|$ is the distance between the points $e^{i\theta}$ and 1 , give a geometric argument to find a value of o in the interval $0 \leq \theta \leq 2\pi$ that satisfies the equation $|e^{i\theta} - 1| = 2$.

14. By writing the individual factors on the left in exponential form, performing the needed operations and finally changing back to rectangulat coordinates, show that

(i) $i(1 - \sqrt{3}i)(\sqrt{3} + i) = 2(1 + \sqrt{3}i)$
(ii) $5i/(2 + i) = 1 + 2i$
(iii) $(-1 + i)^7 = -8(1 + i)$
(iv) $(1 + \sqrt{3}i)^{-10} = 2^{-11}(-1 + \sqrt{3}i)$

15. Let z be a non zero complex number and n a negative integer (n= -1, -2, ...). Also, write z = r$e^{i\theta}$ and m = -n = 1,2,...
Using expressions $z^m = r^m e^{im\theta}$ and $z^{-1} = (\frac{1}{r})e^{i(-\theta)}$, verify that $(z^m)^{-1} = (z^{-1})^m$ and hence the definition $z^n = (z^{-1})^m$ could have been alternatively written as $z^n = (z^m)^{-1}$.

16. Find the square roots of (a) 2i and (b) 1-$\sqrt{3}i$ and express them as rectangle coordinates.

17. Find all roots in rectangle coordinates, exhibit them as vertices of certain squares and point out which is the principal root:

(i) $(-16)^{1/4}$
(ii) $(-8 - 8\sqrt{3}i)^{1/4}$.

It's for practice purpose.