INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

Monsoon Semester, 2017
Algorithms (ISC211)

Assign. I
August 8, 2017

Due: **15.08.17** **Instructor: Dr. Pawan Kumar** Maximum Marks: 57

Instructions: Write answers on your own. Don't write unnecessary text. Solutions to this assignment will be shown during the tutorial on 19/08/17.

1. Prove or disprove the following statements. If the information given is insufficient to prove or disprove, state so with an example. [8=2+2+2+2]
   (a) $f(n) = O(g(n))$ implies $f(n)^2 = O(g(n)^2)$
   (b) $f(n) + g(n) = \Theta(\min\{f(n), g(n)\})$
   (c) $f(n) \neq \Omega(g(n))$ implies $g(n) = O(f(n))$
   (d) $\min\{f(n), g(n)\} \in O(f(n) + g(n))$
2. Prove that $\log(n!) = \Theta(n \log(n))$ and that $n! = o(n^n)$. [5]
3. A sorting algorithm is said to be **stable** if two objects with equal keys appear in the same order in sorted output as they appear in the input unsorted array. Are insertion sort and merge sort stable sorts? Explain with one example for each. [5]
4. Is $(n + a)^b = \Theta(n^b)$ true? If not, then find the conditions on a and b for which this is true. Give proof for your answer. [5]
5. For the recurrence relation $T(n) = 2T(\sqrt{n}) + 1$, $T(1) = 1$. Prove that $T(n) = \Theta(\log n)$. [5]
6. State True/False and give reasons. Here iff stands for if and only if. [8=2+2+2+2]
   (a) $T(n)$ is $O(f(n))$ iff for some constants c and n_0, $T(n) \leq cf(n)$ for all $n \geq n_0$
   (b) $T(n)$ is $\Omega(f(n))$ iff for some constants c and n_0, $T(n) \leq cf(n)$ for all $n \geq n_0$
   (c) $T(n)$ is $\Theta(f(n))$ if $T(n)$ is $O(f(n))$ and $T(n)$ is $\Omega(f(n))$
   (d) $T(n)$ is $o(f(n))$ if $T(n)$ is $O(f(n))$ and $T(n)$ is $\Theta(f(n))$
7. Find the complexity of the recurrence relation $T(n) = T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + n$. [5]
8. Suppose you have 2 arrays A_1 and A_2 each containing n sorted elements. How quickly can you find the median of the $2n$ elements? [5]
9. Is $n^{1.004} = O(n \log n)$ true/false? Prove your claim. [5]
10. Which algorithm works better insertion sort or merge sort in general? [6=2+2+2]
    (a) What happens if an extra information is given that array is almost sorted?
    (b) Can we use the result above to improve merge sort complexity?

    Give a proof for all your answers.

Student's name: End of Assignment