Homework Assignment #08 (Constellations)

[Answer, Code]

1. (10 points) General Awareness: StarLink was in news recently with specific reference to Austin. Find the news story, cite it, and summarize it in two to four sentences.

https://www.kvue.com/article/news/local/starlink-satellite-austin-lights-spacex-elon-musk/269-132c4585-2ab8-44bc-ac8f-d4e43be0cbd2

> Reding, A. (2021, April 08). Yes, those lights you saw over Austin were likely connected to Elon Musk. Retrieved April 20, 2021, from https://www.kvue.com/article/news/local/starlink-satellite-austin-lights-spacex-elon-musk/269-132c4585-2ab8-44bc-ac8f-d4e43be0cbd2

Strange lights were seen flying over Austin on April 7th, 2021. These lights are likely from SpaceX's starlink satellite constellation meant to deliver high speed internet. SpaceX launched 60 starlink satellites that same day.

2. (15 points) Ground Stations: We will consider the station mask from three ground station sites: One on top of the ASE building on campus; a second at Ny Älesund on Svalbard Island (somewhere near the air strip); and a third at Kourou, French Guiana (near the Space Museum). Find the geographical coordinates of the stations – you can use Google Earth or other such mapping tools. Write one to three sentences for the connection of each site to space exploration.

- The ASE building on Campus is known for producing the future of space explorers, engineers, and scientists.
- Svalbard Island is home to the Svalbard Rocket Range. It is well suited to launching sounding rockets that can investigate things such as the Earth's magnetic field.
- Kourou is home to a French and European Spaceport as well as a space Museum.

3. (75 points) Station Visibility: The SpaceX satellites are spread over five different inclinations [Table 1, Portillo et al.]. Pick one inclination such that a satellite in that plane will overfly all three chosen ground stations. Model a secularly precessing ellipse orbit for one satellite in that plane. For the duration of one day, tabulate the satellite passes at the three ground stations. For each pass, include:

SpaceX Satellite #4: 1275 km (81 degrees)

a. start and stop time to within 5 seconds,

ASE Building: (30 passes)

	Rise Time	Set Time
1.	5961.89809490475	6419.84099204960
2.	6851.86259312966	7417.81089054453
3.	9210.70053502675	10800.5400270014
4.	12511.3455672784	14157.3478673934
5.	15820.6310315516	17509.8354917746
6.	19134.2367118356	20862.3231161558
7.	22447.8423921196	24214.8107405370
8.	25761.4480724036	27571.6185809290
9.	29070.7335366768	30928.4264213211
10.	32371.3785689284	34298.1949097455
11.	35659.0629531477	37689.5644782239
12.	38929.4664733237	39879.9139956998
13.	40108.8854442722	41111.1755587779
14.	42160.9880494025	43033.6716835842
15.	43647.1423571179	44593.2696634832
16.	45340.6670333517	46191.7495874794
17.	47155.1577578879	48122.8861443072
18.	48494.4247212361	49401.6700835042
19.	50602.6901345067	52654.7927396370
20.	54007.0203510176	55933.8366918346
21.	57385.4292714636	59230.1615080754
22.	60746.5573278664	62535.1267563378
23.	64103.3651682584	65844.4122206110
24.	67460.1730086504	69158.0179008951
25.	70808.3404170209	72471.6235811791
26.	74160.8280414021	75785.2292614631
27.	77517.6358817941	79090.1945097255
28.	80883.0841542077	81626.1613080654
29.	81665.0432521626	82377.8788939447
30.	84317.6558827941	84563.9081954098

Ny Ålesund: (52 passes)

	Rise Time	Set Time
1.	410.420521026051	1049.81249062453
2.	2276.75383769188	2916.14580729036
3.	3749.94749737487	4397.97989899495
4.	5594.67973398670	6238.39191959598
5.	7098.11490574529	7746.14730736537

6. 8908.28541427071 9551.99759988000
7. 10450.6025301265 11098.6349317466
8. 12217.5708785439 12861.2830641532
9. 13803.0901545077 14455.4427721386
10. 15526.8563428171 16166.2483124156
11. 17164.2182109105 17816.5708285414
12. 18831.8215910796 19475.5337766888
13. 20521.0260513026 21173.3786689335
14. 22141.1070553528 22780.4990249513
15. 23877.8338916946 24530.1865093255
16. 25446.0723036152 26089.7844892245
17. 27234.6417320866 27886.9943497175
18. 28755.3577678884 29403.3901695085
19. 30587.1293564678 31239.4819740987
20. 32073.2836641832 32716.9958497925
21. 33935.2967648382 34583.3291664583
22. 35395.5297764888 36034.9217460873
23. 37283.4641732087 37922.8561428071
24. 38722.0961048052 39357.1678583929
25. 40622.9911495575 41253.7426871344
26. 42057.3028651433 42679.4139706985
27. 43962.5181259063 44575.9887994400
28. 45401.1500575029 46010.3005150258
29. 47293.4046702335 47889.5944797240
30. 48753.6376818841 49345.5072753638
31. 50624.2912145607 51198.8799439972
32. 52114.7657382869 52685.0342517126
33. 53946.5373268663 54503.8451922596
34. 55475.8937946897 56033.2016600830
35. 57260.1430071504 57804.4902245112
36. 58837.0218510926 59385.6892844642
37. 60573.7486874344 61109.4554727736
38. 62198.1499074954 62742.4971248562
39. 63878.7139356968 64414.4207210361
40. 65554.9577478874 66103.6251812591
41. 67183.6791839592 67728.0264013201
42. 68907.4453722686 69464.7532376619
43. 70484.3242162108 71045.9522976149
44. 72251.2925646282 72825.8812940647
45. 73789.2894644732 74368.1984099205

46.	75586.4993249663	76182.6891344567
47.	77098.5749287464	77699.0849542477
48.	78921.7060853043	79535.1767588379
49.	80416.5008250413	81034.2917145857
50.	82252.5926296315	82874.7037351868
51.	83738.7469373469	84373.8186909346
52.	85574.8387419371	86209.9104955248

Kourou: (28 passes)

	Rise Time	Set Time
1.	0	743.077153857693
2.	2566.20831041552	4112.84564228211
3.	5871.17355867793	7473.97369868493
4.	9176.13880694035	10826.4613230662
5.	12489.7444872244	14178.9489474474
6.	15799.0299514976	17535.7567878394
7.	19108.3154157708	20896.8848442422
8.	22408.9604480224	24266.6533326666
9.	25700.9650482524	27662.3431171559
10.	28949.7674883744	31140.1170058503
11.	32099.2049602480	33308.8654432722
12.	33343.4271713586	34920.3060153008
13.	35041.2720636032	36229.3314665733
14.	37166.8183409170	39357.1678583929
15.	40661.8730936547	42605.9702985149
16.	44061.8830941547	45889.3344667233
17.	47431.6515825791	49189.9794989750
18.	50792.7796389820	52499.2649632482
19.	54149.5874793740	55808.5504275214
20.	57502.0751037552	59122.1561078054
21.	60854.5627281364	62431.4415720786
22.	64215.6907845392	65732.0866043302
23.	67581.1390569528	69024.0912045602
24.	70976.8288414421	72272.8936446822
25.	74493.4846742337	75340.2470123506
26.	79595.6597829892	80494.2647132357
27.	82650.0525026251	84015.2407620381
28.	85894.5347267363	0

b. the pass duration (in units of minutes and seconds) to within 5 seconds;

	ASE Building	Ny Alesund	Kourou
1.	7.63238161908095	10.6565328266413	12.3846192309616
2.	9.43247162358117	10.6565328266413	25.7772888644432
3.	26.4973248662433	10.8005400270013	26.7133356667833
4.	27.4333716685834	10.7285364268213	27.5053752687634
5.	28.1534076703835	10.8005400270013	28.1534076703835
6.	28.8014400720036	10.7285364268214	28.9454472723636
7.	29.4494724736237	10.8005400270013	29.8094904745238
8.	30.1695084754238	10.7285364268213	30.9615480774039
9.	30.9615480774039	10.8725436271814	32.6896344817241
10.	32.1136056802840	10.6565328266413	36.5058252912646
11.	33.8416920846042	10.8725436271813	20.1610080504026
12.	15.8407920396020	10.7285364268213	26.2813140657033
13.	16.7048352417621	10.8725436271814	19.8009900495024
14.	14.5447272363618	10.6565328266413	36.5058252912645
15.	15.7687884394220	10.8725436271814	32.4016200810040
16.	14.1847092354618	10.7285364268214	30.4575228761439
17.	16.1288064403221	10.8725436271813	29.3054652732637
18.	15.1207560378019	10.8005400270013	28.4414220711036
19.	34.2017100855044	10.8725436271813	27.6493824691234
20.	32.1136056802841	10.7285364268213	27.0013500675034
21.	30.7455372768639	10.8005400270014	26.2813140657033
22.	29.8094904745237	10.6565328266413	25.2732636631833
23.	29.0174508725437	10.6565328266413	24.0492024601231
24.	28.2974148707435	10.5845292264613	21.6010800540028
25.	27.7213860693034	10.5125256262813	14.1127056352818
26.	27.0733536676834	10.3685184259214	14.9767488374418
27.	26.2093104655231	10.2245112255612	22.7531376568829
28.	12.3846192309616	10.1525076253813	-1431.57557877894
29.	11.8805940297015	9.93649682484126	0
30.	4.10420521026050	9.86449322466118	0
31.	0	9.57647882394122	0
32.	0	9.50447522376114	0
33.	0	9.28846442322113	0
34.	0	9.28846442322113	0
35.	0	9.07245362268113	0
36.	0	9.14445722286109	0
37.	0	8.92844642232109	0
38.	0	9.07245362268113	0
39.	0	8.92844642232121	0

40.	0	9.14445722286109	0
41.	0	9.07245362268101	0
42.	0	9.28846442322101	0
43.	0	9.36046802340134	0
44.	0	9.57647882394110	0
45.	0	9.64848242412118	0
46.	0	9.93649682484126	0
47.	0	10.0085004250211	0
48.	0	10.2245112255613	0
49.	0	10.2965148257414	0
50.	0	10.3685184259213	0
51.	0	10.5845292264615	0
52.	0	10.5845292264613	0

c. the highest elevation (in degrees) attained by the satellite during the pass; and

	ASE Building	Ny Alesund	Kourou
1.	1.52921497968213	80.3717481324584	18.8068962434322
2.	3.05756710919692	83.6482259152063	16.3734173189610
3.	8.60803835211015	74.4999274000928	30.4759304492394
4.	14.0882002874175	63.6130989025304	23.0244347556112
5.	18.9914057546159	57.6148200247986	24.3894558067209
6.	23.3029804859766	51.5056038418446	23.6864023927913
7.	26.6277493620474	47.9944414862437	22.0077218421266
8.	28.5414930615693	44.8868736481020	19.1807234154504
9.	29.1525975538730	43.0731176726924	15.7501889886276
10.	67.1991321960424	41.8771240912464	11.9596729374204
11.	35.7672510565016	41.4571104262242	7.95771466548189
12.	22.5625937494731	41.7103880595252	6.49658654496171
13.	40.4099960026859	42.3872333089352	6.27084750831179
14.	15.6349917444713	44.3878659533478	10.1938041363574
15.	15.1650394222125	44.6232727694859	15.4160345481485
16.	12.6502341628163	50.7522021481288	21.2797500708544
17.	12.2889487643110	48.7245541362231	30.4258099319397
18.	0	62.7100749917325	24.2188447625918
19.	0	53.4838853932949	23.3924751551323
20.	0	82.8441833061632	20.7418369521548
21.	0	60.5437534221552	16.7509579112659
22.	0	70.9951829553951	11.7333432995318

23.	0	56.3116444096725	6.83860798397815
24.	0	48.8015182545331	2.18318401990391
25.	0	39.2620799885175	9.65703358812115
26.	17.8232386164342	34.5941492426561	3.44596637005078
27.	0	28.8885404382409	6.51936440903645
28.	0	25.9128012944294	10.2809319887596
29.	28.2841740045281	22.3945385853893	0
30.	0.701505500205407	20.4024240822855	0
31.	0	18.1701132013755	0
32.	0	16.8189430834333	0
33.	0	15.4398307150806	0
34.	0	14.5811338326248	0
35.	0	13.8379926538171	0
36.	0	13.4204466419275	0
37.	0	13.2074405035013	0
38.	0	12.7291558437475	0
39.	0	13.5111227626773	0
40.	0	11.1101385066355	0
41.	0	14.8159040311247	0
42.	0	8.26397914555626	0
43.	0	17.3119967718880	0
44.	0	1.82350462559977	0
45.	0	21.4176758739592	0
46.	0	15.6107957653625	0
47.	0	28.0155715197583	0
48.	0	14.5260522955536	0
49.	0	14.2278263913262	0
50.	0	48.6119657494671	0
51.	0	57.0711075174412	0
52.	0	71.9492741962457	0
53.	0	0	0

d. the values of the satellite argument of latitude at start and stop times of each pass.

	ASE Building		**Ny Alesund**		**Kourou**	
	Rise	**Set**	**Rise**	**Set**	**Rise**	**Set**
1.	0.0010	0.0011	0.0001	0.0002	0	0.0001
2.	0.0011	0.0012	0.0004	0.0005	0.0004	0.0007
3.	0.0015	0.0018	0.0006	0.0007	0.0010	0.0012

4.	0.0021	0.0024	0.0009	0.0010	0.0015	0.0018
5.	0.0026	0.0029	0.0012	0.0013	0.0021	0.0024
6.	0.0032	0.0035	0.0015	0.0016	0.0026	0.0029
7.	0.0037	0.0040	0.0017	0.0018	0.0032	0.0035
8.	0.0043	0.0046	0.0020	0.0021	0.0037	0.0040
9.	0.0048	0.0051	0.0023	0.0024	0.0043	0.0046
10.	0.0054	0.0057	0.0026	0.0027	0.0048	0.0052
11.	0.0059	0.0063	0.0029	0.0030	0.0053	0.0055
12.	0.0065	0.0066	0.0031	0.0032	0.0055	0.0058
13.	0.0067	0.0068	0.0034	0.0035	0.0058	0.0060
14.	0.0070	0.0072	0.0037	0.0038	0.0062	0.0065
15.	0.0073	0.0074	0.0040	0.0041	0.0068	0.0071
16.	0.0075	0.0077	0.0042	0.0043	0.0073	0.0076
17.	0.0078	0.0080	0.0045	0.0046	0.0079	0.0082
18.	0.0081	0.0082	0.0048	0.0049	0.0084	0.0087
19.	0.0084	0.0088	0.0051	0.0052	0.0090	0.0093
20.	0.0090	0.0093	0.0053	0.0054	0.0096	0.0098
21.	0.0095	0.0098	0.0056	0.0058	0.0101	0.0104
22.	0.0101	0.0104	0.0059	0.0060	0.0107	0.0109
23.	0.0107	0.0109	0.0062	0.0063	0.0112	0.0115
24.	0.0112	0.0115	0.0064	0.0065	0.0118	0.0120
25.	0.0118	0.0121	0.0068	0.0069	0.0124	0.0125
26.	0.0123	0.0126	0.0070	0.0071	0.0132	0.0134
27.	0.0129	0.0132	0.0073	0.0074	0.0137	0.0140
28.	0.0135	0.0136	0.0075	0.0077	0.0143	0
29.	0.0136	0.0137	0.0079	0.0080	0	0
30.	0.0140	0.0141	0.0081	0.0082	0	0
31.	0	0	0.0084	0.0085	0	0
32.	0	0	0.0087	0.0088	0	0
33.	0	0	0.0090	0.0091	0	0
34.	0	0	0.0092	0.0093	0	0
35.	0	0	0.0095	0.0096	0	0
36.	0	0	0.0098	0.0099	0	0
37.	0	0	0.0101	0.0102	0	0
38.	0	0	0.0103	0.0104	0	0
39.	0	0	0.0106	0.0107	0	0
40.	0	0	0.0109	0.0110	0	0
41.	0	0	0.0112	0.0113	0	0
42.	0	0	0.0115	0.0116	0	0
43.	0	0	0.0117	0.0118	0	0

44.	0	0	0.0120	0.0121	0	0
45.	0	0	0.0123	0.0124	0	0
46.	0	0	0.0126	0.0127	0	0
47.	0	0	0.0128	0.0129	0	0
48.	0	0	0.0131	0.0132	0	0
49.	0	0	0.0134	0.0135	0	0
50.	0	0	0.0137	0.0138	0	0
51.	0	0	0.0139	0.0140	0	0
52.	0	0	0.0142	0.0143	0	0

Write a few sentences noting what you find interesting about the contents of the table.

Firstly, I would have thought that they all have the same number of passes before this assignment. In hindsight, it is interesting to note that the more passes the ground station had the quicker the satellite went past it. It is also interesting to know that the satellite doesn't "Rise in the East and Set in the West", but has the potential to disappear over the same horizon.

Code:

```
clear;clc;
%Constants
mu = 3.986e14;
ae = 6378136.3; %m
J2 = 1.082e-3;

%SpaceX Sat 4
a = 1275e3+ae; %m
e = 0; % Assume Circular Orbit and Spherical Earth
I = 81; %deg

oe0 = [a e I 0 0 0];

n = sqrt(mu/oe0(1)^3);
%Ground Station Geogrpahical Coordinates
gsLon1Lat2 = [30.1728, -97.4414; 78.5539 11.5228; 5.2846 -53.1224];
%Timeset
t = linspace(0,3600*24, 20000);

%Preallocation
RiseSet = zeros(1,6);
table = [0 0 0];
InSky = true;
MaxElevation = zeros(1,3);
LANatRiseSet = zeros(1,6);

for i = 1:(length(t)-1)
    %Define Satellite Orbital Elements
    M = n*t(i) + M_dot(a,e,I)*t(i);
    nu = E2nu(kepler(M,e),e); %in rad
    oe = [a e I/180*pi w_dot(a,e,I)*t(i) oe0(5)/180*pi-bigW_dot(a,e,I)*t(i) nu];
    rv = hw6oe2rv(oe,mu);
    rECI = rv(1:3)';

    %Define OE one step forward
    M = n*t(i+1) + M_dot(a,e,I)*t(i+1);
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```
    nuNext = E2nu(kepler(M,e),e); %in rad
    oeNext = [a e I/180*pi w_dot(a,e,I)*t(i+1) oe0(5)/180*pi-bigW_dot(a,e,I)*t(i+1) nuNext];
    rv = hw6oe2rv(oeNext,mu);
    rECI_next = rv(1:3)';

    for gs = 1:3
        %Define slant range now and one step forward
        pSEZ_now = getSEZ(t(i), rECI, gsLon1Lat2(gs,1), gsLon1Lat2(gs,2));
        pSEZ_next = getSEZ(t(i+1), rECI_next, gsLon1Lat2(gs,1), gsLon1Lat2(gs,2));

        %Z-coord
        Z = pSEZ_now(3);
        Z_next = pSEZ_next(3);

        %Test to see if satellite crosses horizon:
        %Rise (Negative to Positive)
        if Z < 0 && Z_next > 0
            RiseSet(table(gs)+1, 2*gs-1) = t(i+1);
            LANatRiseSet(table(gs)+1, 2*gs-1) = oeNext(5);
            MaxElevation(table(gs)+1,gs) = asind(Z/norm(pSEZ_now));
            InSky = true;
        %Set (Positive to Negative)
        elseif Z > 0 && Z_next < 0
            RiseSet(table(gs)+1, 2*gs) = t(i);
            LANatRiseSet(table(gs)+1, 2*gs) = oe(5);
            table(gs) = table(gs)+1;
            MaxElevation(table(gs)+1,1) = 0;
        %InSky marked as no longer visible
            InSky = false;
        end
        %Determine Max Elevation
        if InSky && asind(Z/norm(pSEZ_now))>MaxElevation(table(gs)+1,gs)
            MaxElevation(table(gs)+1,gs) = asind(Z/norm(pSEZ_now));
        end
    end
end

PassDuration = zeros(length(RiseSet),3);
%In minutes w/ decimals representing seconds
for j = 1:length(RiseSet)
    PassDuration(j,1) = abs(RiseSet(j,2) - RiseSet(j,1));
    PassDuration(j,2) = RiseSet(j,4) - RiseSet(j,3);
    PassDuration(j,3) = RiseSet(j,6) - RiseSet(j,5);
end
PassDuration = PassDuration./60;


%% Functions:
% SEZ Frame
function SEZ = getSEZ(t, r_ECI, gs_lat, gs_lon)
    ae = 6378136.3; %m
    we = 7.292115e-5; %rad/s
    rECEF = R3(we*t*180/pi)*r_ECI';
    rSEZ = R2(90-gs_lat)*R3(gs_lon)*rECEF;
    SEZ = rSEZ - ae*[cosd(gs_lon)*cosd(gs_lat);sind(gs_lon)*cosd(gs_lat);sind(gs_lat)];
end

%Precession Rates
function rate = bigW_dot(a,e,i) %i in degrees
    mu = 3.986e14;
    ae = 6378136.3; %m
    j2 = 1.082e-3;
    rate = -1.5*sqrt(mu/(a^3))*(ae/a)^2*j2*(1/sqrt(1-e^2))*cos((i*pi/180));
end
function rate = w_dot(a,e,i)
```

```
    mu = 3.986e14;
    ae = 6378136.3; %m
    j2 = 1.082e-3;
    rate = -.75*sqrt(mu/(a^3))*(ae/a)^2*j2*(1/(1-e^2)^2)*(1-5*(cos((i*pi/180))^2));
end
function rate = M_dot(a,e,i)
    mu = 3.986e14;
    ae = 6378136.3; %m
    j2 = 1.082e-3;
    rate = sqrt(mu/(a^3))*(1-.75*(ae/a)^2*j2*(1/sqrt((1-e^2)^3))*(1-3*(cos((i*pi/180))^2)));
end

% Rotation Matrices
function R1 = R1(theta)
    R1  = [1,       0,         0;
              0, cosd(theta),  sind(theta);
              0, -sind(theta), cosd(theta)];
end
function R2 = R2(theta)
    R2  = [cosd(theta),  0 , -sind(theta);
                  0,      1,      0 ;
              sind(theta),  0, cosd(theta)];
end
function R3 = R3(theta)
    R3  = [cosd(theta),   sind(theta) , 0;
             -sind(theta), cosd(theta),  0;
                0,          0 ,       1];
end
```