Homework Assignment #5 (Drag)

[Answer, Code]

1.

JD = 2451545.0 for Noon Jan 1st, 2000

Problem 1. $\theta_g = (18.697374458 + 24.06570982\, D)\frac{15^\circ}{hour}$

$$\frac{d\theta_g}{dD} = 24.06570982\left(\frac{15^\circ}{hr}\right)\left(\frac{\pi\, rad}{180^\circ}\right)\left(\frac{1\, hr}{3600\, s}\right)\left(\frac{1\, day}{24\, hr}\right)$$

• convert to rad/s → D was in days →

$$\boxed{\dot{\theta}_g = 7.292115 \times 10^{-5}\ rad/s = \omega_e}\ (\text{Eqn } 1.e)$$

2. Epoch: Noon Jan 1, 2000

A. Calculate the mean longitude of the sun (L):

L = 4.8950 rad = 280.463 degrees

```
% Epoch Noon Jan 1, 2000
P2epoch = [2000 1 1 12 0 0];
P3epoch = [2021 2 15 12 0 0];
%A. (10) Calculate the mean longitude of the sun (L)
JD = juliandate(P2epoch);
D = JD - 2451545.0; %in units of days
%Mean Longitude of Sun:
L = (pi/180)*(280.460 + 0.9856474*D) % rad
```

B. Calculate the RA and Dec of the Sun:

RA = -1.3738 rad = -78.7142 deg

Declination = -0.4020 rad = -23.0334 deg

```
%B. Calculate the RA and Dec of the Sun
%Mean Anomaly:
g = pi/180*(357.528 + 0.9856003*D); % rad
%ecliptic longitude:
lambda = L + deg2rad(1.915)*sin(g) + deg2rad(0.020)*sin(2*g);
```

```
%Ecliptic Obliquity:
eps = (pi/180)*(23.439 - (4e-7)*D);
%Right Ascension:
RA = atan(cos(eps)*tan(lambda))
RAdeg = rad2deg(RA)
Dec = asin(sin(eps)*sin(lambda))
Decdeg = rad2deg(Dec)
```

C. Calculate GMST

L = 4.8950 rad = 280.463 degrees

```
%C. Calculate GMST
GMST = 15*(pi/180)*(18.697374458 + 24.06570982*D); %in units of rad
```

D. Why are L and GMST so close to each other

For the given epoch of Noon, Jan 1, 2000. When calculating n or D it is equal 0 as this epoch is the adopted reference time or it could be described as a “starting point”. It is also noon so the sun would be directly above the “observer” at the Greenwich meridian. Thus the Sun’s ‘longitude’ coincides with the longitude of the observer.

E. What is the nearest city? Swakopmund, Namibia

3. (20 points) On the day it snowed in Austin during the recent ice-storm, answer the following:

A. Calculate the RA and Dec of the sun.

Using the time Noon Feb 15th, 2021 UTC:

RA = -0.5369 rad = -30.7619 deg

Declination =-0.2182 rad = -12.5011 deg

```
% Epoch Noon Jan 1, 2000
P2epoch = [2000 1 1 12 0 0];
P3epoch = [2021 2 15 21 0 0];
%A. (10) Calculate the mean longitude of the sun (L)
JD = juliandate(P3epoch);
D = JD - 2451545.0; %in units of days
%Mean Longitude of Sun:
L = (pi/180)*(280.460 + 0.9856474*D) % rad
```

```
Ldeg = rad2deg(L);
%B. Calculate the RA and Dec of the Sun
%Mean Anomaly:
g = pi/180*(357.528 + 0.9856003*D); % rad
%ecliptic longitude:
lambda = L + deg2rad(1.915)*sin(g) + deg2rad(0.020)*sin(2*g);
%Ecliptic Obliquity:
eps = (pi/180)*(23.439 - (4e-7)*D);
%Right Ascension:
RA = atan(cos(eps)*tan(lambda))
RAdeg = rad2deg(RA)
Dec = asin(sin(eps)*sin(lambda))
Decdeg = rad2deg(Dec)
```

B. Calculate the solar hour angle in Austin at noon on that day

Assuming it refers to noon in Austin the solar hour angle is 0 degrees as the sun is directly overhead.

C. Why was it necessary to use the phrase "at noon" in 3.b but not in 3.a?

3b depends on the time of the day in order to reference where the Sun's position is as an angle of the earth's rotation to bring the Sun above that point on Earth. In contrast, the impact that the time of the day has on the right ascension and declination calculations is trivial.

4. (20 points) The position/velocity of GRACE-FO satellite was provided for a certain epoch in HW#01. Assume that those initial conditions define a fixed (orbital) plane. We wish to calculate the RA and Dec of the sun at the epoch. Furthermore, we wish to calculate the longitude and latitude of the sun relative to the orbit plane.

A. (15 points) Make a nice sketch labeling all needed symbol and write down the algorithmic steps. You need not write down the steps for pos/vel to orbit element conversion or the formulas for calculating the sun's position. All other formulas and sequence of calculations should be clearly written out.

1. Find φ from w, f, and I using the formula from slide 23 of "The Angles" lecture
2. Find Λ from w, f, I using the formula from slide 23
3. Find λ from Λ, $\boldsymbol{\Omega}$, and Θ_g

(Sketch on next page) p.s I'm sorry I am nowhere near an artist

POS Unit vector = [0.796, -0.5832, -0.161]

Orbital Elements

$a = 6.8917e+06$

$e = 0.0026$

$I = 92.6714°$

$\omega = -159.8274°$

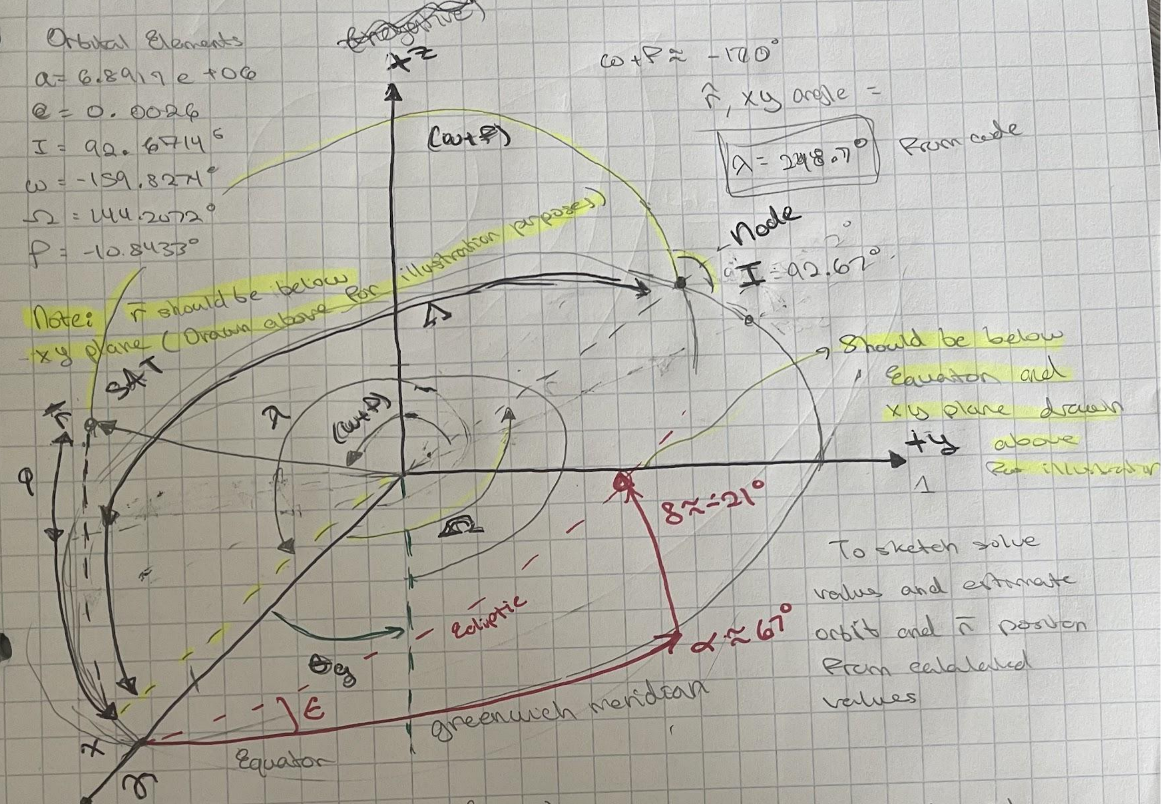
$\Omega = 144.2072°$

$f = -10.8433°$

$\omega + f \approx -170°$

$\hat{r}$, xy angle =

$\boxed{\lambda = 248.7°}$ From code



To sketch solve values and estimate orbit and $\bar{r}$ position from calculated values

$$\frac{\sin\phi}{\sin I} = \frac{\sin(\omega+f)}{\sin 90°} \rightarrow \sin\phi = \sin I \sin(\omega+f)$$

$$\phi = \sin^{-1}\left(\sin(92.6714°)\sin(-170.6707)\right)$$

$$\boxed{\phi = -9.319°}$$

Sun:

$$\boxed{\begin{array}{l} RA = 1.1695 \text{ rad} = 67.0102 \text{ deg} \\ Dec = -0.3797 \text{ rad} = -21.7549 \text{ deg} \end{array}}$$

$$\cos\Delta = \frac{\cos(\omega+f)}{\cos\phi} \rightarrow \Delta = \cos^{-1}\left(\frac{\cos(\omega+f)}{\cos\phi}\right)$$

$$= \cos^{-1}\left(\frac{\cos(-170.6707)}{\cos(-9.319°)}\right)$$

$$\boxed{\Delta = -179.5596°}$$

$$\lambda = \Delta - (\Omega - \theta_g)$$

$$\boxed{\lambda \approx -74°}$$

```
P4epoch = [2018.0, 11.0, 30.0, 23.0, 59.0, 42.0];
%A. (10) Calculate the mean longitude of the sun (L)
JD = juliandate(P4epoch);
D = JD - 2451545.0; %in units of days
%Mean Longitude of Sun:
L = (pi/180)*(280.460 + 0.9856474*D) % rad
Ldeg = rad2deg(L);
%B. Calculate the RA and Dec of the Sun
%Mean Anomaly:
g = pi/180*(357.528 + 0.9856003*D); % rad
%ecliptic longitude:
lambda = L + deg2rad(1.915)*sin(g) + deg2rad(0.020)*sin(2*g);
%Ecliptic Obliquity:
eps = (pi/180)*(23.439 - (4e-7)*D);
%Right Ascension:
RA = atan(cos(eps)*tan(lambda))
RAdeg = rad2deg(RA)
Dec = asin(sin(eps)*sin(lambda))
Decdeg = rad2deg(Dec)
%C. Calculate GMST
GMST = 15*(pi/180)*(18.697374458 + 24.06570982*D); %in units of rad
```

B. (5 points) calculate the solar longitude and latitude relative to the orbit plane – clearly define the reference point for reckoning the longitude.

Reference point is the x-axis from the ECI coordinate system and defined by J2000 reference frame.

4b. RA = 67.0102° Dec = −21.7549°

sun in ECI
$$\begin{cases} x = \sin(111.7549)\cos(67.0102) = 0.363 \\ y = \sin(111.7549)\sin(67.0102) = 0.855 \\ z = \cos(90° - (-21.7549°)) = -0.37 \end{cases}$$

$$\boxed{\overline{sun}_{ECI} = (0.363, 0.855, -0.371)}$$

$R_3(\Omega)$: rotate around z axis by Ω

$R_1(I)$: rotate around new x axis by inclination

$$\overline{sun}_{orbit} = R_3(\Omega)\, R_1(I)\, \overline{sun}_{ECI}$$

$$= \begin{bmatrix} \cos(144.2) & \sin(144.2) & 0 \\ -\sin(144.2) & \cos(144.2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(92.67) & \sin(92.67) \\ 0 & -\sin(92.67) & \cos(92.67) \end{bmatrix} \overline{sun}_{ECI}$$

Plugging into code...

$$\overline{sun}_{orbit} = (0.5424, -0.0709, 0.8371)$$

$$\text{Dec } (\phi) = \arctan \frac{\sqrt{x^2 + y^2}}{z} = 33.163° = 0.579 \text{ rad}$$

$$\text{RA } (\theta) = \arctan\left(\frac{y}{x}\right) = -7.447° = -0.13 \text{ rad}$$

```
rvGraceFO = [5471639.55639308   -4009260.88949393   -1113125.19797190,
-1210.66329250225     440.63844202350   -7515.04479126903];
oeGFO = hw6rv2oe(rvGraceFO,mu);
oeDeg = [oeGFO(1:2)' oeGFO(3:6)'.*180./pi]

sunECIhat = [sin(111.7549)*cos(67.0102), sin(111.7549)*sin(67.0102), cos(111.7549)];
R3CapOmega = [cos(oeDeg(5)) sin(oeDeg(5)) 0; -sin(oeDeg(5)) cos(oeDeg(5)) 0; 0 0 1];
R1Inclination = [1 0 0; 0 cos(oeDeg(3)) sin(oeDeg(3)); 0 -sin(oeDeg(3)) cos(oeDeg(3))];

sunOrbitHat = R3CapOmega*R1Inclination*sunECIhat'
```