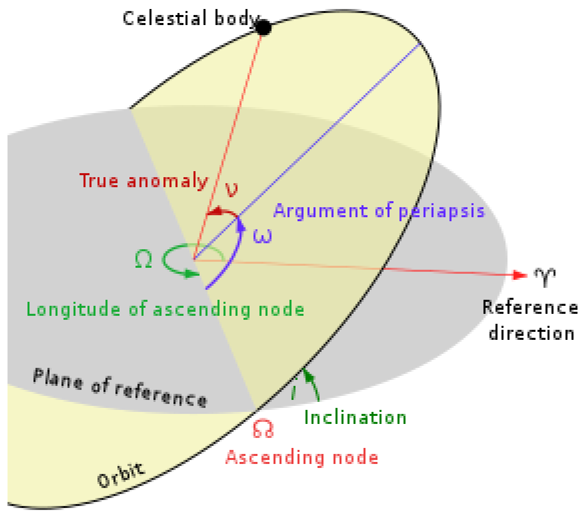
1.(80points) The SPE Approximation:The purpose of this question is to illustrate:

a. How a non-Keplerian orbit can be represented using time-variable orbital elements.

b.How the secularly precessing ellipse is a better approximation to the non-Keplerian orbit than a Keplerian ellipse.

**a.(5 points)** Define a metric for the difference between any two orbits (recall–"orbits" can refer to an ephemeris, or a time-series of position & velocity).




The metric I am most acclimated with to define the differences between orbits is the orbital elements.

- **Semimajor Axis - a:** determines the size of the orbit.
- **Eccentricity - e:** determines the shape of the orbit
- **True Anomaly - $\nu$:** determines location on the orbit
- **Argument of Periapsis - :** determines orientation of the ellipse (orbit plane)
- **Inclination - i:** also orients the orbit plane
- **Longitude of the Ascending Node - Ω:** also orients the orbit plane

**b.** Difference between IPE and TE

This is at the final time 3 days later.

rvDifference =

1.0e+06 *

1.8097 0.9899 -1.0549 -0.0011 -0.0005 -0.0023

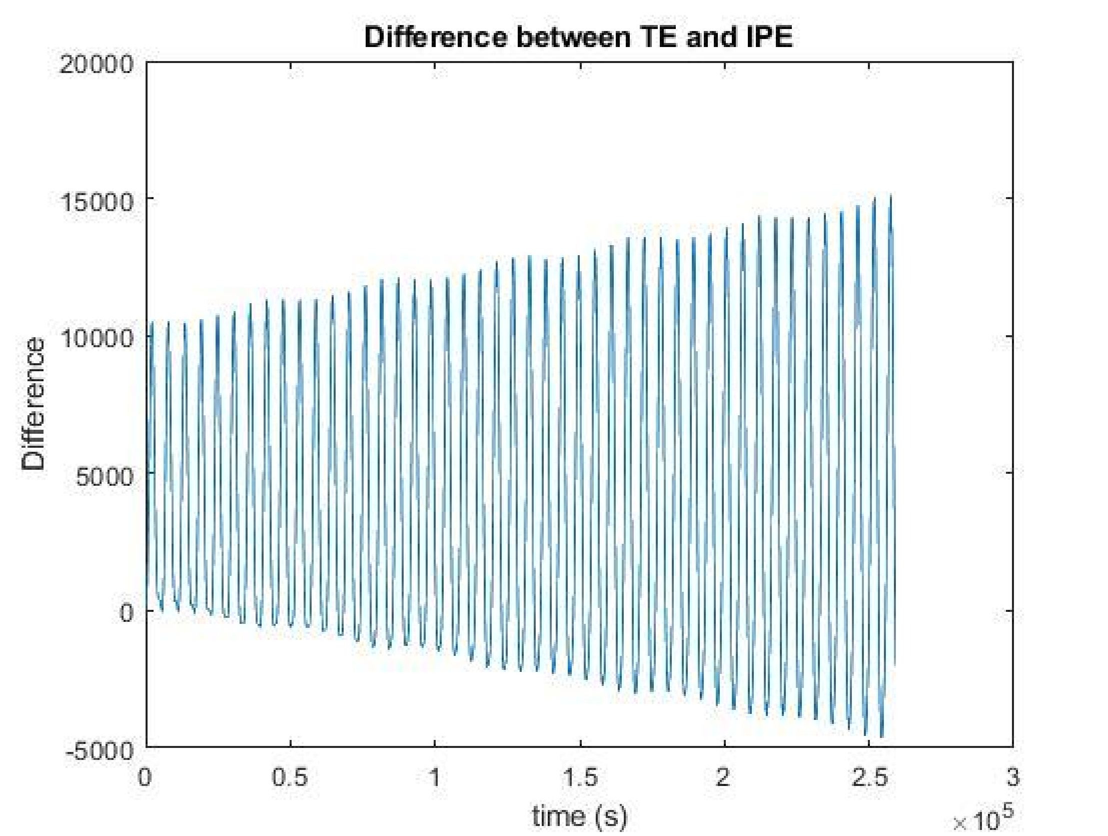
oeDifference =

1.0e+03 *

8.9975 0.0000 -0.0000 0.0002 0.0000 0.0001

**C.** Difference between oscillating elements of IPE and TE with time-scales

Assumed the timescale set as the same found in the data text file, 3 minute intervals for 3 days or 259200 seconds.

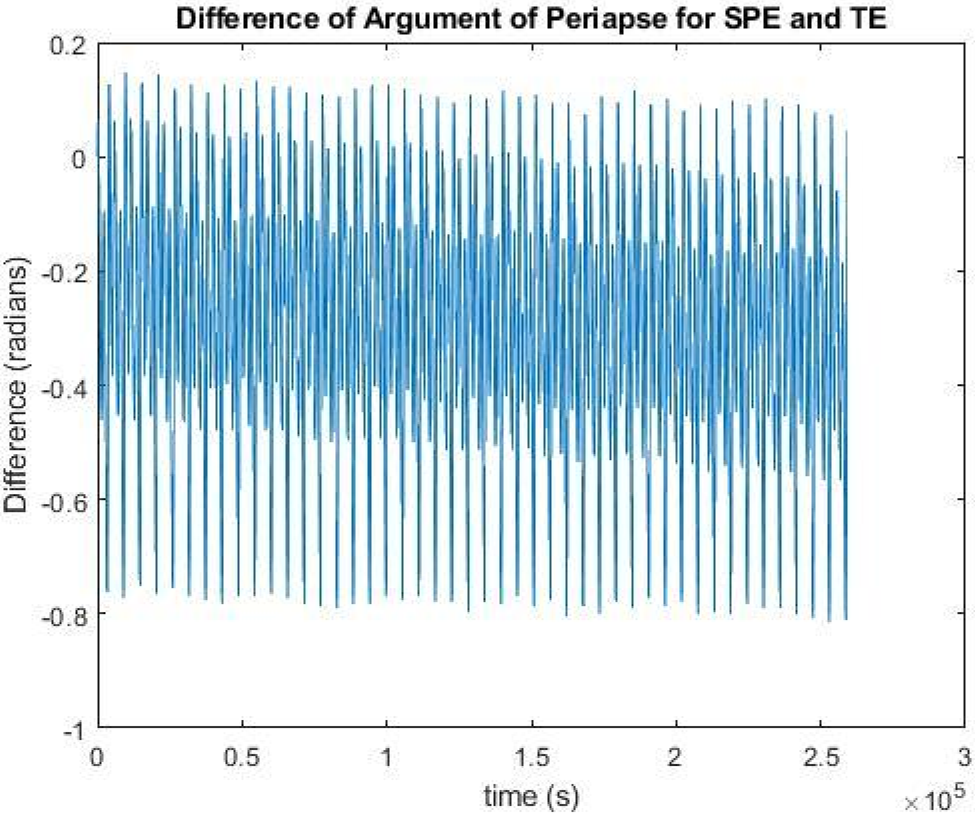



**D.** Difference between SPE and TE after 3 day propagation (final position and velocity in terms of orbital elements)

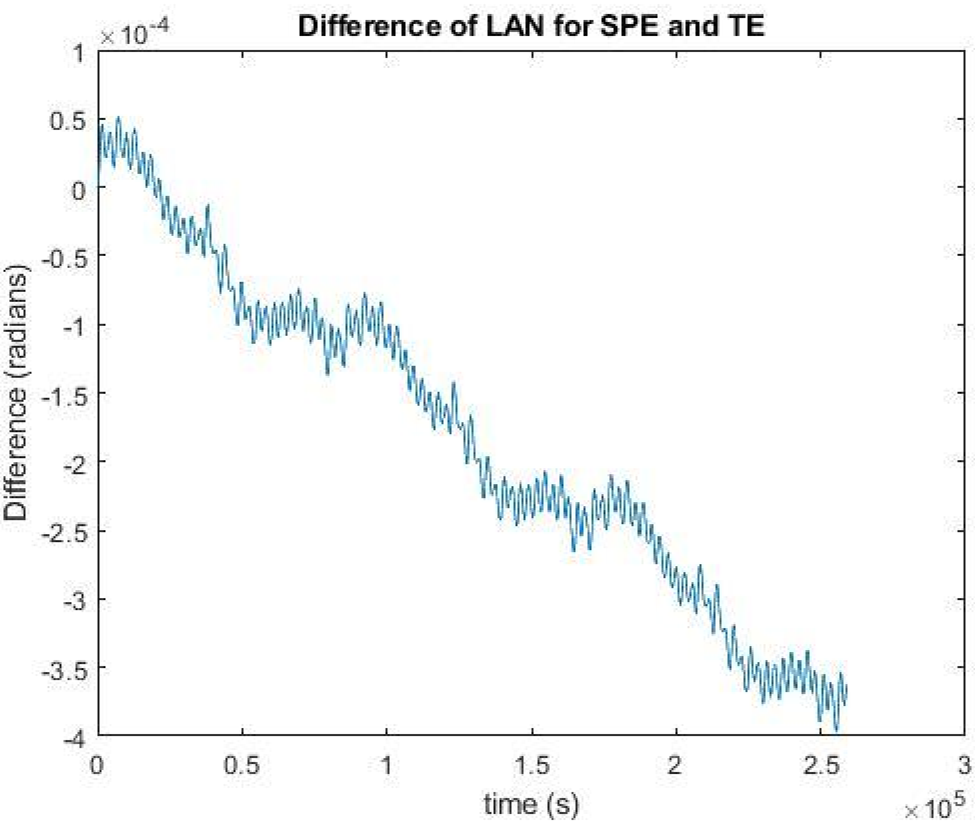
SPEdifference =

0 0 0 -0.2006 -0.0074 287.1639

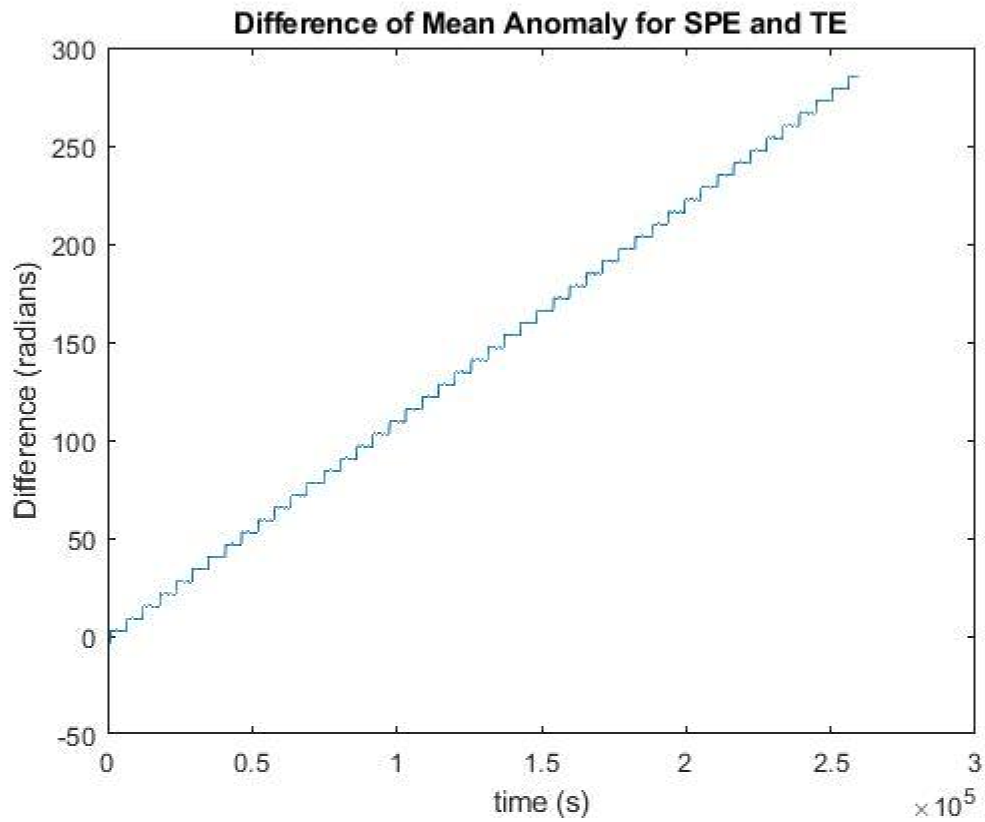
**e.** Plot the difference of the osculating elements



Difference of Argument of Periapse for SPE and TE
Difference (radians)
time (s)
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
3
×10^5

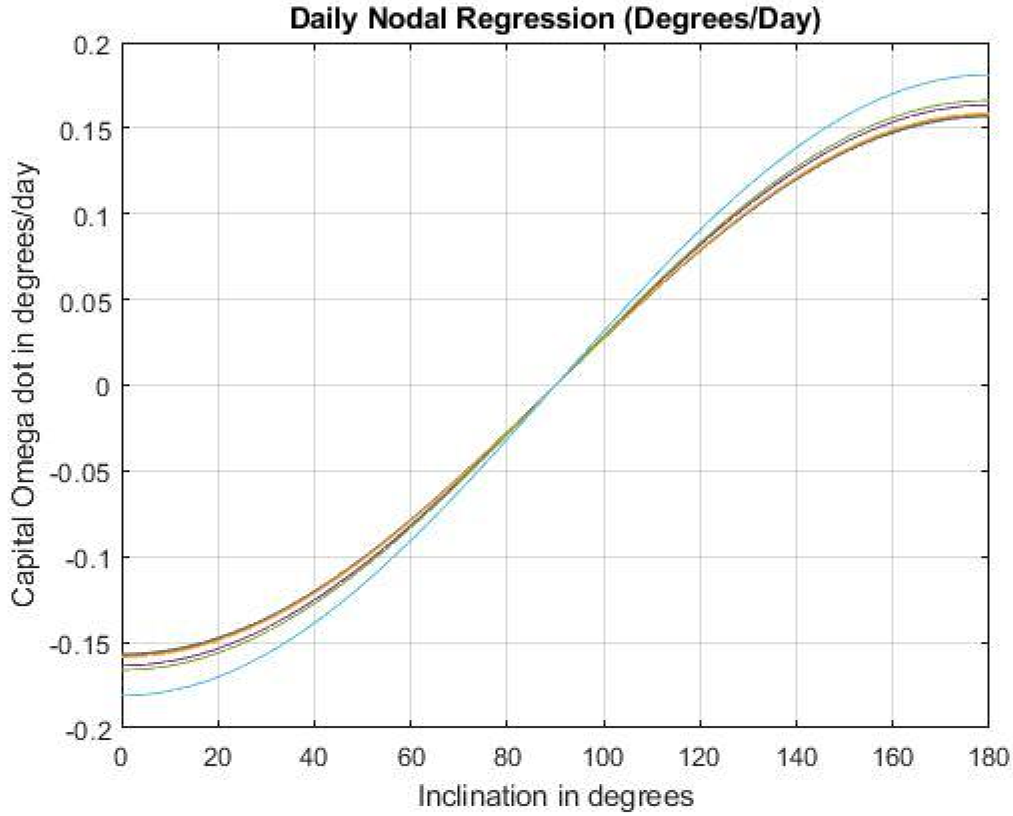



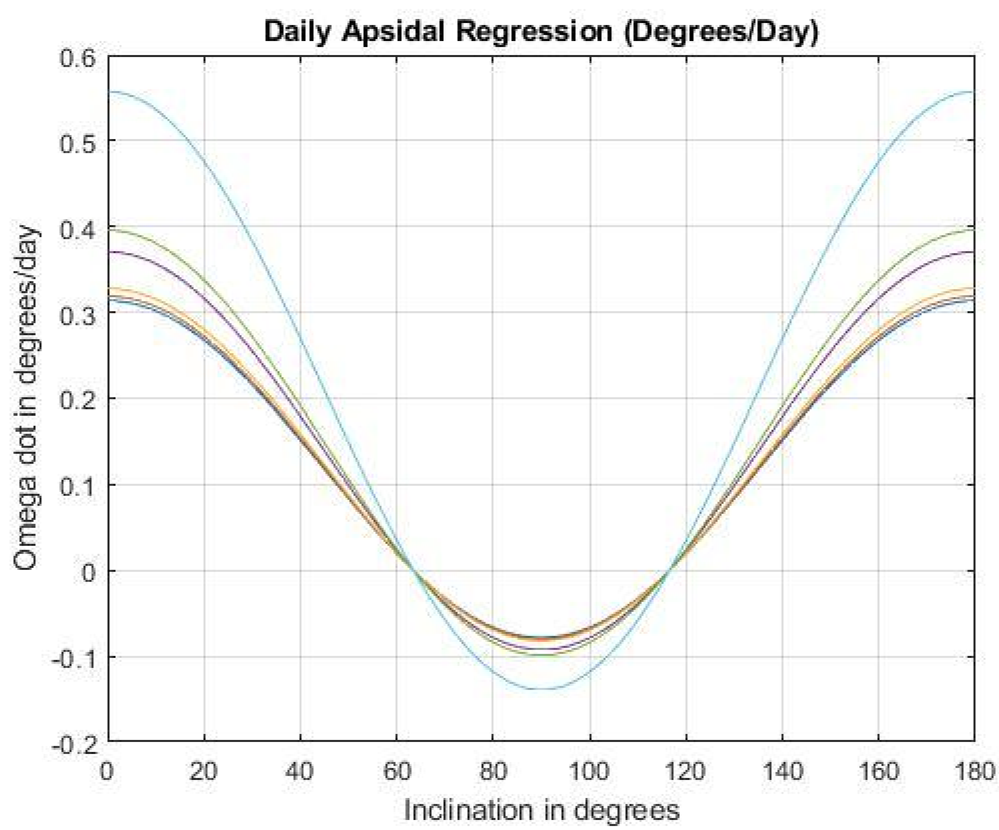
Difference of LAN for SPE and TE
×10^-4
Difference (radians)
time (s)
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
0
0.5
1
1.5
2
2.5
3
×10^5




**F.** The difference found in parts b and c was massive compared to parts d and e. Although part c was calculated as a norm of position and velocity vectors, it has a difference magnitude 1e4 to 1e8 greater than the difference associated with Argument of Periapse, Longitude of Ascending Node, and Argument of Periapse. This was to be expected as parts c and d take into account the oblateness of the Earth and the J2 scalar in order to produce a far more accurate propagation.

**Problem 2.**


Daily Nodal Regression (Degrees/Day)
Capital Omega dot in degrees/day
Inclination in degrees
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
20
40
60
80
100
120
140
160
180


Daily Apsidal Regression (Degrees/Day)
Omega dot in degrees/day
Inclination in degrees
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
20
40
60
80
100
120
140
160
180

CODE APPENDIX:

```
%% Applied Orbital Mechanics HW#3

%Mo & to specified by initial conditions

%constants
mu = 3.986004415*10^14;
ae = 6378136.3;
we = 7.292115*10^-5;
g=9.81;
j2=1.082*10^-3;
%

%% PART B
%t = 0.0 sec
rv0=[0.447927332846917398E+07  0.245170029746151669E+07 -0.461804936123361625E+07 -0.45505549
oe0 = hw6rv2oe(rv0,mu);


options = odeset('RelTol', 1e-14, 'AbsTol', 1e-14);
[t,rvf_prop]=ode45(@(t,r) body2(t,r,mu),[0:180:259200], rv0, options);

rvf_propagated = [rvf_prop(end,1), rvf_prop(end,2), rvf_prop(end,3), rvf_prop(end,4), rvf_prop(end,5), rvf_prop
oef_prop = hw6rv2oe(rvf_propagated,mu);

%t = 259200.0 sec
rvf =[0.187794820127492165E+07  0.835255100187982782E+06  0.653630524325667322E+07  0.64516544
oef = hw6rv2oe(rvf,mu);
%[a e i w bigOmega v(nu)]

rvDifference = rvf_propagated - rvf;
oeDifference = oef_prop - oef;

%% PART C
file = load('hw2-p01.txt');
timescale = (file(:,1));
rvTE = (file(:,2:7));
```

```
for b = 1:size(rvTE,1)
    range_prop = norm(rvf_prop(b,1:3));
    range_TE = norm(rvTE(b,1:3));
    rvSPEdiff_plot(b) = range_TE - range_prop;
end

figure(1)
plot(timescale,rvSPEdiff_plot)
xlabel('time (s)')
ylabel('Difference')
title('Difference between TE and IPE')
%The graph diverges, IPE value becomes less and less accurate as the orbit
%is propagated.

%% PART D
a0=oe0(1);
e0=oe0(2);
i0=oe0(3);
w0=oe0(4);
bigW0=oe0(5);

E0=2*atan(sqrt((1-oe0(2))/(1+oe0(2)))*tan(oe0(6)));
M0=E0-e0*sin(E0);
nu0= oe0(6);

oe0_M=oe0;
oe0_M(6) = M0;
n = sqrt(mu/a0^3);

% T = 2*pi*sqrt(oe0(1)^3/mu);
% Tquarter = T/4;
%
% E0=2*atan(sqrt((1-oe0(2))/(1+oe0(2)))*tan(oe0(6)));
% M0=sqrt(mu/(oe0(1)^3))*T-2*pi+E0-oe0(2)*sin(E0);

bigWbar = -1.5*n*(ae/a0)^2*j2/sqrt(1-e0^2)*cos(i0);
wbar = -0.75*n*(ae/a0)^2*j2/(1-e0^2)^2*(1-5*(cos(i0))^2);
Mbar = n*(1-(0.75*(ae/a0)^2*j2/(1-e0^2)^(3/2)*(1-3*(cos(i0))^2)));

bigWspe = bigW0 + bigWbar*(timescale);
```

```
wspe = w0 + wbar*(timescale);
Mspe = M0 + Mbar*(timescale);

spe=zeros(1441,6);
spe(:,1)=a0;
spe(:,2)=e0;
spe(:,3)=i0;
spe(:,4)=wspe;
spe(:,5)=bigWspe;
spe(:,6)=Mspe;

% Ef_M=zeros(1441,1)
% for iter=1:1441
%     Ef_M(i)=hw7kepler(Mspe(iter),e0);
% end
%
% nu_M = 2*atan(sqrt((1+e0)/(1-e0))*tan(Ef_M/2));
% speNu=spe
oeTE=zeros(1441,6);
for i = 1:1441
   oeTE(i,:)= hw6rv2oe(rvTE(i,:), mu);
end
Ee=2*atan(sqrt((1-e0)/(1+e0)))*tan(oeTE(:,6)/2);
Me=Ee-e0*sin(Ee);
oeTE_M(:,6)=Me;

speFinal = [a0 e0 i0 wspe(end) bigWspe(end) Mspe(end)];
SPEdifference = speFinal - oe0_M'

diffwe=spe(:,4)-oeTE(:,4);
diffWe=spe(:,5)-oeTE(:,5);
diffMe=spe(:,6)-oeTE(:,6);
diffMe=diffMe+2*pi*(diffMe<-5);
% speInitial=zeros(1440,6);
% speInitial(:,1)=a0;
% speInitial(:,2)=e0;
% speInitial(:,3)=i0;
% speInitial(:,4)=wspe;
% speInitial(:,5)=bigWspe;
% speInitial(:,6)=Mspe;
```

```
% spe=[oe0,speInitial];

%% PROBLEM E

figure(2)
plot(timescale,diffwe)
title('Difference of Argument of Periapse for SPE and TE')
xlabel('time (s)')
ylabel('Difference (radians)')

figure(3)
plot(timescale,diffWe)
title('Difference of LAN for SPE and TE')
xlabel('time (s)')
ylabel('Difference (radians)')

figure(4)
plot(timescale,diffMe)
title('Difference of Mean Anomaly for SPE and TE')
xlabel('time (s)')
ylabel('Difference (radians)')
% rv_spe(1,:) = rv0;
%
% w_spe = zeros(1441, 1);
% M_spe = zeros(1441, 1);
% W_spe = zeros(1441, 1);
% E_spe = zeros(1441, 1);
% nu_spe = zeros(1441, 1);
% for k = 2:size(timescale,1)
%     w_spe(k) = w0 + wbar*timescale(k);
%     M_spe(k) = M0 + Mbar*timescale(k);
%     W_spe(k) = bigW0 + bigWbar*timescale(k);
%     E_spe(k) = keplerEqn(M_spe(k),e0);
%     nu_spe(k) = 2*atan(sqrt((1+e0)/(1-e0))*tan(E_spe(k)/2));
%     oe_spe = [a0 e0 i0 w_spe(k) W_spe(k) nu_spe(k)];
%     rv_spe(k,:) = hw6oe2rv(oe_spe,mu)';
% end
%
% for k = 1:size(rvTE,1)
%     range_spe = norm(rv_spe(k,1:3));
```

```
%     range_TE = norm(rvTE(k,1:3));
%     rvSPEdiff_plot(b) = range_TE - range_spe;
% end
%
% figure(2)
% plot(timescale,rvSPEdiff_plot)
% xlabel('time (s)')
% ylabel('Difference')
% title('Difference between TE and SPE')

%% PROBLEM 2
muMars = 4.282837e13;
aMars = 3396190;
J2Mars = 1.932;
am2 = aMars + 100000;
I2 = linspace(0,pi,100)';
e2 = [0 .09 .148 .283 .331 .5];


nM = sqrt(muMars/(am2^3));


O2 = zeros(100,6);
w2 = zeros(100,6);


for p = 1:6
   O2(:,p) = (-3/2)*nM*((aMars/am2)^2)*J2Mars/sqrt(1-(e2(p)^2))*cos(I2);
   w2(:,p) = (-3/4)*nM*((aMars/am2)^2)*J2Mars/((1-(e2(p)^2))^2)*(1-5*(cos(I2).^2));
end


I2plot = I2*180/pi;
Omega2plot = O2*180/pi;
w2plot = w2*180/pi;


figure(5)
plot(I2plot, Omega2plot)
grid on
```

```
title('Daily Nodal Regression (Degrees/Day)')
xlabel('Inclination in degrees')
ylabel('Capital Omega dot in degrees/day')


figure(6)
plot(I2plot, w2plot)
grid on
title('Daily Apsidal Regression (Degrees/Day)')
xlabel('Inclination in degrees')
ylabel('Omega dot in degrees/day')
```