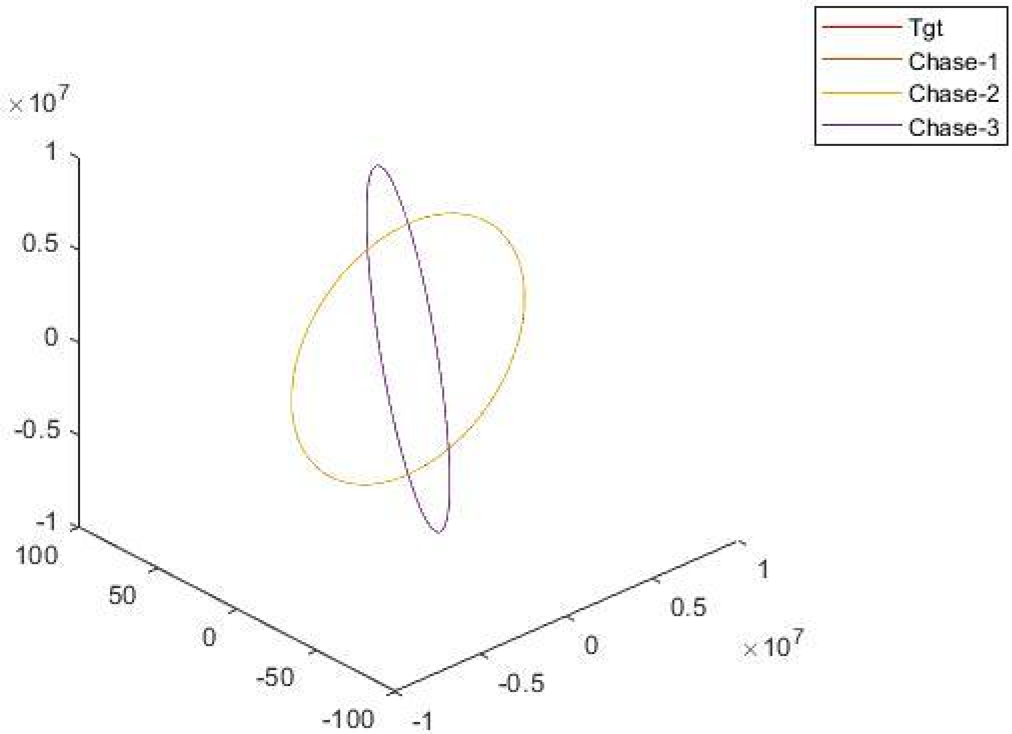
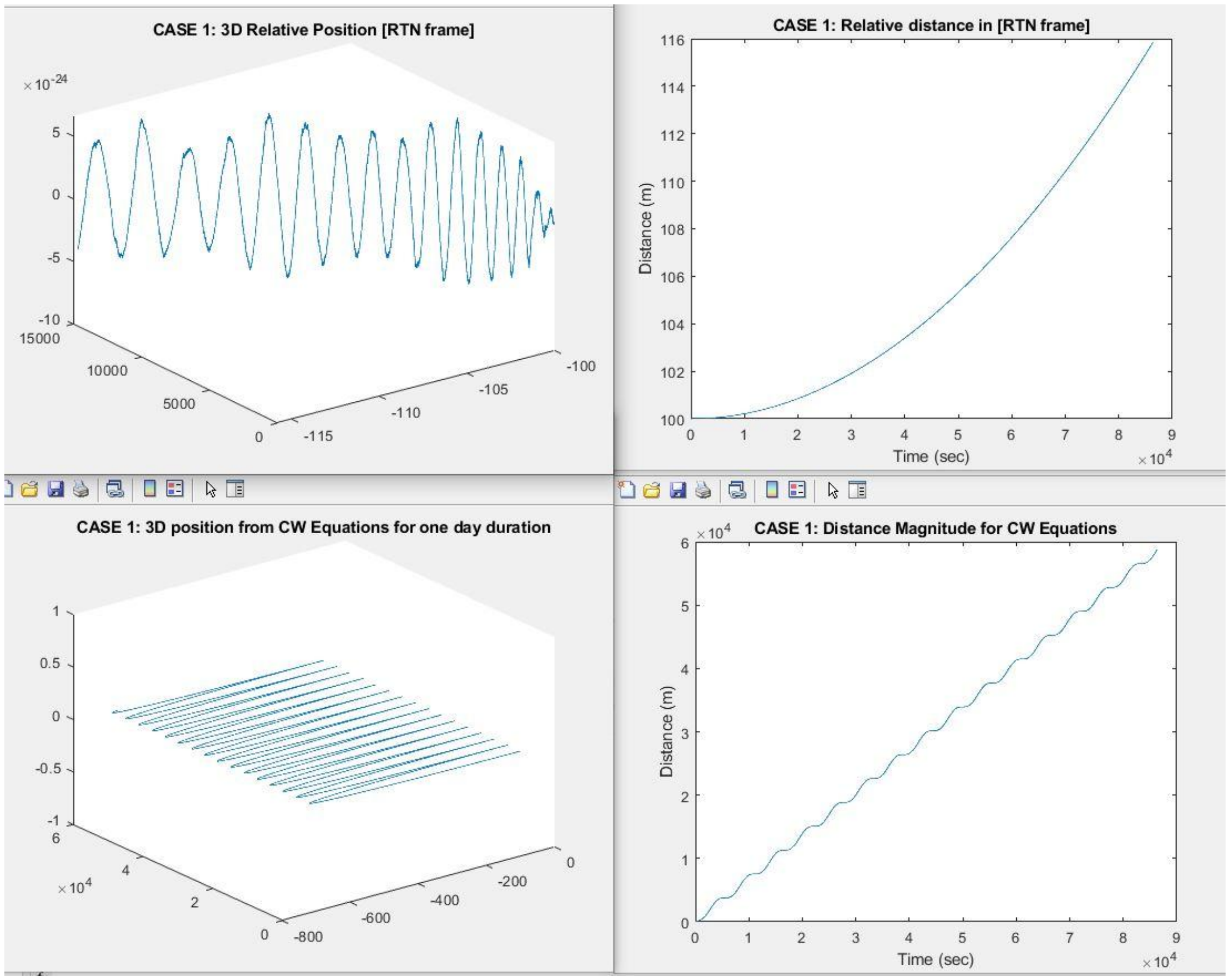
Homework Assignment #10
[Answer, Code]

Problem 1.



From plotting all the true orbits, it can be seen that the target, Case 1, and Case 2 are all virtually on the same path. For Case 3, the plot can be misleading in shape alone, but it can be seen that the max distance apart from the target in the N-frame is about 100 meters.

Case 1: Chaser orbit radius 100 meters less than Target orbit radius.



The 3D relative position in the RTN frame is misleading at first due to the oscillation in the z-frame, but on closer inspection it can be seen that its on the magnitude of 10^-24 so it can be summed up to numerical error as the 2 satellites orbit through space. There is however movement shown in the x-y plane which can be highlighted in the RTN distance magnitude plot as it increases exponentially from 100 meters to 116 meters. The CW equations and their plots displayed the expected results. It is shown to oscillate in the x-frame and have secular displacement in the y-frame this can be expected because of the third term in CW y(t) equation being multiplied by the time itself. The CW distance magnitude plot then shows this secular displacement the oscillation in the x-frame as the "squiggly" line follows the linear trend.

```
clear all, clc
%% Applied Orbital HW#10
%constants
mu = 3.986004415*10^14;
ae = 6378136.3;
we = 7.292115*10^-5;
g=9.81;
j2=1.082*10^-3;
```

```
d2r = pi/180;
r2d2 = 180/pi; %r2d2 cause why not lol (rad2deg)
%% Target Satellite "True" Orbit:
%Set up
t = 0:(60*60*24); %duration of one day
rtgt = 6778*1000;
w = sqrt(mu/rtgt^3);
e = 0; %circular orbit
i = 90*d2r; %polar orbit
% 2 body propagation
OEtgt=[rtgt e i 0 0 0];
RVtgt= hw6oe2rv(OEtgt, mu);
options = odeset('RelTol', 1e-14, 'AbsTol', 1e-14);
[t,RVtgt_prop]=ode45(@(t,r) body2(t,r,mu),[0:86400], RVtgt, options);
%% CASE 1: "True" Orbit
rint = rtgt-100;
eint = 0;
iint = 90;
% 2 body propagation
OEint1 =[rint e i 0 0 0];
RVint1 = hw6oe2rv(OEint1, mu);
options = odeset('RelTol', 1e-14, 'AbsTol', 1e-14);
[t,RVint1_prop]=ode45(@(t,r) body2(t,r,mu),[0:86400], RVint1, options);
%% "True" Orbit difference in ECI frame
RVdiff1 = RVint1_prop - RVtgt_prop;
figure(1)
plot3(RVdiff1(:,1),RVdiff1(:,2),RVdiff1(:,3))
title('CASE 1: 3D difference between "True" Orbits [ECI frame]')

posMagListTrue1 = zeros(1,length(t));
posVecVecTrue1 = [RVdiff1(:,1),RVdiff1(:,2),RVdiff1(:,3)];
for j = 1:length(t)
   posMagListTrue1(j) = norm(posVecVecTrue1(j,:));
end
figure(2)
plot(t,posMagListTrue1);
title("CASE 1: Distance Magnitude difference for True Orbits [ECI frame]")
xlabel("Time (sec)")
ylabel("Distance (m)")
%% RTN Conversion
```

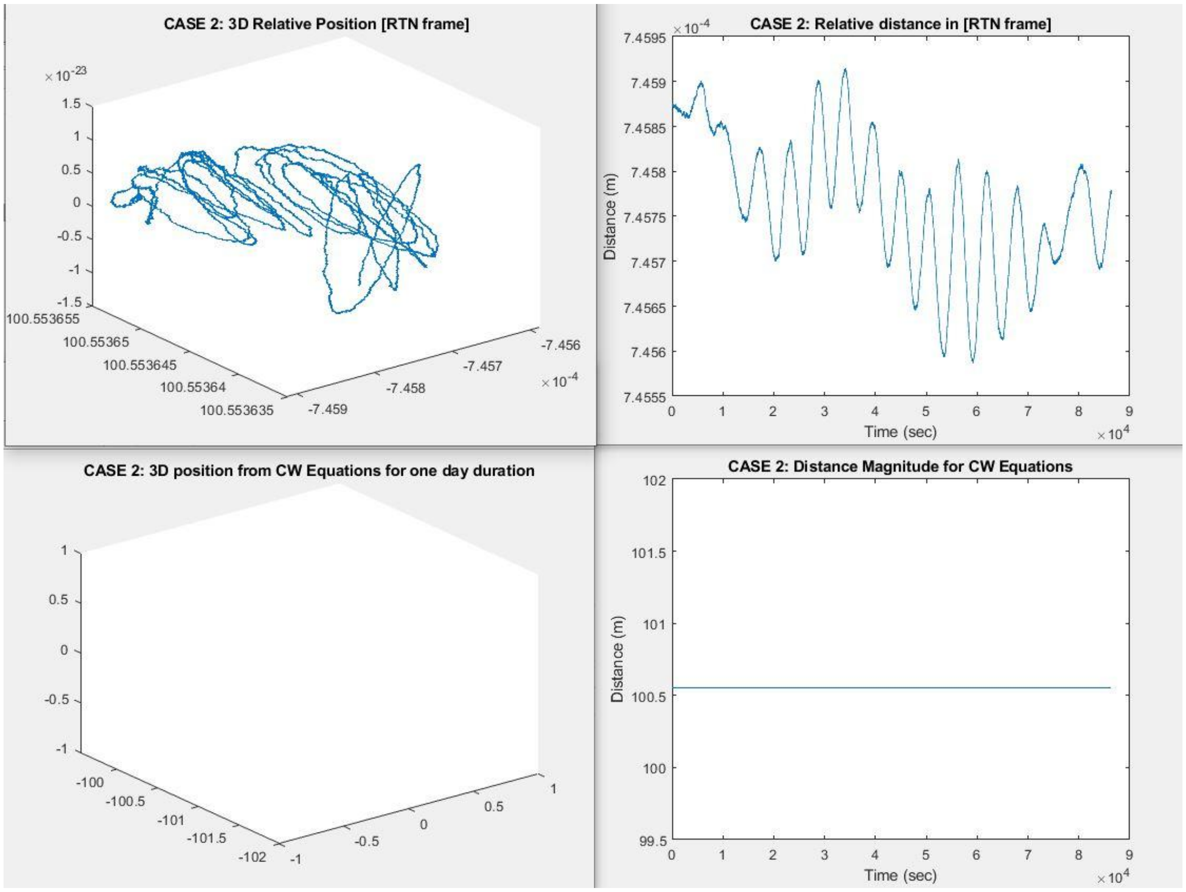
```
R_rel1 = zeros(3,length(t));
Rt = RVtgt_prop(:,1:3)';
Vt = RVtgt_prop(:,4:6)';
Rc1 = RVint1_prop(:,1:3)';
for k= 1:length(t)
   rh = Rt(:,k) / norm(Rt(:,k));
   H = cross(Rt(:,k),Vt(:,k));
   Nh = H/norm(H);
   Th = cross(Nh,rh);
   RTNtoECI = [rh, Th, Nh];
   R_rel1(:,k) = transpose(RTNtoECI) * (Rc1(:,k) - Rt(:,k));
   %relative position of chase vehicle wrt to tgt vehicle
end
figure(3)
plot3(R_rel1(1,:),R_rel1(2,:),R_rel1(3,:))
title('CASE 1: 3D Relative Position [RTN frame]')

posMagListRel1 = zeros(1,length(t));
posVecVecRel1 = [R_rel1(1,:),R_rel1(2,:),R_rel1(3,:)];
for j = 1:length(t)
   posMagListRel1(j) = norm(posVecVecRel1(:,j));
end
figure(4)
plot(t,posMagListRel1);
title("CASE 1: Relative distance in [RTN frame]")
xlabel("Time (sec)")
ylabel("Distance (m)")
%% CASE 1: CW Equations
%vrel = (rint*sqrt(mu/rint^3))-(rtgt*w);
xo = -100;
yo = 0;
zo =0;
vxo = 0;
vyo = 0;
vzo = 0;
xt = (vxo/w)*sin(w*t)-(3*xo+(2*vyo/w))*cos(w*t)+(4*xo+(2*vyo/w));
yt = (6*xo+(4*vyo/w))*sin(w*t)+(2*vxo/w)*cos(w*t)-(6*w*xo+3*vyo)*t+(yo-(2*vxo/w));
zt = -zo*w*sin(w*t)+vzo*cos(w*t);
vxt = vxo*cos(w*t)+(3*w*xo+2*vyo)*sin(w*t);
vyt = (6*w*xo+4*vyo)*cos(w*t)-2*vxo*sin(w*t)-(6*w*xo+3*vyo);
```

```
vzt = -zo*w*sin(w*t)+vzo*cos(w*t);
% Grows infinitely from secular growth.
figure(5)
plot3(xt,yt,zt)
title("CASE 1: 3D position from CW Equations for one day duration")

posMagListCW1 = zeros(1,length(t));
posVecVecCW1 = [xt,yt,zt];
for j = 1:length(t)
   posMagListCW1(j) = norm(posVecVecCW1(j,:));
end
figure(6)
plot(t,posMagListCW1);
title("CASE 1: Distance Magnitude for CW Equations")
xlabel("Time (sec)")
ylabel("Distance (m)")
```

Case 2: Chaser mean anomaly 0.00085 degrees behind the Target satellite initial mean anomaly



For Case 2 the RTN graphs are sporadic and show change, but on closer inspection the orders of magnitude are so small that they may as well be negligible. The same can be seen for the CW equations and their plots, no change in distance whatsoever. These results are expected as it is in the same orbit with all the same conditions. The chaser simply starts off approximately 100 meters behind the target. They then continue their path through space at this same distance from each other.

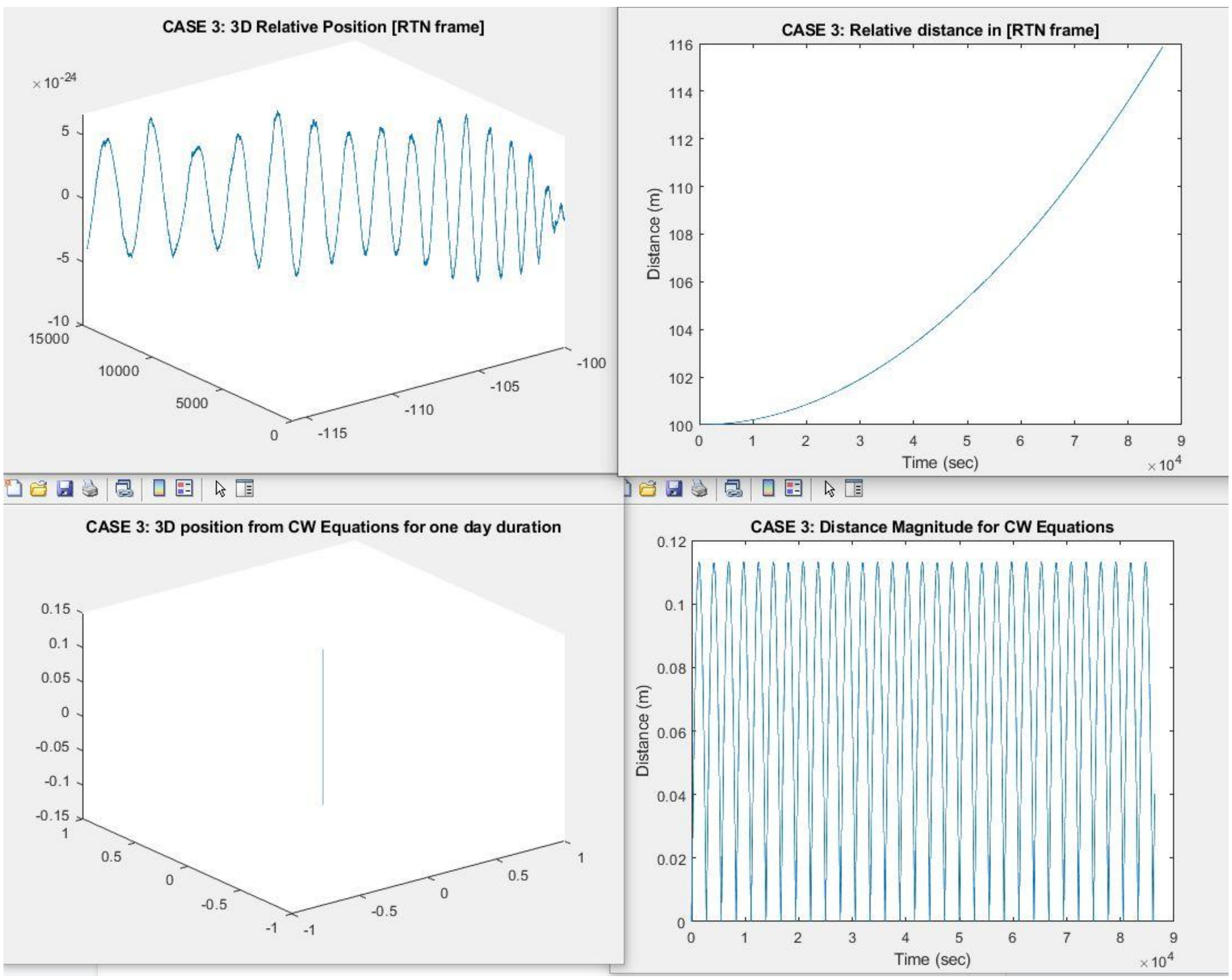
```
%% CASE 2: "True" Orbit
M2 = (360-0.00085)*d2r;
nu2 = E2nu(kepler(M2,e),e); %in rad
OEint2 = [rtgt e i 0 0 nu2];
RVint2 = hw6oe2rv(OEint2, mu);
options = odeset('RelTol', 1e-14, 'AbsTol', 1e-14);
[t,RVint2_prop]=ode45(@(t,r) body2(t,r,mu),[0:86400], RVint2, options);
R_rel2 = zeros(3,length(t));
Rc2 = RVint2_prop(:,1:3)';
for k= 1:length(t)
```

```
    rh = Rt(:,k) / norm(Rt(:,k));
    H = cross(Rt(:,k),Vt(:,k));
    Nh = H/norm(H);
    Th = cross(Nh,rh);
    RTNtoECI = [rh, Th, Nh];
    R_rel2(:,k) = transpose(RTNtoECI) * (Rc2(:,k) - Rt(:,k));
    %relative position of chase vehicle wrt to tgt vehicle
end
figure(7)
plot3(R_rel2(1,:),R_rel2(2,:),R_rel2(3,:))
title('CASE 2: 3D Relative Position [RTN frame]')

posMagListRel2 = zeros(1,length(t));
posVecVecRel2 = [R_rel2(1,:),R_rel2(2,:),R_rel2(3,:)];
for j = 1:length(t)
    posMagListRel2(j) = norm(posVecVecRel2(:,j));
end
figure(8)
plot(t,posMagListRel2);
title("CASE 2: Relative distance in [RTN frame]")
xlabel("Time (sec)")
ylabel("Distance (m)")
%% CASE 2: CW Equations
xo2 = 0;
yo2 = -0.00085*d2r*rtgt;
zo2 = 0;
vxo2 = 0;
vyo2 = 0;
vzo2 = 0;
xt2 = (vxo2/w)*sin(w*t)-(3*xo2+(2*vyo2/w))*cos(w*t)+(4*xo2+(2*vyo2/w));
yt2 =
(6*xo2+(4*vyo2/w))*sin(w*t)+(2*vxo2/w)*cos(w*t)-(6*w*xo2+3*vyo2)*t+(yo2-(2*vxo2/w));
zt2 = -zo2*w*sin(w*t)+vzo2*cos(w*t);
vxt2 = vxo2*cos(w*t)+(3*w*xo2+2*vyo2)*sin(w*t);
vyt2 = (6*w*xo2+4*vyo2)*cos(w*t)-2*vxo2*sin(w*t)-(6*w*xo2+3*vyo2);
vzt2 = -zo2*w*sin(w*t)+vzo2*cos(w*t);
% Grows infinitely from secular growth.
figure(9)
plot3(xt2,yt2,zt2)
title("CASE 2: 3D position from CW Equations for one day duration")
```

```
posMagListCW2 = zeros(1,length(t));
posVecVecCW2 = [xt2,yt2,zt2];
for j = 1:length(t)
   posMagListCW2(j) = norm(posVecVecCW2(j,:));
end
figure(10)
plot(t,posMagListCW2);
title("CASE 2: Distance Magnitude for CW Equations")
xlabel("Time (sec)")
ylabel("Distance (m)")
```

Case 3: Chaser separated by 100 meters along the positive orbit normal of the Target satellite.



The plots for Case 3 are not what I expected. The RTN frame plots are virtually the same as they are for Case 1. The CW equations do show the expected movement in the z-frame to be expected from the 100 meter offset. However the distance is not on the order of 100 meters and is generally smaller. This is likely due to some numerical error. It is worth noting that the oscillation in the z-frame can be extrapolated from the movement shown in the 3D plot and the oscillation in the distance magnitude plot for the CW equations.

```
%% CASE 3: "True" Orbit
OEint3 = [rtgt e i 0 0 0];
RVint3 = hw6oe2rv(OEint3, mu);
RVint3 = [RVint3(1) RVint3(2)+100 RVint3(3) RVint3(4) RVint3(5) RVint3(6)];
%approx seperated by 100m along positive orbit normal?
[t,RVint3_prop]=ode45(@(t,r) body2(t,r,mu),[0:86400], RVint3, options);
R_rel3 = zeros(3,length(t));
Rc3 = RVint1_prop(:,1:3)';
for k= 1:length(t)
   rh = Rt(:,k) / norm(Rt(:,k));
   H = cross(Rt(:,k),Vt(:,k));
   Nh = H/norm(H);
   Th = cross(Nh,rh);
   RTNtoECI = [rh, Th, Nh];
   R_rel3(:,k) = transpose(RTNtoECI) * (Rc3(:,k) - Rt(:,k));
   %relative position of chase vehicle wrt to tgt vehicle
end
figure(11)
plot3(R_rel3(1,:),R_rel3(2,:),R_rel3(3,:))
title('CASE 3: 3D Relative Position [RTN frame]')

posMagListRel3 = zeros(1,length(t));
posVecVecRel3 = [R_rel3(1,:),R_rel3(2,:),R_rel3(3,:)];
for j = 1:length(t)
   posMagListRel3(j) = norm(posVecVecRel3(:,j));
end
figure(12)
plot(t,posMagListRel3);
title("CASE 3: Relative distance in [RTN frame]")
xlabel("Time (sec)")
ylabel("Distance (m)")

%% CASE 3: CW Equations
xo3 = 0;
yo3 = 0;
zo3 = 100;
```

```
vxo3 = 0;
vyo3 = 0;
vzo3 = 0;
xt3 = (vxo3/w)*sin(w*t)-(3*xo3+(2*vyo3/w))*cos(w*t)+(4*xo3+(2*vyo3/w));
yt3 =
(6*xo3+(4*vyo3/w))*sin(w*t)+(2*vxo3/w)*cos(w*t)-(6*w*xo3+3*vyo3)*t+(yo3-(2*vxo3/w));
zt3 = -zo3*w*sin(w*t)+vzo3*cos(w*t);
vxt3 = vxo3*cos(w*t)+(3*w*xo3+2*vyo3)*sin(w*t);
vyt3 = (6*w*xo3+4*vyo3)*cos(w*t)-2*vxo3*sin(w*t)-(6*w*xo3+3*vyo3);
vzt3 = -zo3*w*sin(w*t)+vzo3*cos(w*t);
% Grows infinitely from secular growth.
figure(13)
plot3(xt3,yt3,zt3)
title("CASE 3: 3D position from CW Equations for one day duration")

posMagListCW3 = zeros(1,length(t));
posVecVecCW3 = [xt3,yt3,zt3];
for j = 1:length(t)
   posMagListCW3(j) = norm(posVecVecCW3(j,:));
end
figure(14)
plot(t,posMagListCW3);
title("CASE 3: Distance Magnitude for CW Equations")
xlabel("Time (sec)")
ylabel("Distance (m)")
%% All "True" Orbits
figure(15)
plot3(RVtgt_prop(:,1),RVtgt_prop(:,2),RVtgt_prop(:,3),"r")
hold on
%figure(10)
plot3(RVint1_prop(:,1),RVint1_prop(:,2),RVint1_prop(:,3))
%figure(11)
plot3(RVint2_prop(:,1),RVint2_prop(:,2),RVint2_prop(:,3))
% figure(12)
plot3(RVint3_prop(:,1),RVint3_prop(:,2),RVint3_prop(:,3))
hold off
legend("Tgt","Chase-1","Chase-2","Chase-3")
%% Functions
%Precession Rates
function rate = bigW_dot(a,e,i) %i in degrees
```

```
    mu = 3.986e14;
    ae = 6378136.3; %m
    j2 = 1.082e-3;
    rate = -1.5*sqrt(mu/(a^3))*(ae/a)^2*j2*(1/sqrt(1-e^2))*cos((i*pi/180));
end
function rate = w_dot(a,e,i)
    mu = 3.986e14;
    ae = 6378136.3; %m
    j2 = 1.082e-3;
    rate = -.75*sqrt(mu/(a^3))*(ae/a)^2*j2*(1/(1-e^2)^2)*(1-5*(cos((i*pi/180))^2));
end
function rate = M_dot(a,e,i)
    mu = 3.986e14;
    ae = 6378136.3; %m
    j2 = 1.082e-3;
    rate = sqrt(mu/(a^3))*(1-.75*(ae/a)^2*j2*(1/sqrt((1-e^2)^3))*(1-3*(cos((i*pi/180))^2)));
end
%Keplerian Motion
function drdt=body2(t,r,mu)
drdt(1)=r(4);
drdt(2)=r(5);
drdt(3)=r(6);
drdt(4)=-1*mu*r(1)/(r(1)^2+r(2)^2+r(3)^2)^1.5;
drdt(5)=-1*mu*r(2)/(r(1)^2+r(2)^2+r(3)^2)^1.5;
drdt(6)=-1*mu*r(3)/(r(1)^2+r(2)^2+r(3)^2)^1.5;
drdt=drdt';
end
function E = kepler(M,e)
    E0=M;
    deltaE = 1;
    tol = 1e-4;
    n=0;
    while abs(deltaE) > tol
        deltaE = -(M - E0 + e*sin(E0))/(-1 + e*cos(E0));
        E0 = E0 + deltaE;
        n = n + 1;
    end
    E = E0;
end
function nu = E2nu(E,e)
```

```
    %using rad now
    nu = 2*atan2(tan(E/2)*sqrt((1+e)/(1-e)),1);
    if nu < 0
        nu = 2*pi-nu;
    end
End
function[rv] = hw6oe2rv(oe, mu)

rMag = oe(1)*(1-oe(2)^2)/(1+oe(2)*cos(oe(6)));
pHat = [1 0 0];
qHat = [0 1 0];

rPQW = rMag*cos(oe(6))*pHat+rMag*sin(oe(6))*qHat;
RbigW = [cos(oe(5)) -1*sin(oe(5)) 0; sin(oe(5)) cos(oe(5)) 0; 0 0 1];
Ri = [1 0 0;0 cos(oe(3)) sin(-1*oe(3));0 sin(oe(3)) cos(oe(3))];
Rw = [cos(oe(4)) -1*sin(oe(4)) 0; sin(oe(4)) cos(oe(4)) 0; 0 0 1];

rIJK = RbigW*Ri*Rw*rPQW';
rv(1,1)=rIJK(1);
rv(2,1)=rIJK(2);
rv(3,1)=rIJK(3);

p = oe(1)*(1-oe(2)^2);
vPQW = -1*sin(oe(6))*sqrt(mu/p)*pHat+sqrt(mu/p)*(oe(2)+cos(oe(6)))*qHat;
vIJK=RbigW*Ri*Rw*vPQW';
rv(4,1)=vIJK(1);
rv(5,1)=vIJK(2);
rv(6,1)=vIJK(3);

end
```

Problem 2. Find the conditions under which there will be no secular displacement of the Chaser relative to the Target

The secular displacement term can be found in y(t) hill's equation:

$$(6wx_o + 3\dot{y}_o)t$$

There will be no secular displacement when x_o and $\dot{y}_o$ are equal to zero. As well as when…

$$x_o = (-\tfrac{1}{2})(\dot{y}_o/w)$$

Or equivalently …

$$\dot{y}_o = 2wx_o$$

Where w is the mean motion, x_o is the initial radial position offset, and $\dot{y}_o$ is the initial velocity offset in the T-frame.

Problem 3. Consider Case-3 closely. Why is the displacement at time t_o (the maximum displacement in the N direction between the Target and the Chaser orbits? At what argument of latitude will the Target and Chaser orbits intersect?

The initial condition causes the orbit plane to be offset, but they are both still polar orbits that must cross over the poles. With this in mind the 2 satellites must head toward each other to intersect at the same points above the poles. This is why the max displacement in N direction is at t_o.

As previously stated the 2 orbits intersect at the poles being in polar orbits, so the argument of latitude is 90 degrees.