**Homework Assignment #4 (Drag)**

**1. Orders of Magnitude [Answer, Code]**

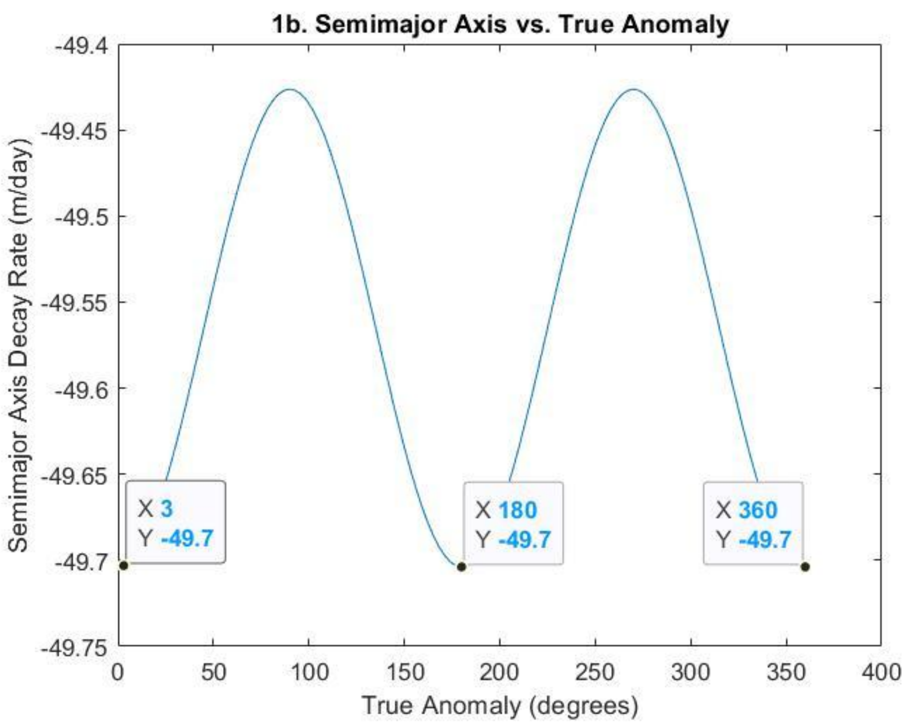
**a. Neglecting both atmospheric co-rotation and wind gusts.**

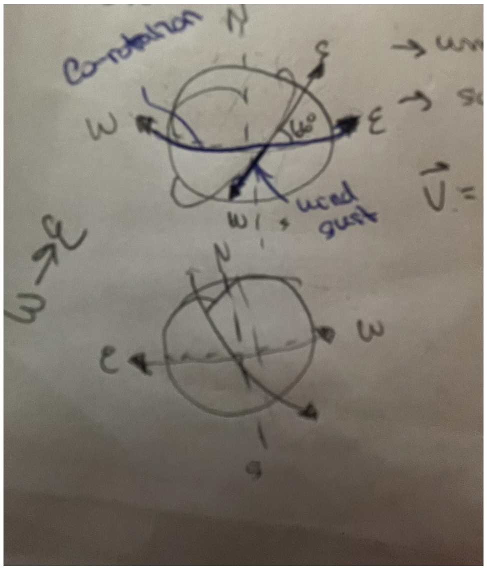
da/dt = -53.574608650503414 meters/day

```
%constants
mu = 3.986004415*10^14;
ae = 6378136.3;
we = 7.292115*10^-5;
g=9.81;
j2=1.082*10^-3;
%A
i = deg2rad(66);
Cd = 2.0;
A = 1.3; %(m^2)
m = 350; %(kg)
rho = 1.6*10^-12; %(kg/m^3)
alt = 450*1000; %(m)
a = ae + alt;
e = 0; %circular orbit
v_1a = sqrt(mu/a);
fd_1a = 0.5*rho*Cd*A*(v_1a^2)/m;
dadt_1a = -2*v_1a*a^2*fd_1a/mu;
dadt_1a_mday = dadt_1a*86400;
```

**b. With atmospheric co-rotation accounted for, the decay rate is maximum at the argument of latitude or true anomaly of…**

0, 180, or 360 degrees with respect to maximum magnitude. The cross product of the earth rotation vector (w_e) and the position vector is subtracted from the inertial velocity calculated with given **Equation 5.** Following this logic, the cross product is largest at the equatorial plane and produces the smallest, v_r. This then produces the smallest magnitude of drag acceleration (f_d) and smallest semimajor axis decay rate (da/dt). The values found correspond with this thought process.

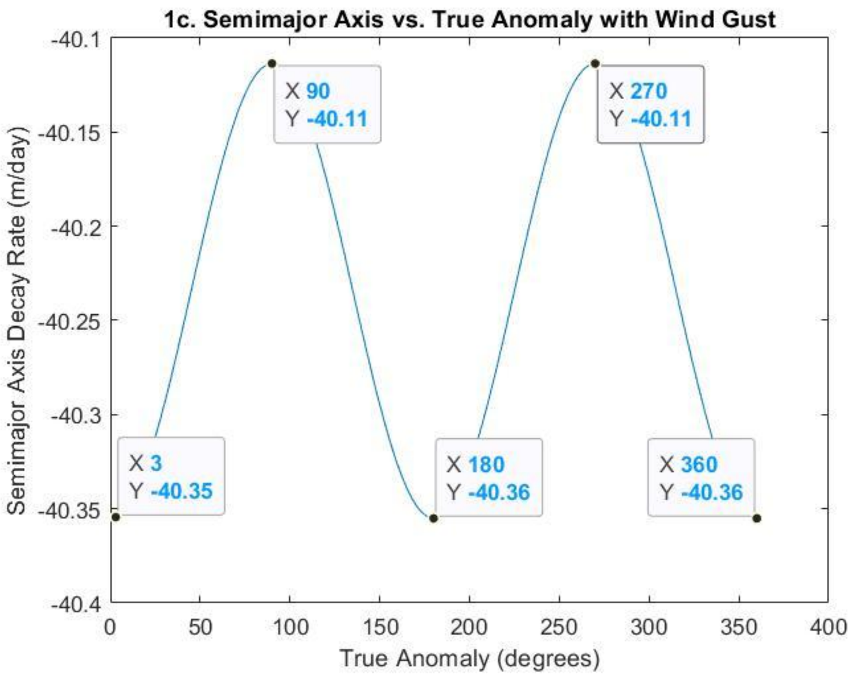


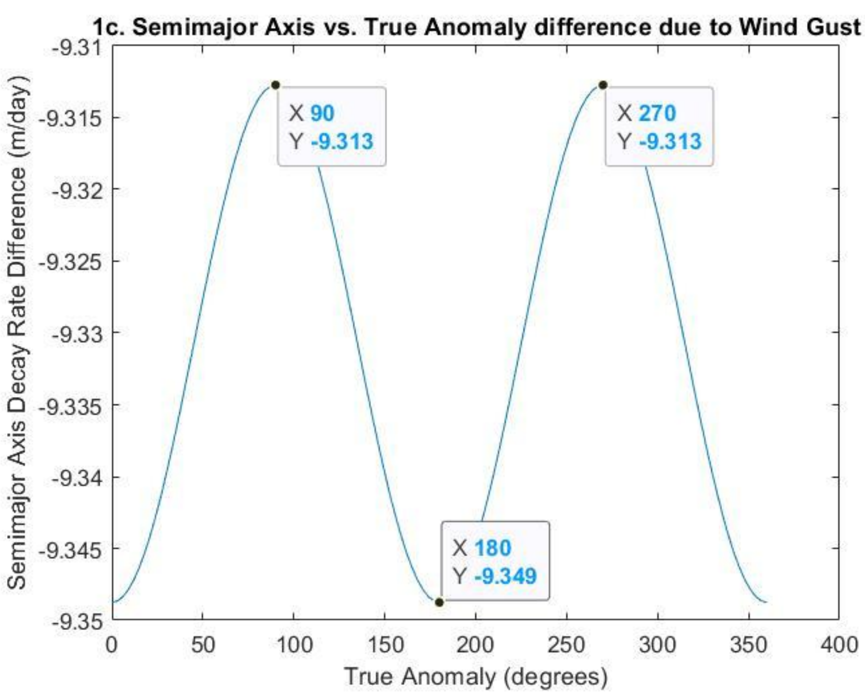



```
%B
f = 0:3:360;
for j = 1:length(f)
oe = [a e i 0 0 f(j)/180*pi];
rv = hw6oe2rv(oe, mu)';
v_1b = norm(rv(4:6) - cross([0 0 we],rv(1:3)));
fd_1b = 0.5*rho*Cd*A*(v_1b^2)/m;
dadt_1b_mday(j) = 86400*-2*v_1b*(a^2)*fd_1b/mu;
end
figure(1)
plot(f,dadt_1b_mday)
title("1b. Semimajor Axis vs. True Anomaly")
xlabel("True Anomaly (degrees)")
ylabel("Semimajor Axis Decay Rate (m/day)")
```

**c. With a sudden wind gust, what is the effect on the decay rate of change?**

The sudden wind gust decreases the magnitude of the decay rate of change by approximately 9.3 meters/day.




%C

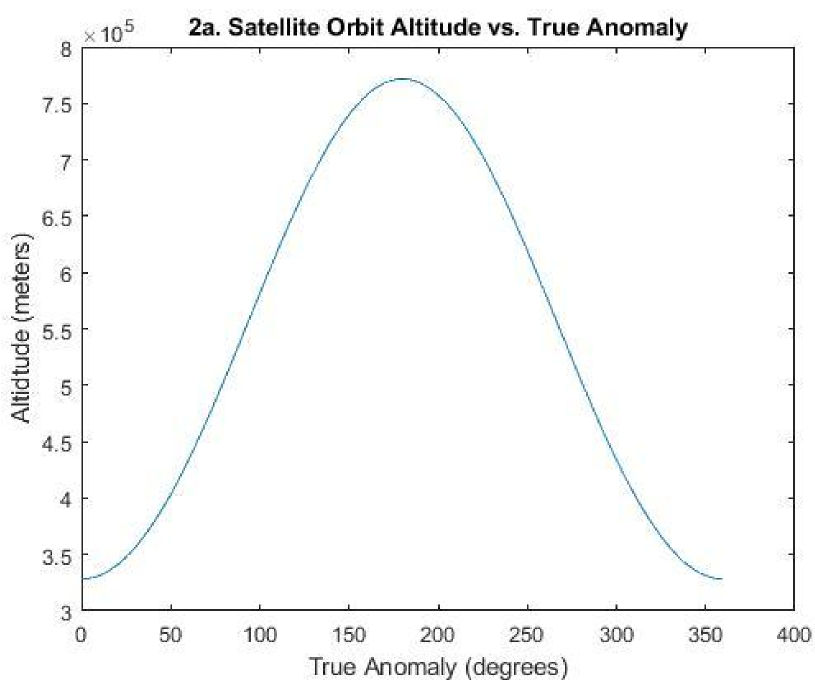
```
for j = 1:length(f)
oe = [a e i 0 0 f(j)/180*pi];
rv = hw6oe2rv(oe, mu)';
v_1b = norm(rv(4:6) - cross([0 0 we],rv(1:3)));
v_1c = v_1b - 500*(v_1b/norm(v_1b));
fd_1c = 0.5*rho*Cd*A*(v_1c^2)/m;
dadt_1c_mday(j) = 86400*-2*v_1c*(a^2)*fd_1c/mu;
End

figure(2)
plot(f,dadt_1c_mday)
title("1c. Semimajor Axis vs. True Anomaly with Wind Gust")
xlabel("True Anomaly (degrees)")
ylabel("Semimajor Axis Decay Rate (m/day)")

dadtCompare = dadt_1b_mday - dadt_1c_mday;
figure(3)
plot(f,dadtCompare)
title("1c. Semimajor Axis vs. True Anomaly difference due to Wind Gust")
xlabel("True Anomaly (degrees)")
ylabel("Semimajor Axis Decay Rate Difference (m/day)")
```

## 2. Eccentric Orbit

**a.**

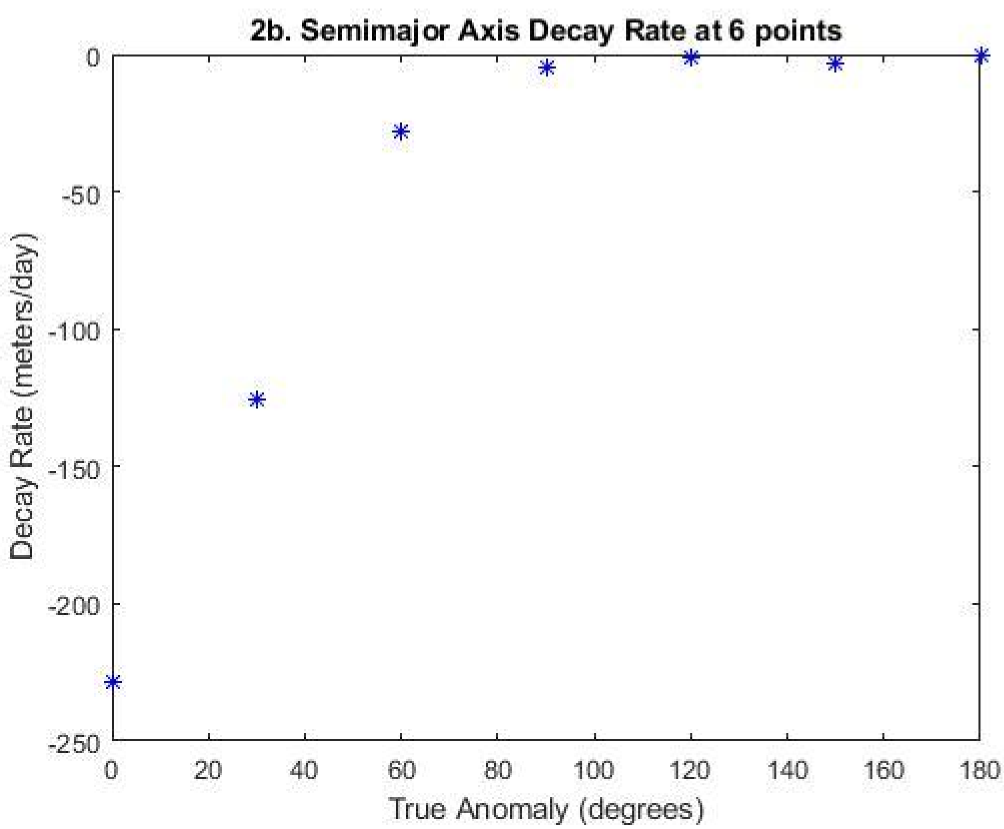


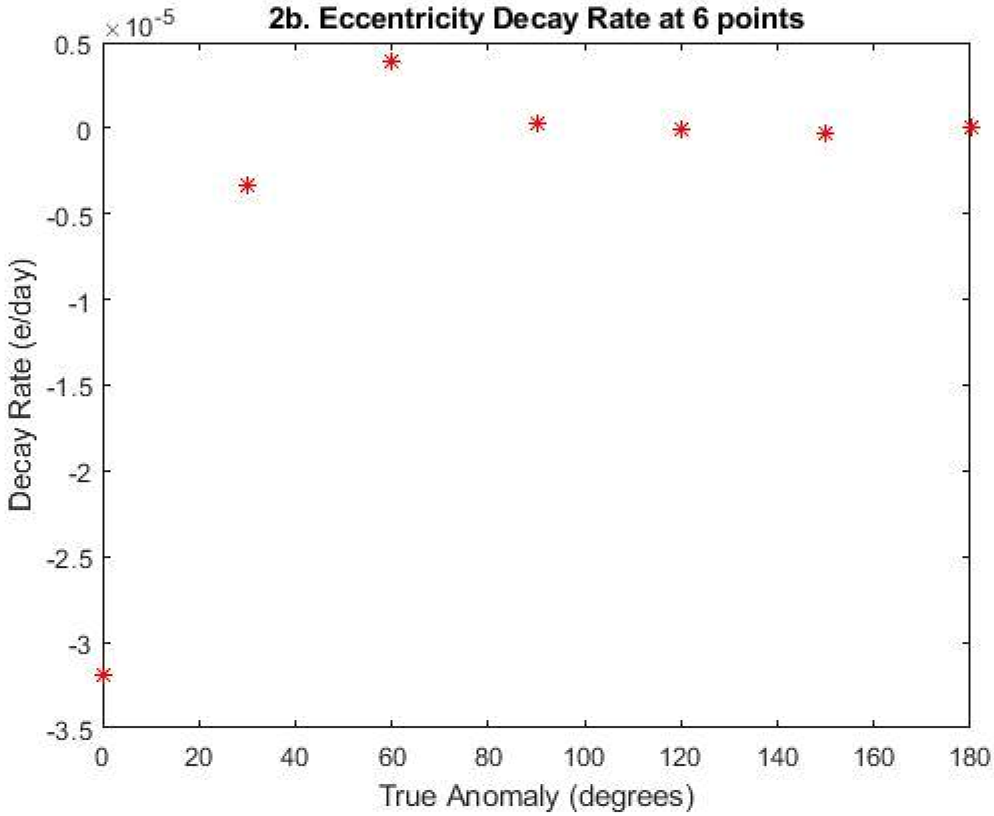
```
%A
e2 = 0.032;
a2 = 6928*1000;
Cd2 = 2.0;
A2 = 0.8;
m2 = 500;

f2=0:360;
for k = 1:length(f2)
p=a2*(1-e2^2);
radsat = p/(1+e2*cosd(f2(k)));
alt2(k) = radsat - ae;
end

figure(4)
plot(f2,alt2)
title("2a. Satellite Orbit Altitude vs. True Anomaly")
xlabel("True Anomaly (degrees)")
ylabel("Altitude (meters)")
```

**b.**






I chose the true anomaly points of 0 degrees to 180 degrees in intervals of 30 degrees, because those values capture the range of maximum decay rate to minimum decay rate and back again. Going from minimum to maximum was the best way to capture the scope of the variation of drag effects, excluding atmospheric co-rotation and wind gusts.. The data points follow the general slope of the graph at their respective points of true anomaly.

```
%B
f3 = [0 30 60 90 120 150 180];
p=a2*(1-e2^2);
for n = 1:length(f3)
radsat = p/(1+e2*cosd(f3(n)));
alt2b(n) = (radsat - ae)/1000; %(km)
end
po = [2.418e-11, 9.518e-12, 3.725e-12, 6.967e-13, 1.454e-13, 3.614e-13, 3.614e-14];
ho = [300, 350, 400, 500, 600, 700, 700];
H = [53.628, 53.298, 58.515, 63.822, 71.835, 88.667, 88.667];
for n = 1:length(f3)
   rho3 = po(n)*exp(-1*((alt2b(n)-ho(n))/H(n)));
```

```
   v3=sqrt(mu/p*(1+e2^2+2*e2*cos(f3(n))));
   fd_2b = 0.5*rho3*Cd2*A2*(v3^2)/m2;
dadt_2b_mday(n) = 86400*-2*v3*(a2^2)*fd_2b/mu;
dedt(n) = 86400*-2/v3*(e2+cos(f3(n)))*fd_2b;
end
figure(5)
plot(f3,dadt_2b_mday,'b*')
title("2b. Semimajor Axis Decay Rate at 6 points")
xlabel("True Anomaly (degrees)")
ylabel("Decay Rate (meters/day)")
avgDadt = mean(dadt_2b_mday)
figure(6)
plot(f3,dedt,'r*')
title("2b. Eccentricity Decay Rate at 6 points")
xlabel("True Anomaly (degrees)")
ylabel("Decay Rate (e/day)")
avgDedt = mean(dedt)
```

**c.**

avgDadt =  -55.9611

Yes, I believe the calculated average of the semi-major decay rate because the value found is within approximately 4% of the decay rate calculated in problem 1b. Additionally, for both c. and d. there is an outlier or extreme at the perigee of the orbit, corresponding to maximum drag, that impacts the average.

```
avgDadt = mean(dadt_2b_mday)
```

**d.**

avgDedt =  -4.5044e-06

I chose the same points for this section for the same reasons I did in part b. In an attempt to capture the maximum variation from going to maximum to minimum 0 - 180 degrees. I stopped there because it simply repeats for 180 - 360 degrees. I do not have a control to compare this part with so I can not ground my answers in any form of accuracy. My calculation method involved preselecting points, calculating the altitude at those points, then manually selecting the corresponding values from the table. Then running these found values through the exponential density model. Finally I calculated fd and de/dt with the same velocity by methods used in previous sections of this homework

avgDedt = mean(dedt)

CODE APPENDIX:

```
%% Applied Orbital Mechanics HW#3

%constants
mu = 3.986004415*10^14;
ae = 6378136.3;
we = 7.292115*10^-5;
g=9.81;
j2=1.082*10^-3;
% Preallocating

%% Intro Functions for syntax and copying:
%fd = 0.5*rho*vr^2*Cd*A/m;
% v = vr + cross(we, r);
% vwind = vr + cross(we, r) + vw;
% dadt = -2*vr*a^2*fd/mu;
% dedt = -2*(e+cos(f))*fd/vr;

%% 1. Orders of Magnitude:
%A
i = deg2rad(66);
Cd = 2.0;
A = 1.3; %(m^2)
m = 350; %(kg)
rho = 1.6*10^-12; %(kg/m^3)
alt = 450*1000; %(m)
a = ae + alt;
e = 0; %circular orbit
v_1a = sqrt(mu/a);
fd_1a = 0.5*rho*Cd*A*(v_1a^2)/m;
dadt_1a = -2*v_1a*a^2*fd_1a/mu;
dadt_1a_mday = dadt_1a*86400;
%B
f = 0:3:360;
for j = 1:length(f)

oe = [a e i 0 0 f(j)/180*pi];
rv = hw6oe2rv(oe, mu)';
```

```
v_1b = norm(rv(4:6) - cross([0 0 we],rv(1:3)));
fd_1b = 0.5*rho*Cd*A*(v_1b^2)/m;
dadt_1b_mday(j) = 86400*-2*v_1b*(a^2)*fd_1b/mu;

end
% avgdadtFull = mean(dadt_1b_mday)
figure(1)
plot(f,dadt_1b_mday)
title("1b. Semimajor Axis vs. True Anomaly")
xlabel("True Anomaly (degrees)")
ylabel("Semimajor Axis Decay Rate (m/day)")

%C
for j = 1:length(f)

oe = [a e i 0 0 f(j)/180*pi];
rv = hw6oe2rv(oe, mu)';
v_1b = norm(rv(4:6) - cross([0 0 we],rv(1:3)));
v_1c = v_1b - 500*(v_1b/norm(v_1b));
fd_1c = 0.5*rho*Cd*A*(v_1c^2)/m;
dadt_1c_mday(j) = 86400*-2*v_1c*(a^2)*fd_1c/mu;

end
figure(2)
plot(f,dadt_1c_mday)
title("1c. Semimajor Axis vs. True Anomaly with Wind Gust")
xlabel("True Anomaly (degrees)")
ylabel("Semimajor Axis Decay Rate (m/day)")

dadtCompare = dadt_1b_mday - dadt_1c_mday;
figure(3)
plot(f,dadtCompare)
title("1c. Semimajor Axis vs. True Anomaly difference due to Wind Gust")
xlabel("True Anomaly (degrees)")
ylabel("Semimajor Axis Decay Rate Difference (m/day)")

%% 2. Eccentric Orbits
%A
e2 = 0.032;
a2 = 6928*1000;
Cd2 = 2.0;
A2 = 0.8;
m2 = 500;

f2=0:360;
for k = 1:length(f2)
```

```
p=a2*(1-e2^2);
radsat = p/(1+e2*cosd(f2(k)));
alt2(k) = radsat - ae;
end

figure(4)
plot(f2,alt2)
title("2a. Satellite Orbit Altitude vs. True Anomaly")
xlabel("True Anomaly (degrees)")
ylabel("Altidtude (meters)")
%B
f3 = [0 30 60 90 120 150 180];
p=a2*(1-e2^2);
for n = 1:length(f3)
radsat = p/(1+e2*cosd(f3(n)));
alt2b(n) = (radsat - ae)/1000; %(km)
end
po = [2.418e-11, 9.518e-12, 3.725e-12, 6.967e-13, 1.454e-13, 3.614e-13, 3.614e-14];
ho = [300, 350, 400, 500, 600, 700, 700];
H = [53.628, 53.298, 58.515, 63.822, 71.835, 88.667, 88.667];
for n = 1:length(f3)
    rho3 = po(n)*exp(-1*((alt2b(n)-ho(n))/H(n)));
    v3=sqrt(mu/p*(1+e2^2+2*e2*cos(f3(n))));
    fd_2b = 0.5*rho3*Cd2*A2*(v3^2)/m2;
dadt_2b_mday(n) = 86400*-2*v3*(a2^2)*fd_2b/mu;
dedt(n) = 86400*-2/v3*(e2+cos(f3(n)))*fd_2b;
end

for n = 1:length(f2)
    radsat = p/(1+e2*cosd(f3(n)));
alt2d(n) = (radsat - ae)/1000; %(km)
    rho3 = po(n)*exp(-1*((alt2d(n)-ho(n))/H(n)));
    v3=sqrt(mu/p*(1+e2^2+2*e2*cos(f3(n))));
    fd_2b = 0.5*rho3*Cd2*A2*(v3^2)/m2;
dadt_2b_mday(n) = 86400*-2*v3*(a2^2)*fd_2b/mu;
dedt(n) = 86400*-2/v3*(e2+cos(f3(n)))*fd_2b;
end
figure(5)
plot(f3,dadt_2b_mday,'b*')
title("2b. Semimajor Axis Decay Rate at 6 points")
xlabel("True Anomaly (degrees)")
ylabel("Decay Rate (meters/day)")
avgDadt = mean(dadt_2b_mday)
figure(6)
plot(f3,dedt,'r*')
title("2b. Eccentricity Decay Rate at 6 points")
```

```
xlabel("True Anomaly (degrees)")
ylabel("Decay Rate (e/day)")
avgDedt = mean(dedt)
```