

Introduction of Machine Learning

Ex5: Dimensionality Reduction

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Problem 1

PCA is an unsupervised method of data dimensionality reduction.

- Write down the general form of data mapping from high to low-dimensionality space according to PCA. Explain the components of the mapping and comment on their orthogonality.
- Compute the principal components of the dataset:

$$\mathbf{X} = \{(1, 2), (3, 3), (3, 5), (5, 4), (5, 6), (6, 5), (8, 7), (9, 8)\}$$

Problem 2

Let $p(x|\omega_i)$ be arbitrary densities with means μ_i and covariance matrices Σ_i , not necessarily normal for $i = 1, 2$. Let $y = w^T x$ be a projection, and let the induced one-dimensional densities $p(y|\omega_i)$ have means μ_i and variances σ_i .

- Write down the formulas for the between-class and inter-class covariance matrices.
- Show that the criterion function:

$$J(\omega) = \frac{(\mu_1 - \mu_2)^2}{\sigma_1^2 + \sigma_2^2}$$

is maximized by:

$$\omega = (\Sigma_1 + \Sigma_2)^{-1}(\mu_1 - \mu_2)$$

We only care about the direction of the projection.