Exercise 1: Regression

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Important Notes

- Due date: **2025/3/5** 23:59
- Formulas must be in **LaTeX format** or inserted using the equation editor in Microsoft Word
- It is not necessary to include the entire calculation process, only the reasoning steps
- Submit as a **PDF**

Problem 1-(a) Linear Regression Without Bias

(a) Let's assume $f(x) = wx$ as a regression model with unknown parameter w. Find w which fits the data best in the sense of the Euclidean norm.

$$\text{Loss}(w) = \sum_{i=1}^{n} (wx_i - y_i)^2$$

Goal: Minimize Loss(w)

Problem 1-(b) Linear Regression With Bias

(b) Let's assume $f(x) = w_0 + w_1 x$ as a regression model with unknown parameter vector $w = [w_0, w_1]^T$. By the use of the normal equation, find the best w.

Goal: Compute best w0 w1 by using Normal equation

Problem 1-(c) Prediction using Both Models

(c) Predict the output value of the system for $x = 1$ using both regression models (a) and (b).

Compute $f(1)$ for:

- Model (a): $f(1) = w(1)$
- Model (b): $f(1) = w_0 + w_1(1)$

Problem 1-(d) Gradient Descent for w

(d) Let's assume the regression model as in (a). Now, compute the unknown parameter w by the gradient descent algorithm. Start with an initial value of $w = 0$ and use the learning rate $\alpha = 0.1$. Compute the first 2 iterations.

Goal: use gradient descent to find w iteratively

Possible Solution:
1.Define Loss Function: $J(w)$

2.Update rule: $w := w - \alpha \frac{dJ}{dw}$

Given: Initial w=0 , Learning rate α = 0.1

Perform two iterations to update w

Problem 2-(a) Finding the Best w Using the Normal Equation

Goal: Use the **normal equation** to find the optimal values of w

Possible solution:
1.Define Input matrix X
2.Define output matrix Y

3.Get the optimal values of w by using normal equation

Problem 2-(b) Predicting Output for x1=7,x2=4

Goal: Get predicted values of y1 and y2

$$\hat{Y} = \begin{bmatrix} w_0 & w_1 \\ w_0' & w_1' \end{bmatrix} \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

Programming Homework 1-1

Important Notes

- **Due date:** 2025/**4/10** 23:59
- Specified programming language: **Python**
- Do not use libraries to complete the task directly (e.g., numpy.polyfit)

Homework 1-1(a)

(a) The order of the polynomial is $m = 2$ for $\{x, y1\}$. Please write down the values of the parameters of the polynomial w_0, w_1, w_2.

- Fit the dataset using a second-degree polynomial (m = 2)
- Calculate and record the values of parameters w0, w1, and w2

Homework 1-1(b)

(b) Choose an appropriate order of the polynomial m so that fitting is successful.

- Determine the best polynomial order m for fitting
- Consider underfitting (too low m) vs. overfitting (too high m)
- The chosen m should balance accuracy and generalization

Homework 1-1(c)

- Fit polynomials with m = 3 and m = 8
- Compute and compare the Mean Squared Error (MSE)
- Analyze how different polynomial orders affect error

Notes and Visualization

- Two figures should be provided for Questions a and b:
  - - Dataset points.
  - - Fitted curve with different colors.
- Polynomial curve fitting must be implemented manually (**no built-in libraries**).