Introduction of Machine Learning

Ex1: Regression

TA: David Lee, david20010603@alum.ccu.edu.tw
Lecturer: Prof. Peggy Lu, peggylu@cs.ccu.edu.tw
Due date: 2025/3/5 23:59

Problem 1

Given the dataset shown in Table 1 and illustrated in Figure 1, we want to predict the output value for $x = 1$. We assume a linear regression model.

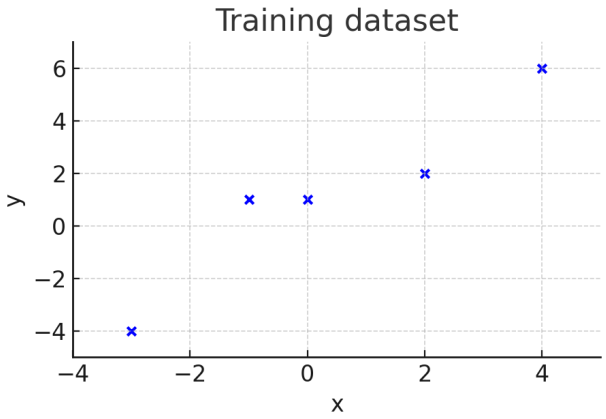

Figure 1: Training dataset

Input x	-3	-1	0	2	4
Output y	-4	1	1	2	6

Table 1: Dataset for Problem 1

Tasks

(a) Let's assume $f(x) = wx$ as a regression model with unknown parameter w. Find w which fits the data best in the sense of the Euclidean norm.

(b) Let's assume $f(x) = w_0 + w_1 x$ as a regression model with unknown parameter vector $w = [w_0, w_1]^T$. By the use of the normal equation, find the best w.

(c) Predict the output value of the system for $x = 1$ using both regression models (a) and (b).

(d) Let's assume the regression model as in (a). Now, compute the unknown parameter w by the gradient descent algorithm. Start with an initial value of $w = 0$ and use the learning rate $\alpha = 0.1$. Compute the first 2 iterations.

Problem 2

Given the dataset shown in Table 2, we want to predict the output values for $x_1 = 7, x_2 = 4$. We assume a linear regression model.

Input x_1	3	2	1	3
Input x_2	5	2	2	3
Output y_1	7	4	2	6
Output y_2	10	5	9	6

Table 2: Dataset for Problem 2

Tasks

(a) Let's assume the regression model:

$$y_1 = w_0 x_1 + w_1 x_2$$
$$y_2 = w_0' x_1 + w_1' x_2$$

with unknown parameters:

$$w = \begin{bmatrix} w_0 & w_1 \\ w_0' & w_1' \end{bmatrix}.$$

By the use of the normal equation, find the best w.

(b) Predict the output value of the system for $x_1 = 7, x_2 = 4$.