ASSIGNMENT-1

6/2/24

A1

b) The steps needed for 8 dishes are as follows:

1	T1 -> T2
2	T1 -> T4
3	T2 -> T4
	T1 -> T2
	T1 -> T3
	T2 -> T3
	T4 -> T1
	T4 -> T3
	T1 -> T3
	T1 -> T4
	T1 -> T2
	T4 -> T2
	T3 -> T4
	T3 -> T1
	T4 -> T1
	T3 -> T4
	T3 -> T2
	T4 -> T2
	T1 -> T3
	T1 -> T2
	T3 -> T2
	T1 -> T3
	T1 -> T4
24	T3 -> T4

25	T2 -> T4
26	T2 -> T3
27	T4 -> T3
	T2 -> T4
	T2 -> T1
	T4 -> T1
	T3 -> T2
	T3 -> T1
	T2 -> T1
	T2 -> T3
	T2 -> T4
	T3 -> T4
	T1 -> T3
	T1 -> T2
	T3 -> T2
	T1 -> T3
	T1 -> T4
	T3 -> T4
	T2 -> T1
	T2 -> T4
45	T1 -> T4

c) for Tower of Hanoi w/ one auxiliary peg.

$$T(n) = 2T(n-1) + 1$$
$$= 2[2T(n-2) + 1] + 1$$
$$= 4T(n-2) + (1+2)$$
$$= 4[2T(n-3) + 1] + (1+2)$$
$$= 8T(n-3) + (1+2+4)$$
$$\vdots$$
$$= 2^k T(n-k) + \sum_{i=0}^{k} 2^i$$

let $n-k=1$, $k=n-1$
$$= 2^{n-1} T(1) + \sum_{i=0}^{n+1} 2^i$$
$$= O(2^{n-1})$$

for Tower of Hanoi w/ two auxiliary pegs,
$$T(n) = 2T(n-2) + c \qquad (c=3 \text{ taken from code})$$
$$= 2[2T(n-4) + c] + c$$
$$= 4T(n-4) + c(1+2)$$
$$\vdots$$
$$= 2^k T(n-2k) + c \sum_{i=0}^{k} 2^i$$
let $n-2k=1$, $k=\frac{n-1}{2}$
$$= 2^{\frac{n-1}{2}} T(1) + \text{summation}$$
$$= O(2^{\frac{n-1}{2}})$$

A2 b) mergesort_iterative ({1,3,6,4,2,5}, 6)

0	1	2	3	4	5
1	3	6	4	2	5

current_size = 1

left = 0

middle = min (left + current_size -1, n-1) = min (0+1-1, 6-1) = 0
right = min (left + 2 current_size -1, n-1) = min(0+2-1, 6-1) = 1
merge (array, 0, 0, 1)

1	3	6	4	2	5

left = 2

middle = min (2+1-1, 6-1) = 2
right = min (2+2-1, 6-1) = 3
merge (array, 2, 2, 3)

1	3	4	6	2	5

left = 4

middle = 4, right = 5, merge (array, 4, 4, 5)

1	3	4	6	2	5

current_size = 2

left = 0

middle = 1, right = 3, merge (array, 0, 1, 3)

1	3	4	6	2	5

left = 4

middle = 5, right = 5, merge (array, 4, 5, 5)

1	3	4	6	2	5

current_size = 3

left = 0

middle = 2, right = 5, merge (array, 0, 2, 5)

1	2	3	4	5	6

The array is sorted, and future calls to merge() output the same array.

A3

less than constant	$\frac{1}{n}, \frac{1}{\sqrt{n}}, \frac{\log n}{n}$
constant	$2^{2^{100}}$
logarithmic	$\log_2 n, \log_{10} n$
linear	$2^{100} n, 2n, 3n$
log-linear	$n \log n, \log n!$
polynomial	$n^{64}, n^{65}, \binom{n}{64}$
exponential	$n!, 2^{2^n}, 2^n, 3^n, n2^n, 2^{n+1}, 2 \cdot 1^{\sqrt{n}}, 2^{2n},$
and beyond	$4^n, n^n$

The required sequence is

$n^{-1}, \log n/n, n^{-\frac{1}{2}}, 2^{2^{100}}, \log_{10} n, \log_2 n, 2n, 3n, 2^{100}n, \log n!,$
$n \log n, \binom{n}{64}, n^{64}, n^{65}, 2 \cdot 1^{\sqrt{n}}, 2^n, 2^{n+1}, n2^n, 3^n, 2^{2n}, 4^n, n!, n^n, 2^{2^n}.$