

The Law of Large Numbers

Lecturer: John Guttag

Law of Large Numbers

In repeated **independent tests** with the same **actual probability** p of a particular outcome in each test, the chance that the **fraction of times** that outcome occurs differs from p converges to zero as the number of trials goes to infinity.



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Law of Large Numbers

Gambler's Fallacy

If deviations from expected behavior occur, these deviations are likely to be evened out by opposite deviations in the future.



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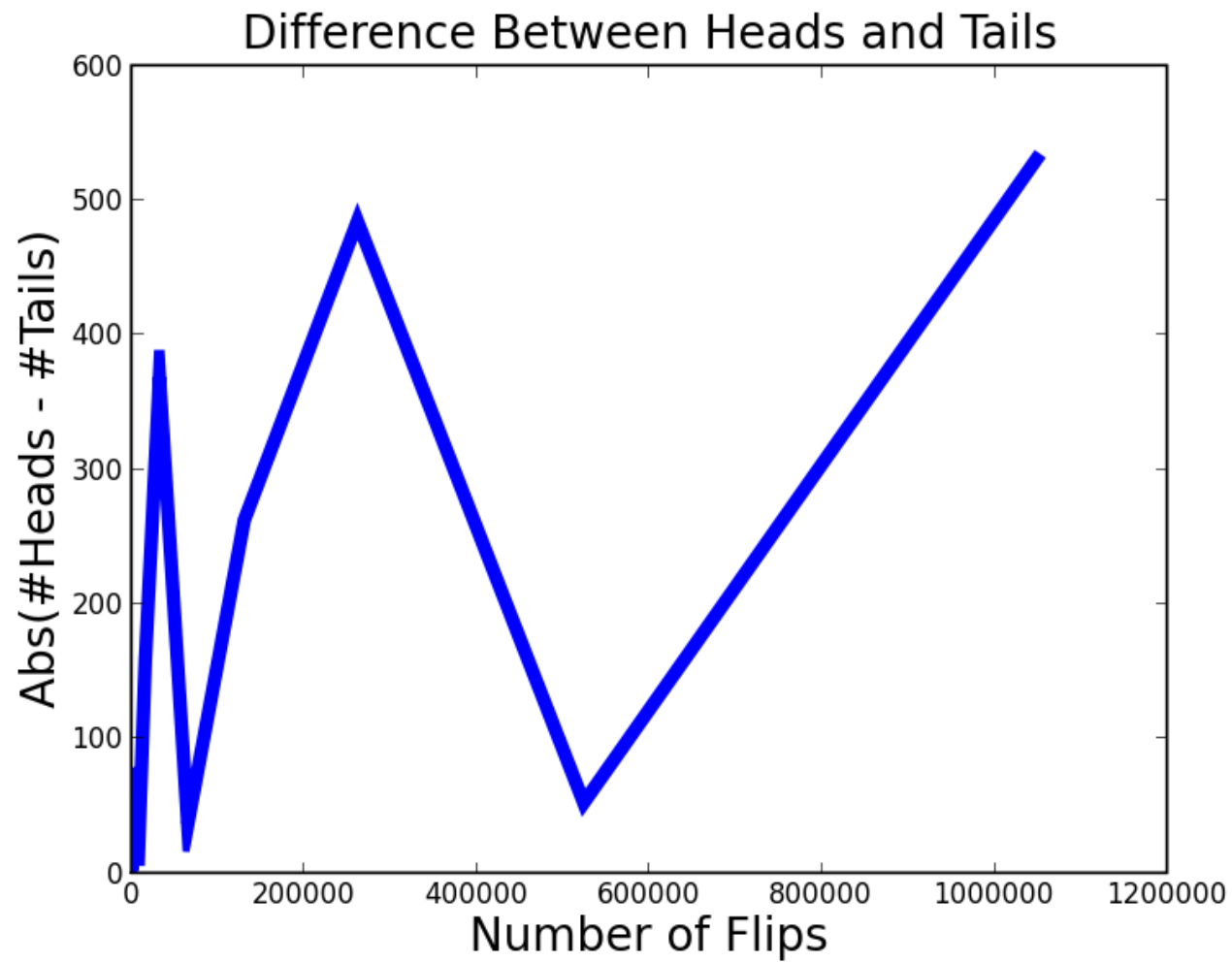
Law of Large Numbers

```
def flipPlot(minExp, maxExp):  
    """Assumes minExp and maxExp positive  
        integers; minExp < maxExp  
        Plots results of 2**minExp to  
        2**maxExp coin flips"""  
    ratios = []  
    diffs = []  
    xAxis = []  
    for exp in range(minExp, maxExp + 1):  
        xAxis.append(2**exp)  
  
    . . .
```

```
for numFlips in xAxis:
    numHeads = 0
    for n in range(numFlips):
        if random.random() < 0.5:
            numHeads += 1
    numTails = numFlips - numHeads
    ratios.append(numHeads/float(numTails))
    diffs.append(abs(numHeads - numTails))
```

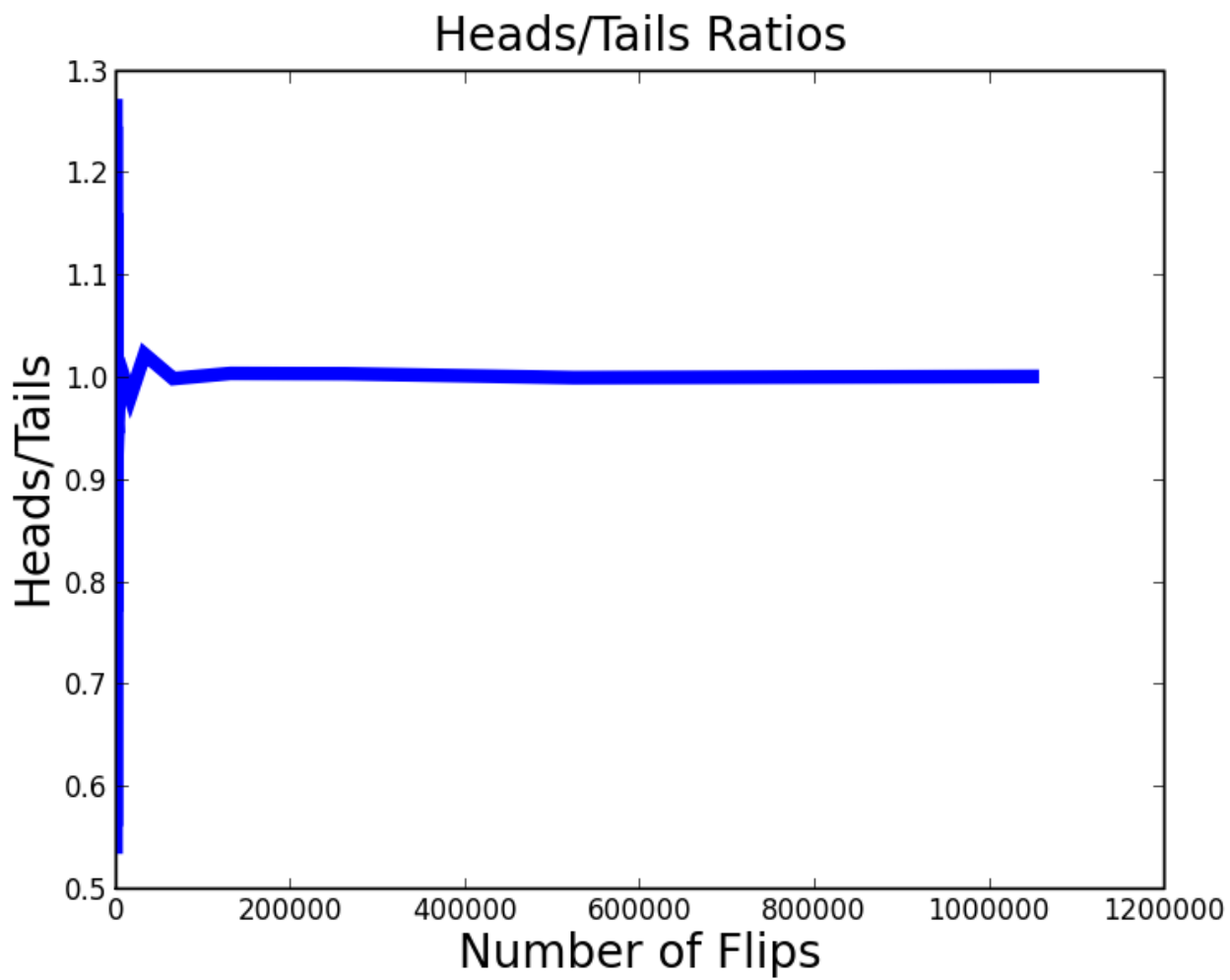
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```
pylab.title('Difference Between Heads and Tails')
pylab.xlabel('Number of Flips')
pylab.ylabel('Abs(#Heads - #Tails)')
pylab.plot(xAxis, diffs)
pylab.figure()
pylab.title('Heads/Tails Ratios')
pylab.xlabel('Number of Flips')
pylab.ylabel('Heads/Tails')
pylab.plot(xAxis, ratios)
```



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How Much Is Enough?

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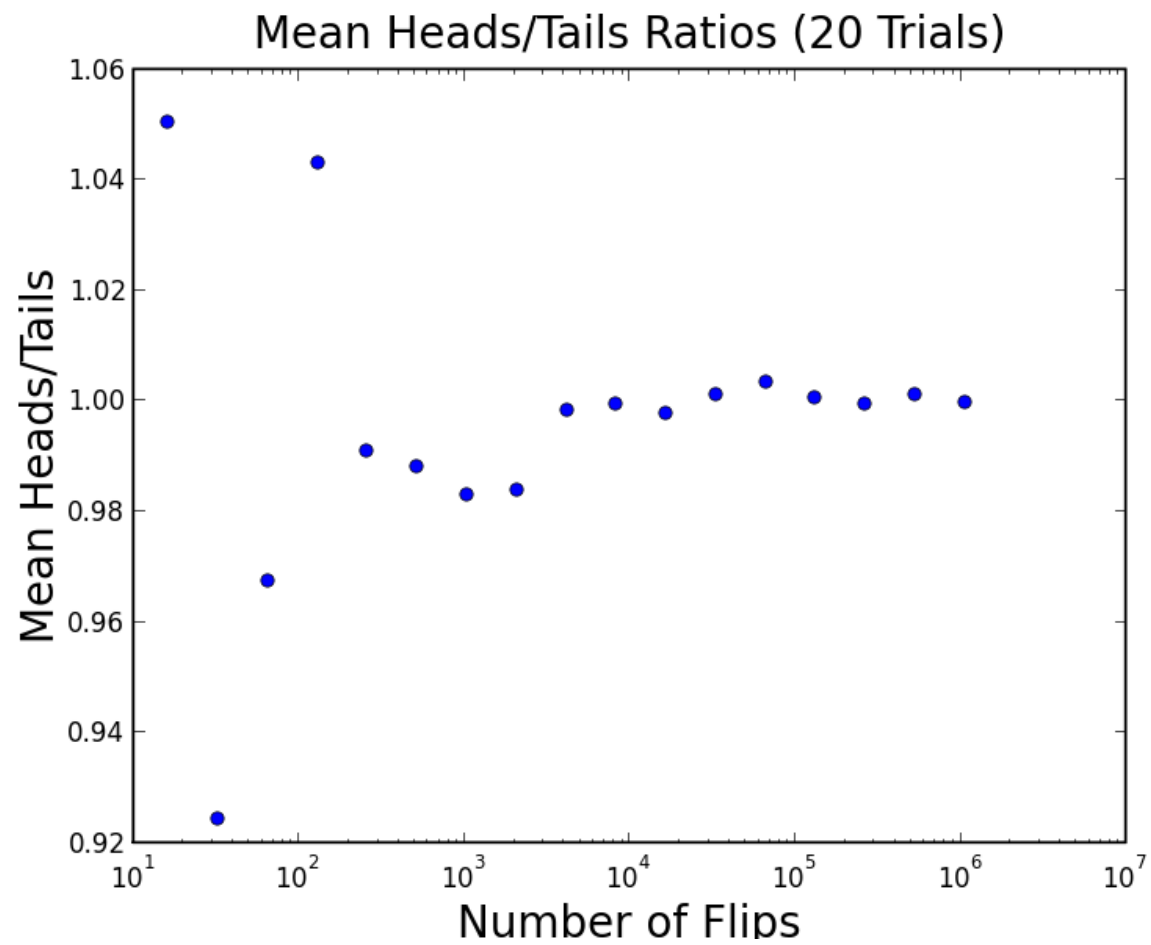
How many samples do we need to look at in order to have a justified confidence that something that is true about the population of samples is also true about the population from which the samples were drawn?

Depends upon the variance in the underlying distribution

Variance

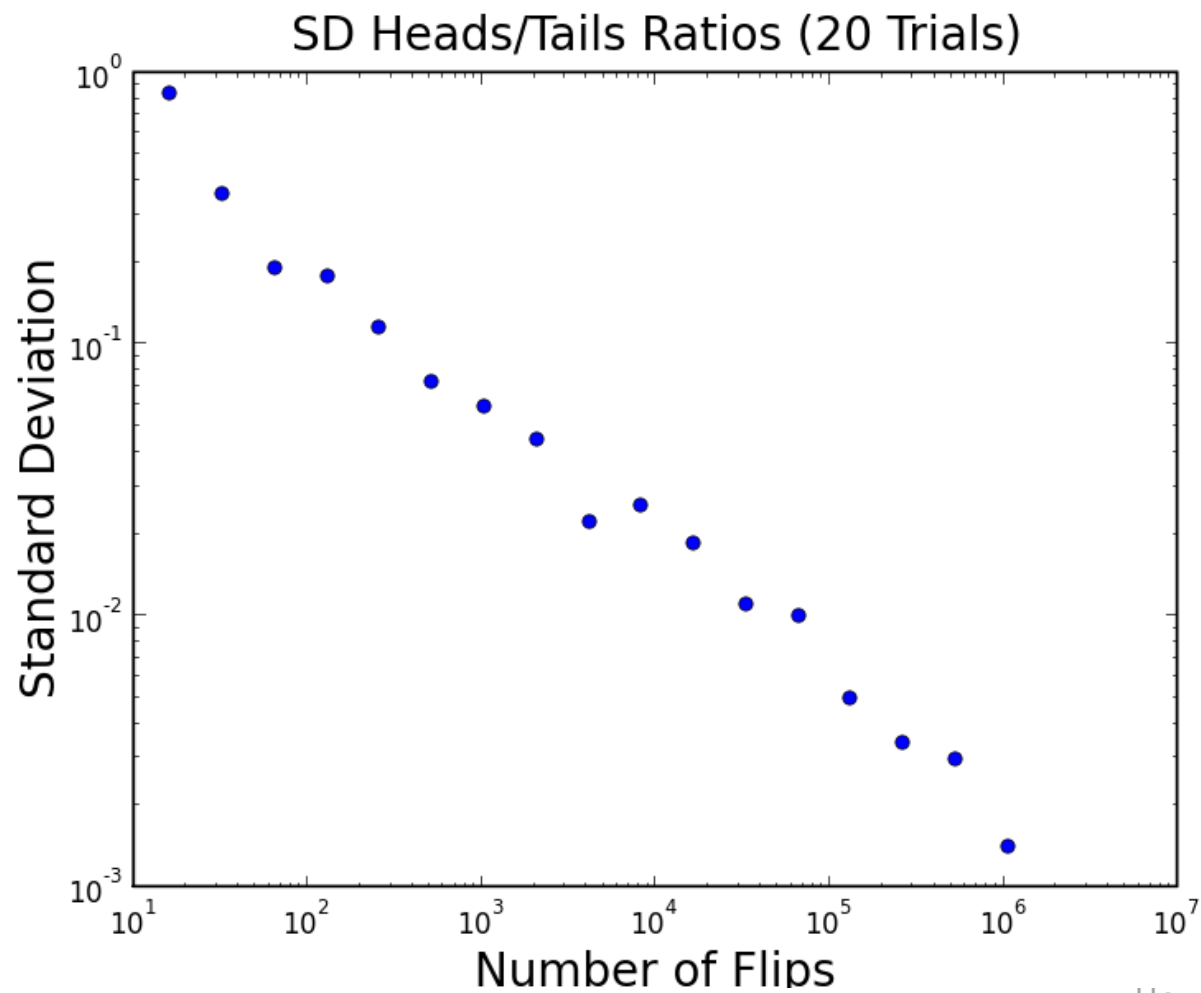
We measure the amount of variance in the outcomes of multiple trials.

```
def stdDev(X):  
    mean = sum(X)/float(len(X))  
    tot = 0.0  
    for x in X:  
        tot += (x - mean)**2  
    return (tot/len(X))**0.5
```



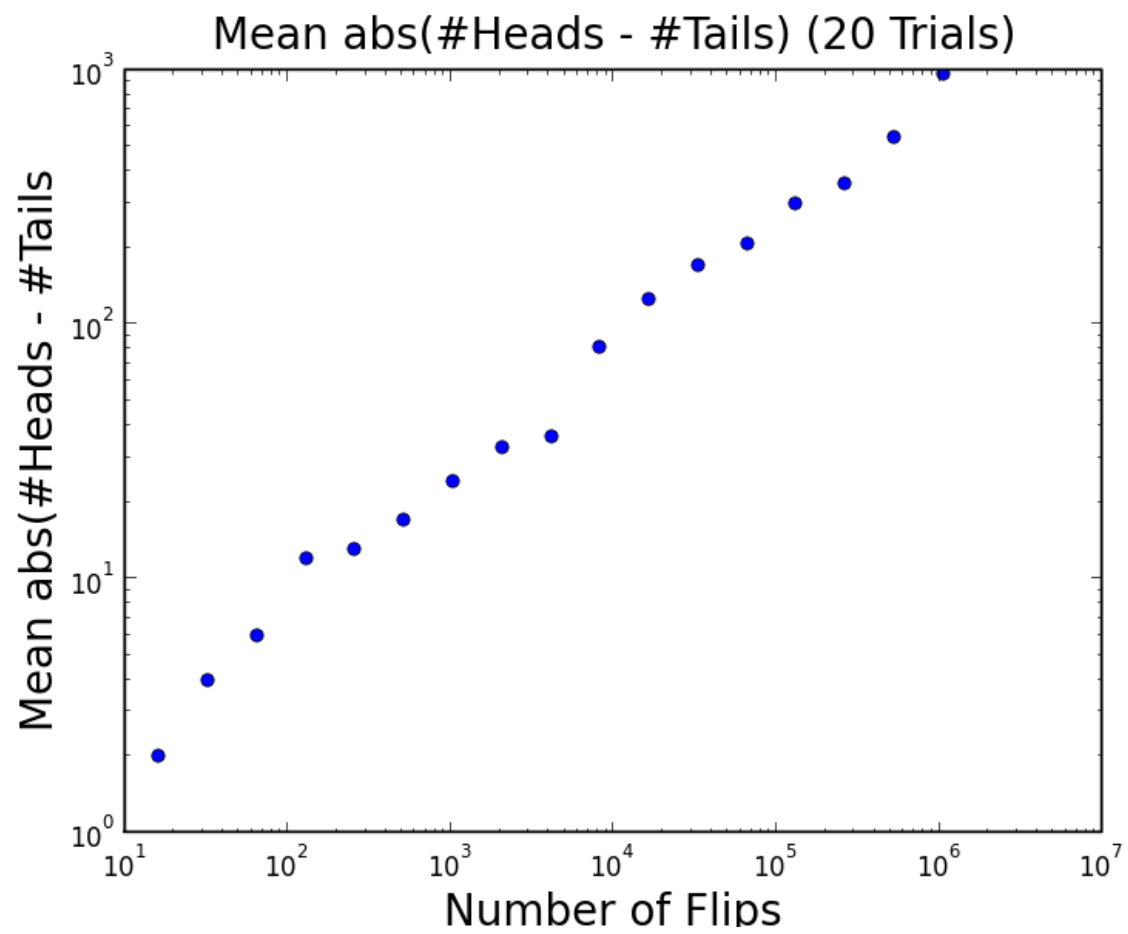
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How Much Is Enough?



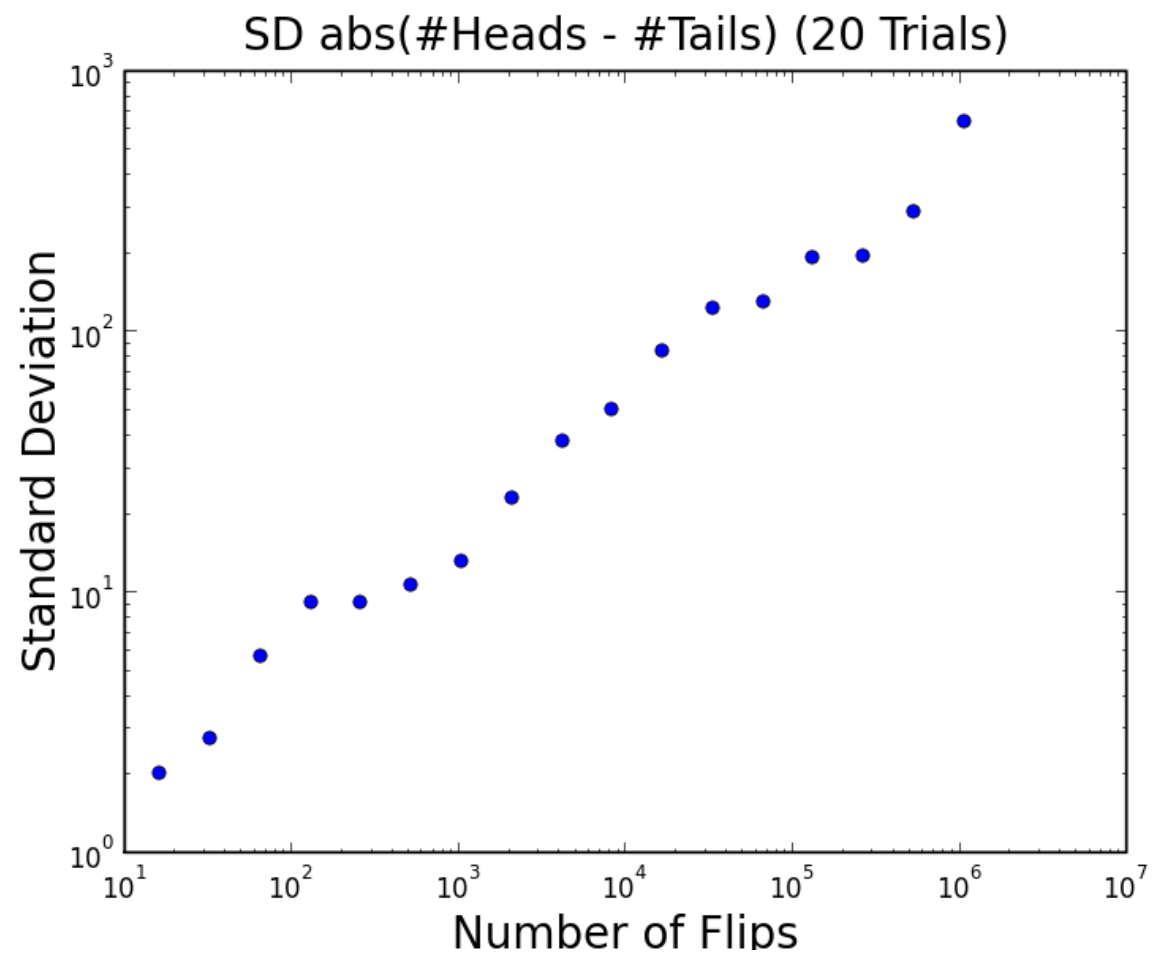
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How Much Is Enough?



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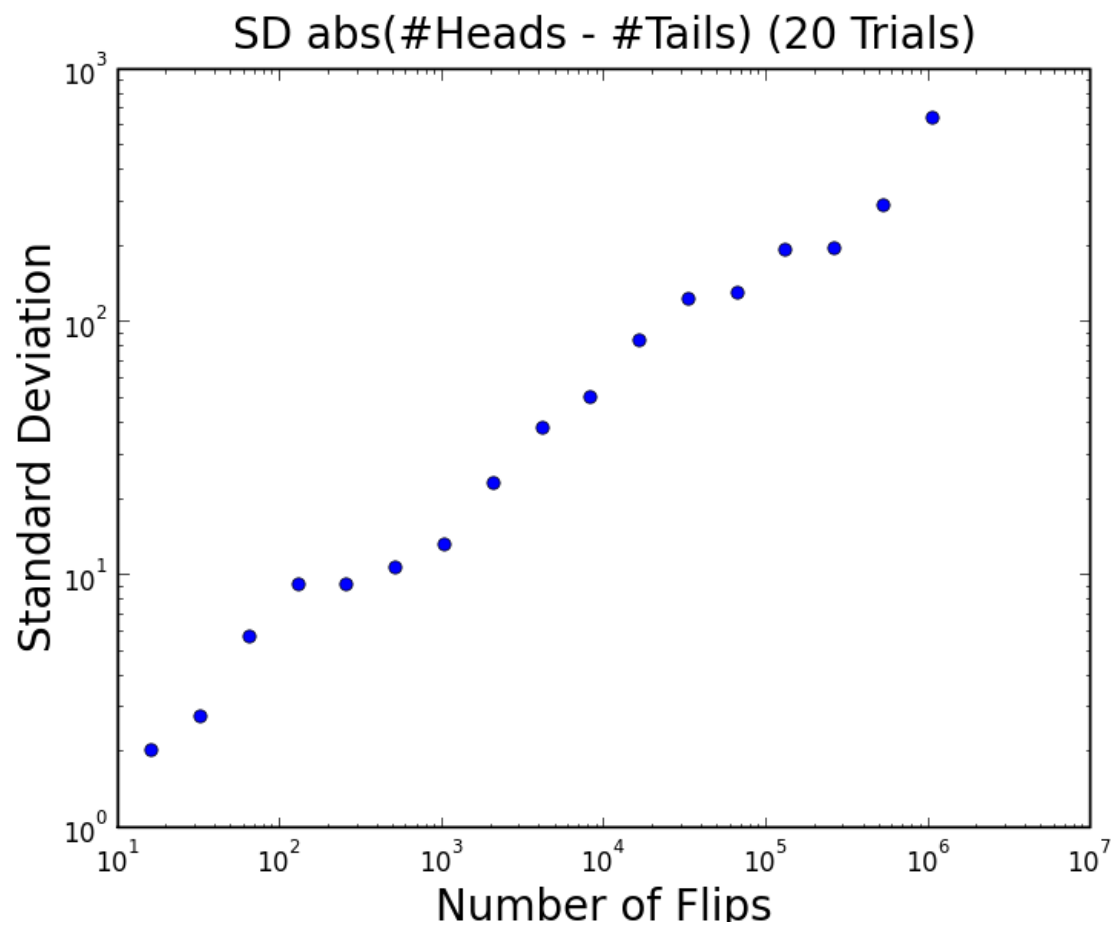


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How Much Is Enough?

Standard Deviations and Histograms

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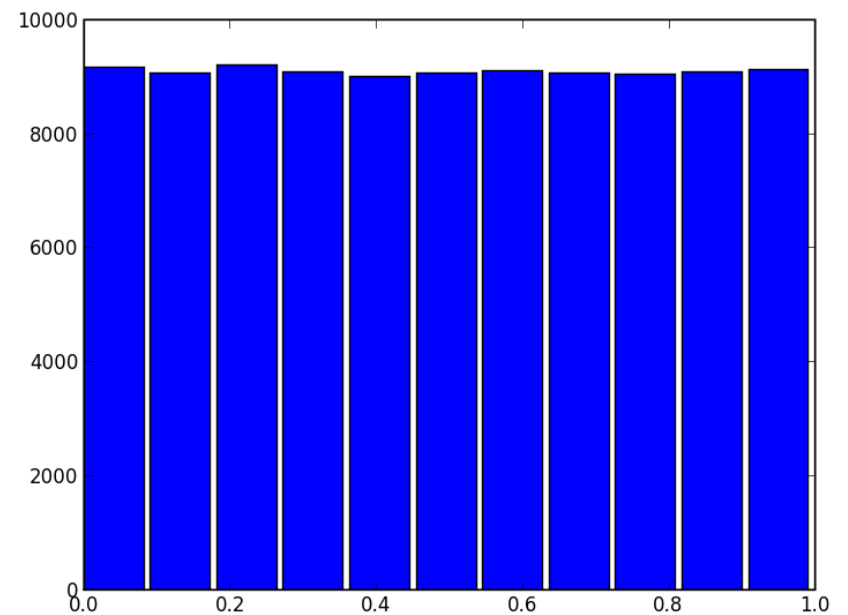
Standard Deviations and Histograms

```
def CV(X):  
    mean = sum(X)/float(len(X))  
    try:  
        return stdDev(X)/mean  
    except ZeroDivisionError:  
        return float('NaN')
```

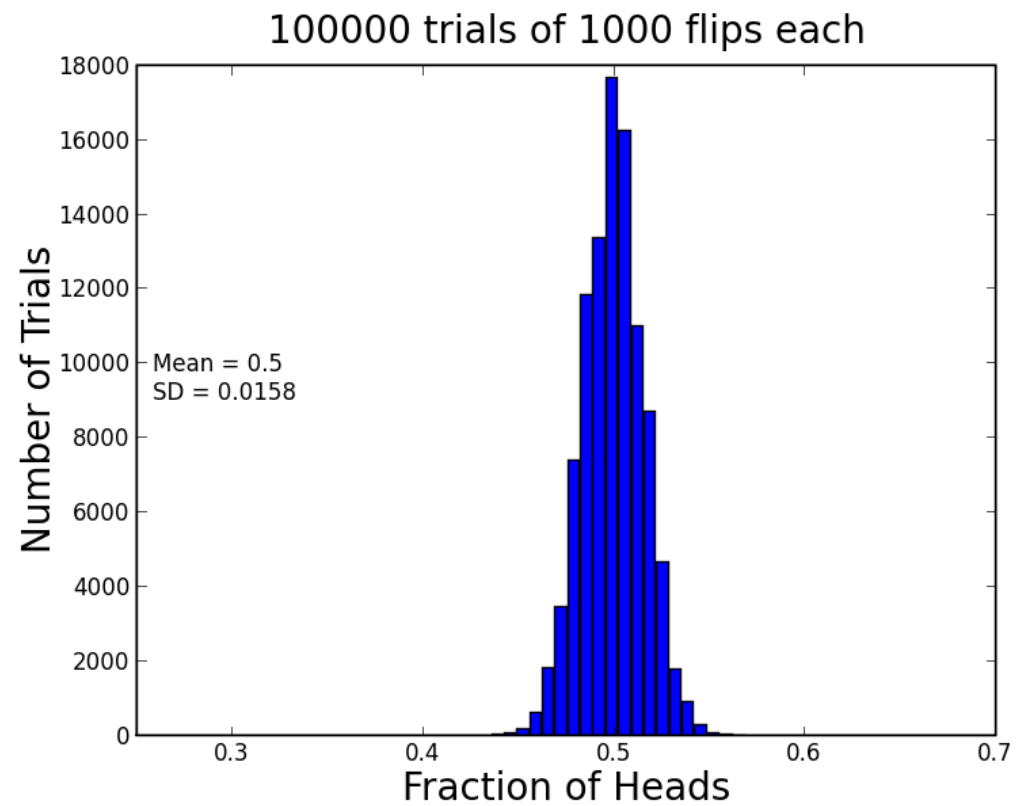
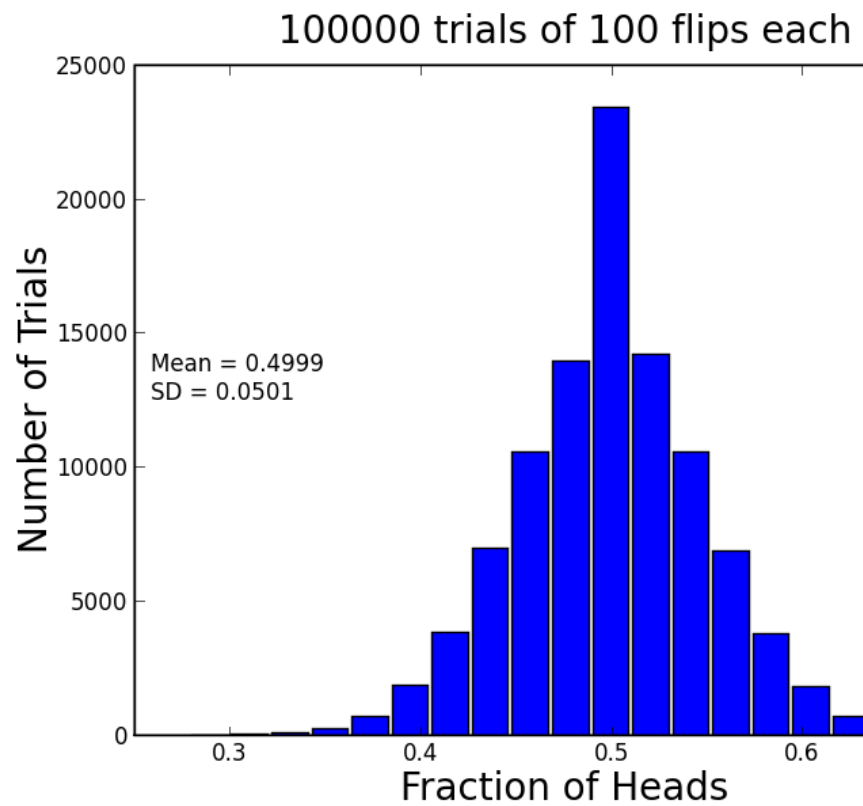
```
def flip(numFlips):  
  
def flipSim(numFlipsPerTrial, numTrials):  
  
def labelPlot(nf, nt, mean, sd):  
  
def makePlots(nf1, nf2, nt):  
    """nt = number of trials per experiment  
       nf1 = number of flips 1st experiment  
       nf2 = number of flips 2nd experiment"""
```

```
def makePlots(numFlips1, numFlips2, numTrials):  
    val1, mean1, sd1 = flipSim(numFlips1, numTrials)  
    pylab.hist(val1, bins = 20)  
    xmin, xmax = pylab.xlim()  
    ymin, ymax = pylab.ylim()  
    labelPlot(numFlips1, numTrials, mean1, sd1)  
    pylab.figure()  
    val2, mean2, sd2 = flipSim(numFlips2, numTrials)  
    pylab.hist(val2, bins = 20)  
    pylab.xlim(xmin, xmax)  
    ymin, ymax = pylab.ylim()  
    labelPlot(numFlips2, numTrials, mean2, sd2)
```

```
vals = []  
for i in range(100000):  
    num = random.random()  
    vals.append(num)  
pylab.hist(vals, bins = 11)
```



```
vals = []
for i in range(100000):
    num = random.random()
    vals.append(num)
pylab.hist(vals, bins = 11)
xmin, xmax = pylab.xlim()
ymin, ymax = pylab.ylim()
print 'x-range =', xmin, '-', xmax
print 'y-range =', ymin, '-', ymax
pylab.figure
pylab.hist(vals, bins = 11)
#pylab.xlim(-1.0, 2.0)
```

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Standard Deviations and Histograms